Title: Introduction to AdS/CFT with Flavour

Date: Sep 12, 2008 02:00 PM

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Abstract: TBA

Outline

- Introduction to AdS/CFT with Flavour
- $B_{t,\vec{x}}$: Electric/Magnetic Background
  - Magnetic Field
  - Electric Field
- B_{u,S^5}: Noncommutativity & FI-Terms
  - Commutative Instantons & Coulomb-Higgs Branch
  - Noncommutative Instantons and the FI term
- Summary & Outlook

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1. Introduction to AdS/CFT with Flavour
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The AdS/CFT Correspondence

The Original Correspondence (weakest form)

IIB Supergravity on $AdS_5 \times S^5$ with $(R/l_s)^4 = 4\pi\lambda \gg 1$

$$\mathrm{d}s^2_{AdS_5 \times S^5} = R^2 \left(\frac{\mathrm{d}x^{\mu 2} + \mathrm{d}u^2}{u^2} + \mathrm{d}\Omega_5^2 \right)$$

$$\Leftrightarrow$$

large N_c limit of $\mathcal{N} = 4$ $SU(N_c)$ Super Yang-Mills with $\lambda = g^2_{YM} N_c$

1. Strong-Weak Coupling Duality
2. Operator-Field Dictionary:

$$\phi_{m^2=\Delta(\Delta-4)} \simeq u^{4-\Delta} J_{\mathcal{O}} + u^{\Delta} \langle \mathcal{O} \rangle$$

3. Symmetries are important: $SO(4,2) \times SU(4)$

ntroducing Flavour

Extended Correspondence: N_f D7 Branes in $AdS_5 \times S^5$

4d $\mathcal{N} = 4$ SU(N) Super Yang-Mills theory
coupled to
4d N_f $\mathcal{N} = 2$ hypermultiplets

$\leftrightarrow$

type IIB SUGRA on $AdS_5 \times S^5$
+
Dirac-Born-Infeld theory on D7

The Right Embedding

$$\mathrm{d}s^2_{AdS_5 \times S^5} = \frac{\rho^2 + L^2}{R^2} \mathrm{d}x^{\mu 2} + \frac{R^2}{\rho^2 + L^2} \left(\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega_3^2 + \mathrm{d}L^2 + L^2 \mathrm{d}\Phi^2 \right)$$

Embedding: $\xi^\alpha = (x^\mu, \rho, S^3)\,, \quad L = L(\rho)$

$$\mathcal{L}_{DBI} \propto \rho^3 \sqrt{1 + L'^2} \Rightarrow L = 2\pi\alpha' m_q = const.$$

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ntroducing Flavour: Field Theory

$\mathcal{N} = 4$ Glue & N_f $\mathcal{N} = 2$ Quarks

$$\mathcal{L} = \Im\Big[\tau \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \left(\mathrm{tr}(\bar{\Phi}_i e^V \Phi_i e^{-V}) + Q_I^\dagger e^V Q^I + \tilde{Q}_I e^V \tilde{Q}^{I\dagger}\right) +$$
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2. Global Symmetries ($m = 0$):
   $SO(4,2) \times SU(2)_\Phi \times SU(2)_\mathcal{R} \times U(1)_\mathcal{R} \times U(N_f)$
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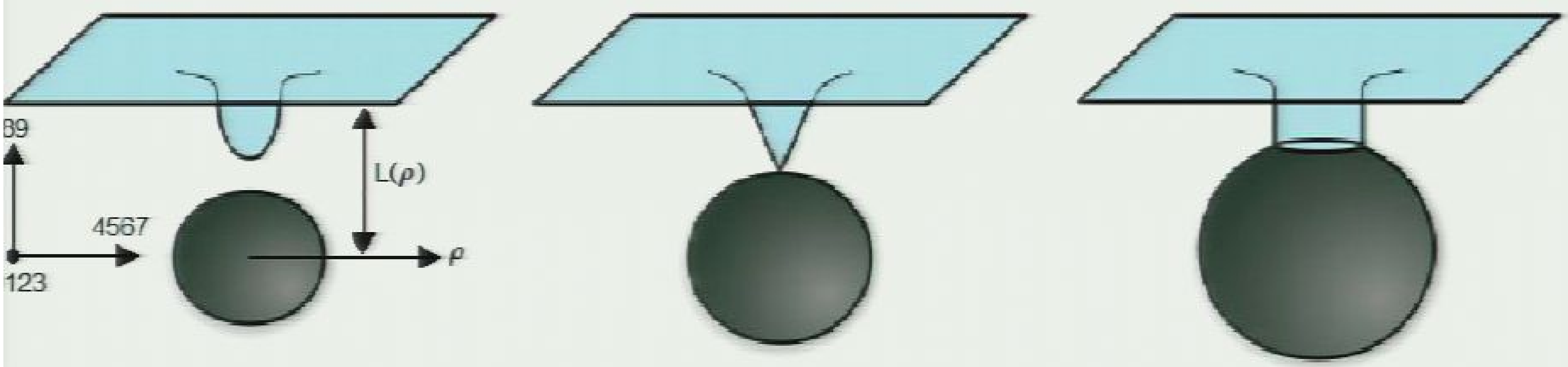
Flavour at Finite Temperature

Flavour Physics at Finite Temperature

AdS-Schwarzschild $\times$ S^5 (Black brane) , $T = T_{\text{Hawking}} \propto r_s$



icture: [hep-th/0611099]

1. Embedding: $L(\rho) \overset{\rho\to\infty}{\sim} 2\pi\alpha' m_q + \frac{(2\pi\alpha')^3}{\rho^2}\langle\bar{\psi}\psi\rangle$
2. Fluctuations: Mesons $\rightarrow$ Melting transition is topological!
3. No Spontaneous CSB: $\langle\bar{\psi}\psi\rangle(m_q = 0) = 0$

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Electric/Magnetic B-Field [see also 0709.1547, 0709.1554 (hep-th)]

Ansatz for the Kalb-Ramond Field

$$B_{el} = B\mathrm{d}t \wedge \mathrm{d}x\,, \quad B_{mag} = B\mathrm{d}y \wedge \mathrm{d}z$$

- $\mathrm{d}B = 0 \Rightarrow$ No deformation of AdS-Schwarzschild

$$-\frac{T_7}{g_2}\sqrt{-det\left(P[G+B]+2\pi\alpha' F\right)} + \sum_p P[C_p \wedge e^B] \wedge e^{2\pi\alpha' F}$$

- Affects Brane (Flavour) physics: D7 embeddings , thermodynamics, phase transitions , meson spectra ...
- Effect: Background for $U(1)_B$ gauge field, mimics constant $U(1) \in U(N_c)$ field strength

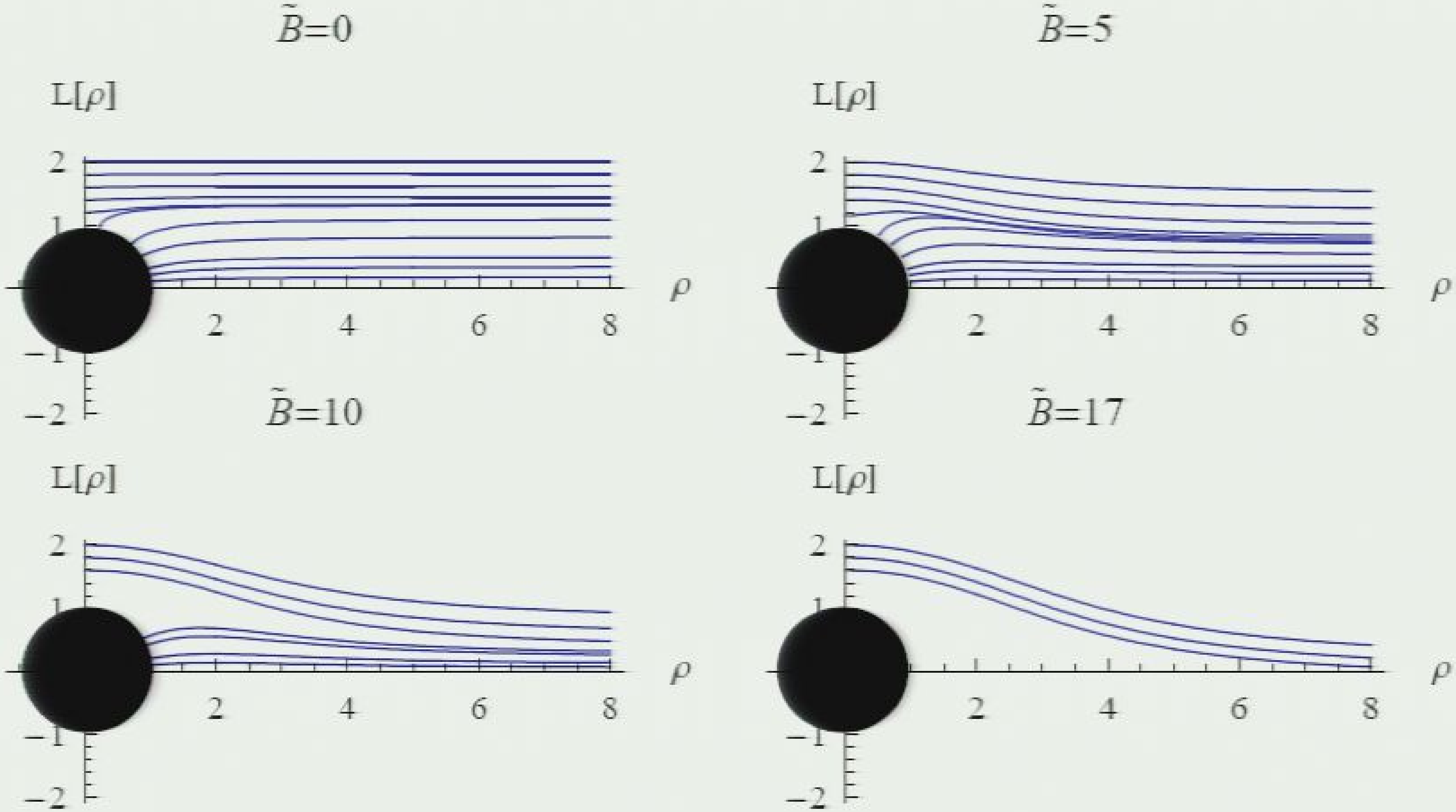
Magnetic $B_{\mu\nu}$

Magnetic Field

$$B_{mag} = B\mathrm{d}y \wedge \mathrm{d}z$$

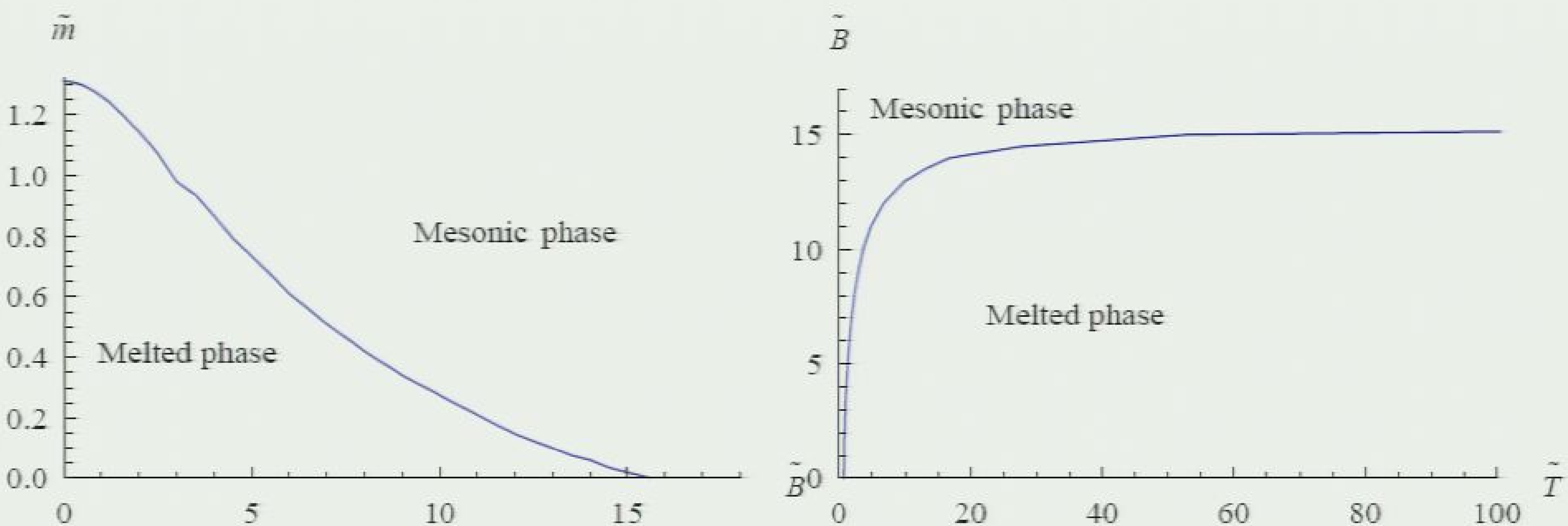
- Zero Temperature: [Filev etal.hep-th/0701001] CSB, Goldstone Boson with $M \propto \sqrt{m_q}$ for small m_q, Zeeman splitting
- Finite Temperature:
  - Small B: Meson Melting Transition
  - No molten phase & CSB above a critical magnetic field strength (GMOR)
  - Phase diagram
  - Spectrum of Pseudoscalar Mesons

