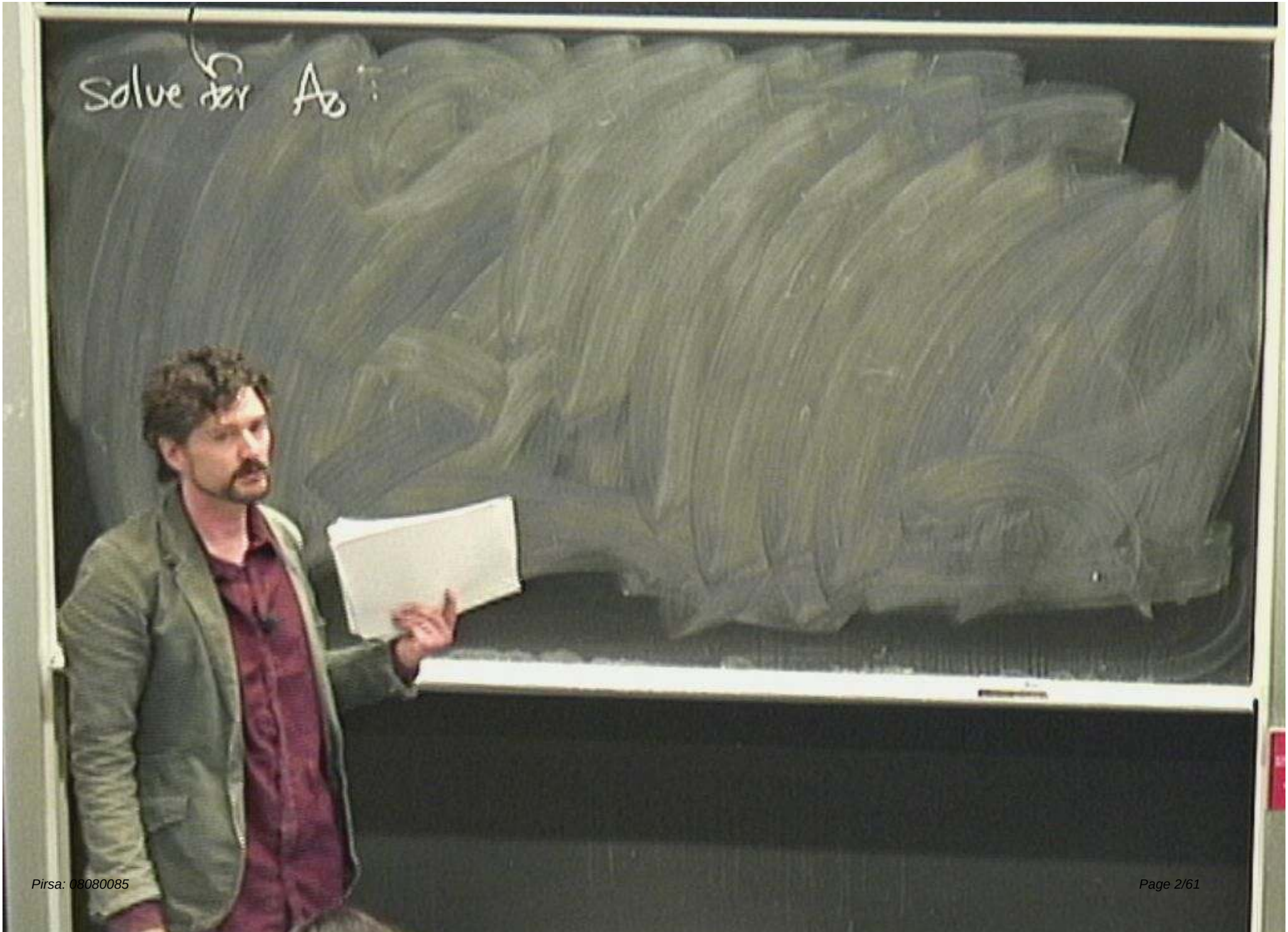
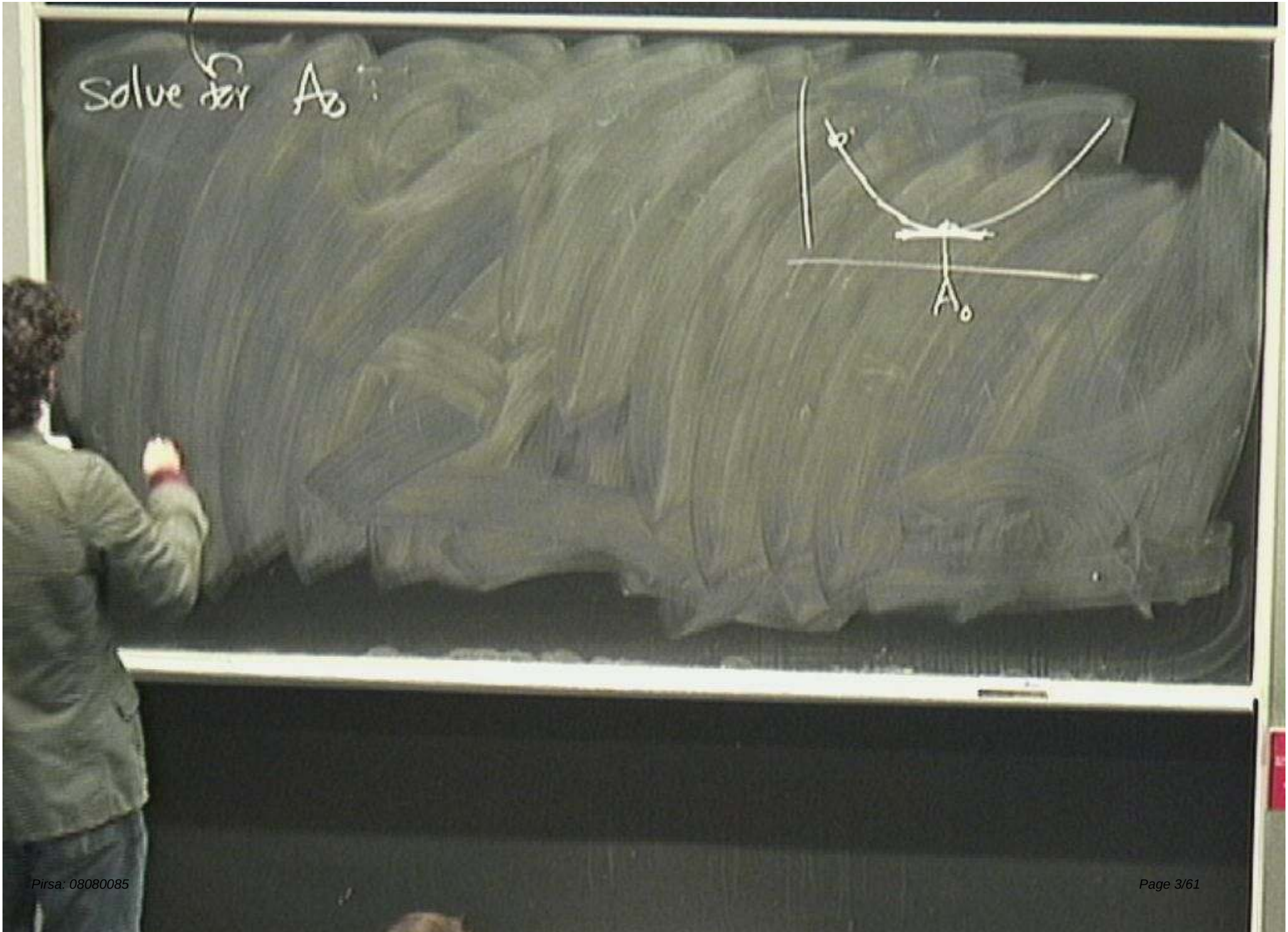
Title: Quantum Mechanics 10 - Stability of the Atom

Date: Aug 13, 2008 10:30 AM

URL: http://pirsa.org/08080085

Abstract: A discussion of how the zero point energy of atoms is what makes possible their existence in our universe â€“ atoms are purely quantum mechanical objects.
Learning Outcomes:
â€¢ Continuation of QM-9: A calculus-based derivation of the zero point energy of the quantum harmonic oscillator.
â€¢ How our previous understanding of energy quantization and zero point energy can be applied also to the hydrogen atom.
â€¢ Why classical atoms cannot exist in our universe, and how the Heisenberg Uncertainty Principle and associated zero point energy stabilizes atoms, making their existence possible.


Solve for A₀


Solve for A_0:
A_0

Solve for A_0:

$$\frac{d\langle E\rangle}{dA} =$$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 r m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum: KE

↑ classical PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum: KE

↑ classical PE

$$\frac{d}{dx}x^n =$$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum: KE

↑ classical PE

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dA}(A^2) =$$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum: KE

↑ classical PE

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dA}(A^2) = 2A$$

$$\frac{d}{dA}\left(\frac{1}{A^2}\right) =$$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum: KE

$\frac{1}{A^2} : A^{-2}$

↑ classical PE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dA}(A^2) = 2A$$

$$\frac{d}{dA}\left(\frac{1}{A^2}\right) = -2\frac{1}{A^3}$$

Solve for A_0:

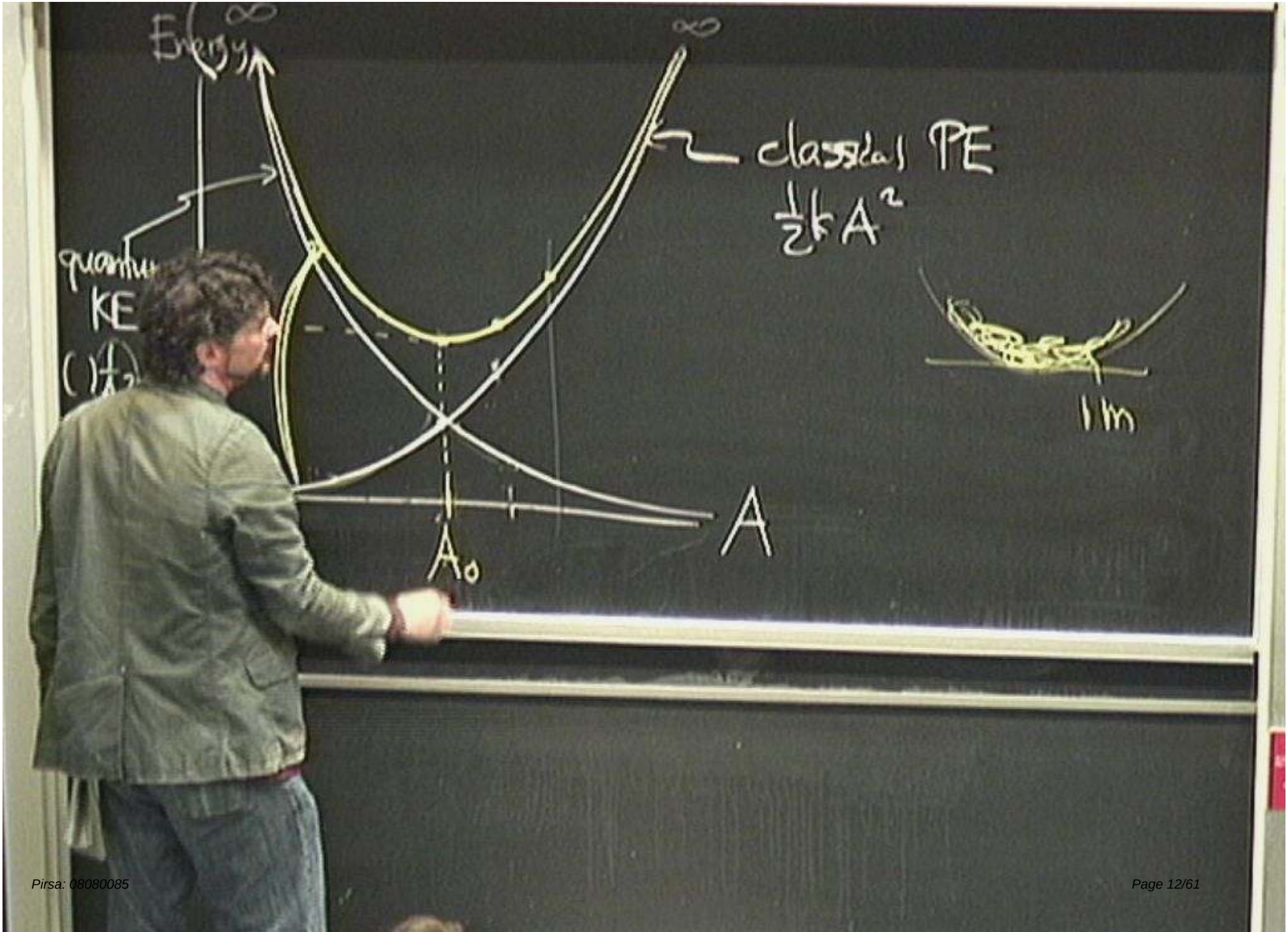
$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

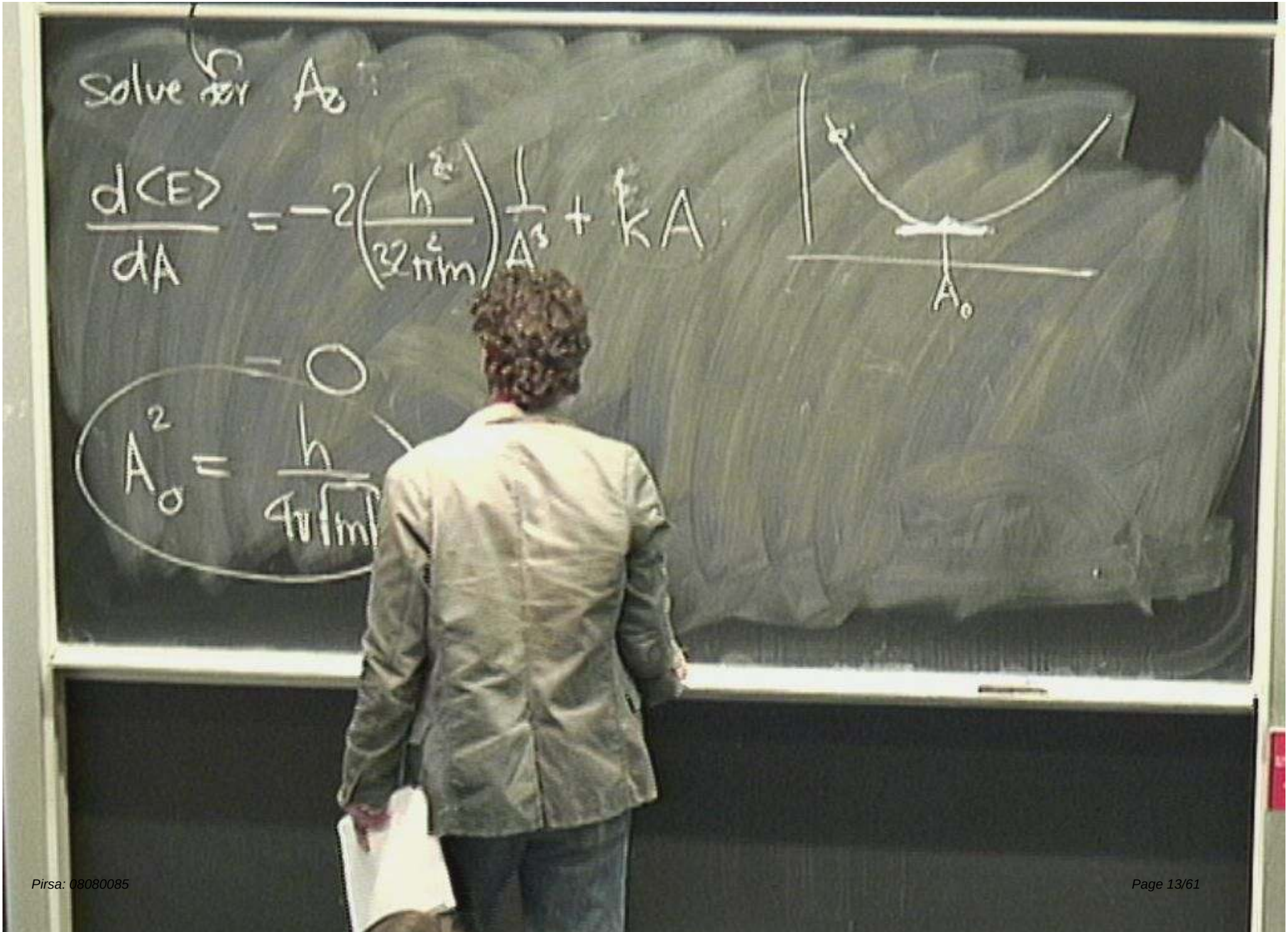
Solve for A_0:

$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

$$= 0$$

$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$


Energy
classical PE
$\frac{1}{2}kA^2$
quantum
KE
A
A_0

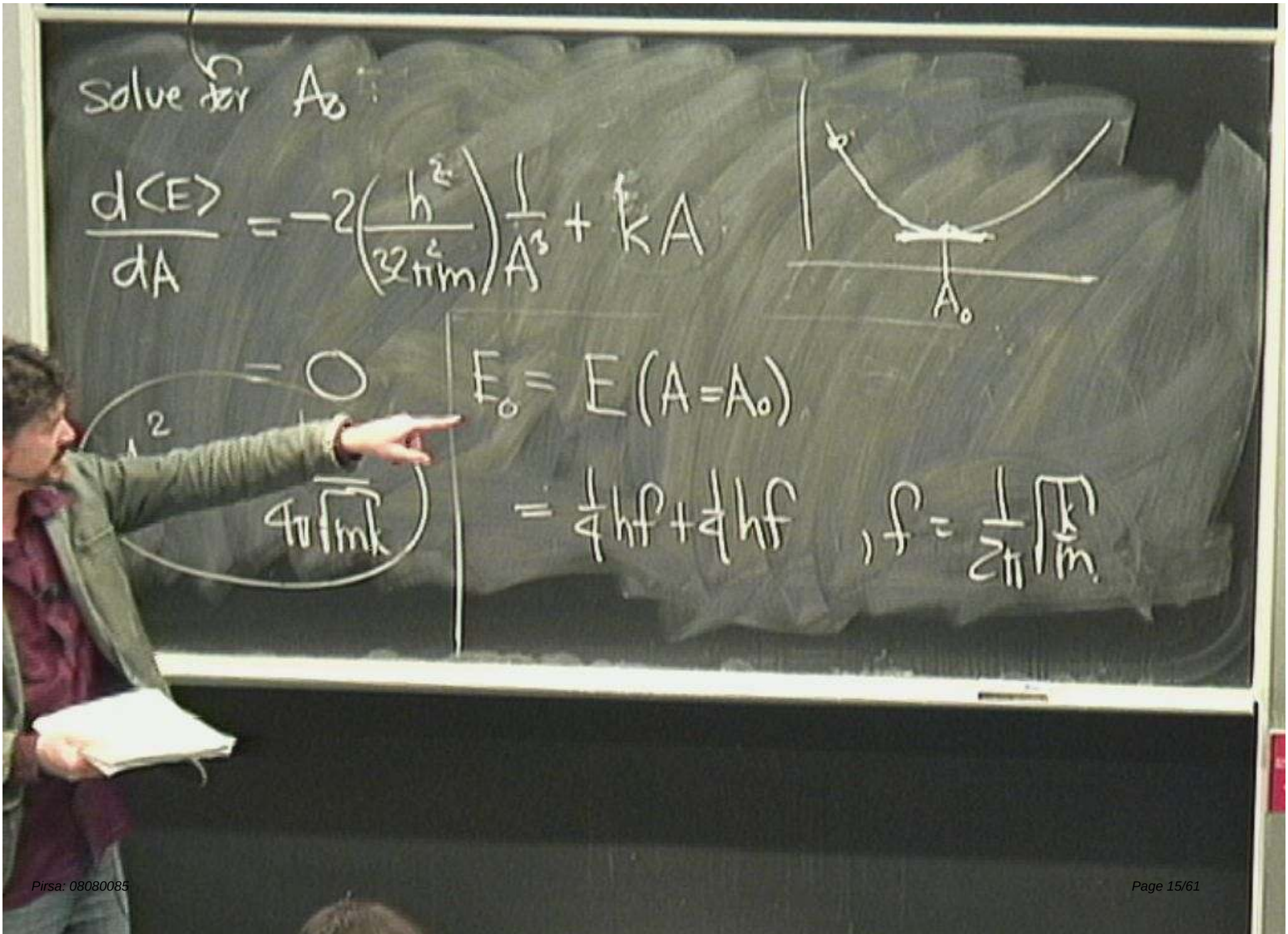

Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
$= 0$
$A_0^2 = \frac{h}{4\pi\sqrt{km}}$
A_0

