Title: Orientifolds and Twisted KR Theory

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Abstract: I will report on some work in progress with Dan Freed and Greg Moore. In an orientifold background, D-brane charge takes values in a certain twisted version of KR Theory. Moreover, there is a nontrivial background charge (\'tadpole\'). Up \'til now, this background charge has only been calculated rationally -- i.e., ignoring torsion. We derive a formula for it, over the integers. Only after \'inverting 2\', does the charge localize to the fixed point sets of the orientifold action, and we can give a compact formula for it. This reproduces the previously known rational results, but contains new information.

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Background charge, over the integers

Jacques Distler
University of Texas at Austin

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# Why Orientifold?

Like orbifolds: procedure for generating new string theory backgrounds from old.

Has proven very useful for model building: IIB (and IIA) orientifold, "with fluxes" (KKLT et al ...).

- Evades the Gibbons-Maldacena-Nuñez Theorem because orientifold fixed-planes have negative tension, violating the SEC.
- We ***think*** we understand moduli stabilization in this context.

Key feature: orientifolds have "background" D-brane charge. On a compact space, must be cancelled for consistency (Gauss's Law).

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Subsidiary objective: clean up a bit of the bestiary of Orientifolds.

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2. ***Only*** after inverting 2 does the formula for the background charge ***localize*** to the f.p.s.

3. Taking Chern characters, can compare with existing formulæ in the literature

$$(d + u^{-1} H\wedge) G = \underbrace{\check{j}}_{\text{D-branes}} + \underbrace{\check{\mu}}_{\text{orientifold}}$$

N.b.: putting the $H \wedge G$ term on the RHS, as is often done is morally wrong!

# Orbifolds

## Worldsheet description

- $\sigma$-model: $\Sigma \stackrel{\phi}{\to} Y$
- $\Gamma$: discrete group of isometries of $Y$
- Gauge the $\Gamma$-symmetry
  - $\tilde{\Sigma} \to \Sigma$, a principal $\Gamma$-bundle
  - $\tilde{\phi} : \tilde{\Sigma} \to Y$ an equivariant map.
  - $\gamma_{\tilde{\Sigma}}$ is fixed-point free, so our original surface $\Sigma = \tilde{\Sigma} / \Gamma$. This defines a map $\phi : \Sigma \to \mathscr{X}$, where $\mathscr{X} = Y /\!/ \Gamma$ is a "stack".)

$$\begin{array}{ccc} \tilde{\Sigma} & \stackrel{\tilde{\phi}}{\to} & Y \\ \gamma_{\tilde{\Sigma}} \downarrow & & \downarrow \gamma_Y \\ \tilde{\Sigma} & \underset{\tilde{\phi}}{\to} & Y \end{array}$$

- To describe states (CFT operators), allow $\Sigma$ to have in/out boundaries.
  - Restricting to a boundary circle, $S$, get a principal $\Gamma$-bundle, $\tilde{S} \to S$.
  - These are classified by holonomy

$$[\gamma] \in \mathrm{Hom}(\pi_1(S), \Gamma) / \mathrm{conj}$$

Twisted sectors = nontrivial $\Gamma$-bundles $\tilde{S} \to S$ = conjugacy classes of $\Gamma$.

# Orientifolds

Manifold, $Y$, with a discrete group of isometries, $\Gamma$.

$$1 \to \Gamma_0 \to \Gamma \stackrel{\omega}{\to} \mathbb{Z}_2 \to 1$$

(As a set, $\Gamma = \Gamma_0 \amalg \Gamma_1$, but $\Gamma$ can be nonabelian, even if $\Gamma_0$, is abelian.) Gauge $\Gamma$. But, this time, accompany $\gamma \in \Gamma_1$ by orientation-reversal on worldsheet.

$$\begin{array}{cccc} & \tilde{\Sigma} & \stackrel{\tilde{\phi}}{\to} & Y & \\ \gamma_{\tilde{\Sigma}} & \downarrow & & \downarrow & \gamma_Y \\ & \tilde{\Sigma} & \underset{\tilde{\phi}}{\to} & Y & \end{array}$$

- $\tilde{\Sigma}$ ***oriented*** surface. $\tilde{\Sigma} \to \Sigma$ a $\Gamma$-principal bundle.
- $\gamma_{\tilde{\Sigma}}$ fixed point free $\begin{cases} \text{orientation preserving for} & \gamma \in \Gamma_0 \\ \text{orientation reversing for} & \gamma \in \Gamma_1 \end{cases}$

Again, restricting to in/out circles, get $\Gamma$-bundle $\tilde{S} \to S$.

