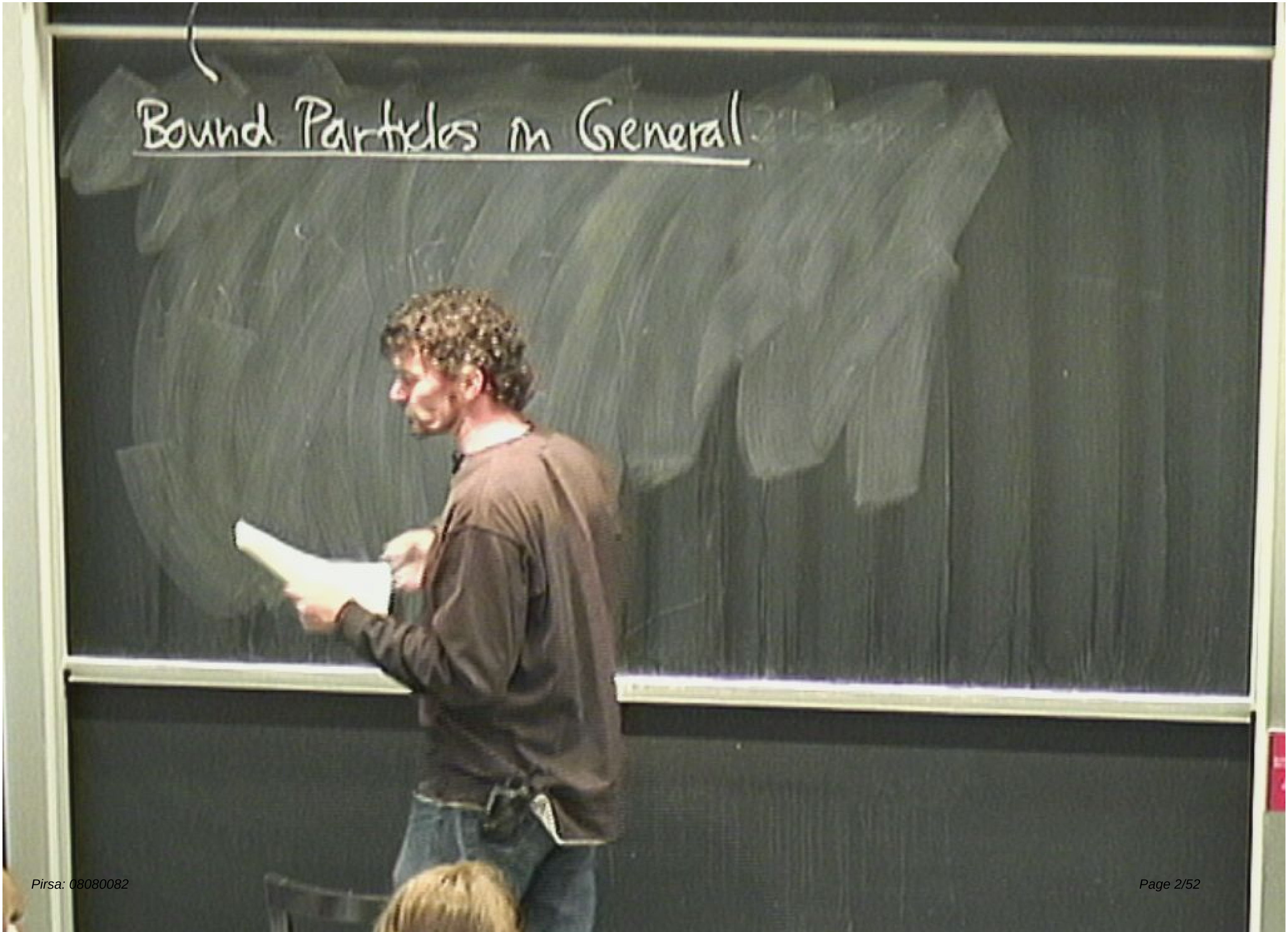
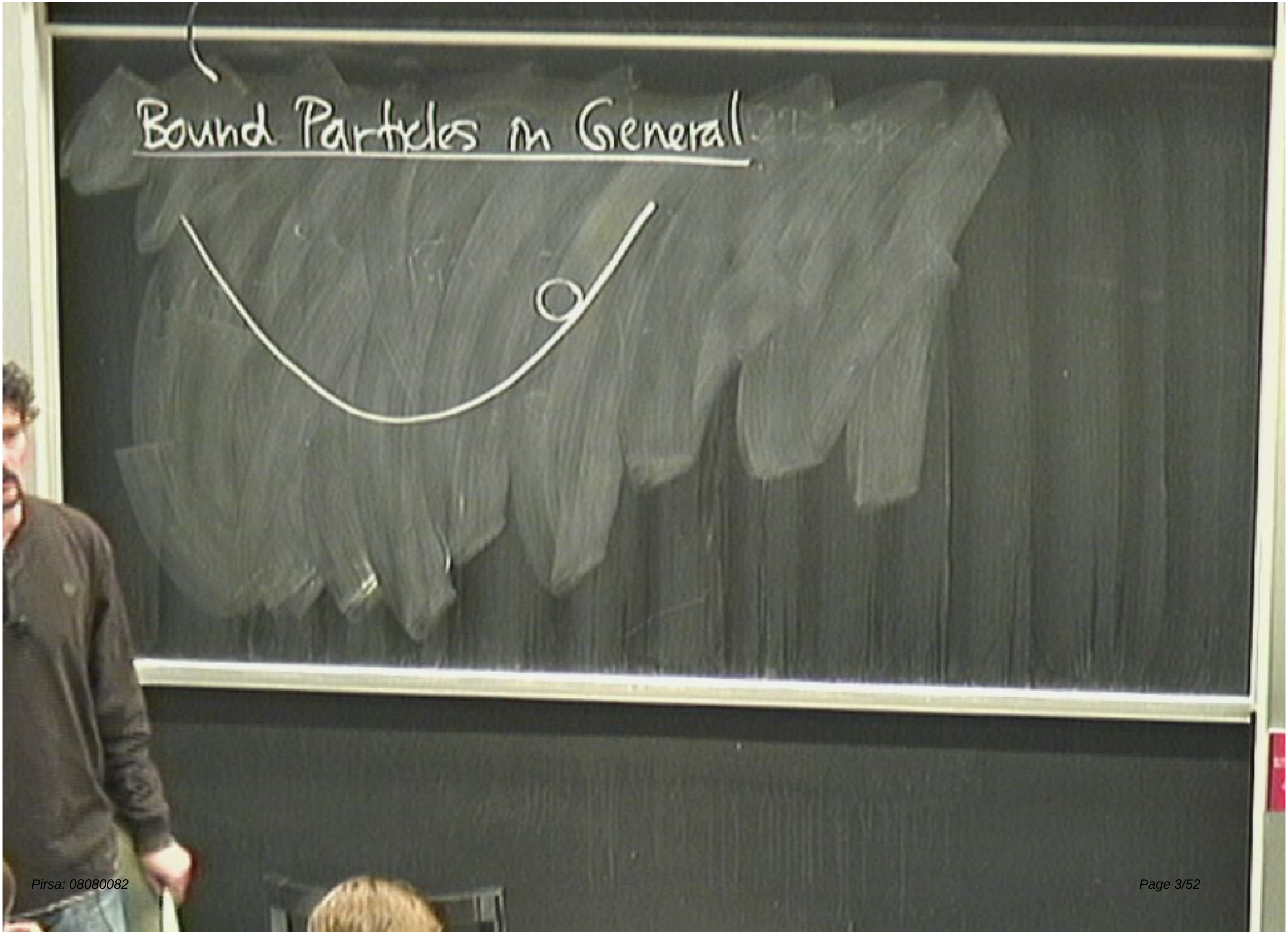
Title: Quantum Mechanics 7 - The Quantum Harmonic Oscillator

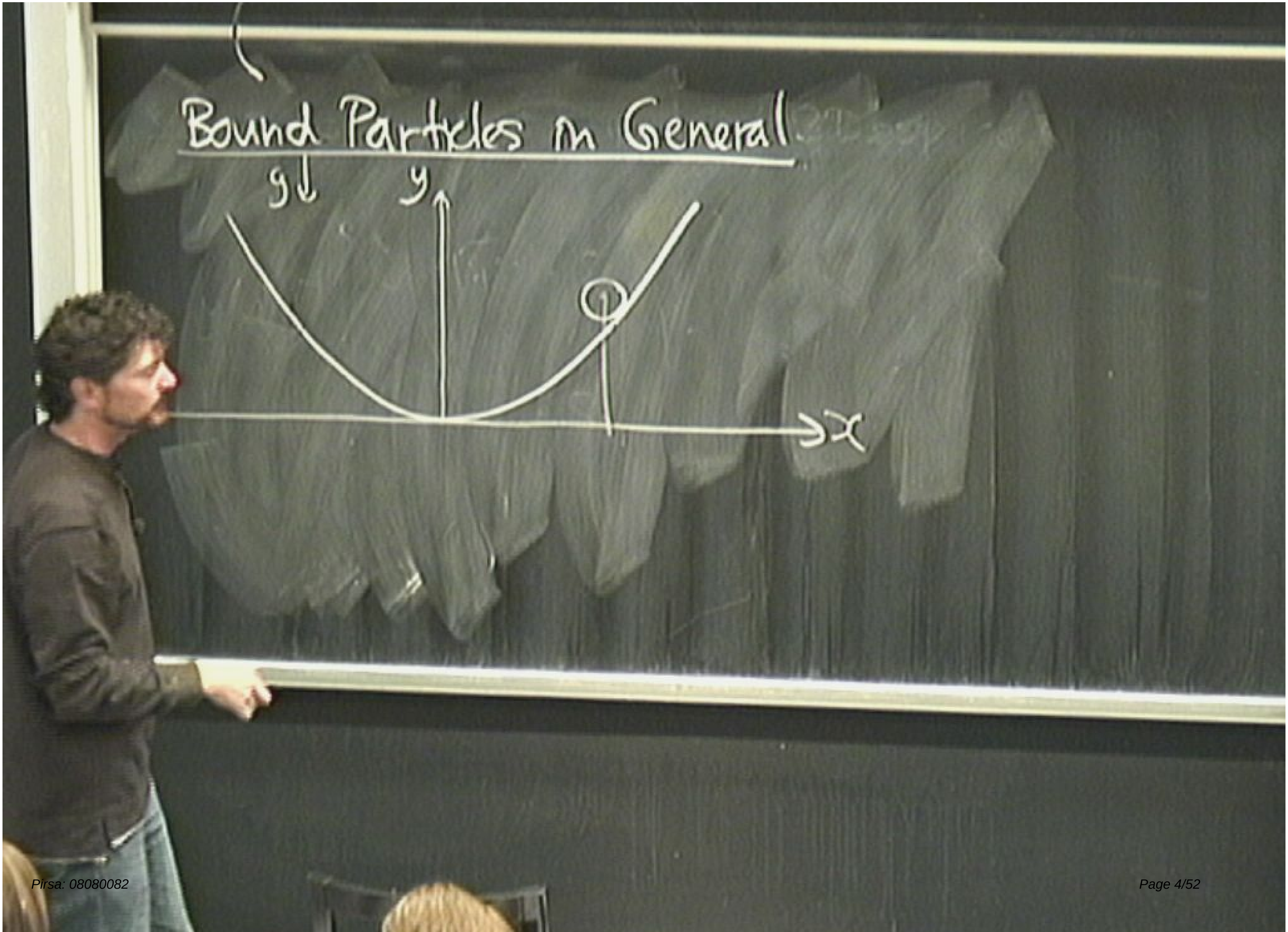
Date: Aug 12, 2008 10:30 AM

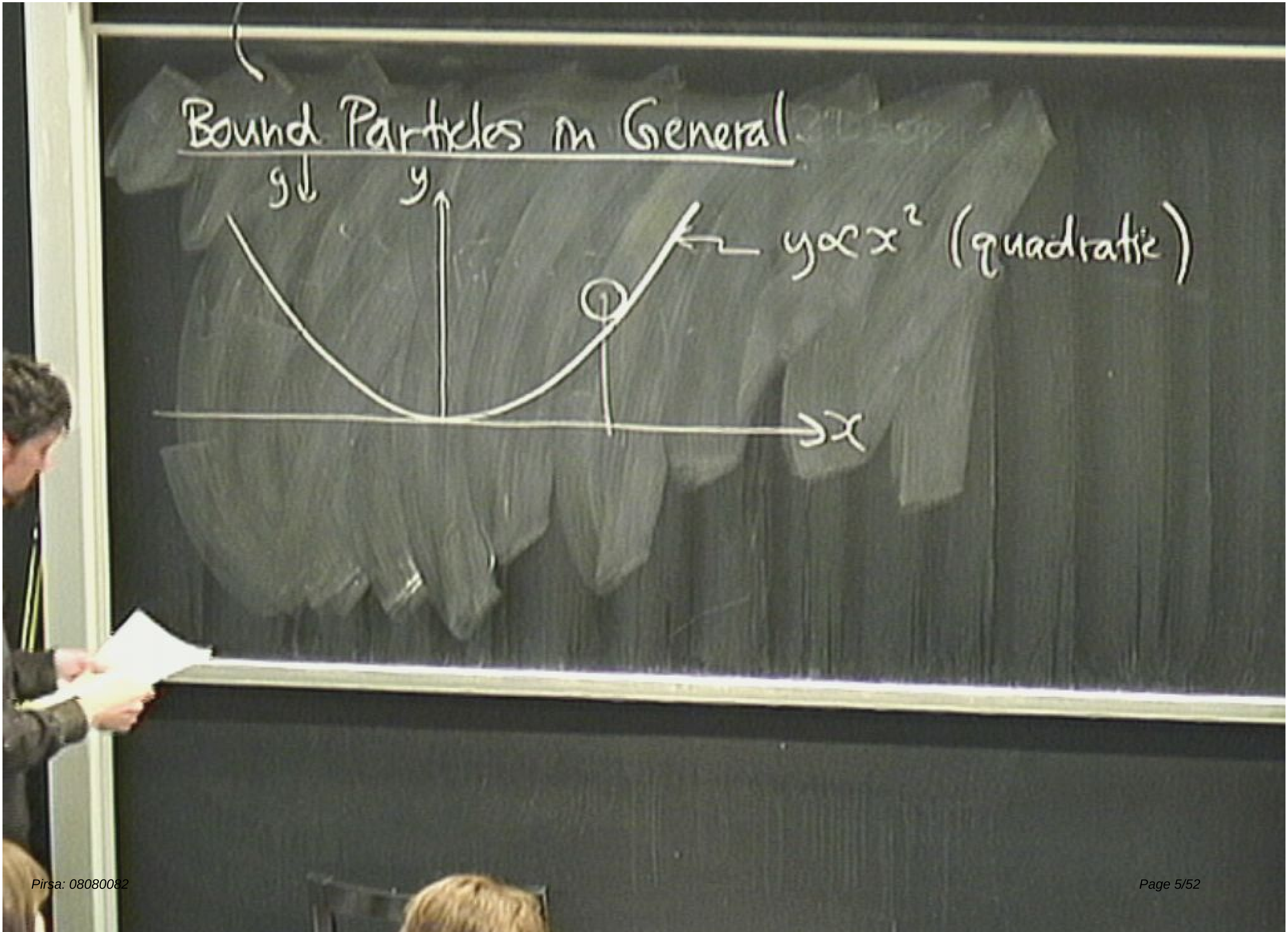
URL: http://pirsa.org/08080082

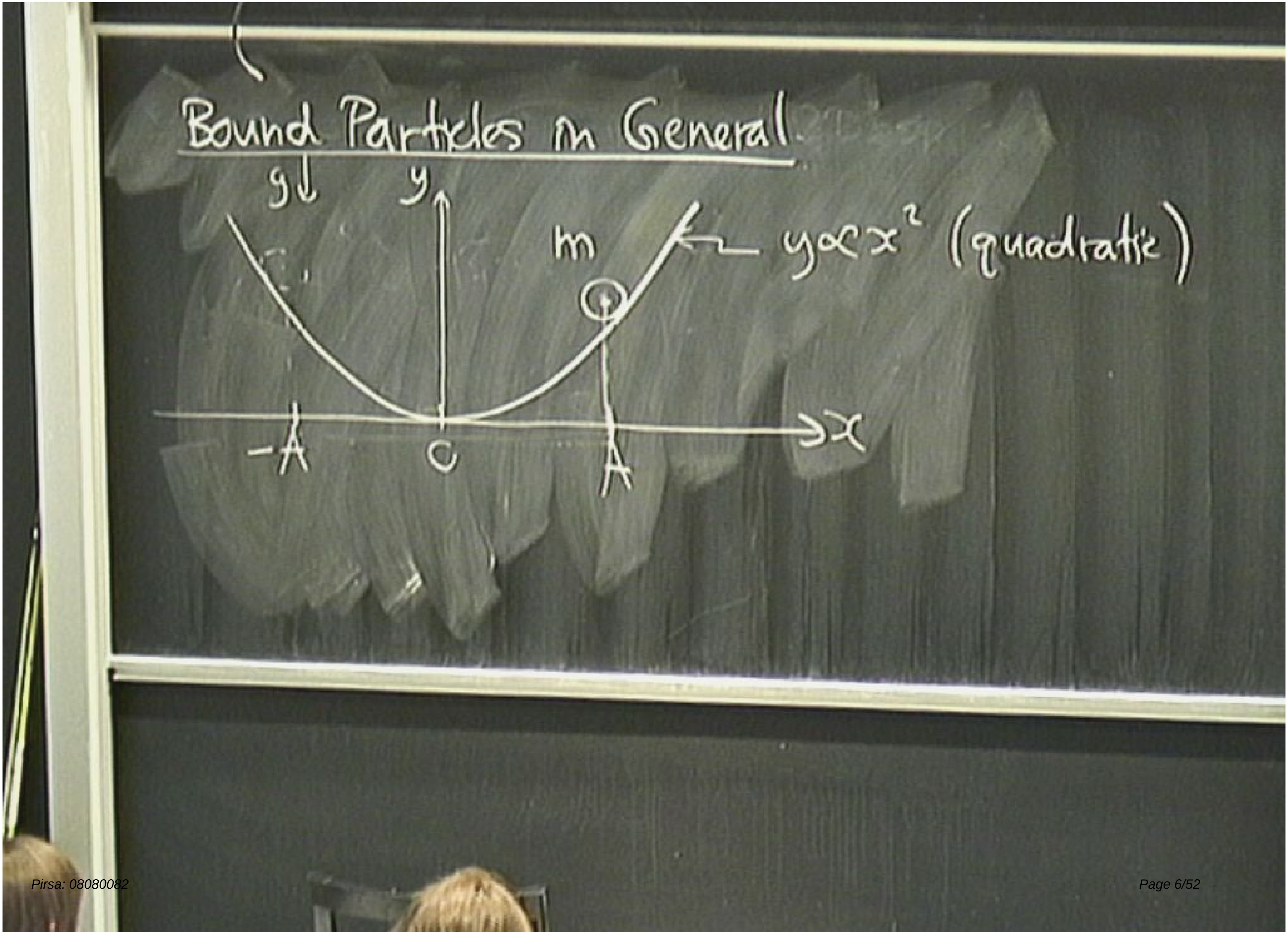
Abstract: Taking our intuitive understanding of the quantum world gained by studying a particle in a one-dimensional box, we generalize to understand a quantum harmonic oscillator.
Learning Outcomes:
â€¢ Introduction to the classical physics of a ball rolling back and forth in a bowl, a simple example of a very important type of bounded motion called a â€œharmonic oscillator.â€•
â€¢ The quantization of allowed energies of a harmonic oscillator: even spacing between energy levels, and zero point energy.
â€¢ Being able to sketch the allowed wavefunctions and particle probability patterns of a quantum harmonic oscillator, including a new phenomenon called â€œtunnelling.â€•