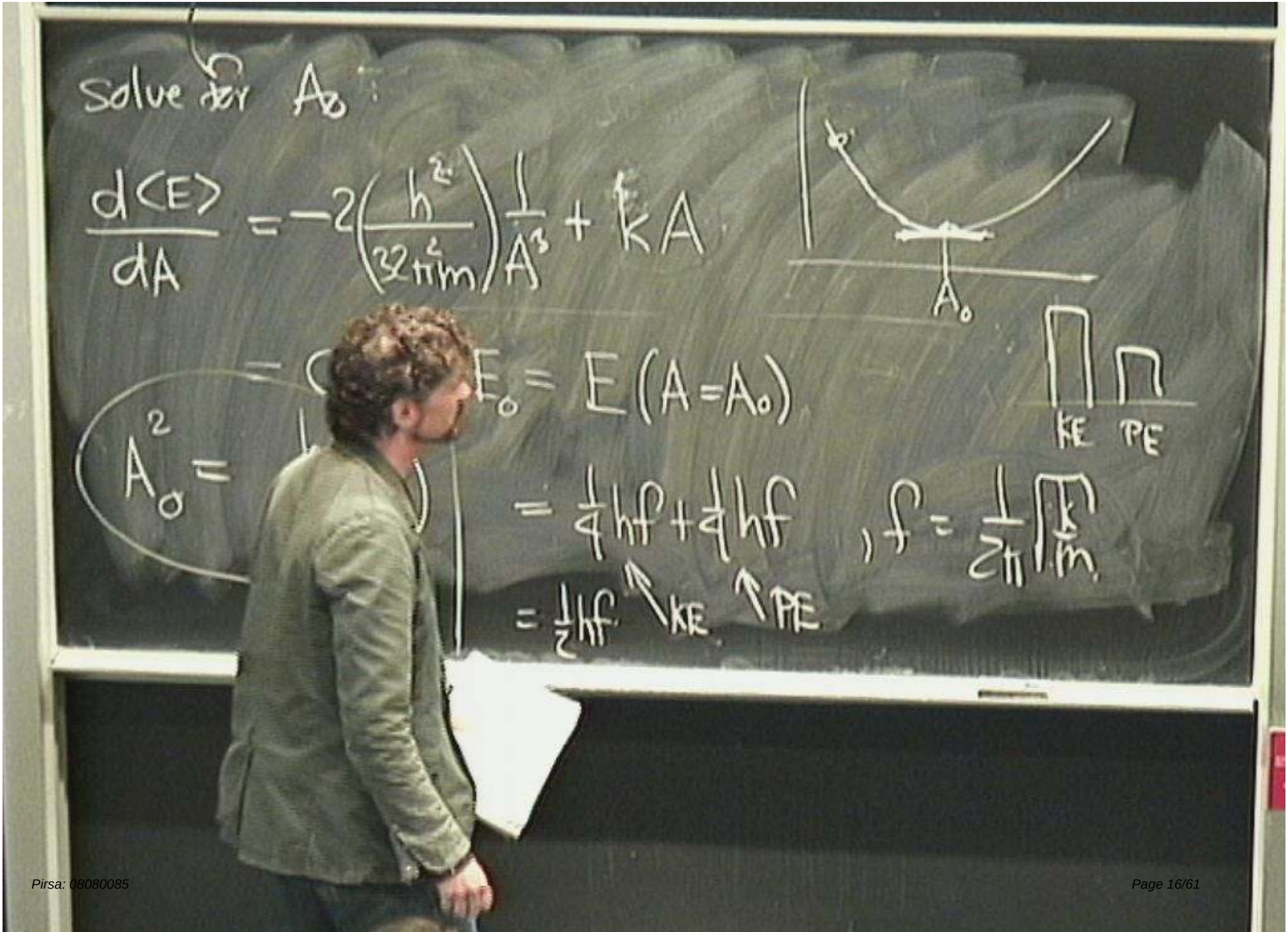
Solve for A_0:

$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

$$= 0$$

$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$


Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + \frac{1}{2}kA$
$= 0$
A_0
$E_0 = E(A = A_0)$
$= \frac{1}{4}hf + \frac{1}{4}hf$, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$


Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
A_0
$E_0 = E(A = A_0)$
$A_0^2 =$
$= \frac{1}{4}hf + \frac{1}{4}hf$
$= \frac{1}{2}hf$
KE
PE
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Solve for A_0:

$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

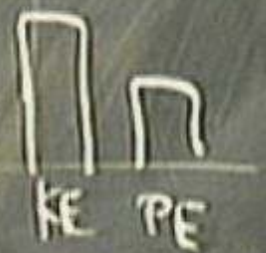
$$= 0$$

$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$

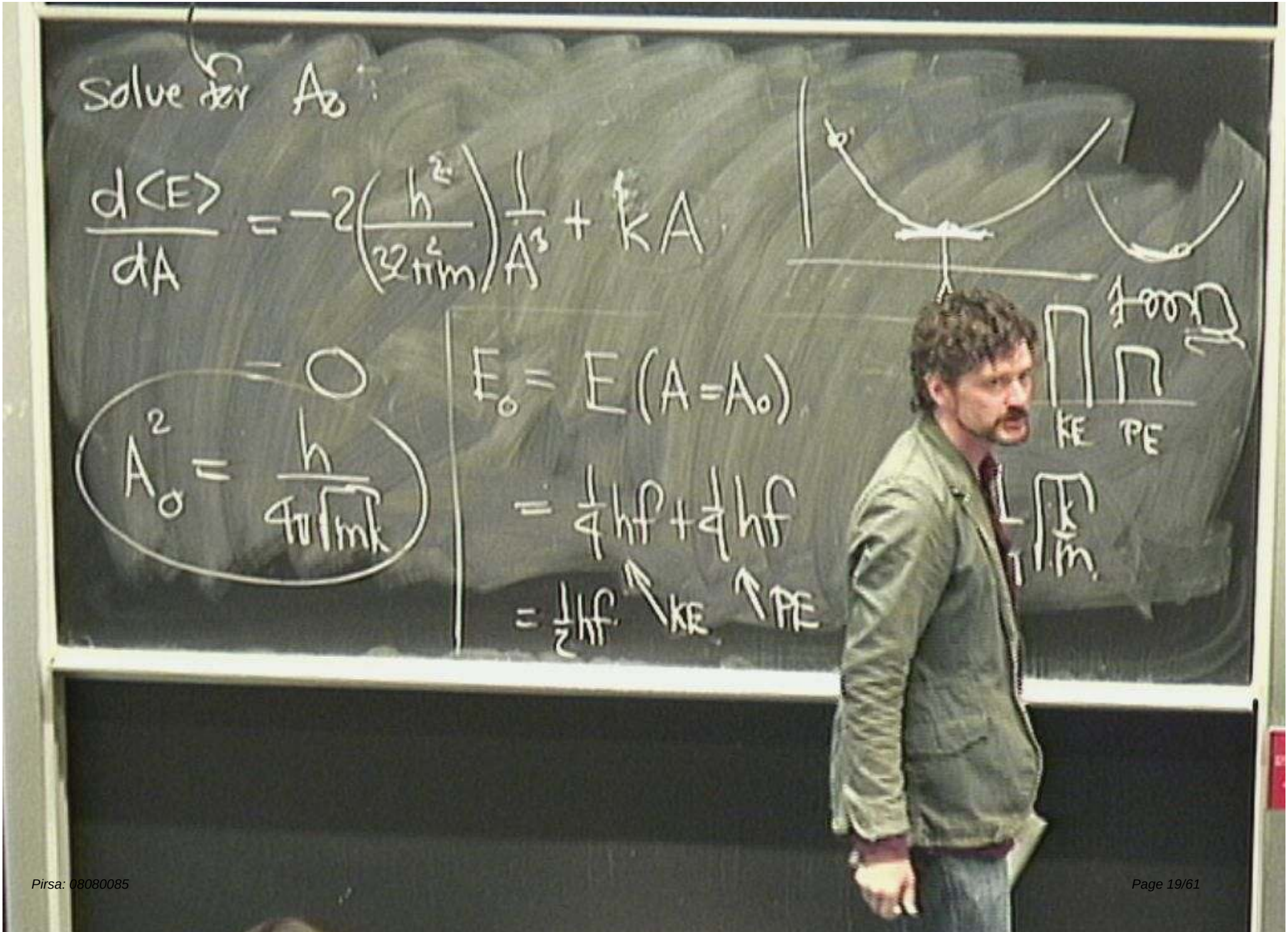
$E_0 = E(A = A_0)$

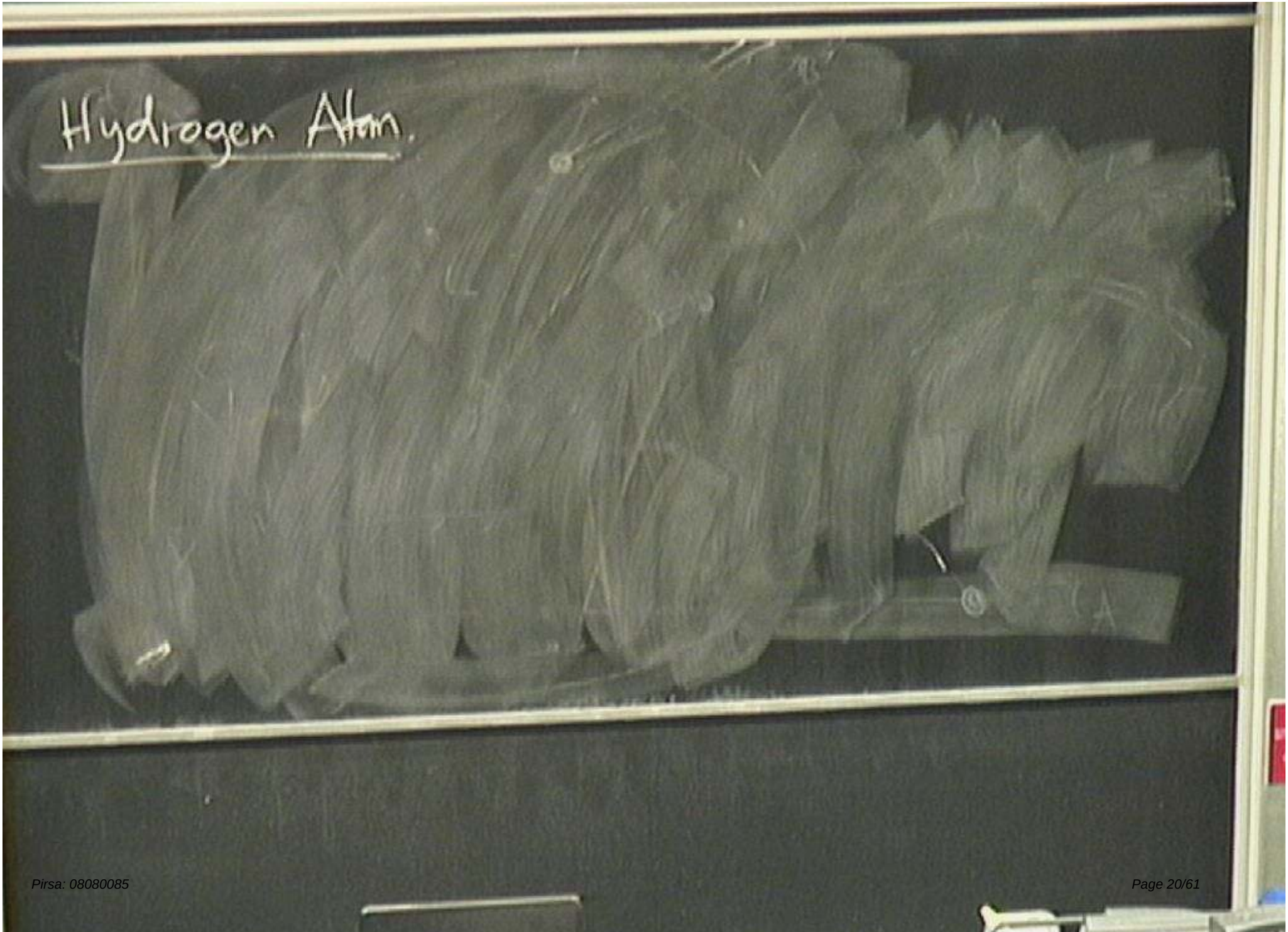
$= \frac{1}{4}hf + \frac{1}{4}hf$, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

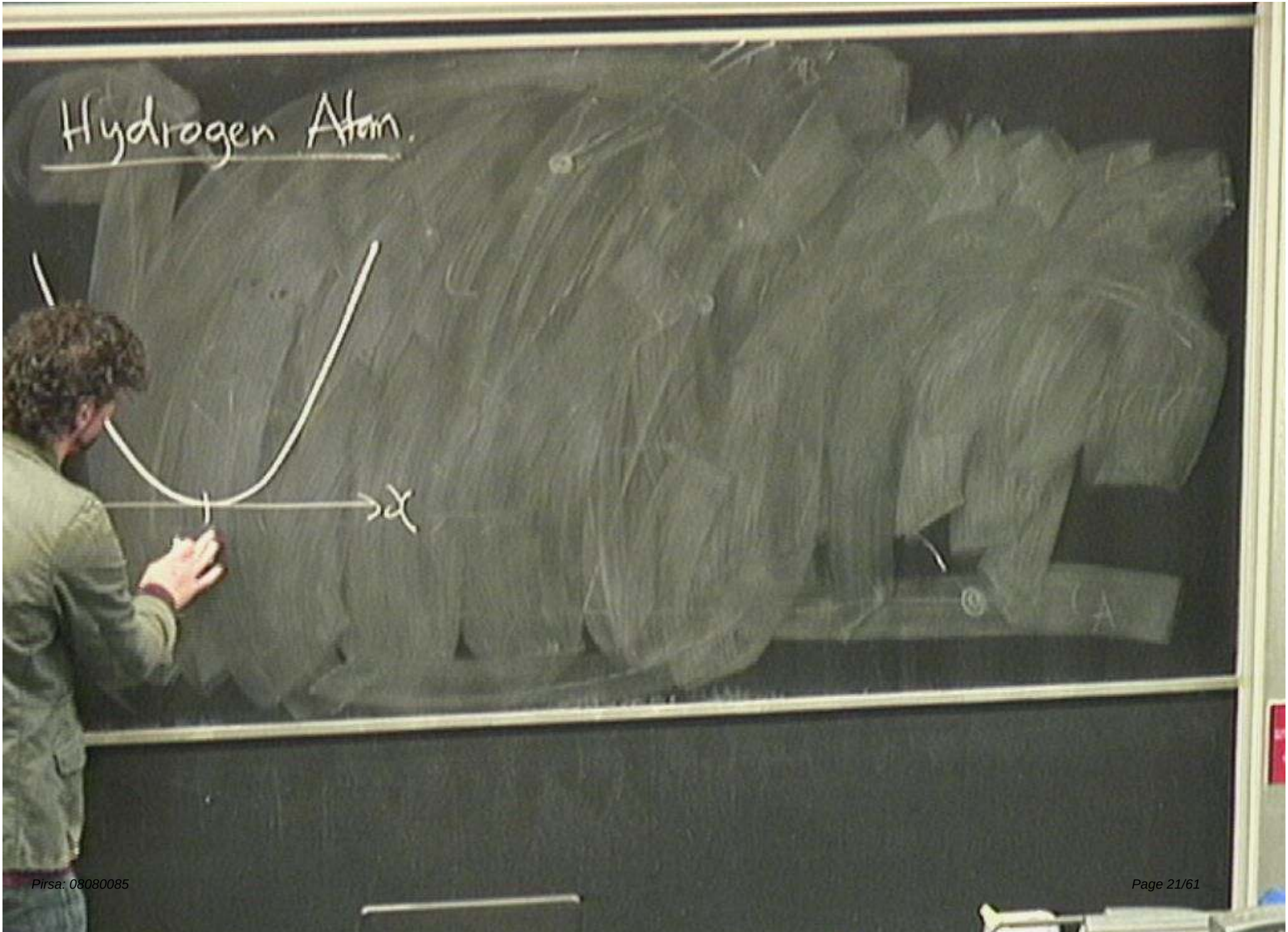
$= \frac{1}{2}hf$ (KE, PE)



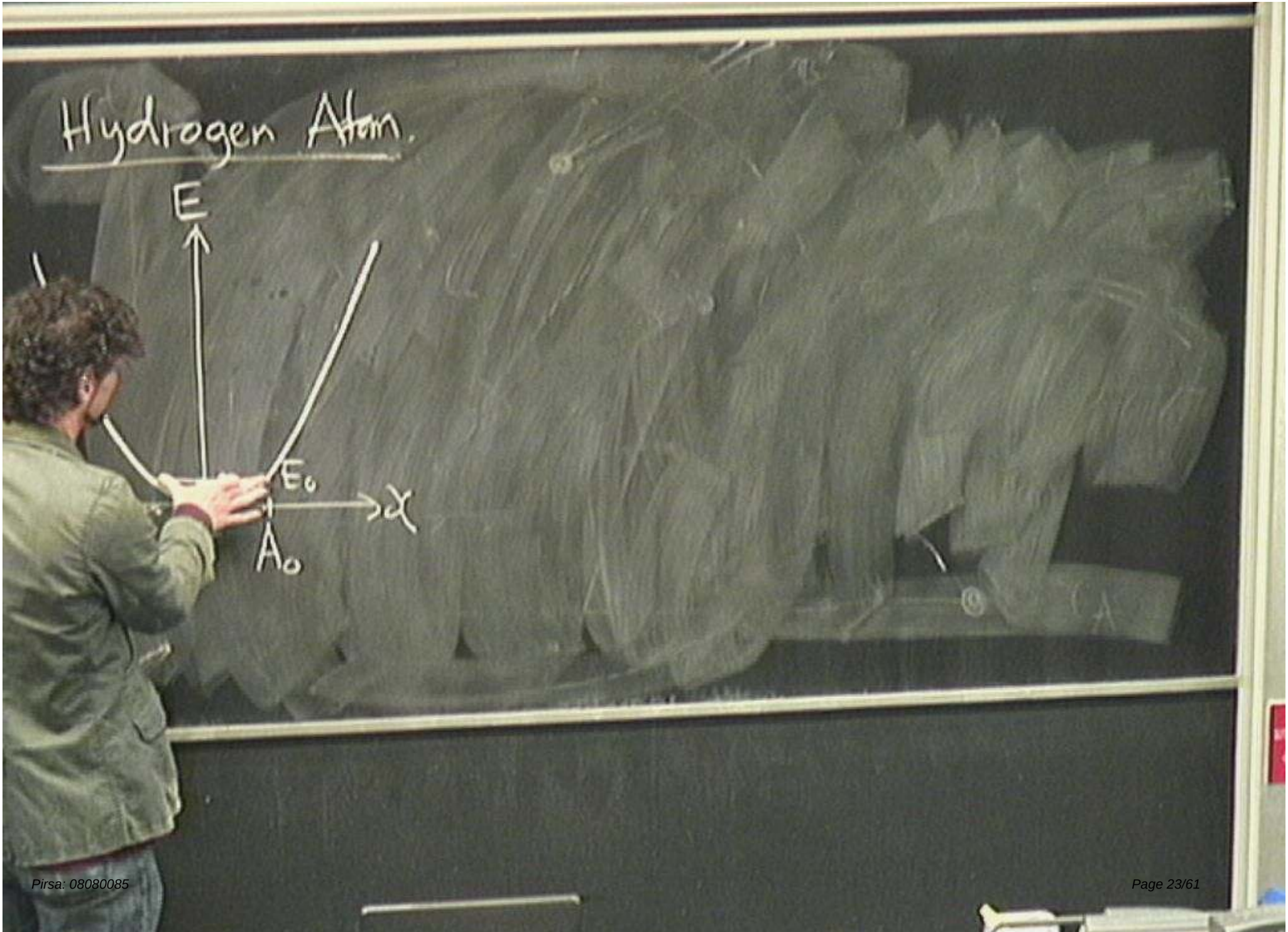
3/4
1/4
1/4
1/4
0

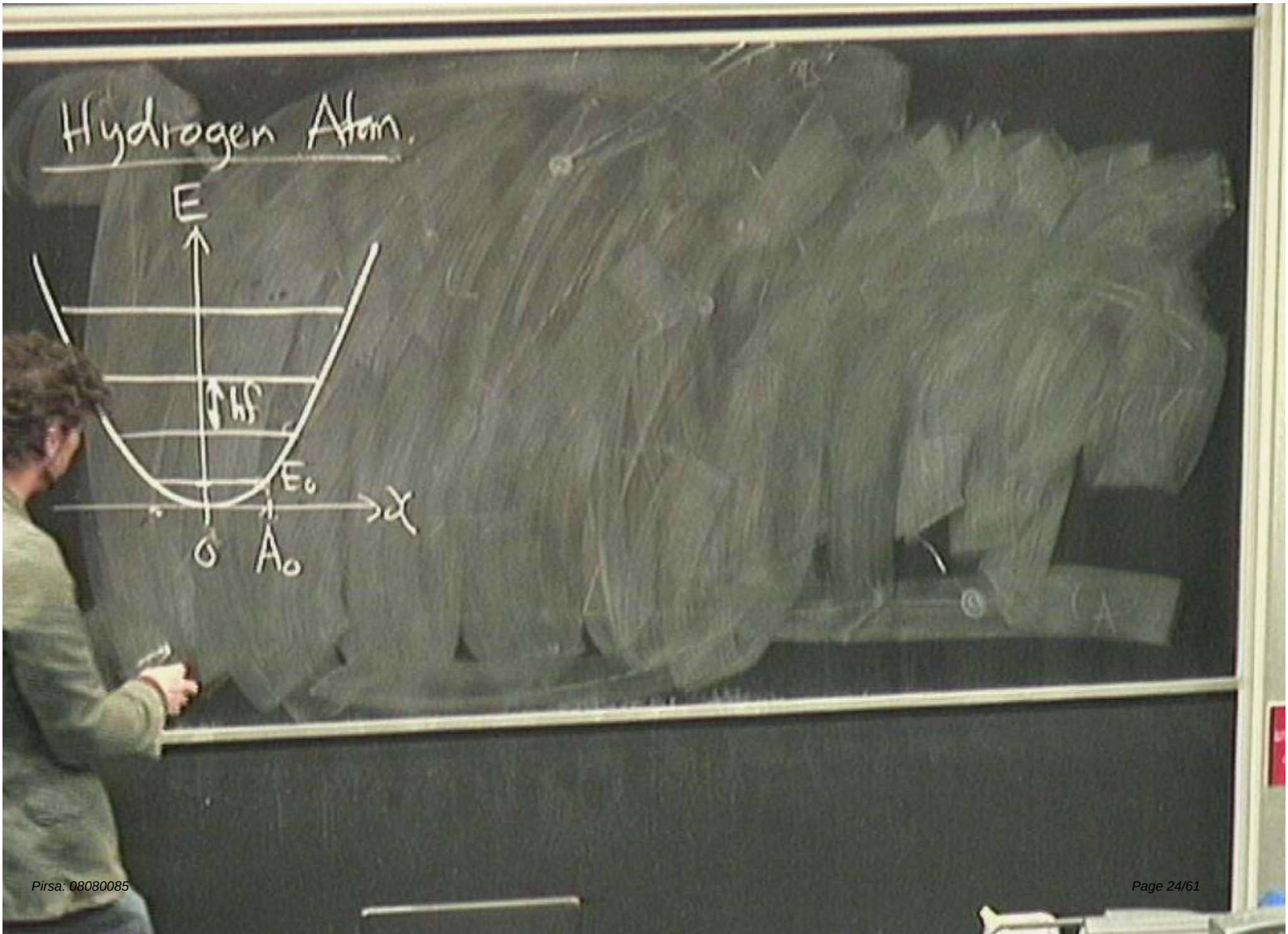

Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
$= 0$
$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$
$E_0 = E(A = A_0)$
$= \frac{1}{4}hf + \frac{1}{4}hf$
$= \frac{1}{2}hf$
KE
PE
KE
PE
L
m


Hydrogen Atom.

