Title: Direct simulation of multiply-concatenated fault-tolerant quantum error correction

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Abstract: I will report on efforts to implement a new method for simulating concatenated quantum error correction, where many levels of concatenation are simulated together explicitly. That is, the approach involves a Monte Carlo simulation of a noisy circuit involving many thousands of qubits, rather than tens of qubits previously. The new approach allows the threshold and resource usage of concatenated quantum error correction to be determined more accurately than before. Also, the approach makes it possible to better study the effects of circuit optimizations and message-passing algorithms [Poulin, PRA, 2006] on the performance of fault-tolerant concatenated quantum error correction. Such studies are necessary in order make a proper comparison with competing families of error-correction protocols, such as those involving surface codes. In the talk, a range of new numerical results will be presented.

Direct simulation of multiply-concatenated fault-tolerant quantum error correction

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Introduction

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Important properties of any QEC protocol:-

Noise threshold

- The maximum amount of noise, per gate, that a QEC circuit can reliably correct

Resource overhead

- How many gates it takes to implement an error correction circuit

Two broad types of QEC protocol:

1. Concatenation-based,
2. Surface codes

- Both types of protocol are efficient in principle

  But numbers can look daunting in practice: 10^{13} – 10^{19} gates per EC step *[desired noise rate 10^{-9}, physical noise rate = half threshold]*

- Not clear yet which type of protocol if either will be practical to implement

- My work focuses on concatenated QEC

Aims of this work

1. To calculate the noise threshold and resource overhead for concatenated quantum error correction, more accurately than has been done before

and

2. Find improvements to concatenated quantum error correction protocols. Reduce the resource overhead, increase the noise threshold.

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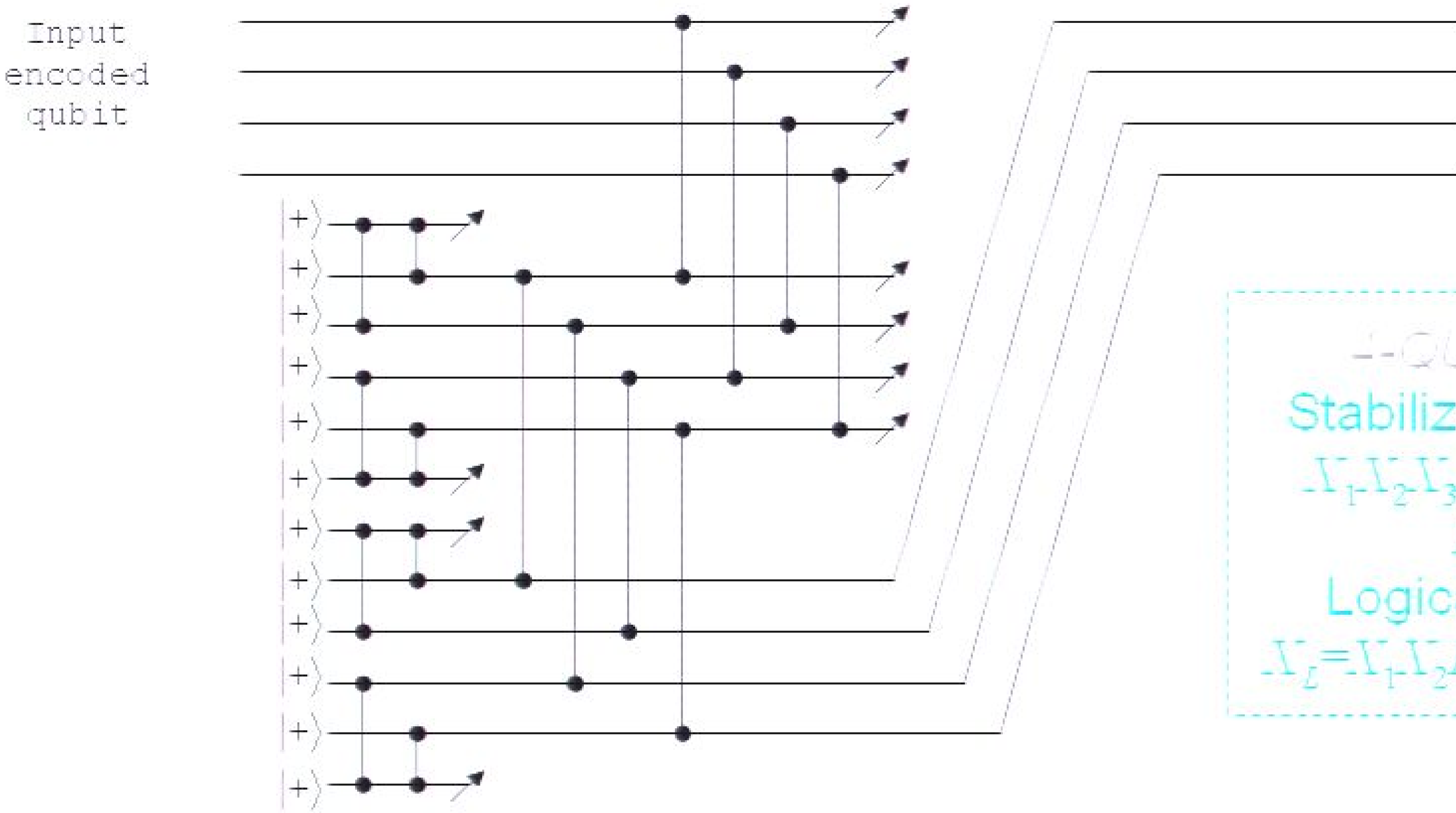
These aims can be achieved by developing a new type of simulator, one that simulates many levels of concatenation explicitly.

Circuits, codes, and concatenation

Simple fault-tolerant QEC circuit, based on a 4-qubit error detecting code:



4-QUBIT CODE

Stabilizer generators:

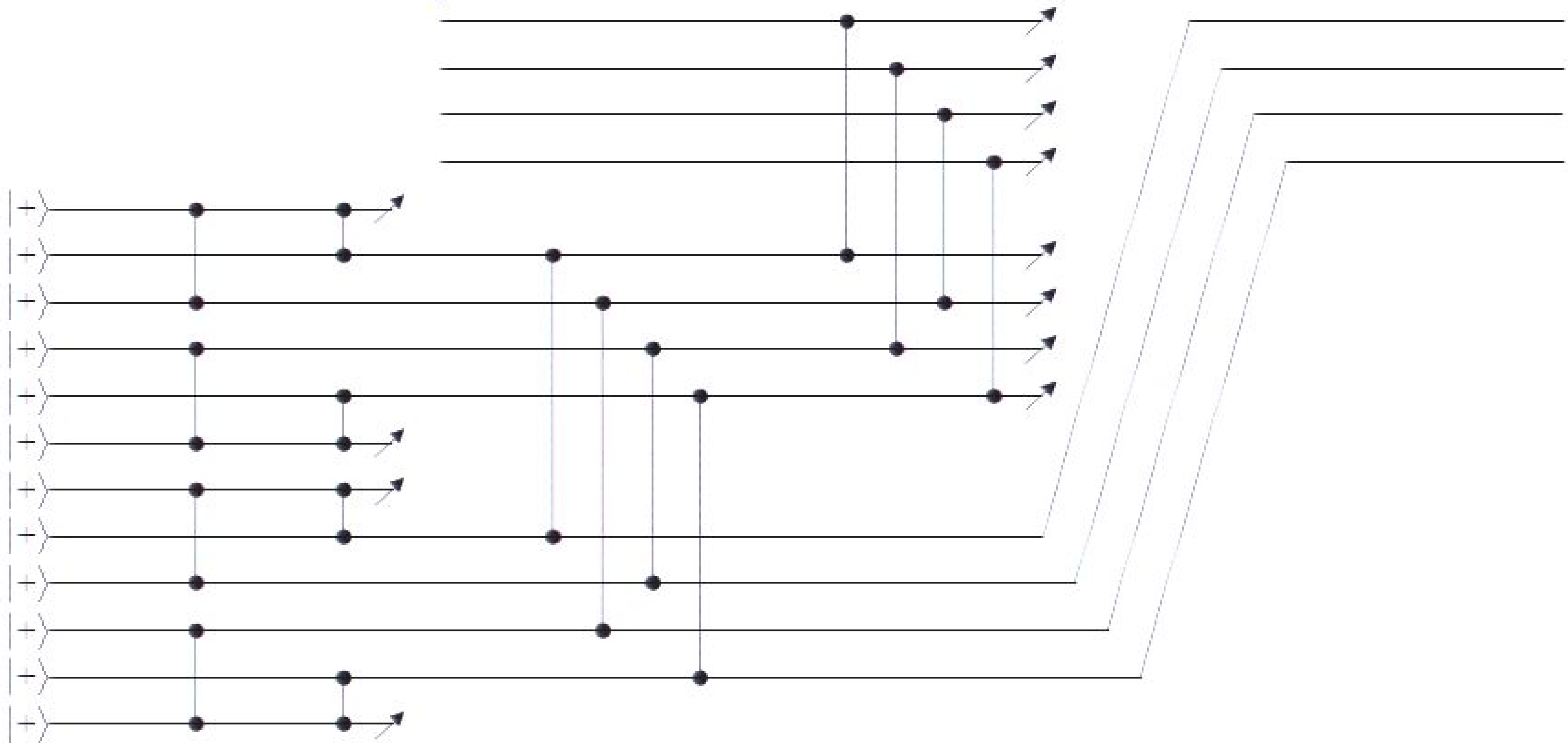
$X_1X_2X_3X_4$, $Z_1Z_2Z_3Z_4$, $I_1I_2Z_3Z_4$

