

Title: Vacuum Energy and Fine Tuning

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Abstract: I will discuss fine tuning in modified gravity models that can account for today's dark energy. I will introduce some models where the underlying cosmological constant may be Planck scale but starts as a redundant coupling which can be eliminated by a field redefinition. The observed vacuum energy arises when the redundancy is explicitly broken. I will give a recipe for constructing models that realize this mechanism and satisfy all solar system constraints on gravity, including one based on Gauss-Bonnet gravity which provides a technically natural explanation for dark energy.

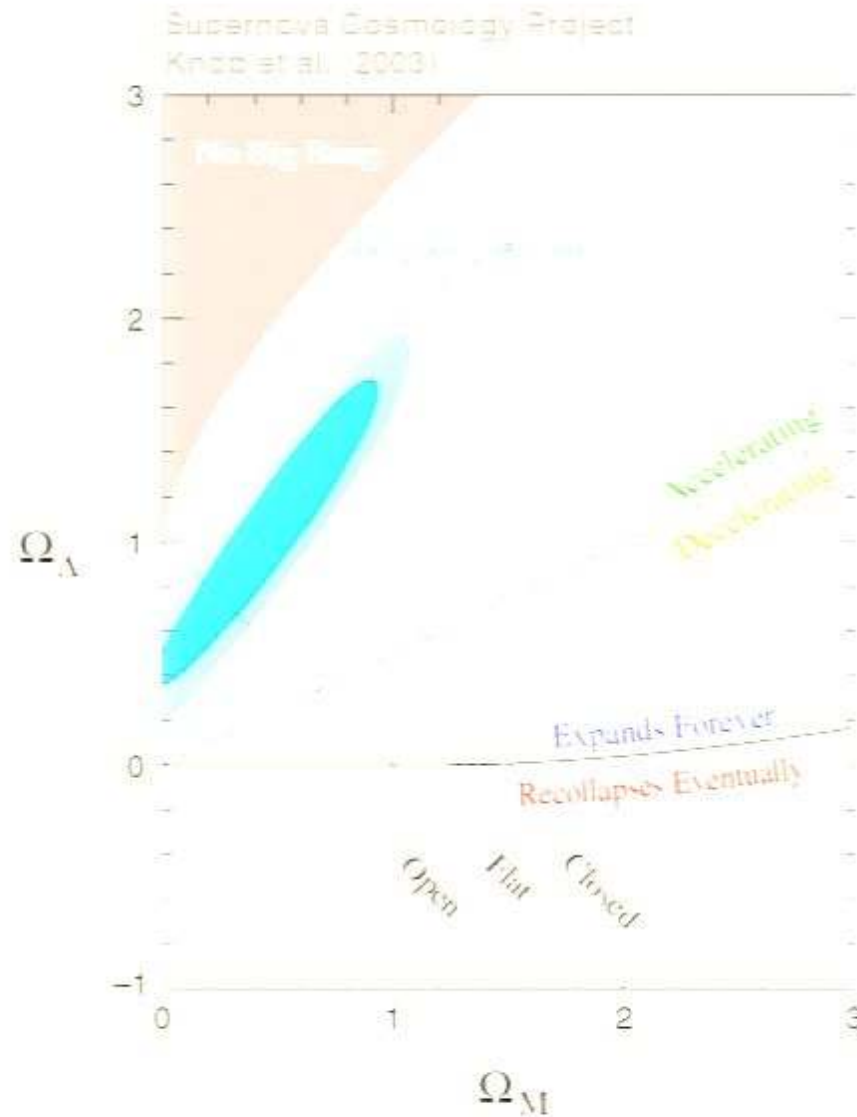
Vacuum Energy and Fine Tuning

Kurt Hinterbichler

Perimeter Institute, Sept. 30, 2008



The universe is accelerating



Einstein's equations don't work

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \neq \frac{1}{m_P^2}T_{\mu\nu}$$

Put in observed expansion history

Put in all observed mass/energy
(including dark matter)

Easy to fix

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Alter right hand side:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{m_P^2} [T_{\mu\nu} - m_P^2 \Lambda g_{\mu\nu}] \quad \text{“Mysterious dark energy”}$$

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Or alter left hand side:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{m_P^2} T_{\mu\nu} \quad \text{“Modified gravity”}$$

Easy to Fix

$$S = \frac{m_p^2}{2} \int d^4x \sqrt{-g} (R - 2\Lambda) + \mathcal{L}_M$$

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- Can reproduce any expansion history $a(t)$ (with $dH/dt < 0$) by adding a minimally coupled scalar with the right potential.
- The issue is that of plausibility of the theory.

