Title: General Relativity 2 - Gravity is a Curvature

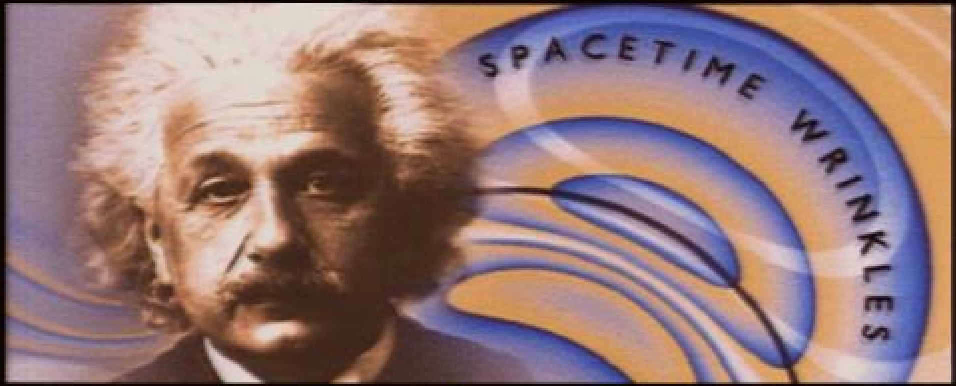
Date: Aug 01, 2006 09:00 AM

URL: http://pirsa.org/08080072

Abstract: Spacetime tells matter how to move, and matter tells spacetime how to curve.
Learning Outcomes:
â€¢ Why gravity can be seen as a curvature of spacetime.
â€¢ That Einsteinâ€™s field equations describe how matter curves spacetime.
â€¢ How Sir Arthur Eddington verified Einsteinâ€™s theory of general relativity by measuring the change in position of stars during a solar eclipse.

Gravity as a curvature of Spacetime

The early 1900's changed the way gravity is looked at. Einstein didn't think of gravity as a force between objects, but as a curving of "straight lines" due to mass. Light always follows straight lines, but these may look curved near masses. Time also slows down near masses (space and time are different parts of "spacetime", which is what gets bent).



$$G_{uv} + \Lambda g_{uv} = 8\pi T_{uv}$$

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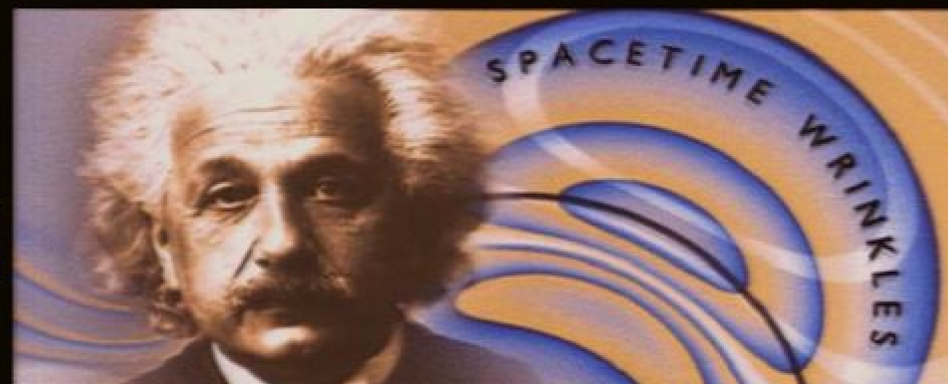

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Encodes geometry (curving) of Space Time

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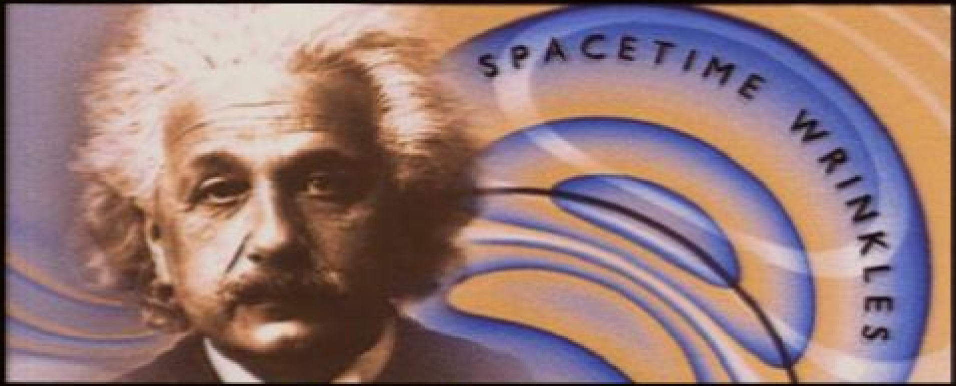

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Cosmological Constant

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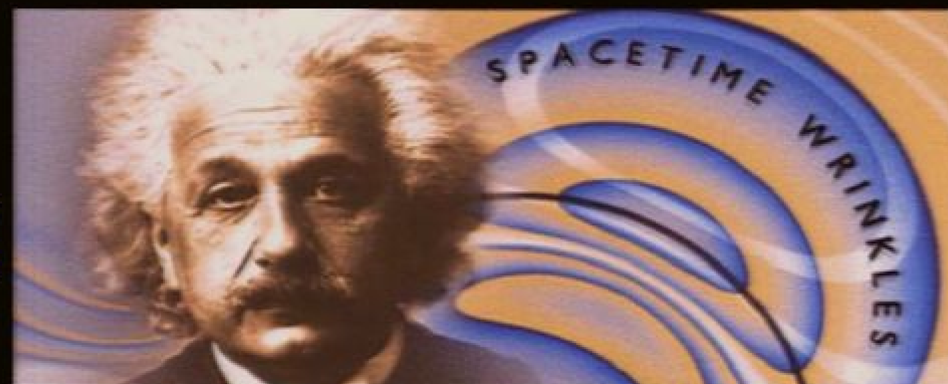

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$

Encodes geometry (curving) of Space Time

Cosmological Constant

Encodes the distribution of matter and its motion in Space Time

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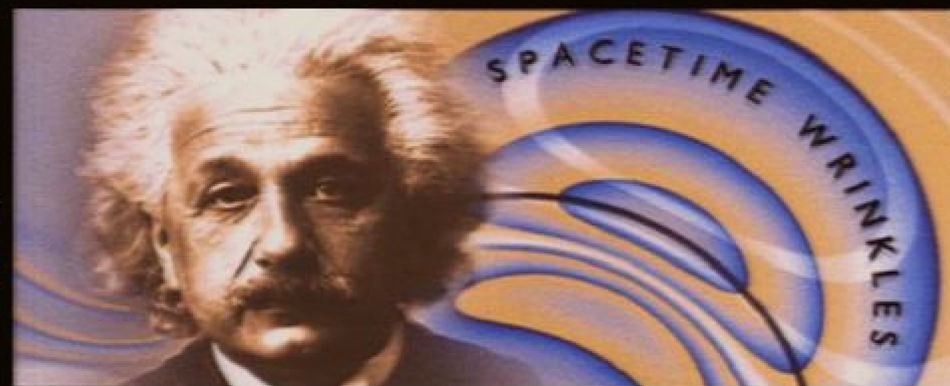

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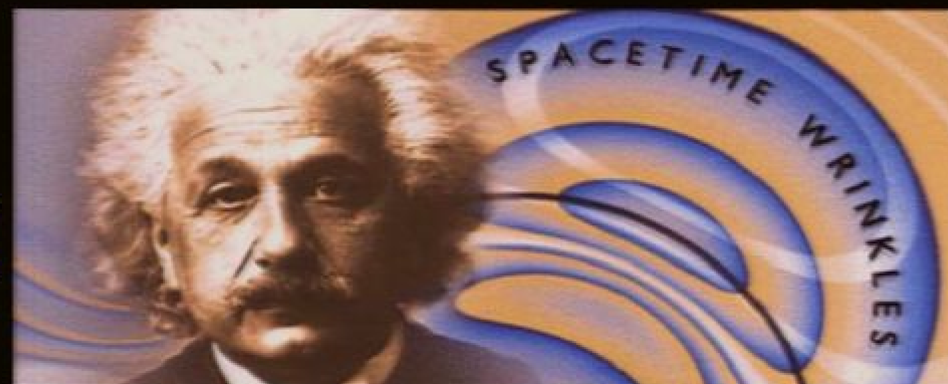


$$G_{uv} = 8\pi T_{uv}$$

Encodes geometry (curving) of Space Time

Encodes the distribution of matter and its motion in Space Time

Let's Review

Let's Review

Let's Review

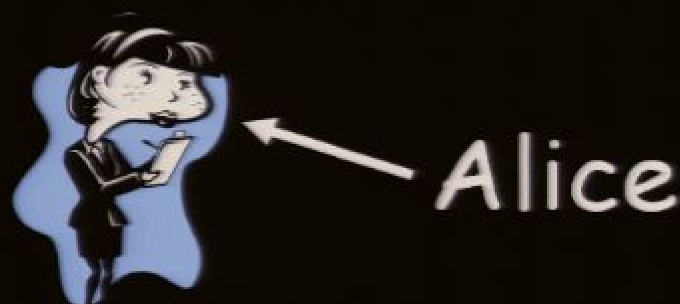
SPACETIME

Space Diagram

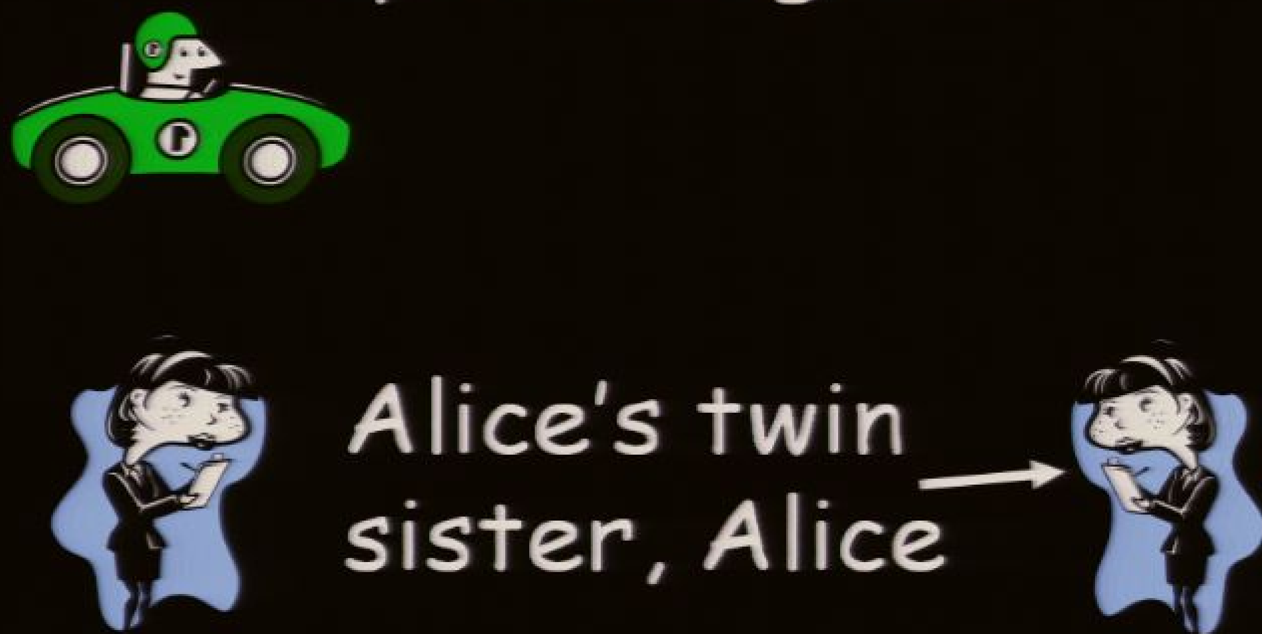
Bob

Space Diagram

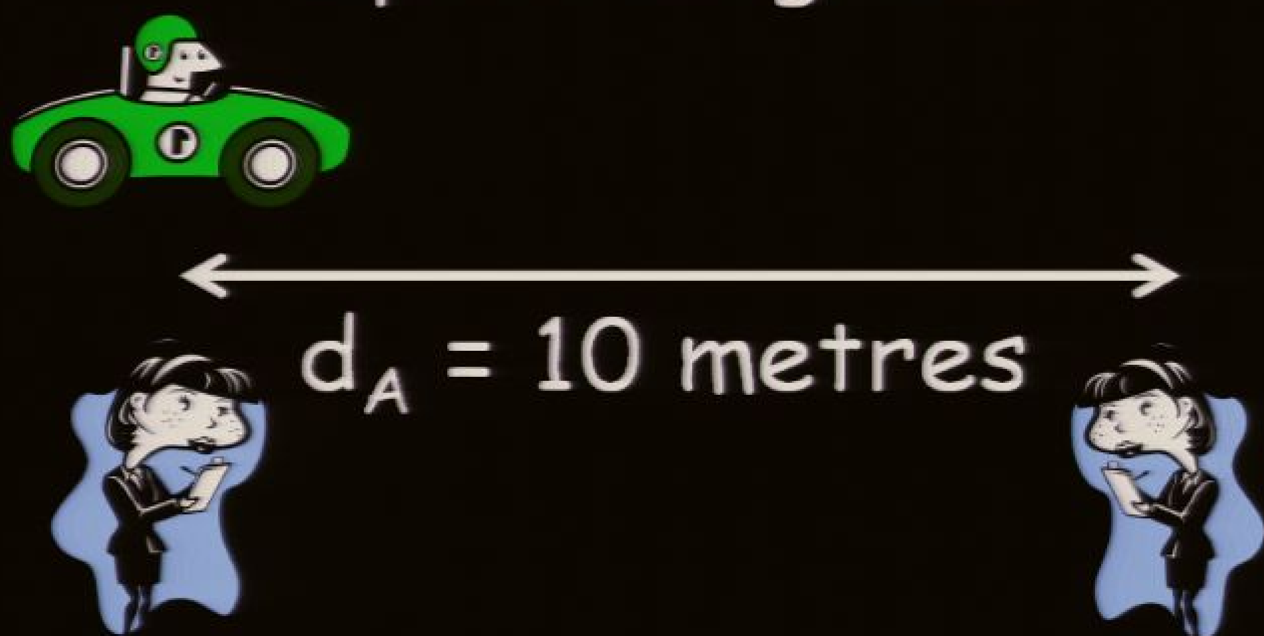
Space Diagram



Space Diagram



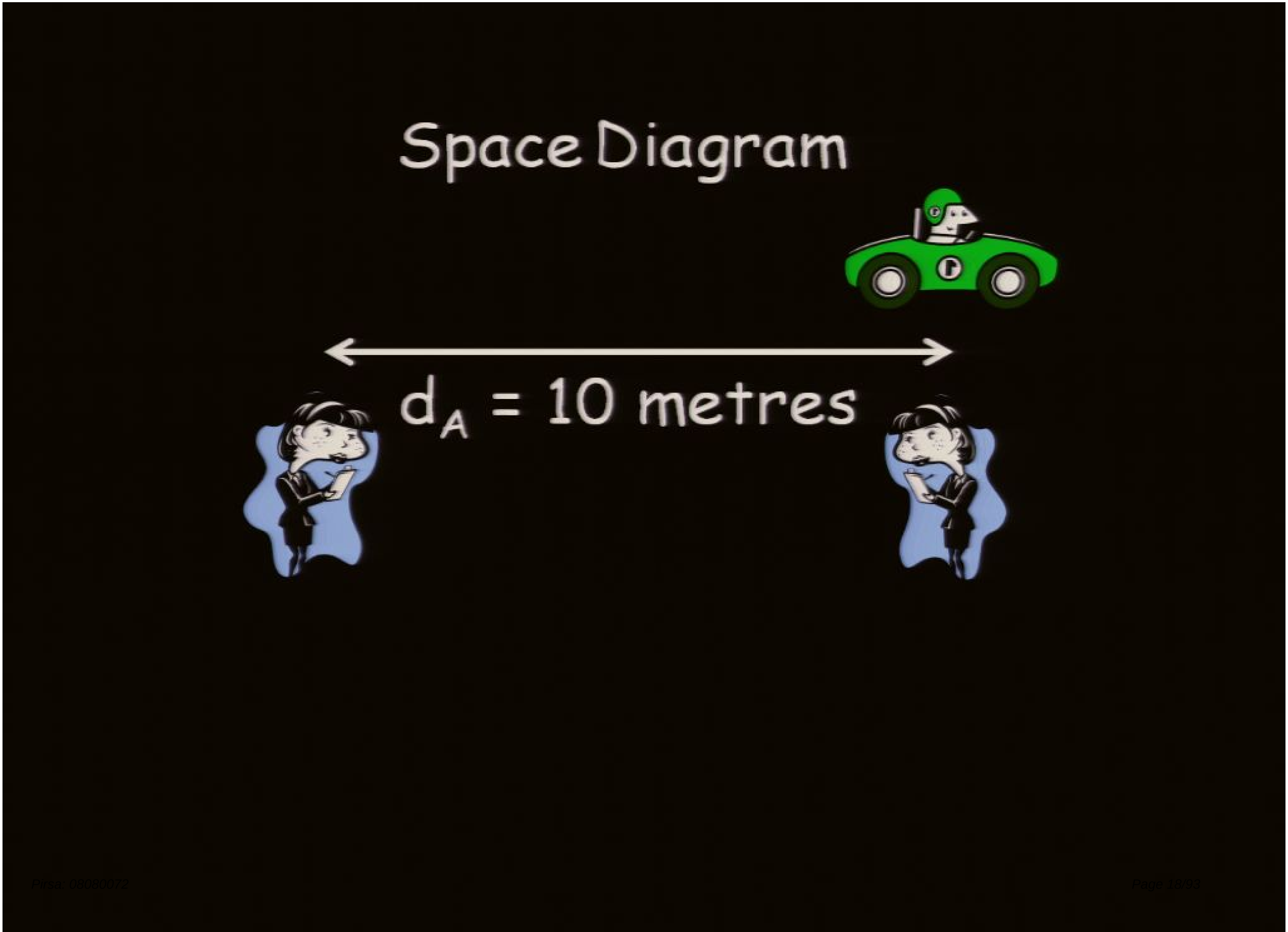
Space Diagram



Space Diagram

d_A = 10 metres

$t_A = 0$ $t_A = 0$


Space Diagram
d_A = 10 metres

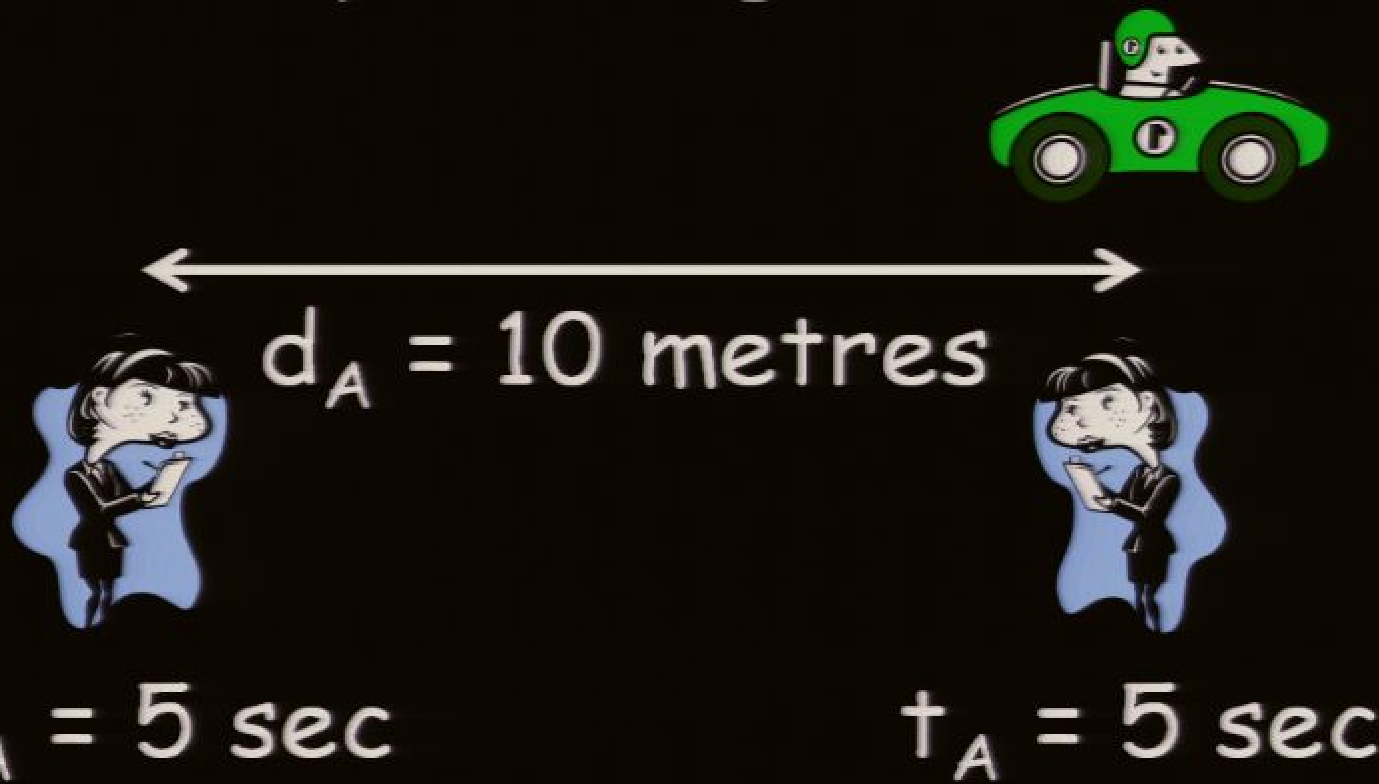
Space Diagram

$d_A = 10$ metres

$t_A = 5$ sec

$t_A = 5$ sec

Space Diagram



Question: How much time has elapsed for Bob?

Draw a "Spacetime Diagram"

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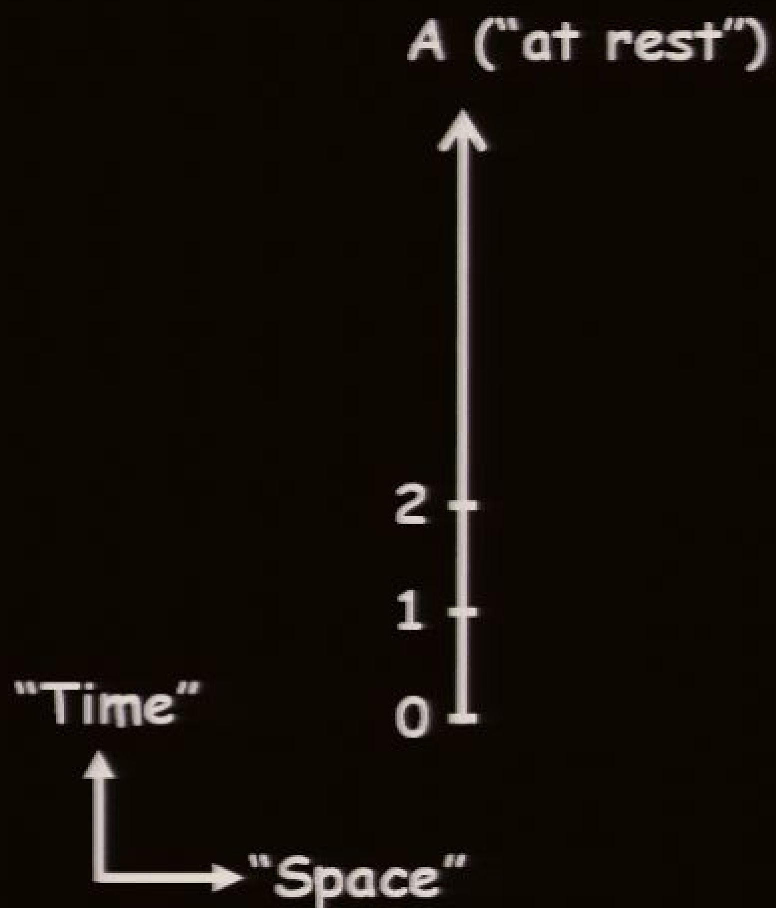
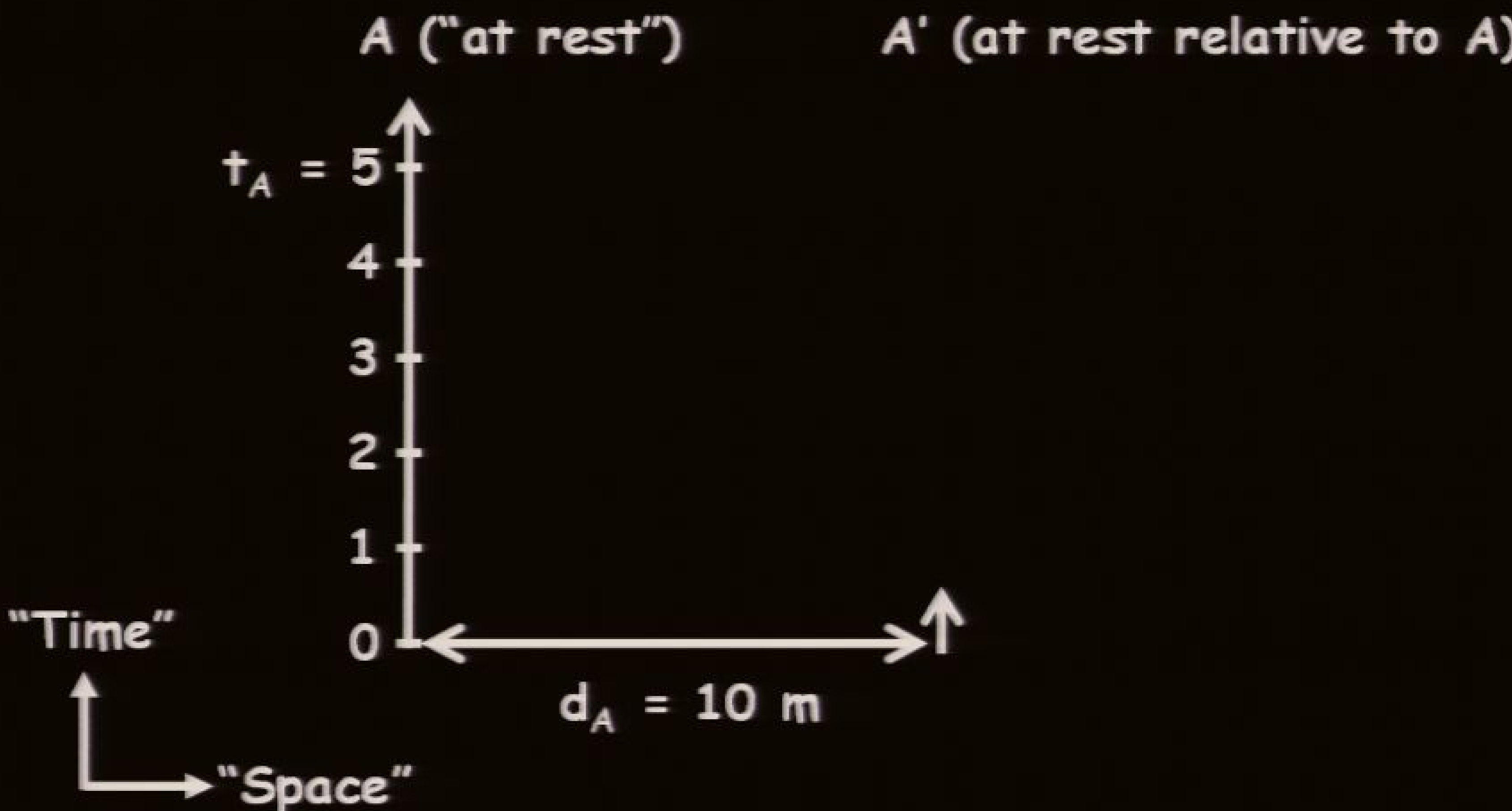


