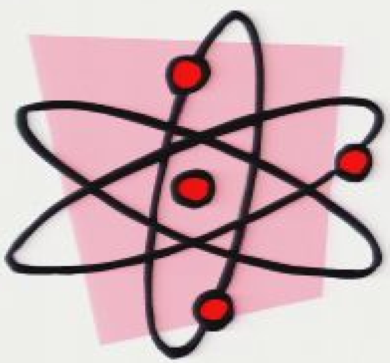
Title: Quantum Information & Entanglement

Date: Aug 18, 2008 09:00 AM

URL: http://pirsa.org/08080098

Abstract: One simple way to think about physics is in terms of information. We gain information about physical systems by observing them, and with luck this data allows us to predict what they will do next. Quantum mechanics doesn't just change the rules about how physical objects behave - it changes the rules about how information behaves. In this talk we explore what quantum information is, and how strangely it differs from our intuitions. In particular we see how information about quantum particles can become entangled, leading to seemingly impossibly coordinated behaviour for separate objects, and to phenomena such as quantum teleportation.

Measurement, Information and Teleportation



PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

Physical Information

Quantum theory is *not* a fully formed model of the physical world.

It is a framework – a mathematical structure we can colour in with whatever physics we need at for any situation.

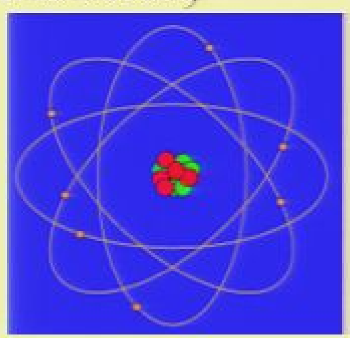
It tells us something remarkable:

Every physical system follows identical rules of behaviour.

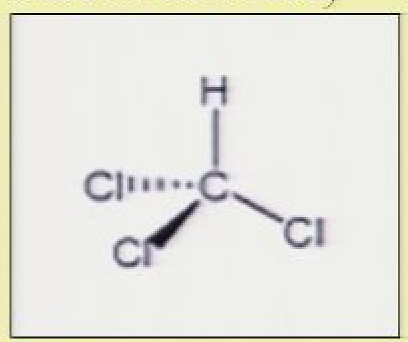
Photons,

Electrons,

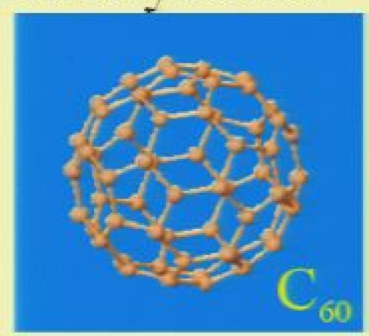
Atoms,



Chloroform,



Buckyballs.



Physical Information

Studying and measuring physical systems is attempting to get information about them. We interact with them to see what they'll do.

Every physical system follows the same, counterintuitive rules for how we can learn about to them.

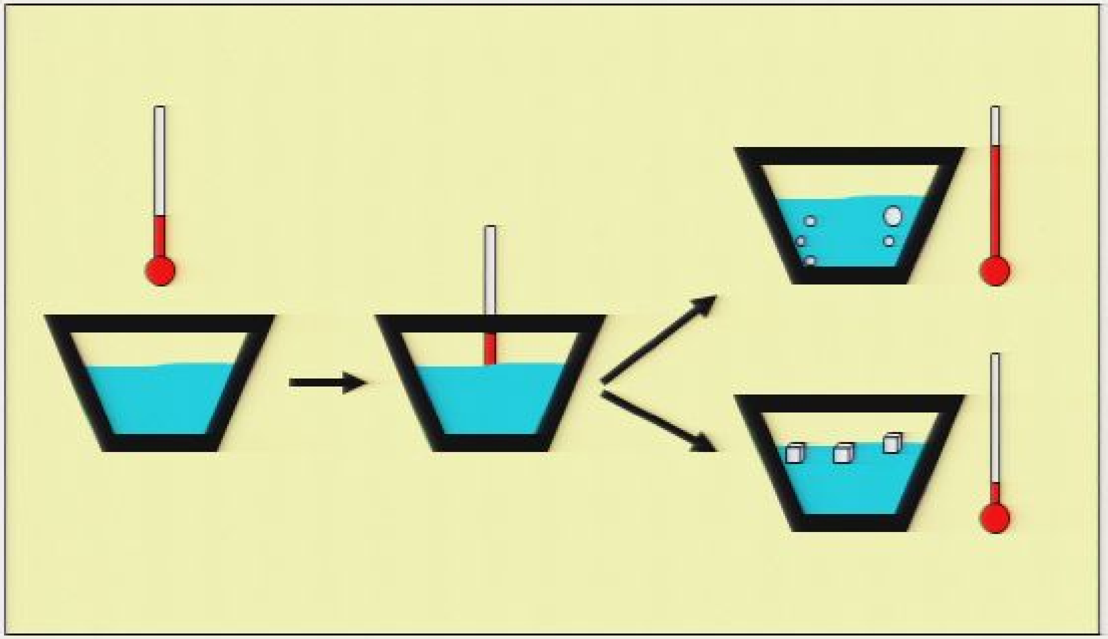
Quantum information theory is the study of these rules.

What do these rules let us do?

- Quantum Cryptography
- Quantum Computing
- Teleportation

What do these systems know that we don't?

Empirical science = measurements



Let S = unknown bath.

Let T = thermometer.

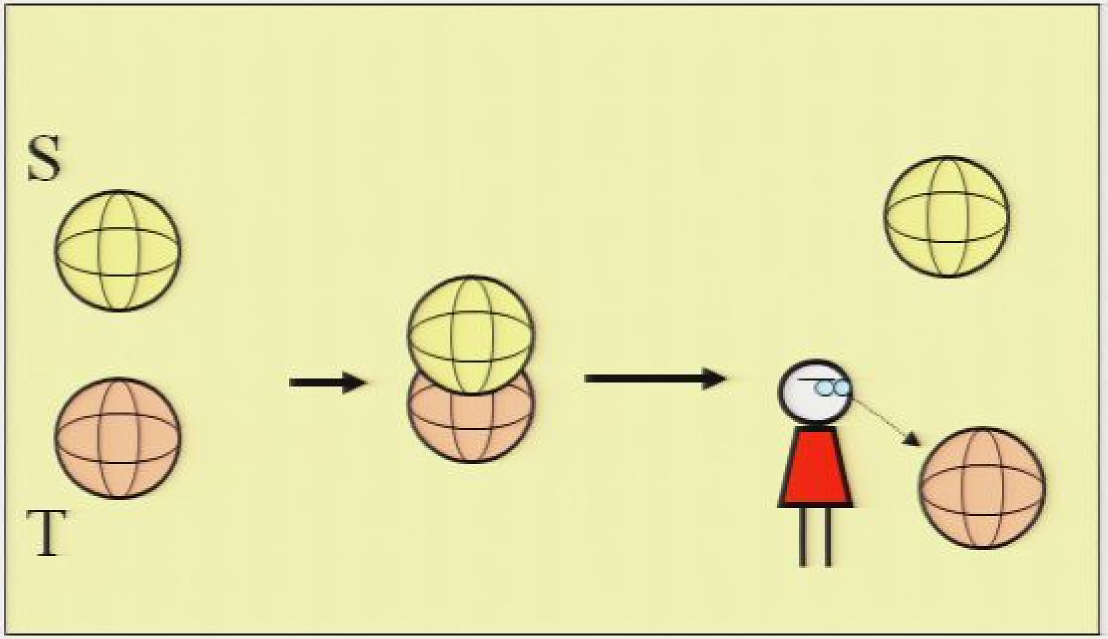
We know how our thermometer behaves.

We can describe the possible future states of the thermometer - e.g. by the integers 1 to 99 (C).

To perform a temperature measurement on system S:

- Prepare system T in a known state (e.g. 20C, not cracked).
- Allow system T to interact with system S for a fixed time.
- Observe the actual new state of system T.

Non-demolition measurements



Quantum measurements work just the same way.

Let S = measured system.

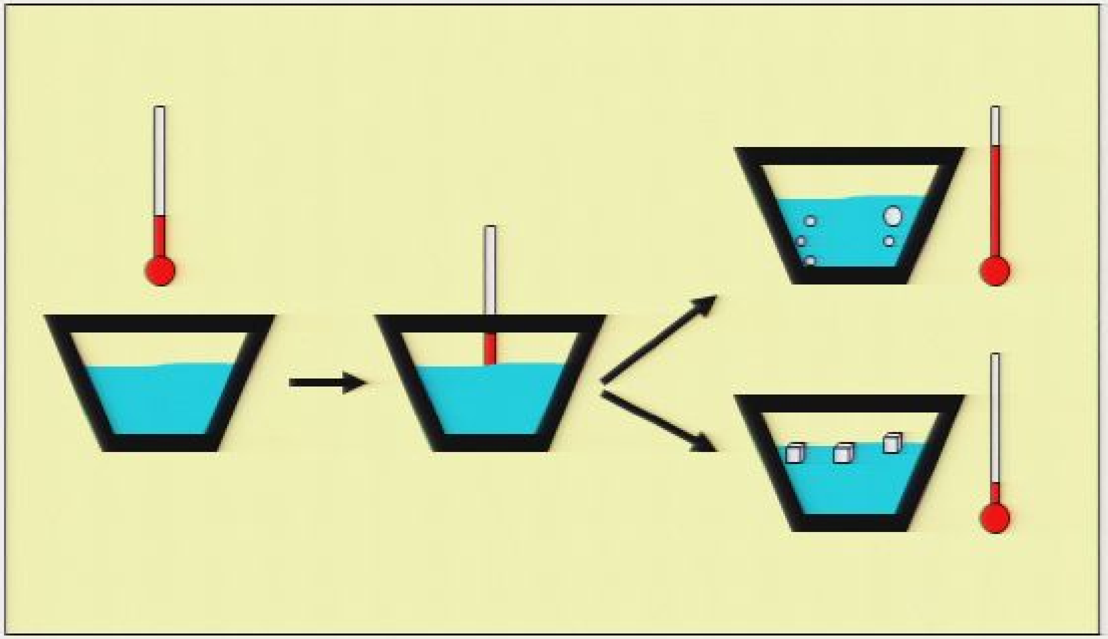
Let T = known system.

The interaction of S and T is known, as are the possible future states of T.

To measure S:

- Prepare system T in a known state.
- Allow system T to interact with system S for a fixed time.
- Observe the actual new state of system T (this observation can be destructive).
- Because we know how T interacted with S, extrapolate to the state of S.

Empirical science = measurements



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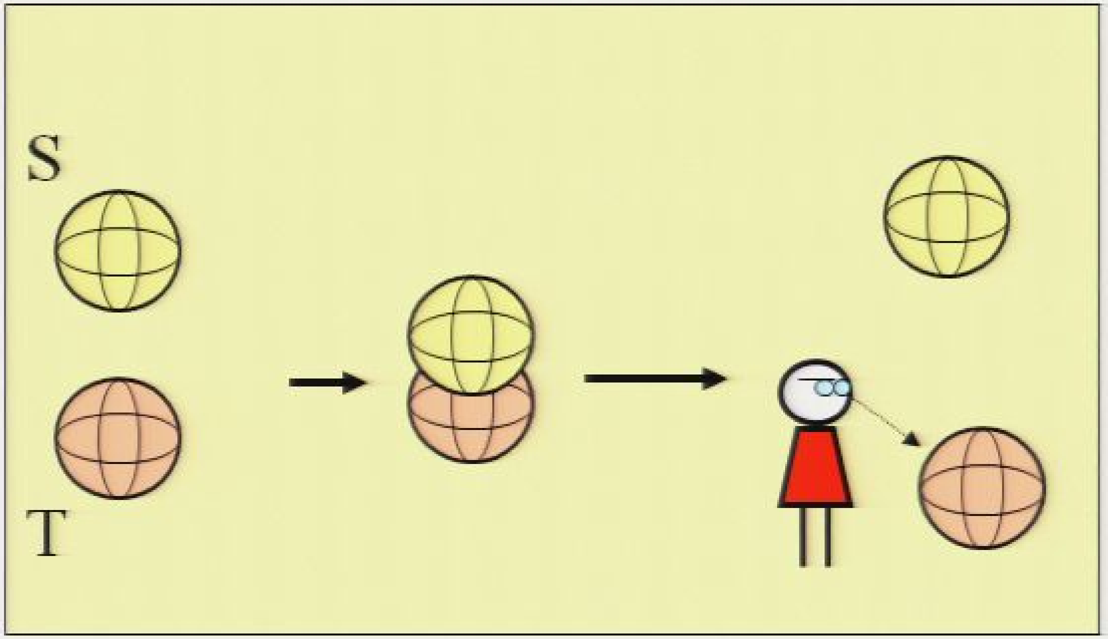

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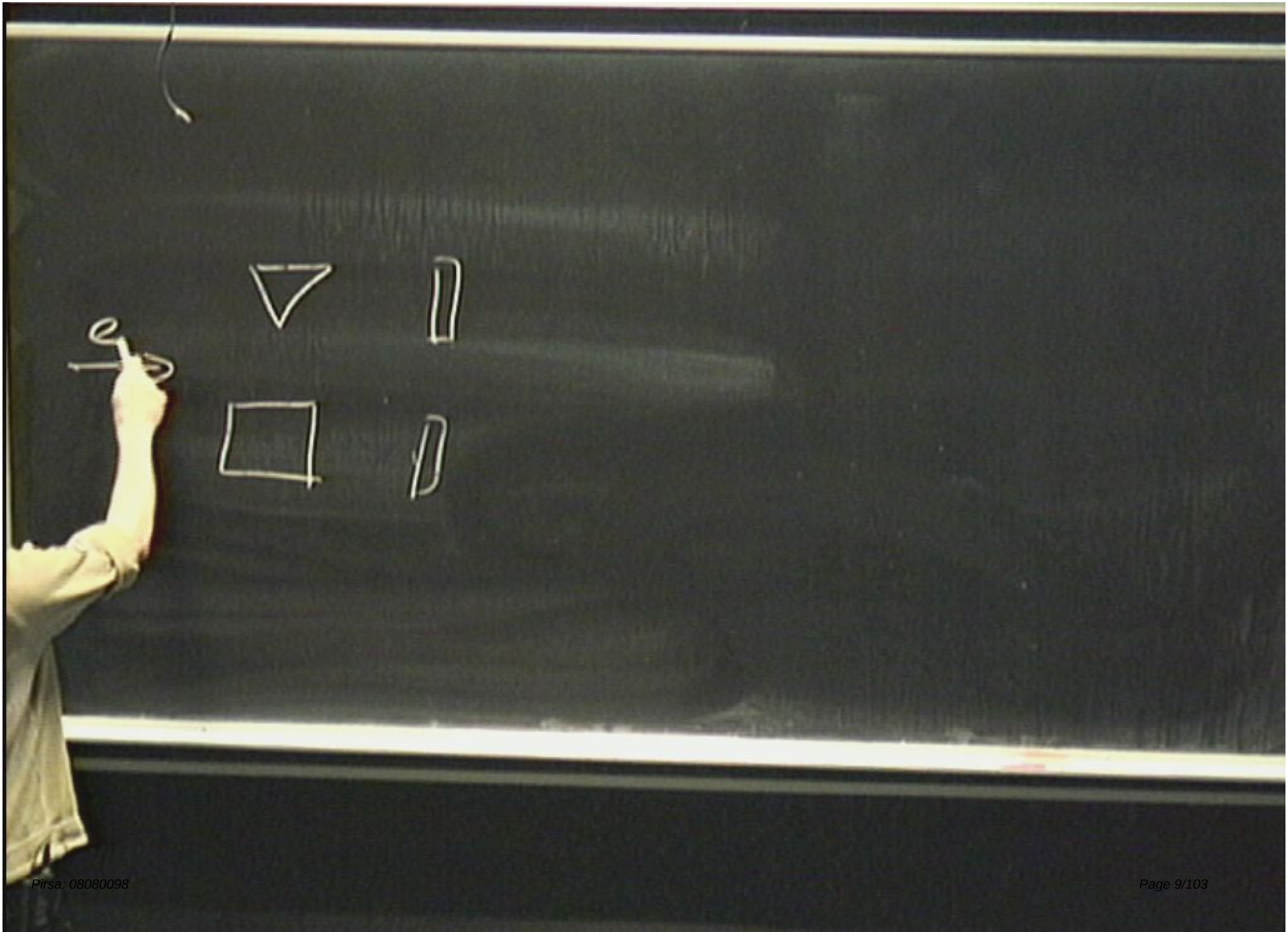

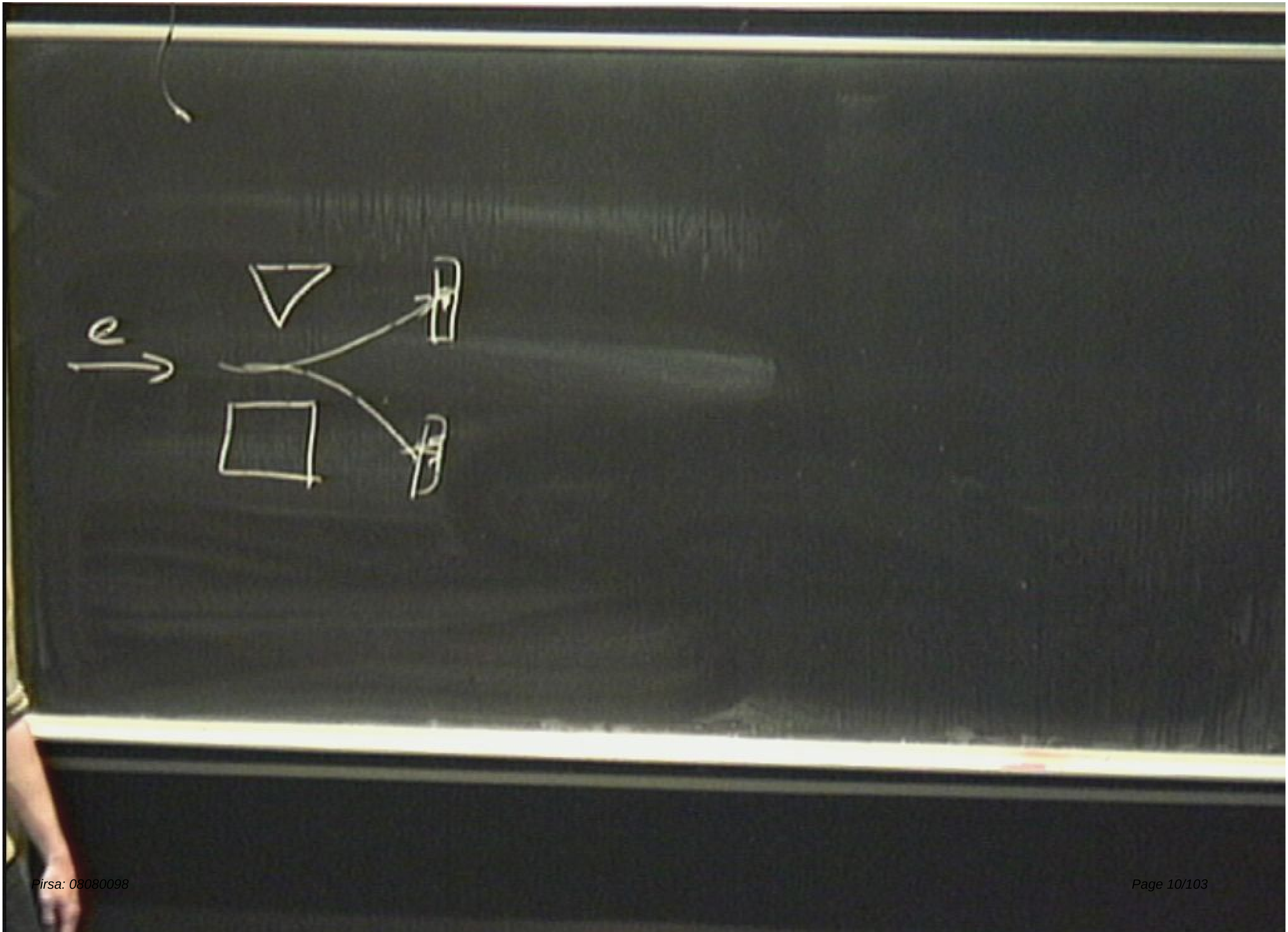
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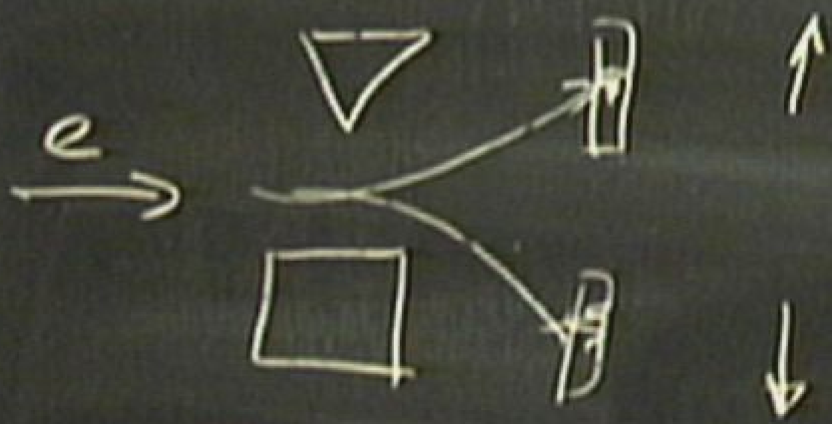
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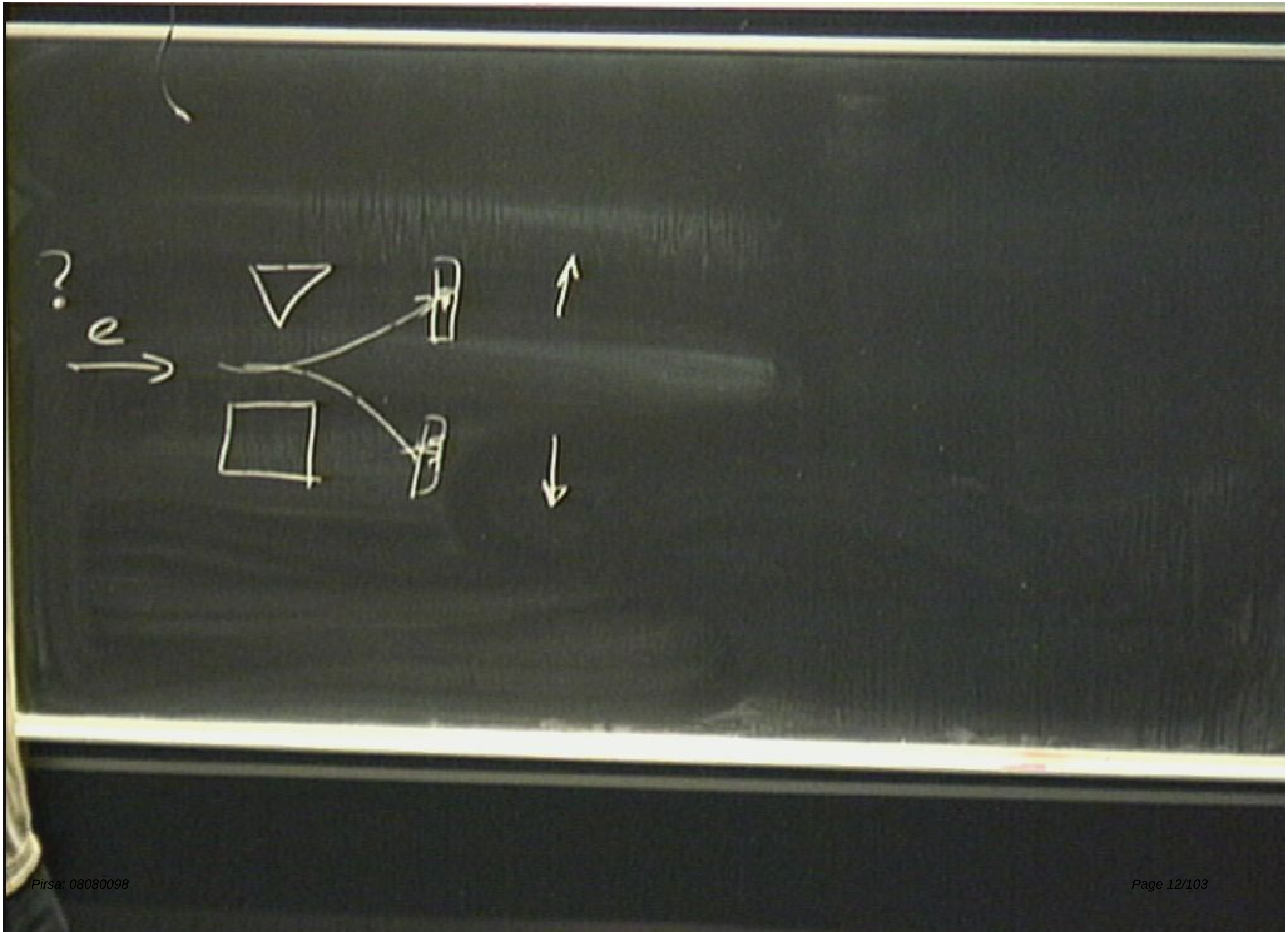
?
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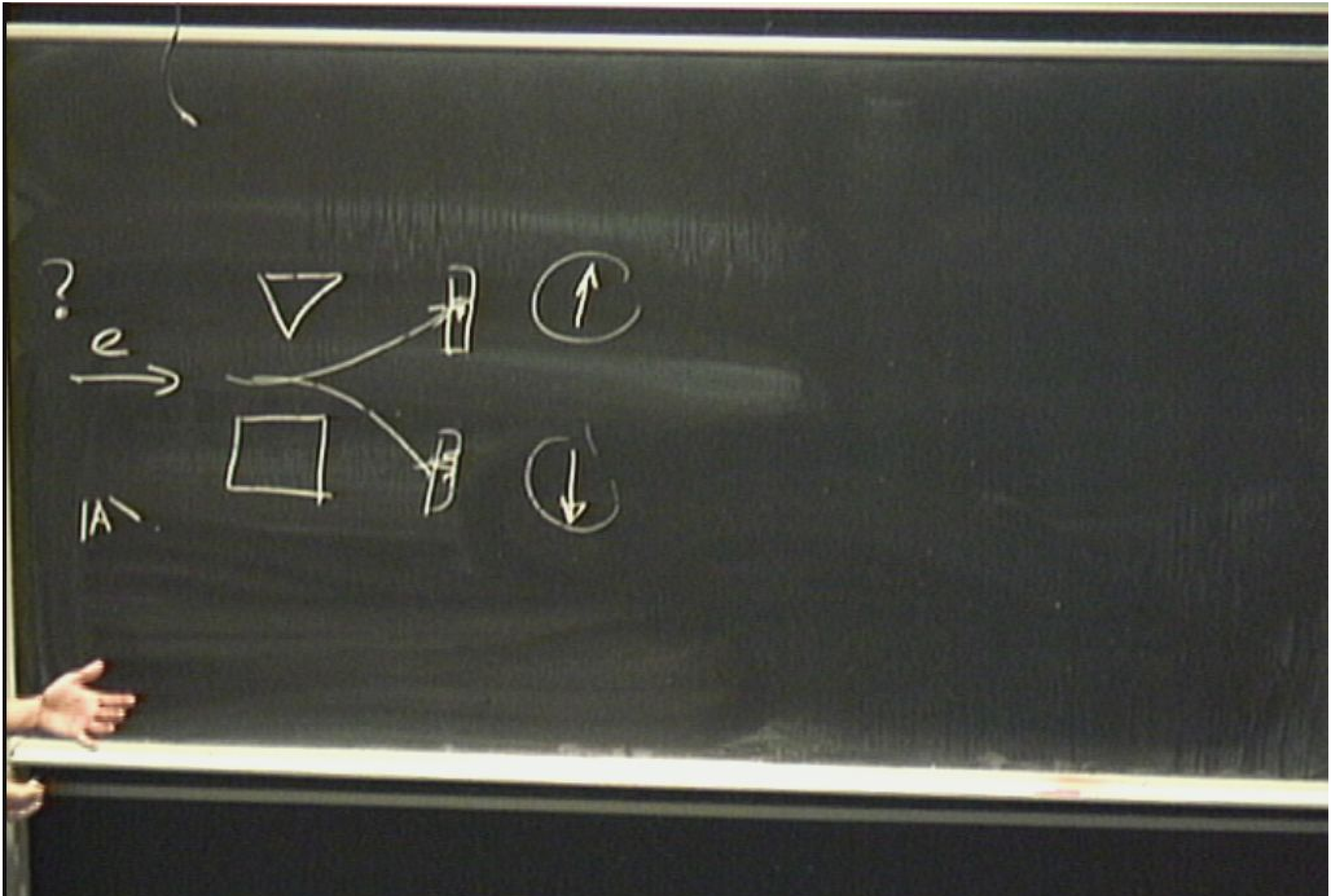
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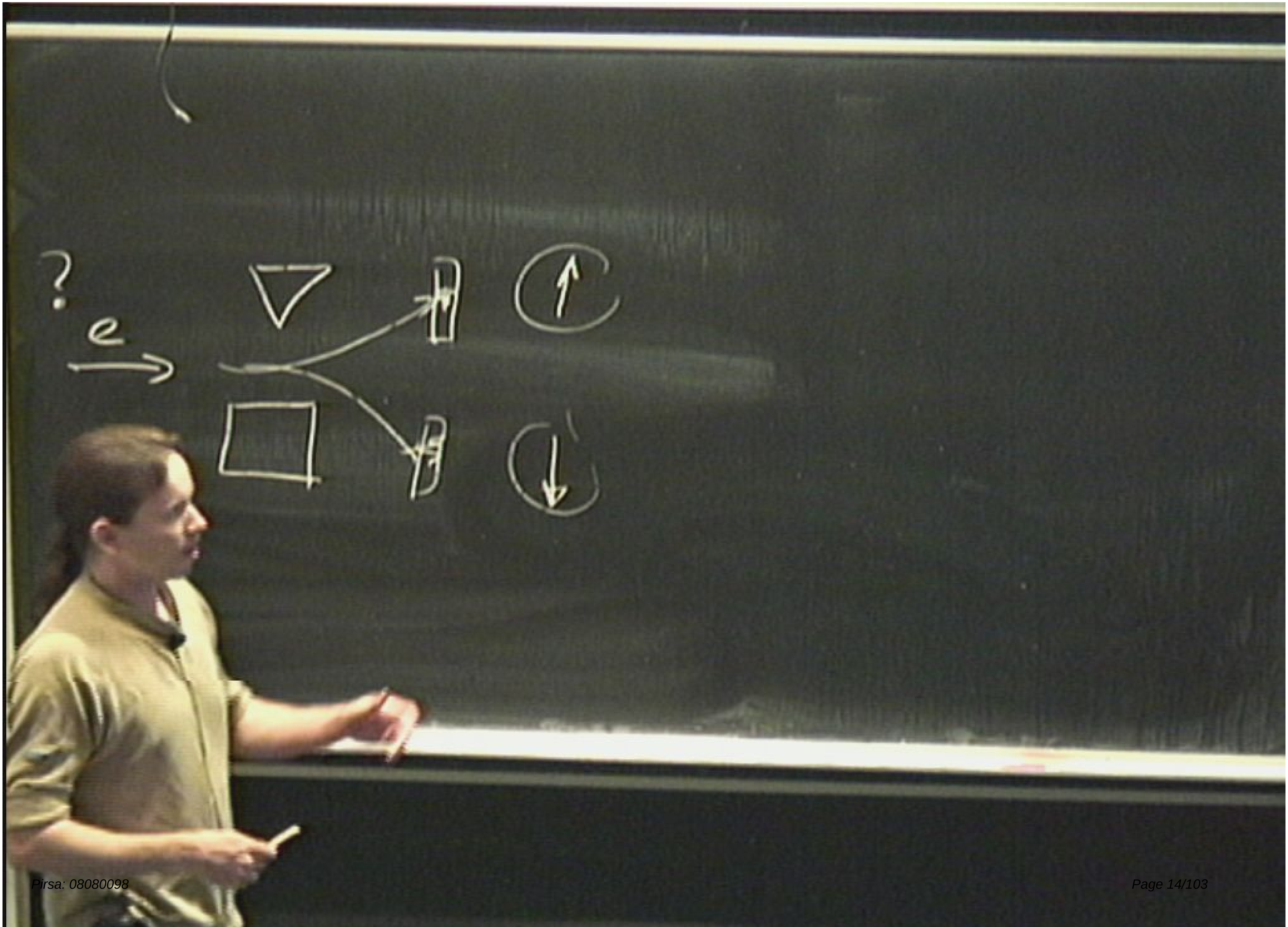
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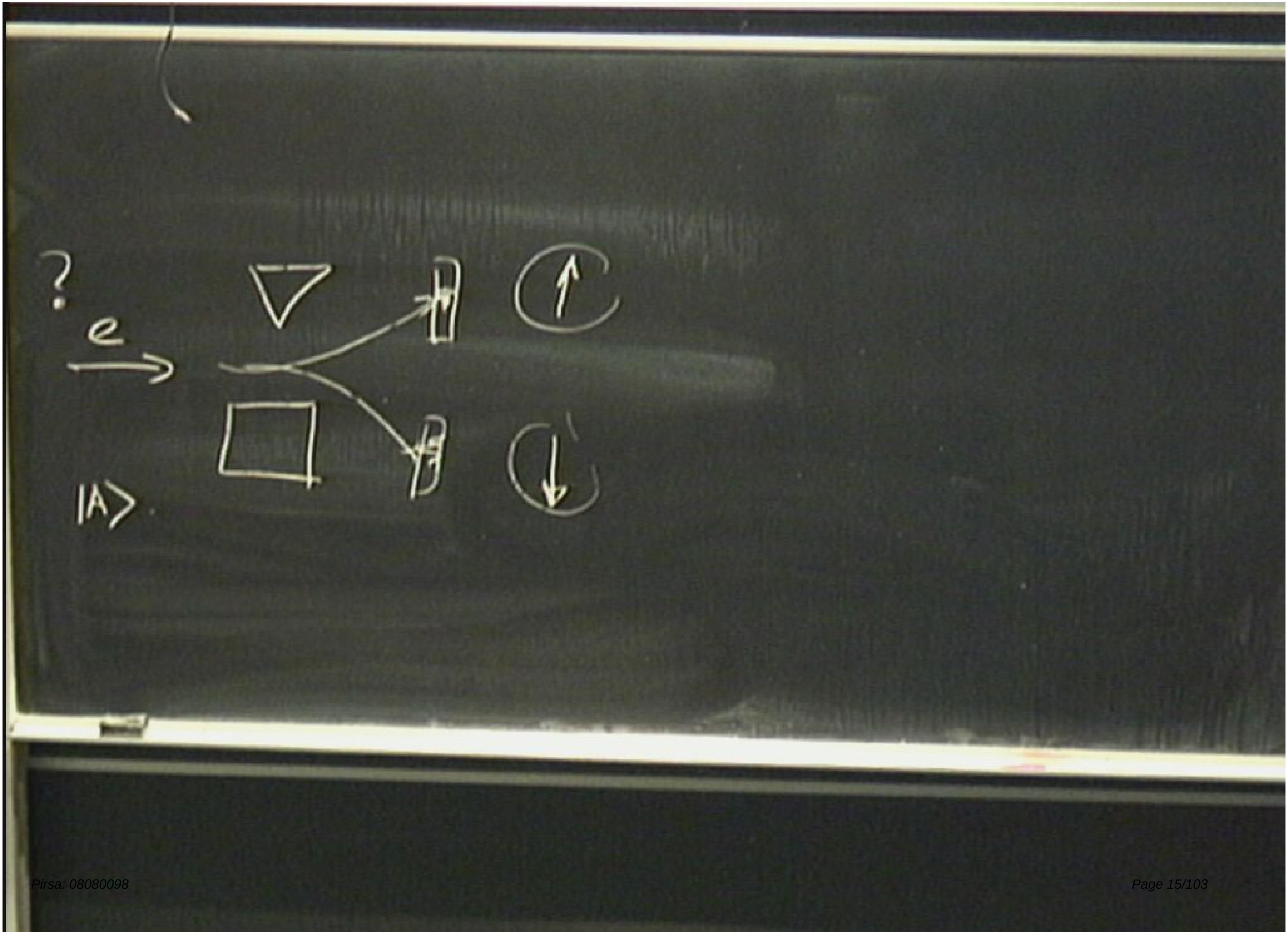
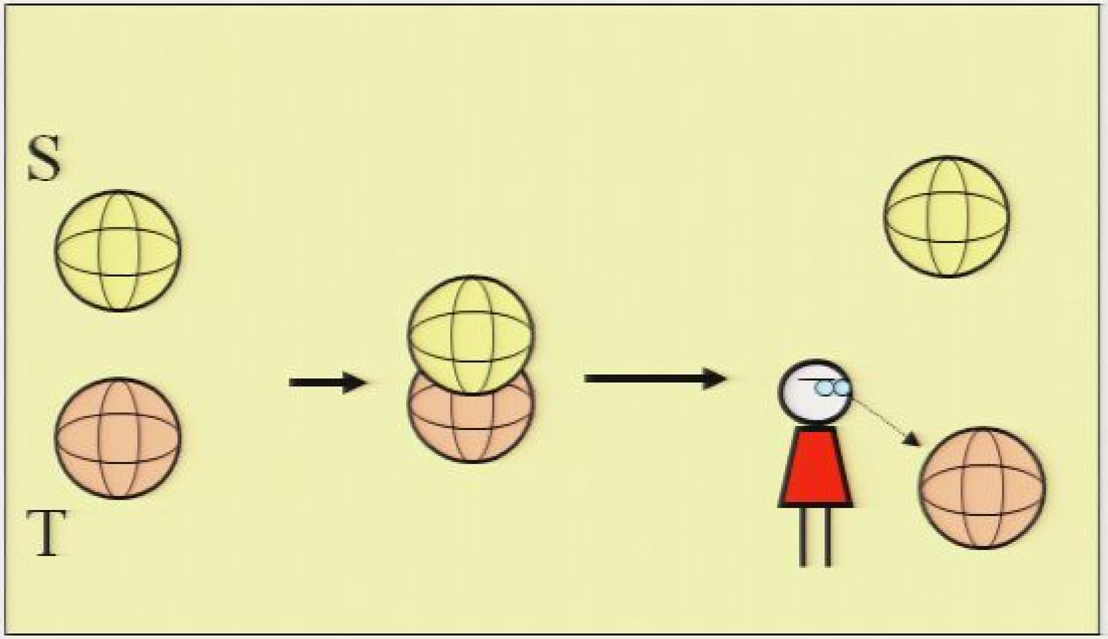