**Claim:** reduction of structure group of $\tilde{S}$ from $\Gamma$ to $\Gamma_0$.

No fixed point free orientation-reversing map from $S^1 \to S^1 \Rightarrow$
$\mathrm{Or}(\Sigma = \tilde{\Sigma}/\Gamma) = \tilde{\Sigma}/\Gamma_0$ and $\mathrm{Or}(S) = S \amalg S$. Restricting to one copy of $S$ gives explicit reduction of str group.

# Abelian Gauge Theory

RR fields in String Theory: an exotic type of abelian gauge theory.

Recall electromagnetism.

- $j_e \in H^{d-1}_{\mathrm{cpt}}(X)$, $j_m \in H^3_{\mathrm{cpt}}(X)$.
- Electromagnetic field trivializes these: $dF = j_m, \quad d * F = j_e$.
- In quantum theory, need Dirac quantization condition.

RR field: replace de Rham cohomology by generalized cohomology theory, $E^\bullet$.
Dirac condition: integrality of bilinear form

$$b(\cdot,\ \cdot) : E^\bullet \times E^\bullet \to H^2(S)$$

$$\mathbb{X} \downarrow X \quad S$$

# Self-Duality

In self-dual theory, electric current determines magnetic current (and vice versa).

$$j_E = \theta(j_M)$$

where $\theta : E^\bullet \to E^{d+2-\bullet}$ is an isomorphism.

To define the quantum theory, need a quadratic refinement, $q(\cdot)$, of the bilinear form

$$b(x_1, x_2) = q(x_1 + x_2) - q(x_1) - q(x_2) \qquad (q(0) = 0)$$

# The Center

A quadratic function is symmetric about its center. Let $\psi(x) = q(x) - q(-x)$, a linear function. Since $b(\cdot, \ \cdot)$ is a perfect pairing, $\psi(x) = -b(\lambda, x)$ for some $\lambda$.

$$q(\lambda) = q(0) = 0$$

The center of $q$ is $\mu$ such that $2\mu = \lambda$. This determines $\mu$ up to 2-torsion. Can do better, but this will suffice for today's lecture.

Type I is an orientifold of IIB (with trivial action of $\Gamma = \mathbb{Z}_2$). The charge group is $KR^0(X) = KO^0(X)$. Freed and Hopkins wrote down $q(\cdot)$ and computed its center

$$-\mu = T + 22 \qquad \text{(up to codimension 8)}$$

which is, indeed, the background charge of this orientifold. (Compare with heterotic dual.)

**Objective:** generalize this.

# Differential Cohomology

$\check{H}^q(X)$: add geometrical data to topological data in $H^q(X, \mathbb{Z})$.

Examples:

- $\check{H}^0(X) = H^0(X, \mathbb{Z})$
- $\check{H}^1(X) = \mathrm{Maps}(X, S^1)$: circle-valued scalar, $\varphi$. $\omega = d\varphi$ closed 1-form, de Rham representative of a class in $H^1(X, \mathbb{Z})$.
- $\check{H}^2(X) = \{\mathrm{isom}(L, \nabla)\}$: $c(L) \in H^2(X, \mathbb{Z})$, the first Chern class of $L$. $F = dA$ a de Rham representative of $c(L)$.
- $\check{H}^3(X)$: where the $B$-field in String Theory lives.
- ***etc.***

# Exact Sequences

More generally, the differential cohomology groups, $\check{H}^q(X)$, fit into the exact sequences

$$0 \to \overbrace{H^{q-1}(X, \mathbb{R}/\mathbb{Z})}^{\text{flat}} \to \check{H}^q(X) \to \Omega^q_{\text{closed}, \mathbb{Z}}(X) \to 0$$

$$0 \to \underbrace{\Omega^{q-1}(X)/\Omega^{q-1}_{\text{closed}, \mathbb{Z}}(X)}_{\text{topologically trivial}} \to \check{H}^q(X) \to H^q(X, \mathbb{Z}) \to 0$$

N.b.: $\Omega^{q-1}_{\text{closed}, \mathbb{Z}}(X)$ is the group of gauge transformations.

