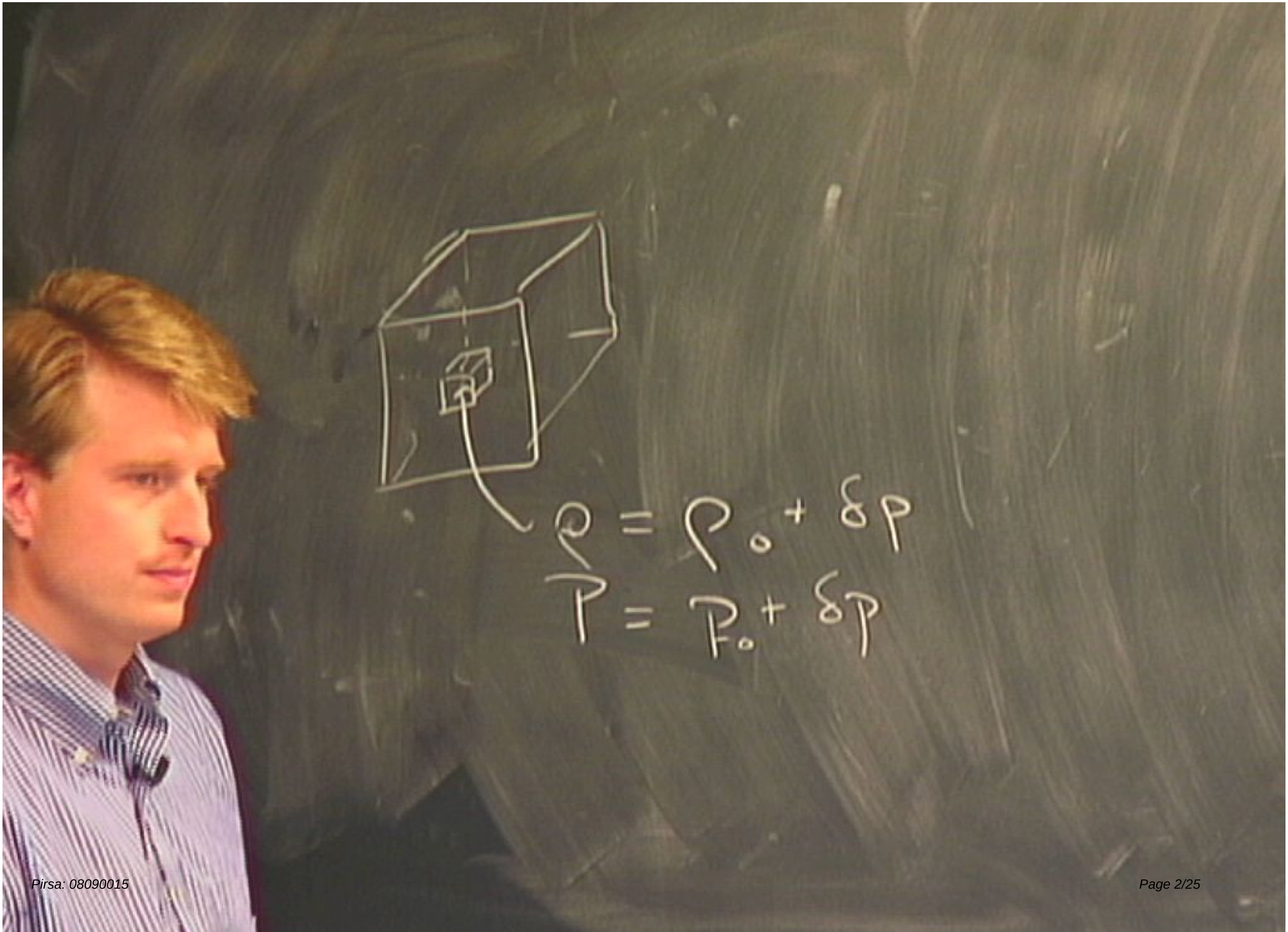
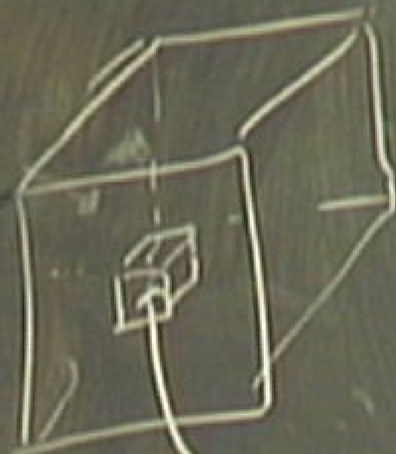
Title: Lecture 3B

