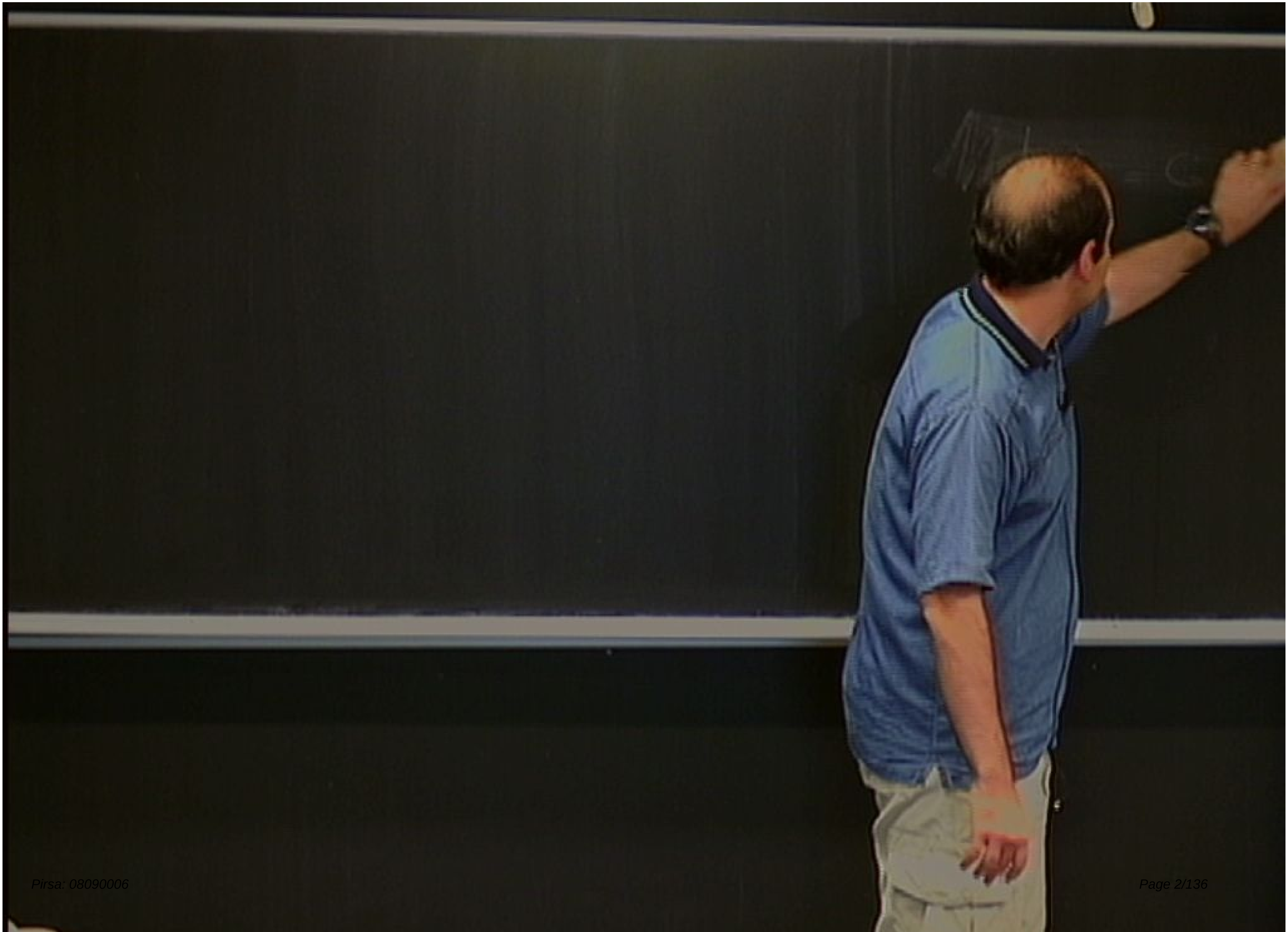
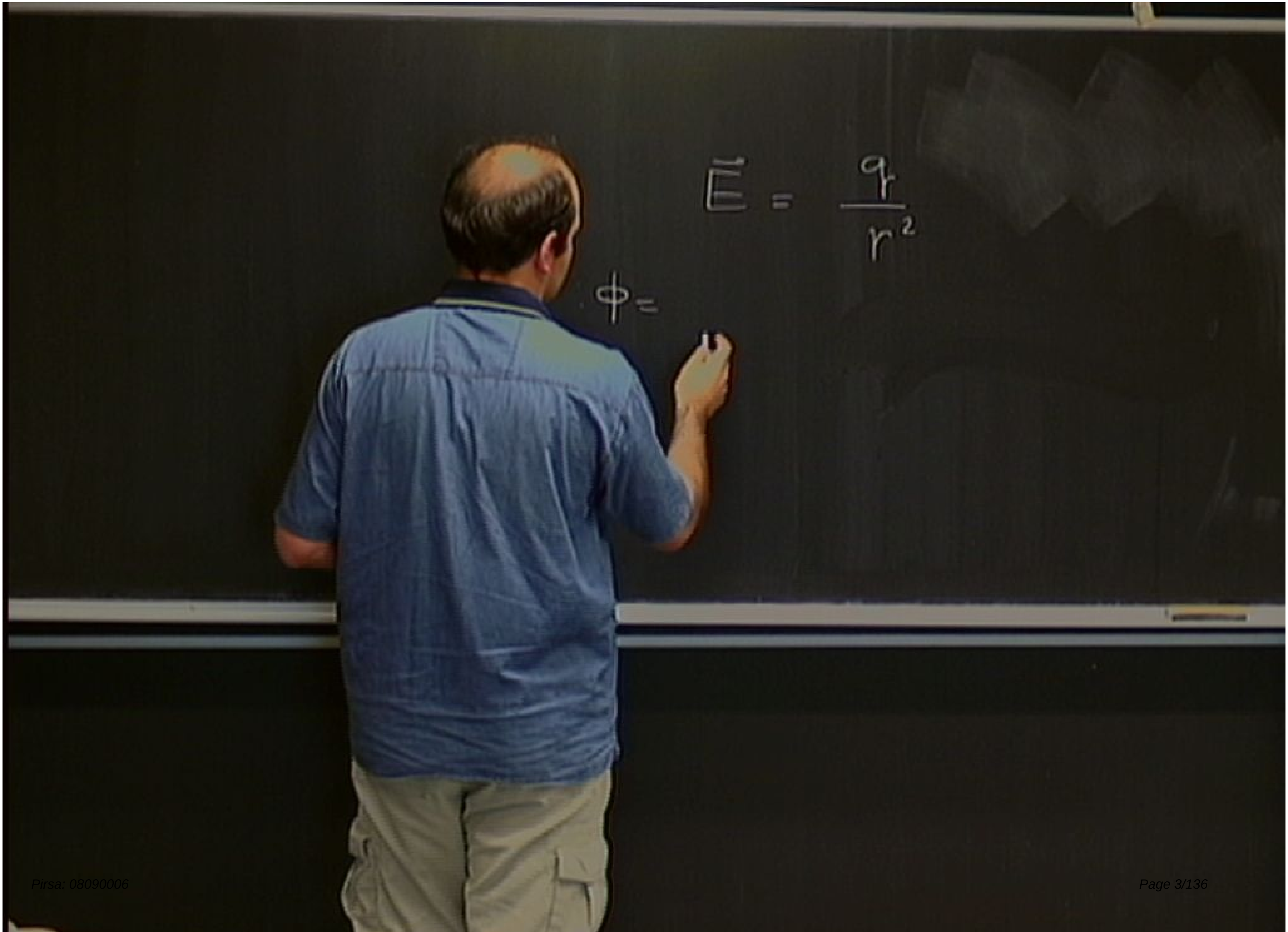
Title: Astrophysics and Cosmology through Problems - 1A

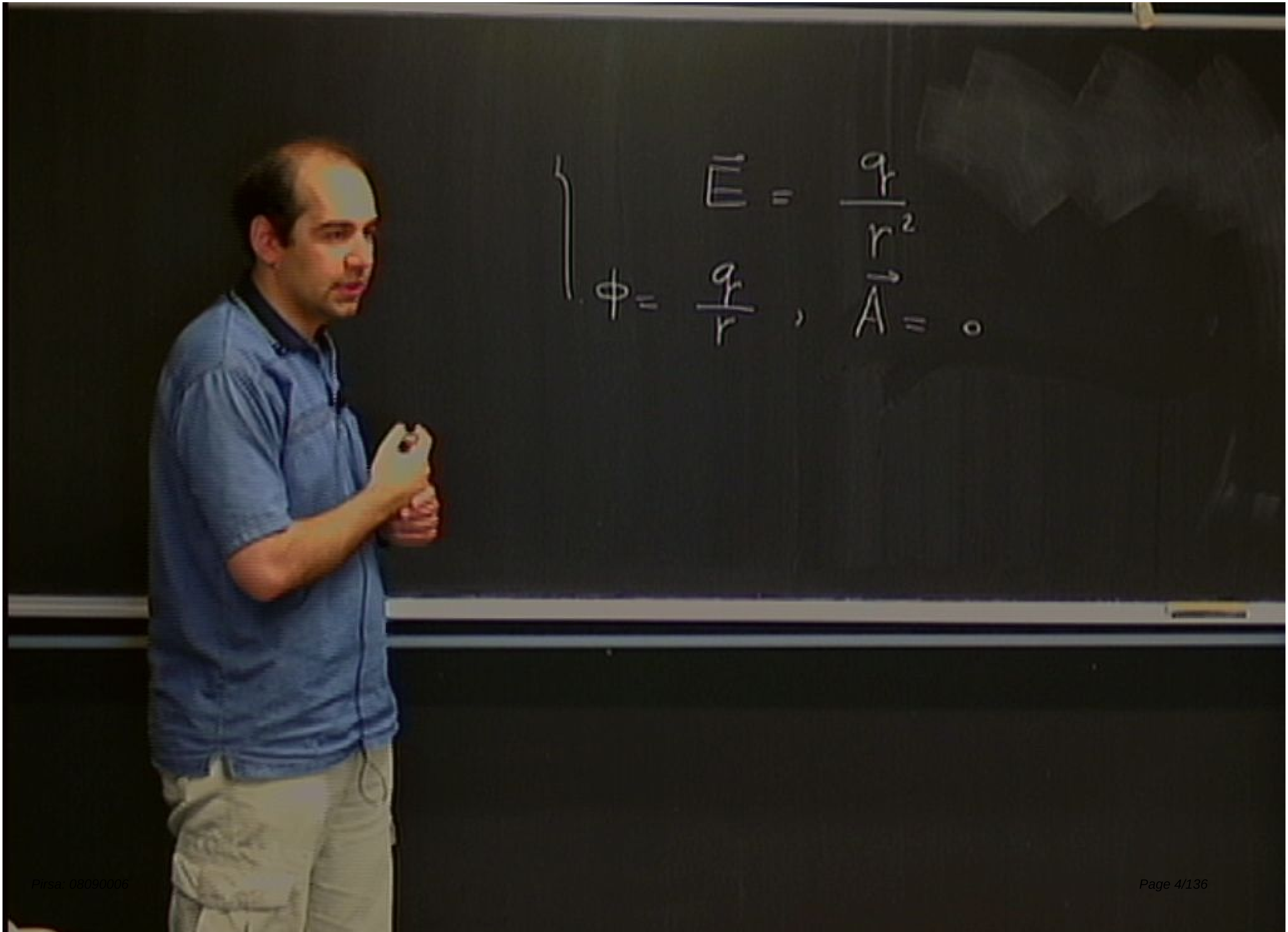
Date: Sep 04, 2008 10:00 AM

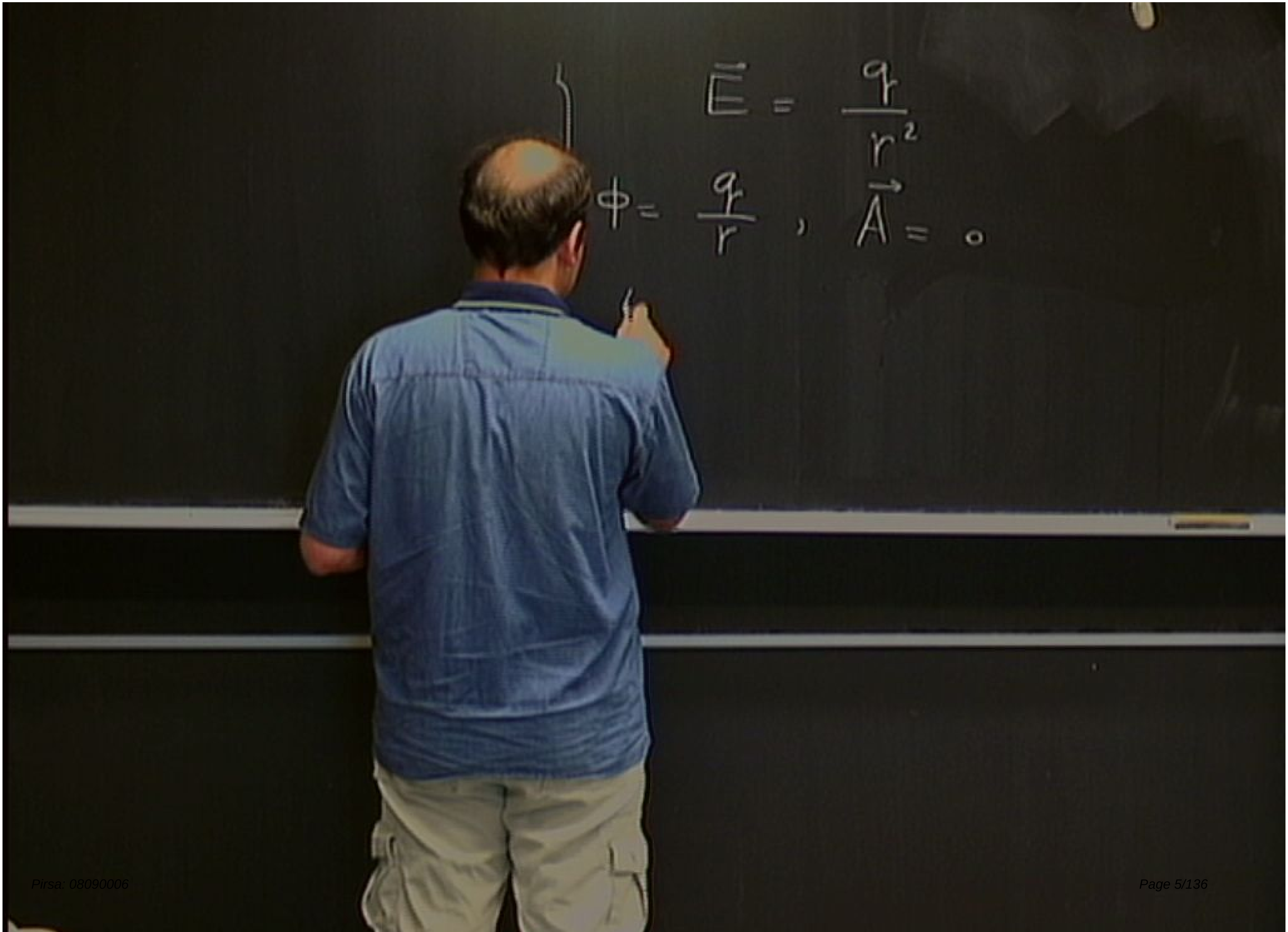
URL: http://pirsa.org/08090006

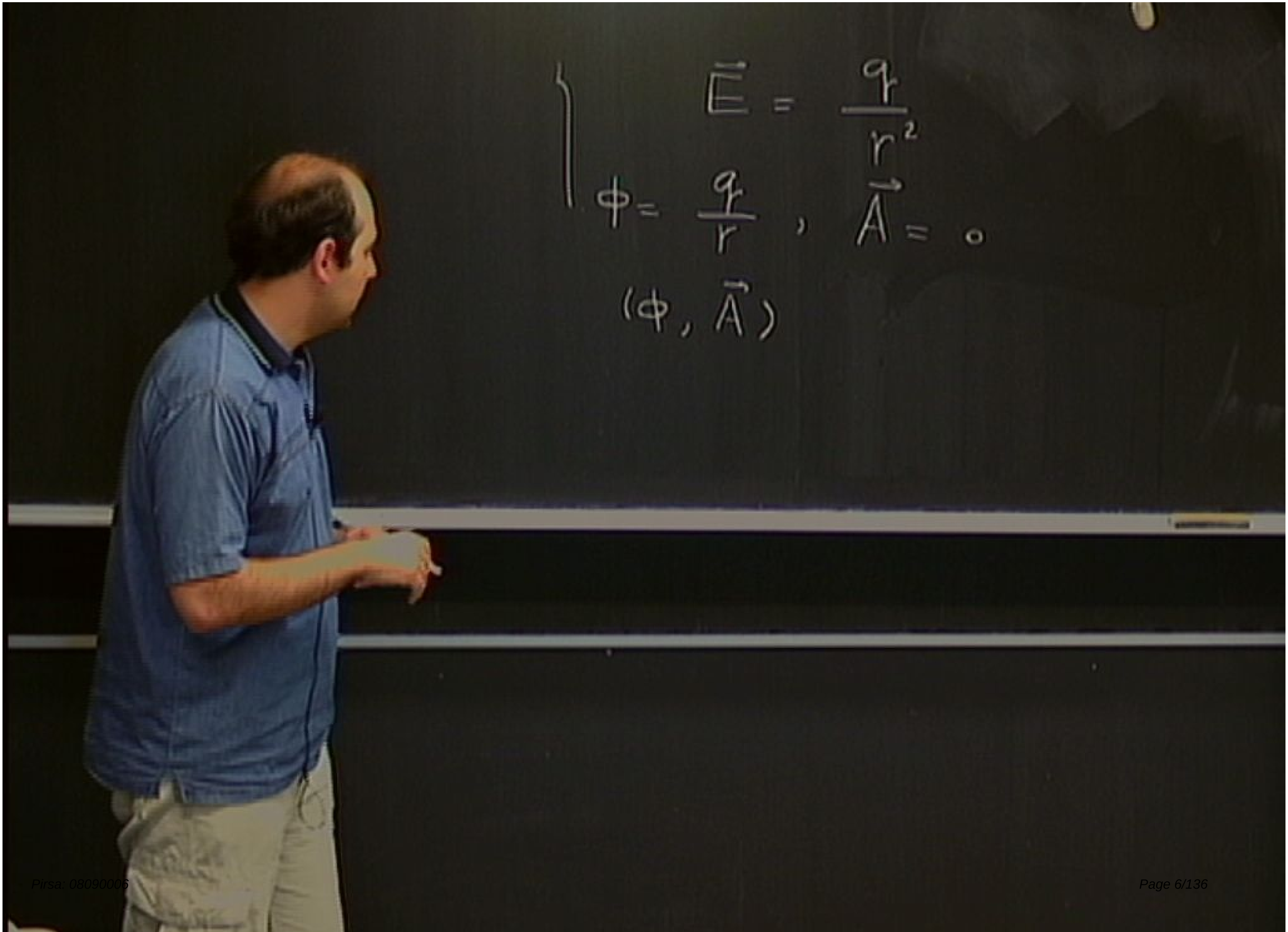
Abstract: This course is aimed at advanced undergraduate and beginning graduate students, and is inspired by a book by the same title, written by Padmanabhan. Each session consists of solving one or two pre-determined problems, which is done by a randomly picked student. While the problems introduce various subjects in Astrophysics and Cosmology, they do not serve as replacement for standard courses in these subjects, and are rather aimed at educating students with hands-on analytic/numerical skills to attack new problems.

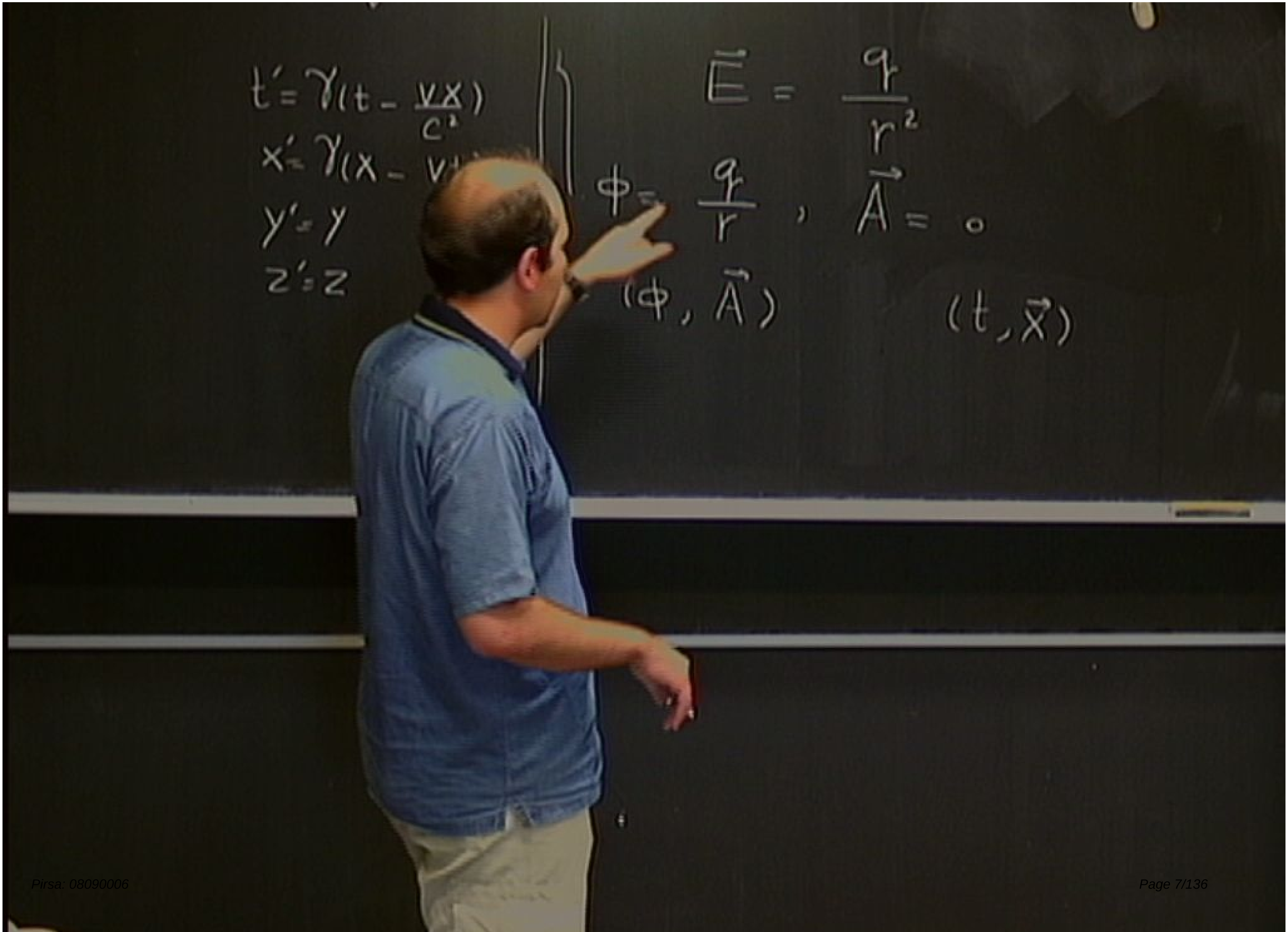


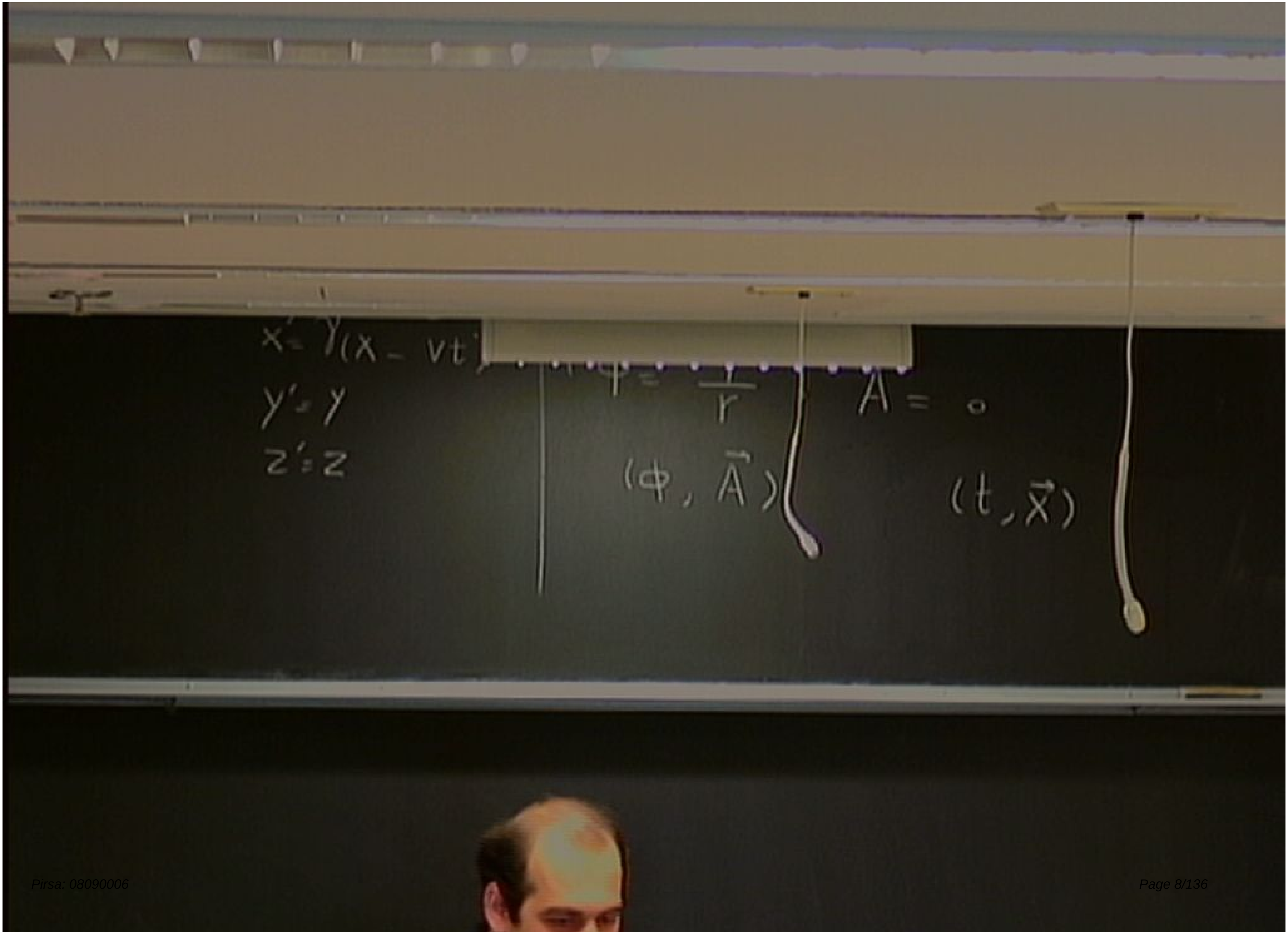

$\vec{E} = \frac{q}{r^2}$
$\phi =$


$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$

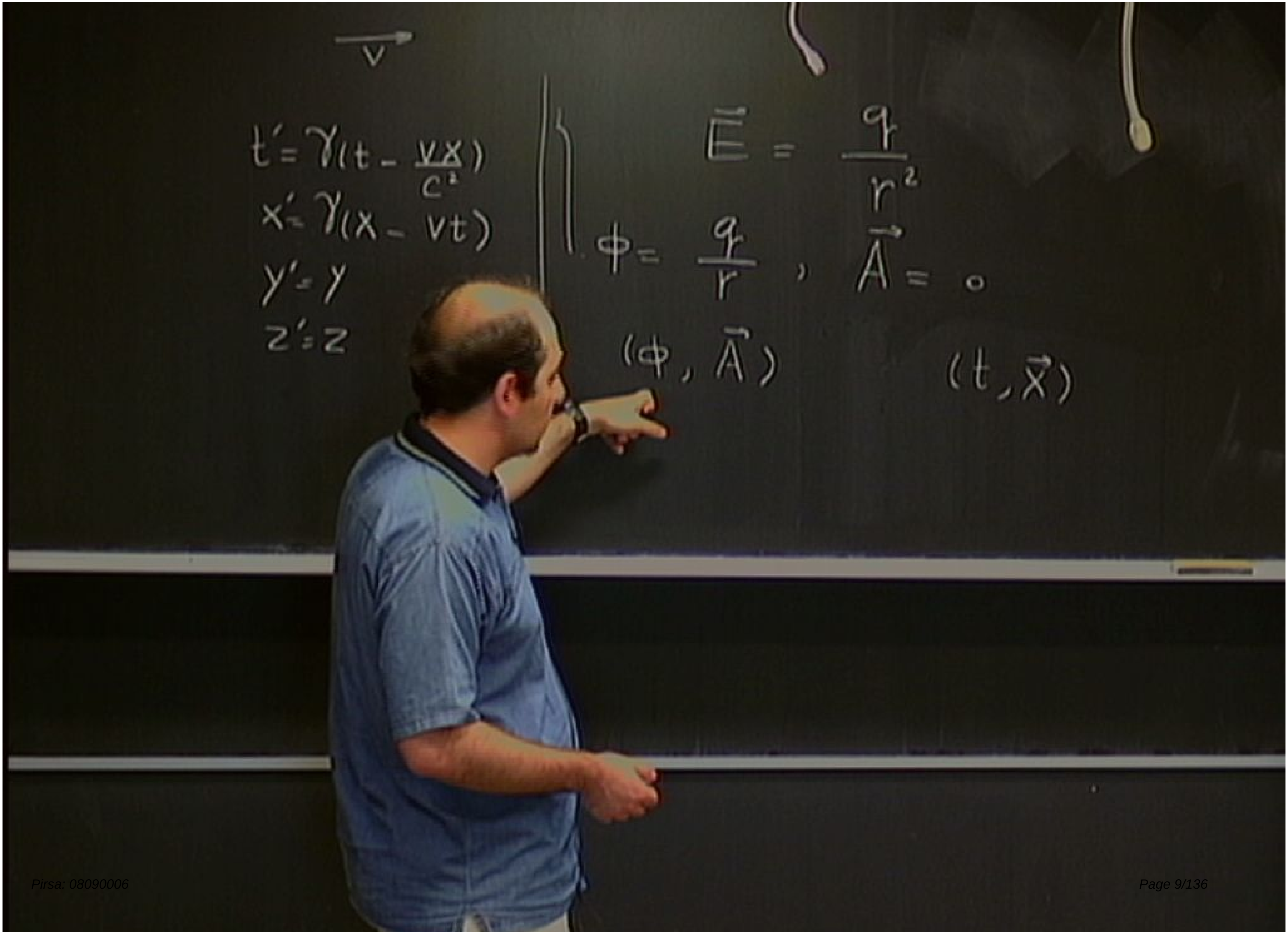

$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$


$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$
$(\phi, \vec{A})$


t' = γ(t − vx/c²)
x' = γ(x − v
y' = y
z' = z
E⃗ = q/r²
φ = q/r , A⃗ = 0
(φ, A⃗)
(t, x⃗)



x' = γ(x − vt)
y' = y
z' = z
A = 0
(φ, A⃗)
(t, x⃗)



$\vec{v}$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$
$(\phi, \vec{A})$
$(t, \vec{x})$

$\vec{v}$

$t' = \gamma(t - \frac{vx}{c^2})$

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$\vec{E} = \frac{q}{r^2}$

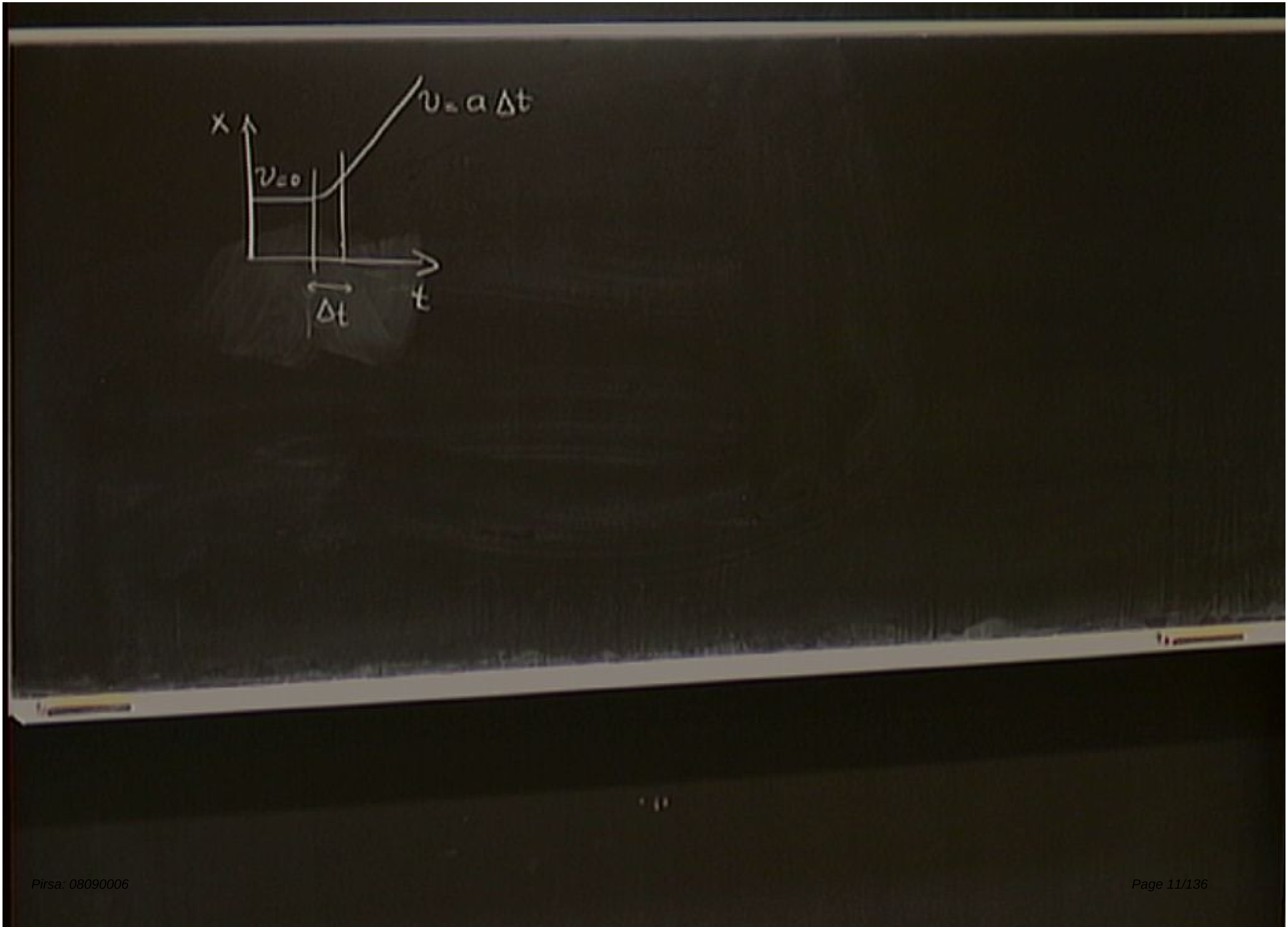
$\phi = \frac{q}{r}, \quad \vec{A} = 0$

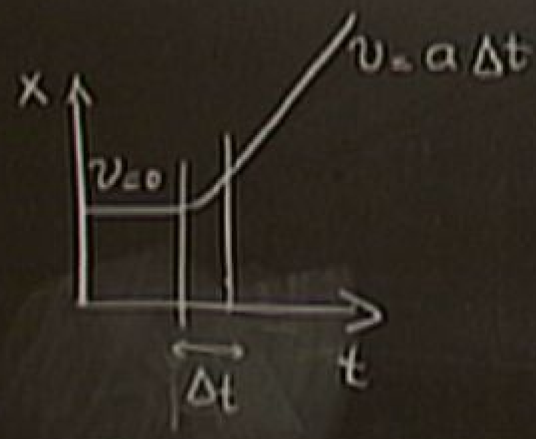
$(\phi, \vec{A})$

A^μ

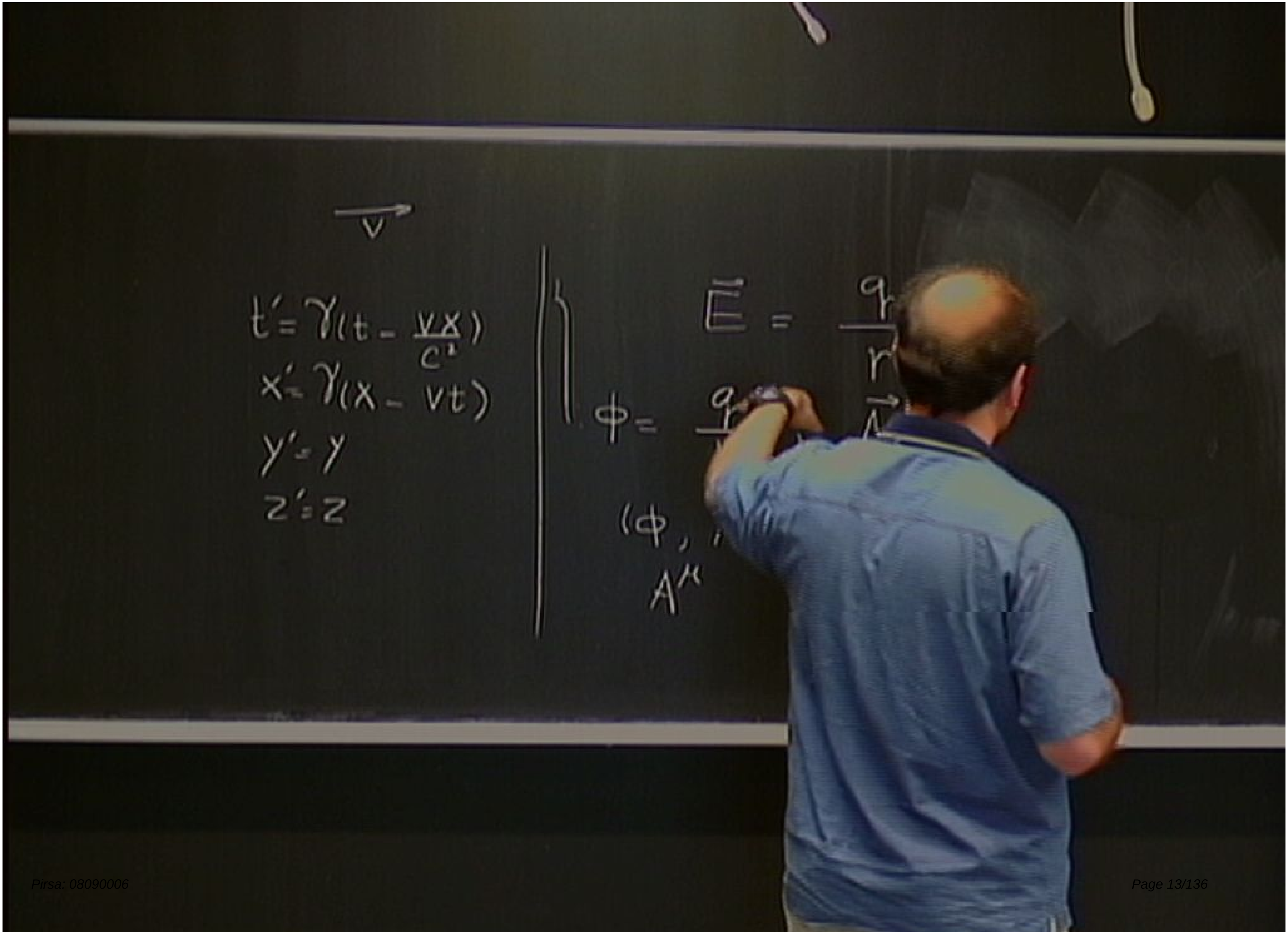
$\underline{(t, \vec{x})}$

x^μ


x
v=0
v=aΔt
Δt
t



x
v=0
v= a Δt
Δt
t


v
t' = γ(t - vx/c²)
x' = γ(x - vt)
y' = y
z' = z
E =
φ =
A^μ

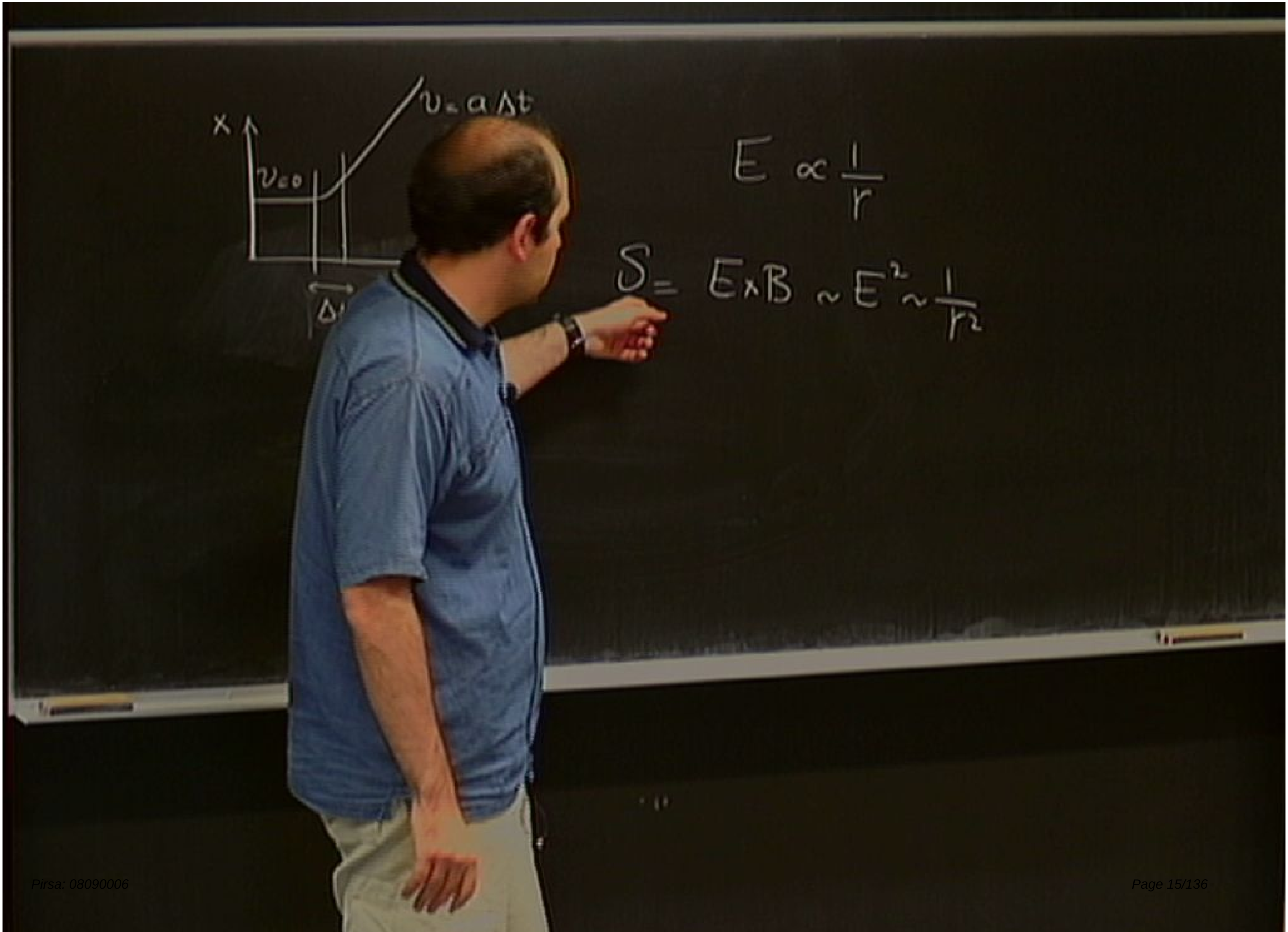
$\overrightarrow{v}$

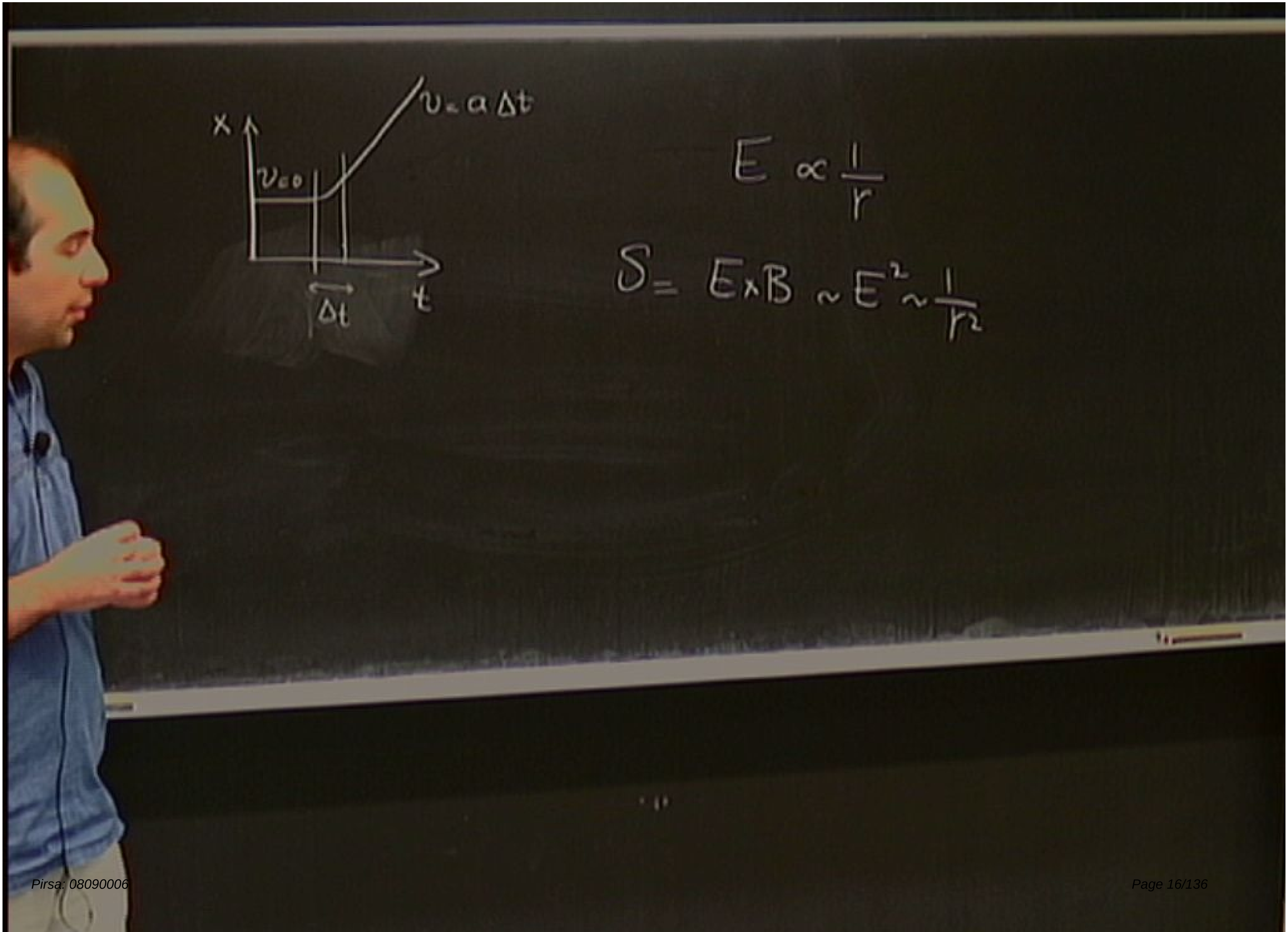
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

$$\vec{E} = \frac{q}{r^2}$$

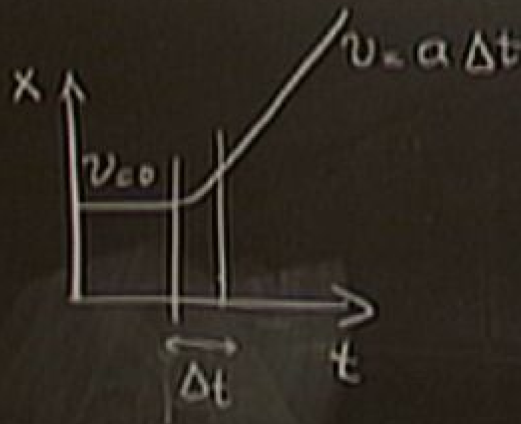
$$\phi = \frac{q}{r}, \quad \vec{A} = 0$$

$$(\phi, \vec{A}) \qquad (t, \vec{x})$$
$$A^\mu \qquad x^\mu$$


x
v=0
v= a Δt
Δt
E ∝ 1/r
S = E x B ~ E² ~ 1/r²

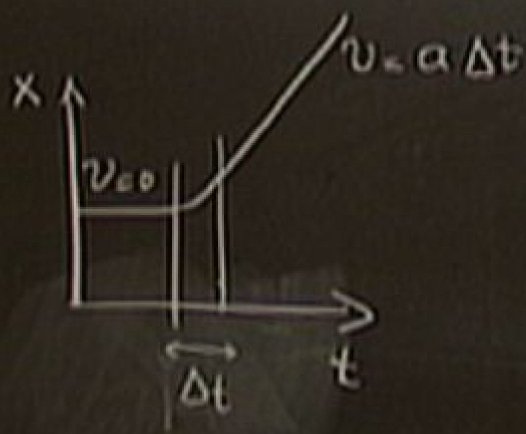


x
v=0
v= a Δt
Δt
t
E ∝ 1/r
S= E×B ~ E² ~ 1/r²



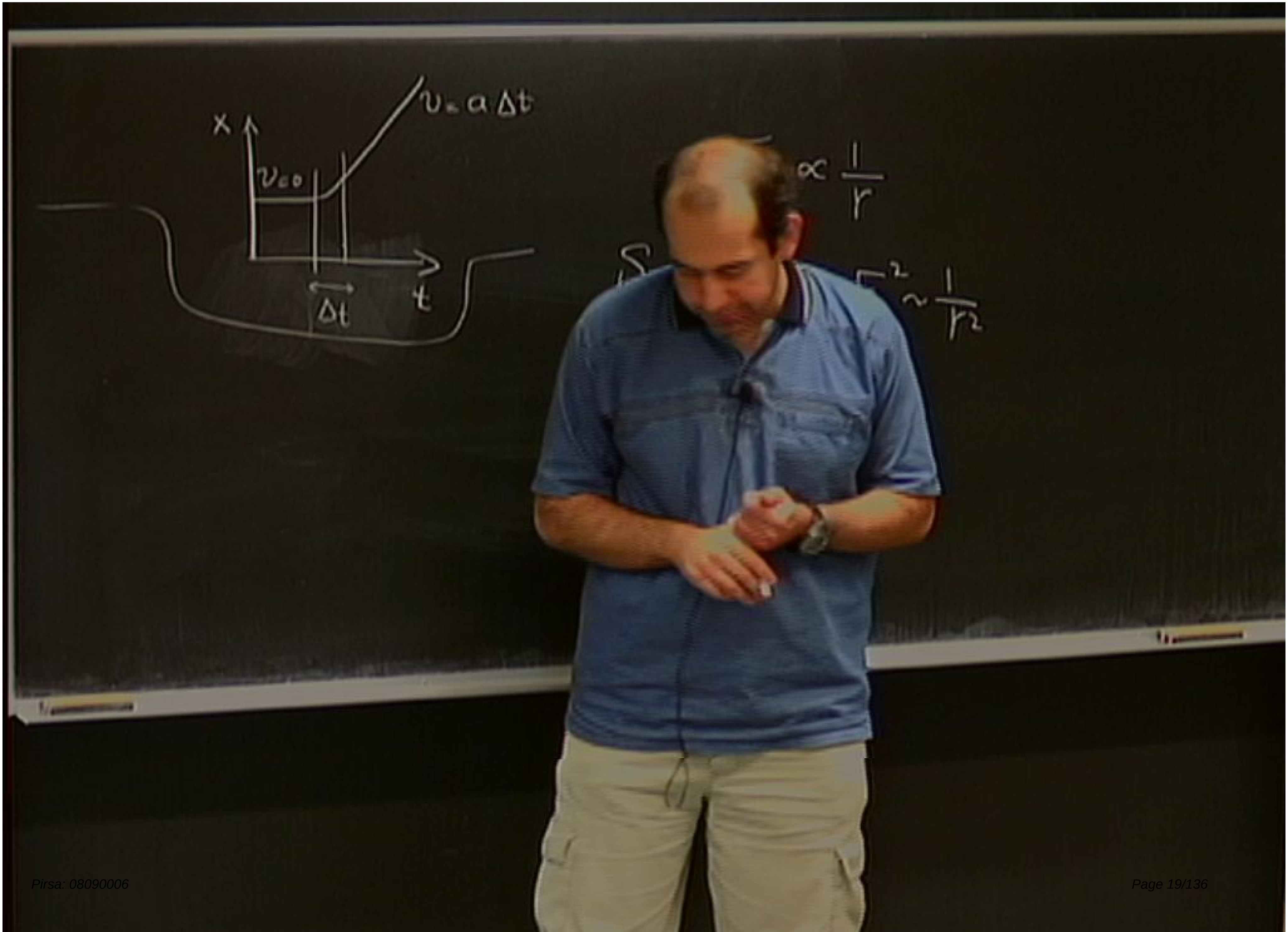
$$E_{rad} \propto \frac{1}{r}$$

$$S = E \times B \sim E^2 \sim \frac{1}{r^2}$$

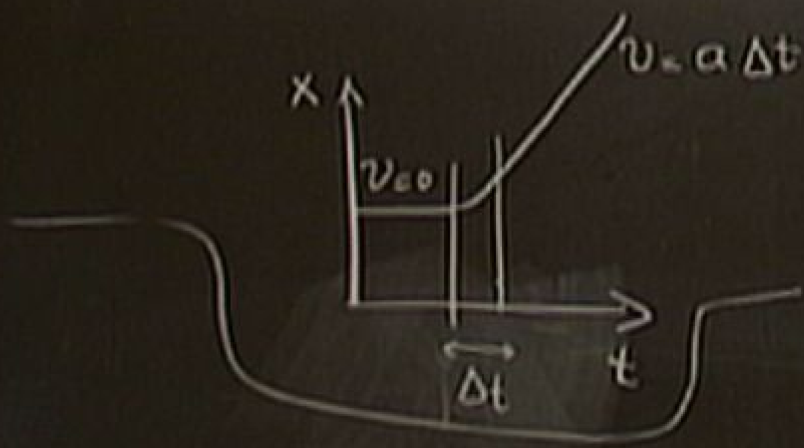


$$E_{rad} \propto \frac{1}{r}$$

$$S = E \times B \sim E^2 \sim \frac{1}{r^2}$$

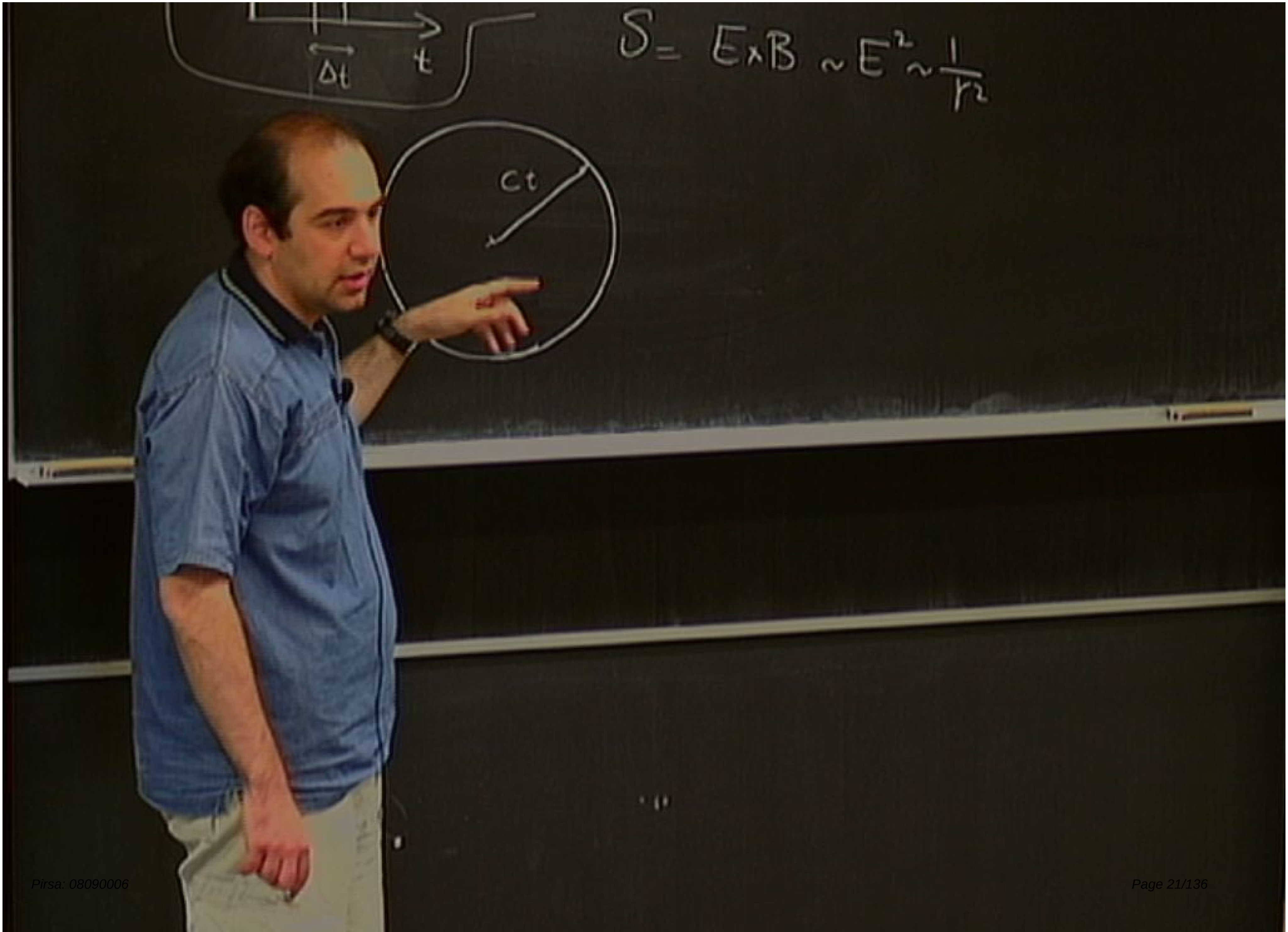


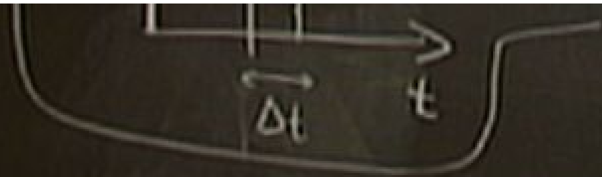
v = aΔt
x
v=0
Δt
t



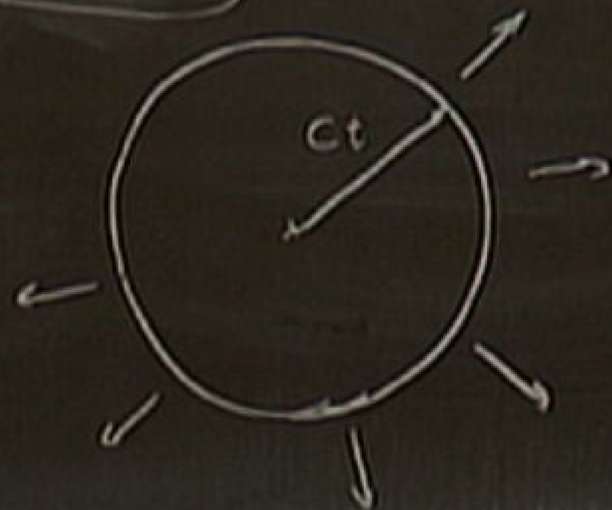
$$E_{rad} \propto \frac{1}{r}$$

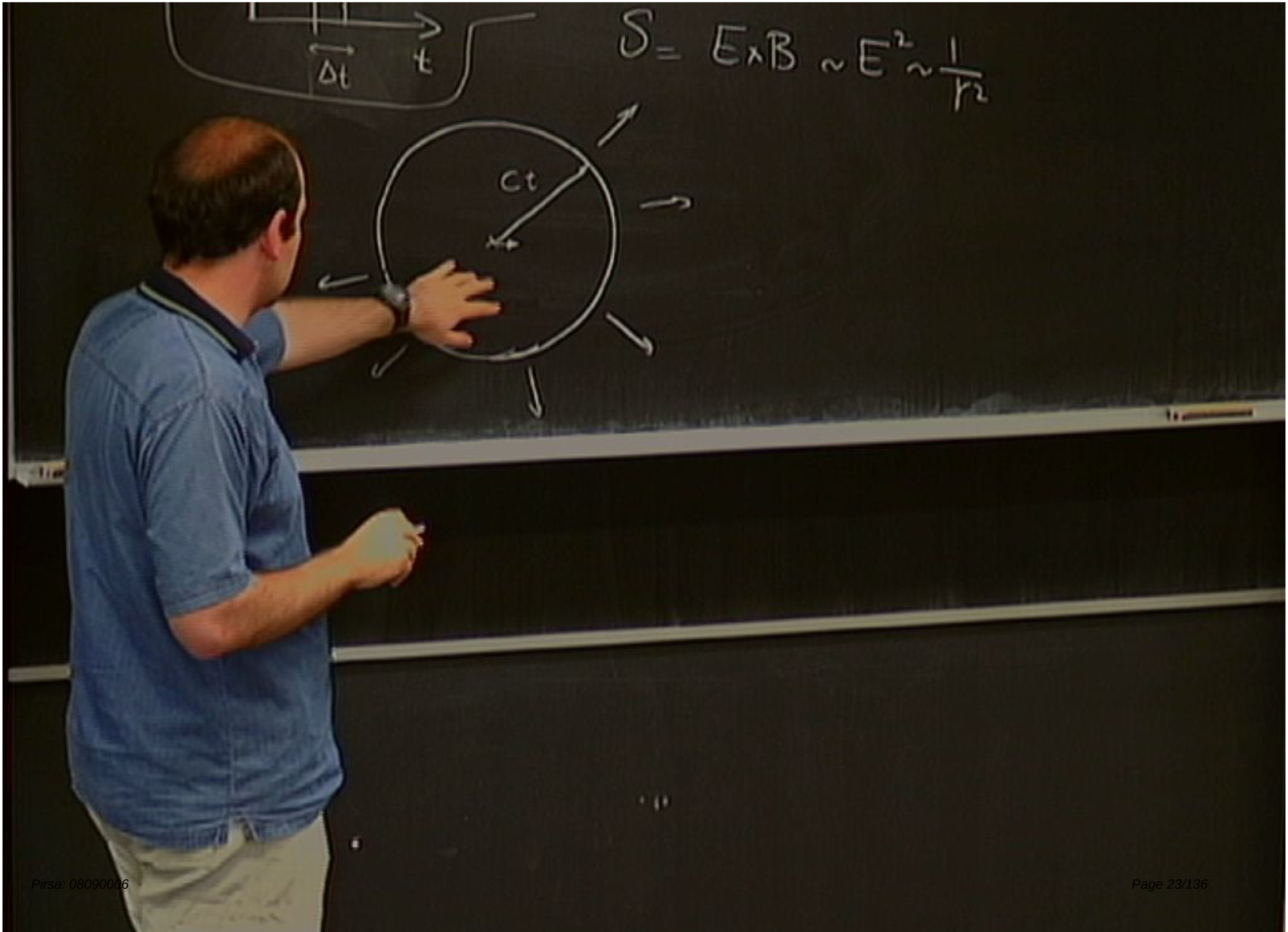
$$S = E \times B \sim E^2 \sim \frac{1}{r^2}$$