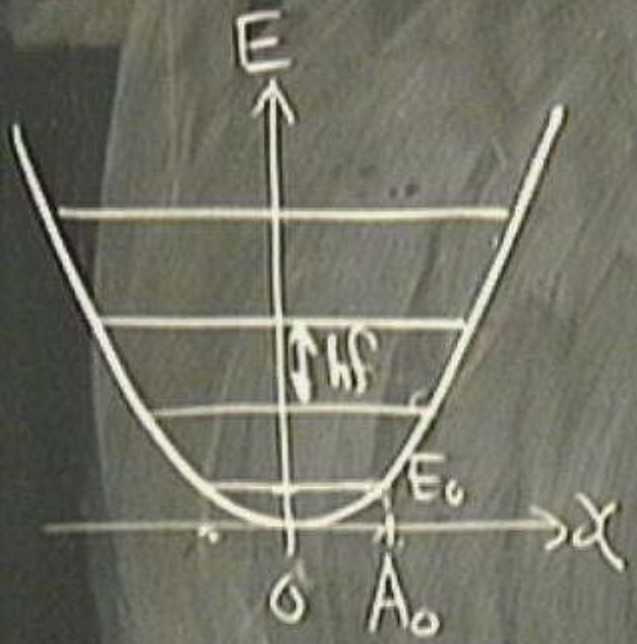

Hydrogen Atom.
x

Hydrogen Atom.

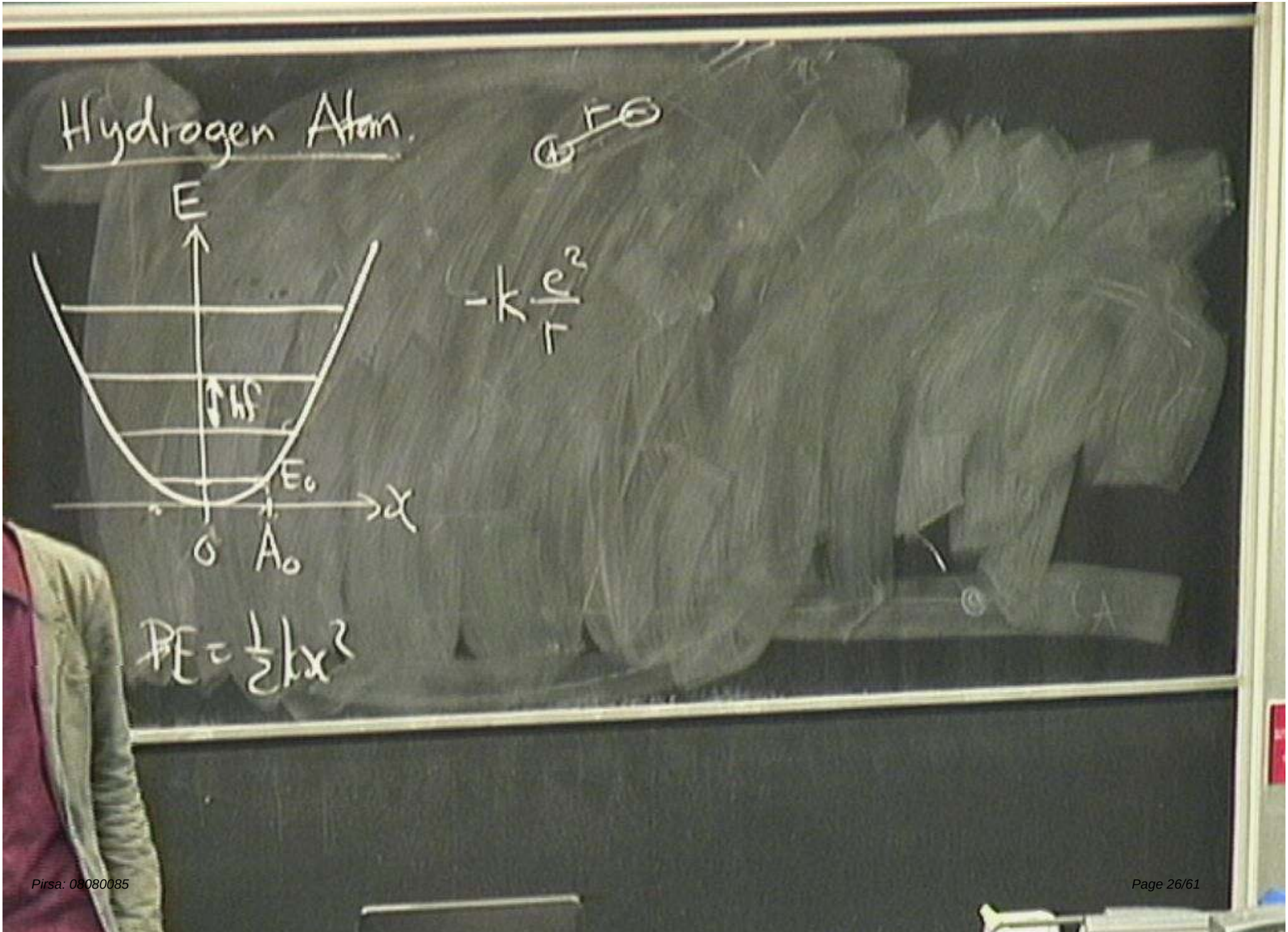

Hydrogen Atom.
E
E_0
x
A_0


Hydrogen Atom.
E
hf
E₀
x
0
A₀

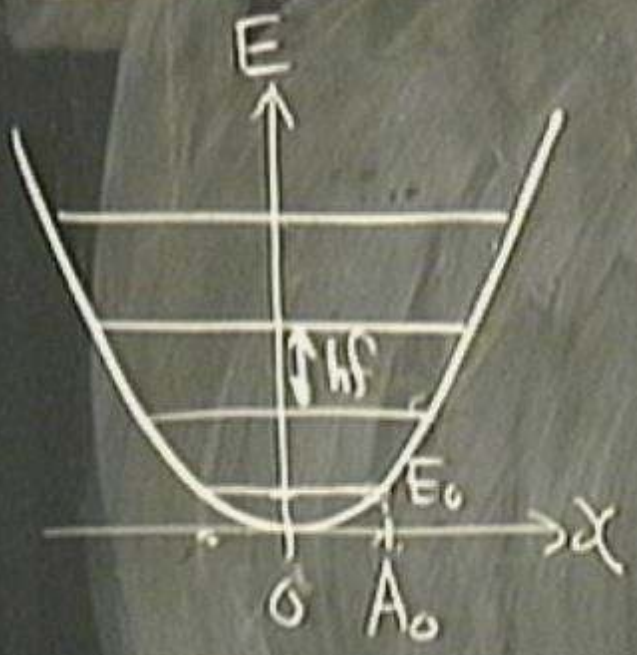
Hydrogen Atom.



$PE = \frac{1}{2}kx^2$

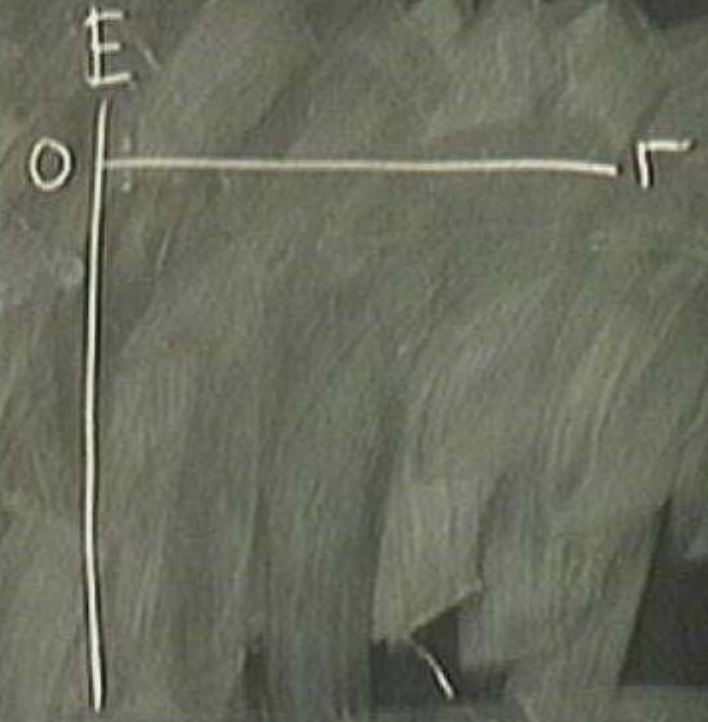

Hydrogen Atom.
E
hf
E_0
x
0
A_0
$PE = \frac{1}{2}kx^2$
$-k\frac{e^2}{r}$

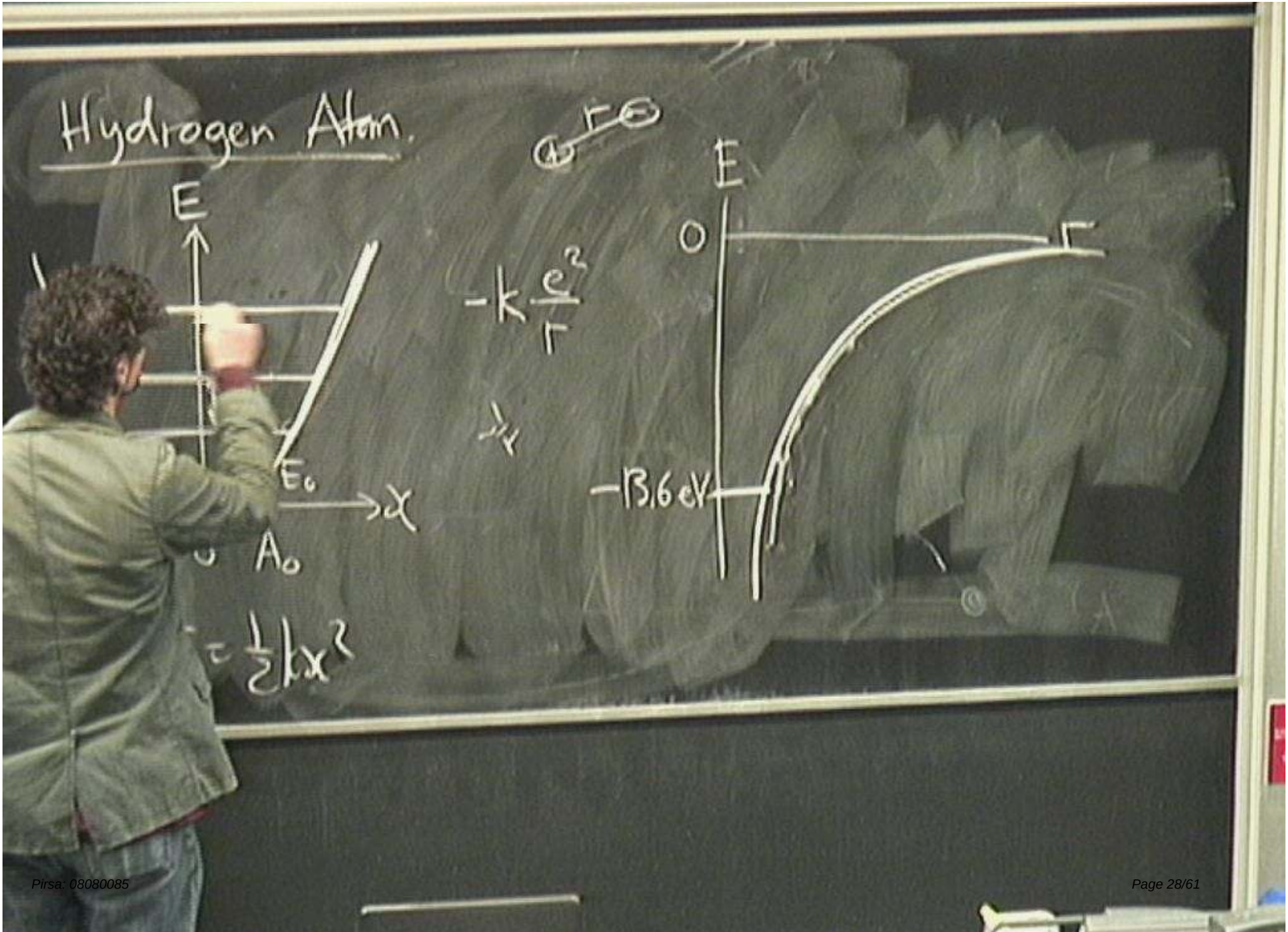
Hydrogen Atom



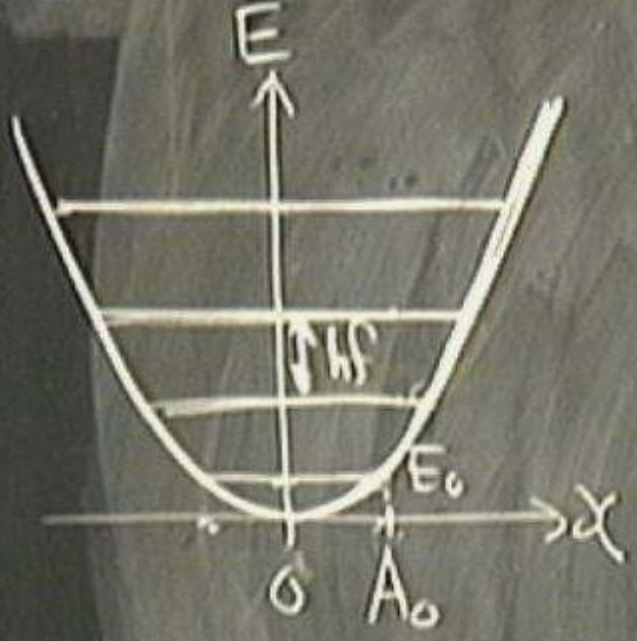
$PE = \frac{1}{2} k x^2$

$-k\frac{e^2}{r}$



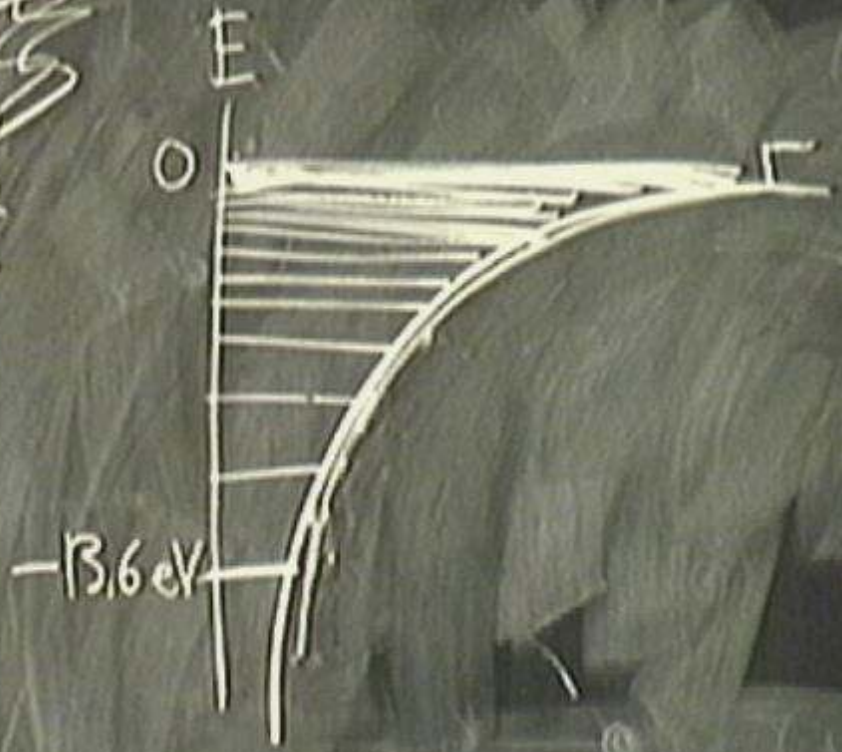

Hydrogen Atom.
E
E_0
x
A_0
$-k\frac{e^2}{r}$
E
0
r
-13.6 eV

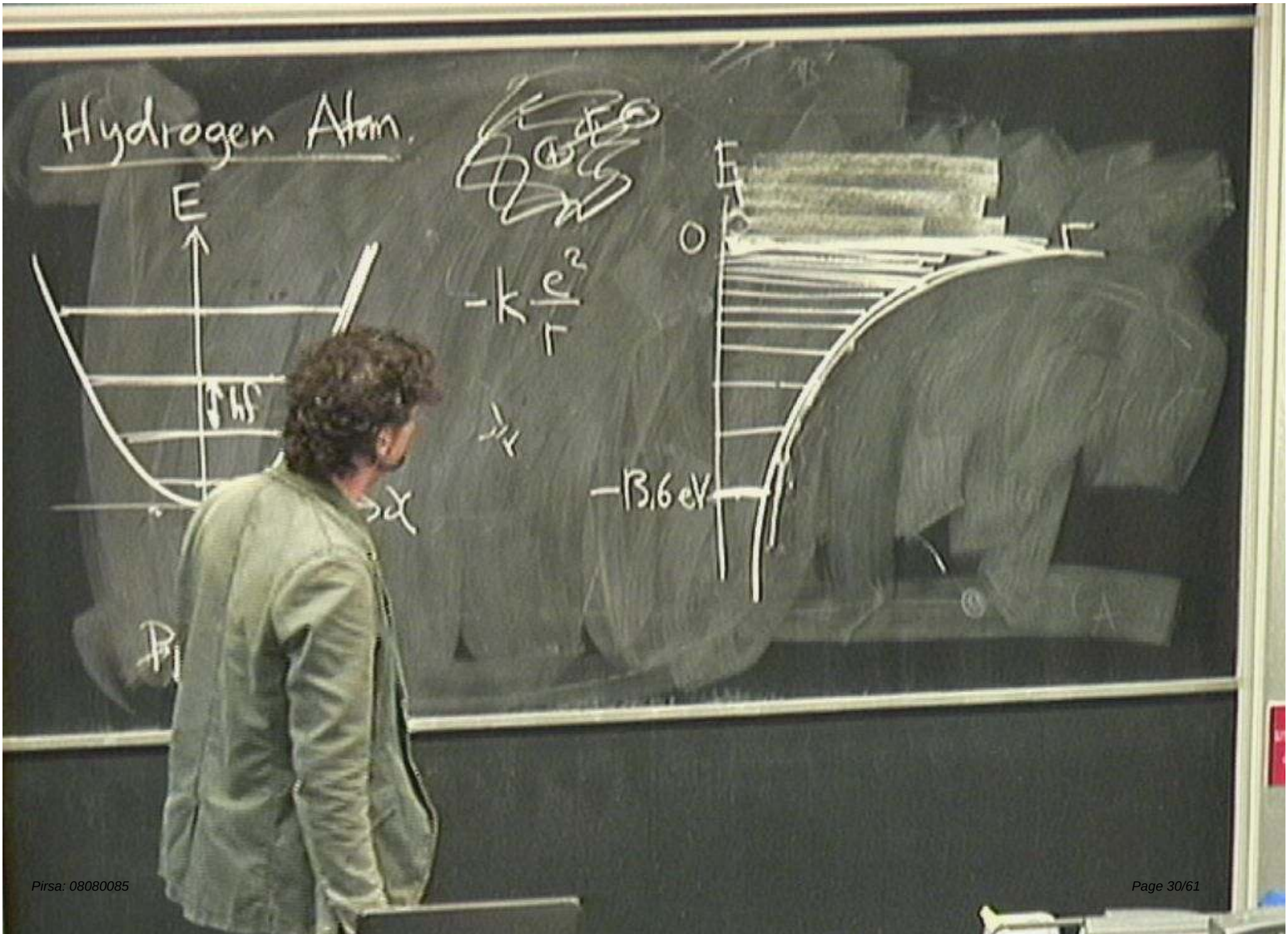
Hydrogen Atom.

