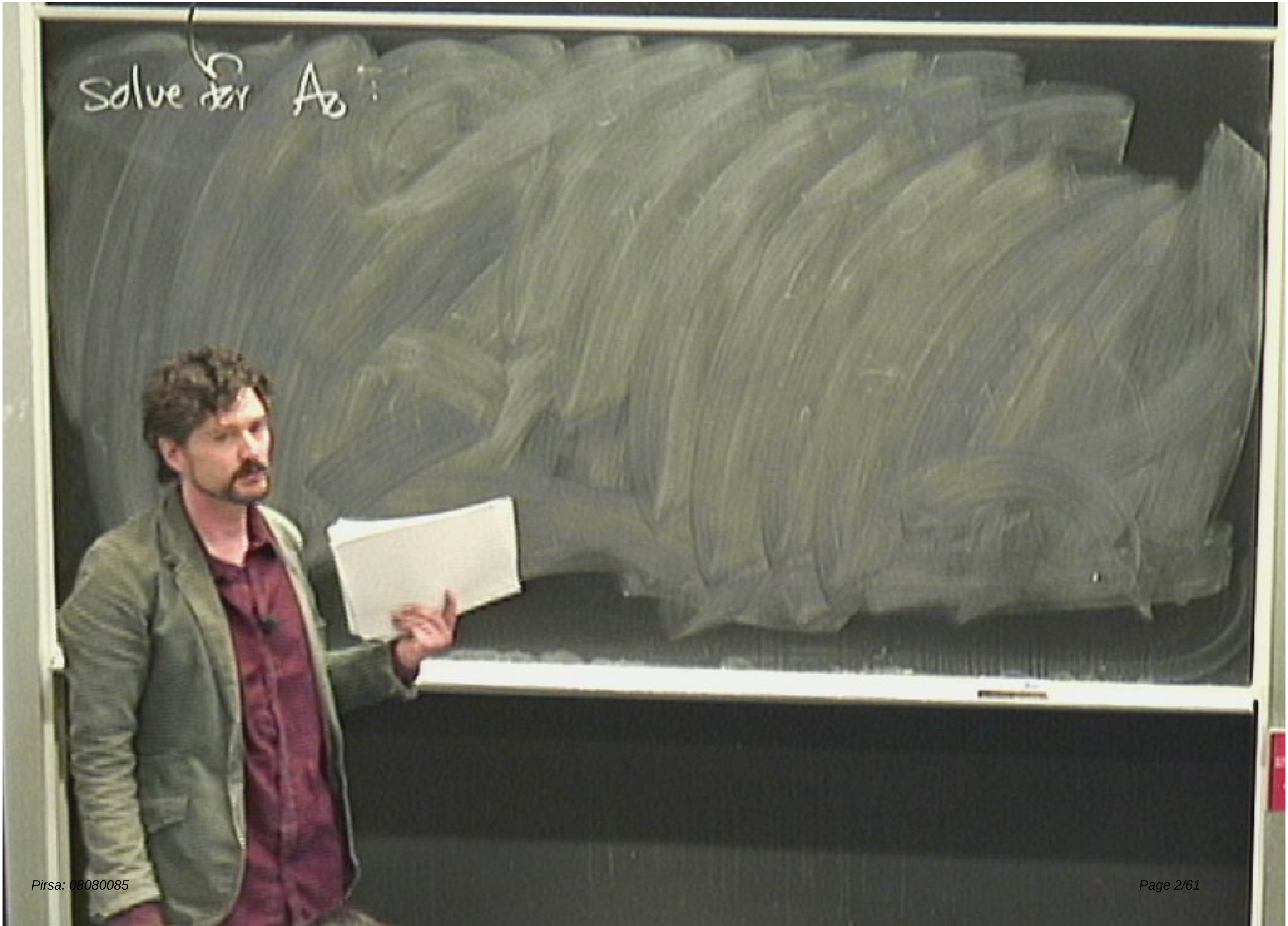
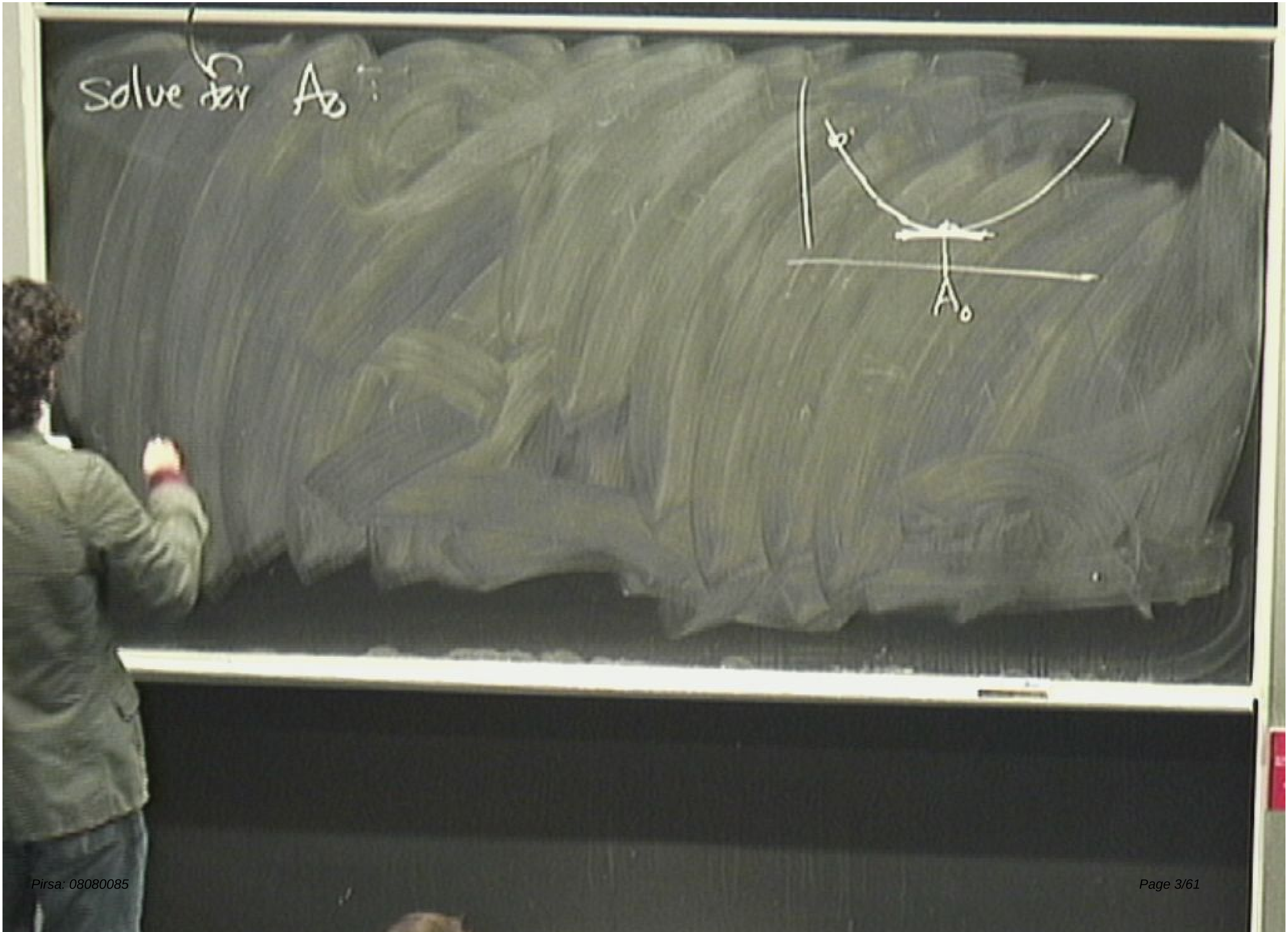
Title: Quantum Mechanics 10 - Stability of the Atom

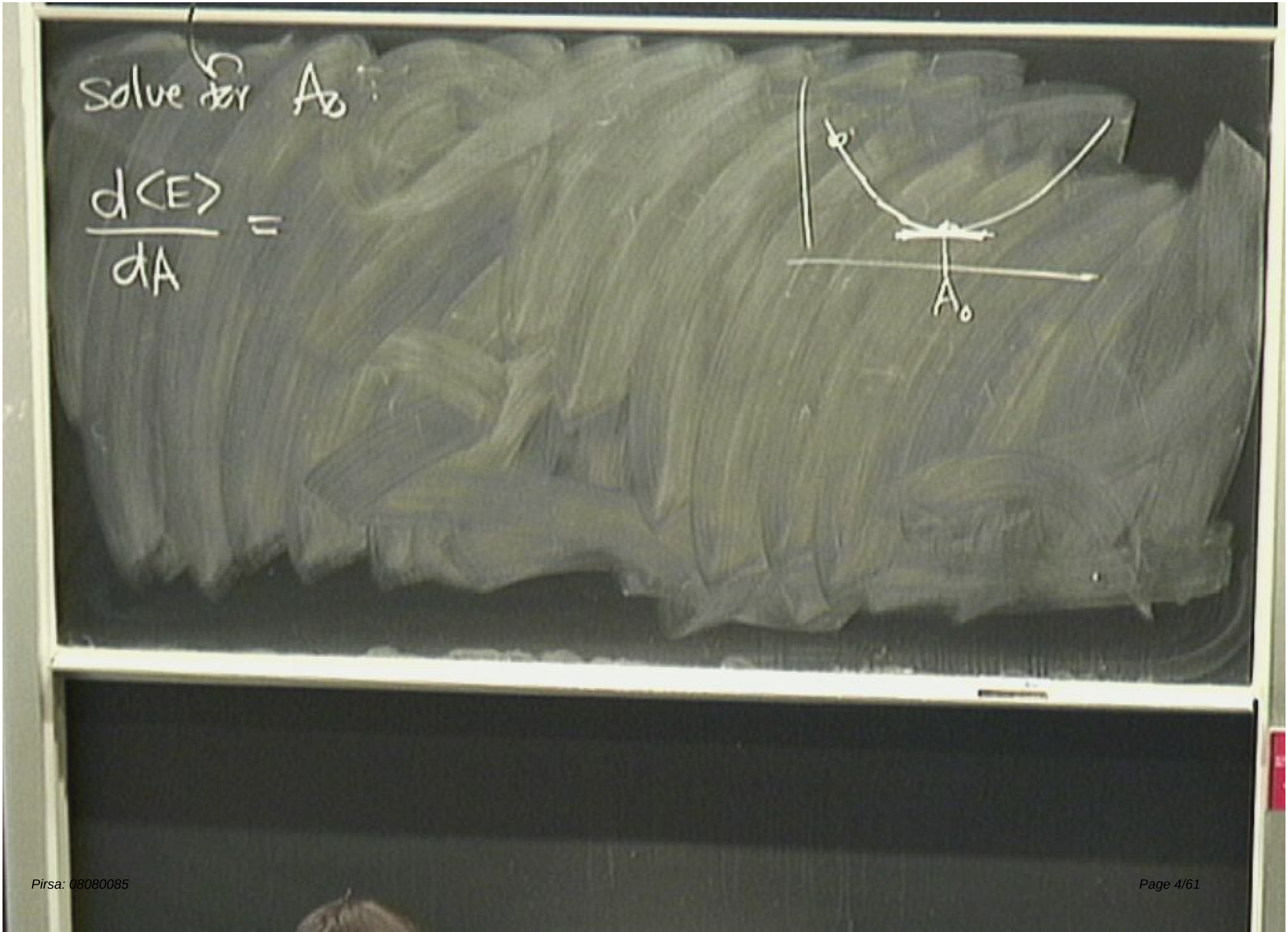
Date: Aug 13, 2008 10:30 AM

URL: http://pirsa.org/08080085

Abstract: A discussion of how the zero point energy of atoms is what makes possible their existence in our universe â€“ atoms are purely quantum mechanical objects.
Learning Outcomes:
â€¢ Continuation of QM-9: A calculus-based derivation of the zero point energy of the quantum harmonic oscillator.
â€¢ How our previous understanding of energy quantization and zero point energy can be applied also to the hydrogen atom.
â€¢ Why classical atoms cannot exist in our universe, and how the Heisenberg Uncertainty Principle and associated zero point energy stabilizes atoms, making their existence possible.


Solve for A0

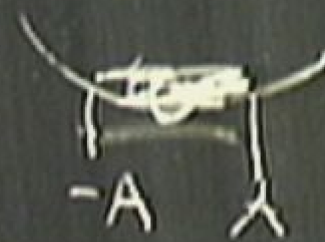

Solve for A0
A0


Solve for A0 :
d⟨E⟩/dA =
A0

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2} k A^2$$

$\uparrow$ quantum KE

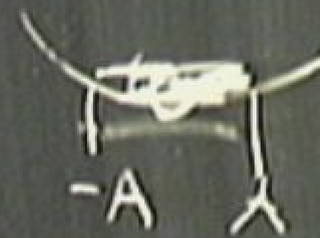
$\uparrow$ classical PE



$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2} k A^2$$

↑ quantum KE

↑ classical PE

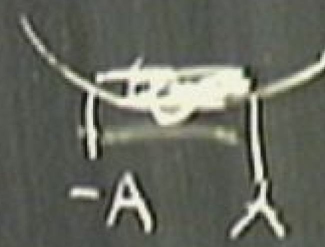


$$\frac{d}{dx}x^n =$$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE

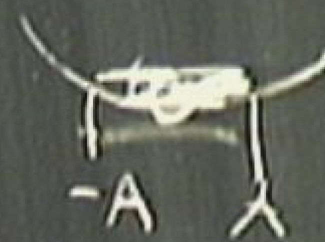


$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dA}(A^2) =$$

$$\langle E\rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE



$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dA}(A^2) = 2A$$

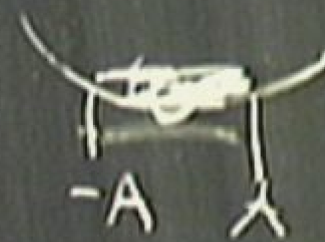
$$\frac{d}{dA}\left(\frac{1}{A^2}\right) =$$

$$\langle E\rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE

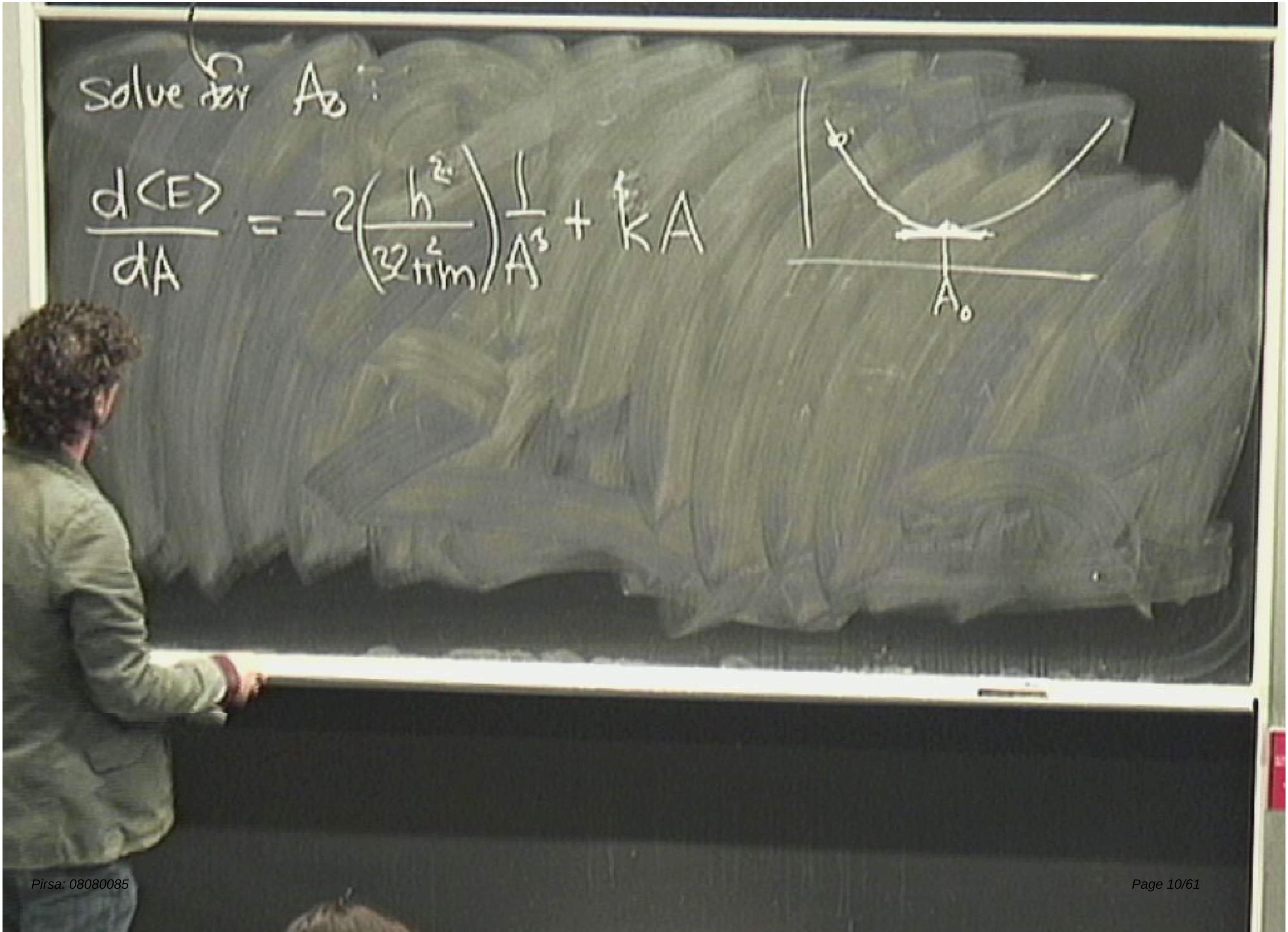
$\frac{1}{A^2} = A^{-2}$



$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dA}(A^2) = 2A$$

$$\frac{d}{dA}\left(\frac{1}{A^2}\right) = -2\frac{1}{A^3}$$

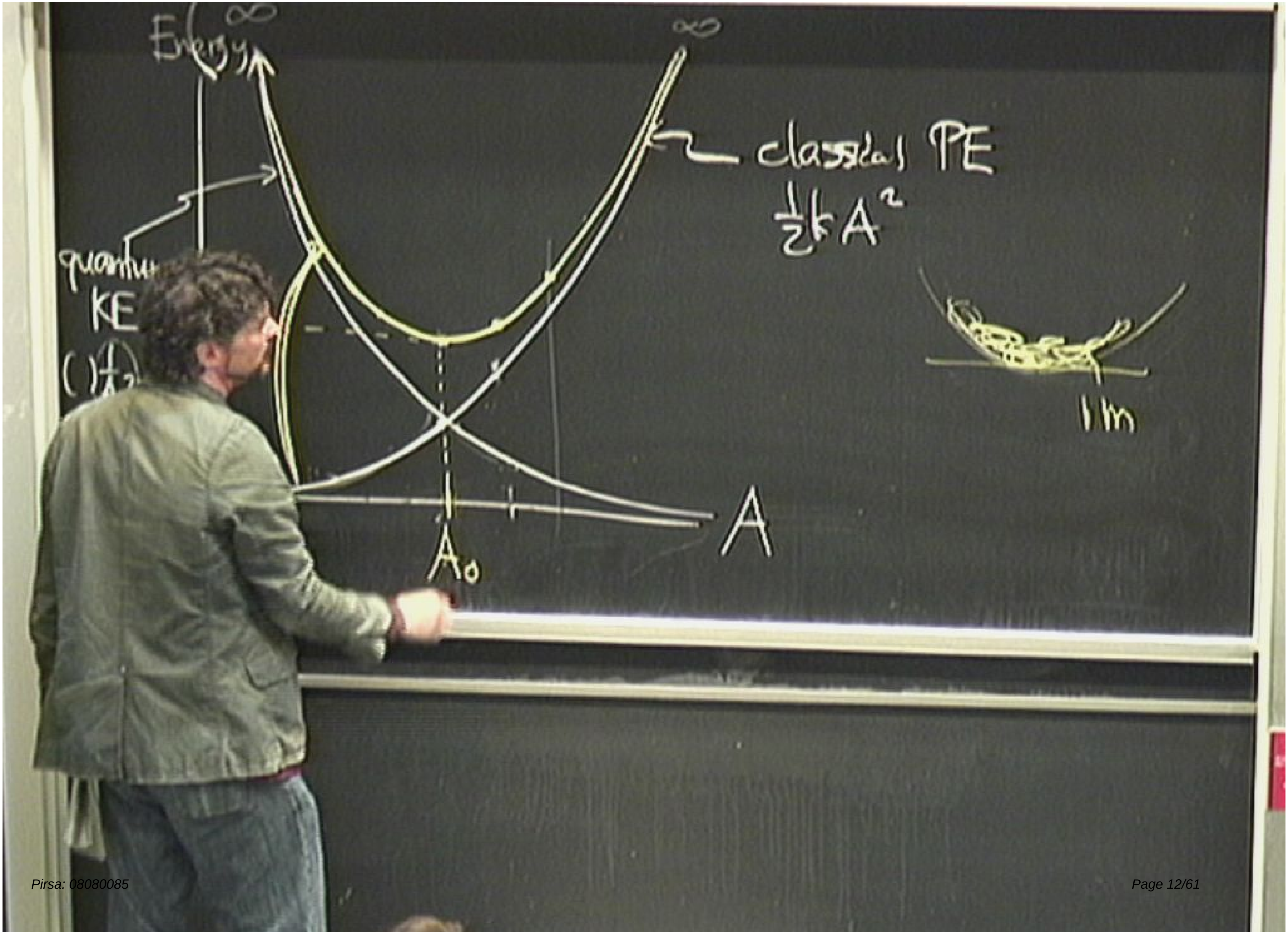

Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
A_0

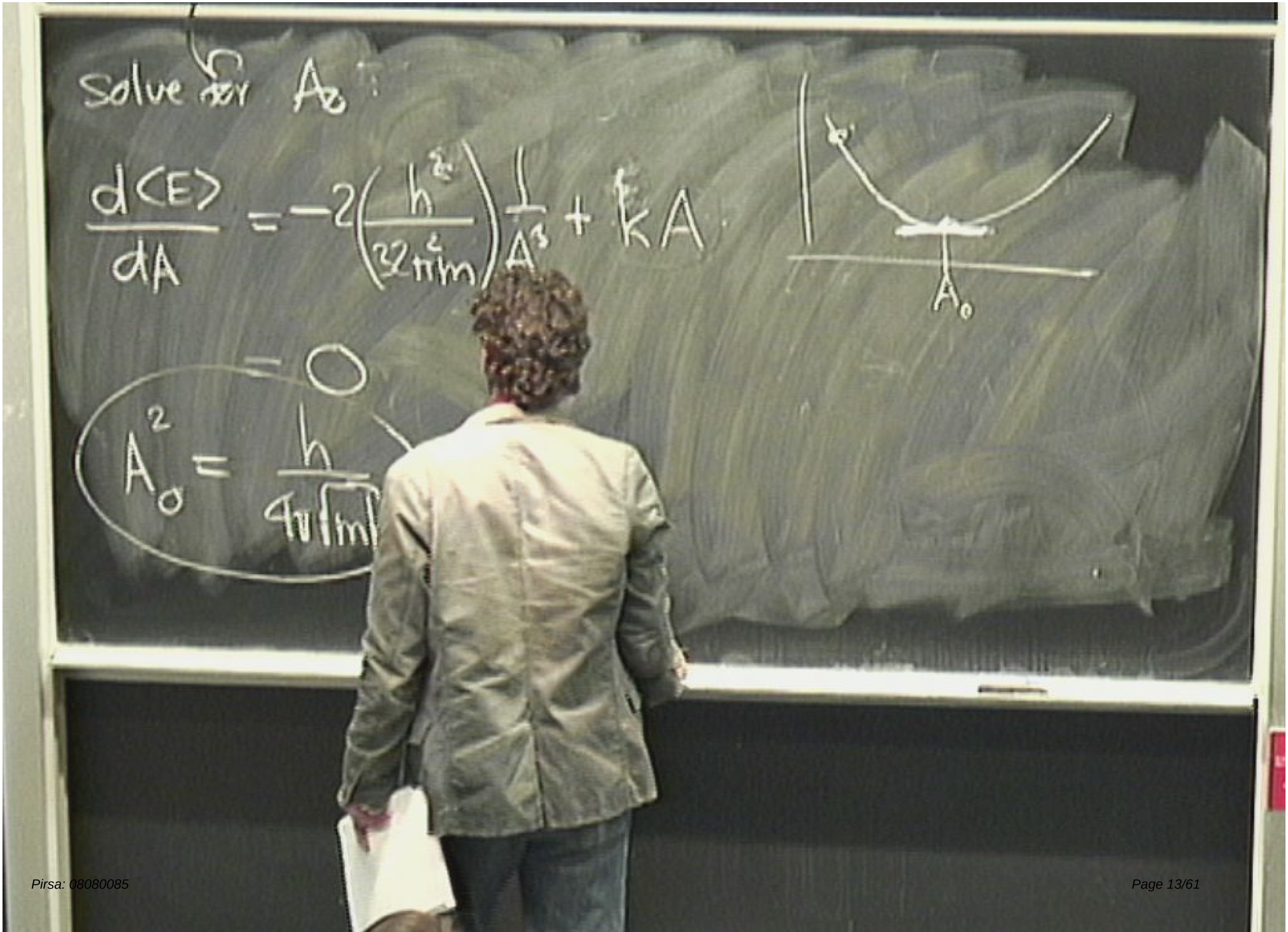
Solve for A_0:

$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

$$= 0$$

$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$


Energy
∞
∞
classical PE
$\frac{1}{2}kA^2$
quantum
KE
A
A_0
1m

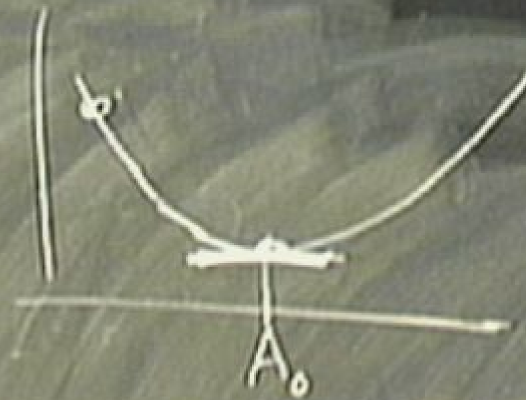

Solve for A0
d⟨E⟩/dA = −2(h²/(32π²m)) 1/A³ + kA
= 0
A0² = h/(4π√m
A0

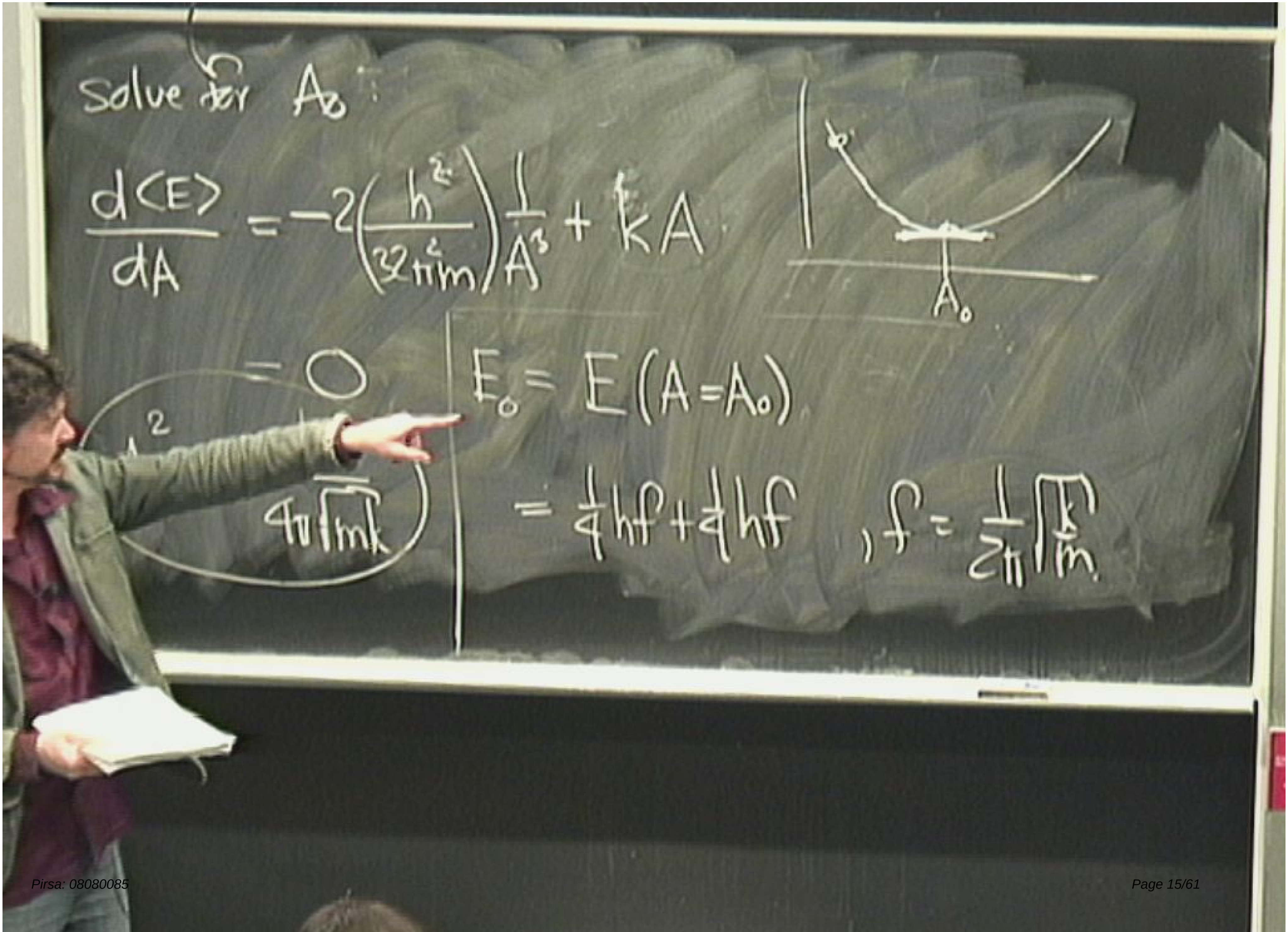
Solve for A_0:

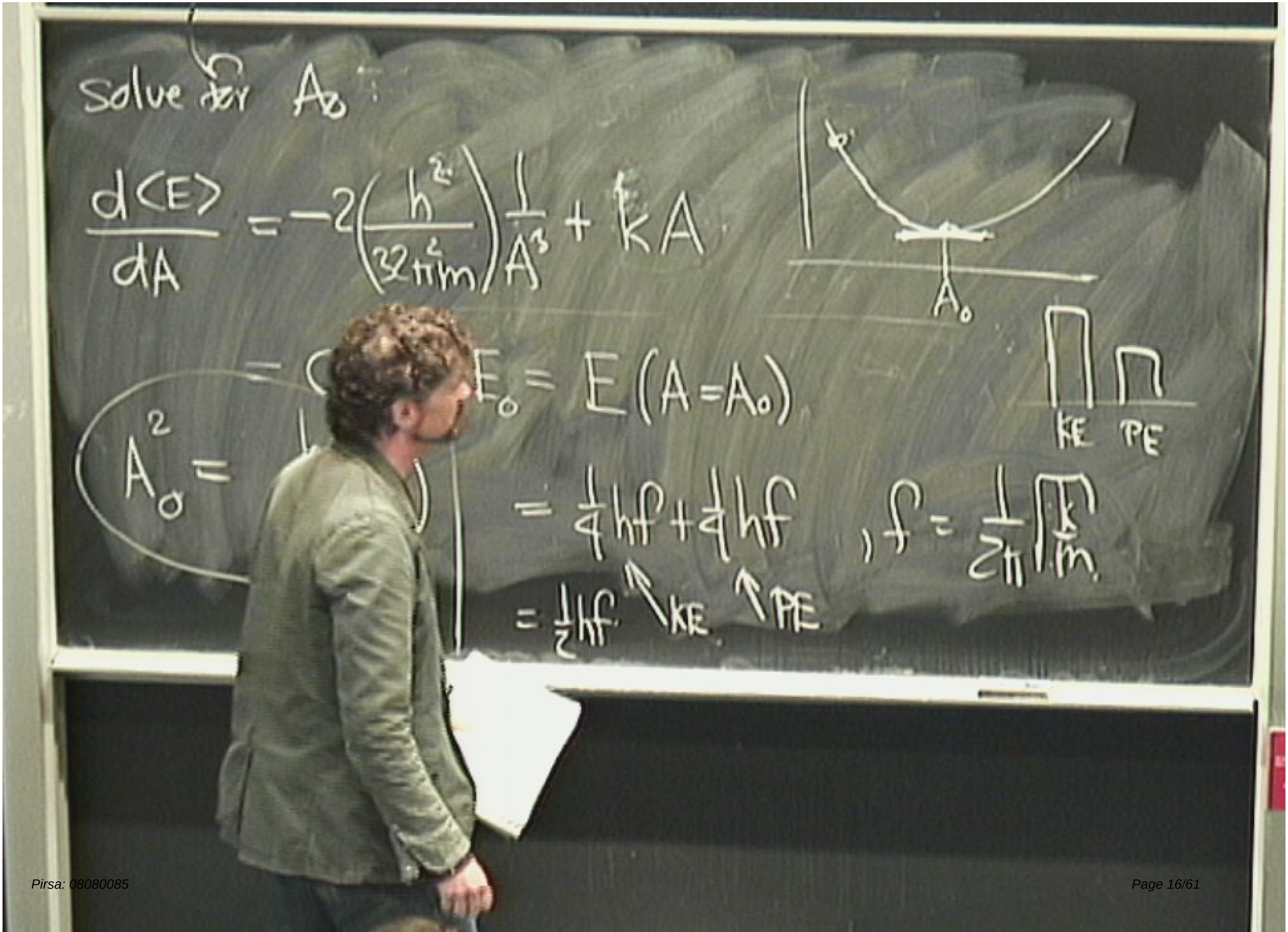
$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

$$= 0$$

$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$




Solve for A0:
d<E>/dA = −2(h²/32π²m) 1/A³ + kA
= 0
A0
E0 = E(A=A0)
= ¼hf + ¼hf , f = 1/2π √(k/m)


Solve for A₀
d⟨E⟩/dA = −2(h²/32π²m) 1/A³ + kA
A₀
E₀ = E(A=A₀)
KE PE
= ¼hf + ¼hf , f = 1/2π √(k/m)
= ½hf KE PE

Solve for A_0:

$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

$$= 0$$

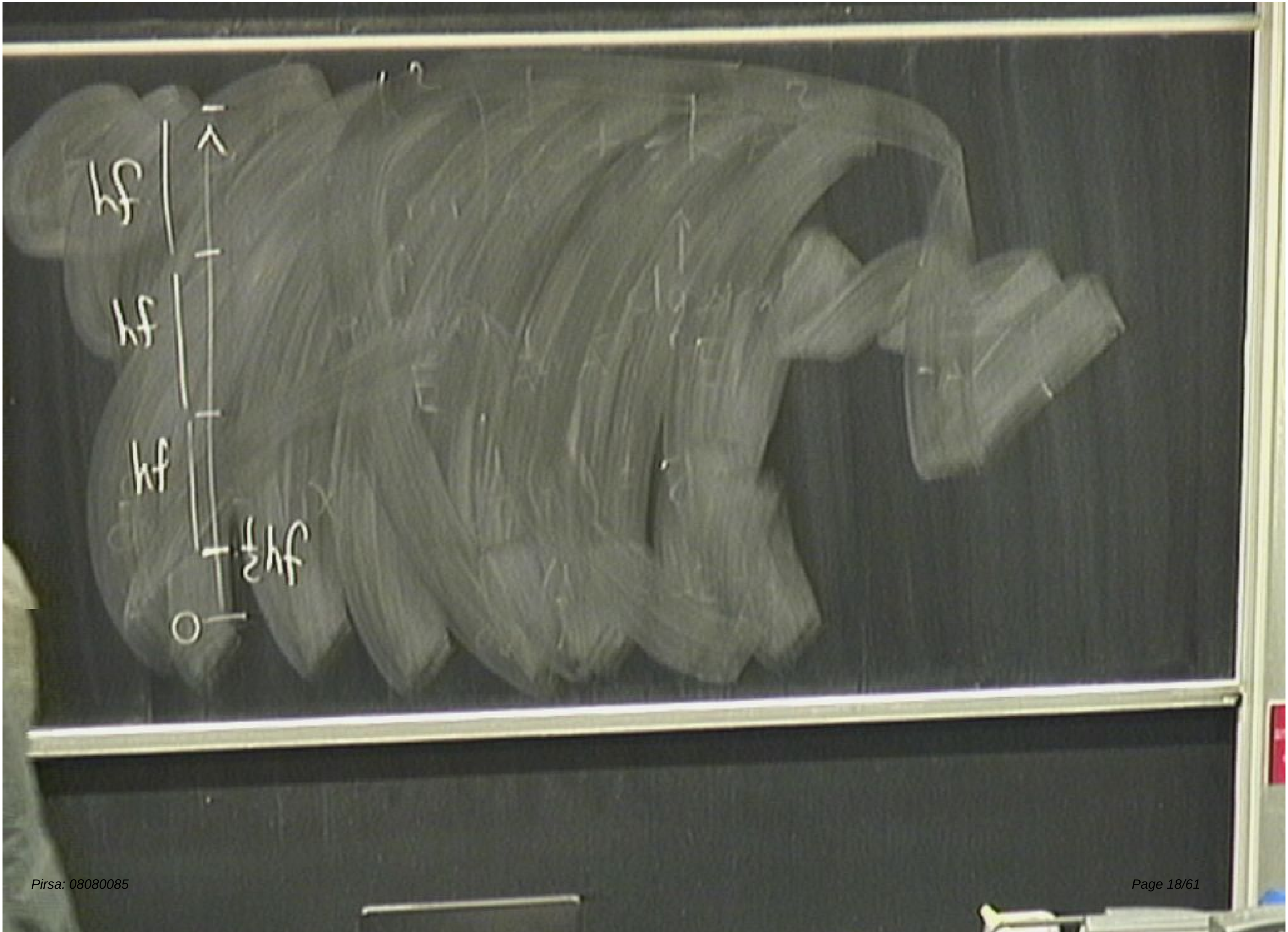
$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$

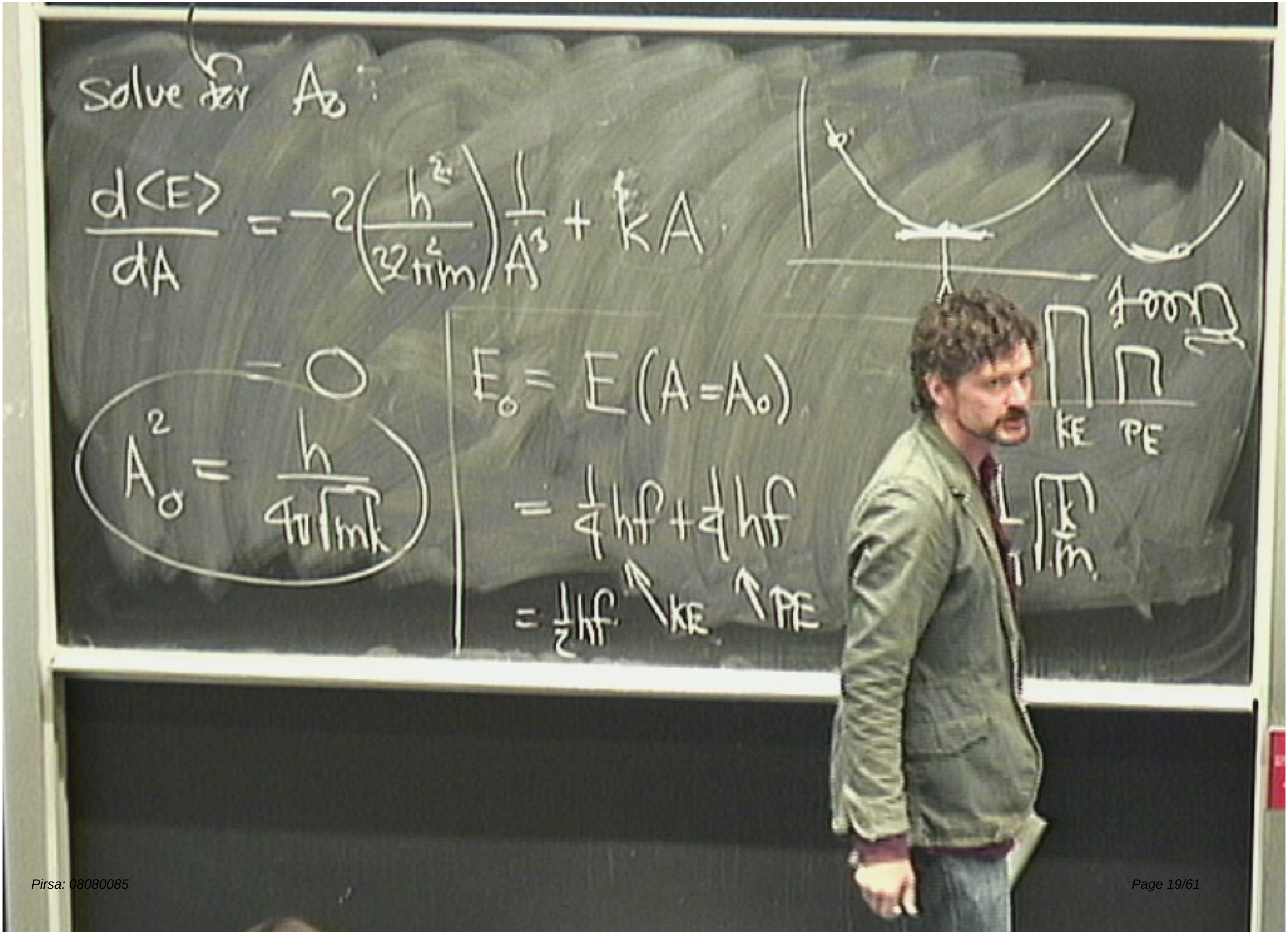
$$E_0 = E(A = A_0)$$

$$= \frac{1}{4}hf + \frac{1}{4}hf, \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