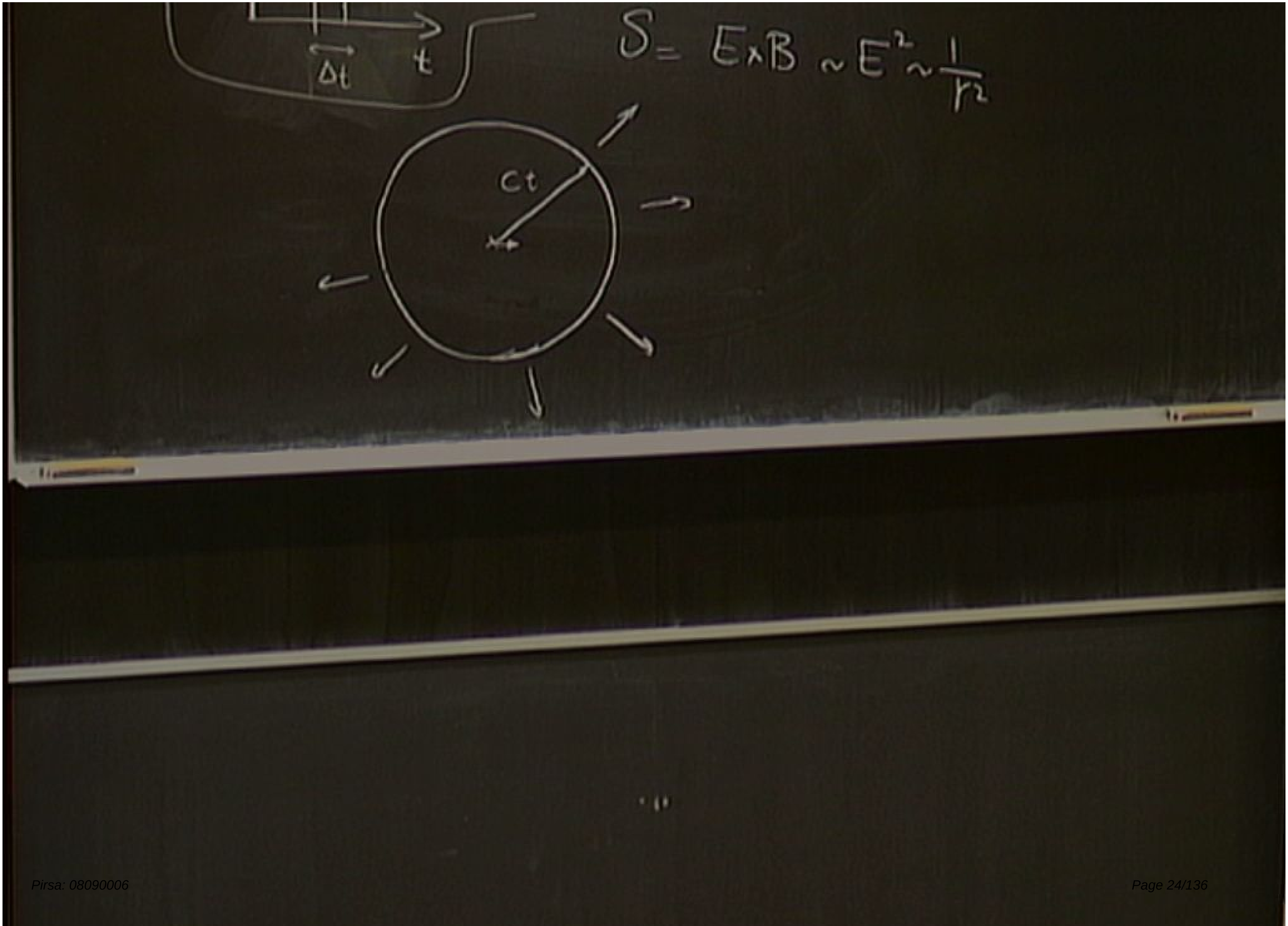

S = E x B ~ E² ~ 1/r²
ct
Δt
t

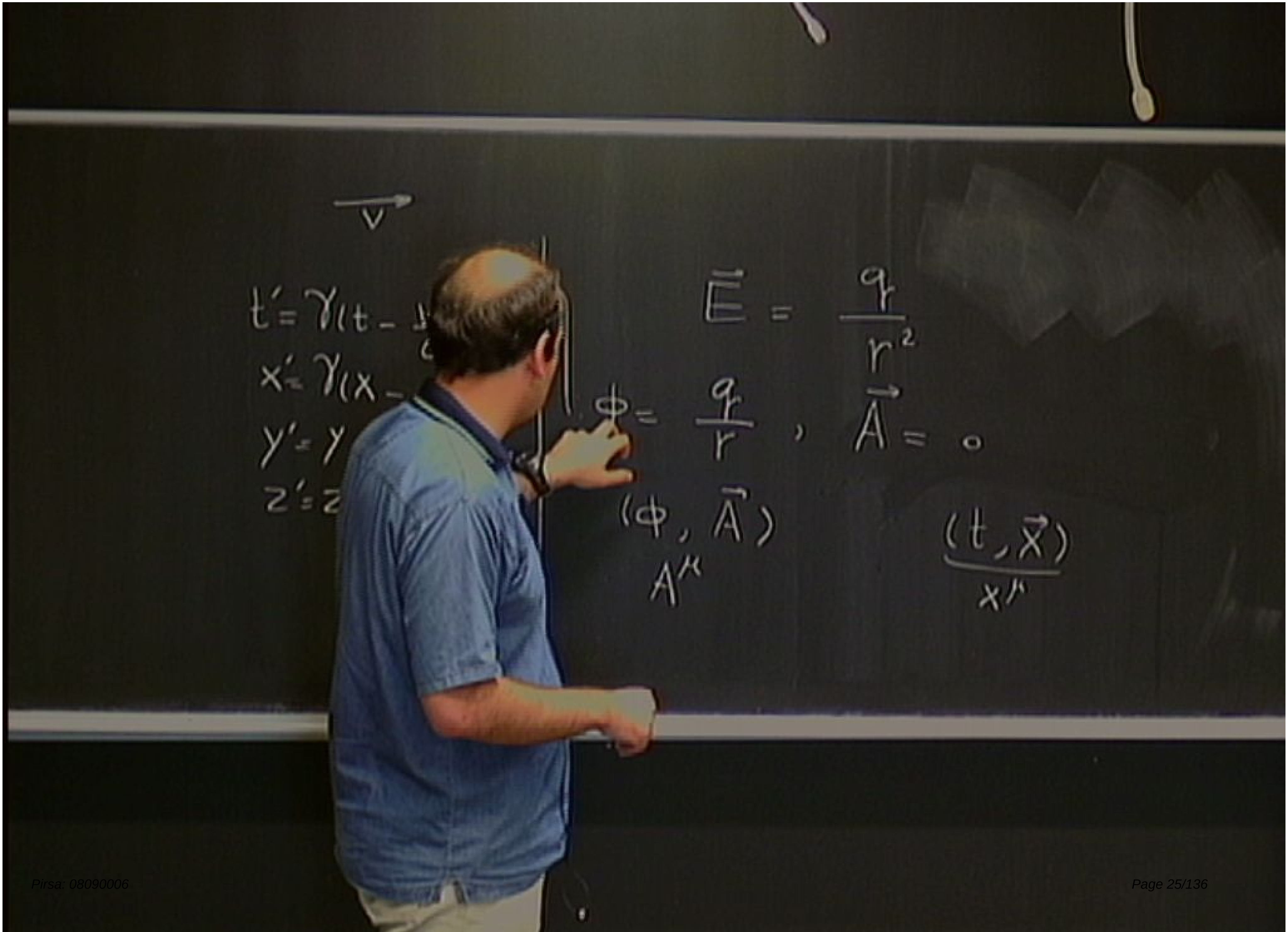


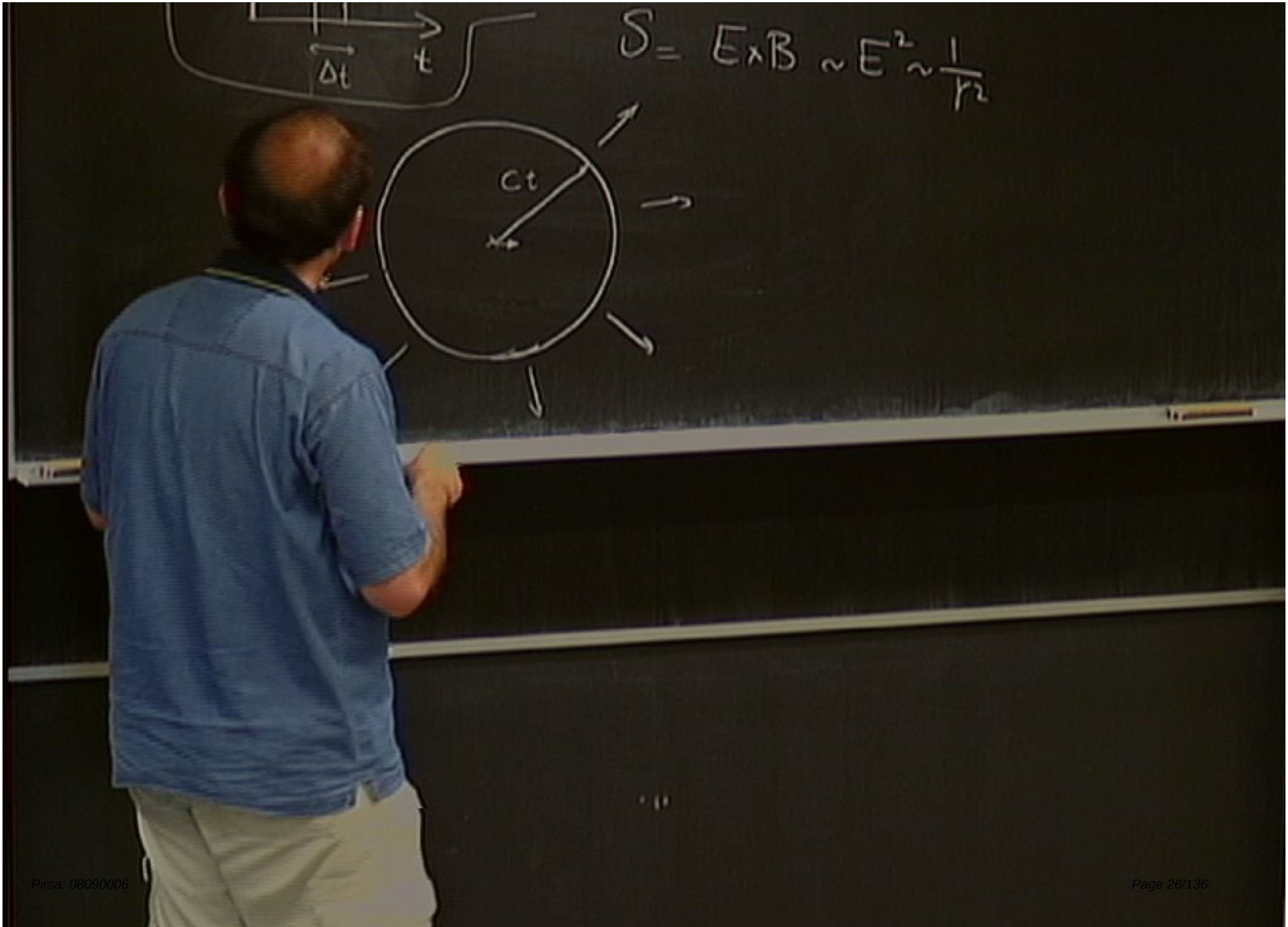
$$S = E \times B \sim E^2 \sim \frac{1}{r^2}$$



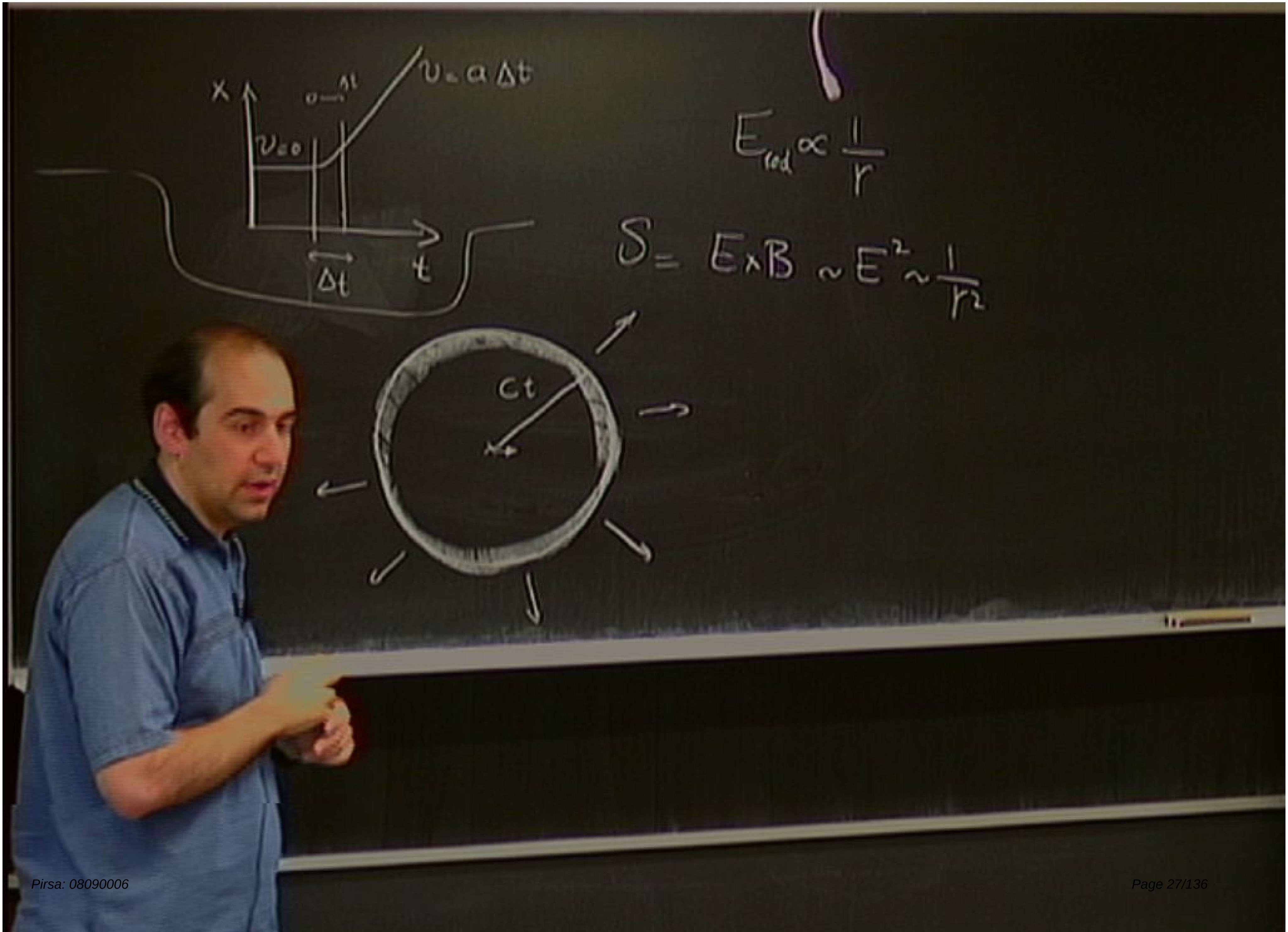

Δt
t
S = E×B ~ E² ~ 1/r²
ct

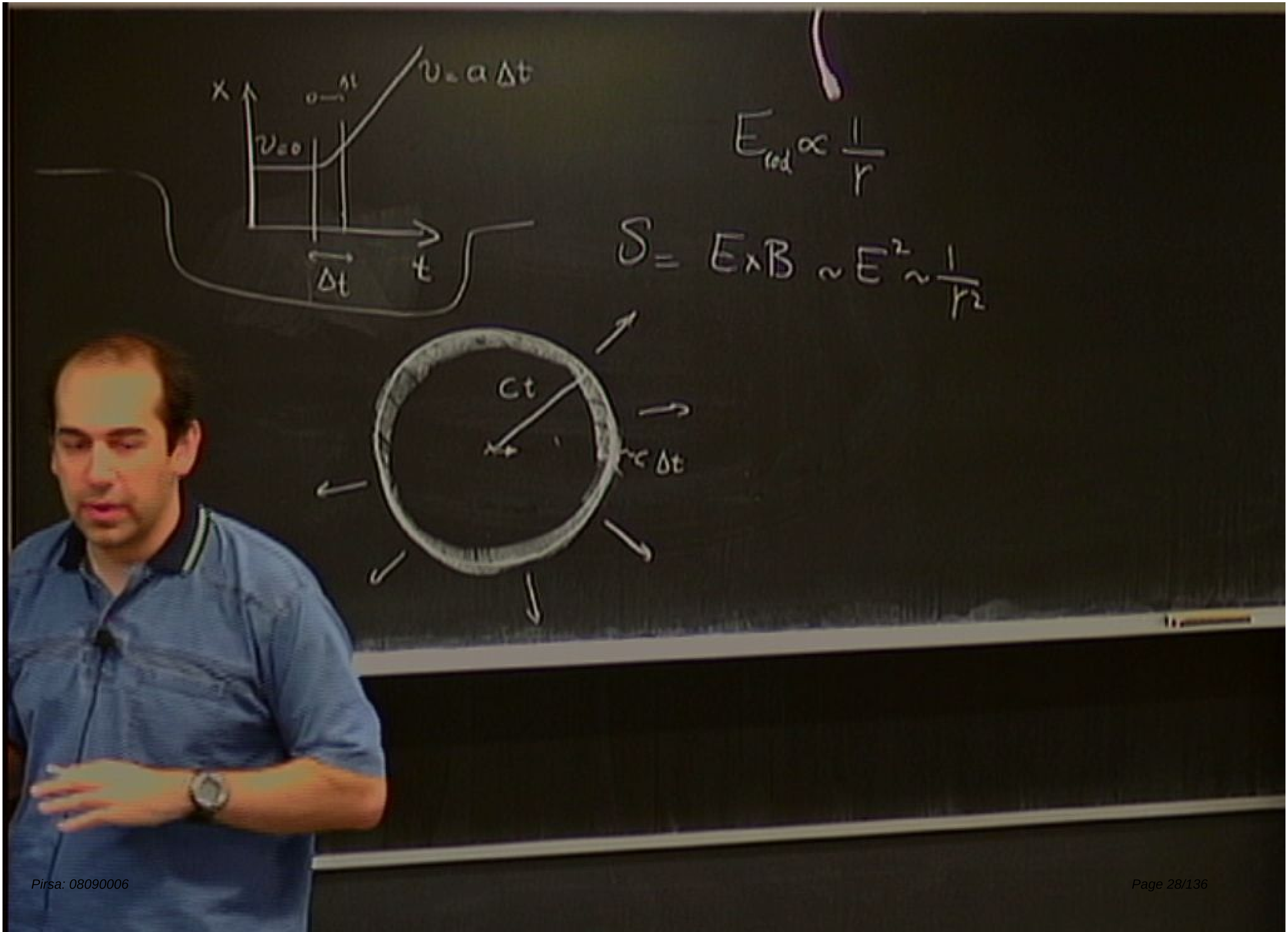

Δt
t
S = E×B ~ E² ~ 1/r²
ct

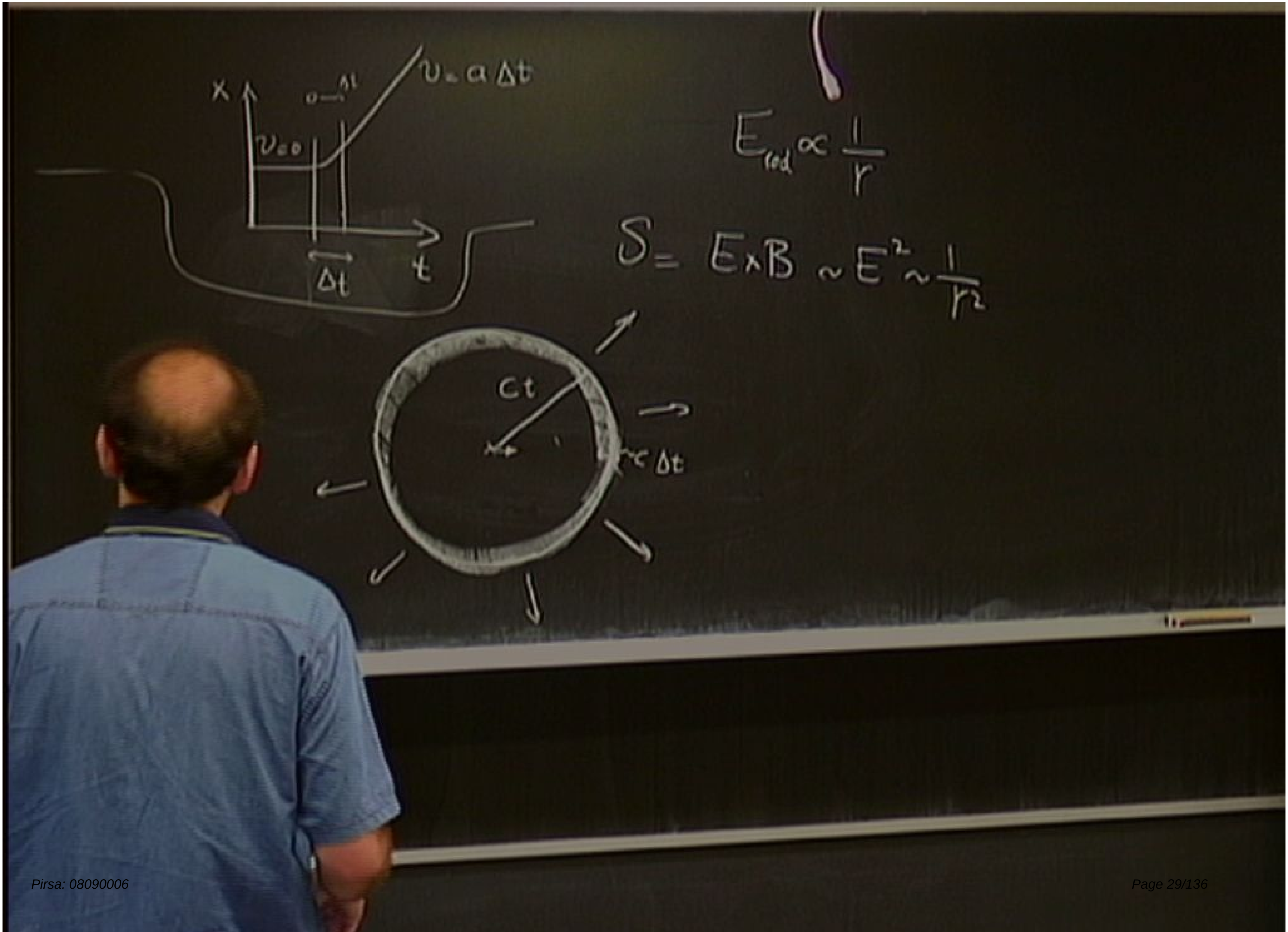

v
t' = γ(t -
x' = γ(x -
y' = y
z' = z
E = q / r²
φ = q / r , A = 0
(φ, A)
A^μ
(t, x)
x^μ

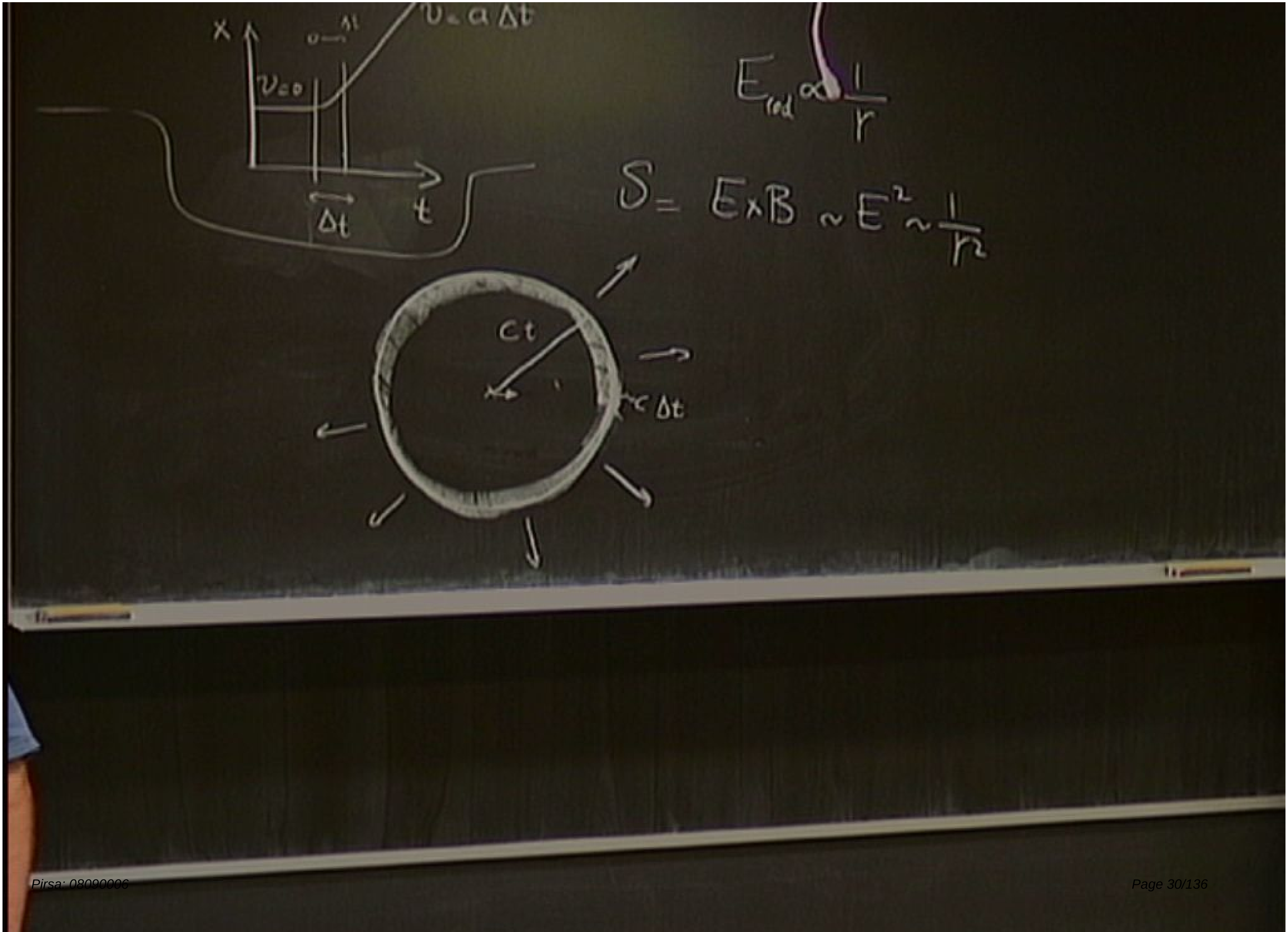


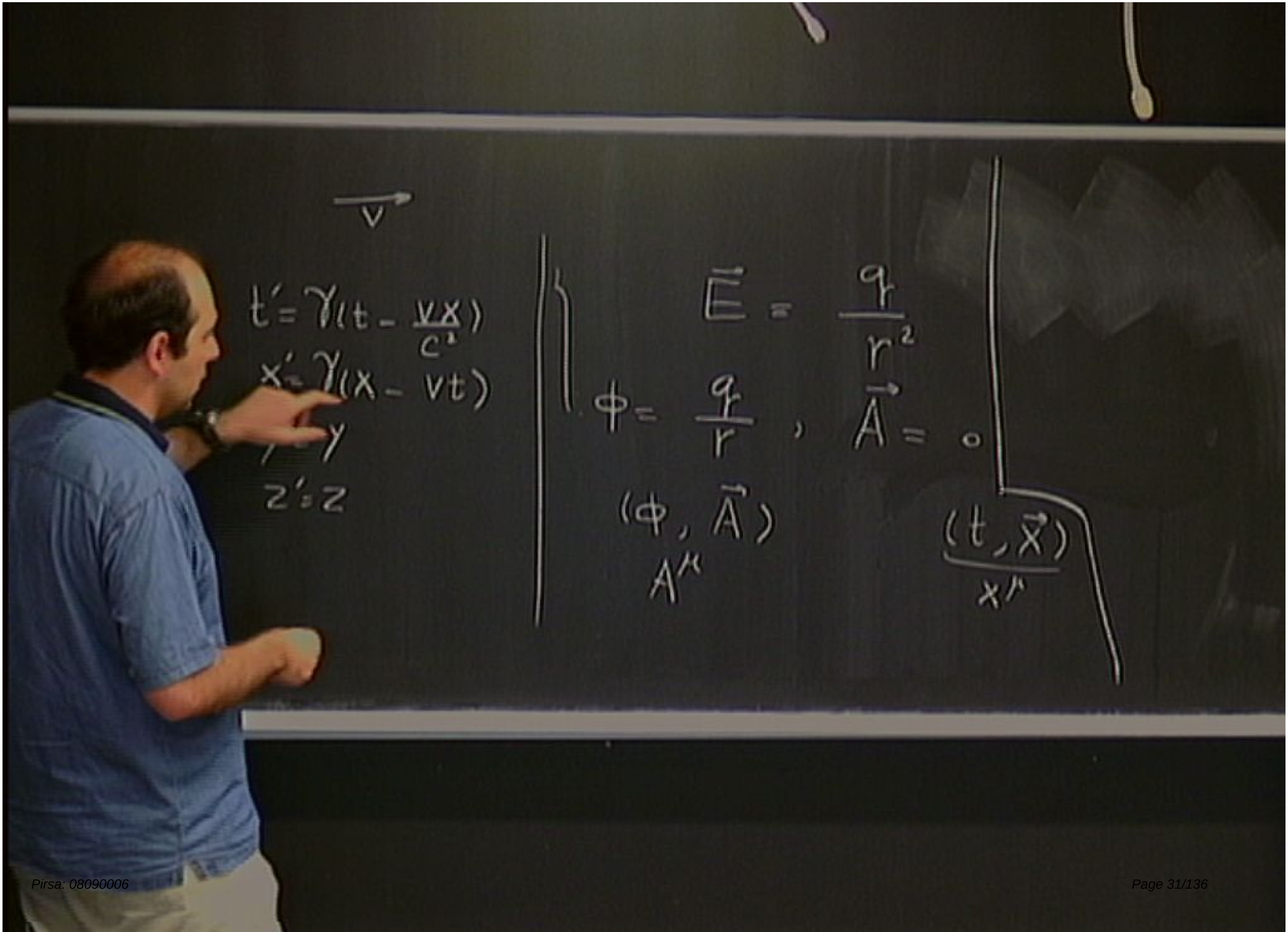
Δt
t
$S = E \times B \sim E^2 \sim \frac{1}{r^2}$
ct

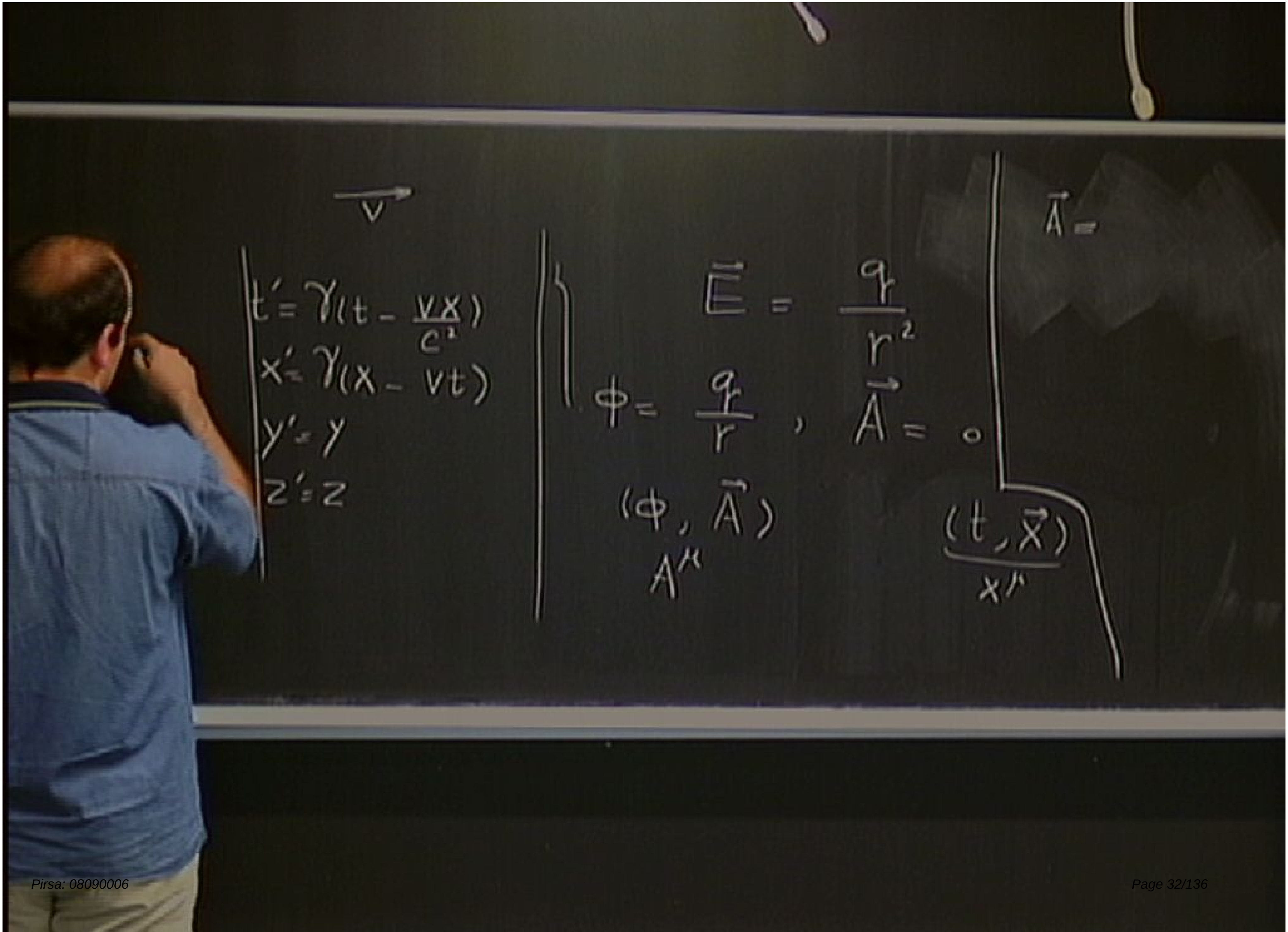

x
v = aΔt
v = 0
Δt
t
E_rad ∝ 1/r
S = E x B ~ E² ~ 1/r²
ct

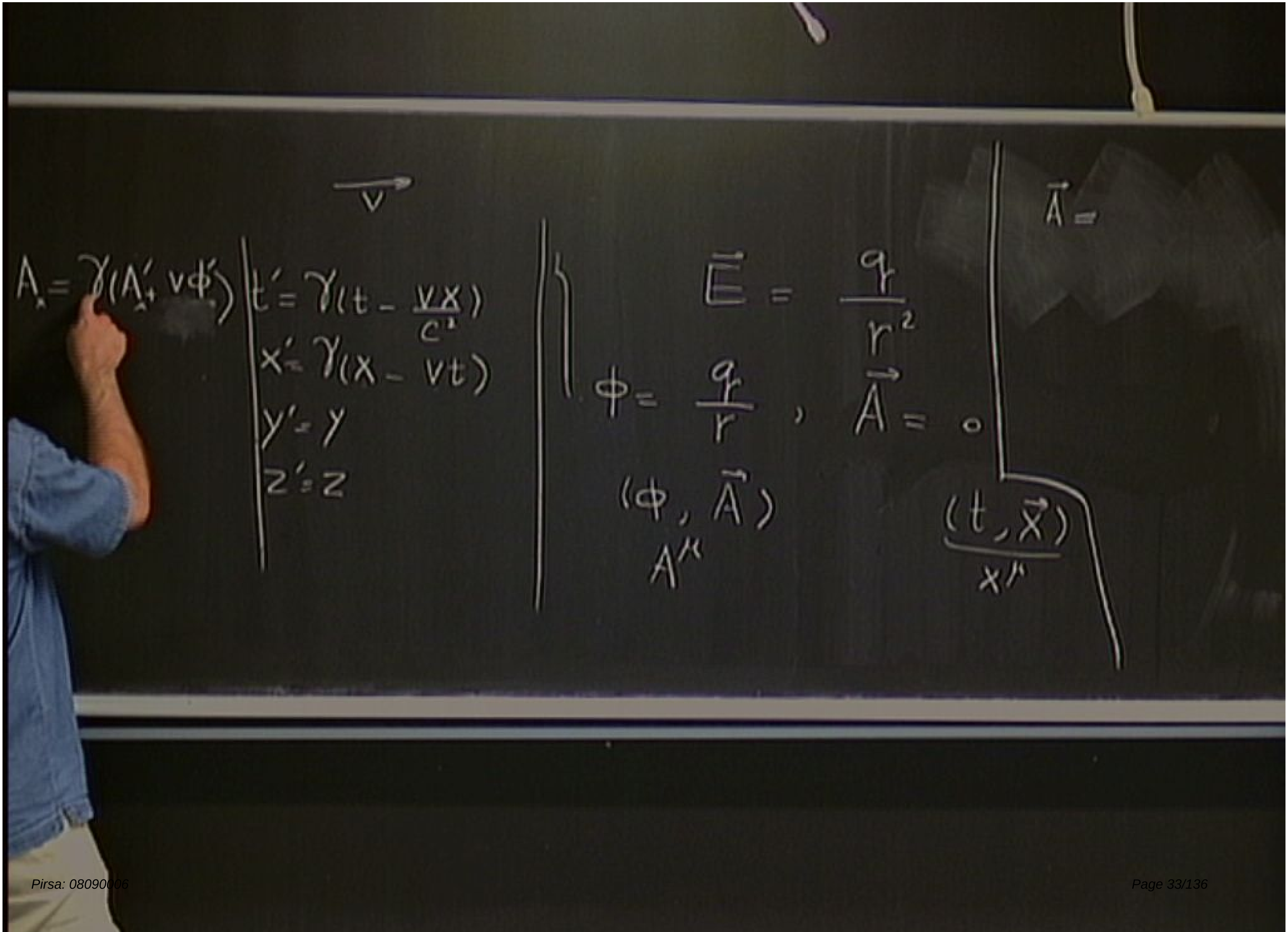

x
v = a Δt
v=0
Δt
t
E_rad ∝ 1/r
S = E x B ~ E^2 ~ 1/r^2
ct
~c Δt

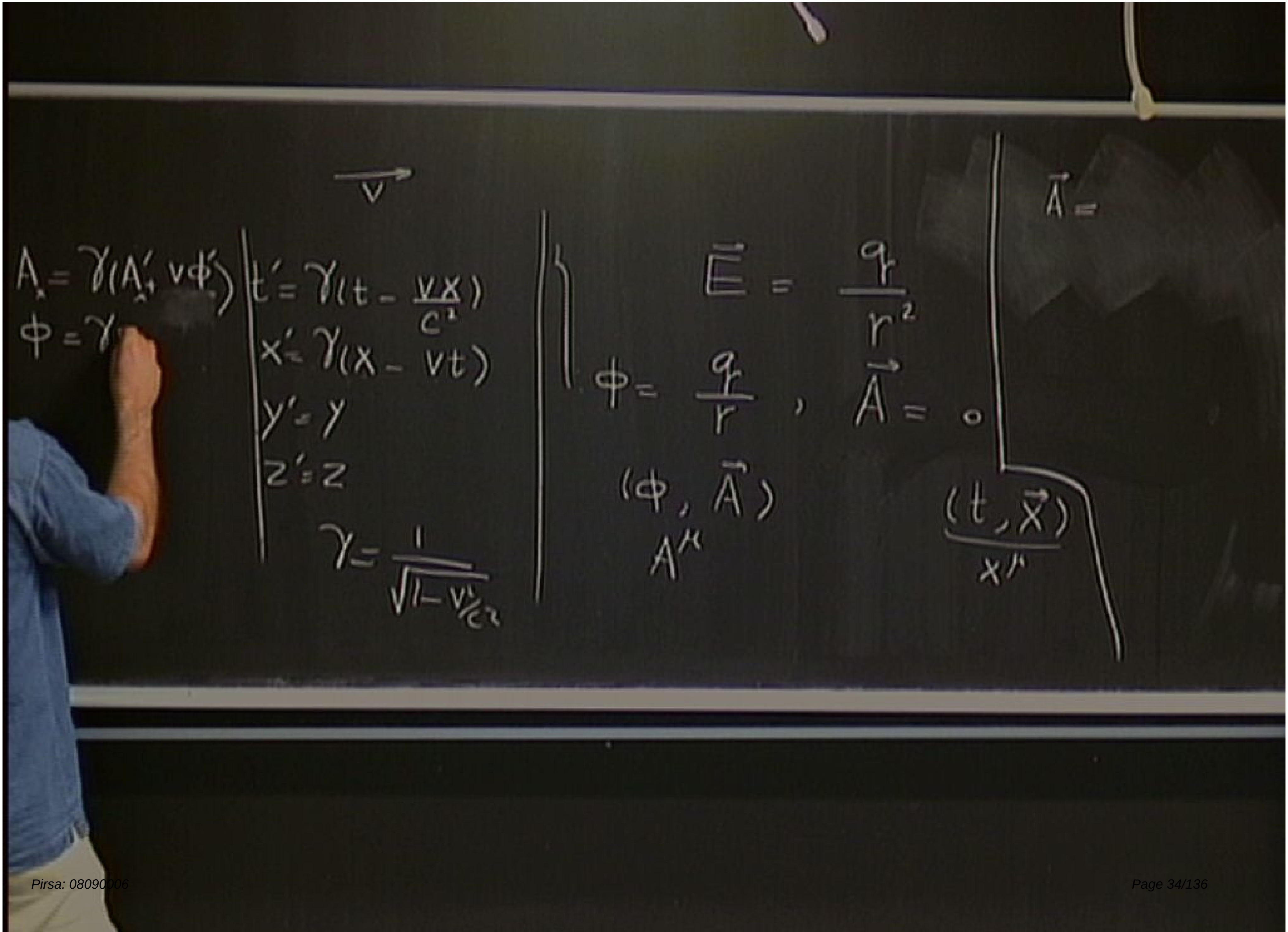

x
v = a Δt
v = 0
Δt
t
E_rad ∝ 1/r
S = E x B ~ E^2 ~ 1/r^2
ct
~c Δt

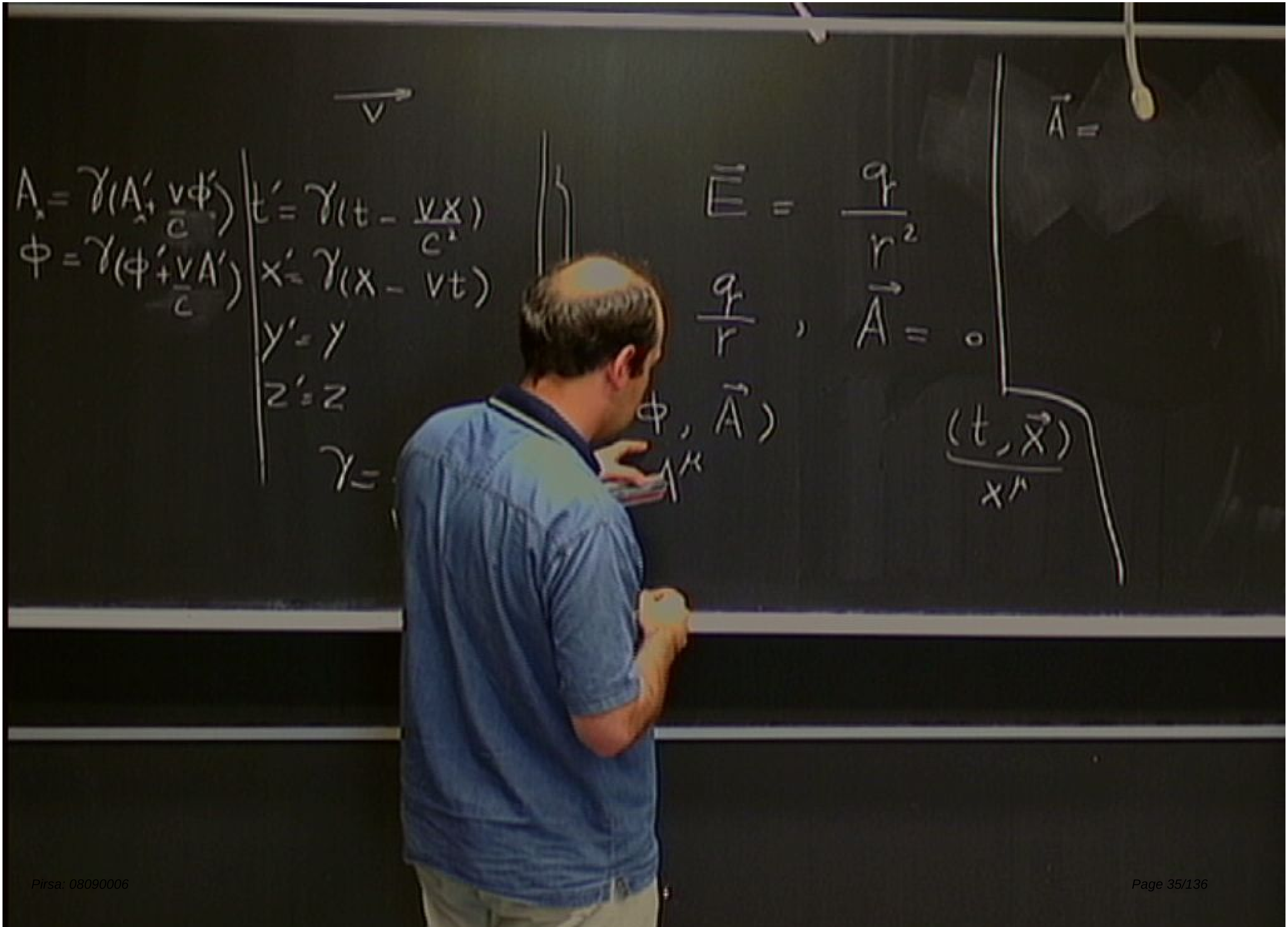

v = a Δt
x
v = 0
Δt
t
E_rad ∝ 1/r
S = E×B ~ E^2 ~ 1/r^2
ct
c Δt


$\vec{v}$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$z' = z$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$
$(\phi, \vec{A})$
A^μ
$(t, \vec{x})$
x^μ

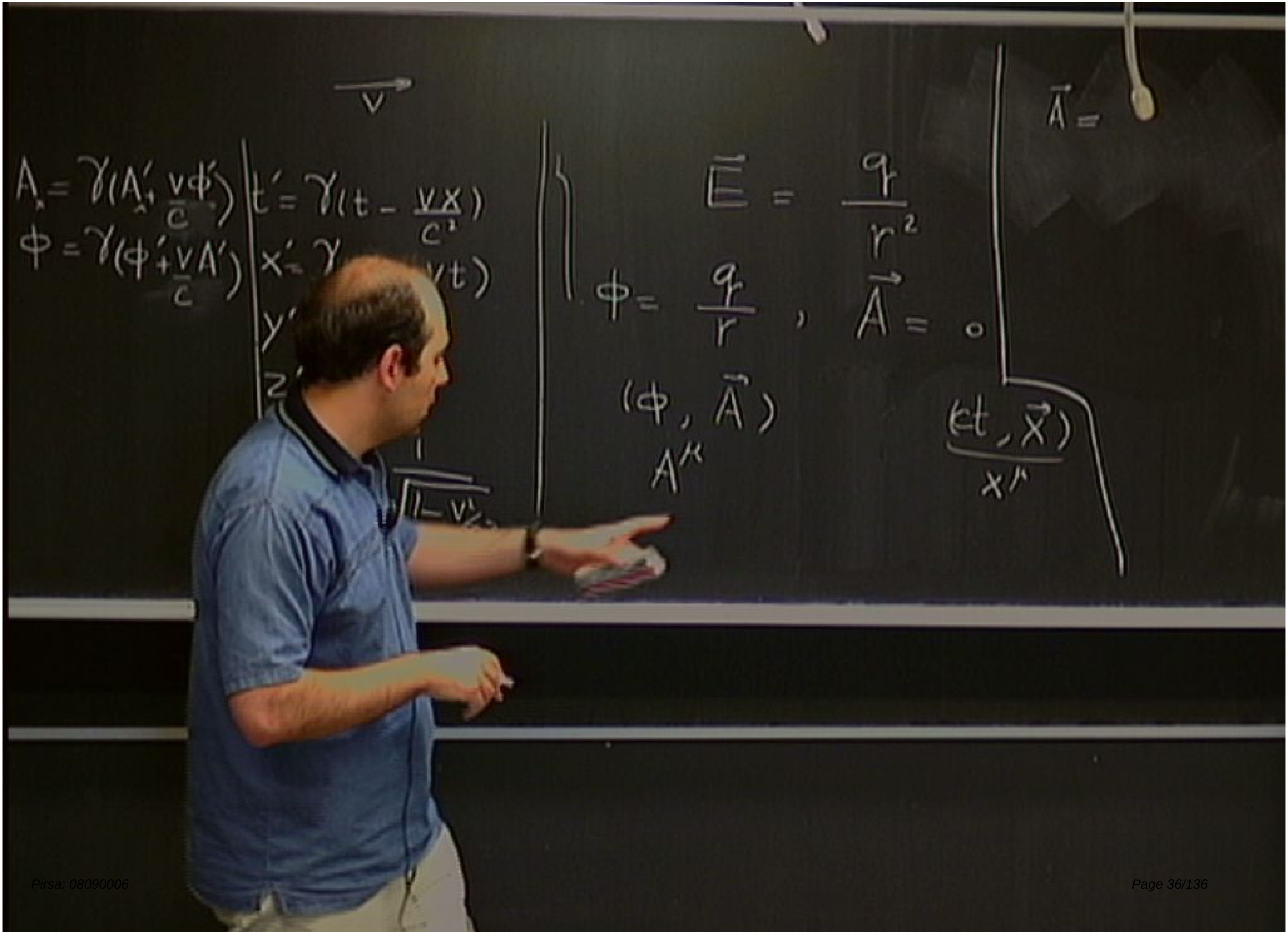

$\vec{v}$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$
$(\phi, \vec{A})$
A^μ
$(t, \vec{x})$
x^μ
$\vec{A} =$

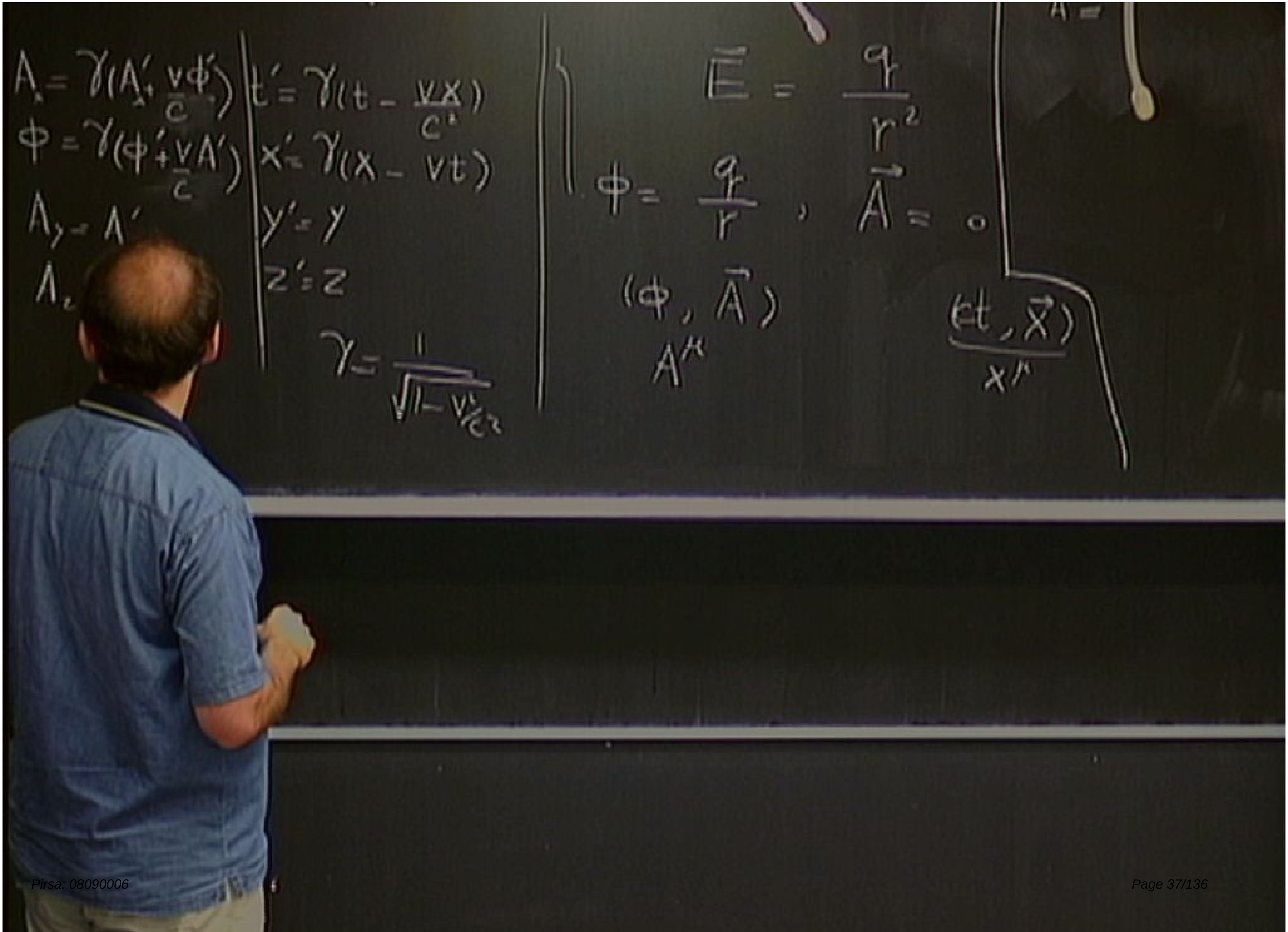

$\vec{v}$
$A_x = \gamma(A'_x + v\phi')$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}$, $\vec{A} = 0$
$(\phi, \vec{A})$
A^μ
$\vec{A} =$
$(t, \vec{x})$
x^μ

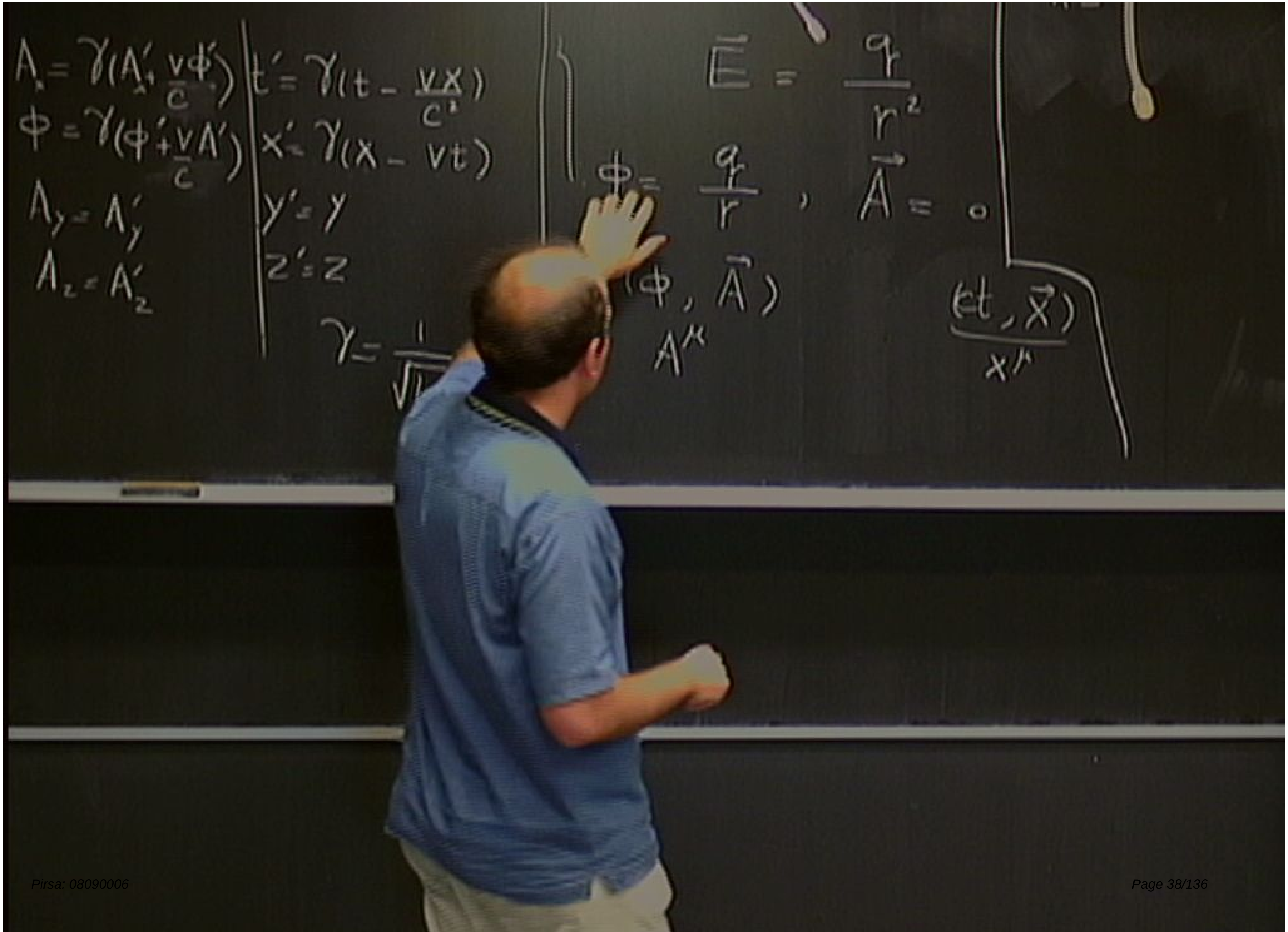

$\vec{v}$
$A_x = \gamma(A'_x + v\phi')$
$\phi = \gamma$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}, \ \vec{A} = 0$
$(\phi, \vec{A})$
A^μ
$\vec{A} =$
$(t, \vec{x})$
x^μ



