

Title: Astrophysics and Cosmology through Problems - 4A

Date: Sep 25, 2008 10:00 AM

URL: <http://pirsa.org/08090009>

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

$$n = \frac{2}{x^2} \quad m = 2x \quad y = 2 \ln x \rightarrow \frac{1}{x^2} \frac{d}{dx}(2x) = \frac{1}{x^2}$$

$$\frac{1}{x^2} \frac{d}{dx} \left( x^2 \frac{dy}{dx} \right) = e^{-y}$$

$$\frac{1}{x^2} \frac{d}{dx} \left( x^2 \frac{2}{x} \right) = \frac{1}{x^2}$$

$$\frac{2}{x^2} \neq \frac{1}{x^2}$$

$$n = \frac{z}{x^2} \quad m = 2x \quad y = 2 \ln x \rightarrow \frac{1}{x^2} \frac{d}{dx}(2x) = \frac{1}{x^2}$$

$$\frac{1}{x^2} \frac{d}{dx} \left( x^2 \frac{dy}{dx} \right) = e^{-y}$$

$$\frac{1}{x^2} \frac{d}{dx} \left( x^2 \frac{z}{x} \right) = \frac{1}{x^2}$$

$$y = 2 \ln x + \ln z$$

$$\frac{z}{x^2} \neq \frac{1}{x^2}$$

$$= e^{-(2 \ln x - \ln z)}$$

$$= e^{-2 \ln x} e^{\ln z}$$

$$= \frac{1}{x^2} \cdot z = \frac{z}{x^2}$$



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$$= e^{-2 \ln x} e^{\ln z}$$

$$= \frac{1}{x^2} \cdot z = \frac{z}{x^2}$$

$$x \rightarrow X = \ln x$$



$$x \rightarrow X = \ln x \Rightarrow \frac{d^2 y}{dX^2} + \frac{dy}{dX} = e^{2X-y}$$

$$y \rightarrow Y = 2X - y \Rightarrow \frac{d^2 Y}{dX^2} + \frac{dY}{dX} = -(e^Y - 2)$$

$$\frac{d^2 Y}{dX^2} = -(e^Y - 2) - \frac{dY}{dX}$$

$$x \rightarrow X = \ln x \Rightarrow \frac{d^2 y}{dX^2} + \frac{dy}{dX} = e^{2X-y}$$

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$$\frac{d^2 Y}{dX^2} = -(e^Y - 2) - \frac{dY}{dX}$$

$$\int e^Y - 2 dY = e^Y - 2Y$$



$$x \rightarrow X = \ln x \Rightarrow \frac{d^2 y}{dX^2} + \frac{dy}{dX} = e^{2X-y}$$

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$$\frac{d^2 Y}{dX^2} = -(e^Y - 2) - \frac{dY}{dX}$$

$$\int e^Y - 2 dY = \int e^Y - 2 Y$$

$$e^Y - 2 = 0$$

$$Y = \ln 2$$

$$2X - y = \ln 2$$

$$2 \ln x - y = \ln 2$$

$$y = 2 \ln x - \ln 2$$



$$V = \frac{m}{x} \quad u = \frac{nx^3}{m}$$

$$\frac{u}{V} \frac{dV}{du} = - \frac{u+1}{u+V-3}$$

$$V = \frac{m}{x} \quad u = \frac{n x^3}{m} \quad V' = \frac{m'}{x} - \frac{m}{x^2} - \frac{m \cdot 3n}{x^2} = \frac{1}{x} \frac{m}{x}$$

$$\frac{u}{V} \frac{dV}{du} = - \frac{u+1}{u+V-3}$$

$$V' = \frac{m}{x} - \frac{1}{x} V$$

$$V' = \frac{uV}{x} - \frac{V}{x}$$



$$\psi \rightarrow \psi + a \quad X \rightarrow e^{a/2} X$$

$$\psi(0) \rightarrow \psi(0) + a$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = g$$

