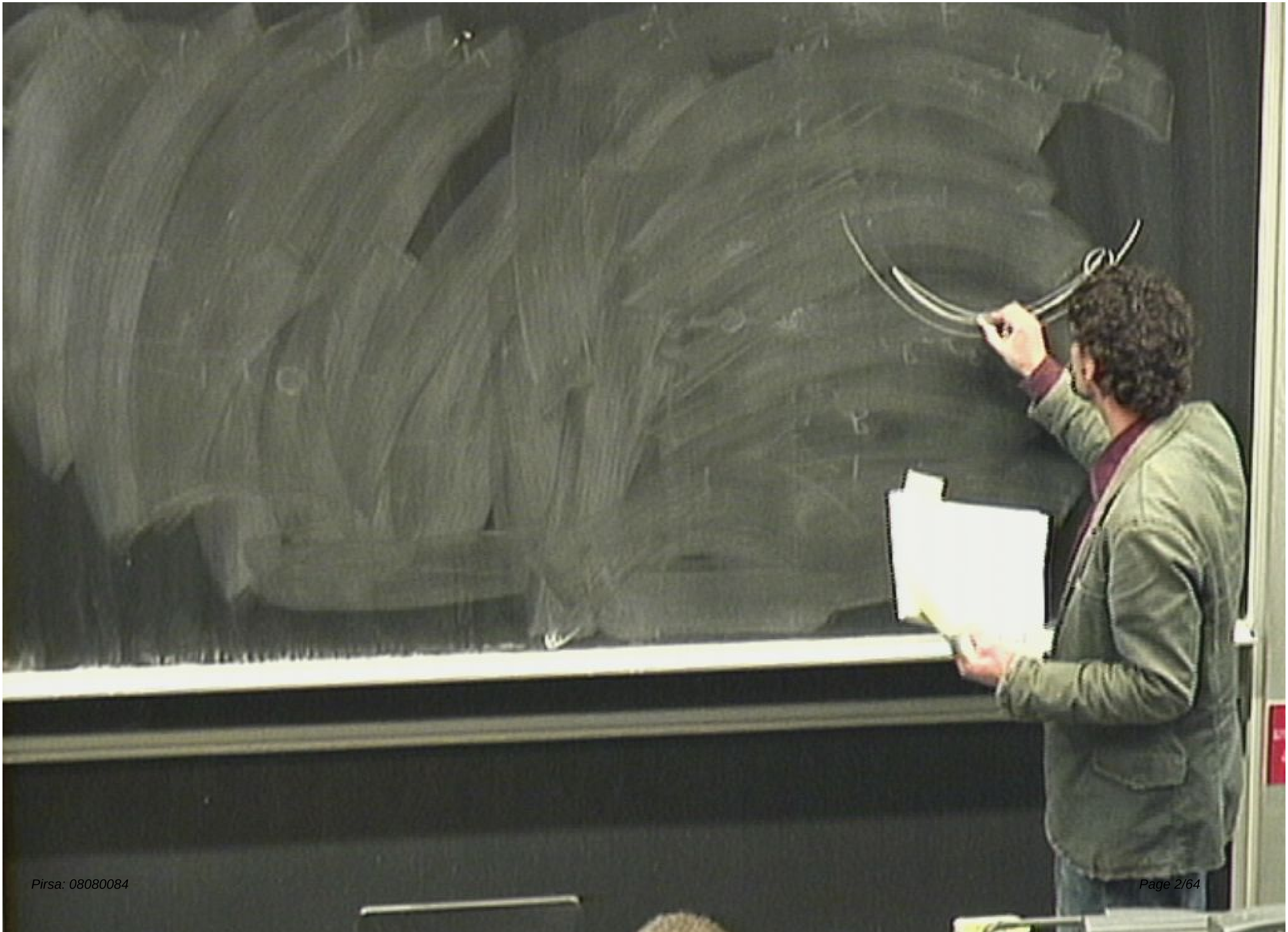
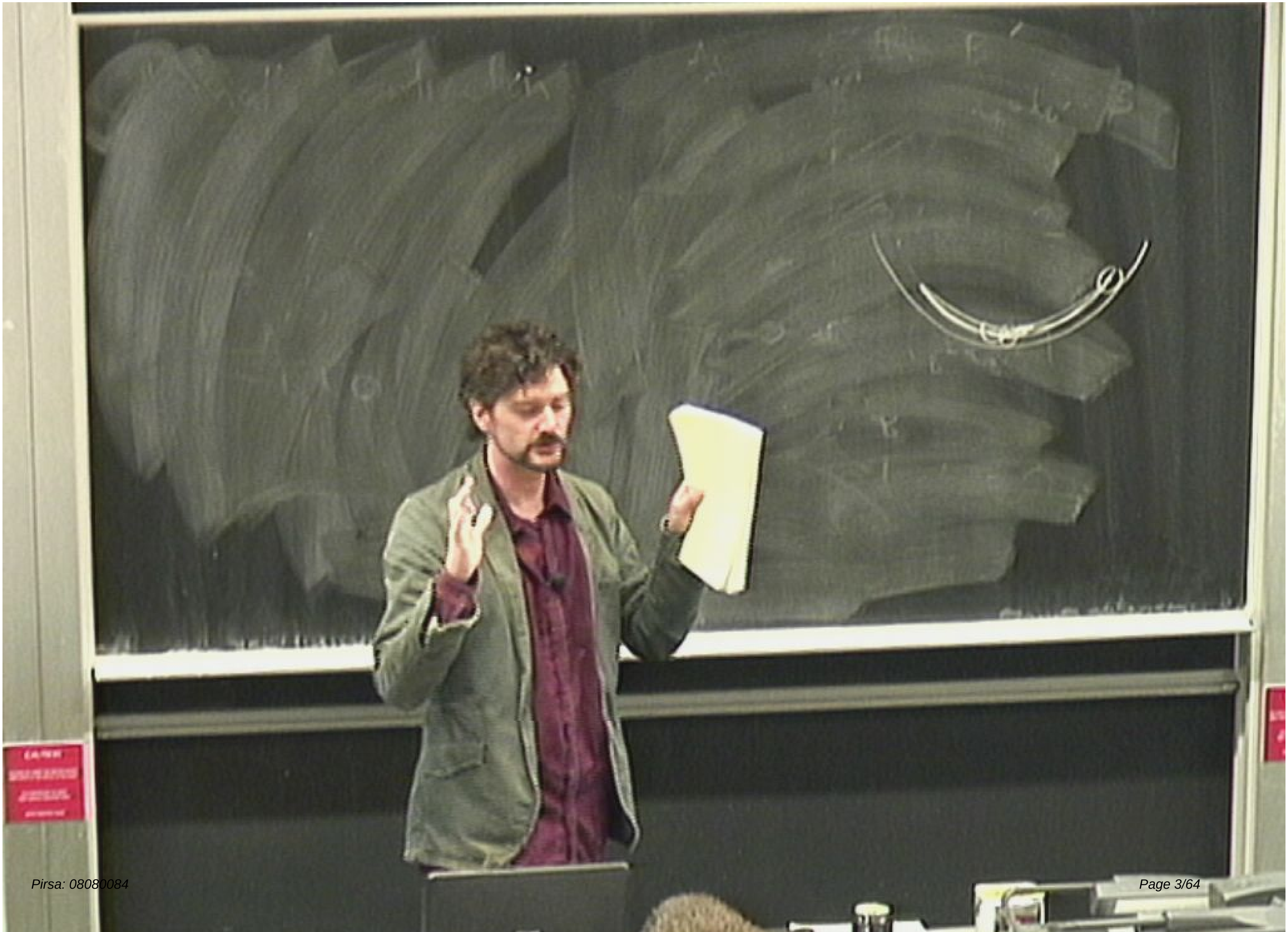
Title: Quantum Mechanics 9 - Zero Point Energy

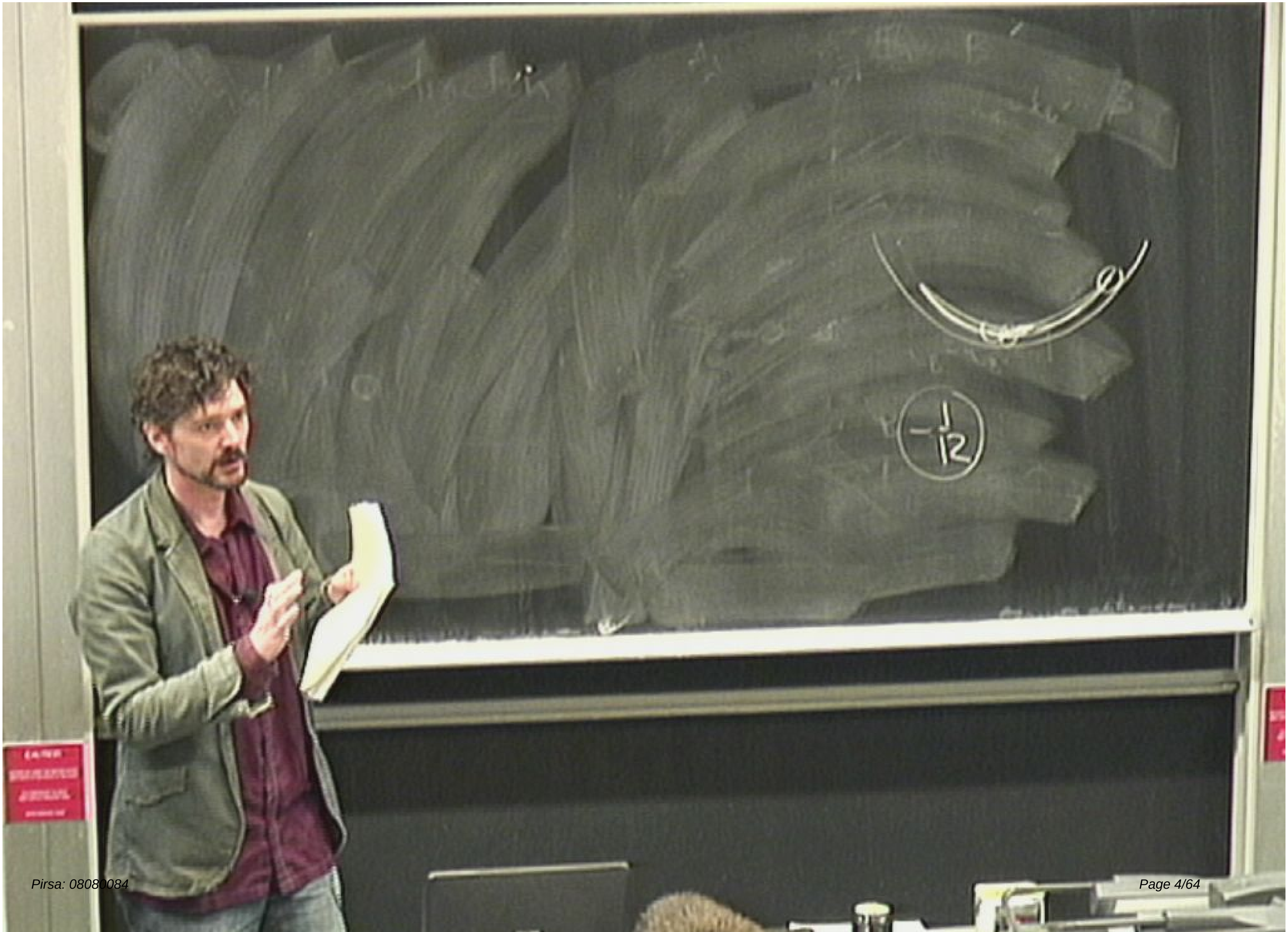
Date: Aug 13, 2008 10:30 AM

URL: http://pirsa.org/08080084

Abstract: Understanding the zero point energy of the quantum harmonic oscillator as a consequence of the Heisenberg Uncertainty Principle.
Learning Outcomes:
â€¢ Understanding why the minimum energy of a ball in a bowl must be greater than zero based on the Heisenberg Uncertainty Principle.
â€¢ How the Heisenberg Uncertainty Principle adds a purely quantum mechanical kinetic energy to the ball, in addition to its classical potential energy.
â€¢ Understanding graphically how the total energy â€“ the sum of the classical potential energy and the new quantum kinetic energy â€“ has a minimum that is greater than zero: the zero point energy.