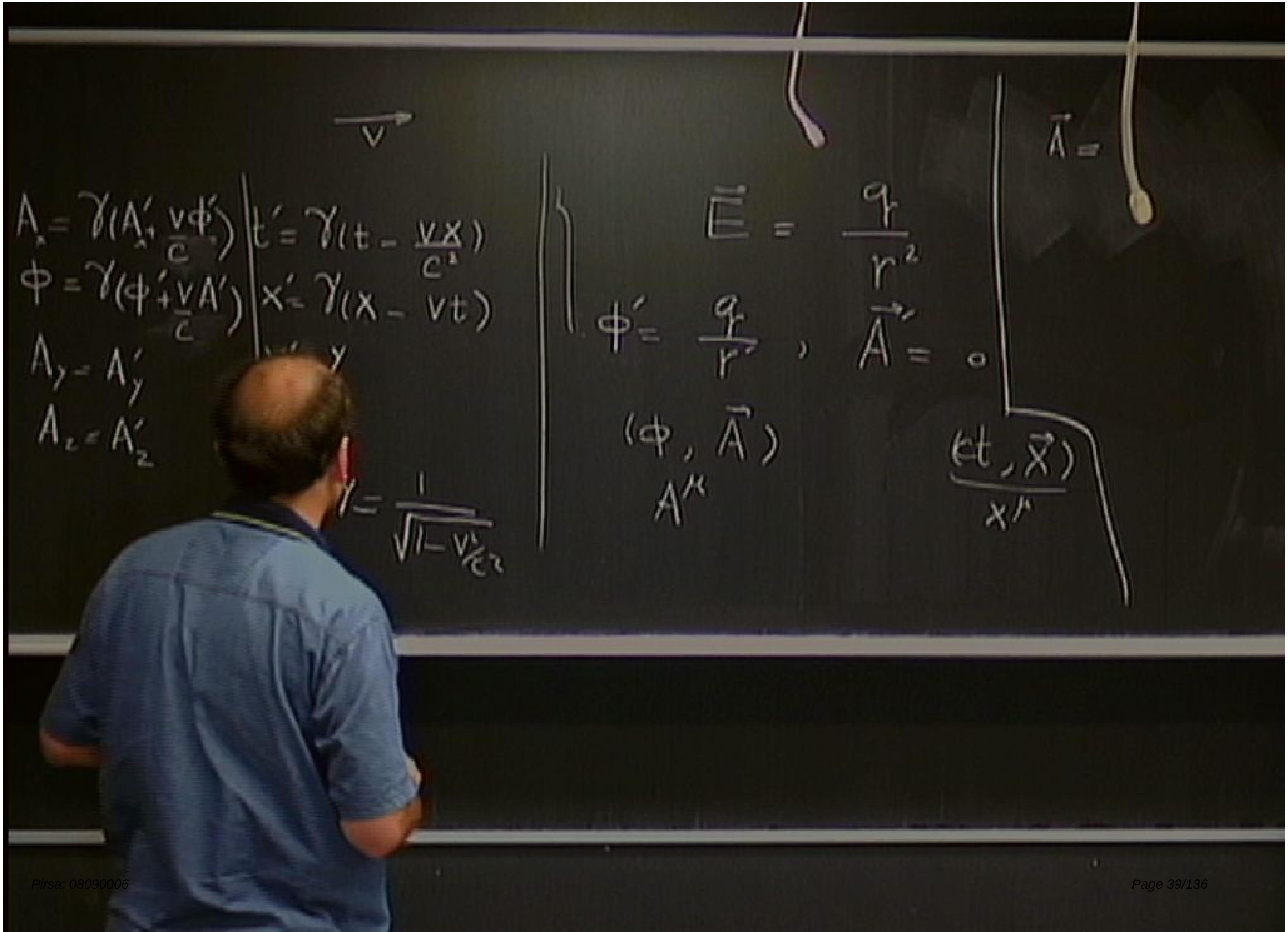
v
A_x = γ(A'_x + vφ'/c)
φ = γ(φ' + vA'/c)
t' = γ(t − vx/c²)
x' = γ(x − vt)
y' = y
z' = z
γ =
E = q/r²
q/r , A = 0
(φ, A)
A^μ
(t, x)
x^μ
A =

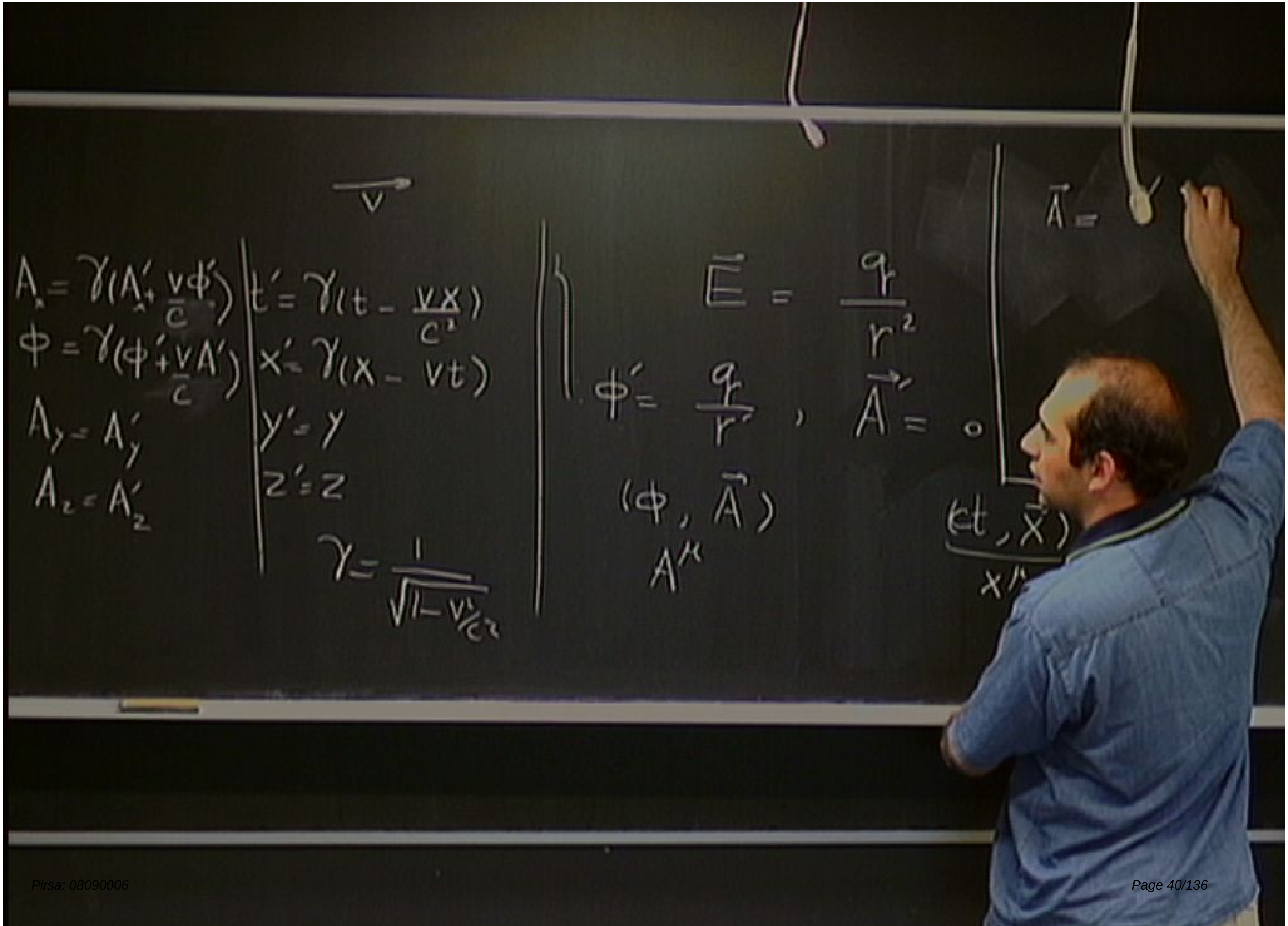

E = q / r²
φ = q / r , A = 0
(φ, A)
A^μ
(ct, x)
x^μ

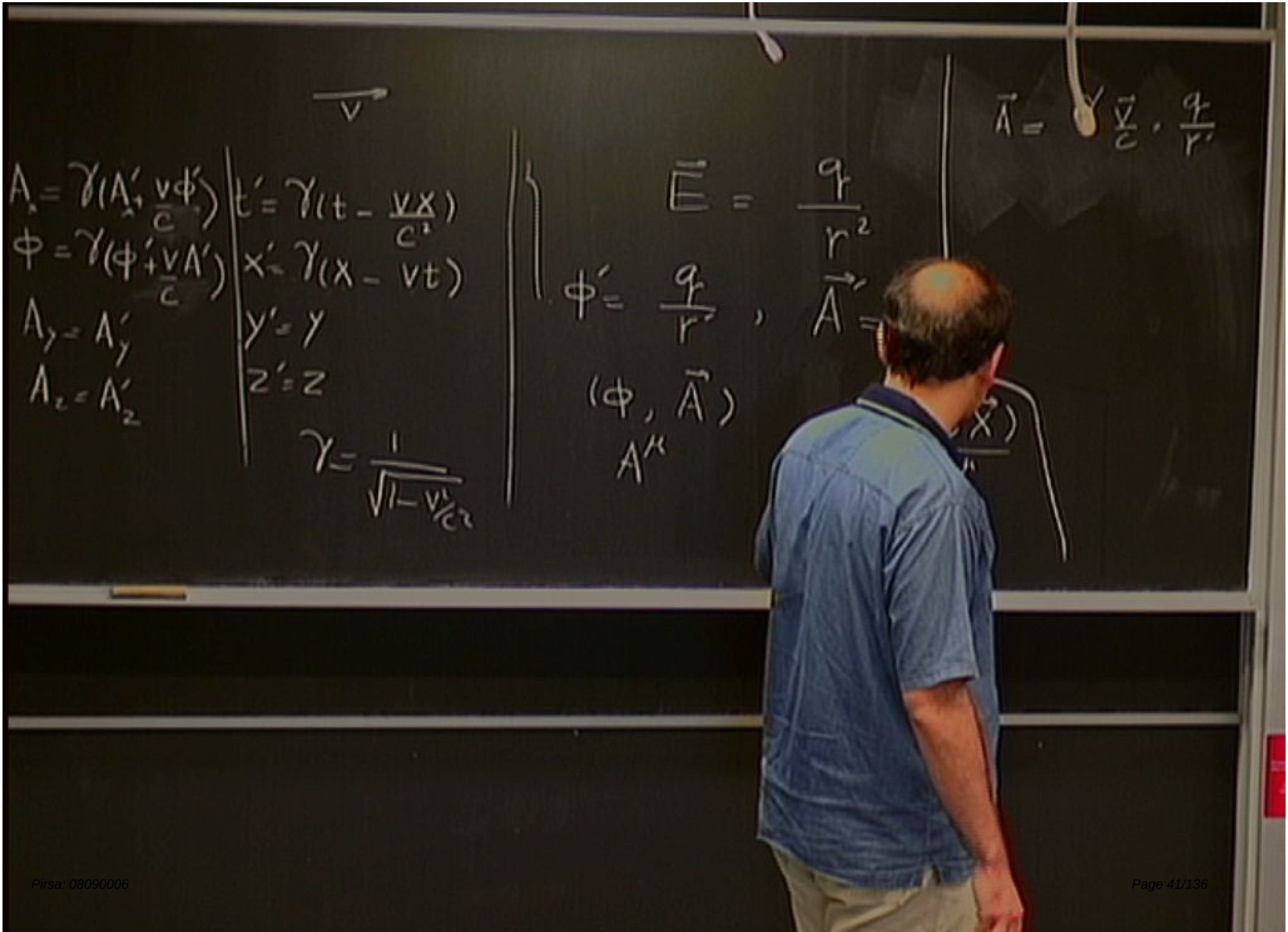

$A_x = \gamma(A'_x + \frac{v\phi'}{c})$
$\phi = \gamma(\phi' + \frac{v}{c}A')$
$A_y = A'$
A_z
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}, \quad \vec{A} = 0$
$(\phi, \vec{A})$
A^μ
$(ct, \vec{x})$
x^μ

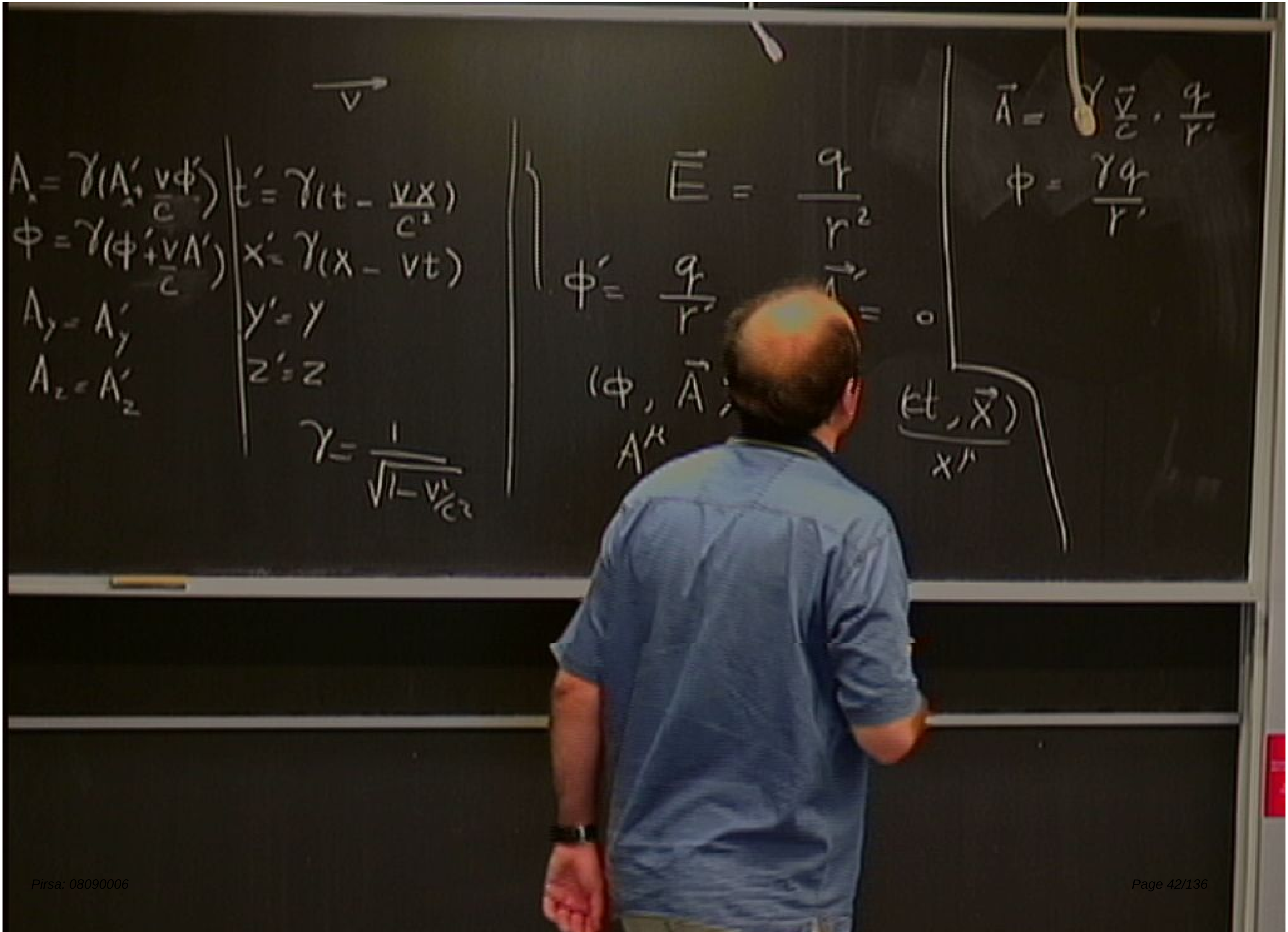


$A_x = \gamma(A'_x + \frac{v\phi'}{c})$
$\phi = \gamma(\phi' + \frac{v}{c}A')$
$A_y = A'_y$
$A_z = A'_z$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\vec{E} = \frac{q}{r^2}$
$\phi = \frac{q}{r}, \vec{A} = 0$
$(\phi, \vec{A})$
A^μ
$(ct, \vec{x})$
x^μ

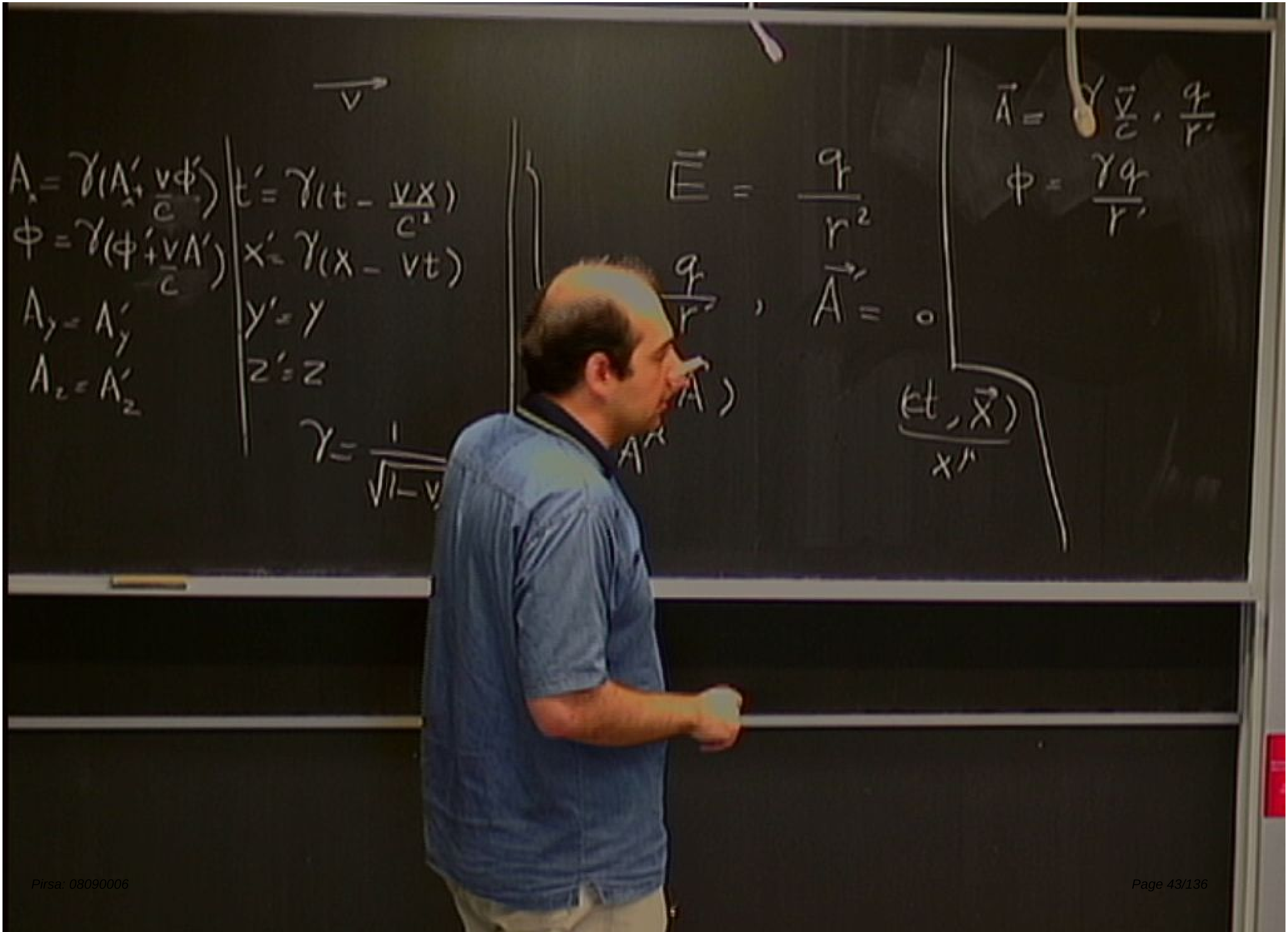

$\vec{v}$
$A_x = \gamma(A_x' + \frac{v\phi'}{c})$
$\phi = \gamma(\phi' + \frac{v}{c}A_x')$
$A_y = A_y'$
$A_z = A_z'$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
$\vec{E} = \frac{q}{r^2}$
$\phi' = \frac{q}{r'}, \quad \vec{A}' = 0$
$(\phi, \vec{A})$
A^μ
$(ct, \vec{x})$
x^μ
$\vec{A} =$

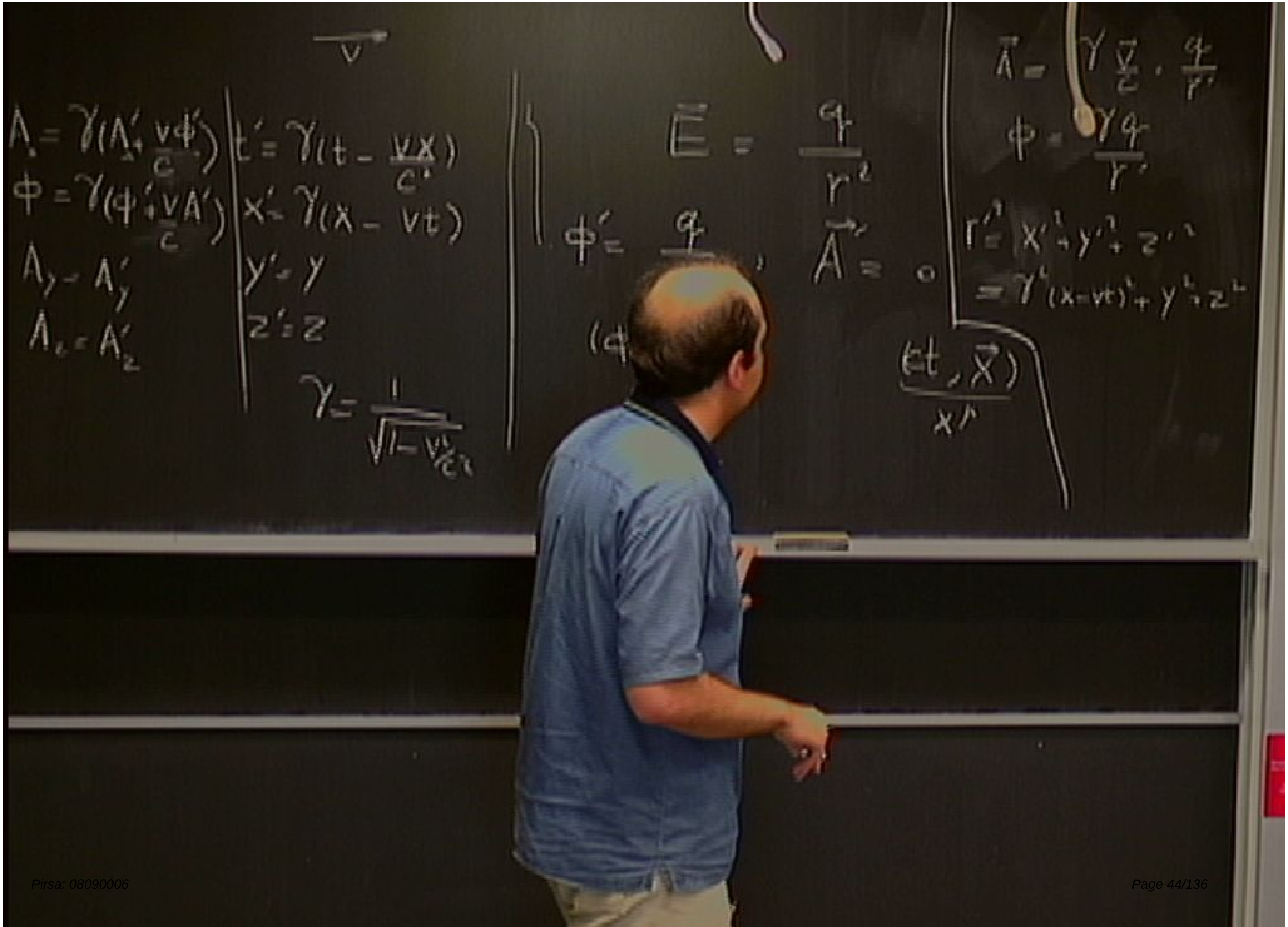

v
A_x = γ(A'_x + vφ'/c)
φ = γ(φ' + vA'_x/c)
A_y = A'_y
A_z = A'_z
t' = γ(t − vx/c²)
x' = γ(x − vt)
y' = y
z' = z
γ = 1/√(1 − v²/c²)
E = q/r²
φ' = q/r' , A' = 0
(φ, A)
A^μ
(ct, x)
x^μ
A =


v
A_x = γ(A'_x + vφ'/c)
φ = γ(φ' + v A'/c)
A_y = A'_y
A_z = A'_z
t' = γ(t − vx/c²)
x' = γ(x − vt)
y' = y
z' = z
γ = 1/√(1 − v²/c²)
E = q/r²
φ' = q/r' , A' =
(φ, A)
A^μ
A = γ v/c · q/r'

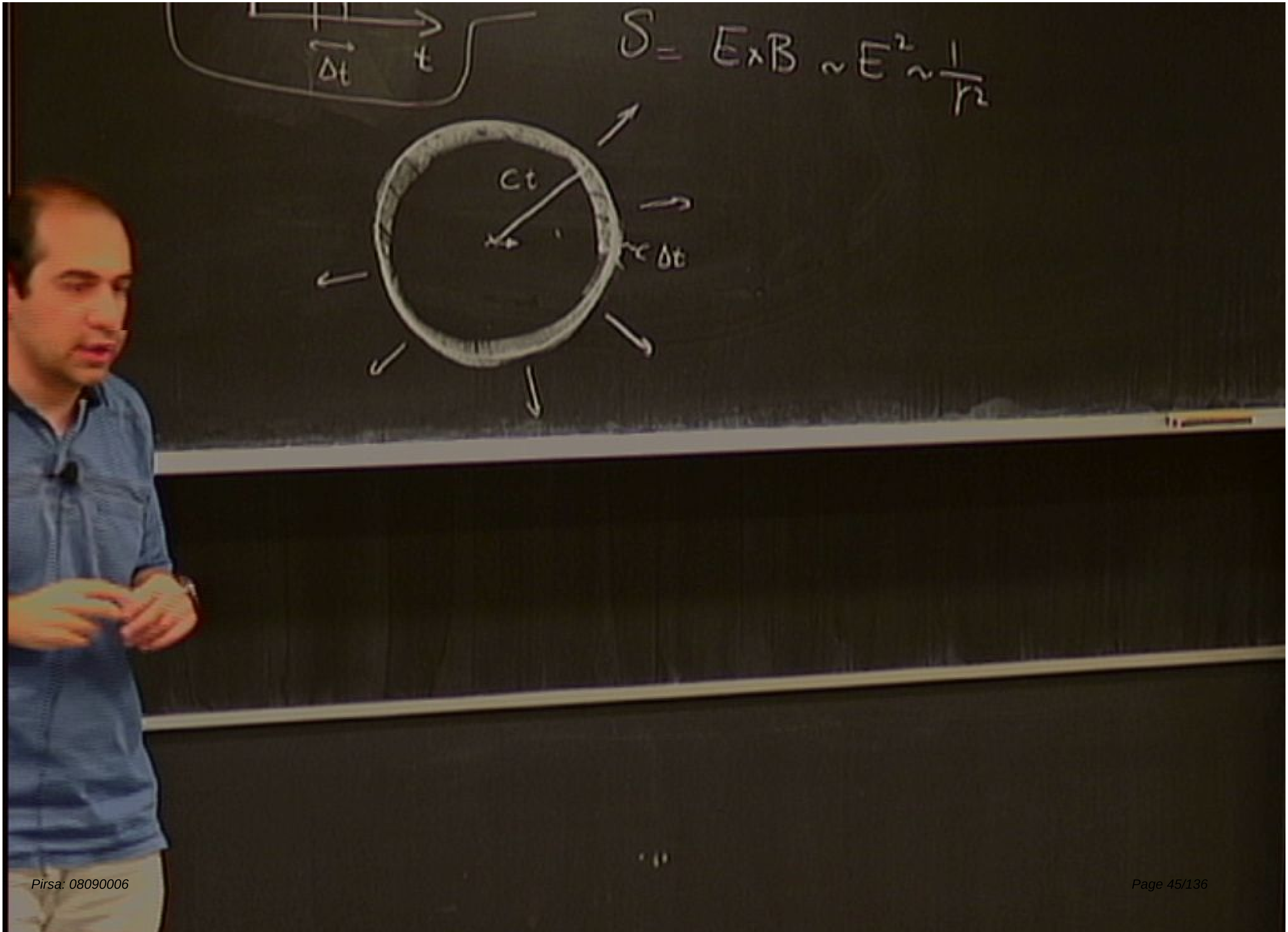


$\vec{v}$
$A_x = \gamma(A'_x + \frac{v\phi'}{c})$
$\phi = \gamma(\phi' + \frac{v}{c}A'_x)$
$A_y = A'_y$
$A_z = A'_z$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
$\vec{E} = \frac{q}{r^2}$
$\phi' = \frac{q}{r'}$
$= 0$
A^μ
$(ct, \vec{x})$
x^μ
$\vec{A} = \gamma \frac{\vec{v}}{c} \cdot \frac{q}{r'}$
$\phi = \frac{\gamma q}{r'}$

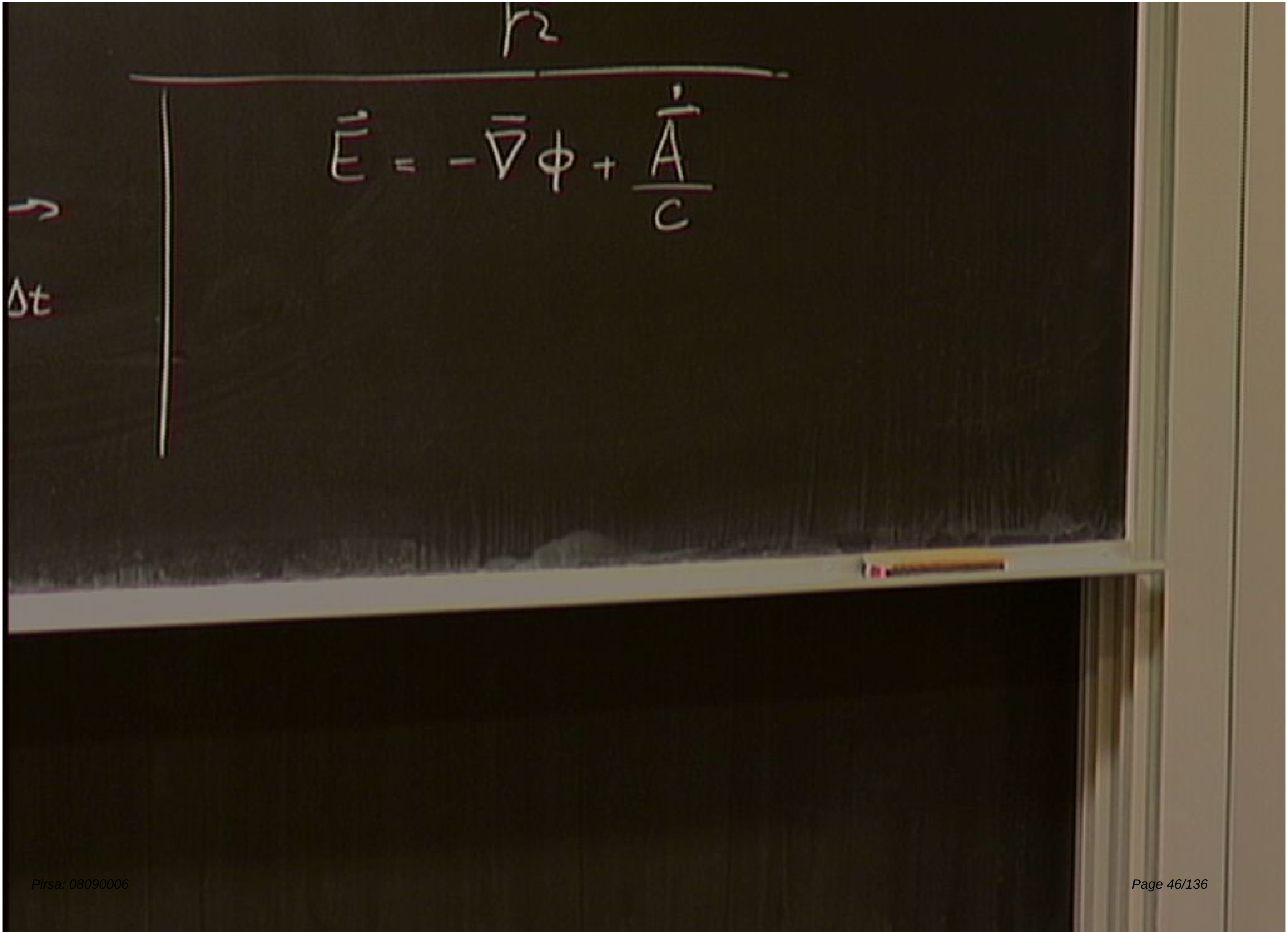

v
A_x = γ(A'_x + vφ'/c)
φ = γ(φ' + vA'/c)
A_y = A'_y
A_z = A'_z
t' = γ(t − vx/c²)
x' = γ(x − vt)
y' = y
z' = z
γ = 1/√(1 − v
E = q/r²
q/r' , A' = 0
A = γ v/c · q/r'
φ = γq/r'
(ct, x)
x^μ



$A_x = \gamma(A'_x + \frac{v\phi'}{c})$
$\phi = \gamma(\phi' + \frac{vA'_x}{c})$
$A_y = A'_y$
$A_z = A'_z$
$t' = \gamma(t - \frac{vx}{c^2})$
$x' = \gamma(x - vt)$
$y' = y$
$z' = z$
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
$\vec{E} = \frac{q}{r^2}$
$\phi' = \frac{q}{r'}, \quad \vec{A}' = 0$
$\vec{A} = \gamma \frac{\vec{v}}{c} \cdot \frac{q}{r'}$
$\phi = \frac{\gamma q}{r'}$
$r'^2 = x'^2 + y'^2 + z'^2$
$= \gamma^2 (x - vt)^2 + y^2 + z^2$
$(ct, \vec{x})$



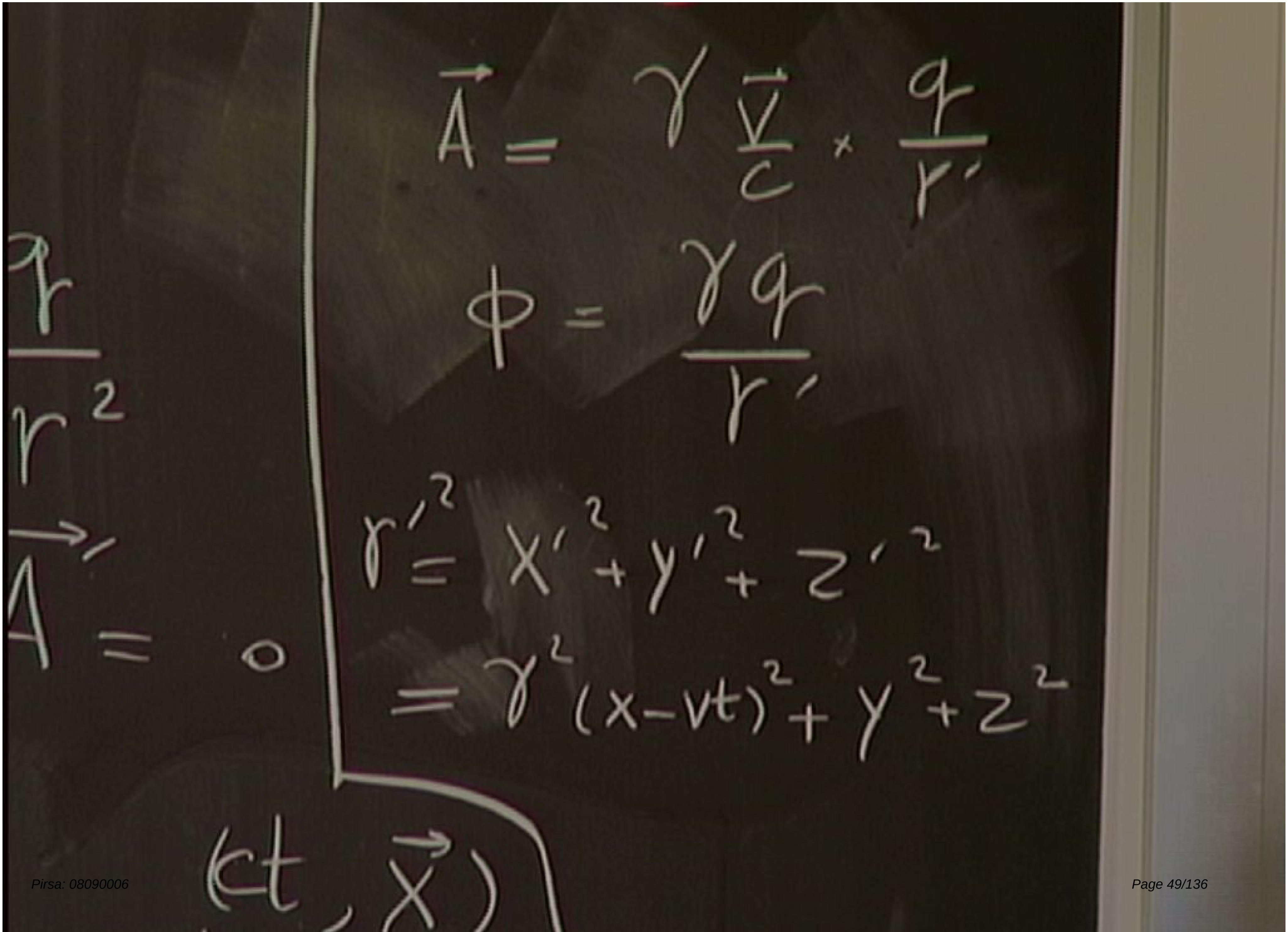
Δt
t
S = E×B ~ E² ~ 1/r²
ct
~c Δt

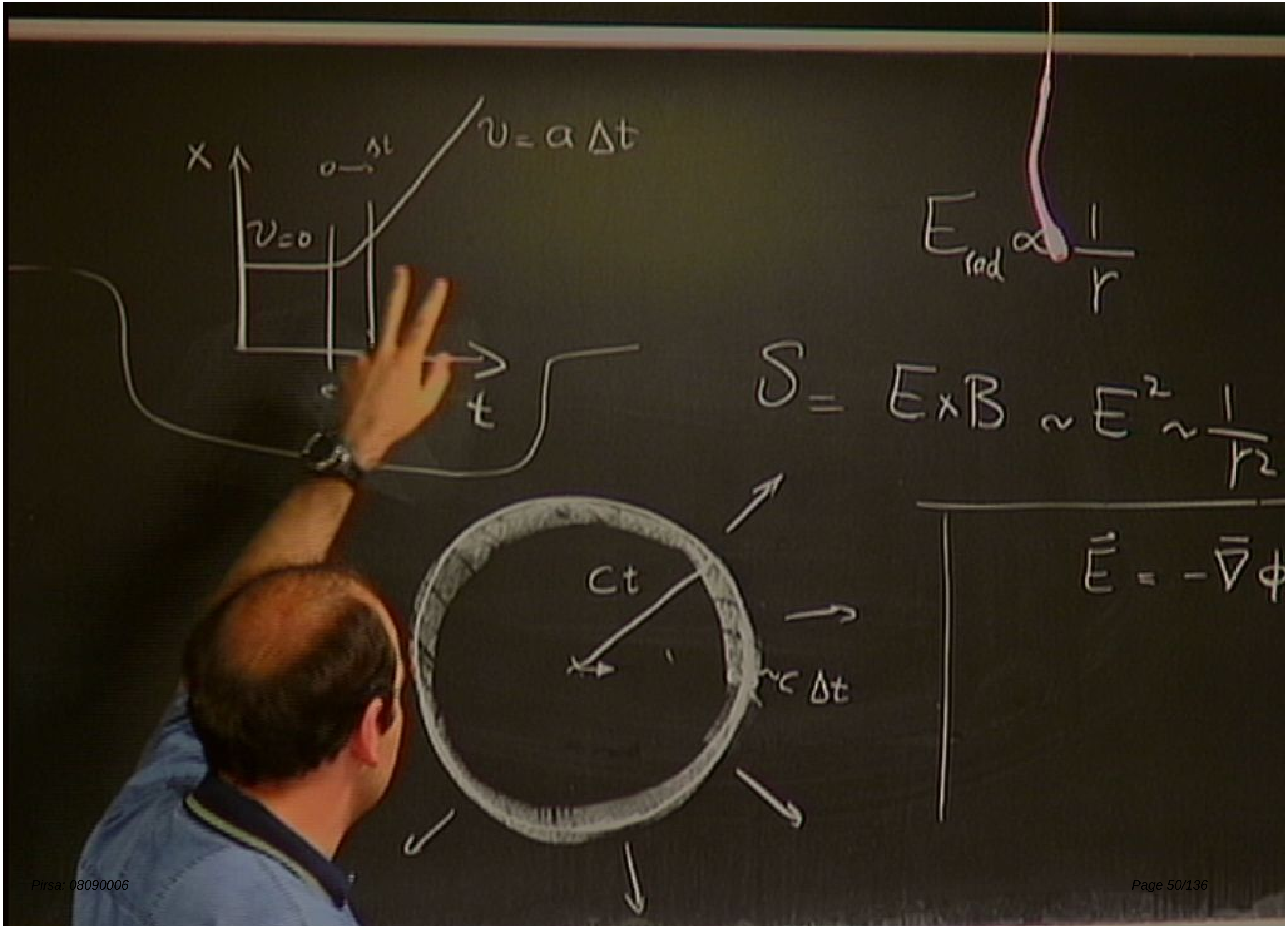

$\vec{E} = -\bar{\nabla}\phi + \frac{\dot{\vec{A}}}{c}$

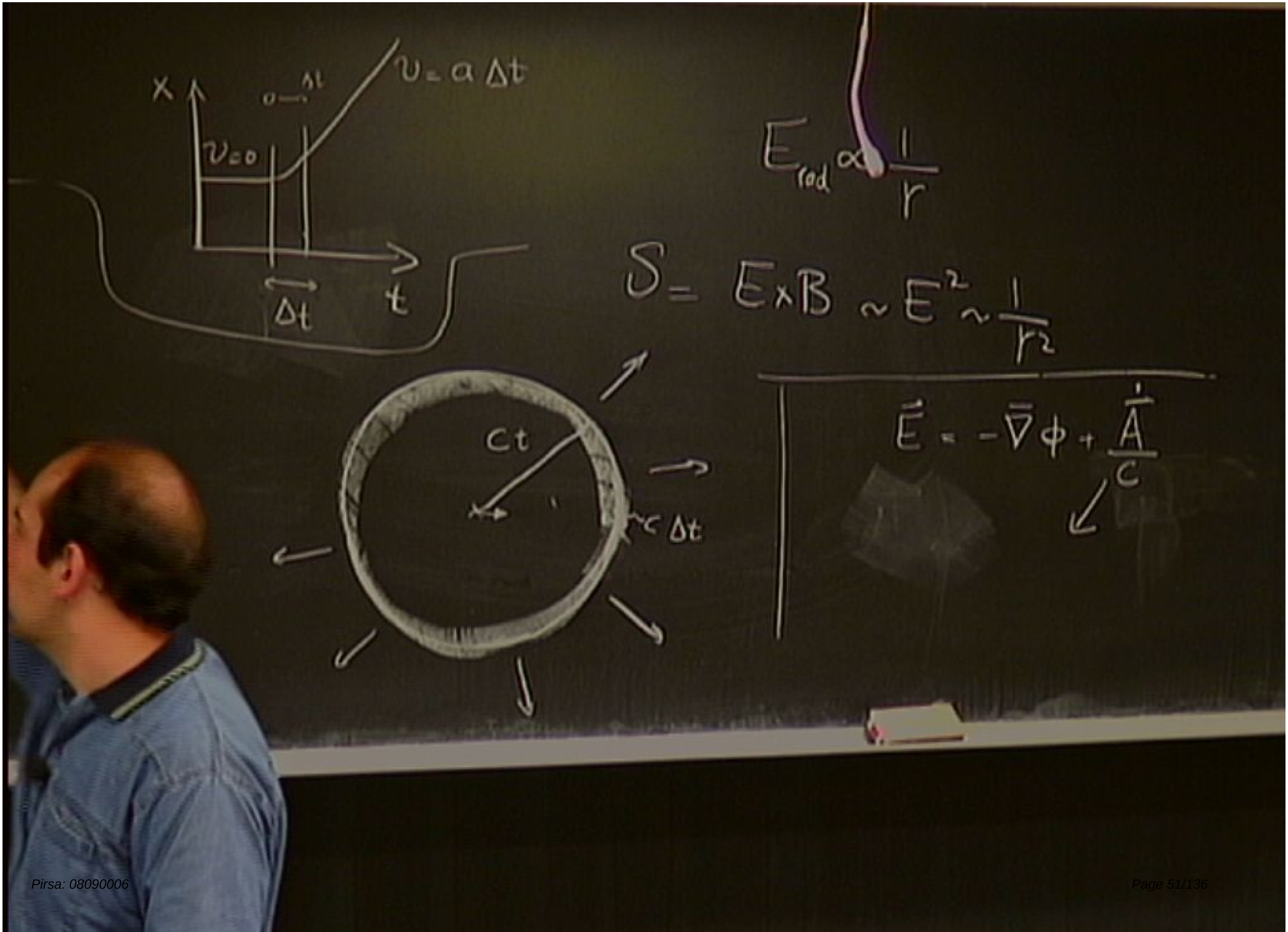
$$\sim e \sim \frac{\cdots}{\hbar}$$

$$\vec{E} = -\vec{\nabla}\phi + \dot{\vec{A}}$$

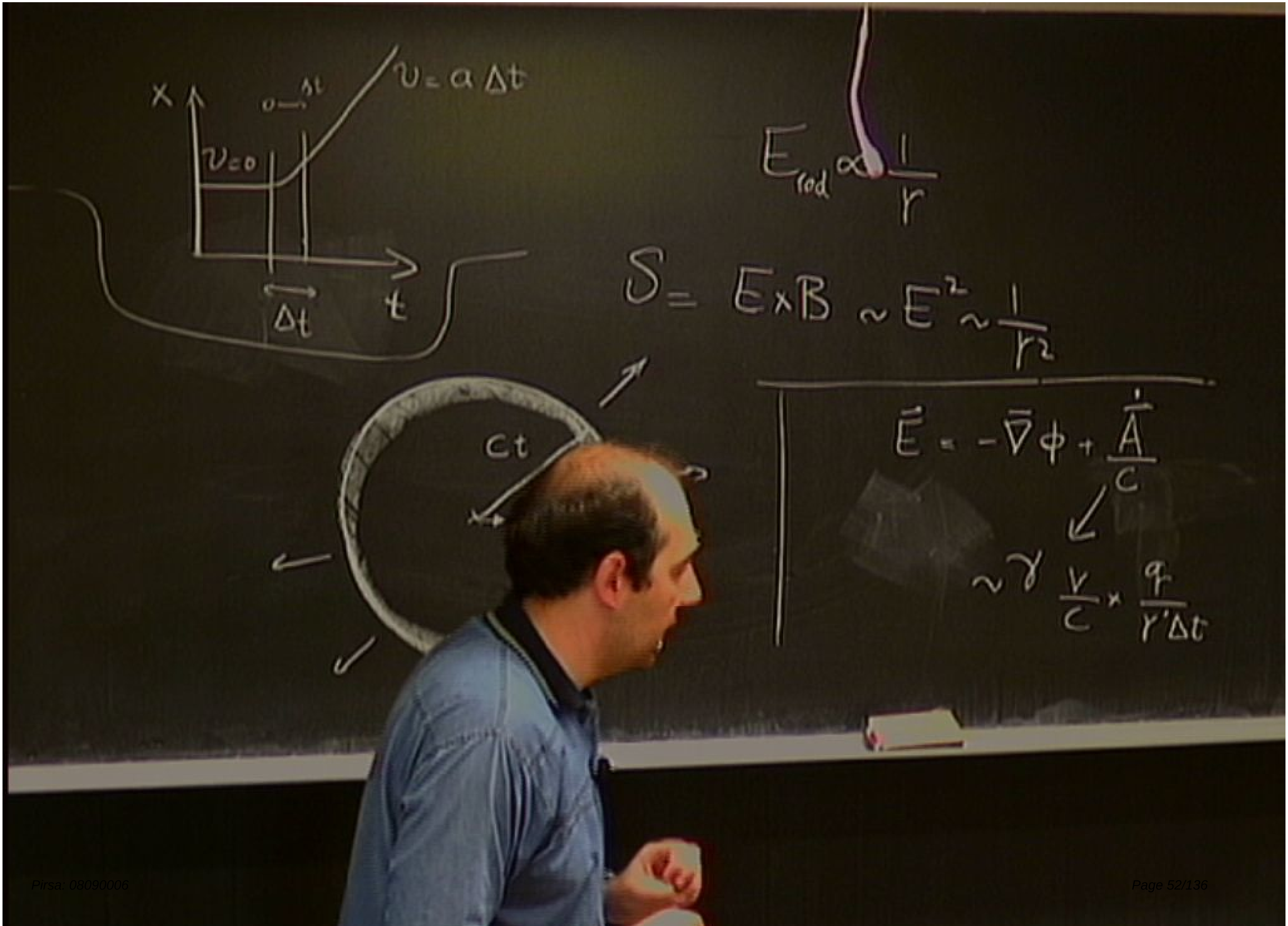
$$\vec{E} = -\bar{\nabla}\phi + \frac{\dot{\vec{A}}}{c}$$

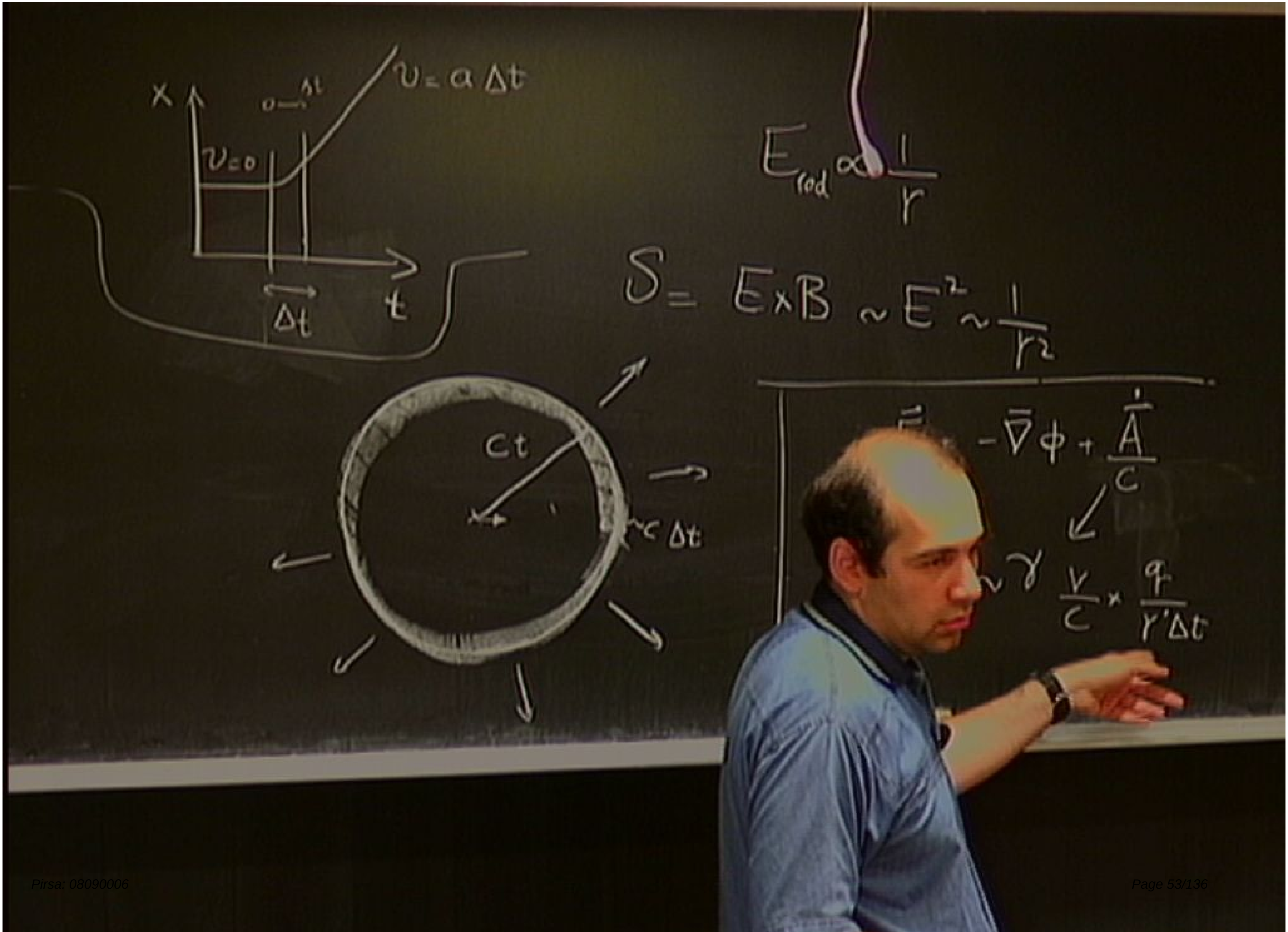

$\vec{A} = \gamma \frac{\vec{v}}{c} \times \frac{q}{r'}$
$\phi = \frac{\gamma q}{r'}$
$r'^2 = x'^2 + y'^2 + z'^2$
$= \gamma^2 (x-vt)^2 + y^2 + z^2$
$\frac{q}{r^2}$
$\vec{A} = 0$
$(ct, \vec{x})$

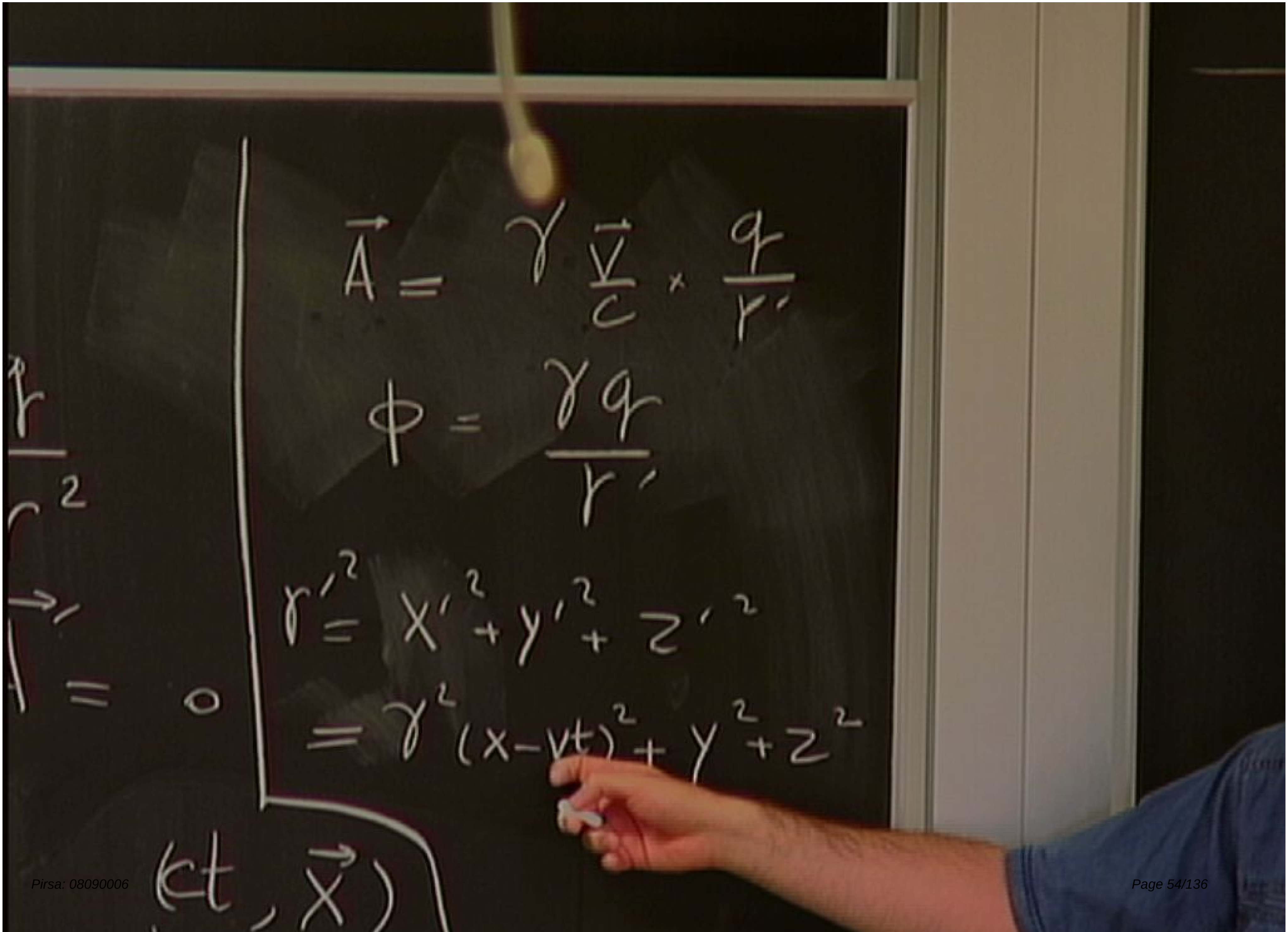

x
v = a Δt
v = 0
t
E_rad ∝ 1/r
S = E×B ~ E² ~ 1/r²
ct
~c Δt
E = -∇φ



x
t
v = 0
v = a Δt
Δt
E_rad ∝ 1/r
S = E×B ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
ct
~c Δt


x
v = a Δt
v = 0
t
Δt
ct
E_rad ∝ 1/r
S = E×B ~ E² ~ 1/r²
E = -∇φ + Ȧ/c
~ γ v/c × q/r'Δt


x
v = a Δt
v = 0
Δt
t
E_rad ∝ 1/r
S = E×B ~ E² ~ 1/r²
ct
~c Δt
Ȧ/c
~γ v/c × q/γ'Δt


