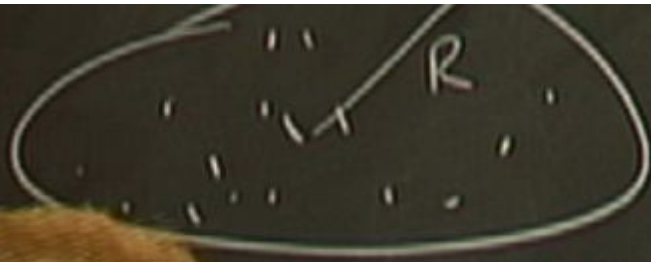


Title: Astrophysics and Cosmology through Problems - 1B

Date: Sep 04, 2008 12:30 PM

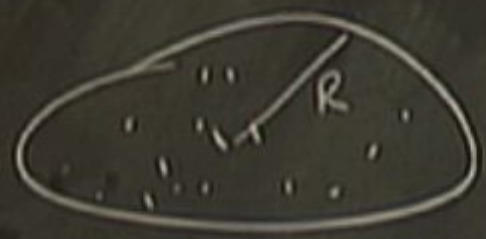
URL: <http://pirsa.org/08090013>

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.



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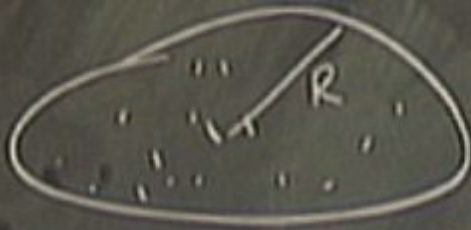
1.14



Z

$$\supset \frac{\#Z}{\#R} = \frac{\#Z}{\#R}$$

1.14

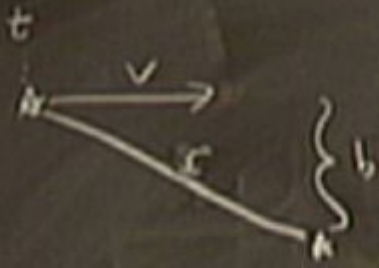


Z

$\mathbb{Z} \cong \mathbb{Z} \oplus \mathbb{Z}$

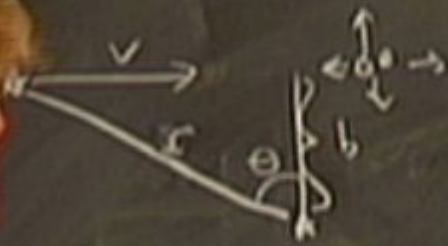


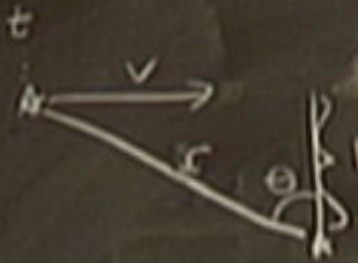
Δt

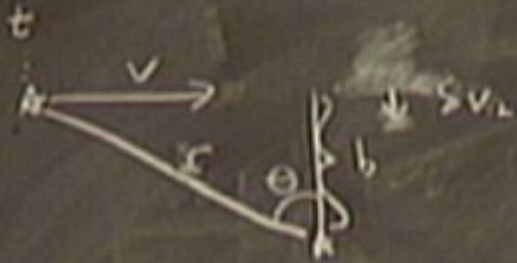


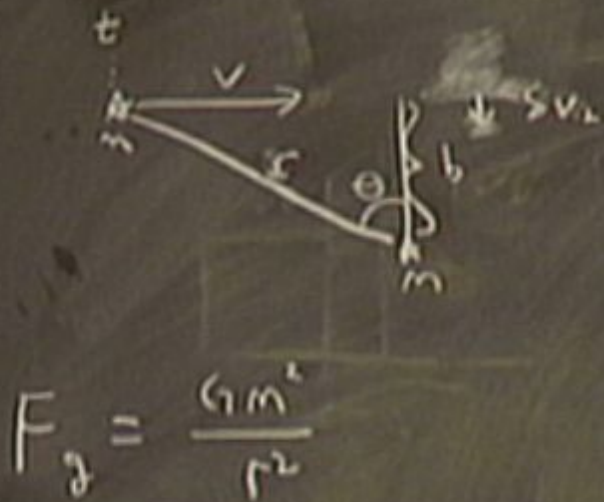
$$s = v \Delta t$$

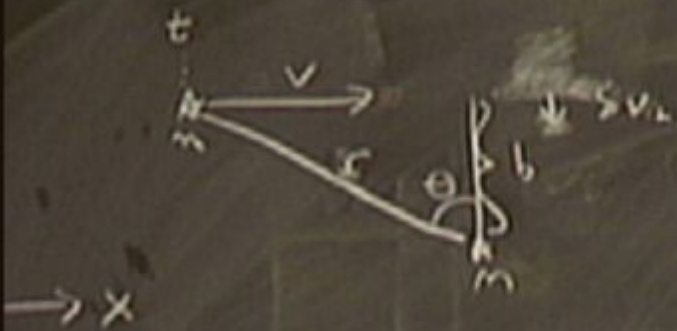




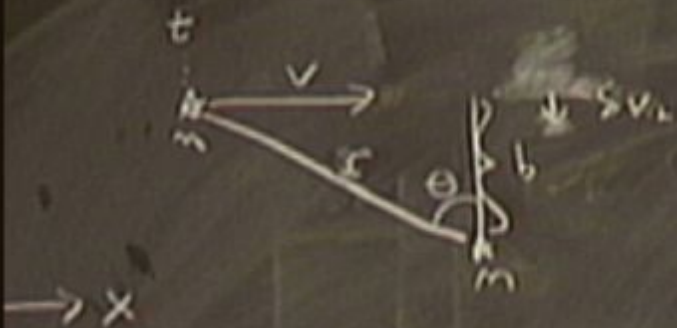






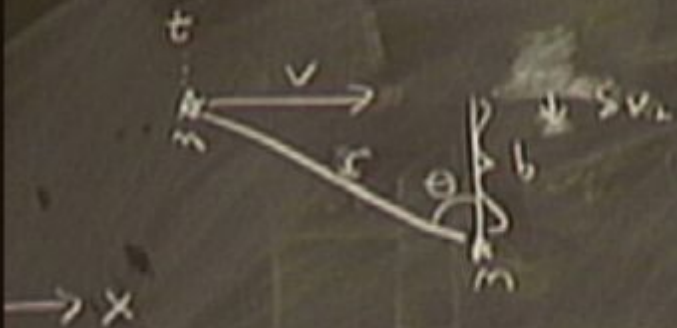


$$F_2 = \frac{5}{2} \hat{z}$$



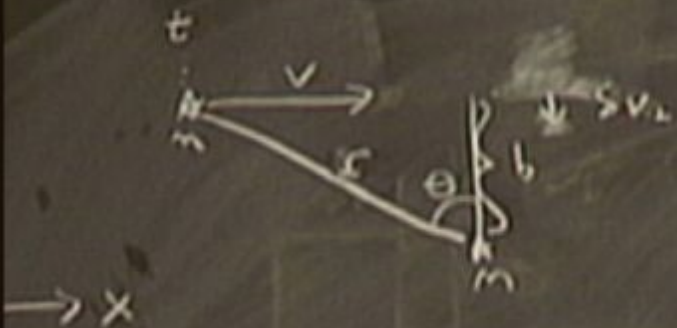
$$\int \sqrt{x^2 + a^2}$$

$$\frac{1}{2} \ln \left| \frac{5}{3} \right|$$



$$r = \sqrt{x^2 + y^2}$$

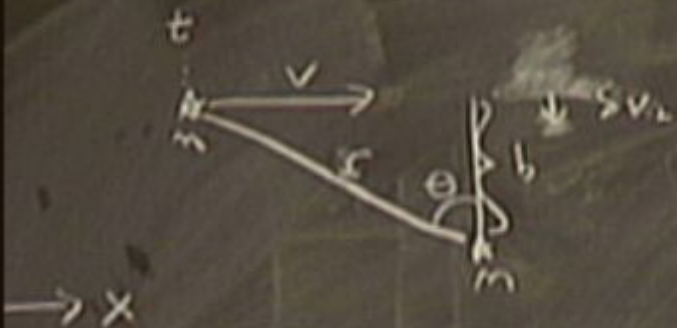
$$\frac{r}{2} = \frac{\sqrt{3^2}}{2} = \frac{\sqrt{3^2}}{x^2 + y^2}$$



$$r = \sqrt{x^2 + b^2}$$

$$F_g = \frac{GMm}{r^2} = \frac{GMm}{x^2 + b^2}$$

α α_{tot}

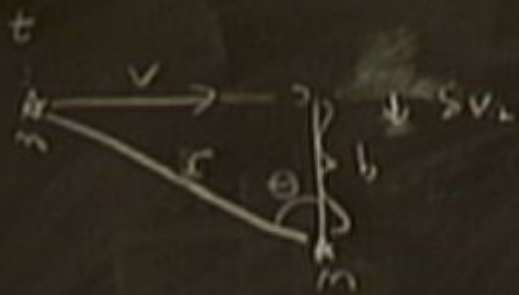


$$r = x^2 + y^2$$

$$\frac{F}{a} = \frac{F y}{r} = \frac{F y}{x^2 + y^2}$$

$\propto a_{rel}$

$$\frac{F}{a} = \frac{F}{a} \cos \theta$$

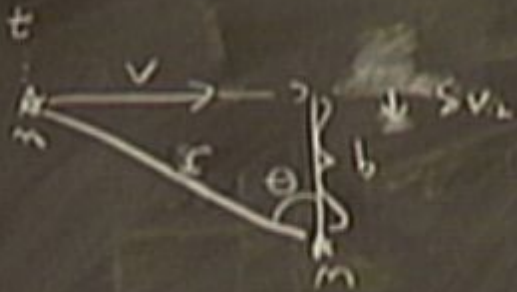


$r = \sqrt{x^2 + b^2}$

$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$\frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$\frac{b}{r} = \cos \theta$$



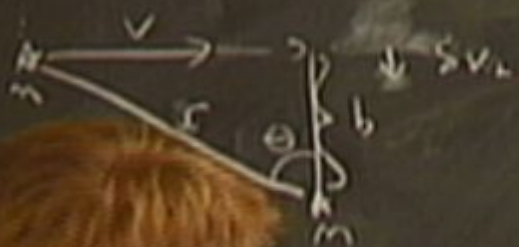
$$r = \sqrt{x^2 + b^2}$$

$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$F_x = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2 + b^2}$$