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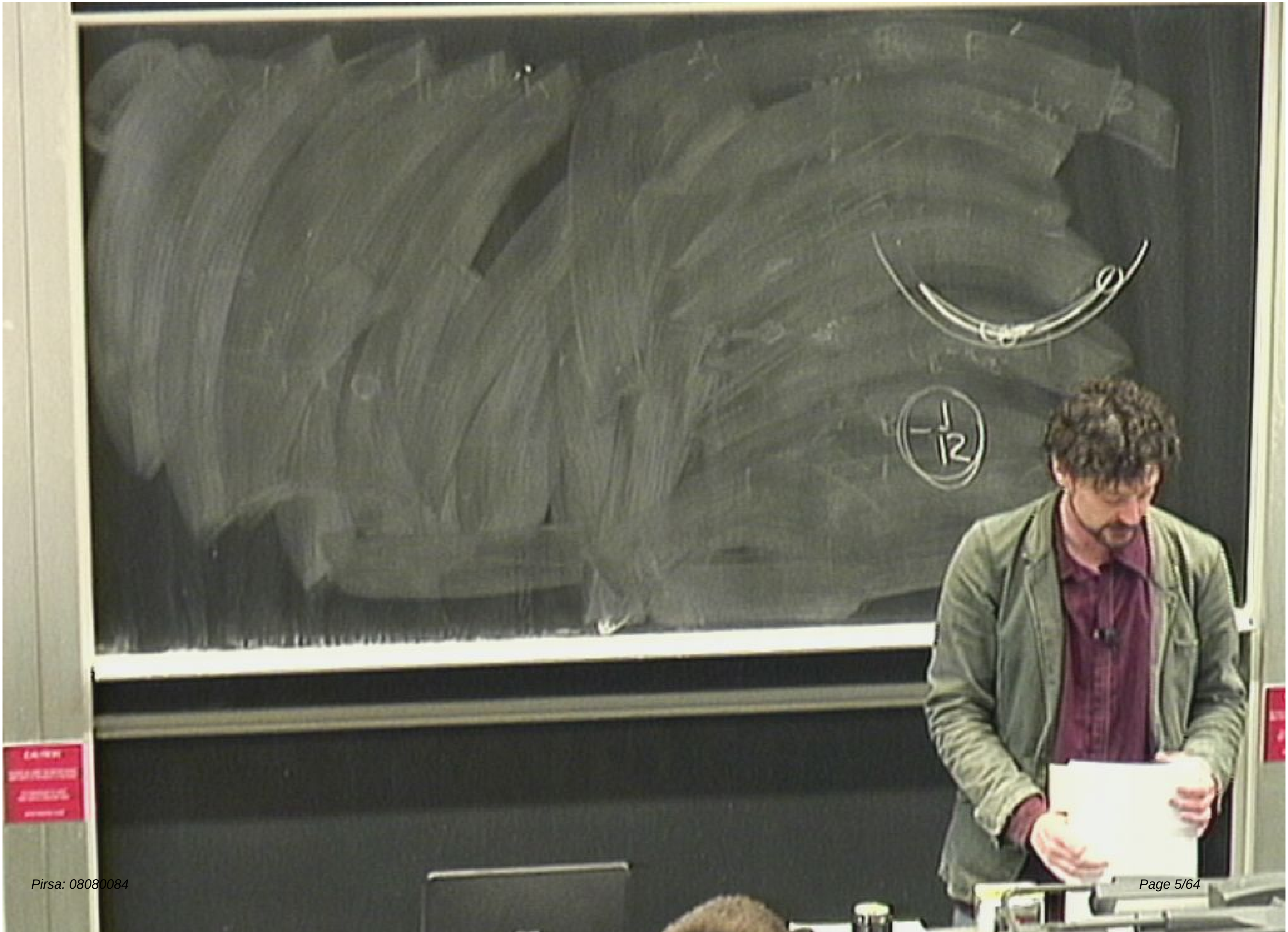
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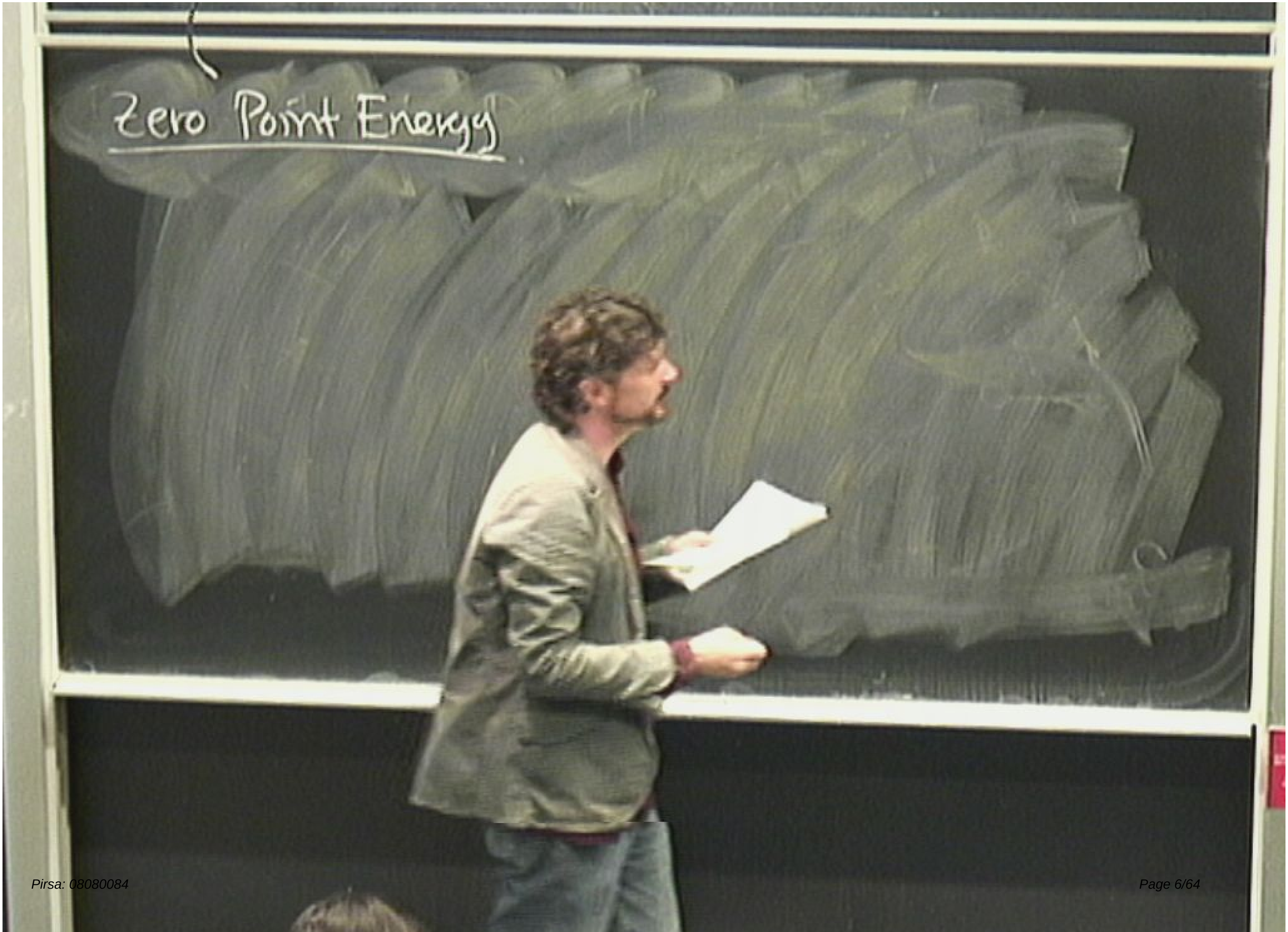
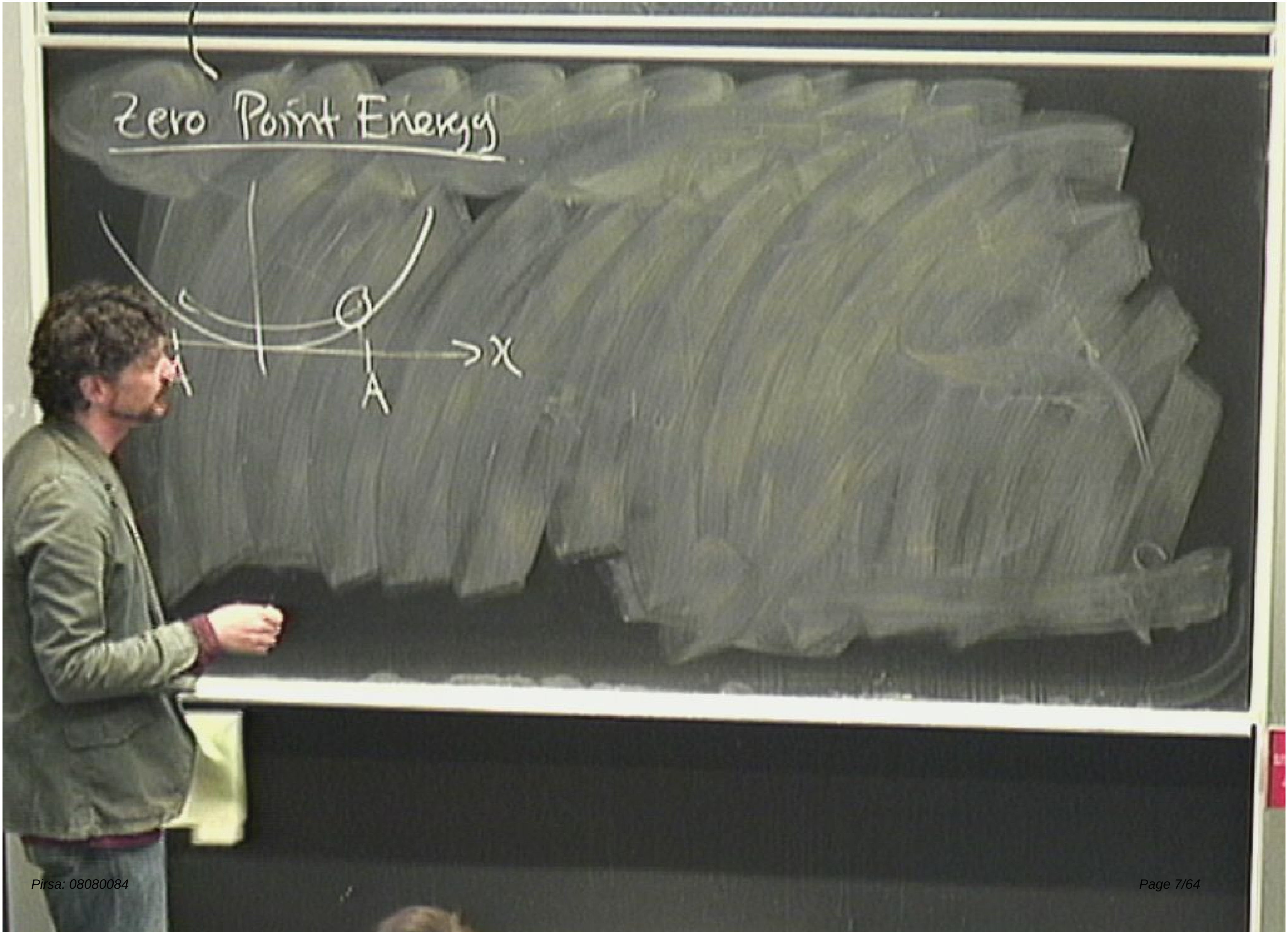
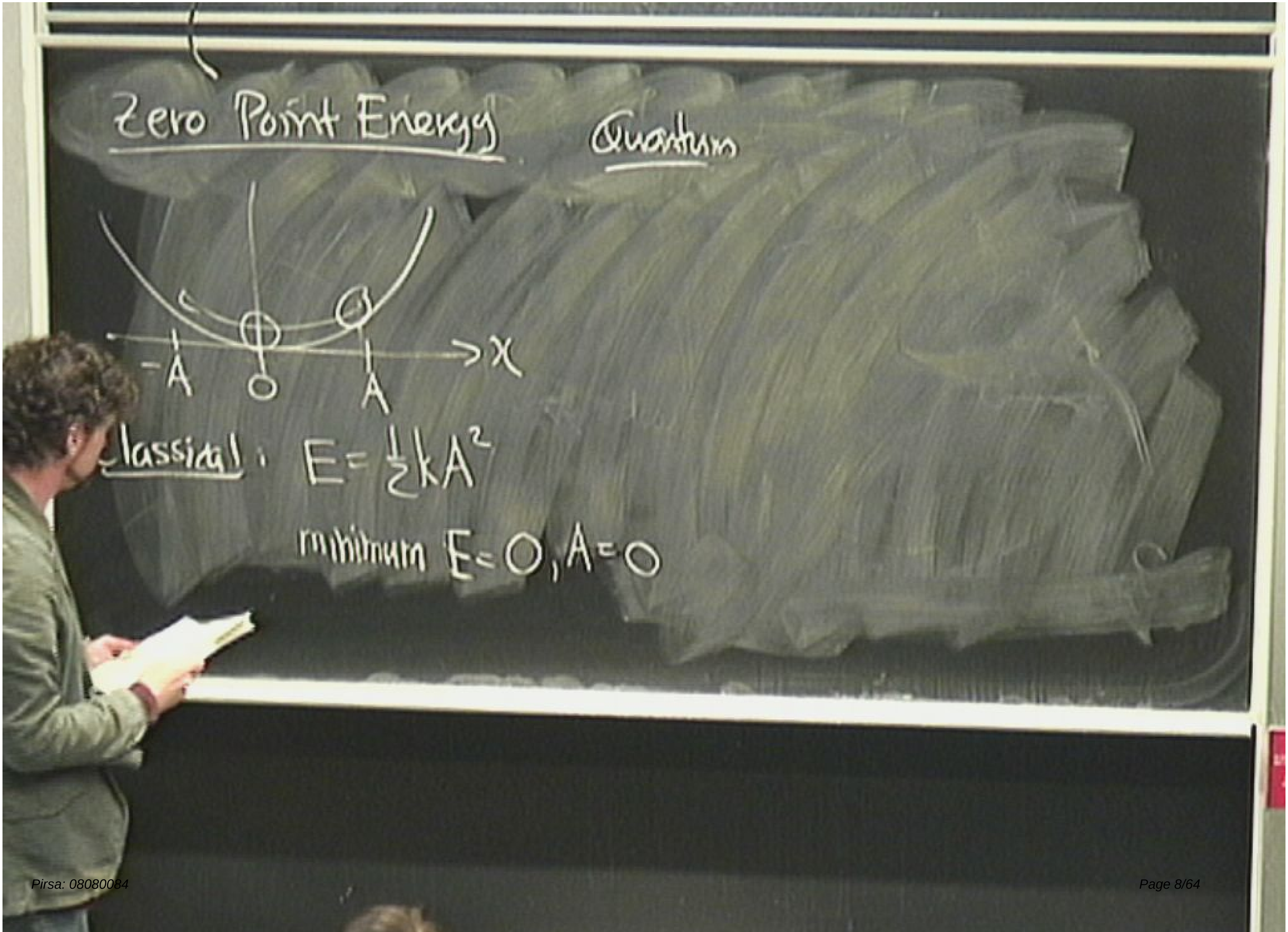
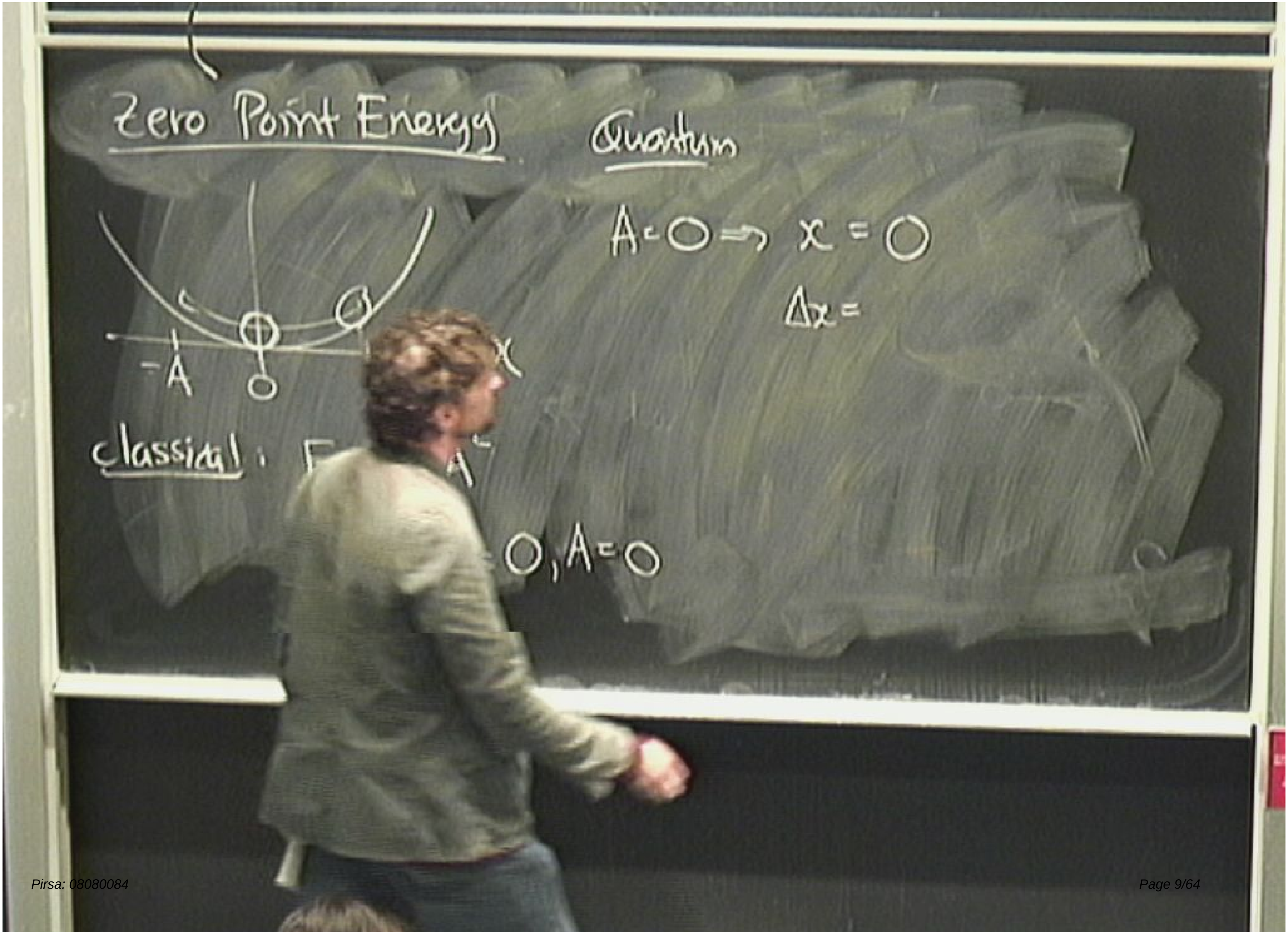
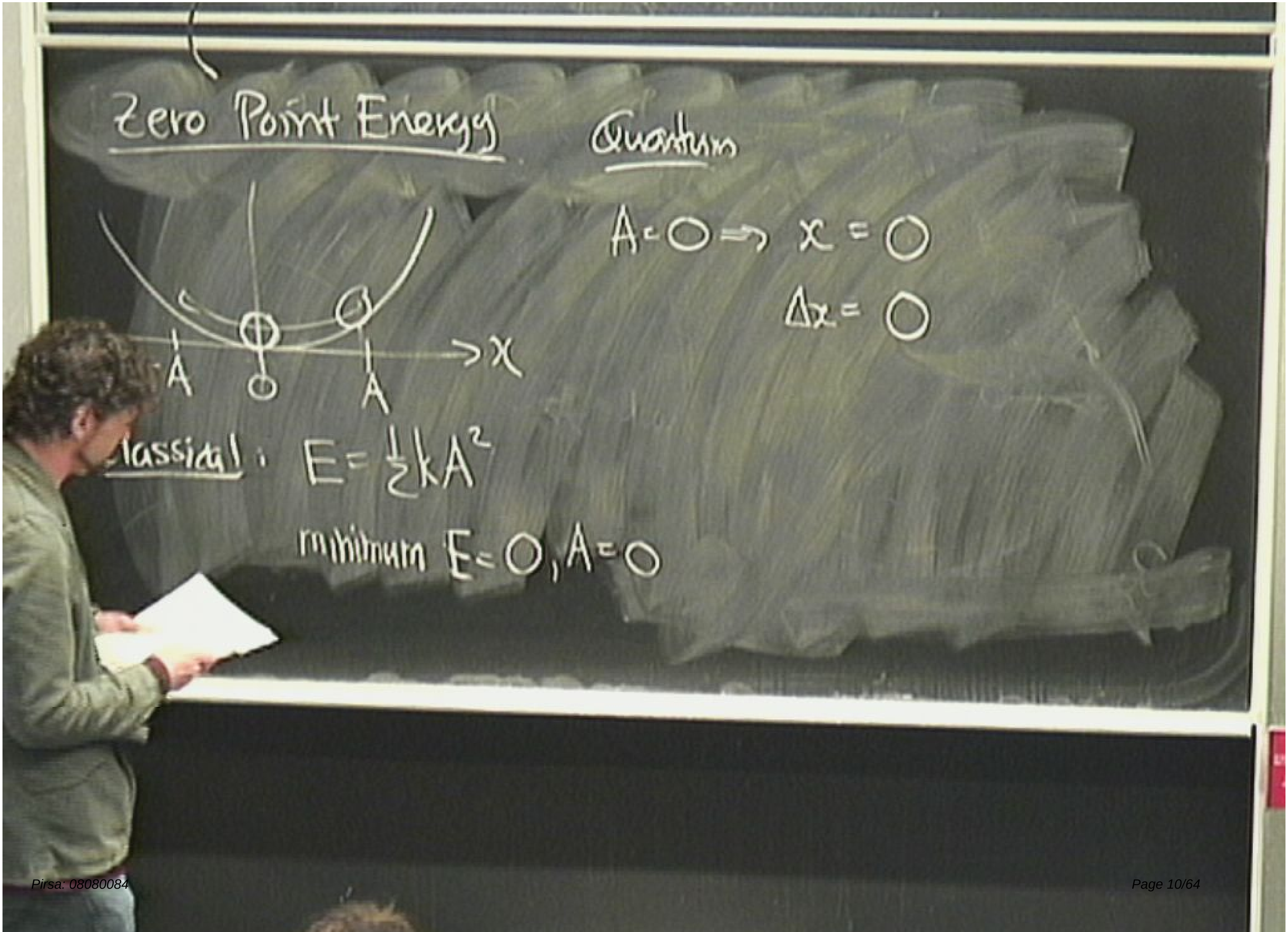
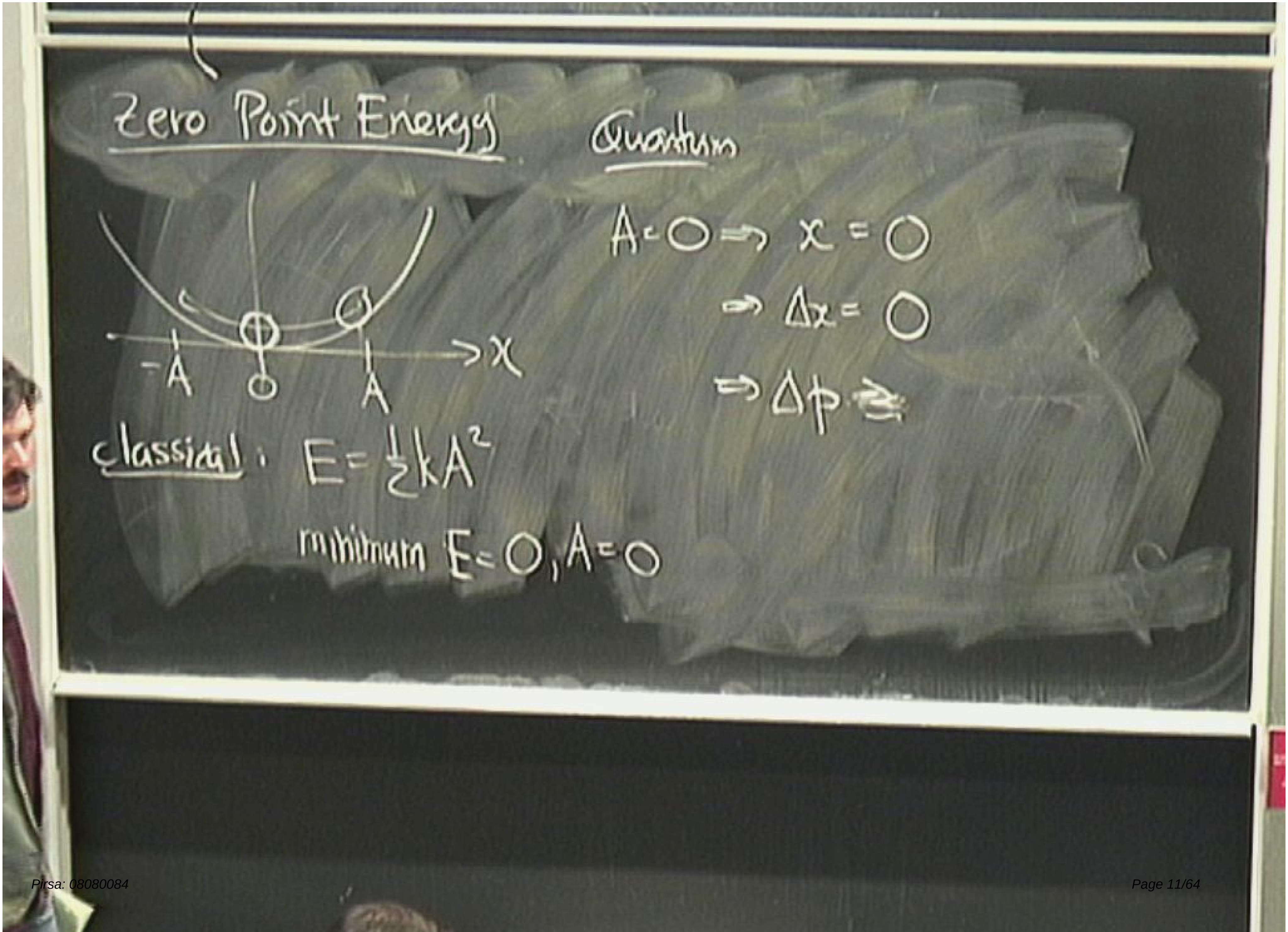

Zero Point Energy

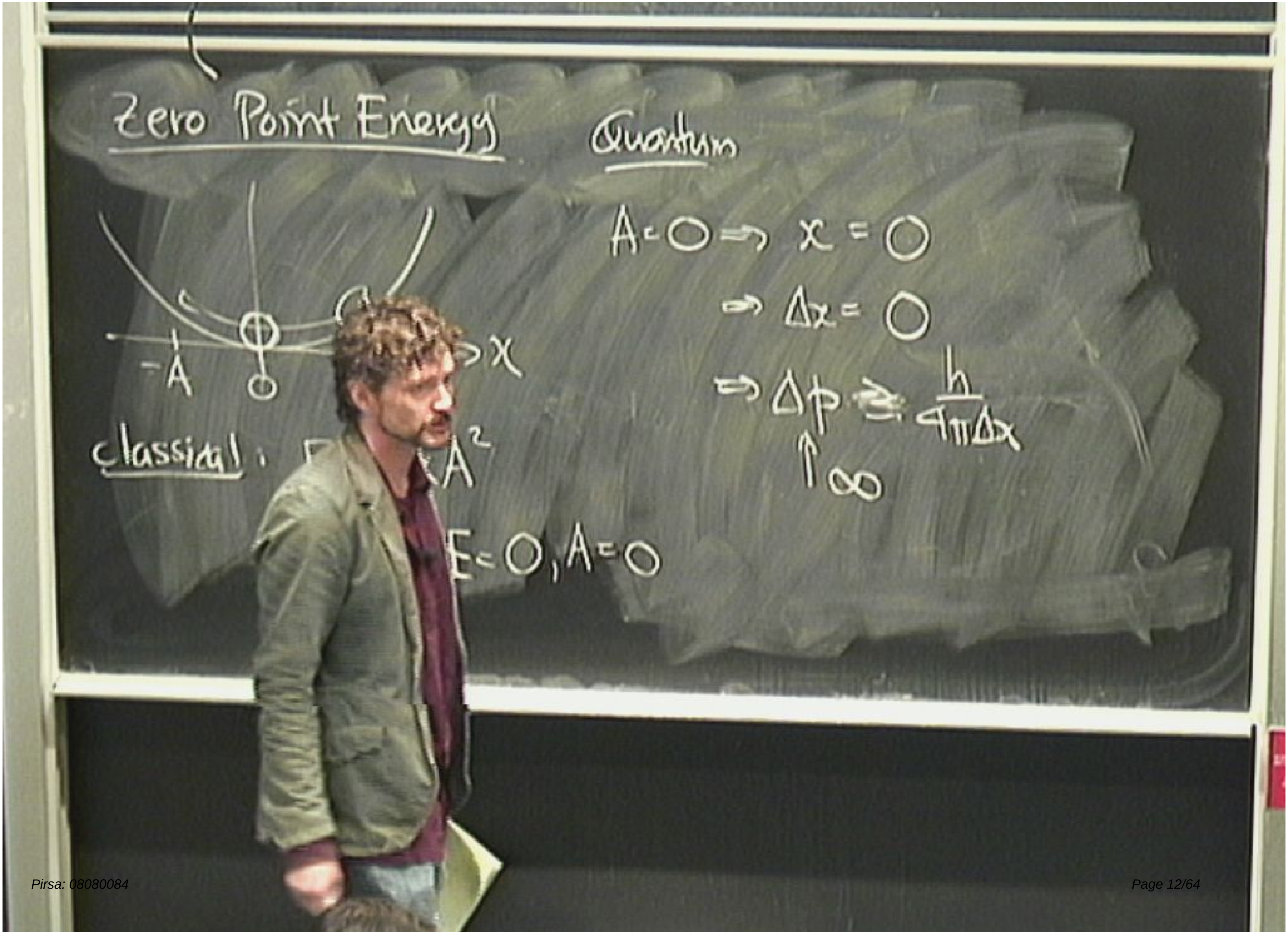

Zero Point Energy
x
A

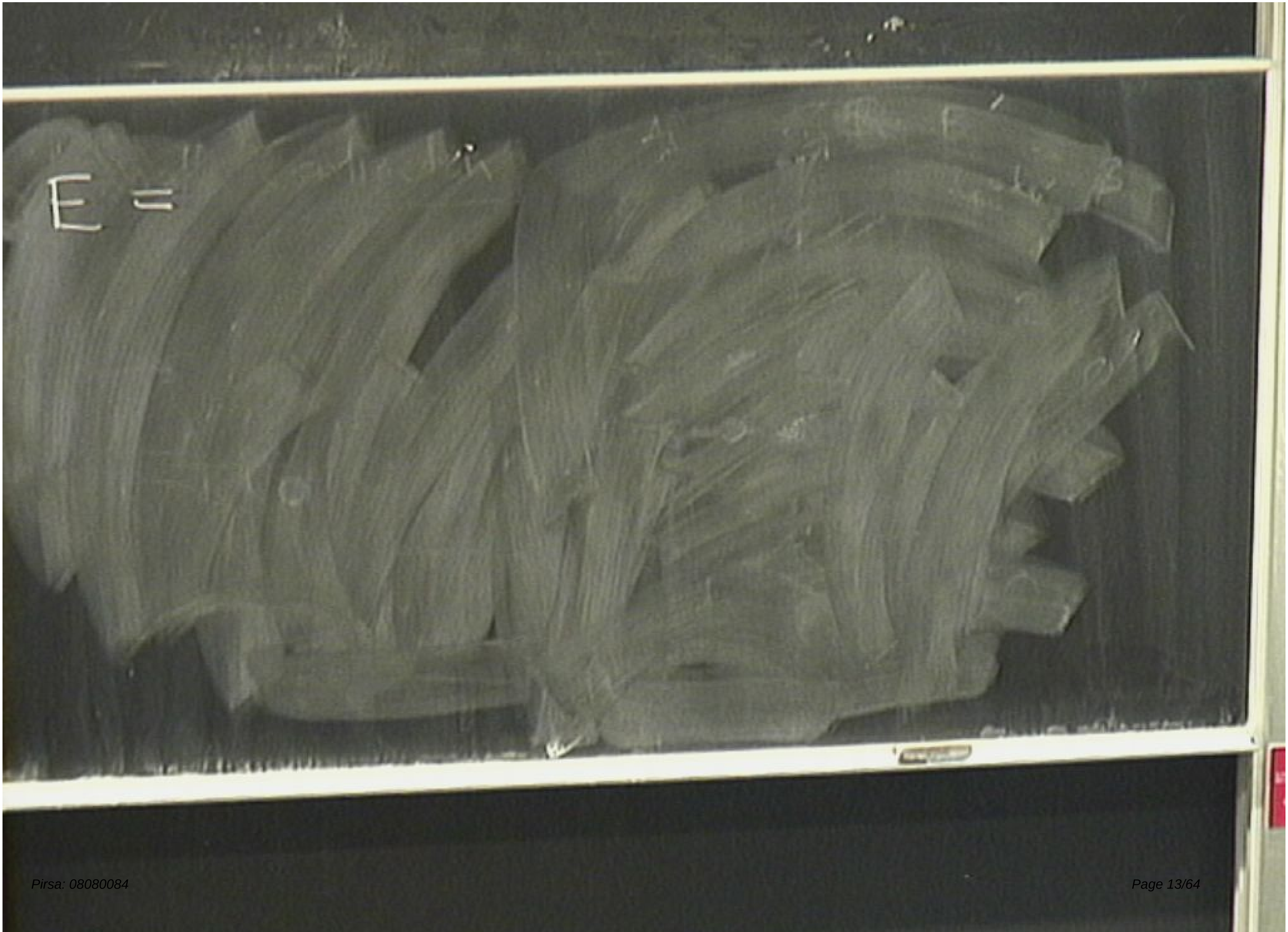

Zero Point Energy
Quantum
x
-A
0
A
lassical : $E = \frac{1}{2}kA^2$
minimum $E = 0, A = 0$