$\vec{A} = \gamma \frac{\vec{v}}{c} \times \frac{q}{r'}$
$\phi = \frac{\gamma q}{r'}$
$r'^2 = x'^2 + y'^2 + z'^2$
$= \gamma^2 (x-vt)^2 + y^2 + z^2$
$= 0$
$(ct, \vec{x})$

$$\vec{A} = \gamma \frac{\vec{V}}{c} \cdot \frac{q}{r'}$$

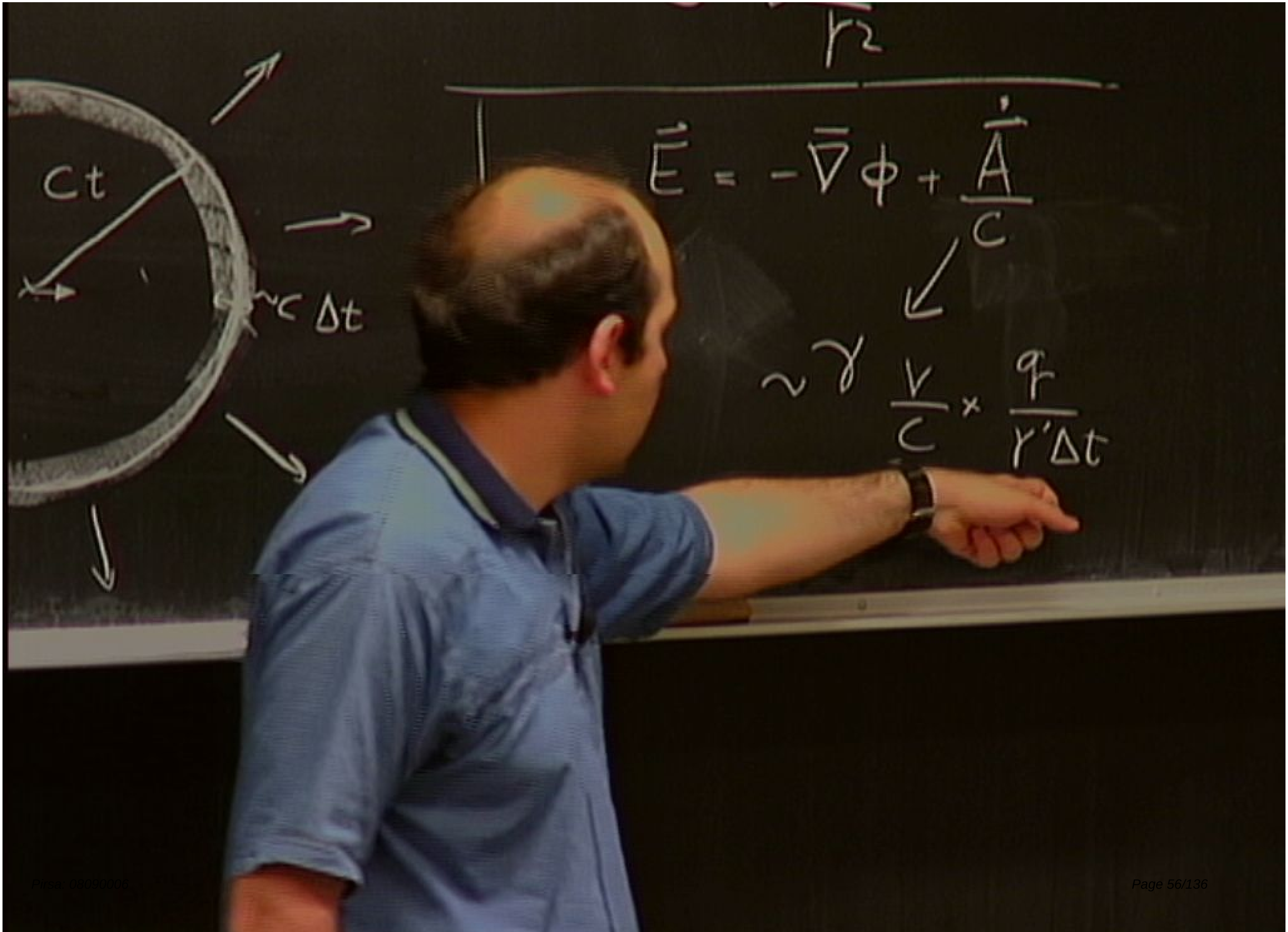
$$\phi = \frac{\gamma q}{r'}$$

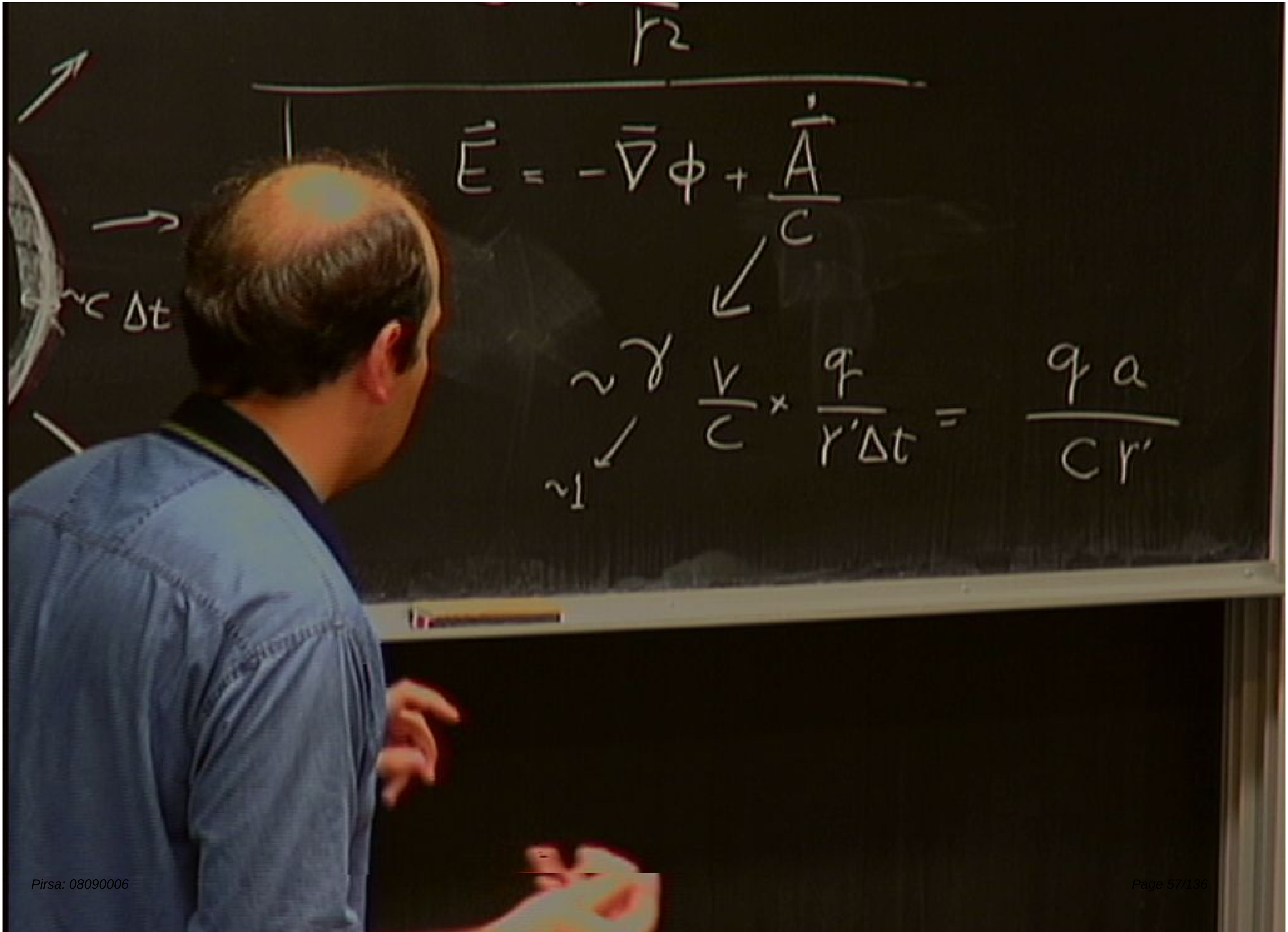
$$r'^2 = x'^2 + y'^2 + z'^2$$

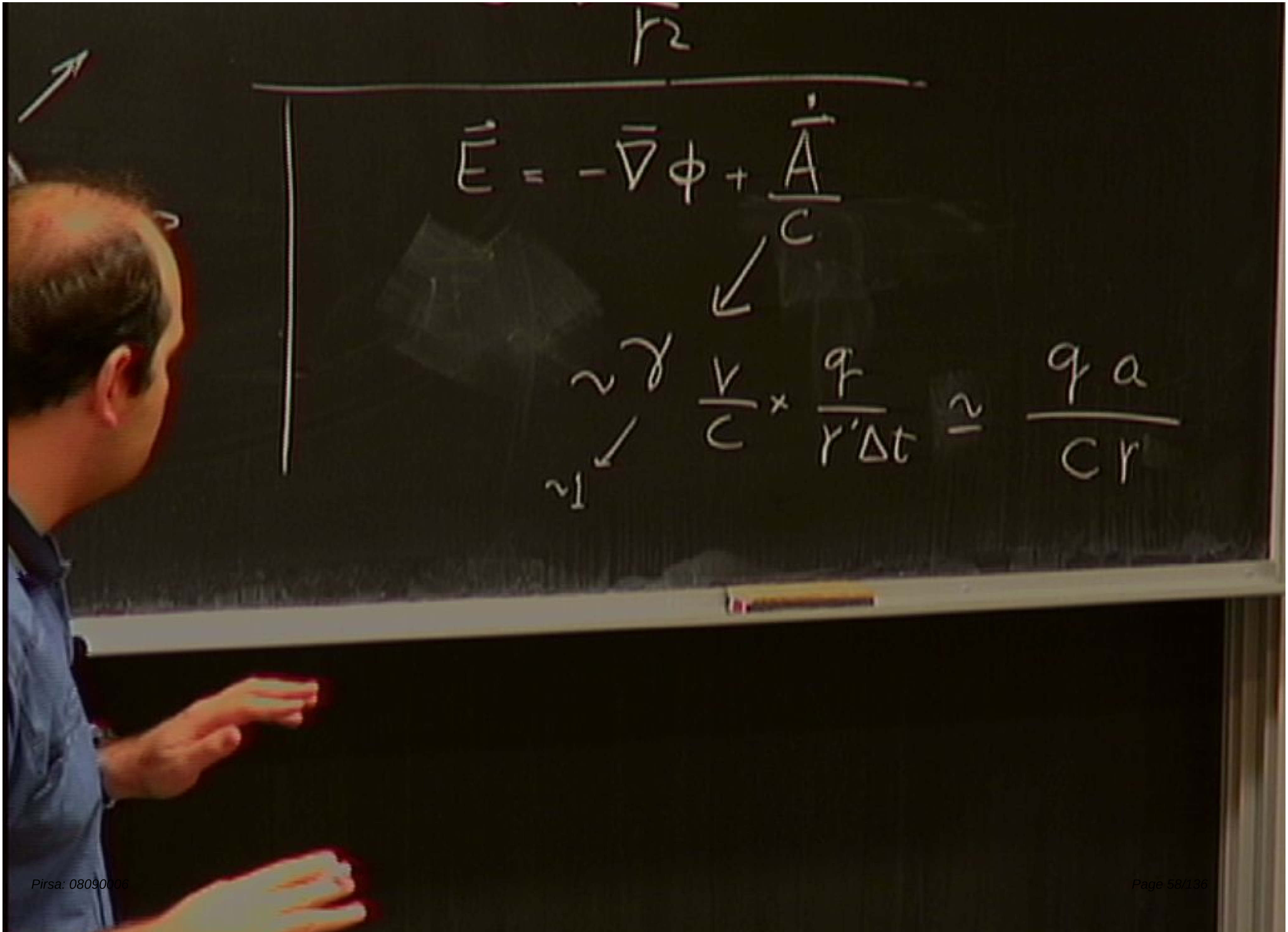
$$= \gamma^2 (x - vt)^2 + y^2 + z^2$$

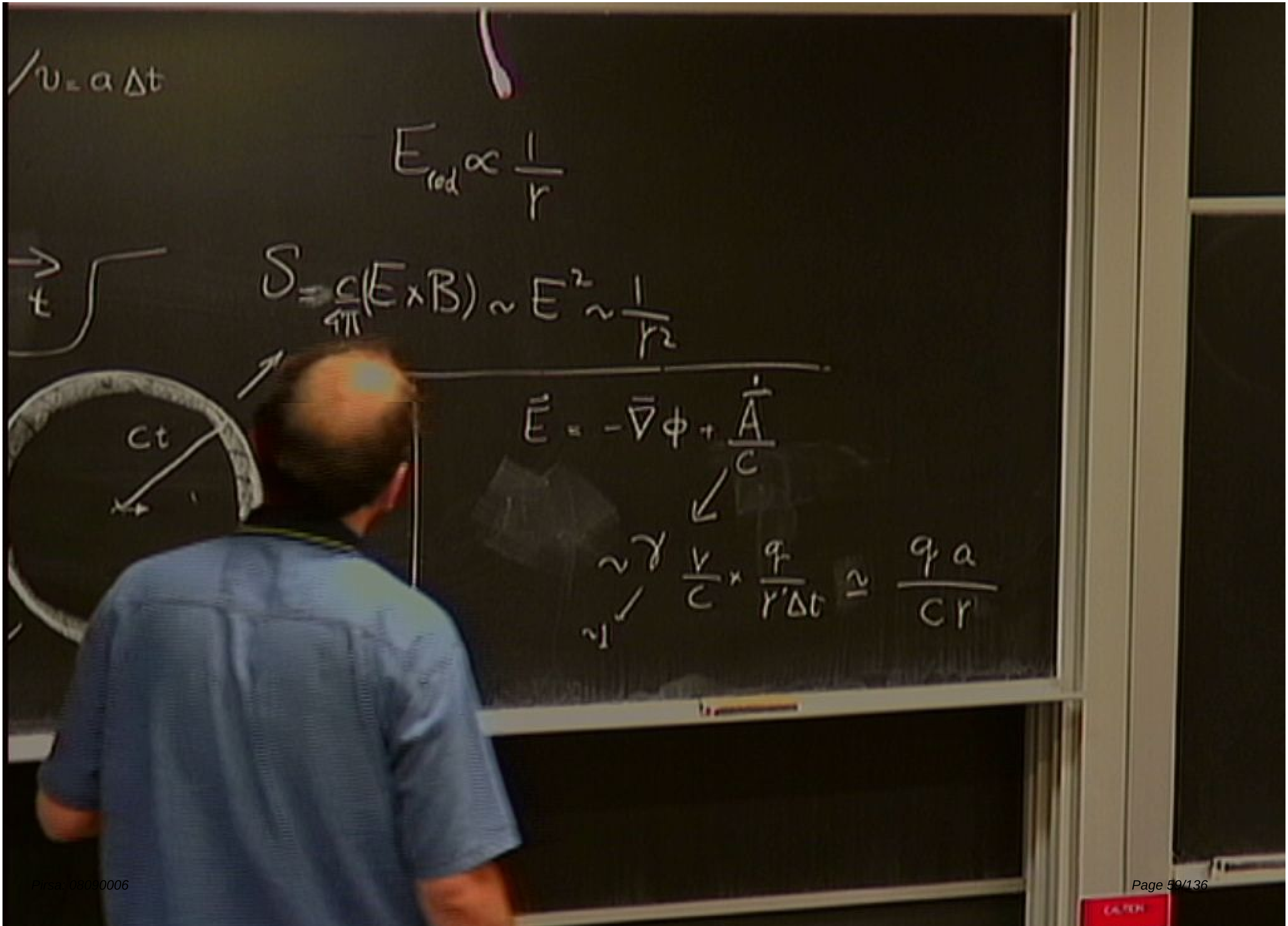
$$\gamma \simeq 1$$

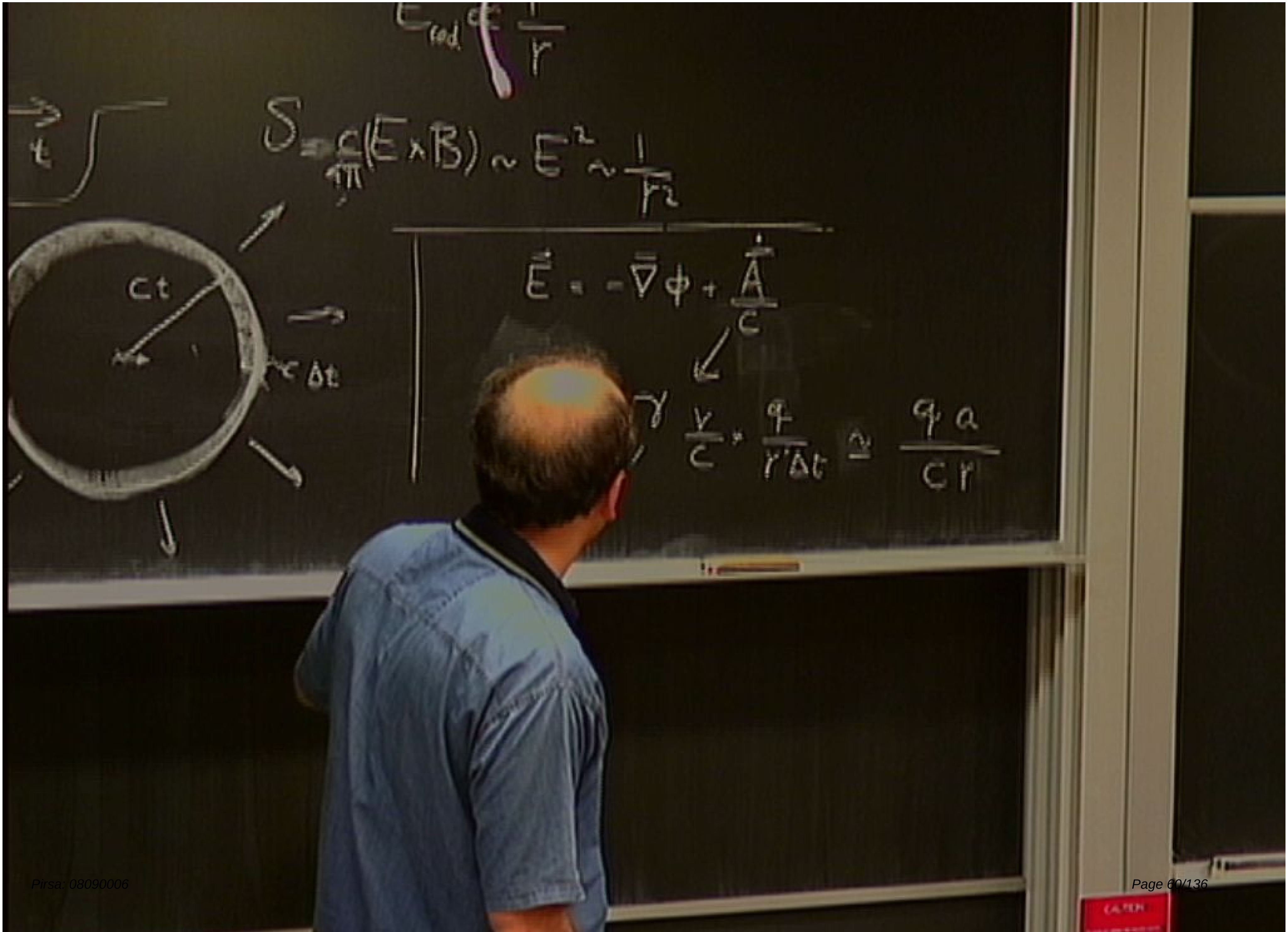
$$(ct, \vec{X})$$

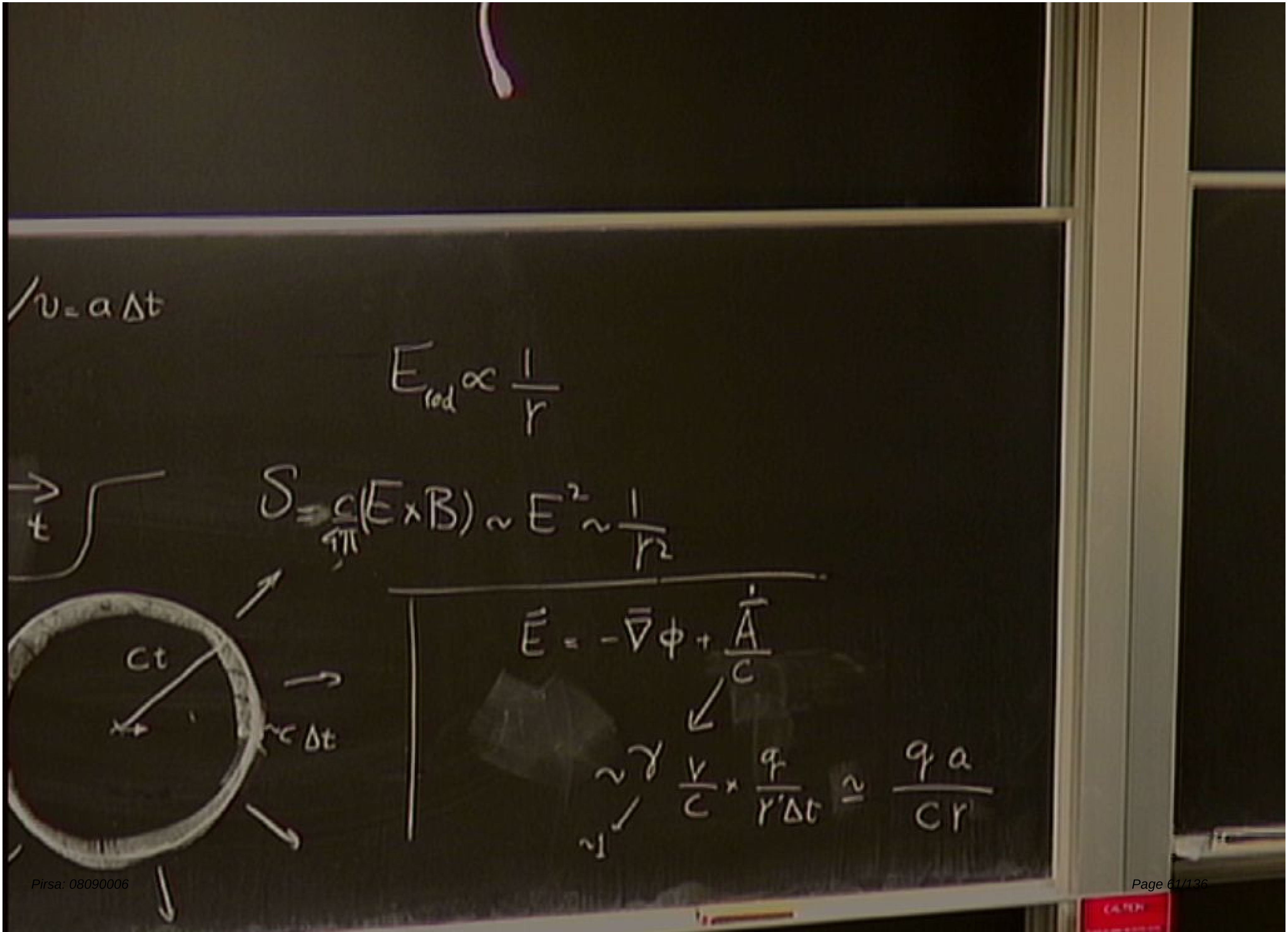

ct
~c Δt
r2
E = -∇φ + Ȧ/c
~γ v/c × q/γ'Δt

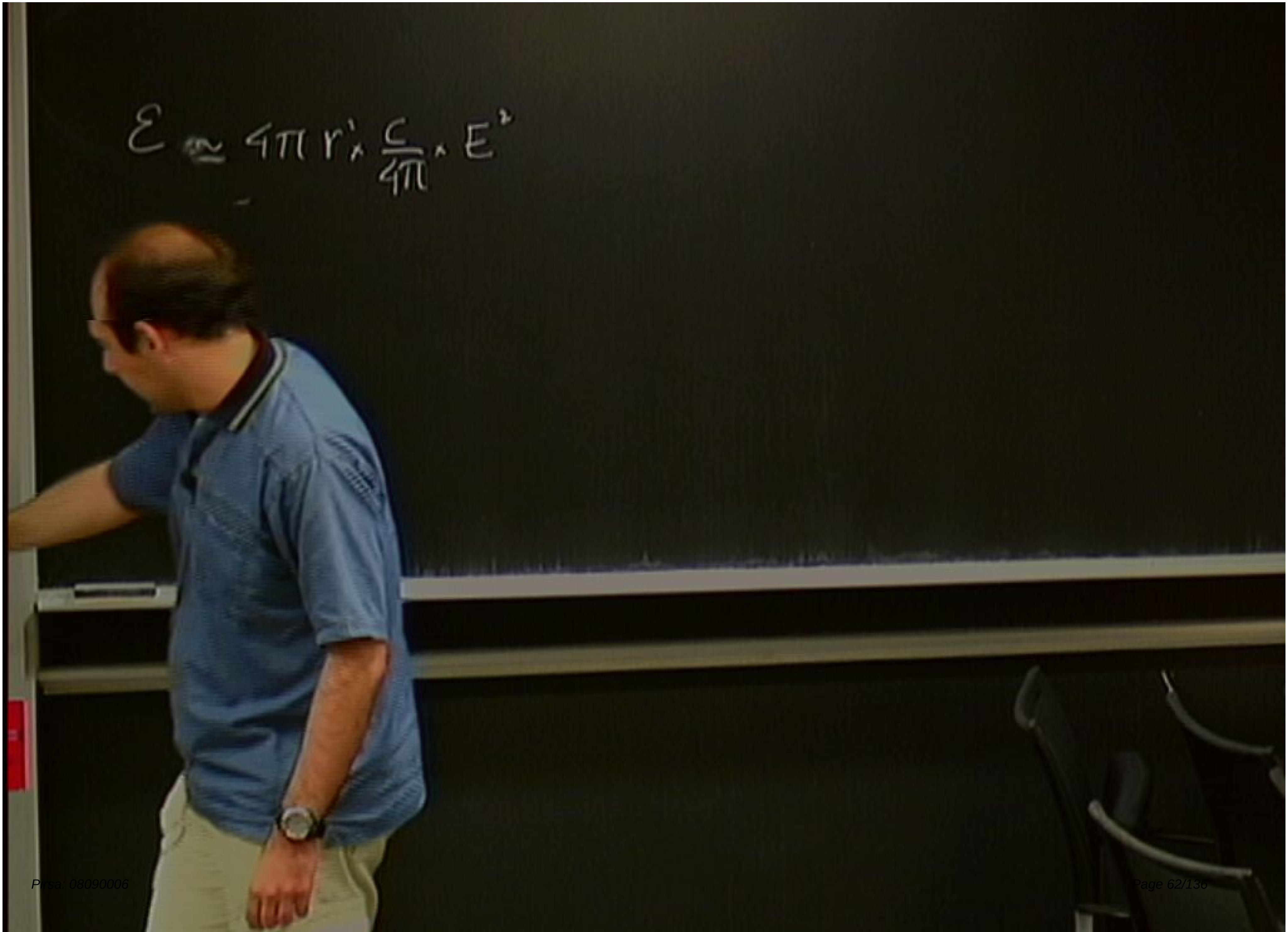

r2
~c Δt
E = -∇φ + Ȧ/c
~γ v/c × q/r'Δt = q a/c r'
~1

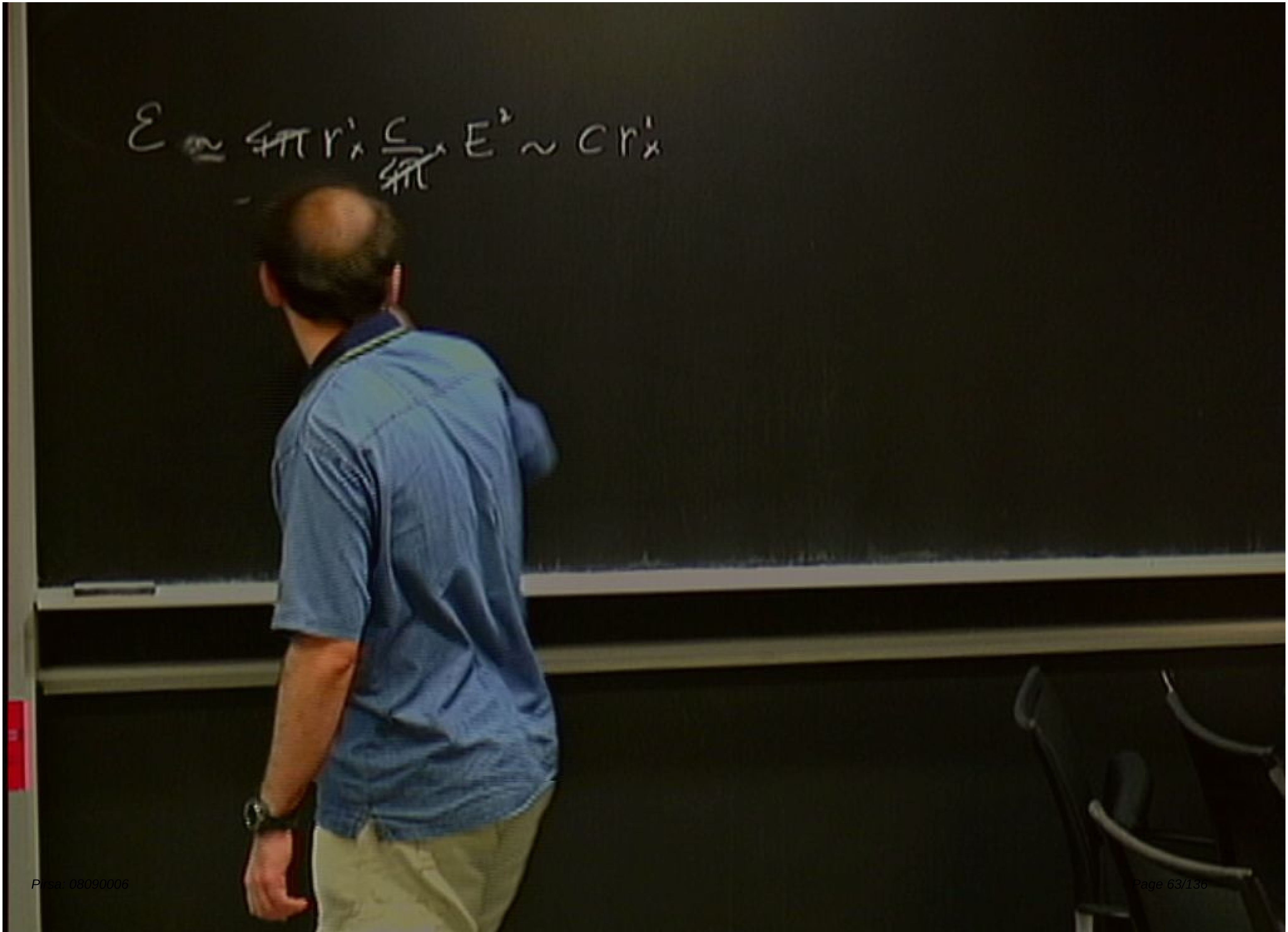

$\vec{E} = -\vec{\nabla}\phi + \frac{\dot{\vec{A}}}{c}$
$\sim \gamma \frac{v}{c} \times \frac{q}{r'\Delta t} \simeq \frac{q a}{c r}$
~ 1

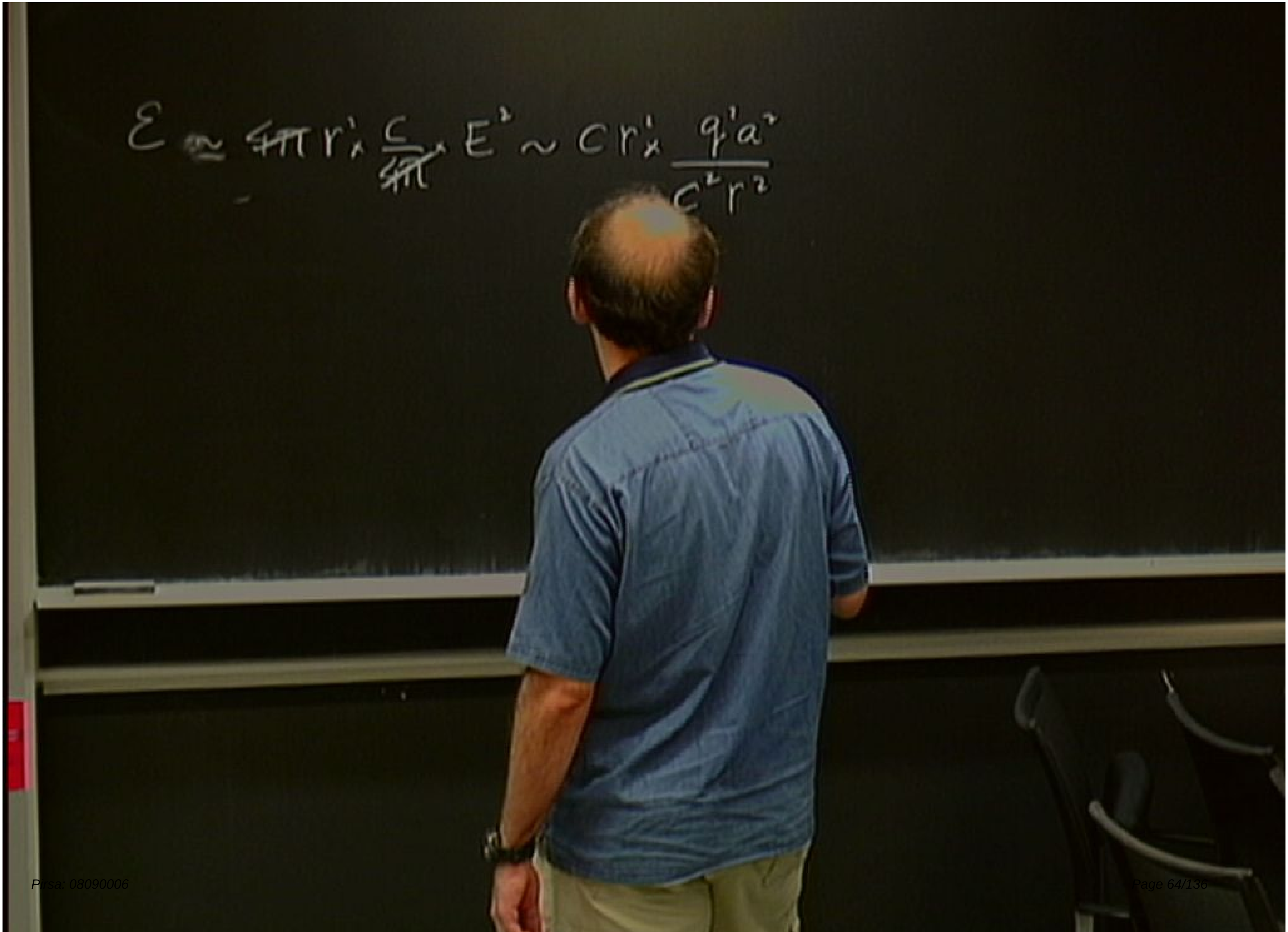

v = a Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
ct
E = −∇φ + Ȧ/c
~ γ v/c × q/(r′Δt) ≃ qa/(cr)
~1

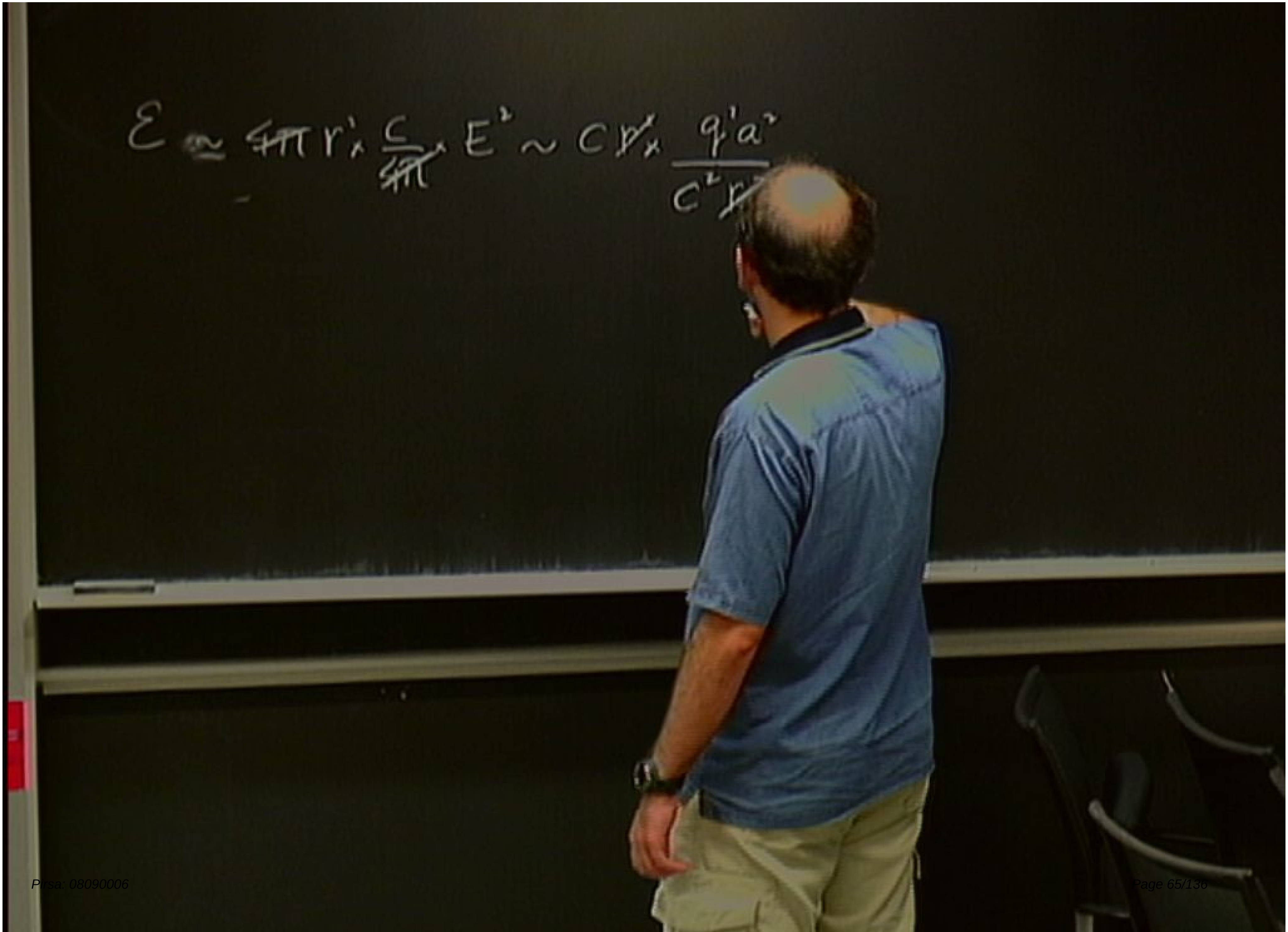

ct
c Δt

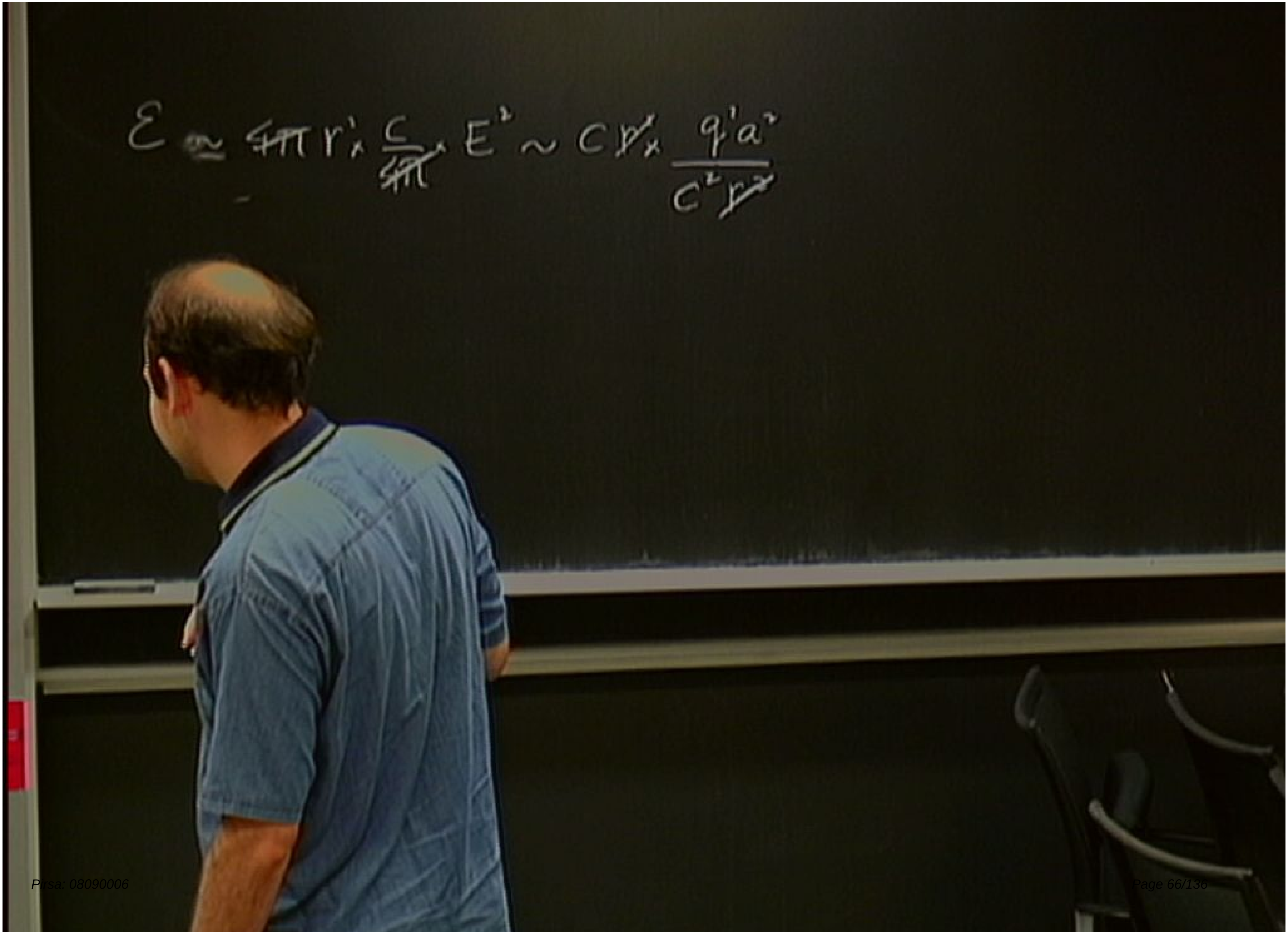

v = a Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
ct
~c Δt
E = −∇φ + Ȧ/c
~γ v/c × q/r'Δt ≃ qa/cr
~1


$\mathcal{E} \simeq 4\pi r^2 \times \frac{c}{4\pi} \times E^2$


ε ~ 4π r_x² × c/4π × E² ~ c r_x²

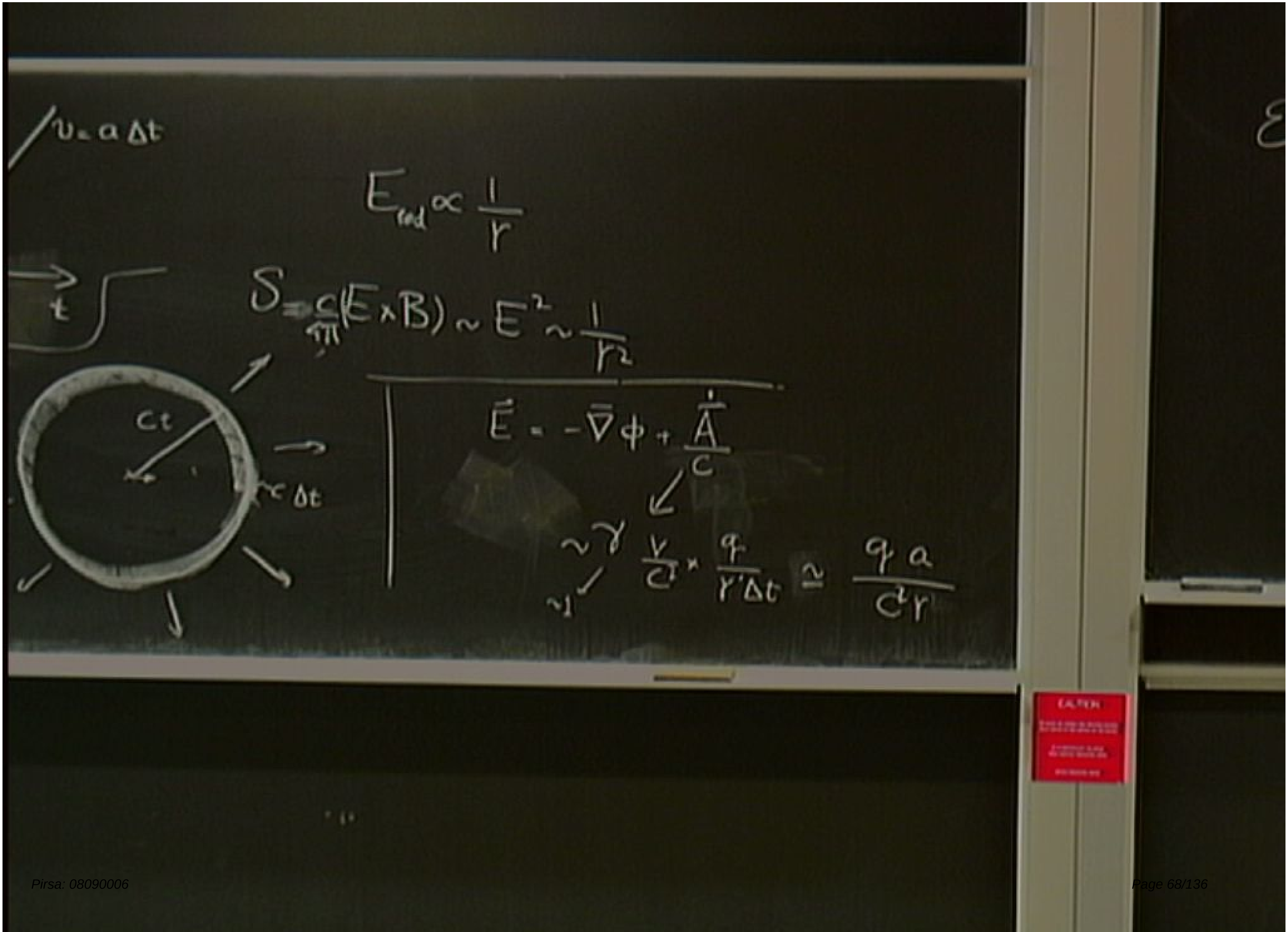

$\mathcal{E} \simeq 4\pi r^2 \times \frac{c}{4\pi} \times E^2 \sim c r^2 \times \frac{q^2 a^2}{c^2 r^2}$

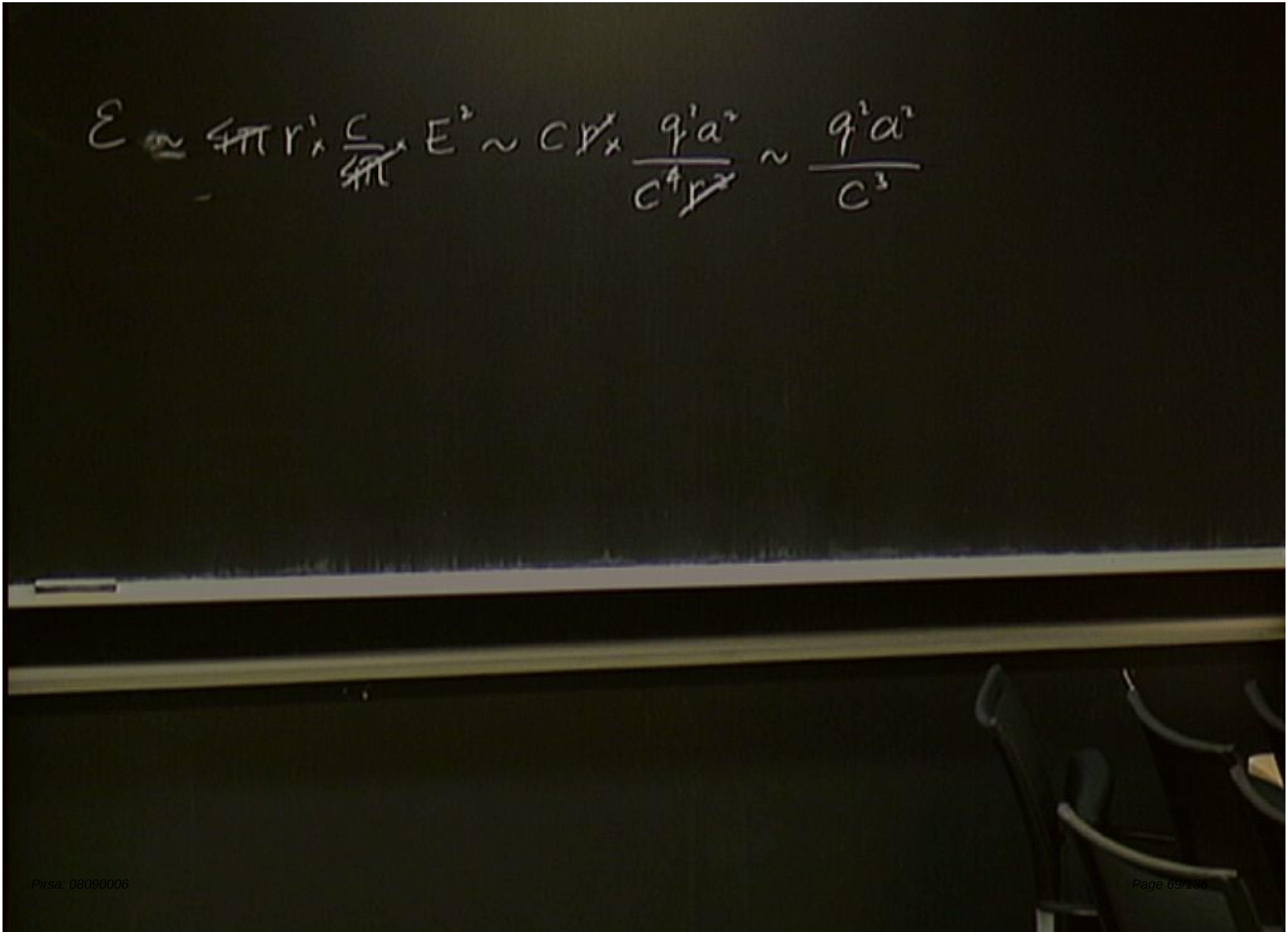


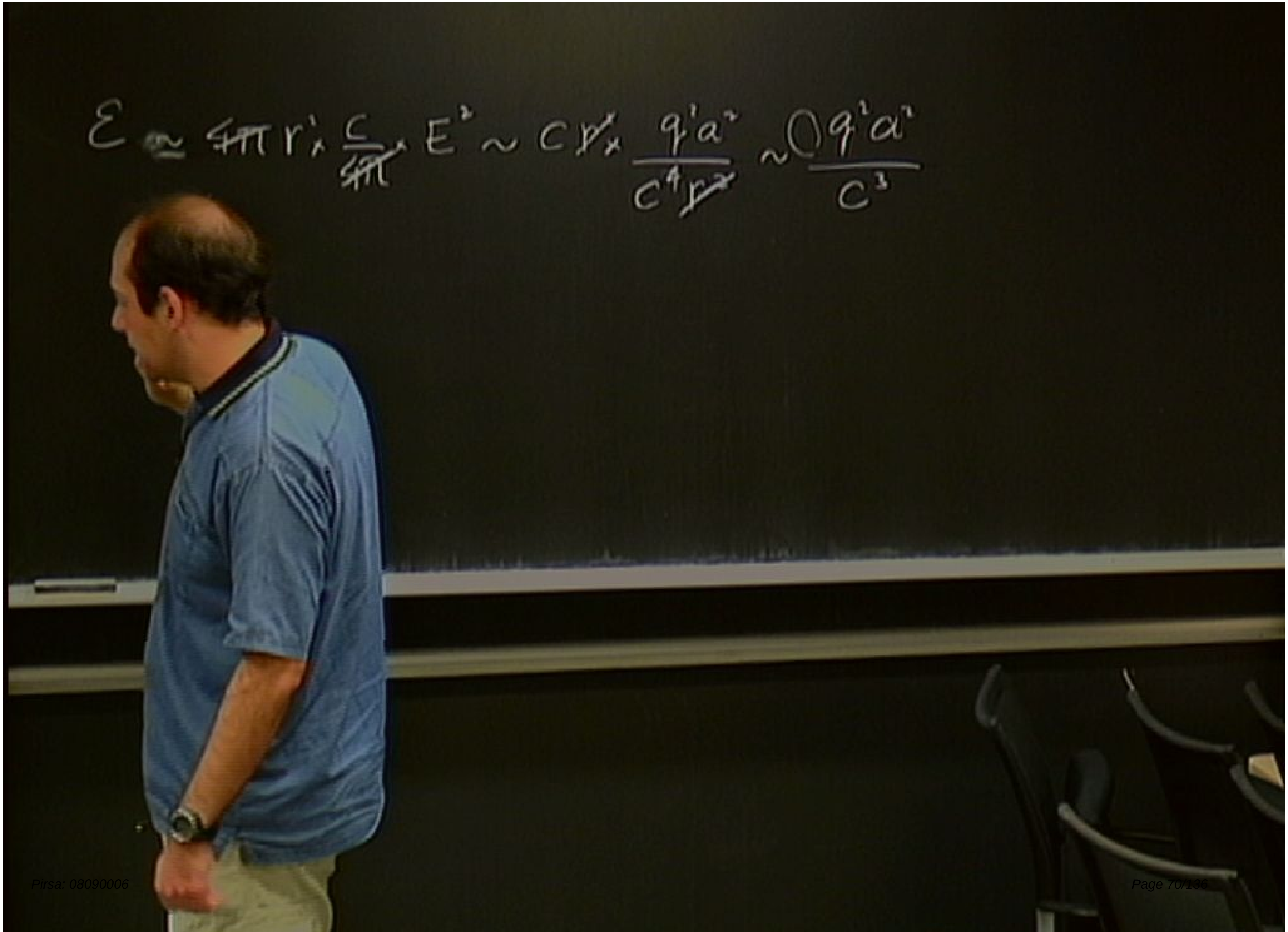


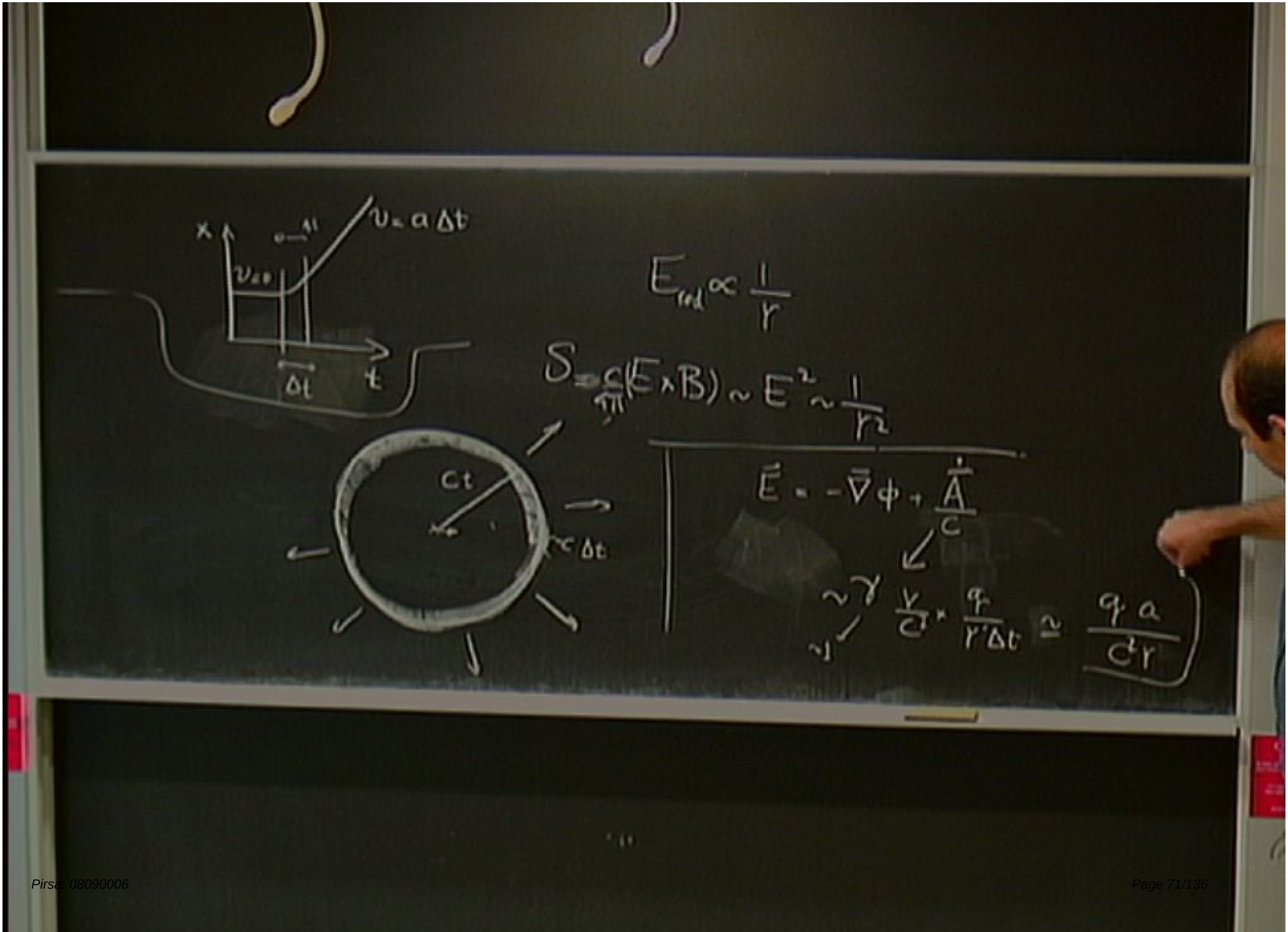
$\mathcal{E} \simeq 4\pi r^2 \times \frac{c}{4\pi} \times E^2 \sim c\, r^2 \times \frac{q^2 a^2}{c^2 r^2}$