Draw a "Spacetime Diagram"



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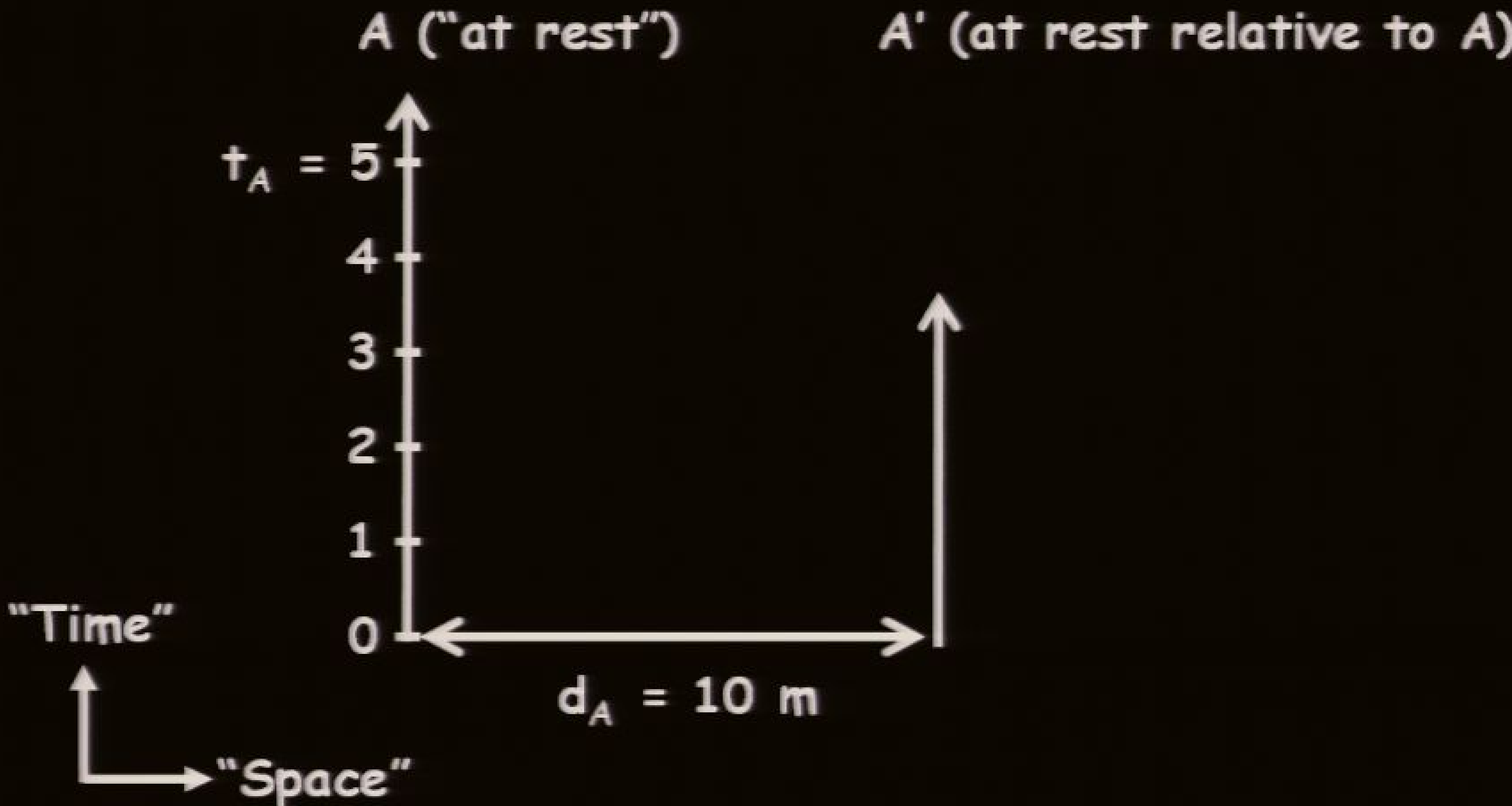

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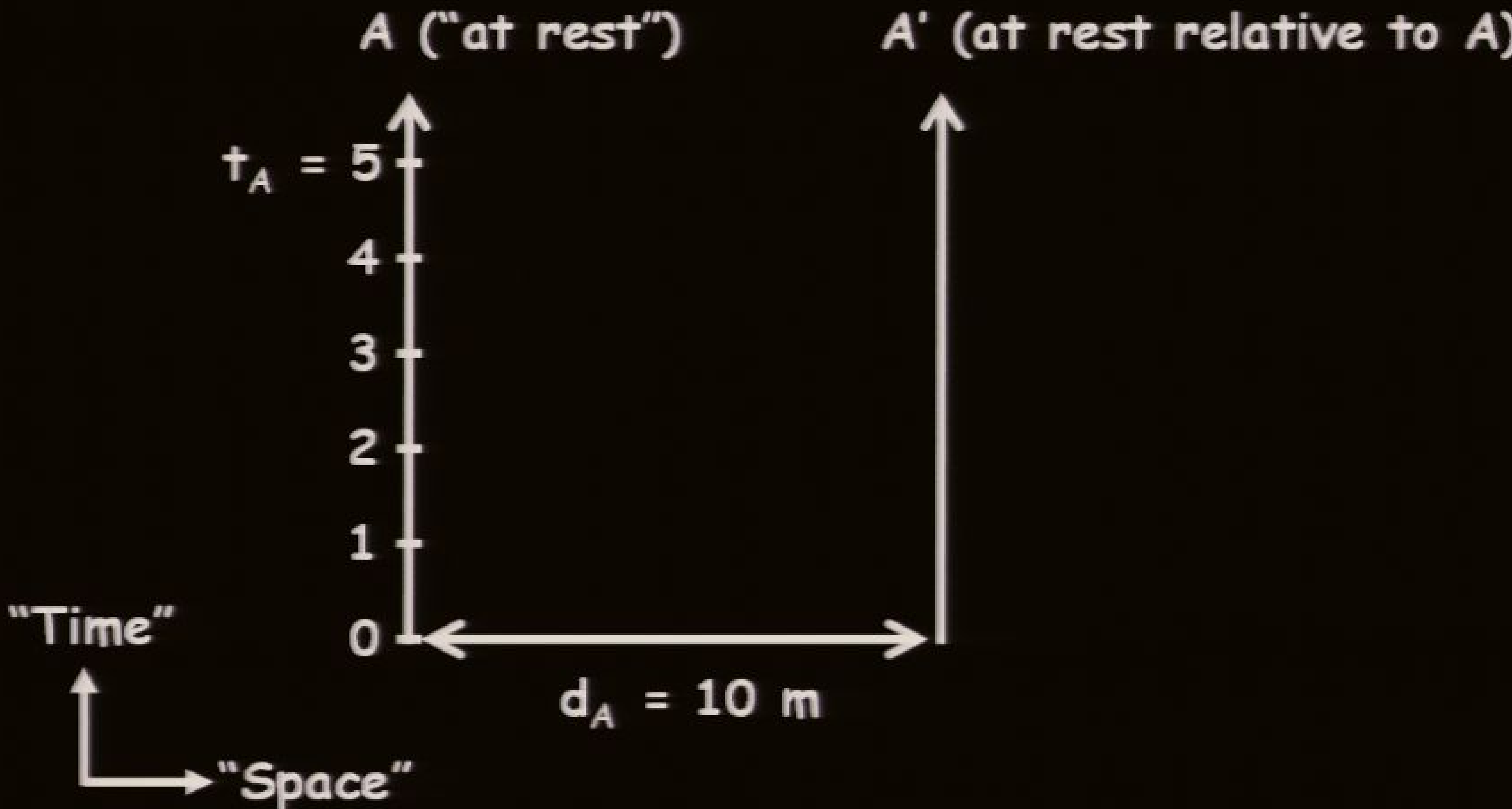

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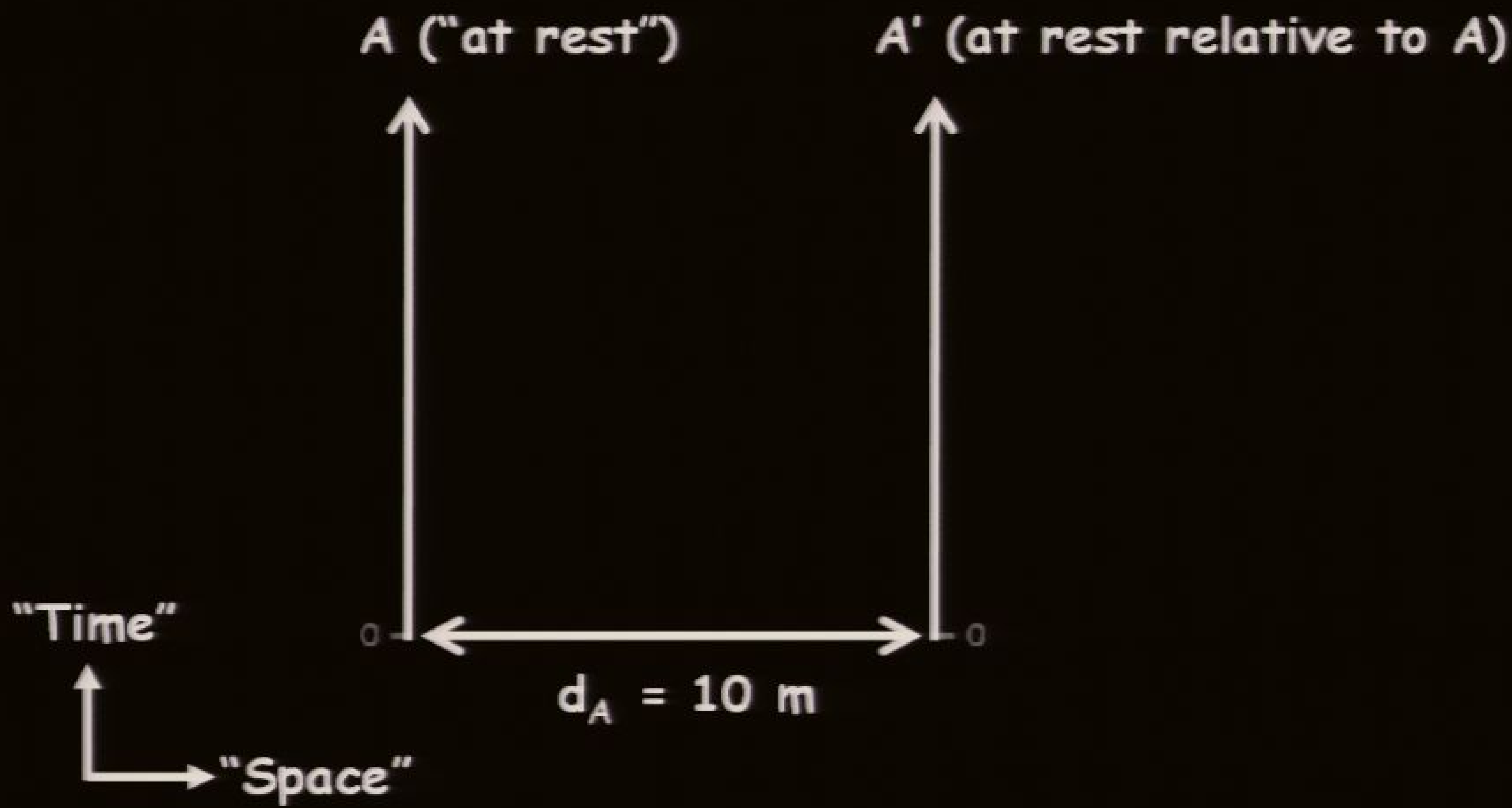

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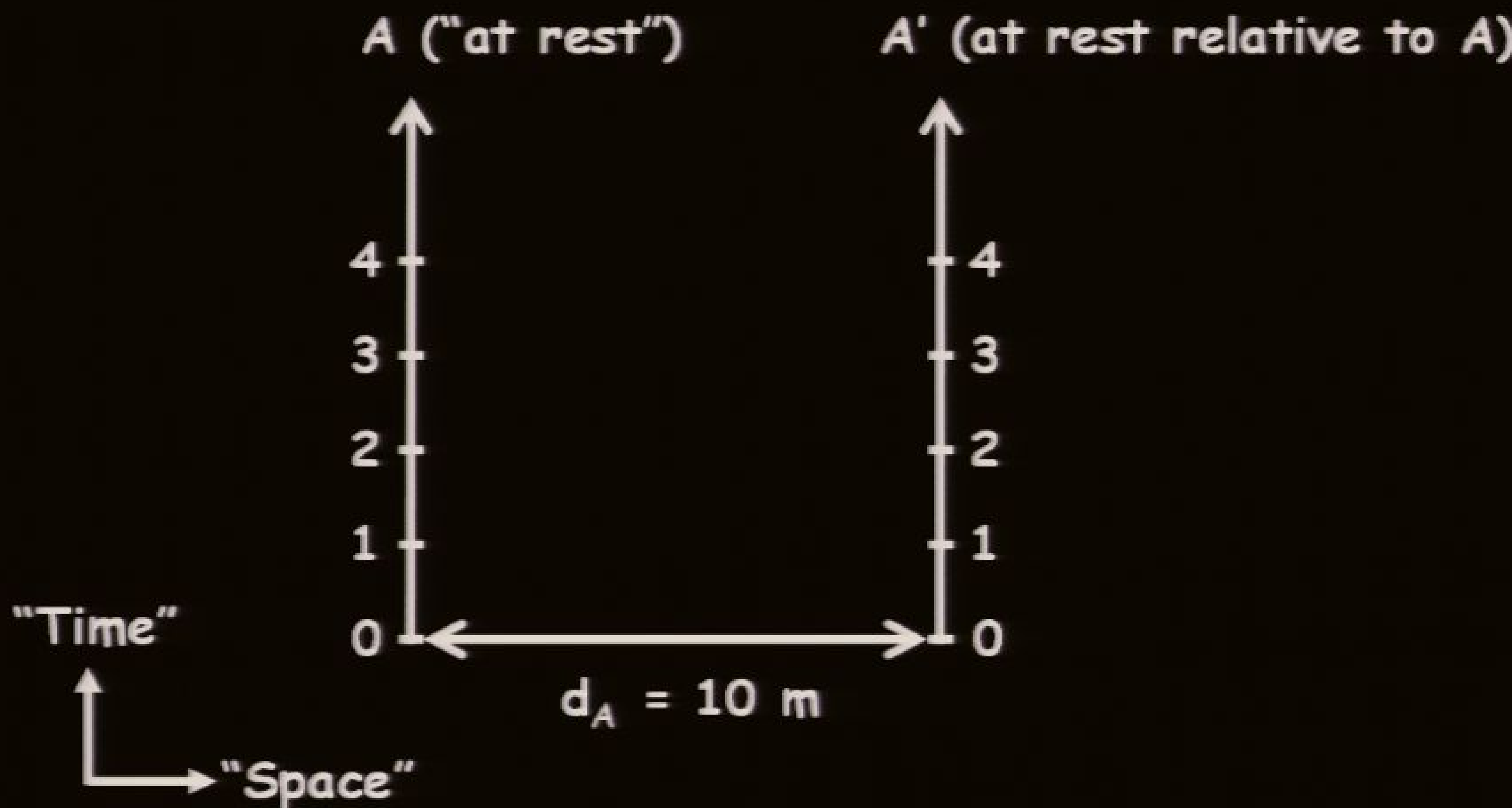

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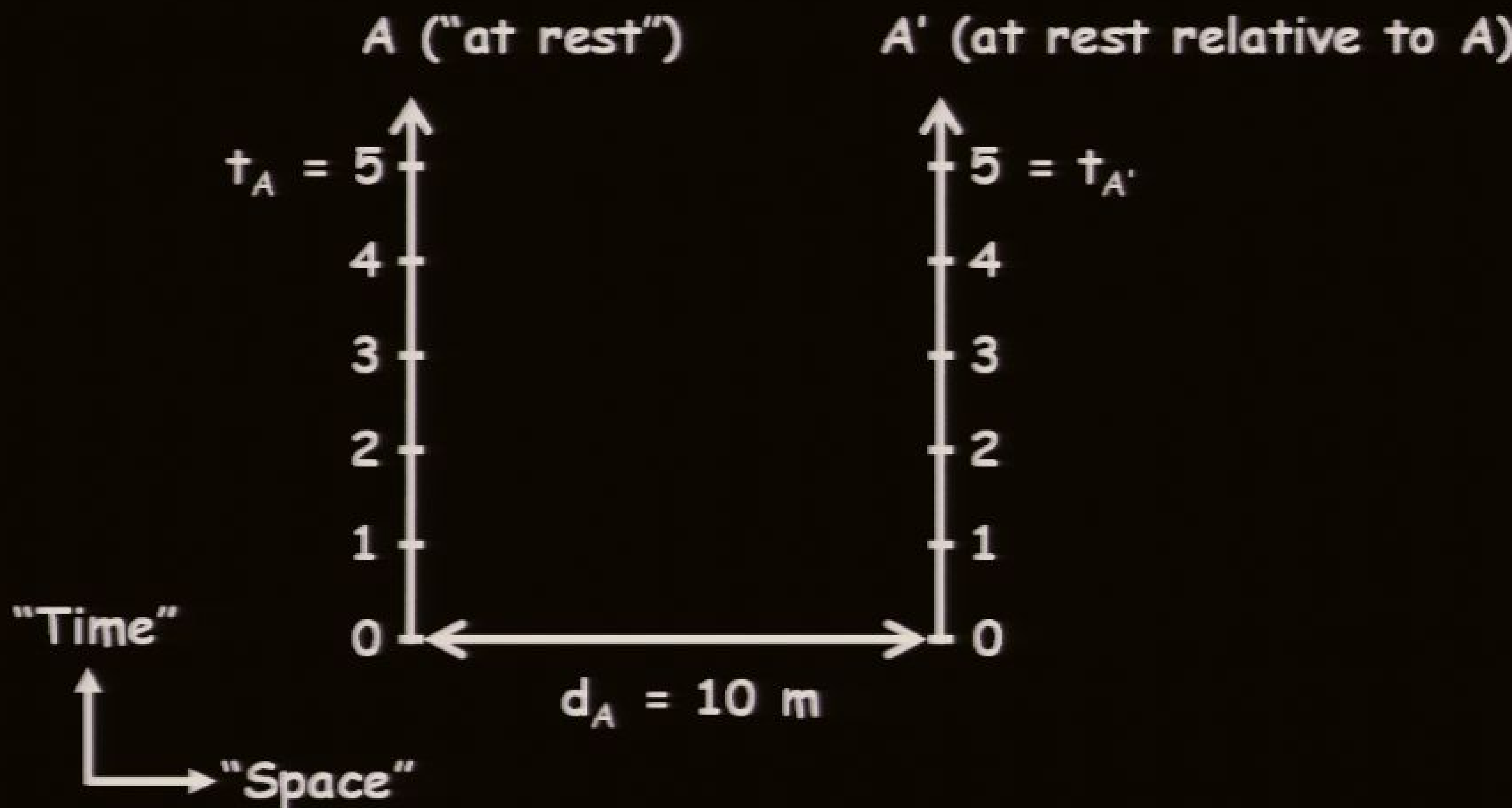

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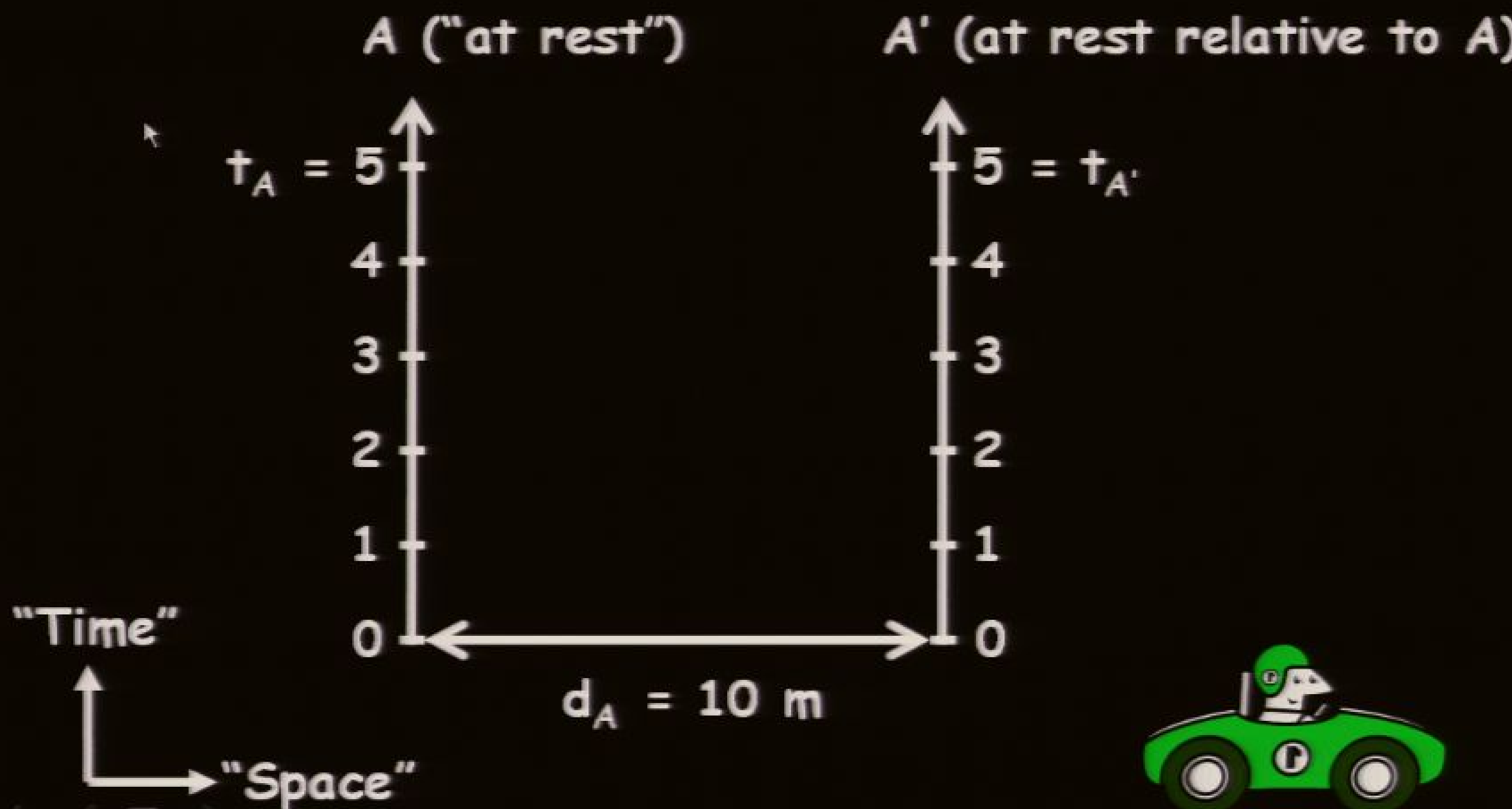

