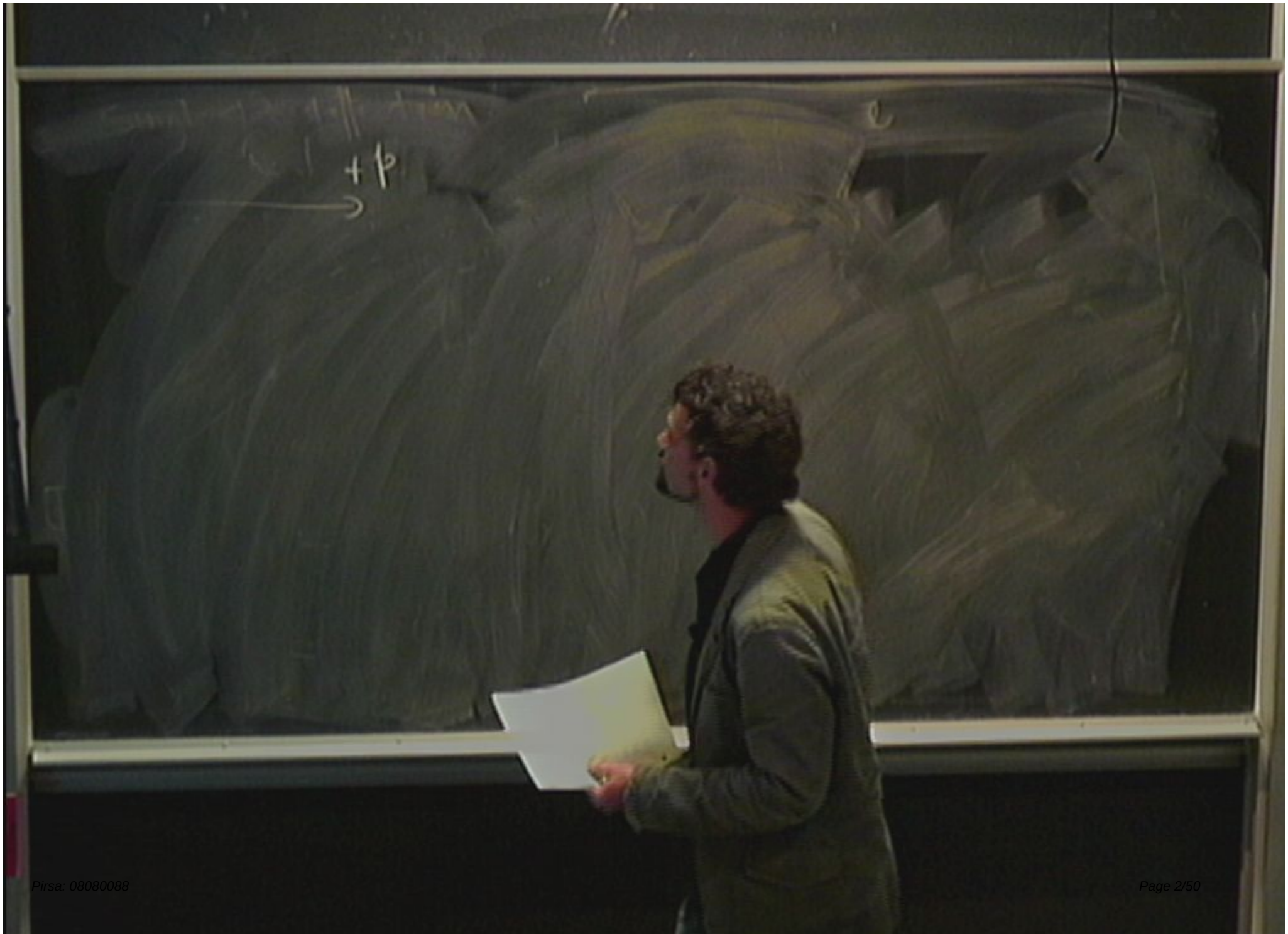
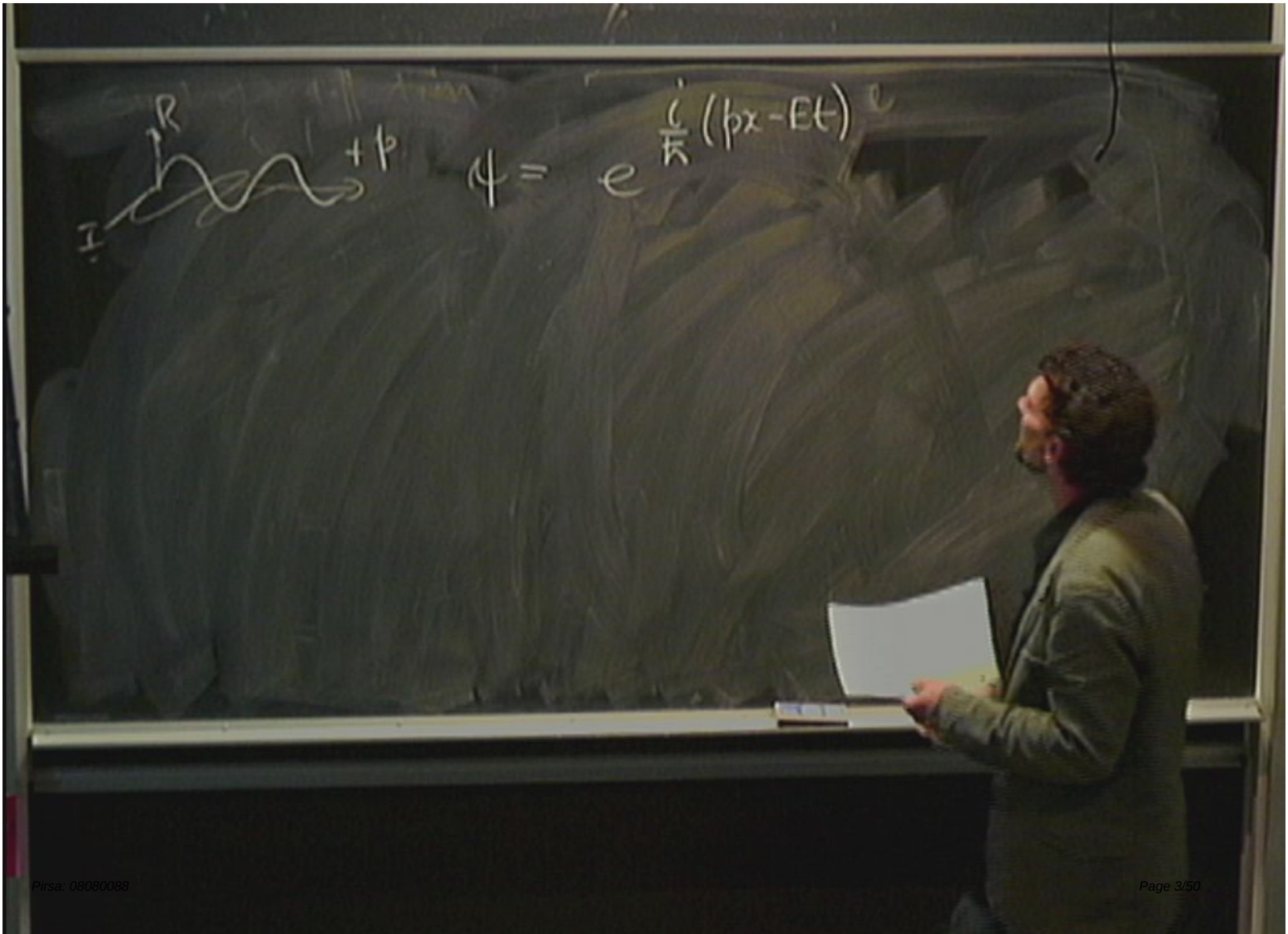
Title: Quantum Mechanics 13 - Schrodinger Wave Equation

Date: Aug 16, 2008 03:30 PM

URL: http://pirsa.org/08080088

Abstract: A â€œderivationâ€• of the Schrodinger wave equation based on simple calculus.
Learning Outcomes:
â€¢ How to express the de Broglie wave of a free particle, i.e. a complex traveling wave, in terms of the particleâ€™s energy and momentum, and how to differentiate this wave with respect to its space and time variables (x and t).
â€¢ How to combine the above mathematical results with the Newtonian expression for the total energy of a particle to get Schrodingerâ€™s wave equation.
â€¢ Diracâ€™s extension of these ideas to Einsteinâ€™s expression for the total energy of a particle: introduction to spin, antimatter, and the Standard Model of particle physics.


+p


R
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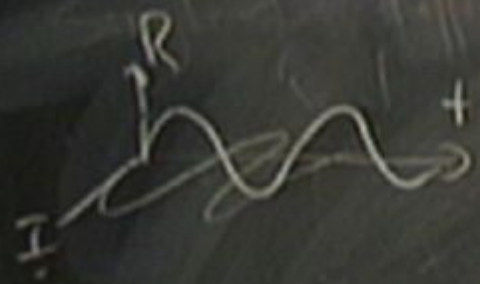
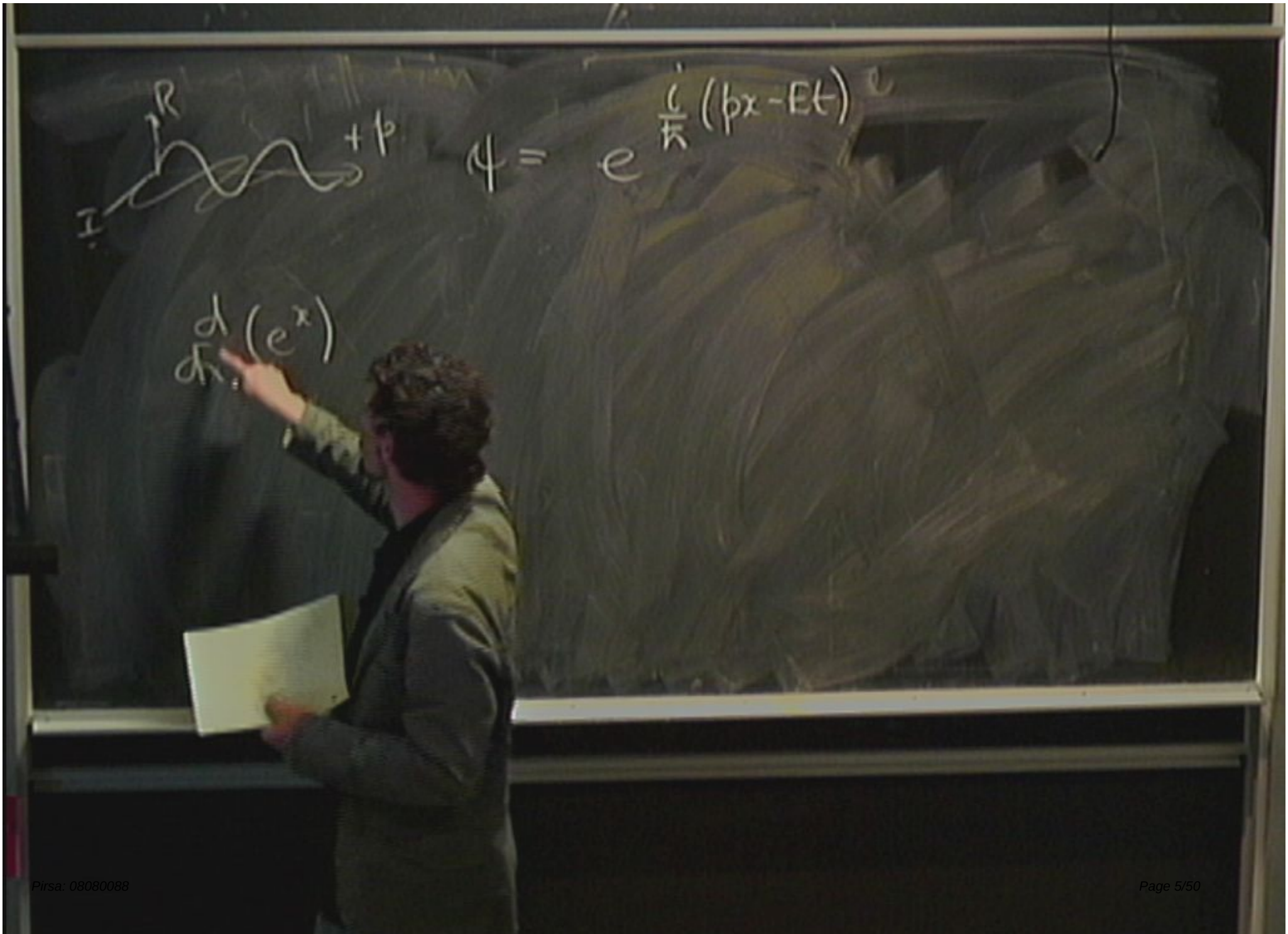
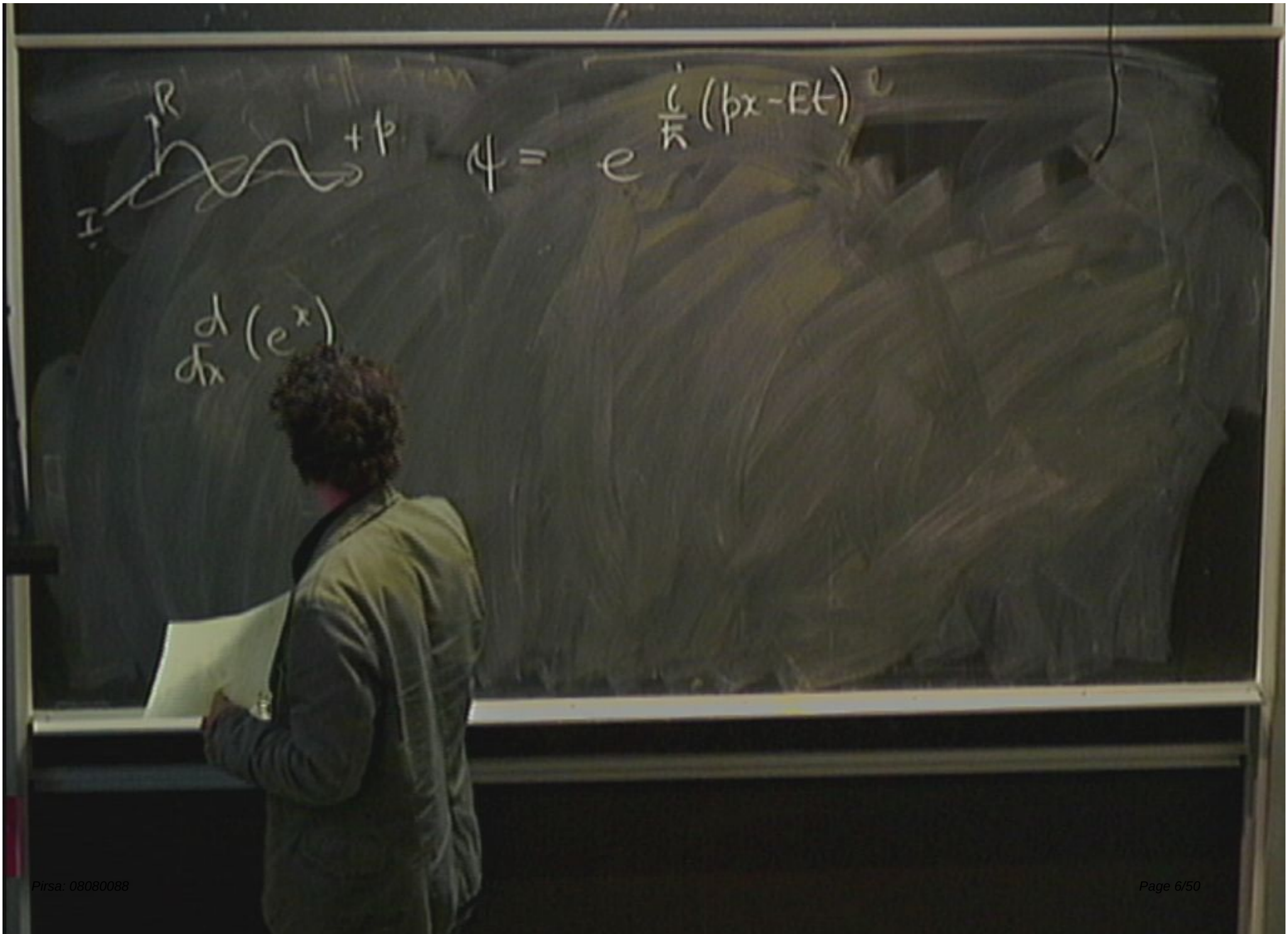
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$