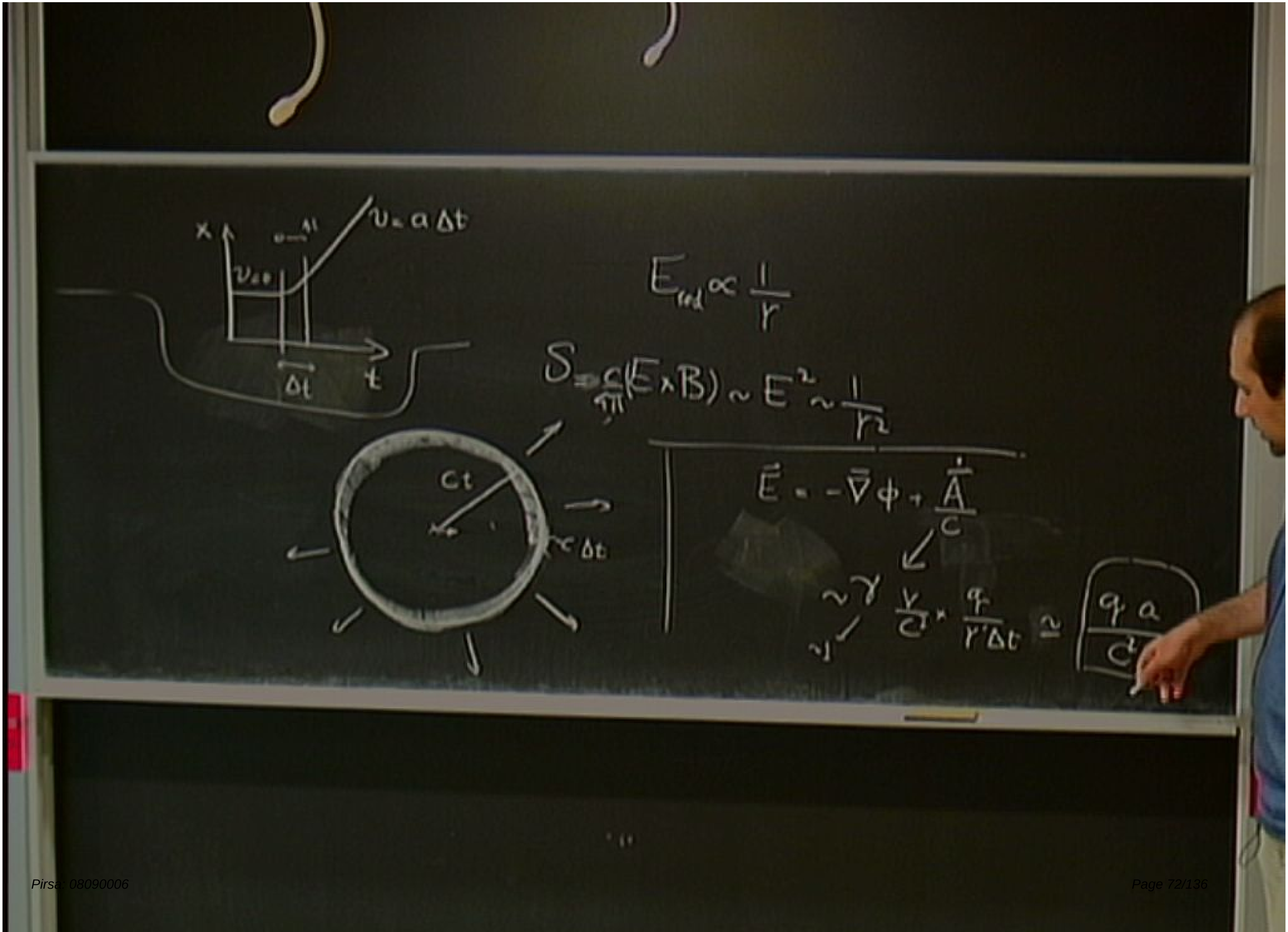
$$\mathcal{E} \simeq \cancel{4\pi} r^2 \times \frac{c}{\cancel{4\pi}} \times E^2 \sim c \cancel{r^2} \times \frac{q^2 a^2}{c^2 \cancel{r^2}}$$


v = a Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
ct
c Δt
t
E = −∇φ + Ȧ/c
~γ v/c² × q/r'Δt ≃ qa/c²r
~1

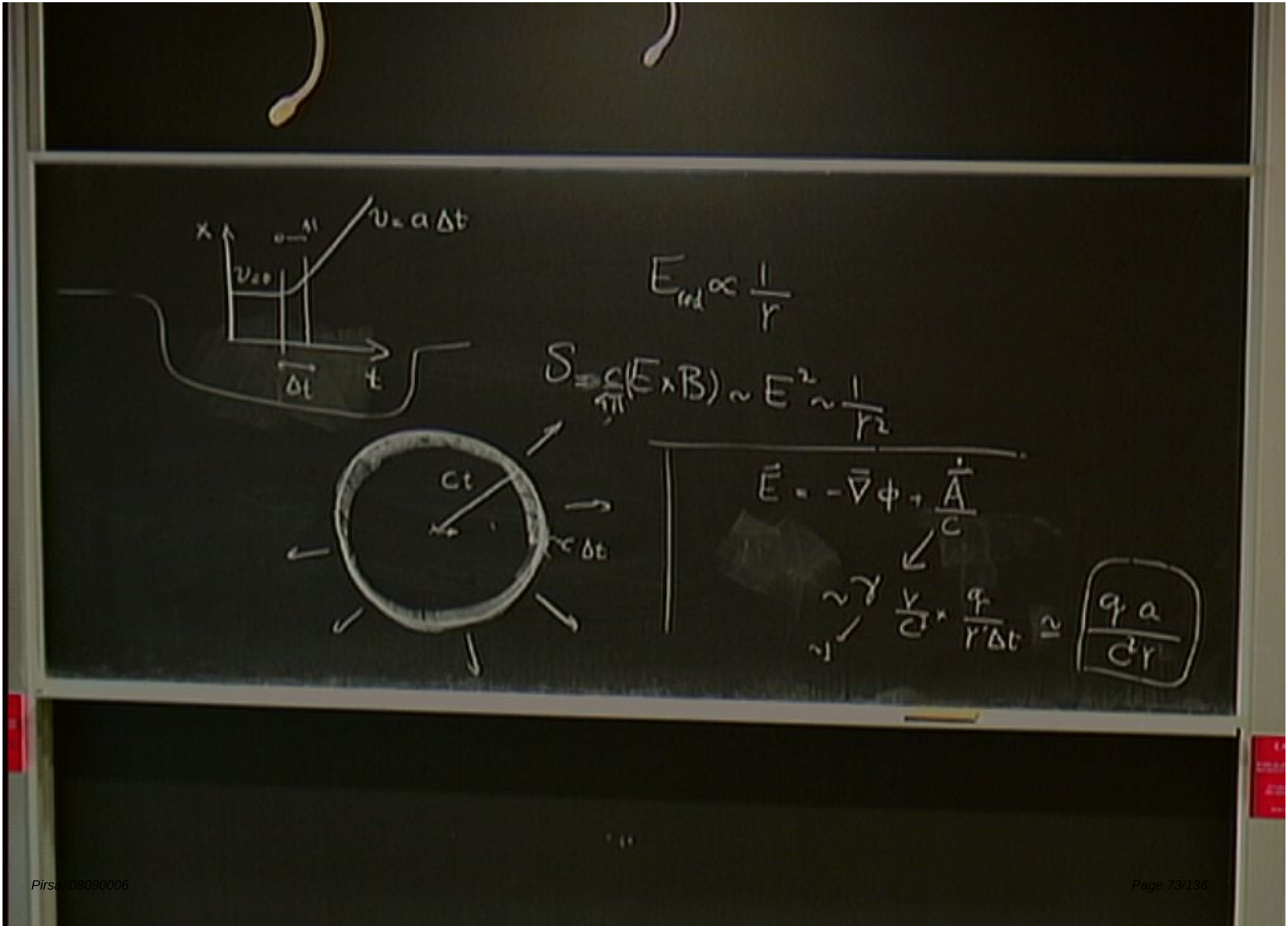

$\mathcal{E} \simeq 4\pi r^2 \times \frac{c}{4\pi} \times E^2 \sim c\, r^2 \times \frac{q^2 a^2}{c^4 r^2} \sim \frac{q^2 a^2}{c^3}$

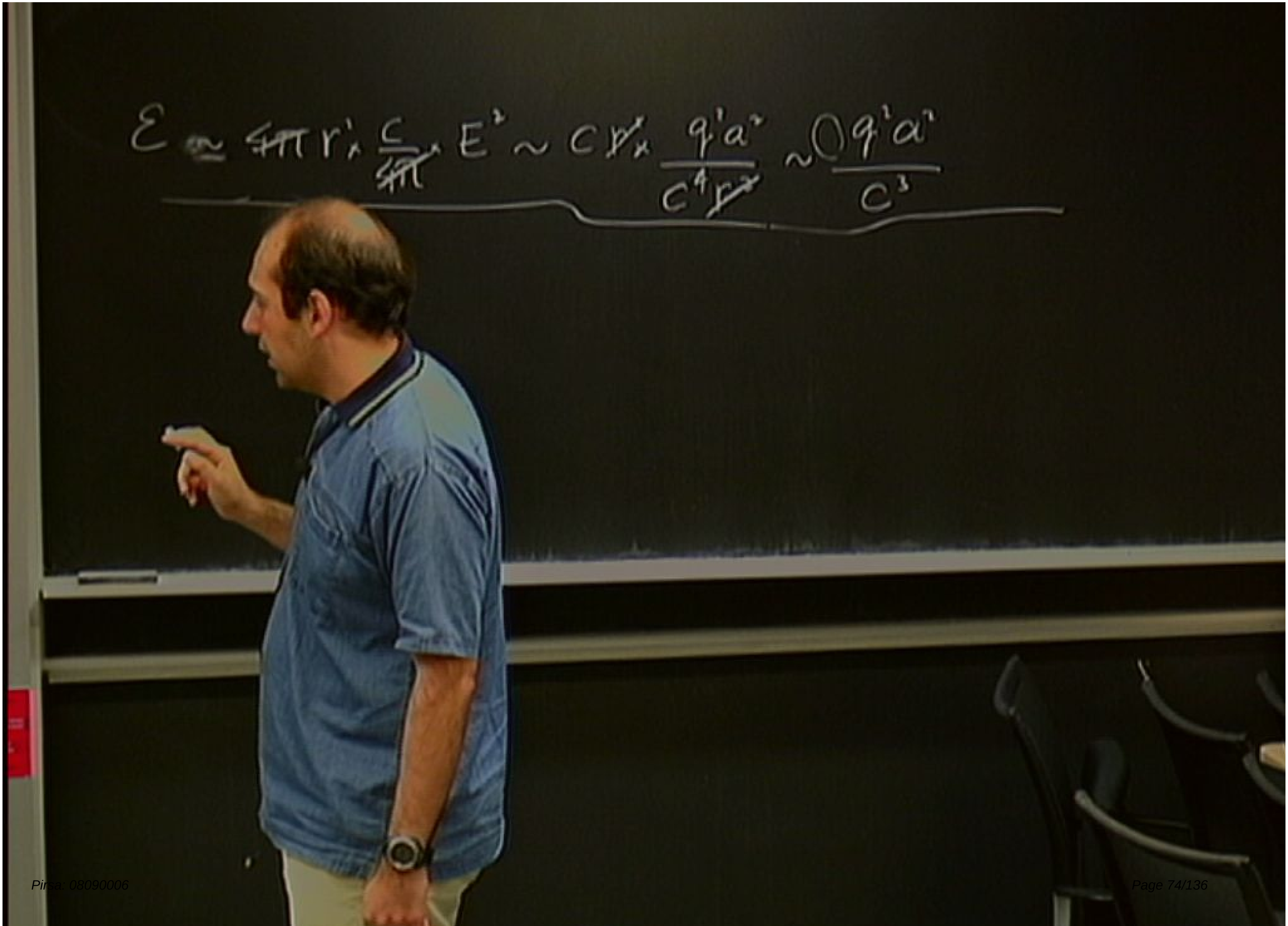

ℰ ≈ 4π r² × c/4π × E² ~ c r² × q²a²/(c⁴ r²) ~ q²a²/c³


x
v=0
v = a Δt
Δt
t
E_rad ∝ 1/r
S = c/4π (E×B) ~ E² ~ 1/r²
ct
cΔt
E = −∇φ + Ȧ/c
q a / c² r

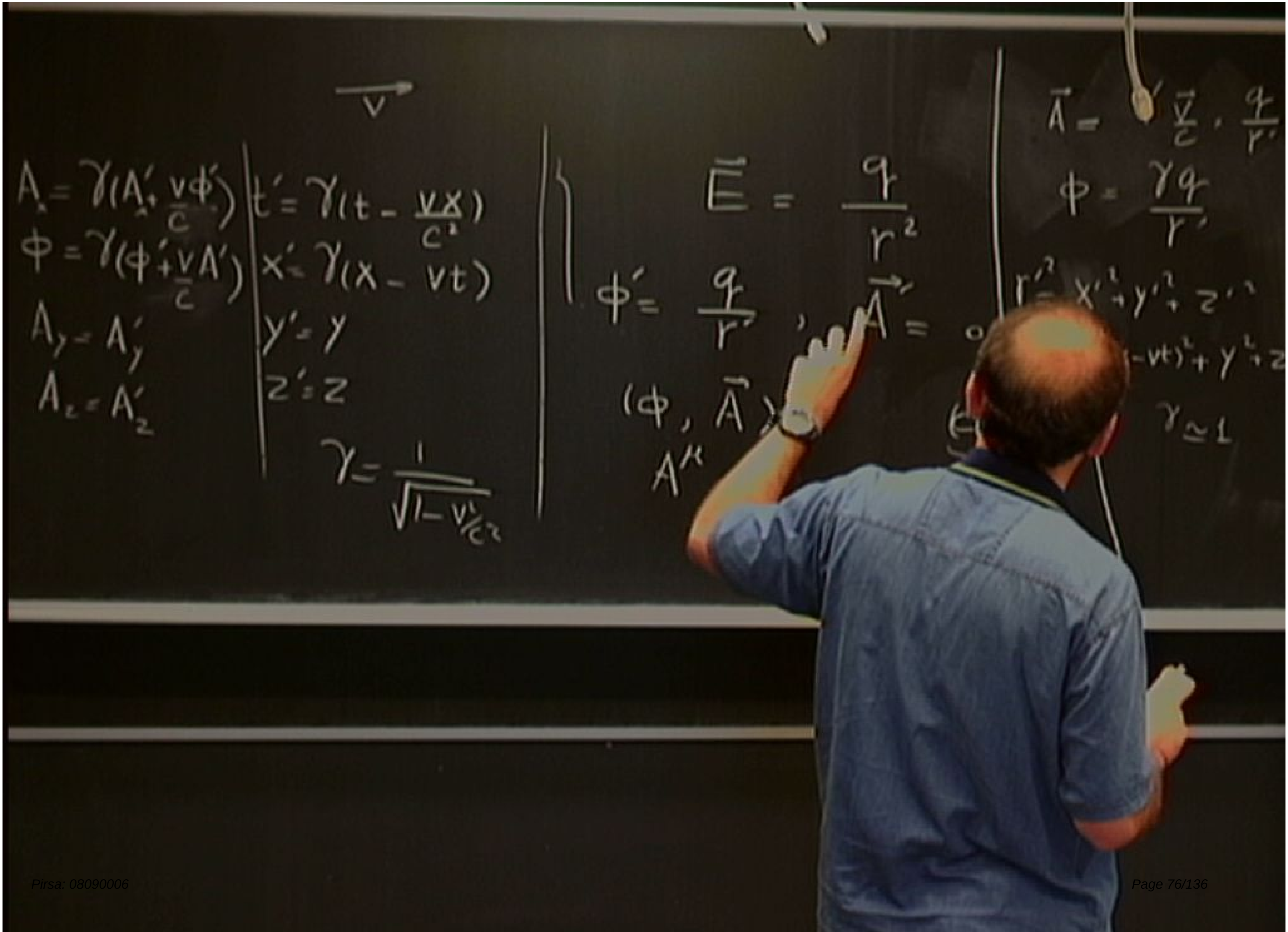


v = a Δt
x
v = 0
Δt
t
E ∝ 1/r
S = c/4π (E×B) ~ E² ~ 1/r²
ct
cΔt
E = −∇φ + Ȧ/c
q a / c²


x
Δt
v = a Δt
v = 0
Δt
t
$E_{rad} \propto \frac{1}{r}$
$S = \frac{c}{4\pi}(E \times B) \sim E^2 \sim \frac{1}{r^2}$
ct
c Δt
$\vec{E} = -\vec{\nabla}\phi + \frac{\dot{\vec{A}}}{c}$
$\sim \gamma \frac{v}{c^2} \times \frac{q}{r^2 \Delta t} \simeq \frac{q\,a}{c^2 r}$
~1


ℰ ≃ 4πr² × c/4π × E² ~ c r² × q²a²/c⁴r² ~ q²a²/c³

$$\mathcal{E} \simeq 4\pi r_*^2 \frac{c}{4\pi} E^2 \sim c r_*^2 \frac{q^2 a^2}{c^4 r_*^2} \sim \frac{q^2 a^2}{c^3}$$


v⃗
A_x = γ(A'_x + vφ'/c)
φ = γ(φ' + v/c A'_x)
A_y = A'_y
A_z = A'_z
t' = γ(t − vx/c²)
x' = γ(x − vt)
y' = y
z' = z
γ = 1/√(1 − v²/c²)
E⃗ = q/r²
φ' = q/r' , A⃗' = 0
(φ, A⃗)
A^μ
A⃗ = γ v⃗/c · q/r'
φ = γq/r'
γ ≃ 1

$\vec{v}$

$A_x = \gamma\left(A'_x + \frac{v\phi'}{c}\right)$

$\phi = \gamma\left(\phi' + \frac{v}{c}A'_x\right)$

$A_y = A'_y$

$A_z = A'_z$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$\vec{E} = \frac{q}{r^2}$

$\phi' = \frac{q}{r'}, \quad \vec{A}' = 0$

$(\phi, \vec{A})$

A^μ

$(ct, \vec{x})$

x^μ

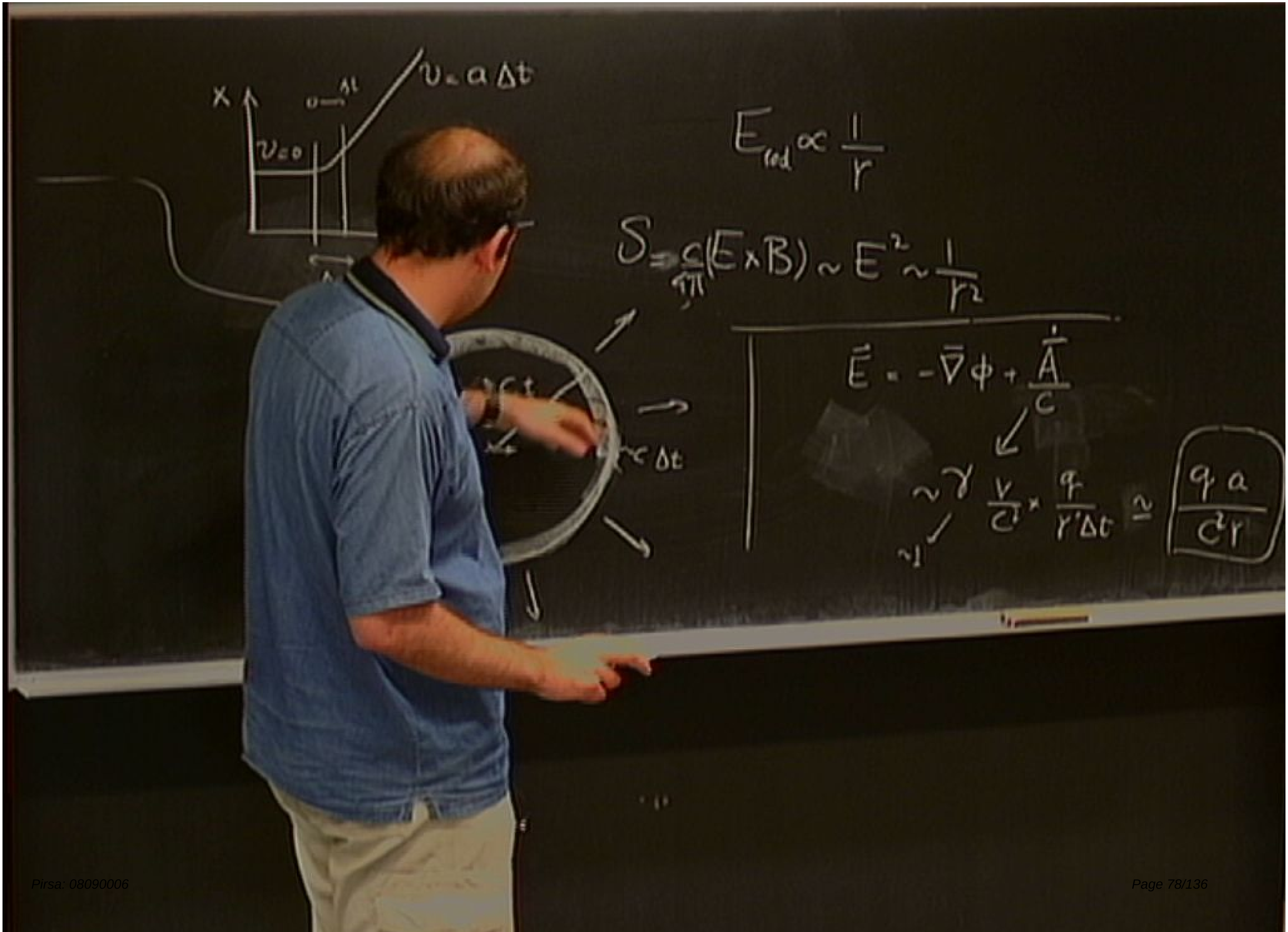
$\vec{A} = \gamma \frac{\vec{v}}{c} \cdot \frac{q}{r'}$

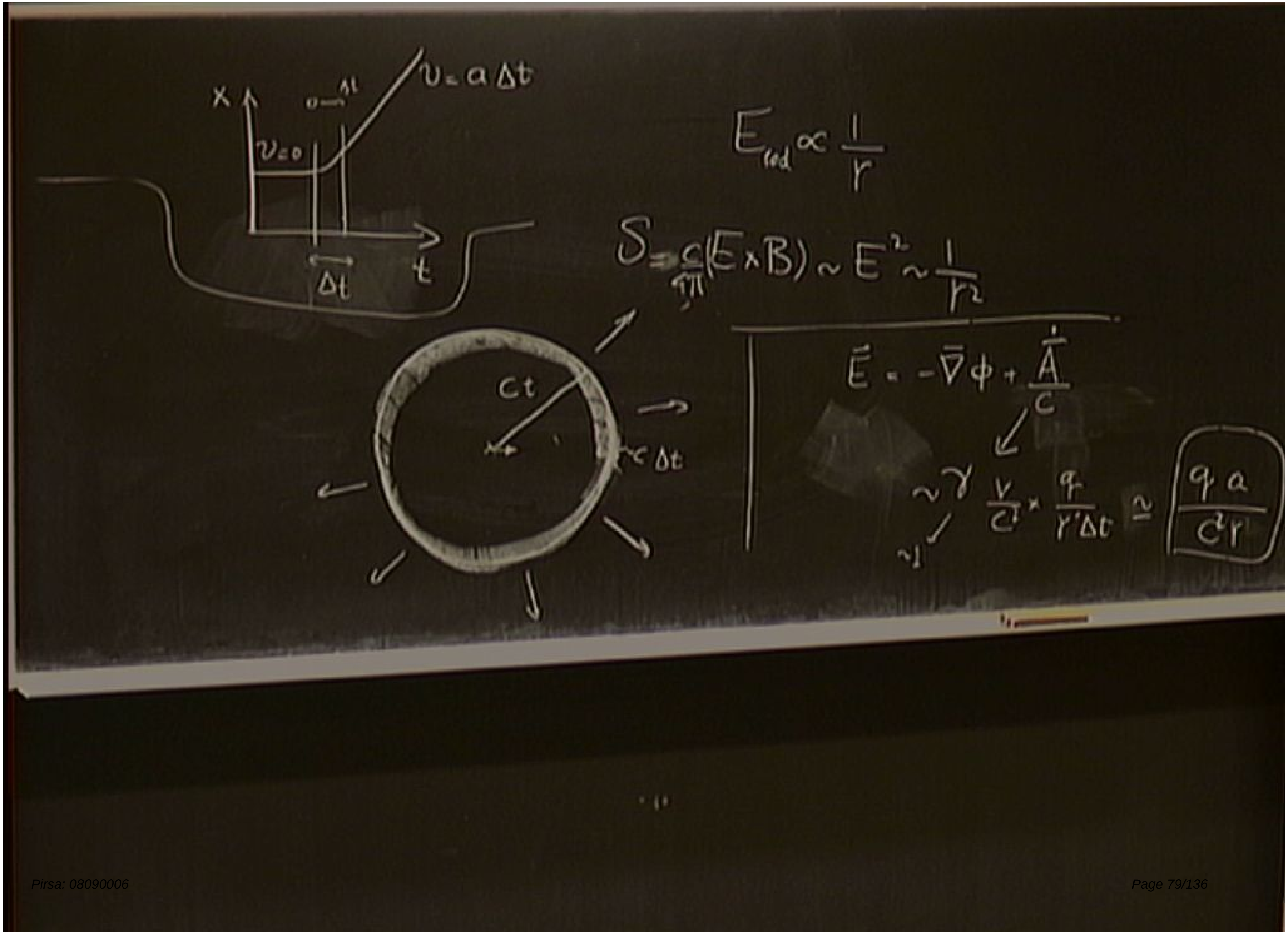
$\phi = \frac{\gamma q}{r'}$

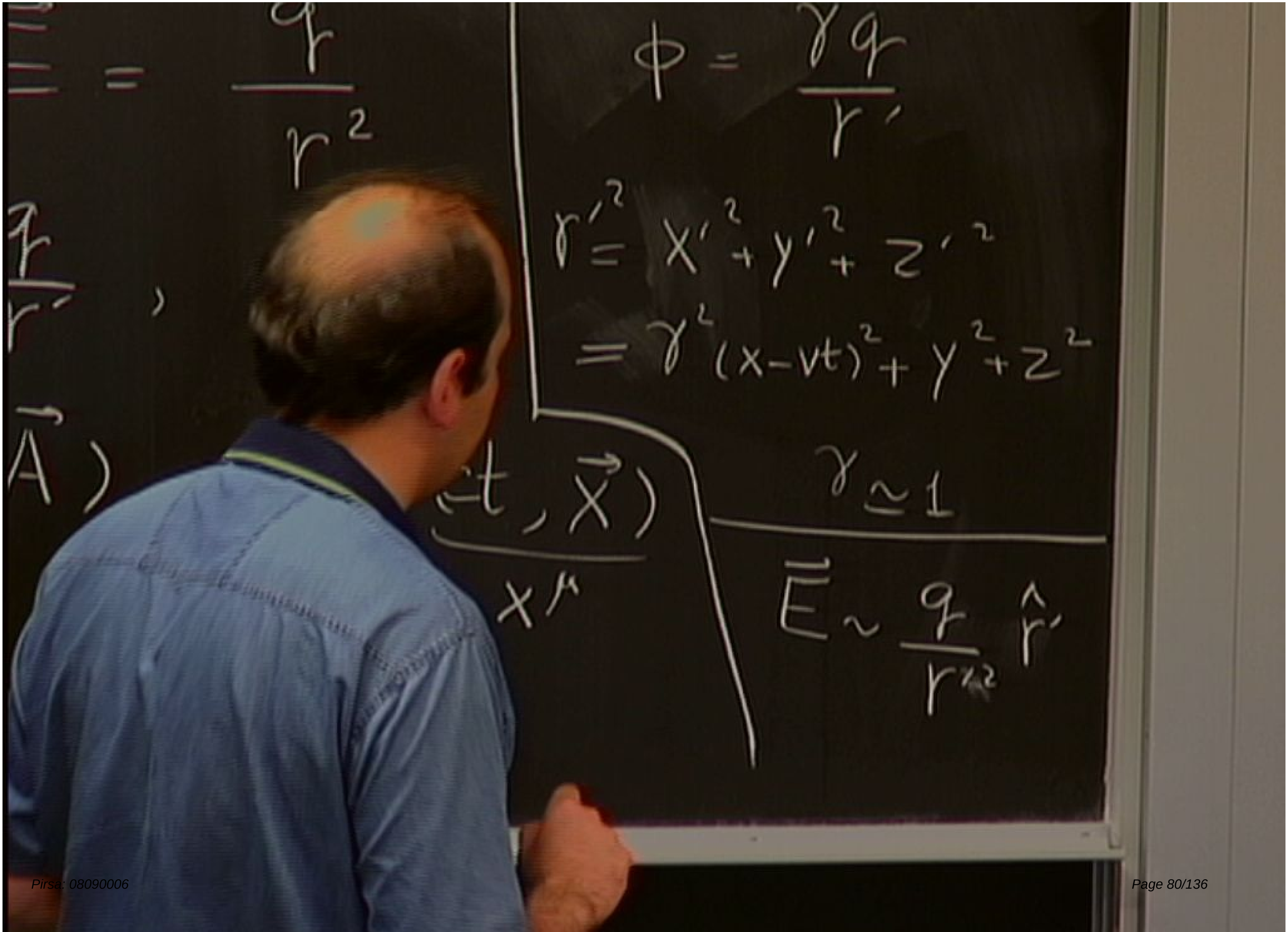
$r'^2 = x'^2 + y'^2 + z'^2$

$= \gamma^2 (x - vt)^2 + y^2 + z^2$

$\gamma \simeq 1$


v = a Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
~ γ v/c² × q/r'Δt ≃ qa/c²r


x
v = a Δt
v = 0
t
Δt
$E_{rad} \propto \frac{1}{r}$
$S = \frac{c}{4\pi}(E \times B) \sim E^2 \sim \frac{1}{r^2}$
ct
c Δt
$\vec{E} = -\vec{\nabla}\phi + \frac{\dot{A}}{c}$
$\sim \gamma \frac{v}{c^2} \times \frac{q}{r'\Delta t} \simeq \frac{q a}{c^2 r}$


E = q / r^2
φ = γq / r'
r'^2 = x'^2 + y'^2 + z'^2
= γ^2 (x-vt)^2 + y^2 + z^2
γ ≃ 1
E ~ q / r'^2 r̂'

$$E = \frac{q}{r^2}$$

$$\frac{q}{r'} , \quad \vec{A} = 0$$

$$\vec{A})$$

$$\phi = \frac{\gamma q}{r'}$$

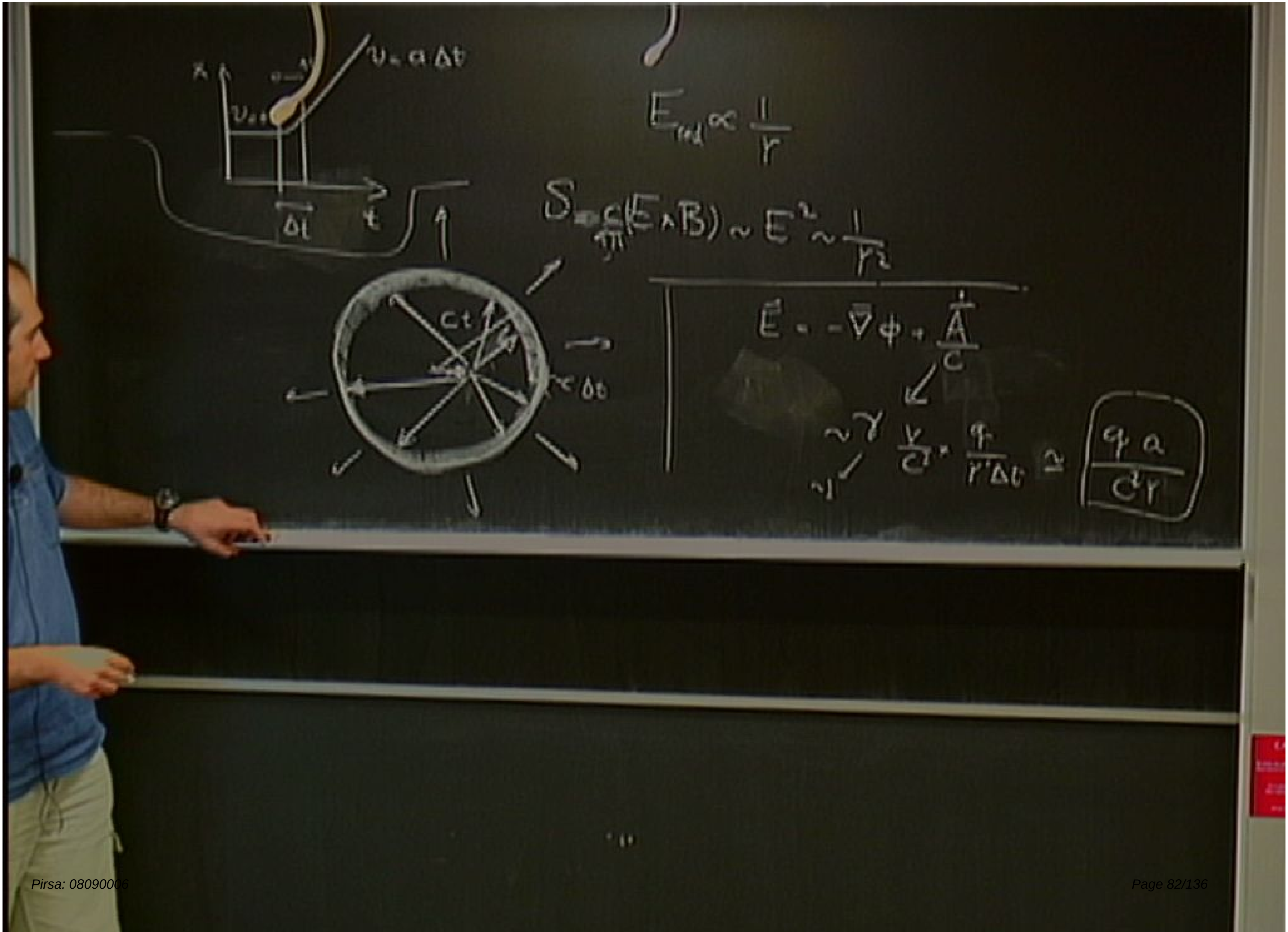
$$r'^2 = x'^2 + y'^2 + z'^2$$

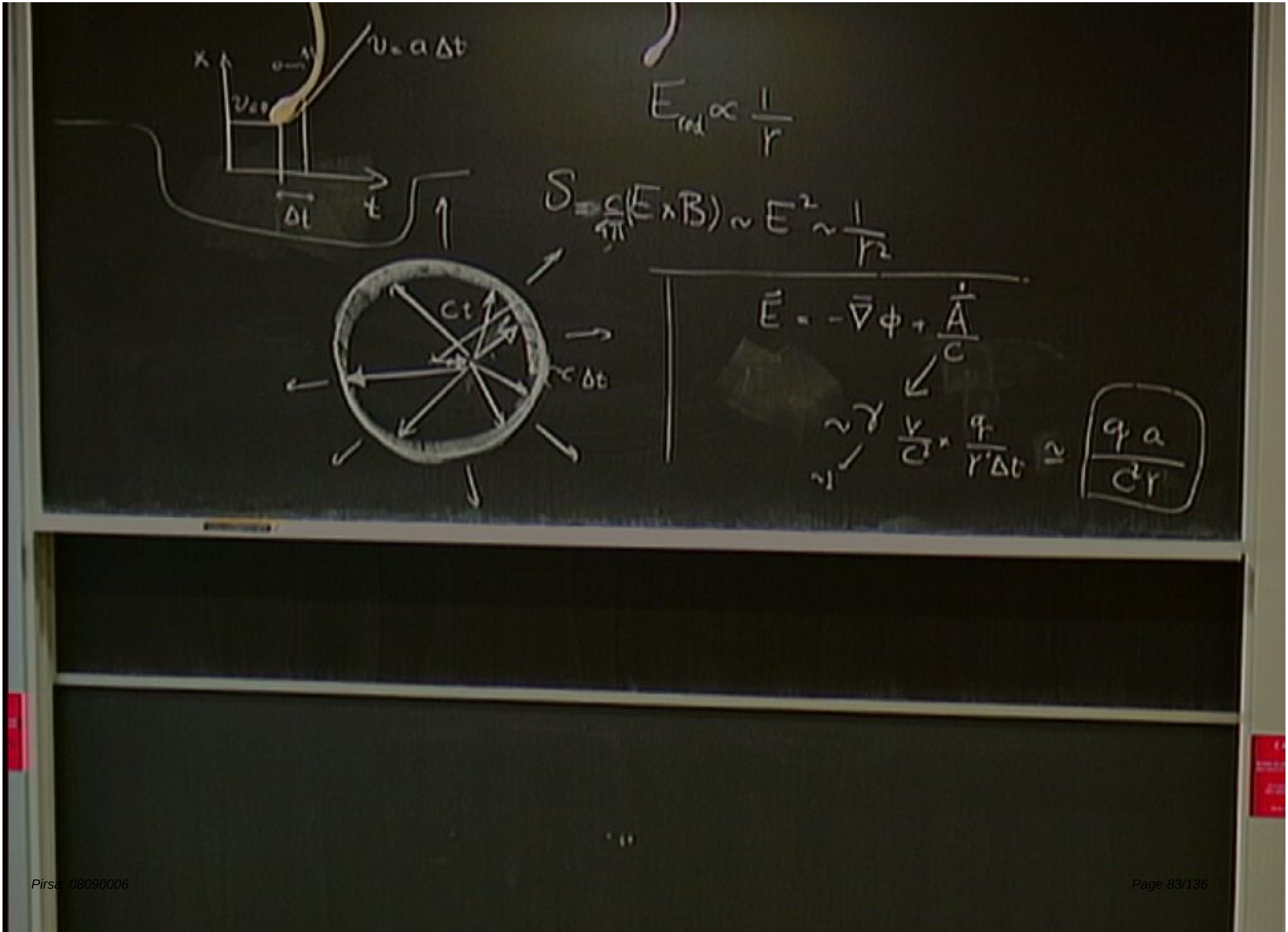
$$= \gamma^2 (x - vt)^2 + y^2 + z^2$$

$$\underbrace{(ct, \vec{x})}_{x^\mu}$$

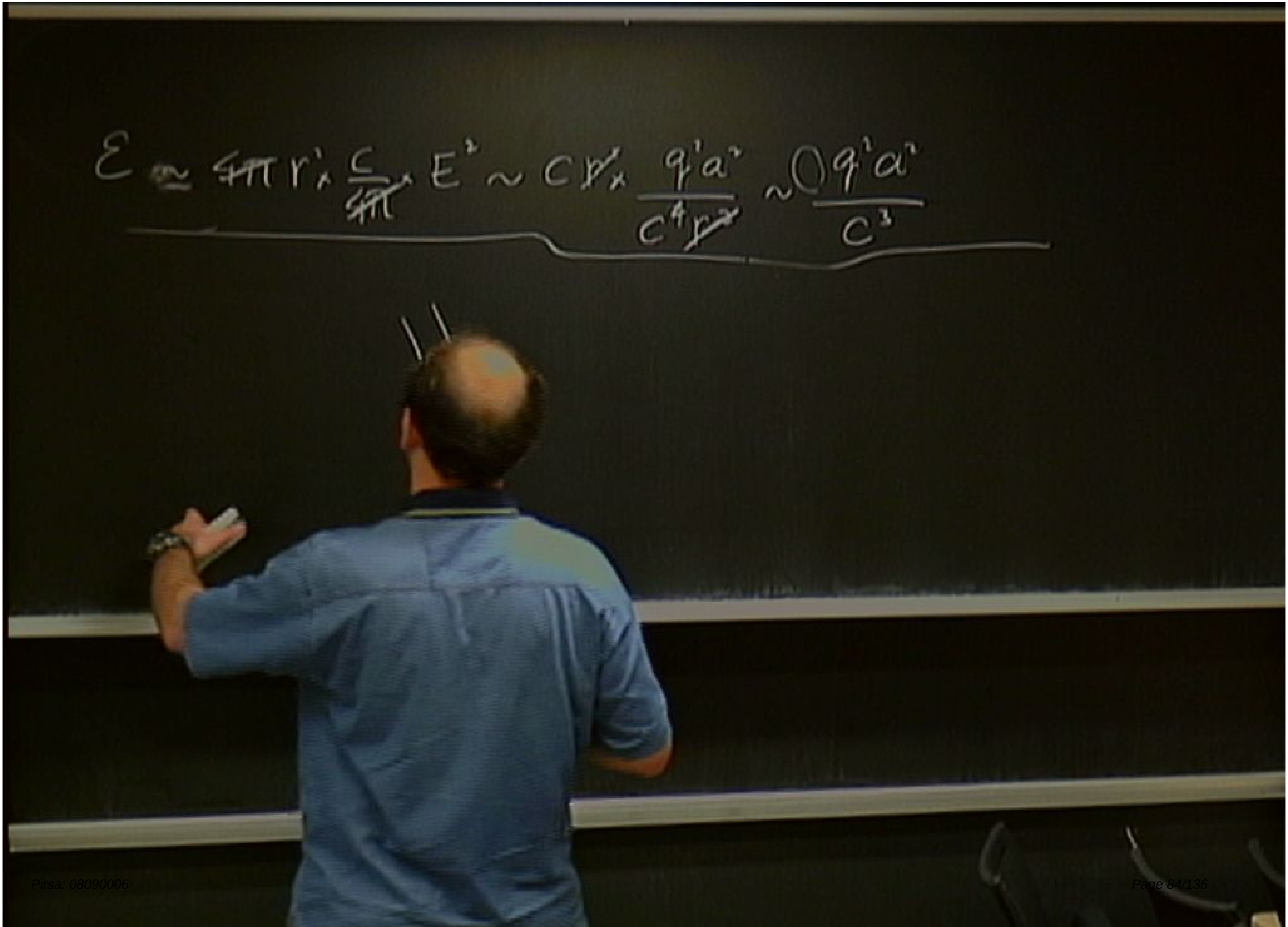
$$\gamma \simeq 1$$

$$E \sim \frac{q}{r'^2} \hat{r}'$$

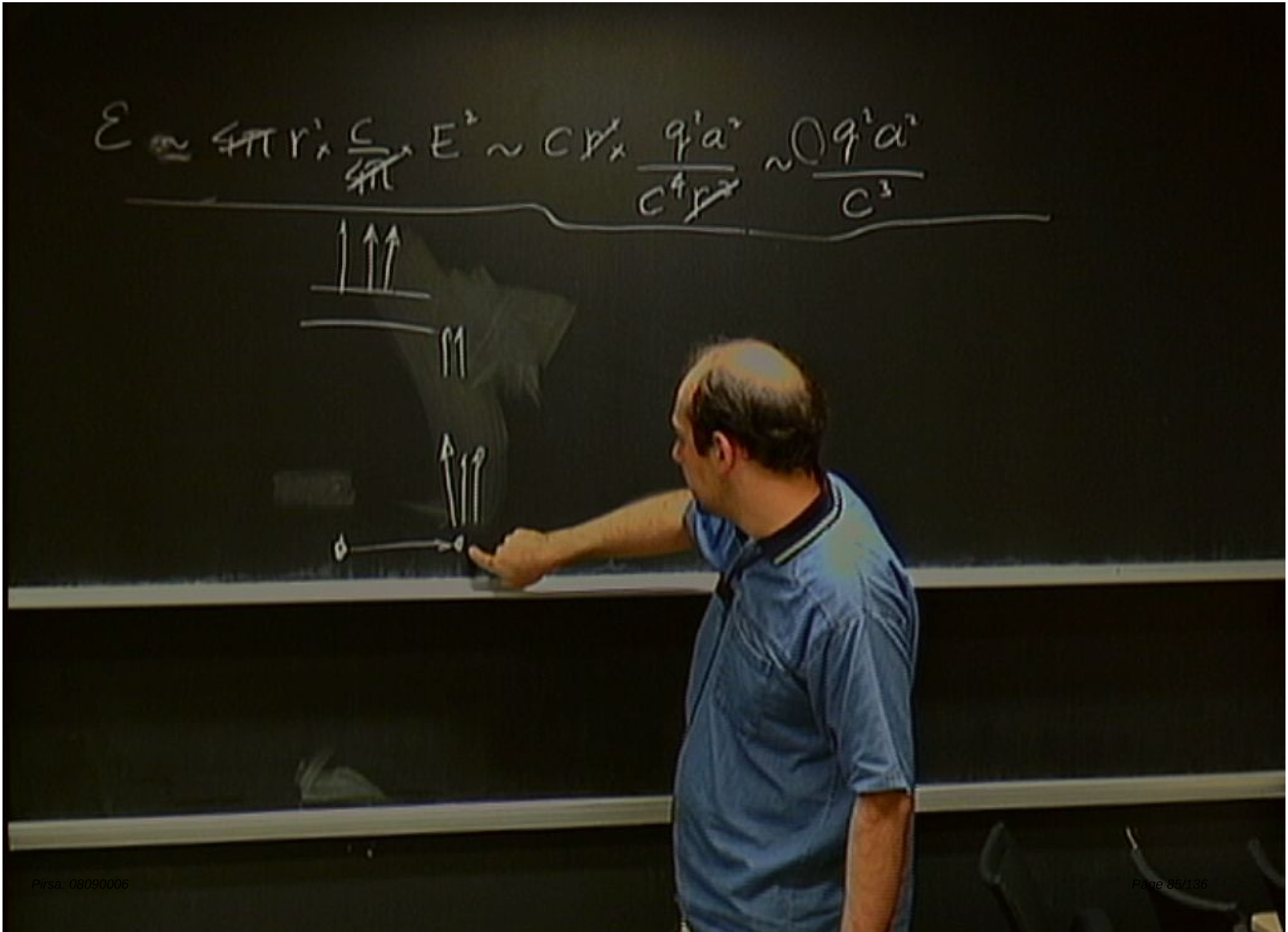

v = a Δt
Δt
t
ct
c Δt

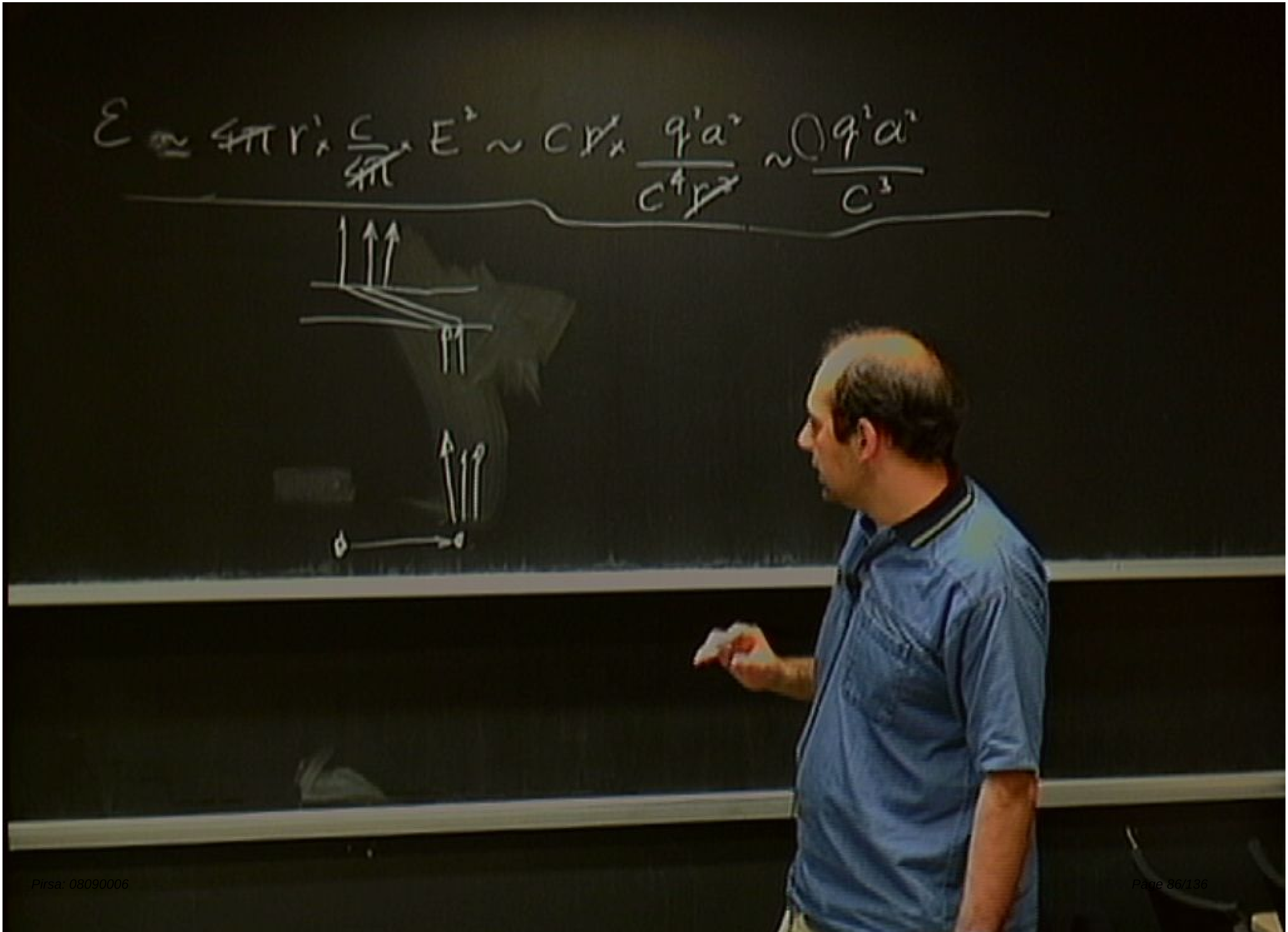


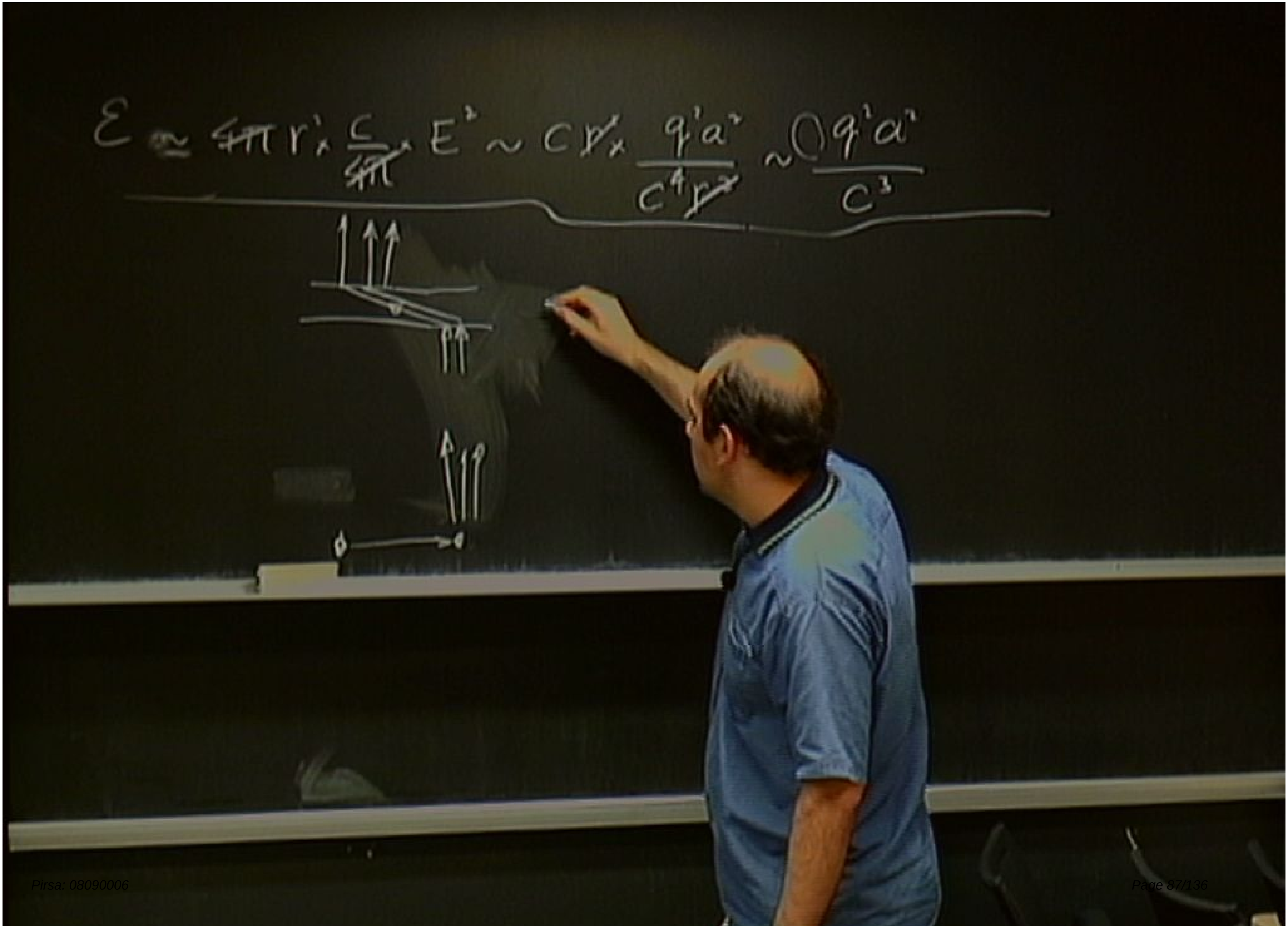
v = aΔt
x
t
Δt
ct
cΔt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
~ γ v/c² × q/r'Δt ≃ qa/c²r

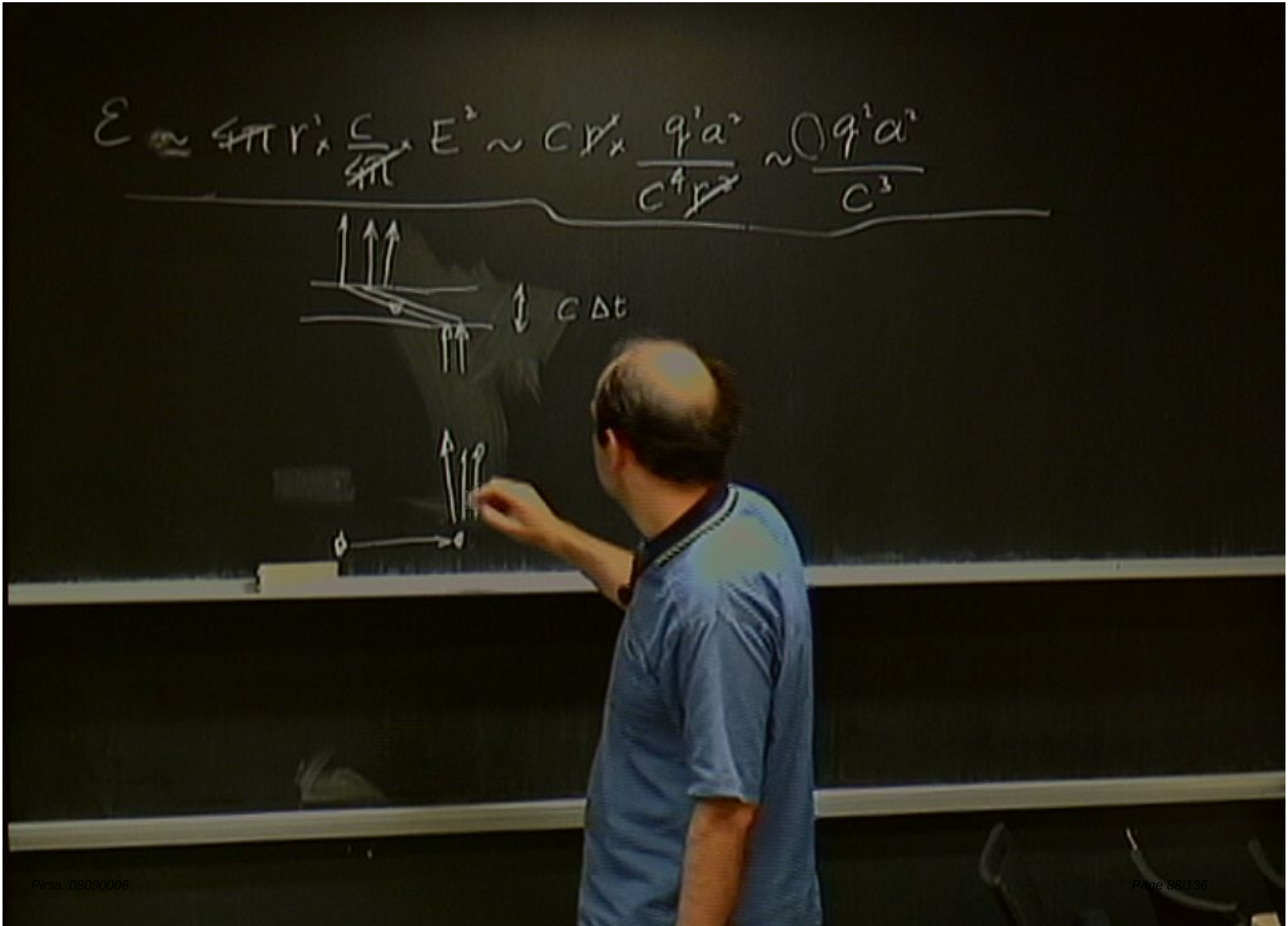


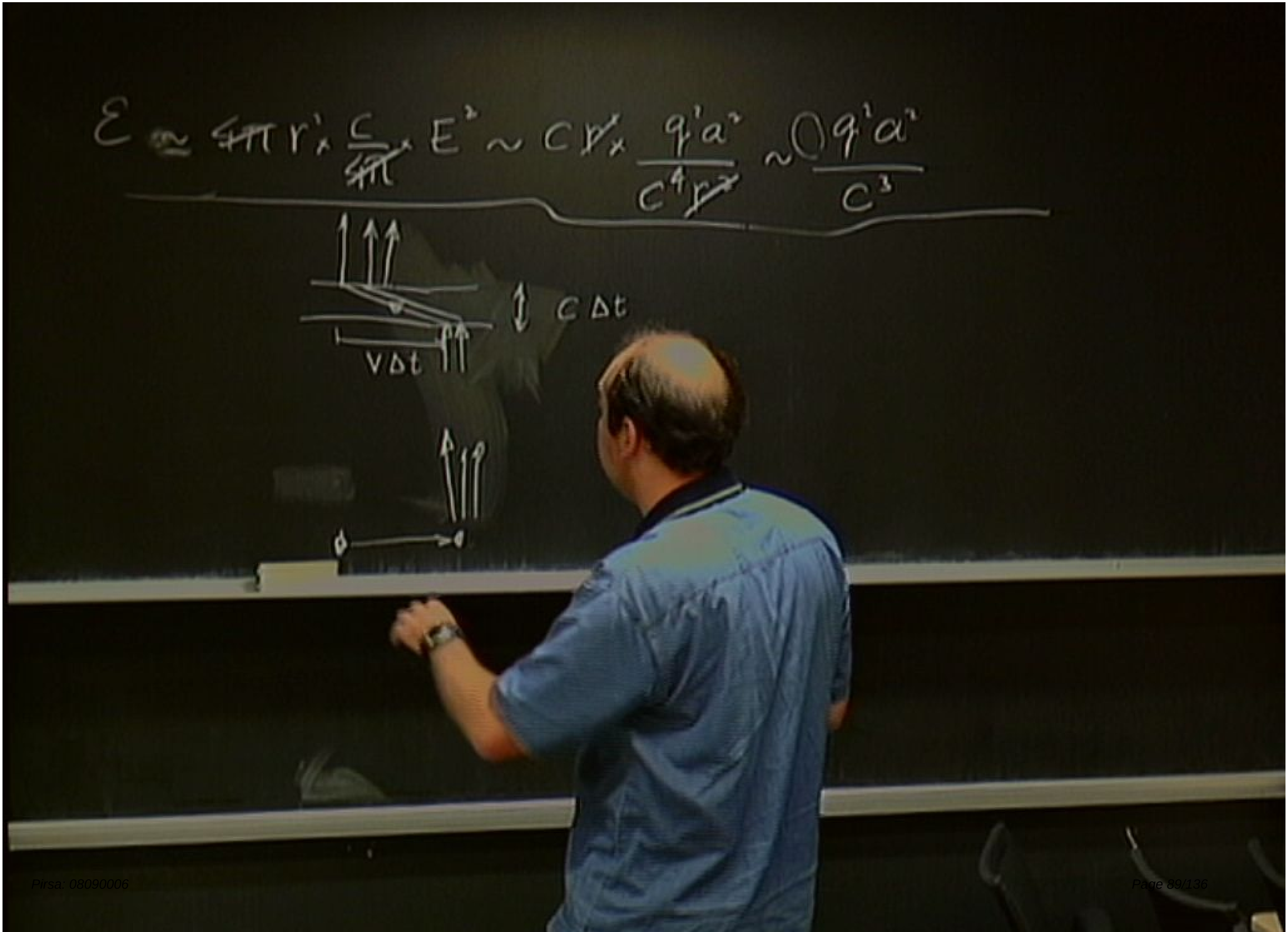
ℰ ≃ 4πr² × c/4π × E² ~ c r² × q²a²/(c⁴r²) ~ q²a²/c³