$\propto a_{rel}$

$$F_x = \frac{F}{\cos \theta}$$



$$r^2 = x^2 + b^2$$

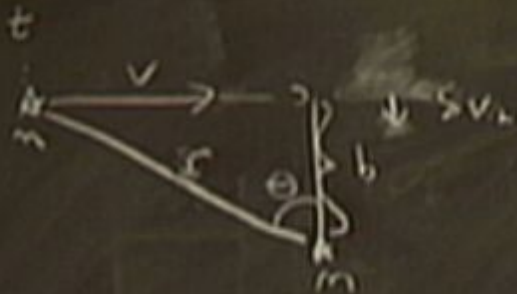
$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$x = x(t)$$

$$F = \frac{G M m^2}{x^2 + b^2}$$

$$a = a_{rel}$$

$$\cos \theta$$



$$r^2 = x^2 + b^2$$

$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

→ x

$$\frac{F}{g} = \frac{G M^2}{r^2} = \frac{G M^2}{x^2 + b^2}$$

α α_{tot}

$$\frac{F}{g} = \frac{F}{g} \cos \theta$$

$$= x(t)$$

$$= x_0 + v t$$

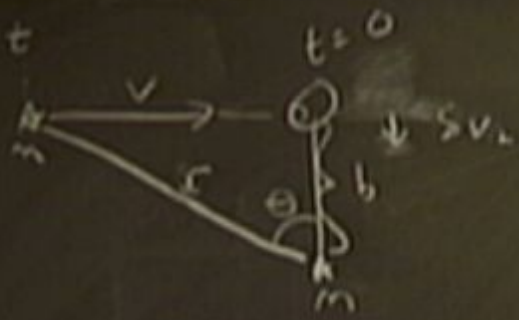
$$F \propto \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2 + b^2} \propto \alpha_{...}$$

$$F \propto \frac{1}{r^2}$$

$$= x_0 + v t$$

$$t=0, x_0 \rightarrow 0$$

$$x(t) = v t$$



$$r^2 = x^2 + b^2$$

$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$x = x(t) = x_0 + vt$$

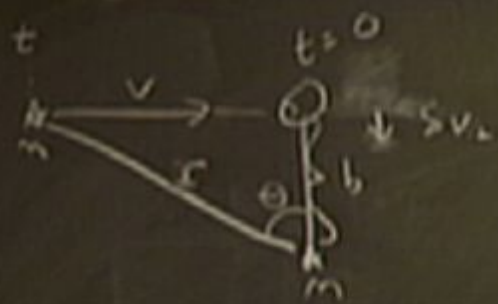
$$t=0, x_0 = 0$$

$$x(t) = vt$$

$$F_g = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2 + b^2}$$

$\propto a_{rel}$

$$F_{\perp} = F_g \cos \theta$$



$$r^2 = x^2 + b^2$$

$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$x = x(t) = x_0 + vt$$

$$t=0, x_0=0$$

$$x(t) = vt$$

$$F_y = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2 + b^2}$$

$\propto a_{y1}$

$$F_y^\perp = F_y \cos \theta$$

$$F = m \quad F_y^\perp = m$$

$$F_{\perp} = F_{\parallel} \cos \theta$$

$$t=0, x_0=0$$

$$x(t) = vt$$

$$F = ma \Rightarrow F_{\perp} = ma_{\perp} = m \dot{v}_{\perp}$$

$$\dot{S}_v = \frac{dv}{dt}$$

$$v = F_{\perp} = \frac{Gm^2}{x^2 + b^2} \cos\theta = \frac{Gm^2}{(x^2 + b^2)^{3/2}} b$$

CAUTION

DO NOT TOUCH THE BOARD
OR THE MARKERS

$$\dot{\delta v} = \frac{dv}{dt}$$

$$m \delta v$$

$$= \frac{G M^2}{a^2 + b^2} \cos \theta = \frac{G m^2}{(x^2 + b^2)^{3/2}} b$$

$$\rightarrow = \frac{G m^2}{(v^2 + b^2)^{3/2}} b$$

$$\frac{\delta v_x}{\delta t} = G m$$

CAUTION
DO NOT TOUCH THE BOARD
OR THE MARKERS
OR THE ERASER
OR THE WIPER

$$m \delta v = F_{\perp} = \frac{G M^2}{x^2 + b^2} \cos \theta = \frac{G M^2}{(x^2 + b^2)^{3/2}} b$$

$$x = vt \longrightarrow = \frac{G M^2}{(v^2 t^2 + b^2)^{3/2}} b$$

$$\frac{\delta v_{\perp}}{\delta t} = \frac{G M b}{b^2 \left(\frac{v^2}{b^2} + 1 \right)^{3/2}}$$



$$r^2 = x^2 + b^2$$

$$\cos \Theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$\frac{\sin^2 \theta}{x^2 + b^2}$$

α a_{\dots}

$$x = x(t)$$

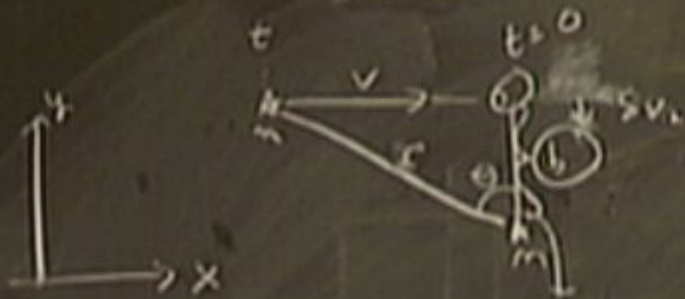
$$= x_0 + v t$$

$$t = 0, x_0 = 0$$

$$x(t) = v t$$

$$\frac{1}{\cos \theta}$$

$$F = m a \Rightarrow F_{\perp} = m a_{\perp} = m \dot{\delta v}_{\perp}$$



$$F_z = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2 + b^2}$$

$$F_z = F_z \cos \theta$$

$$r^2 = x^2 + b^2$$

α a_{rel}

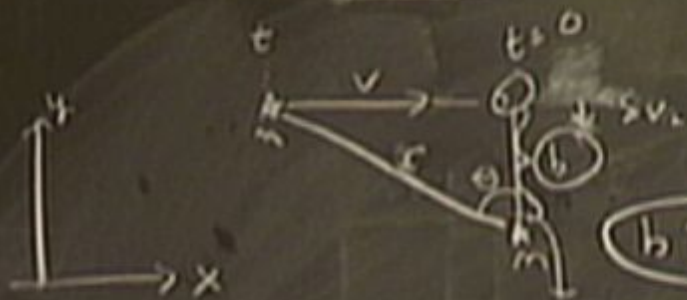
$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$x = x(t) \\ = x_0 + vt$$

$$t=0, x_0$$

$$x(t) = vt$$

$$F = ma \Rightarrow F_z = ma_z = m \dot{\delta z}$$



$$b > b_{min}$$

$$r^2 = x^2 + b^2$$

$$\cos \theta = \frac{b}{r} = \frac{b}{\sqrt{x^2 + b^2}}$$

$$F_{\parallel} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2 + b^2} \quad \alpha \quad a_{\parallel}$$

