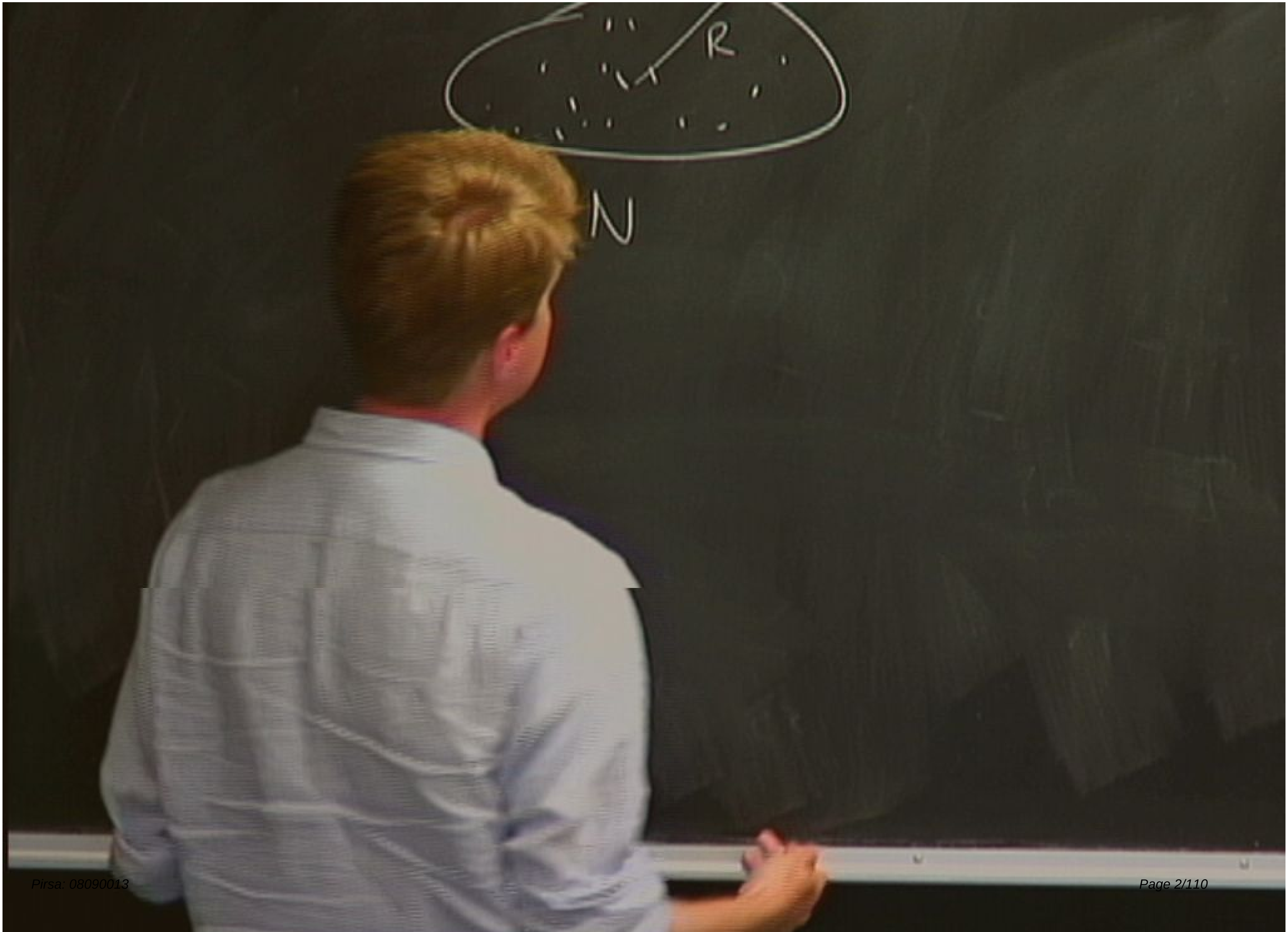
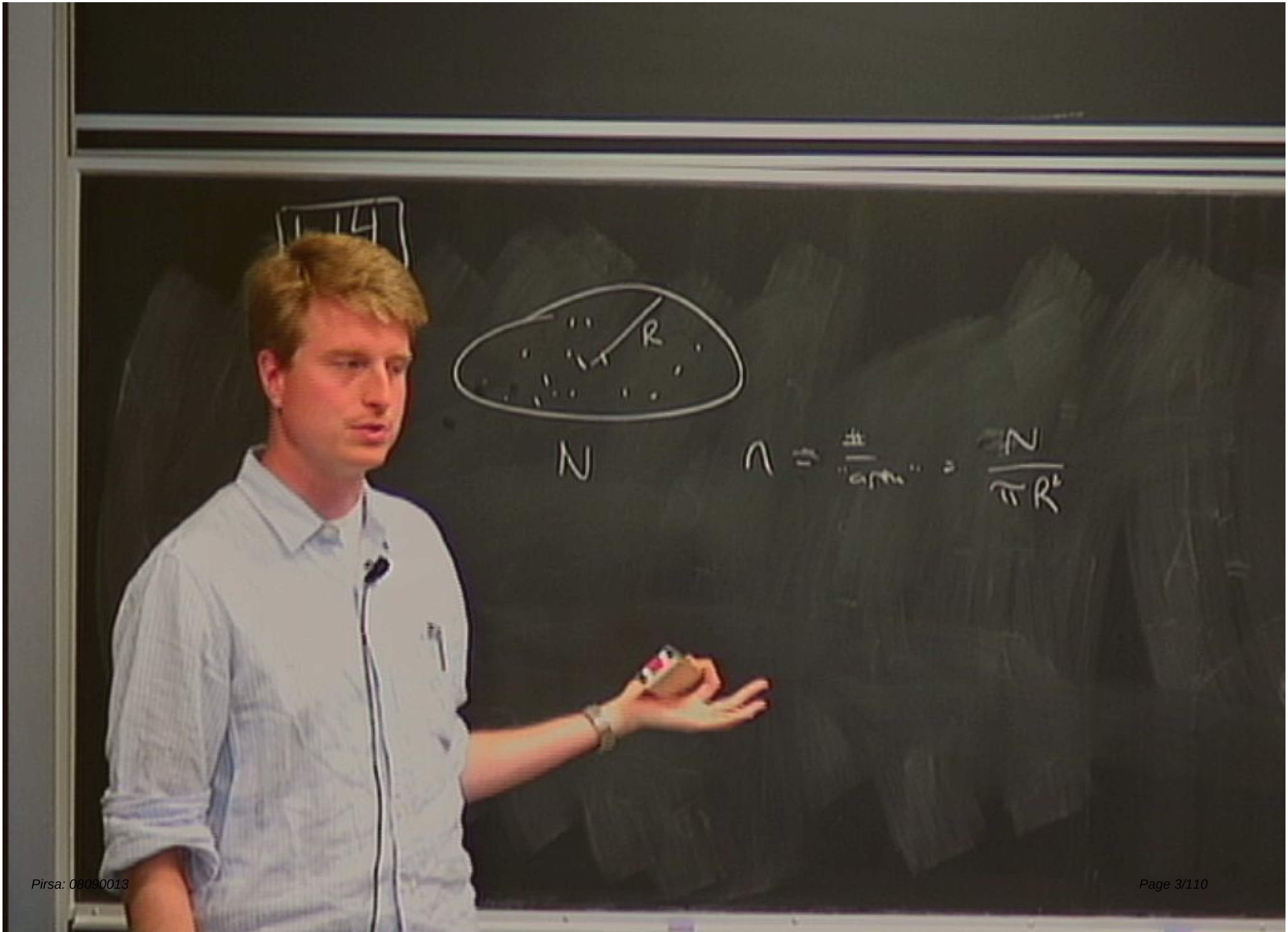
Title: Astrophysics and Cosmology through Problems - 1B

Date: Sep 04, 2008 12:30 PM

URL: http://pirsa.org/08090013

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

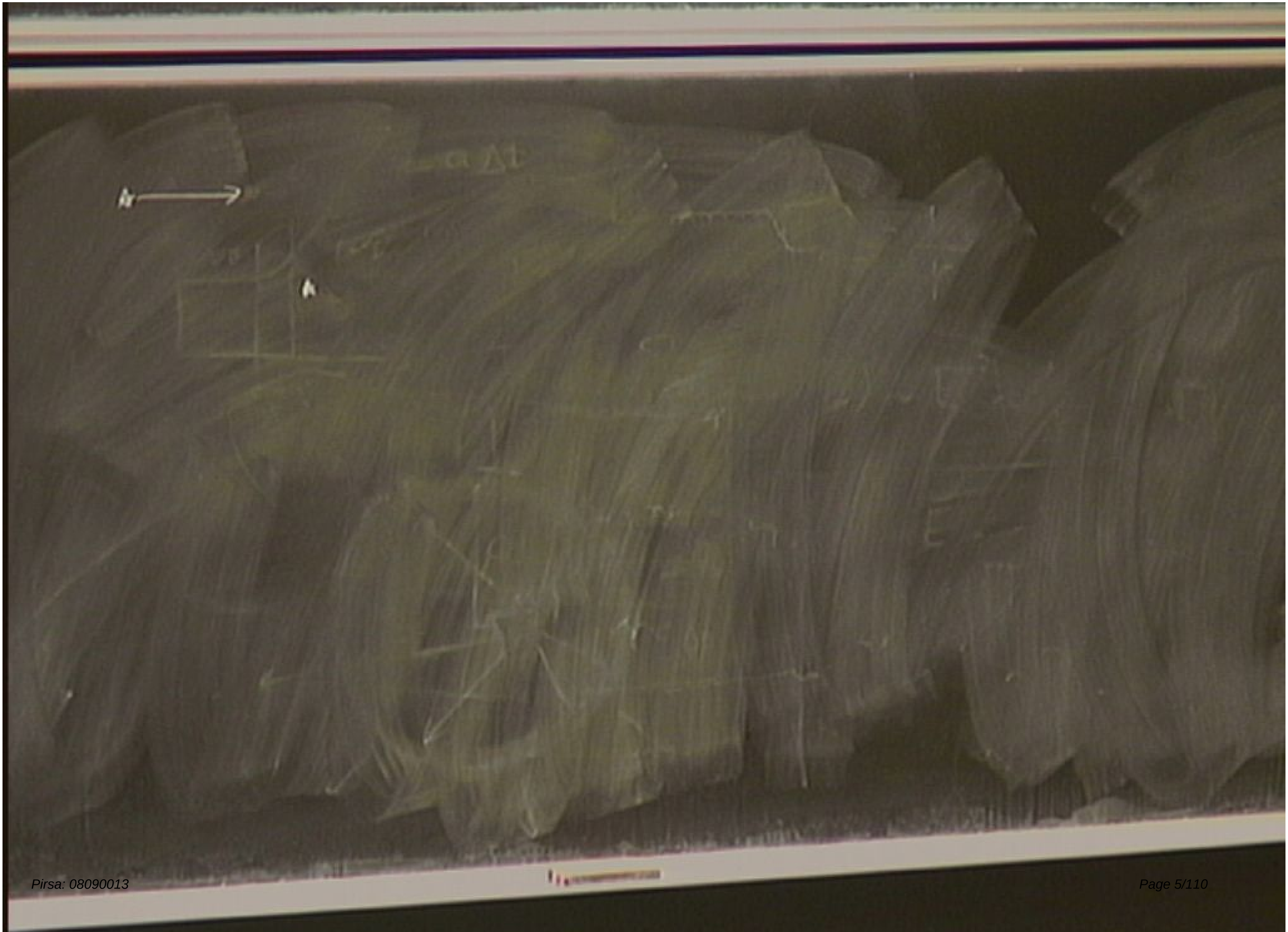

R
N

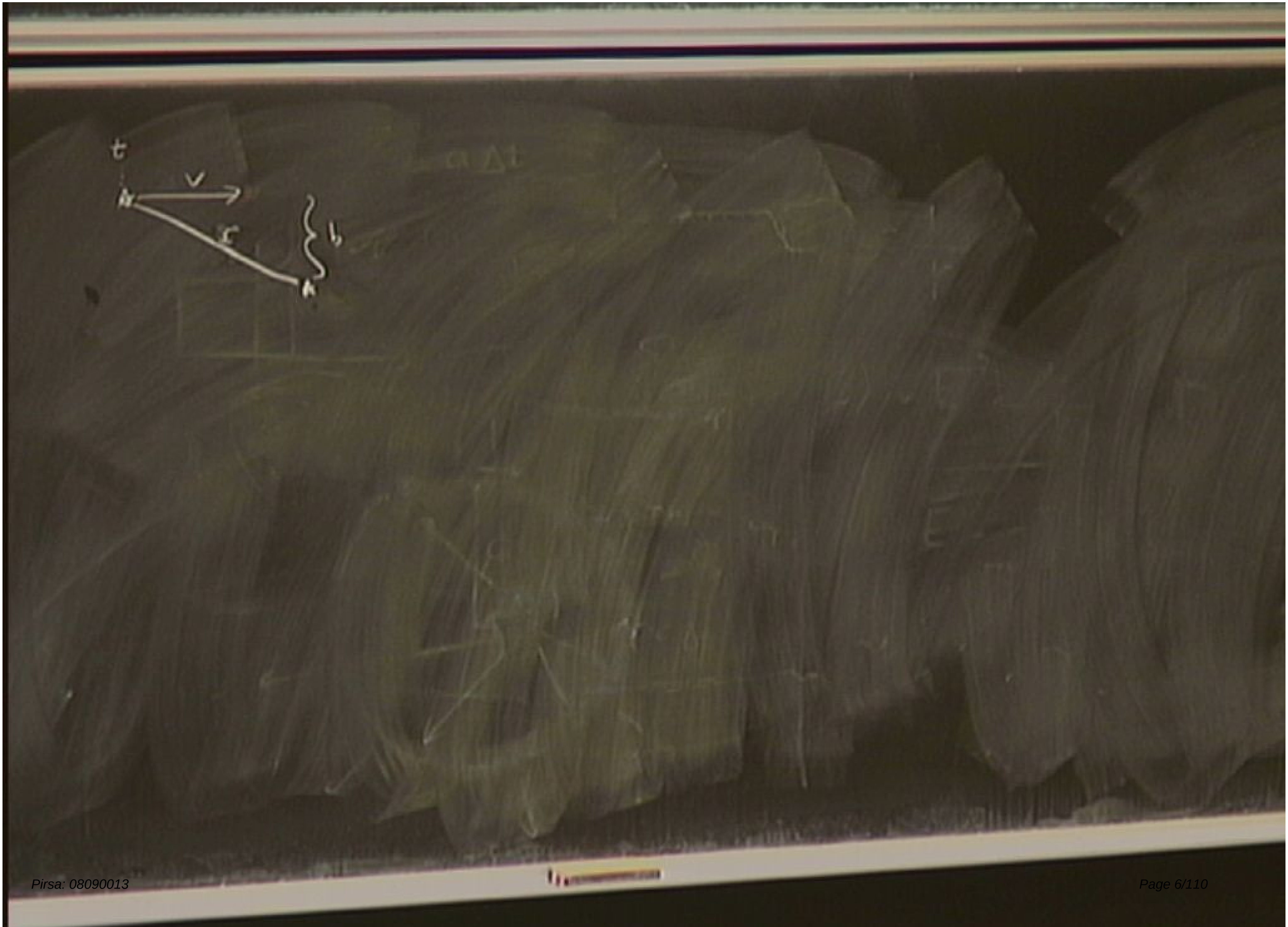

R
N
n = "#" / "area" = N / πR²

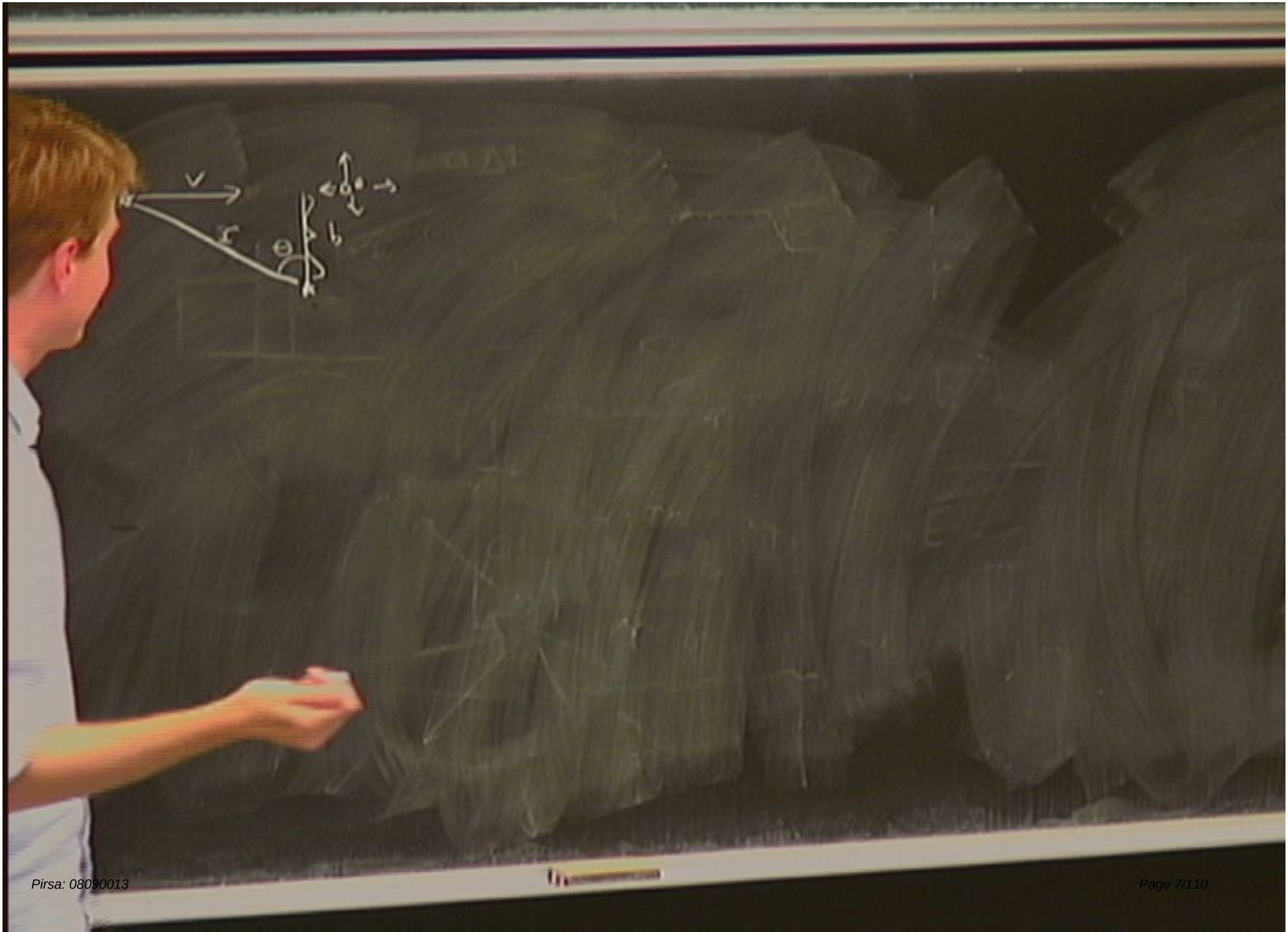
$\boxed{1.14}$

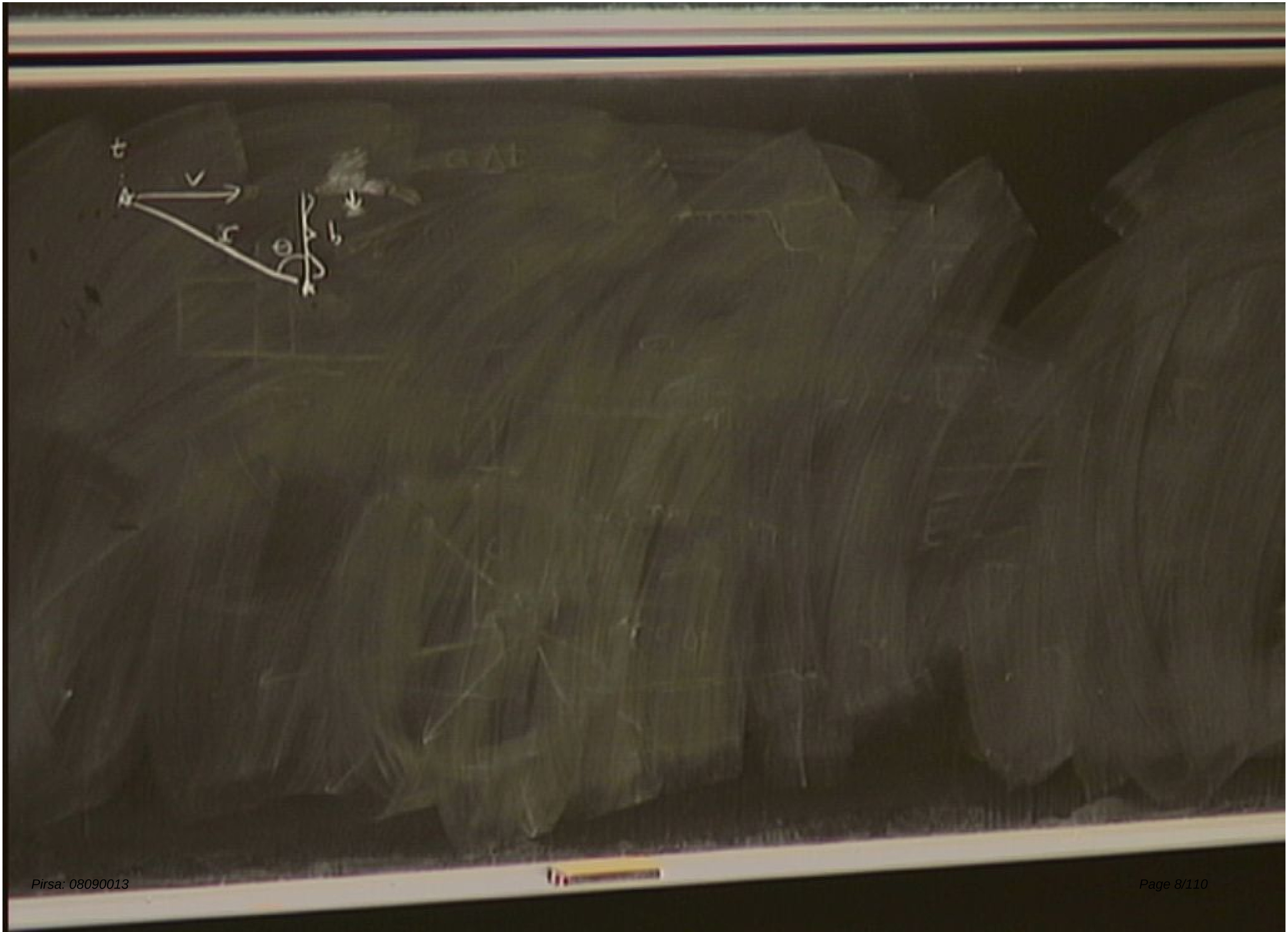
N

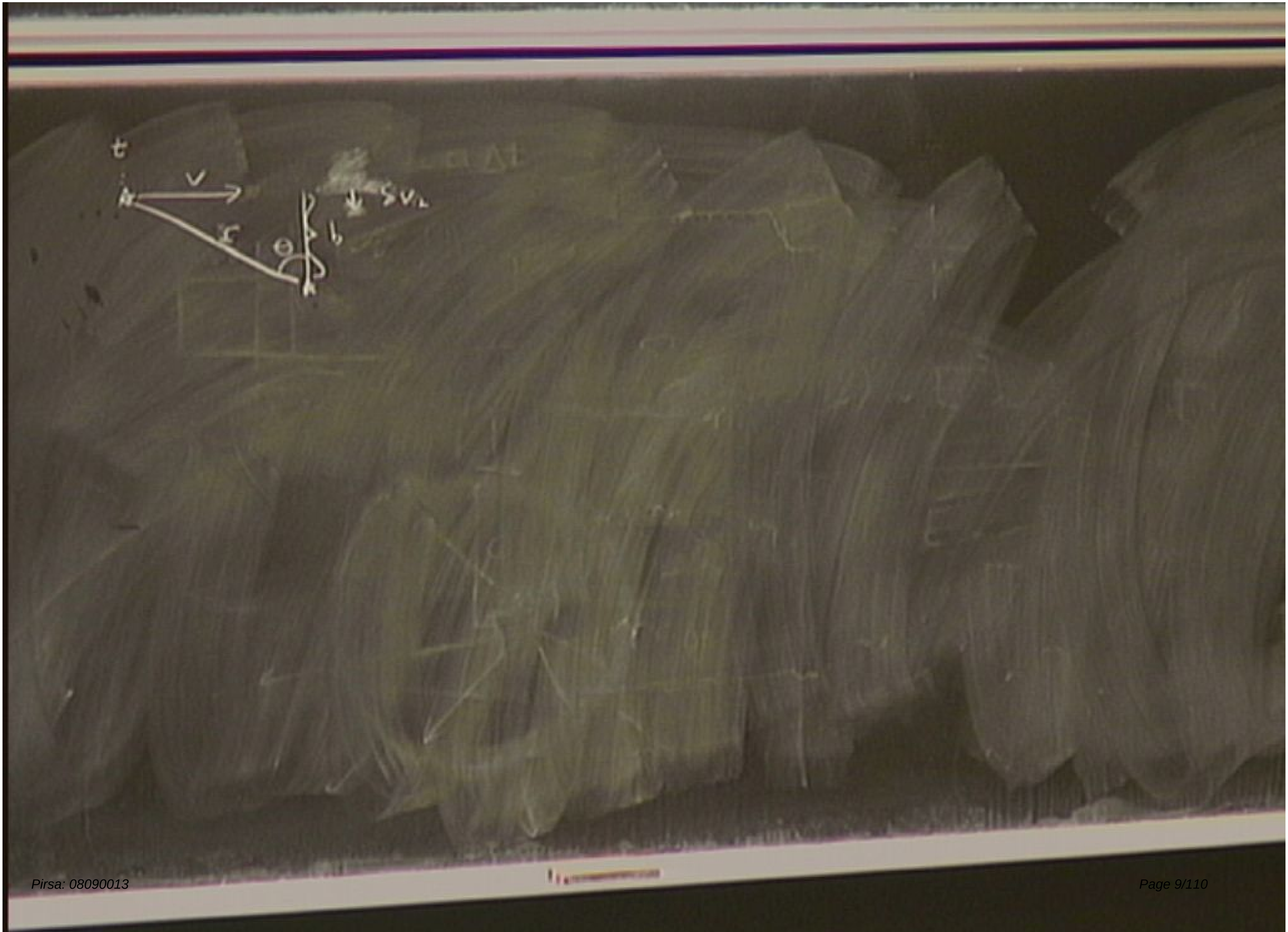
$$n = \frac{\#}{\text{"area"}} = \frac{N}{\pi R^2}$$