$$\psi = e^{\frac{i}{\hbar}(px - Et)}$$


R
I
+p
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(e^x)$


R
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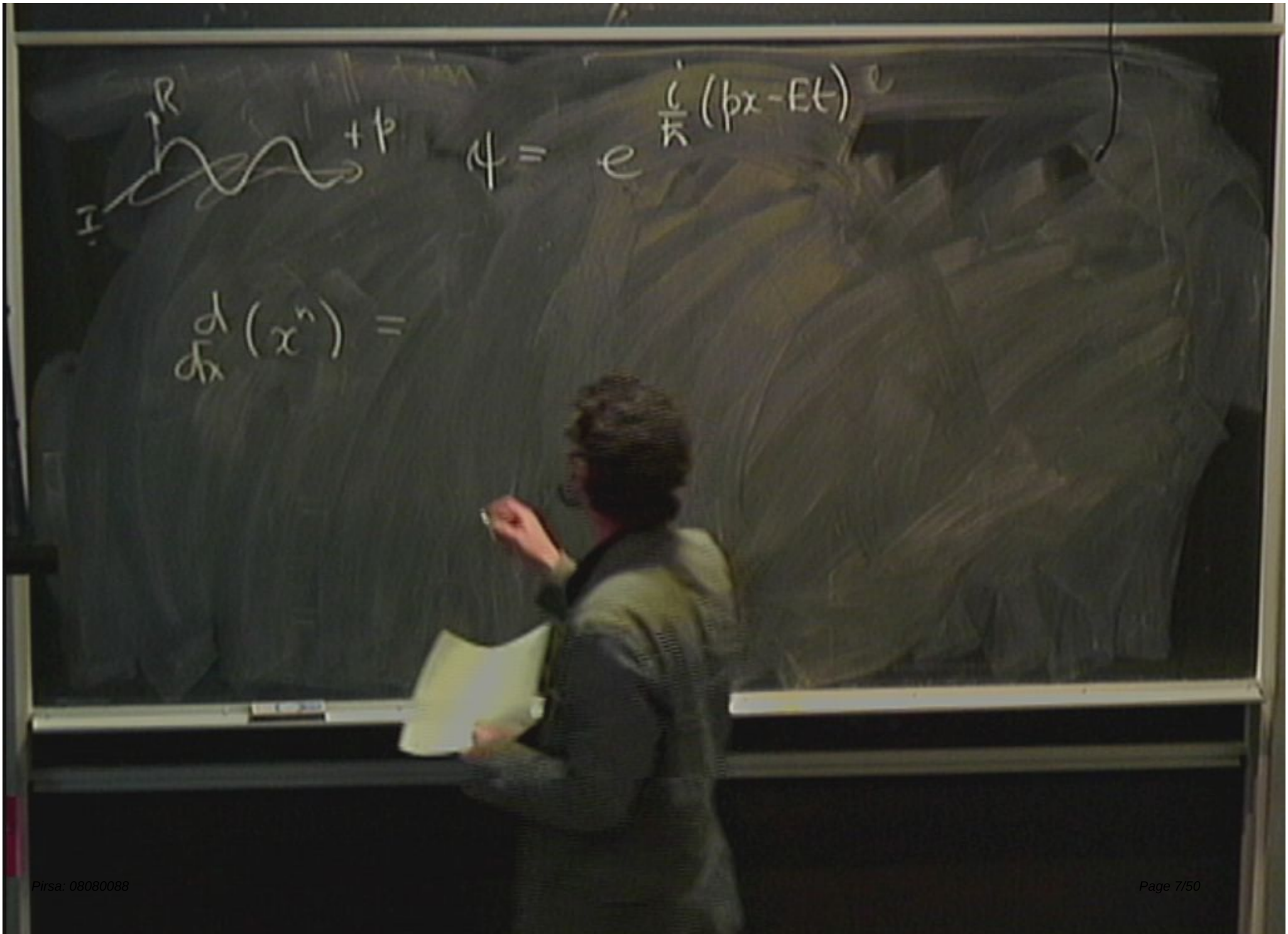
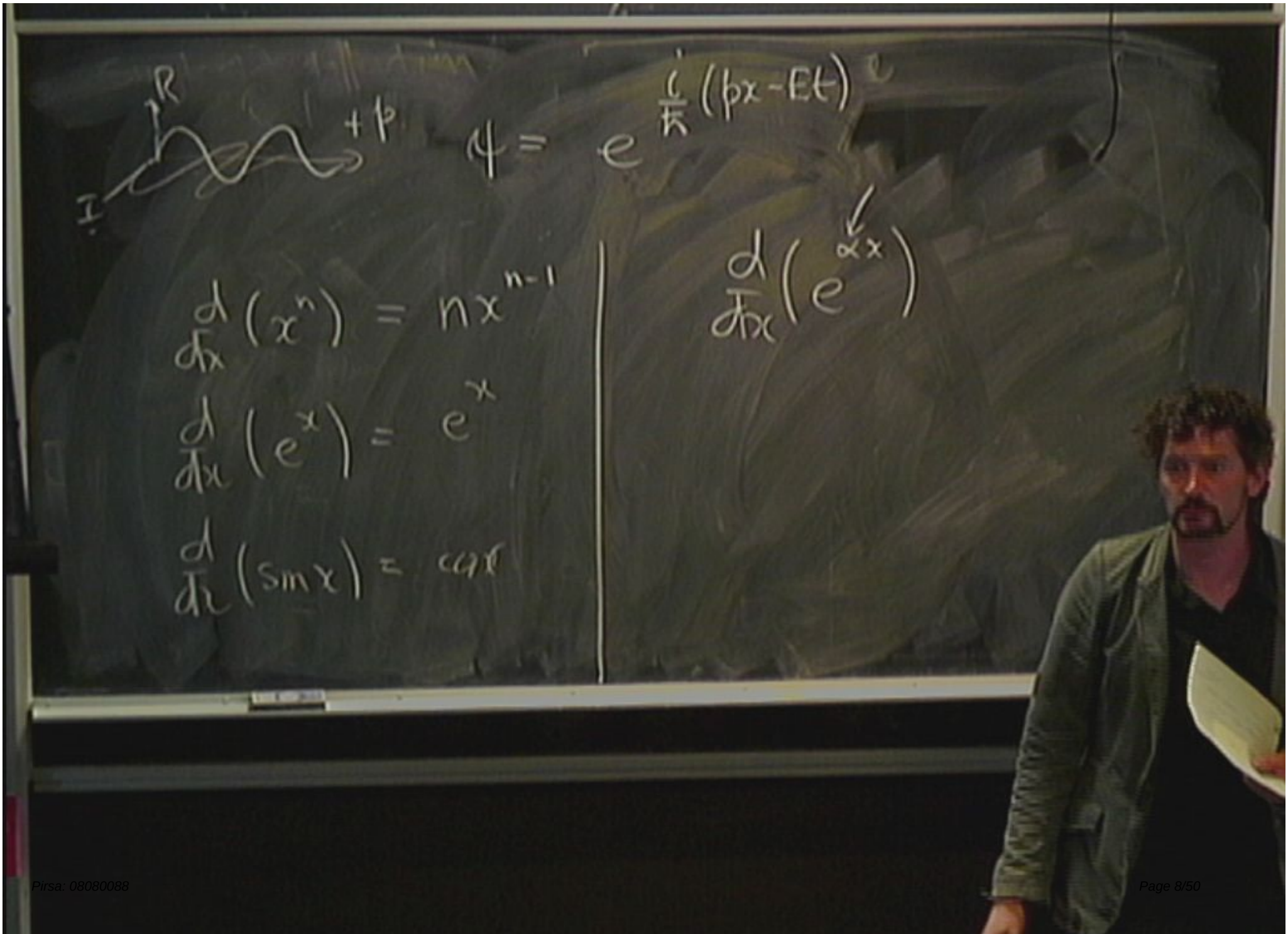
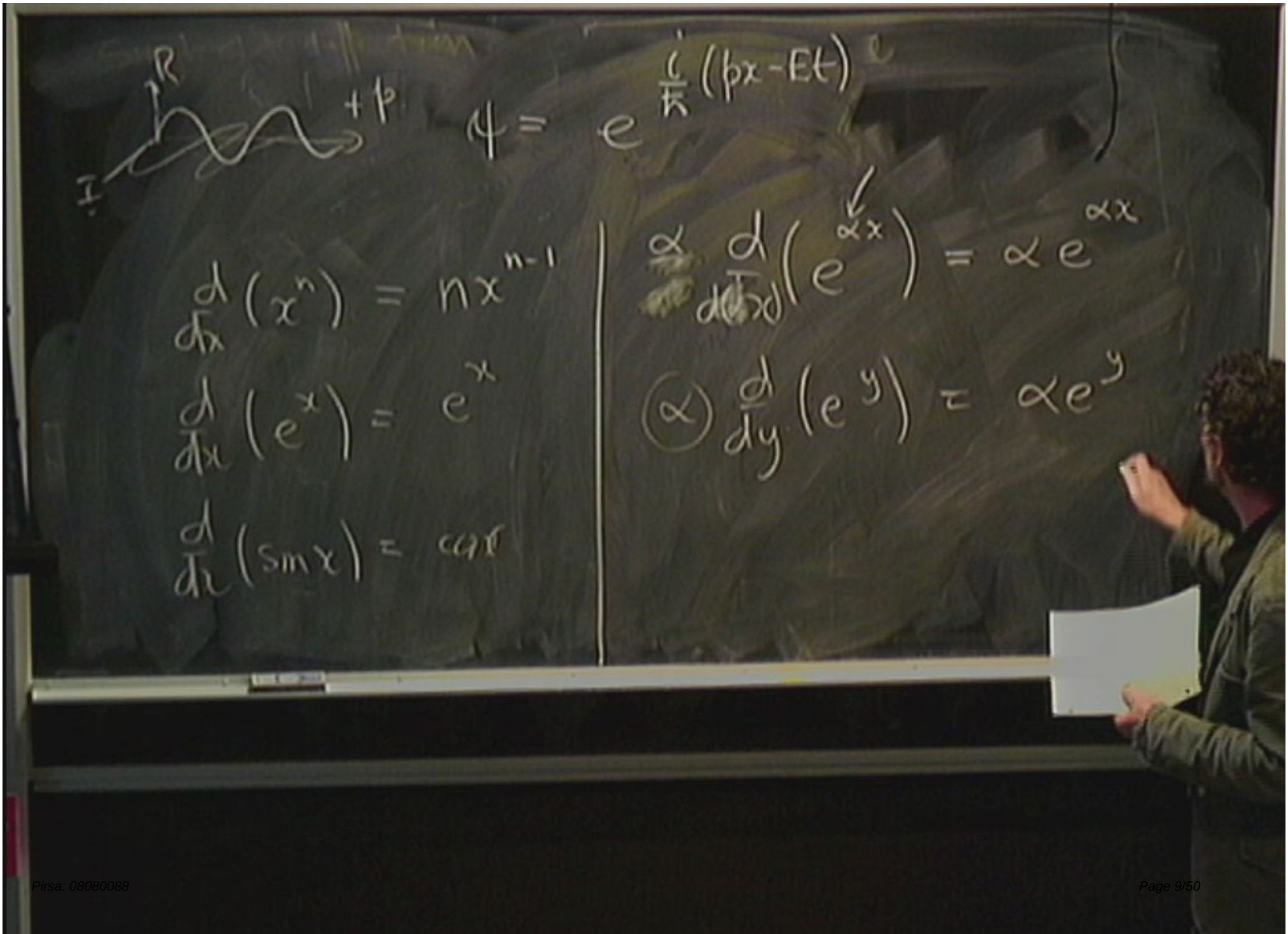
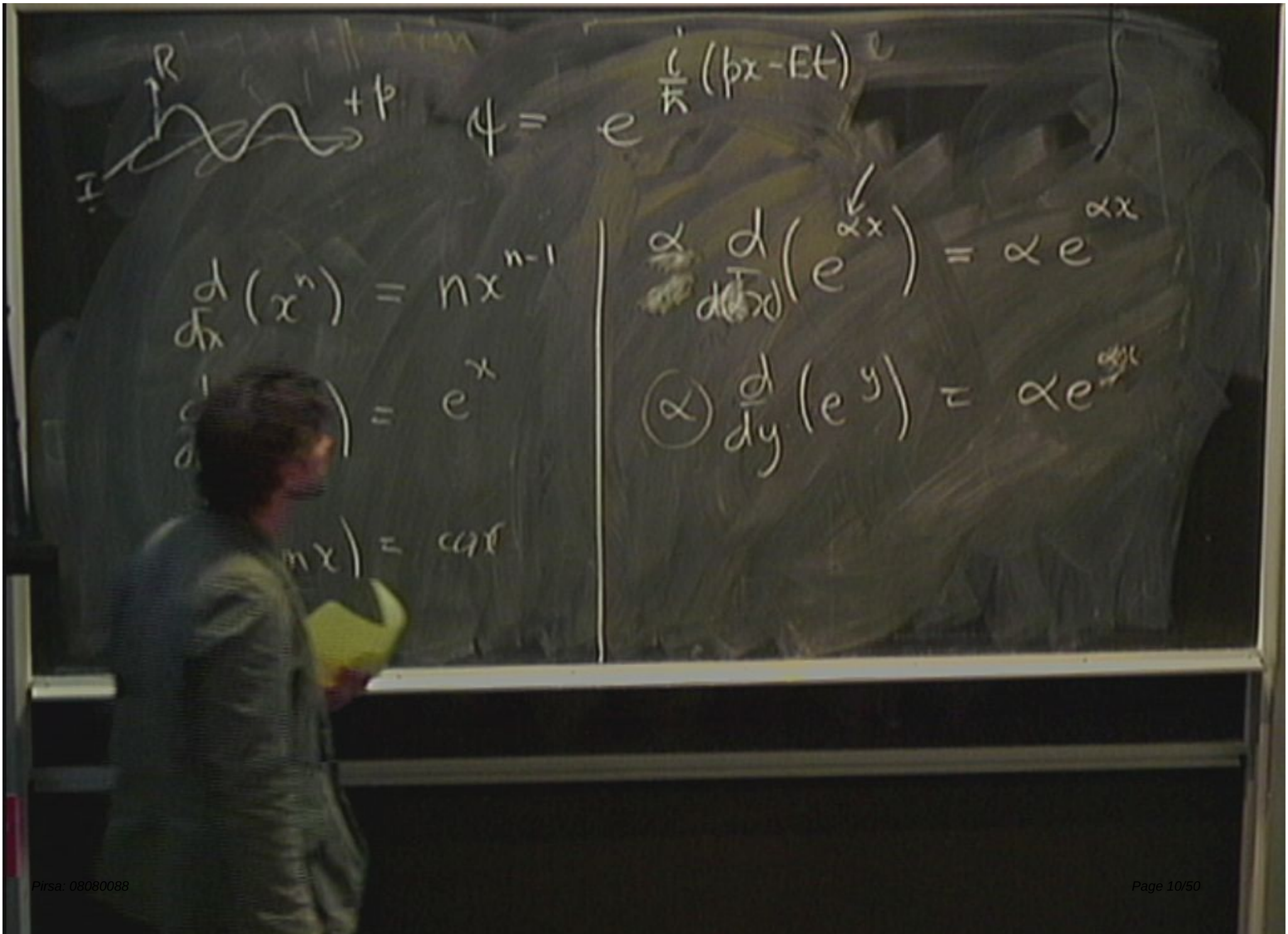
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(e^x)$


