Title: Averaging Robertson-Walker Cosmologies

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Abstract: The so-called cosmological backreaction arises when one directly averages the Einstein equations to recover cosmology. While usually applied to avoid employing dark energy models, strictly speaking any cosmological model should be built from such an averaging procedure rather than an assumed background. We apply the Buchert formalism to Einstein-de Sitter, Lambda CDM and quintessence cosmologies, and as a first approach to the full problem, evaluate numerically the discrepancies arising from linear perturbation theory between the averaged behaviour and the assumed behaviour. (References: J. Behrend, IB and G. Robbers, JCAP01(2008)013, aXiv:0710.4964; IB, G. Robbers and J. Behrend, in preperation)

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Institut für Theoretische Physik, Universität Heidelberg

"Backreaction from Perturbations", J. Behrend, IB and G. Robbers, JCAP 0801 013

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Perimeter Institute, Waterloo, 9th September 2008

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Standard Cosmology

- Copernican Principle: There are no unique places in the universe;

  CMB observations: The universe is highly isotropic about the Earth

  $\Rightarrow$ Universe homogeneous and isotropic.

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  $\Rightarrow$ Universe homogeneous and isotropic.
- Robertson-Walker cosmology: foliate spacetime with maximally-symmetric three-spaces
  - Line element: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
  - Friedmann equation: $(\dot{a}/a)^2 = (8\pi G/3)\overline{\rho} + \Lambda/3$
  - Raychaudhuri equation: $\ddot{a}/a = -(4\pi G/3)(\overline{\rho} + \overline{p}) + \Lambda/3$
  - Perturb metric with $\mathcal{O}(\epsilon) \approx 10^{-5}$
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  - Perturb metric with $\mathcal{O}(\epsilon) \approx 10^{-5}$
  - Inclusion of Λ ensures observed acceleration
- We have assumed the existence of an average and added perturbations

Averaging in Cosmology

- An implicit averaging in cosmology transfers local equations to global cosmology; should be made explicit
- Coincidence problem ties Λ to structure formation $\Rightarrow$ less fine-tuning?
- Averaging in G.R. difficult to define but $\langle \partial_t \rho \rangle \neq \partial_t \langle \rho \rangle$

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  $\Rightarrow$ We are using wrong large-scale EFE
- We should be using

$$\langle G_{\mu\nu}(g_{\mu\nu}) \rangle = 8\pi G \langle T_{\mu\nu} \rangle + \Lambda \langle g_{\mu\nu} \rangle$$

  instead of

$$G_{\mu\nu}(\langle g_{\mu\nu} \rangle) = 8\pi G \langle T_{\mu\nu} \rangle + \Lambda \langle g_{\mu\nu} \rangle ,$$

  averaged in a domain $\mathcal{D}$ on some 3-surface $\Sigma(t)$
- The difference is called "backreaction"; in principle, can resemble dark energy

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- Currently Zalaletdinov's macroscopic gravity is the only "covariant" averaging procedure and is extremely complex
- Studies of averaging in cosmology date to the 60s (Shirokov and Fisher) and 80s (e.g. Ellis, Futamase, Kasai)
- Explosion of interest in cosmology, chiefly to address the dark energy problem (Buchert, Räsänen, Wetterich, Li and Schwarz, Martineau and Brandenberger, Kolb *et. al.*,. . .)
- Also applications of macroscopic gravity (e.g. Zalaletdinov, Coley, Paranjape and Singh. . .)

Cosmological Averaging: “Backreaction”

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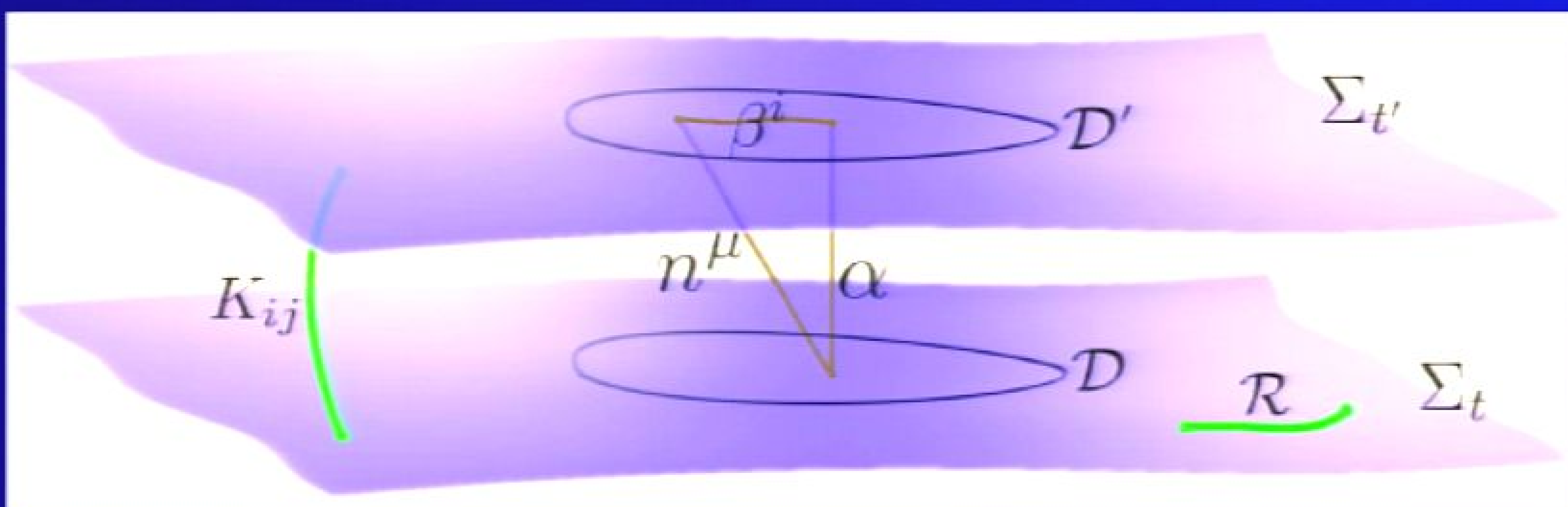
- Effect from linear perturbations (e.g. Wetterich, Räsänen) $\approx 10^{-5}$
  - Vanderveld, Flanagan and Wasserman 07 $\approx 10^{-5}$ perturbatively
  - Khosravi *et. al.* 08 $\approx 10^{-5}$ in a “structured FLRW” model
  - Li and Schwarz 07, significant results ($\approx 10^{-1}$) (EdS, high order)
  - Behrend, IB and Robbers 08 $\approx 10^{-5}$ numerically (EdS, ΛCDM)
  - Räsänen 08 $\approx 10^{-3}$ in “peak structure” model (EdS)
  - Similar results from Paranjape 08 (EdS, simplified radiation+CDM)
- Equations of state similar to dust are usual - but these are usually matter models

Cosmological Averaging: “Backreaction”

- Authors tend to focus on dark energy, but averaging problem is distinct from the dark energy problem
- While “backreaction” may not be dark energy, ***all*** cosmological models should be properly averaged (e.g. Wetterich 02, $\sim \mathcal{O}(1)$ impact from clustered cosmon)
- Aim: Express Buchert equations in general form, apply to range of perturbed Robertson-Walker models from radiation domination to present day.

Formalism: 3+1 Split

- Buchert's averaging recovers Friedmann- and Raychaudhuri-like equations from averaging spatial 3-surfaces
- Employ 3+1 split with vanishing shift vector, non-vanishing lapse function: $ds^2 = -\alpha^2 dt^2 + h_{ij} dx^i dx^j$
- Choose slices with normal n^μ and induced metric $h_{ij} = g_{ij} + n_i n_j$



Formalism: Multifluids

- Backreaction studies normally consider only CDM: EdS is $\sim 95\%$ CDM
- Otherwise they focus on scalar fields

Formalism: 3+1 Split

$$ds^2 = -\alpha^2 dt^2 + h_{ij} dx^i dx^j$$

- Will work with scalar perturbations in Newtonian gauge: retain lapse
- Extrinsic curvature

$$K_{ij} = -\frac{1}{2}\mathcal{L}_n h_{ij} = -\frac{1}{2\alpha}\dot{h}_{ij}$$

- Stress-energy tensor on the slice:

$$T_{\mu\nu} = \varrho n_\mu n_\nu + 2n_{(\mu}j_{\nu)} + S_{\mu\nu},$$

So

$$\varrho = n^\mu n^\nu T_{\mu\nu},\ j_i = -n^\mu T_{i\mu},\ S_{ij} = T_{ij}$$

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- Backreaction studies normally consider only CDM: EdS is $\sim 95\%$ CDM
- Otherwise they focus on scalar fields
- For generally viable cosmology, must retain Λ, scalar fields, baryons, radiation
- Model fluids as perfect fluids, $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$

$$
\begin{aligned}
\varrho &= n^\mu n^\nu T_{\mu\nu} = \rho\,(n^\mu u_\mu)^2 + p\left((n^\mu u_\mu)^2 - 1\right), \\
j^i &= -n^\mu T_{\mu i} = -\,(n^\mu u_\mu)\,(\rho + p)\,u_i, \\
S &= T^i_i = 3p + (\rho + p)\,u^i u_i.
\end{aligned}
$$

Formalism: Buchert Averaging

- Select simple scalar average

$$\langle A \rangle = \frac{1}{V} \int_{\mathcal{D}} A \sqrt{h} d^3\mathbf{x},$$

Define averaged "scale factor" and Hubble rate by

$$3H_{\mathcal{D}} = 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{\dot{V}}{V} = -\frac{1}{V}\int_{\mathcal{D}} \alpha K \sqrt{h} d^3\mathbf{x} = -\langle \alpha K \rangle = \langle \mathcal{H} \rangle\,,$$

with commutation relation

$$\left\langle \dot{A} \right\rangle = \frac{\partial}{\partial t}\langle A \rangle + 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\langle A \rangle - \langle A \alpha K \rangle$$

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- MG has also been applied to cosmology (e.g. Zalaletdinov 07, Paranjape and Singh 07, Paranjape 08), but highly complex

Formalism: Buchert Equations

- Hamiltonian constraint:

$$\mathcal{R} + K^2 - K^i_j K^j_i = 16\pi G \varrho + 2\Lambda$$

- Averages to a “Friedmann” equation with additional terms:

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\langle \alpha^2 \varrho \rangle + \frac{\Lambda}{3}\langle \alpha^2 \rangle - \frac{1}{6}\left(\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}}\right)$$

Formalism: Buchert Equations

- Evolution equation:

$$\frac{1}{\alpha}\dot{K}_{ij} = 8\pi G S_{ij} + 4\pi G h_{ij}\left(\varrho - S\right) + \Lambda h_{ij} + 2K_{in}K^{n}_{j} - KK_{ij} - \mathcal{R}_{ij} + \frac{1}{\alpha}D_i D_j \alpha$$

- Averages to a "Raychaudhuri" equation with additional terms:

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3}\langle\alpha^2(\varrho + S)\rangle + \frac{\Lambda}{3}\langle\alpha^2\rangle + \frac{1}{3}\left(\mathcal{Q}_{\mathcal{D}} + \mathcal{P}_{\mathcal{D}}\right)$$

Formalism: Modifications to Standard Cosmology

- Kinematical "backreaction":

$$\mathcal{Q}_{\mathcal{D}} = \left\langle \alpha^2 \left(K^2 - K^i_j K^j_i \right) \right\rangle - \frac{2}{3} \langle \alpha K \rangle^2$$

- Dynamical "backreaction":

$$\mathcal{P}_{\mathcal{D}} = \langle \dot{\alpha} K \rangle + \langle \alpha D^i D_i \alpha \rangle$$

- Curvature contribution:

$$\mathcal{R}_{\mathcal{D}} = \langle \alpha^2 \mathcal{R} \rangle$$

- Deviation from average density and pressure:

$$3\mathcal{T}_{\mathcal{D}}^{(a)} \; = \langle \alpha^2 \rho_{(a)} \rangle - \overline{\rho}_{(a)}, \quad 3\mathcal{S}_{\mathcal{D}}^{(a)} \; = \langle \alpha^2 S_{(a)} \rangle - \overline{S}_{(a)}$$

Formalism: Modifications to Standard Cosmology

- The Buchert equations can then be written

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\sum_a \overline{\rho}_{(a)} + \frac{\Lambda}{3} + \frac{8\pi G}{3}\overline{\rho}_{\rm eff},$$

$$\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = -\frac{4\pi G}{3}\sum_a \left(\overline{\rho}_{(a)} + \overline{S}_{(a)}\right) + \frac{\Lambda}{3} - \frac{4\pi G}{3}\left(\overline{\rho}_{\rm eff} + \overline{S}_{\rm eff}\right)$$

- Standard Friedmann equations plus corrective fluid
- Observational issues: what do these actually mean? (e.g. Larena *et. al.* 08)

Formalism: Buchert Averaging

- Select simple scalar average

$$\langle A\rangle = \frac{1}{V}\int_{\mathcal{D}} A\sqrt{h}d^3\mathbf{x},$$

Define averaged "scale factor" and Hubble rate by

$$3H_{\mathcal{D}} = 3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} = \frac{\dot{V}}{V} = -\frac{1}{V}\int_{\mathcal{D}} \alpha K\sqrt{h}d^3\mathbf{x} = -\langle \alpha K\rangle = \langle \mathcal{H}\rangle\,,$$

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Formalism: Modifications to Standard Cosmology

- Effective correction fluid given by

$$\frac{8\pi G}{3}\overline{\rho}_{\rm eff} = \sum_a \mathcal{T}_{\mathcal{D}}^{(a)} + \left\langle \alpha^2 - 1 \right\rangle \frac{\Lambda}{3} - \frac{1}{6}\left(\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}}\right),$$

$$16\pi G\overline{p}_{\rm eff} = 4\sum_a \mathcal{S}_{\mathcal{D}}^{(a)} - 2\left\langle \alpha^2 - 1 \right\rangle \Lambda + \frac{1}{3}\left(\mathcal{R}_{\mathcal{D}} - 3\mathcal{Q}_{\mathcal{D}} - 4\mathcal{P}_{\mathcal{D}}\right),$$

$$w_{\rm eff} = -\frac{1}{3}\frac{\mathcal{R}_{\mathcal{D}} - 3\mathcal{Q}_{\mathcal{D}} - 4\mathcal{P}_{\mathcal{D}} + 12\sum_a \mathcal{S}_{\mathcal{D}}^{(a)} - 6\Lambda\left\langle \alpha^2 - 1 \right\rangle}{\mathcal{R}_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}} - 6\sum_a \mathcal{T}_{\mathcal{D}}^{(a)} - 2\Lambda\left\langle \alpha^2 - 1 \right\rangle}.$$

- For carefully chosen model, resembles effective dark energy
- Buchert equations fully general for any irrotational system – but:
  - Forced to average only scalar quantities
  - Useless without a concrete model

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Formalism: What Next?