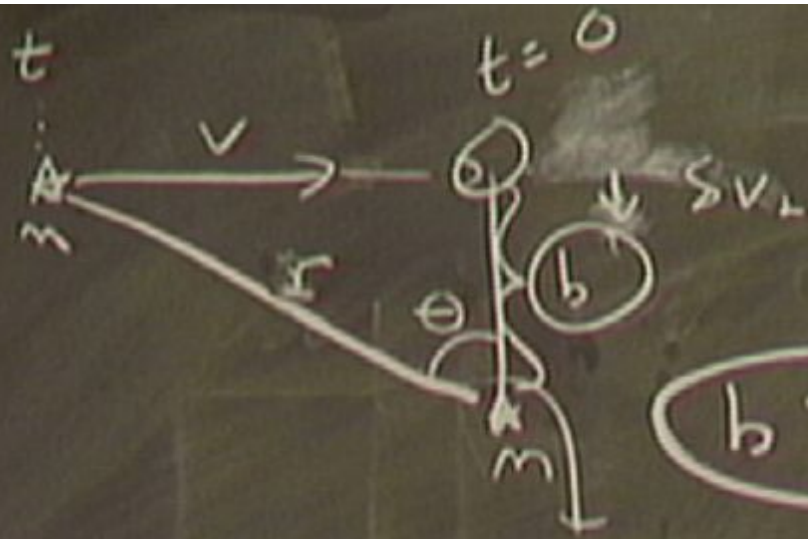
$$x = x(t) = x_0 + vt$$

$$t=0, x_0 = 0$$

$$F_{\perp} = F_{\parallel} \cos \theta$$

$$x(t) = vt$$

$$F = ma \Rightarrow F_{\perp} = ma_{\perp} = m \dot{\delta v}_{\perp}$$



$$r^2 = x^2 + b^2$$

$$b > b_{min}$$

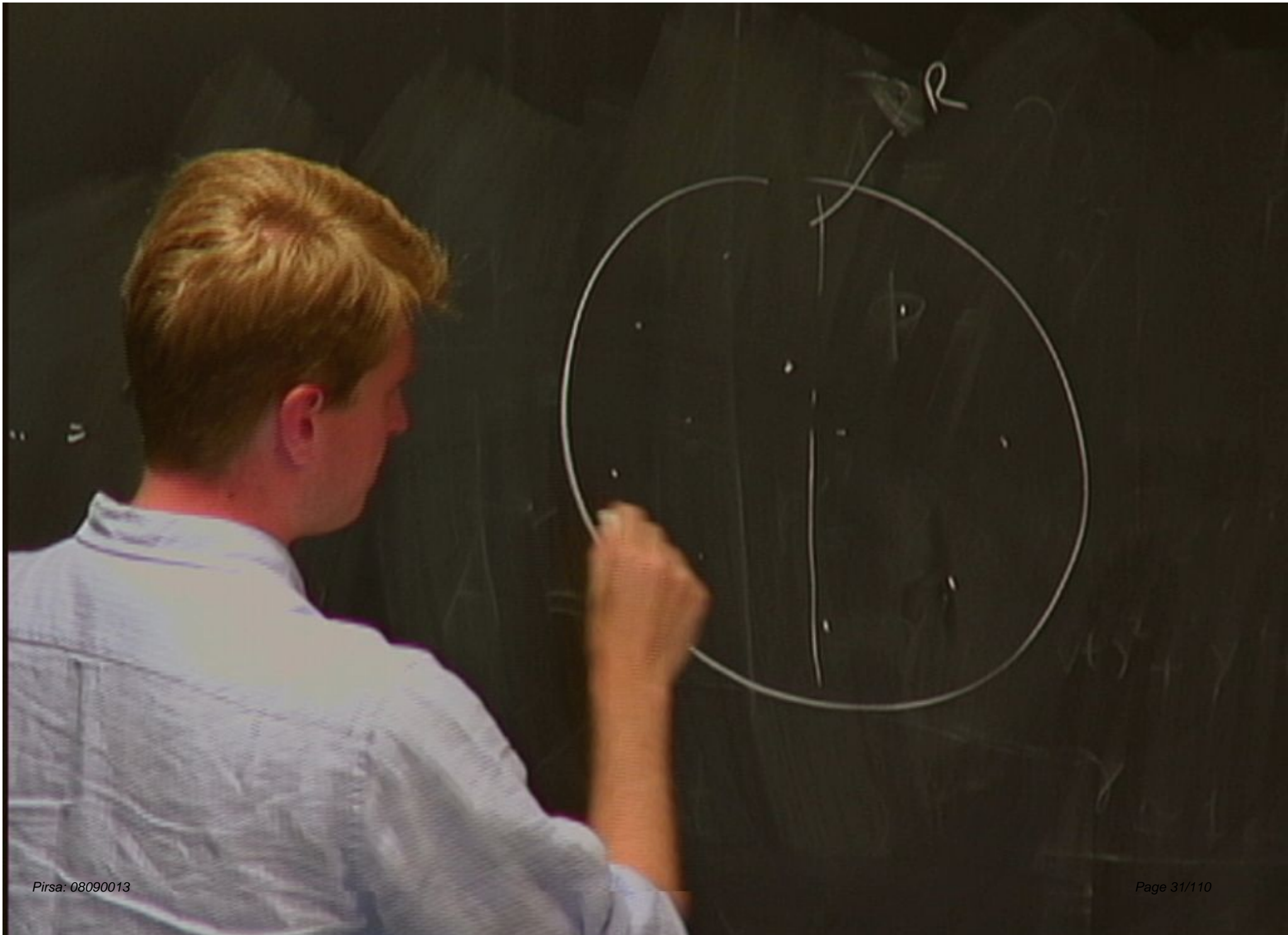
$$\cos(\theta) =$$

$$a = \frac{GM^2}{r^2} = \frac{GM^2}{x^2 + b^2}$$

$$\propto a_{tot}$$

$$a = \frac{F}{m} \cos \theta$$

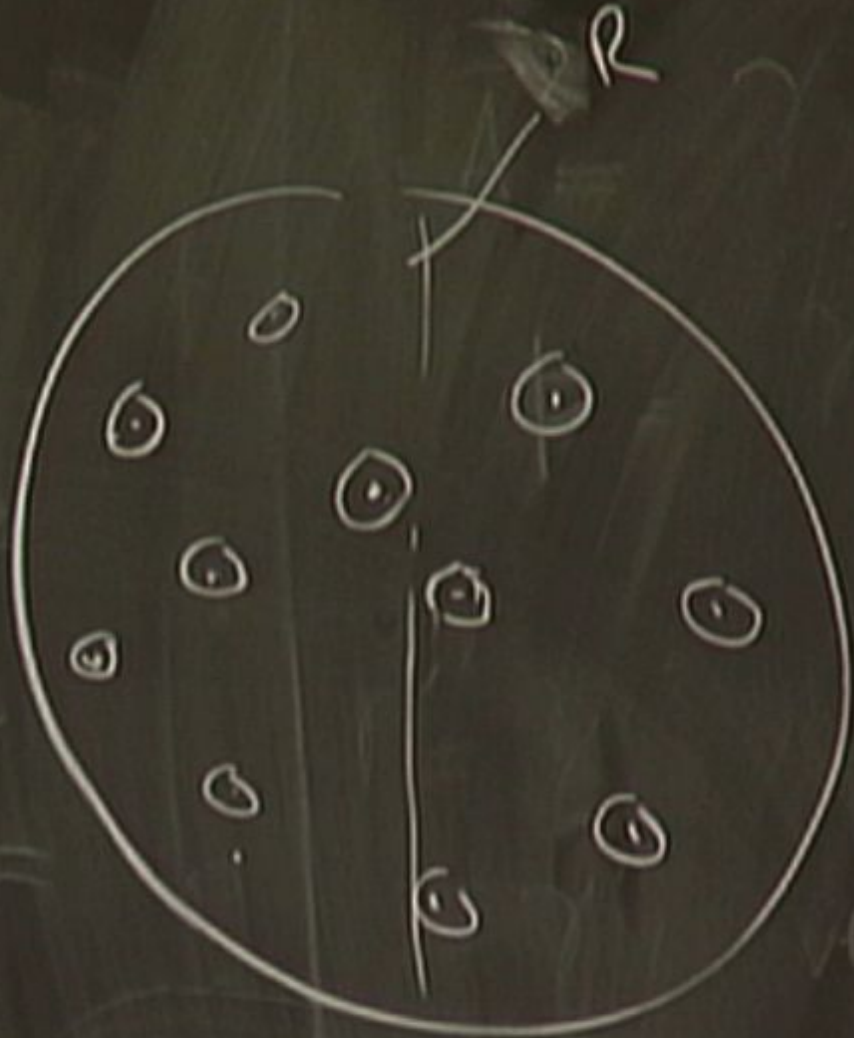
$$F = ma \Rightarrow F_{\perp} = ma_{\perp}$$



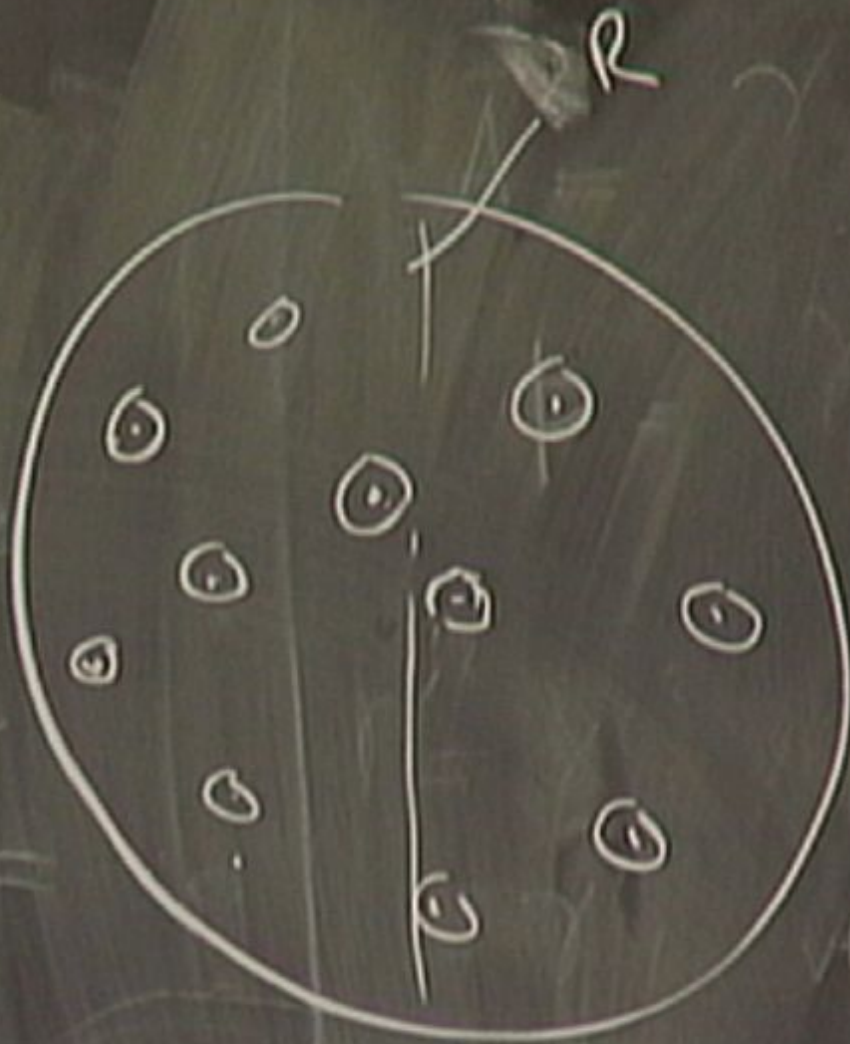
$$\mathbb{R}^2 / \mathbb{Z}$$



$$\mathbb{R}^2 / \mathbb{Z}$$



$$\frac{\mathbb{Z}}{\mathbb{R}^2}$$



$$v_{\perp}(b) = \int dt \left(\frac{GM}{b^2} \right) \frac{1}{\left(\frac{3}{2} + 1 \right)^{3/2}}$$

$$\delta v_{\perp}(b) = \int_{-\infty}^{\infty} dt \left(\frac{GM}{b^2} \right) \left(\frac{1}{\frac{v^2}{c^2} + 1} \right)^{3/2}$$

$$= \frac{GM}{bv} \int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}}$$

2

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b} dt$$

$$\begin{aligned}
 \delta v_{\perp}(b) &= \int_{-\infty}^{\infty} dt \left(\frac{GM}{b^2} \right) \left(\frac{1}{\left(\frac{v^2}{c^2} + 1 \right)^{3/2}} \right) \\
 &= \frac{GM}{bv} \int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}} \\
 &= \frac{2GM}{bv}
 \end{aligned}$$

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b} dt$$

$$\delta v_{\perp}(b) = \int_{-\infty}^{\infty} dt \left(\frac{GM}{b^2} \right) \left(\frac{1}{\left(\frac{v^2}{c^2} + 1 \right)^{3/2}} \right)$$

$$= \frac{GM}{bv} \int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}}$$

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b} dt$$

$$\boxed{\delta v_{\perp}(b) = \frac{2GM}{bv}}$$

$$\frac{\delta v_{\perp}}{v} \ll 1$$

$$\delta v_{\perp}(b) = \int_{-\infty}^{\infty} dt \left(\frac{GM}{b^2} \right) \left(\frac{1}{\frac{v^2}{c^2} + 1} \right)^{3/2}$$

$$= \frac{GM}{bv} \int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}}$$

$$\delta v_{\perp}(b) = \frac{2GM}{bv}$$

$$\frac{\delta v_{\perp}}{v} \ll 1$$

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b} dt$$

$$F = \left(\frac{GM}{b^2} \right)$$

$$\Delta t = \frac{b}{v}$$

$$\int_{-\infty}^{\infty} dt \left(\frac{GM}{b^2} \right) \left(\frac{1}{\frac{v^2}{c^2} + 1} \right)^{3/2}$$