R
+p
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(x^n) =$


R
+p
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\sin x) = \cos$
$\frac{d}{dx}(e^{\alpha x})$


R
+p
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\sin x) = \cos x$
$\alpha \frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\alpha \frac{d}{dy}(e^y) = \alpha e^y$


R
+p
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$= e^x$
$nx) = \alpha x$
$\alpha \frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\alpha \frac{d}{dy}(e^y) = \alpha e^{\alpha y}$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

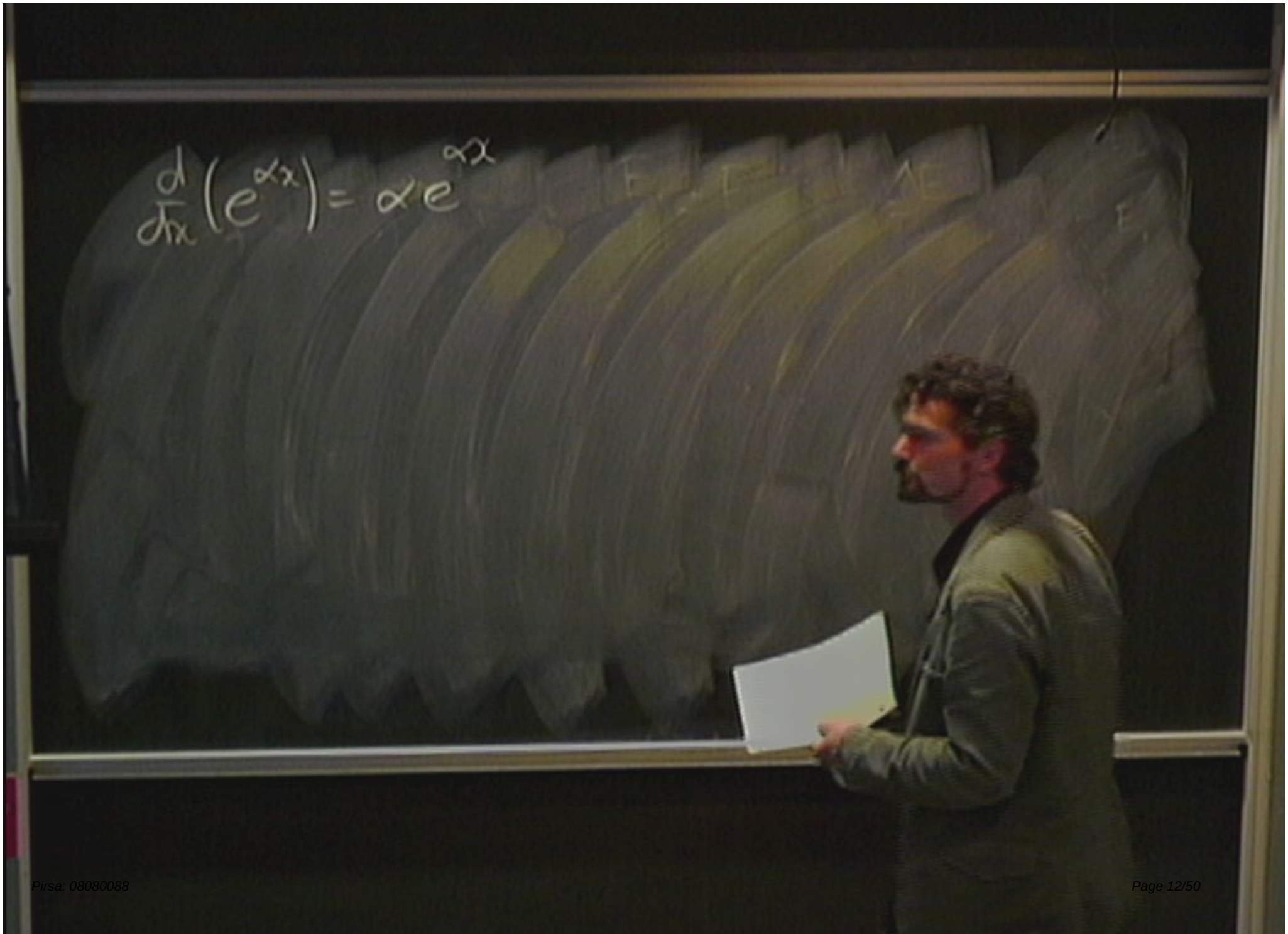
$$\frac{d}{dx}(e^x) = e^x$$

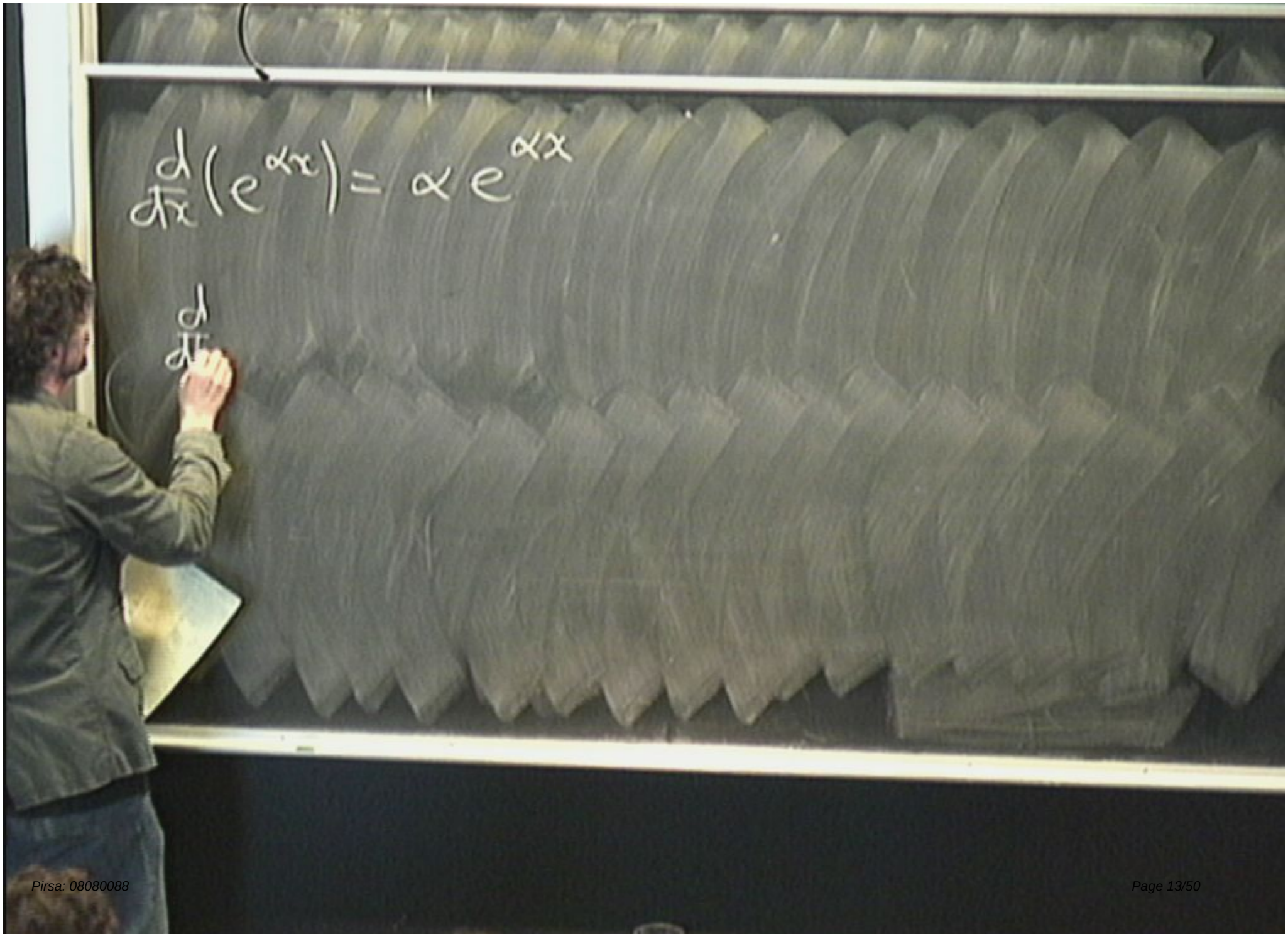
$$\frac{d}{dx}(\sin x) = \cos x$$

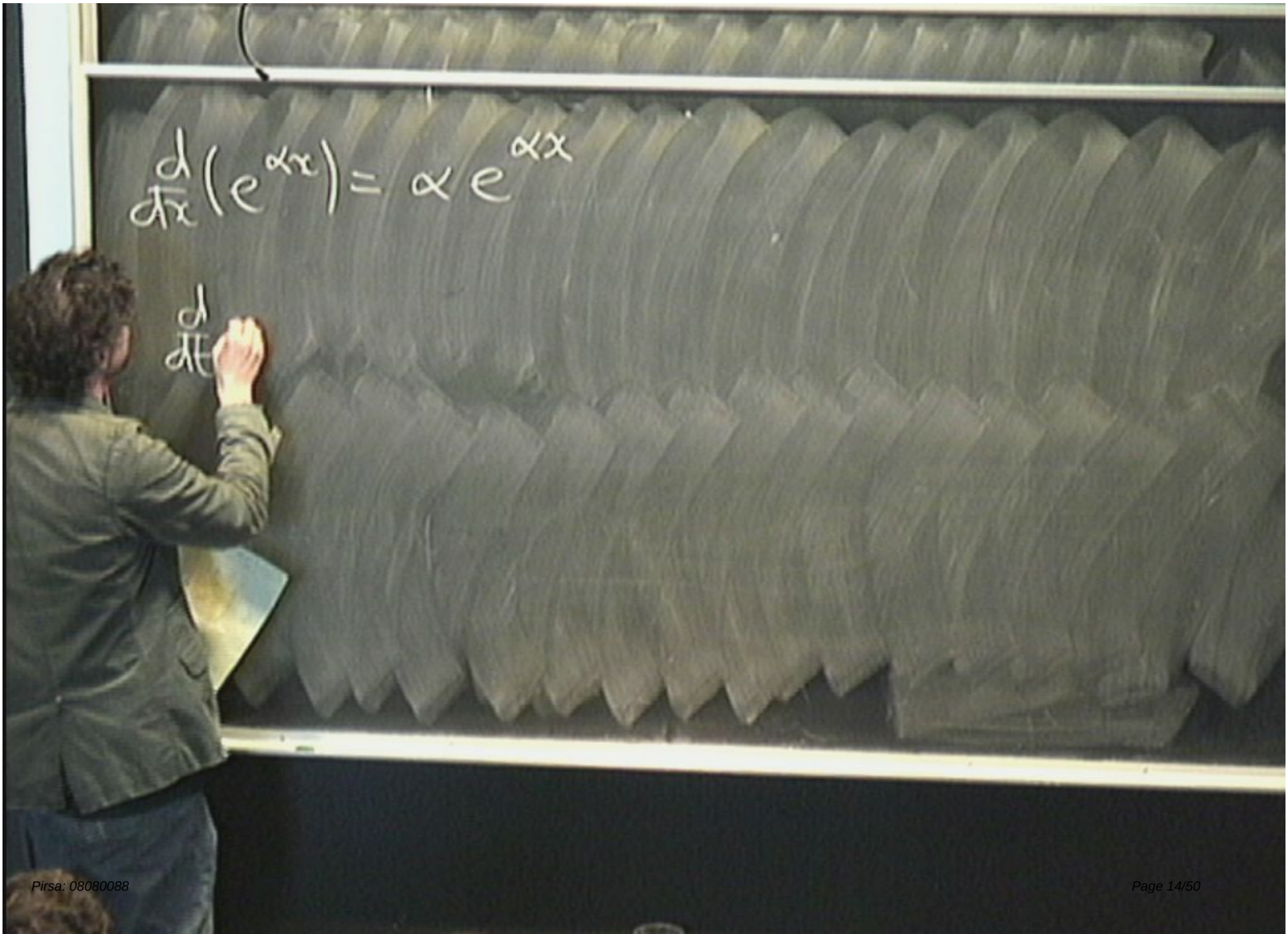
$$\frac{d}{d(\alpha x)}(e^{\alpha x}) = \alpha e^{\alpha x}$$

