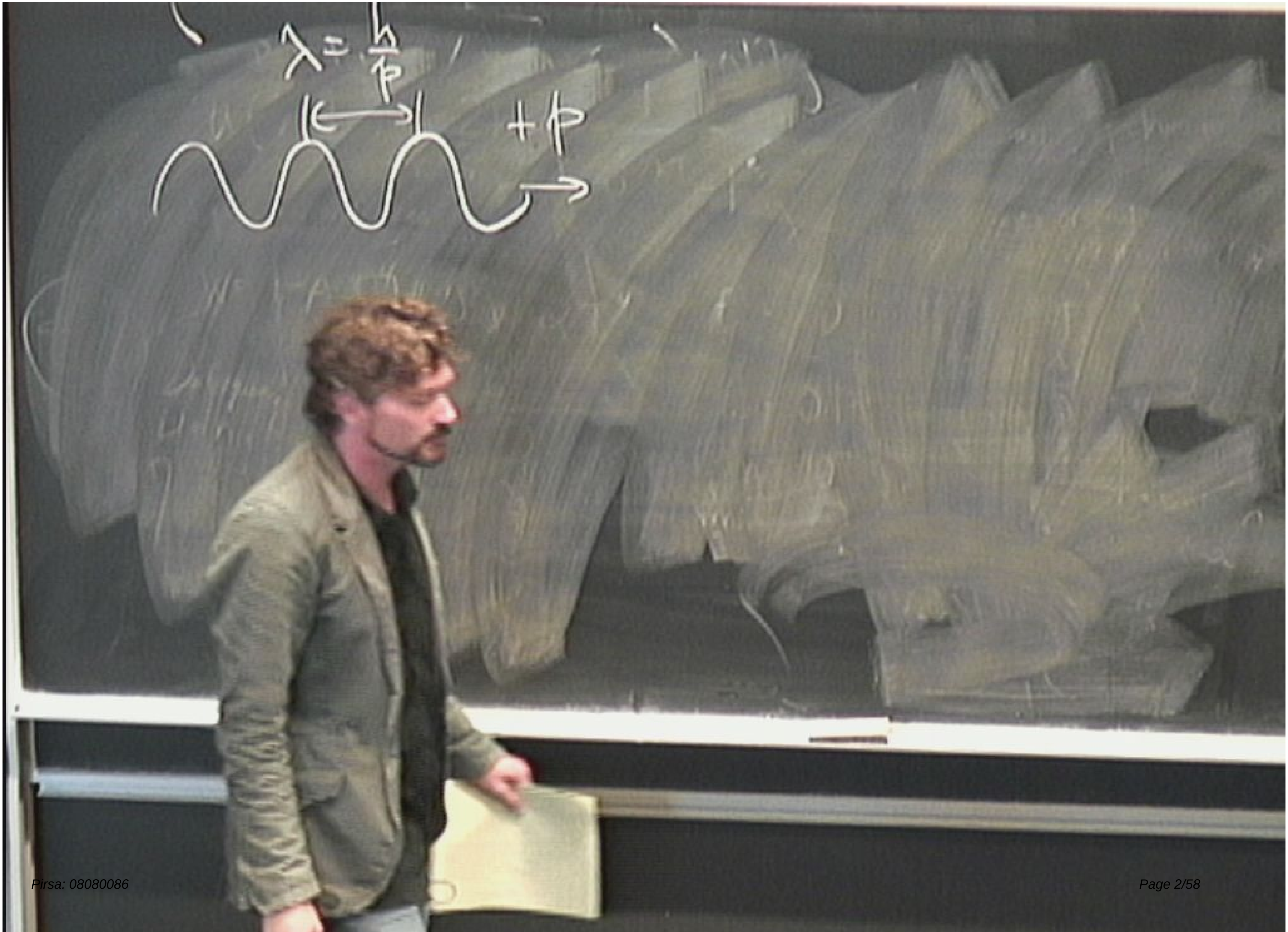
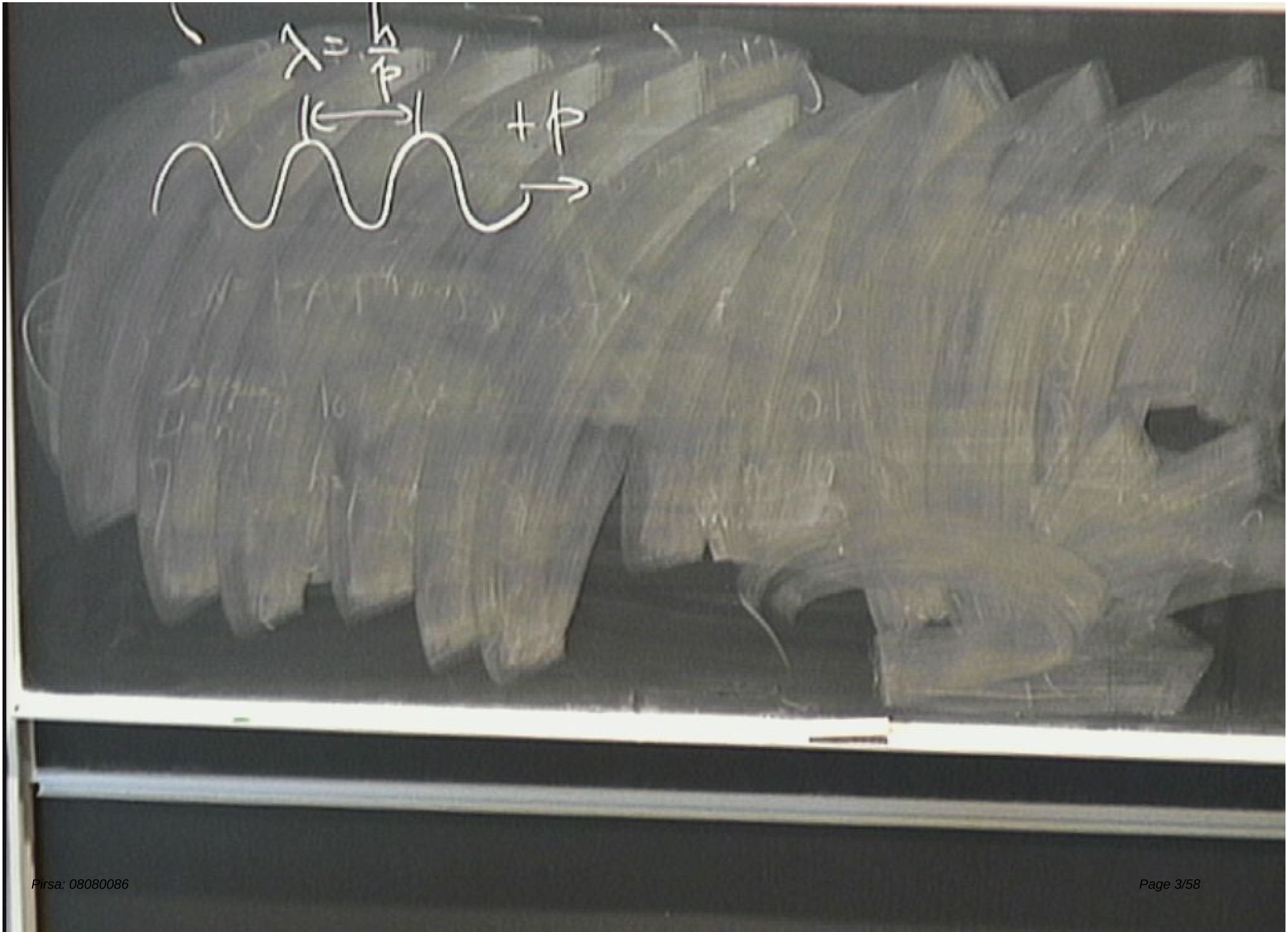
Title: Quantum Mechanics 11 - De Broglie Waves Are Complex

Date: Aug 16, 2008 02:30 PM

URL: http://pirsa.org/08080086

Abstract: The de Broglie waves we have been using thus far were assumed to be real functions; we discuss why this is wrong and how to fix the problem.
Learning Outcomes:
â€¢ Understanding why there is a serious flaw with using real de Broglie waves, and how using a complex wave (one with both a real and an imaginary part) solves the problem.
â€¢ Understanding how the de Broglie wave corresponding to a free particle is like a moving corkscrew, with a magnitude that is uniform across space and constant in time.
â€¢ When right- and left-travelling de Broglie waves (â€œcorkscrewsâ€•) are added, as happens for a particle in a box, we get a complex standing wave whose magnitude is constant in time.


λ = h/p
+p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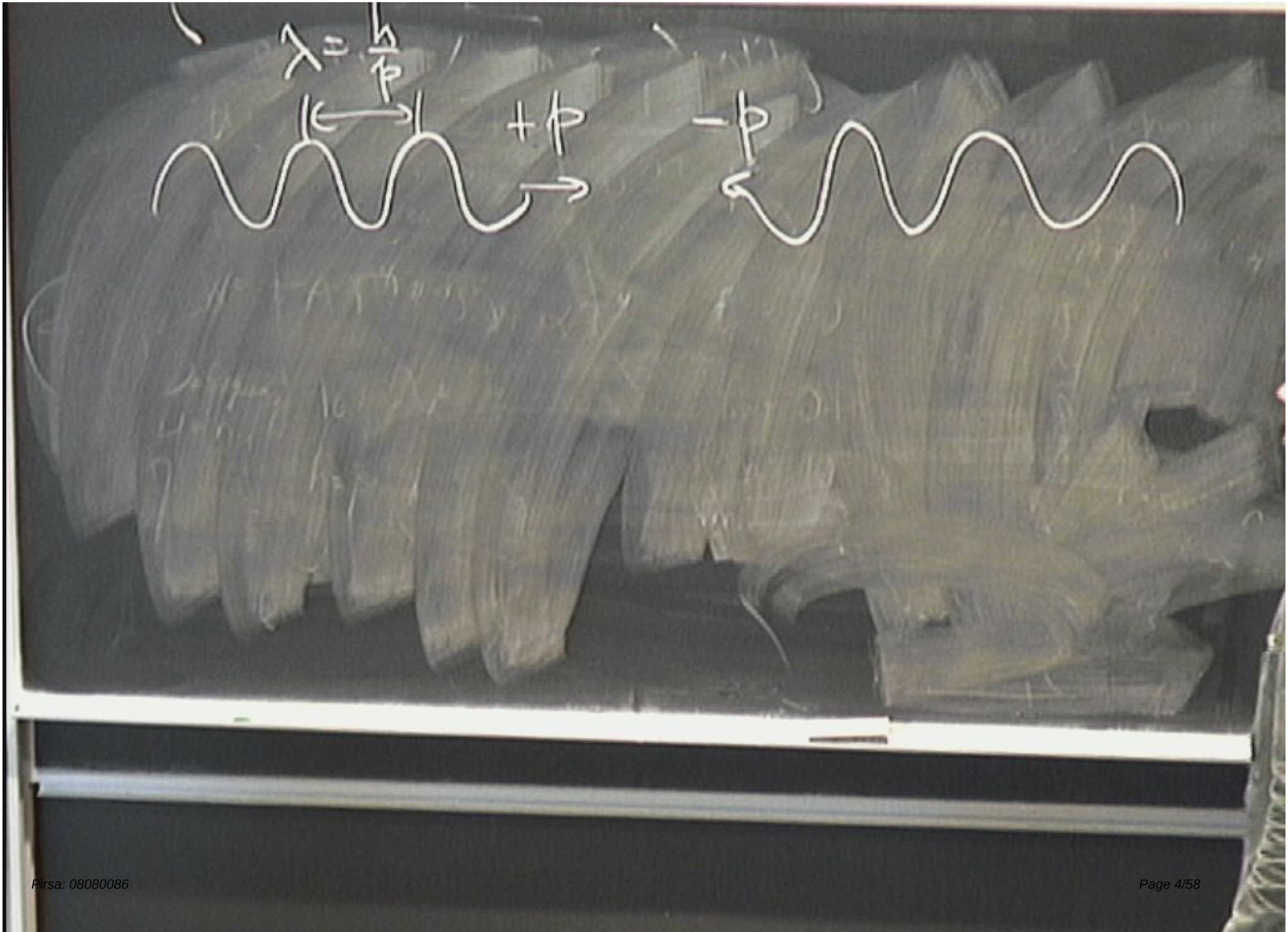
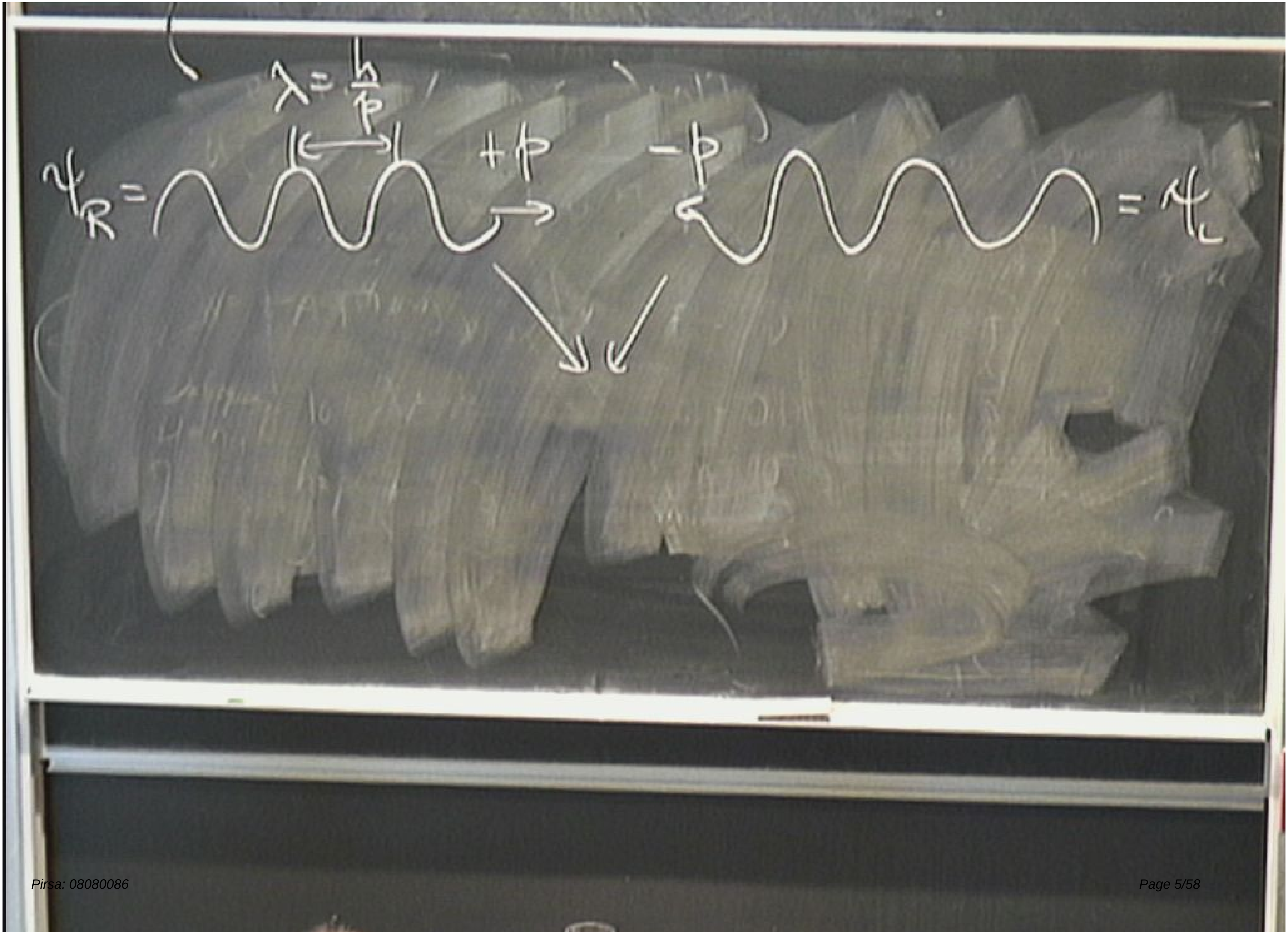
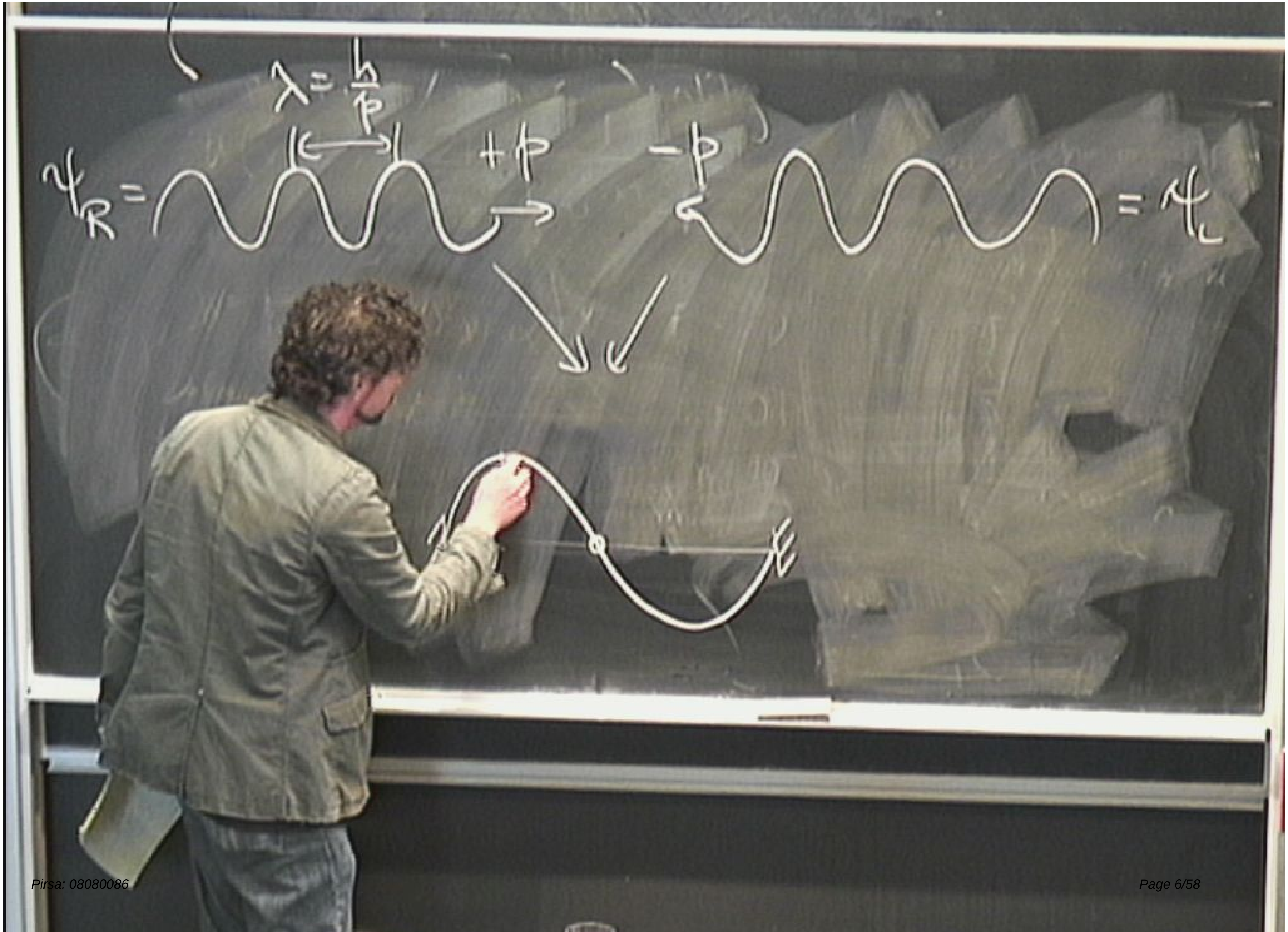
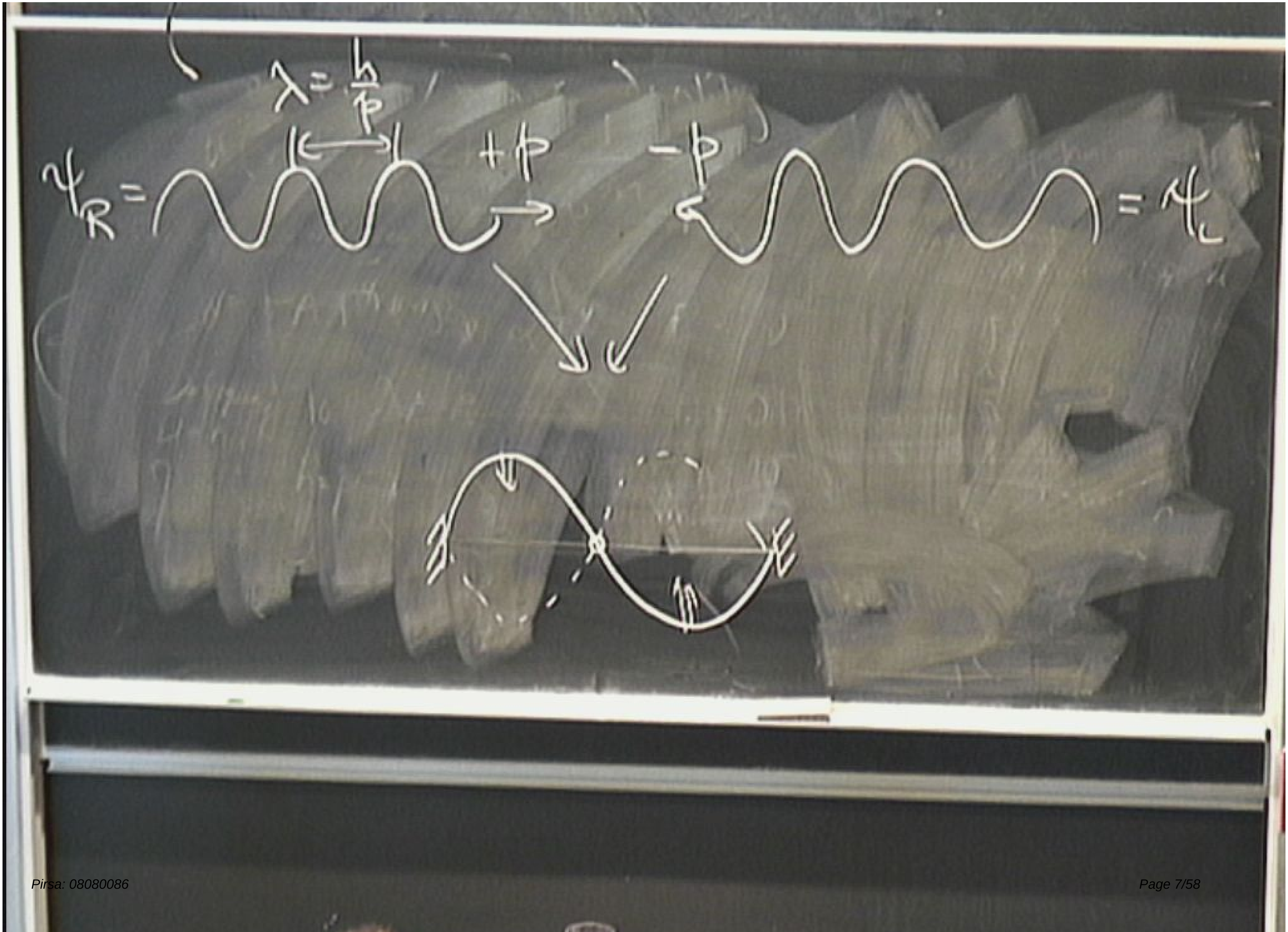
+p