$$\frac{GM}{bv} \int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}}$$

$$= \frac{2GM}{bv}$$

$$\frac{\Delta v_{\perp}}{v} \ll 1$$

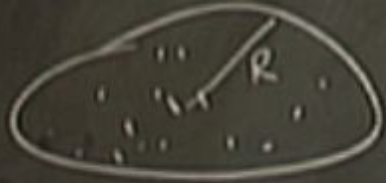
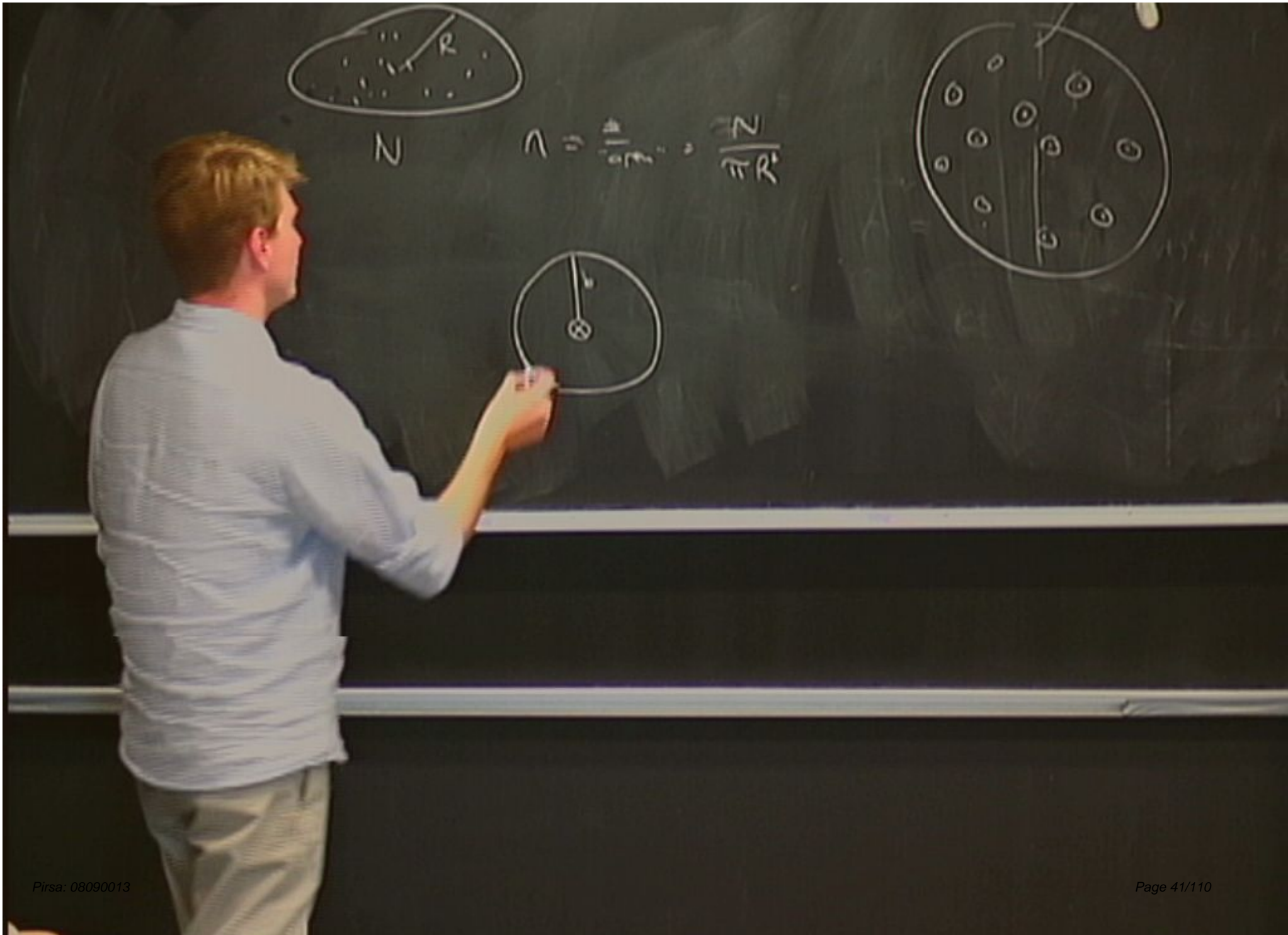
$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b} dt$$