Date: Sep 18, 2008 12:30 PM

URL: http://pirsa.org/08090015

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

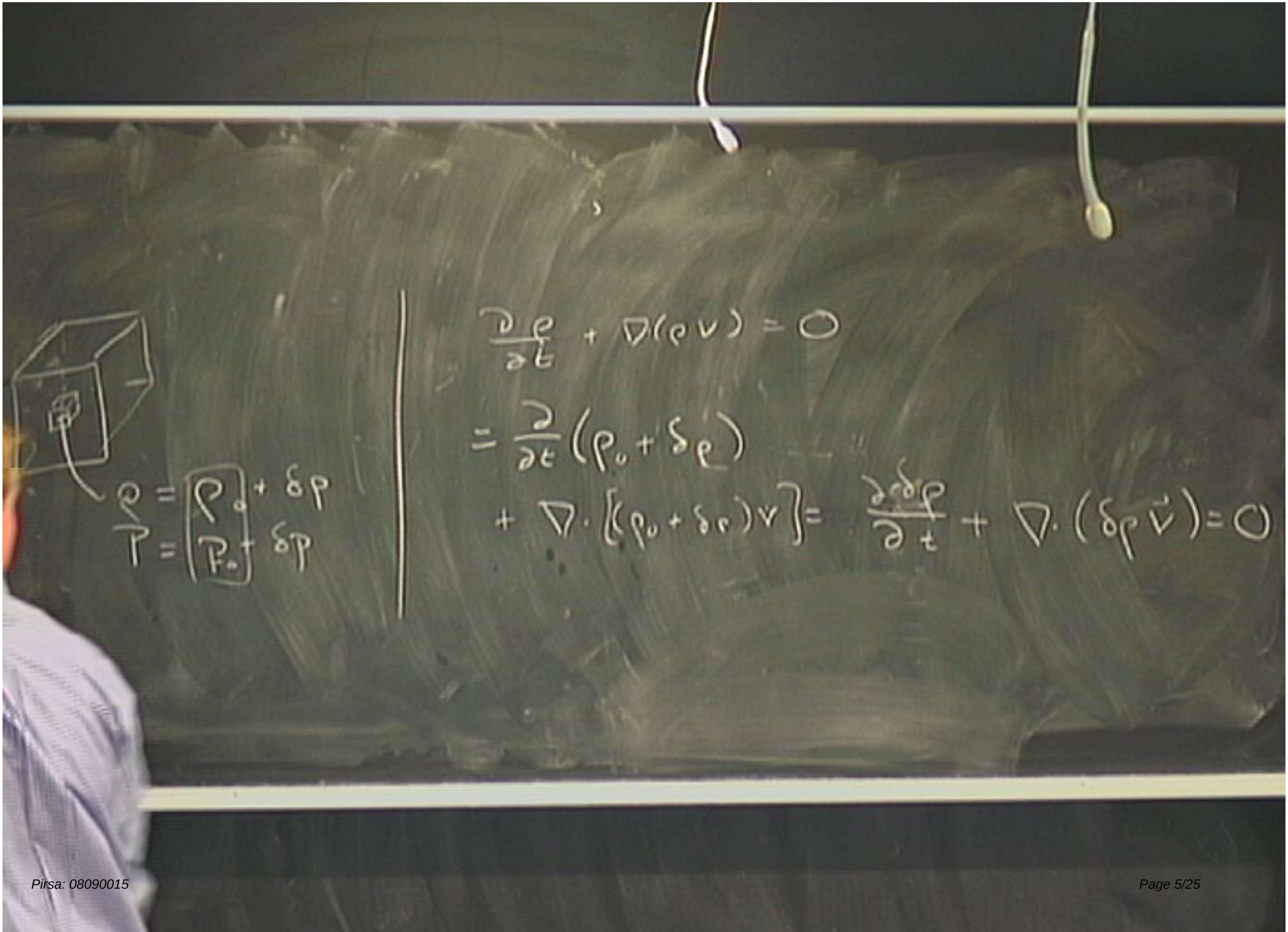

$\rho = \rho_0 + \delta\rho$
$P = P_0 + \delta P$

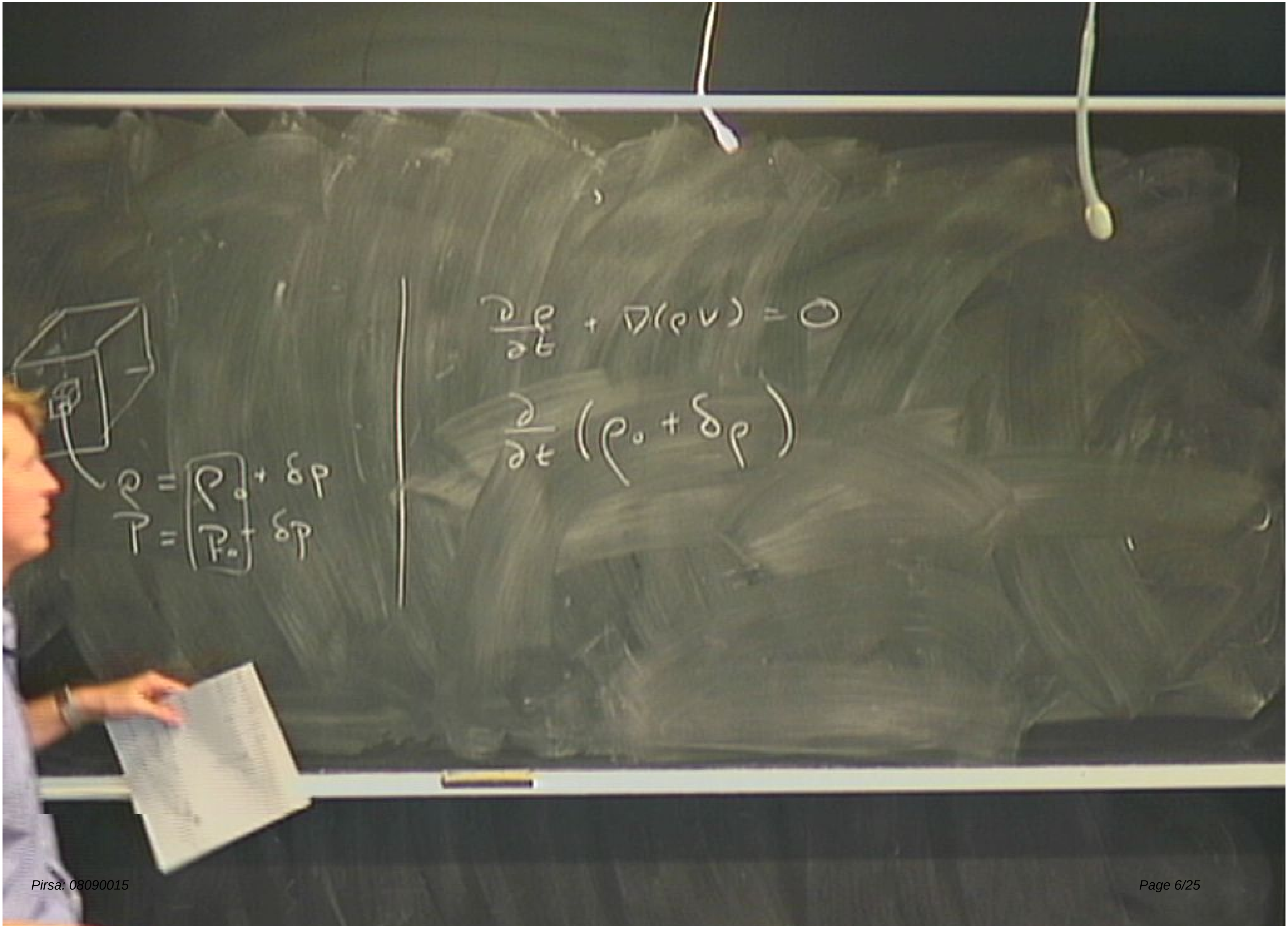


$$\rho = \rho_0 + \delta\rho$$

$$P = P_0 + \delta P$$


$\rho = \rho_0 + \delta\rho$
$P = P_0 + \delta P$
$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$


$\rho = \boxed{\rho_0} + \delta\rho$
$P = \boxed{P_0} + \delta p$