?
e
|A>

Non-demolition measurements



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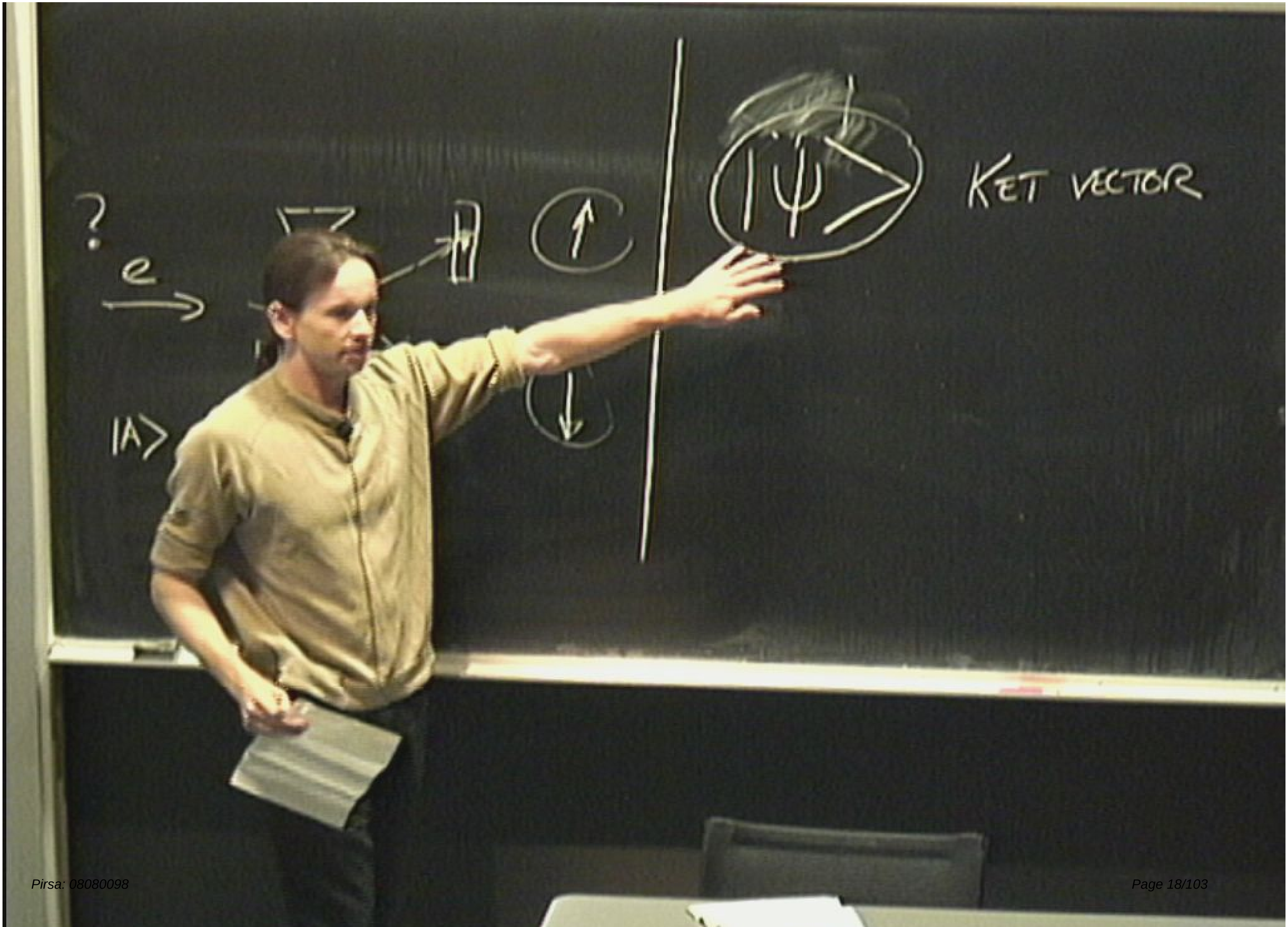
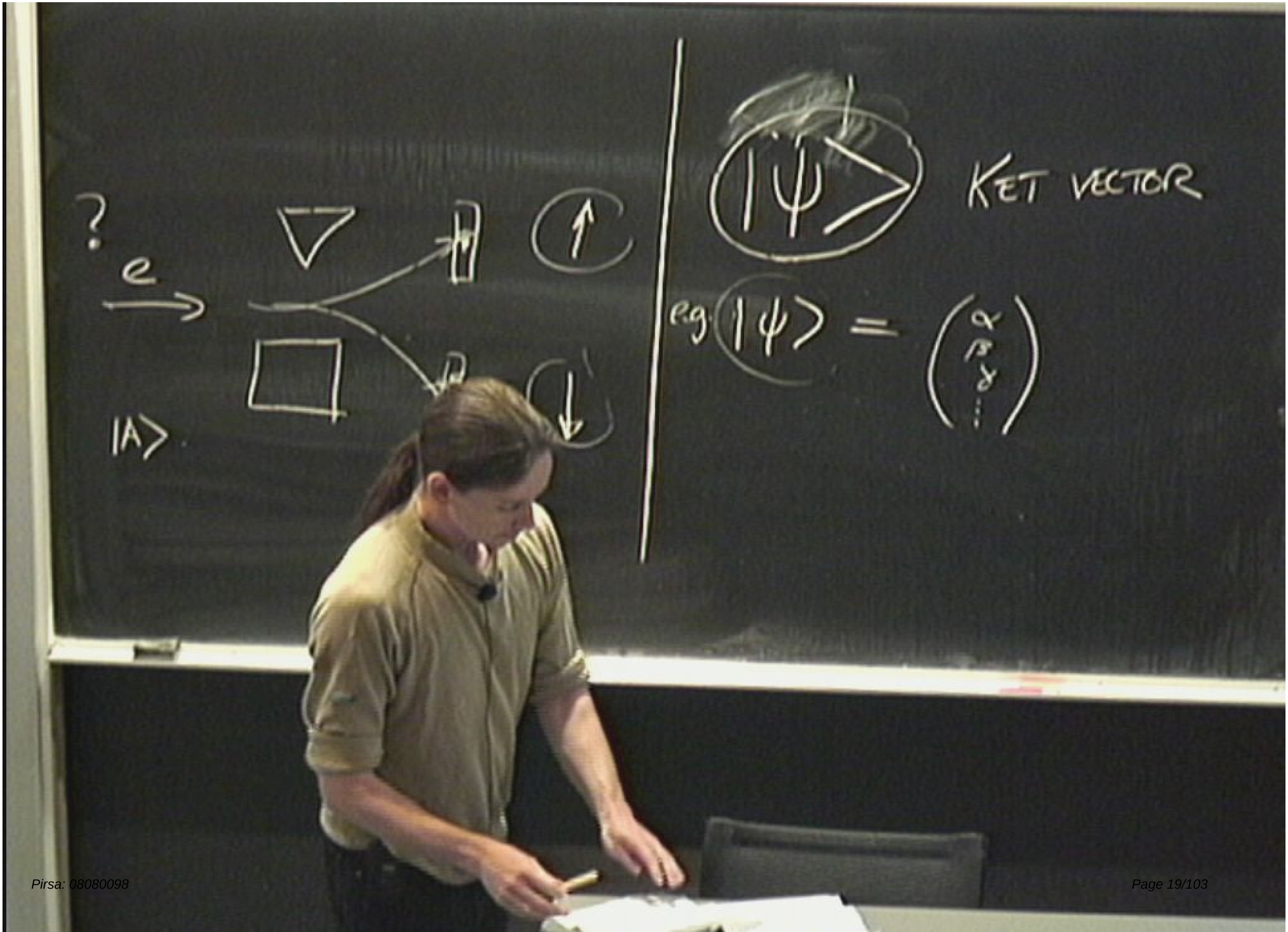
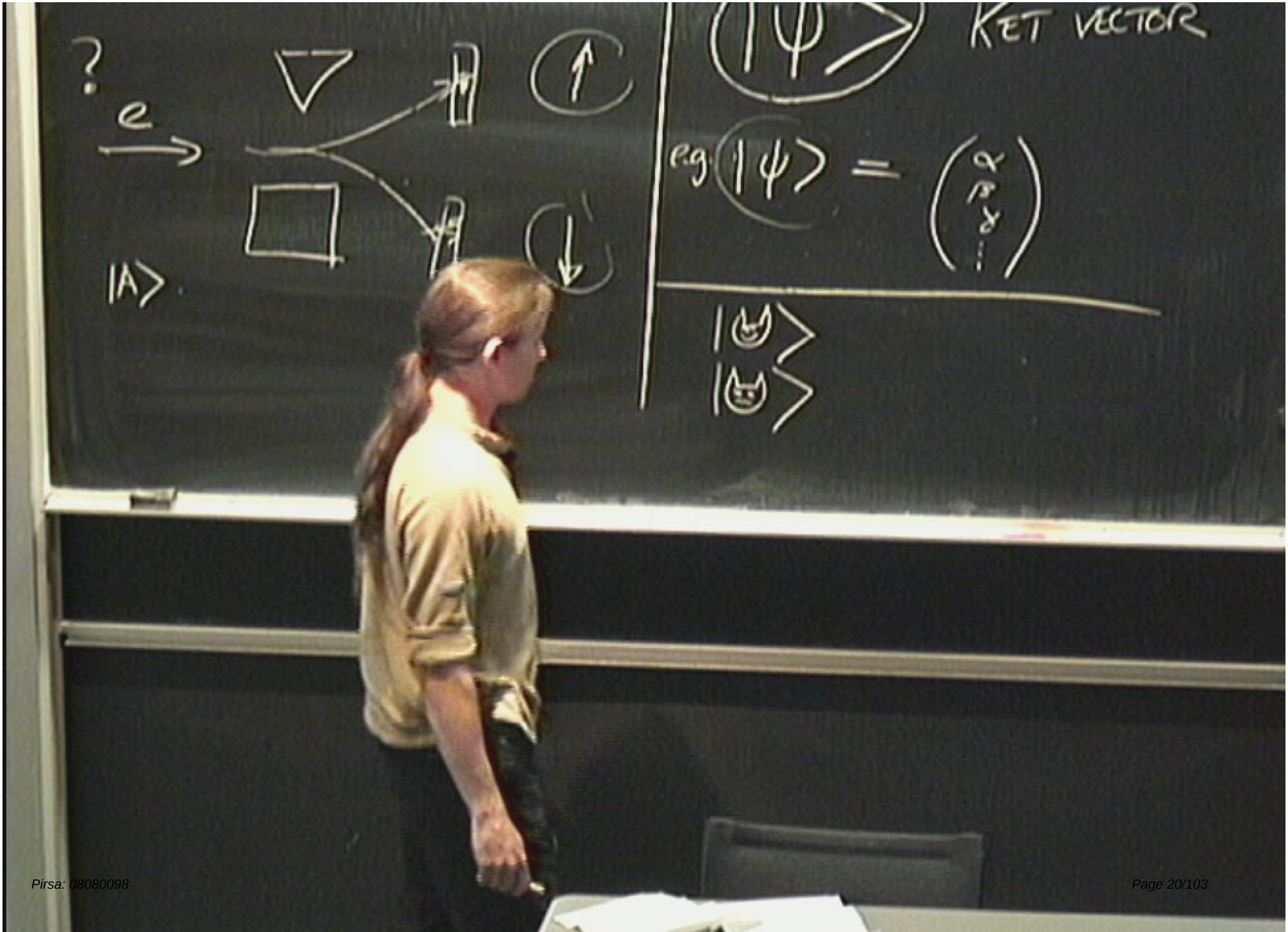

?
e
|A>
|ψ>
KET VECTOR


?
e
|A>
|ψ> KET VECTOR
e.g. |ψ> = (α β γ ⋮)


?
e
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?
e
|A>
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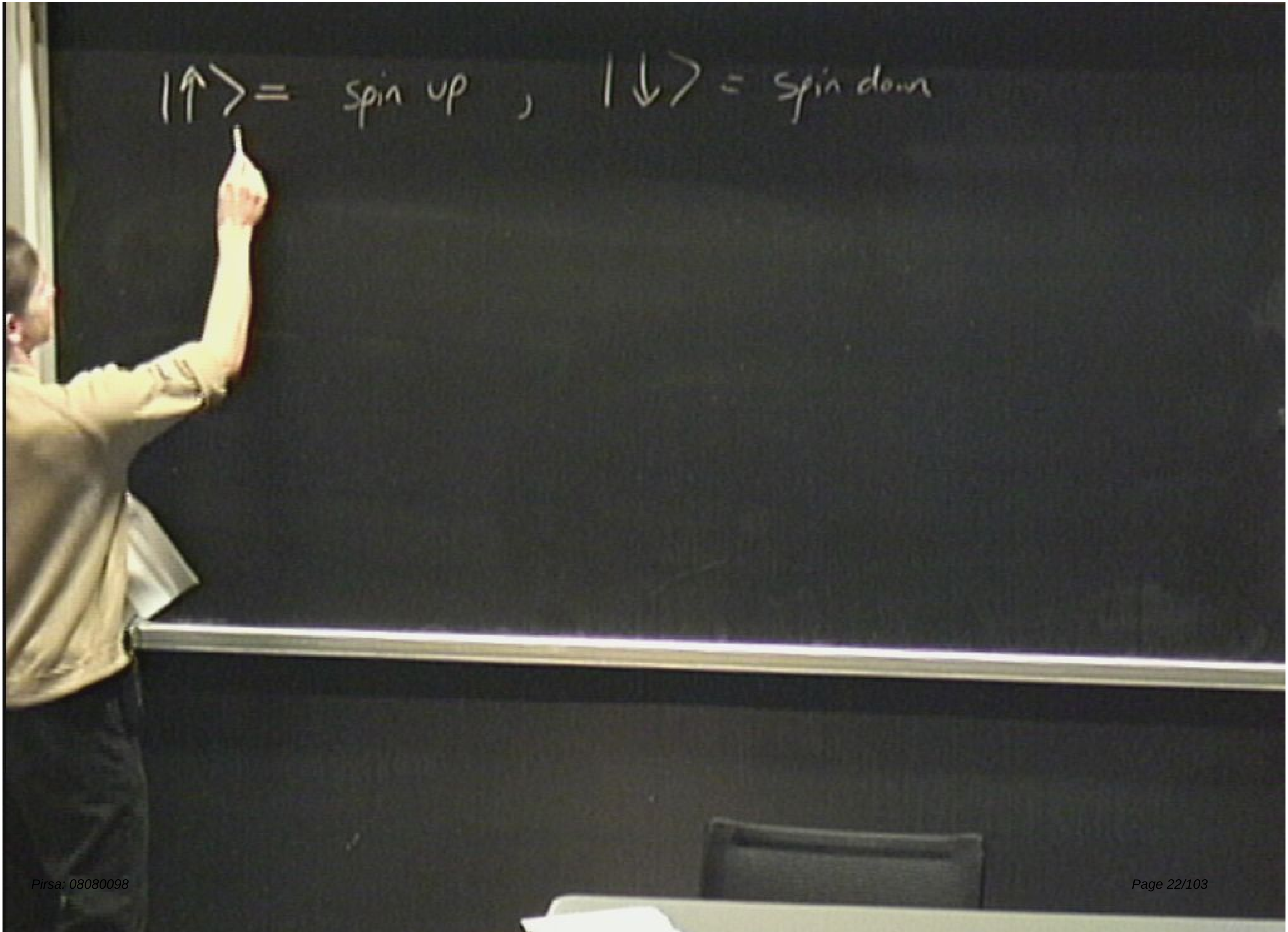
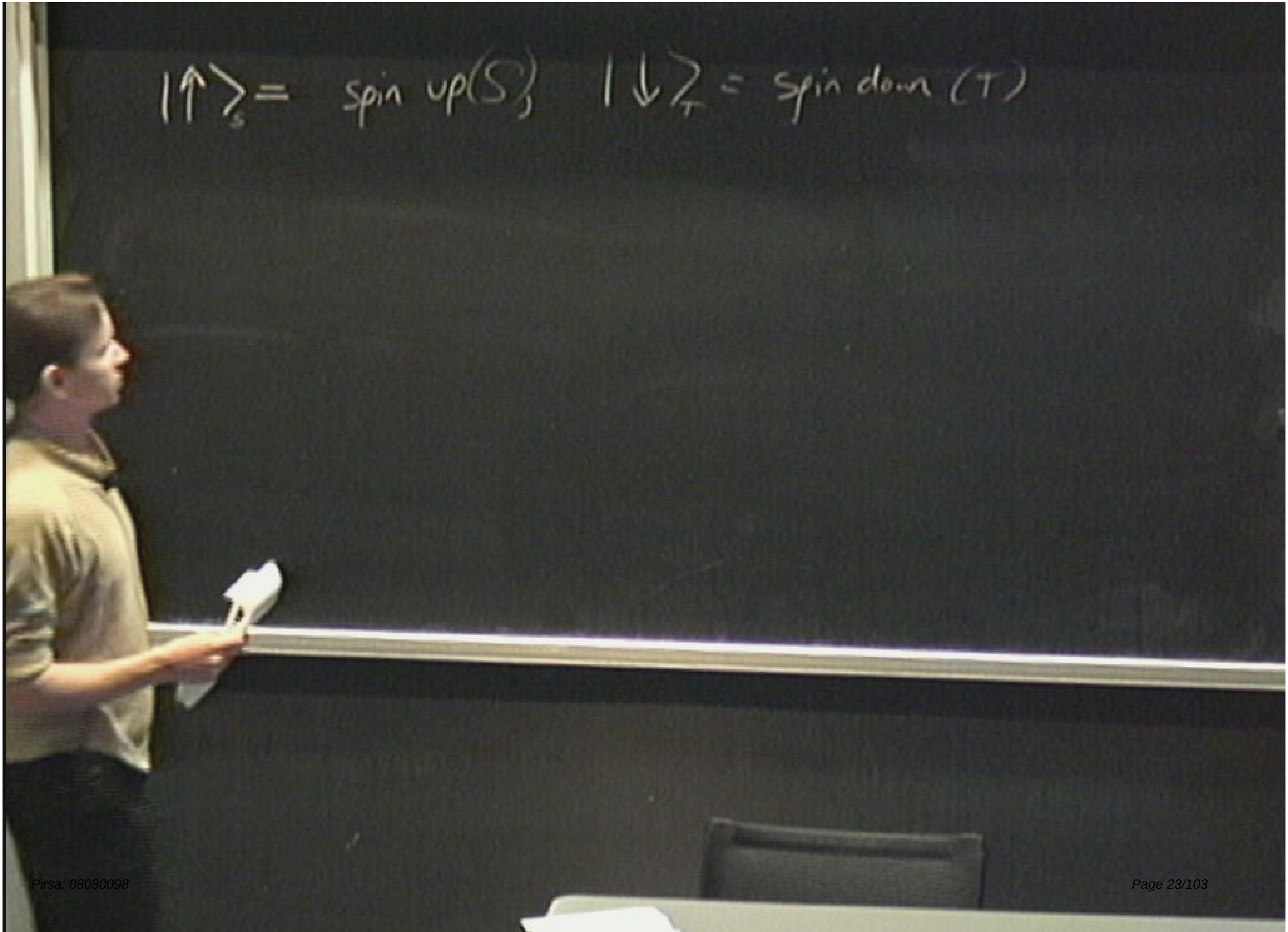
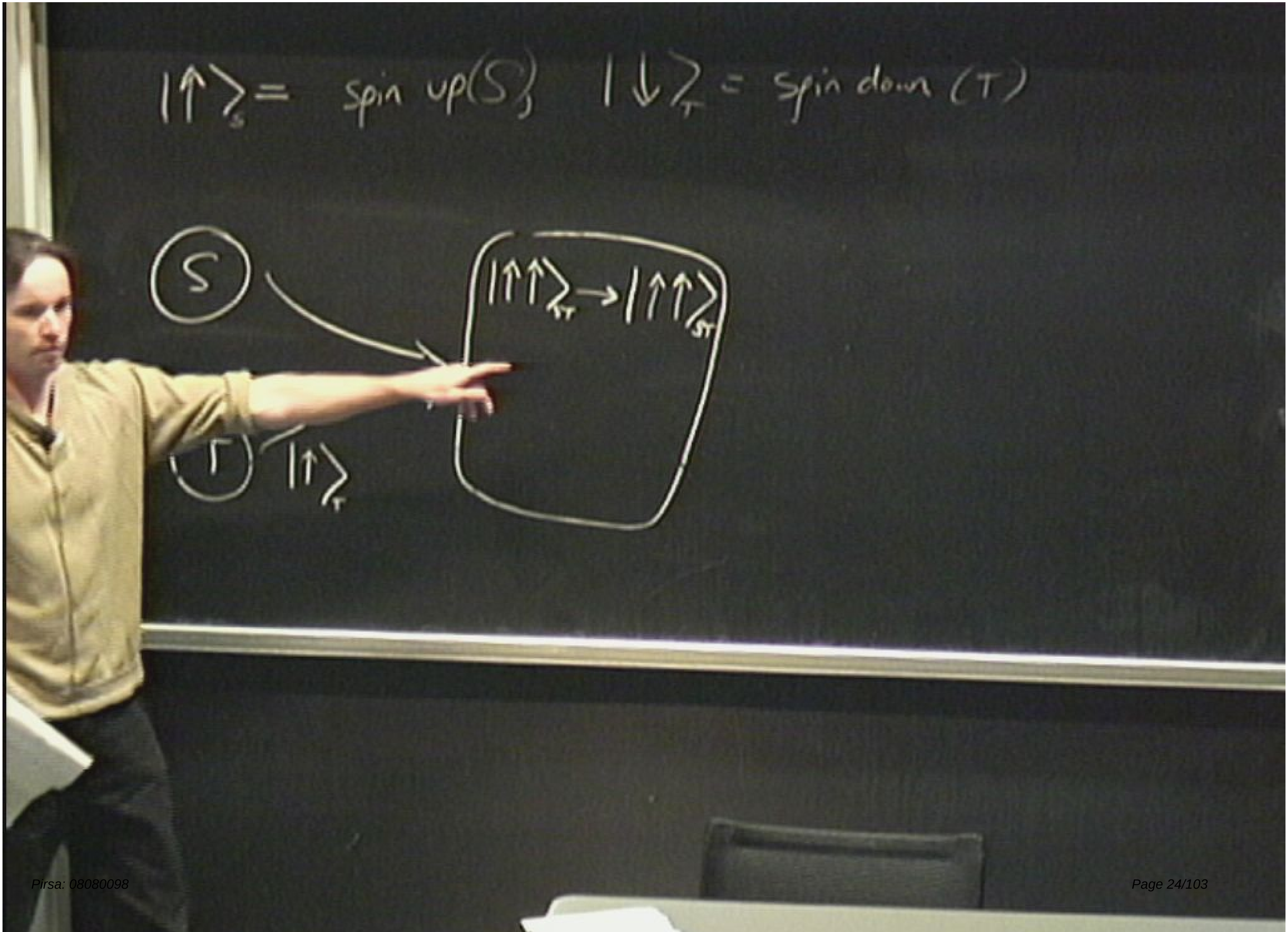
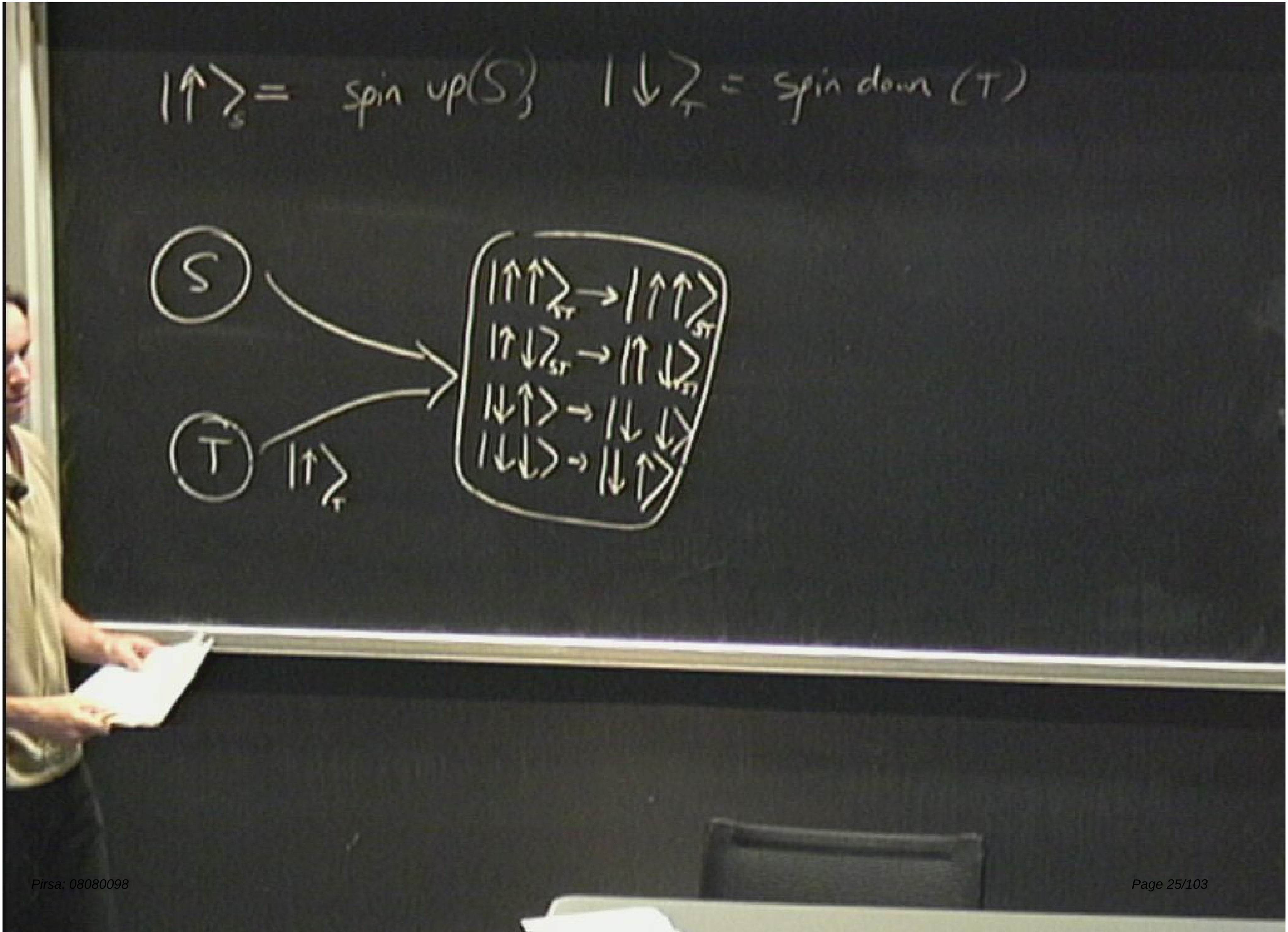
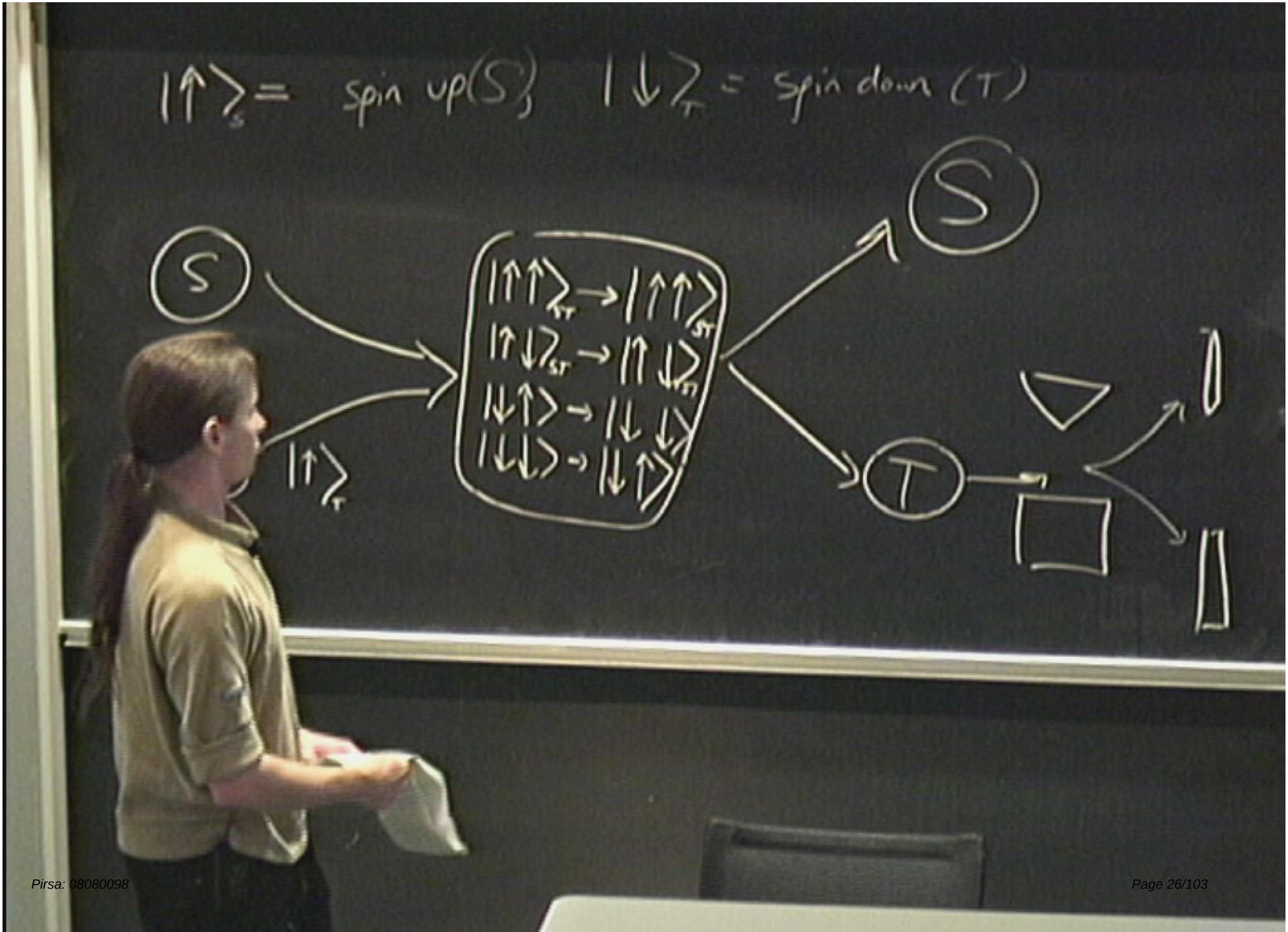
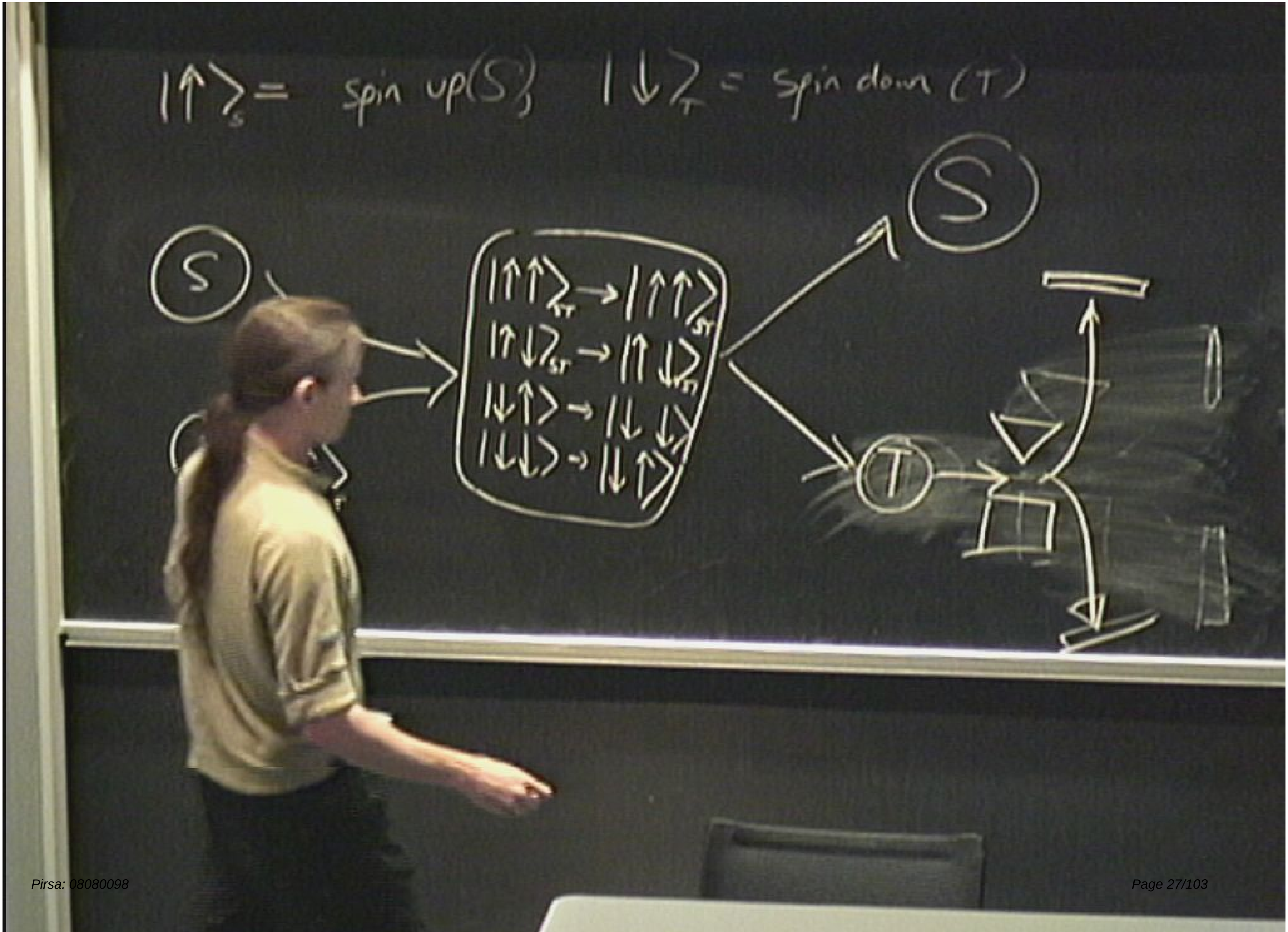