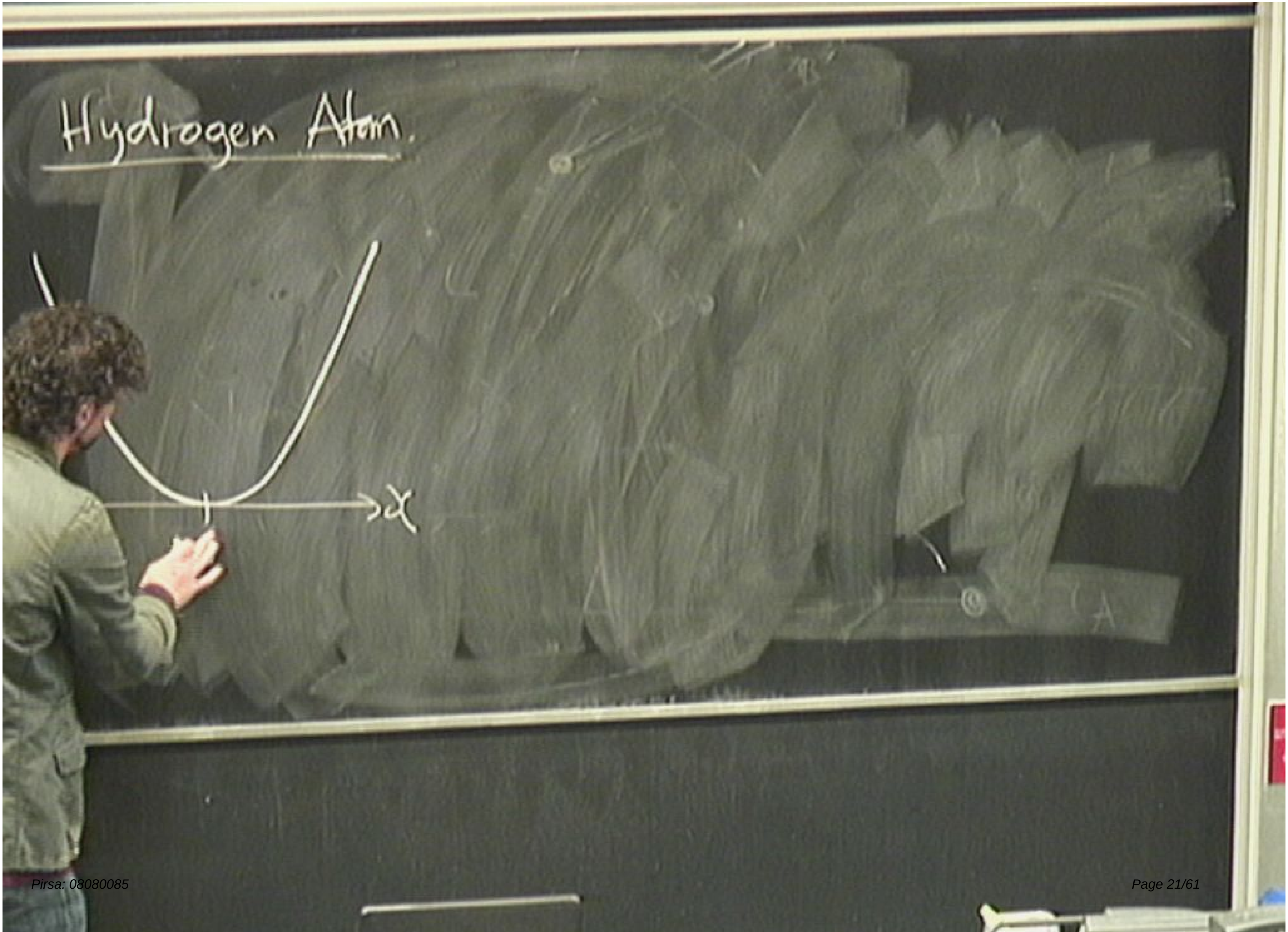
↑KE ↑PE

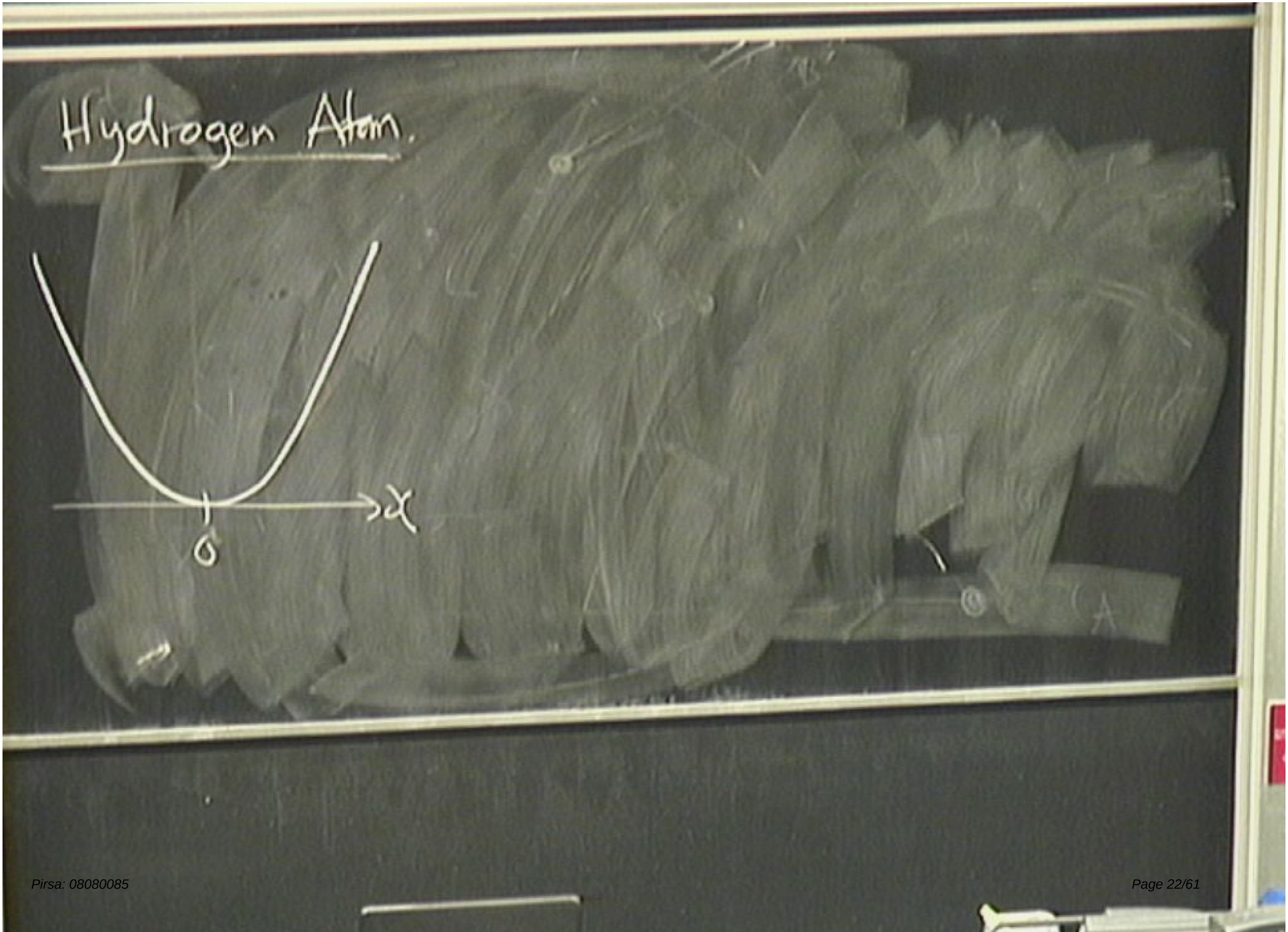
$$= \frac{1}{2}hf$$

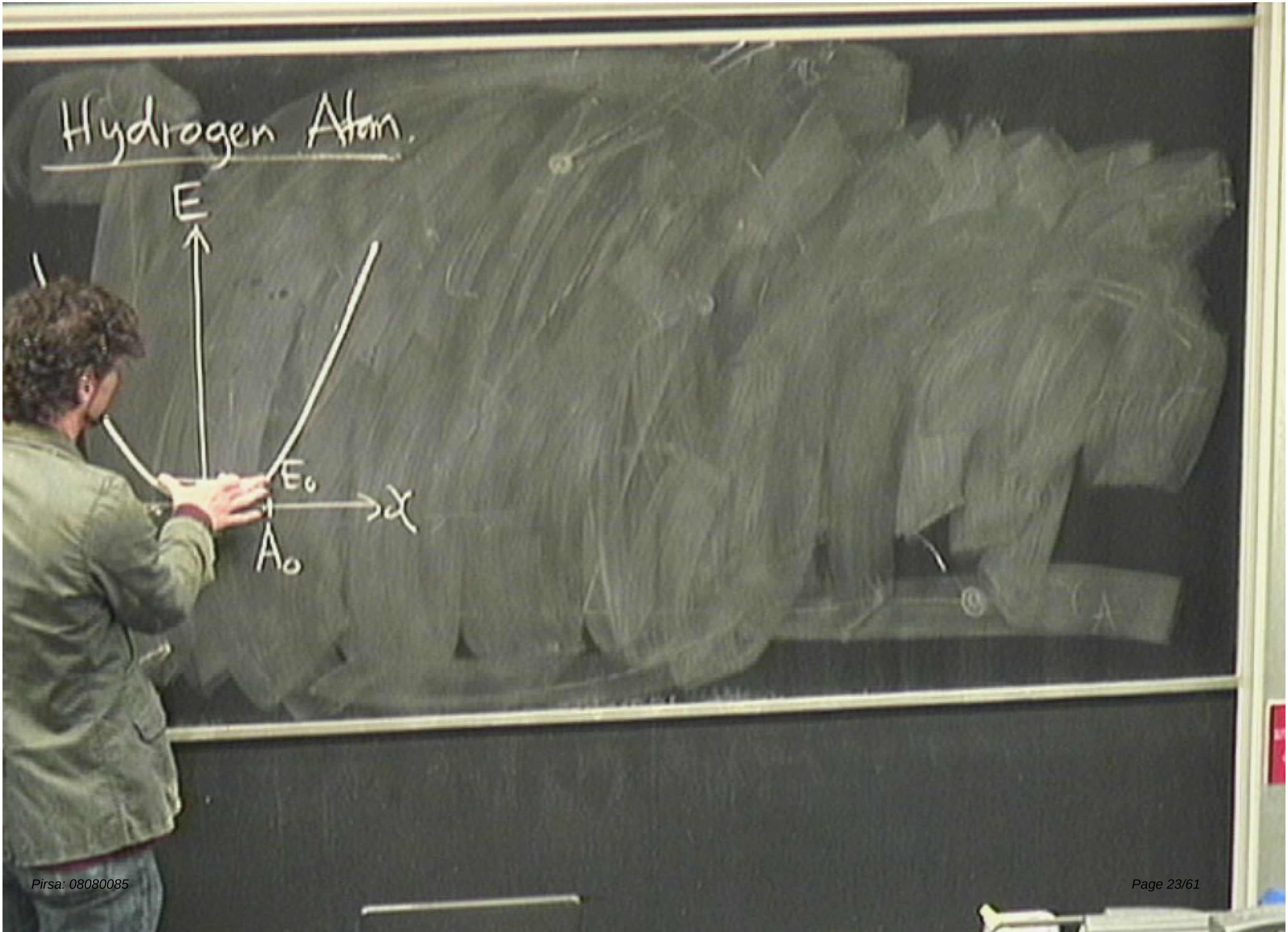

hf
hf
hf
½hf
0

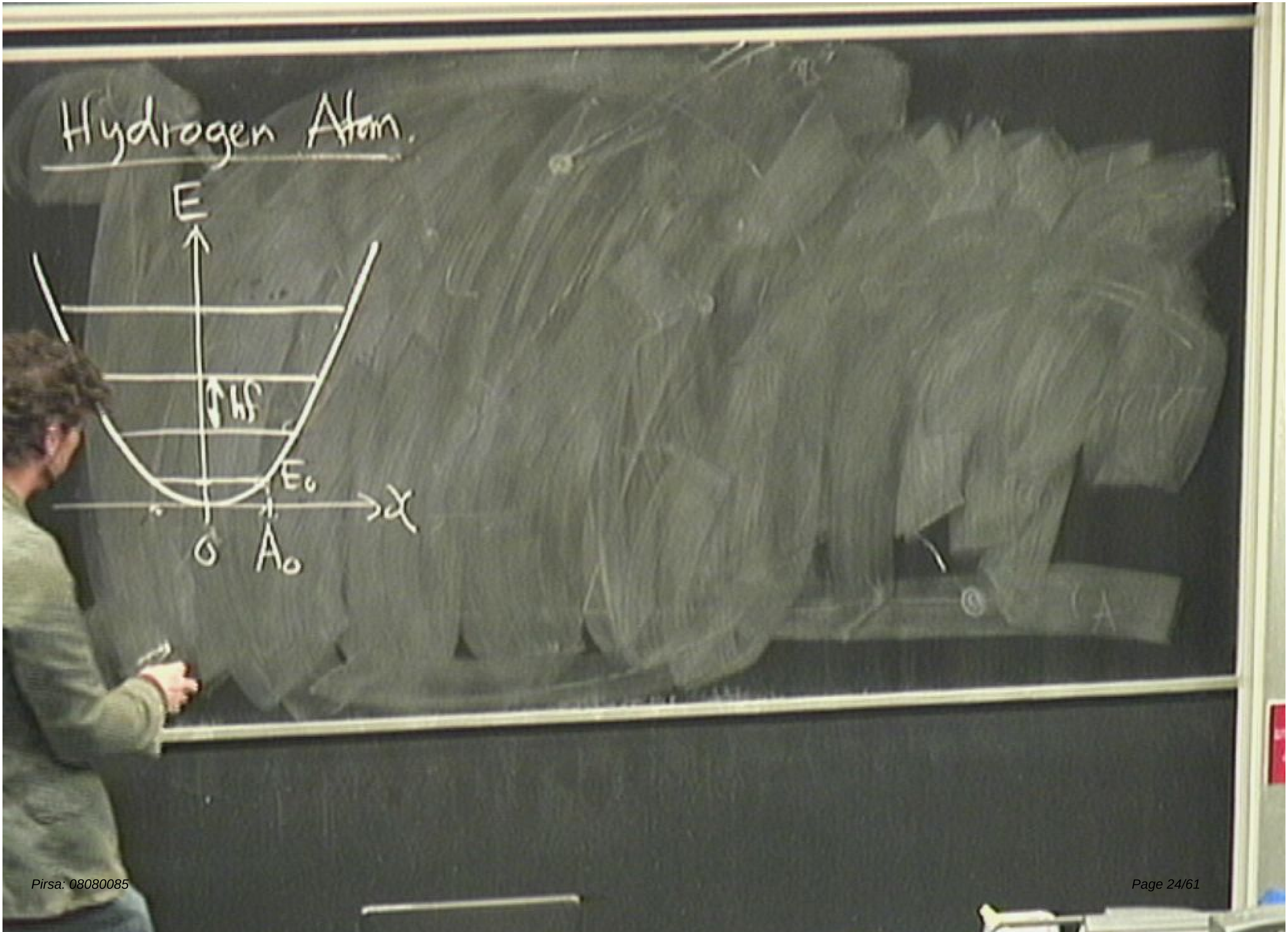

Solve for A0:
d<E>/dA = −2(h²/(32π²m)) 1/A³ + kA
= 0
A0² = h/(4π√mk)
E0 = E(A=A0)
= ¼hf + ¼hf
= ½hf
KE
PE
KE PE

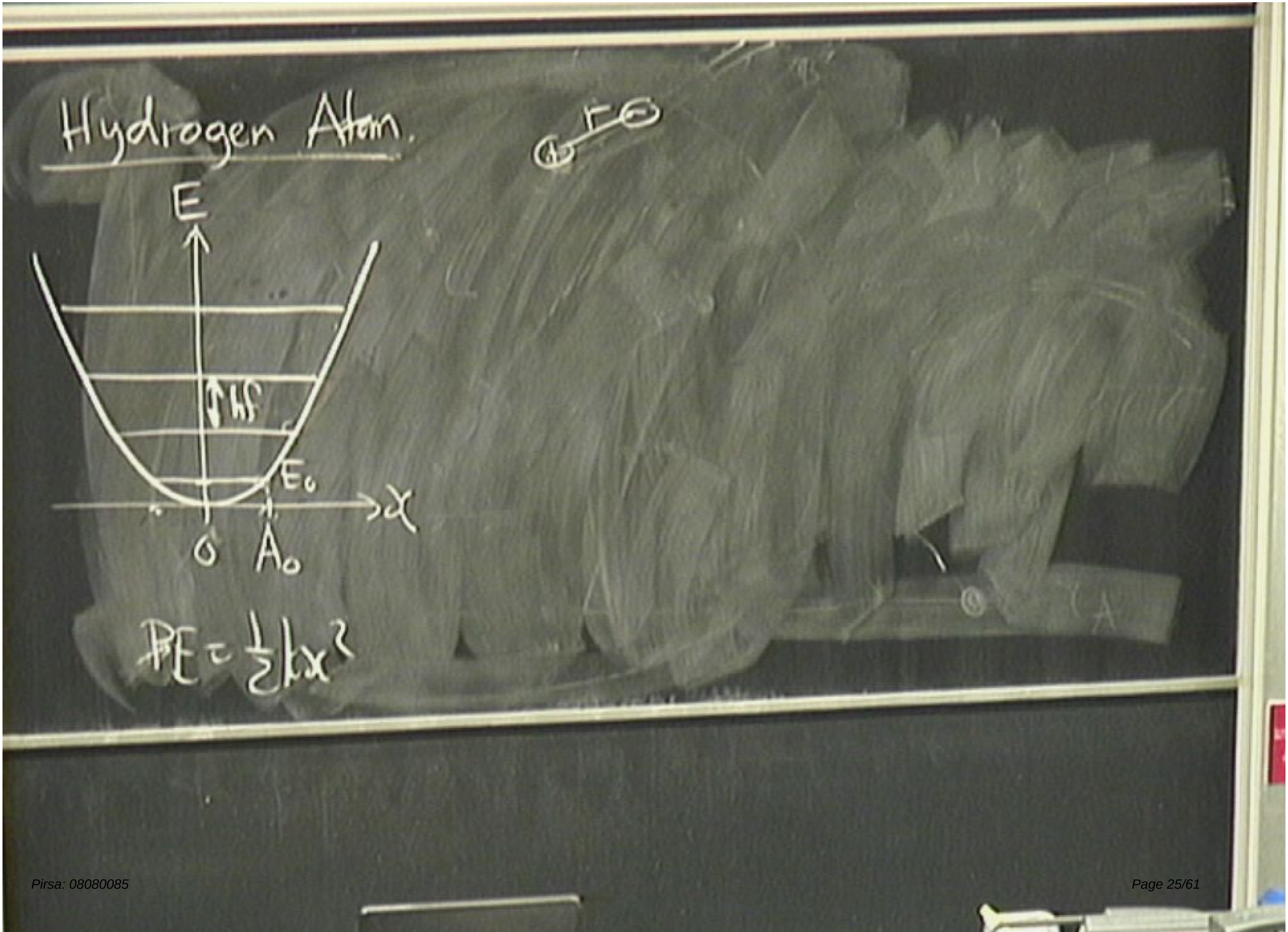
Hydrogen Atom.