Magnetic Finite Temperature Embeddings



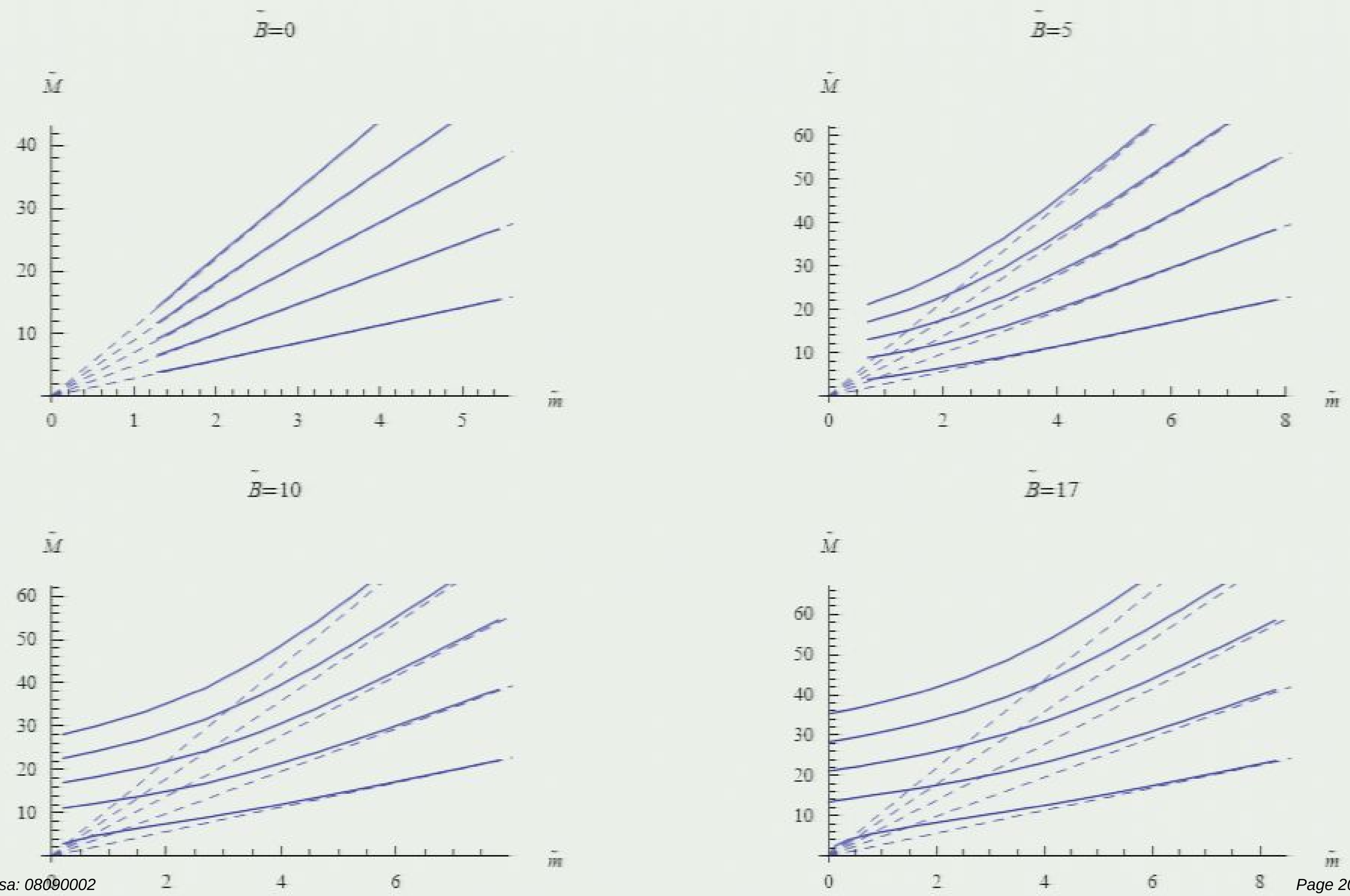
$$\chi SB : \quad \tilde{B}_{crit} \approx 16 , \quad \tilde{B} = \frac{B\lambda}{2\pi^2 R^2 m_q^2}$$

Phase Diagram $\tilde{m} = 2m_q/(\sqrt{\lambda}T)\,,\, \tilde{T} = \tilde{m}^{-1}$

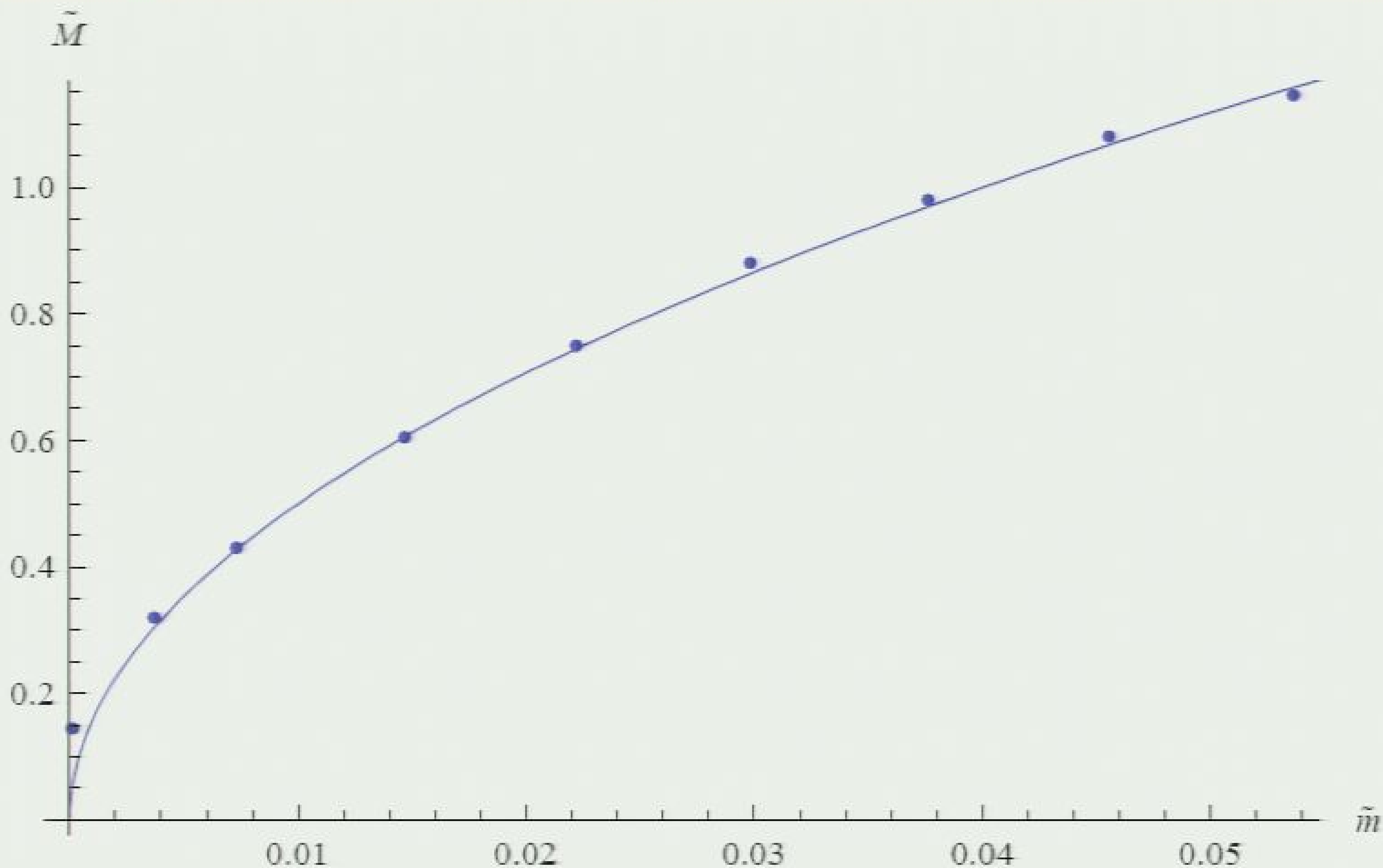


- Meson Melting Transition below $\tilde{B}_{crit}$
- No molten phase and spontaneous CSB above $\tilde{B}_{crit}$
- Magnetic KR-Field acts repells the D7s from the origin

Meson Spectrum (Upper Branch, $\tilde{M} = \frac{M\sqrt{\lambda}}{\sqrt{\pi} m_q}$)



Goldstone Boson ($\tilde{B} = 16$)



$$\tilde{M} = 5\sqrt{\tilde{m}}$$

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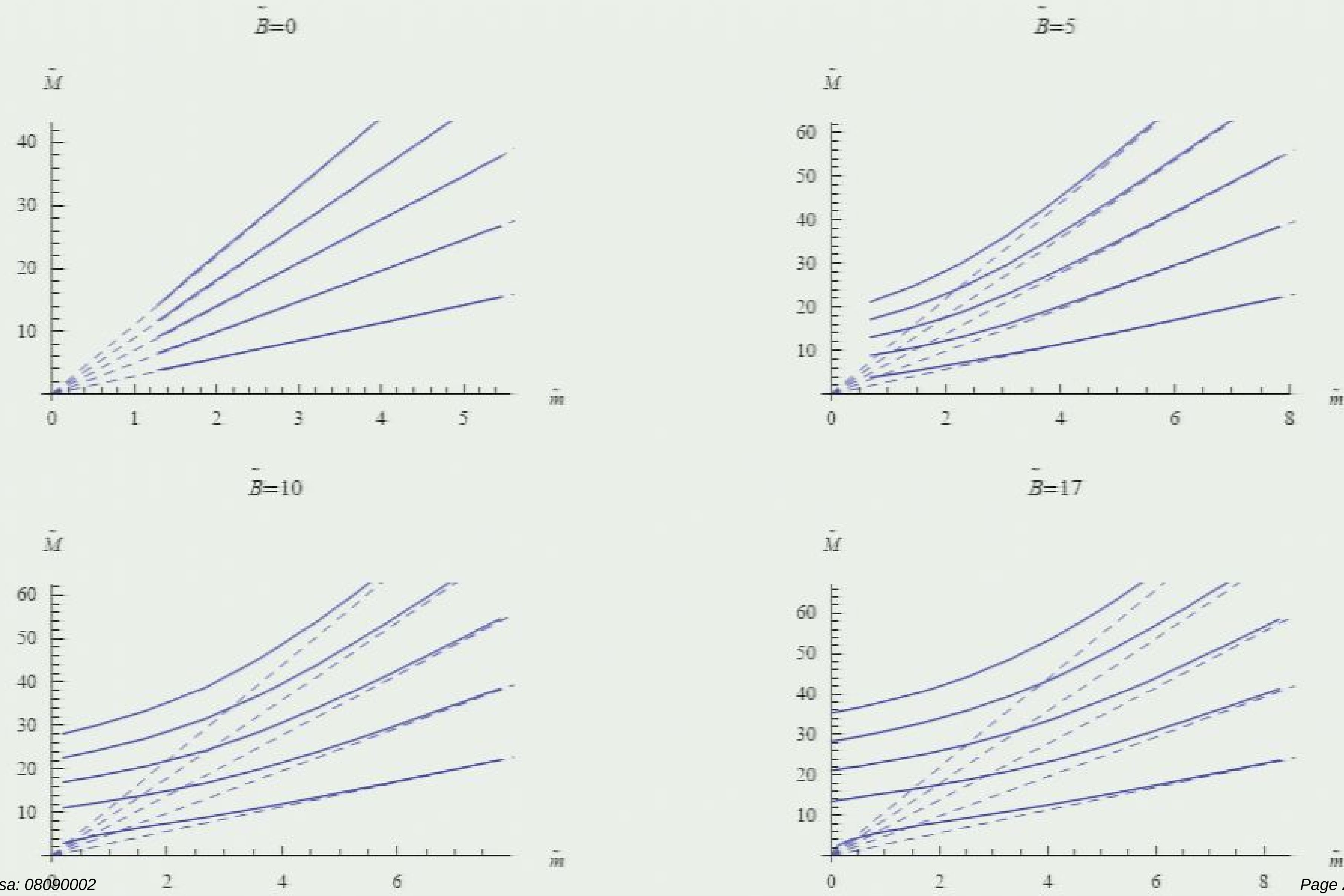

