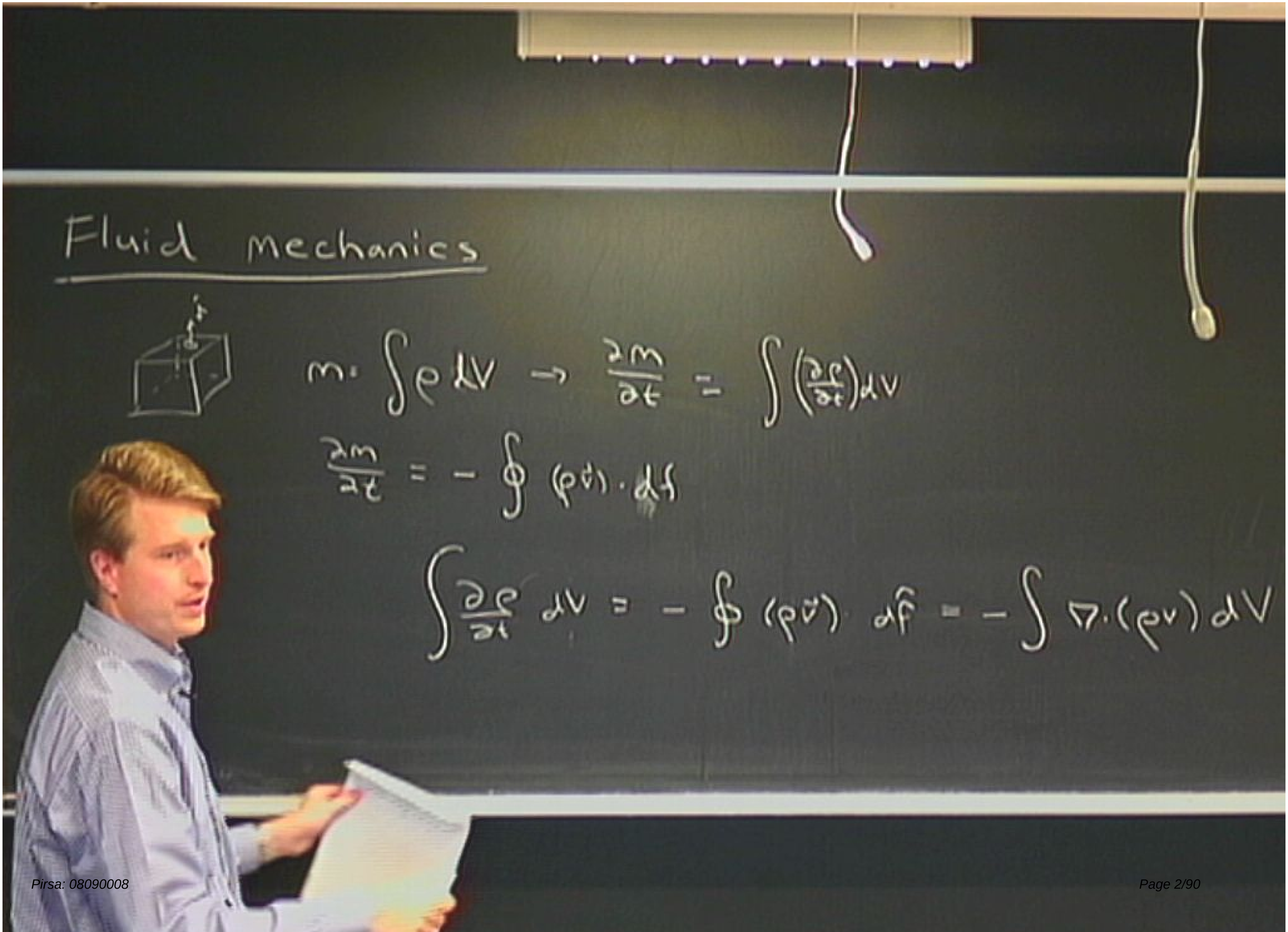
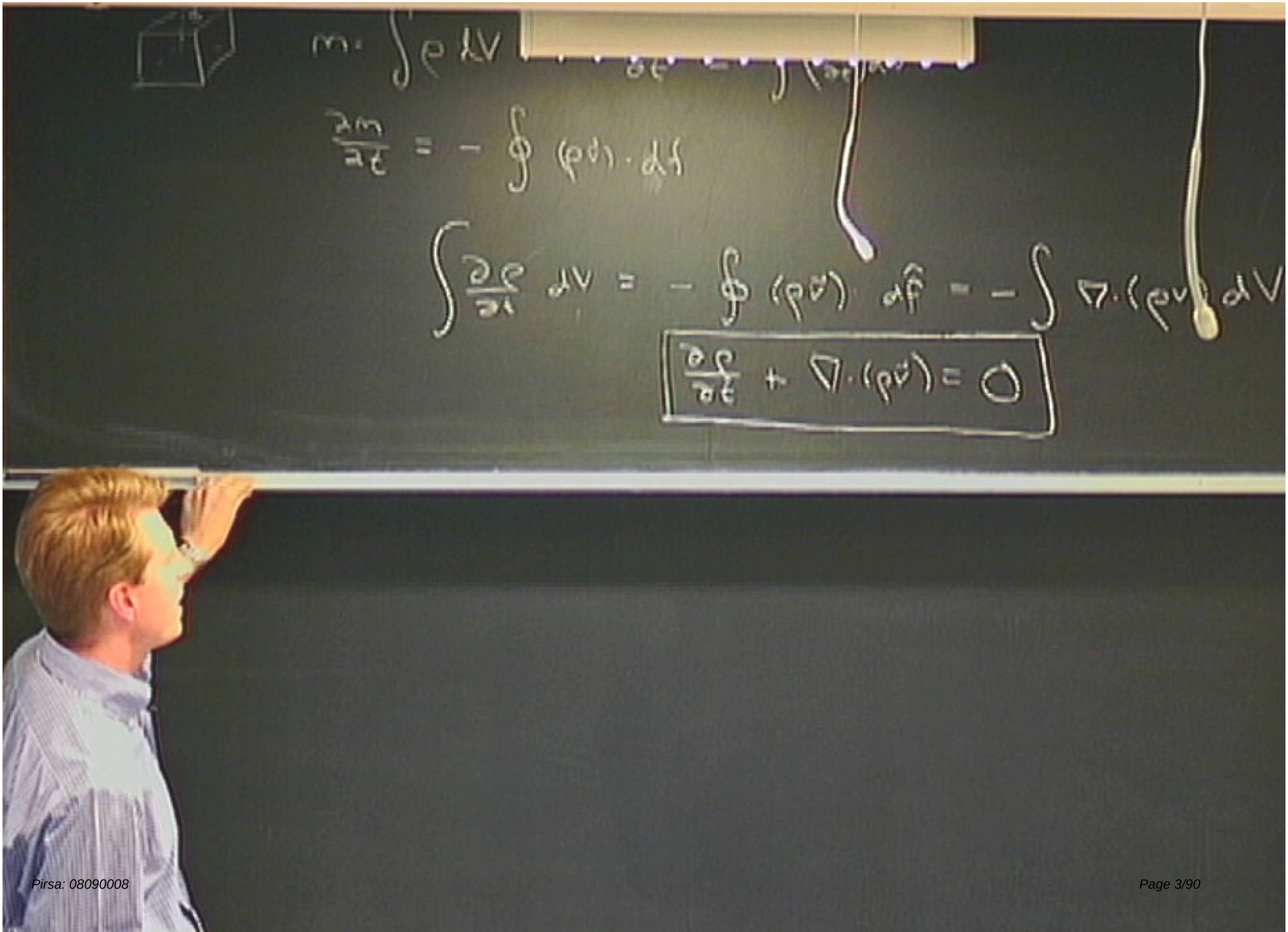
Title: Astrophysics and Cosmology through Problems - 3A

Date: Sep 18, 2008 10:00 AM

URL: http://pirsa.org/08090008

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

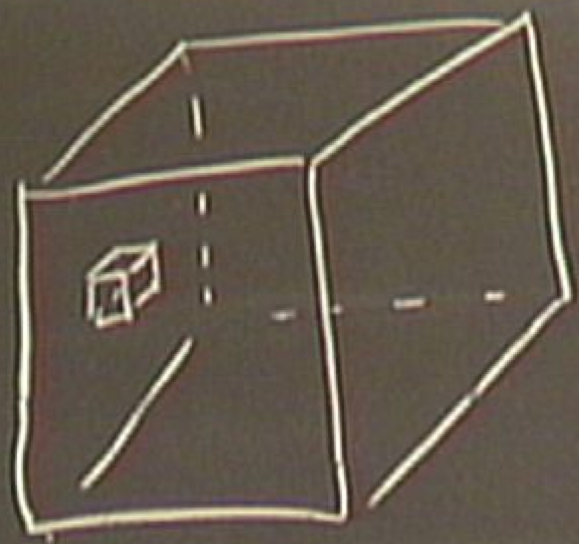

Fluid mechanics
$m = \int \rho \, dV \rightarrow \frac{\partial m}{\partial t} = \int \left(\frac{\partial \rho}{\partial t}\right) dV$
$\frac{\partial m}{\partial t} = - \oint (\rho \vec{v}) \cdot d\vec{f}$
$\int \frac{\partial \rho}{\partial t} \, dV = - \oint (\rho \vec{v}) \, d\vec{f} = - \int \nabla \cdot (\rho v) \, dV$



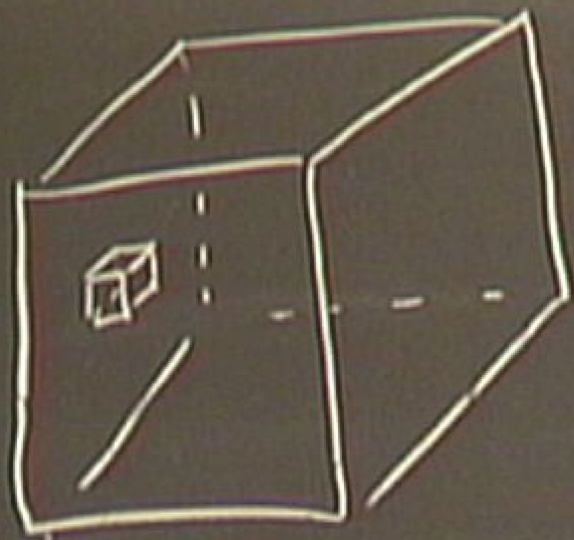
$m = \int \rho \, dV$
$\frac{\partial m}{\partial t} = - \oint (\rho \vec{v}) \cdot d\vec{f}$
$\int \frac{\partial \rho}{\partial t} dV = - \oint (\rho \vec{v}) \cdot d\vec{f} = - \int \nabla \cdot (\rho \vec{v}) \, dV$
$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0}$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0}$$

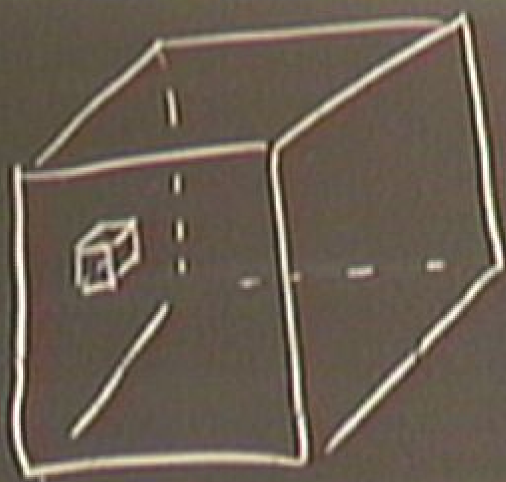
$$\frac{\partial \rho}{\partial t} + \rho(\nabla \cdot v) + (v \cdot \nabla)\rho = 0$$



$$\rho \frac{dv}{dt} =$$



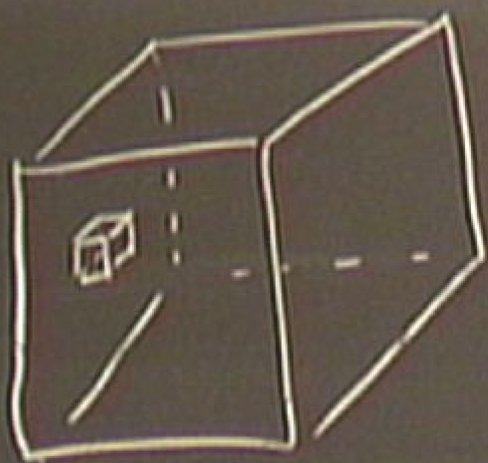
$$\rho \frac{dv}{dt} = - \oint \vec{F} \cdot d\vec{A}$$



$$\rho \frac{d\vec{v}}{dt} = -\oint F \cdot d\vec{A} = -\int (\nabla F)\, dV$$

pressure

$$F = P$$

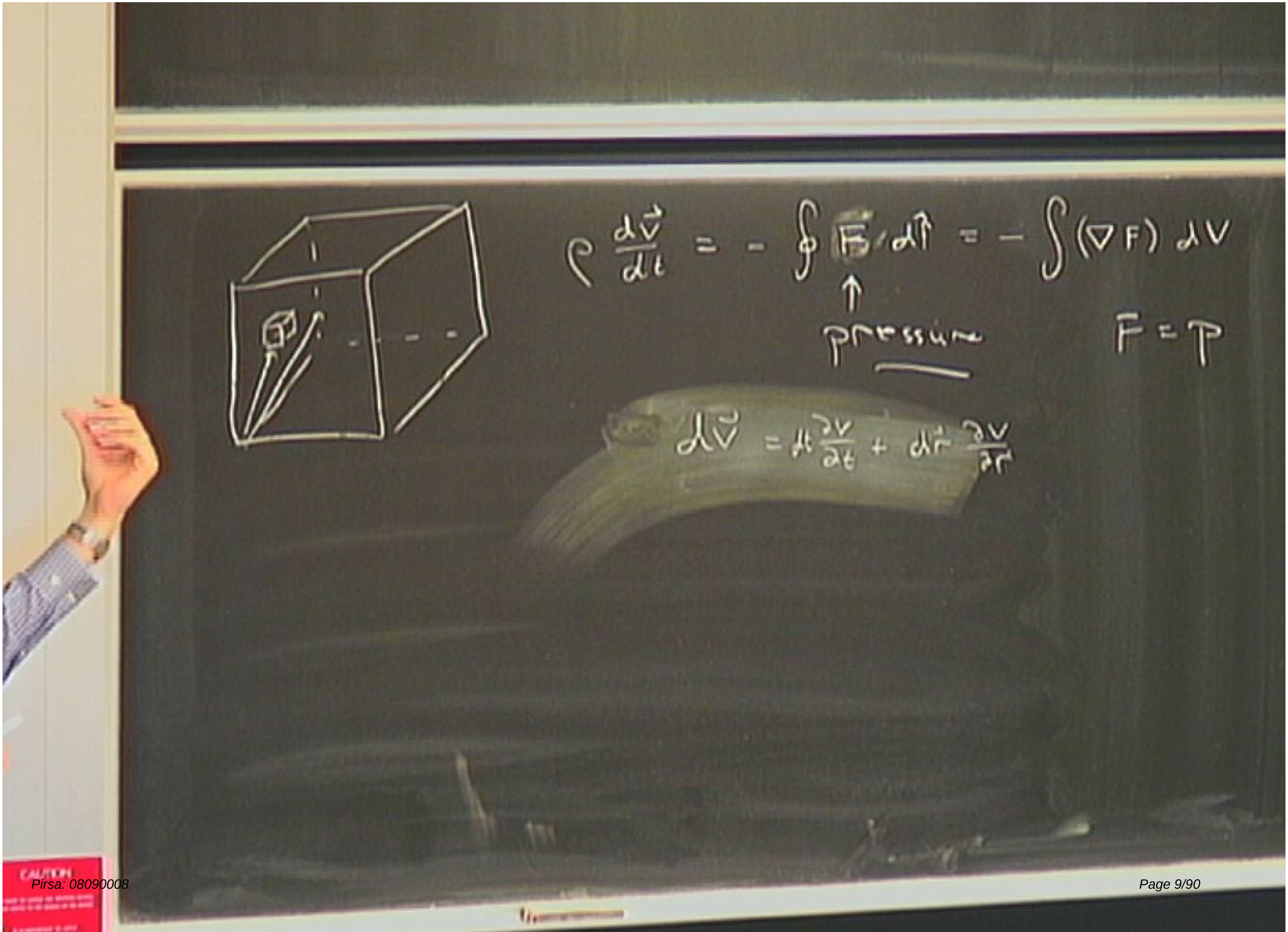


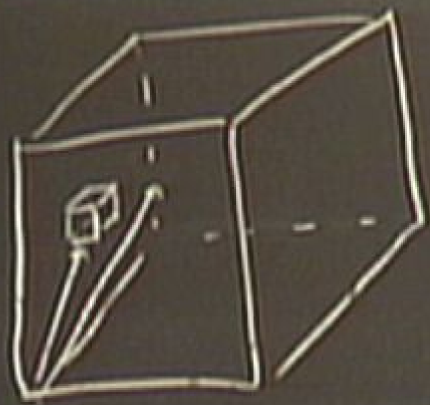
$$\rho \frac{d\vec{v}}{dt} = -\oint F \, d\vec{A} = -\int (\nabla F) \, dV$$

