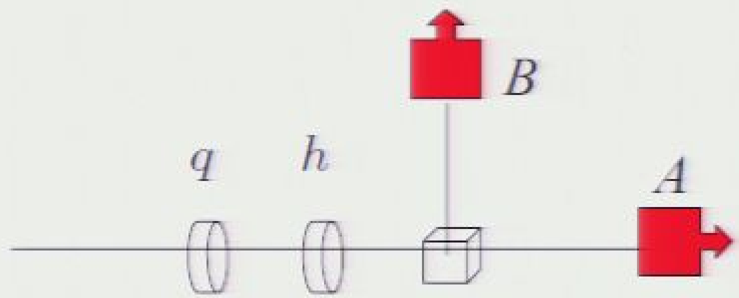
Title: News Flash: adapting compressed sensing to quantum systems

Date: Aug 29, 2008 11:45 AM

URL: http://pirsa.org/08080057

Abstract:

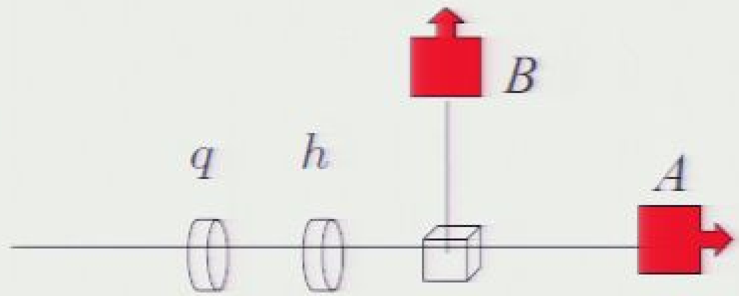
example



- input configurations $\rho \in \left\{ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\}$
- wave plate configurations (degrees) $h = 0, \quad q = \begin{cases} 5,\ 30,\ 60 \\ 5,\ 30 \\ 5 \end{cases}$
- basis for $\mathbf{C}^{2\times 2}$: $\left\{ \frac{1}{\sqrt{2}} I_2,\ \frac{1}{\sqrt{2}} X,\ \frac{1}{\sqrt{2}} Y,\ \frac{1}{\sqrt{2}} Z \right\} \Longrightarrow X_{\text{true}} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

  (sparsity pattern is not known)
- here there are 2 outcomes and at most 6 configurations; standard QPT requires $2^4 - 2^2 = 12$ parameters

