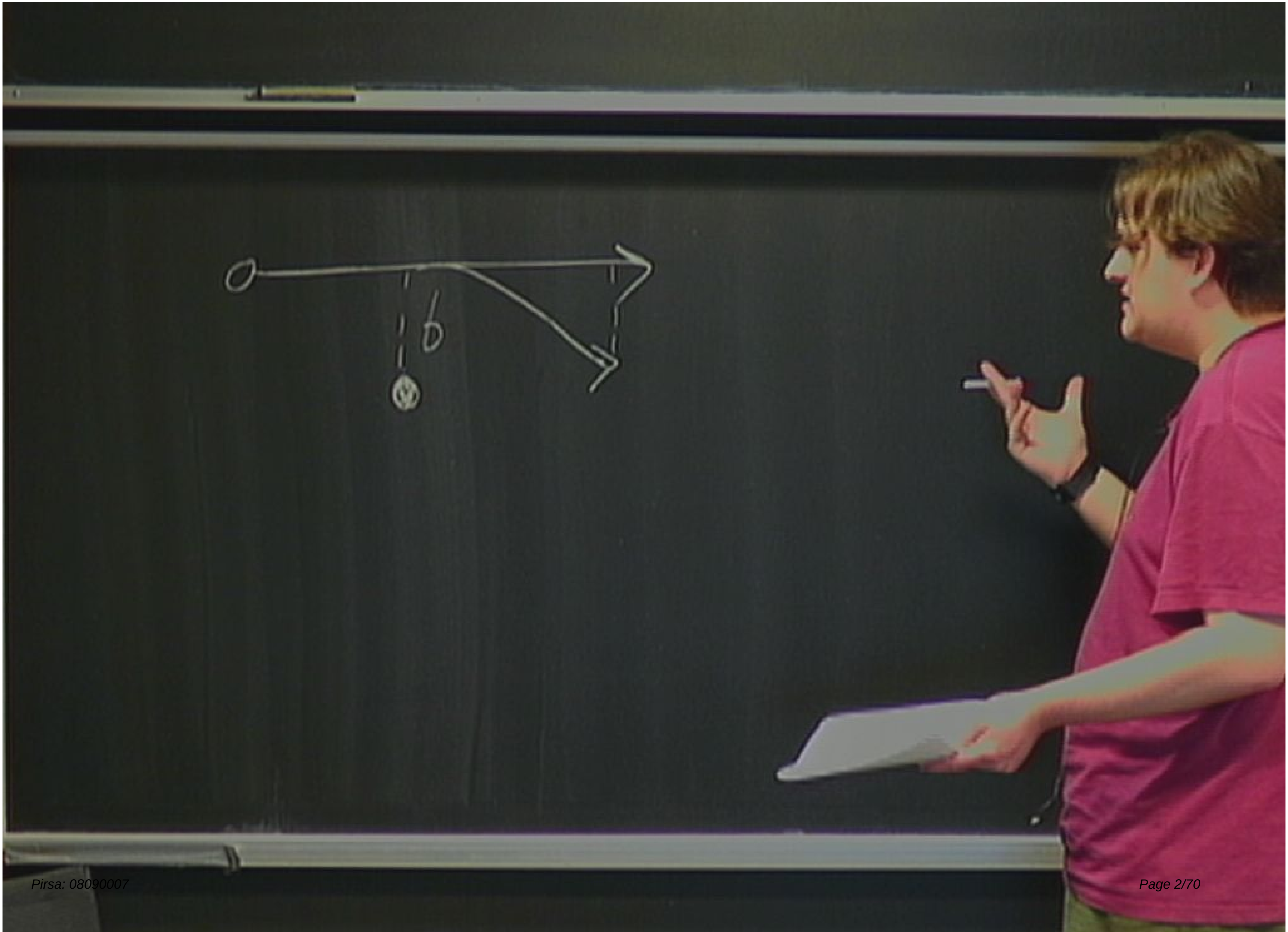
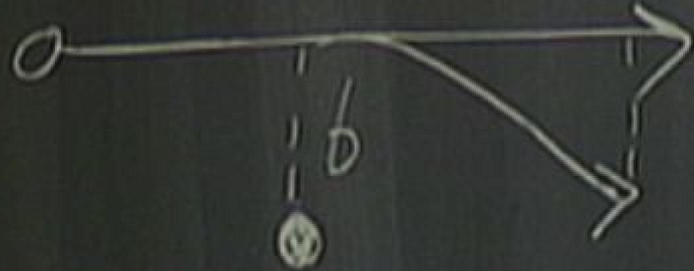
Title: Astrophysics and Cosmology through Problems - 2A