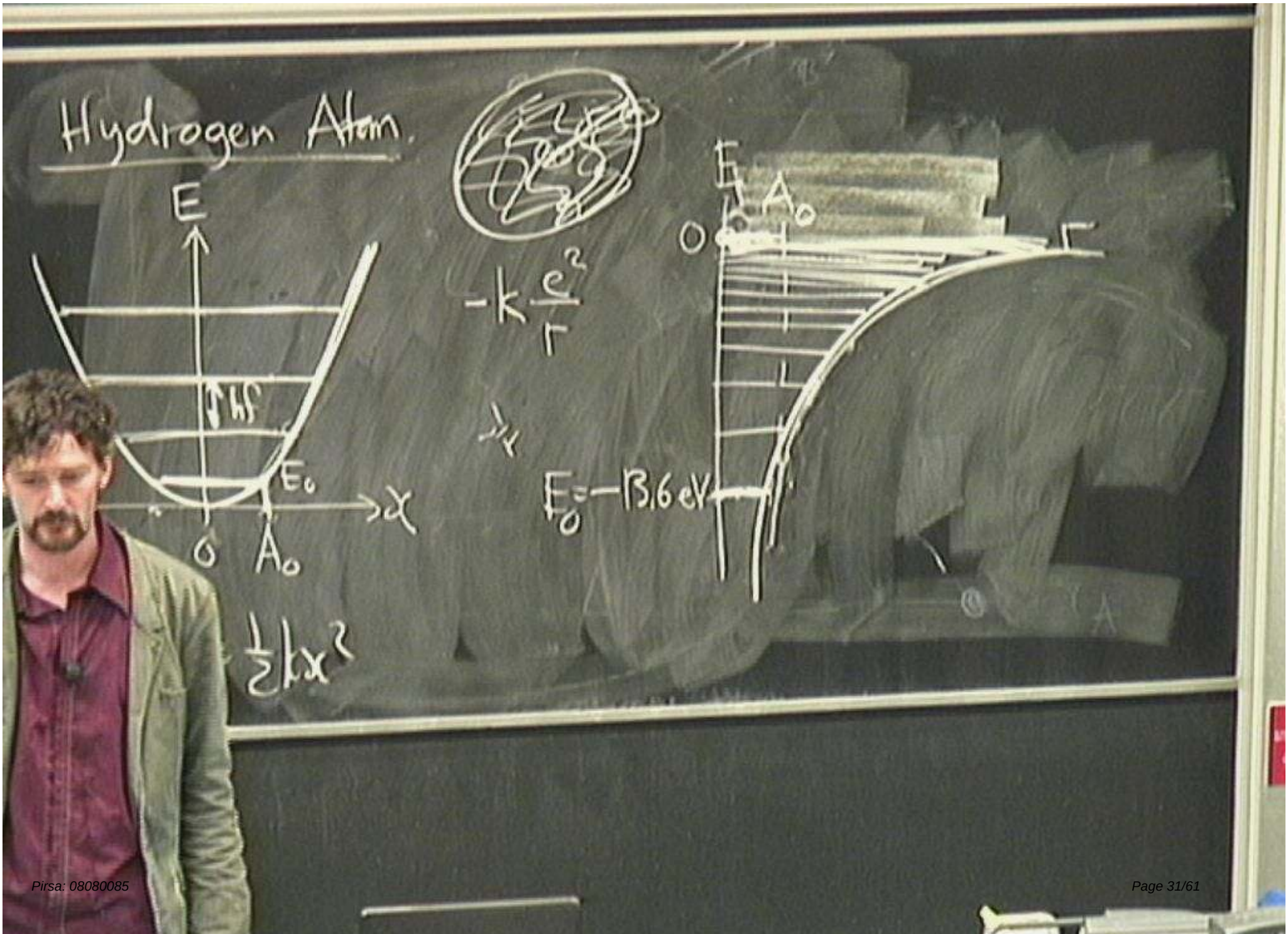


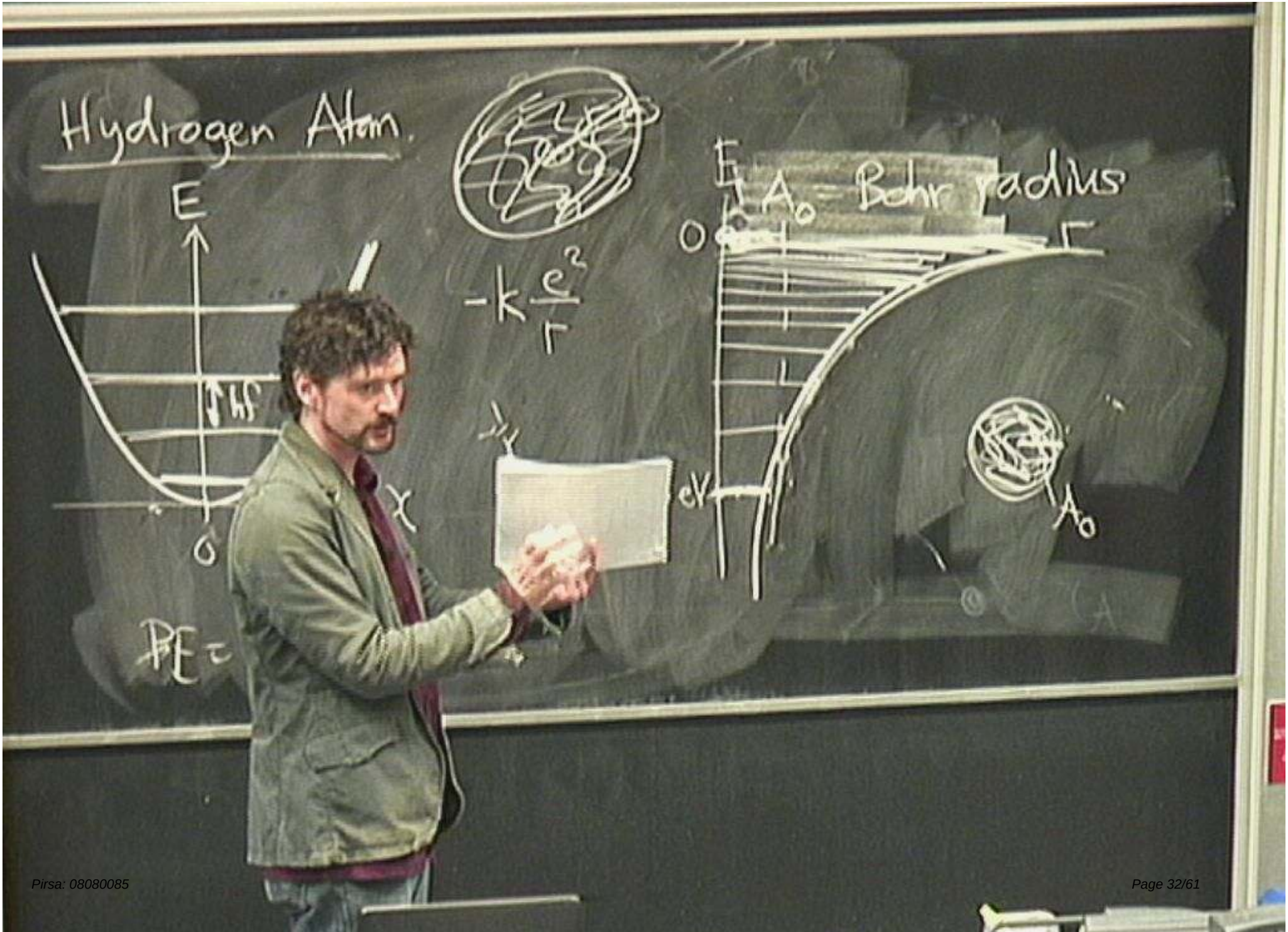
$PE = \frac{1}{2}kx^2$

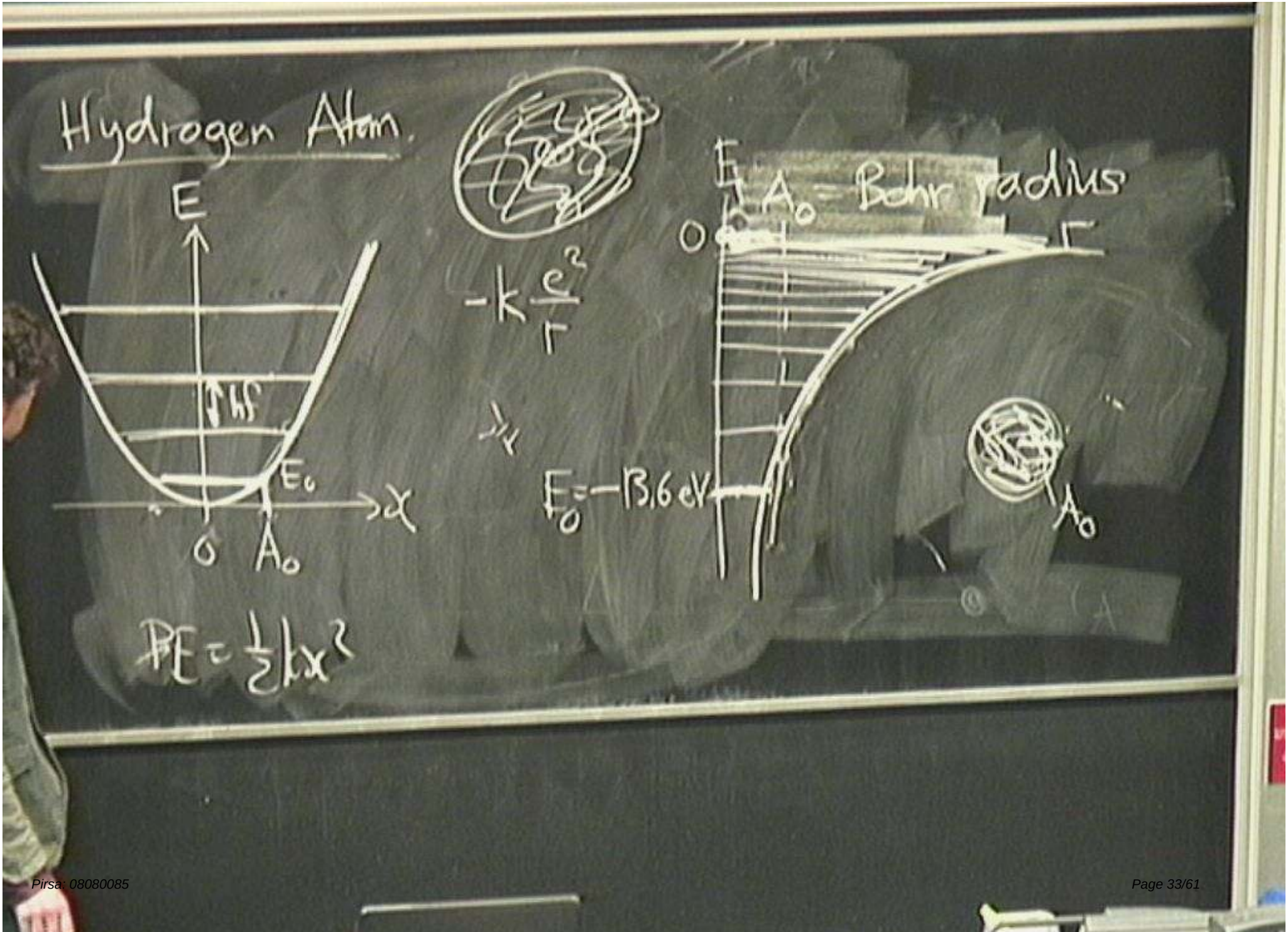
$-k\frac{e^2}{r}$



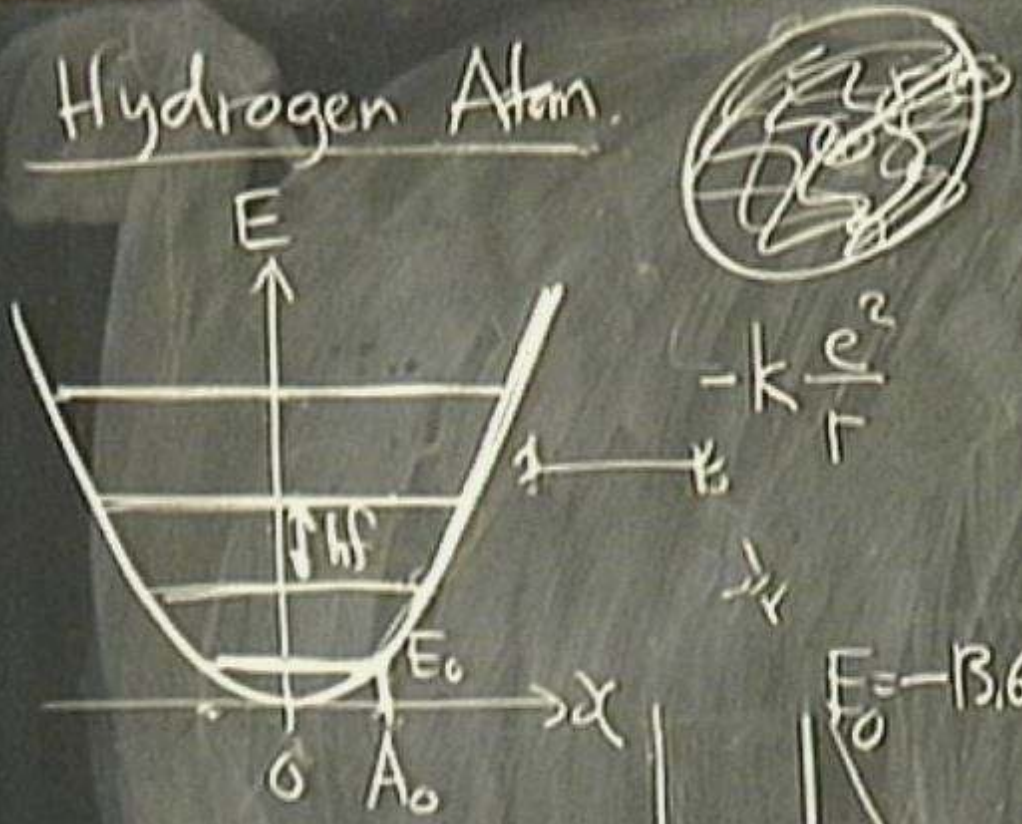

Hydrogen Atom.
E
$-k\frac{e^2}{r}$
E
0
Γ
$\frac{1}{2}\hbar\omega$
$-13.6\,eV$


Hydrogen Atom.
E
$\frac{1}{2}\hbar\omega$
E_0
x
0
A_0
$\frac{1}{2}kx^2$
$-k\frac{e^2}{r}$
$E_0 = -13.6\ eV$
E
A_0
0
r


Hydrogen Atom
E
0
$-k\frac{e^2}{r}$
E
A_0 = Bohr radius
0
r
eV
A_0

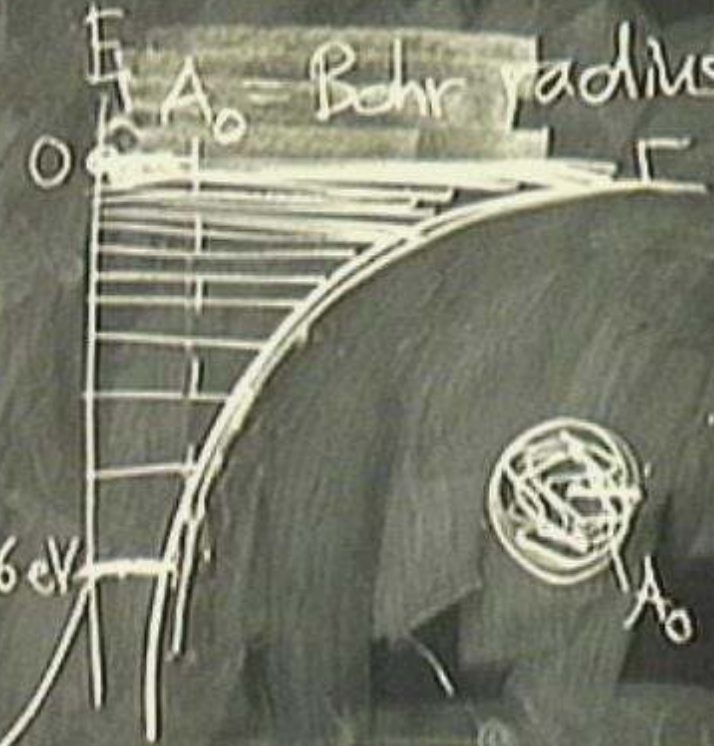

Hydrogen Atom.
E
hf
E_0
x
0
A_0
$PE = \frac{1}{2}kx^2$
$-k\frac{e^2}{r}$
$E_0 = -13.6\ eV$
E
A_0 = Bohr radius
0
r
A_0