$$\alpha \frac{d}{dy}(e^y) = \alpha e^{\alpha y}$$

$$\frac{d}{dx}($$

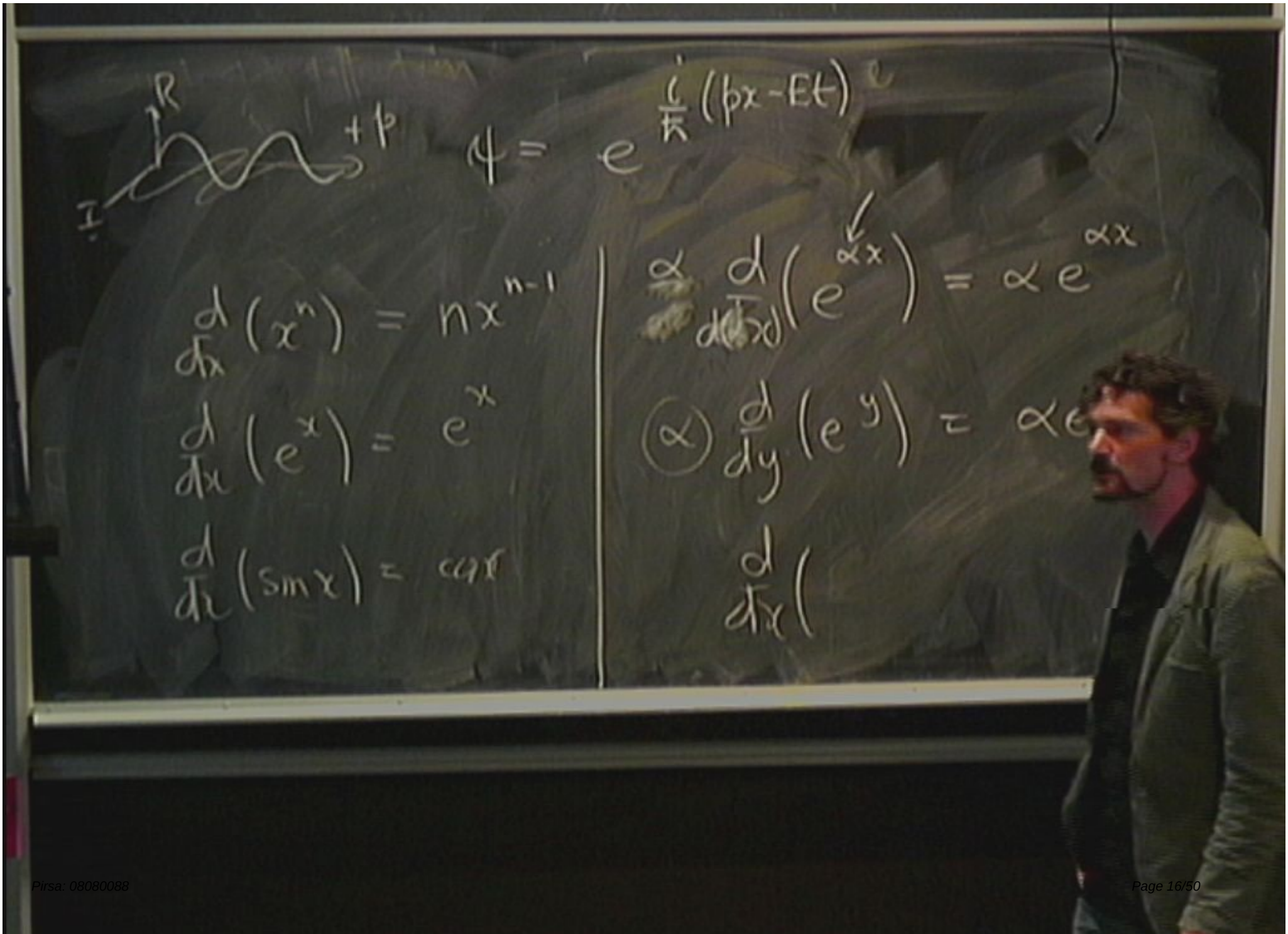

$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$

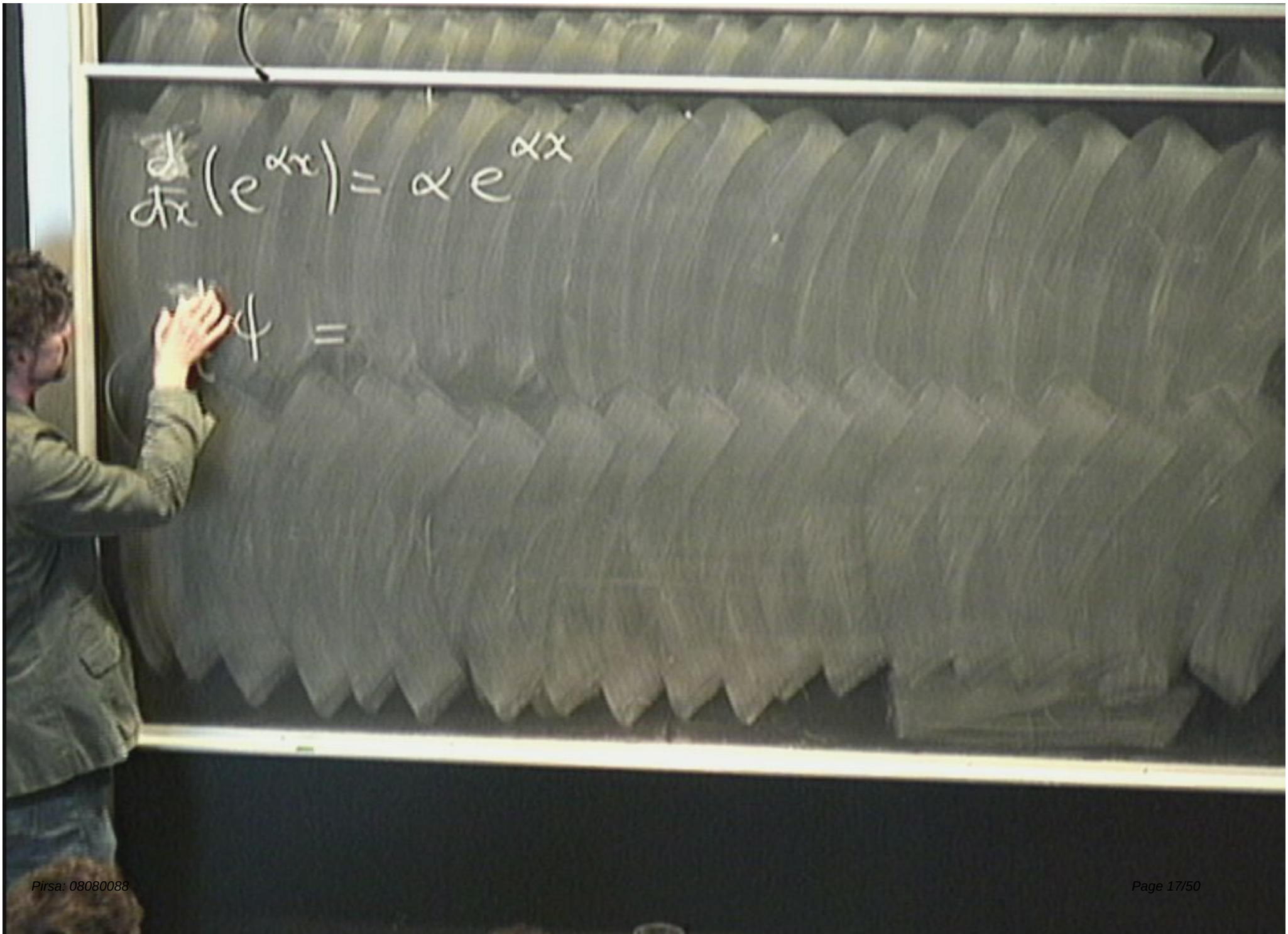

$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\frac{d}{dx}$

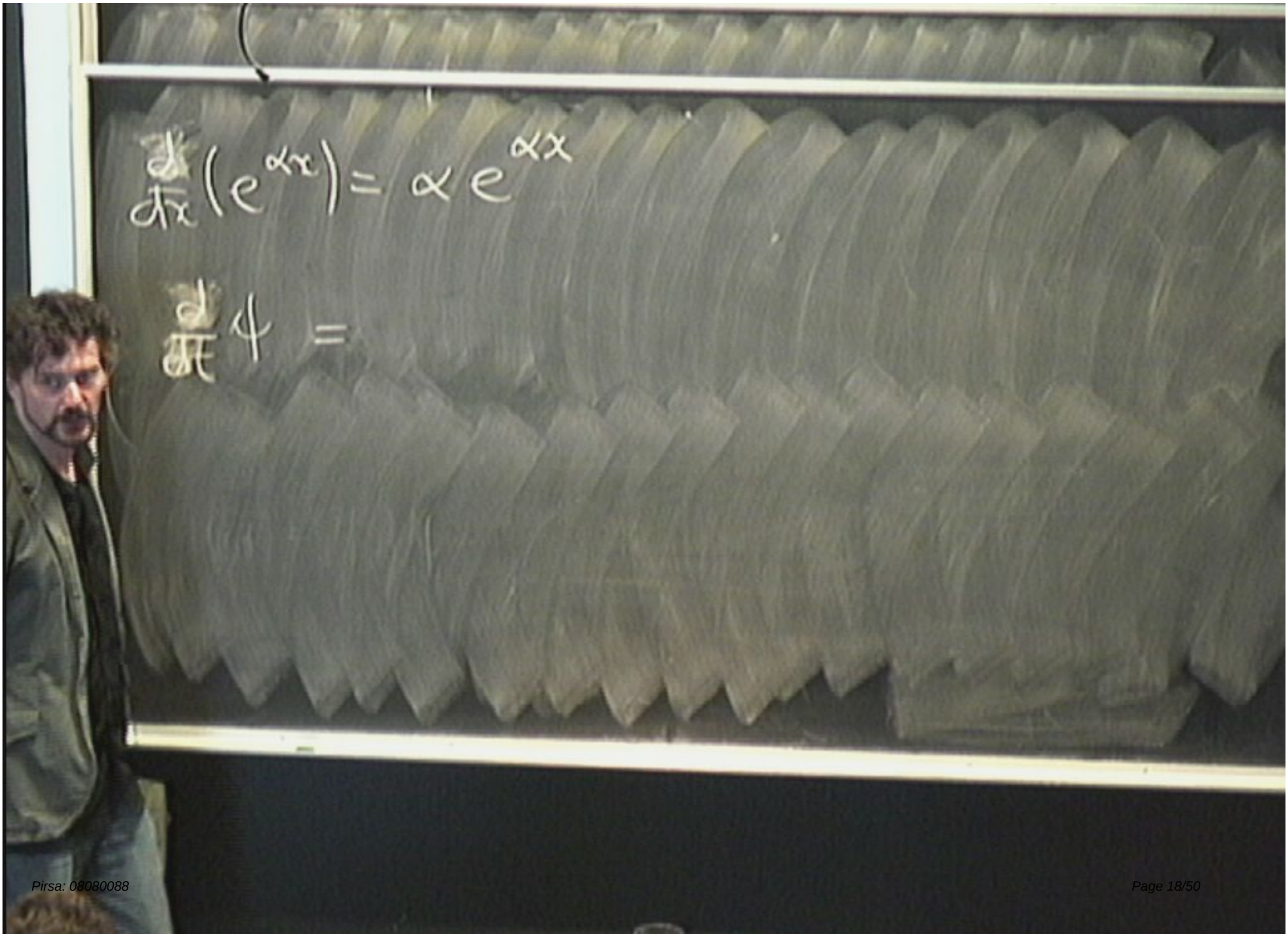

$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\frac{d}{dt}$

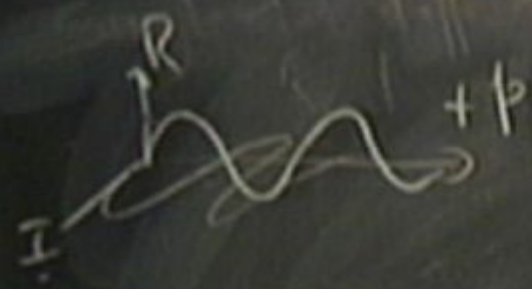
$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

$$\frac{d}{dt}\psi =$$


R
+p
I
$\psi = e^{\frac{i}{\hbar}(px - Et)}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\sin x) = \cos x$
$\alpha \frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\alpha \frac{d}{dy}(e^y) = \alpha e$
$\frac{d}{dx}($


$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\psi =$


$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\frac{d}{dt}\psi =$



$$\psi = e^{\frac{i}{\hbar}(px - Et)}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

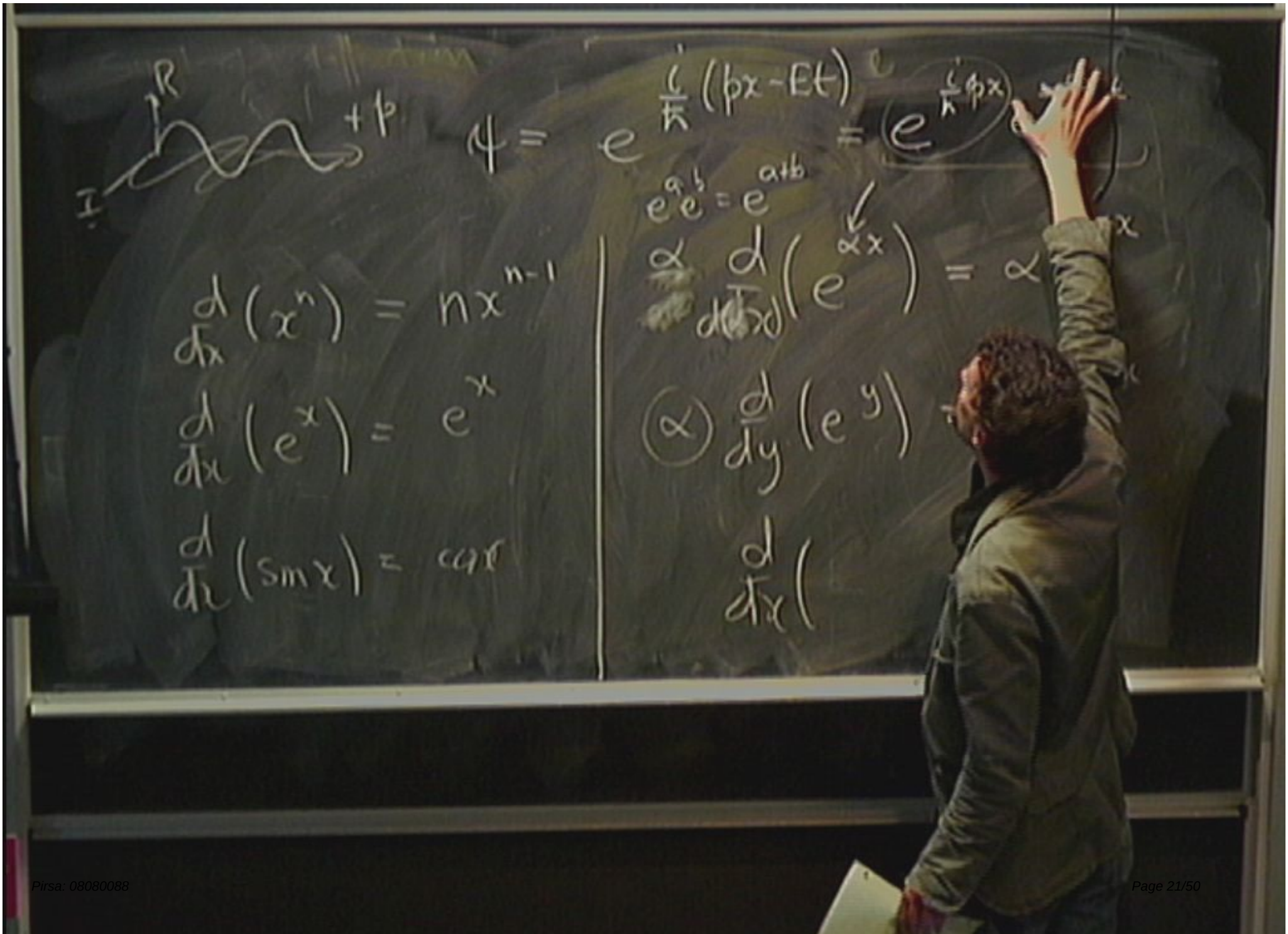
$$\alpha \frac{d}{d(\alpha x)}(e^{\alpha x}) = \alpha e^{\alpha x}$$