$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$
$= \frac{\partial}{\partial t}(\rho_0 + \delta\rho)$
$+ \nabla\cdot[(\rho_0 + \delta\rho) v] = \frac{\partial \delta\rho}{\partial t} + \nabla\cdot(\delta\rho \vec{v}) = 0$


$\rho = \boxed{\rho_0} + \delta\rho$
$P = \boxed{P_0} + \delta p$
$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$
$\frac{\partial}{\partial t}(\rho_0 + \delta\rho)$

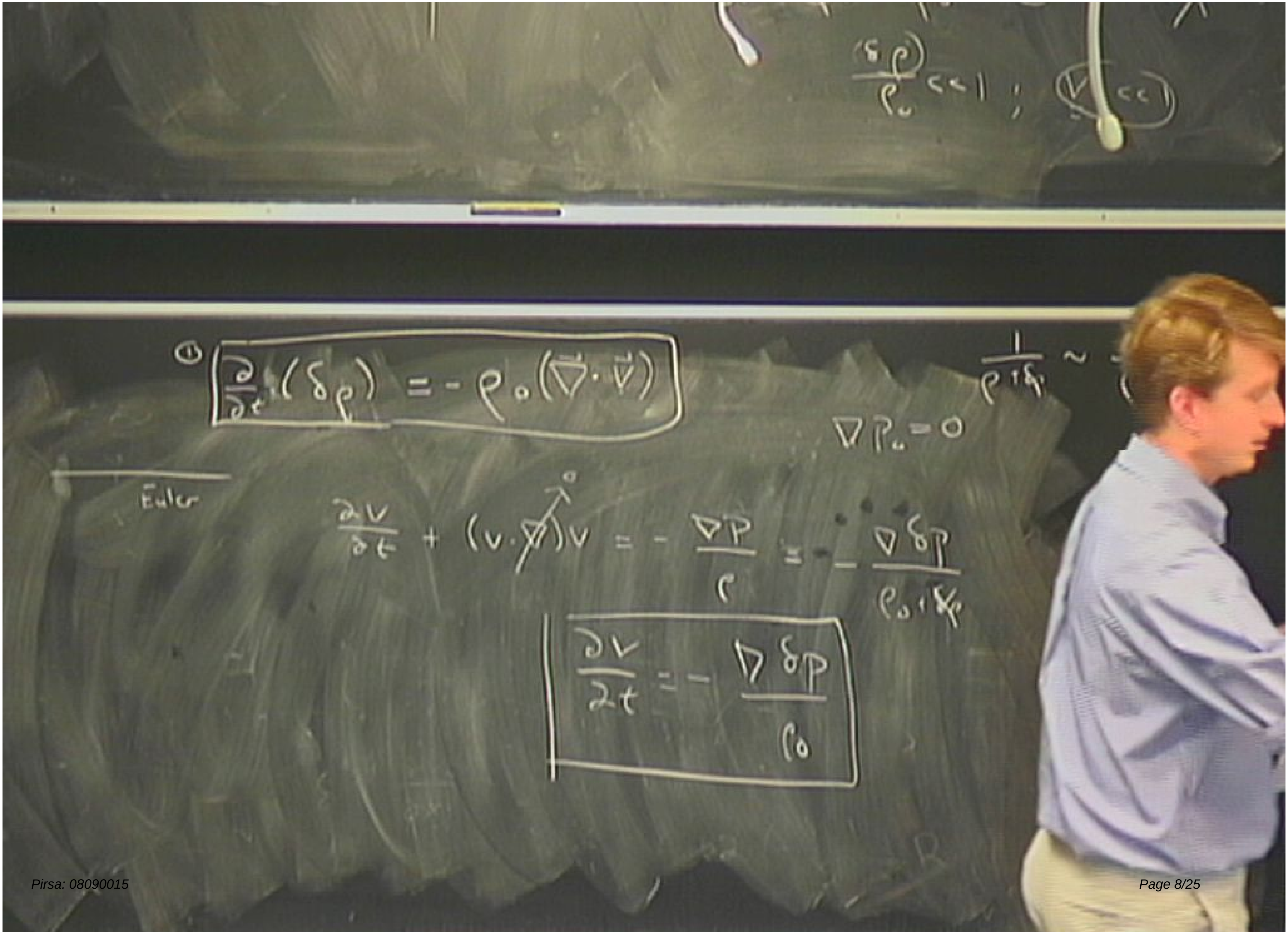
$$= \boxed{\rho_0} + \delta\rho$$
$$= \boxed{P_0} + \delta p$$