$$\left[ \frac{dv}{dx} = \frac{\dot{v}}{\dot{x}} = \frac{\dot{v}}{v} = \frac{g}{v} \right]$$



$$y \rightarrow y+a \quad x \rightarrow e^{a/2} x$$

$$y(0) \rightarrow y(0)+a$$

$$\frac{dv}{du} = -\frac{v(u-1)}{u(u+v-3)}$$

$$\frac{d^2 x}{dt^2} = \frac{dv}{dt} = g$$

$$(v, u) = (0, 3)$$



$$(v, u) = (0, 3)$$

$$\frac{dv}{du} = -5/3$$

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$$\frac{dv}{du} = -5/3$$

$$u \rightarrow u - 10^{-5}$$

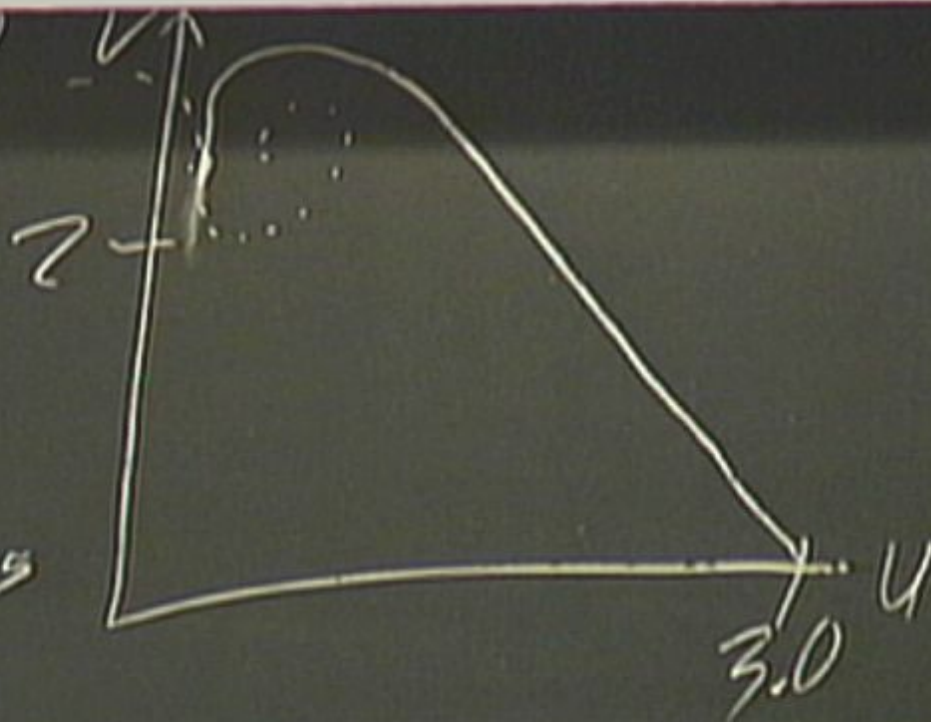
$$v \rightarrow v + \frac{5}{3} 10^{-5}$$