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  - Analytical approaches, e.g. LTB, swiss-cheese, scaling ansatzes etc.
  - Numerical approaches, e.g. simulated or mock cluster distributions
  - Cosmological perturbation theory – but impact is $\sim 10^{-5}$
  - Observational approaches, e.g. reconstruction, cluster surveys
  - Analysis based on physical observables (e.g. d_A, d_L – see e.g. Räsänen, Marra et. al.)

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- Choose perturbative approaches for quantitativity

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Perturbative Models

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- Background: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
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- Newtonian gauge well-controlled on sub-horizon scales $\Rightarrow$ no gauge worries, $\phi \ll 1$ across scales considered
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- Average of second-order quantity hard to define; assume $\langle \phi^{(2)} \rangle = 0$, retain $\langle \phi^{(1)}\phi^{(1)} \rangle$
- Linear theory implies very large scales $\Rightarrow$ small results expected; results strictly unobservable; care should be taken with domain boundary
- Perturbation theory not necessarily valid as $z \to 0$ – should be tested

Perturbative Models

- Identify ADM and Newtonian co-ordinates (c.f. Mukhanov et. al.)

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j = -\alpha^2 dt^2 + h_{ij}dx^i dx^j$$

- Note: alternative gauges – uniform density to simplify $\mathcal{T}_\mathcal{D}$ and $\mathcal{S}_\mathcal{D}$, uniform curvature to remove $\mathcal{R}_\mathcal{D}$, synchronous gauge to remove $\mathcal{P}_\mathcal{D}$. $\mathcal{Q}_\mathcal{D}$ cannot be entirely removed except in EdS matter domination.

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- Fluid corrections, $w = p/\rho$, $c_s^2 = \partial p/\partial\rho$:

$$\mathcal{T}_{\mathcal{D}} = \frac{8\pi G}{3}\overline{\rho}\left\langle \delta + 2\Psi + a^2(1+\overline{w})v^2 + 2\Psi\delta \right\rangle ,$$

$$\mathcal{S}_{\mathcal{D}} = \frac{4\pi G}{3}\overline{\rho}\left\langle 3c_s^2\delta + 6\overline{w}\Psi + (1+\overline{w})a^2v^2 + 6c_s^2\Psi\delta \right\rangle$$

Perturbative Models

- Kinematical and dynamical backreactions:

$$\mathcal{Q}_{\mathcal{D}} = 6\left(\left\langle\dot{\Phi}^2\right\rangle - \left\langle\dot{\Phi}\right\rangle^2\right),$$

$$\mathcal{P}_{\mathcal{D}} = \frac{1}{a^2}\langle\nabla^2\Psi - (\nabla\Psi)^2 + 2\Phi\nabla^2\Psi - (\nabla\Phi)\cdot(\nabla\Psi)\rangle + 3\frac{\dot{a}}{a}\left\langle\dot{\Psi} - 2\Psi\dot{\Psi}\right\rangle - 3\left\langle\dot{\Psi}\dot{\Phi}\right\rangle$$

- Curvature correction:

$$\mathcal{R}_{\mathcal{D}} = \frac{2}{a^2}\langle 2\nabla^2\Phi + 3(\nabla\Phi)^2 + 4(2\Phi+\Psi)\nabla^2\Phi\rangle\,.$$

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$$\mathcal{P}_{\mathcal{D}} = \frac{1}{a^2}\left\langle \nabla^2\Psi - (\nabla\Psi)^2 + 2\Phi\nabla^2\Psi - (\nabla\Phi)\cdot(\nabla\Psi)\right\rangle + 3\frac{\dot{a}}{a}\left\langle \dot{\Psi} - 2\Psi\dot{\Psi}\right\rangle - 3\left\langle \dot{\Psi}\dot{\Phi}\right\rangle$$

- Curvature correction:

$$\mathcal{R}_{\mathcal{D}} = \frac{2}{a^2}\left\langle 2\nabla^2\Phi + 3(\nabla\Phi)^2 + 4(2\Phi + \Psi)\nabla^2\Phi\right\rangle.$$

Perturbative Models: Large Scales

- Take $\mathcal{D}$ large enough to neglect first-order averages:
  - Fluid and curvature modifications:

$$\mathcal{T}_{\mathcal{D}}^{(a)} = \frac{8\pi G\overline{\rho}_{(a)}}{3}\left\langle(1+\overline{w}_{(a)})a^2v_{(a)}^2 + 2\Psi\delta_{(a)}\right\rangle,$$

$$\mathcal{S}_{\mathcal{D}}^{(a)} = \frac{4\pi G\overline{\rho}_{(a)}}{3}\left\langle(1+\overline{w}_{(a)})a^2v_{(a)}^2 + 6c_{s(a)}^2\Psi\delta_{(a)}\right\rangle,$$

$$\mathcal{R}_{\mathcal{D}} = \frac{2}{a^2}\langle 3(\nabla\Phi)^2 + 4(2\Phi+\Psi)\nabla^2\Phi\rangle$$

Perturbative Models: Large Scales

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- Dynamical and kinematical backreactions:

$$\mathcal{P}_{\mathcal{D}} = \frac{1}{a^2}\langle 2\Phi\nabla^2\Psi - (\nabla\Psi)^2 - (\nabla\Phi)\cdot(\nabla\Psi)\rangle$$
$$-6\frac{\dot{a}}{a}\left\langle \Psi\dot{\Psi}\right\rangle - 3\left\langle \dot{\Psi}\dot{\Phi}\right\rangle,$$
$$\mathcal{Q}_{\mathcal{D}} = 6\left\langle \dot{\Phi}^2\right\rangle$$

Numerical Study

Ergodic Averaging

- Boltzmann codes are 1-d, averages are 3-d, so take $\mathcal{D}$ large enough to employ ergodic principle

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Perturbative Models: Large Scales

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$$-6\frac{\dot{a}}{a}\left\langle \Psi\dot{\Psi}\right\rangle - 3\left\langle \dot{\Psi}\dot{\Phi}\right\rangle,$$
$$\mathcal{Q}_{\mathcal{D}} = 6\left\langle \dot{\Phi}^2\right\rangle$$

20:36:49

Ergodic Averaging

- Boltzmann codes are 1-d, averages are 3-d, so take $\mathcal{D}$ large enough to employ ergodic principle

Ergodic Averaging

- Boltzmann codes are 1-d, averages are 3-d, so take $\mathcal{D}$ large enough to employ ergodic principle
- Linear theory, so modes decouple $\Rightarrow$ statistics imprinted in primordial era
- Then

$$\langle AB \rangle = \int_k \mathcal{P}_\psi(k) A(k) B^*(k) \frac{dk}{k}$$

with primordial power spectrum

$$\left\langle \Psi(\mathbf{k}) \Psi^*(\mathbf{k}') \right\rangle = \mathcal{P}_\psi(k) \delta(\mathbf{k} - \mathbf{k}') = \frac{2\pi^2}{k^3} A_s \left(\frac{k}{k_*} \right)^{n_s - 1} \delta(\mathbf{k} - \mathbf{k}')$$

- Corrections to standard case of form

$$\mathcal{Q}_\mathcal{D} = \int 6\mathcal{P}_\psi(k) \left| \dot{\Phi} \right|^2 \frac{dk}{k} = \int \mathcal{Q}_\mathcal{D}(k) \frac{dk}{k}$$

Cosmological Models

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- BBN constraints imply $\Omega_b \approx 0.05$
- Structure formation and prejudice suggest $\Omega \approx 1$
- Consider:
  - EdS ($\Omega_c = 0.95$, $h \approx 0.45$; fits CMB but not LSS), main model pre-SNIa
  - ΛCDM ($\Omega_\Lambda \approx 0.73$, $h \approx 0.71$; fits all data), main model post-SNIa
  - Early dark energy ($\Omega_\Lambda^0 = 0.73$, $\Omega_\Lambda^\infty = 0.05$; fits all data)
  - Tracker Quintessence, $V(\phi) \propto \exp(-\phi)$ ($\Omega_\phi = 0.2$, $w_0 \approx 0$), resembles EdS
  - Inverse-Power Law Quintessence, $V(\phi) \propto 1/\phi^2$ ($\Omega_\phi = 0.12$, $w_0 \approx -0.52$), resembles tracker

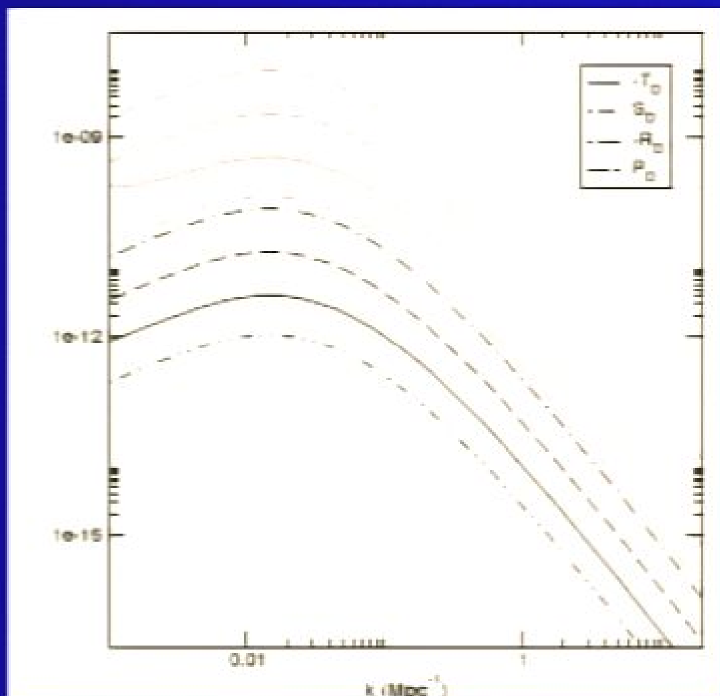
Approximations

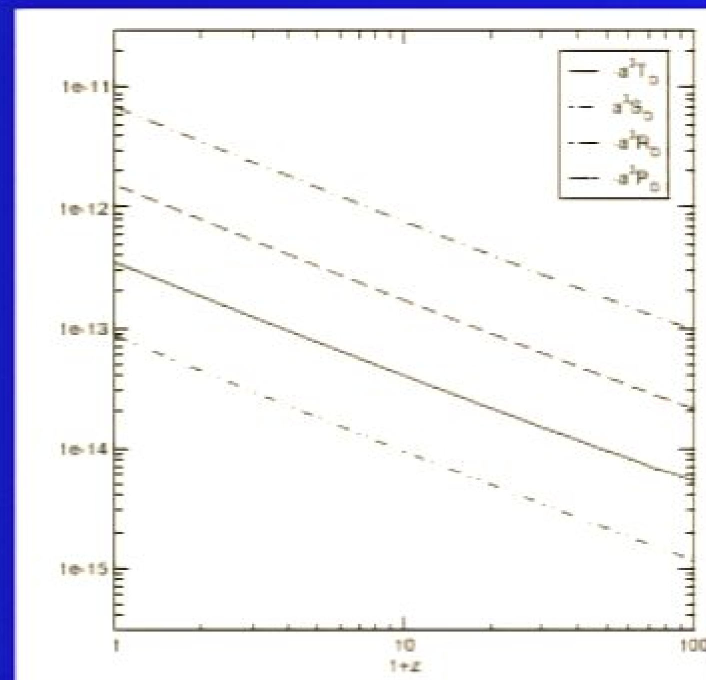
- Poisson equation implies $\phi \propto \delta/k^2$
- Euler equation for matter implies $|v| \propto \delta/k$
- So with scale-invariant primordial spectrum and $k^{-4} \ll k^{-2}$,

$$\begin{aligned} \mathcal{Q}_{\mathcal{D}}(k) &\propto \delta^2/k^4, \\ \mathcal{P}_{\mathcal{D}}(k) \propto \mathcal{T}_{\mathcal{D}}(k) \propto \mathcal{Q}_{\mathcal{D}}(k) &\propto \delta^2/k^2 \end{aligned}$$

- Corrections closely related to matter power spectrum $P(k) = |\delta(k)|^2$
- Expect kinematical backreaction to be negligible and the others to be proportional to one-another except on largest scales
- Integrate to comoving Hubble horizon, $k \in (1/\eta, 100\mathrm{Mpc}^{-1})$