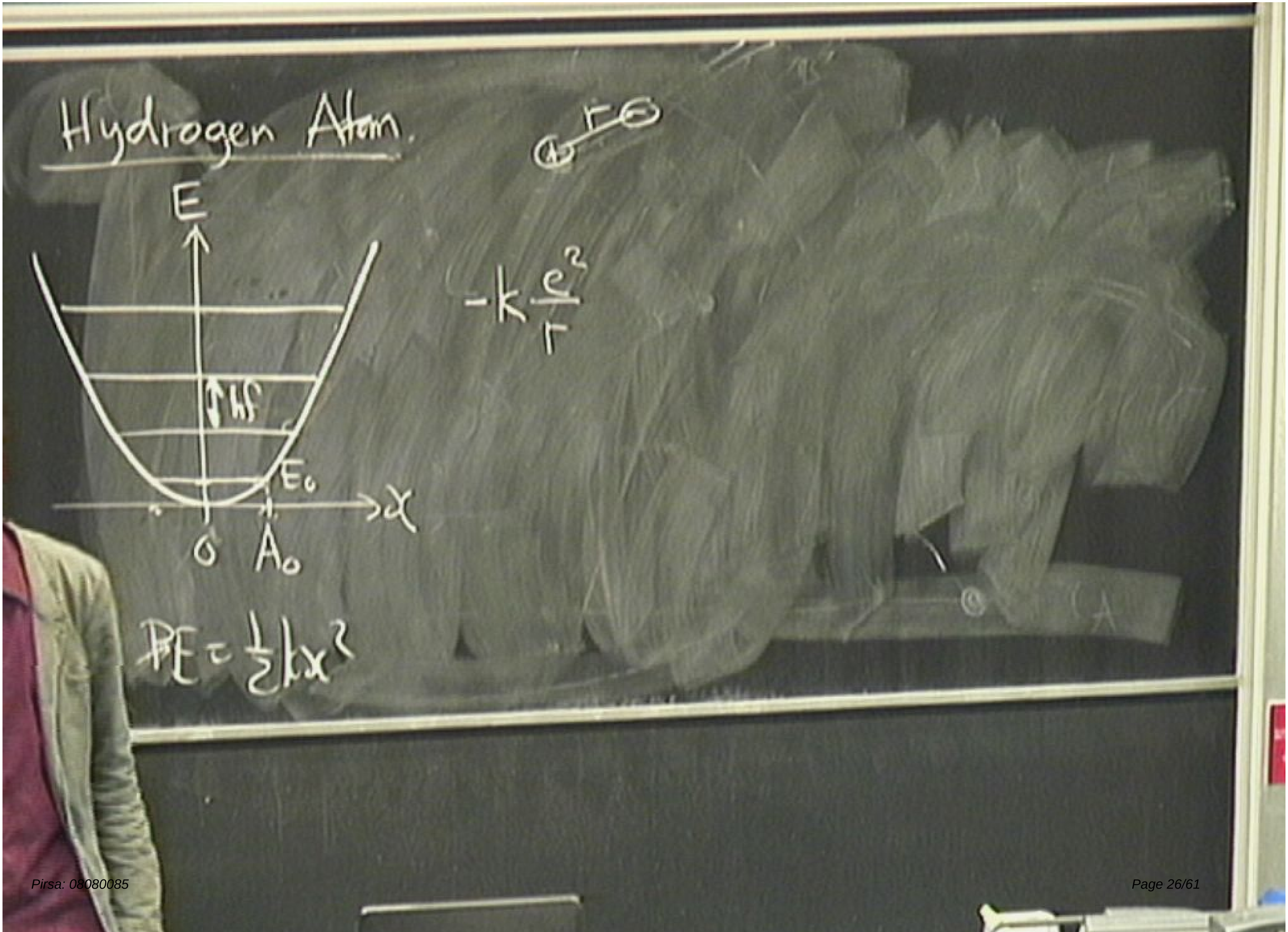

Hydrogen Atom.
x

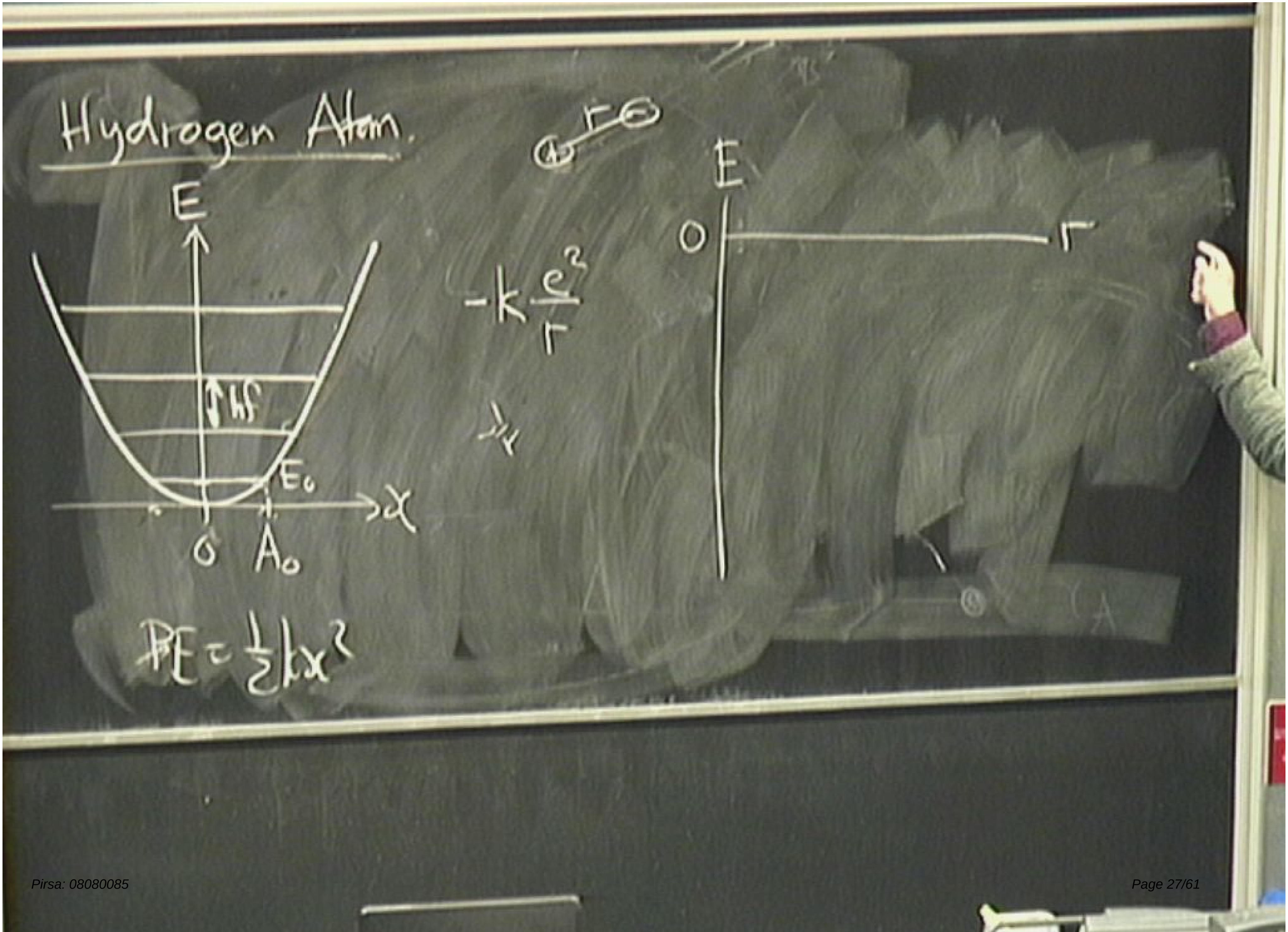

Hydrogen Atom.
x
0

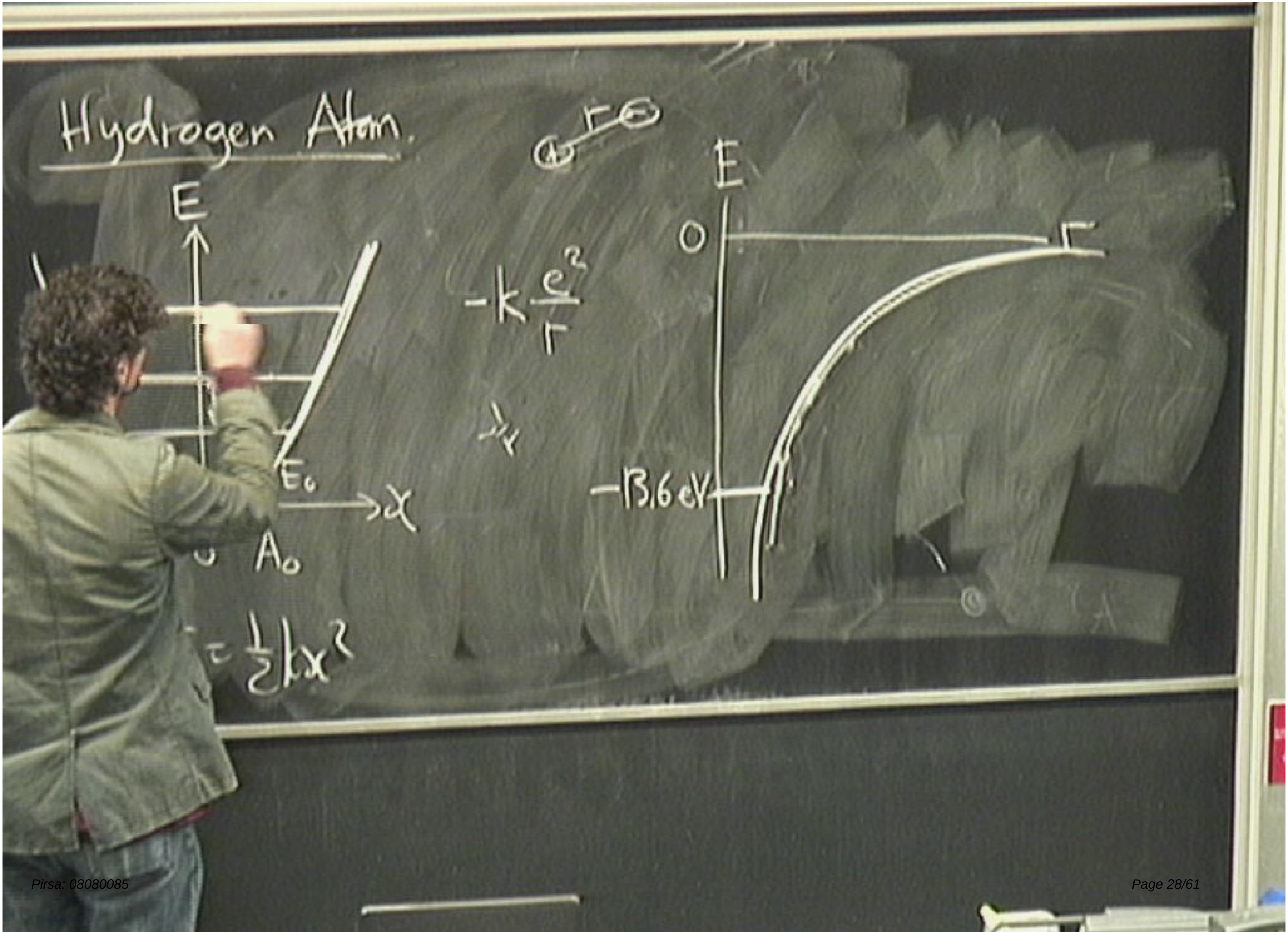

Hydrogen Atom.
E
E_0
x
A_0

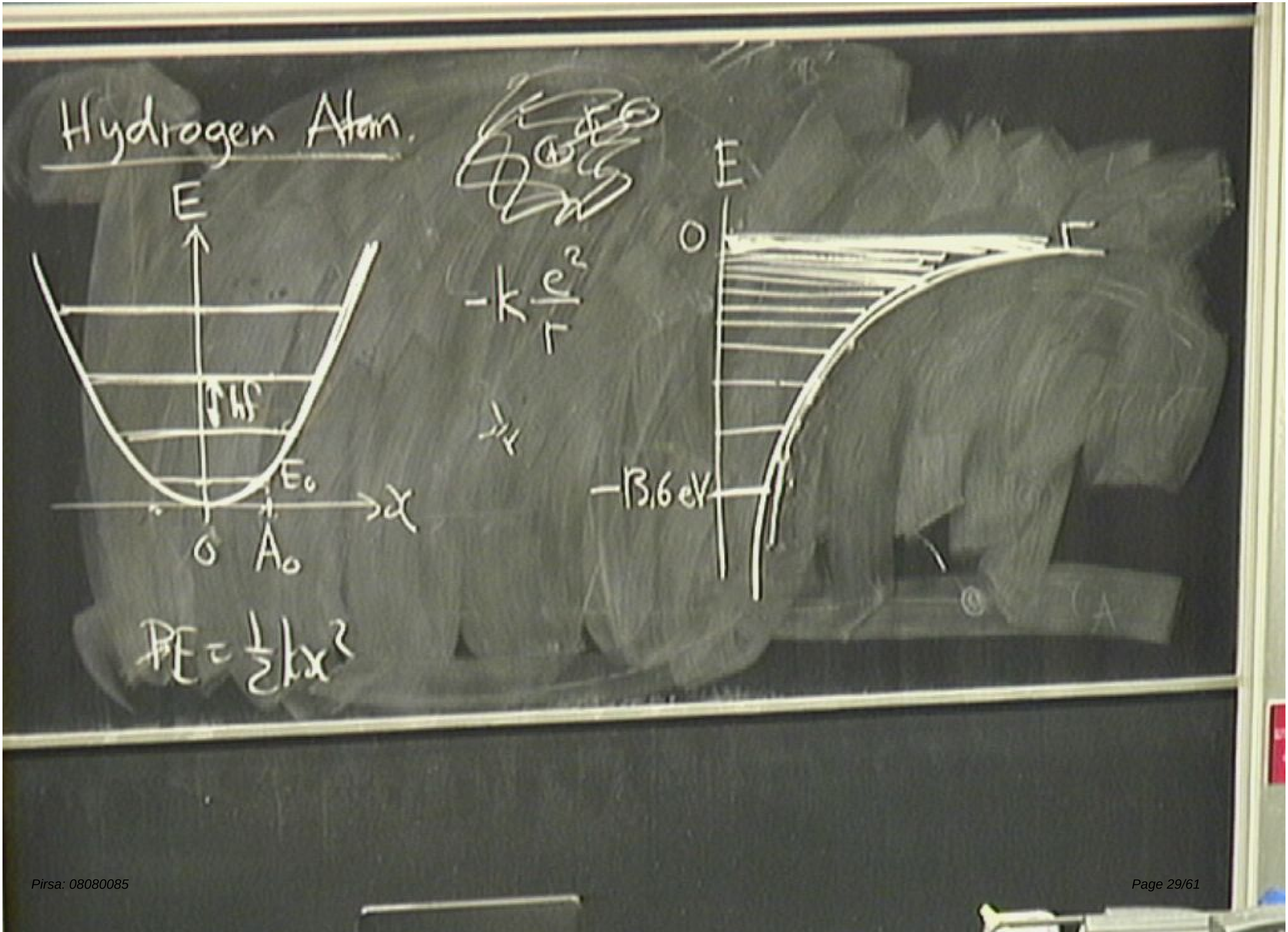

Hydrogen Atom.
E
hf
E0
x
0
A0

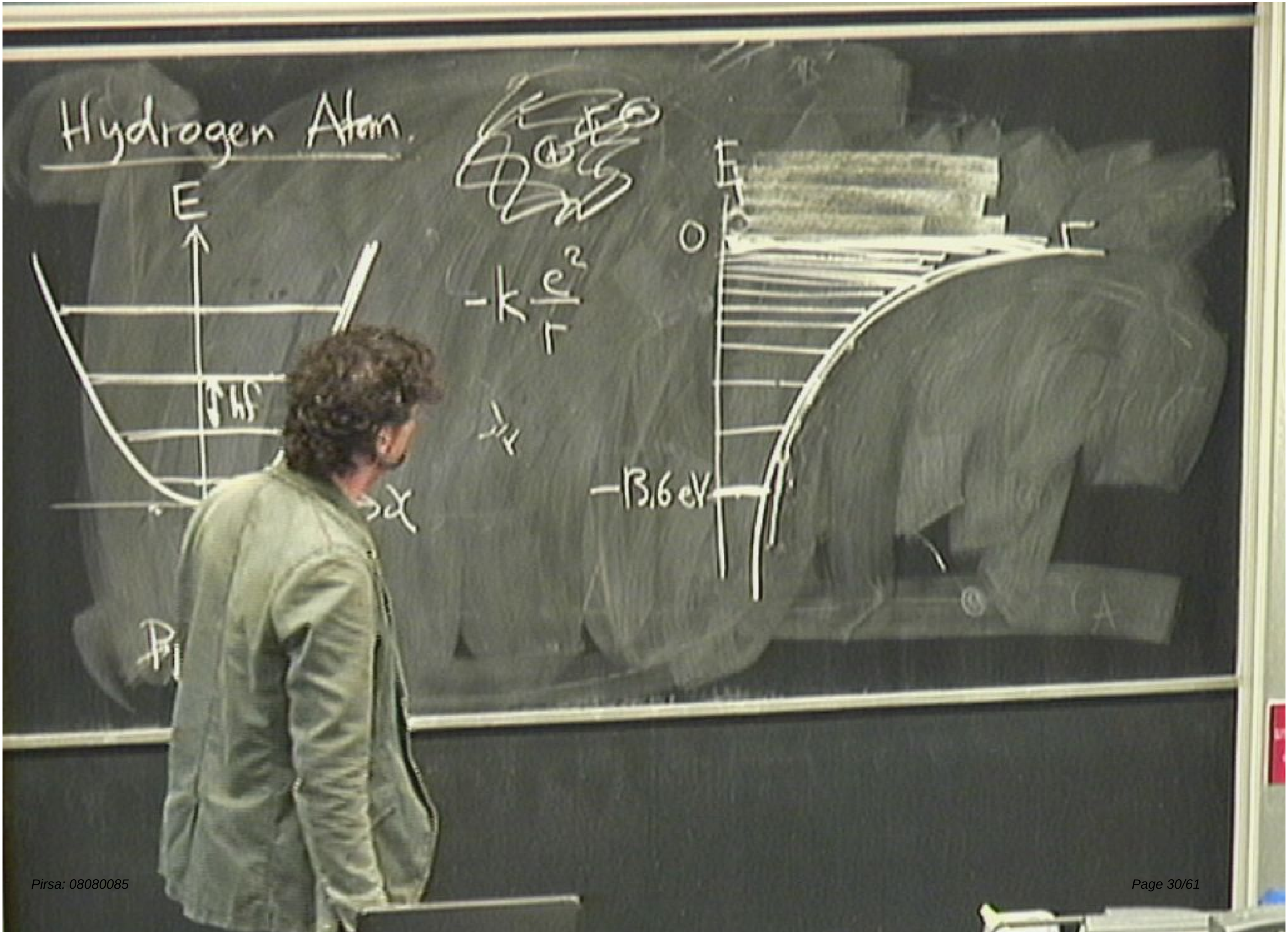

Hydrogen Atom.
E
hf
E0
x
0
A0
PE = 1/2 kx2
r

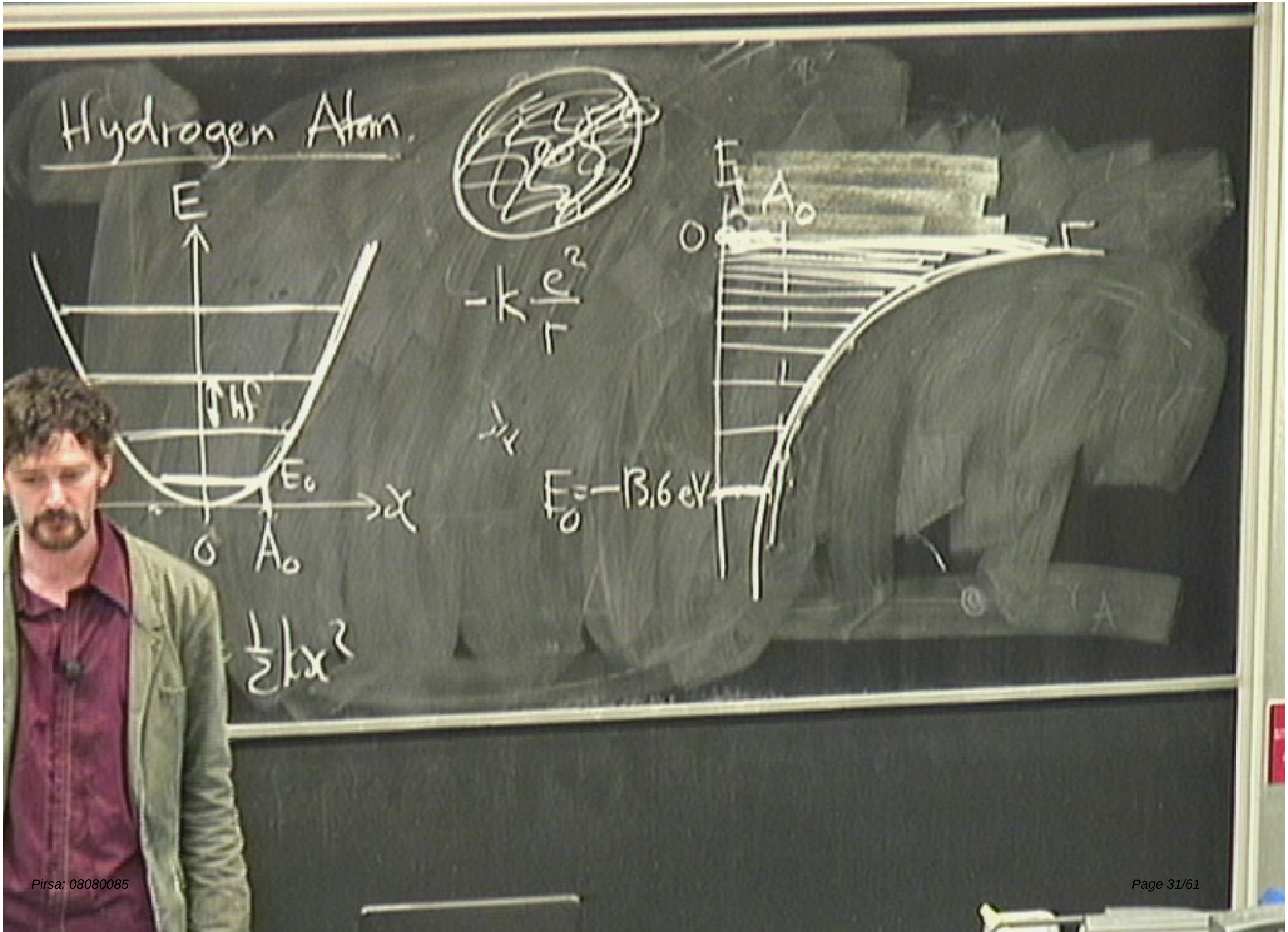

Hydrogen Atom
r
E
hf
E0
x
0
A0
-k e^2/r
PE = 1/2 kx^2

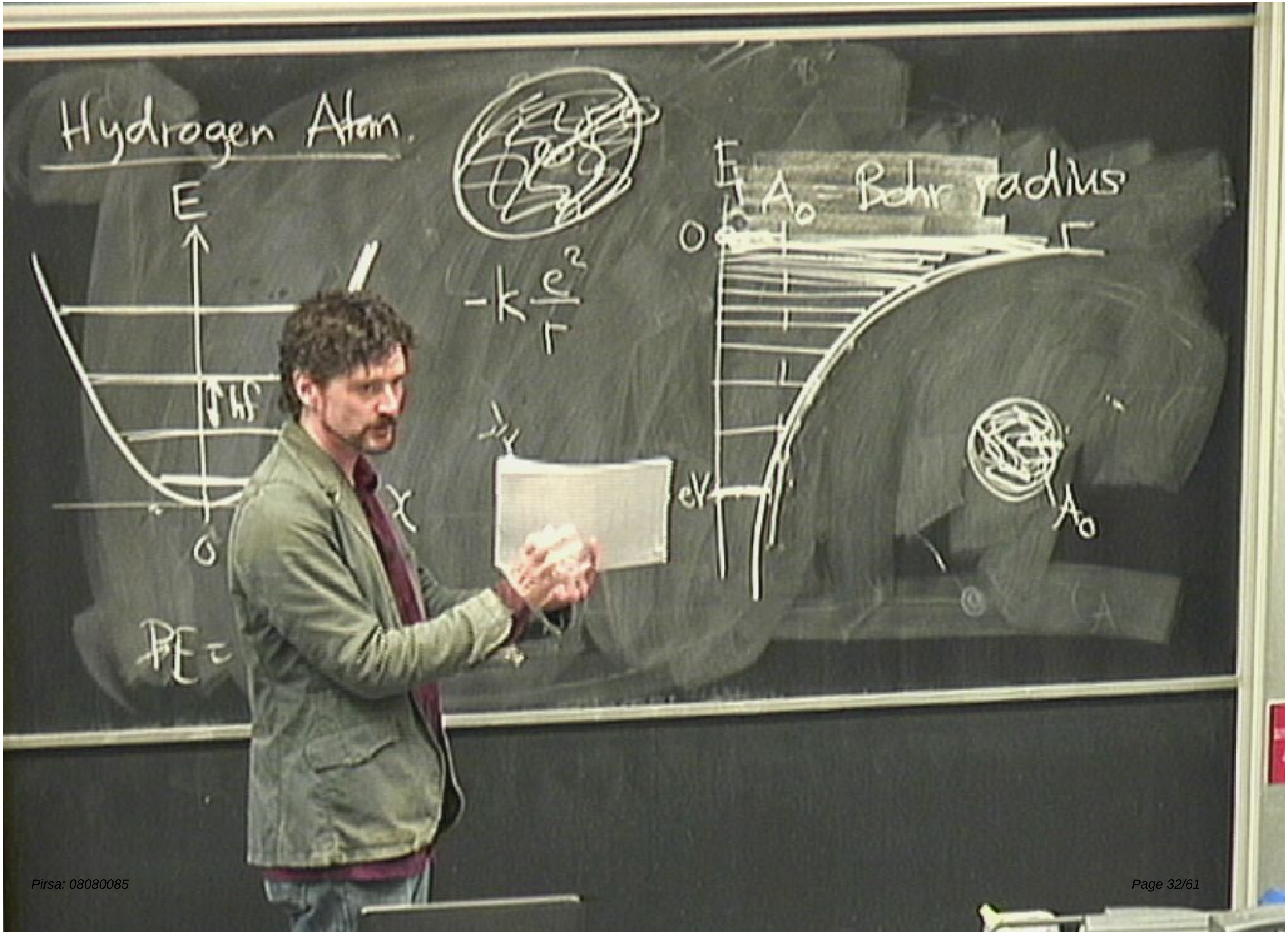

Hydrogen Atom
E
hf
E0
x
0
A0
PE = 1/2 kx^2
r
-k e^2/r
E
0
r

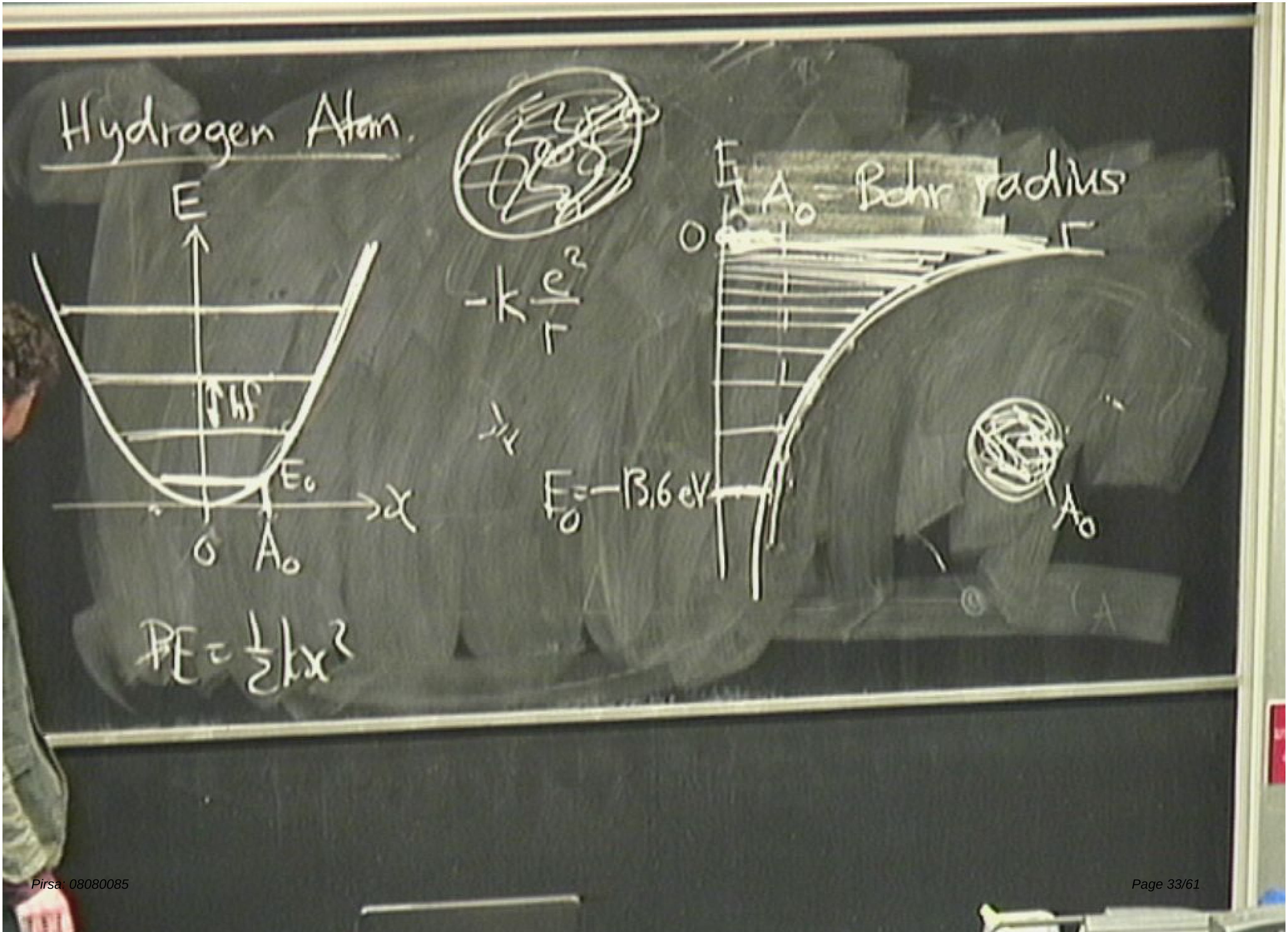

Hydrogen Atom
E
E₀
x
A₀
= ½kx²
-k e²/r
E
0
r
-13.6 eV

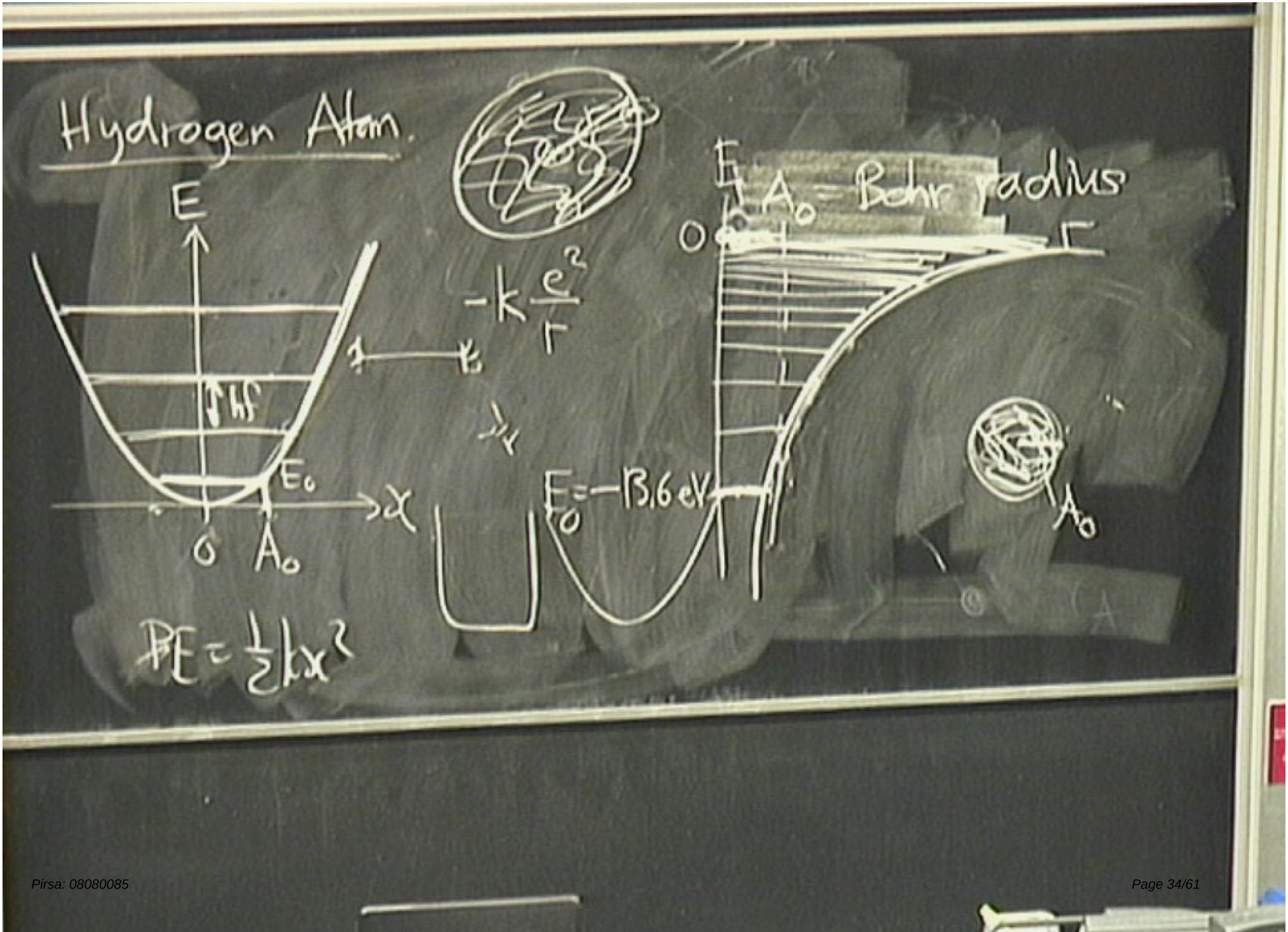