$$(v, u) = (0, 3)$$

$$\frac{dv}{du} = -5/3$$

$$u \rightarrow u + 10^{-5}$$

$$v \rightarrow v + \frac{5}{3} 10^{-5}$$





$$y \rightarrow y+a \quad x \rightarrow e^{a/2} x$$

$$y(0) \rightarrow y(0)+a$$

$$\frac{dv}{du} = \frac{v(u-1)}{u(u+v-3)}$$

$$\frac{dv}{dt} = -u(u-1) +$$

$$\frac{du}{dt} = u(u+v-3) +$$

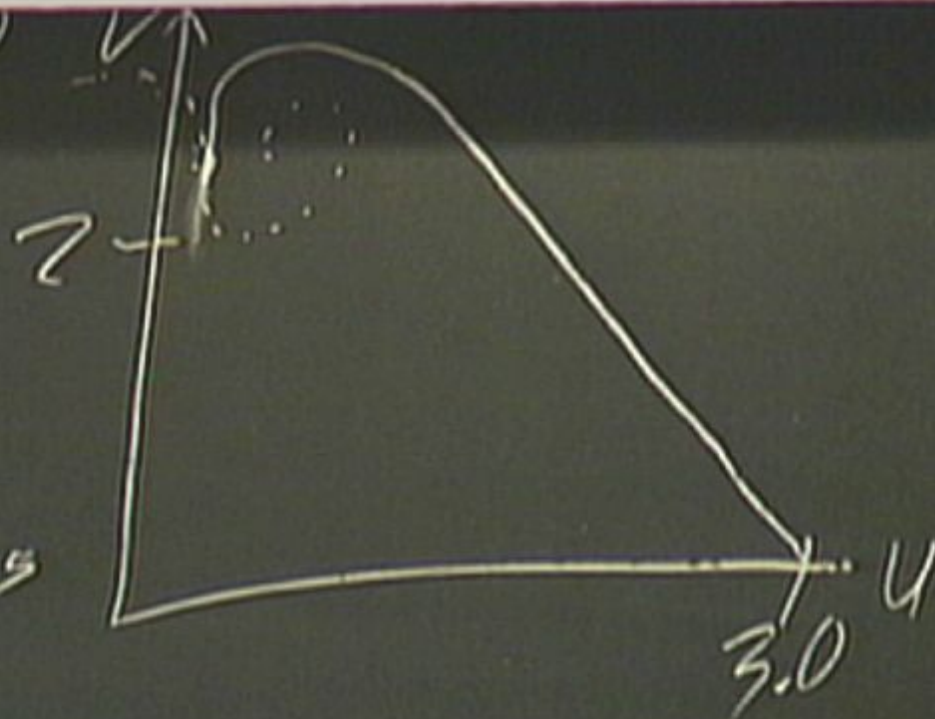
$$\frac{d^2 x}{dt^2} = \frac{dv}{dt} = g$$

$$(v, u) = (0, 3)$$

$$\frac{dv}{du} = -5/3$$

$$u \rightarrow u + 10^{-5}$$

$$v \rightarrow v + \frac{5}{3}10^{-5}$$





$$V = \frac{m}{x} \quad u = \frac{n x^3}{m} \quad V' = \frac{m'}{x} - \frac{m}{x^2} \frac{m x^3 n}{x^2 m} - \frac{1}{x} \frac{m}{x}$$