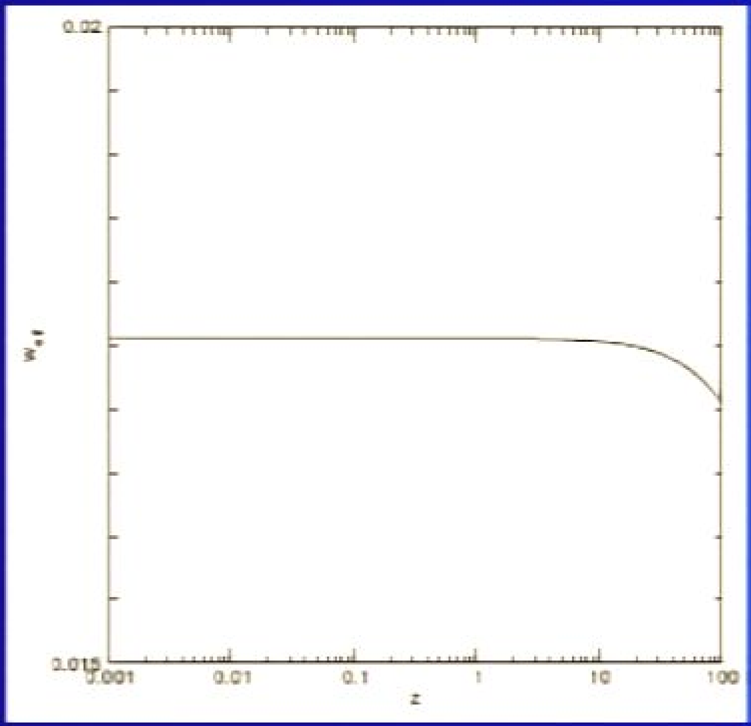
Einstein-de Sitter

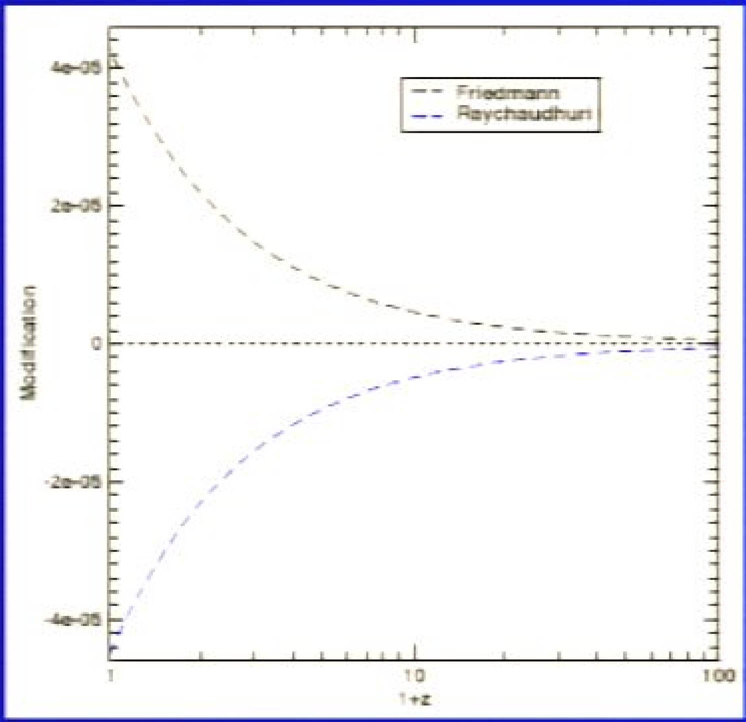




- Modifications at $z = 10, 0$ (left) and integrated (right)
- Exhibit expected behaviour: $\mathcal{R}_\mathcal{D}$ dominates, $\mathcal{P}_\mathcal{D} \approx 2\mathcal{R}_\mathcal{D}/9$, $\mathcal{T}_\mathcal{D} \approx \mathcal{R}_\mathcal{D}/20$, $\mathcal{S}_\mathcal{D} \approx -\mathcal{R}_\mathcal{D}/80$, $\mathcal{Q}_\mathcal{D} \approx 0$

EdS: Low-z

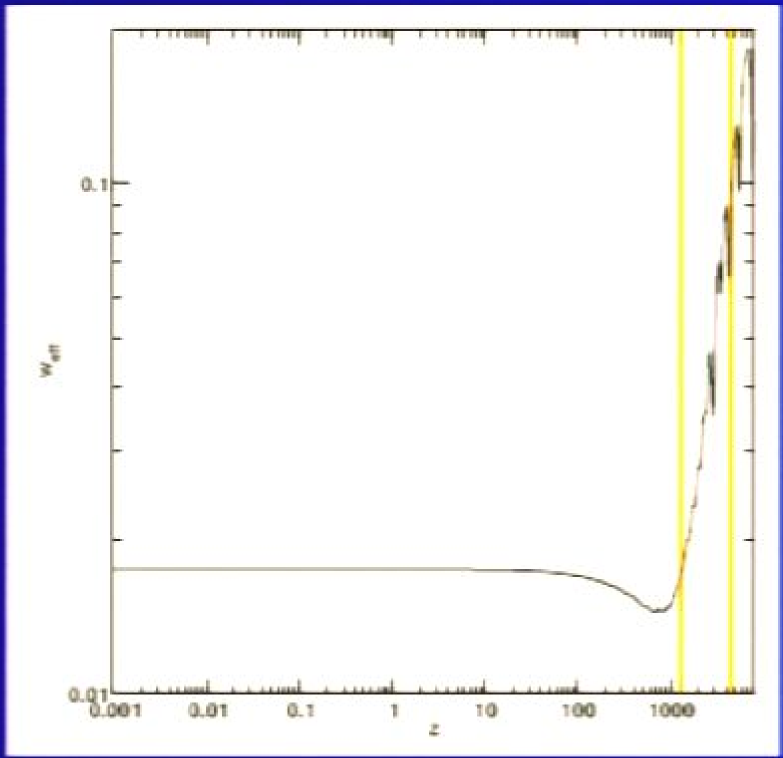


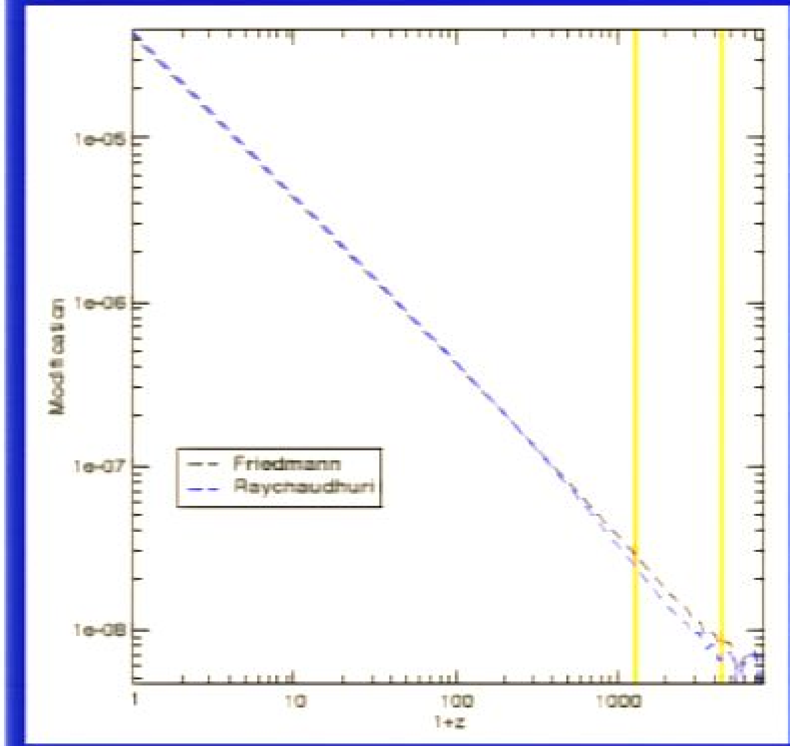


- $w_{\rm eff}$ (left) and $\Delta F/F$, $\Delta R/R$ (right)
- $\sim 10^{-5}$ as predicted
- $w_{\rm eff} > 0$ – acts as dark matter

20:42:09

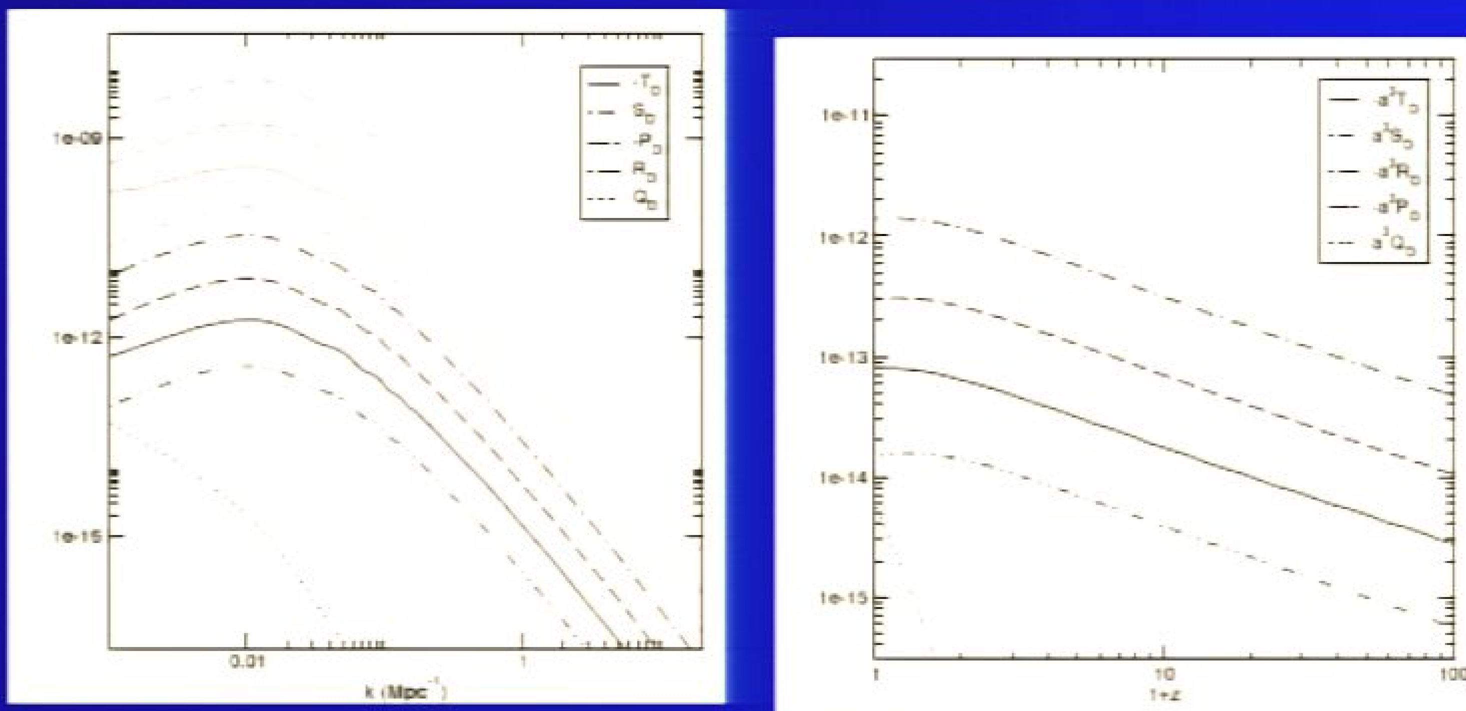
EdS: High-z





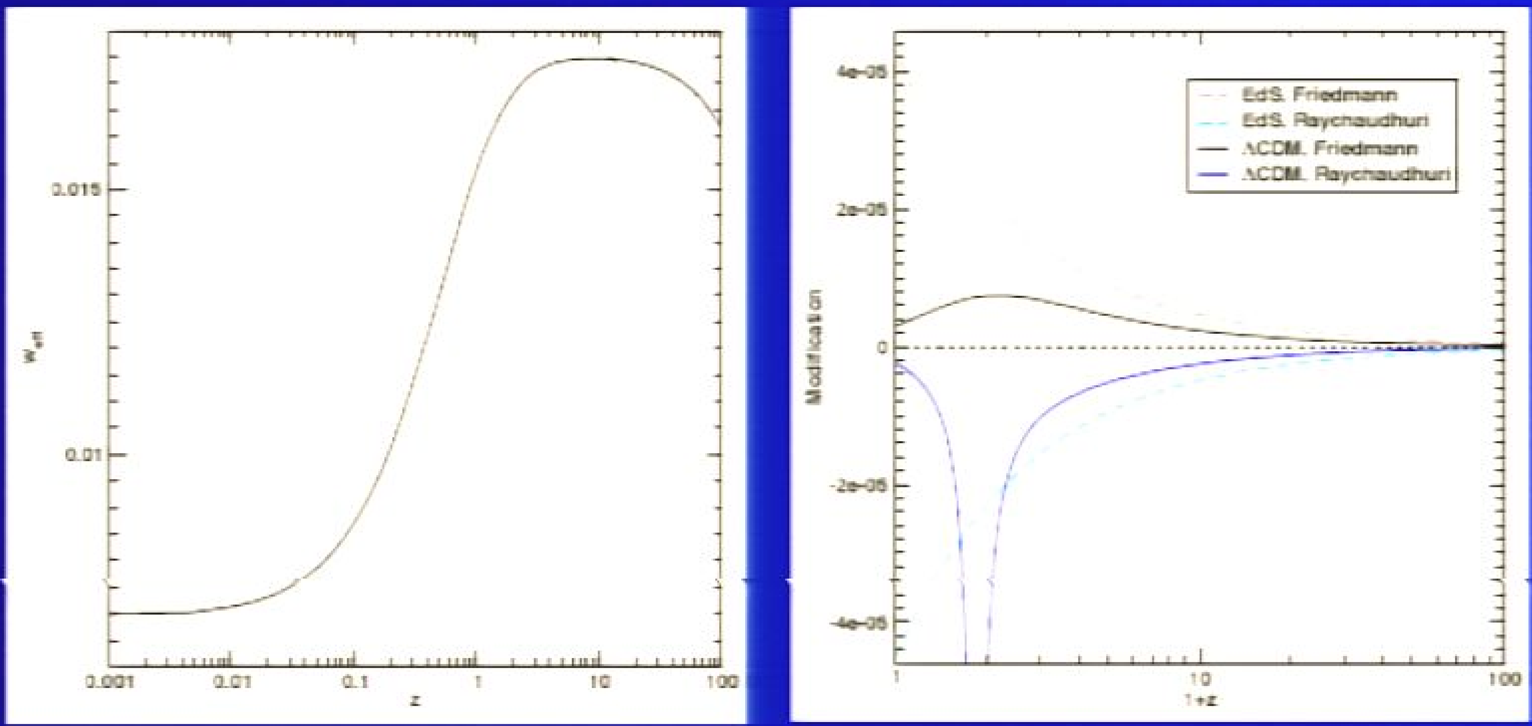
- $w_{\rm eff} \rightarrow 0.2$ for $z \rightarrow 8000$
- Impact at recombination $\sim 10^{-8}$ – potentially observable with Planck?