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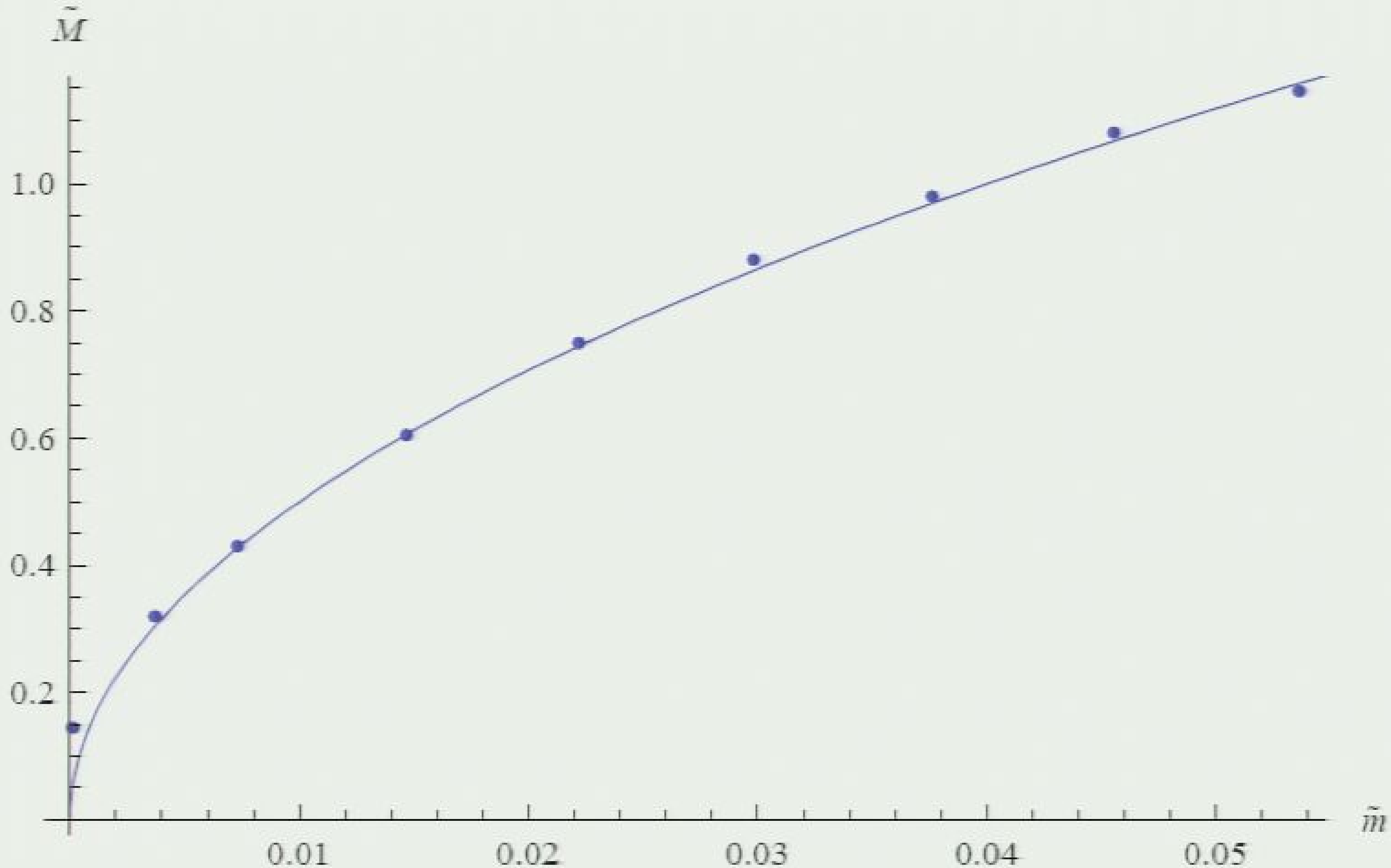

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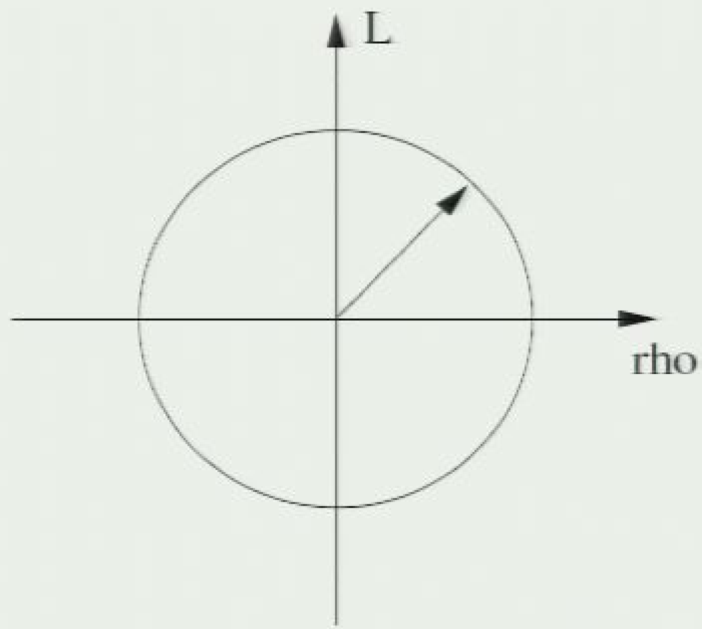
Electric $B_{\mu\nu}$

$$B_{el} = B\mathrm{d}t \wedge \mathrm{d}x$$

- Problem: Zero Locus of DBI action

$$\sqrt{-\det P[G+B]} = 0 \quad \text{for } \rho_{IR}^2 + L(\rho_{IR})^2 = \frac{BR^2}{2} + \frac{1}{2}\sqrt{4b^4 + B^2R^4}$$

Zero Locus



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- Solution: $U(1)_f$ gauge field $A_x(\rho), A_t(\rho)$ [hep-th:0705.3890]

$$\partial_a \left(\frac{\delta \mathcal{L}_{D7}}{\delta \partial_a A_b} \right) = 0 \Rightarrow \underline{\text{Two Conserved Quantities}}$$

$$A_t(\rho) \simeq \mu - \frac{\mathcal{D}}{\rho^2} \leftrightarrow \text{finite baryon number density } \langle J_t \rangle = \mathcal{D}$$

$$A_x(\rho) \simeq \frac{\mathcal{B}}{\rho^2} \leftrightarrow \text{baryon number current in } x\text{-direction } \langle J_x \rangle = \mathcal{B}$$

$$\Rightarrow A'_x(\rho) = f'(\mathcal{D}, \mathcal{B}, \rho)\,,\ A'_t(\rho) = g'(\mathcal{D}, \mathcal{B}, \rho) \Rightarrow \text{Legendre transform} \Rightarrow$$

Require $\tilde{S}_{D7}[L(\rho), \mathcal{D}, \mathcal{B}]$ to be well-defined :

$$\mathcal{B} = \mathcal{B}(\rho_{IR}, T, B, \mathcal{D})$$

$\Rightarrow$ Well-defined EOMs for $L(\rho)$!

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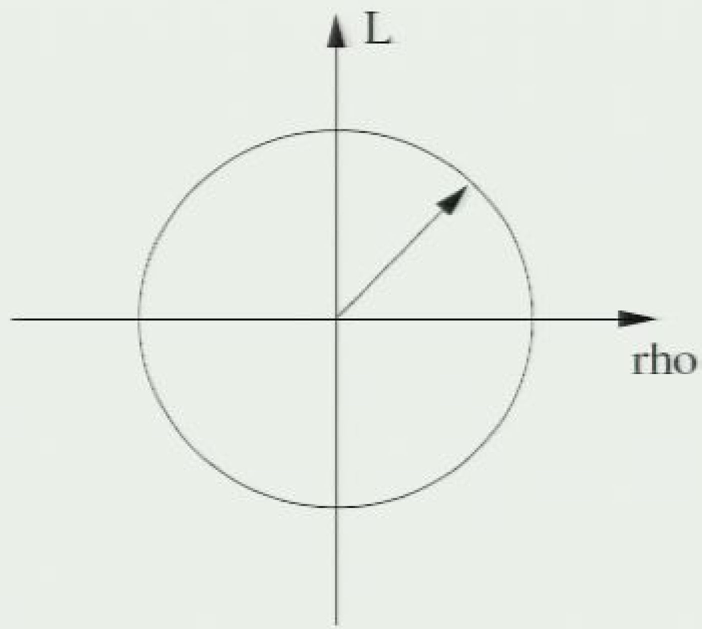

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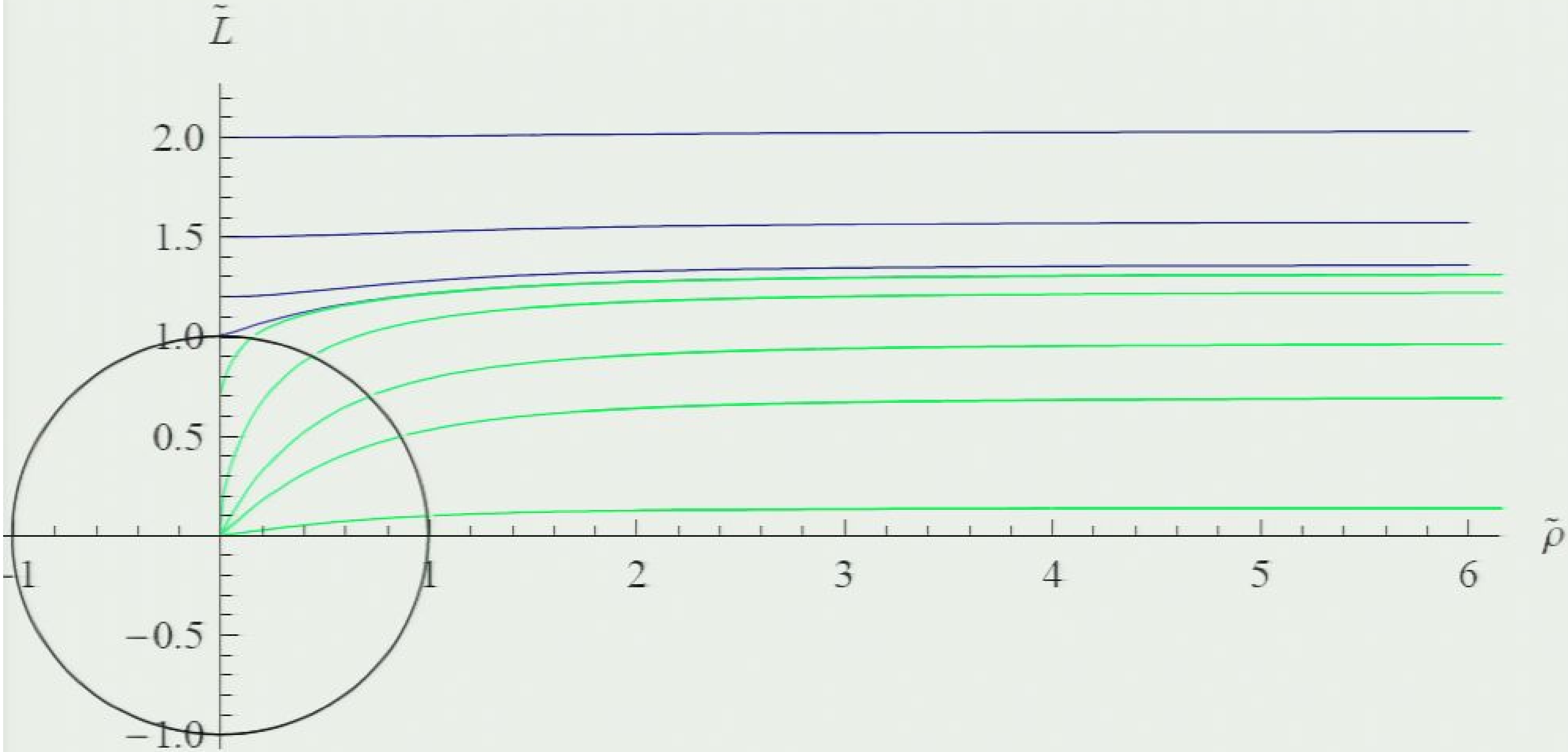
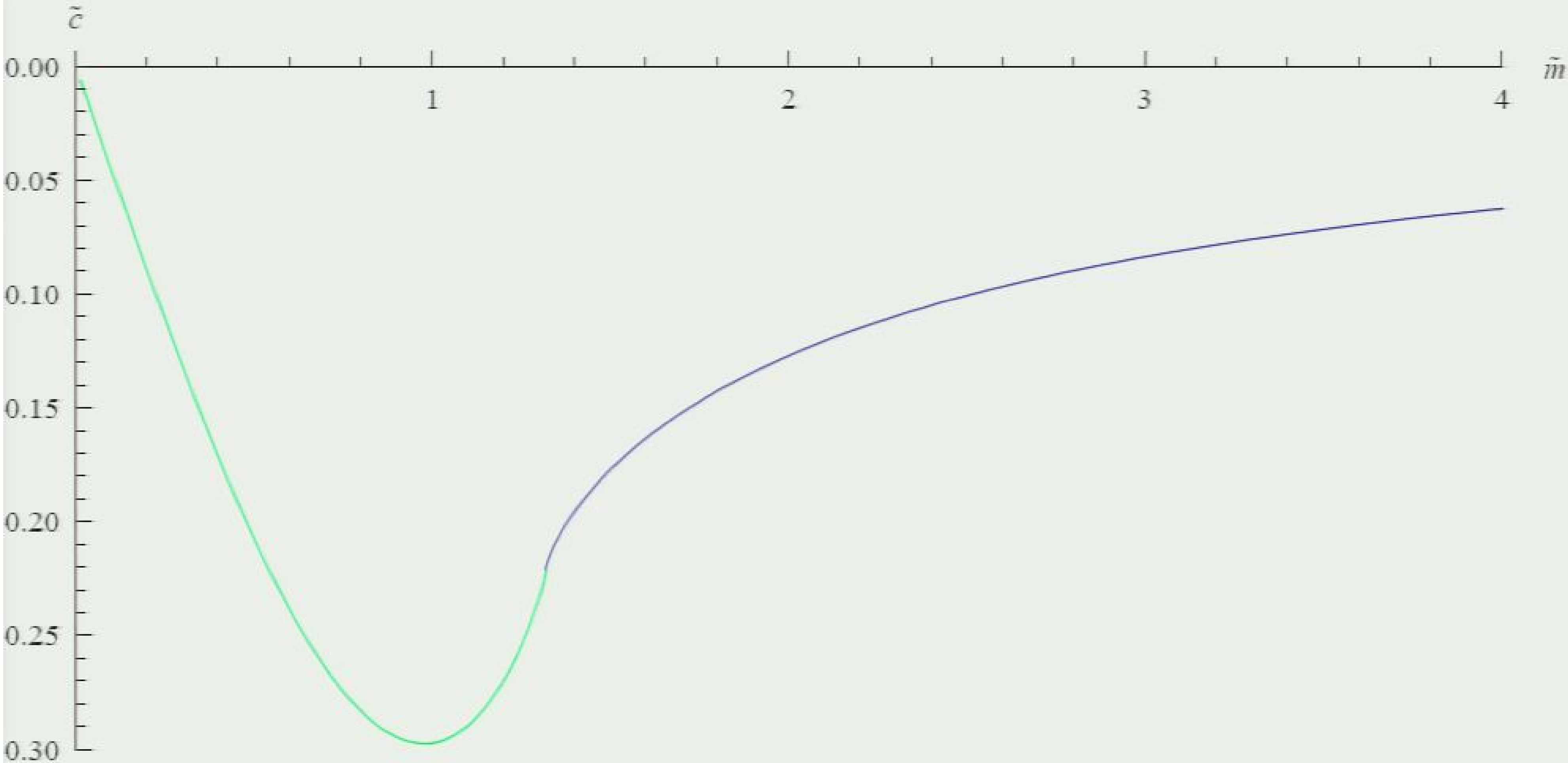
Electric Embeddings at $T = 0$: No CSB, Phase Transition