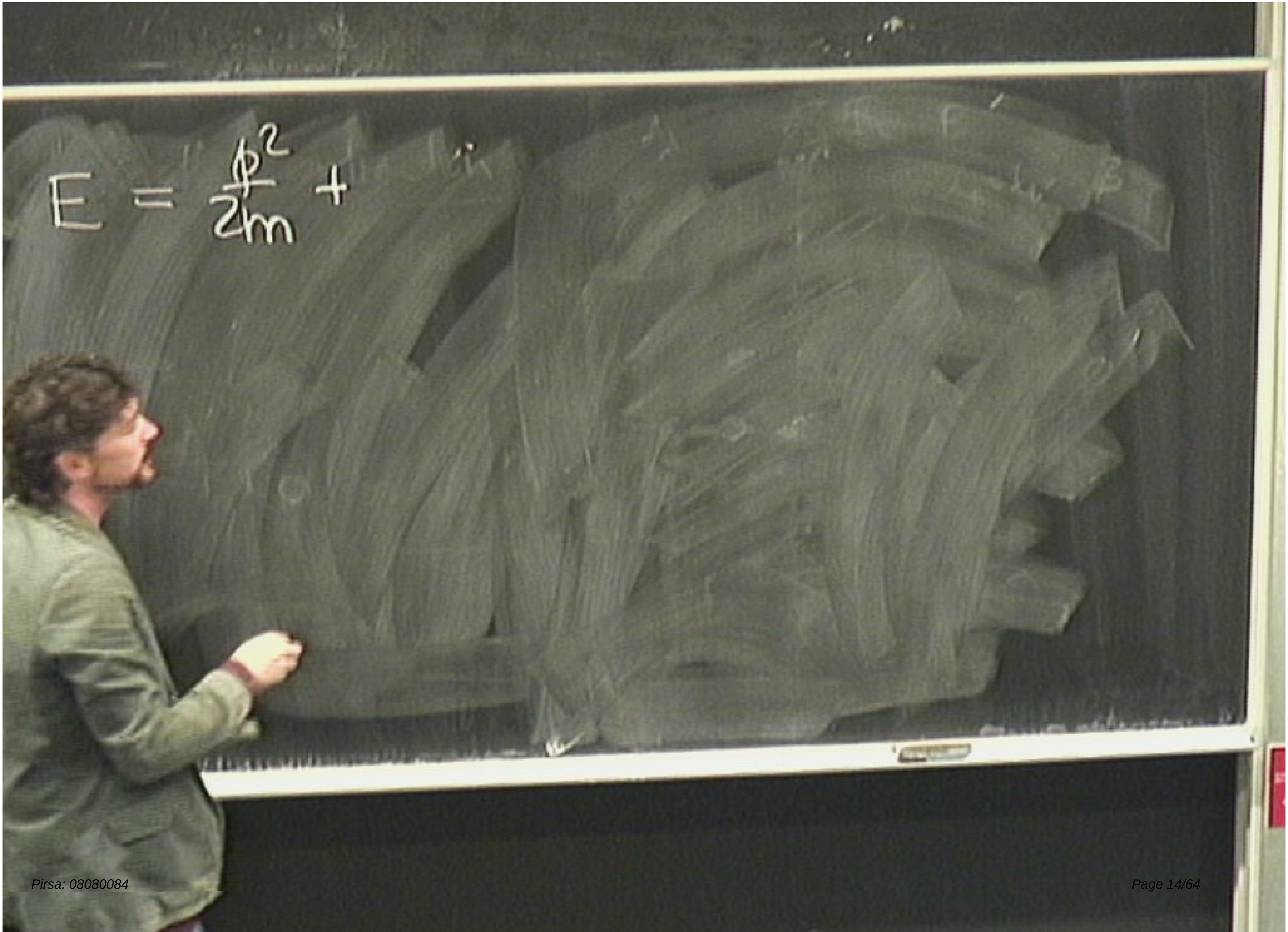

Zero Point Energy
Quantum
A=0 ⇒ x=0
Δx =
-A
0
x
classical:
0, A=0

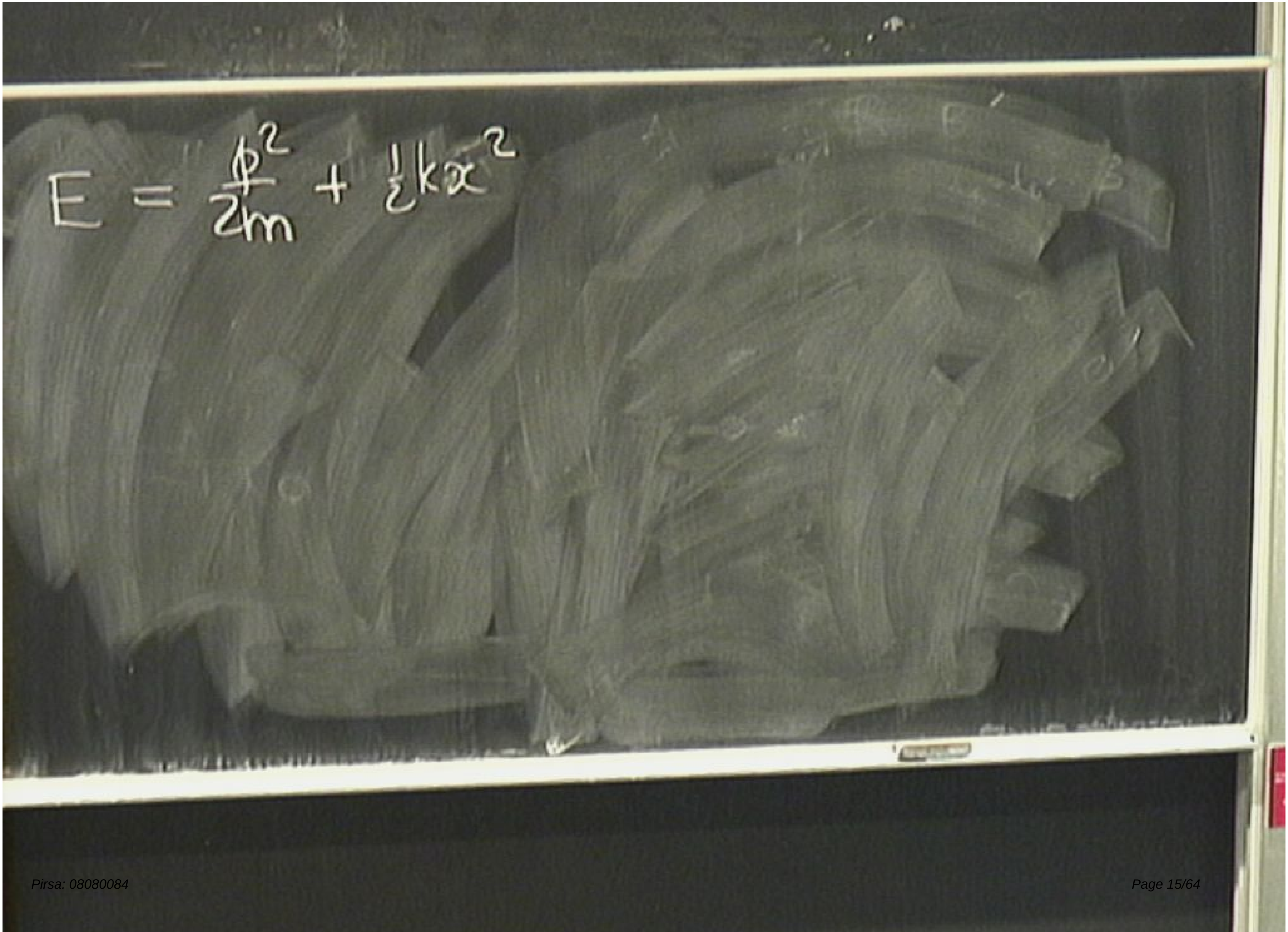

Zero Point Energy
Quantum
$A=0 \Rightarrow x=0$
$\Delta x = 0$
$-A$ 0 A x
Classical: $E = \frac{1}{2}kA^2$
minimum $E=0, A=0$


Zero Point Energy
Quantum
A=0 ⇒ x=0
⇒ Δx=0
⇒ Δp
-A
0
A
x
classical: E=½kA²
minimum E=0, A=0

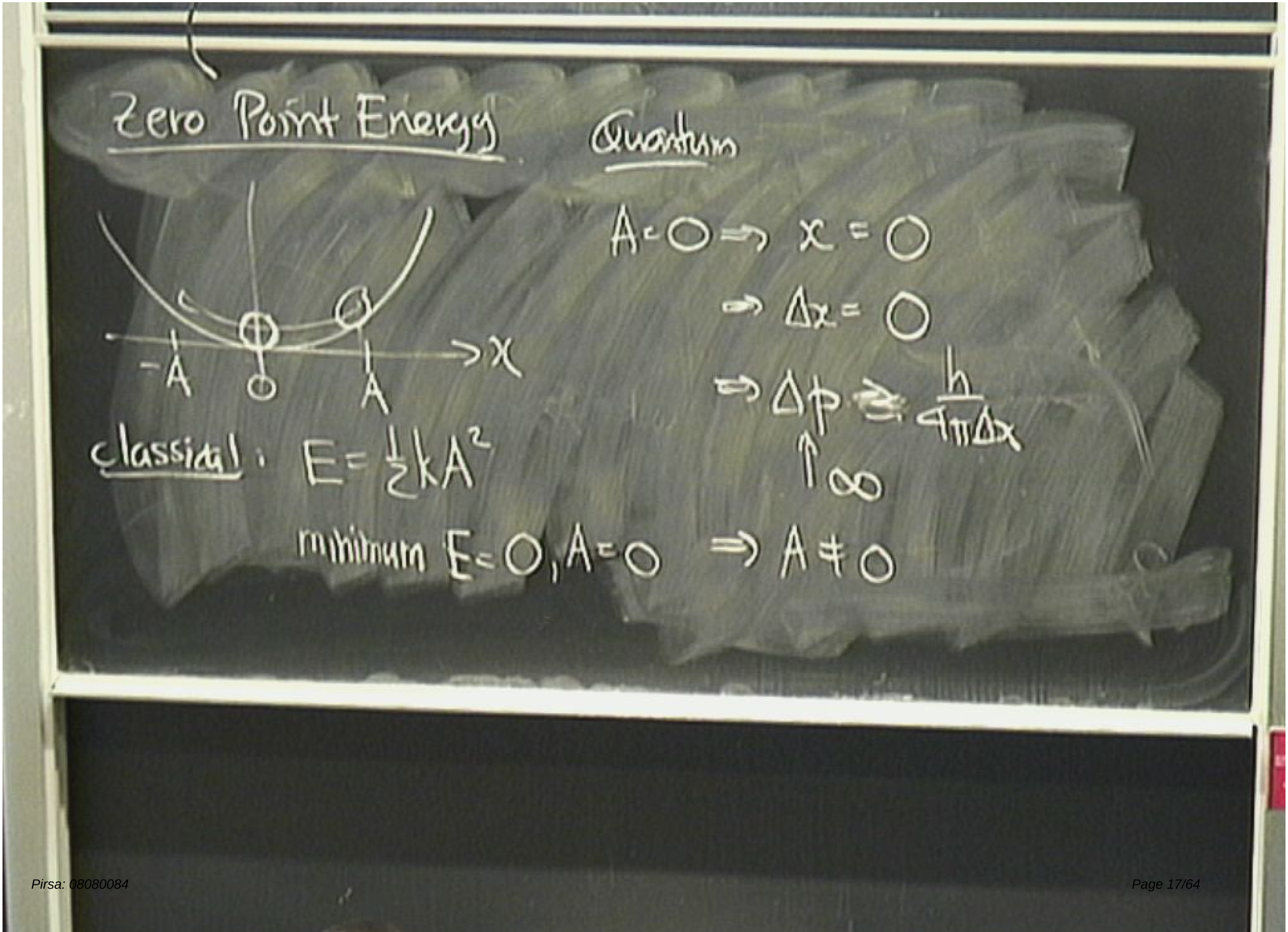

Zero Point Energy
Quantum
A=0 ⇒ x=0
⇒ Δx=0
⇒ Δp ≥ h/4πΔx
∞
−A
0
x
classical:
E=0, A=0

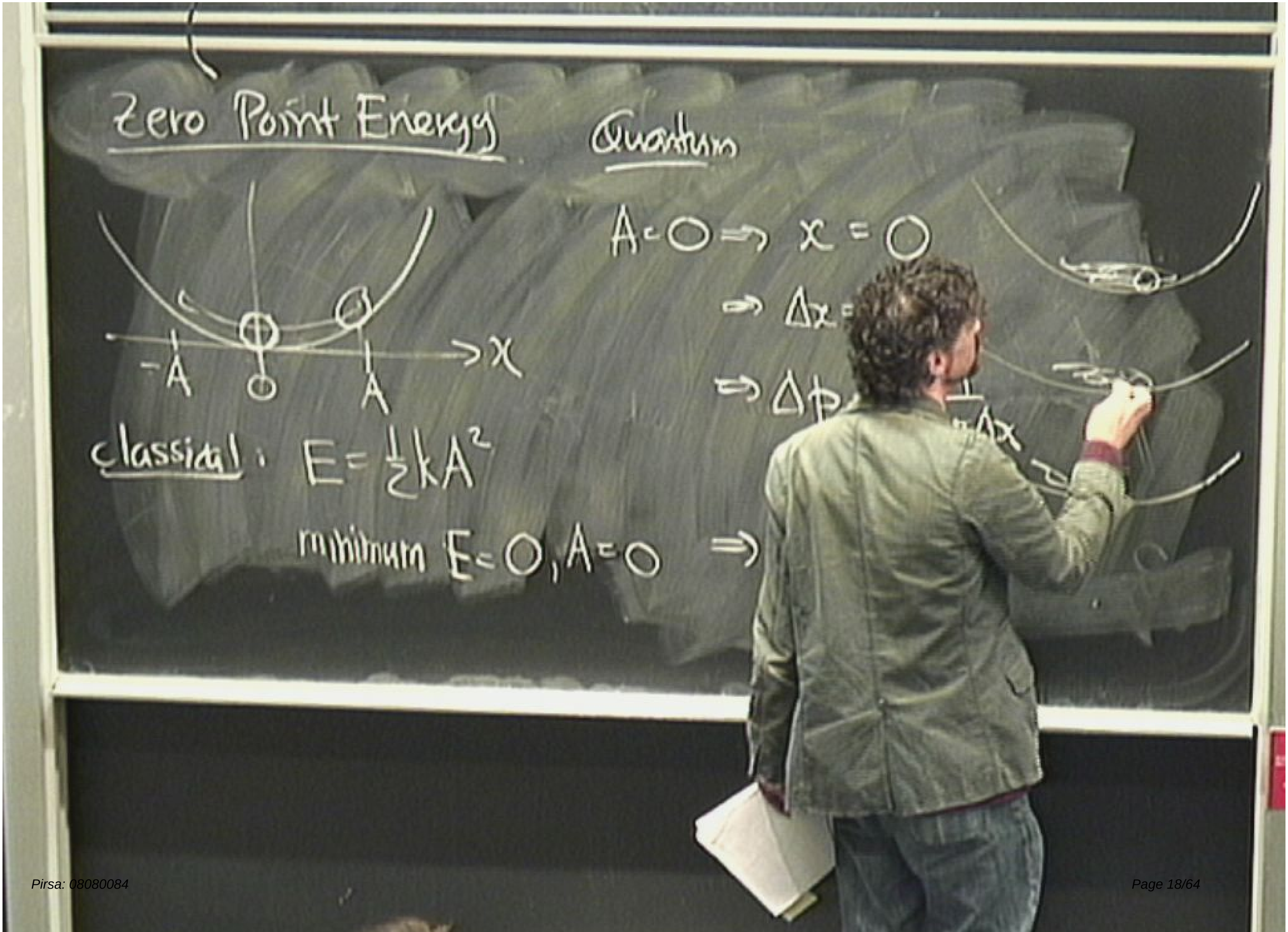

E =

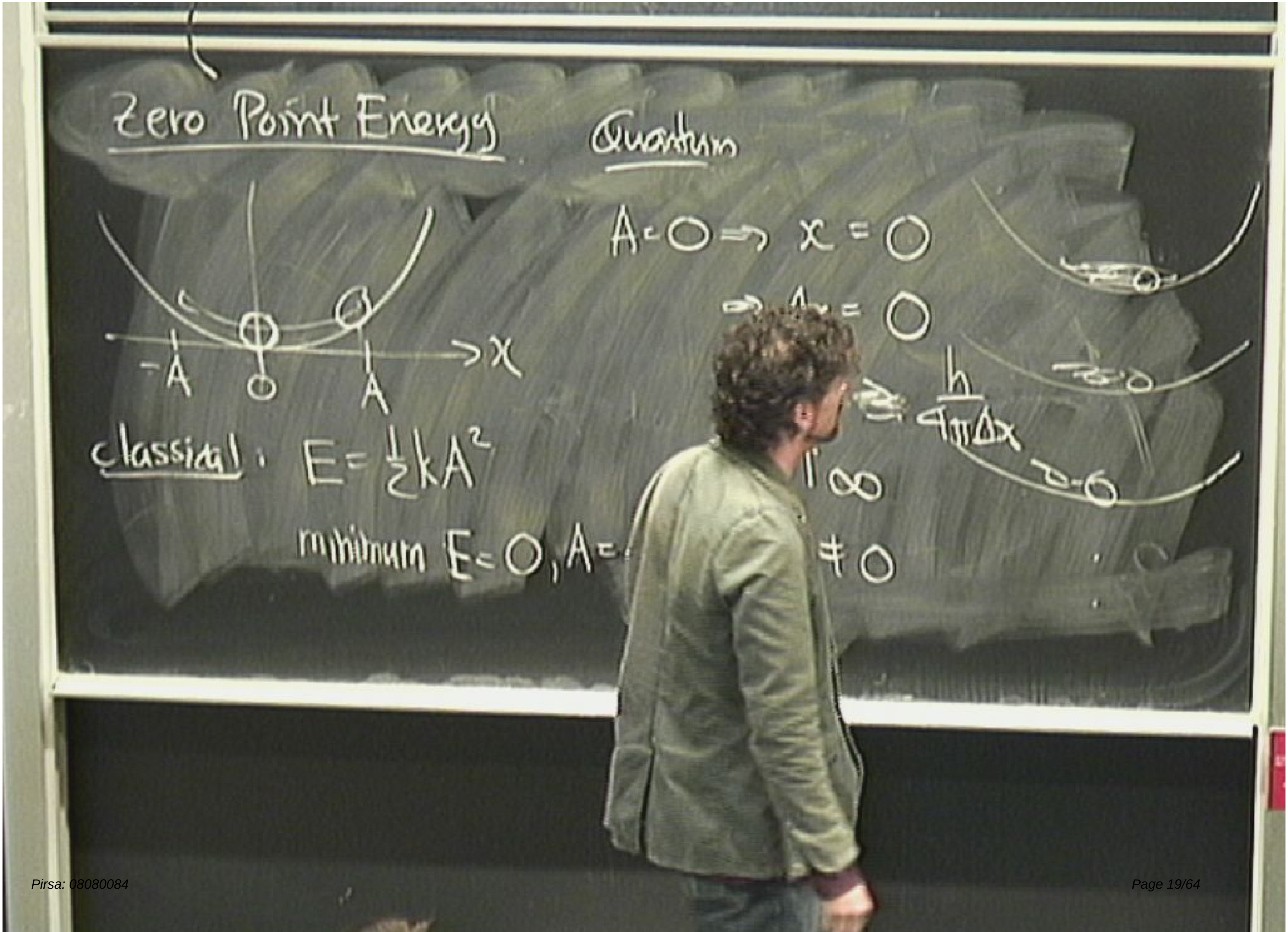

$E = \frac{p^2}{2m} +$


$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$


Zero Point Energy
Quantum
-A
0
A
x
classical: $E = \frac{1}{2}kA^2$
minimum $E=0$, $A=0$
$A=0 \Rightarrow x=0$
$\Rightarrow \Delta x = 0$
$\Rightarrow \Delta p \geq \frac{h}{4\pi\Delta x}$
$\uparrow \infty$
$\Rightarrow A \neq 0$


Zero Point Energy
Quantum
A=0 ⇒ x=0
x
-A
0
A
classical: E = ½kA²
minimum E=0, A=0 ⇒


Zero Point Energy
Quantum
A=0 ⇒ x=0
-A
0
A
x
classical:
E = ½kA²
minimum E=0,
∞
≠0