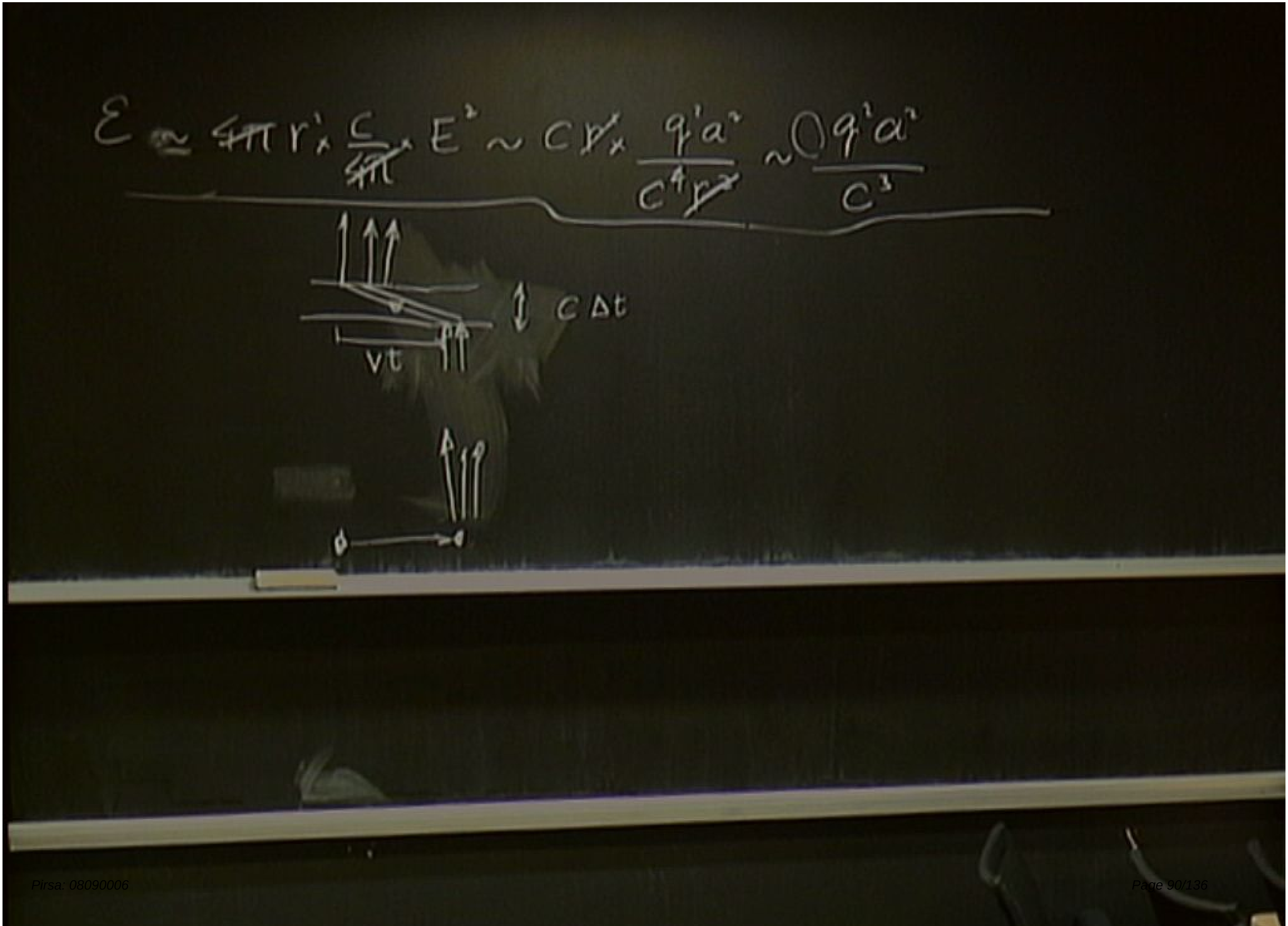


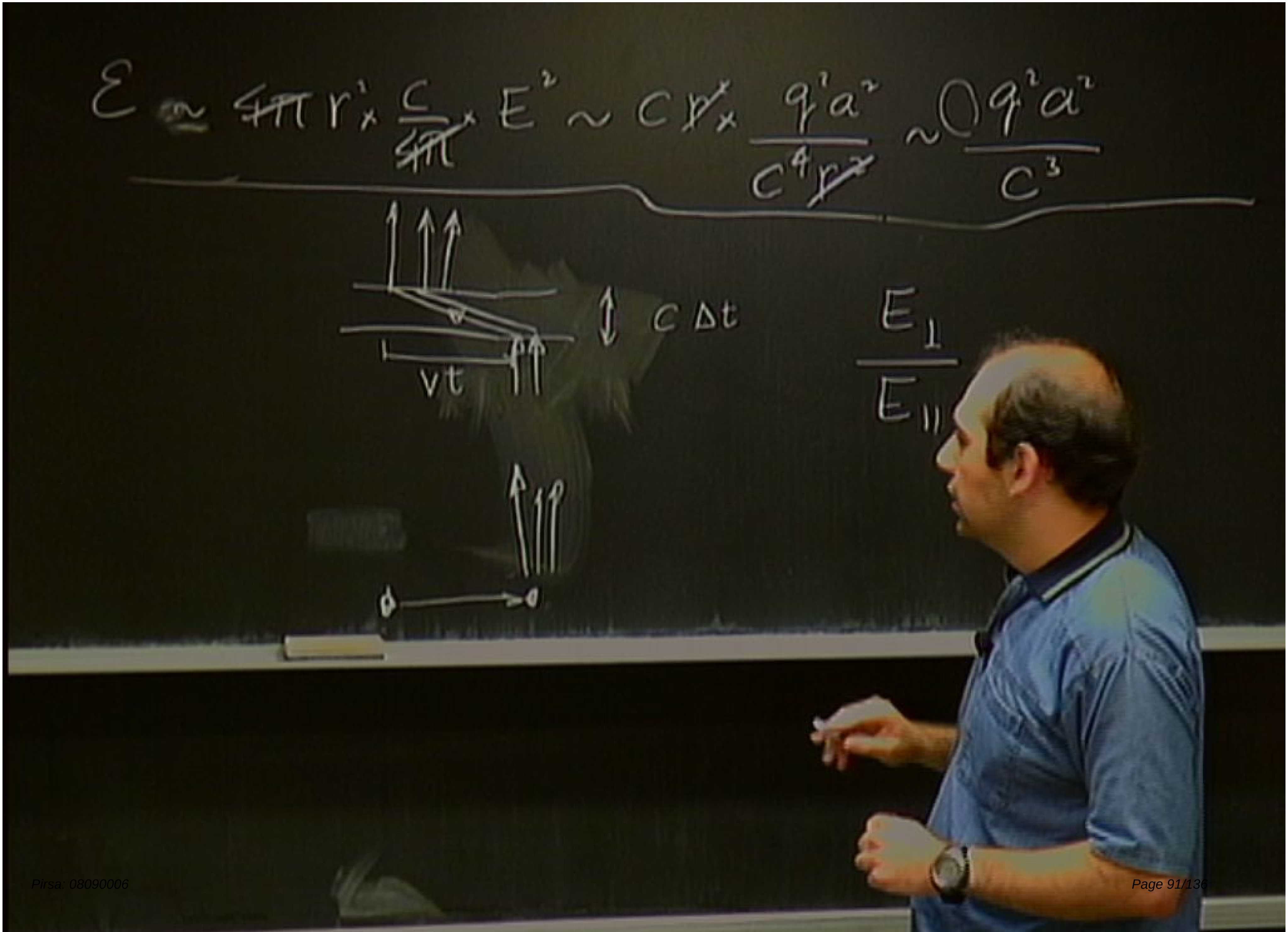


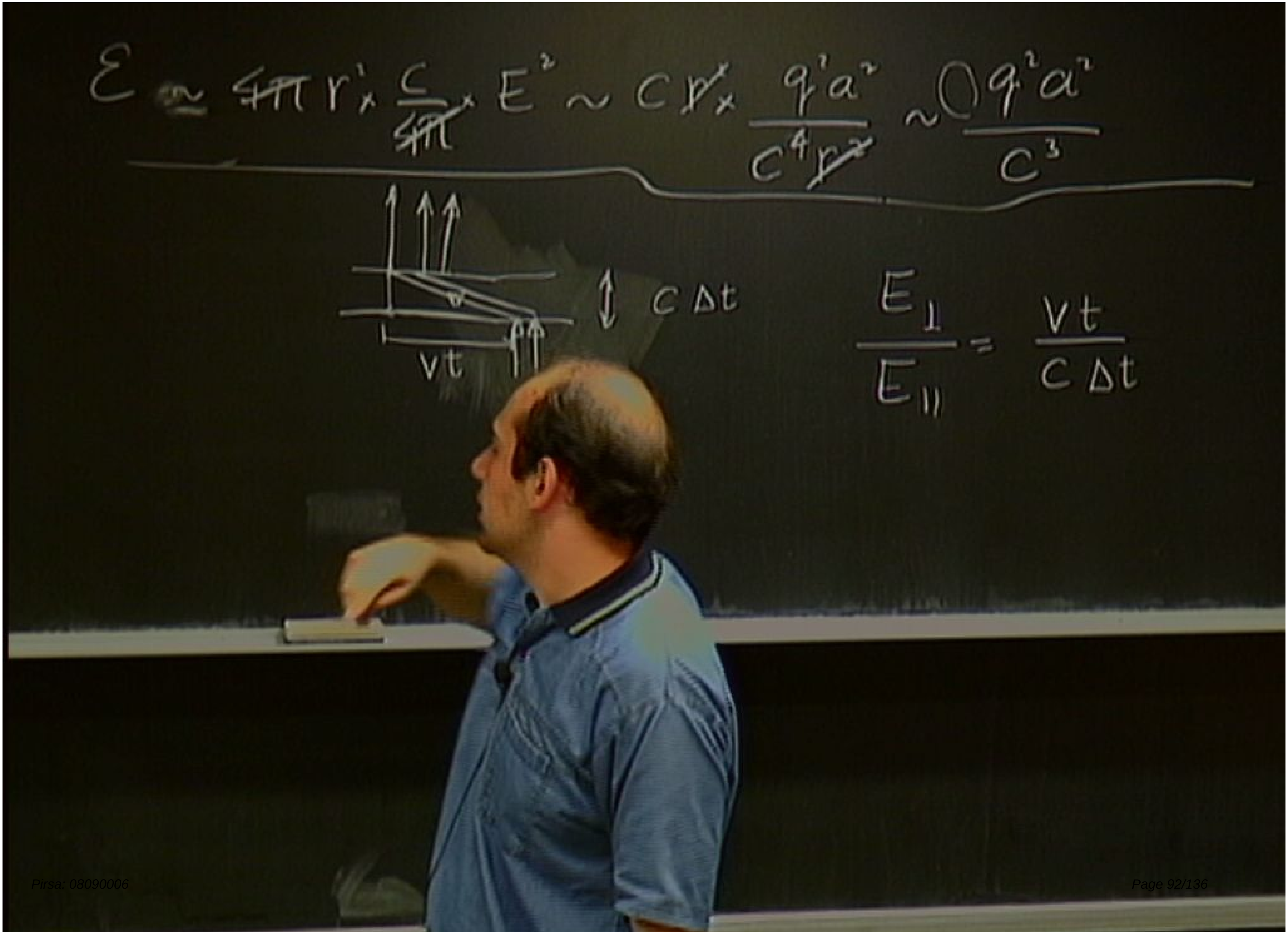

ℰ ≃ 4π r² × c/4π × E² ~ c r² × q²a²/(c⁴r²) ~ q²a²/c³

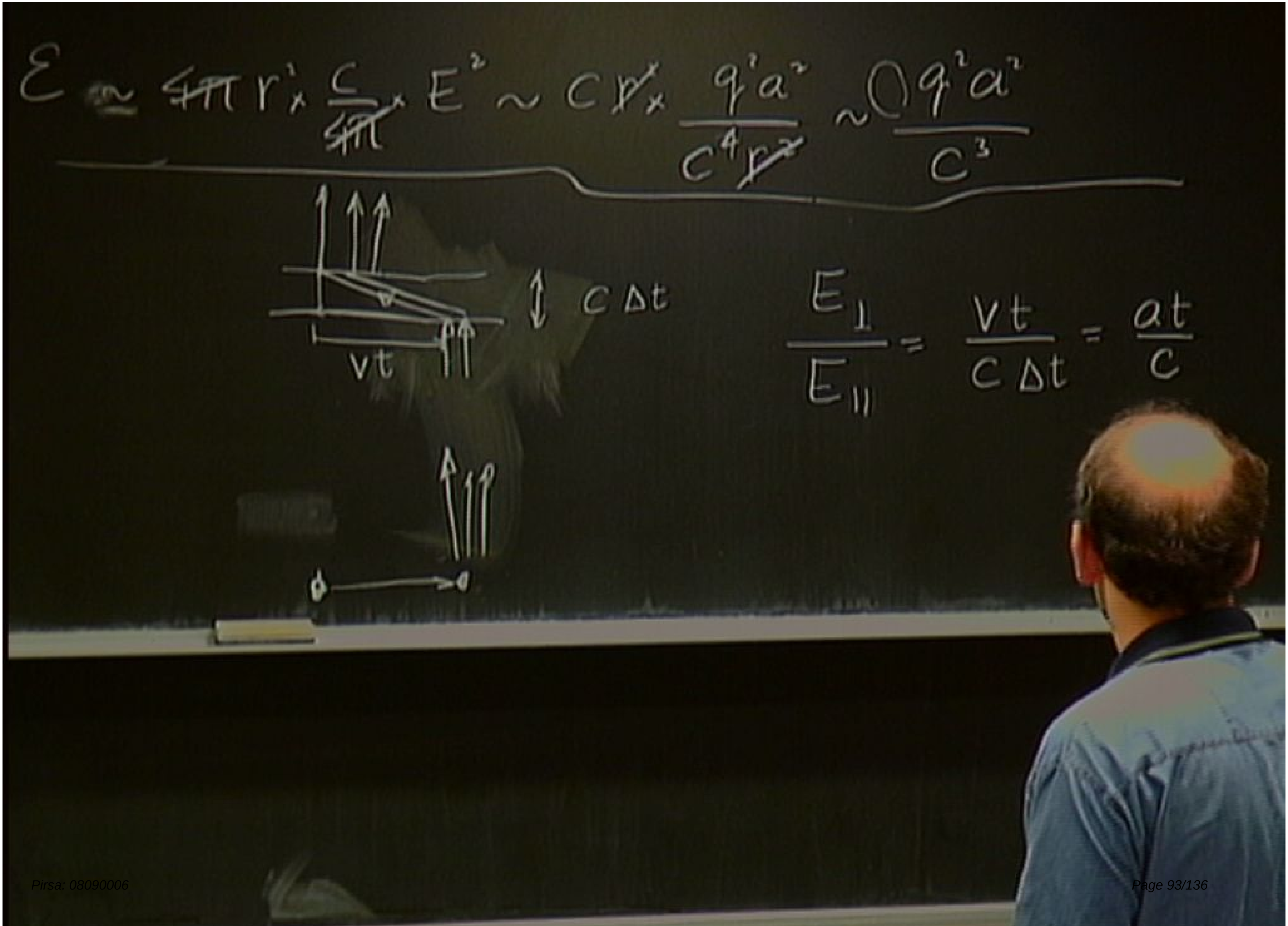

c Δt

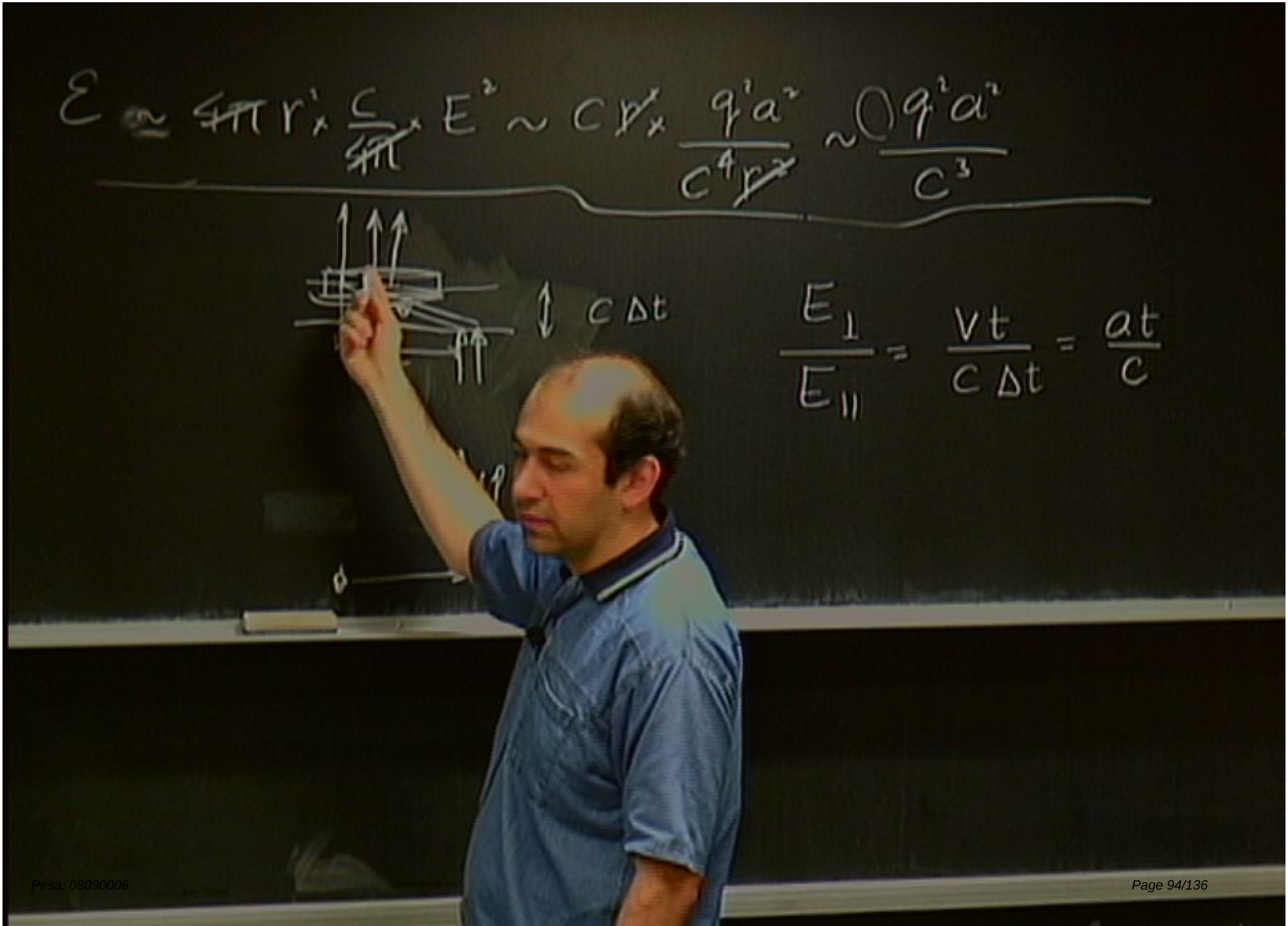

c Δt
v Δt

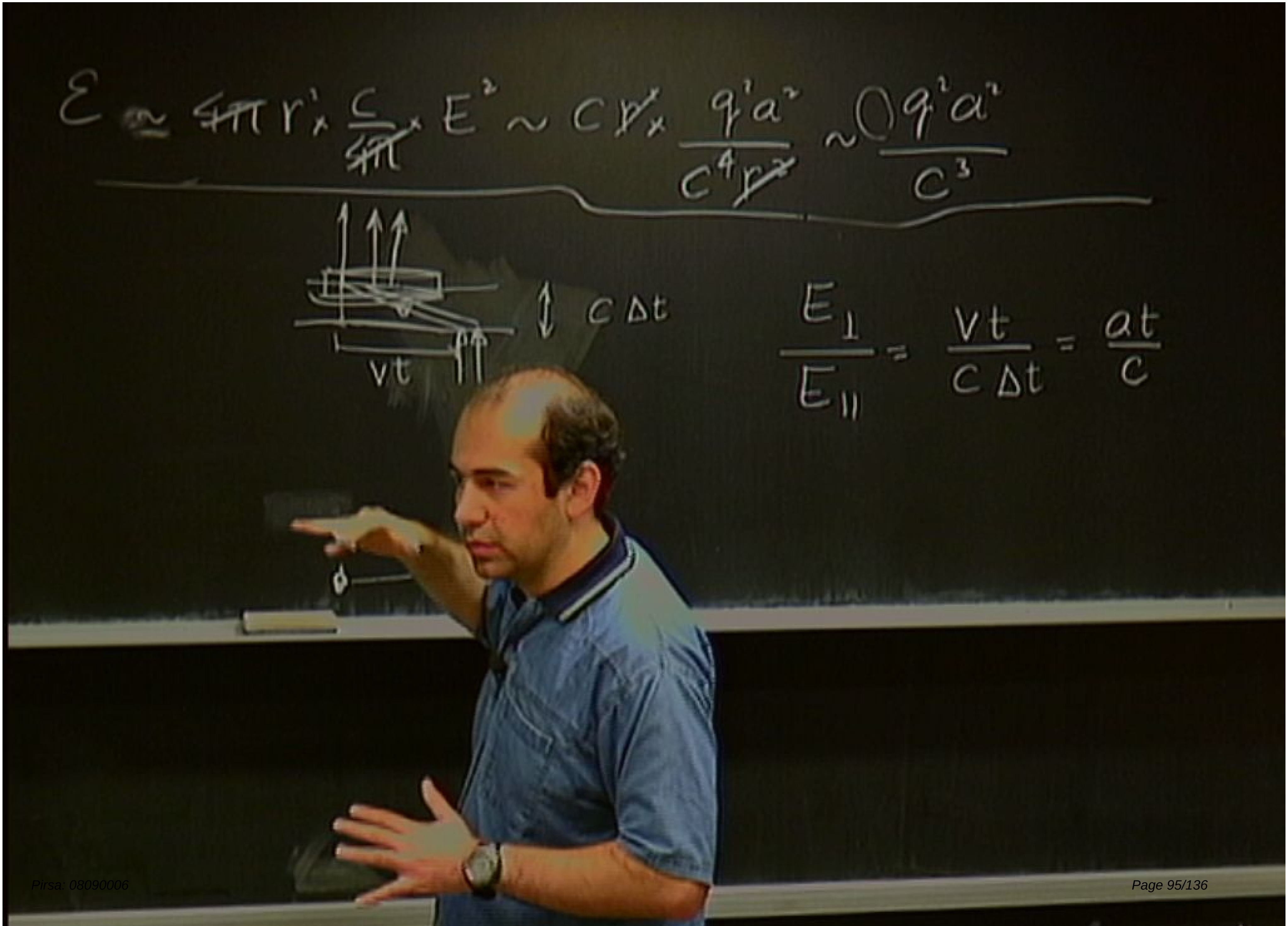

$\mathcal{E} \sim 4\pi r^2 \times \frac{c}{4\pi} \times E^2 \sim c r^2 \times \frac{q^2 a^2}{c^4 r^2} \sim \frac{q^2 a^2}{c^3}$
$c\Delta t$
vt

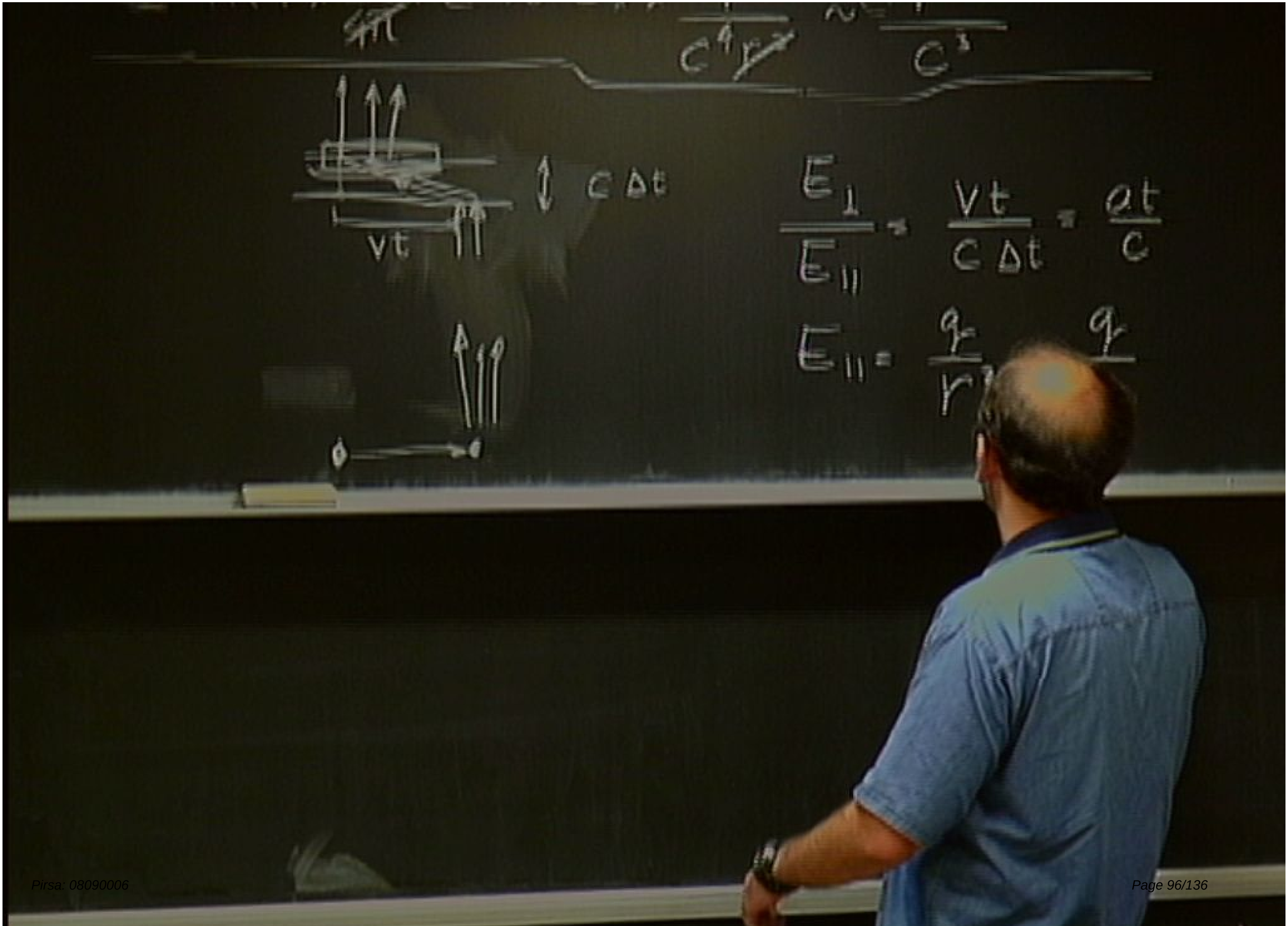

c Δt
vt
E⊥
E∥

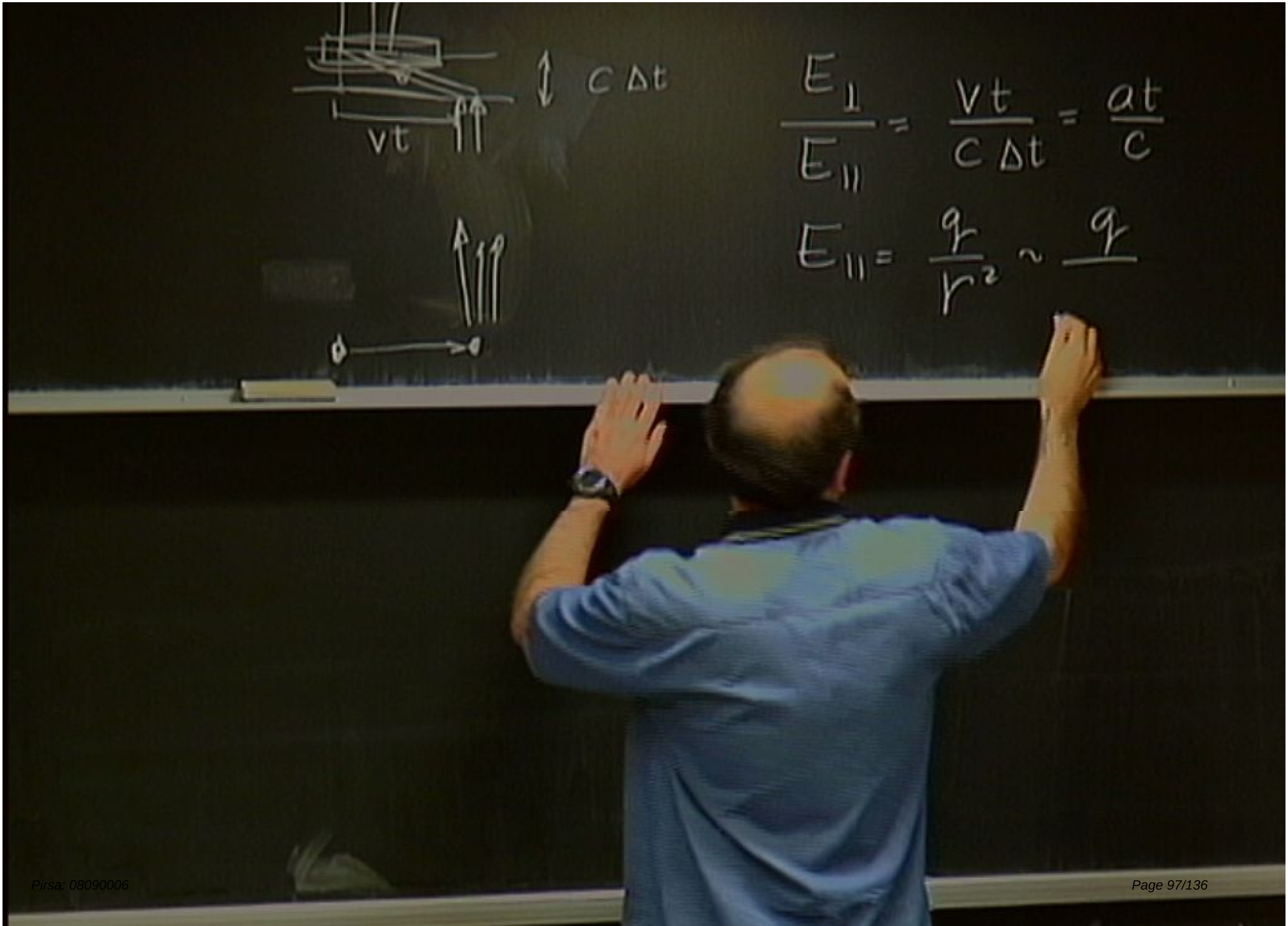

c Δt
vt
E⊥ / E∥ = vt / c Δt

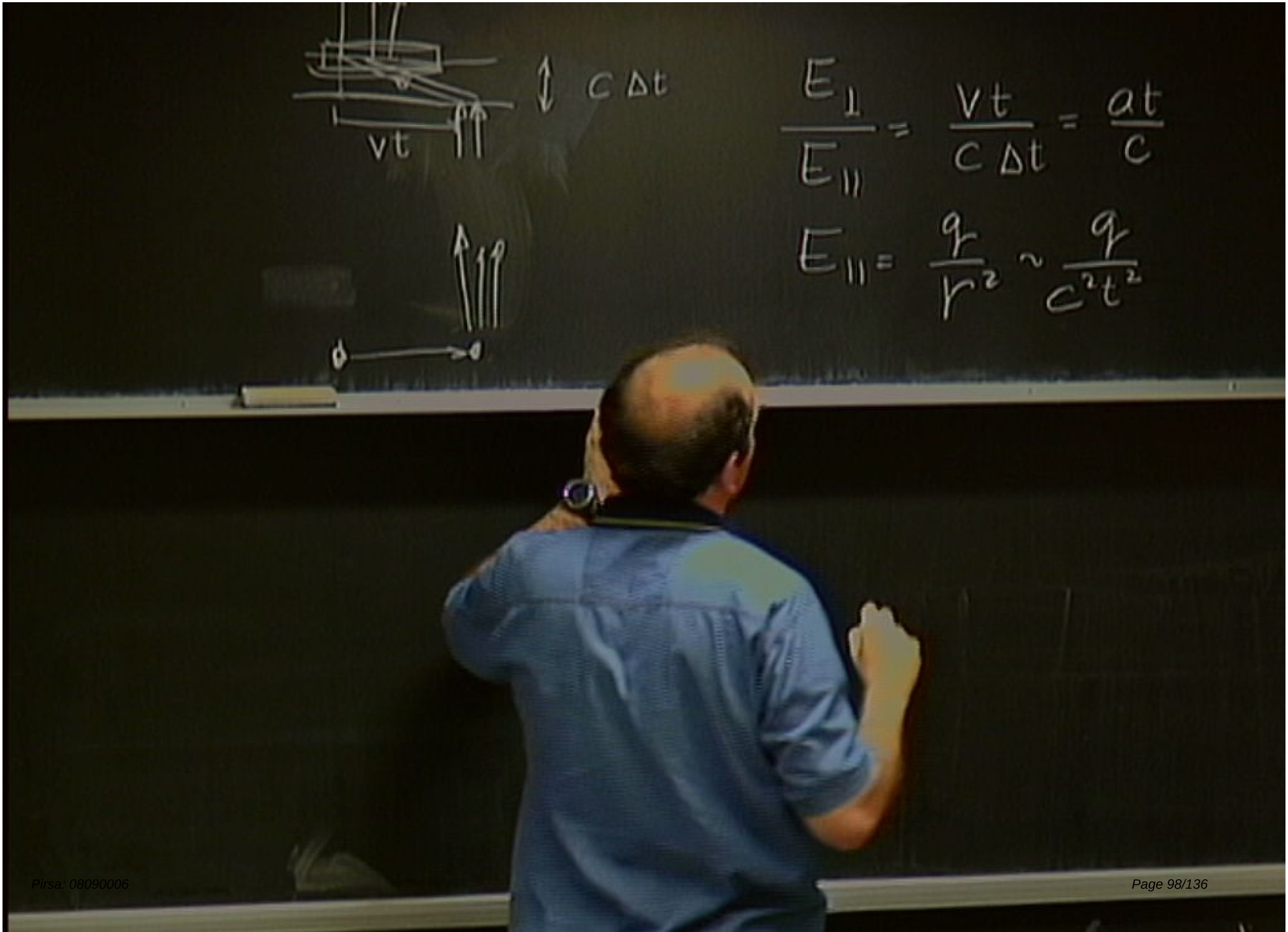

$\mathcal{E} \sim 4\pi r^2 \times \frac{c}{4\pi} \times E^2 \sim c\, r^2 \times \frac{q^2 a^2}{c^4 r^2} \sim \frac{q^2 a^2}{c^3}$
$c\Delta t$
vt
$\frac{E_\perp}{E_\parallel} = \frac{vt}{c\Delta t} = \frac{at}{c}$

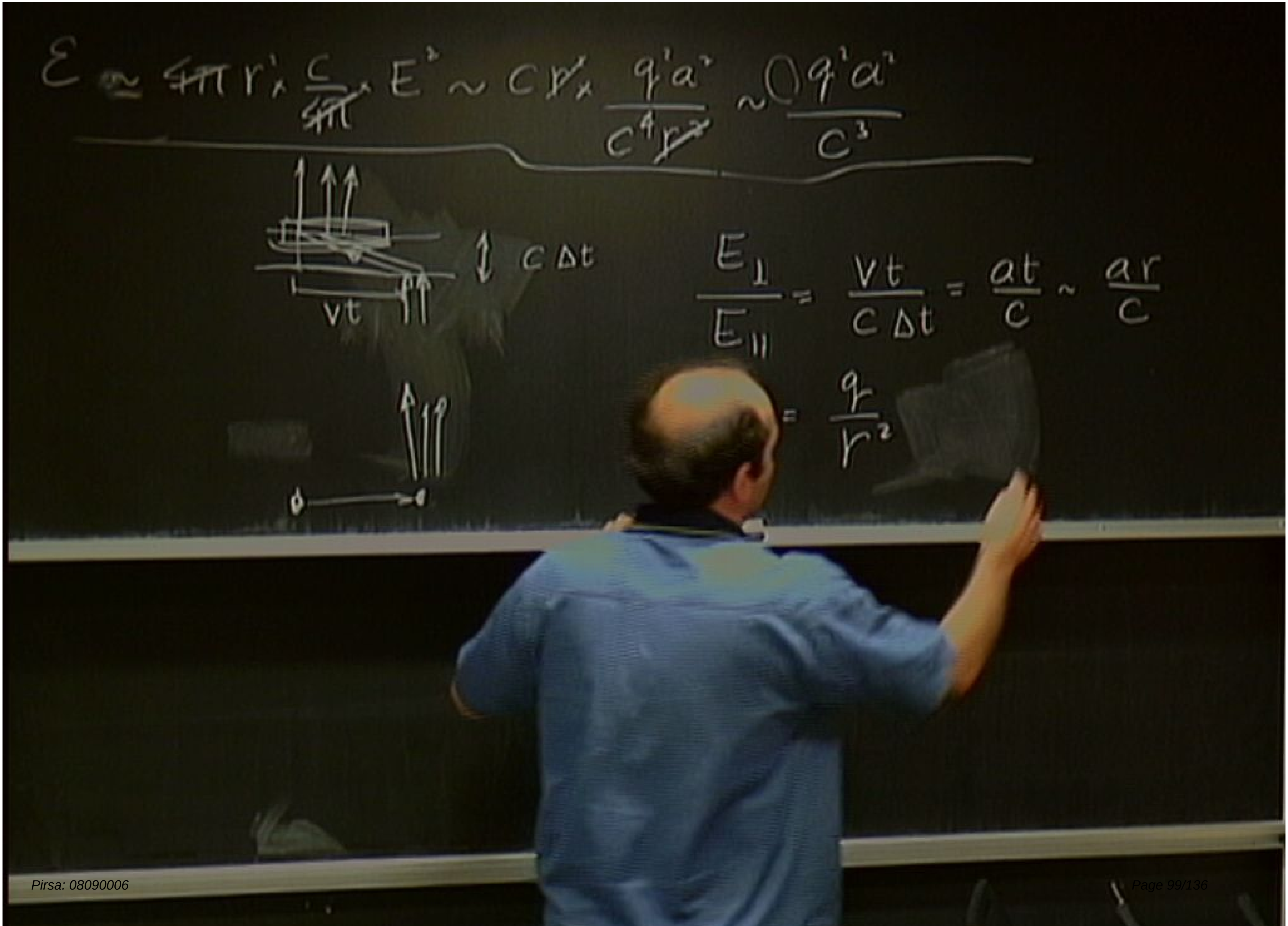

c Δt
E⊥ / E∥ = vt / (c Δt) = at / c

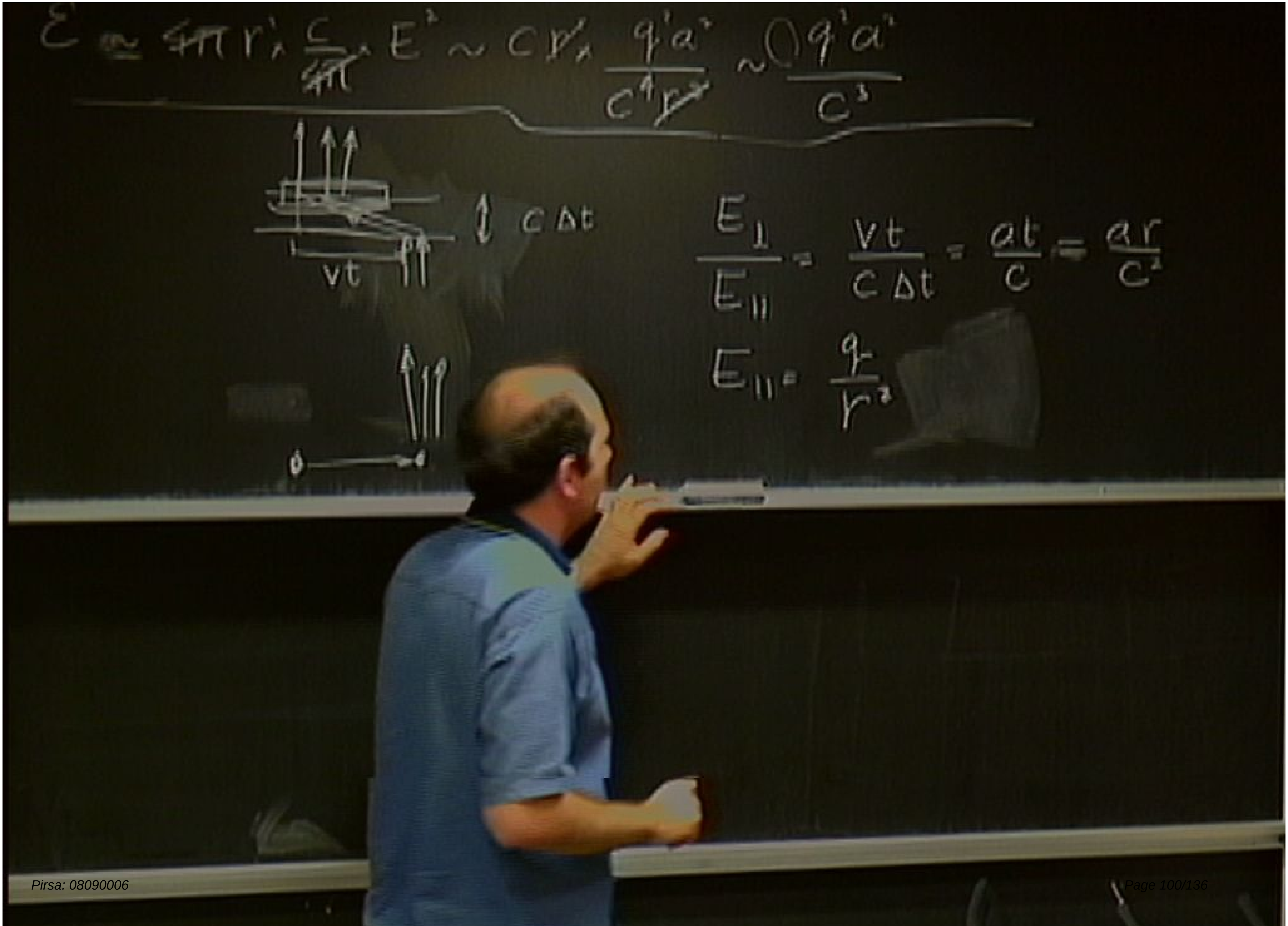

c Δt
vt
E⊥ / E∥ = vt / c Δt = at / c

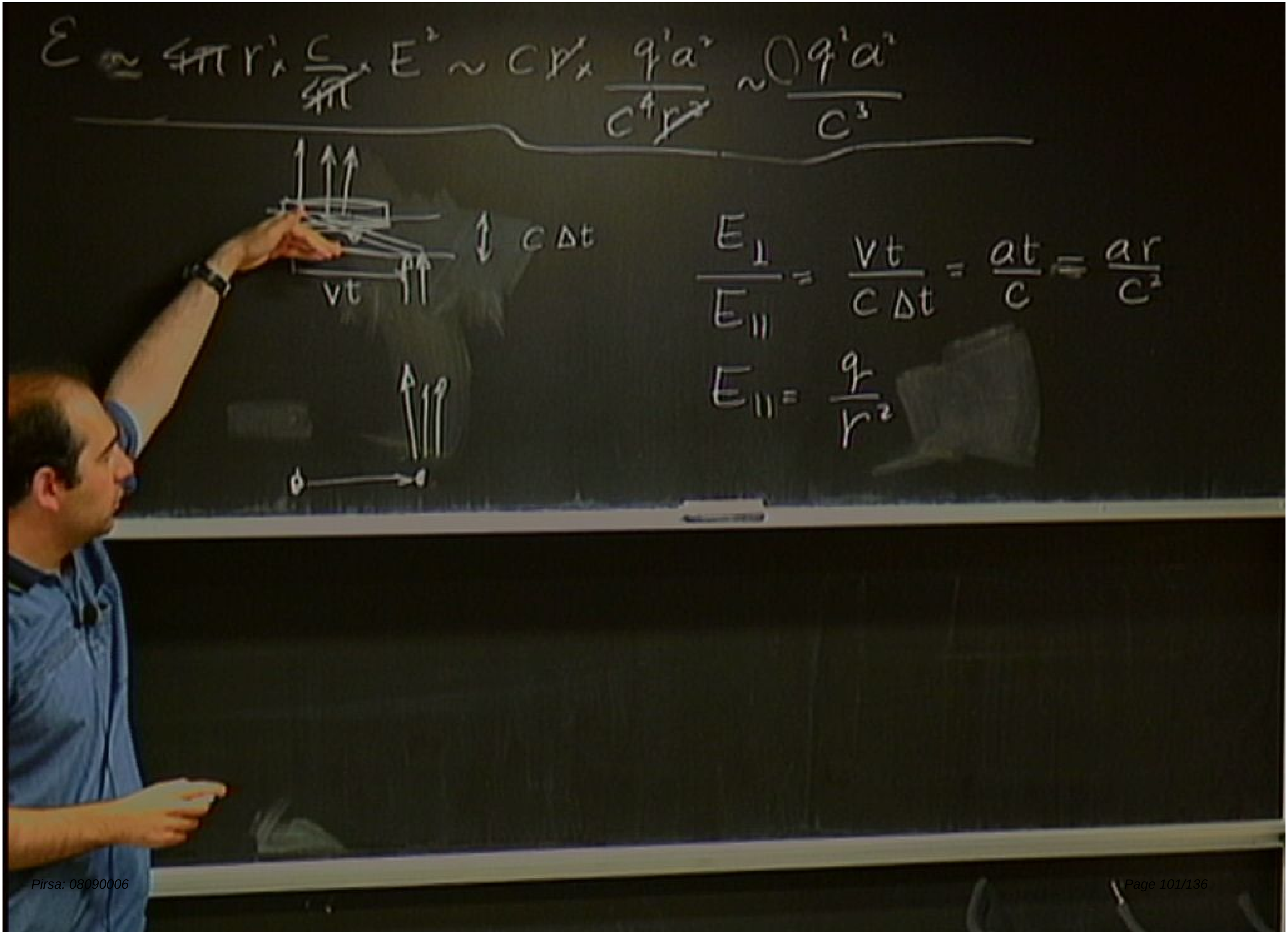

c Δt
vt
E⊥
E∥
vt
c Δt
at
c
E∥ =
q
q


cΔt
vt
$\frac{E_\perp}{E_\parallel} = \frac{vt}{c\Delta t} = \frac{at}{c}$
$E_\parallel = \frac{q}{r^2} \sim \frac{q}{}$

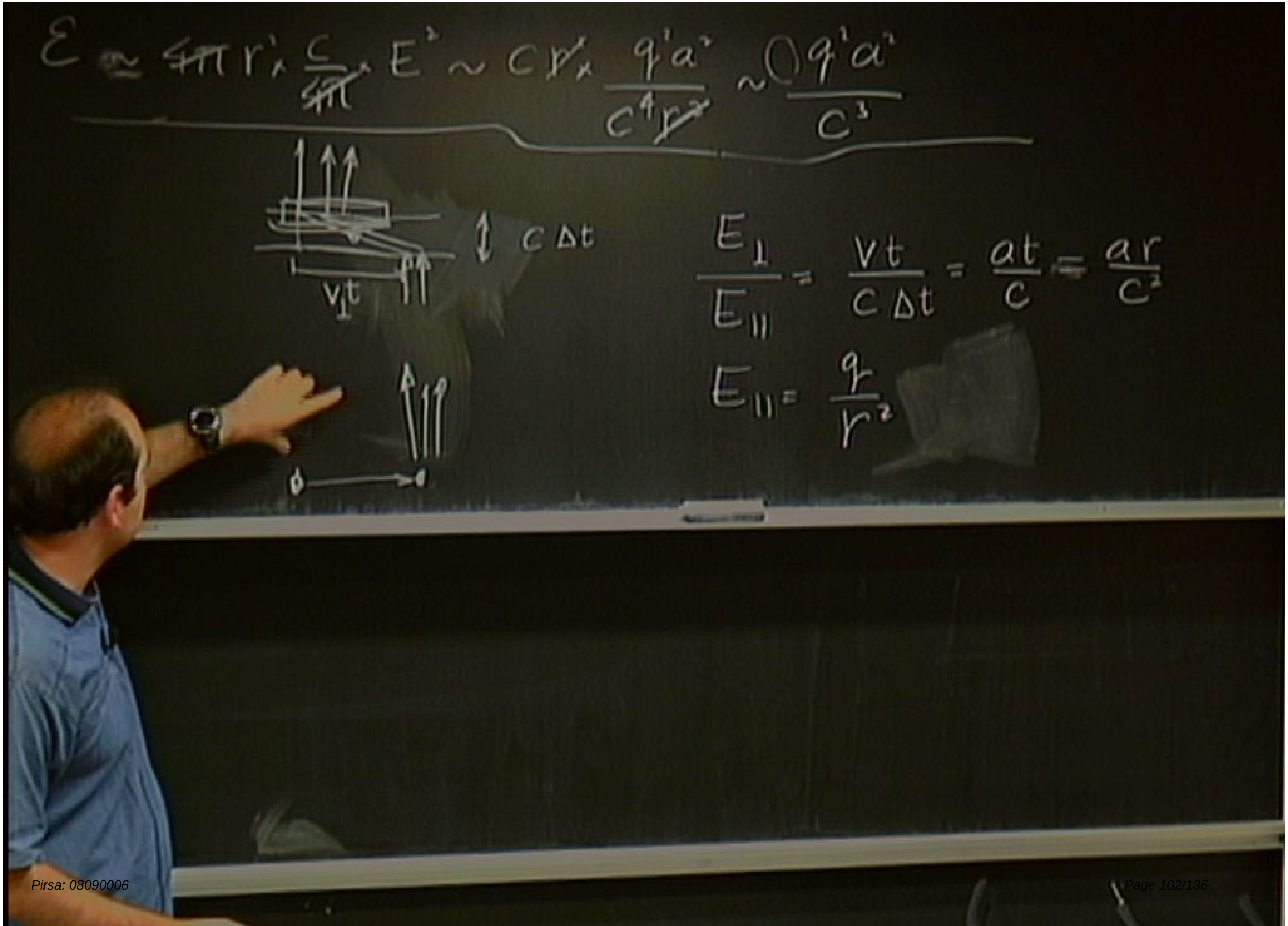

c Δt
vt
$\frac{E_\perp}{E_\parallel} = \frac{vt}{c\Delta t} = \frac{at}{c}$
$E_\parallel = \frac{q}{r^2} \sim \frac{q}{c^2t^2}$

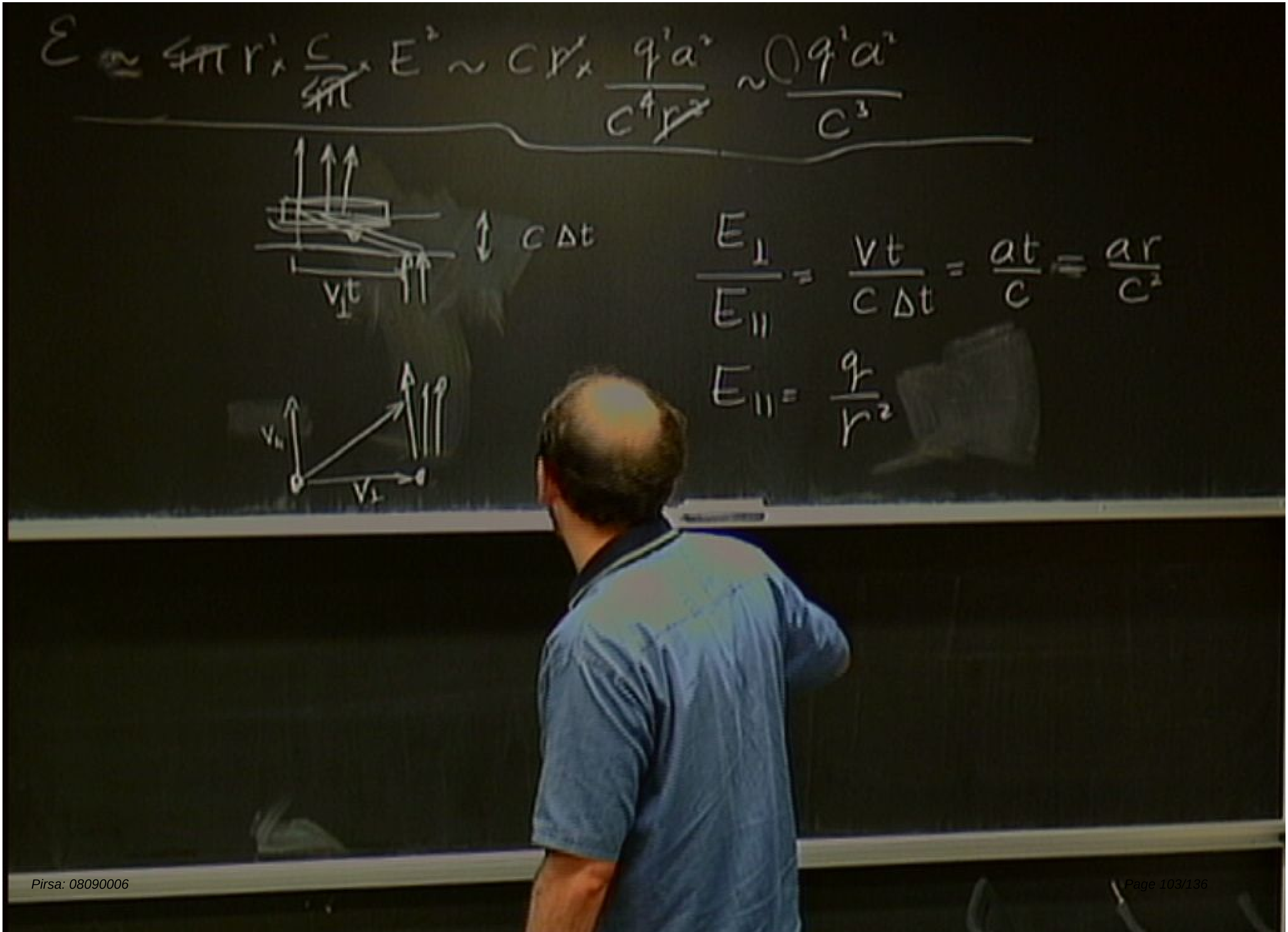

c Δt
vt

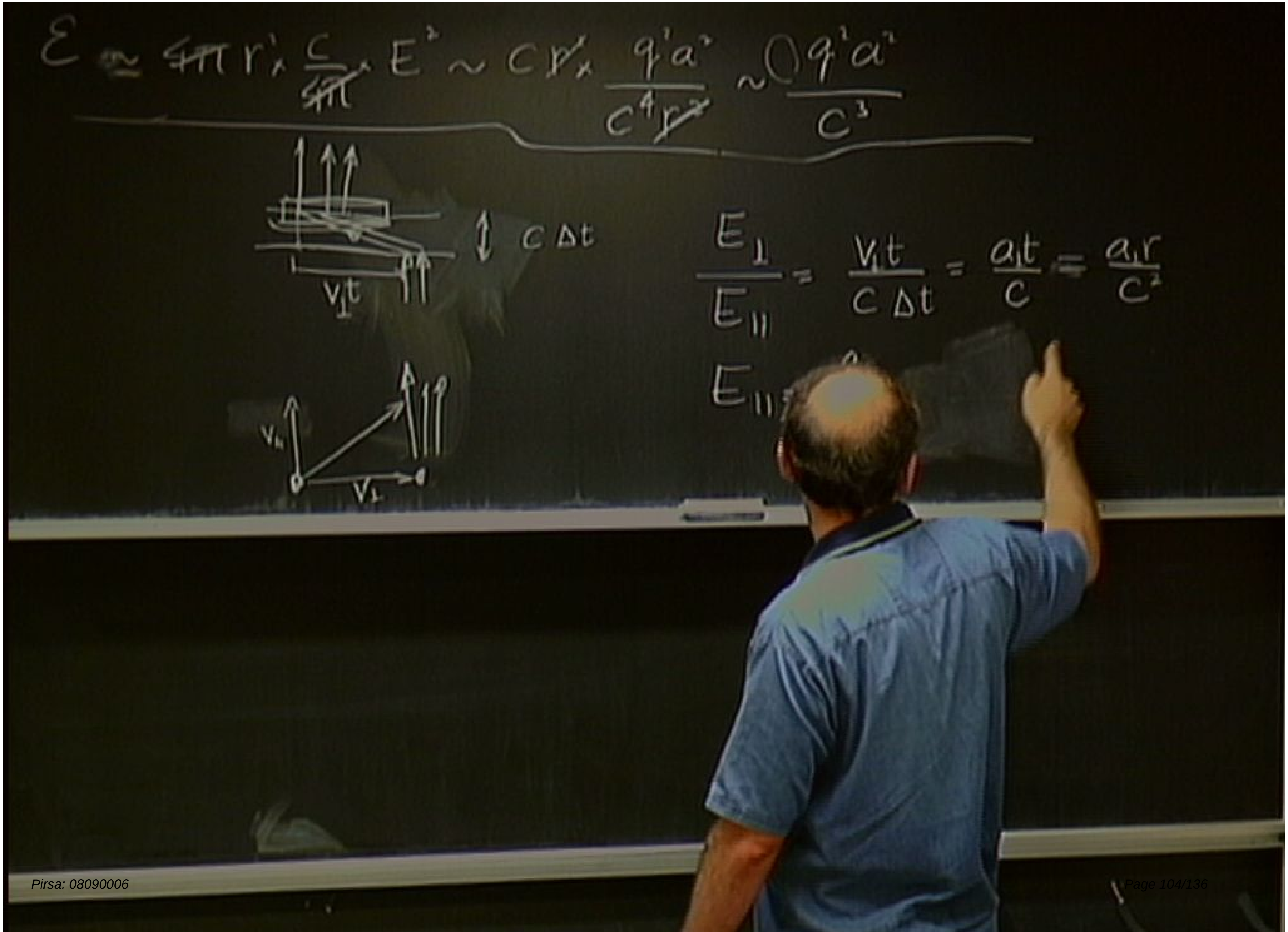

c Δt
vt
E⊥ / E∥ = vt / c Δt = at / c = ar / c²
E∥ = q / r²

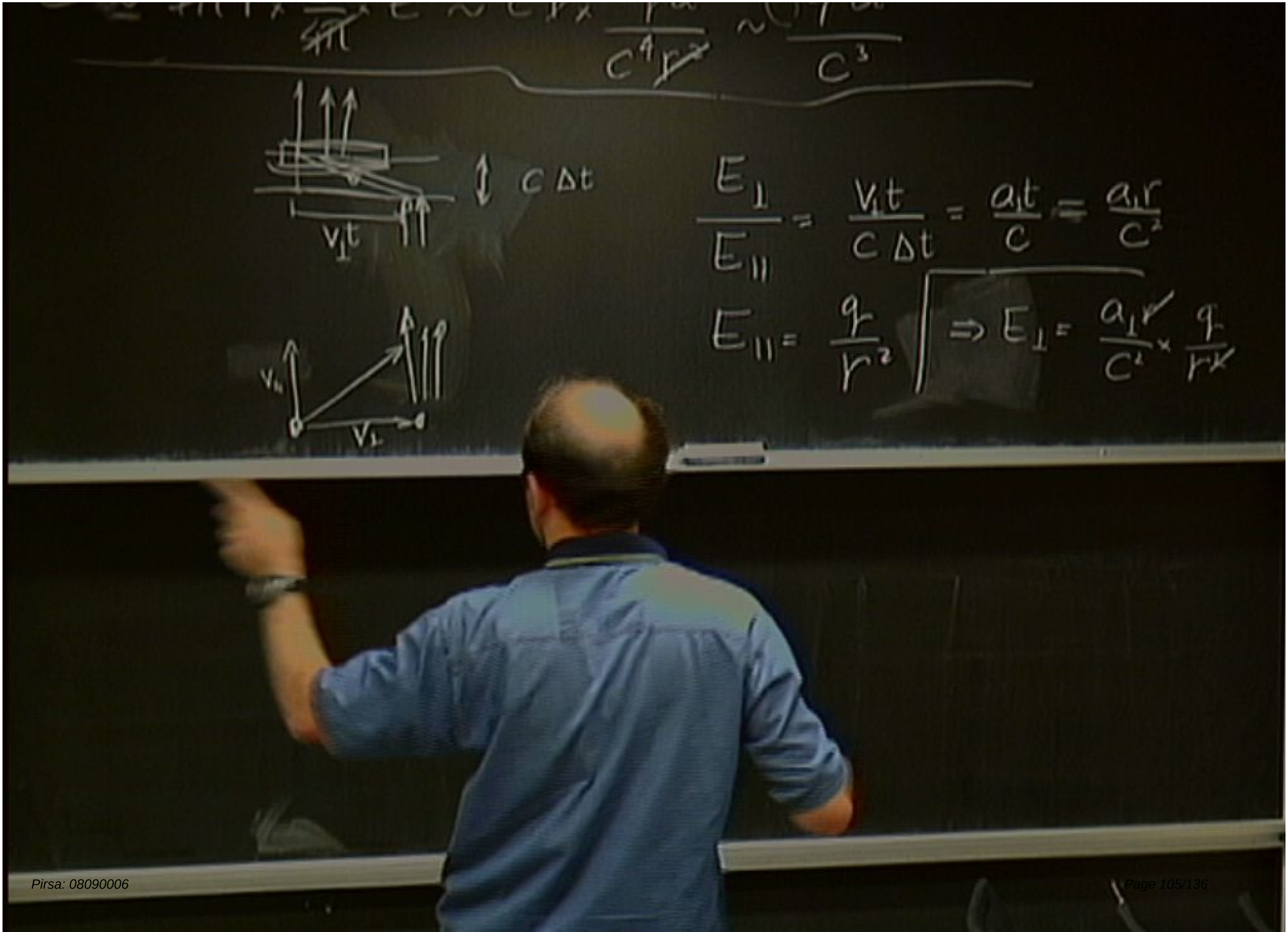


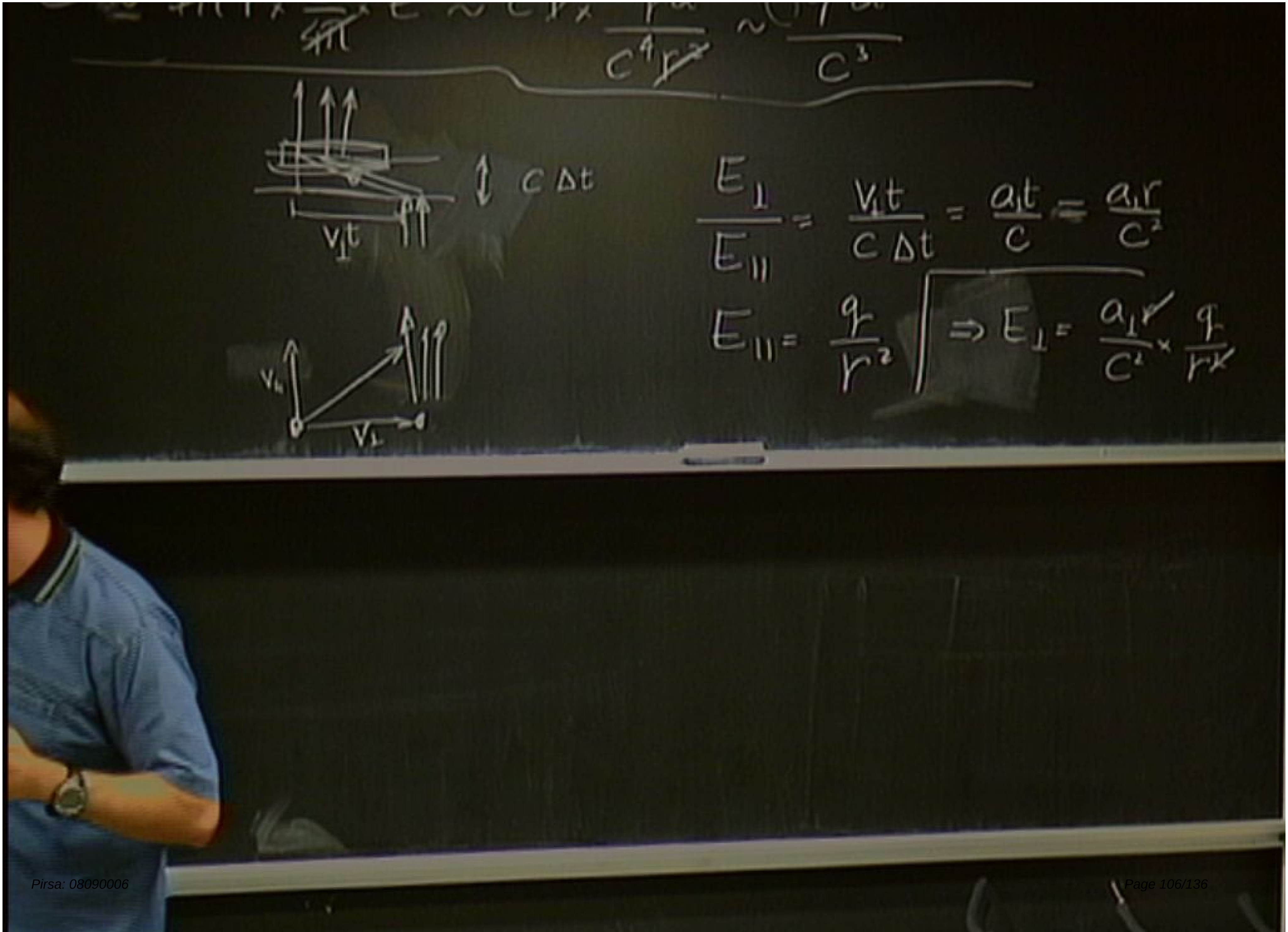
ℰ ~ 4πr² × c/4π × E² ~ c r² × q²a²/(c⁴r²) ~ q²a²/c³
c Δt
vt
E⊥/E∥ = vt/(c Δt) = at/c = ar/c²
E∥ = q/r²

