Title: Collapsing and Expanding Multiverse

Date: Sep 02, 2008 03:30 PM

URL: http://pirsa.org/08090052

Abstract:

Expanding and Contracting Multi-verses

Expanding Isotropic Universe: scalar field Inflation, scalar instability, Inflationary multi-verse

Contracting Anisotropic Universe: BKL regime, new Kasner tensor mode instability, Contracting multi-verse

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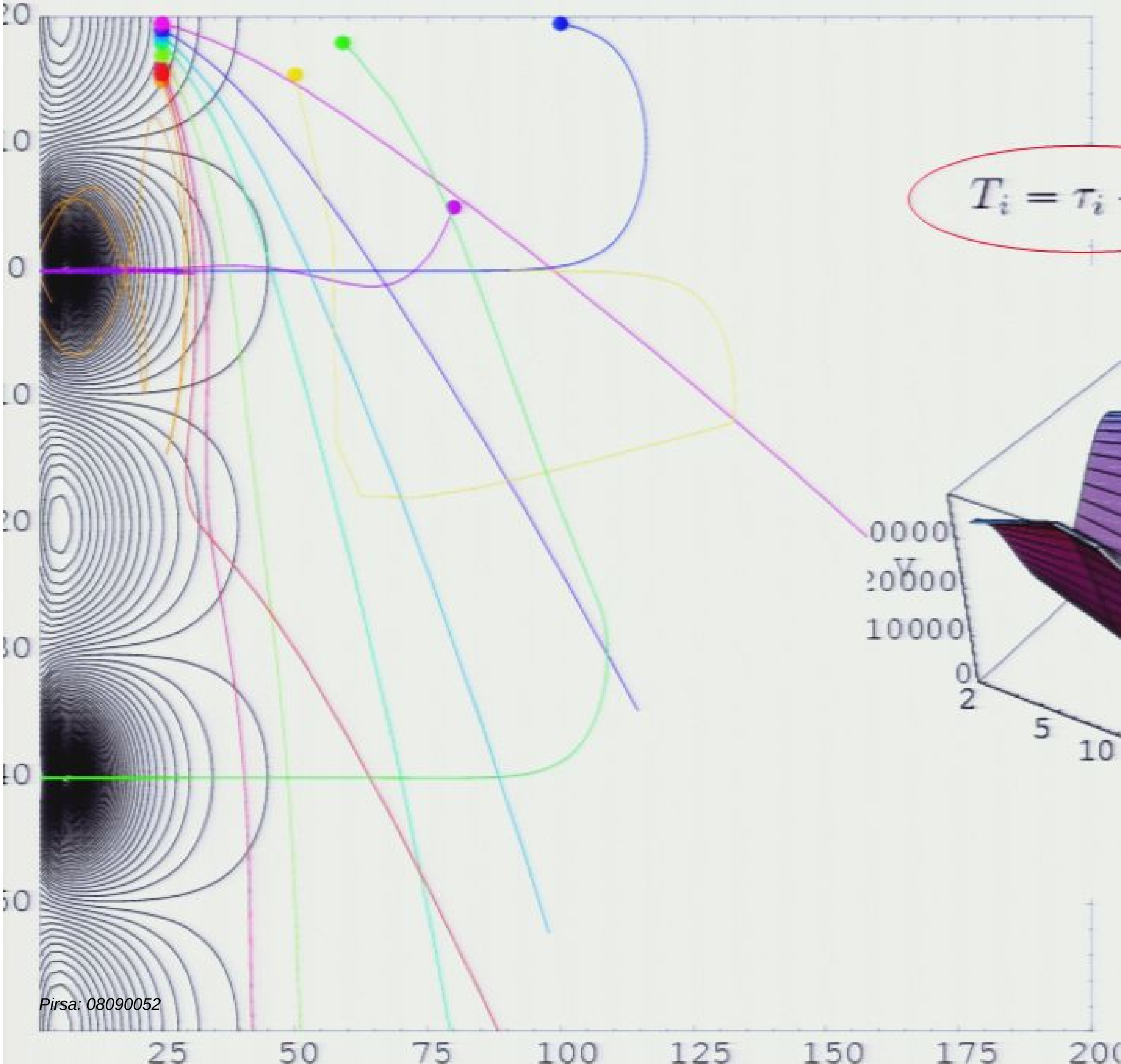
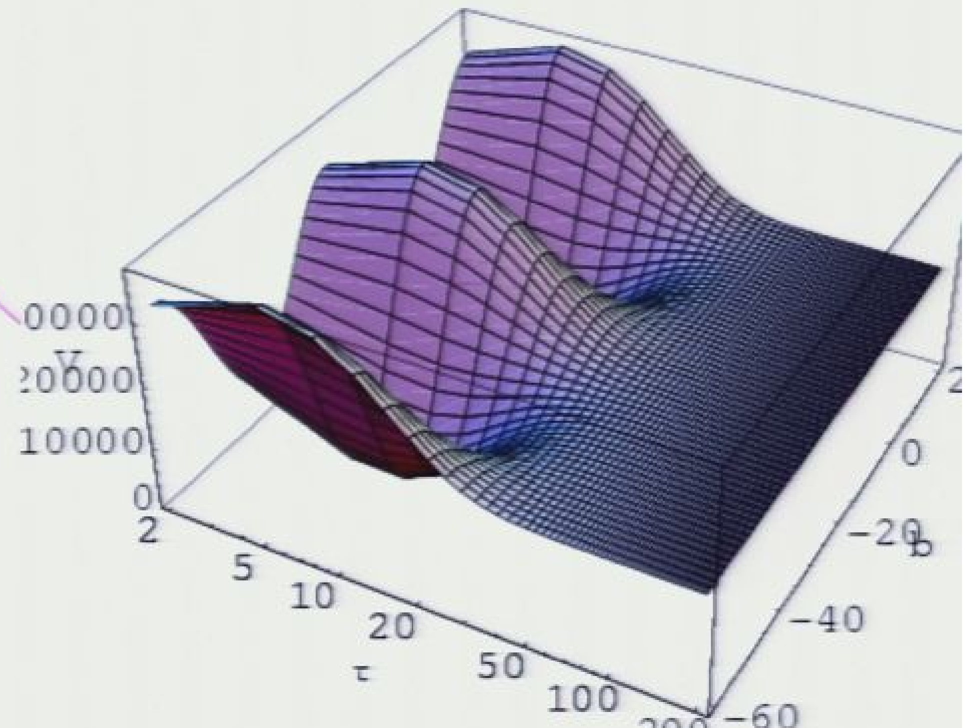
PI, September 2, 2008

String theory landscape of the Kahler moduli/axion Inflation



$$T_i = \tau_i + i\theta_i$$



Lessons:

Multiple fields Inflation

Ensemble of acceleration histories (trajectories) for the same underlying theory

Prior probabilities of trajectories P(H(t))

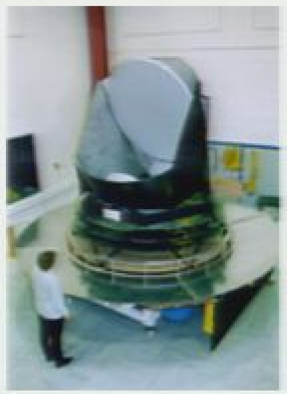
Small amplitude of gravity waves r from inflation $r \sim 10^{-10}$

Some exceptions

$r \sim 10^{-2}$

Measurement of GW from CMB anisotropy polarization

< 0.34 — Current limit



Planck
Satellite
2008.11

0.1

.Spider
Balloon
2009.9

0.01 — CMBPol 2017

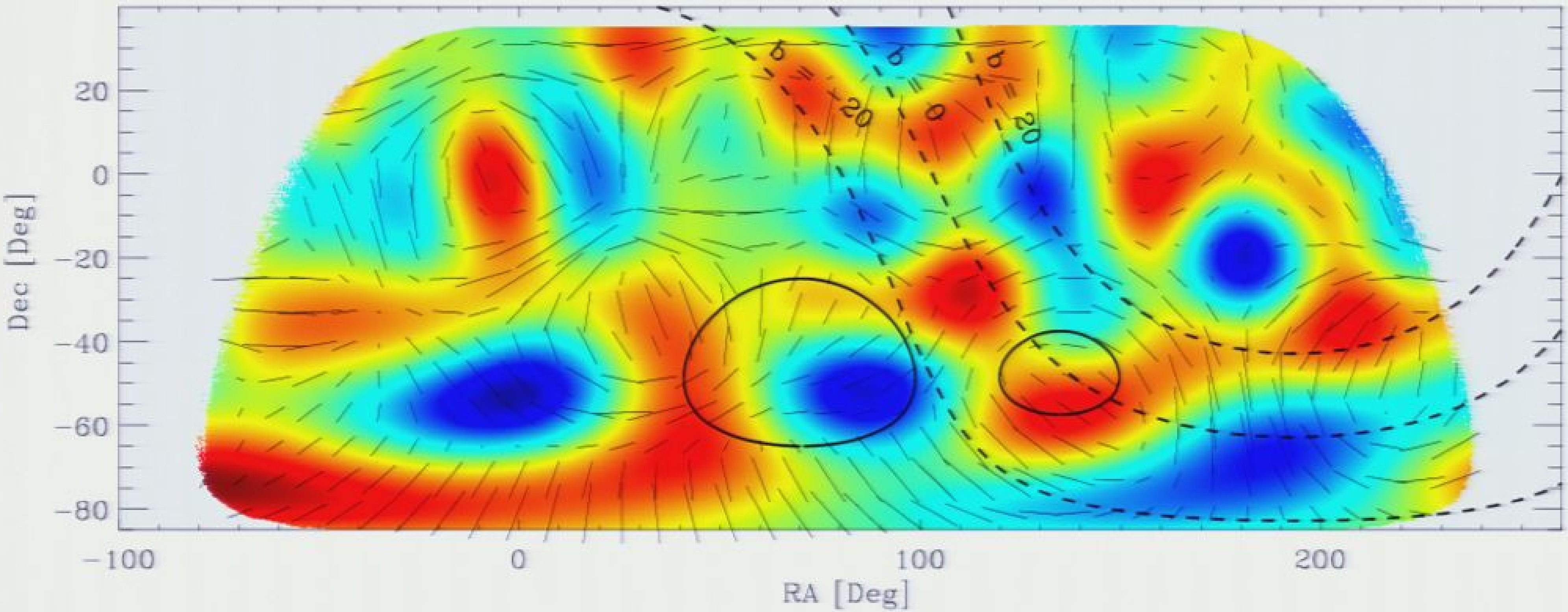
0.001 — CMB Ultimate limit

10^{-6} — GW astronomy

SPIDER Tensor Signal

- **Simulation of large scale polarization signal** $\frac{A_T}{A_S} = 0.1$

No Tensor



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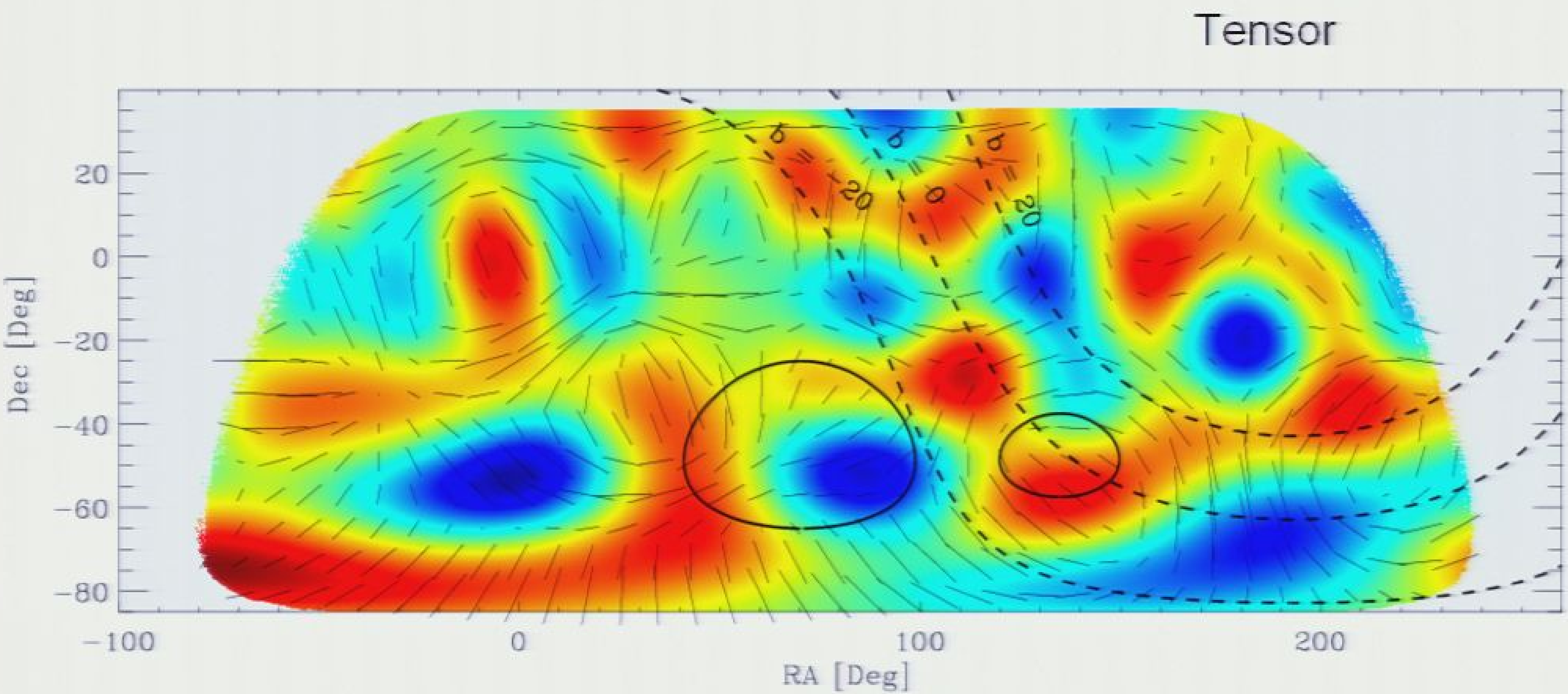