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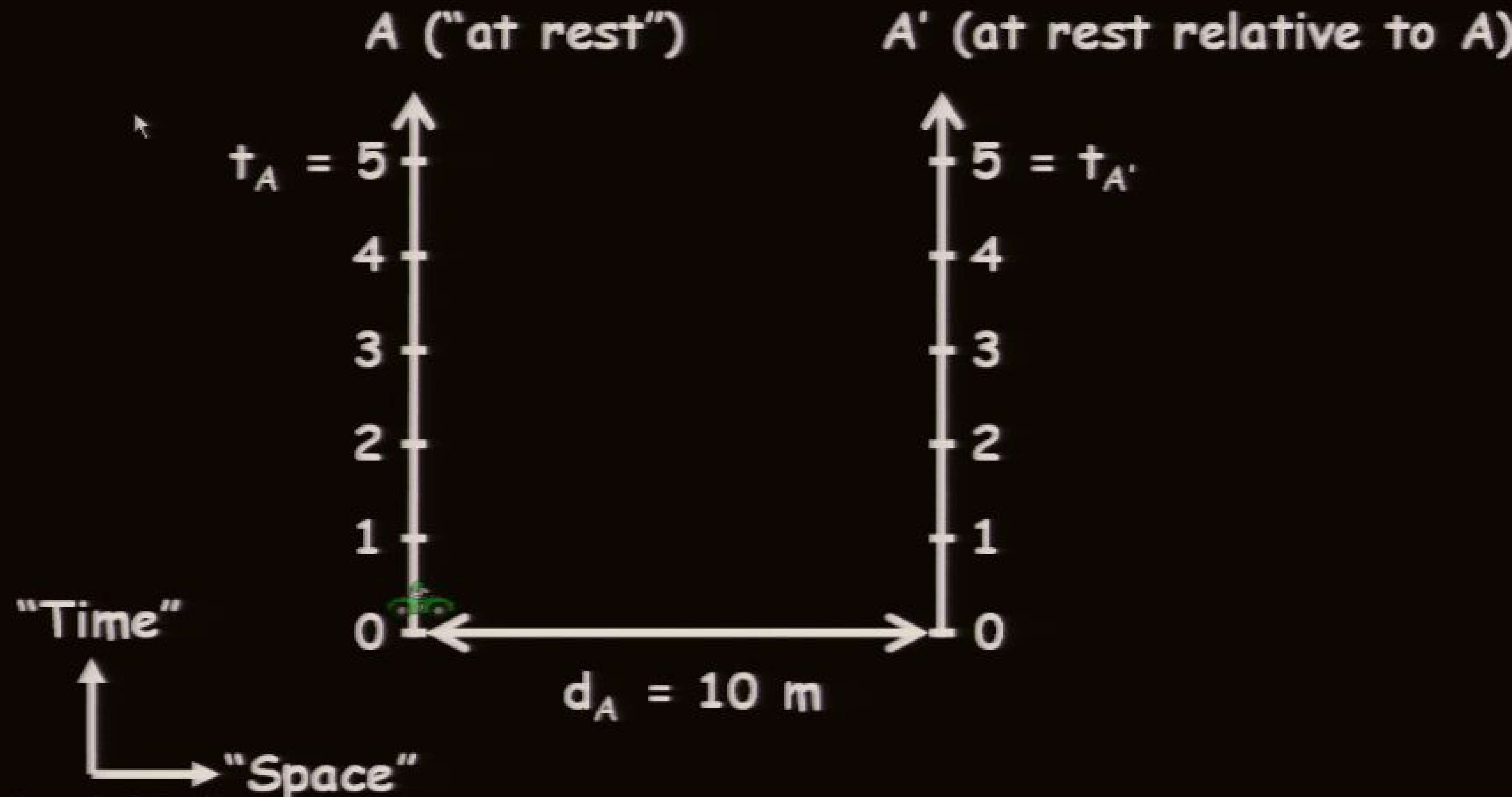

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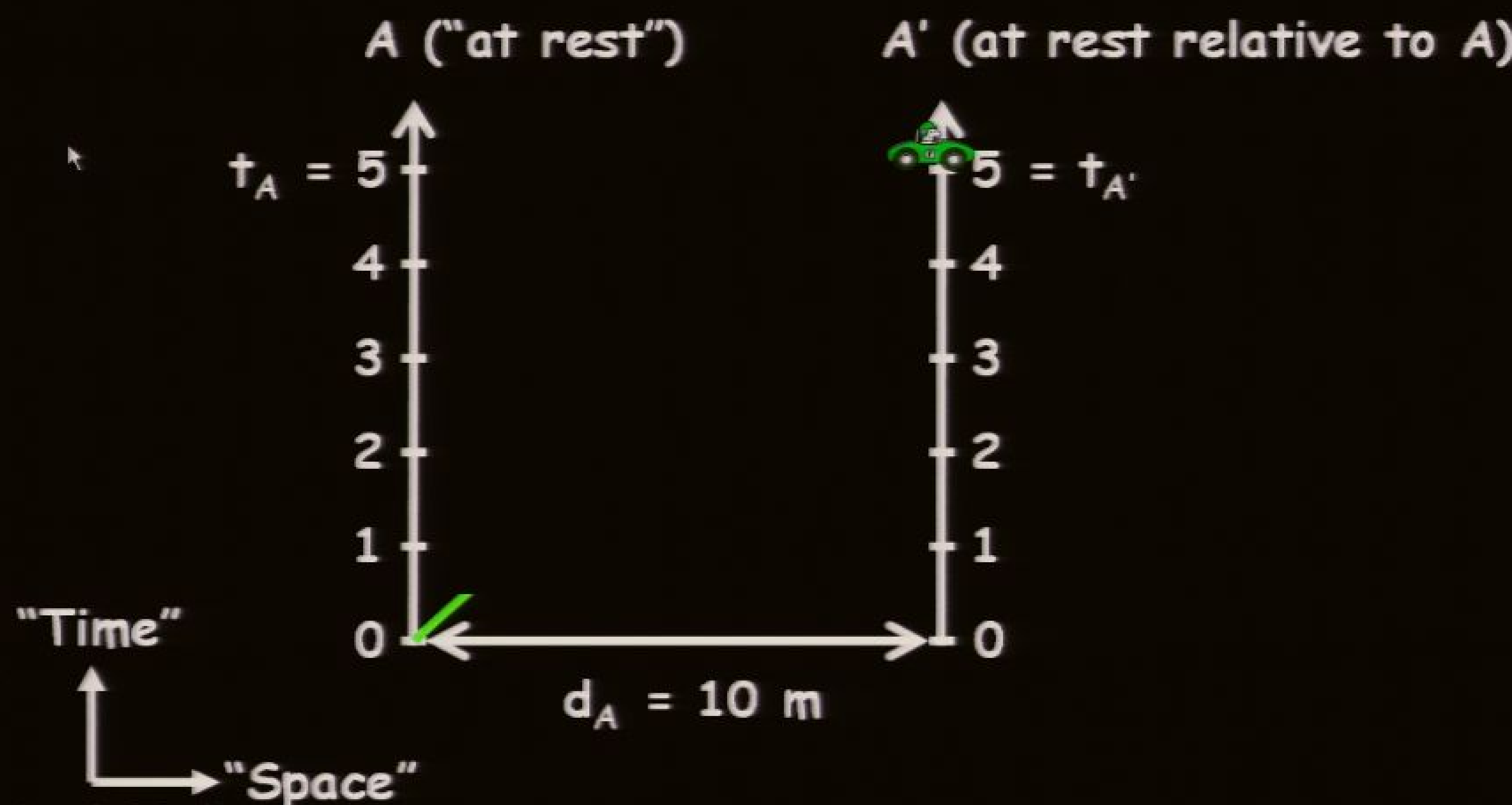

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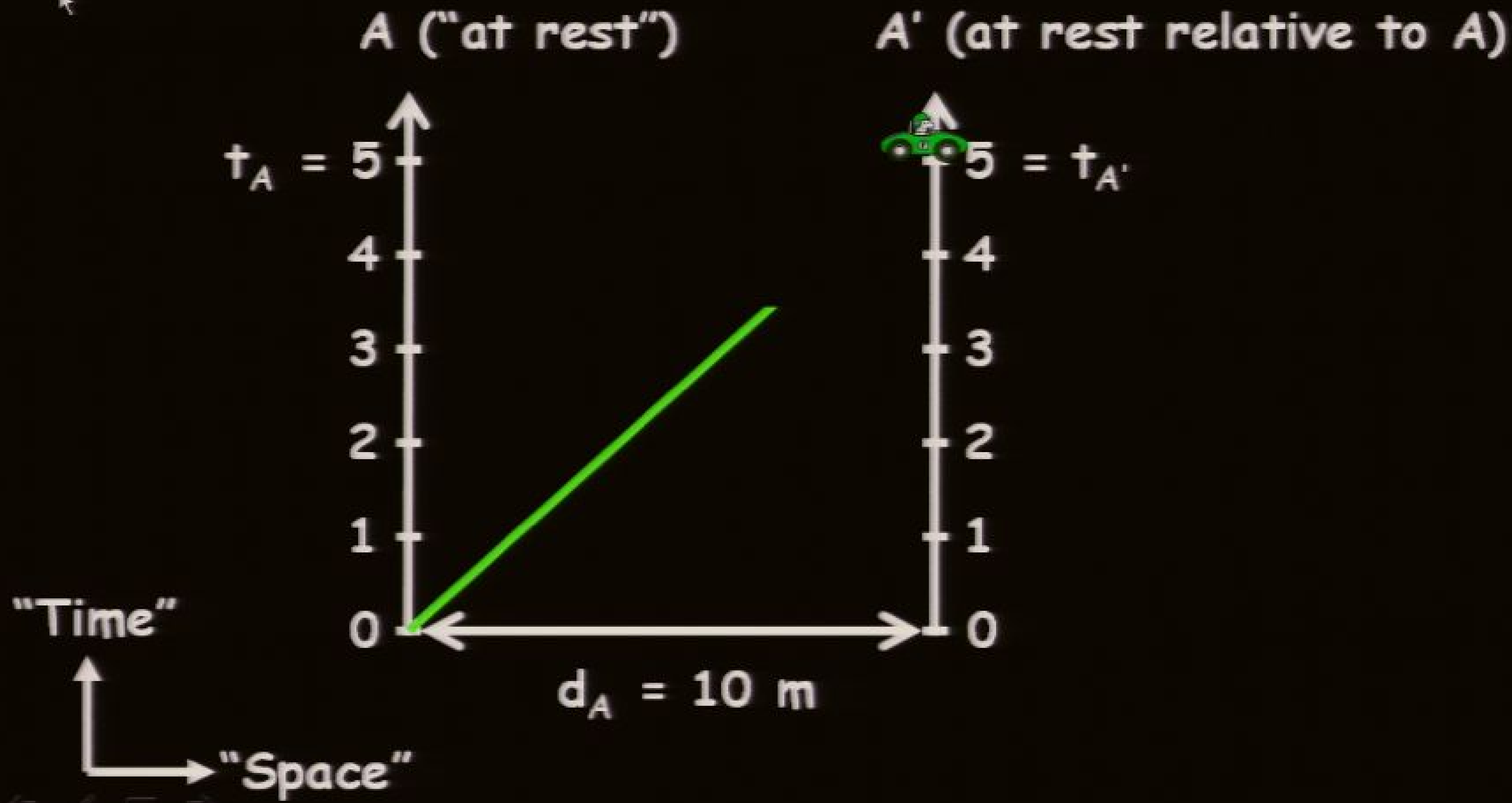

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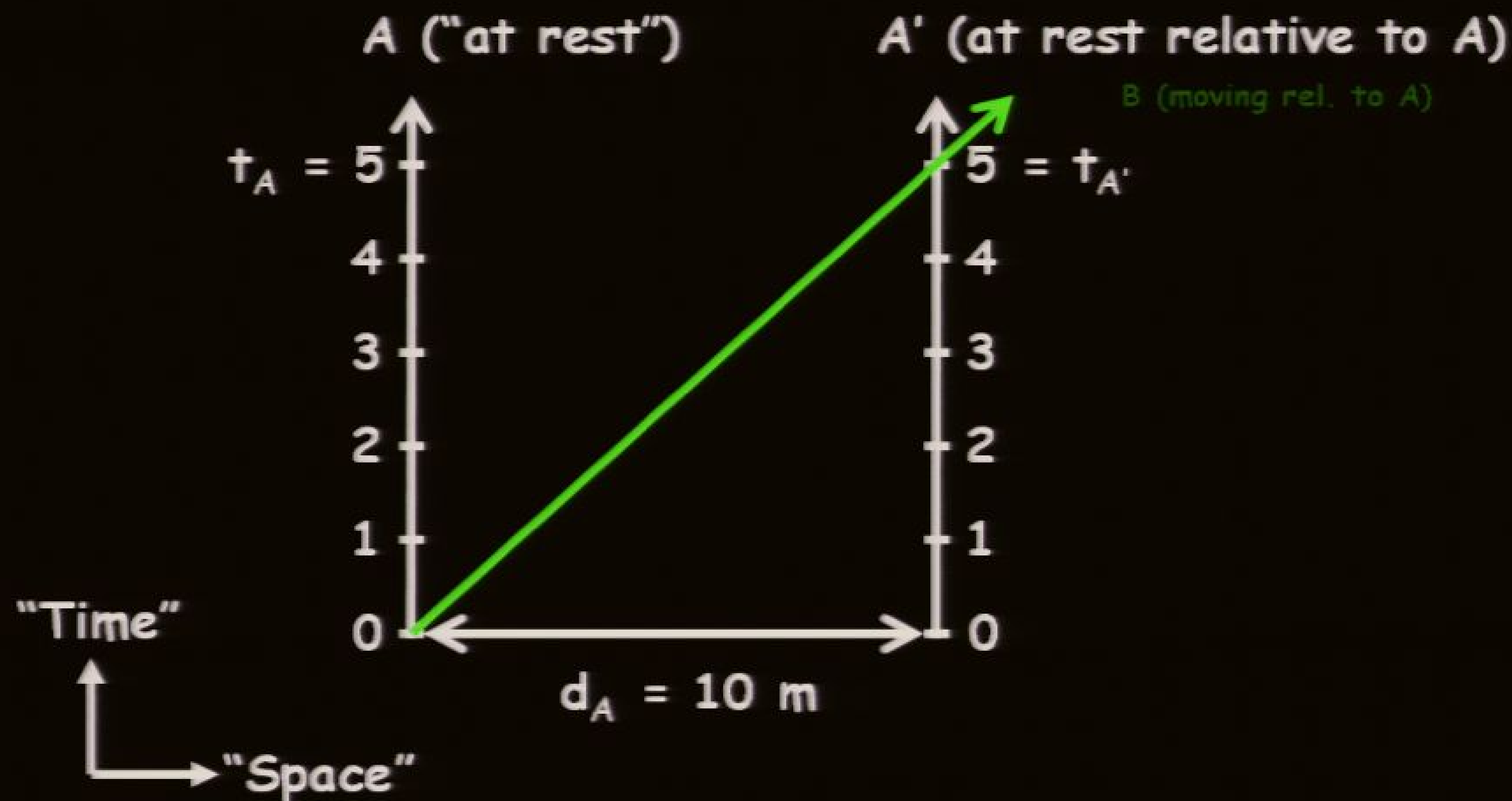

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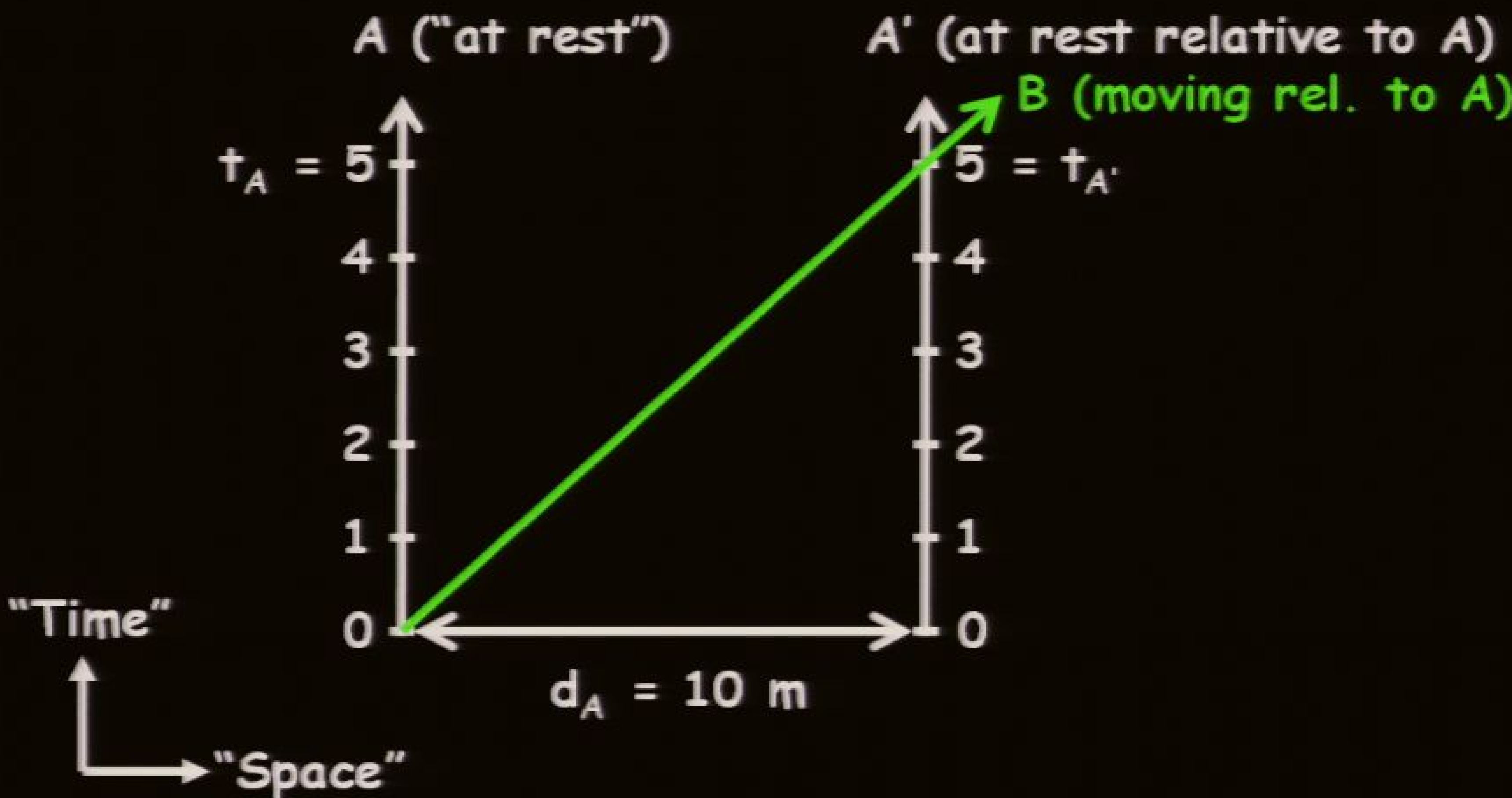

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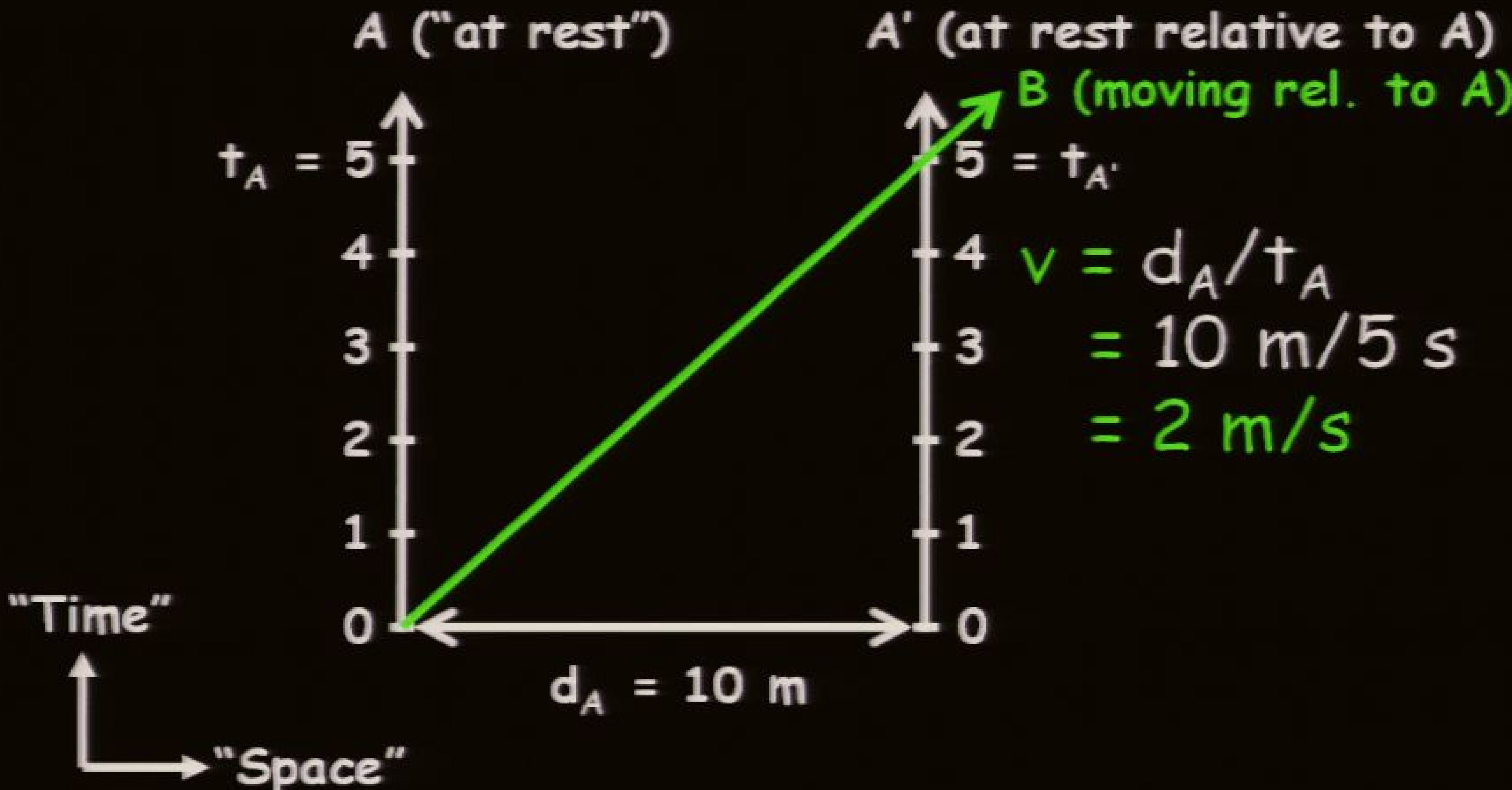

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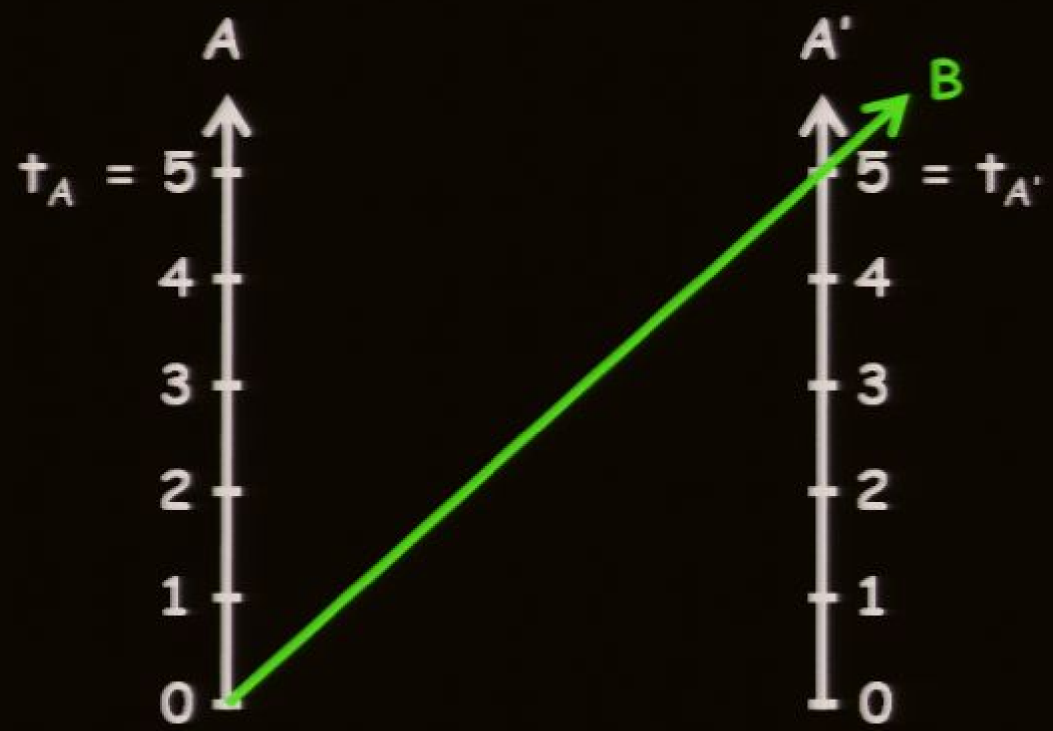

How do we indicate the ticking of B's clock?