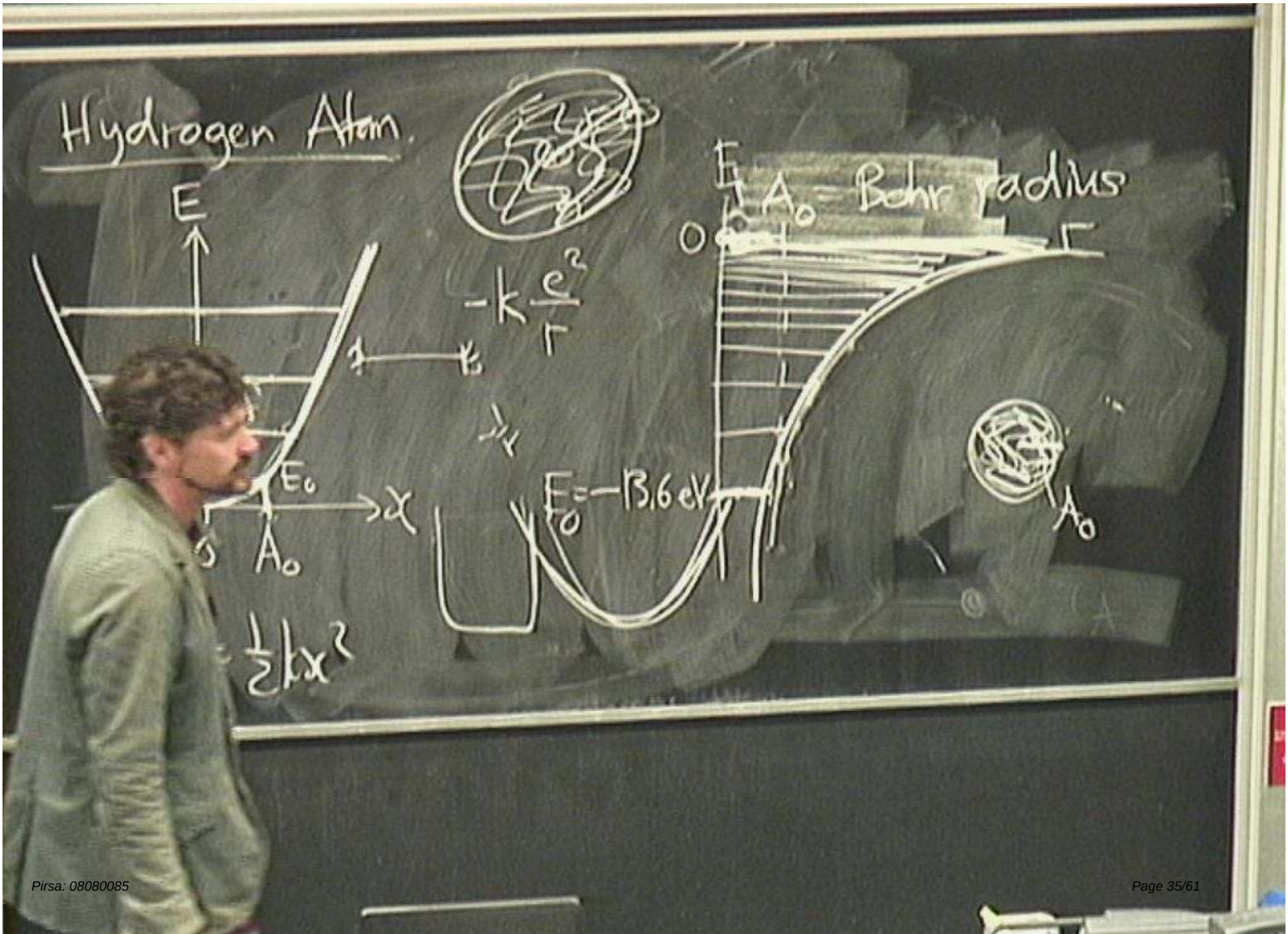
Hydrogen Atom

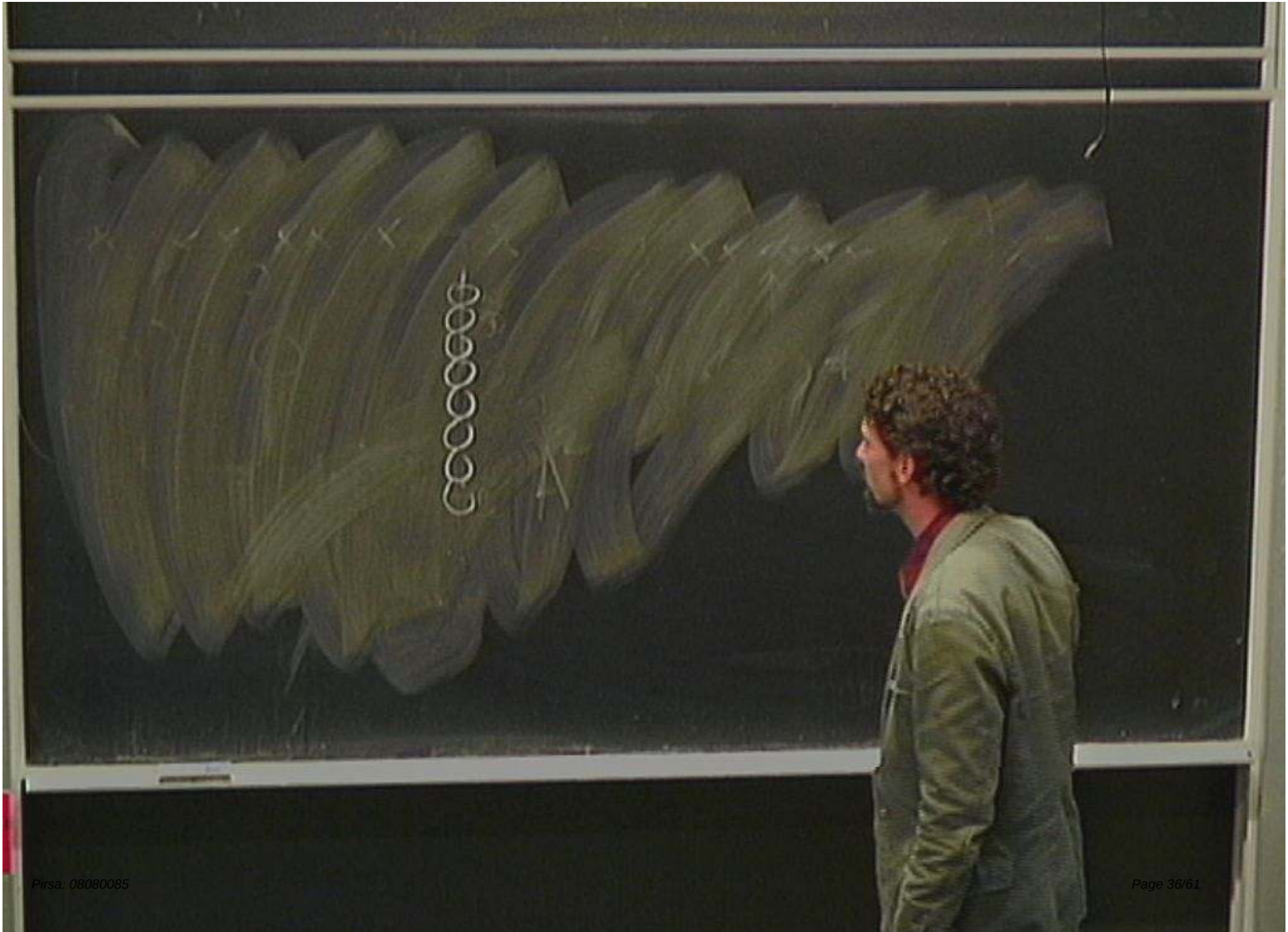


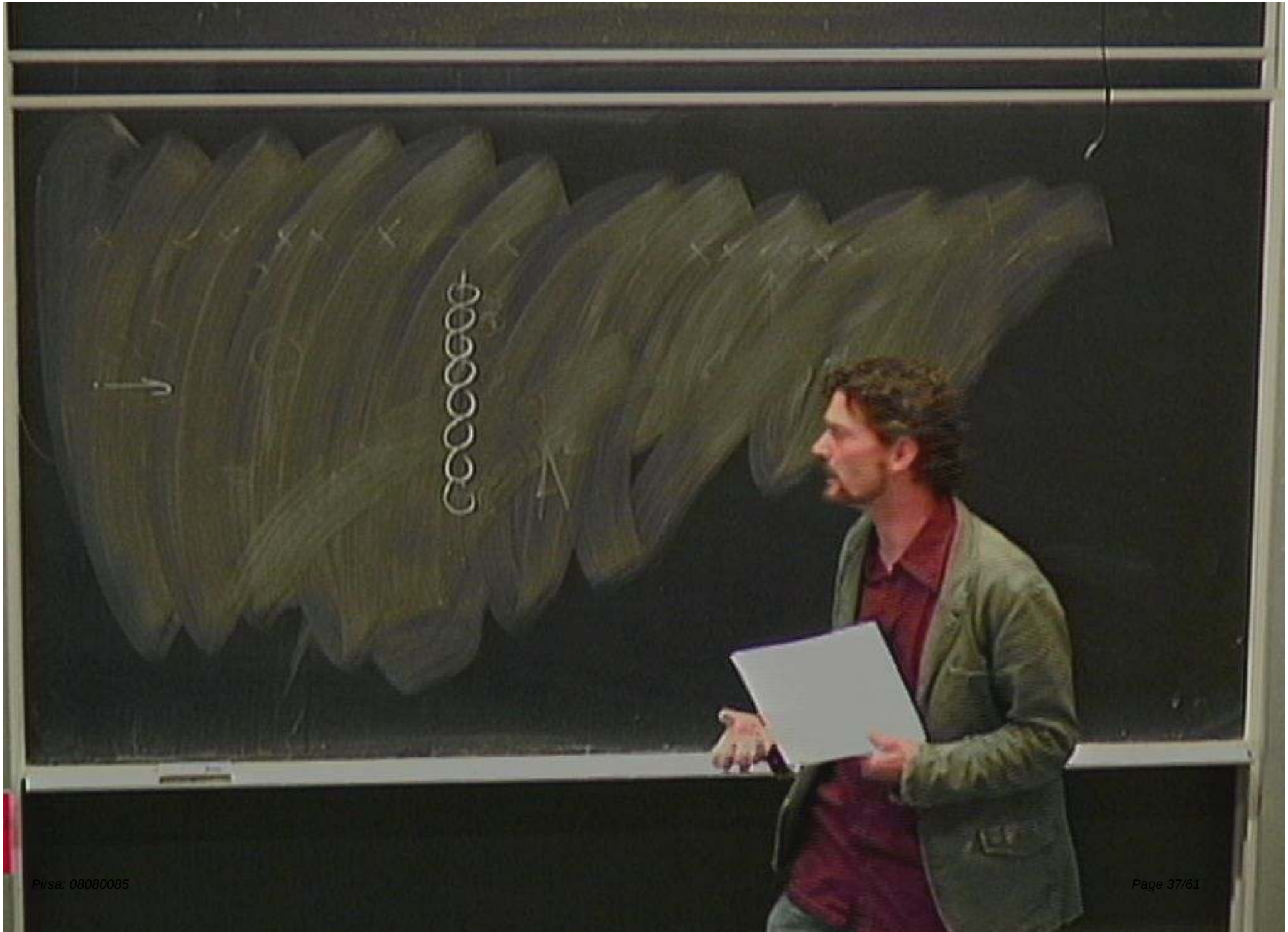
$PE = \frac{1}{2}kx^2$

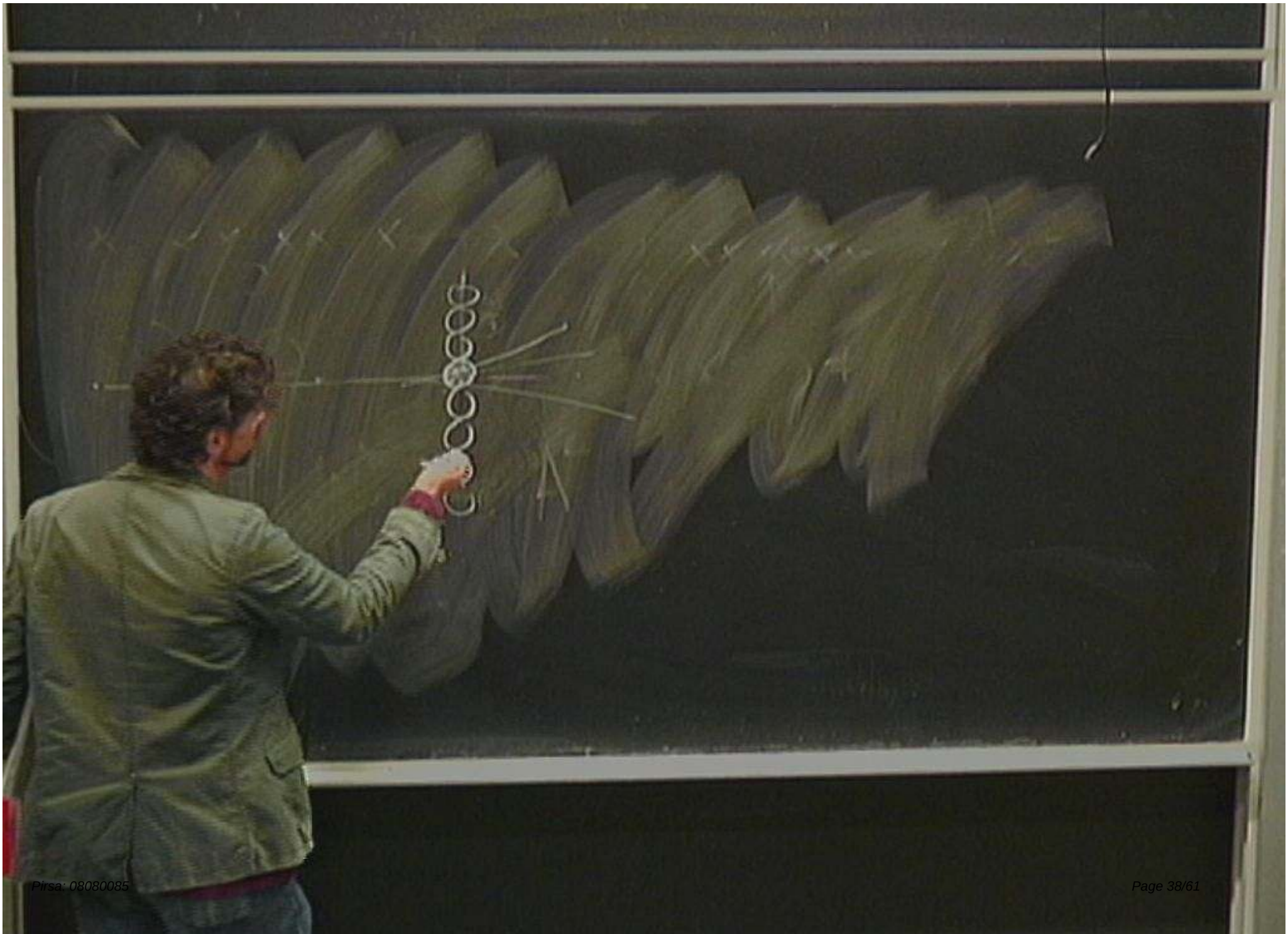
$-k\frac{e^2}{r}$

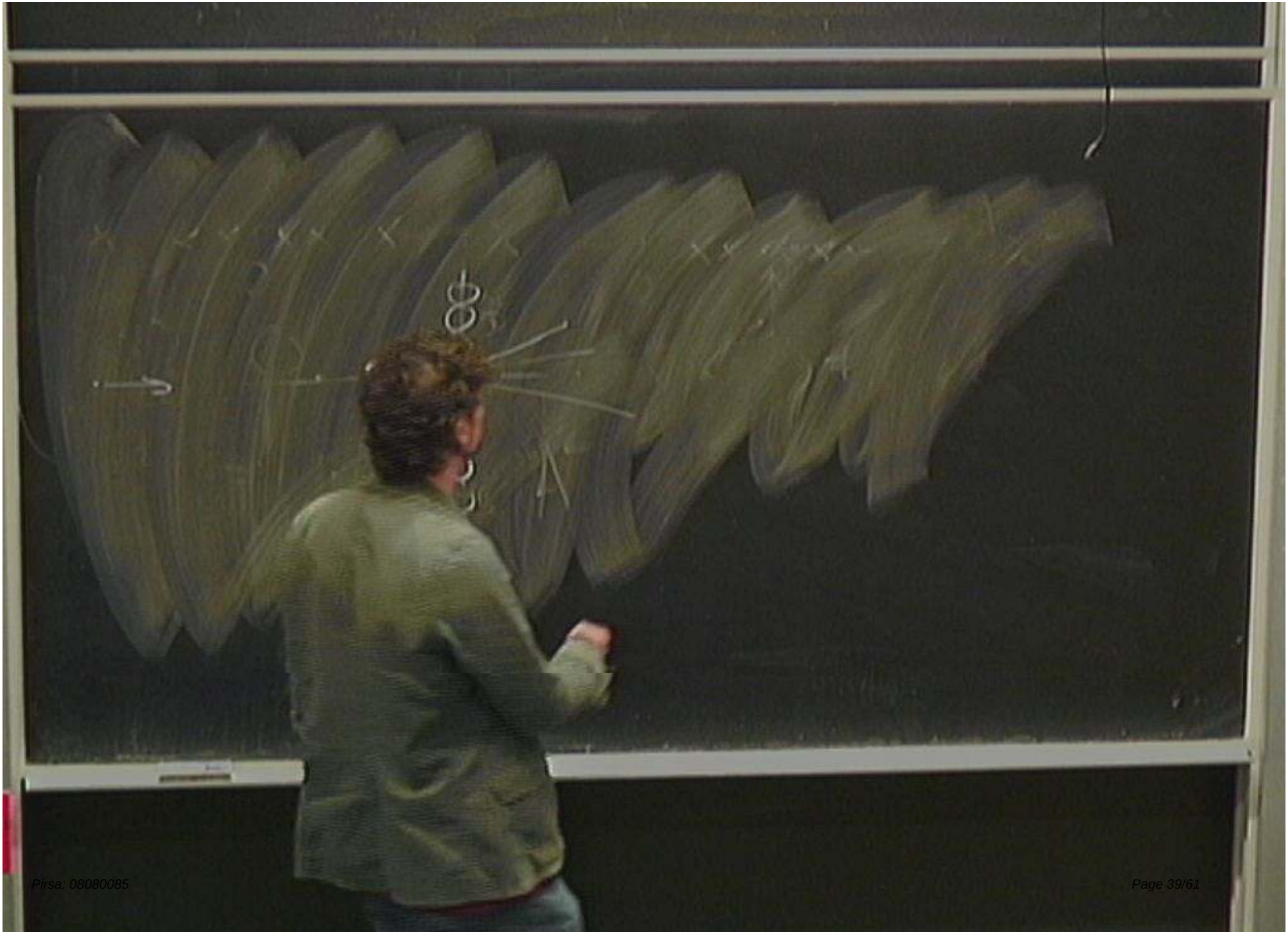


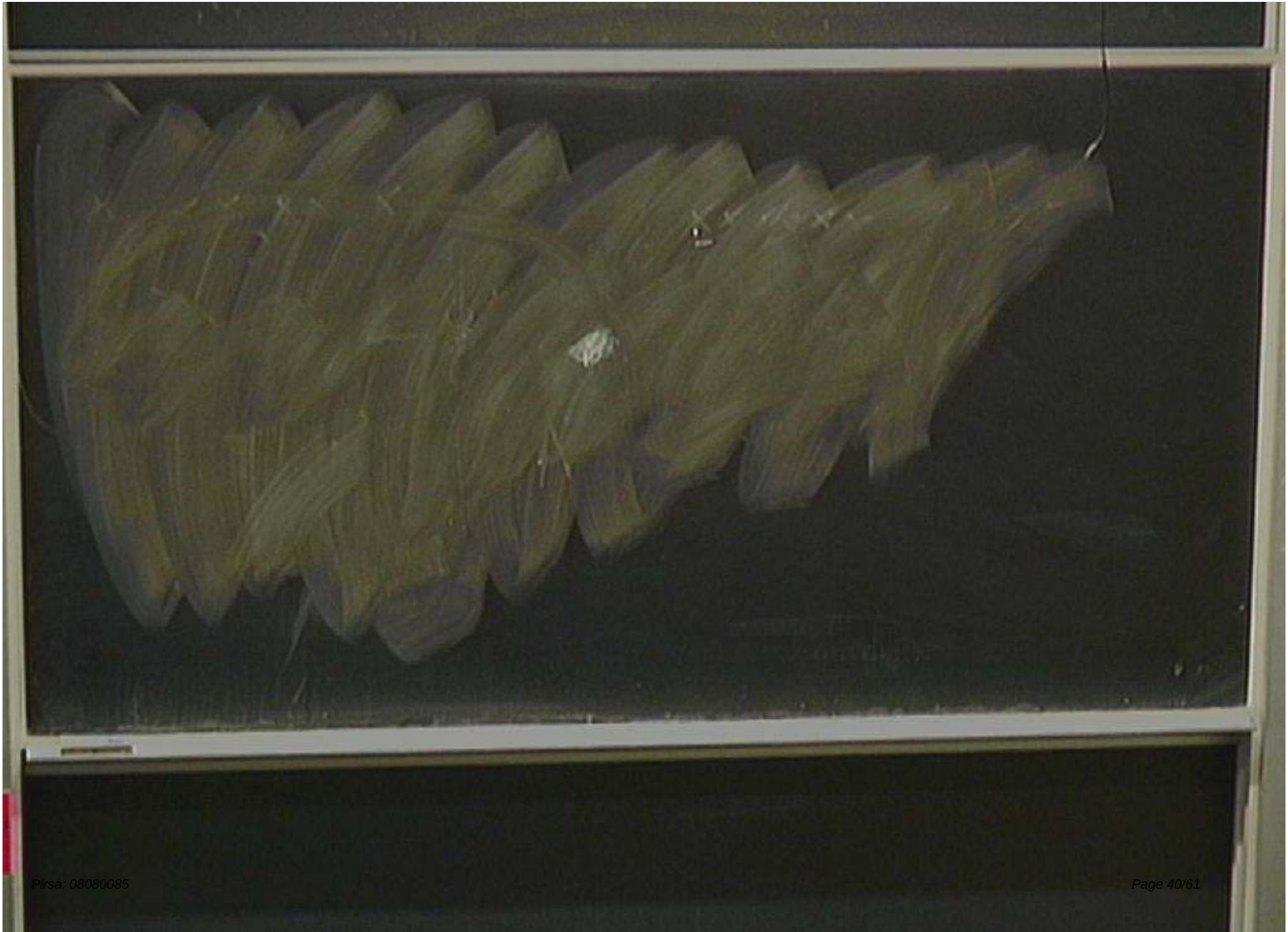

Hydrogen Atom.
E
$-k\frac{e^2}{r}$
E_0
A_0
$\frac{1}{2}kx^2$
E
A_0 = Bohr radius
0
r
$E_0 = -13.6$ eV
A_0