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

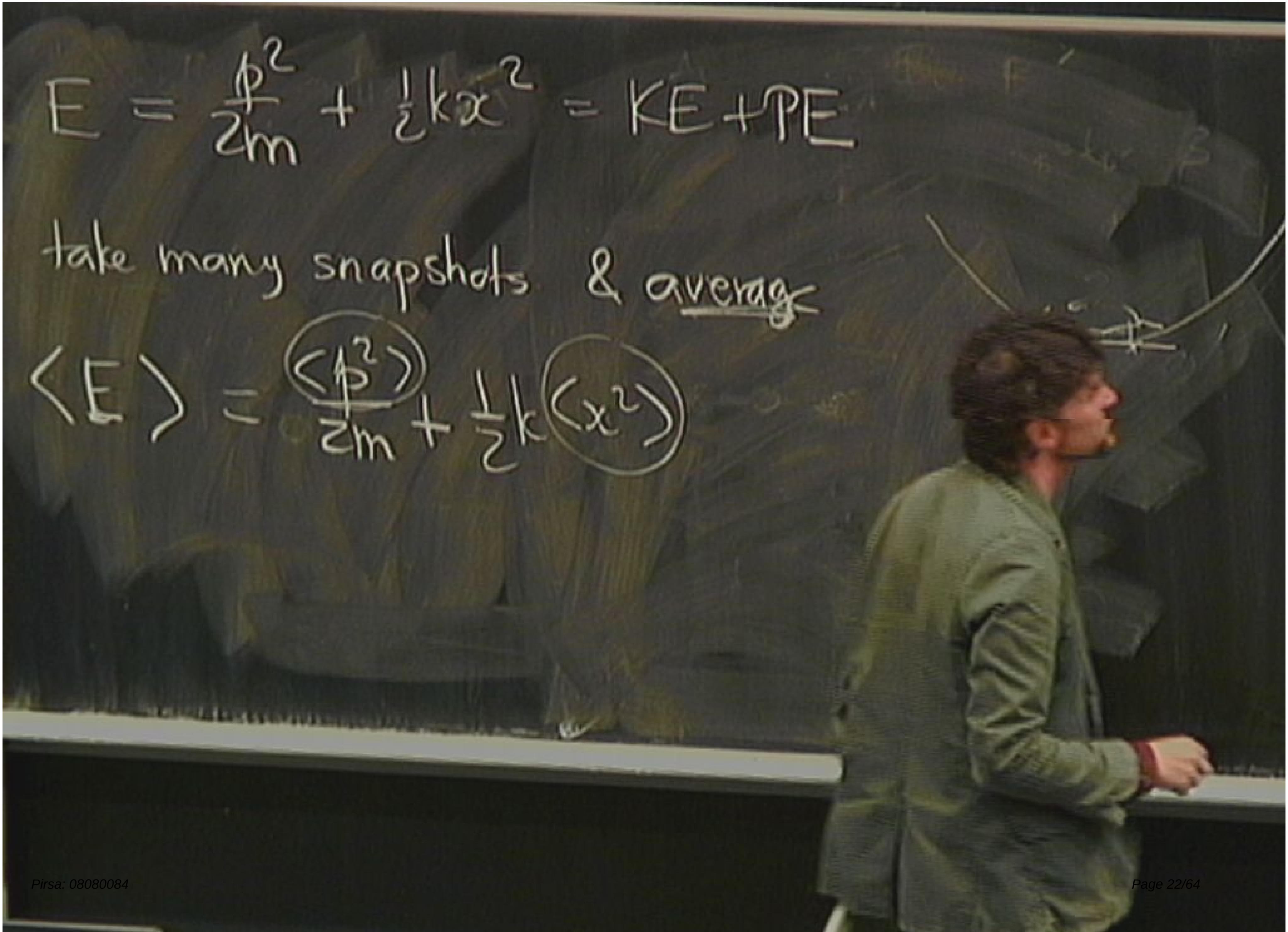
take many snapshots & average

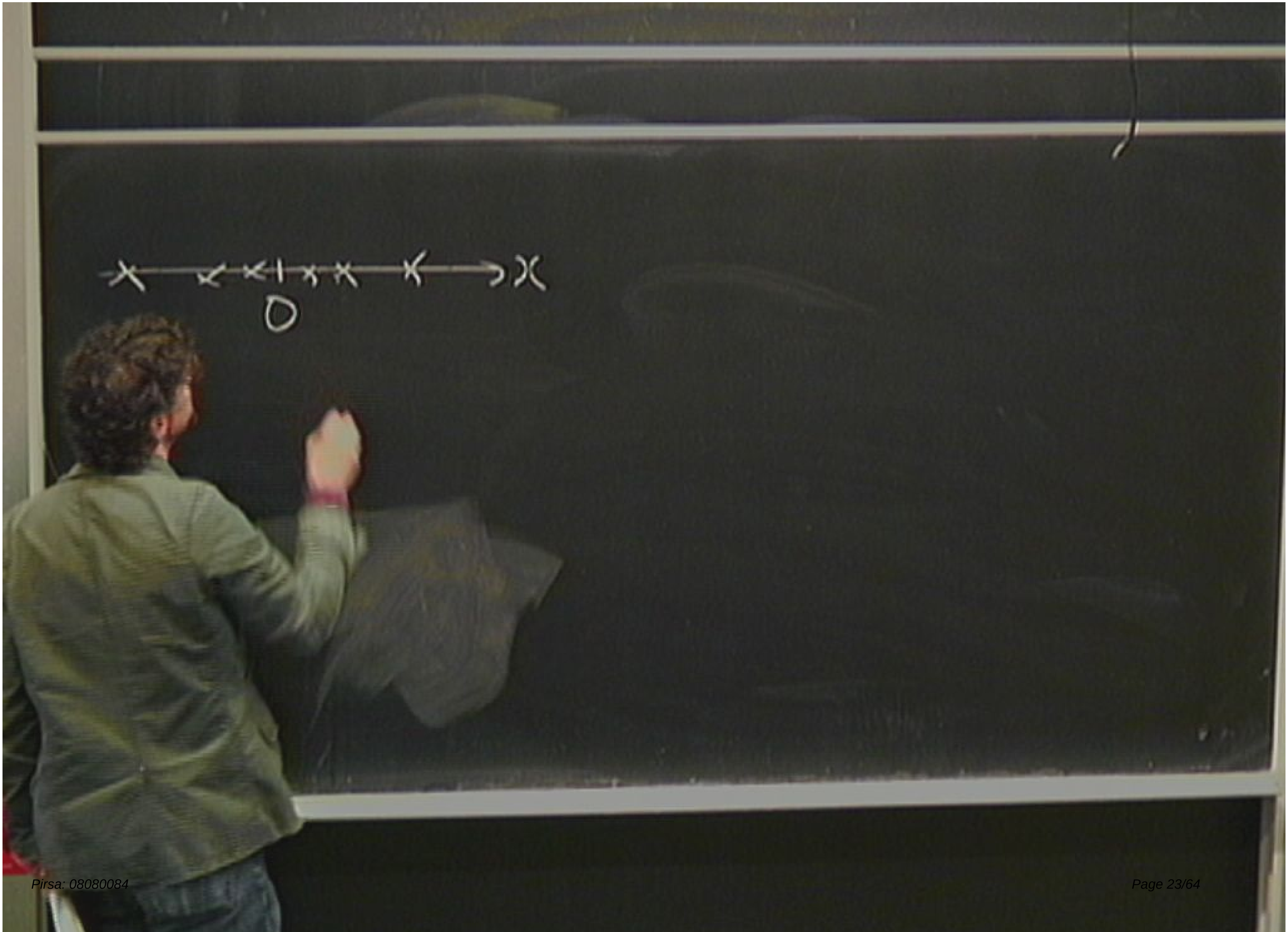
$\langle E \rangle$

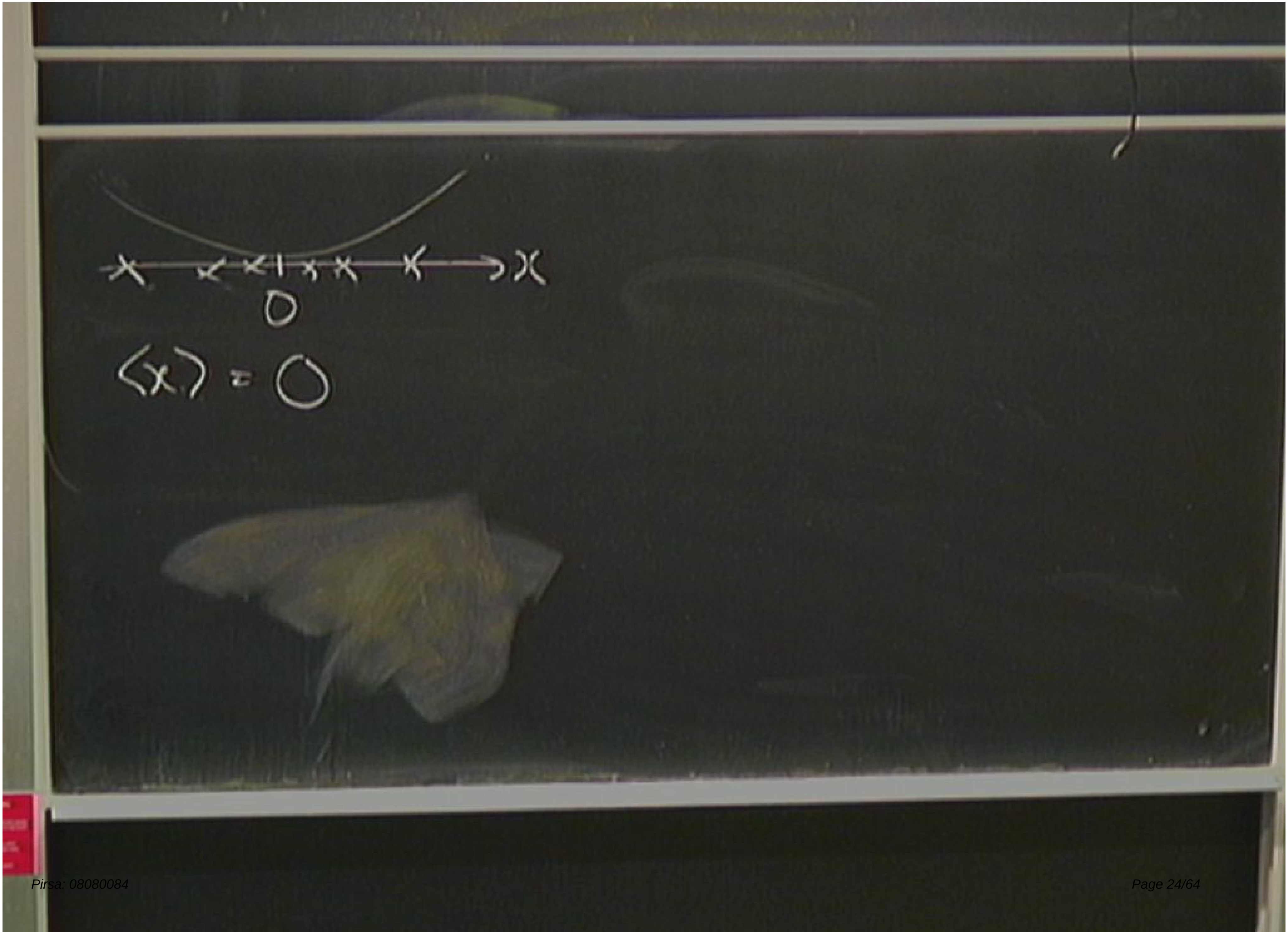
$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