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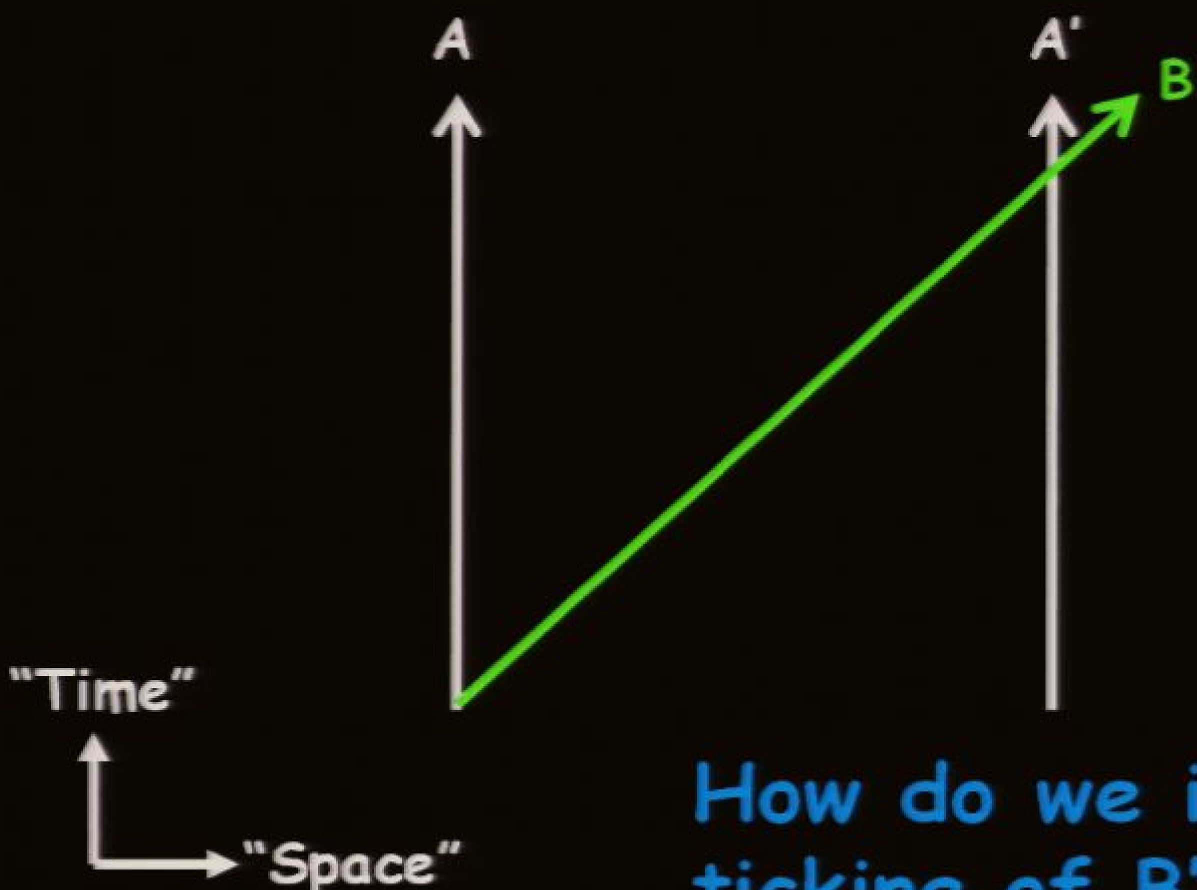

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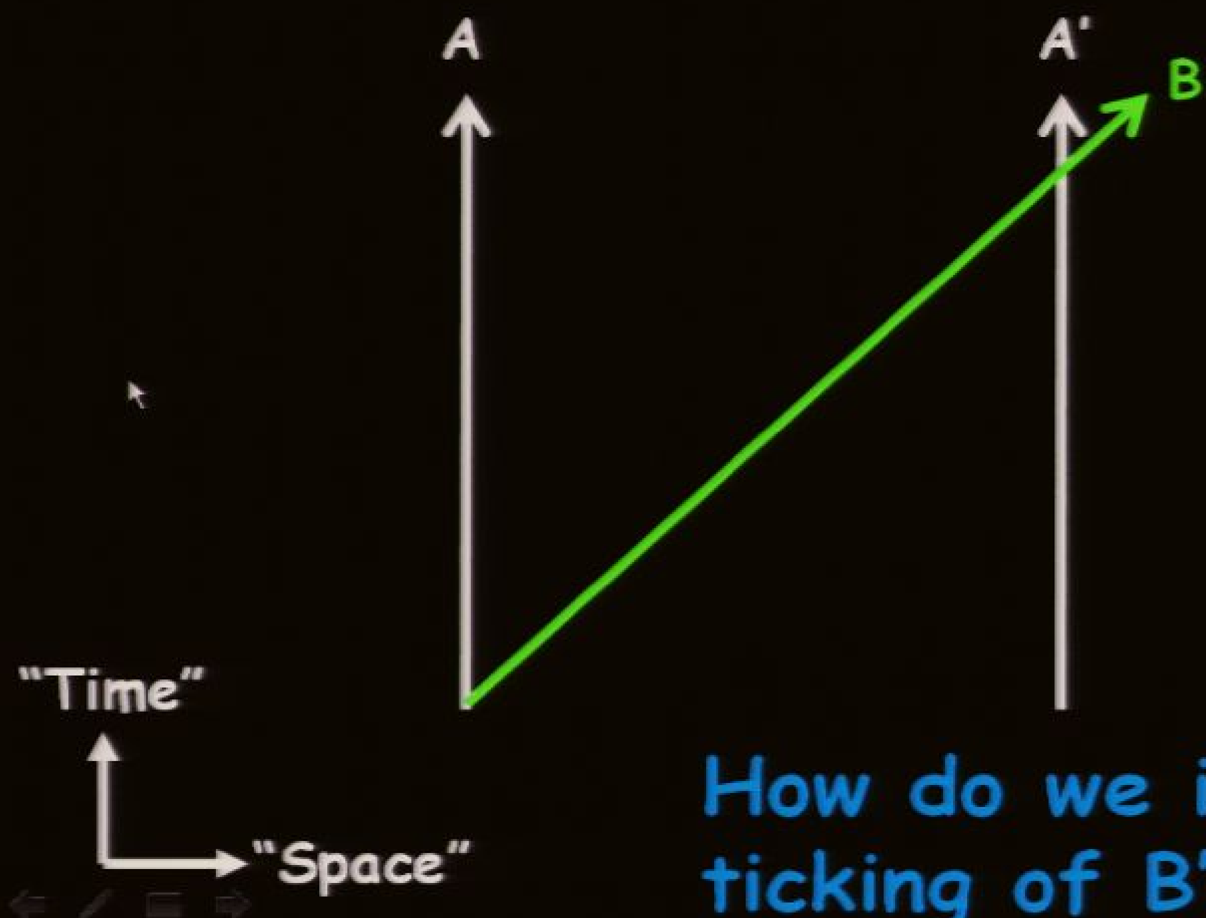

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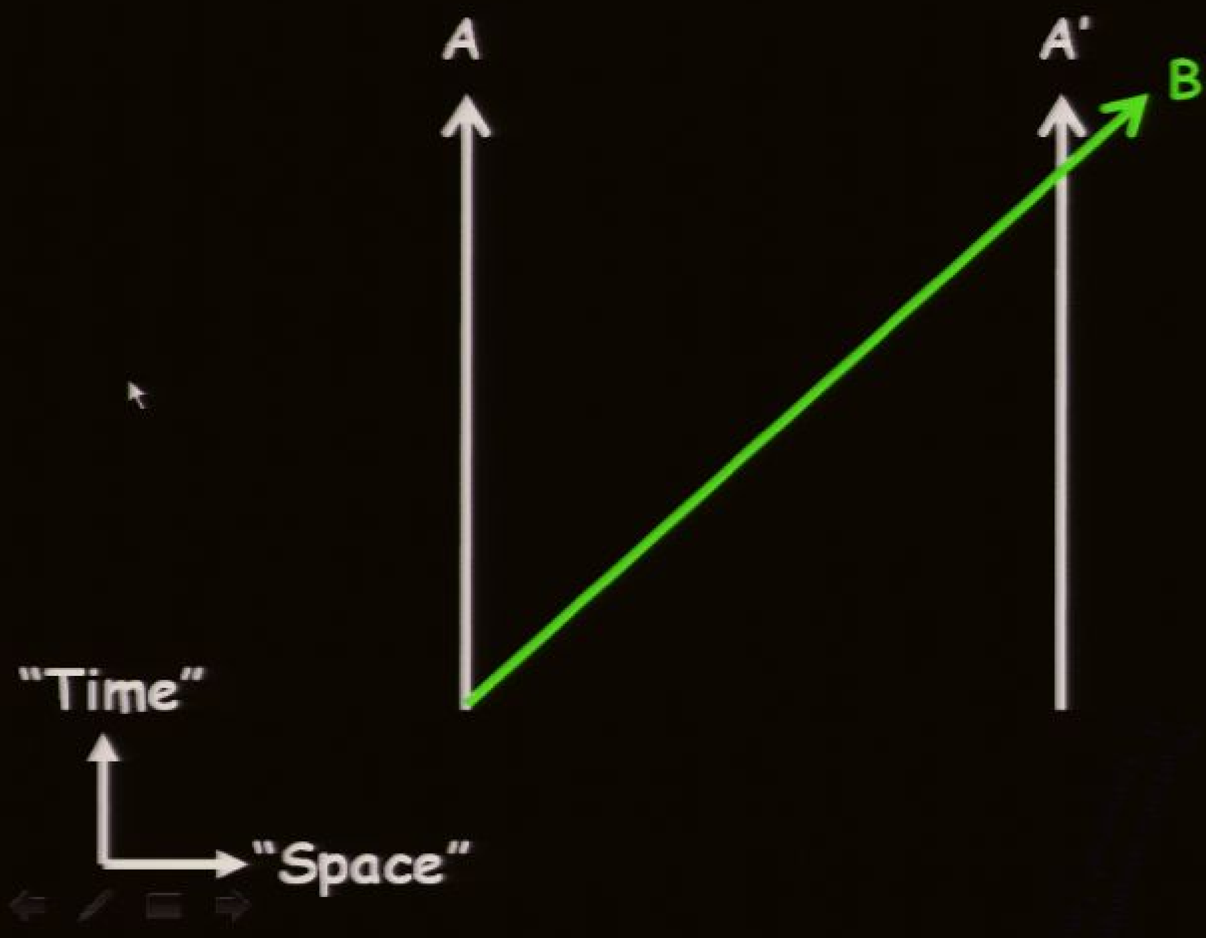

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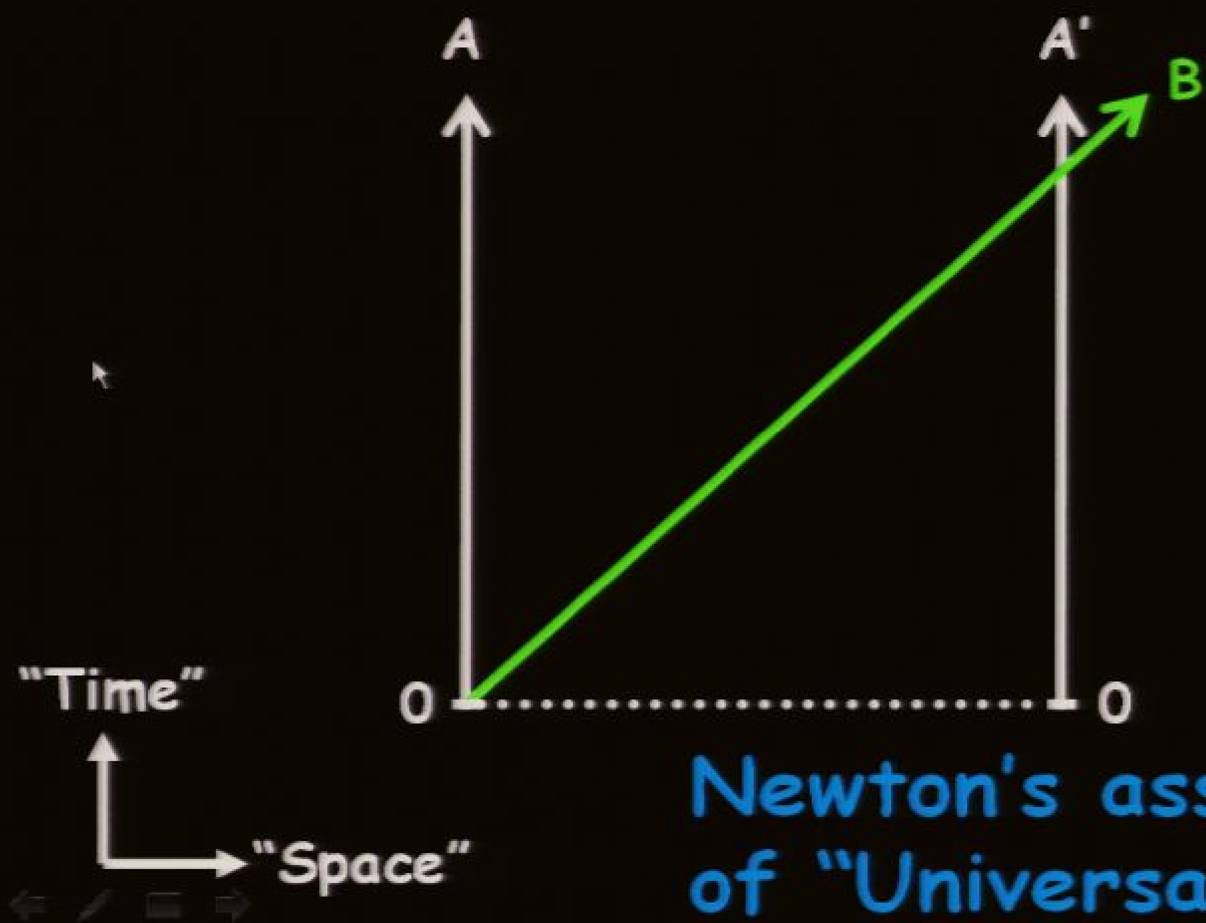
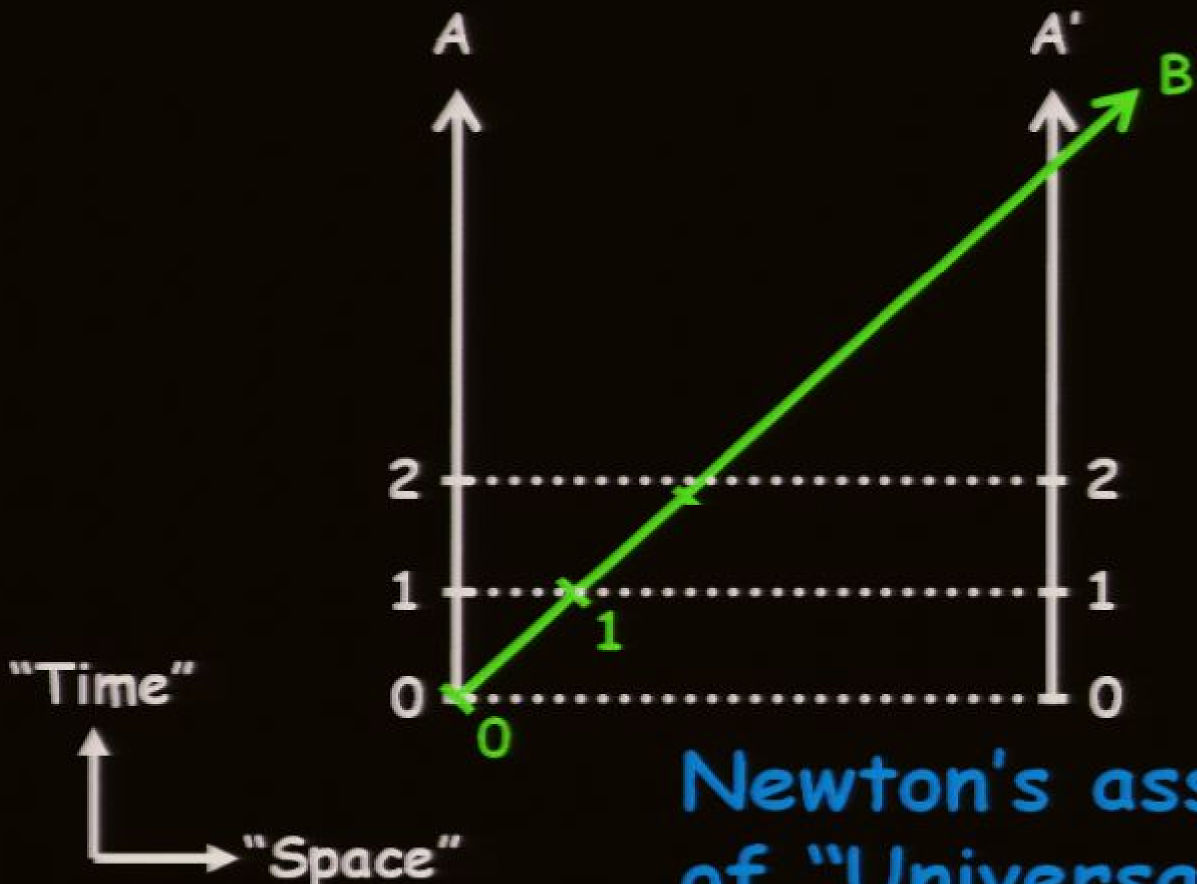


Newton's assumption
of “Universal Time”

Draw a "Spacetime Diagram"



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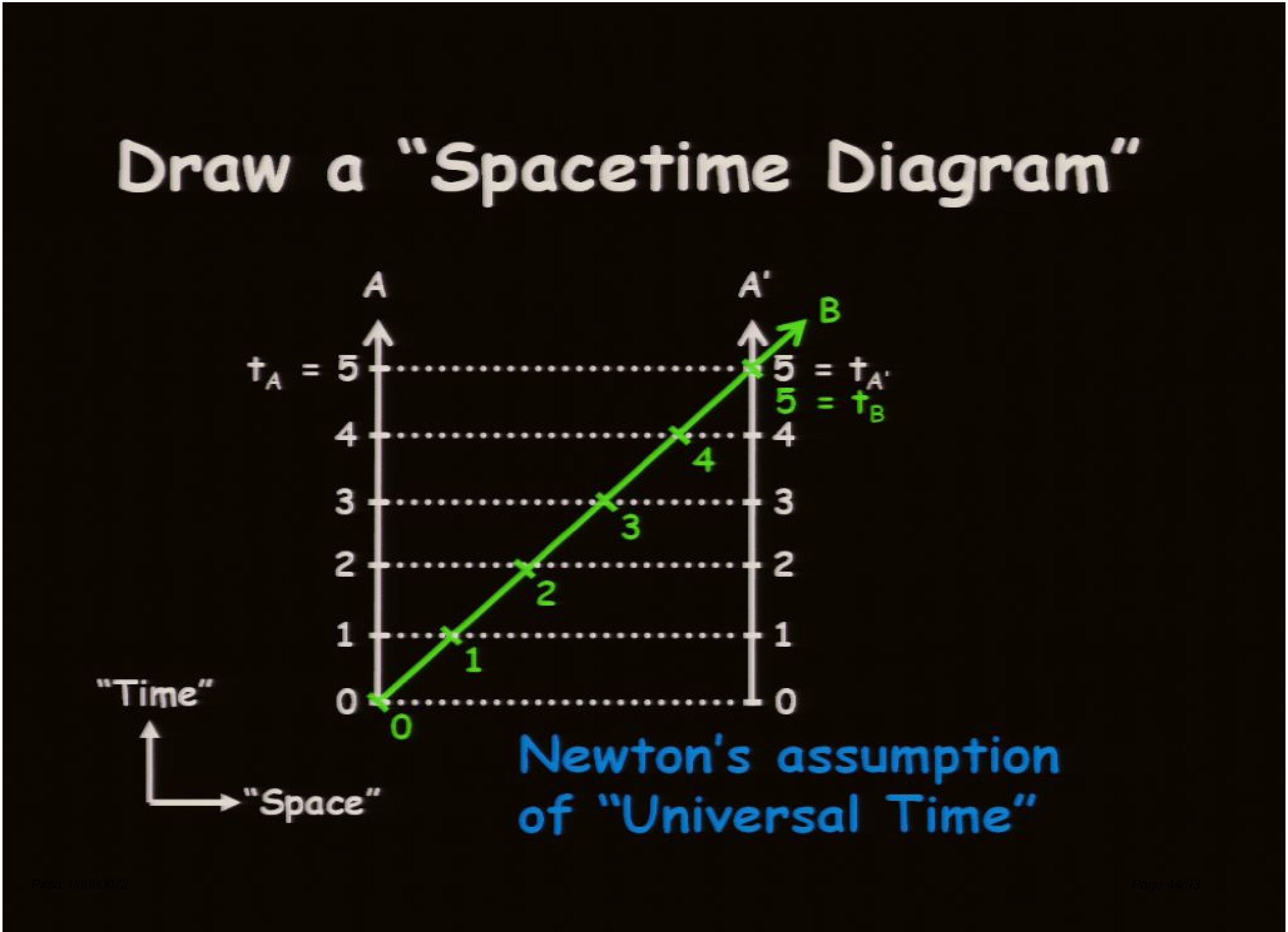

Draw a "Spacetime Diagram"
A
A'
B
t_A = 5
5 = $t_{A'}$
5 = t_B
4
3
2
1
0
"Time"
"Space"
Newton's assumption of "Universal Time"

Let's Have Spacetime Fun!

Sketch spacetime diagrams for each:

t rest relative to Alice
tossing a baseball up
noving Fast
noving Slow
th revolving around the Sun

Let's Have Spacetime Fun!

Sketch spacetime diagrams for each:

1: Bob at rest relative to Alice

2: Alice tossing a baseball up

3: Bob moving Fast

4: Bob moving Slow

5: The Earth revolving around the Sun

Let's Have Spacetime Fun!

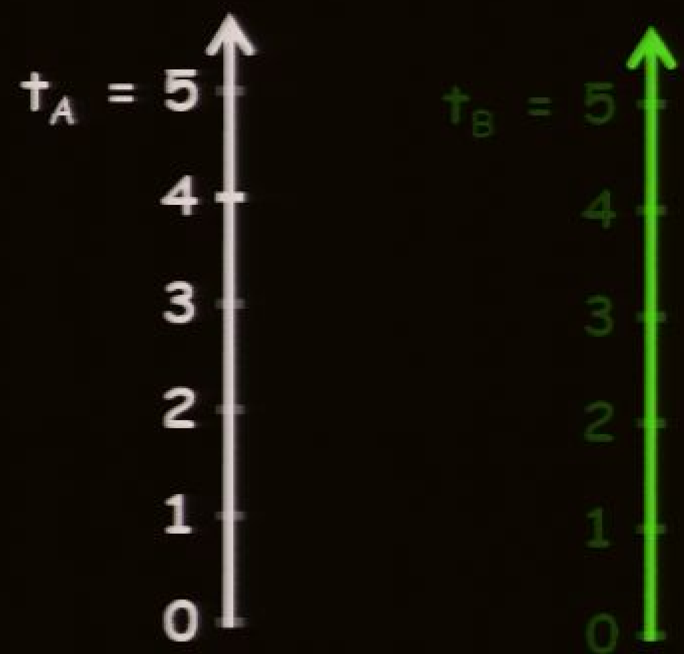
Sketch spacetime diagrams for each:

1: Bob at rest relative to Alice
2: Alice tossing a baseball up
3: Bob moving Fast
4: Bob moving Slow
5: The Earth revolving around the Sun