example



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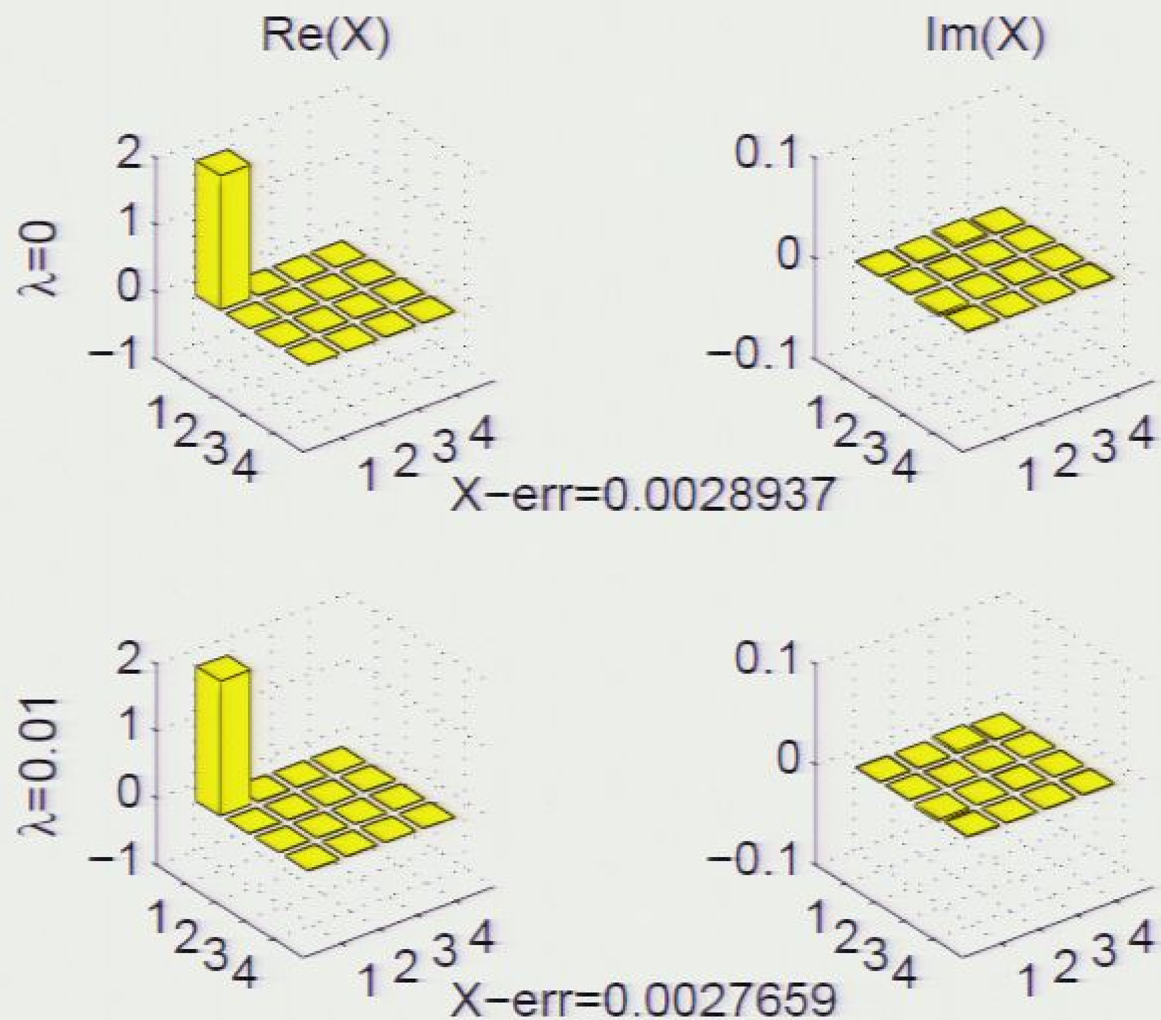
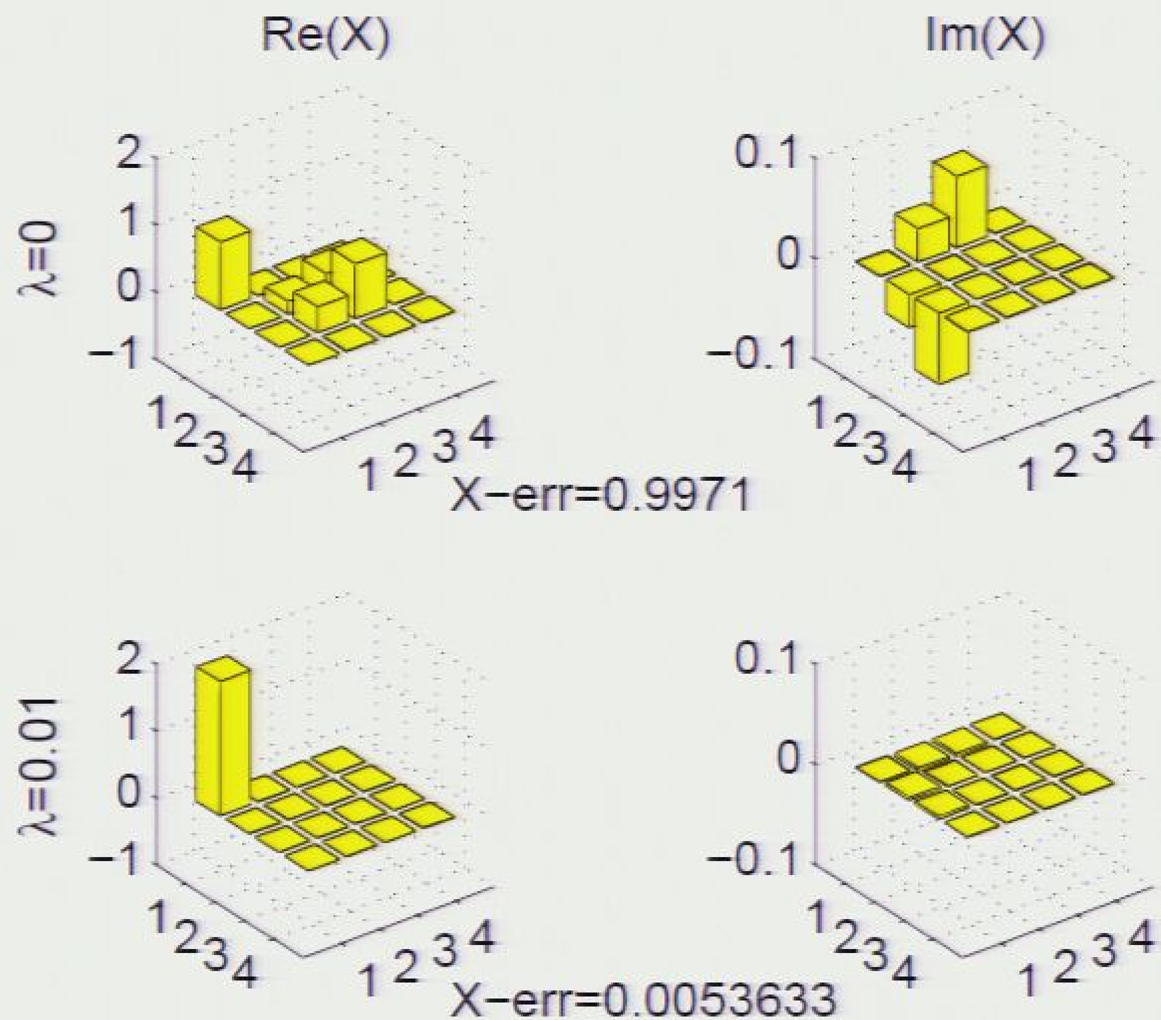
$q = 5,\ 30,\ 60$