→ Dissociation of Mesons?

→ Conical singularities?

Condensate vs. Mass



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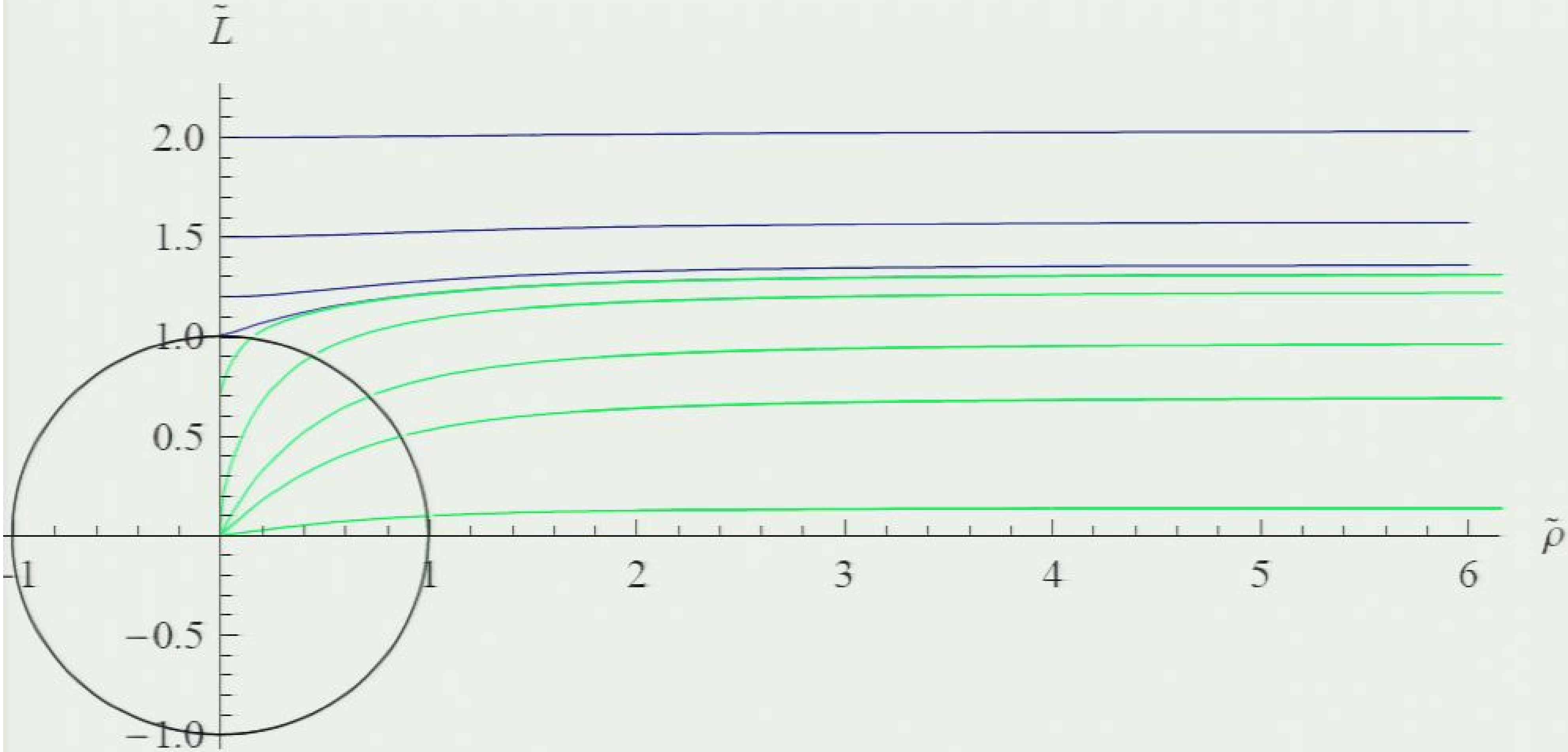

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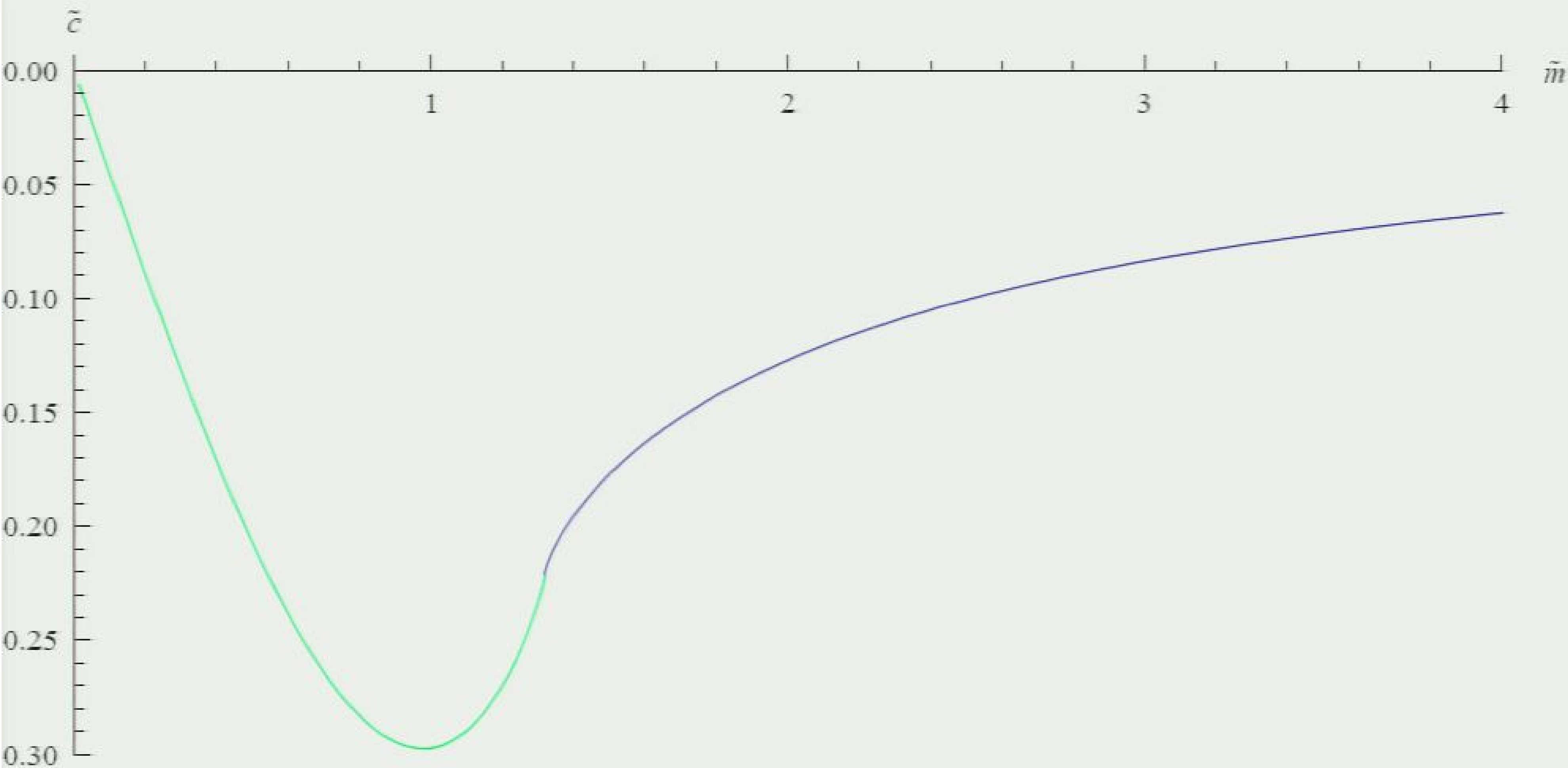