Hydrogen Atom
E
hf
E0
x
0
A0
PE = 1/2 kx^2
-k e^2/r
E
0
r
-13.6 eV

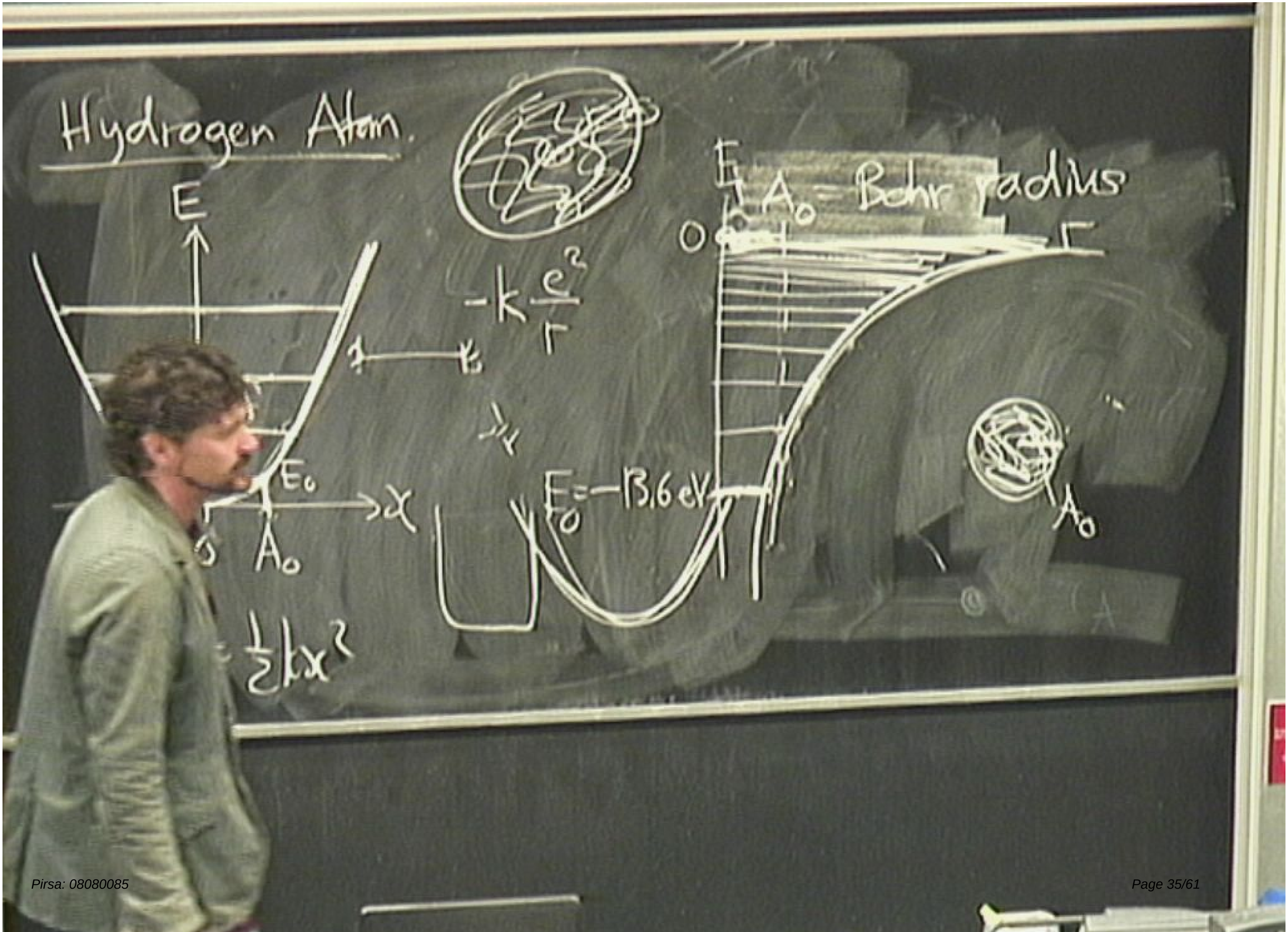

Hydrogen Atom.
E
ħω
x
$-k\frac{e^2}{r}$
E
0
r
-13.6 eV

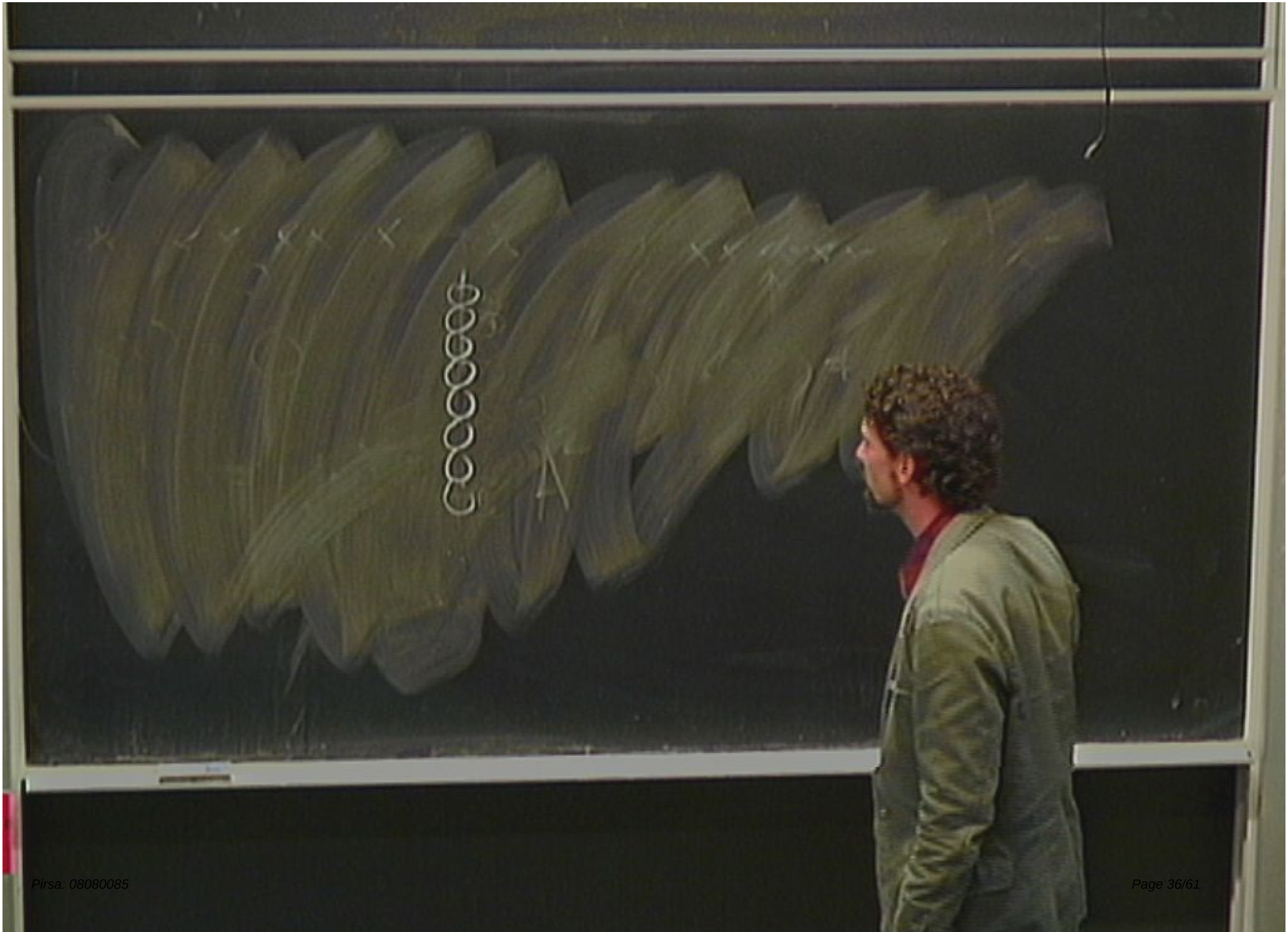

Hydrogen Atom
E
hf
E0
x
0
A0
½kx²
-k e²/r
E0 = -13.6 eV
E
A0
0
r

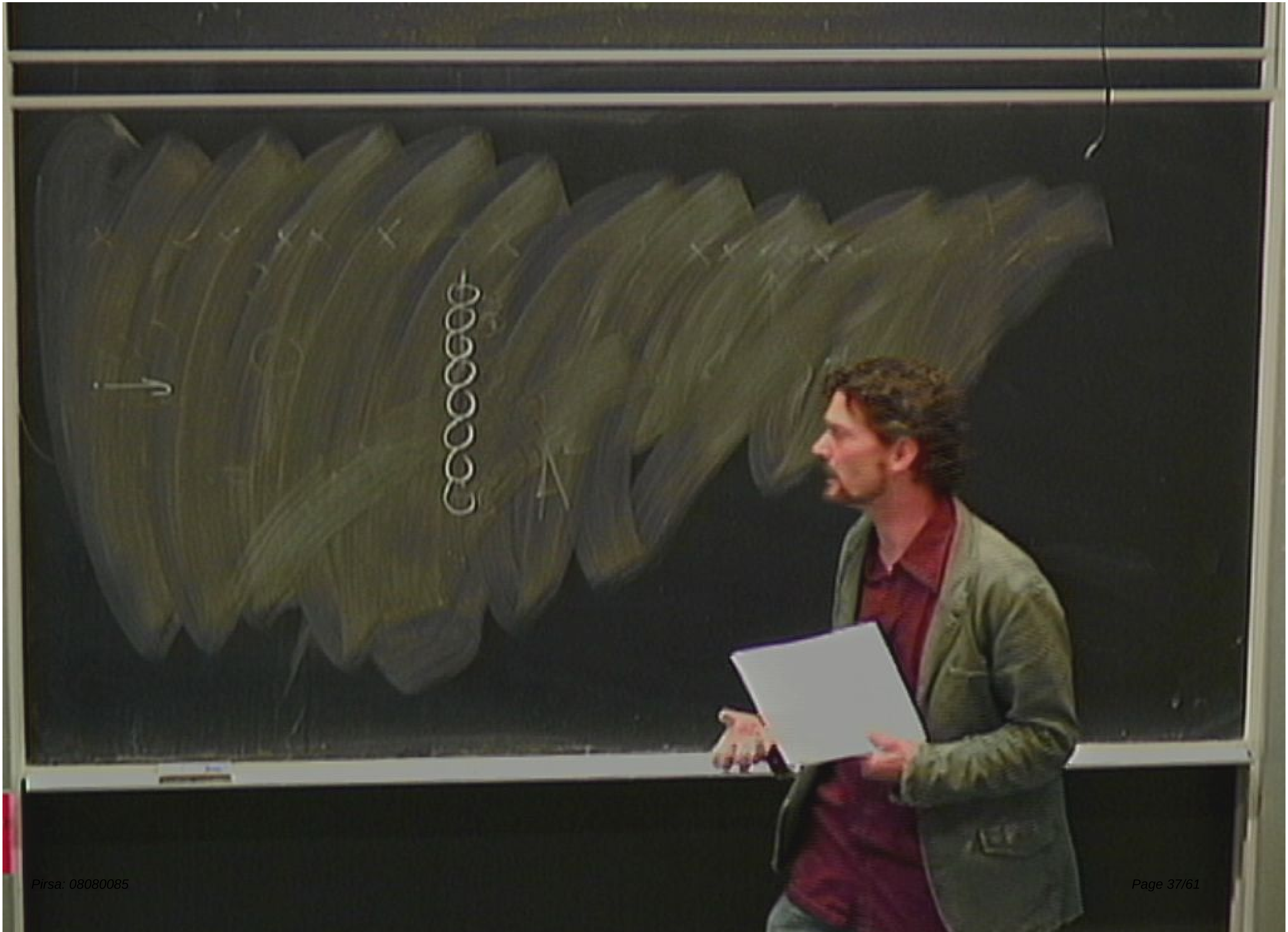

Hydrogen Atom
E
hf
x
0
PE =
$-k\frac{e^2}{r}$
E
A_0 = Bohr radius
0
r
eV
A_0

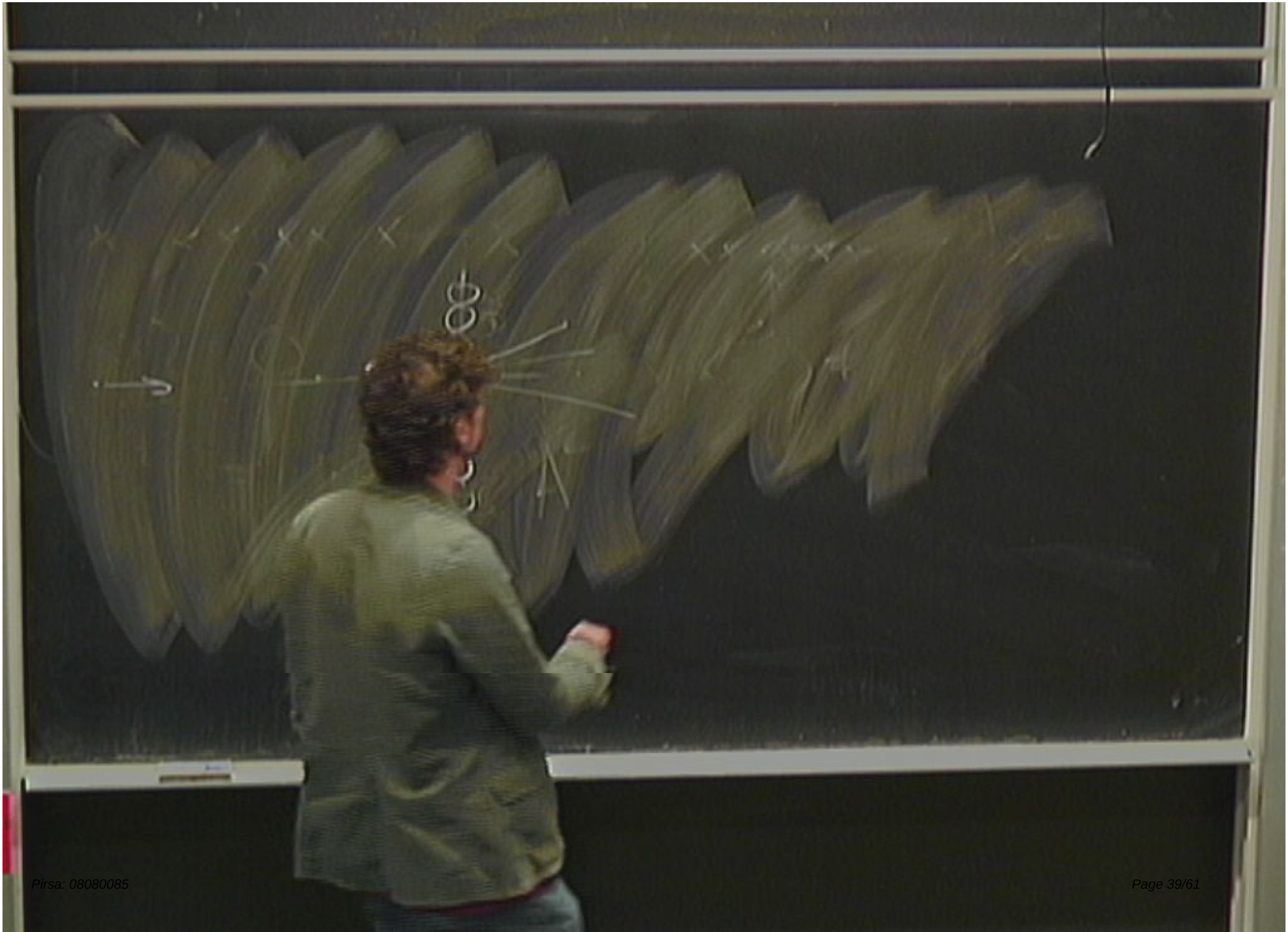

Hydrogen Atom
E
hf
E0
x
0
A0
PE = 1/2 kx^2
-k e^2/r
E0 = -13.6 eV
E
A0 = Bohr radius
0
r
A0

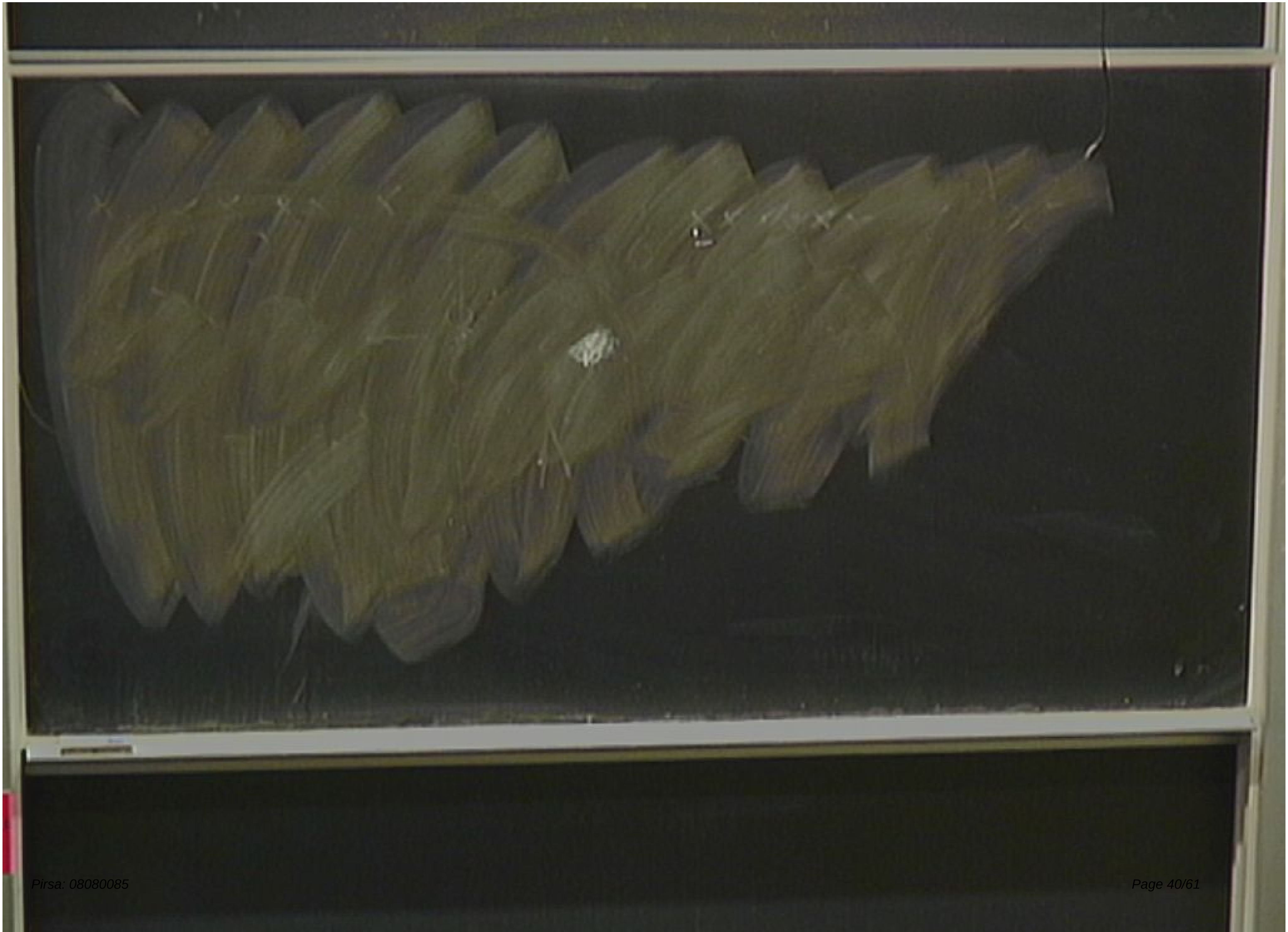

Hydrogen Atom
E
hf
E0
x
0
A0
PE = 1/2 kx^2
-k e^2/r
1/r
E0 = -13.6 eV
E
A0 = Bohr radius
0
r
A0