Fine tuning?

$$S = \int d^4x \sqrt{-g} \left(\frac{m_P^2}{2} R - \rho_{vac} \right) + \mathcal{L}_M$$

Measured parameters

$$\left\{ \begin{array}{l} m_p \sim 10^{19} \text{ GeV} \\ H^2 \sim \Lambda \sim (10^{-33} \text{ eV})^2 \end{array} \right. \quad \frac{m_P^2}{\Lambda} \sim 10^{122}$$

$$\rho_{vac} \sim m_P^2 H^2 \sim (10^{-3} \text{ eV})^4$$

Huge

What's the problem with large/small numbers in a theory?

Why the vacuum energy scale should be m_P

UV theory: scalar with mass M

$$\int d^4x \sqrt{-g} \left[\frac{m_P^2}{2} R - \rho_{vac} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} M^2 \phi^2 \right]$$

Integrate out a scalar, match to UV theory

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Integrate out a scalar, match to UV theory

$$\begin{aligned} \frac{1}{\sqrt{-g}} \mathcal{L}_{eff} &= \overbrace{\left[\rho_{vac} + \frac{M^4}{4(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right) \right]}^{\rho_{vac}^{eff}} + \frac{1}{2} \overbrace{\left[m_p^2 + \frac{M^2}{6(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right) \right]}^{m_p^{eff}} R \\ &+ \frac{1}{12(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right) \left[\frac{1}{30} R_{\mu\nu} R^{\mu\nu} - \frac{1}{30} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - \frac{1}{12} R^2 + \frac{1}{5} \square R \right] + \dots \end{aligned}$$

Technical naturalness

Suppose some symmetry ensures $\rho_{\text{vac}}=0$. Quantum corrections to ρ_{vac} will vanish.

Now add some term with a small parameter ξ that breaks the symmetry. Quantum corrections are proportional to ξ , since they must vanish as $\xi \rightarrow 0$.

$$\frac{1}{\sqrt{-g}} \mathcal{L}_{eff} = \left[\rho_{\text{vac}} + \xi \frac{M^4}{4(4\pi)^2} \log \left(\frac{M^2}{\mu^2} \right) \right] + \dots$$

We can then hope to find a UV mechanism to make the bare ρ_{vac} small. Quantum mechanics won't ruin it.