$|\uparrow\rangle =$ spin up , $|\downarrow\rangle =$ spin down

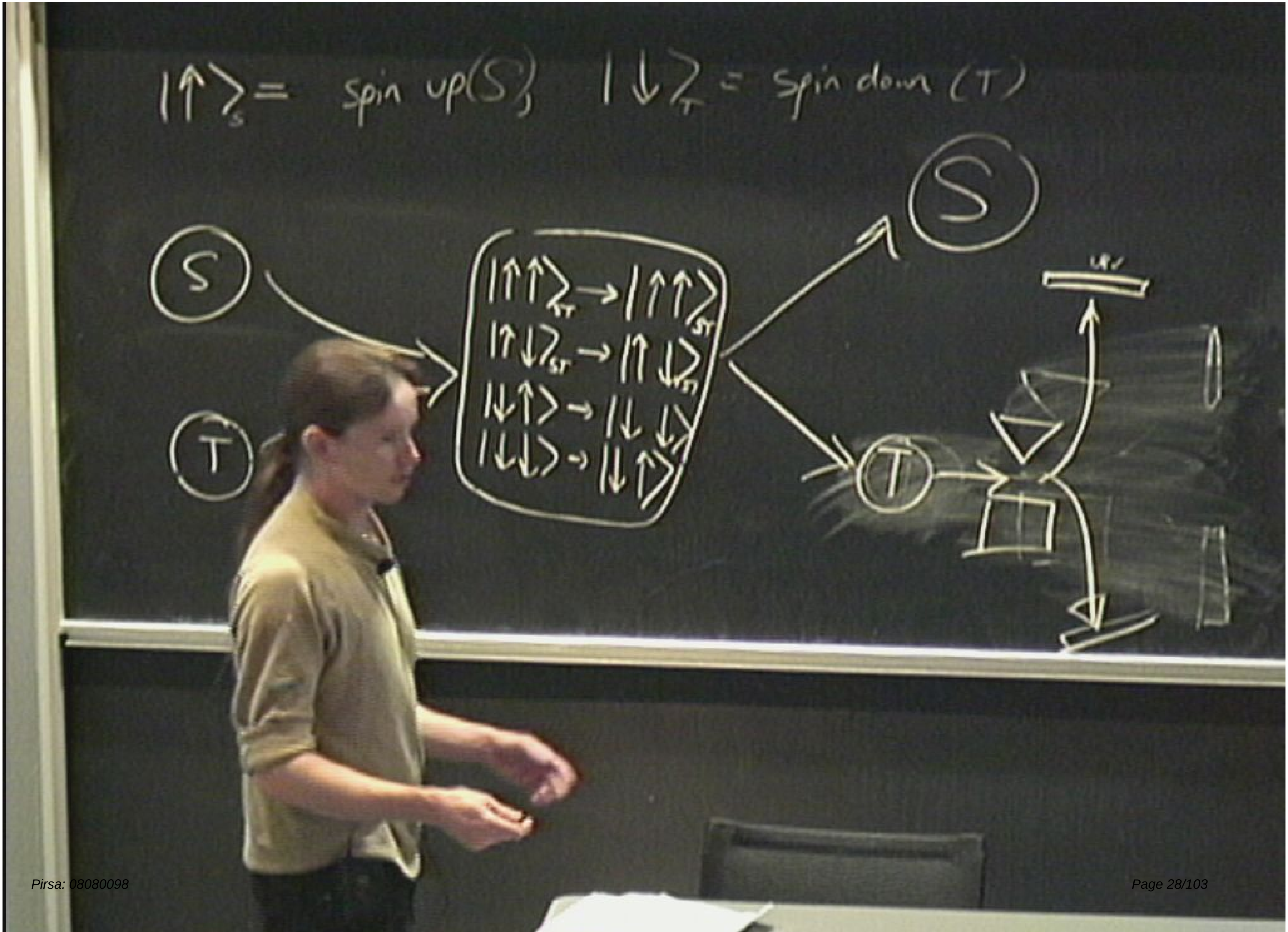

$|\uparrow\rangle_S =$ spin up(S)
$|\downarrow\rangle_T =$ spin down (T)

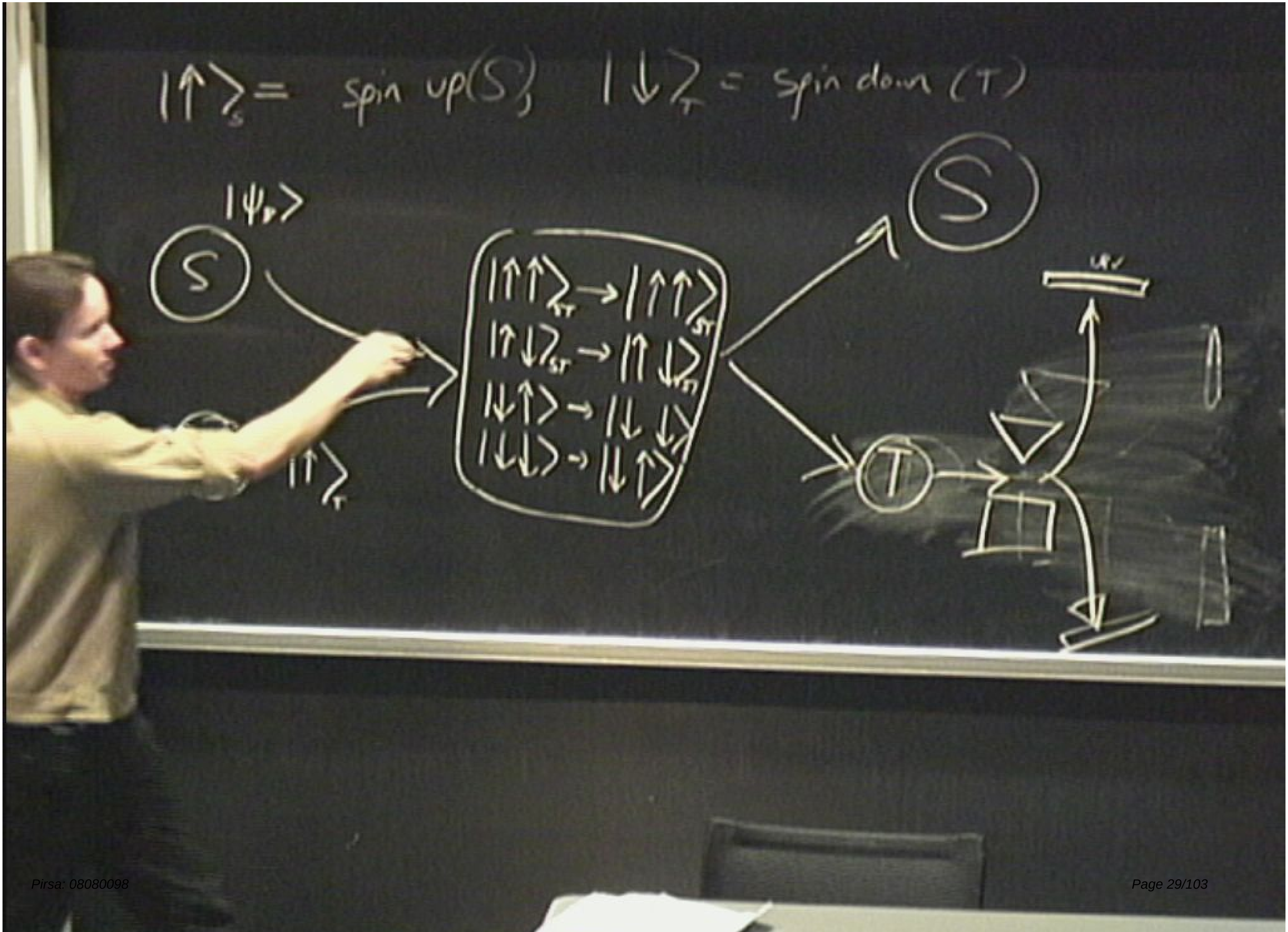

$|\uparrow\rangle_S$ = spin up (S),
$|\downarrow\rangle_T$ = spin down (T)
S
T
$|\uparrow\uparrow\rangle_{ST} \rightarrow |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\rangle_T$


$|\uparrow\rangle_S =$ spin up (S); $|\downarrow\rangle_T =$ spin down (T)
S
T
$|\uparrow\rangle_T$
$|\uparrow\uparrow\rangle_{ST} \to |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\downarrow\rangle_{ST} \to |\uparrow\downarrow\rangle_{ST}$
$|\downarrow\uparrow\rangle \to |\downarrow\downarrow\rangle$
$|\downarrow\downarrow\rangle \to |\downarrow\uparrow\rangle$

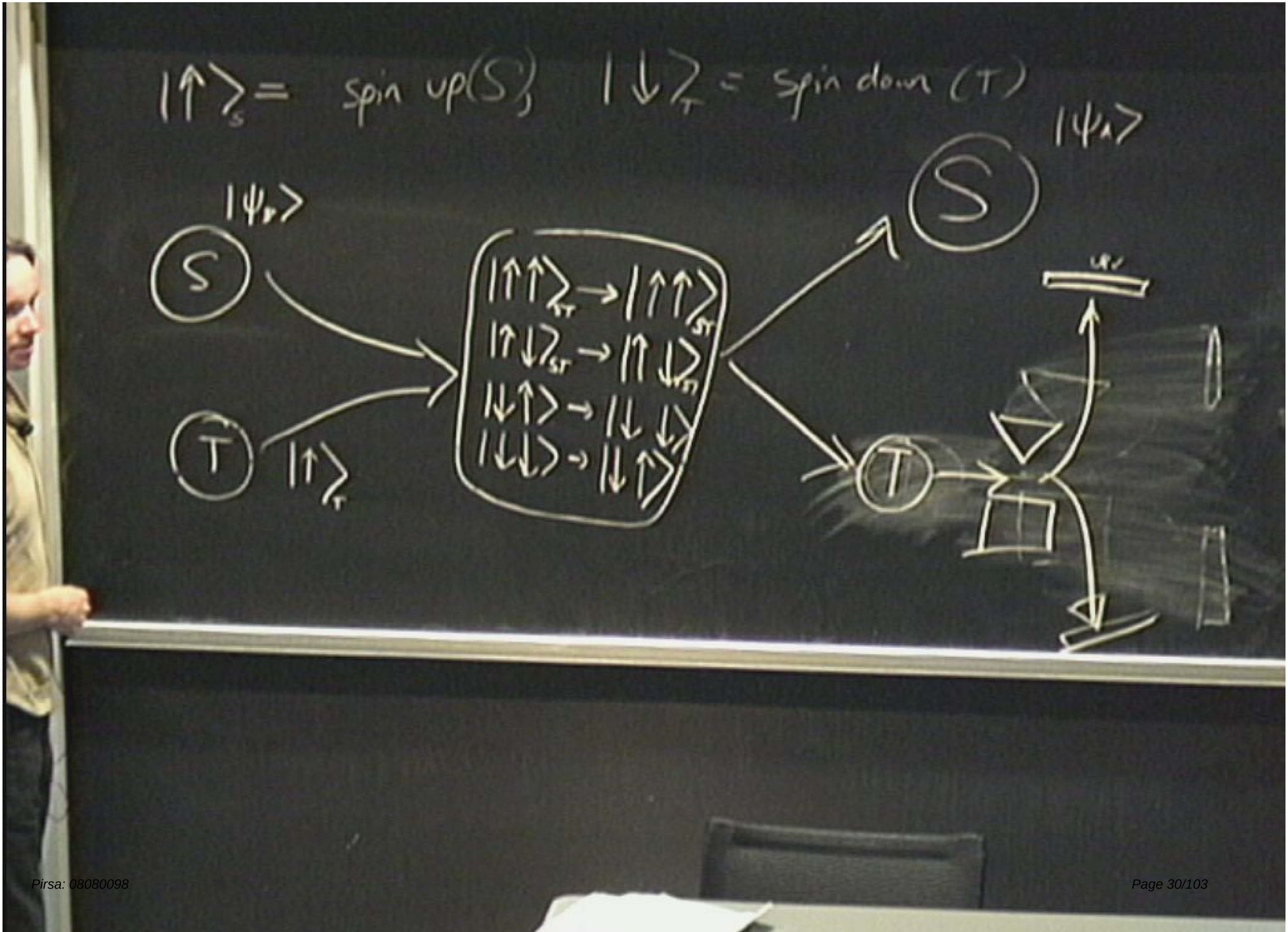

$|\uparrow\rangle_S$ = spin up (S); $|\downarrow\rangle_T$ = spin down (T)
S
$|\uparrow\rangle_T$
$|\uparrow\uparrow\rangle_{ST} \to |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\downarrow\rangle_{ST} \to |\uparrow\downarrow\rangle_{ST}$
$|\downarrow\uparrow\rangle \to |\downarrow\downarrow\rangle$
$|\downarrow\downarrow\rangle \to |\downarrow\uparrow\rangle$
S
T


$|\uparrow\rangle_S$ = spin up (S); $|\downarrow\rangle_T$ = spin down (T)
S
$|\uparrow\uparrow\rangle_{ST} \to |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\downarrow\rangle_{ST} \to |\uparrow\downarrow\rangle_{ST}$
$|\downarrow\uparrow\rangle \to |\downarrow\downarrow\rangle$
$|\downarrow\downarrow\rangle \to |\downarrow\uparrow\rangle$
S
T

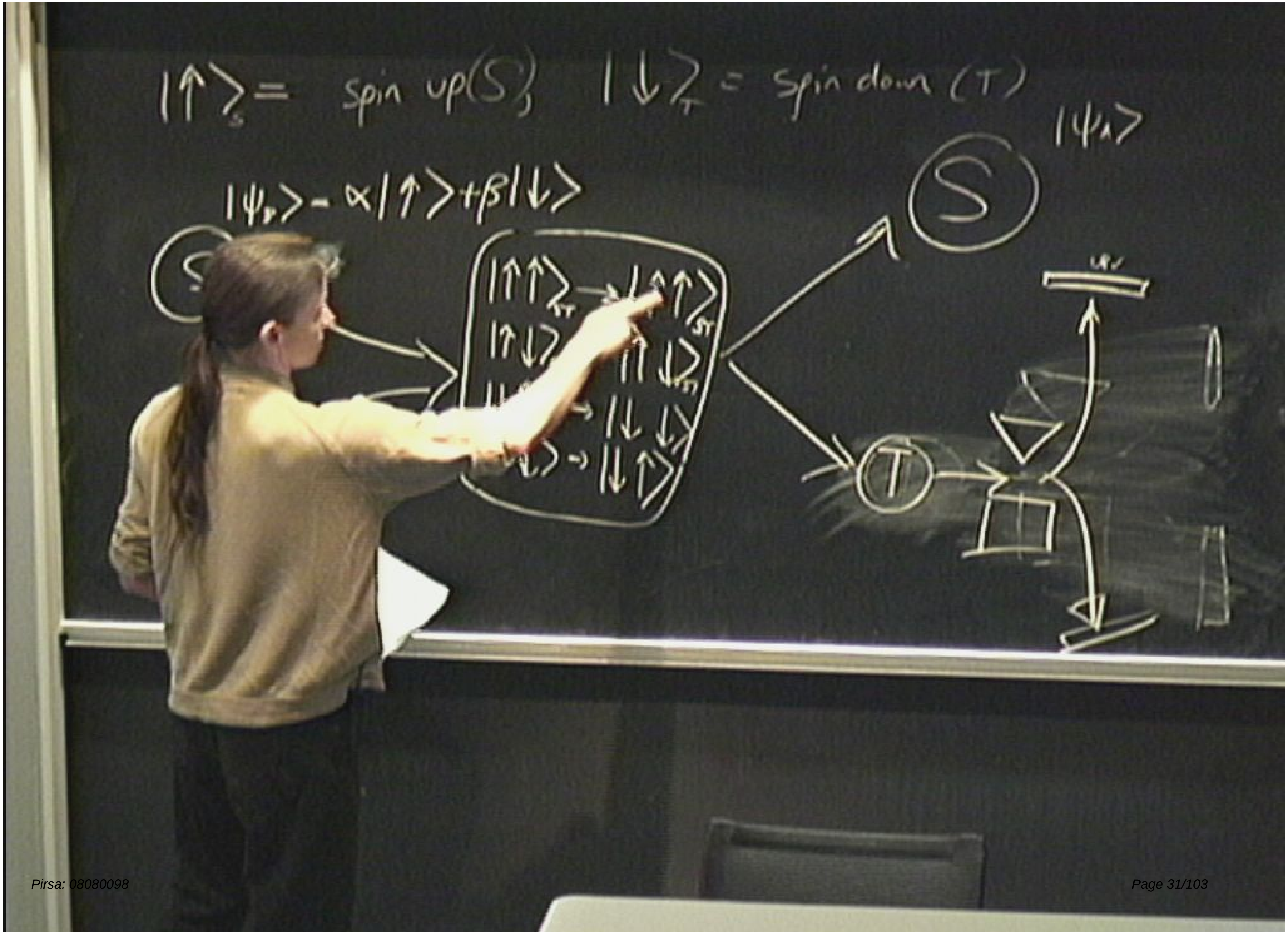

|↑⟩_S = spin up(S); |↓⟩_T = spin down (T)
S
T
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
S
T

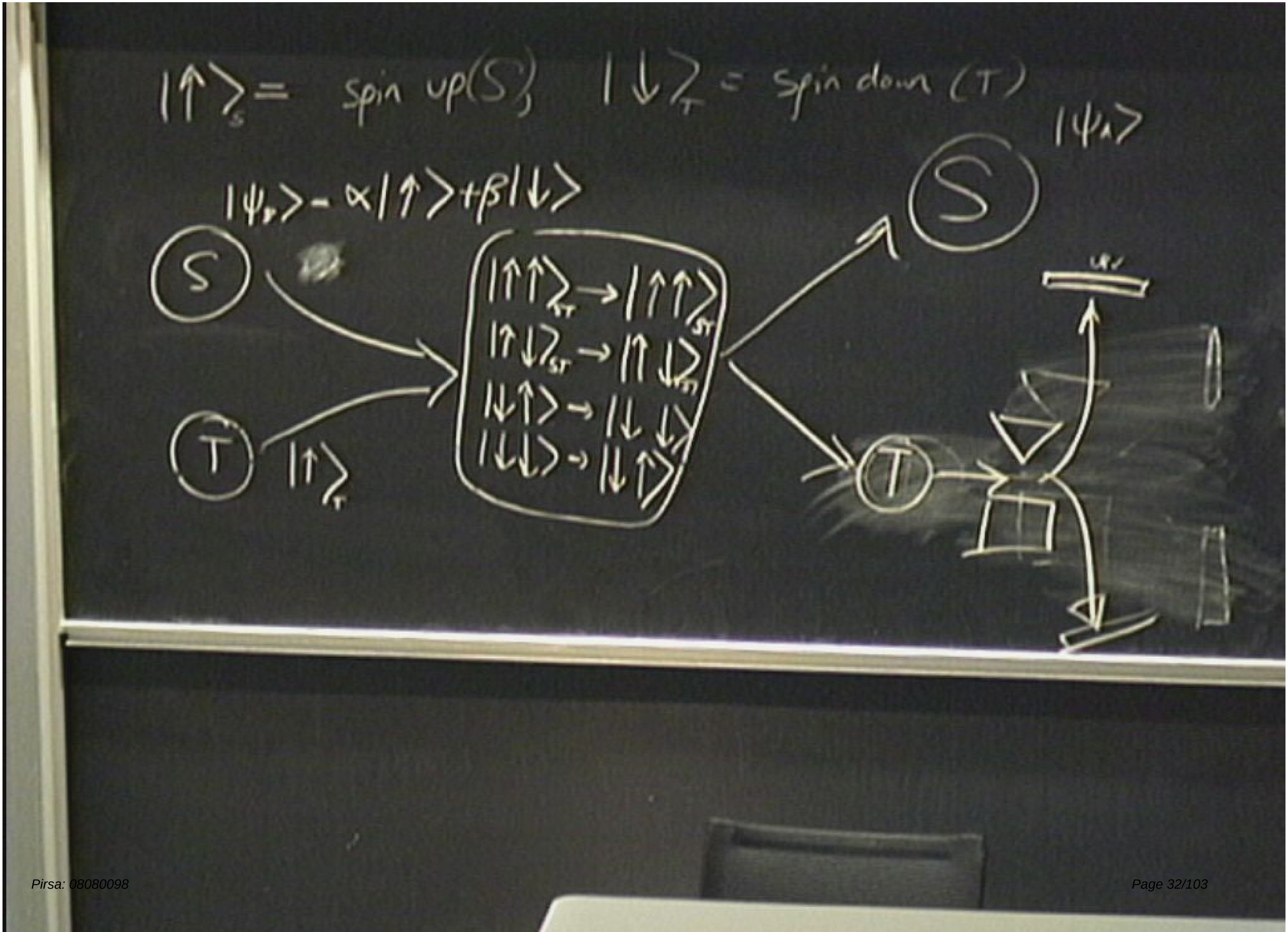


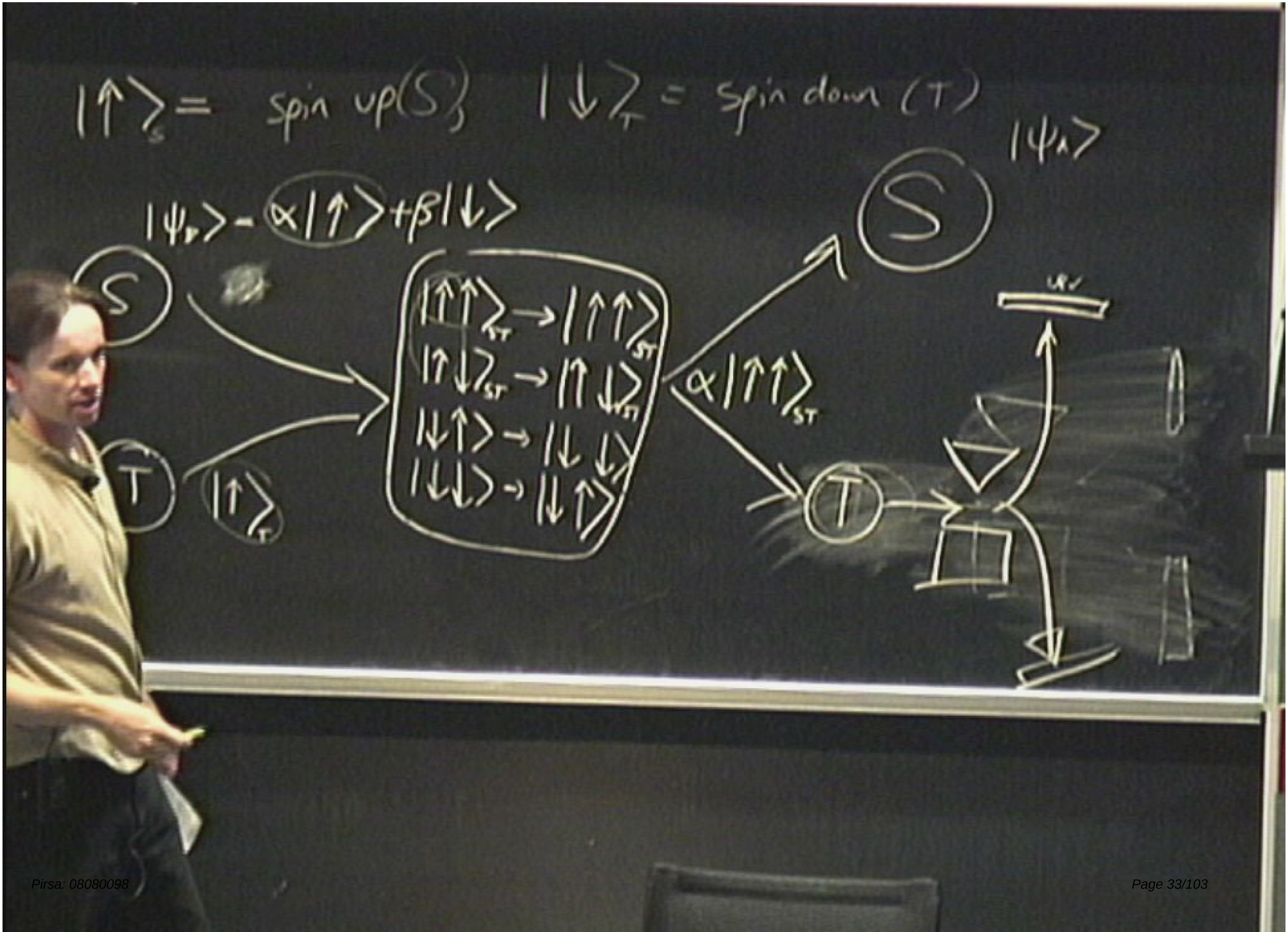
|↑⟩_S = spin up(S); |↓⟩_T = spin down (T)
|ψ_P⟩
S
|↑⟩_T
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
S
T