ΛCDM



- Modifications at $z = 10, 0$ (left) and integrated (right)
- $\mathcal{R}_\mathcal{D}$ dominates, $\mathcal{P}_\mathcal{D} \approx 2\mathcal{R}_\mathcal{D}/9$, $\mathcal{T}_\mathcal{D} \approx \mathcal{R}_\mathcal{D}/20$, $\mathcal{S}_\mathcal{D} \approx -\mathcal{R}_\mathcal{D}/95$, $\mathcal{Q}_\mathcal{D} \approx 0$

ΛCDM: Low-z



- w_{eff} (left) and $\Delta F/F$, $\Delta R/R$ (right)
- $\sim 10^{-5}$ as predicted
- Again $w_{\text{eff}} > 0$

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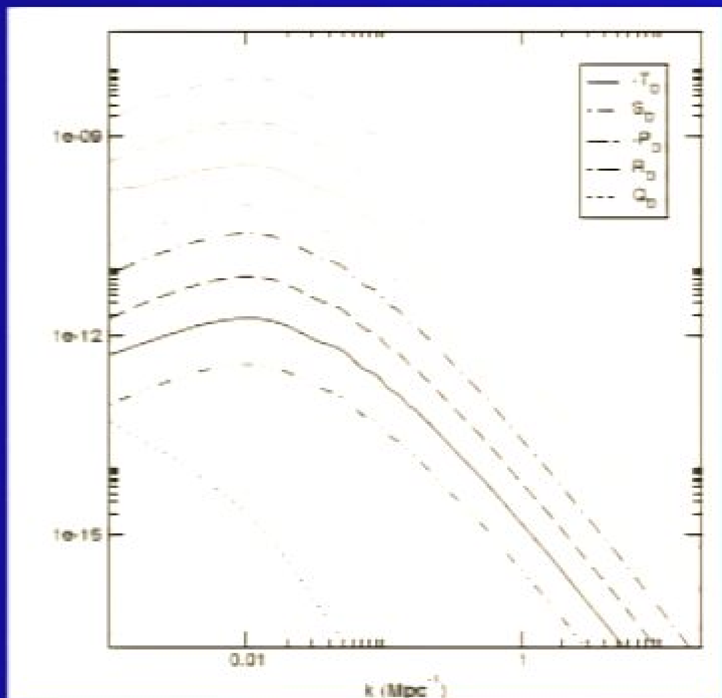
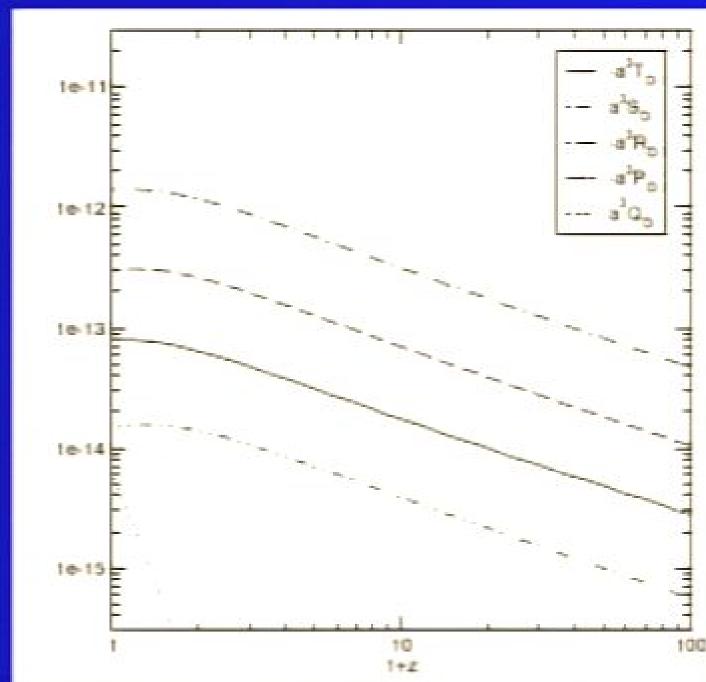
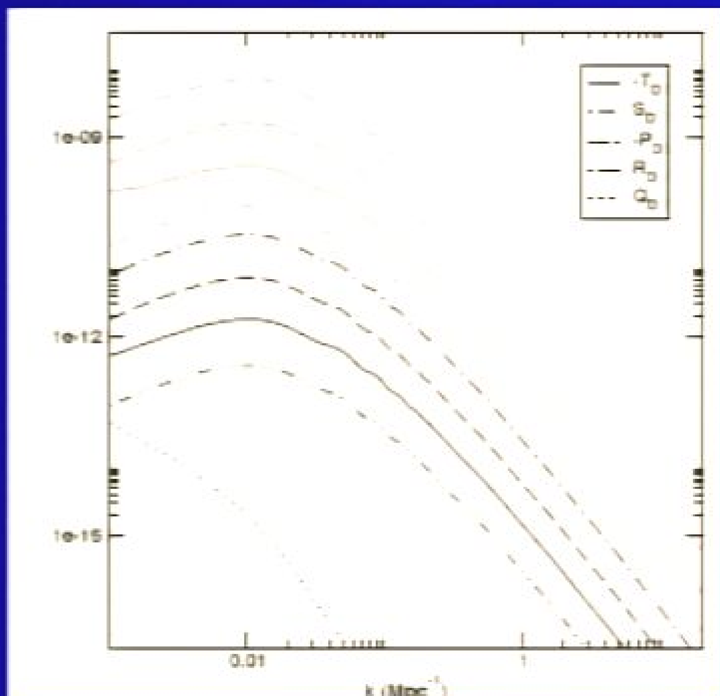
ΛCDM

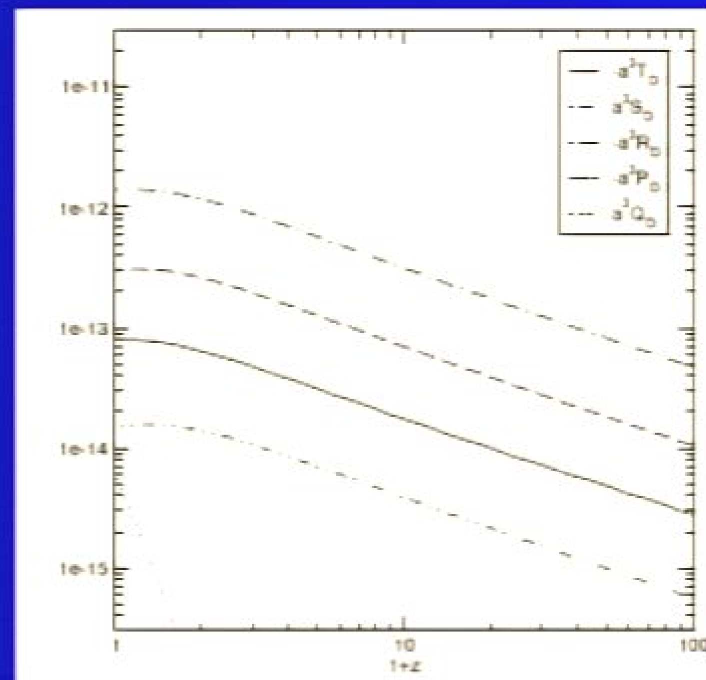




- Modifications at $z = 10, 0$ (left) and integrated (right)
- $\mathcal{R_D}$ dominates, $\mathcal{P_D} \approx 2\mathcal{R_D}/9$, $\mathcal{T_D} \approx \mathcal{R_D}/20$, $\mathcal{S_D} \approx -\mathcal{R_D}/95$, $\mathcal{Q_D} \approx 0$

ΛCDM





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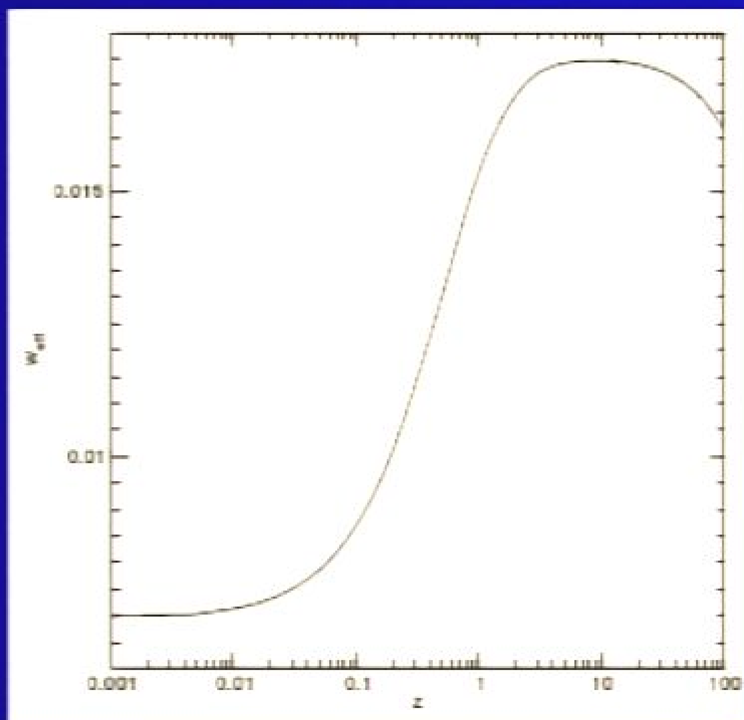
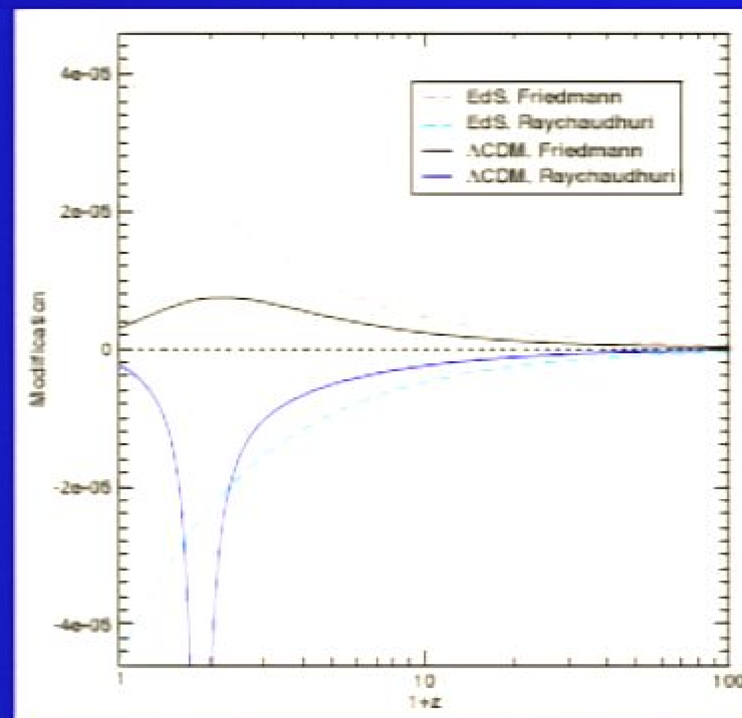
ΛCDM: Low-z





- $w_{\rm eff}$ (left) and $\Delta F/F$, $\Delta R/R$ (right)
- $\sim 10^{-5}$ as predicted
- Again $w_{\rm eff} > 0$

