Title: Vacuum Energy and Fine Tuning

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Abstract: I will discuss fine tuning in modified gravity models that can account for todayâ€™s dark energy. I will introduce some models where the underlying cosmological constant may be Planck scale but starts as a redundant coupling which can be eliminated by a field redefinition. The observed vacuum energy arises when the redundancy is explicitly broken. Iâ€™ll give a recipe for constructing models that realize this mechanism and satisfy all solar system constraints on gravity, including one based on Gauss-Bonnet gravity which provides a technically natural explanation for dark energy.

Vacuum Energy and Fine Tuning

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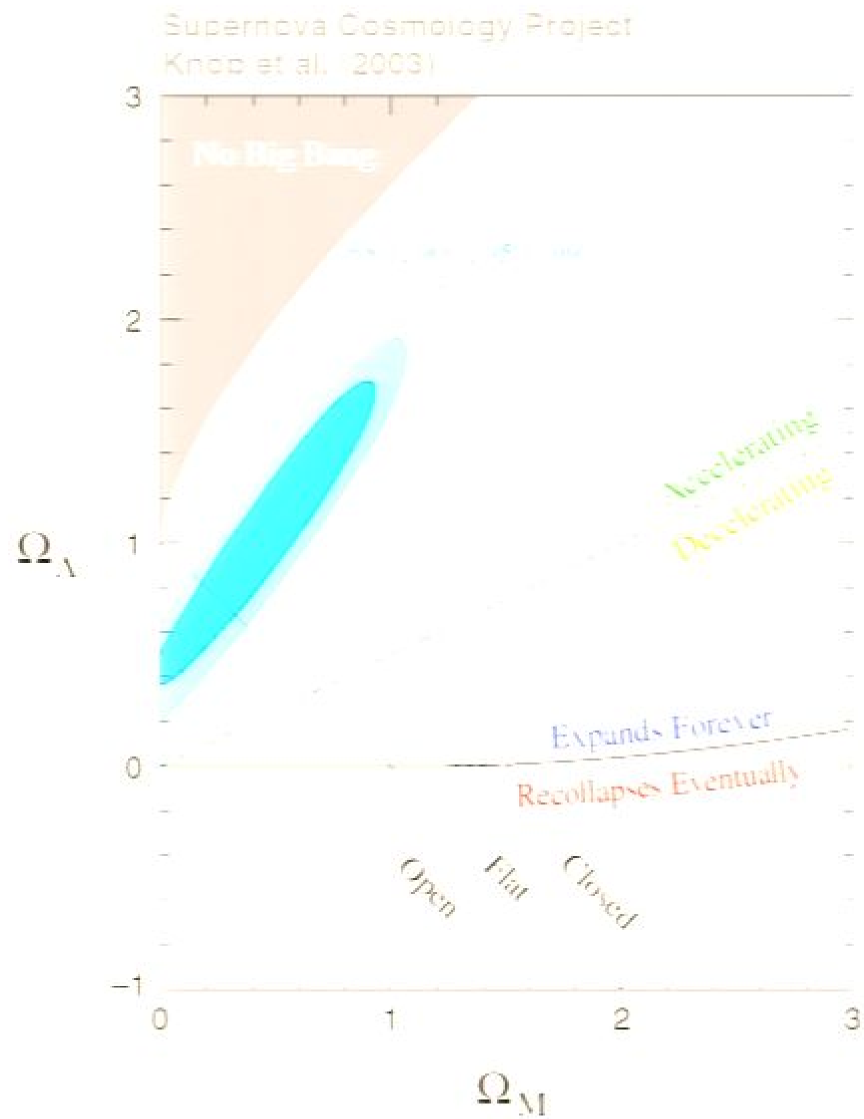
IN THE CITY OF NEW YORK

ISCAP

The universe is accelerating



Einstein's equations don't work

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \neq \frac{1}{m_P^2} T_{\mu\nu}$$

Put in observed expansion history

Put in all observed mass/energy
(including dark matter)

Easy to fix

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Alter right hand side:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{m_P^2}\left[T_{\mu\nu} - m_P^2\Lambda g_{\mu\nu}\right]$$ "Mysterious dark energy

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"Mysterious dark energy

Or alter left hand side:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{m_P^2}T_{\mu\nu}$$

"Modified gravity"

Easy to Fix

$$S = \frac{m_p^2}{2} \int d^4x \, \sqrt{-g} \, (R - 2\Lambda) + \mathcal{L}_M$$

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- Can reproduce any expansion history *a(t)* (with $dH/dt<0$) by adding a minimally coupled scalar with the right potential.