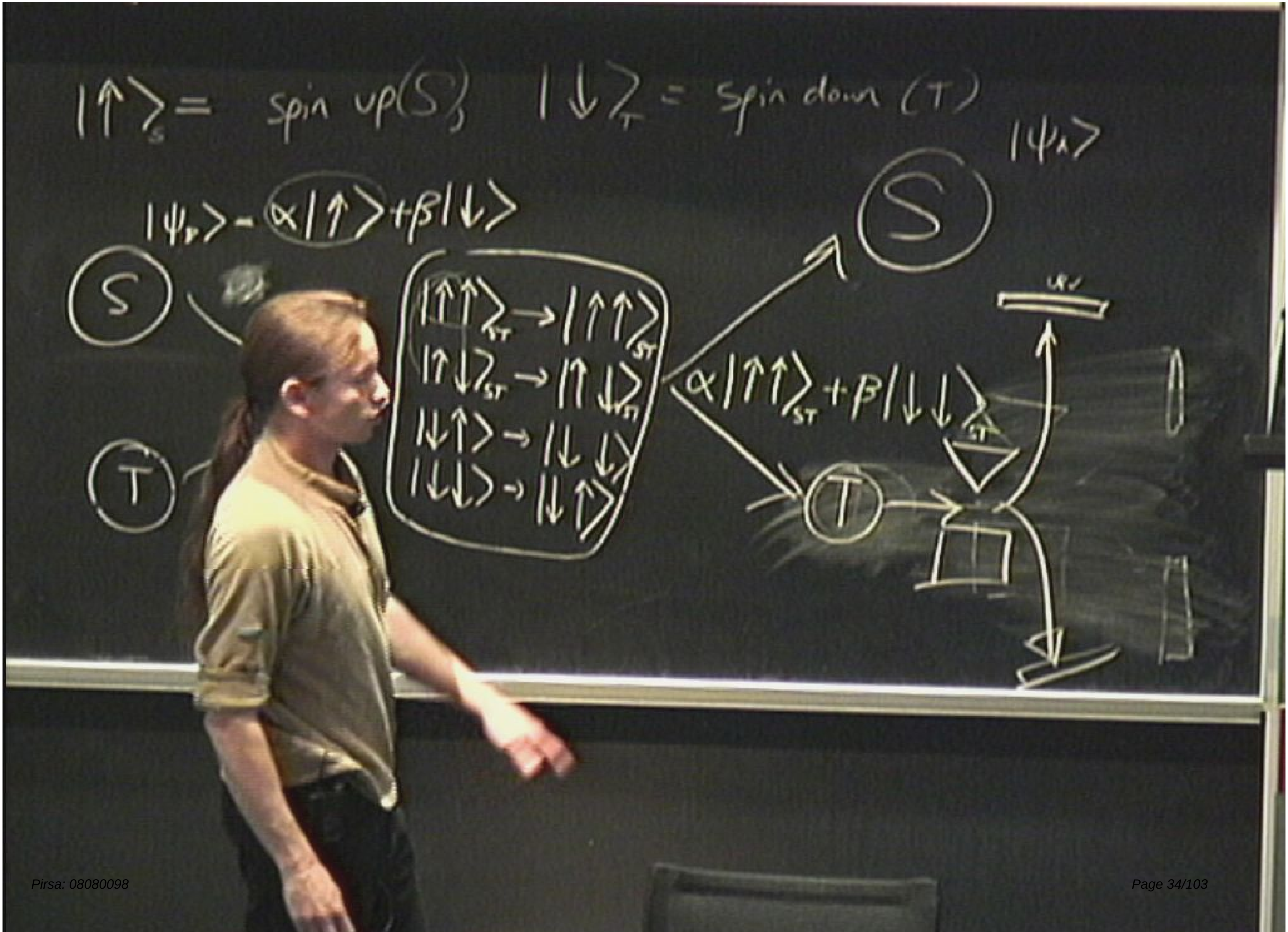
$|\uparrow\rangle_S$ = spin up(S); $|\downarrow\rangle_T$ = spin down (T)
$|\psi_A\rangle$
S
S
T
T
$|\uparrow\rangle_T$
$|\uparrow\uparrow\rangle_{ST} \rightarrow |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\downarrow\rangle_{ST} \rightarrow |\uparrow\downarrow\rangle_{ST}$
$|\downarrow\uparrow\rangle \rightarrow |\downarrow\downarrow\rangle$
$|\downarrow\downarrow\rangle \rightarrow |\downarrow\uparrow\rangle$


|↑⟩_S = spin up (S), |↓⟩_T = spin down (T)
|ψ_A⟩
|ψ_T⟩ = α|↑⟩ + β|↓⟩
S
T


$|\uparrow\rangle_S$ = spin up(S); $|\downarrow\rangle_T$ = spin down (T)
$|\psi_A\rangle$
$|\psi_{i}\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$
S
T
$|\uparrow\rangle_T$
$|\uparrow\uparrow\rangle_{ST} \rightarrow |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\downarrow\rangle_{ST} \rightarrow |\uparrow\downarrow\rangle_{ST}$
$|\downarrow\uparrow\rangle \rightarrow |\downarrow\downarrow\rangle$
$|\downarrow\downarrow\rangle \rightarrow |\downarrow\uparrow\rangle$
S
T



|↑⟩_S = spin up (S) ; |↓⟩_T = spin down (T)
|ψ_P⟩ = α|↑⟩ + β|↓⟩
S
T
|↑⟩_T
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST
|ψ_A⟩
S
T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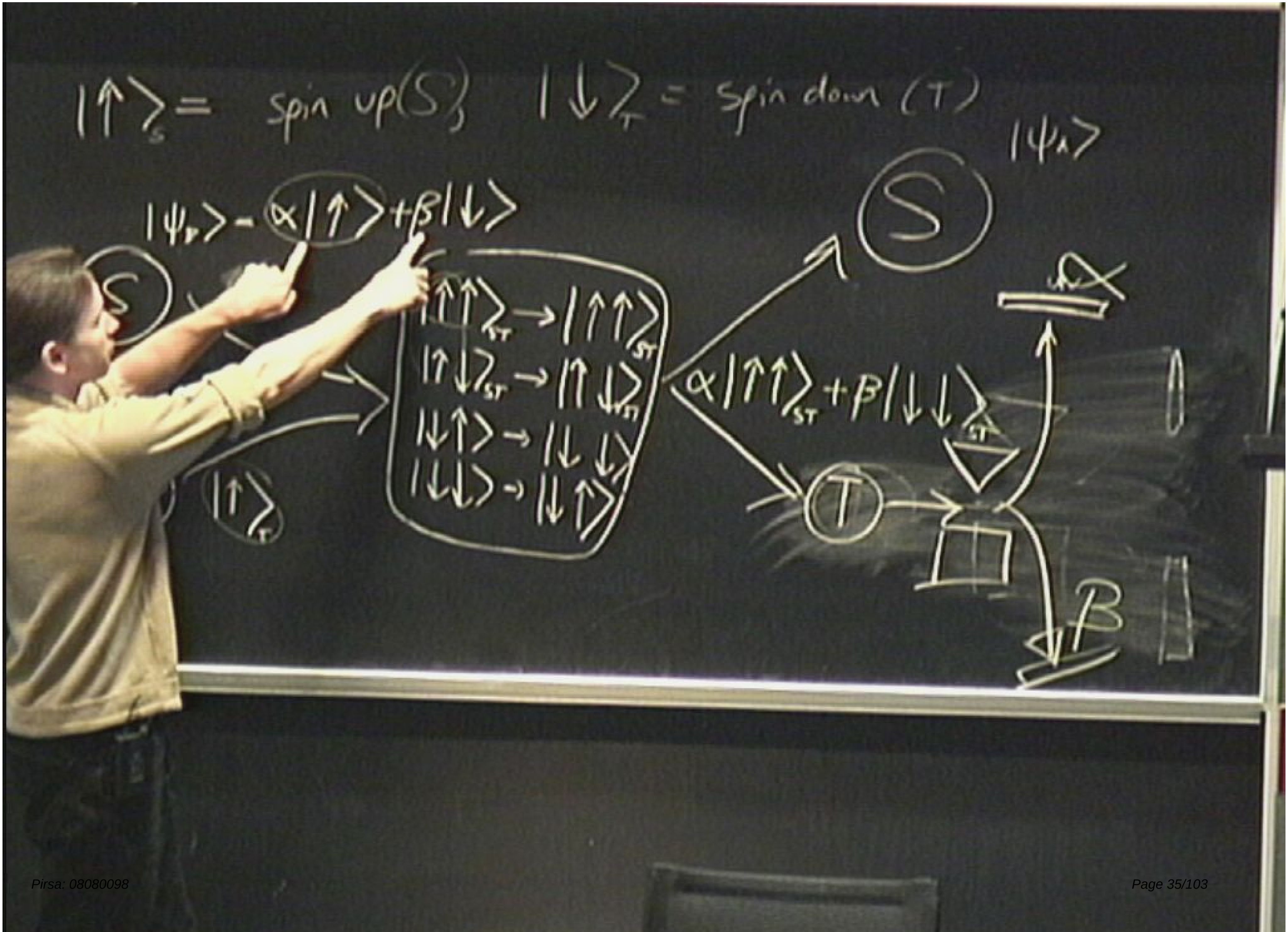
|ψ_A⟩
|ψ_P⟩ = α|↑⟩ + β|↓⟩
S
T
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST + β|↓↓⟩_ST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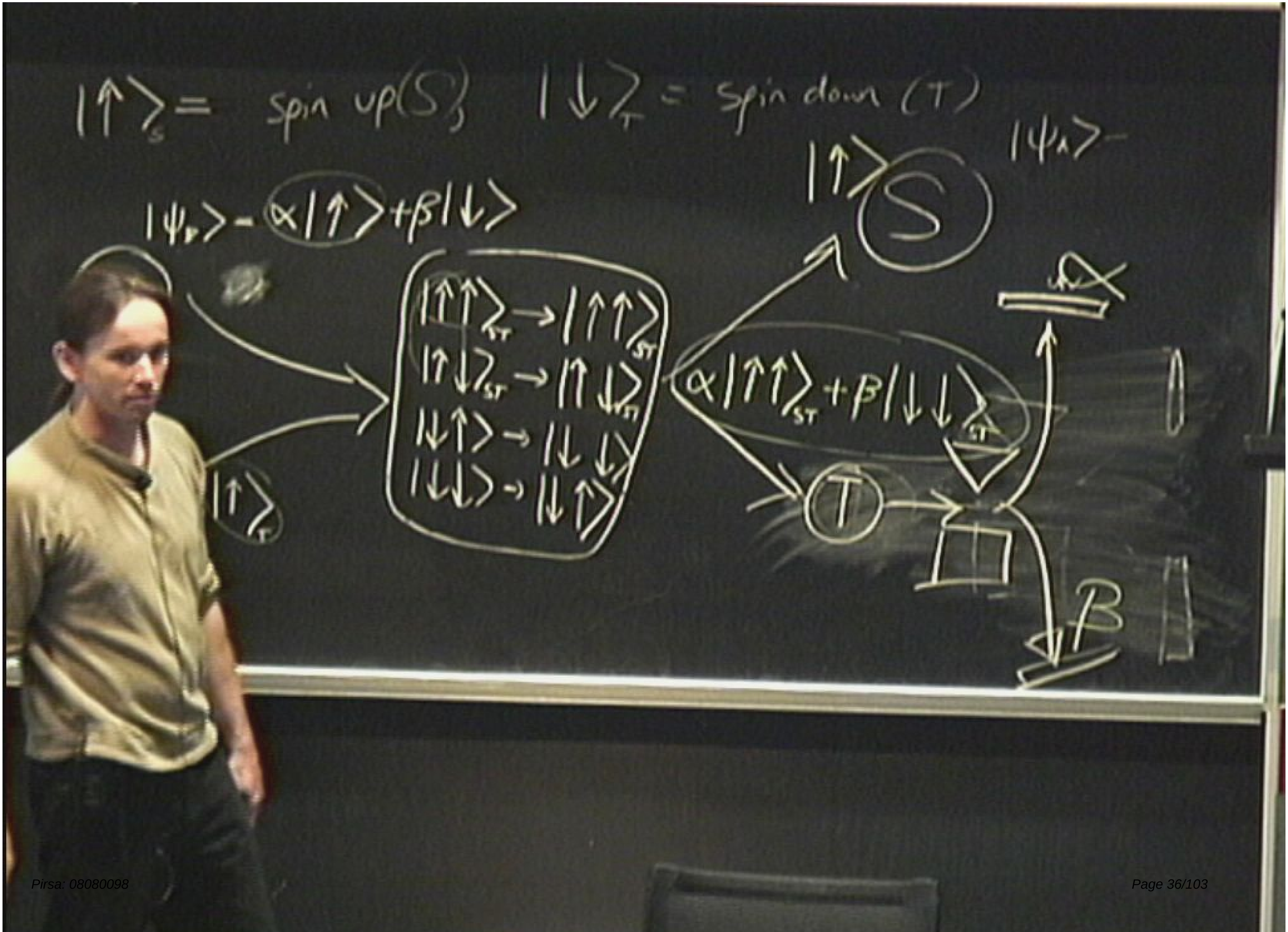
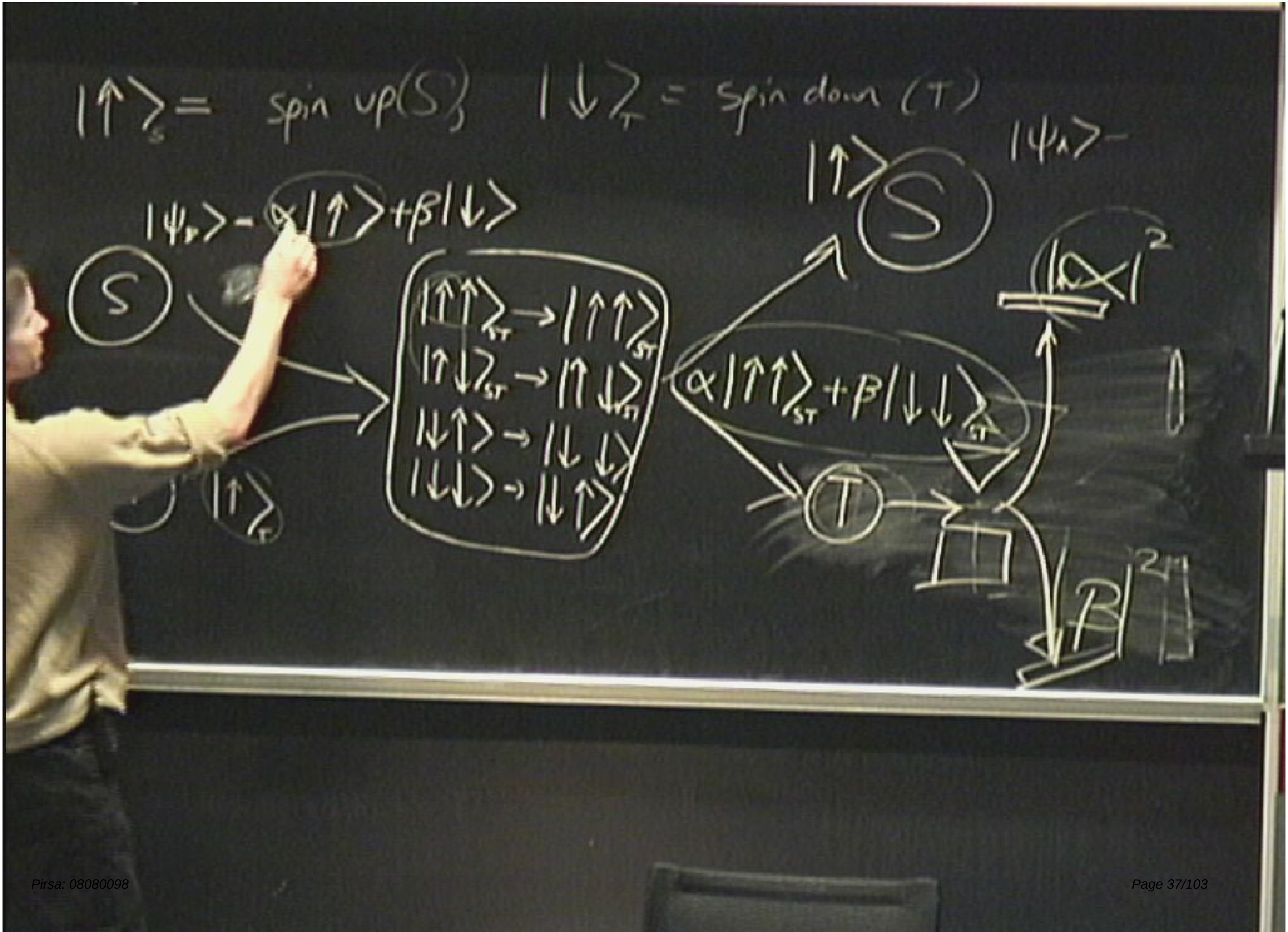
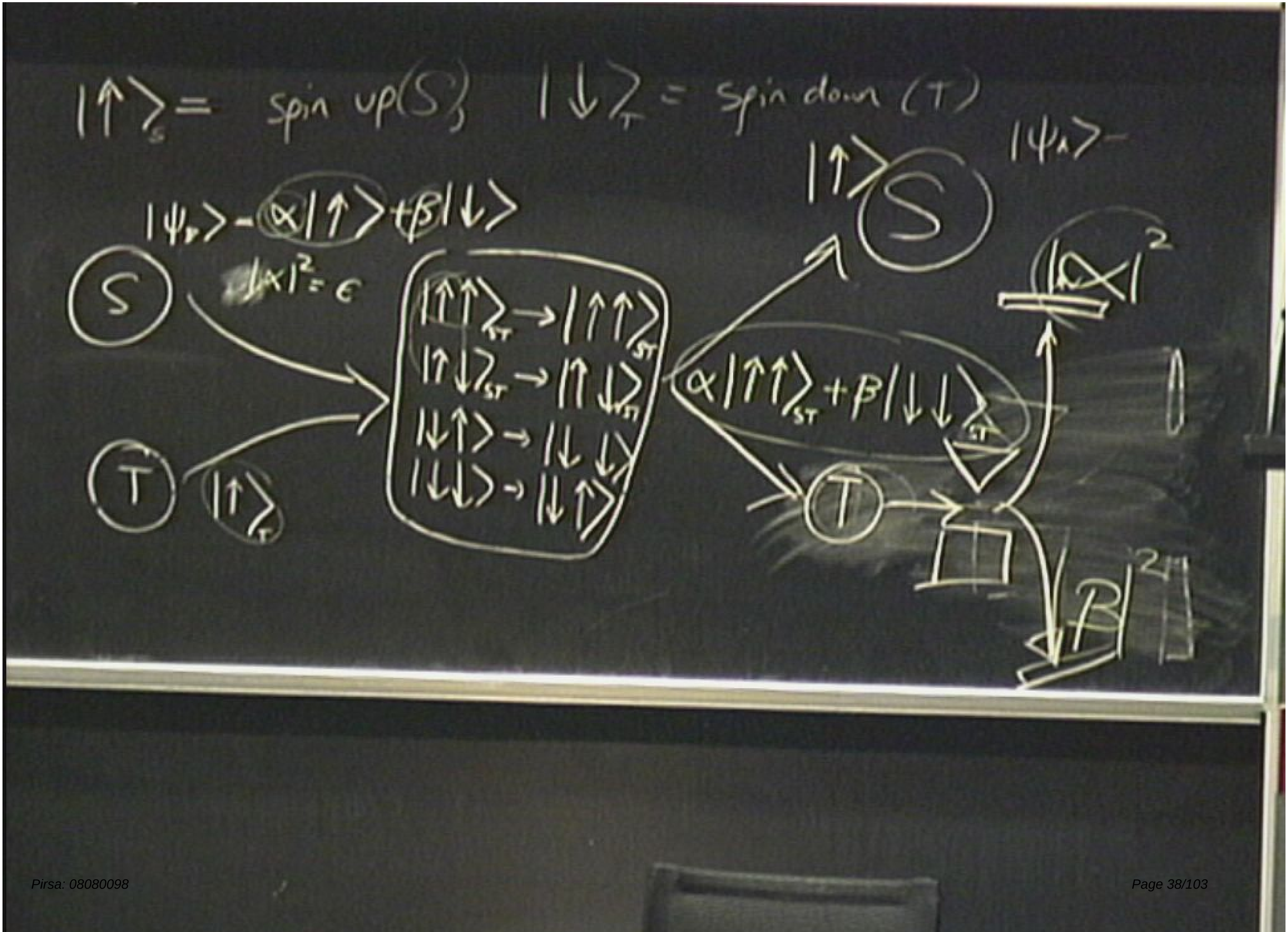
|↓⟩_T = spin down (T)
|ψ_A⟩
|ψ_P⟩ = α|↑⟩ + β|↓⟩
S
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST + β|↓↓⟩_ST
T
|↑⟩_T
α
β

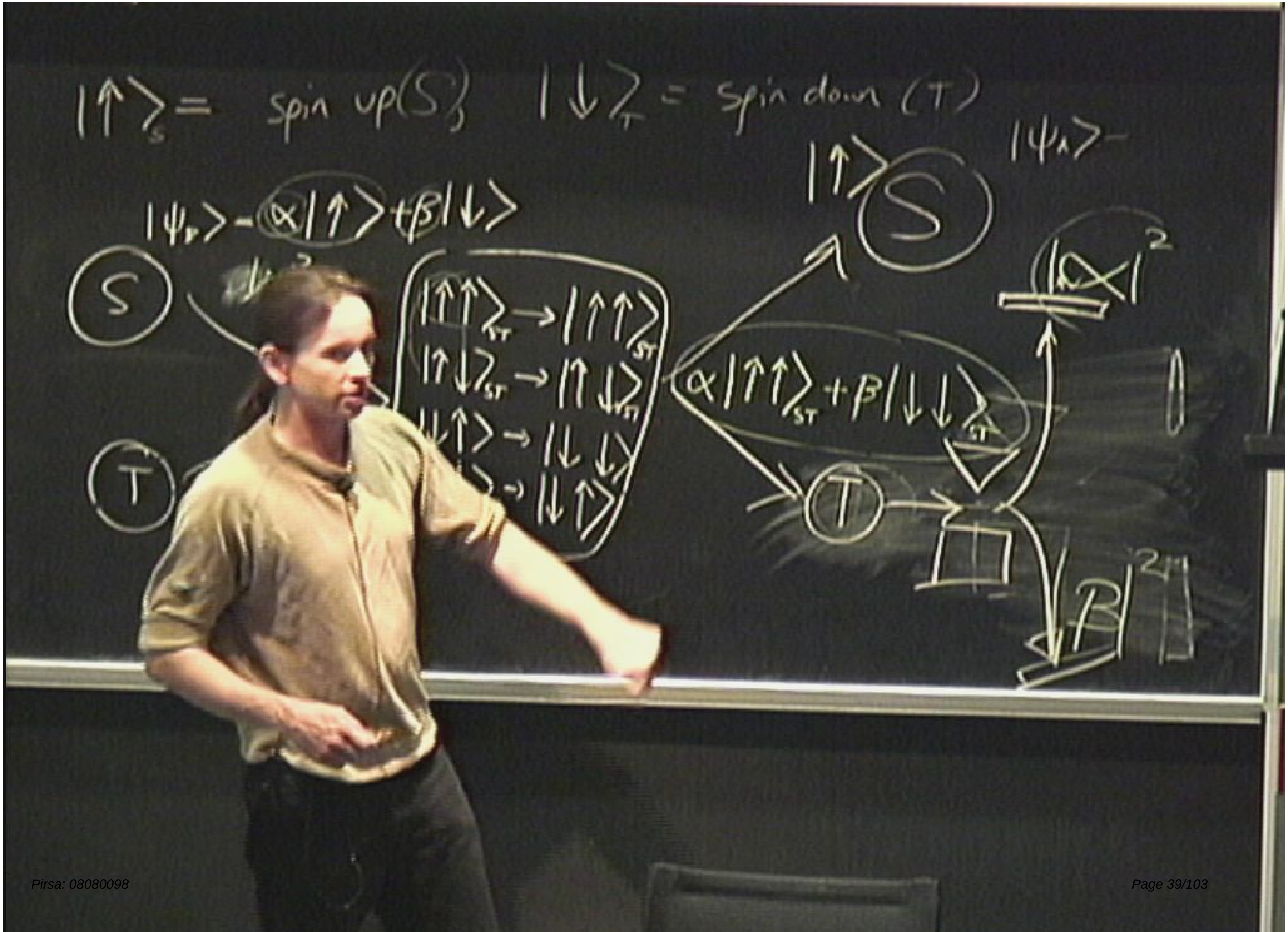

|↑⟩_S = spin up (S); |↓⟩_T = spin down (T)
|ψ_P⟩ = α|↑⟩ + β|↓⟩
|ψ_A⟩ =
|↑⟩ S
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST + β|↓↓⟩_ST
|↑⟩_T
T
α
β

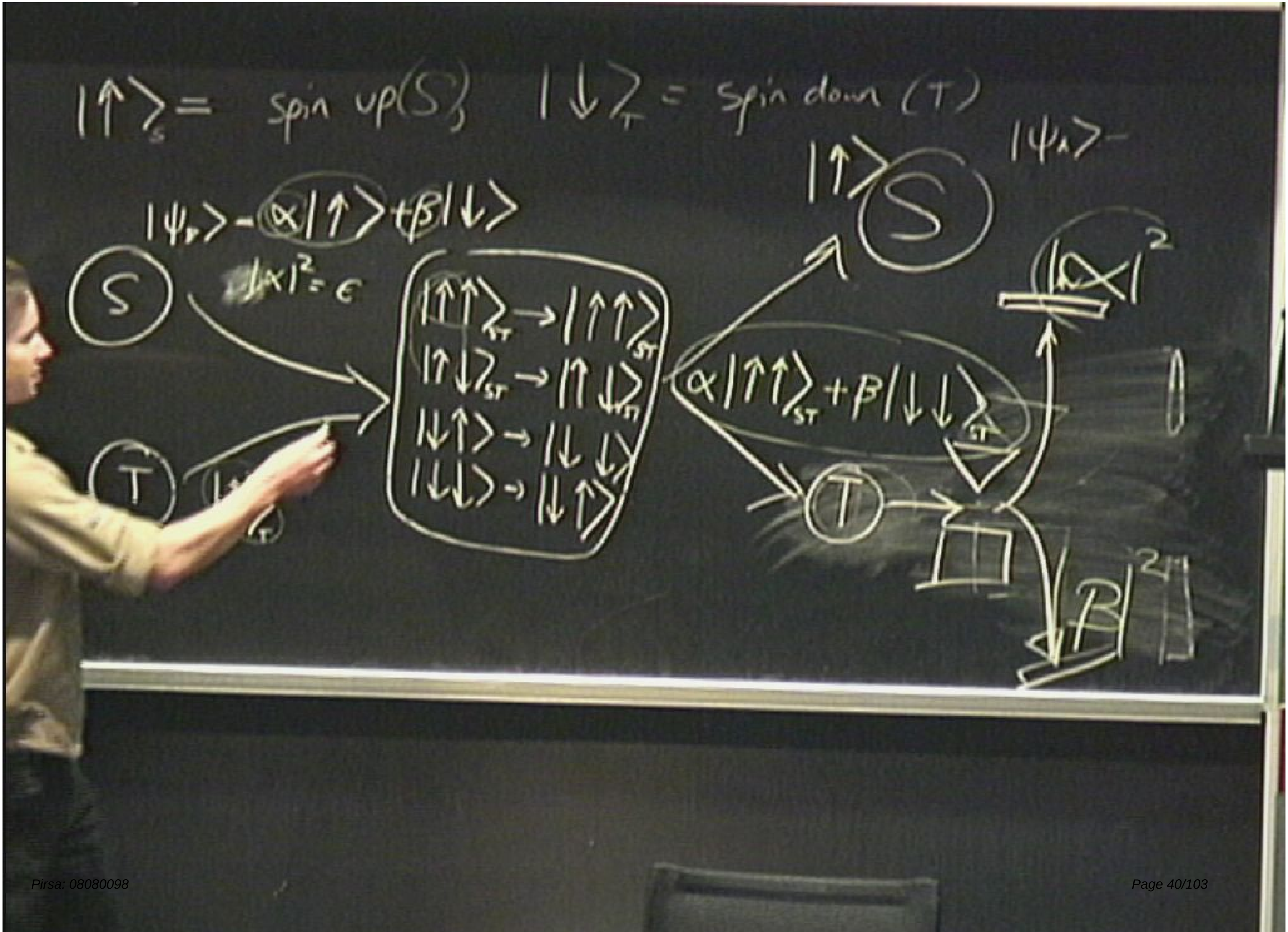


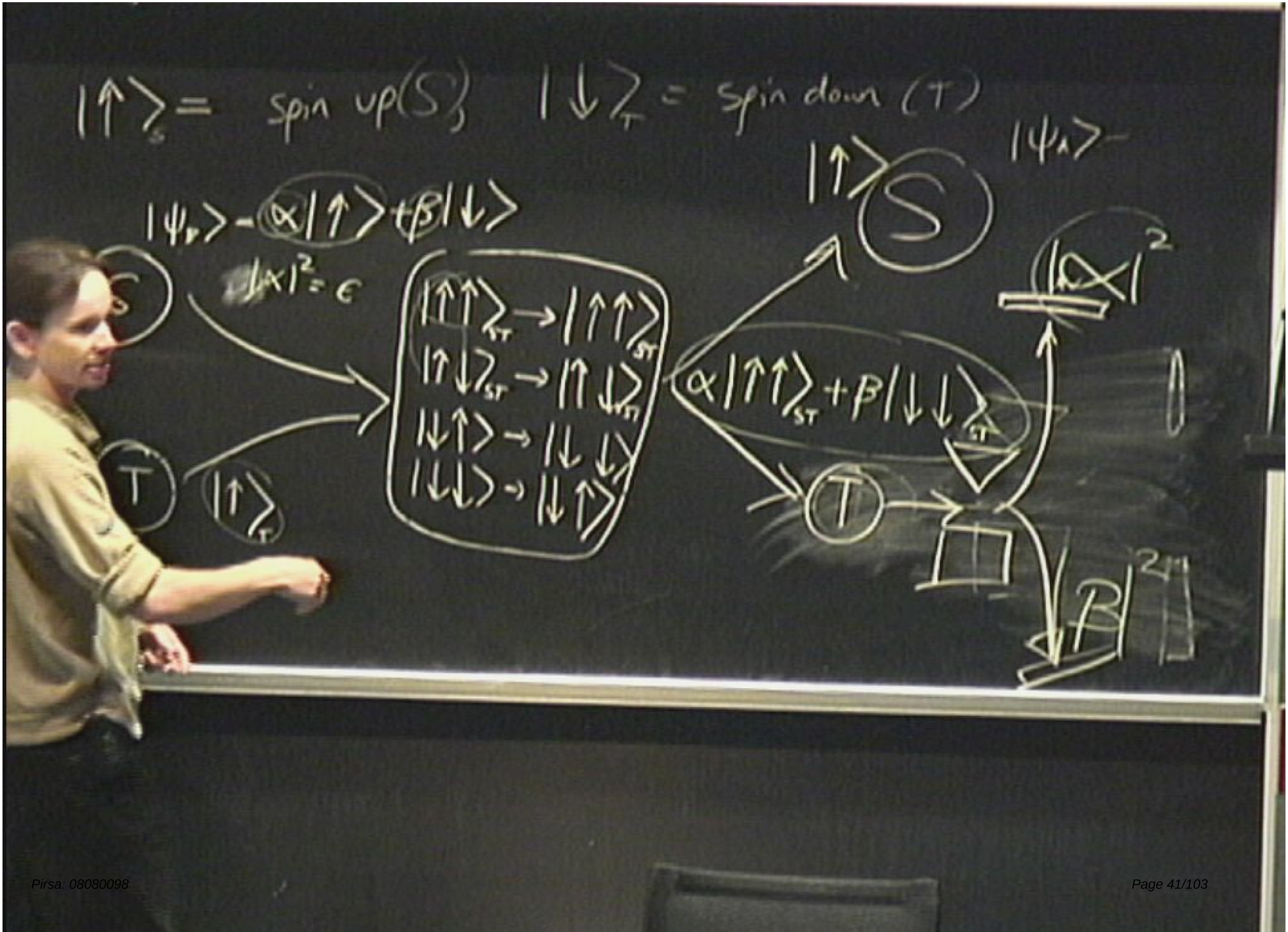
|↑⟩_S = spin up (S), |↓⟩_T = spin down (T)
|ψ_P⟩ = α|↑⟩ + β|↓⟩
|ψ_A⟩ =
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST + β|↓↓⟩_ST
|α|²
|β|²



|↑⟩_S = spin up (S); |↓⟩_T = spin down (T)
|ψ_P⟩ = α|↑⟩ + β|↓⟩
|α|² = ε
S
T
|↑⟩_T
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST + β|↓↓⟩_ST
|↑⟩
|ψ_A⟩ =
|α|²
|β|²


$|\uparrow\rangle_S$ = spin up (S)
$|\downarrow\rangle_T$ = spin down (T)
$|\psi_F\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$
$|\uparrow\uparrow\rangle_{ST} \rightarrow |\uparrow\uparrow\rangle_{ST}$
$|\uparrow\downarrow\rangle_{ST} \rightarrow |\uparrow\downarrow\rangle_{ST}$
$\alpha|\uparrow\uparrow\rangle_{ST} + \beta|\downarrow\downarrow\rangle_{ST}$
$|\alpha|^2$
$|\beta|^2$


|↑⟩_S = spin up (S); |↓⟩_T = spin down (T)
|ψ_P⟩ = α|↑⟩ + β|↓⟩
|α|² = ε
S
T
|↑↑⟩_ST → |↑↑⟩_ST
|↑↓⟩_ST → |↑↓⟩_ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
α|↑↑⟩_ST + β|↓↓⟩_ST
|↑⟩
|ψ_A⟩ =
|α|²
|β|²


|↑⟩S = spin up (S)
|↓⟩T = spin down (T)
|ψ⟩ = α|↑⟩ + β|↓⟩
|α|² = ε
S
T
|↑⟩T
|↑↑⟩ST → |↑↑⟩ST
|↑↓⟩ST → |↑↓⟩ST
|↓↑⟩ → |↓↓⟩
|↓↓⟩ → |↓↑⟩
|↑⟩ S
|ψ⟩
α|↑↑⟩ST + β|↓↓⟩ST
T
|α|²
|β|²

Measurements = questions

Measurements are questions asked about the system S:

Interact S with T. Observe the state of T. What state will S be in afterwards?

