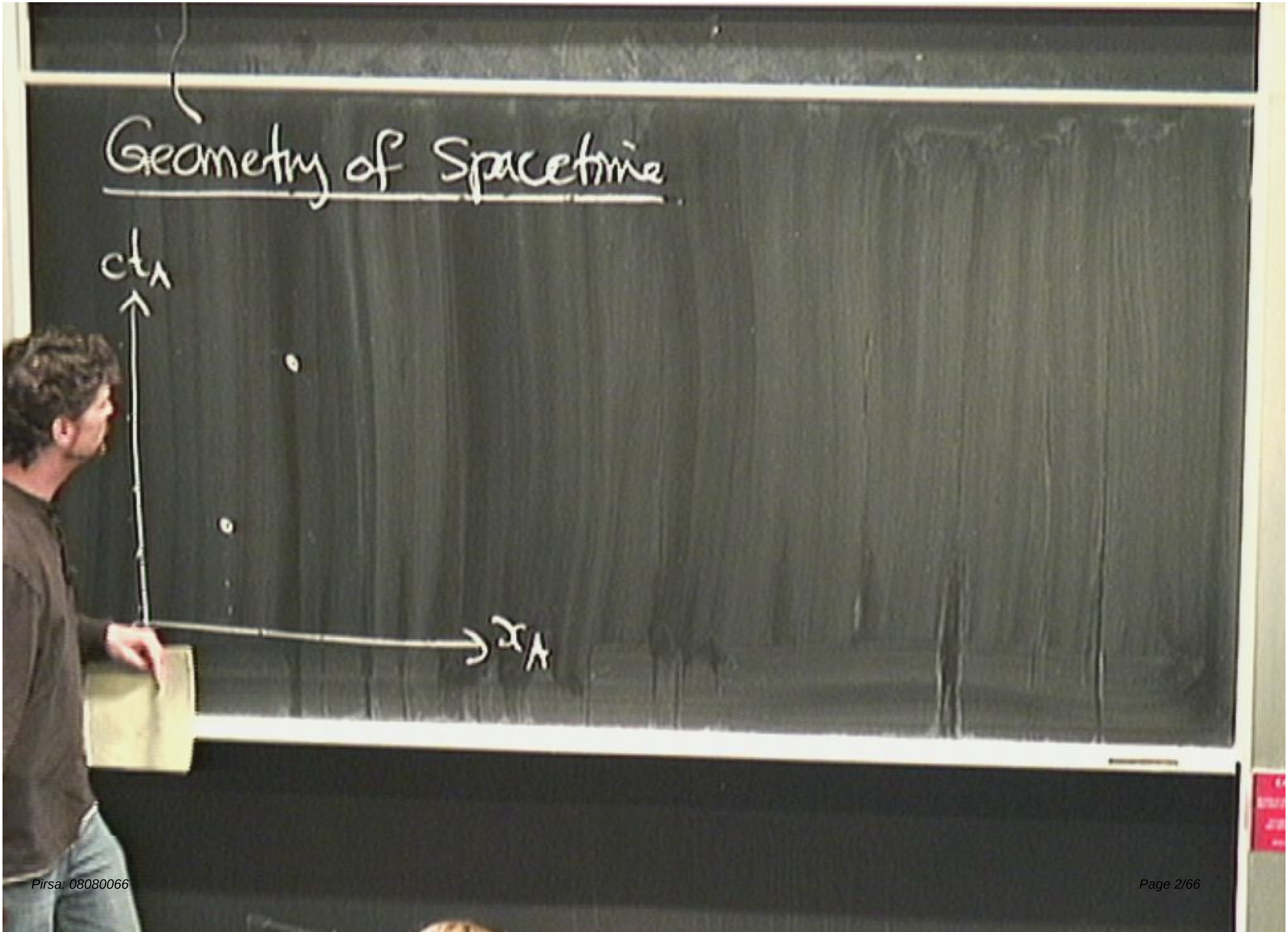
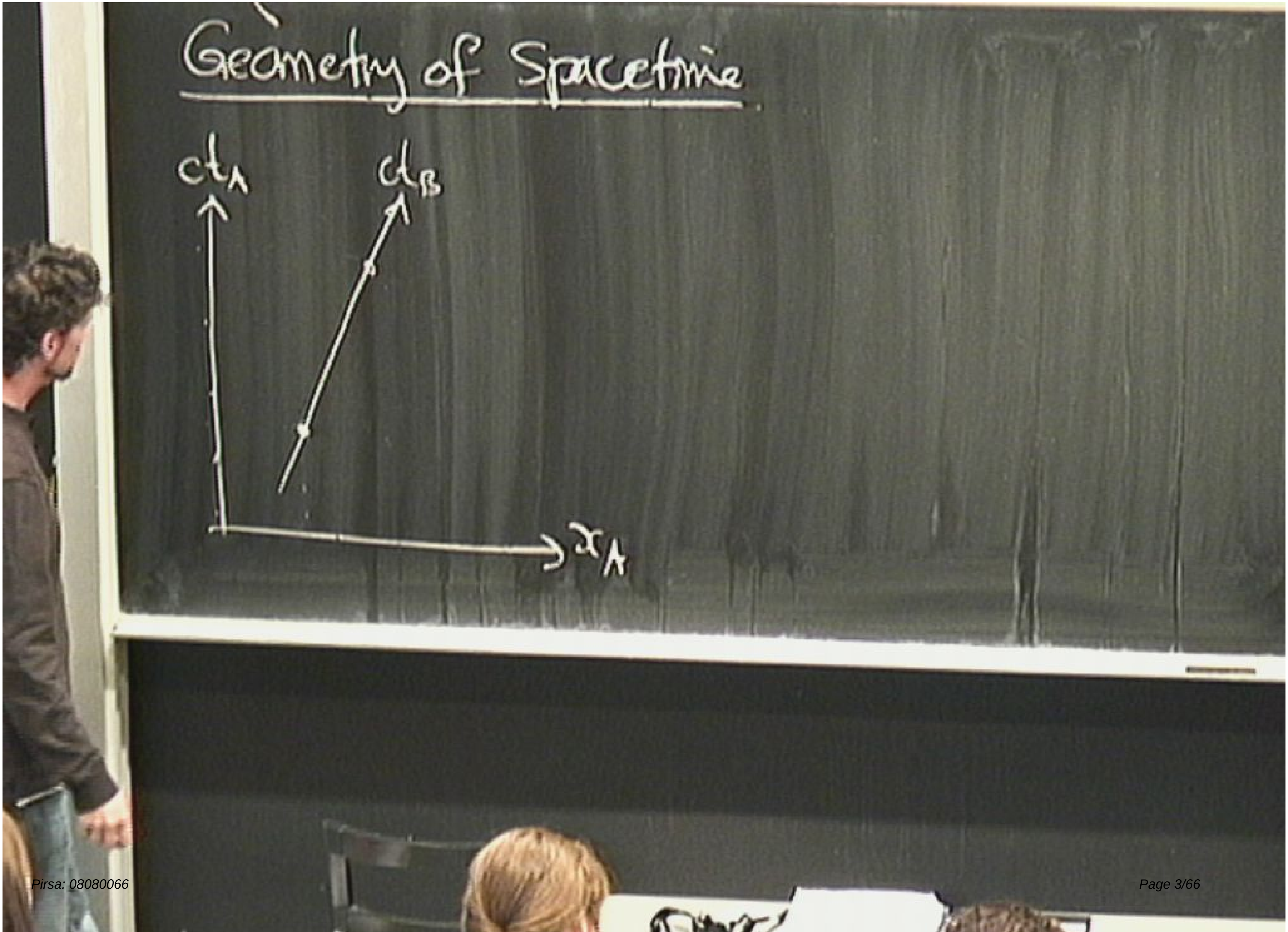
Title: Special Relativity 7 - Minkowskian Geometry

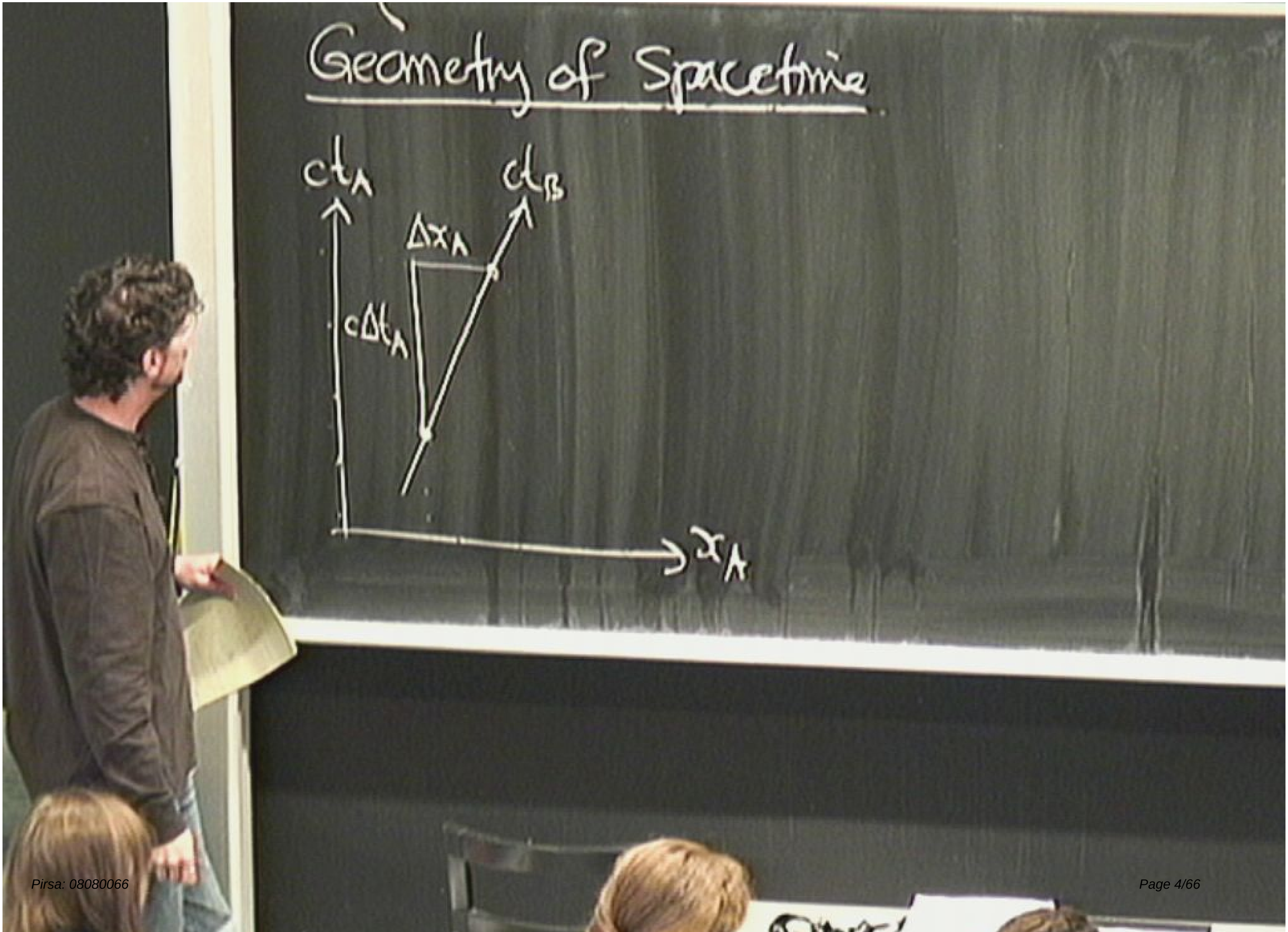
Date: Aug 12, 2008 09:00 AM

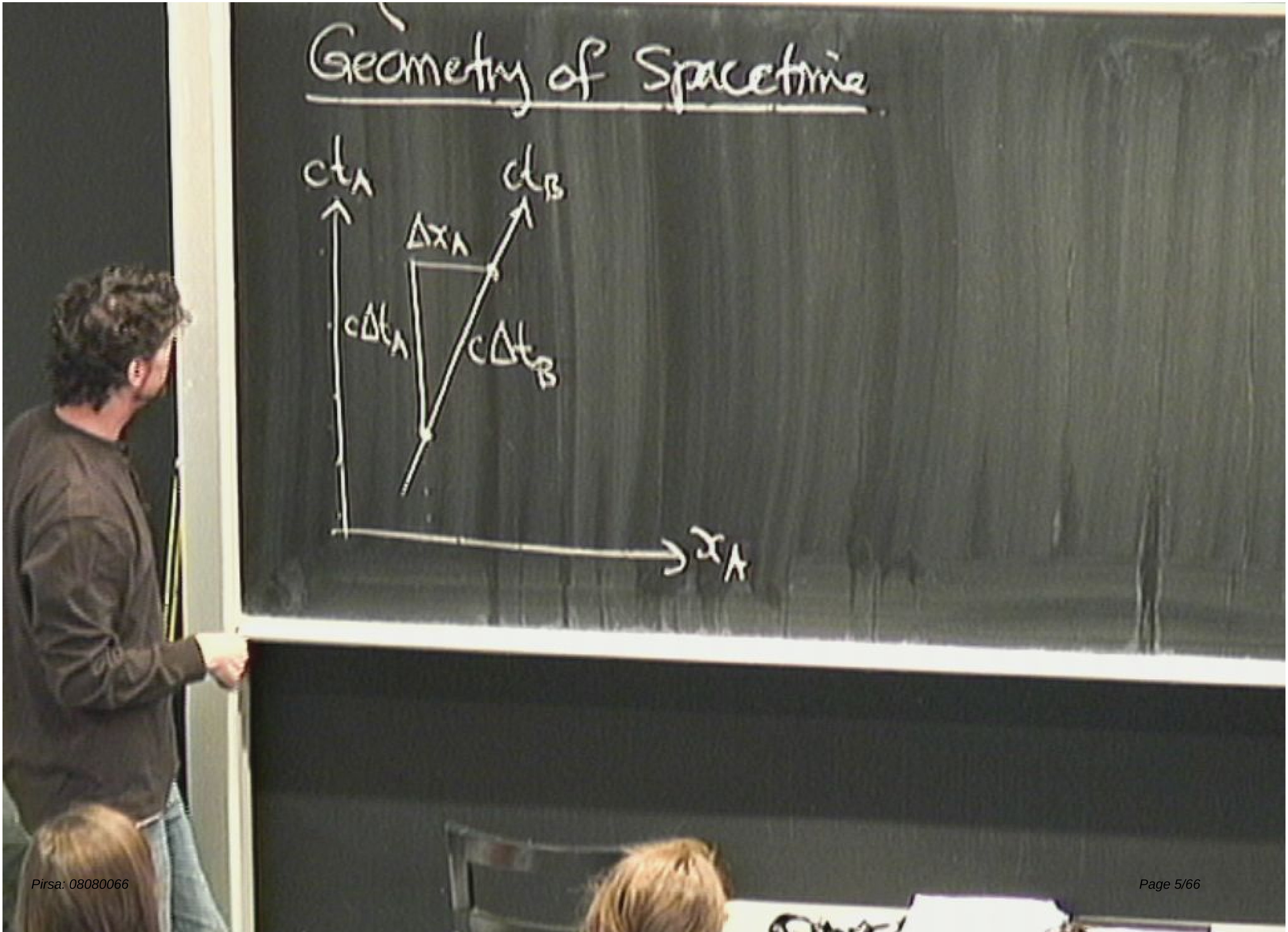
URL: http://pirsa.org/08080066

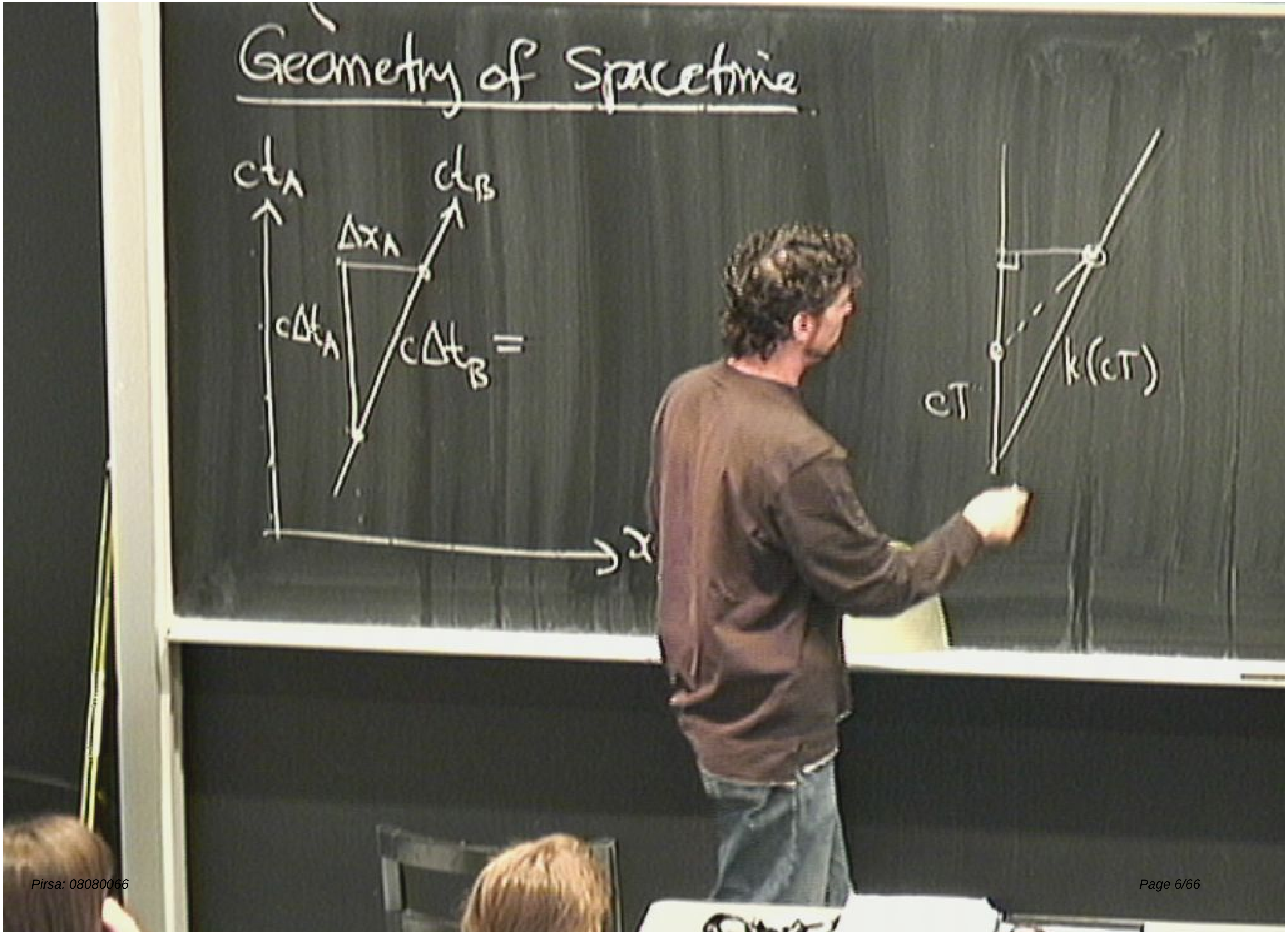
Abstract: Space obeys the rules of Euclidean geometry. Spacetime obeys the rules of a new kind of geometry called Minkowskian geometry.
Learning Outcomes:
â€¢ Triangles in spacetime obey a Pythagoras-like theorem, but with an unusual minus sign.
â€¢ The true nature of time as geometrical distance in spacetime.
â€¢ How to analyse and resolve the Twinsâ€™ Paradox using spacetime diagrams in combination with Minkowskian geometry.

