

Title: Astrophysics and Cosmology through Problems - 2B

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URL: <http://pirsa.org/08090014>

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

$$\sigma = \frac{d}{R}$$

$$L = 4\pi d^2 \sigma$$

$$\Omega = \frac{A_c}{A_E}$$

$$\Omega = \frac{A_c}{A_E} = \frac{\pi \left(\frac{d}{2}\right)^2}{4\pi R^2} = \frac{d^2}{16R^2}$$

$$L_0 = 4\pi R^2 \epsilon_0$$

$$L = 4\pi d^2 \epsilon_0$$

$$\Omega_e = \frac{A_c}{A_E} = \frac{\pi \left(\frac{d}{2}\right)^2}{4\pi R^2} = \frac{d^2}{16R^2}$$

ber

$$F_0 = a T^4$$



$$D_{im} = \left( \frac{d_0}{R} \right)^2$$



number

$$F_0 = \sigma T^4$$

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$$E_{\text{leaving sun}} = \bar{F} \cdot 4\pi R_0^2$$



$$D_{in} = \left( \frac{d_0}{R} \right)^2 \bar{F}$$

---

ber

$$F_0 = \sigma T^4$$

---

$$E_{\text{leaving sun}} = \bar{F} \cdot 4\pi R_{\odot}^2$$

$$E_{\text{lens}} = \frac{E_0}{(4\pi R_i^2)} \approx r_{\text{sun}}^2$$



$$D_{\text{im}} = \left( \frac{d_0}{R} \right) F$$

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number

$$F_0 = \sigma T^4$$

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$$E_{\text{leaving sun}} = \bar{F} \cdot 4\pi R_0^2$$

$$E_{\text{lens}} = \frac{E_0}{(4\pi R_i^2)} \approx r_{\text{sun}}^2$$
$$= E_0 \Omega$$



$$D_{\text{im}} = \left( \frac{d_0}{R} \right) F$$

---



number

$$F_0 = aT^4$$


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$$D_{im} = \left( \frac{d_0}{R} \right) F$$


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$$E_{\text{leaving sun}} = \bar{F} \cdot 4\pi R_0^2$$

$$E_{\text{lens}} = \frac{E_0}{(4\pi R_i^2)} \Rightarrow r_{\text{sun}}^2 = \frac{aT_0^4 R_0^2 d_e^2}{4R_E^2}$$

$$= E_0 \Omega_0$$

$$F_{\text{ward}} = \frac{E_{\text{lens}}}{A_{\text{image}}}$$
$$= \frac{F_e}{\pi \left(\frac{D_{\text{im}}}{2}\right)^2}$$

$$F_{\text{ward}} = \frac{E_{\text{lens}}}{A_{\text{image}}}$$

$$= \frac{F_e}{\pi \left(\frac{D_{\text{im}}}{2}\right)^2}$$

$$= \frac{4}{\left(\frac{d_o}{R} \cdot F\right)} \left( \frac{a \pi T_o^4 R^2 d_e^2}{4 R_e^2} \right)$$



$F \leftarrow$

$$F_{\text{wood}} = \frac{E_{\text{lens}}}{A_{\text{image}}}$$

$$= \frac{F_e}{\pi \left( \frac{D_{\text{im}}}{2} \right)^2}$$

$$f = \frac{F}{d}$$

$$= \frac{4}{\pi \left( \frac{d_0}{R} \right)^2} \left( \frac{a \pi T_0^4 R^2 d_e^2}{4 R^2} \right)$$