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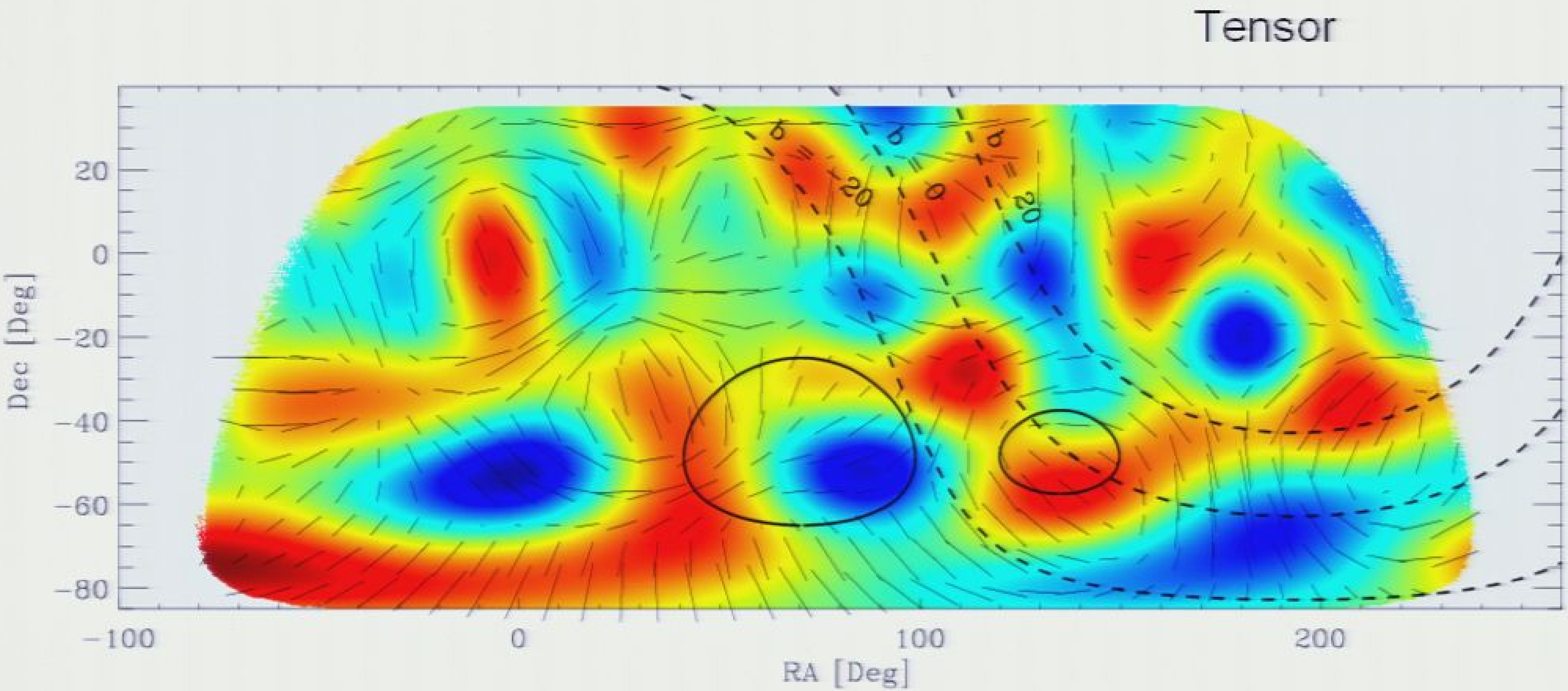

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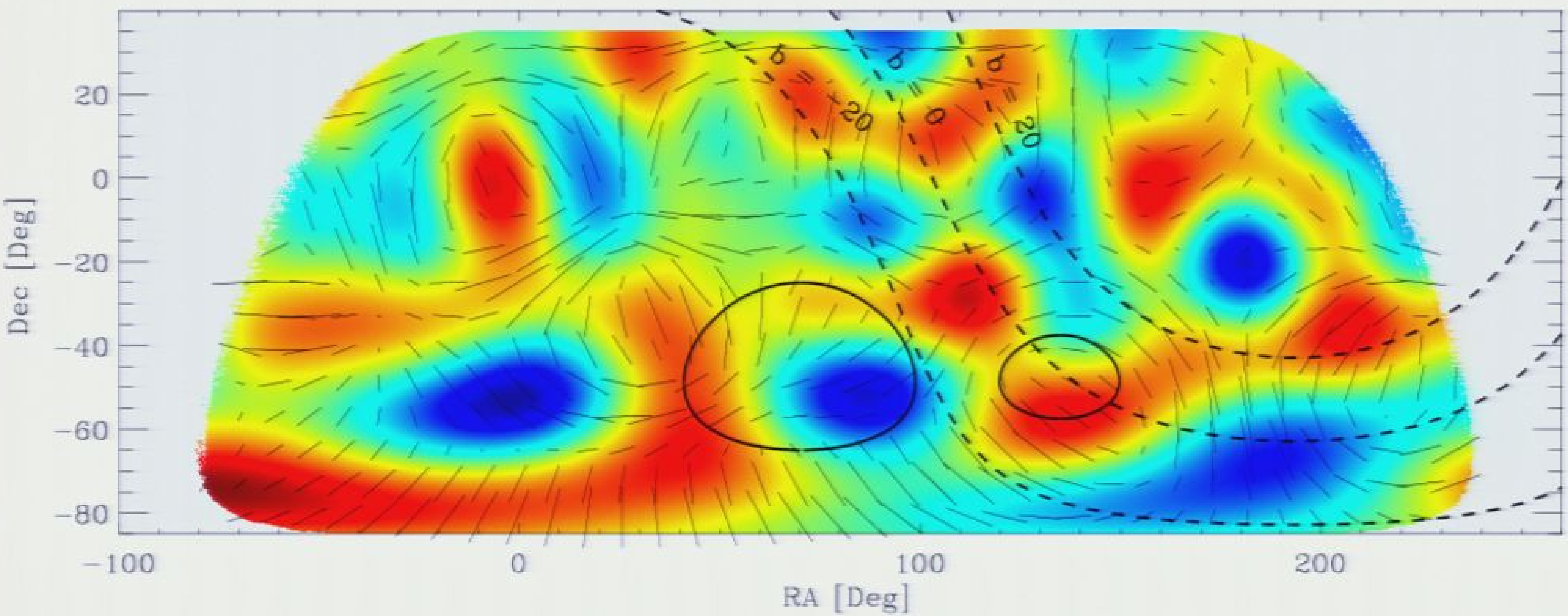

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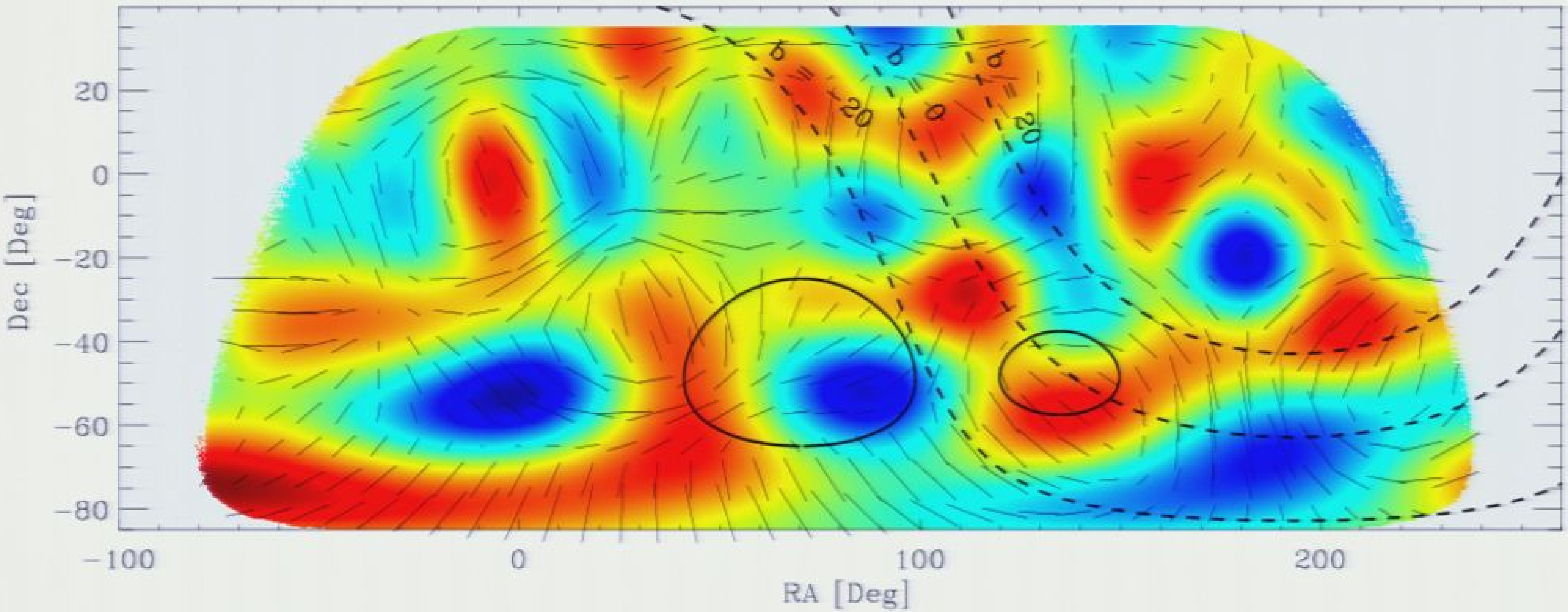

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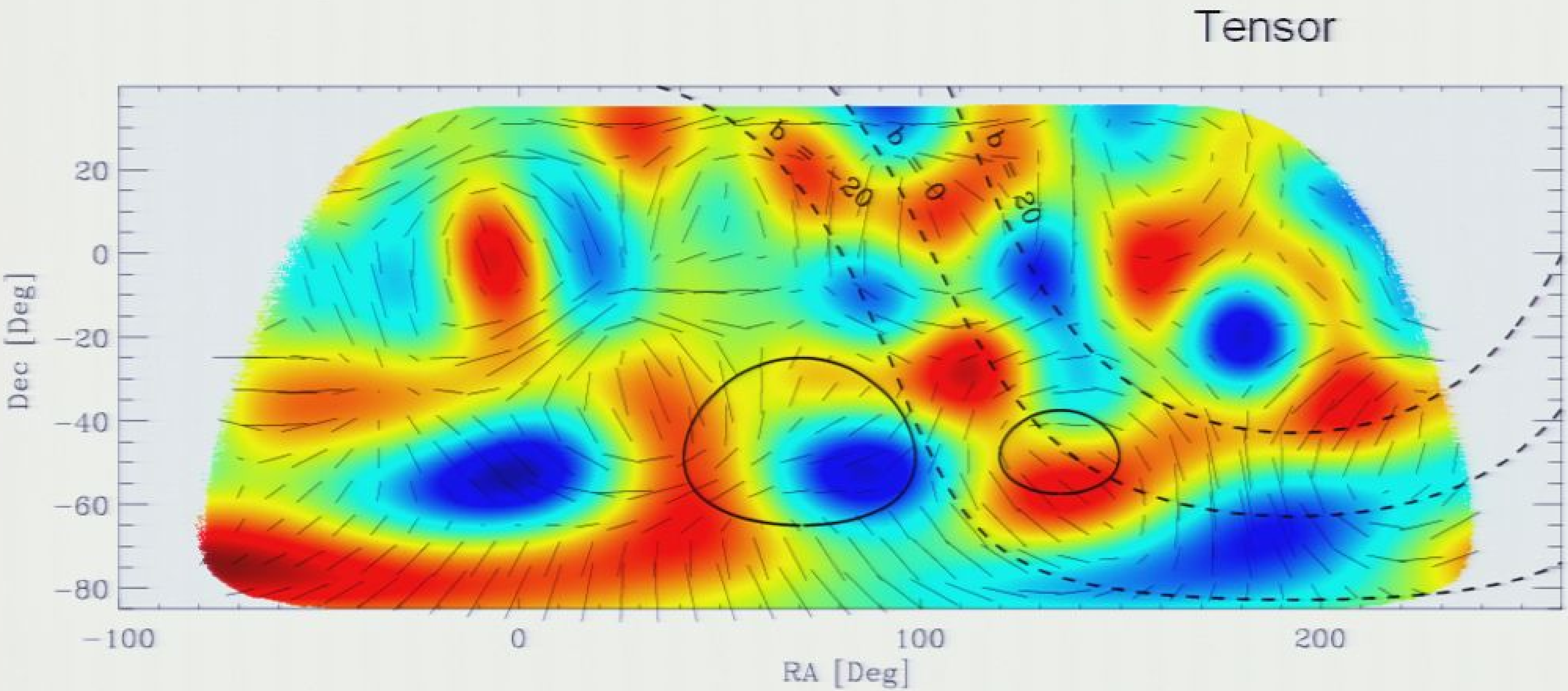


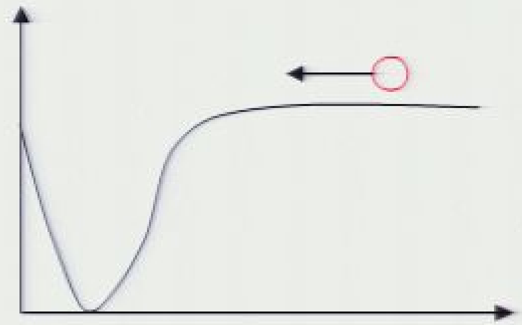
http://www.astro.caltech.edu/~lgg/spider_front.htm

Gravity Waves Signatures from Pre-inflation?

Gumrukcuoglu, LK, Peloso 08

$$ds^2 = -dt^2 + a^2 dx^2 + b^2 dy^2 + c^2 dz^2$$

$$V = V_0 \left(1 - \mathrm{e}^{-\phi/\phi_0}\right)^2$$



$$H_0^2 = \frac{8\pi V_0}{3 M_p^2}$$

$$\Big(a(t), b(t), c(t)\Big) = (a_*, b_*, c_*) \left[\sinh(3H_0 t)\right]^{1/3} \left[\tanh\left(\frac{3}{2} H_0 t\right)\right]^{p_i - 1/3}$$

$t \ll 1/H_0$ $\quad \Big(a(t), b(t), c(t)\Big) = (a_0, b_0, c_0) \cdot t^{p_i}$ $\quad \sum p_i = 1$, $\sum p_i^2 = 1$

$t \gg 1/H_0$ $\quad \Big(a(t), b(t), c(t)\Big) = (a_0, b_0, c_0) \cdot e^{H_0 t}$

The most general metric perturbations around (11) can be written as

$$g_{\mu\nu} = \begin{pmatrix} -a_{\rm av}^2\,(1+2\,\Phi) & a_{\rm av}\,a\,\partial_1\chi & a_{\rm av}\,b\,(B_{,i}+B_i) \\ & a^2\,(1-2\,\Psi) & a\,b\,\partial_1\left(\tilde{B}_{,i}+\tilde{B}_i\right) \\ & & b^2\left[(1-2\,\Sigma)\,\delta_{ij}+2\,E_{,ij}+E_{(i,j)}\right] \end{pmatrix}$$

$$\tilde{B} \,=\, \Sigma \,=\, E \,=\, E_i \,=\, 0$$

$$k^2 = k_L^2 + k_T^2 \quad , \quad p^2 = p_L^2 + p_T^2 = \left(\frac{k_L}{a}\right)^2 + \left(\frac{k_T}{b}\right)^2$$

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The canonical variables are

$$V \equiv b\left[\delta\phi + \frac{p_T^2\,\phi}{H_a\,p_T^2 + H_b\,(2p_L^2+p_T^2)}\Psi\right] \quad , \quad H_+ \equiv \frac{\sqrt{2}\,b\,M_p\,p_T^2\,H_b}{H_a\,p_T^2 + H_b\,(2p_L^2+p_T^2)}\Psi$$

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$$H_\times \equiv \frac{M_p}{\sqrt{2}}\,\frac{p_L}{p}\,a_{\rm av}\,\epsilon_{ij}\,p_i\,\tilde{B}_j$$

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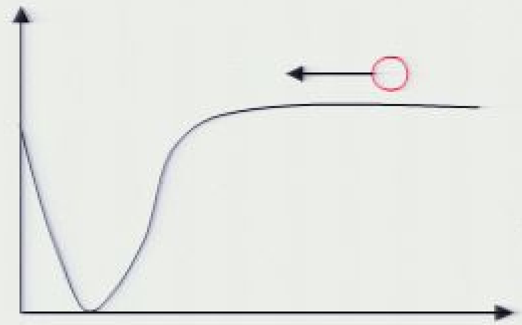