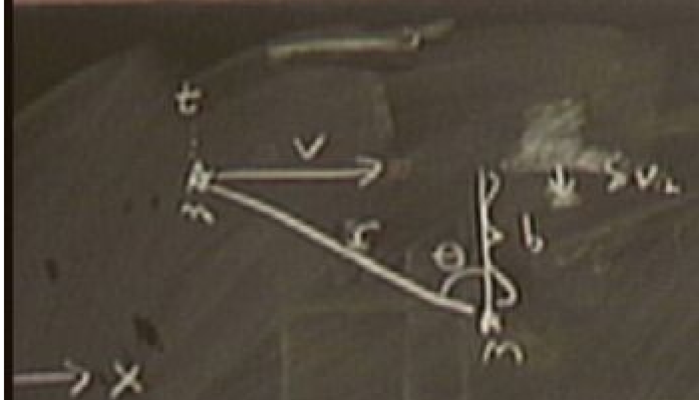

t
v
c
b


v
r
θ
b

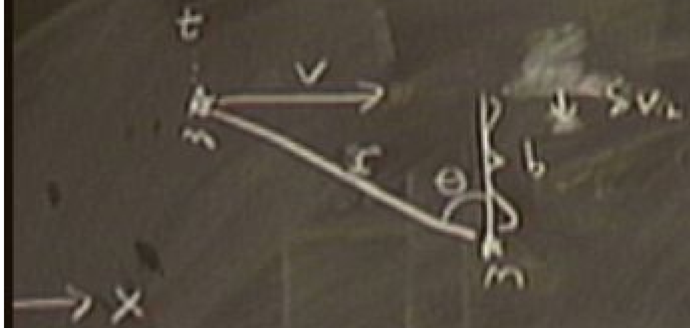

t
v
r
θ
b


t
v
r
θ
b

$$F_g = \frac{Gm^2}{r^2}$$

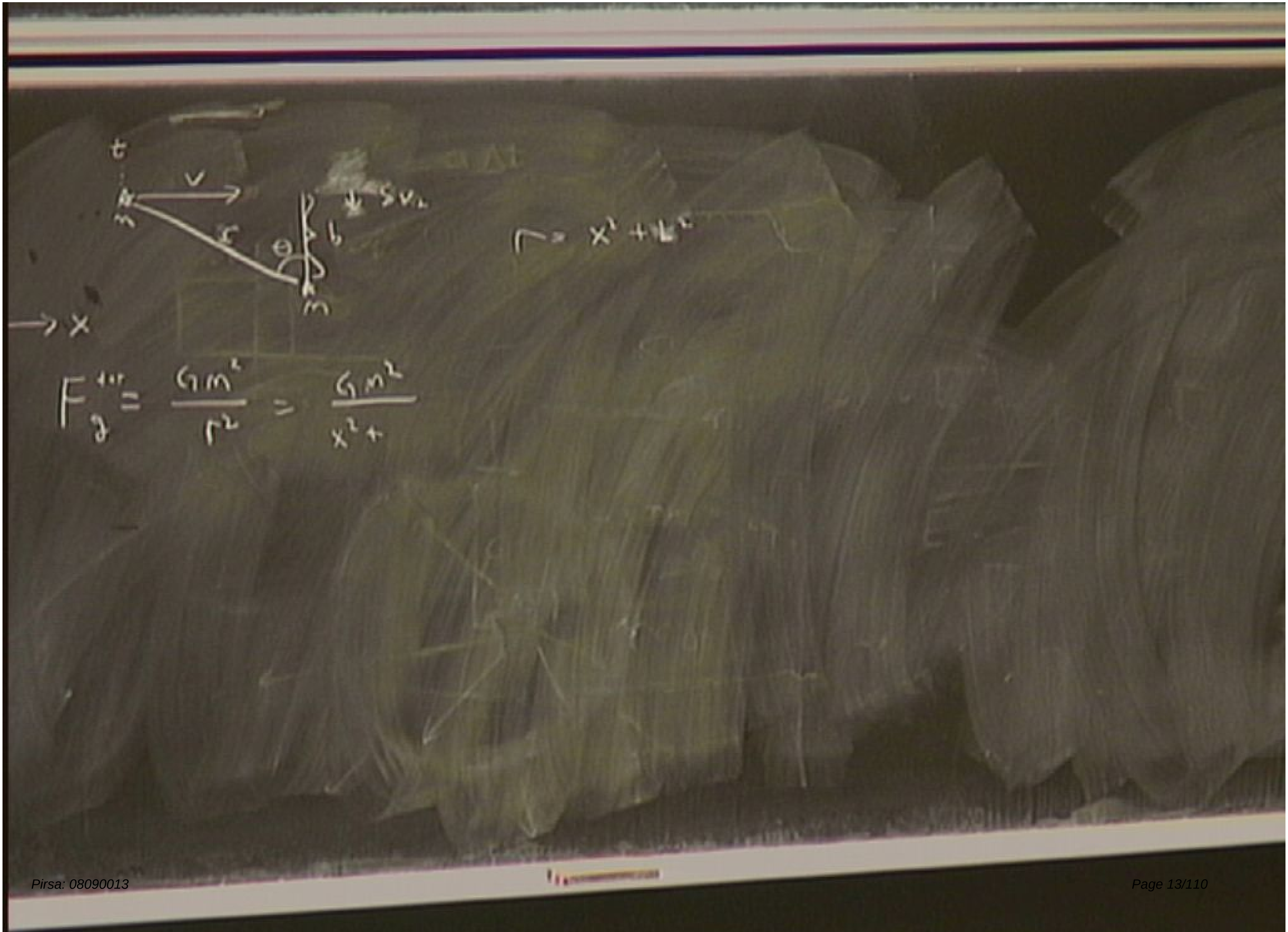


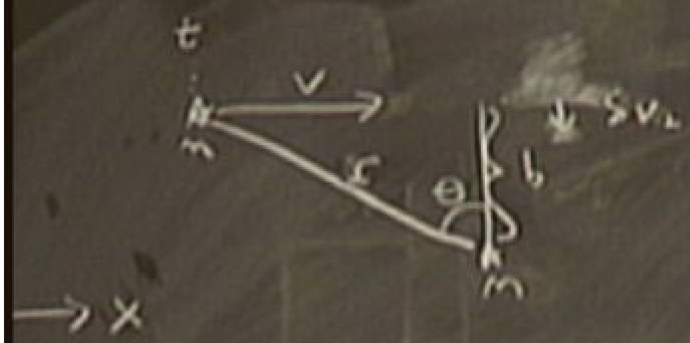
$$F_g = \frac{Gm^2}{r^2}$$



$r \to x^2 + r^2$

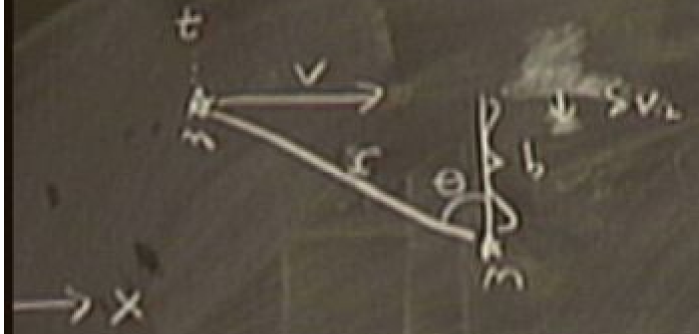
$$F_g = \frac{Gm^2}{r^2}$$


t
v
δv
r
θ
b
m
m
x
r = x² + b²
F_g = Gm²/r² = Gm²/(x² +



$r = x^2 + b^2$

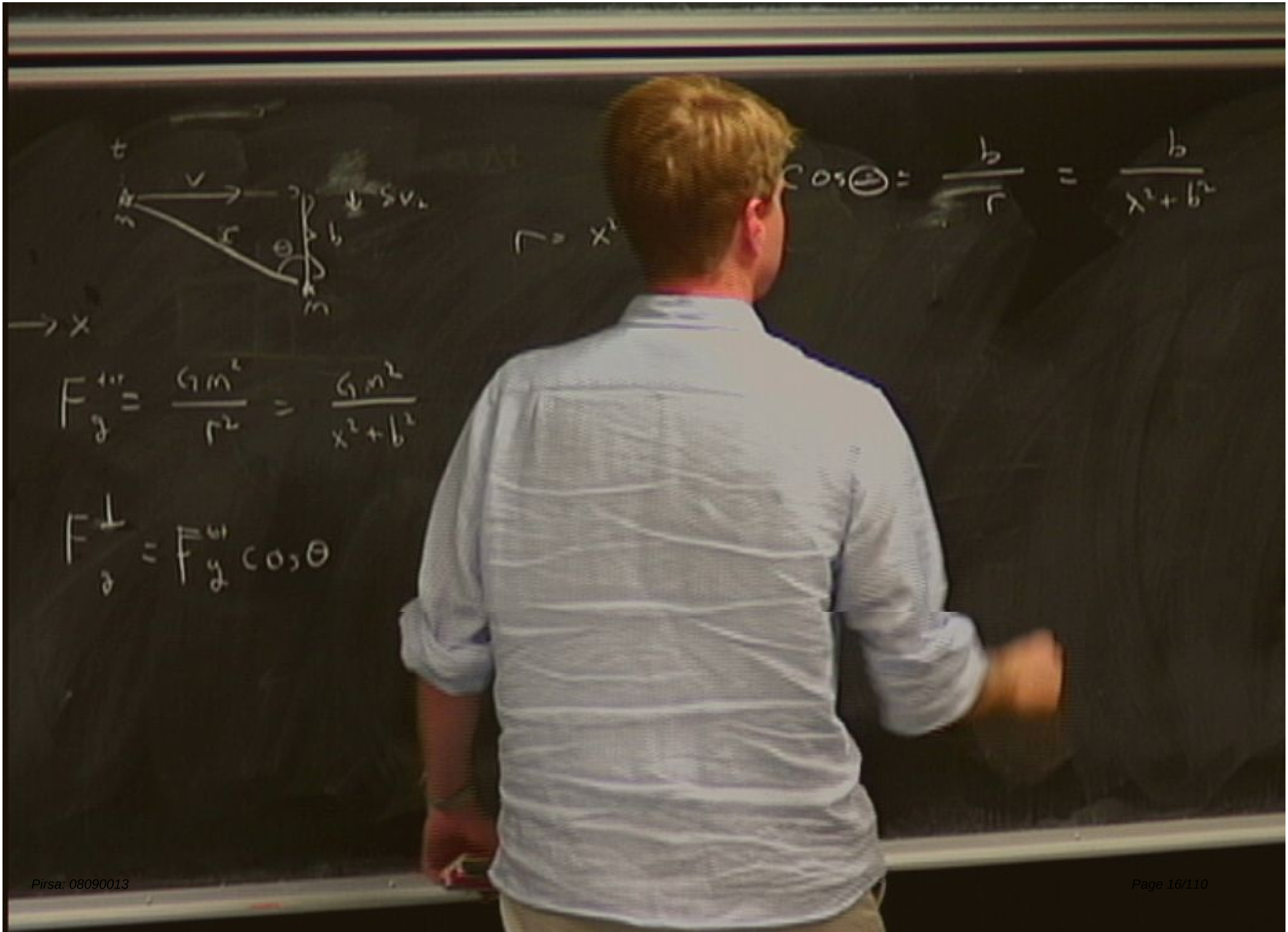
$$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \propto a_{tot}$$

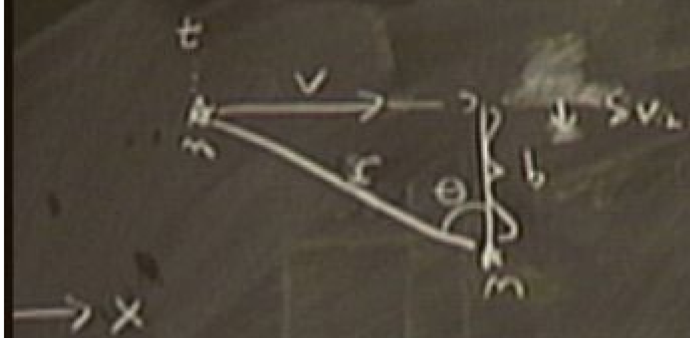


$r = x^2 + b^2$

$$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \quad \propto a_{tot}$$

$$F_g^{\perp} = F_g^{tot} \cos\theta$$


t
v
$\delta v_\perp$
m
r
θ
b
m
x
$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2}$
$F_g^{\perp} = F_g^{tot} \cos\theta$
$r = x^2$
$\cos\theta = \frac{b}{r} = \frac{b}{x^2+b^2}$

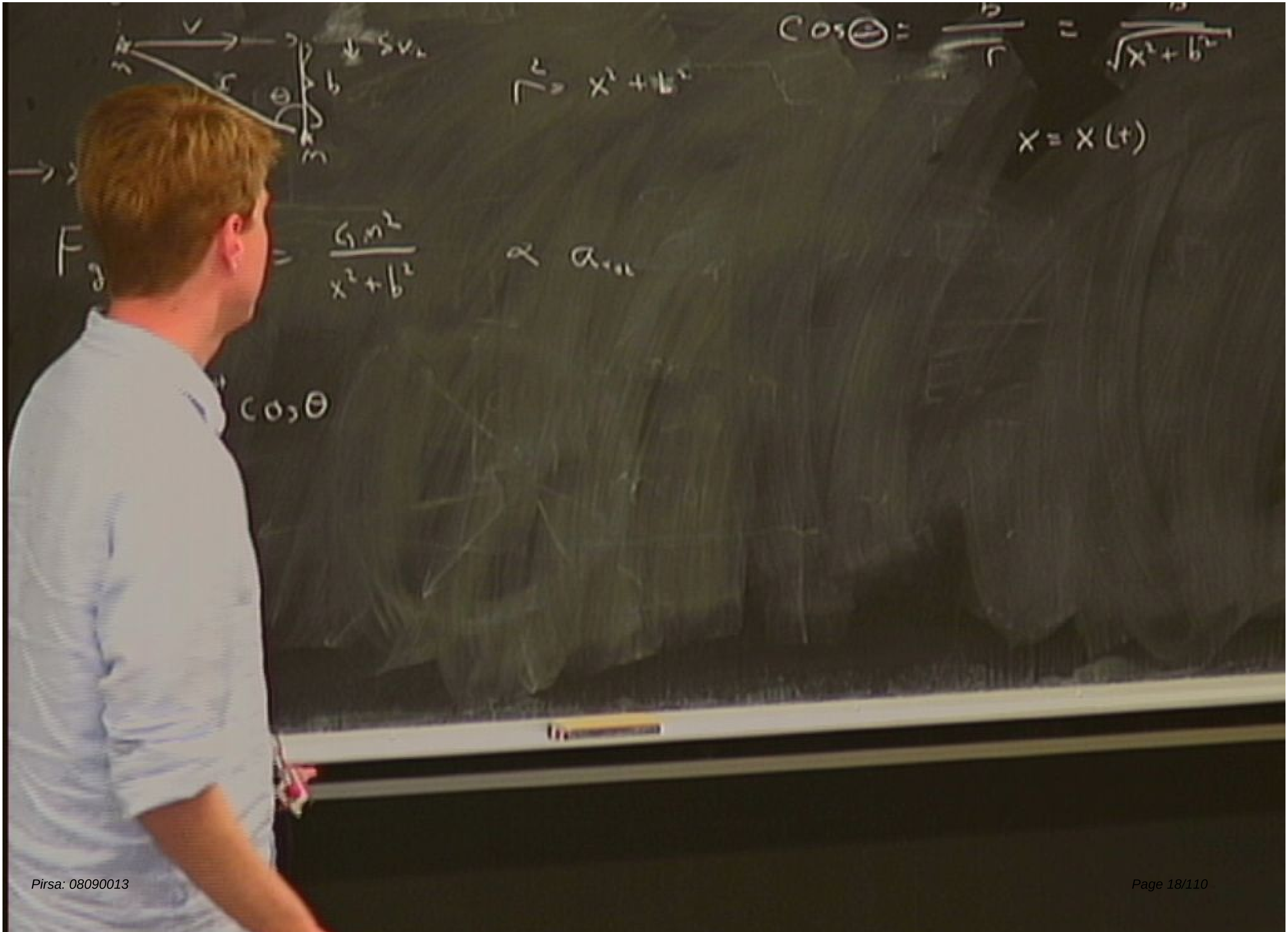


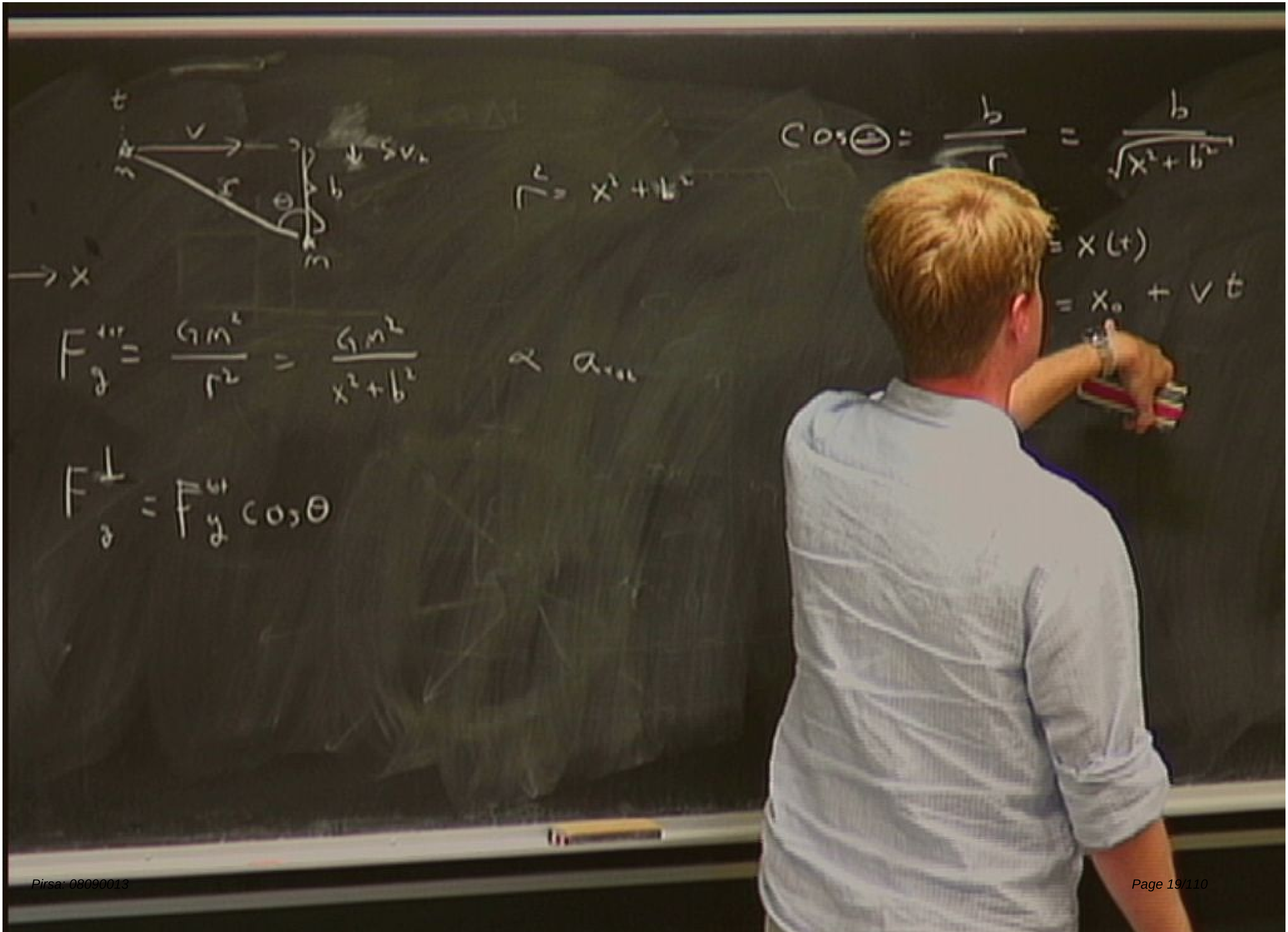
$$\cos\Theta = \frac{b}{r} = \frac{b}{\sqrt{x^2+b^2}}$$

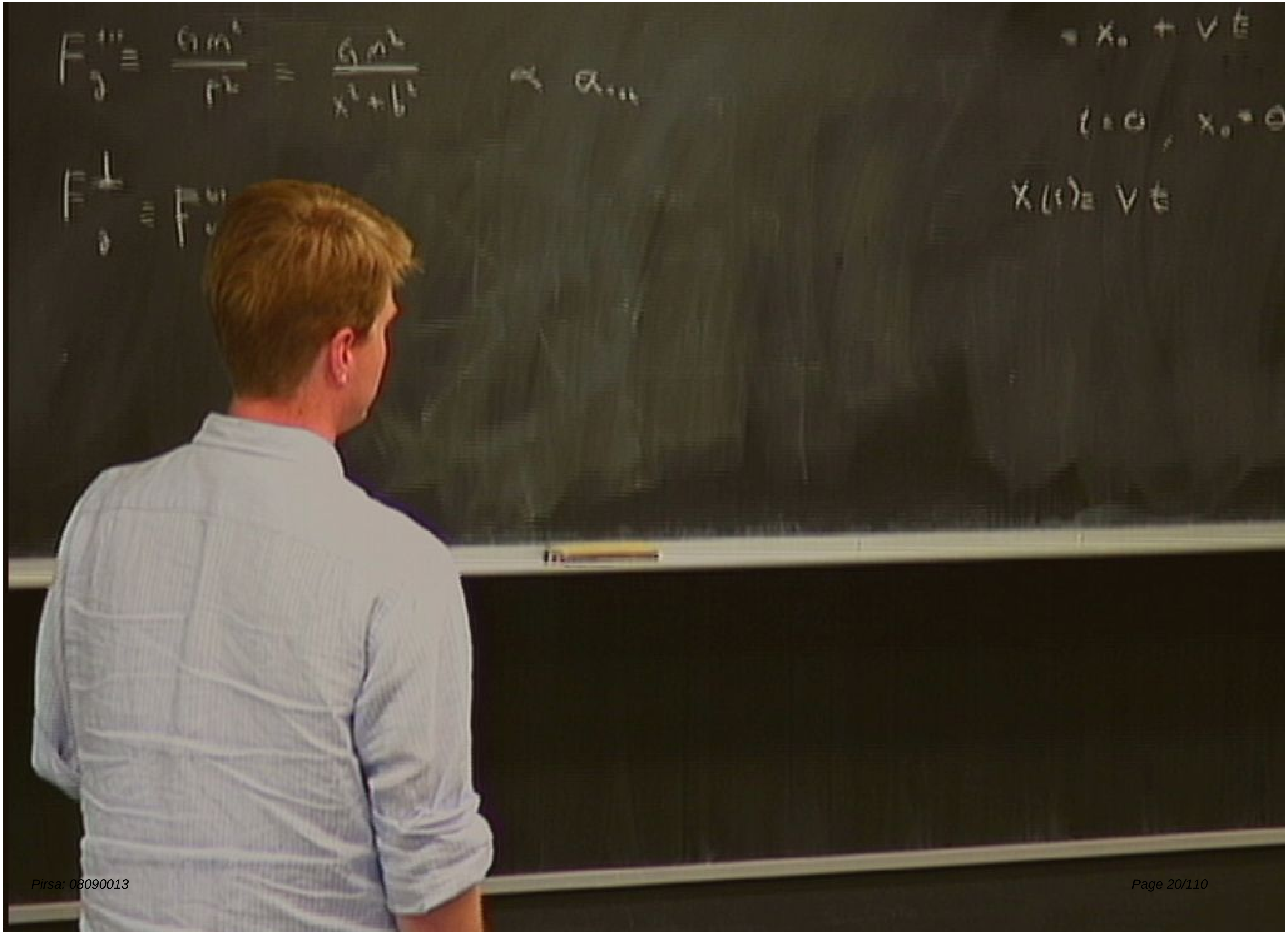
$$r = x^2 + b^2$$

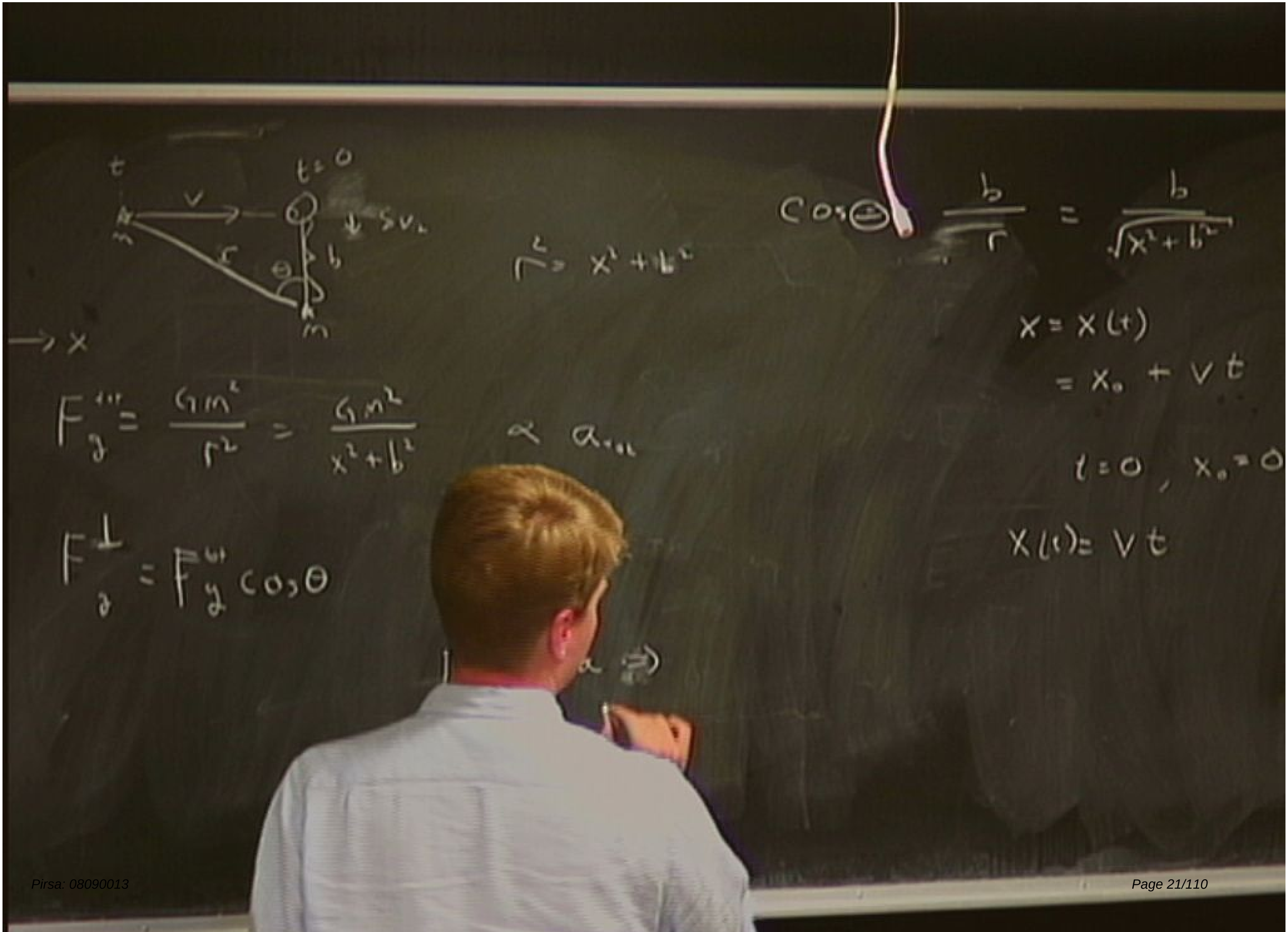
$$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \quad \propto a_{tot}$$

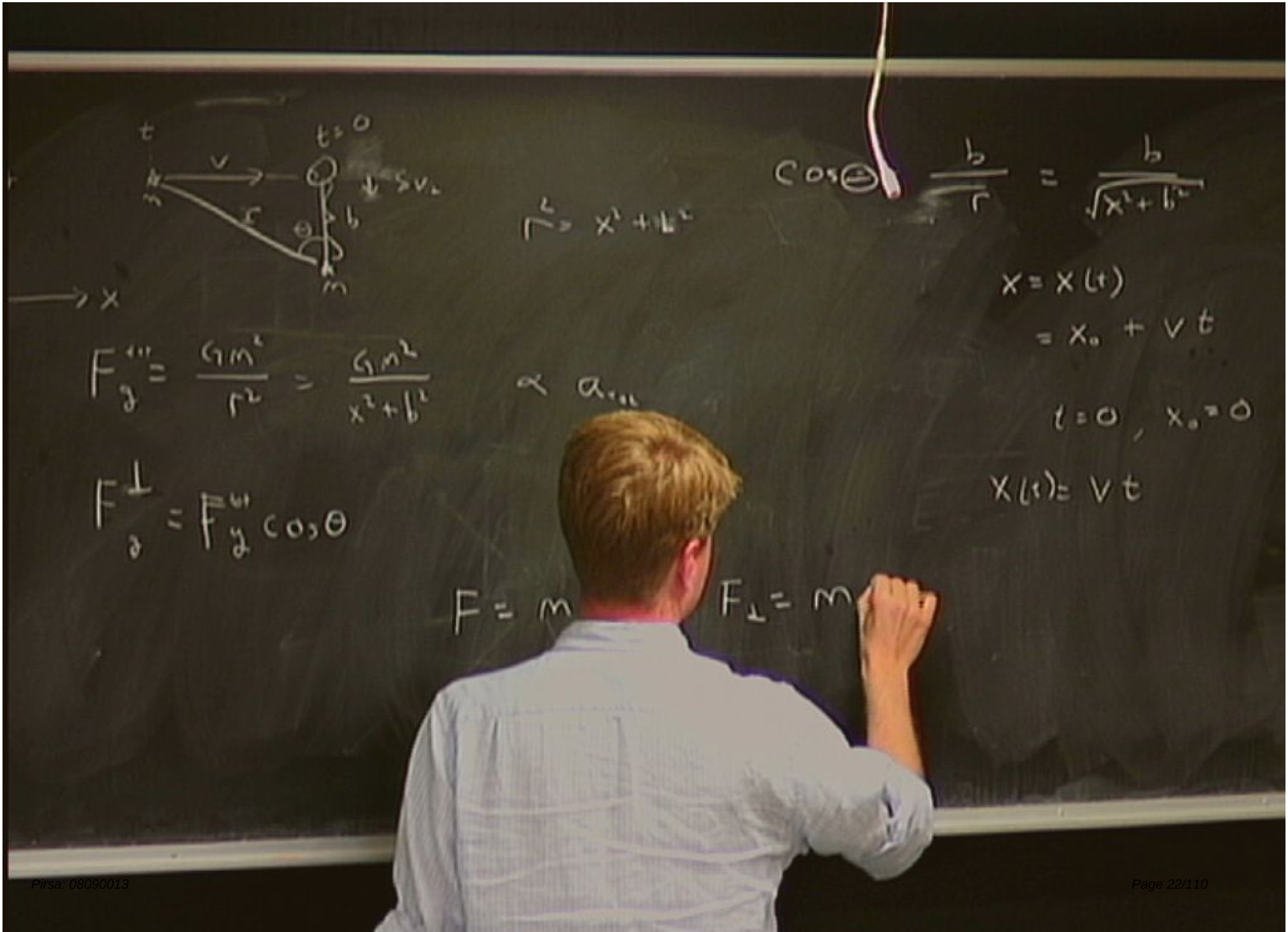
$$F_g^{\perp} = F_g^{tot} \cos\Theta$$


$\cos\theta = \frac{b}{r} = \frac{b}{\sqrt{x^2+b^2}}$
$r^2 = x^2 + b^2$
$x = x(t)$
$\frac{Gm^2}{x^2+b^2}$
$\cos\theta$

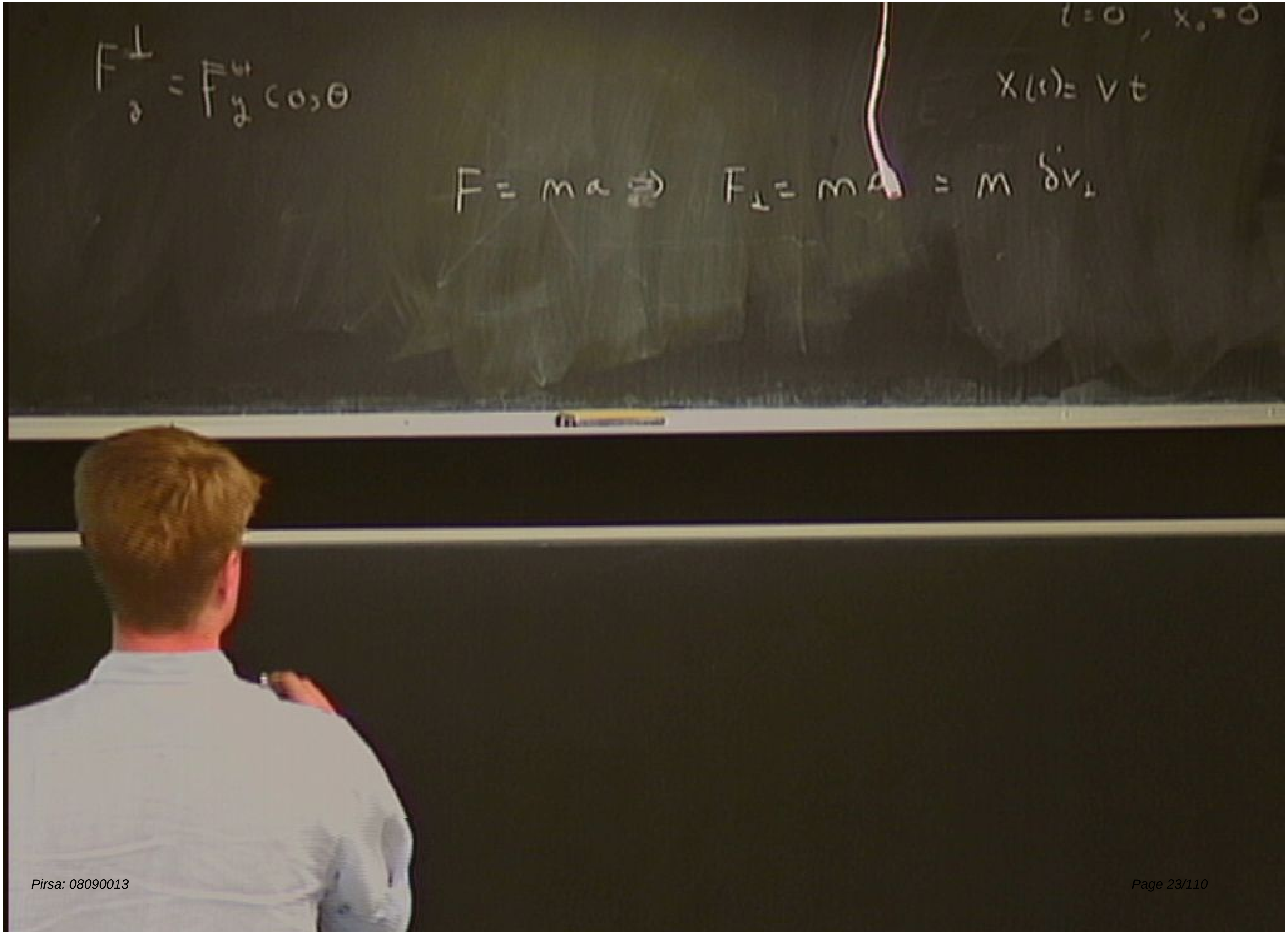

cos θ = b / r = b / √(x² + b²)
r² = x² + b²
= x(t)
= x₀ + v t
F_g^tot = Gm²/r² = Gm²/(x² + b²)
F_g^⊥ = F_g^tot cos θ


$F_g^{\parallel} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2}$
$= x_0 + Vt$
$t=0, x_0=0$
$X(t) = Vt$

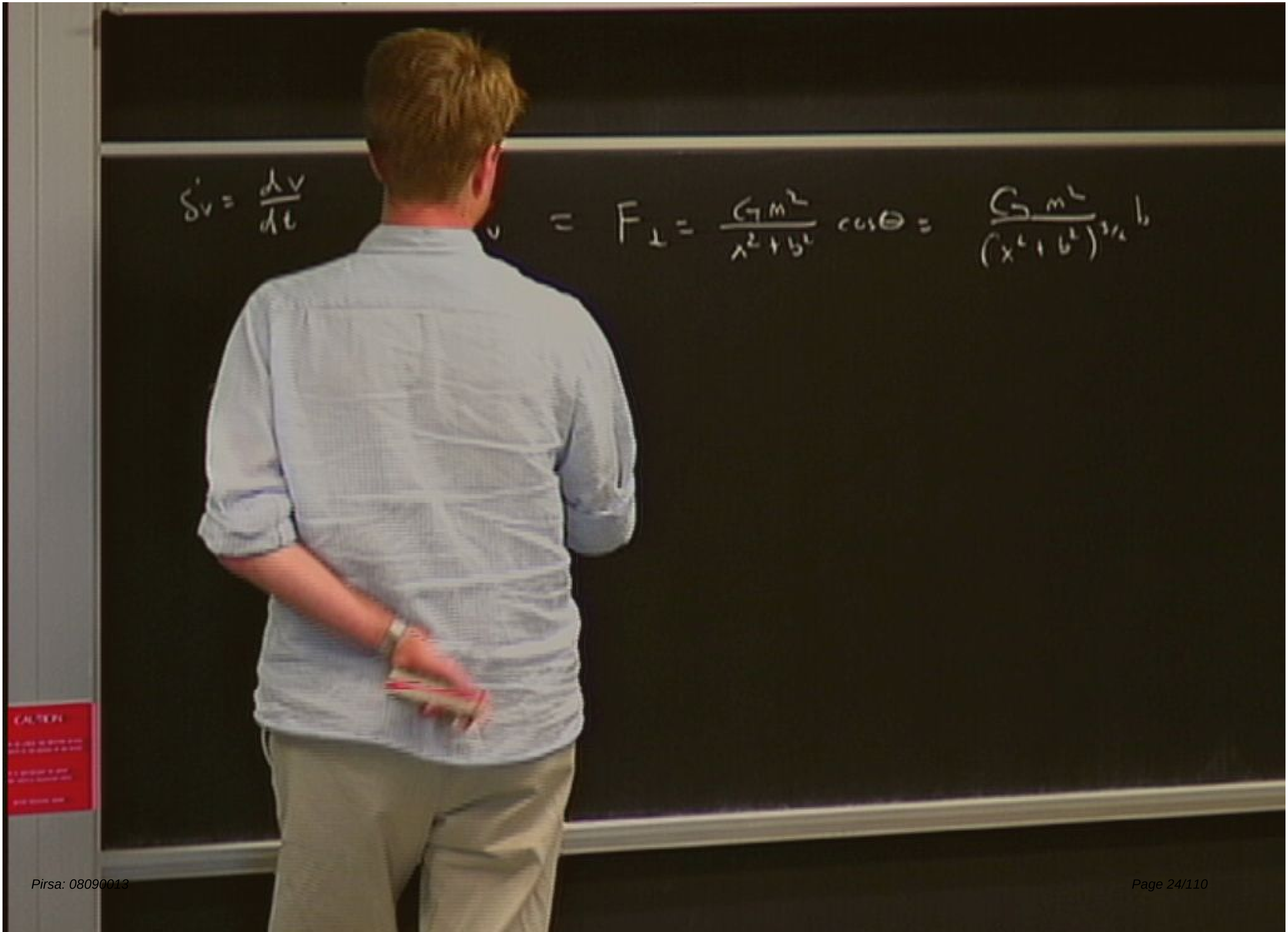

t
t = 0
v
r
θ
b
m
x
r² = x² + b²
cos θ = b / r = b / √(x² + b²)
x = x(t)
= x₀ + v t
t = 0 , x₀ = 0
x(t) = v t
F_g^tot = Gm² / r² = Gm² / (x² + b²)
F_g^⊥ = F_g^tot cos θ