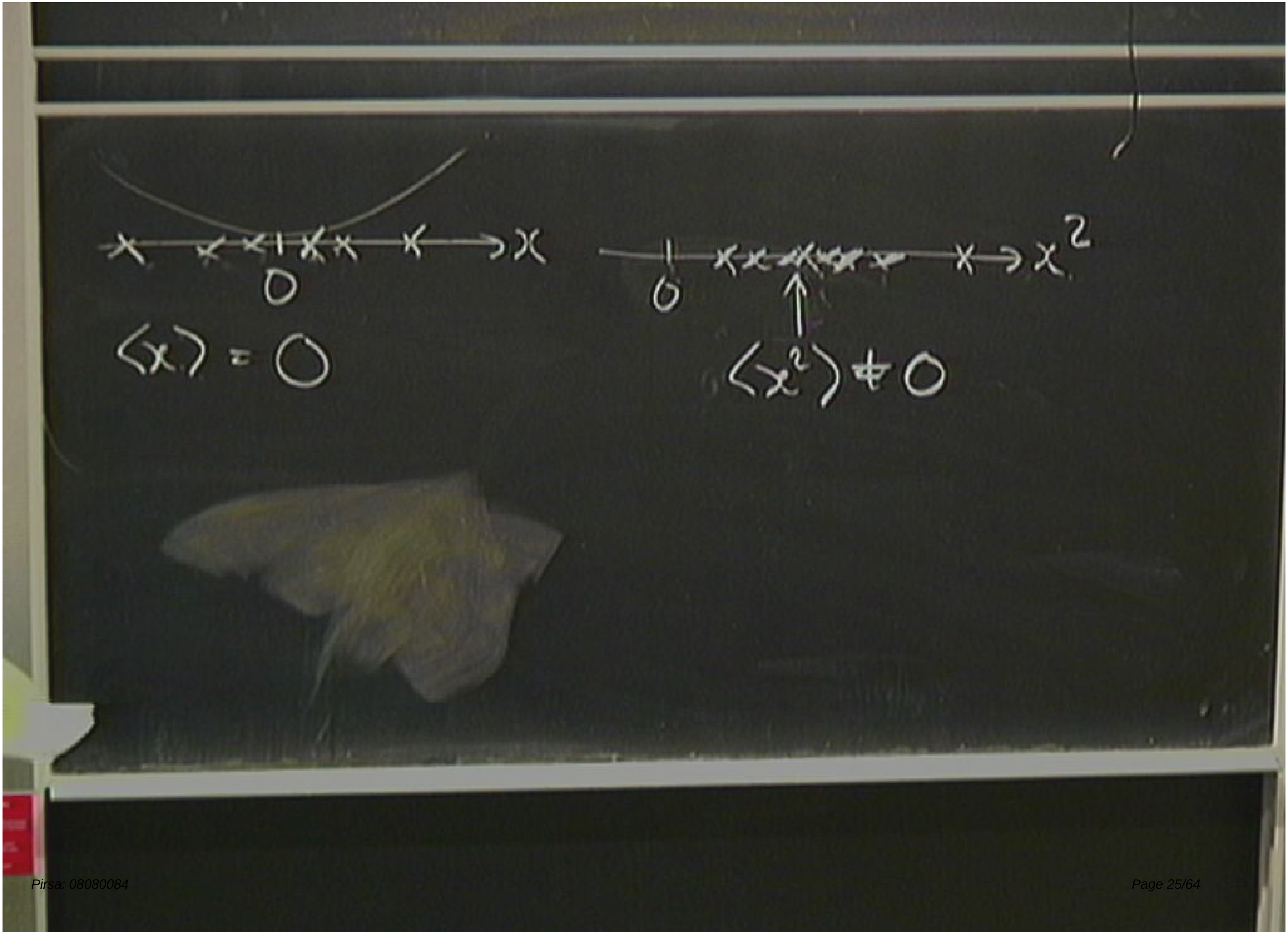
take many snapshots & average

$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$$

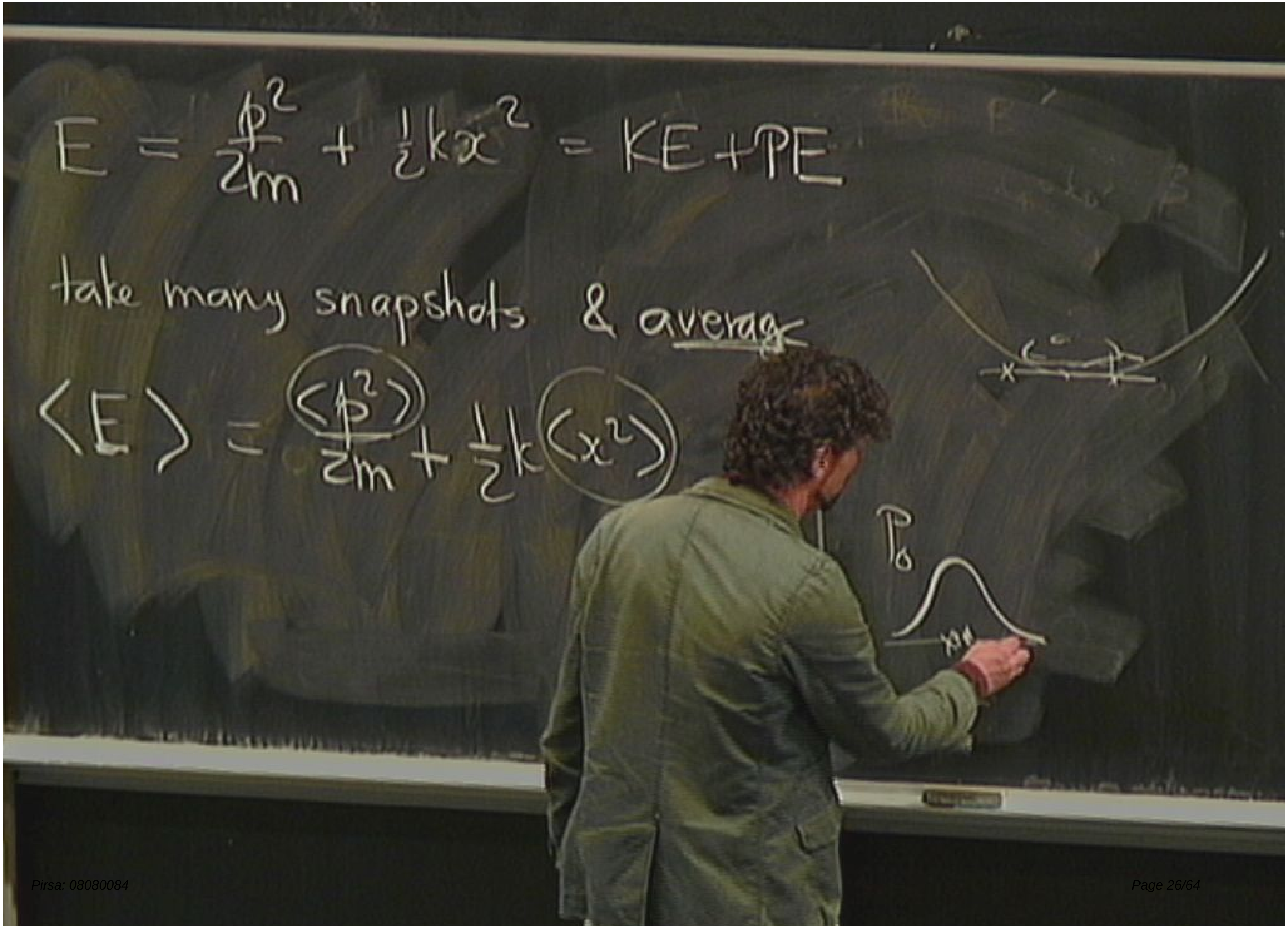

$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$

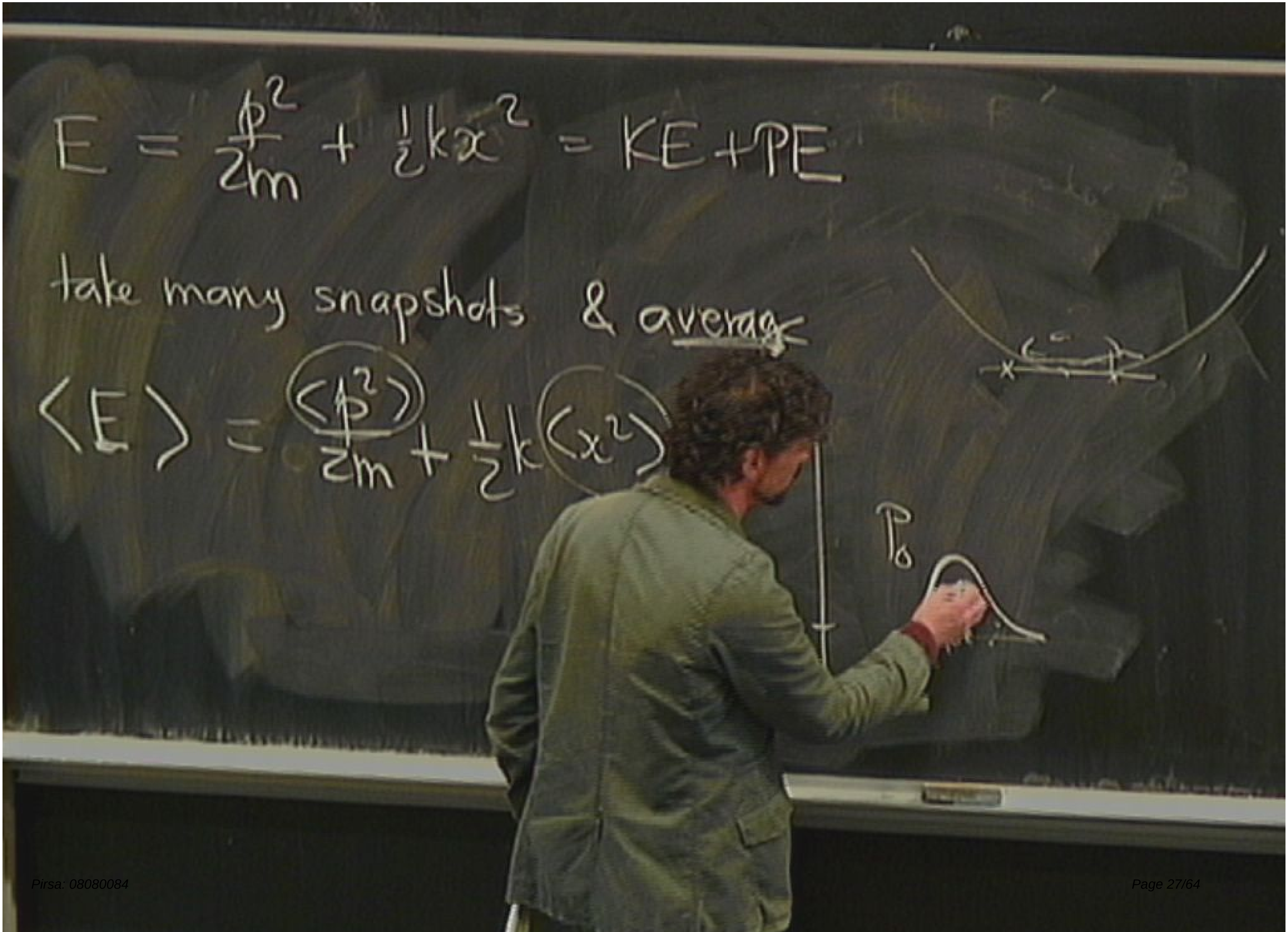

x
0

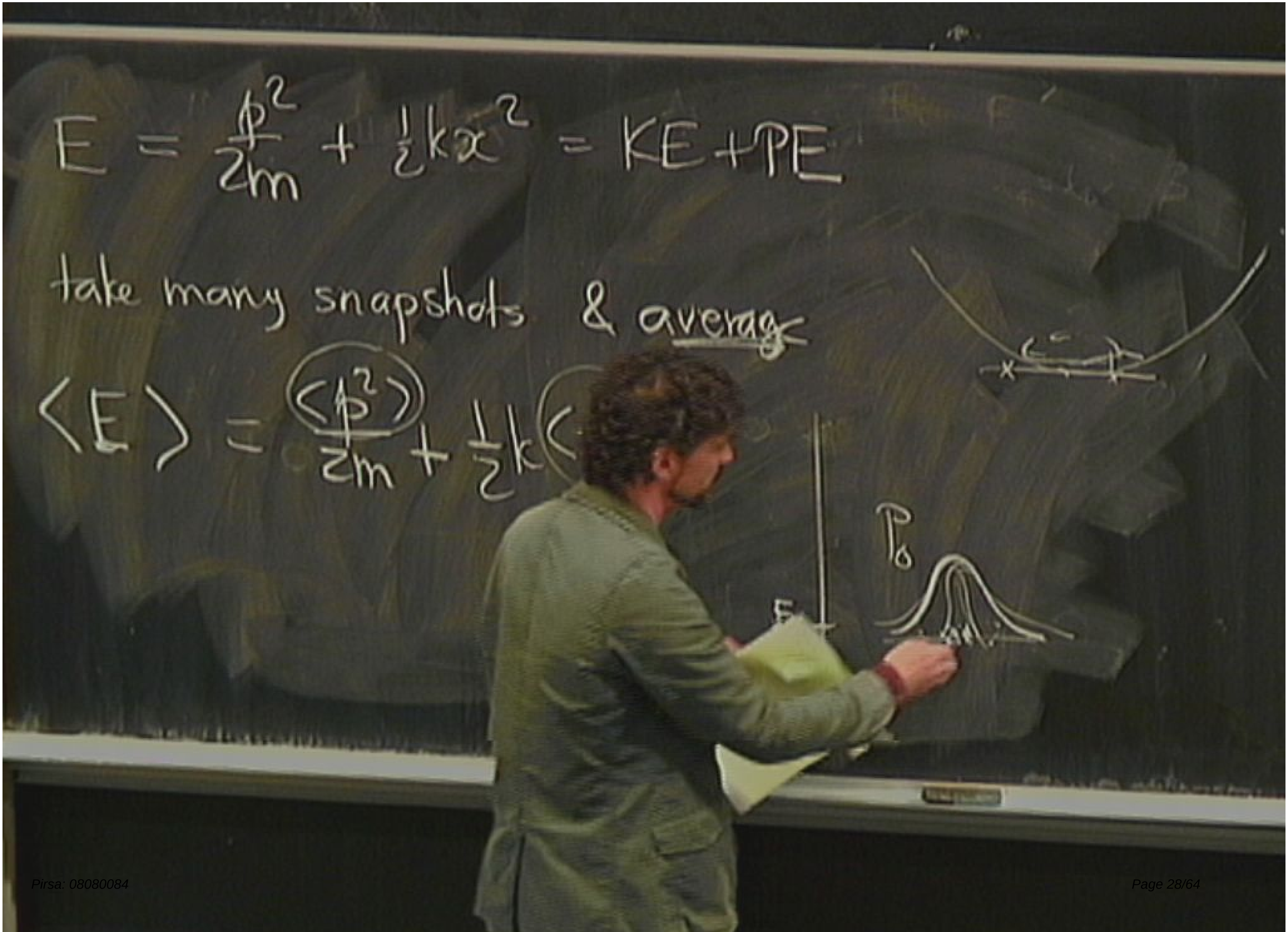

x
0
$\langle x \rangle = 0$



x
0
$\langle x \rangle = 0$
x^2
0
$\langle x^2 \rangle \neq 0$


$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$
P_0


$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$
P_0

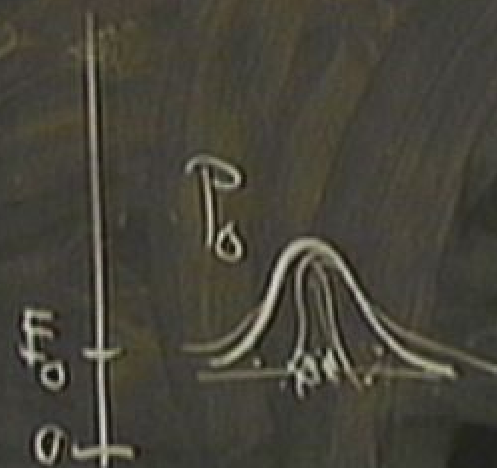


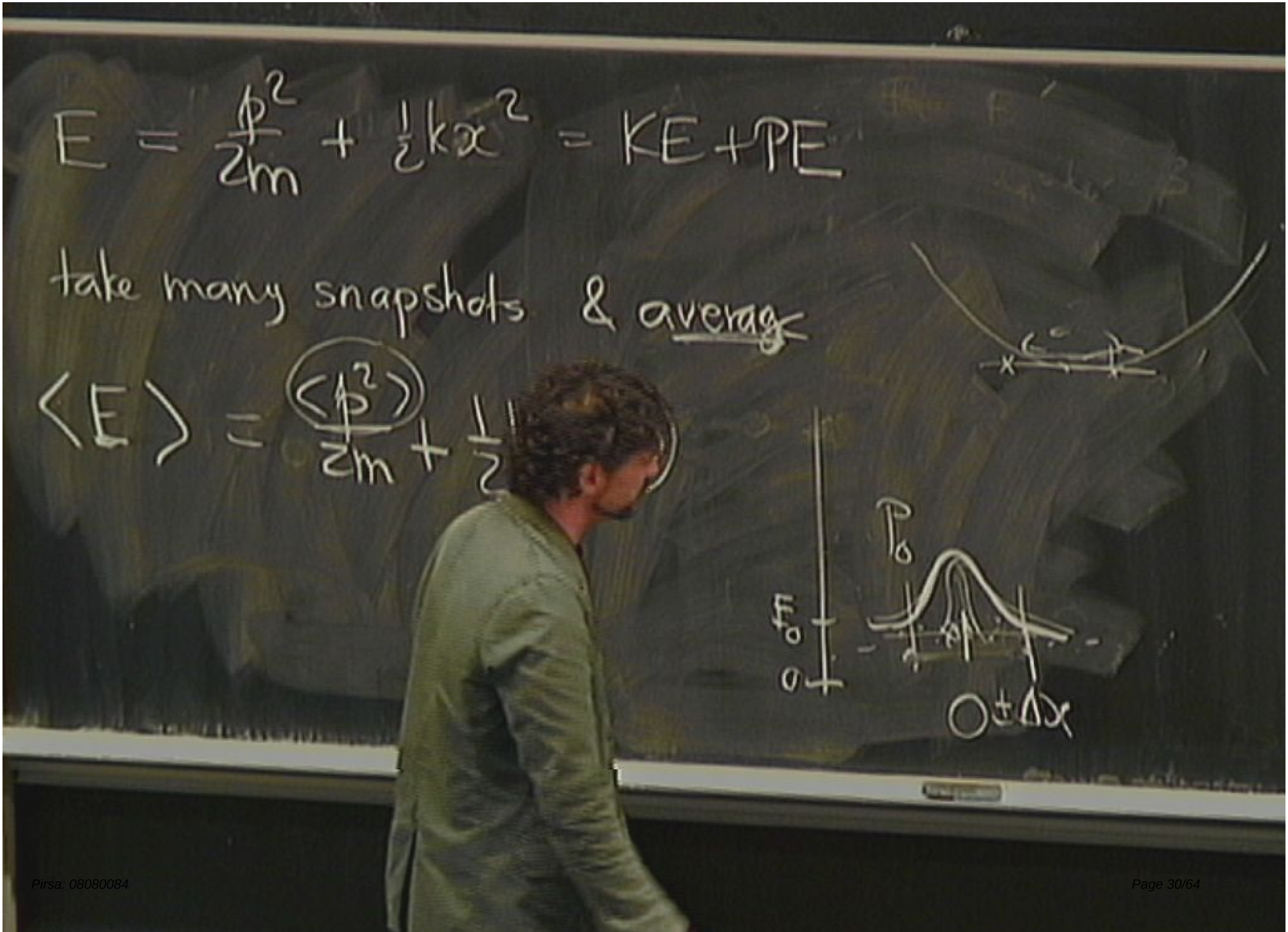
$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle$
P_0

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

take many snapshots & average

$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$$

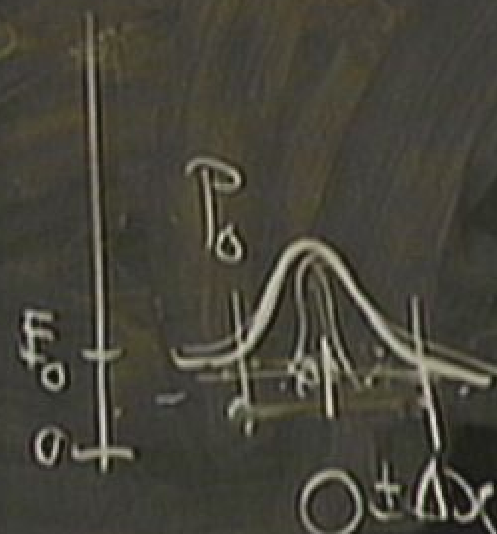


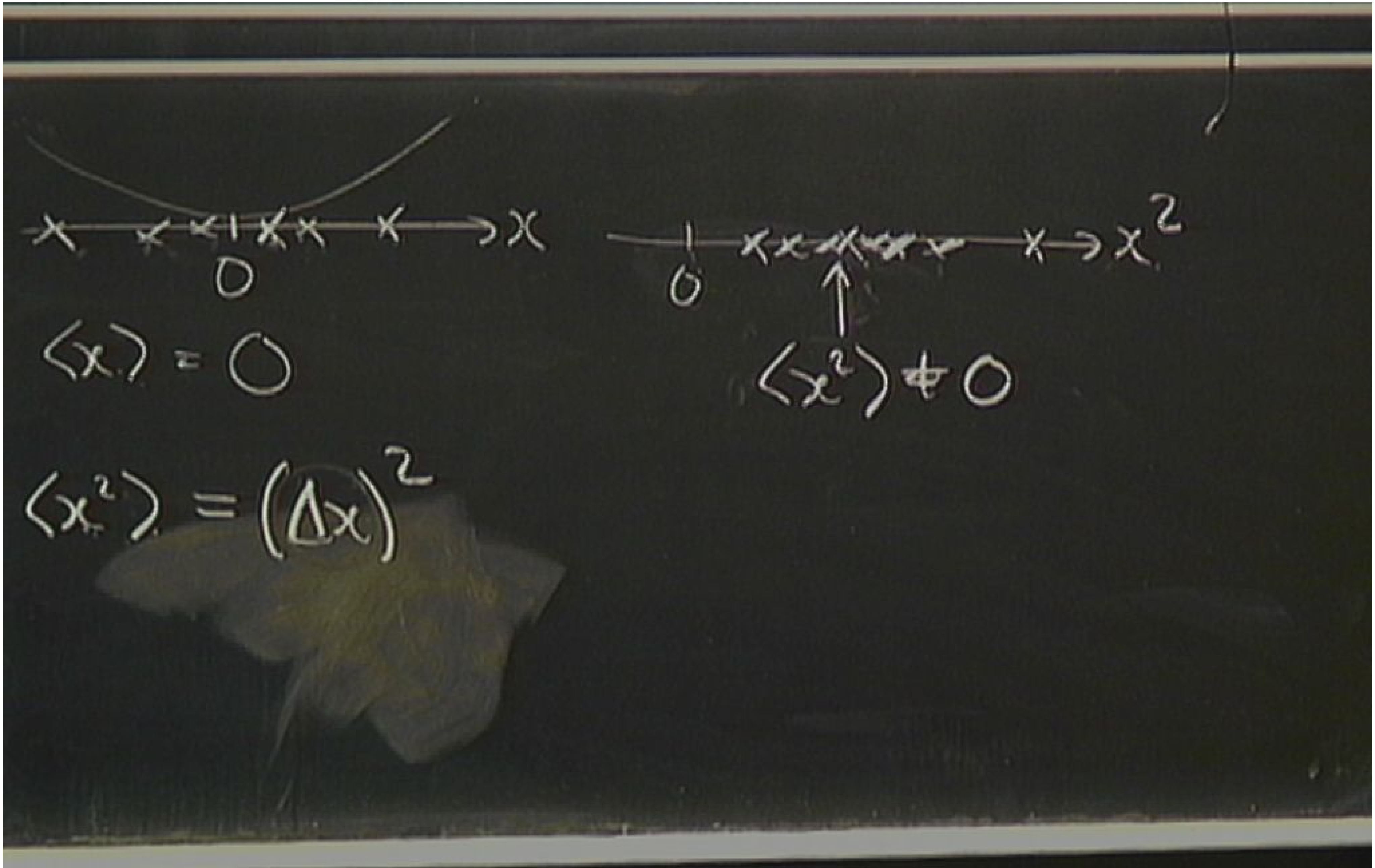

E = p²/2m + ½kx² = KE+PE
take many snapshots & average
⟨E⟩ = ⟨p²⟩/2m + ½
P₀
E₀
0
0 ± Δx

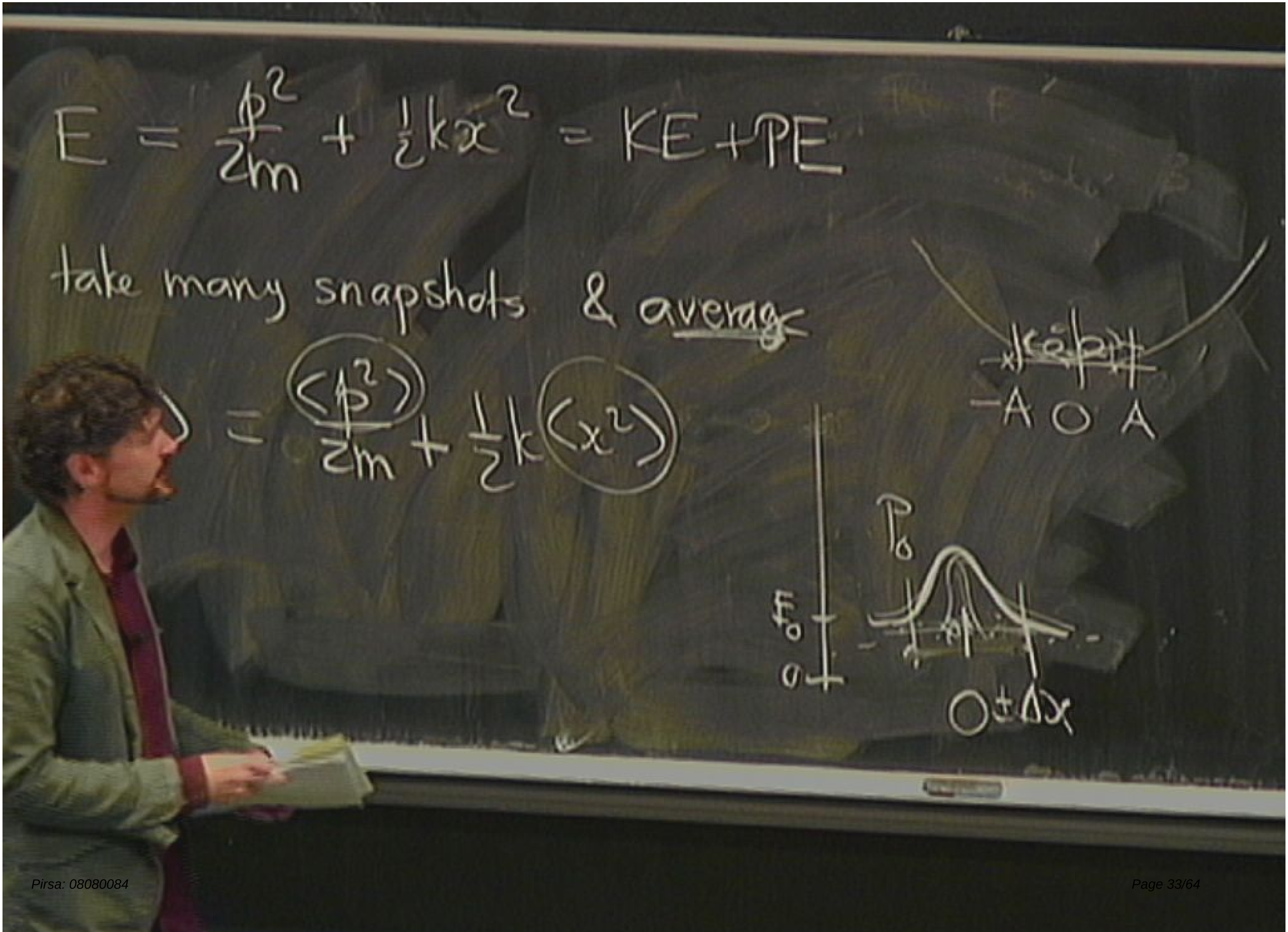
$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

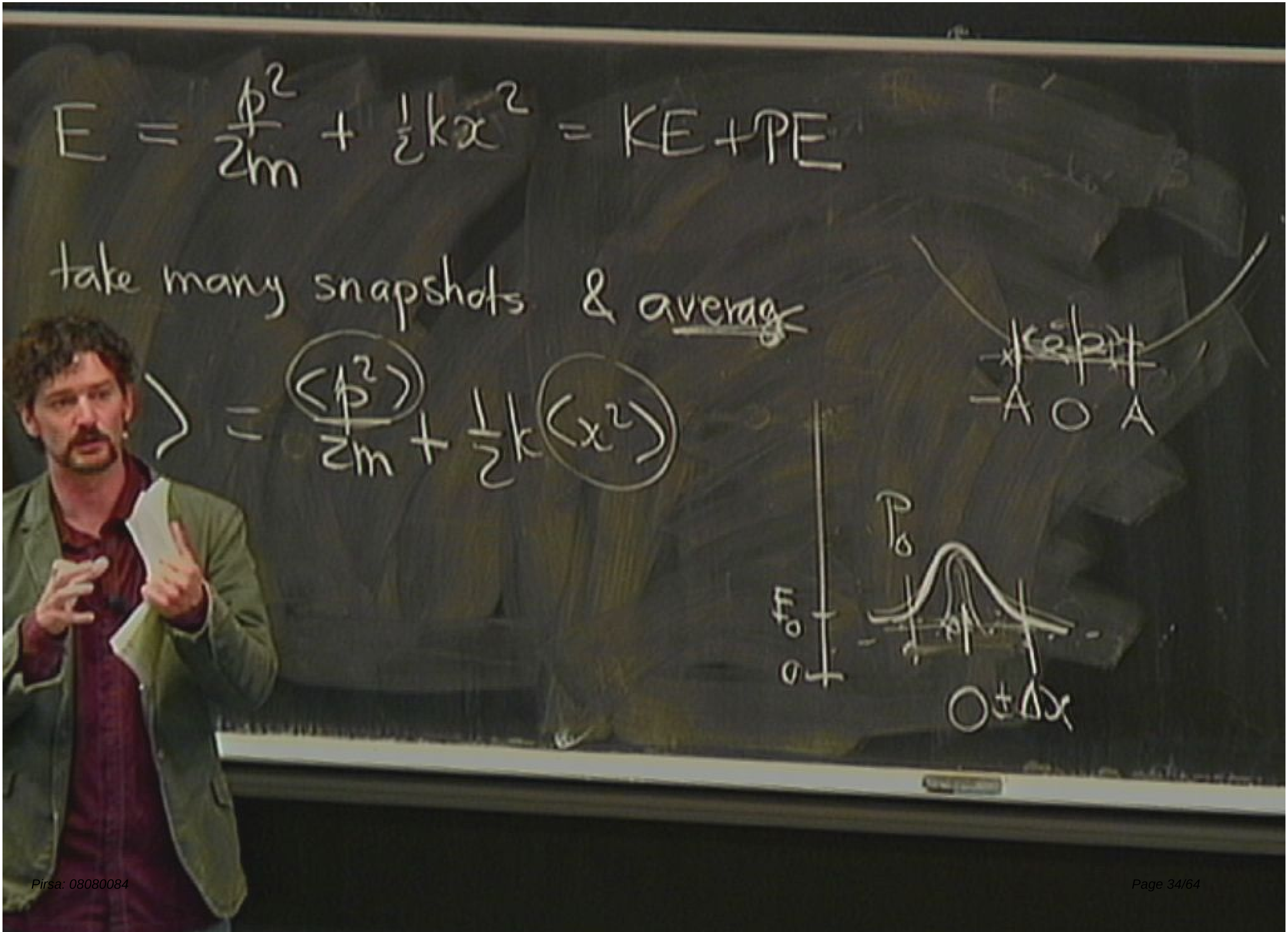
take many snapshots & average

$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$$

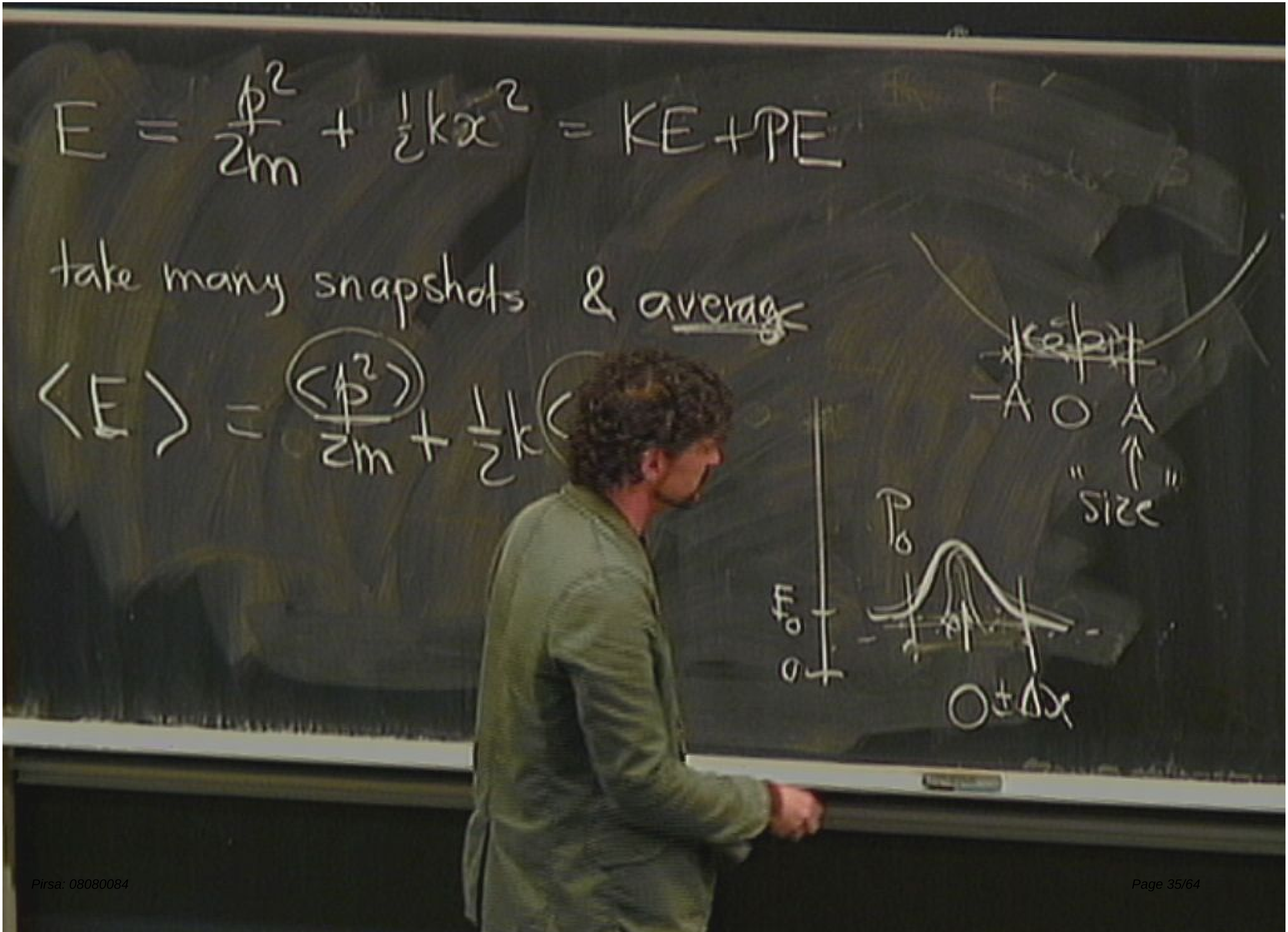


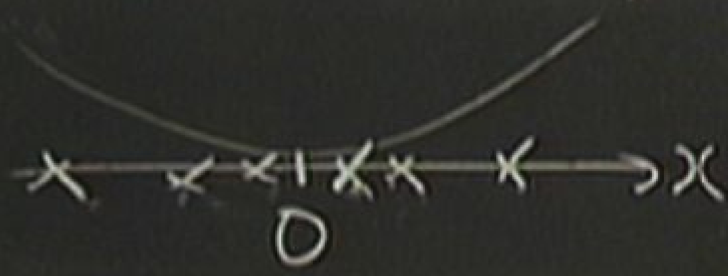

x
0
$\langle x \rangle = 0$
$\langle x^2 \rangle = (\Delta x)^2$
x^2
0
$\langle x^2 \rangle \neq 0$


E = p²/2m + ½kx² = KE+PE
take many snapshots & average
= ⟨p²⟩/2m + ½k⟨x²⟩
-A O A
P0
E0
0
0±Δx



$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$
$-A \quad O \quad A$
P_0
E_0
0
$0 \pm \Delta x$