# Differential K-Theory

Similarly, for $\check{K}^q(X)$.

$$0 \to K^{q-1}(X, \mathbb{R}/\mathbb{Z}) \to \check{K}^q(X) \to \overbrace{\Omega^q_{\text{closed}, \mathbb{Z}}(X, R)}^{\text{currents}} \to 0$$

$$0 \to \underbrace{\Omega^{q-1}(X, R)/\Omega^{q-1}_{\text{closed}, \mathbb{Z}}(X, R)}_{\text{RR field strength}} \to \check{K}^q(X) \to K^q(X) \to 0$$

where $R = K^\bullet(\text{pt}) = \mathbb{R}[u, u^{-1}]$, $\deg(u) = 2$ and

$$0 \to K^q(X, \mathbb{R})/K^q(X) \to K^q(X, \mathbb{R}/\mathbb{Z}) \to K^{q+1}_{\text{tors}}(X) \to 0$$

# K Theory

Will help to have a specific model in mind.

Usually think of $K^0$ as represented by formal differences of vector bundles $E_0 \ominus E_1$. Instead, consider $\mathbb{Z}_2$-graded vector bundles $E = E_0 \oplus E_1$ with an odd, skew-adjoint endomorphism, $T$ (requires a Hermitian metric on $E$ — a contractible choice).

Some applications will require $E$ $\infty$-dimensional and $T$ Fredholm.

$\mathrm{Cliff}_n^{\pm}$ (real Clifford algebra):

$$\gamma_i\gamma_j + \gamma_j\gamma_i = \pm 2\delta_{ij}, \qquad i,\ j = 1,\ \ldots,\ n$$

$K^{\pm n}$:

represented by a pair, $(E,\ T)$, as before, carrying a left $\mathrm{Cliff}_n^{\pm}$ action, which graded-commutes with $T$ (the $\gamma_i$ are odd).

# KR Theory

KR theory:

$\sigma : X \circlearrowleft$. Consider complex, $\mathbb{Z}_2$-graded vector bundles, $E \to X$, s.t. $\sigma$ lifts to an even, ***antilinear*** action on fibers, which commutes with $T$ and with the Clifford action. K-theory of such gadgets is called KR theory.

N.b., if $\sigma$ acts trivially, this is just an antilinear involution for the fibers, i.e. a real structure $\Rightarrow$ KO theory.

Equivariant K-theory, $K_G^\bullet(X)$:

$G$ acts on $X$, lifts to an ***even*** linear action on $E$, which commutes with $T$ and with the Clifford action.

Hybrid, ${}^{\omega}K_{\Gamma}^{\bullet}(Y)$:

$1 \to \Gamma_0 \to \Gamma \overset{\omega}{\to} \mathbb{Z}_2 \to 1$

lift of $\gamma \in \Gamma_0$ is even-linear; lift of $\gamma \in \Gamma_1$ is even-antilinear.

${}^{\omega}K_{\Gamma}^{\bullet}(Y) = \mathrm{KR}^{\bullet}(\mathcal{X}_w)$

where $\mathcal{X}_w$ is a certain double cover of the ***stack***, $\mathcal{X} = Y /\!/ \Gamma$.

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- $\mathscr{X}_0, \mathscr{X}_1$ smooth manifolds.
- $p_{0,1}$ local diffeomorphisms. $p_0 \times p_1 : \mathscr{X}_1 \to \mathscr{X}_0 \times \mathscr{X}_0$ proper.

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- $p_{0,1}$ local diffeomorphisms. $p_0 \times p_1 : \mathscr{X}_1 \to \mathscr{X}_0 \times \mathscr{X}_0$ proper.
- $\mathrm{Stab}(x) = \{ f \in \mathscr{X}_1 \mid p_1(f) = p_0(f) = x \}$.
  - Deligne-Mumford Stack: $\mathrm{Stab}(x)$ finite group, $\forall\, x \in \mathscr{X}_0$.
  - Artin Stack: relax this condition.

# Examples

1. Main example: $\mathscr{X} = Y /\!\!/ \Gamma$.

   $\mathscr{X}_0 = Y$, $\mathscr{X}_1 = Y \times \Gamma$ with $p_0(y, \gamma) = y$, $p_1(y, \gamma) = \gamma \cdot y$.