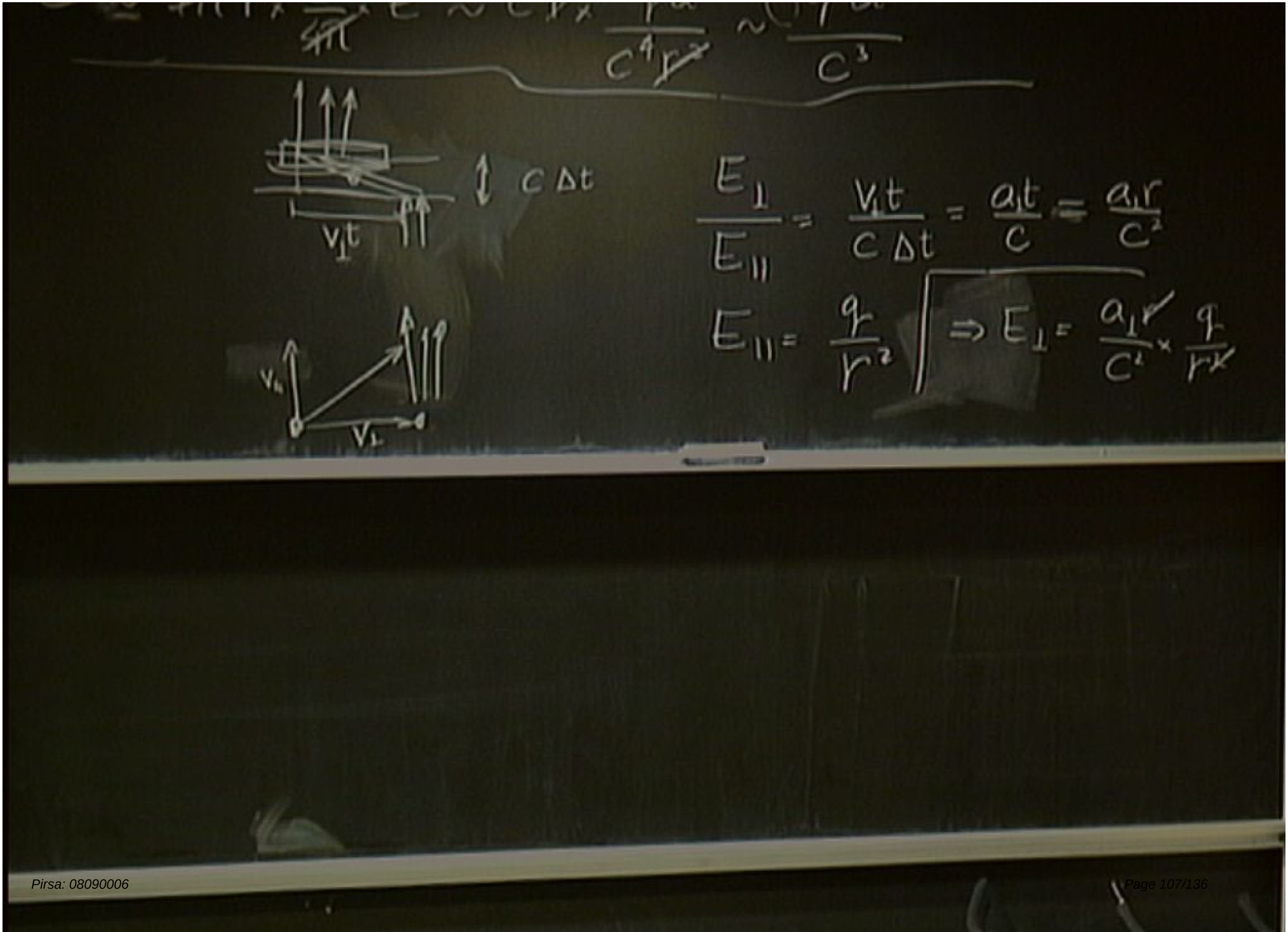

ℰ ~ 4πr² × c/4π × E² ~ c r² × q²a²/c⁴r² ~ q²a²/c³
c Δt
v⊥t
E⊥/E∥ = vt/cΔt = at/c = ar/c²
E∥ = q/r²

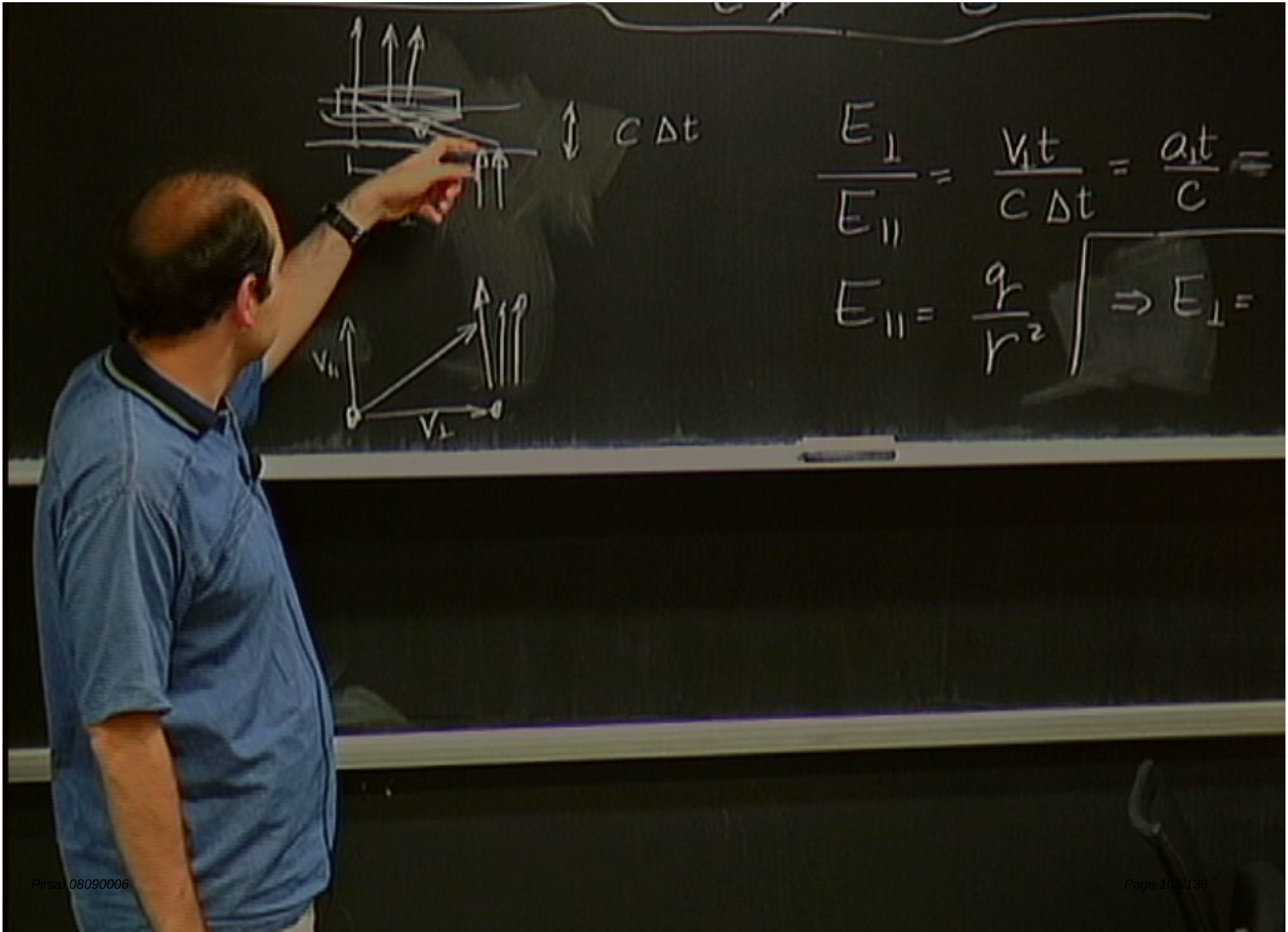

c Δt
E⊥ / E∥ = vt / c Δt = at / c = ar / c²
E∥ = q / r²
v⊥t
v⊥

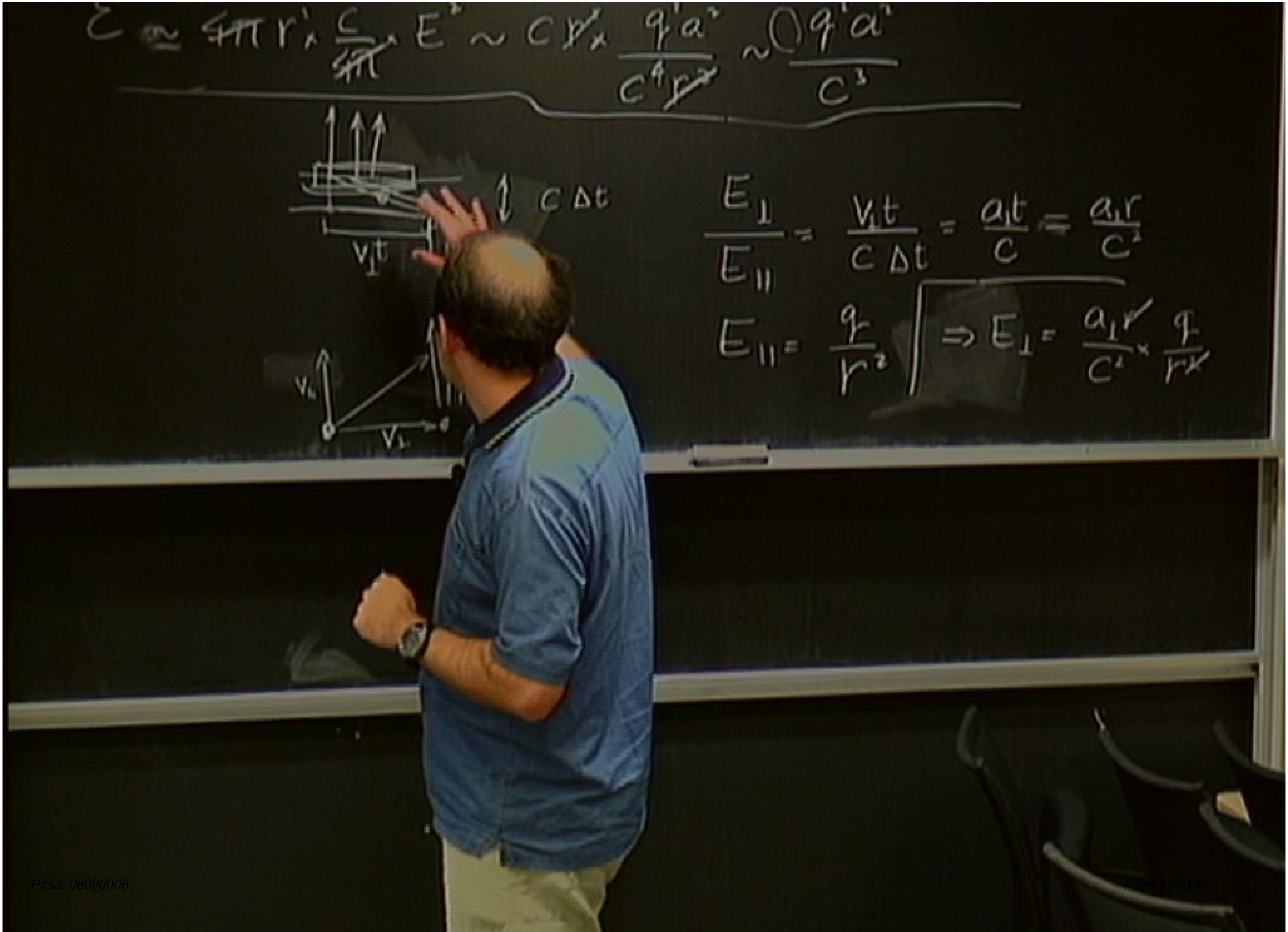

c Δt
v⊥t
v∥
v⊥
E⊥ / E∥ = v⊥t / c Δt = a⊥t / c = a⊥r / c²
E∥

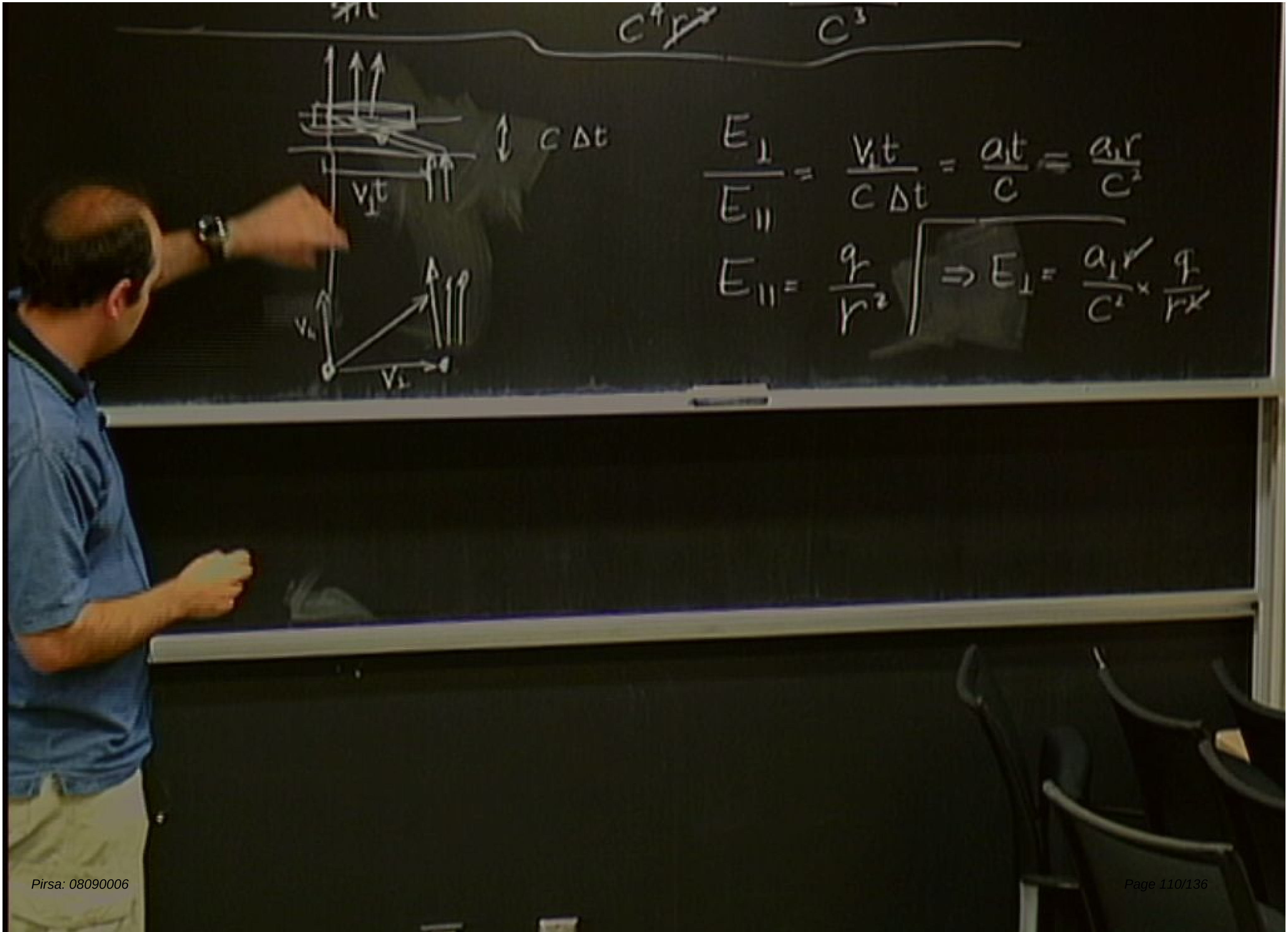

c Δt
v⊥t
E⊥ / E∥ = v⊥t / cΔt = a⊥t / c = a⊥r / c²
E∥ = q / r²
⇒ E⊥ = a⊥r / c² × q / r²

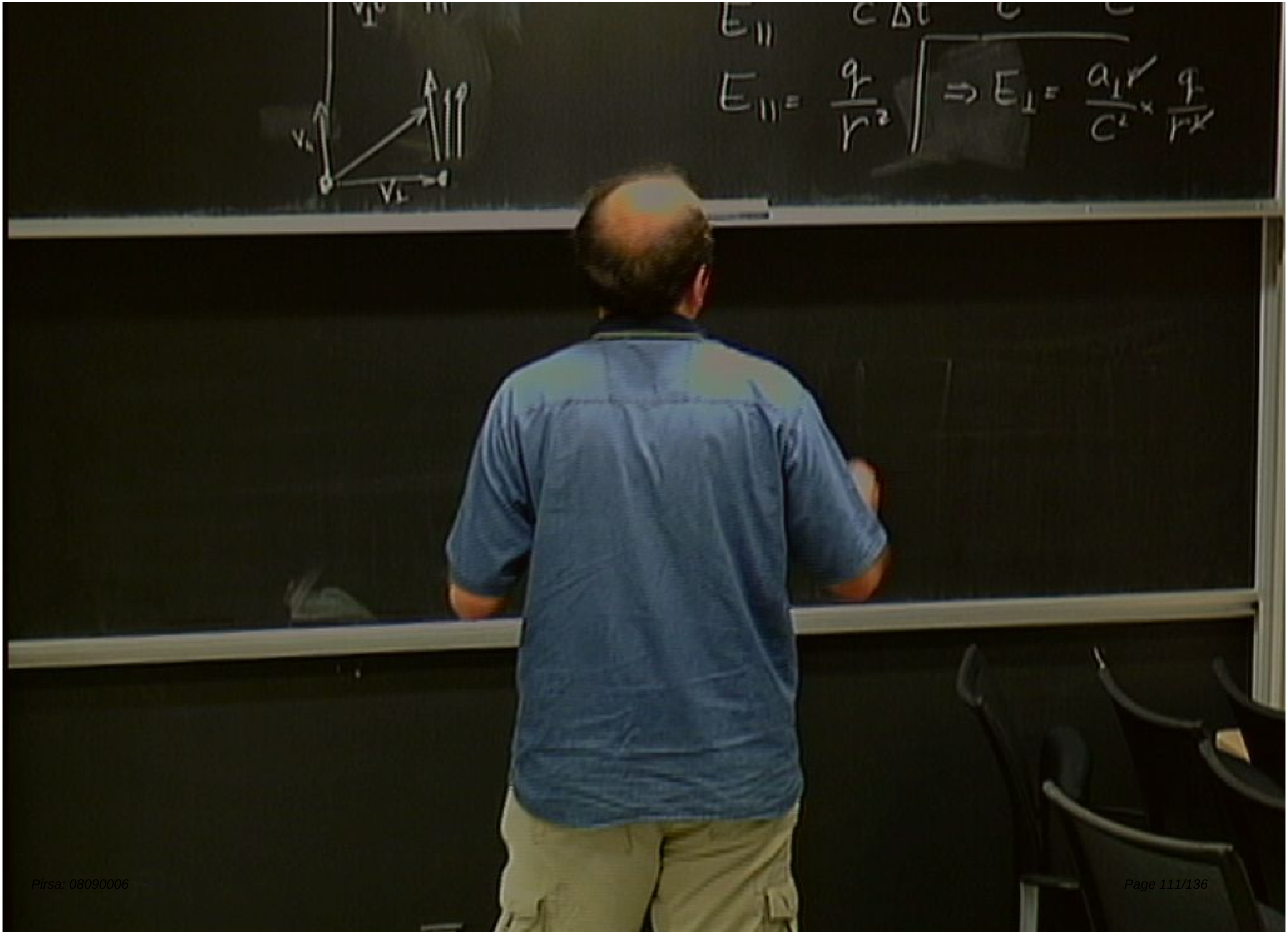

c Δt
v⊥t
E⊥ / E∥ = v⊥t / c Δt = a⊥t / c = a⊥r / c²
E∥ = q / r²
⇒ E⊥ = a⊥r / c² × q / r²
V∥
V⊥

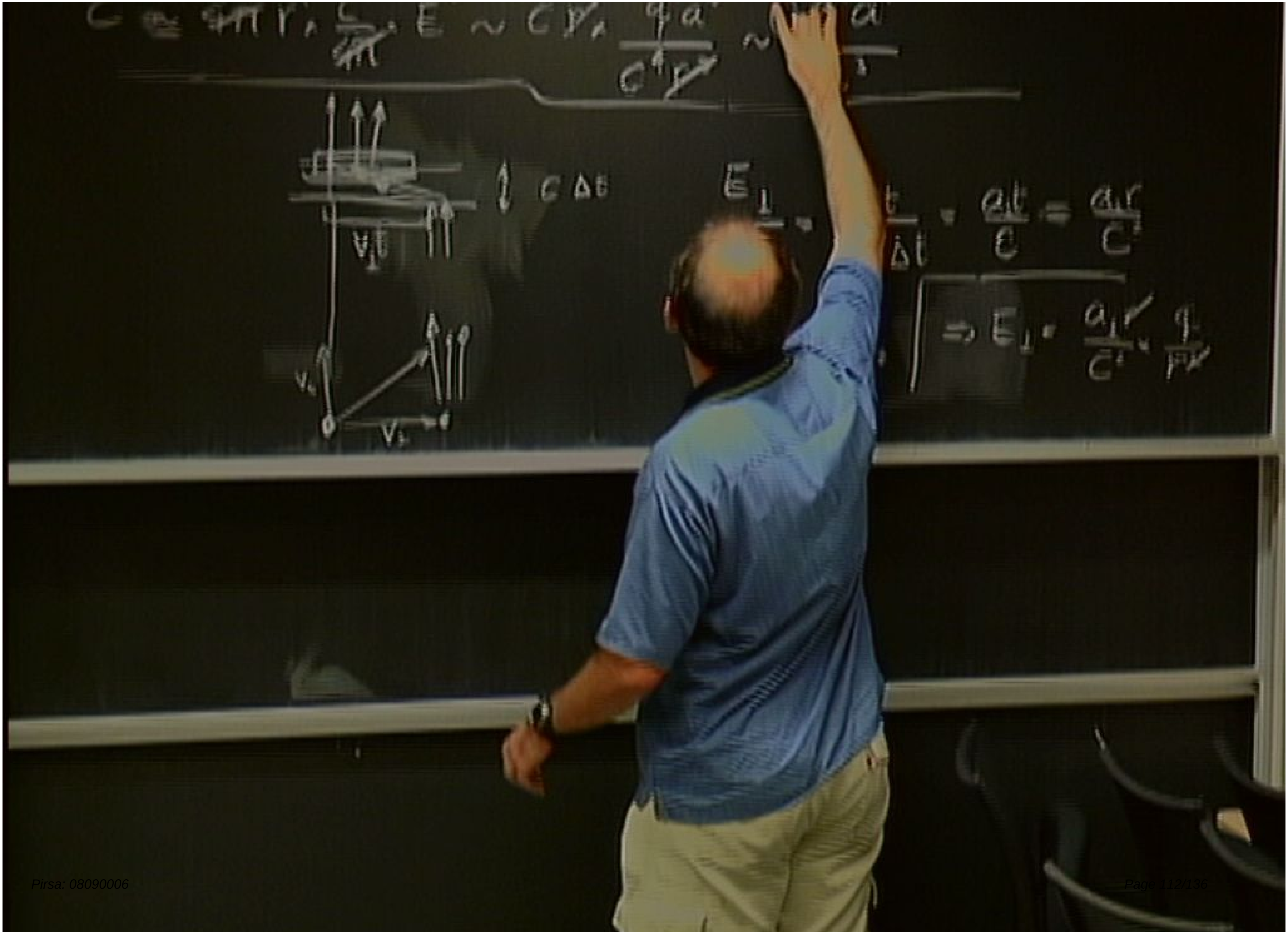

c Δt
v⊥t
E⊥ / E∥ = v⊥t / c Δt = a⊥t / c = a⊥r / c²
E∥ = q / r²
⇒ E⊥ = a⊥r / c² × q / r²
v∥
v⊥

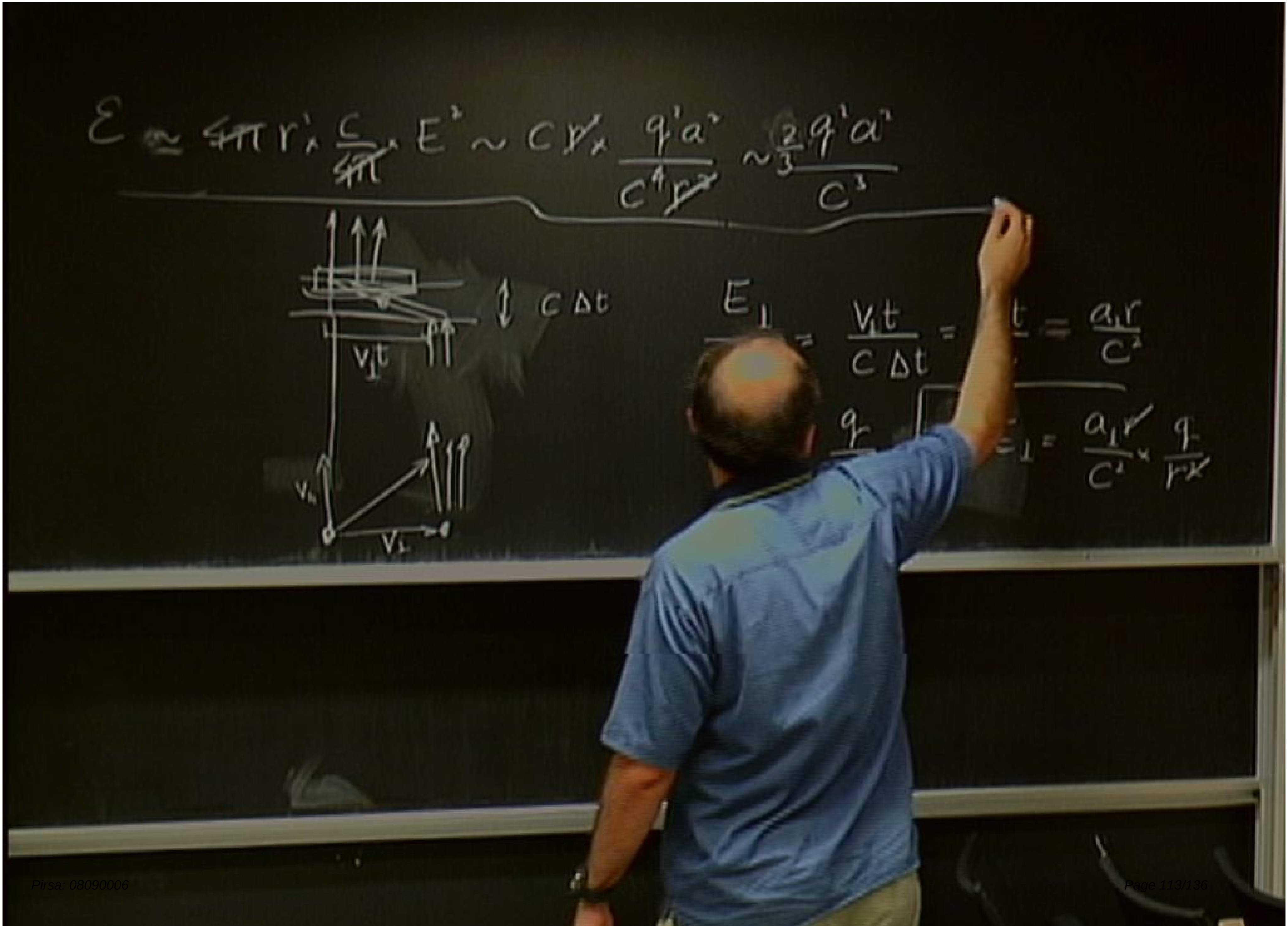

c Δt
E⊥ / E∥ = v⊥t / c Δt = a⊥t / c =
E∥ = q / r²
⇒ E⊥ =
v∥
v⊥

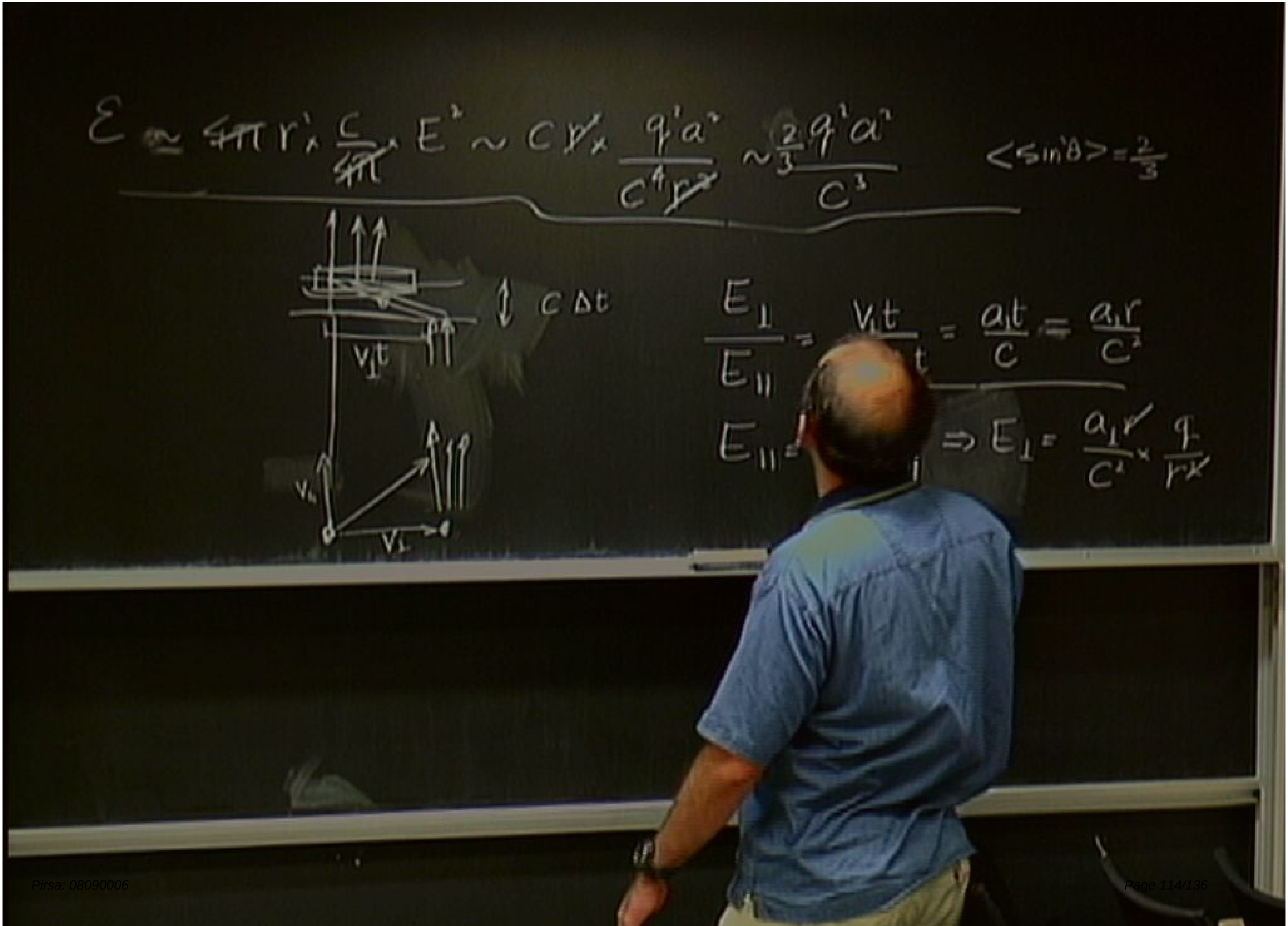

c Δt
E⊥ / E∥ = v⊥t / c Δt = a⊥t / c = a⊥r / c²
E∥ = q / r²
⇒ E⊥ = a⊥r / c² × q / r²
v⊥t
v⊥

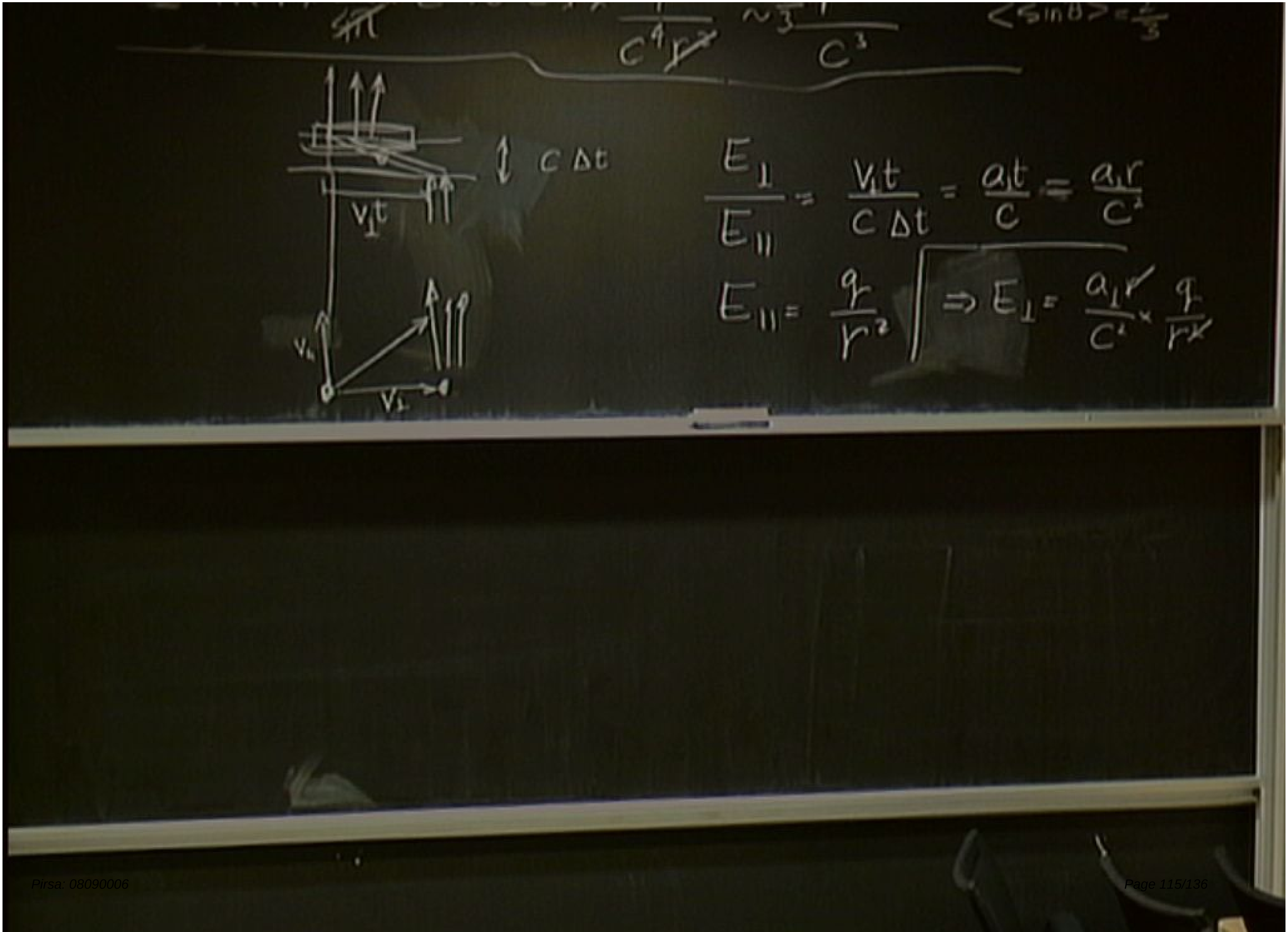

$c\Delta t$
$v_\perp t$
$v_\perp$
$\frac{E_\perp}{E_\parallel} = \frac{v_\perp t}{c\,\Delta t} = \frac{a_\perp t}{c} = \frac{a_\perp r}{c^2}$
$E_\parallel = \frac{q}{r^2}$
$\Rightarrow E_\perp = \frac{a_\perp r}{c^2} \times \frac{q}{r^2}$



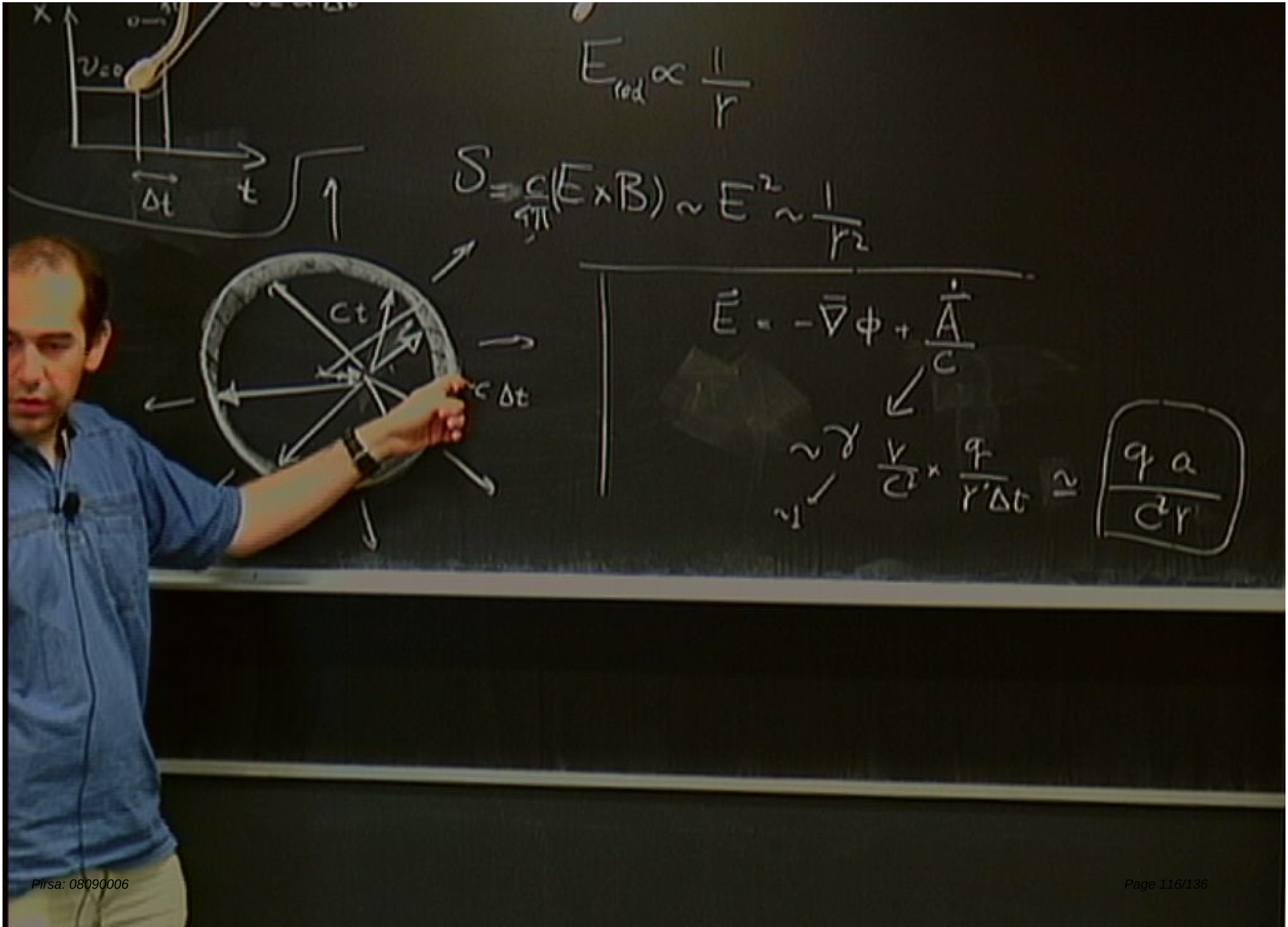


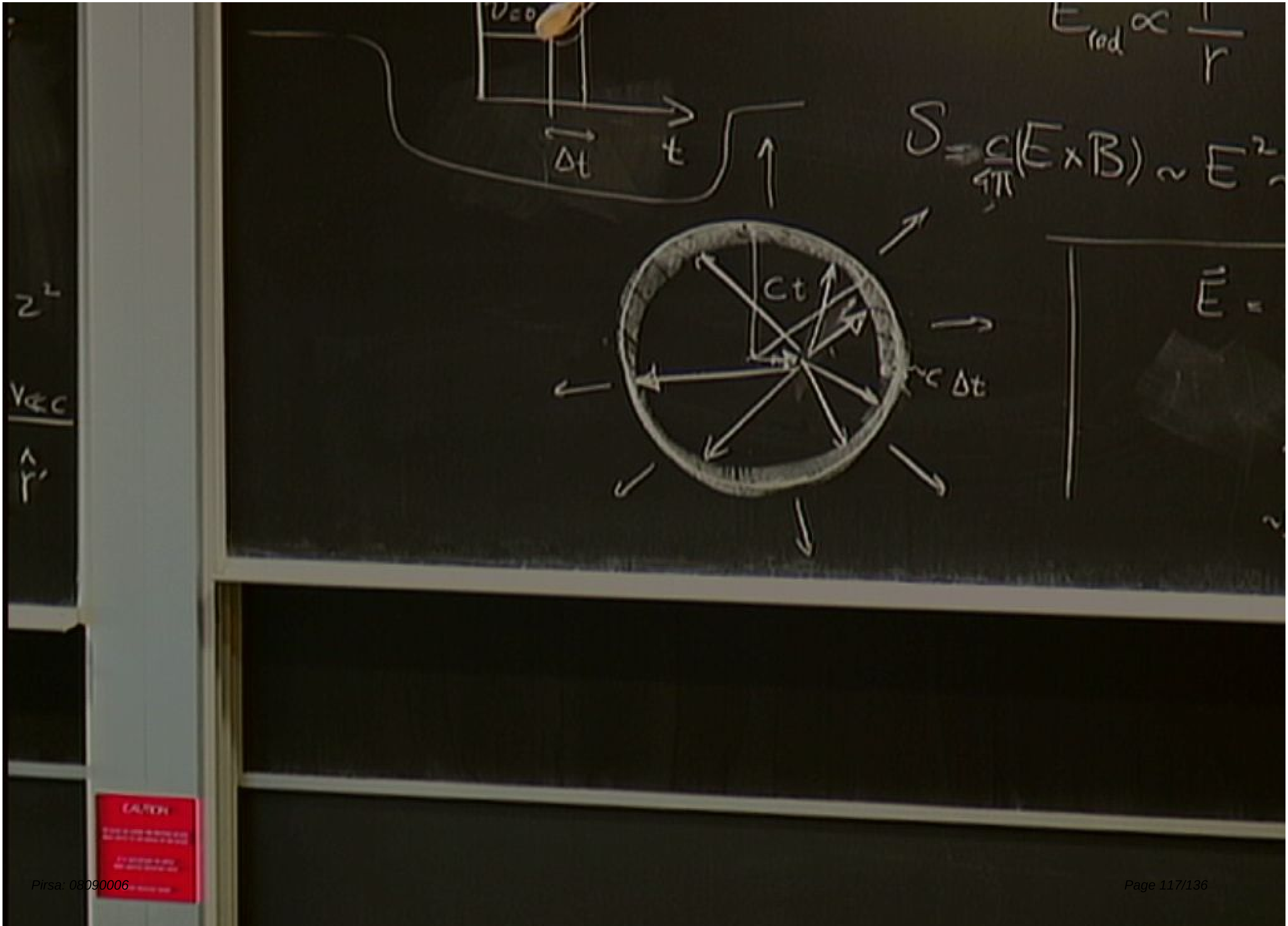


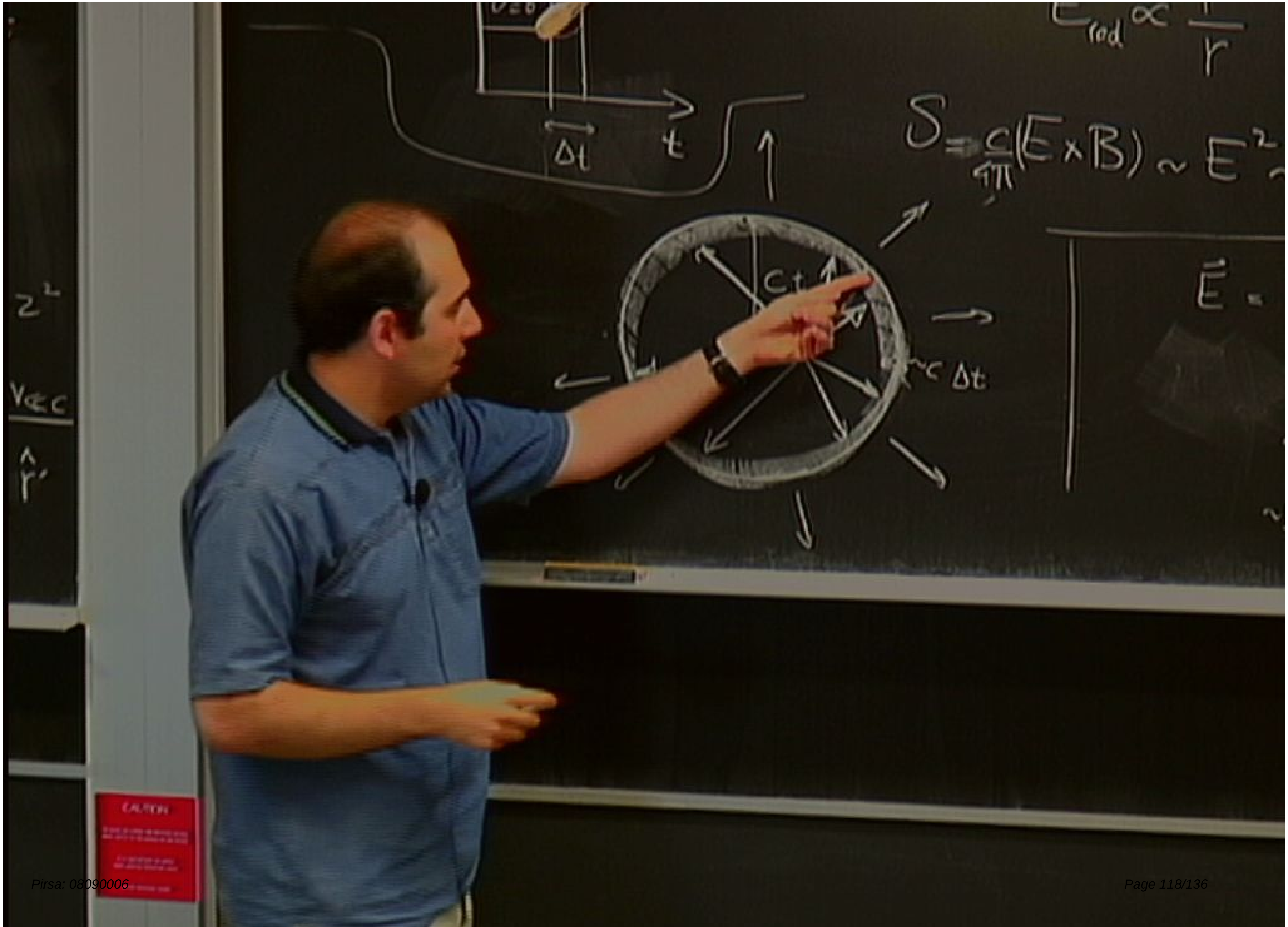

c Δt
<sin²θ> = 2/3

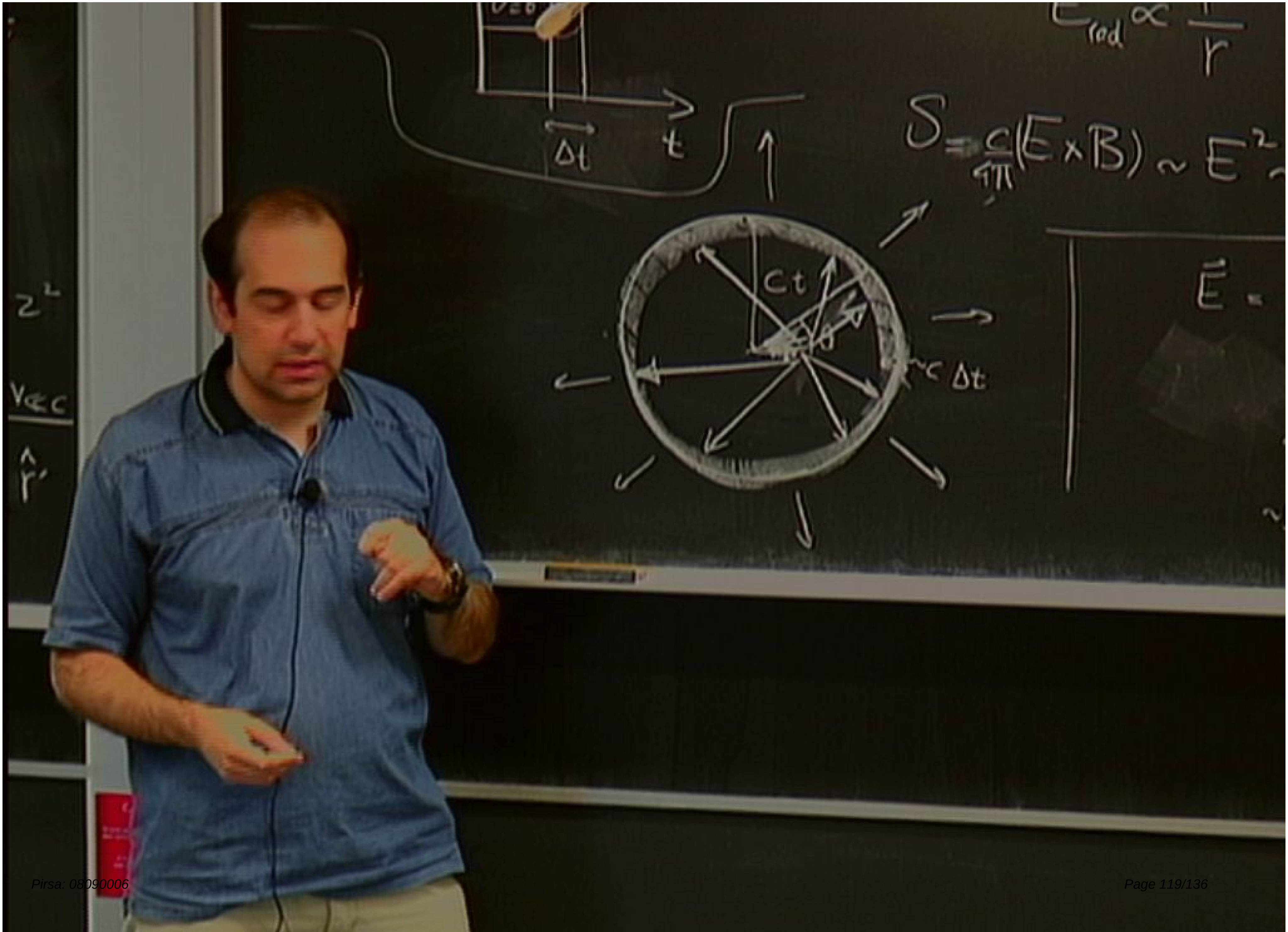


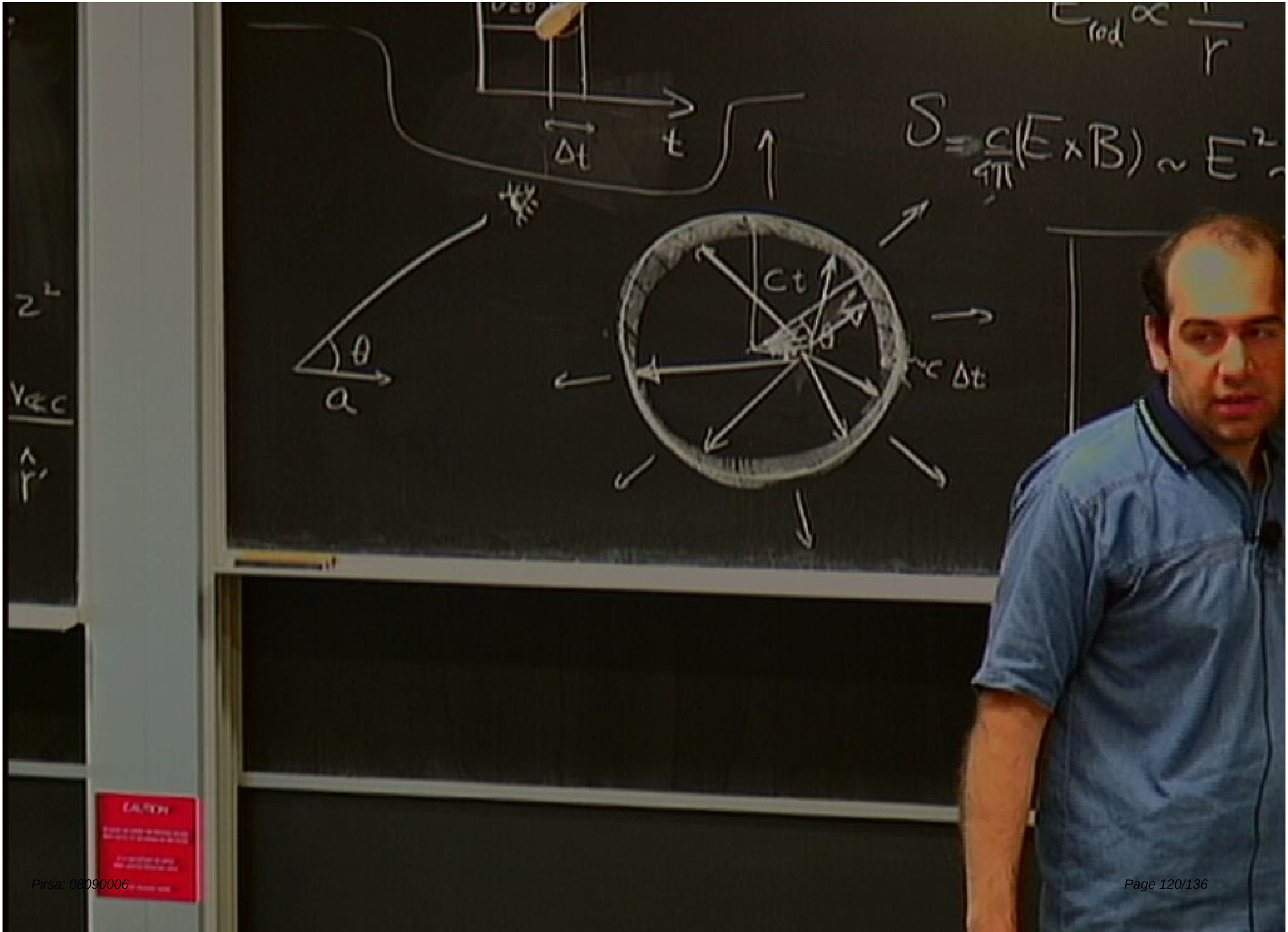
c Δt
v⊥t
v∥
v⊥
E⊥ / E∥ = v⊥t / c Δt = a⊥t / c = a⊥r / c²
E∥ = q / r²
⇒ E⊥ = a⊥r / c² × q / r²


E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
~ γ v/c² × q/(γ²Δt) ≃ q a/(c² r)
ct
c Δt
Δt
t
x