(i.e. What does S do when we kick it?)

The number of possible answers to these questions depends only on how many different possible states T can be found in, not on S directly.

What kind of measurements?

We tend to think of properties like 'position' and 'momentum' as the simple, fundamental building blocks of the world.

Information-wise, a position measurement is a huge undertaking.

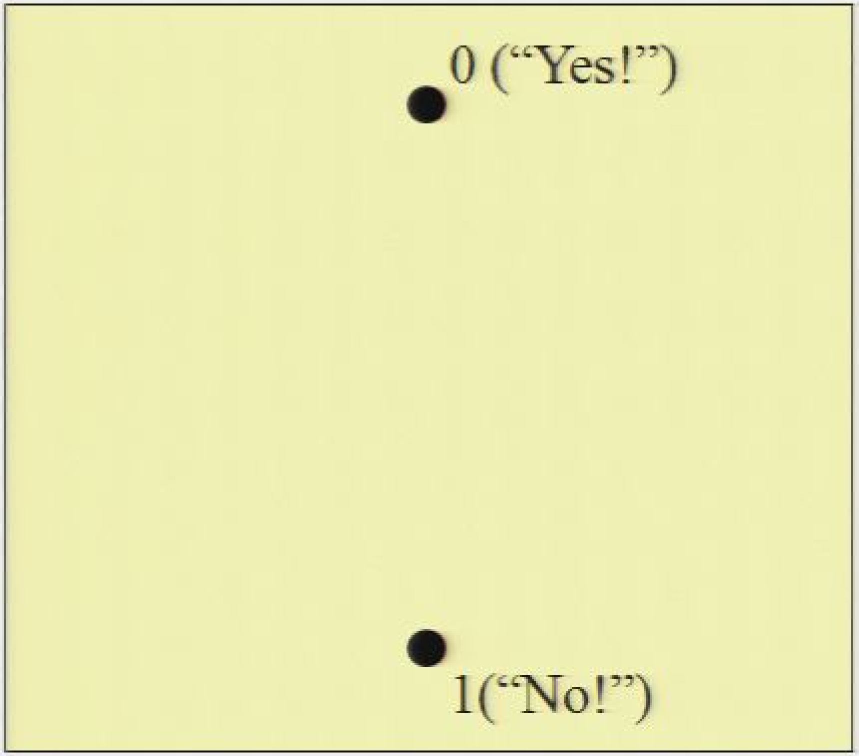
With better and better equipment, position measurements yield *arbitrarily large* amounts of information.

43 N

80 W

90 x 2 x 180 x 2 = 64800 possible states

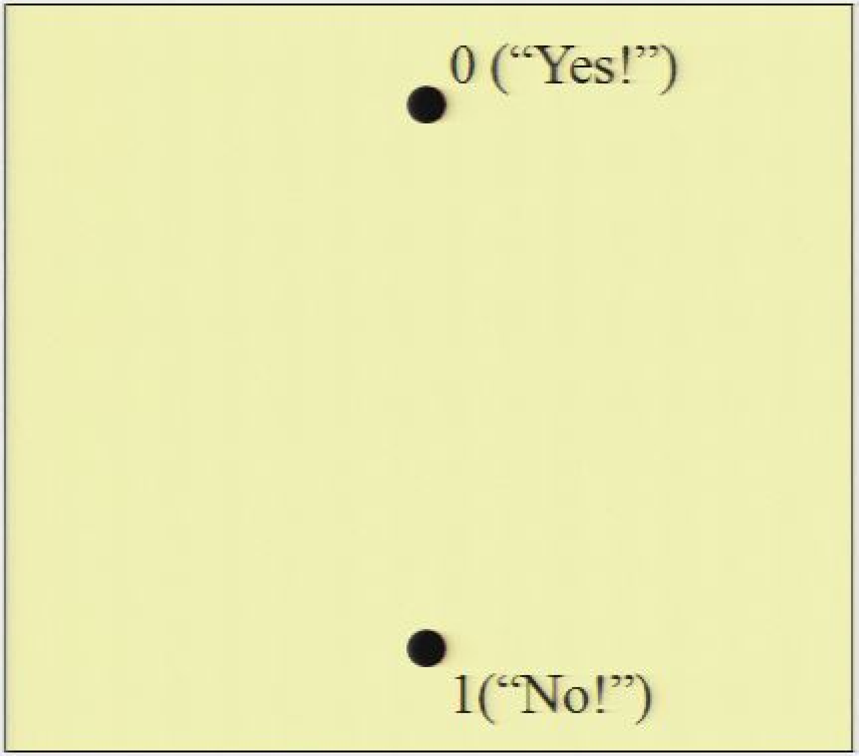
Bits and Qubits



In a classical worldview, we think of the world as being in one of two possible states, each corresponding to the possible answers “Yes” and “No”.

Possible answers:	2
Possible states:	2

Bits and Qubits



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Possible answers:	2
Possible states:	2

Bits and Qubits

Quantum mechanics does things differently.

Still the same two possible answers to the simplest questions:

"**0**" and "**1**".

But a quantum system can exist in any complex superposition of these possibilities:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

There are infinitely many of these states!

Possible answers:	2
Possible states:	*lots*

Bits and Qubits

The quantum states of the simplest system form the surface of a sphere.

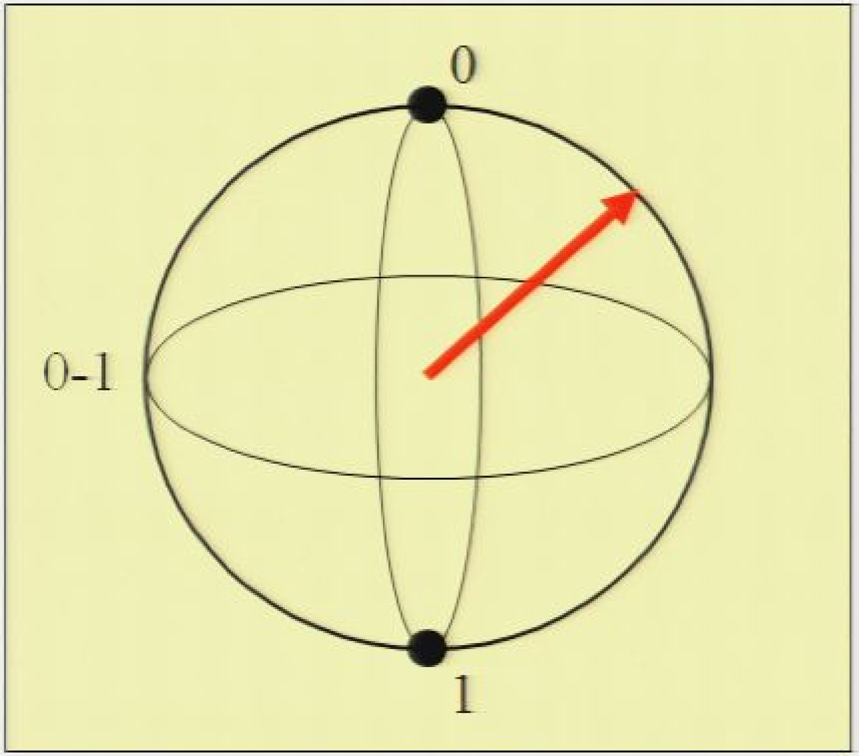
We call $|0\rangle$ and $|1\rangle$ *basis vectors* for the system.

There is nothing special about the points $|0\rangle$ **and** $|1\rangle$ **.**

Different measurements are possible as well. **We can also ask " Is the state** $|0\rangle + |1\rangle$ **or is it** $|0\rangle - |1\rangle$ **? "**

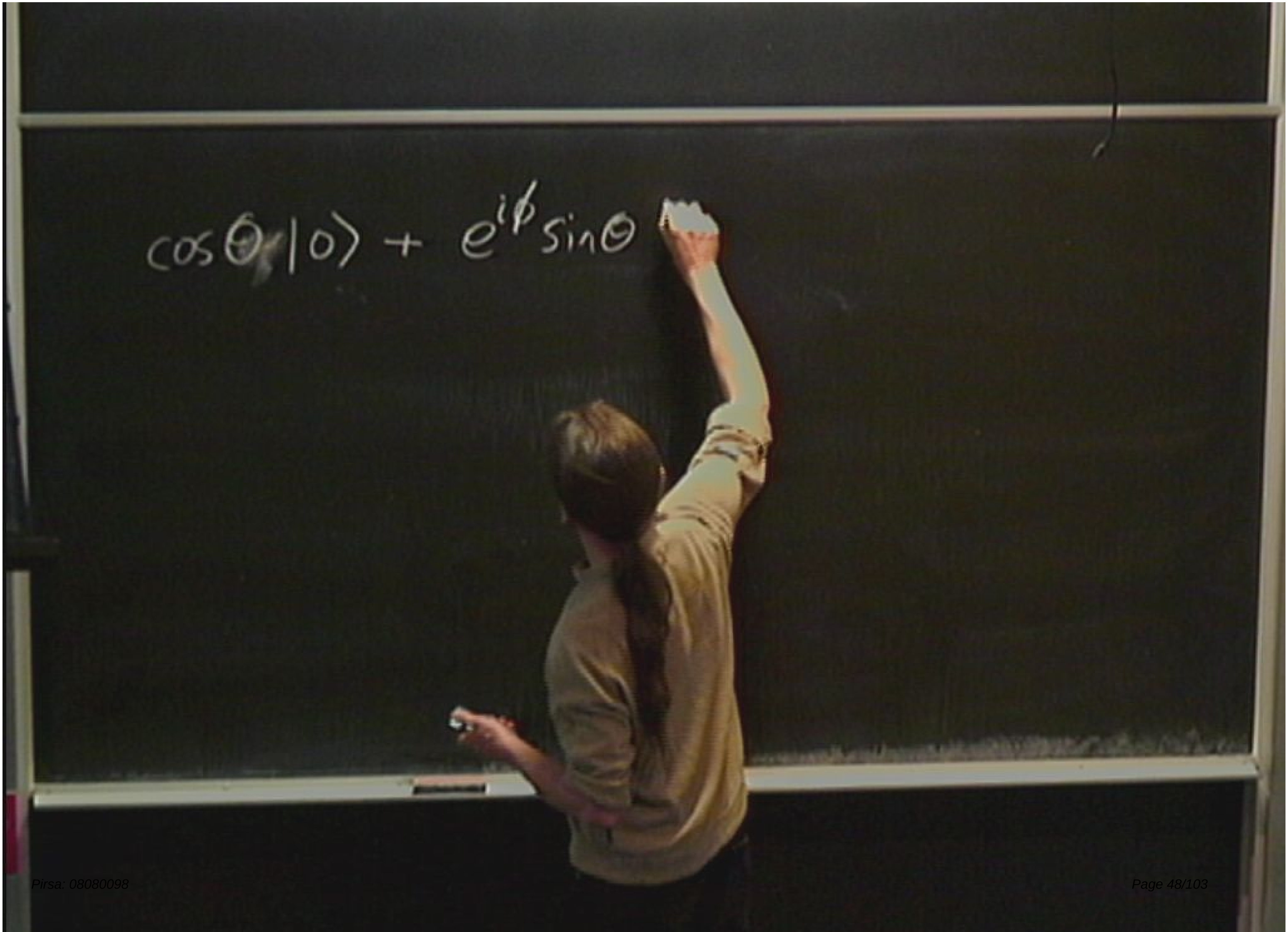
But we have to ask a binary question!

We can't learn α and β directly.



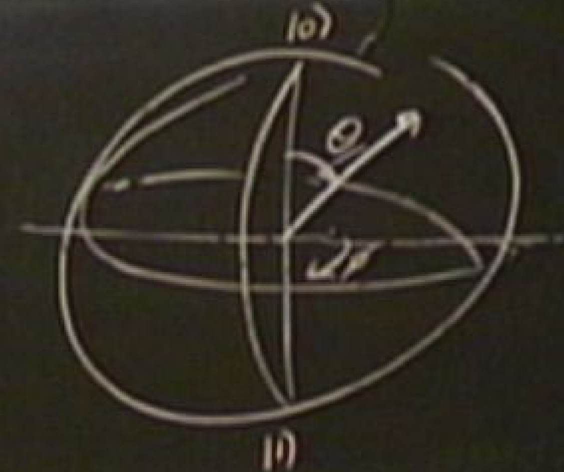
Qubit

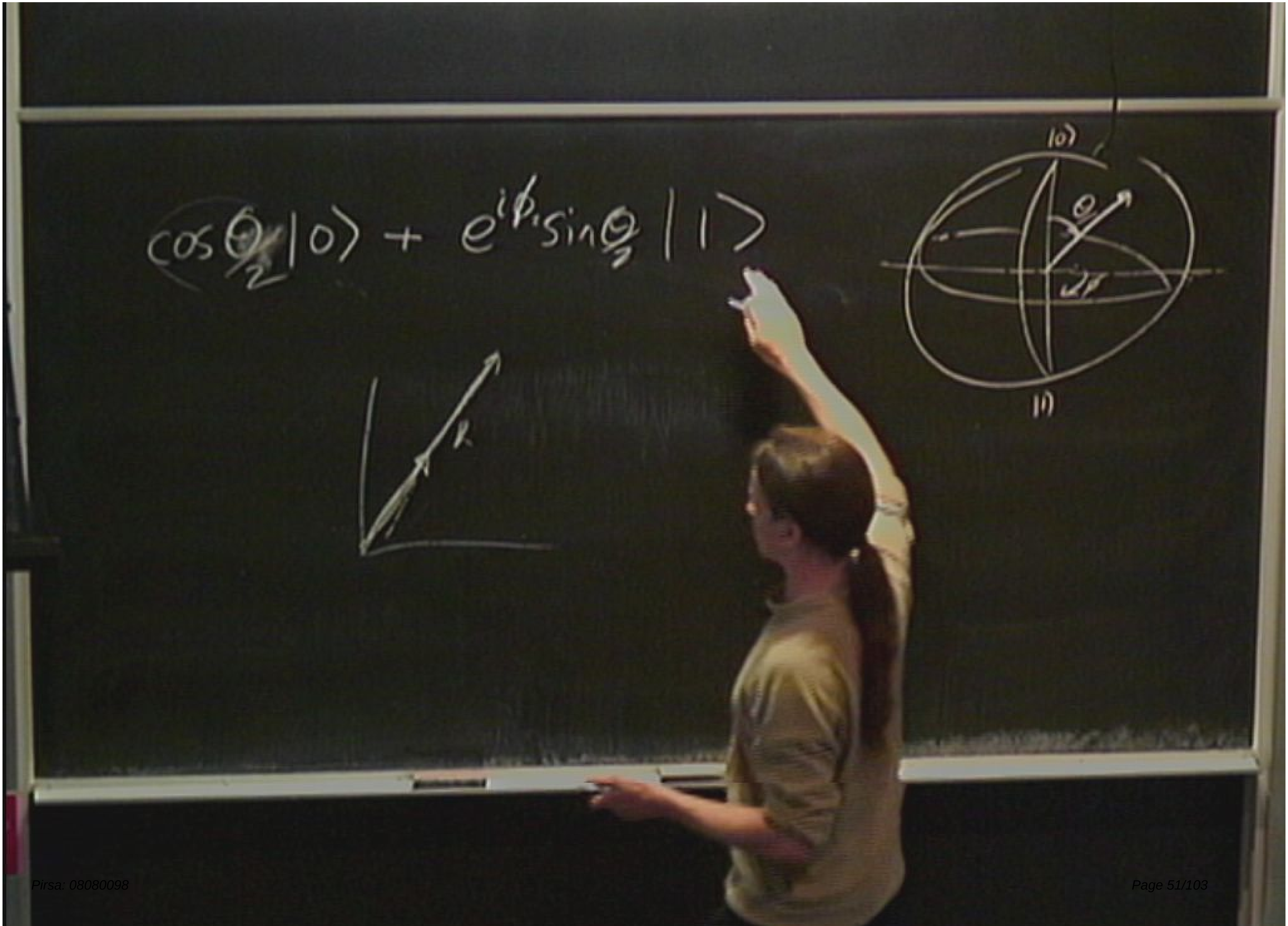
$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$


$\cos\theta\,|0\rangle + e^{i\phi}\sin\theta$

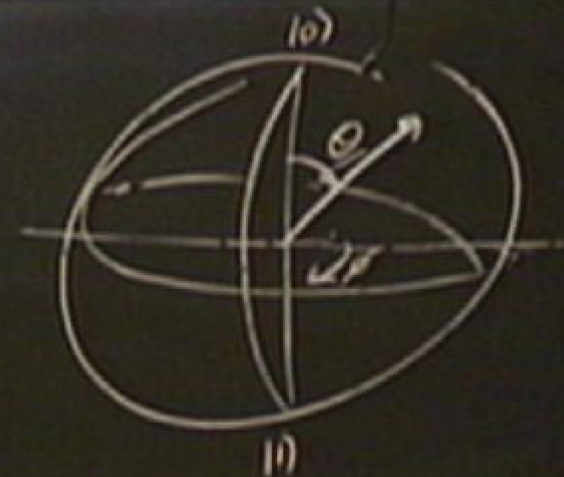
$$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$




$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$
θ
ϕ
$|1\rangle$
R

$$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



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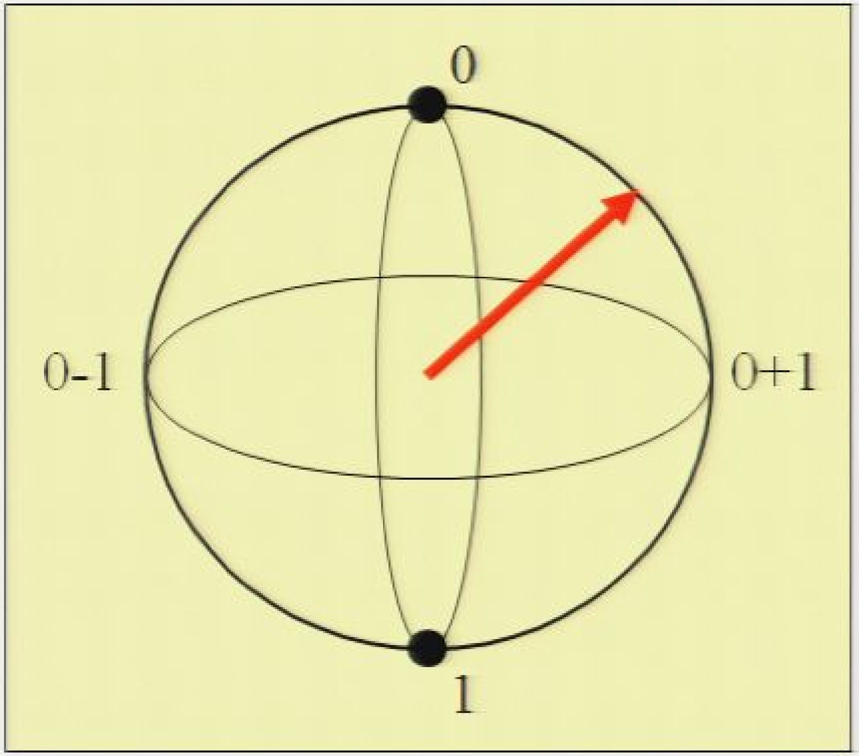

Qubit

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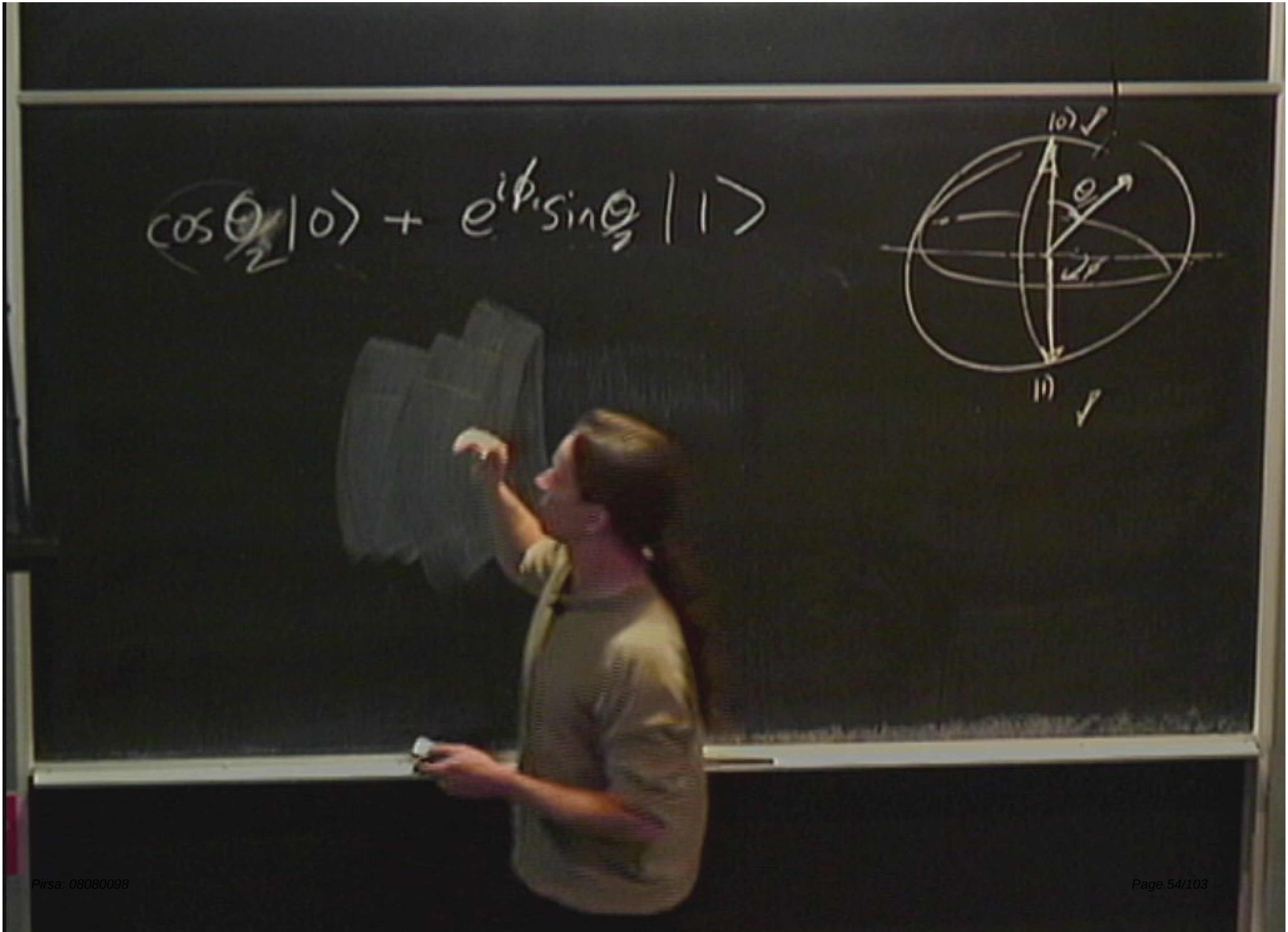
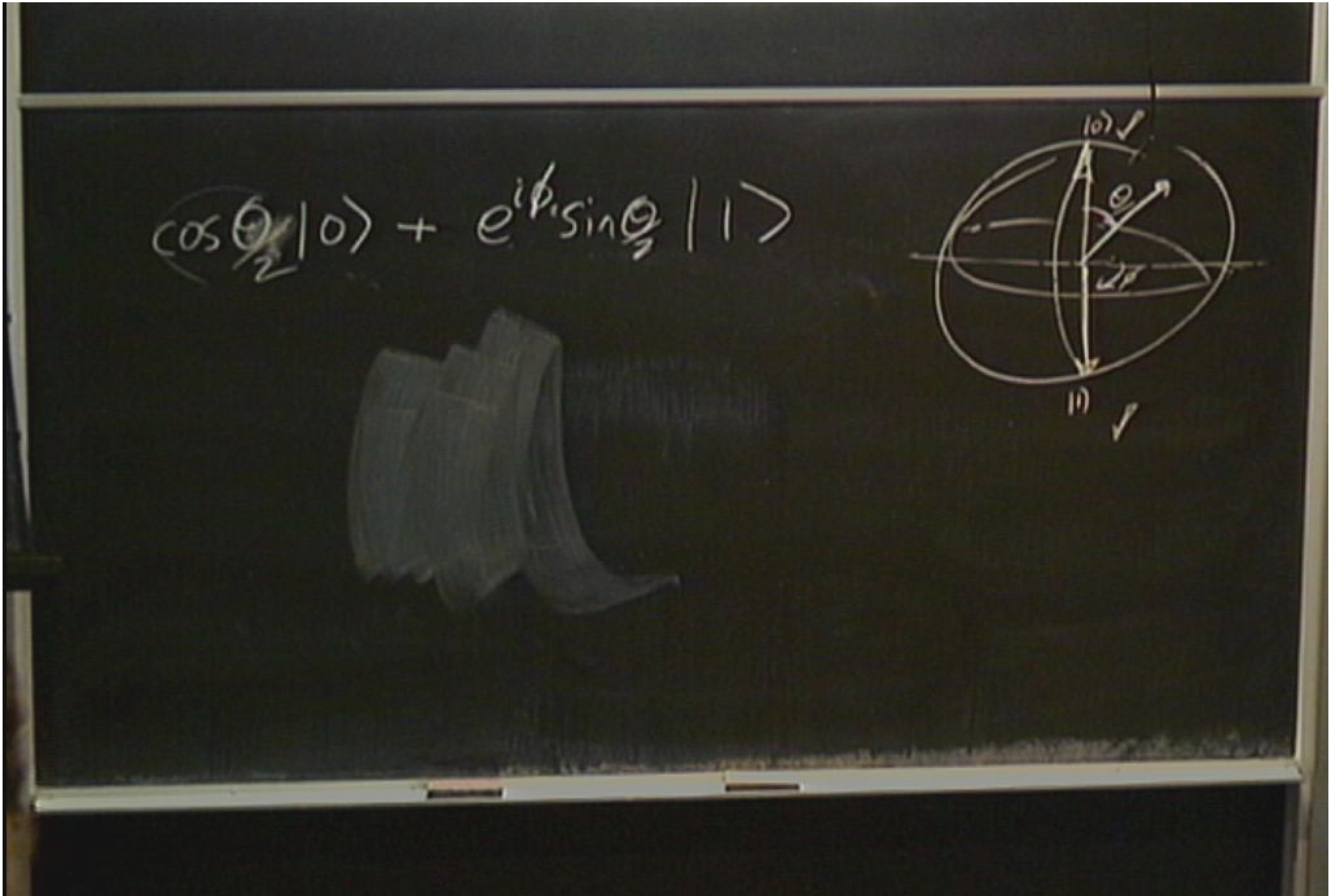