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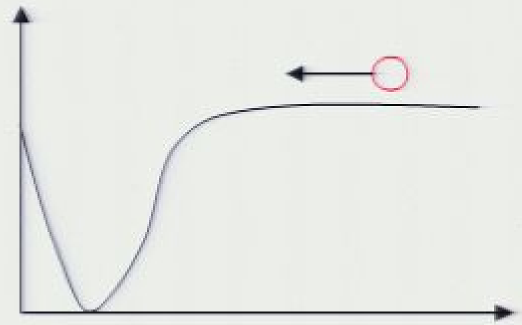

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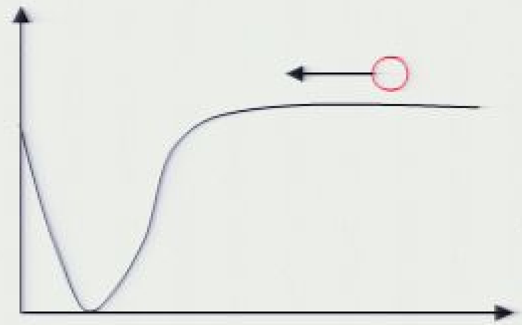

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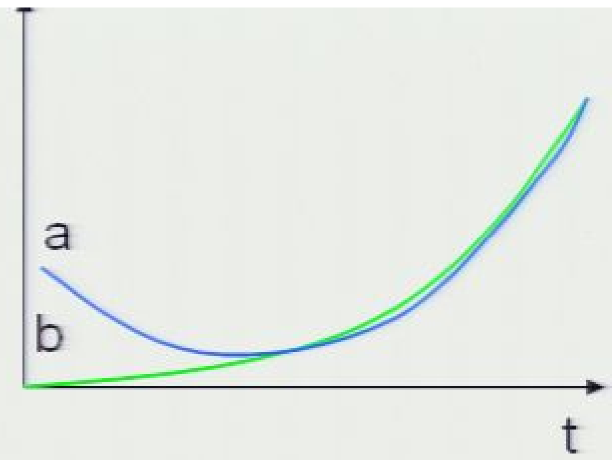
$\delta g_{\mu\nu}$



$$H''_\times + \omega^2_\times H_\times = 0$$

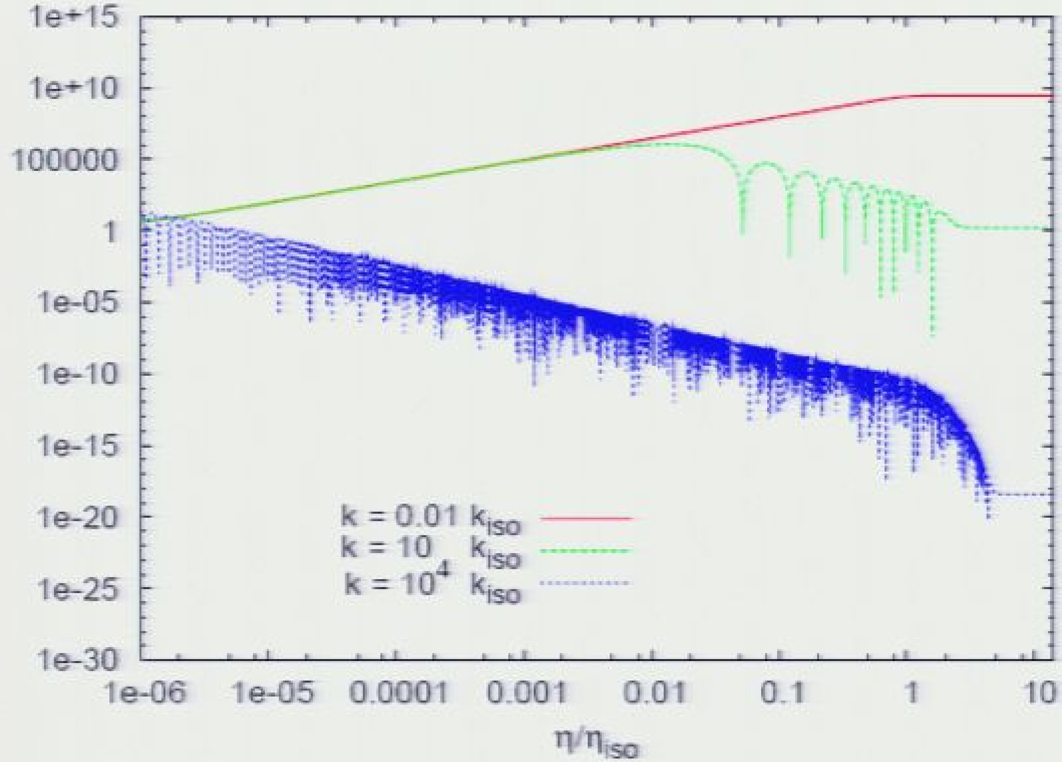
$$\begin{pmatrix} V \\ H_+ \end{pmatrix}'' + \begin{pmatrix} \omega^2_{11} & \omega^2_{12} \\ \omega^2_{12} & \omega^2_{22} \end{pmatrix} \begin{pmatrix} V \\ H_+ \end{pmatrix} = 0$$

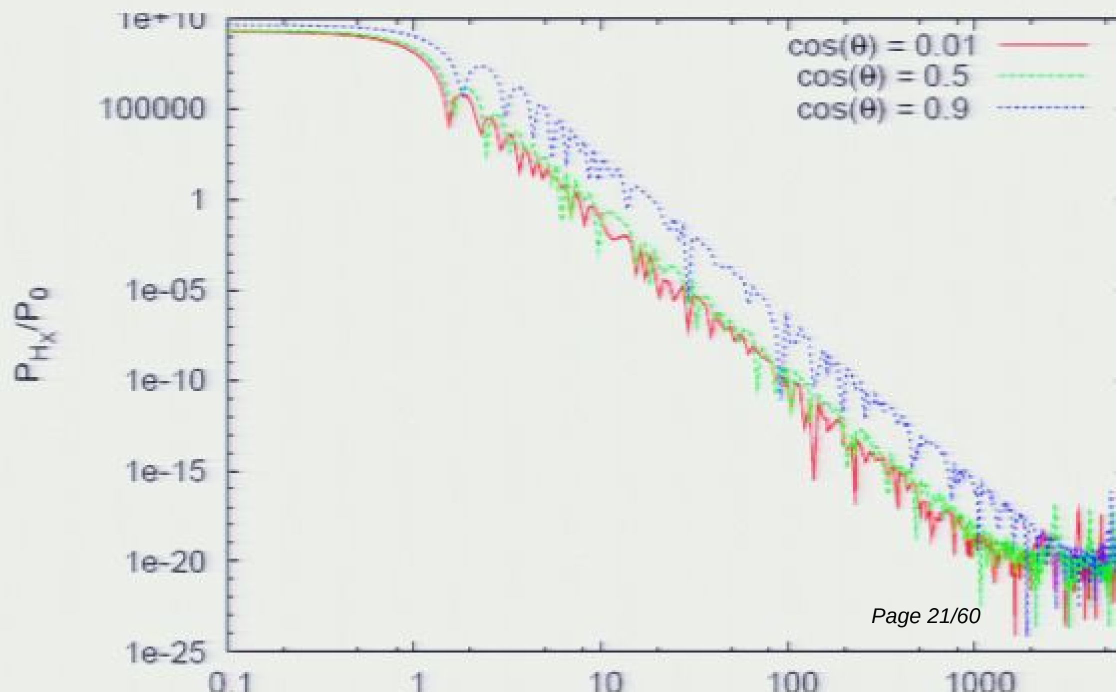
$$S = \frac{1}{2} \int d\eta \, d^3k \left[|H'|^2 + |V'_+|^2 - (H^*, V^*_+) \omega^2 \begin{pmatrix} H \\ V_+ \end{pmatrix} \right]$$

$$\frac{\omega^2_\times}{(a\, b^2)^{2/3}} = p_1^2 + p_2^2 + \frac{H_a^2 - 14\, H_a H_b - 5\, H_b^2}{9} + \frac{\dot{\phi}^2}{2M_p^2} - (H_a - H_b)^2 \frac{p_2^2 \left(-2p_1^2 + p_2^2\right)}{\left(p_1^2 + p_2^2\right)^2}$$

$$H_a = \frac{\dot{a}}{a} \, , H_b = \frac{\dot{b}}{b}$$

$\frac{\Delta T}{T}$ from residual classical GW

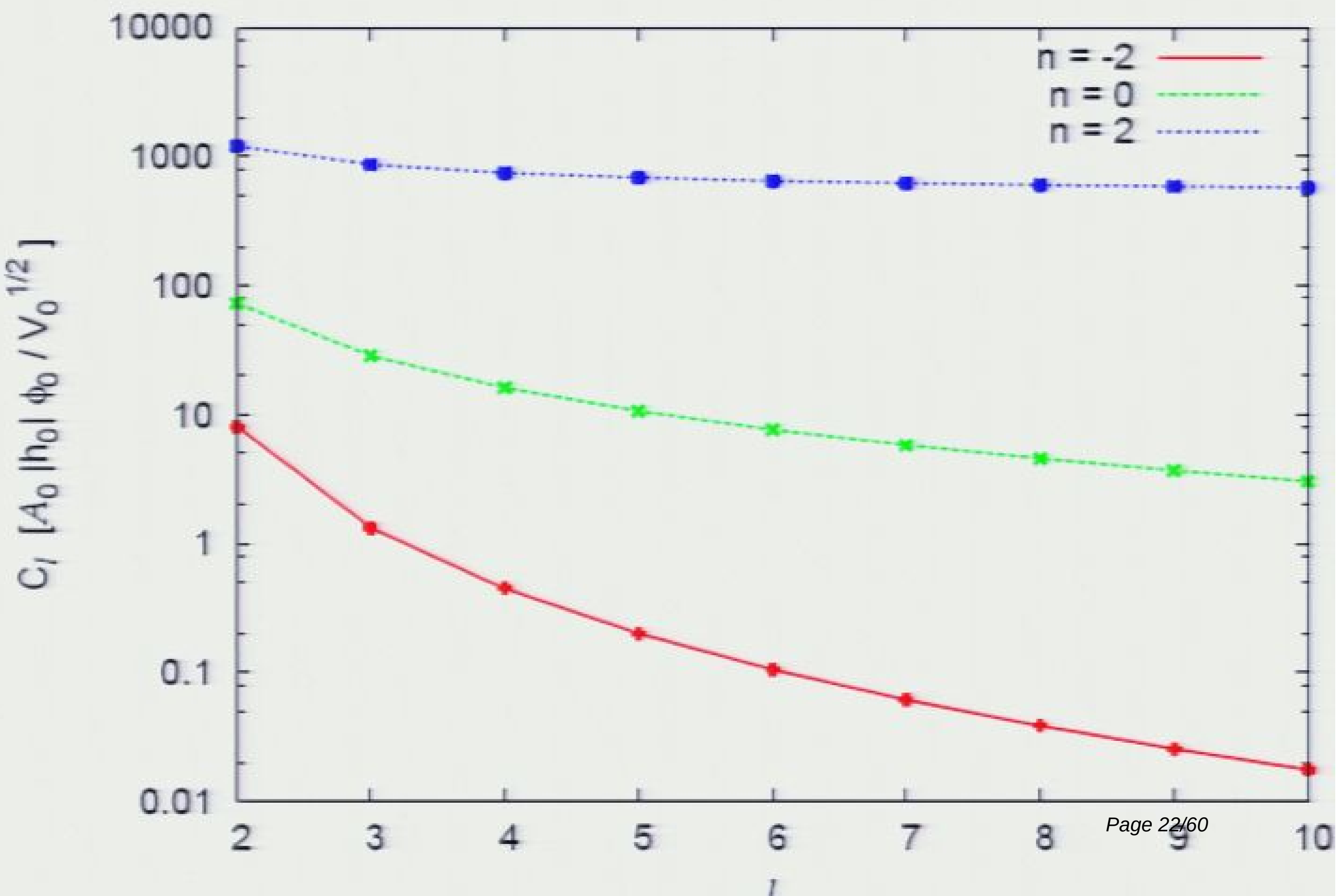




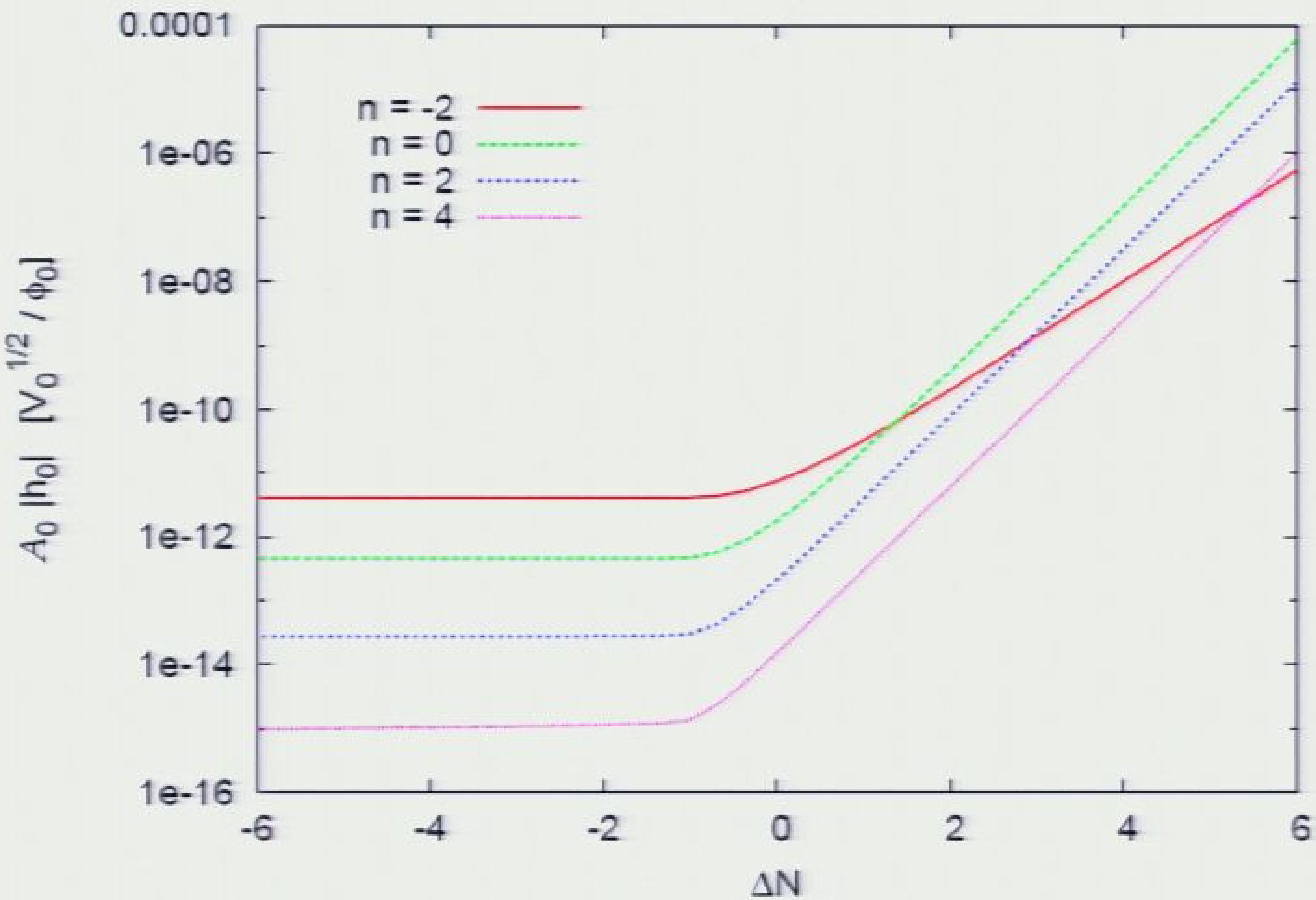
$$C_{\ell\ell' m m'} \equiv \langle a_{\ell m} a^\star_{\ell' m'} \rangle = \int d\Omega_{\hat{p}}\, d\Omega_{\hat{p}'} \left\langle \frac{\delta T}{T}(\hat{\mathbf{p}})\, \frac{\delta T}{T}(\hat{\mathbf{p}}') \right\rangle Y^\star_{\ell m}(\hat{\mathbf{p}}) Y_{\ell' m'}(\hat{\mathbf{p}}')$$

$$C_\ell = \frac{9\,\pi^3}{8}\,\frac{(\ell+2)!}{(\ell-2)!} \int_0^\infty \frac{d(k\,\eta_0)}{(k\,\eta_0)}\, I_\ell^2(k\,\eta_0) \int_{-1}^{+1} \frac{d\xi}{2} \int_0^{2\pi} \frac{d\phi_k}{2\,\pi} \left[P_{H_+}(\mathbf{k}) + P_{H_\times}(\mathbf{k})\right]$$