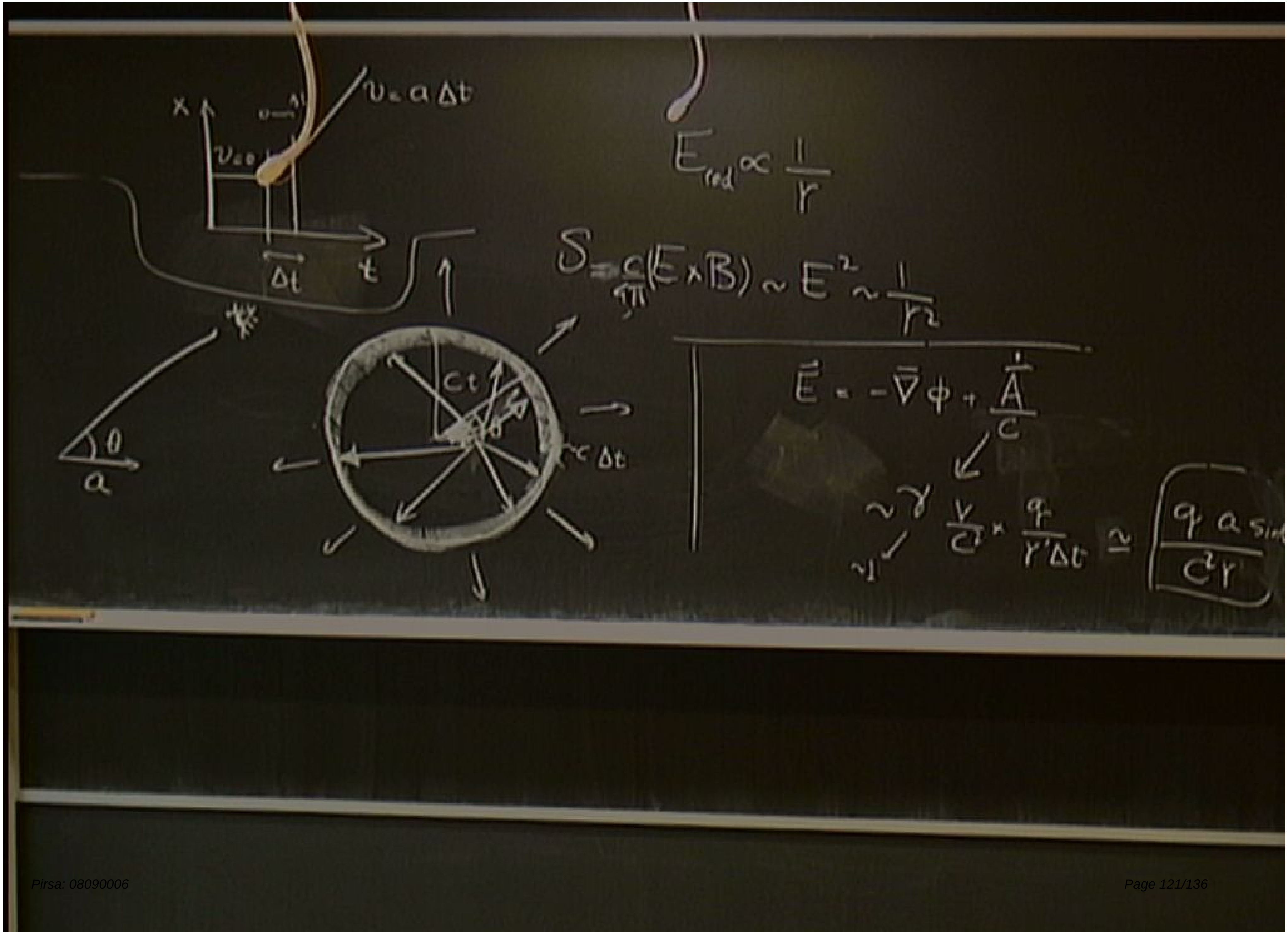

E_rad ∝ 1/r
S = c/4π (E × B) ~ E²
Δt
t
ct
~c Δt
E =
v≪c
CAUTION

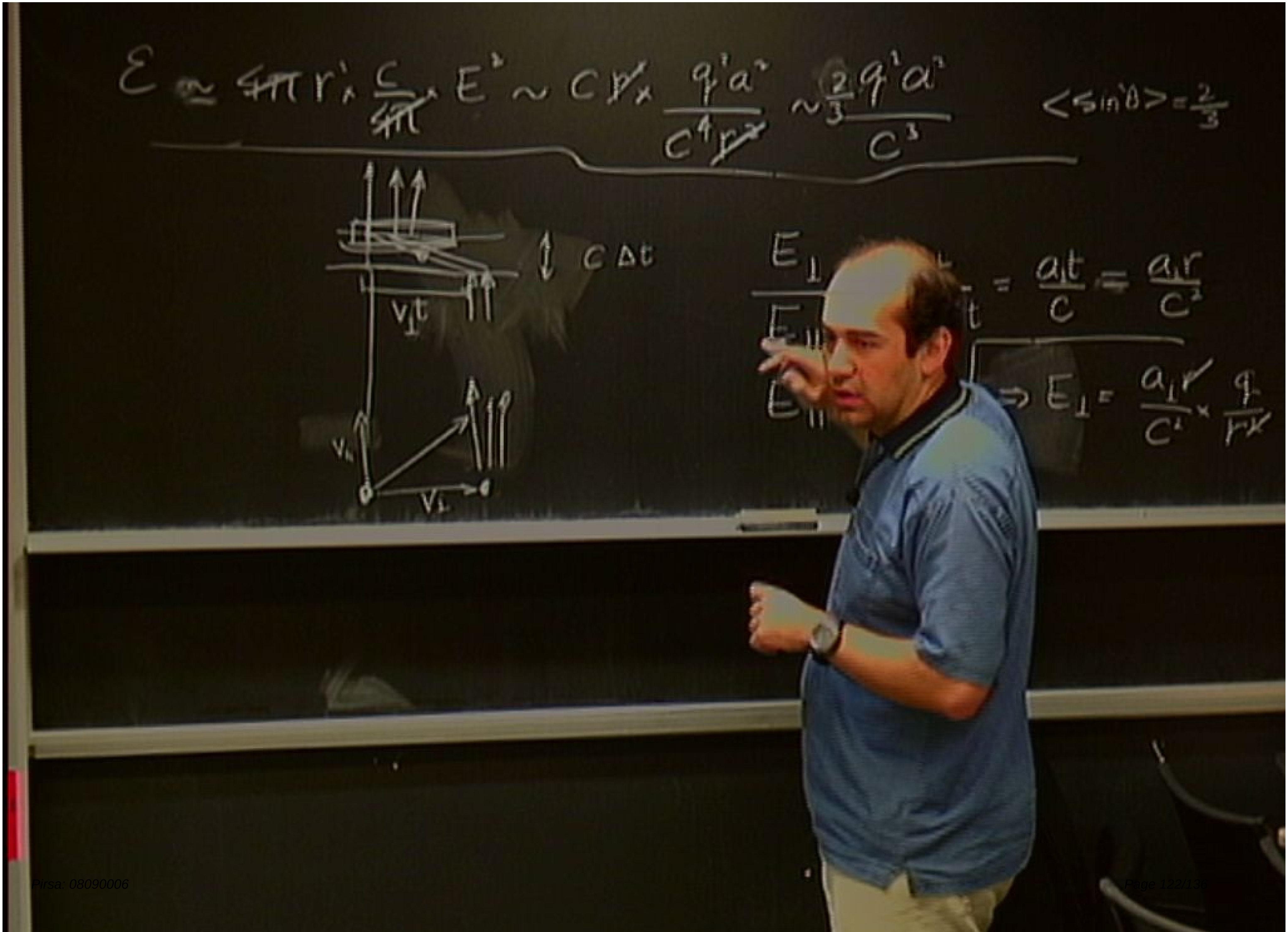

E_rad ∝ 1/r
S = c/4π (E×B) ~ E²
Δt
t
ct
~c Δt
v≪c
CAUTION


E_rad ∝ 1/r
S = c/4π (E × B) ~ E²
Δt
t
ct
~c Δt
v≪c



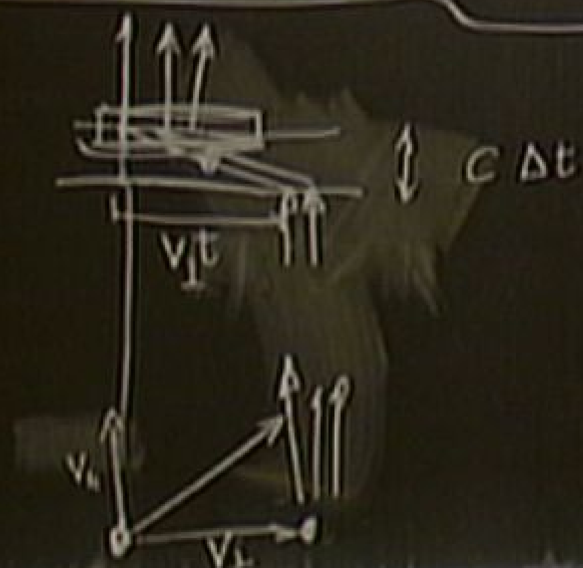
E_rad ∝ 1/r
S = c/4π (E × B) ~ E²
Δt
t
ct
θ
a
~c Δt
v≪c
z²
r̂'


x
v = a Δt
v = 0
Δt
t
θ
a
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
ct
c Δt
E = −∇φ + Ȧ/c
~ γ v/c² × q/(r Δt) ≃ q a sin/(c² r)


<sin²θ> = 2/3
c Δt

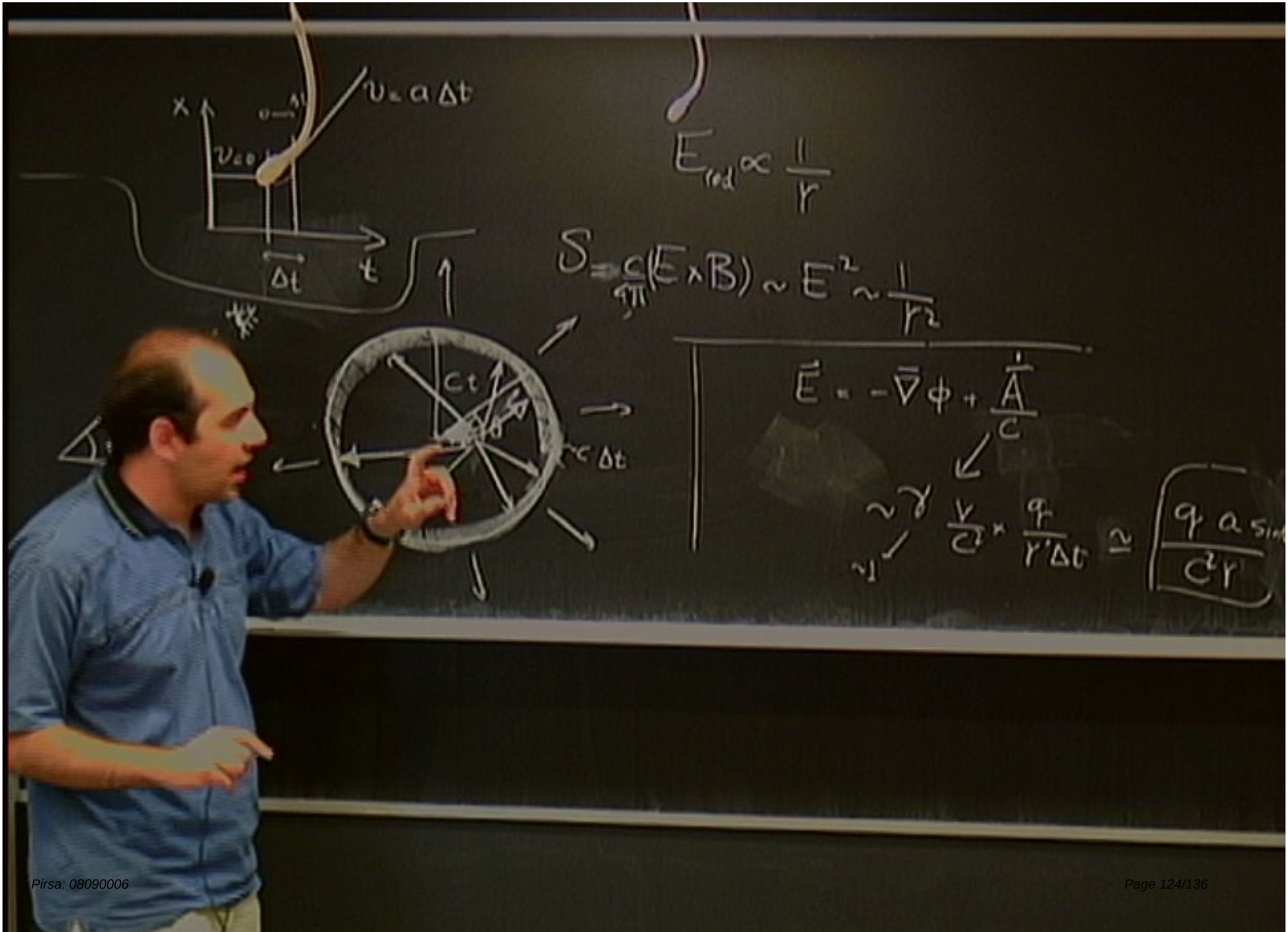
$$\mathcal{E} \simeq 4\pi r^2 \times \frac{c}{4\pi} \times E^2 \sim c\, r^2 \times \frac{q^2 a^2}{c^4 r^2} \sim \boxed{\frac{2}{3}\frac{q^2 a^2}{c^3}}$$

$\langle \sin^2\theta \rangle = \frac{2}{3}$

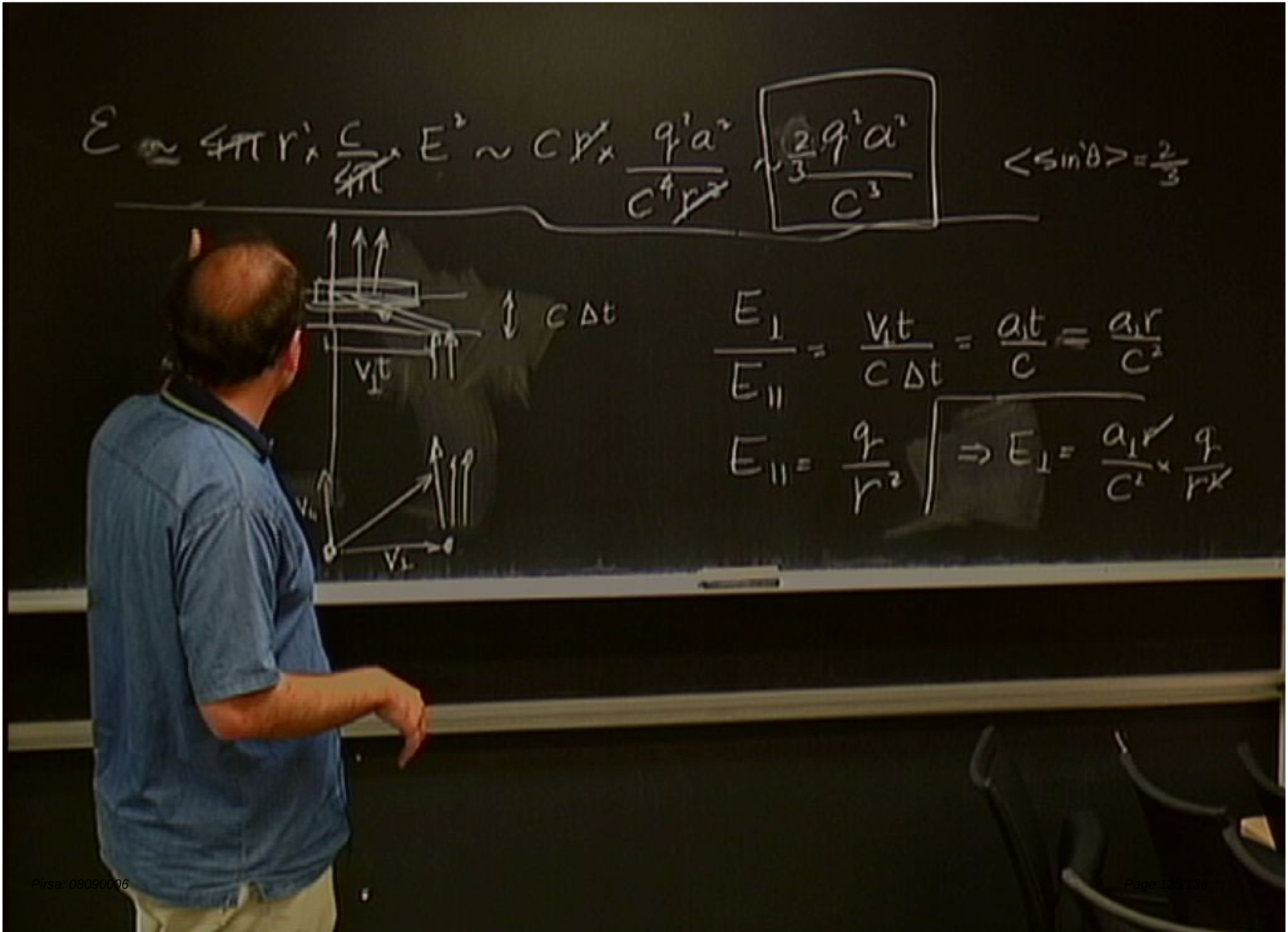


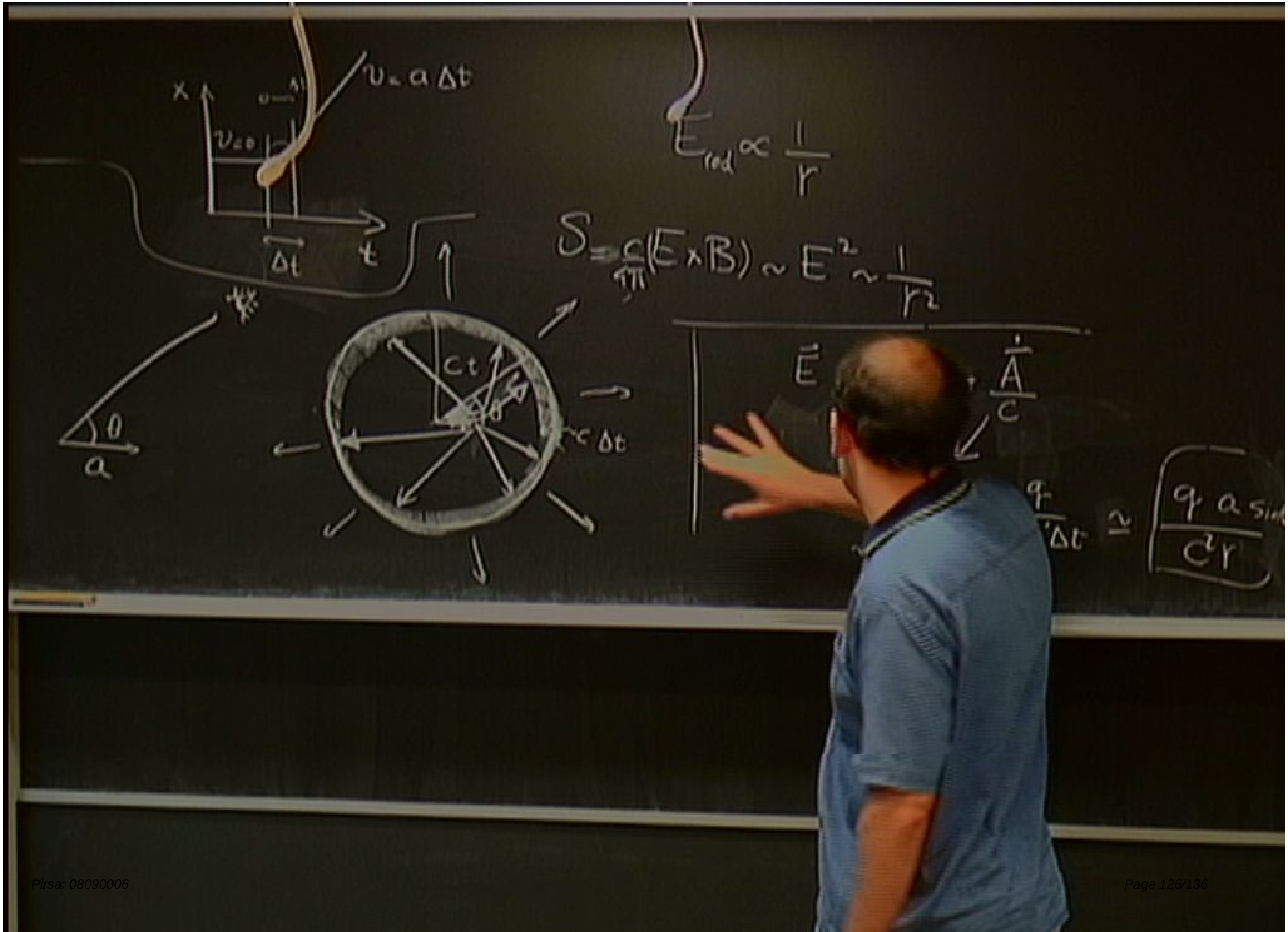
$$\frac{E_\perp}{E_\parallel} = \frac{v_\perp t}{c\,\Delta t} = \frac{a_\perp t}{c} = \frac{a_\perp r}{c^2}$$

$$E_\parallel = \frac{q}{r^2} \Rightarrow E_\perp = \frac{a_\perp r}{c^2} \times \frac{q}{r^2}$$

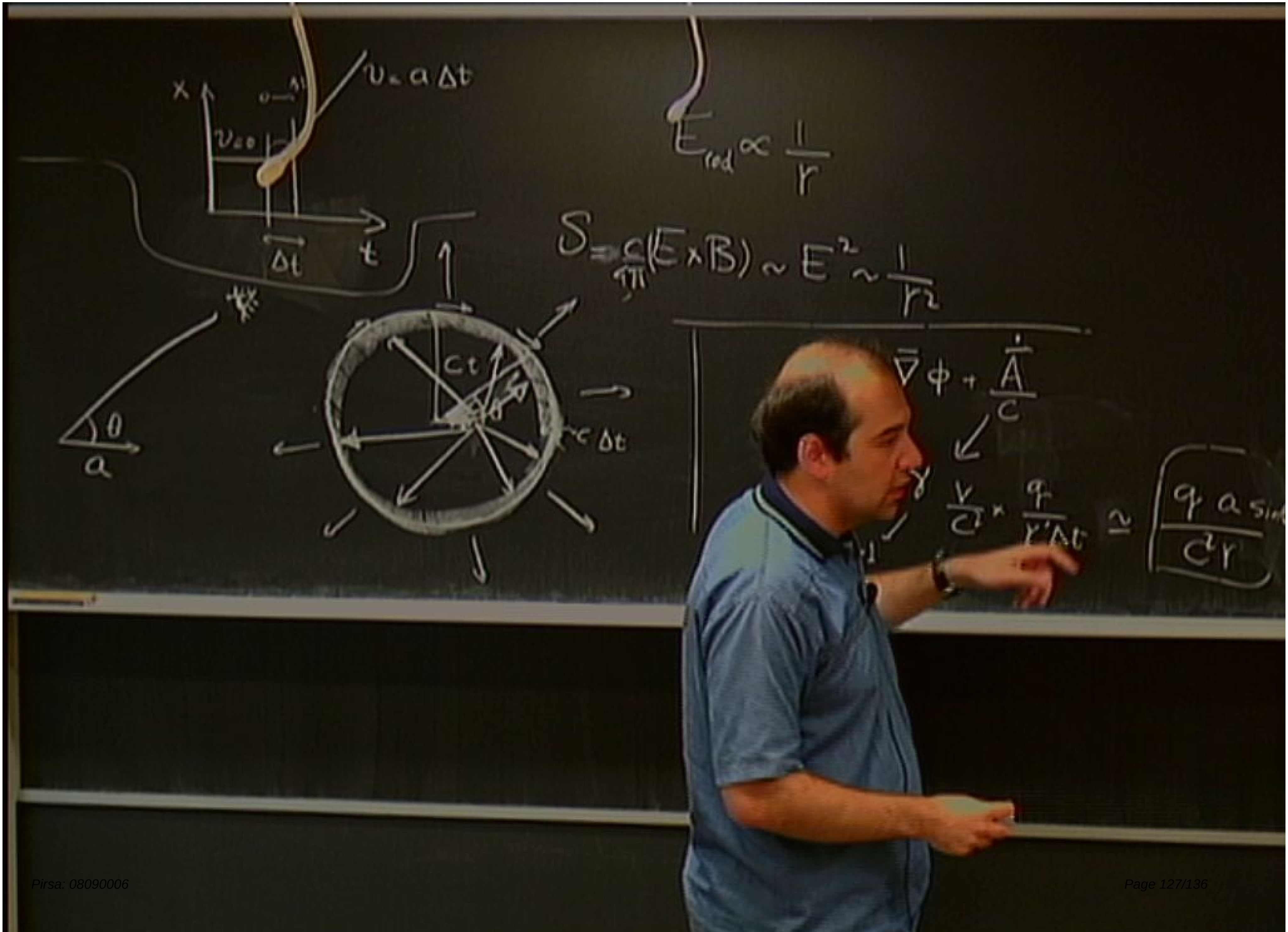


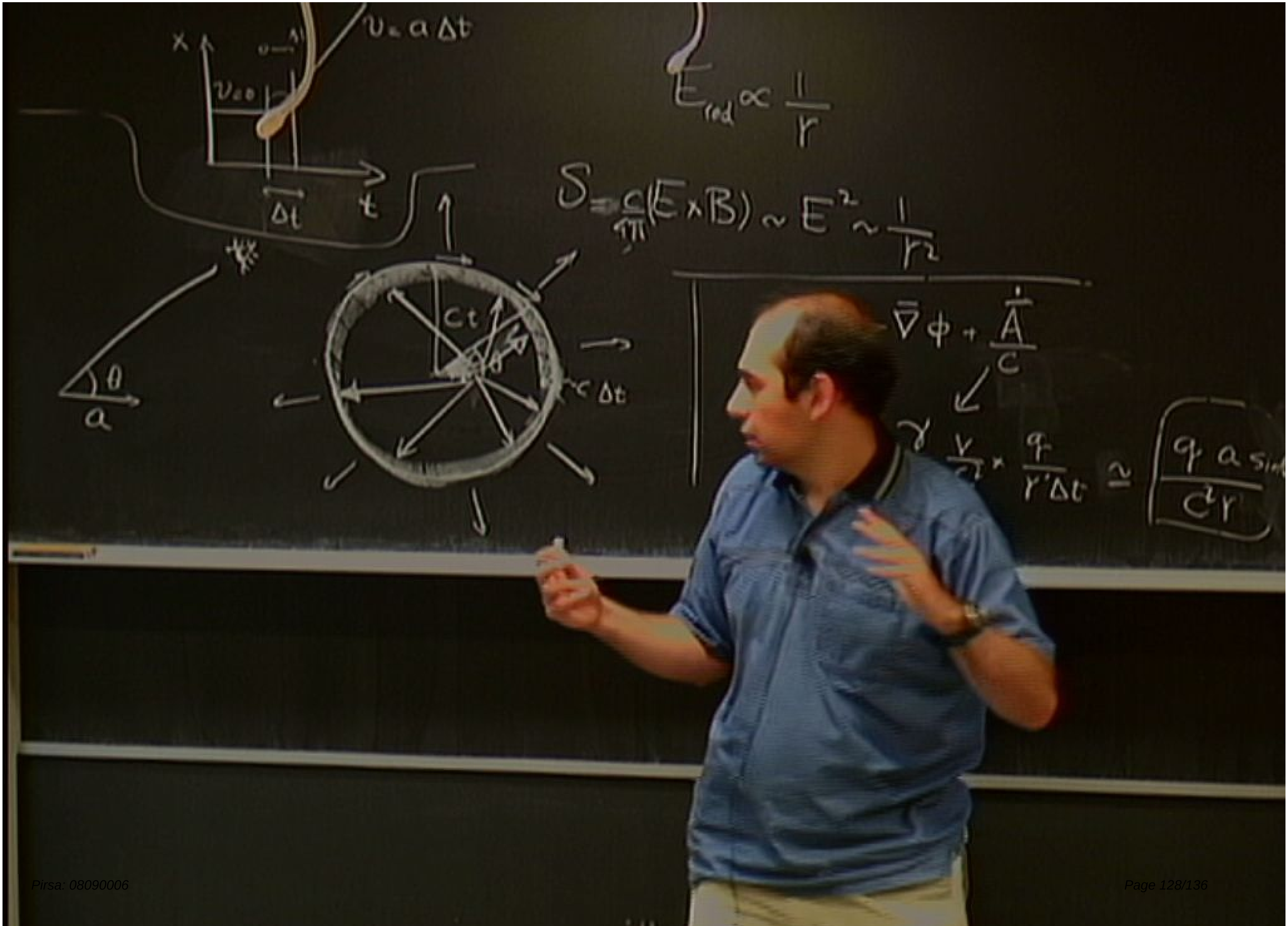
v = a Δt
x
t
Δt
ct
c Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c


<sin²θ> = 2/3
c Δt

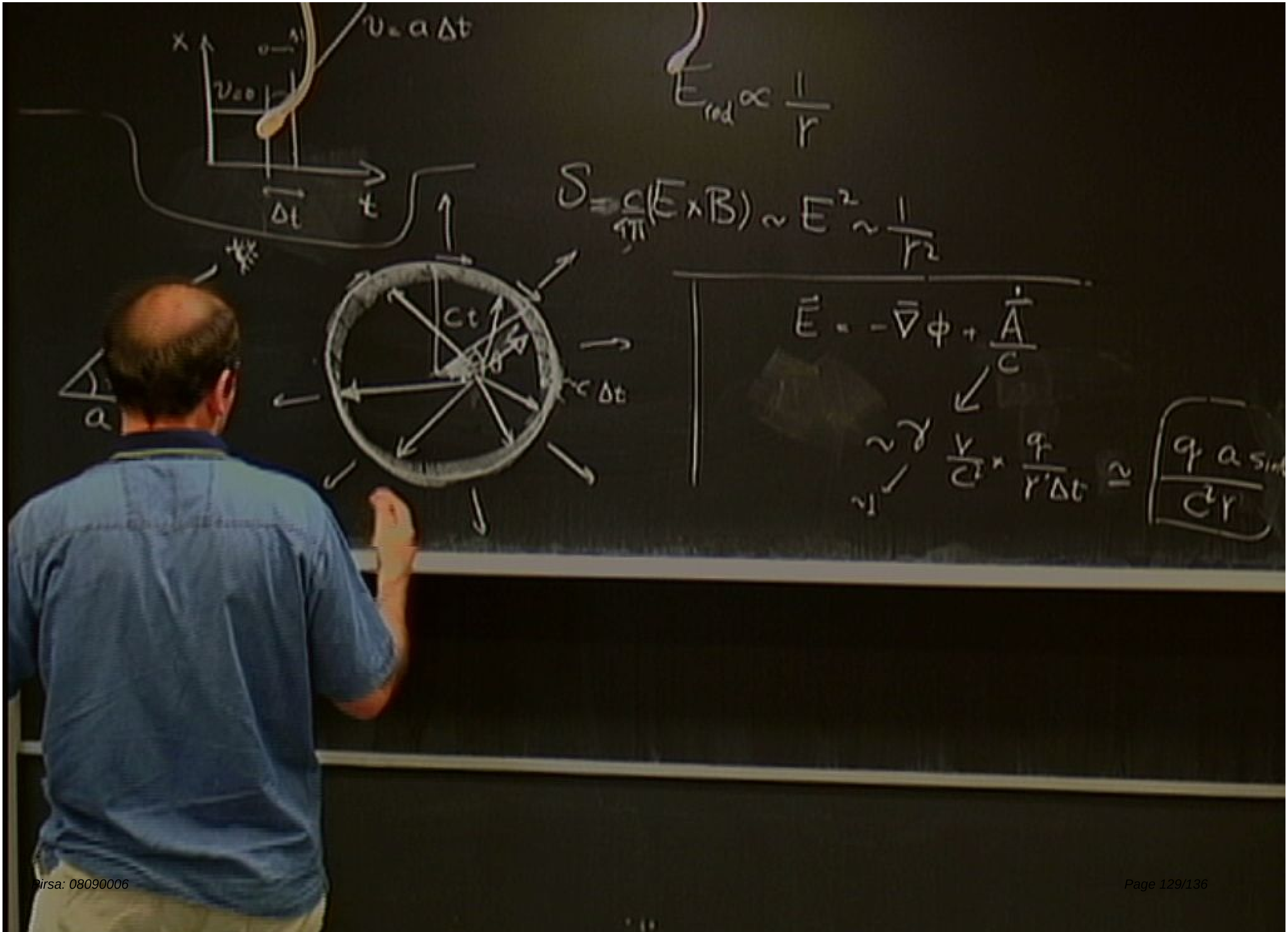


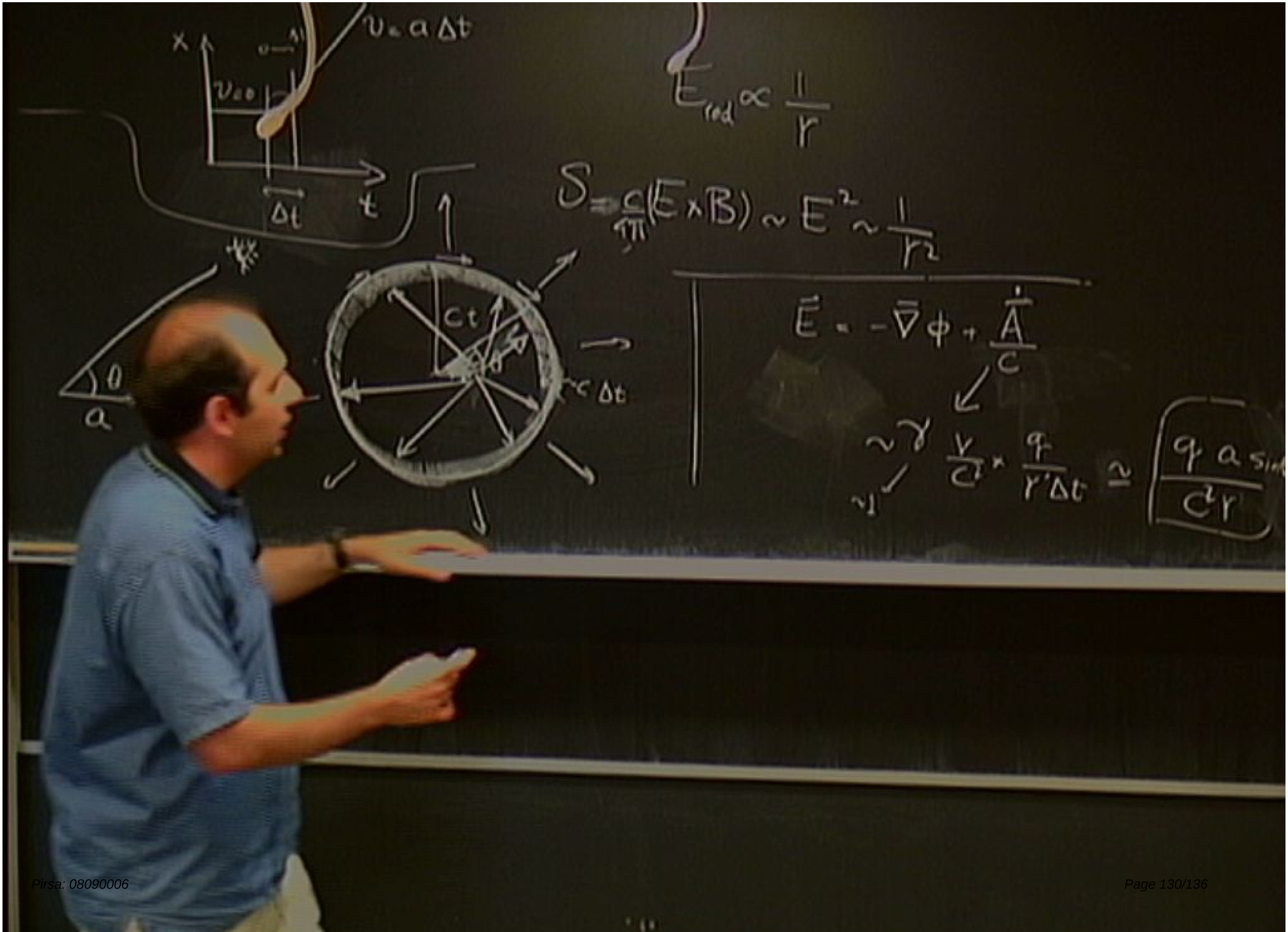
x
v = a Δt
v=0
t
Δt
E_rad ∝ 1/r
S = c/4π (E×B) ~ E² ~ 1/r²
ct
c Δt
θ
a
E
A/c
q a sin
c²r

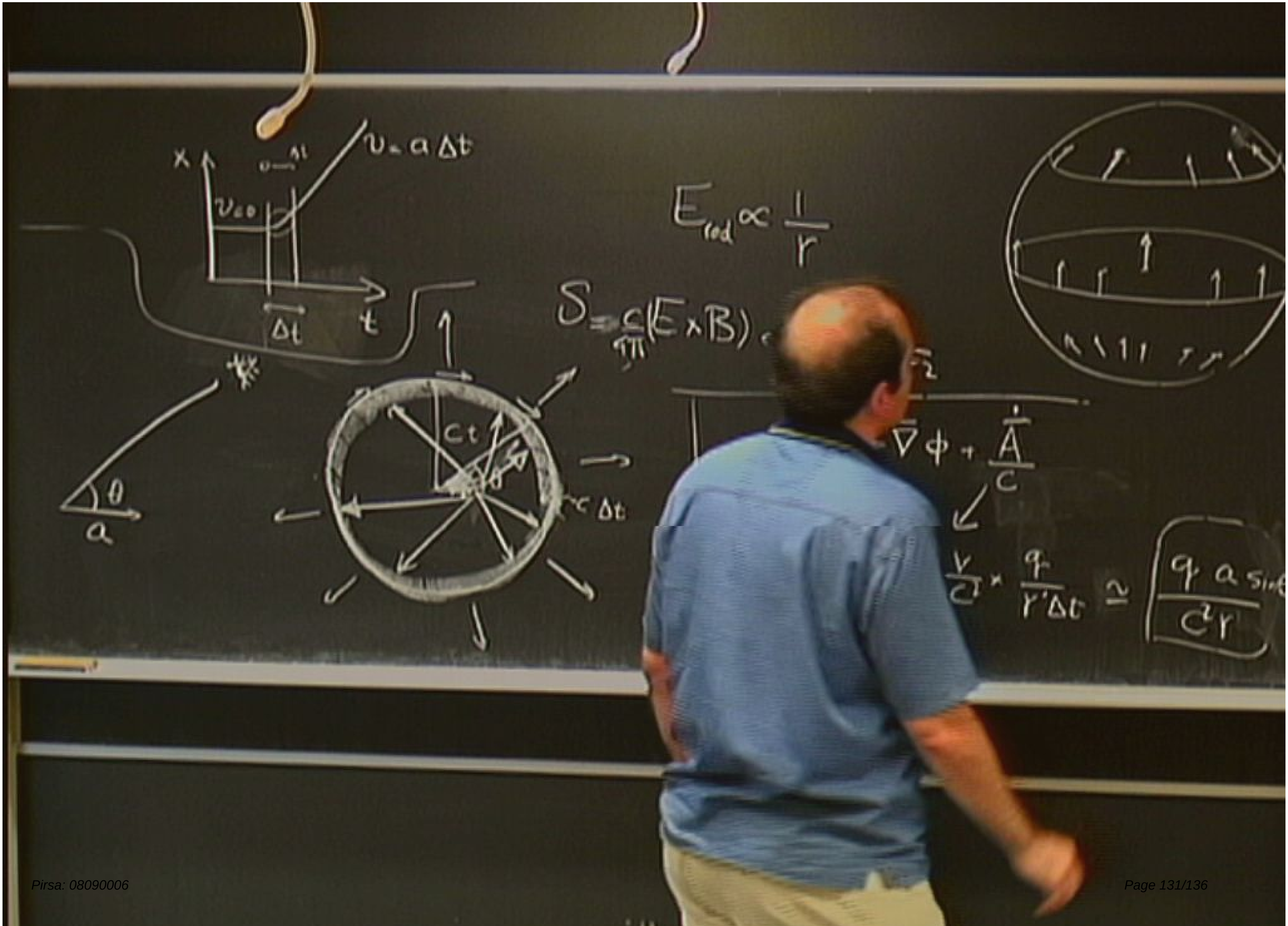

v = a Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E^2 ~ 1/r^2
ct
c Δt
θ
a
Δt
t

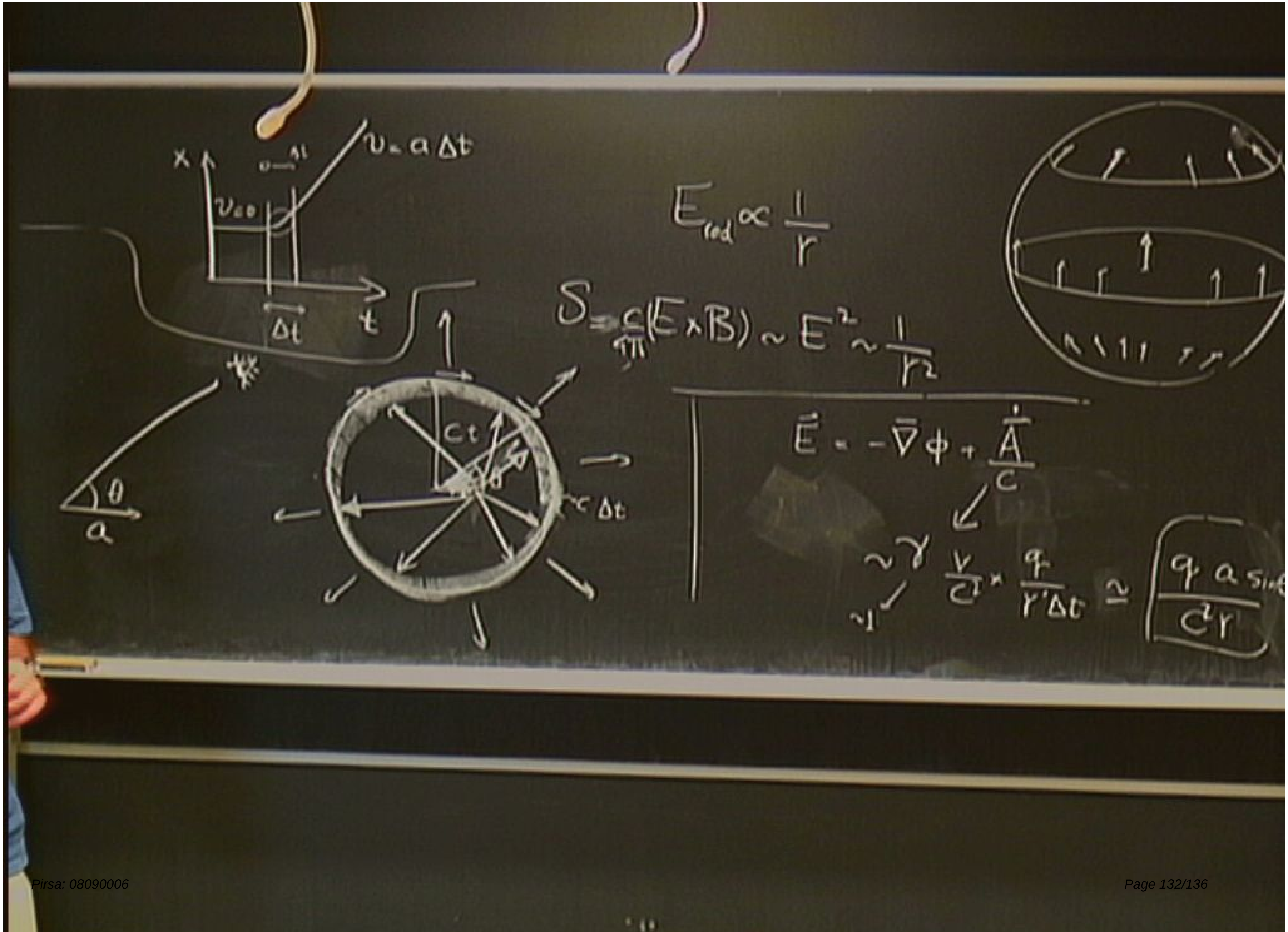


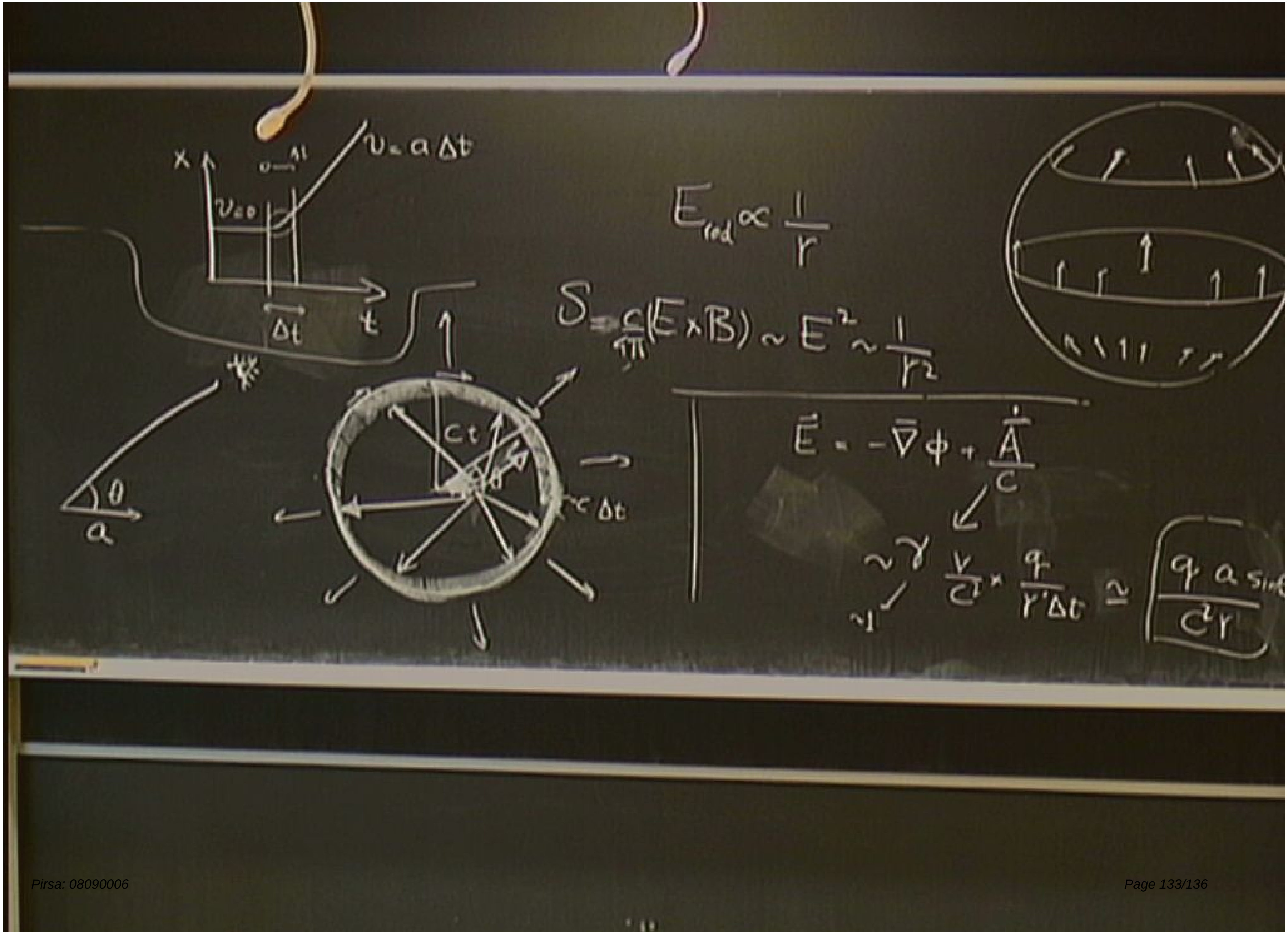
v = a Δt
x
Δt
t
E_rad ∝ 1/r
S = c/4π (E x B) ~ E^2 ~ 1/r^2
ct
c Δt
θ
a


v = a Δt
x
t
Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
ct
c Δt
E = −∇φ + Ȧ/c
q a sin / c² r

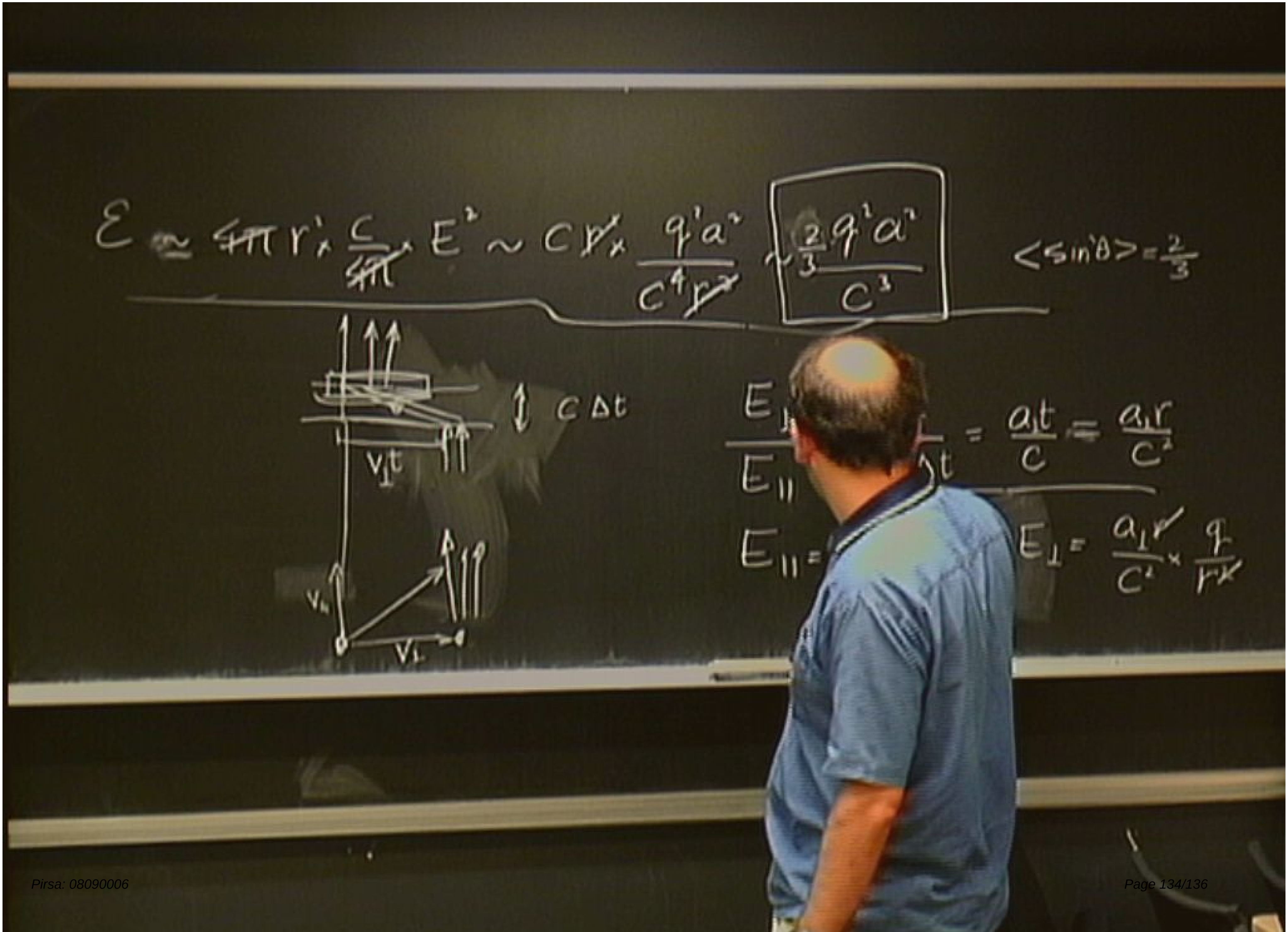

v = a Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
ct
c Δt
Δt
t
x
θ
a
~ γ v/c² × q/(r′Δt) ≈ q a sin / c²r

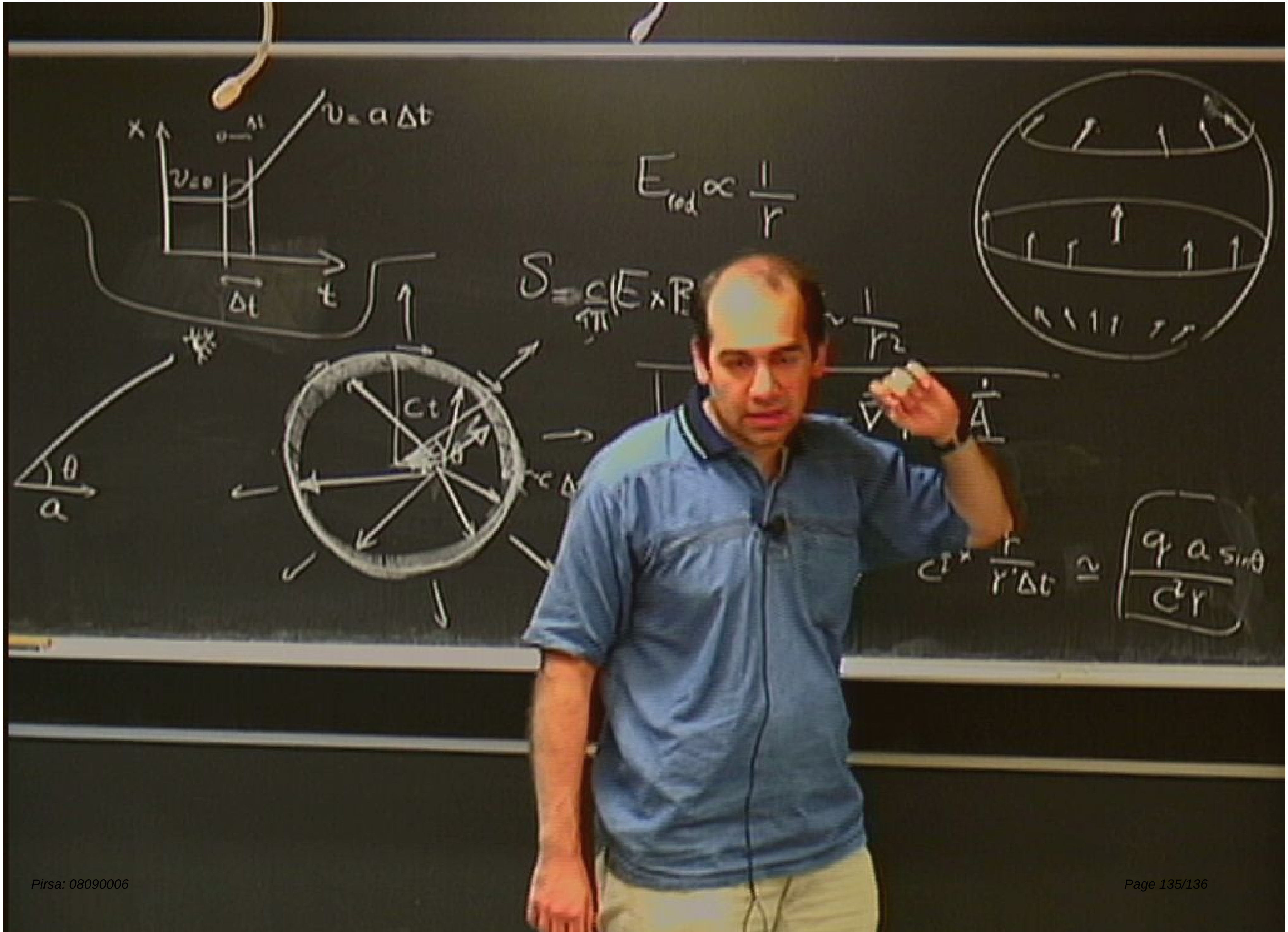

v = a Δt
x
t
Δt
E_rad ∝ 1/r
S = c/4π (E × B)
ct
θ
a
c Δt
∇φ + Ȧ/c
q a sin θ / c² r

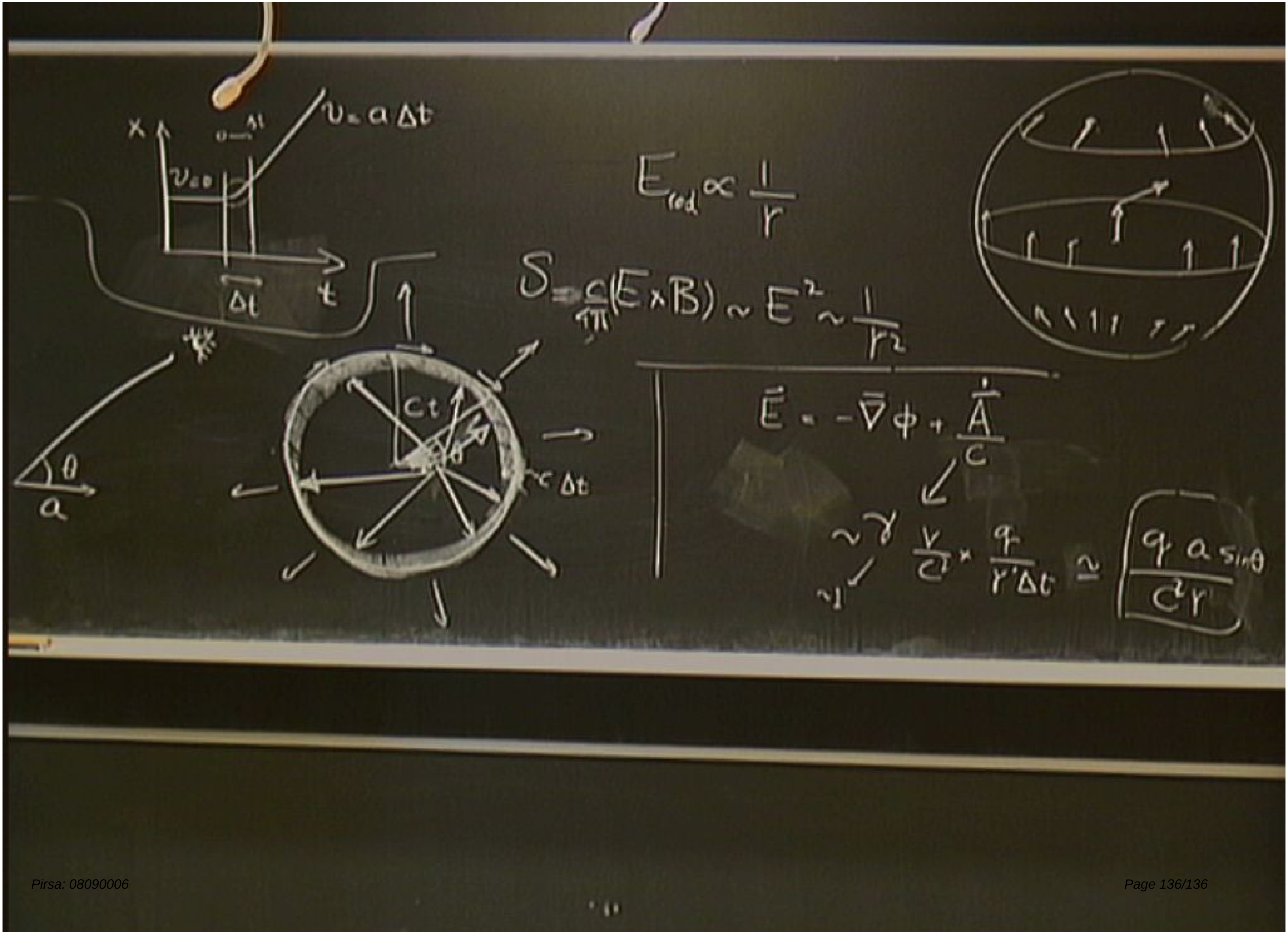

v = a Δt
x
t
Δt
θ
a
ct
c Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
q a sin
c² r



x
v = a Δt
v = 0
t
Δt
θ
a
ct
c Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
~ γ v/c² × q/(r Δt) ≃ q a sin/(c² r)




v = a Δt
E_rad ∝ 1/r
ct
θ
a
q a sinθ / c²r


v = a Δt
x
v=0
t
Δt
θ
a
ct
c Δt
E_rad ∝ 1/r
S = c/4π (E × B) ~ E² ~ 1/r²
E = −∇φ + Ȧ/c
~ γ v/c² × q/(r Δt) ≃ q a sin θ / (c² r)