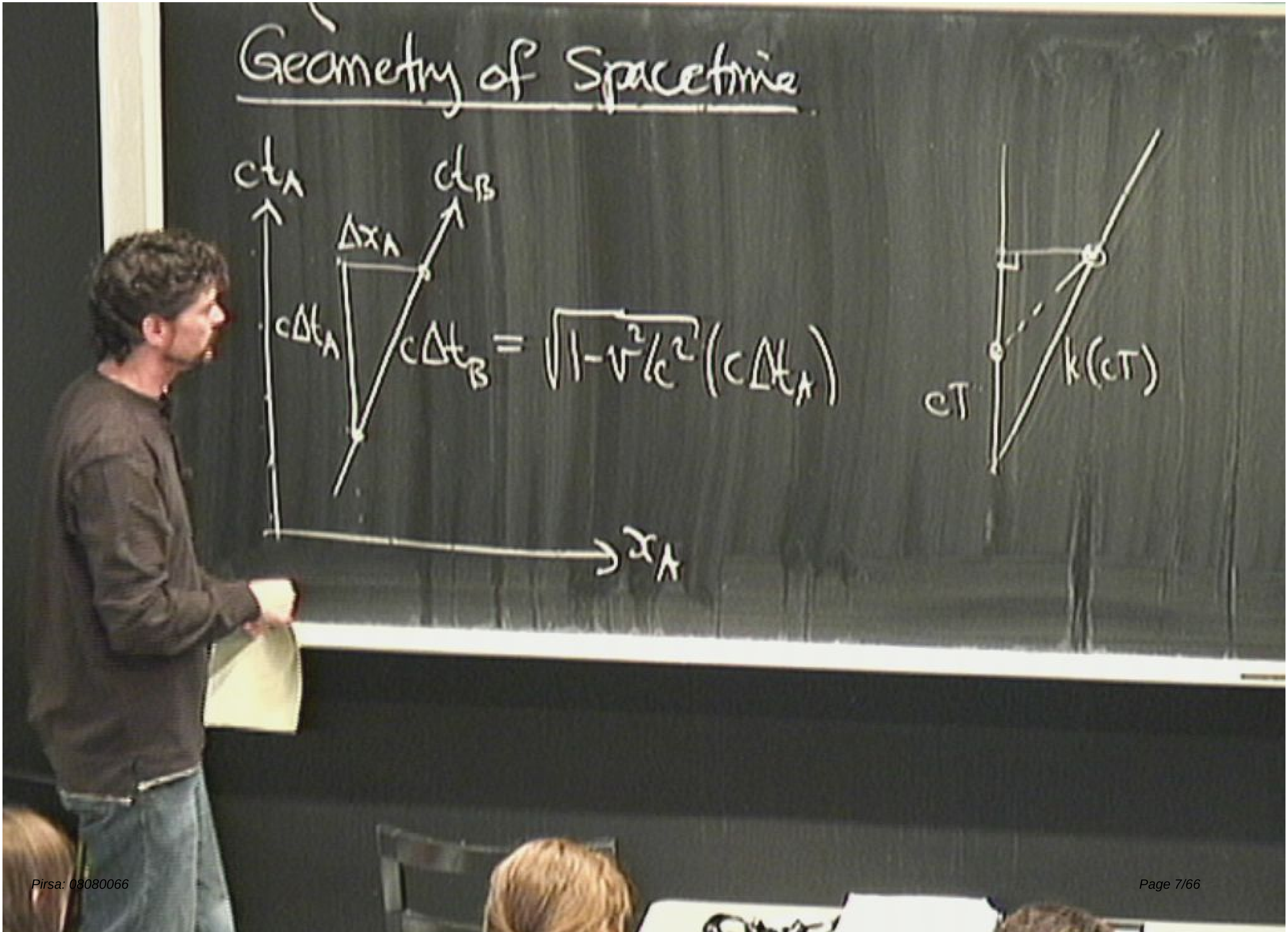

Geometry of Spacetime
ct_A
x_A

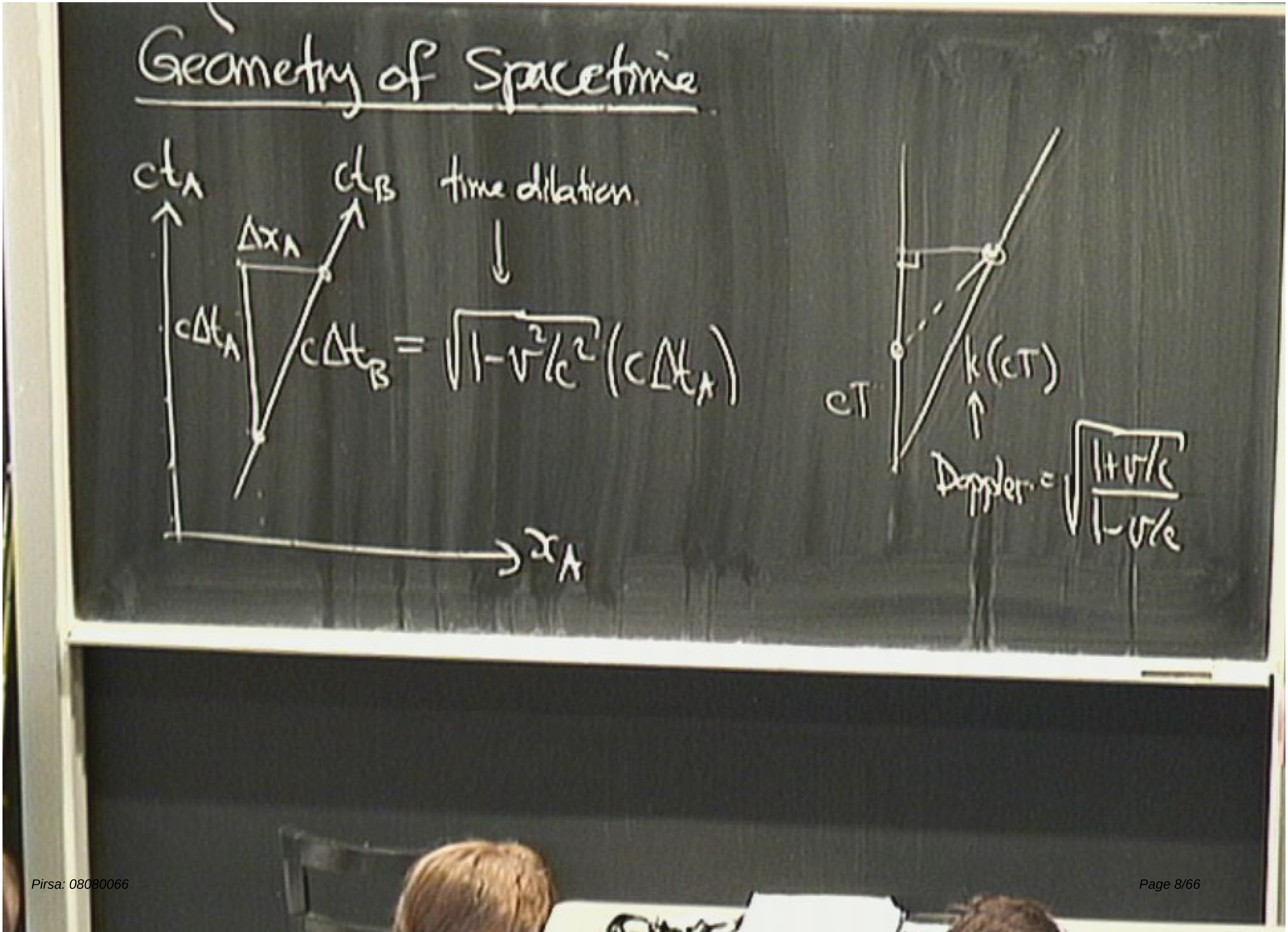

Geometry of Spacetime
ct_A
ct_B
x_A


Geometry of Spacetime
ct_A
ct_B
Δx_A
$c\Delta t_A$
x_A

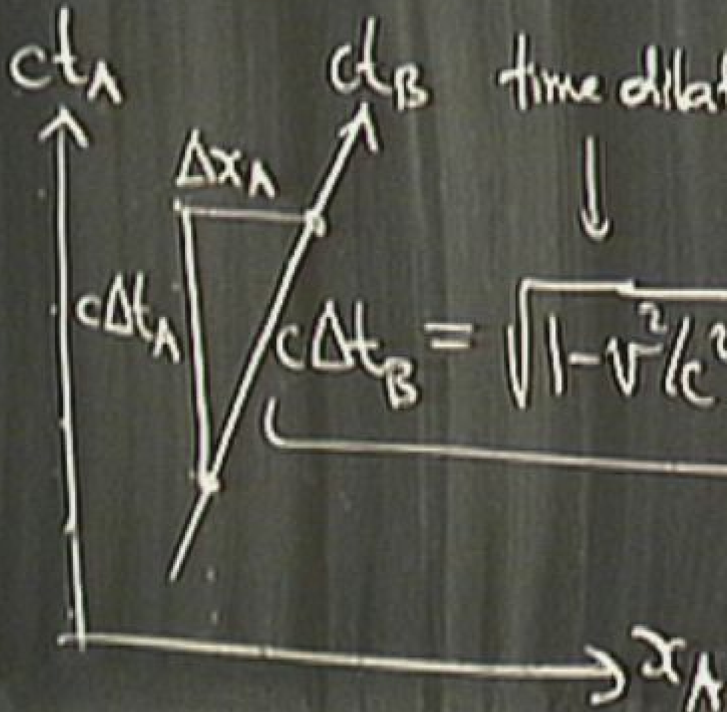

Geometry of Spacetime
ct_A
ct_B
Δx_A
$c\Delta t_A$
$c\Delta t_B$
x_A


Geometry of Spacetime
ctA
ctB
ΔxA
cΔtA
cΔtB =
x
cT
k(cT)


Geometry of Spacetime
ct_A
ct_B
Δx_A
$c\Delta t_A$
$c\Delta t_B = \sqrt{1-v^2/c^2}\,(c\Delta t_A)$
x_A
cT
$k(cT)$

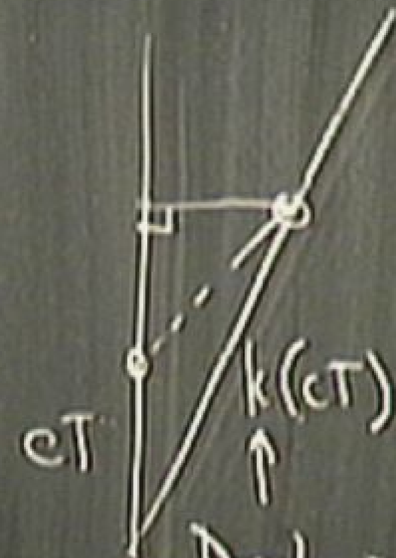

Geometry of Spacetime
ct_A
ct_B
time dilation
Δx_A
$c\Delta t_A$
$c\Delta t_B = \sqrt{1-v^2/c^2}\,(c\Delta t_A)$
x_A
cT
$k(cT)$
Doppler $= \sqrt{\frac{1+v/c}{1-v/c}}$

Geometry of Spacetime



time dilation

$$c\Delta t_B = \sqrt{1 - v^2/c^2}\,(c\Delta t_A)$$



$$\text{Doppler} = \sqrt{\frac{1+v/c}{1-v/c}}$$

$$c^2 \Delta t_B^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t_A^2$$

$$c^2 \Delta t_B^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t_A^2$$

$$v = \frac{\text{dist.}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

$$c^2 \Delta t_B^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t_A^2$$

$$v = \frac{\text{dist.}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

$$=$$

$$c^2\Delta t_B^2 = \left(1-\frac{v^2}{c^2}\right)c^2\Delta t_A^2$$

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

$$= \left(1-\frac{\Delta x_A^2}{c^2\Delta t_A^2}\right)c^2\Delta t_A^2$$

$$c^2 \Delta t_B^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t_A^2$$

$$v = \frac{\text{dist.}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

$$= \left(1 - \frac{\Delta x_A^2}{c^2 \Delta t_A^2}\right) c^2 \Delta t_A^2$$

$$= c^2 \Delta t_A^2 -$$

$$c^2 \Delta t_B^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t_A^2$$

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

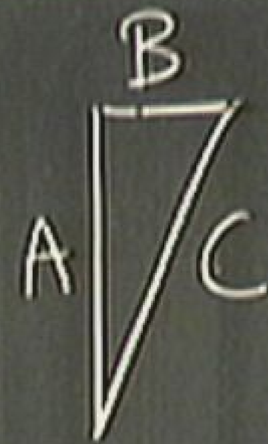
$$= \left(1 - \frac{\Delta x_A^2}{c^2 \Delta t_A^2}\right) c^2 \Delta t_A^2$$

$$\boxed{c^2 \Delta t_B^2 = c^2 \Delta t_A^2 - \Delta x_A^2}$$

$$c^2\Delta t_B^2 = \left(1-\frac{v^2}{c^2}\right)c^2\Delta t_A^2$$

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

$$= \left(1-\frac{\Delta x_A^2}{c^2\Delta t_A^2}\right)c^2\Delta t_A^2$$



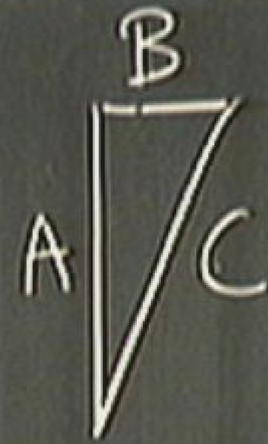
$$\boxed{c^2\Delta t_B^2 = c^2\Delta t_A^2 - \Delta x_A^2}$$

$$c^2\Delta t_B^2 = \left(1-\frac{v^2}{c^2}\right)c^2\Delta t_A^2$$

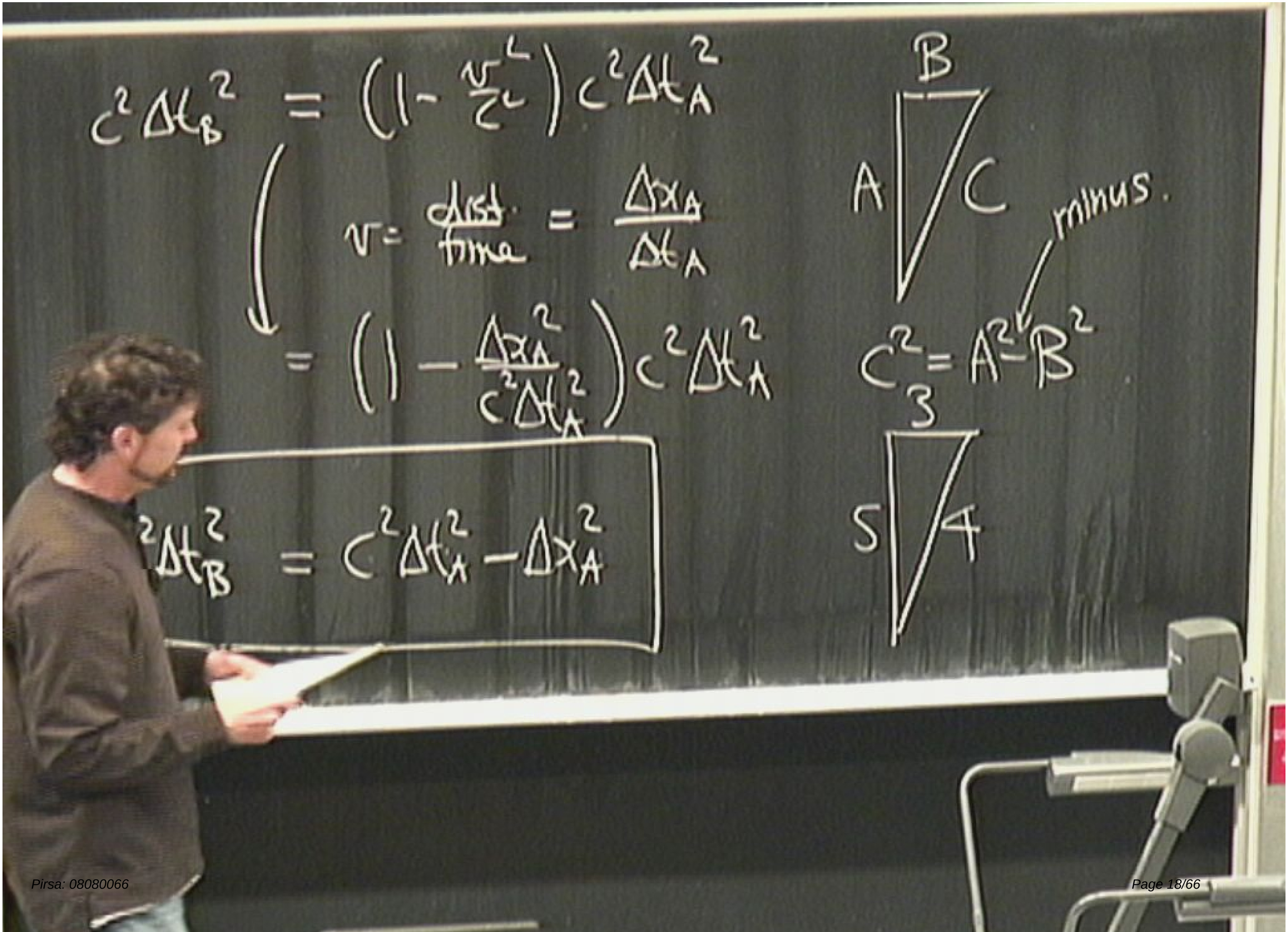
$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

$$= \left(1-\frac{\Delta x_A^2}{c^2\Delta t_A^2}\right)c^2\Delta t_A^2$$

$$\boxed{c^2\Delta t_B^2 = c^2\Delta t_A^2 - \Delta x_A^2}$$



$$C^2 = A^2 - B^2$$


$c^2\Delta t_B^2 = \left(1 - \frac{v^2}{c^2}\right) c^2 \Delta t_A^2$
$v = \frac{\text{dist.}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$
$= \left(1 - \frac{\Delta x_A^2}{c^2 \Delta t_A^2}\right) c^2 \Delta t_A^2$
$^2\Delta t_B^2 = c^2 \Delta t_A^2 - \Delta x_A^2$
B
A
C
minus.
$C^2 = A^2 - B^2$
3
5
4

$$c^2\Delta t_B^2 = \left(1-\frac{v^2}{c^2}\right)c^2\Delta t_A^2$$

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

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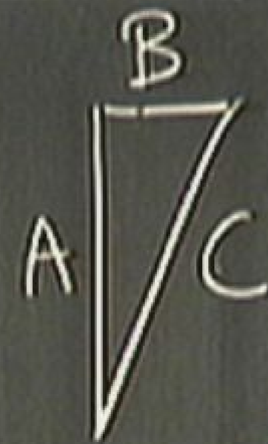
$$\boxed{c^2\Delta t_B^2 = c^2\Delta t_A^2 - \Delta x_A^2}$$



minus.

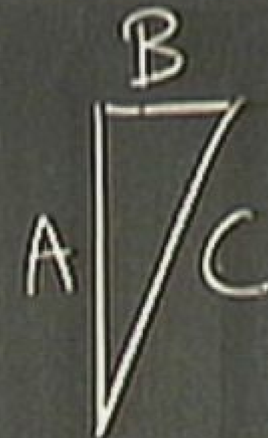
$$C^2 = A^2 - B^2$$

$$c^2\Delta t_B^2 = \left(1-\frac{v^2}{c^2}\right)c^2\Delta t_A^2$$

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

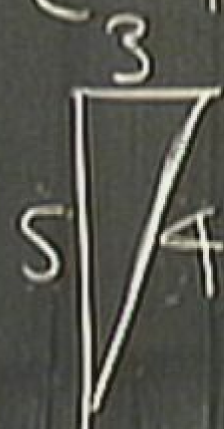
$$= \left(1-\frac{\Delta x_A^2}{c^2\Delta t_A^2}\right)c^2\Delta t_A^2$$

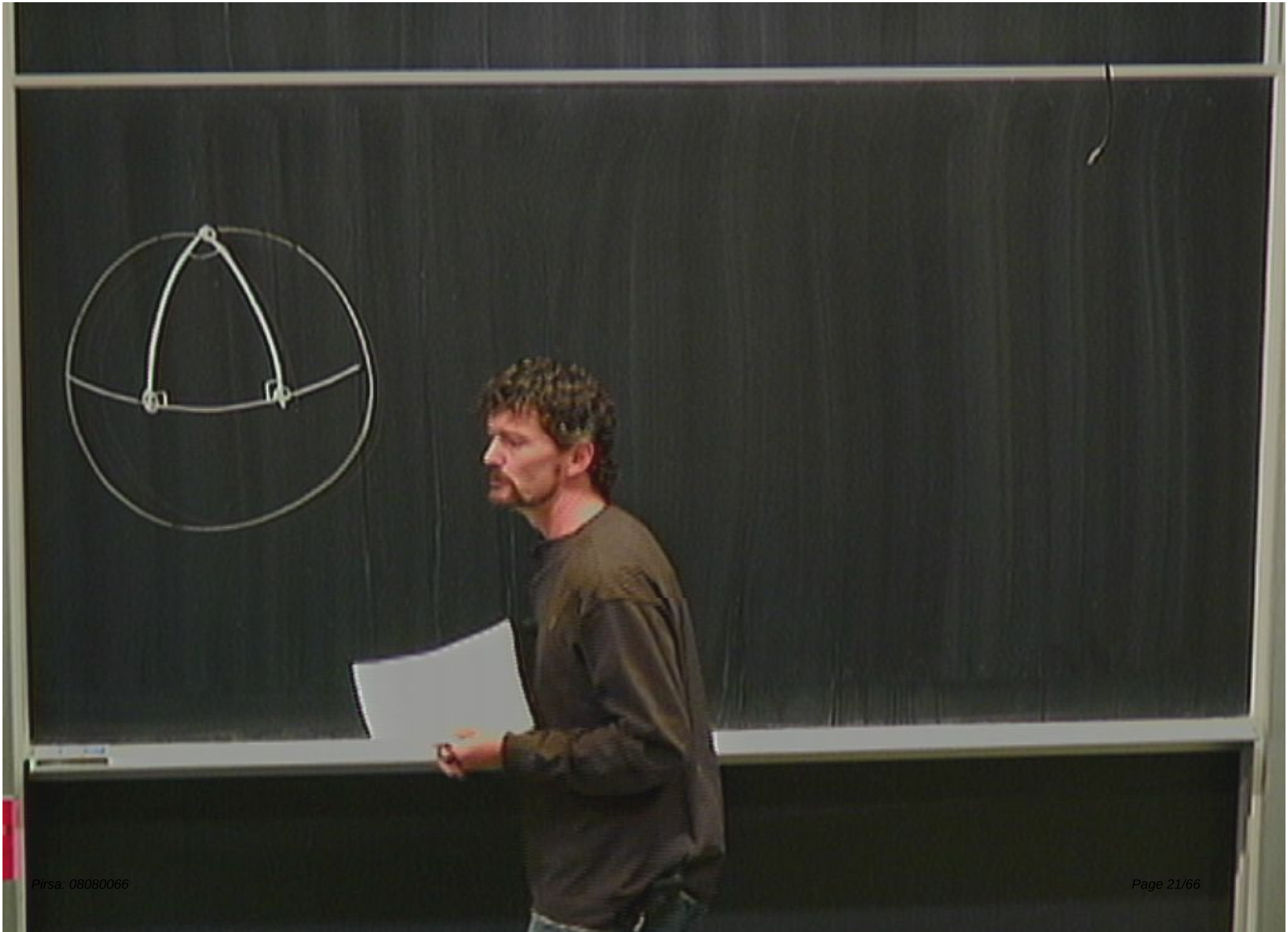
$$\boxed{c^2\Delta t_B^2 = c^2\Delta t_A^2 - \Delta x_A^2}$$



minus.