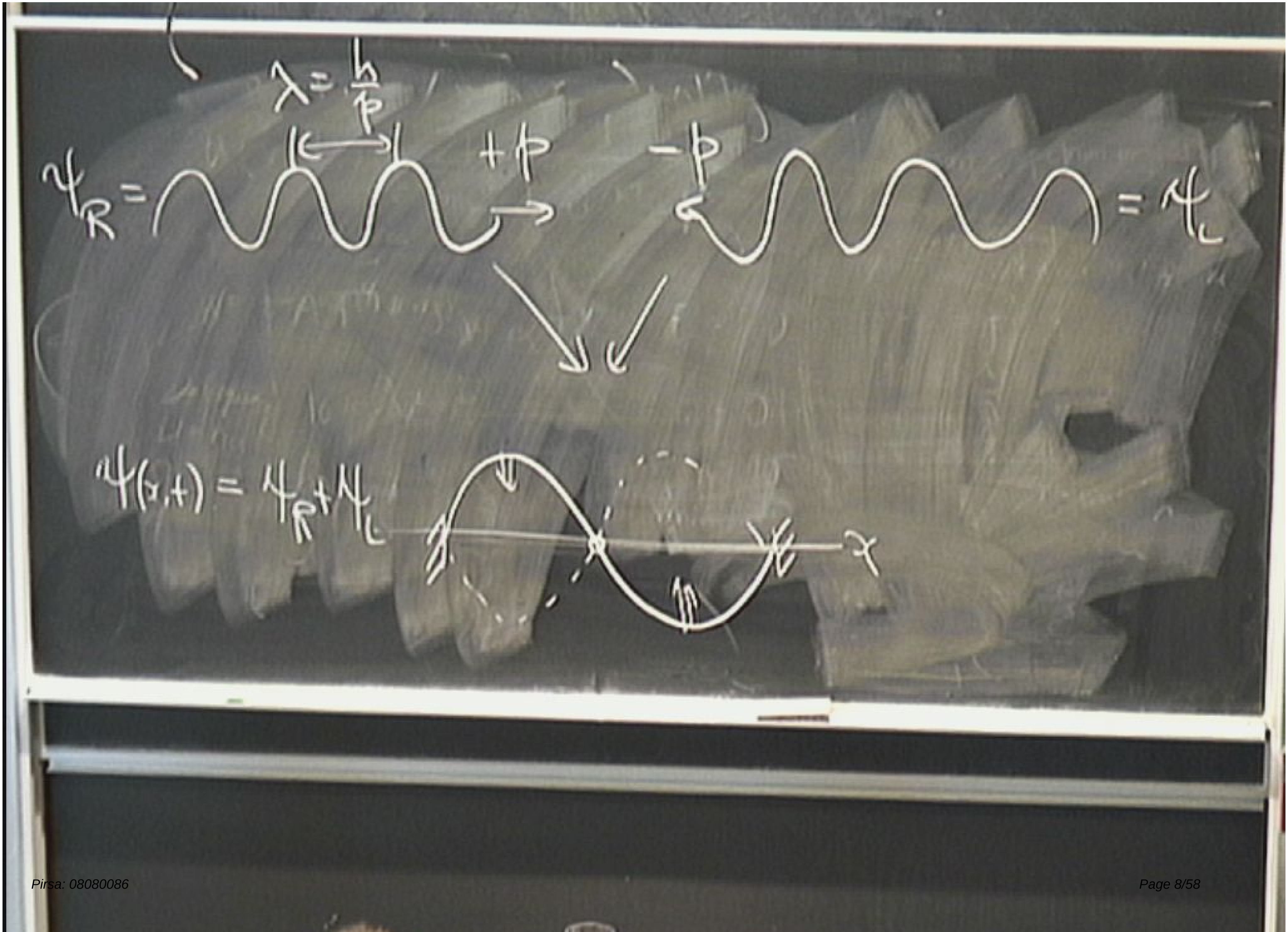

$\lambda = \frac{h}{p}$
$+p$
$-p$

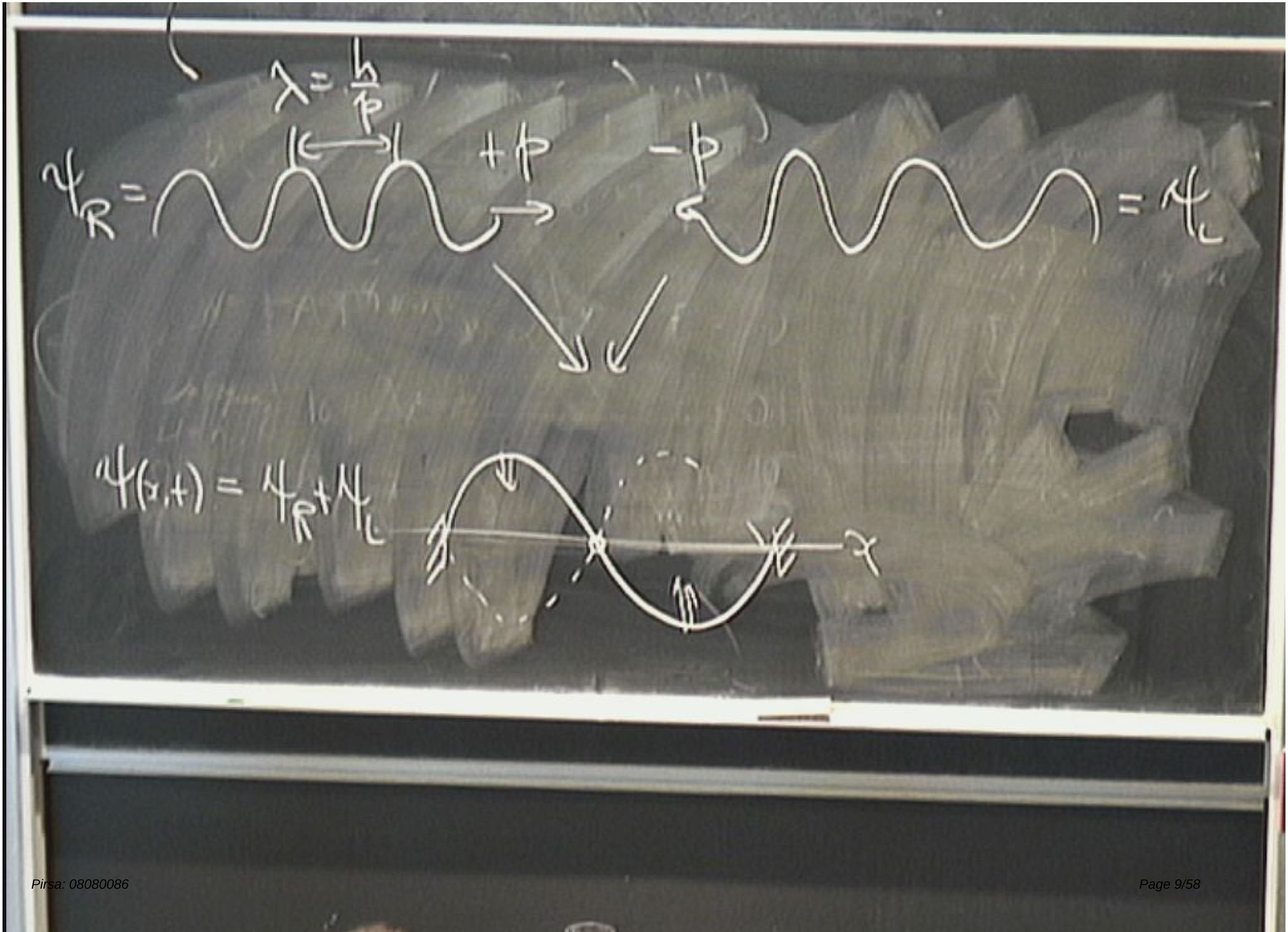

λ = h/p
+p
−p
ψ_R =
= ψ_L

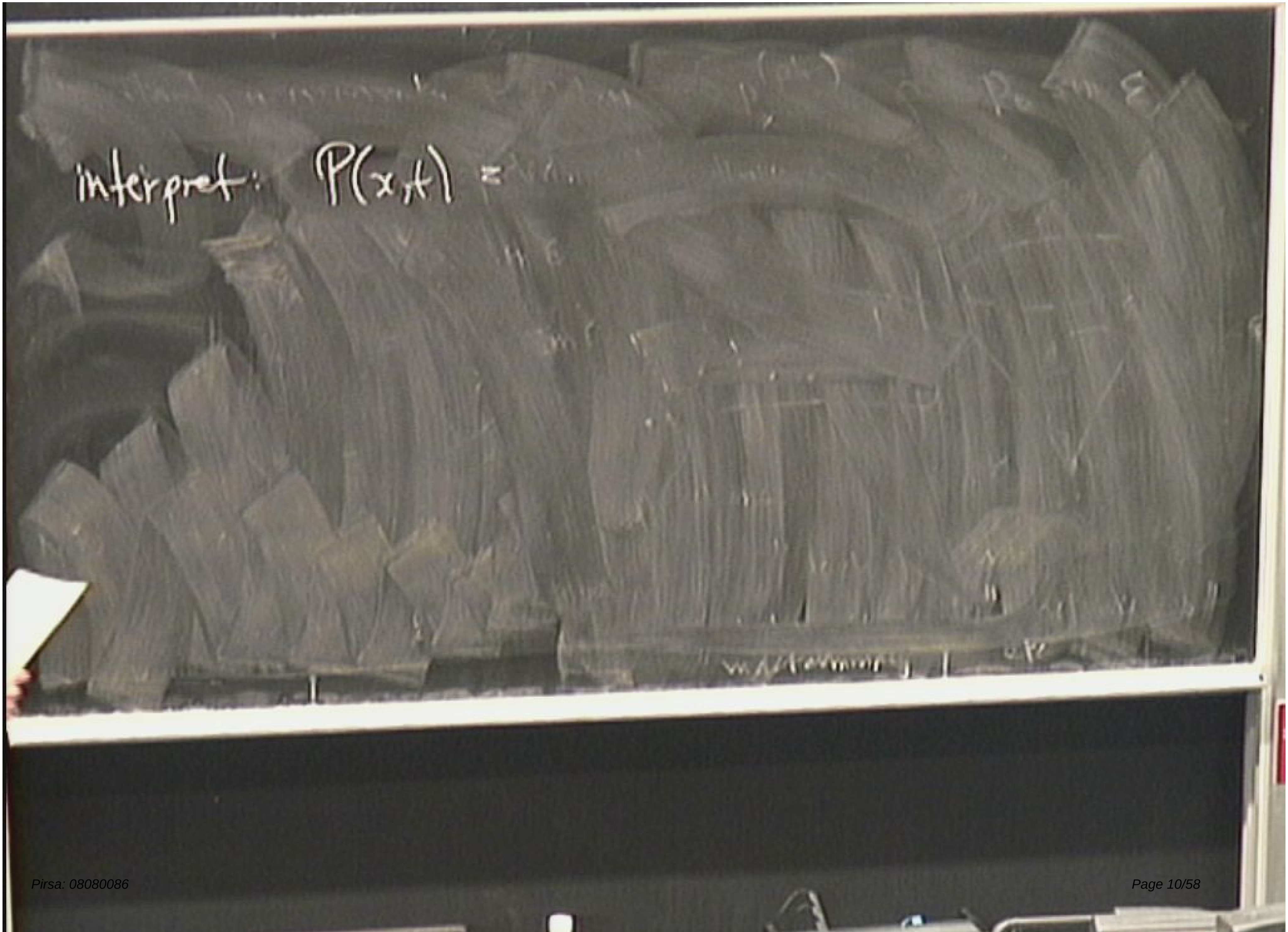


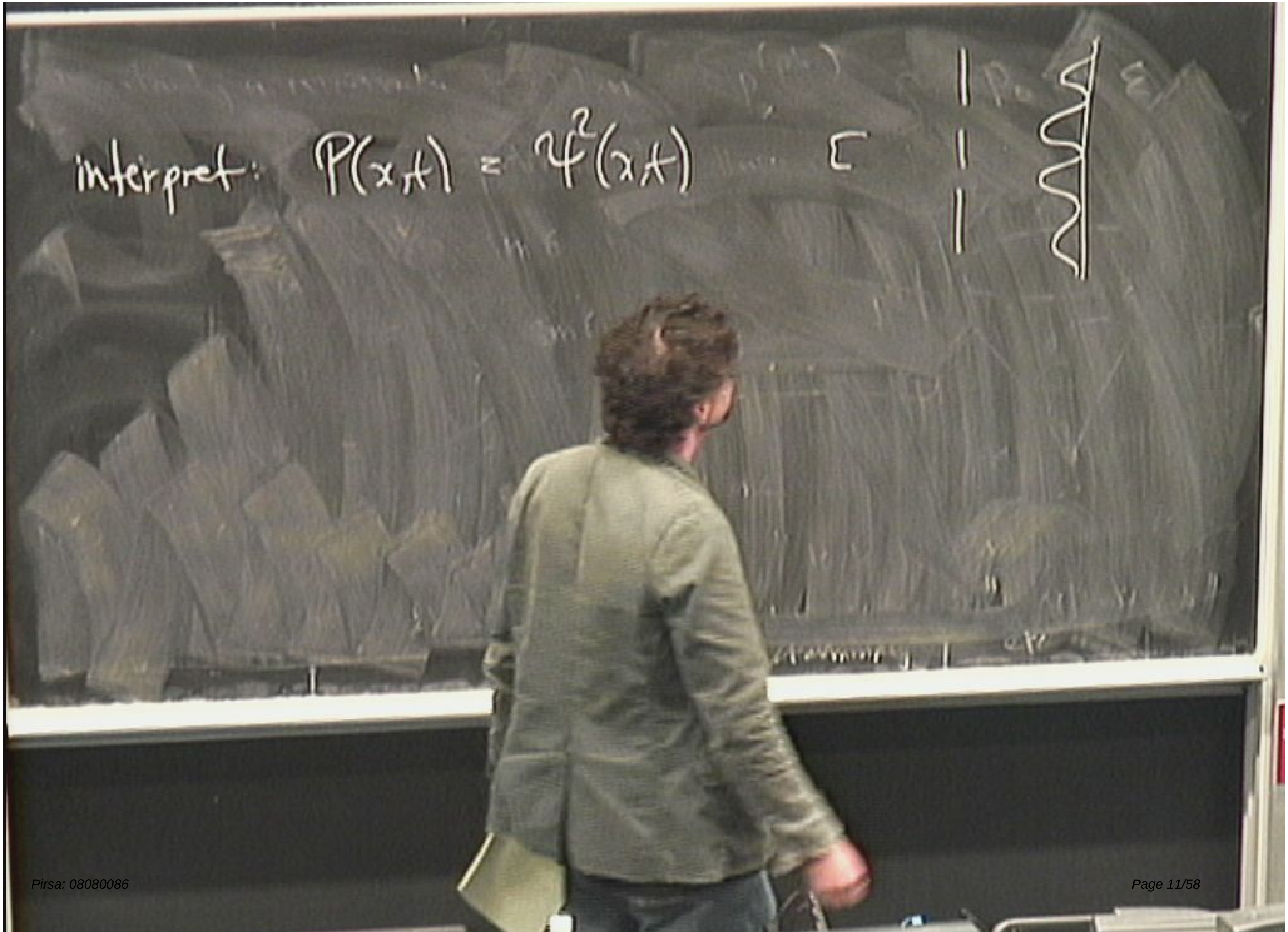
λ = h/p
+p
−p
ψ_R =
= ψ_L
E


λ = h/p
ψR =
+p
−p
= ψL

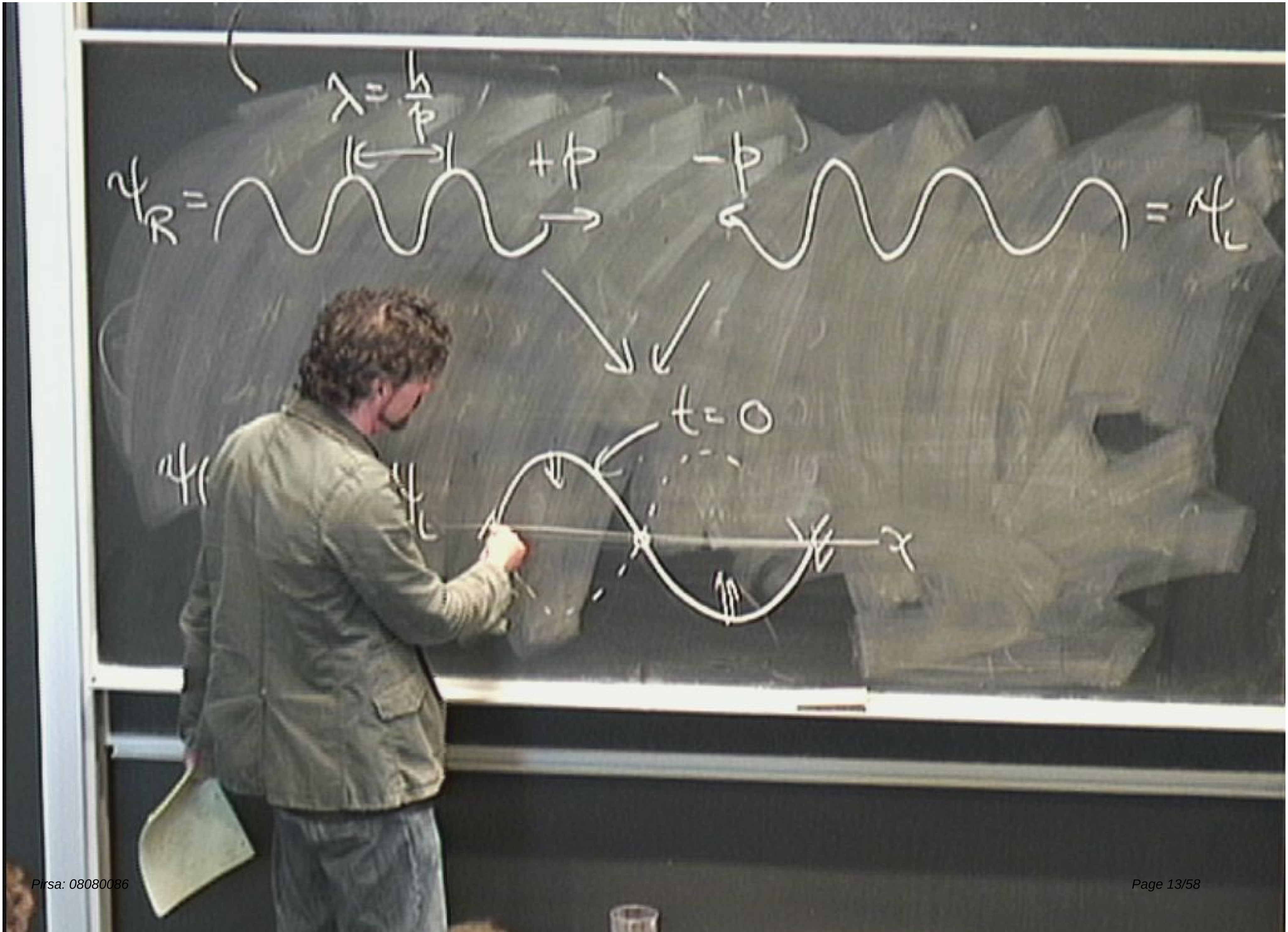

$\lambda = \frac{h}{p}$
$+p$
$-p$
$\psi_R =$
$= \psi_L$
$\psi(x,t) = \psi_R + \psi_L$
x

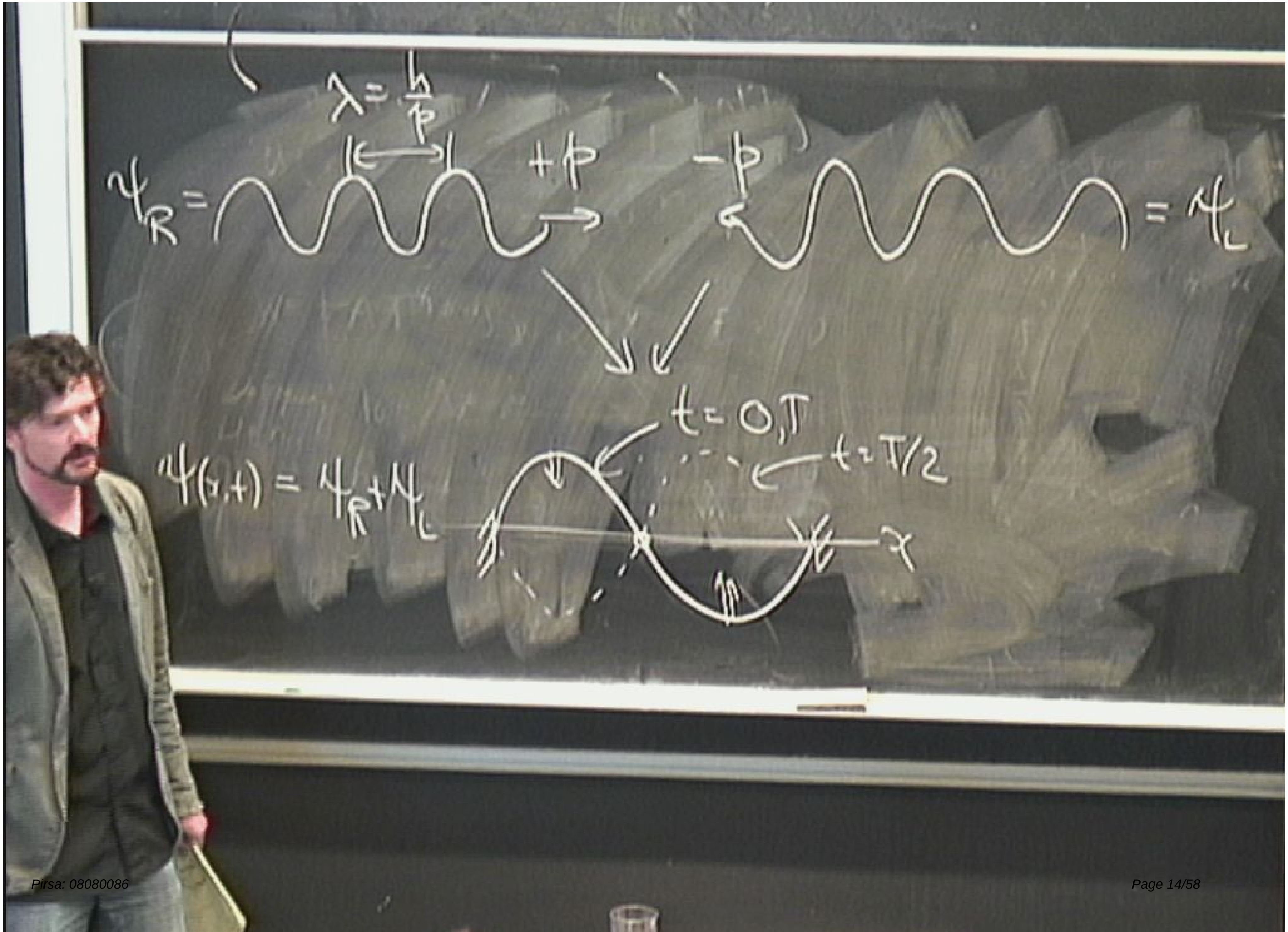

$\lambda = \frac{h}{p}$
$\psi_R =$
$+p$
$-p$
$= \psi_L$
$\psi(x,t) = \psi_R + \psi_L$
x