Date: Sep 11, 2008 10:00 AM

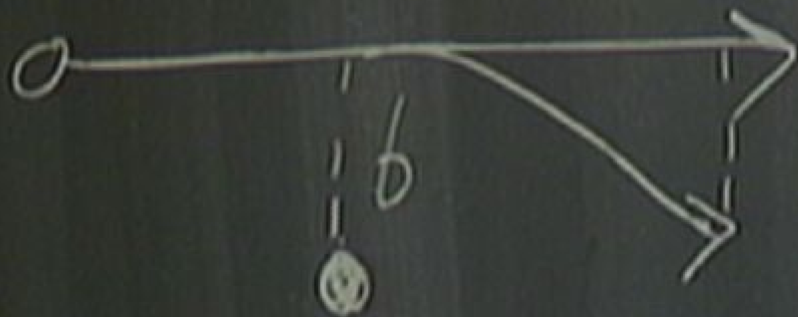
URL: http://pirsa.org/08090007

Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

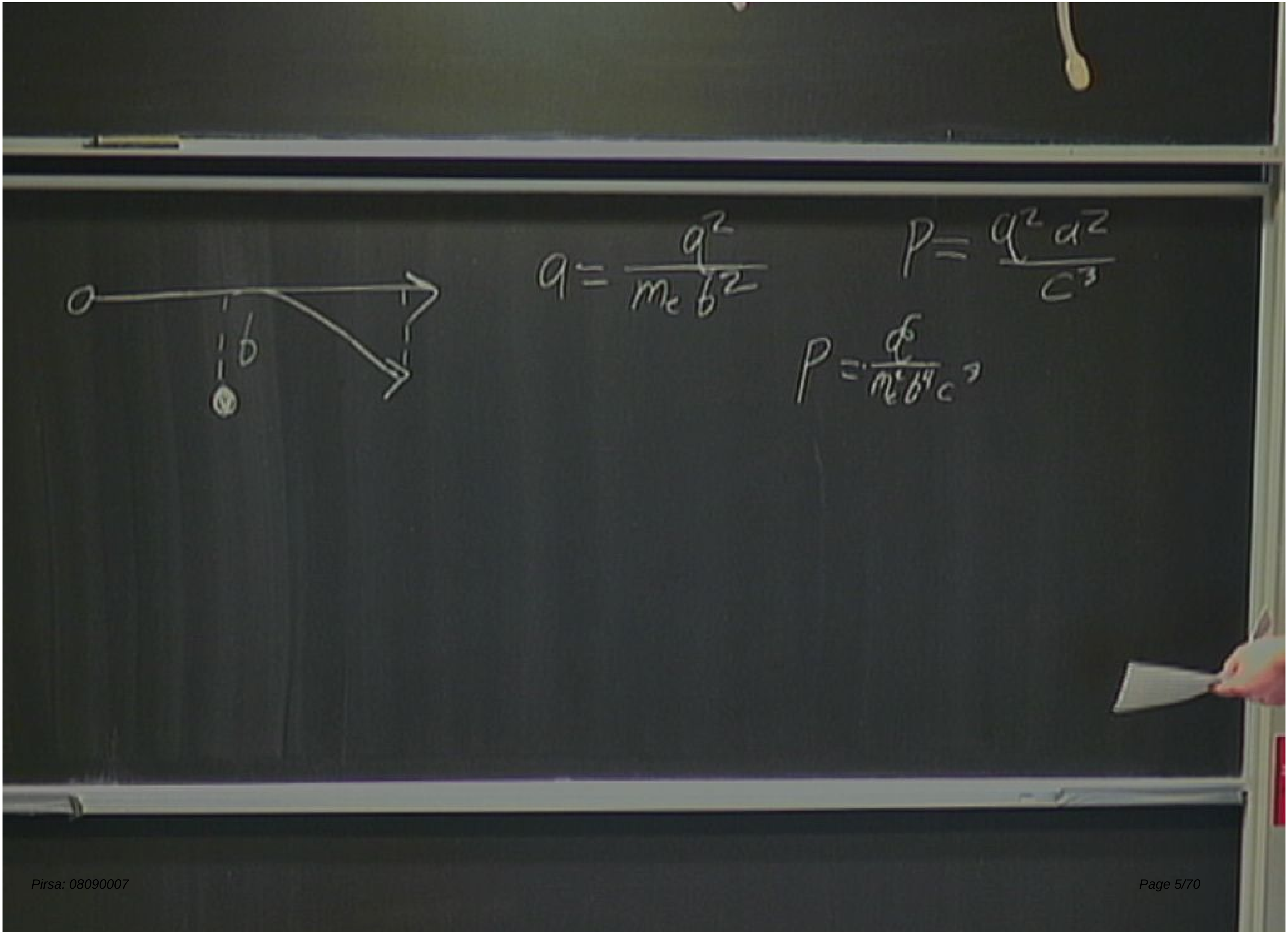

b

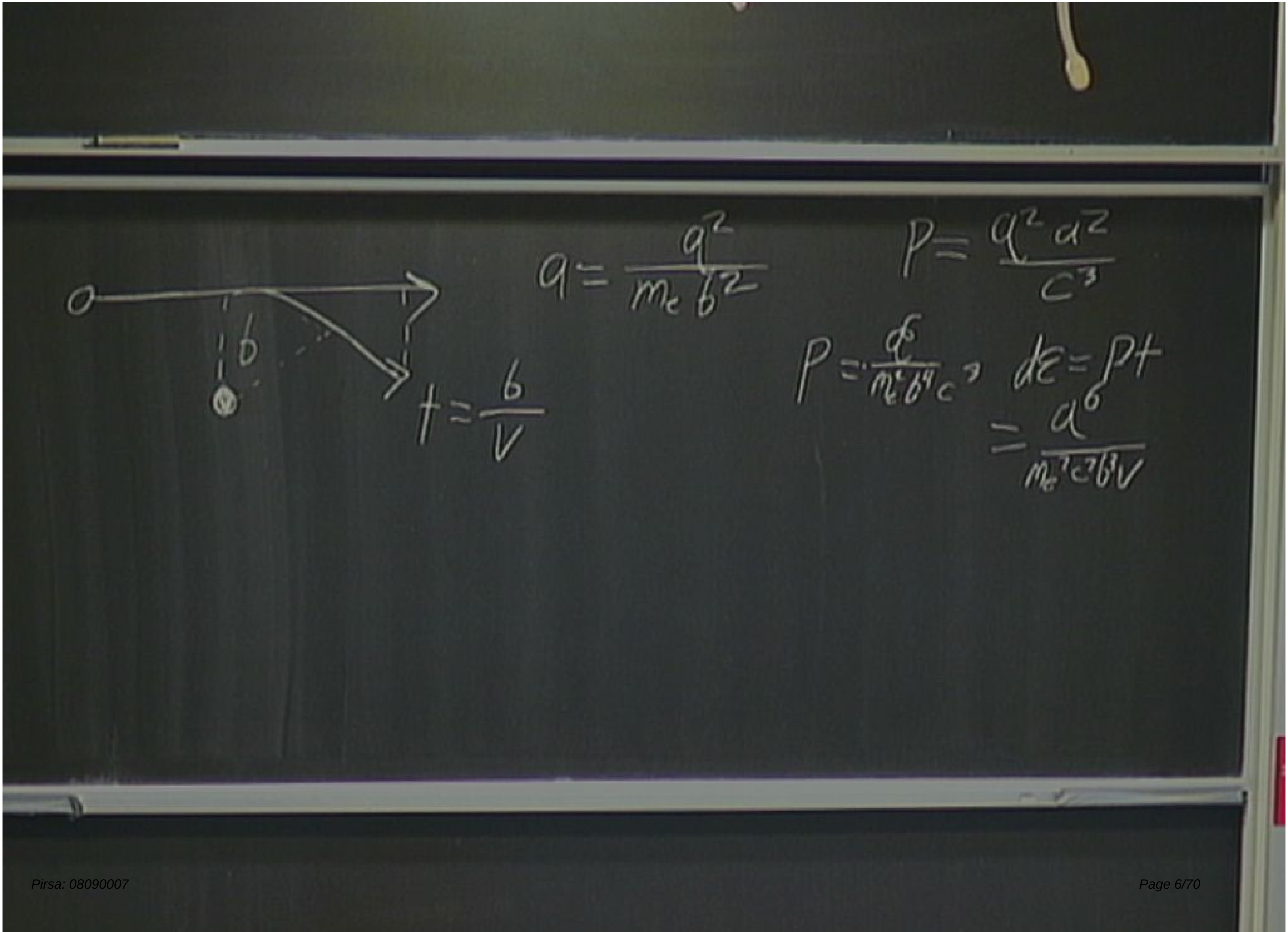


$$a = \frac{q^2}{m_e b^2}$$



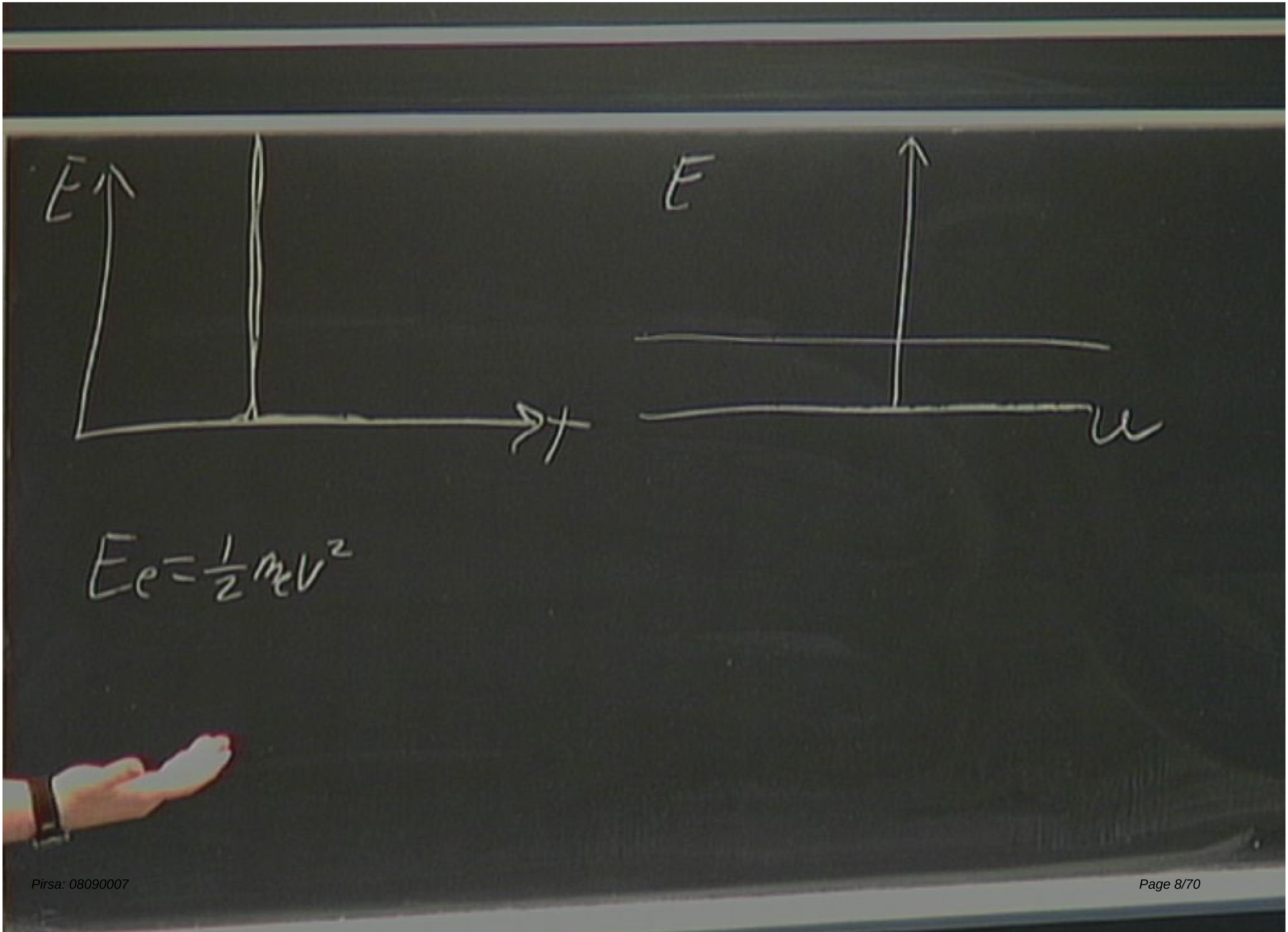
$$a = \frac{q^2}{m_e b^2}$$

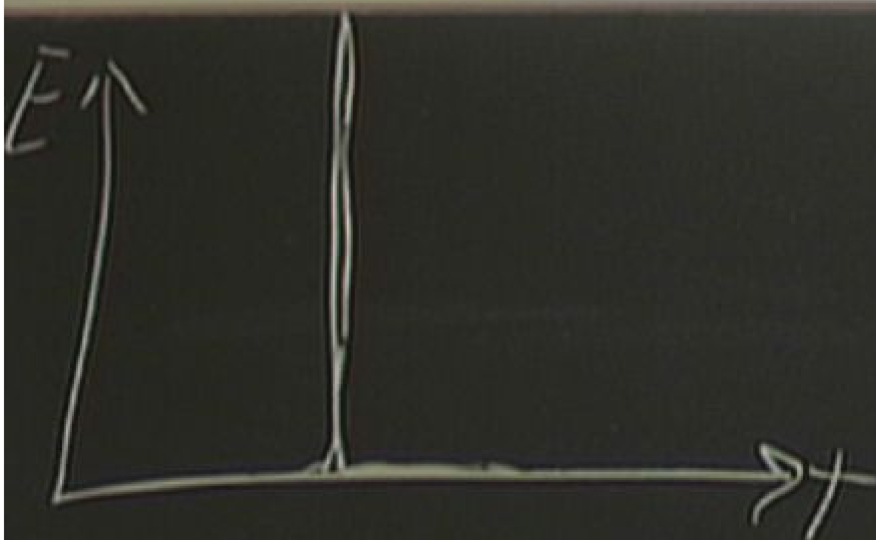

b
$a = \frac{q^2}{m_e b^2}$
$P = \frac{q^2 a^2}{c^3}$
$P = \frac{q^6}{m_e^2 b^4 c^3}$

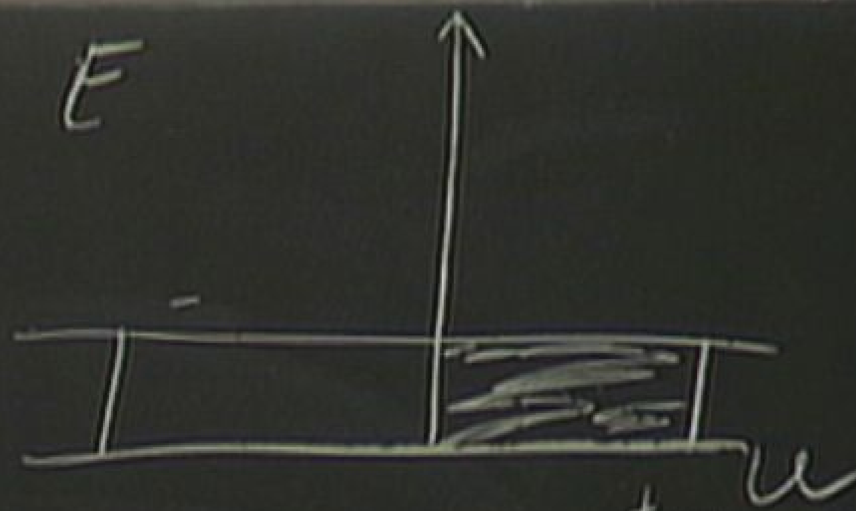

b
$t = \frac{b}{v}$
$a = \frac{q^2}{m_e b^2}$
$P = \frac{q^2 a^2}{c^3}$
$P = \frac{q^6}{m_e^2 b^4 c^3}$
$dE = Pt = \frac{q^6}{m_e^2 c^3 b^3 v}$

$$(dt)\,\frac{d\mathcal{E}}{dV} = \frac{q^6\, n}{m_t^2 c^3 b^3 v} \qquad \frac{d\mathcal{E}}{dt\,V} = \int d\omega \frac{q^6 n}{m_t^2 c^3 b^3 v}$$

$$\frac{d\mathcal{E}}{d\omega\, dt\, dV} = \frac{q^6\, n}{m_t^2 c^3 b^3 v}$$


E
t
E
u
$E_e = \frac{1}{2} m_e v^2$





$$\hbar\omega_{max} = \frac{1}{2} m_e v^2$$

$$\omega_{max} = \frac{m_e v^2}{\hbar}$$

$$E_e = \frac{1}{2} m_e v^2$$

$$(dt)\,\frac{d\mathcal{E}}{dV} = \frac{q^6\, n}{m_e^2 c^3 b^3 v}$$

$$\frac{d\mathcal{E}}{d\omega\, dt\, dV} = \frac{q^6\, n}{m_e^2 c^3 b^3 v}$$

$$\frac{d\mathcal{E}}{dt\,V} = \int_0^{\frac{m_e v^2}{\hbar}} d\omega\, \frac{q^6 n}{m_e^2 c^3 b^3 v}$$

$$= \frac{q^6\, n}{m_e^2 c^3 b^3 v} \cdot \frac{m_e v^2}{\hbar}$$

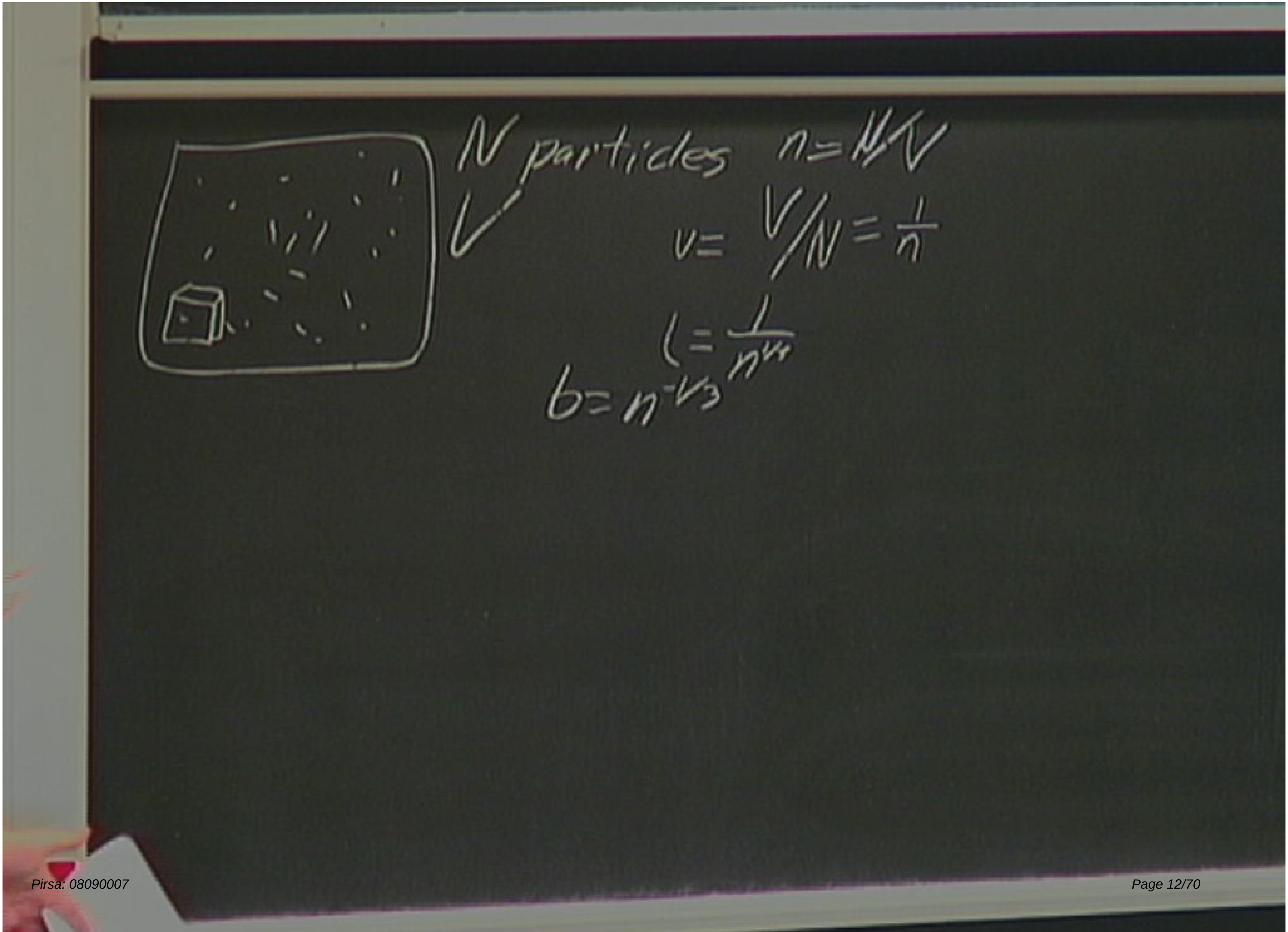
$$= \frac{q^6\, n\, v}{m_e c^3 b^3 v \hbar}$$

N particles $n = N/V$

V

$v = V/N = \frac{1}{n}$

$l = \frac{1}{n^{1/4}}$


N particles n = N/V
V
v = V/N = 1/n
l = 1/n^1/4
b = n^-1/3

N particles $n = N/V$

V

$v = V/N = \frac{1}{n}$

$\ell = \frac{1}{n b^2}$

$b = n^{-1/3}$

$$\frac{d\mathcal{E}}{dt\,dV} = \frac{q^6 n^2 v}{m_e c^3 \hbar}$$

$E = KT = \frac{1}{2} m_e v^2$

$v = \sqrt{\frac{KT}{m_e}}$

$$\frac{d\mathcal{E}}{dt\,dV} = \frac{q^6 \sqrt{KT}}{m_e^{3/2} c^3 \hbar}$$

N particles $\quad n = N/V$

V

$v = V/N = \frac{1}{n}$

$l = \frac{1}{n^{1/3}}$

$b = n^{-1/3}$

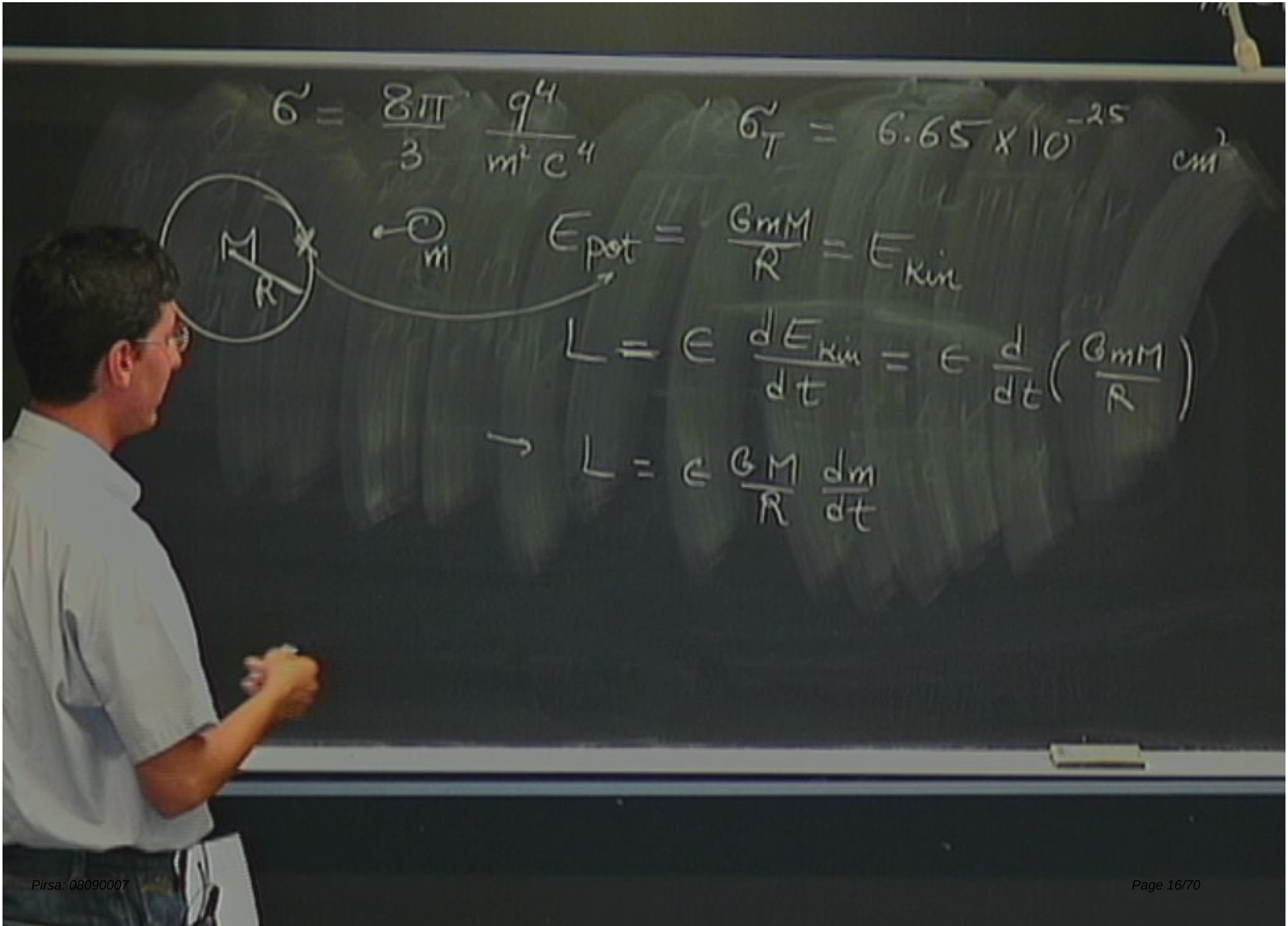
$$\frac{d\varepsilon}{dt\,dV} = \frac{q^6 n^2 v}{m_e c^3 \hbar}$$