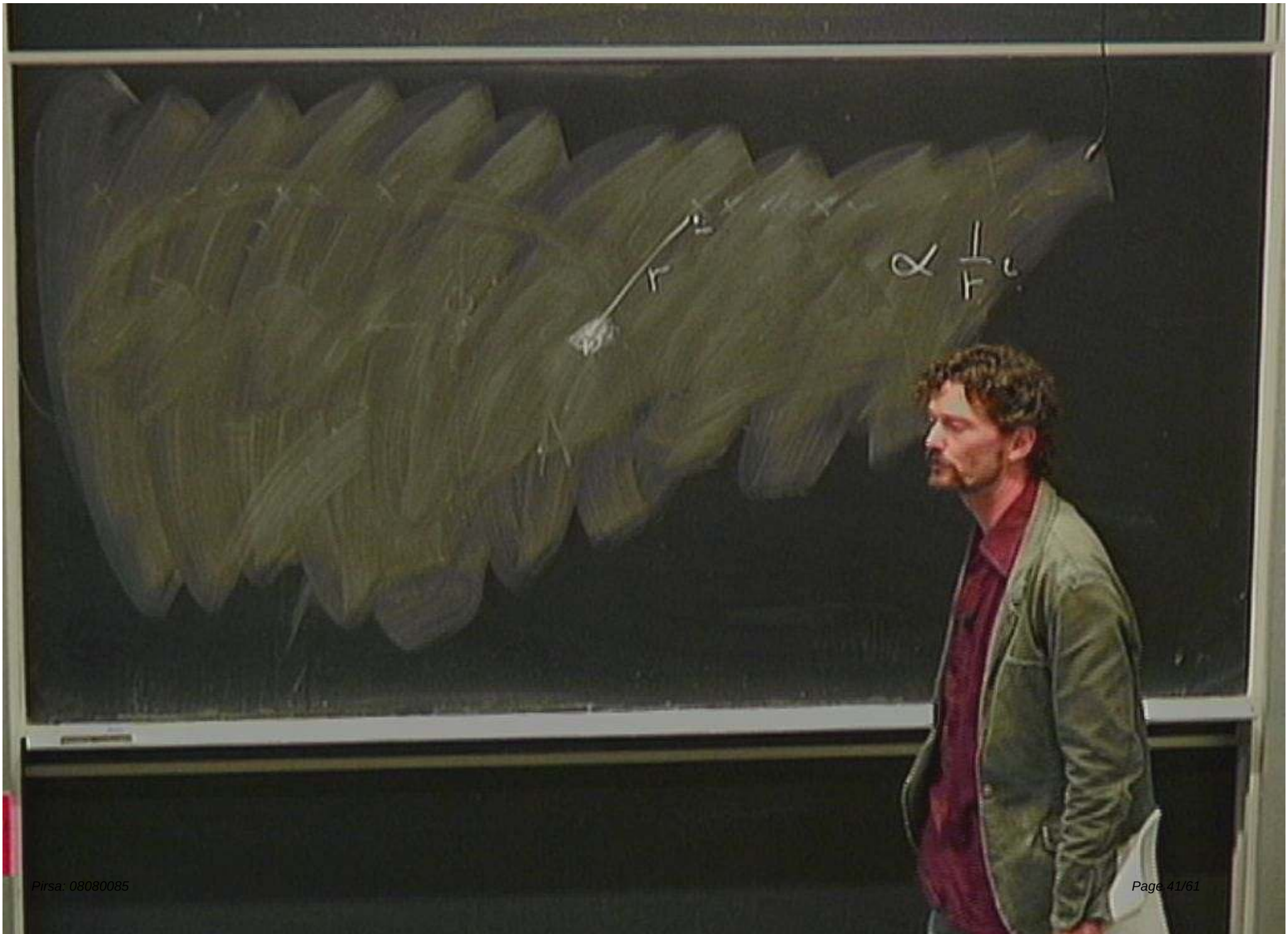


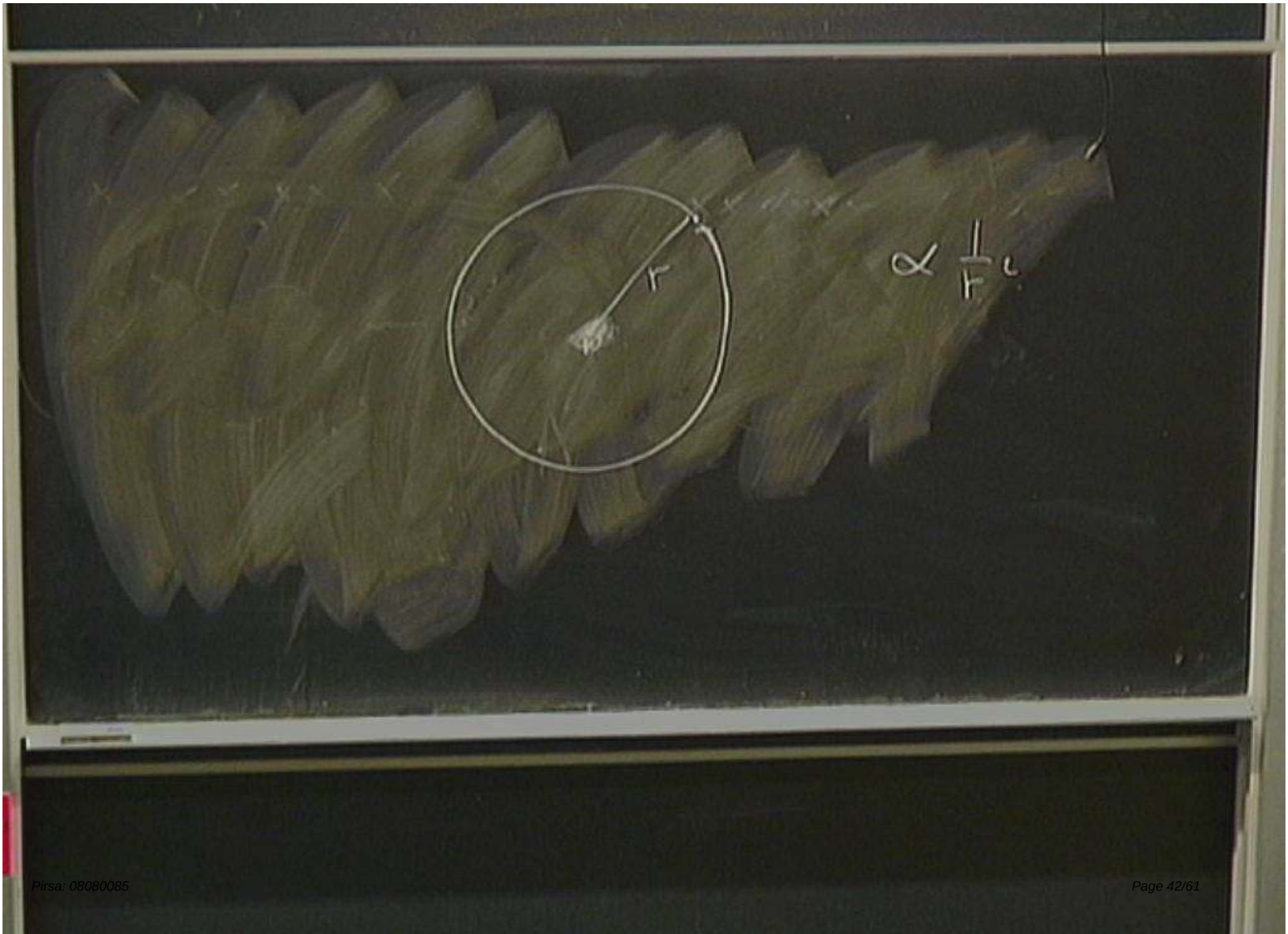


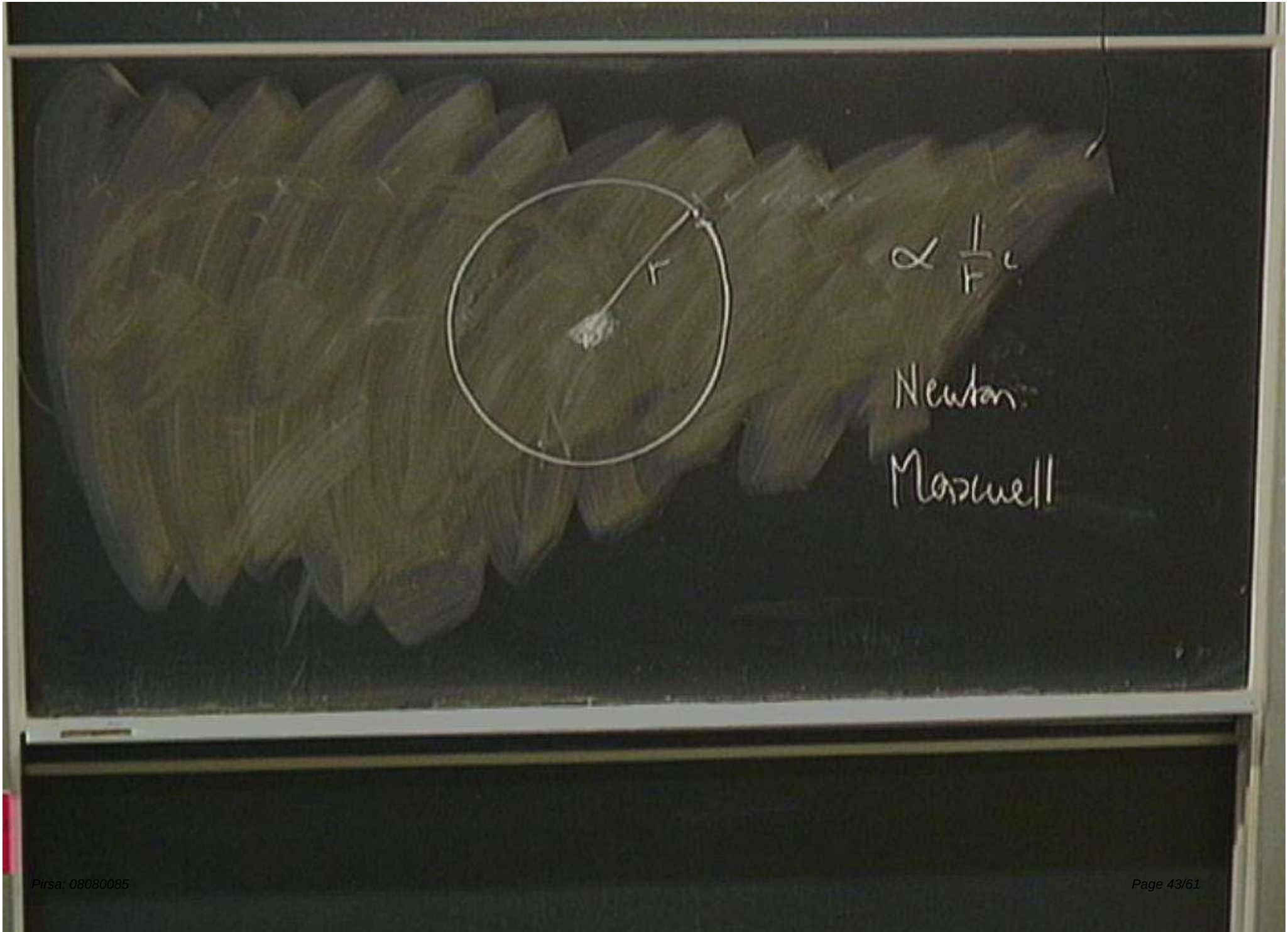


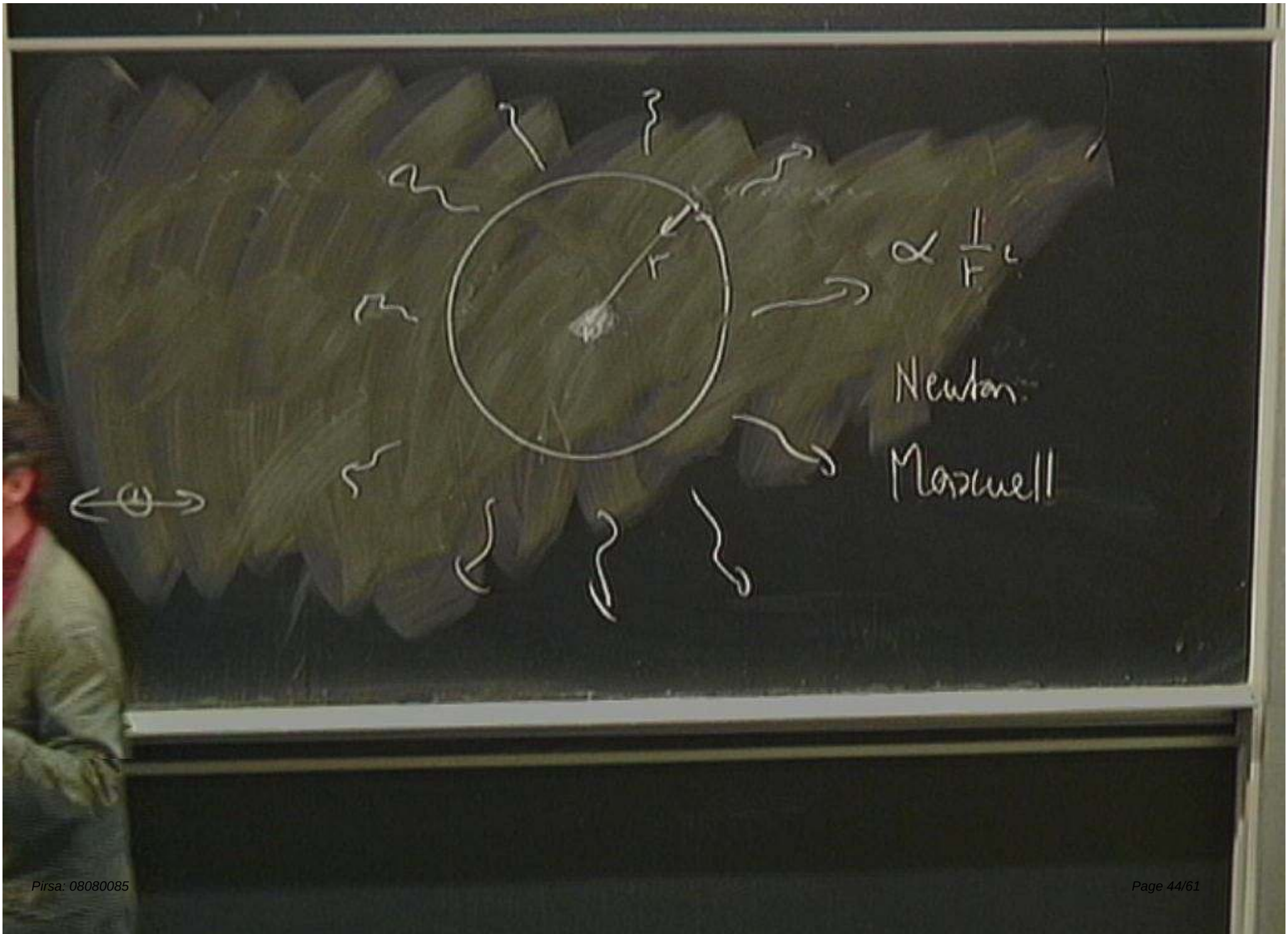


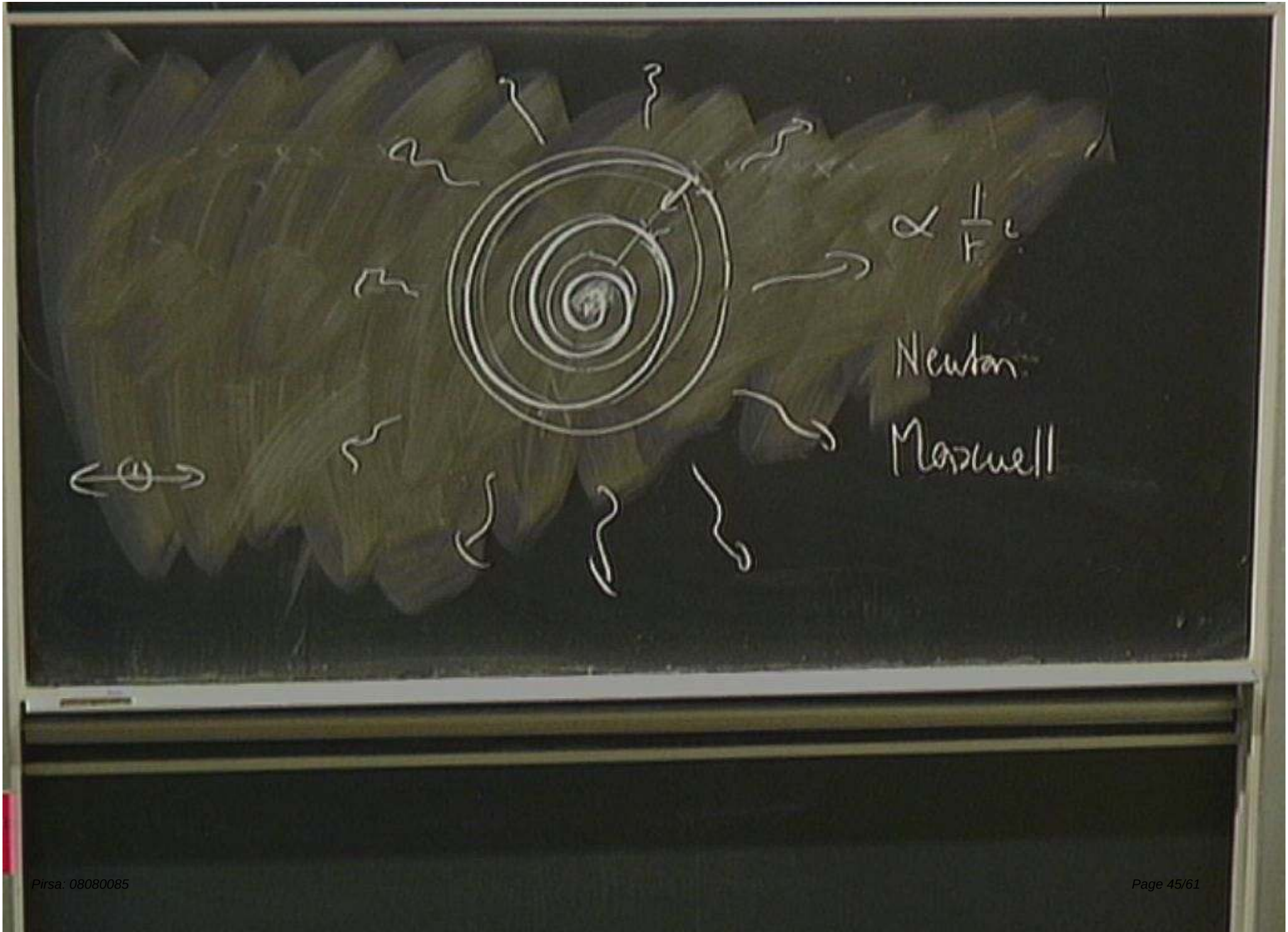



∝ 1/r²


r
$\propto \frac{1}{r^2}$

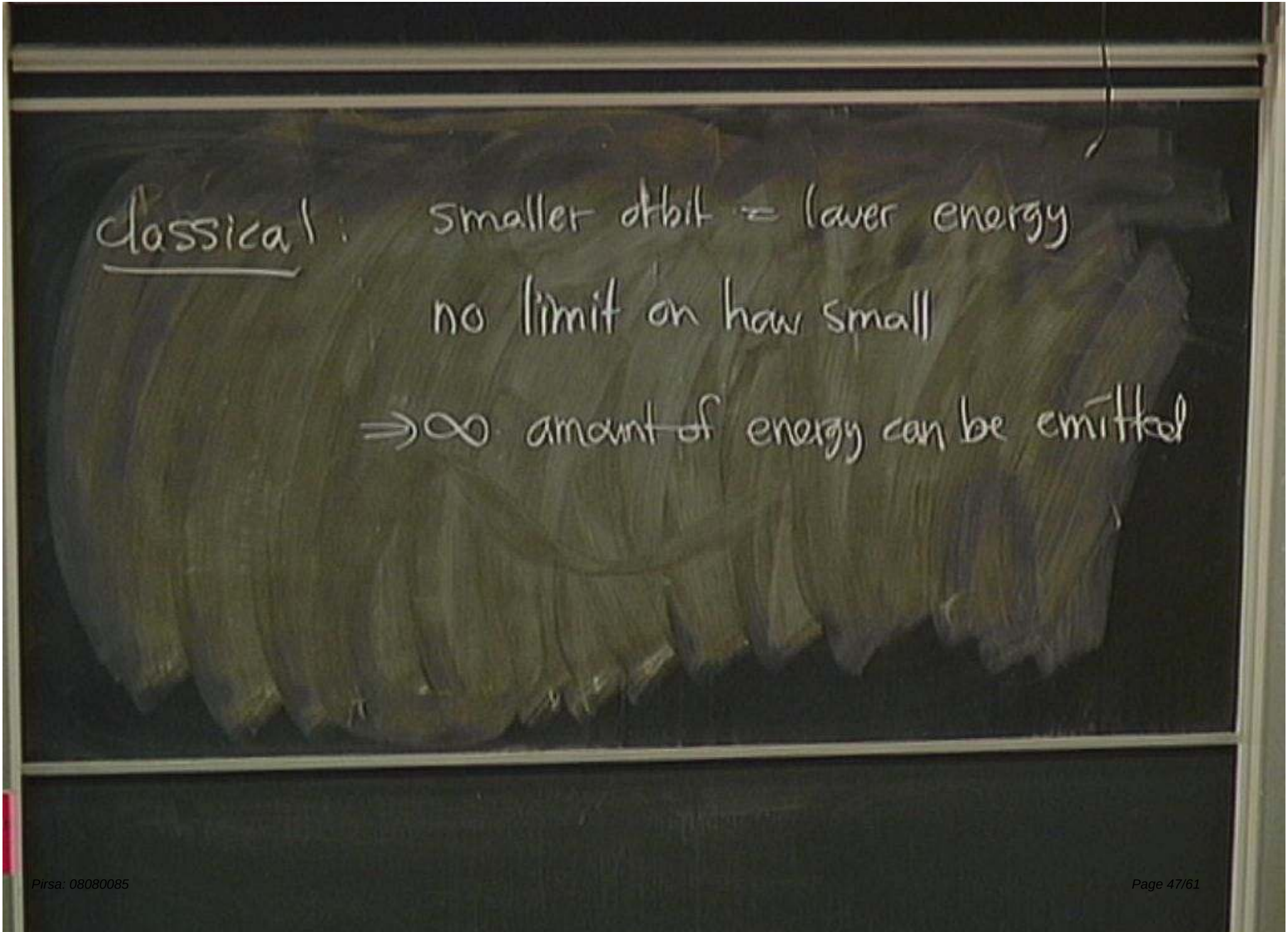

r
$\propto \frac{1}{r^2}$
Newton:
Maxwell

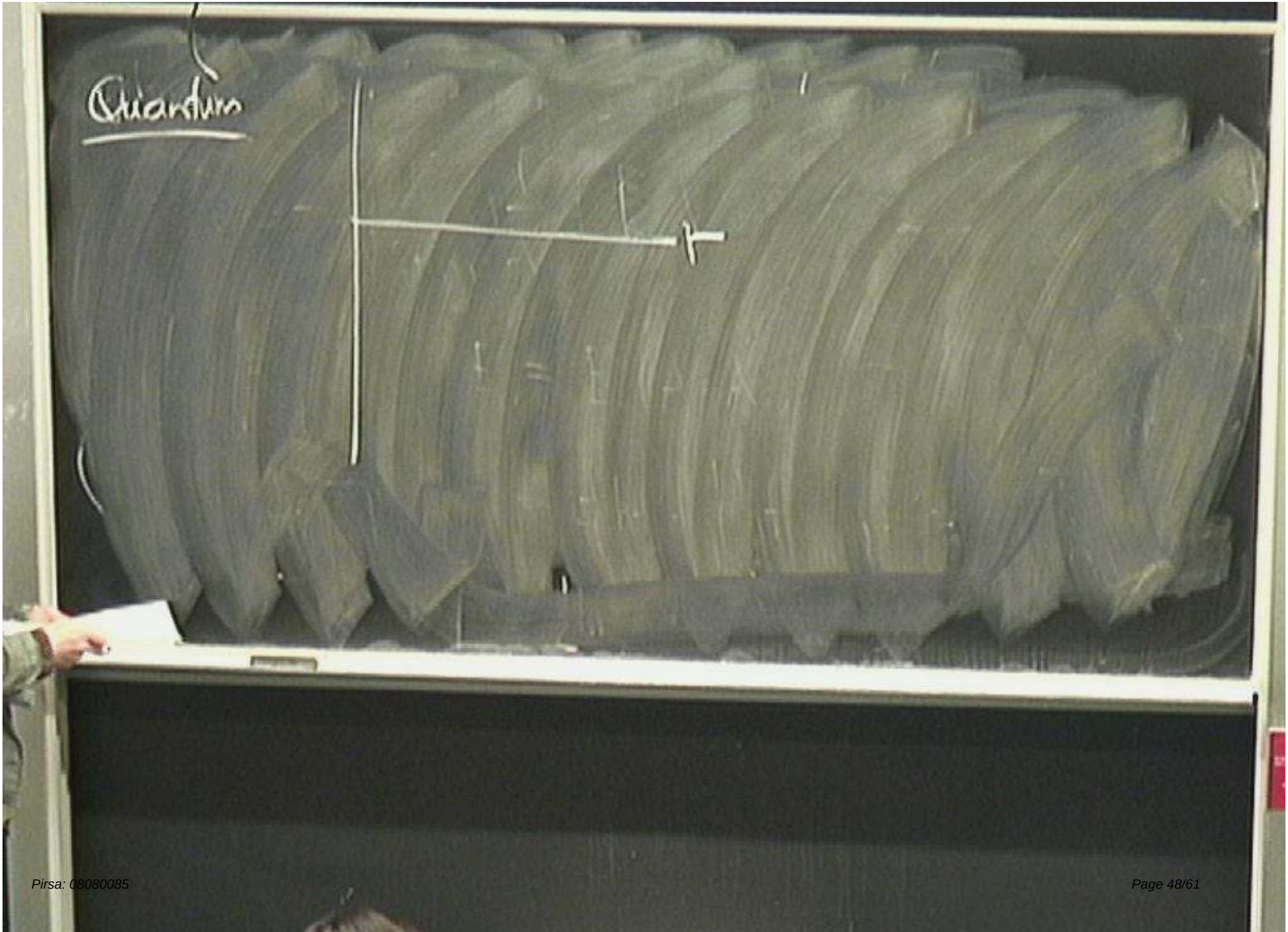

$\propto \frac{1}{r^2}$