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- Can reproduce any expansion history *a(t)* (with $dH/dt<0$) by adding a minimally coupled scalar with the right potential.

- The issue is that of plausibility of the theory.

Fine tuning?

$$S = \int d^4x \sqrt{-g} \left(\frac{m_p^2}{2} R - \rho_{vac} \right) + \mathcal{L}_M$$

Measured parameters

$$\begin{cases} m_p \sim 10^{19}\ GeV \\ H^2 \sim \Lambda \sim (10^{-33}\ eV)^2 \end{cases}$$

$$\frac{m_P^2}{\Lambda} \sim 10^{122}$$

$$\rho_{vac} \sim m_P^2 H^2 \sim (10^{-3}\ eV)^4$$

Huge

What's the problem with large/small numbers in a theory?

Why the vacuum energy scale should be m_P

UV theory: scalar with mass M

$$\int d^4x\,\sqrt{-g}\left[\frac{m_p^2}{2}R-\rho_{vac}-\frac{1}{2}(\partial\phi)^2-\frac{1}{2}M^2\phi^2\right]$$

Integrate out a scalar, match to UV theory

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Integrate out a scalar, match to UV theory

$$\begin{aligned}\frac{1}{\sqrt{-g}} \mathcal{L}_{eff} &= \overbrace{\left[\rho_{vac} + \frac{M^4}{4(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right)\right]}^{\rho_{vac}^{\text{eff}}} + \frac{1}{2} \overbrace{\left[m_p^2 + \frac{M^2}{6(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right)\right]}^{m_p^{\text{eff}}} R \\ &+ \frac{1}{12(4\pi)^2} \log\left(\frac{M^2}{\mu^2}\right) \left[\frac{1}{30} R_{\mu\nu} R^{\mu\nu} - \frac{1}{30} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - \frac{1}{12} R^2 + \frac{1}{5} \Box R \right] + \cdots\end{aligned}$$

Technical naturalness

Suppose some symmetry ensures ρ_{vac}=0. Quantum corrections to ρ_{vac} will vanish.

Now add some term with a small parameter ξ that breaks the symmetry. Quantum corrections are proportional to ξ, since they must vanish as $\xi \rightarrow 0$.

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{eff} = \left[\rho_{\text{vac}} + \xi\frac{M^4}{4(4\pi)^2}\log\left(\frac{M^2}{\mu^2}\right)\right] + \cdots$$

We can then hope to find a UV mechanism to make the bare ρ_{vac} small. Quantum mechanics won't ruin it.

Modified gravity

Modified gravity = change the lagrangian

$$\mathcal{L} = \sqrt{-g}F(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}, \ldots, \nabla_{\mu}R\nabla^{\mu}R, R_{\mu\nu}R^{\mu}{}_{\lambda}R^{\lambda\nu}, \ldots$$

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Stay in 4-D?

yes

no

Compactification, Brane worlds, DGP, Randall-Sundrum, ADD, etc..

Break diff invariance ?
(i.e. introduce background fields)

no

yes

Scalar-tensor theory, TeVeS, etc.

Massive gravity, etc.

$F(R)$ gravity

$$S = \int d^4x \ \sqrt{-g} F(R) + \mathcal{L}_m(g_{\mu\nu})$$

Fourth order equations of motion

$$F'(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} + g_{\mu\nu}\nabla^2 F'(R) - \nabla_\mu \nabla_\nu F'(R) = \frac{1}{2}T_{\mu\nu}$$

Introduce auxiliary scalar

$$\int d^4x\sqrt{-g}\,[F(\phi) + F'(\phi)(R-\phi)] + \mathcal{L}_m(g_{\mu\nu})$$

$$F''(\phi)\,(R-\phi) = 0$$

(Teyssandier, Tourrenc, Whitt, Barrow, Cotsakis, Wands, Chib 1983-2003)

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Change of variables to Einstein frame

$$\tilde{g}_{\mu\nu}=2\kappa^2F'(\phi)g_{\mu\nu}\qquad V(\phi)=\frac{\phi F'-F}{(2\kappa^2)^2F'^2}\qquad F'(\phi)=e^{\sqrt{\frac{2}{3}}\kappa\varphi}$$

Ghost free scalar-tensor theory

$$\int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}(\partial\varphi)^2-V\left(\phi(\varphi)\right)\right]+\mathcal{L}_m(g_{\mu\nu})$$