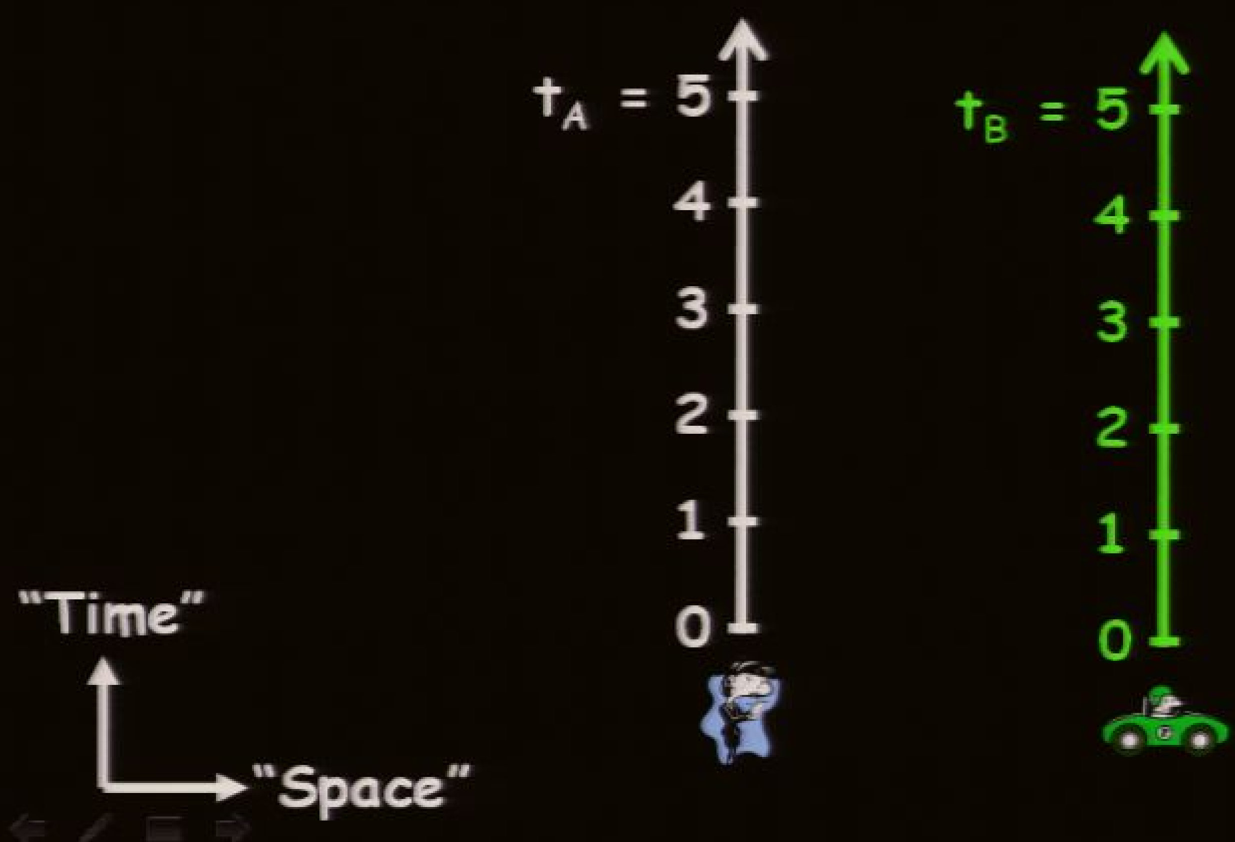
Bob at re

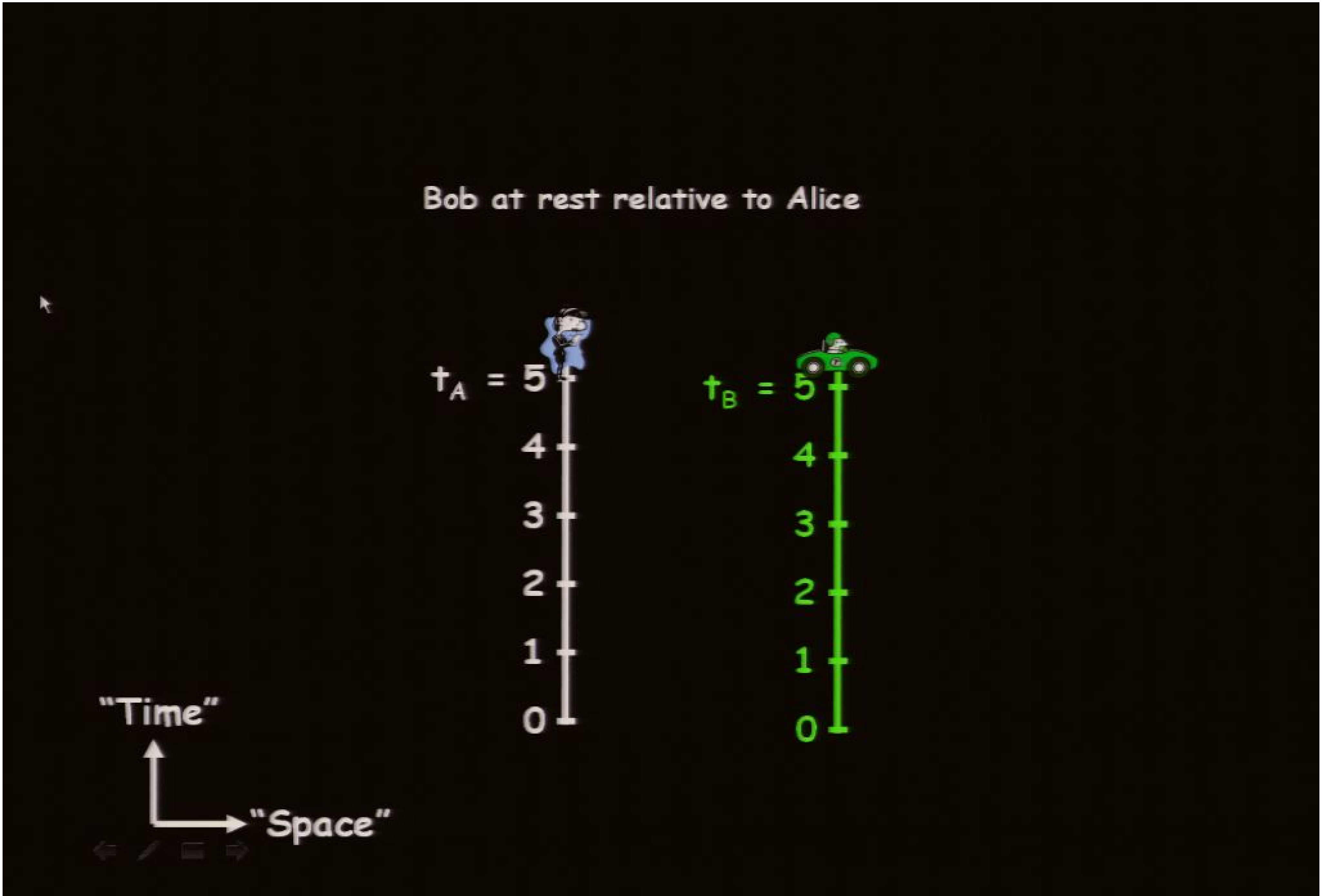
Bob at rest relative to Alice

Bob at rest relative to Alice



Bob at rest relative to Alice

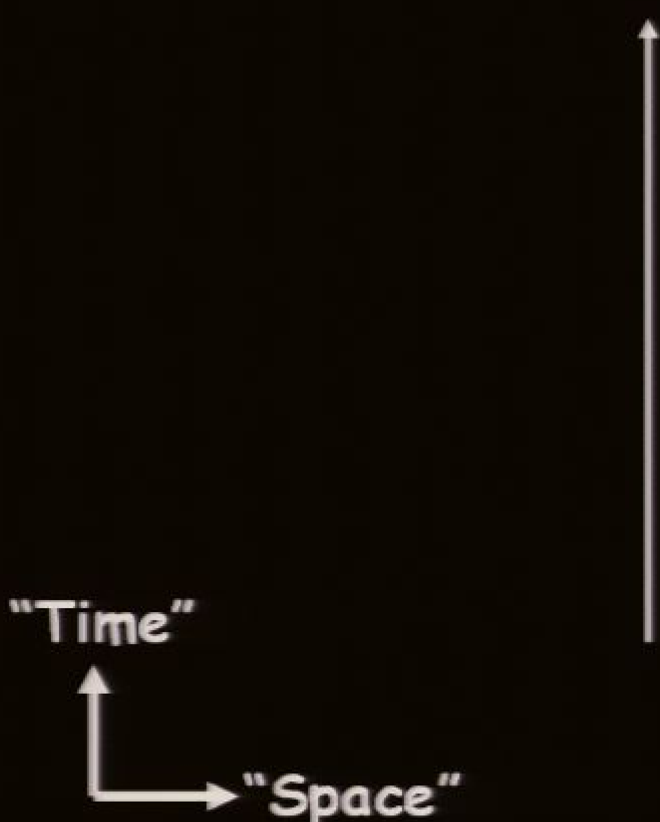




Bob at rest relative to Alice
t_A = 5
4
3
2
1
0
t_B = 5
4
3
2
1
0
"Time"
"Space"

Alice tossing a baseball up

Alice tossing a baseball up



Alice tossing a baseball up

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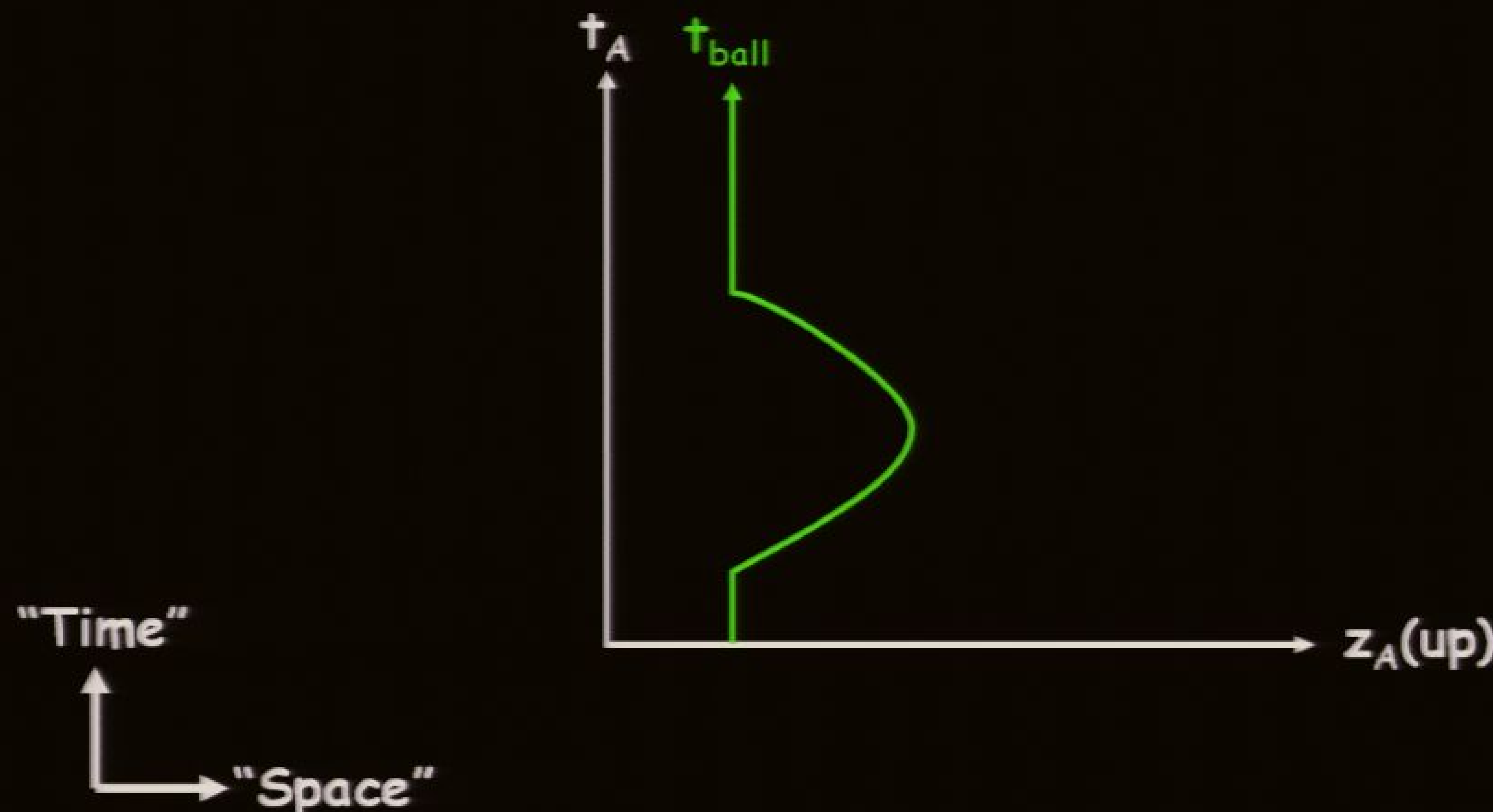
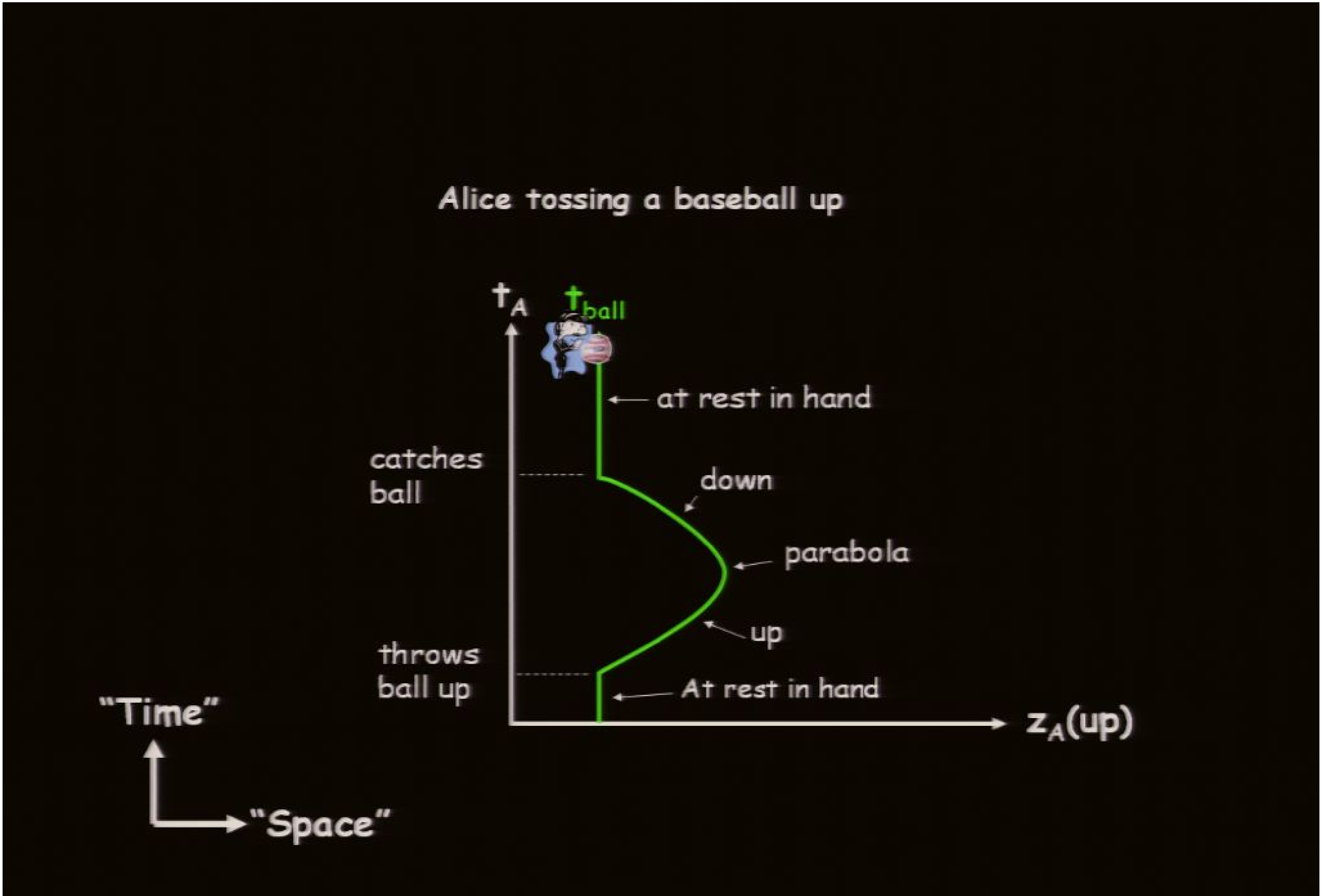
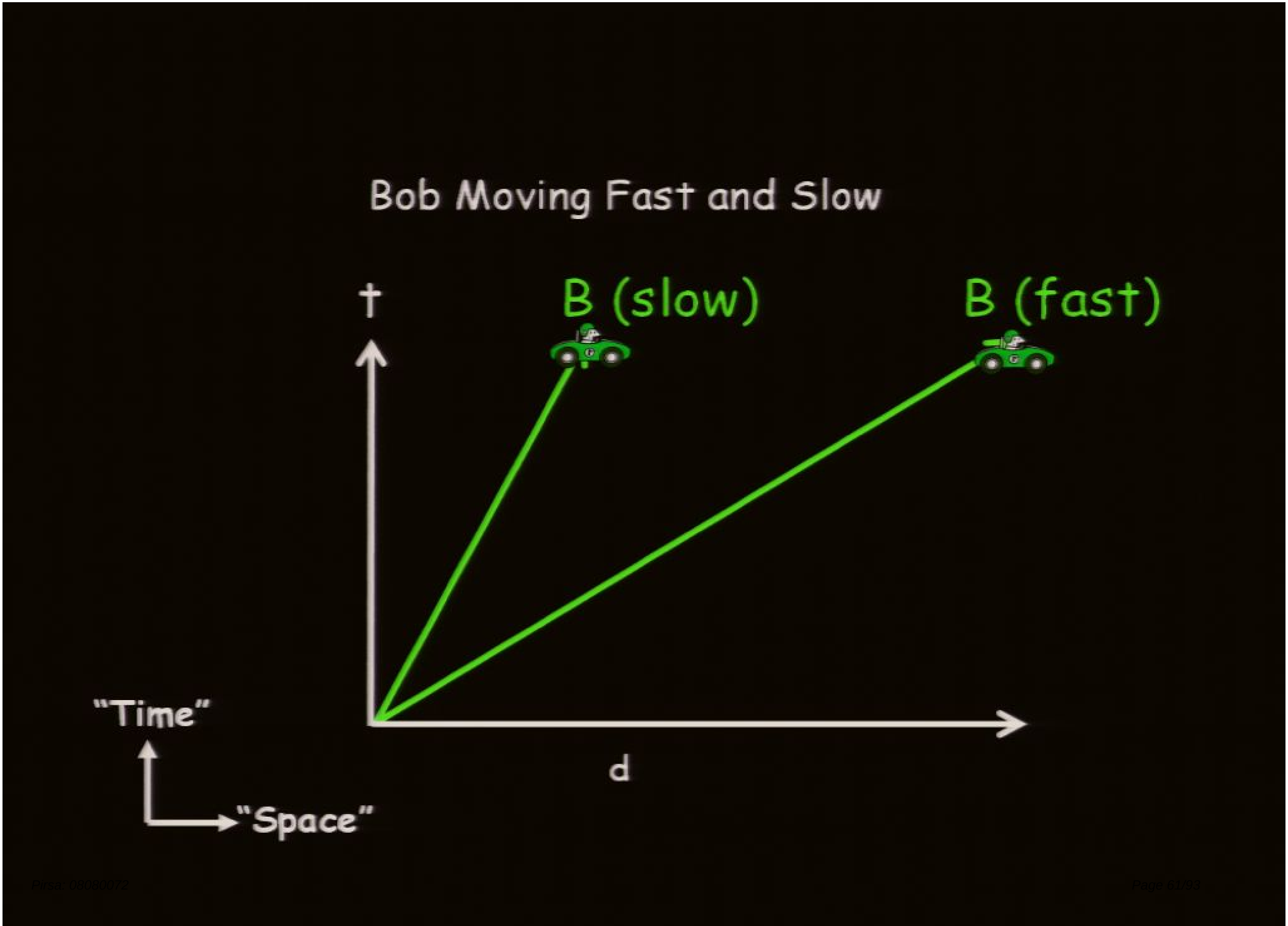
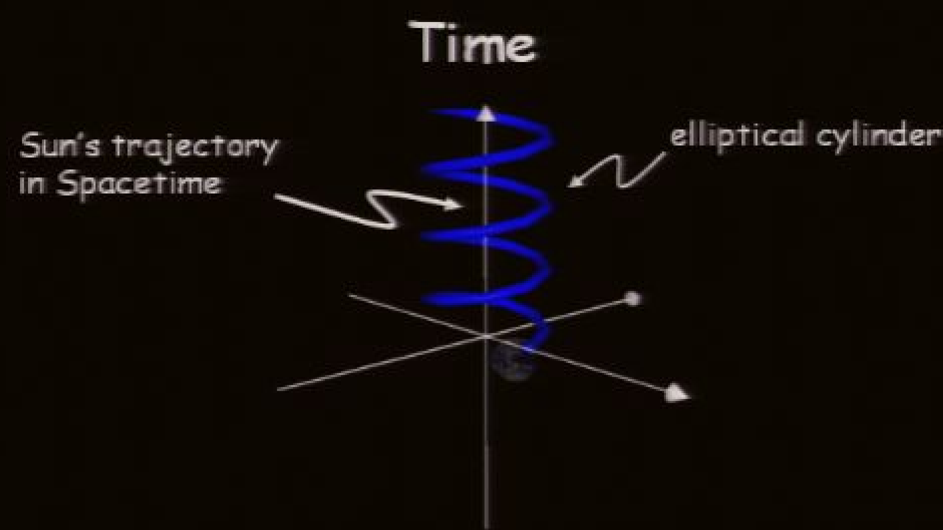



Alice tossing a baseball up
t_A
t_{ball}
at rest in hand
catches ball
down
parabola
up
throws ball up
At rest in hand
z_A(up)
"Time"
"Space"

Bob Moving Fast and Slow

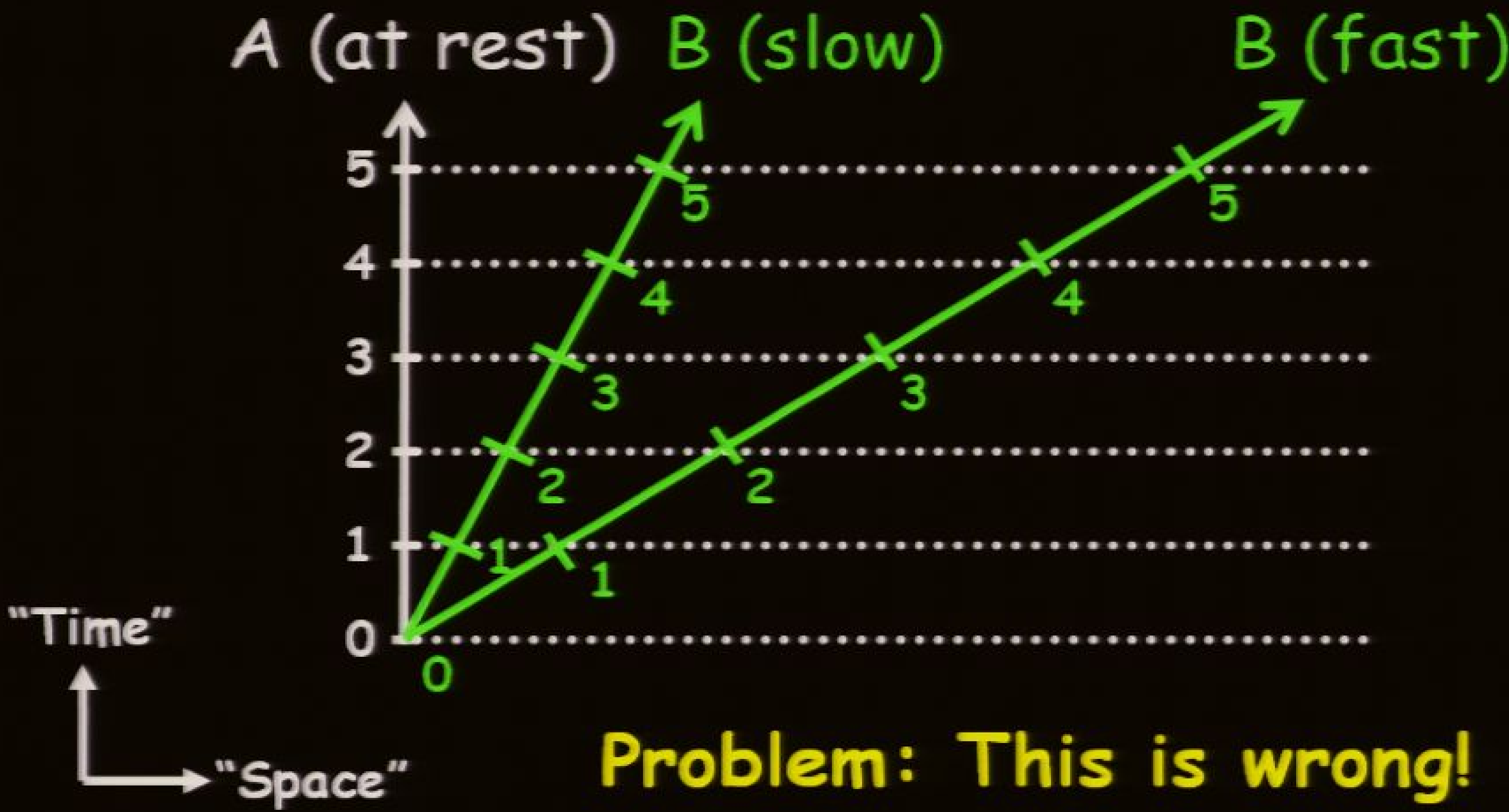

Bob Moving Fast and Slow
t
B (slow)
B (fast)
d
"Time"
"Space"

Earth Revolving Around Sun



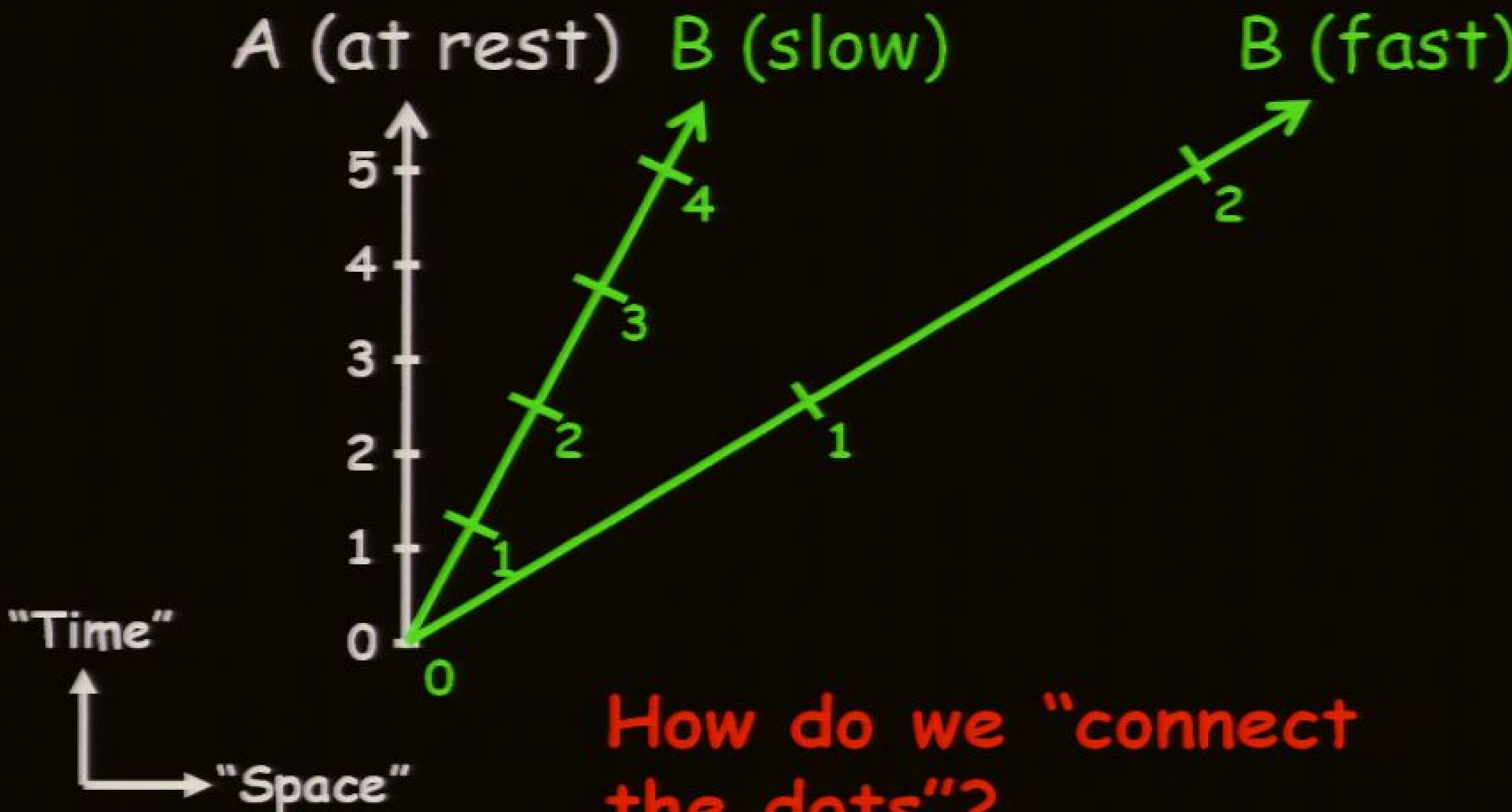
Earth's Trajectory

Newton's "Universal Time"