$$C^2 = A^2 - B^2$$



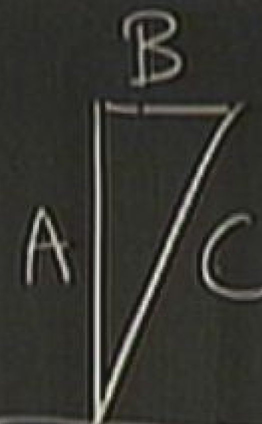


$$c^2\Delta t_B^2 = \left(1-\frac{v^2}{c^2}\right)c^2\Delta t_A^2$$

$$v = \frac{\text{dist}}{\text{time}} = \frac{\Delta x_A}{\Delta t_A}$$

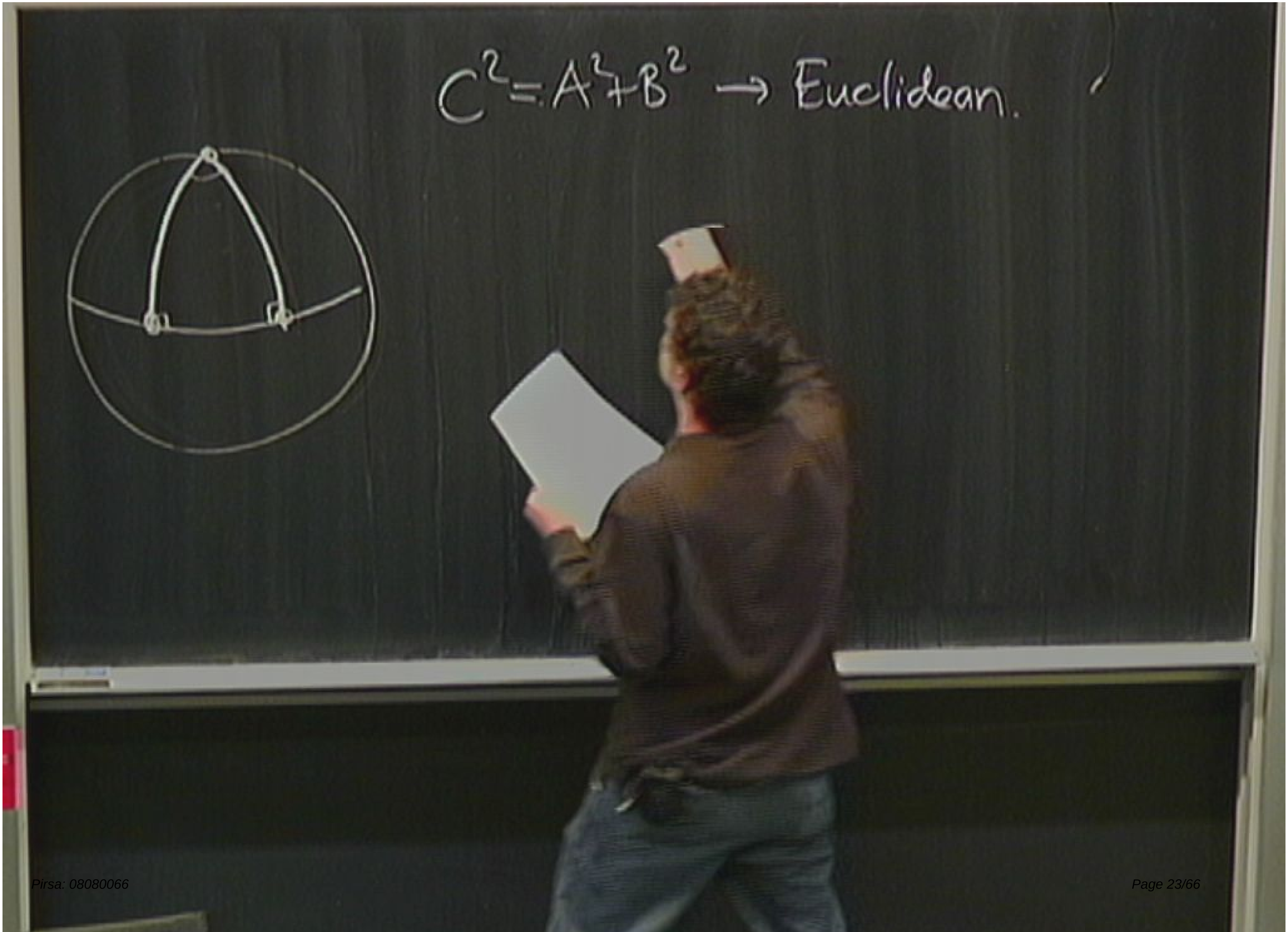
$$= \left(1-\frac{\Delta x_A^2}{c^2\Delta t_A^2}\right)c^2\Delta t_A^2$$

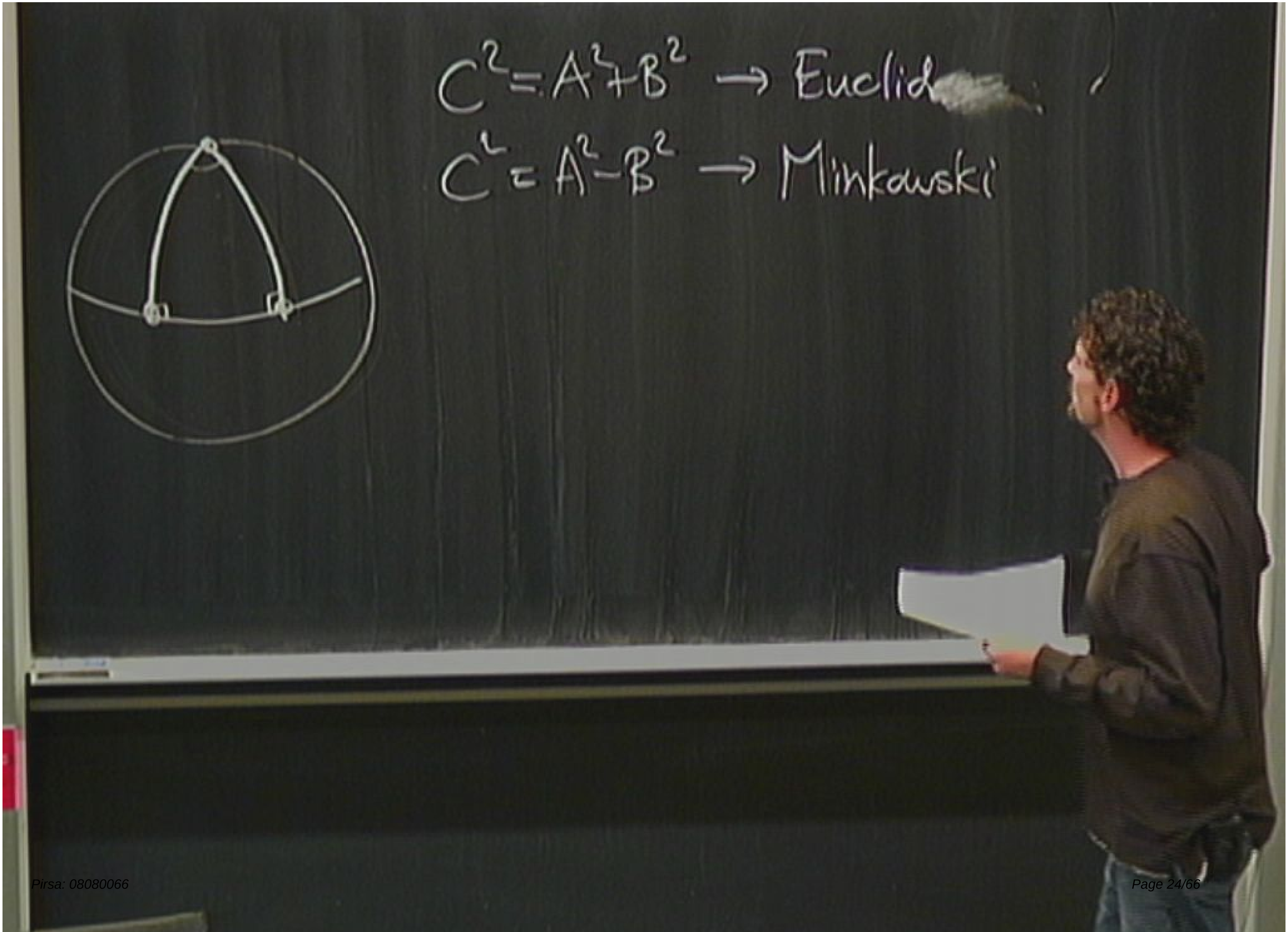
$$\boxed{c^2\Delta t_B^2 = c^2\Delta t_A^2 - \Delta x_A^2}$$

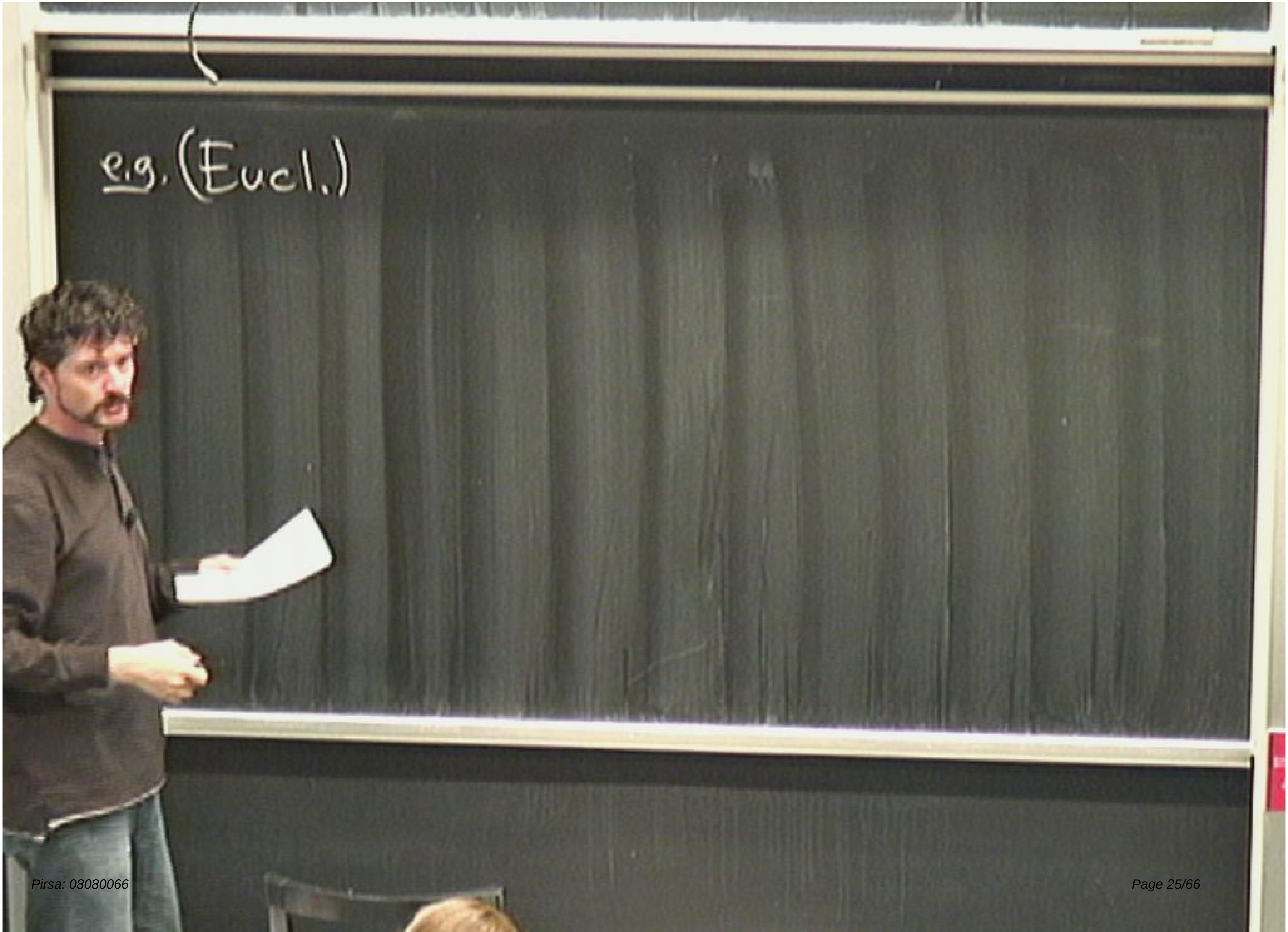


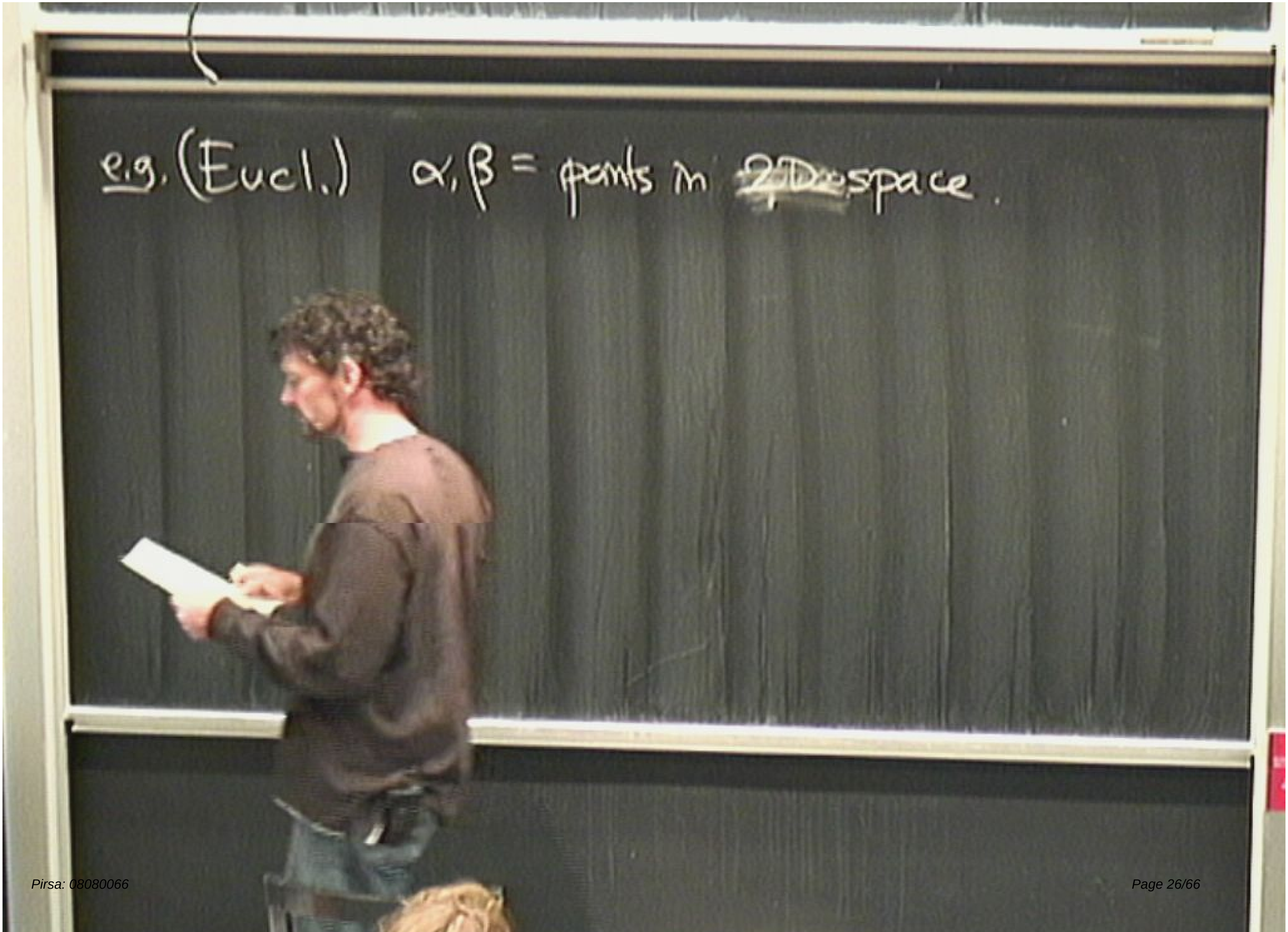
minus

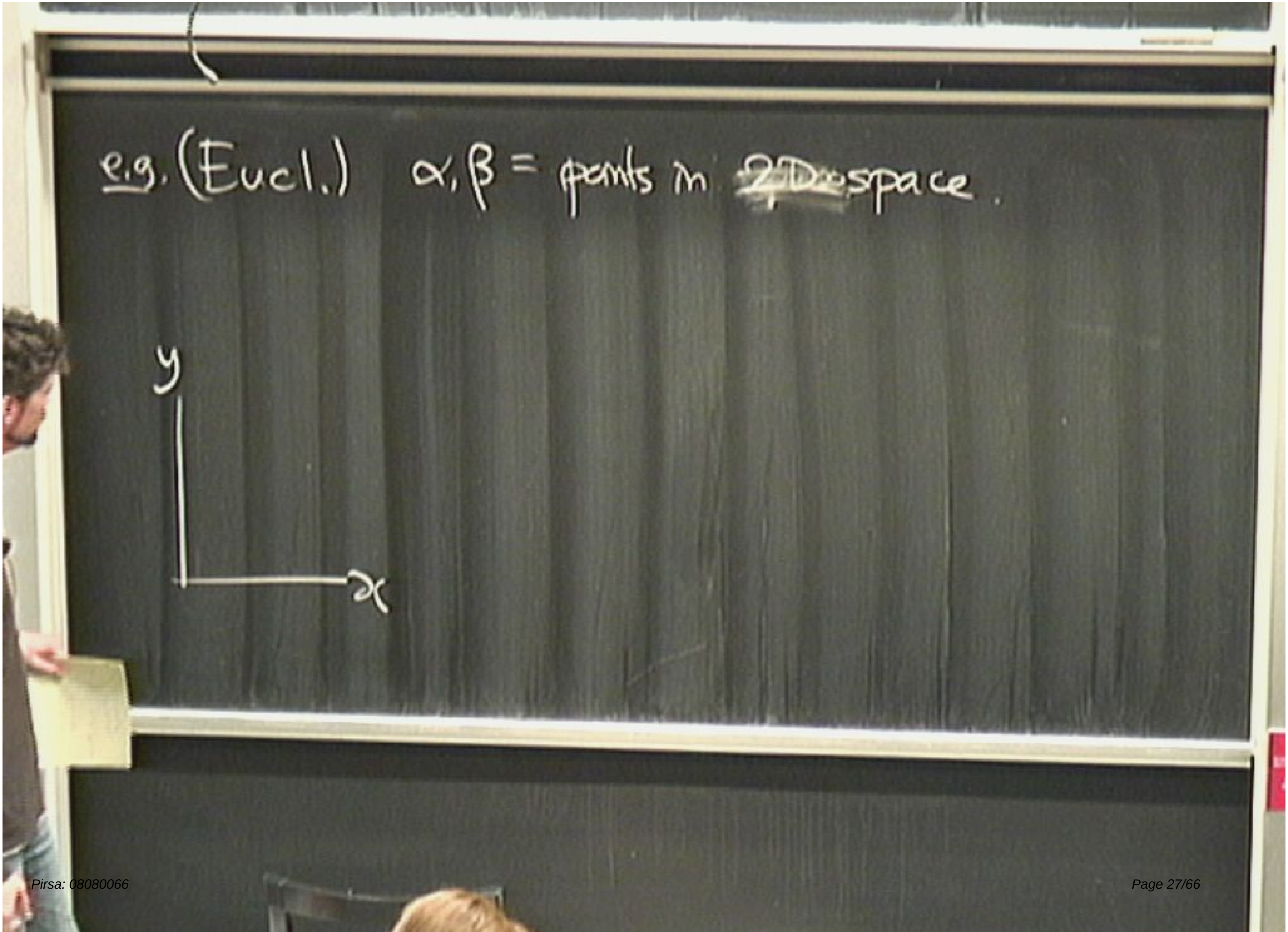
$$\boxed{C^2 = A^2 - B^2}$$

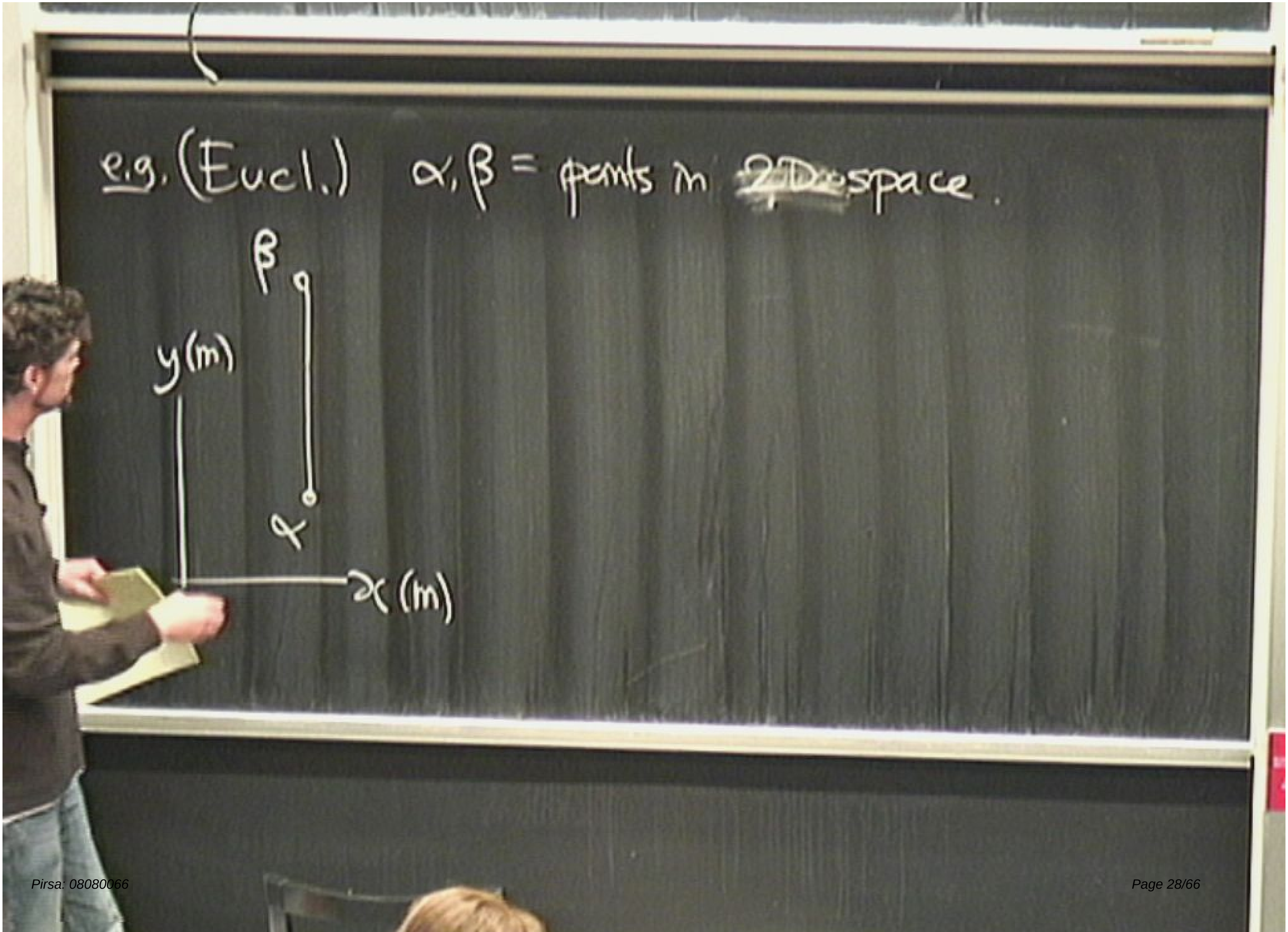

$C^2=A^2+B^2 \rightarrow$ Euclidean.


$C^2 = A^2 + B^2 \rightarrow$ Euclid
$C^2 = A^2 - B^2 \rightarrow$ Minkowski

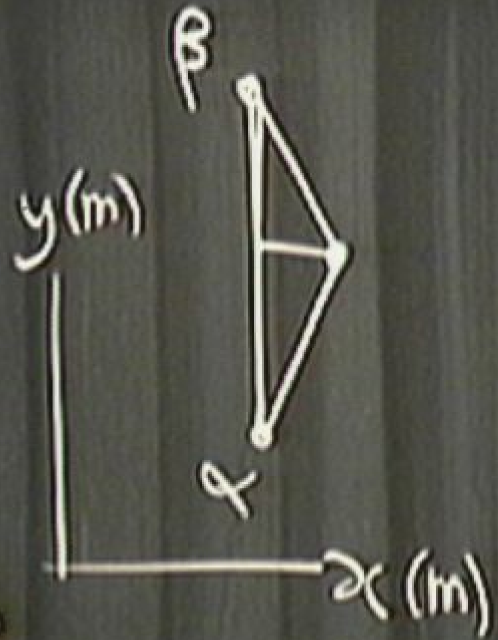

e.g. (Eucl.)