Modified gravity

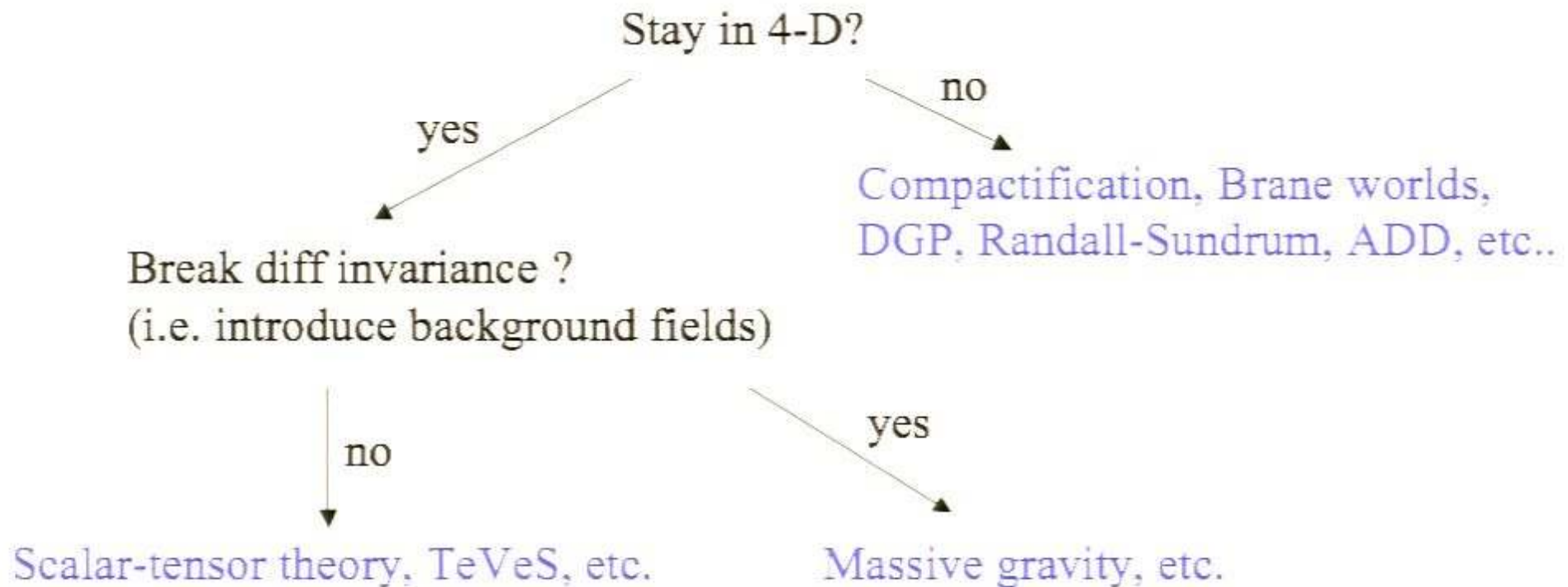
Modified gravity = change the lagrangian

$$\mathcal{L} = \sqrt{-g}F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}, \dots, \nabla_{\mu}R\nabla^{\mu}R, R_{\mu\nu}R^{\mu}_{\lambda}R^{\lambda\nu}, \dots)$$

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$F(R)$ gravity

$$S = \int d^4x \sqrt{-g} F(R) + \mathcal{L}_m(g_{\mu\nu})$$

Fourth order equations of motion

$$F'(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} + g_{\mu\nu}\nabla^2 F'(R) - \nabla_\mu \nabla_\nu F'(R) = \frac{1}{2}T_{\mu\nu}$$

Introduce auxiliary scalar

$$\int d^4x \sqrt{-g} [F(\phi) + F'(\phi)(R - \phi)] + \mathcal{L}_m(g_{\mu\nu})$$

$$F''(\phi) (R - \phi) = 0$$

(Teyssandier, Tourrenc, Whitt,
Barrow, Cotsakis, Wands, Chic
1983-2003)

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Change of variables to Einstein frame

$$\tilde{g}_{\mu\nu} = 2\kappa^2 F'(\phi) g_{\mu\nu} \quad V(\phi) = \frac{\phi F' - F}{(2\kappa^2)^2 F'^2} \quad F'(\phi) = e^{\sqrt{\frac{2}{3}}\kappa\varphi}$$

Ghost free scalar-tensor theory

$$\int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\partial\varphi)^2 - V(\phi(\varphi)) \right] + \mathcal{L}_m(g_{\mu\nu})$$

More complicated lagrangians

$$\int d^4x \sqrt{-g} F (R, R_{\mu\nu} R^{\mu\nu}, R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta})$$

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Equivalent to scalar-tensor theory plus possible spin-2 ghost

$$\int \sqrt{-g} d^4x \left[\frac{1}{3} (3F_1 + 2(F_2 + F_3)\phi_4) R + \frac{1}{2} (F_2 + 4F_3) C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right. \\ \left. - \frac{1}{2} (F_2 + 2F_3) (R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}) \right. \\ \left. F - \phi_1 F_1 - \phi_2 F_2 - \phi_3 F_3 - \frac{1}{3} (F_2 + F_3) \phi_4^2 \right].$$