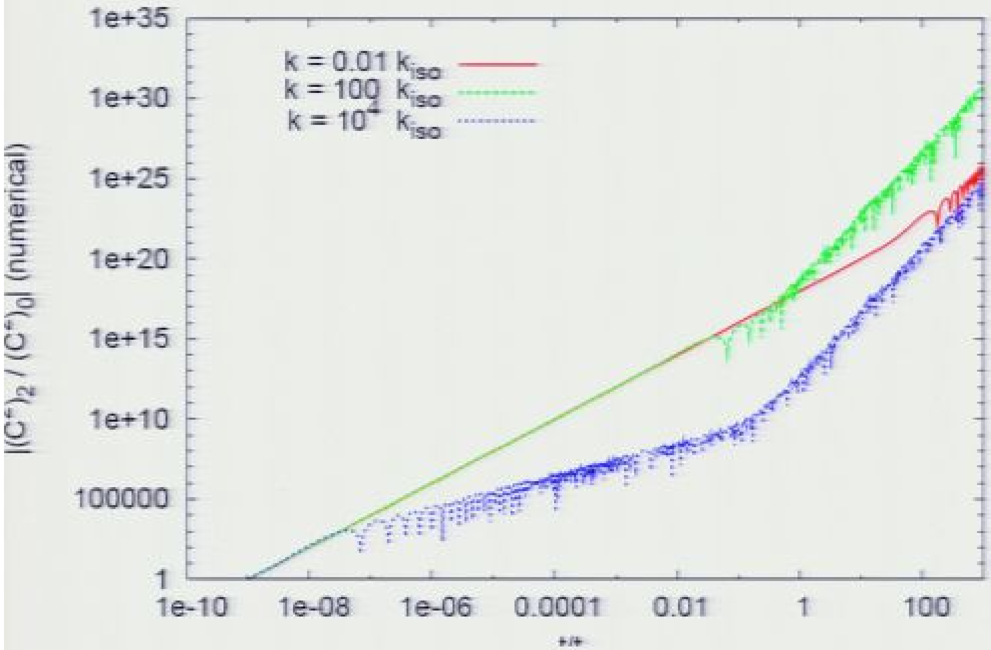
$$C_\ell \simeq \begin{cases} 80\,\ell^{-3.8} \\ 248\,\ell^{-1.9} \\ 1430\,\ell^{-0} \end{cases}$$



limits on the total duration of inflation and the initial GW amplitude.



Expanding and contracting Kasner solutions are instable against GW



$$C_{\alpha\beta}{}^{\gamma\delta} C_{\gamma\delta}{}^{\alpha\beta} = C^2 + C\delta C + \delta C^2 + \cdots$$

$$C^2 = \frac{64}{27t^4}$$

	$\delta C/C$ for H_+	$\delta C^2/C^2$ for H_+	$\delta C^2/C^2$ for $H_\times$	
EXPANDING	const.	const.	t^2	$\leftarrow$ large scales
$(0 < t < \infty)$	const.	$t^{16/3}$	t^4	$\leftarrow$ short scales
CONTRACTING	const.	$\|t\|^{16/3}$	$\|t\|^4$	$\leftarrow$ short scales
$(-\infty < t < 0)$	$\ln\|t\|$	$\|t\|^{-2/3}$	$\|t\|^{-2}$	$\leftarrow$ large scales

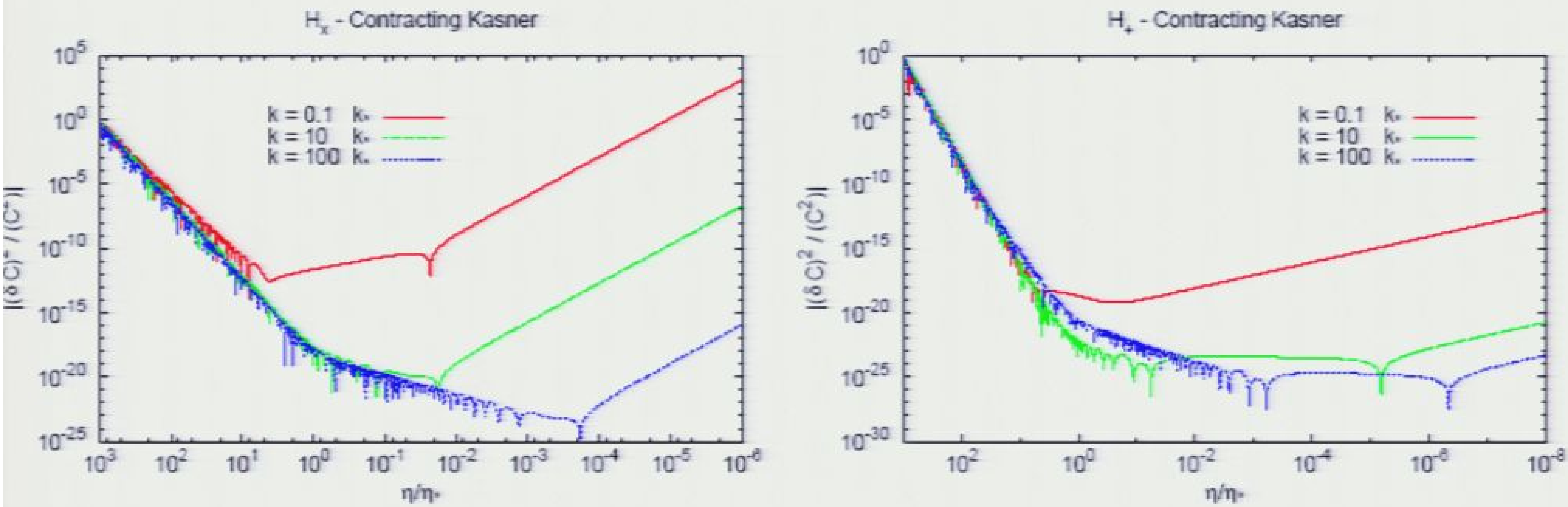


FIG. 7: Contribution to $\delta C^2/C^2$, defined in eq. (40), from $H_\times$ (left panel) and H_+ (right panel) modes with some fixed comoving momentum. The times and momenta are chosen as in Figure 6. While in Figure 6 the background geometry is expanding (from the singularity at $\eta = 0^+$), the background geometry is contracting (towards the singularity at $\eta = 0^-$) in the evolutions shown here.

BKL story

$$ds^2 = dt^2 - h_{\alpha\beta}\,(t, x^\gamma)\,dx^\alpha dx^\beta$$

$$\kappa_{\alpha\beta} = \frac{\partial h_{\alpha\beta}}{\partial t}, \quad h \equiv | h_{\alpha\beta} |$$

$$R_0^0 = 0$$

$$R_b^a = 0$$

$$\dot{\kappa}_a^a + \frac{1}{2}\kappa_a^b \kappa_b^a = 0$$

$$\frac{1}{\sqrt{\eta}}\frac{\partial}{\partial t}\left(\sqrt{\eta}\kappa_a^b\right) = P_b^a$$

$$P_{\alpha\beta} = \partial_\gamma \lambda^\gamma_{\alpha\beta} - \partial_\alpha \lambda^\gamma_{\beta\gamma} + \lambda^\gamma_{\alpha\beta}\lambda^\delta_{\gamma\delta} - \lambda^\gamma_{\alpha\delta}\lambda^\delta_{\beta\gamma}$$

$$\lambda^\gamma_{\alpha\beta} \equiv \frac{1}{2} h^{\gamma\delta}(\partial_\alpha h_{\delta\beta} + \partial_\beta h_{\alpha\delta} - \partial_\delta h_{\alpha\beta}$$

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$$\begin{aligned} \dot{\kappa}_a^a + \frac{1}{2}\kappa_a^b \kappa_b^a &= 0 \\ \frac{1}{\sqrt{\eta}}\frac{\partial}{\partial t}\left(\sqrt{\eta}\kappa_a^b\right) &= P_b^a \end{aligned}$$

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$$h_{\alpha\beta} = a^2(t) l_\alpha l_\beta + b^2(t) m_\alpha m_\beta + c^2(t) n_\alpha n_\beta \, .$$

$$-R_l^l = \frac{(\dot{a}bc)^{\cdot}}{abc} + \frac{1}{2a^2b^2c^2}\left[\lambda^2 a^4 - \left(\mu b^2 - \nu c^2\right)^2\right] = 0$$

$$-R_m^m = \frac{(a\dot{b}c)^{\cdot}}{abc} + \frac{1}{2a^2b^2c^2}\left[\mu^2 b^4 - \left(\lambda a^2 - \nu c^2\right)^2\right] = 0$$

$$-R_n^n = \frac{(ab\dot{c})^{\cdot}}{abc} + \frac{1}{2a^2b^2c^2}\left[\nu^2 c^4 - \left(\lambda a^2 - \mu b^2\right)^2\right] = 0$$

(λ, μ, ν) for Bianchi type II (1, 0, 0), VII (1, 1, 0), VIII (1, 1, -1) and IX (1, 1, 1).

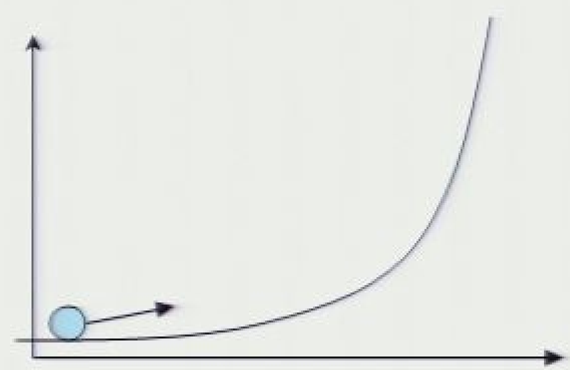
$$\alpha = \ln a \, , \quad \beta = \ln b \, , \quad \gamma = \ln c \qquad\qquad dt = abc \, d\tau$$

$$
\begin{aligned}
2\alpha_{\tau\tau} &= \left(\mu b^2 - \nu c^2\right)^2 - \lambda^2 a^4 \\
2\beta_{\tau\tau} &= \left(\lambda a^2 - \nu c^2\right)^2 - \mu^2 b^4 \\
2\gamma_{\tau\tau} &= \left(\lambda a^2 - \mu b^2\right)^2 - \nu^2 c^4
\end{aligned}
$$

Bianchi type II

$$\alpha_{\tau\tau} = -\frac{1}{2}e^{4\alpha}$$

$$\beta_{\tau\tau} = \gamma_{\tau\tau} = \frac{1}{2}e^{4\alpha}$$



BKL billiard

$$h_{\alpha\beta} = a^2(t) l_\alpha l_\beta + b^2(t) m_\alpha m_\beta + c^2(t) n_\alpha n_\beta \, .$$

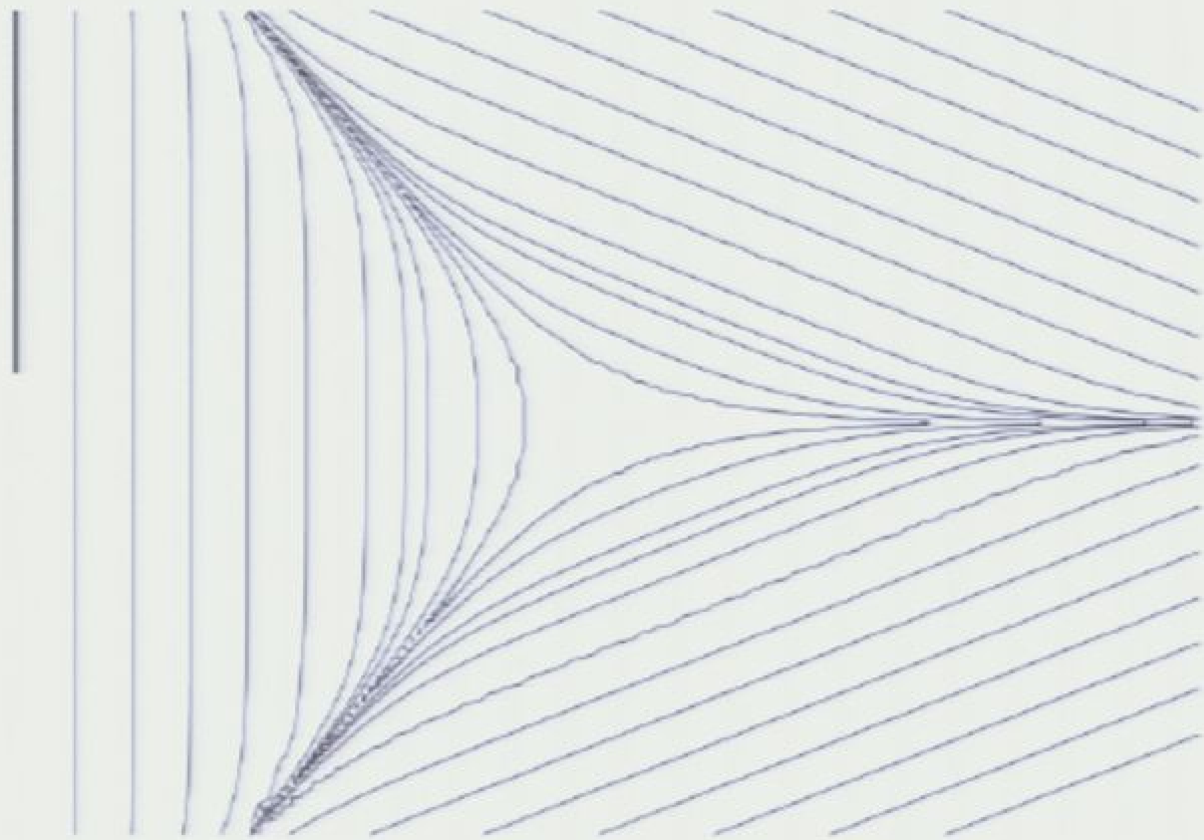
$$p_l = p_1 \, , \quad p_m = p_2 \, , \quad p_n = p_3 \, ,$$