E = p²/2m + ½kx² = KE+PE
take many snapshots & average
⟨E⟩ = ⟨p²⟩/2m + ½k⟨
−A 0 A
"size"
P0
E0
0
0±Δx



$$\langle x \rangle = 0$$

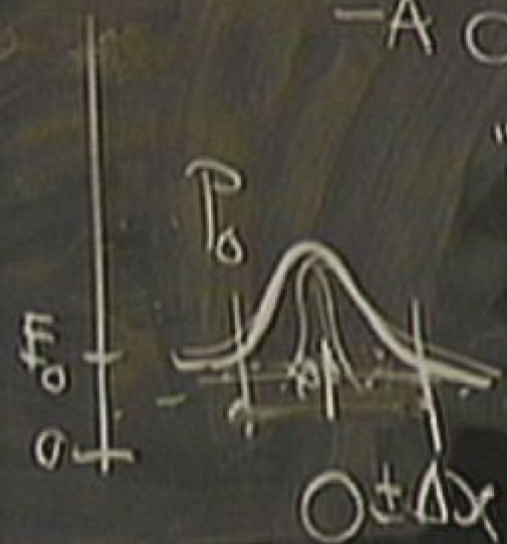
$$\langle x^2 \rangle \neq 0$$

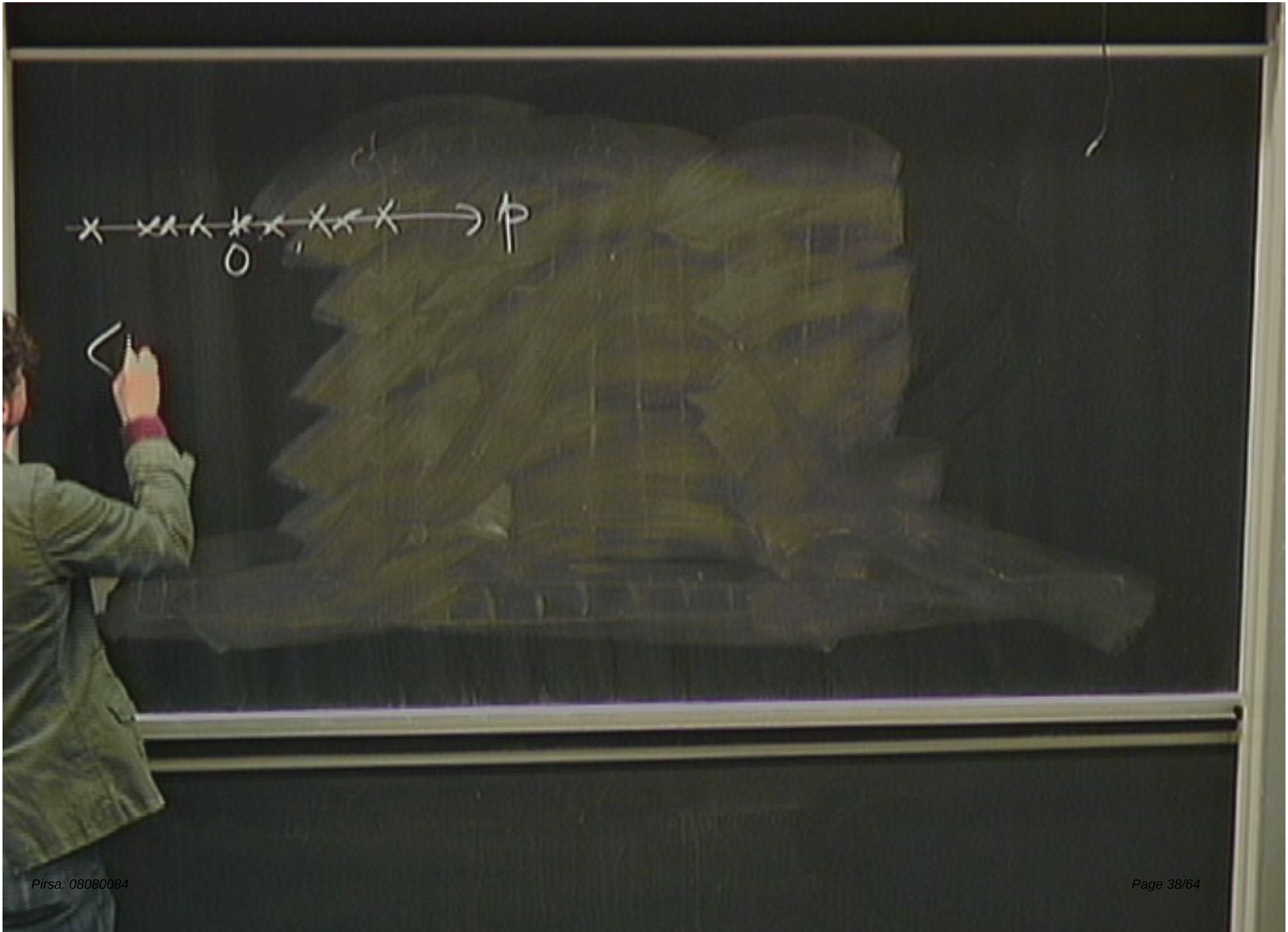
$$\langle x^2 \rangle = (\Delta x)^2 = A^2$$

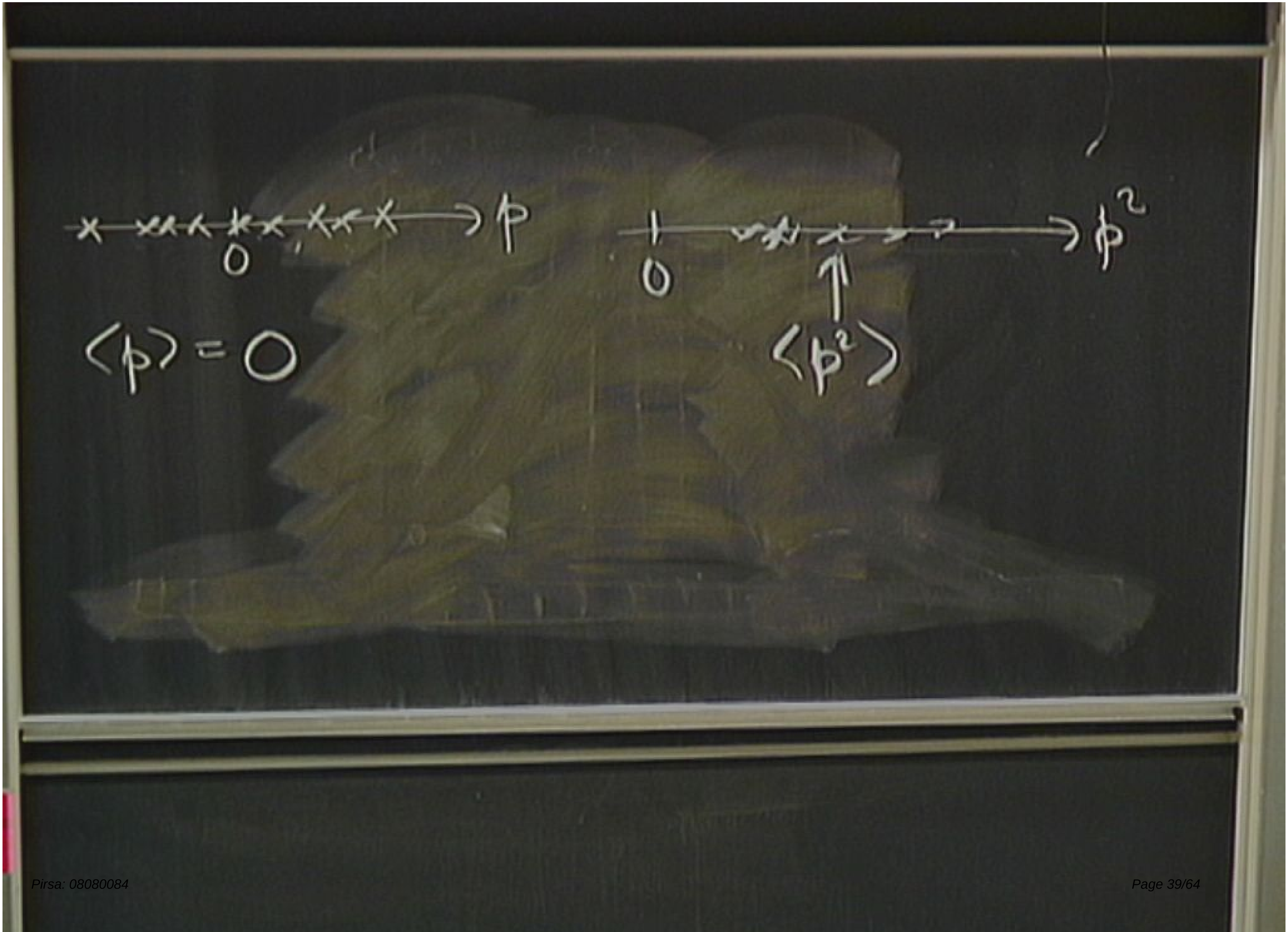
$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

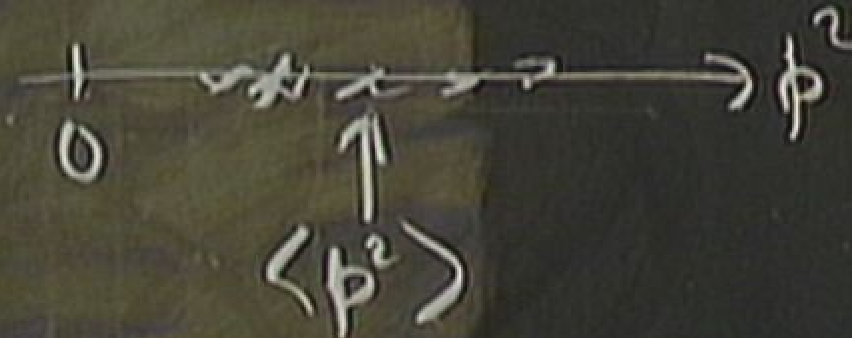
take many snapshots & average

$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$$



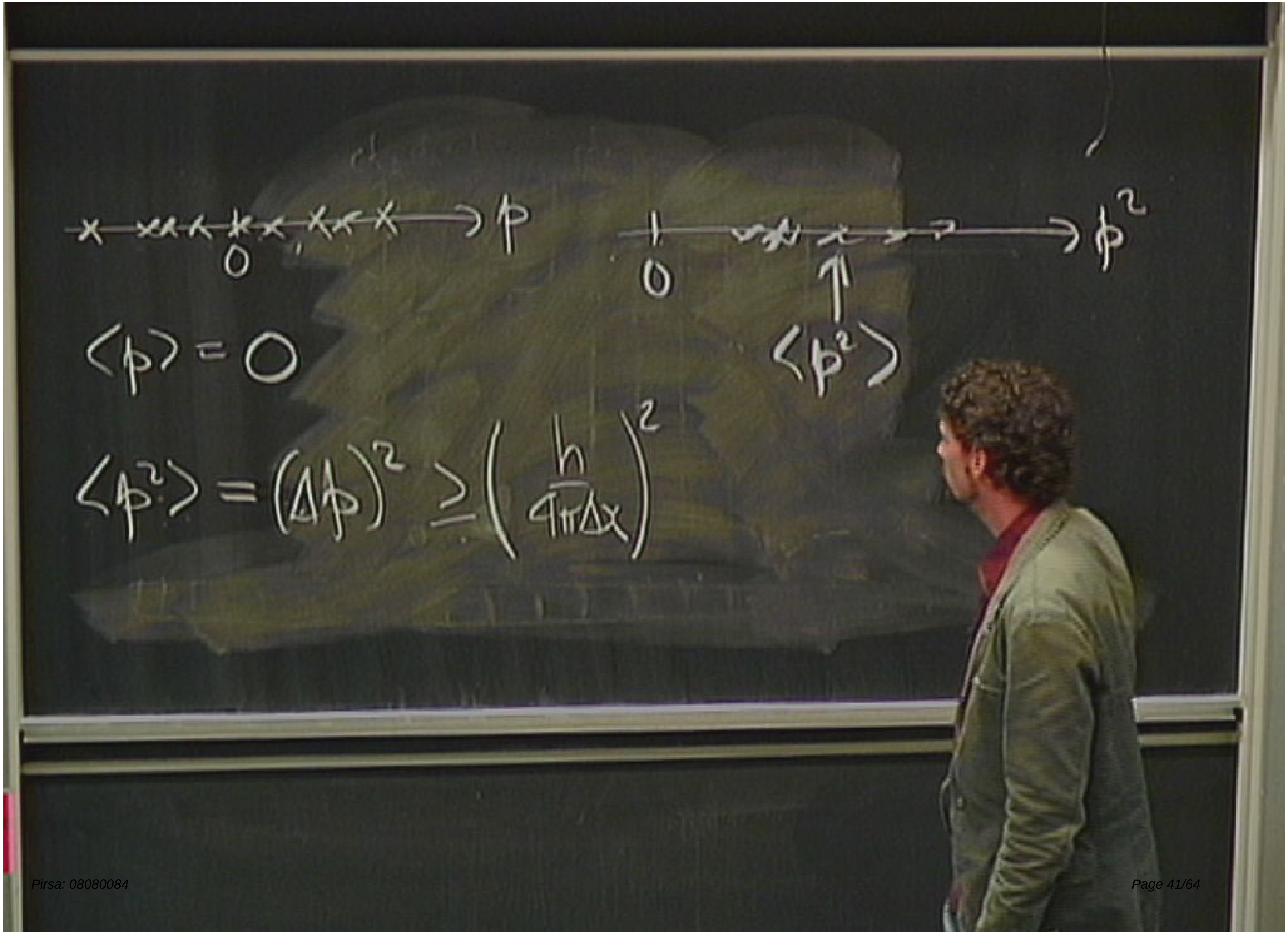



p
0
$\langle p \rangle = 0$
p^2
0
$\langle p^2 \rangle$



$$\langle p \rangle = 0$$

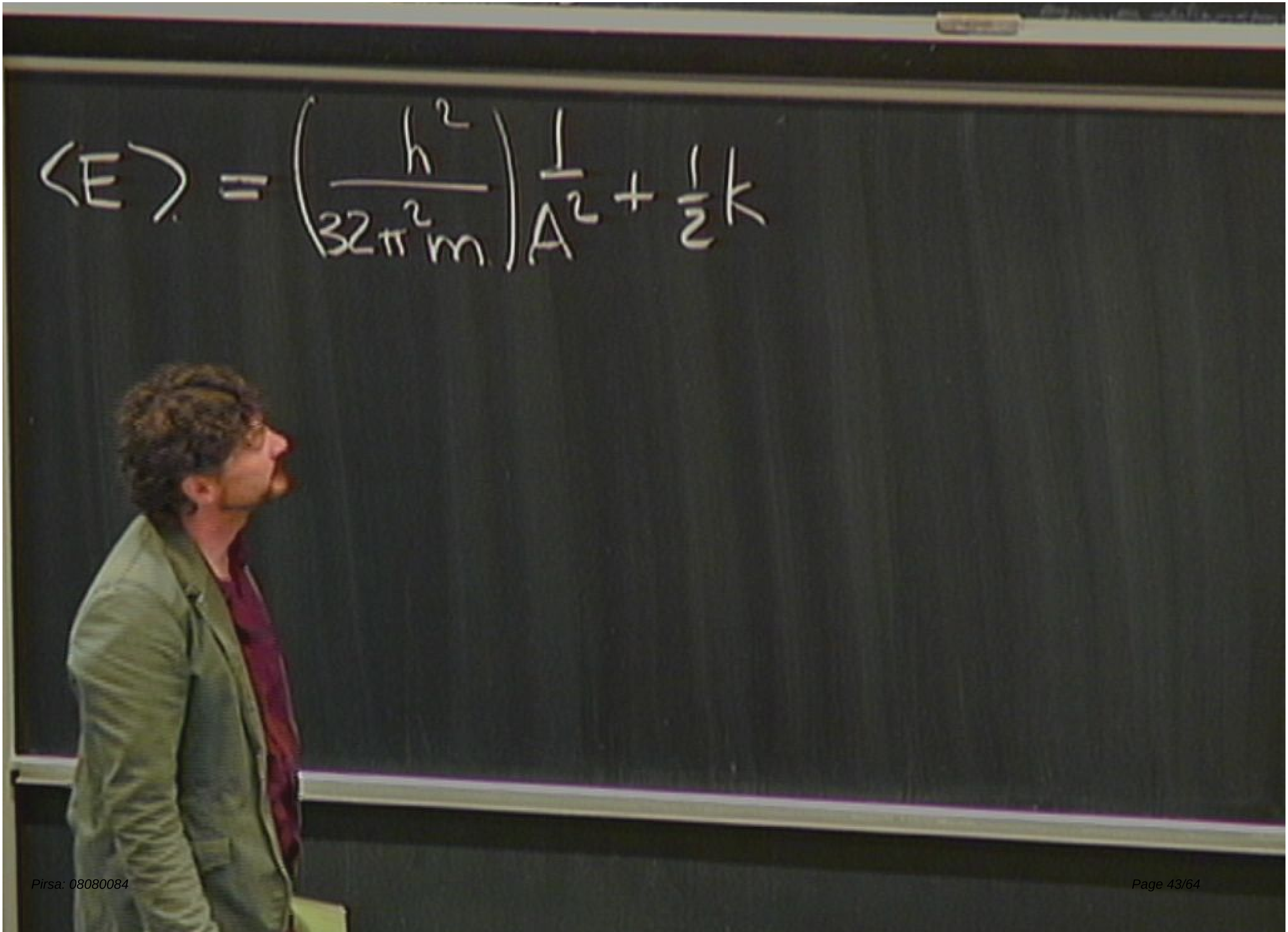
$$\langle p^2 \rangle = (\Delta p)^2$$

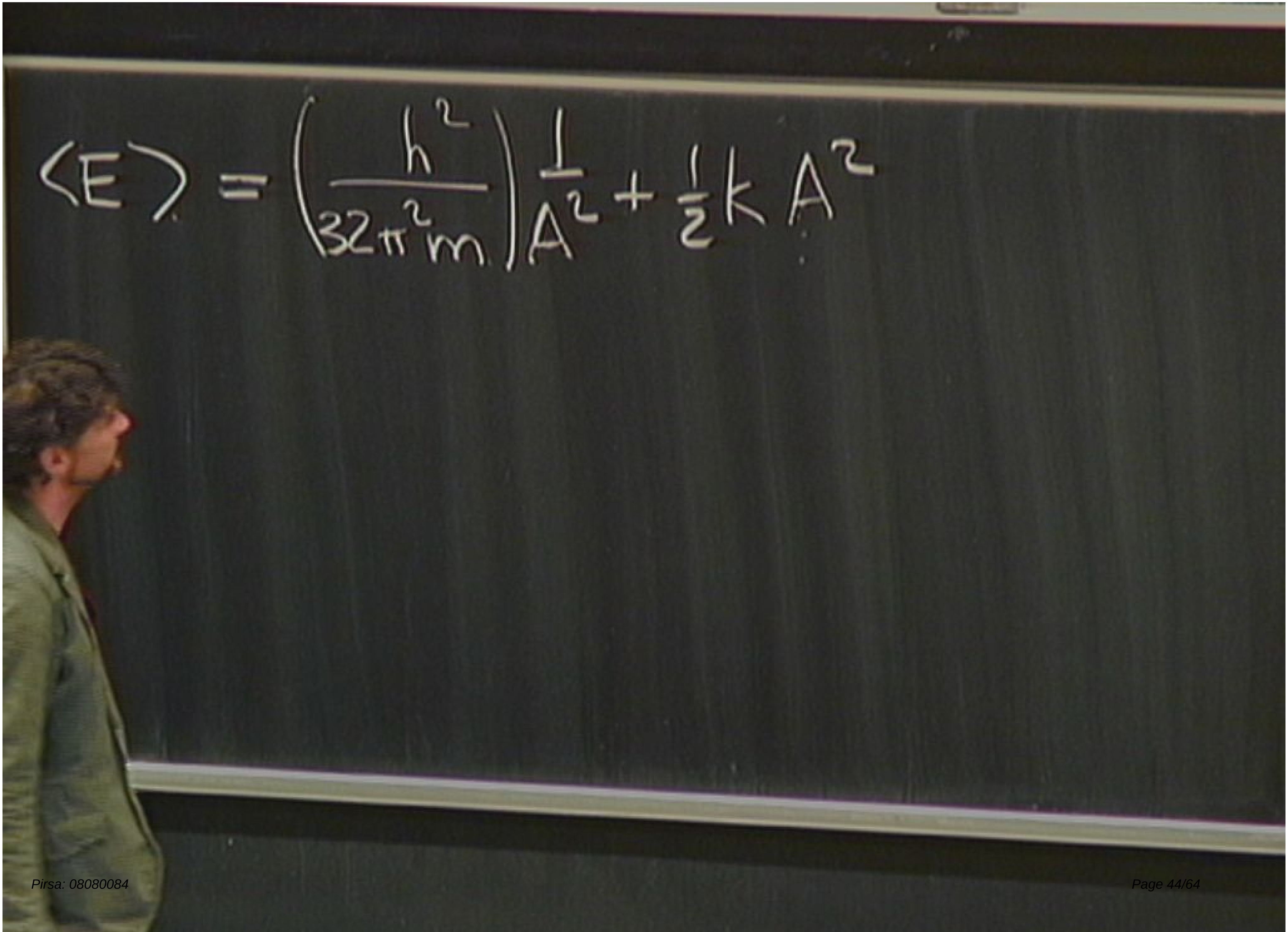

p
0
p^2
0
⟨p^2⟩
⟨p⟩ = 0
⟨p^2⟩ = (Δp)^2 ≥ (h / 4πΔx)^2

$$\langle p \rangle = 0$$

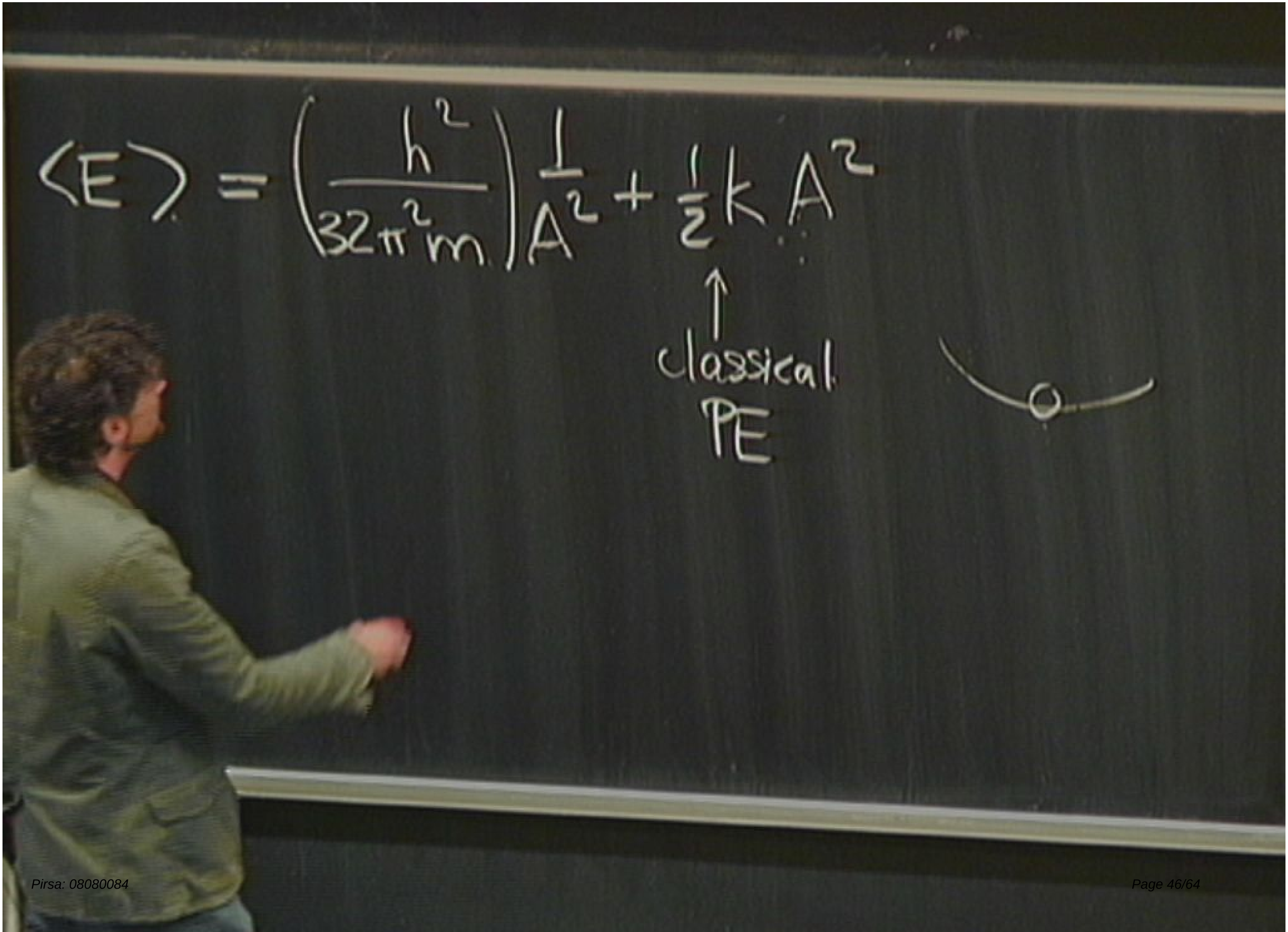
$$\langle p^2 \rangle$$

$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x} \right)^2 = \left(\frac{h}{4\pi A} \right)^2$$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}k$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \underbrace{\frac{1}{2}kA^2}_{\text{classical PE}}$$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$
classical
PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