$$\frac{\partial}{\partial t}(\rho_0 + \delta\rho) + \nabla\cdot(\rho_0 v) + \nabla\cdot(\delta\rho\, v)$$

$$= \frac{\partial}{\partial t}(\delta\rho) + \rho_0 \nabla\cdot V + \nabla\cdot(\cancel{\delta\rho \vec{V}})^{0} = 0$$

$$\frac{(\delta\rho)}{\rho_0} \ll 1 \; ; \quad V \ll 1$$

∂



δρ/ρ₀ << 1 ; v << 1
① ∂/∂t (δρ) = − ρ₀ (∇ · v)
∇P₀ = 0
Euler
∂v/∂t + (v · ∇)v = − ∇P/ρ = − ∇δP/(ρ₀ + δρ)
∂v/∂t = − ∇δP/ρ₀

$$\frac{\delta\rho}{\rho_0} \ll 1 \; ; \quad v \ll 1$$

$$① \quad \boxed{\frac{\partial}{\partial t}(\delta\rho) = -\rho_0(\vec{\nabla}\cdot\vec{v})}$$

$$\frac{1}{\rho + \delta\rho} \sim \frac{1}{\rho}\left(1 - \frac{\delta\rho}{\rho} \ldots\right)$$

$$\nabla P_0 = 0$$

Euler

$$\frac{\partial v}{\partial t} + \cancel{(v\cdot\nabla)v}^{\,0} = -\frac{\nabla P}{\rho} = -\frac{\nabla \delta P}{\rho_0 + \delta\rho}$$