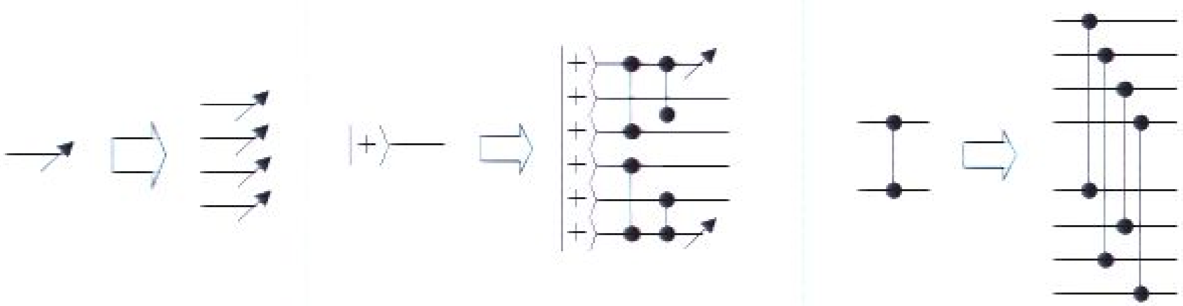
Logical operators:

$X_L = X_1X_2I_3I_4$, $Z_L = Z_1I_2Z_3I_4$

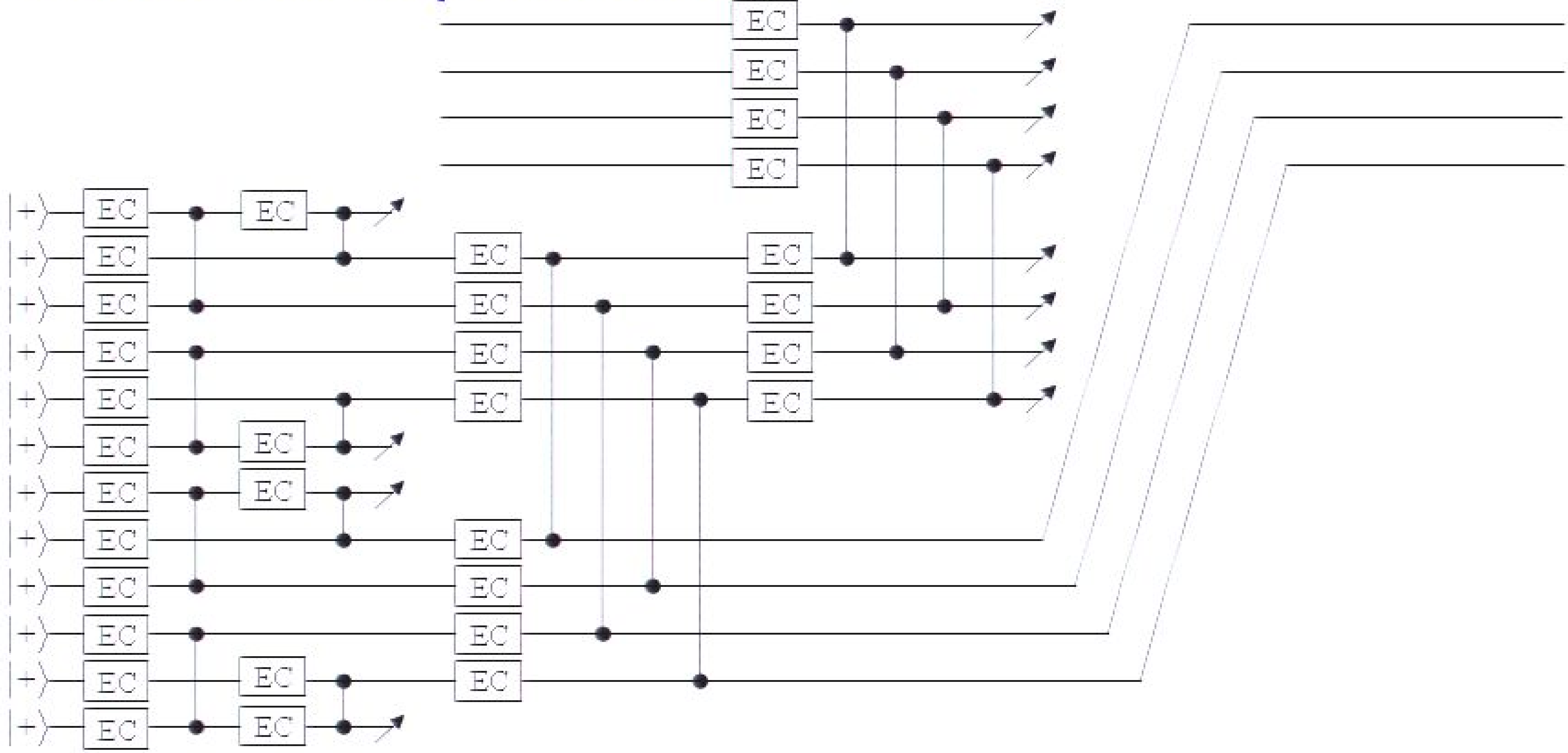
Concatenation is a procedure for transforming a QEC circuit into a new circuit that corrects more errors (but is more complicated)