20:47:11

ΛCDM: High-z

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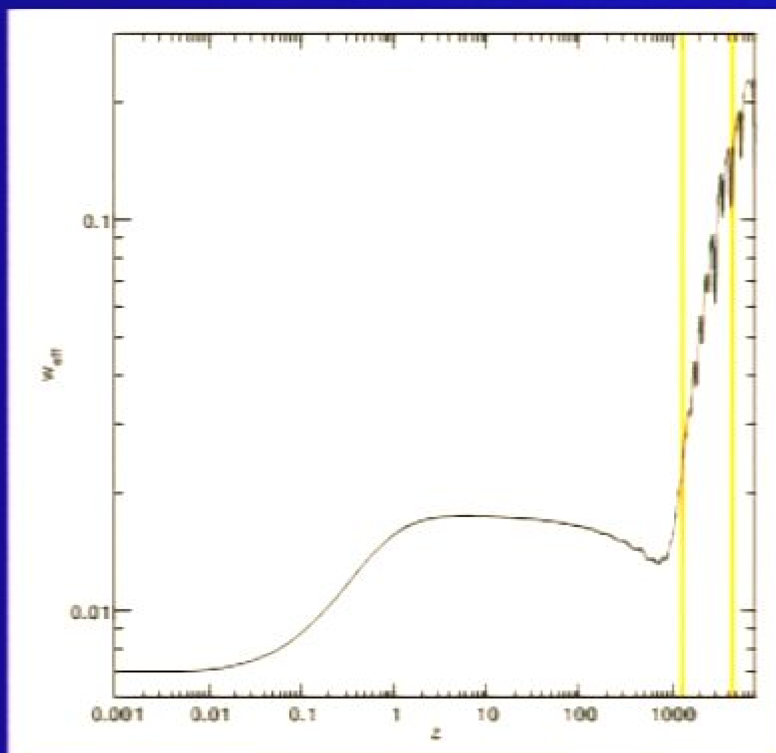
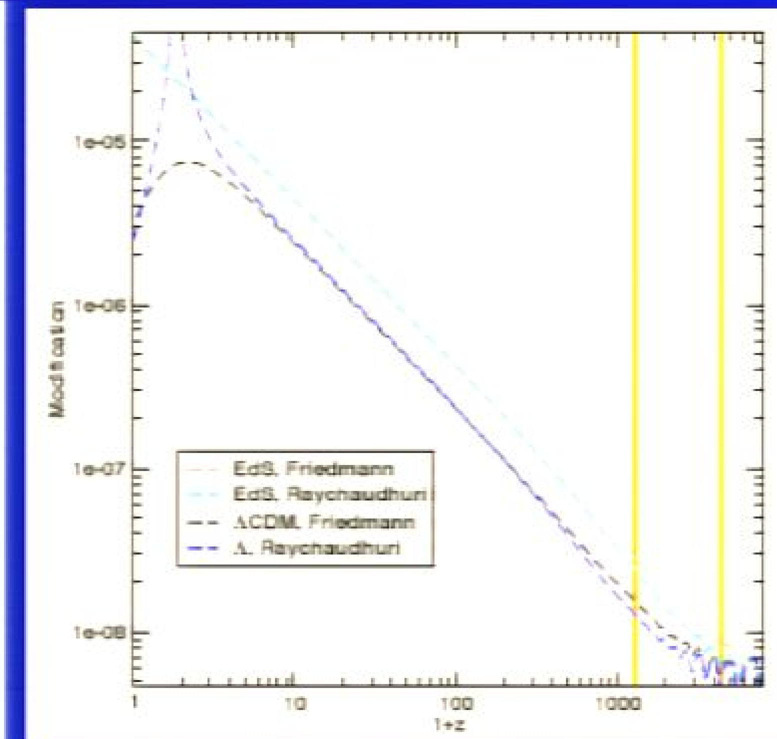
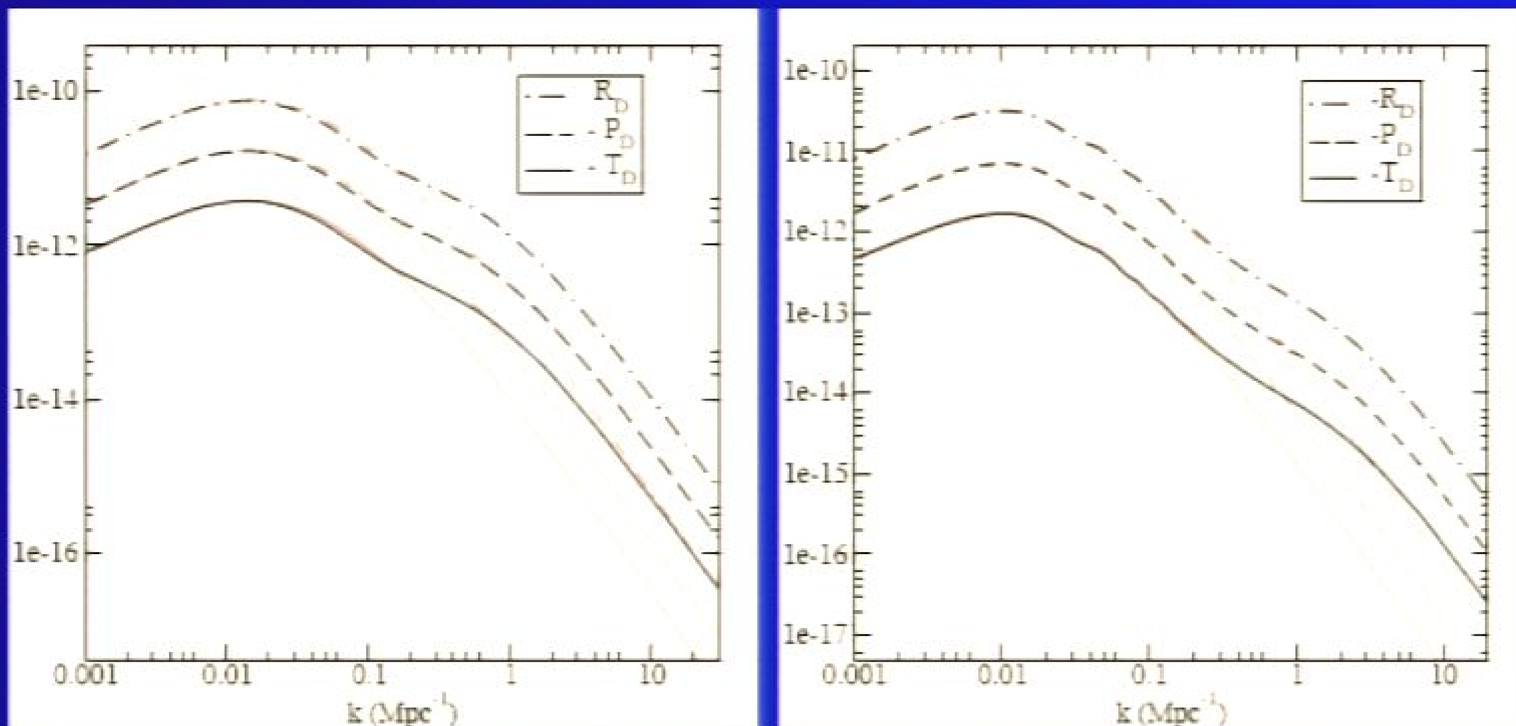




- $w_{\rm eff} \to 0.3$ for $z \to 8000$
- Impact at recombination still $\sim 10^{-8}$ – Planck?

ΛCDM and EdS: Halofit



- Phenomenological nonlinear model of $P(k)$ from galaxy clustering
- EdS (left) and ΛCDM (right)
- Corrections minor, $w_{\rm eff}$ unaltered

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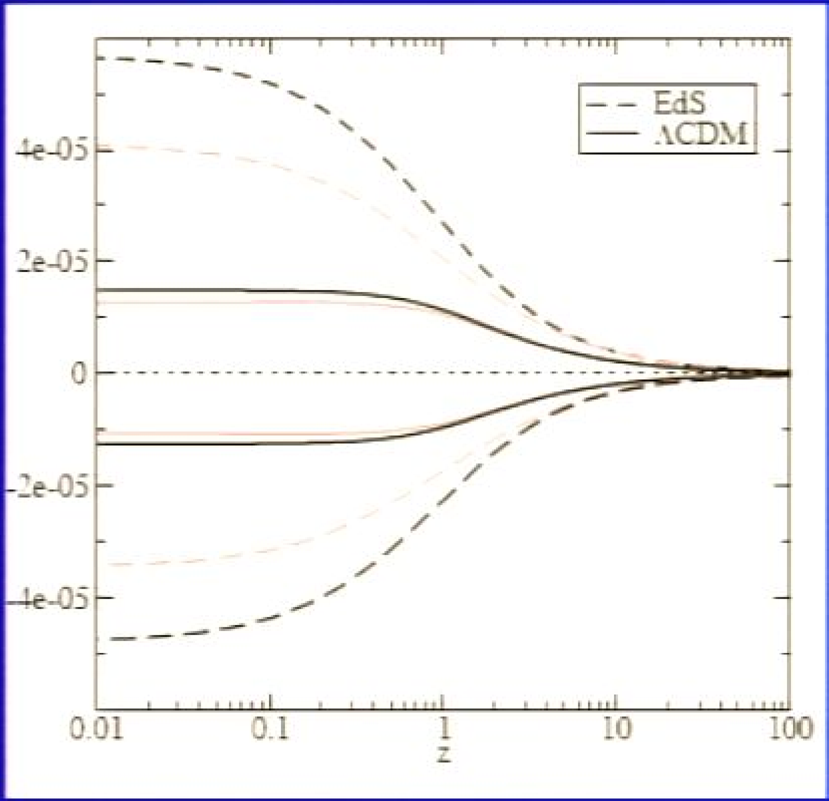
ΛCDM and EdS: Halofit



- Modifications ~ 1.5

Quintessence Cosmology

- Why study them?
- Linear analysis $\Rightarrow$ still small impacts on the observed evolution
- Expect $w_{\rm eff}$ to increase with dark energy perturbations – so $w_{\rm eff}$ clearest discriminant
- $\mathcal{T}_{\mathcal{D}}, \mathcal{S}_{\mathcal{D}}$ include $\langle V(\phi)\rangle - V(\langle\phi\rangle)$ terms – important for exponentials?
- Wetterich '02: $\mathcal{O}(1)$ impact, $w_{\rm eff} \approx -1/15$
- $w_{\rm eff} > -1/3$ at linear and non-linear scales for quintessence $\Rightarrow$ smaller-scale study might be needed…

Numerical Study

Perturbative Models: Large Scales

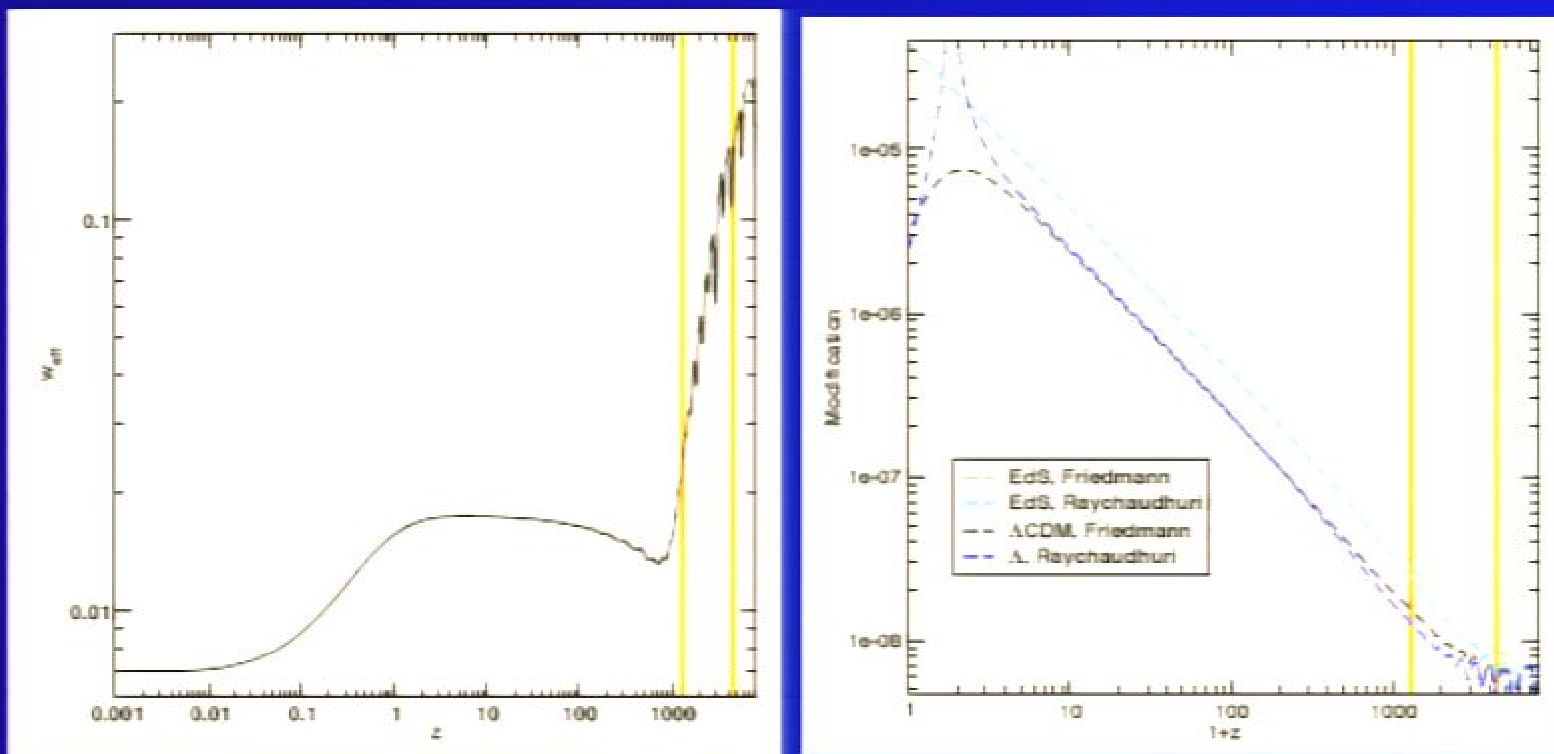
- Take $\mathcal{D}$ large enough to neglect first-order averages:
  - Fluid and curvature modifications:

$$\mathcal{T}_{\mathcal{D}}^{(a)} = \frac{8\pi G\overline{\rho}_{(a)}}{3}\left\langle(1+\overline{w}_{(a)})a^2v_{(a)}^2 + 2\Psi\delta_{(a)}\right\rangle,$$

$$\mathcal{S}_{\mathcal{D}}^{(a)} = \frac{4\pi G\overline{\rho}_{(a)}}{3}\left\langle(1+\overline{w}_{(a)})a^2v_{(a)}^2 + 6c_{s(a)}^2\Psi\delta_{(a)}\right\rangle,$$

$$\mathcal{R}_{\mathcal{D}} = \frac{2}{a^2}\langle 3(\nabla\Phi)^2 + 4(2\Phi+\Psi)\nabla^2\Phi\rangle$$