Re(X)
Im(X)
λ=0
2
1
0
−1
1 2 3 4
1 2 3 4
0.1
0
−0.1
X−err=0.0028937
λ=0.01
X−err=0.0027659

$q = 5,\ 30$



Re(X)
Im(X)
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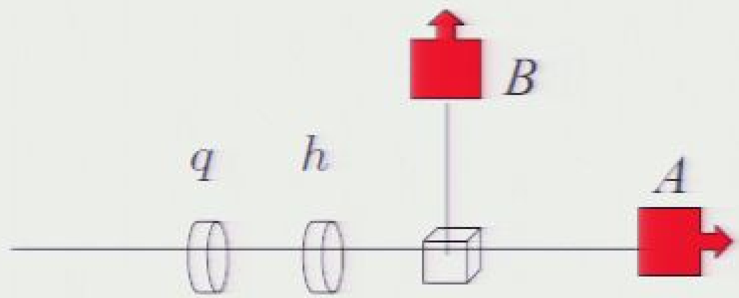
example



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sparse process matrix tomography

- LS process tomography

$$\text{minimize} \quad \|P(X_{\text{true}}) - P(X)\|_{\text{fro}}$$

$$\text{subject to} \quad \sum_{\alpha,\beta} X_{\alpha\beta} B_\alpha^\dagger B_\beta = I_n, \quad X \geq 0$$

- the **cardinality** of $X \in \mathbf{C}^{n^2 \times n^2}$ is the number of non-zero elements

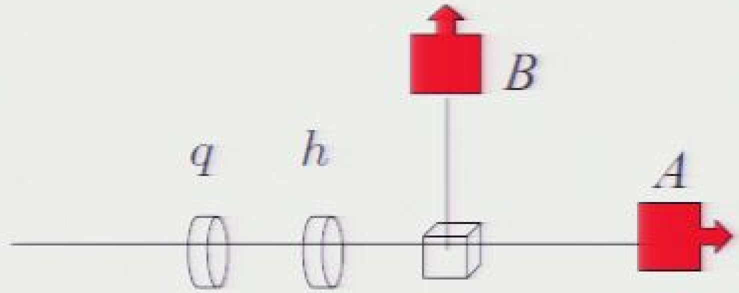
- a heuristic for cardinality is the 1-norm, *i.e.*, $\|\text{vec}(X)\|_1 = \sum_{i,j} |X_{ij}|$