$$\frac{u}{V} \frac{dV}{du} = - \frac{u+1}{u+V-3}$$

$$V' = \frac{m}{x} - \frac{1}{x} V$$

$$Z = \frac{m}{x} \quad 1 = \frac{n x^3}{m}$$

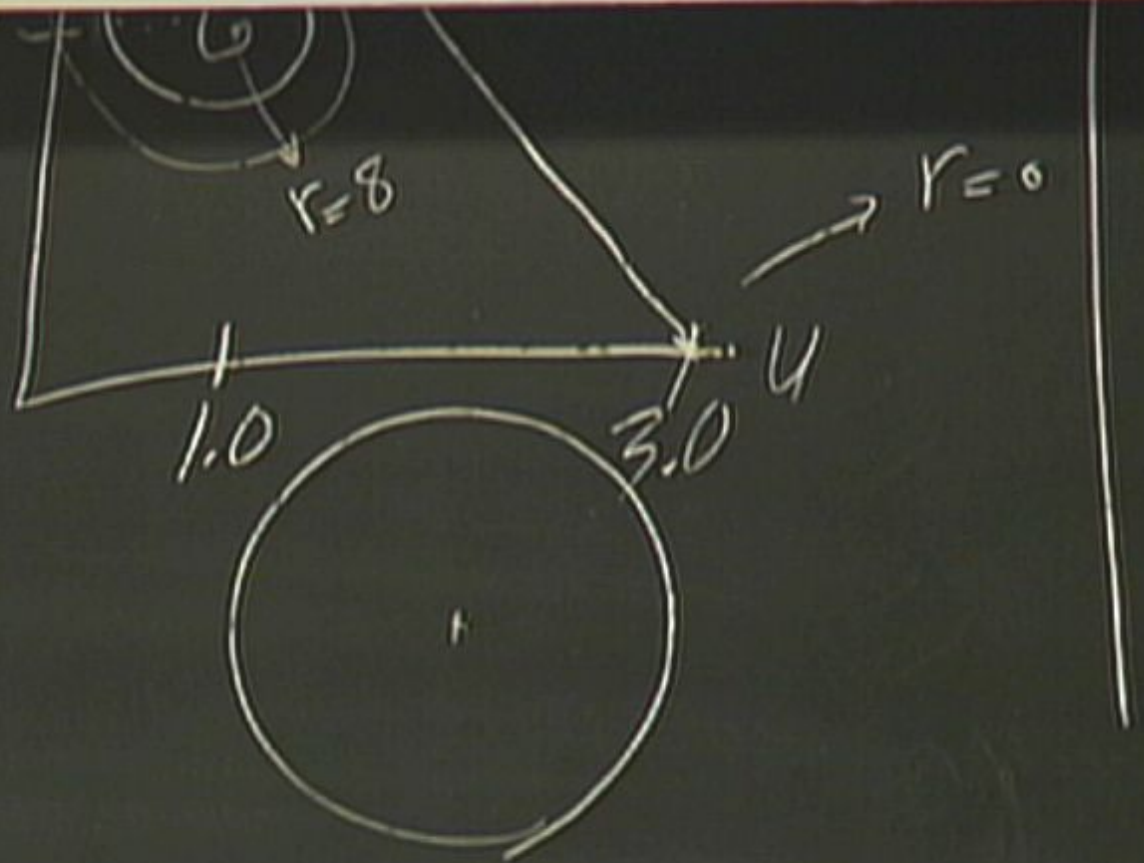
$$V' = \frac{uV}{x} - \frac{V}{x}$$

$$m = Zx \quad n = \frac{m}{x^3} = \frac{Z}{x^2}$$

$$\frac{dV}{dU} =$$

$$U \rightarrow U - 10^{-5}$$

$$V \rightarrow V + \frac{5}{3} 10^{-5}$$





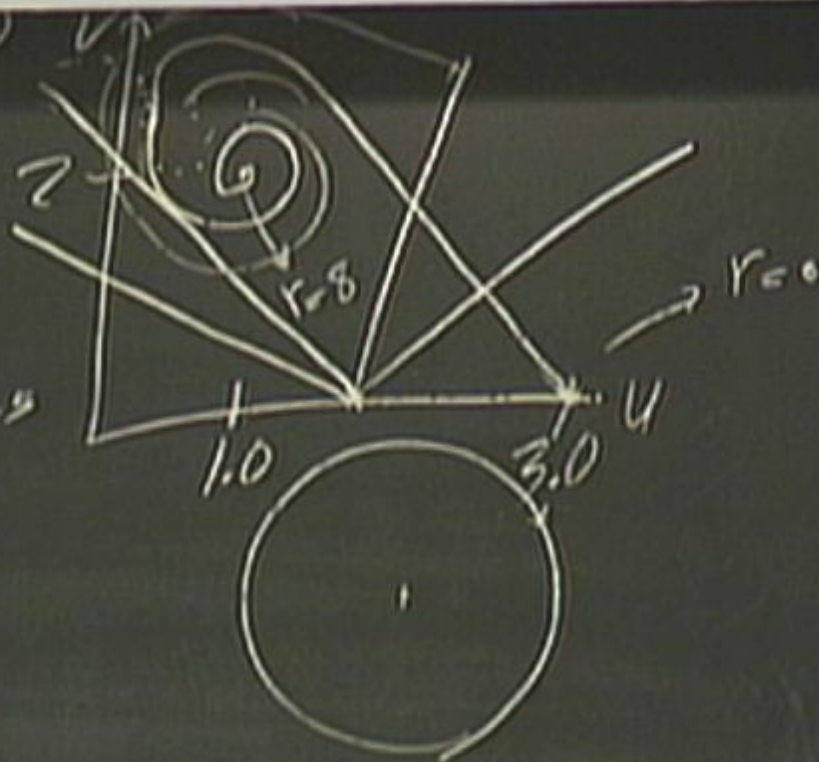
$$\frac{du}{dt} = v(u + v - 3)$$

$$(v, u) = (0, 3)$$

$$\frac{dv}{du} = -5/3$$

$$u \rightarrow u - 10^{-5}$$

$$v \rightarrow v + \frac{5}{3} 10^{-5}$$



$$E = K + U$$

$$\sim \int F(m, n)$$

$$\lambda = \frac{RE}{GM^2} = \frac{1}{v_0} (u_0 - \frac{3}{2})$$

$$u = \lambda v_0 + \frac{3}{2}$$

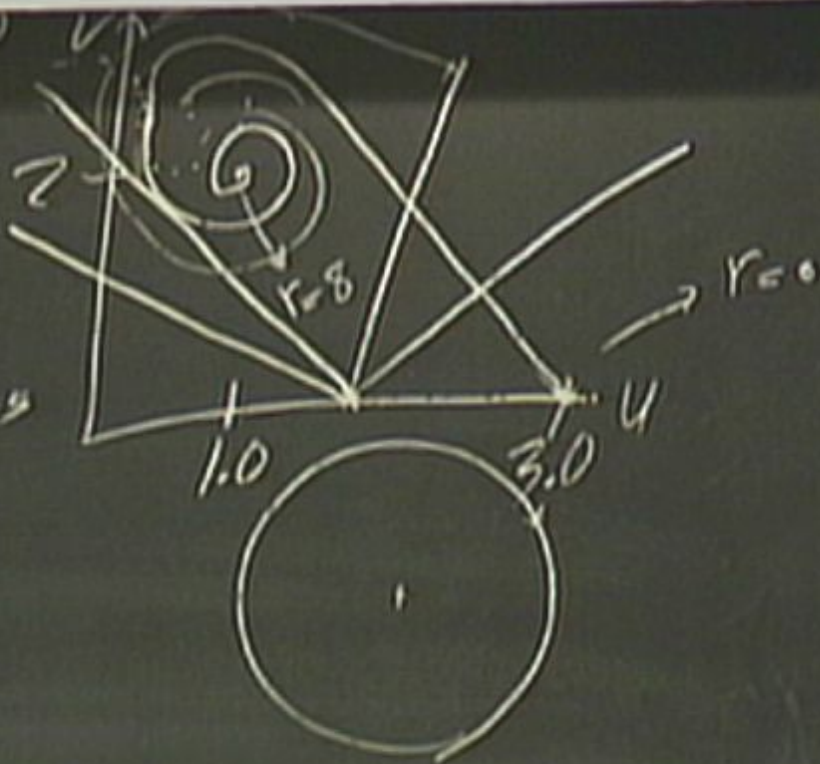
$$\frac{du}{dt} = v(u + v - 3)$$

$$(v, u) = (0, 3)$$

$$\frac{dv}{du} = -5/3$$

$$u \rightarrow u + 10^{-5}$$

$$v \rightarrow v + \frac{5}{3} 10^{-5}$$



$$E = K + U$$