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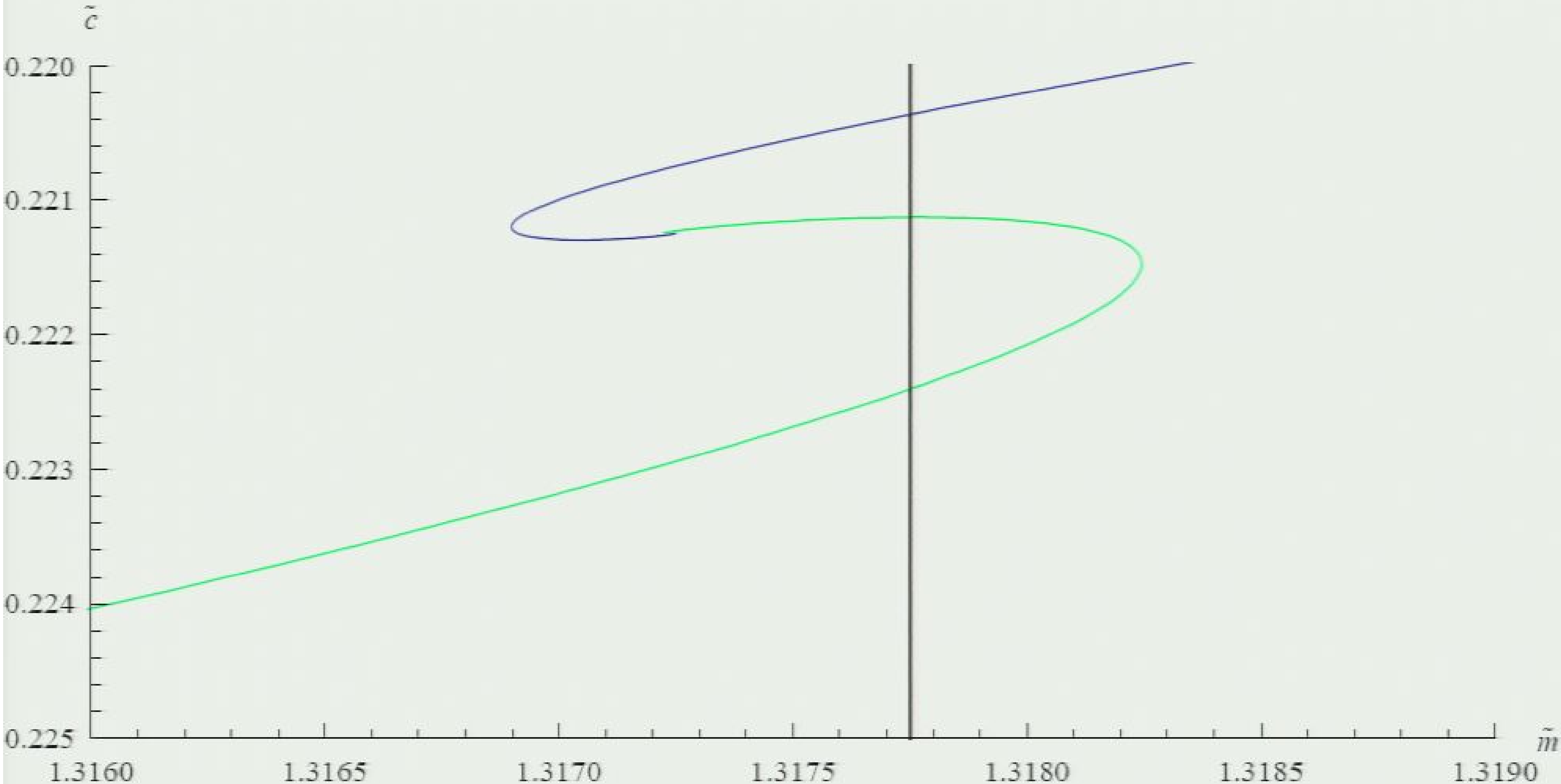
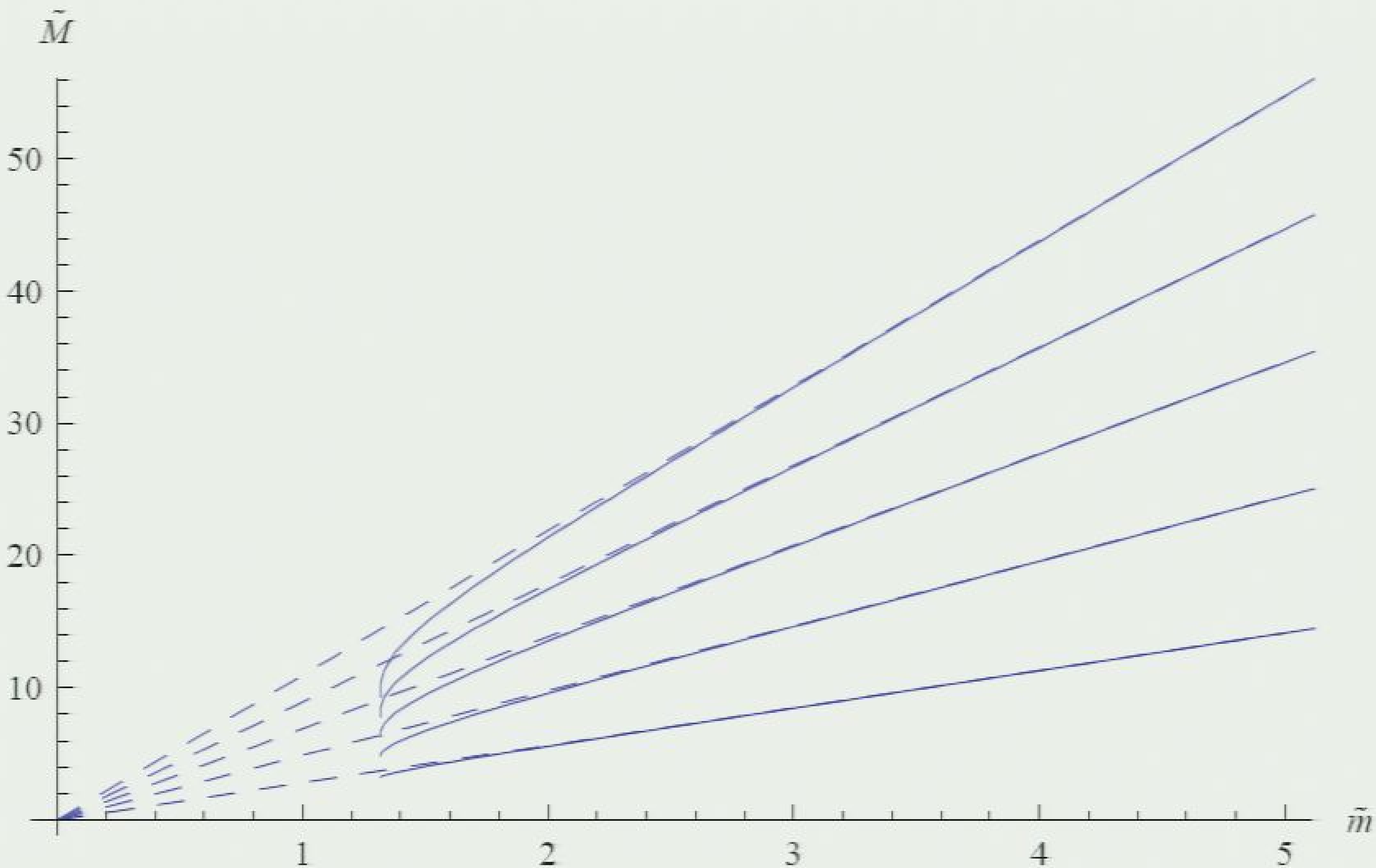


Area $\propto$ F

(l=0) Meson Spectrum at $T = 0$: $\Delta M < 0$



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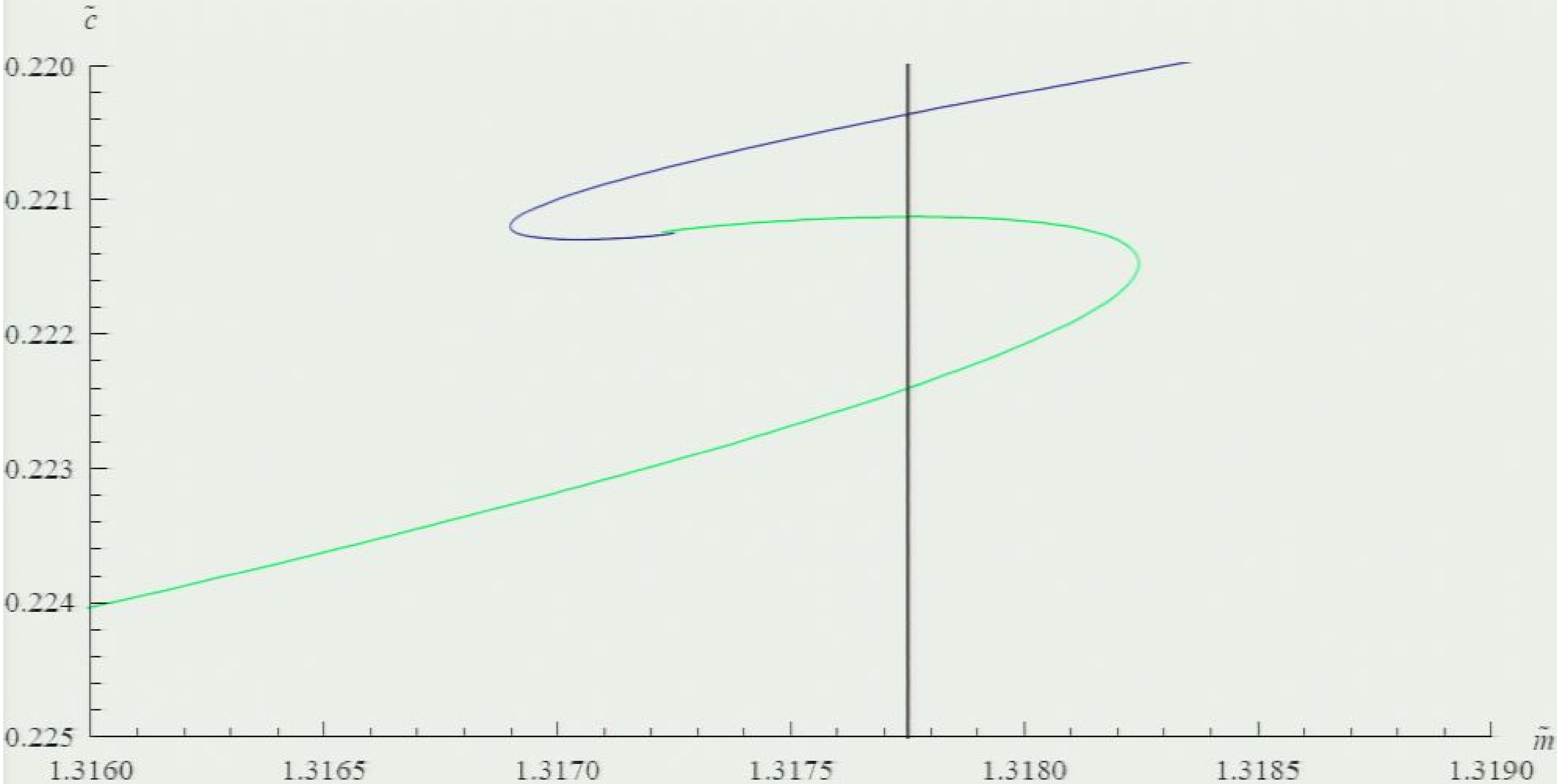

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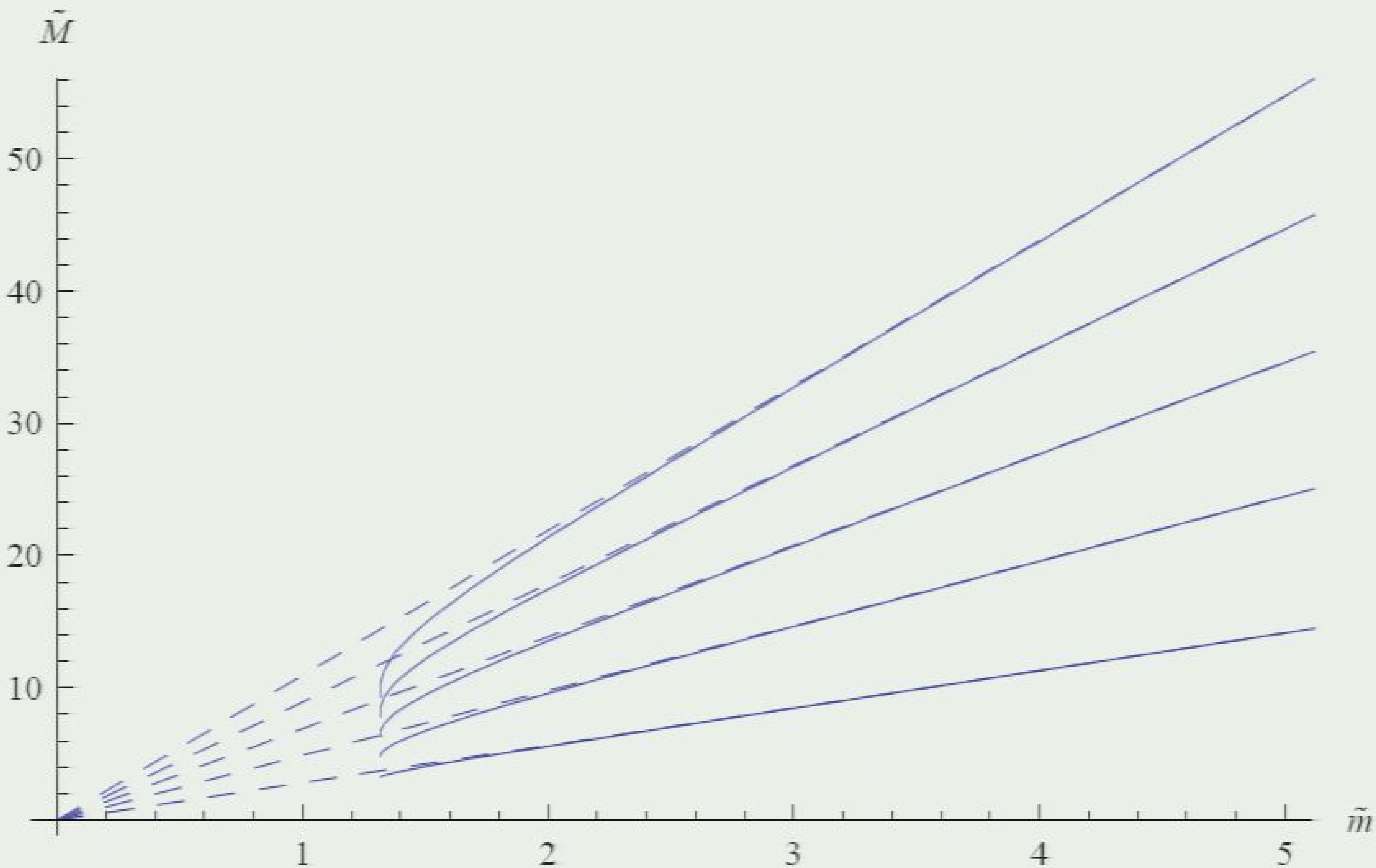

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Electric $B_{\mu\nu}$: Finite Temperature

What to expect at Finite Temperature?