- "compressed sensing" $\Rightarrow$ $N_{\text{data}} \geq C \, \mathbf{card}(X) \log(n^4), \; n = 2^q$

- add this as a penalty to the objective:

$$\text{minimize} \quad \|P(X_{\text{true}} - P(X)\|_{\text{fro}} + \lambda \, \|\text{vec}(X)\|_1$$

example



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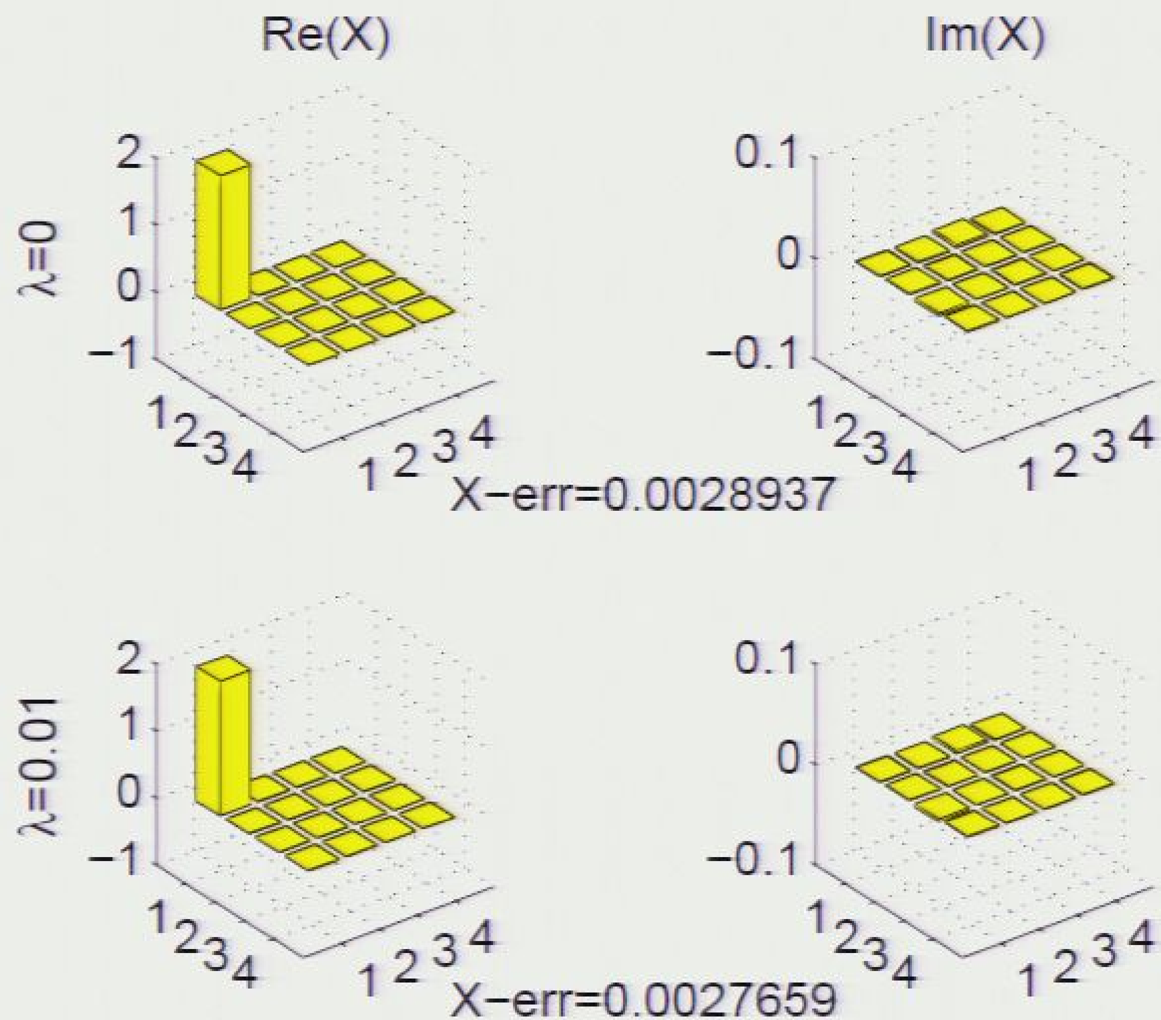