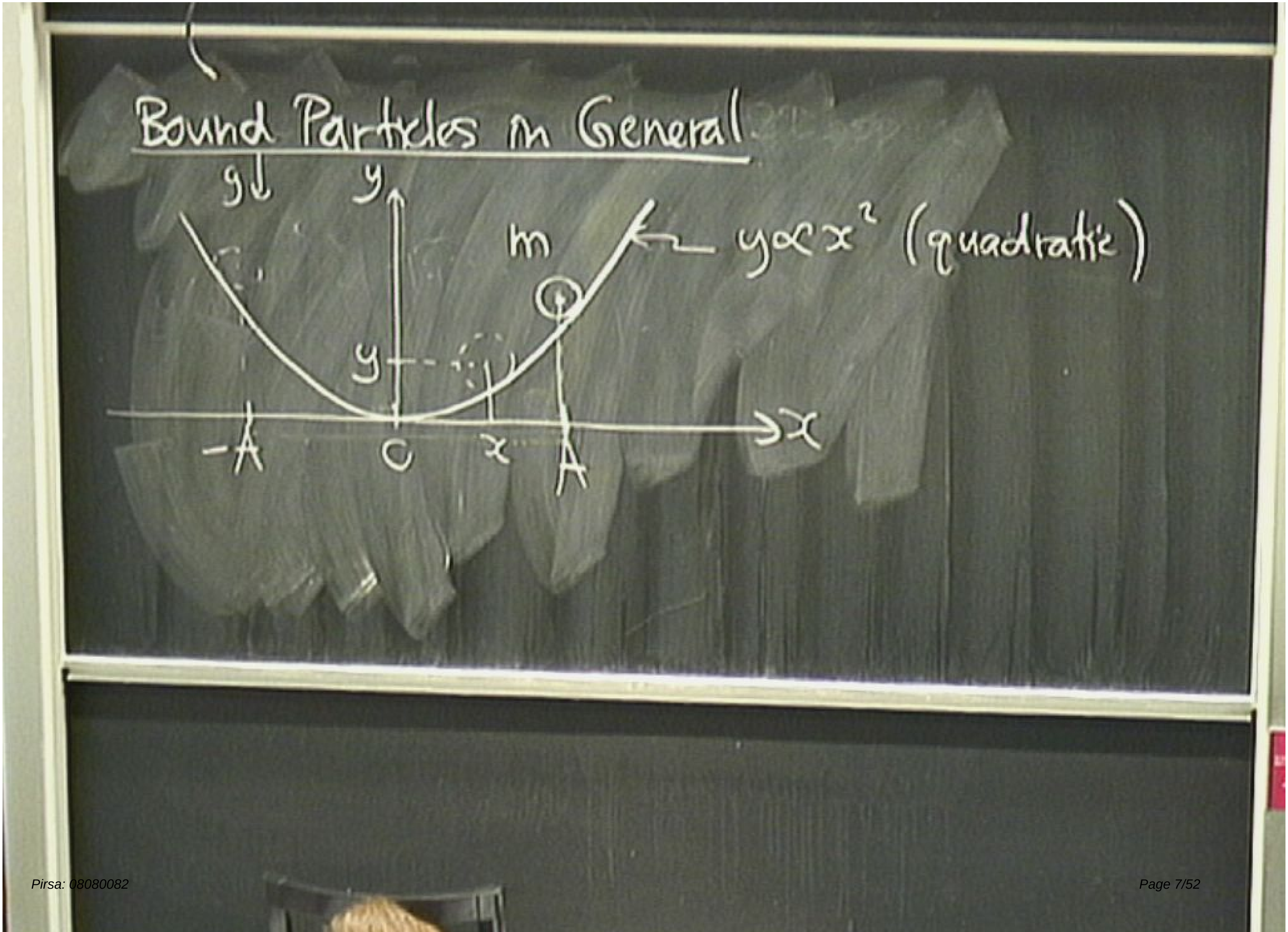

Bound Particles in General

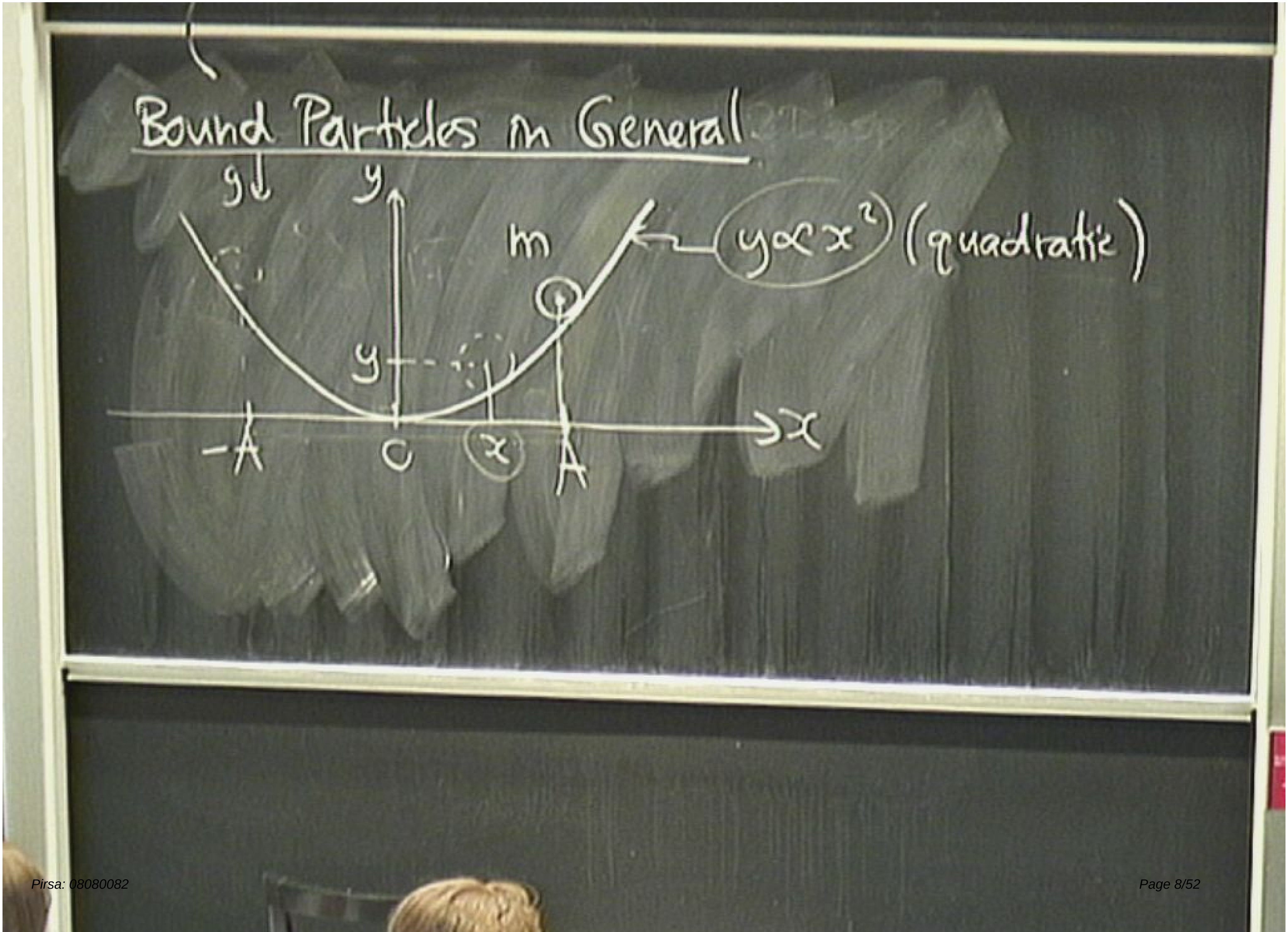

Bound Particles in General


Bound Particles in General
g
y
x

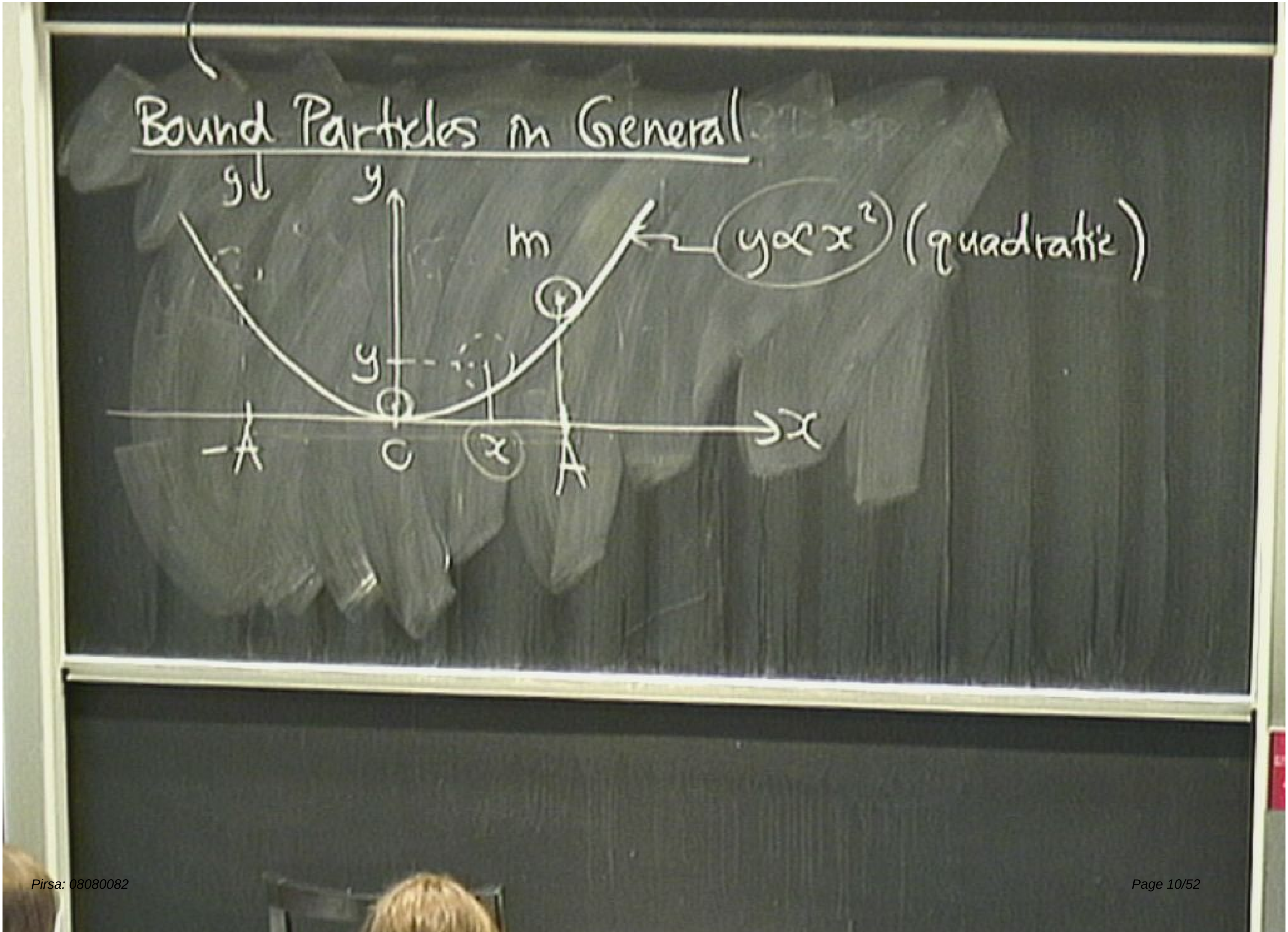

Bound Particles in General
g
y
y ∝ x² (quadratic)
x


Bound Particles in General
g
y
m
$y \propto x^2$ (quadratic)
$-A$
0
A
x


Bound Particles in General
g
y
m
y ∝ x² (quadratic)
y
x
-A
0
x
A


Bound Particles in General
g
y
m
y ∝ x² (quadratic)
y
-A
0
x
A
x

$$PE = mgy = \frac{1}{2}kx^2$$


Bound Particles in General
g
y
m
y
-A
0
x
A
x
y ∝ x²
(quadratic)

$$PE = mgy = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

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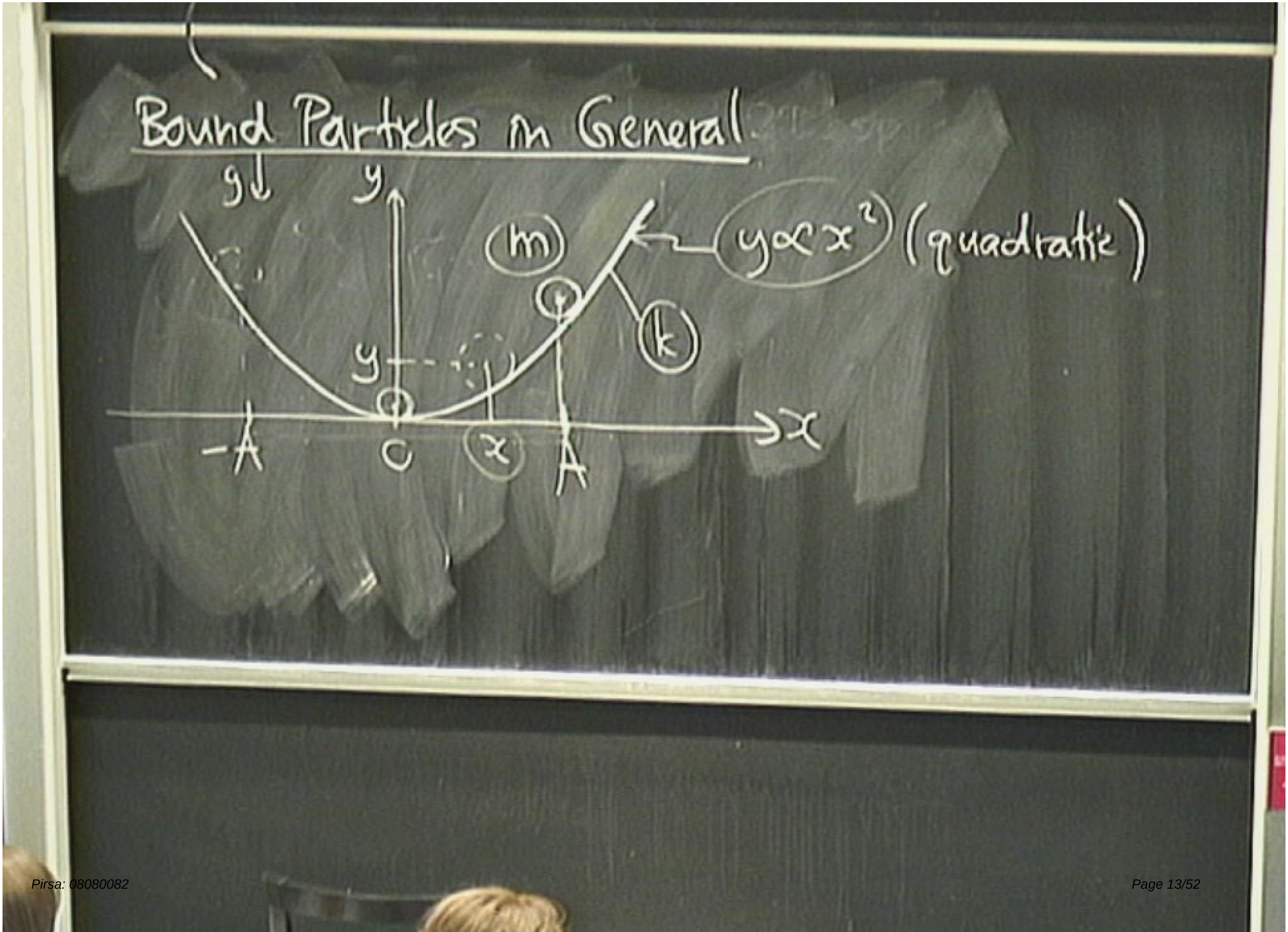
$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