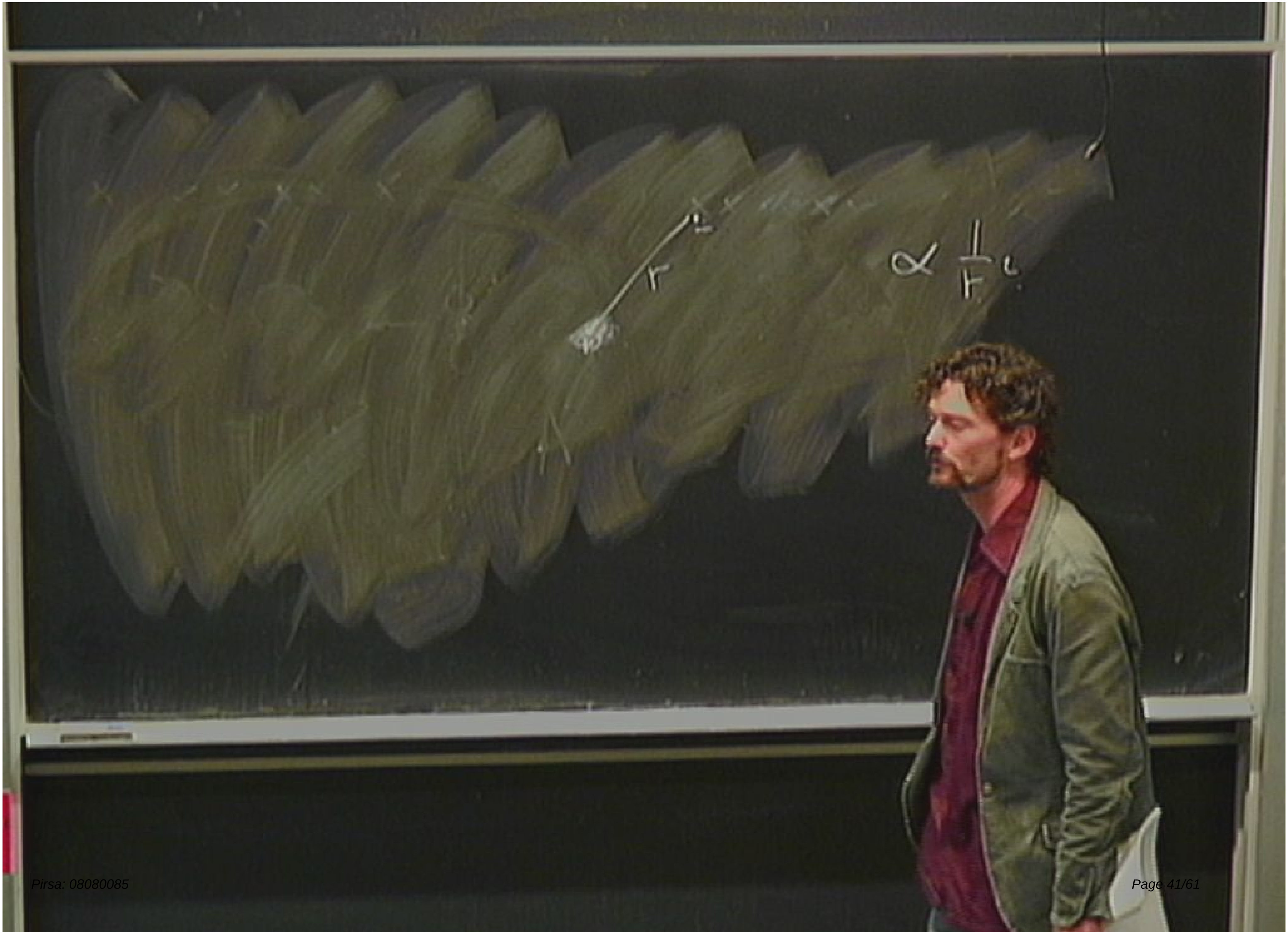

Hydrogen Atom
E
E0
x
A0
-k e^2/r
A0 = Bohr radius
E0 = -13.6 eV
r
1/2 kx^2

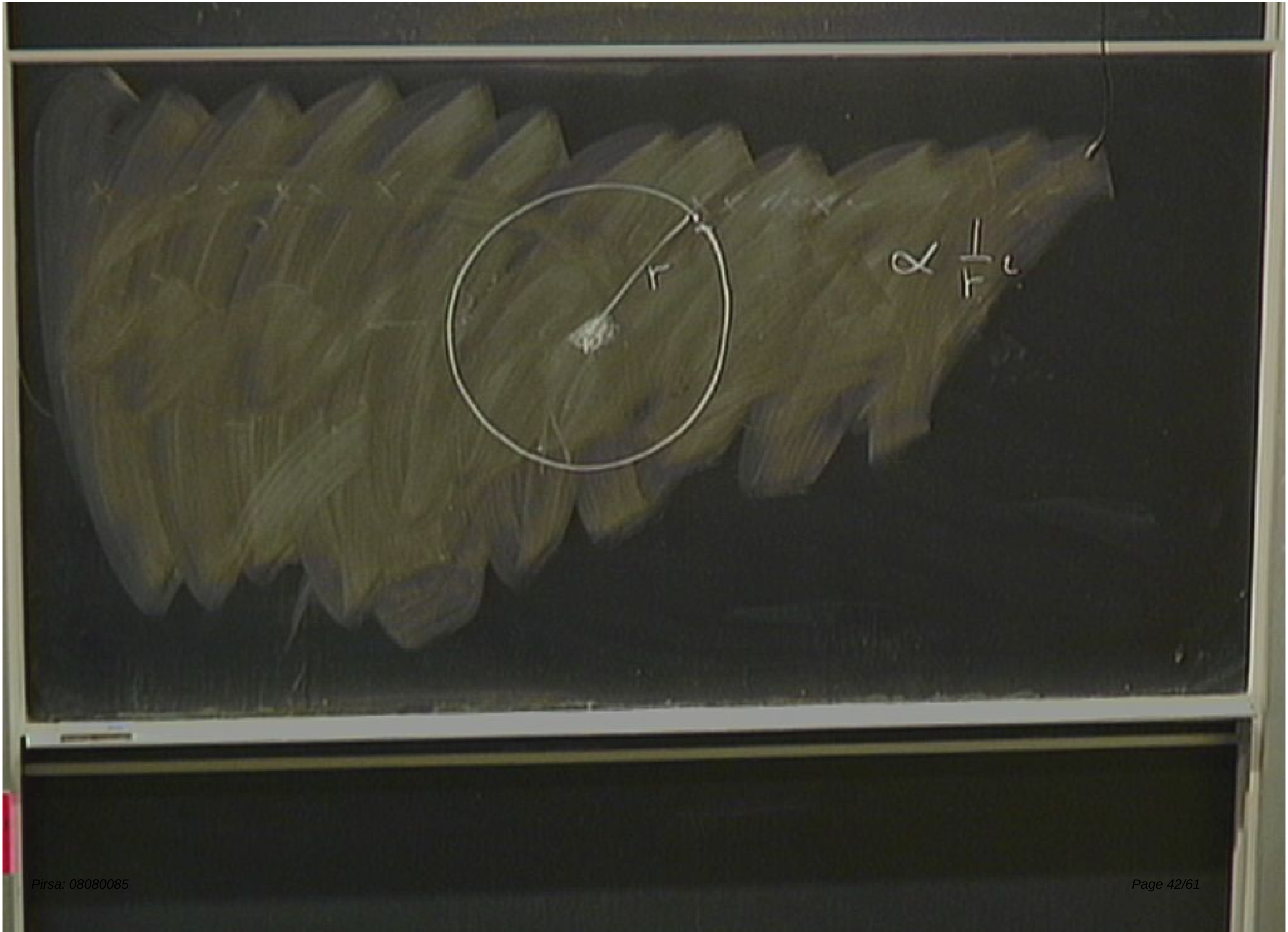


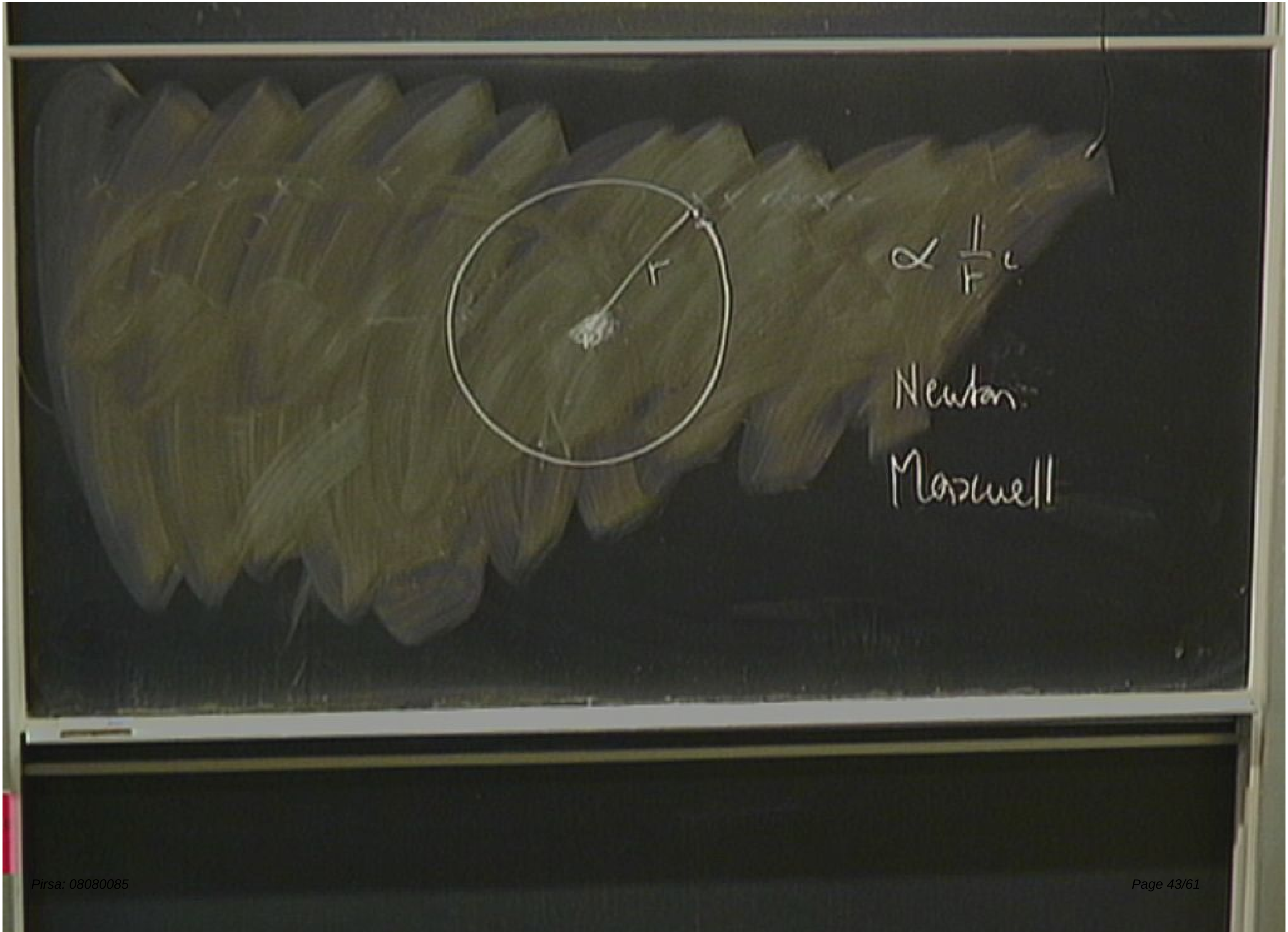


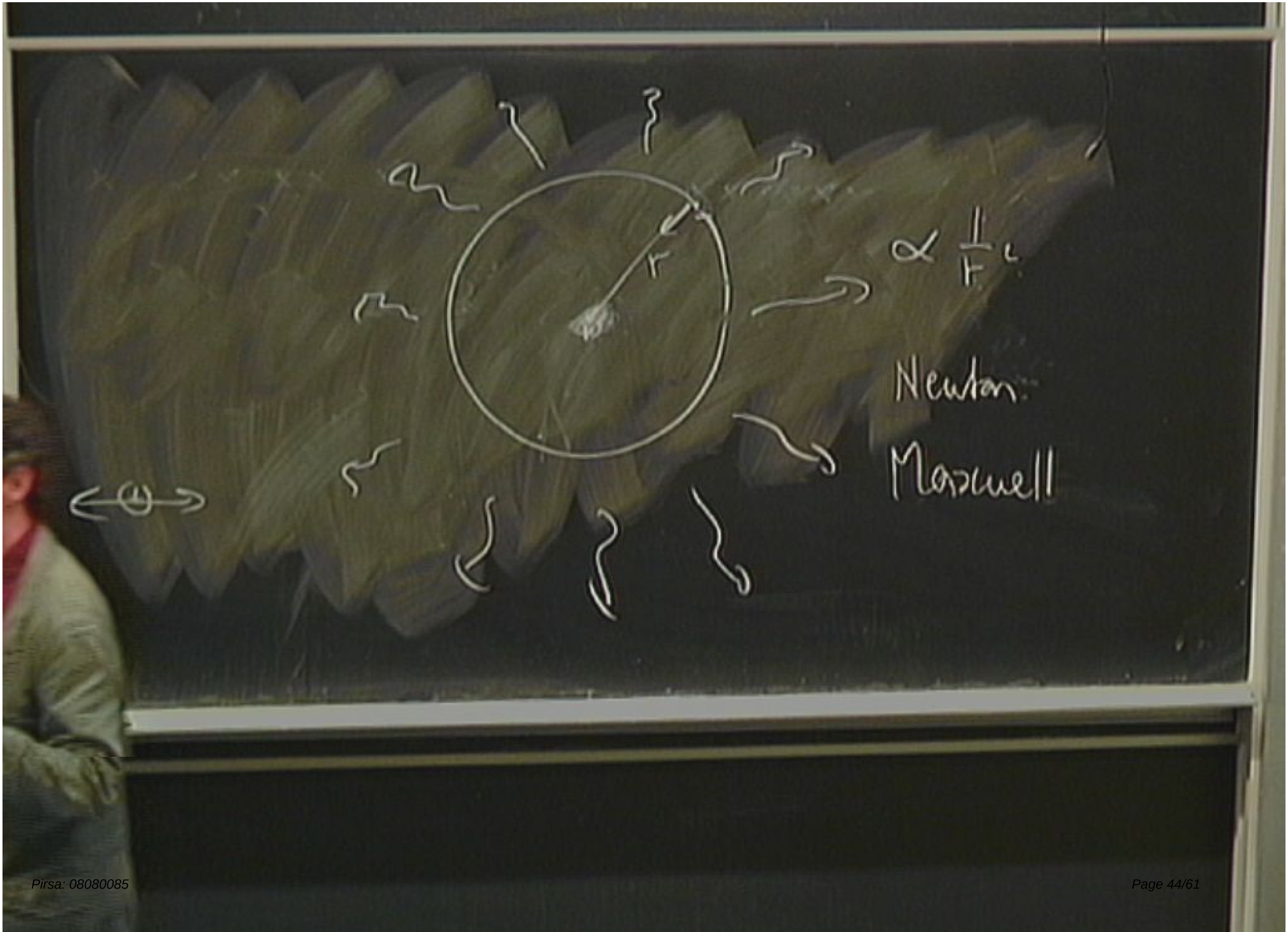


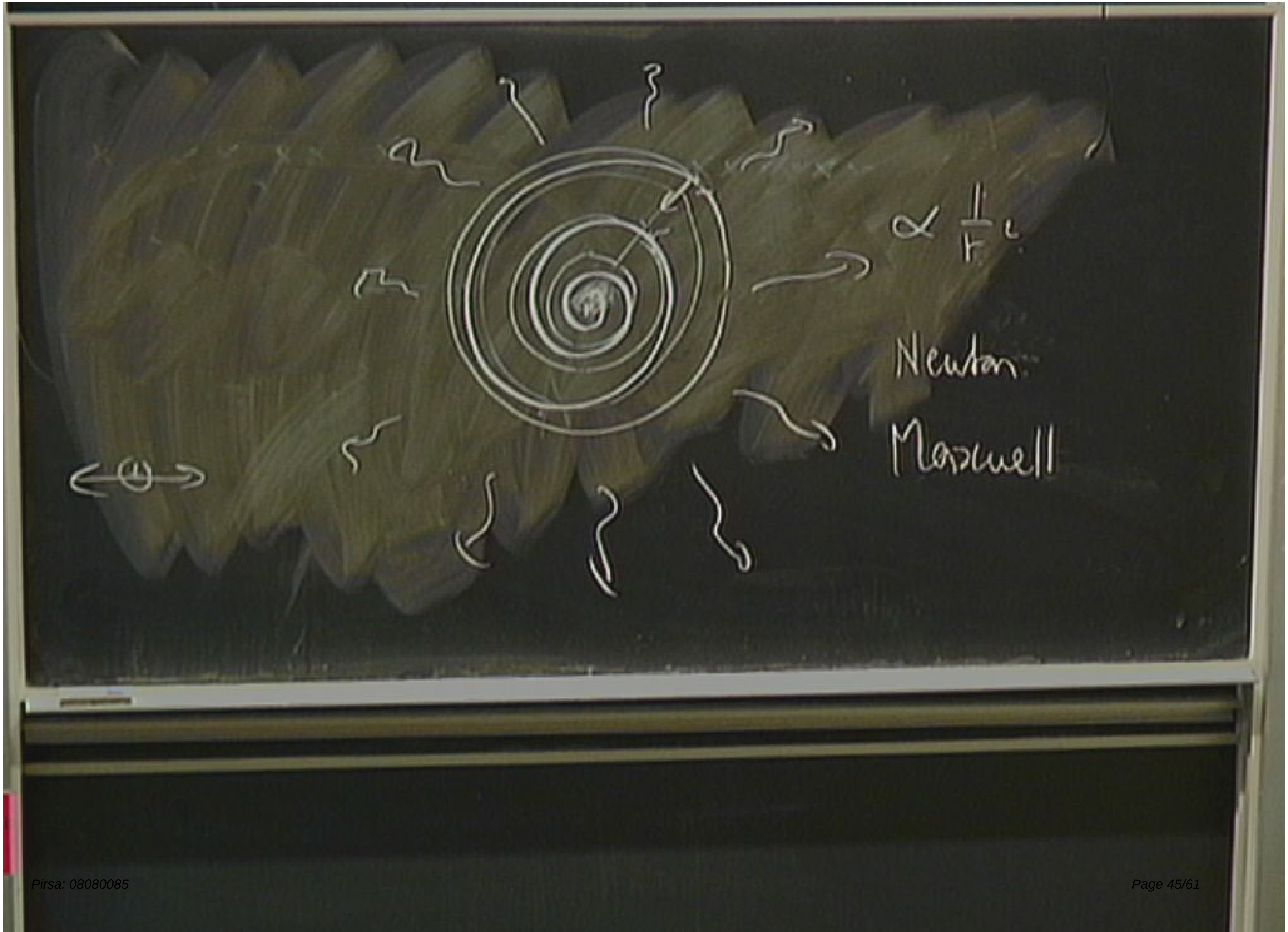




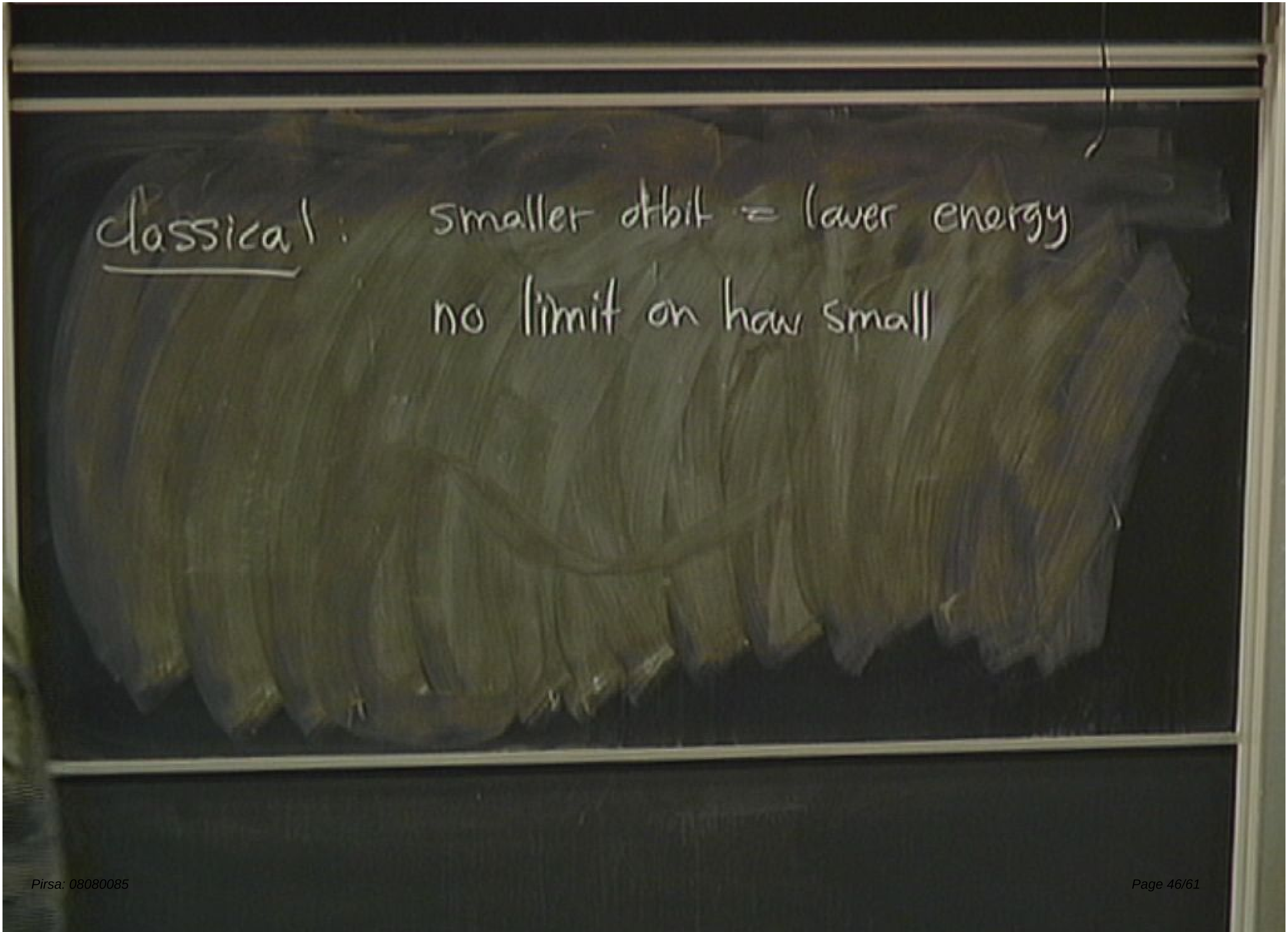

r
∝ 1/r

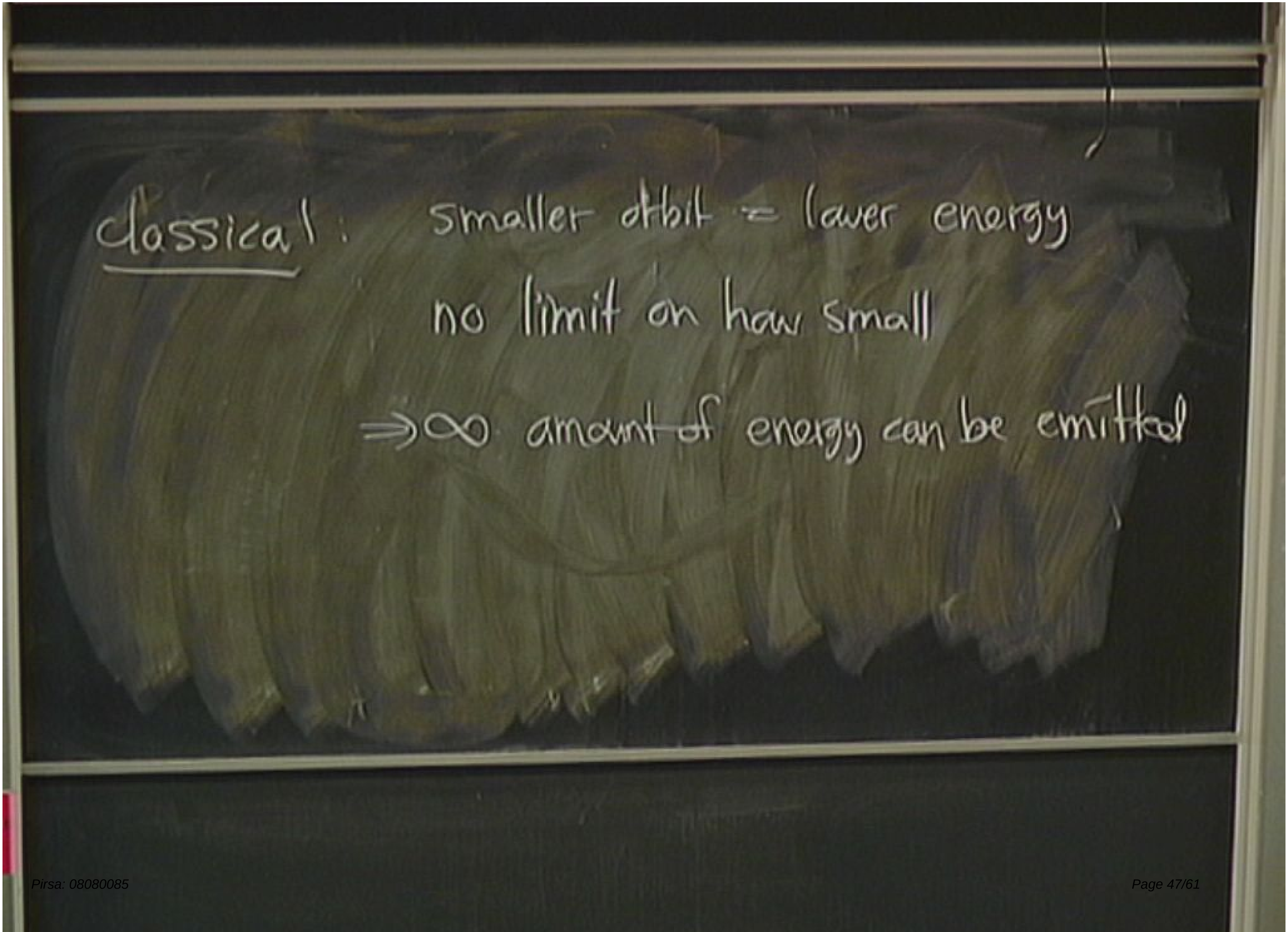

r
∝ 1/r²
Newton
Maxwell