$$F = \left(\frac{GM}{b^2} \right)$$

$$\Delta t = \frac{b}{v}$$

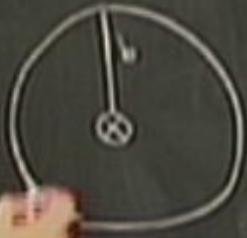
$$F \Delta t = \frac{GM}{bv}$$

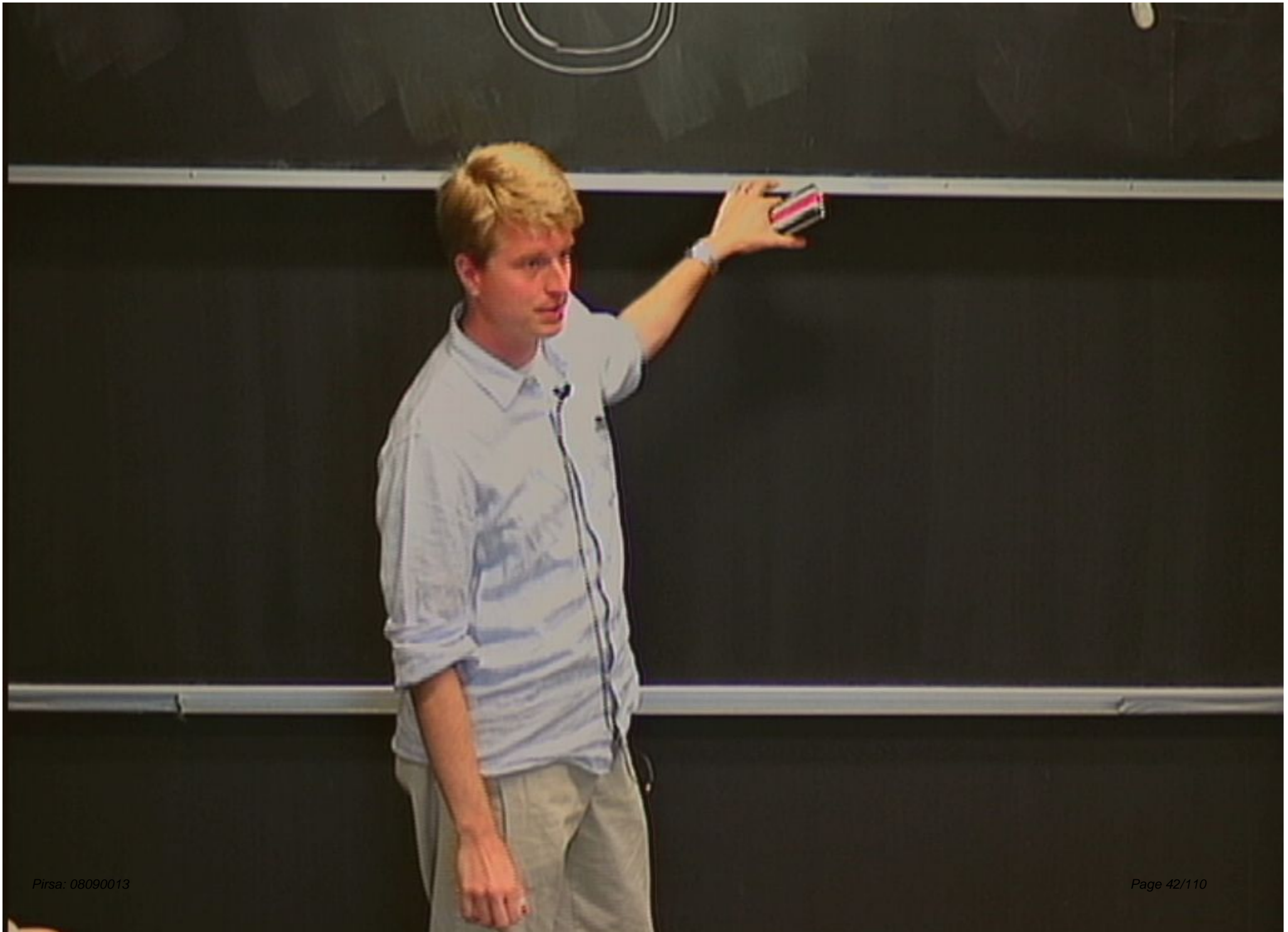


Z

$\mathbb{Z} \cong \mathbb{Z}/n\mathbb{Z}$

\mathbb{R}/\mathbb{Z}



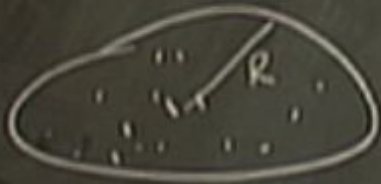




$$A = \pi b db$$



1.14

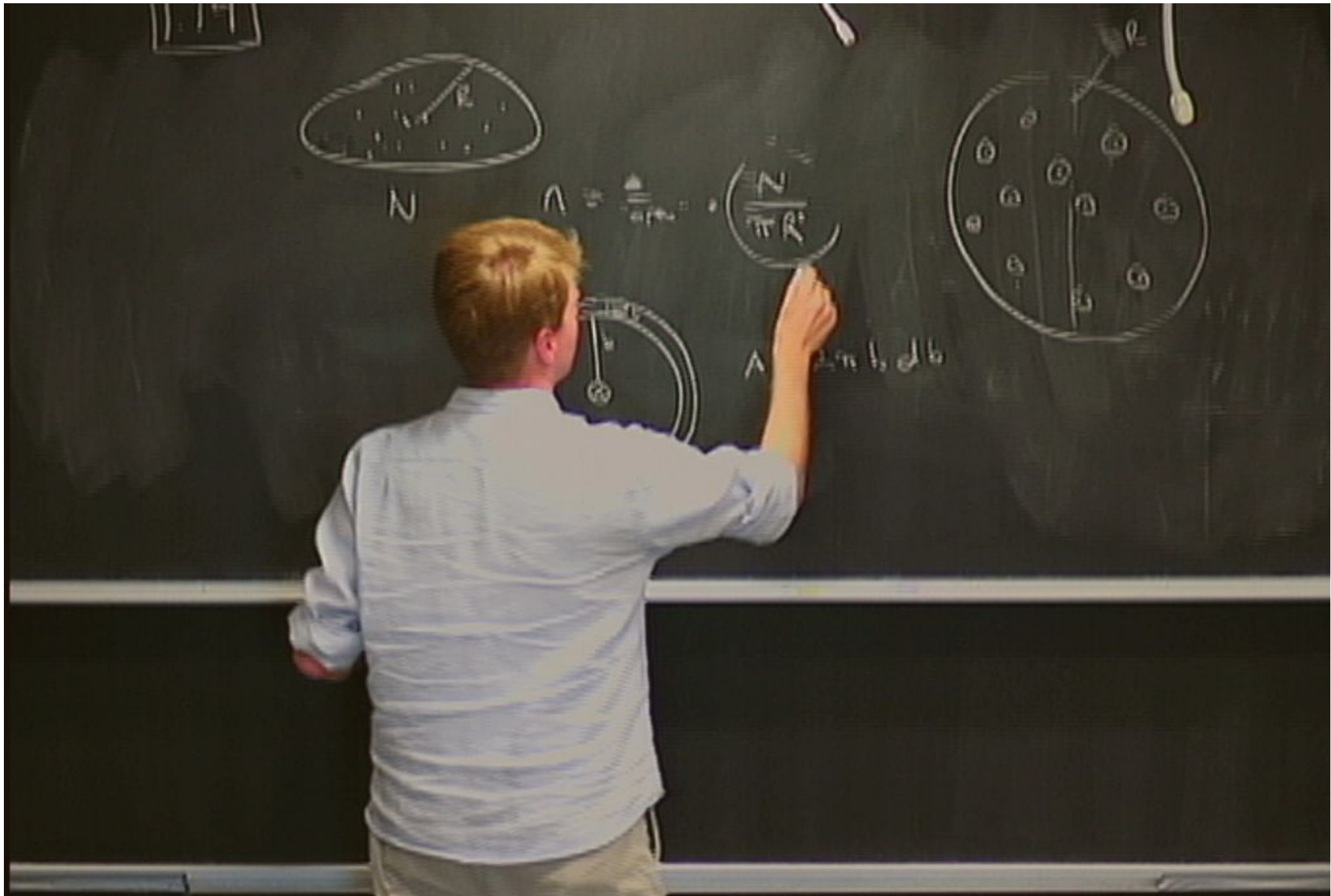


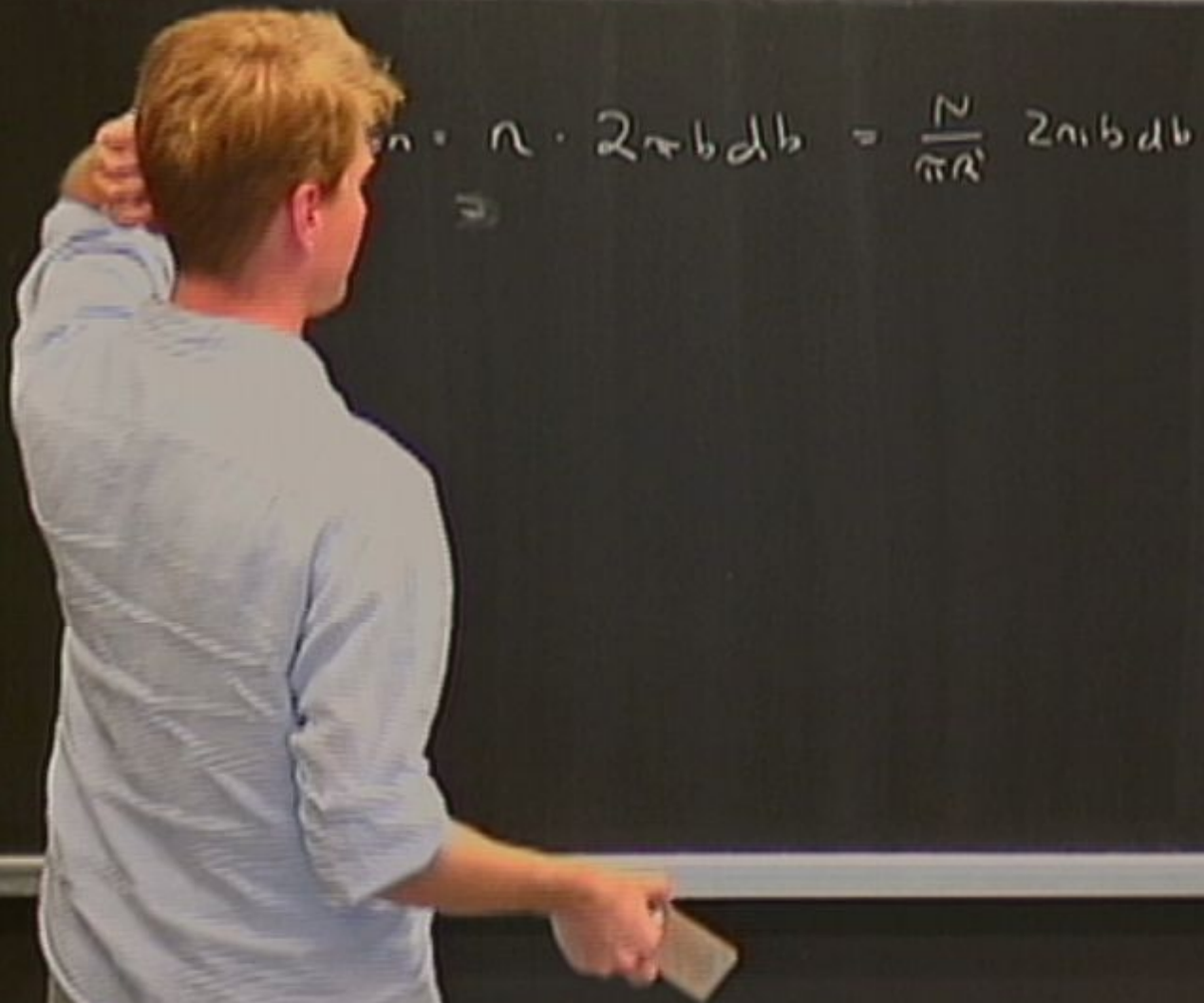
Z

$$\Rightarrow \frac{1}{s} = \left(\frac{1}{R} \right)$$



$$A = 2\pi b, db$$





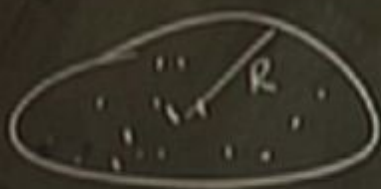
$$n \cdot n \cdot 2 \pi b db = \frac{2}{\pi a^2} 2 \pi b db$$

$$s_n = n \cdot 2\pi b db = \frac{N}{\pi R^2} 2\pi r b db = \frac{2N}{R^2} b db$$

$$s_n = n \cdot 2\pi b \text{ db} = \frac{N}{\pi R^2} 2\pi b \text{ db} = \frac{2N}{R^2} b \text{ db}$$

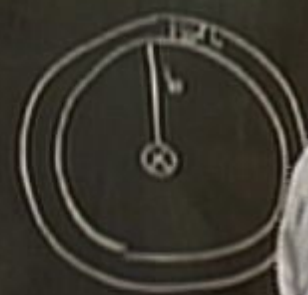
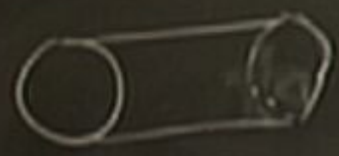
$\langle s_{V_2} \rangle$ per passage?

1.14



N

$$N = \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \right\}$$



b, db

$$s_n = n \cdot 2\pi \text{ bdb} = \frac{N}{2\pi R} 2\pi \text{ bdb} = \frac{2N}{R} \text{ bdb}$$

$$\langle \delta v_x \rangle \text{ per passage?} = 0$$

$$\langle (\delta v_x)^2 \rangle$$

$$s_n = n \cdot 2\pi b \text{ dB} = \frac{N}{2\pi R^2} 2\pi b \text{ dB} = \frac{2N}{R^2} b \text{ dB}$$

$$\langle \delta v_x \rangle \text{ per passage?} = 0$$

$$\langle (\delta v_x)^2 \rangle$$

$$s_n = n \cdot 2\pi b \text{ dB} = \frac{N}{2\pi R^2} 2\pi b \text{ dB} = \frac{2N}{R^2} b \text{ dB}$$

$\langle \delta v_{\perp} \rangle$ per position = 0

$\langle (\delta v_{\perp})^2 \rangle$

$$s_n = n \cdot 2\pi b db = \frac{N}{\pi R^2} 2\pi b db = \frac{2N}{R^2} b db$$

$$\langle \delta v_x \rangle \text{ per passage?} = 0$$

$$\langle (\delta v_x)^2 \rangle = n \langle (\delta v_x)^2 \rangle = \frac{4\pi n \langle v^2 \rangle}{b^2 v^2} 2\pi b db$$

$$\delta n = n \cdot 2\pi b db = \frac{N}{\pi R^2} 2\pi b db = \frac{2N}{R^2} b db$$

$$\langle \delta v_x \rangle \text{ per passage?} = 0$$

$$\langle (\delta v_x)^2 \rangle = n \langle (\delta v_x)^2 \rangle = \frac{4n \langle v_x^2 \rangle}{b^3 v^2} 2\pi b db$$

$$= \frac{8\pi \langle v_x^2 \rangle}{v^2} \frac{db}{b}$$

$$\Delta V_{\perp} = \int_{b_{min}}^{b_{max}} db \left(\right) \frac{1}{r^2}$$

$\cos \theta$

$$E \Delta v_{\perp} = \int_{b_{min}}^{b_{max} = r} db \left(\right) \frac{1}{s}$$

$$\frac{2GM}{bv} = \Delta v_{\perp} \sim v$$

$$\Delta v_{\perp}^2 = \int_{b_{\min}}^{b_{\max} = R} db \left(\right) \frac{1}{b}$$

$$\frac{2GM}{b_{\min} v} = \Delta v_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

$$\Delta v_{\perp} = \int_{b_{\min}}^{b_{\max} = R} db \left(\right) \frac{1}{s} = \frac{8\pi G^2 M^2}{c^3} \ln\left(\frac{R}{b_{\min}}\right)$$

$$\frac{2GM}{c^2} = \Delta v_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

$$\Delta v_{\perp}^2 = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G^2 M^2}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

$$\frac{2GM}{b_{\min} v} = \Delta v_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

$$\Delta v_{\perp} = \int_{b_{\min}}^{b_{\max} = R} db \left(\right) \frac{1}{b} = \frac{8\pi \epsilon_0^2 m^2 R}{v^2} \ln \left(\frac{R}{b_{\min}} \right)$$

$$\frac{2\epsilon_0 m}{b_{\min}^2 v} = \Delta v_{\perp}$$

$$b_{\min} \sim \frac{2\epsilon_0 m}{\Delta v_{\perp} v}$$

$$\Delta V_{\perp}^2 = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G^2 M^2 R}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

(P_⊥ = 11g)

$$\frac{2GM}{b_{\min} v} = \Delta V_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

cos θ

$$\Delta v_{\perp}^2 = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G^2 M^2 R}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

(P = 11.3)

$$\Delta v_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM} \cos\theta$$



$$\Delta V_{\perp} = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G^2 M^2 N}{v^2} \ln \left(\frac{R}{b_{\min}} \right)$$

$$\frac{2G^2 M^2 N}{b_{\min} v^2}$$

$$\sim v$$

$$\sim \frac{v^2}{2GM}$$

Virial theorem

$$2KE = PE$$

$$\Delta V_{\perp}^2 = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G^2 M^2 N}{v^2} \ln \left(\frac{R}{b_{\min}} \right)$$

(P = 113g)

$$\frac{2GM}{b_{\min}} = \Delta V_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

cos θ

Virial theorem

$$2KE = PE$$

$$\Delta V_{\perp} = \int_{b_{\min}}^{b_{\max} = R} db \left(\right) \frac{1}{b} = \frac{8\pi G^2 M^2 R}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

(P = 113g)

$$\frac{GM}{b_{\min}} = \Delta V_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