Newton:
Maxwell

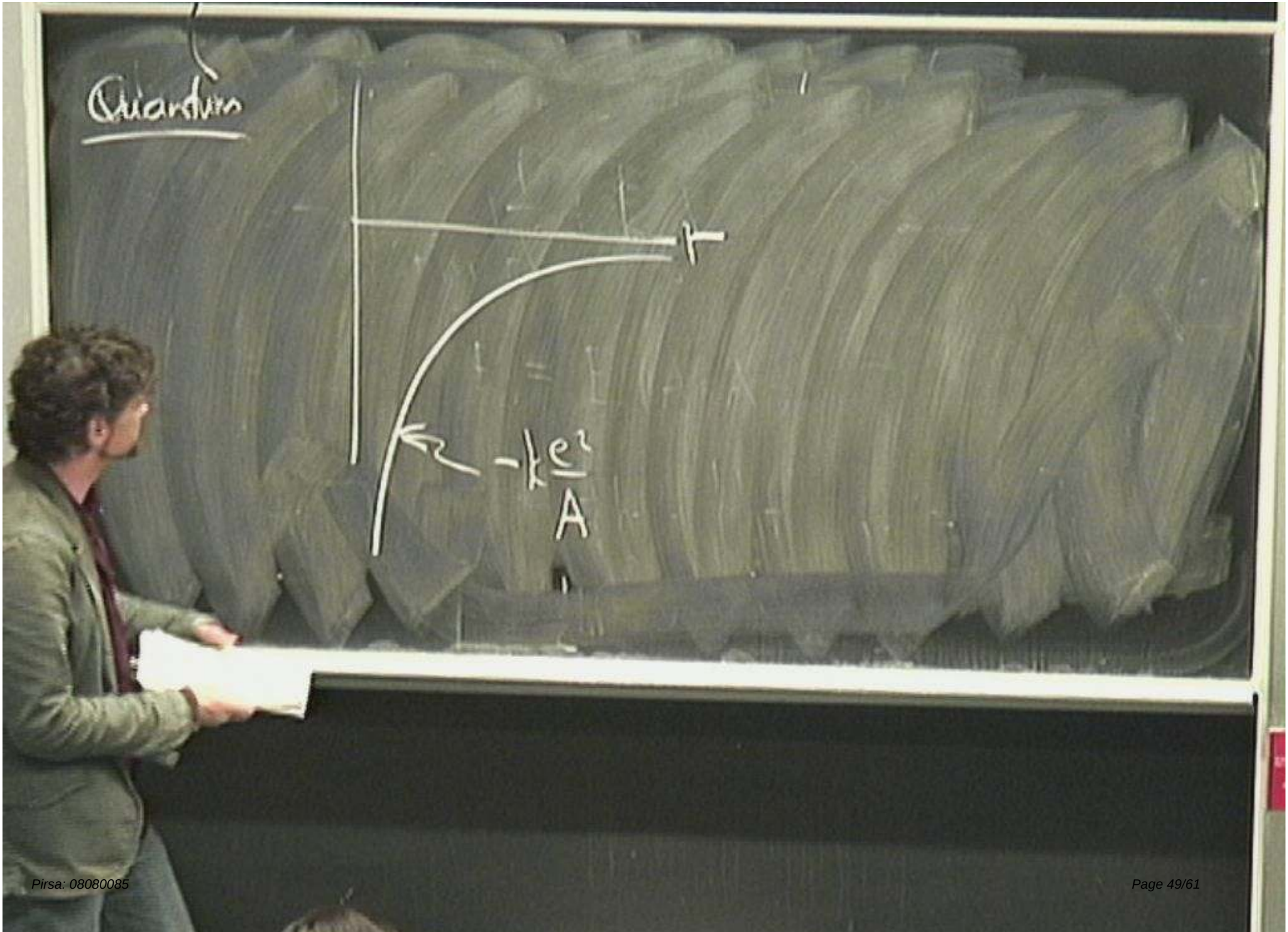

$\propto \frac{1}{r^2}$
Newton
Maxwell

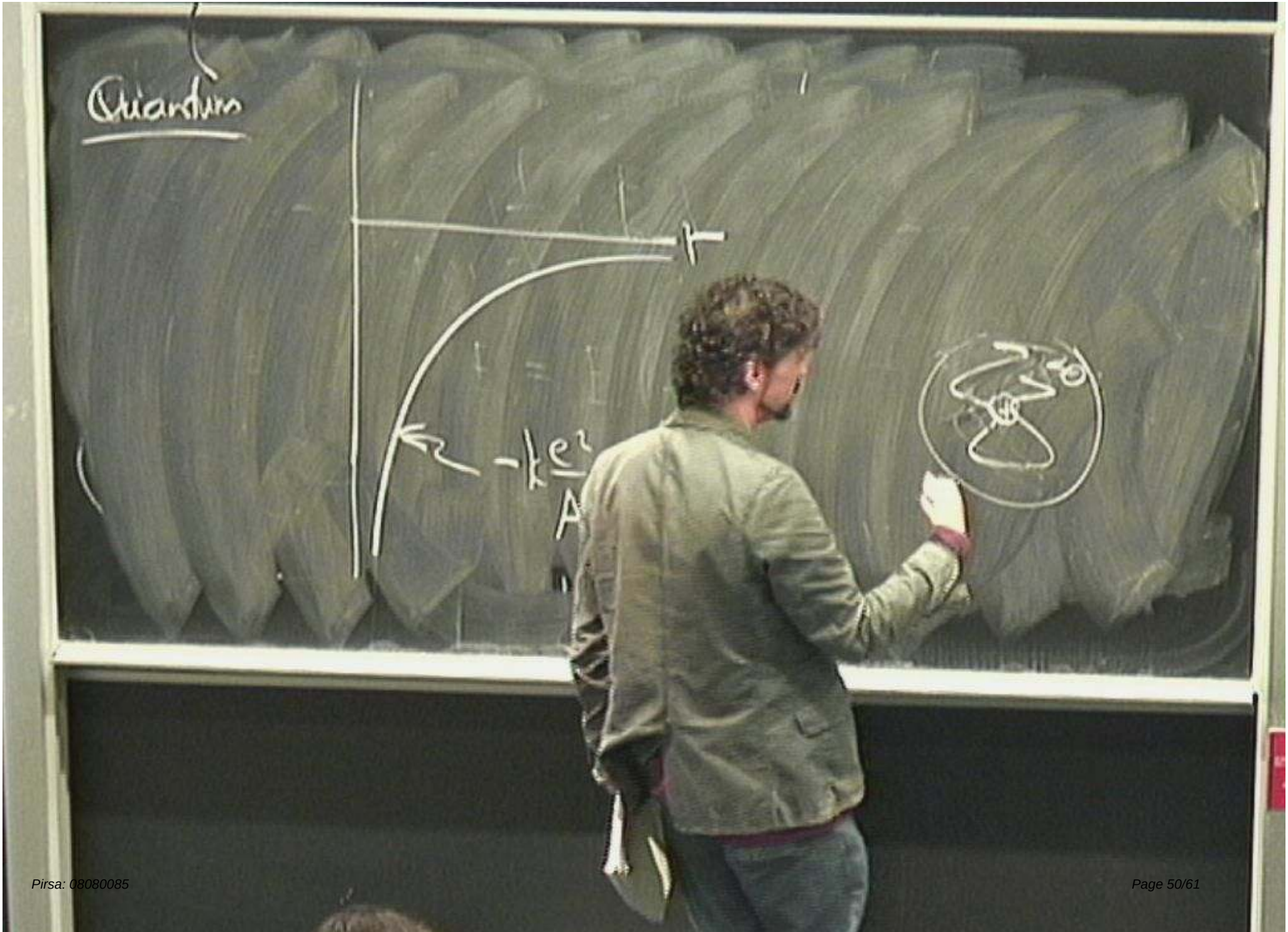
classical: smaller orbit = lower energy

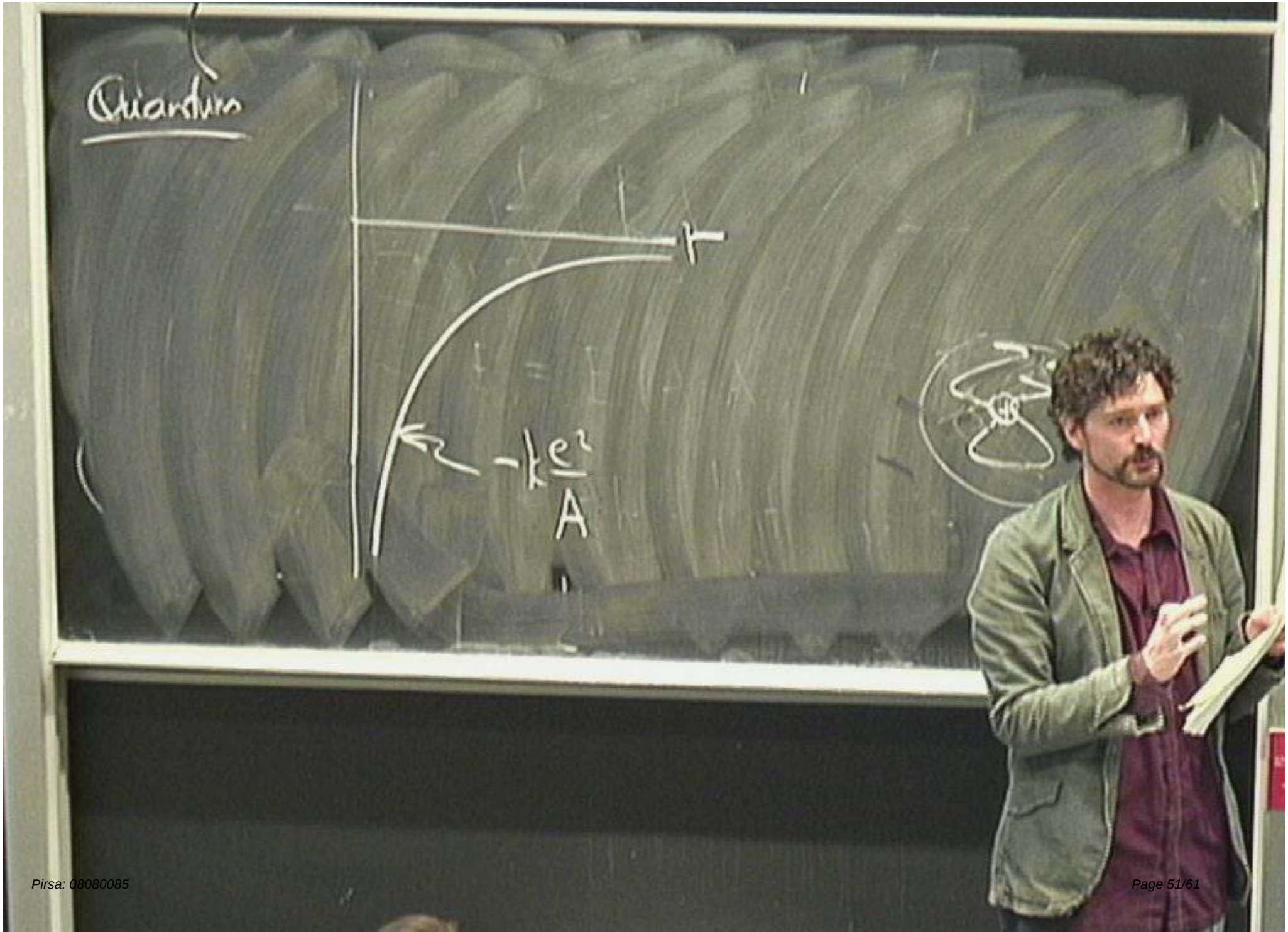
no limit on how small

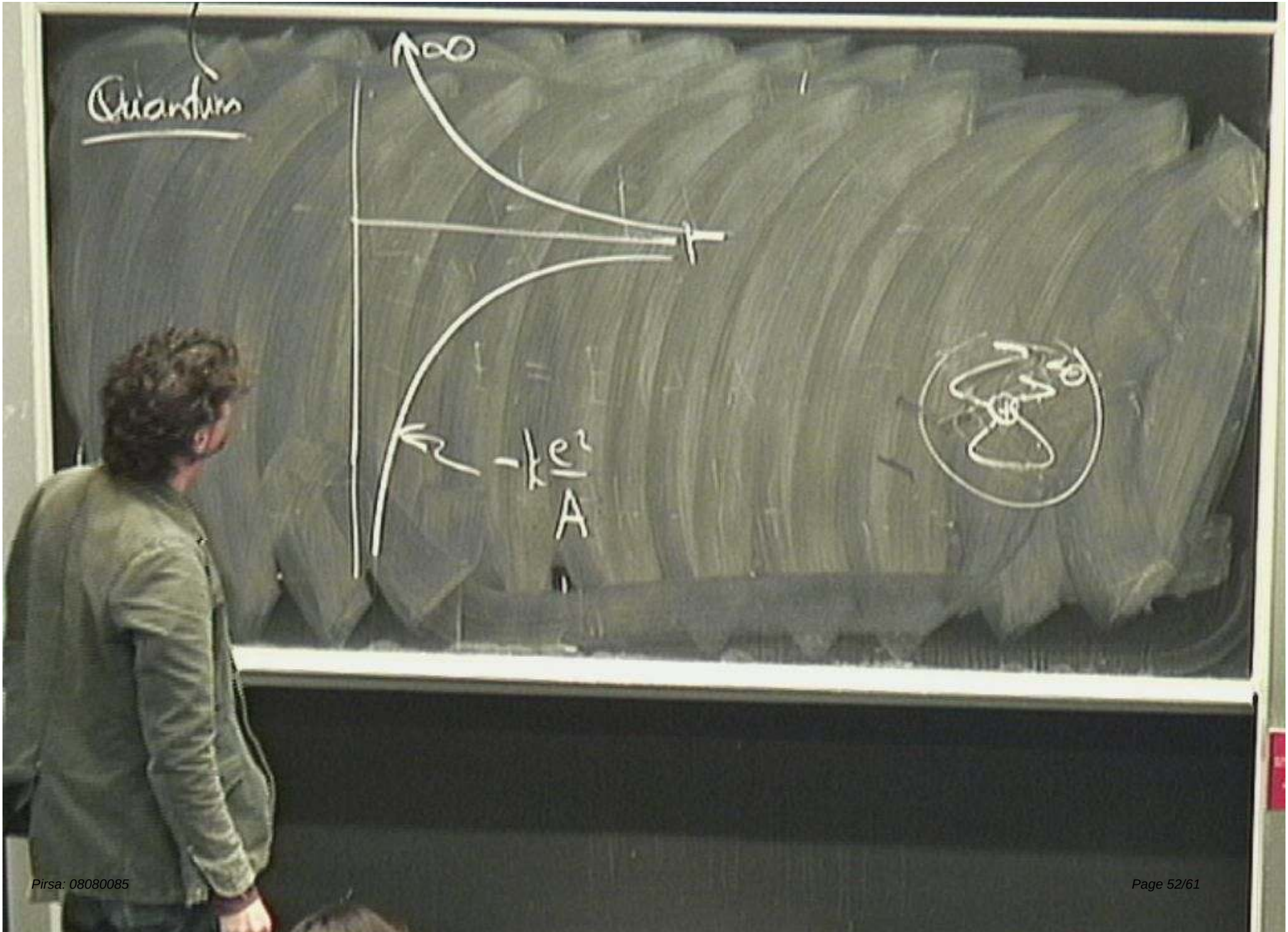

classical: smaller orbit = lower energy
no limit on how small
⇒ ∞ amount of energy can be emitted