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3. An equivalent groupoid is given by an open cover, $\{U_i\}$, of $X$. $\mathcal{X}_0 = \coprod_i U_i$, $\mathcal{X}_1 = \coprod_{i \neq j} U_{ij}$. For each point on $X$, on the overlap between patches, we have an extra pair of morphisms, identifying the corresponding points in each patch. I.e. $U_{ij} \begin{array}{l} \overset{p_0}{\nearrow} U_i \\ \underset{p_1}{\searrow} U_j \end{array}$ inclusions.

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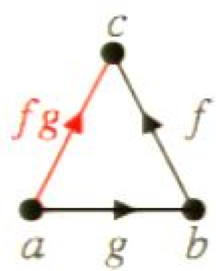
Key property: a map $M \xrightarrow{\phi} Y /\!/ \Gamma$ "is" an equivariant map $P \xrightarrow{\tilde{\phi}} Y$, where $P \to M$ is a $\Gamma$-principal bundle.

# Simplicial Space

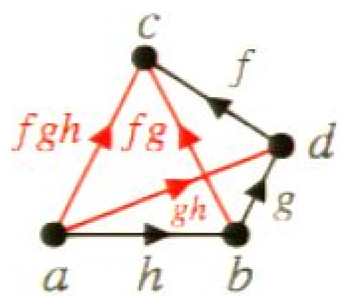
Motivated by example 3, define simplicial space

$$\ldots \mathscr{X}_3 \overset{\rightrightarrows}{\rightrightarrows} \mathscr{X}_2 \substack{\rightarrow\\\rightarrow\\\rightarrow} \mathscr{X}_1 \rightrightarrows \mathscr{X}_0$$

where the points of $\mathscr{X}_n$ are $n$-simplices generated by $n$-composable morphisms




$\in \mathscr{X}_2$




$\in \mathscr{X}_3$

A Vector Bundle on $\mathscr{X}$:

- A vector bundle, $E \to \mathscr{X}_0$.
- Isomorphism, $\theta : p_0^*(E) \xrightarrow{\sim} p_1^*(E)$ on $\mathscr{X}_1$.
- Compatibility condition on $\mathscr{X}_2$. (For example 3, this is the cocycle condition.)

# Cohomology

Can define cohomology, K-theory, ... on stacks.

$$H^j(Y /\!/ \Gamma) = H^j_\Gamma(Y)$$

- $H^1_\Gamma(Y, \mathbb{Z}/2)$ classifies double covers $\hat{Y} \to Y$, with a lift of $\Gamma$ action to $\hat{Y}$.
- $w_1(Y /\!/ \Gamma) = w_1^{\mathrm{eq}}(Y)$ measures whether $Y$ is orientable, and $\Gamma$ preserves an orientation.
- $w_2(Y /\!/ \Gamma) = w_2^{\mathrm{eq}}(Y)$ measures whether $Y$ admits a spin structure, with a spin action of $\Gamma$.

$$K(Y /\!/ \Gamma) = K_\Gamma(Y)$$

For orientifolds, we need the slightly more exotic ${}^{\omega}K_\Gamma(Y)$.

# Orientifold: Spacetime Data

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- Double cover: $\mathcal{X}_w \to \mathcal{X}$, for $w \in H^1(\mathcal{X}, \mathbb{Z}/2)$. If $\mathcal{X} = Y/\!\!/\Gamma$, $\mathcal{X}_w = Y/\!\!/\Gamma_0$.

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- A $B$-field, $\check{B} \in \check{E}^3(\mathcal{X})$, where

$$0 \to \overbrace{\check{H}^3(\mathcal{X}, \tilde{\mathbb{Z}})}^{\text{twisted coefs}} \to \check{E}^3(\mathcal{X}) \to H^1(\mathcal{X}, \mathbb{Z}/2) \to 0$$

and $\tilde{\mathbb{Z}}$ indicates coefficients twisted by the double cover $\mathcal{X}_w \to \mathcal{X}$.

Topologically, $[\check{B}] = (h, a)$.

$h \in H^3(\mathcal{X}, \tilde{\mathbb{Z}})$ is cohomology class of $H$.
$a \in H^1(\mathcal{X}, \mathbb{Z}/2)$ tells you when $(-1)^{F_L}$ accompanies $\gamma \in \Gamma$ in the worldsheet theory.