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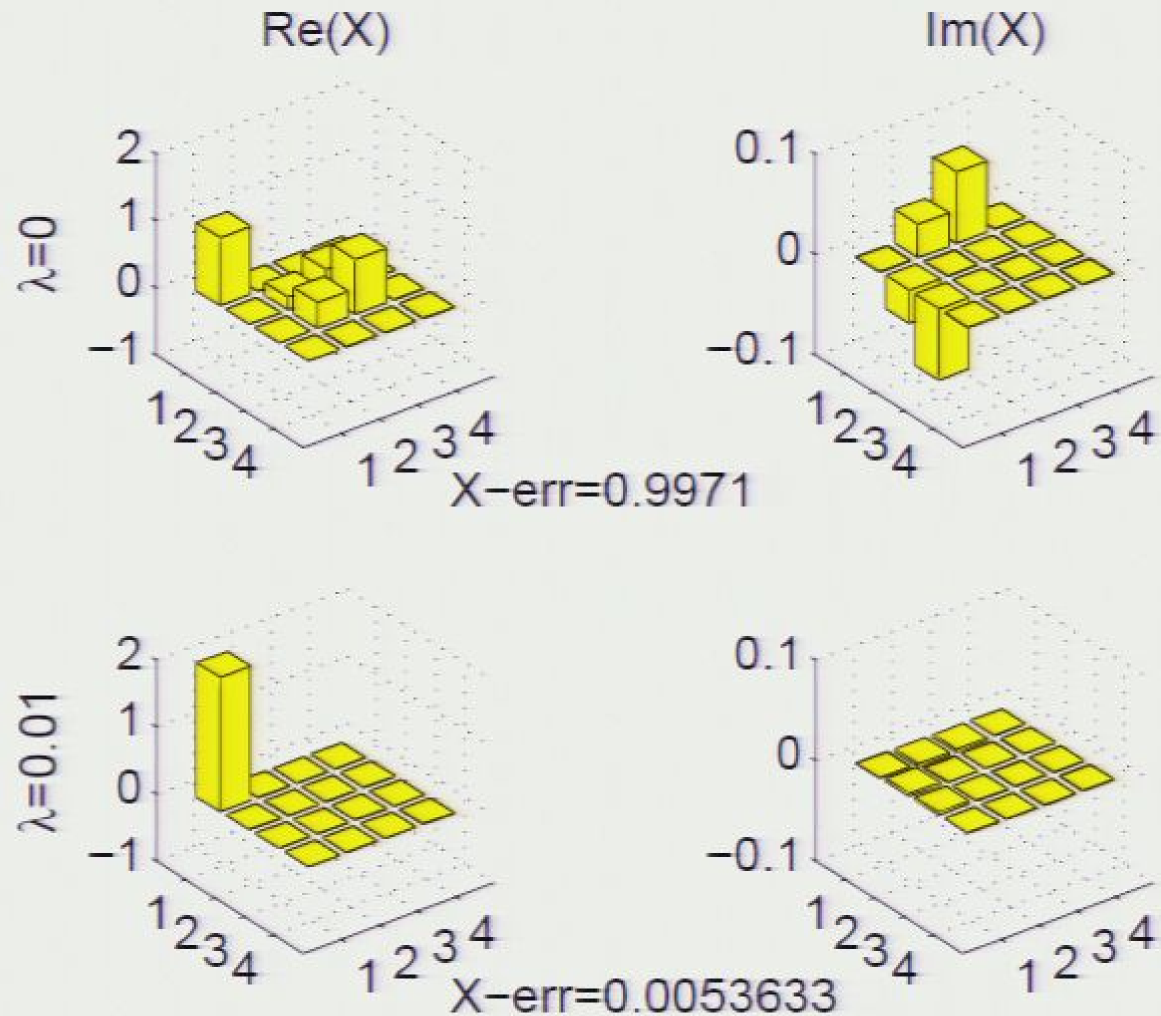