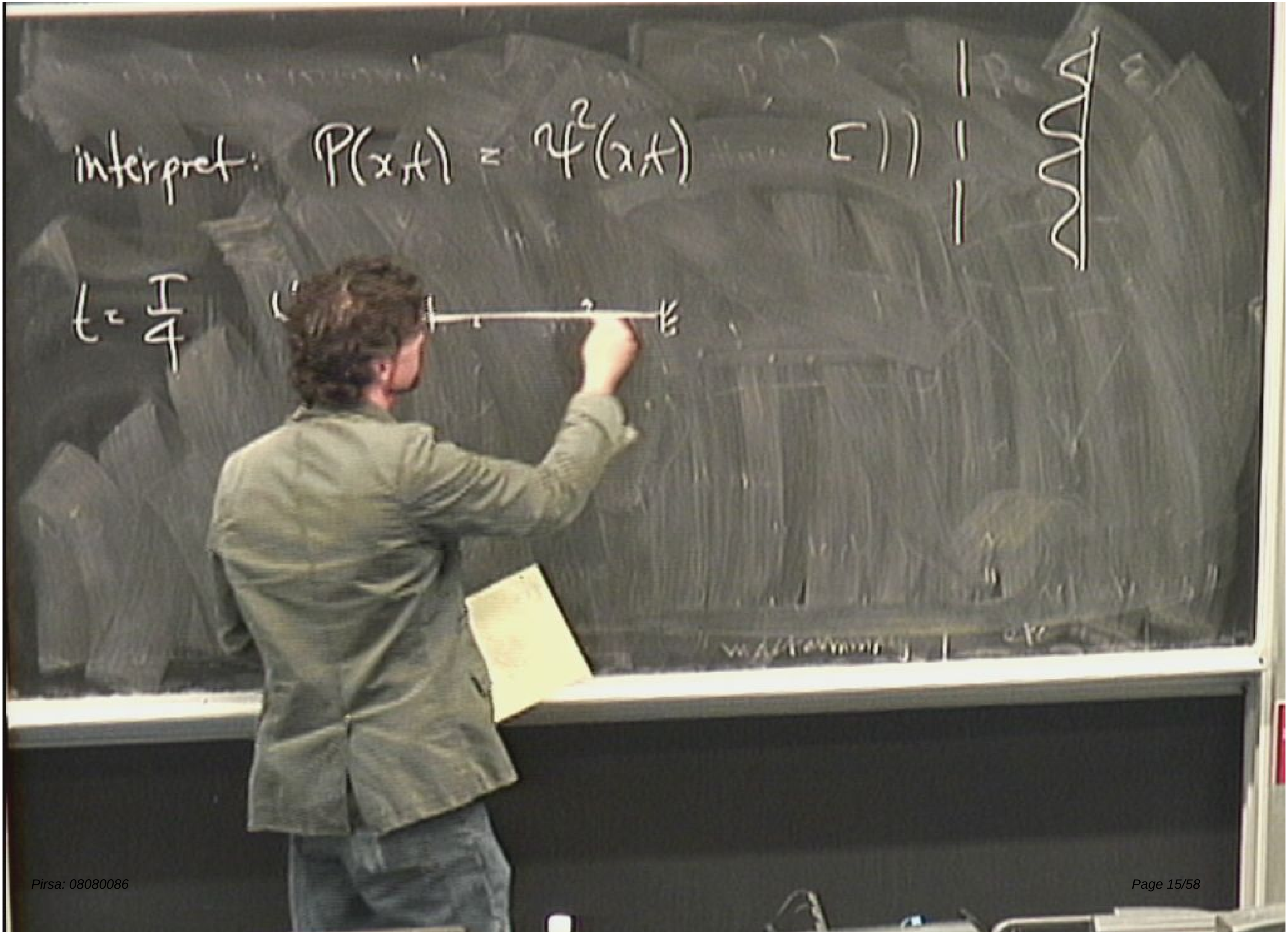

interpret: P(x,t) =


interpret: $P(x,t) = \psi^2(x,t)$
E

interpret: $P(x,t) = \psi^2(x,t)$

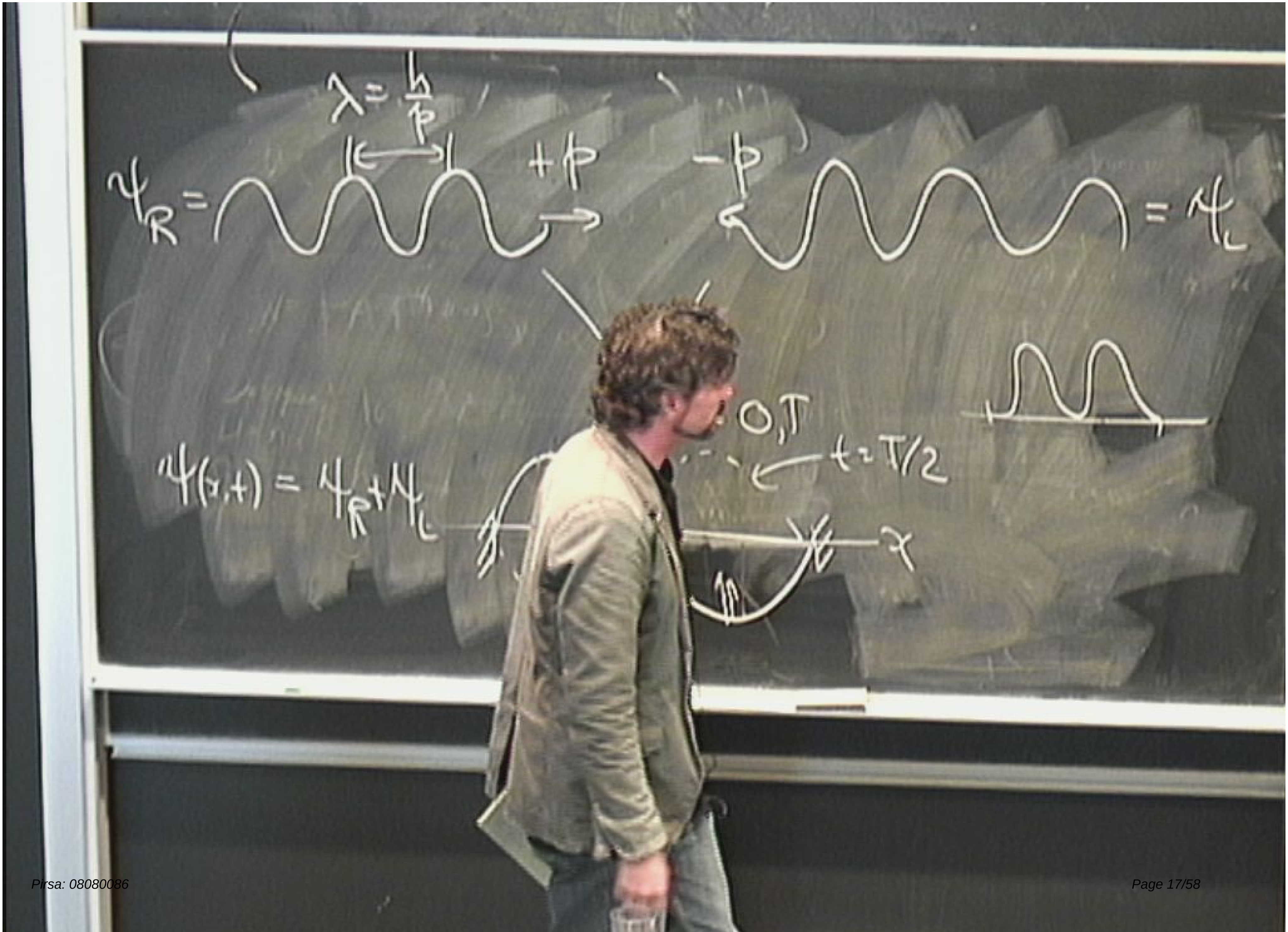

λ = h/p
+p
−p
ψ_R =
= ψ_L
t = 0
x


λ = h/p
+p
−p
ψ_R =
= ψ_L
t = 0, T
t = T/2
ψ(x,t) = ψ_R + ψ_L
x

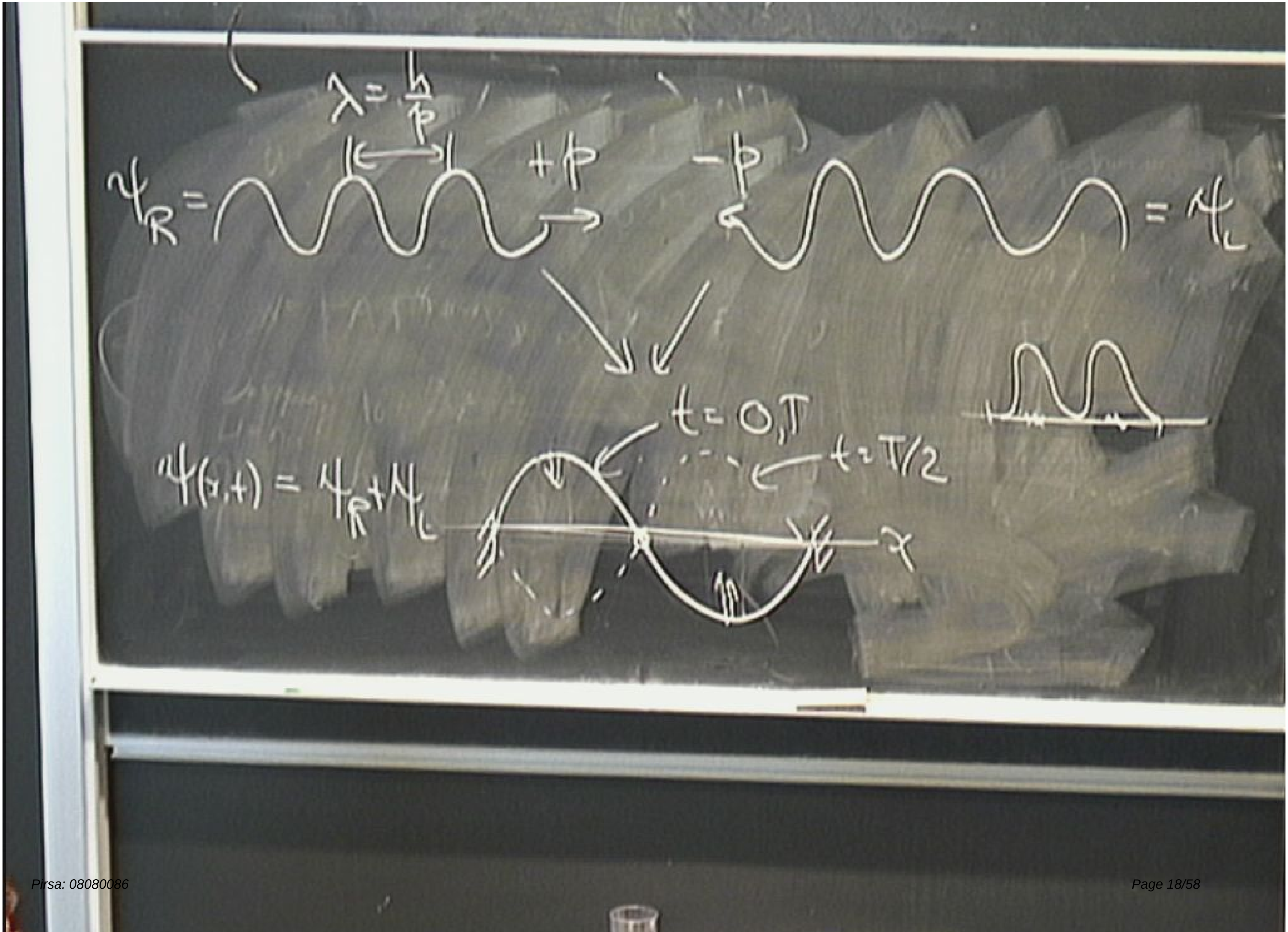

interpret: P(x,t) = ψ²(x,t)
t = T/4

interpret: $P(x,t) = \psi^2(x,t)$

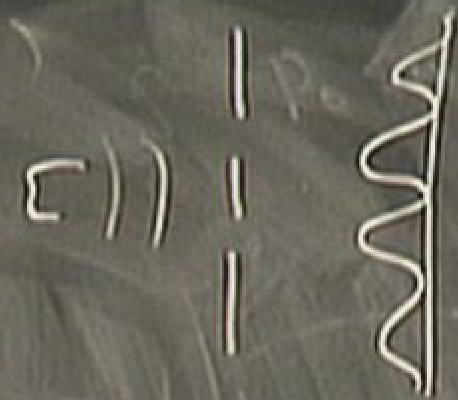
$t = \frac{T}{4}$ $\psi =$ $P =$

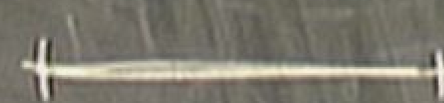


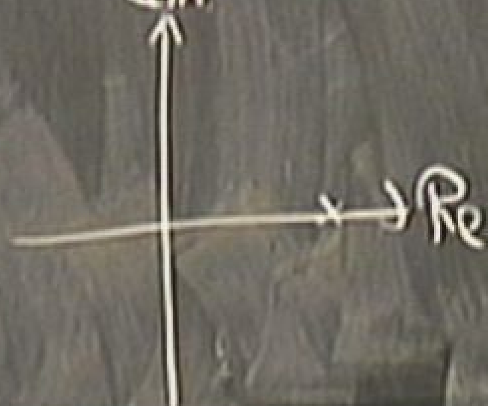
λ = h/p
+p
−p
ψR =
= ψL
ψ(x,t) = ψR + ψL
0,T
t = T/2
x


λ = h/p
+p
−p
ψ_R =
= ψ_L
t = 0, T
t = T/2
ψ(x,t) = ψ_R + ψ_L
x

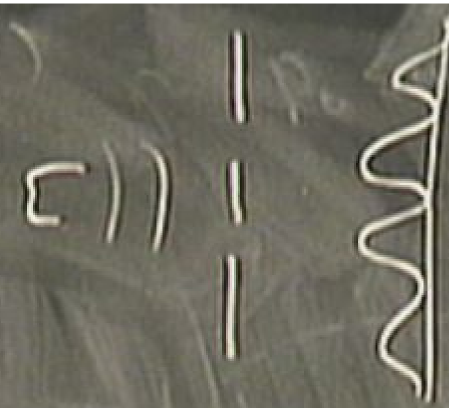
interpret: $P(x,t) = \psi^2(x,t)$

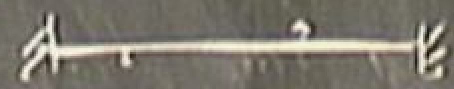


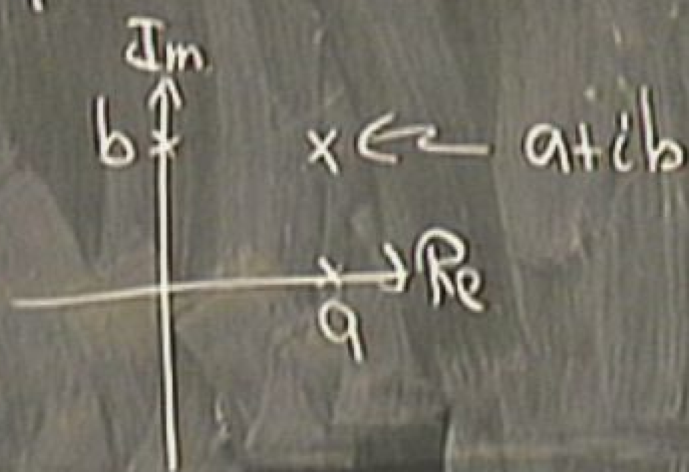
$t = \frac{T}{4}$ $\psi =$  $P =$ 

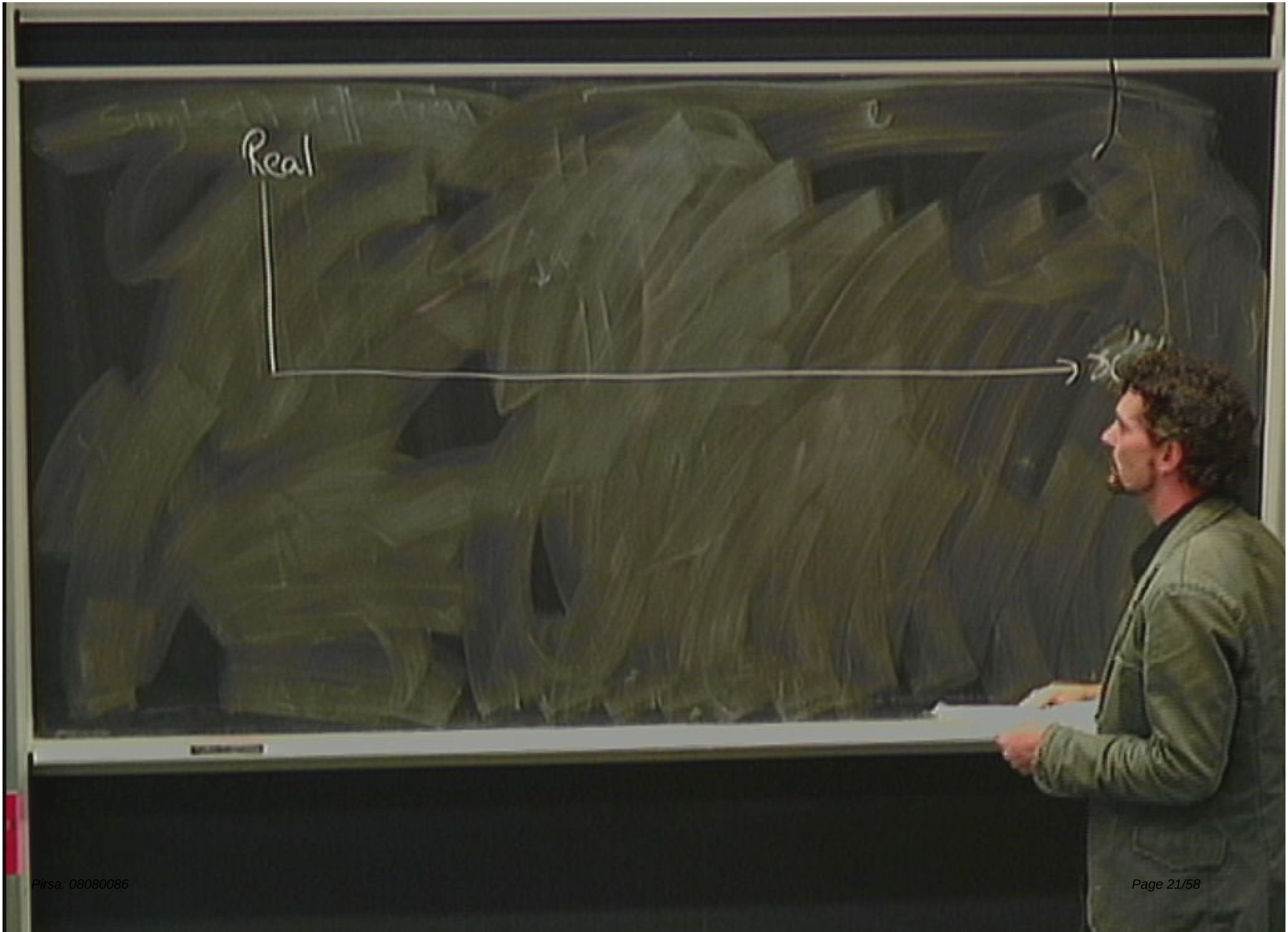


interpret: $P(x,t) = \psi^2(x,t)$



$t = \frac{T}{4}$ $\psi =$  $P =$ 

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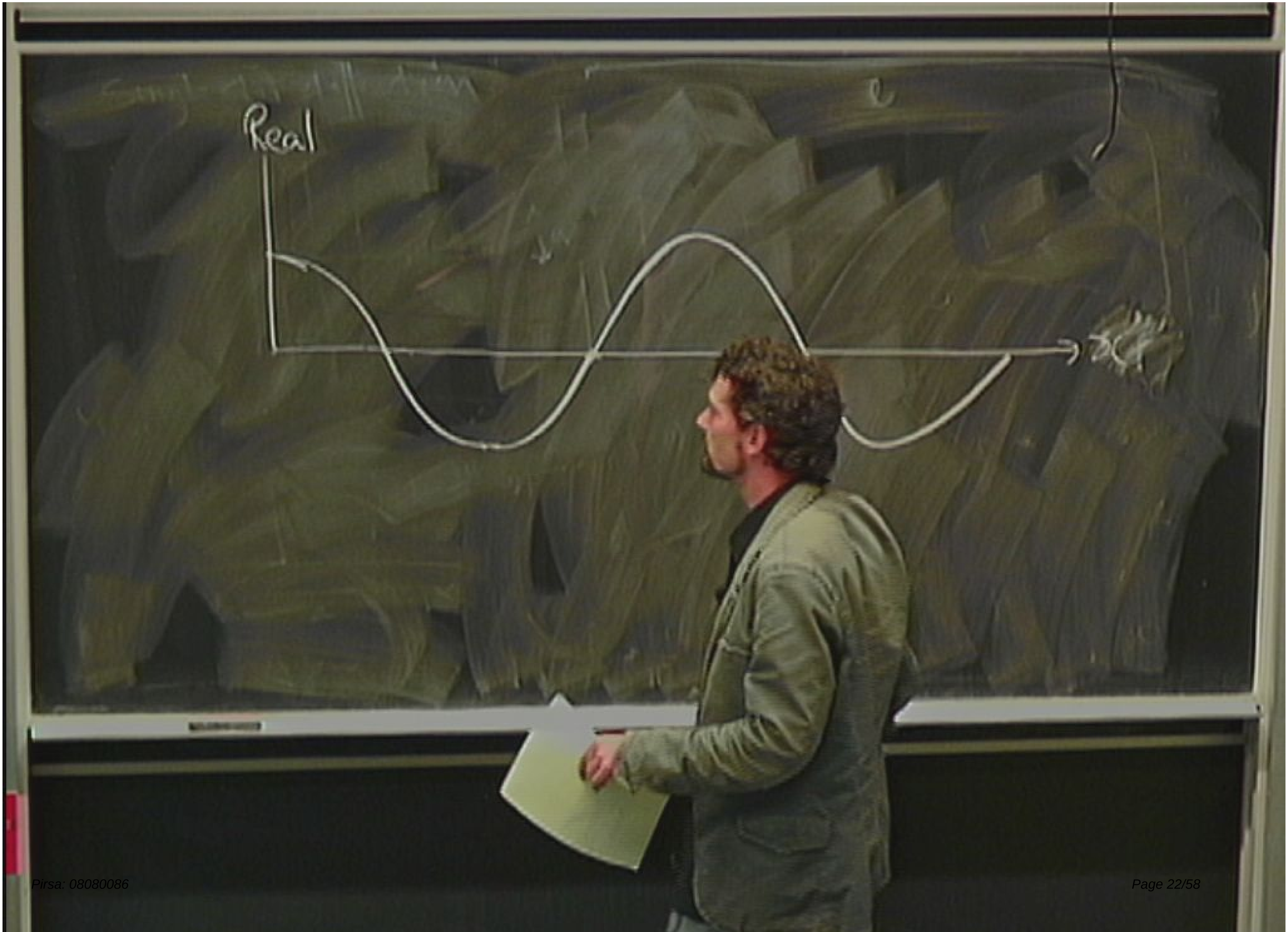
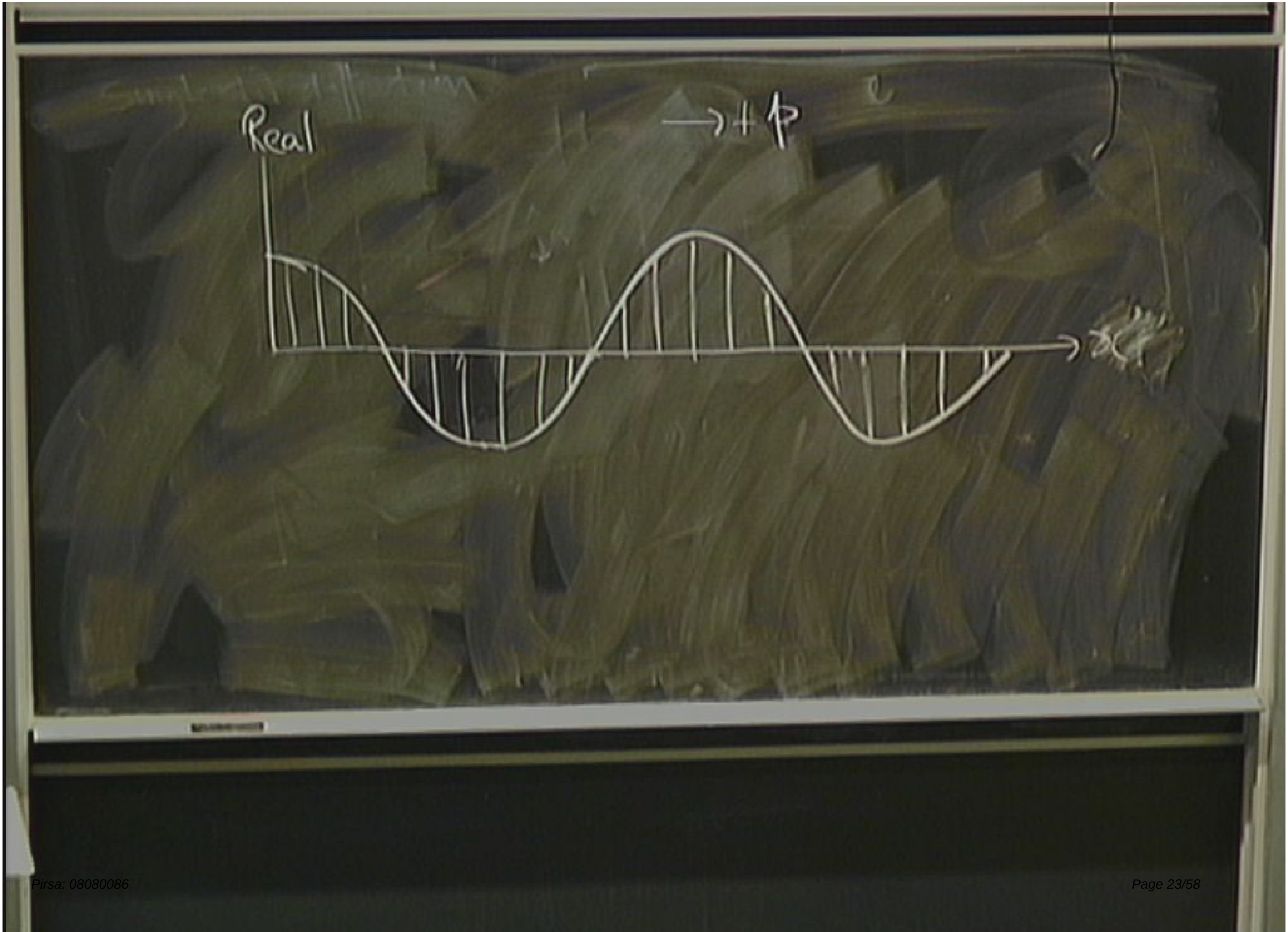
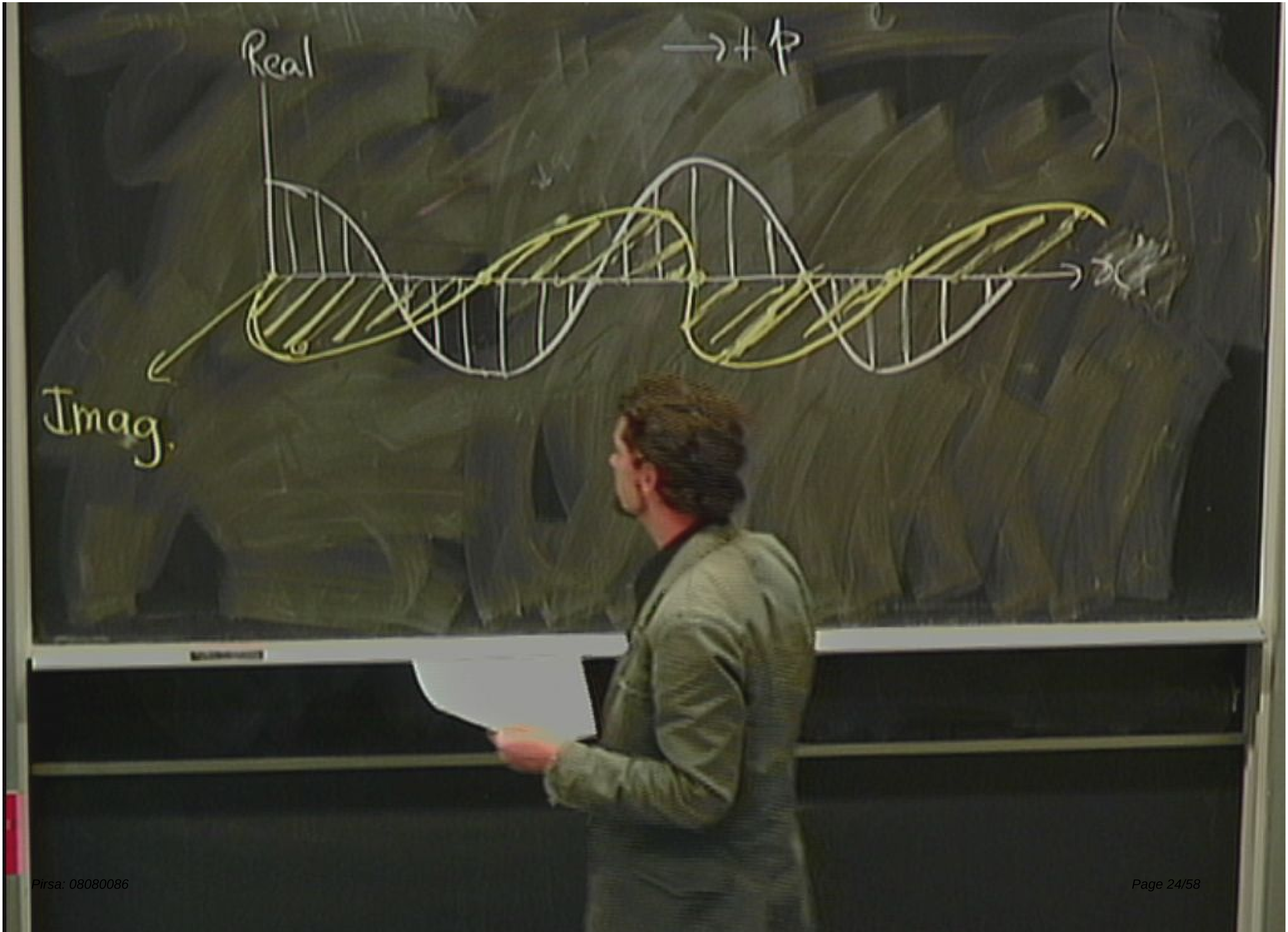
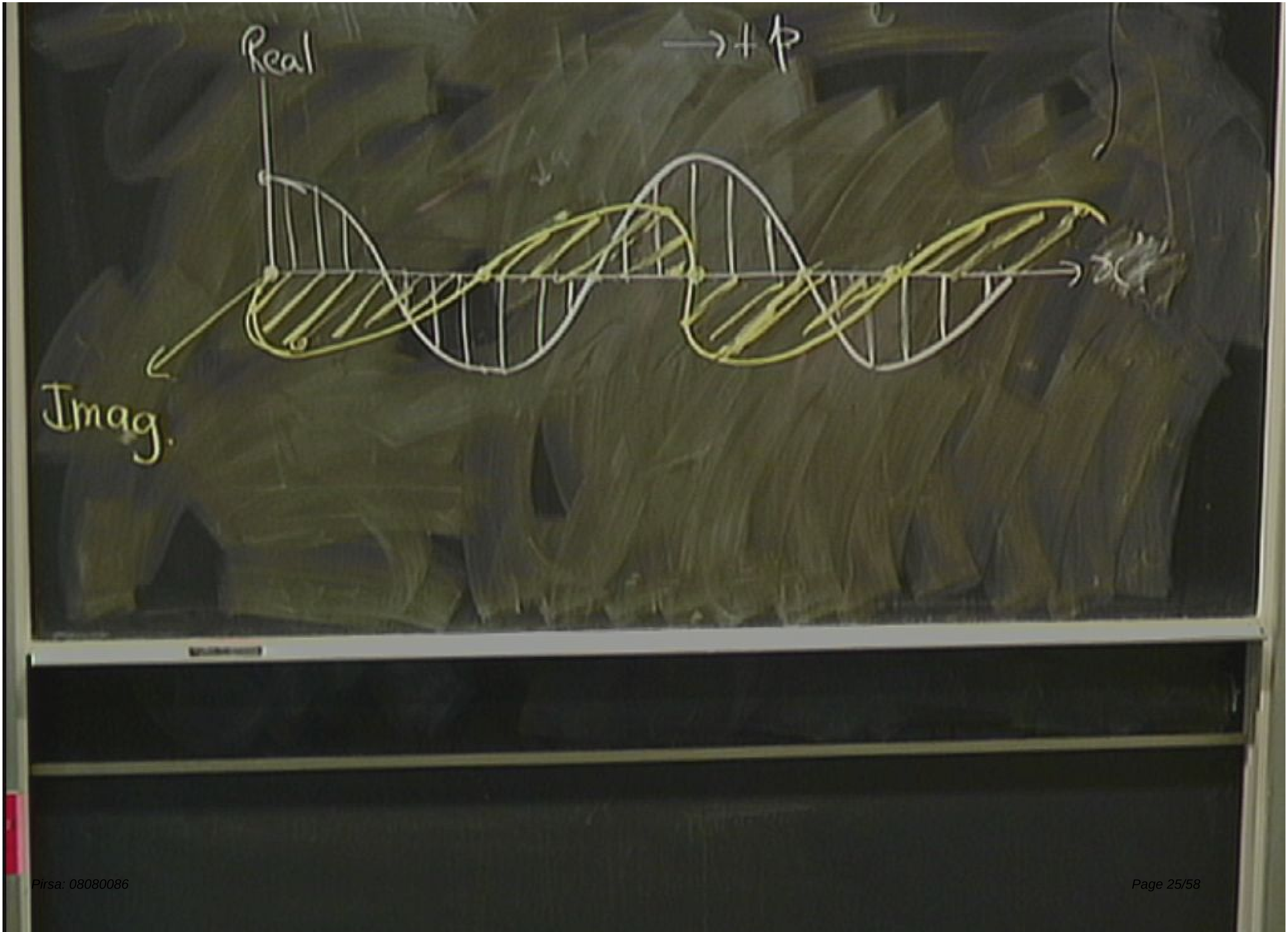