$$a \sim t^{p_1} \, , \quad b \sim t^{p_2} \, , \quad c \sim t^{p_3} \, ,$$

$$a \sim t^{p'_l} \, , \quad b \sim t^{p'_m} \, , \quad c \sim t^{p'_n} \, .$$

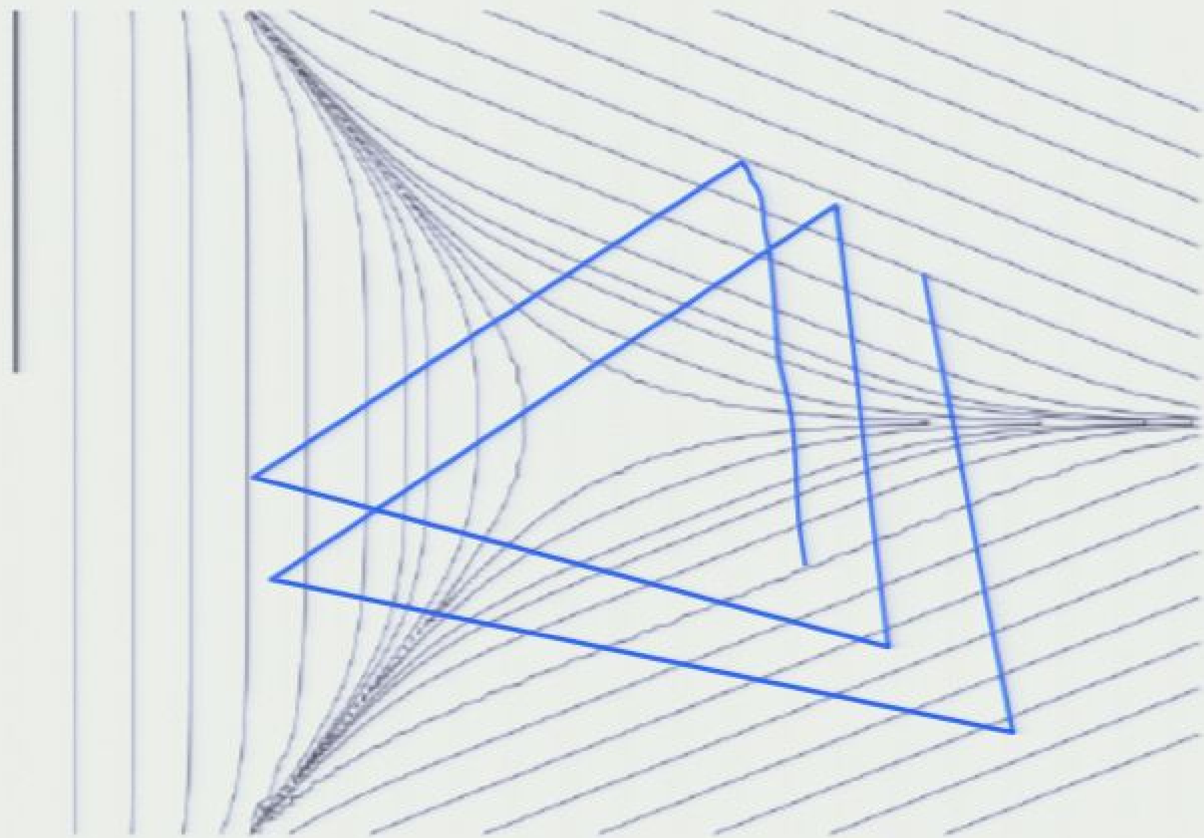
$$p'_l = \frac{|\, p_1 \,|}{1 - 2 \,|\, p_1 \,|} \qquad p'_m = -\frac{2 \,|\, p_1 \,| - p_2}{1 - 2 \,|\, p_1 \,|} \, . \qquad p'_n = \frac{p_3 - 2 \,|\, p_1 \,|}{1 - 2 \,|\, p_1 \,|}$$

BKL billiard



Equipotential lines of the Bianchi type IX model in the β_+, β_- plane.

BKL billiard



Equipotential lines of the Bianchi type IX model in the β_+, β_- plane.

The Generalized Kasner Solution

$$p_l(x^\gamma) + p_m(x^\gamma) + p_n(x^\gamma) = p_l^2(x^\gamma) + p_m^2(x^\gamma) + p_n^2(x^\gamma) = 1$$

Unstable mode

$$l \cdot \nabla \wedge l = 0$$

Generalized solution is valid as far as

$$P_l^l,\ P_m^m,\ P_n^n \ll t^{-2}$$

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$$-R^l_l = \frac{(\dot{a}bc)^{\cdot}}{abc} + \lambda^2 \frac{a^2}{2b^2c^2} = 0\,,$$

$$-R^m_m = \frac{(a\dot{b}c)^{\cdot}}{abc} - \lambda^2 \frac{a^2}{2b^2c^2} = 0\,,$$

$$-R^l_l = \frac{(ab\dot{c})^{\cdot}}{abc} - \lambda^2 \frac{a^2}{2b^2c^2} = 0\,,$$

$$-R^0_0 = \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} = 0\,,$$

$$\lambda(x) = \frac{l \cdot \nabla \wedge l}{l \cdot [m \times n]}$$

BKL billiard

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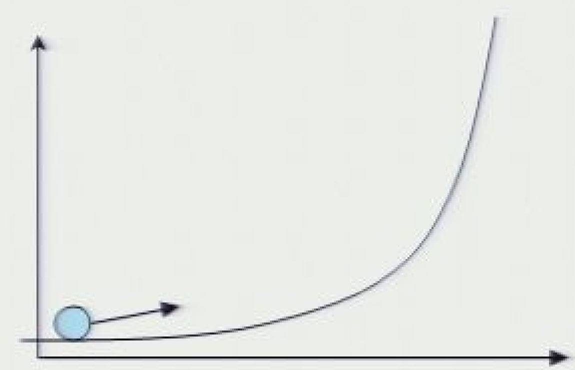
$$
\begin{aligned}
2\alpha_{\tau\tau} &= \left(\mu b^2 - \nu c^2\right)^2 - \lambda^2 a^4 \\
2\beta_{\tau\tau} &= \left(\lambda a^2 - \nu c^2\right)^2 - \mu^2 b^4 \\
2\gamma_{\tau\tau} &= \left(\lambda a^2 - \mu b^2\right)^2 - \nu^2 c^4
\end{aligned}
$$

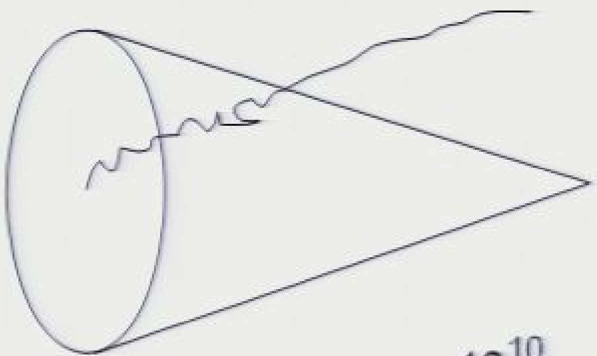
Bianchi type II

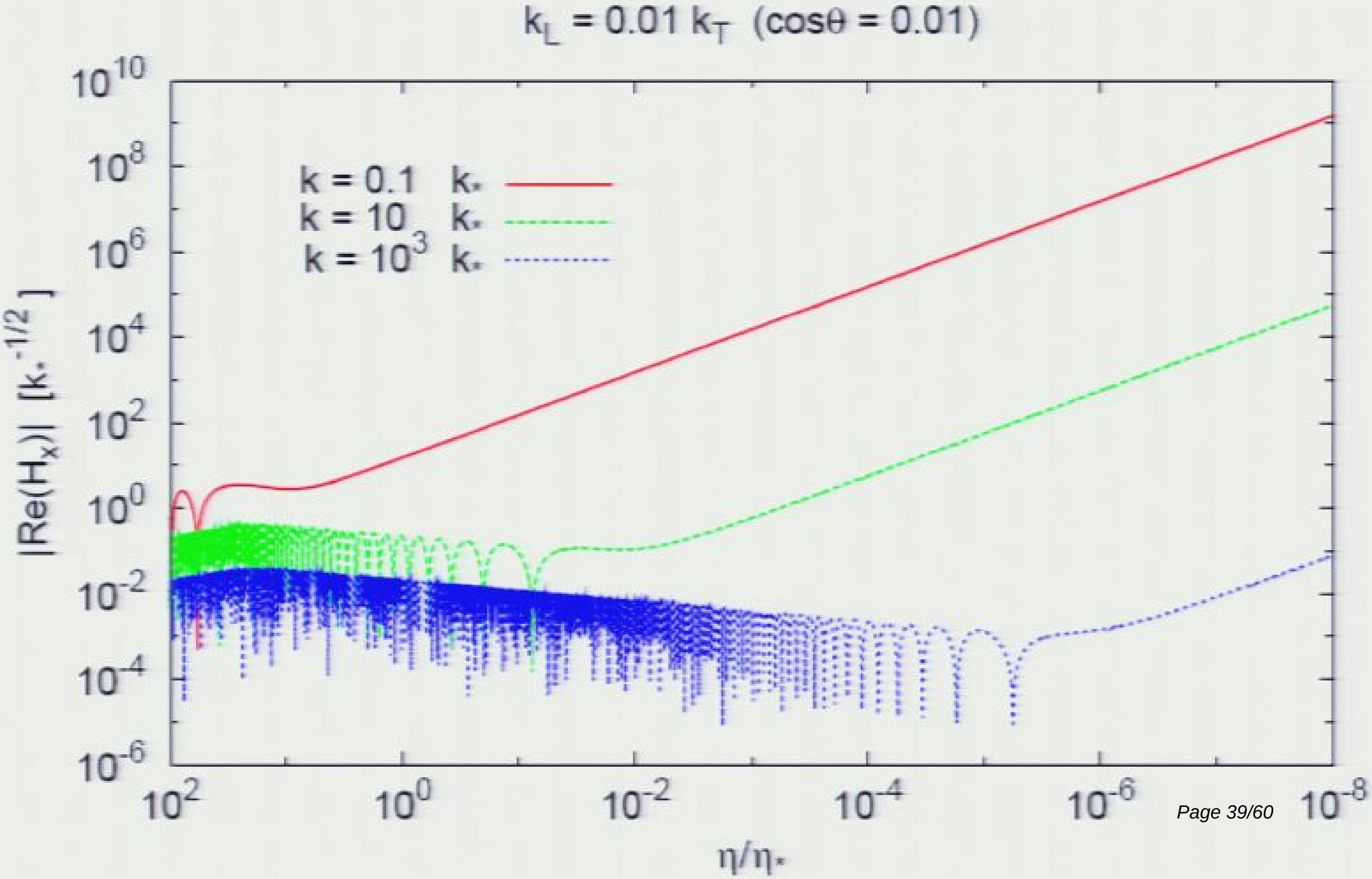
$$\alpha_{\tau\tau} = -\frac{1}{2}e^{4\alpha}$$

$$\beta_{\tau\tau} = \gamma_{\tau\tau} = \frac{1}{2}e^{4\alpha}$$



nstability of contracting Kasner against vacuum fluctuations of gravitons





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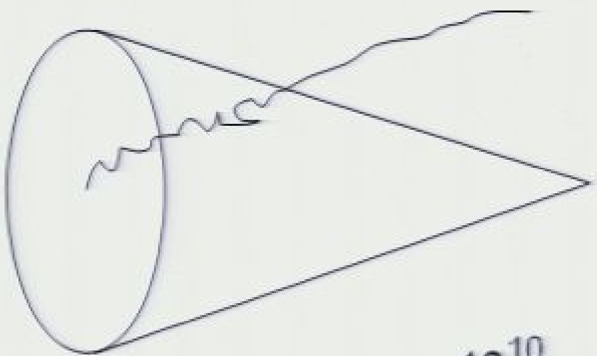

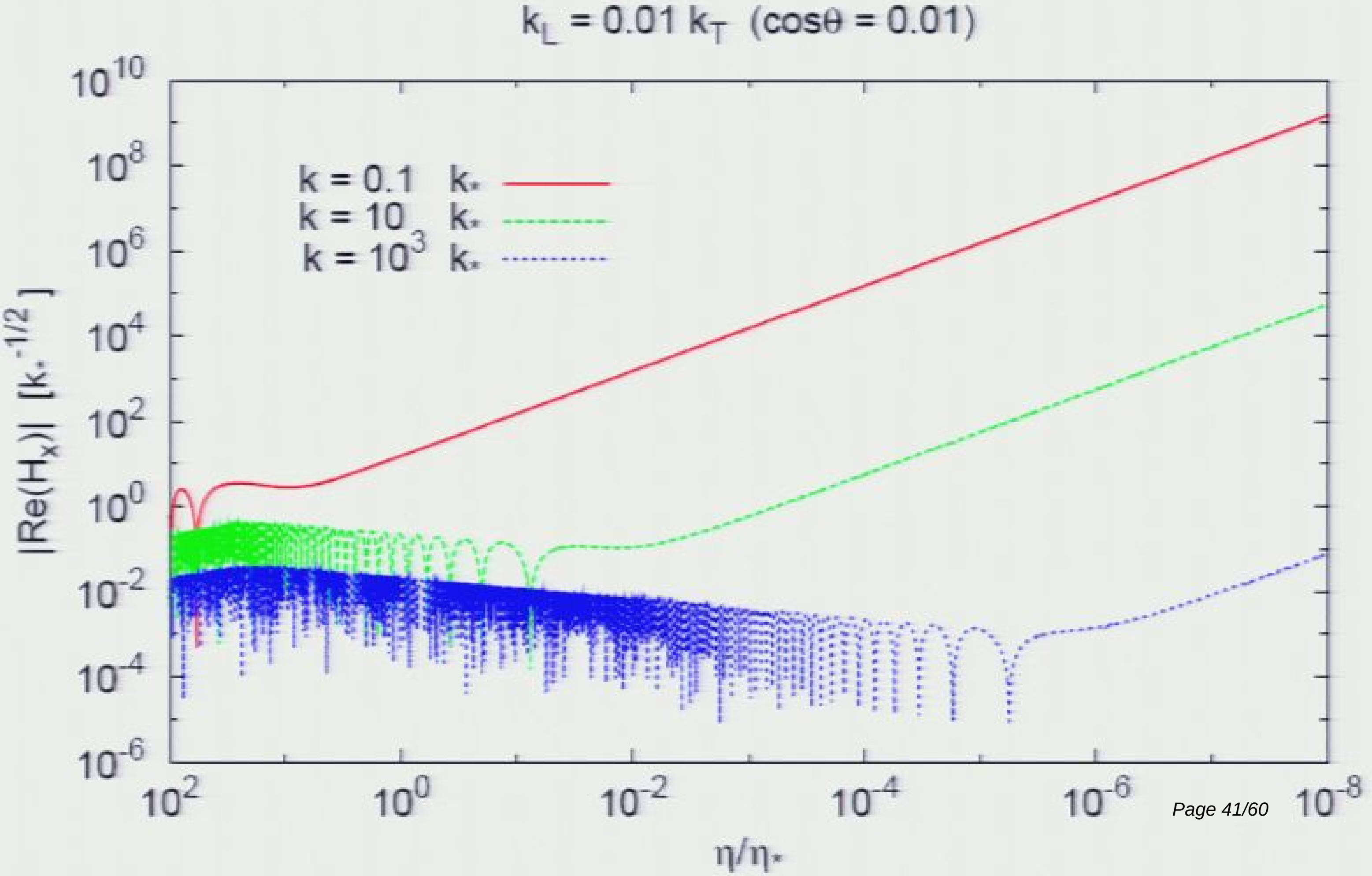

Stochastic equation for generalized Kasner contraction collapse

$$\begin{aligned} \dot{\kappa}_a^a + \frac{1}{2}\kappa_a^b\kappa_b^a &= 0 \\ \frac{1}{\sqrt{\eta}}\frac{\partial}{\partial t}\left(\sqrt{\eta}\kappa_a^b\right) &= \widehat{P}_b^a(H_+, H_\times) \end{aligned}$$