Getting a small Λ

CDTT model $F(R) = \frac{1}{2\kappa^2} \left(R - \frac{\mu^4}{R} \right)$ Solution $R = \sqrt{3}\mu^2$

(Carroll, Duvvuri, Trodden, Turner, 2004)

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Effective scalar potential

$$V(\varphi) = \frac{e^{\sqrt{\frac{2}{3}}\kappa\varphi} \mu^{8/3}}{\kappa^{4/3} \left((\kappa\mu)^{4/3} + e^{(\frac{2}{3})^{3/2}\kappa\varphi} \right)^2}$$

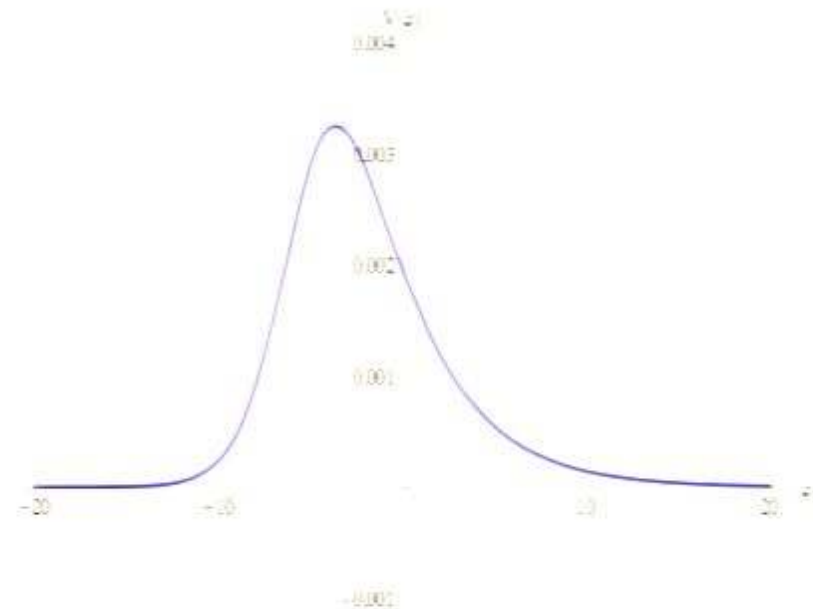


Figure 2: $V(\varphi)$ for the values $\kappa = 1$, $\mu = 0.1$.

$$V(\varphi) = \frac{3\sqrt{3}\mu^2}{16\kappa^2} - \frac{\mu^2}{32\sqrt{3}}\varphi'^2 + \dots$$

Higher interactions go like $\mu^2 \kappa^n \phi^{n+2}$

As EFT it is valid up to the Planck scale

UV “completion” of CDTT

$R\phi^2$ model (Batra, Hinterbichler, Hut, Kabat, 2007)

$$\int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \xi R \phi^2 + J\phi - \rho_{\text{vac}} \right)$$

\swarrow
 $\langle \bar{\psi} \psi \rangle$

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There are now two different vacuum solutions

$$H^2 = \frac{\kappa^2}{6} \left[\rho_{\text{vac}} \pm \sqrt{\rho_{\text{vac}}^2 - \frac{3J^2}{4\xi\kappa^2}} \right]$$



Assuming $J^2 \ll \xi\kappa^2\rho_{\text{vac}}^2$

High curvature

$$H^2 \approx \frac{1}{6} \kappa^2 \rho_{\text{vac}}$$

Low curvature

$$H^2 \approx J^2 / (16\xi\rho_{\text{vac}})$$

$$\langle \phi \rangle = \frac{J}{\xi R}$$

If J is reasonable, the scalar VEV is huge

$$(m_P^{\text{eff}})^2 = \frac{1}{\kappa^2} - \xi \langle \phi \rangle^2$$

The $R\phi^2$ term contributes to the effective Planck mass, so the bare Planck mass must be tuned to nearly cancel the large VEV

$$\rho_{\text{vac}}^{\text{eff}} = \rho_{\text{vac}} - J \langle \phi \rangle$$

The source nearly cancels the large bare CC

$$\left(\frac{1}{\kappa^2} - \xi \langle \phi \rangle^2 \right) H^2 = \frac{1}{3} (\rho_{\text{vac}} - J \langle \phi \rangle)$$