$d_e \approx \frac{1}{2} \frac{1}{f}$



$$aT^4 = \left(1 - \frac{1}{2}\right) \frac{aT_0^4}{f^2}$$

$$T_{\text{wind}} = T_0 \sqrt[4]{\frac{1}{2f^2}}$$

6000

$$T_{\text{wind}} = 600$$

$$\frac{T}{T_0} = \sqrt[4]{\frac{1}{2f^2}}$$

$$f = \frac{1}{\sqrt{2}} 10^2$$

$$= \frac{100}{\sqrt{2}} = 70$$

$$\frac{1}{10^4} = \frac{1}{2f^2}$$

$$aT^4 = \left(1 - \frac{1}{2}\right) \frac{aT_0^4}{f^2}$$

$$T_{\text{wind}} = T_0 \sqrt[4]{\frac{1}{2f^2}}$$

6000

$$T_{\text{wind}} = 600$$

$$\frac{7}{10} = \sqrt[4]{\frac{1}{2f^2}}$$

$$f = \frac{1}{\sqrt{2}} 10^2$$

$$= \frac{100}{\sqrt{2}} = 70$$

$$\frac{1}{10^4} = \frac{1}{2f^2}$$





$$\nabla^2 \phi = 4\pi G \rho_c \times \frac{1}{r^2}$$
$$\times \exp \left\{ \beta [\phi(\vec{x}) - \phi(0)] \right\}$$
$$\beta = \frac{1}{H^2}$$

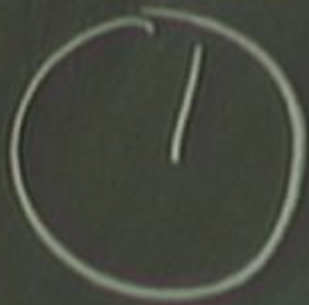


$$\nabla^2 \phi = 4\pi G \rho_c \times \frac{1}{r^2} \times \exp \left\{ \beta [\phi(\vec{x}) - \phi(0)] \right\}$$
$$\beta = \frac{1}{\hbar^2}$$

$$\rightarrow \nabla^2 \phi = 4\pi G \rho_c \times \left. \begin{aligned} & \times \exp \left\{ \beta [\phi(\vec{x}) - \phi(0)] \right\} \end{aligned} \right\}$$

$$\beta = \frac{1}{T}$$

white dwarfs



white dwarfs



Fermions

$f(p)$





white dwarfs

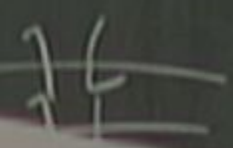
Fermions



$f(p)$



$E$



white dwarfs

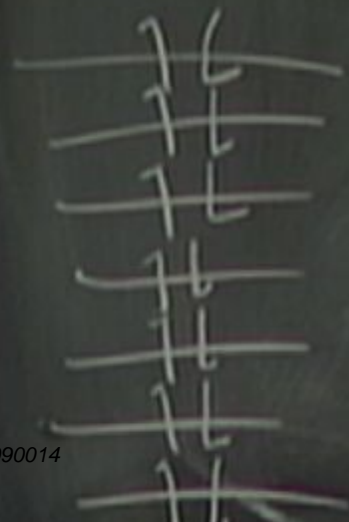


Fermions

$f(p)$



$\rightarrow E$



urda

# Fermions



$$F(p) = \frac{g}{(2\pi)^3}$$

$$e^{-\frac{E - E_F}{k_B T}}$$

12



$$\frac{g}{(2\pi)^3} \quad p < p_F$$

$$0 \quad p > p_F$$

$$= \frac{g}{(h^3)}$$

$$e^{-\frac{E-\mu}{kT}}$$

12

$$g / (2\pi h)^3$$

$$N_{tot} = \int f(p) d^3p = \frac{g}{h^3} \left( \frac{4}{3} \pi \right) p^3$$



$\beta$

$$T^{\mu\nu} \rightarrow \ddot{p} \propto F^{ij}$$

$$P = \int \frac{p^2}{E_{tot}} f(p) d^3p$$

$$T = \begin{pmatrix} p \\ - \\ (p) \end{pmatrix}$$

$$T^{\mu\nu} \rightarrow \text{"P"} \propto T^{ij}$$

$$P = \int \frac{P^2}{E_{\text{tot}}} f(P) d^3P$$

$$T = \begin{pmatrix} P \\ P \\ P \end{pmatrix}$$

$$T = \begin{cases} m \\ pc \end{cases}$$

$\textcircled{NR}$

$\rho_r$

"