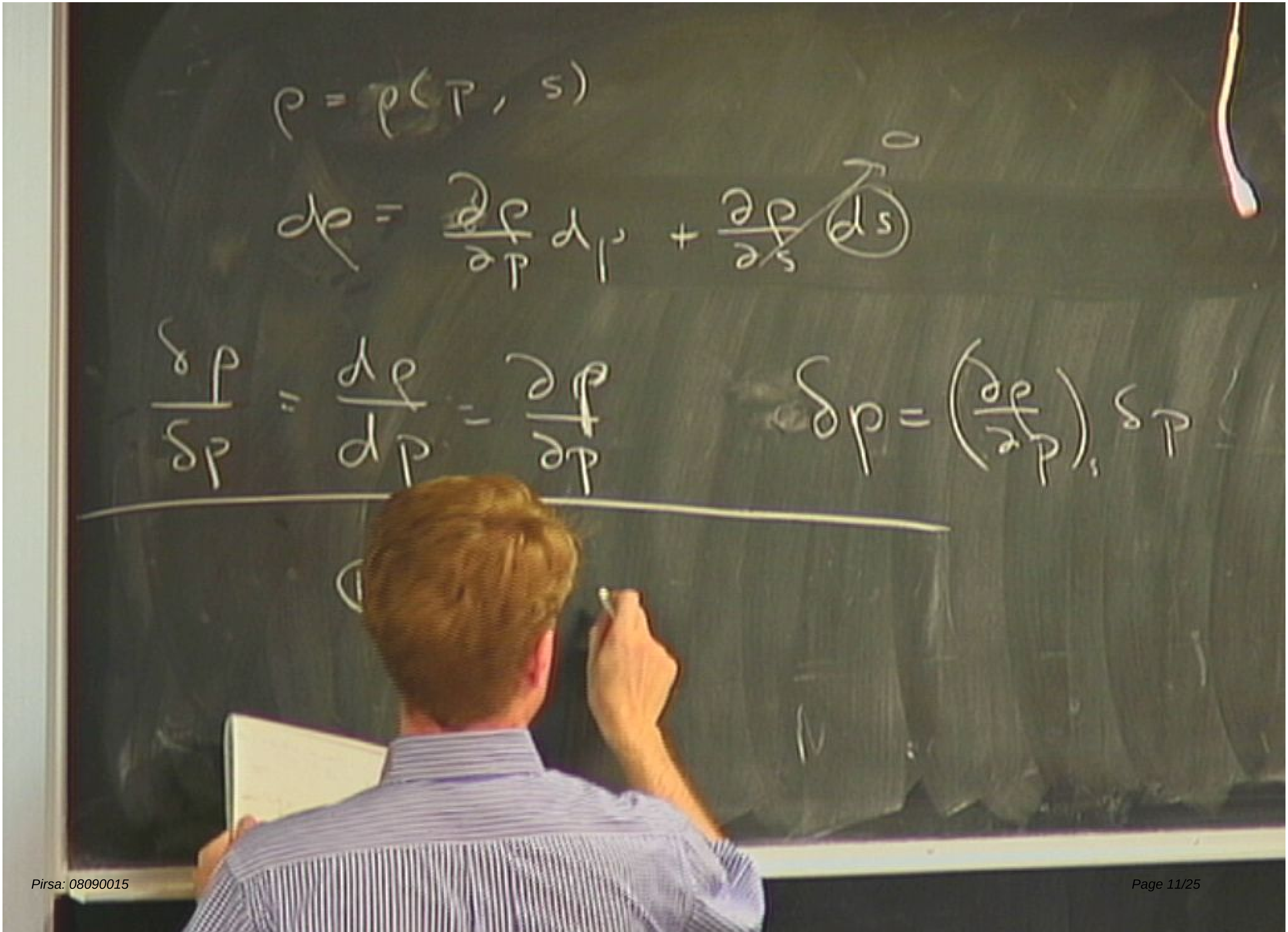
$$\boxed{\frac{\partial v}{\partial t} = -\frac{\nabla \delta P}{\rho_0}}$$

$$\rho = \rho(p, s)$$

$$d\rho = \frac{\partial \rho}{\partial p} dp + \frac{\partial \rho}{\partial s} \cancel{ds}^{\,0}$$

$$\frac{\delta \rho}{\delta p} = \frac{d\rho}{dp} = \frac{\partial \rho}{\partial p}$$

$$\delta\rho = \left(\frac{\partial \rho}{\partial p}\right)_s \delta p$$


$\rho = \rho(p, s)$
$d\rho = \frac{\partial \rho}{\partial p} dp + \cancel{\frac{\partial \rho}{\partial s} ds}$
0
$\frac{\delta \rho}{\delta p} = \frac{d\rho}{dp} = \frac{\partial \rho}{\partial p}$
$\delta \rho = \left(\frac{\partial \rho}{\partial p}\right)_s \delta p$

$$= \frac{\partial \rho}{\partial p} d_1 p + \cancel{\frac{\partial \rho}{\partial s} ds}^{\,0}$$

$$\frac{\rho}{p} = \frac{\partial \rho}{\partial p} \qquad \delta \rho = \left(\frac{\partial \rho}{\partial p}\right)_s \delta p$$

$$\frac{\partial}{\partial t}\left(\delta p \left(\frac{\partial \rho}{\partial p}\right)\right) + \rho_0 \nabla \cdot v = 0 \implies \frac{\partial(\delta p)}{\partial t} + \rho_0 \left(\frac{1}{\left(\frac{\partial \rho}{\partial p}\right)_s}\right) \nabla \cdot v$$

$$\vec{v} = \vec{\nabla}\phi \quad \Longrightarrow \quad \frac{\partial(\delta\rho)}{\partial t} + \rho_0\left(\frac{\partial P}{\partial \rho}\right)\nabla^2\phi = 0$$

$$\vec{\nabla}\left(\frac{\partial\phi}{\partial t} + \frac{\delta P}{\rho_0}\right) = 0$$

$$\boxed{\delta P = \rho\frac{\partial\phi}{\partial t}}$$

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{\partial \phi}{\partial t}\right) = \rho_0 \left(\frac{\partial P}{\partial \rho}\right) \nabla^2 \phi$$

$$\frac{\partial^2 \phi}{\partial t^2} = \left(\frac{\partial P}{\partial \rho}\right) \nabla^2 \phi$$

$$\frac{\partial}{\partial t}\left(\rho_0 \frac{\partial \phi}{\partial t}\right) = \rho_0 \left(\frac{\partial P}{\partial \rho}\right) \nabla^2 \phi$$

$$\frac{\partial^2 \phi}{\partial t^2} = \underbrace{\left(\frac{\partial P}{\partial \rho}\right)}_{c_s^2} \nabla^2 \phi$$