$\cos \theta$

Virial theorem

$$2KE = PE$$

$$2\left(\frac{1}{2} M v^2\right) = \frac{GMm^2}{r}$$

$$\Delta V_{\perp}^2 = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G^2 M^2 R}{v^2} \ln \left(\frac{R}{b_{\min}} \right)$$

$$\frac{2GM}{b_{\min}} = \Delta V_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

Virial theorem

$$2KE = PE$$

$$2 \left(\frac{1}{2} M v^2 \right) = \frac{GMm^2}{r} \rightarrow$$

Virial theorem

$$2KE = PE$$

$$2 \left(\frac{1}{2} m v^2 \right) = \frac{G M m}{r} \rightarrow v^2 = \frac{G M}{r}$$

Virial theorem

$$2KE = PE$$

$$2 \left(\frac{1}{2} m v^2 \right) = \frac{GMm}{r}$$

$$\rightarrow v^2 = \frac{GM}{r}$$

Virial theorem

$$(KE = kT)$$

$$2KE = PE$$

$$2 \left(\frac{1}{2} m v^2 \right) = \frac{GMm}{r}$$

$$\rightarrow v^2 = \frac{GM}{r}$$

$$\Delta V_{\perp} = \int_{b_{\min}}^{\infty} db \left(\frac{1}{b} \right) = \frac{8\pi G M^2 R}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

$$\frac{2GM}{b_{\min} v} = \Delta V_{\perp} \sim v$$

b_{\min}

Virial theorem

($KE = KT$)

$$2KE = PE$$

$$2\left(\frac{1}{2} m v^2\right) = \frac{GMm}{R} \rightarrow v^2 = \frac{GM}{R}$$

$$\Delta V_{\perp} = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G M^2 R}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

$$\frac{2GM}{b_{\min} v} = \Delta V_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

Virial theorem

(KE = KT)

$$2KE = PE$$

$$2\left(\frac{1}{2}mv^2\right) = \frac{GMm^2}{R} \rightarrow v^2 = \frac{GM}{R}$$

$$\Delta V_{\perp} = \int_{b_{\min}}^{b_{\max} = R} db \left(\frac{1}{b} \right) = \frac{8\pi G M^2 R}{v^2} \ln\left(\frac{R}{b_{\min}}\right)$$

$$\frac{2GM}{b_{\min} v} = \Delta V_{\perp} \sim v$$

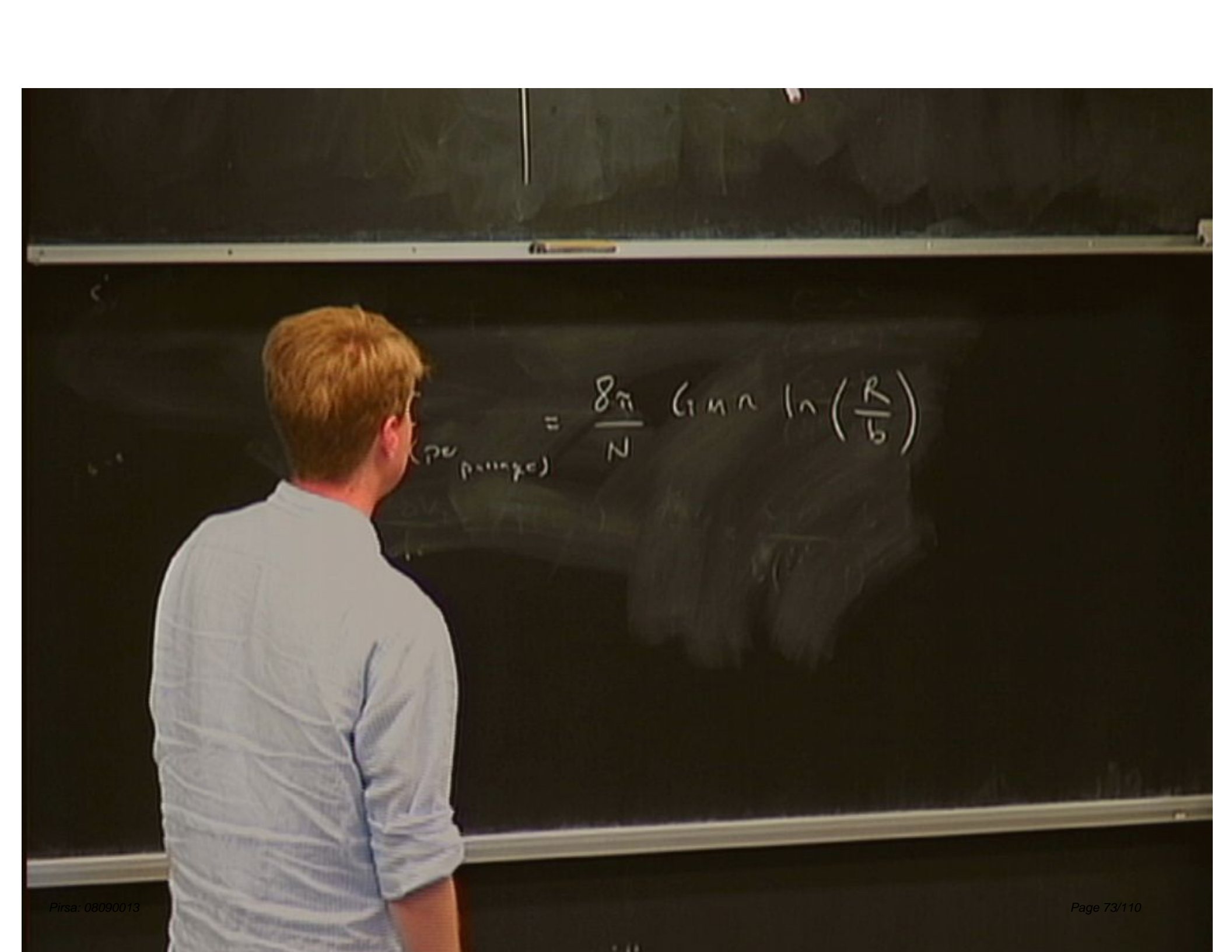
$$b_{\min} \sim \frac{v^2}{2GM}$$

Virial theorem

(KE = KT)

$$\rightarrow \begin{cases} 2KE = PE \\ 2\left(\frac{1}{2}mv^2\right) = \frac{GMm^2}{R} \end{cases}$$

$$\rightarrow v^2 = \frac{GMm}{R}$$



A man with short brown hair, wearing a light blue button-down shirt, stands with his back to the camera, looking at a chalkboard. The chalkboard is dark and has a handwritten equation in white chalk. The equation is:
$$\text{(per passage)} = \frac{\delta_{\text{in}}}{N} \sum_{i=1}^N \ln\left(\frac{R}{b}\right)$$

$$b_{\min} \sim \frac{v^2}{2GM}$$

Virial theorem
 $\rightarrow \begin{cases} 2KE = PE \end{cases}$

$$2\left(\frac{1}{2}mv^2\right) = \frac{GMm^2}{R}$$

$$\rightarrow v^2 = \frac{GM}{R}$$

$$\frac{2Gm}{b^2 v} = \delta v_{\perp} \sim v$$

$$b_{\min} \sim \frac{v^2}{2Gm}$$

Virial theorem

(KE = KT)

$$\rightarrow \begin{cases} 2KE = PE \\ 2 \left(\frac{1}{2} m v^2 \right) = \frac{G M m}{R} \end{cases}$$

$$\rightarrow v^2 = \frac{GM}{R}$$

$$v^2 = \frac{GM}{R}$$

$$C_{TM} = \frac{R}{v^2}$$

$$\Delta V_{\perp}^2 \text{ (per passage)} = \frac{8\pi}{N} C_{TM} \frac{1}{\sigma} \left(\frac{R}{\sigma} \right)$$

$\frac{R}{v^2}$

R

$$= \frac{8\pi}{N} \left(\frac{R v^2}{N} \right) \left(\frac{N}{\pi R^2} \right) \frac{1}{\sigma} \left(\frac{R}{\sigma} \right)$$

$$\frac{2C_{TM}}{h^2 v} = \sum v_i^2 \sim v$$

$$h \sim \frac{v^2}{2C_{TM}}$$

Virial theorem

(KE = kT)

$$\rightarrow \begin{cases} 2KE = PE \\ 2 \left(\frac{1}{2} m v^2 \right) = \frac{C_{TM} N v^2}{R} \end{cases}$$

$$\rightarrow v^2 = \frac{C_{TM}}{R}$$

$$C_{TM} = \frac{R v^2}{N}$$

$$\frac{2C_{TM}}{3V} = \sum \delta v_z \sim v$$

$$b_{min} \sim \frac{v^2}{2C_{TM}}$$

$$\sim \frac{N}{2R}$$

Virial theorem

(KE = KT)

$$\rightarrow \begin{cases} 2KE = PE \end{cases}$$

$$2 \left(\frac{1}{2} m v^2 \right) = \frac{C_{TM} N v^2}{R}$$

$$\rightarrow v^2 = \frac{C_{TM}}{R}$$

$$C_{TM} = \frac{R v^2}{N}$$

$$\frac{2C_{TM}}{b_{min}} = \sum v_i^2 \sim v^2$$

$$b_{min} \sim \frac{v^2}{2C_{TM}}$$

$$\sim \frac{N}{2R}$$

$$\frac{R}{b_{min}} \sim \frac{R}{N/2}$$

Virial theorem

($K_E = K_T$)

$$\rightarrow \begin{cases} 2K_E = PE \\ 2\left(\frac{1}{2}mv^2\right) = \frac{C_{TM}N}{R} \end{cases}$$

$$2\left(\frac{1}{2}mv^2\right) = \frac{C_{TM}N}{R}$$

$$\rightarrow v^2 = \frac{C_{TM}}{R}$$

$$C_{TM} = \frac{Rv^2}{N}$$

$$\frac{2Gm}{b_{min}} = \sum v_{\perp} \sim v$$

$$b_{min} \sim \frac{2Gm}{v^2}$$

$$\sim \frac{2R}{N}$$

$$\frac{R}{b_{min}} \sim \frac{R}{R/N} \sim N$$

Virial theorem

(KE = KT)

$$\rightarrow \begin{cases} 2KE = PE \end{cases}$$

$$2 \left(\frac{1}{2} m v^2 \right) = \frac{G M m}{R}$$

$$\rightarrow v^2 = \frac{GM}{R}$$

$$G M = \frac{R v^2}{N}$$

$$\frac{K}{b_{min}} \sim \frac{K}{R/N} \sim N$$

$$C_{TM} = \frac{R v^2}{N}$$

$$\begin{aligned} \Delta v_{\perp}^2 &= \frac{8\pi}{N} C_{TM} \frac{1}{2} \left(\frac{R}{b} \right) \\ &= \frac{8\pi}{N} \left(\frac{R v^2}{\Delta} \right) \left(\frac{N}{\pi R^2} \right) \frac{1}{2} (N) \\ &= v^2 \left(\frac{8}{2} \frac{1}{2} N \right) \end{aligned}$$