$$\sim \int F(m, n)$$

$$\lambda = \frac{RE}{GM^2} = \frac{1}{v_0} (u_0 - \frac{3}{2})$$

$$u = \lambda v_0 + \frac{3}{2}$$



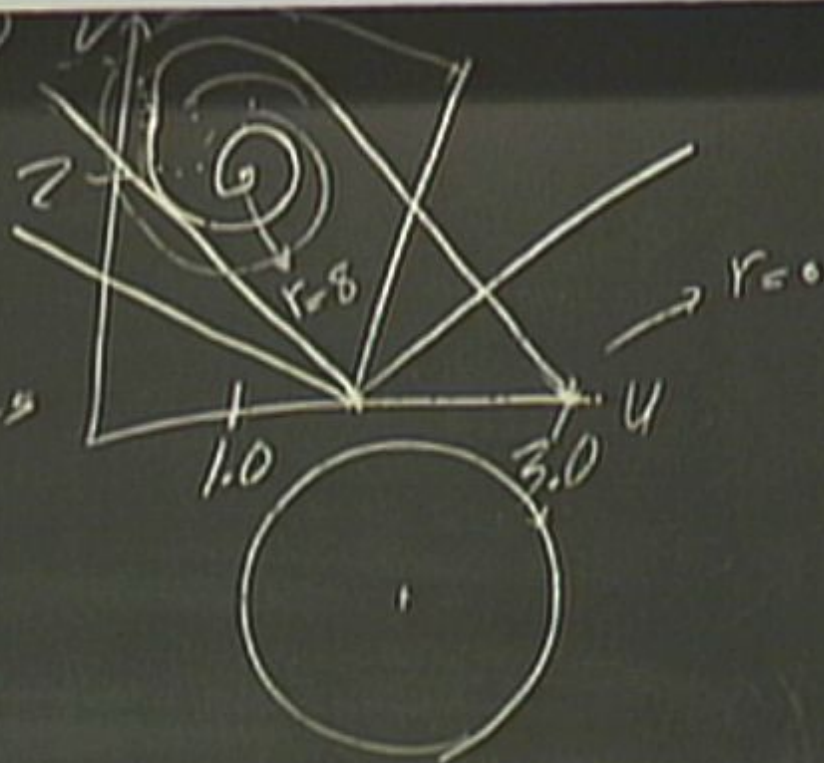
$$\frac{dU}{d\tau} = v(u + v - 3)$$

$$(v, u) = (0, 3)$$

$$\frac{dv}{du} = -5/3$$

$$u \rightarrow u + 10^{-5}$$

$$v \rightarrow v + \frac{5}{3} 10^{-5}$$



$$E = K + U$$

$$\sim \int F(m, n)$$

$$\lambda = \left( \frac{RE}{GM^2} \right) = \frac{1}{v_0} \left( u_0 - \frac{3}{2} \right)$$

$$\boxed{u_0 = \lambda v_0 + \frac{3}{2}}$$

$$\nabla = \frac{\partial}{\partial x}$$

$$x \rightarrow a(t) = vt$$

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho} \nabla p$$



$$\nabla = \frac{\partial}{\partial x}$$

$$x \rightarrow a(t) = vt$$

$$\frac{\partial v}{\partial t} + (v \nabla) v = -\frac{1}{\rho} \nabla p$$

$$dv = dt \frac{\partial v}{\partial t} + da \frac{\partial v}{\partial a}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \cdot \nabla v \rightarrow \frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + v \cdot \nabla \phi$$

$$\frac{dv}{dt} - \cancel{v \nabla v} + \cancel{v \nabla v} = -\frac{1}{\rho} \nabla p$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \nabla p = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{dv}{dt} = -\frac{1}{\rho v} \left( \frac{dp}{dt} - v \nabla p \right) = -\frac{1}{\rho v} \left( \frac{dp}{dt} - v \nabla p \right)$$

$$\Rightarrow \frac{dv}{dt} = -\frac{1}{v(1+v^2)} \frac{dp}{dt}$$

$$\nabla = \frac{\partial}{\partial x}$$

$$x \rightarrow a(t) = vt$$

$$\frac{\partial v}{\partial t} + (v \nabla) v = -\frac{1}{\rho} \nabla p$$

$$dv = dt \frac{\partial v}{\partial t} + da \frac{\partial v}{\partial a}$$

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \cdot \nabla v \rightarrow \frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + v \cdot \nabla \phi$$