Real
x

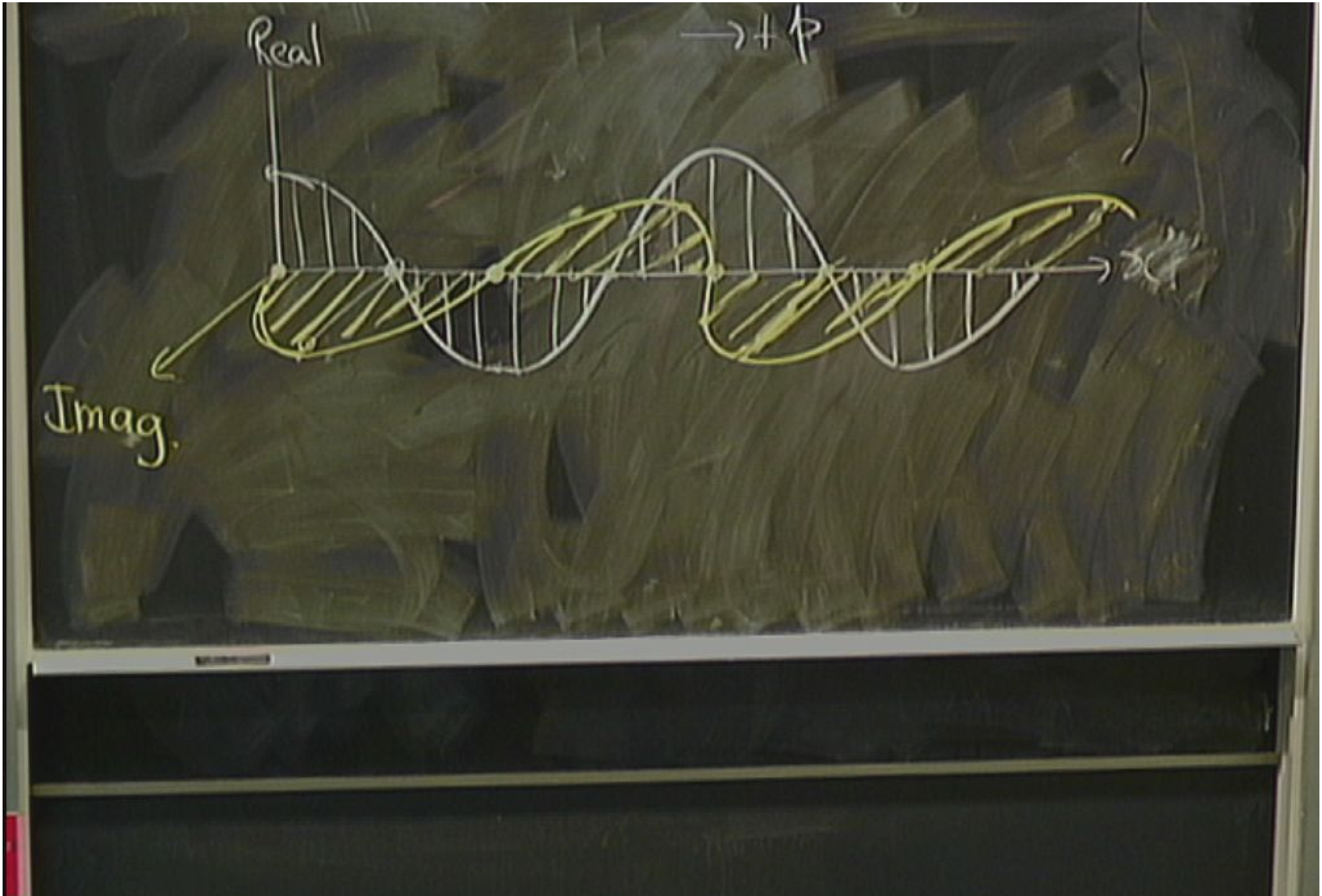


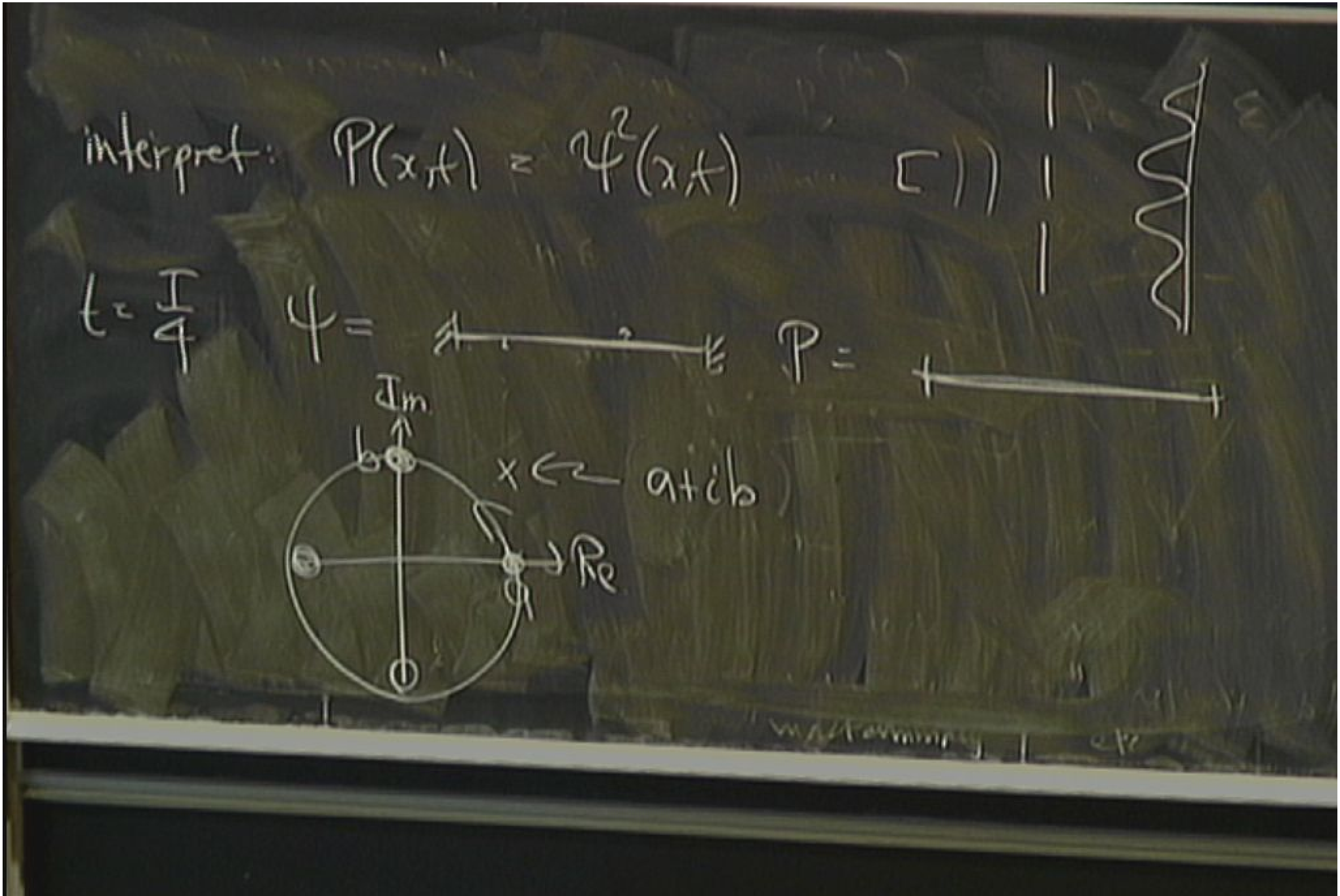
Real
→ +p
x

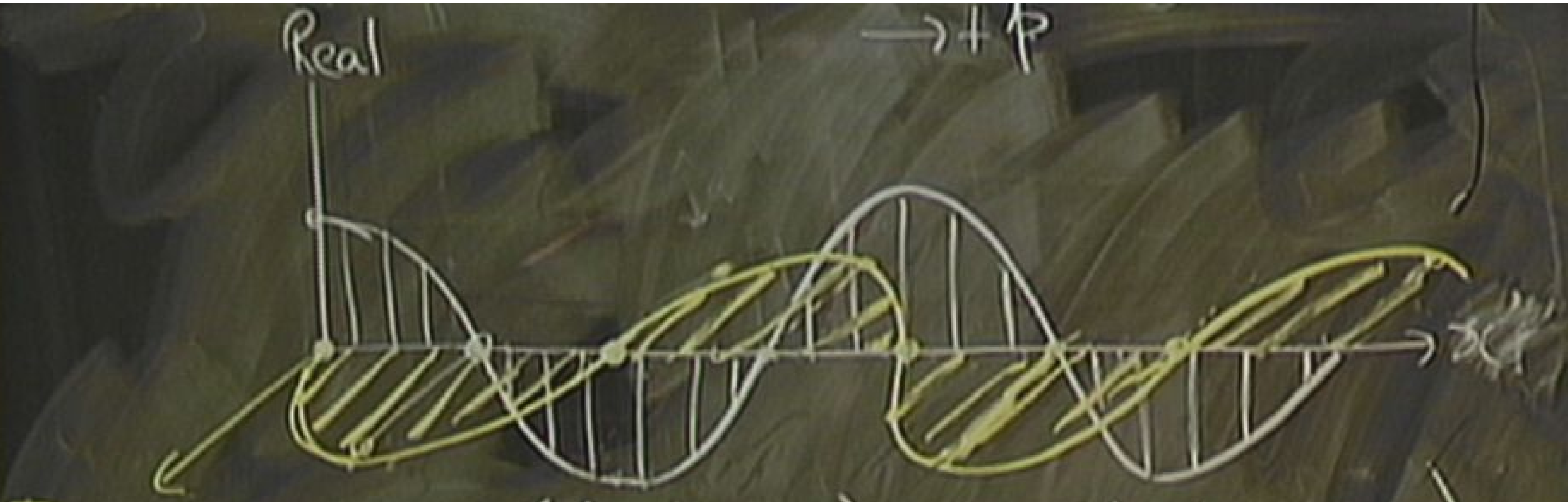


Real
$\rightarrow +p$
x
Imag.


Real
→ +p
x
Imag.

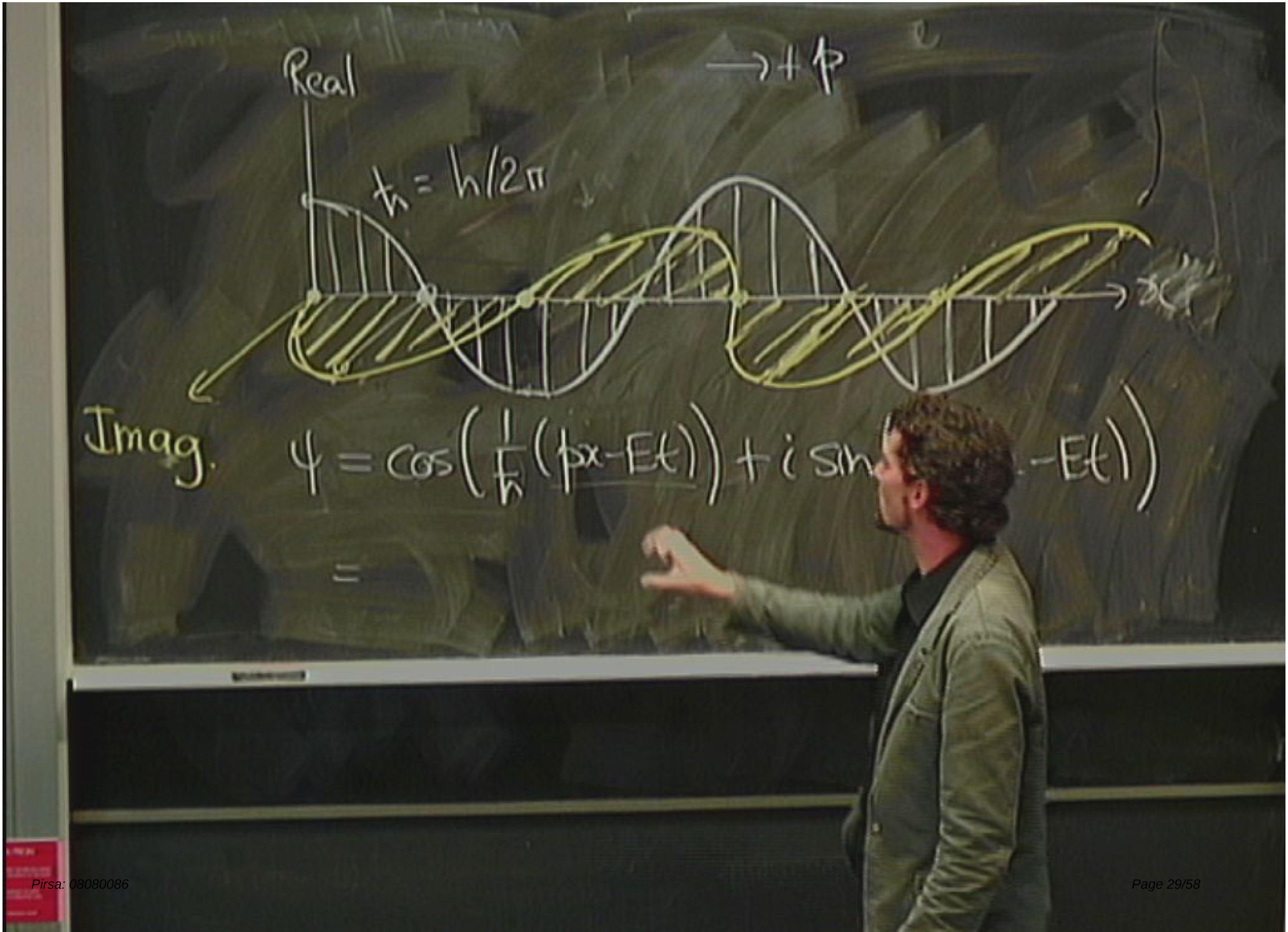

Real
$\longrightarrow +p$
x
Imag.

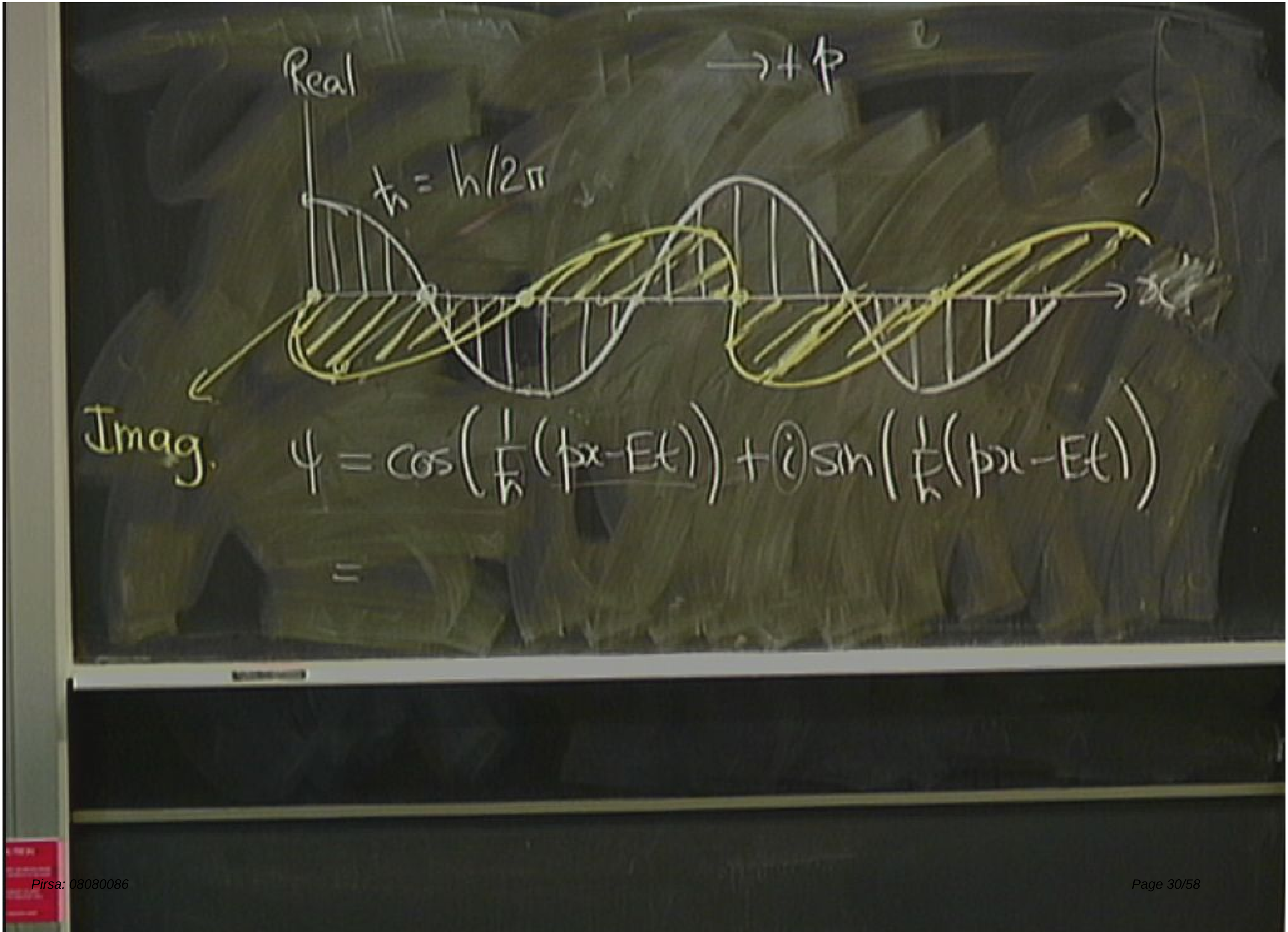

interpret: P(x,t) = ψ²(x,t)
t = T/4
ψ =
P =
Im
b
x ⇐ a+ib
Re
a

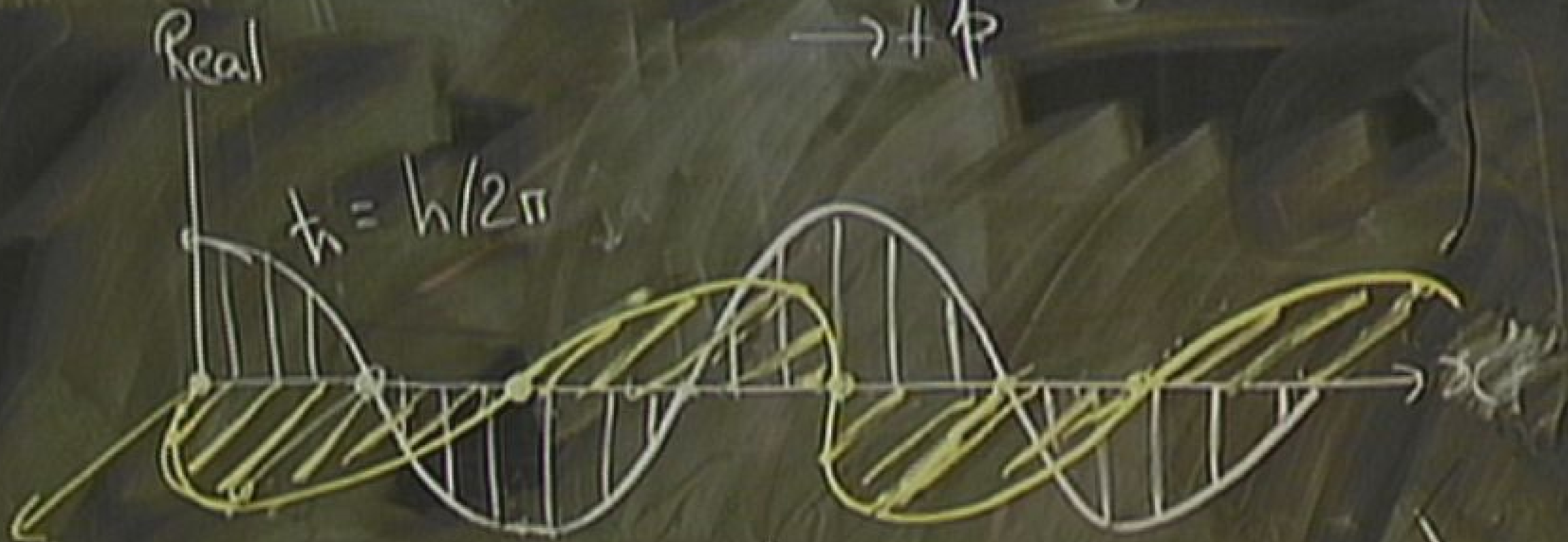


Imag.

$$\psi = \cos\left(\frac{1}{\hbar}(px - Et)\right) + i\sin\left(\frac{1}{\hbar}(px - Et)\right)$$