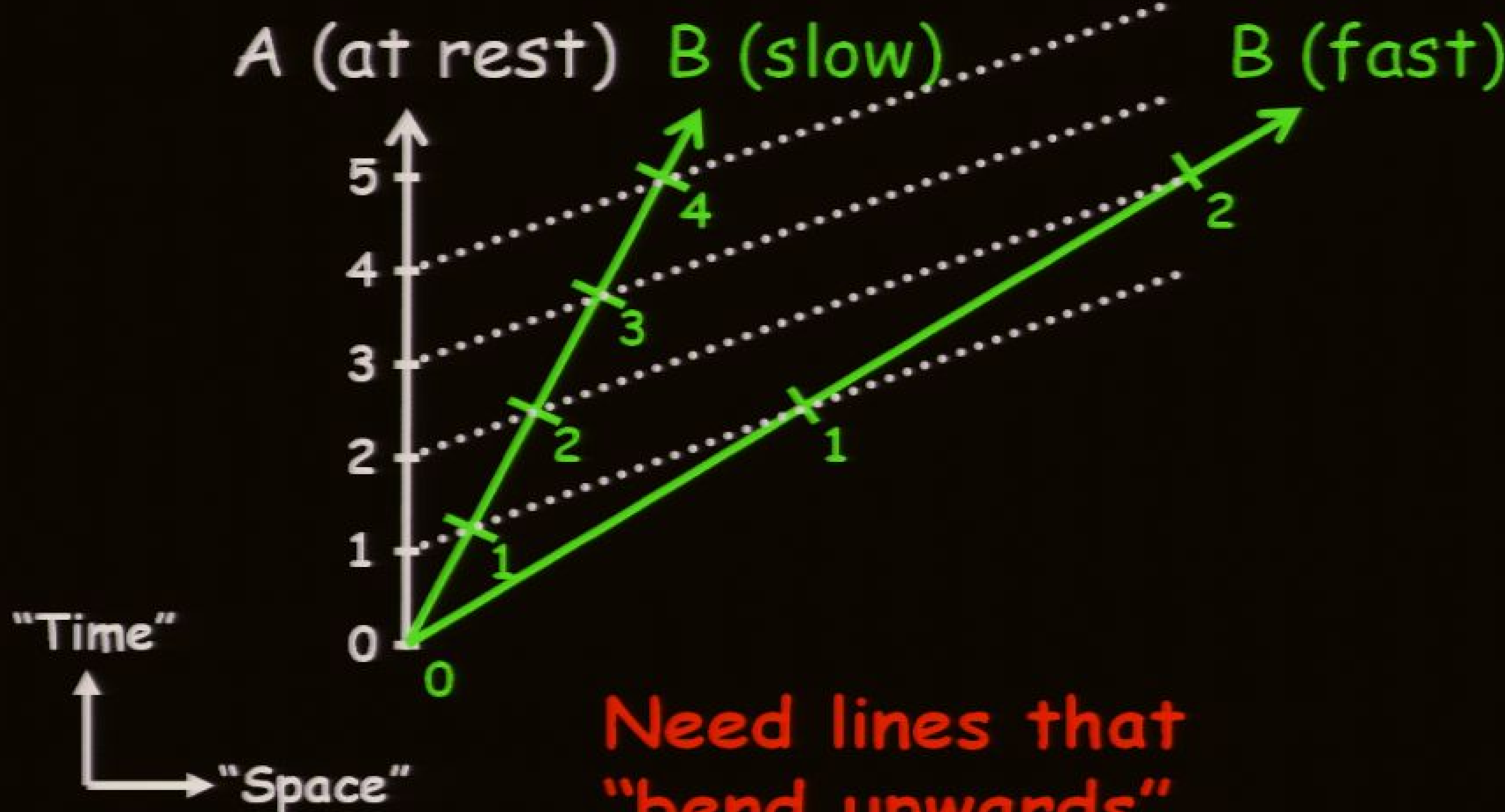
Problem: This is wrong!

Newton's "Universal Time"



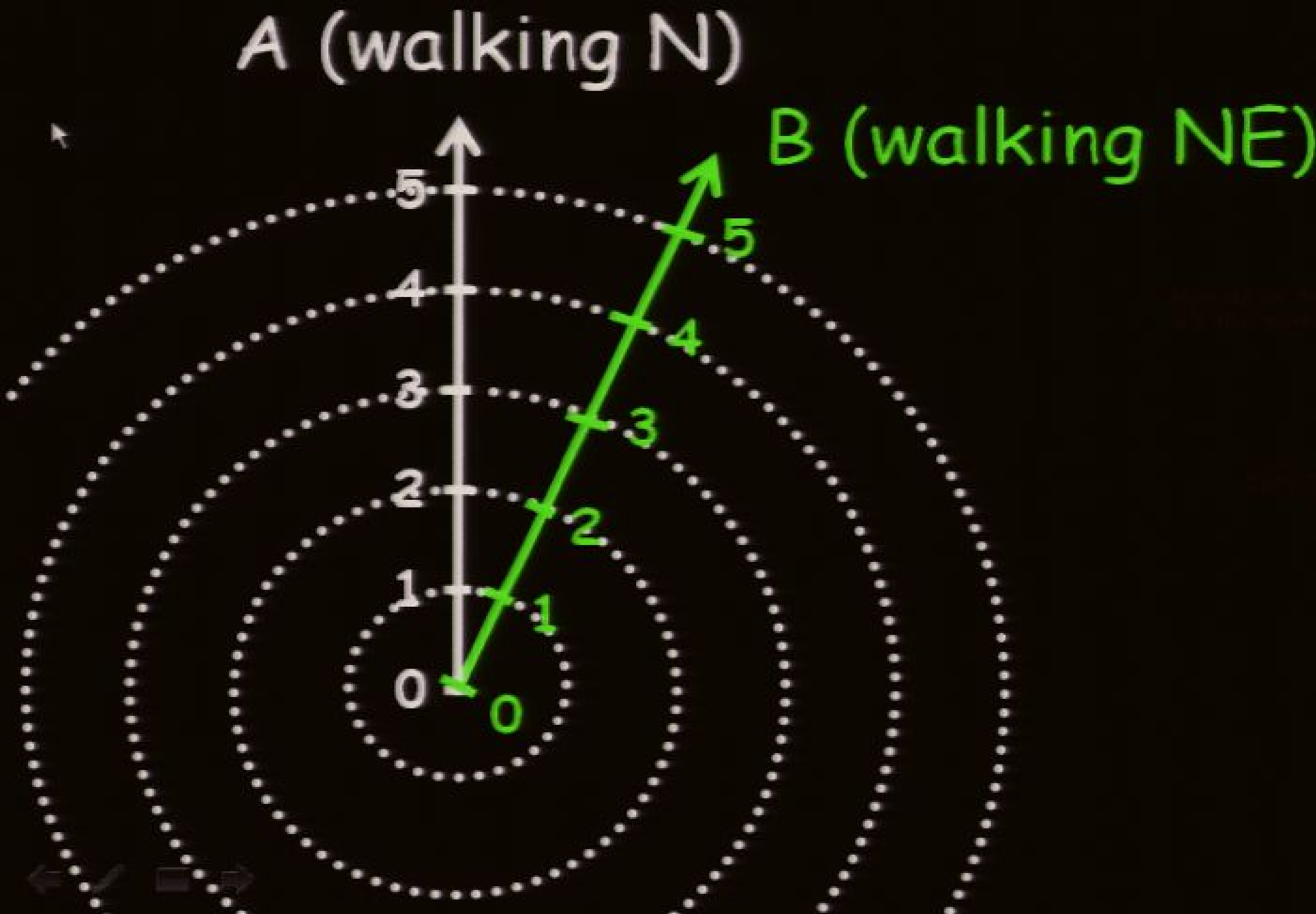
How do we "connect the dots"?

Newton's "Universal Time"

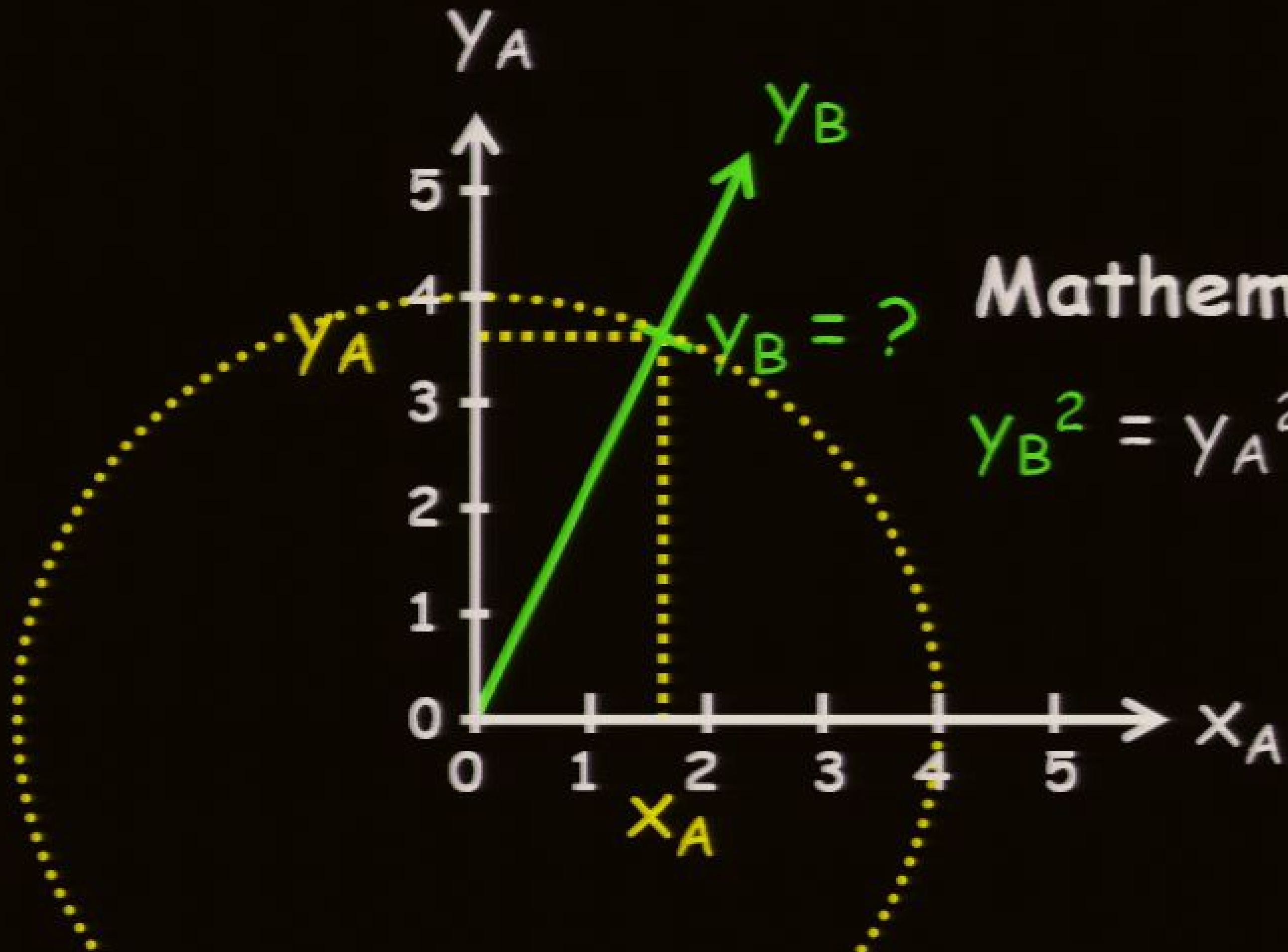


Need lines that "bend upwards"

The Geometry of Space



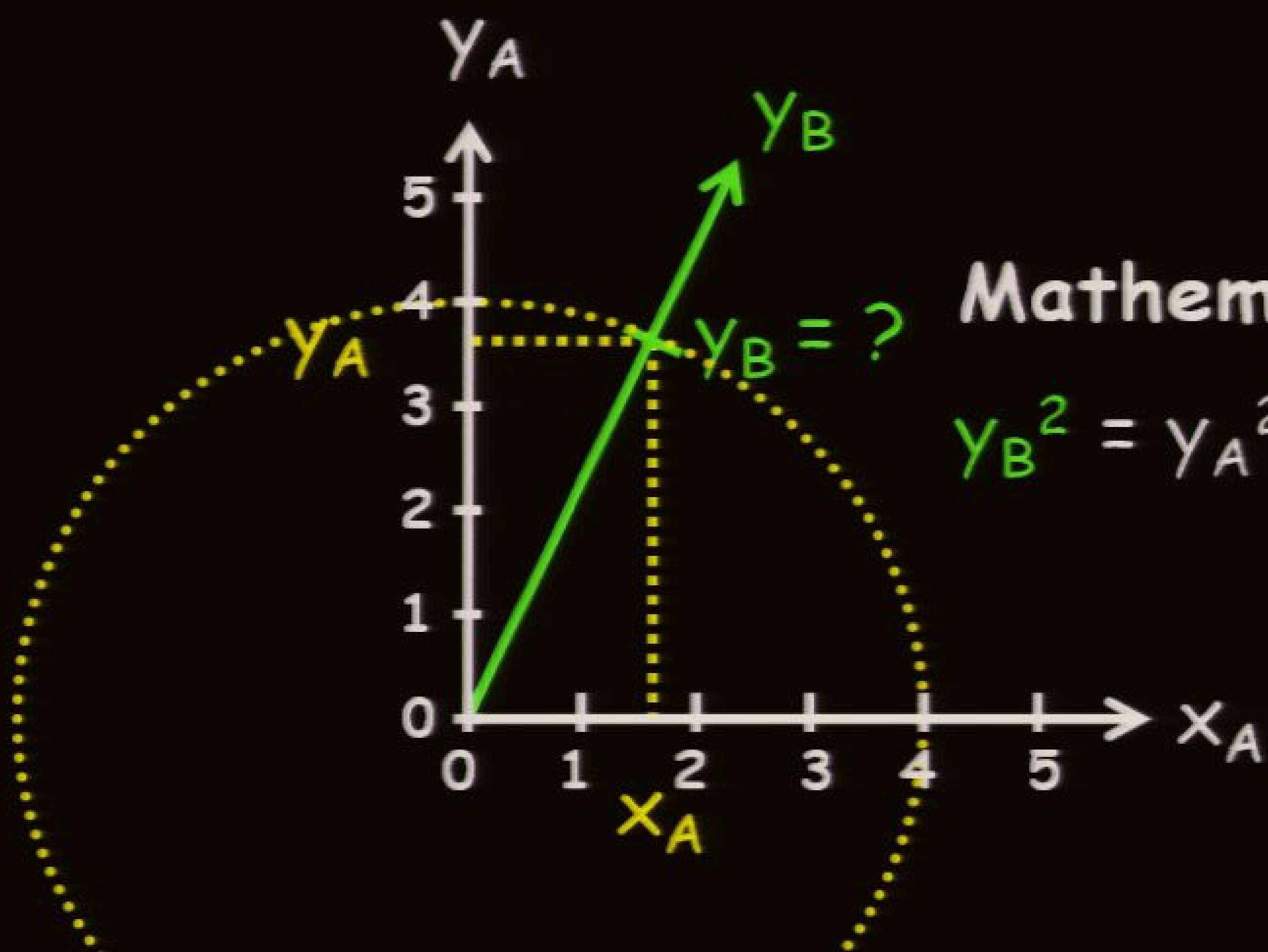
The Geometry of Space



Mathematically...

$$y_B^2 = y_A^2 + x_A^2$$

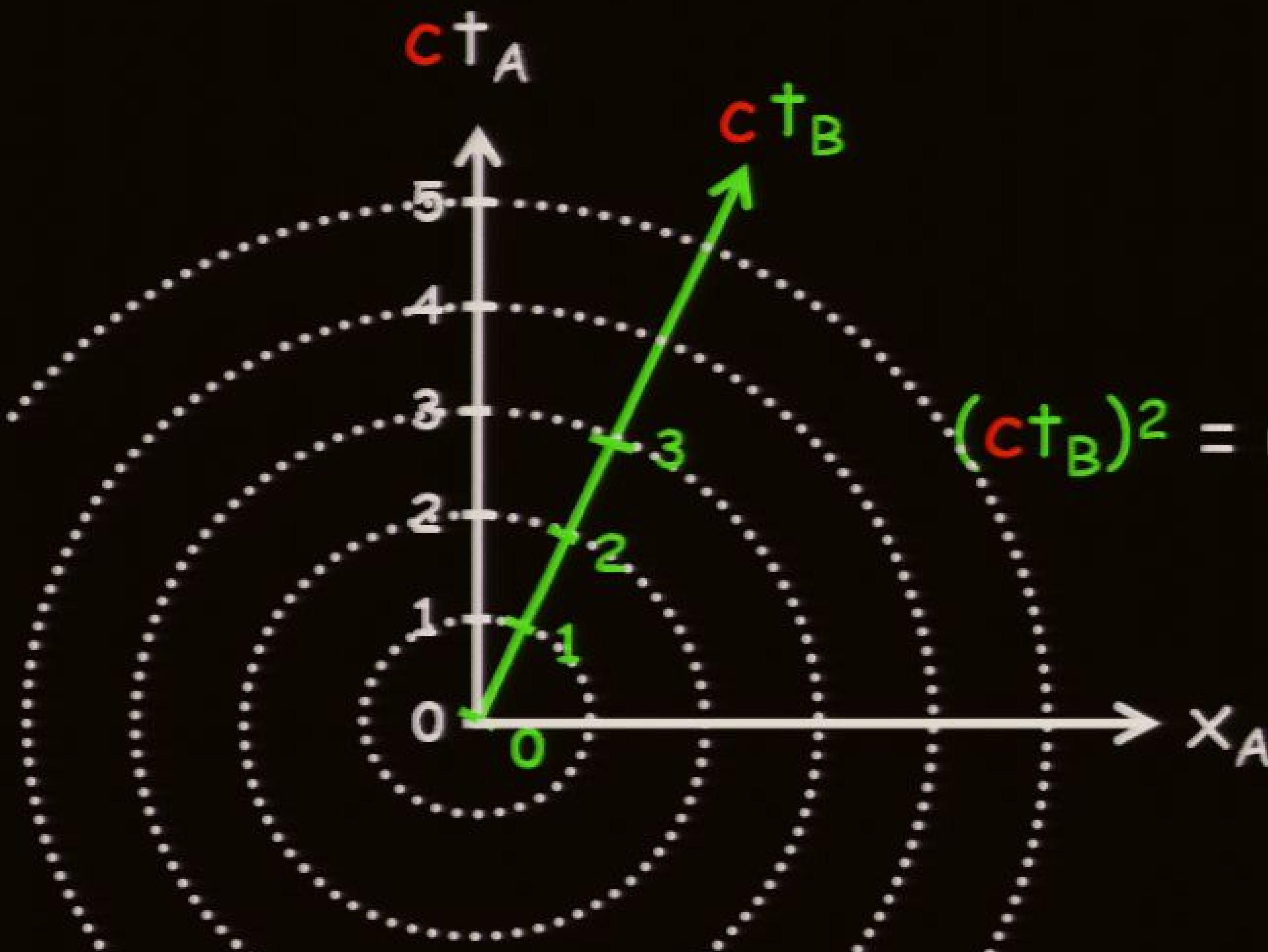
The Geometry of Space



Mathematically...