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-
- The fine tuning of the CC has been shifted into other parameters.
 - The low curvature solution is classically unstable.
 - The model can pass solar system and stability constraints, but only by making ξ very small (10^{-125}).
 - There are large ϕ VEVs. This requires that the scalar potential be extremely flat.
 - Under radiative corrections, small values of ξ and J are technically natural since $\xi = J = 0$ has enhanced symmetry $\phi \rightarrow \phi + \text{const.}$
 - Radiative correction destabilize $m_{\text{P}}^{\text{eff}}$.

There are now two tunings. The model is secretly more finely tuned than a bare CC.

$$\rho_{\text{vac}}^{\text{eff}} \ll \rho_{\text{vac}}$$

$$\rho_{\text{vac}}^{\text{eff}} \ll (m_P^{\text{eff}})^4$$



$$\frac{\rho_{\text{vac}}}{m_P^4} \sim 10^{-240}$$

$$\frac{J^2 m_P^2}{\xi \rho_{\text{vac}}^2} \sim 1 + \mathcal{O}(10^{-120})$$

More general perspective

General scalar-tensor action

$$S = \int d^4x \sqrt{-g} \left[f(\phi) R - \frac{1}{2} h(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_{\text{matter}}$$

Constant curvature solutions

$$R = \frac{2V}{f}, \quad R = \frac{dV}{df}$$

Effective vacuum energy and Planck mass

$$\rho_{\text{vac}}^{\text{eff}} = \langle V(\phi) \rangle, \quad (m_P^{\text{eff}})^2 = 2 \langle f(\phi) \rangle$$

Build models with desired CC and Planck mass

(Barra, Hinterbichler, Hu, Kabat, 2007)

Given an $F(R)$ theory with the right solutions, we can put back the scalar

$$S = \int d^4x \sqrt{-g} \mathcal{L}(R), \quad \mathcal{L}(R) = fR - V$$
$$f = d\mathcal{L}/dR, \quad V = fR - \mathcal{L}$$

Make things dimensionless for convenience

$$y = R/4 (m_P^{\text{eff}})^2, \quad \mathcal{L}(R) = (m_P^{\text{eff}})^4 F(y), \quad \epsilon = \frac{R_0}{4} (m_P^{\text{eff}})^2$$

Any function $F(y)$ satisfying the following two conditions will work

$$F(\epsilon) = \epsilon, \quad F'(\epsilon) = 2$$

Some examples

$$F(y) = \exp(y^2/\epsilon) - 1, \sqrt{1 + 2y^2/\epsilon} - 1, 2\epsilon \log(y/\epsilon) + \epsilon, \dots$$

Expanding in powers of ϵ $F(y) = \sum_{n=-\infty}^{\infty} a_n y^n$

Conditions become $\sum_n a_n \epsilon^n = \epsilon, \sum_n n a_n \epsilon^{n-1} = 2$

Some examples

$$F(y) = \exp(y^2/\epsilon) - 1, \sqrt{1 + 2y^2/\epsilon} - 1, 2\epsilon \log(y/\epsilon) + \epsilon, \dots$$

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Conditions become $\sum_n a_n \epsilon^n = \epsilon, \sum_n n a_n \epsilon^{n-1} = 2$

$$F(y) = 2y - \epsilon \implies \text{Einstein gravity + CC}$$

$$F(y) = y^2/\epsilon \implies \int d^4x \sqrt{-g} \left(\phi R - \frac{1}{2} h(\partial\phi)^2 - 4\epsilon\phi^2 \right)$$

$$a_0 = \text{arbitrary}, a_1 = \frac{3\epsilon - a_0}{2\epsilon}, a_{-1} = -\epsilon(a_0 + \epsilon)/2 \implies R\phi^2 \text{ model}$$

$$a_0 = \text{arbitrary}, a_{-1} = -2\epsilon(a_0 - 2\epsilon), a_{-2} = \epsilon^2(a_0 - 3\epsilon)$$

$$\hookrightarrow S = (m_P^{\text{eff}})^4 \int d^4x \sqrt{-g} \left(\frac{4(m_P^{\text{eff}})^2 a_{-1}}{P} + \frac{16(m_P^{\text{eff}})^4 a_{-2}}{P^2} + a_0 \right)$$