pressure

$$F = P$$

$$\frac{d\vec{v}(\vec{r},t)}{dt}$$


ρ dv/dt = − ∮ F dA = − ∫(∇F) dV
pressure
F = P
dV = dt ∂v/∂t + dr ∂v/∂r



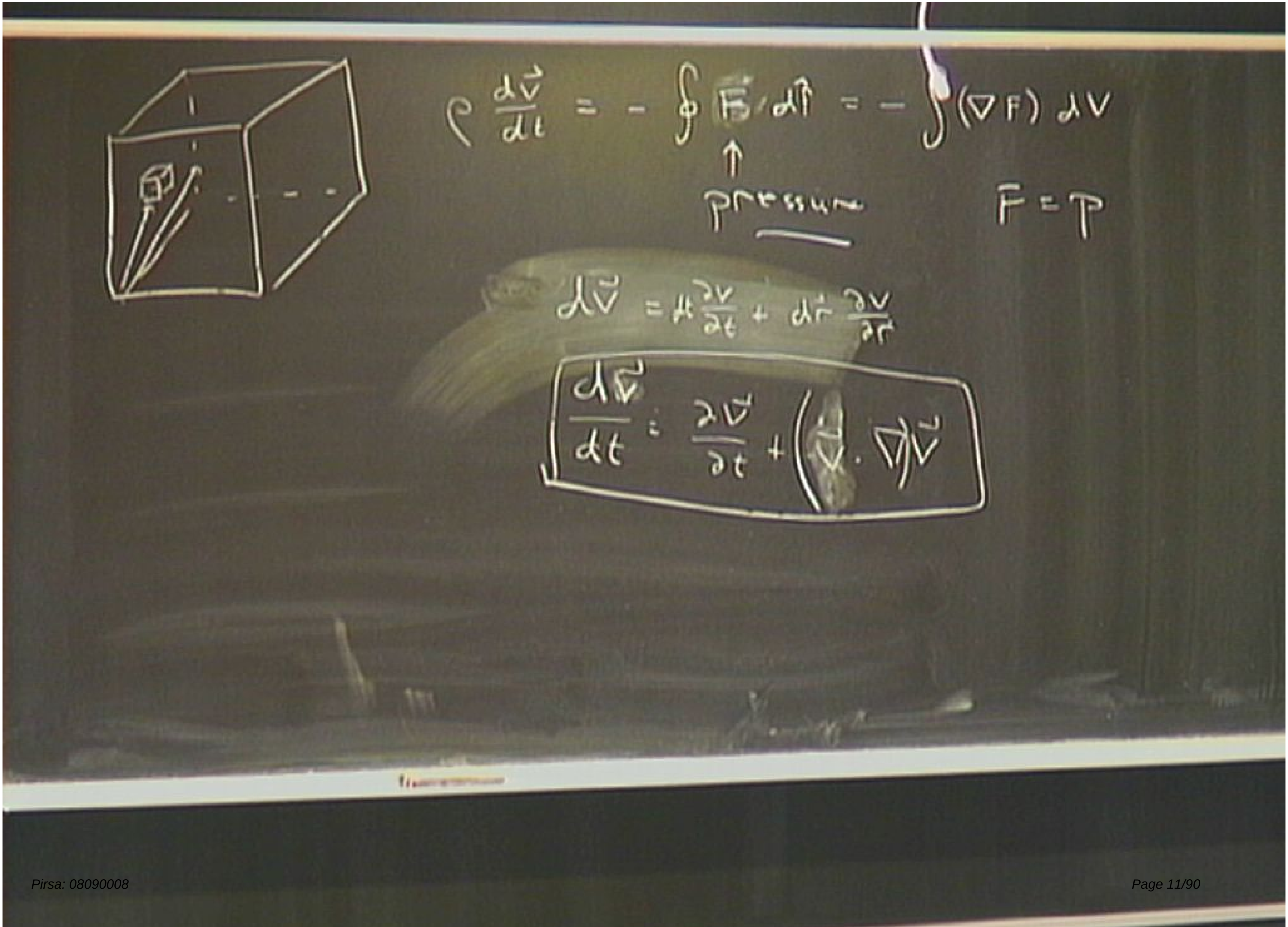
$$\rho \frac{d\vec{v}}{dt} = -\oint P\, d\vec{A} = -\int (\nabla P)\, dV$$

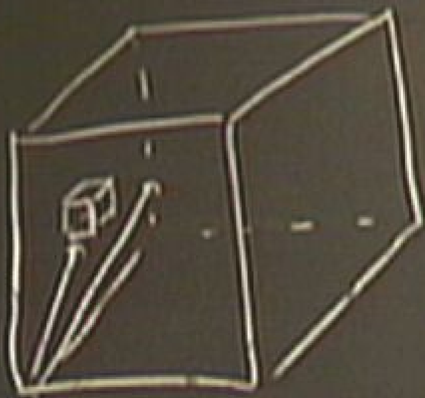
$\uparrow$ pressure

$P = P$

$$d\vec{v} = dt \frac{\partial v}{\partial t} + d\vec{r} \cdot \frac{\partial v}{\partial r}$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}$$


$\rho \frac{d\vec{v}}{dt} = -\oint F \, d\vec{A} = -\int (\nabla F) \, dV$
pressure
$F = P$
$d\vec{v} = dt \frac{\partial v}{\partial t} + d\vec{r} \frac{\partial v}{\partial \vec{r}}$
$\boxed{\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}}$



$$\rho \frac{d\vec{v}}{dt} = -\oint F \, d\vec{A} = -\int (\nabla F)\, dV$$

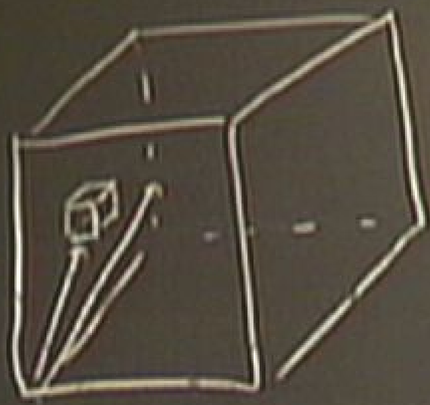
pressure

$F = P$

$$d\vec{v} = dt \frac{\partial v}{\partial t} + d\vec{r} \frac{\partial v}{\partial \vec{r}}$$

$$\boxed{\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}}$$

$$\boxed{\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\,\vec{v} = -\frac{\nabla P}{\rho}}$$



$$\rho \frac{d\vec{v}}{dt} = -\oint \underbrace{F}_{\text{pressure}} \, d\vec{A} = -\int (\nabla F)\, dV$$

$$F = P$$

$$d\vec{v} = dt \frac{\partial v}{\partial t} + d\vec{r} \frac{\partial v}{\partial \vec{r}}$$

$$\boxed{\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}}$$

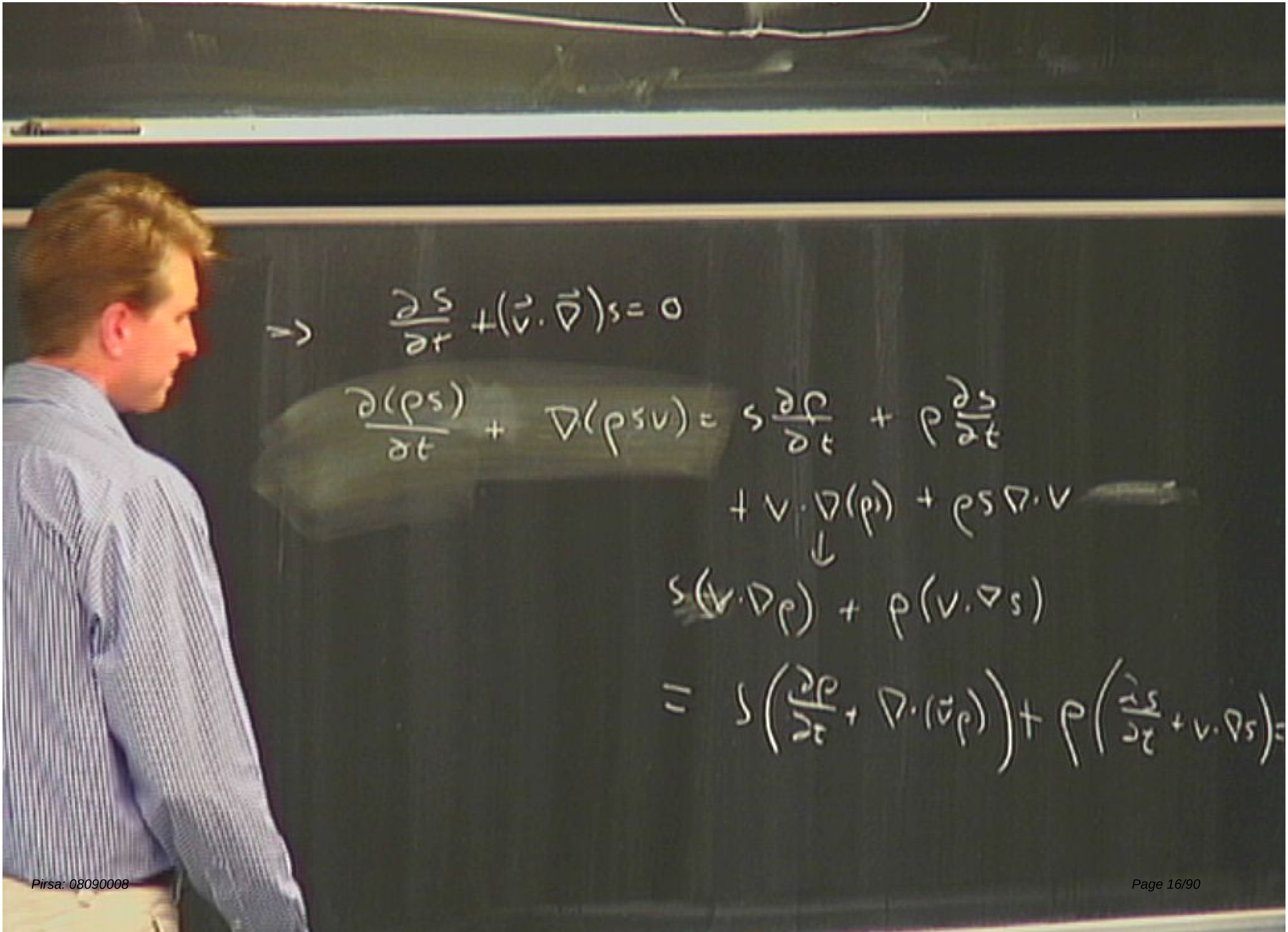
Euler's Eqn.

$$\boxed{\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\, \vec{v} = -\frac{\vec{\nabla} P}{\rho} \left(+ \vec{g} \right)}$$

$$\frac{dS}{dt} = 0 \quad \rightarrow$$

$$\frac{ds}{dt} = 0 \quad \rightarrow \quad \frac{\partial s}{\partial t} + (\vec{v} \cdot \vec{\nabla}) s = 0$$

$(\rho s \vec{v}$


$\Rightarrow \quad \frac{\partial s}{\partial t} + (\vec{v}\cdot\vec{\nabla})s = 0$
$\frac{\partial(\rho s)}{\partial t} + \nabla\cdot(\rho s v) = s\frac{\partial \rho}{\partial t} + \rho\frac{\partial s}{\partial t}$
$+ v\cdot\nabla(\rho s) + \rho s\nabla\cdot v$
$\downarrow$
$s(v\cdot\nabla\rho) + \rho(v\cdot\nabla s)$
$= s\left(\frac{\partial \rho}{\partial t} + \nabla\cdot(\vec{v}\rho)\right) + \rho\left(\frac{\partial s}{\partial t} + v\cdot\nabla s\right) =$

$$\frac{ds}{dt} = 0 \quad \to \quad \frac{\partial s}{\partial t} + (\vec{v}\cdot\vec{\nabla})s = 0$$

$$\boxed{0 = \frac{\partial(\rho s)}{\partial t} + \nabla(\rho s v)} = s\frac{\partial \rho}{\partial t} + \rho\frac{\partial s}{\partial t}$$

$$+ v\cdot\nabla(\rho) + \rho s \nabla\cdot v$$

$$\Downarrow$$

$$s(v\cdot\nabla\rho) + \rho(v\cdot\nabla s)$$

$$= s\left(\frac{\partial \rho}{\partial t} + \nabla\cdot(\vec{v}\rho)\right) + \rho\left(\frac{\partial s}{\partial t} + v\cdot\nabla s\right) =$$

$$\frac{ds}{dt} = 0 \quad \Rightarrow \quad \frac{\partial s}{\partial t} + (\vec{v}\cdot\vec{\nabla})s = 0$$

$$\boxed{0 = \frac{\partial(\rho s)}{\partial t} + \nabla(\rho s v)} = s\frac{\partial \rho}{\partial t} + \rho\frac{\partial s}{\partial t}$$

$$+ v\cdot\nabla(\rho) + \rho s \nabla\cdot v$$

$$\Downarrow$$

$$s(v\cdot\nabla\rho) + \rho(v\cdot\nabla s)$$

$$= s\left(\frac{\partial\rho}{\partial t} + \nabla\cdot(\vec{v}\rho)\right) + \rho\left(\frac{\partial s}{\partial t} + v\cdot\nabla s\right) = 0$$

$S = \text{const.}$

$$dw = T\cancel{dS}^{0} + V\,dp \qquad V \equiv \frac{1}{\rho}$$

$$dw = \frac{dp}{\rho}$$

$S = \text{const.}$

$$dw = T\,\overset{0}{dS} + V\,dp \qquad V \equiv \frac{1}{\rho}$$

$$dw = \frac{dp}{\rho}$$

$$\frac{\partial \vec{v}}{\partial t} + (v \cdot \nabla)\vec{v} = -\nabla w$$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\nabla v^2 - v \times \nabla \times v = -\nabla w$$

$$v \times \nabla \times v = \frac{1}{2}\nabla v^2 - (v \cdot \nabla)v$$

$$(v \cdot \nabla)v = \frac{1}{2}\nabla v^2 - v \times \nabla \times v$$

$S = \text{const.}$

$$dw = T\,\overset{0}{\cancel{dS}} + V\,dp \qquad V \equiv \frac{1}{\rho}$$

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$$\frac{\partial \vec{v}}{\partial t} + (v\cdot\nabla)\vec{v} = -\nabla w$$