$$\frac{\partial}{\partial t}\left(\cancel{\rho_0}\,\frac{\partial \phi}{\partial t}\right) = \cancel{\rho_0}\left(\frac{\partial p}{\partial \rho}\right)\nabla^2 \phi$$

$$\frac{\partial^2 \phi}{\partial t^2} = \underbrace{\left(\frac{\partial p}{\partial \rho}\right)}_{c_s^2}\nabla^2 \phi$$

$$c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)}$$

$$\frac{\partial}{\partial t}\left(\cancel{\rho_0}\,\frac{\partial \phi}{\partial t}\right) = \cancel{\rho_0}\left(\frac{\partial p}{\partial \rho}\right)\nabla^2 \phi$$

$$\frac{\partial^2 \phi}{\partial t^2} = \underbrace{\left(\frac{\partial p}{\partial \rho}\right)}_{c_s^2}\nabla^2 \phi$$

$$c_s = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_0}$$

$$\phi \simeq e^{i(kx + \omega t)}$$

c_s^2

$\phi \simeq e^{i(kx+\omega t)}$

$x' \Rightarrow x \qquad \nabla = \frac{\partial}{\partial x}$

$\partial_t^2 \phi$

c_s^2

$\phi \simeq e^{i(kx + \omega t)}$

$\vec{x} \Rightarrow x \qquad \nabla = \frac{\partial}{\partial x}$

$$\frac{\partial^2 \phi}{(c_s^2 \partial t^2 - \partial x^2)} = 0$$

$$(-c_s^2 \partial t^2 + \partial x^2) = (\partial x - c_s \partial t)(\partial x + c_s dt)$$

c_s^2

$\phi \simeq e^{i(kx \pm \omega t)}$

$\vec{x} \Rightarrow x \qquad \nabla = \frac{\partial}{\partial x}$

$$\frac{\partial^2 \phi}{(c_s^2 \partial t^2 - \partial x^2)} = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial x^+ \partial x^-}}$$

$$(-c_s^2 \partial t^2 + \partial x^2) = (\partial x - c_s \partial t)(\partial x + c_s \partial t)$$

$$X^+ = x + c_s t \quad ; \quad X^- = x - c_s t$$

$$\partial x^+ = \partial x + c_s \partial t \qquad \partial x^- = \partial x - c_s \partial t$$

$$c_s^2 \qquad \phi \simeq e^{i(kx + \omega t)}$$

$$\vec{x} \Rightarrow x \qquad \nabla = \frac{\partial}{\partial x}$$

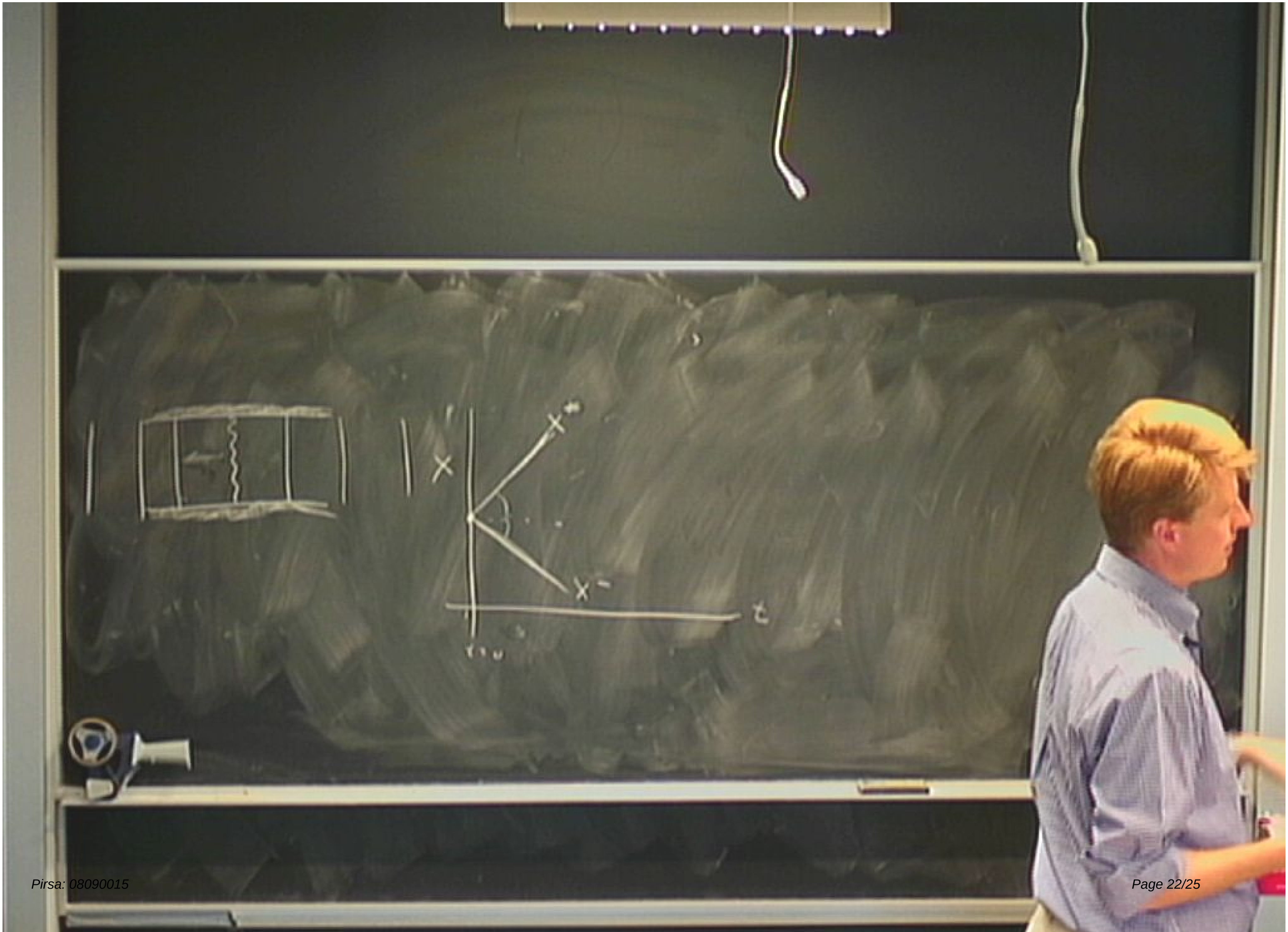
$$\frac{\partial^2 \phi}{(c_s^2 \partial t - \partial x^2)} = 0$$