Still double tuned

Conditions are $F(\epsilon) = \epsilon, F'(\epsilon) = 2$

← One tuning

Which suggests $F(0) \sim -\epsilon$

Having Planck scale bare CC means $F(y) \sim -1 + \dots$

Which suggests $F(0) \sim -1$

} Second tuning

- In $F(R)$ gravity it is unnatural to have a small effective vacuum energy. It is doubly unnatural to obtain a small effective vacuum energy by nearly canceling off a large underlying vacuum energy.
- These tunings are not generally technically natural.

Lovelock terms

$$\mathcal{L}_m = \frac{1}{2^m} \delta_{\mu_1 \nu_1 \dots \mu_m \nu_m}^{\alpha_1 \beta_1 \dots \alpha_m \beta_m} R_{\alpha_1 \beta_1}{}^{\mu_1 \nu_1} \dots R_{\alpha_m \beta_m}{}^{\mu_m \nu_m}$$

(Lovelock, 1971)

$$\mathcal{L}_0 = 1$$

$$\mathcal{L}_1 = R$$

$$\mathcal{L}_2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$

$$\begin{aligned} \mathcal{L}_3 = & 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu} \\ & + 3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} - 12RR^{\mu\nu}R_{\mu\nu} + R^3 \end{aligned}$$

- Integral is a topological invariant \Rightarrow total derivative in $2m$ dimensions.
- Gives second order equations in higher dimensions, vanishes identically in lower dimensions (does not add new DOF).
- Gives second order equations when multiplied by scalars.
- Only terms where metric and Palatini variational approaches are equivalent.
- Comes from α' correction of string theory

Gauss-Bonnet model

(Barra, Hinterbichler, Hui, Kabat 2007)

$$\int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} \xi \kappa^2 \mathcal{G} \phi^2 + J\phi - \rho_{\text{vac}} \right)$$

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$

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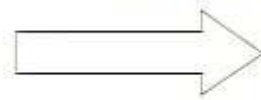
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Vacuum equations of motion

$$R = 4\kappa^2 (\rho_{\text{vac}} - J\phi)$$

$$\phi = \frac{6J}{\xi \kappa^2 R^2}$$

$$J^2 \ll \xi \kappa^6 \rho_{\text{vac}}^3$$



Large curvature solution

$$R \approx 4\kappa^2 \rho_{\text{vac}}$$

$$R \approx \pm \sqrt{\frac{6J^2}{\xi \kappa^2 \rho_{\text{vac}}}}$$

Small curvature solutions

Total derivative structure of the non-minimal coupling ensures:

$$m_P^{\text{eff}} = m_P$$

Only one small parameter needed:

$$\left(\rho_{\text{vac}}^{\text{eff}}\right)^2 = \frac{J^2 m_P^6}{6\xi\rho_{\text{vac}}}$$

Same tuning as a bare CC:

$$\frac{J^2 m_P^6}{\xi\rho_{\text{vac}}^3} \sim \left(\frac{\rho_{\text{vac}}^{\text{eff}}}{\rho_{\text{vac}}}\right)^2 \sim 10^{-240}$$

Low curvature solution is unstable, but is stable on cosmological time scales provided $\xi < O(1)$.