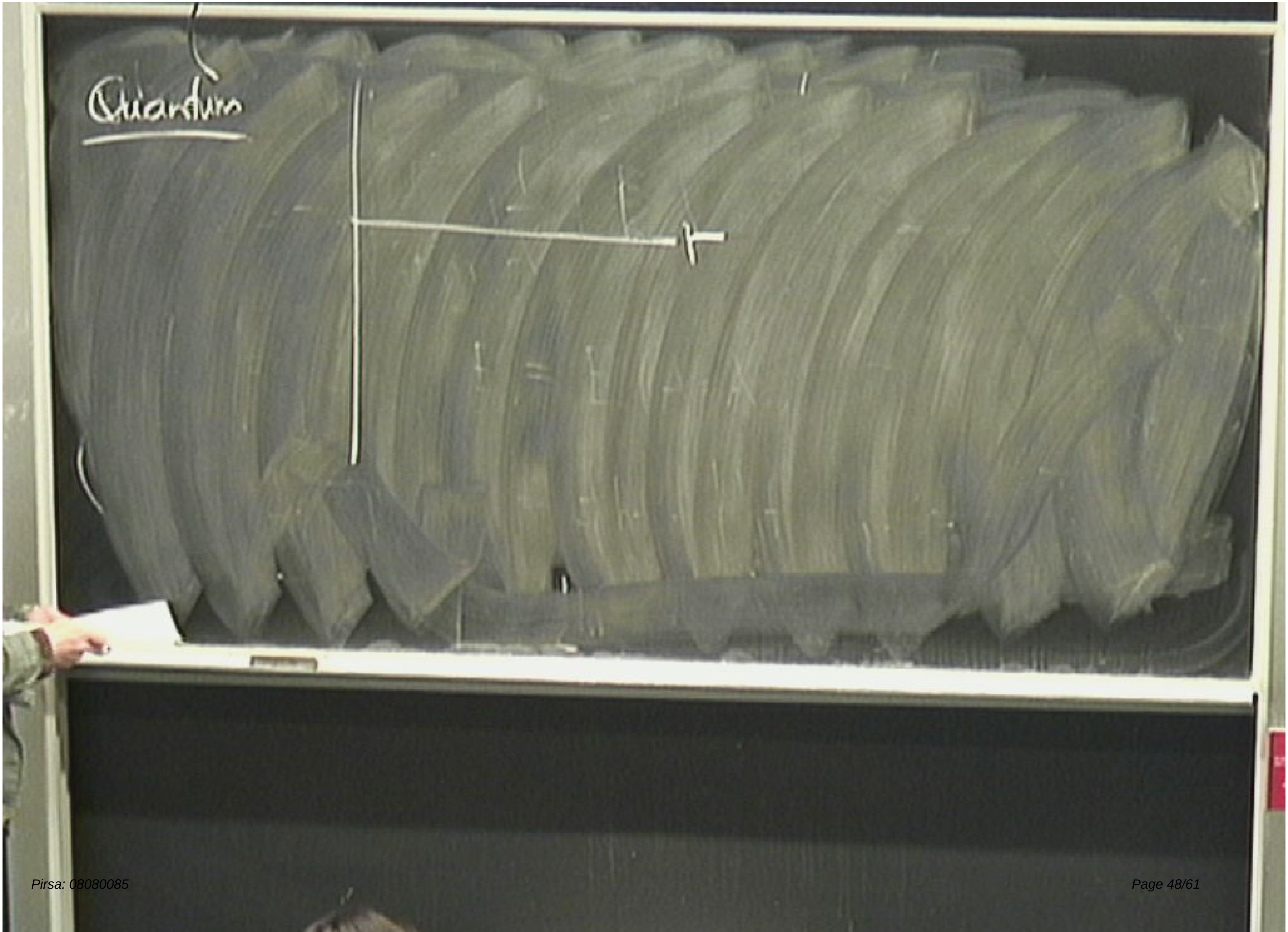

r
∝ 1/r²
Newton
Maxwell

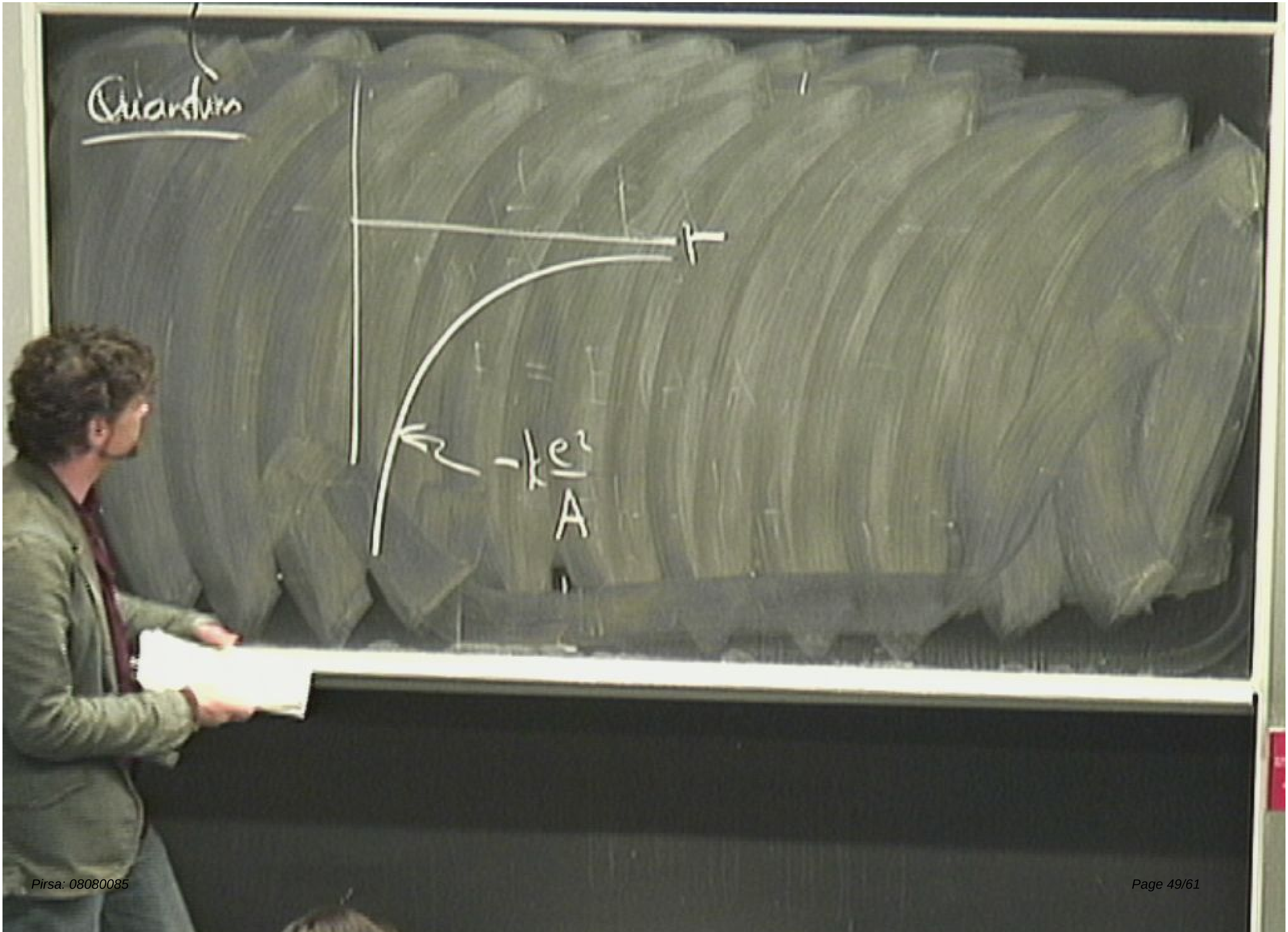


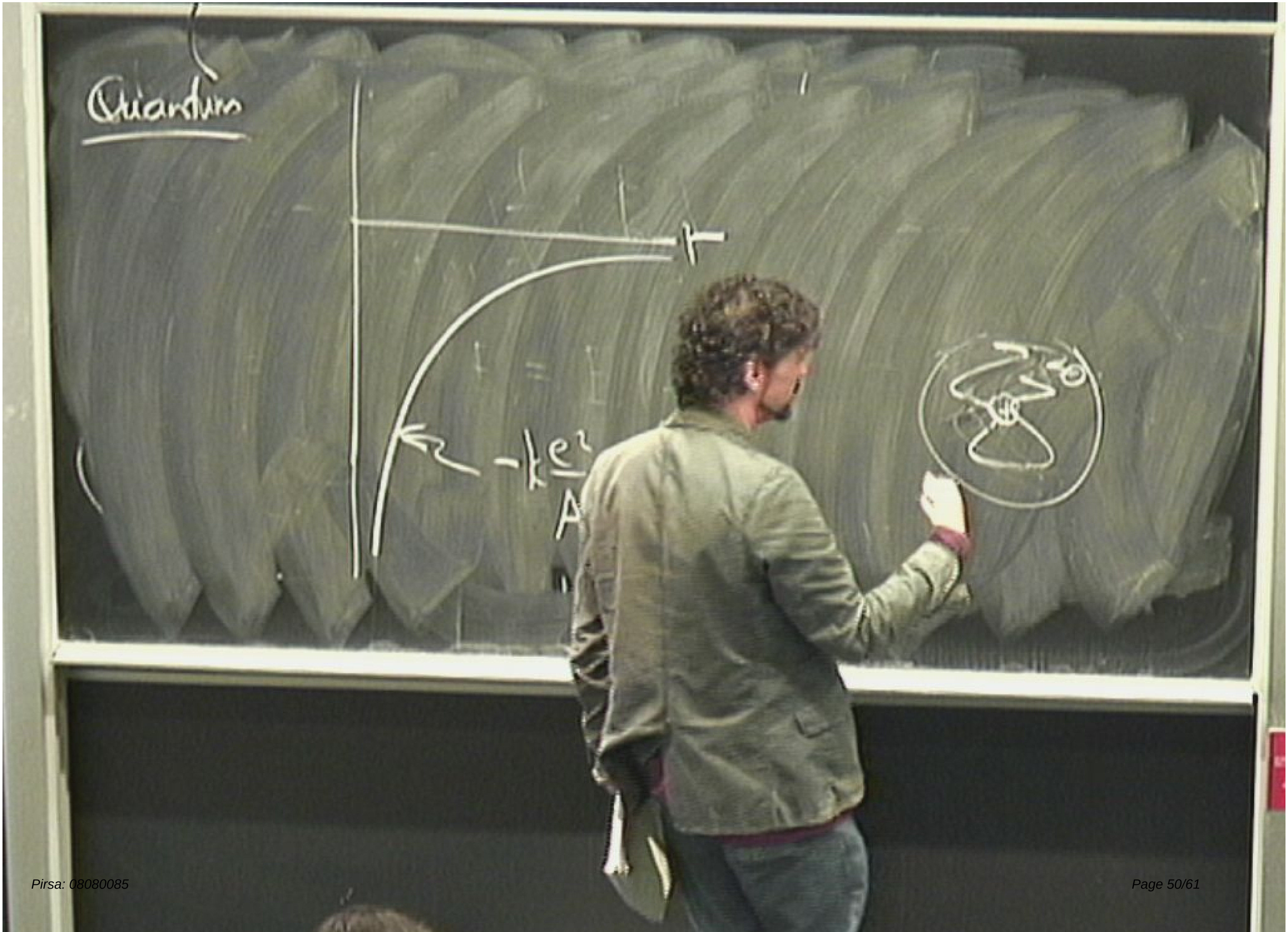
∝ 1/r²
Newton
Maxwell

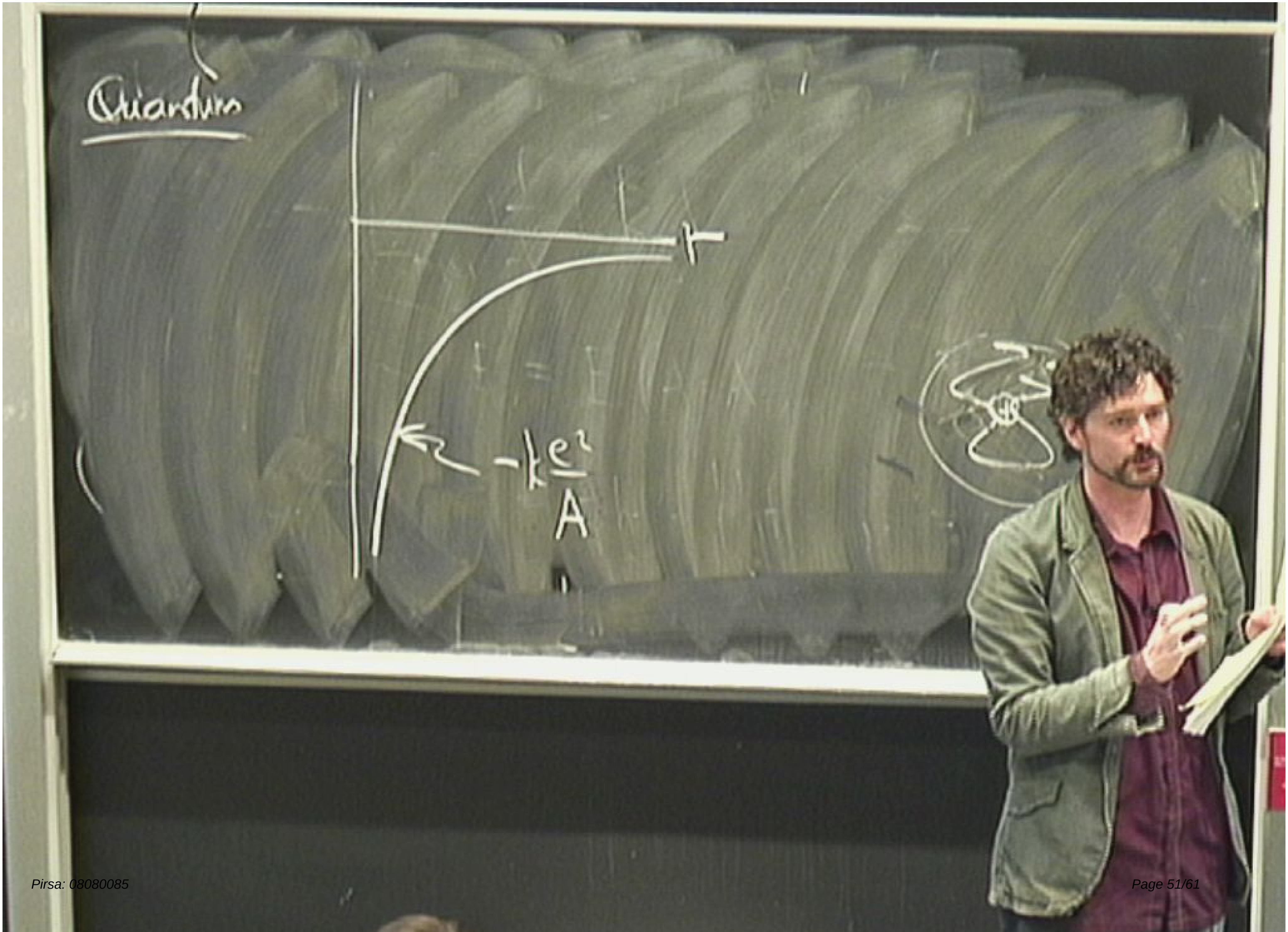

classical: smaller orbit = lower energy
no limit on how small

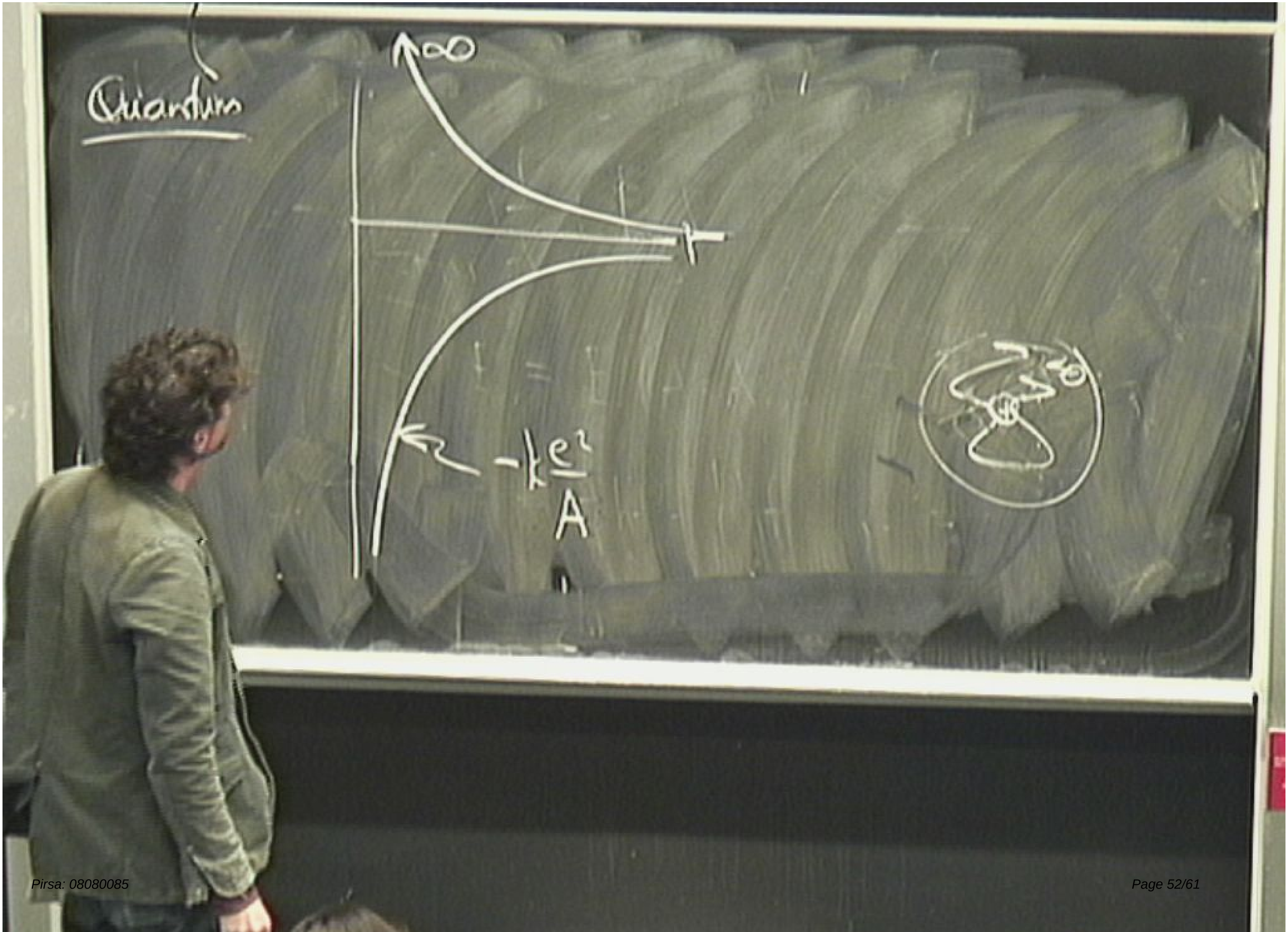

classical:
smaller orbit = lower energy
no limit on how small
⇒ ∞ amount of energy can be emitted