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- "Twisted Spin structure" ($t = 0$ for IIB, $t = 1$ for IIA)

$$\begin{aligned} w_1(\mathcal{X}) &= tw \\ w_2(\mathcal{X}) &= tw^2 + aw \end{aligned}$$

  where $a \in H^1(\mathcal{X}, \mathbb{Z}/2)$ is part of the $B$-field.

# Mother Of All Type I/II Theories

- Choose $\mathcal{X} = X$ and $w = 0$. ($\mathcal{X}_w \to \mathcal{X}$ trivial double-cover.) Then
  $$\begin{cases} t = 0 & \Rightarrow \text{Type IIB} \\ t = 1 & \Rightarrow \text{Type IIA} \end{cases}$$
  Choice of $a$ gives $2^{\dim(H^1(X, \mathbb{Z}/2))}$ theories, with
  $$S' = \begin{cases} S \otimes L(a), & \text{Type IIB} \\ S^{\mathrm{op}} \otimes L(a), & \text{Type IIA} \end{cases}, \text{ where } w_1(L) = a.$$

- Choose $\mathcal{X} = X \times (\mathrm{pt}/\!/\mathbb{Z}_2)$ and $w = \alpha$, where $\alpha \in H^1(\mathcal{X}, \mathbb{Z}/2)$ is the "universal" element, pulled back from $\mathrm{pt}/\!/\mathbb{Z}_2$ ($\mathcal{X}_w = X$).
  $\exists$ twisted spin structure $\Rightarrow t = 0$ and $a = 0 \Rightarrow$ Type I.

- Choose $\mathcal{X} = \mathbb{R}^{10-n} \times (\mathbb{R}^n /\!/ \mathbb{Z}_2)$, reflection on $n$ dimensions. $w = \alpha$.
  $T\mathcal{X} = 1^{\oplus(10-n)} \oplus L(\alpha)^{\oplus n} \Rightarrow w_1(\mathcal{X}) = n\alpha,\ w_2(\mathcal{X}) = \binom{n}{2}\alpha^2$.
  $$\text{Twisted spin str: } \begin{aligned} w_1(\mathcal{X}) &= tw \\ w_2(\mathcal{X}) &= tw^2 + aw \end{aligned} \Rightarrow \begin{cases} t &= n \pmod 2 \\ a &= \begin{cases} 0 & n = 0,3 \\ \alpha & n = 1,2 \end{cases} \pmod 4 \end{cases}$$

- etc. ...

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# Twisting KR

We wrote $[\check{B}]$ as a pair, $(h, a)$. But addition law in the group $E^3(\mathcal{X})$ is

$$(h_1, a_1) + (h_2, a_2) = (h_1 + h_2 + \tilde{\beta}(a_1 \cup a_2), a_1 + a_2)$$

where $\tilde{\beta} : H^2(\mathcal{X}, \mathbb{Z}/2) \to H^3(\mathcal{X}, \tilde{\mathbb{Z}})$ is the twisted Bockstein associated to $0 \to \tilde{\mathbb{Z}} \stackrel{\times 2}{\to} \tilde{\mathbb{Z}} \to \mathbb{Z}/2 \to 0$

As in ordinary Type II, turning on a nontrivial B-field twists the K-Theory where D-brane charge lives. In our case, we want a twisting of $\check{KR}^t(\mathcal{X}_w)$ by $\check{B} \in \check{E}^3(\mathcal{X})$.

The quadratic refinement uses the fact that, for $x \in \check{KR}^{t+\check{B}}(\mathcal{X}_w)$, $u^{-t} x \bar{x} \in \check{KO}^{\check{\tau}}(\mathcal{X})$ where

$$\check{\tau} \simeq \check{B} + \overline{\check{B}} + t\alpha^2$$

is a twisting of $\check{KO}$. (This isomorphism ***requires*** a twisted spin structure.)

We'll frequently write $\check{KO}(\mathcal{X}) = \check{KO}_{\mathbb{Z}_2}(\mathcal{X}_w)$.

# Specialize

At this point, will specialize to $\mathscr{X} = X /\!/ \mathbb{Z}_2$, an ordinary manifold with involution. I.e., $\Gamma = \mathbb{Z}_2$, with generator $\sigma$.

Let $F$ be f.p.s. of $\sigma$ (could be all of $X$, or could be empty).

A priori:

$\sigma$ orientation-preserving $\Rightarrow$ $\mathrm{codim}(F) = \text{even}$.