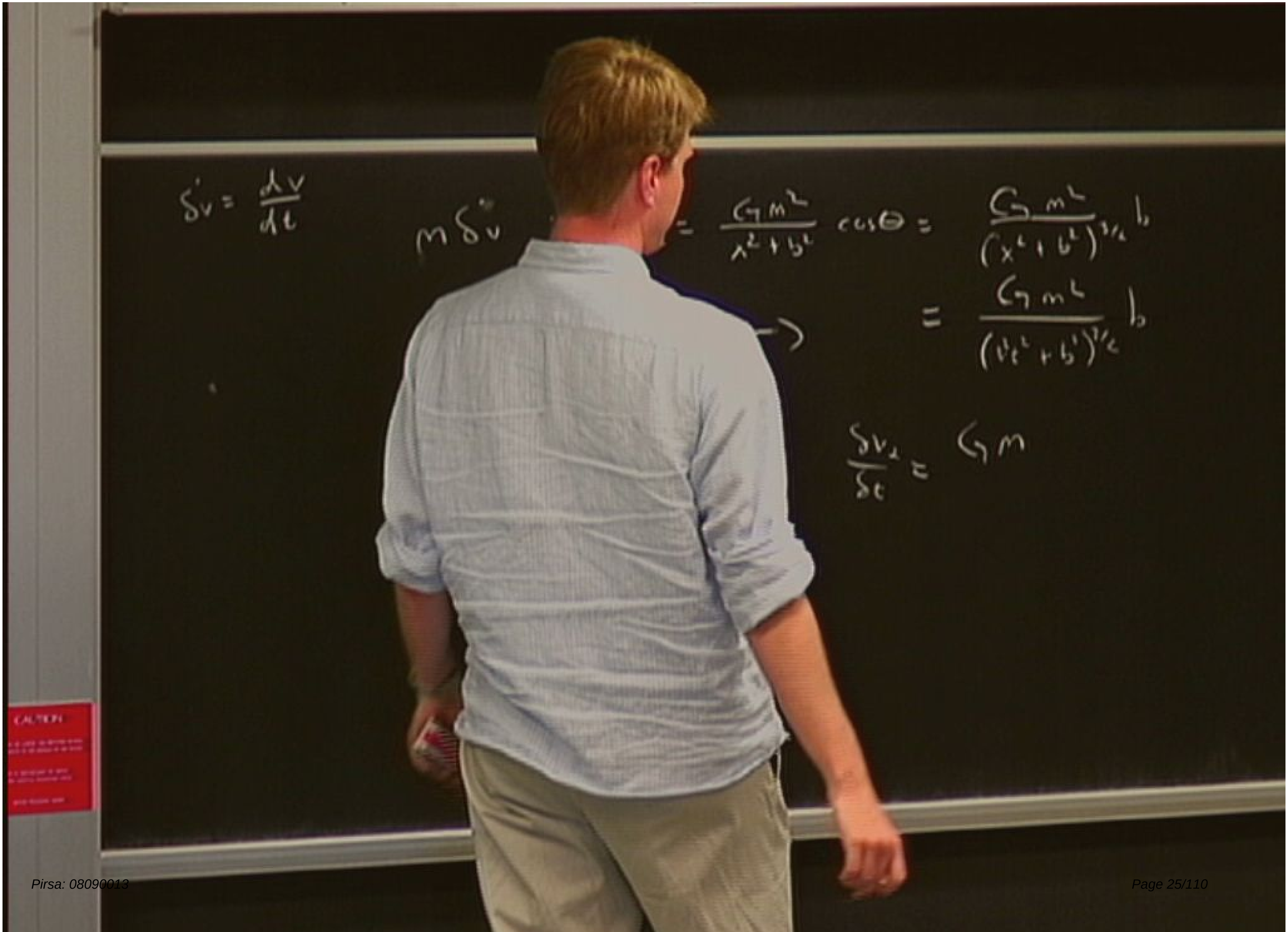


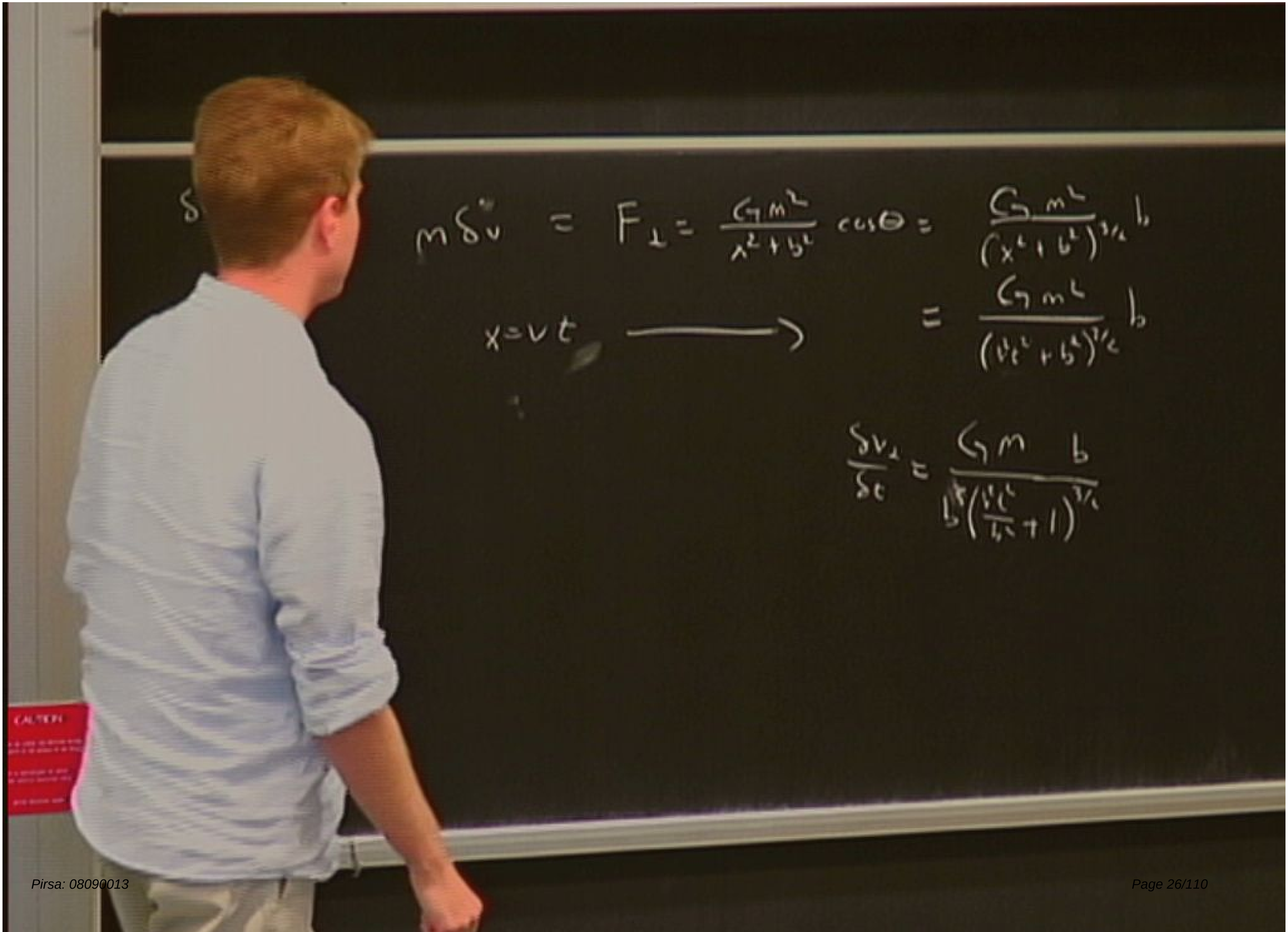
t
$t=0$
v
$\delta v_\perp$
r
θ
b
m
x
$\cos\theta = \frac{b}{r} = \frac{b}{\sqrt{x^2+b^2}}$
$r^2 = x^2 + b^2$
$x = x(t)$
$= x_0 + vt$
$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \propto a_{tot}$
$t=0,\ x_0=0$
$x(t) = vt$
$F_g^\perp = F_g^{tot}\cos\theta$
$F = m$
$F_\perp = m$

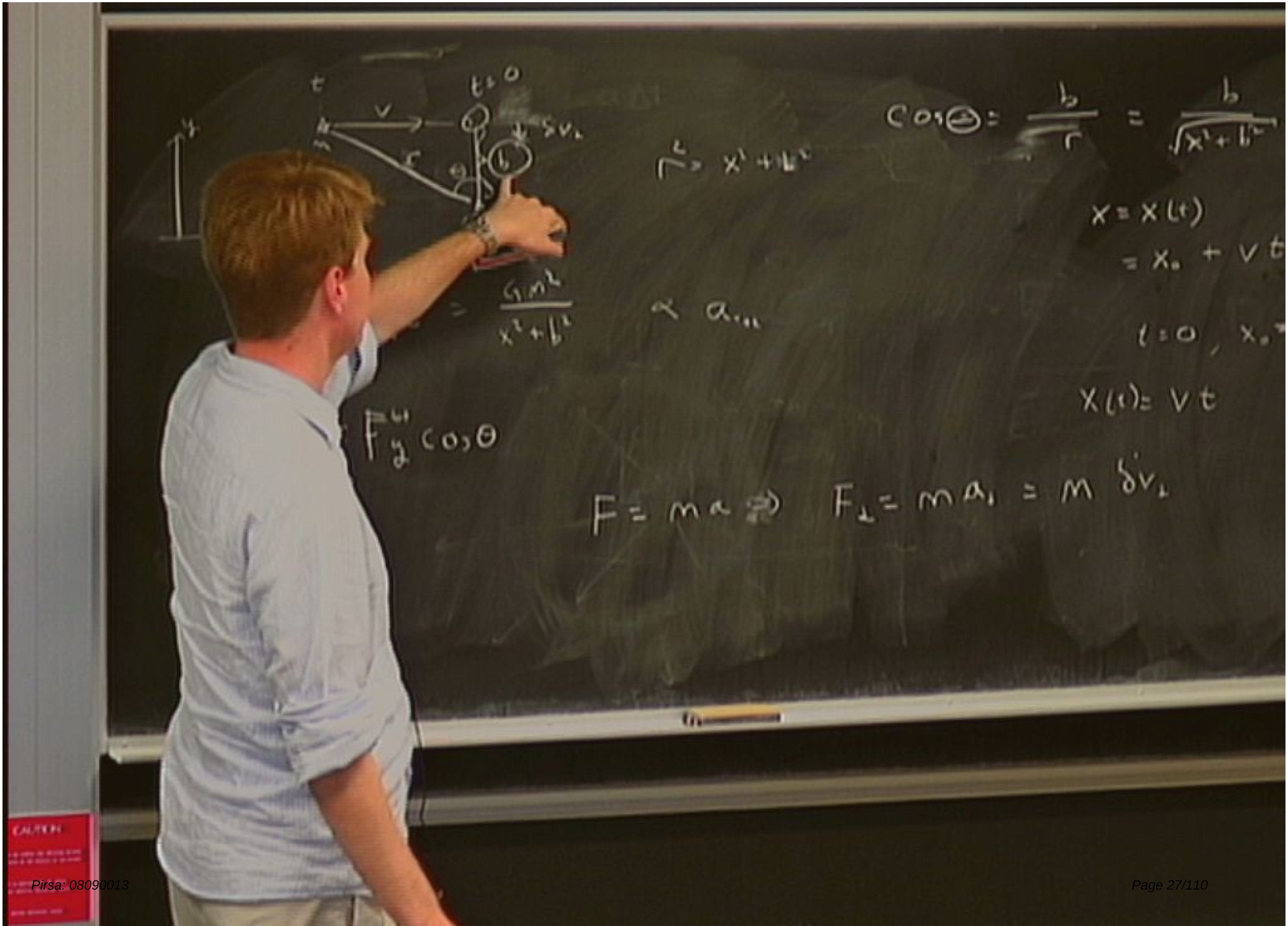


F⊥g = Fwtg cos θ
t = 0 , x₀ = 0
x(t) = v t
F = ma ⇒ F⊥ = ma = m δv⊥

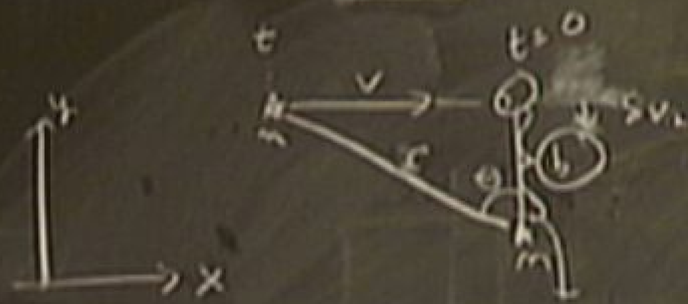

$\delta v = \frac{dv}{dt}$
$= F_\perp = \frac{G m^2}{x^2 + b^2} \cos\Theta = \frac{G m^2}{(x^2 + b^2)^{3/2}} b$


$\dot{\delta v} = \frac{dv}{dt}$
$m\dot{\delta v}$
$= \frac{G m^2}{x^2+b^2} \cos\Theta = \frac{G m^2}{(x^2+b^2)^{3/2}} b$
$= \frac{G m^2}{(v^2t^2+b^2)^{3/2}} b$
$\frac{\delta v_\perp}{\delta t} = G m$
CAUTION


$m\,\delta\dot{v} = F_\perp = \frac{G m^2}{x^2+b^2}\cos\theta = \frac{G m^2}{(x^2+b^2)^{3/2}}\, b$
$x = vt \longrightarrow \quad = \frac{G m^2}{(v^2t^2+b^2)^{3/2}}\, b$
$\frac{\delta v_\perp}{\delta t} = \frac{G m \quad b}{b^3\left(\frac{v^2t^2}{b^2}+1\right)^{3/2}}$



t
t = 0
v
δv⊥
r
θ
b
cos θ = b / r = b / √(x² + b²)
r² = x² + b²
x = x(t)
= x₀ + v t
t = 0 , x₀ =
X(t) = V t
= Gm² / (x² + b²)
∝ a
F_y cos θ
F = ma ⇒ F⊥ = m a⊥ = m δv⊥



$$\cos\theta = \frac{b}{r} = \frac{b}{\sqrt{x^2+b^2}}$$

$$r^2 = x^2 + b^2$$

$$x = x(t)$$
$$= x_0 + vt$$

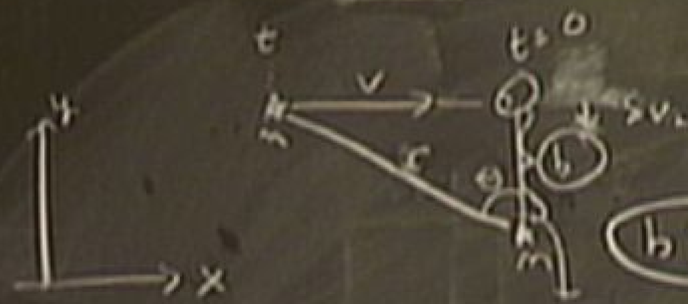
$$t = 0,\ x_0 =$$

$$x(t) = vt$$

$$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \quad \propto a_{tot}$$

$$F_g^{\perp} = F_g^{tot}\cos\theta$$

$$F = ma \Rightarrow F_\perp = m a_\perp = m\,\dot{\delta v}_\perp$$



$\boxed{b > b_{min}}$

$r^2 = x^2 + b^2$

$\cos\theta = \frac{b}{r} = \frac{b}{\sqrt{x^2+b^2}}$

$x = x(t)$
$= x_0 + vt$

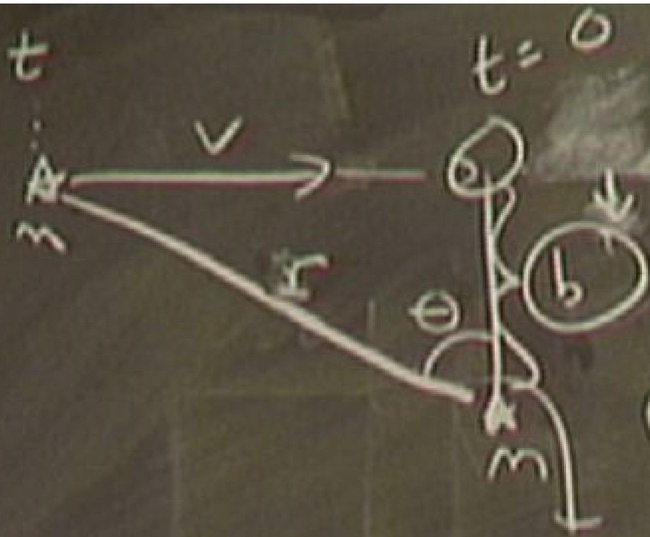
$t = 0,\ x_0 =$

$x(t) = vt$

$$F_g^{tot} = \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \quad \propto a_{tot}$$

$$F_g^{\perp} = F_g^{tot} \cos\theta$$

$$F = ma \Rightarrow F_\perp = m a_\perp = m\,\delta v_\perp$$



$\cos\theta =$

$r^2 = x^2 + b^2$

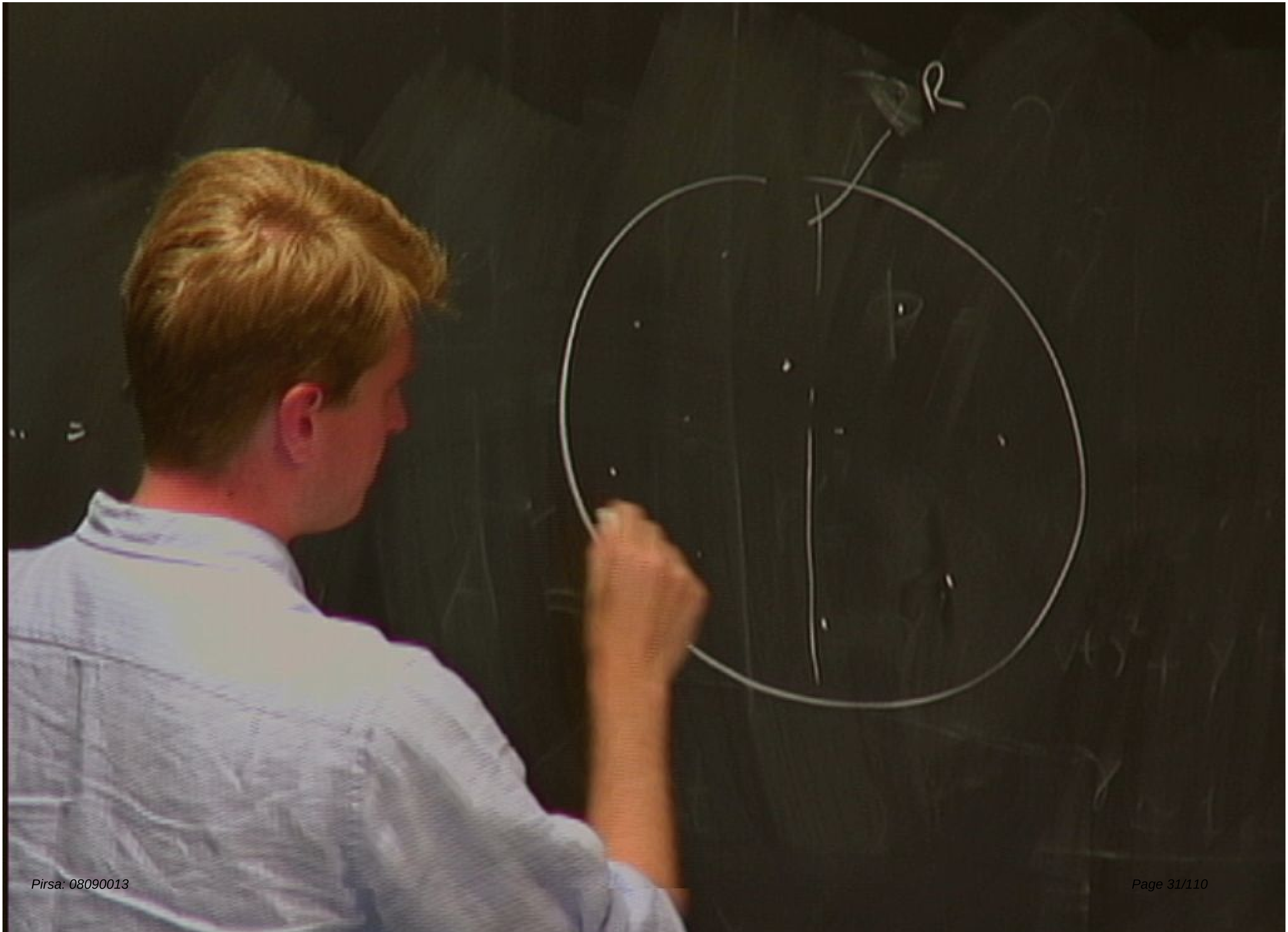
$b > b_{min}$

$$= \frac{Gm^2}{r^2} = \frac{Gm^2}{x^2+b^2} \propto a_{tot}$$

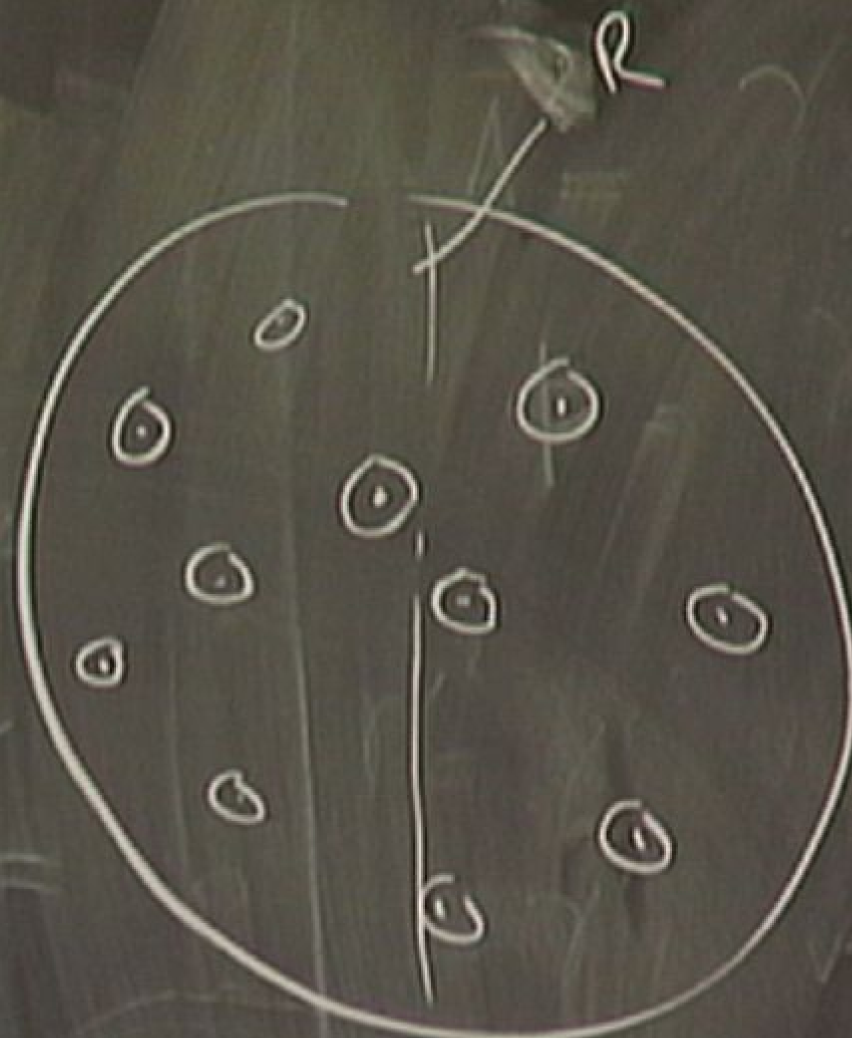
$\perp$

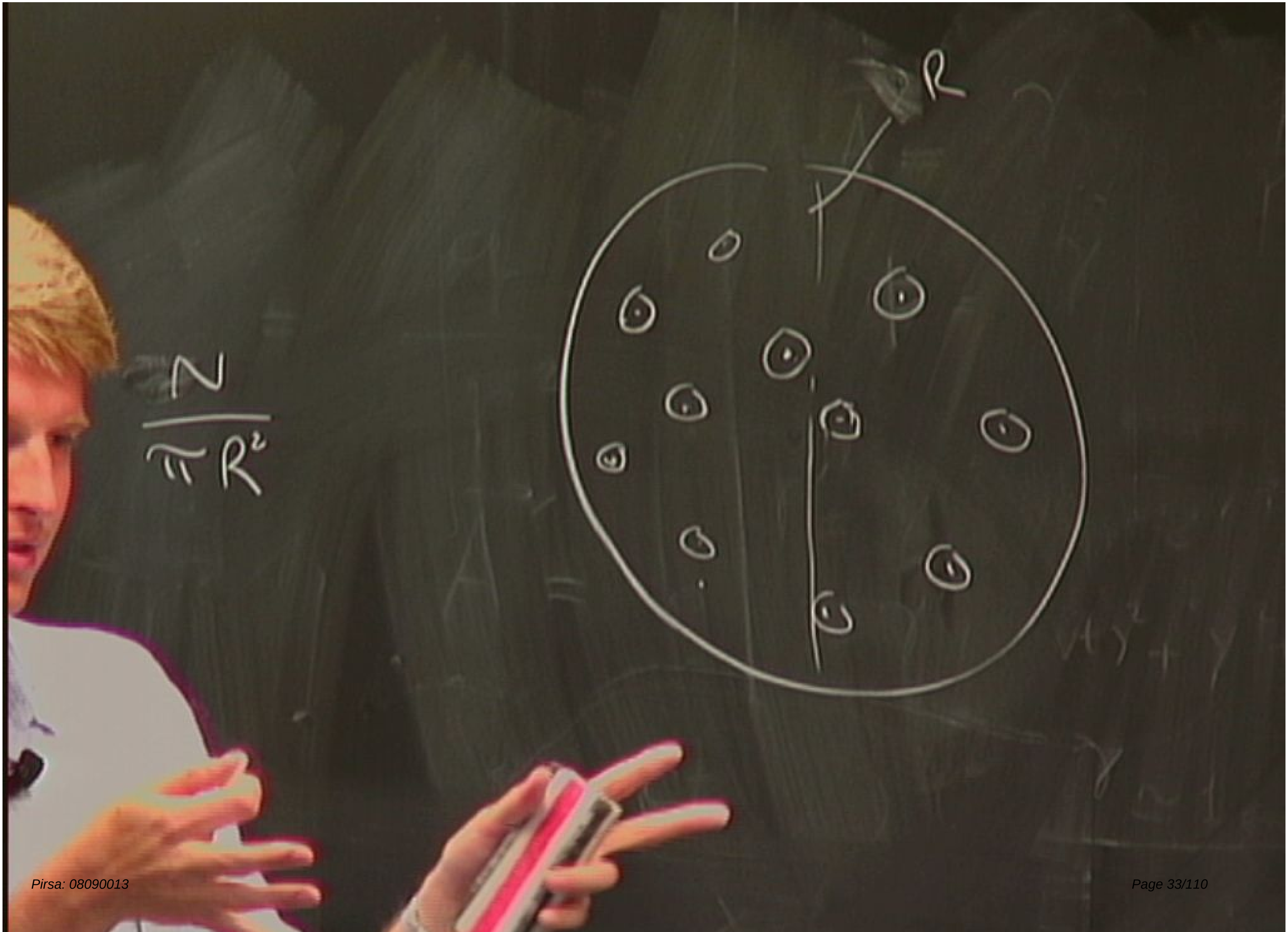
$$= F_y^{tot} \cos\theta$$

$$F = ma \Rightarrow F_\perp = m a_\perp$$


R

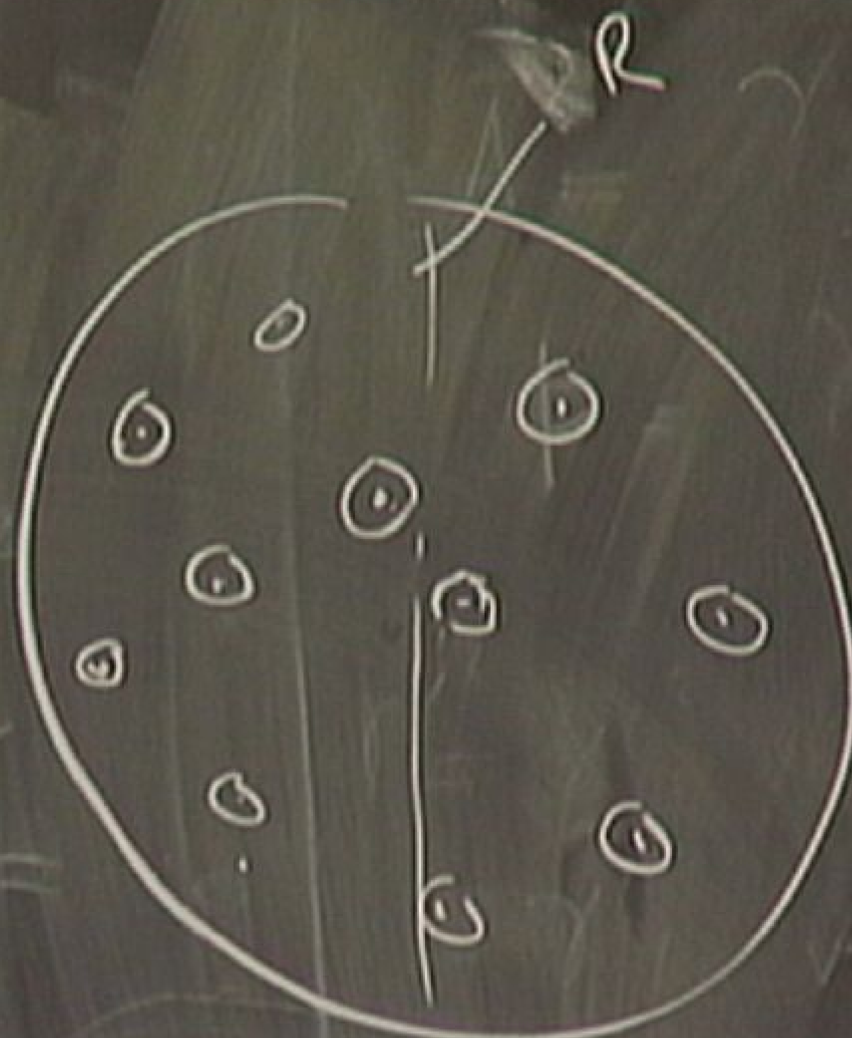
$$\ldots = \frac{N}{\pi R^2}$$

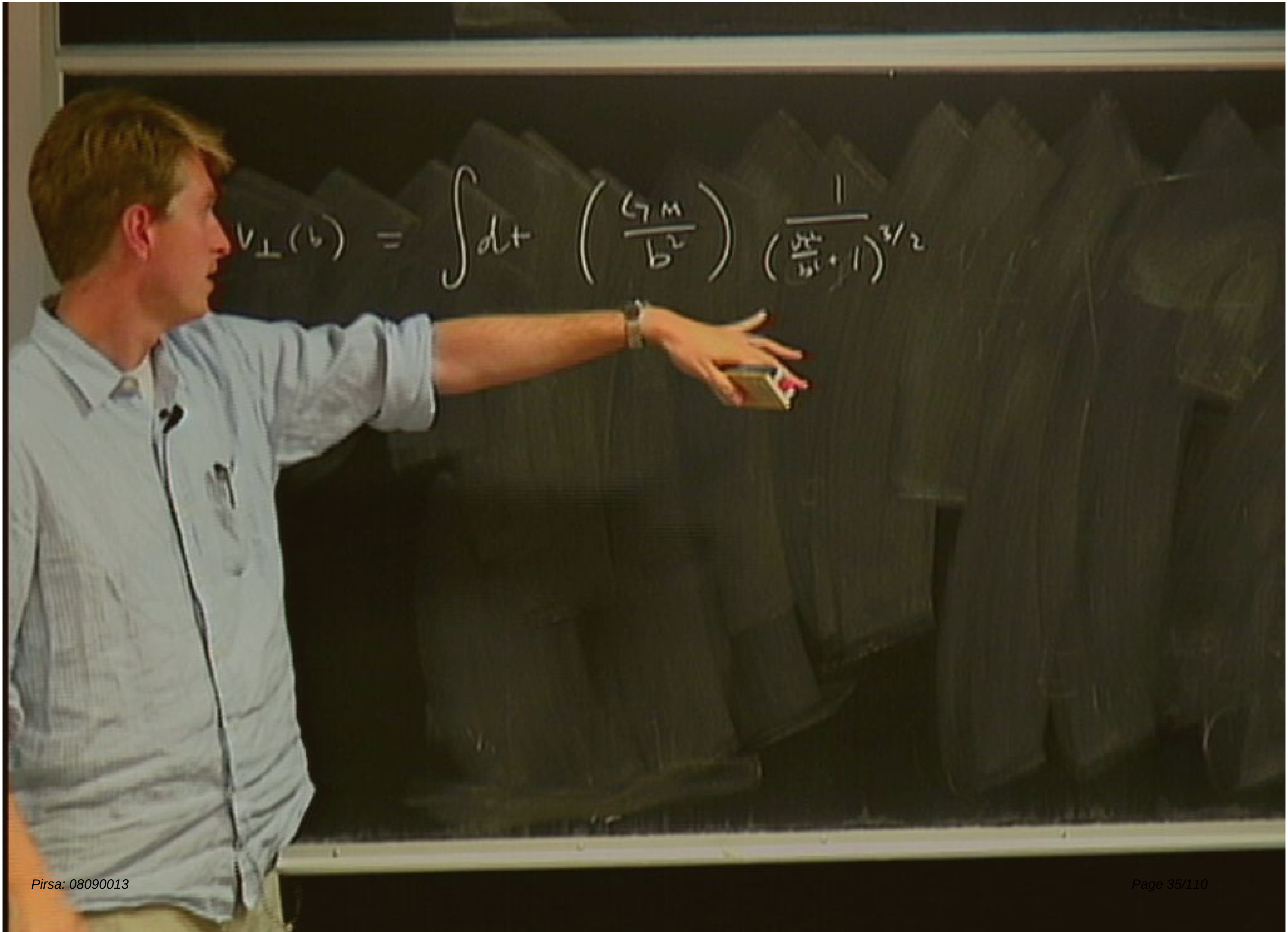




R
N
πR²

$$= \frac{N}{\pi R^2}$$




$v_\perp(b) = \int dt \left(\frac{Gm}{b^2}\right) \frac{1}{\left(\frac{v^2t^2}{b^2} + 1\right)^{3/2}}$

$$\delta v_\perp(b) = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right) \frac{1}{\left(\frac{v^2t^2}{b^2}+1\right)^{3/2}}$$

$$= \frac{Gm}{bv} \underbrace{\int_{-\infty}^{\infty} ds \, \frac{1}{(1+s^2)^{3/2}}}_{2}$$