More complicated lagrangians

$$\int d^4x \, \sqrt{-g} F\left(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right)$$

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Equivalent to scalar-tensor theory plus possible spin-2 ghost

$$\int \sqrt{-g}d^4x\Big[\frac{1}{3}\left(3F_1 + 2(F_2+F_3)\phi_4\right)R + \frac{1}{2}(F_2+4F_3)C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$$
$$-\frac{1}{2}(F_2+2F_3)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right)$$
$$F - \phi_1F_1 - \phi_2F_2 - \phi_3F_3 - \frac{1}{3}(F_2+F_3)\phi_4^2\Big].$$

(Chiba, 2005)

Getting a small Λ

CDTT model $F(R) = \frac{1}{2\kappa^2}\left(R - \frac{\mu^4}{R}\right)$ Solution $R = \sqrt{3}\mu^2$

(Carroll, Duvvuri, Trodden, Turner, 2004)

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Effective scalar potential

$$V(\varphi) = \frac{e^{\sqrt{\frac{2}{3}}\kappa\varphi}\mu^{8/3}}{\kappa^{4/3}\left((\kappa\mu)^{4/3} + e^{\left(\frac{2}{3}\right)^{3/2}\kappa\varphi}\right)^2}$$

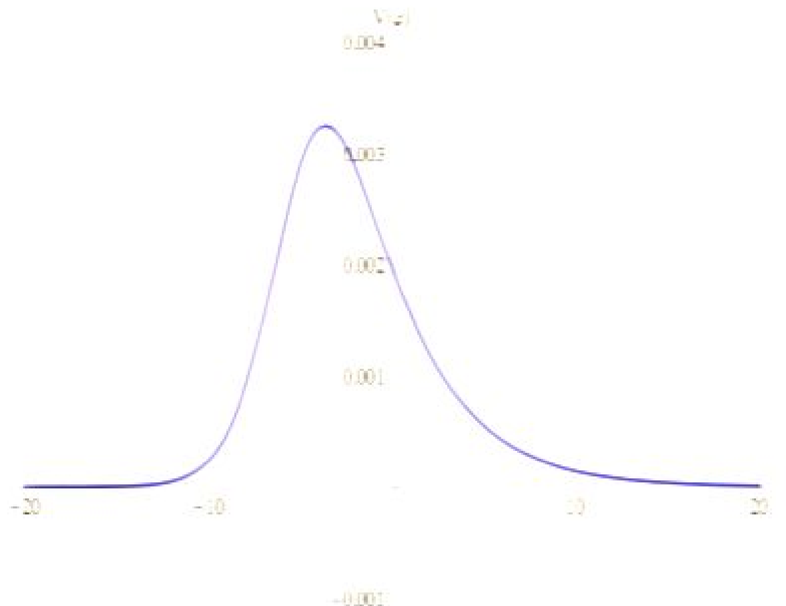

Figure 2: $V(\varphi)$ for the values $\kappa = 1$, $\mu = 0.1$.

$$V(\varphi') = \frac{3\sqrt{3}\mu^2}{16\kappa^2} - \frac{\mu^2}{32\sqrt{3}}\varphi'^2 + \cdots$$

Higher interactions go like $\mu^2\kappa^n\phi^{n+2}$

As EFT it is valid up to the Planck scale

UV "completion" of CDTT

$R\phi^2$ model (Batra, Hinterbichler, Hui, Kabat, 2007)

$$\int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\xi R\phi^2 + J\phi - \rho_{\rm vac}\right)$$

$$\langle\bar{\psi}\psi\rangle$$

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Integrate out the scalar $\langle\phi\rangle = \frac{J}{\xi R}$

$$\int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2}R + \frac{J^2}{2\xi R} - \rho_{\rm vac}\right)$$

There are now two different vacuum solutions

$$H^2 = \frac{\kappa^2}{6}\left[\rho_{\rm vac} \pm \sqrt{\rho_{\rm vac}^2 - \frac{3J^2}{4\xi\kappa^2}}\right]$$

$\Downarrow$ Assuming $J^2 \ll \xi\kappa^2\rho_{\rm vac}^2$

High curvature

$$H^2 \approx \frac{1}{3}\kappa^2\rho_{\rm vac}$$

Low curvature

$$H^2 \approx J^2/(16\xi\rho_{\rm vac})$$

$$\langle\phi\rangle = \frac{J}{\xi R}$$

If J is reasonable, the scalar VEV is huge

$$\left(m_P^{\rm eff}\right)^2 = \frac{1}{\kappa^2} - \xi\langle\phi\rangle^2$$

The $R\phi^2$ term contributes to the effective Planck mass, so the bare Plank mass must be tuned to nearly cancel the large VEV

$$\rho_{\rm vac}^{\rm eff} = \rho_{\rm vac} - J\langle\phi\rangle$$

The source nearly cancels the large bare CC

$$\left(\frac{1}{\kappa^2} - \xi\langle\phi\rangle^2\right) H^2 = \frac{1}{3}\left(\rho_{\rm vac} - J\langle\phi\rangle\right)$$

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• The fine tuning of the CC has been shifted into other parameters.