$\sigma$ orientation-reversing $\Rightarrow$ $\mathrm{codim}(F) = \text{odd}$.

In fact:

$\exists$ twisted spin structure $\Rightarrow \mathrm{codim}(F)$ well-defined $\bmod (4)$.

Accompany $\sigma$ by reversal of orientation on worldsheet.

There are "universal" twistings pulled back from $H^3(\mathrm{pt}/\!/\mathbb{Z}_2, \tilde{\mathbb{Z}}) \rtimes H^1(\mathrm{pt}/\!/\mathbb{Z}_2, \mathbb{Z}_2)$. Depending on $\mathrm{codim}(F) \bmod 4$, only 2 of 4 are compatible with a twisted spin structure, and lead to $Op^{\pm}$.

# Integration

Integration in equivariant KO-theory given by Dirac Index (for families).

$$\int_X : \mathrm{KO}^n_{\mathbb{Z}_2}(\mathbb{X}) \to \mathrm{KO}^{n-\dim X}_{\mathbb{Z}_2}(S) \qquad \begin{array}{c}\mathbb{X}\\ \downarrow X\\ S\end{array}$$

I'll take a shortcut: consider 12-manifold (2-parameter family). So we map $\mathrm{KO}^0_{\mathbb{Z}_2} \to \mathrm{KO}^{-12}_{\mathbb{Z}_2}(\mathrm{pt}) = \mathrm{KO}^{-4}(\mathrm{pt}) \otimes \mathrm{RO}(\mathbb{Z}_2) = \mathbb{Z} \oplus \epsilon\,\mathbb{Z}$

# Quadratic Refinement

Let $x \in \mathrm{KR}^{0+\tau}(X)$ (IIB) or $x \in \mathrm{KR}^{1+\tau}(X)$ (IIA). Can lift $x\bar{x}$ (or $u^{-1}x\bar{x}$) to $\mathrm{KO}^{0}_{\mathbb{Z}_2}(X)$. Then we integrate over a 12-manifold, and pick off the coefficient of $\epsilon$ in the result.

$$q(x) = \Big(\int_{\mathbb{X}} x\bar{x}\Big)\Big|_{\epsilon}$$

The center (or, rather, twice the center) can be computed from

$$\psi(x) = q(x) - q(\pi x) = \Big(\int_{\mathbb{X}} ((x\bar{x})_{+} - (\pi x \overline{\pi x})_{+})\Big)\Big|_{\epsilon}$$

# Localization

After inverting 2, the formula for $\psi(x)$ localizes to the f.p.s, $F$

$$\psi(x) = \int_F^{KO[1/2]} \frac{2\psi_2(x|_F)}{\Delta(\nu)}$$

where $\Delta(\nu)$ is the spinor bundle of the normal bundle, and $\psi_2(V) = \mathrm{Sym}^2 V \ominus \wedge^2 V$ is the Adams operation.

In KO[1/2], $\psi_2$ has an inverse, $\psi_{1/2}$,[1] with which we can write

$$\psi(x) = 2\int_F^{KO} \psi_2\left(\frac{x|_F}{\psi_{1/2}(\Delta(\nu))}\right)$$

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1. Use splitting principle and the fact that $\psi_2$ is a ring homomorphism. $\psi_2(L) = L^2$. Let $L = 1 + x$, with $x$ nilpotent. (Note: only powers of 2 in the denominators!)

$$\psi_{1/2}(L) = (1+x)^{1/2} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 - \ldots$$

# Cannibalistic Class

The Bott Cannibalistic Class is the KO analogue of the Wu class

$$\int_M^{\mathrm{KO}[1/2]} \psi_2(y) = \int_M^{\mathrm{KO}[1/2]} y \cup \rho(M)$$

It can be written as (at least, for $M$ even-dim and spin)

$$\rho(M) = \prod_{i=1}^{\dim(M)/2} \left(l_i^{1/4} \oplus l_i^{-1/4}\right) = \psi_{1/2}(\Delta(M))$$

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$$\mathrm{TM} \otimes \mathbb{C} = \oplus_{i=1}^{\dim(M)/2} (l_i \oplus l_i^{-1})$$

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# The Background Charge (After Inverting 2)

Using this, we can finally write ($i : F \hookrightarrow X$)

$$\mu = i_* \mu_F, \qquad \mu_F = -\psi_{1/2}\left(\frac{\Delta(F)}{\Delta(\nu)}\right)$$

Note that, even though, in the derivation I presented, I assumed that $F$ was spin, the ratio, $\Delta(F)/\Delta(\nu)$, makes sense even when neither the numerator nor the denominator makes sense separately (say, because $F$ is only $\mathrm{Spin}_{\mathbb{C}}$).