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$$v\times\nabla\times v = \frac{1}{2}\nabla v^2 - (v\cdot\nabla)v$$

$$(v\cdot\nabla)v = \frac{1}{2}\nabla v^2 - v\times\nabla\times v$$

$S = \text{const.}$

$$dw = T\,\overset{0}{\cancel{dS}} + V\,dp \qquad V \equiv \frac{1}{\rho}$$

$$dw = \frac{dp}{\rho}$$

$\times \nabla \phi = 0$

$$\frac{\partial \vec{v}}{\partial t} + (v \cdot \nabla)\vec{v} = -\nabla w$$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\cancel{\nabla}(v^2) - v \times \nabla \times v = -\cancel{\nabla}(w)$$

$$\frac{\partial}{\partial t}(\nabla \times v)$$

$$v \times \nabla \times v = \frac{1}{2}\nabla v^2 - (v \cdot \nabla) v$$

$$(v \cdot \nabla) v = \frac{1}{2}\nabla v^2 - v \times \nabla \times v$$

$S = \text{const.}$

$$dw = T\,\overset{0}{\cancel{dS}} + V\,dp \qquad V \equiv \frac{1}{\rho}$$

$$dw = \frac{dp}{\rho}$$

$\nabla \times \nabla \phi = 0$

$$\frac{\partial \vec{v}}{\partial t} + (v \cdot \nabla)\vec{v} = -\nabla w$$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\cancel{\nabla} v^2 - v \times \nabla \times v = -\cancel{\nabla} w$$

$$\frac{\partial}{\partial t}(\nabla \times v) = \nabla \times v \times \nabla \times v$$

$$v \times \nabla \times v = \frac{1}{2}\nabla v^2 - (v \cdot \nabla) v$$

$$(v \cdot \nabla) v = \frac{1}{2}\nabla v^2 - v \times \nabla \times v$$

$S = \text{const.}$

$$dw = T\,\overset{0}{dS} + V\,dp \qquad V \equiv \frac{1}{\rho}$$

$$dw = \frac{dp}{\rho}$$

$\nabla \times \nabla \phi = 0$

$$\frac{\partial \vec{v}}{\partial t} + (v \cdot \nabla)\vec{v} = -\nabla w$$

$$\frac{\partial v}{\partial t} + \frac{1}{2}\cancel{\nabla} v^2 - v \times \nabla \times v = -\cancel{\nabla} w$$

$$\boxed{\frac{\partial}{\partial t}(\nabla \times v) = \nabla \times v \times \nabla \times v}$$

$$v \times \nabla \times v = \frac{1}{2}\nabla v^2 - (v \cdot \nabla)v$$

$$(v \cdot \nabla)v = \frac{1}{2}\nabla v^2 - v \times \nabla \times v$$

$S = \text{const.}$

$dw = T\cancel{dS}^{\,0} + V\,dp \qquad V = \frac{1}{\rho}$

$dw = \frac{dp}{\rho}$

$\nabla\times\nabla\phi = 0 \qquad \frac{\partial \vec{v}}{\partial t} + (v\cdot\nabla)\vec{v} = -\nabla w$

$\frac{\partial v}{\partial t} + \frac{1}{2}\cancel{\nabla} v^2 - v\times\nabla\times v = -\cancel{\nabla} w$

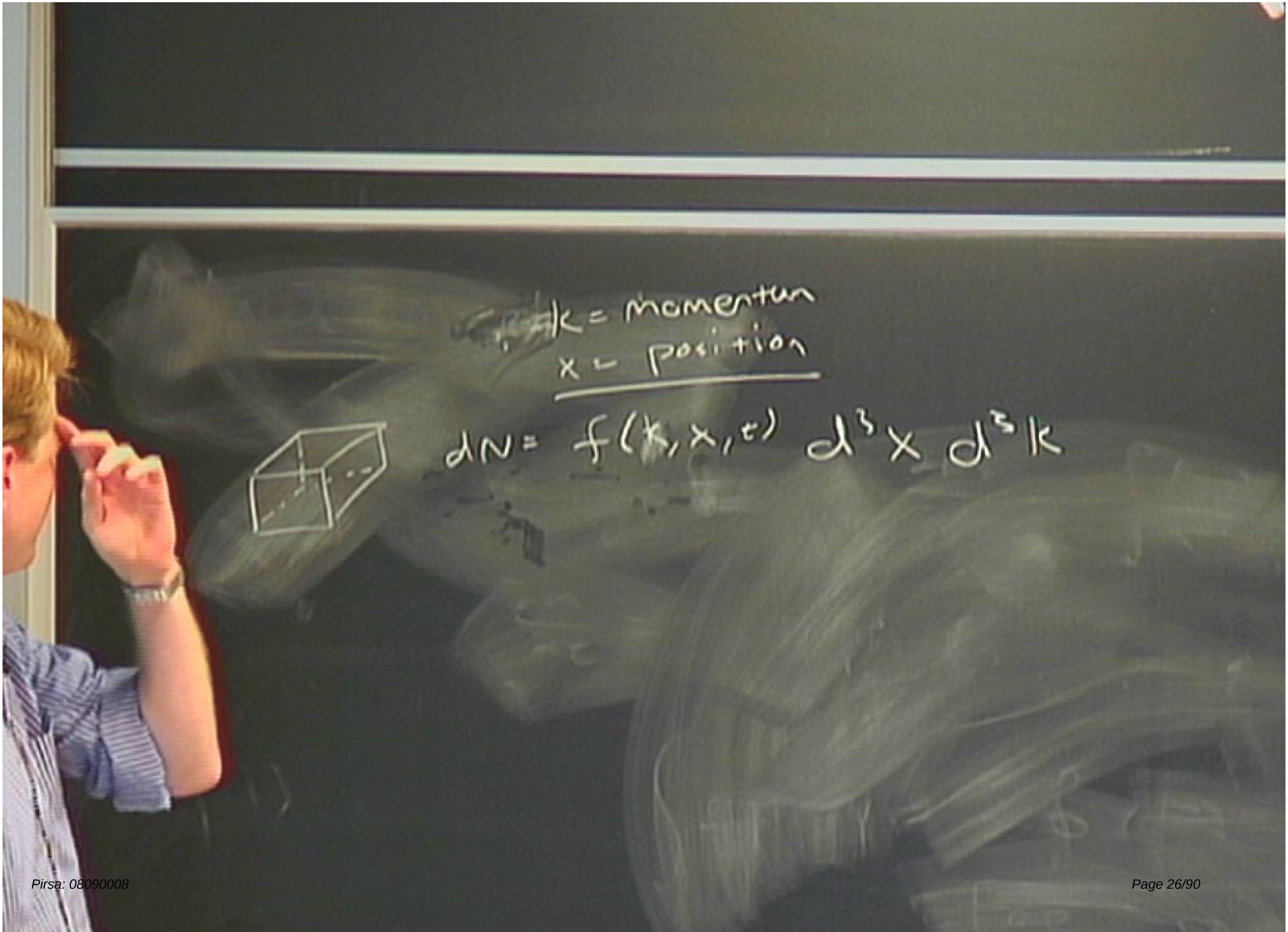
$\boxed{\frac{\partial}{\partial t}(\nabla\times v) = \nabla\times v\times\nabla\times v}$

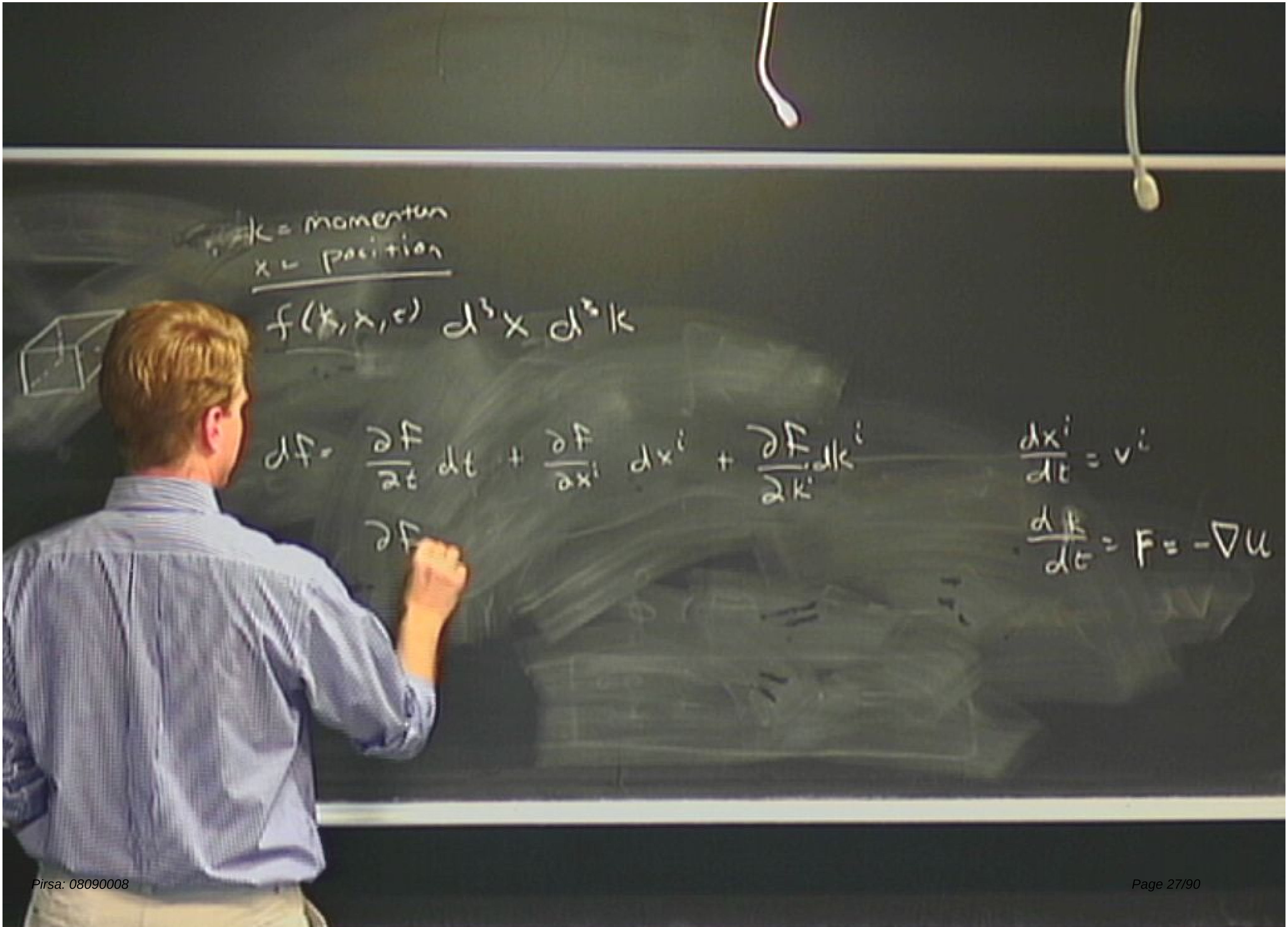
$\nabla\times v = \Omega$

$\boxed{\frac{\partial\Omega}{\partial t} = \nabla\times v\times\Omega}$

$v\times\nabla\times v = \frac{1}{2}\nabla v^2 - (v\cdot\nabla)v$

$(v\cdot\nabla)v = \frac{1}{2}\nabla v^2 - v\times\nabla\times v$


k = momentum
x = position
dN = f(k, x, t) d³x d³k


k = momentum
x = position
f(k, x, t) d³x d³k
df = ∂f/∂t dt + ∂f/∂xⁱ dxⁱ + ∂f/∂kⁱ dkⁱ
∂f
dxⁱ/dt = vⁱ
dk/dt = F = −∇U

k = momentum

x = position

$$dN = f(k, x, t)\, d^3x\, d^3k$$

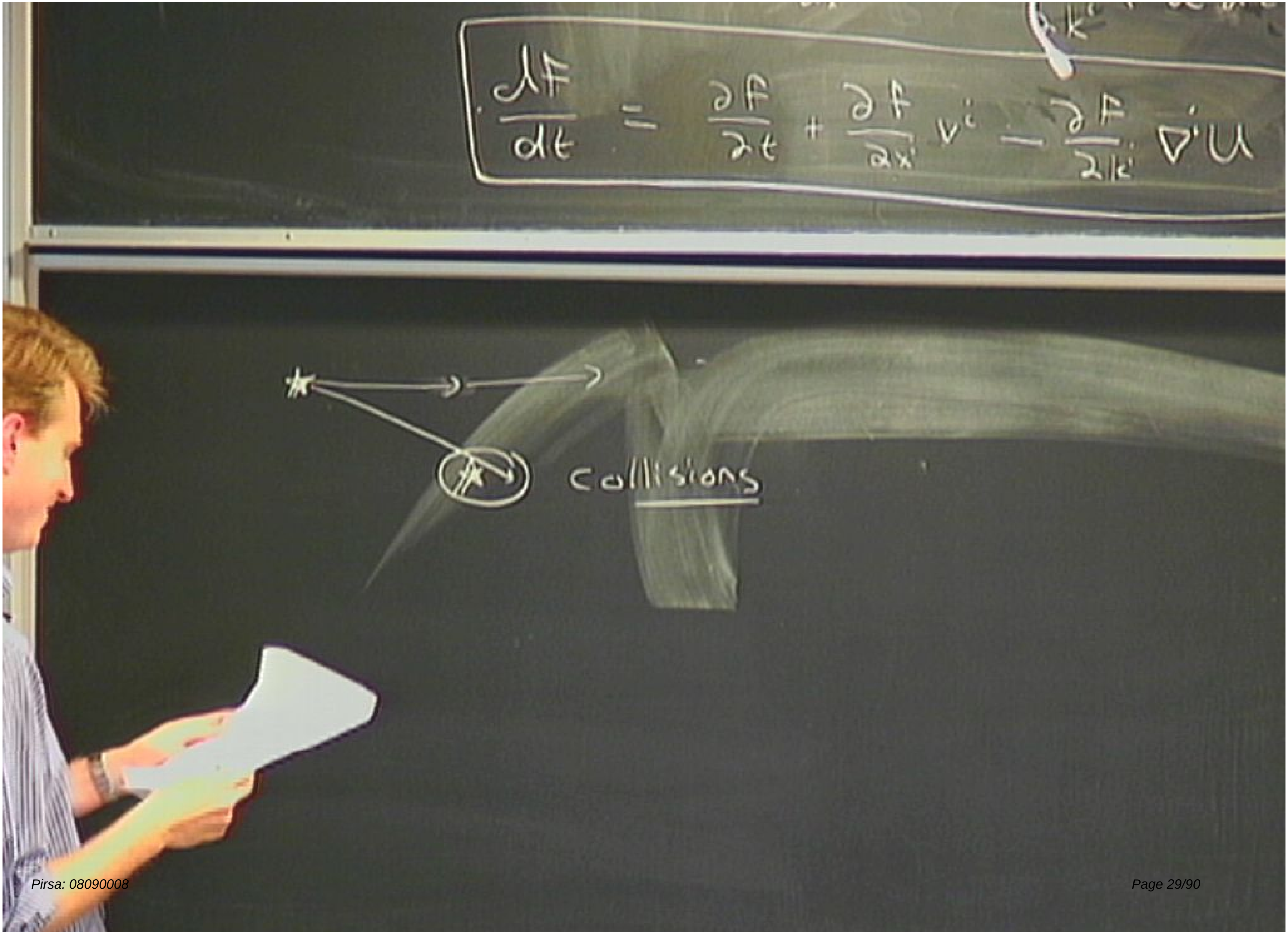
$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x^i} dx^i + \frac{\partial f}{\partial k^i} dk^i$$