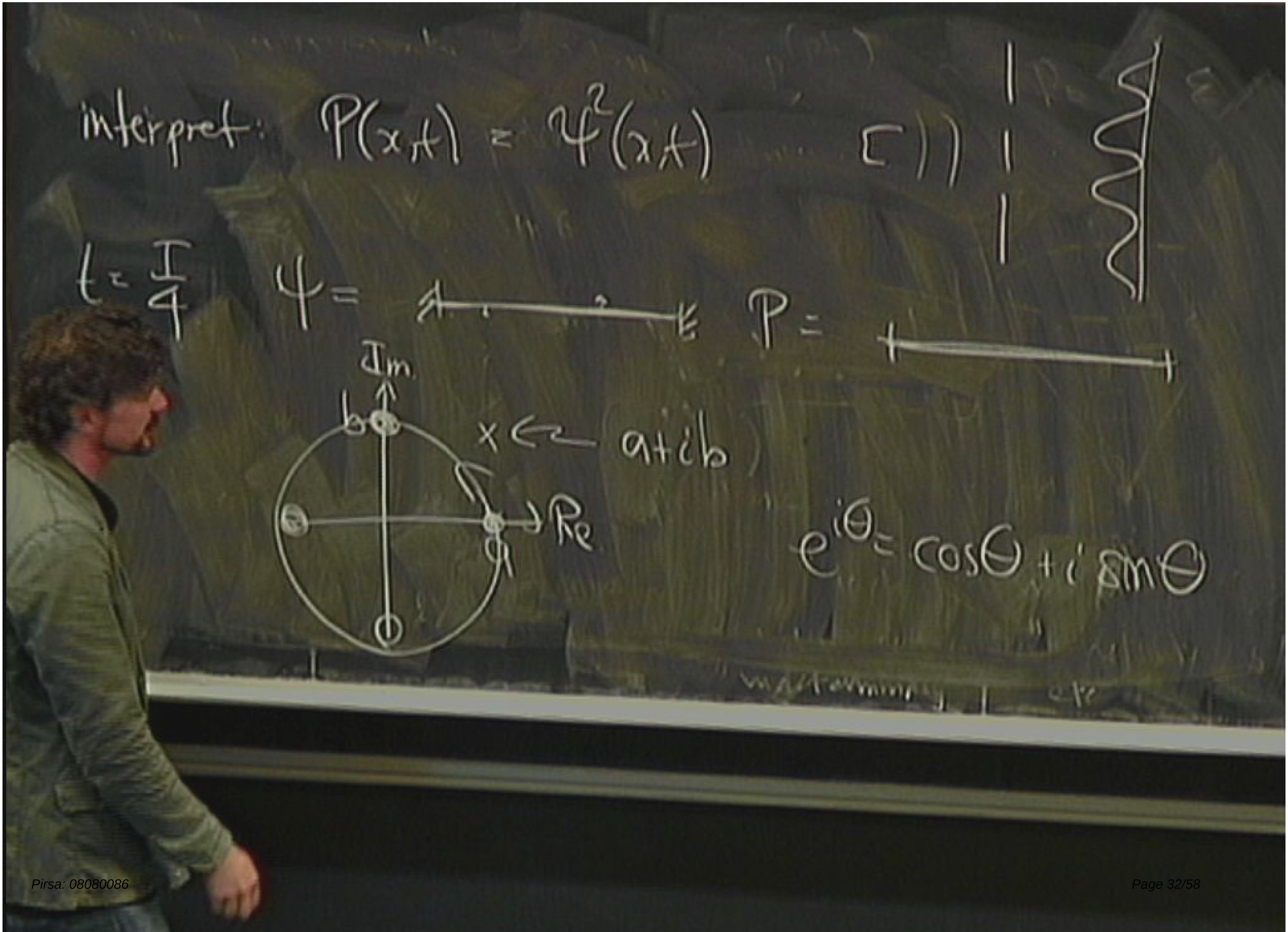

Real
→ + p
ħ = h/2π
x
Imag.
ψ = cos(1/ħ (px − Et)) + i sin
−Et))
=

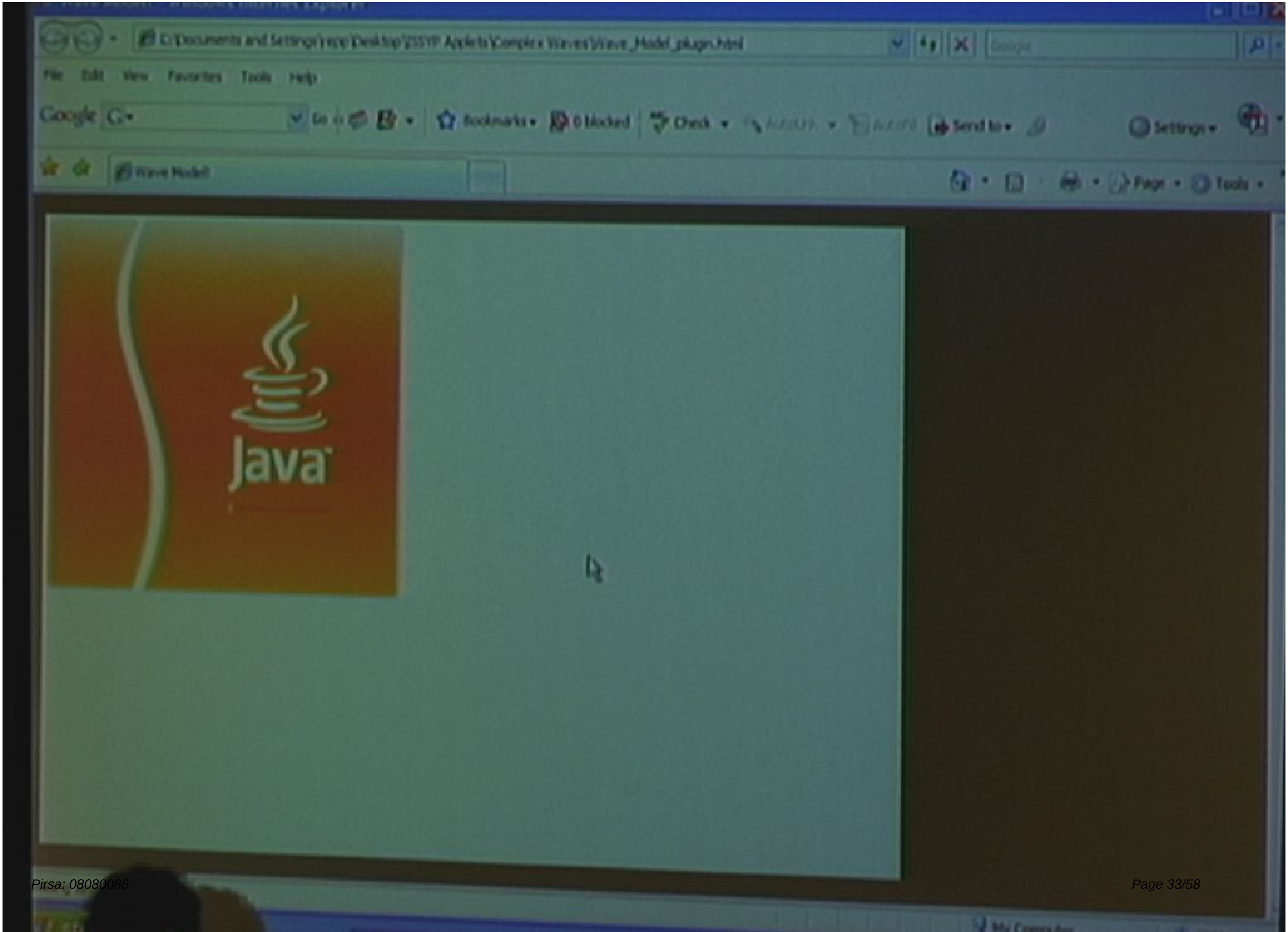

Real
$\hbar = h/2\pi$
$\rightarrow +p$
x
Imag.
$\psi = \cos\left(\frac{1}{\hbar}(px - Et)\right) + i\sin\left(\frac{1}{\hbar}(px - Et)\right)$
$=$



$$\psi = \cos\left(\frac{1}{\hbar}(px - Et)\right) + i\sin\left(\frac{1}{\hbar}(px - Et)\right)$$

$$= e^{i\frac{1}{\hbar}(px - Et)}$$


interpret: P(x,t) = ψ²(x,t)
t = T/4
ψ =
P =
Im
b
x ⟵ a+ib
Re
a
e^{iθ} = cosθ + i sinθ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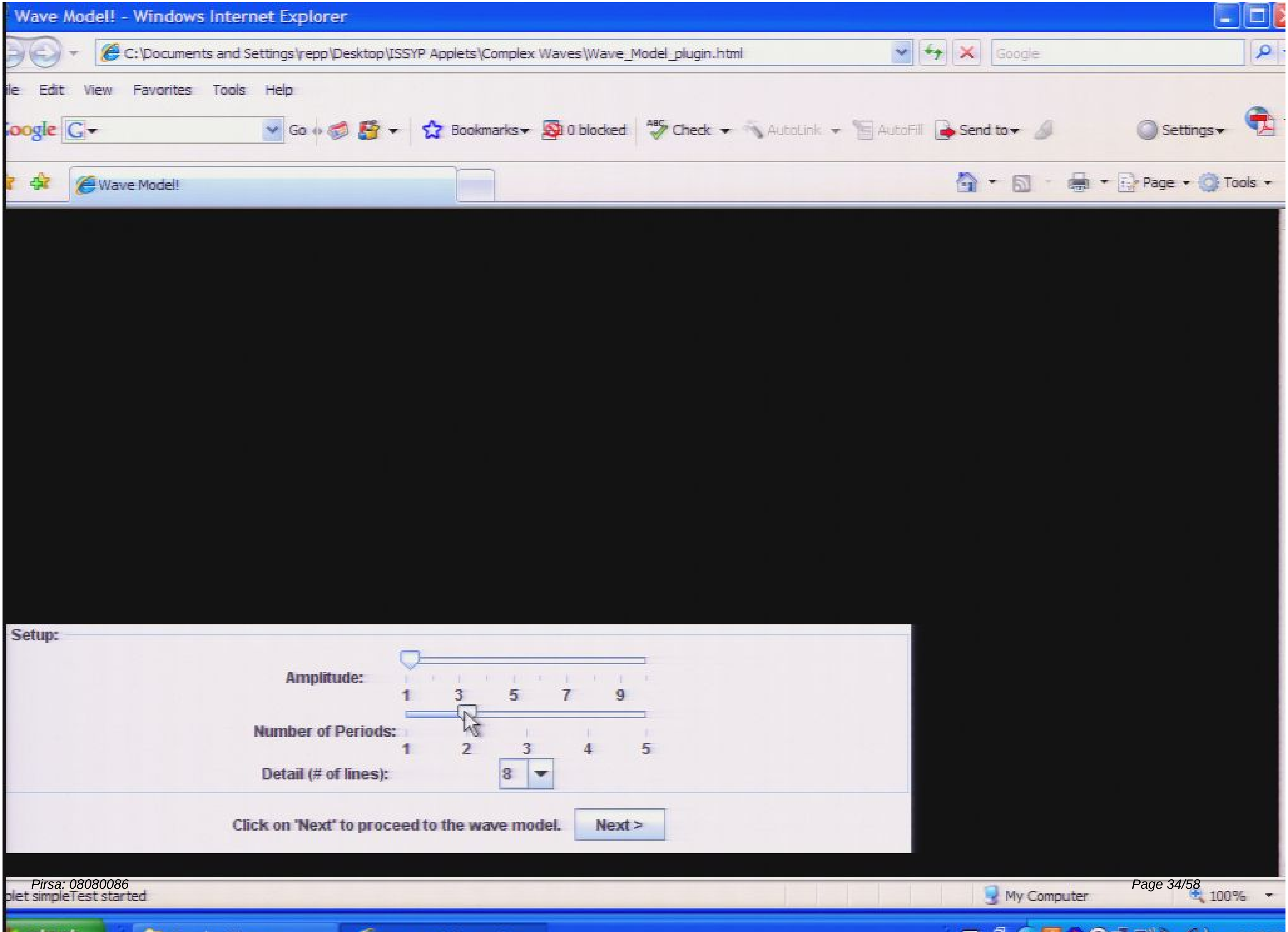
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AutoLink
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Send to
Settings
Wave Model!
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Setup:
Amplitude:
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Number of Periods:
1 2 3 4 5
Detail (# of lines):
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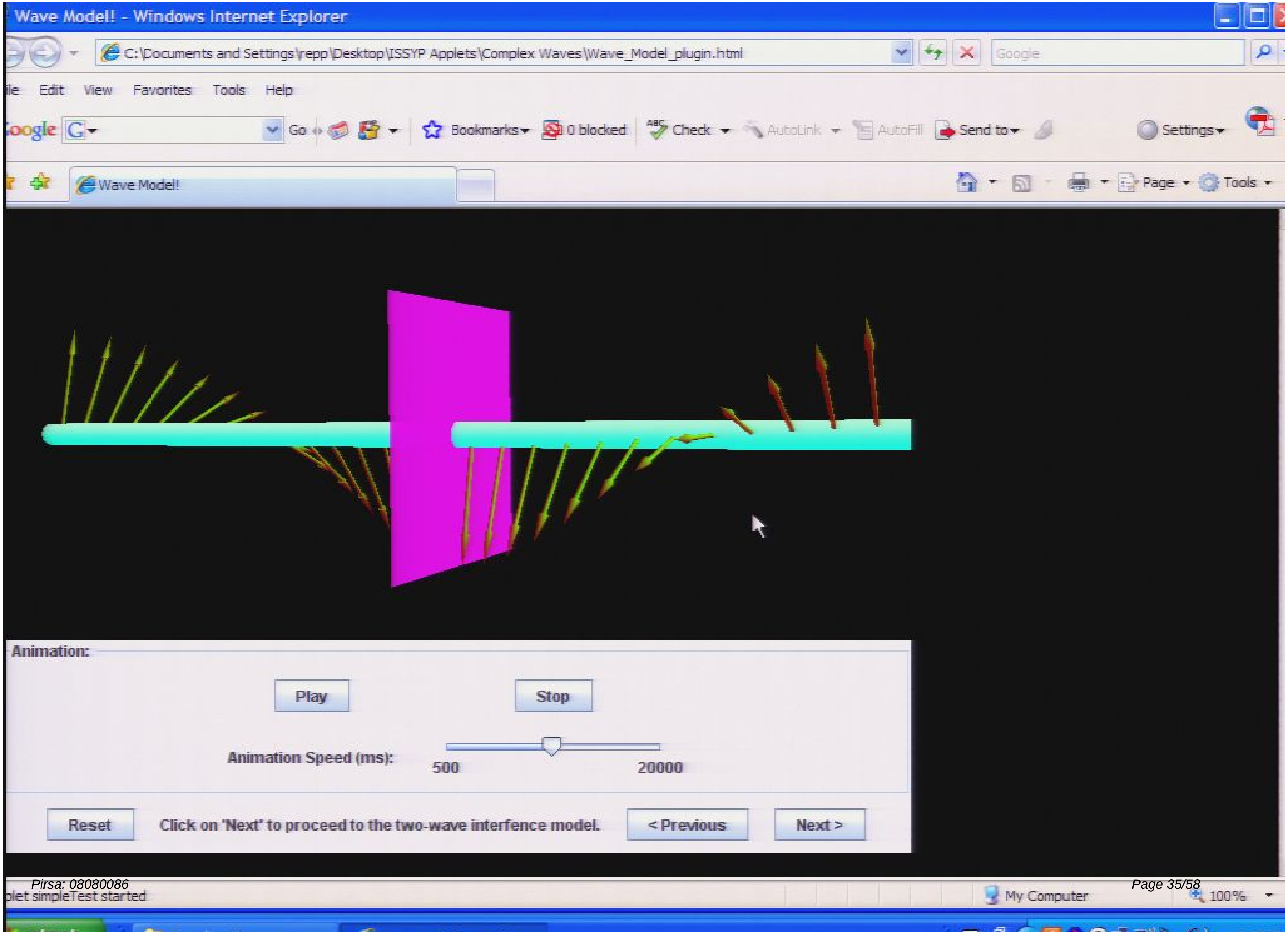
Wave Model! - Windows Internet Explorer
C:\Documents and Settings\repp\Desktop\ISSYP Applets\Complex Waves\Wave_Model_plugin.html
Edit View Favorites Tools Help
Wave Model!
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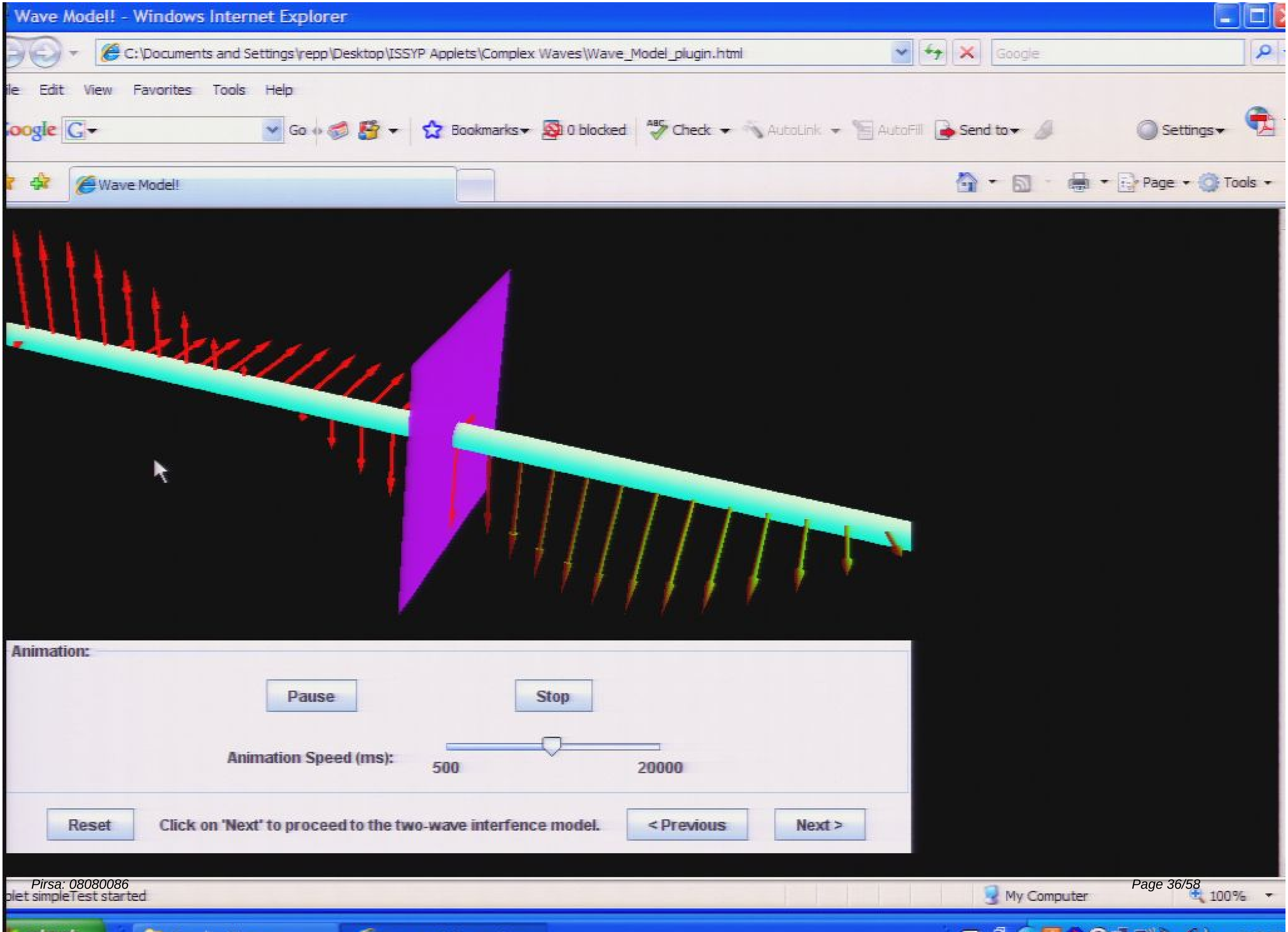
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Wave Model!
Page
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Animation:
Pause
Stop
Animation Speed (ms):
500
20000
Reset
Click on 'Next' to proceed to the two-wave interfence model.
< Previous
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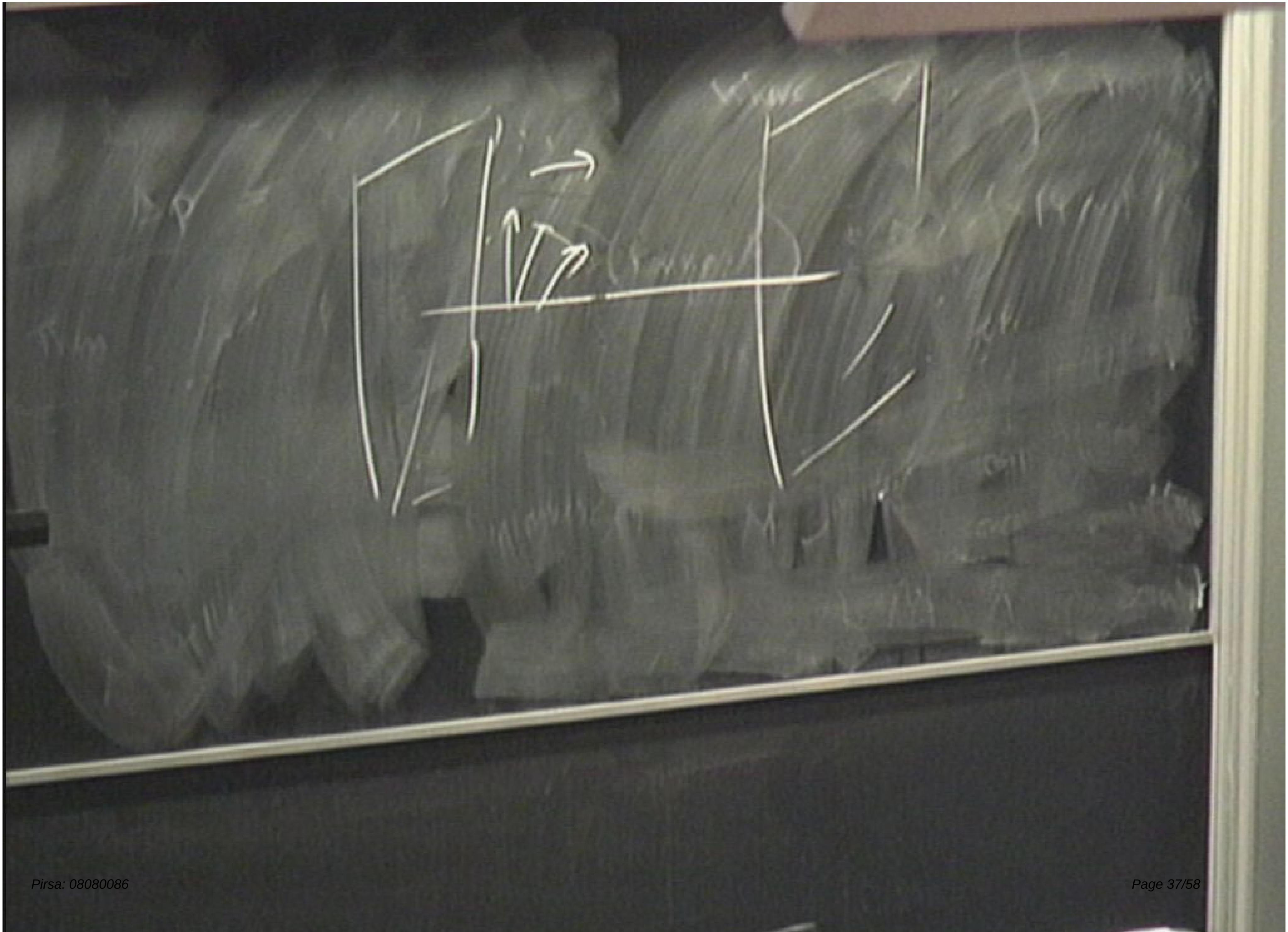