$$\frac{dv}{dt} - \cancel{v \cdot \nabla v} + \cancel{v \cdot \nabla v} = -\frac{1}{\rho} \nabla p$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \nabla p = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial t} \frac{\partial t}{\partial x}$$

$$\frac{dv}{dt} = -\frac{1}{\rho v} \left( \frac{dp}{dt} - v \cdot \nabla p \right) = -\frac{1}{\rho v} \left( \frac{dp}{dt} + v \frac{dv}{dt} \right)$$

$$\Rightarrow \frac{dv}{dt} = -\frac{1}{v(1+v^2)} \frac{dp}{dt}$$



$$\nabla = \frac{\partial}{\partial x}$$

$$x \rightarrow a(t) = vt$$

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$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + v \cdot \nabla v \rightarrow \frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + v \cdot \nabla \phi$$

$$\frac{dv}{dt} - \cancel{v \cdot \nabla v} + \cancel{v \cdot \nabla v} = -\frac{1}{\rho} \nabla p$$

$$\frac{dv}{dt} = -\frac{1}{\rho} \nabla p = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial t} \frac{dt}{dx}$$

$$\frac{dv}{dt} = -\frac{1}{\rho v} \left( \frac{dp}{dt} - v \cdot \nabla p \right) = -\frac{1}{\rho v} \left( \frac{dp}{dt} + v \frac{dv}{dt} \right)$$

$$\Rightarrow \frac{dv}{dt} = -\frac{1}{v(1+v^2)} \frac{dp}{dt}$$

$$\frac{\partial}{\partial x} = \frac{\partial t}{\partial x} \frac{\partial}{\partial t}$$

$$\nabla = \frac{\partial}{\partial x} = \left[ \frac{1}{v} \frac{\partial}{\partial t} \right]$$

$$x =$$

3.0



$$\frac{\partial}{\partial x} = \frac{\partial t}{\partial x} \frac{\partial}{\partial t} + \frac{\partial v}{\partial x} \frac{\partial}{\partial v} = \frac{v}{t} \frac{\partial}{\partial v}$$

$$\nabla = \frac{\partial}{\partial x} = \left[ \frac{1}{v} \frac{\partial}{\partial t} \right]$$

$$\frac{dF(u,v)}{dt} = \left. \frac{\partial F}{\partial u} \right|_v \dot{u} + \left. \frac{\partial F}{\partial v} \right|_u \dot{v}$$

$$\frac{\partial}{\partial x} = \frac{\partial t}{\partial x} \frac{\partial}{\partial t} + \frac{\partial v}{\partial x} \frac{\partial}{\partial v} = \frac{\partial}{\partial v}$$

$$\nabla = \frac{\partial}{\partial x} = \left[ \frac{1}{v} \frac{\partial}{\partial t} \right]$$

$$\frac{dF(u,v)}{dt} = \left. \frac{\partial F}{\partial u} \right|_v \dot{u} + \left. \frac{\partial F}{\partial v} \right|_u \dot{v}$$

$r=0$



$$\frac{\partial}{\partial x} = \frac{\partial t}{\partial x} \frac{\partial}{\partial t} + \frac{\partial v}{\partial x} \frac{\partial}{\partial v}$$

$$\nabla = \frac{\partial}{\partial x} = \left[ \frac{1}{v} \frac{\partial}{\partial t} \right]$$

$$(u, v) \rightarrow (t, x)$$

$$r=0$$

$$\frac{dF(u, v)}{dt} = \left. \frac{\partial F}{\partial u} \right|_v \dot{u} + \left. \frac{\partial F}{\partial v} \right|_u \dot{v}$$

$$\left. \frac{\partial F}{\partial t} \right|_x = \left. \frac{\partial F}{\partial u} \right|_v \times \left. \frac{\partial u}{\partial t} \right|_x + \left. \frac{\partial F}{\partial v} \right|_u \left. \frac{\partial v}{\partial t} \right|_x$$