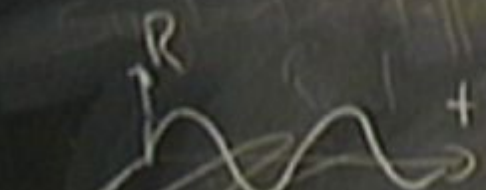
$$\alpha \frac{d}{dy}(e^y) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}($$

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}\psi =$$


R
+p
I
$\psi = e^{\frac{i}{\hbar}(px - Et)} = e^{\frac{i}{\hbar}px}$
$e^a e^b = e^{a+b}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\sin x) = \cos$
$\alpha \frac{d}{dx}(e^{\alpha x}) = \alpha$
$\alpha \frac{d}{dy}(e^y)$
$\frac{d}{dx}($



$$\psi = e^{\frac{i}{\hbar}(px - Et)} = e^{\frac{i}{\hbar}px}\, e^{-\frac{i}{\hbar}Et}$$

$$e^a e^b = e^{a+b}$$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

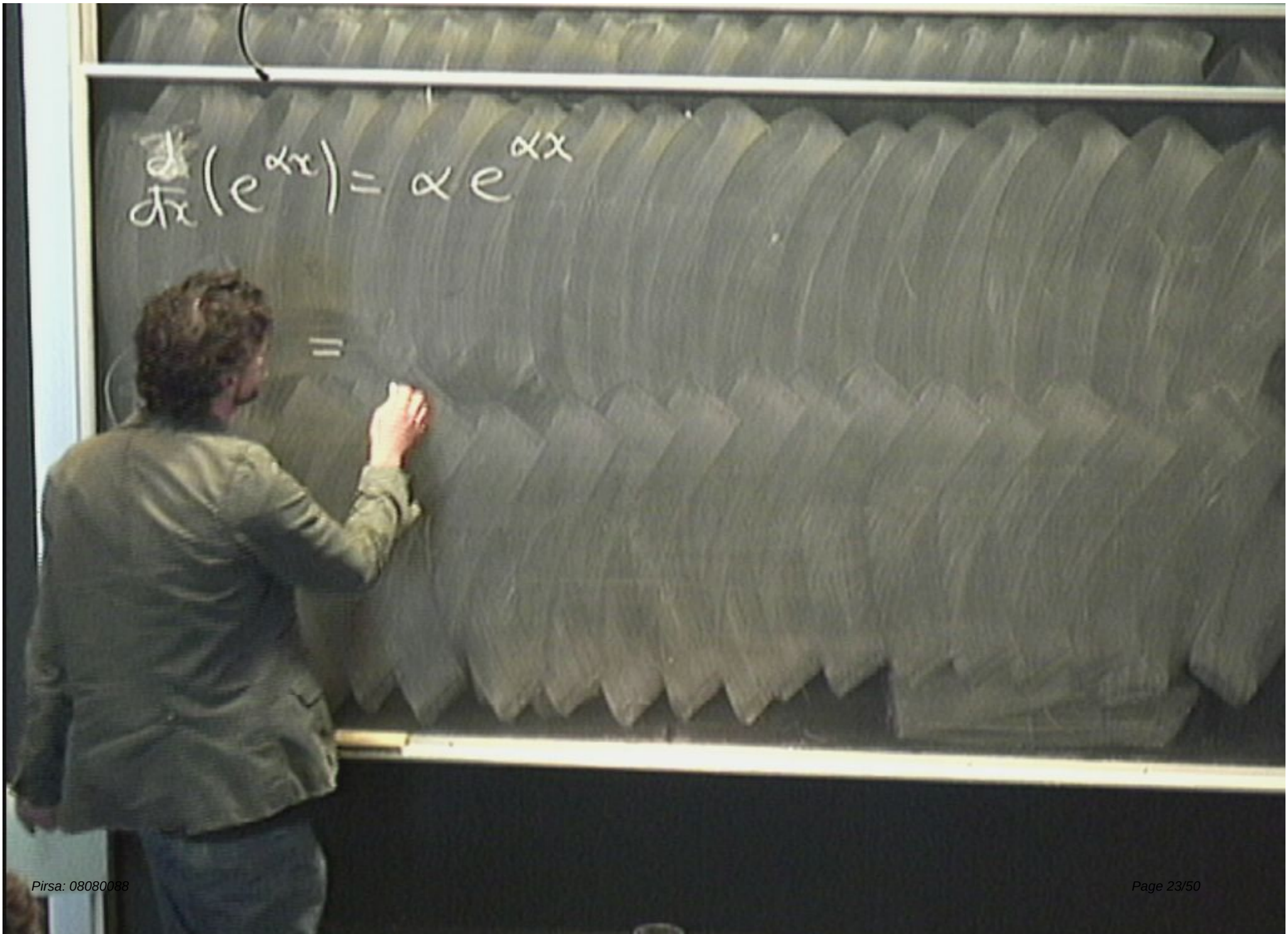
$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\alpha \frac{d}{d(\alpha x)}(e^{\alpha x}) = \alpha e^{\alpha x}$$

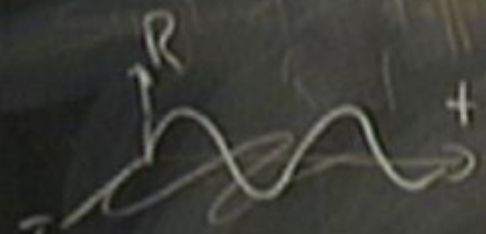
$$\alpha \frac{d}{dy}(e^y) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}($$


$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
=

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}\psi = \frac{i}{\hbar}p\psi$$



$$\psi = e^{\frac{i}{\hbar}(px - Et)} = e^{\frac{i}{\hbar}px}\, e^{-\frac{i}{\hbar}Et}$$