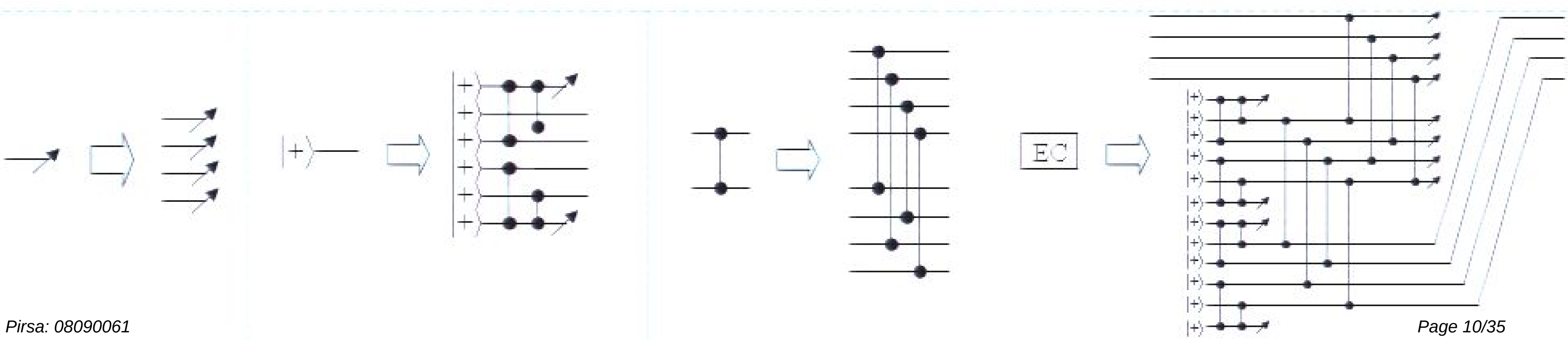
Concatenation procedure

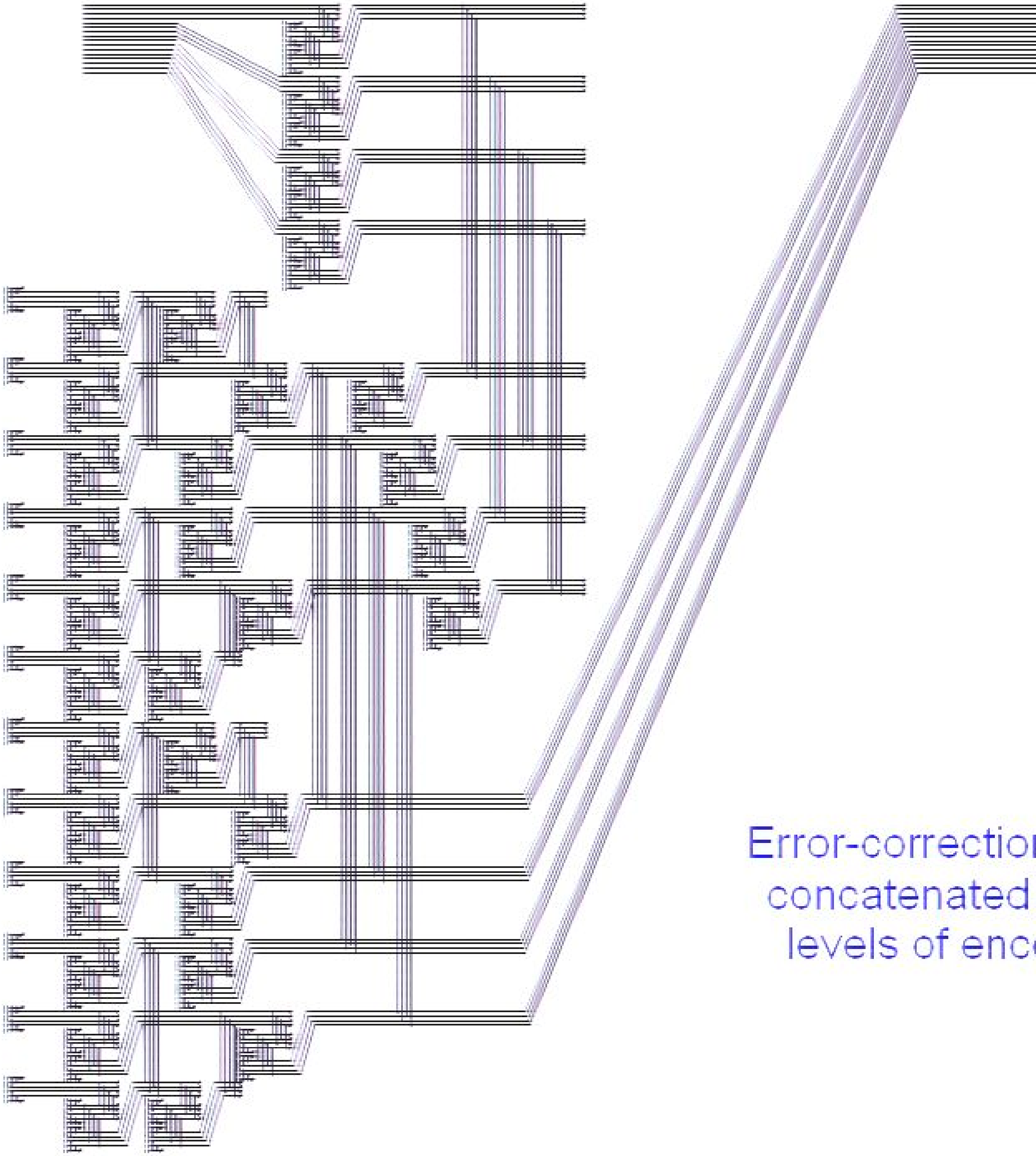




Concatenation procedure







Error-correction circuit concatenated to two levels of encoding

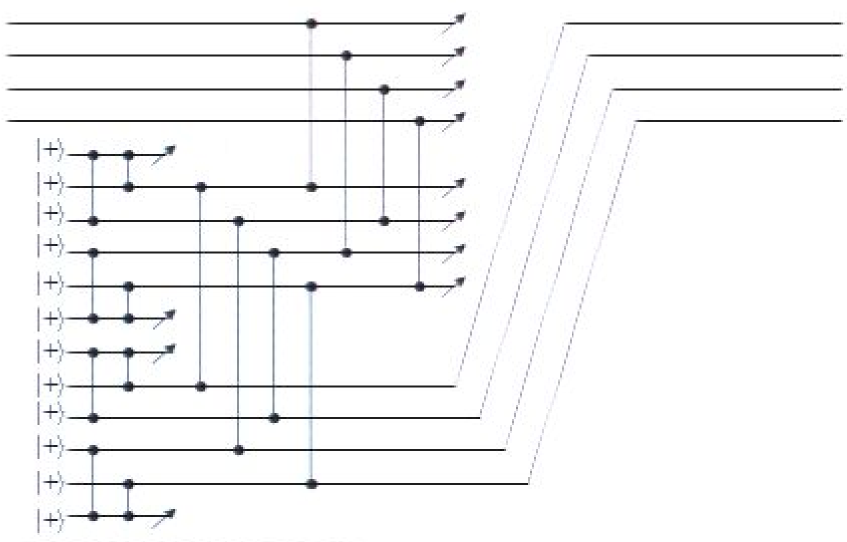
More about the 4-qubit QEC protocol:

Located & unlocated errors

- When an error is detected (a nonzero syndrome) the whole code block is marked as containing an error.
- At the next highest level of encoding: this code block is treated as a qubit containing a *located* error
- Any code that can *detect* one ordinary error can *correct* one located error

Post-selection

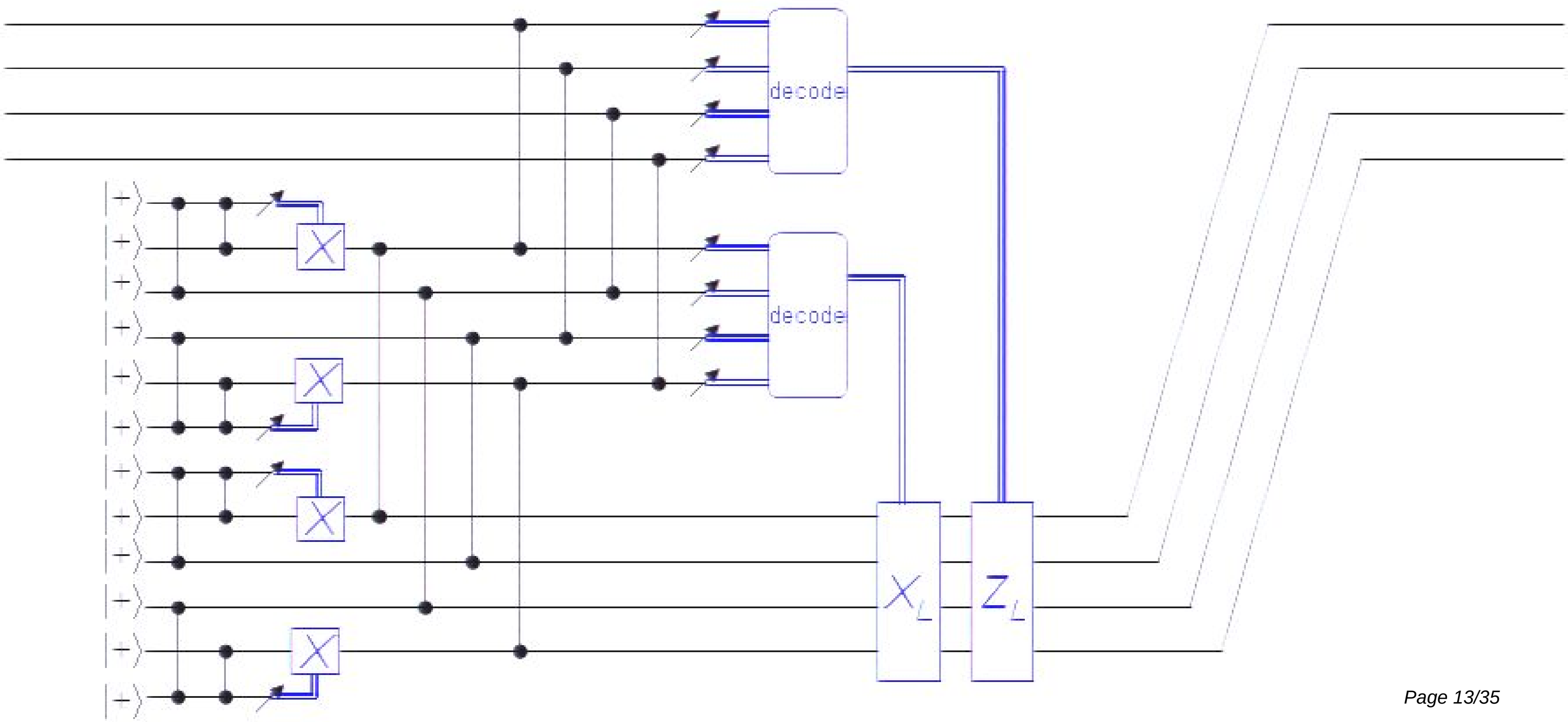
- If a located error occurs inside the shaded region, the circuit is re-started (required for fault-tolerance)



More about the 4-qubit QEC protocol: (cont'd)

Implied classical feed-forward

- Measurement results control Pauli operations as shown.
- The Pauli operations can be omitted if accounted for with a *Pauli frame*.
- Act as a source of noise, regardless.



Previous methods for analysing concatenated QEC

One-level-at-a-time numerical method

1. Simulate just one level of concatenation using Monte Carlo simulation techniques, then
2. Use a series of assumptions/approximations to extrapolate the results to many levels of concatenation

   – Assume: that noise on an encoded qubit can be described in the same way as noise on physical qubit (*"course-graining"*)
   – Assume: that the error probability on an encoded qubit is determined by the error-correction part of the circuit at the lower level

Analytical bounds

The "Ex-Rec" method provides tight-ish lower bounds to the noise threshold.

Reasons for performing explicit simulations of multiply-concatenated QEC

Old methods are inaccurate

Flexibility. Can now simulate:

- Message-passing algorithms for decoding [D. Poulin, PRA 74, 2006].
- A variety of circuit rearrangements and optimisations

The simulator

Concatenations of four-qubit code only, so far

Practical up to ≈ 6 levels (limited only by speed)

- 6 levels gives: 4096-qubit code, and a error-correction circuit containing ≈ 1,000,000 gates
- Naturally limited to sizes which might be practical to implement

Monte Carlo techniques

- Errors are introduced stochastically at each physical gate
- Simulator measures the probability that an EC circuit introduces and uncorrectable error at the highest level.

Simulator keeps track of Pauli errors (not actual quantum state, or stabilizers)

Implementation

- C, approx 3000 lines

EXPERIMENT 1: Comparing old and new simulation methods

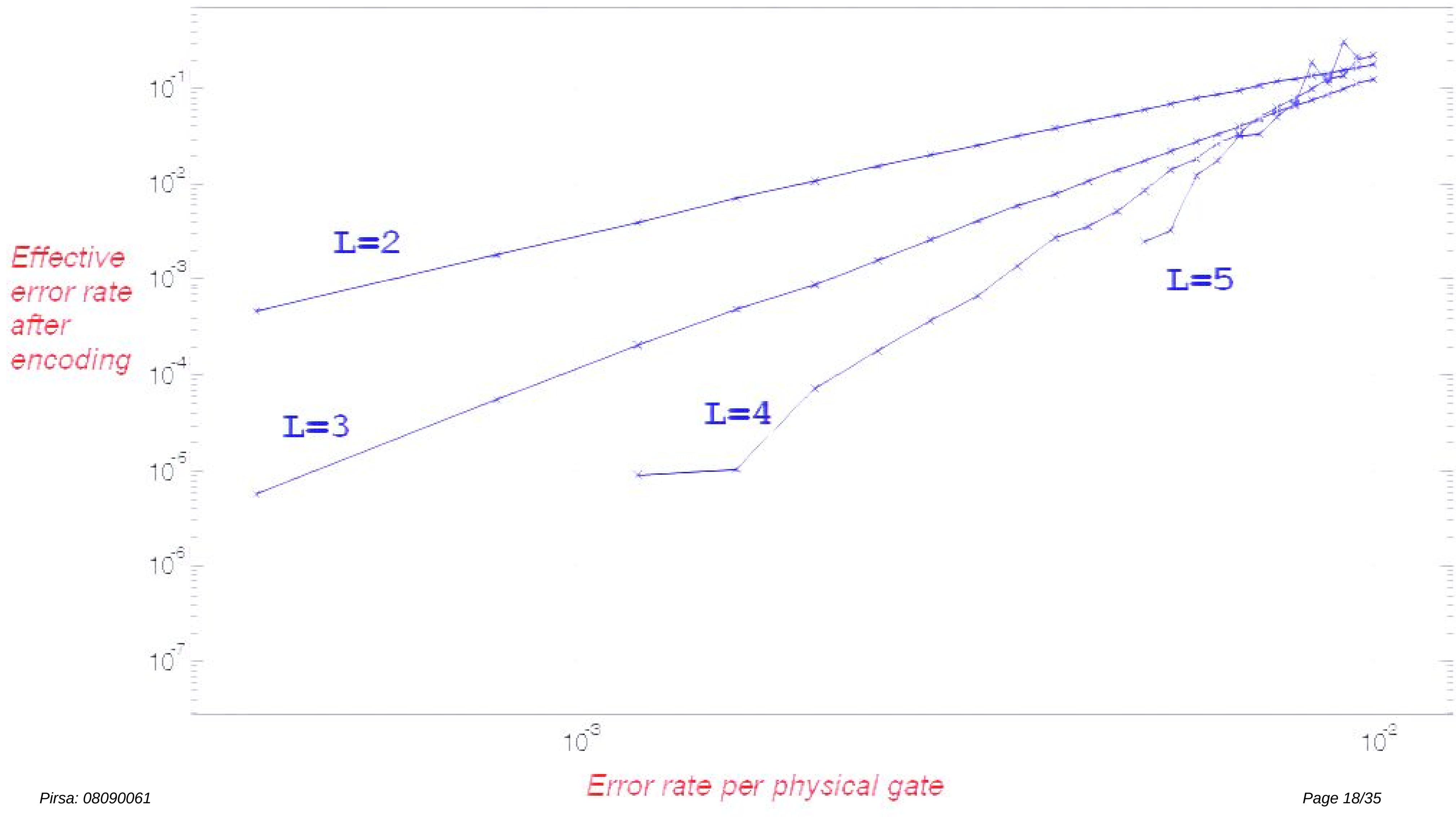
We simulate the same concatenated 4-qubit protocol (with same noise model, etc) two different ways:

1) Using the old one-level-at-a-time numerical techniques

- For one level of concatenation of the protocol, simulations are performed for a range of unlocated and located error rates (p_i, q_i).
- The corresponding rates of unlocated and located uncorrectable errors are measured. (P_i, Q_i).
- The mapping of error rates from one level to the next is characterized by fitting a polynomial to $(p_i, q_i) \rightarrow P$ and to $(p_i, q_i) \rightarrow Q$

2) Explicit simulation, for a range of concatenation levels.