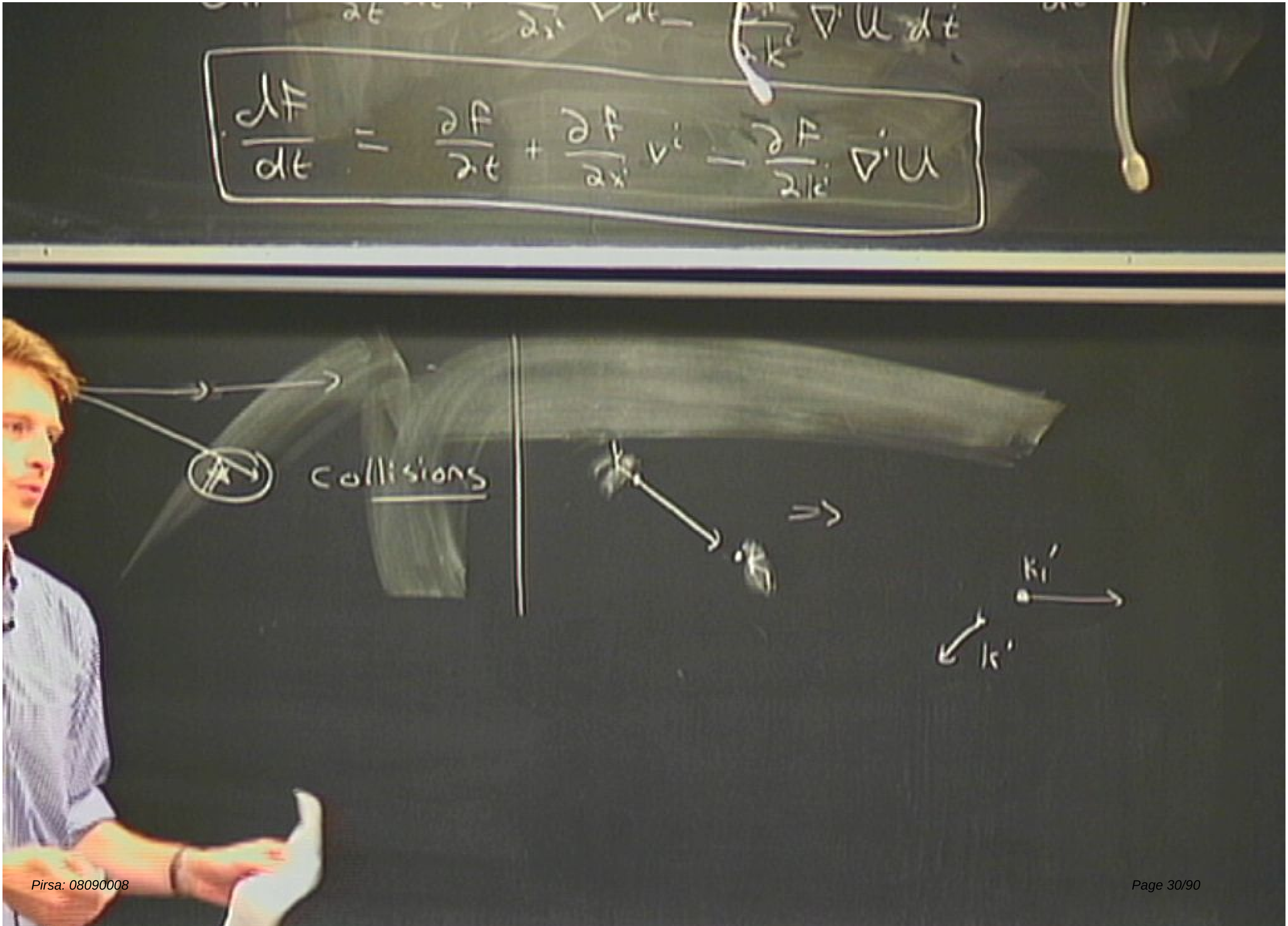
$$\frac{dx^i}{dt} = v^i$$

$$\frac{dk}{dt} = F = -\nabla U$$

$$df = \frac{\partial f}{\partial t} dt + \frac{\partial f}{\partial x^i} v^i dt - \frac{\partial f}{\partial k^i} \nabla^i U\, dt$$

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} v^i - \frac{\partial f}{\partial k^i} \nabla^i U}$$


$\frac{dF}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} v^i - \frac{\partial F}{\partial k^i} \nabla^i U$
collisions


$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} v^i - \frac{\partial f}{\partial k^i} \nabla^i U$
collisions
k
$\Rightarrow$
k_1'
k'

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} v^i - \frac{\partial f}{\partial k^i} \nabla^i U}$$

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} v^i - \frac{\partial f}{\partial k^i} \nabla^i U}$$

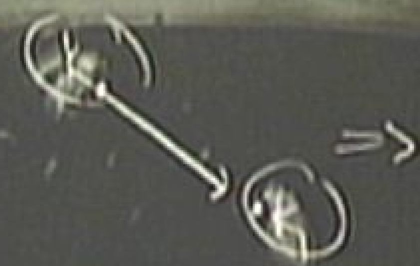
$\underline{dR_{out}}$

Collisions

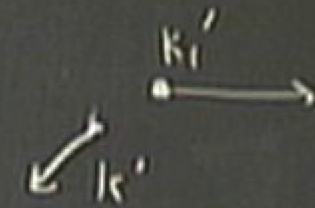
$$\frac{dR_{out}}{d^3x_1\, d^3p_1\, d^3x\, d^3p}$$

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} v^i - \frac{\partial f}{\partial k^i} \nabla^i U}$$

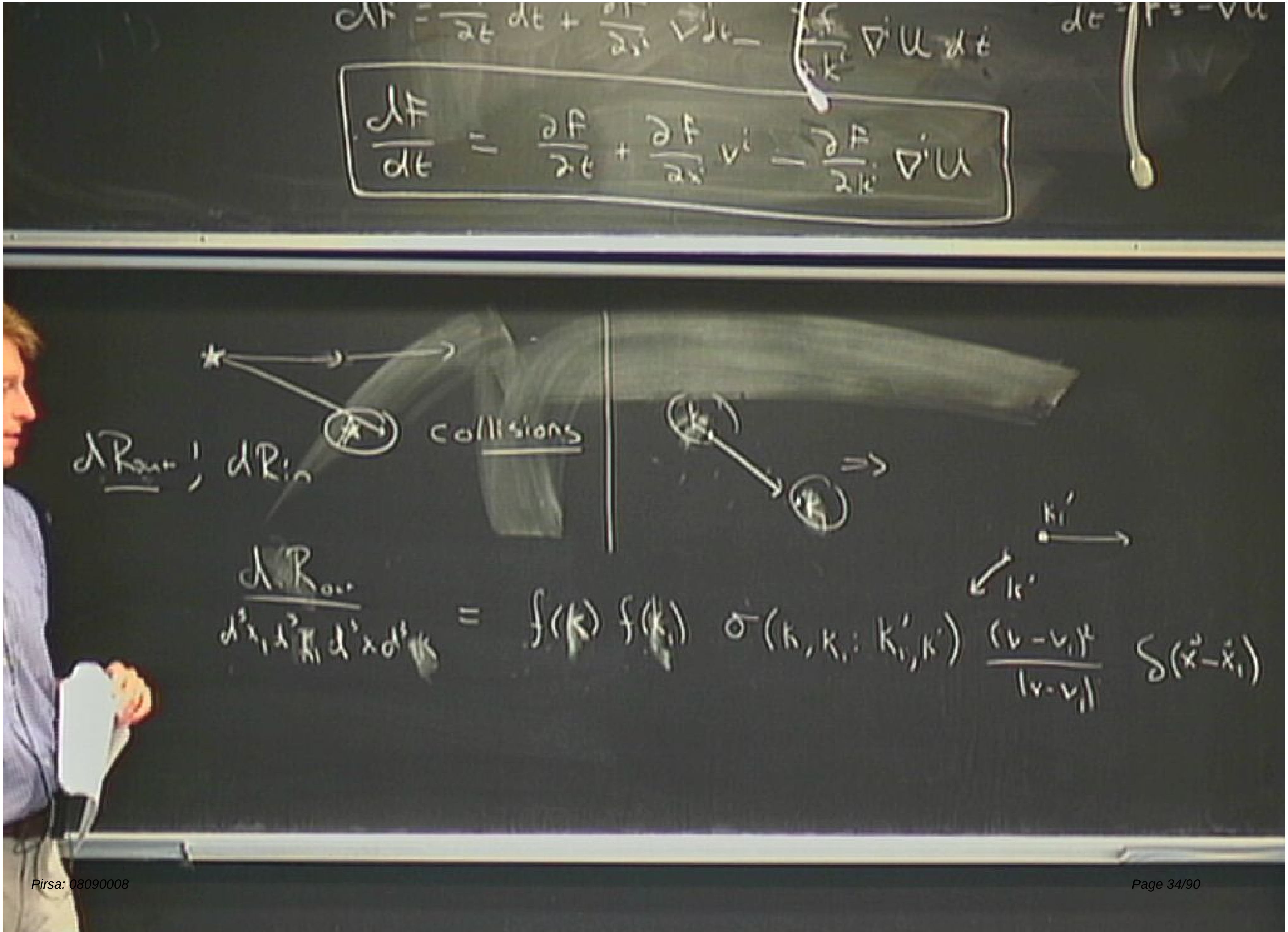
collisions

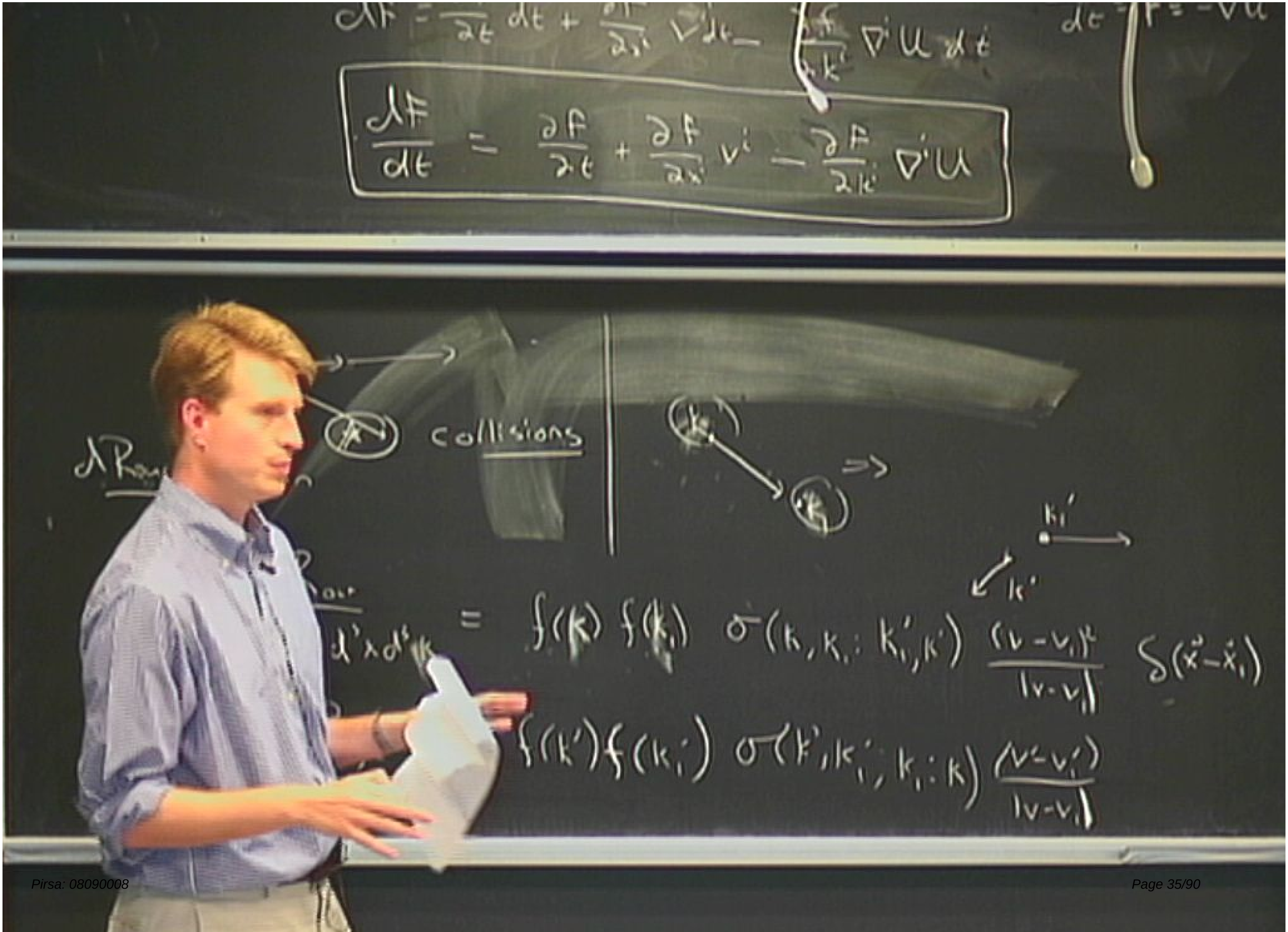


dR_{out} ; dR_{in}



$$\frac{dR_{out}}{d^3x_1\, d^3k_1\, d^3x\, d^3k} = f(k)\, f(k_1)$$


dF = ∂F/∂t dt + ∂F/∂x^i v^i dt − ∂F/∂k^i ∇^i U dt
dF/dt = ∂F/∂t + ∂F/∂x^i v^i − ∂F/∂k^i ∇^i U
collisions
dR_out ; dR_in
k_1'
k'
dR_out / (d^3x_1 d^3k_1 d^3x d^3k) = f(k) f(k_1) σ(k, k_1 : k_1', k') (v − v_1)^2 / |v − v_1| δ(x − x_1)