$$\frac{\partial^2 \phi}{\partial t^2} = \left( \frac{\partial P}{\partial \rho} \right)_s \nabla^2 \phi$$

$$c_s = \sqrt{\left( \frac{\partial P}{\partial \rho} \right)_s}$$

$$P_n = \frac{(3\pi^4)^{1/3} \hbar^2}{5m} \left( \frac{Z}{A} \right)^{2/3} \left( \frac{\rho}{m_p} \right)^{2/3} = \lambda_n \rho^{5/3} \Rightarrow c_{s,n} = \frac{5}{3} \lambda_n \rho^{2/3}$$

$$P_r = \lambda_r \rho^{4/3} \Rightarrow c_{s,r} = \frac{4}{3} \lambda_r \rho^{1/3}$$



$$\frac{\partial^2 \varphi}{\partial t^2} = \left( \frac{\partial P}{\partial \rho} \right)_s \nabla^2 \varphi$$

$$c_s = \sqrt{\left( \frac{\partial P}{\partial \rho} \right)_s}$$

$$P_{nr} = \frac{(3\pi^4)^{1/3} \hbar^2}{5m} \left( \frac{z}{\Lambda} \right)^{2/3} \left( \frac{\rho}{m\rho} \right)^{2/3} \equiv \lambda_{nr} \rho^{5/3} \Rightarrow C_{5, nr}^{2/3} \lambda_{nr} \rho^{2/3}$$

$$P_r \equiv \lambda_r \rho^{4/3} \Rightarrow C_{5, r}^2 = \frac{4}{3} \lambda_r \rho^{1/3}$$

$$C_{5, nr} \sim \rho^{1/6} \quad C_{5, r} \sim \rho^{1/6}$$

$$dt \frac{\partial V}{\partial t} + da \frac{\partial V}{\partial a} = \frac{\partial V}{\partial t} + V \cdot \nabla V \rightarrow \frac{d\mathcal{E}}{dt} = \frac{\partial \mathcal{E}}{\partial t} + V \cdot \nabla \mathcal{E} \Rightarrow \frac{dV}{dt} = -\frac{1}{V(1+V^2)} \frac{dP}{dt}$$

$$\frac{\partial^2 \phi}{\partial t^2} = c_s^2 \nabla^2 \phi$$

$$\nabla^2 \phi \rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right)$$



$$\nabla^2 \phi \rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right)$$

$$\phi(r, t) = \frac{1}{r} \psi(r, t)$$

$$\begin{aligned} &= e^{-2 \ln x} \ln z \\ &= e^{-2 \ln x} e^{\ln z} \\ &= \frac{1}{x^2} \cdot z = \frac{z}{x^2} \end{aligned}$$

$$\frac{\partial^2 \psi}{\partial t^2} = c_s^2 \frac{\partial^2 \psi}{\partial r^2}$$

$$\nabla^2 \phi \rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) = \frac{\partial^2 \phi}{\partial r^2}$$

$$\phi(r, t) = \frac{1}{r} \psi(r, t)$$



$$r^+ = r - c_s t$$

$$r^- = r + c_s t$$

$$\frac{\partial^2 \psi}{\partial t^2} = c_s^2 \frac{\partial^2 \psi}{\partial r^2}$$

$$\nabla^2 \phi \rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) = \frac{\partial^2 \phi}{\partial r^2}$$

$$\phi(r, t) = \frac{1}{r} \psi(r, t)$$

$$\psi(r, t) = F(r_+) + G(r_-)$$

$$r^+ = r - c_s t$$

$$r^- = r + c_s t$$

$$\frac{\partial^2 \psi}{\partial t^2} = c_s^2 \frac{\partial^2 \psi}{\partial r^2}$$

$$\nabla^2 \phi \rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) = \frac{\partial^2 \phi}{\partial r^2}$$

$$\phi(r, t) = \frac{1}{r} \psi(r, t)$$