EXPERIMENT 1: Comparing old and new simulation methods



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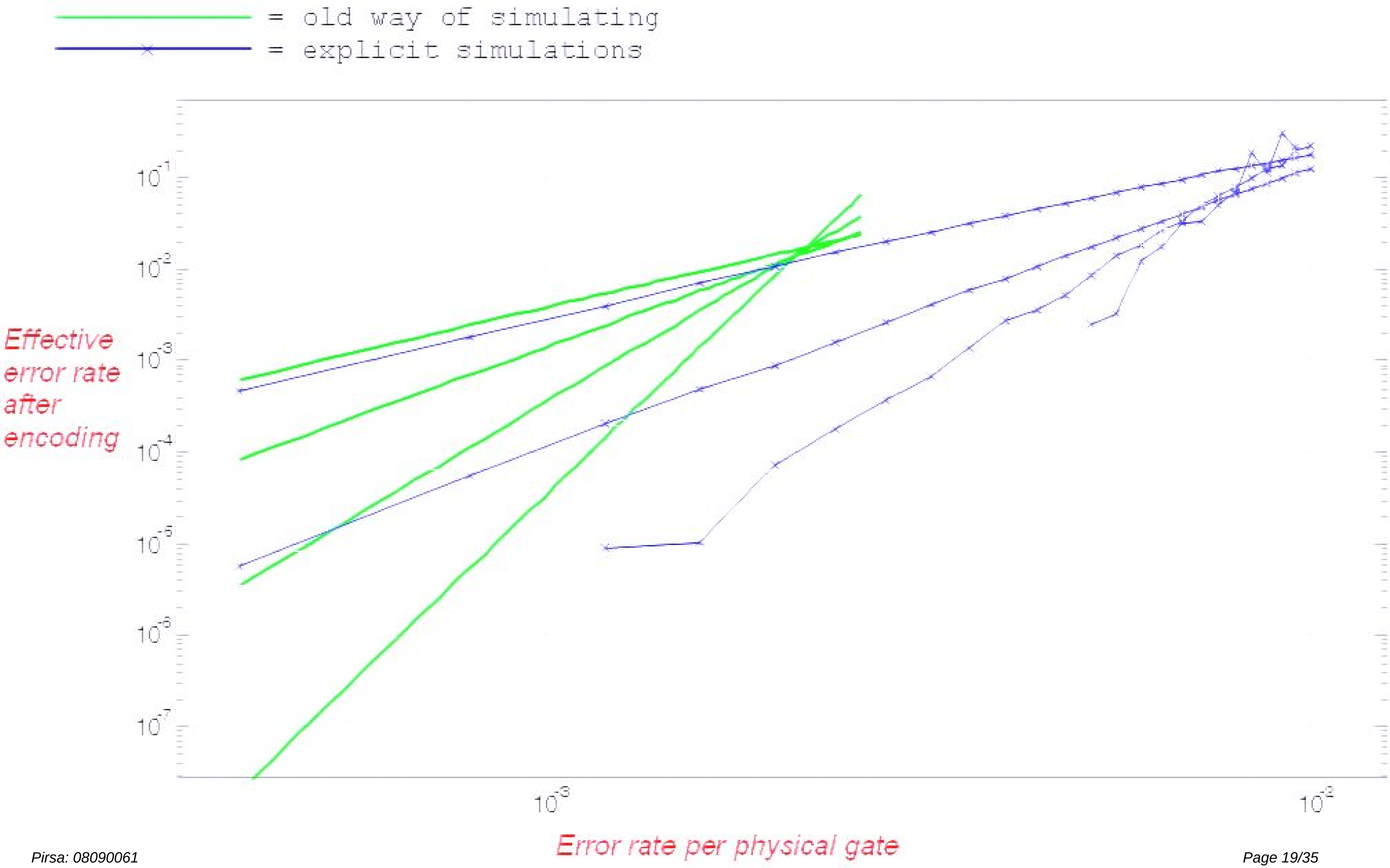


EXPERIMENT 2: Soft decisions and message-passing decoding

Modified protocol:

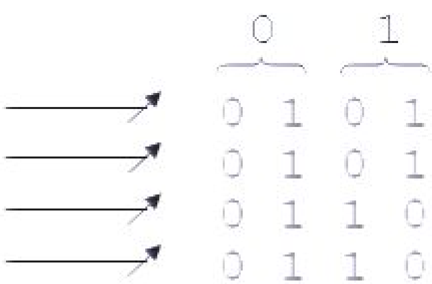
- Each qubit (at every concatenation level) has associated with it an error probability vector, $[p_I, p_X, p_Y, p_Z]$, which the imagined experimenter keeps track of
- Replaces the notion of located and unlocated errors.

- These values are updated according to each syndrome measurement
- A message-passing decoding algorithm [D. Poulin, PRA 74, 2006] is used to take advantage of the probability information

Does this improve the performance of the 4-qubit FTQEC protocol?