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b}\,dt$$

$$\delta v_\perp(b) = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right) \frac{1}{\left(\frac{v^2t^2}{b^2}+1\right)^{3/2}}$$

$$= \frac{Gm}{bv} \underbrace{\int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}}}_{2}$$

$$= \frac{2Gm}{bv}$$

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b}\,dt$$

$$\delta v_\perp(b) = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right) \frac{1}{\left(\frac{v^2t^2}{b^2}+1\right)^{3/2}}$$

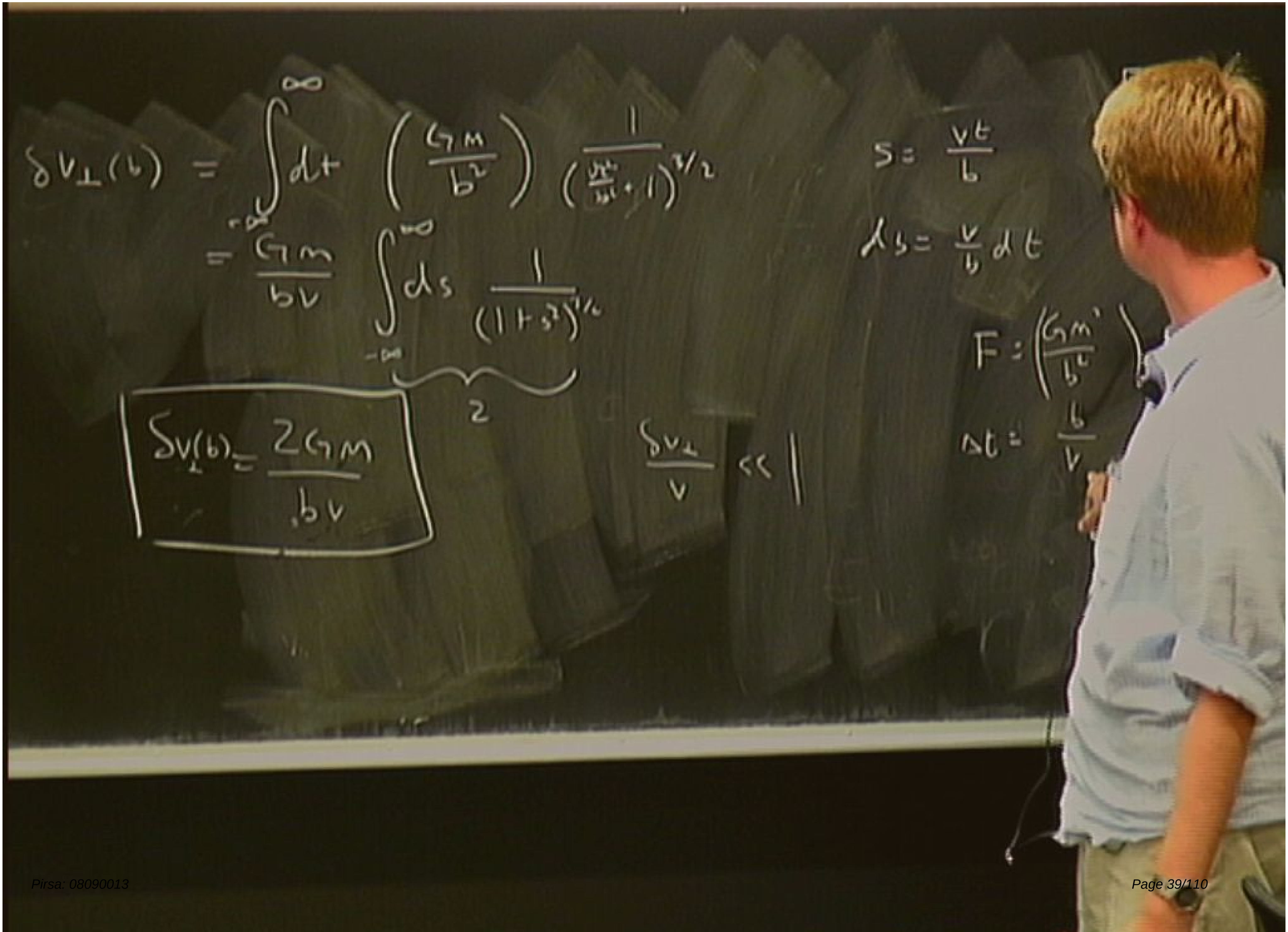
$$= \frac{Gm}{bv} \underbrace{\int_{-\infty}^{\infty} ds \, \frac{1}{(1+s^2)^{3/2}}}_{2}$$

$$s = \frac{vt}{b}$$

$$ds = \frac{v}{b}\, dt$$

$$\boxed{\delta v_\perp(b) = \frac{2Gm}{bv}}$$

$$\frac{\delta v_\perp}{v} \ll 1$$



$\delta v_\perp(b) = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right) \frac{1}{\left(\frac{v^2t^2}{b^2}+1\right)^{3/2}}$
$= \frac{Gm}{bv} \underbrace{\int_{-\infty}^{\infty} ds \frac{1}{(1+s^2)^{3/2}}}_{2}$
$\boxed{\delta v_\perp(b) = \frac{2Gm}{bv}}$
$\frac{\delta v_\perp}{v} \ll 1$
$s = \frac{vt}{b}$
$ds = \frac{v}{b} dt$
$F = \left(\frac{Gm}{b^2}\right)$
$\Delta t = \frac{b}{v}$

$$\int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right)\frac{1}{\left(\frac{v^2t^2}{b^2}+1\right)^{3/2}}$$

$$= \frac{Gm}{bv}\underbrace{\int_{-\infty}^{\infty} ds\, \frac{1}{(1+s^2)^{3/2}}}_{2}$$