To compare with the existing formulæ we take Chern characters. The coupling to the RR "connection" is

$$\int_X C \wedge \mathrm{Ch}(\mu)\sqrt{\hat{A}(X)} = \int_F i^* C \wedge \frac{\mathrm{Ch}(\mu_F)\, i^* \sqrt{\hat{A}(X)}}{\hat{A}(\nu)}$$

$$= \int_F i^* C \wedge \mathrm{Ch}(\mu_F)\sqrt{\frac{\hat{A}(F)}{\hat{A}(\nu)}}$$

# Scruca-Serone

It's now a very pretty little computation[1], using the splitting principle, to check that this is

$$-2^{5-\mathrm{codim}(F)} \int_F i^* C \wedge \sqrt{\frac{L(R_F/4)}{L(R_\nu/4)}}$$

which agrees with the standard formulæ that you find in the physics literature.

---

1. You'll need the characteristic polynomials $\hat{A}(R) = \prod \frac{x_i/2}{\sinh(x_i/2)}$ and $L(R) = \prod \frac{x_i}{\tanh(x_i)}$.

↩

# So What?

After inverting 2, I presented you with a pretty nice, ***computable*** formula for the background charge

$$\mu = -i_*\left(\Psi_{1/2}\left(\frac{\Delta(F)}{\Delta(\nu)}\right)\right)$$

Though we've lost 2-torsion, there are still interesting examples (with 3-torsion, etc) where one can compute this explicitly.

Can be generalized to cases with non-torsion flux ($H \neq 0$ rationally). "RR flux" is not a separate issue. From our point of view:

- You ***prescribe*** $H$ (which determines what twisted KR theory we need).
- You ***compute*** $\check{\mu}$.
- You ***prescribe*** $\check{j}$.
- If $\mu + j$ is trivial in the relevant twisted KR group, then you can ***solve*** for the RR field. Multiple solutions ⇔ "choices of RR flux"
- If it's nontrivial, then no solution for $G$ (over $\mathbb{Z}$!). The background is ***inconsistent***.

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The rubric of twisted (differential) KR-theory seems to be a powerful organizing principle for this bestiary of orientifolds. Perhaps you'll find some of the ideas to be useful in other contexts as well.

# Orientifolds

Manifold, $Y$, with a discrete group of isometries, $\Gamma$.

$$1 \to \Gamma_0 \to \Gamma \overset{\omega}{\to} \mathbb{Z}_2 \to 1$$

(As a set, $\Gamma = \Gamma_0 \amalg \Gamma_1$, but $\Gamma$ can be nonabelian, even if $\Gamma_0$, is abelian.) Gauge $\Gamma$. But, this time, accompany $\gamma \in \Gamma_1$ by orientation-reversal on worldsheet.

$$\begin{array}{ccccc} & \tilde{\Sigma} & \overset{\tilde{\phi}}{\to} & Y & \\ \gamma_{\tilde{\Sigma}} & \downarrow & & \downarrow & \gamma_Y \\ & \tilde{\Sigma} & \underset{\tilde{\phi}}{\to} & Y & \end{array}$$

- $\tilde{\Sigma}$ ***oriented*** surface. $\tilde{\Sigma} \to \Sigma$ a $\Gamma$-principal bundle.
- $\gamma_{\tilde{\Sigma}}$ fixed point free $\begin{cases} \text{orientation preserving for} & \gamma \in \Gamma_0 \\ \text{orientation reversing for} & \gamma \in \Gamma_1 \end{cases}$

Again, restricting to in/out circles, get $\Gamma$-bundle $\tilde{S} \to S$.

**Claim:** reduction of structure group of $\tilde{S}$ from $\Gamma$ to $\Gamma_0$.

No fixed point free orientation-reversing map from $S^1 \to S^1 \Rightarrow$

$\mathrm{Or}(\Sigma = \tilde{\Sigma}/\Gamma) = \tilde{\Sigma}/\Gamma_0$ and $\mathrm{Or}(S) = S \amalg S$. Restricting to one copy of $S$ gives explicit reduction of str group.