Ordinary decoding

1 level of concatenation



Ordinary decoding

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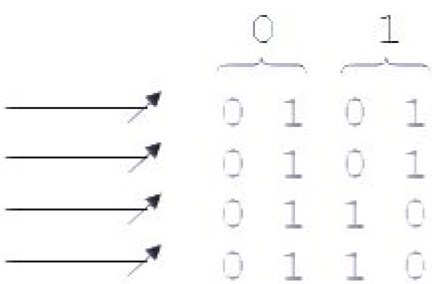
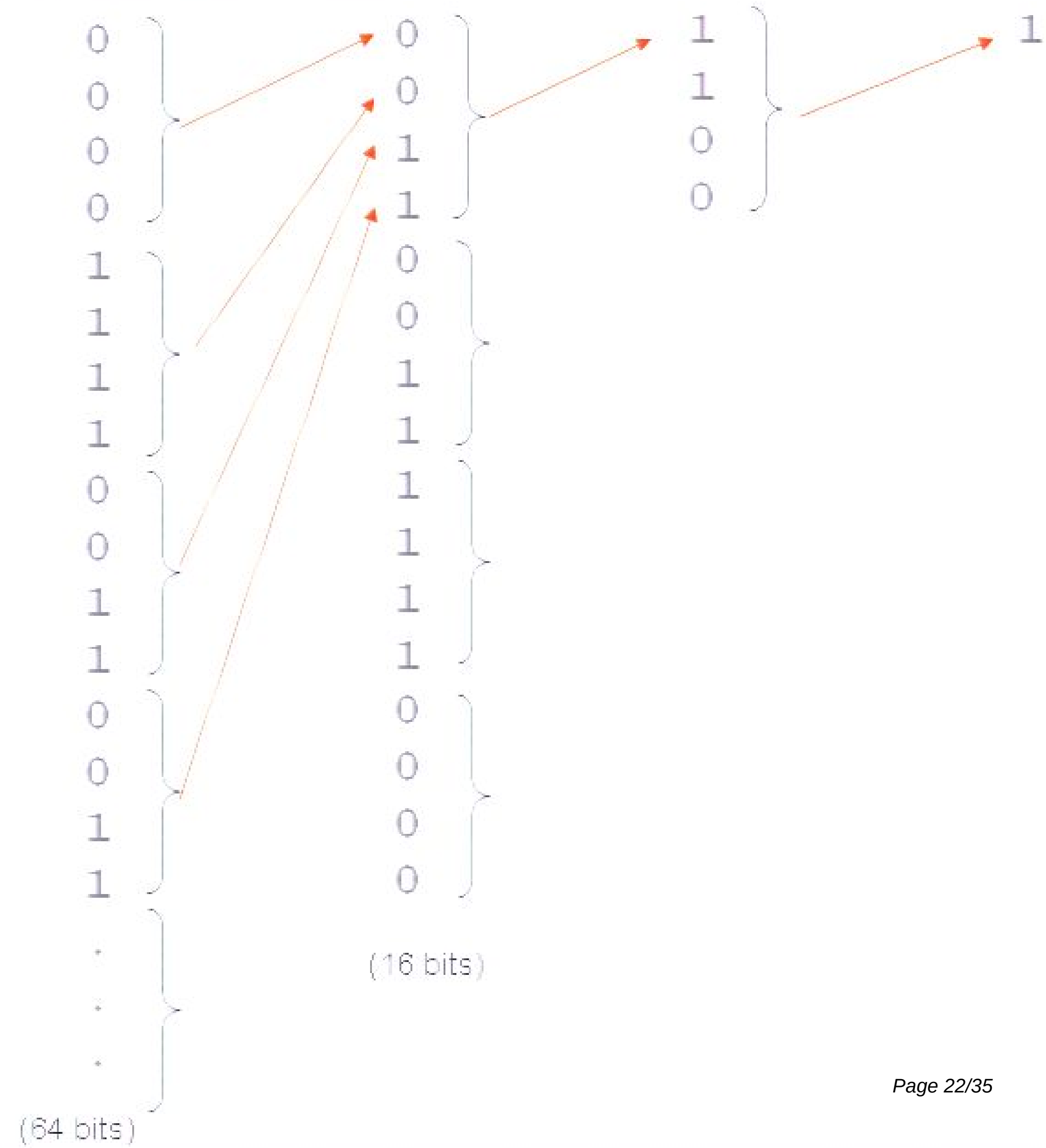
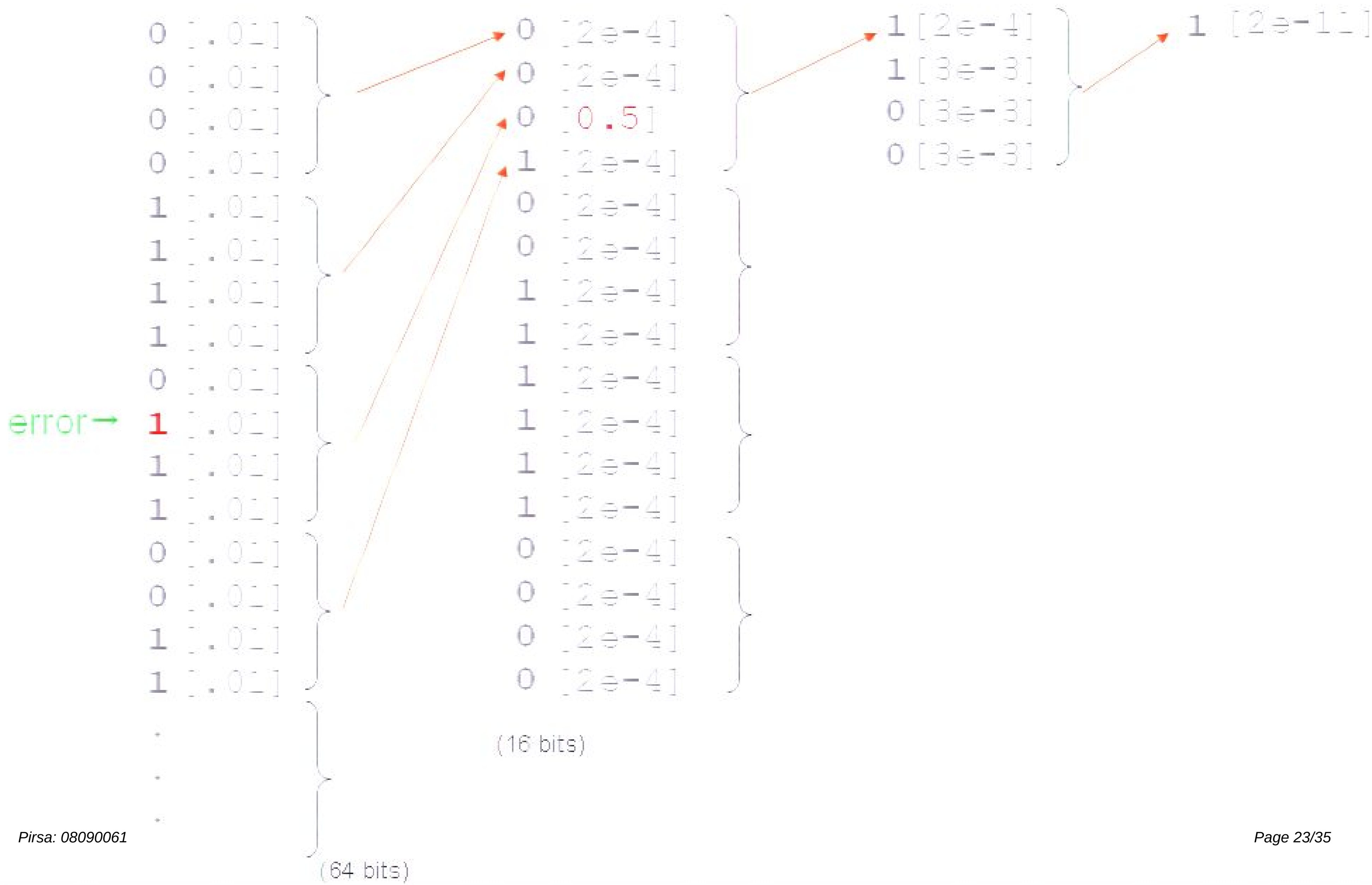


3 levels of concatenation



Message-passing decoding



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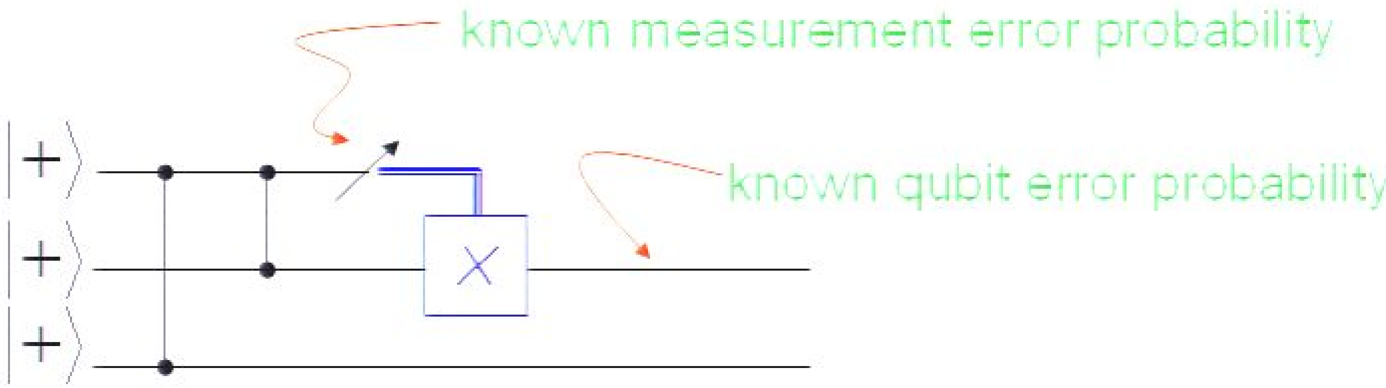


Poulin showed:

- for idealized quantum communication (Alice and Bob have perfect operations, only channel is noisy), message passing increases threshold from 0.0969 to 0.1885 (concatenated 5-qubit code).

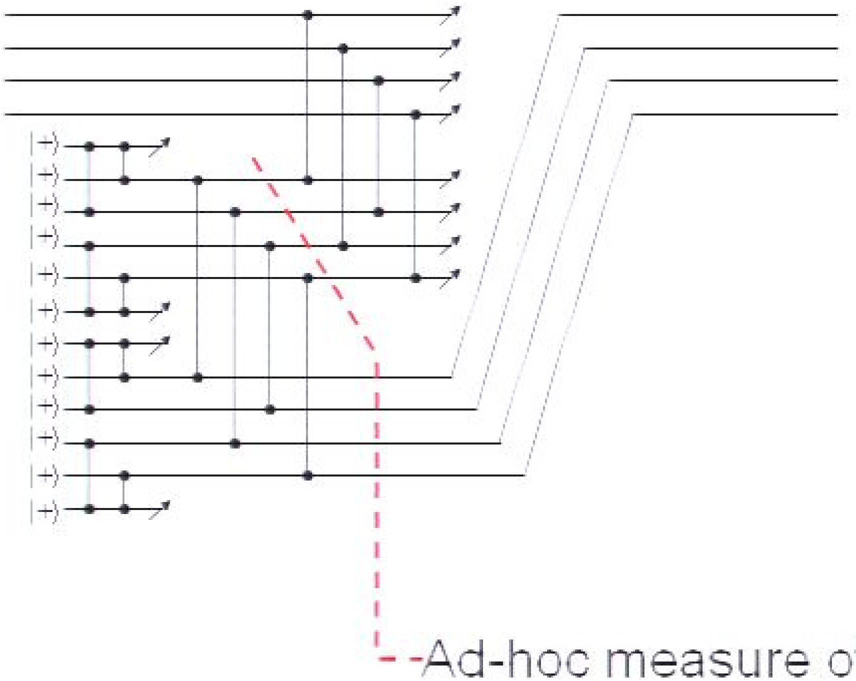
What about fault-tolerant QC? Not easy to try before.

Note: message passing has only a *classical* cost.

EXPERIMENT 2: Soft decisions and message-passing decoding

A complication...

If there is no such thing as a located error any more, how do you do postselection?



Ad-hoc measure of "noisiness", η, calculated here. If $\eta > \eta_{\text{thresh}}$, repeat circuit.

$$\eta = \sum_{\text{the 8 qubits}} H\left(\left[p_I, p_X, p_Y, p_Z\right]\right)$$