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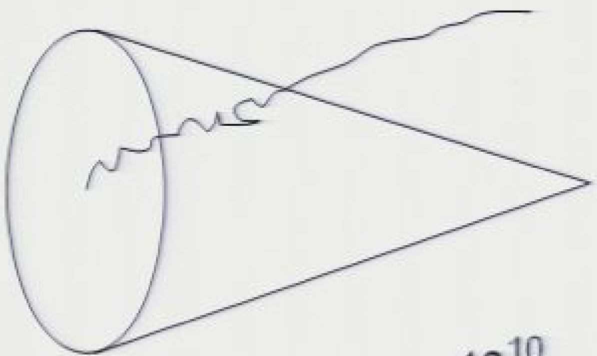

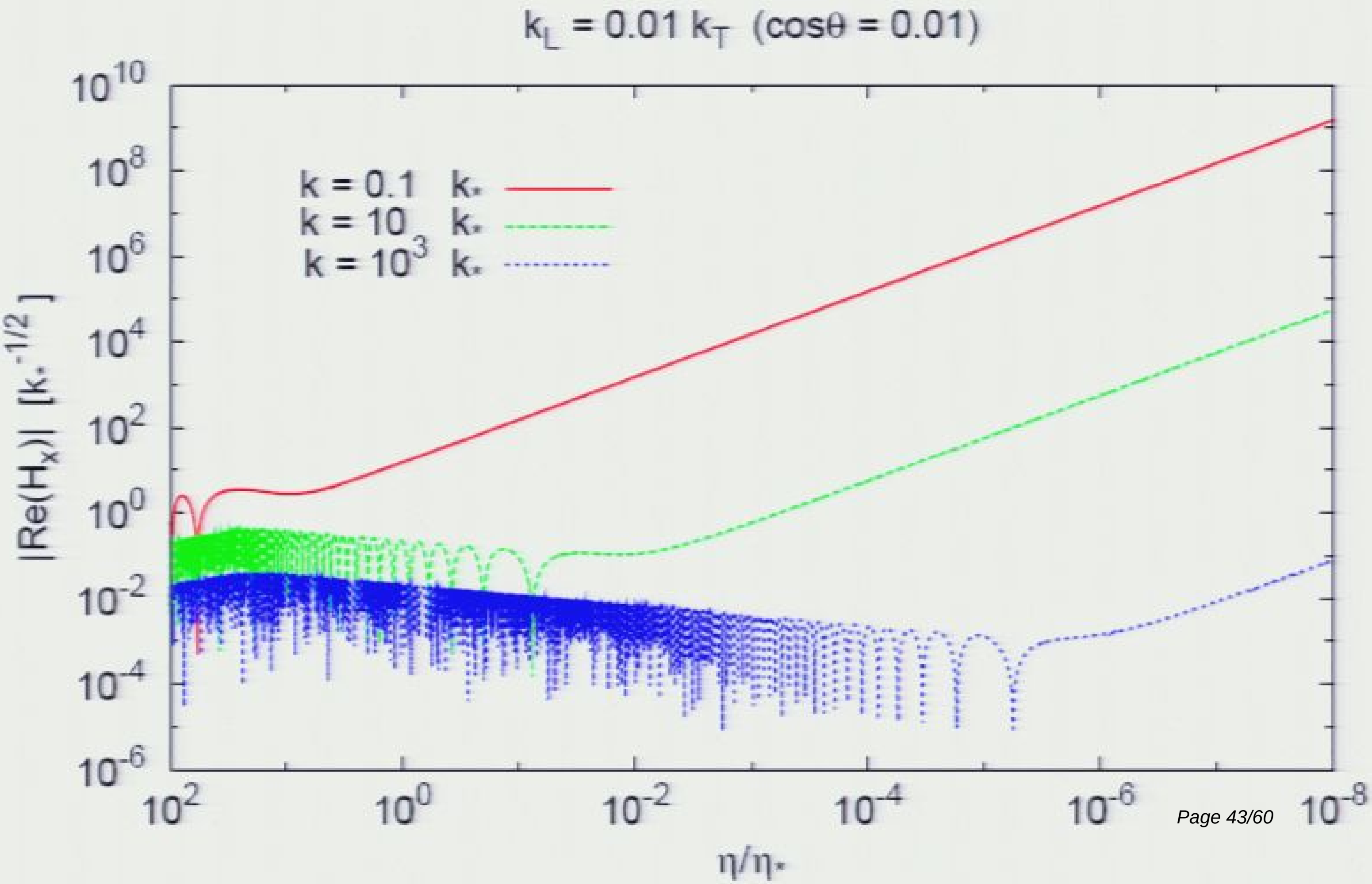

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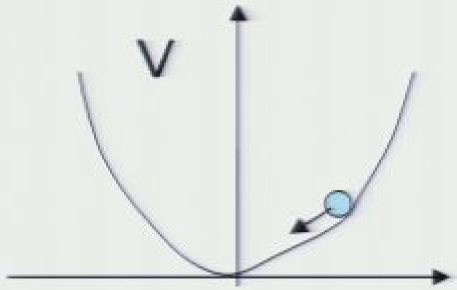
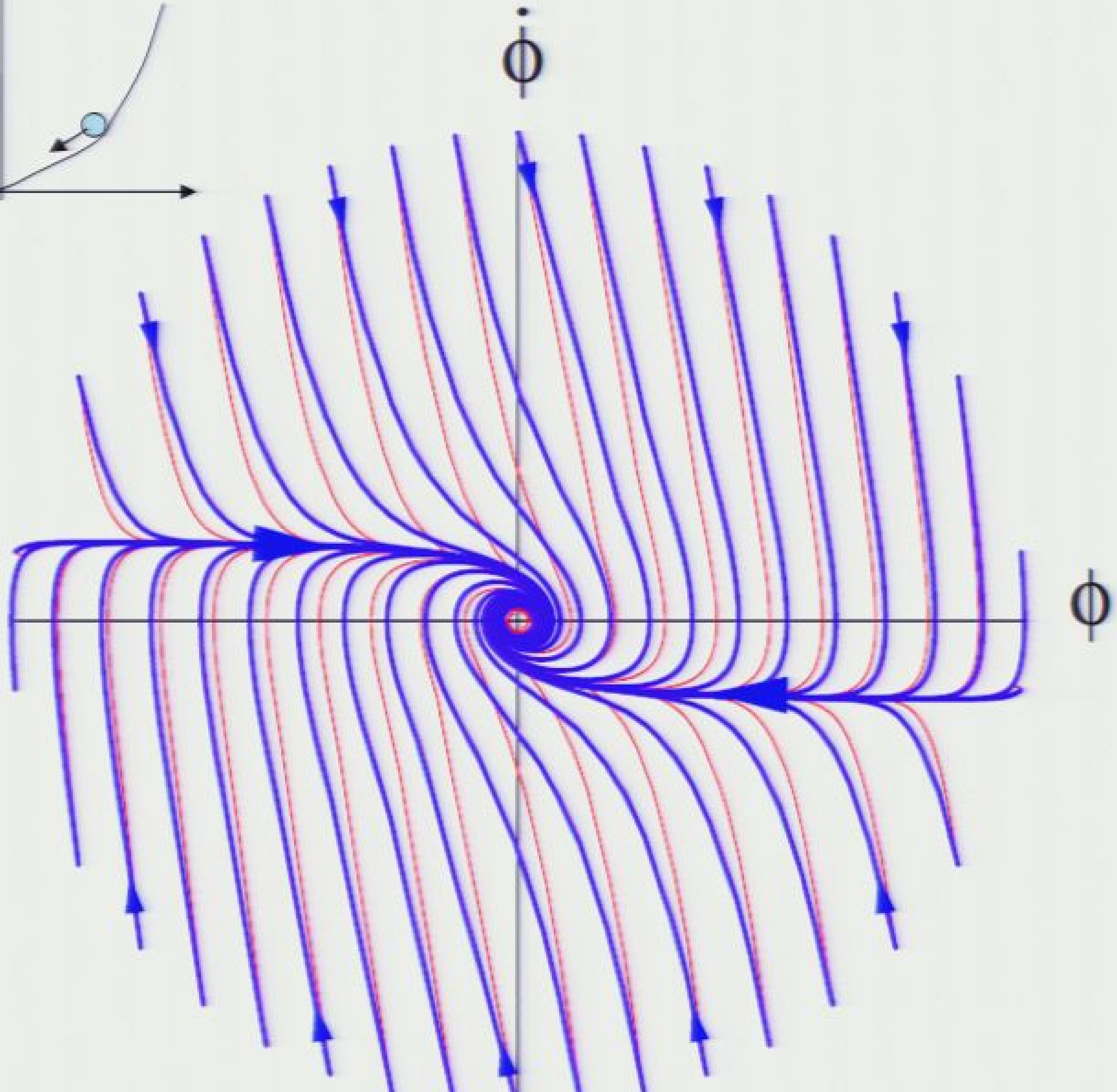
\end{aligned}
$$

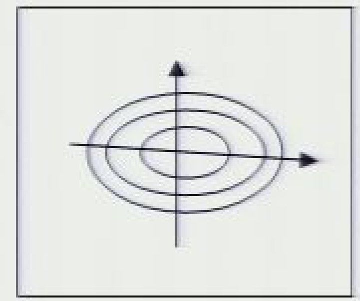
Family phase portrait of inflation

$$\ddot{\phi} + 3H\phi + V_{,\phi} = 0$$

$$3H^2 = \frac{8\pi}{M_p^2}\left(\dot{\phi}^2/2 + V\right)$$







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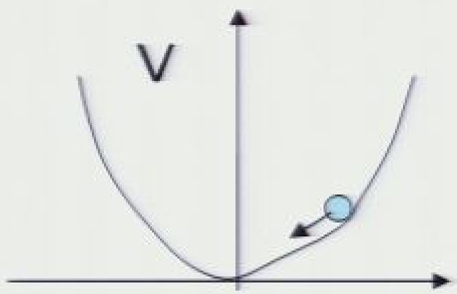
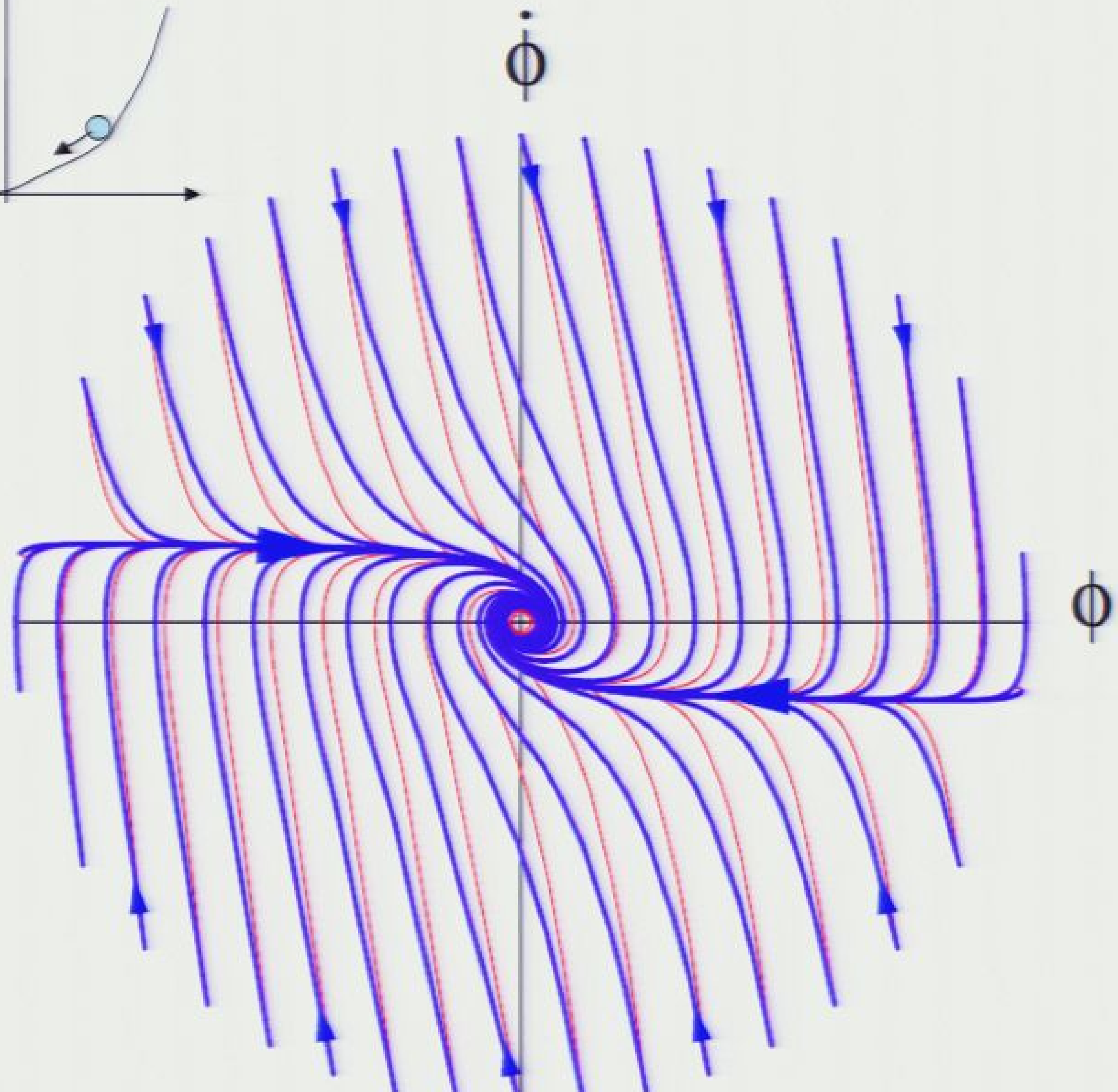
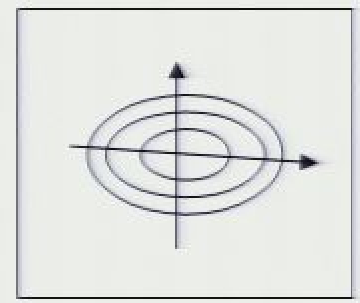
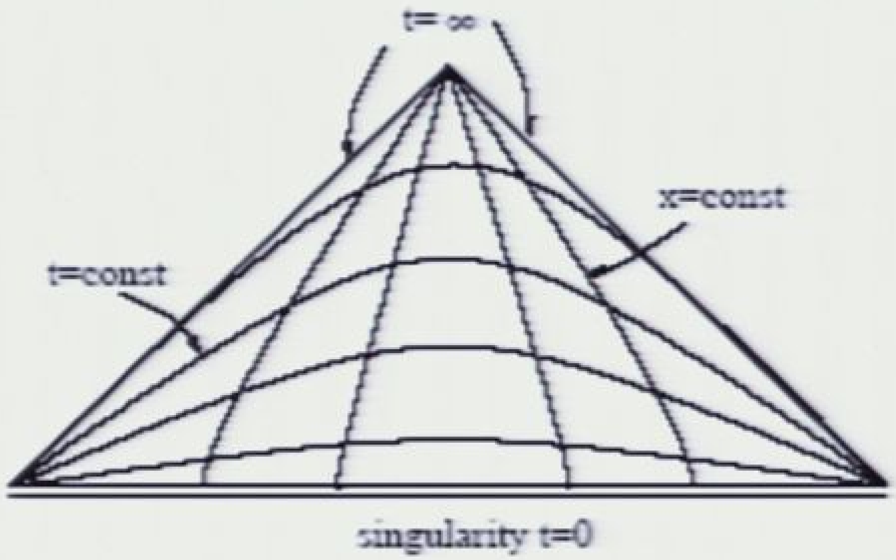
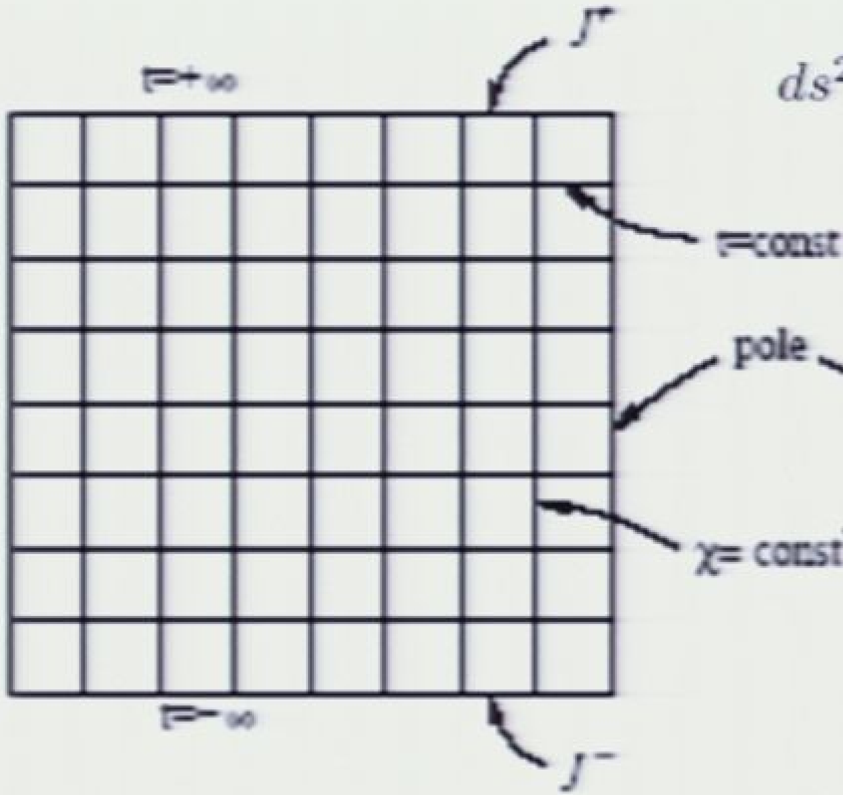