$$\boxed{= \frac{2Gm}{bv}}$$

$$\frac{\delta v_\perp}{v} \ll 1$$

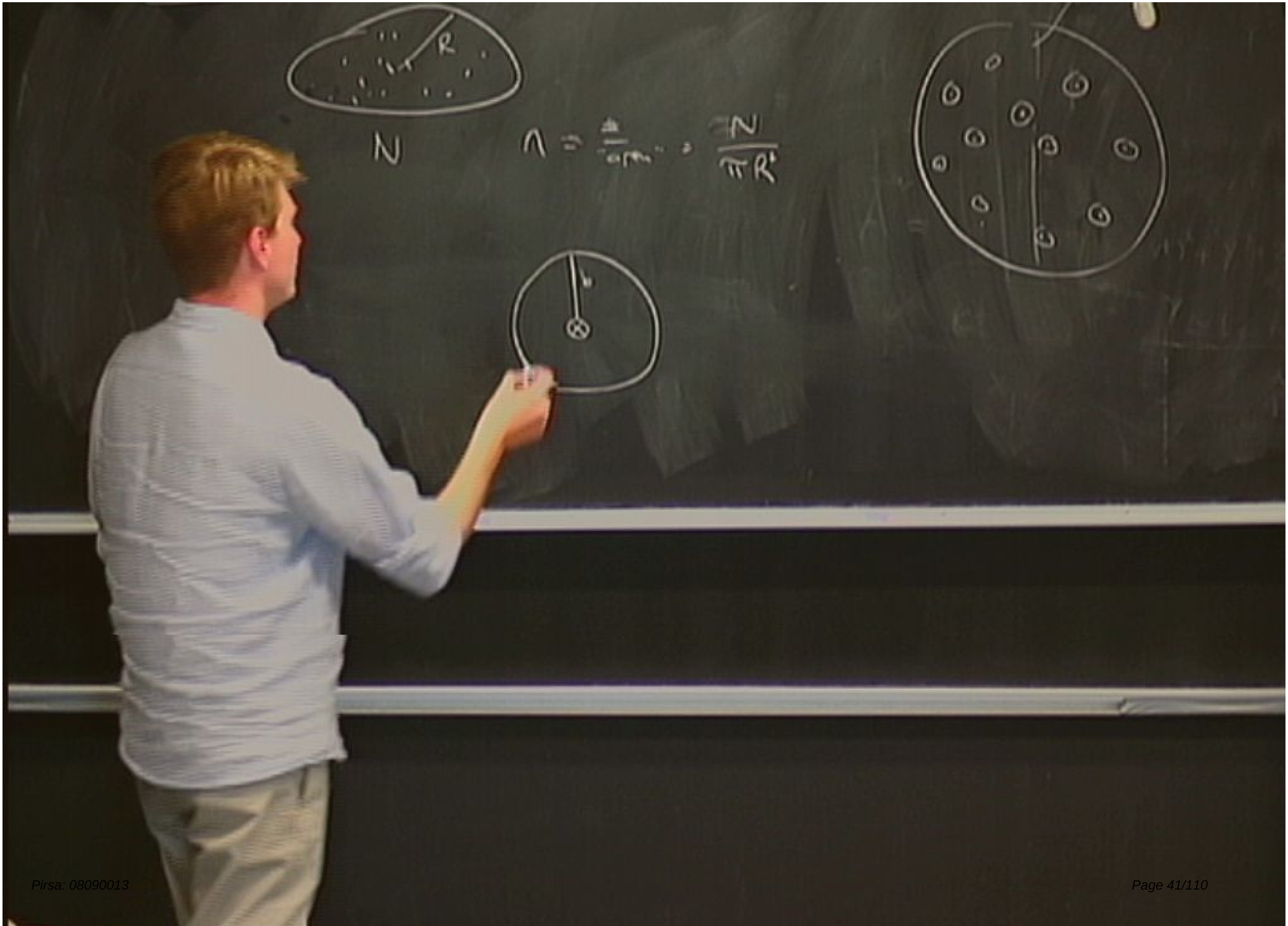
$$s = \frac{vt}{b}$$

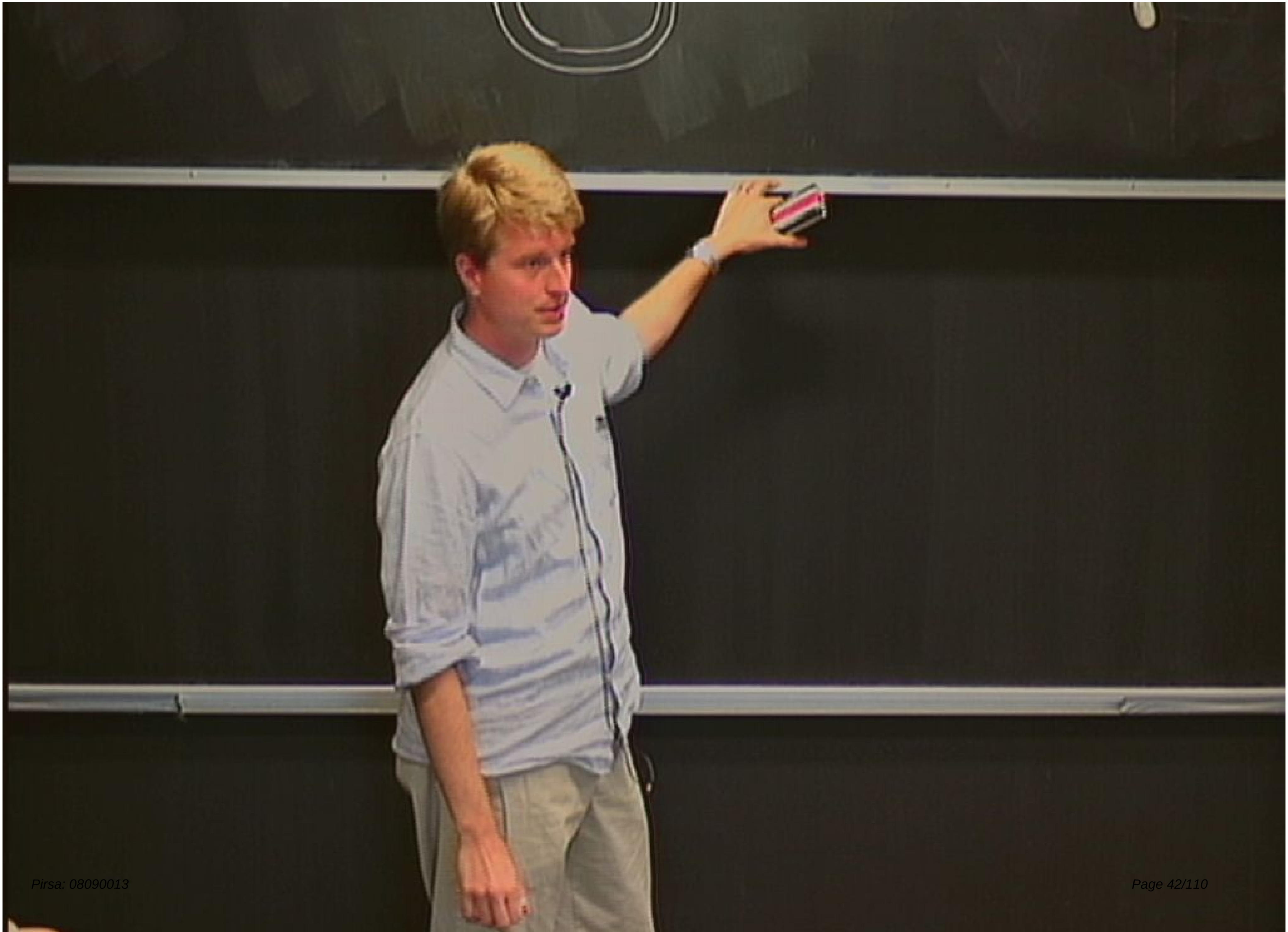
$$ds = \frac{v}{b}\,dt$$

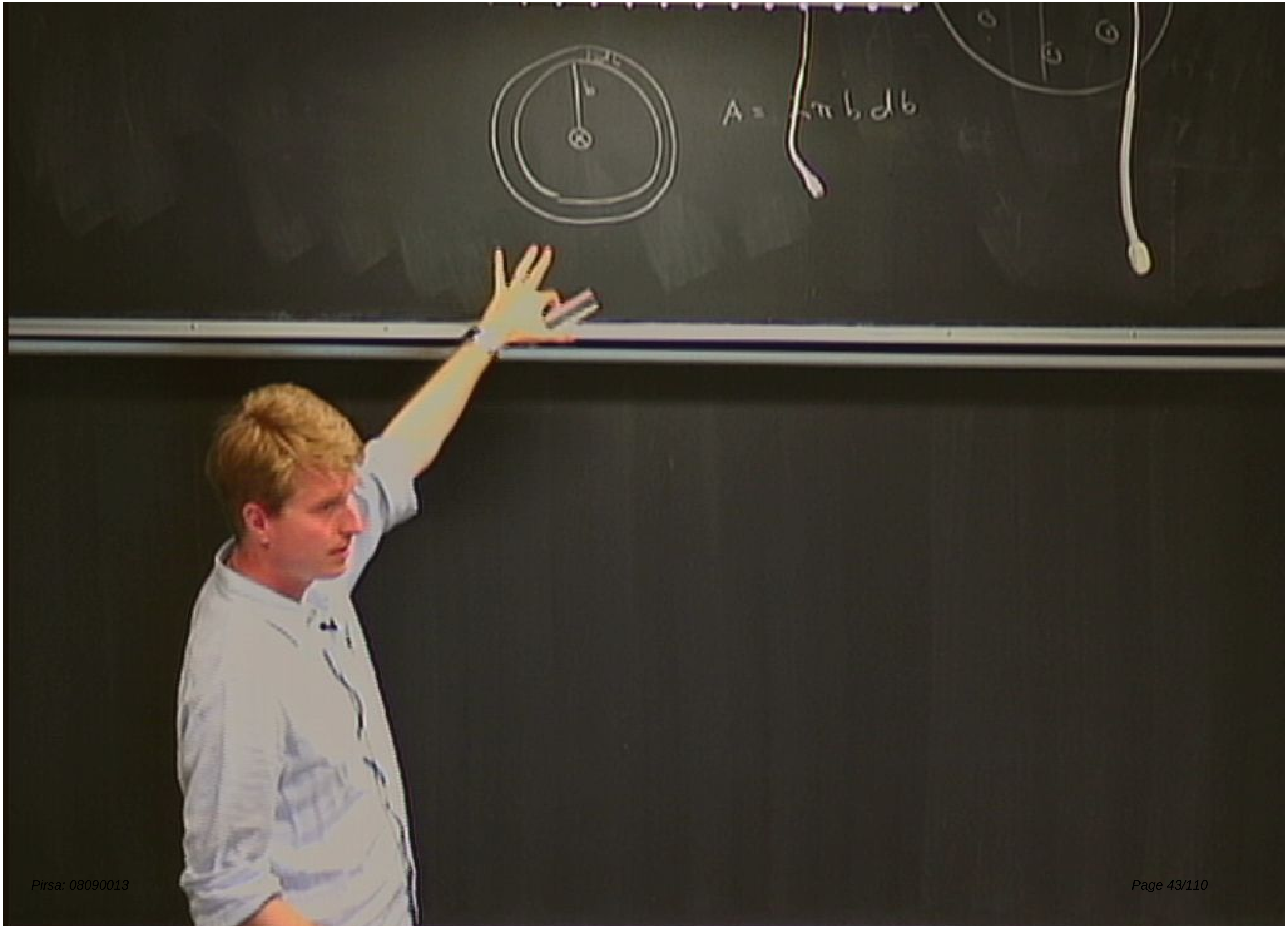
$$F\Delta t = \frac{Gm^2}{bv}$$

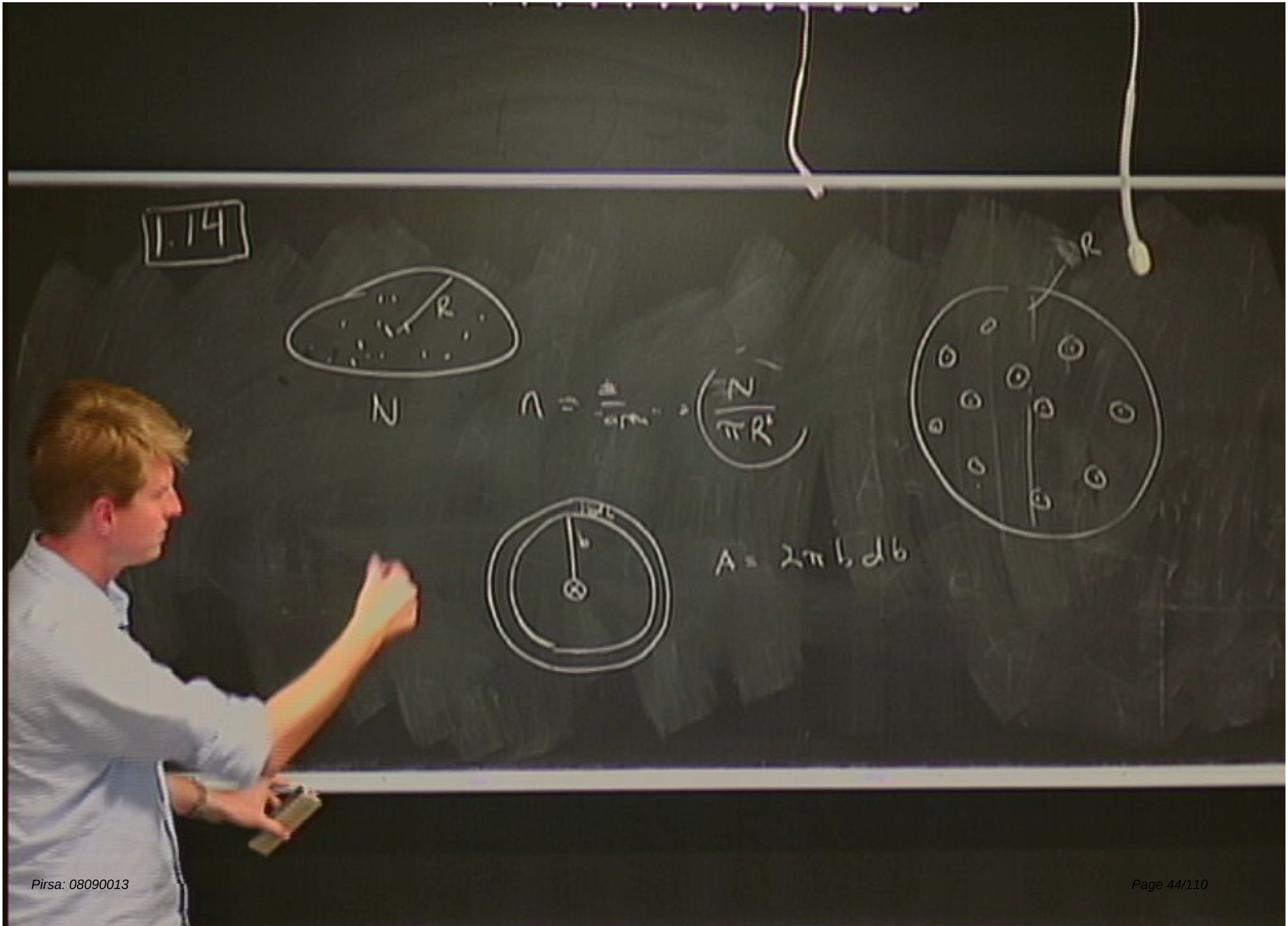
$$F = \left(\frac{Gm^2}{b^2}\right)$$

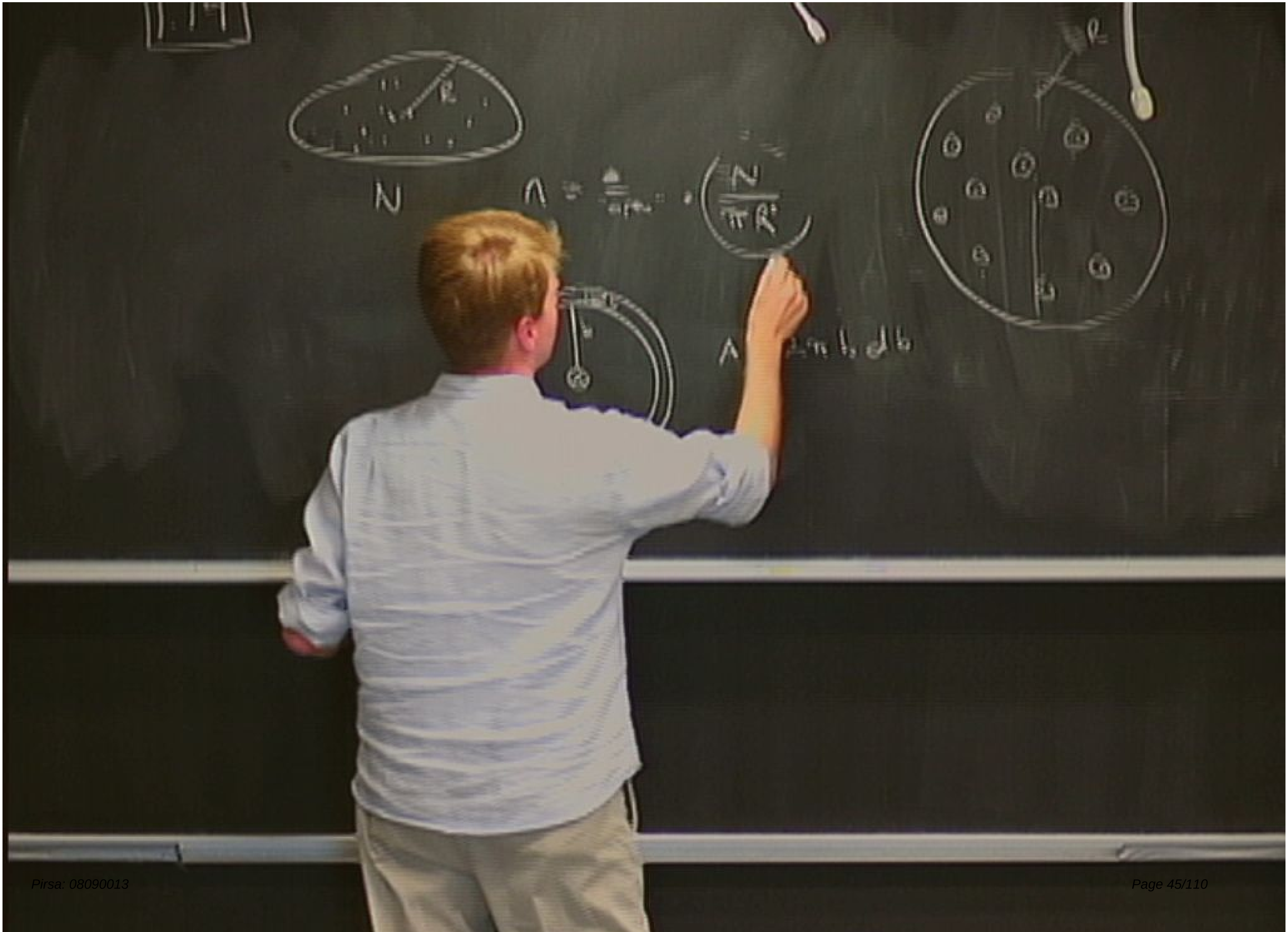
$$\Delta t = \frac{b}{v}$$

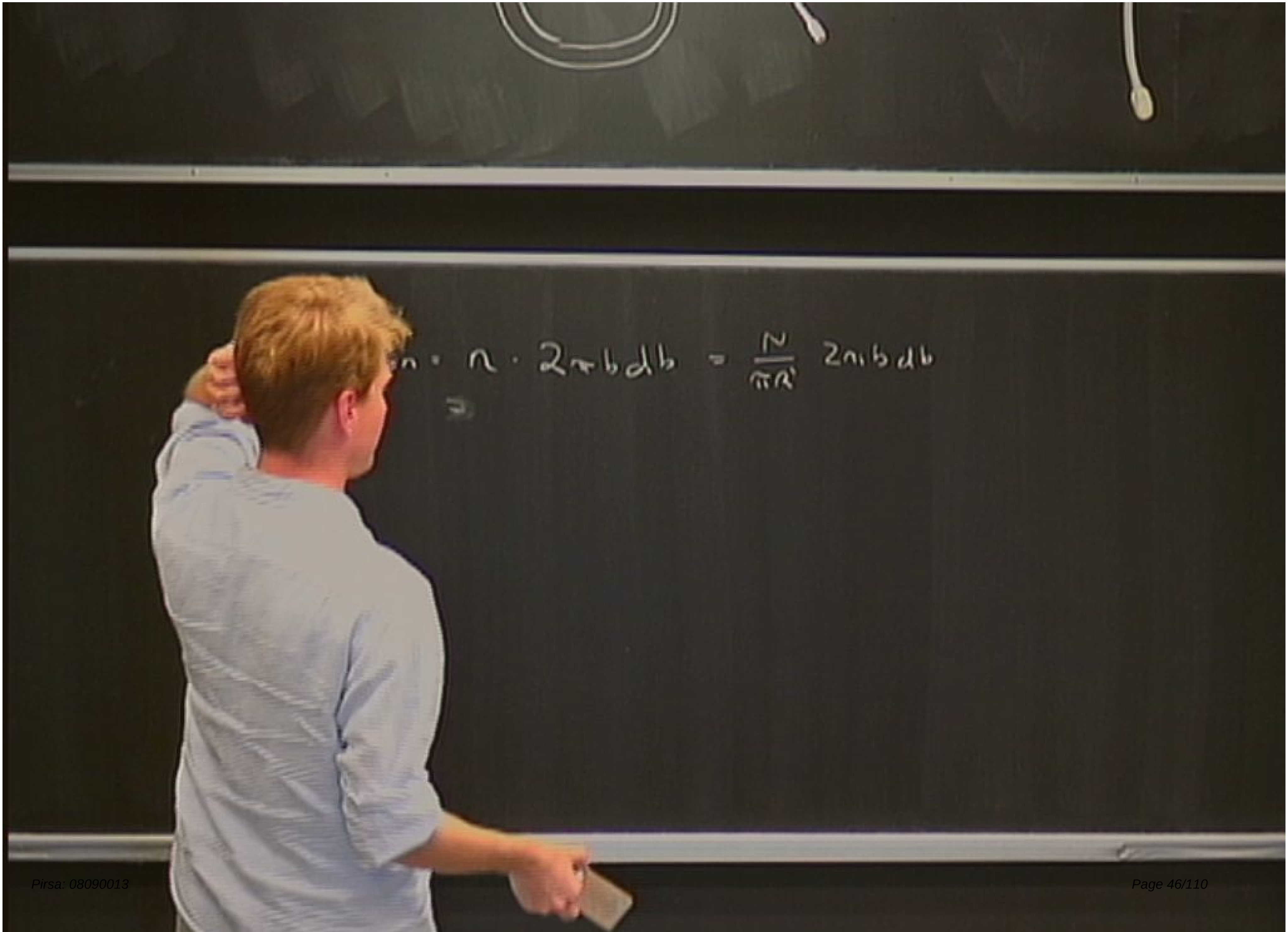

R
N

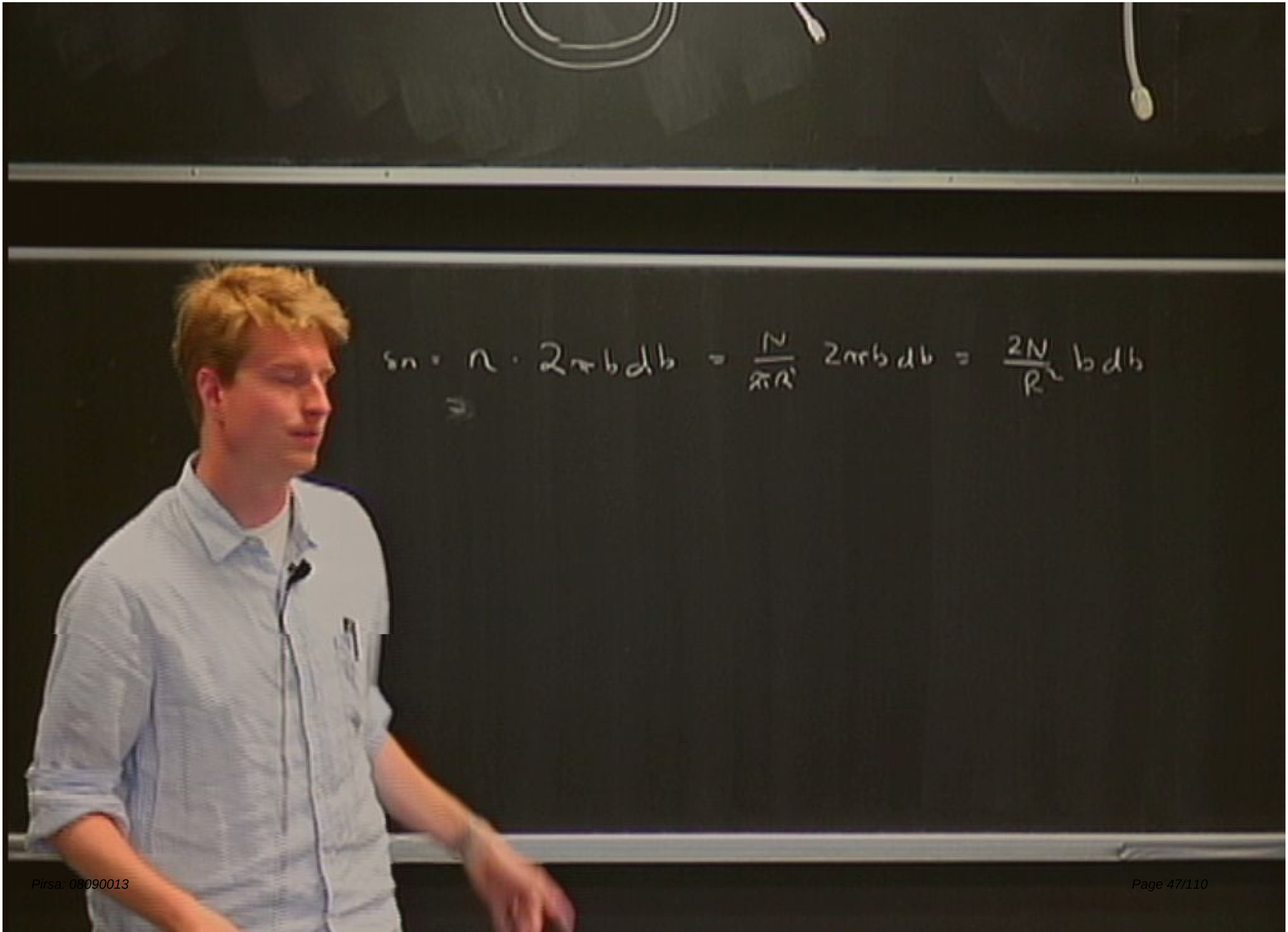


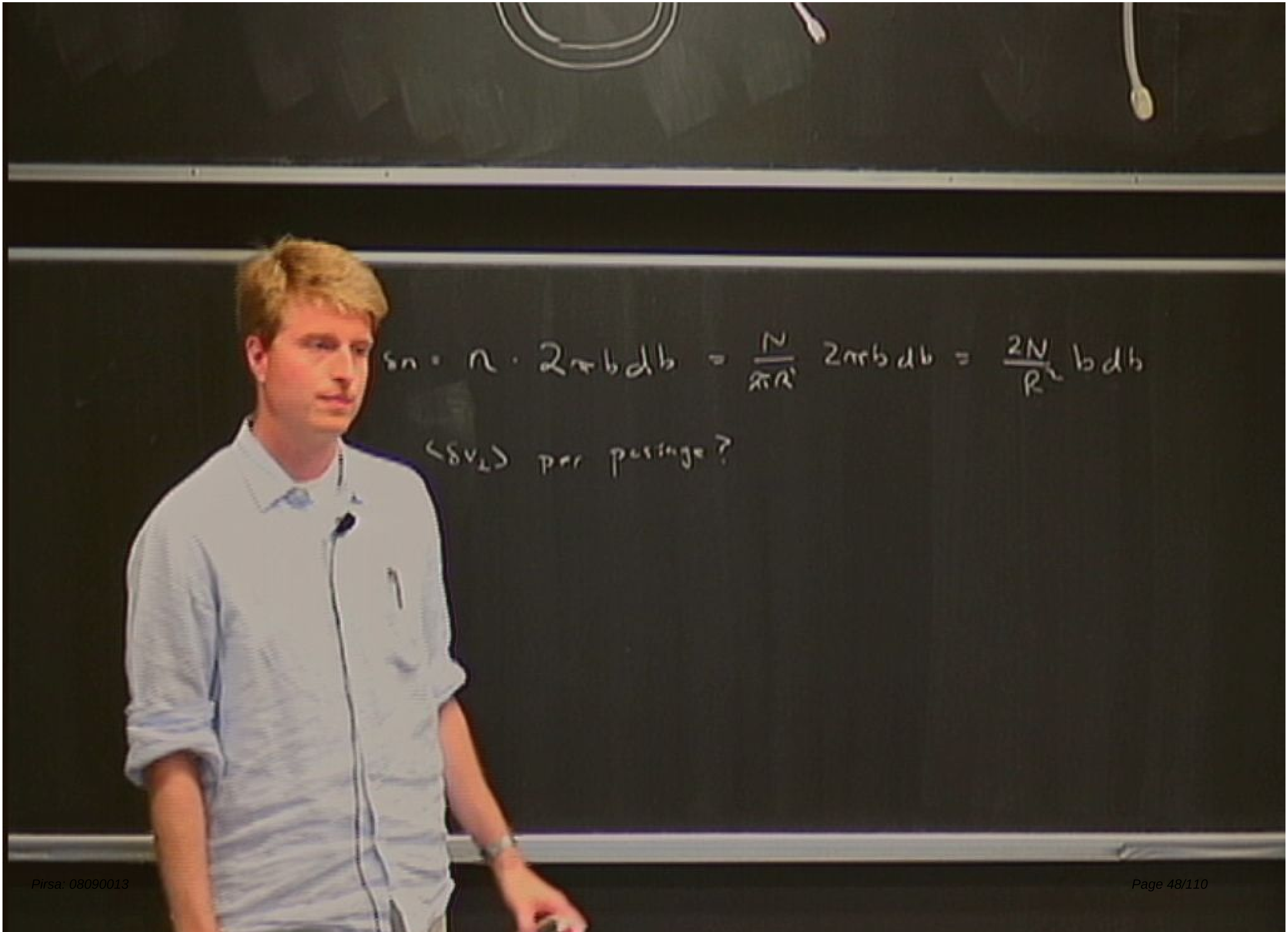



1.14
R
N
$n = \frac{N}{\pi R^2}$
$A = 2\pi b\, db$
R

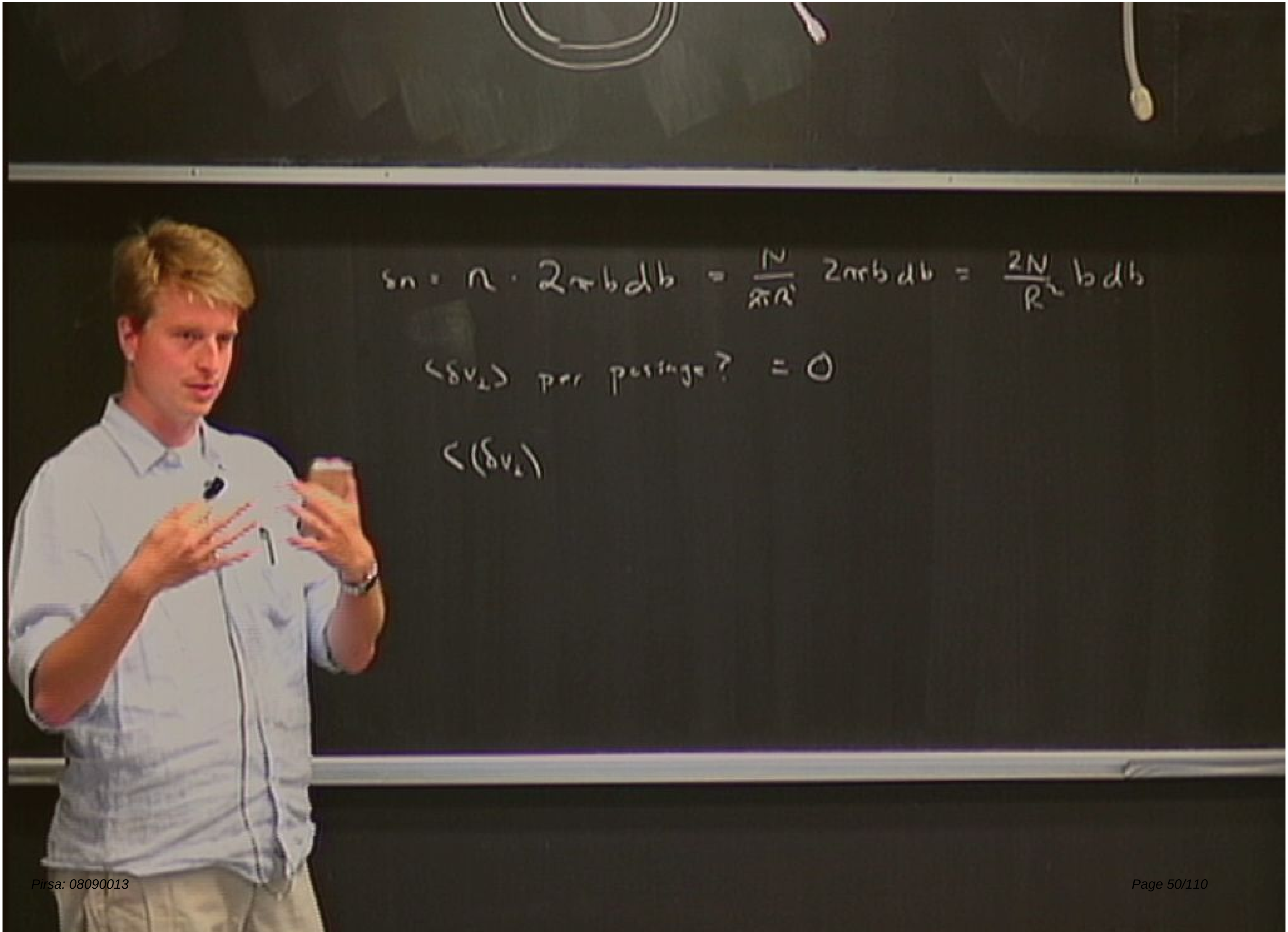


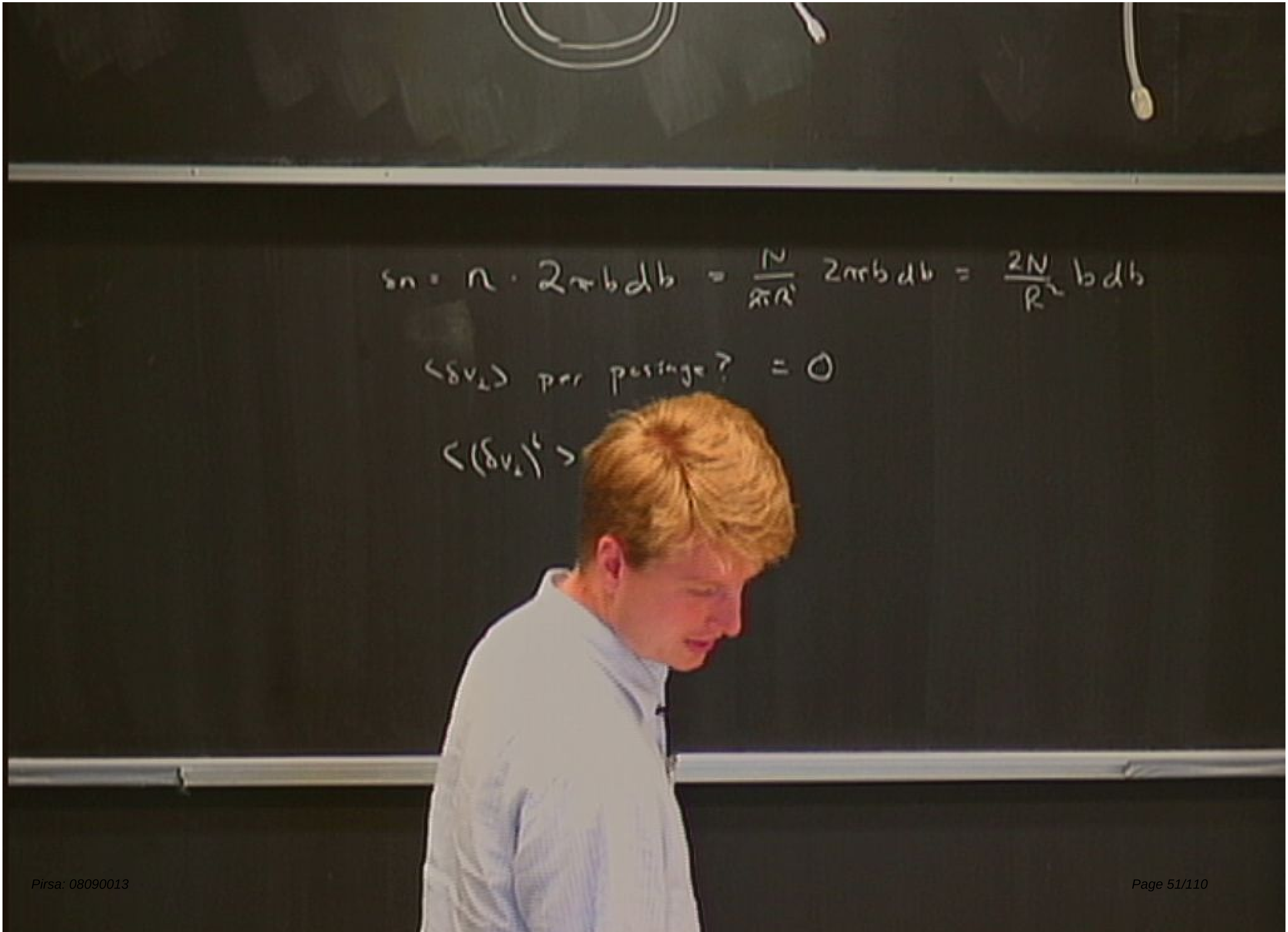

n · n · 2πb db = N/πR² 2πb db

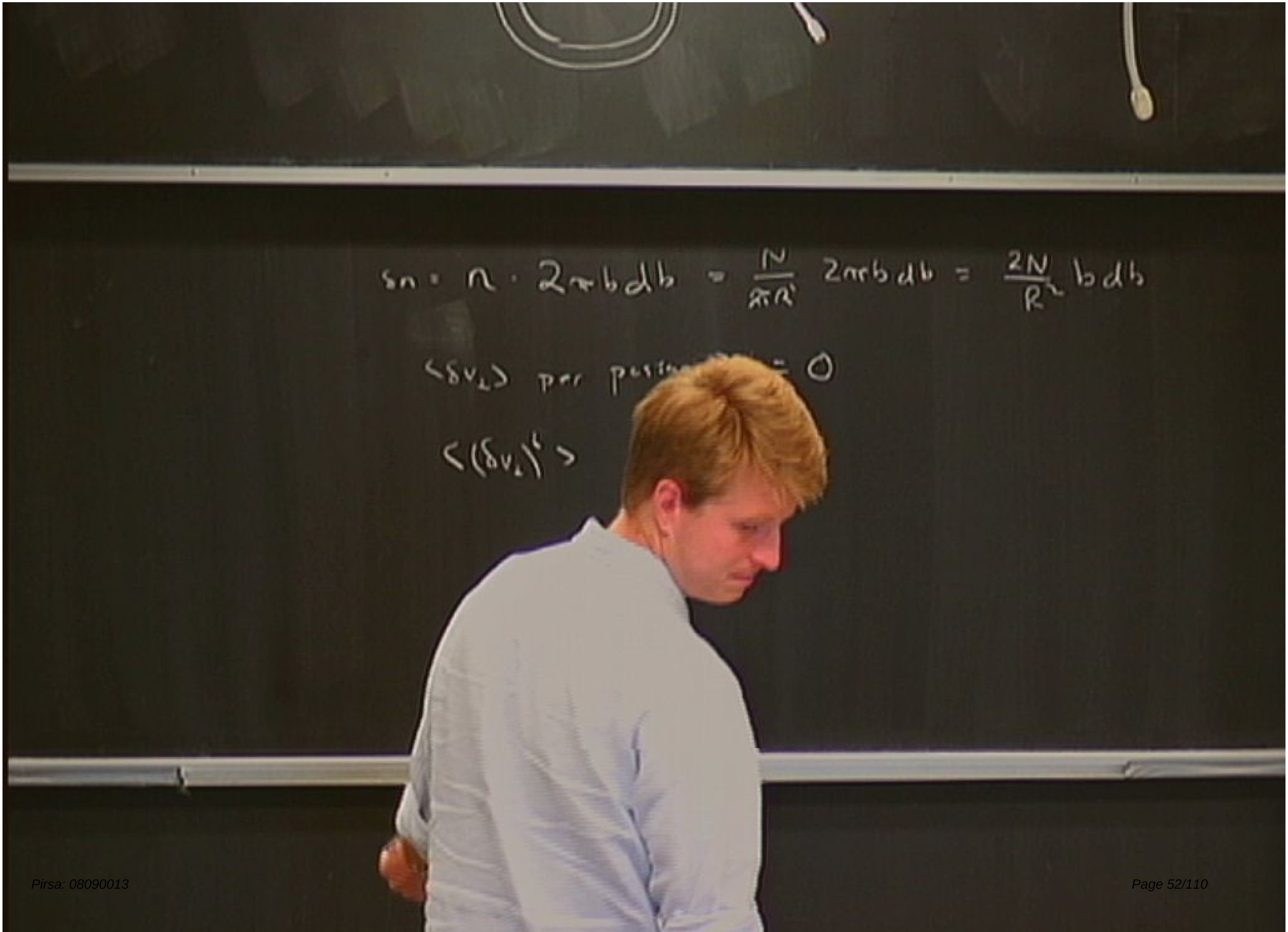

$\delta n = n \cdot 2\pi b\, db = \frac{N}{\pi R^2} 2\pi b\, db = \frac{2N}{R^2} b\, db$
$\Rightarrow$


$\delta n = n \cdot 2\pi b\,db = \frac{N}{\pi R^2} 2\pi b\,db = \frac{2N}{R^2} b\,db$
$\langle \delta v_\perp \rangle$ per passage?

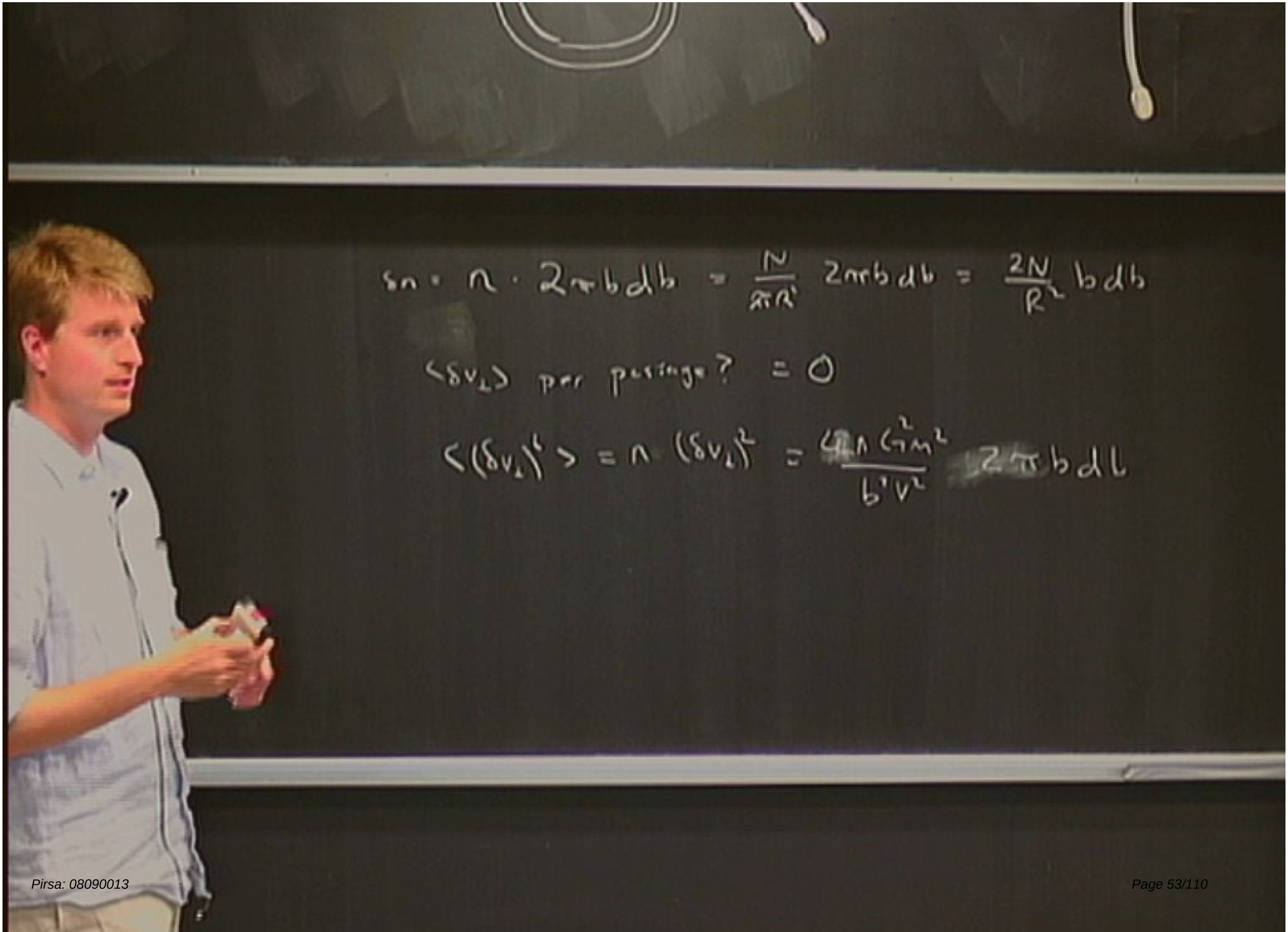

1.14
R
N
R
R

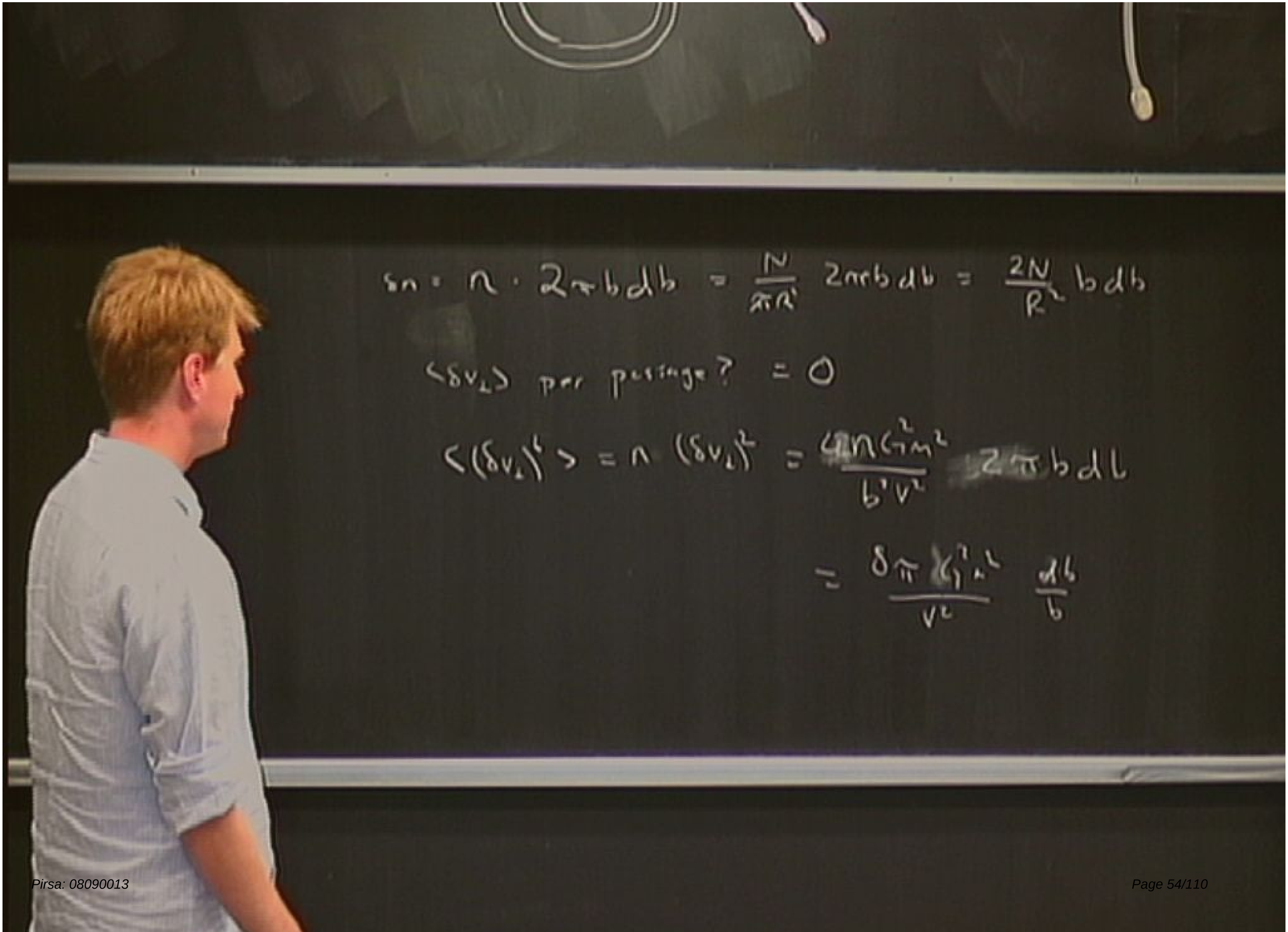

$\delta n = n \cdot 2\pi b\, db = \frac{N}{\pi R^2} 2\pi b\, db = \frac{2N}{R^2} b\, db$
$\langle \delta v_\perp \rangle$ per passage? $= 0$
$\langle (\delta v_\perp)$

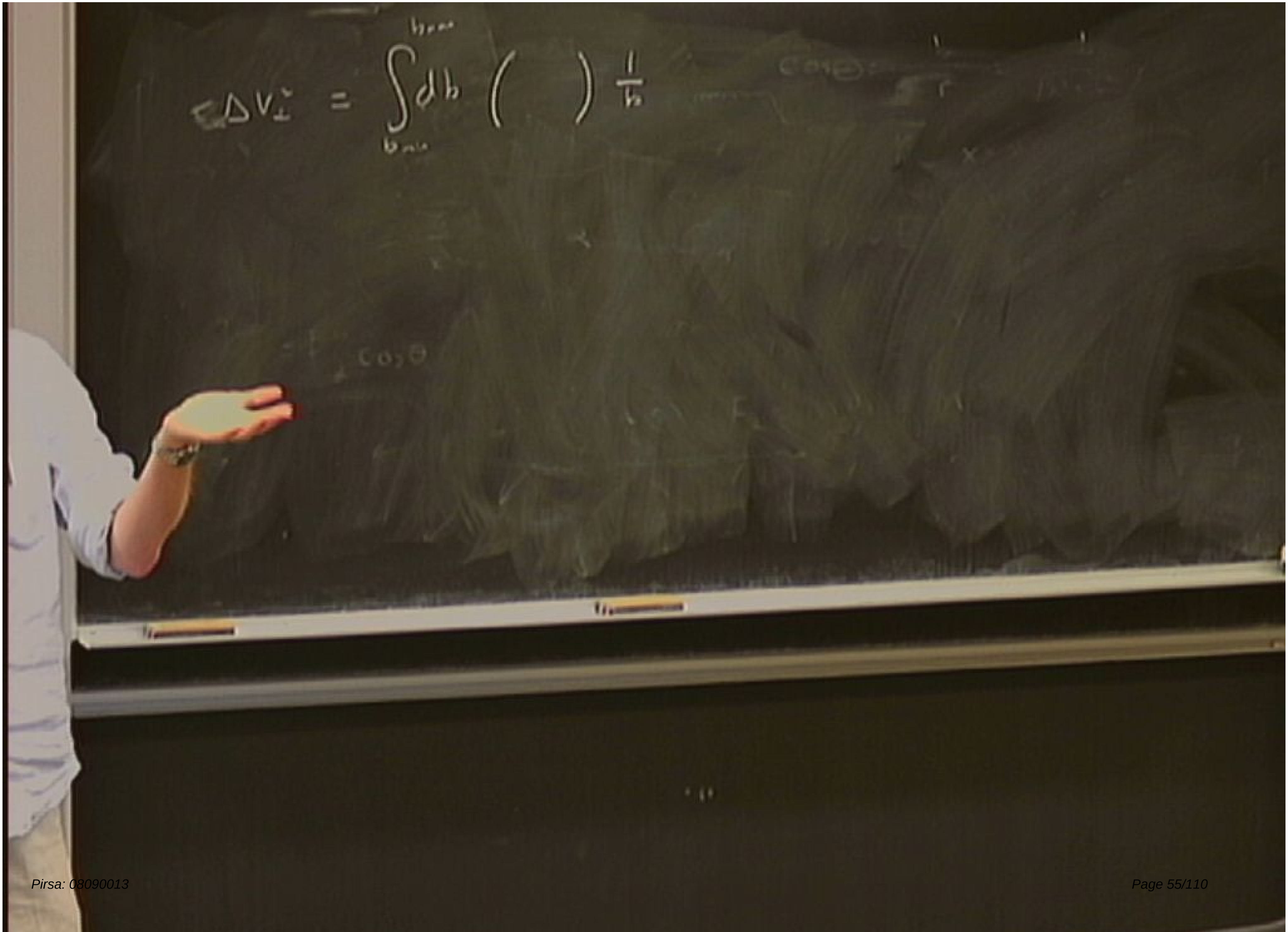

$\delta n = n \cdot 2\pi b\, db = \frac{N}{\pi R^2} 2\pi b\, db = \frac{2N}{R^2} b\, db$
$\langle \delta v_\perp \rangle$ per passage? $= 0$
$\langle (\delta v_\perp)^2 \rangle$

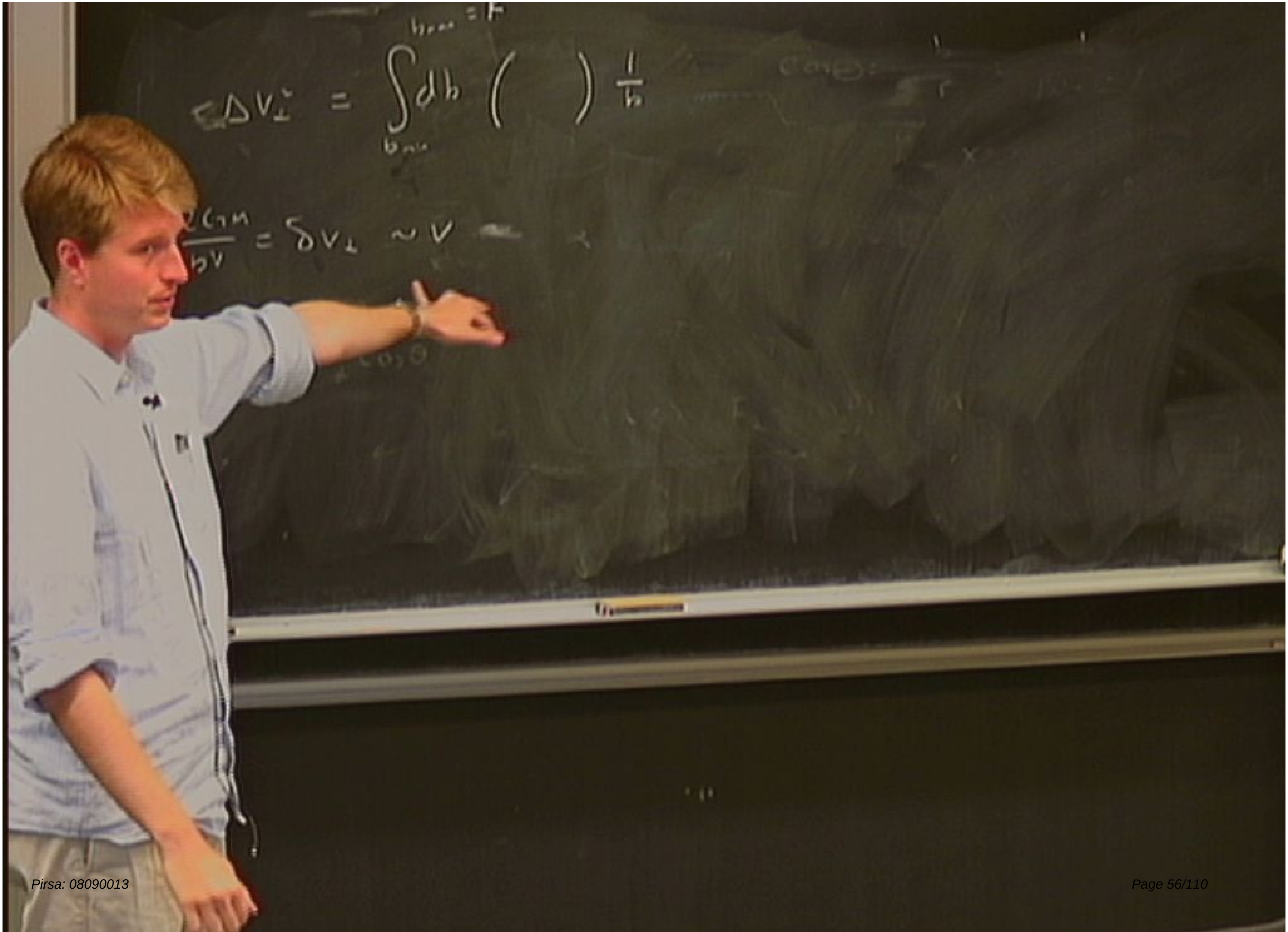


$\delta n = n \cdot 2\pi b\,db = \frac{N}{\pi R^2} 2\pi b\,db = \frac{2N}{R^2} b\,db$
$\langle \delta v_\perp \rangle$ per
$= 0$
$\langle (\delta v_\perp)^2 \rangle$