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$$\frac{d}{dx}($$

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

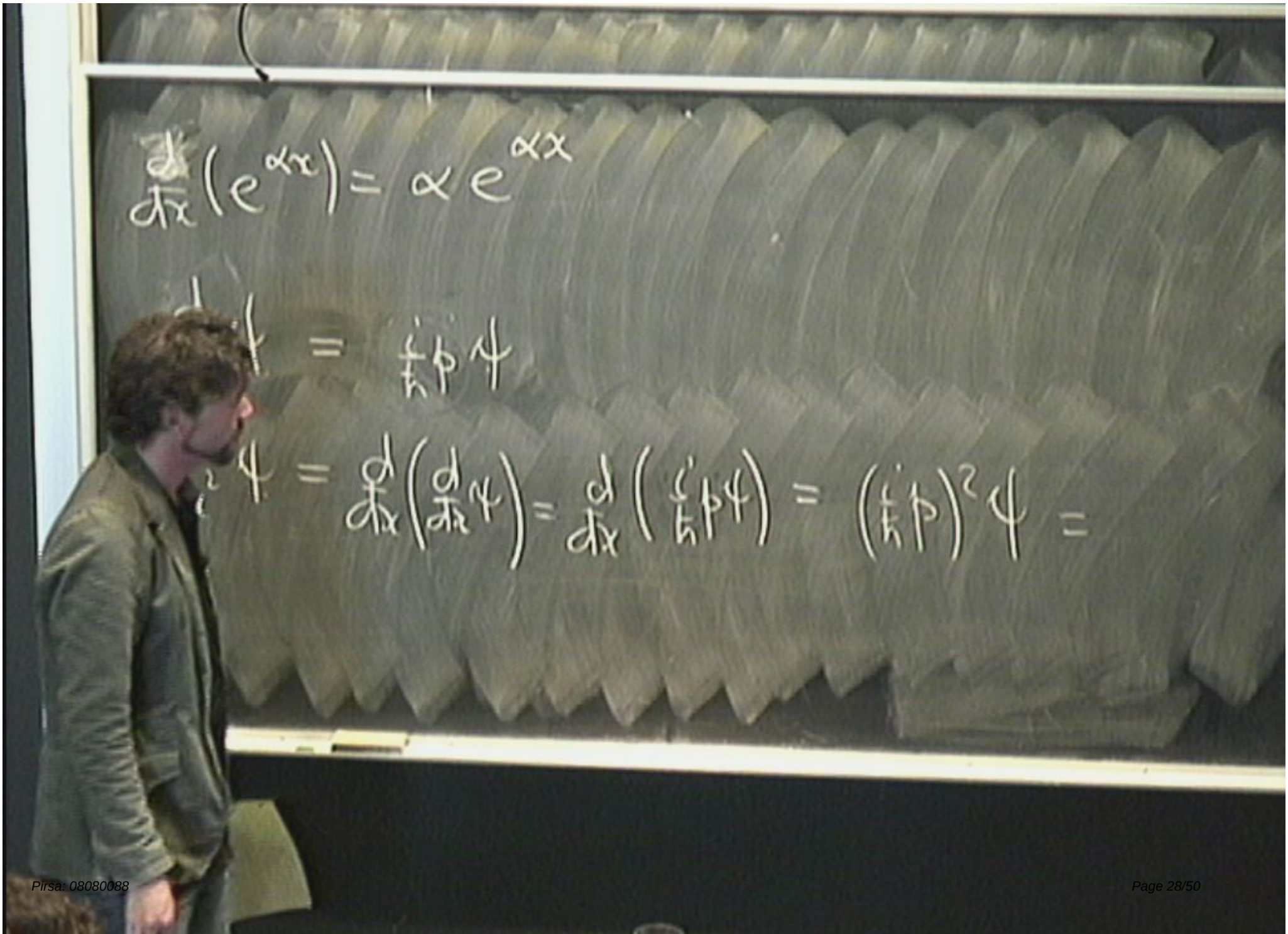
$$\frac{d}{dx}\psi = \frac{i}{\hbar}p\psi$$

$$\psi = \frac{d}{dx}\left(\frac{d}{dx}\psi\right)$$

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}\psi = \frac{i}{\hbar}p\psi$$

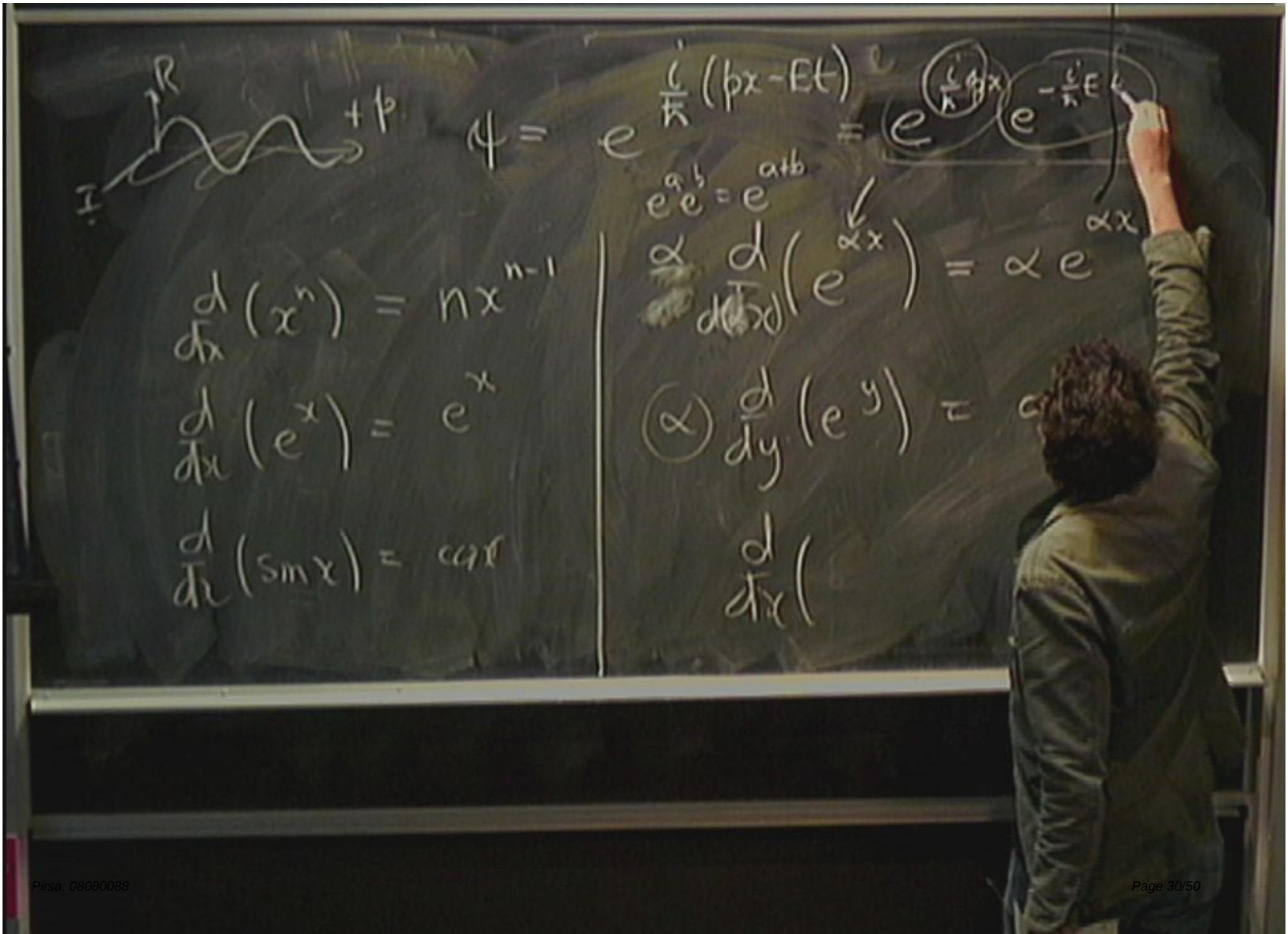
$$\frac{d^2}{dx^2}\psi = \frac{d}{dx}\left(\frac{d}{dx}\psi\right) = \frac{d}{dx}\left(\frac{i}{\hbar}p\psi\right) =$$


$\frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$
$\psi = \frac{i}{\hbar} p \psi$
$\psi = \frac{d}{dx}\left(\frac{d}{dx}\psi\right) = \frac{d}{dx}\left(\frac{i}{\hbar} p \psi\right) = \left(\frac{i}{\hbar} p\right)^2 \psi =$

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}\psi = \frac{i}{\hbar}p\psi$$

$$\frac{d^2}{dx^2}\psi = \frac{d}{dx}\left(\frac{d}{dx}\psi\right) = \frac{d}{dx}\left(\frac{i}{\hbar}p\psi\right) = \left(\frac{i}{\hbar}p\right)^2\psi = -\frac{p^2}{\hbar^2}\psi$$


$\psi = e^{\frac{i}{\hbar}(px - Et)} = e^{\frac{i}{\hbar}px} e^{-\frac{i}{\hbar}Et}$
$e^a e^b = e^{a+b}$
$\frac{d}{dx}(x^n) = nx^{n-1}$
$\frac{d}{dx}(e^x) = e^x$
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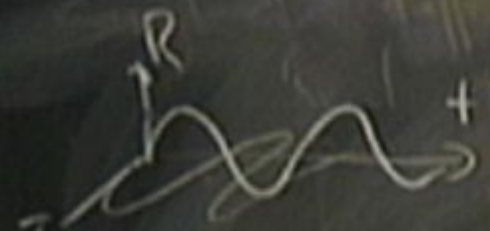
$\alpha \frac{d}{dy}(e^y) =$
$\frac{d}{dx}($



$$\psi = e^{\frac{i}{\hbar}(px - Et)} = e^{\frac{i}{\hbar}px}\, e^{-\frac{i}{\hbar}Et}$$

$$e^{a}e^{b} = e^{a+b}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^x) = e^x$$

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$$\alpha \frac{d}{dx}(e^{\alpha x}) = \alpha e^{\alpha x}$$

$$\alpha \frac{d}{dy}(e^{y}) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}($$

$$\frac{d}{dx}\left(e^{\alpha x}\right) = \alpha e^{\alpha x}$$

$$\frac{d}{dx}\psi = \frac{i}{\hbar}p\psi$$

$$\frac{d^2}{dx^2}\psi = \frac{d}{dx}\left(\frac{d}{dx}\psi\right) = \frac{d}{dx}\left(\frac{i}{\hbar}p\psi\right) = \left(\frac{i}{\hbar}p\right)^2\psi = -\frac{p^2}{\hbar^2}\psi$$

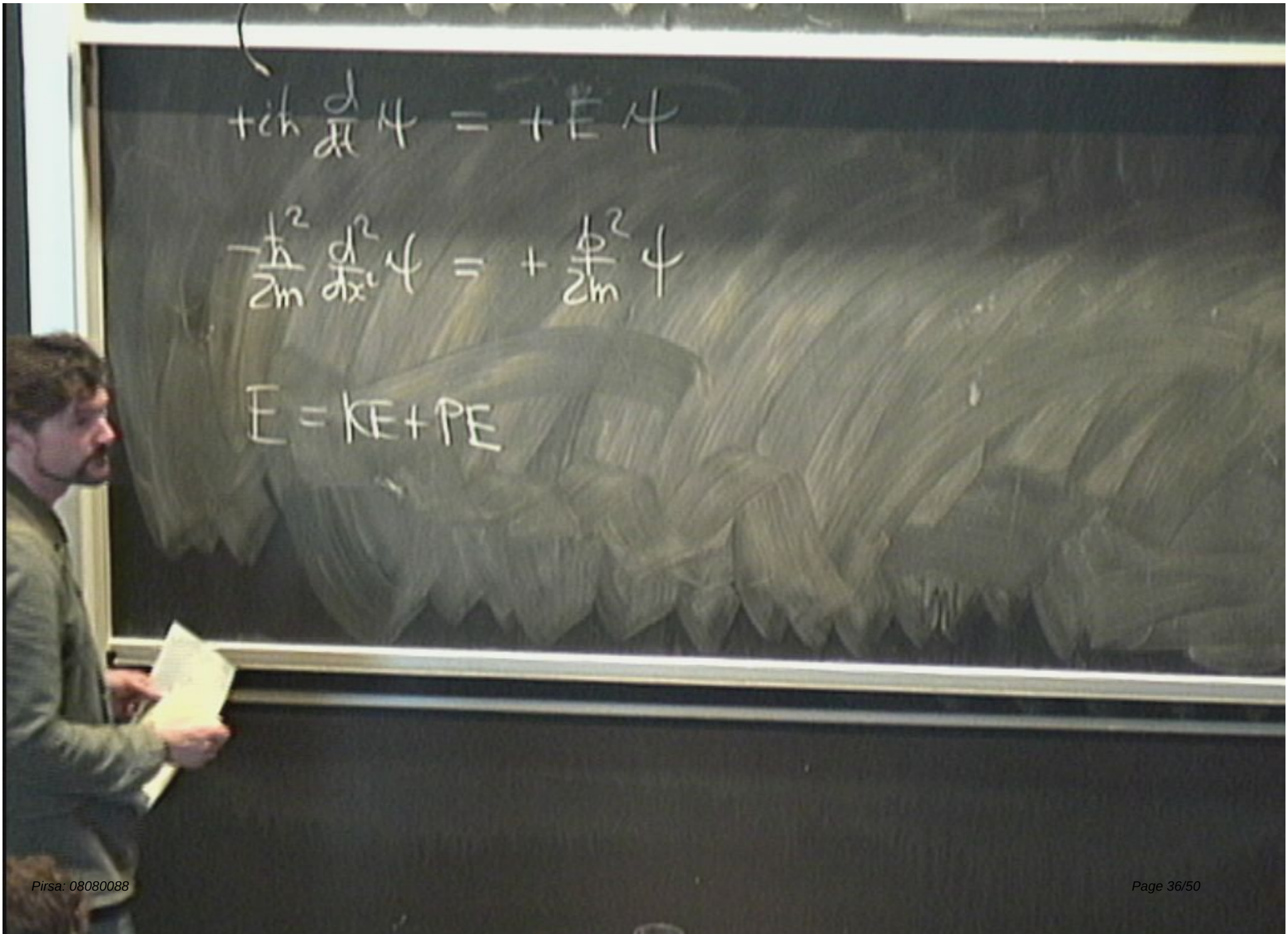
$$\frac{d}{dt}\psi = -\frac{i}{\hbar}E\psi$$

$$-i\hbar \frac{d}{dt} \Psi =$$

$$+i\hbar \frac{d}{dt}\psi = +E\psi$$

$$+i\hbar \frac{\partial}{\partial t}\psi = +E\psi$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi =$$