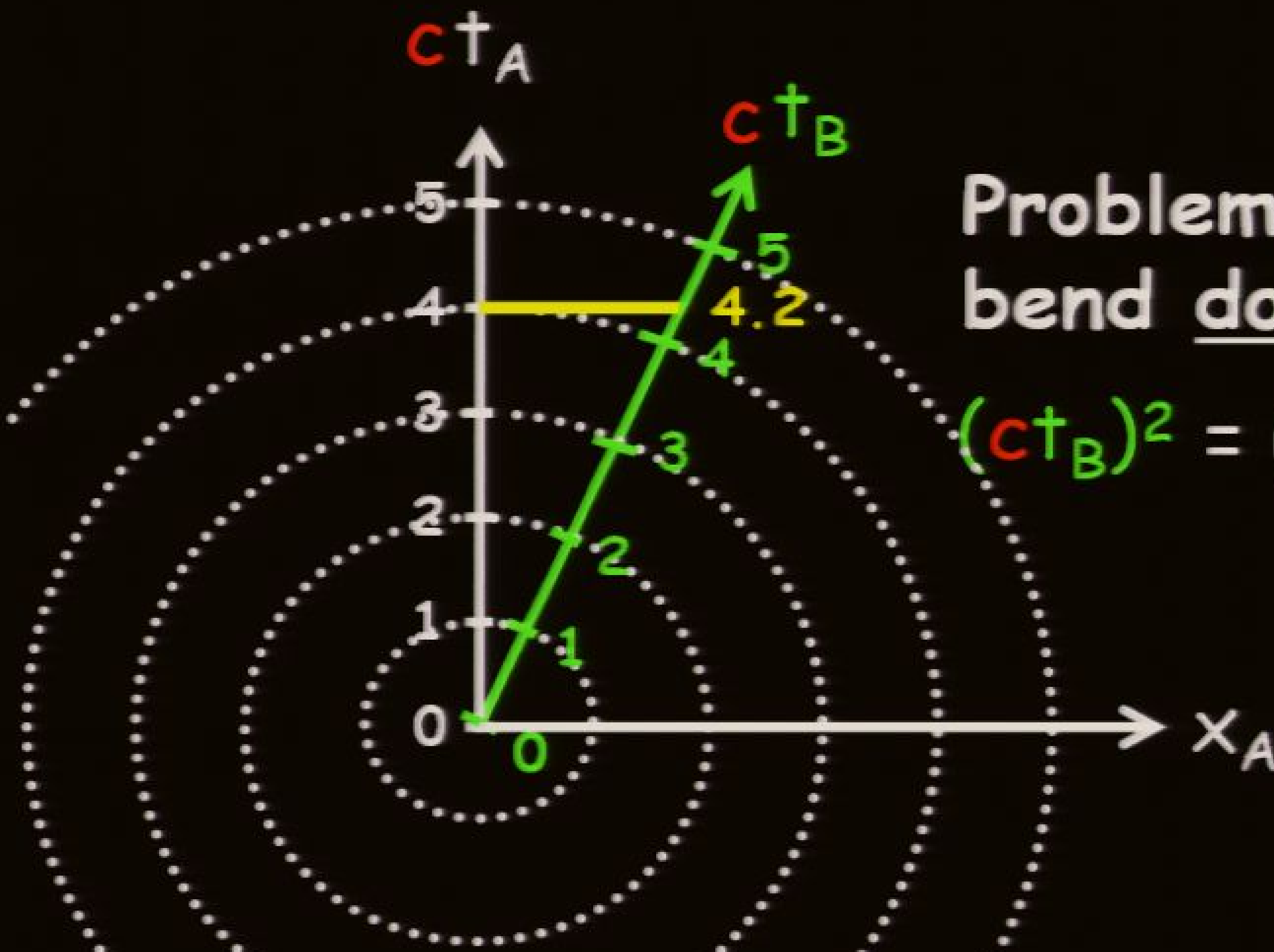
$$y_B^2 = y_A^2 + x_A^2$$

The Geometry of Spacetime



$$(ct_B)^2 = (ct_A)^2 + x_A{}^2$$

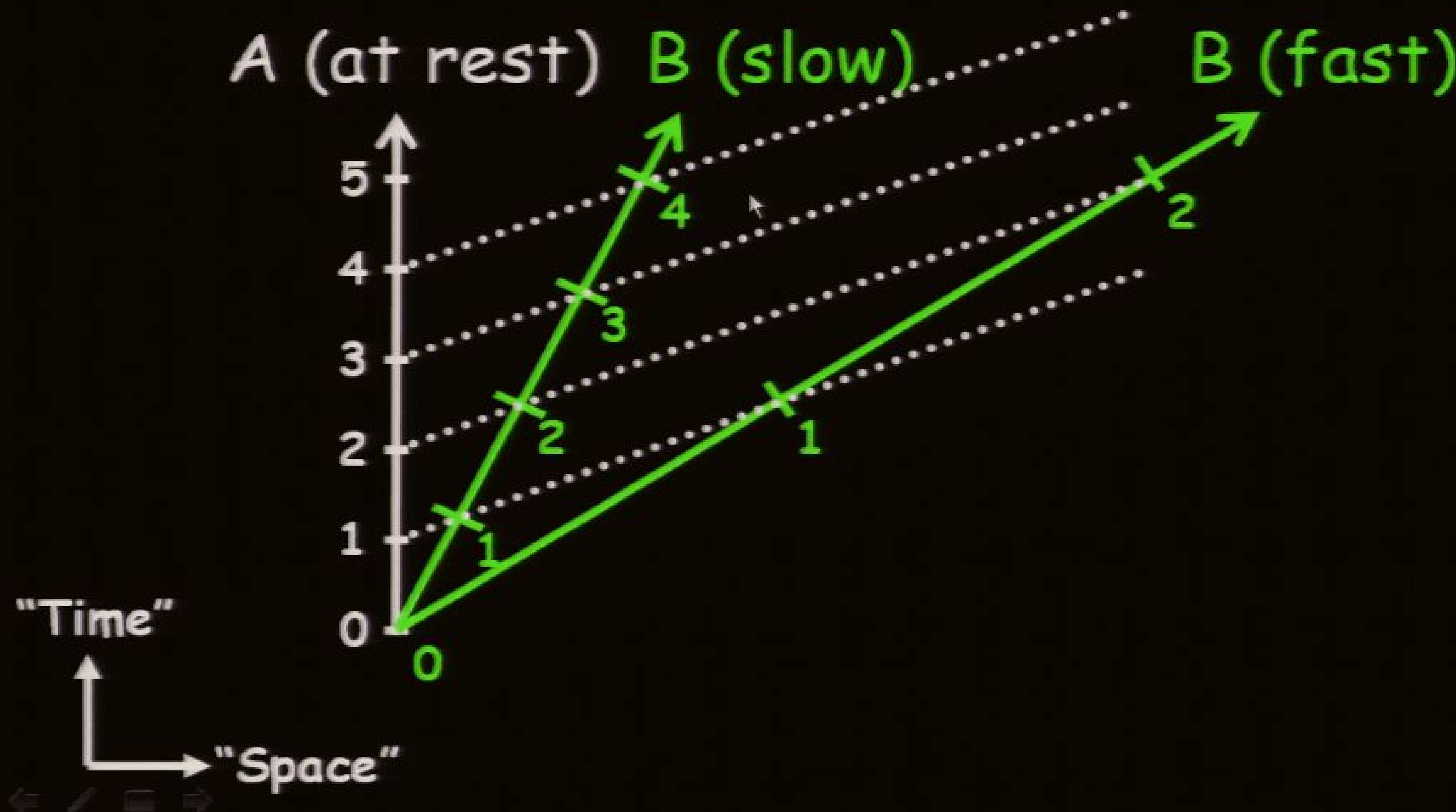
The Geometry of Spacetime



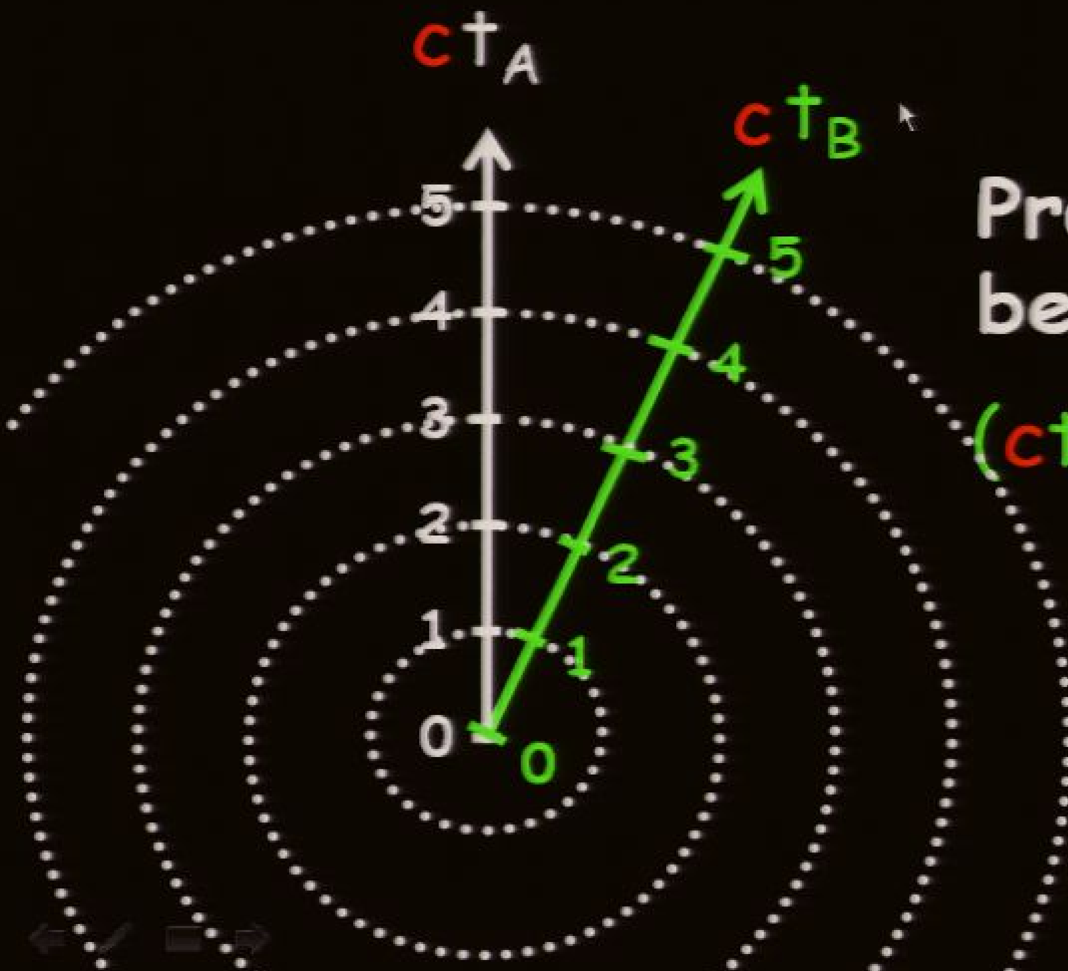
Problem 2: Curves bend <u>down</u> not <u>up</u>

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

Experimental Data:



The Geometry of Spacetime



Problem 2: Curves bend down not up

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

The Geometry of Spacetime

Try <u>hyperbolas</u> instead of circles:

$$(ct_B)^2 = (ct_A)^2 + x_A^2$$

The Geometry of Spacetime

Try hyperbolas instead of circles:

$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

Minus sign

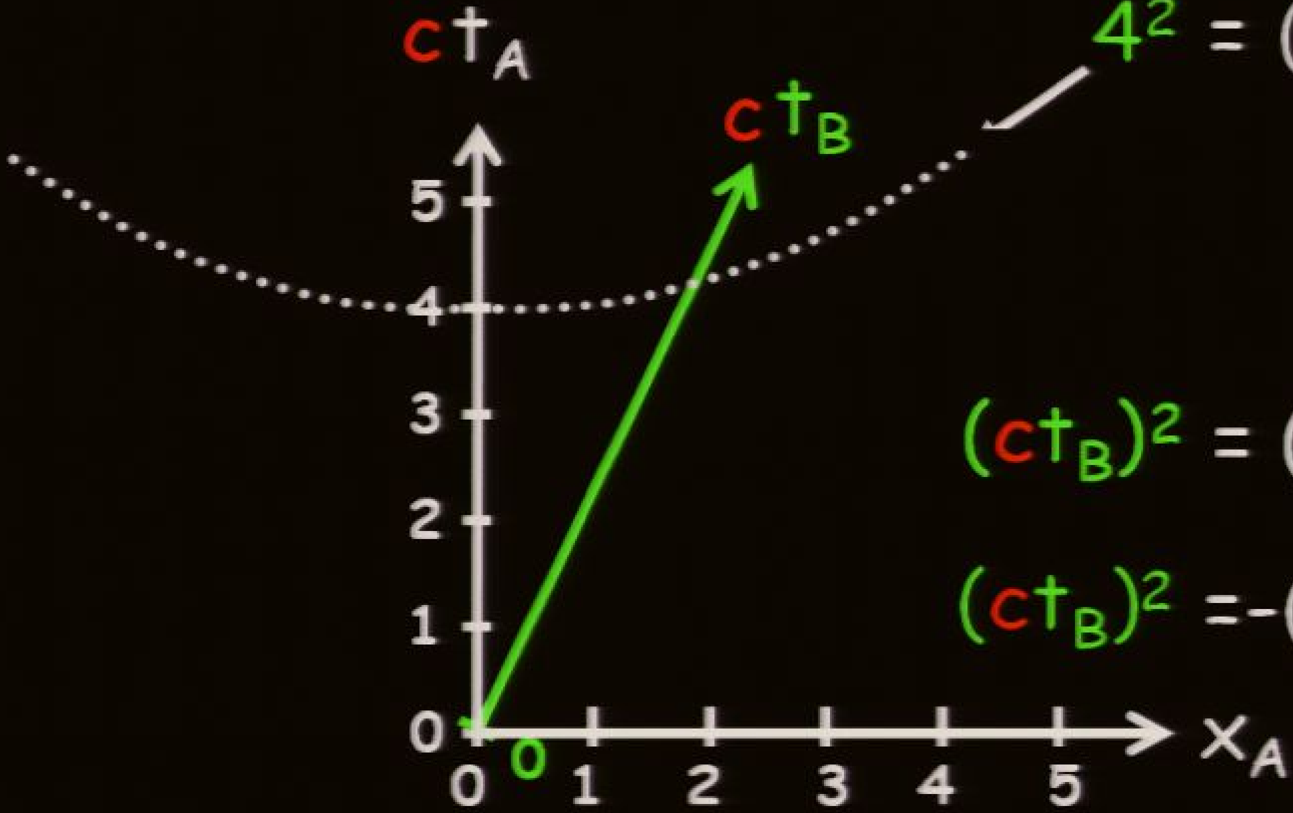
The Geometry of Spacetime

Try <u>hyperbolas</u> instead of circles:

$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

Minus sign

The Geometry of Spacetime



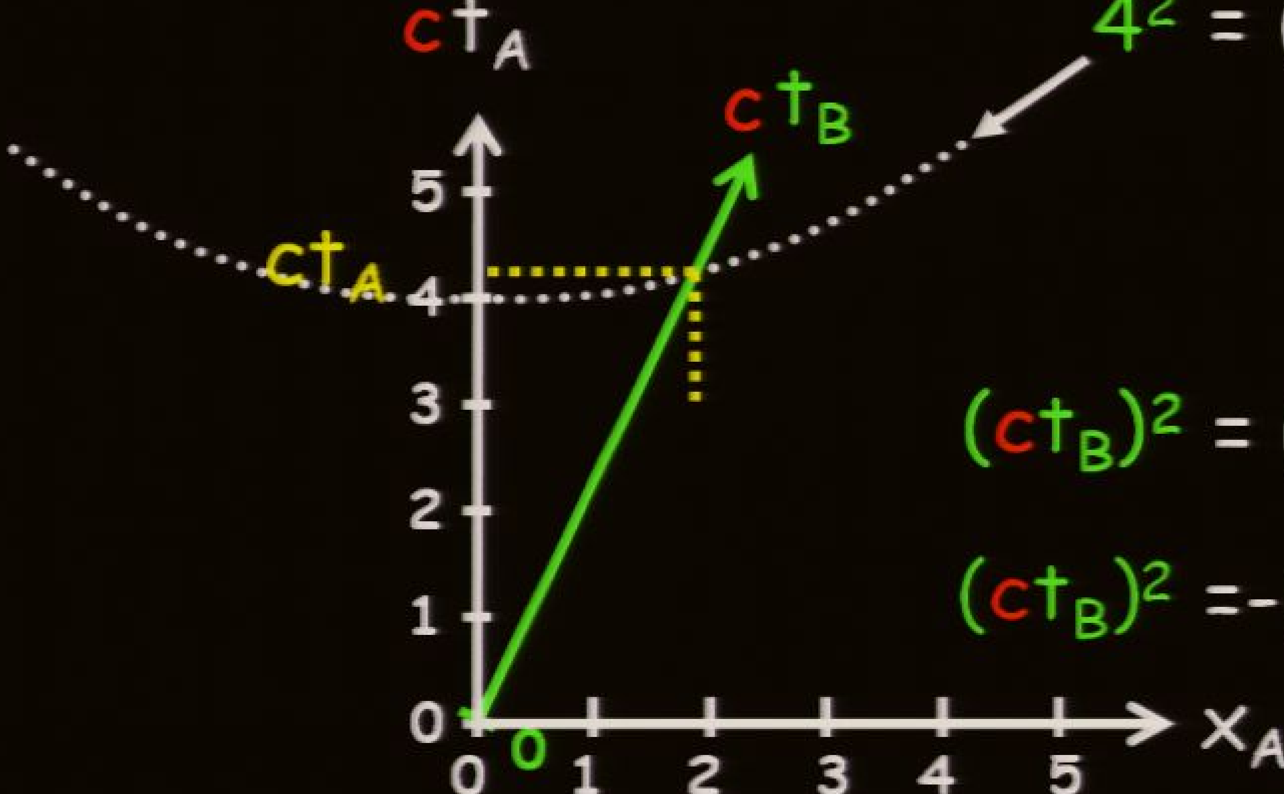
$$4^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$

The Geometry of Spacetime

$$4^2 = (ct_A)^2 - x_A^2$$

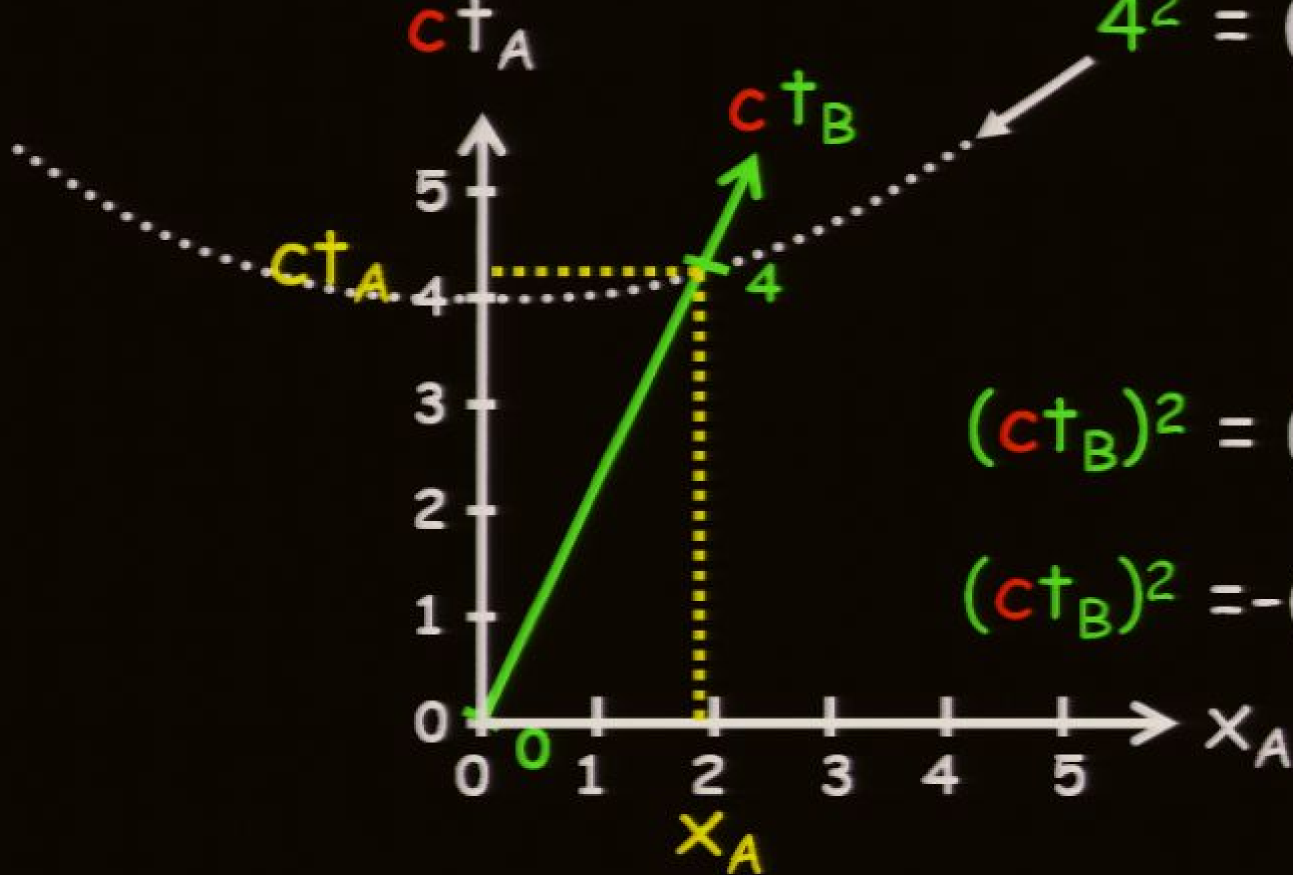


$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$

The Geometry of Spacetime

$$4^2 = (ct_A)^2 - x_A^2$$

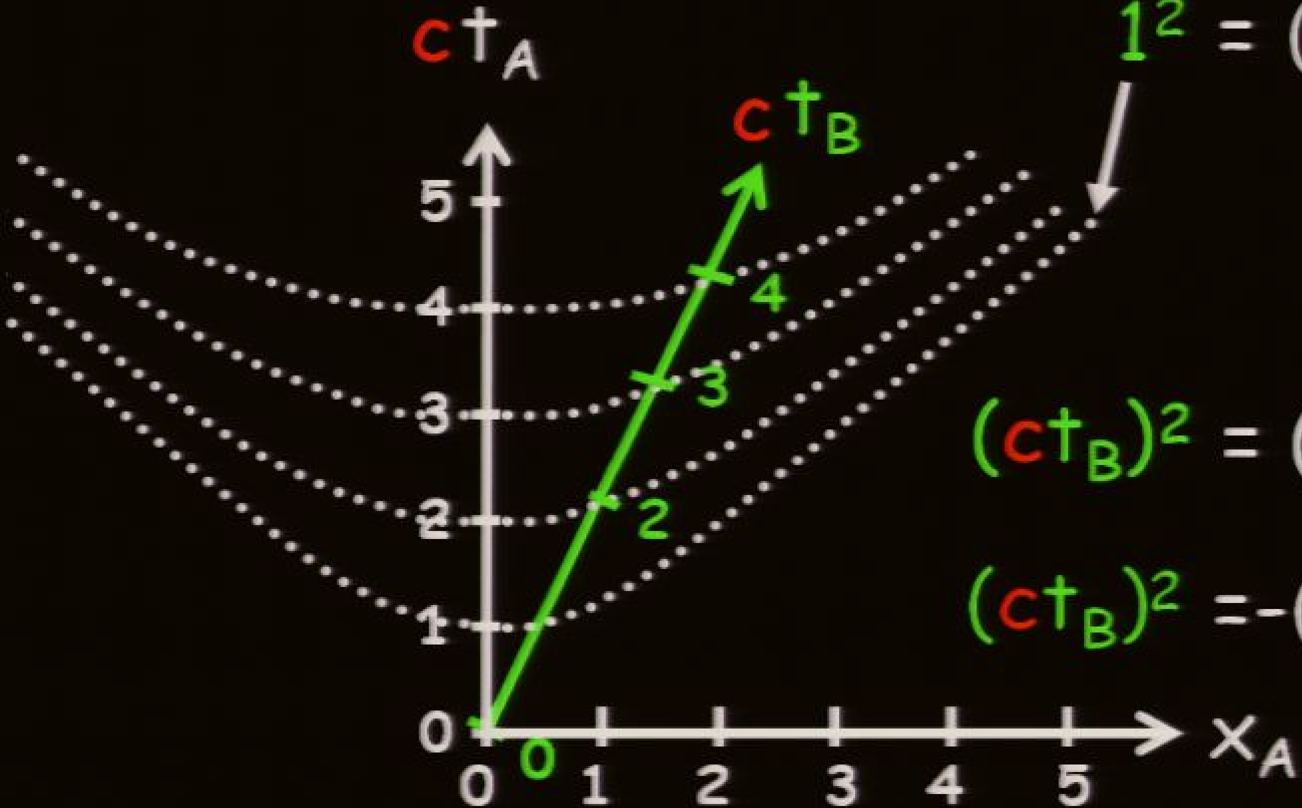


$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$

The Geometry of Spacetime

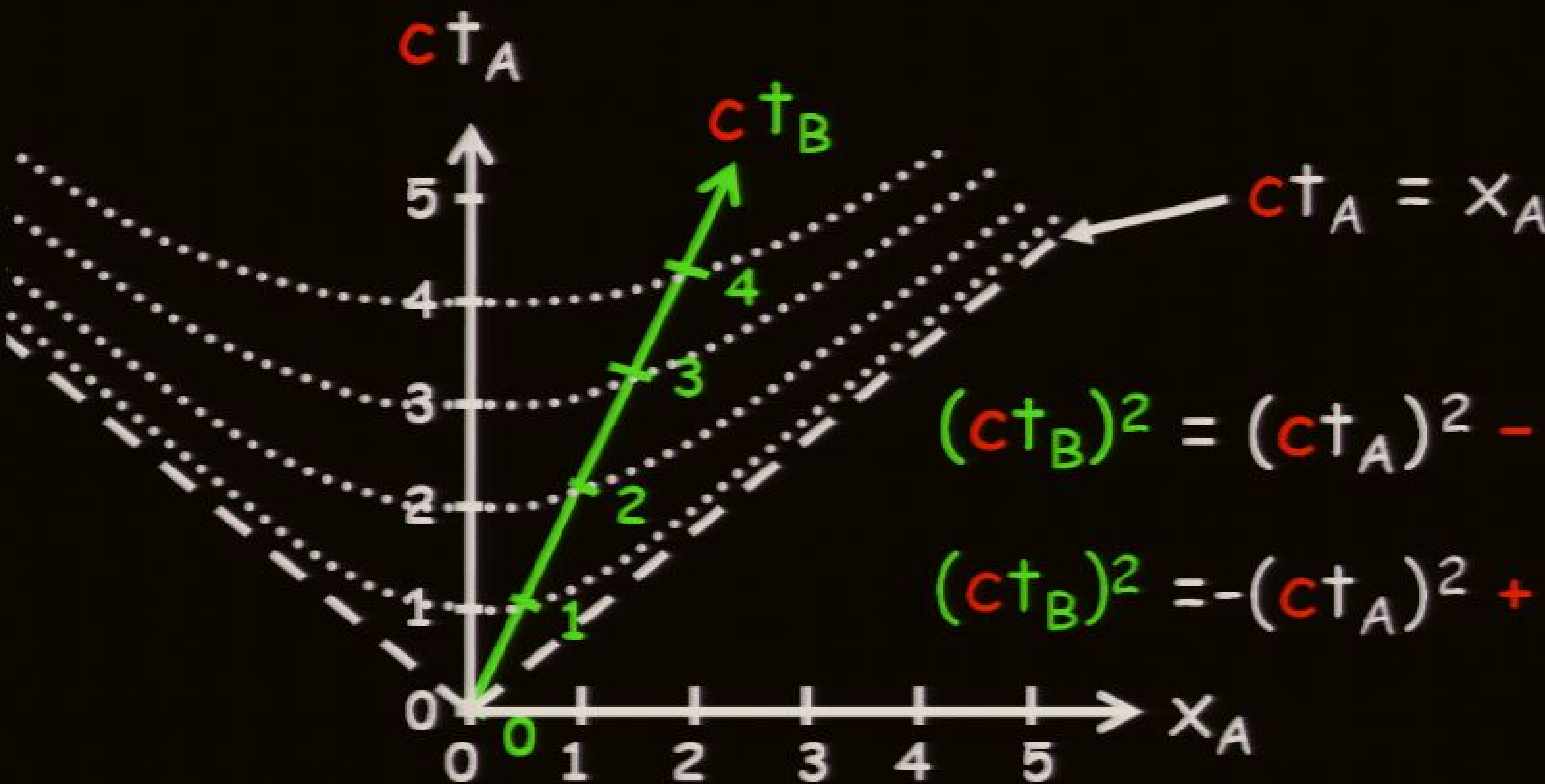
$$1^2 = (ct_A)^2 - x_A^2$$



$$(ct_B)^2 = (ct_A)^2 - x_A^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A^2$$

The Geometry of Spacetime



$$(ct_B)^2 = (ct_A)^2 - x_A{}^2$$

$$(ct_B)^2 = -(ct_A)^2 + x_A{}^2$$

Einstein's Spacetime

- *Define a metric that handles both Space and Time P(t, x, y, z)*
- *Example two dimensional Euclidean Space*

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2$$ *Cartesian coordinates*

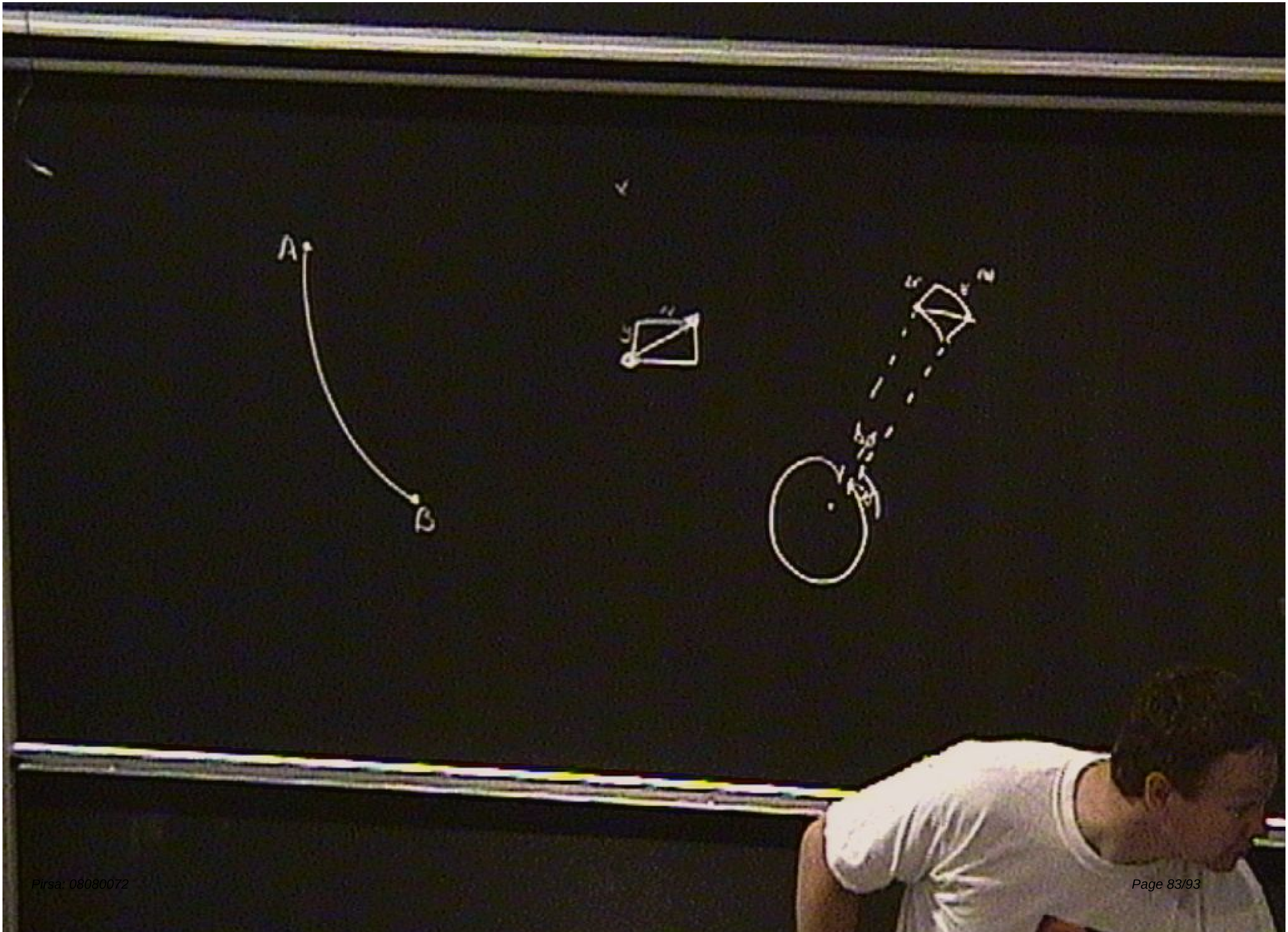
$$(\Delta s)^2 = (\Delta r)^2 + r^2 (\Delta \phi)^2$$ *Polar coordinates*