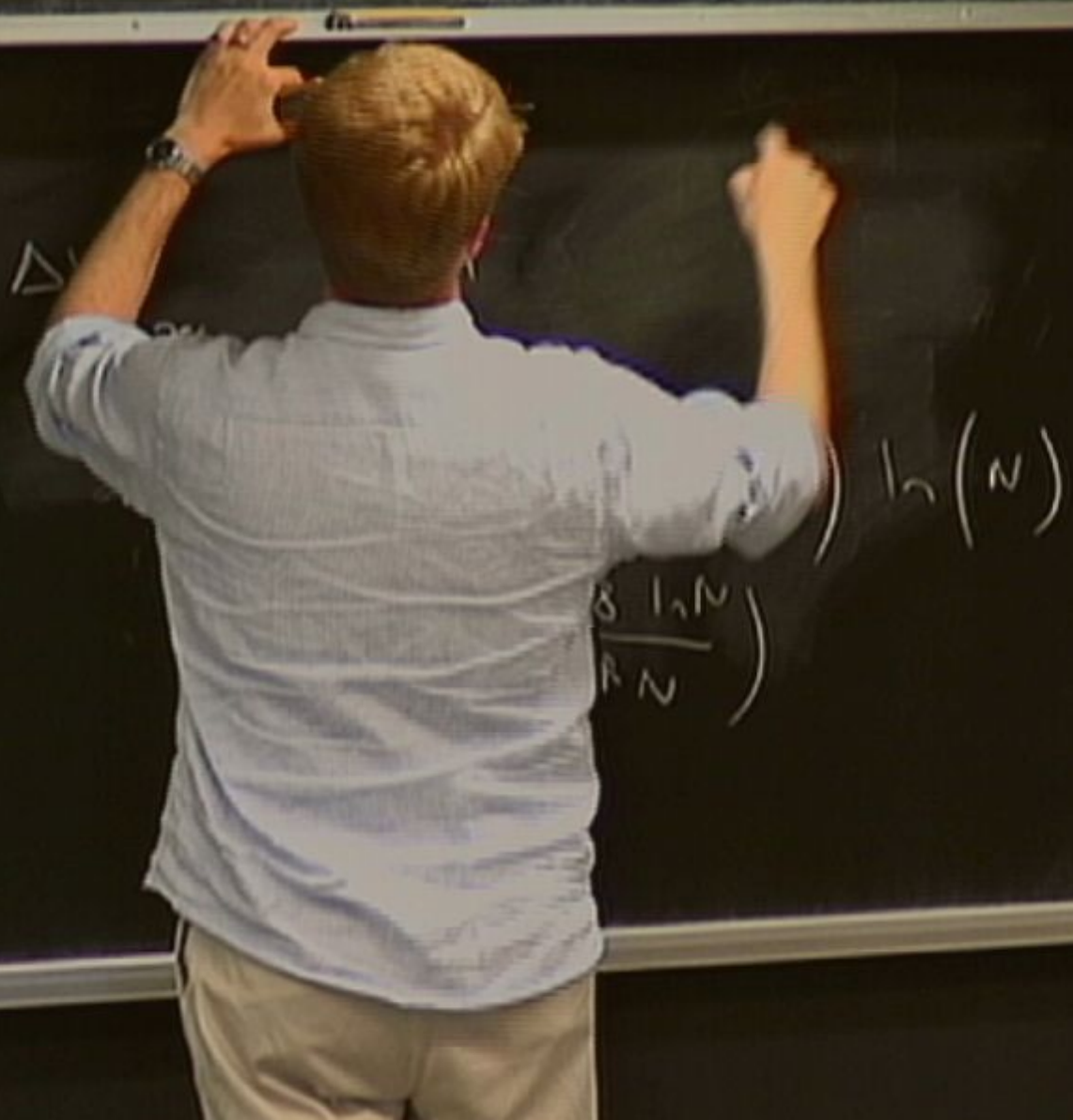
$$\frac{K}{b_{min}} \sim \frac{K}{R/N} \sim N$$

$$C_{TM} = \frac{R v^L}{N}$$

$$\begin{aligned} \Delta v_{\perp}^2 &= \frac{8\pi}{N} C_{TM} \ln\left(\frac{R}{b}\right) \\ &\quad \text{(per passage)} \quad \swarrow \frac{R}{\pi R^L} \\ &= \frac{8\pi}{N} \left(\frac{R v^L}{\Delta}\right) \left(\frac{N}{\pi R^L}\right) \ln(N) \\ &= v^L \left(\frac{8 \ln N}{\pi N}\right) \end{aligned}$$

$$\frac{R}{b_{min}} \sim \frac{R}{R/N} \sim N$$

$$C_{TM} = \frac{R}{v_c}$$
$$C_{TM} = \frac{R^2 N}{v_c}$$



$$\frac{2GM}{b_{min}^2 v} = \delta v_{\perp} \sim v$$

$$b_{min} \sim \frac{2GM}{v^2}$$

$$\sim \frac{2R}{Z}$$

$$\frac{R}{b_{min}} \sim \frac{R}{R/Z} \sim Z$$

Virial theorem (1/2 KE)

$$\rightarrow \begin{cases} 2KE \approx PC \\ 2 \left(\frac{1}{2} m v^2 \right) = \frac{GMm}{R} \end{cases}$$

$$\frac{GMm}{R}$$

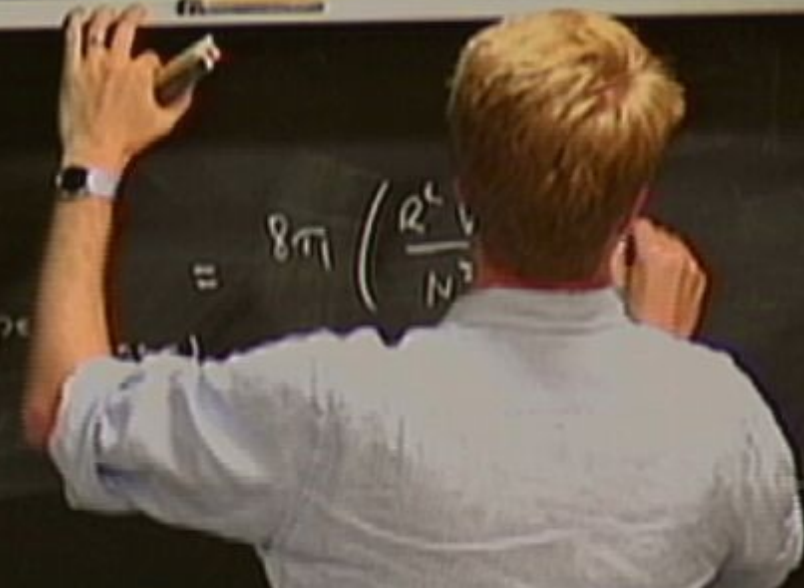
$$\rightarrow v^2 = \frac{GM}{R}$$

$$GM = \frac{R v^2}{Z}$$

$$GM = \frac{R^2 v^2 N}{Z^2}$$

$$\Delta v_{\perp}^2 = 8\pi \left(\frac{R^2 v^2}{Z^2} \right)$$

$$\left(Z \right)$$



$$\Delta V_{\perp}^2 \text{ (per particle)} = 8\pi \left(\frac{R^2 v_{\perp}^2}{N^2} \right) \frac{N}{v_{\perp}^2} \frac{1}{2} \left(\frac{R}{b_n} \right)$$

$$= 8\pi \left(\frac{R^2 v_{\perp}^2}{N^2} \right) \left(\frac{A}{R^2} \right) \frac{1}{2} \left(\frac{R}{b_n} \right)$$

$$= \frac{8 \ln(N)}{2} \left(\frac{R}{b_n} \right)$$

$$\Delta v_L(b) = \frac{2GM}{bv}$$

$$\frac{\Delta v_L}{v} \ll 1$$

$$\Delta t = \frac{b}{v}$$

$$\Delta v_L \sim v$$

$$\sum (\Delta v_i)^2 \sim (v)^2 \quad N (\Delta v_i)^2 = v^2 \quad \Delta v_i^2 = \frac{v^2}{N}$$

$$\frac{v^2}{N} = v^2 \left(\frac{\delta \ln N}{N} \right)$$

$$N = \frac{N}{\delta \ln N}$$

$$\sum (\Delta v_i)^2 \sim$$

$$(\Delta v_i)^2 \approx v^2$$

$$\Delta v_i \approx \frac{v}{N}$$

$\frac{v}{N}$

$$\left(\frac{\delta \ln N}{N} \right)$$

$$t_{\text{one passage}} = \frac{R}{v}$$

$$\sum (\Delta v_i)^2 \sim (v)^2$$

$$N (\Delta v_i)^2 \sim v^2$$

$$\Delta v_i \sim \frac{v}{\sqrt{N}}$$

$$\frac{v}{\sqrt{N}} = v \left(\frac{\delta \ln N}{N} \right)$$

$$N = \frac{N}{\delta \ln N}$$

$$t_{\text{one passage}} = \frac{R}{v}$$

$$t = N t = N \frac{R}{v} = \frac{N}{\delta \ln N} \left(\frac{R}{v} \right)$$

$$G^2 M^2 = \frac{R^2 v^2 N}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^7$$

60

$$C_1 m^2 = \frac{R^2 N}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

$$\text{Etag} = 10^{14}$$

$$C^2 m^2 = \frac{R^2 v^2 N}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^7$$

$t_{cross} \sim 10^7$ yrs

Wavelength $\sim 10^{16}$

$$C^2 m^2 = \frac{R^2 v^2}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

$$t_{\text{cross}} \sim 10^7 \text{ yrs}, \quad t_{\text{age}} \sim 10^{10} \text{ yrs} \sim 10^3$$

$$C^2 m^2 = \frac{R^2 v^2}{2}$$

$$\underline{\text{Galaxy}} \quad N \sim 10^{11}$$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

$$t_{\text{cross}} \sim 10^7 \text{ yrs}, \quad \text{Age} \sim 10^{10} \text{ yrs} \sim 10^3$$

$$C_1^2 M^2 \sim \frac{R^2 v^2 N}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^7$$

$$t_{\text{cross}} \sim 10^7 \text{ yrs}, \quad t_{\text{age}} \sim 10^{10} \text{ yrs} \sim 10^3$$

Galactic clusters

$$N = 10^5, \quad t_{\text{cross}} = 10^5 \text{ yrs} \quad N_{\text{enc}} = \frac{10^5}{10(5)} \sim 10^{3.5}$$

$$C_1^2 m^2 = \frac{12^2 v^2 N}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

$t_{\text{cross}} \sim 10^7 \text{ yrs}$, $t_{\text{drag}} \sim 10^{10} \text{ yrs} \sim 10^3$