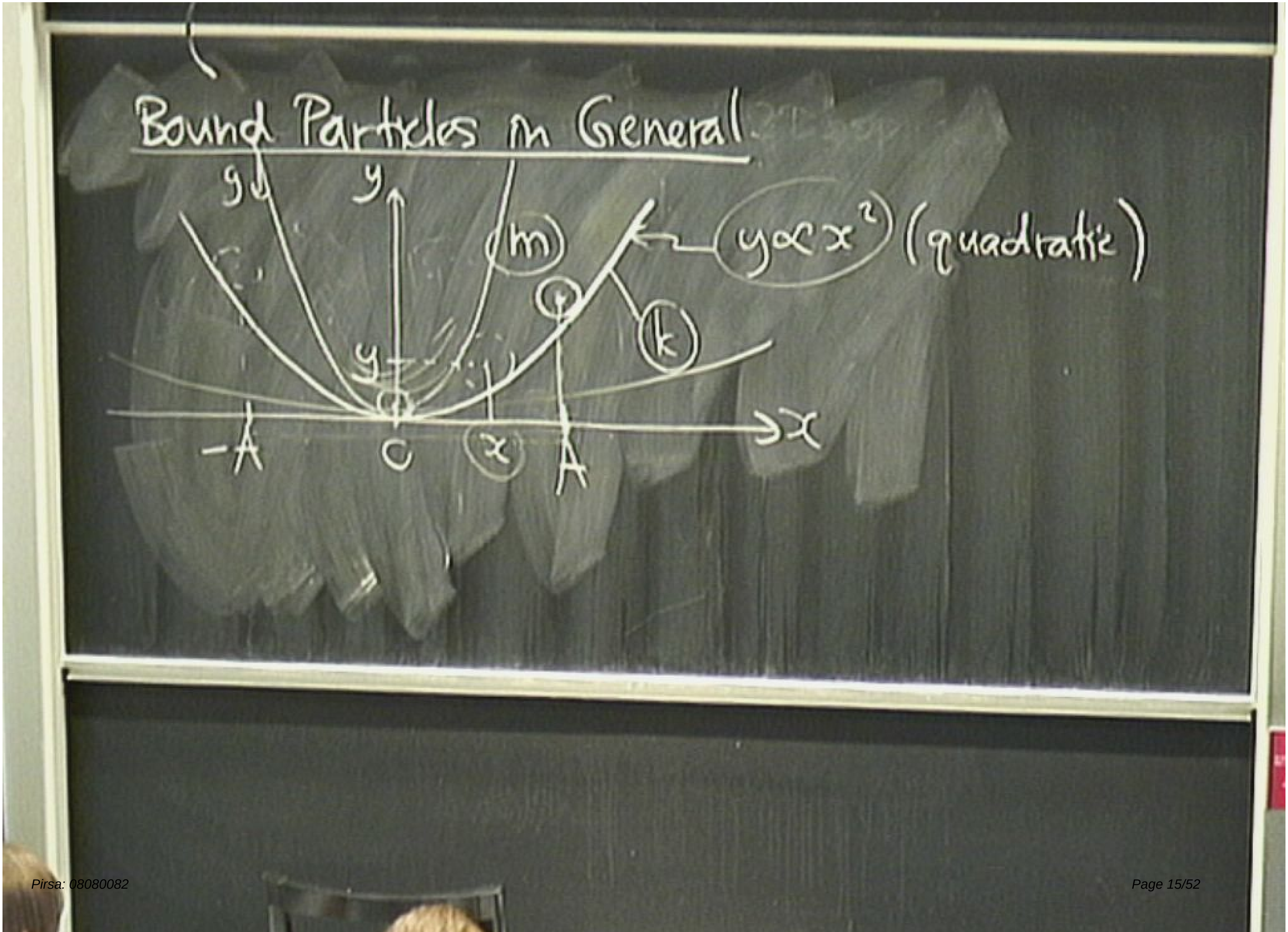


Bound Particles in General
g
y
m
y ∝ x²
(quadratic)
k
y
−A
0
x
A
x

$$PE = mgy = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$


Bound Particles in General
g
y
m
y ∝ x²
(quadratic)
k
y
-A
0
x
A
x

$y = kx^2$

$$PE = mgy = \frac{1}{2}kx^2$$

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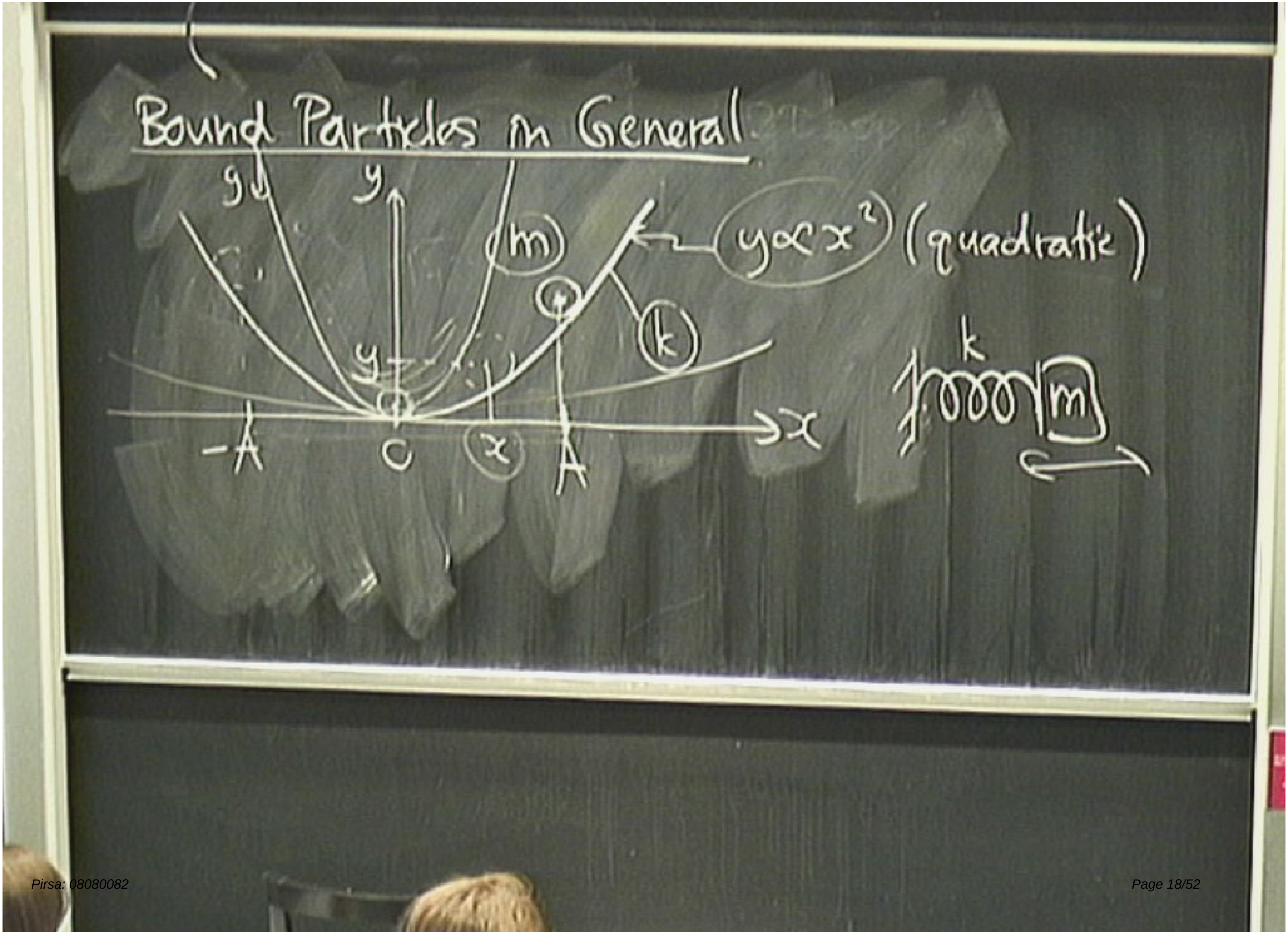
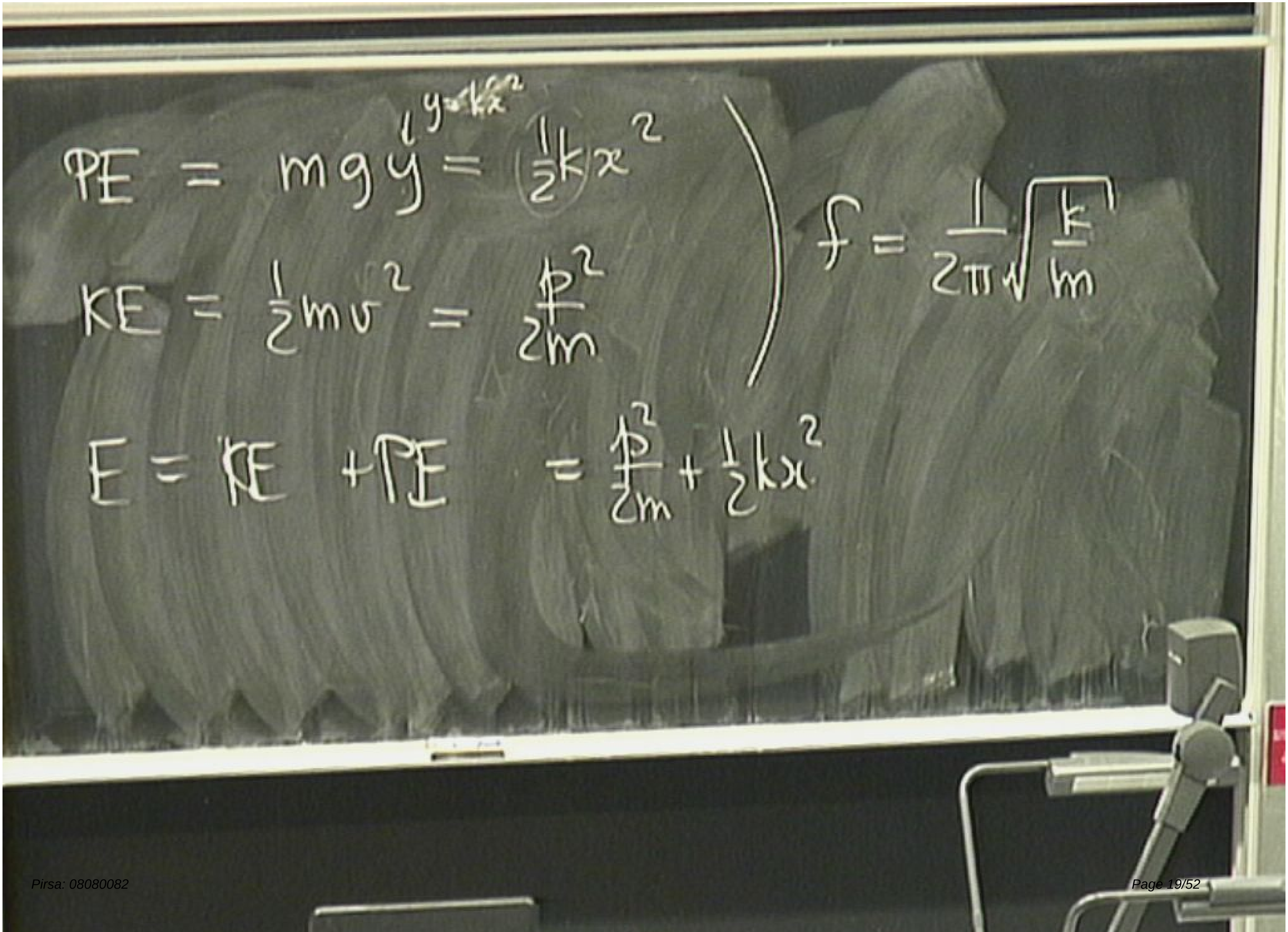
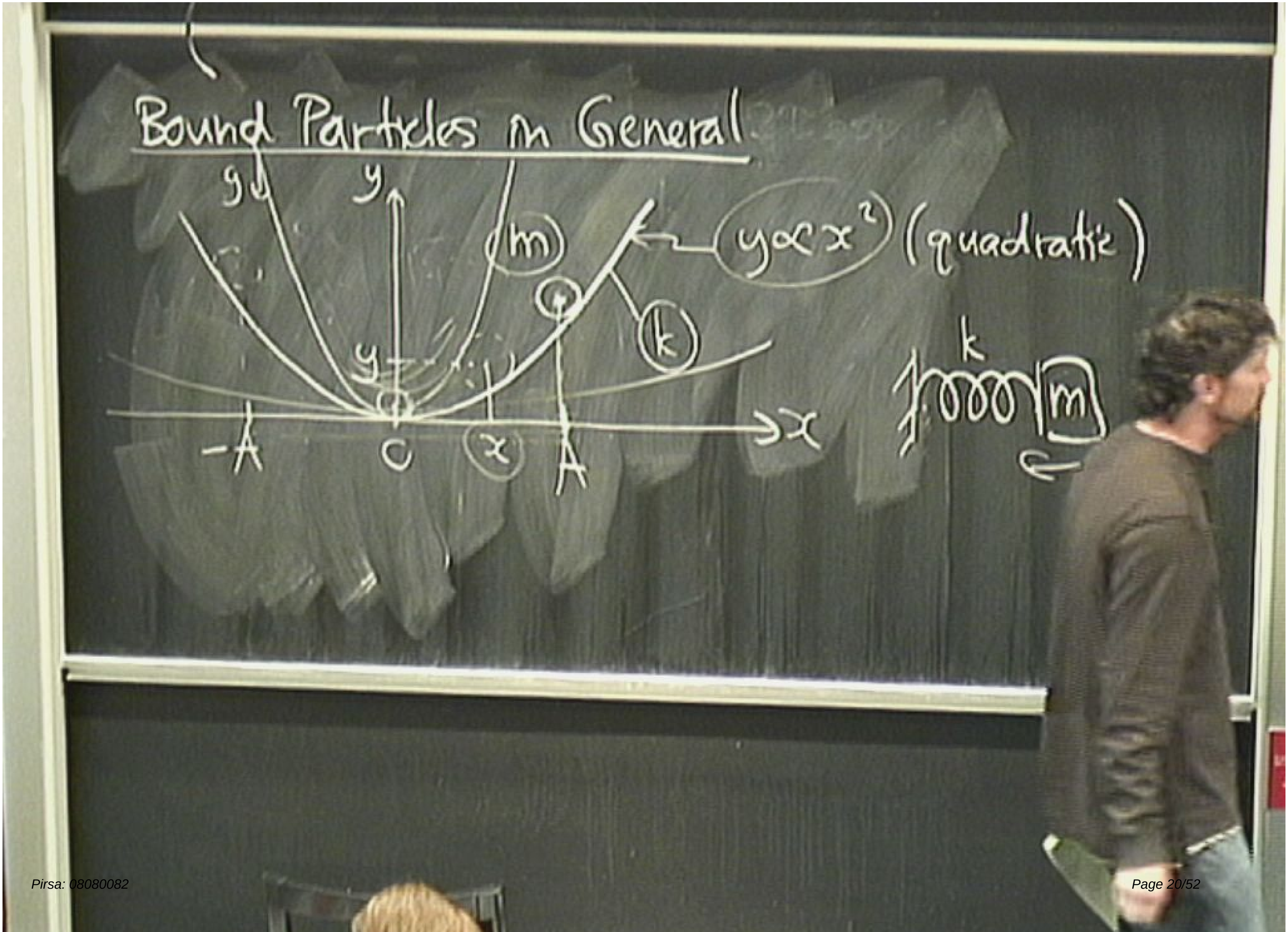

Bound Particles in General
y
y
m
y ∝ x² (quadratic)
k
k
m
-A
0
x
A
x


$y = kx^2$
$PE = mgy = \frac{1}{2}kx^2$
$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
$E = KE + PE = \frac{p^2}{2m} + \frac{1}{2}kx^2$


Bound Particles in General
y ∝ x² (quadratic)
m
k
y
-A
0
x
A

$$y = kx^2$$

$$PE = mgy = \frac{1}{2}kx^2$$

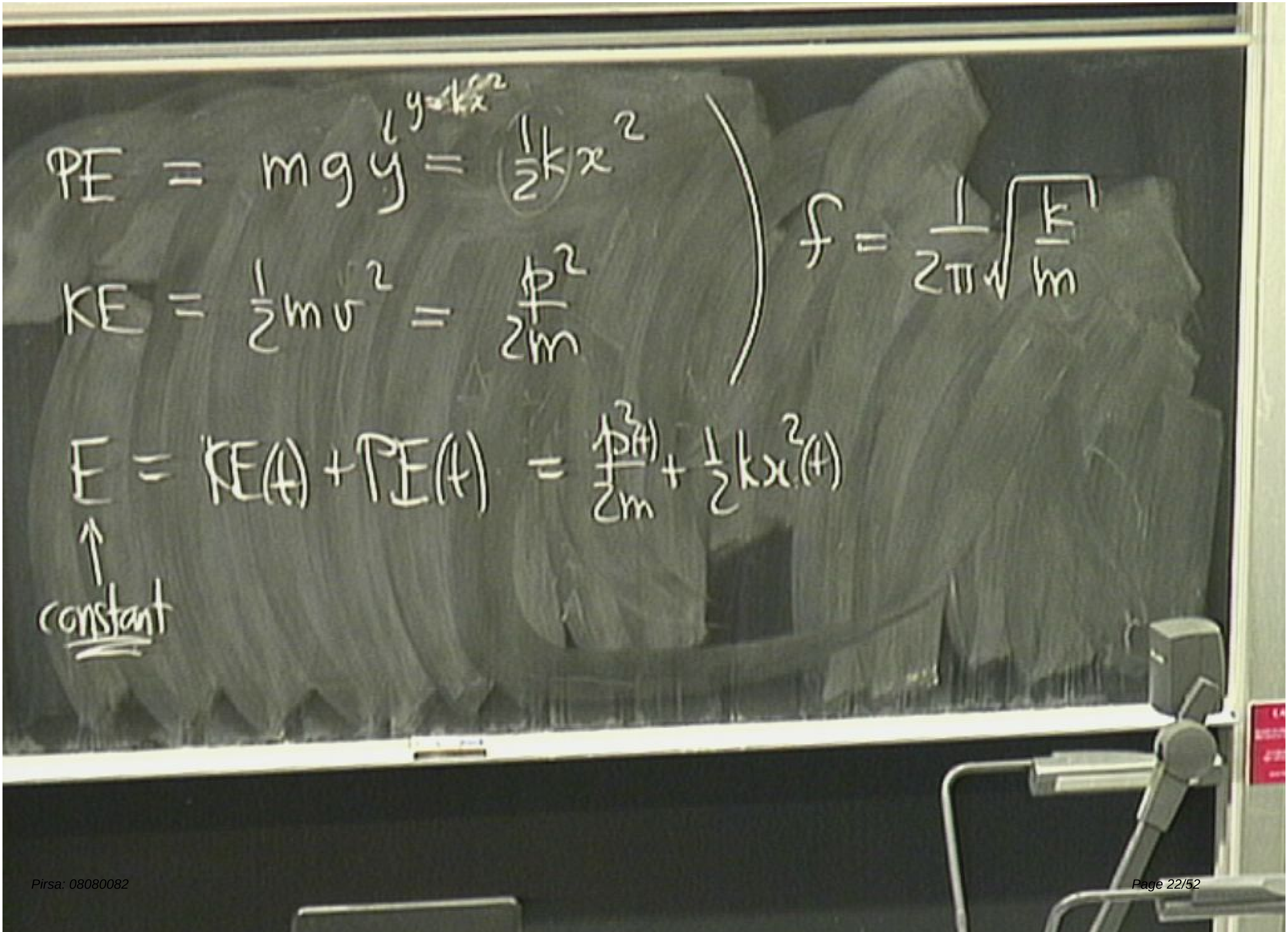
$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi\sqrt{\frac{k}{m}}}$$

$$E = KE(t) + PE(t) = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$\uparrow$

constant


$y = kx^2$
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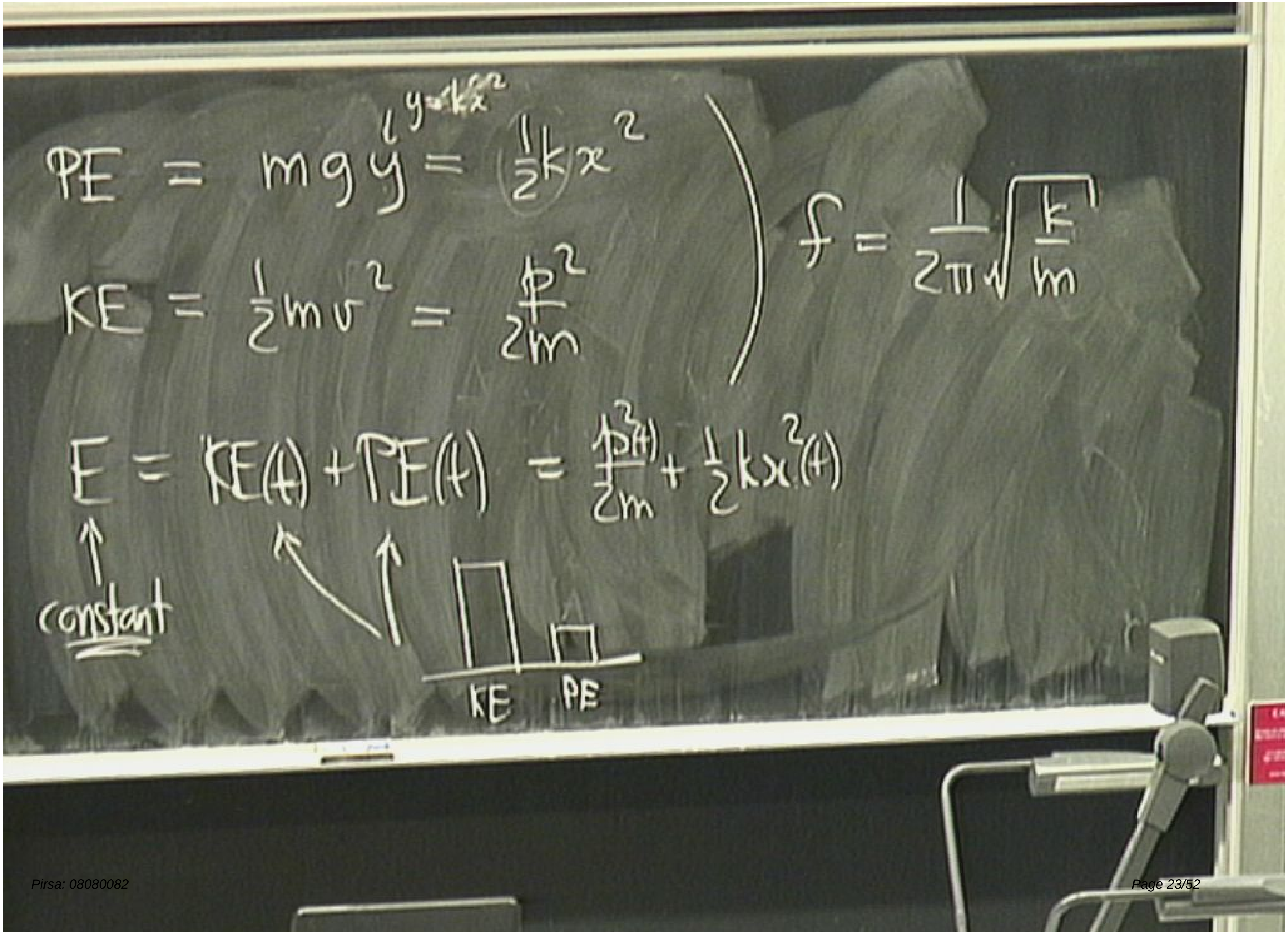
$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
$E = KE(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$
constant


$y = \frac{1}{2}kx^2$
$PE = mgy = \frac{1}{2}kx^2$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
$E = KE(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$
constant
KE
PE