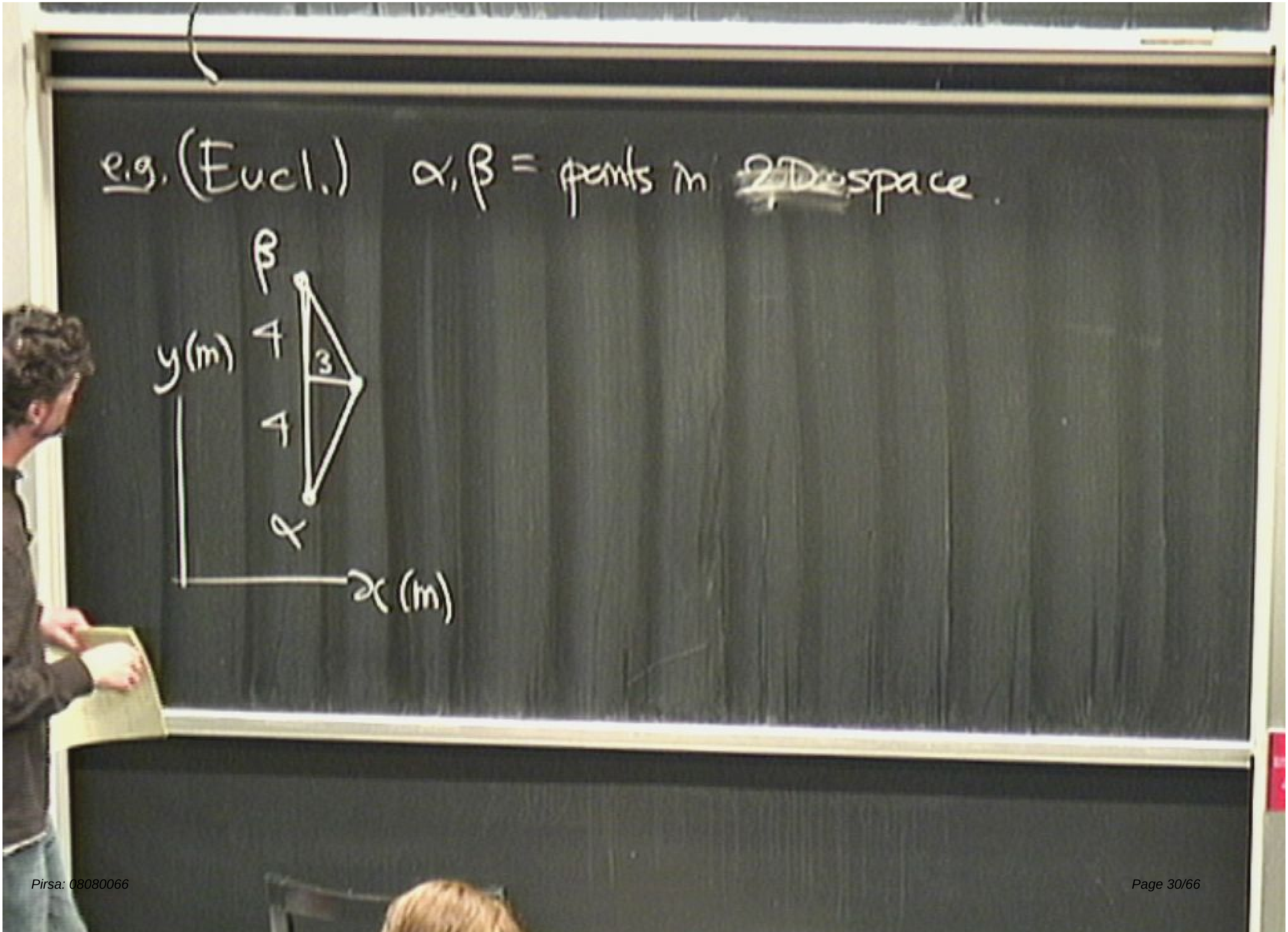

e.g. (Eucl.) α, β = points in space.


e.g. (Eucl.) α, β = points in 2D space.
y
x

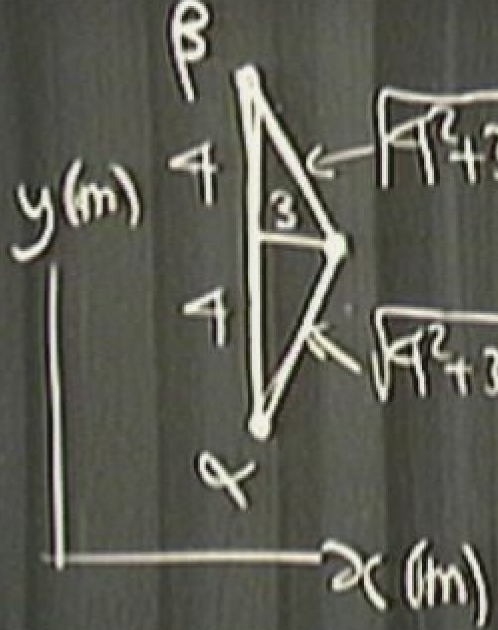

e.g. (Eucl.) α, β = points in space
β
y(m)
α
x(m)

e.g. (Eucl.) α, β = points in 2D space.




e.g. (Eucl.)
α, β = points in space
β
4
3
4
α
y (m)
x (m)

e.g. (Eucl.) α, β = points in ~~2D~~ space.

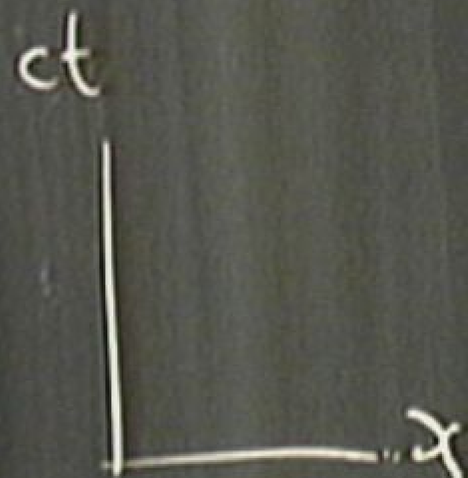


$\sqrt{4^2+3^2} = 5$

$\sqrt{4^2+3^2} = 5$

$5+5 = 10 > 8$

e.g. (Mink.) α, β = points in 2D spacetime



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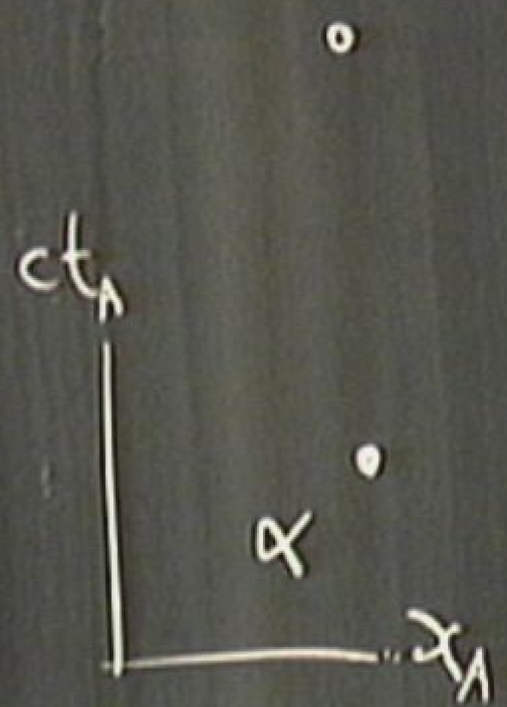
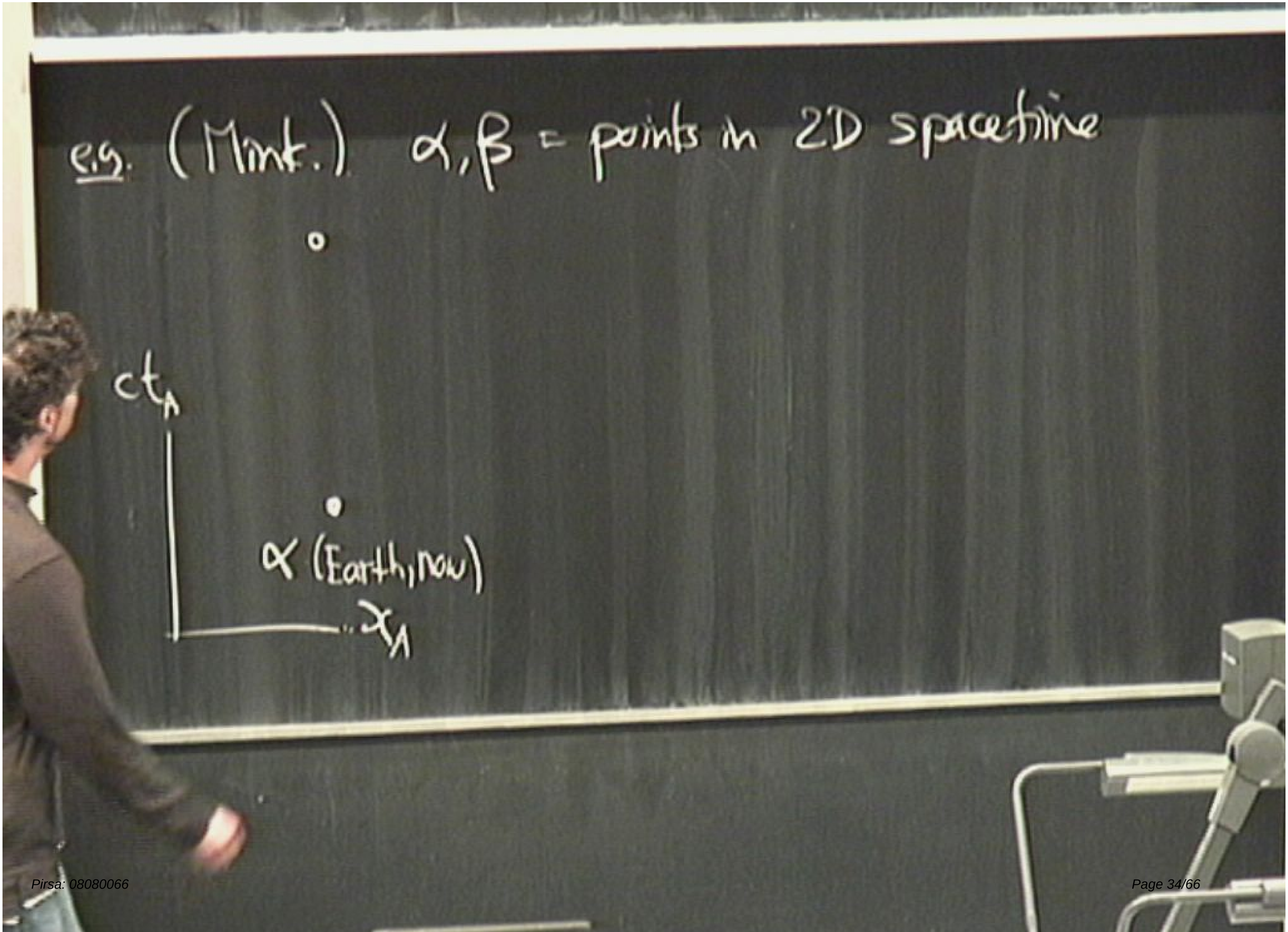
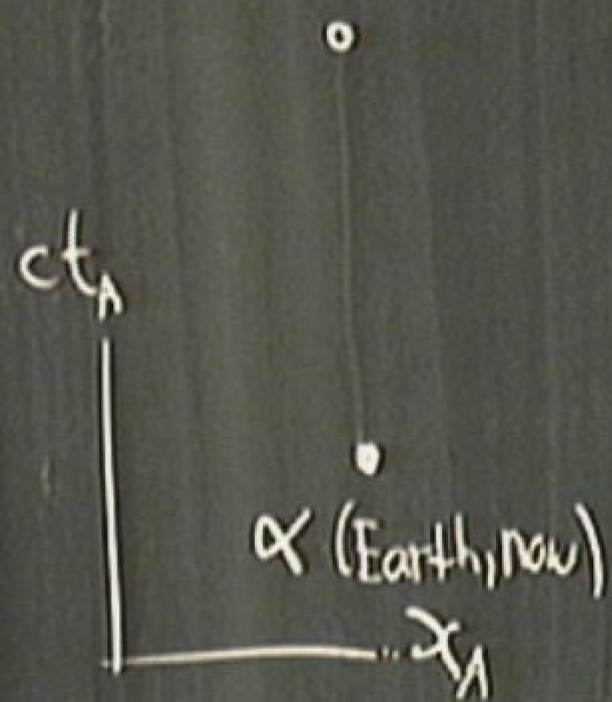



e.g. (Mink.) α, β = points in 2D spacetime
ct_A
α (Earth, now)
x_A

e.g. (Mink.) α, β = points in 2D spacetime



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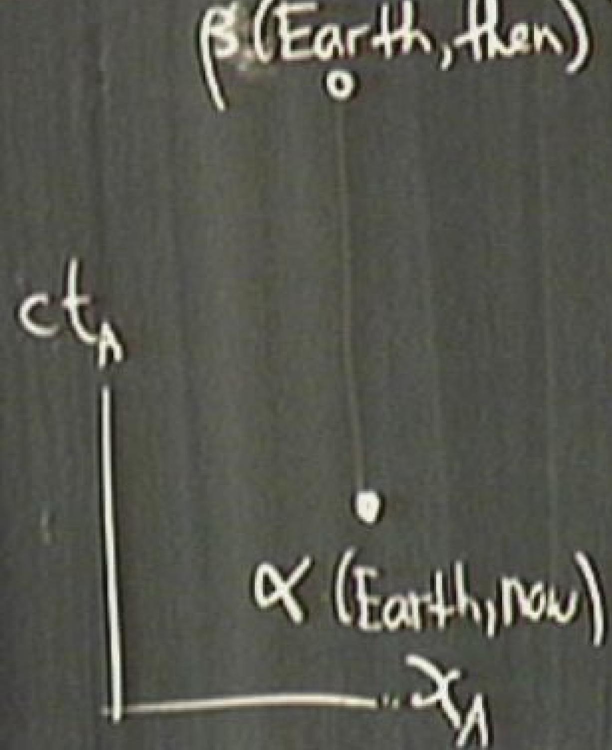

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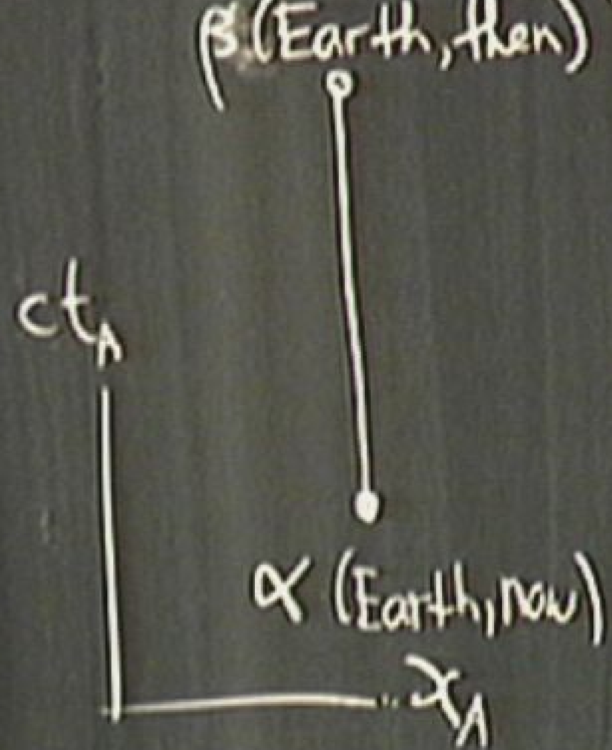

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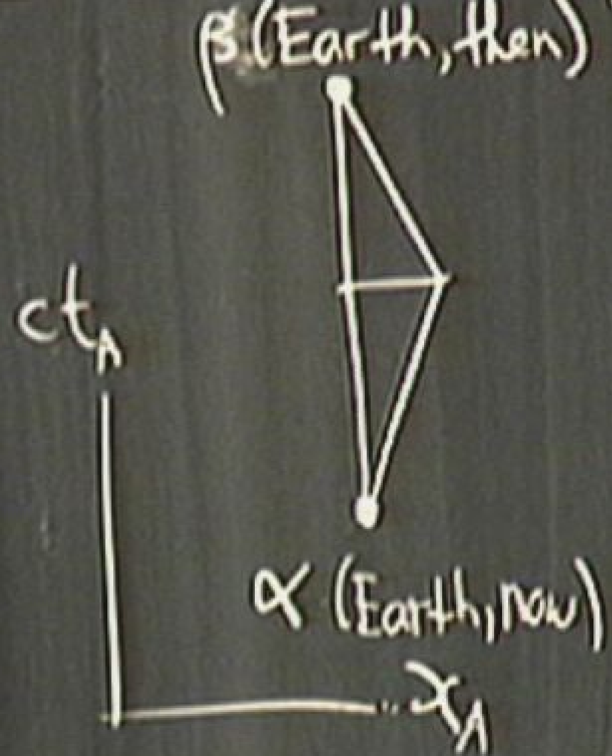

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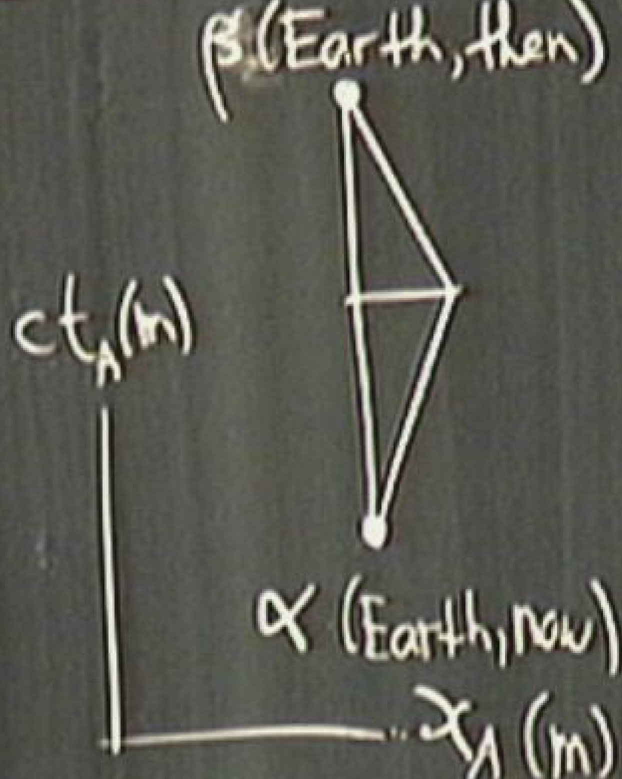
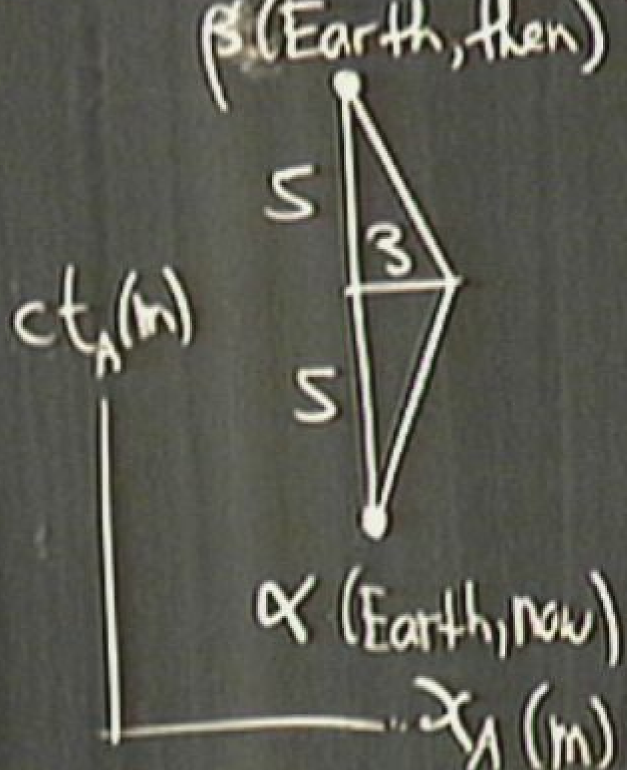


e.g. (Mink.) α, β = points in 2D spacetime



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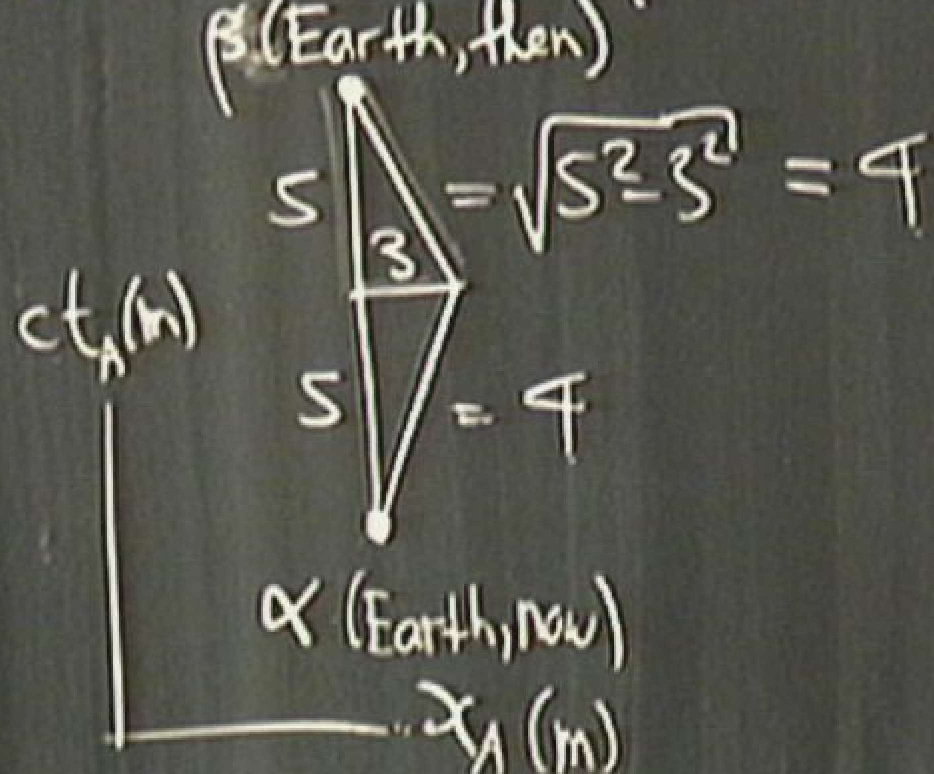
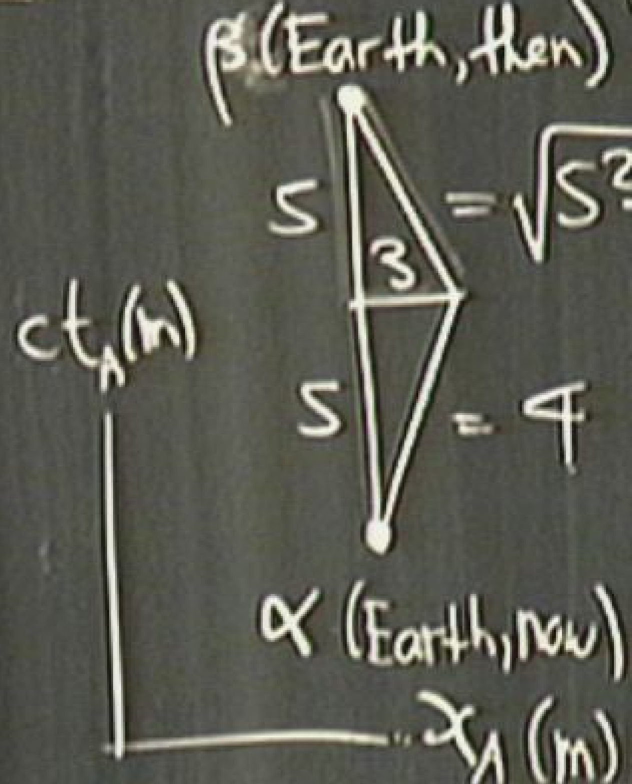


e.g. (Mink.) α, β = points in 2D spacetime



$$\left.\begin{array}{l} = \sqrt{5^2-3^2} = 4 \\ = 4 \end{array}\right\} = 8 < 10$$

e.g. (Mink.) α, β = points in 2D spacetime

β (Earth, then)