$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\partial F}{\partial x^i} v^i - \frac{\partial F}{\partial k^i} \nabla^i U$
collisions
$= f(k) f(k_1)\ \sigma(k, k_1 : k_1', k')\ \frac{(v - v_1)^2}{|v - v_1|}\ \delta(\vec{x} - \vec{x}_1)$
$f(k') f(k_1')\ \sigma(k', k_1' ; k_1 : k)\ \frac{(v' - v_1')}{|v - v_1|}$

$$dF = \frac{\partial f}{\partial t}\,dt + \frac{\partial f}{\partial x^i}\,v^i\,dt - \frac{\partial f}{\partial k^i}\,\nabla^i U\,dt \qquad dt \quad F = -\nabla U$$

$$\boxed{\frac{dF}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\,v^i - \frac{\partial F}{\partial k^i}\,\nabla^i U}$$

collisions

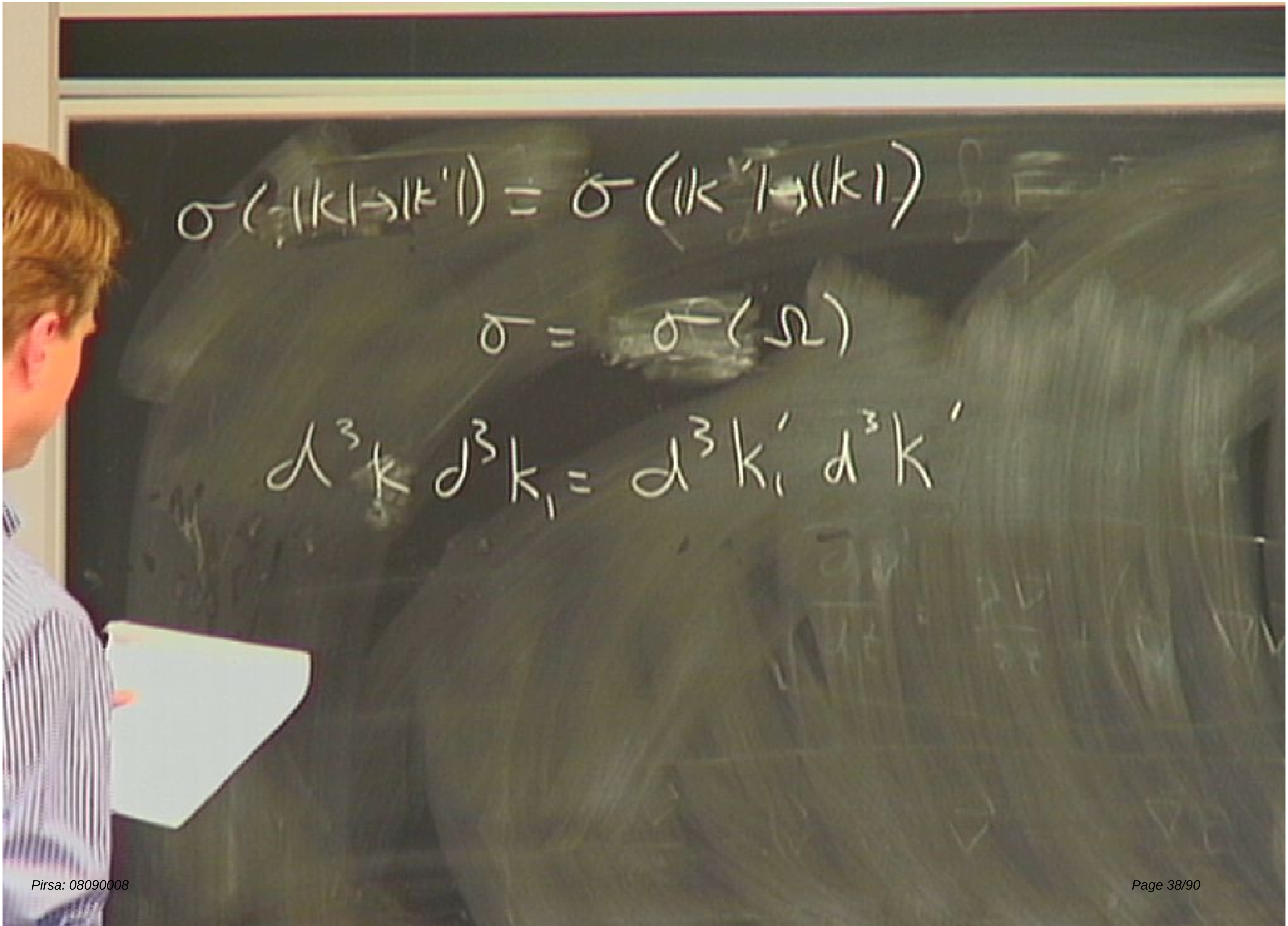
dR_{out} ; dR_{in}

$$\frac{dR_{out}}{d^3x_1\,d^3k_1\,d^3x\,d^3k} = f(k)\,f(k_1)\;\sigma(k, k_1 : k_1', k')\;\frac{(v-v_1)^2}{|v-v_1|}\;\delta(\vec{x}-\vec{x}_1)$$

$$\frac{dR_i}{(\;\;\;\;\;\;)} = f(k')\,f(k_1')\;\sigma(k', k_1'; k_1 : k)\;\frac{(v'-v_1')}{|v-v_1|}\;\delta(\vec{x}'-\vec{x}_1')$$

$$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$


$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$
$\sigma = \sigma(\Omega)$
$d^3k \, d^3k_1 = d^3k_1' \, d^3k'$

$$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$

$$d^3k\, d^3k_1 = d^3k_1'\, d^3k'$$

$$\text{Collision}[f] = d^3x\, d^3k$$

$$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$

$$d^3k\, d^3k_1 = d^3k_1'\, d^3k'$$

$$dR_{in} - dR_{out} = d^3x\, d^3k$$

$$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$

$$d^3k\, d^3k_1 = d^3k_1'\, d^3k'$$

$$dR_{in} - dR_{out} = d^3x\, d^3k \int$$

$$\sigma(\,|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$

$$\to \; d^3k \, d^3k_1 = d^3k_1' \, d^3k' \;\to\; \text{Momentum conservation}$$

$$dR_{in} - dR_{out} = d^3k \int |v - v_1| \, \sigma(\Omega) \, d\Omega \left[f_1' f_1' - f f_1 \right] d^3k_1$$

$$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$

$$\to d^3k\, d^3k_1 = d^3k_1'\, d^3k' \to \text{Momentum conservation}$$

$$dR_{in} - dR_{out} = d^3k \int |v - v_1|\, \sigma(\Omega)\, d\Omega \left[f_1' f_1' - f f_1 \right] d^3k_1$$

$$\sigma(|k| \to |k'|) = \sigma(|k'| \to |k|)$$

$$\sigma = \sigma(\Omega)$$

$$\to d^3k\, d^3k_1 = d^3k_1'\, d^3k' \to \text{Momentum conservation}$$

$$\left[dR_{in} - dR_{out}\right](k) = d^3k \int |v - v_1|\, \sigma(\Omega)\, d\Omega\, \left[f' f_1' - f f_1\right] d^3k_1$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^i}(v^i f) - \frac{\partial}{\partial k^i}\left(f(\nabla^i U)\right) = C[f]$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^i}(v^i f) - \frac{\partial}{\partial k^i}\left(f(\nabla^i U)\right) = C[f]$$

$$\frac{\partial v^i}{\partial x^i} = 0$$

$$\frac{\partial}{\partial k} u = 0$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^i}(v^i f) - \frac{\partial}{\partial k^i}\left(f(\nabla^i u)\right) = C[f]$$

$$\int m\,[LHS]\,d^3k = m\int C[f] = 0$$

$$\frac{\partial v^i}{\partial x^i} = 0$$

$$\frac{\partial}{\partial k} U = 0$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^i}(v^i f) - \frac{\partial}{\partial k^i}\left(f(\nabla^i U)\right) = C[f]$$

$$\int m\,[LHS]\,d^3k = m\int C[f] = 0$$

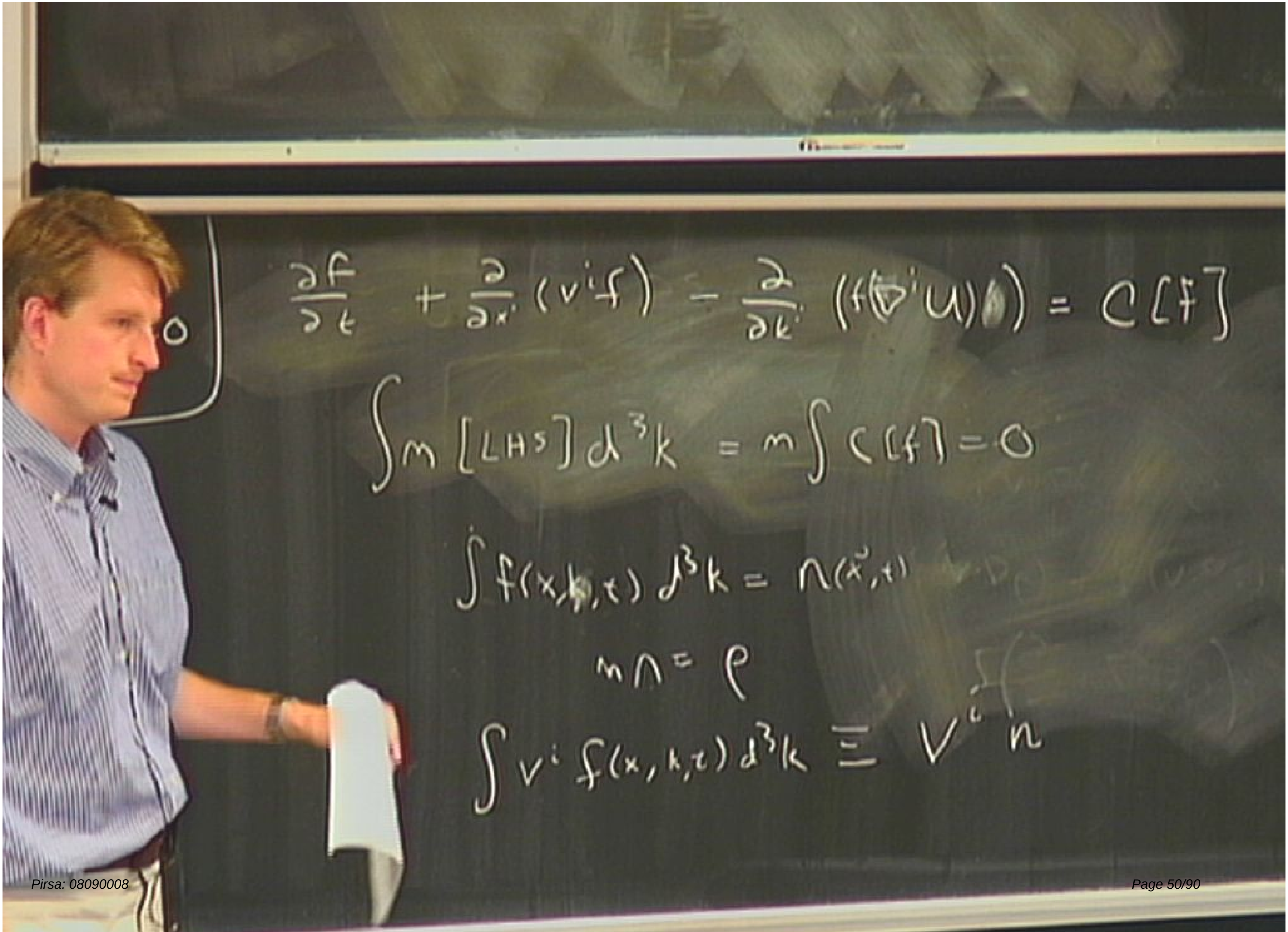
$$\frac{\partial v^i}{\partial x^i}=0$$

$$\frac{\partial}{\partial k}U=0$$

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^i}(v^i f) - \frac{\partial}{\partial k^i}\left(f \nabla^i U\right) = C[f]$$

$$\int m\,[LHS]\,d^3k = m\int C[f] = 0$$

$$\int f(x,k,t)\,d^3k = n$$


$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x^i}(v^i f) - \frac{\partial}{\partial k^i}\left((f \nabla^i U)\right) = C[f]$
$\int m\,[\mathrm{LHS}]\, d^3k = m \int C[f] = 0$
$\int f(x,k,t)\, d^3k = n(\vec{x},t)$
$mn = \rho$
$\int v^i f(x,k,t)\, d^3k \equiv V^i n$

$$\int \frac{\partial}{\partial k}(\quad)\, d^3k = 0$$

$$(x)\quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x^i}(v^i \rho) = 0$$

$$\int m v^j \, [\text{LHS}] \, d^3k = 0$$

① $-\frac{\partial}{\partial t}\int m v^j \cdot f(x,k,t)\, d^3k = \frac{\partial}{\partial t}(v^j \rho)$

②

$$\int m v^j \, [\text{LHS}] \, d^3k = 0$$

① $$\frac{\partial}{\partial t} \int m v^j \, f(x, k, t) \, d^3k = \frac{\partial}{\partial t} (v^j \rho)$$

②

$$\int m v^j \,[LHS]\, d^3k = 0$$

(1) $$\frac{\partial}{\partial t}\int m v^j f(x,k,t)\, d^3k = \frac{\partial}{\partial t}(V^j \rho)$$

(2) $$\frac{\partial}{\partial x^i}\int m v^i v^j f(x,k,t)\, d^3k = \frac{\partial}{\partial x^i} T^{ij}$$

$$\int m v^j [LHS]\, d^3k = 0$$

① $\frac{\partial}{\partial t} \int m v^j \cdot f(x,k,t)\, d^3k = \frac{\partial}{\partial t} (V^j \rho)$

② $\frac{\partial}{\partial x^i} \int m v^i v^j f(x,k,t)\, d^3k = \frac{\partial}{\partial x^i} T^{ij}$

③ $\frac{\partial}{\partial k^i} \int \frac{\partial u}{\partial x^i} f(x,k,t)\, k_j\, d^3k = 0 \quad , \quad \delta^i{}_j$

② $\frac{\partial}{\partial x^j} \int m v^i v^j f(x,k,t)\, d$ [illegible] $\frac{\partial}{\partial x^i}$

③ $\frac{\partial}{\partial k^i} \int \frac{\partial u}{\partial x^j} f(x,k,t)\, k_j\, d^3k = 0, \quad \delta^i_j$

$= \frac{\partial u}{\partial x^i} \rho$

① + ② − ③ = 0

$\partial x^i \; ($

$① + ② - ③ = 0$

$$\frac{\partial}{\partial t}(v^j \rho) + \frac{\partial}{\partial x^i} T^{ij} - \frac{\partial u}{\partial x^j}\rho = 0$$

∂x^i (

(1) + (2) - (3) = 0

$$\frac{\partial}{\partial t}(v^j \rho) + \frac{\partial}{\partial x^i} T^{ij} - \frac{\partial u}{\partial x^j}\rho = 0$$

∂x^i (

① + ② − ③ = 0

$$\boxed{\frac{\partial}{\partial t}(v^j \rho) + \frac{\partial}{\partial x^i} T^{ij} - \frac{\partial u}{\partial x^j} \rho = 0}$$

∂x^i (

① + ② − ③ = 0

$$\boxed{\frac{\partial}{\partial t}(v^j \rho) + \frac{\partial}{\partial x^i} T^{ij} - \frac{\partial u}{\partial x^j}\rho = 0}$$