$y = kx^2$

$$PE = mgy = \frac{1}{2}kx^2$$

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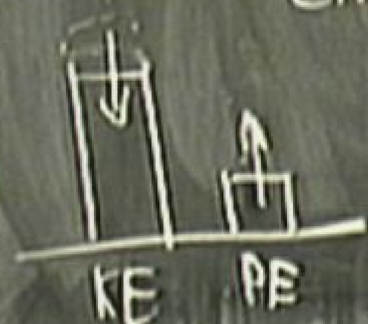
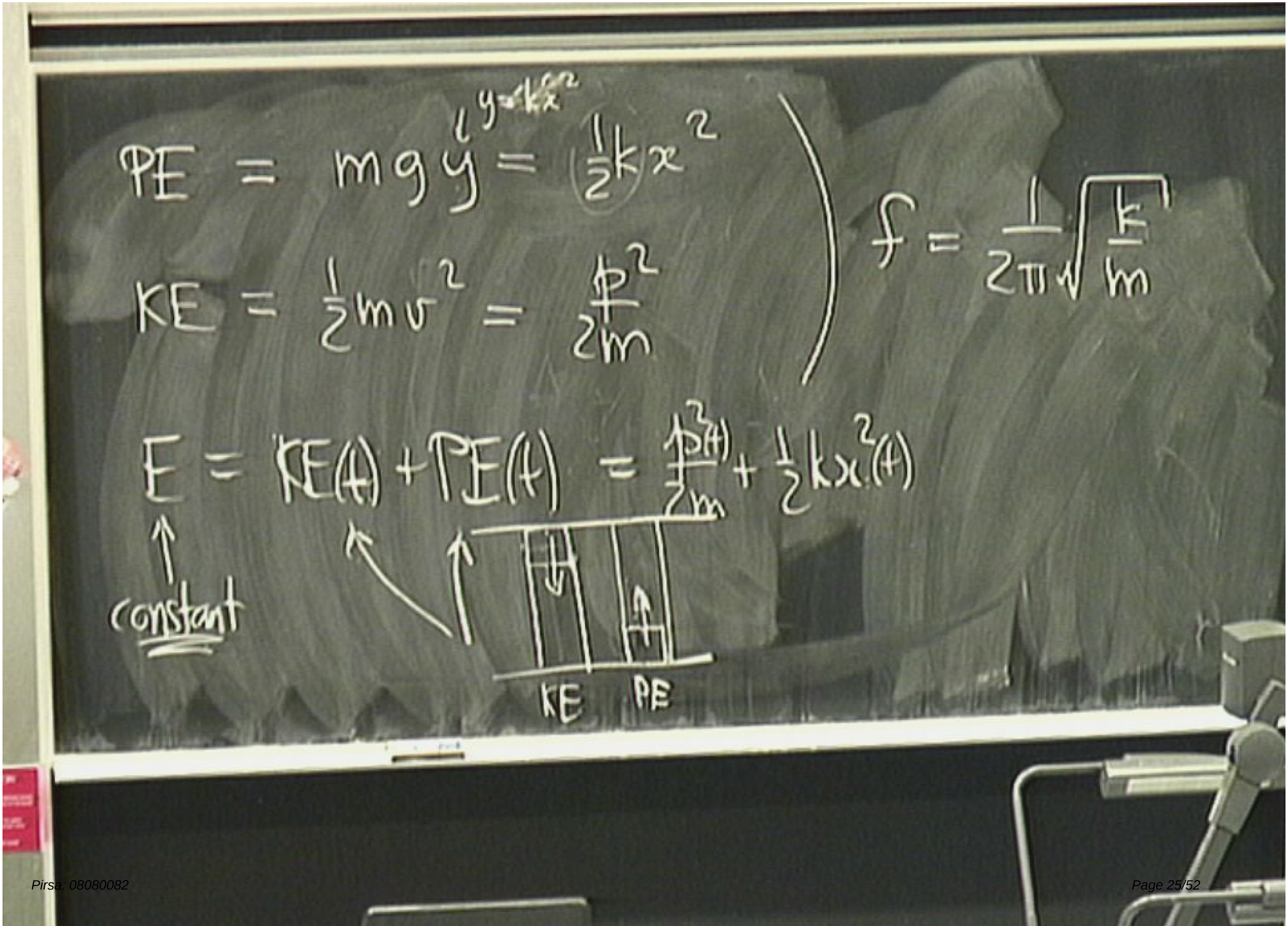
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$E = KE(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$$

constant

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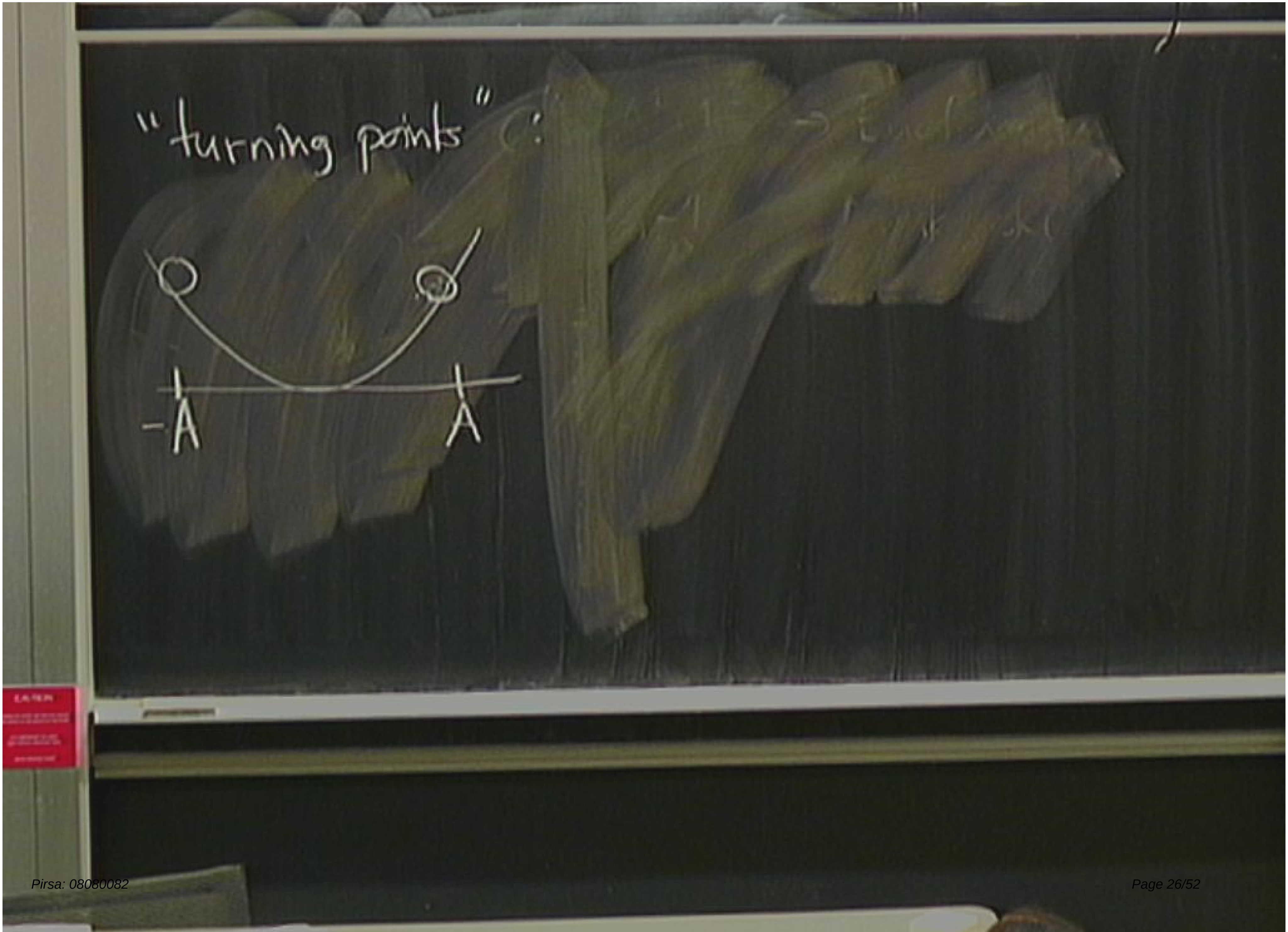
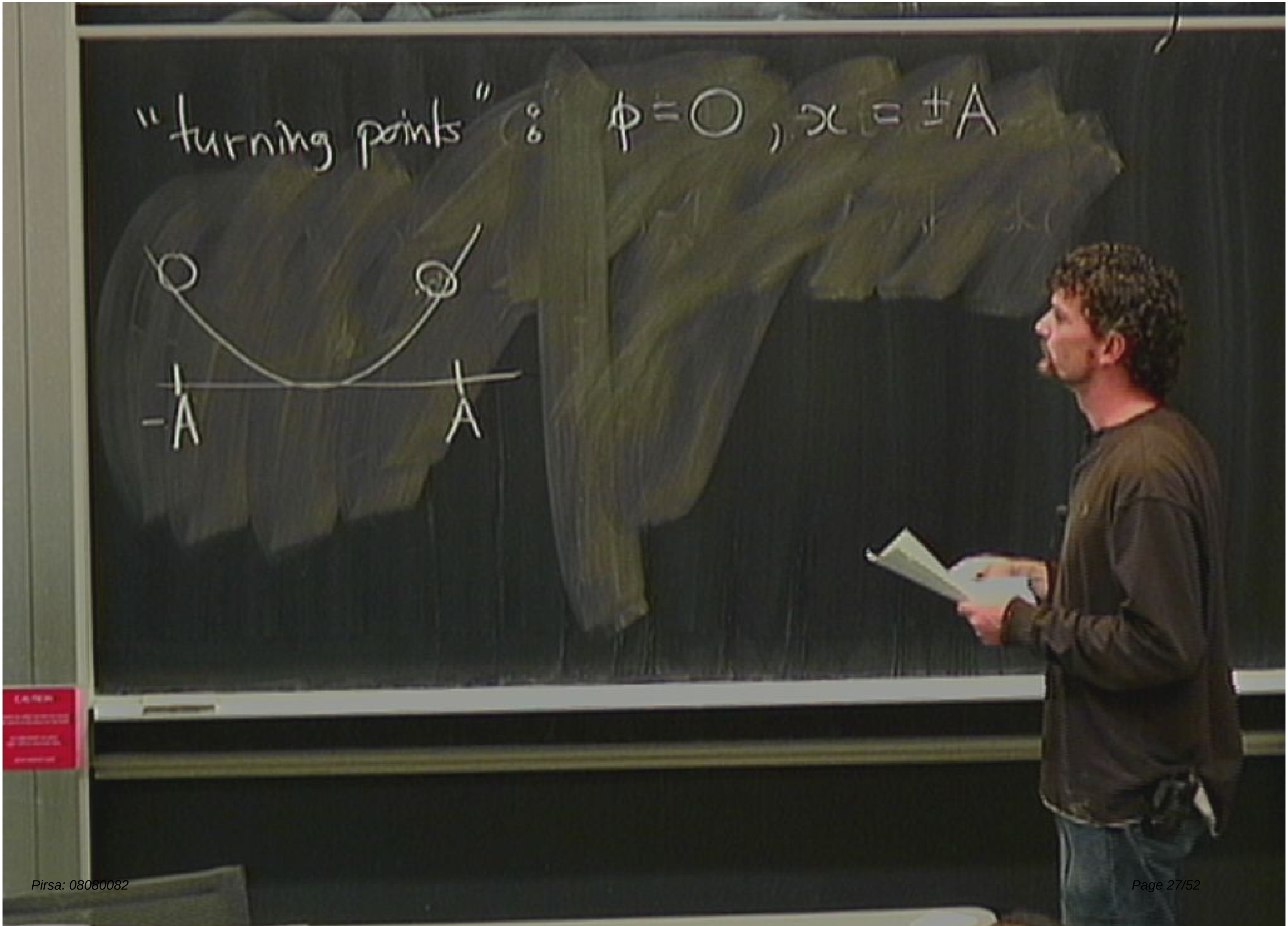
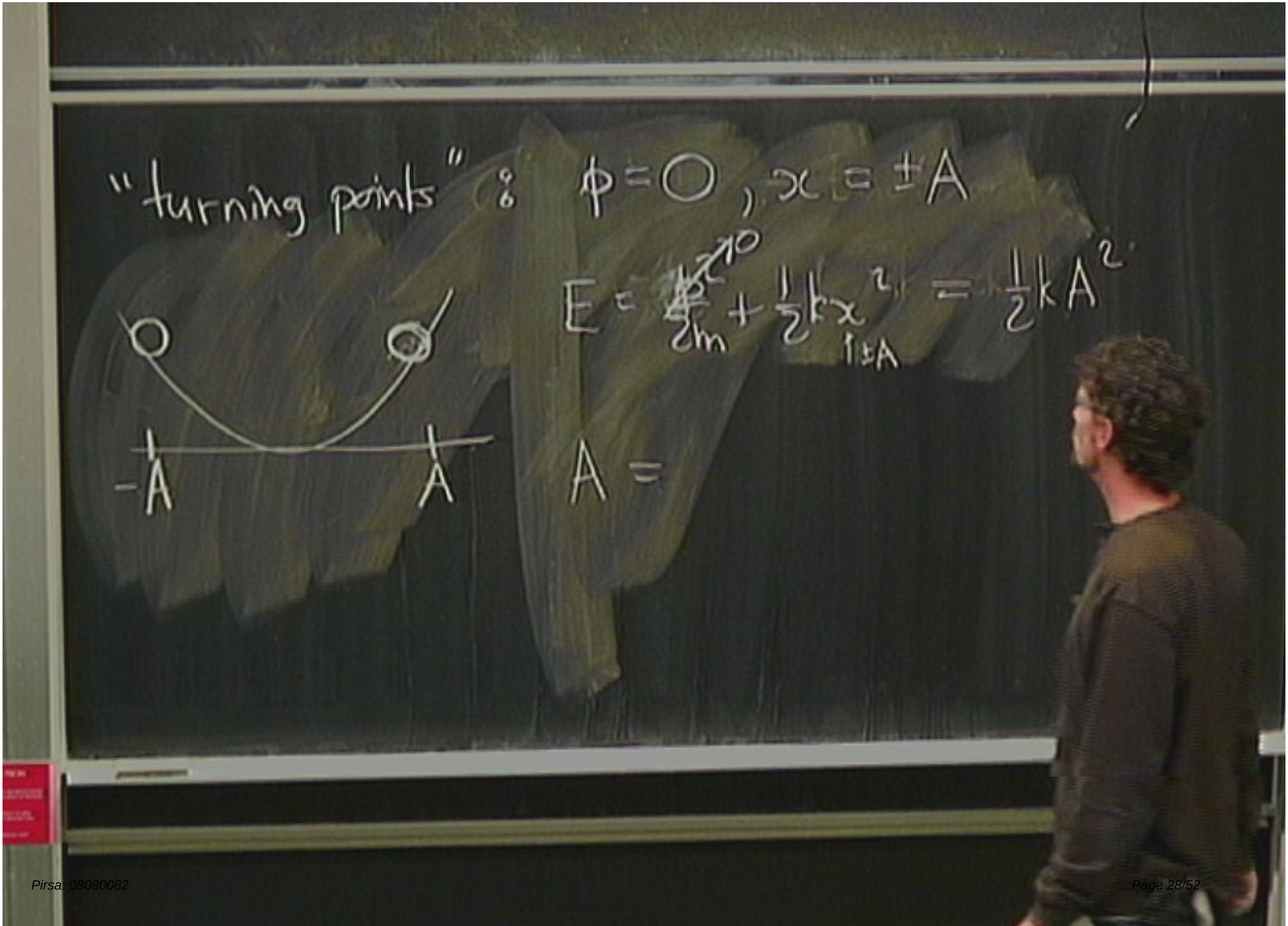
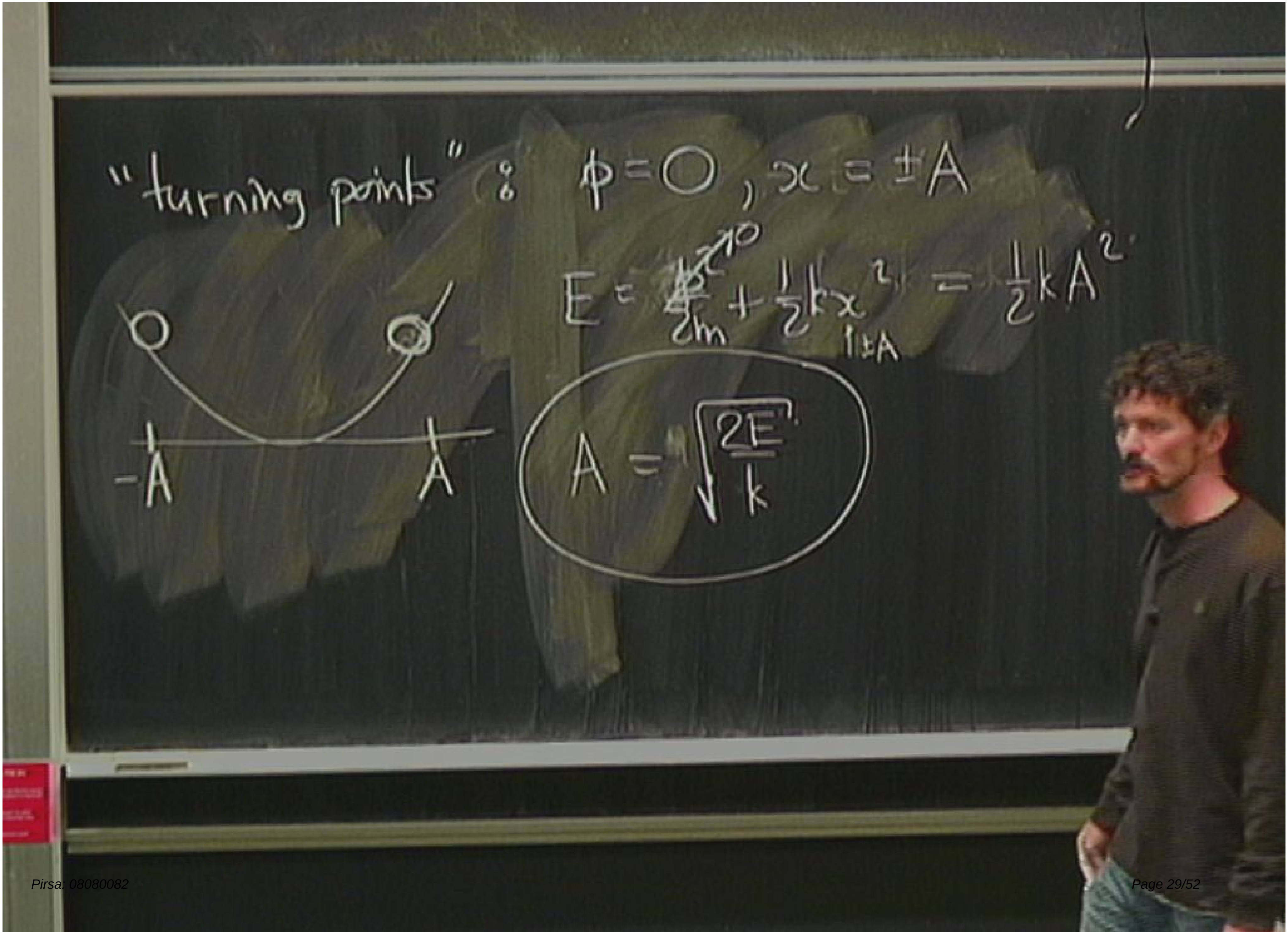
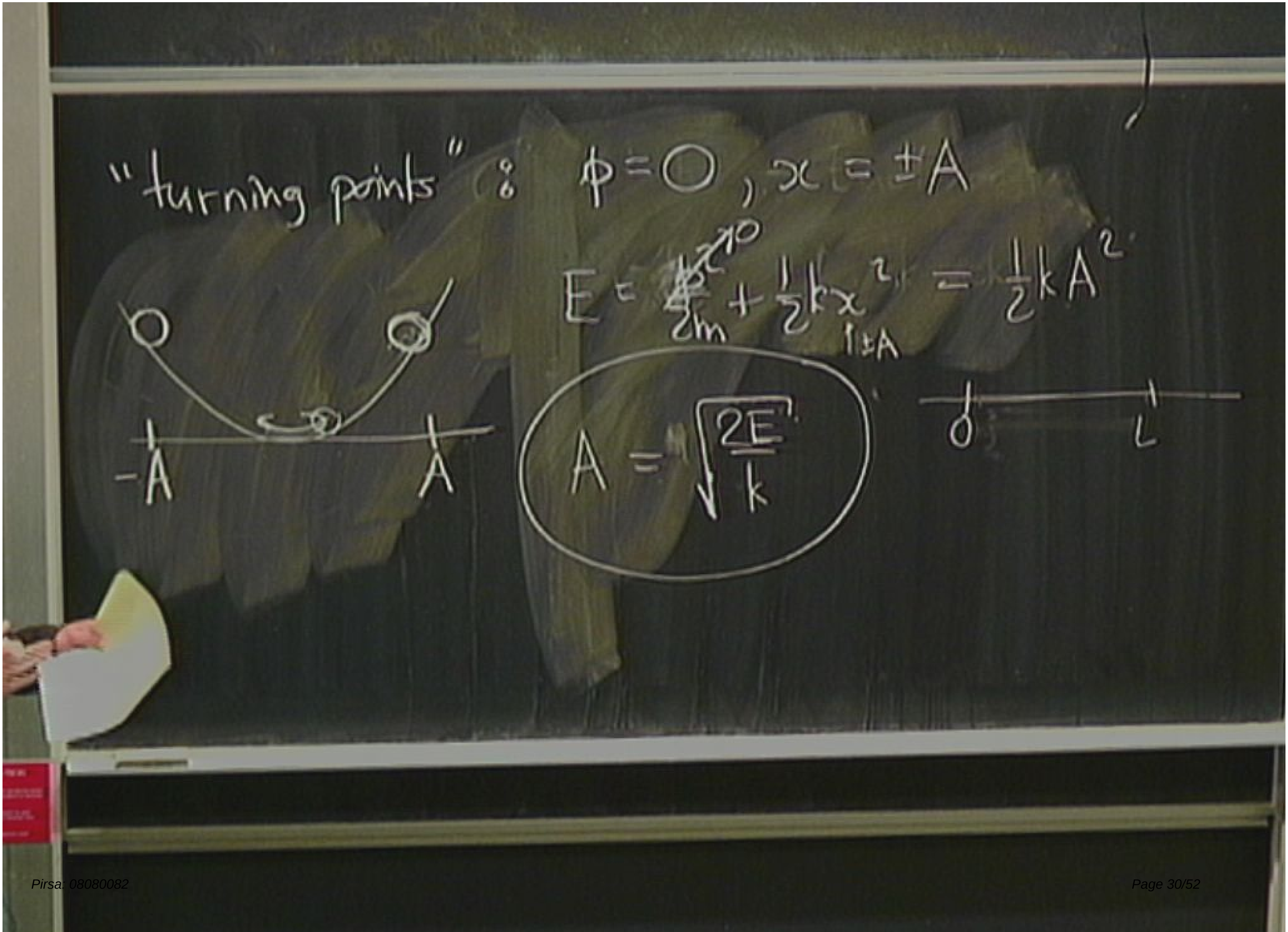
$E = KE(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$
constant
KE
PE