$$(-c_s^2 \partial t^2 + \partial x^2) = (\partial x - c_s \partial t)(\partial x + c_s \partial t)$$

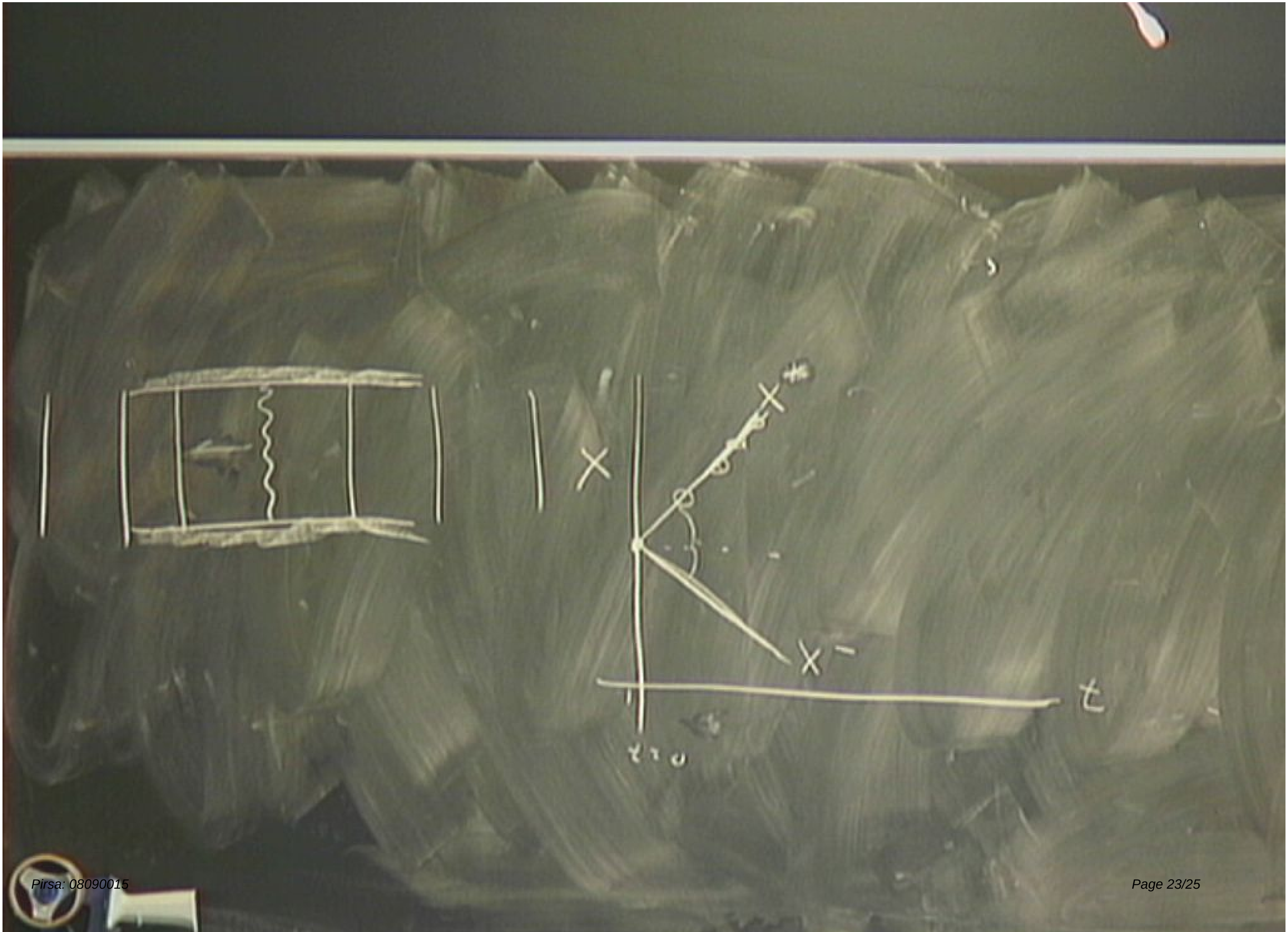
$$\boxed{X^+ = x + c_s t \quad ; \quad X^- = x - c_s t}$$