ΛCDM: High-z



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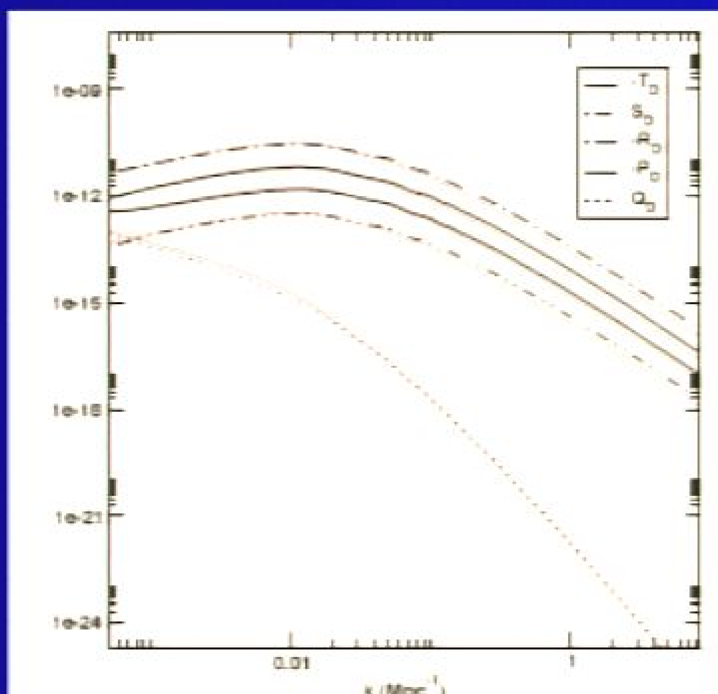
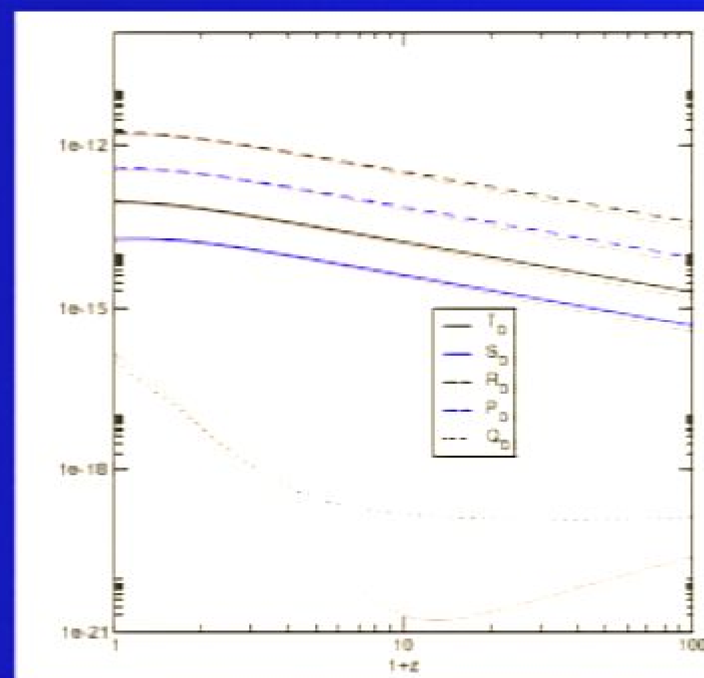
- $w_{\mathrm{eff}} > -1/3$ at linear and non-linear scales for quintessence $\Rightarrow$ smaller-scale study might be needed. . .

Early Dark Energy

- Rough model of early dark energy: $\Omega_\phi^{z=0} = 0.7$, $\Omega_\phi^{z=\infty} = 0.05$, $w_0 = -0.95$
- Very similar to ΛCDM; larger at present day, smaller at peak





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Exponential Potential

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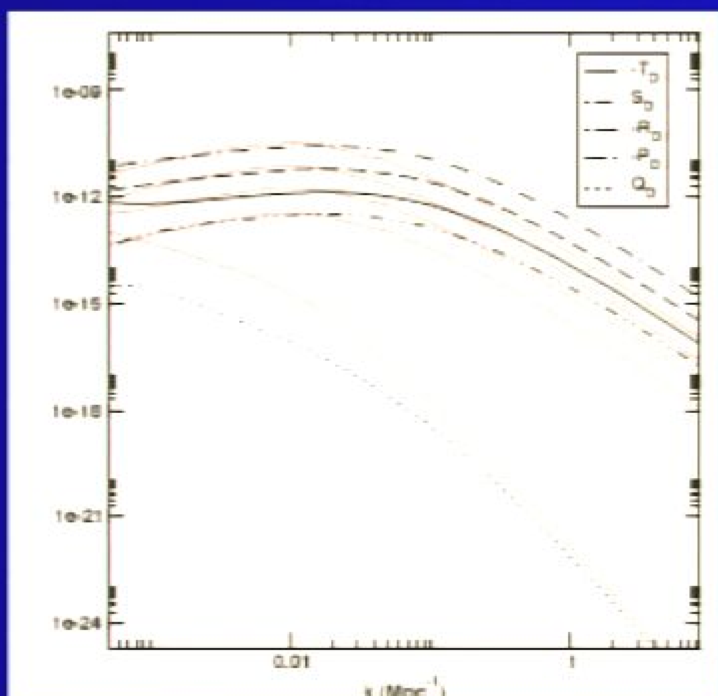
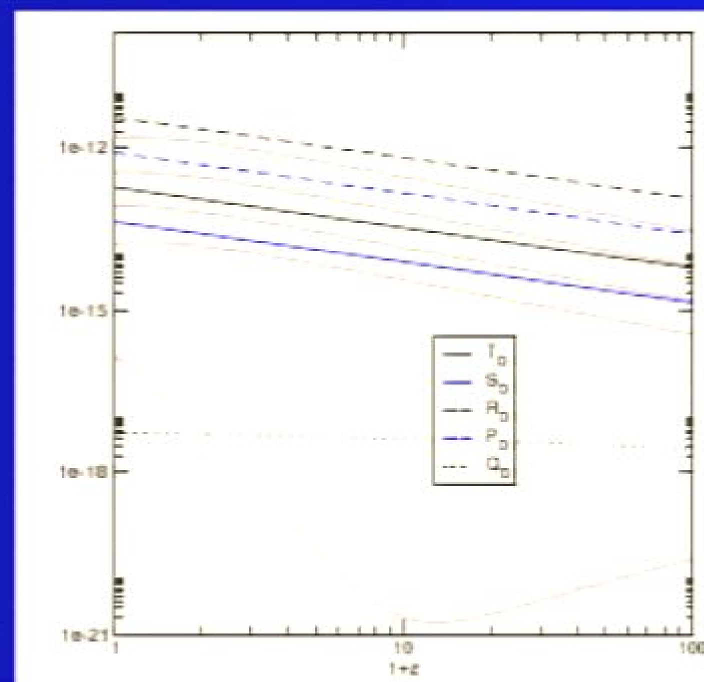
- Exponential Potential: $\Omega_\phi = 0.2, \Omega_b = 0.04, \Omega_m = 0.76$
- Similar to, but much smaller than, EdS





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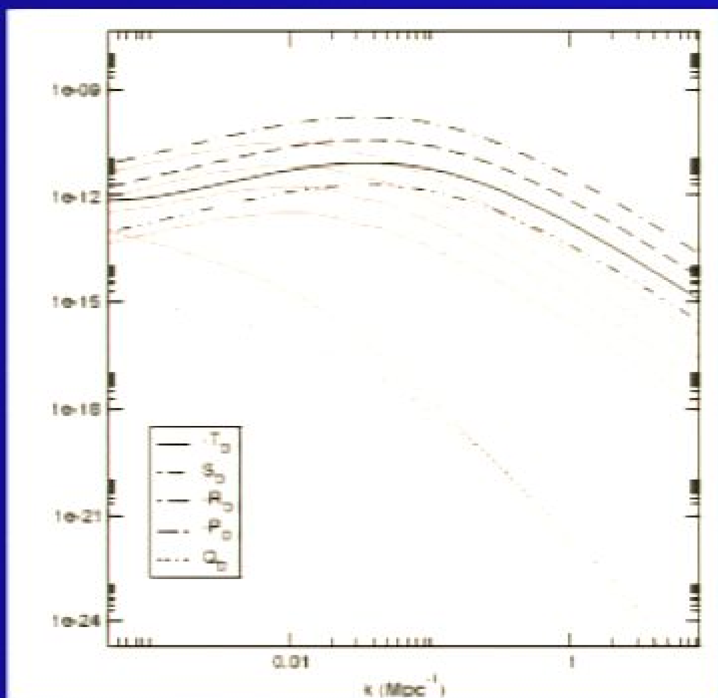
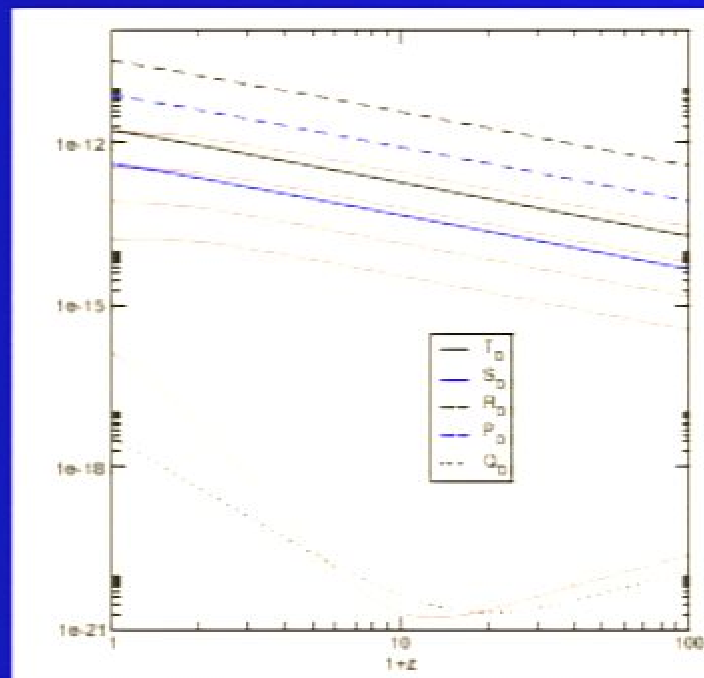
Inverse Power Law Potential

- Inverse-Power Law Potential: $\Omega_\phi = 0.12$, $\Omega_b = 0.05$, $\Omega_c = 0.84$
- Similar to exponential, but larger





Exponential Potential

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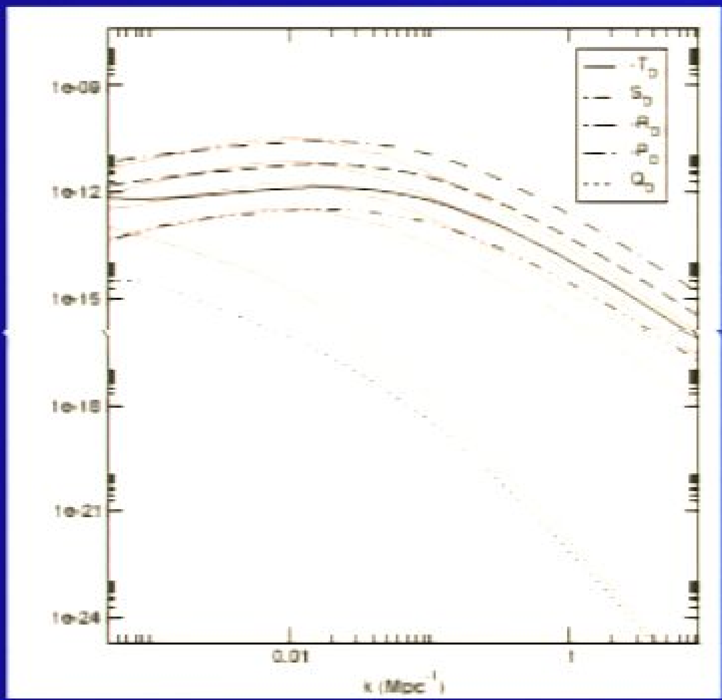
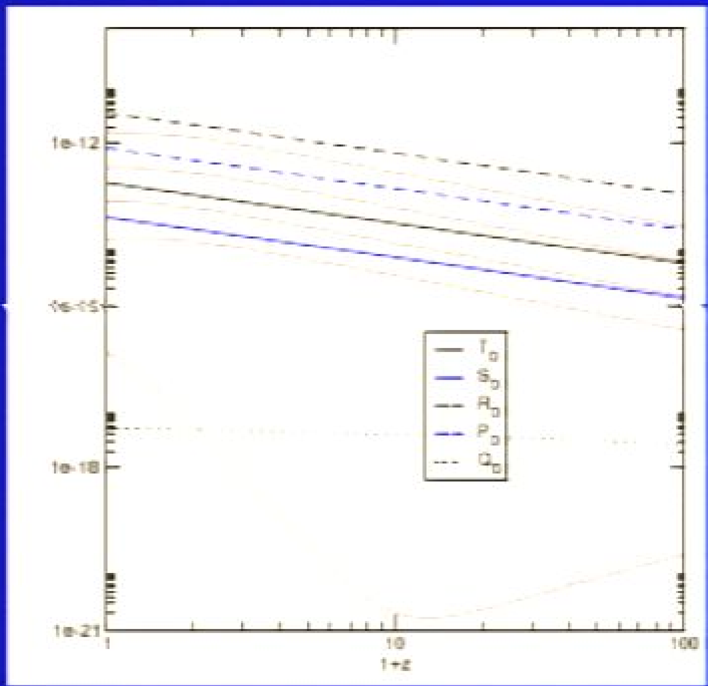
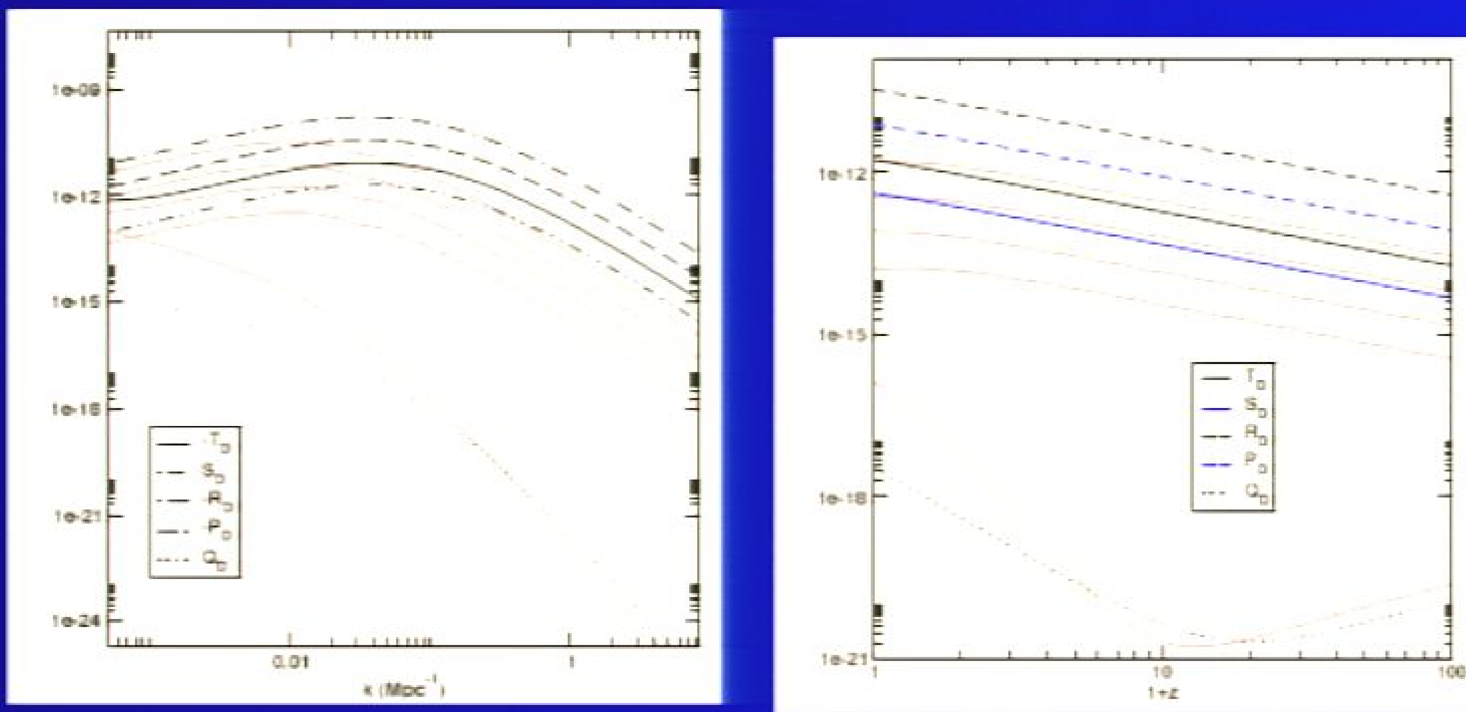
- Similar to, but much smaller than, EdS