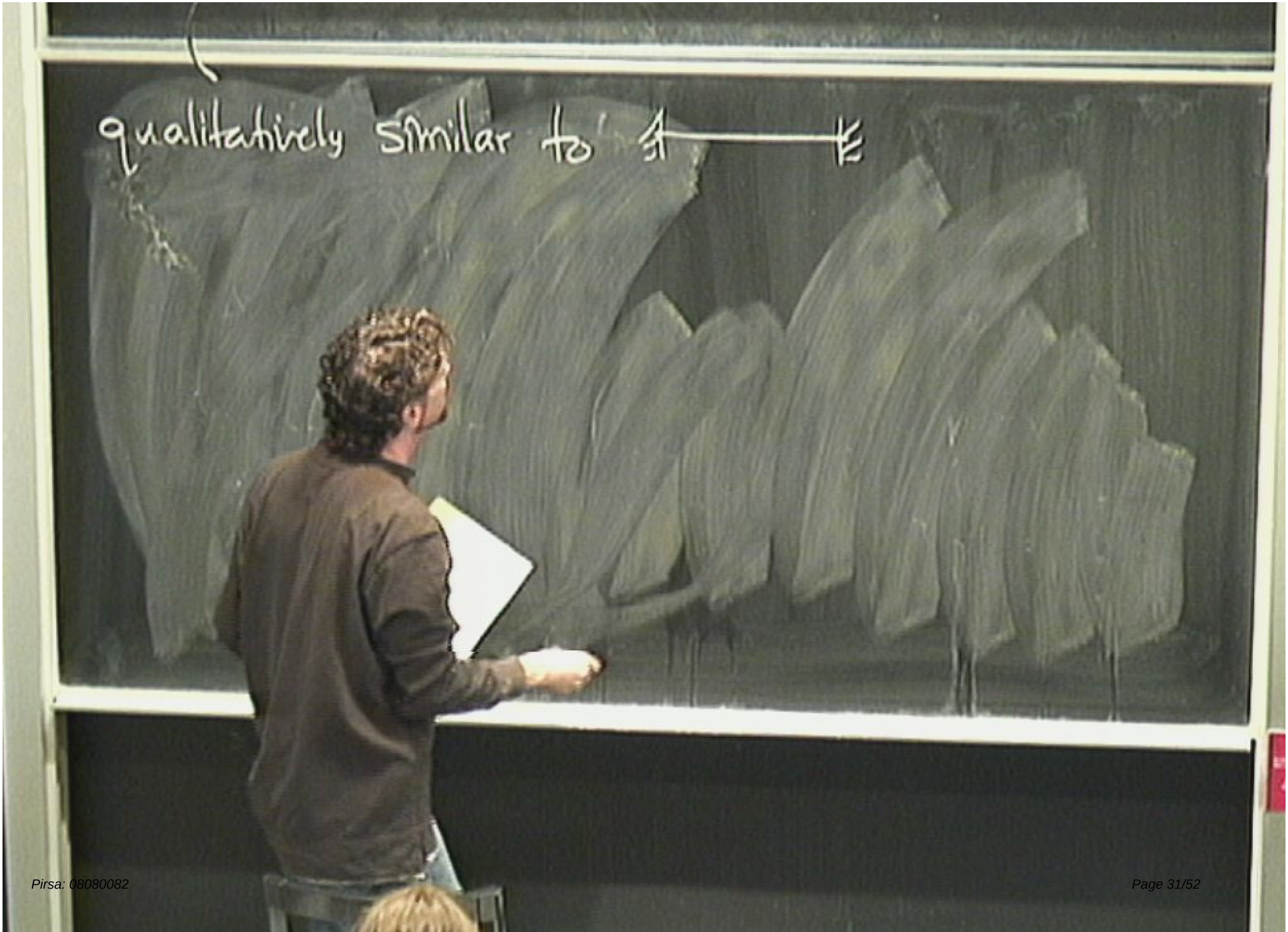

"turning points"
-A
A

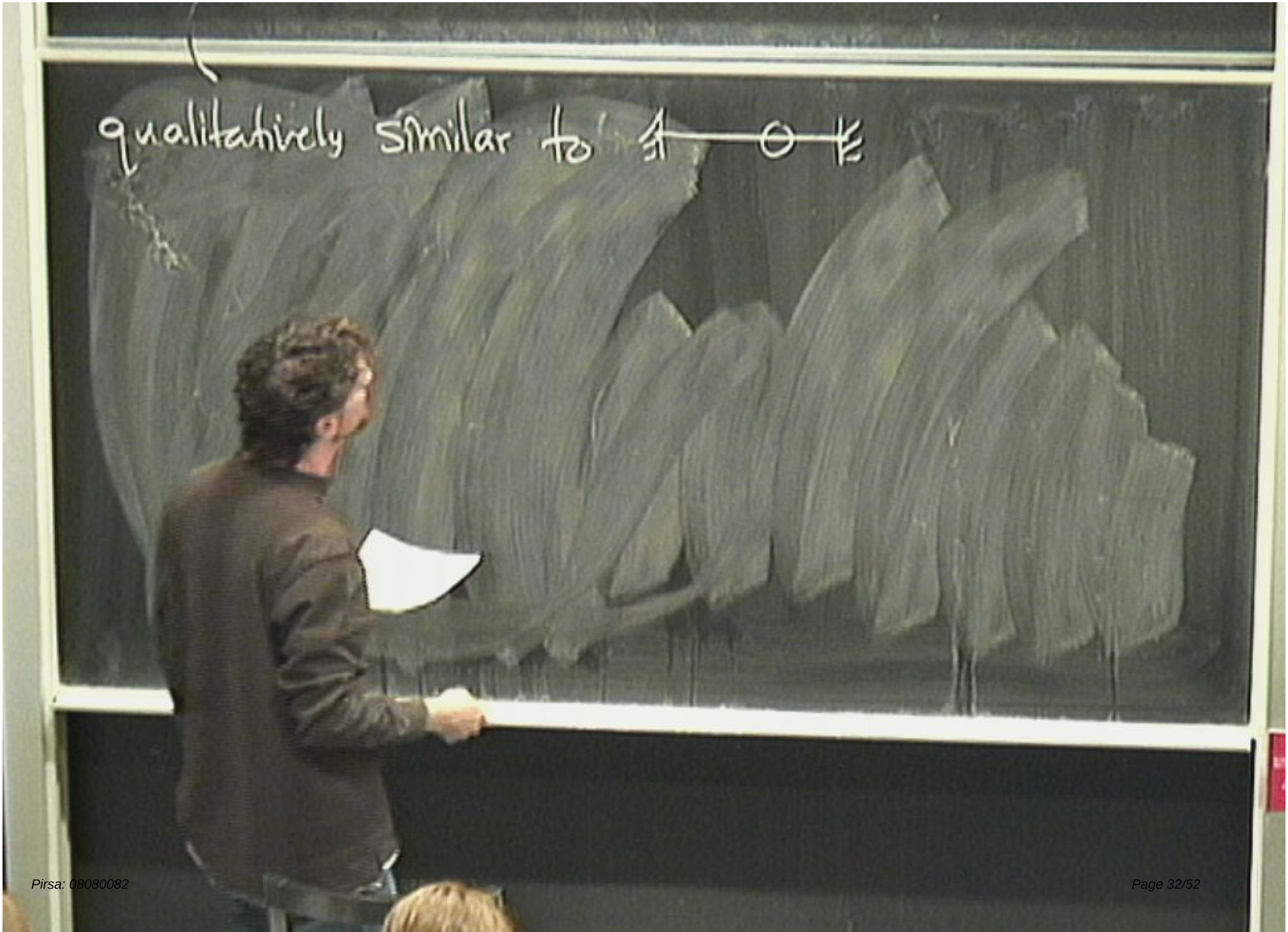

"turning points" : $\phi = 0$, $x = \pm A$
$-A$
A


"turning points": $\phi = 0$, $x = \pm A$
$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$
$\pm A$
$-A$
A
$A =$


"turning points": $p = 0$, $x = \pm A$
$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$
$\pm A$
$-A$
A
$A = \sqrt{\frac{2E}{k}}$


"turning points" : p = 0 , x = ±A
E = p²/2m + ½kx² = ½kA²
-A
A
A = √(2E/k)
0
L


qualitatively similar to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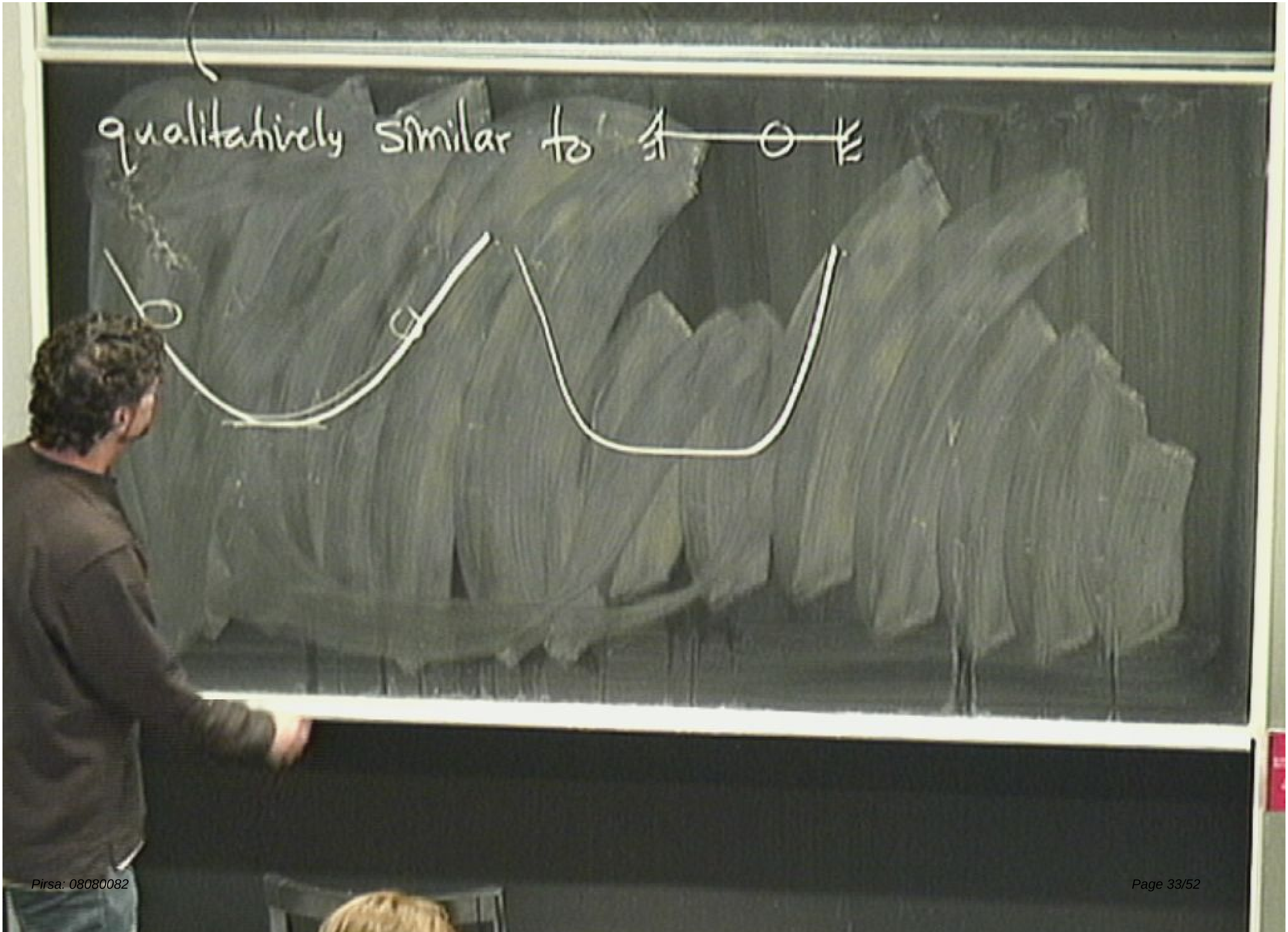
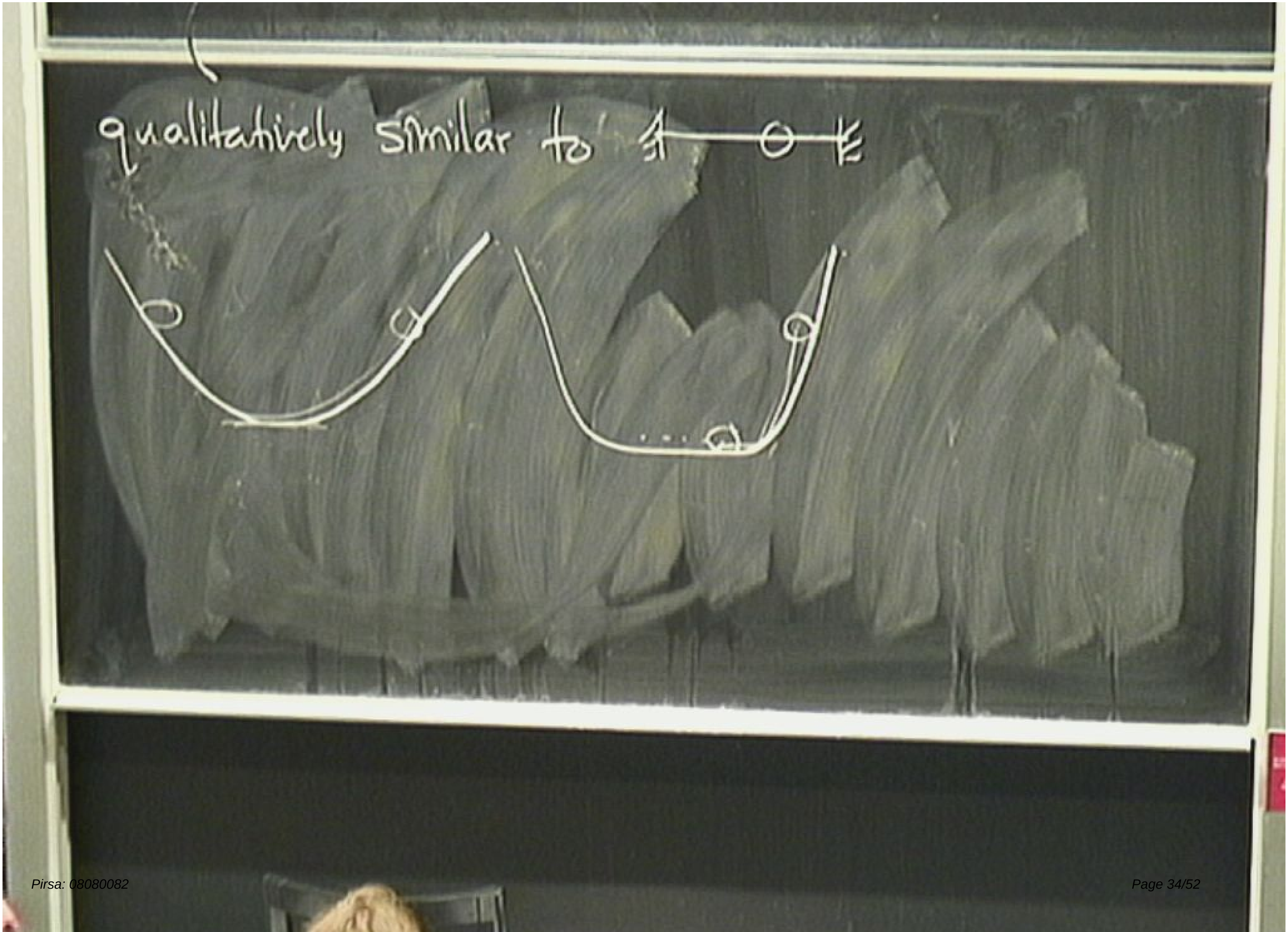
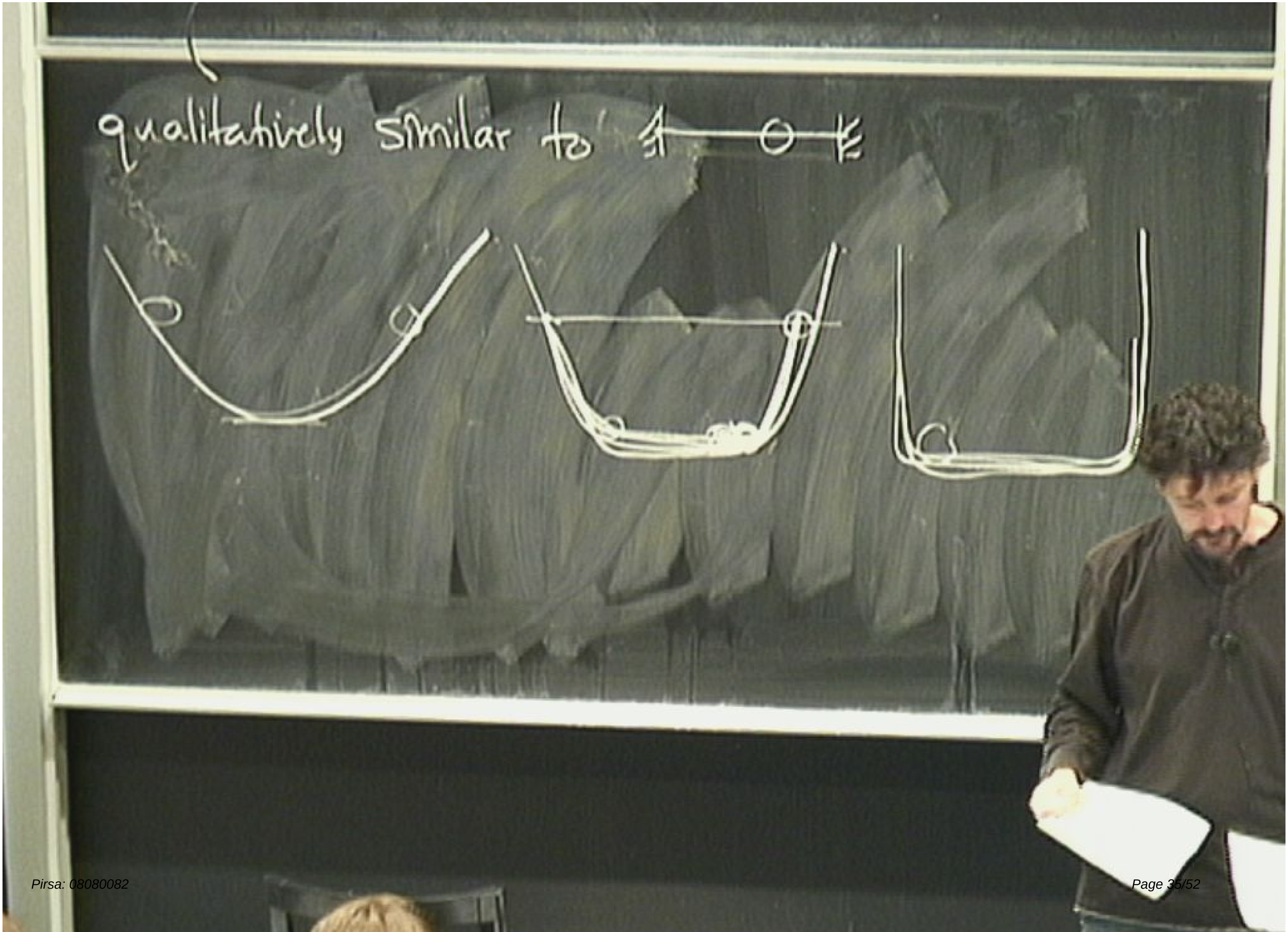
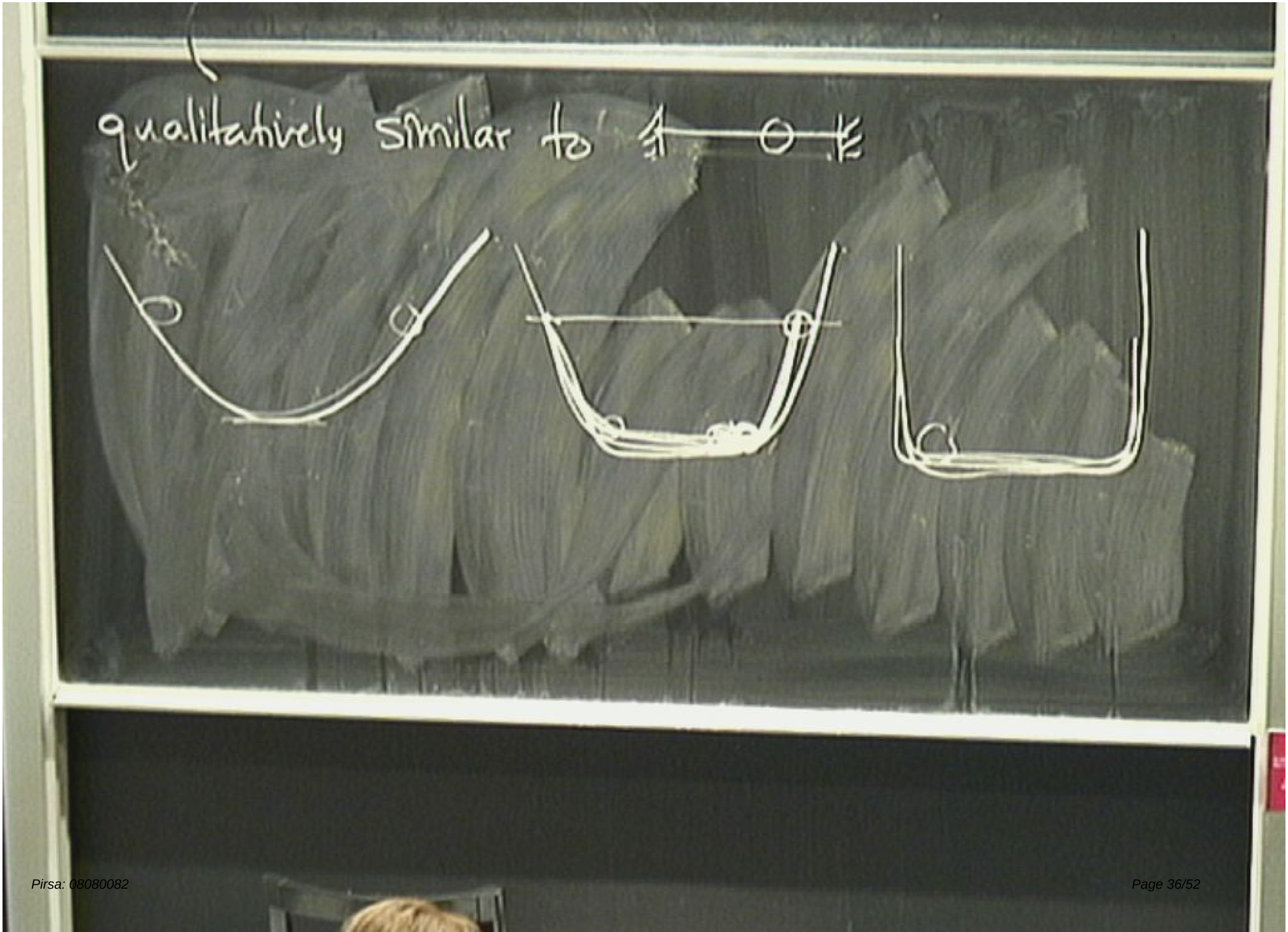
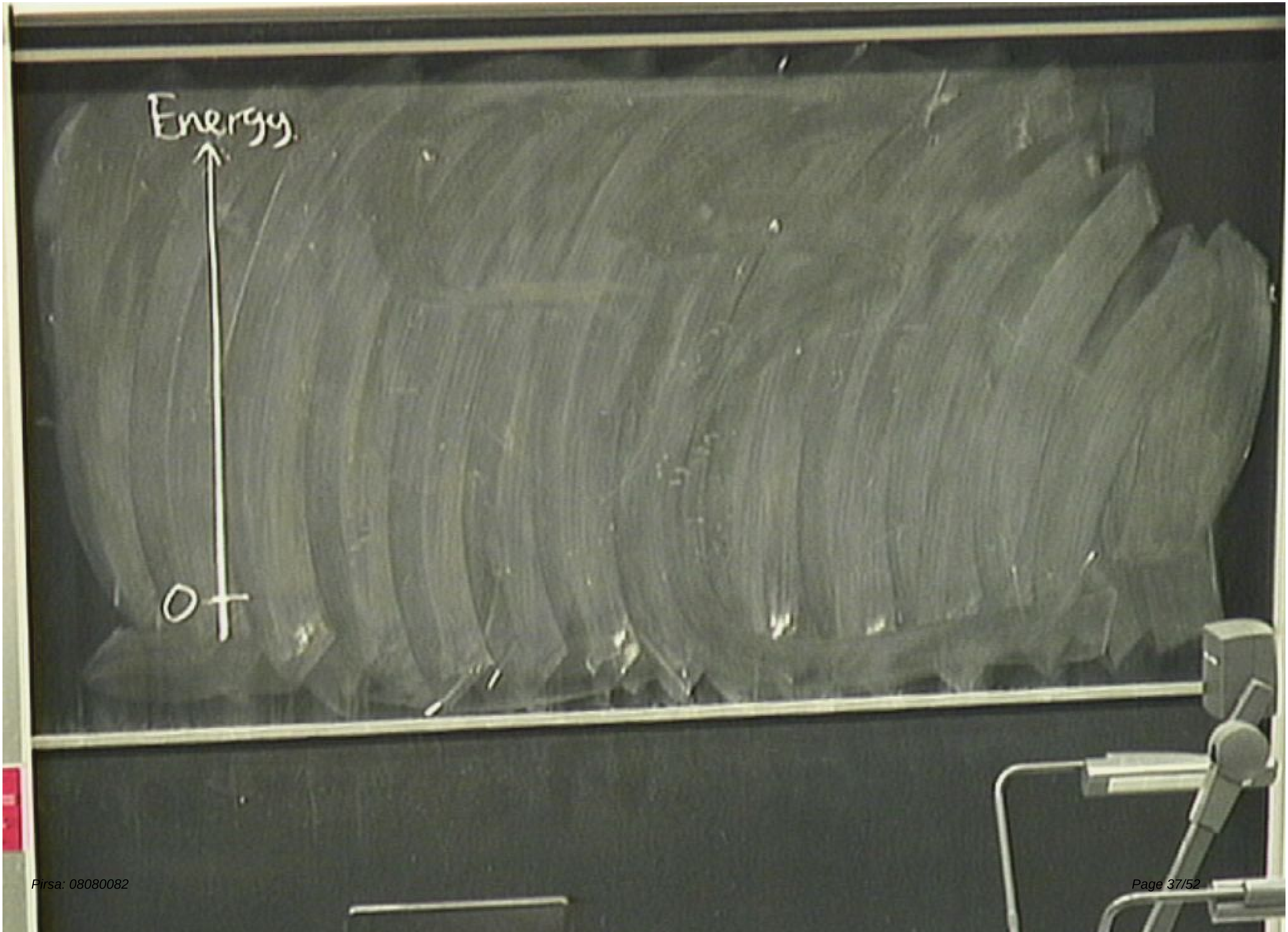
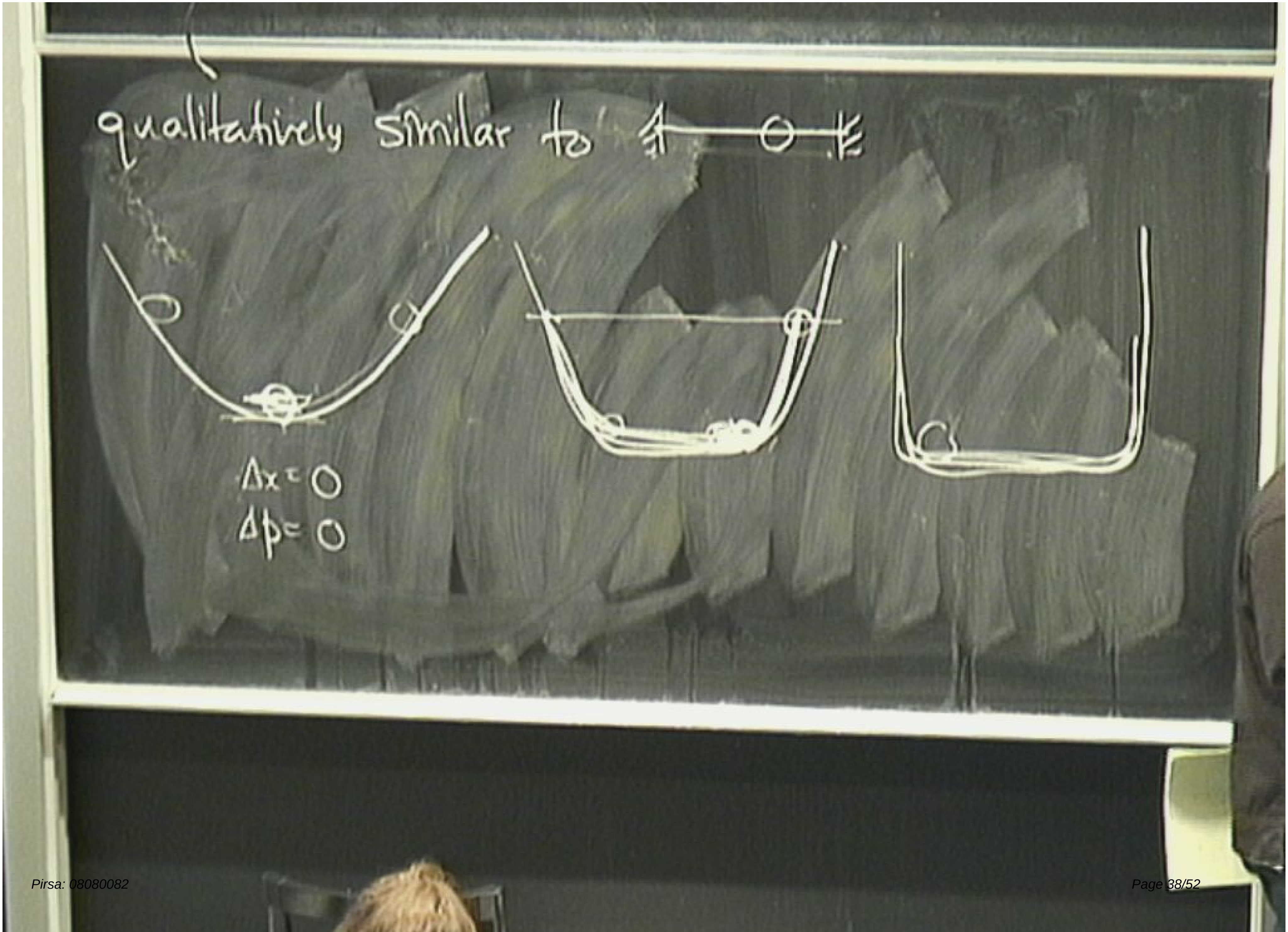