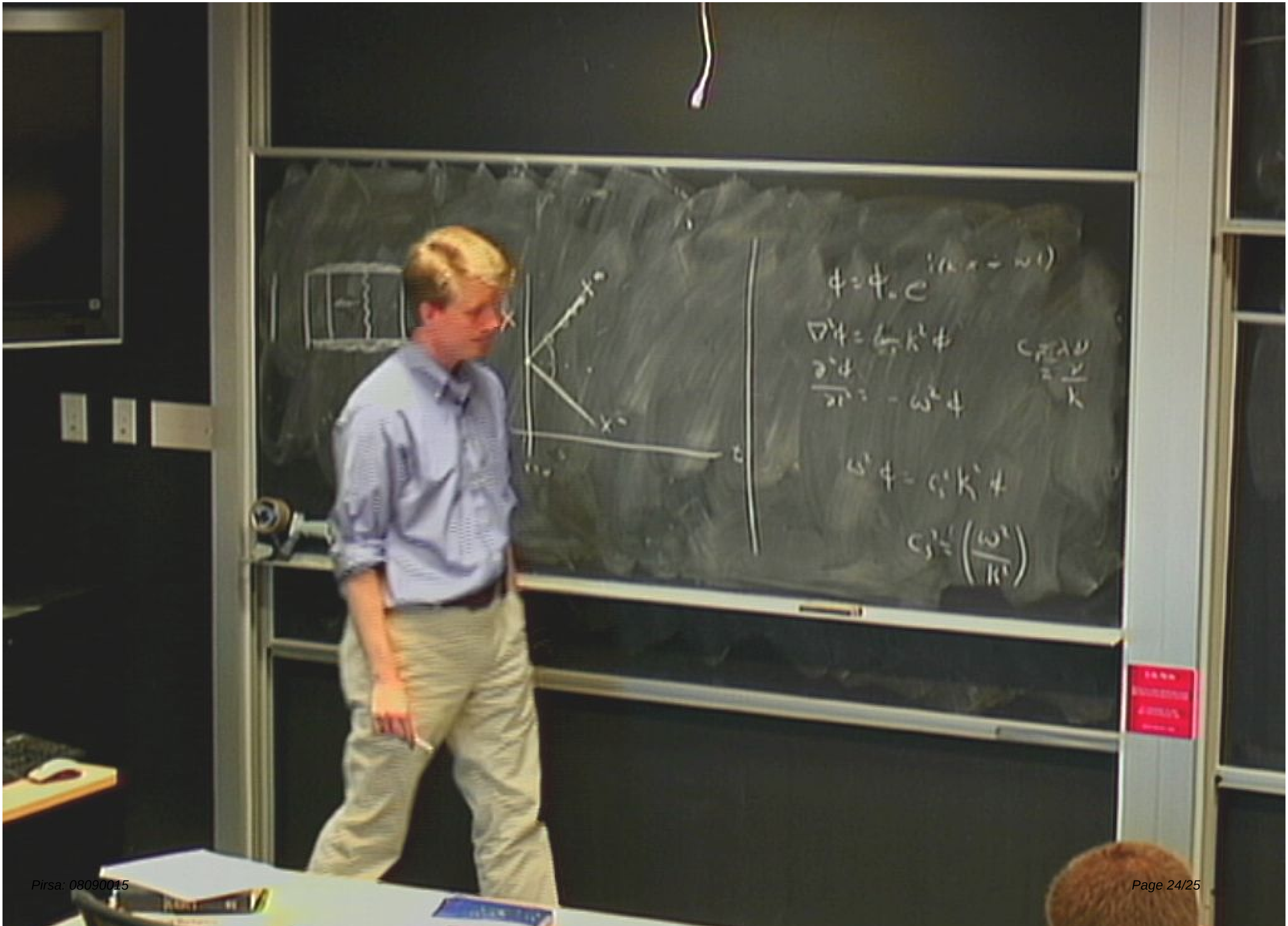
$$\partial X^+ = \partial x + c_s \partial t \qquad \partial X^- = \partial x - c_s \partial t$$

$$\boxed{\frac{\partial^2 \phi}{\partial X^+ \partial X^-} = 0}$$



x
x⁺
x⁻
t


X
X+
X−
t
t = 0



t

$$\phi = \phi_0 e^{i(k \cdot x \mp \omega t)}$$

$$\nabla^2 \phi = -k^2 \phi$$

$$c = \lambda \nu = \frac{\nu}{k}$$

$$\frac{\partial^2 \phi}{\partial t^2} = -\omega^2 \phi$$

$$\omega^2 \phi = c_s^2 k^2 \phi$$

$$c_s^2 = \left(\frac{\omega^2}{k^2}\right)$$