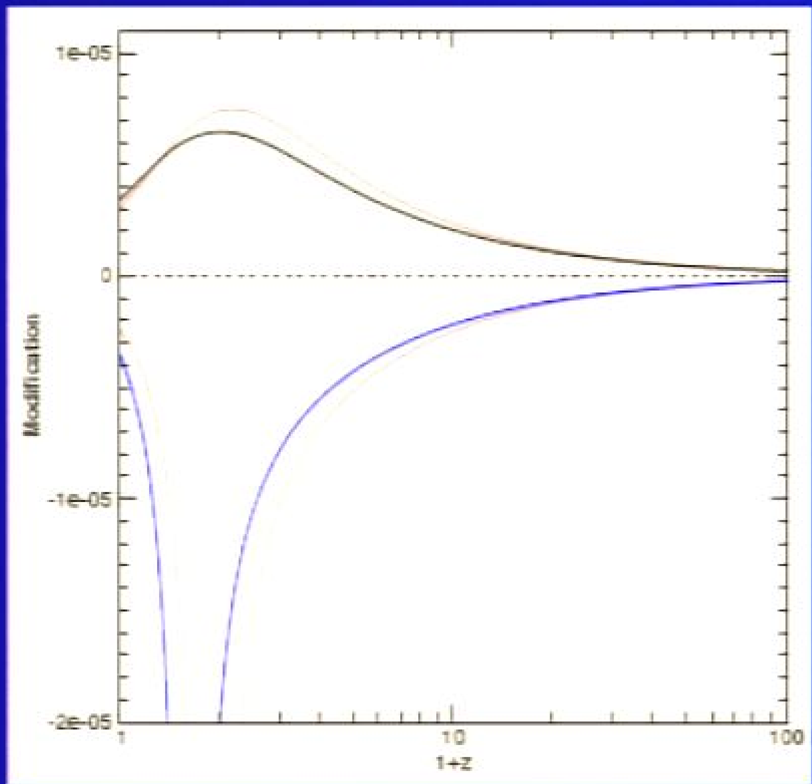
Inverse Power Law Potential

- Inverse-Power Law Potential: $\Omega_\phi = 0.12$, $\Omega_b = 0.05$, $\Omega_c = 0.84$
- Similar to exponential, but larger



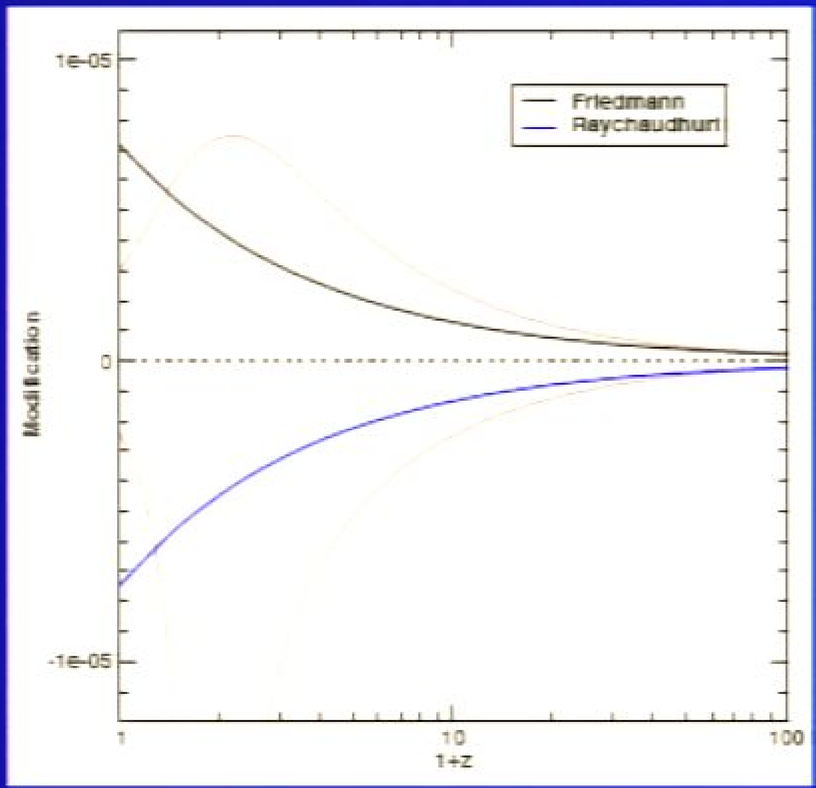
Quintessence: Friedmann and Raychaudhuri Equations

- Impact small as expected
- Early dark energy:



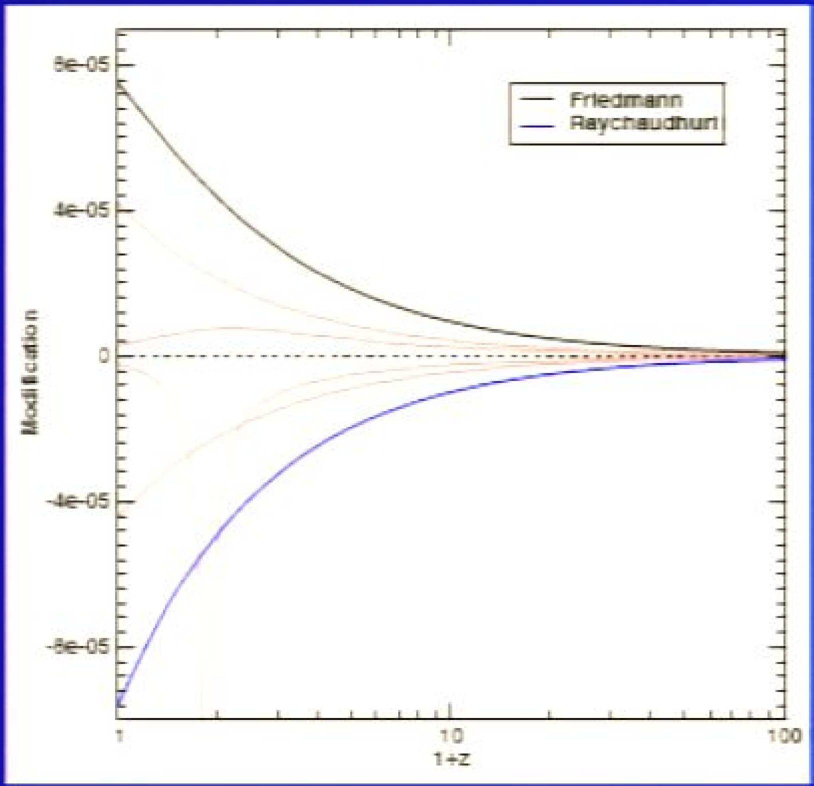
Quintessence: Friedmann and Raychaudhuri Equations

- Impact small as expected
- Exponential – similar to EdS but much smaller



Quintessence: Friedmann and Raychaudhuri Equations

- Impact small as expected
- Inverse-power law – $\sim 2\times$EdS!



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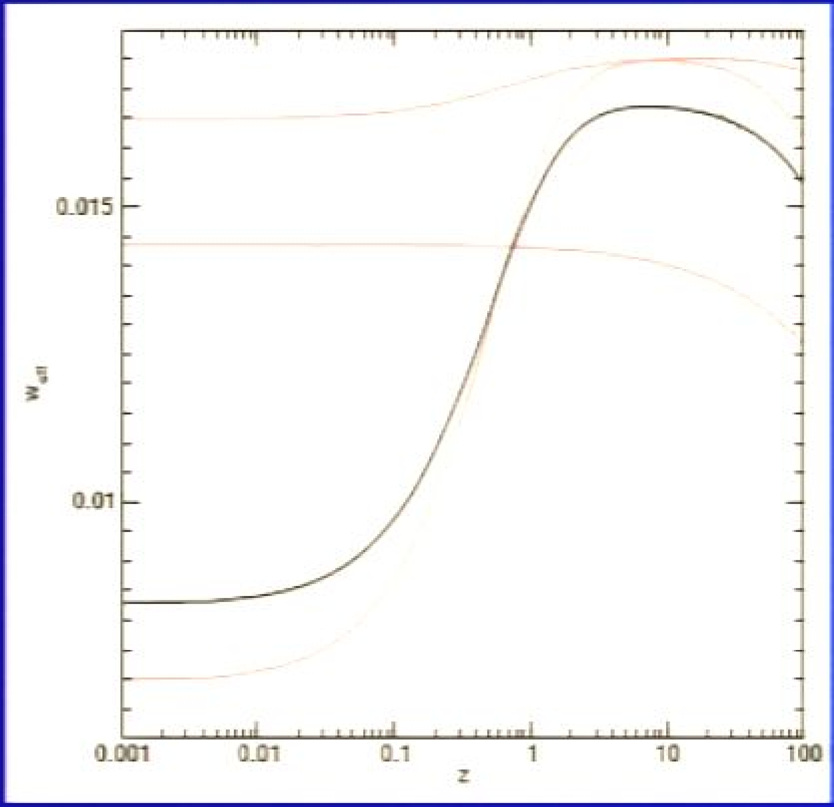
Quintessence: Equations of State



- Early dark energy: broadly similar to ΛCDM

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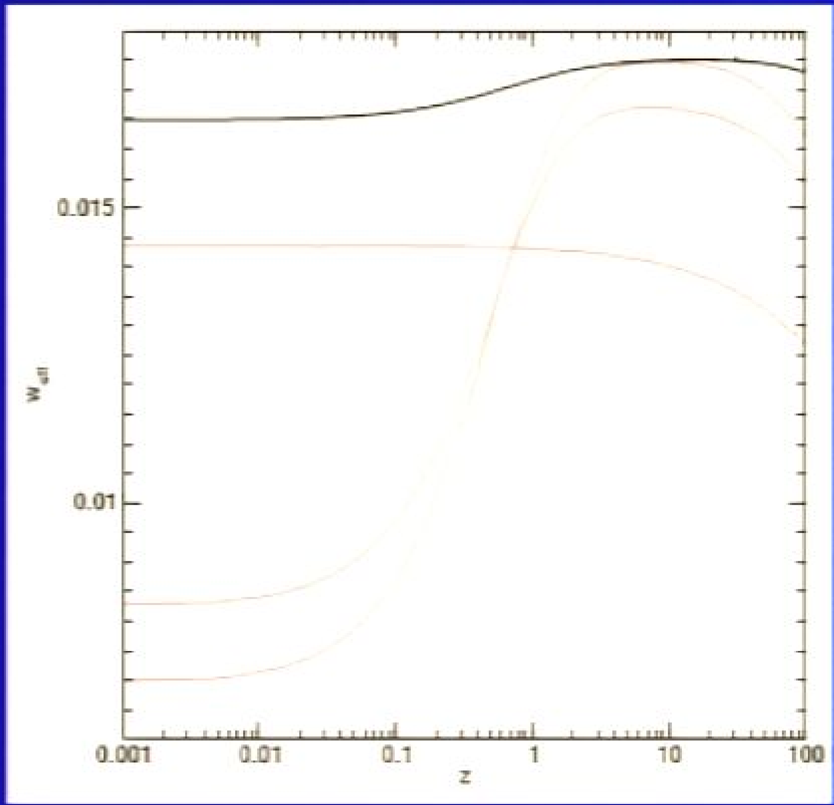
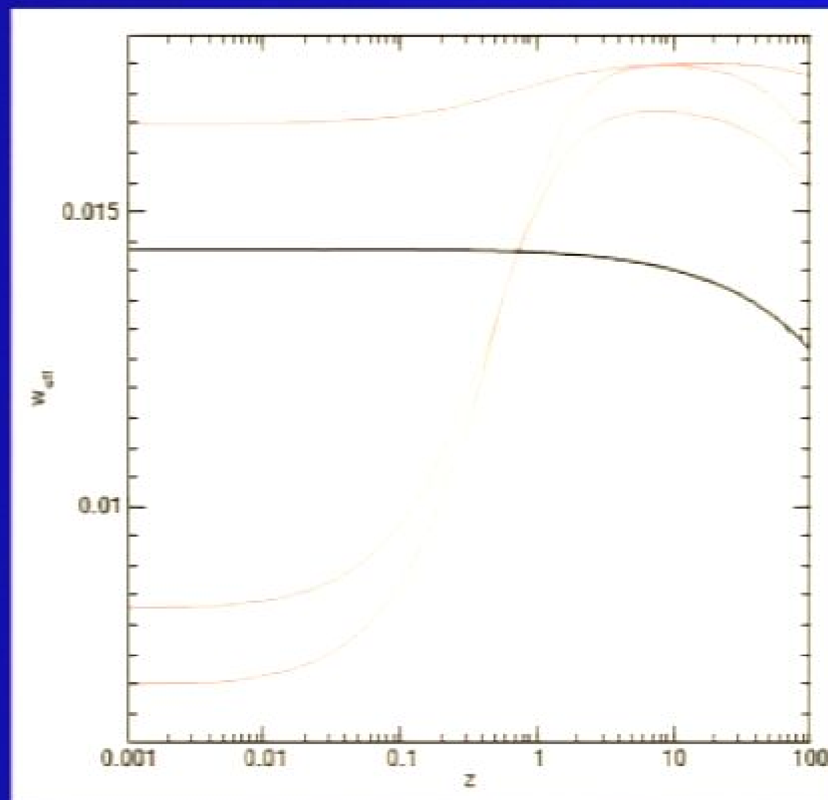
Equations of State