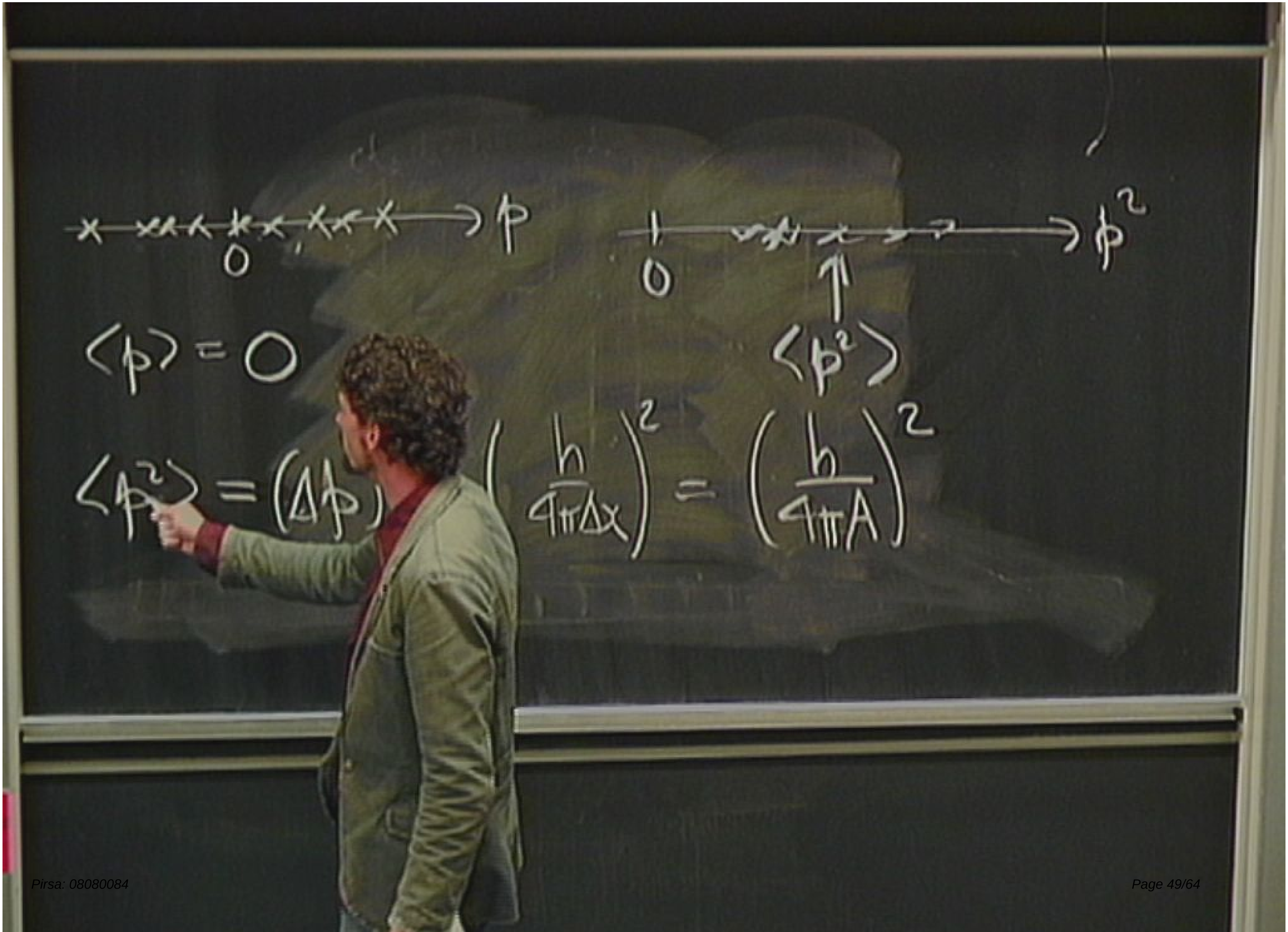
quantum

classical PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

quantum KE

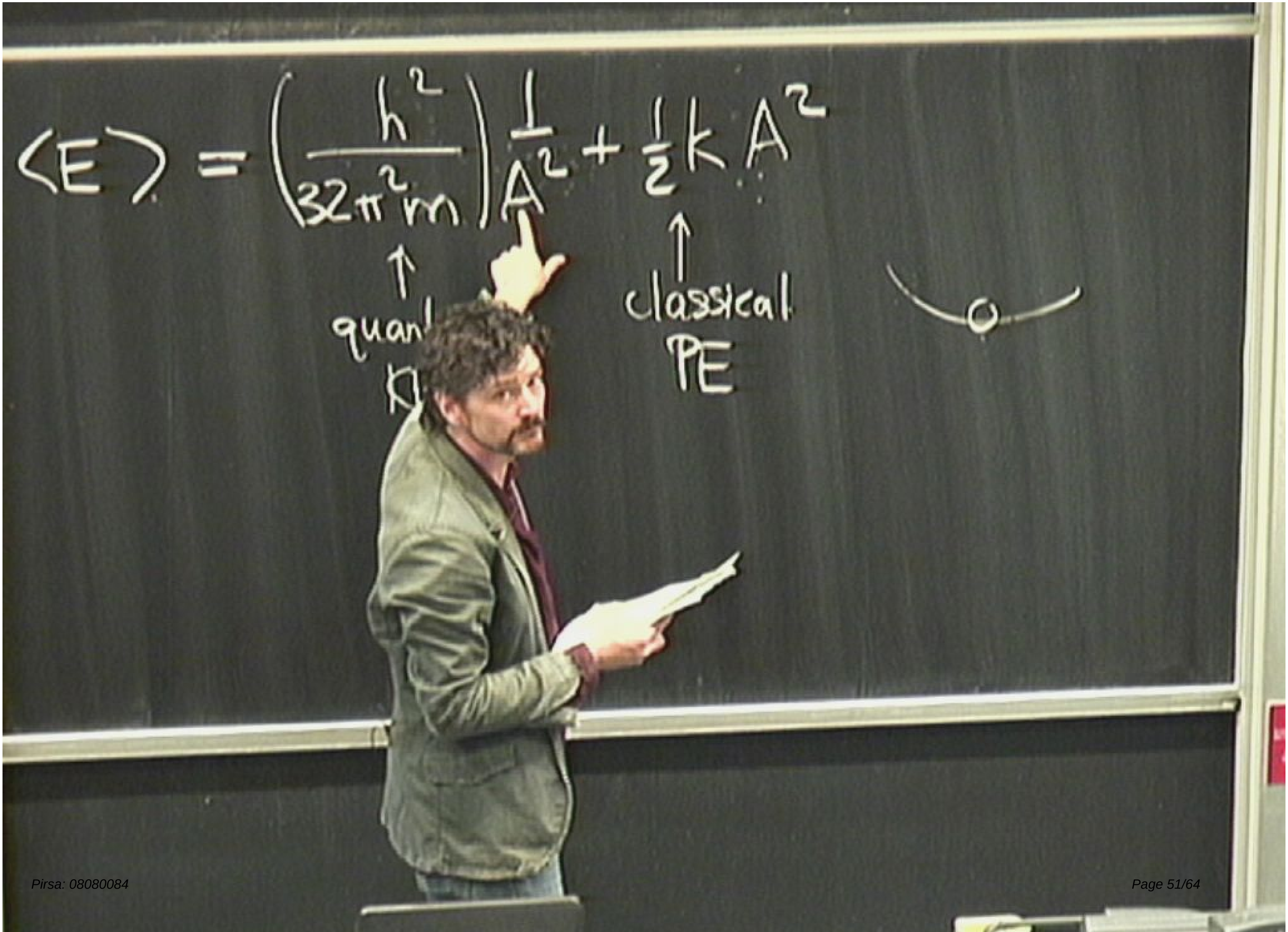
classical PE


p
0
p²
0
⟨p⟩ = 0
⟨p²⟩
⟨p²⟩ = (Δp)
(h / 4πΔx)² = (h / 4πA)²

$\langle p^2 \rangle$

$$\langle p \rangle = 0$$

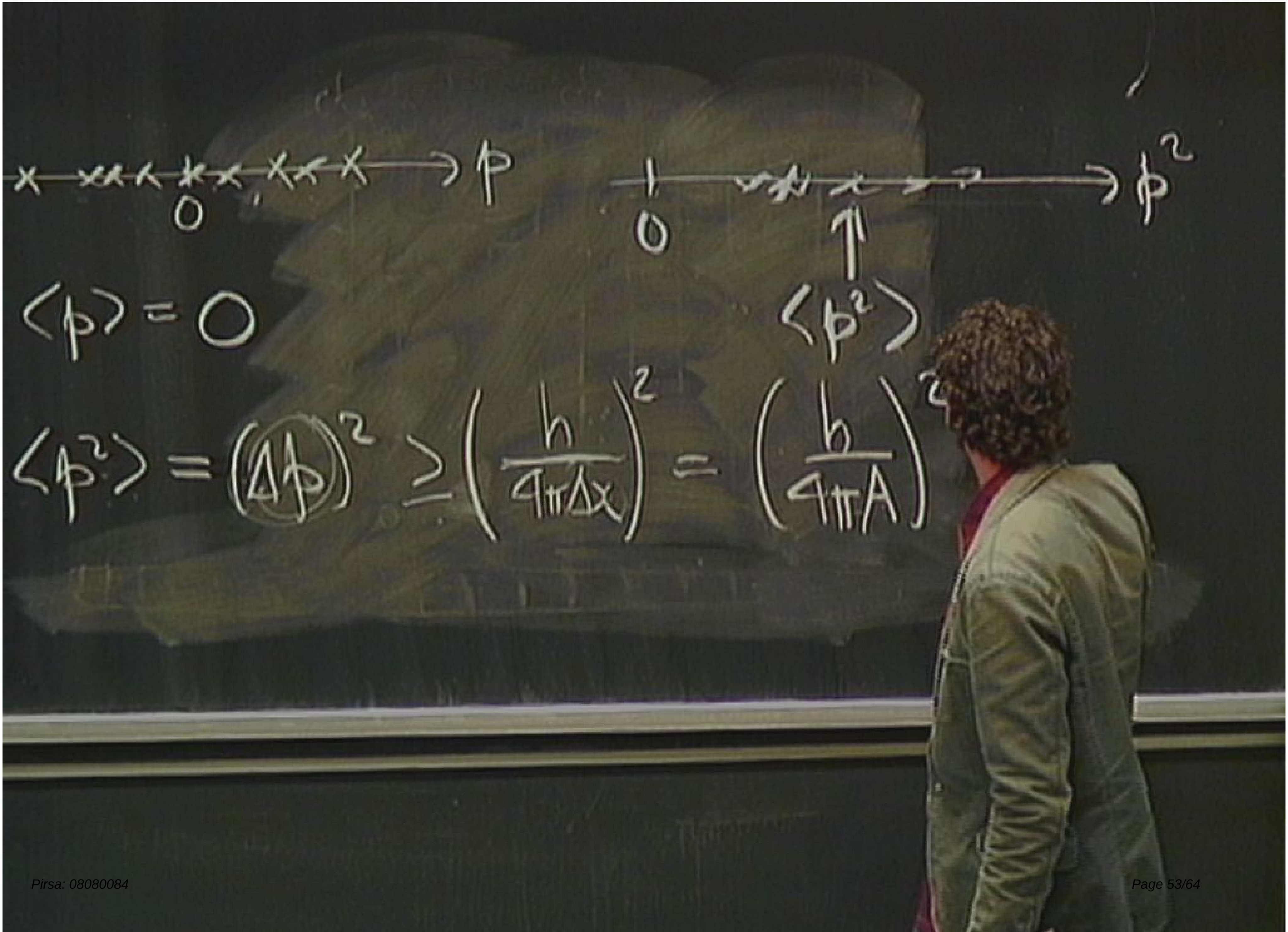
$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x}\right)^2 = \left(\frac{h}{4\pi A}\right)^2$$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2} k A^2$
classical
PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

quantum KE

classical PE

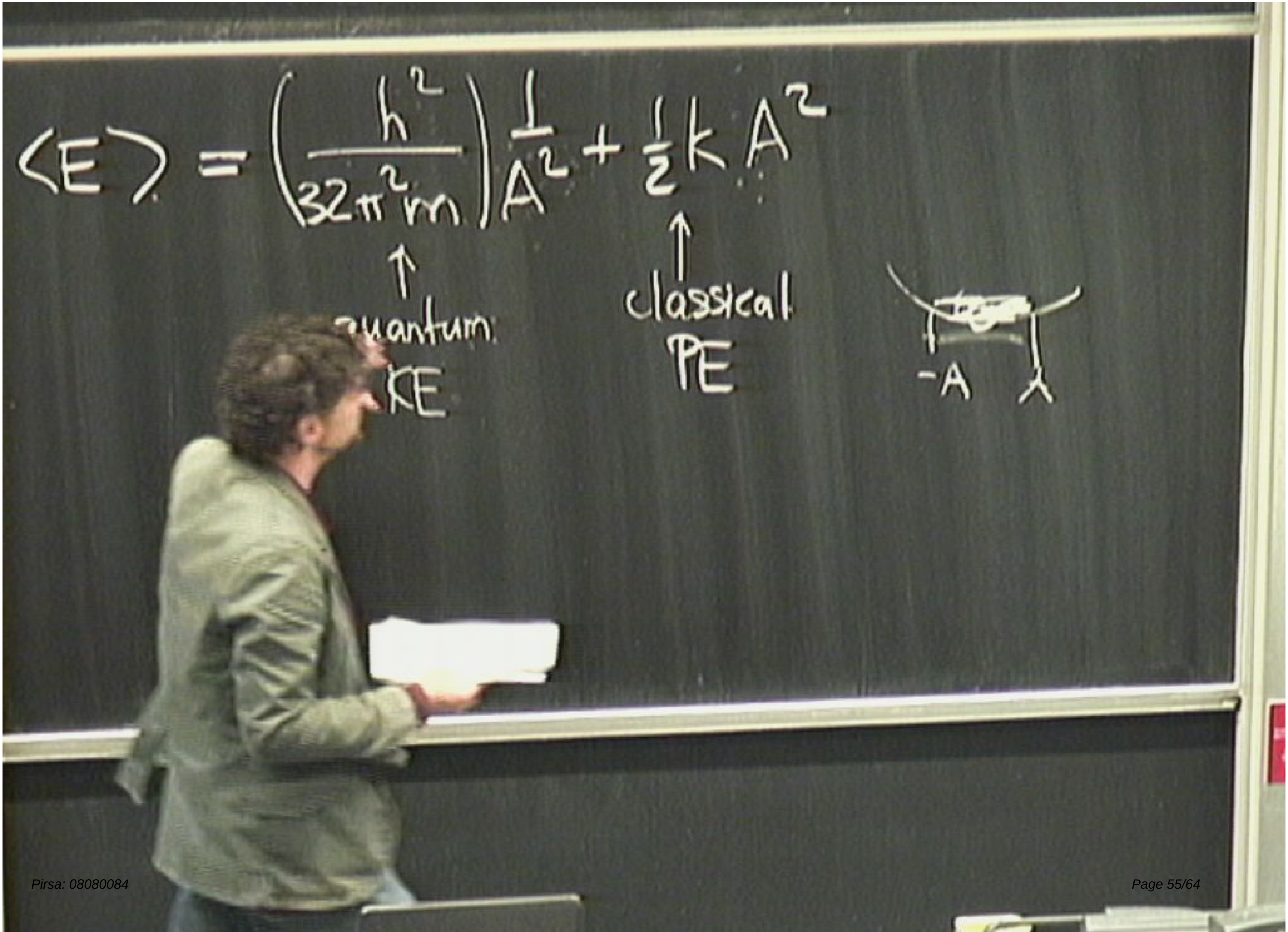

p
0
p^2
0
⟨p⟩ = 0
⟨p^2⟩
⟨p^2⟩ = (Δp)^2 ≥ (h / 4πΔx)^2 = (h / 4πA)^2

$$\langle p \rangle = 0$$

$$\langle p^2 \rangle$$

$$\Delta x = A$$

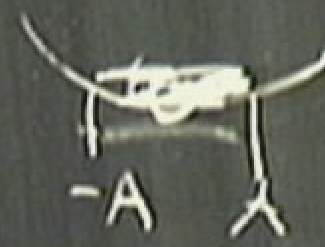
$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x} \right)^2 = \left(\frac{h}{4\pi A} \right)^2$$

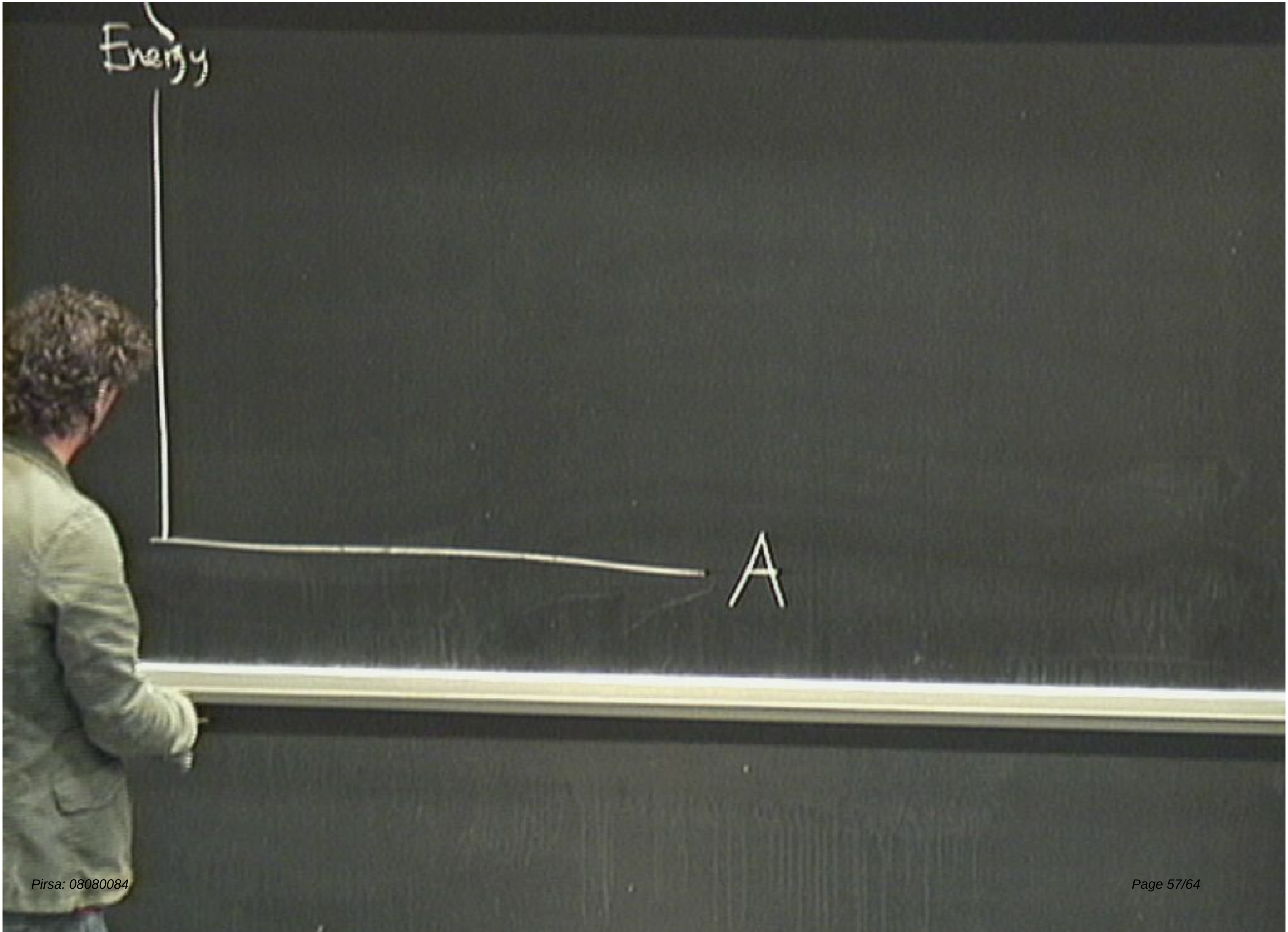

$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$
quantum
KE
classical
PE
$-A$
A