$\delta n = n \cdot 2\pi b\, db = \frac{N}{\pi R^2} 2\pi b\, db = \frac{2N}{R^2} b\, db$
$\langle \delta v_\perp \rangle$ per passage? $= 0$
$\langle (\delta v_\perp)^2 \rangle = n\, (\delta v_\perp)^2 = \frac{4 n G^2 m^2}{b^2 v^2} 2\pi b\, db$


$\delta n = n \cdot 2\pi b\, db = \frac{N}{\pi R^2} 2\pi b\, db = \frac{2N}{R^2} b\, db$
$\langle \delta v_\perp \rangle$ per passage? $= 0$
$\langle (\delta v_\perp)^2 \rangle = n\ (\delta v_\perp)^2 = \frac{4 n G^2 m^2}{b^2 v^2}\ 2\pi b\, db$
$= \frac{8\pi G^2 m^2}{v^2} \frac{db}{b}$

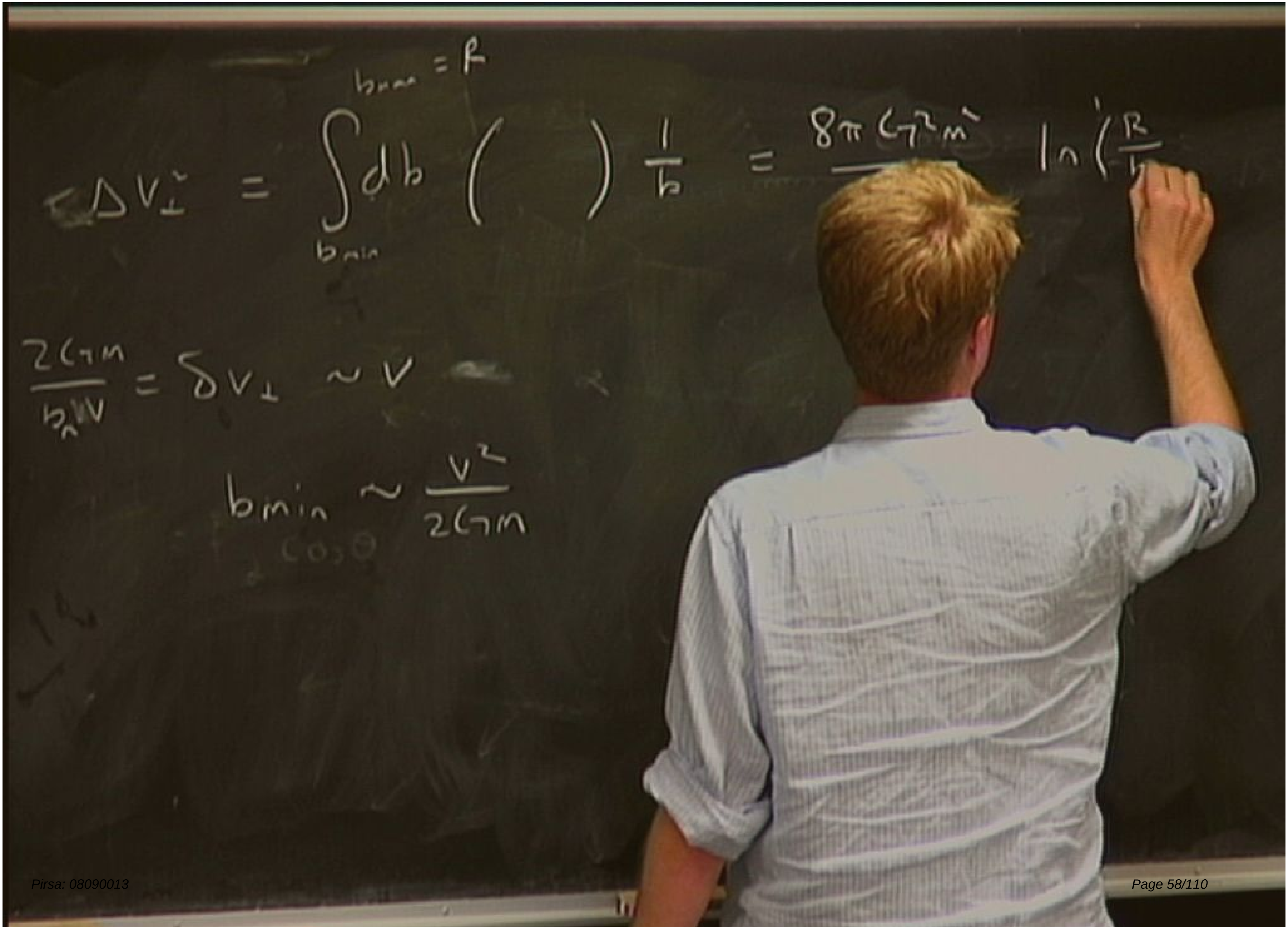

$\Sigma \Delta v_\perp^2 = \int_{b_{min}}^{b_{max}} db \, (\quad) \frac{1}{b}$



$$\Delta v_\perp^2 = \int_{b_{min}}^{b_{max} = R} db \left(\quad \right) \frac{1}{b}$$

$$\frac{2Gm}{b_{min} v} = \delta v_\perp \sim v$$

$$b_{min} \sim \frac{v^2}{2Gm}$$



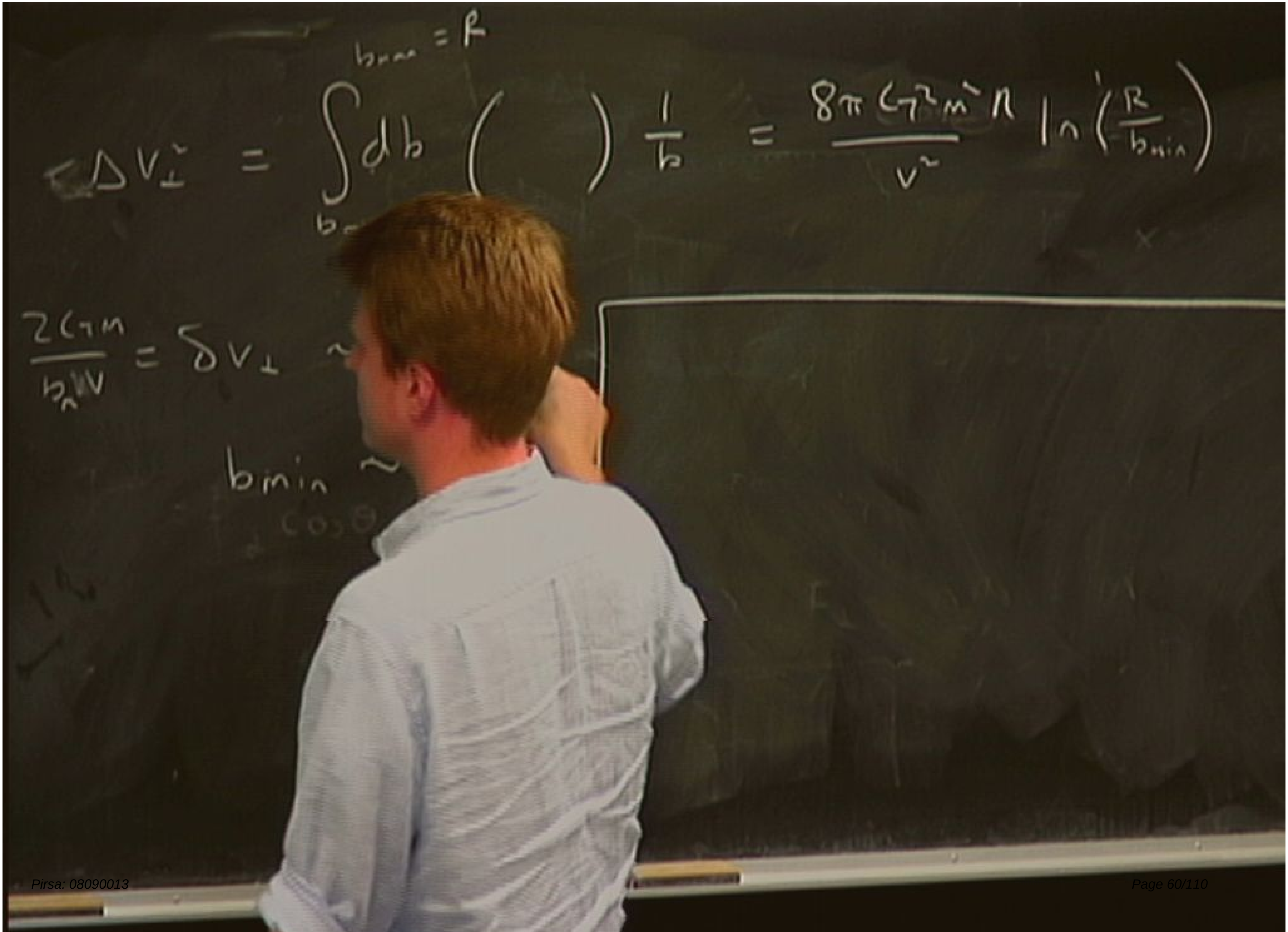
$\Delta V_\perp^2 = \int_{b_{min}}^{b_{max}=R} db \, (\quad) \frac{1}{b} = 8\pi G^2 m^2 \ln(\frac{R}{b}$
$\frac{2Gm}{b_n V} = \delta V_\perp \sim V$
$b_{min} \sim \frac{V^2}{2Gm}$

$$\Delta v_\perp^2 = \int_{b_{min}}^{b_{max} = R} db \left(\quad \right) \frac{1}{b} = \frac{8\pi G^2 m^2}{v^2} \ln\left(\frac{R}{b_{min}}\right)$$

$$\frac{2Gm}{b_{min} v} = \delta v_\perp \sim v$$

$$b_{min} \sim \frac{v^2}{2Gm}$$

$\cos\theta$

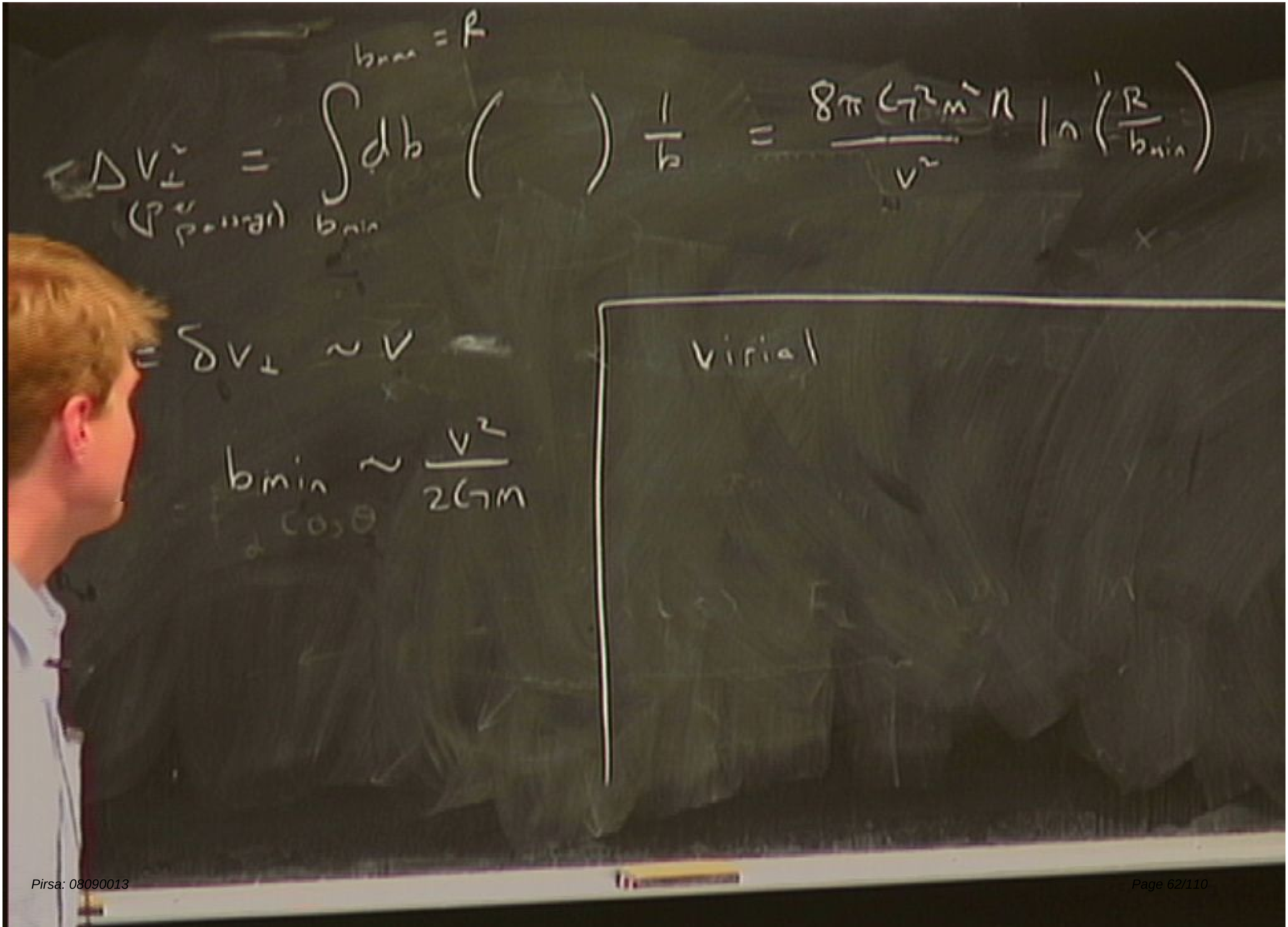


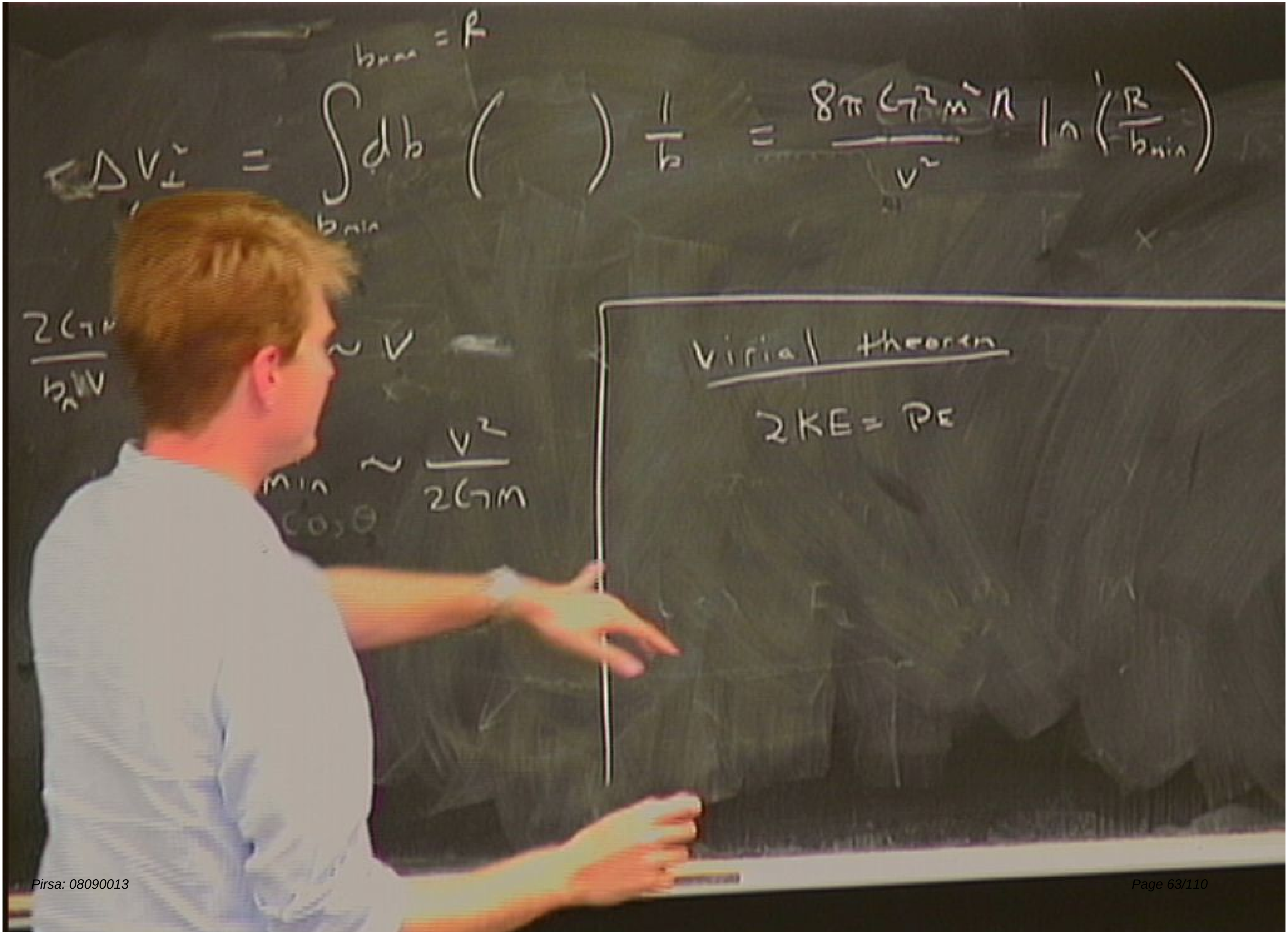
$b_{max} = R$
$\Delta v_\perp^2 = \int db \, (\quad) \, \frac{1}{b} = \frac{8\pi G^2 m^2 R}{v^2} \ln\left(\frac{R}{b_{min}}\right)$
$\frac{2Gm}{b_{min} v} = \delta v_\perp \sim$
$b_{min} \sim$

$$\Delta v_\perp^2 \ (\text{per passage}) = \int_{b_{min}}^{b_{max}=R} db \, \left(\quad \right) \frac{1}{b} = \frac{8\pi G^2 m^2 n}{v^2} \ln\left(\frac{R}{b_{min}}\right)$$

$$\frac{2Gm}{b_{min} v} = \delta v_\perp \sim v$$

$$b_{min} \sim \frac{v^2}{2Gm}$$


$b_{max} = R$
$\Delta v_\perp^2 = \int_{b_{min}}^{b_{max}} db \, (\quad) \, \frac{1}{b} = \frac{8\pi G^2 m^2 R}{v^2} \ln\left(\frac{R}{b_{min}}\right)$
(per passage)
$\delta v_\perp \sim v$
$b_{min} \sim \frac{v^2}{2Gm}$
virial



b_max = R
ΔV⊥² = ∫ db ( ) 1/b = 8π G² m² R / V² ln(R / b_min)
b_min
~ V
V² / 2Gm
Virial theorem
2KE = PE

$$\Delta v_\perp^2 \underset{(\text{per passage})}{} = \int_{b_{min}}^{b_{max}=R} db \left(\quad \right) \frac{1}{b} = \frac{8\pi G^2 m^2 n}{v^2} \ln\left(\frac{R}{b_{min}}\right)$$

$$\frac{2Gm}{b_n v} = \delta v_\perp \sim v$$

$$b_{min} \sim \frac{v^2}{2Gm}$$

$\cos\theta$

Virial theorem

$2KE = PE$

$$\Delta v_\perp^2 \text{ (per passage)} = \int_{b_{min}}^{b_{max}=R} db \left(\quad \right) \frac{1}{b} = \frac{8\pi G^2 m^2 n}{v^2} \ln\left(\frac{R}{b_{min}}\right)$$

$$\frac{Gm}{b_n v} = \delta v_\perp \sim v$$

$$b_{min} \sim \frac{v^2}{2Gm}$$

$\cos\theta$

Virial theorem

$$2KE = PE$$

$$2\left(\frac{1}{2} m v^2\right) = \frac{G m^2}{r}$$

$$\Delta V_\perp^2 \underset{(\text{per passage})}{} = \int_{b_{min}}^{b_{max}=R} db \left(\quad \right) \frac{1}{b} = \frac{8\pi G^2 m^2 n}{V^2} \ln\left(\frac{R}{b_{min}}\right)$$

$$\frac{2Gm}{b_{min} V} = \delta V_\perp \sim V$$

$$b_{min} \sim \frac{V^2}{2Gm}$$

$\cos\theta$

Virial theorem

$2KE = PE$

$$2\left(\frac{1}{2} m v^2\right) = \frac{G m n}{r} \rightarrow$$

Virial theorem

$$2KE = PE$$

$$2\left(\frac{1}{2} m v^2\right) = \frac{G N m^2}{r} \rightarrow v^2 = \frac{G N m}{r}$$

Virial theorem

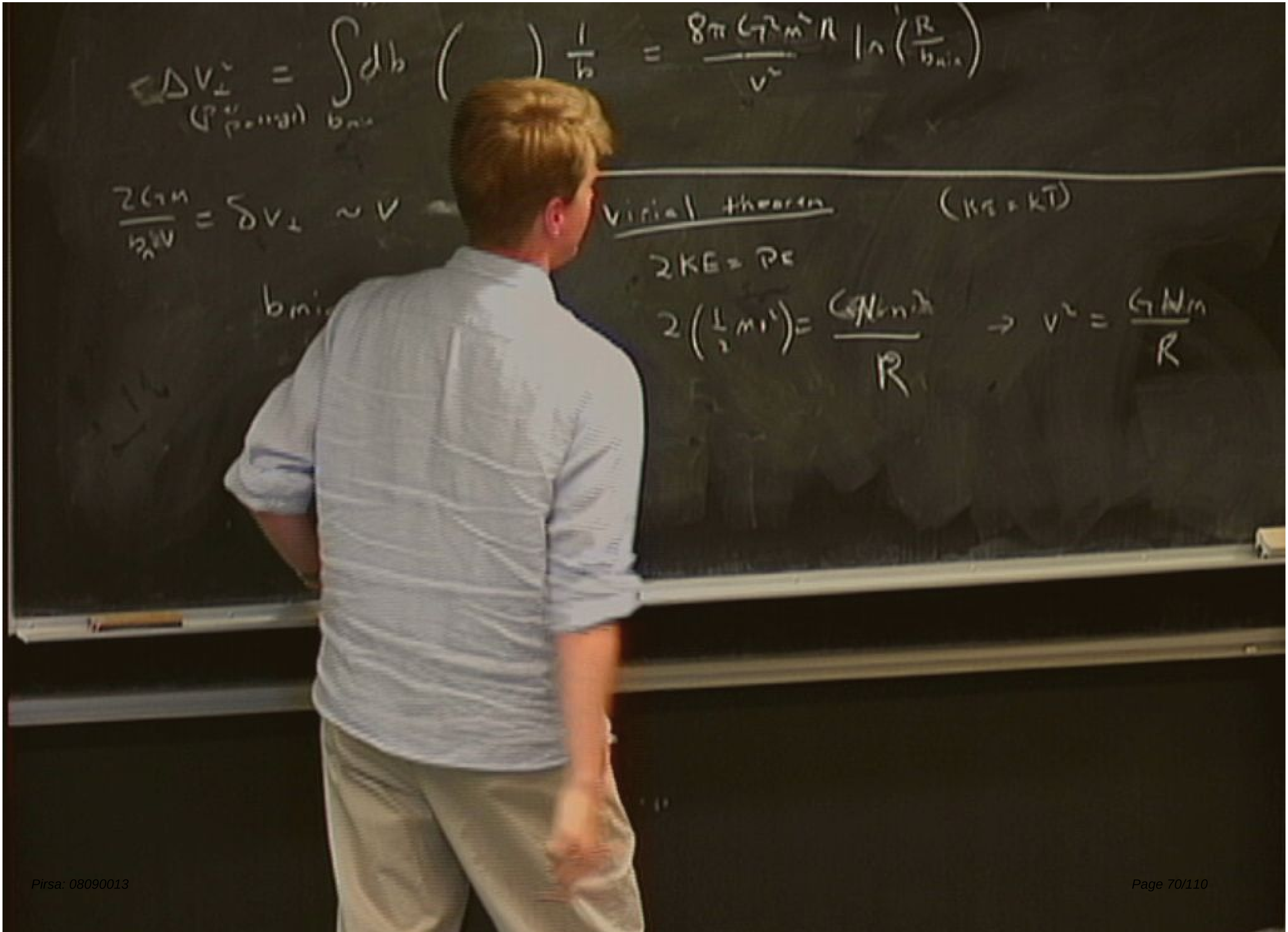
$$2KE = PE$$

$$2\left(\frac{1}{2}mv^2\right) = \frac{GNm^2}{r} \rightarrow v^2 = \frac{GNm}{r}$$

Virial theorem $(KE = kT)$

$$2KE = PE$$

$$2\left(\frac{1}{2}mv^2\right) = \frac{GNm^2}{r} \rightarrow v^2 = \frac{GNm}{r}$$


virial theorem
2KE = PE

$$\Sigma \Delta V_\perp^2 = \int_{b_{min}}^{b_{max}=R} db \left(\quad \right) \frac{1}{b} = \frac{8\pi G^2 m^2 R}{V^2} \ln\left(\frac{R}{b_{min}}\right)$$

$(\rho \frac{\#}{\text{pc}^3})$

$$\frac{2Gm}{b_{min} V} = \delta V_\perp \sim V$$

$$b_{min} \sim \frac{V^2}{2Gm}$$

$\perp \cos\theta$

Virial theorem $(KE \approx kT)$

$2KE = PE$

$$2\left(\frac{1}{2} m v^2\right) = \frac{G N m^2}{R} \rightarrow v^2 = \frac{G N m}{R}$$

$$\Sigma \Delta V_\perp^2 \underset{\text{(per crossing)}}{} = \int_{b_{min}}^{b_{max}=R} db \left(\qquad \right) \frac{1}{b} = \frac{8\pi G^2 m^2 R}{V^2} \ln\left(\frac{R}{b_{min}}\right)$$

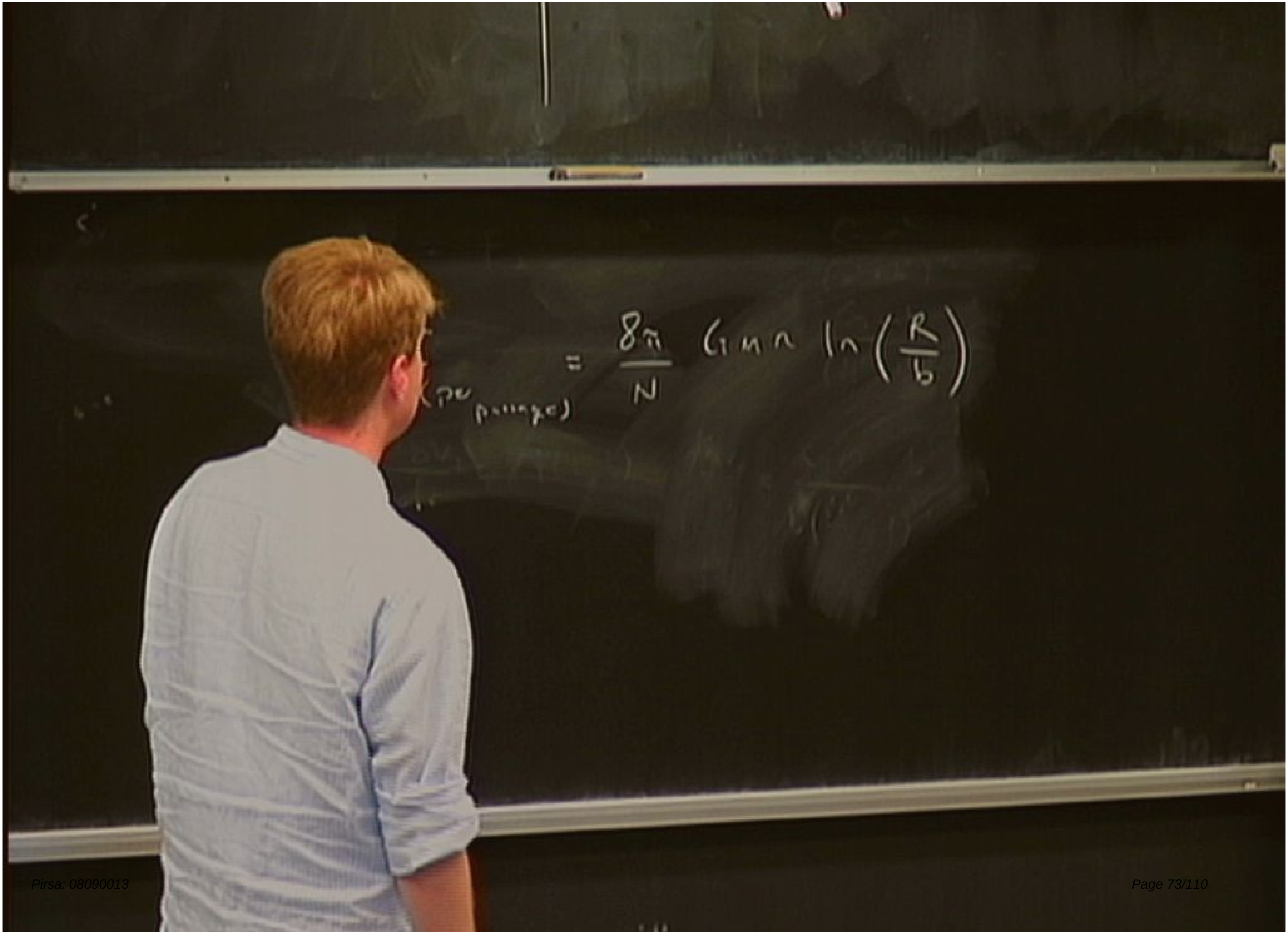
$$\frac{2Gm}{b_{min} V} = \delta V_\perp \sim V$$