$+i\hbar \frac{d}{dt}\psi = +E\psi$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$
$E = KE + PE$

$$+i\hbar \frac{d}{dt}\psi = +E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$$

$$E = KE + \cancel{PE} =$$

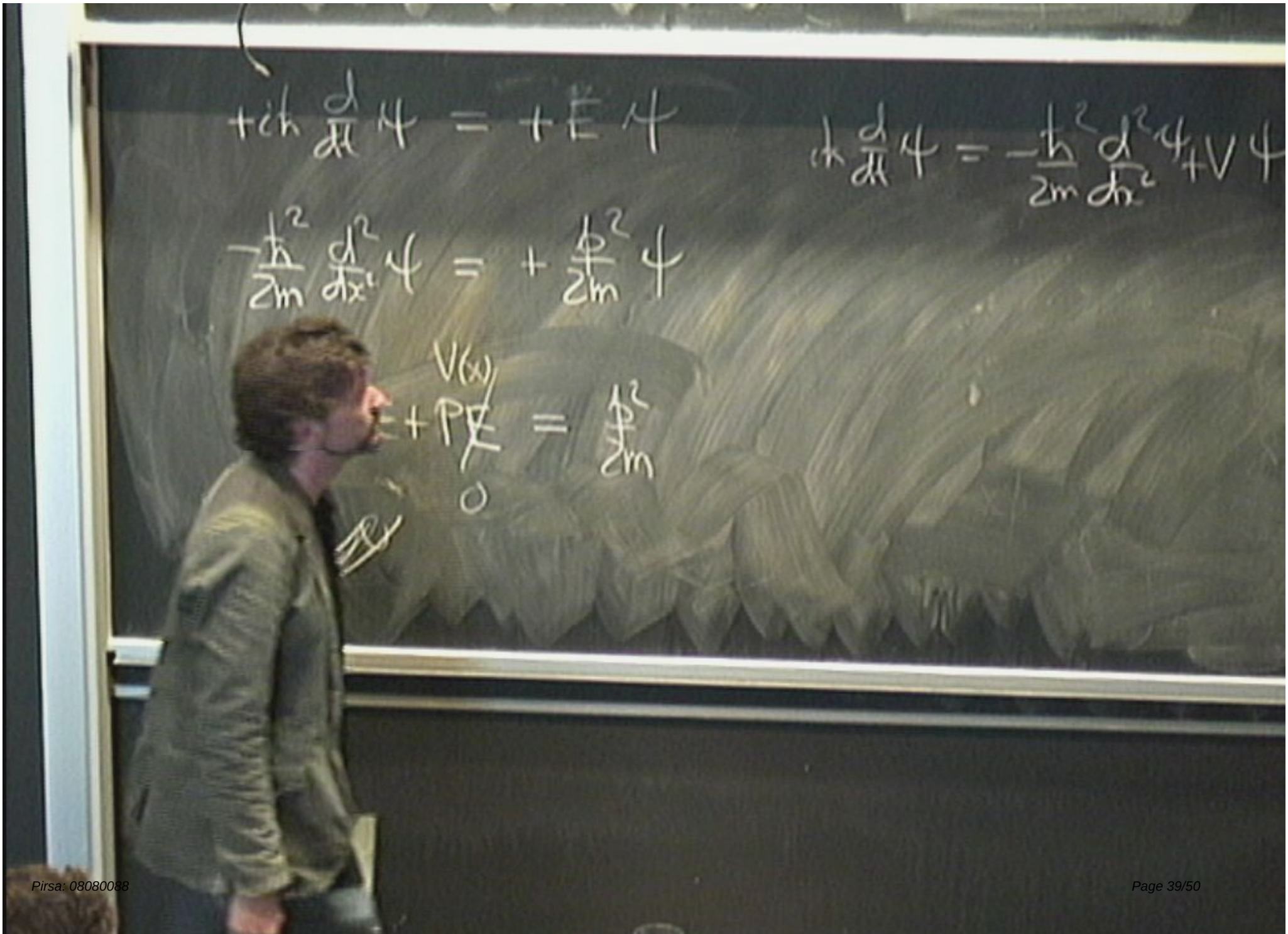
$$0$$

$$+i\hbar \frac{d}{dt}\psi = +E\psi$$

$$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$$

$$E = KE + \underbrace{\cancel{PE}}_{0} = \frac{p^2}{2m}$$


$+i\hbar \frac{d}{dt}\psi = +E\psi$
$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$
$V(x)$
$E + PE = \frac{p^2}{2m}$
0

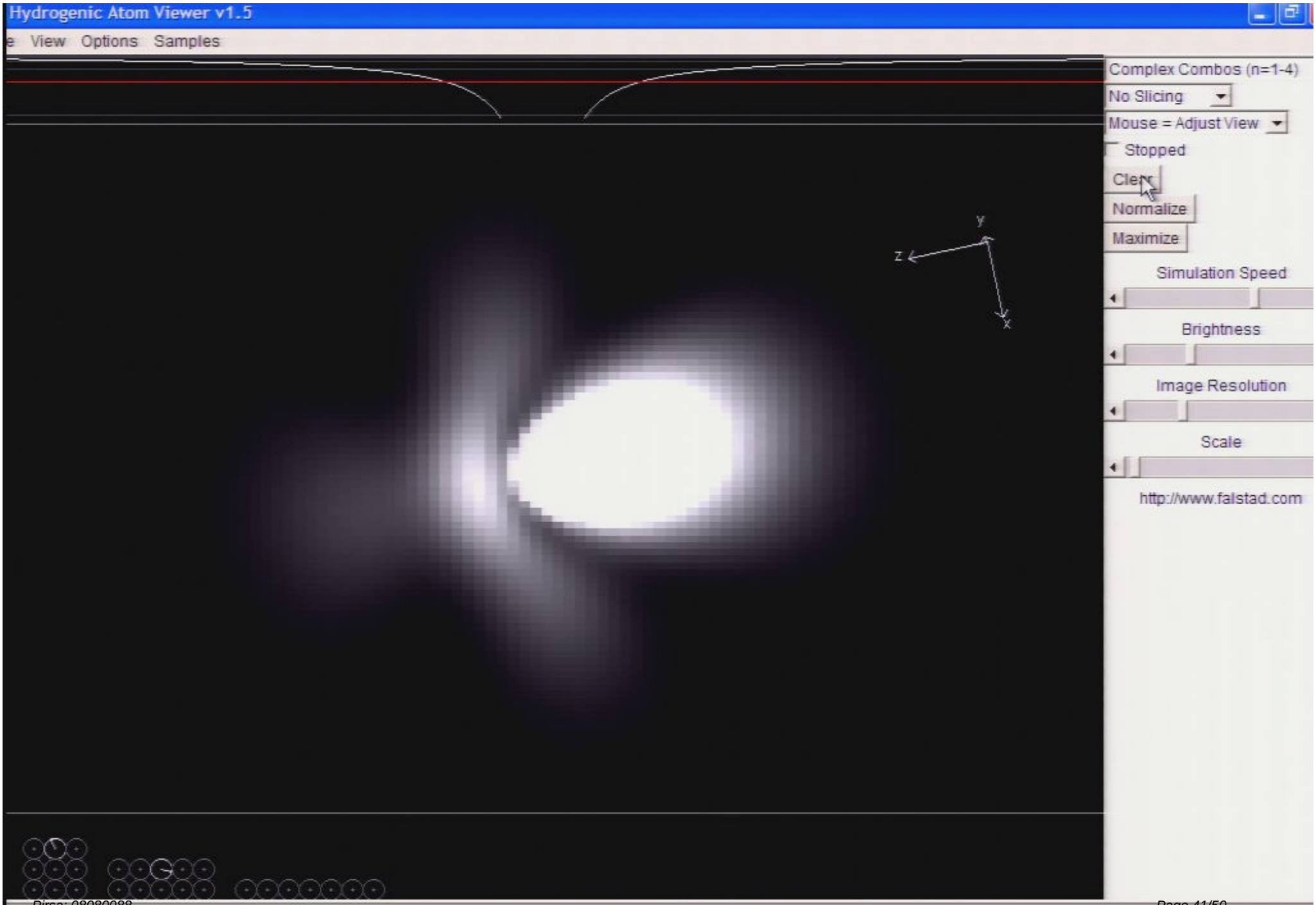
$$+i\hbar \frac{d}{dt}\psi = +E\psi$$

$$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$$

$$E = KE + \overset{V(x)}{\cancel{PE}} = \frac{p^2}{2m}$$

$$0$$


Hydrogenic Atom Viewer v1.5
View
Options
Samples
Complex Combos (n=1-4)
No Slicing
Mouse = Adjust View
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Clear
Normalize
Maximize
Simulation Speed
Brightness
Image Resolution
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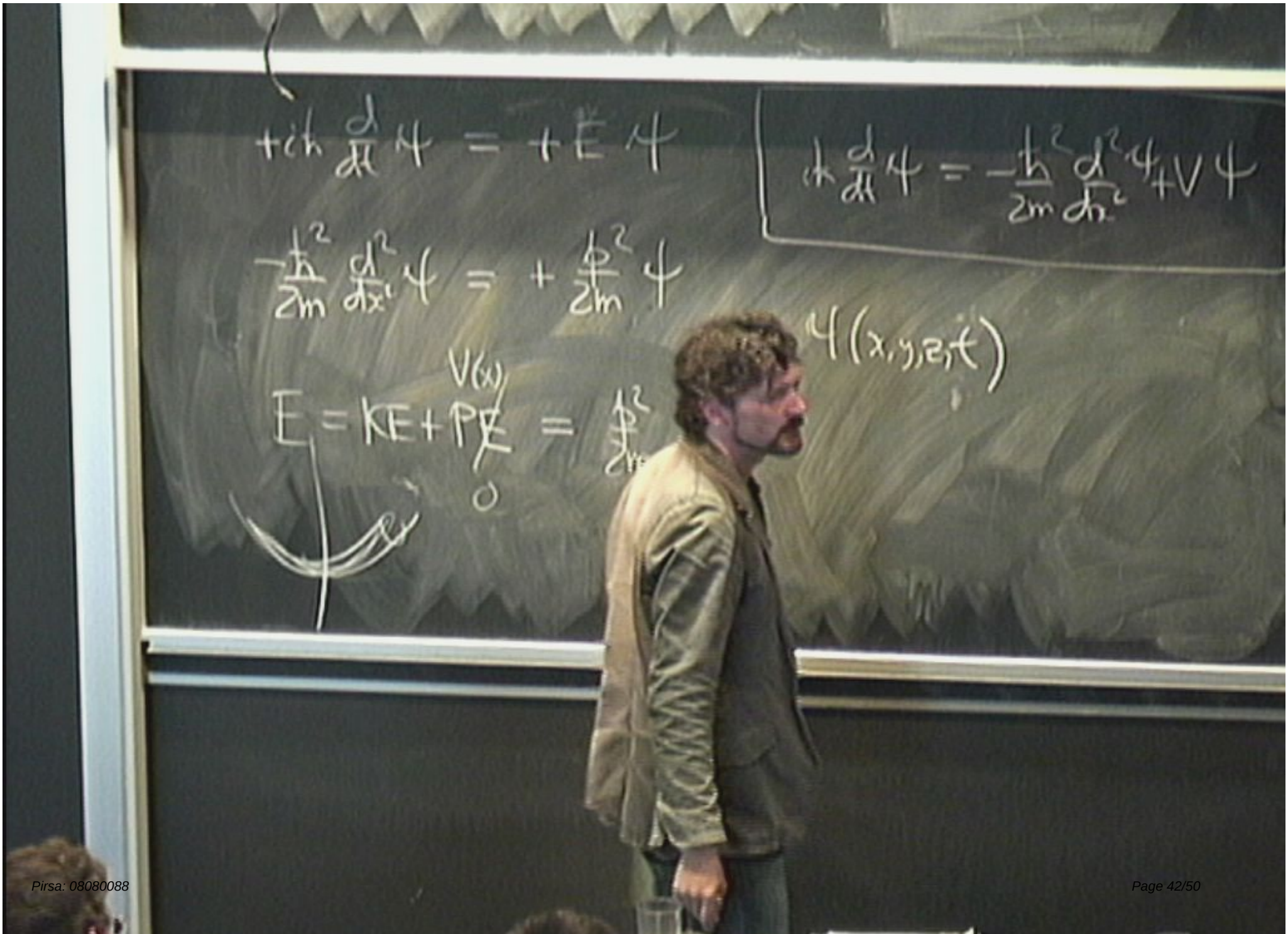