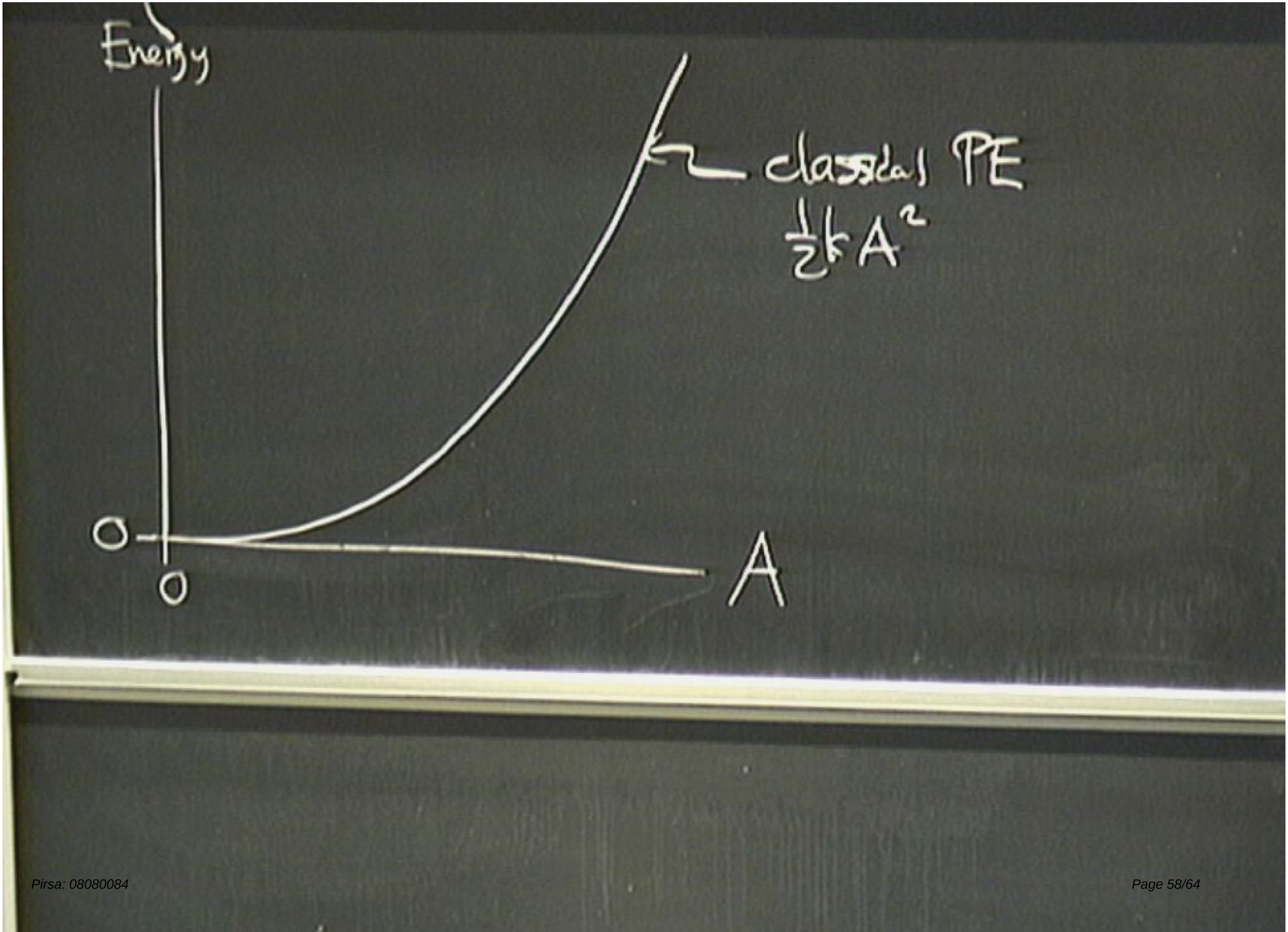
$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

quantum KE

classical PE




Energy
A

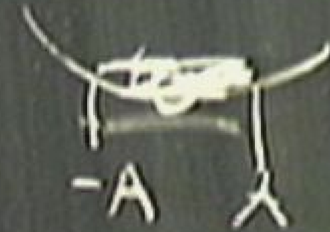


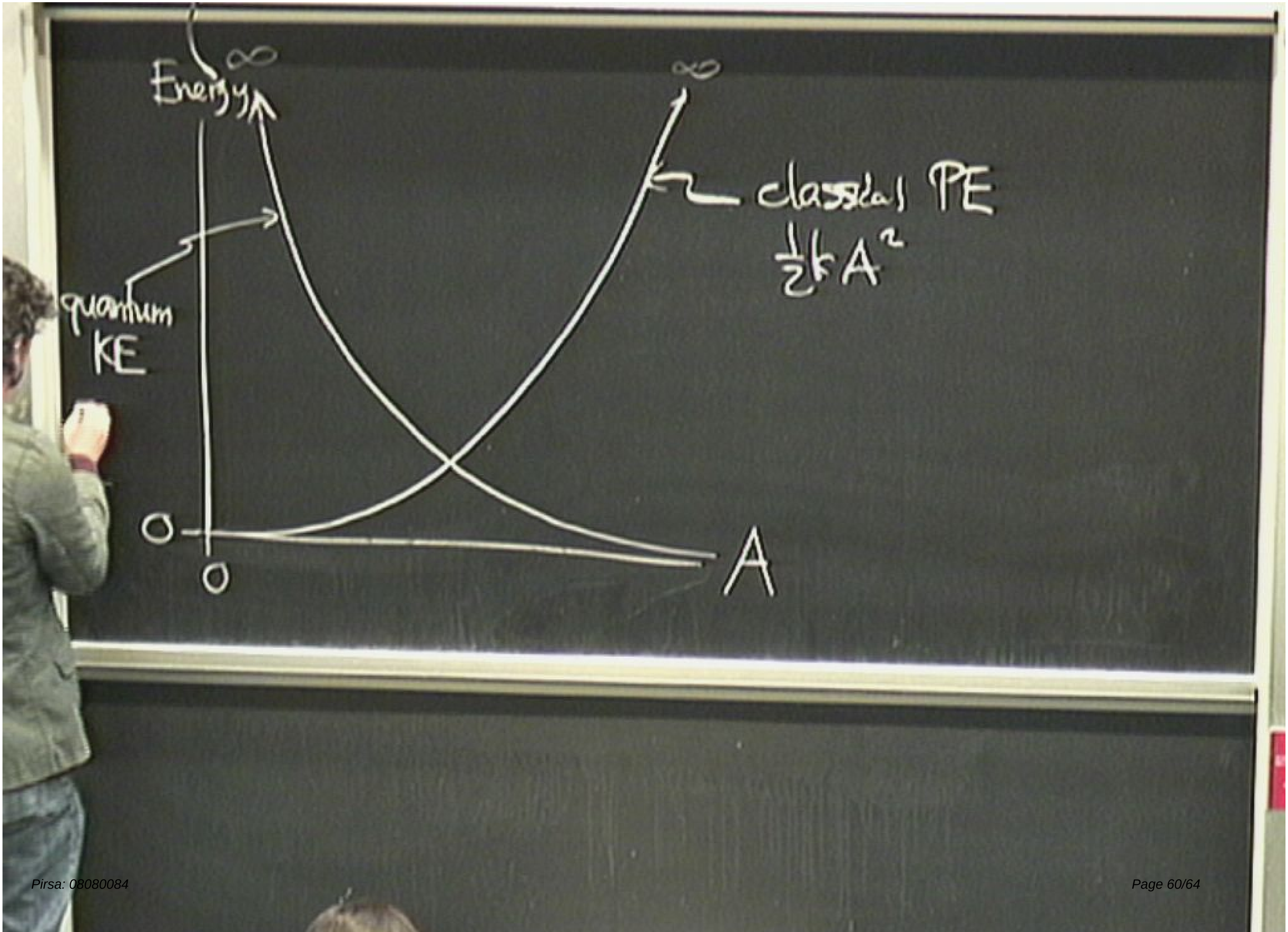
Energy
classical PE
$\frac{1}{2}kA^2$
0
0
A

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

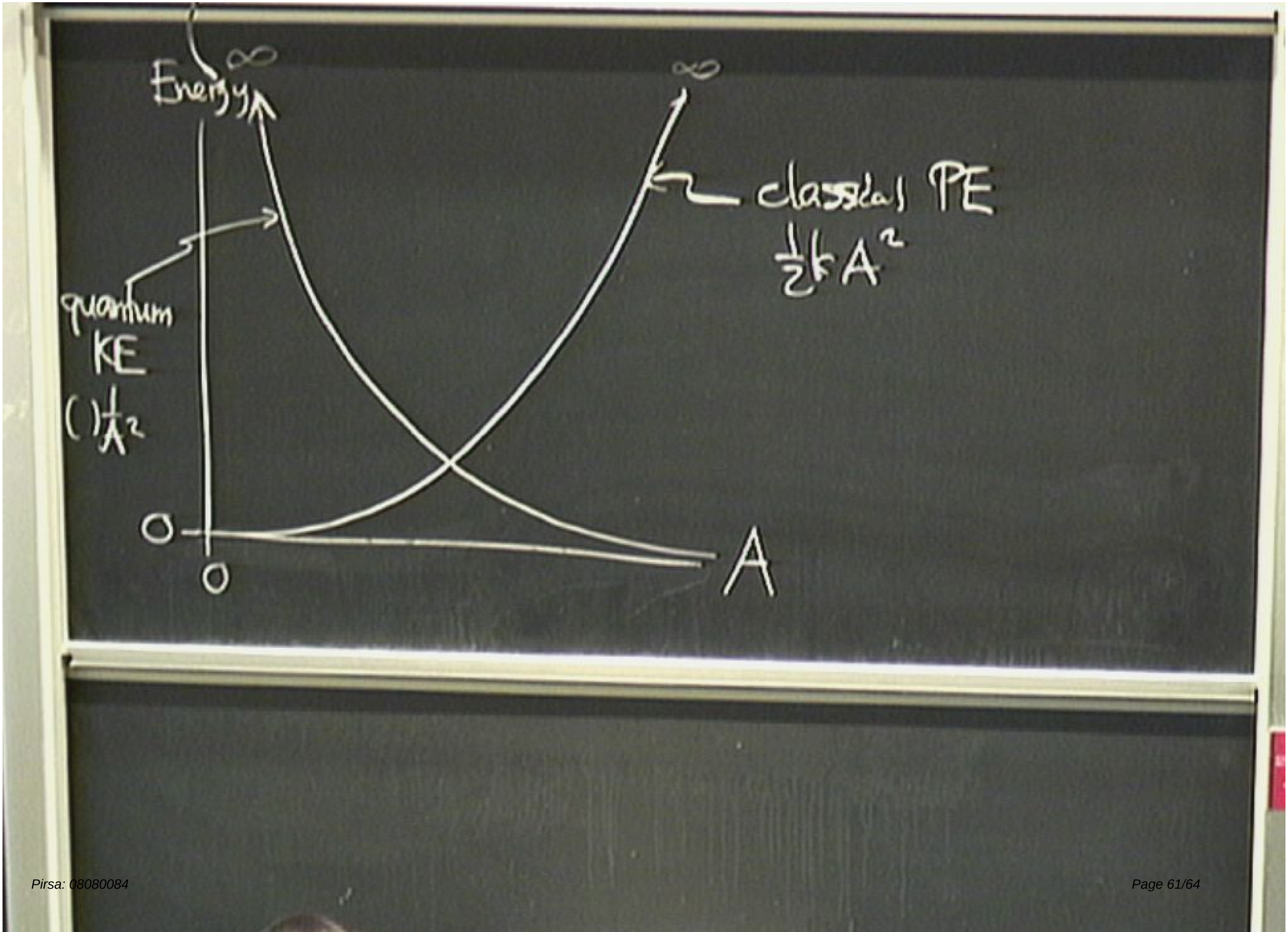
quantum KE

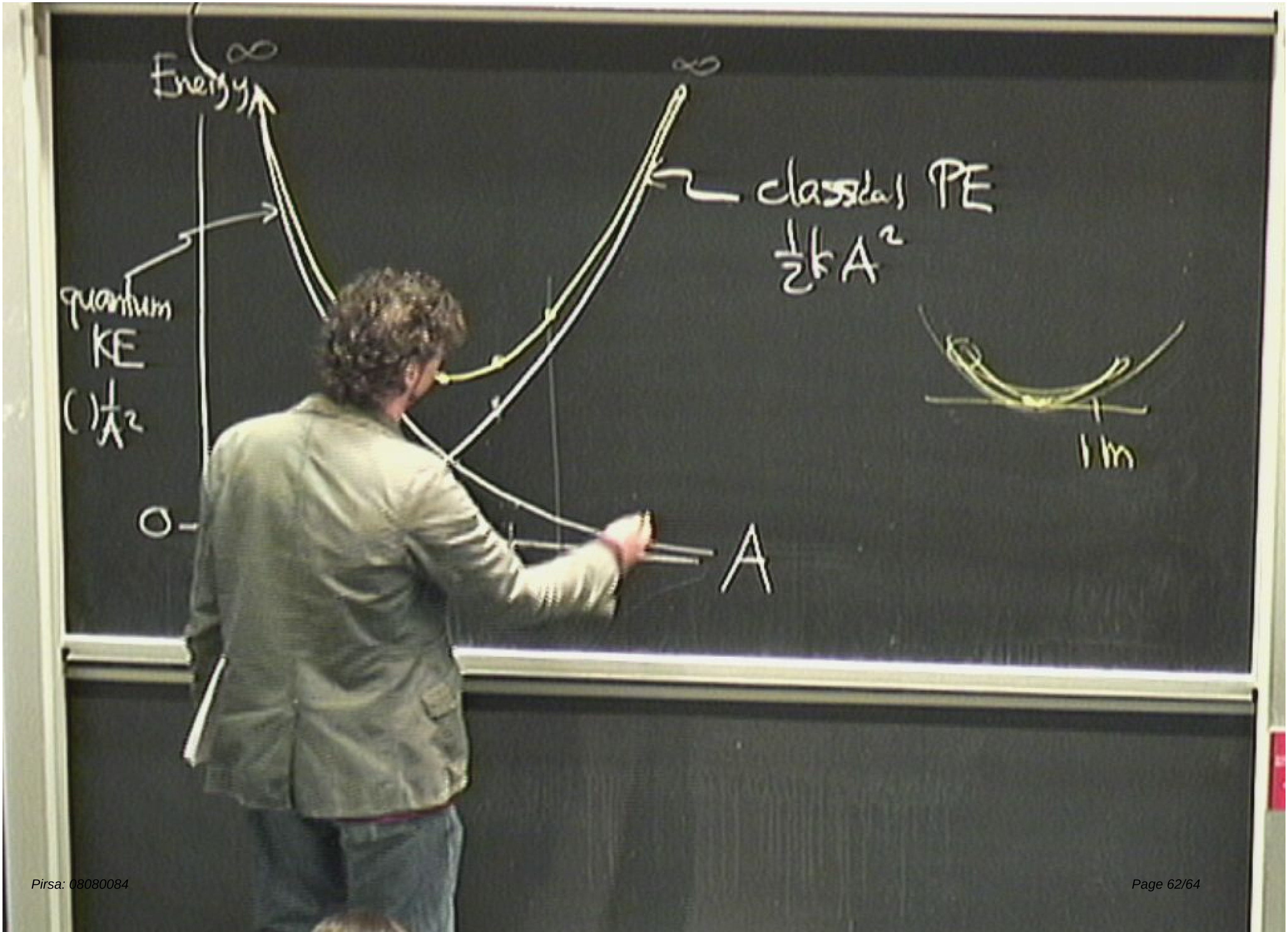
classical PE

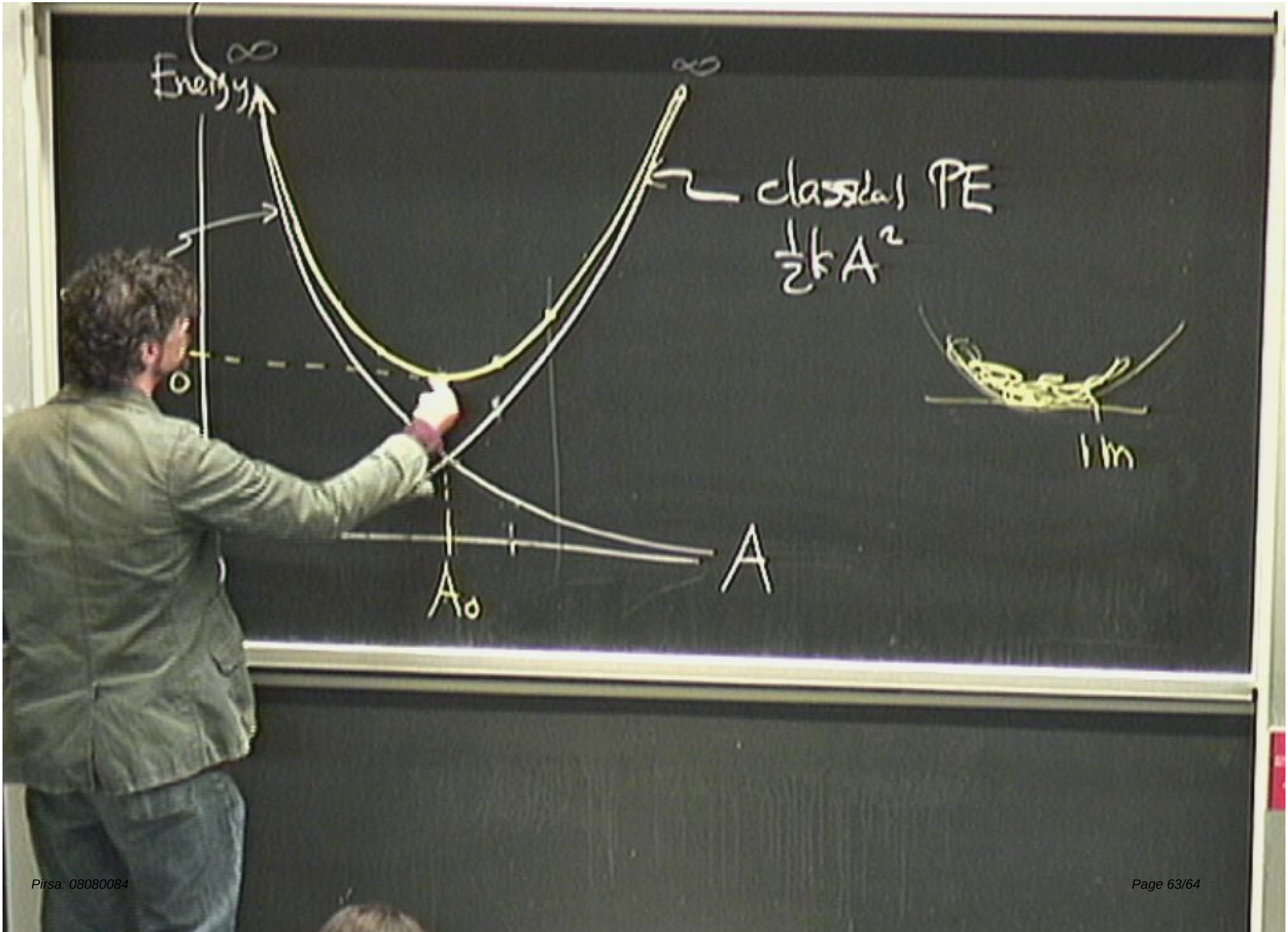


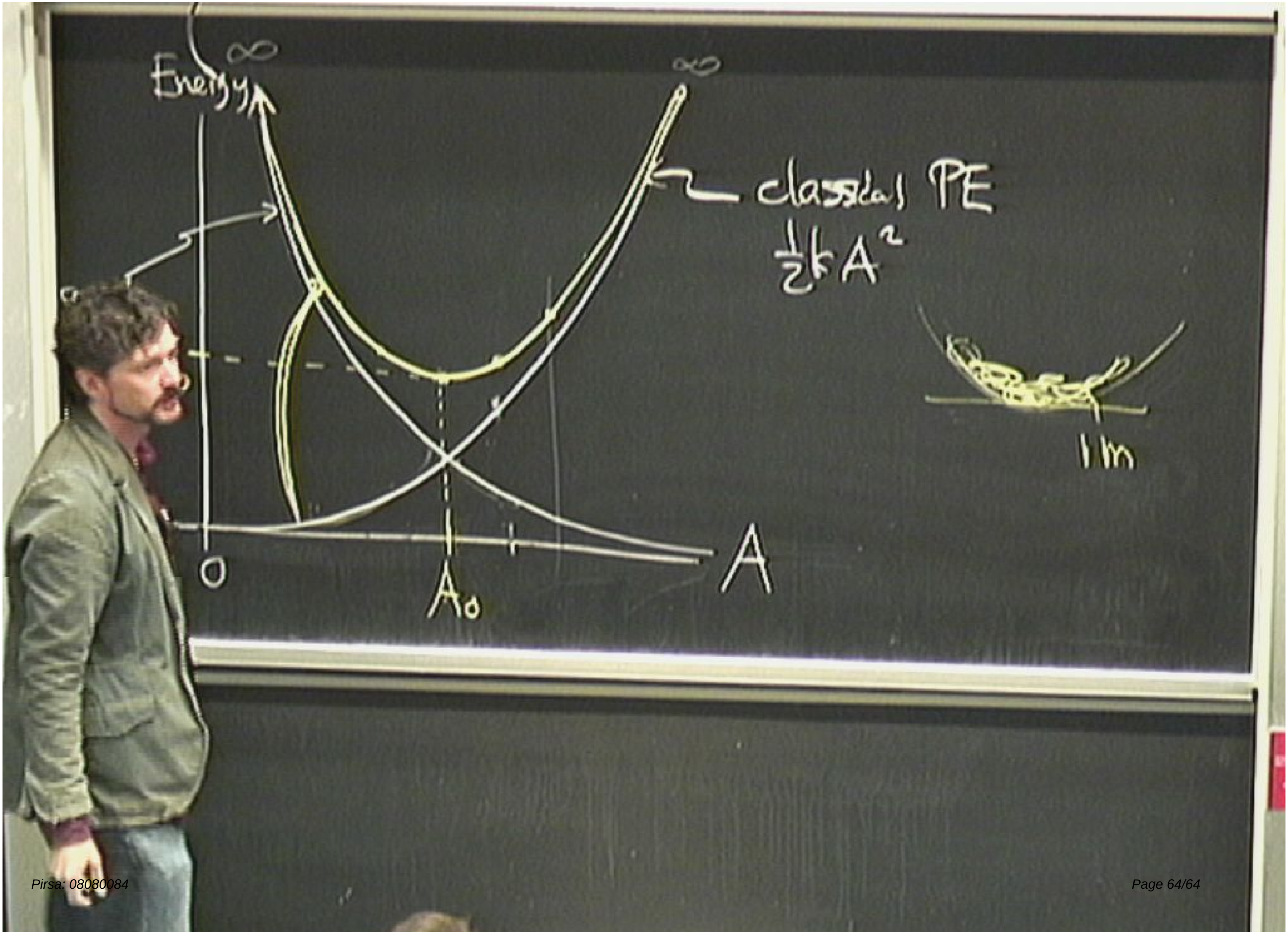


Energy
∞
∞
classical PE
$\frac{1}{2}kA^2$
quantum KE
0
0
A


∞
Energy
∞
classical PE
$\frac{1}{2}kA^2$
quantum
KE
$(\ )\frac{1}{A^2}$
0
0
A


∞
Energy
∞
classical PE
½kA²
quantum KE
( ) 1/A²
0
A
1 m


∞
Energy
∞
classical PE
$\frac{1}{2}kA^2$
0
A
A_0
1m


∞
Energy
∞
classical PE
$\frac{1}{2}kA^2$
1m
0
A_0
A