- Meson melting enhanced by dissociation
- Any nonzero electric field will decrease the melting temperature
- No SCSB
- Finite T: One or two transitions?

Open Questions

- Fate of the conically singular solutions? They are either
  - physical $\rightarrow$ What creates the singularity?
  - unphysical $\rightarrow$ What happens in that mass range?
- Energy of the system is time-dependent $\rightarrow$
  - Is this still equilibrium physics?
  - Thermodynamics in the presence of external currents?

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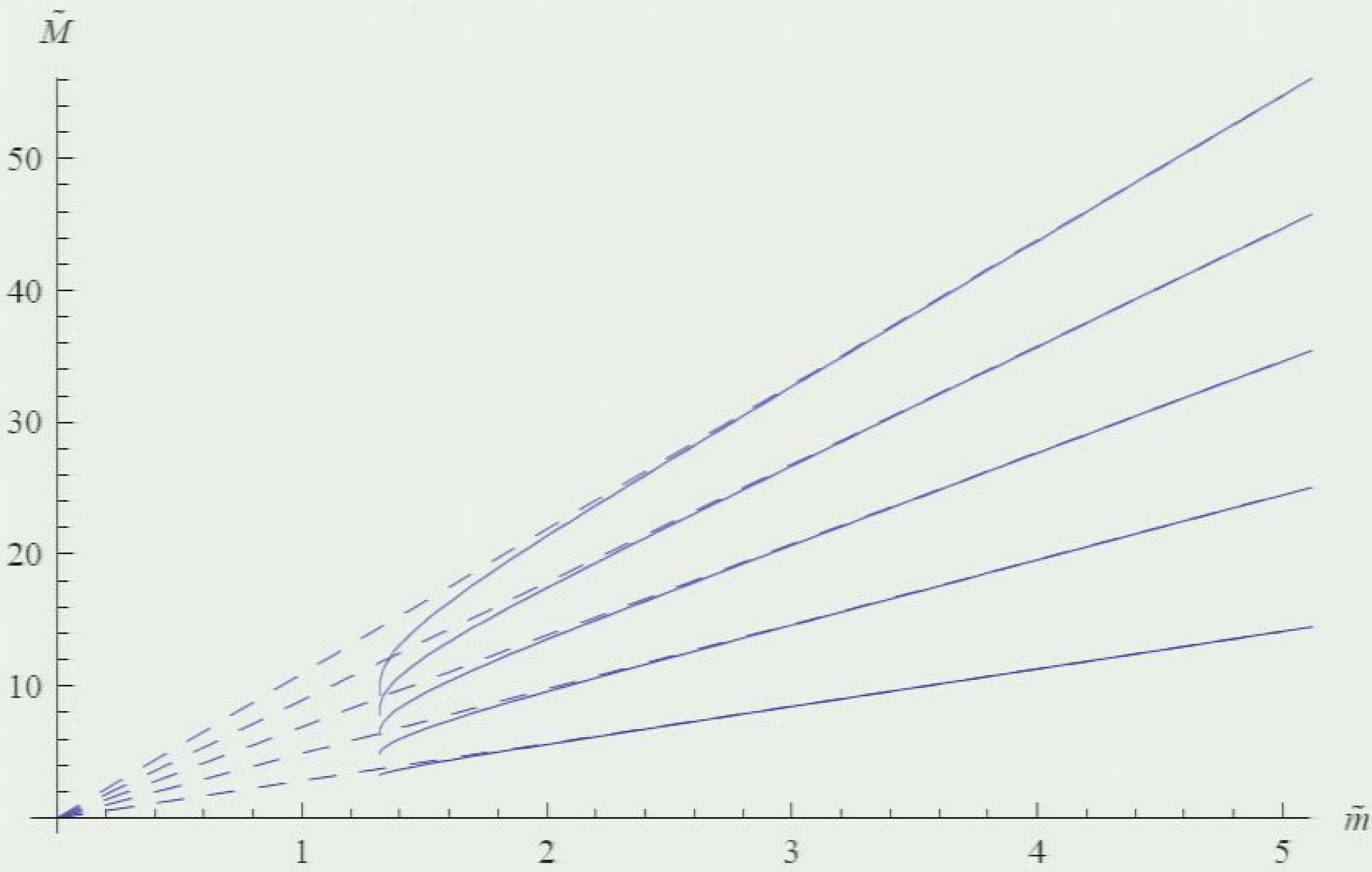

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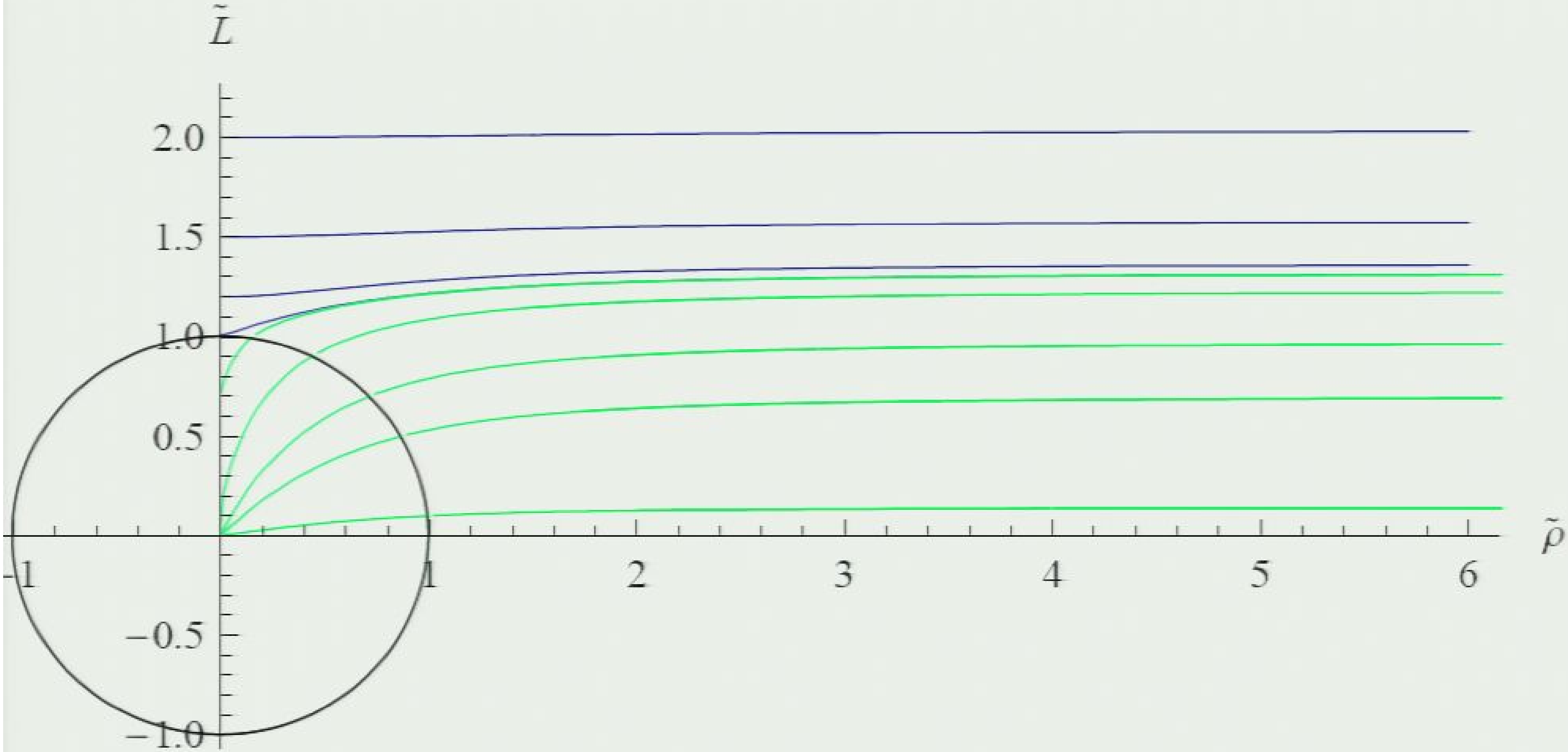

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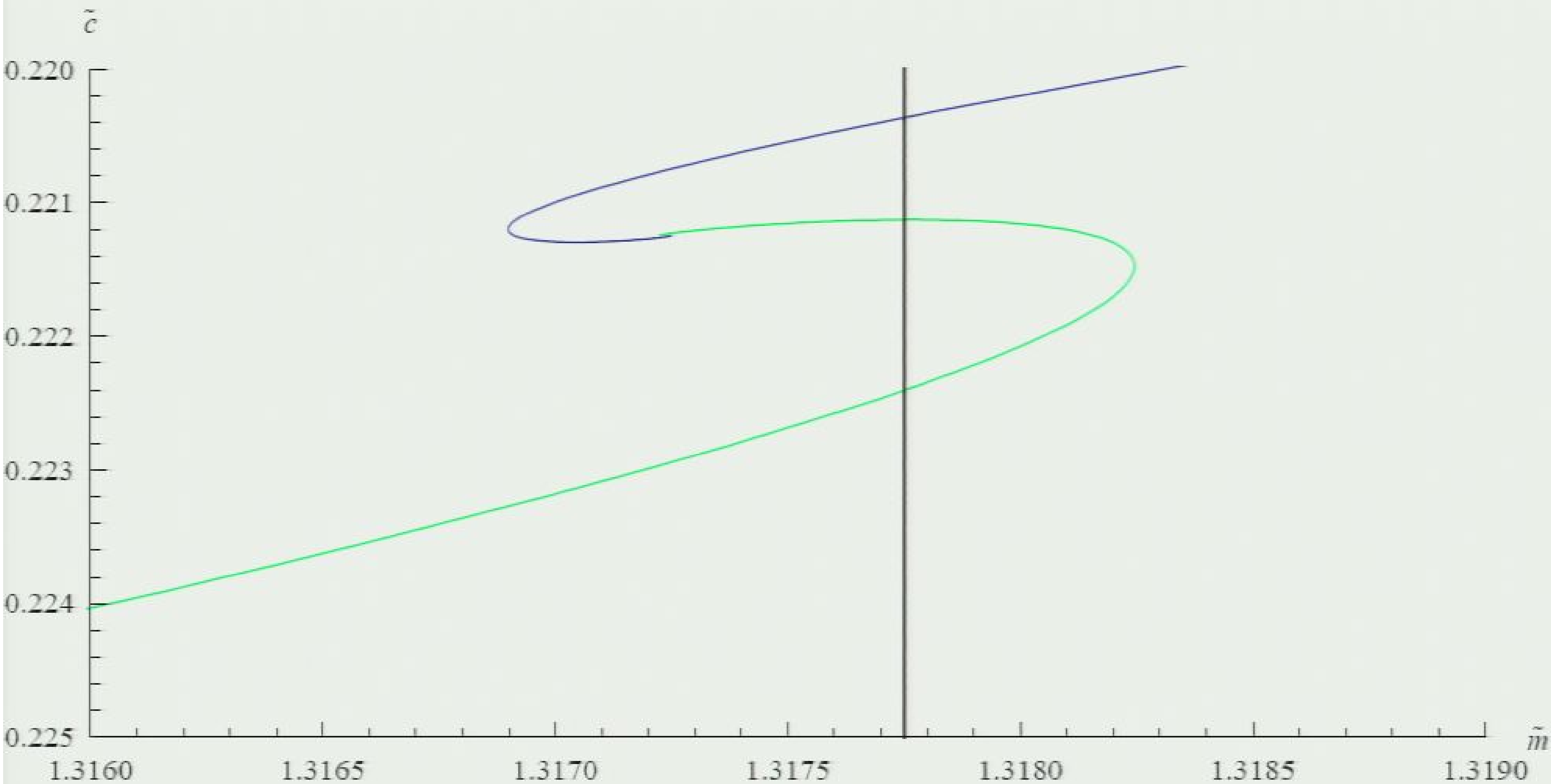
  - physical $\rightarrow$ What creates the singularity?
  - unphysical $\rightarrow$ What happens in that mass range?
- Energy of the system is time-dependent $\rightarrow$
  - Is this still equilibrium physics?
  - Thermodynamics in the presence of external currents?