• The low curvature solution is classically unstable.

• The model can pass solar system and stability constraints, but only by making ξ very small (10^{-125}).

• There are large ϕ VEVs. This requires that the scalar potential be extremely flat.

• Under radiative corrections, small values of ξ and J are technically natural since $\xi = J = 0$ has enhanced symmetry $\phi \to \phi + const$.

• Radiative correction destabilize $m_{\mathrm{P}}^{\mathrm{eff}}$.

There are now two tunings. The model is secretly more finely tuned than a bare CC.

$$\rho_{\text{vac}}^{\text{eff}} \ll \rho_{\text{vac}}$$

$$\rho_{\text{vac}}^{\text{eff}} \ll \left(m_P^{\text{eff}}\right)^4$$

$$\Longrightarrow$$

$$\frac{\rho_{\text{vac}}}{m_P^4} \sim 10^{-240}$$

$$\frac{J^2 m_P^2}{\xi \rho_{\text{vac}}^2} \sim 1 + \mathcal{O}(10^{-120})$$

More general perspective

General scalar-tensor action

$$S = \int d^4x \sqrt{-g} \left[f(\phi) R - \frac{1}{2} h(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + S_{\mathrm{matter}}$$

Constant curvature solutions

$$R = \frac{2V}{f}, \; R = \frac{dV}{df}$$

Effective vacuum energy and Planck mass

$$\rho_{\mathrm{vac}}^{\mathrm{eff}} = \langle V(\phi) \rangle, \;\; \left(m_P^{\mathrm{eff}} \right)^2 = 2 \langle f(\phi) \rangle$$

Build models with desired CC and Planck mass

(Batra, Hinterbichler, Hui, Kabat, 2007)

Given an *F(R)* theory with the right solutions, we can put back the scalar

$$S = \int d^4x\sqrt{-g}\,\mathcal{L}(R), \quad \mathcal{L}(R) = fR - V$$

$$f = d\mathcal{L}/dR, \quad V = fR - \mathcal{L}$$

Make things dimensionless for convenience

$$y = R/4\left(m_P^{\text{eff}}\right)^2, \quad \mathcal{L}(R) = \left(m_P^{\text{eff}}\right)^4 F(y), \quad \epsilon = \frac{R_0}{4}\left(m_P^{\text{eff}}\right)^2$$

Any function *F(y)* satisfying the following two conditions will work

$$F(\epsilon) = \epsilon, \quad F'(\epsilon) = 2$$

Some examples

$$F(y) = \exp\left(y^2/\epsilon\right) - 1,\ \sqrt{1 + 2y^2/\epsilon} - 1,\ 2\epsilon \log(y/\epsilon) + \epsilon, \ldots$$

Expanding in powers of R $\quad F(y) = \sum_{n=-\infty}^{\infty} a_n y^n$

Conditions become $\quad \sum_n a_n \epsilon^n = \epsilon,\ \sum_n n a_n \epsilon^{n-1} = 2$

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$F(y) = 2y - \epsilon \quad \Longrightarrow \quad$ Einstein gravity + CC

$$F(y) = y^2/\epsilon \quad \Longrightarrow \quad \int d^4x \sqrt{-g} \left(\phi R - \frac{1}{2} h (\partial \phi)^2 - 4 \epsilon \phi^2 \right)$$

$a_0 = \text{arbitrary},\ a_1 = \dfrac{3\epsilon - a_0}{2\epsilon},\ a_{-1} = -\epsilon(a_0 + \epsilon)/2 \quad \Longrightarrow \quad$ $R\phi^2$ model

$$a_0 = \text{arbitrary},\ a_{-1} = -2\epsilon(a_0 - 2\epsilon),\ a_{-2} = \epsilon^2 (a_0 - 3\epsilon)$$

$$S = \left(m_P^{\text{eff}}\right)^4 \int d^4x \sqrt{-g} \left(\frac{4 \left(m_P^{\text{eff}}\right)^2 a_{-1}}{R} + \frac{16 \left(m_P^{\text{eff}}\right)^4 a_{-2}}{R^2} + a_0$$