$E = kT = \frac{1}{2} m_e v^2$

$v = \sqrt{\frac{kT}{m_e}}$

$$\frac{d\varepsilon}{dt\,dV} = \frac{q^6 \sqrt{kT}}{m_e^{1/2} c^3 \hbar}$$

$$\sigma = \frac{8\pi}{3} \frac{q^4}{m^2 c^4} \qquad \sigma_T = 6.65 \times 10^{-25} \ cm^2$$


$\sigma = \frac{8\pi}{3} \frac{q^4}{m^2 c^4}$
$\sigma_T = 6.65 \times 10^{-25}$ cm^2
M
R
m
$E_{pot} = \frac{GmM}{R} = E_{kin}$
$L = \epsilon \frac{dE_{kin}}{dt} = \epsilon \frac{d}{dt}\left(\frac{GmM}{R}\right)$
$\rightarrow L = \epsilon \frac{GM}{R} \frac{dm}{dt}$

$$\sigma = \frac{8\pi}{3}\frac{q^4}{m^2c^4} \qquad \sigma_T = 6.65 \times 10^{-25}\ \text{cm}^2$$

$$E_{pot} = \frac{GmM}{R} = E_{Kin}$$

$$L = \epsilon \frac{dE_{Kin}}{dt} = \epsilon \frac{d}{dt}\left(\frac{GmM}{R}\right)$$

$$\rightarrow \quad L = \epsilon \frac{GM}{R}\frac{dm}{dt}$$

number density of photons in the r : $\frac{L}{4\pi r^2} \times \frac{1}{\hbar\omega}$

number density of photons in the r : $\frac{L}{4\pi r^2} \times \frac{1}{\hbar\omega} = n(r)$

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$\sigma_T \longrightarrow$ collisional cross section

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$\sigma_T \longrightarrow$ collisional cross section

rate of collisions: $n(r) \times \sigma_T$

$$f_{rad} = f_{Grav}$$

$$\frac{L \sigma_T}{4 \pi r^2 c} = G$$

$$f_{rad} = f_{Grav}$$

$$\frac{L \sigma_T}{4\pi r^2 c} = \frac{G m_p M}{r^2}$$

$$f_{rad} = f_{Grav}$$

$$\frac{L \sigma_T}{4\pi r^2 c} = \frac{G m_p M}{r^2} \longrightarrow L_E = \frac{4\pi c \, G m_p}{\sigma_T} M$$

$$f_{rad} = f_{Grav}$$

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$$\frac{dm}{dt} = ?$$

$$L = \epsilon \frac{GM}{R} \frac{dm}{dt} \Longrightarrow \frac{dm}{dt} = \frac{L_E R}{\epsilon G M}$$

$$f_{rad} = f_{Grav}$$

$$\frac{L \sigma_T}{4\pi r^2 c} = \frac{G m_p M}{r^2} \longrightarrow L_E = \frac{4\pi c\, G m_p}{\sigma_T} M$$

$$\frac{dm}{dt} = ?$$

$$L = \epsilon \frac{GM}{R} \frac{dm}{dt} \Longrightarrow \frac{dm}{dt} = \frac{L_E R}{\epsilon G M} \qquad R = \frac{GM}{c^2}$$

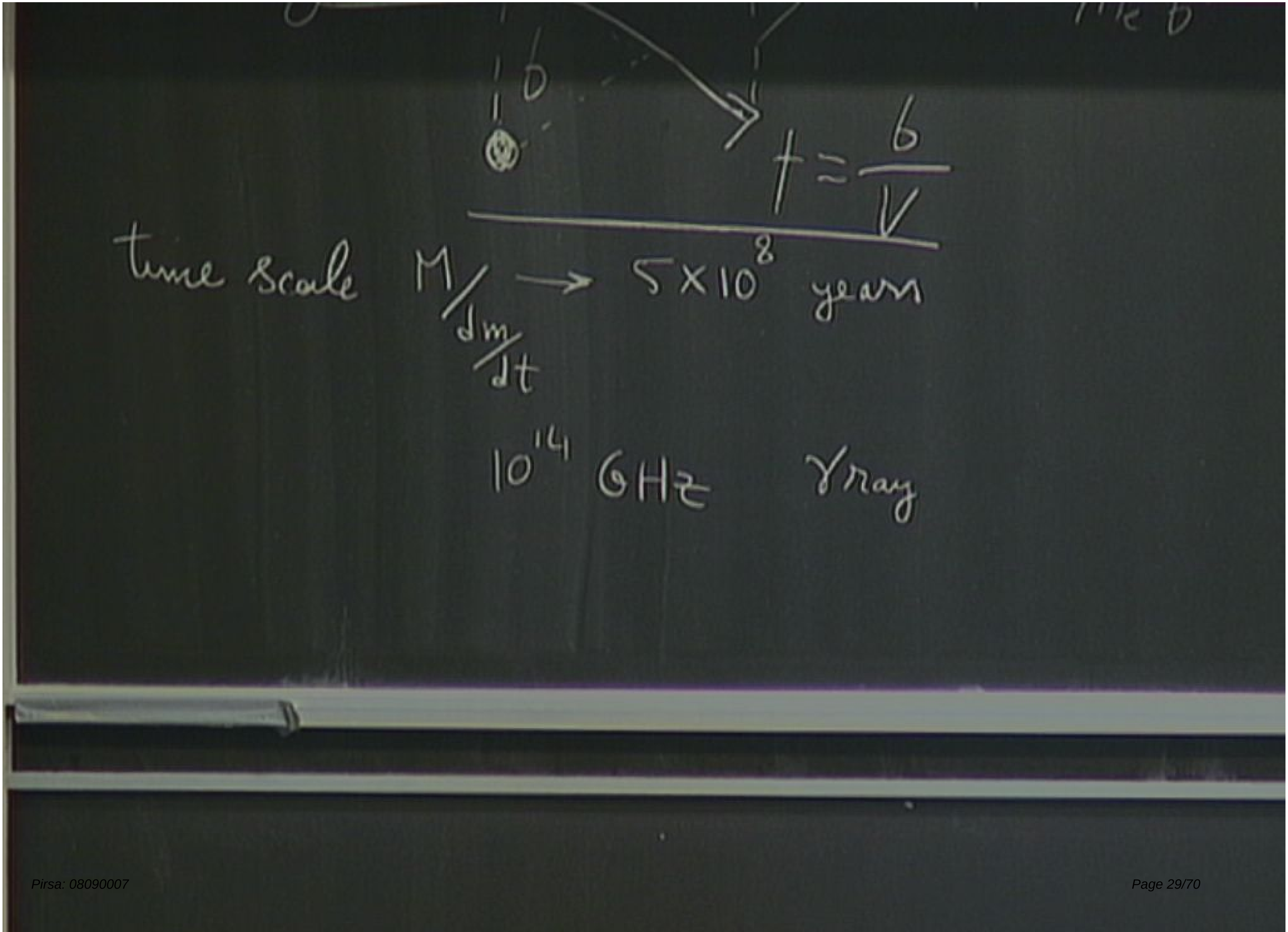
$$\Longrightarrow \frac{dm}{dt} = \frac{4\pi G m_p}{\sigma_T c}$$

time scale $M / \frac{dm}{dt} \rightarrow 5 \times 10^{8}$ years

$$t \approx \frac{b}{v}$$

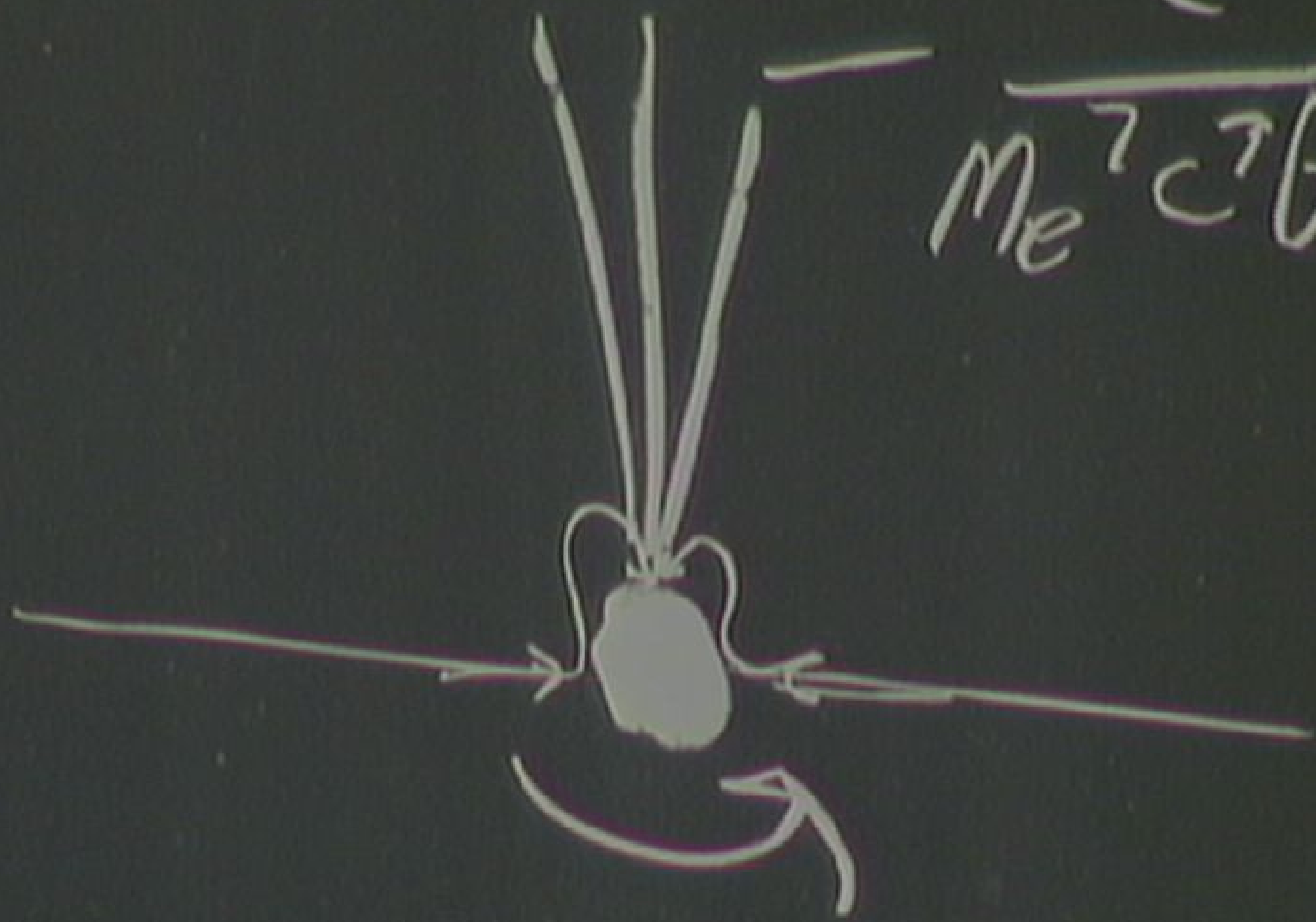
time scale $M / \frac{dm}{dt} \rightarrow 5 \times 10^{8}$ years