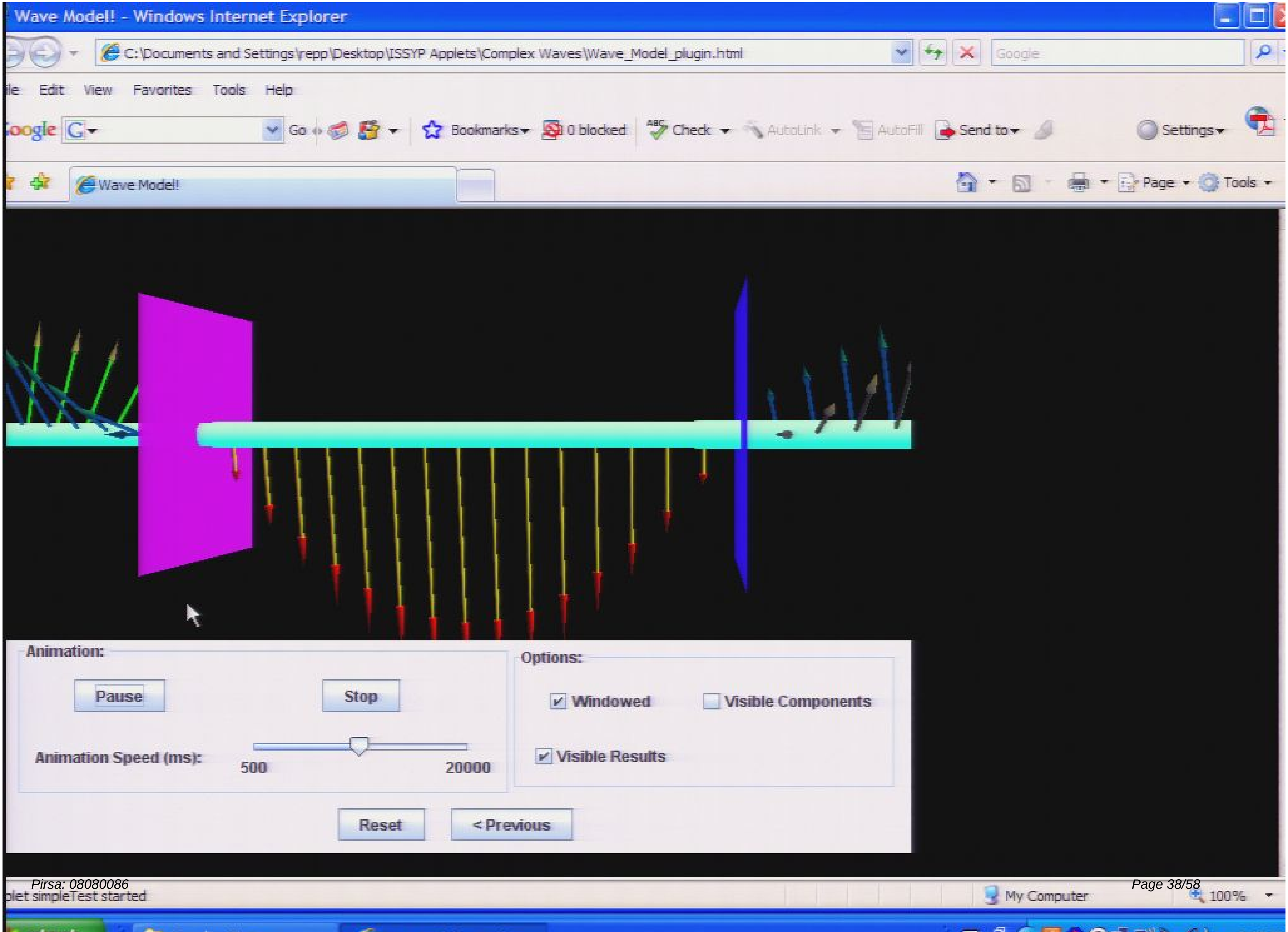
Wave Model! - Windows Internet Explorer
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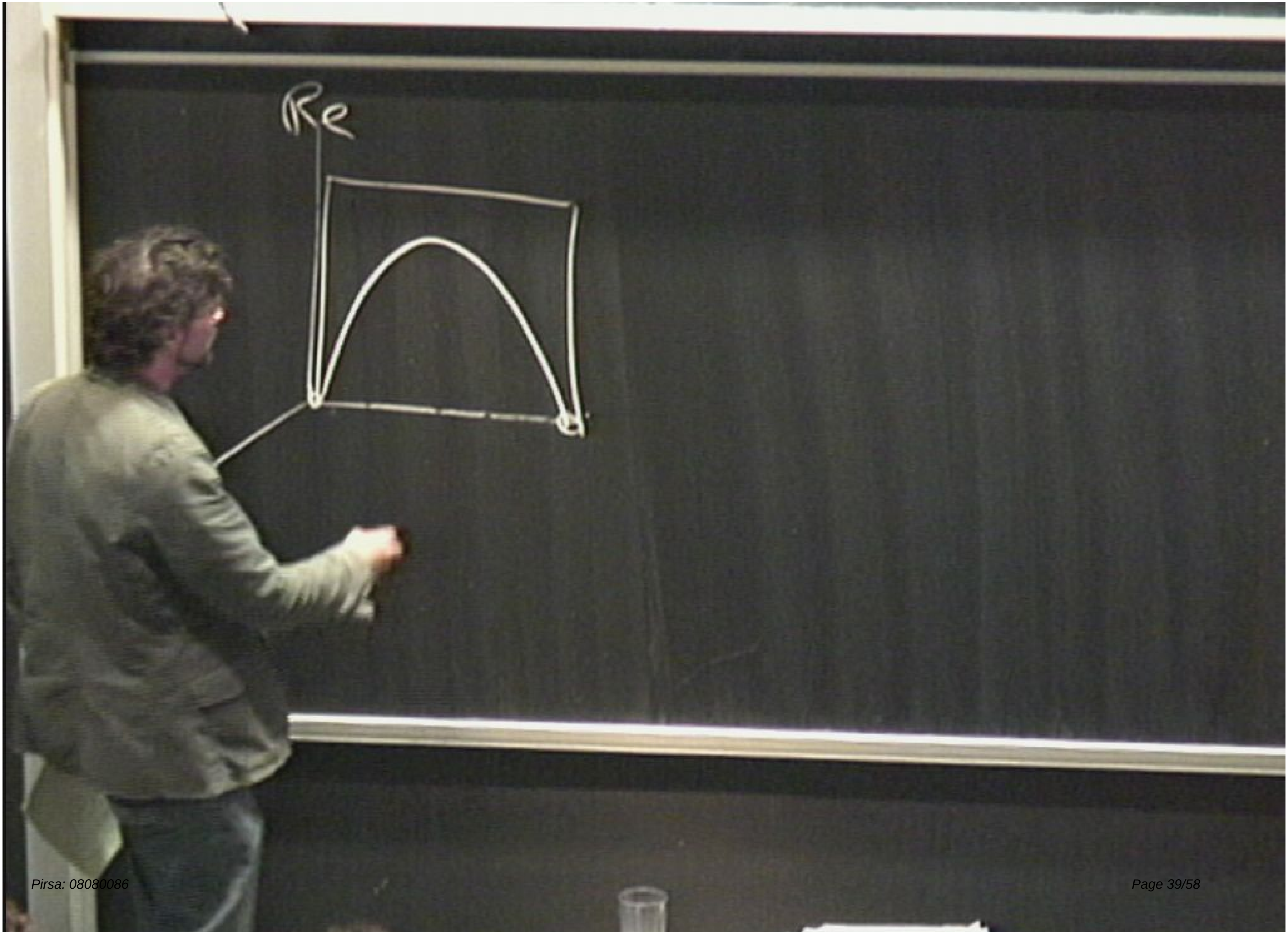
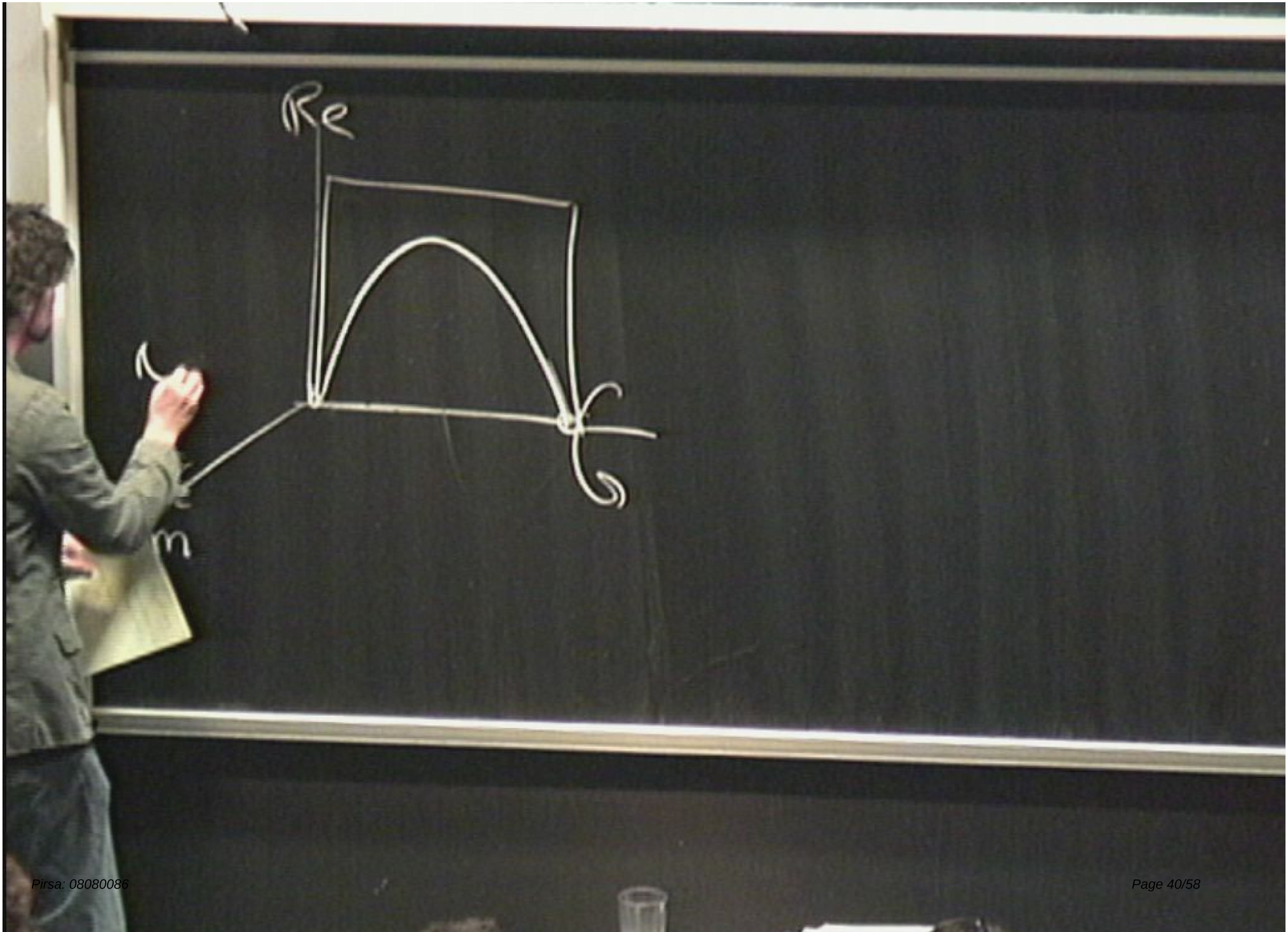
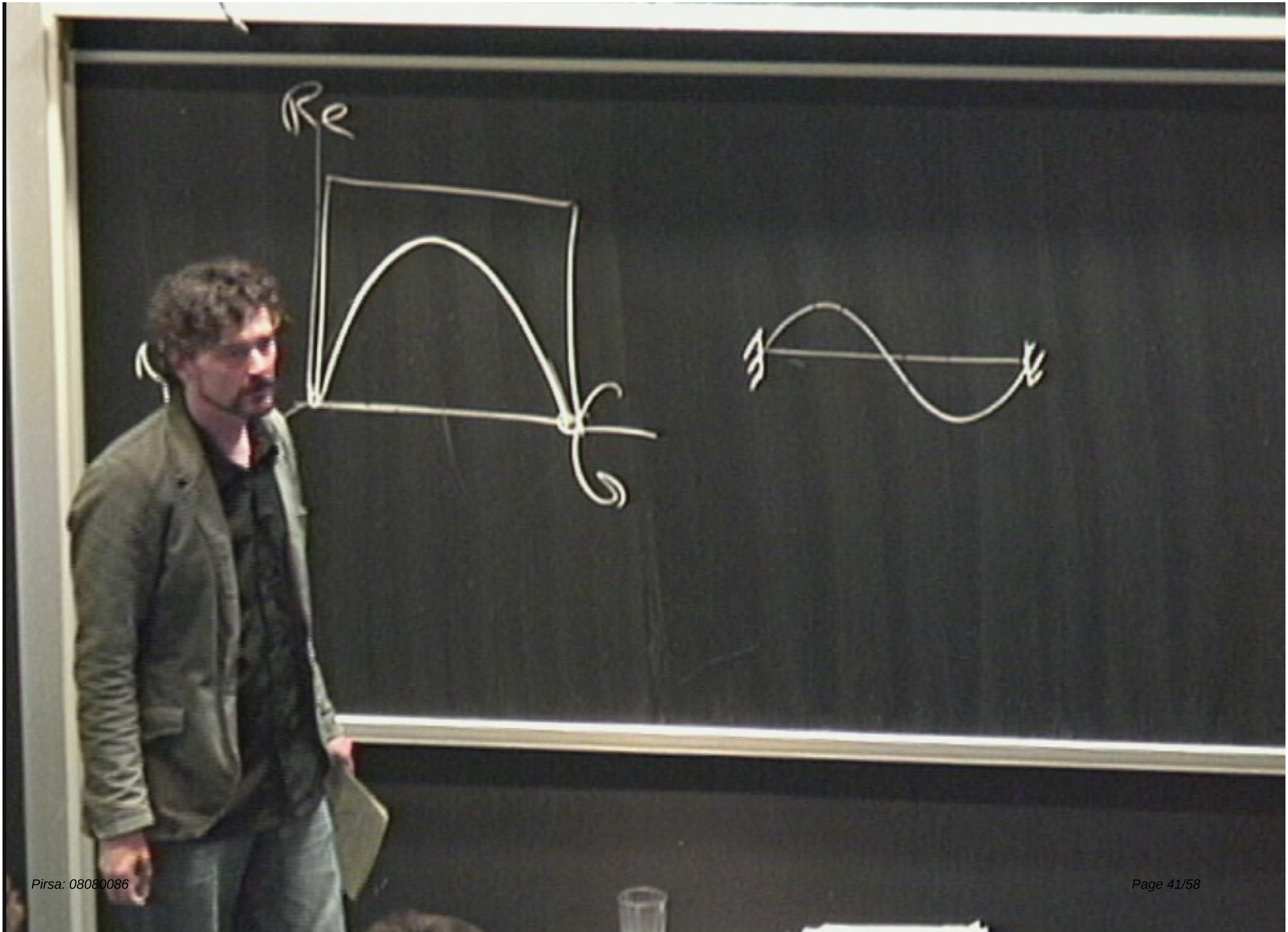
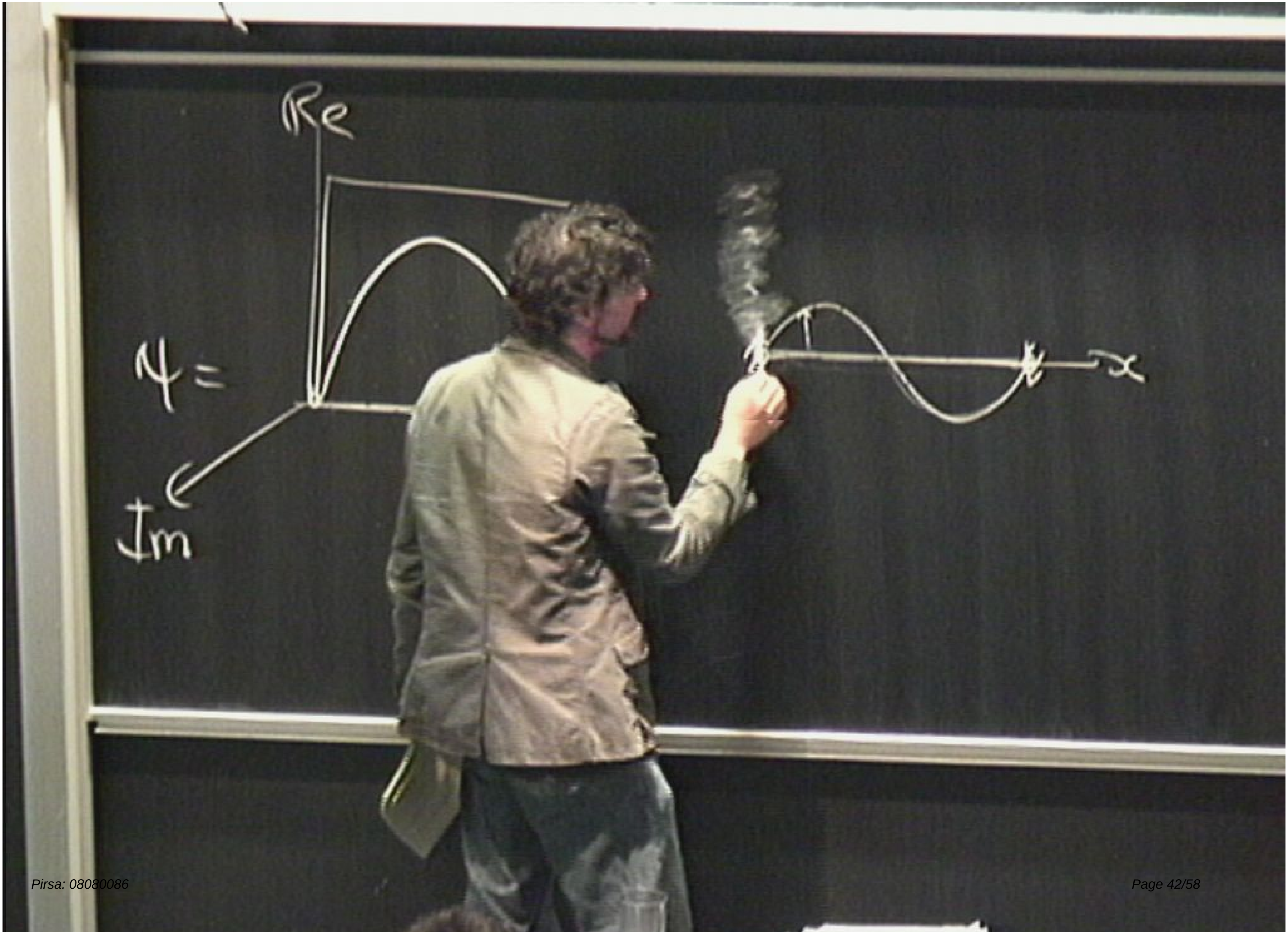
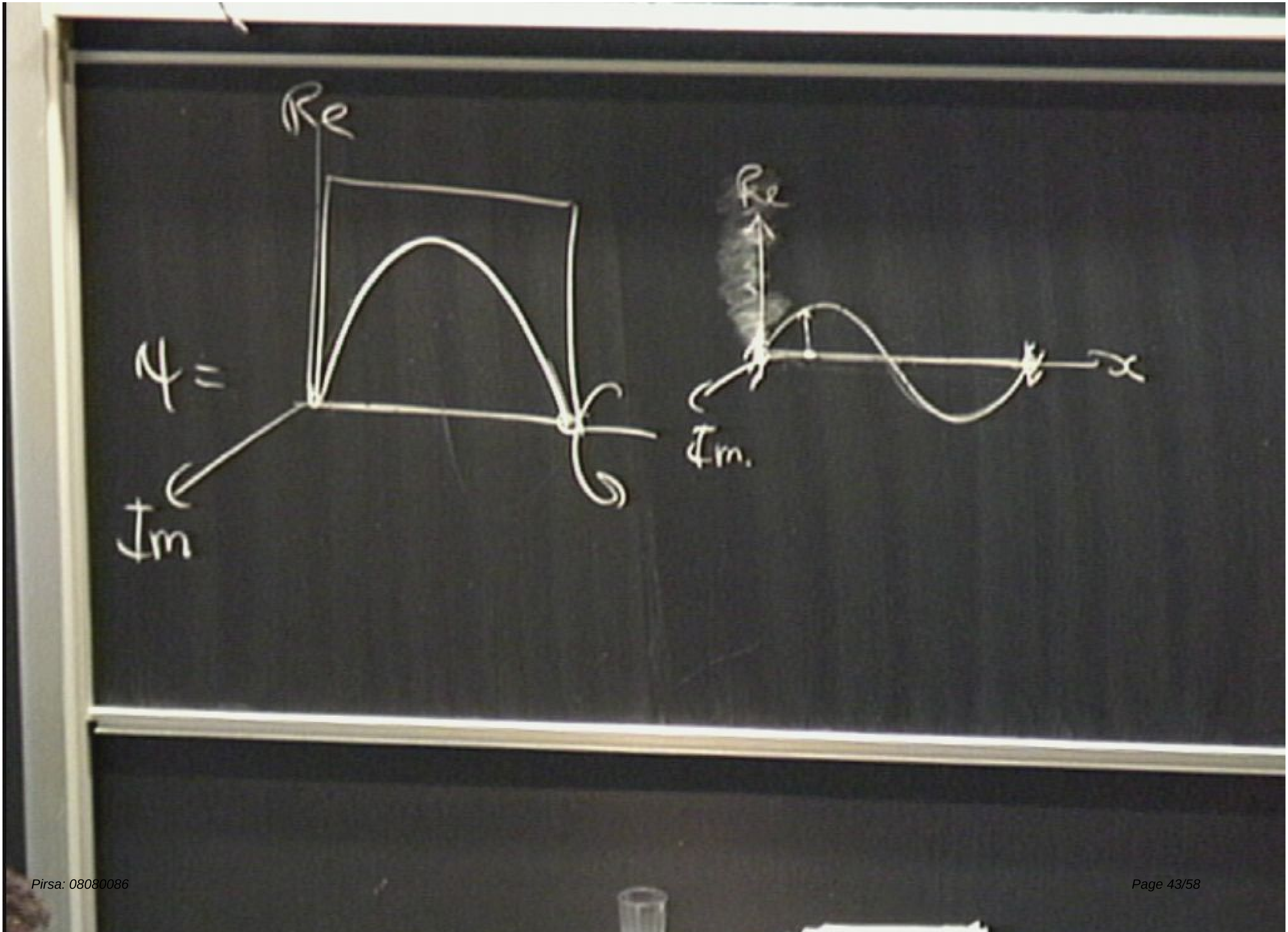
100%