$+i\hbar \frac{d}{dt}\psi = +E\psi$
$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi$
$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$
$\psi(x,y,z,t)$
$V(x)$
$E = KE + PE = \frac{p^2}{2m}$
0

$$+i\hbar \frac{d}{dt}\psi = +E\psi$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$$

$$E = KE + \underset{0}{\cancel{PE}}^{\,V(x)} = \frac{p^2}{2m}$$

$$\boxed{i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi}$$

$$\psi(x,y,z,t)$$

$$\boxed{E^2 = m^2c^4 + c^2p^2}$$

$$+i\hbar \frac{d}{dt}\psi = +E\psi$$

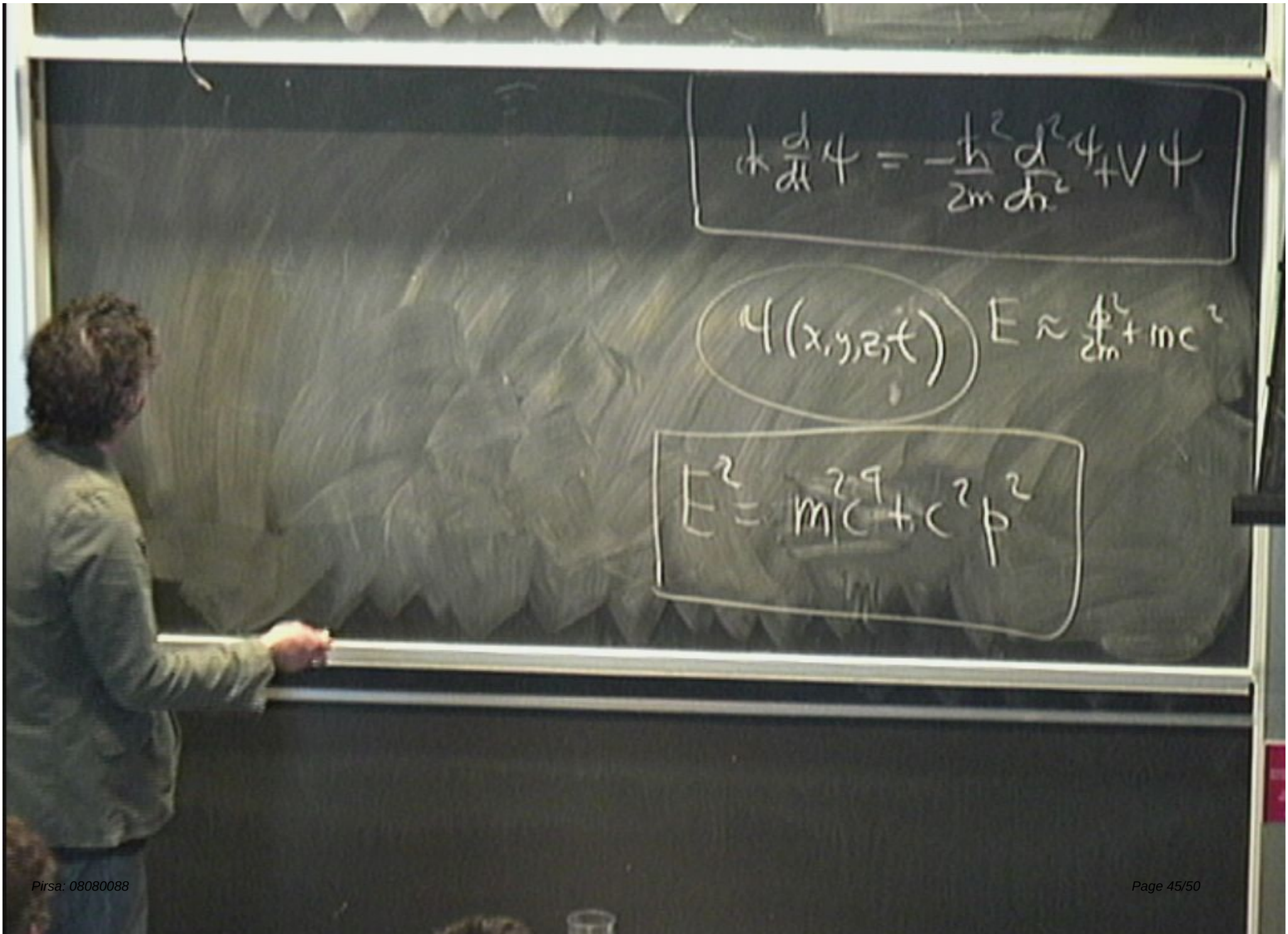
$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = +\frac{p^2}{2m}\psi$$

$$KE + \underset{0}{\overset{V(x)}{\cancel{PE}}} = \frac{p^2}{2m}$$

$$\boxed{i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi}$$

$$\psi(x,y,z,t) \qquad E \approx \frac{p^2}{2m} + mc^2$$

$$\boxed{E^2 = m^2c^4 + c^2p^2}$$


$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$
$\psi(x,y,z,t)$
$E \approx \frac{p^2}{2m} + mc^2$
$E^2 = m^2c^4 + c^2p^2$

$$\psi \to \psi_R + i\psi_I$$

$$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$$

$$\psi(x,y,z,t)$$

$$E \approx \frac{p^2}{2m} + mc^2$$

$$E^2 = m^2c^4 + c^2p^2$$

$$\psi \to \psi_R + i\psi_I$$

$$\to \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

$$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi$$

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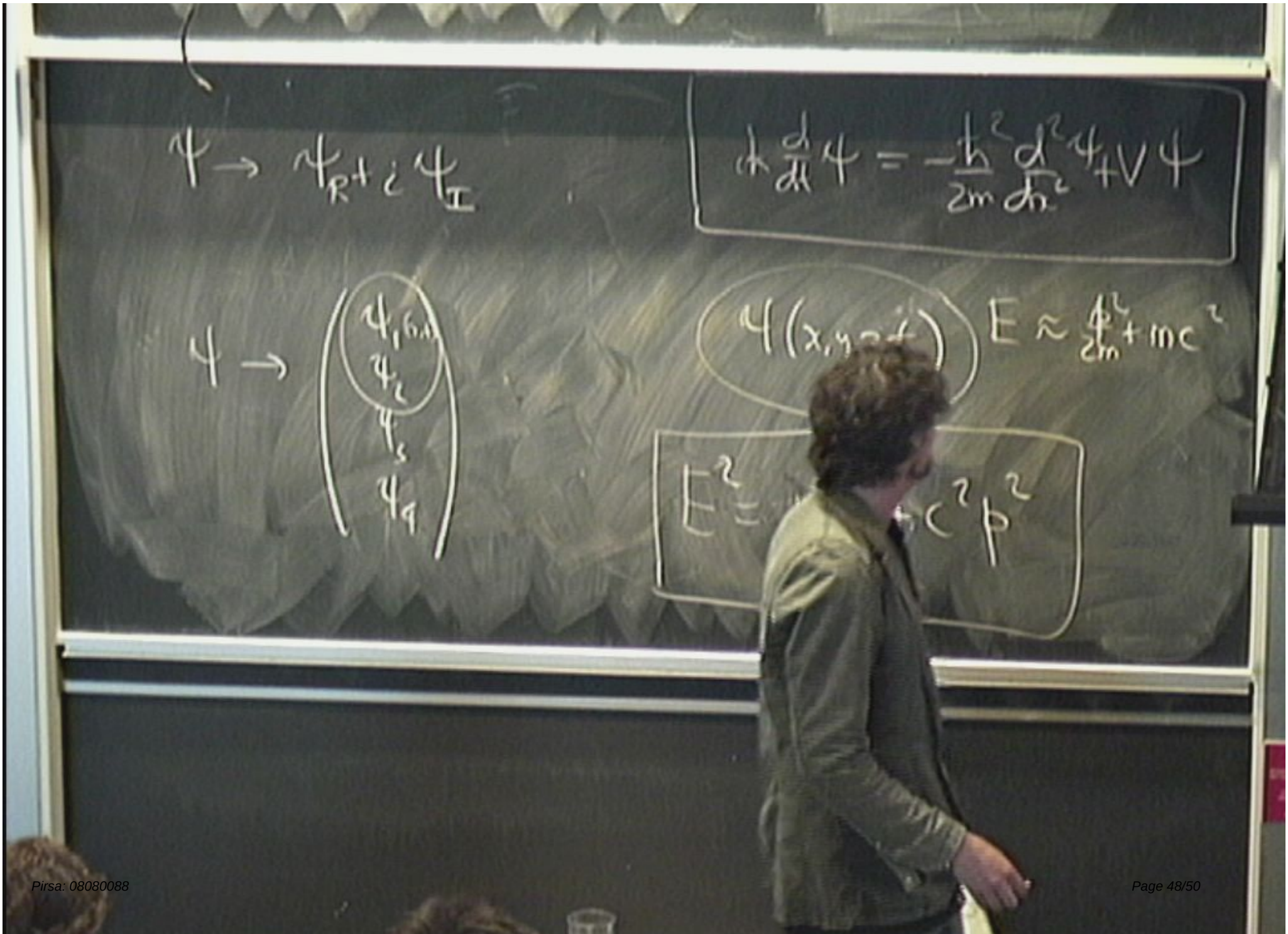
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$$\psi(x,y,z,t) \qquad E \approx \frac{p^2}{2m} + mc^2$$

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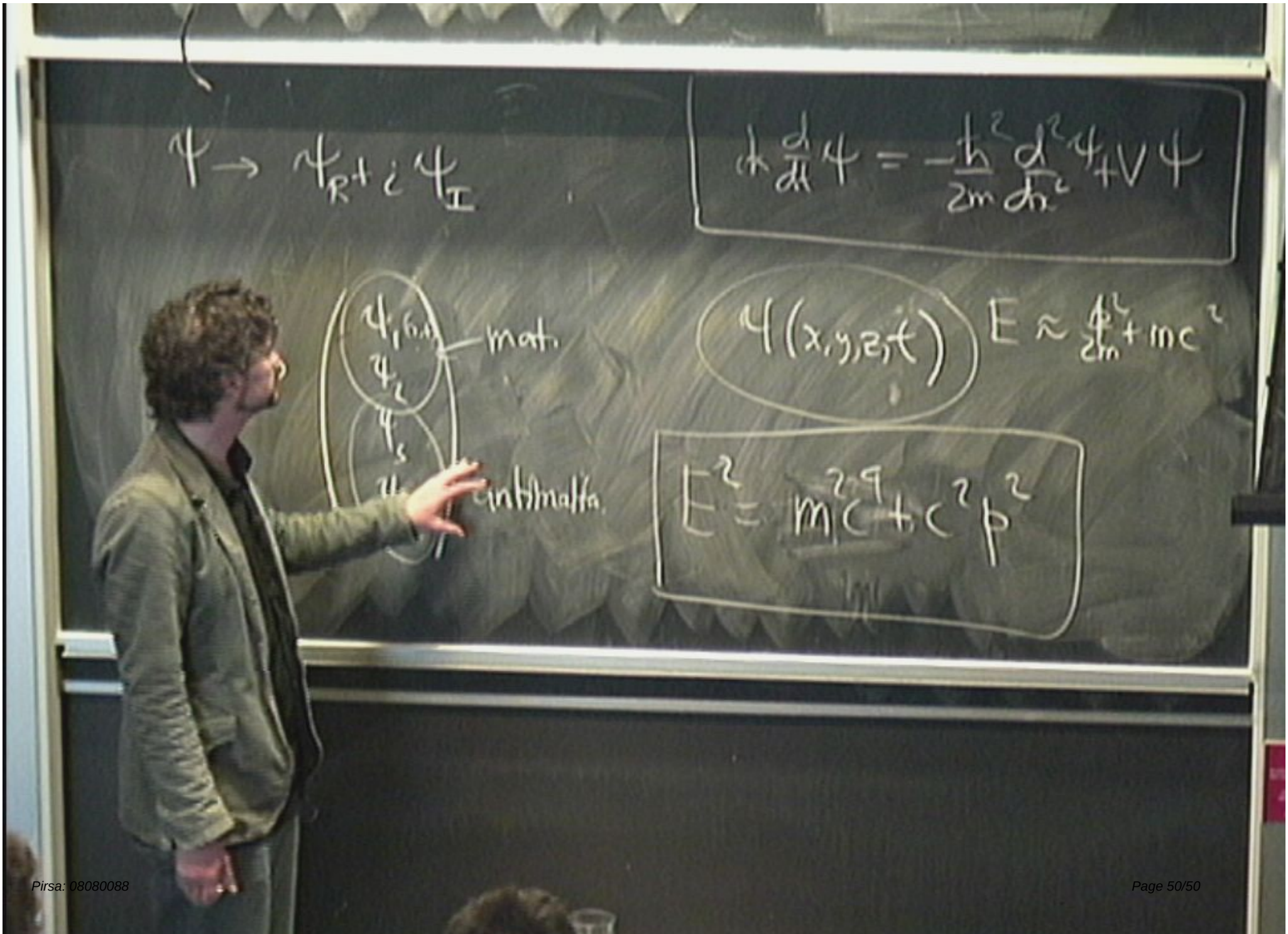
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$i\hbar \frac{d}{dt}\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V\psi$
ψ_1, ψ_2
mat.
ψ_3
ψ_4
antimatter
$\psi(x,y,z,t)$
$E \approx \frac{p^2}{2m} + mc^2$
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