10^{14} GHz

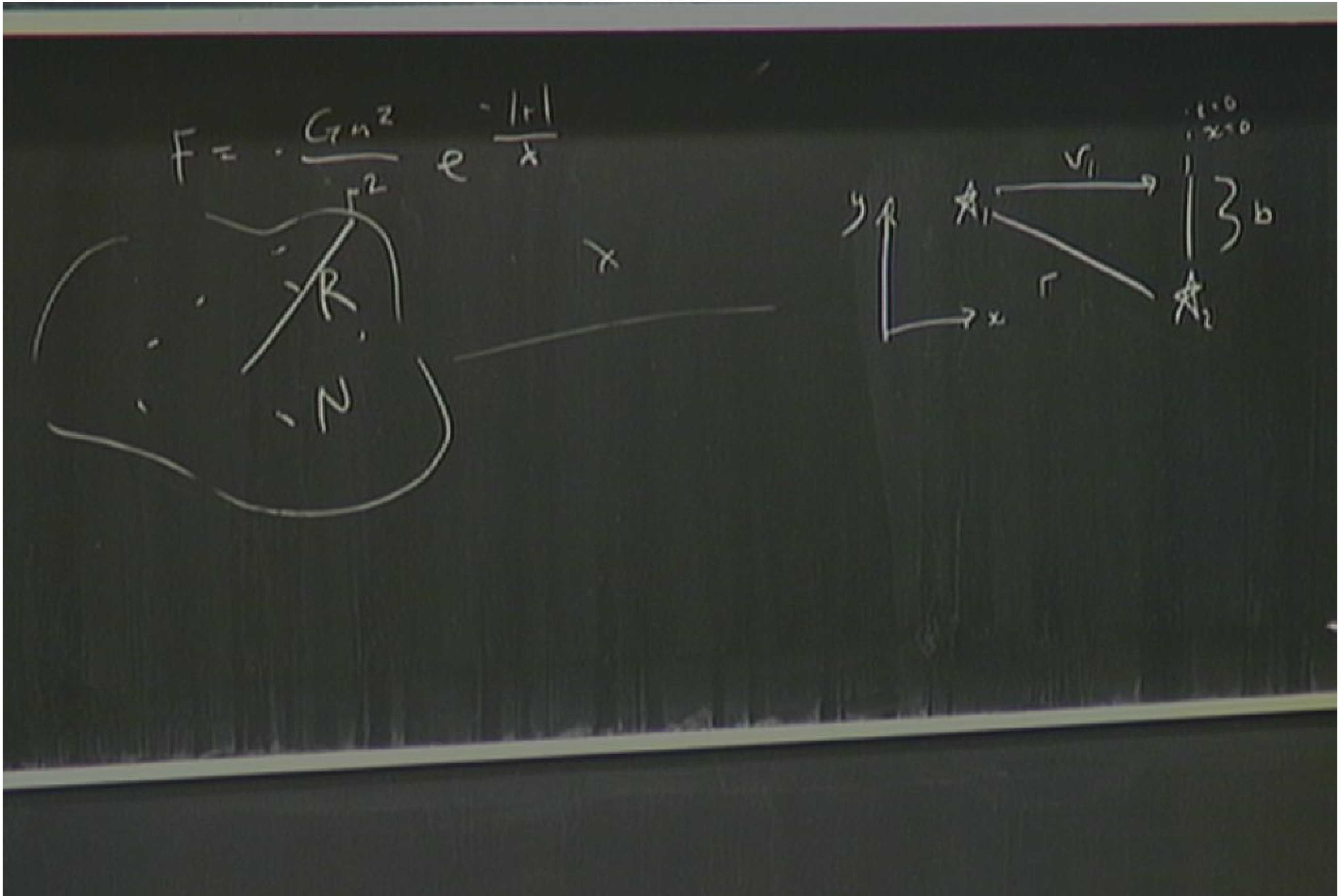

b
t = b/V
time scale M/(dm/dt) → 5 × 10^8 years
10^14 GHz γ ray

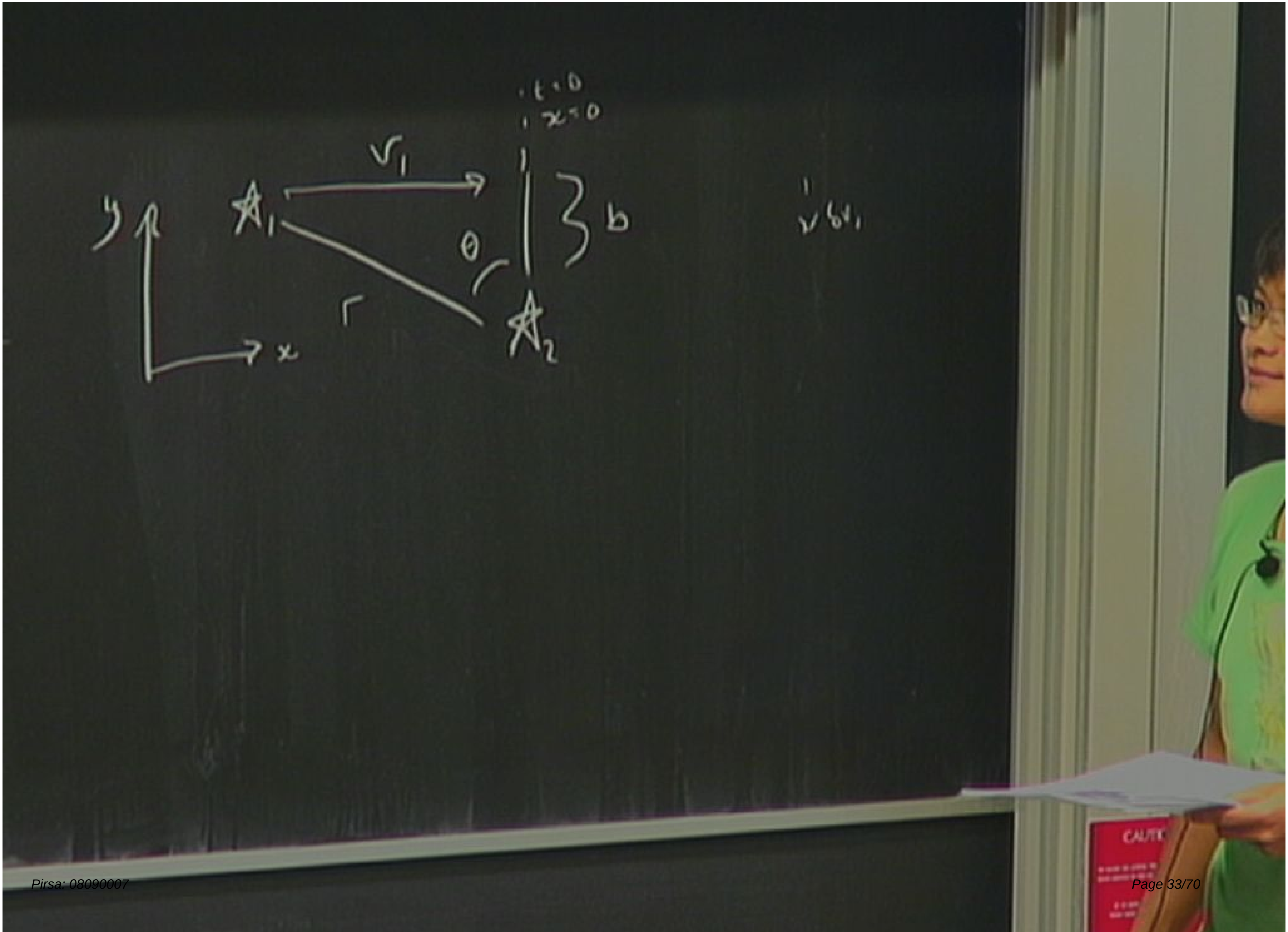
$m_e b^4 c$

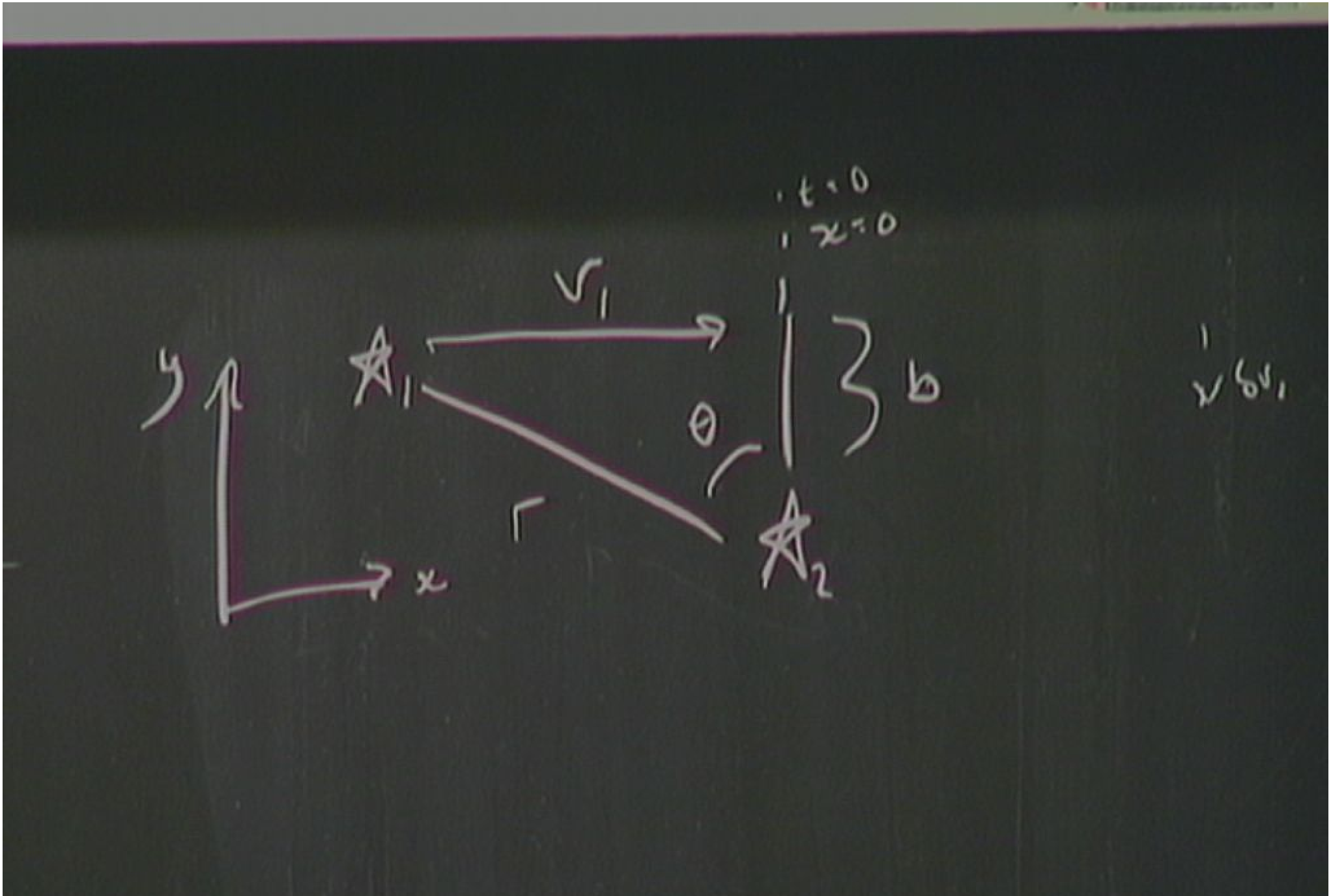
$$= \frac{a^6}{m_e^7 c^7 b^3 V}$$



$$F = -\frac{Gm^2}{r^2} e^{-\frac{|r|}{\lambda}}$$


$F = -\frac{Gm^2}{r^2} e^{-\frac{|r|}{\lambda}}$
R
N
λ
y
x
r
v_1
b
$t=0$
$x=0$


t = 0
x = 0
v1
b
θ
r
y
x


t = 0
x = 0
v1
A1
A2
b
θ
r
y
x

$$F_\perp = m\,\delta a_\perp$$

$$\delta a_\perp = \frac{1}{m} F_\perp$$

$$\frac{\delta v_\perp}{\delta t} = \frac{1}{m} F_{tot} \cos\theta$$

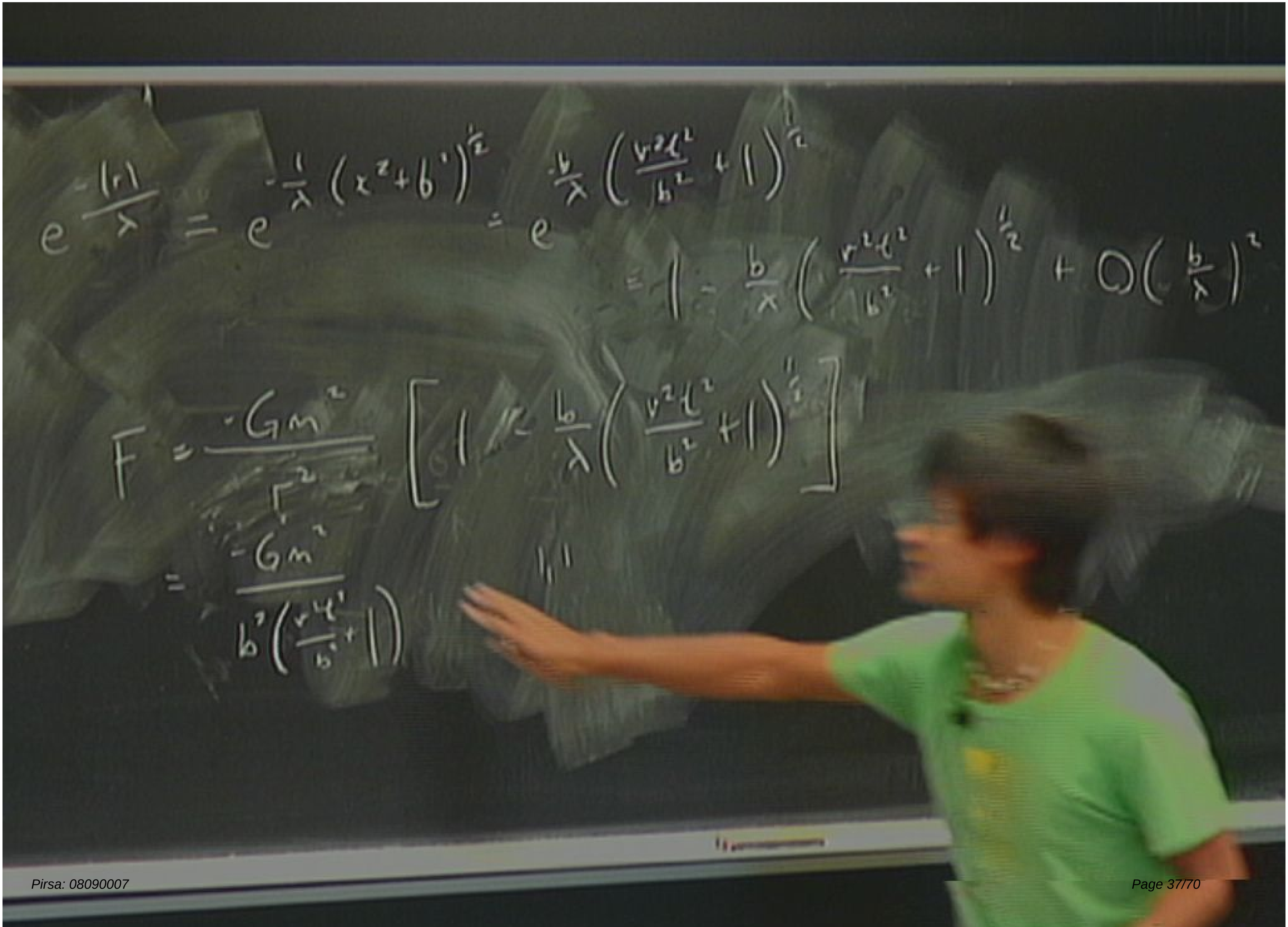
$$= \frac{1}{m} F_{tot} \frac{b}{r} \quad , \; r = (x^2 + b^2)^{1/2}$$

$$x(t) = x_0 + vt$$

$$= vt$$

$$e^{-\frac{|r|}{\lambda}} = e^{-\frac{1}{\lambda}\left(x^2+b^2\right)^{\frac{1}{2}}} = e^{-\frac{b}{\lambda}\left(\frac{r^2+l^2}{b^2}+1\right)^{\frac{1}{2}}}$$

$$= 1 - \frac{b}{\lambda}\left(\frac{r^2+l^2}{b^2}+1\right)^{\frac{1}{2}} + O\left(\frac{b}{\lambda}\right)^2$$