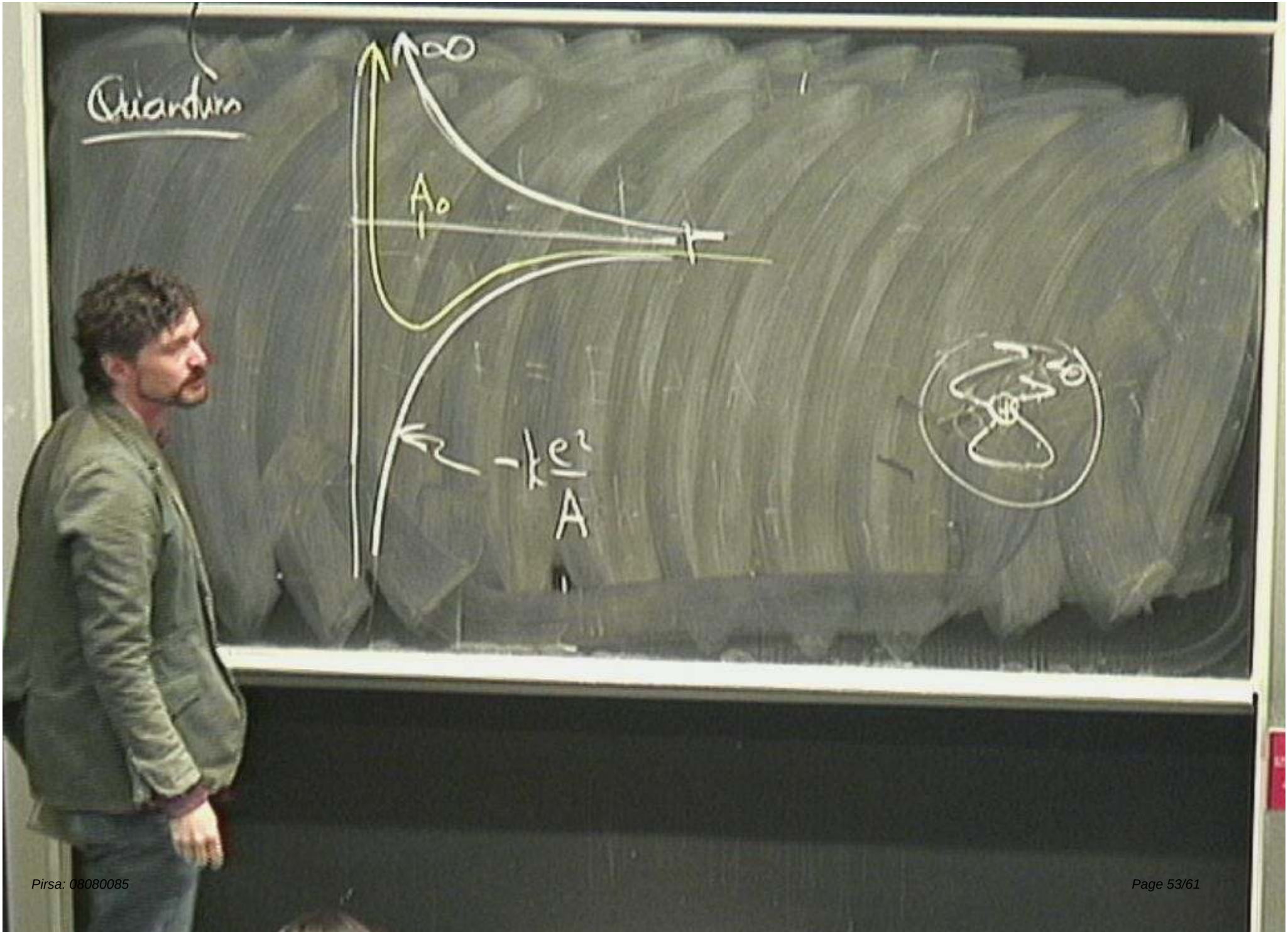

Quantum

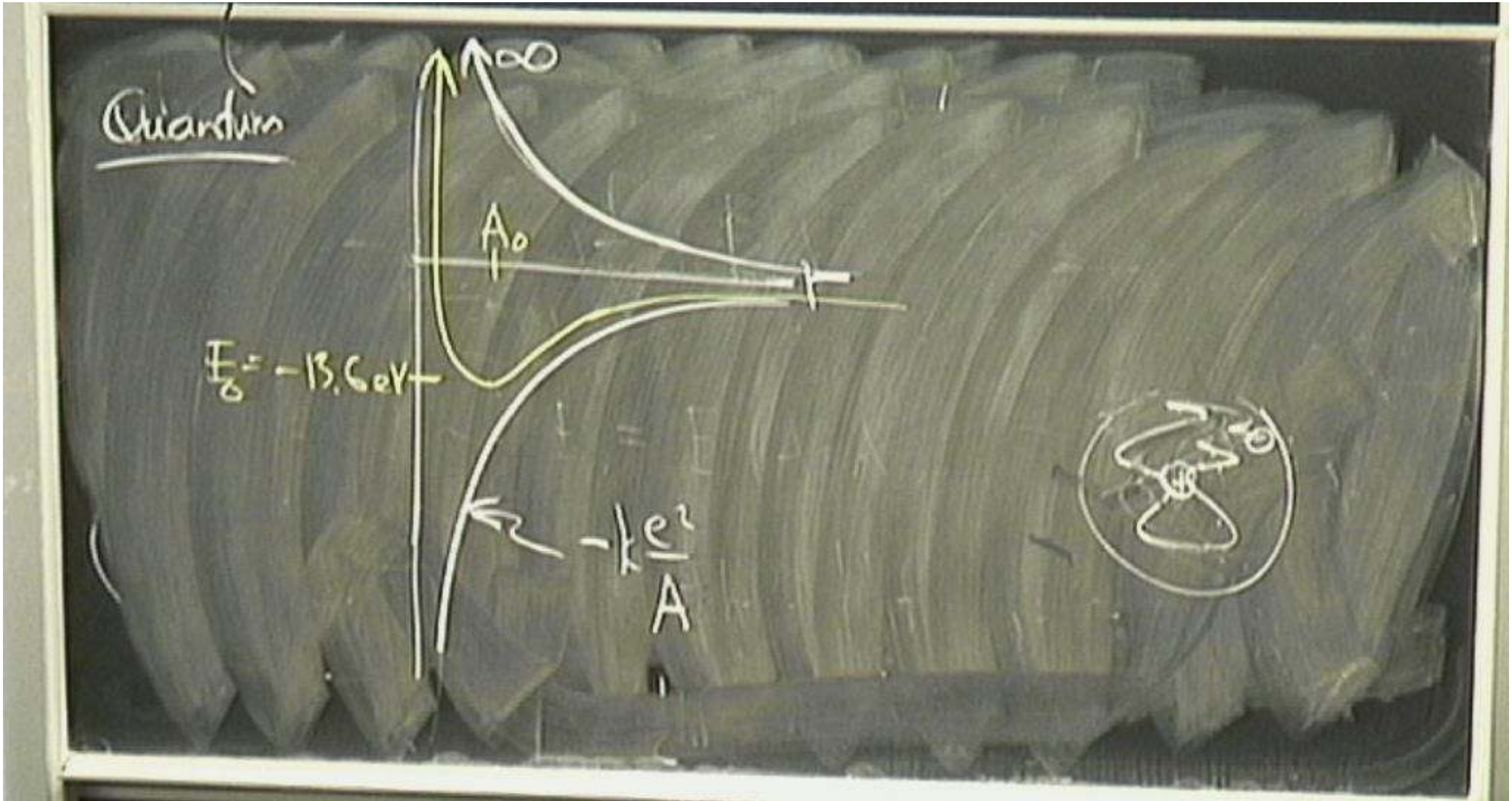

Quantum
$-\frac{ke^2}{A}$

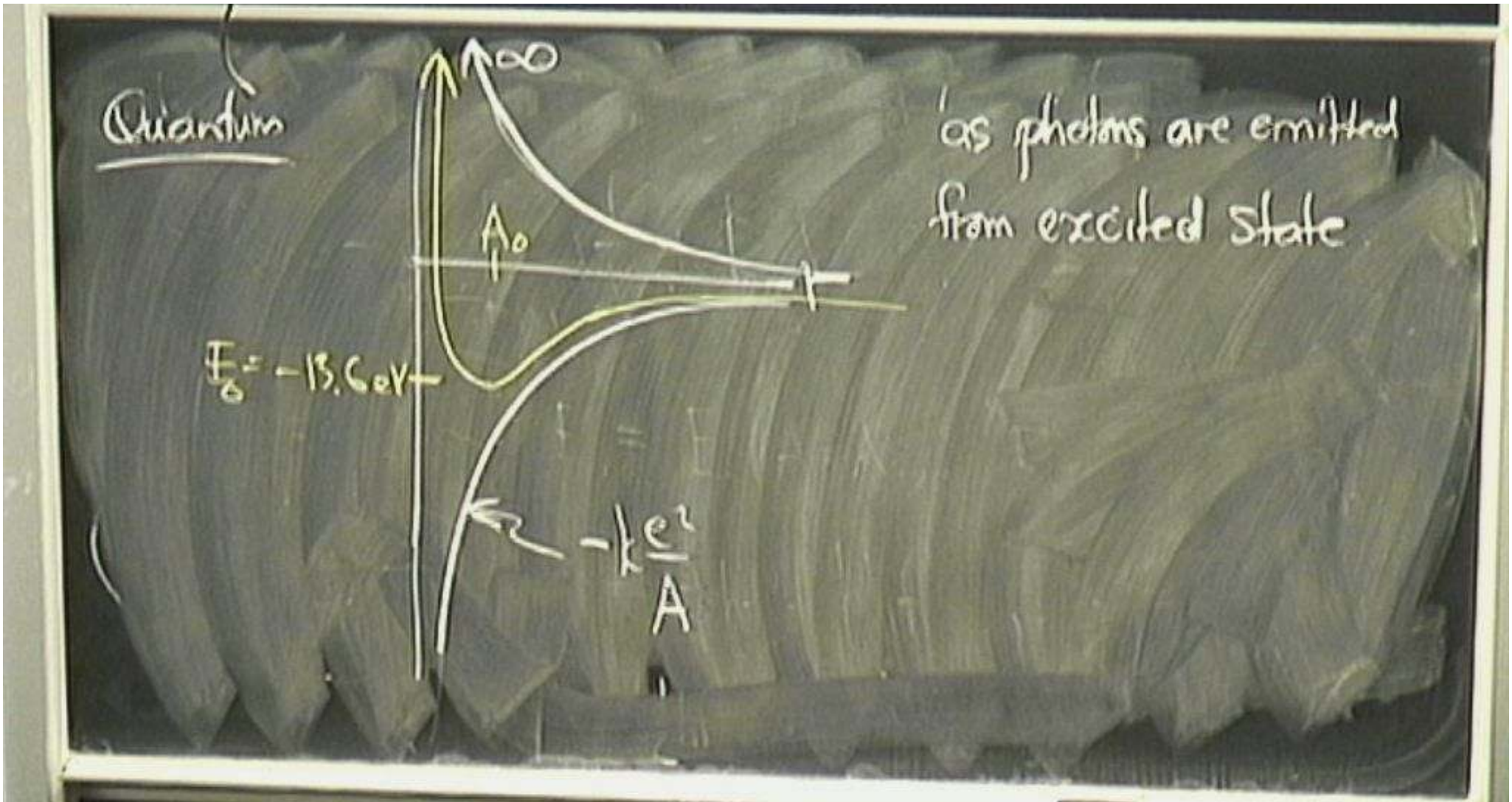

Quantum

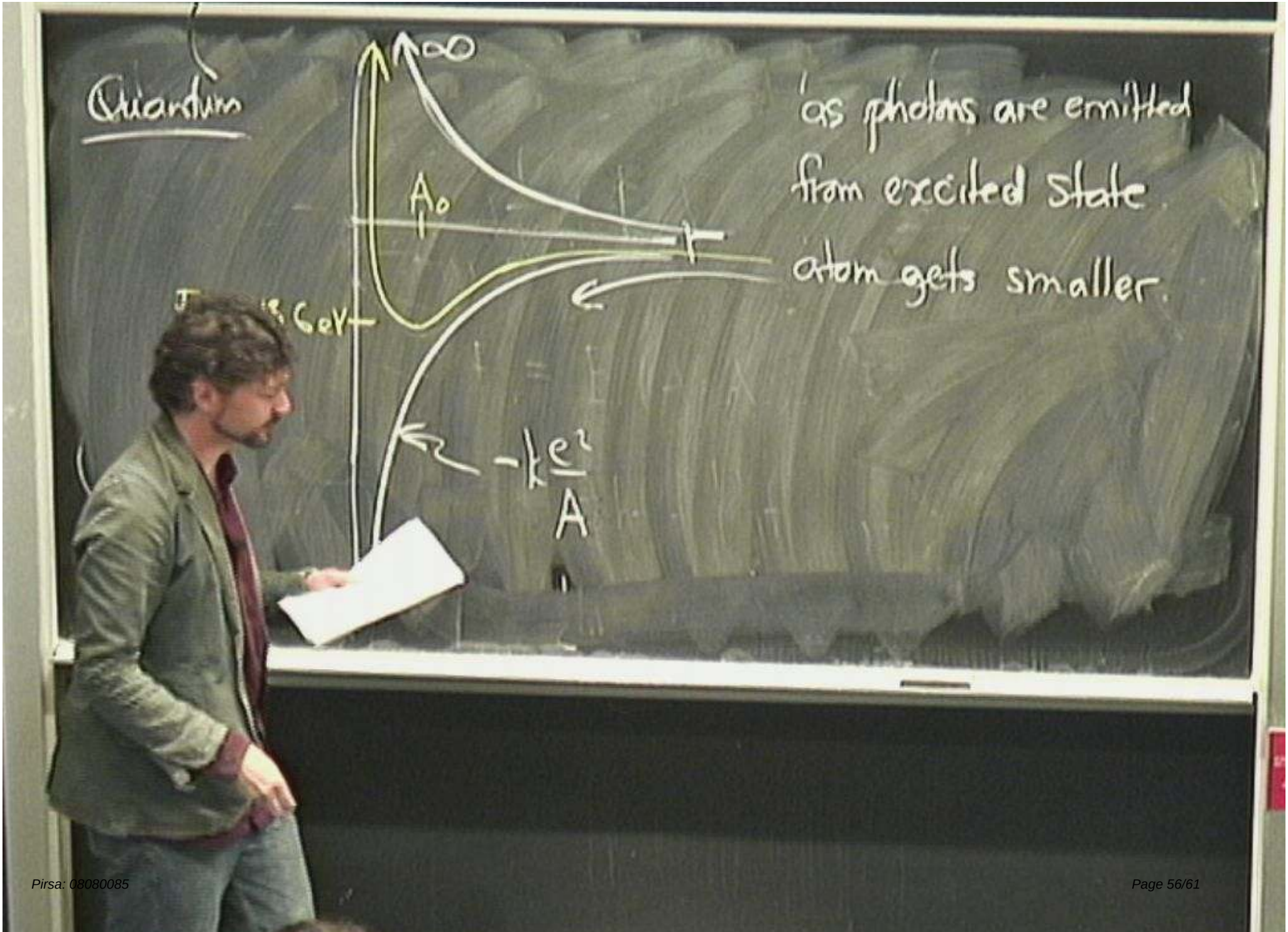

Quantum
$-\frac{ke^2}{A}$

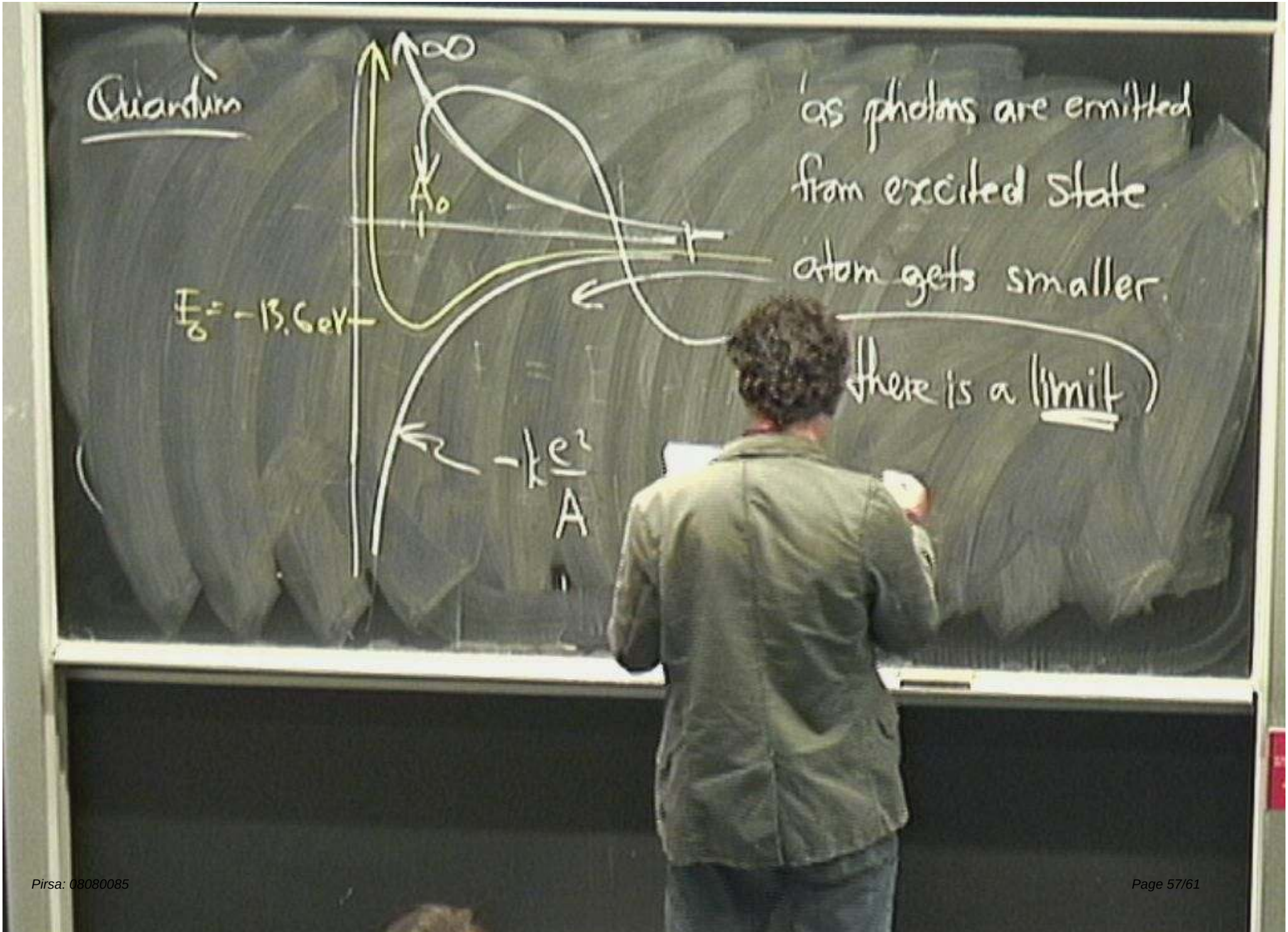

Quantum
∞
$-\frac{1}{4}e^2$
A

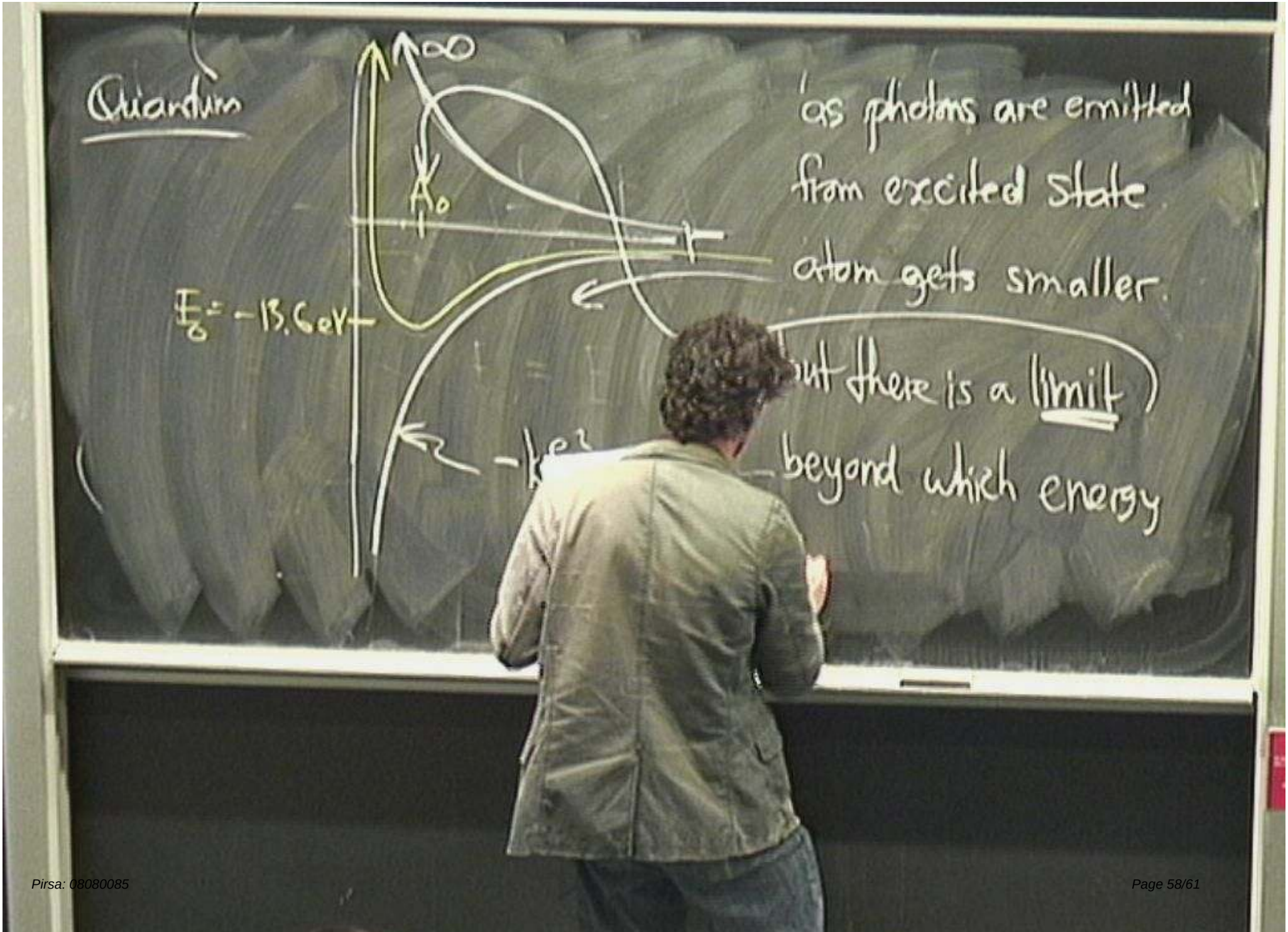

Quantum
∞
A_0
$-\frac{ke^2}{A}$

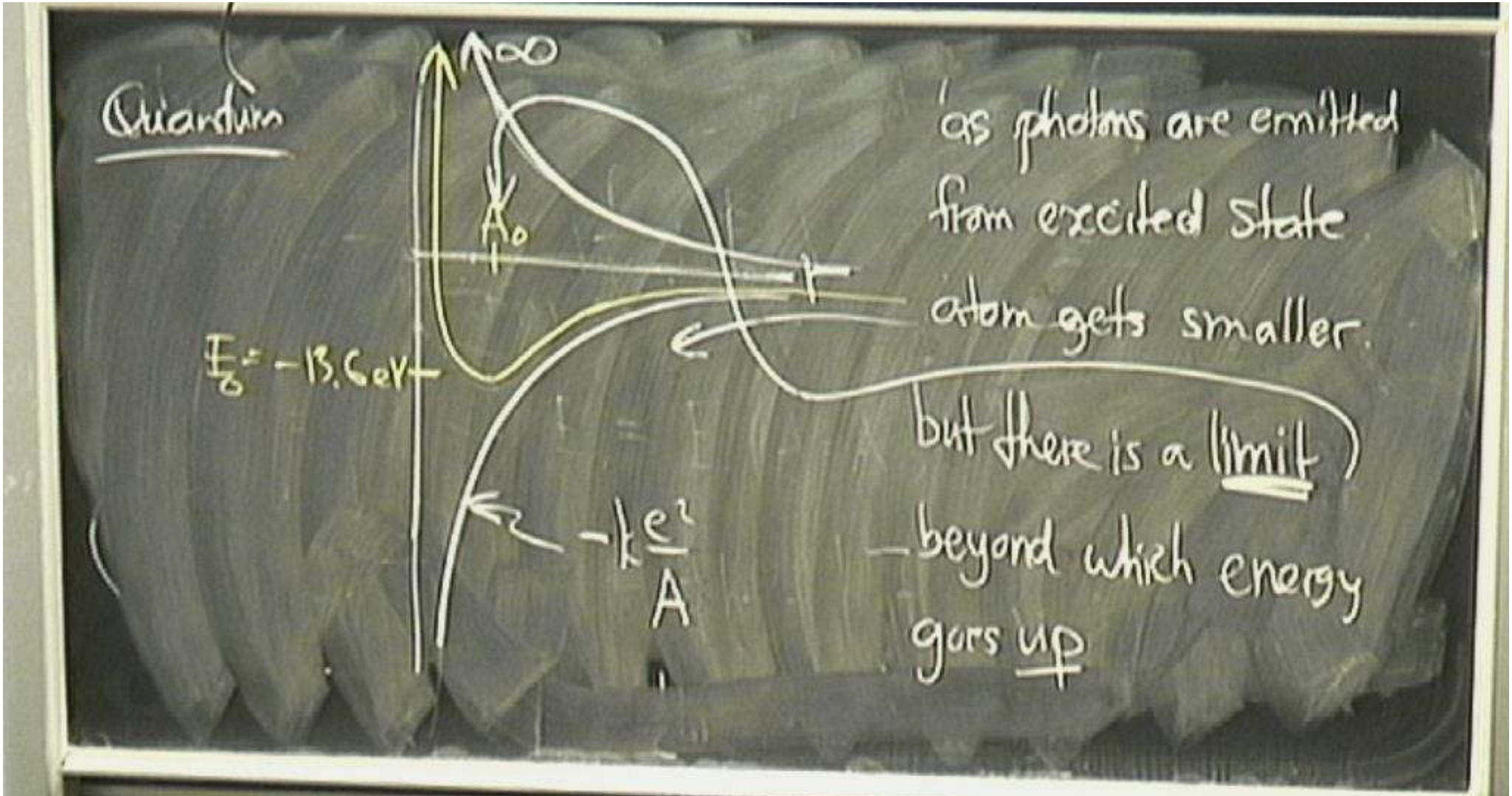

Quantum
∞
A_0
$E_0 = -13.6\,eV$
$-\frac{ke^2}{A}$

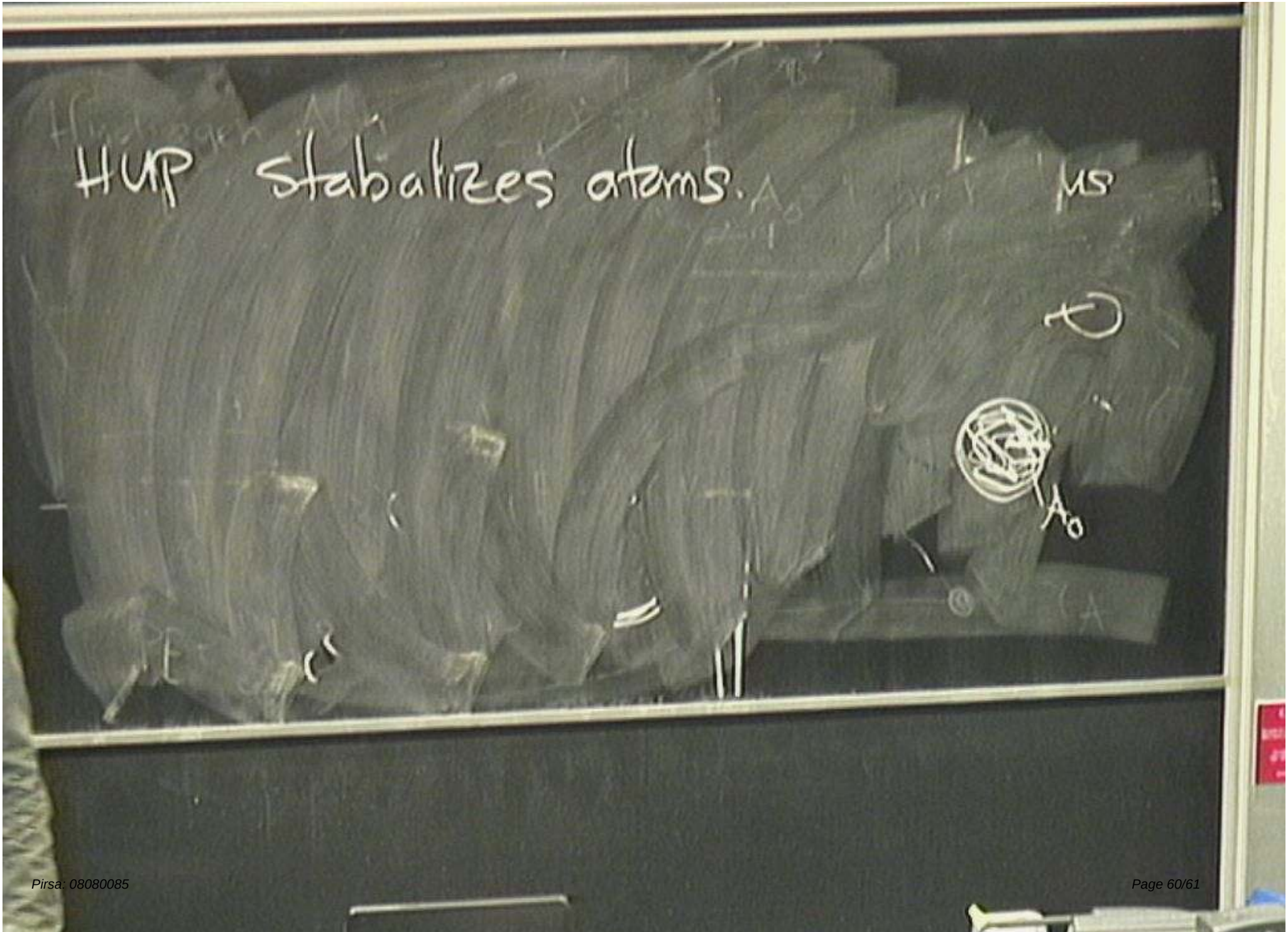
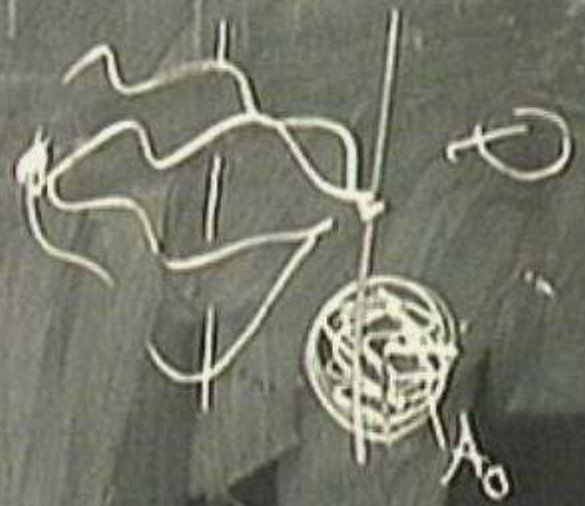

Quantum
∞
A_0
$E_0 = -13.6\,eV$
$-\frac{ke^2}{A}$
as photons are emitted
from excited state


Quantum
∞
A_0
$-\frac{ke^2}{A}$
as photons are emitted
from excited state
atom gets smaller.


Quantum
∞
A_0
$E_0 = -13.6eV$
$-ke^2/A$
as photons are emitted
from excited state
atom gets smaller.
there is a limit


Quantum
∞
A_0
$E_0 = -13.6eV$
as photons are emitted
from excited state
atom gets smaller.
but there is a limit
beyond which energy


Quantum
∞
A_0
$E_0 = -13.6eV$
$-\frac{ke^2}{A}$
as photons are emitted
from excited state
atom gets smaller.
but there is a limit
beyond which energy
goes up


HUP stabalizes atoms.
MS
Ao

HUP stabalizes atoms. MS