$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$
θ
ϕ
$|1\rangle$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ϕ
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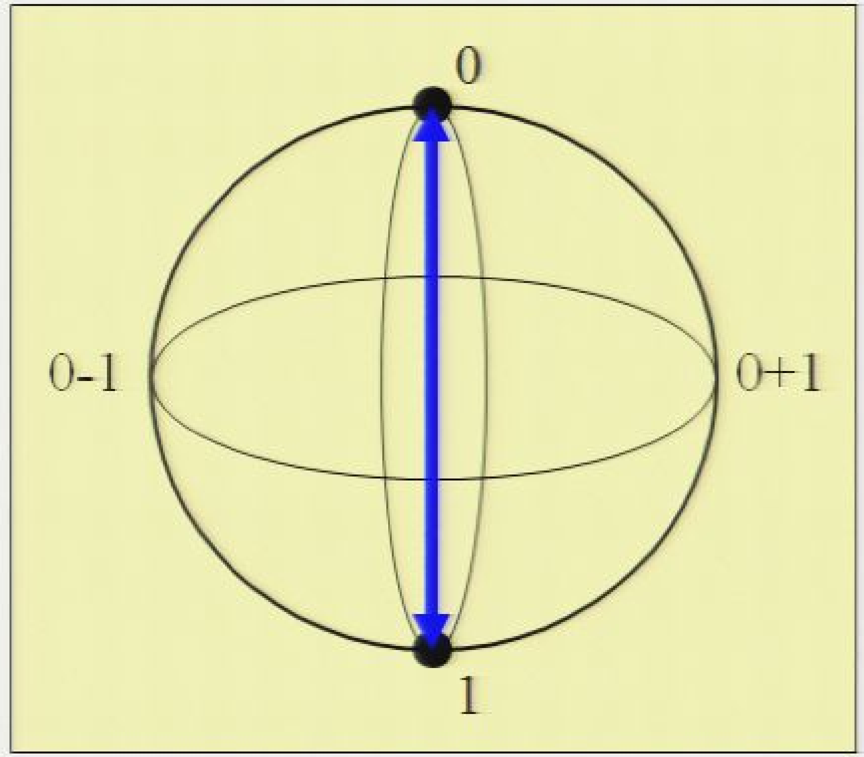
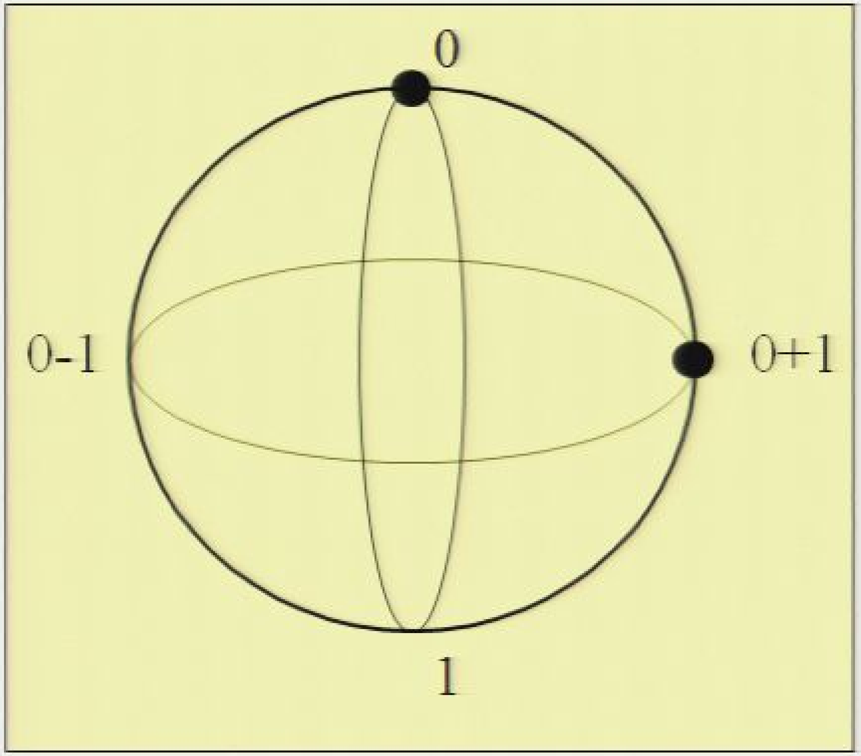


Qubit

Uncertainty and nonorthogonality

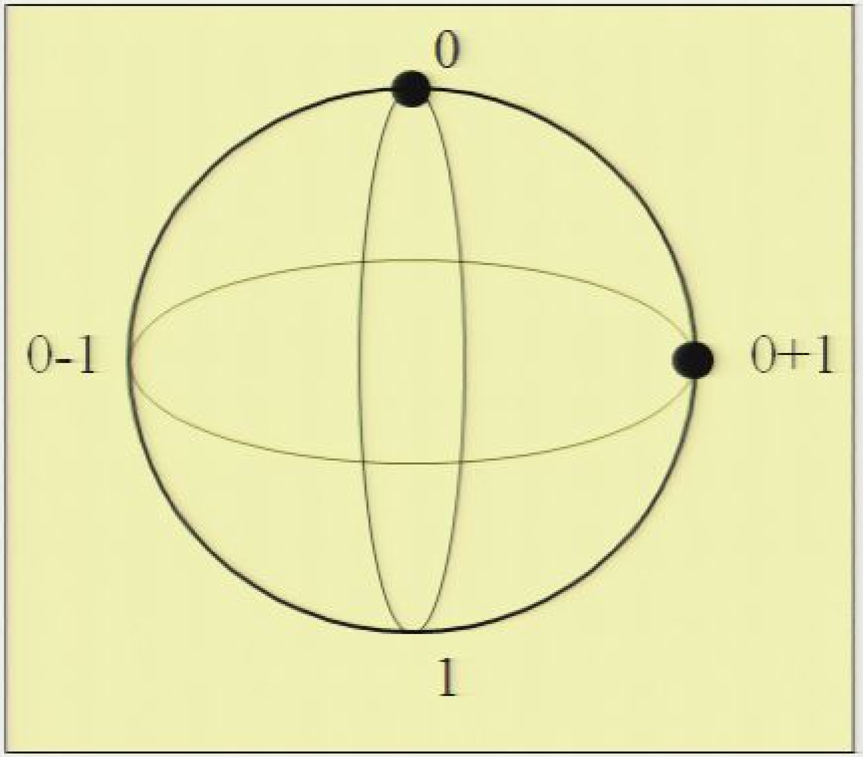


This is one way to approach Heisenberg's uncertainty principle.

Having a definite answer to one question (e.g. "0 or 1?") necessarily entails having a maximally uncertain answer to other questions (e.g. "0+1 or 0-1?").

$$|0\rangle \qquad \qquad \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Uncertainty and nonorthogonality



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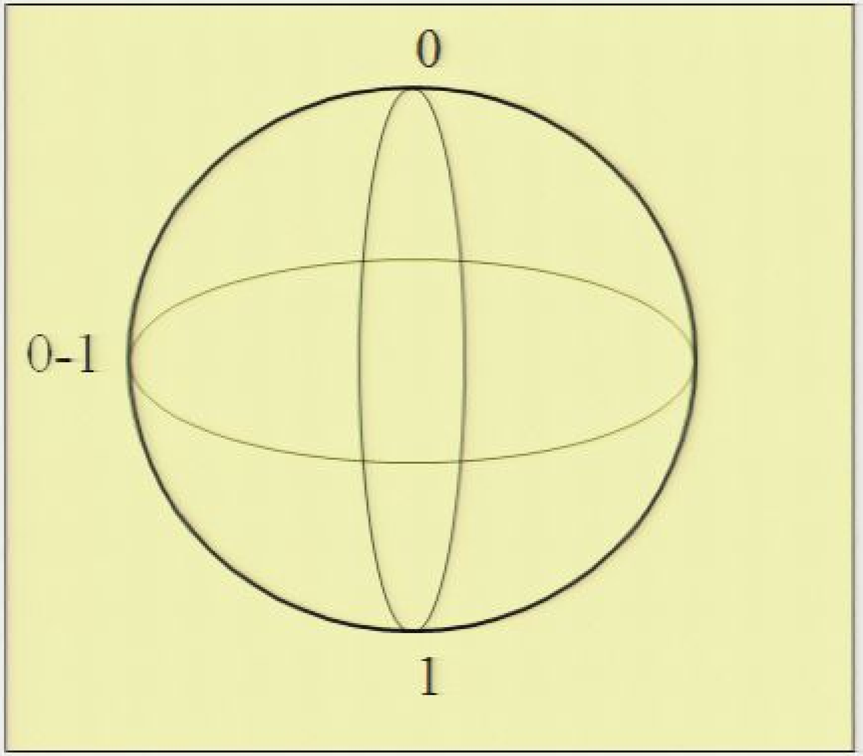
No cloning of quantum information



Qubits cannot be cloned.

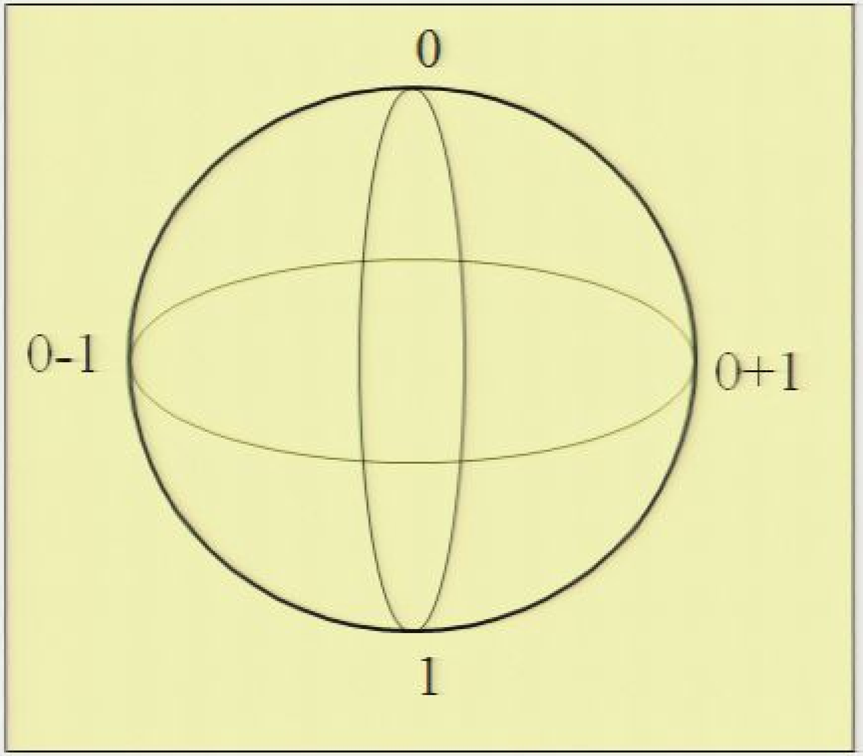
If they could, we could measure the quantum state directly by producing lots of clones and asking different questions of all of them.

So cloning of quantum information would imply measurement without disturbance, and would violate the uncertainty principle.

How much information is there in the simplest quantum system?

- It can reveal only one bit of classical data… but…
- Its state requires an infinite number of bits to specify.

No cloning of quantum information



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- Its state requires an infinite number of bits to specify.

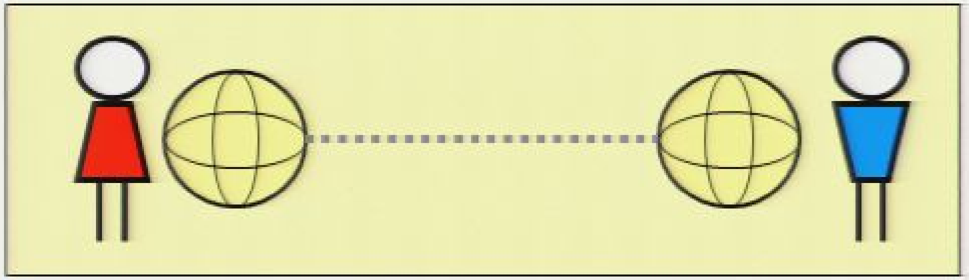
Quantum information

So simply by considering how information behaves, without even considering *what the information is about,* we've got the some quantum weirdness:

nonorthogonality, no cloning, uncertainty

In fact, thinking about information will get us the full range of distinctly quantum phenomena, including entanglement and nonlocality.

Entanglement



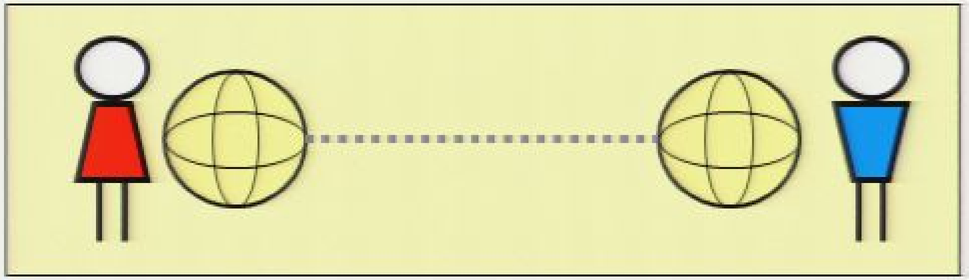
Consider not one, but *two* elementary quantum systems.

Each system really corresponds to some possible binary question we ask of reality. For example, these could be two atoms, in two different places, and of each atom we could ask "are you in the ground state or the exited state?"

There are four possible answers:

- •A and B are both in the ground state.
- •A is in the excited state and B is in the ground state.
- •A is in the ground state B is in the excited state.
- •A and B are both in the the excited state.

Entanglement



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There are four possible answers:

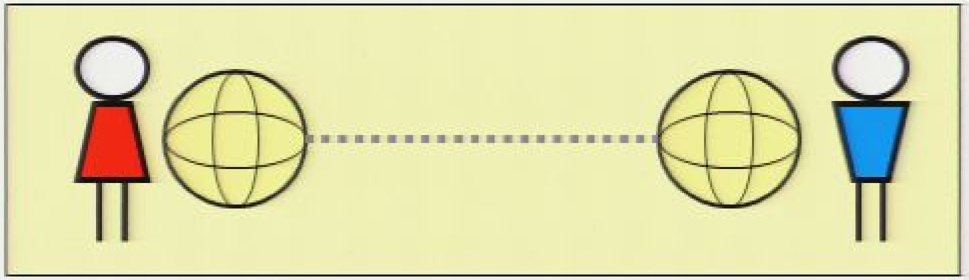
$$|\psi_1\rangle = |0\rangle_A \; |0\rangle_B$$

$$|\psi_2\rangle = |1\rangle_A \; |0\rangle_B$$

$$|\psi_3\rangle = |0\rangle_A \; |1\rangle_B$$

$$|\psi_4\rangle = |1\rangle_A \; |1\rangle_B$$

Entanglement



We can apply the superposition principle exactly as before, but now the space of possible states has *four* complex dimensions.

Possible state:

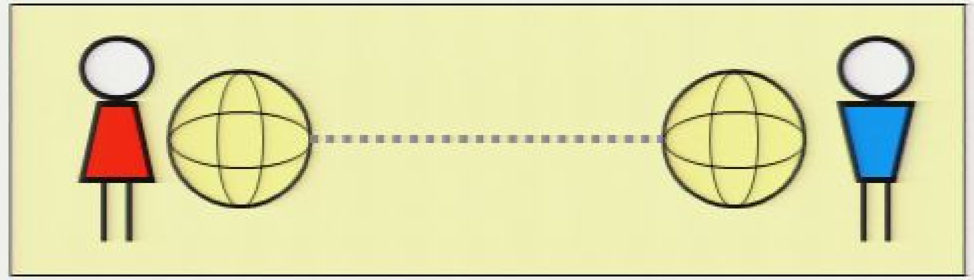
$$\alpha|0\rangle_A|0\rangle_B + \beta|0\rangle_A|1\rangle_B + \gamma|1\rangle_A|0\rangle_B + \delta|1\rangle_A|1\rangle_B$$

Consider the state:

$$\frac{1}{2}|0\rangle_A|0\rangle_B + \frac{1}{2}|0\rangle_A|1\rangle_B + \frac{1}{2}|1\rangle_A|0\rangle_B + \frac{1}{2}|1\rangle_A|1\rangle_B$$

There is an equal chance of all four results. Seeing one atom in a given state yields no information about the other atom at all. The two systems are not at all entangled.

Entanglement



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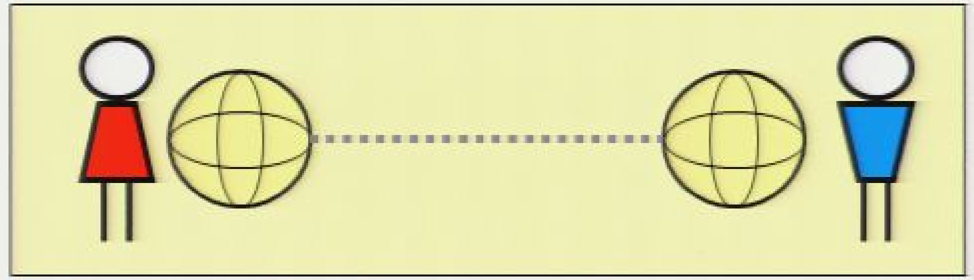
Consider the state:

$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

The two atoms are correlated. They are either both in the ground state, or both in the excited state. When measuring their energy, we will never see them in different states.

Entanglement



$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

This kind of correlation isn't too impressive at first. Many objects seem to be able to do this. For example, socks:

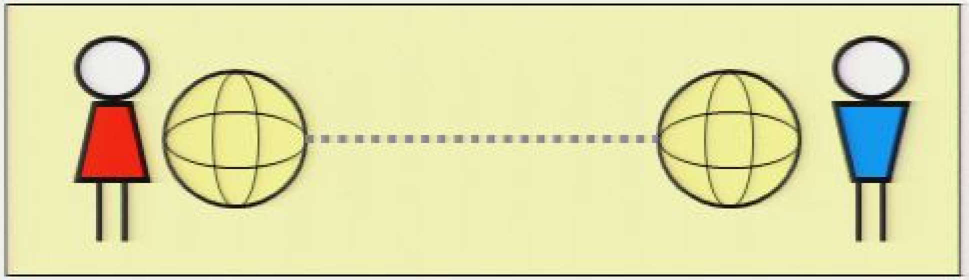
Probability (1/2) = = | 0 > | 0 >

Probability (1/2) = = | 1 > | 1 >

And this is true, so long as we stick to asking the same classical question "Is it **0** or is it **1** ? "

But what if we ask a different question?

Entanglement



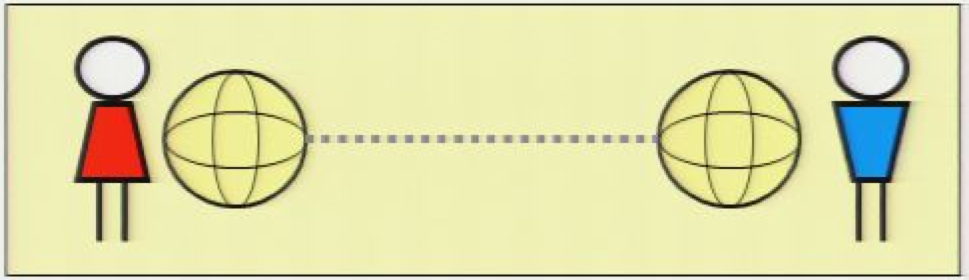
$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

According to quantum mechanics, we should be able to ask each system

"Are you in state $|0\rangle + |1\rangle$ or in state $|0\rangle - |1\rangle$? "

What happens then?

Entanglement



$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

If we have a classical mixture, the correlations are lost when we ask our system a different question.

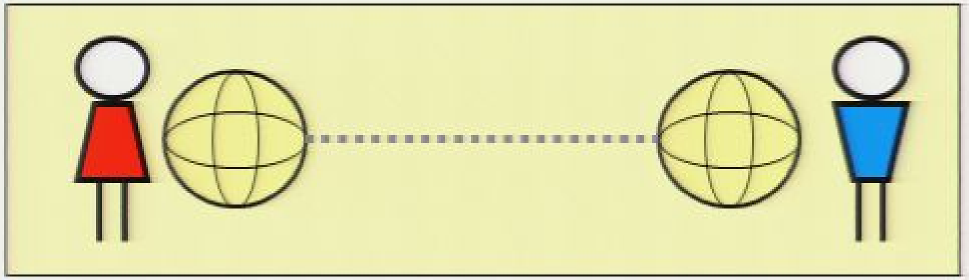
With an entangled state, the correlations are so strong that they exist *for all possible questions we might ask.*

Whatever question we ask system A, and whatever the result, system B will always behave as if it were in exactly the same state.

But B couldn't possibly know what measurement we performed on A. How does B know whether to act like $|0\rangle$ or act like $|0\rangle + |1\rangle$?

But how does one system seem to know what question we asked the other?

Randomness



$$\frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

Despite the very strong correlations, no information can be transmitted.

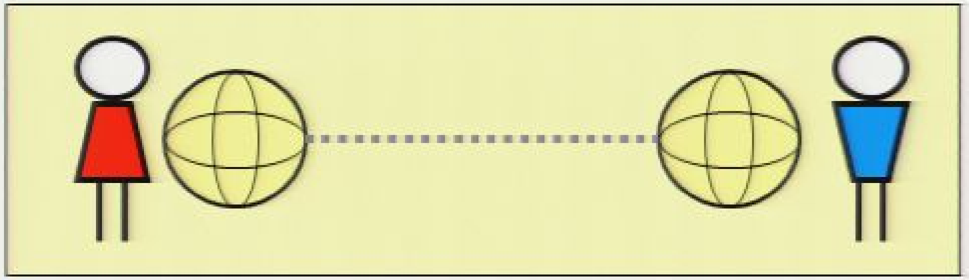
Alice's measurement produces a *completely random* result.

Although she now knows what Bob's system will do next, as far as Bob is concerned his system is behaving completely randomly too.

The correlations only appear when they compare notes later.

So relativity survives!

Entanglement



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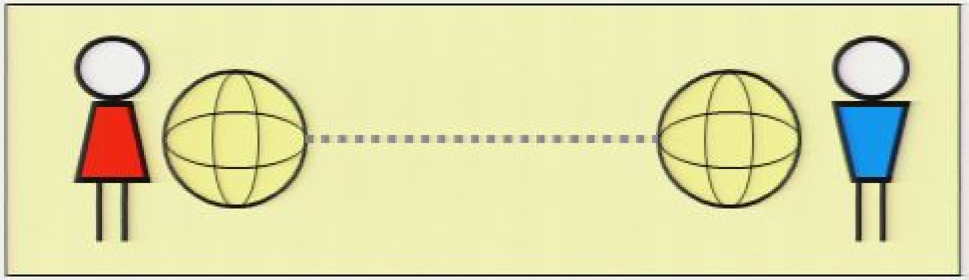

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Quantum teleportation

Teleportation shows that quantum physics is all about information.

The state of a physical system and the information it contains are one and the same thing.

But…

Is quantum teleportation at all interesting?

Classical teleportation is trivially easy!

Telephones.

Some classical devices are even more powerful:

Xerox machines.

Why should we care that with quantum theory, we can (with great difficulty) build fax machines that self destruct after one use?

Quantum teleportation

Why should we care that with quantum theory, we can (with great difficulty) build fax machines that self destruct after one use?

A couple of reasons:

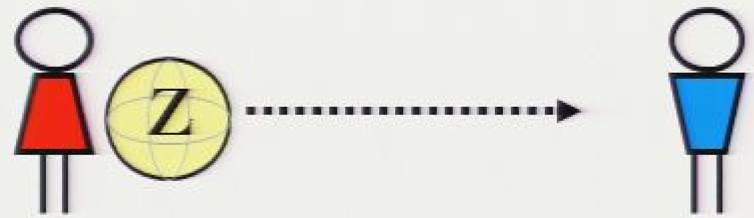
1. The world is made of quantum stuff, not classical stuff, so the quantum rules are the ones that count.

2. Classically, information is not identity. Multiple copies of a newspaper are not all the same object – they are different objects carrying the same information.

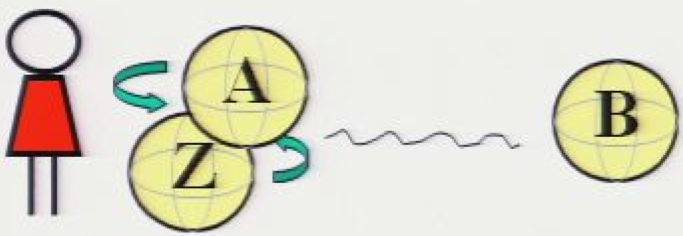
   Quantum mechanically, the issue is more subtle. Particles are identical, and their information (in other words, how they're going to behave) *cannot* be copied. So what matters most? Who you are, or what you do?

   Does it even make sense to ask who you are independently of what you do?

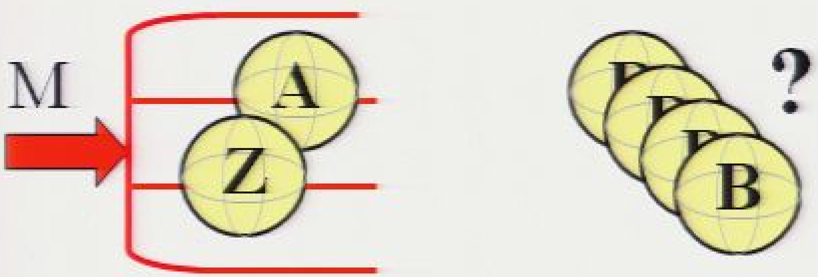
Quantum teleportation



Alice and Bob share a maximally entangled state of qubits **A** and **B**.

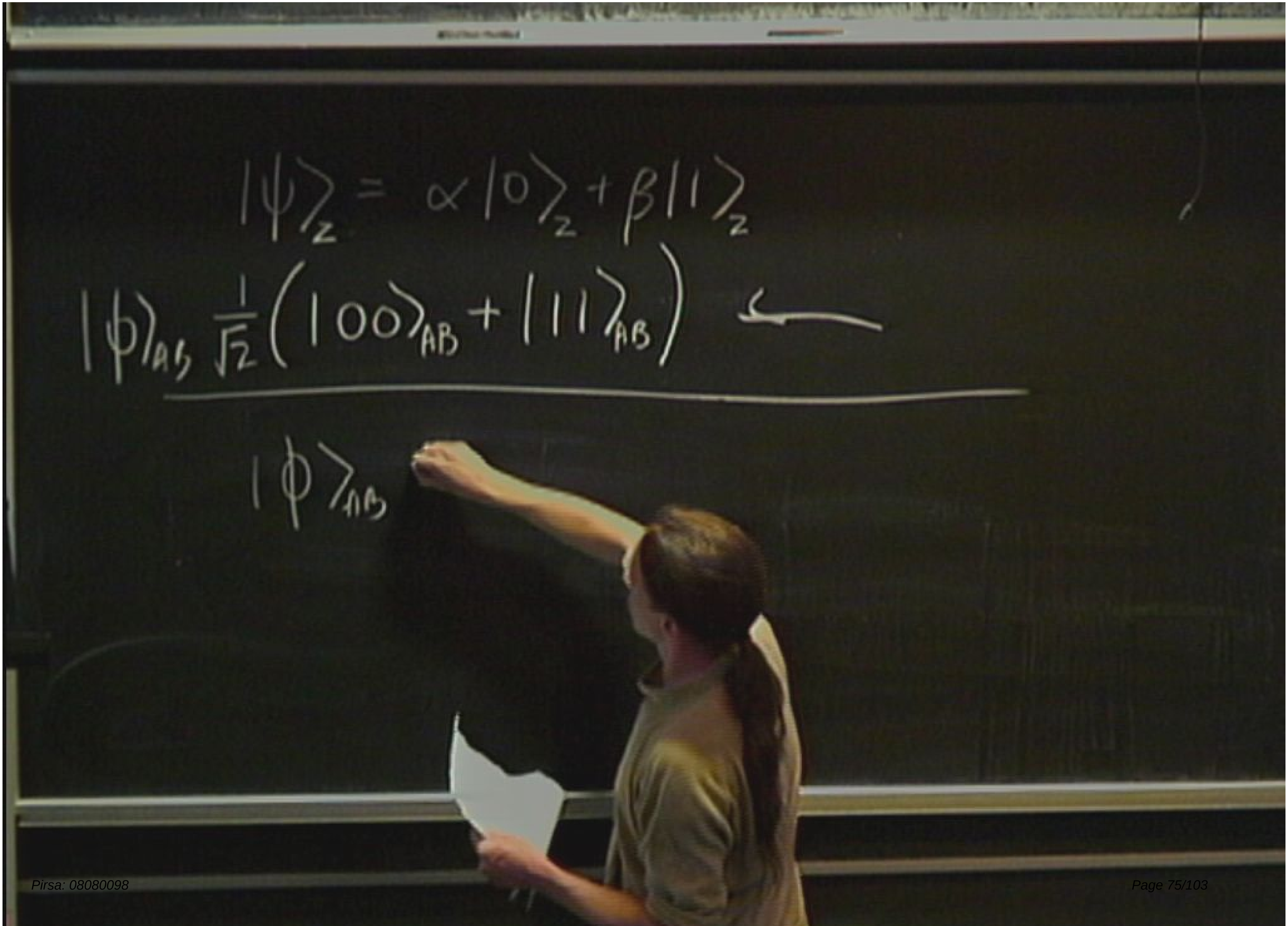


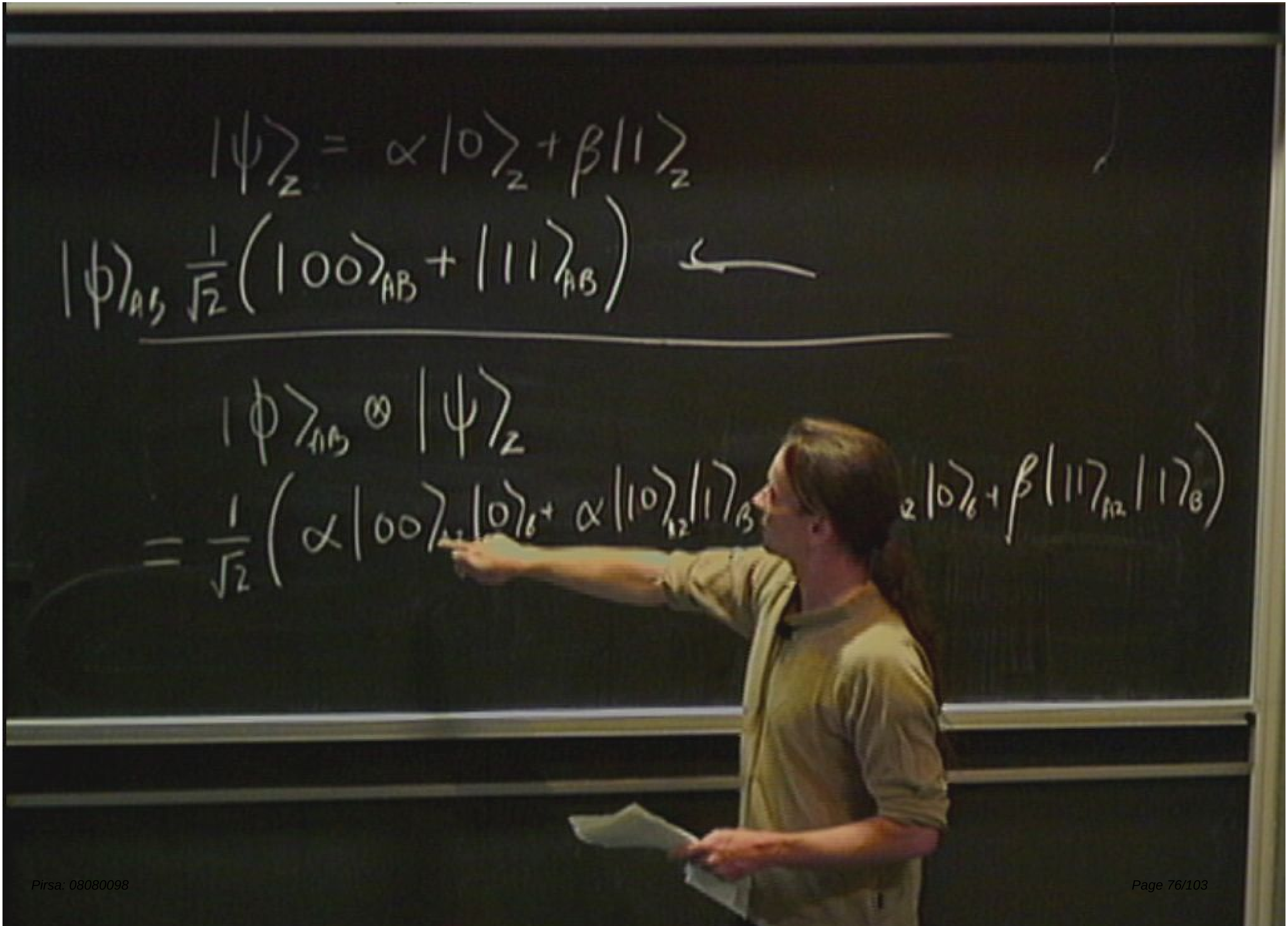
Alice interacts **A** with the qubit she wants to send Bob, **Z**.