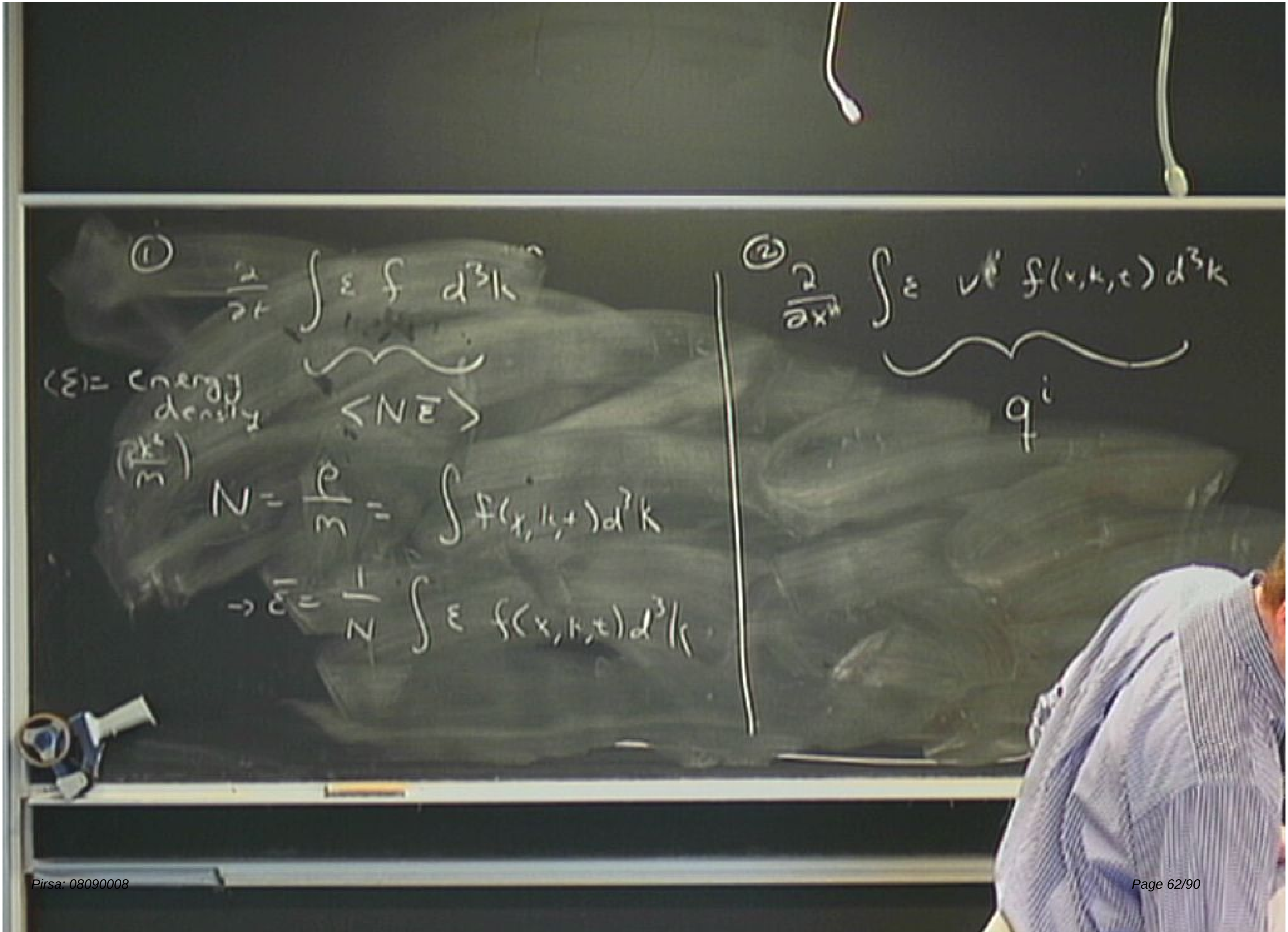
$$\int \left(\frac{k^2}{m}\right) [1 + S]\, d^3k$$

$\frac{\partial}{\partial x^i}$ (

① + ② − ③ = 0

$$\boxed{\frac{\partial}{\partial t}(v^j \rho) + \frac{\partial}{\partial x^i} T^{ij} - \frac{\partial u}{\partial x^j} \rho = 0}$$

$$\int \left(\frac{k^2}{m}\right) [L + S]\, d^3k$$


①
$\frac{\partial}{\partial t} \int \varepsilon f \, d^3k$
$\langle N \bar{\varepsilon} \rangle$
$\langle \varepsilon \rangle$ = energy density
$\left(\frac{\hbar k^2}{m}\right)$
$N = \frac{\rho}{m} = \int f(x,k,t) \, d^3k$
$\rightarrow \bar{\varepsilon} = \frac{1}{N} \int \varepsilon \, f(x,k,t) \, d^3k$
②
$\frac{\partial}{\partial x^i} \int \varepsilon \, v^i f(x,k,t) \, d^3k$
q^i

①

$$\frac{\partial}{\partial t}\underbrace{\int \varepsilon f \, d^3k}_{\langle N\bar{\varepsilon}\rangle}$$

$(\varepsilon) =$ energy density

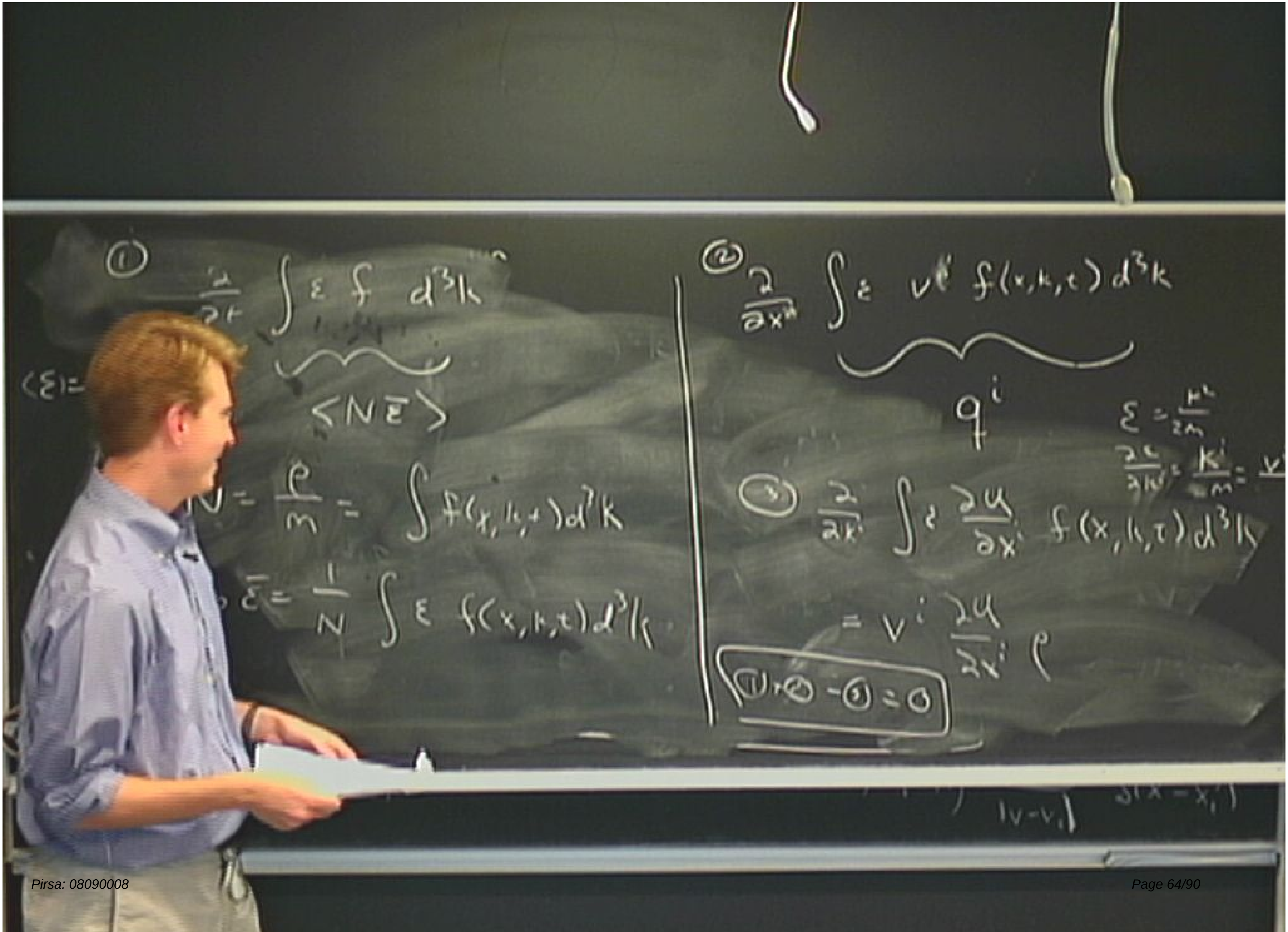
$\left(\frac{\hbar k^2}{m}\right)$

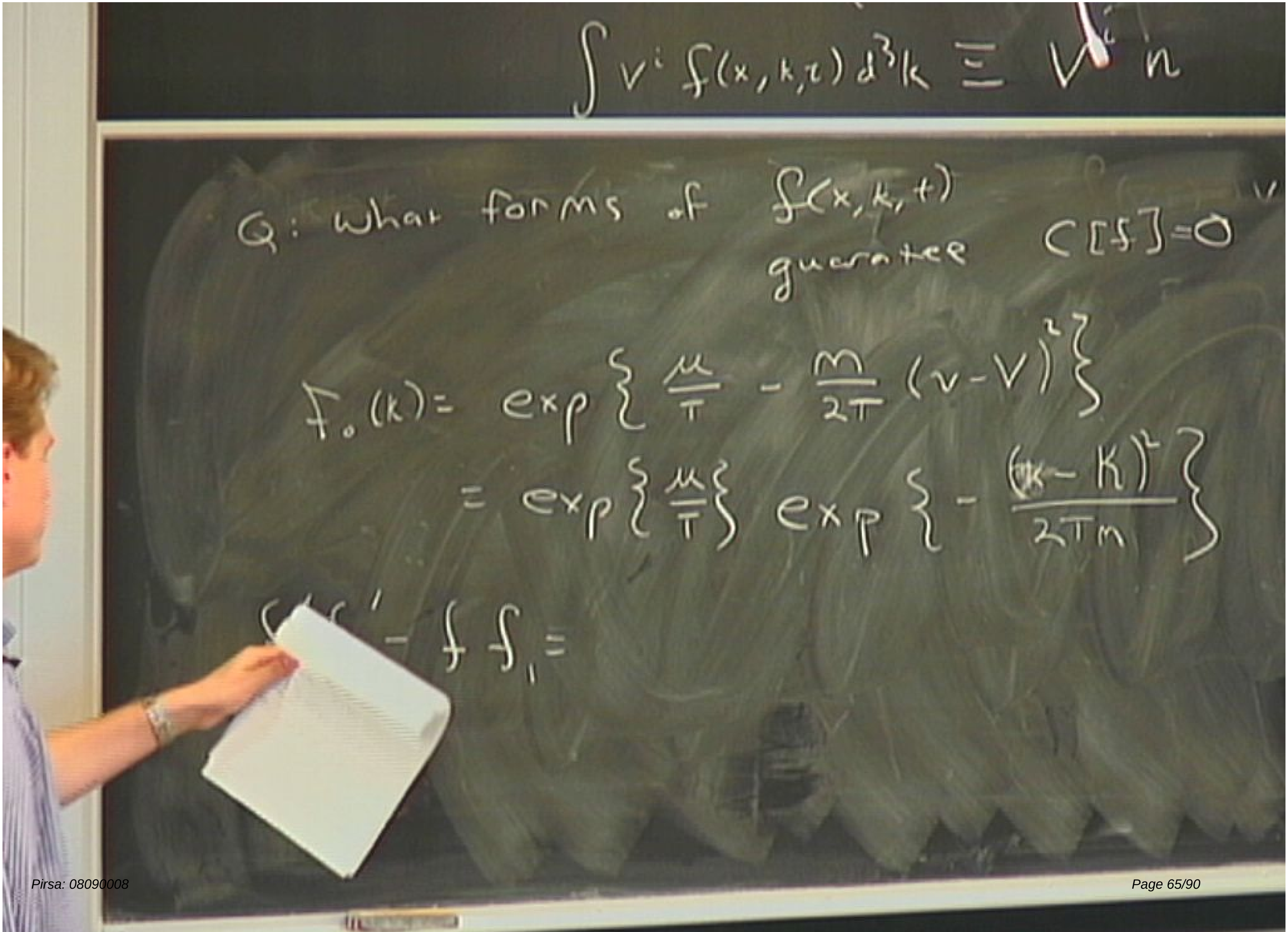
$$N = \frac{\rho}{m} = \int f(x,k,t)\, d^3k$$

$$\rightarrow \bar{\varepsilon} = \frac{1}{N}\int \varepsilon\, f(x,k,t)\, d^3k$$

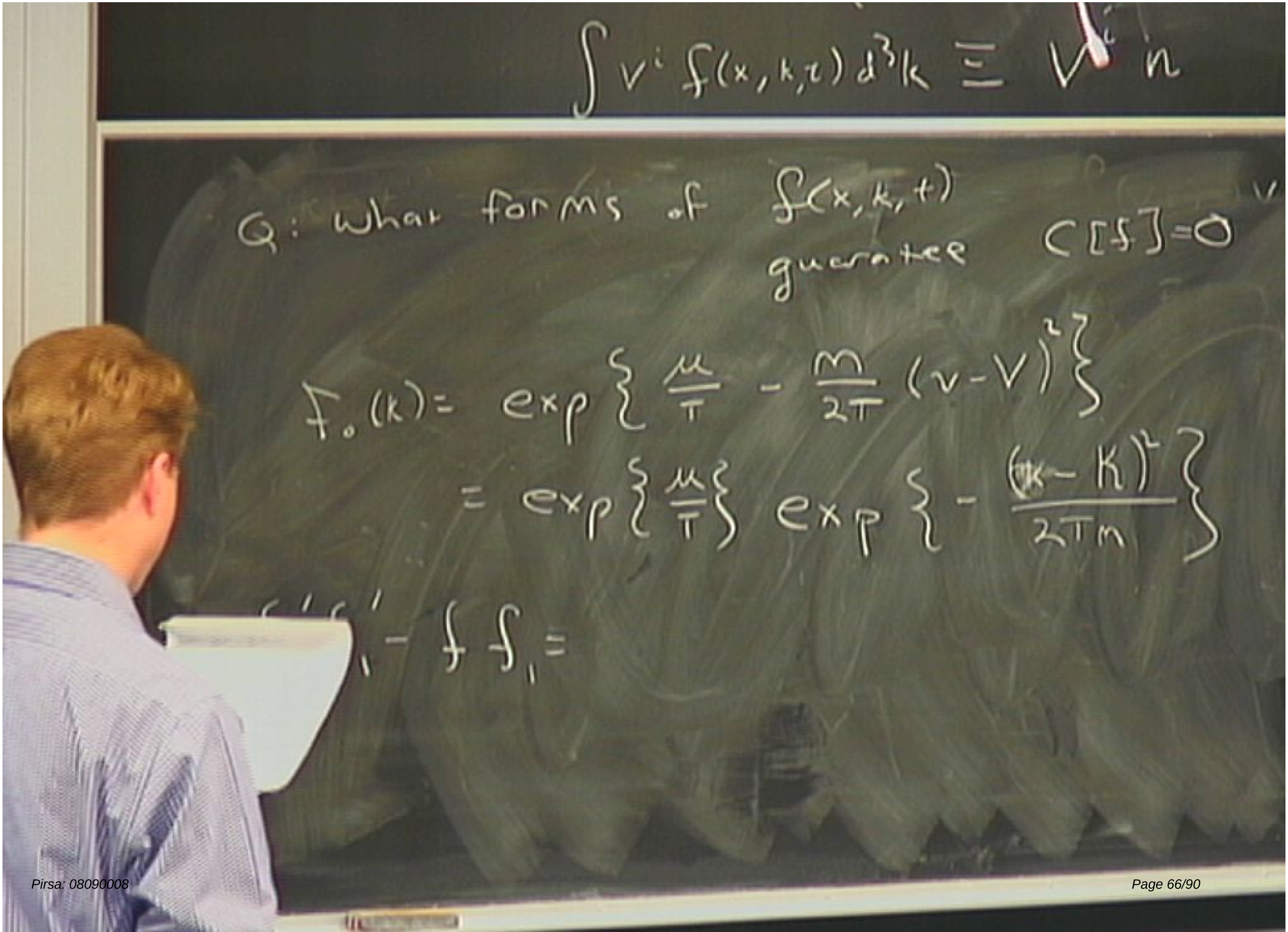
②

$$\frac{\partial}{\partial x^i}\underbrace{\int \varepsilon\, v^i f(x,k,t)\, d^3k}_{q^i}$$


① ∂/∂t ∫ ε f d³k
⟨NE⟩
N = ρ/m = ∫ f(x,k,t) d³k
ε̄ = 1/N ∫ ε f(x,k,t) d³k
② ∂/∂x^i ∫ ε v^i f(x,k,t) d³k
q^i
ε = k²/2m
∂ε/∂k^i = k^i/m = v^i
③ ∂/∂k^i ∫ ε ∂U/∂x^i f(x,k,t) d³k
= v^i ∂U/∂x^i ρ
①+②−③=0