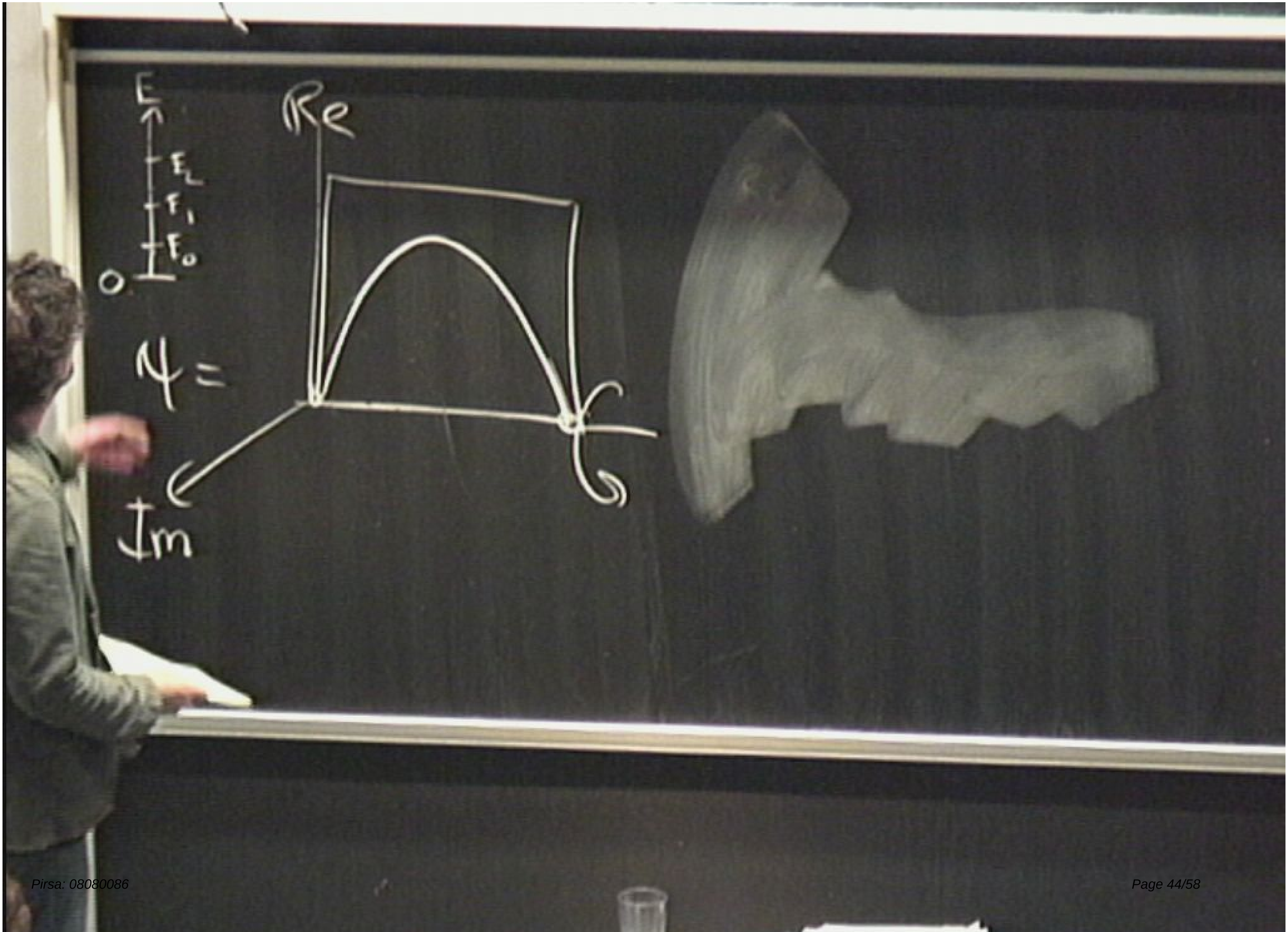

Re

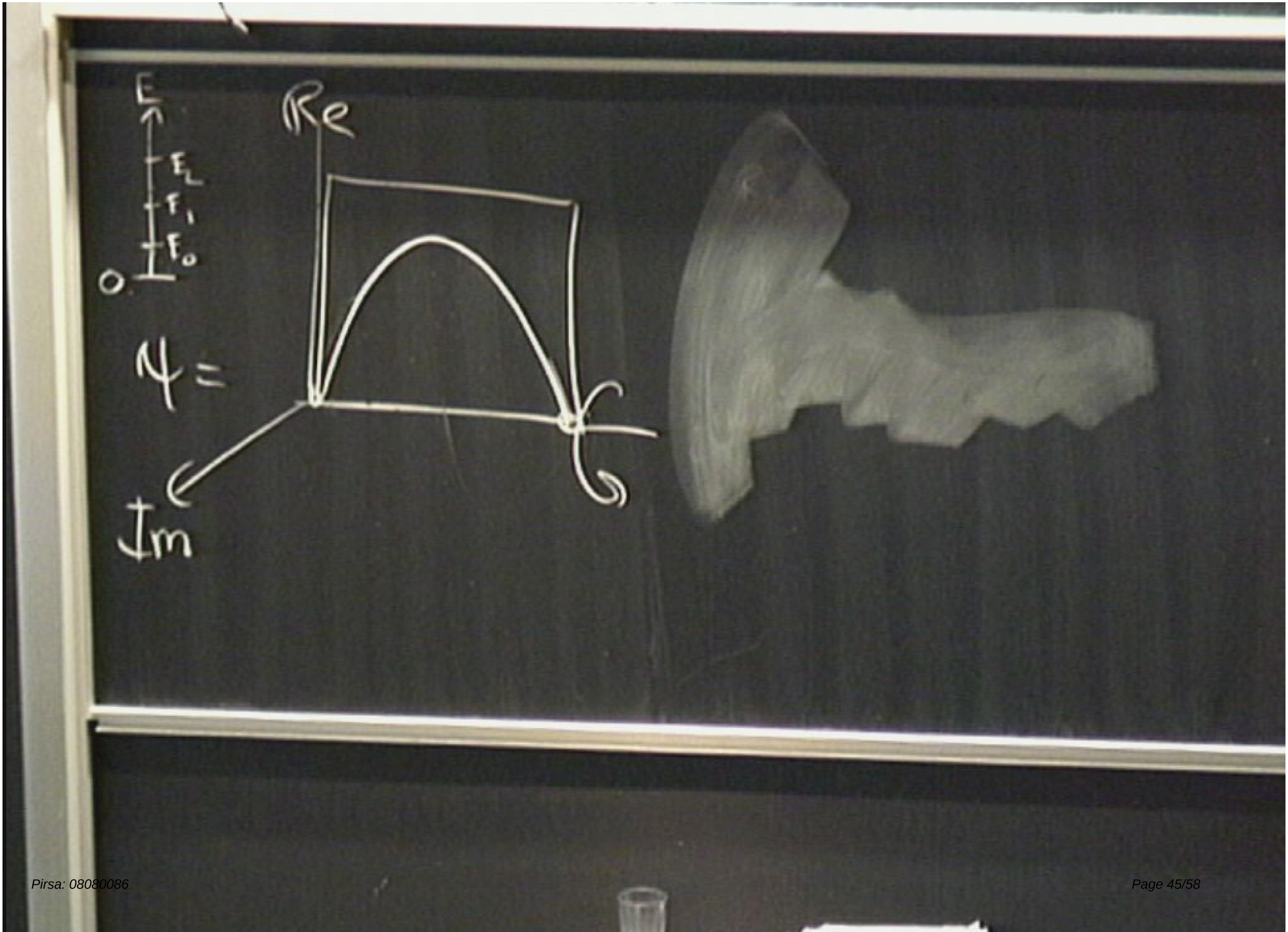

Re

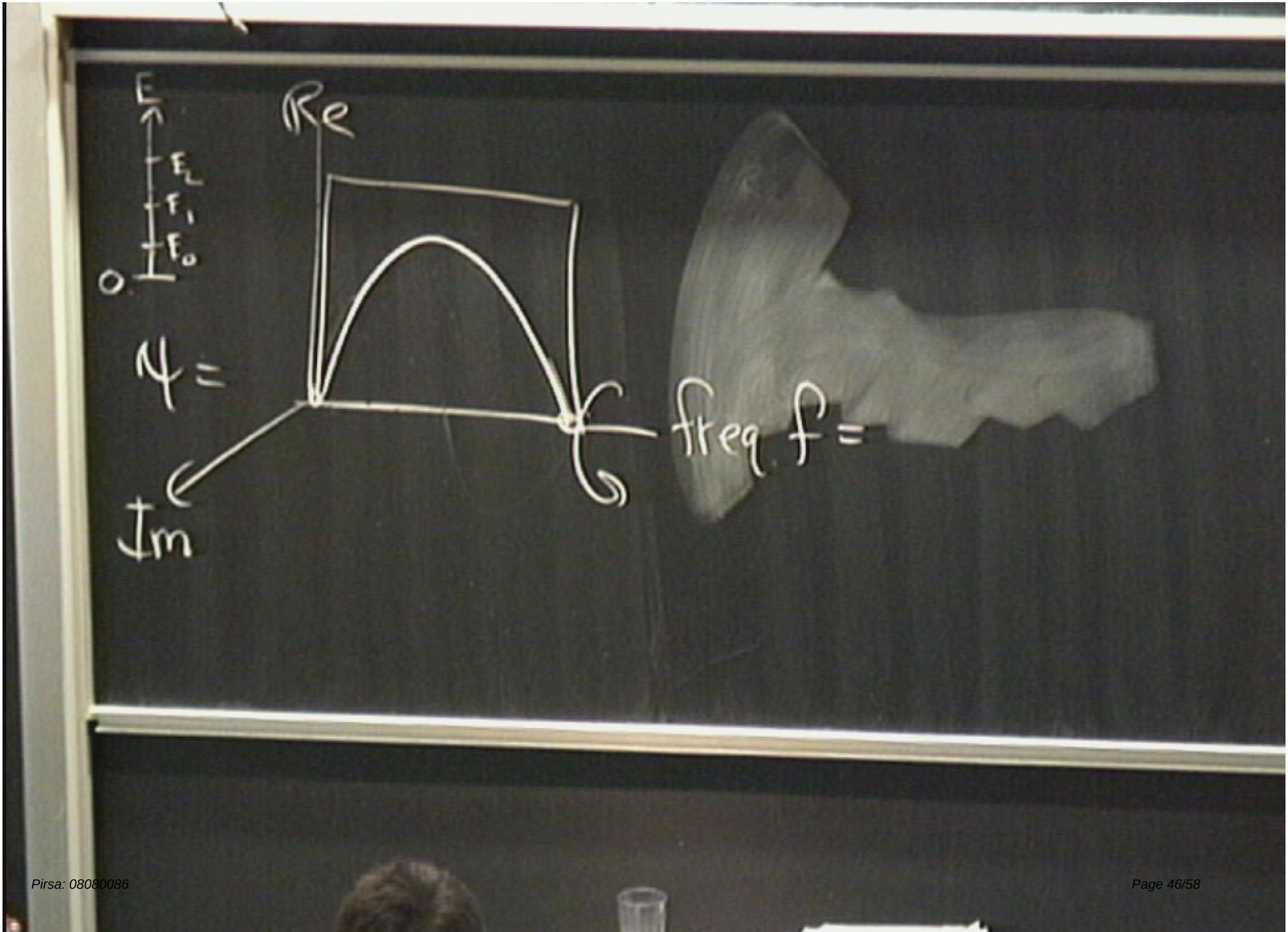

Re


Re
Ψ =
Im
x

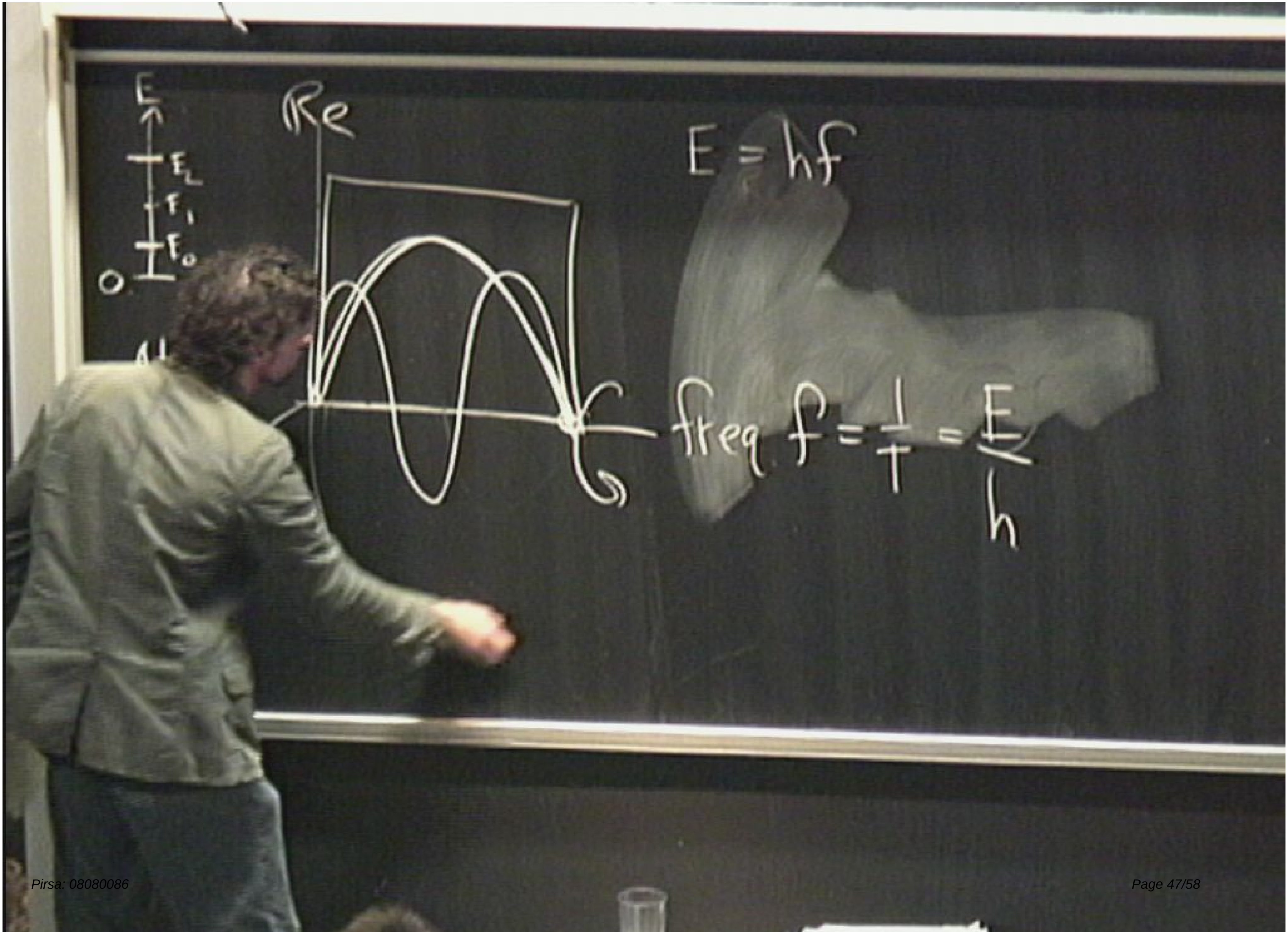

Re
Ψ =
Im
Re
x
Im.