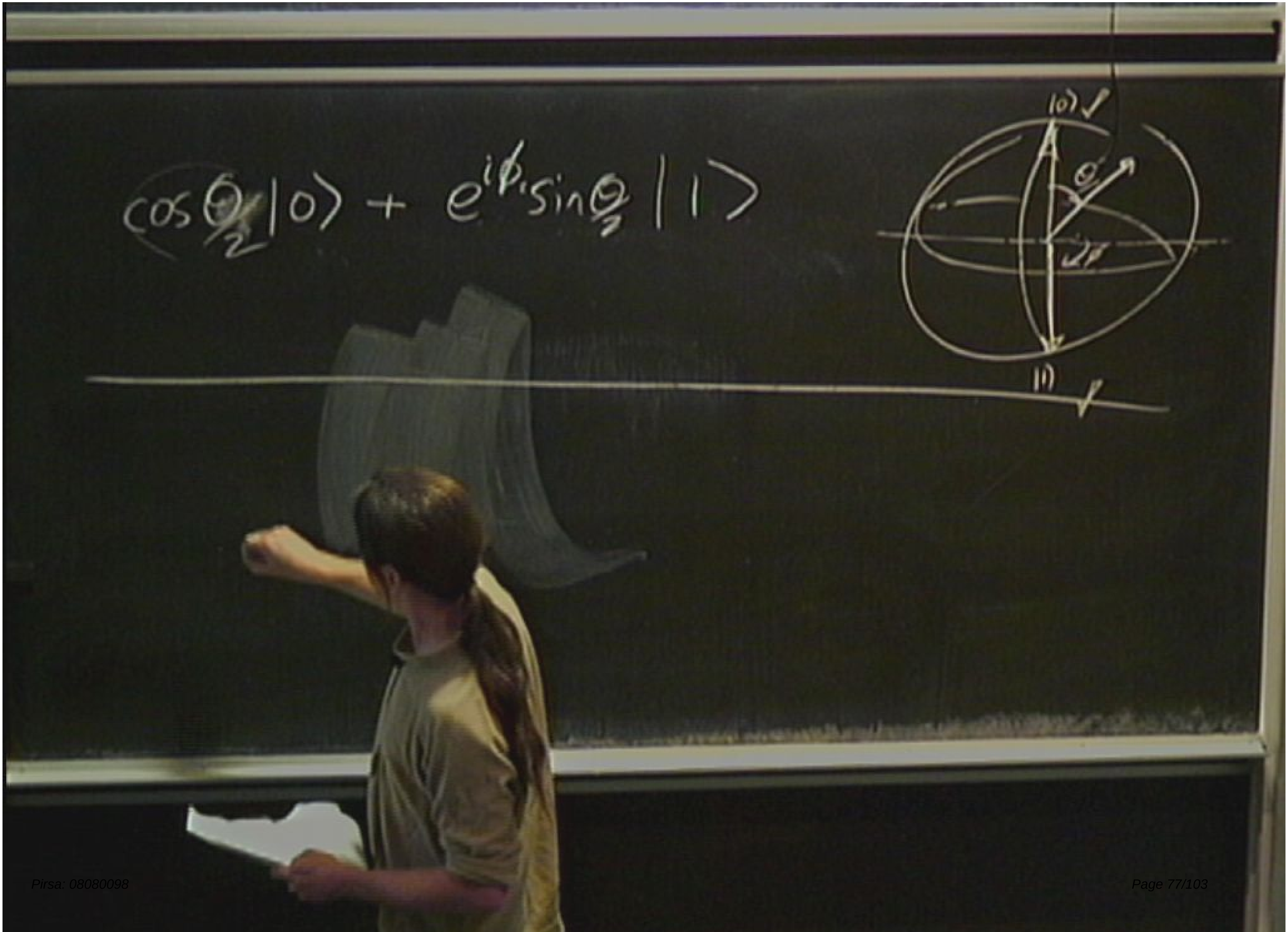


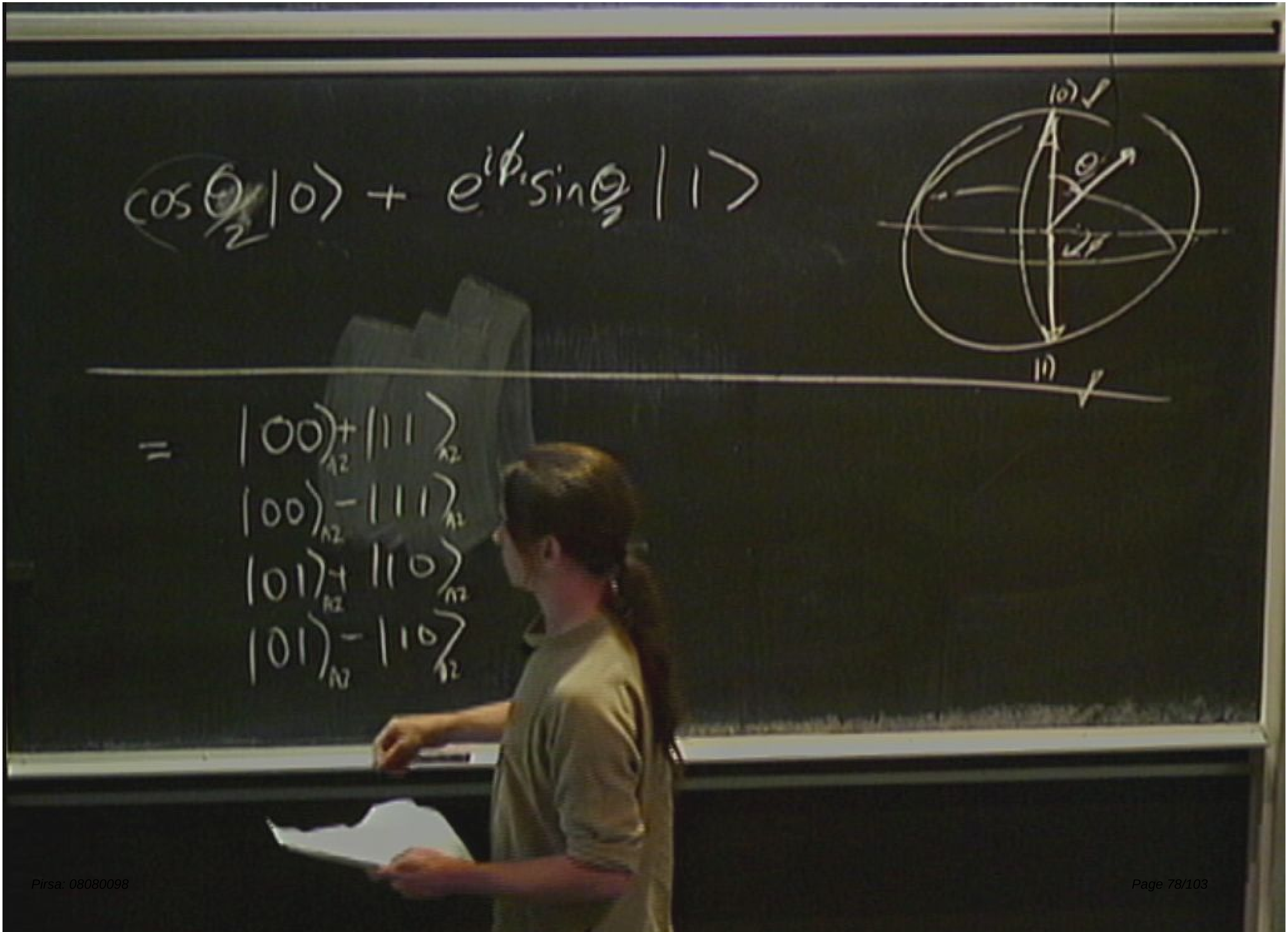
Alice measures the larger **AZ** system, allowing four outcomes. The entanglement with Bob is lost, and his system collapses into one of four possible states. **A** and **Z** become entangled instead.

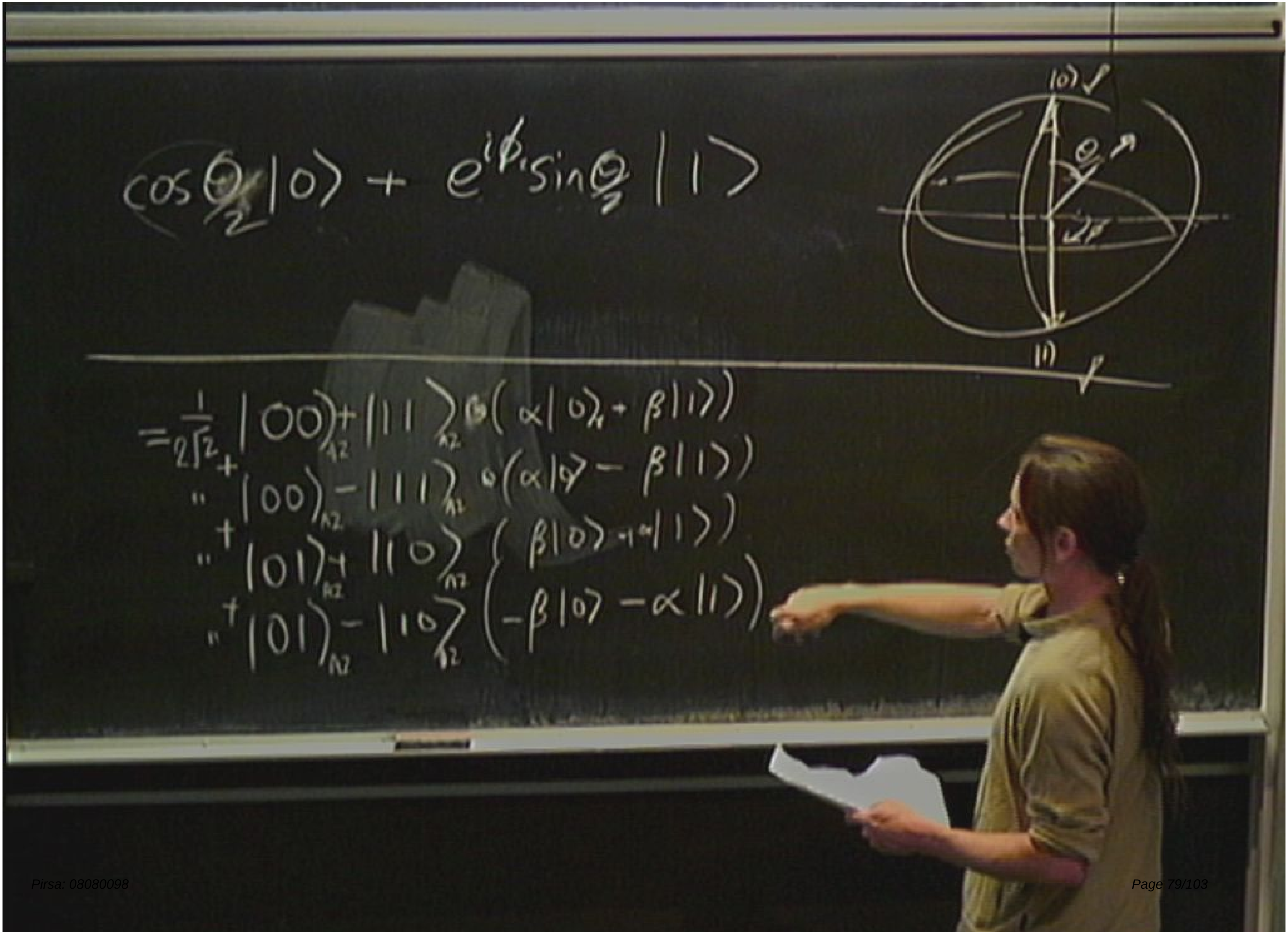
Alice tells Bob the result of her measurement. After a rotation based on Alice's message, Bob's qubit is magically in the exact same state as **Z** was initially.

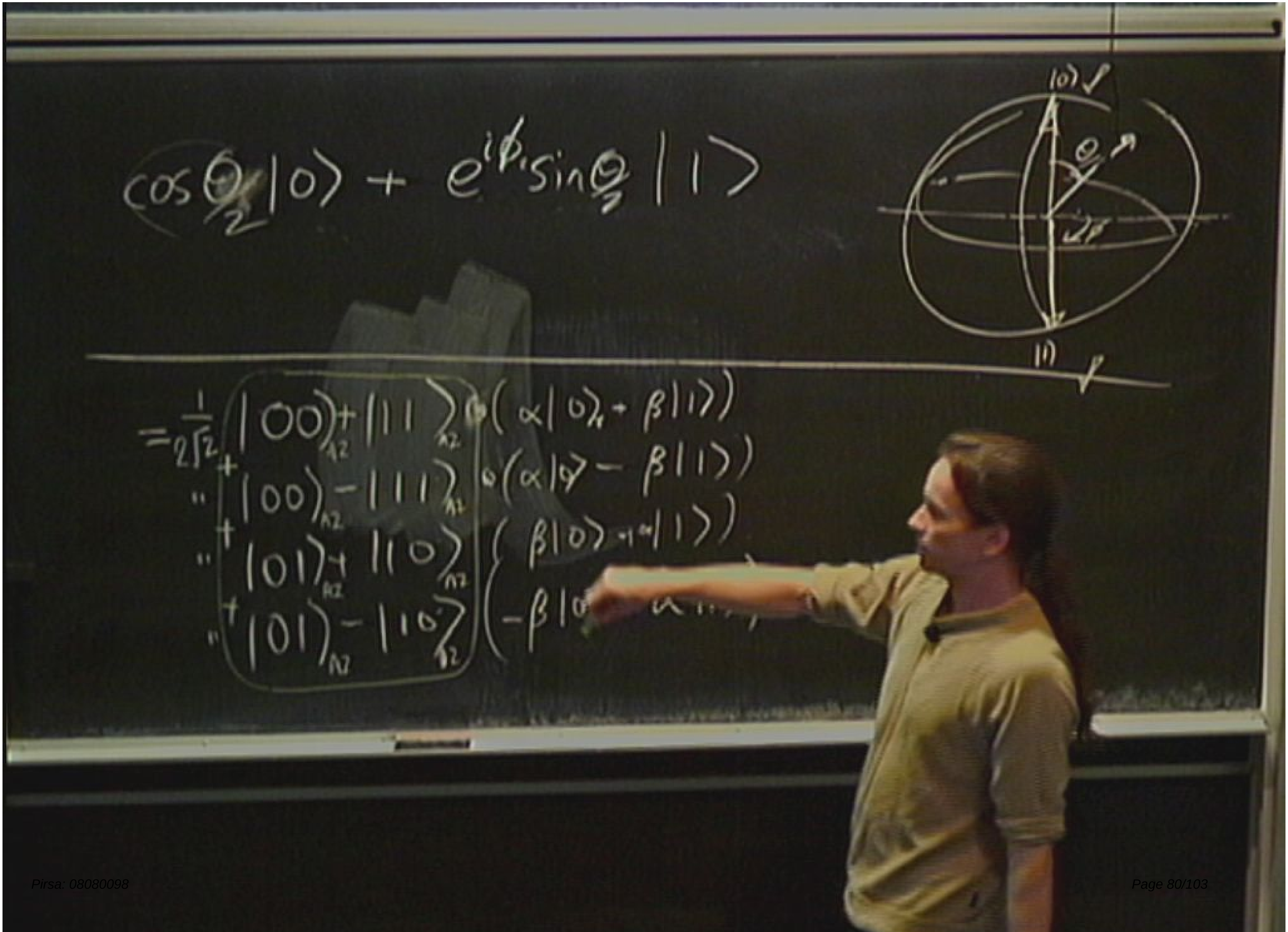

$|\psi\rangle_z = \alpha|0\rangle_z + \beta|1\rangle_z$
$|\psi\rangle_{AB} \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$
$|\phi\rangle_{AB}$

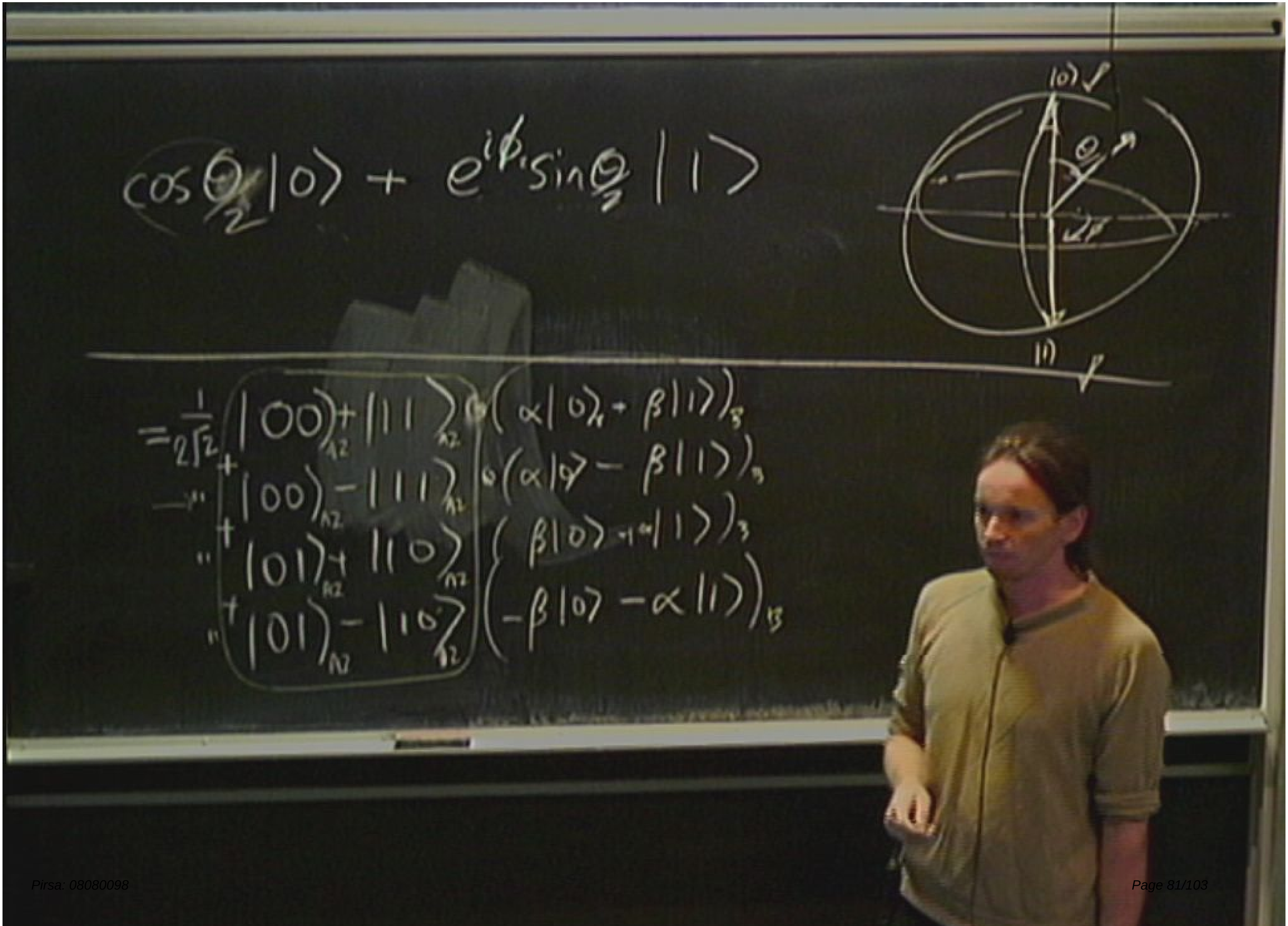

$|\psi\rangle_2 = \alpha|0\rangle_2 + \beta|1\rangle_2$
$|\phi\rangle_{AB}\ \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$
$|\phi\rangle_{AB} \otimes |\psi\rangle_2$
$= \frac{1}{\sqrt{2}}\left(\alpha|00\rangle_{A2}|0\rangle_B + \alpha|10\rangle_{A2}|1\rangle_B \ldots |0\rangle_B + \beta|11\rangle_{A2}|1\rangle_B\right)$


$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$ ✓
θ
ϕ
$|1\rangle$ ✓

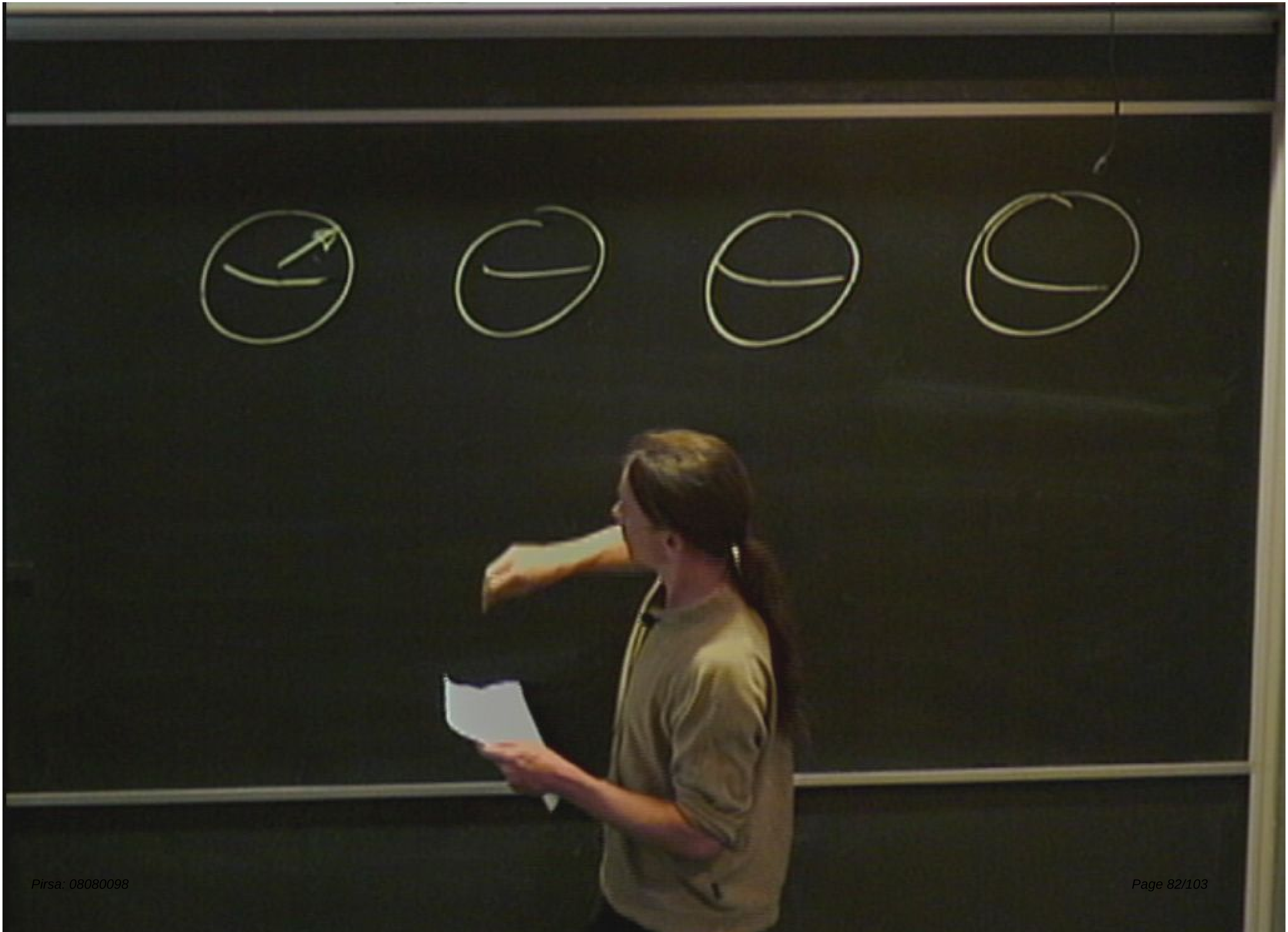

$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$
θ
ϕ
$|1\rangle$
$= |00\rangle_{12} + |11\rangle_{12}$
$|00\rangle_{12} - |11\rangle_{12}$
$|01\rangle_{12} + |10\rangle_{12}$
$|01\rangle_{12} - |10\rangle_{12}$

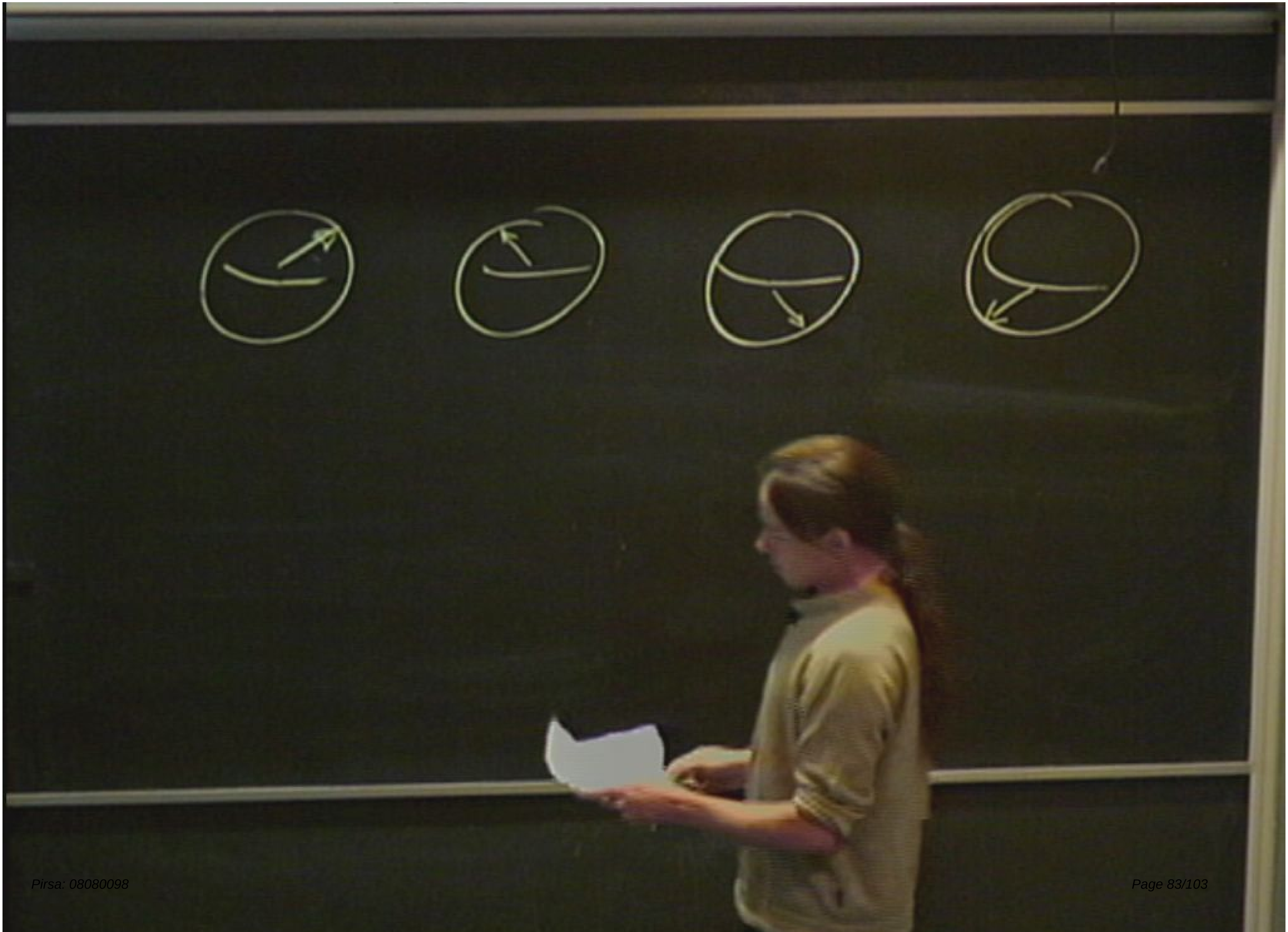

$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$
$|1\rangle$
$= \frac{1}{2\sqrt{2}} |00\rangle_{A2} + |11\rangle_{A2} \otimes (\alpha|0\rangle_1 + \beta|1\rangle)$
$+ |00\rangle_{A2} - |11\rangle_{A2} \otimes (\alpha|0\rangle - \beta|1\rangle)$
$+ |01\rangle_{A2} + |10\rangle_{A2} (\beta|0\rangle + \alpha|1\rangle)$
$+ |01\rangle_{A2} - |10\rangle_{A2} (-\beta|0\rangle - \alpha|1\rangle)$

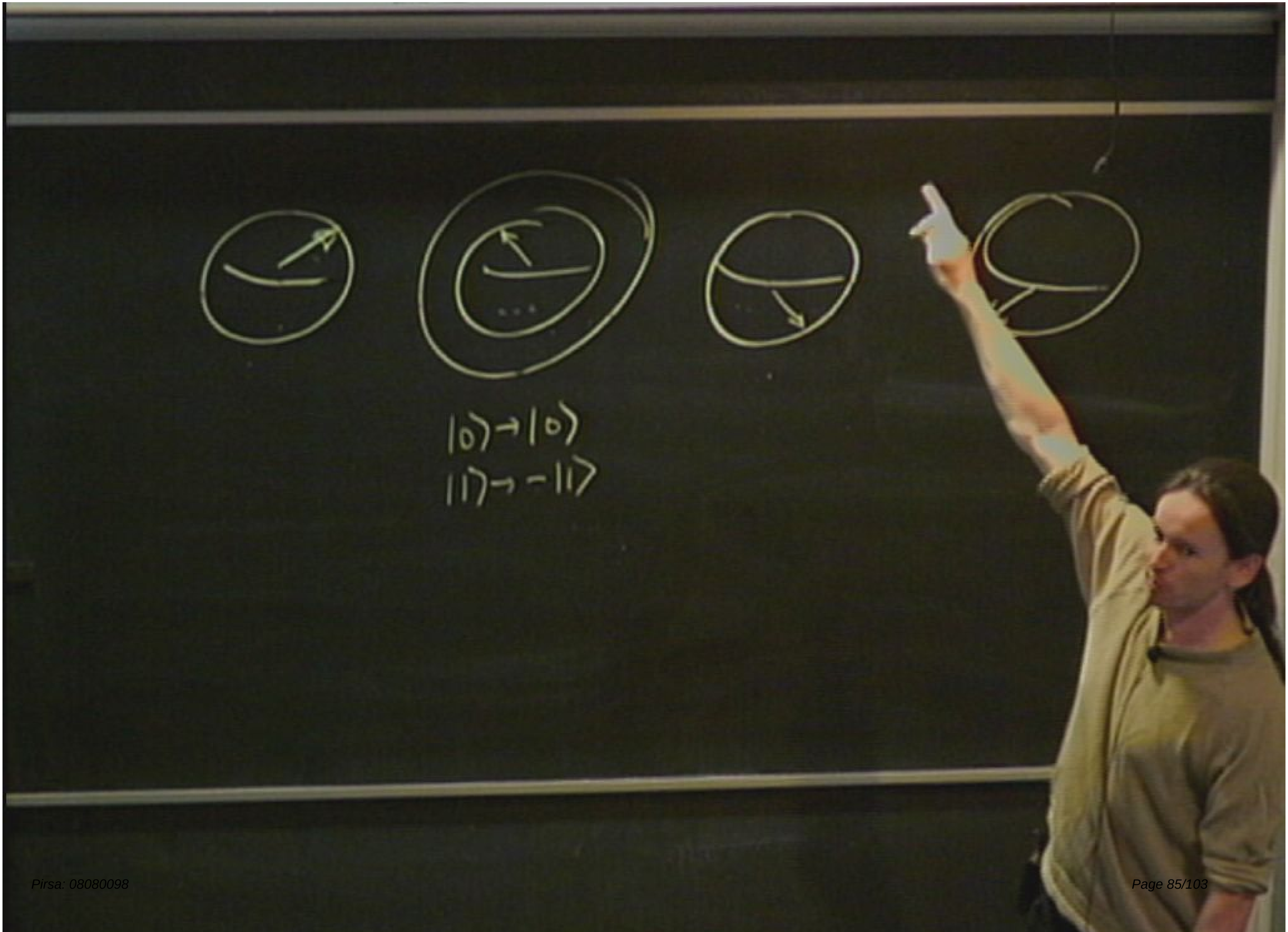

cos θ/2 |0⟩ + e^{iφ} sin θ/2 |1⟩
|0⟩ ✓
|1⟩ ✓
θ
φ
= 1/(2√2) [|00⟩_{A2} + |11⟩_{A2}] ⊗ (α|0⟩_1 + β|1⟩)
+ " [|00⟩_{A2} − |11⟩_{A2}] ⊗ (α|0⟩ − β|1⟩)
+ " [|01⟩_{A2} + |10⟩_{A2}] (β|0⟩ + α|1⟩)
+ " [|01⟩_{A2} − |10⟩_{A2}] (−β|0⟩ + α|1⟩)