Condensate vs. Mass: Phase Transition



Area $\propto$ F

Electric $B_{\mu\nu}$: Finite Temperature

What to expect at Finite Temperature?

- Meson melting enhanced by dissociation
- Any nonzero electric field will decrease the melting temperature
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- Finite T: One or two transitions?

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D3-D7 field theory

$$\mathcal{L} = \Im\Big[\tau \int d^2\theta d^2\bar{\theta} \left(\mathrm{tr}(\bar{\Phi}_i e^V \Phi_i e^{-V}) + Q_I^\dagger e^V Q^I + \tilde{Q}_I e^V \tilde{Q}^{I\dagger}\right) +$$
$$+ \tau \int d^2\theta \left(\mathrm{tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) + \mathrm{tr}(\epsilon_{ijk}\Phi_i\Phi_j\Phi_k) + \tilde{Q}_I(m + \Phi_3)Q^I\right)\Big],$$

$$\Phi_1 = \phi_1 + i\phi_2\,,\ \Phi_2 = \phi_3 + i\phi_4\,,\ \Phi_3 = \phi_5 + i\phi_6\,; \quad I = 1,\ldots,N_f\,.$$

Global Symmetries:

	components	spin	$SU(2)_\Phi \times SU(2)_\mathcal{R}$	$U(1)_\mathcal{R}$	Δ	$U(N_f)$	$U(1)_B$
Φ_1, Φ_2	X^4, X^5, X^6, X^7	0	$(\frac{1}{2}, \frac{1}{2})$	0	1	1	0
	λ_1, λ_2	$\frac{1}{2}$	$(\frac{1}{2}, 0)$	-1	$\frac{3}{2}$	1	0
Φ_3, W_α	$X_V^A = (X^8, X^9)$	0	$(0,0)$	$+2$	1	1	0
	λ_3, λ_4	$\frac{1}{2}$	$(0, \frac{1}{2})$	$+1$	$\frac{3}{2}$	1	0
	v_μ	1	$(0,0)$	0	1	1	0
	(D, F_1, F_2)	0	$(0,1)$	0	2	1	0
$Q, \tilde{Q}$	$q^m = (q, \bar{\tilde{q}})$	0	$(0, \frac{1}{2})$	0	1	N_f	$+1$
	$\psi_i = (\psi, \tilde{\psi}^\dagger)$	$\frac{1}{2}$	$(0,0)$	∓ 1	$\frac{3}{2}$	N_f	$+1$

N_f D7s wrapping $AdS_5 \times S^3$ inside $AdS_5 x S^5$ ($m = \mathrm{VEVs} = 0$):

$$SO(4,2) \times SU(2)_L \times SU(2)_R \times U(1)_z \times U(N_f)$$

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Outline

- Introduction to AdS/CFT with Flavour
- $B_{t,\vec{x}}$: Electric/Magnetic Background
  - Magnetic Field
  - Electric Field
- B_{u,S^5}: Noncommutativity & FI-Terms
  - Commutative Instantons & Coulomb-Higgs Branch
  - Noncommutative Instantons and the FI term
- Summary & Outlook

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D3-D7 field theory: Mixed Coulomb-Higgs branch

F-Terms

$$0 = \tilde{Q}_I(\Phi_3 + m) = (\Phi_3 + m)Q^I \quad (1)$$

$$0 = [\Phi_1, \Phi_3] = [\Phi_2, \Phi_3], \quad (2)$$

$$0 = Q^I\tilde{Q}_I + [\Phi_1, \Phi_2] \quad \longleftarrow \text{N x N} \quad (3)$$

$$(1) \Rightarrow \Phi_3 = \begin{pmatrix} \tilde{m}_1 & & & & & \\ & \ddots & & & & \\ & & \tilde{m}_{N-k} & & & \\ & & & -m & & \\ & & & & \ddots & \\ & & & & & -m \end{pmatrix}, \quad Q^I = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ q_1^i \\ \vdots \\ q_k^i \end{pmatrix},$$

$$\tilde{Q}_i = (0 \cdots 0, \tilde{q}_i^1 \cdots, \tilde{q}_i^k), \quad \text{with} \sum_{j=1}^{N-k} \tilde{m}_j = km$$

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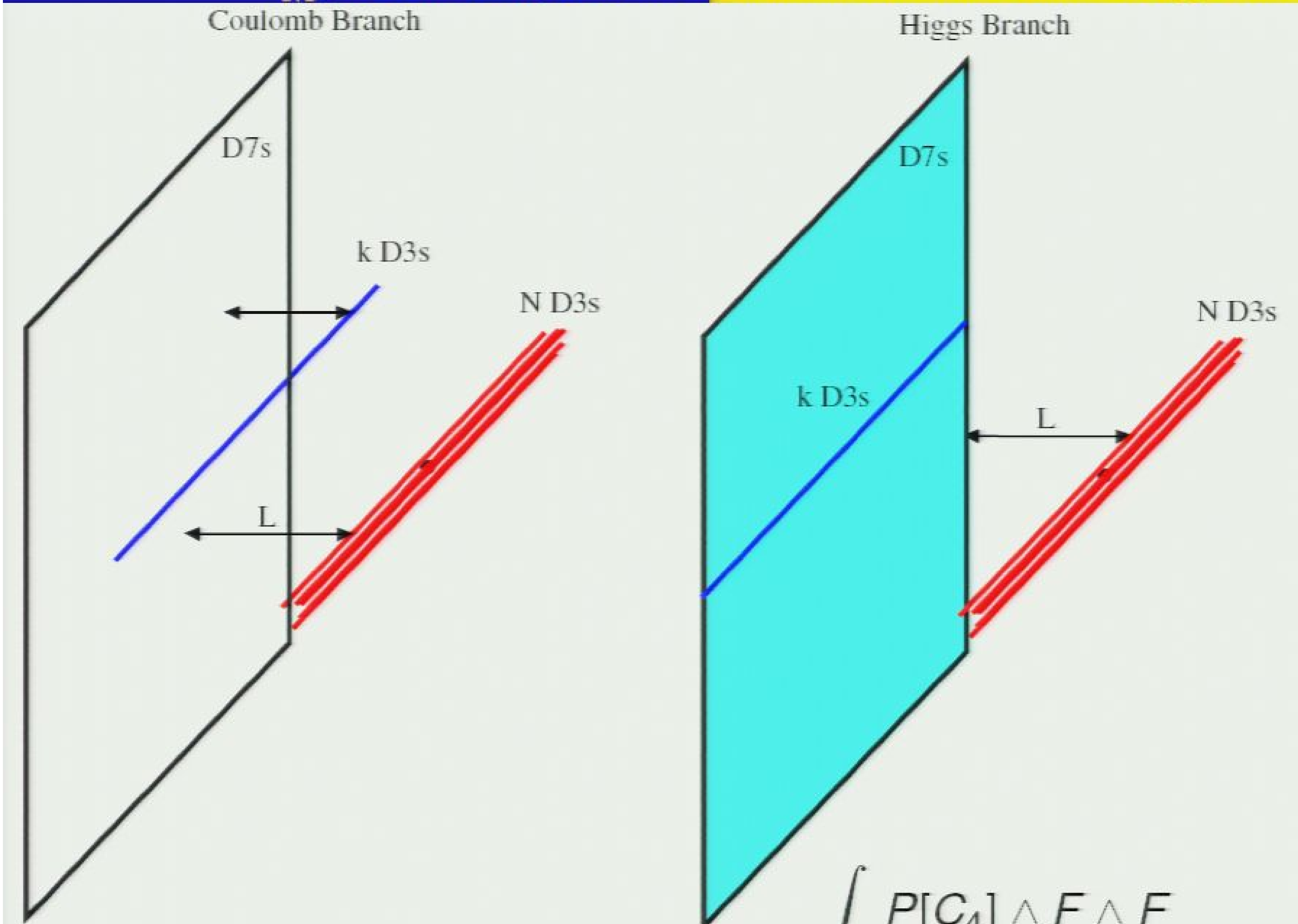

$$\int_{\mathcal{M}_8} P[C_4] \wedge F \wedge F$$

3-D7 field theory: Mixed Coulomb-Higgs branch

ADHM equations & Higgs Branch

$$
\begin{aligned}
(2) \Rightarrow \qquad\qquad 0 &= [\Phi_1, \Phi_3] = [\Phi_2, \Phi_3]\,, \\
(3) \Rightarrow \mu_c = 0 &= q^I \tilde{q}_I + [\Phi_1, \Phi_2]\,, \\
\mu_r = 0 &= |q^I|^2 - |\tilde{q}_I|^2 + [\Phi_1, \Phi_1^\dagger] + [\Phi_2, \Phi_2^\dagger] \qquad \text{D-Term}
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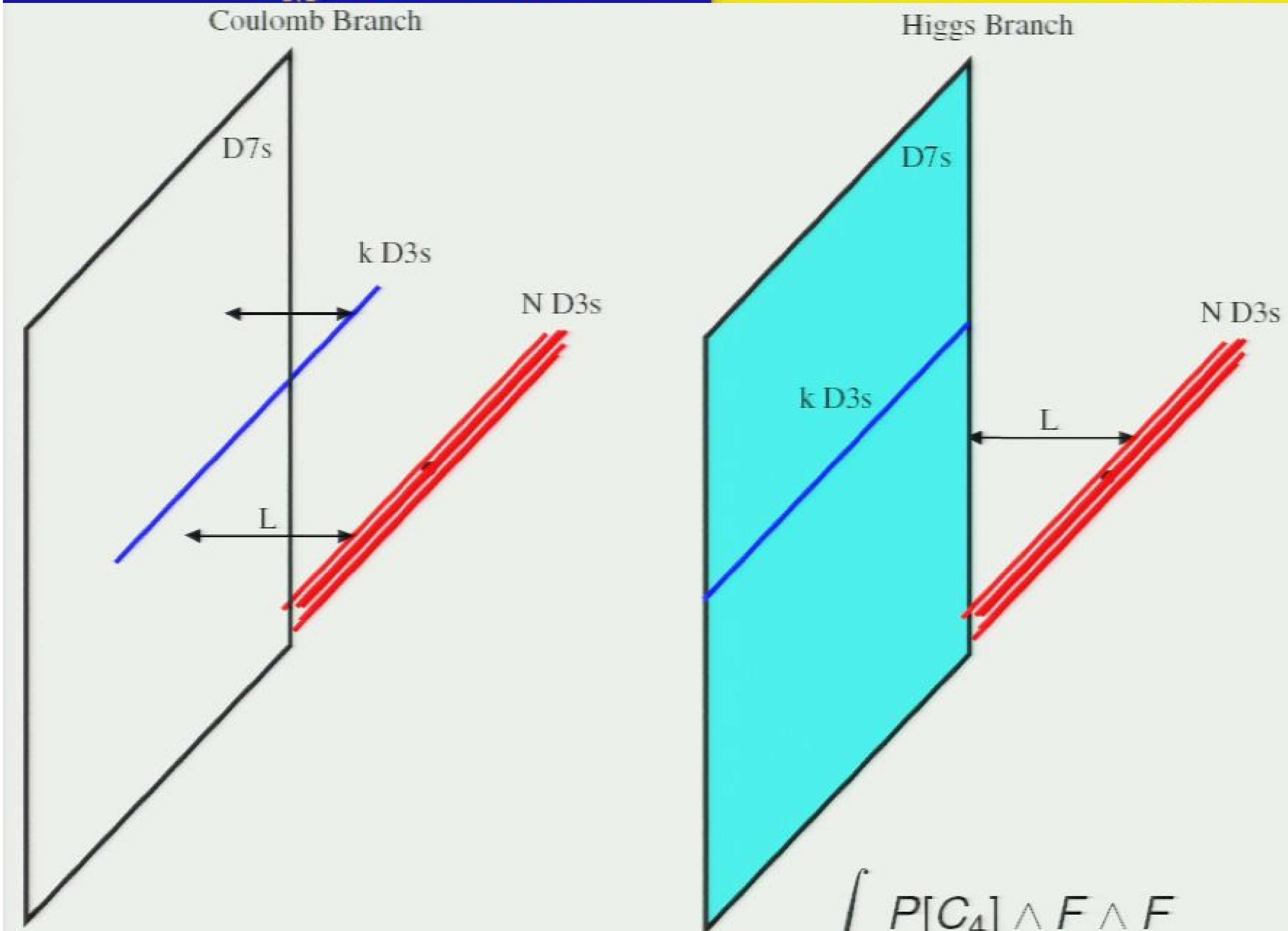
$\Rightarrow \exists$ Nontrivial gauge field configuration in the D7 directions perpendicular to the D3 branes

Gravity Dual of Coulomb-Higgs branch [hep-th/0405127,0412074,0502224]

The gravity dual of the mixed Coulomb-Higgs branch of the field theory on the N_c D3/N_f D7 intersection is given by identifying the Higgs VEVs with the size moduli of instantons in the directions of the D7 branes perpendicular to the D3 branes.