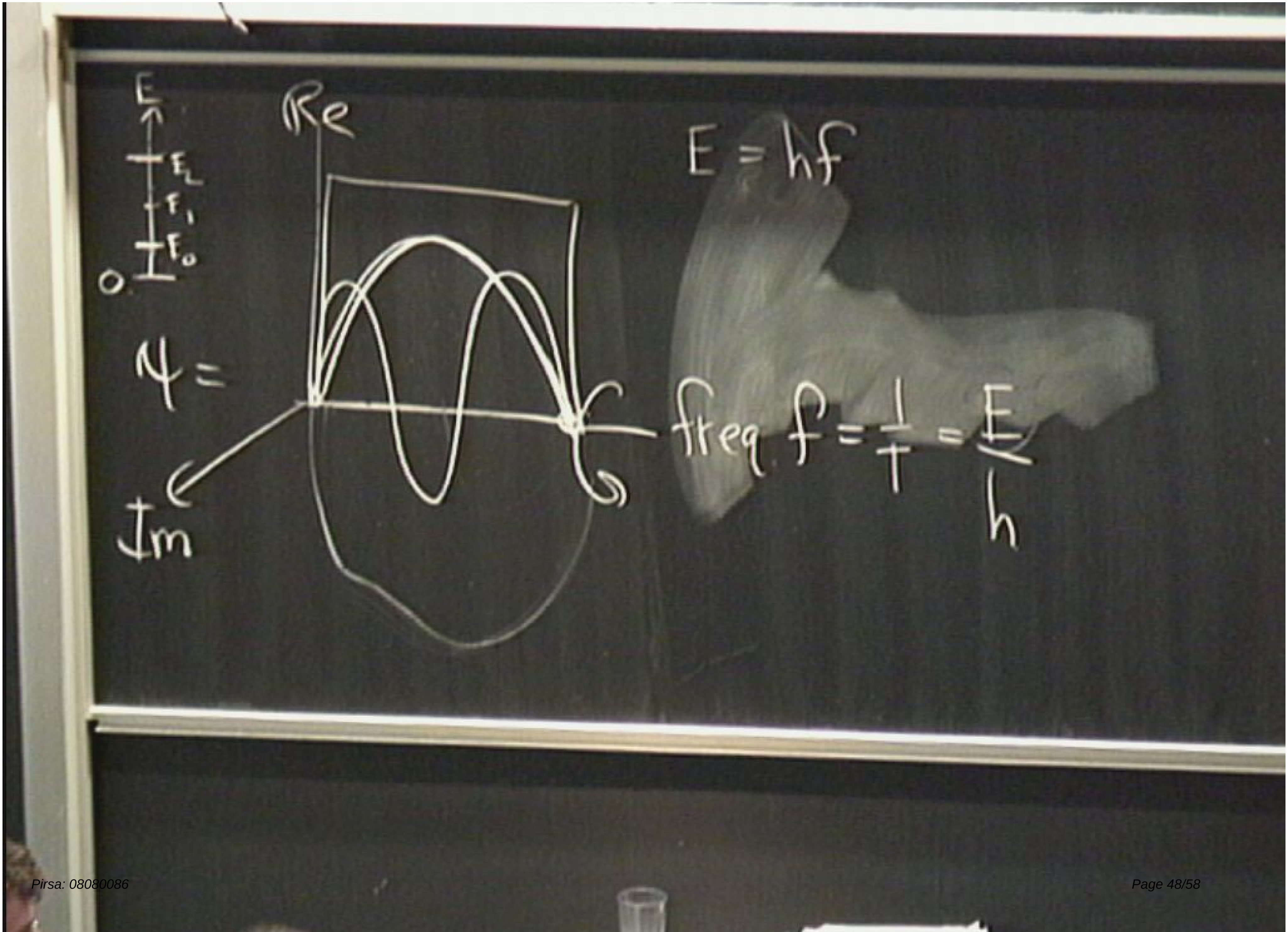

E
E_2
F_1
F_0
0
Re
Ψ =
Im

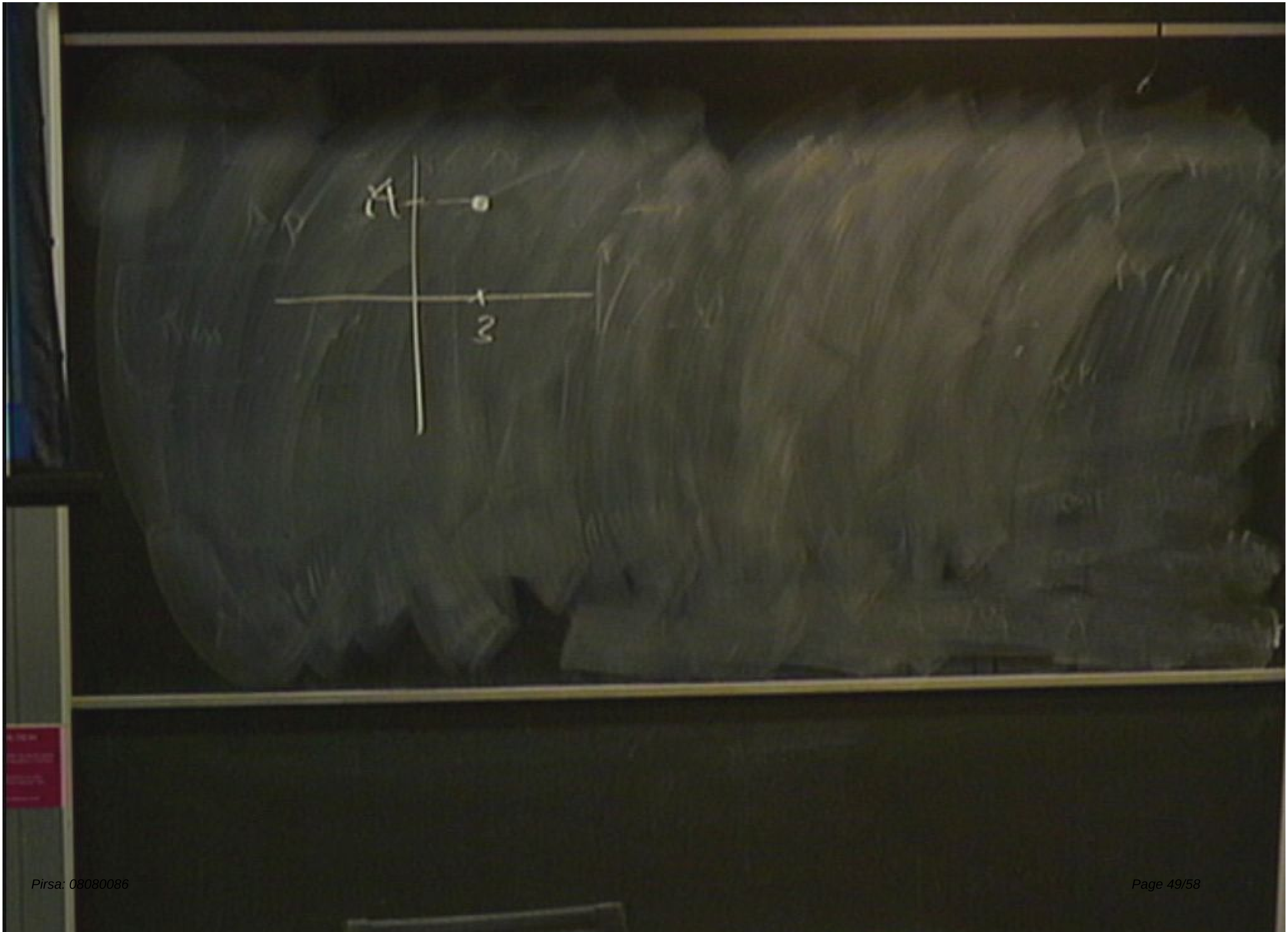

E
E_2
E_1
E_0
0
Re
$\Psi =$
Im



E
E_2
E_1
E_0
0
Re
Ψ =
freq. f =
Im


E
E_2
E_1
E_0
0
Re
E = hf
freq $f = \frac{1}{T} = \frac{E}{h}$


E
E2
E1
E0
0
Re
Ψ =
Im
E = hf
freq f = 1/T = E/h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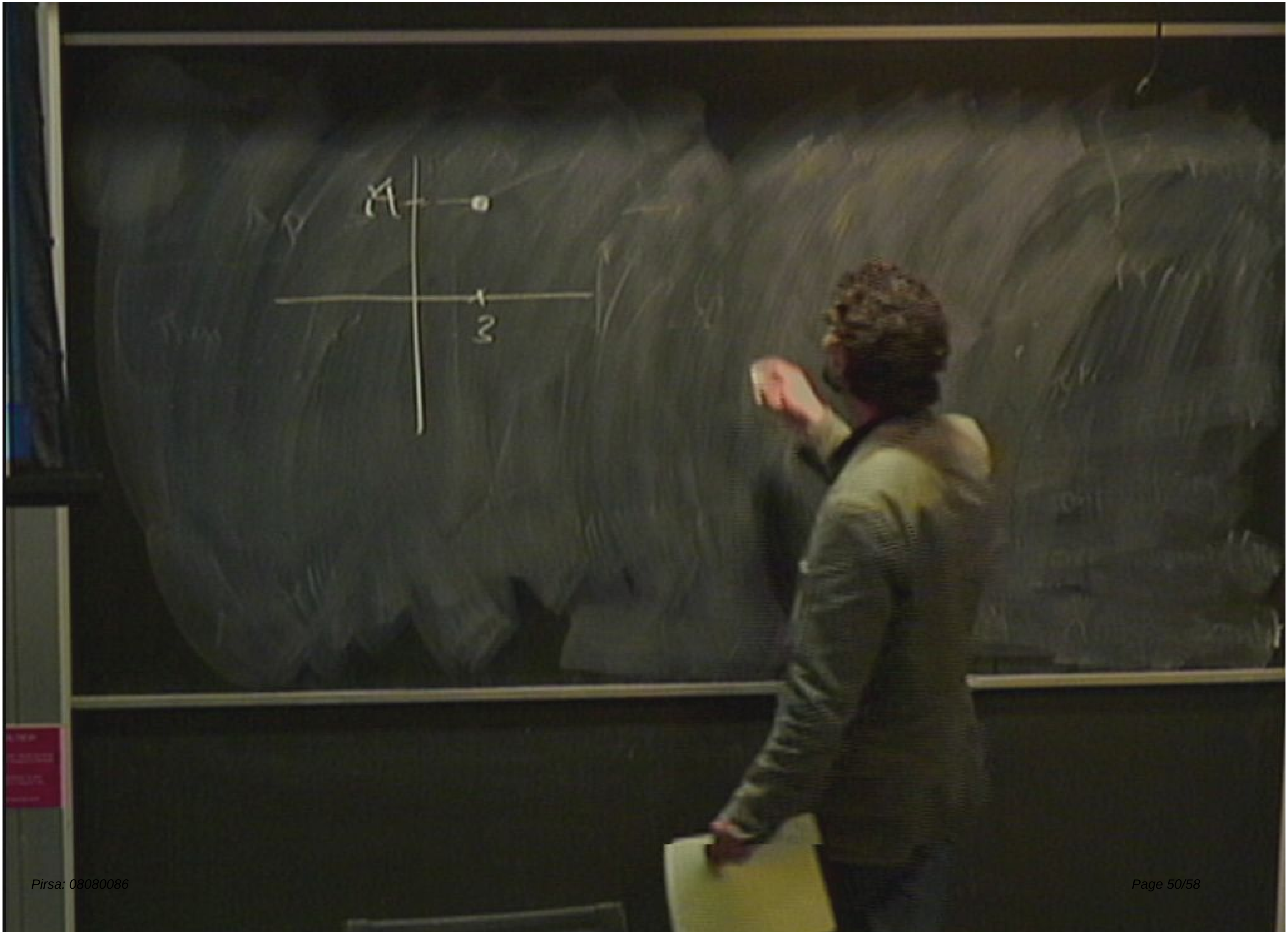
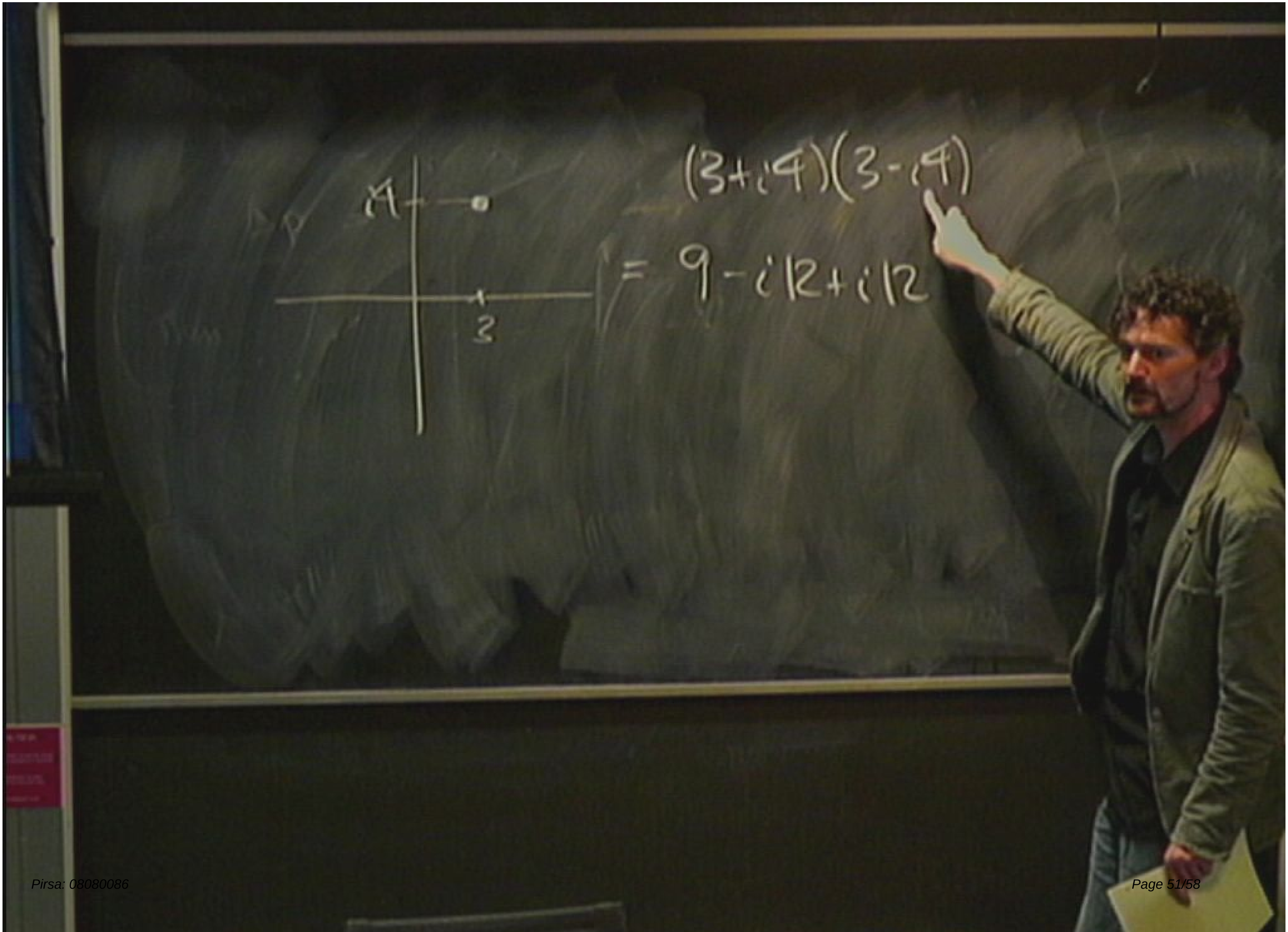
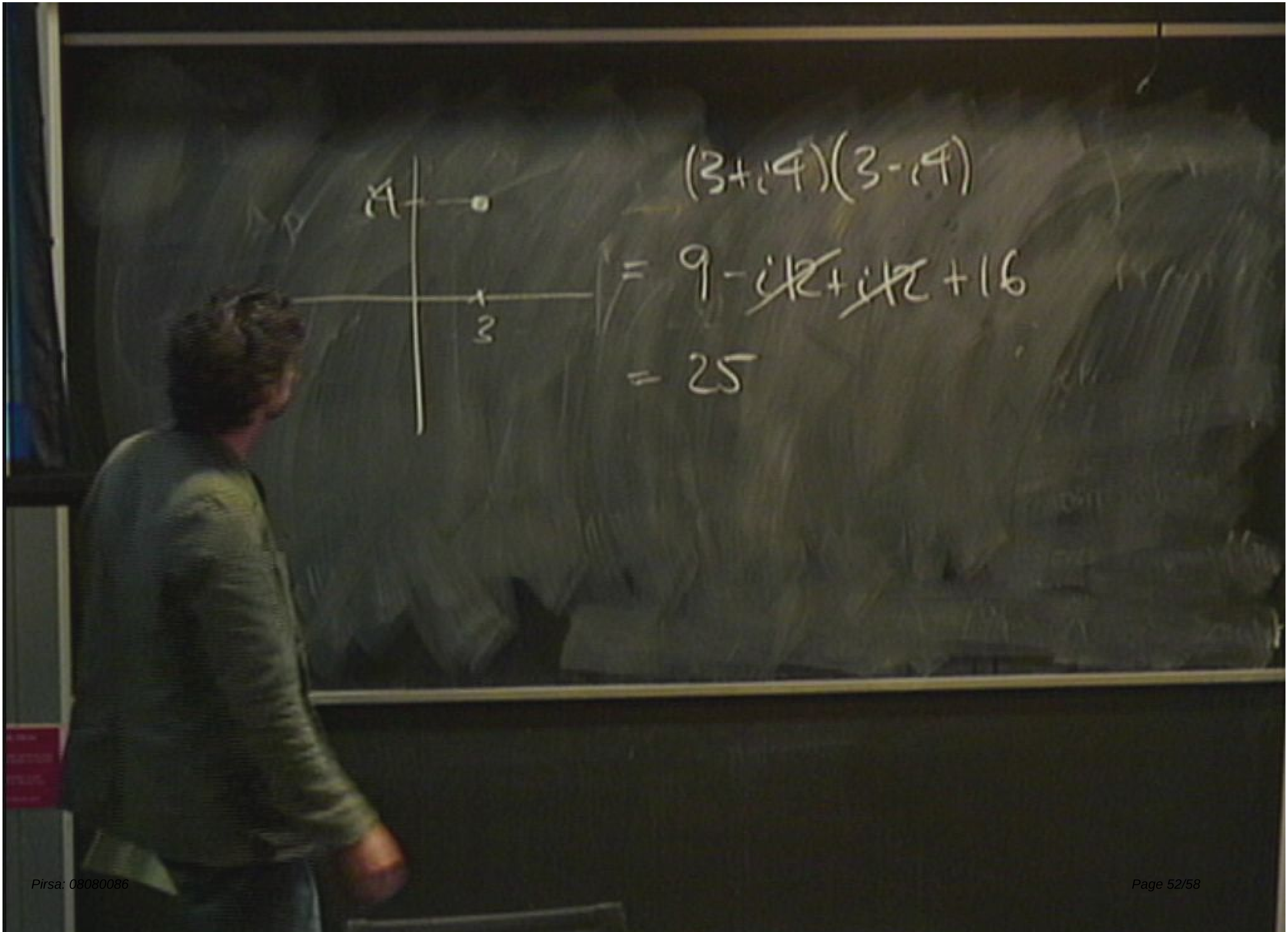
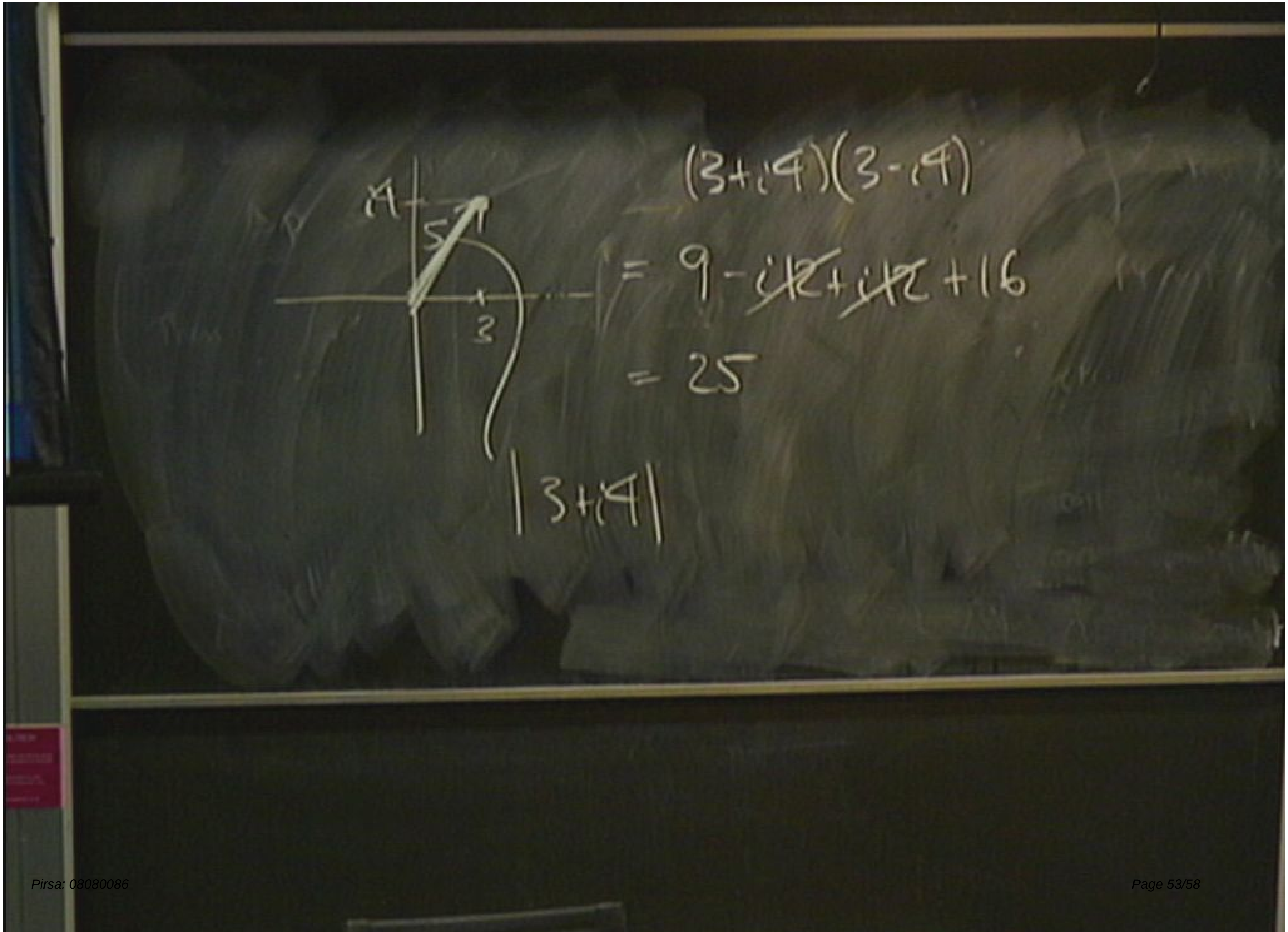
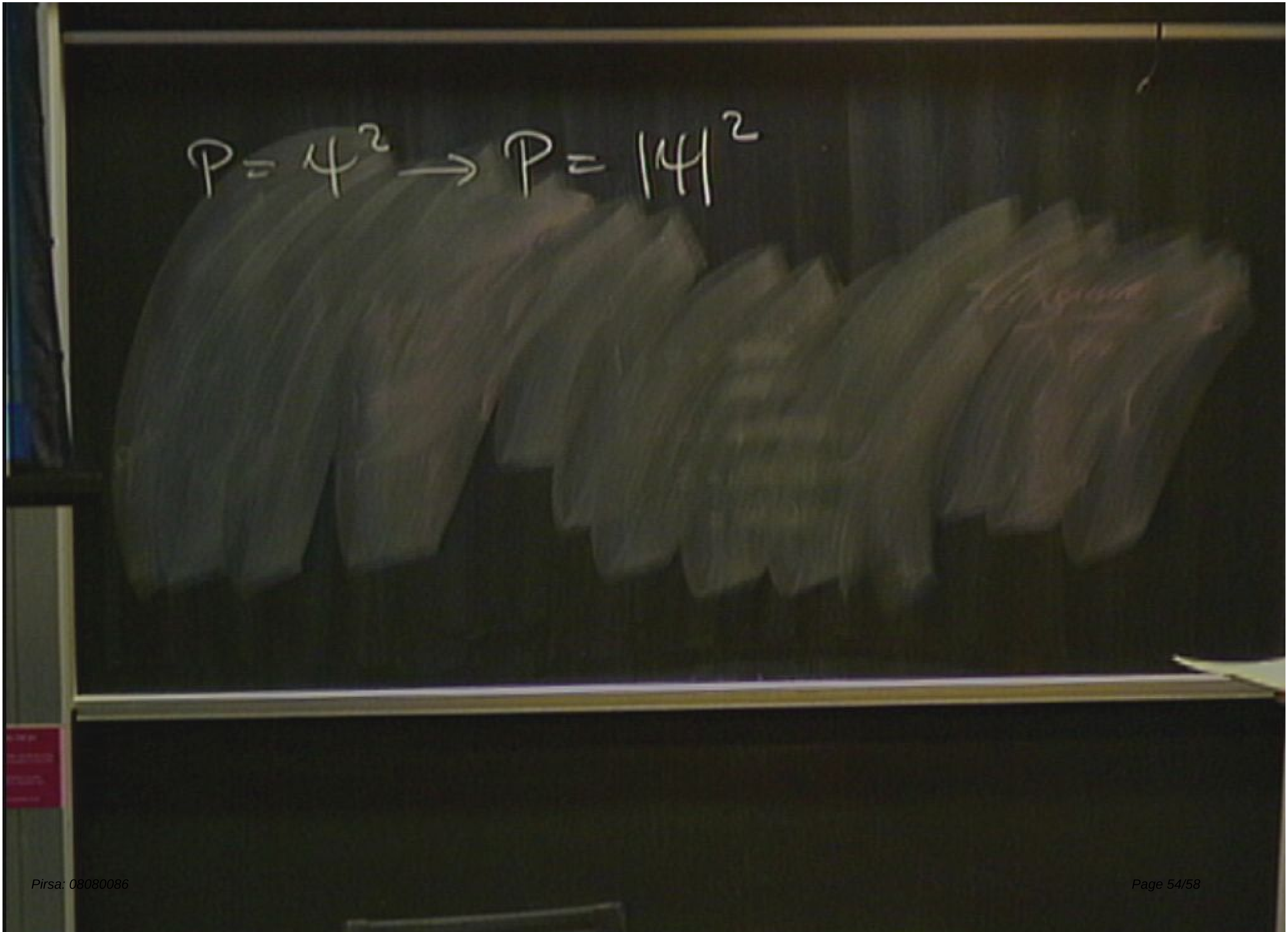
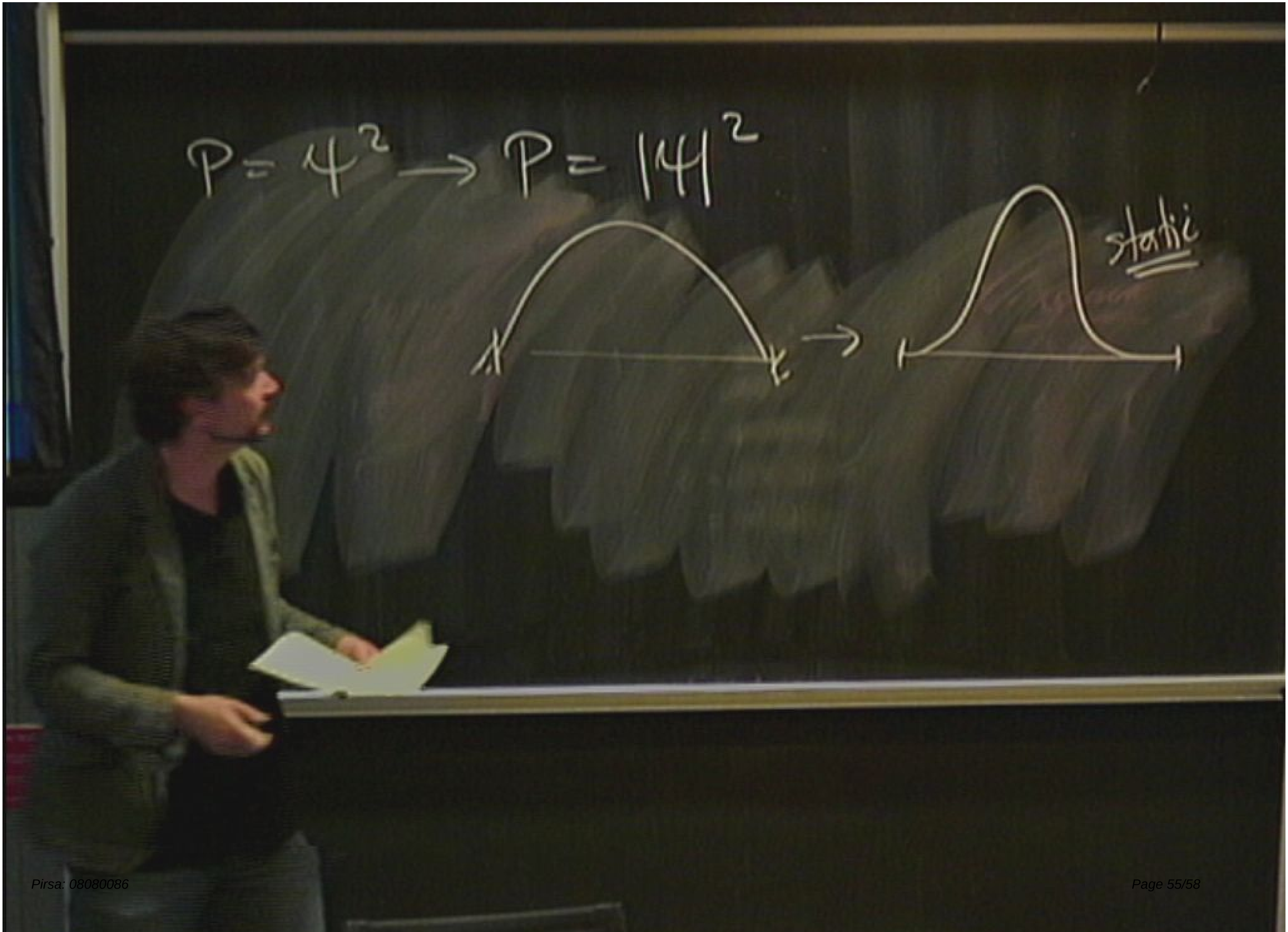

A
1
3


i4
3
(3+i4)(3-i4)
= 9 - i12 + i12

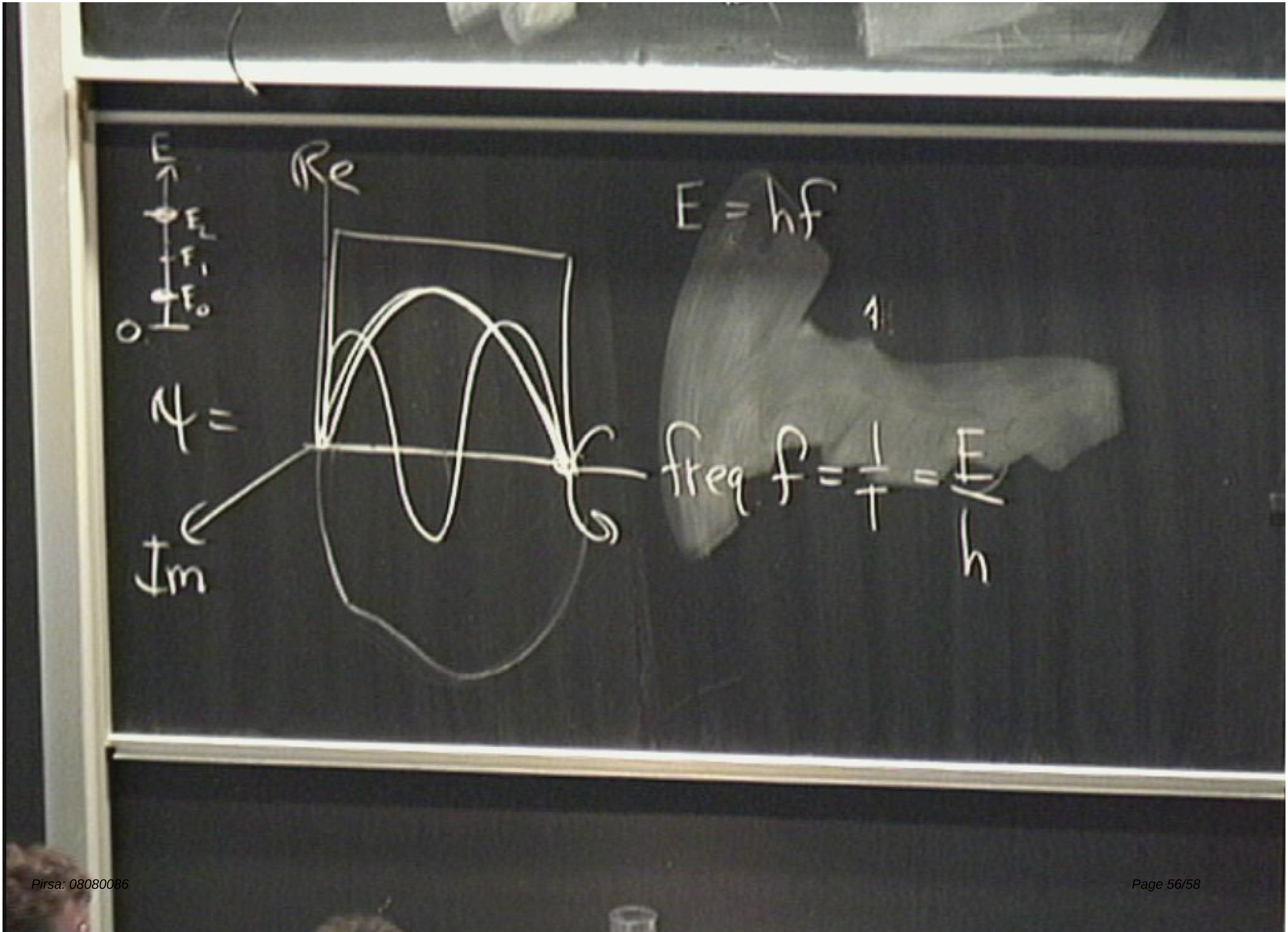

i4
3
(3+i4)(3-i4)
= 9 - i12 + i12 + 16
= 25

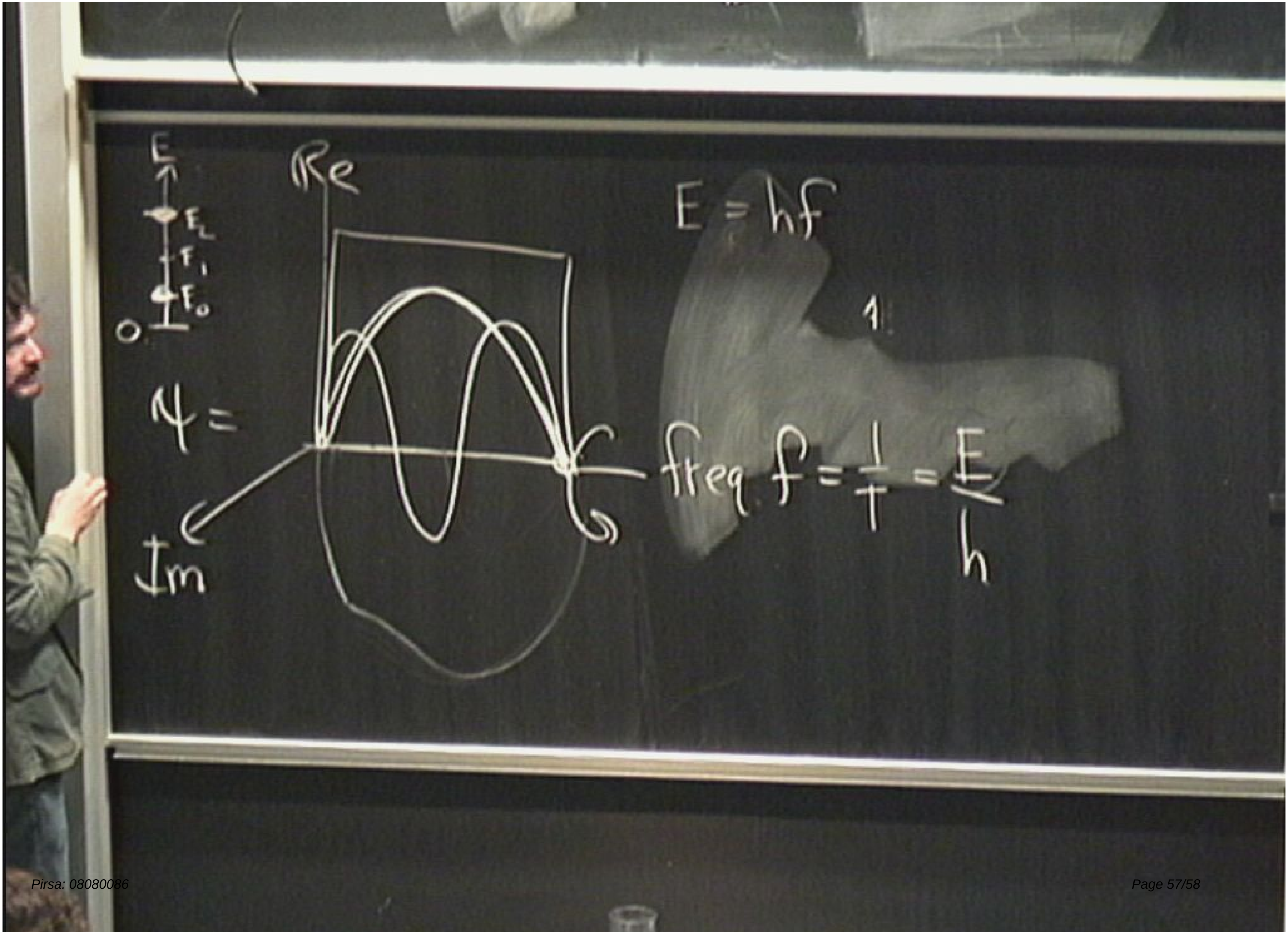

i4
5
3
(3+i4)(3-i4)
= 9 - ~~i12~~ + ~~i12~~ + 16
= 25
|3+i4|

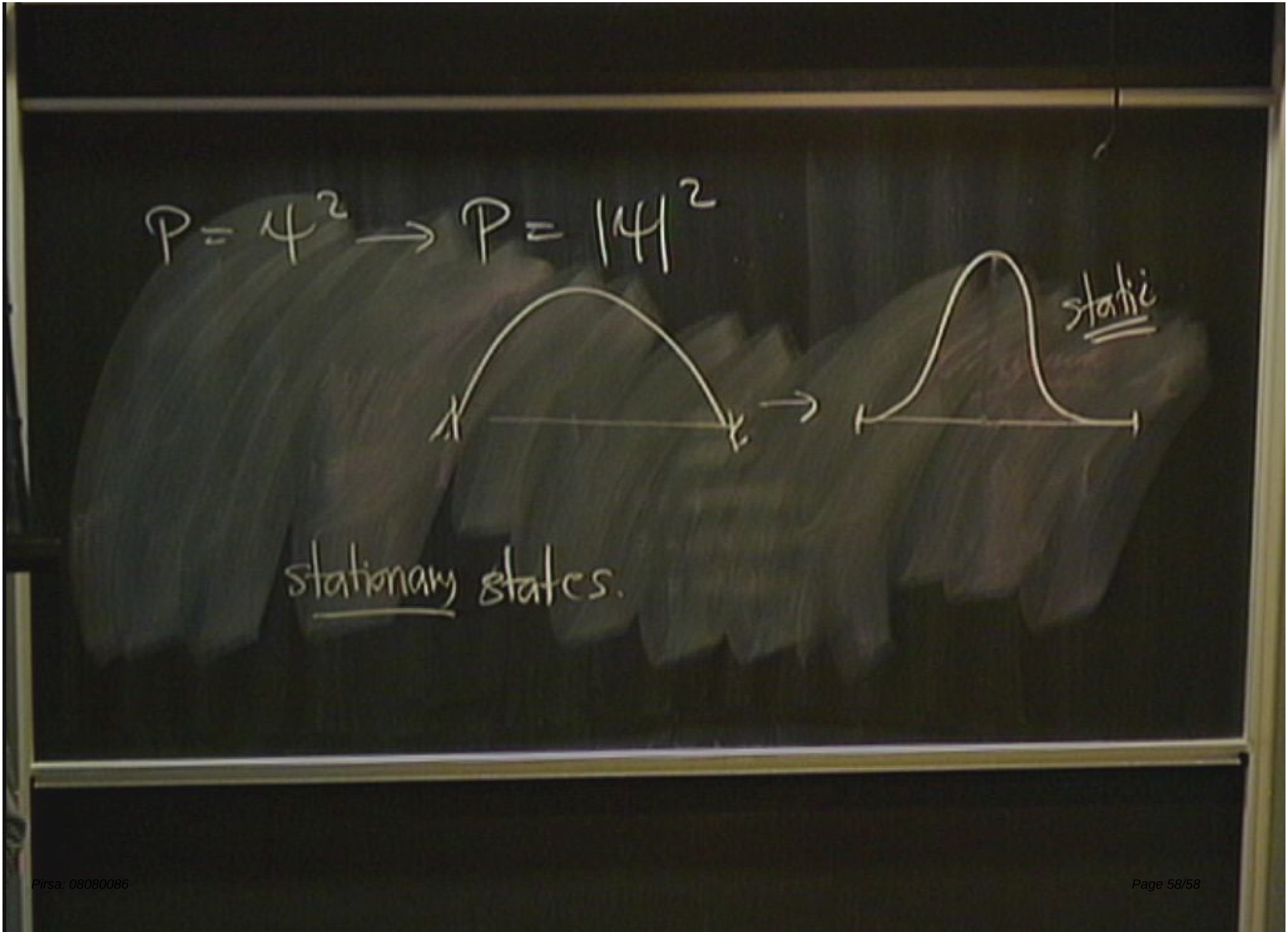

$P = \psi^2 \longrightarrow P = |\psi|^2$



P = ψ² → P = |ψ|²
x
t
static


E
E_2
E_1
E_0
0
Re
$\Psi =$
Im
$E = hf$
freq $f = \frac{1}{T} = \frac{E}{h}$


E
E₂
E₁
E₀
0
Re
Ψ =
Im
E = hf
freq. f = 1/T = E/h


$P = \psi^2 \longrightarrow P = |\psi|^2$
static
stationary states.