qualitatively similar to

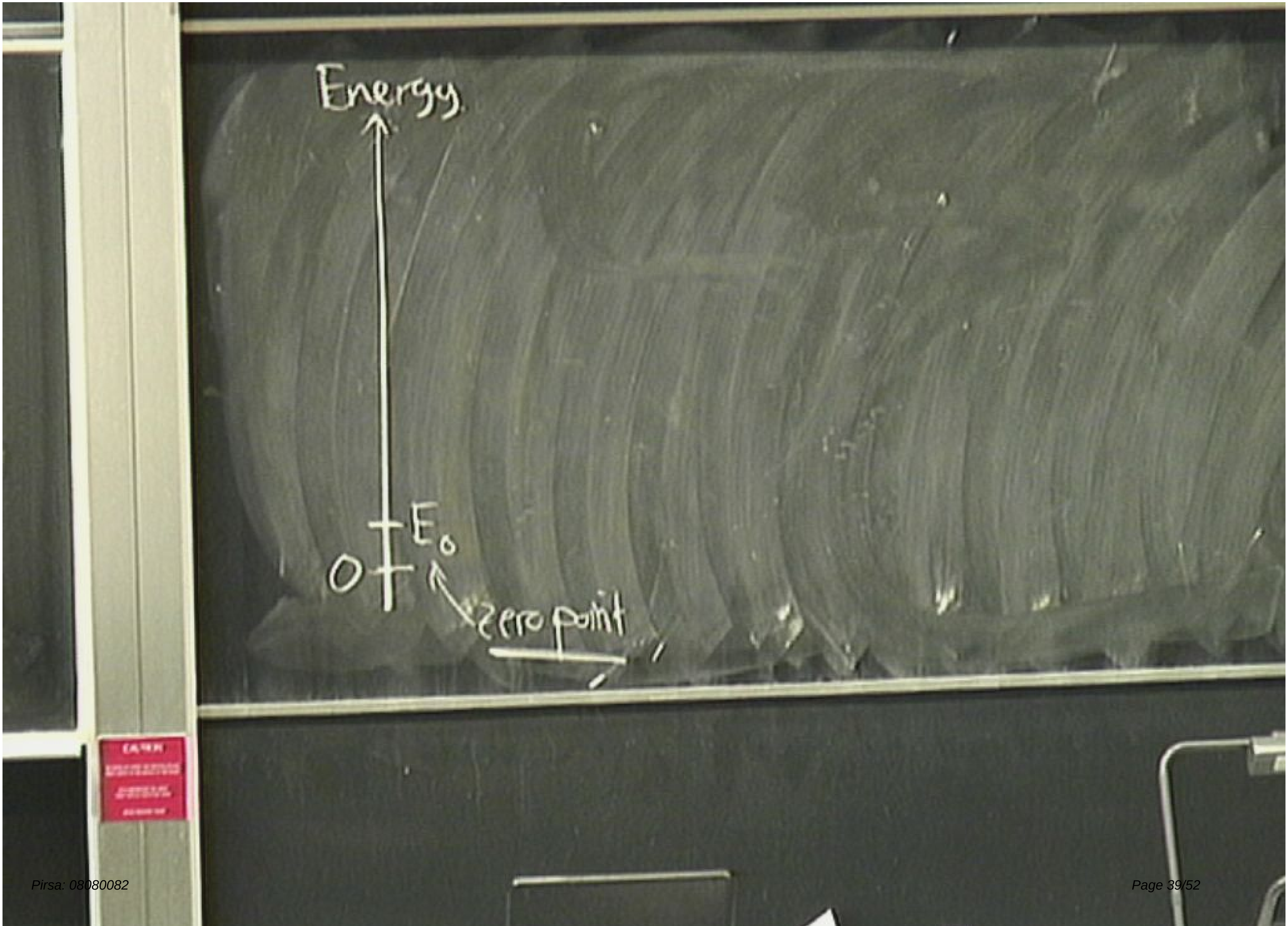

qualitatively similar to


qualitatively similar to

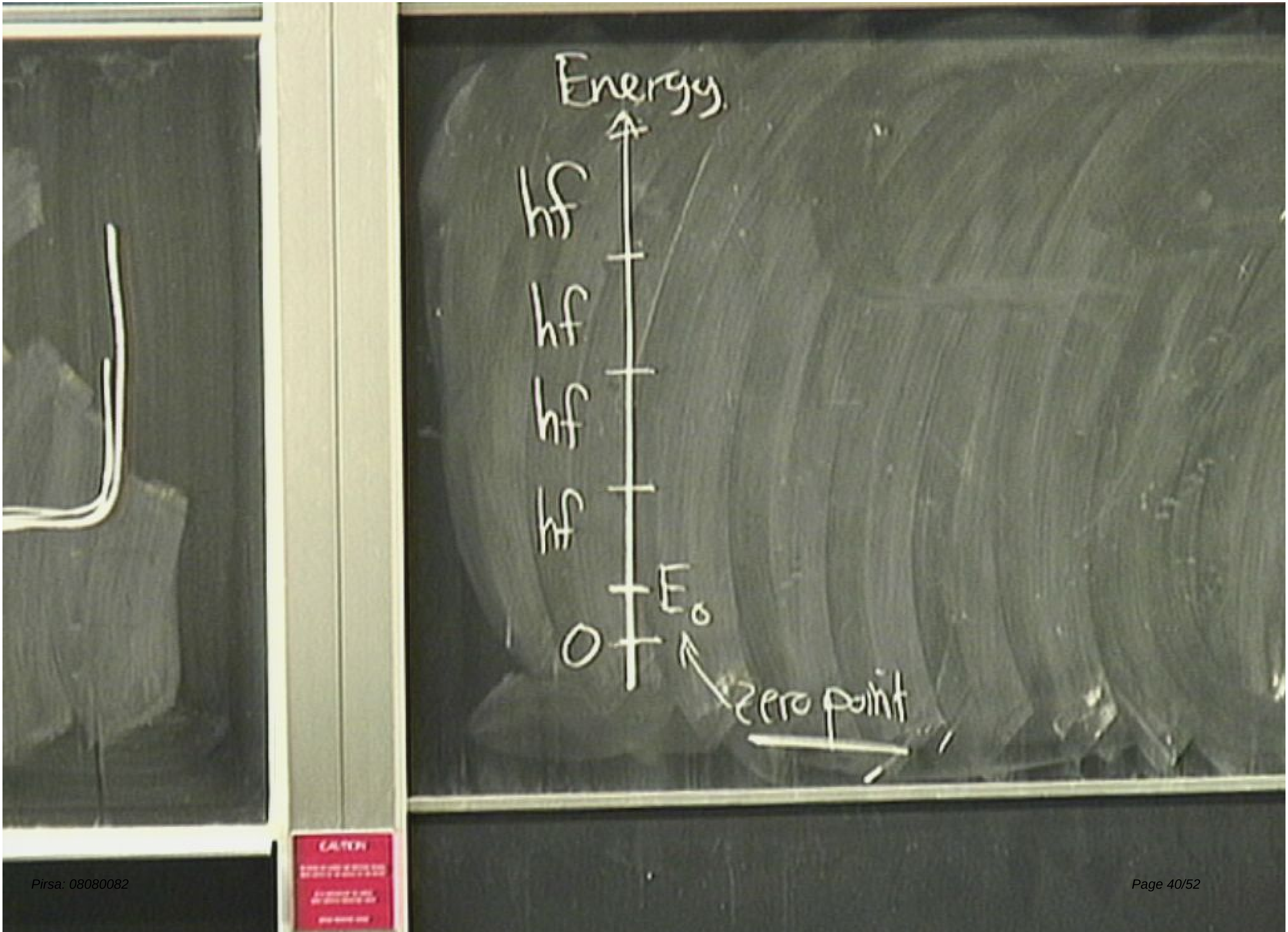

qualitatively similar to

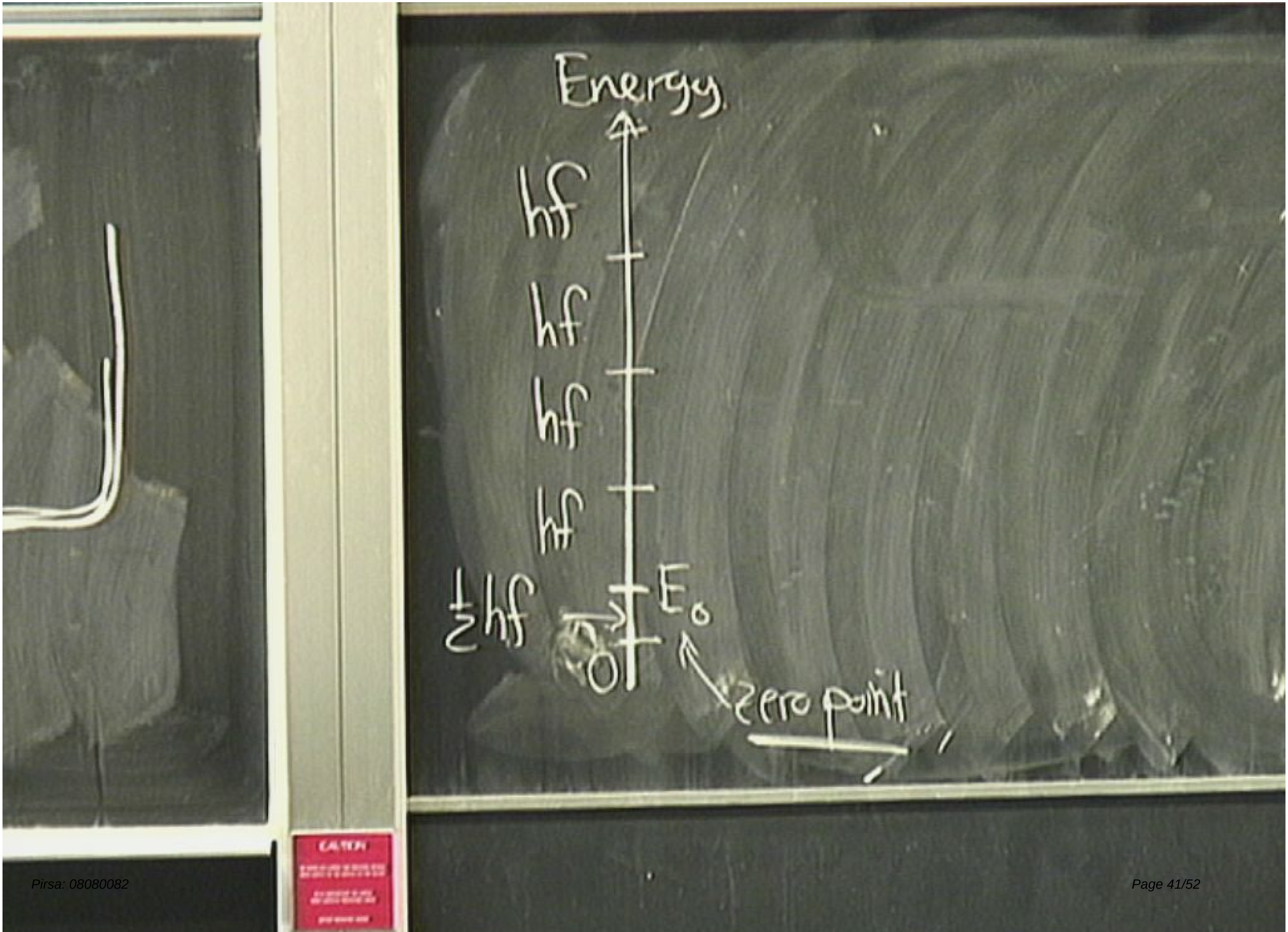

Energy
0

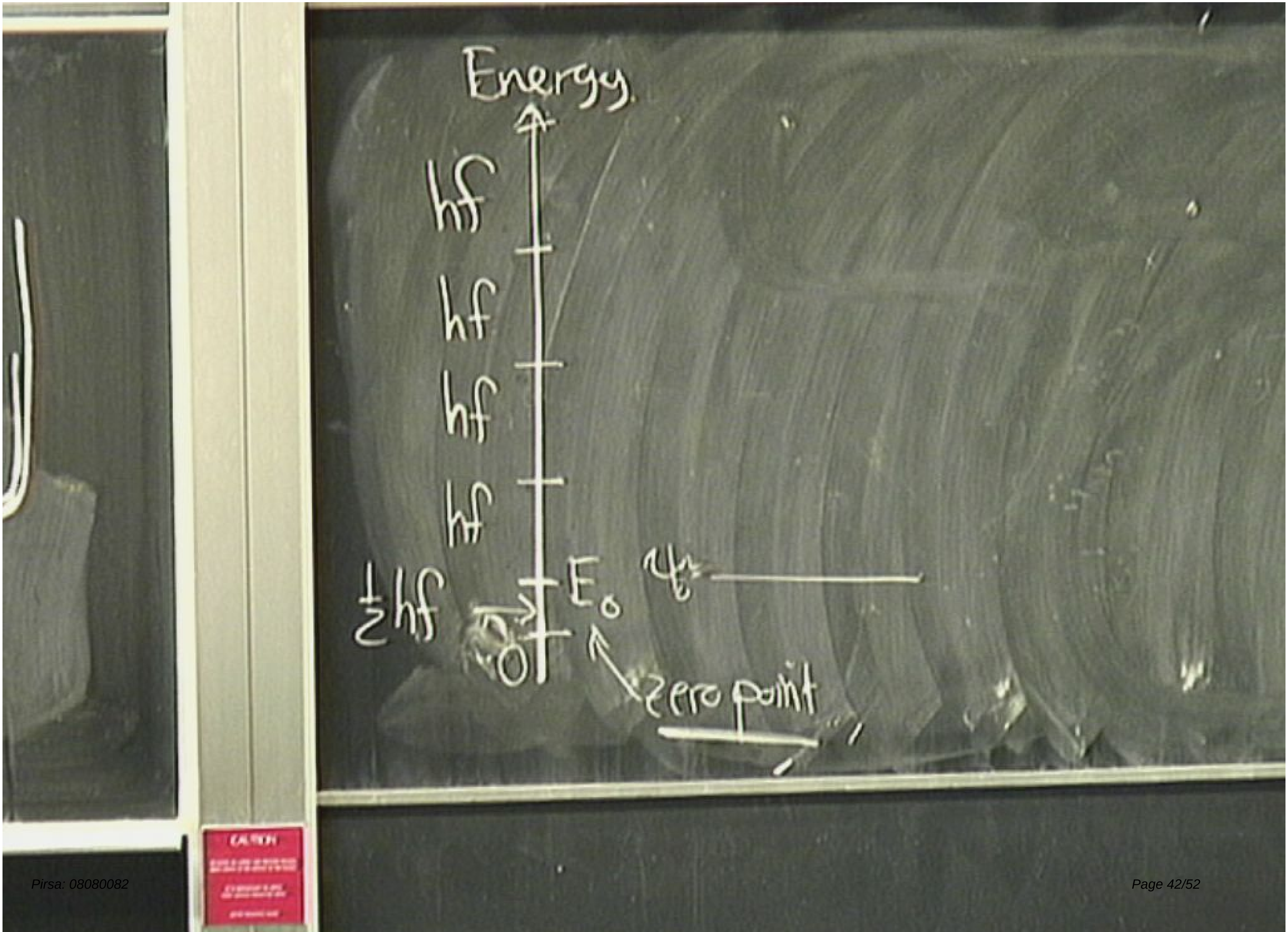

qualitatively similar to
Δx = 0
Δp = 0

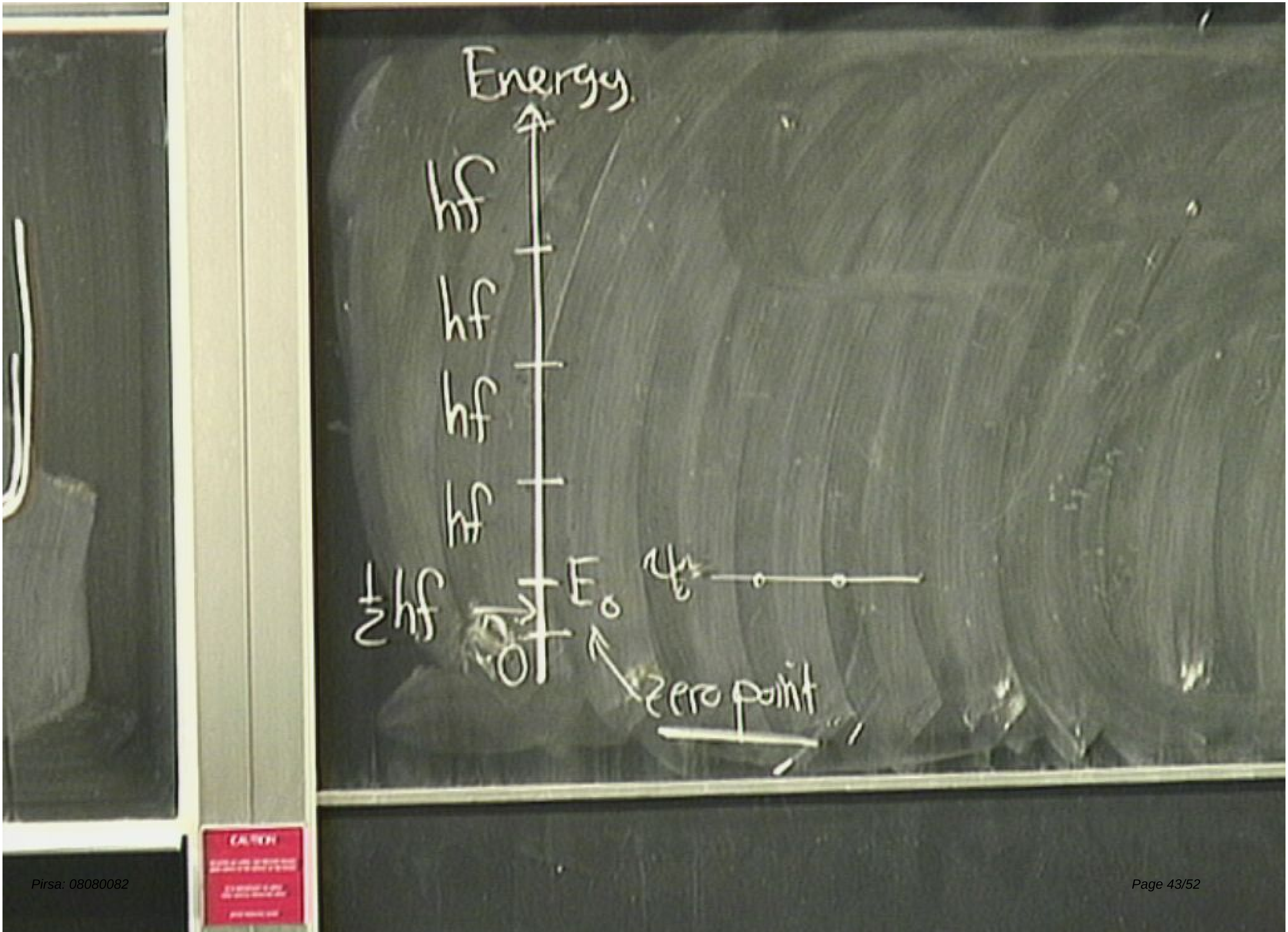


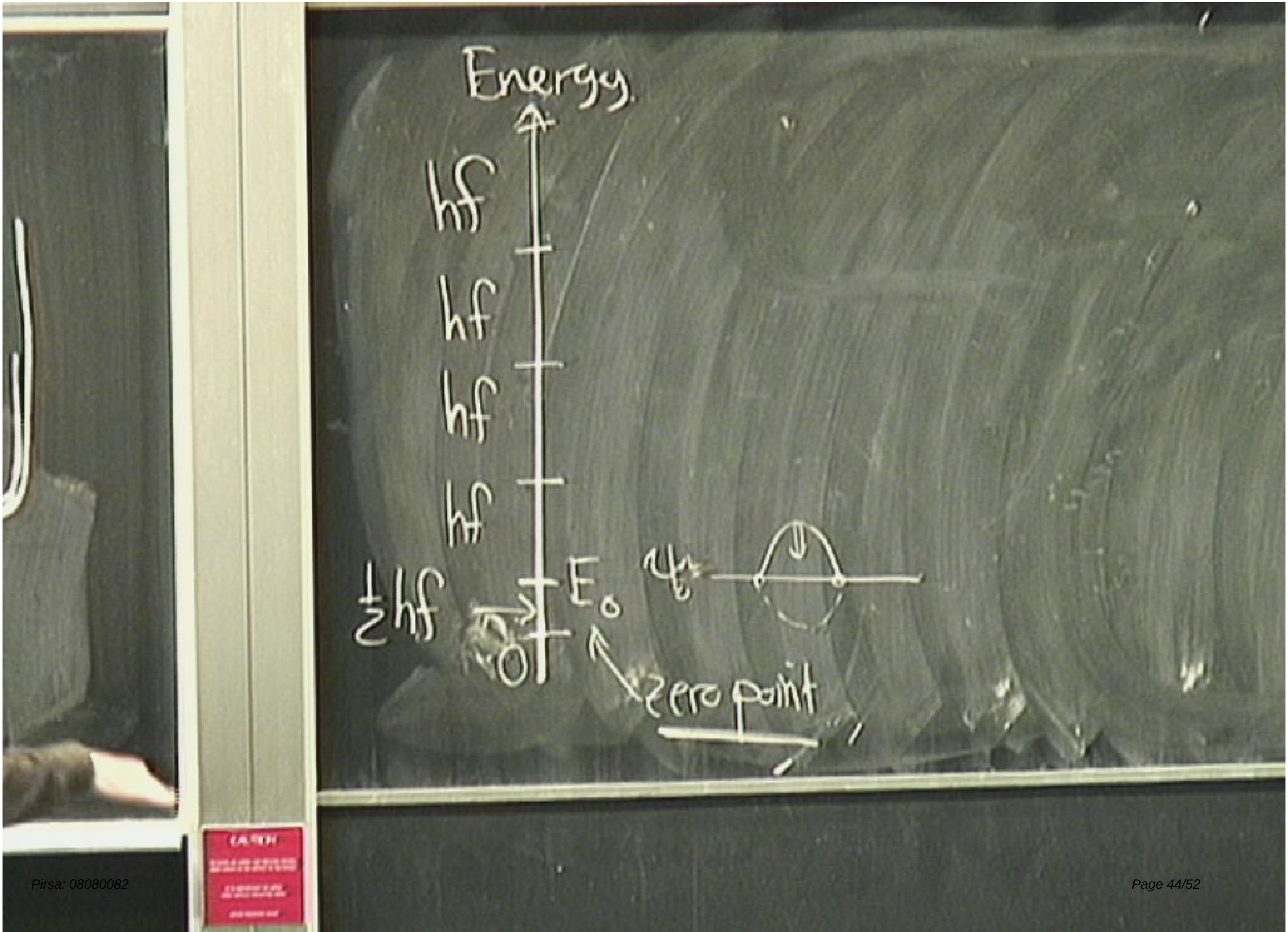
Energy
E_0
0
zero point
CAUTION

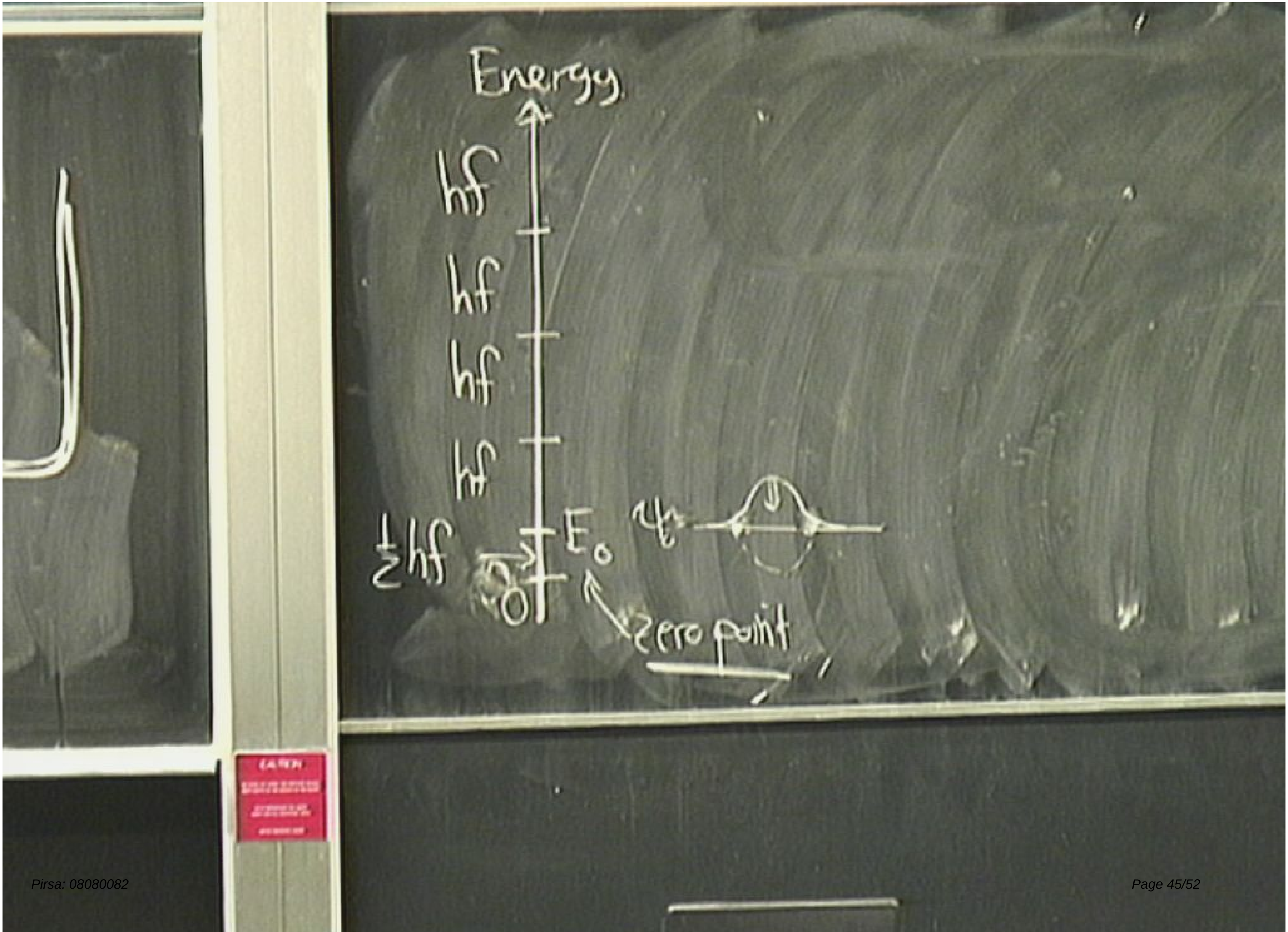

Energy
hf
hf
hf
hf
E0
0
zero point
CAUTION


Energy
hf
hf
hf
hf
E0
1/2 hf