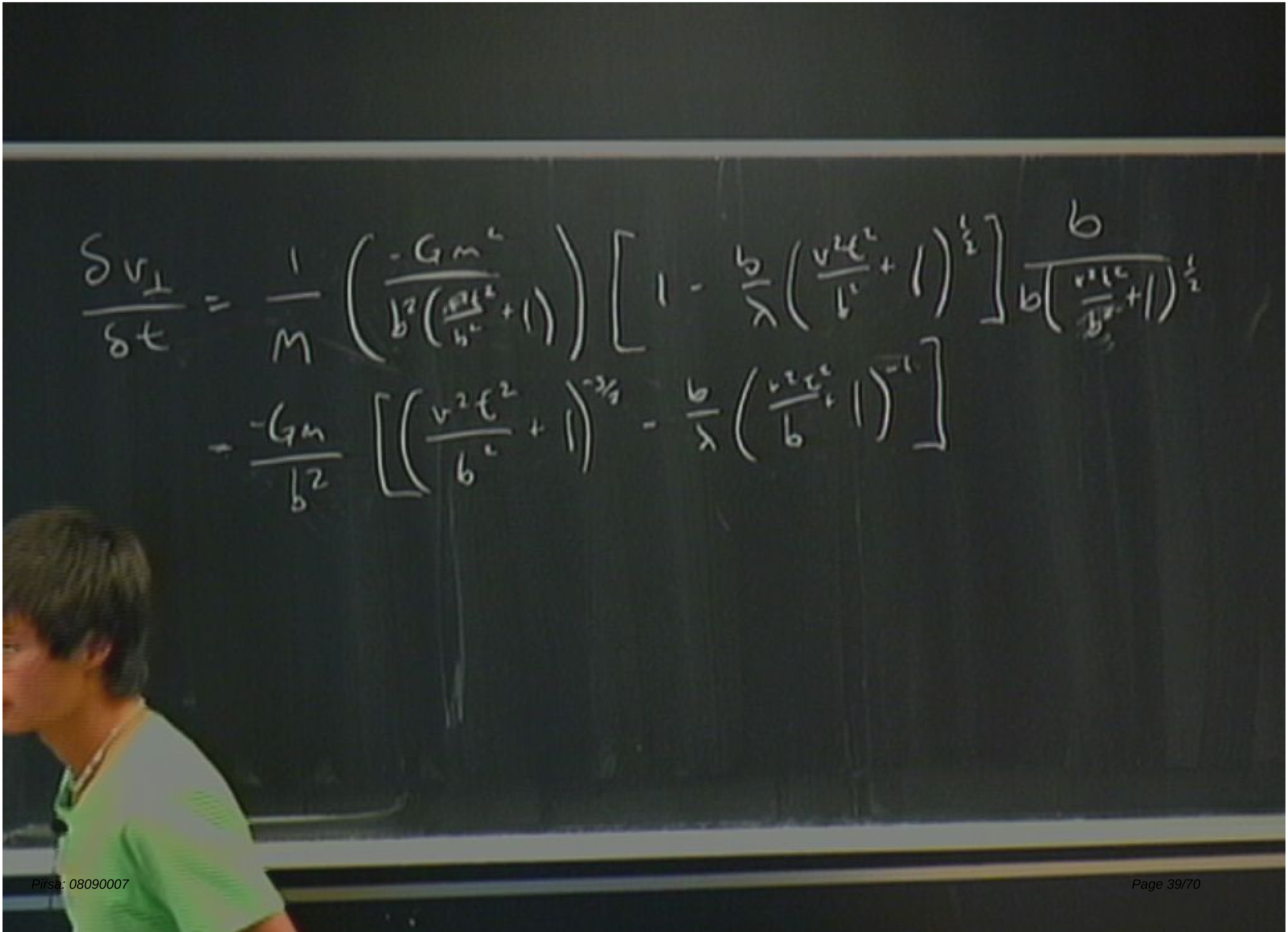

$e^{-\frac{|r|}{\lambda}} = e^{-\frac{1}{\lambda}(x^2+b^2)^{\frac{1}{2}}} = e^{-\frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}}}$
$= 1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}} + O\left(\frac{b}{\lambda}\right)^2$
$F = \frac{-Gm^2}{r^2}\left[1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}}\right]$
$= \frac{-Gm^2}{b^2\left(\frac{v^2t^2}{b^2}+1\right)}$

$$e^{-\frac{|r|}{\lambda}} = e^{-\frac{1}{\lambda}(x^2+b^2)^{1/2}} = e^{-\frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{1/2}}$$

$$= 1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{1/2} + O\left(\frac{b}{\lambda}\right)^2$$

$$F = \frac{-Gm^2}{r^2}\left[1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{1/2}\right]$$

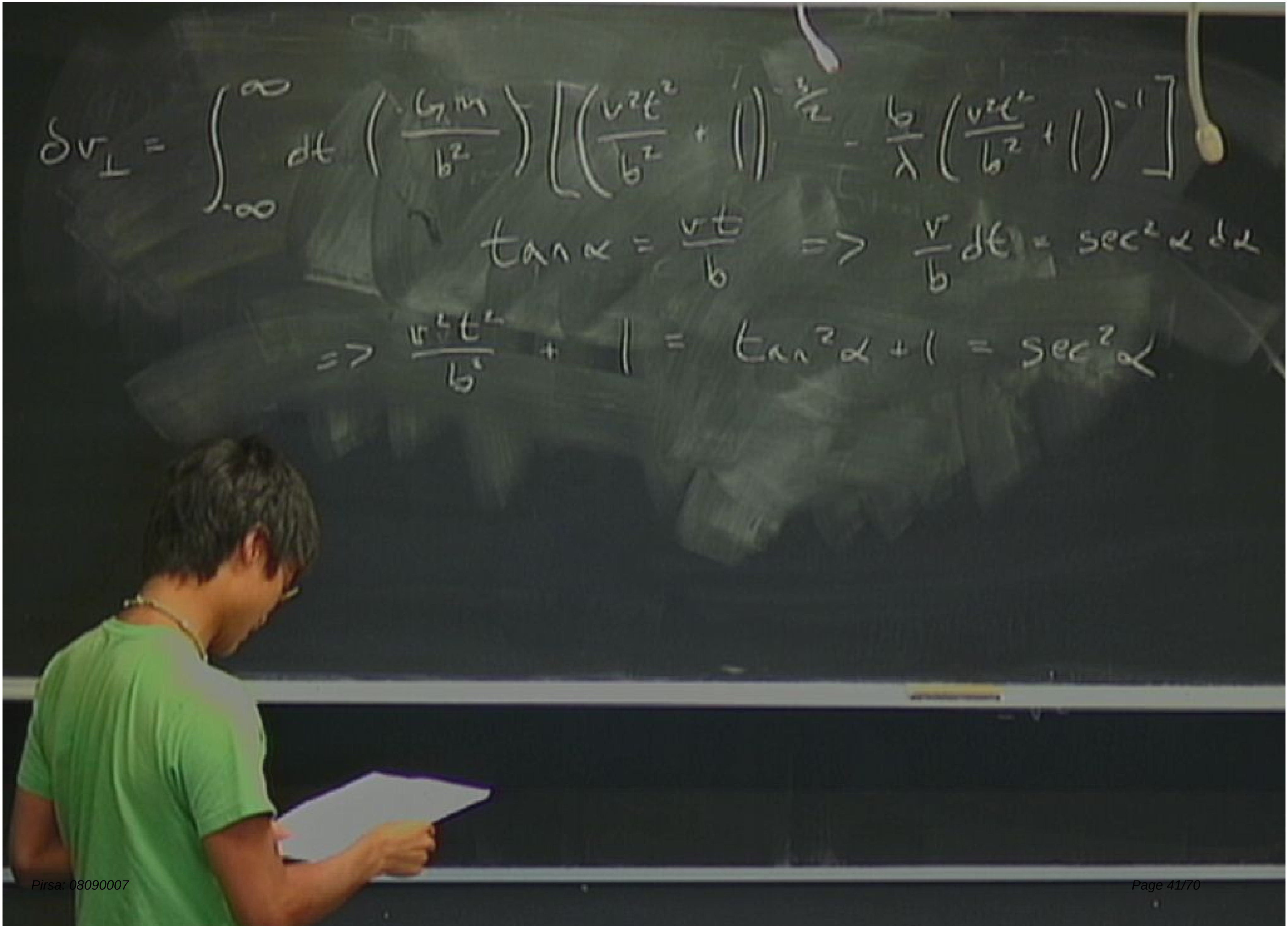
$$= \frac{-Gm^2}{b^2\left(\frac{v^2t^2}{b^2}+1\right)}$$



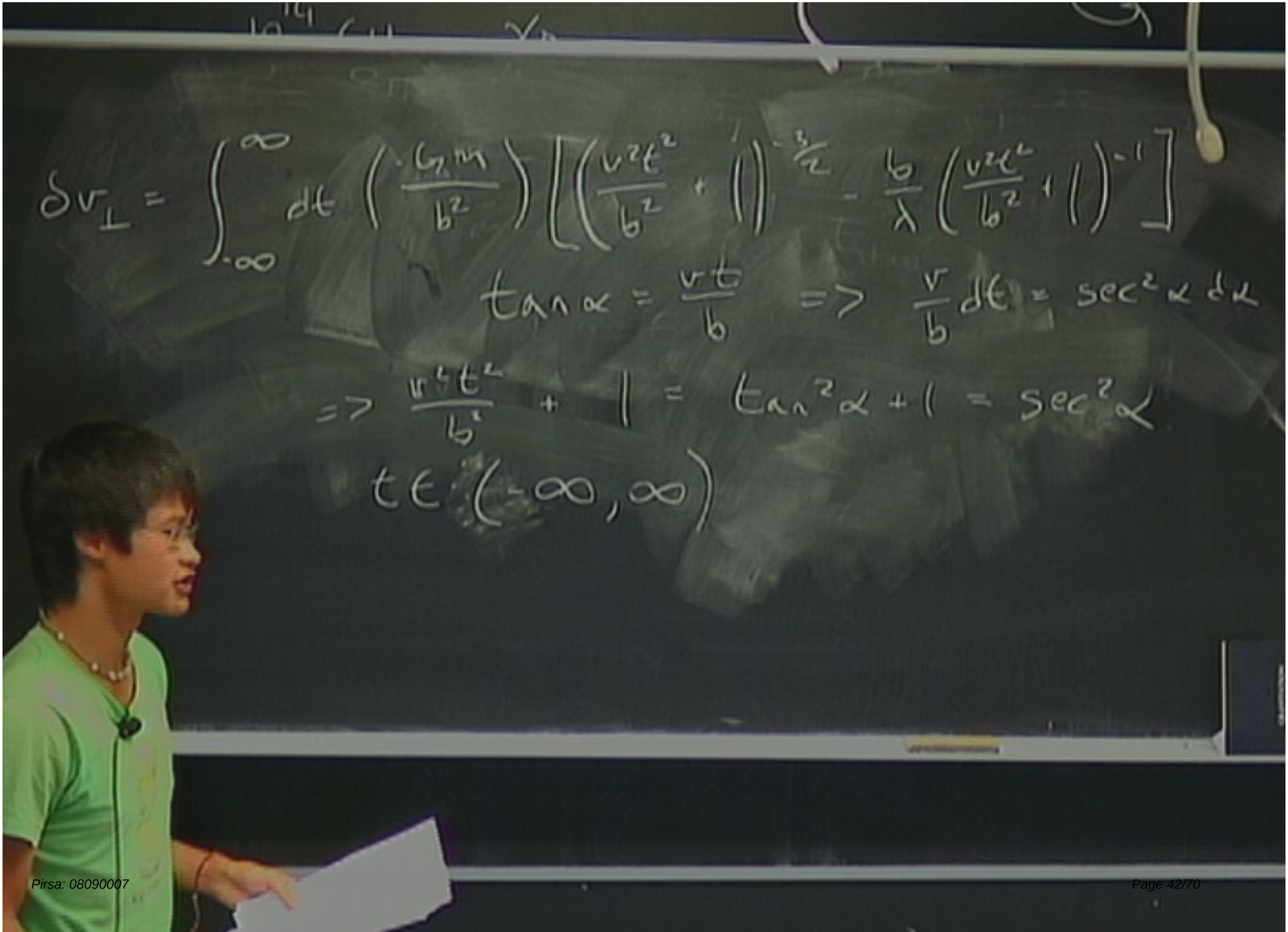
$\frac{\delta v_\perp}{\delta t} = \frac{1}{m}\left(\frac{-Gm^2}{b^2\left(\frac{v^2t^2}{b^2}+1\right)}\right)\left[1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}}\right]\frac{b}{b\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}}}$
$= \frac{-Gm}{b^2}\left[\left(\frac{v^2t^2}{b^2}+1\right)^{-3/2} - \frac{b}{\lambda}\left(\frac{v^2t^2}{b}+1\right)^{-1}\right]$

10^{14} GHz γ ray

$$\delta v_\perp = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2} \right) \left[\left(\frac{v^2 t^2}{b^2} + 1 \right)^{-3/2} - \frac{b}{\lambda} \left(\frac{v^2 t^2}{b^2} + 1 \right)^{-1} \right]$$



$\delta v_\perp = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right)\left[\left(\frac{v^2t^2}{b^2}+1\right)^{-3/2} - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{-1}\right]$
$\tan\alpha = \frac{vt}{b} \Rightarrow \frac{v}{b}dt = \sec^2\alpha\, d\alpha$
$\Rightarrow \frac{v^2t^2}{b^2} + 1 = \tan^2\alpha + 1 = \sec^2\alpha$


$\delta v_\perp = \int_{-\infty}^{\infty} dt \left(\frac{-Gm}{b^2}\right)\left[\left(\frac{v^2t^2}{b^2}+1\right)^{-3/2} - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{-1}\right]$
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$\Rightarrow \frac{v^2t^2}{b^2} + 1 = \tan^2\alpha + 1 = \sec^2\alpha$
$t \in (-\infty, \infty)$

$$\delta v_\perp = \int_{-\infty}^{\infty} dt \left(\frac{G_N m_4}{b^2}\right)\left[\left(\frac{v^2t^2}{b^2}+1\right)^{-3/2} - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{-1}\right]$$

$$\tan\alpha = \frac{vt}{b} \Rightarrow \frac{v}{b}dt = \sec^2\alpha\, d\alpha$$

$$\Rightarrow \frac{v^2t^2}{b^2} + 1 = \tan^2\alpha + 1 = \sec^2\alpha$$

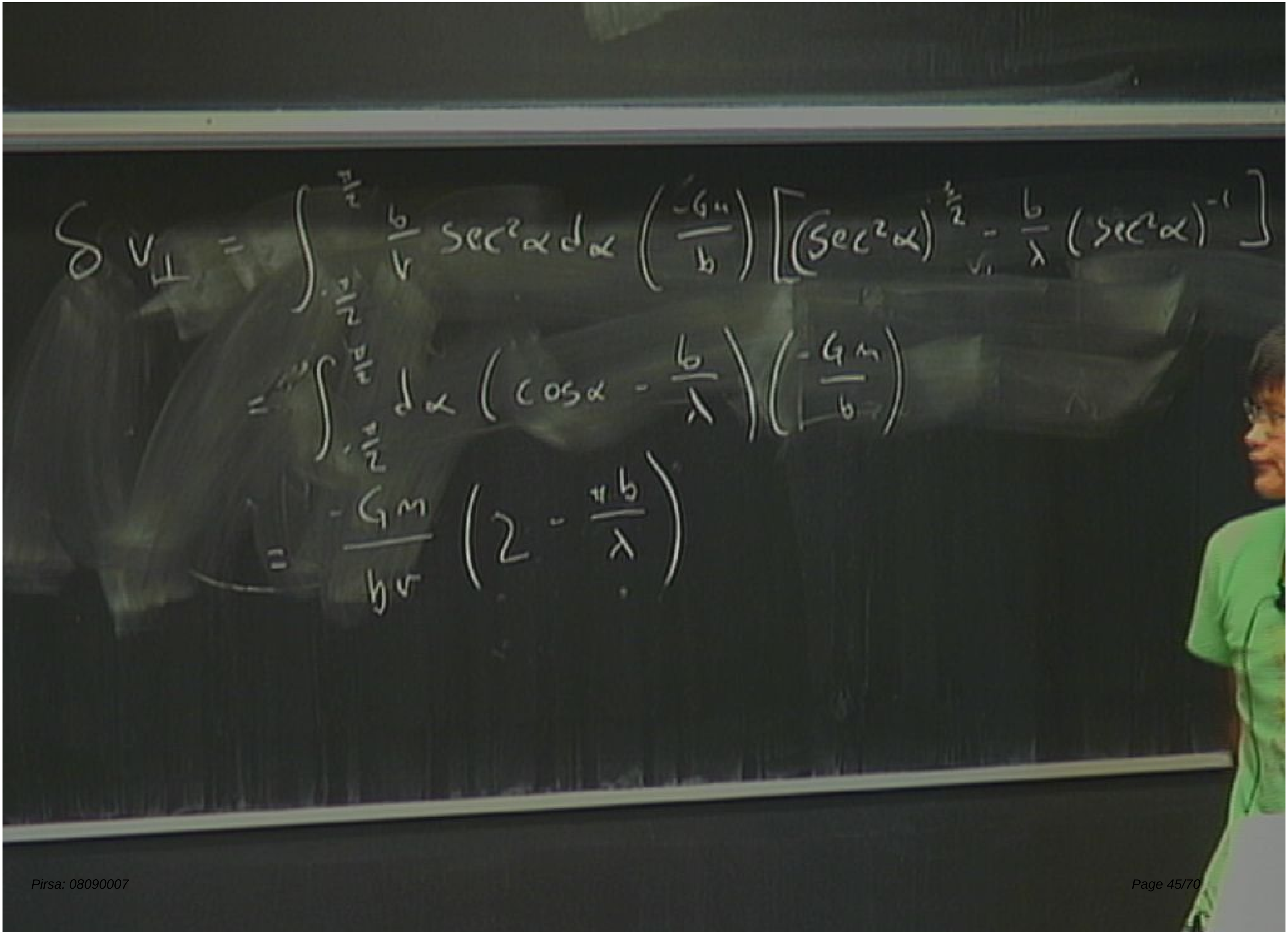
$$t \in (-\infty, \infty)$$

$$\delta v_\perp = \int_{-\infty}^{\infty} dt \left(\frac{Gm}{b^2}\right)\left[\left(\frac{v^2t^2}{b^2}+1\right)^{-3/2} - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{-1}\right]$$