Still double tuned

Conditions are $F(\epsilon) = \epsilon, \; F'(\epsilon) = 2$

One tuning

Which suggests $F(0) \sim -\epsilon$

Having Planck scale bare CC means $F(y) \sim -1 + \cdots$

Which suggests $F(0) \sim -1$

Second tuning

- In *F(R)* gravity it is unnatural to have a small effective vacuum energy. It is doubly unnatural to obtain a small effective vacuum energy by nearly canceling off a large underlying vacuum energy.

- These tunings are not generally technically natural.

Lovelock terms

$$\mathcal{L}_m = \frac{1}{2^m}\delta^{\alpha_1\beta_1\ldots\alpha_m\beta_m}_{\mu_1\nu_1\ldots\mu_m\nu_m}\, R_{\alpha_1\beta_1}{}^{\mu_1\nu_1} \ldots . R_{\alpha_m\beta_m}{}^{\mu_m\nu_m}$$

(Lovelock, 1971)

$$\mathcal{L}_0 = 1$$

$$\mathcal{L}_1 = R$$

$$\mathcal{L}_2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$$

$$\mathcal{L}_3 = 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu}$$
$$+3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} - 12RR^{\mu\nu}R_{\mu\nu} + R^3$$

- Integral is a topological invariant $\Rightarrow$ total derivative in $2m$ dimensions.
- Gives second order equations in higher dimensions, vanishes identically in lower dimensions (does not add new DOF).
- Gives second order equations when multiplied by scalars.
- Only terms where metric and Palatini variational approaches are equivalent.
- Comes from α' correction of string theory

Gauss-Bonnet model

(Batra, Hinterbichler, Hui, Kabat, 2007)

$$\int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\xi\kappa^2\mathcal{G}\phi^2 + J\phi - \rho_{\rm vac}\right)$$

$$\boxed{\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}}$$

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Vacuum equations of motion

$$R=4\kappa^2(\rho_{\rm vac}-J\phi)$$

$$\phi=\frac{6J}{\xi\kappa^2R^2}$$

$$J^2\ll\xi\kappa^6\rho_{\rm vac}^3 \quad\Longrightarrow$$

Large curvature solution

$$R\approx4\kappa^2\rho_{\rm vac}$$

$$R\approx\pm\sqrt{\frac{6J^2}{\xi\kappa^2\rho_{\rm vac}}}$$

Small curvature solutions

Total derivative structure of the non-minimal coupling ensures:

$$m_P^{\text{eff}} = m_P$$

Only one small parameter needed:

$$\left(\rho_{\text{vac}}^{\text{eff}}\right)^2 = \frac{J^2 m_P^6}{6\xi\rho_{\text{vac}}}$$

Same tuning as a bare CC:

$$\frac{J^2 m_P^6}{\xi\rho_{\text{vac}}^3} \sim \left(\frac{\rho_{\text{vac}}^{\text{eff}}}{\rho_{\text{vac}}}\right)^2 \sim 10^{-240}$$

Low curvature solution is unstable, but is stable on cosmological time scales provided ξ<O(1).

Drawbacks:

- Large scalar VEV requires the potential to be extremely flat
- Passing solar system tests requires taking ξ small