- Inverse power law: similar to EdS, different evolution

Equations of State



- Exponential: simlar to EdS but suppressed

Equations of State

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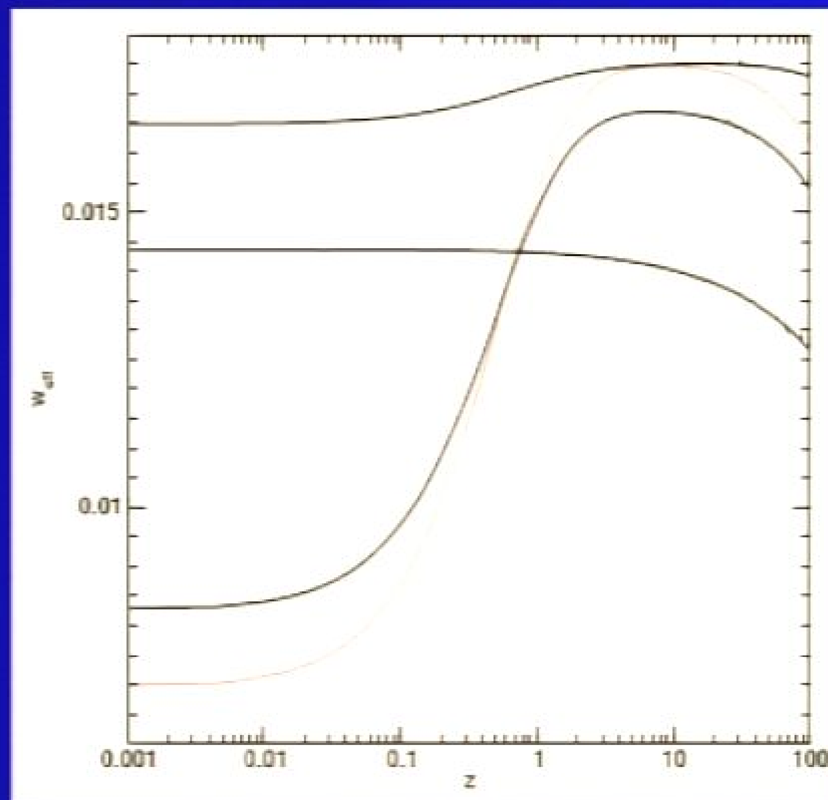

- $w_{\rm eff} > 0$ – as before acts against acceleration
- But: this includes quintessence perturbations!
- These differences far too small to observe, but smaller-scale study looks vital

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Summary

Summary

- Have expressed Buchert equations in form easily incorporated into general Boltzmann codes for wide variety of models
- Backreaction is a real physical effect with maximum contribution from linear modes at $z \approx 1.4$

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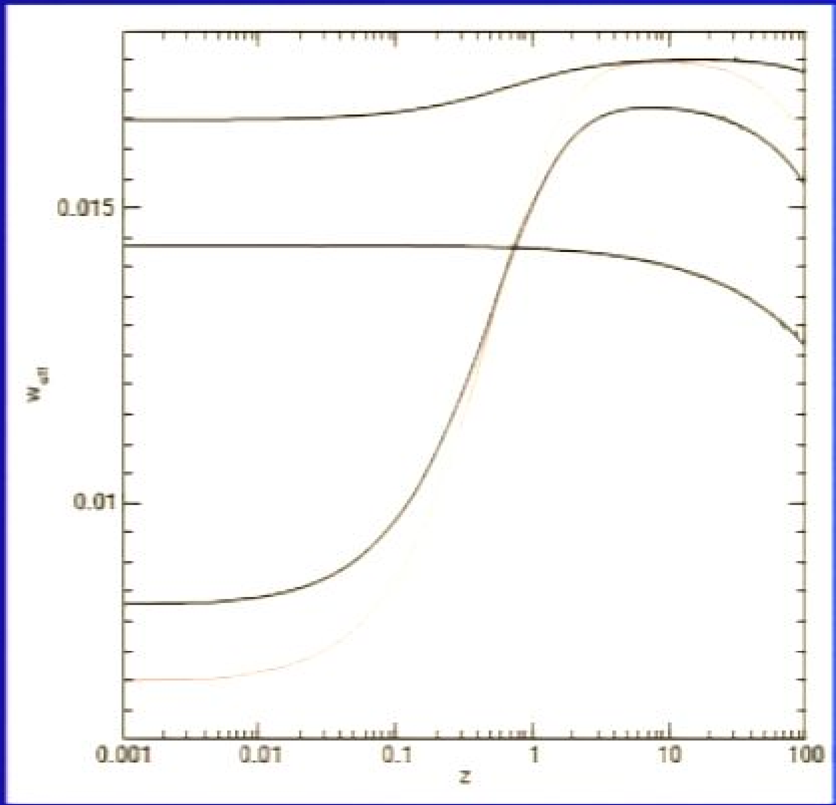
Equations of State



- $w_{\rm eff} > 0$ – as before acts against acceleration
- But: this includes quintessence perturbations!
- These differences far too small to observe, but smaller-scale study looks vital

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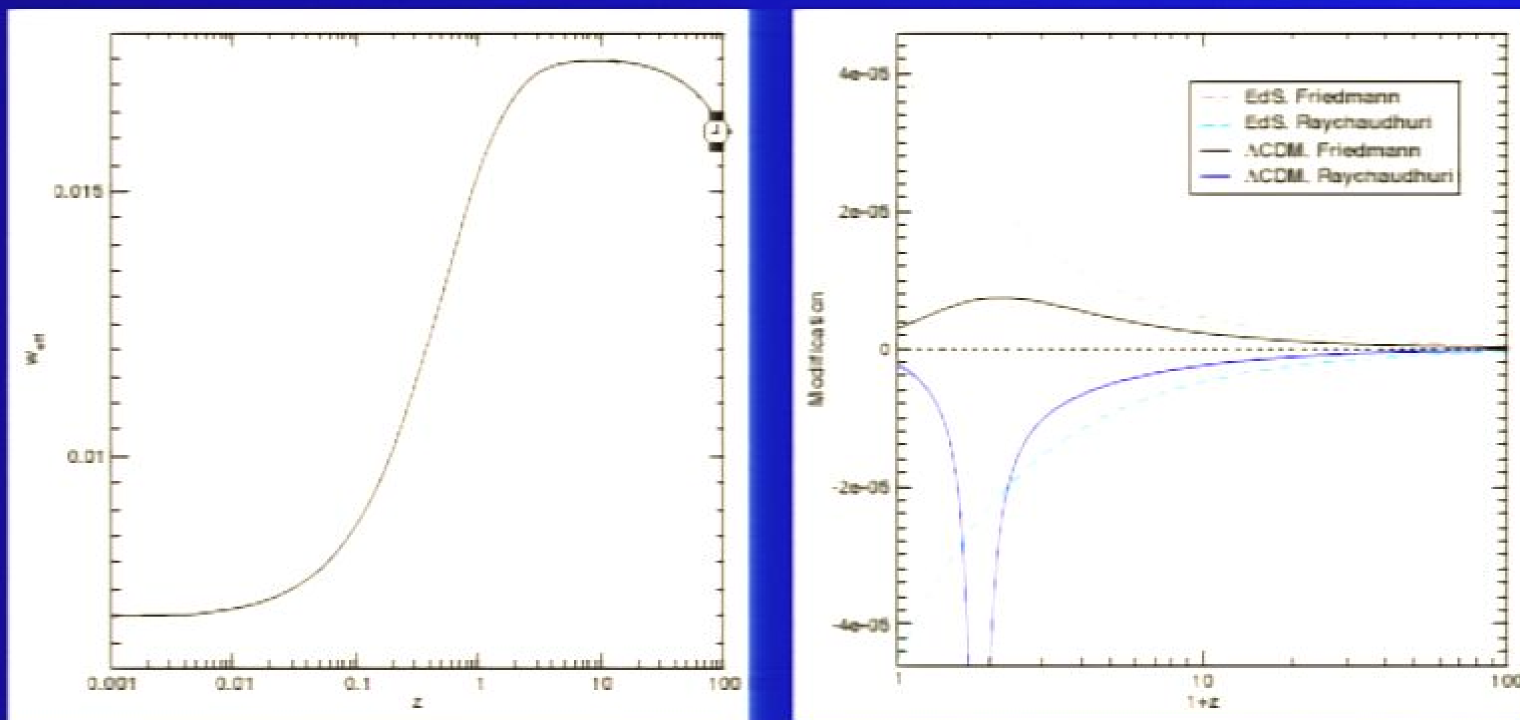
Summary

ΛCDM: Low-z



- $w_{\rm eff}$ (left) and $\Delta F/F$, $\Delta R/R$ (right)
- $\sim 10^{-5}$ as predicted
- Again $w_{\rm eff} > 0$

20:57:07

Numerical Study

Perturbative Models

- Background: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
- Work in Newtonian gauge with Λ, b, c, γ, massless ν
- Newtonian gauge well-controlled on sub-horizon scales $\Rightarrow$ no gauge worries, $\phi \ll 1$ across scales considered
- $a_{\mathcal{D}}(t)$ is "observational", $a(t)$ is "physical" – drawback of re-averaging assumed average

Formalism: Modifications to Standard Cosmology

- Effective correction fluid given by

$$\frac{8\pi G}{3}\overline{\rho}_{\rm eff} = \sum_a \mathcal{T}_\mathcal{D}^{(a)} + \langle\alpha^2 - 1\rangle\frac{\Lambda}{3} - \frac{1}{6}\left(\mathcal{Q}_\mathcal{D} + \mathcal{R}_\mathcal{D}\right),$$

$$16\pi G\overline{p}_{\rm eff} = 4\sum_a \mathcal{S}_\mathcal{D}^{(a)} - 2\langle\alpha^2 - 1\rangle\Lambda + \frac{1}{3}\left(\mathcal{R}_\mathcal{D} - 3\mathcal{Q}_\mathcal{D} - 4\mathcal{P}_\mathcal{D}\right),$$

$$w_{\rm eff} = -\frac{1}{3}\frac{\mathcal{R}_\mathcal{D} - 3\mathcal{Q}_\mathcal{D} - 4\mathcal{P}_\mathcal{D} + 12\sum_a \mathcal{S}_\mathcal{D}^{(a)} - 6\Lambda\langle\alpha^2 - 1\rangle}{\mathcal{R}_\mathcal{D} + \mathcal{Q}_\mathcal{D} - 6\sum_a \mathcal{T}_\mathcal{D}^{(a)} - 2\Lambda\langle\alpha^2 - 1\rangle}.$$

- For carefully chosen model, resembles effective dark energy
- Buchert equations fully general for any irrotational system – but:
  - Forced to average only scalar quantities
  - Useless without a concrete model

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Summary

- Have expressed Buchert equations in form easily incorporated into general Boltzmann codes for wide variety of models
- Backreaction is a real physical effect with maximum contribution from linear modes at $z \approx 1.4$
- Magnitude of effect is small on such large scales, $\rho_{\rm eff} \lesssim 5 \times 10^{-5} \rho_m$
- $w_{\rm eff} \approx 1/100$, so acts as dark matter not dark energy
- c.f. Wetterich 02, $w_{\rm eff} \approx -1/27$
- c.f. Khosravi *et. al.* 07, Vanderveld *et. al.* 07, $\rho_{\rm eff} \approx 10^{-5} \rho_m$; Räsänen 08, $\rho_{\rm eff} \approx \times 10^{-3} \rho_m$; Li and Schwarz 07, $\rho_{\rm eff} \approx \rho_m/10$

Summary

- Magnitude of effects is unsurprisingly small
- Impact at recombination is close to observable anisotropies $\Rightarrow$ possible chance of detection?
- Inverse-power law has largest linear impact yet, ~ 1.5EdS+halofit
- w_{eff} is the clearest discriminant, both form and magnitude:
  - ΛCDM: $w_{\text{eff}} \approx 0.007$
  - Early dark energy: $w_{\text{eff}} \approx 0.009$
  - Exponential: $w_{\text{eff}} \approx 0.014$
  - Inverse power law: $w_{\text{eff}} \approx 0.016$
- Equation of state from quintessence perturbations $>> -1$: does the average of a clumped cosmon act like a homogeneous cosmon?

Future Directions

- Open universes, ISW, CMB observables, breakdown of perturbation theory
- Adoption to other models: $f(R)$, scalar/tensor theories, coupled dark energy models etc.
- Non-Linear models:
  - Second-order perturbations
  - Fully inhomogeneous models, analytical (e.g. LTB, currently well-studied), numerical (simulated clusters) and observational (SDSS)
  - Detailed models of networks of cosmon clumps (analytical and/or numerical)
- Modified averaging procedures (e.g. Behrend/Nachtmann):
  - Metric recovery from perturbed 3-plane, simulated data, SDSS data
  - Application to general tensors: alternative to Zalaletdinov's MG