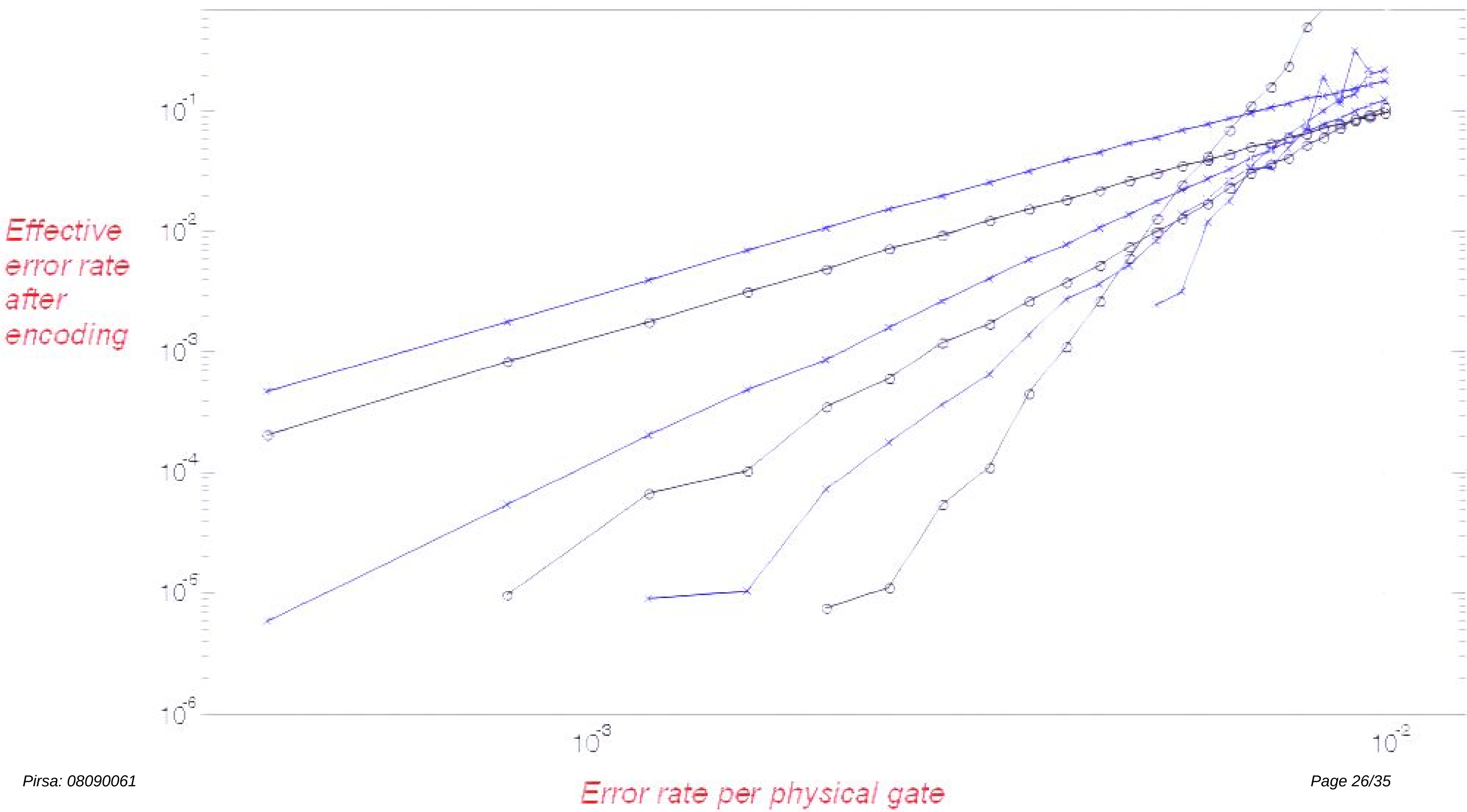
Introduces extra parameter in protocol: lowering η_{thresh} gives greater resource usage, but better noise performance

EXPERIMENT 2: Soft decisions and message-passing decoding

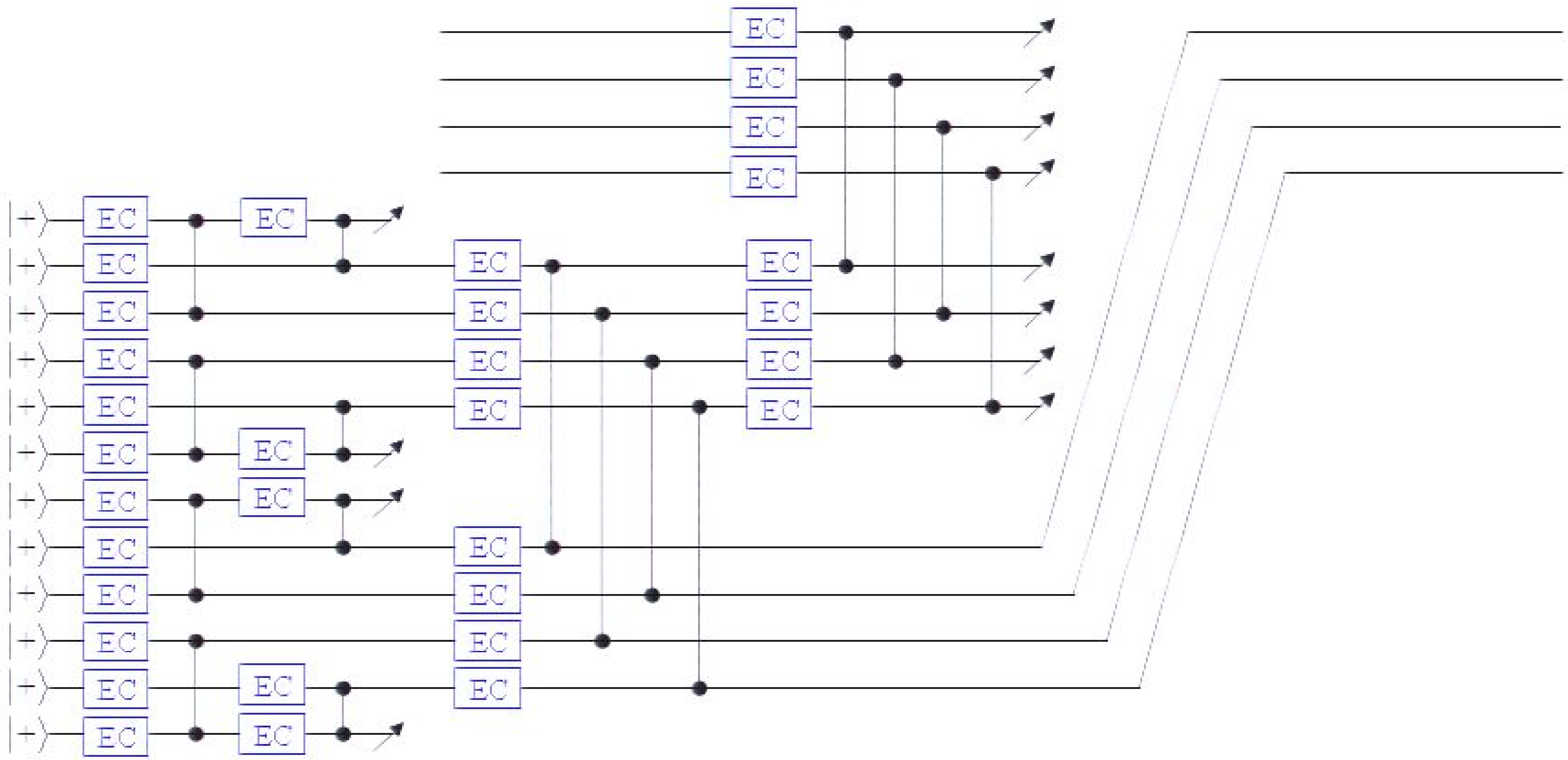
= no message passing

= message passing enabled

NB: η_{thresh} chosen so that resource usage same in both cases



EXPERIMENT 3: Testing simplifications to the QEC circuit



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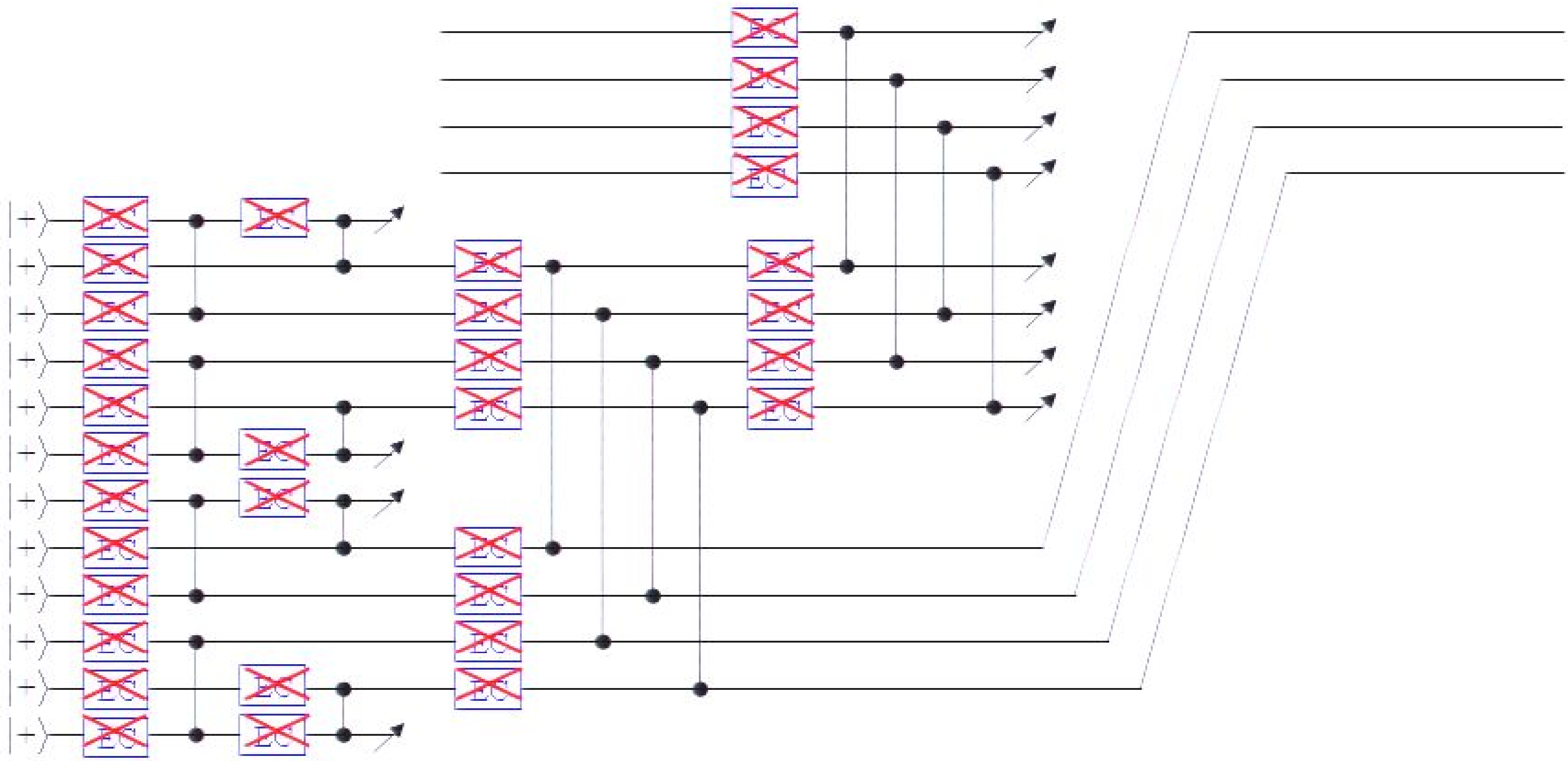