∫ v^i f(x, k, t) d^3k ≡ V^i n
Q: what forms of f(x, k, t) guarantee C[f] = 0
f_0(k) = exp{ μ/T − m/2T (v − V)^2 }
= exp{ μ/T } exp{ − (k − K)^2 / 2Tm }
f' f_1' − f f_1 =


$\int v^i f(x,k,t)\, d^3k \equiv V^i n$
Q: what forms of $f(x,k,t)$ guarantee $C[f]=0$
$f_0(k) = \exp\left\{ \frac{\mu}{T} - \frac{m}{2T}(v-V)^2 \right\}$
$= \exp\left\{\frac{\mu}{T}\right\} \exp\left\{ - \frac{(k-K)^2}{2Tm} \right\}$
$f' f_1' - f f_1 =$

$$\int v^i f(x,k,t)\, d^3k \equiv V^i n$$

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$$f_0(k) = \exp\left\{\frac{\mu}{T} - \frac{m}{2T}(v-V)^2\right\}$$

$$= \exp\left\{\frac{\mu}{T}\right\} \exp\left\{-\frac{(k-K)^2}{2Tm}\right\}$$

$f' f_1' - f f_1 =$ plug in : $(k+k_1)^2 = (k'+k_1')^2$

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$f'f_1' - f f_1 =$ plug in: $(k+k_1)^2 = (k'+k_1')^2$

$$e^{\frac{\mu}{T}}\left[e^{(-)} - e^{(\text{same})}\right] = 0$$

$$\int v^i f(x,k,t)\, d^3k \equiv V^i n$$

Q: what forms of $f(x,k,t)$ guarantee $C[f]=0$

$$f_0(k) = \exp\left\{\frac{\mu}{T} - \frac{m}{2T}(v-V)^2\right\}$$

$$= \exp\left\{\frac{\mu}{T}\right\} \exp\left\{-\frac{(k-K)^2}{2Tm}\right\}$$

$$\boxed{0 = f' f_1' - f f_1}$$ plug in: $(k+k_1)^2 = (k'+k_1')^2$

$$\left[e^{(-)} - e^{(\text{same})} \right] = 0$$

LHS of Boltzmann is non-trivial, but RHS = 0

→ Local Thermal Equilibrium

$$\int v^i f(x,k,t)\, d^3k \equiv V^i n$$

Q: what forms of $f(x,k,t)$ guarantee $C[f]=0$

$$f_0(k) = \exp\left\{ \frac{\mu}{T} - \frac{m}{2T}(v-V)^2 \right\}$$

$$= \exp\left\{\frac{\mu}{T}\right\} \exp\left\{ - \frac{(k-K)^2}{2Tm} \right\}$$

$$\boxed{0 = f' f_1' - f f_1} \text{ plug in :} \quad (k+k_1)^2 = (k'+k_1')^2$$

$$e^{\frac{\mu}{T}} \left[e^{(-)} - e^{(\text{same})} \right] = 0$$

LHS of Boltzmann is non-trivial, but RHS $=0$

$\rightarrow$ Local Thermal Equilibrium

$$\int v^i f(x,k,t)\, d^3k \equiv V^i n$$

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$$f_0(k) = \exp\left\{\frac{\mu}{T} - \frac{m}{2T}(v-V)^2\right\}$$

$$= \exp\left\{\frac{\mu}{T}\right\} \exp\left\{-\frac{(k-K)^2}{2Tm}\right\}$$

$$\boxed{0 = f' f_1' - f f_1} \Rightarrow \text{plug in}: \quad (k+k_1)^2 = (k'+k_1')^2$$

$$e^{\frac{\mu}{T}}\left[e^{(-)} - e^{(\text{same})}\right] = 0$$

LHS of Boltzmann is non-trivial, but RHS = 0

$\rightarrow$ Local Thermal Equilibrium

$$\int v^i f(x,k,t)\, d^3k \equiv V^i n$$

$$\int \frac{\partial}{\partial k}(\quad)\, d^3k = 0$$

Q: What forms of $f(x,k,t)$ guarantee $C[f]=0$

$$f_0(k) = \exp\left\{\frac{\mu}{T} - \frac{m}{2T}(v-V)^2\right\}$$

$$= \exp\left\{\frac{\mu}{T}\right\} \exp\left\{-\frac{(k-K)^2}{2Tm}\right\}$$

$\boxed{0 = \int f'f_1' - f f_1}$ = plug in :

$(k+k_1)^2 = (k'+k_1')^2$

$$e^{\frac{\mu}{T}}\left[e^{(-)} - e^{(\text{same})}\right] \stackrel{?}{=} 0$$

LHS of Boltzmann is non-trivial, but RHS = 0

$\rightarrow$ Local Thermal Equilibrium

$$\langle \star \rangle = \int \star \, f(-) \, d^3k$$

$$T^{ij} = m N \langle v^i v^j \rangle$$

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$(\vec{x}, t)$

$$\langle \star \rangle = \int \star \, f(-) \, d^3k$$
$$(\vec{x}, t)$$

$$T^{ij} = m N \langle v_i^i v^j \rangle$$

$$= m N \langle (v_i' + V_i)(v_j' + V_k) \rangle$$

$$= m N V^i V^k + m N \langle v_i' v_j' \rangle$$

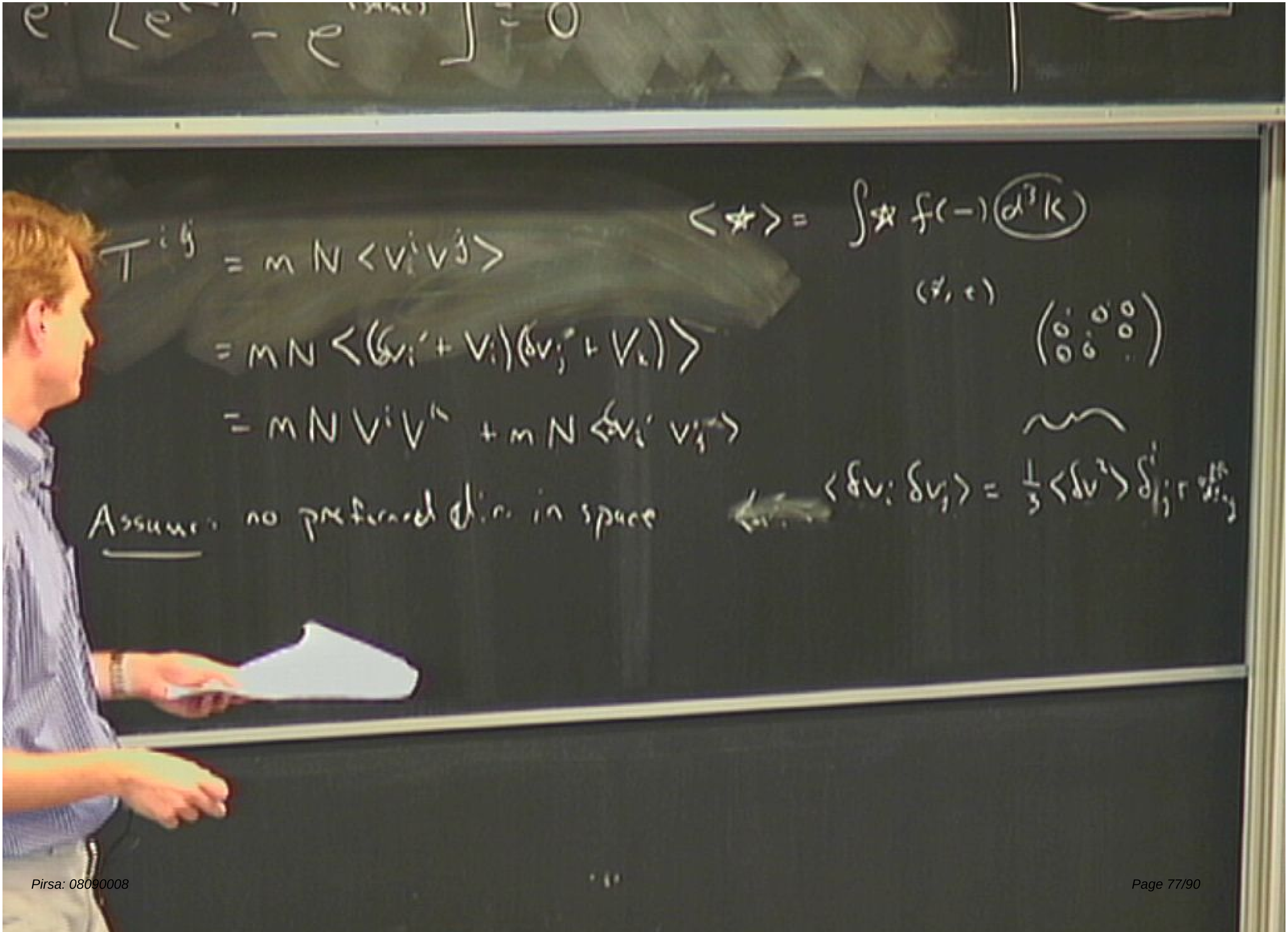
$$\langle \star \rangle = \int \star \, f(-)\, d^3k$$

$$(\vec{x}, t)$$

$$T^{ij} = mN \langle v_i^i v^j \rangle$$

$$= mN \langle (v_i' + V_i)(v_j' + V_k) \rangle$$

$$= mN\, V^i V^k + mN \langle v_i' \, v_j' \rangle$$


$\langle \star \rangle = \int \star f(-) d^3k$
$T^{ij} = m N \langle v^i v^j \rangle$
$= m N \langle (\delta v_i' + V_i)(\delta v_j' + V_k) \rangle$
$= m N V^i V^k + m N \langle \delta v_i' \, v_j' \rangle$
$\langle \delta v_i \delta v_j \rangle = \frac{1}{3} \langle \delta v^2 \rangle \delta_{ij}$
Assume: no preferred dir. in space

$PV = NkT$

$$\langle \star \rangle = \int \star \, f(-) \, d^3k$$

$$T^{ij} = mN \langle v^i v^j \rangle$$

$(\vec{x}, t)$

$$= mN \langle (\delta v_i' + V_i)(\delta v_j' + V_k) \rangle$$

$$= mN V^i V^k + mN \langle \delta v_i' \, v_j' \rangle$$

$$\langle \delta v_i \, \delta v_j \rangle = \frac{1}{3} \langle \delta v^2 \rangle \delta_{ij}$$

Assume: no preferred dir. in space

$$\frac{1}{2} m \langle \delta v^2 \rangle = \frac{3}{2} kT \qquad \frac{1}{3} \langle \delta v^2 \rangle = \frac{T}{m} = \frac{P}{Nm}$$

$$T^{ij} = mNV^iV^j + mN($$

$$T^{ij} = mNV^iV^j + \cancel{mN}\left(\frac{P}{\cancel{mN}}\right)\delta^{ij}$$

$$= mNV^iV^j + P\,\delta^{ij}$$

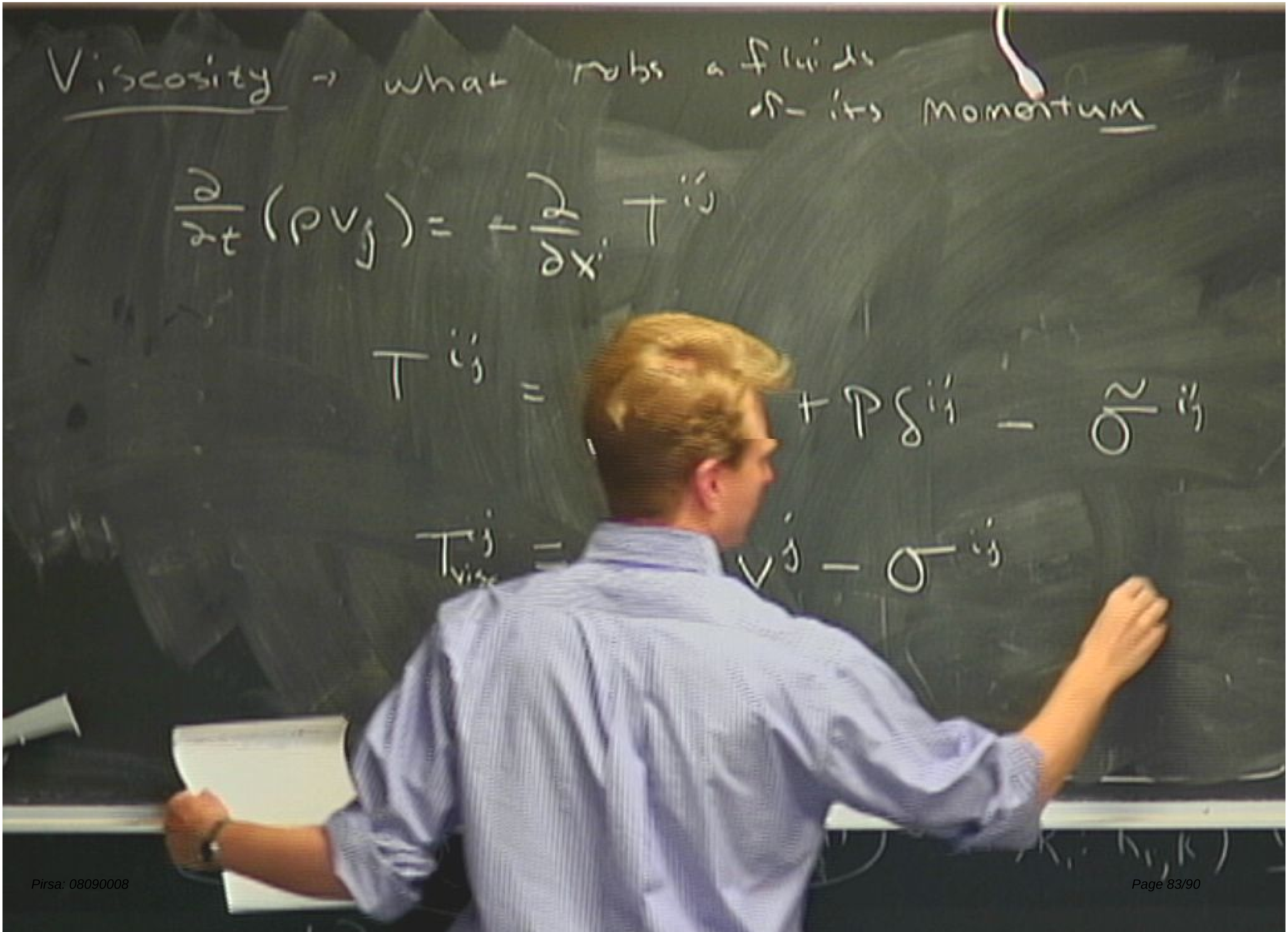
$$T^{ij} = mNV^iV^j + \cancel{mN}\left(\frac{P}{\cancel{N}m}\right)\delta^{ij}$$