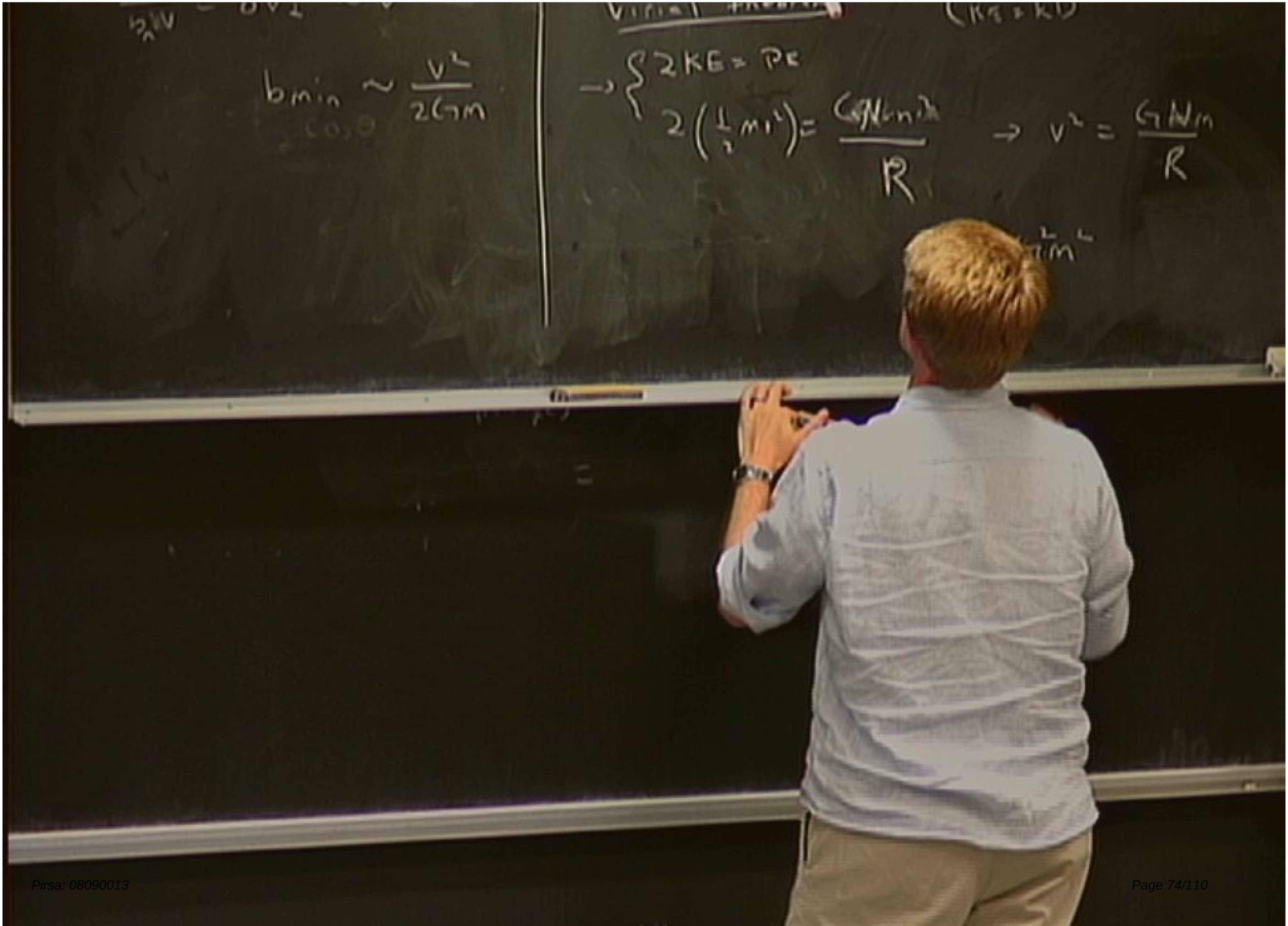
$$b_{min} \sim \frac{V^2}{2Gm}$$

$\cos\theta$

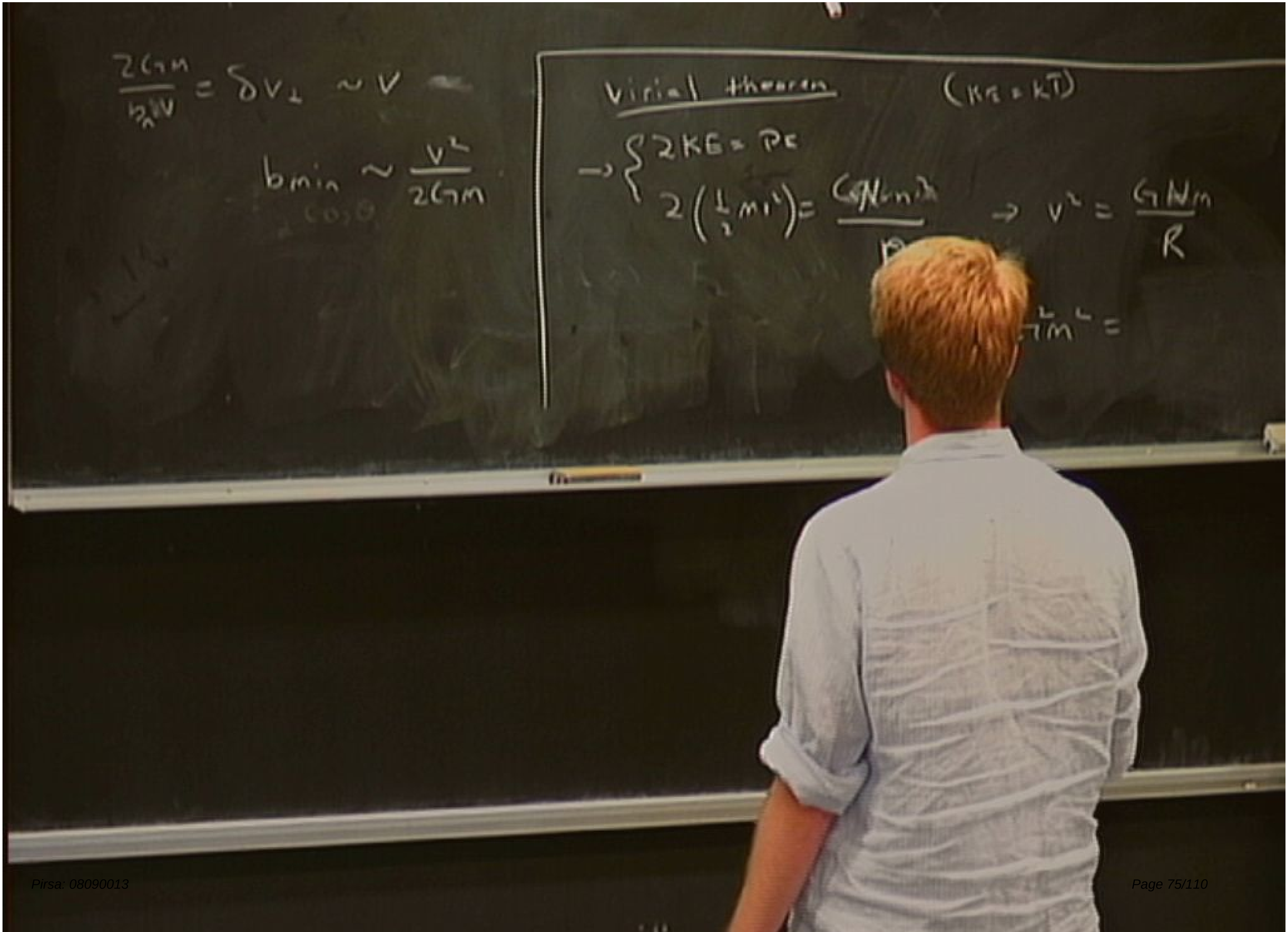
Virial theorem $\quad (KE \approx kT)$

$$\rightarrow \begin{cases} 2\,KE = PE \\ 2\left(\frac{1}{2} m v^2\right) = \frac{G N m m}{R} \end{cases} \rightarrow v^2 = \frac{G N m}{R}$$


(per passage) = 8π/N Gmn ln(R/b)



$b_{min} \sim \frac{v^2}{2Gm}$
$\rightarrow$ 2KE = PE
$2(\frac{1}{2}mv^2) = \frac{GNm^2}{R}$ $\rightarrow$ $v^2 = \frac{GNm}{R}$



2Gm / b v = δv⊥ ~ v
bmin ~ v² / 2Gm
Virial theorem
(KE = kT)
2KE = PE
2(½ mv²) = GNm² / R
→ v² = GNm / R

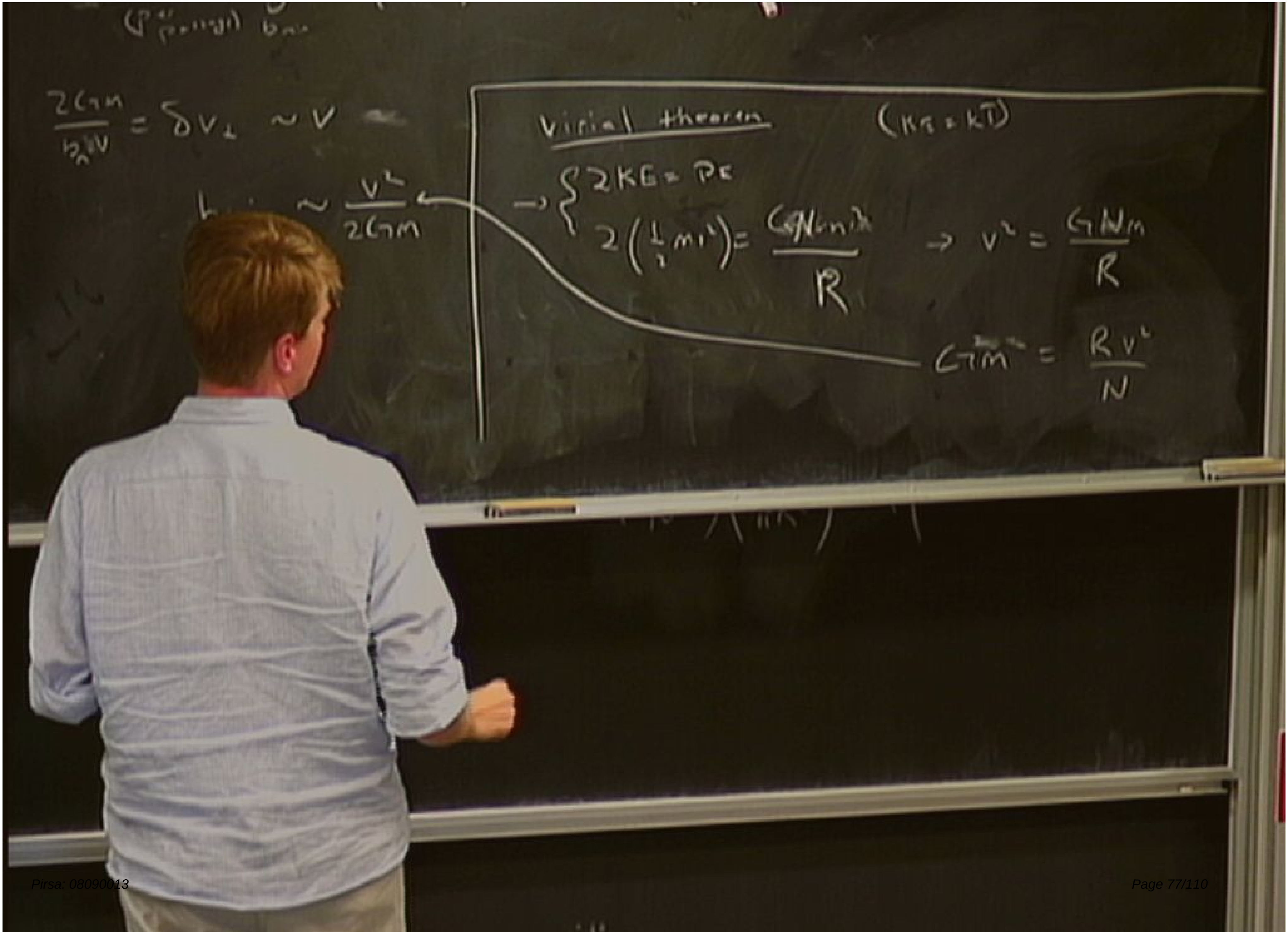
$$Gm = \frac{Rv^2}{N}$$

$$R$$

$$\frac{N}{\pi R^2}$$

$$\Delta v_{\perp}^{2} \;(\text{per passage}) = \frac{8\pi}{N}\, G m n \ln\left(\frac{R}{b}\right)$$

$$= \frac{8\pi}{N}\left(\frac{Rv^2}{N}\right)\left(\frac{N}{\pi R^2}\right)\ln\Big($$


Virial theorem
(KE = kT)
2KE = PE
2(½mv²) = GNm²/R
v² = GNm/R
Gm = Rv²/N

$(\rho^{\text{at}}_{\text{pellegr}})\ b_{\text{min}}$

$$\frac{2Gm}{b_n V} = \delta v_\perp \sim V$$

$$b_{\text{min}} \sim \frac{V^2}{2Gm}$$

$\cos\theta$

$$\sim \frac{N}{2R}$$

Virial theorem $(KE = kT)$

$$\rightarrow \begin{cases} 2KE = PE \\ 2\left(\frac{1}{2}mv^2\right) = \dfrac{GNm^2}{R} \end{cases}$$

$$\rightarrow v^2 = \frac{GNm}{R}$$

$$Gm = \frac{Rv^2}{N}$$

$$\frac{2Gm}{b_{min} V} = \delta v_\perp \sim V$$

$$b_{min} \sim \frac{V^2}{2Gm}$$

$$\sim \frac{N}{2R}$$

$$\frac{R}{b_{min}} \sim \frac{R}{N/}$$

Virial theorem (KE = kT)

$$\rightarrow \begin{cases} 2KE = PE \\ 2\left(\frac{1}{2} m v^2\right) = \frac{GNm^2}{R} \end{cases} \rightarrow v^2 = \frac{GNm}{R}$$

$$Gm = \frac{Rv^2}{N}$$

$$\frac{2Gm}{b_n v} = \delta v_\perp \sim v$$

$$b_{min} \sim \frac{2Gm}{v^2}$$

$$\sim \frac{2R}{N}$$

$$\frac{R}{b_{min}} \sim \frac{R}{R/N} \sim N$$

Virial theorem (KE = kT)

$$\rightarrow \begin{cases} 2KE = PE \\ 2\left(\frac{1}{2}mv^2\right) = \frac{GNm^2}{R} \end{cases}$$

$$\rightarrow v^2 = \frac{GNm}{R}$$

$$Gm = \frac{Rv^2}{N}$$

$$\frac{R}{b_{min}} \sim \frac{R}{R/N} \sim N$$

$$G m = \frac{R v^2}{N}$$

$$\Delta v_\perp^2 \underset{(\text{per passage})}{=} \frac{8\pi}{N} G m n \ln\left(\frac{R}{b}\right) \qquad n = \frac{N}{\pi R^2}$$

$$= \frac{8\pi}{N}\left(\frac{R v^2}{N}\right)\left(\frac{N}{\pi R^2}\right)\ln(N)$$

$$= v^2\left(\frac{8 \ln N}{N}\right)$$

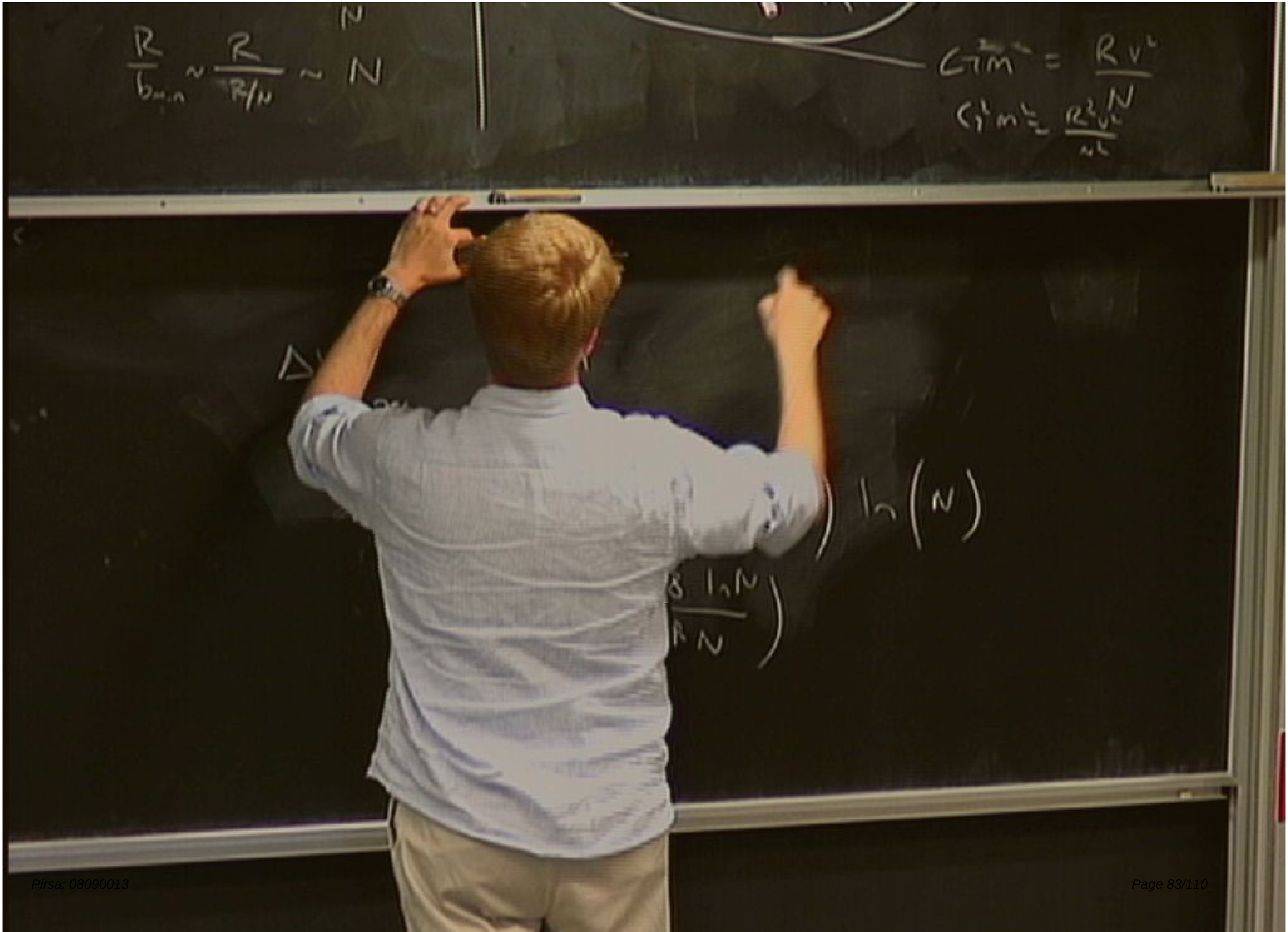
$$\frac{R}{b_{min}} \sim \frac{R}{R/N} \sim N$$

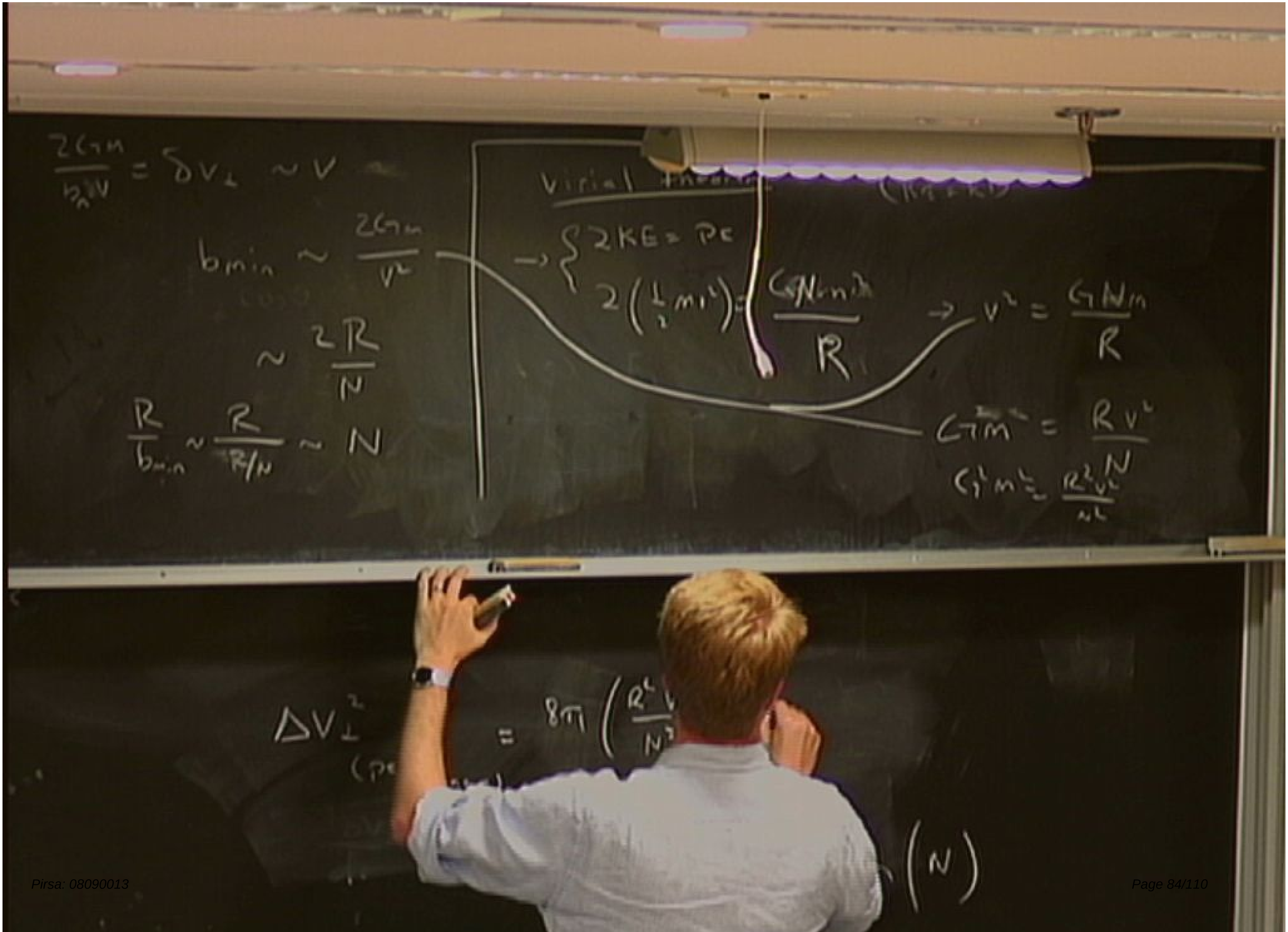
$$G m^2 = \frac{R v^2}{N}$$

$$\Delta v_\perp^2 \; (\text{per passage}) = \frac{8\pi}{N} G m \overset{\frac{N}{\pi R^2}}{n} \ln\left(\frac{R}{b}\right)$$

$$= \frac{8\pi}{N}\left(\frac{\cancel{R} v^2}{\cancel{N}}\right)\left(\frac{\cancel{N}}{\pi R^{\cancel{2}}}\right)\ln(N)$$

$$= v^2\left(\frac{8 \ln N}{R N}\right)$$


R/b_min ~ R/(R/N) ~ N
ln(N)


2Gm / b_n²V = δv⊥ ~ v
b_min ~ 2Gm / v²
~ 2R / N
R / b_min ~ R / (R/N) ~ N
Virial theorem
2KE = PE
2(½mv²) = GNm² / R
v² = GNm / R
Gm = Rv² / N
G²m² = R²v⁴ / N²
ΔV⊥²
=
8π
(N)

$$n = N/\pi R^2$$

$$\Delta v_{\perp}^2 \;(\text{per passage}) = 8\pi \left(\frac{R^2 v^{\cancel{4}\,2}}{N^2}\right) \frac{n}{\cancel{x^2}} \ln\left(\frac{R}{b_m}\right)$$

$$= 8\cancel{\pi}\left(\frac{\cancel{R^2} v^2}{N^{\cancel{2}}}\right)\left(\frac{\cancel{N}}{\cancel{\pi}\cancel{R^2}}\right) \ln\left(\frac{R}{b_m}\right)$$

$$\simeq v^2 \left(\frac{8 \ln(N)}{N}\right)$$

$$\boxed{\delta v_\perp(b) = \frac{2Gm}{bv}}$$

$$\frac{\delta v_\perp}{v} \ll 1$$

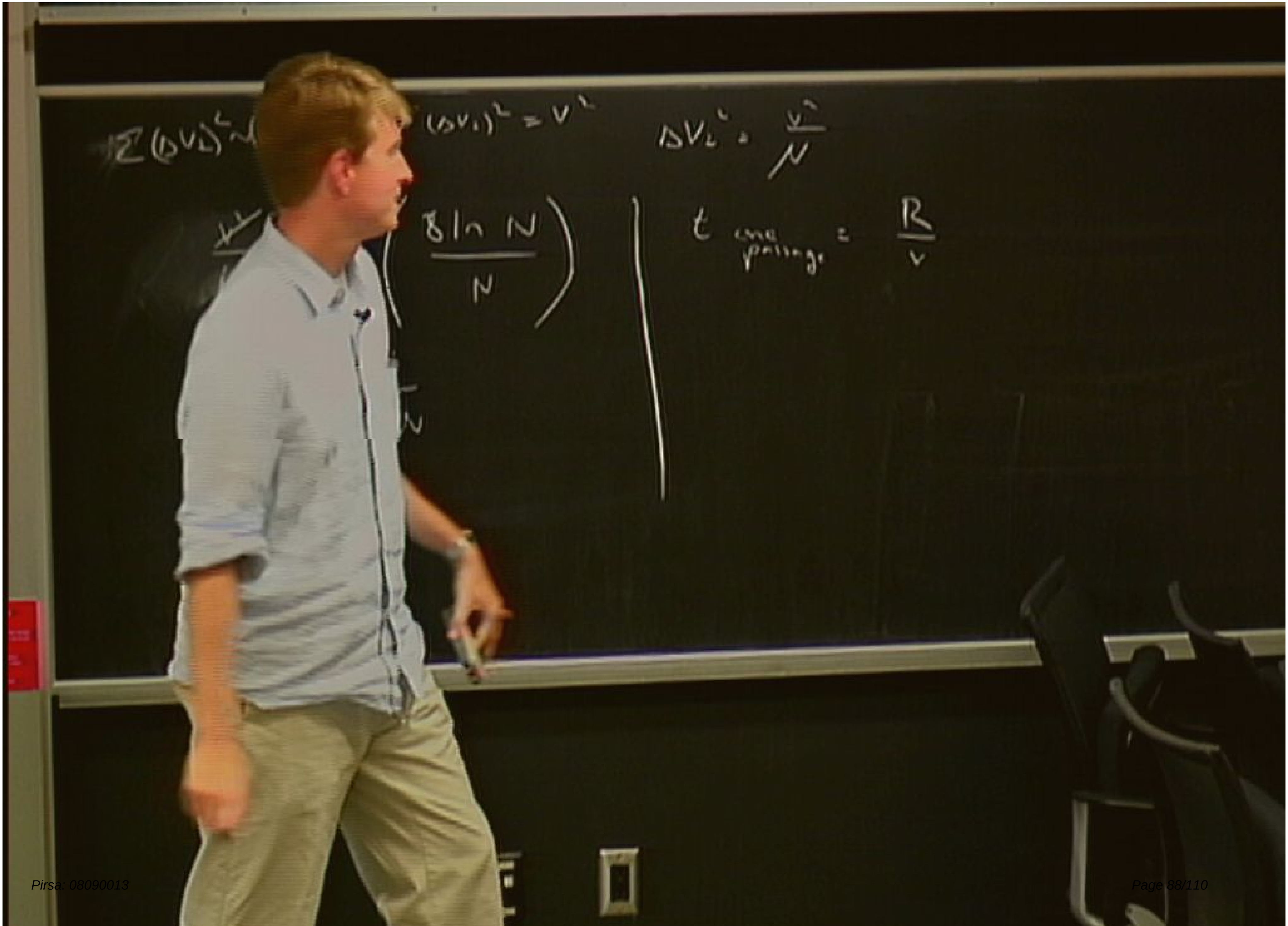
$$\Delta t = \frac{b}{v}$$

$$\Delta v_\perp \sim v$$

$$\sum (\Delta v_i)^2 \sim (v)^2 \qquad \mathcal{N} (\Delta v_i)^2 = v^2 \qquad \Delta v_i^2 = \frac{v^2}{\mathcal{N}}$$

$$\frac{v^2}{\mathcal{N}} = v^2 \left(\frac{8 \ln N}{N} \right)$$

$$\mathcal{N} = \frac{N}{8 \ln N}$$


$(\Delta v_1)^2 \approx v^2$
$\Delta v_\perp^2 \approx \frac{v^2}{N}$
$\frac{8 \ln N}{N}$
$t_{\text{one passage}} = \frac{R}{v}$

$$\Sigma (\Delta v_i)^2 \sim (v)^2 \qquad \mathcal{N} (\Delta v_i)^2 = v^2 \qquad \Delta v_i^2 = \frac{v^2}{\mathcal{N}}$$

$$\frac{v^2}{\mathcal{N}} = v^2 \left(\frac{8 \ln N}{N} \right)$$

$$\mathcal{N} = \frac{N}{8 \ln N}$$

$$t_{\text{one passage}} = \frac{R}{v}$$

$$t = \mathcal{N} \tau = \mathcal{N} \frac{R}{v} = \frac{N}{8 \ln N} \overbrace{\left(\frac{R}{v} \right)}^{\text{time}}$$

$$G^2 m^2 \approx \frac{R^2 v^4 N}{N^2}$$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} \approx \frac{10^{11}}{10(11)} \sim 10^{9}$$

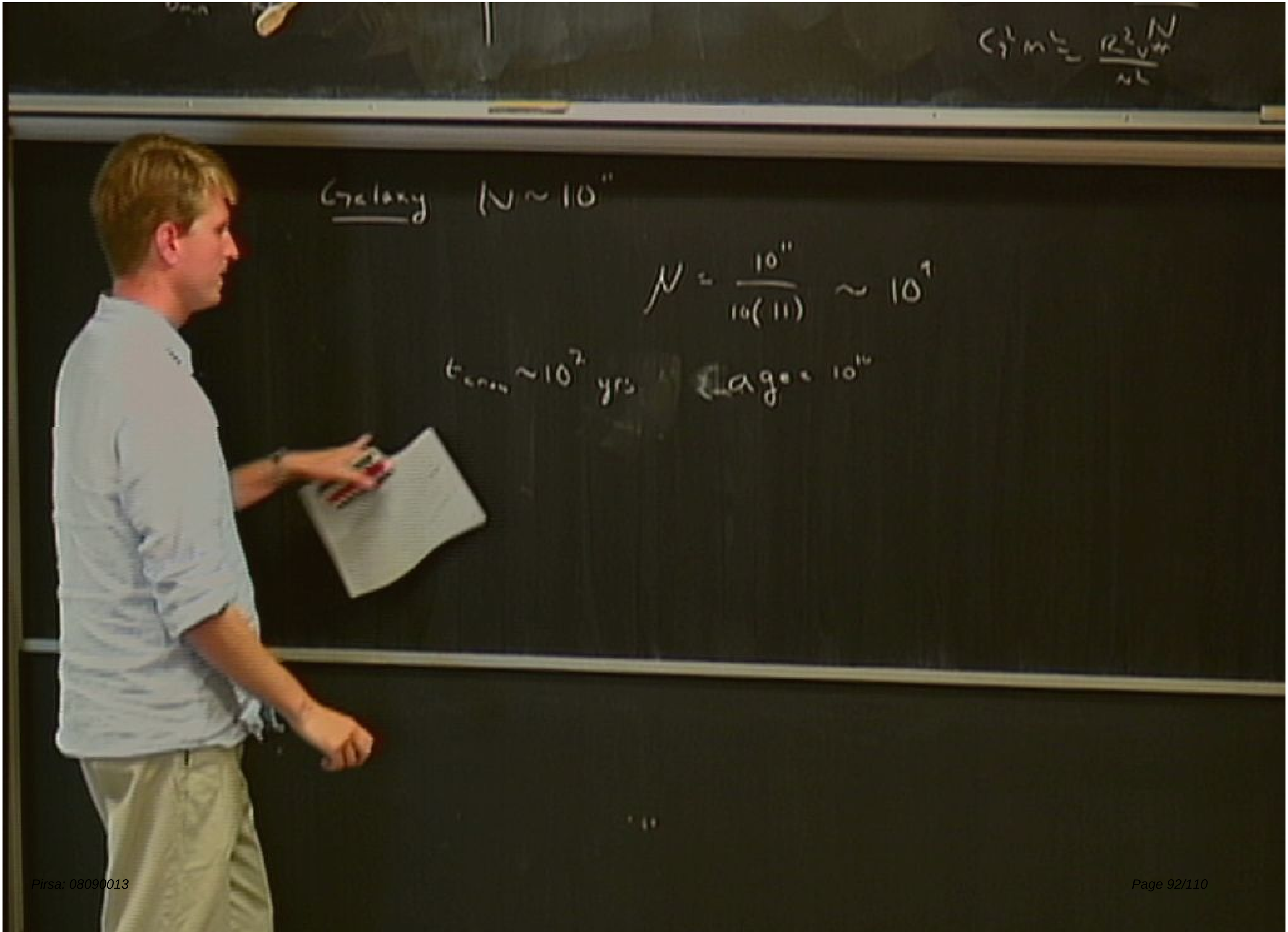
N

$G_T^2 m^2 \simeq \frac{R^2 v^4}{N^2}$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} \simeq \frac{10^{11}}{10(11)} \sim 10^{9}$$

$t_{age} \simeq 10^{16}$


Galaxy N ~ 10^11
N = 10^11 / 10(11) ~ 10^9
age ~ 10^10

$$G^2 m^2 \simeq \frac{R^2 v^4}{N^2}$$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} \simeq \frac{10^{11}}{10(11)} \sim 10^{9}$$