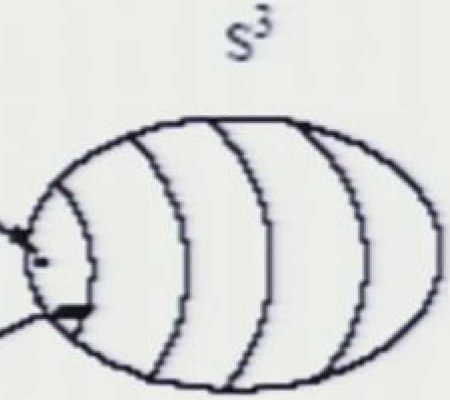


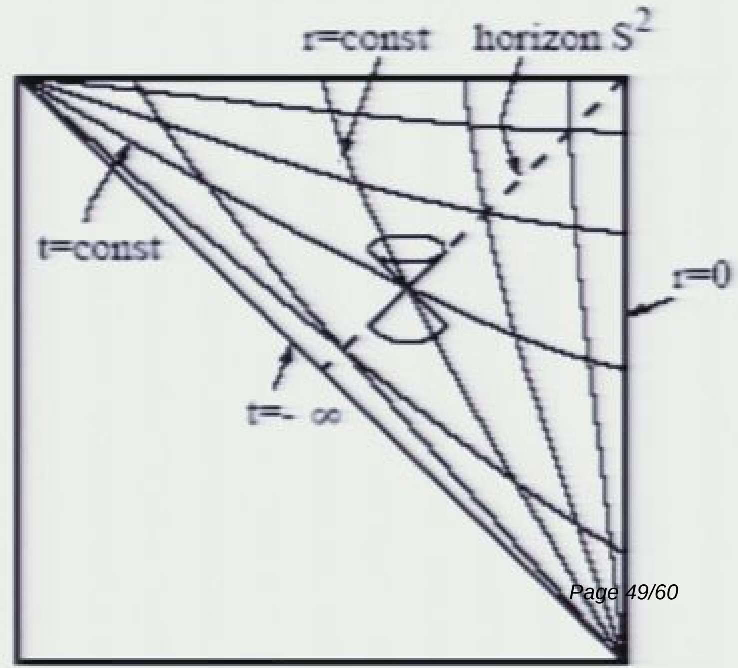



t= ∞
x=const
t=const
singularity t=0

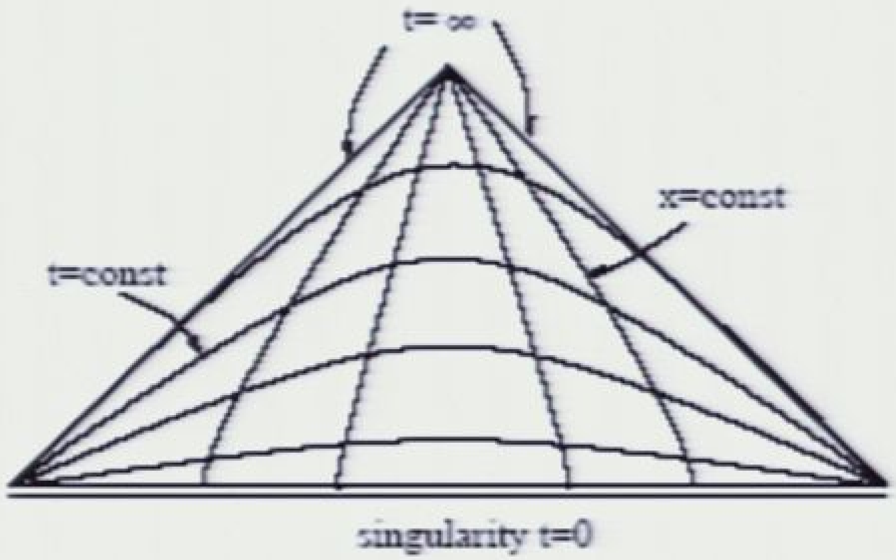


$$ds^2 = dt^2 - \frac{1}{H^2}\cosh^2(Ht)\,\left(d\chi^2 + \sin^2\chi\, d\Omega_2^2\right)$$





$$ds^2 = dt^2 - e^{\pm 2Ht}\left(dr^2 + r^2 d\Omega_2^2\right)$$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x=const
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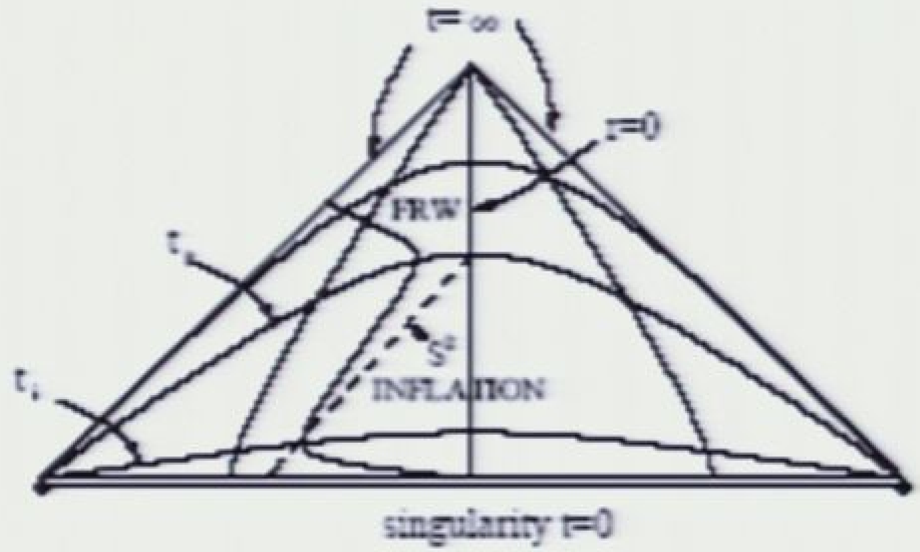
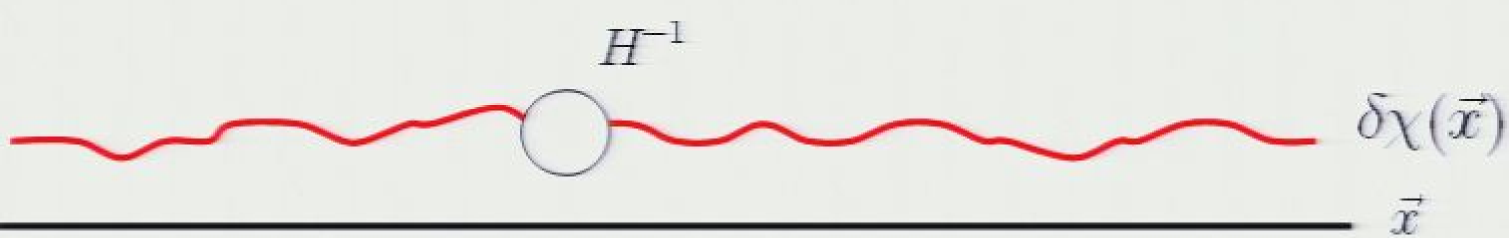
singularity t=0


t= ∞
r=0
FRW
INFLATION
singularity t=0

Light field at inflation

$$\delta\chi = \int d^3k (a_k \chi_k(t) e^{i\vec{k}\vec{x}} + h.c.)$$

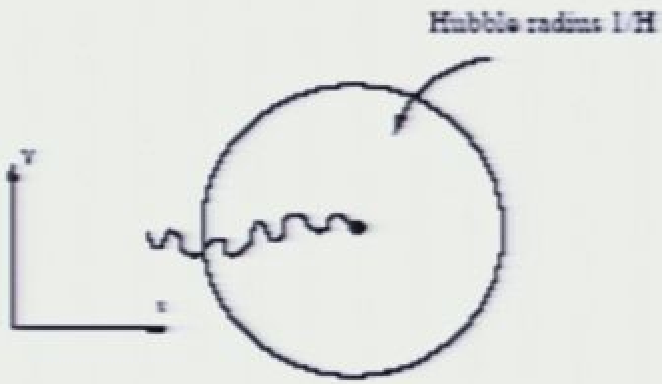
$$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = 0$$

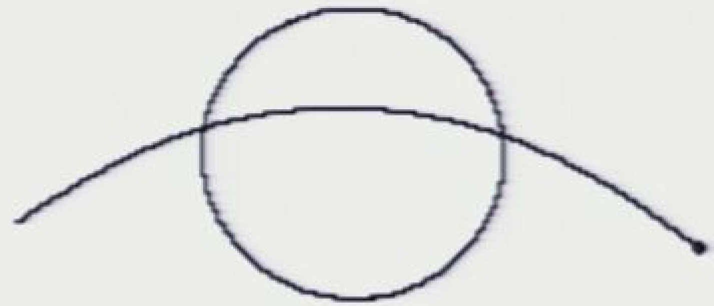


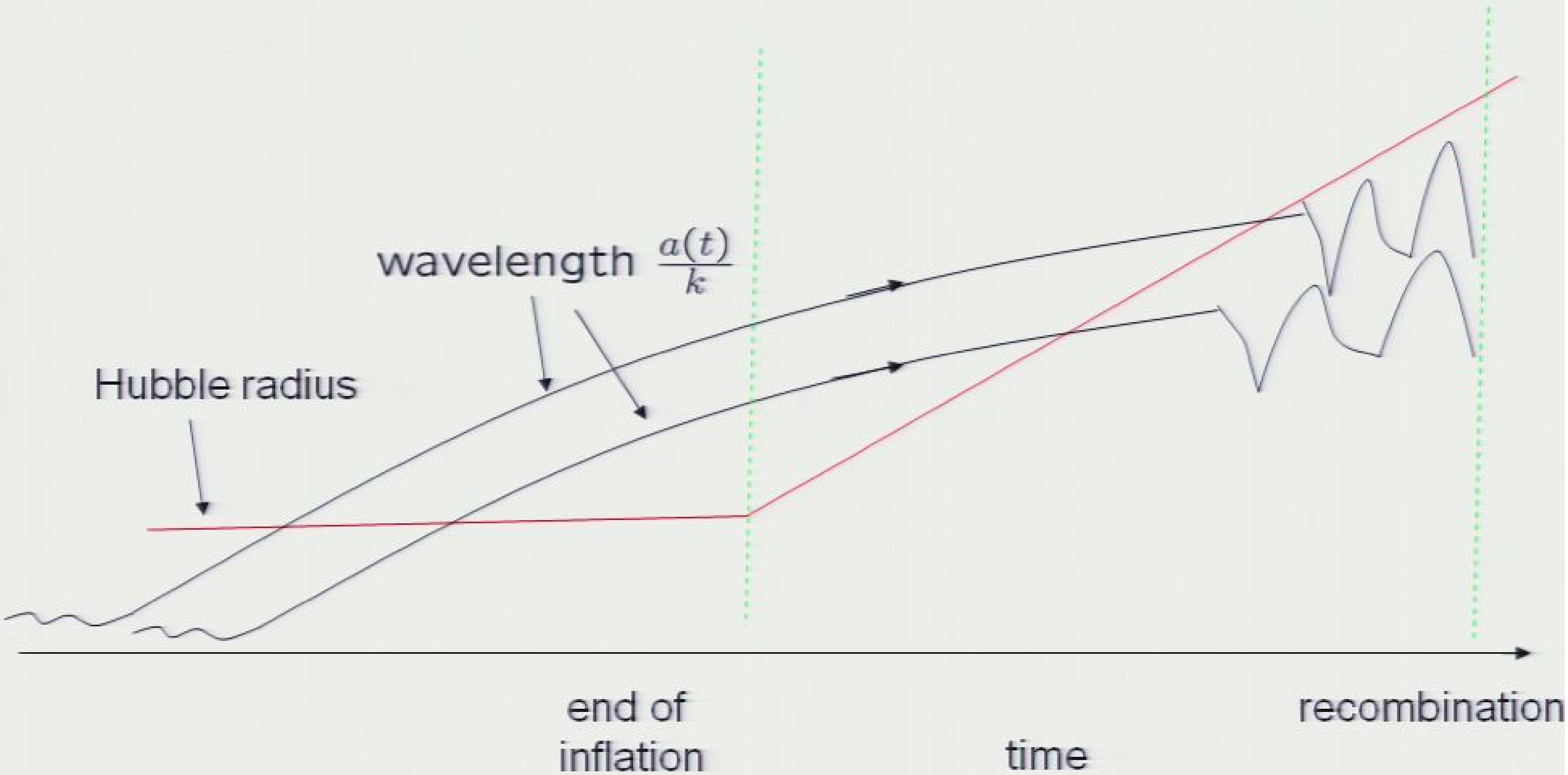
$$f_k(\eta) = \frac{iH}{\sqrt{2}k^{3/2}}\, e^{-ik\eta}\,(1-ik\eta)$$