5, 3: $= \sqrt{5^2 - 3^2} = 4$

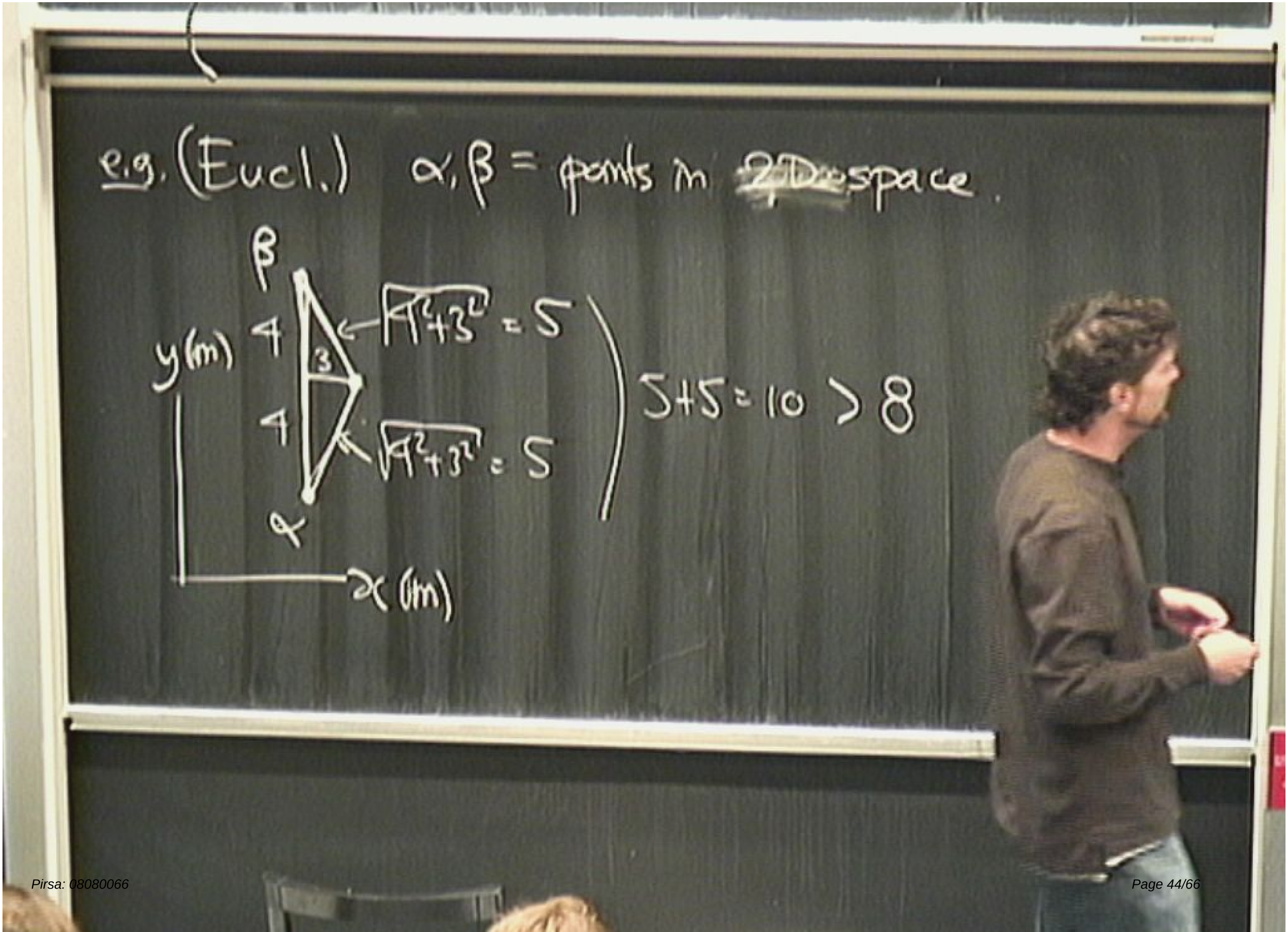
5: $= 4$

$\Bigg\} = 8 < 10$

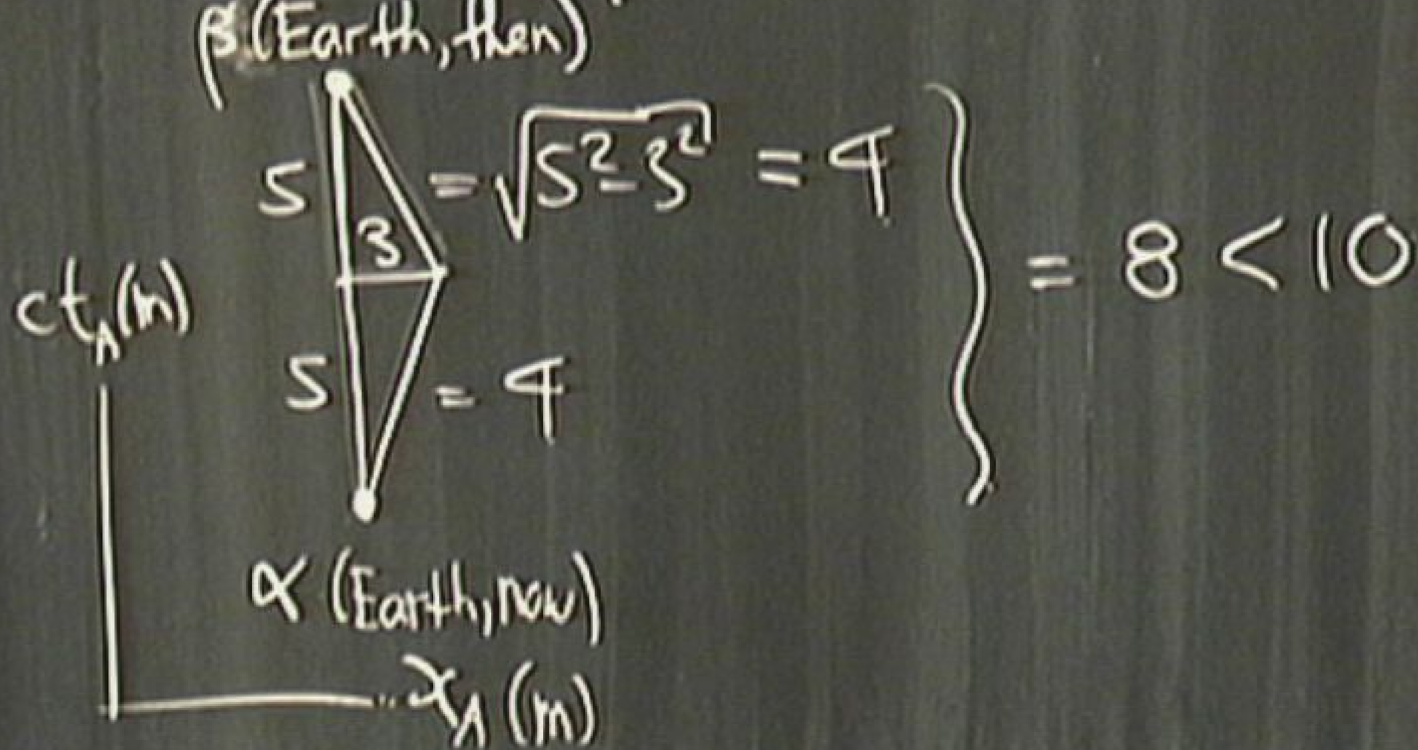
ct_A (m)

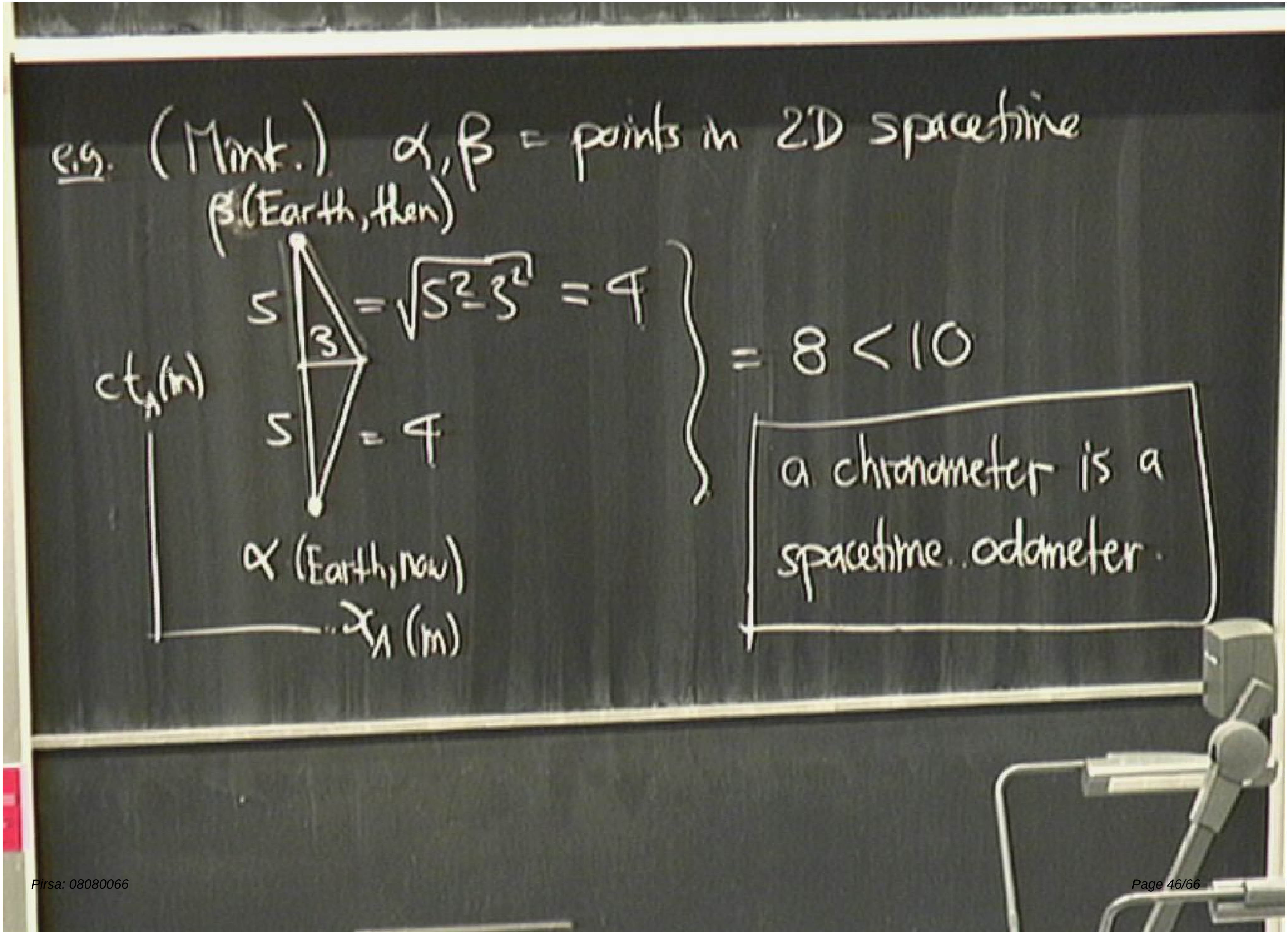
α (Earth, now)

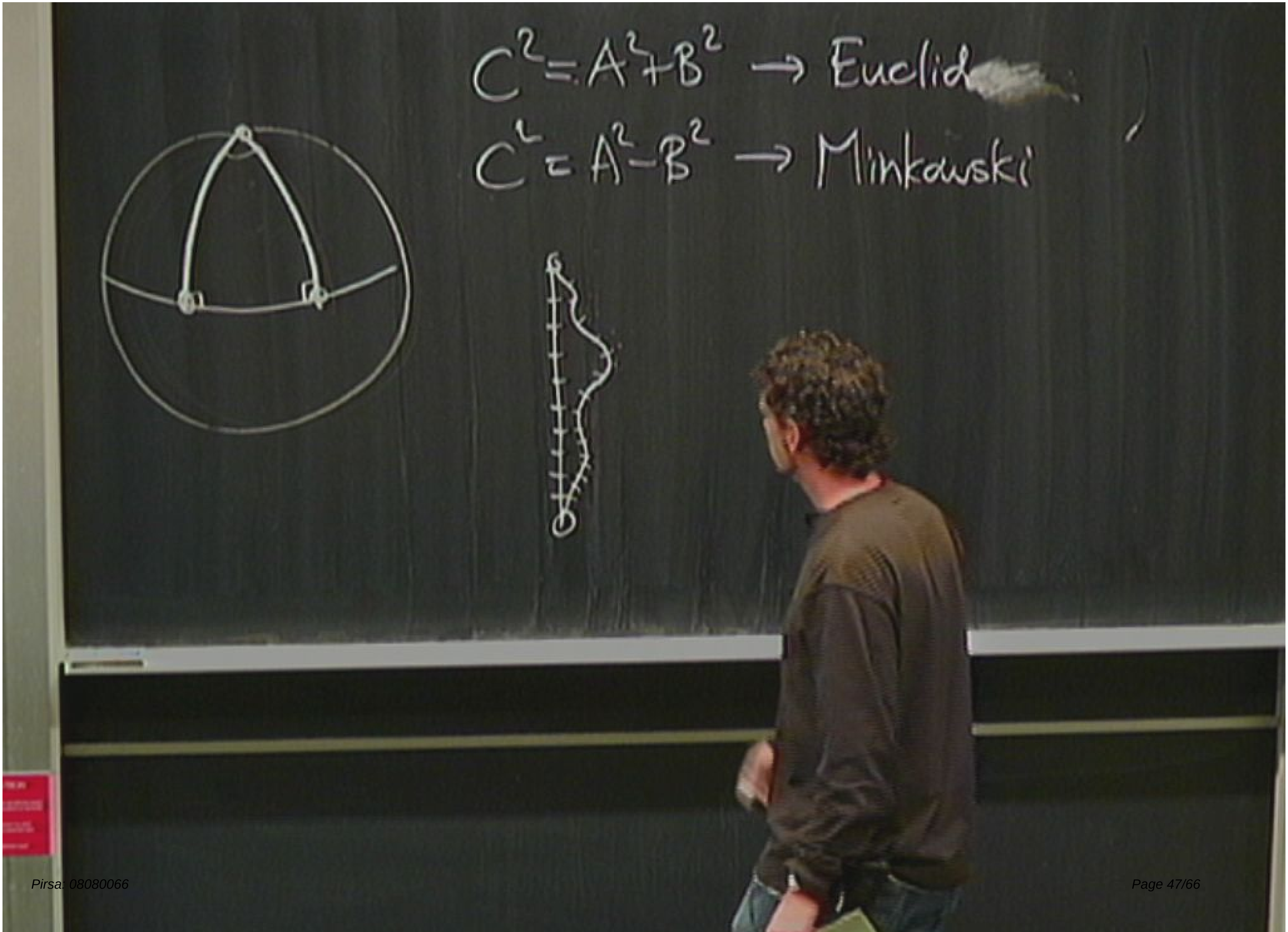
x_A (m)

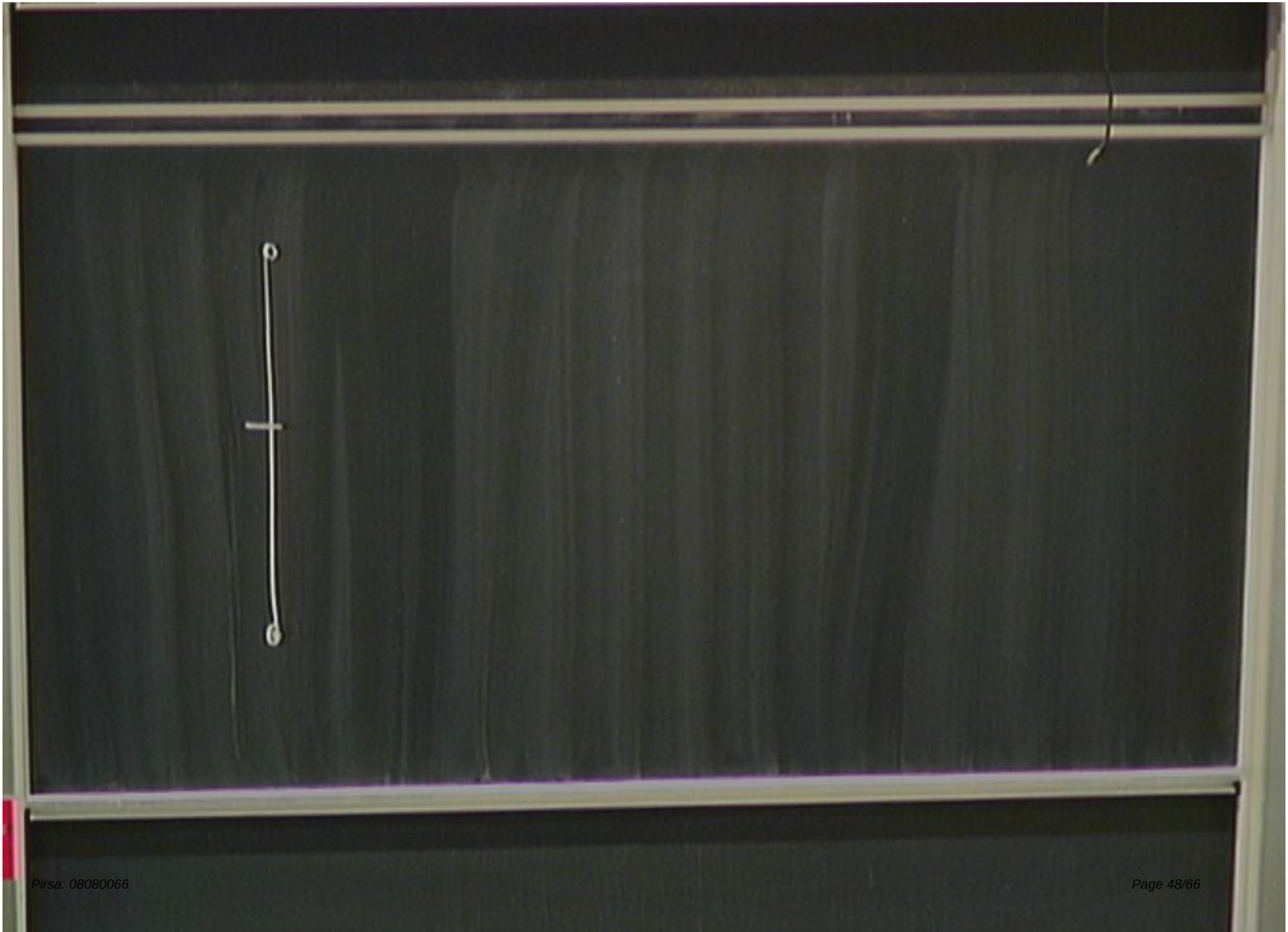

e.g. (Eucl.) α, β = points in space
β
4
3
4
α
y (m)
x (m)
√(4²+3²) = 5
√(4²+3²) = 5
5+5 = 10 > 8