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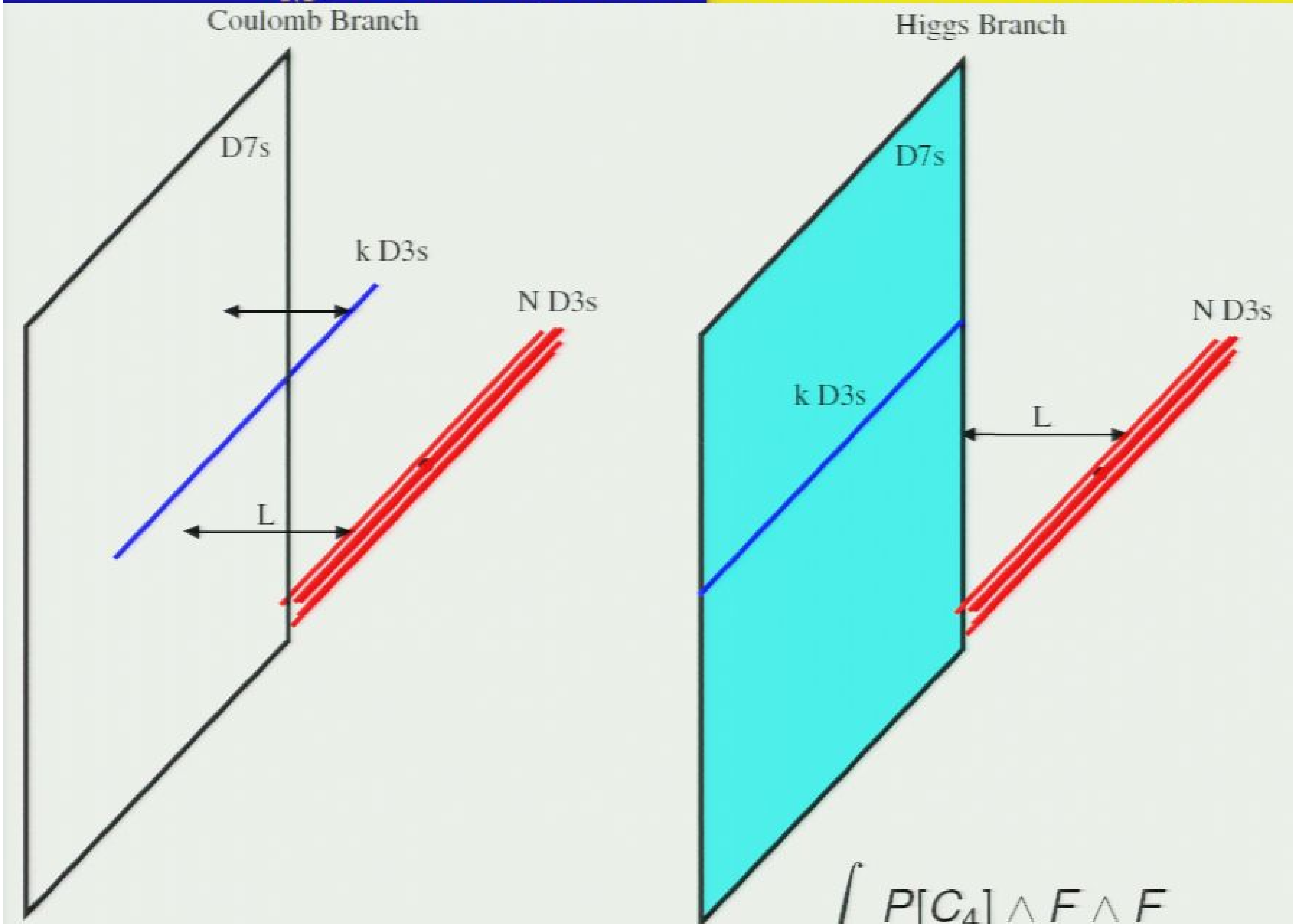

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Two Checks

BPS Property of N_f D7s in $AdS_5 \times S^5$

$$S_{D7} = \frac{(2\pi\alpha')^2 T_7}{2} \int \mathrm{d}^4x \mathrm{d}^4y \frac{(\vec{y}^2 + m^2)^2}{R^4} \mathrm{Tr} F_-^2$$

Higgs VEV $q_{I\alpha} = v\varepsilon_{I\alpha} \Leftrightarrow$ Instanton Size: $v = \Lambda/2\pi\alpha'$ [hep-th/0502224]

The SU(2) Instanton in singular gauge

$$A_\mu = 0\,, \quad A_m = \frac{2\Lambda^2 \bar{\sigma}_{nm} y_n}{y^2(y^2 + \Lambda^2)}$$

breaks global symmetries on the gravity side to

$$SO(1,3) \times SU(2)_L \times \mathrm{diag}(SU(2)_R \times SU(2)_f)\,.$$

Noncommutative Instantons and FI terms [JHEP 0807:068,2008]

Goal

Describe the relation between noncommutative instantons on flavour branes, FI-terms in the ADHM equations,

$$|q^I|^2 - |\tilde{q}_I|^2 + [\Phi_1, \Phi_1^\dagger] + [\Phi_2, \Phi_2^\dagger] = \zeta \mathrm{Id}\,,$$

and find their dual field theory description.

Guess

A constant anti-selfdual $B_{\mu\nu}$ in the internal $\mathbb{R}^4$ filled by the D7 branes breaks supersymmetry, and thus induces an FI term for the $U(1) \in U(N_c)$. On the flavour D7 branes it induces noncommutativity, rendering the $U(N_f)$ ADHM construction noncommutative. Noncommutative Instanton configurations on the flavour branes then describe nonsupersymmetric states on the Coulomb-Higgs branch of the dual $\mathcal{N} = 2$ theory.

Noncommutative Instantons and FI terms [JHEP 0807:068,2008]

The Background

$$\mathrm{d}s^2 = \frac{\vec{y}^2 + \vec{z}^2}{R^2}\mathrm{d}x^{\mu\,2} + \frac{R^2}{\vec{y}^2 + \vec{z}^2}\left(\mathrm{d}\vec{y}^2 + \mathrm{d}\vec{z}^2\right)$$

$$B = B_{ij}\mathrm{d}y^i \wedge \mathrm{d}y^j$$

- $B_{ij} = const. \Rightarrow \mathrm{d}B = 0$
- Flat D7 embedding: $(x^\mu, \vec{y})$, $\vec{z} = 0$
- (anti)-selfduality $\Rightarrow$ SUSY

	$*B = +B$	$*B = -B$
D3-D7	SUSY if $*F = +F$	no SUSY, FI
D3-$\overline{D7}$	no SUSY, FI	SUSY if $*F = -F$

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D3-D7, $*B = -B$

FI Term in Flat Space [Lerda et.al. hep-th/0511036]

$$\langle V_D V_B \rangle \propto \frac{1}{g_{-1}^2} D \bar{\eta}^3_{ij} \theta^{ij} \Rightarrow \frac{\bar{\eta}^3_{ij}\theta^{ij}}{g_{YM}^2} \int \mathrm{d}^4 x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \, \mathrm{tr} V$$

- $\theta^{ij} = 2\pi\alpha' B^{ij}$, fixed while $\alpha' \to 0 \Rightarrow B \to \infty$ [Chu&Russo Mod.Phys.Lett.A 16 (2001)]
- $D(-1) - D3 \to D3 - D7$ by dim. oxidation
- Normalization correct for contributing to the D-Term, with constant $\zeta = \bar{\eta}^3_{ij}\theta^{ij}$
- Term is part of $U(N_c)$ $\mathcal{N} = 4$ theory and thus survives the $\alpha' \to 0$ limit which decouples the 7-7 string DOFs.

Check: Symmetries I

Symmetry breaking for $N_f = 1$, One Instanton

Object	Coordinates			
	x^μ	ρ	y_m S^3	z^i
$AdS_5 \times S^5$	$SO(4,2)$		$SO(6) \simeq SU(4)$	
$D7$	$SO(4,2)$		$SU(2)_L \times SU(2)_R$	$U(1)_z$
$*B = -B$	$SO(3,1)$		$SU(2)_L \times U(1)_R$	$U(1)_z$
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- B_{IJ}: 15 of SO(6) $= (0,0)_0 \oplus \overbrace{(1,0)_0}^{SD} \oplus \overbrace{(0,1)_0}^{ASD} \oplus (\frac{1}{2},\frac{1}{2})_2 \oplus (\frac{1}{2},\frac{1}{2})_{-2}$
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Check: Symmetries II

Induced Noncommutative Field Theory on $AdS_5 \times S^5$ not known
$\rightarrow$ Use flat space result $[y^i, y^j] = i\theta^{ij}$

Noncommutative $U(1)$ Instanton [Nekrasov&Schwarz hep-th/9802068]

$$A = \frac{1}{d\left(d + \frac{\zeta}{2}\right)}\left[z_0 \mathrm{d}\bar{z}_0 + \bar{z}_1 \mathrm{d}z_1\right], \quad d = \bar{z}_0 z_0 + \bar{z}_1 z_1$$

$z_0 = y^4 + iy^5,\ z_1 = y^6 + iy^7 \Rightarrow [z_0, \bar{z}_0] = 2\theta^{45},\ [z_1, \bar{z}_1] = 2\theta^{67},\ [z_0, z_1] = 0$

Naive Symmetries

$$y^{i\prime} = y^i + M^i{}_j y^j, \quad M \in su(2)_L \times u(1)_R$$

$$\Rightarrow \forall M : A_i'(z')\mathrm{d}z'^i = A_i(z)\mathrm{d}z$$

$\Rightarrow$ Naively: $SU(2)_L \times U(1)_R \times U(1)_F$ preserved

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Instanton Charge $k = 1$: One squark colour component VEV

$$0 = q\tilde{q}\,, \quad \bar{\eta}^3_{ij}\theta^{ij} = \zeta = |q|^2 - |\tilde{q}|^2$$

Solution: $q = \sqrt{\zeta}\,, \tilde{q} = 0$

$U(1)_F$-Rotations: $q \rightarrow e^{i\alpha_F} q$, $U(1)_R$-Rotations: $q \rightarrow e^{i\alpha_R} q$

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Noncommutative $U(1)$ Instanton [Nekrasov&Schwarz hep-th/9802068]

$$A = \frac{1}{d\left(d + \frac{\varsigma}{2}\right)}\left[z_0 \mathrm{d}\bar{z}_0 + \bar{z}_1 \mathrm{d}z_1\right], \quad d = \bar{z}_0 z_0 + \bar{z}_1 z_1$$

$$z_0 = y^4 + iy^5,\ z_1 = y^6 + iy^7 \Rightarrow [z_0, \bar{z}_0] = 2\theta^{45},\ [z_1, \bar{z}_1] = 2\theta^{67},\ [z_0, z_1] = 0$$

Naive Symmetries

$$y^{i\prime} = y^i + M^i{}_j y^j, \quad M \in su(2)_L \times u(1)_R$$

$$\Rightarrow \forall M : A_i'(z')\mathrm{d}z'^i = A_i(z)\mathrm{d}z$$

$\Rightarrow$ Naively: $SU(2)_L \times U(1)_R \times U(1)_F$ preserved

Check: Symmetries II

Induced Noncommutative Field Theory on $AdS_5 \times S^5$ not known
$\to$ Use flat space result $[y^i, y^j] = i\theta^{ij}$

Field Theory Symmetries

Instanton Charge $k = 1$: One squark colour component VEV

$$0 = q\tilde{q}\,, \quad \bar{\eta}^3_{ij}\theta^{ij} = \zeta = |q|^2 - |\tilde{q}|^2$$

Solution: $q = \sqrt{\zeta}\,, \tilde{q} = 0$

$U(1)_F$-Rotations: $q \to e^{i\alpha_F} q$, $U(1)_R$-Rotations: $q \to e^{i\alpha_R} q$

$\Rightarrow SU(2)_L \times \mathrm{diag}(U(1)_R \times U(1)_F)$ preserved

- $\mathrm{diag}(SU(2)_R \times SU(2)_F)$ for BPST-Instanton should carry over to $N_f = 2$ noncommutative case
- Possible Solutions: 1) Mixing of Rotations and Gauge Symmetries?
  2) Effect of $AdS_5 \times S^5$ on $\theta^{ij}(x)$?

Summary & Outlook

- $B_{\mu\nu}$ in Minkowski directions: constant electric/magnetic field background for quenched flavours, by its coupling to baryon number
  - Magnetic field: Catalyzes SCSB, prevents meson melting
  - Electric field: Dissociation of mesons, no SCSB
- $B_{\mu\nu}$ in internal D7 directions:
  - Induces FI-term in the field theory & Noncommutativity
  - Check 1: Symmetry Breaking by θ^{ij}
  - Noncommutative Instantons on D7 correspond to nontrivial Coulomb-Higgs States in the field theory
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