$H/a(t) < k < H$

$$\langle \delta\phi(t)^2 \rangle = \frac{H^2}{4\pi^2} \int_{H/a}^{H} \frac{dk}{k} = \frac{H^3}{4\pi^2}\, t$$


Hubble radius 1/H




wavelength $\frac{a(t)}{k}$
Hubble radius
end of inflation
time
recombination

Generation of cosmological fluctuations

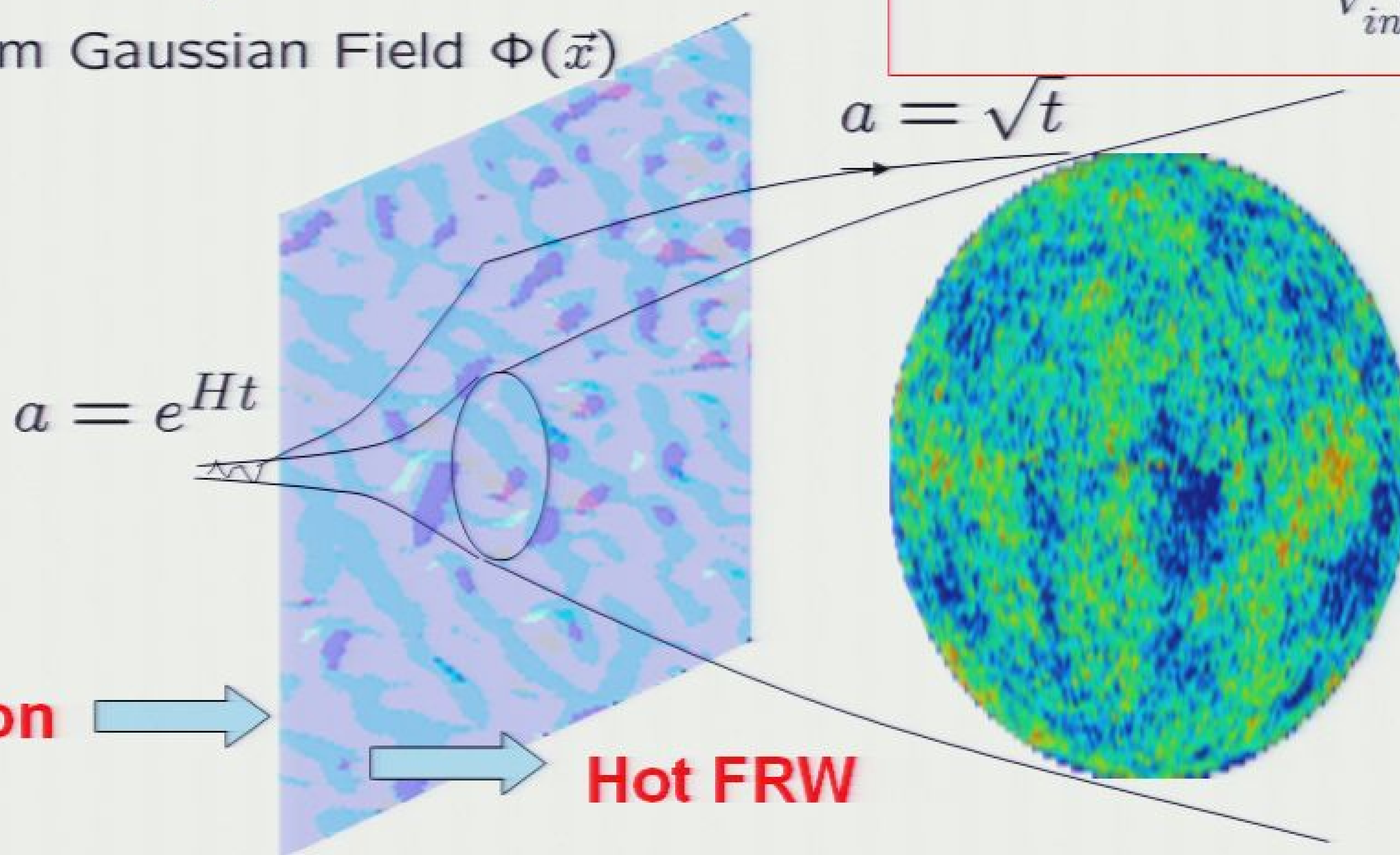
calar metric Fluctuations from Inflation

$$^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 d\vec{x}^2$$

Initial conditions from Inflation ⟶

$$\Omega_{tot} = 1$$
$$k^3\Phi_k^2 \to P_s = A_s k^{n_s - 1}$$
$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$
$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

Random Gaussian Field $\Phi(\vec{x})$



Stochastic equation for slow-rolling inflaton

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

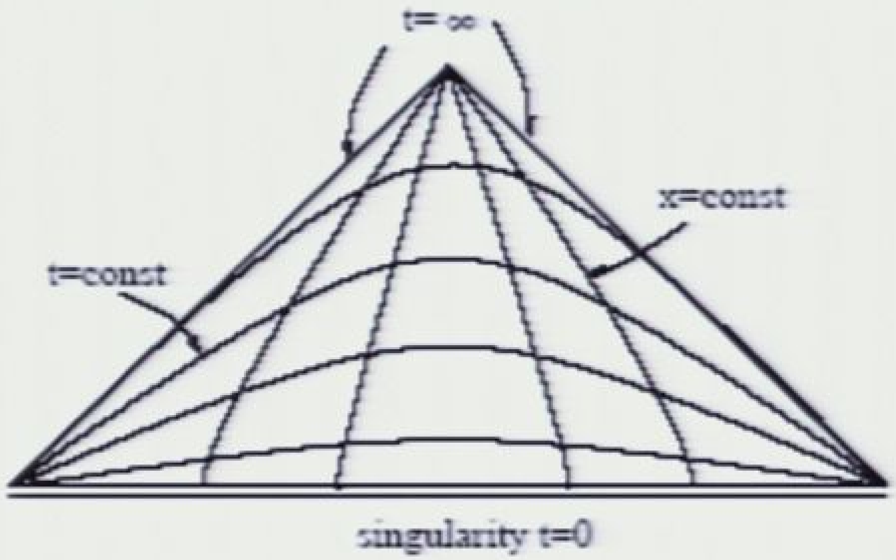
slow roll

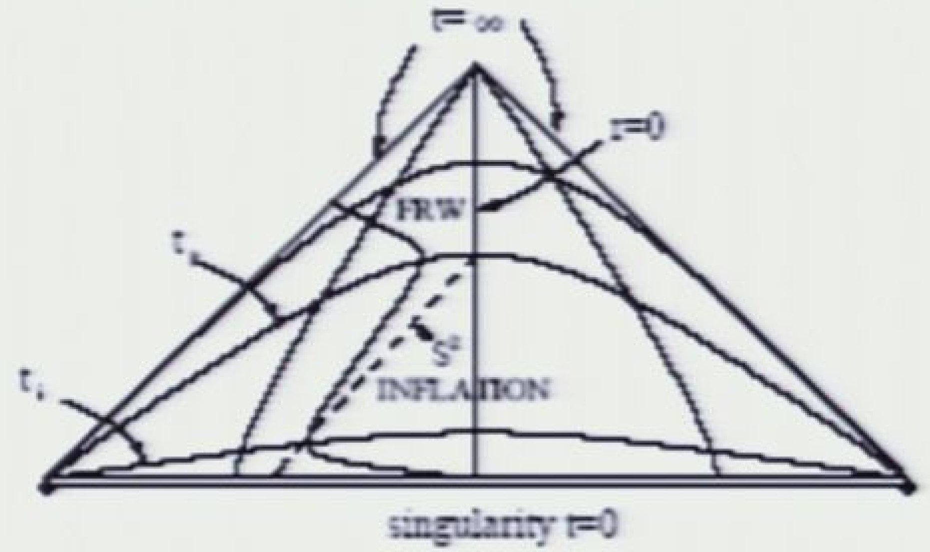
$$\phi(t,\vec{x}) = \phi(t) + \delta\phi(t,\vec{x})$$

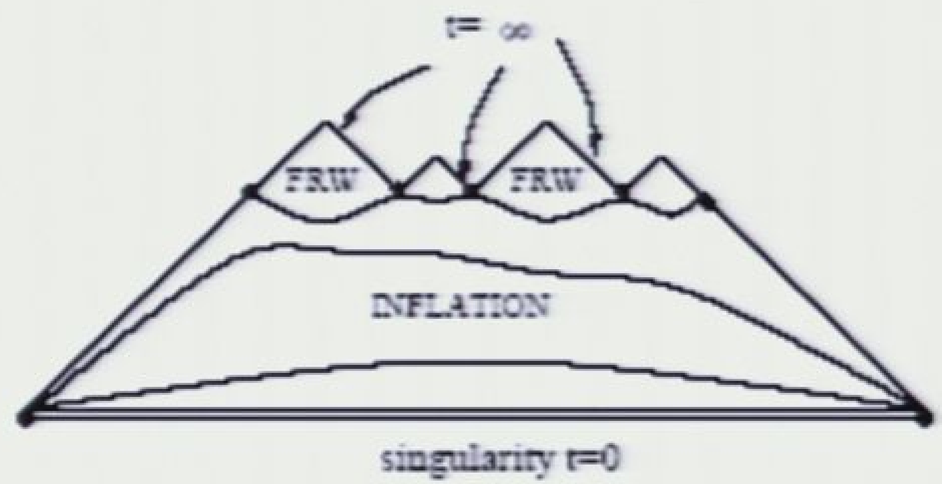
$$\delta\phi = \int_0^{1/H} \theta(k - a(t)H)\phi_k + \text{short wavlength modes}$$

$$3H\dot{\phi} = -V_{,\phi} + \hat{\eta}$$

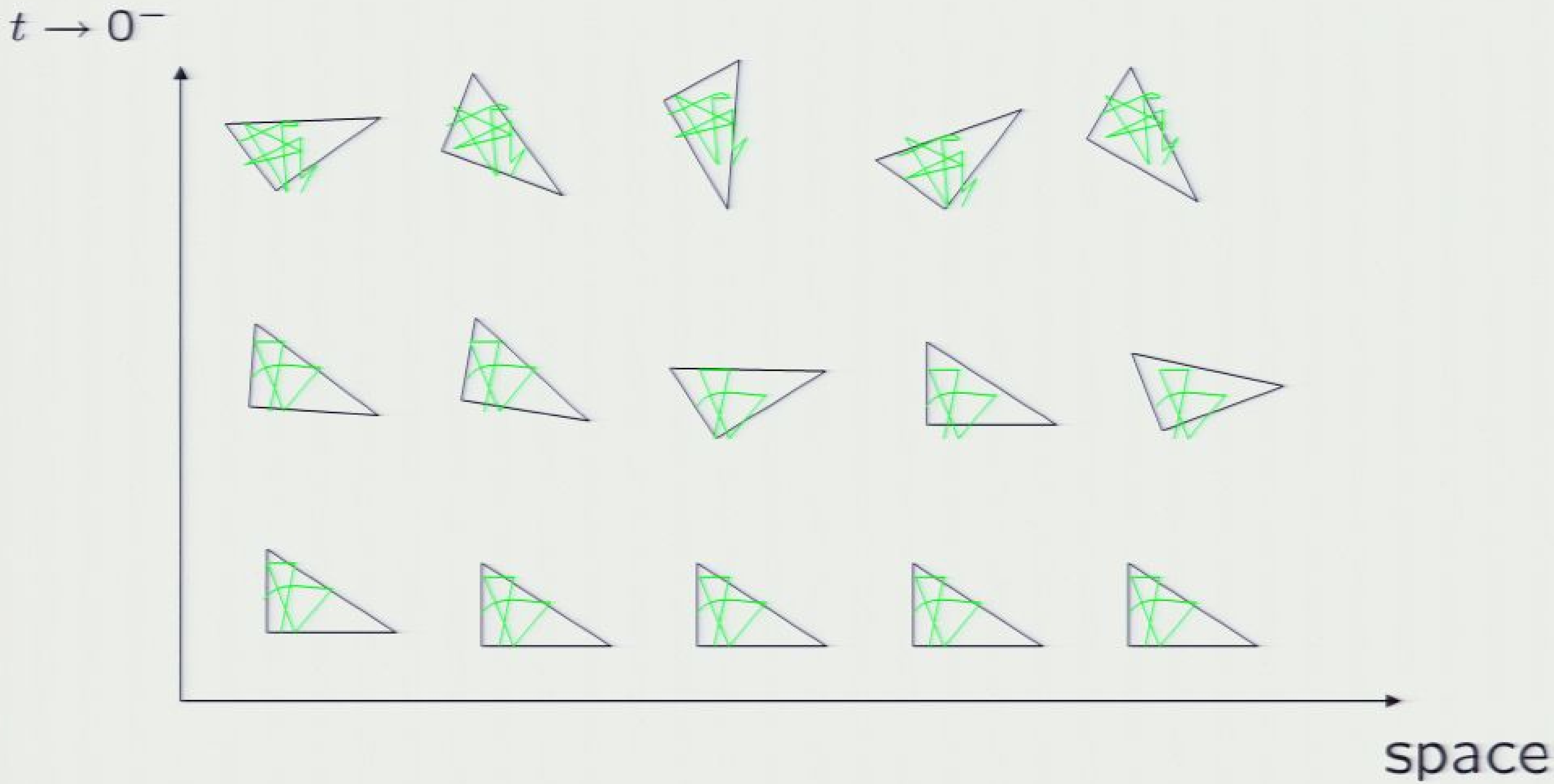
$$< \eta(t)\eta(t') > = \frac{H^3}{4\pi^2}\delta(t - t')$$


t= ∞
x=const
t=const
singularity t=0


t= ∞
r=0
FRW
INFLATION
singularity t=0


t= ∞
FRW
FRW
INFLATION
singularity t=0

$$ds^2 = dt^2 - e^{2\alpha} e^{\beta_{ij}} dx_i dx_j \qquad \beta_{ij} \rightarrow \beta_+, \beta_-$$



Stochastic equation for generalized Kasner contraction collapse

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\end{aligned}
$$