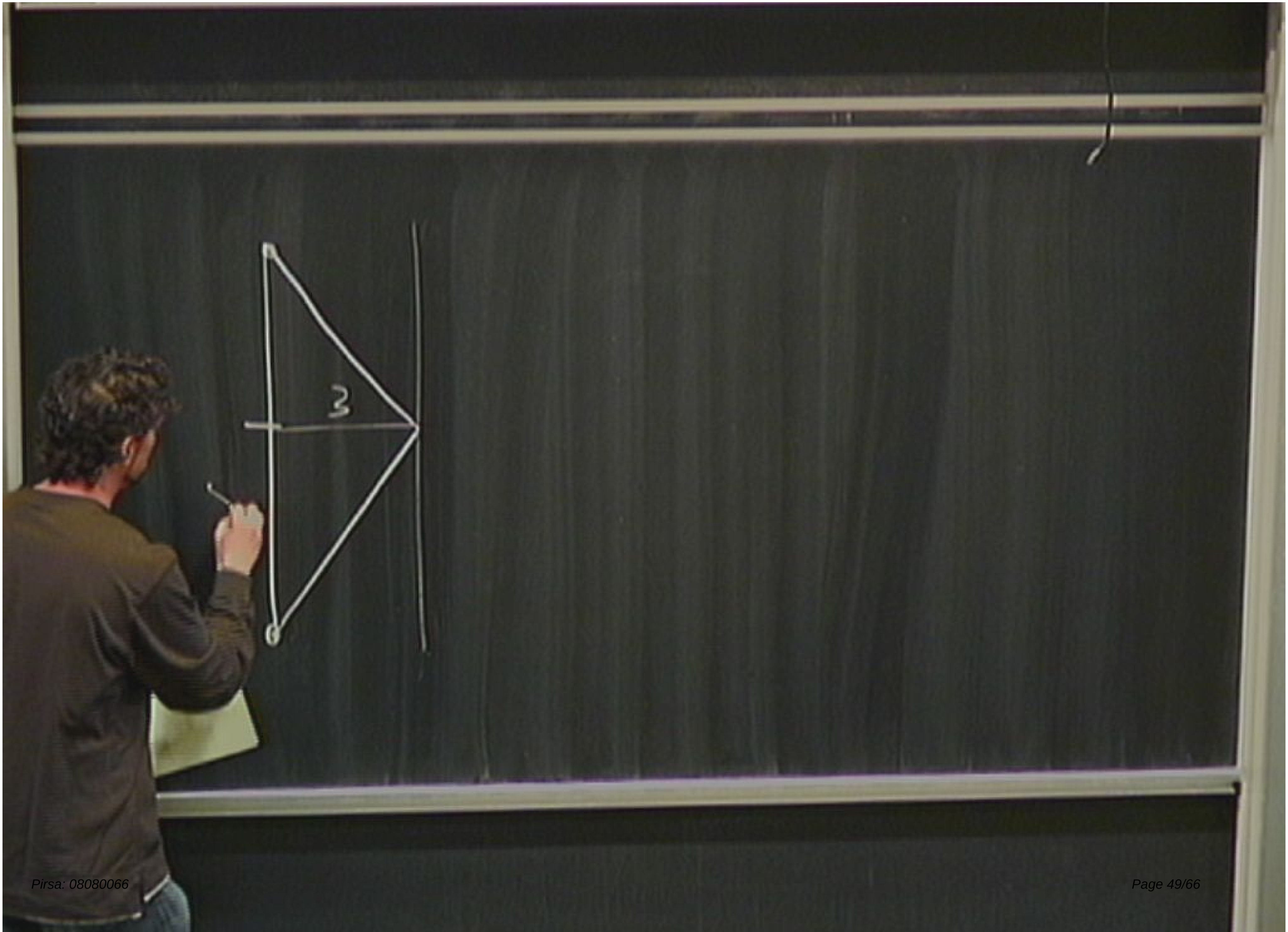
e.g. (Mink.) α, β = points in 2D spacetime

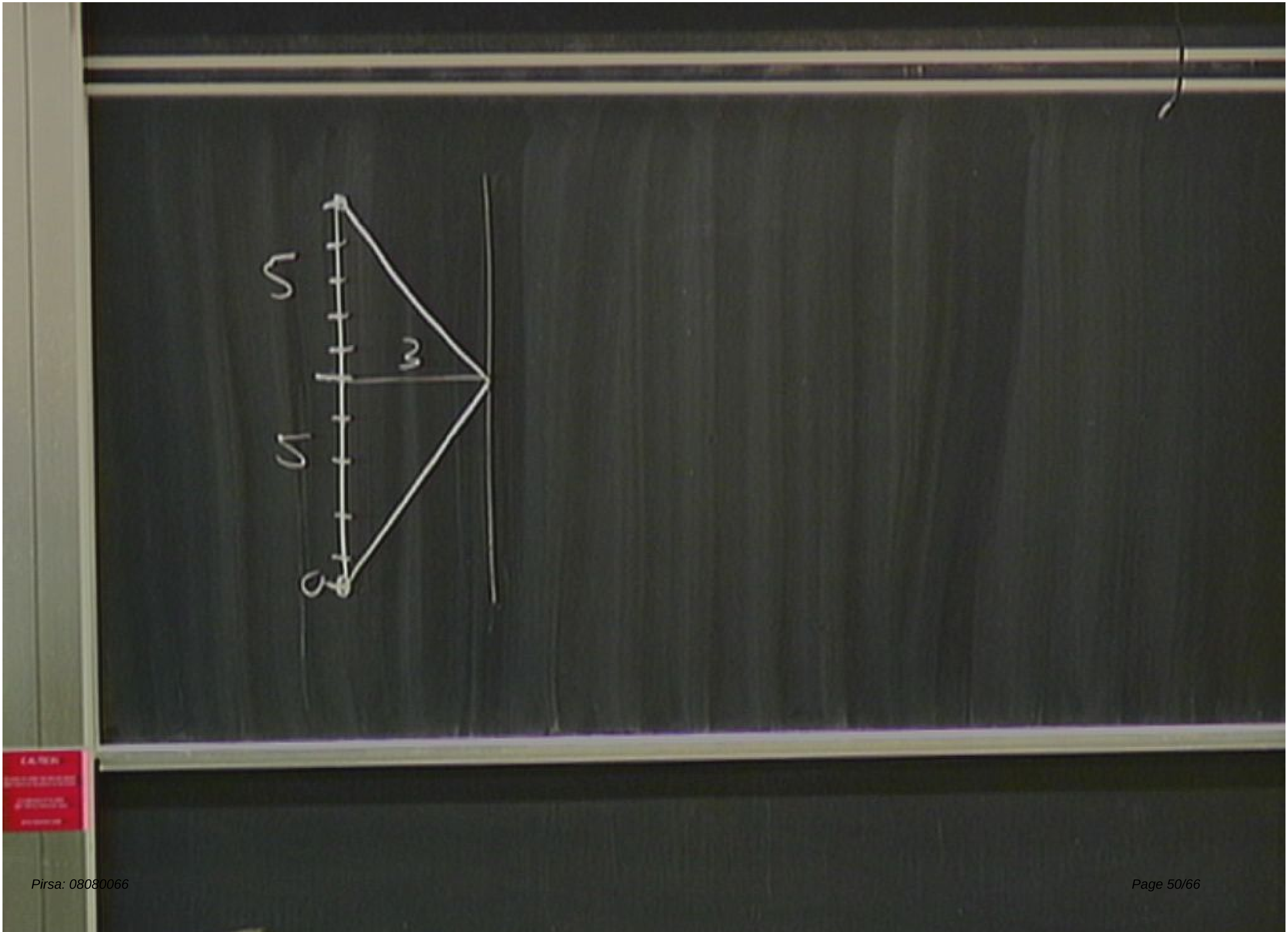


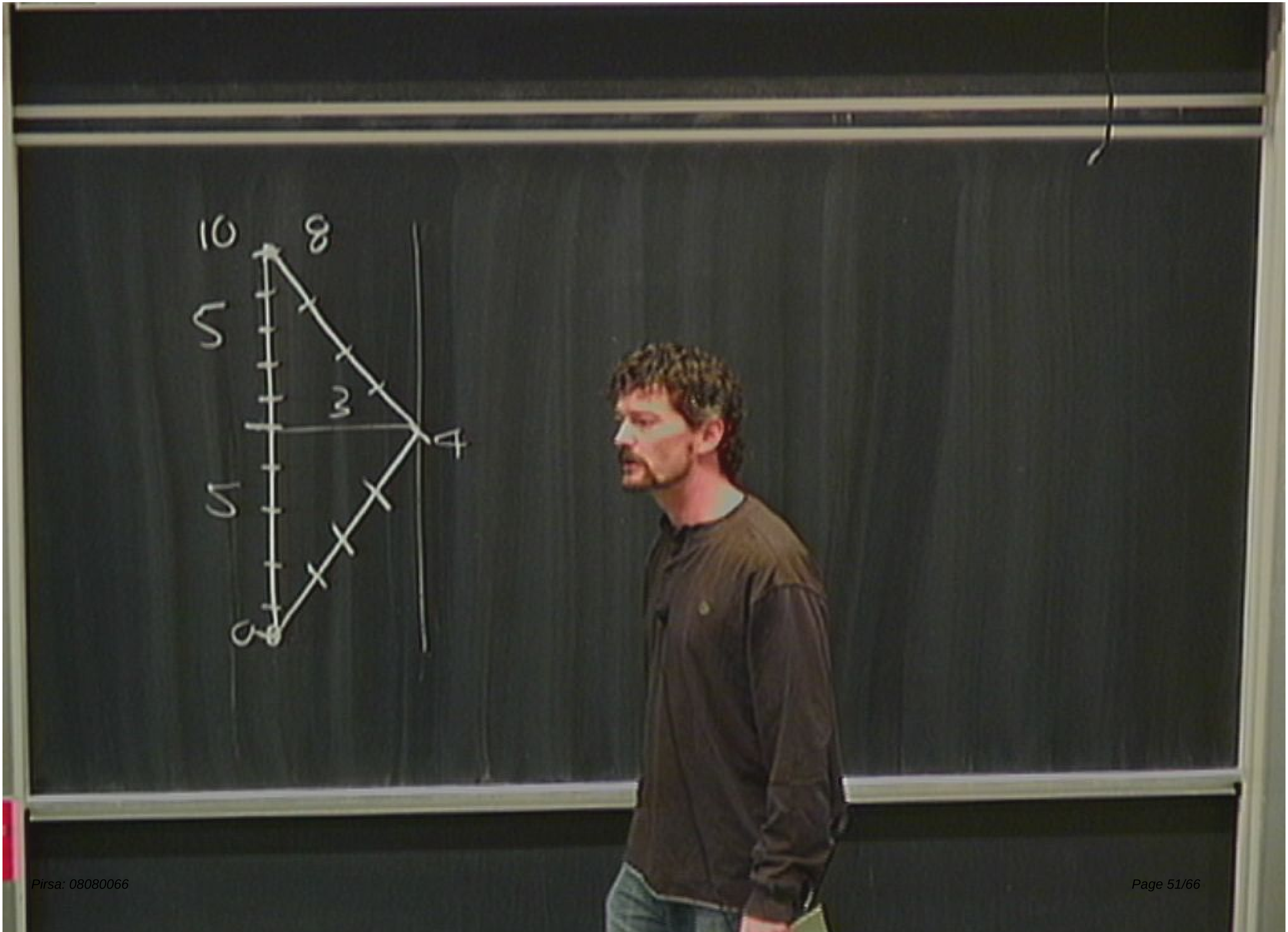

e.g. (Mink.) α, β = points in 2D spacetime
β (Earth, then)
5
3
= √(5²−3²) = 4
5
= 4
= 8 < 10
ct₁ (m)
α (Earth, now)
x₁ (m)
a chronometer is a
spacetime odometer.