$$\tan\alpha = \frac{vt}{b} \Rightarrow \frac{v}{b}dt = \sec^2\alpha\, d\alpha$$

$$\Rightarrow \frac{v^2t^2}{b^2} + 1 = \tan^2\alpha + 1 = \sec^2\alpha$$

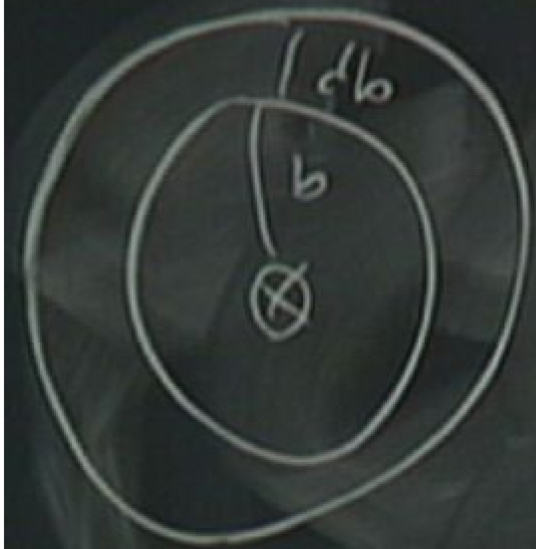
$$t \in (-\infty, \infty) \Rightarrow \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$


$\delta v_\perp = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{b}{v} \sec^2\alpha \, d\alpha \left(\frac{-Gm}{b}\right)\left[(\sec^2\alpha)^{-\frac{3}{2}} - \frac{b}{\lambda}(\sec^2\alpha)^{-1}\right]$
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\alpha \left(\cos\alpha - \frac{b}{\lambda}\right)\left(\frac{-Gm}{b}\right)$
$= \frac{-Gm}{bv}\left(2 - \frac{\pi b}{\lambda}\right)$

$$\delta v_{\perp} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{b}{v} \sec^2\alpha \, d\alpha \left(\frac{-Gm}{b} \right) \left[(\sec^2\alpha)^{\frac{3}{2}} - \frac{b}{\lambda} (\sec^2\alpha)^{-1} \right]$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\alpha \left(\cos\alpha - \frac{b}{\lambda} \right) \left(\frac{-Gm}{b} \right)$$

$$= \frac{-Gm}{bv} \left(2 - \frac{\pi b}{\lambda} \right)$$



$$A_r = \pi (b+db)^2 - \pi b^2$$

$$= 2\pi b\,db + O(db)^2$$

$$\# \text{ per } p = \frac{N}{\pi R^2}(2\pi b\,db)$$

$$n_p = \frac{2N}{R^2}\, b\,db$$

$$\langle (\delta v_\perp)^2 \rangle = n_p \, (\delta v_\perp)^2$$

$$= \frac{2N}{R^2}\, b\, db\, \frac{G^2 m^2}{b^2 v^2} \left[4 - 4\frac{\pi b}{\lambda} + O\left(\frac{b}{\lambda}\right)^2 \right]$$

$$= \frac{8N}{R^2} \frac{G^2 m^2}{v^2} \left(\frac{1}{b} - \frac{\pi}{\lambda} \right)$$

$$2\,K.E = P.E$$

$$F = \frac{-Gm^2}{R^2}\left[1 - \frac{R}{\lambda} + O\left(\frac{R}{\lambda}\right)^2\right]$$

$$= \frac{-Gm^2}{R^2} + \frac{Gm^2}{R\lambda}$$

$$\frac{\ln R}{\lambda} << 1$$

$$V = \frac{Gm^2}{R} - \frac{Gm^2}{\lambda}\ln R$$

$$2\,K.E = P.E$$

$$2\left(\frac{1}{2} m v^2\right) = N \frac{Gm}{R}$$

$$F = -\frac{Gm^2}{R^2}\left[1 - \frac{R}{\lambda} + O\left(\frac{R}{\lambda}\right)^2\right]$$

$$= -\frac{Gm^2}{R^2} + \frac{Gm^2}{R\lambda}$$

$$v^2 = \frac{GmN}{R}$$

$$V = \frac{Gm^2}{R} - \frac{Gm^2}{\lambda} \ln R$$

$$2\,K.E = P.E$$

$$2\left(\tfrac{1}{2} m v^2\right) = N \frac{G m^2}{R}$$

$$F = -\frac{G m^2}{R^2}\left[1 - \frac{R}{\lambda} + O\left(\left(\frac{R}{\lambda}\right)^2\right)\right]$$

$$v^2 = \frac{G m N}{R}$$

$$= -\frac{G m^2}{R^2} + \frac{G m^2}{R \lambda}$$

$$V = \frac{G m^2}{R} - \frac{G m^2}{\lambda} \ln R$$

$$\langle (\delta v_\perp)^2 \rangle = n_p (\delta v_\perp)^2$$

$$= \frac{2N}{R^2}\, b\, db\, \frac{G^2 m^2}{b^2 v^2} \left[4 - 4\frac{\pi b}{\lambda} + O\left(\frac{b}{\lambda}\right)^2 \right]$$

$$= \frac{8N}{R^2} \frac{G^2 m^2}{v^2} \left(\frac{1}{b} - \frac{\pi}{\lambda} \right)$$

$$= \frac{8}{N} v^2 \left(\frac{1}{b} - \frac{\pi}{\lambda} \right) db$$

$$e^{-\frac{|r|}{\lambda}} = e^{-\frac{1}{\lambda}(x^2+b^2)^{\frac{1}{2}}} = e^{-\frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}}}$$

$$= 1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}} + O\left(\frac{b}{\lambda}\right)^2$$

$$\Delta v_\perp^2 = \int_{b_{min}}^{b_{max}} \frac{8}{N} v^2 \left(\frac{1}{b} - \frac{\pi}{\lambda}\right) db$$

$b_{max} = R$ $\quad$ b_{min}: $\quad \delta v_\perp = v \Rightarrow$

$$\frac{-Gm}{b_{min} v}\left(2 - \frac{\pi b_{min}}{\lambda}\right) = v$$

$$e^{-\frac{|r|}{\lambda}} = e^{-\frac{1}{\lambda}(x^2+b^2)^{\frac{1}{2}}} = e^{-\frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}}}$$

$$= 1 - \frac{b}{\lambda}\left(\frac{v^2t^2}{b^2}+1\right)^{\frac{1}{2}} + O\left(\frac{b}{\lambda}\right)^2$$

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$$\frac{-Gm}{b_{min} v}\left(2 - \frac{\pi b_{min}}{\lambda}\right) = v$$

$$e^{-\frac{|r|}{\lambda}} = e^{-\frac{1}{\lambda}}($$

$$\frac{v^2}{Gm} = \frac{2}{b_{min}} - \frac{\pi}{\lambda}$$

$$\Delta v_\perp^2 = \int_{b_{min}}^{b_{max}} \frac{8}{N} v^2 \left(\frac{1}{b} - \frac{\pi}{\lambda}\right) db$$

$$b_{max} = R \qquad b_{min}: \quad \delta v_\perp = v \Rightarrow \frac{-Gm}{b_{min} v}\left(2 - \frac{\pi b_{min}}{\lambda}\right) = v$$

$$\frac{v^2}{Gm} = \frac{2}{b_{min}} - \frac{\pi}{\lambda}$$

$$\frac{2}{b_{min}} = \frac{\lambda v^2 + Gm\pi}{Gm\lambda}$$

$$\left(\frac{1}{b} - \frac{\pi}{\lambda}\right) db$$

b_{min}: $\delta v_\perp = v \Rightarrow$

$$\frac{-Gm}{v}\left(2 - \frac{\quad}{\lambda}\right) = v$$

$$\frac{v^2}{Gm} = \frac{2}{b_{min}} - \frac{\pi}{\lambda}$$

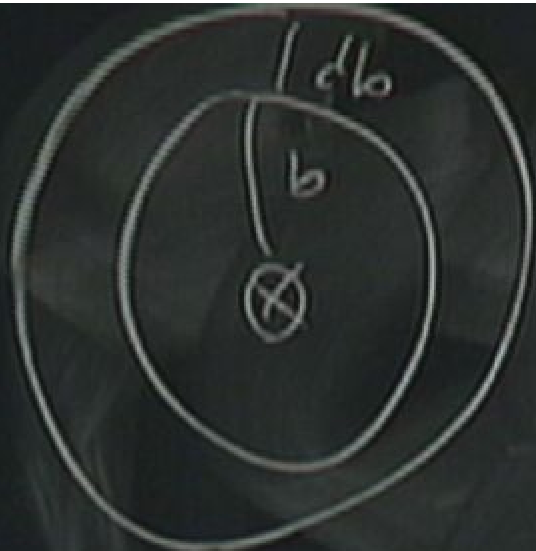
$$\frac{2}{b_{min}} = \frac{\lambda v^2 + Gm\pi}{Gm\lambda}$$

$$b_{min} = 2\left(\frac{Gm\lambda}{\lambda v^2 + Gm\pi}\right)$$

$$\left(\frac{1}{b} - \frac{\pi}{x}\right)db$$

b_{min}: $\delta v_\perp = v \Rightarrow$

$$\frac{-Gm}{b_{min} v}\left(2 - \frac{\pi b_{min}}{\lambda}\right) = v$$



$$A_r = \pi (b+db)^2 - \pi b^2$$

$$= 2\pi b\,db + O(db)^2$$

$$\#:\ \text{per } p = \frac{N}{\pi R^2}(2\pi b\,db)$$

$$b_{min} = 2\left(\frac{Gm\lambda}{\frac{\lambda G m N}{R} + \pi G m}\right) \qquad p = \frac{2N}{R^2}\, b\,db$$

$$\Rightarrow \frac{2\lambda}{\frac{N\lambda}{R} + \pi} \qquad \frac{\lambda}{R} \sim 10,\quad N \gg 1$$

$$b_{min} \simeq \frac{R}{N}$$

$$\Delta v_\perp^2 = \int_{R/N}^{R} \frac{8}{N} v^2 \left(\frac{1}{b} - \frac{\pi}{\lambda} \right) db$$

$$= \frac{8}{N} v^2 \left[\ln(N) - \frac{\pi}{\lambda}\left(R - \frac{R}{N}\right) \right]$$

$$=$$

b_{min}

$$\Delta v_\perp^2 = \int_{\frac{R}{N}}^{R} \frac{8}{N} v^2 \left(\frac{1}{b} - \frac{\pi}{\lambda} \right) db$$

$$= \frac{8}{N} v^2 \left[\ln(N) - \frac{\pi}{\lambda} \left(R - \frac{R}{N} \right) \right]$$

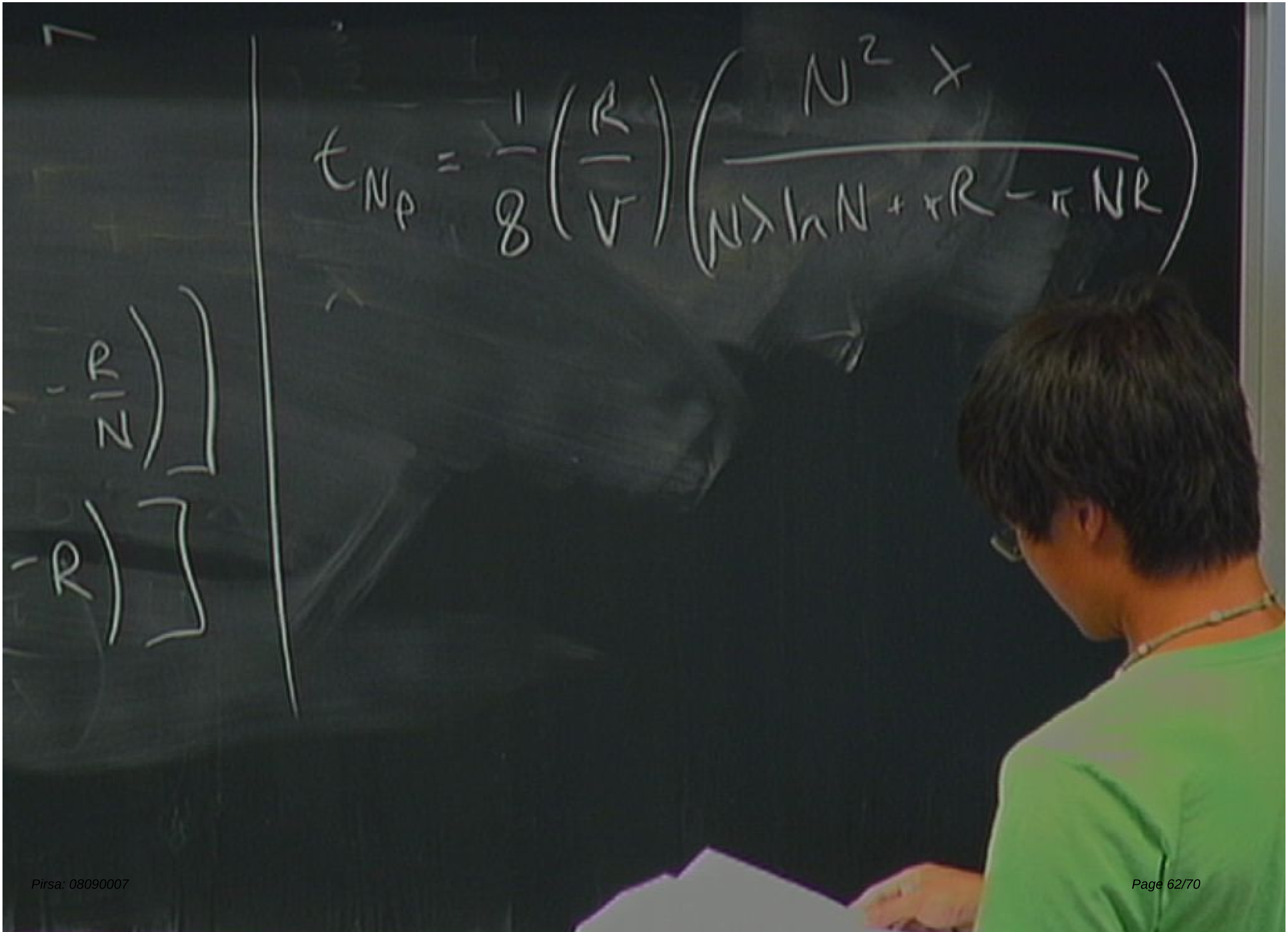
$$= 8 v^2 \left[\frac{\ln N}{N} + \frac{\pi}{N\lambda} \left(\frac{R}{N} - R \right) \right]$$