- *Example of two dimensional Minkowski Space*

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2$$ *Usual representation*

$$(\Delta s)^2 = -(\Delta t)^2 + t^2 (\Delta \phi)^2$$ *Milne representation*


A
B


A
B

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The Einstein field equation (EFE) is usually written in the form

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

Here R_{ab} is the Ricci tensor, R is the Ricci scalar, g_{ab} is the metric tensor, Tab is the stress-energy tensor, and the constants are π , G (the gravitational constant) and c (the speed of light). The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. It is written here using the abstract index notation. Each tensor has 10 independent components. Given the freedom of choice of the four spacetime coordinates, the independent equations reduce to 6 in number

$$R_{\eta\eta} = -\frac{2a^2 \frac{\partial\psi}{\partial\theta}\cot\theta}{\delta\psi} + \frac{2ac\frac{\partial\psi}{\partial\eta}\cot\theta}{\delta\psi} + \frac{a\frac{\partial c}{\partial\eta}\cot\theta}{\delta} - \frac{\frac{\partial a}{\partial\eta}c\cot\theta}{2\delta} - \frac{a\frac{\partial a}{\partial\theta}\cot\theta}{2\delta} - \frac{2a^2\frac{\partial^2\psi}{\partial\theta^2}}{\delta\psi}$$

$$-\frac{2a^2(\frac{\partial\psi}{\partial\theta})^2}{\delta\psi^2} + \frac{4ac\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi^2} - \frac{a^2\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{ac\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{2a\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta\psi}$$

$$-\frac{3a\frac{\partial a}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{2a^2c\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{2a^2b\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a^2\frac{\partial b}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\eta}bc\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{a^3\frac{\partial b}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi}$$

$$+\frac{a^2\frac{\partial a}{\partial\theta}b\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{2ab\frac{\partial^2\psi}{\partial\eta^2}}{\delta\psi} - \frac{2\frac{\partial^2\psi}{\partial\eta^2}}{\psi} + \frac{4ac\frac{\partial^2\psi}{\partial\eta\partial\theta}}{\delta\psi} - \frac{2ab(\frac{\partial\psi}{\partial\eta})^2}{\delta\psi^2} + \frac{6(\frac{\partial\psi}{\partial\eta})^2}{\psi^2}$$

$$+\frac{ac\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{ab\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{2c\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta\psi} - \frac{2a\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial\psi}{\partial\eta}}{\delta\psi}$$

$$+\frac{2a^2b\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{2abc\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a^2\frac{\partial b}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\theta}bc\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a^2b\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a\frac{\partial a}{\partial\eta}b^2\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}$$

$$+\frac{a\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\theta}}{2\delta d} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial d}{\partial\theta}}{4\delta d} - \frac{a\frac{\partial a}{\partial\theta}\frac{\partial d}{\partial\theta}}{4\delta d} - \frac{\frac{\partial^2 d}{\partial\eta^2}}{2d} + \frac{(\frac{\partial d}{\partial\eta})^2}{4d^2} - \frac{c\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\eta}}{2\delta d}$$

$$+\frac{\frac{\partial a}{\partial\theta}c\frac{\partial d}{\partial\eta}}{4\delta d} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial d}{\partial\eta}}{4\delta d} + \frac{a\frac{\partial^2 c}{\partial\eta\partial\theta}}{\delta} - \frac{a\frac{\partial^2 b}{\partial\eta^2}}{2\delta} - \frac{a\frac{\partial^2 a}{\partial\theta^2}}{2\delta} + \frac{ac\frac{\partial c}{\partial\eta}\frac{\partial c}{\partial\theta}}{\delta^2}$$

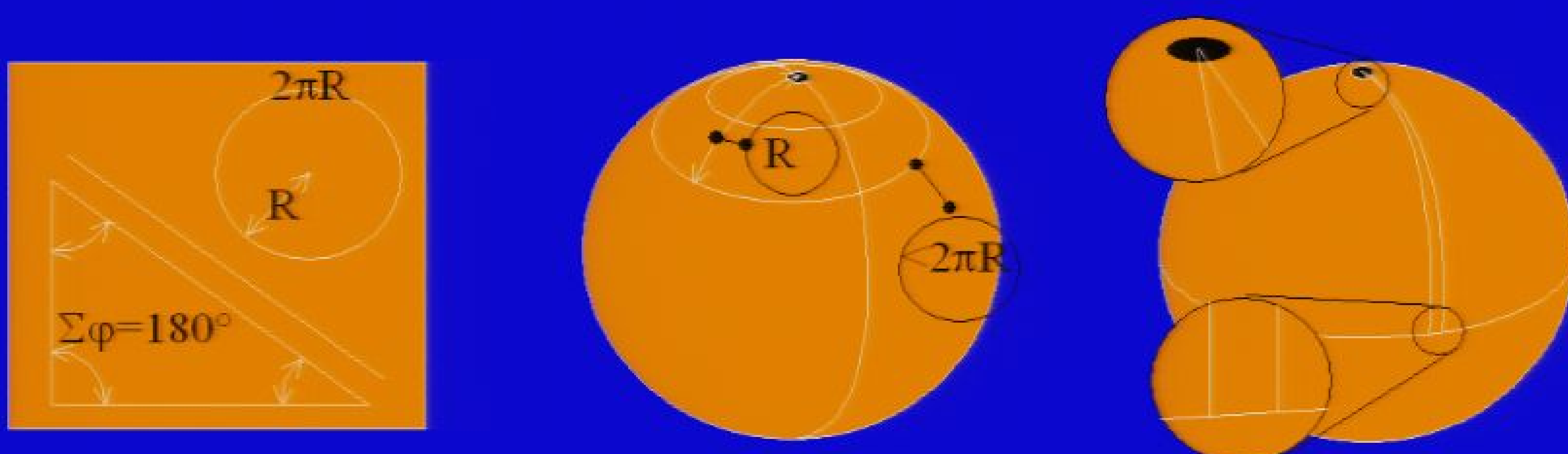
Curvature in 2D…

- In a curved space, Euclidean geometry does not apply:

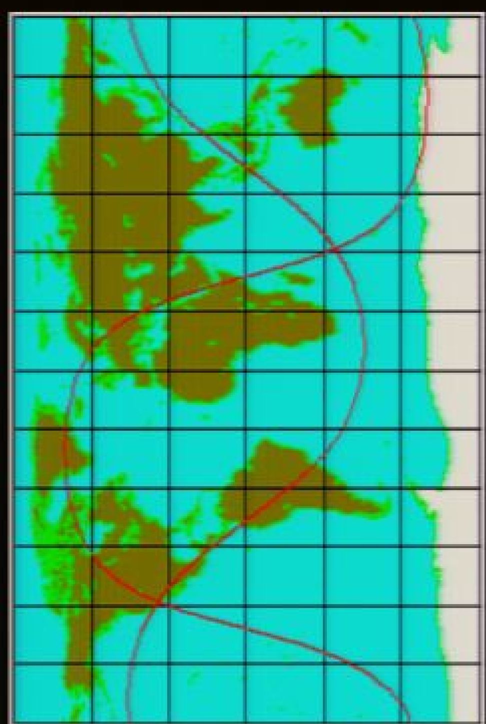
  - circumference $\neq 2\pi$ R

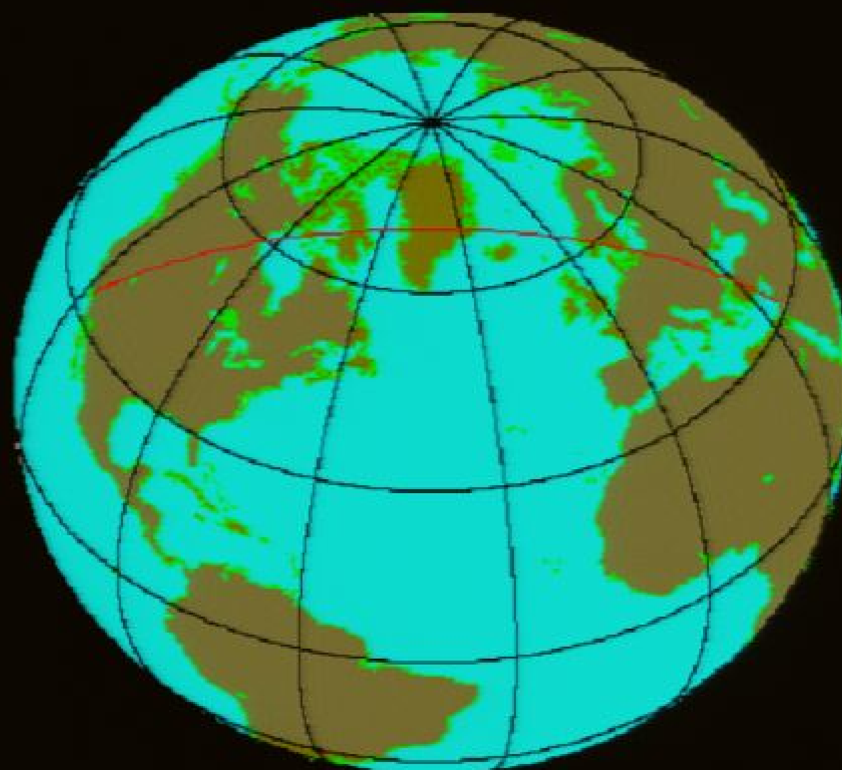
  - triangles $\neq 180°$

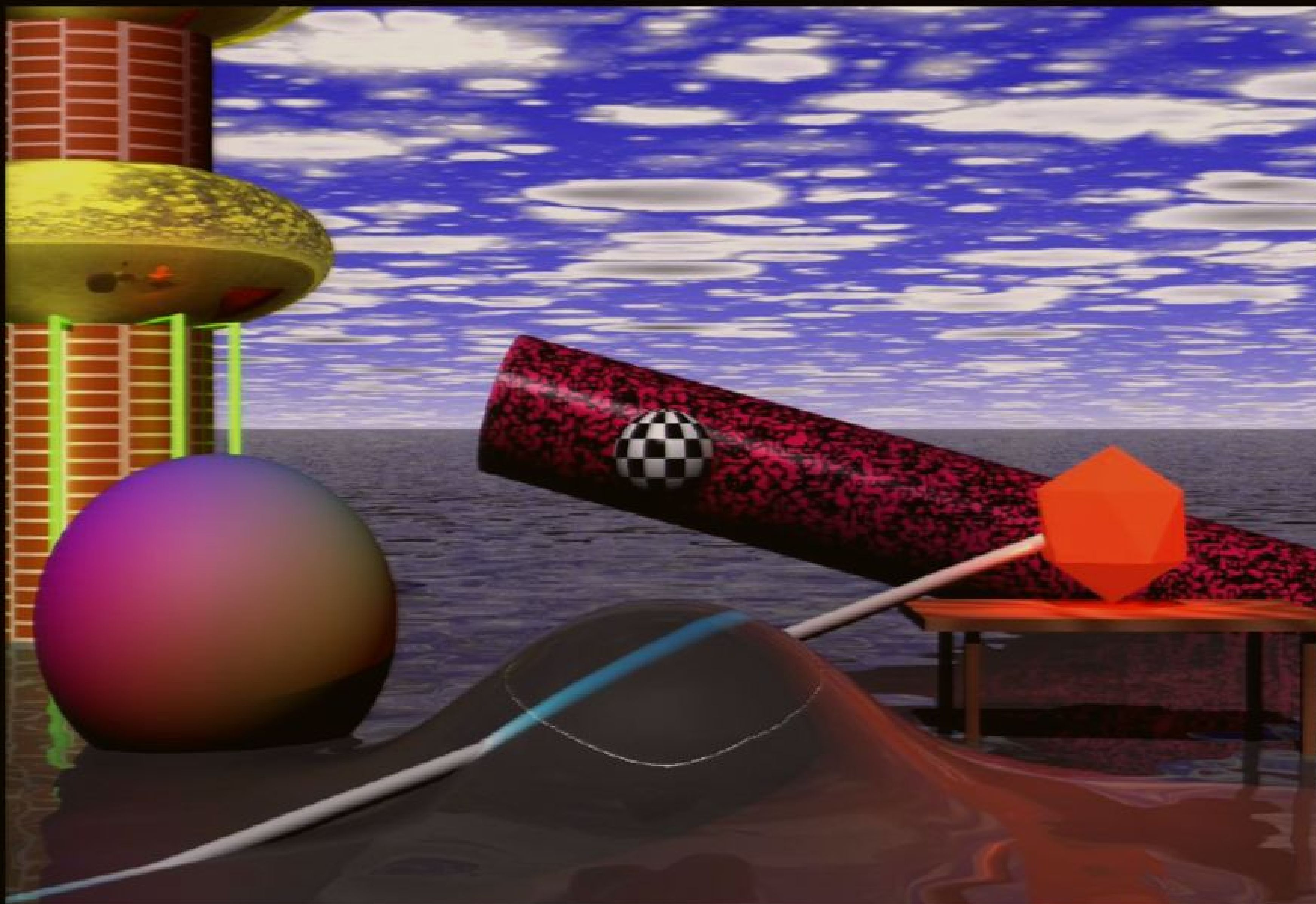
  - parallel lines don't stay parallel

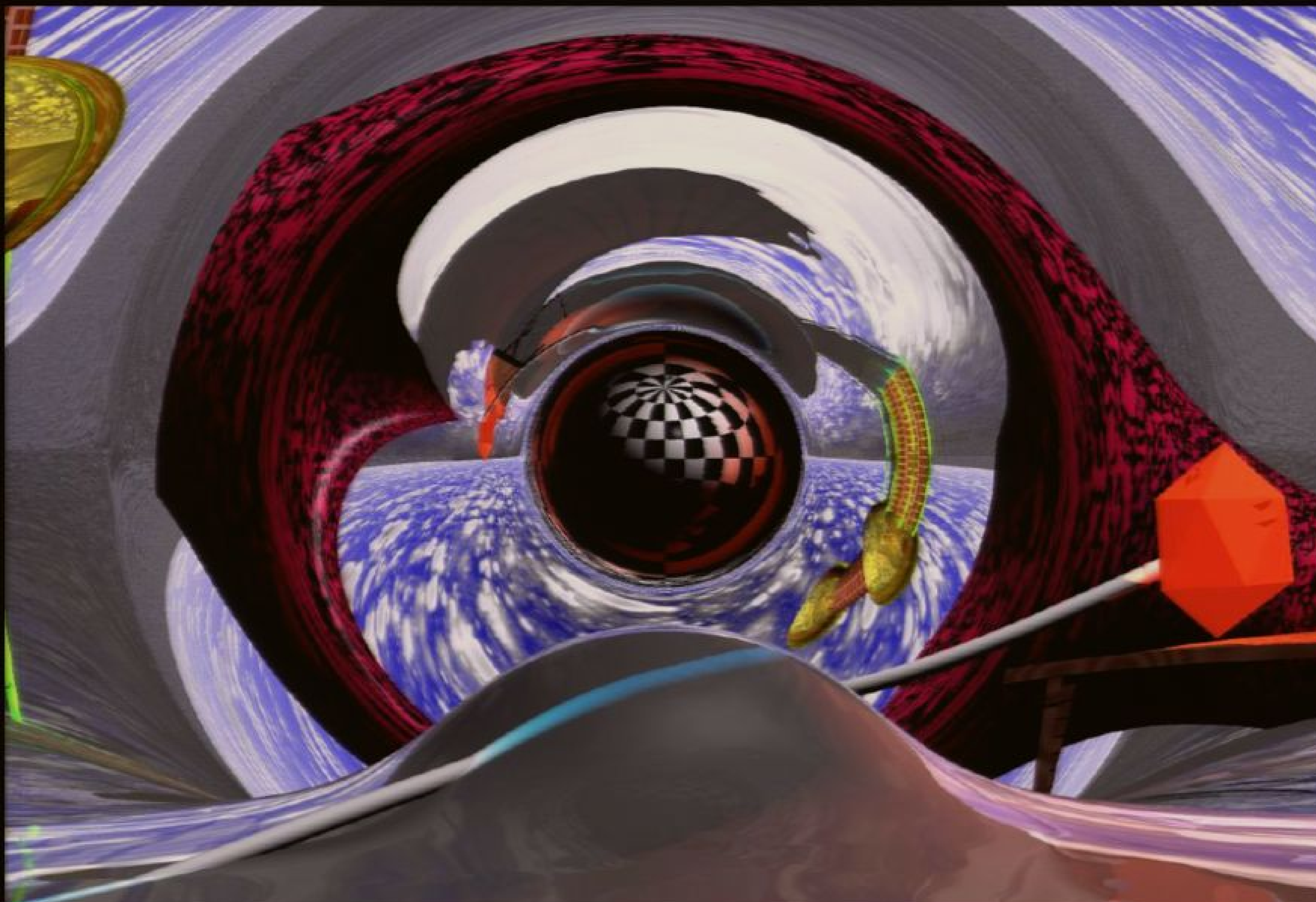


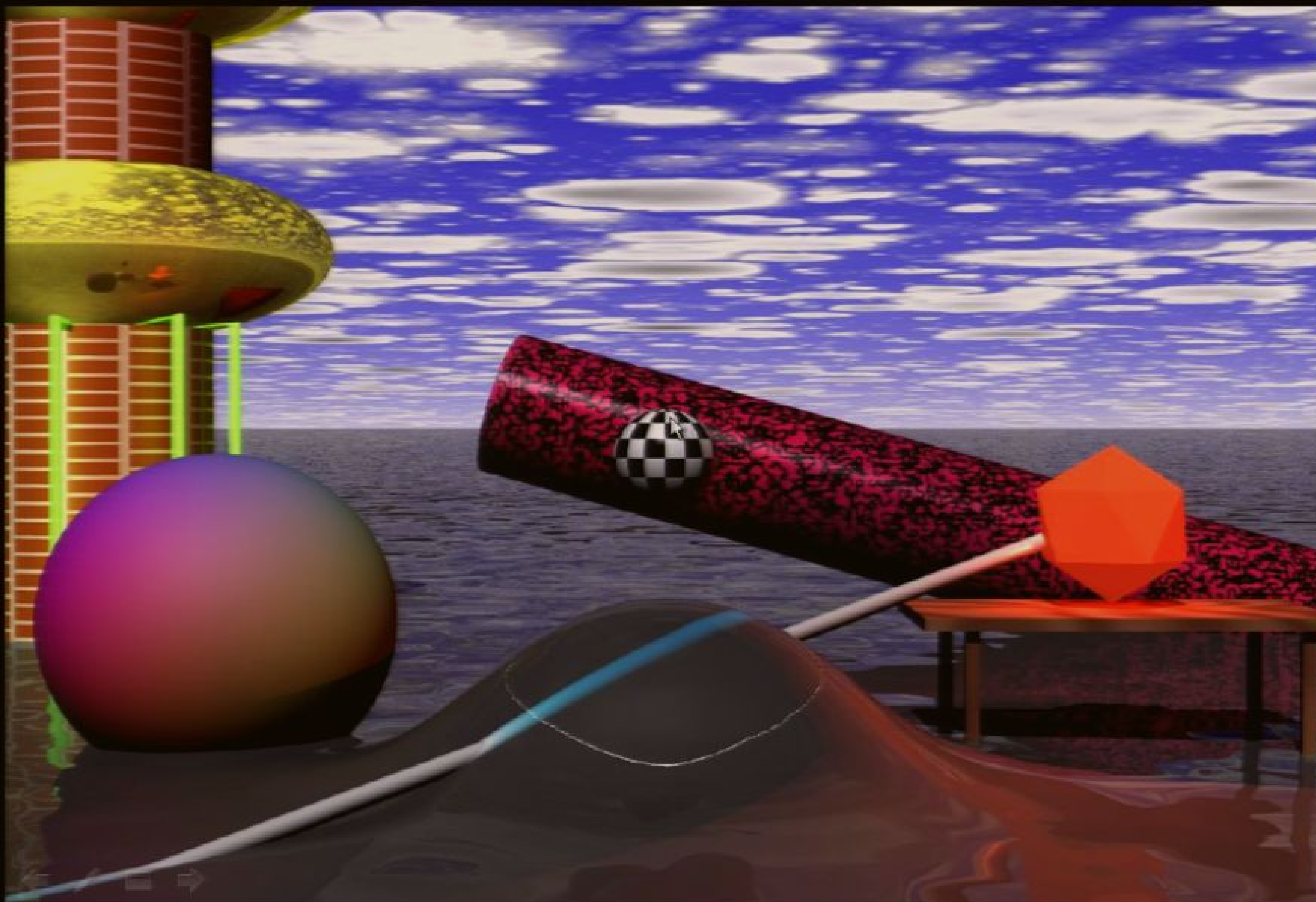
Working with a Curved Geometry

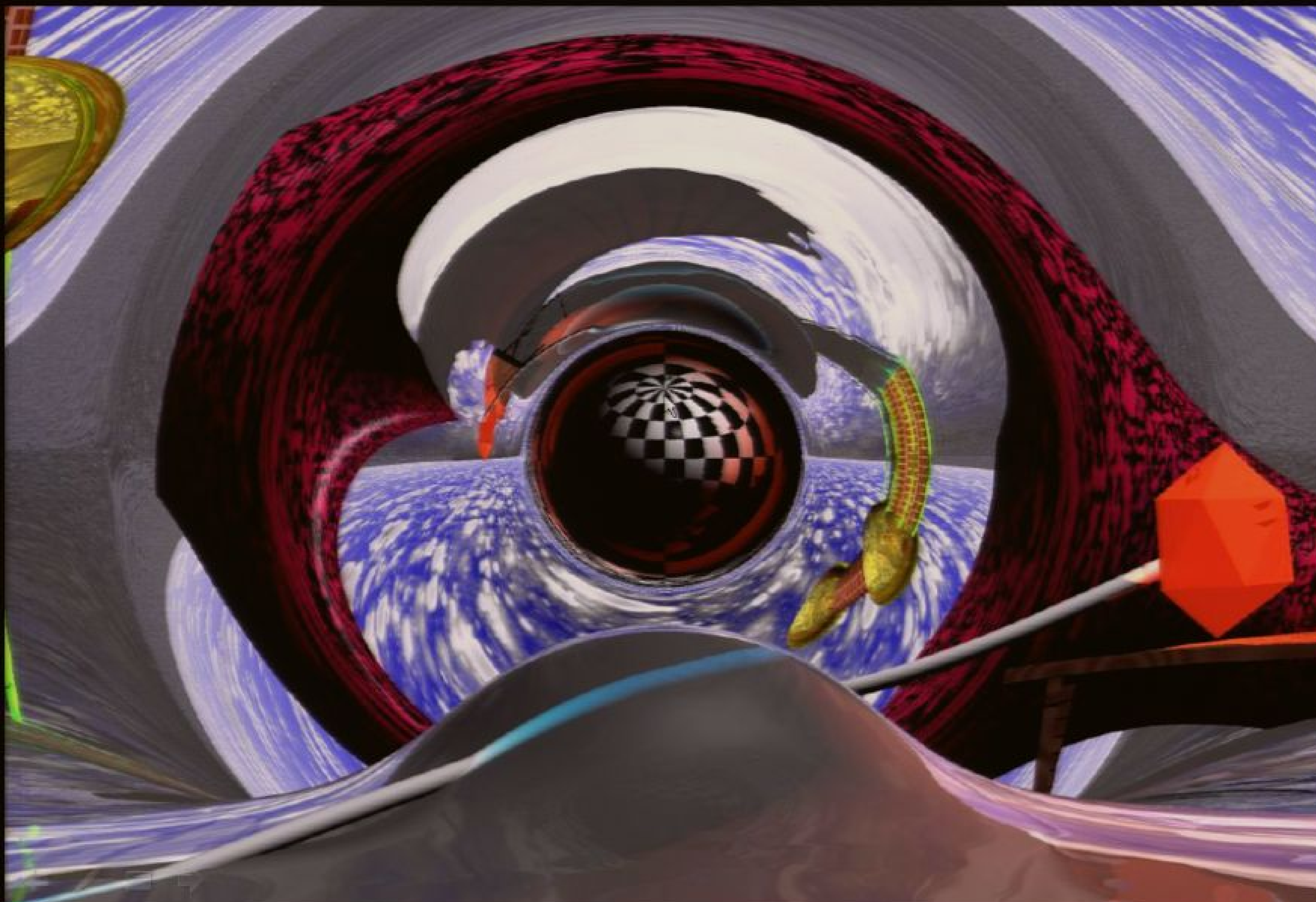




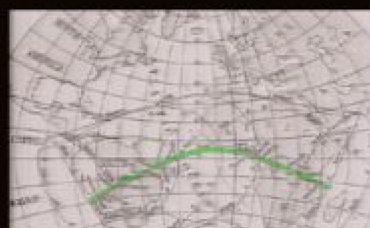








1919 Verification



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.