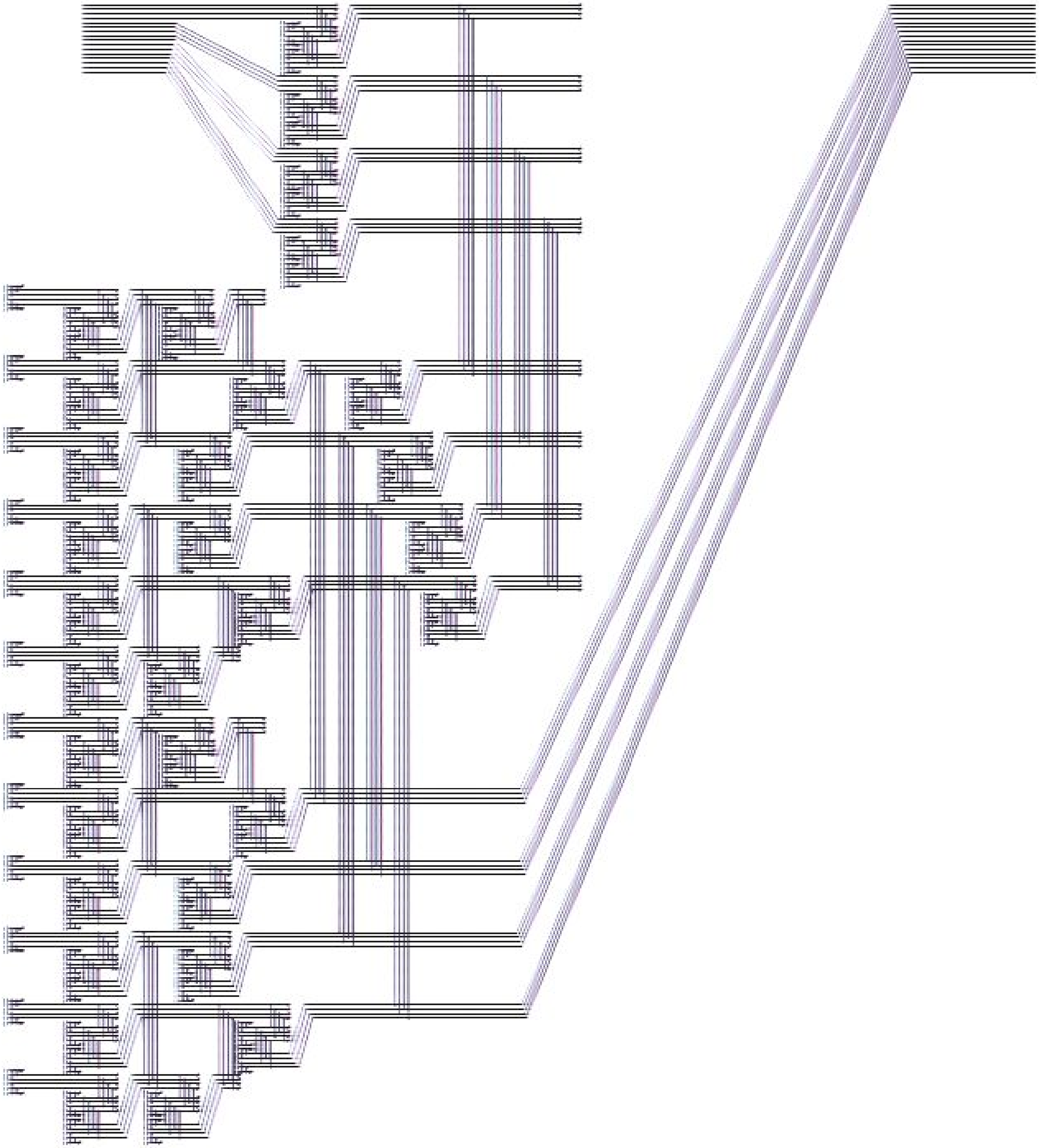

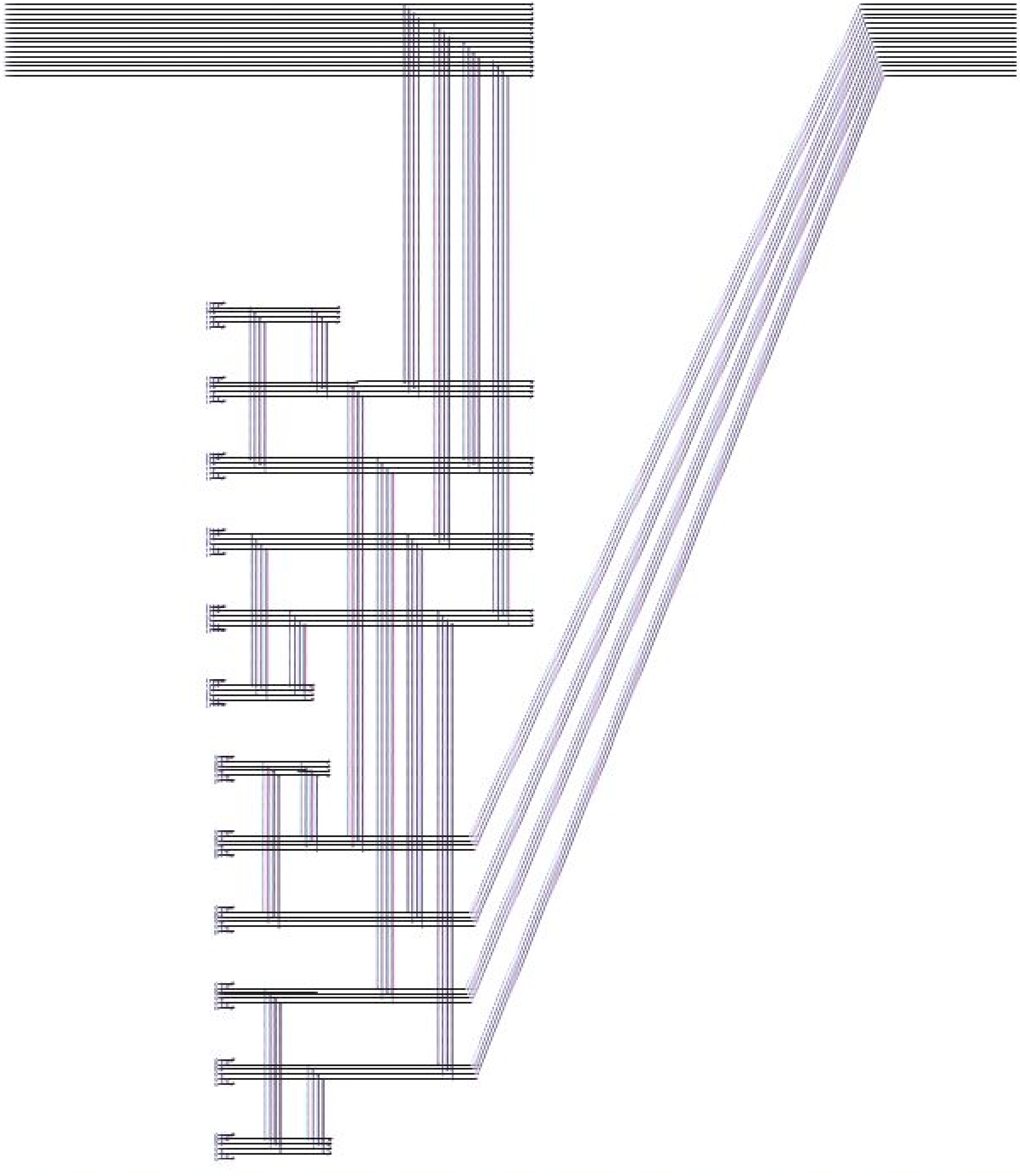

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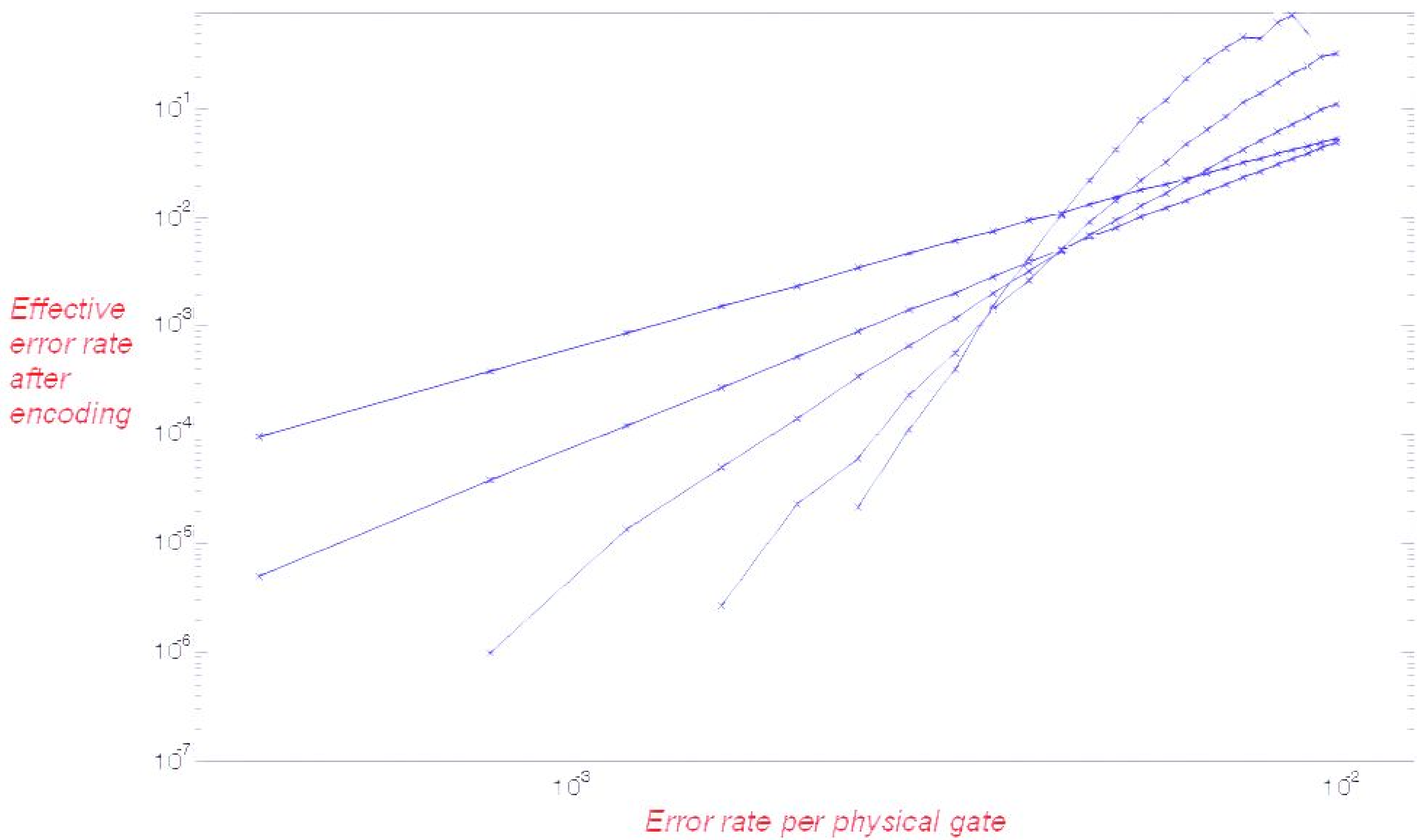
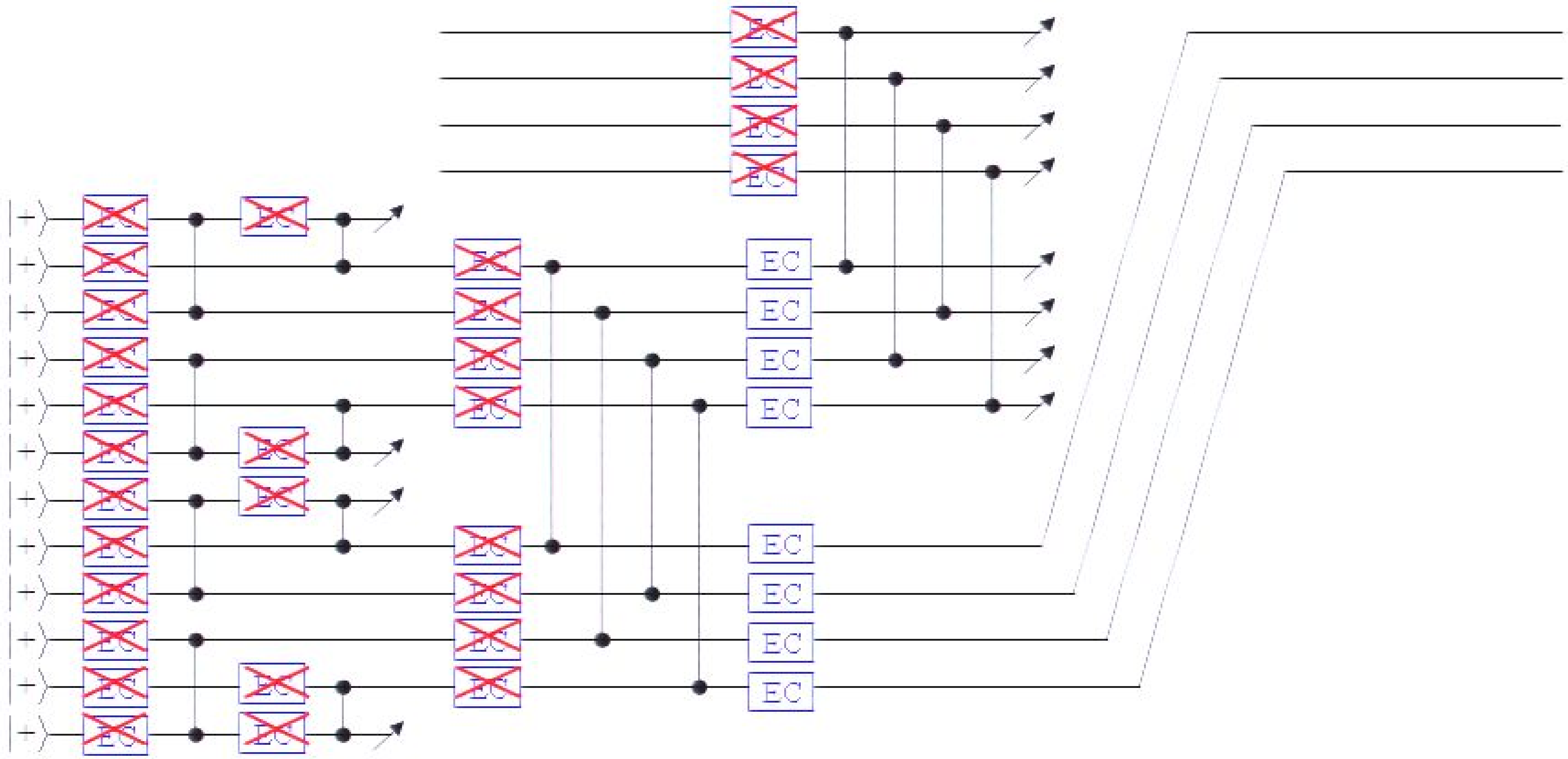
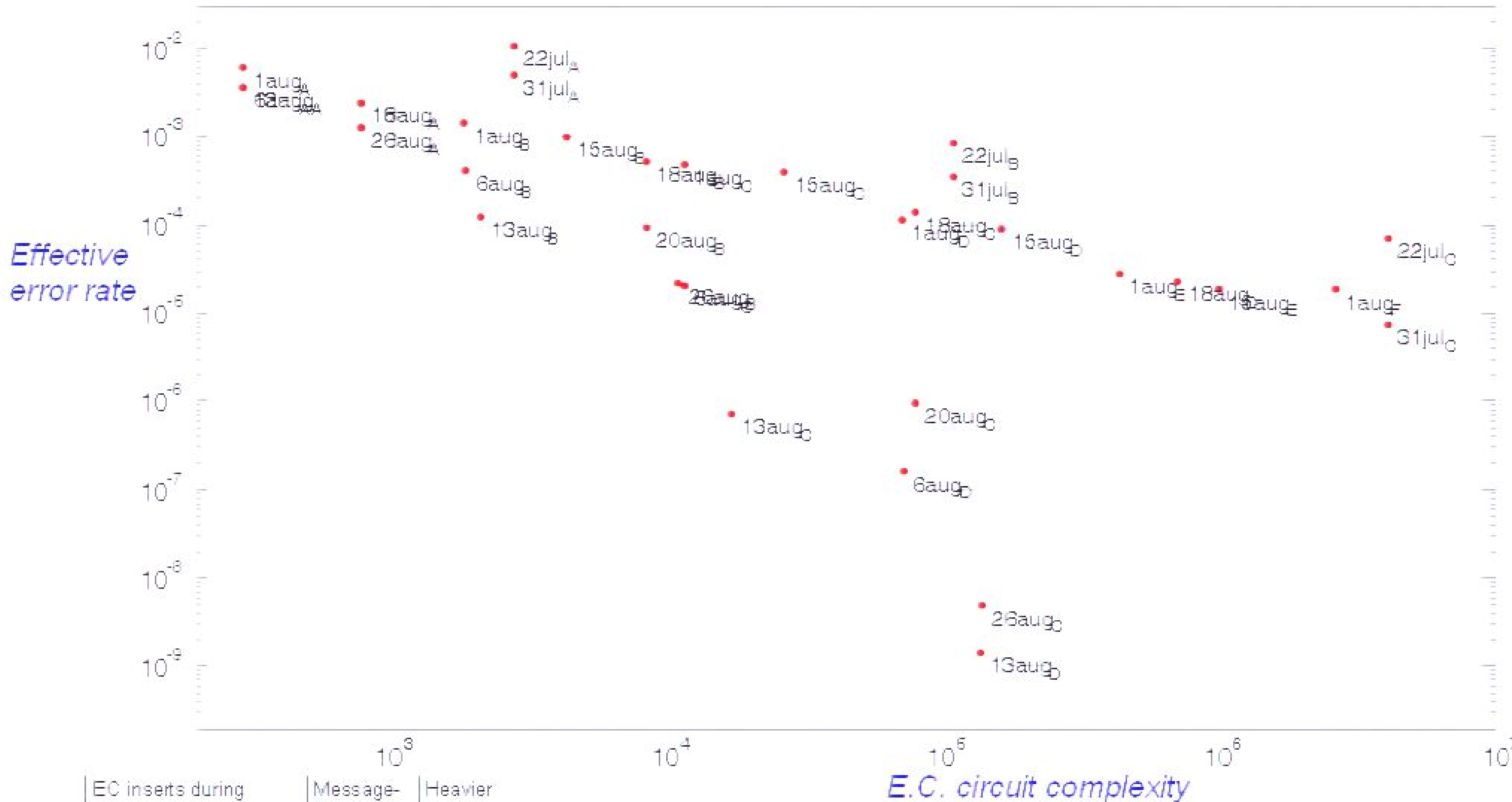


EXPERIMENT: *Some* error correction during concatenation



Results: better noise performance, and a little larger resource overhead



	EC inserts during concatenation			Message-passing decoding	Heavier post-selection
	ALL	SOME	NONE		
22jul	✓				
31jul	✓			✓	
15aug		✓			
18aug		✓			
1aug			✓		
20aug		✓		✓	
6aug			✓	✓	
26aug		✓		✓	✓
13aug			✓	✓	✓

Notes:
- Physical error rate = .002
- Subscript A,B,C,... means L=2,3,4,...

Conclusions

Early results indicate that significant improvements to concatenated QEC are possible.

Many more things to try

- *Different codes*
- *More ways of circuit arrangement*
- *Different methods for post-selection*
- *Non-Clifford group operations?*

Ultimate Aim

Find the most efficient way of doing concatenated QEC, and accurately measure corresponding noise performance and resource overhead

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