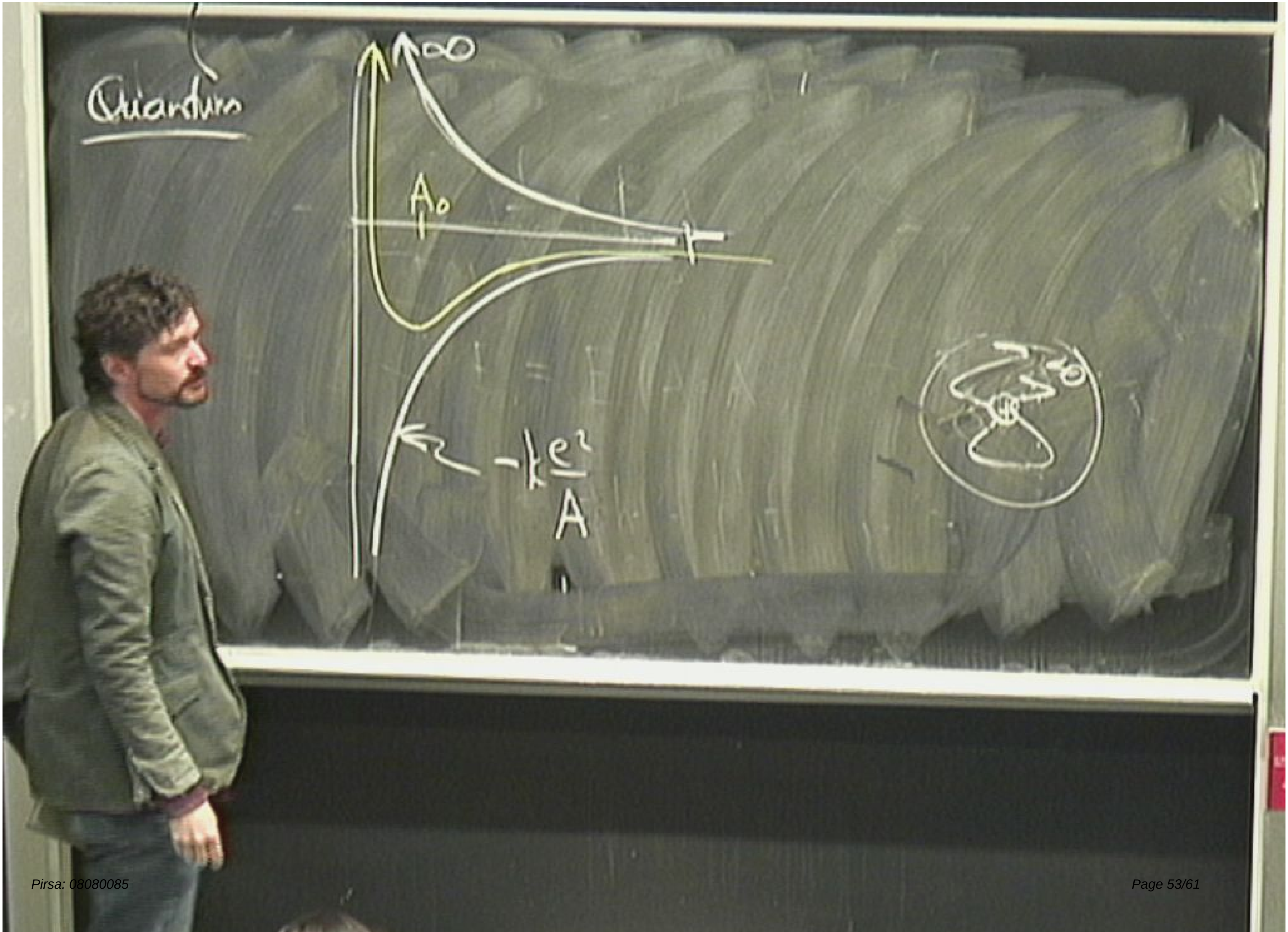

Quantum

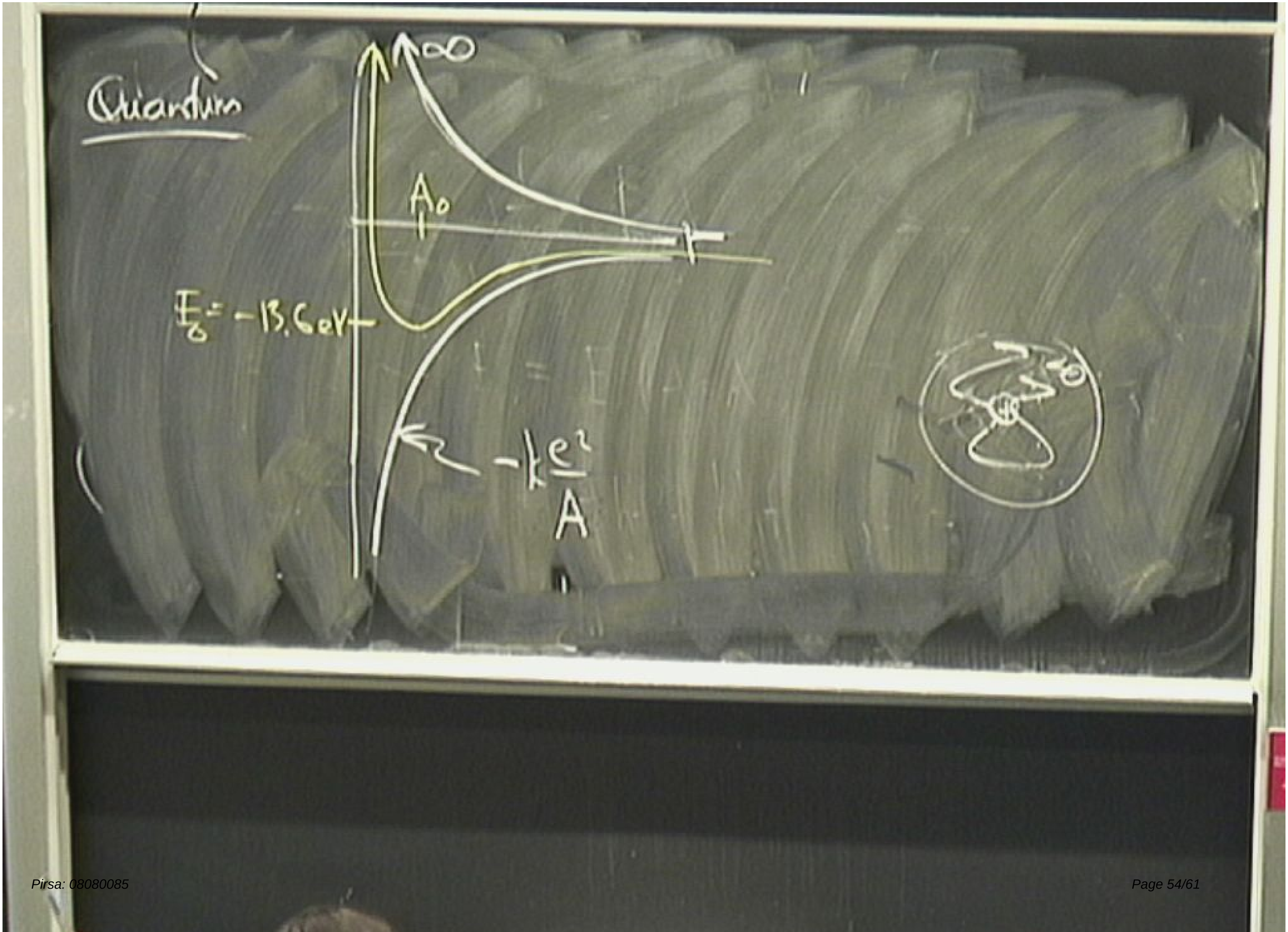

Quantum
r
-ke²/A

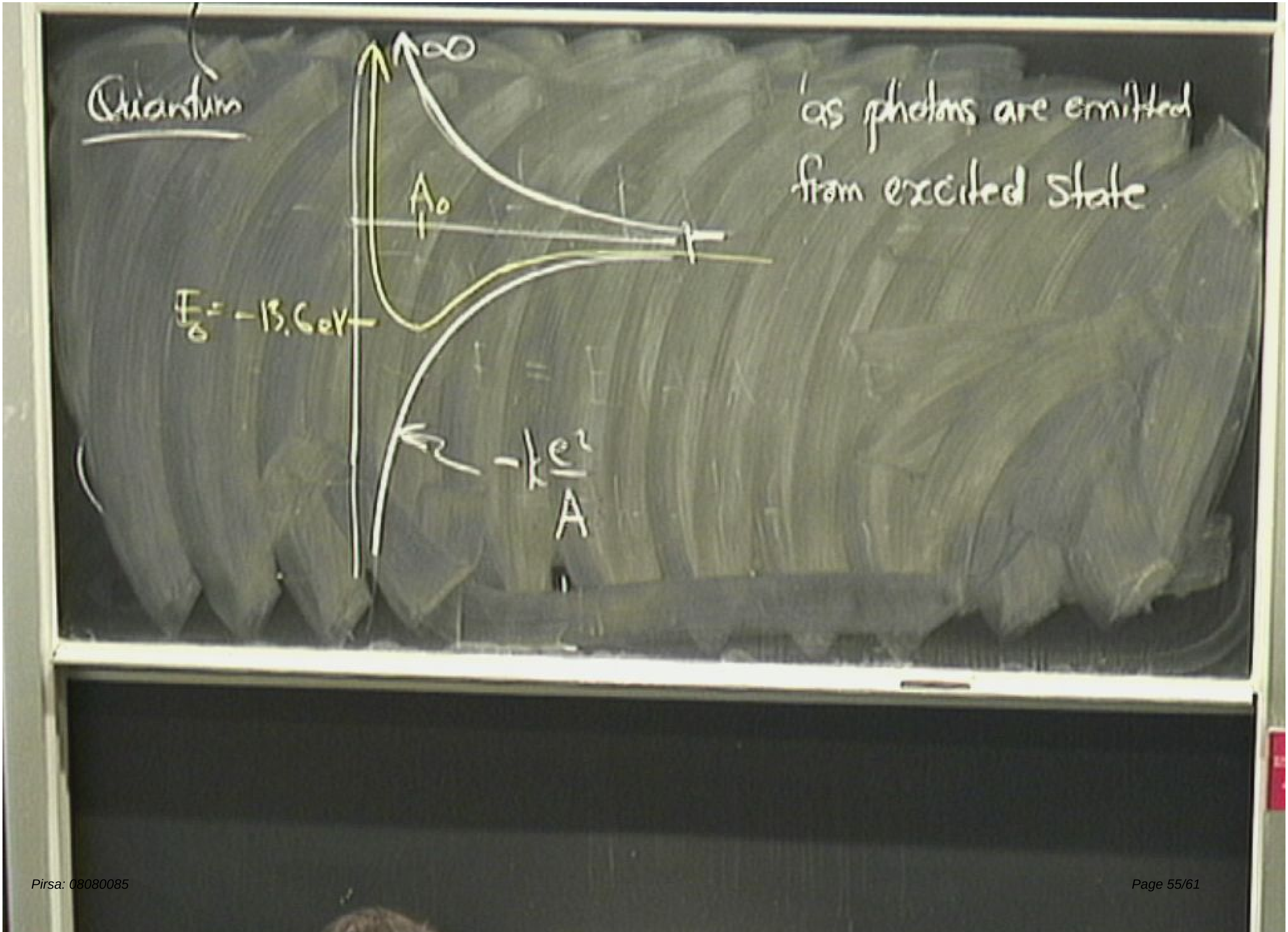

Quantum

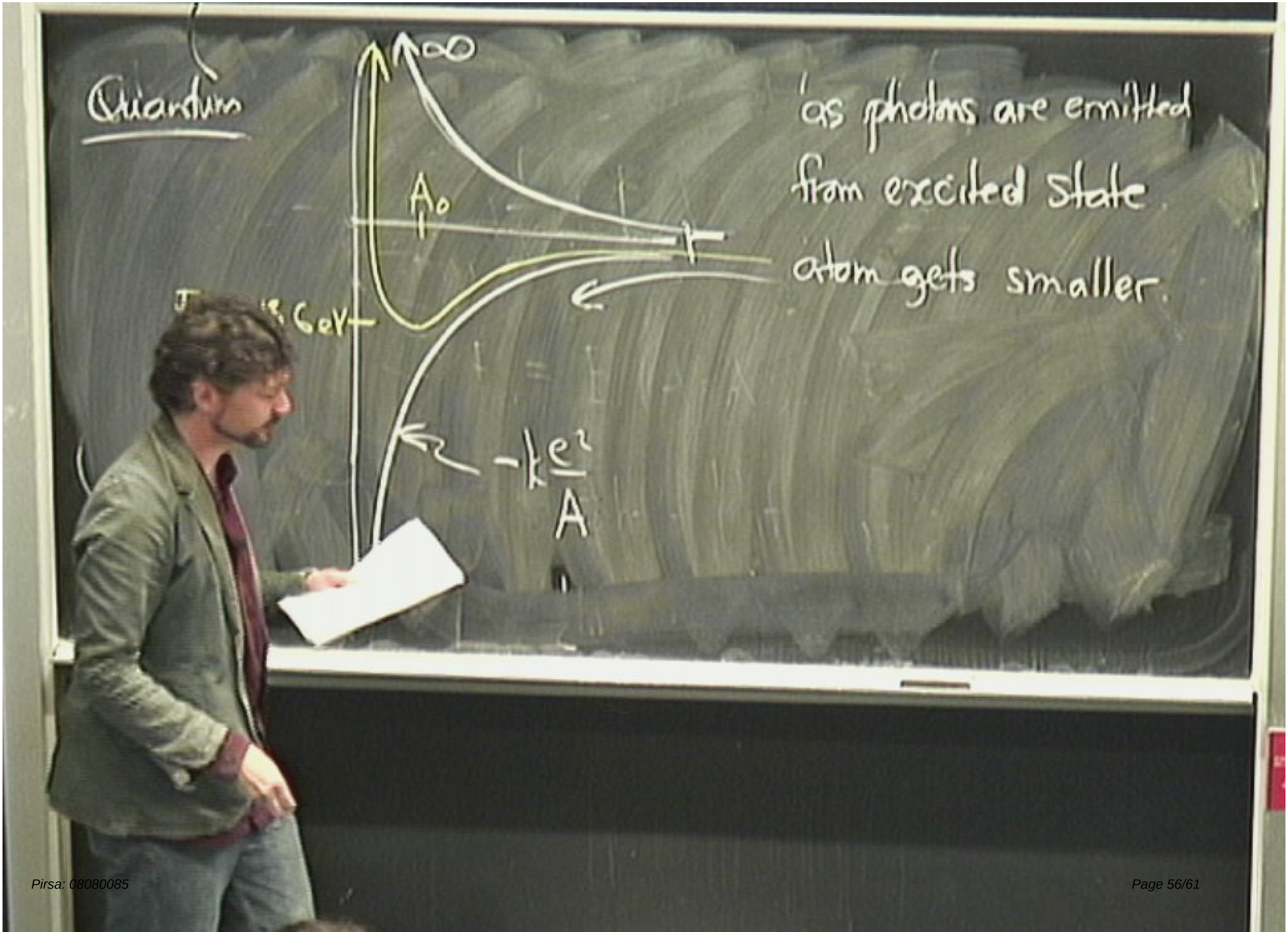

Quantum
$-\frac{ke^2}{r}$
A

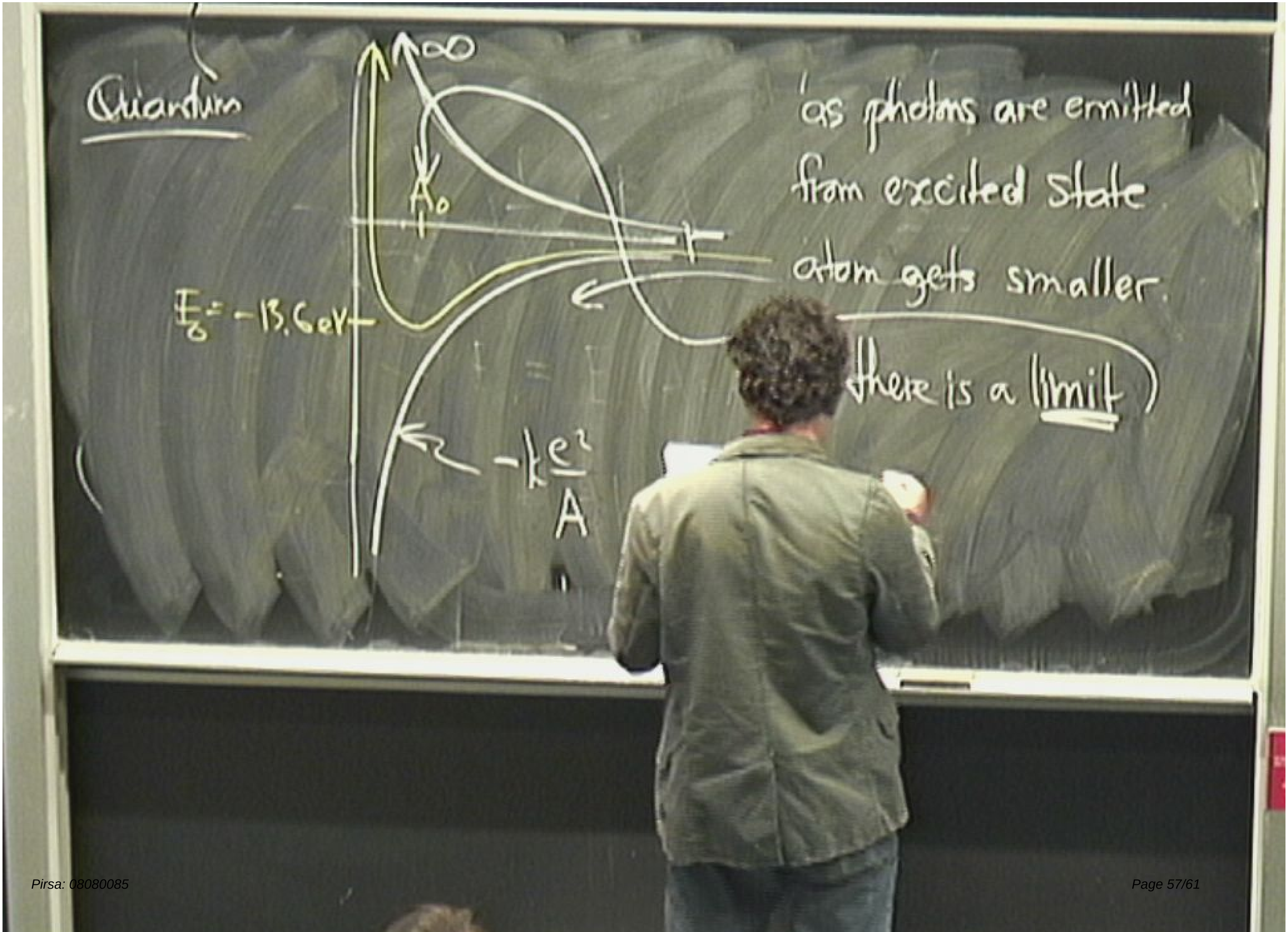

Quantum
∞
-ke²
A

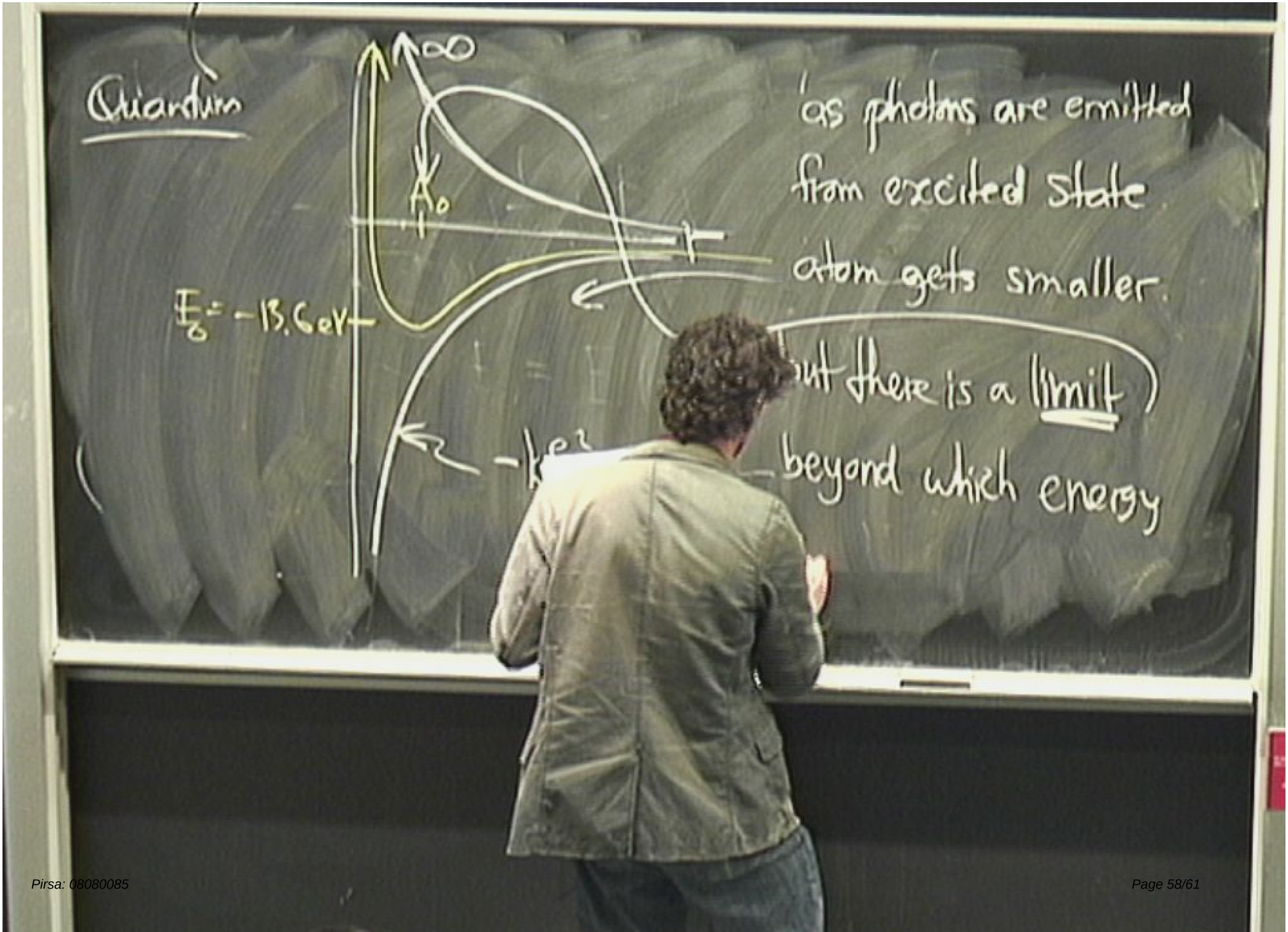

Quantum
∞
A₀
−ke²
A

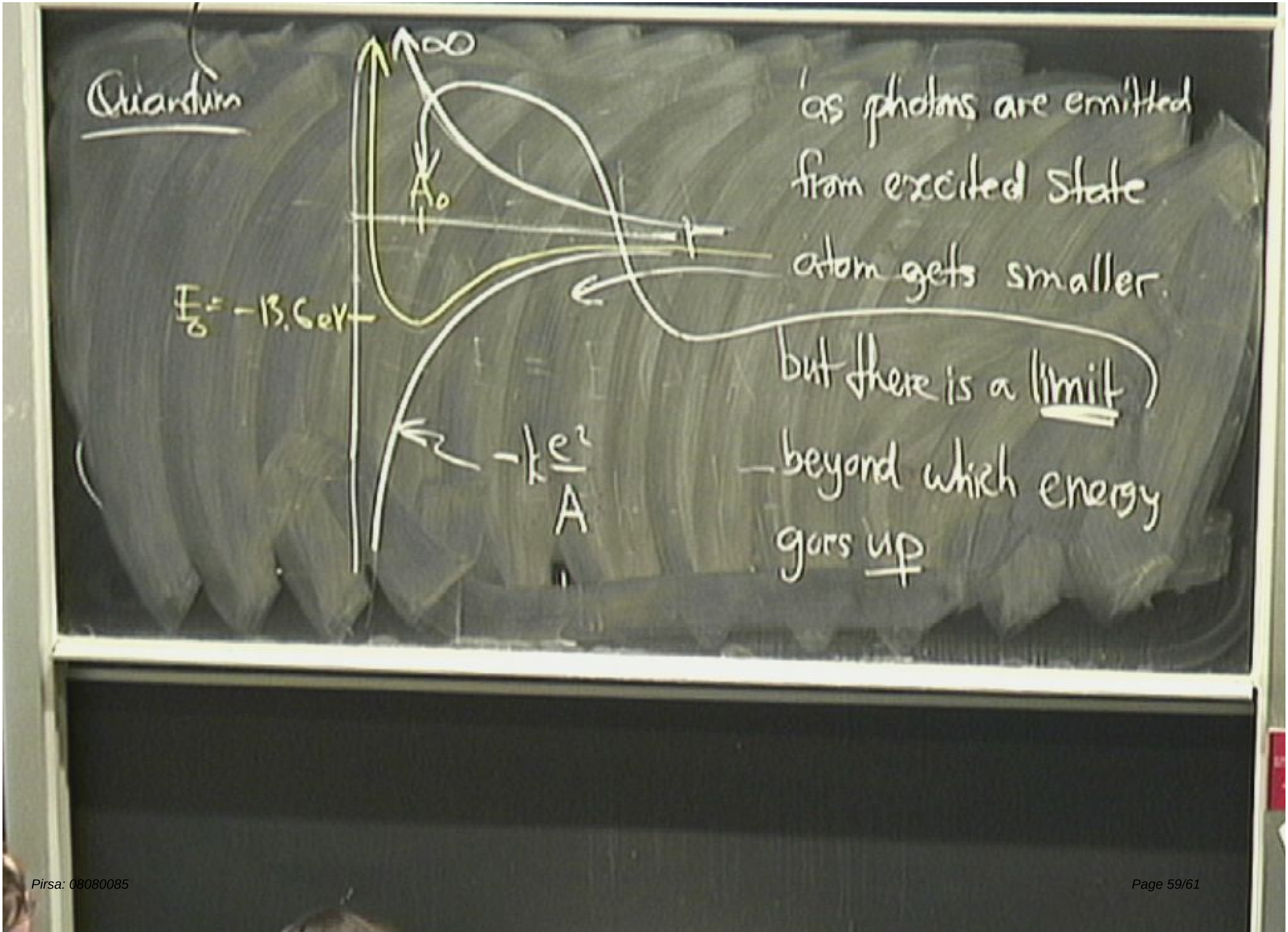

Quantum
A_0
$E_0 = -13.6\,eV$
$-k\frac{e^2}{A}$
∞

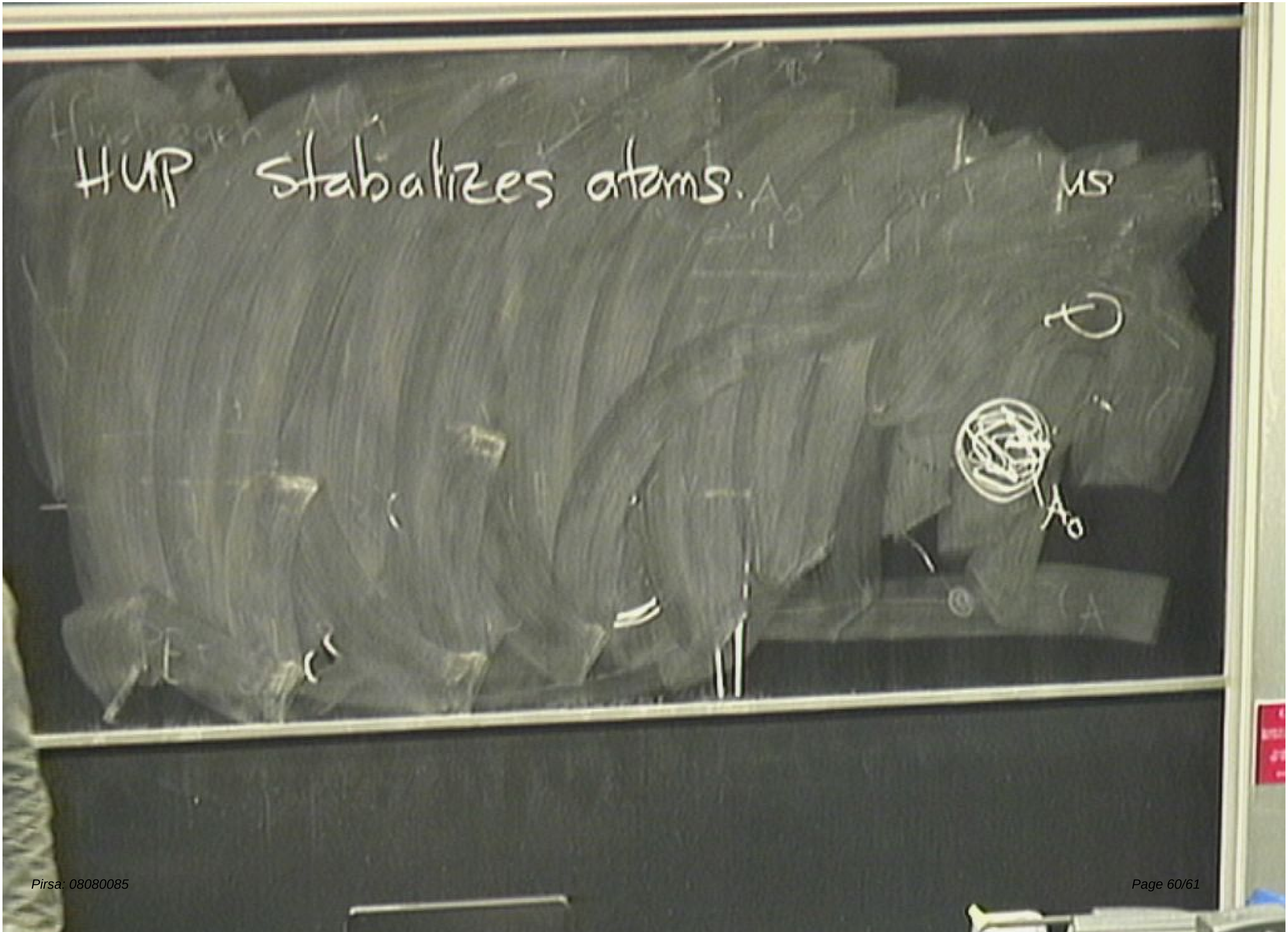

Quantum
as photons are emitted
from excited State
∞
A₀
E₀ = −13.6 eV
−ke²/A

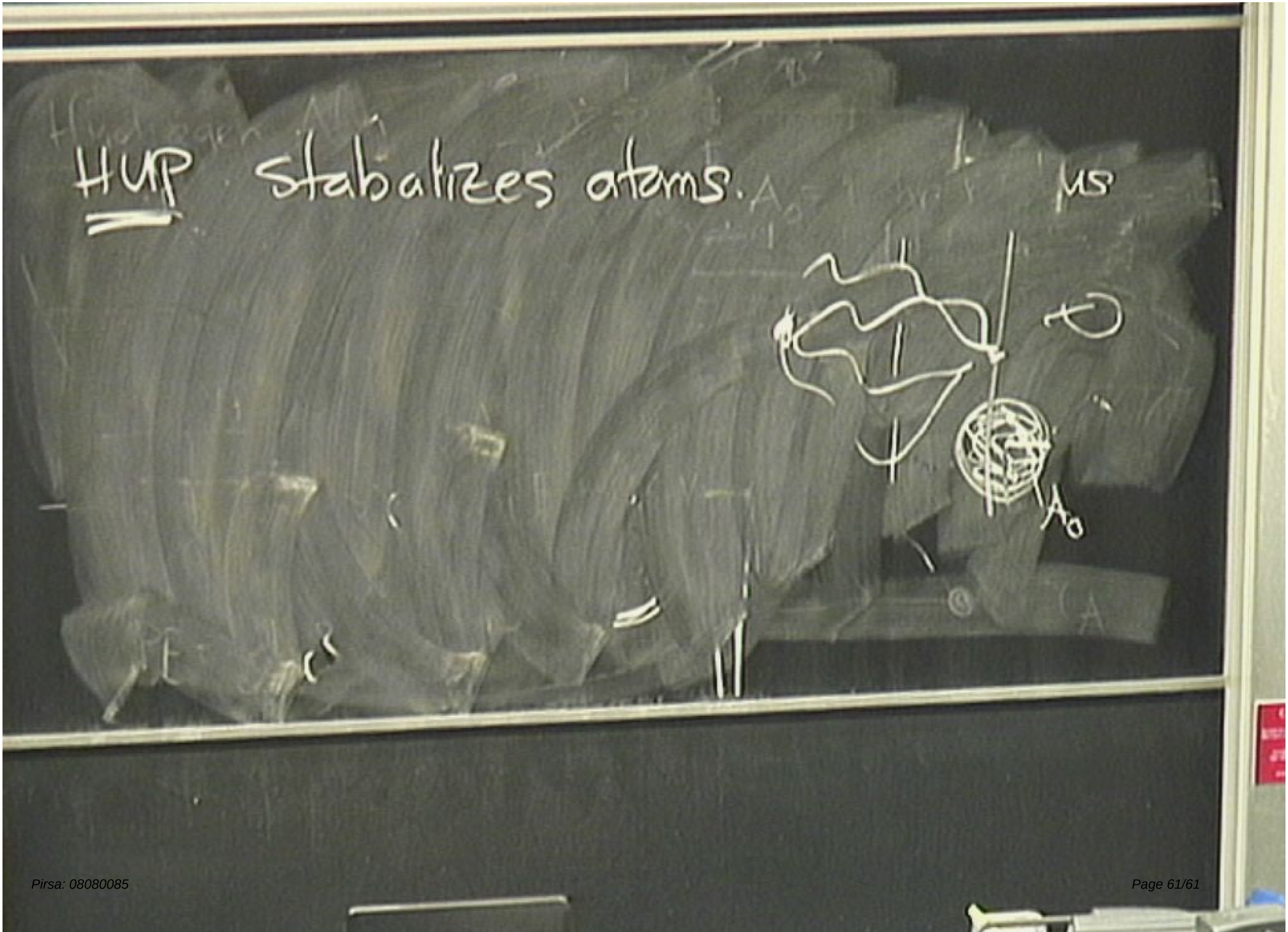

Quantum
∞
A₀
r
as photons are emitted
from excited state
atom gets smaller.
GeV
$-k\frac{e^2}{A}$


Quantum
∞
A_0
r
$E_0 = -13.6$ eV
$-k\frac{e^2}{A}$
as photons are emitted
from excited state
atom gets smaller
there is a limit


Quantum
∞
r
E0 = -13.6eV
as photons are emitted
from excited state
atom gets smaller
ut there is a limit
beyond which energy


Quantum
∞
A_0
r
$E_0 = -13.6$ eV
$-k\frac{e^2}{A}$
as photons are emitted
from excited state
atom gets smaller.
but there is a limit
beyond which energy
goes up


HUP stabalizes atoms.


HUP stabalizes atoms.