Drawbacks:

- Large scalar VEV requires the potential to be extremely flat
- Passing solar system tests requires taking ξ small

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- Under radiative corrections, small values of ξ and J are technically natural since $\xi=J=0$ has enhanced symmetry $\phi \rightarrow \phi + \text{const.}$
- Corrections to scalar mass still dangerous, i.e. $\delta m^2 \sim \xi \Lambda_{UV}^2 \sim \xi m_P^2$ ruins $m^2 < \xi \kappa^2 H^4$

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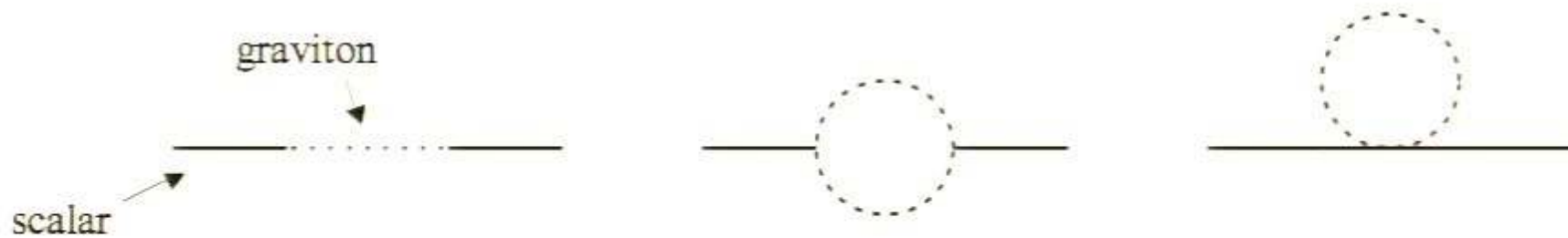
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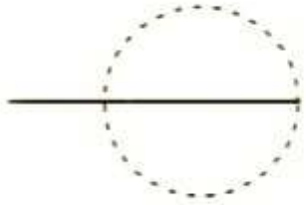
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First correction comes at 2-loops

$$m^2 \sim \xi^2 \kappa^4 \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} p^4 q^4 \frac{\kappa^2}{p^2} \frac{\kappa^2}{q^2} \frac{1}{(p+q)^2} \sim \xi^2 \kappa^8 \Lambda_{UV}^{10} \sim \xi^2 m_P^2$$

Does not spoil see-saw for $\xi^2 m_P^2 < \xi \kappa^2 H^4 \rightarrow \xi < (\rho_{\text{vac}}^{\text{eff}} / m_P^4)^2 \sim 10^{-24}$

Corrections to the scalar source J



$$\delta J \sim \xi \kappa^2 \xi \kappa^2 \langle \phi \rangle \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} p^4 q^4 \frac{\kappa^2}{p^2} \frac{\kappa^2}{q^2} \frac{1}{(p+q)^2} \sim \xi^2 \kappa^8 \langle \phi \rangle \Lambda_{UV}^{10} \sim \langle \phi \rangle \xi^2 m_P^2$$

$$\frac{\delta J}{J} \sim \frac{\xi m_P^4}{H^4}$$

$$\text{Need } \frac{\delta J}{J} < 1 \quad \longrightarrow \quad \xi < 10^{-240}$$

Corrections to the vacuum energy

$$\delta\rho_{vac} \sim \Lambda_{UV}^4 \sim m_P^4 \sim \rho_{vac}$$

- Preserves the tuning.
- The VEV $\langle\phi\rangle$ shifts to maintain a small effective vacuum energy.
$$\delta\rho_{vac}^{\text{eff}} \sim \rho_{vac}^{\text{eff}}$$
- Gauss-Bonnet structure is crucial. Assures that the effective m_P is not shifted as well.
- Technically natural tuning of the CC.

Landscape/swampland

- Easy for fine tunings to be shifted around
- 10^{500} vacuua of the landscape: 10^{100} have small CC, 10^{120} have other mechanisms?

Conclusions

- $F(R)$ models illustrate that today's Hubble expansion energy can be accounted for even in the presence of a Planck scale CC.
- Modified gravity can not really cure fine tuning problems, but it can push tuning into other parameters.
- Pushing the tuning into other parameters can make it technically natural, as in the Gauss-Bonnet model.

Future questions:

- Realistic cosmological solutions with inflation?
High curvature vacuum \rightarrow low curvature vacuum?
- Realization in fundamental theory?