N

$$\frac{v^2}{N_p} = 8v^2\left[\frac{\ln N}{N} + \frac{\pi}{N\lambda}\left(\frac{R}{N} - R\right)\right]$$

$$\frac{1}{N_p} = 8\left[\frac{N\lambda\ln N + \pi R - \pi NR}{N^2\lambda}\right]$$

$$N_p = \frac{1}{8}\left(\frac{N^2\lambda}{N\lambda\ln N + \pi R - \pi NR}\right)$$



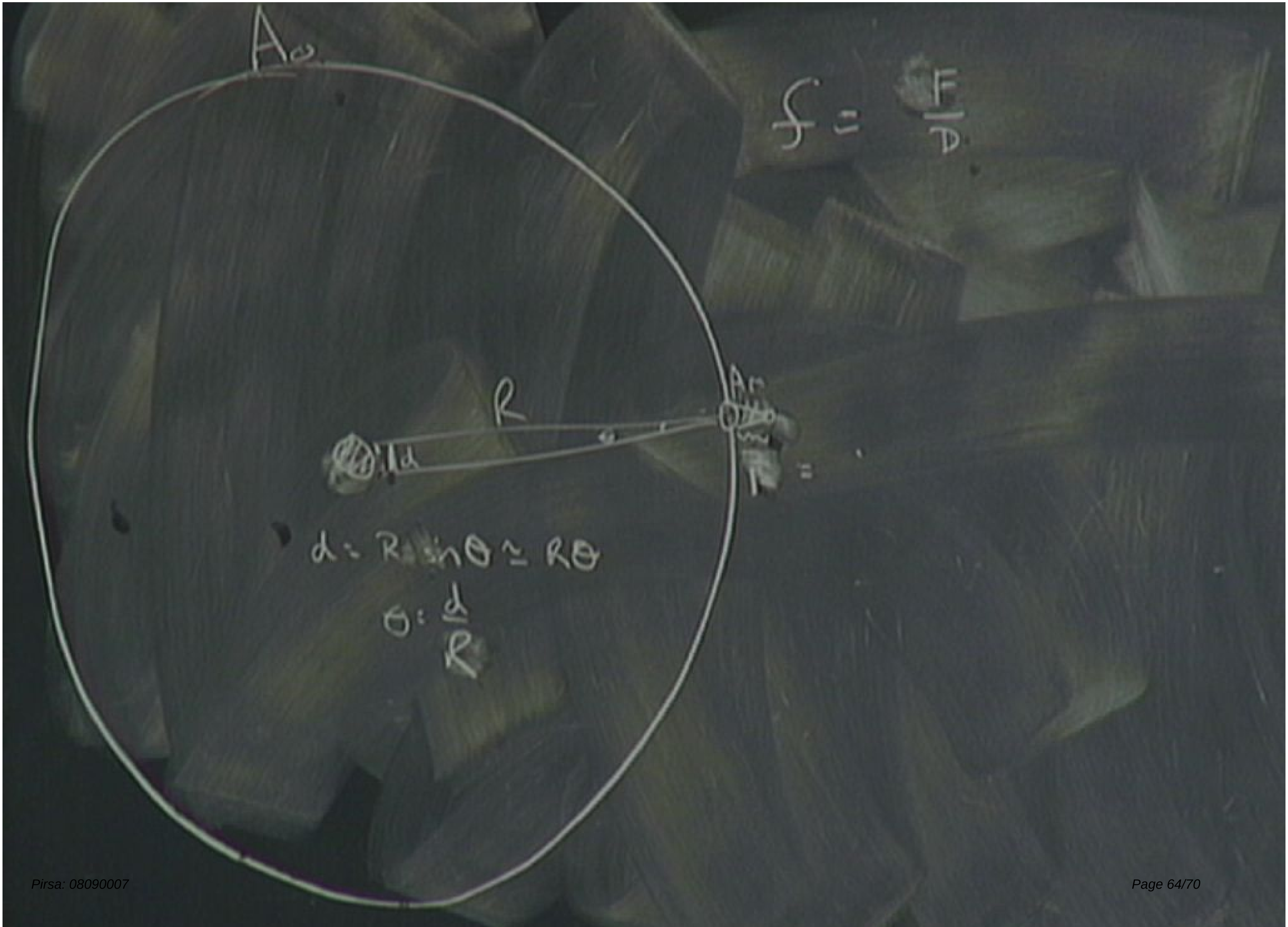
$t_{N_p} = \frac{1}{8}\left(\frac{R}{v}\right)\left(\frac{N^2 x}{N x \ln N + x R - \pi N R}\right)$