0
zero point


Energy
hf
hf
hf
hf
$\frac{1}{2}hf$
E_0
ψ
0
zero point
CAUTION

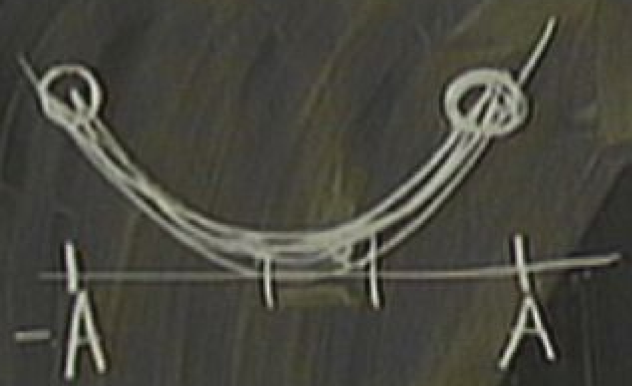

Energy
hf
hf
hf
hf
$\frac{1}{2}hf$
E_0
ψ_0
0
zero point


Energy
hf
hf
hf
hf
½hf
E₀
ψ
0
zero point

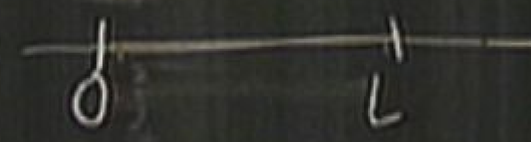

Energy
hf
hf
hf
hf
$\frac{1}{2}hf$
E_0
ψ
0
zero point

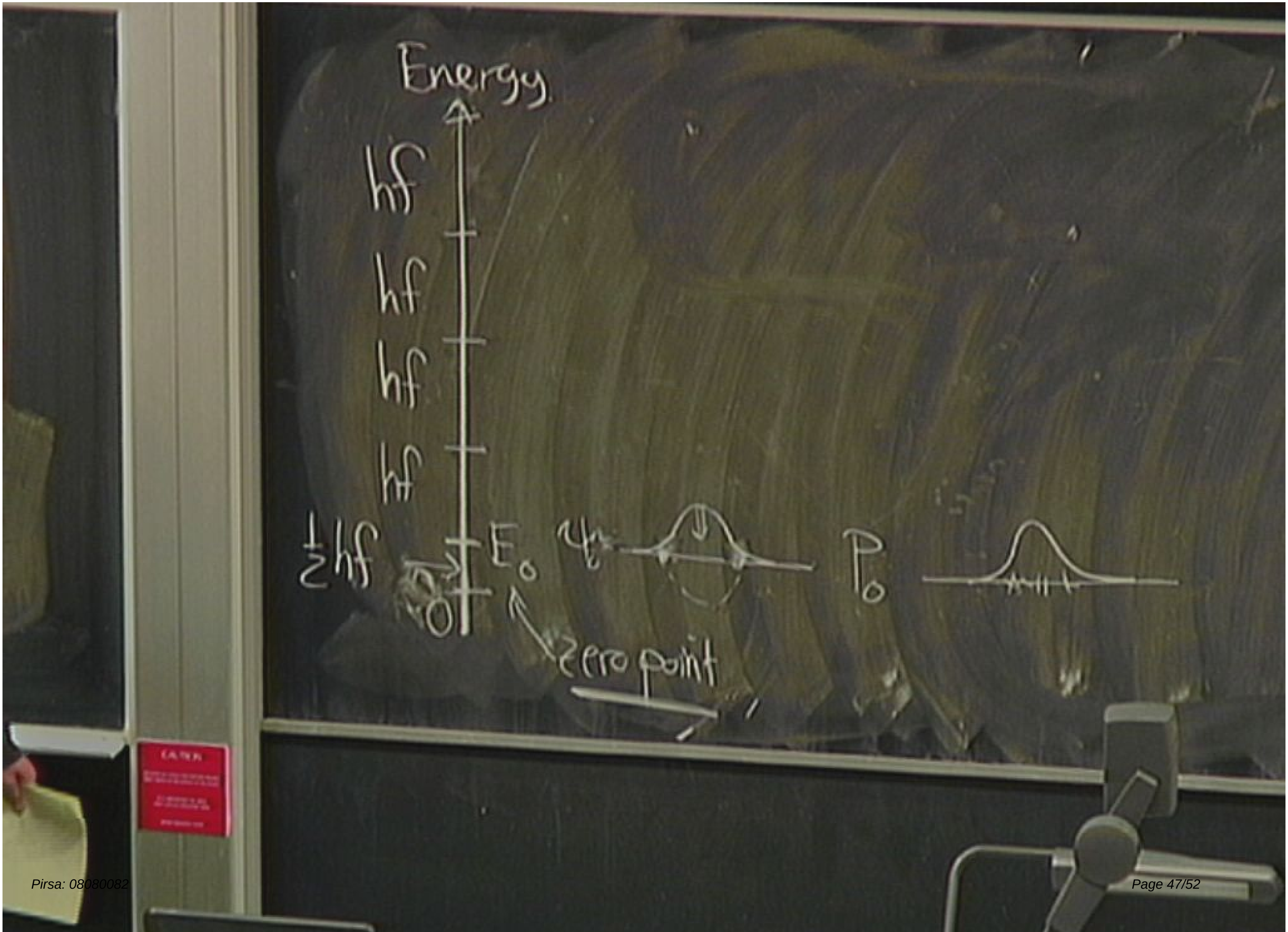
"turning points": $p = 0$, $x = \pm A$

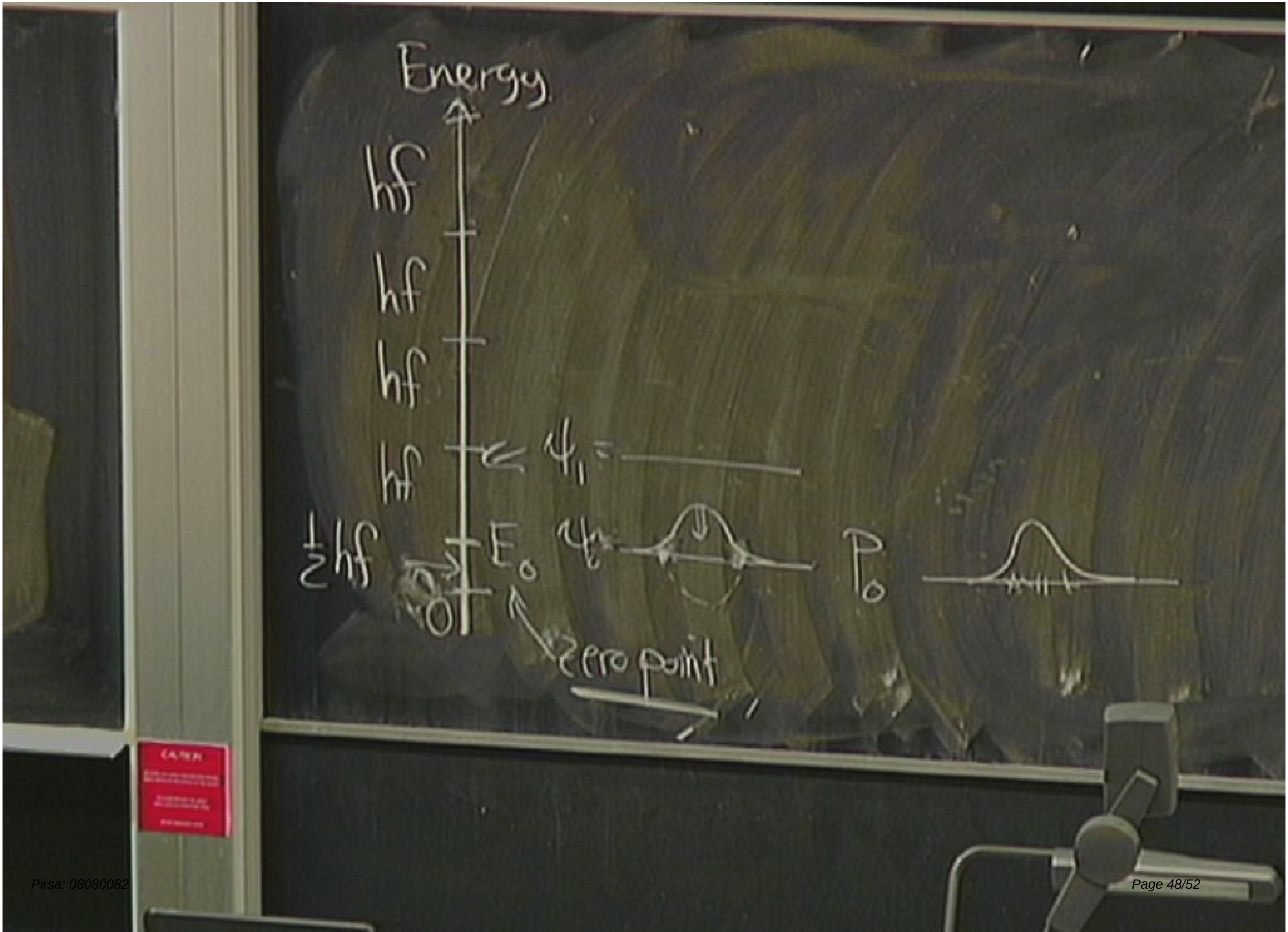
$$E = \frac{\overset{\to 0}{p^2}}{2m} + \frac{1}{2}k\underset{\to \pm A}{x^2} = \frac{1}{2}kA^2$$