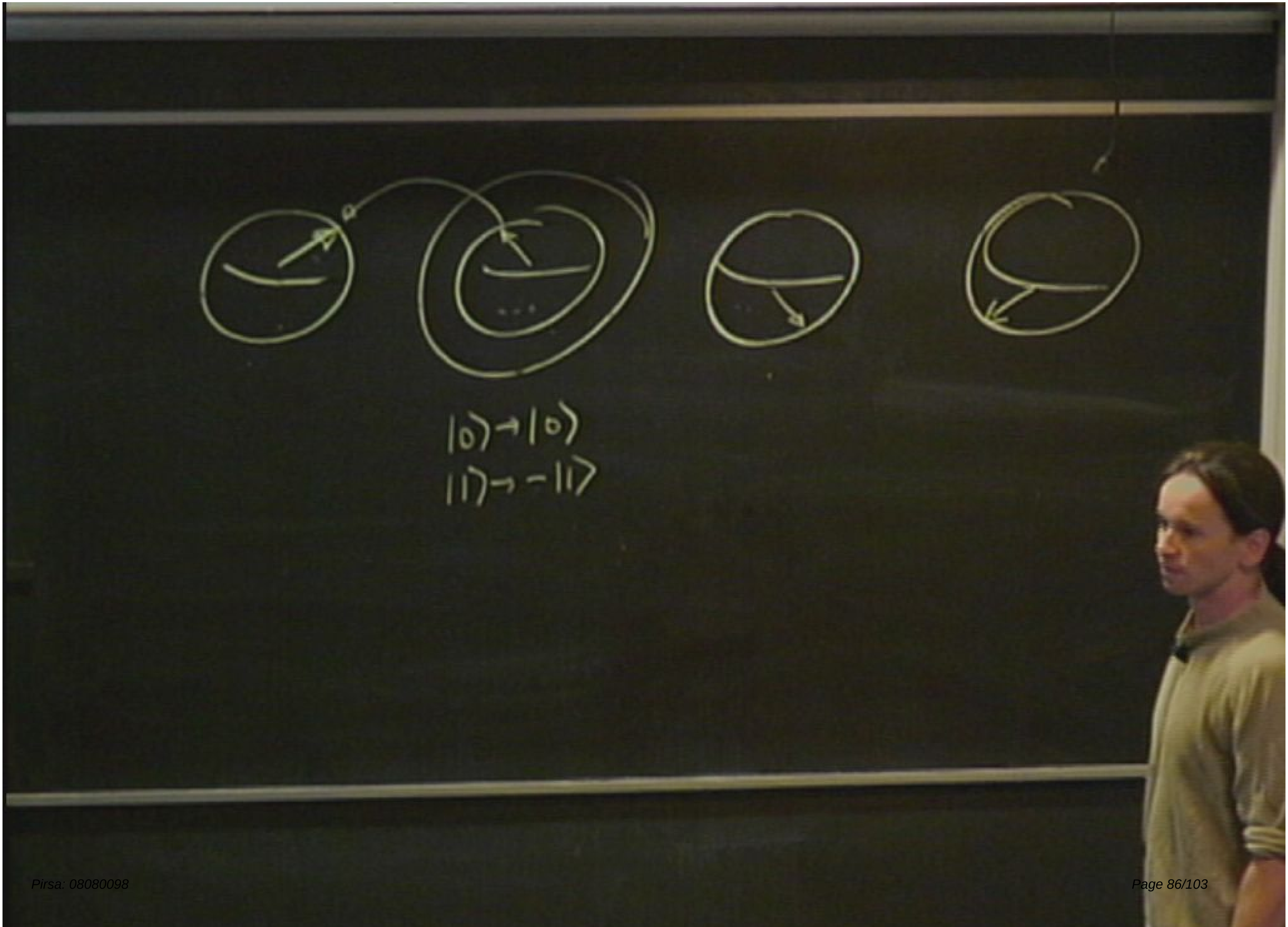


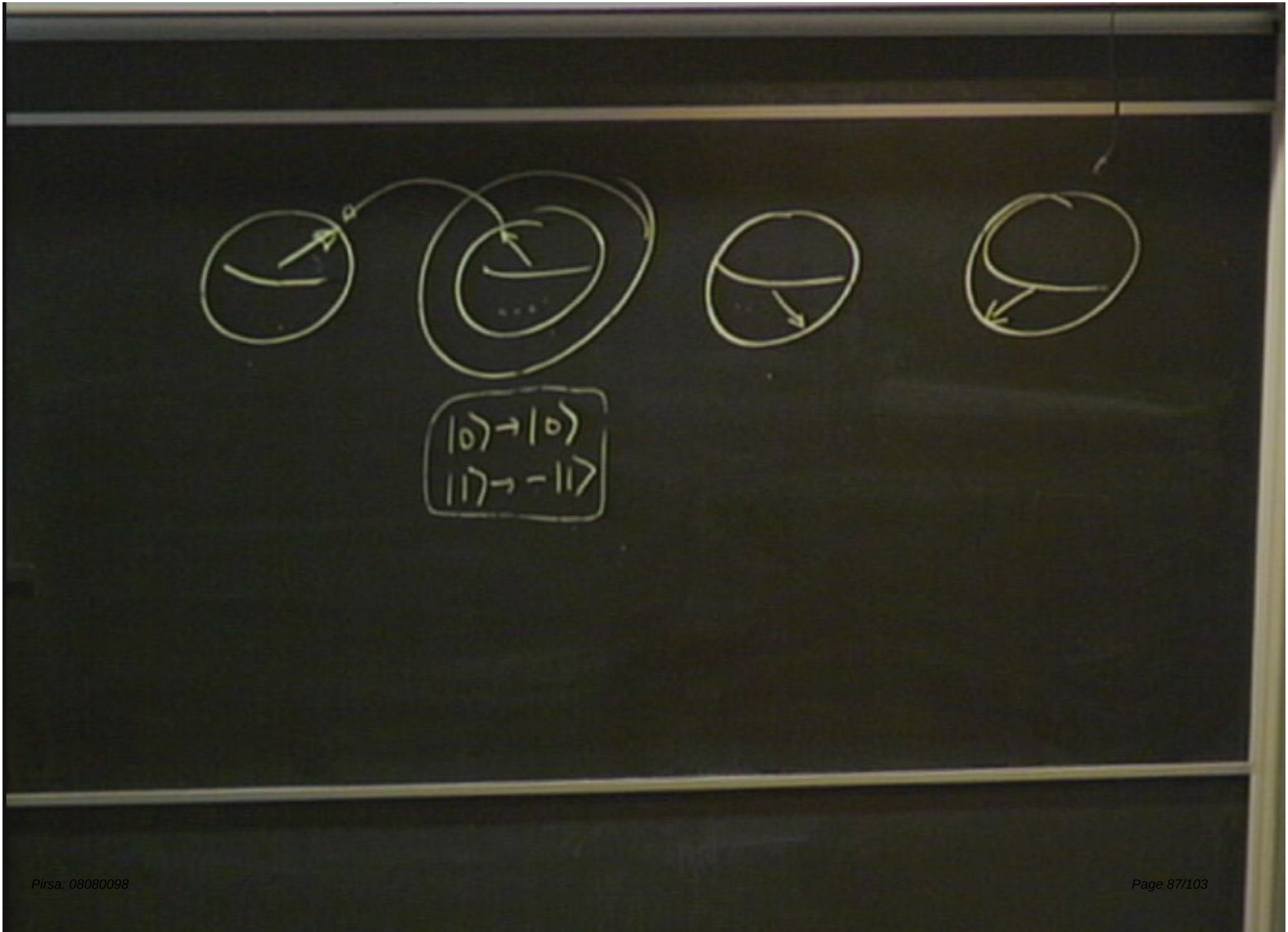
$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$ ✓
θ
ϕ
$|1\rangle$ ✓
$= \frac{1}{2\sqrt{2}} \Big[(|00\rangle + |11\rangle)_{12} \otimes (\alpha|0\rangle_3 + \beta|1\rangle)_3$
$+ (|00\rangle - |11\rangle)_{12} \otimes (\alpha|0\rangle - \beta|1\rangle)_3$
$+ (|01\rangle + |10\rangle)_{12} \otimes (\beta|0\rangle + \alpha|1\rangle)_3$
$+ (|01\rangle - |10\rangle)_{12} \otimes (-\beta|0\rangle - \alpha|1\rangle)_3 \Big]$






|0⟩→|0⟩
|1⟩→ −|1⟩


$|0\rangle \to |0\rangle$
$|1\rangle \to -|1\rangle$


$|0\rangle \to |0\rangle$
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Quantum teleportation

Note the following:

Alice loses the quantum state. The particles are still there, but all information about how they would have behaved vanishes.

Alice learns nothing about the quantum state she didn't already know!

Other than having it arrive in his lab, Bob learns nothing about it either!

Question: Does Bob know when Alice has performed her measurement?

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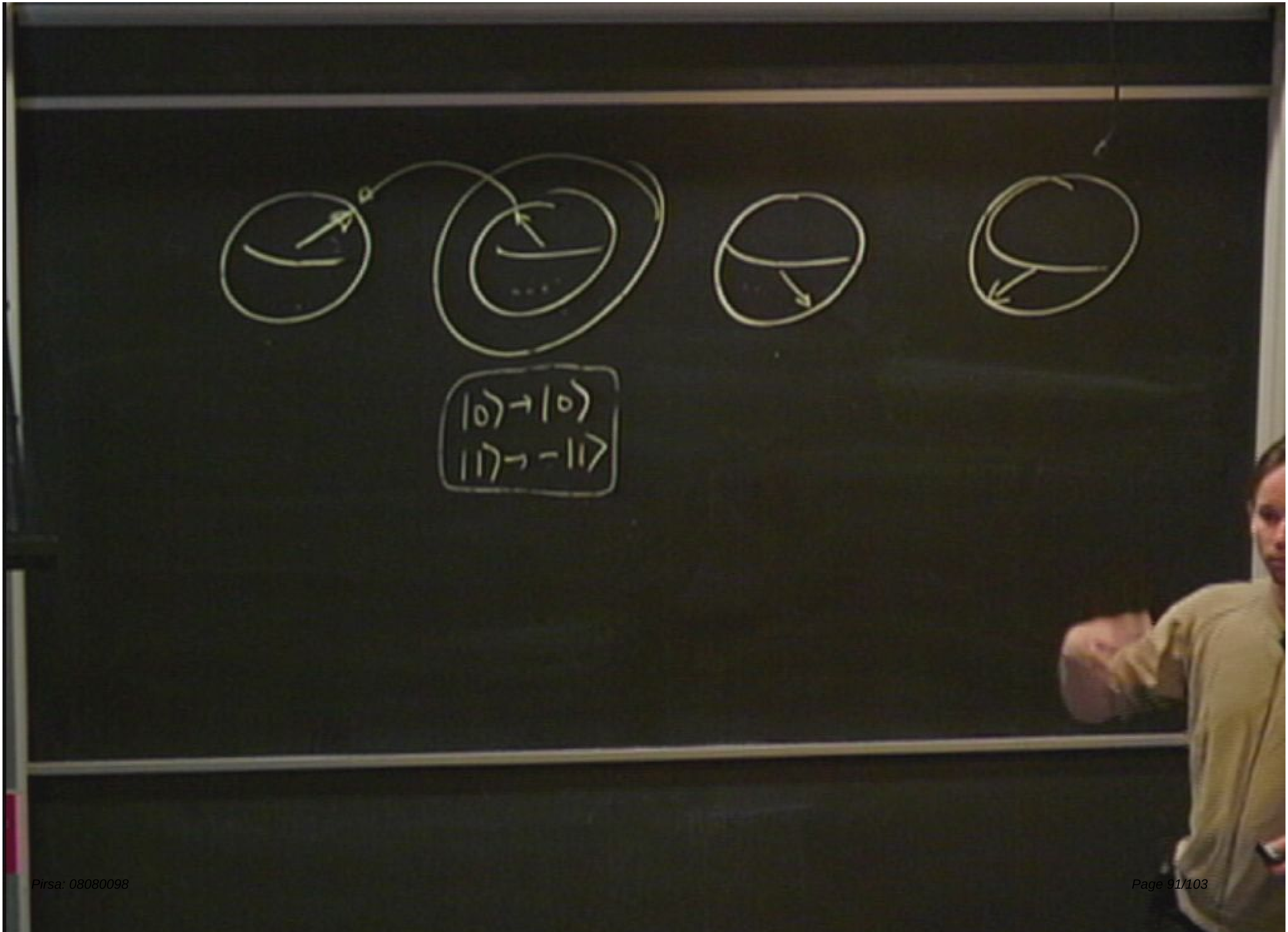
No. There is nothing to distinguish his system before and after its 'collapse'.

So has anything really happened to it?

What happens to it when Alice's message arrives?


|0⟩ → |0⟩
|1⟩ → −|1⟩

$$= \frac{1}{2}\Big(\alpha|00\rangle_{A2}|0\rangle_B + \alpha|10\rangle_{A2}|1\rangle_B + \beta|01\rangle_{A2}|0\rangle_B + \beta$$

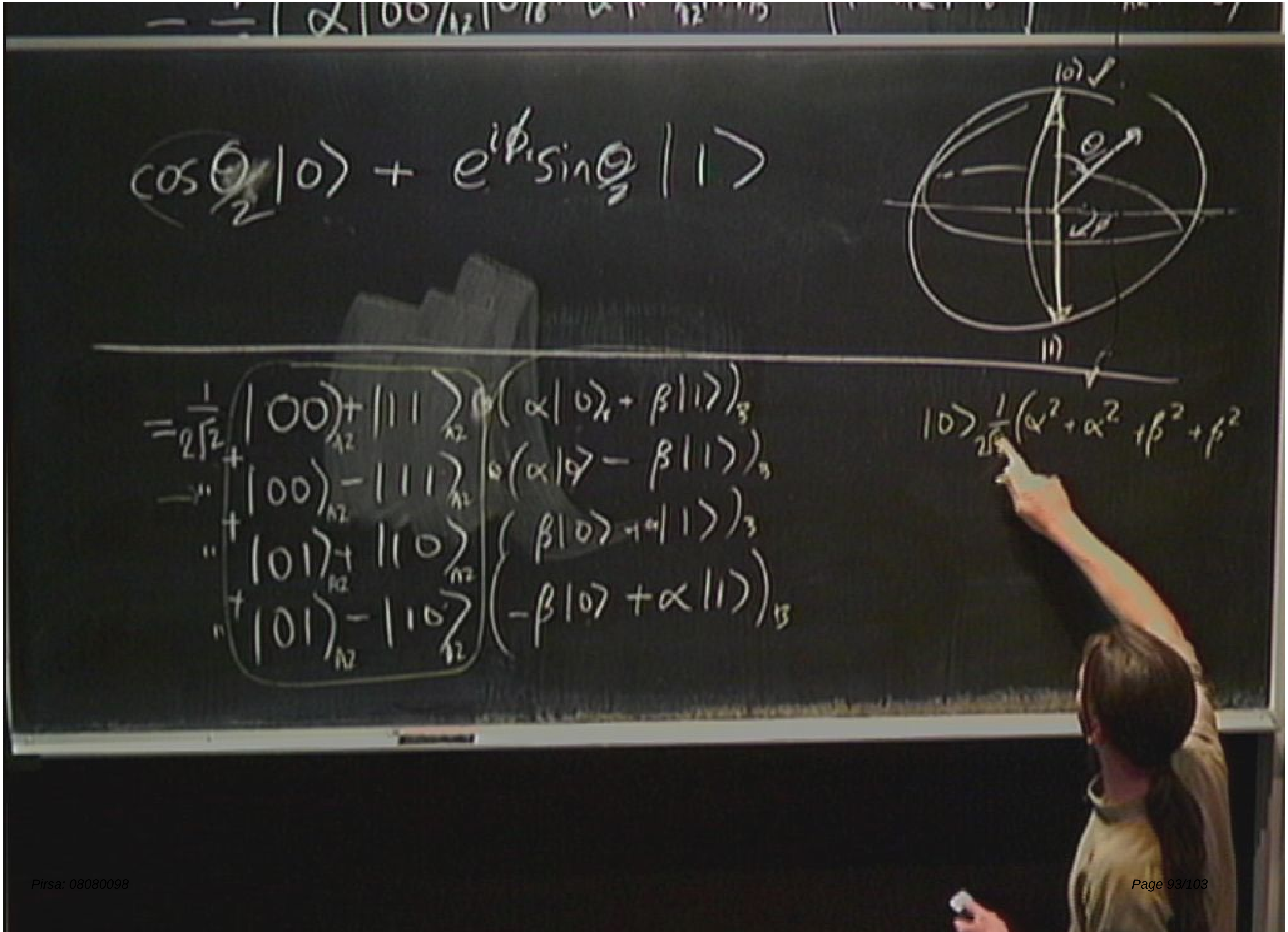
$$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

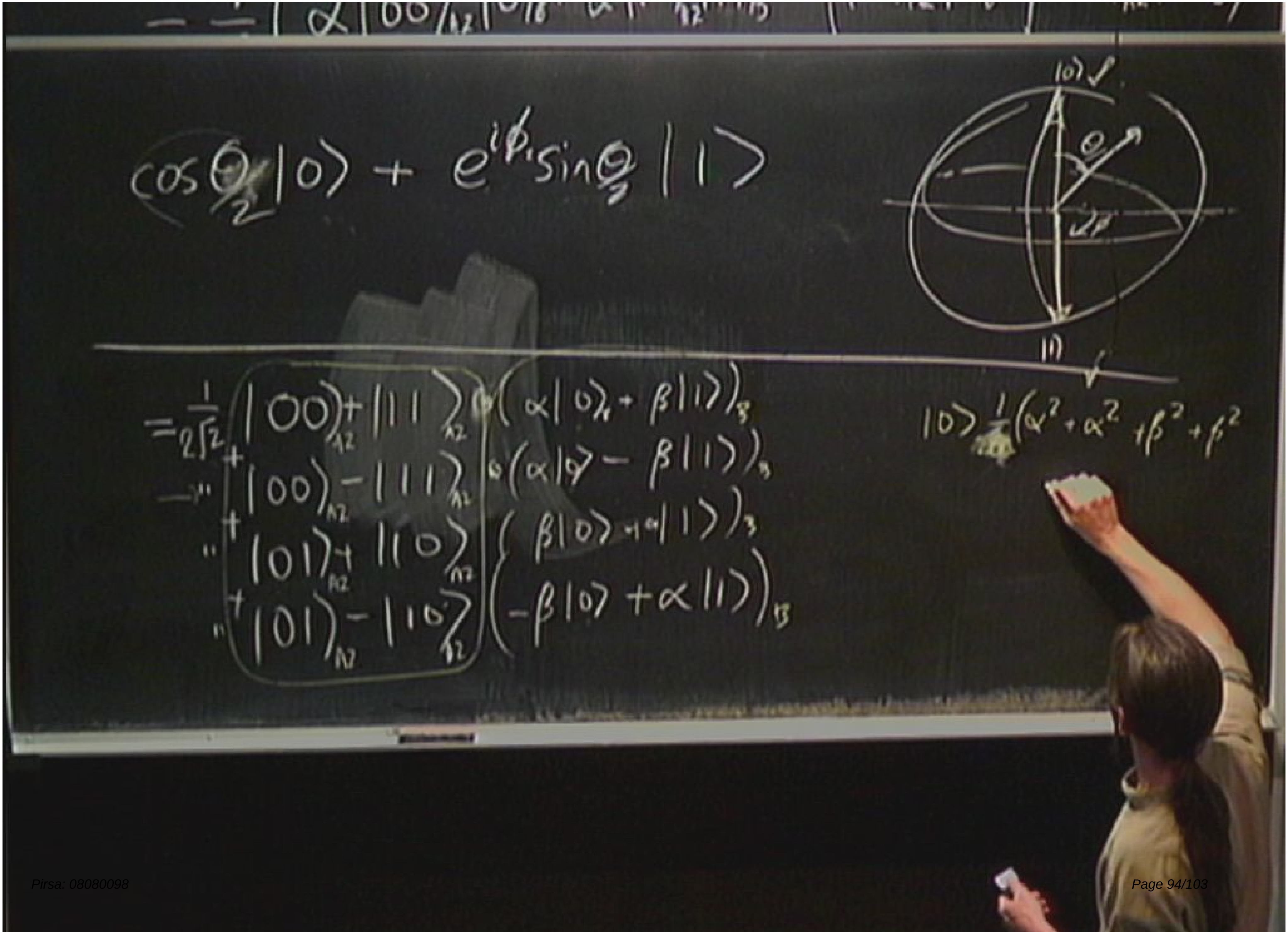
$$= \frac{1}{2\sqrt{2}}\Big[\big(|00\rangle_{A2} + |11\rangle_{A2}\big)\otimes\big(\alpha|0\rangle_B + \beta|1\rangle\big)_B$$

$$+\ \text{''}\ \big(|00\rangle_{A2} - |11\rangle_{A2}\big)\otimes\big(\alpha|0\rangle - \beta|1\rangle\big)_B$$

$$+\ \text{''}\ \big(|01\rangle_{A2} + |10\rangle_{A2}\big)\big(\beta|0\rangle + \alpha|1\rangle\big)_B$$

$$+\ \text{''}\ \big(|01\rangle_{A2} - |10\rangle_{A2}\big)\big(-\beta|0\rangle + \alpha|1\rangle\big)_B\Big]$$


cos θ/2 |0⟩ + e^{iφ} sin θ/2 |1⟩
|0⟩
|1⟩
θ
φ
= 1/(2√2) (|00⟩ + |11⟩)₁₂ (α|0⟩ + β|1⟩)₃
+ (|00⟩ − |11⟩)₁₂ (α|0⟩ − β|1⟩)₃
+ (|01⟩ + |10⟩)₁₂ (β|0⟩ + α|1⟩)₃
+ (|01⟩ − |10⟩)₁₂ (−β|0⟩ + α|1⟩)₃
|0⟩ 1/(2√2) (α² + α² + β² + β²


$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$
$|0\rangle$
$|1\rangle$
$= \frac{1}{2\sqrt{2}} \left(|00\rangle + |11\rangle\right)_{12} \otimes (\alpha|0\rangle + \beta|1\rangle)_3$
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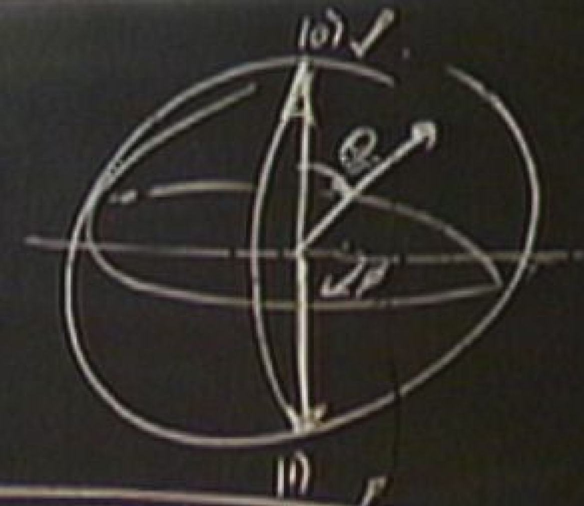
$+ \left(|01\rangle - |10\rangle\right)_{12} (-\beta|0\rangle + \alpha|1\rangle)_3$
$|0\rangle \frac{1}{4}(\alpha^2 + \alpha^2 + \beta^2 + \beta^2$

$$\cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$



$$= \frac{1}{2\sqrt{2}}\Big(|00\rangle + |11\rangle\Big)_{12}(\alpha|0\rangle + \beta|1\rangle)_3$$
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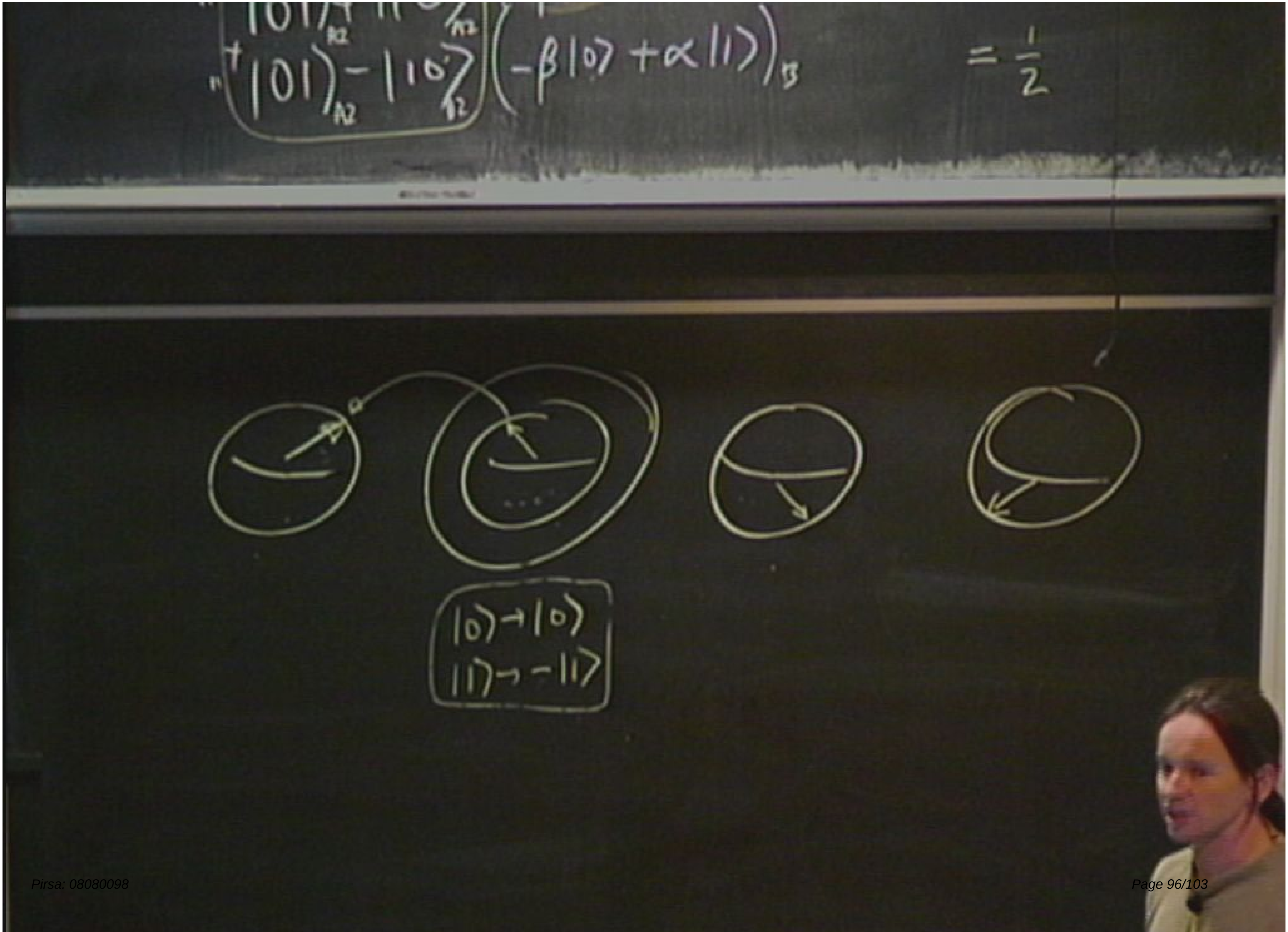
$$+ \Big(|01\rangle + |10\rangle\Big)_{12}(\beta|0\rangle + \alpha|1\rangle)_3$$
$$+ \Big(|01\rangle - |10\rangle\Big)_{12}(-\beta|0\rangle + \alpha|1\rangle)_3$$

$$|0\rangle \ \frac{1}{4}(\alpha^2 + \alpha^2 + \beta^2 + \beta^2)$$
$$\frac{1}{4}(2)$$
$$= \frac{1}{2}$$


(−β|0⟩ + α|1⟩)
= 1/2
|0⟩ → |0⟩
|1⟩ → −|1⟩

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Question: How much information about the quantum state is contained in Alice's classical message?

None. If the classical message contained any information at all about the state, that information would be known to Alice. But if Bob has a perfect copy, Alice must have no information (no cloning).

The classical information is two perfectly random bits, and Bob's system is maximally scrambled. So while the message is in transit, where is the teleportee?

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Question: If Alice tried to send half of an entangled system to Bob, would the entanglement survive?

Yes. Everything gets through.

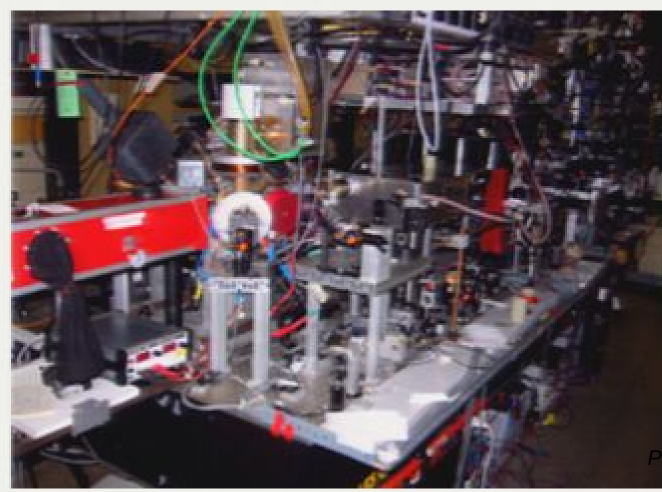
This is actually used in experiments to prove that the teleportation worked. If a nonlocality effect can be detected after teleportation, some of the 'quantumness' of the entangled photon must have got through.

Actual quantum teleportation

- Photons in optical fibres high fidelity.
- Sequential teleportation with fidelity 0.57.
- Teleportation of a two-qubit composite system.
- Teleportation between photons and caesium atoms.
- Teleportation between beryllium atoms.

Photons in Geneva

Atoms in Innsbruck



Encryption using teleportation

Teleportation allows unreasonably strong encryption.

Teleportation allows you to send fully quantum messages securely.

Why?

Because the classical message is completely random, it reveals no information to any eavesdropper.

In fact, eavesdroppers cannot even disrupt communication, because Alice can broadcast the classical message loudly as often as she wants.

Remarkably, even if we allow Eve superhuman powers, allowing her access to adjust the entangled state before it is distributed to Alice and Bob, they can still teleport in perfect security! The routine does not depend on any specifics of the entanglement.

Conclusion

So what is quantum information?

What is this quantum state we're teleporting anyway?

Asher Peres was founder of quantum information theory, and one of the discoverers of teleportation.

He was once asked by a journalist whether it was possible to teleport not only the body, but also the soul.

"*Only* the soul."