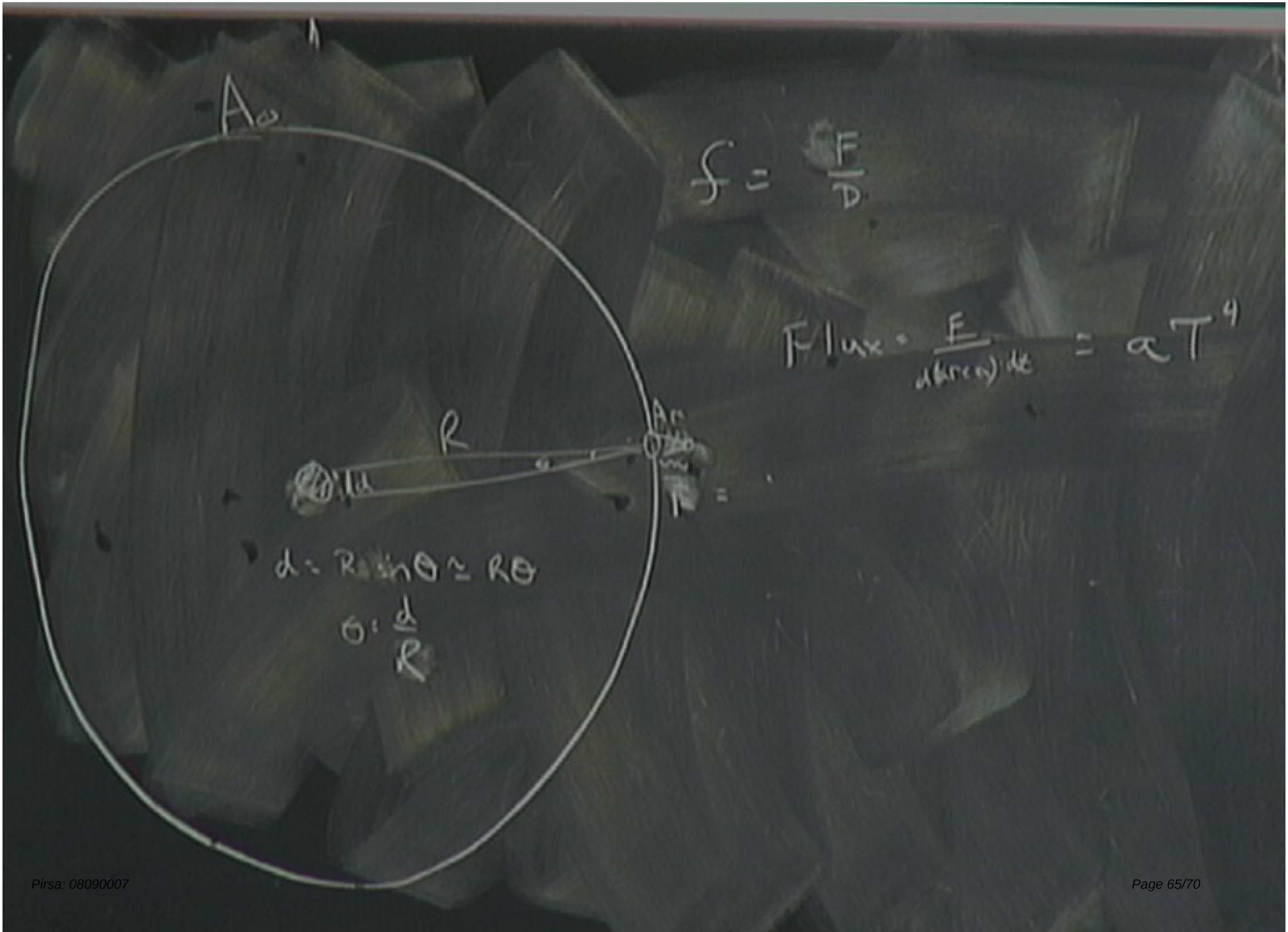
$$- \frac{R}{N} \Big) \Big]$$

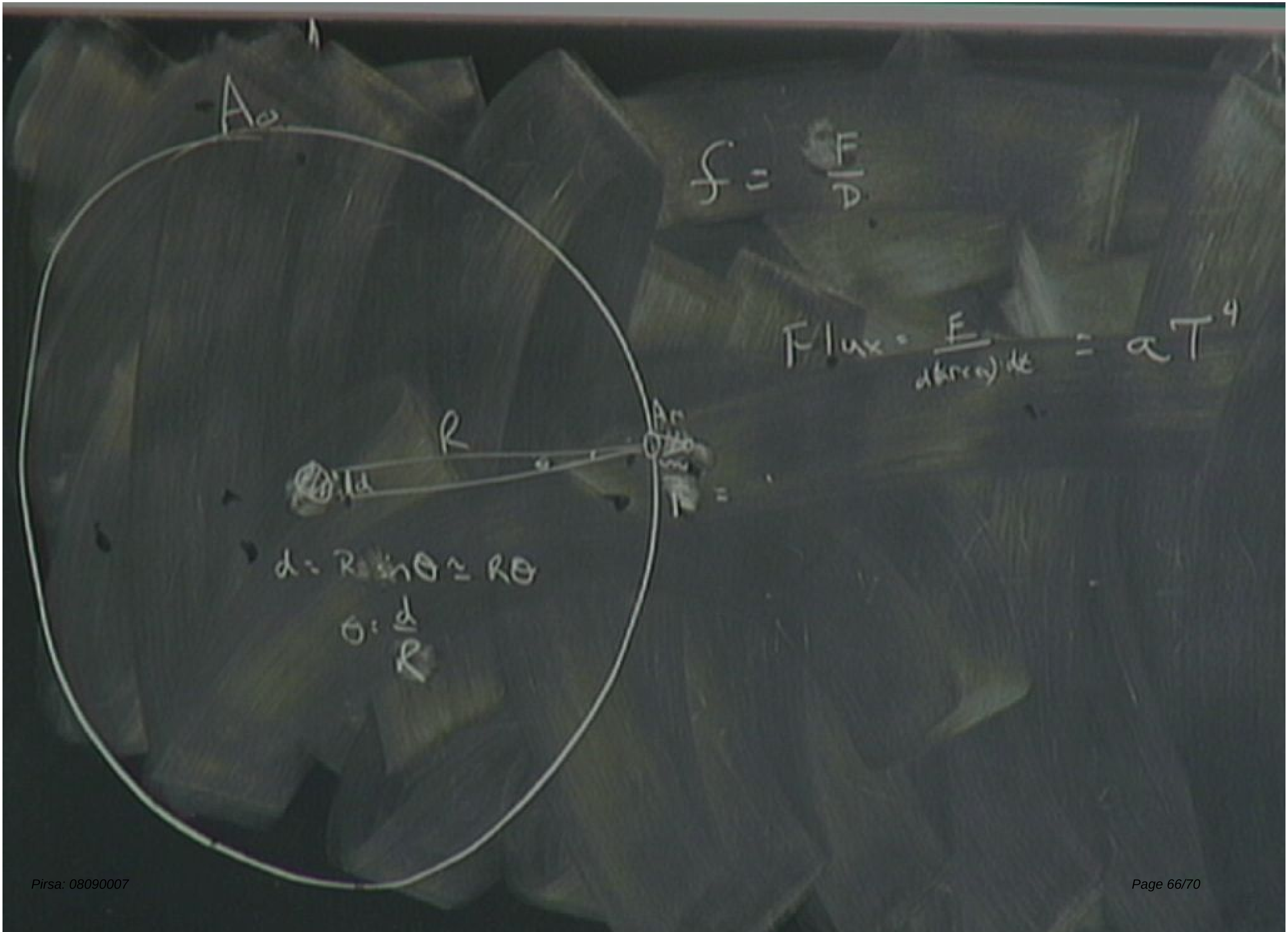
$$- R \Big) \Big]$$

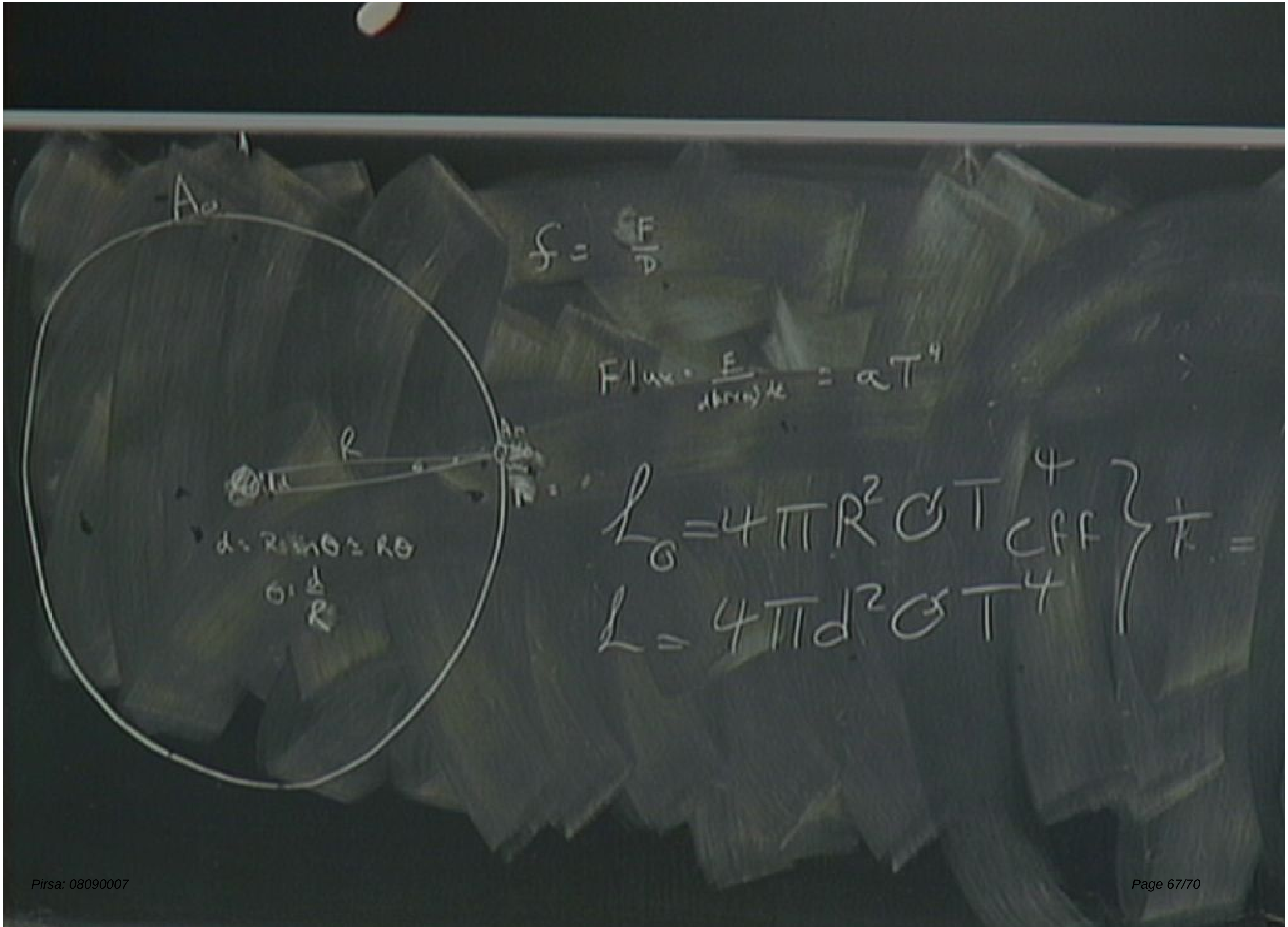
$$t_{N_p} = \frac{1}{8}\left(\frac{R}{V}\right)\left(\frac{N^2 \lambda}{N\lambda \ln N + \pi R - \pi N R}\right)$$

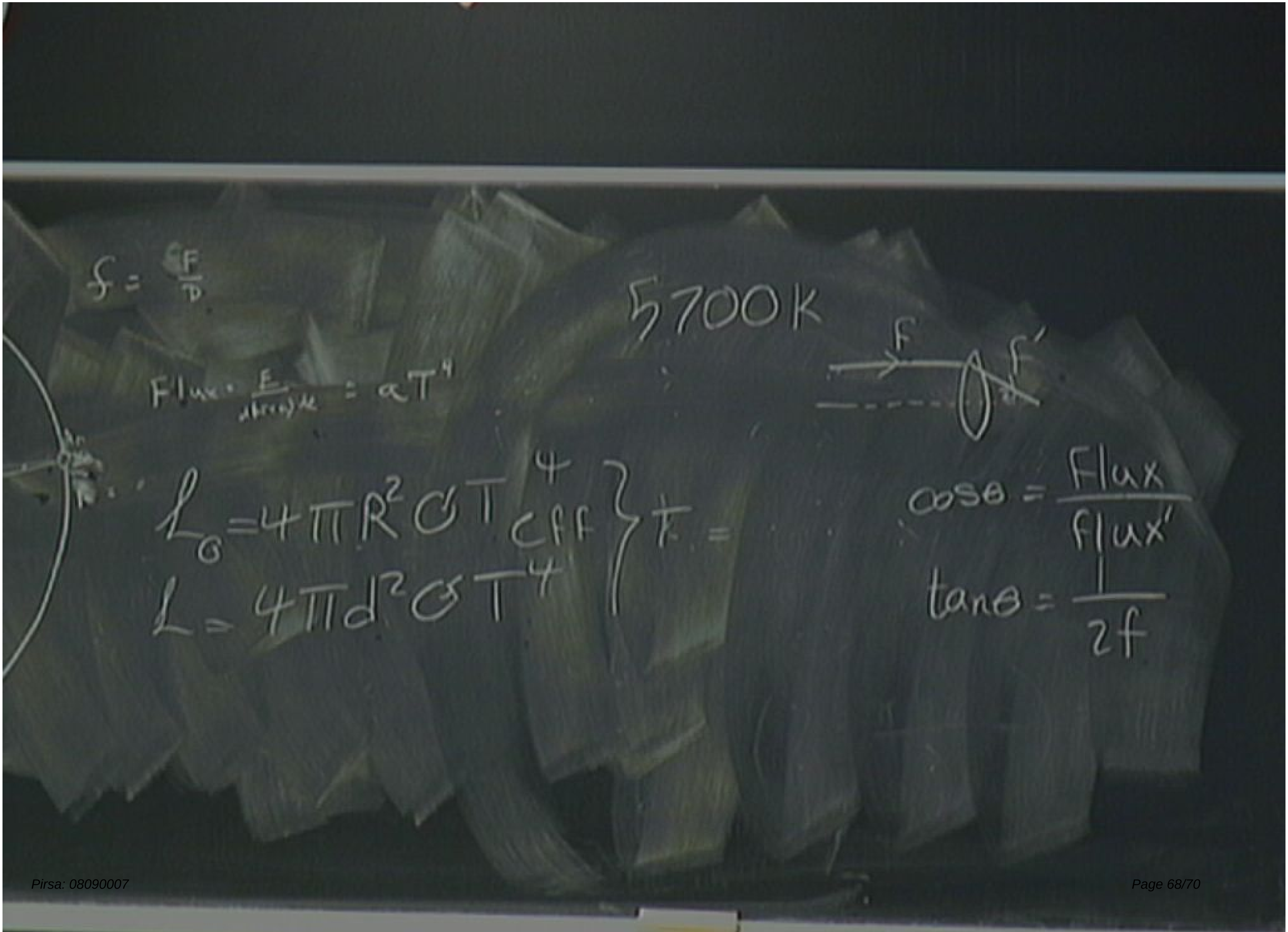
$$t_{N_p\,\mathrm{norm}} = \frac{1}{8}\left(\frac{R}{V}\right)\frac{N}{\ln N}$$

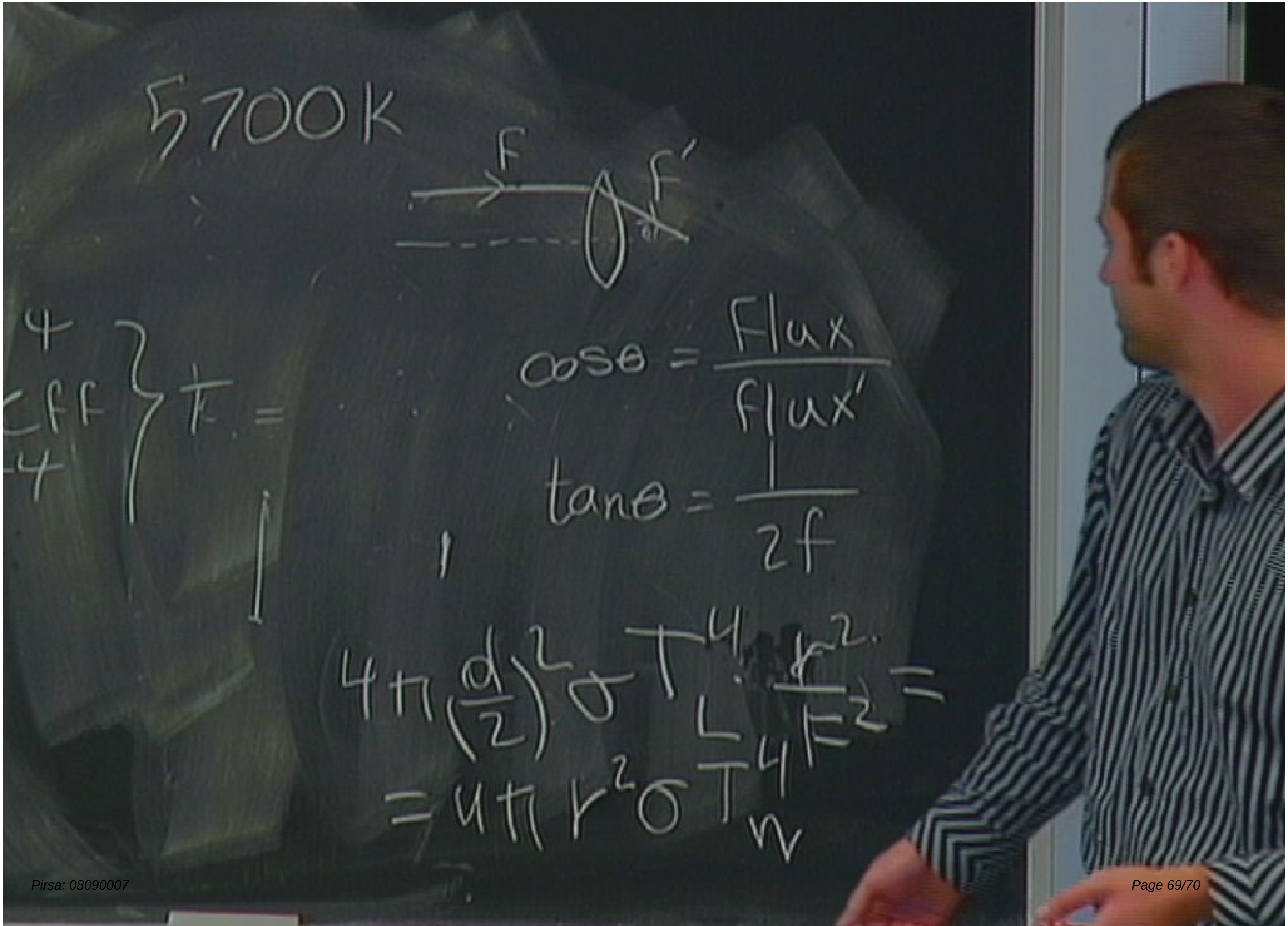

f = F/D
R
d
d = R sinθ ≃ Rθ
θ = d/R

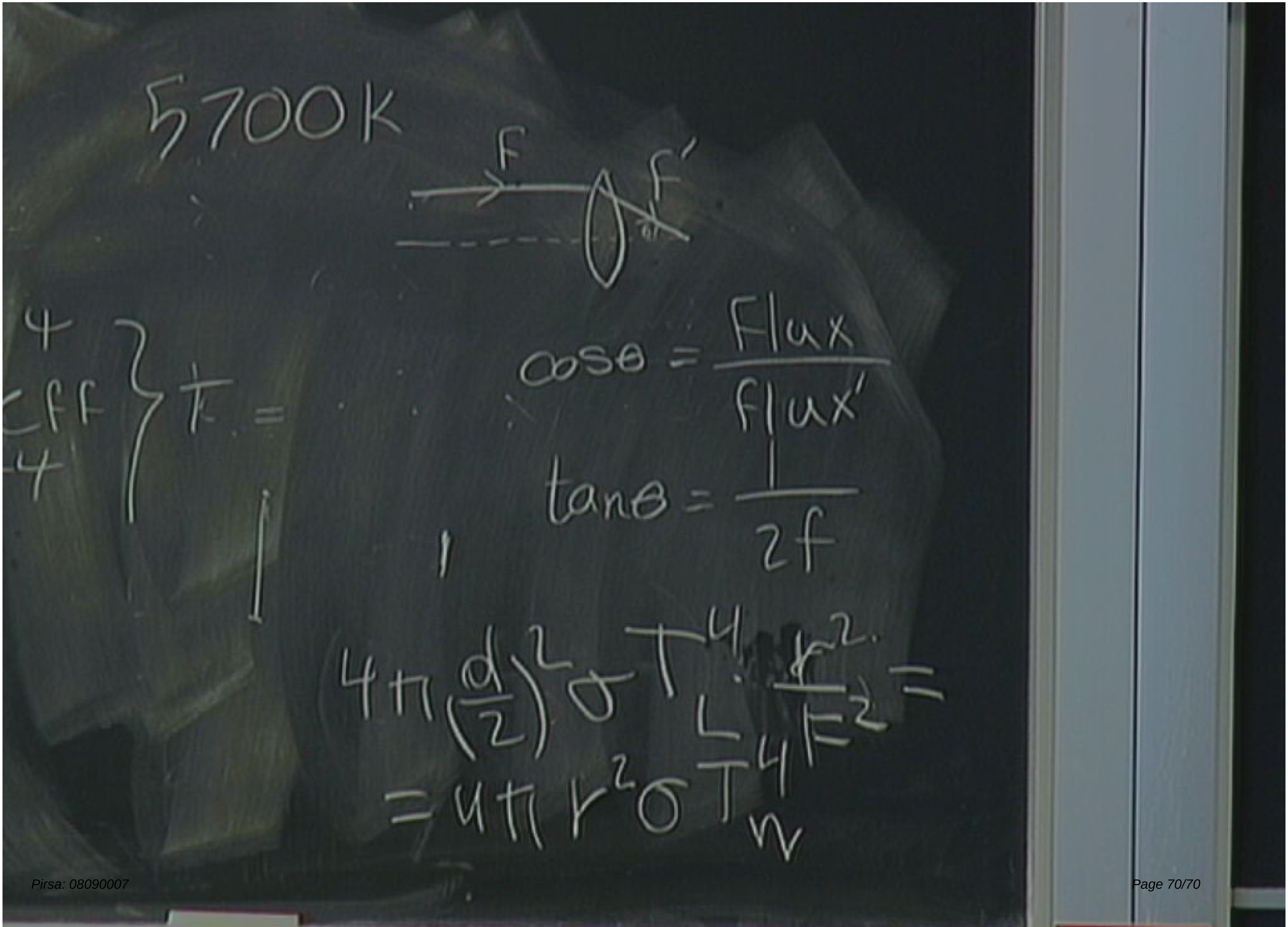

f = F/D
Flux = E/ (d(area) dt) = σT^4
R
d = R sinθ ≈ Rθ
θ = d/R


A⊙
f = F/D
Flux = E / d(4πr²)·dt = aT⁴
R
d
d = R sinθ ≈ Rθ
θ = d/R


A⊙
f = F/D
Flux = E/(d(area) dt) = σT^4
R
d = R sinθ ≈ Rθ
θ = d/R
L⊙ = 4πR²σT_eff^4
L = 4πd²σT^4


f = F/D
Flux = E/(area)(time) = σT^4
5700 K
F
F'
L_⊙ = 4πR²σT_eff^4
L = 4πd²σT^4
F =
cosθ = Flux/flux'
tanθ = 1/2f


5700 K
F
F'
cos θ = Flux / flux'
tan θ = 1 / 2f


5700K
F
F'
cosθ = Flux / flux'
tanθ = 1 / 2f
4π(d/2)²σT⁴
= 4πr²σT⁴