Galactic clusters

$$N = 10^3, t_{\text{cross}} = 10^5 \text{ yrs}, N_{\text{enc}} = \frac{10^5}{10(5)} \sim 10^{3.5}$$

$$t_{\text{enc}} \sim 10^{10} \text{ yrs}$$

$$C_1 m^2 = \frac{R^2 v^2 N}{2L}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

$$t_{\text{cross}} \sim 10^7 \text{ yrs}, \quad t_{\text{age}} \sim 10^{10} \text{ yrs} \sim 10^3$$

Galactic clusters

$$N = 10^3, \quad t_{\text{cross}} = 10^5 \text{ yrs}, \quad N_{\text{enc}} = \frac{10^3}{(10^5)} \sim 10^{-2}$$

$$t_{\text{life}} \sim 10^{10} \text{ yrs}, \quad t_{\text{cross}} \sim 10^5$$

$$C^2 M^2 = \frac{R^2 N}{2L}$$

Cluster of Galaxies

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

$$t_{\text{cross}} \sim 10^7 \text{ yrs}, \quad t_{\text{age}} \sim 10^{10} \text{ yrs} \sim 10^3$$

Galaxy clusters

$$N = 10^3, \quad t_{\text{cross}} = 10^5 \text{ yrs}, \quad N_{\text{enc}} = \frac{10^5}{10(5)} \sim 10^{35}$$

$$t_{\text{enc}} \sim 10^{10} \text{ yrs}, \quad t_{\text{cross}} \sim 10^5$$

$$C_1 M^2 = \frac{12.2 \sqrt{N}}{2L}$$

Cluster of Galaxies

$$N \sim 10^2$$

$$t_{cross} \sim 10^7$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^7$$

$$t_{cross} \sim 10^7 \text{ yrs}, \quad t_{age} \sim 10^{10} \text{ yrs} \sim 10^3$$

Galaxy clusters

$$N = 10^3, \quad t_{cross} = 10^5 \text{ yrs}, \quad N_{enc} = \frac{10^3}{10(5)} \sim 10^{3.5}$$

$$t_{age} \sim 10^{10} \text{ yrs}, \quad t_{cross} \sim 10^5$$

Cluster of Galaxies

$$N \sim 10^3$$

$$t_{\text{cross}} \sim 10^9$$

$$N_{\text{cross}} = \frac{10^3}{30} \sqrt{\frac{1}{10^9}}$$

Galaxy $N \sim 10^{11}$

$$N = \frac{10^{11}}{10(11)} \sim 10^9$$

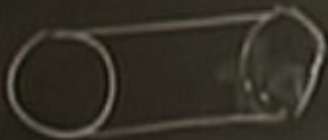
$$t_{\text{cross}} \sim 10^7 \text{ yrs}, \quad \text{Lagrange } 10^{10} \text{ yrs} \sim 10$$

Globular clusters

$$N = 10^5, \quad t_{\text{cross}} = 10^5 \text{ yrs}$$

$$N_{\text{gc}} = \frac{10^5}{10(5)} \sim 10^{3.5}$$

$$t_{\text{life}} \sim 10^{10} \text{ yrs}, \quad N_{\text{cross}} \sim 10^5$$



$$A = 2\pi b db$$

$$A_{\text{min}} b_{\text{min}} = \pi b_{\text{min}}^2$$

$$\langle (\delta v_z)^2 \rangle = n (\delta v_z)^2 = \frac{4\pi n G m^2}{b^2 v^2} 2\pi b db$$

$$= \frac{8\pi G^2 m^2 n}{v^2} \frac{db}{b}$$

$$\Delta v_1(b) = \frac{2Gm}{bv}$$

$$\frac{\Delta v_2}{v} \ll 1$$

$$\Delta C = \frac{b}{v}$$



$$\delta n = n \pi b u^2$$

$$\Delta v(b) = \frac{2Gm}{bv}$$

$$\frac{\Delta v}{v} \ll 1$$

$$\Delta t = \frac{b}{v}$$



$$\begin{aligned} \delta n &= n \pi b v \Delta t \\ &\sim \frac{N}{\pi R^2} \pi b v \Delta t \\ &\sim \frac{N}{R^2} b v \Delta t \end{aligned}$$

$$\Delta v_2(b) = \frac{2Gm}{bv}$$

$$\frac{\Delta v_2}{v} \ll 1$$

$$\Delta C = \frac{v}{c}$$



$$\begin{aligned} \delta m &= n \pi b v \Delta t \\ &\sim \frac{N}{\pi R^2} \pi b v \Delta t \\ &\sim \frac{N}{R^2} b v \Delta t \end{aligned}$$

$$\Delta v_1(b) = \frac{2GM}{bv}$$

$$\frac{\Delta v_2}{v} \ll 1$$

$$\Delta C = \frac{v}{c}$$

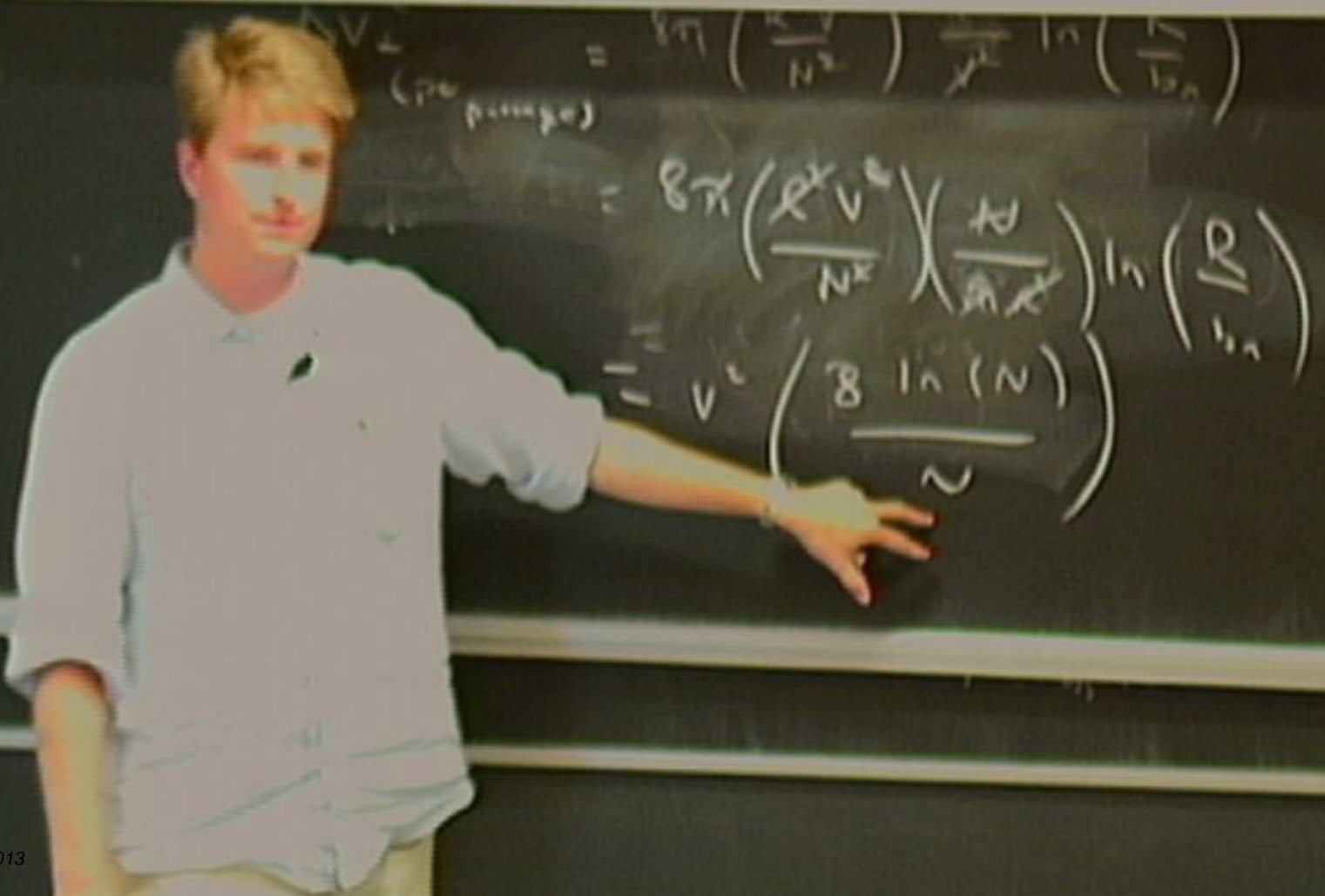


$$\begin{aligned} \delta n &= n \pi b \sin^2 \theta \\ &\sim \frac{N}{\pi R^2} \pi b \sin^2 \theta \\ &\sim \frac{N}{R^2} b \sin^2 \theta \sim \frac{1}{2} \end{aligned}$$

$$v^2 = \frac{GM}{R}$$

$$GM = \frac{Rv^2}{1}$$

$$GM = \frac{R^2 v^2}{N}$$



$$= 8\pi \left(\frac{R^2 v^2}{N} \right) \left(\frac{R}{N} \right) = \left(\frac{R}{N} \right)$$

$$\left(\frac{R}{N} \right)$$



$\sum_{m=1}^N =$

$$\frac{110 \ln 2}{2} + \frac{1}{2}$$

$$\delta n = n \pi b_{min}$$

$$\sim \frac{N}{\pi R^2} \pi b_{min}$$

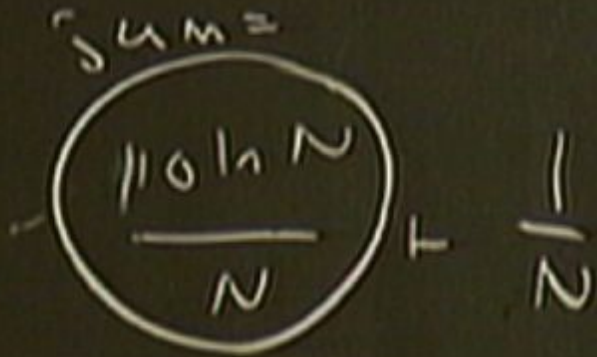
$$\sim \frac{N}{R^2} b_{min}^2 \sim \frac{1}{2}$$



$$\delta n = n \pi b_{min}^2$$

$$\sim \frac{N}{\pi R^2} \pi b_{min}^2$$

$$\sim \frac{N}{R^2} b_{min}^2 \sim \frac{1}{2}$$



$$= \frac{2N}{R^2} b db$$

$$2\pi b dl$$

$$\frac{c^2}{v^2} n \frac{db}{b}$$

$\Delta V \perp$
(per point)

$$\text{Flux} = aT^4$$

$$= \frac{2N}{R^2} b db$$

$$2\pi b dl$$

$$\frac{h db}{b}$$

$\Delta V \perp$
(per point)

$$\text{Flux} = aT^4$$

CAUTION
DO NOT TOUCH THE BOARD WITH
YOUR HANDS OR OBJECTS
IT IS HOT & YOU
MAY BE BURNED