$3|_r$

$\left( \frac{6|_r}{5|_r} \right)$

$\int$

$\rho^4$

$4\pi$

$\textcircled{\rho} d\rho$

"

$3|_r$

$\left( \frac{6|_r}{5|_r} \right)$

$\textcircled{NR}$

$\rho_r$

"

$\frac{3}{4}$

$\left( \frac{5}{4} \right)$

$\int \rho^4$

$4\pi$

$\textcircled{\rho} d\rho$

"

$\frac{3}{4}$

$\left( \frac{5}{4} \right)$



NR

$\frac{1}{r}$

"  $\frac{3}{3}$

$\left( \frac{5}{5} \right)$

$\int \frac{1}{r^4} dr$

$4\pi$

NSDP

"  $\frac{4}{3}$

$\left( \frac{5}{5} \right)$

$\left( \frac{5}{5} \right)$

$$\rho_{\text{avg}} = \int f(\rho) d^3\rho = \frac{1}{V} \left( \frac{4}{3}\pi R^3 \right)$$

$$\rho = \frac{M}{V} = \frac{gM}{\frac{4}{3}\pi R^3}$$

$(NR)$

$$P_r = \frac{1}{3} \left( \frac{6}{5} \right) \int_0^{r_f} r^4 \Omega dr$$

$$= \frac{4\pi}{3} \left( \frac{6}{5} \right) \left( \frac{r_f^5}{5} \right)$$

$$\propto r_f^{5/3}$$

(NR)  $d^3p = p^2 \sin\theta dp$

$$P_r = \frac{1}{3m} \left( \frac{g^2 V}{5} \right) \int_0^{p_f} p^4 (4\pi) dp$$

$$= \frac{4\pi}{3m} \left( \frac{g^2 V}{5} \right) \left( \frac{p_f^5}{5} \right)$$

$\propto p^{5/3}$

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(R)  $P_r \propto \int p^3 dp \propto \frac{1}{4} p_f^4$

$\propto \frac{1}{4} p_f^{4/3}$

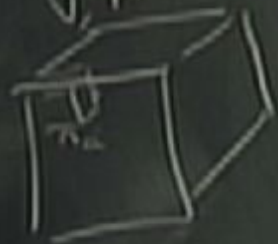


$$\frac{M}{d_m} \rightarrow 5 \times 10^6 \text{ year}$$

$$m_0 c^2 \approx 1.02 \text{ MeV}$$

$N,$

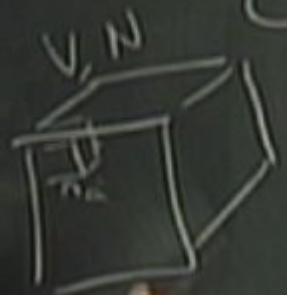
$$e^s = \frac{1}{N!} \int \prod_{i=1}^N d^3x_i d^3p_i \delta(E - H)$$



$$= \frac{1}{N!} \int d^3x \left[ E - \sum_{i=1}^N \left( \frac{1}{2} m_0 v_i^2 + \dots \right) \right]$$

$$\frac{\delta S}{\delta N_i} = 0$$

$N,$



$$e^S = \frac{1}{N!} \int d^{3N}x \int d^{3N}p \delta(E - H)$$

$$\nabla^2 \phi = 4\pi G \rho$$

$$= \frac{A}{N!} \int d^3x \left[ E - \frac{1}{2} \sum_{i=1}^{3N} U(x_i) \right]^{3N/2}$$

$$\frac{\delta S}{\delta N_i} = 0$$

$$\text{const} = \ln(\rho(x)) + \frac{1}{T} \int \dots$$

$$- \frac{GM^2}{R} \ln R$$