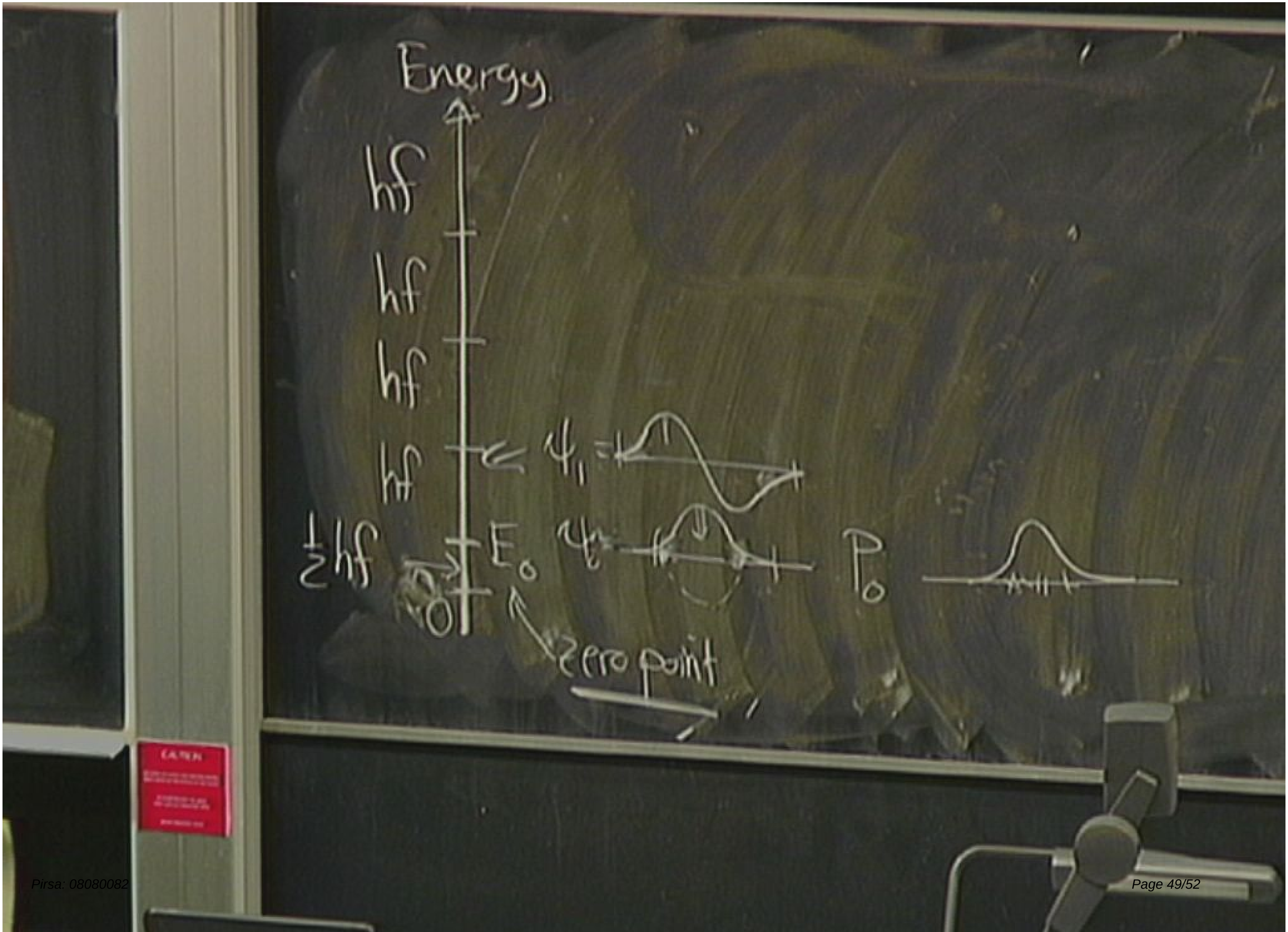


$$A = \sqrt{\frac{2E}{k}}$$

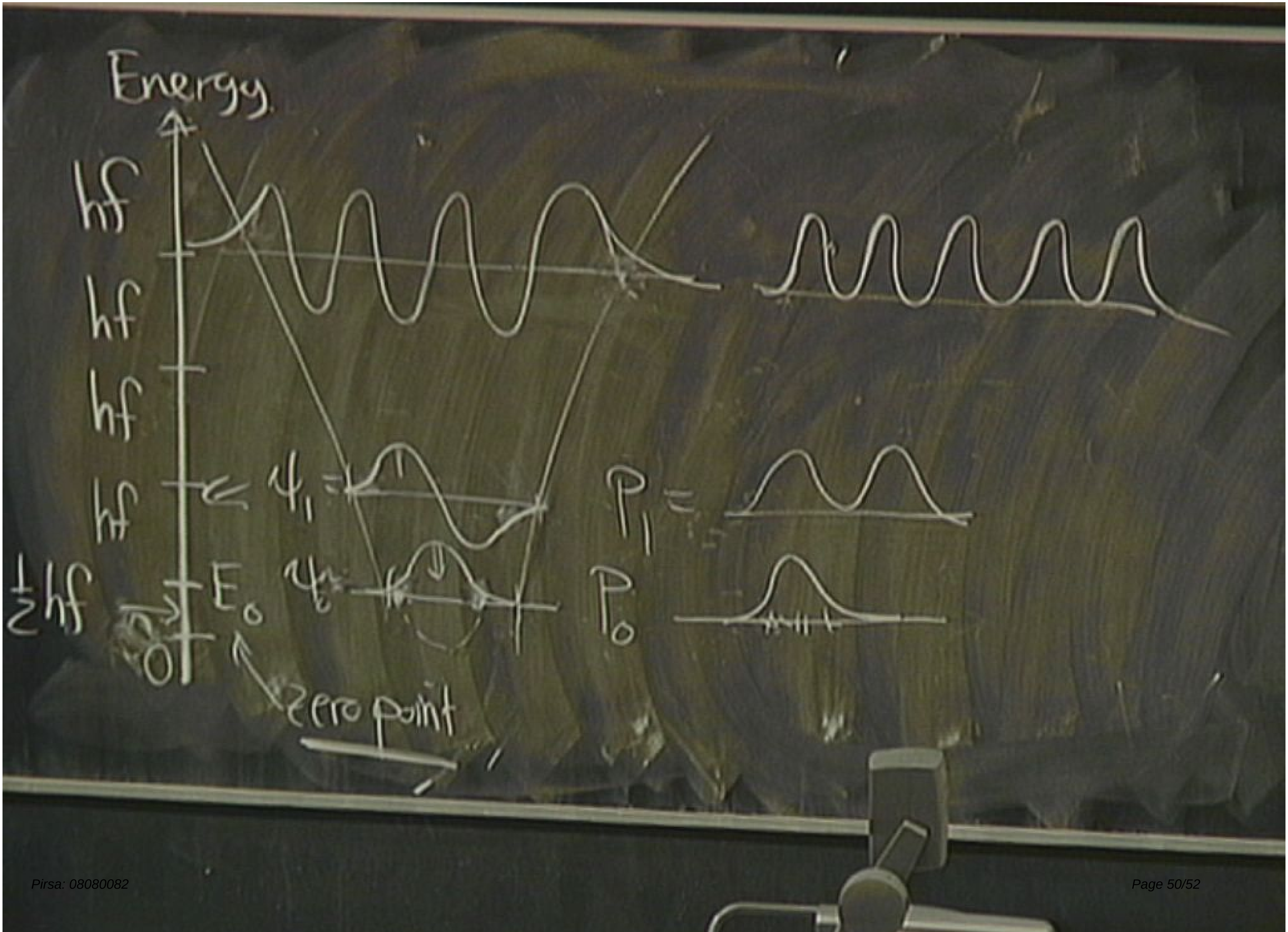


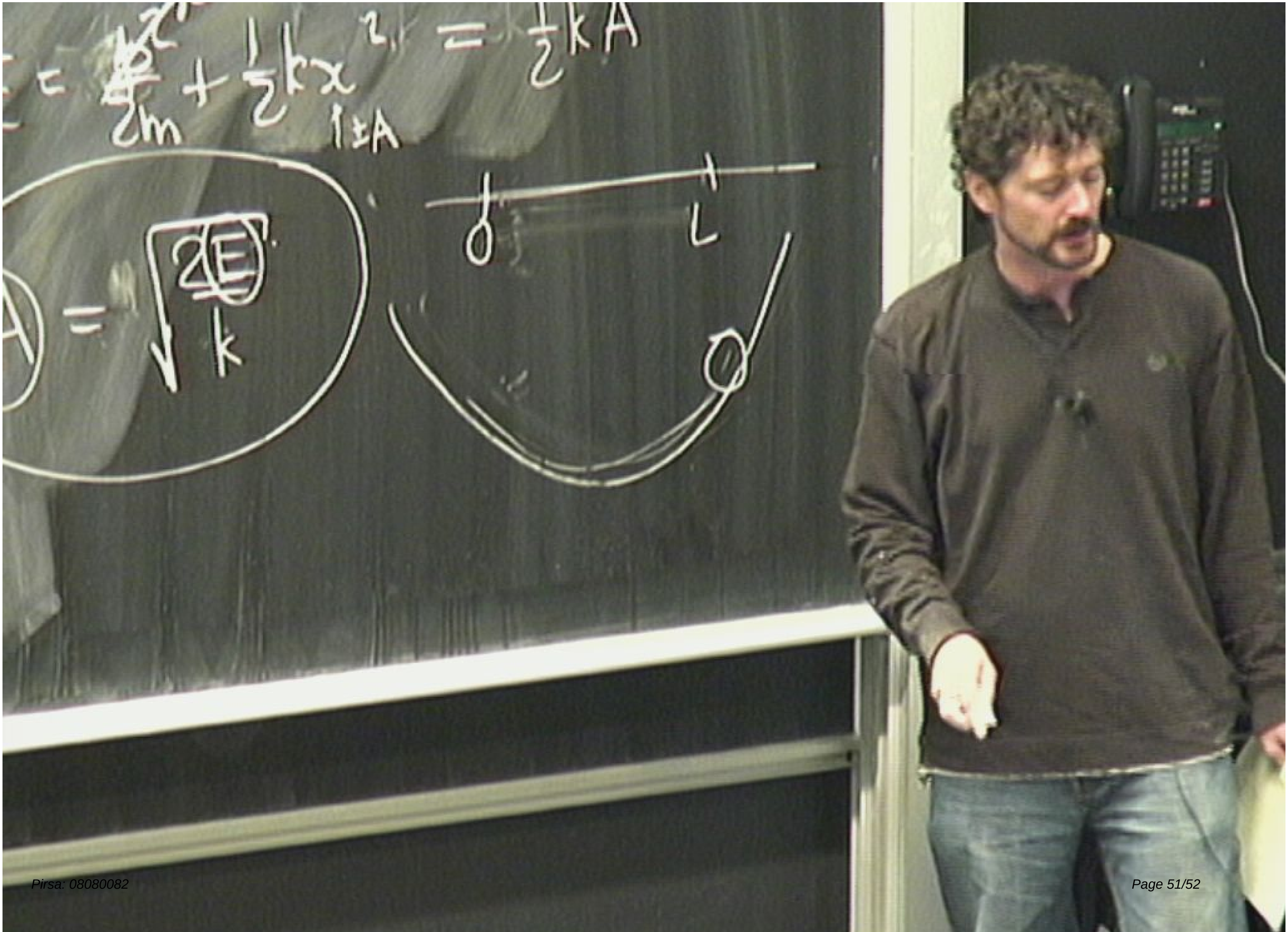

Energy
hf
hf
hf
hf
½hf
E₀
ψ₀
P₀
0
zero point

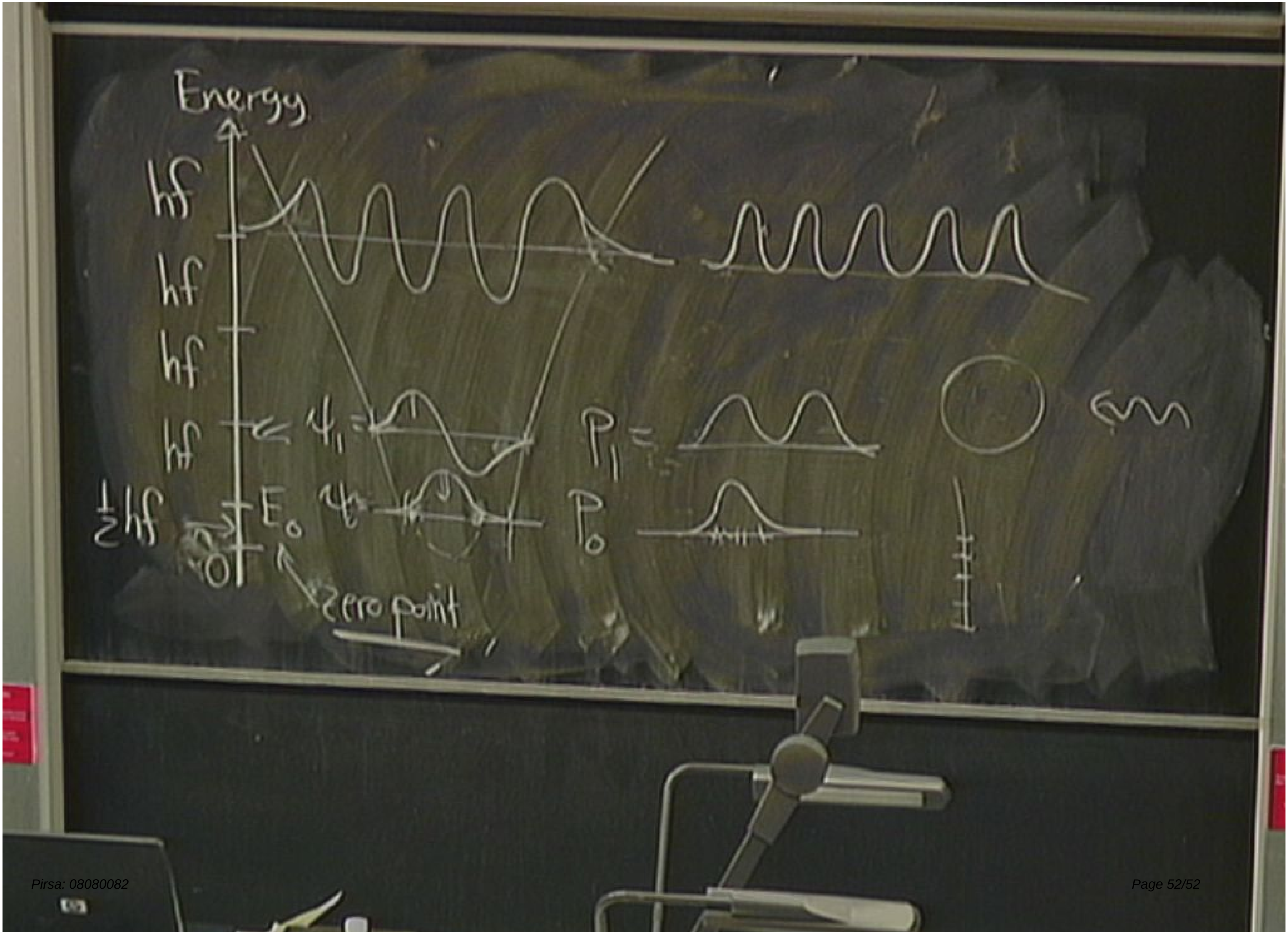

Energy
hf
hf
hf
hf
$\frac{1}{2}hf$
E_0
0
$\psi_1 =$
$\psi_0 =$
P_0
zero point



Energy
hf
hf
hf
hf
$\frac{1}{2}hf$
E_0
0
$\psi_1 =$
$\psi_0 =$
P_0
zero point


Energy
hf
hf
hf
hf
$\frac{1}{2}hf$
0
E_0
$\psi_1 =$
ψ_0
$P_1 =$
P_0
zero point


L


Energy
hf
hf
hf
hf
$\frac{1}{2}$hf
0
E_0
ψ_1 =
ψ_0 =
P_1 =
P_0
zero point