$t_{cross} \sim 10^{7}$ yrs , $t_{age} \simeq 10^{10}$ yrs $\sim 10^{3}$

$$G^2 m^2 \sim \frac{R^2 v^4 N}{N^2}$$

<u>Galaxy</u> $N \sim 10^{11}$

$$\mathcal{N} \approx \frac{10^{11}}{10(11)} \sim 10^{9}$$

$t_{relax} \sim 10^{7}$ yrs , $t_{age} \approx 10^{10}$ yrs $\sim 10^{3}$

$G^2 m^2 \simeq \frac{R^2 v^4}{N^2}$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} = \frac{10^{11}}{10(11)} \sim 10^9$$

$t_{cross} \sim 10^7$ yrs , $t_{age} \approx 10^{10}$ yrs $\sim 10^3$

Globular clusters

$N = 10^5$, $t_{cross} = 10^5$ yrs $\quad \mathcal{N}_{GC} = \frac{10^5}{10(5)} \sim 10^{3.5}$

$G^2 m^2 \approx \frac{R^2 v^4}{N^2}$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} = \frac{10^{11}}{10(11)} \sim 10^{9}$$

$t_{cross} \sim 10^{7}$ yrs , $t_{age} \approx 10^{10}$ yrs $\sim 10^{3}$

Globular clusters

$N = 10^{5}$, $t_{cross} = 10^{5}$ yrs $\quad \mathcal{N}_{relax} = \frac{10^{5}}{10(5)} \sim 10^{3.5}$

$t_{relax} \sim 10^{10}$ yrs

$G^2 m^2 \simeq \frac{R^2 v^4 N}{N^2}$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} \simeq \frac{10^{11}}{10(11)} \sim 10^{9}$$

$t_{cross} \sim 10^{7}$ yrs, $t_{age} \simeq 10^{10}$ yrs $\sim 10^{3}$

Globular clusters

$N = 10^{5}$, $t_{cross} = 10^{5}$ yrs $\quad \mathcal{N}_{rlx} \simeq \frac{10^{5}}{10(5)} \sim 10^{3.5}$

$t_{life} \sim 10^{10}$ yrs, $t_{cross} \sim 10^{5}$

$G^2 m^2 \sim \frac{R^2 v^4}{N^2} N$

Cluster of Galaxies

Galaxy $N \sim 10^{11}$

$$\mathcal{N} \approx \frac{10^{11}}{10(11)} \sim 10^{9}$$

$t_{cross} \sim 10^{7}$ yrs, $t_{age} \approx 10^{10}$ yrs $\sim 10^{3}$

Globular clusters

$N = 10^{5}$, $t_{cross} = 10^{5}$ yrs $\quad \mathcal{N}_{GC} \approx \frac{10^{5}}{10(5)} \sim 10^{3.5}$

$t_{age} \sim 10^{10}$ yrs, $\quad t_{cross} \sim 10^{5}$

$G^2 m^2 \simeq \frac{R^2 v^4}{N^2} N$

Cluster of Galaxies

$N \sim 10^3$

$t_{cross} \sim 10^9$

Galaxy $N \sim 10^{11}$

$$\mathcal{N} \simeq \frac{10^{11}}{10(11)} \sim 10^9$$

$t_{cross} \sim 10^7$ yrs , $t_{age} \simeq 10^{10}$ yrs $\sim 10^3$

Globular clusters

$N \simeq 10^5$, $t_{cross} \simeq 10^5$ yrs $\mathcal{N}_{GC} \simeq \frac{10^5}{10(5)} \sim 10^{3.5}$

$t_{GC} \sim 10^{10}$ yrs , $t_{cross} \sim 10^5$

Cluster of Galaxies

$N \sim 10^3$

$t_{cross} \sim 10^9$

$\mathcal{N}_{cross} = \frac{10^3}{30} \sim 10^{?}$

Galaxy $N \sim 10^{11}$

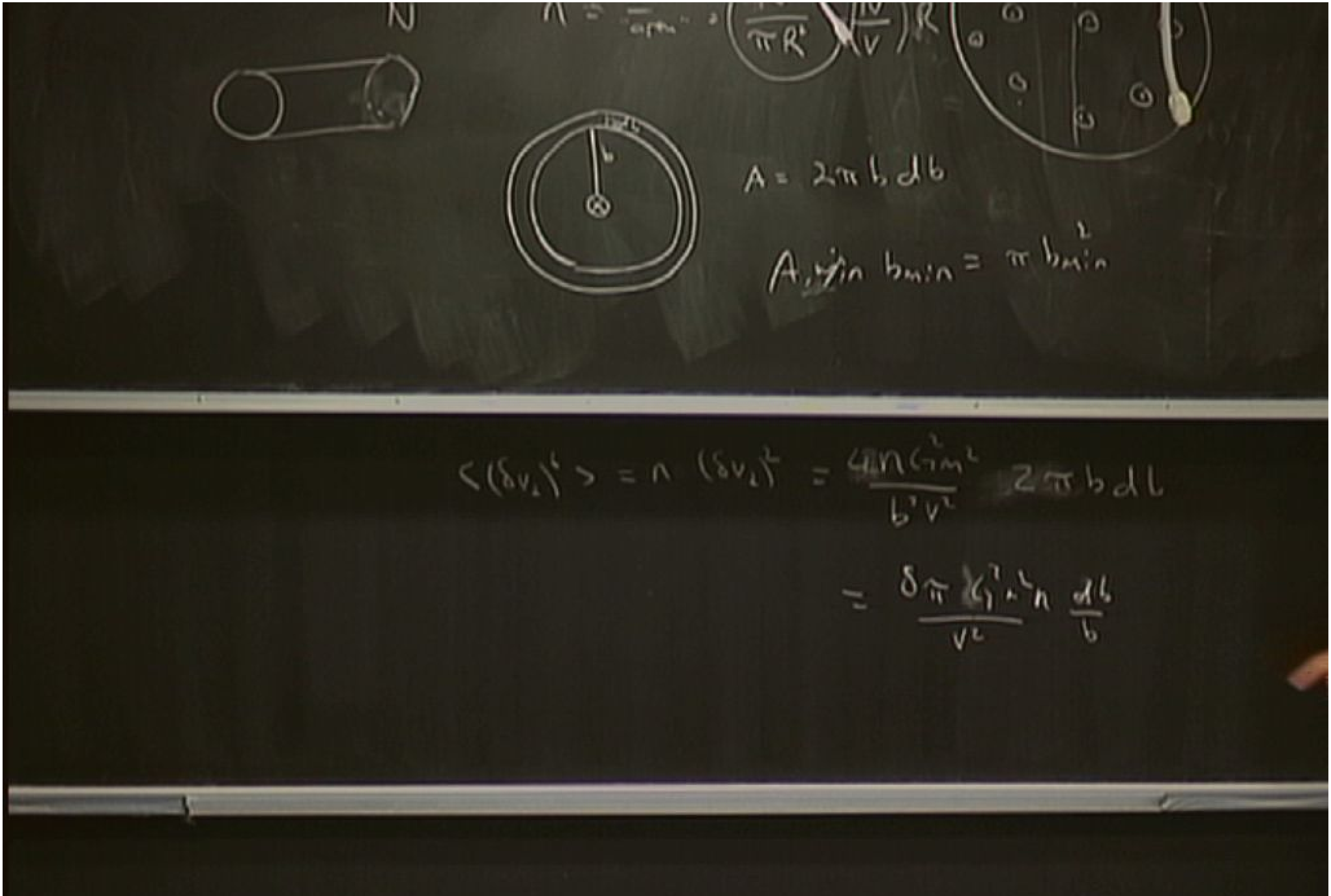
$$\mathcal{N} = \frac{10^{11}}{10(11)} \sim 10^9$$

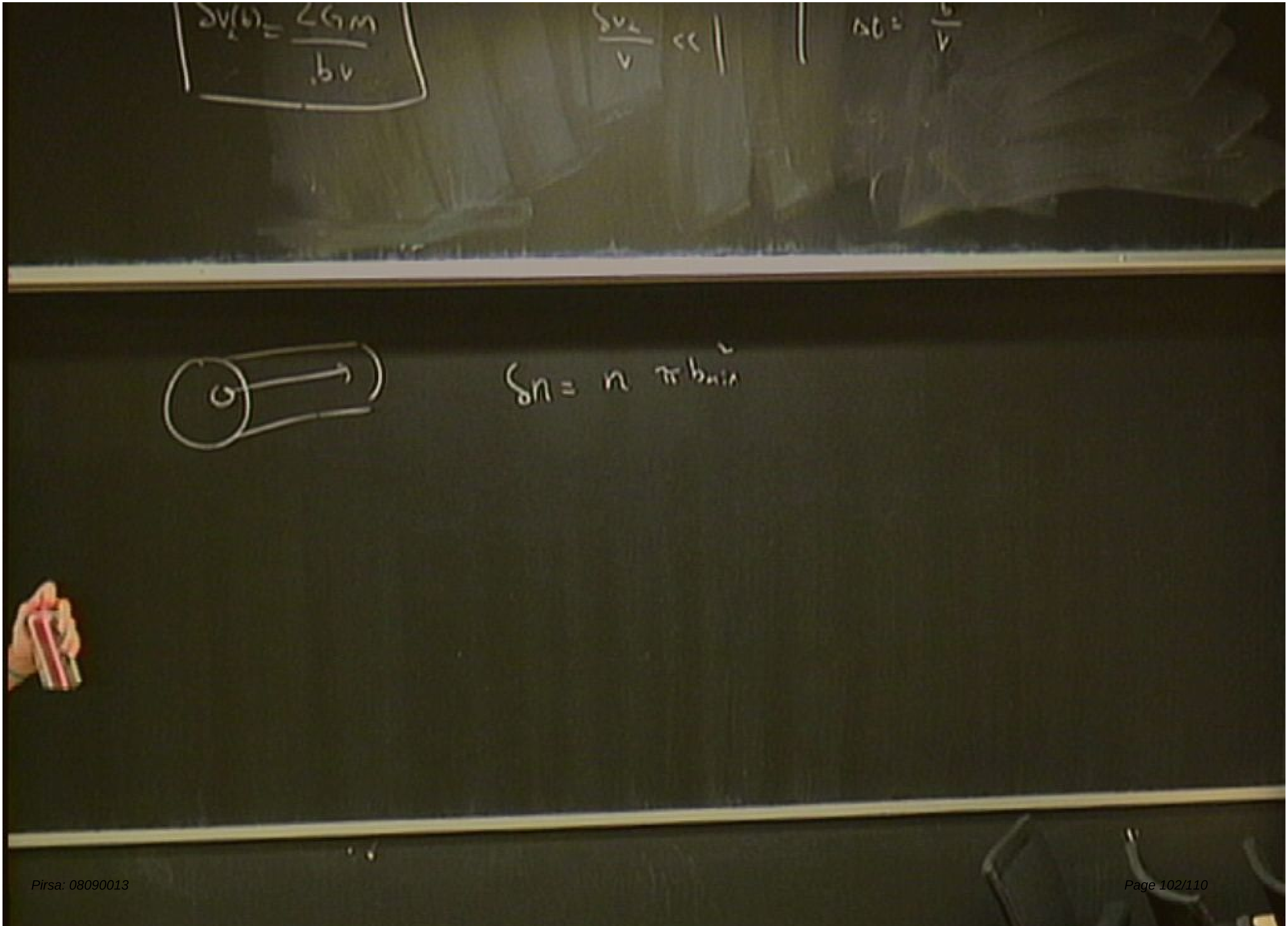
$t_{cross} \sim 10^7$ yrs , age $= 10^{10}$ yrs $\sim 10^{?}$

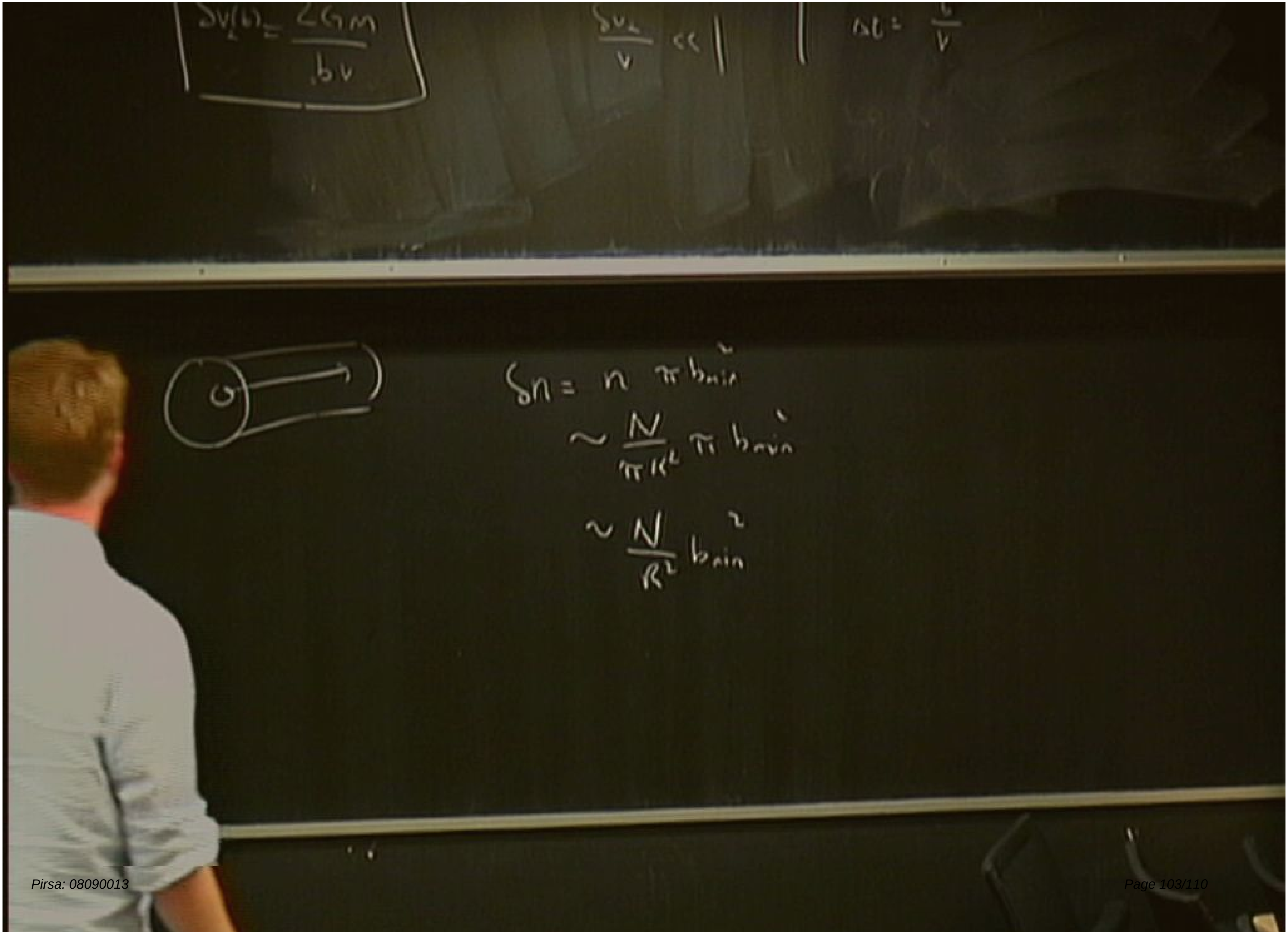
Globular clusters

$N = 10^5$, $t_{cross} = 10^5$ yrs $\quad \mathcal{N}_{GC} = \frac{10^5}{10(5)} \sim 10^{3.5}$

$t_{age} \sim 10^{10}$ yrs , $\mathcal{N}_{cross} \sim 10^5$


$A = 2\pi b\, db$
$A_{min}\, b_{min} = \pi b_{min}^2$
$\langle (\delta v_\perp)^2 \rangle = n\, (\delta v_\perp)^2 = \frac{4 n G^2 m^2}{b^2 v^2}\, 2\pi b\, db$
$= \frac{8\pi G^2 m^2 n}{v^2} \frac{db}{b}$


$\delta v_\perp(b) = \frac{2Gm}{bv}$
$\frac{\delta v_\perp}{v} \ll 1$
$\Delta t = \frac{b}{v}$
$\delta n = n \pi b^2$

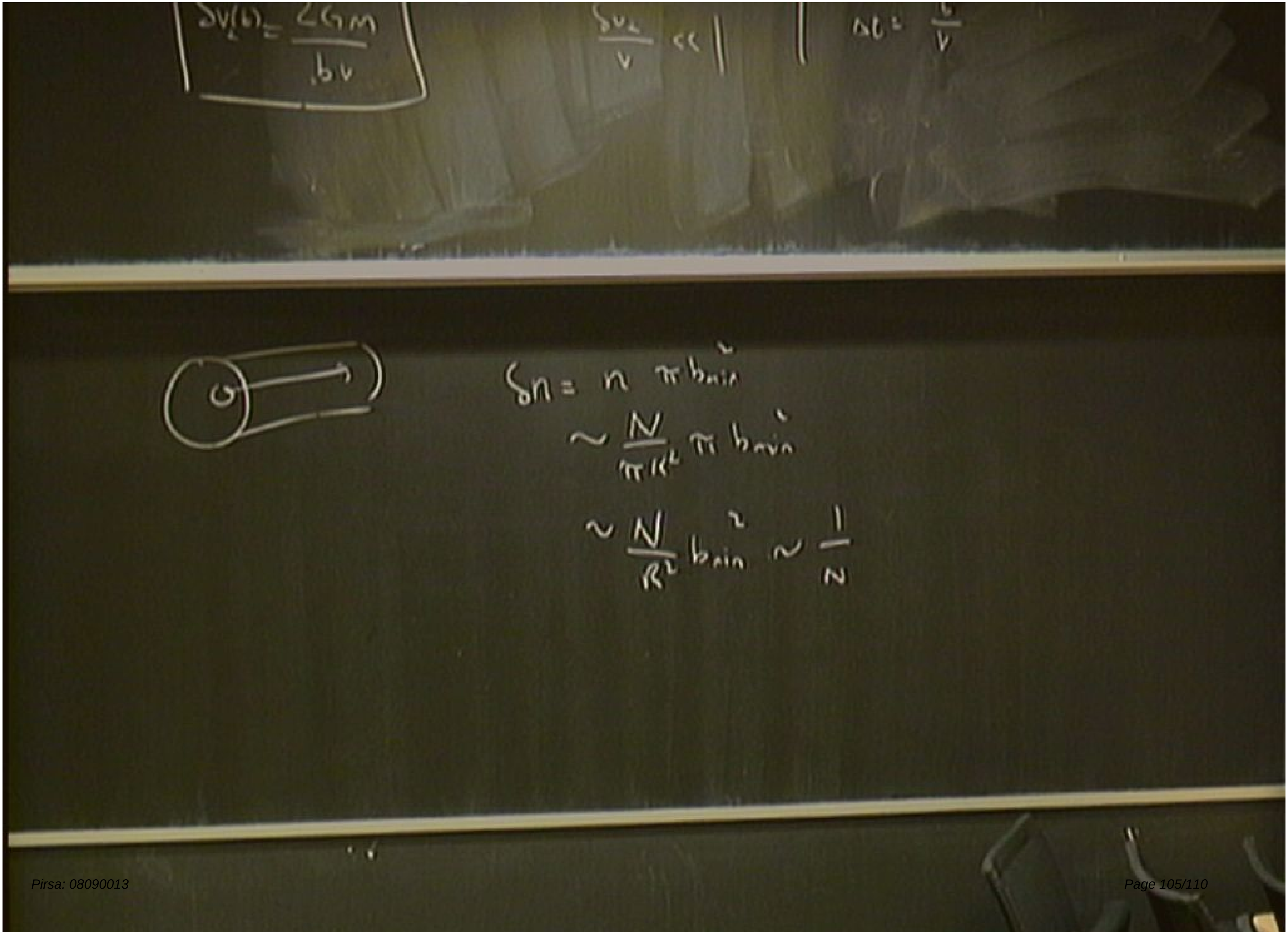

δn = n π b_min^2
~ N/(πR^2) π b_min^2
~ N/R^2 b_min^2

$$\boxed{\delta v_\perp(b) = \frac{2Gm}{bv}} \qquad \frac{\delta v_\perp}{v} \ll 1 \qquad \Delta t = \frac{b}{v}$$

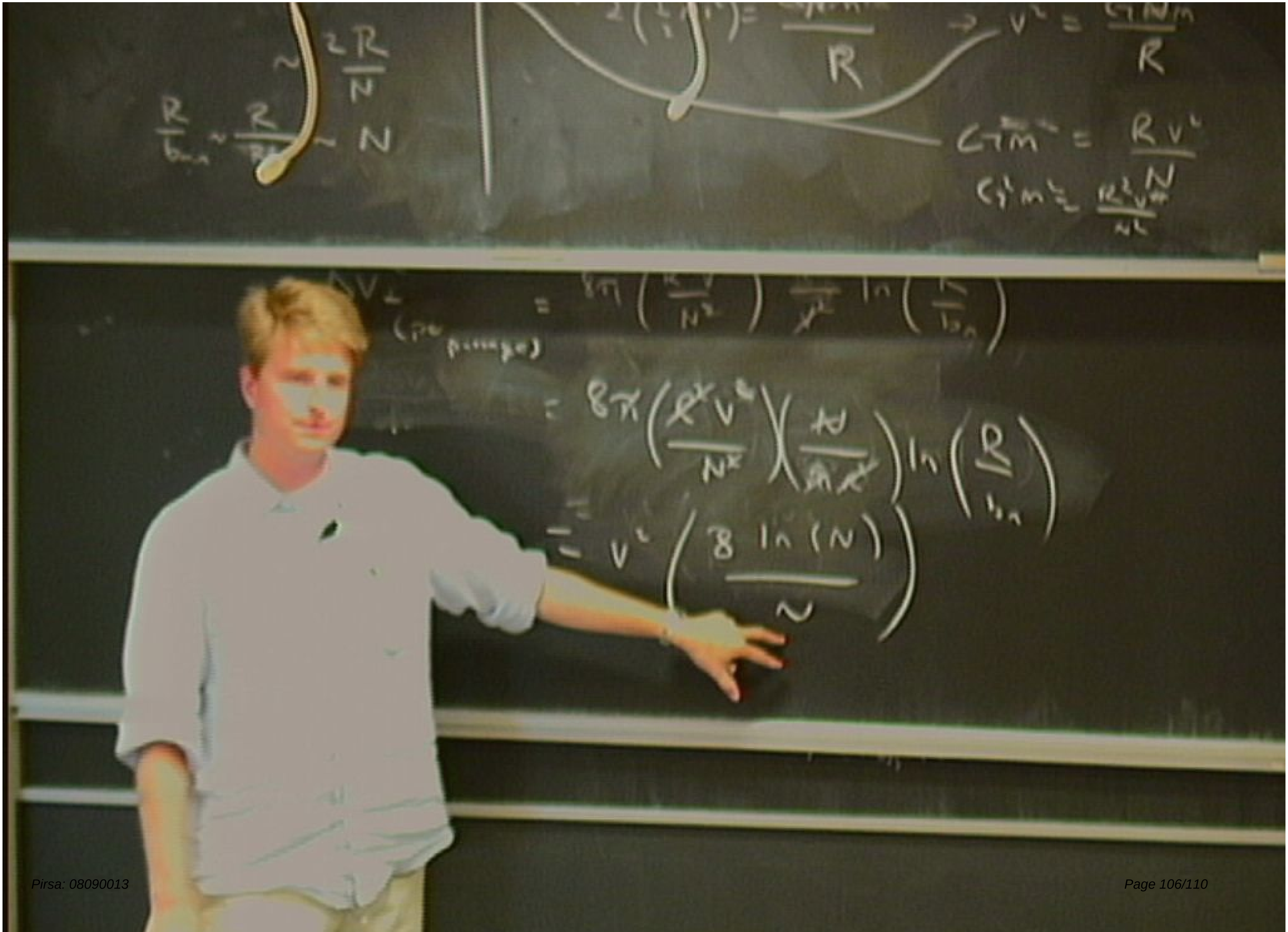
$$\delta n = n \, \pi b_{min}^2$$

$$\sim \frac{N}{\pi R^2} \pi b_{min}^2$$

$$\sim \frac{N}{R^2} b_{min}^2$$



$\delta v_\perp(b) = \frac{2Gm}{bv}$
$\frac{\delta v_\perp}{v} \ll 1$
$\delta n = n \pi b_{min}^2$
$\sim \frac{N}{\pi R^2} \pi b_{min}^2$
$\sim \frac{N}{R^2} b_{min}^2 \sim \frac{1}{N}$



$$\delta n = n \, \pi b_{min}^2$$

$$\sim \frac{N}{\pi R^2} \pi \, b_{min}^2$$

$$\sim \frac{N}{R^2} b_{min}^2 \sim \frac{1}{N}$$

sum =

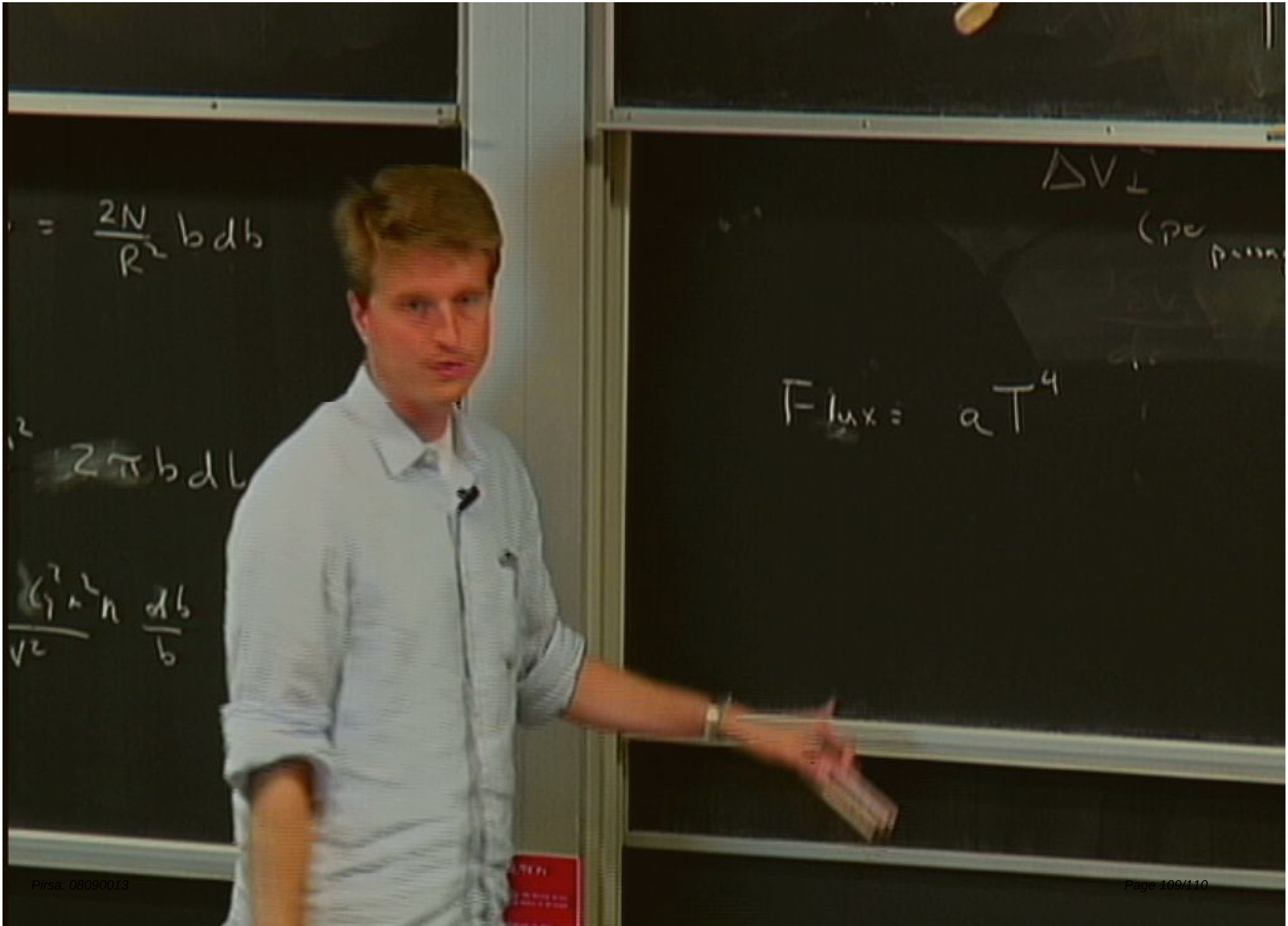
$$\frac{110 \ln N}{N} + \frac{1}{N}$$

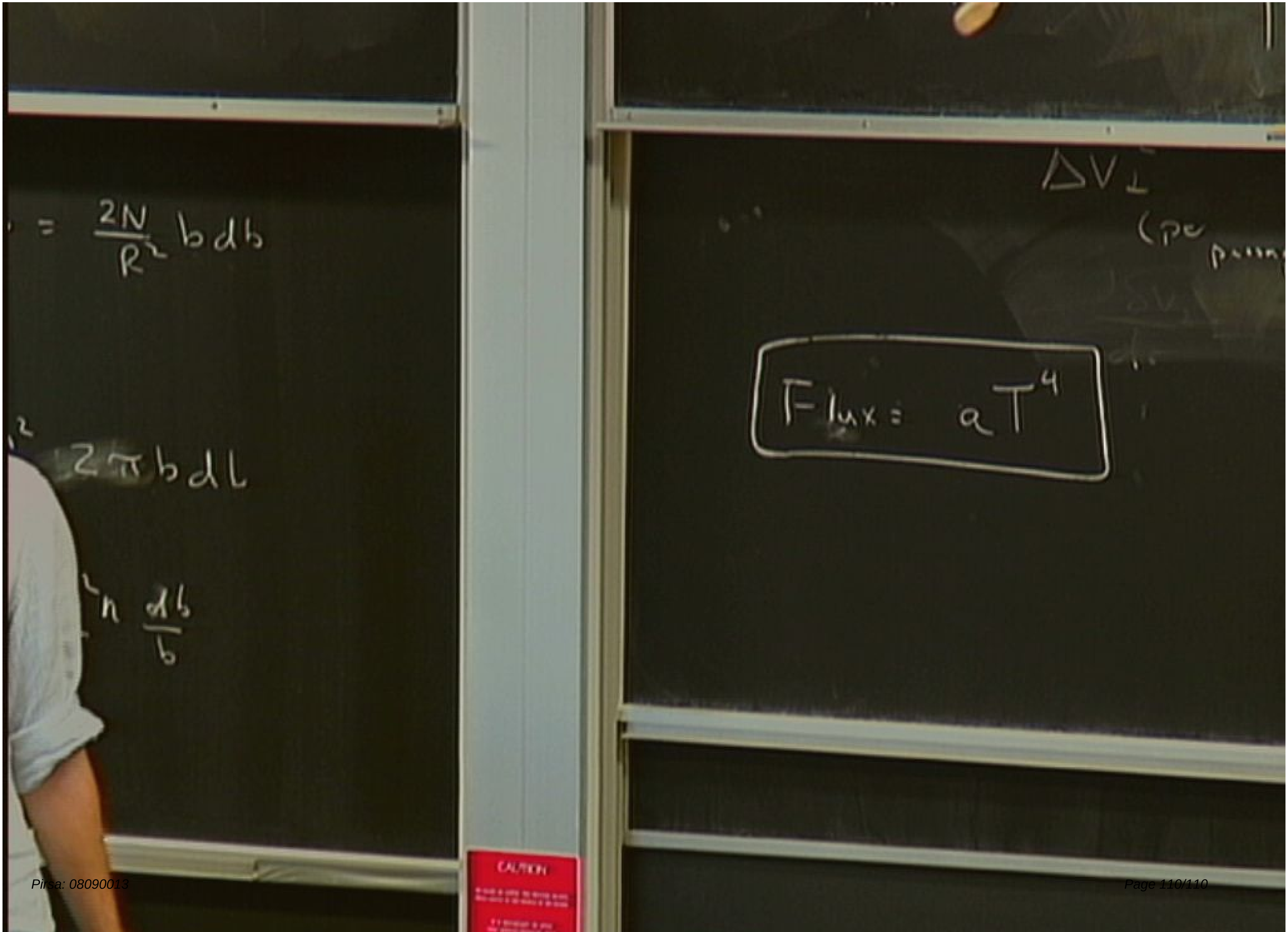
$$\delta n = n\,\pi b_{min}^2$$

$$\sim \frac{N}{\pi R^2}\,\pi\, b_{min}^2$$

$$\sim \frac{N}{R^2}\, b_{min}^2 \sim \frac{1}{N}$$

sum $= \dfrac{10 \ln N}{N} + \dfrac{1}{N}$


= $\frac{2N}{R^2}$ b db
2π b dL
Flux = aT^4
$\Delta V_\perp$


= 2N/R^2 b db
2π b dL
n db/b
ΔV⊥
(pe
Flux = aT^4
CAUTION