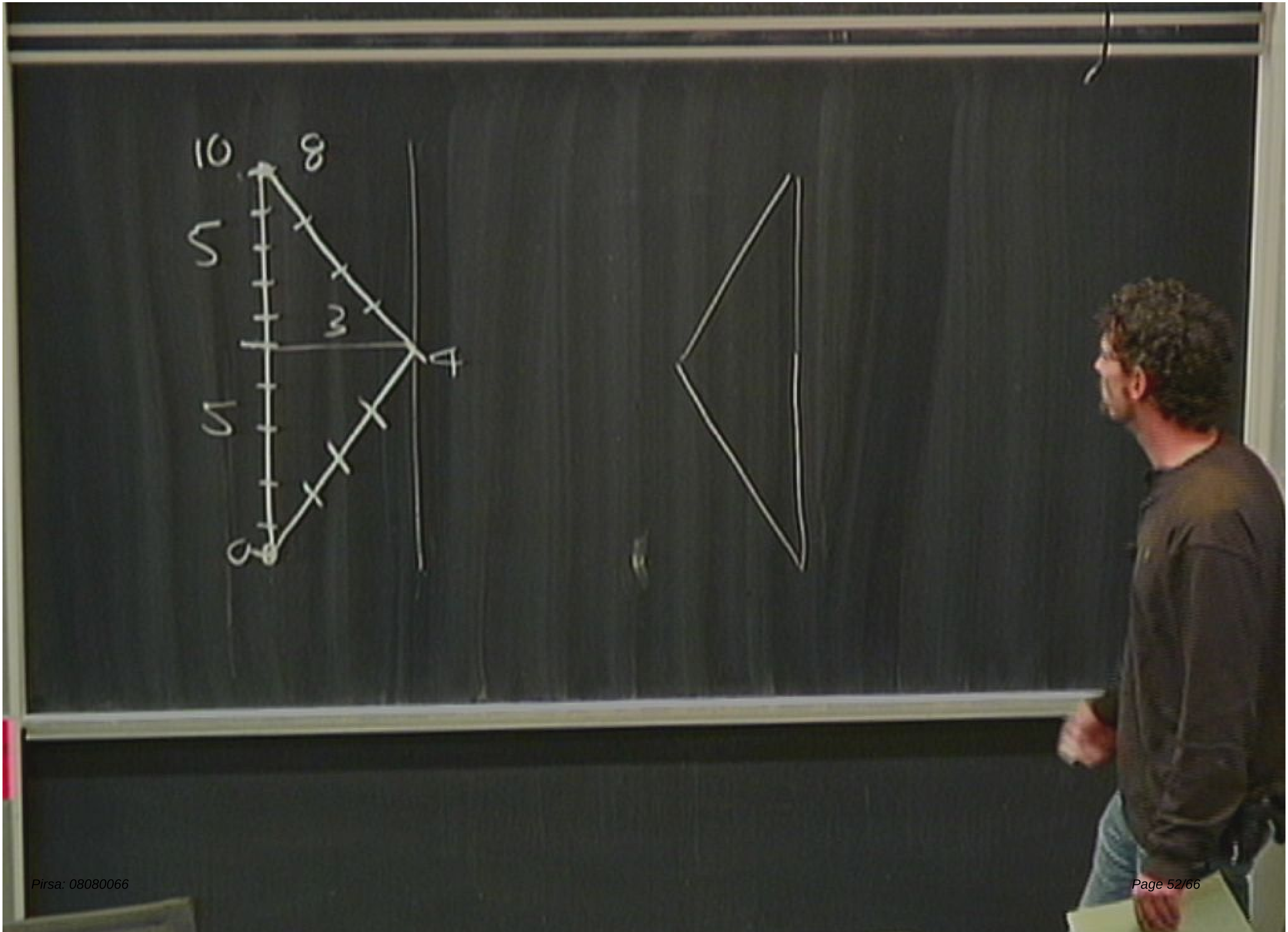

$C^2 = A^2 + B^2 \rightarrow$ Euclid
$C^2 = A^2 - B^2 \rightarrow$ Minkowski




3


5
3
5
0


10
8
5
3
4
5
0


10
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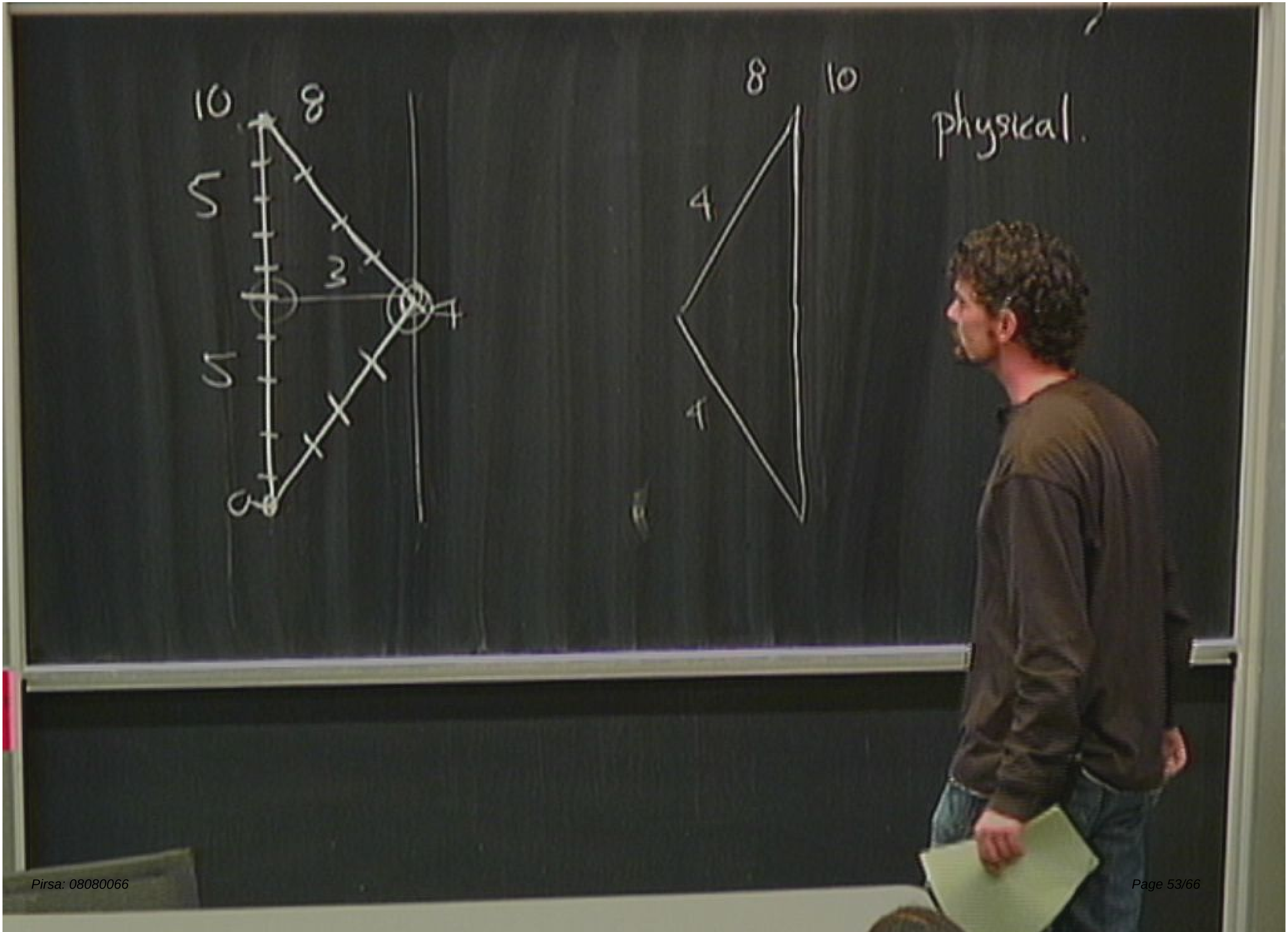
10
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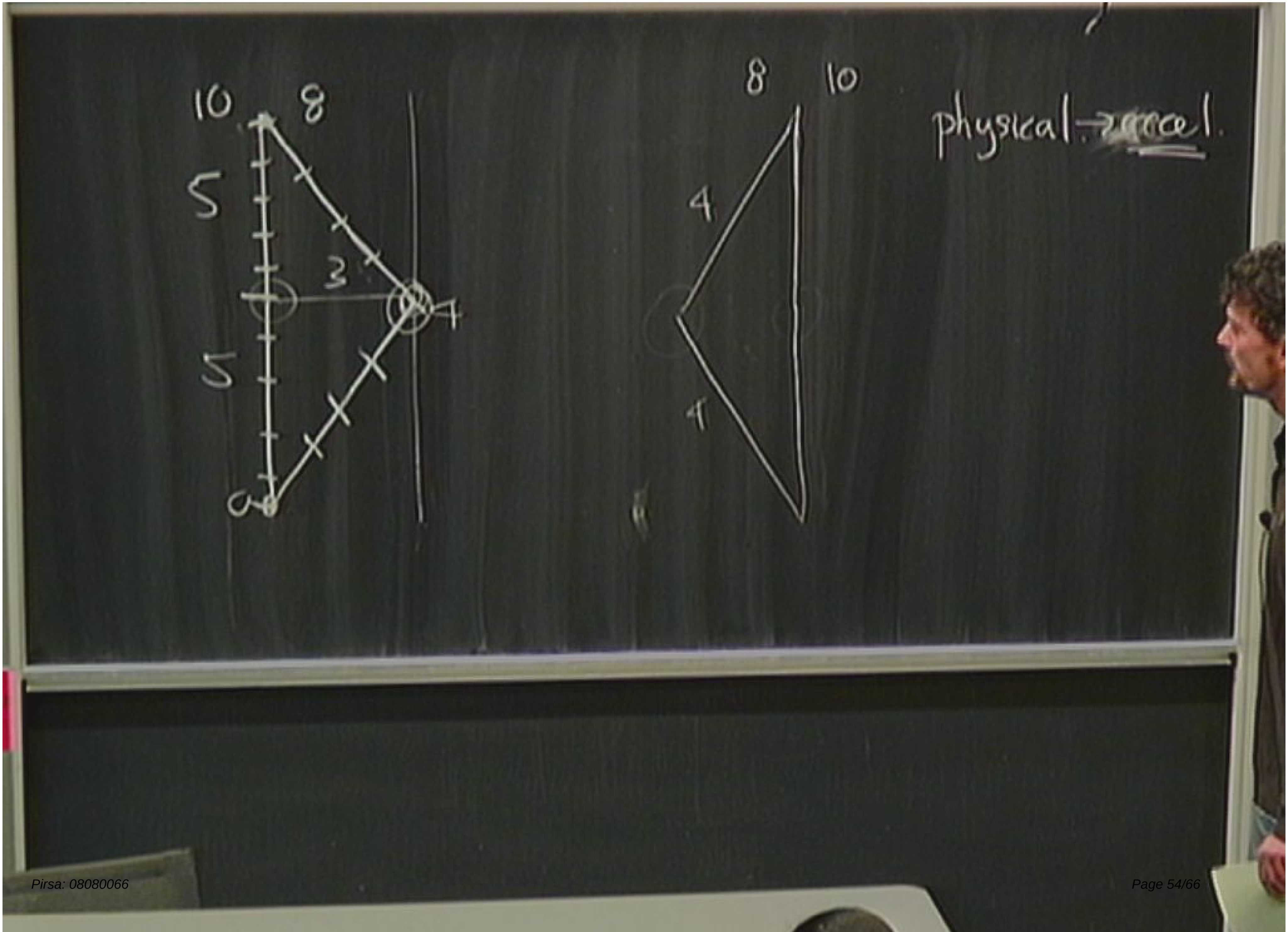
physical.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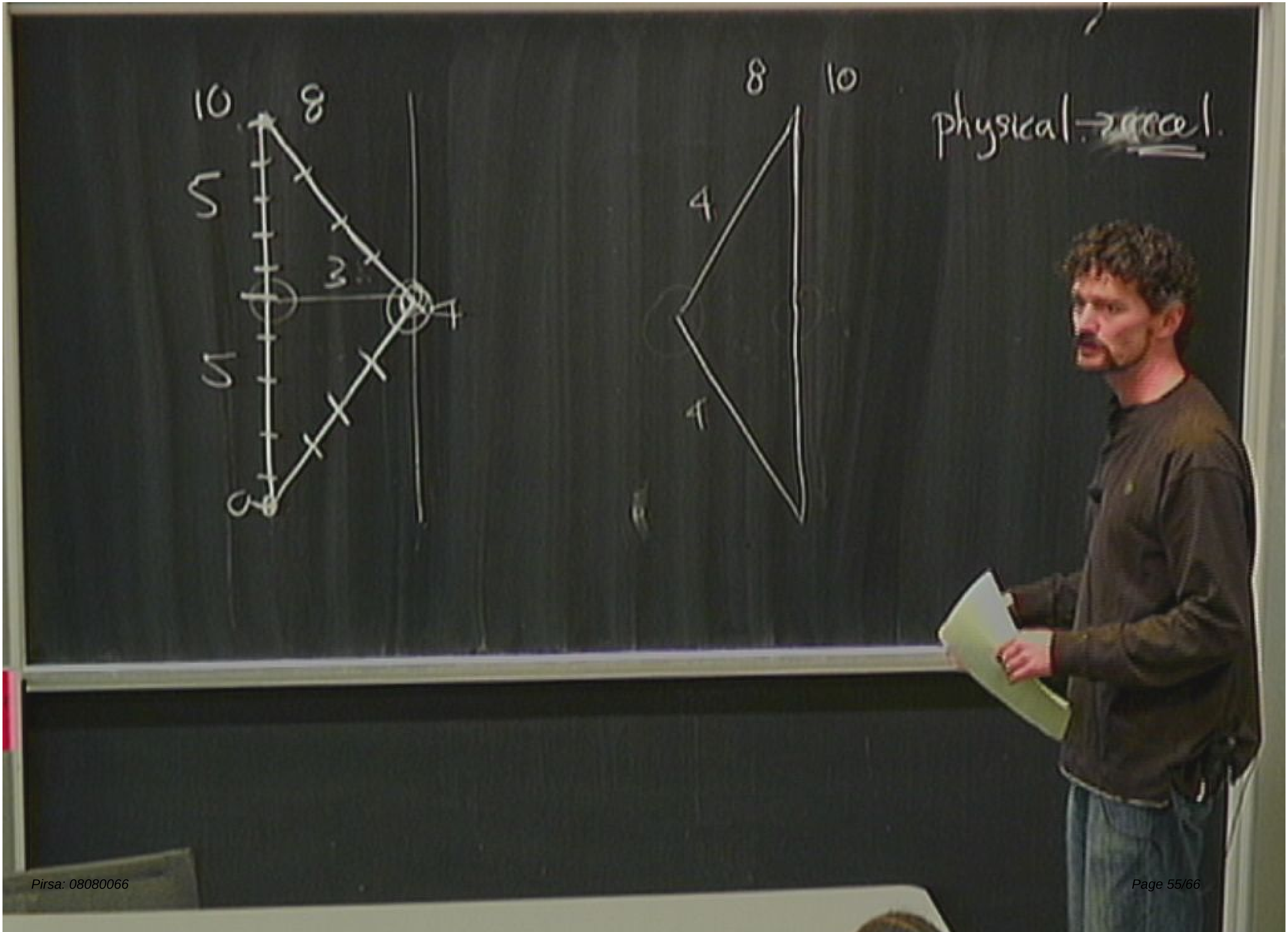
4
physical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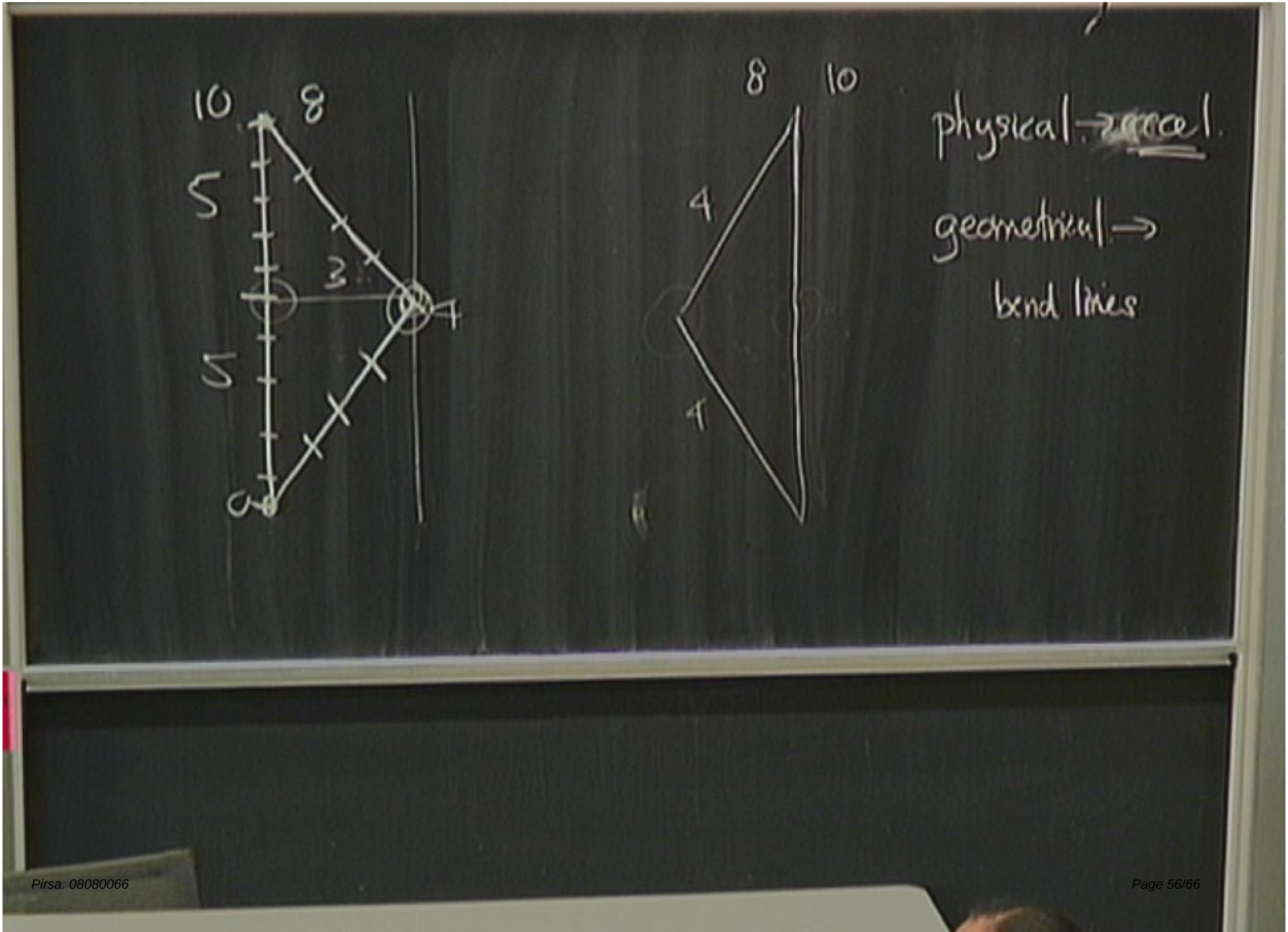
physical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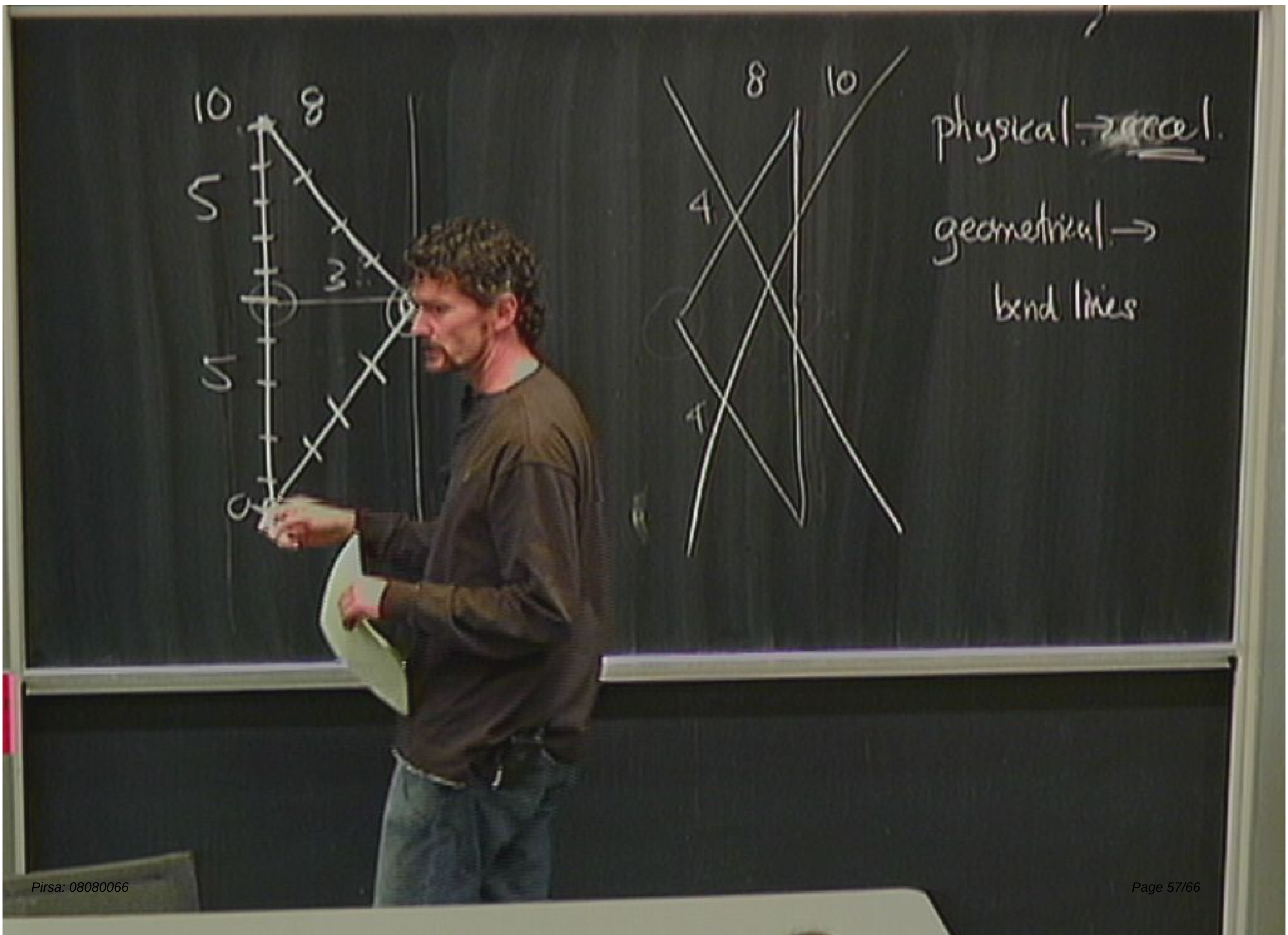
physical
geometrical →
bend lines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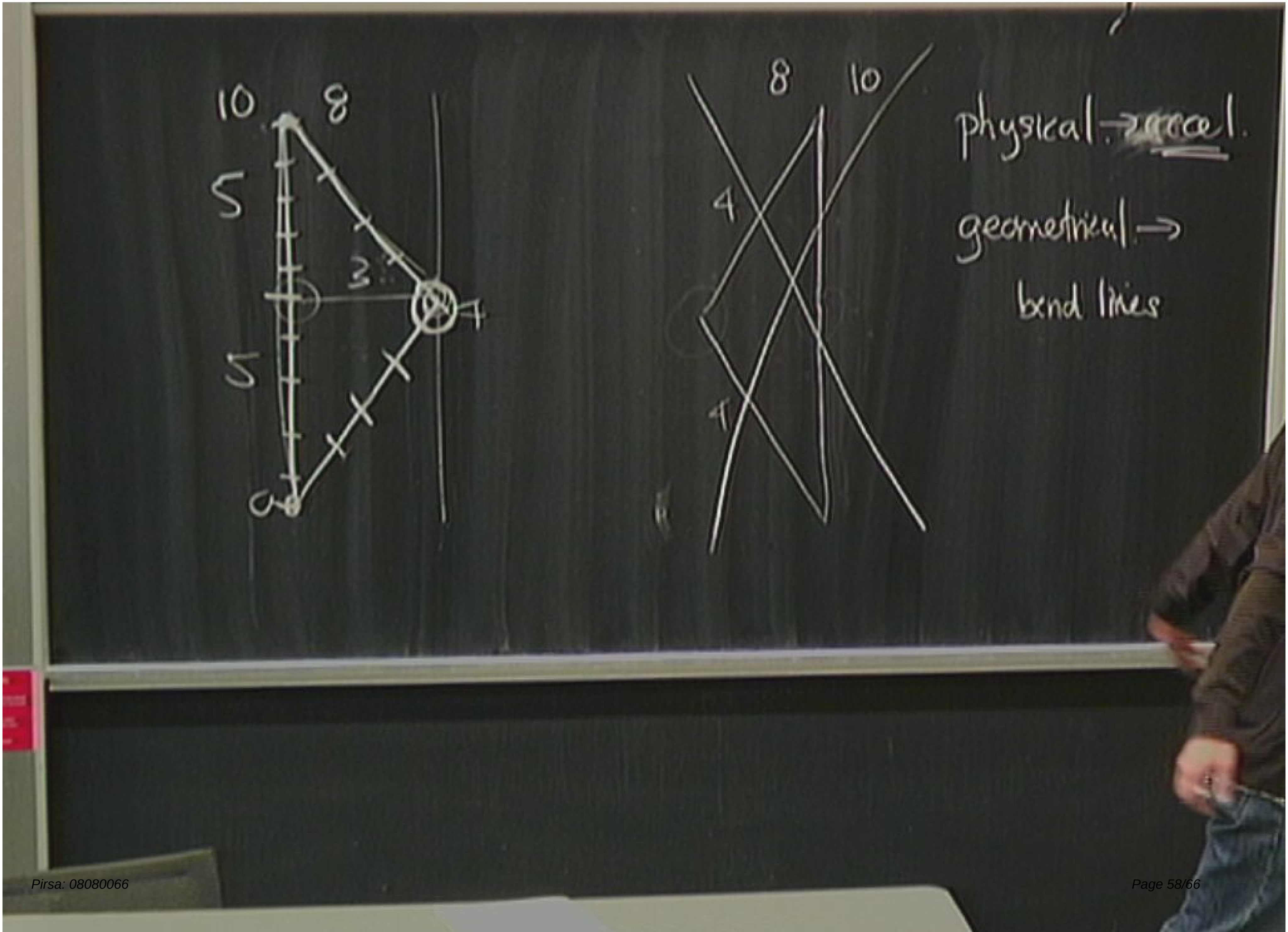
physical → real
geometrical →
bend lines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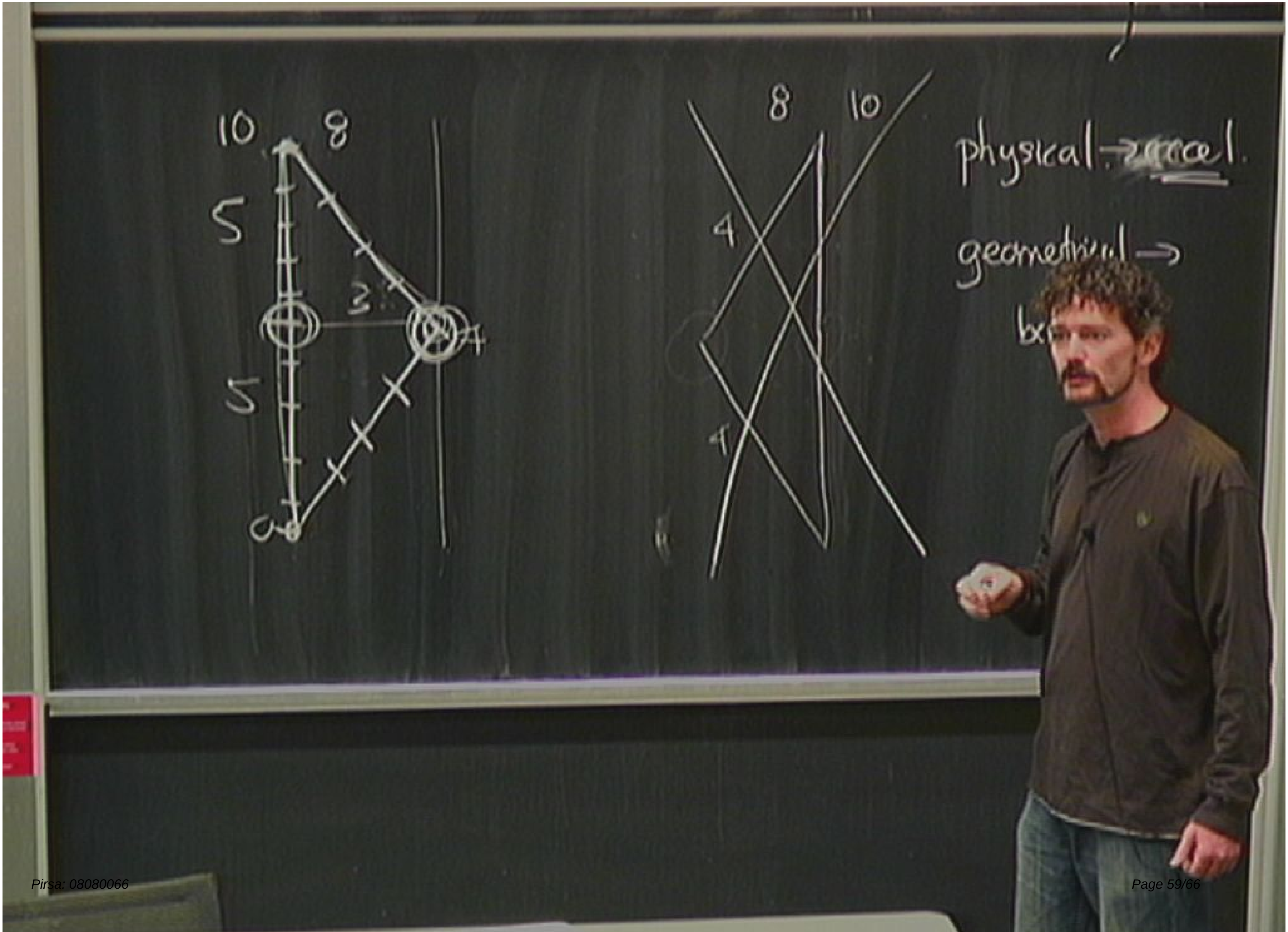
physical →
geometrical →
bend lines