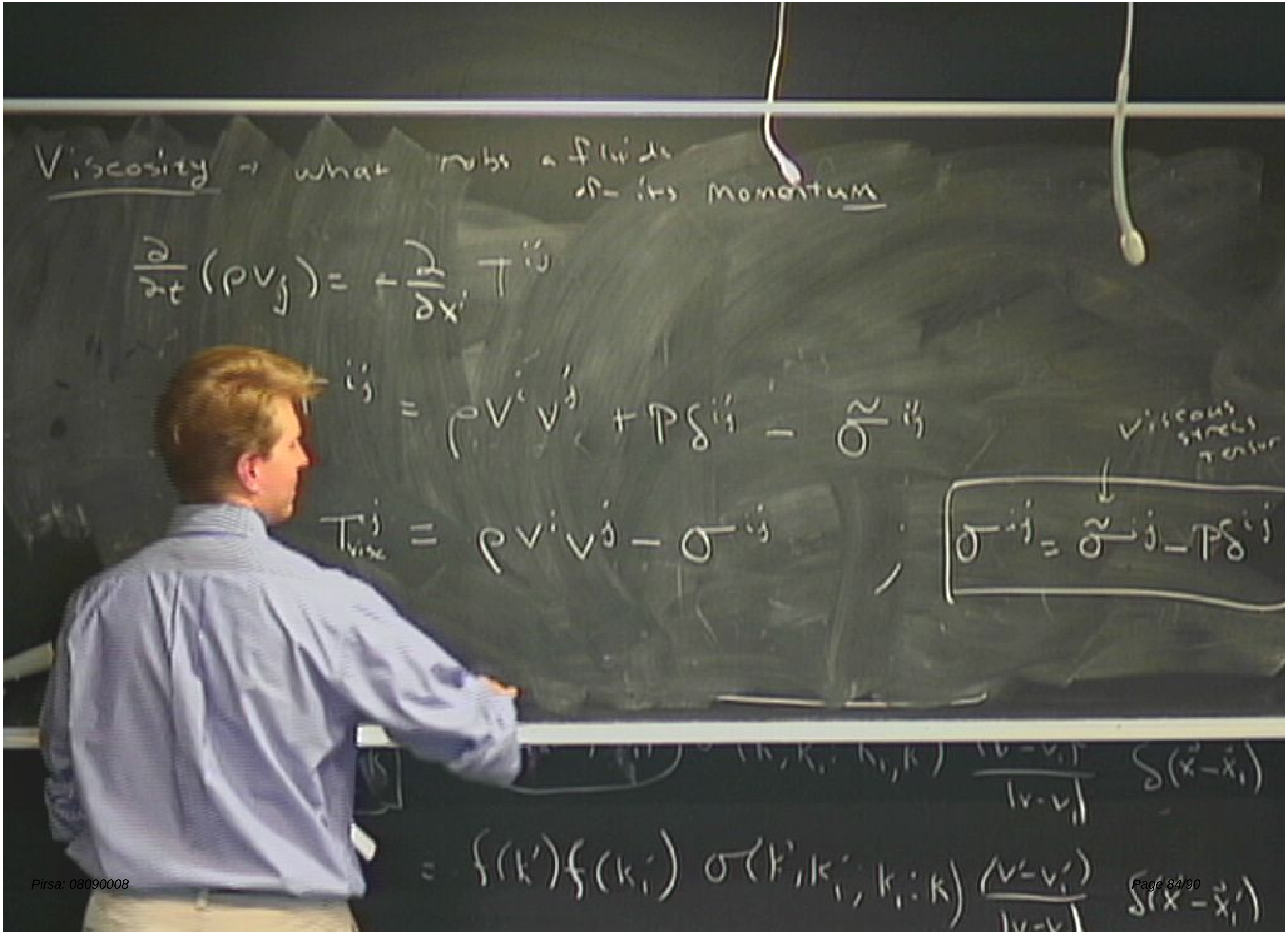
$$= mNV^iV^j + P\,\delta^{ij}$$

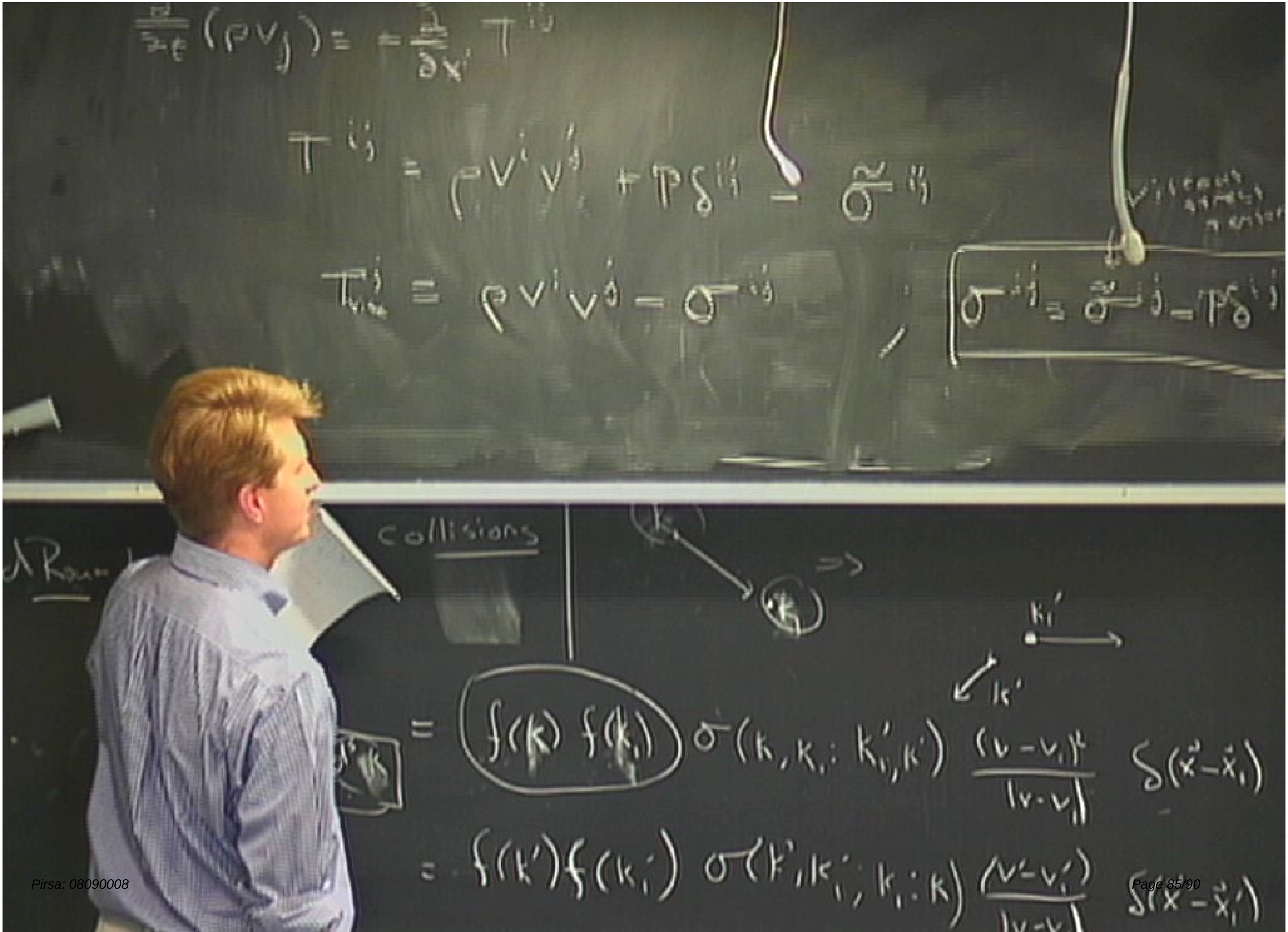
$$T^{ij} = mNV^iV^j + \cancel{mN}\left(\frac{P}{\cancel{N}m}\right)\delta^{ij}$$

$$= mNV^iV^j + P\delta^{ij}$$

HW: same process, find q_i for LTE (?)


Viscosity → what robs a fluid's
of its momentum
$\frac{\partial}{\partial t}(\rho v_j) = -\frac{\partial}{\partial x^i} T^{ij}$


Viscosity → what robs a fluid of its momentum
$\frac{\partial}{\partial t}(\rho v_j) = -\frac{\partial}{\partial x^i} T^{ij}$
$T^{ij} = \rho v^i v^j + P\delta^{ij} - \tilde{\sigma}^{ij}$
viscous stress tensor
$T^{ij}_{visc} = \rho v^i v^j - \sigma^{ij}$; $\sigma^{ij} = \tilde{\sigma}^{ij} - P\delta^{ij}$


T^{ij} = ρv^i v^j + pδ^{ij} − σ̃^{ij}
T^{ij} = ρv^i v^j − σ^{ij}
σ^{ij} = σ̃^{ij} − pδ^{ij}
collisions
= f(k) f(k_1) σ(k, k_1 : k_1', k') (v − v_1)/|v − v_1| δ(x − x_1)
= f(k') f(k_1') σ(k', k_1' ; k_1 : k) (v' − v_1')/|v' − v_1'| δ(x' − x_1')

$$\tilde{\sigma}^{ij} = \,?$$

$$v \neq 100$$

$$\nabla_i v_j$$

$$\nabla \cdot v$$

$$\propto \frac{\partial v_i}{\partial x^j} + \frac{\partial v_j}{\partial x^i}$$

$$= \eta \left\{ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x^i} - \frac{2}{3}\delta^{ij}\frac{\partial v_k}{\partial x_k} \right) + \zeta\, \delta^{ij} \frac{\partial v_k}{\partial x_k}$$

$$\delta_{ij} \frac{\partial v_i}{\partial x_j}$$

$\tilde{\sigma}^{ij} = f_{ij} ?$

$\rightarrow V \neq 100$

$\nabla_i V_j$

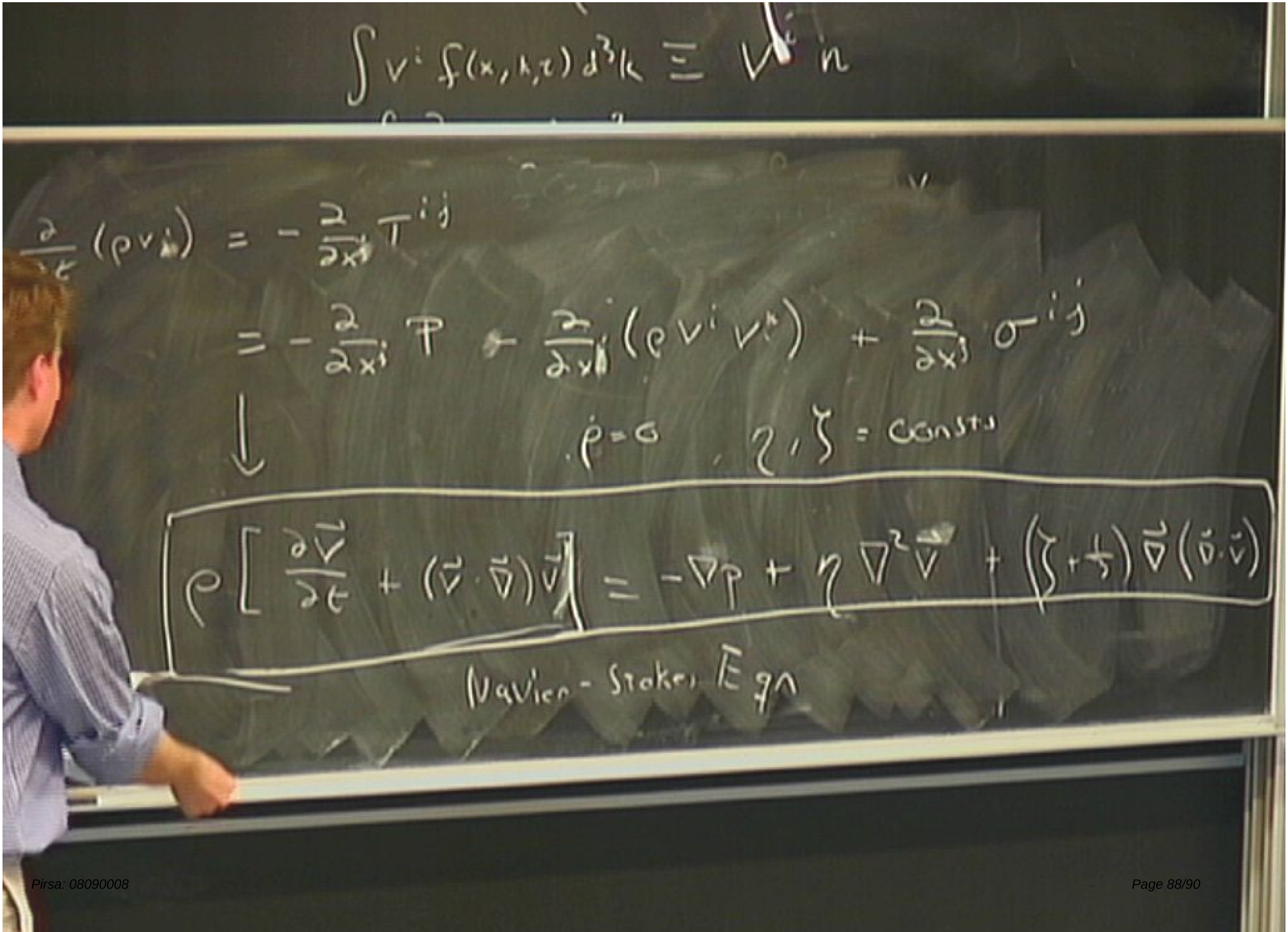
$\nabla \cdot V$

$$\propto \frac{\partial V_i}{\partial x^j} + \frac{\partial V_j}{\partial x^i}$$

$$= \eta \left\{ \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x^i} - \frac{2}{3} \delta^{ij} \frac{\partial V_\ell}{\partial x_\ell} \right) + \zeta \, \delta^{ij} \frac{\partial v_\ell}{\partial x_\ell}$$

$$\delta^{ij} \frac{\partial v_i}{\partial x_j} = \frac{\partial v_\ell}{\partial x_\ell}$$

$$2 \left(\frac{\partial v_\ell}{\partial x_\ell} \right) - 2 \left(\frac{\partial v_\ell}{\partial x_\ell} \right)$$


$\int v^i f(x,k,t)\, d^3k \equiv V^i n$
$\frac{\partial}{\partial t}(\rho v^i) = -\frac{\partial}{\partial x^j} T^{ij}$
$= -\frac{\partial}{\partial x^i} P - \frac{\partial}{\partial x^j}(\rho v^i v^j) + \frac{\partial}{\partial x^j}\sigma^{ij}$
$\dot{\rho} = 0 \quad \eta, \zeta = \text{consts}$
$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\nabla p + \eta \nabla^2 \vec{v} + (\zeta + \frac{\eta}{3})\vec{\nabla}(\vec{\nabla}\cdot\vec{v})$
Navier-Stokes Eqn

$$\int v^i f(x,k,t)\, d^3k \equiv V^i n$$

$$\frac{\partial}{\partial t}(\rho v^i) = -\frac{\partial}{\partial x^j} T^{ij}$$

$$= -\frac{\partial}{\partial x^i} P - \frac{\partial}{\partial x^j}(\rho v^i v^j) + \frac{\partial}{\partial x^j}\sigma^{ij}$$

$$\downarrow \qquad \dot{\rho} = 0 \quad , \quad \eta, \zeta = \text{consts}$$

$$\boxed{\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\nabla p + \eta \nabla^2 \vec{v} + \left(\zeta + \frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})}$$

Navier-Stokes Eqn

$$T^{ij} = mNV^iV^j + \cancel{nkT}\left(\frac{P}{\cancel{Nm}}\right)\delta^{ij}$$

$$= mNV^iV^j + P\delta^{ij}$$

HW: same process, find q_i for LTE (?)

$$\vec{\nabla}\cdot\vec{V} = 0 \quad \text{Incompressible}$$

η = dynamic viscosity ; $\nu = \dfrac{\eta}{\rho}$ Kinematic viscosity

$\eta_{H_2O} = 0.01 \; g/cm\text{-}s$

$\eta_{N^2} = 1.8\times10^{-4} \; \frac{g}{cm\text{-}s}$

$\eta_{Alco} = 0.018 \; \frac{g}{cm\text{-}s}$