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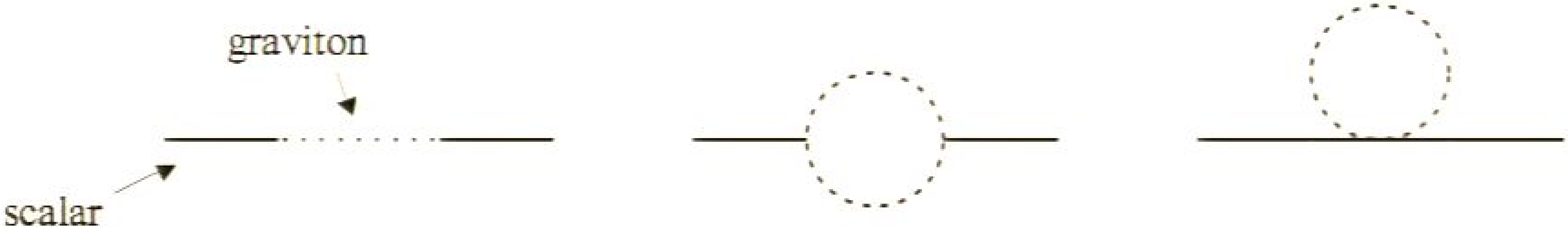
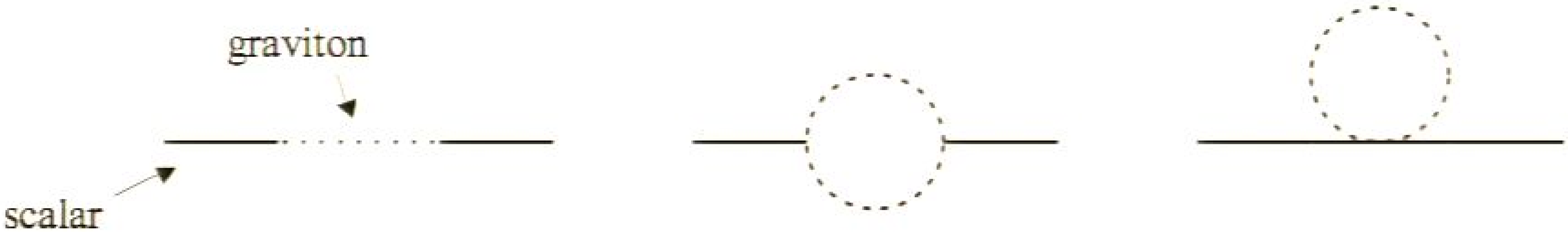
Leading corrections to the scalar mass



Since the Euler density is a total derivative, the vertices vanish when all scalar lines together carry no net momentum into any vertex.

Leading corrections to the scalar mass



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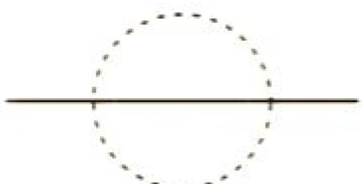
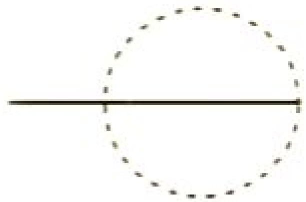
First correction comes at 2-loops



$$m^2 \sim \xi^2 \kappa^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} p^4 q^4 \frac{\kappa^2}{p^2} \frac{\kappa^2}{q^2} \frac{1}{(p+q)^2} \sim \xi^2 \kappa^8 \Lambda_{UV}^{10} \sim \xi^2 m_P^2$$

Does not spoil see-saw for $\xi^2 m_P^2 < \xi \kappa^2 H^4 \longrightarrow \xi < (\rho_{\rm vac}^{\rm eff}/m_P^4)^2 \sim 10^{-24}$

Corrections to the scalar source J



$$\delta J \sim \xi\kappa^2\xi\kappa^2\langle\phi\rangle \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}p^4q^4\frac{\kappa^2}{p^2}\frac{\kappa^2}{q^2}\frac{1}{(p+q)^2} \sim \xi^2\kappa^8\langle\phi\rangle\Lambda_{UV}^{10} \sim \langle\phi\rangle\xi^2 m_P^2$$

$$\frac{\delta J}{J} \sim \frac{\xi m_P^4}{H^4}$$

Need $\quad \dfrac{\delta J}{J} < 1 \quad \longrightarrow \quad \xi < 10^{-240}$

Corrections to the vacuum energy

$$\delta\rho_{vac} \sim \Lambda_{UV}^4 \sim m_P^4 \sim \rho_{vac}$$

- Preserves the tuning.

- The VEV $<\phi>$ shifts to maintain a small effective vacuum energy.
$\delta\rho_{vac}^{\text{eff}} \sim \rho_{vac}^{\text{eff}}$

- Gauss-Bonnet structure is crucial. Assures that the effective m_{P} is not shifted as well.

- Technically natural tuning of the CC.

Landscape/swampland

- Easy for fine tunings to be shifted around

- 10^{500} vacuua of the landscape: 10^{100} have small CC, 10^{120} have other mechanisms?

Conclusions

• *F(R)* models illustrate that today's Hubble expansion energy can be accounted for even in the presence of a Plank scale CC.

• Modified gravity can not really cure fine tuning problems, but it can push tuning into other parameters.

• Pushing the tuning into other parameters can make it technically natural, as in the Gauss-Bonnet model.

Future questions:

• Realistic cosmological solutions with inflation?
High curvature vaccum $\rightarrow$ low curvature vacuum?

• Realization in fundamental theory?