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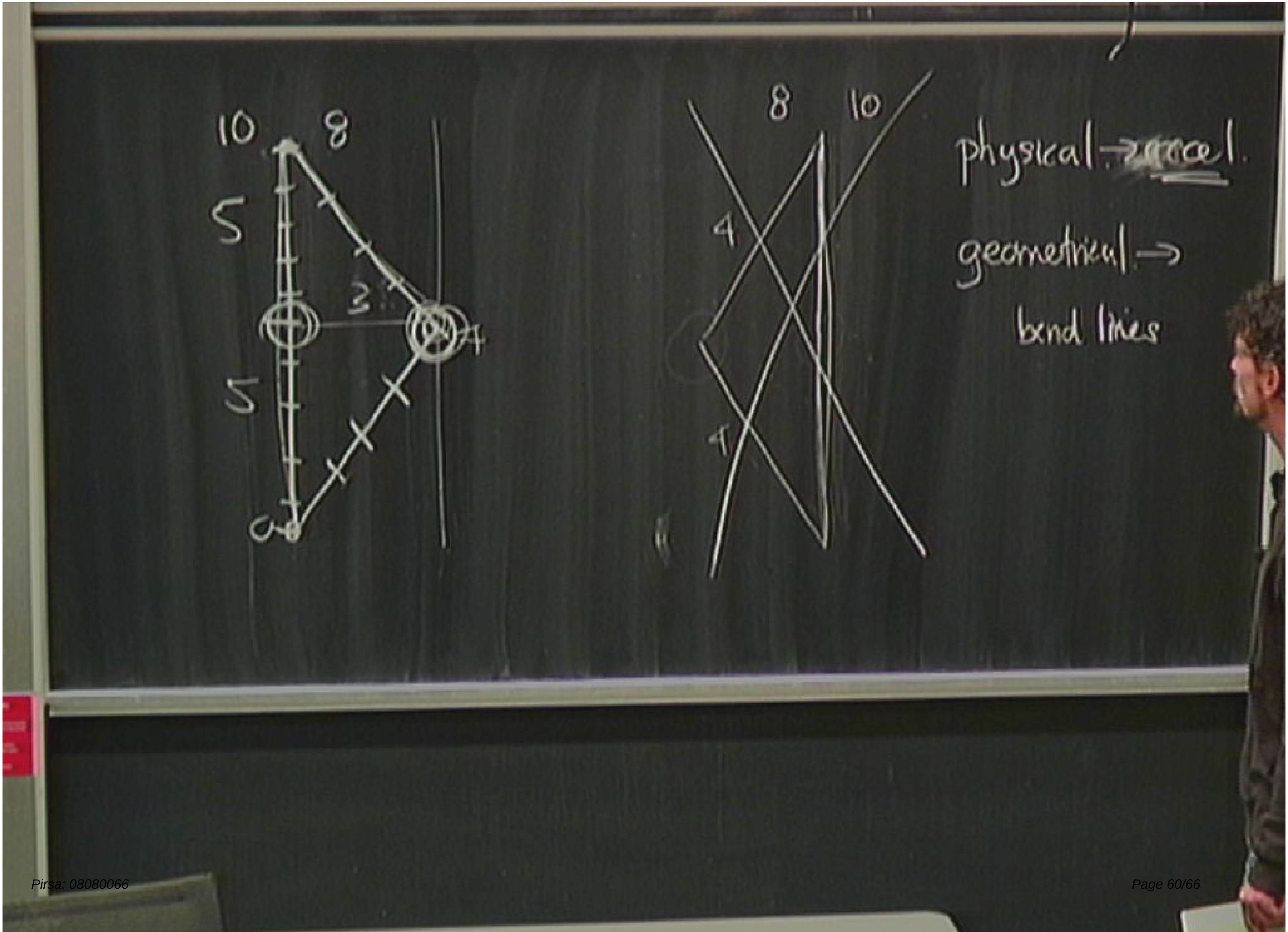
physical → real.
geometrical →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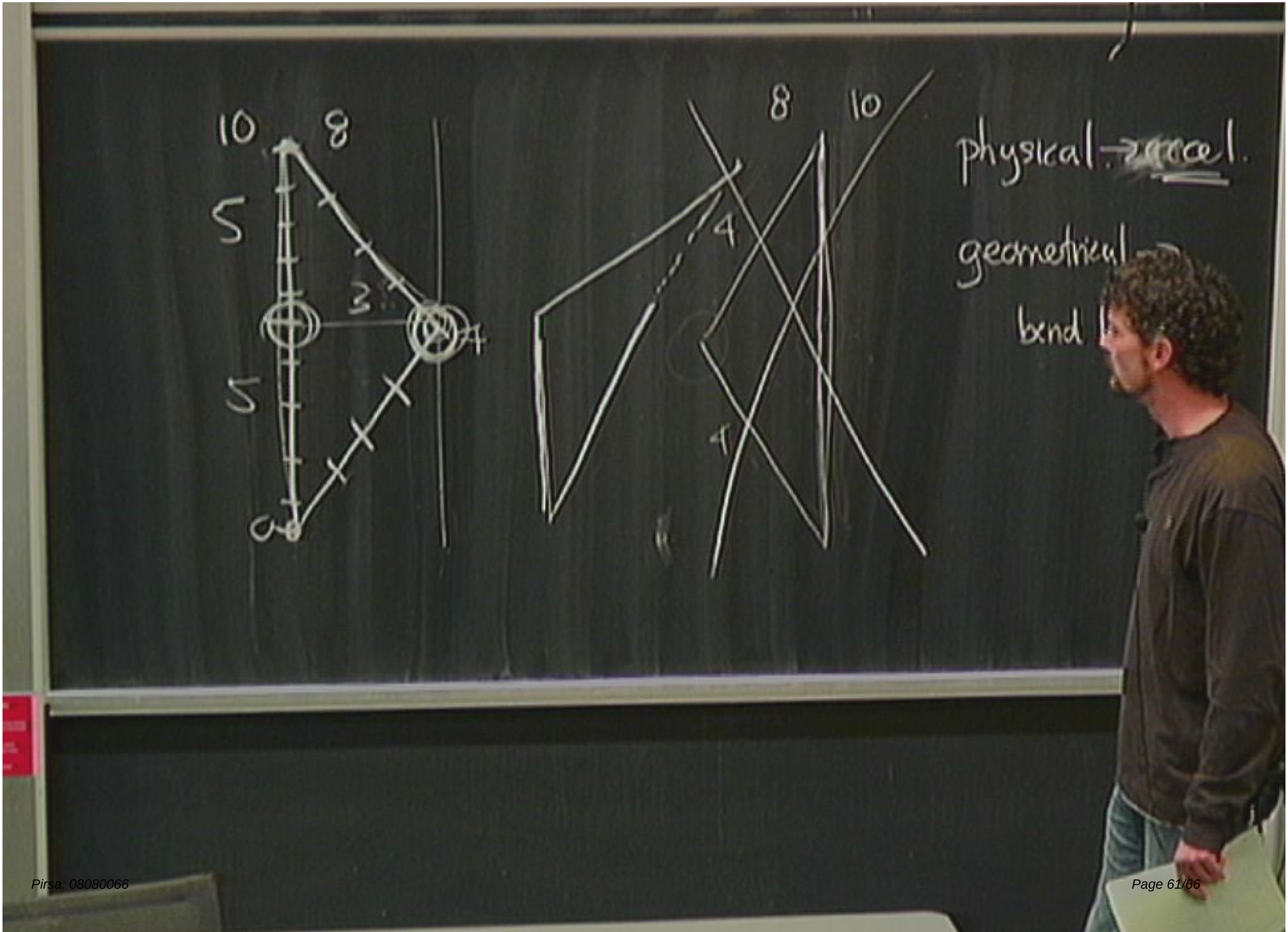
physical →
geometrical →
bend lines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8
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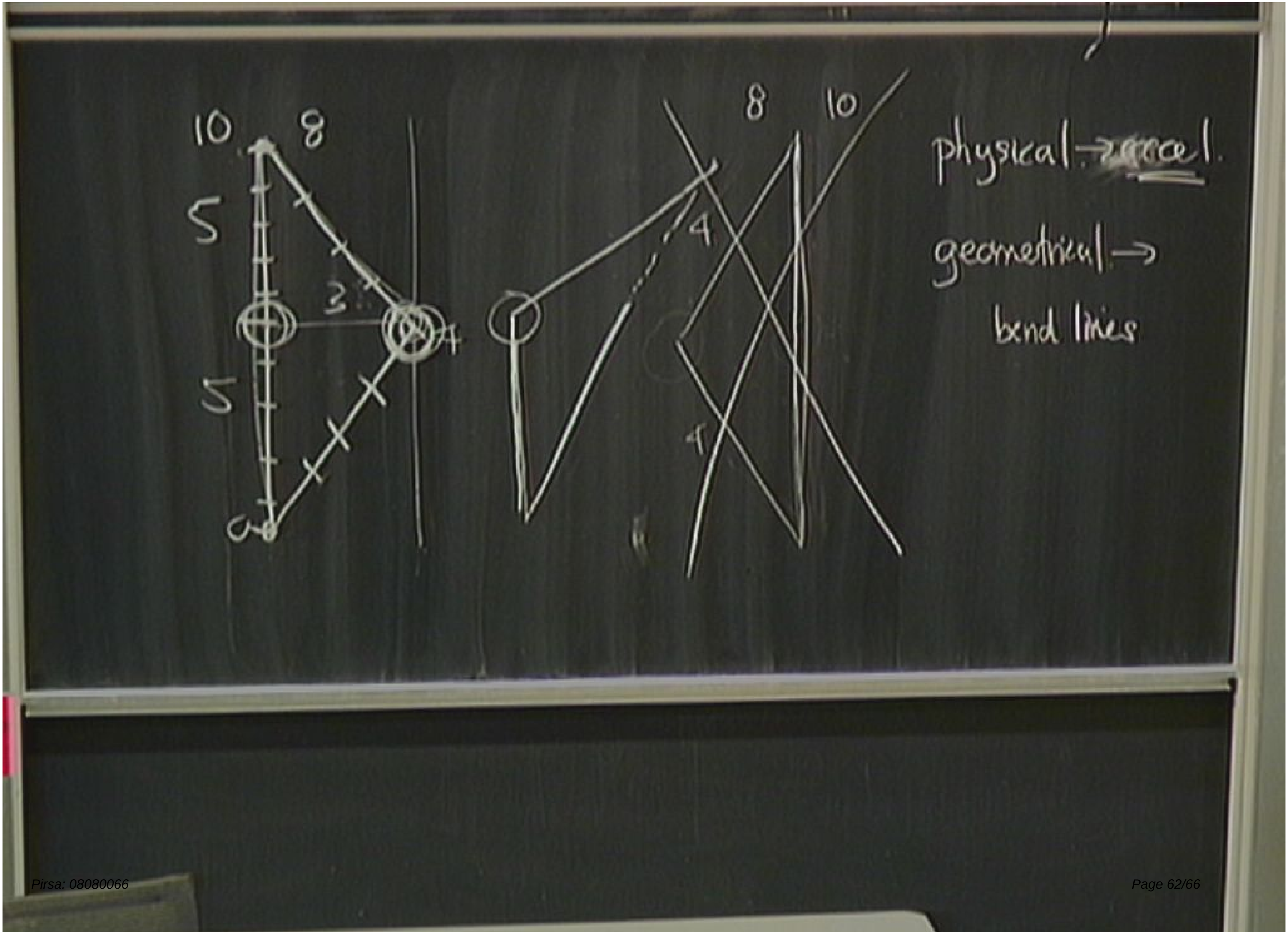
8
10
physical
geometrical
bend


10
8
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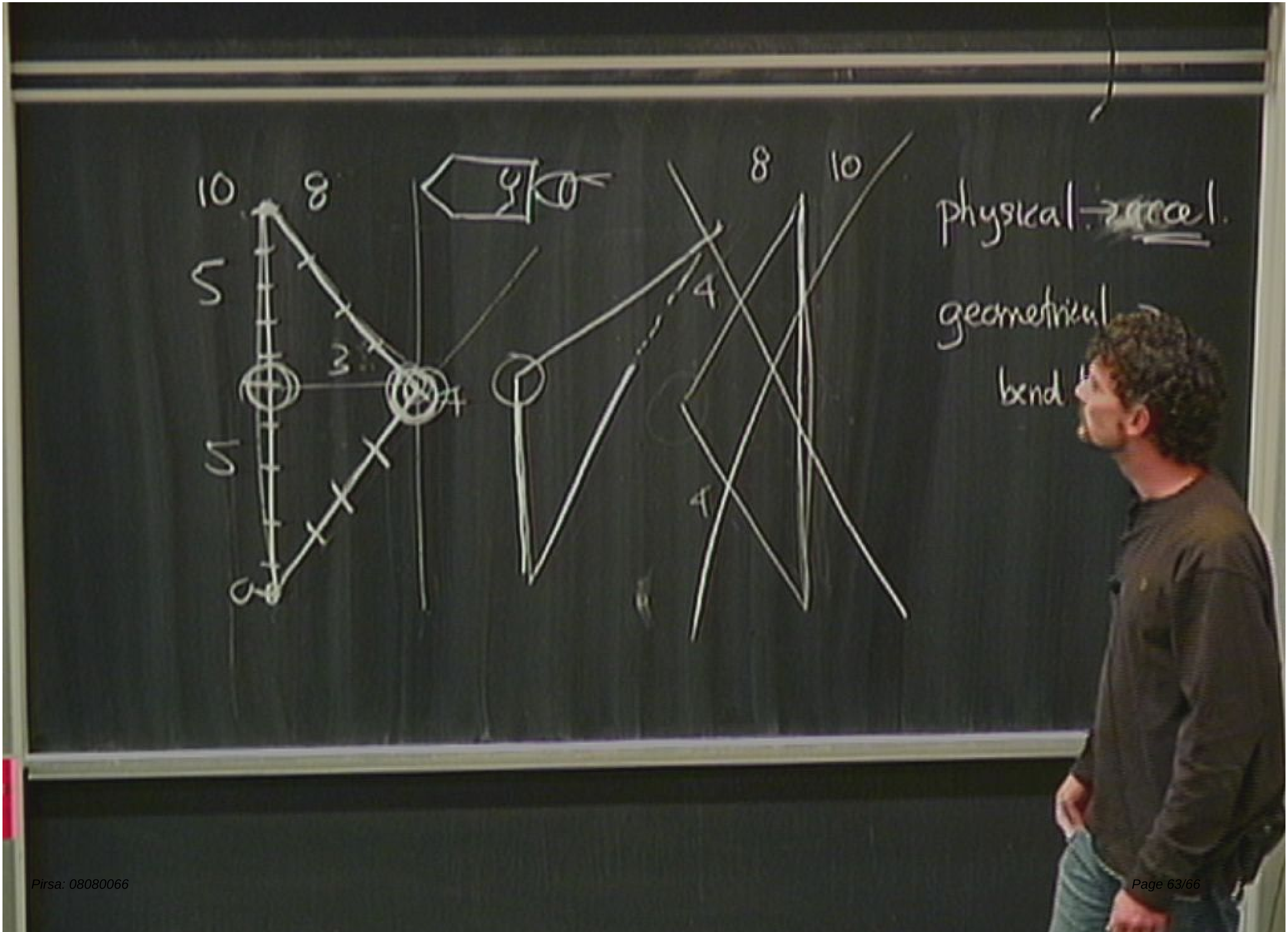
10
4
physical → real
geometrical →
bend lines


10
8
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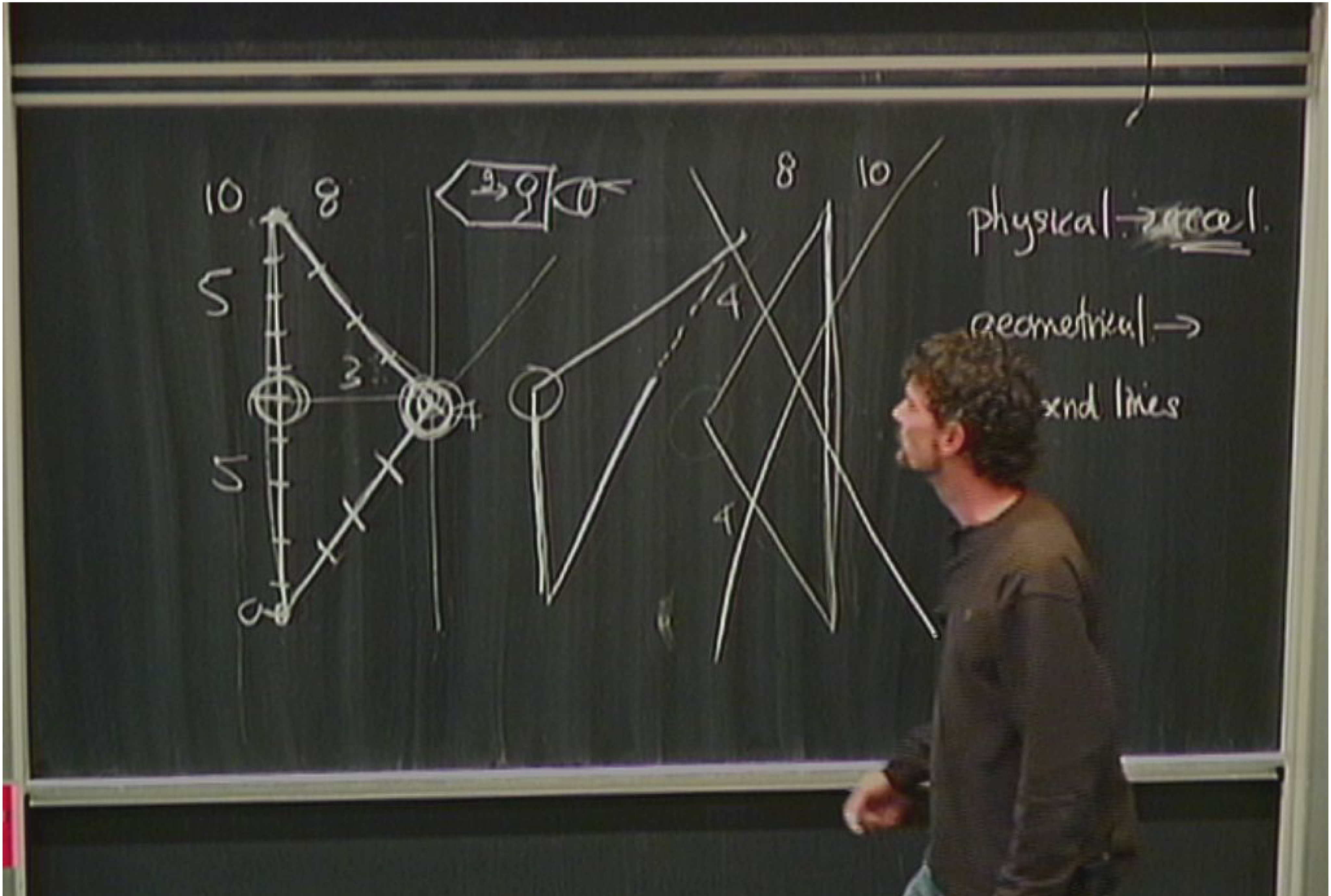
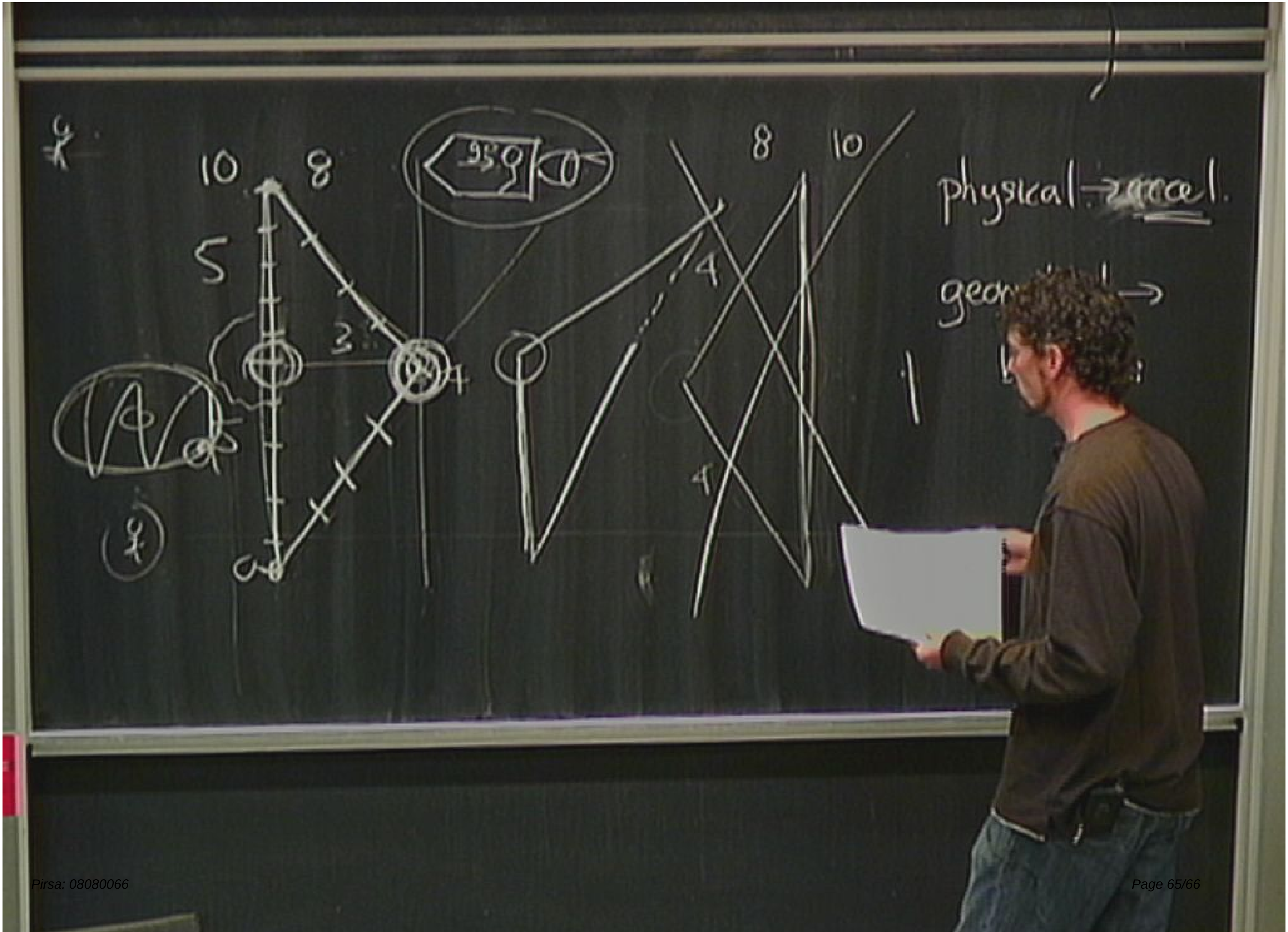
4
physical
geometrical
bend


physical →
geometrical →
xnd lines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8
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3
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8
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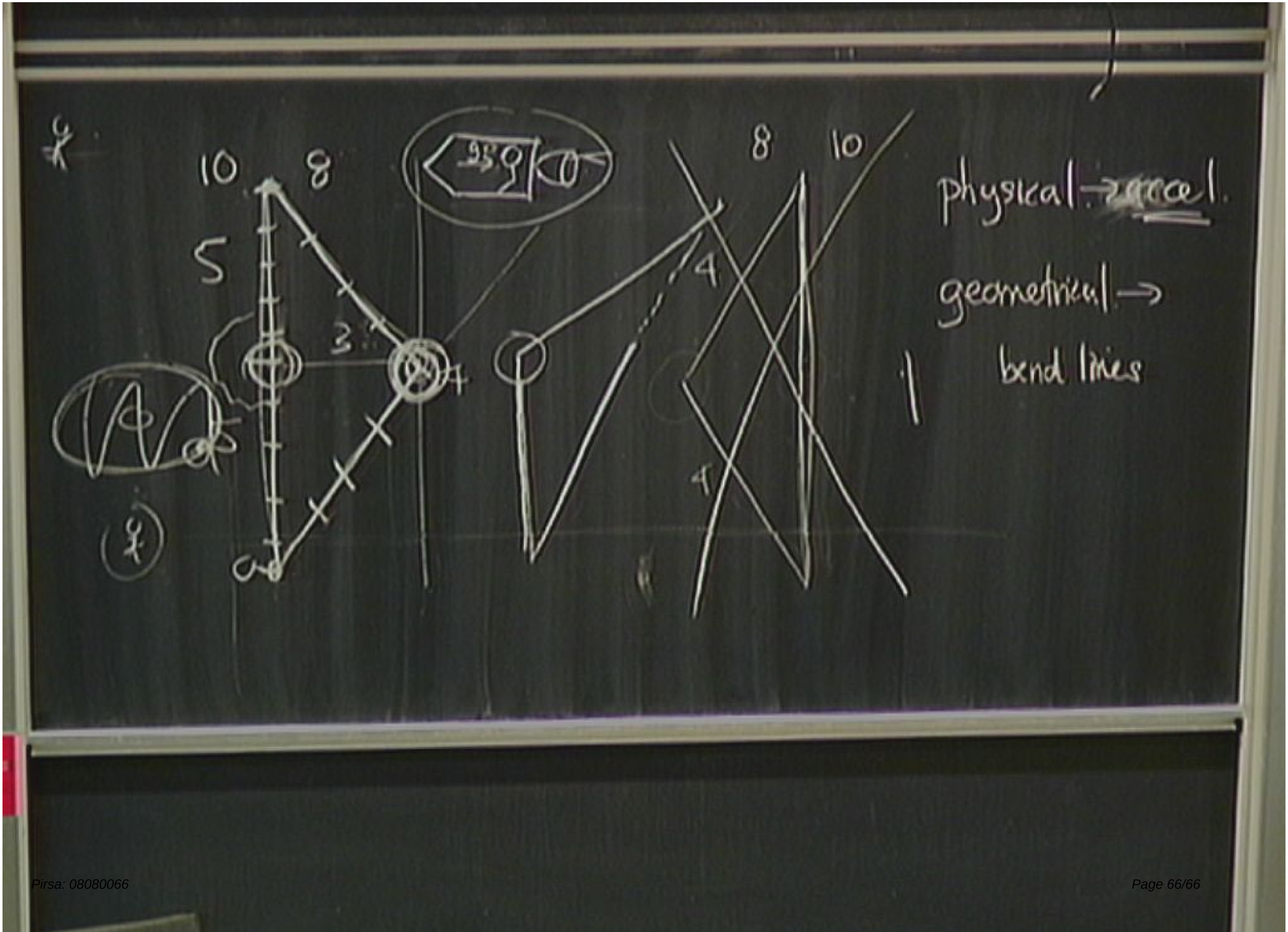
physical
geom


10
8
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3
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8
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physical → accel
geometrical →
bend lines