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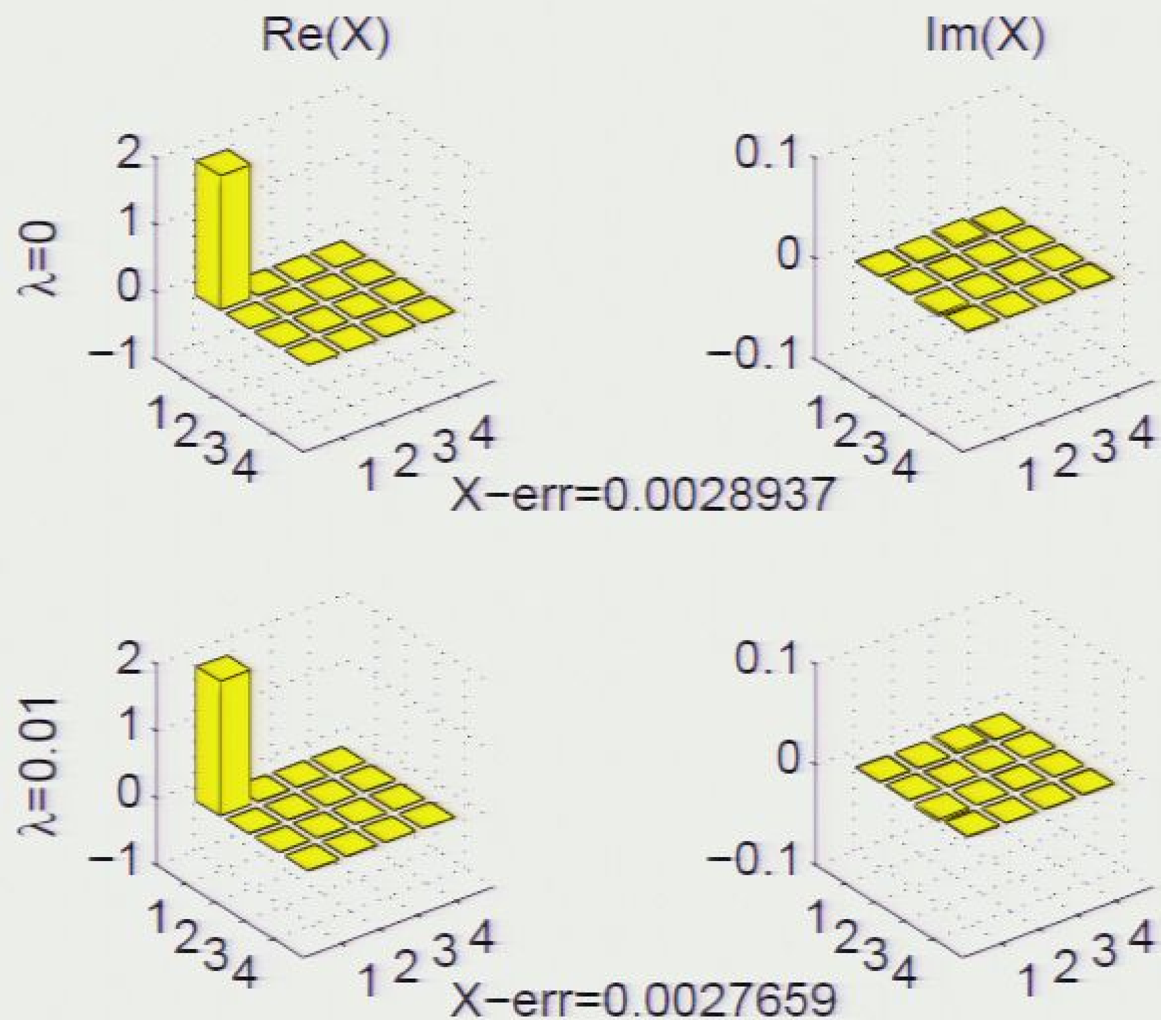
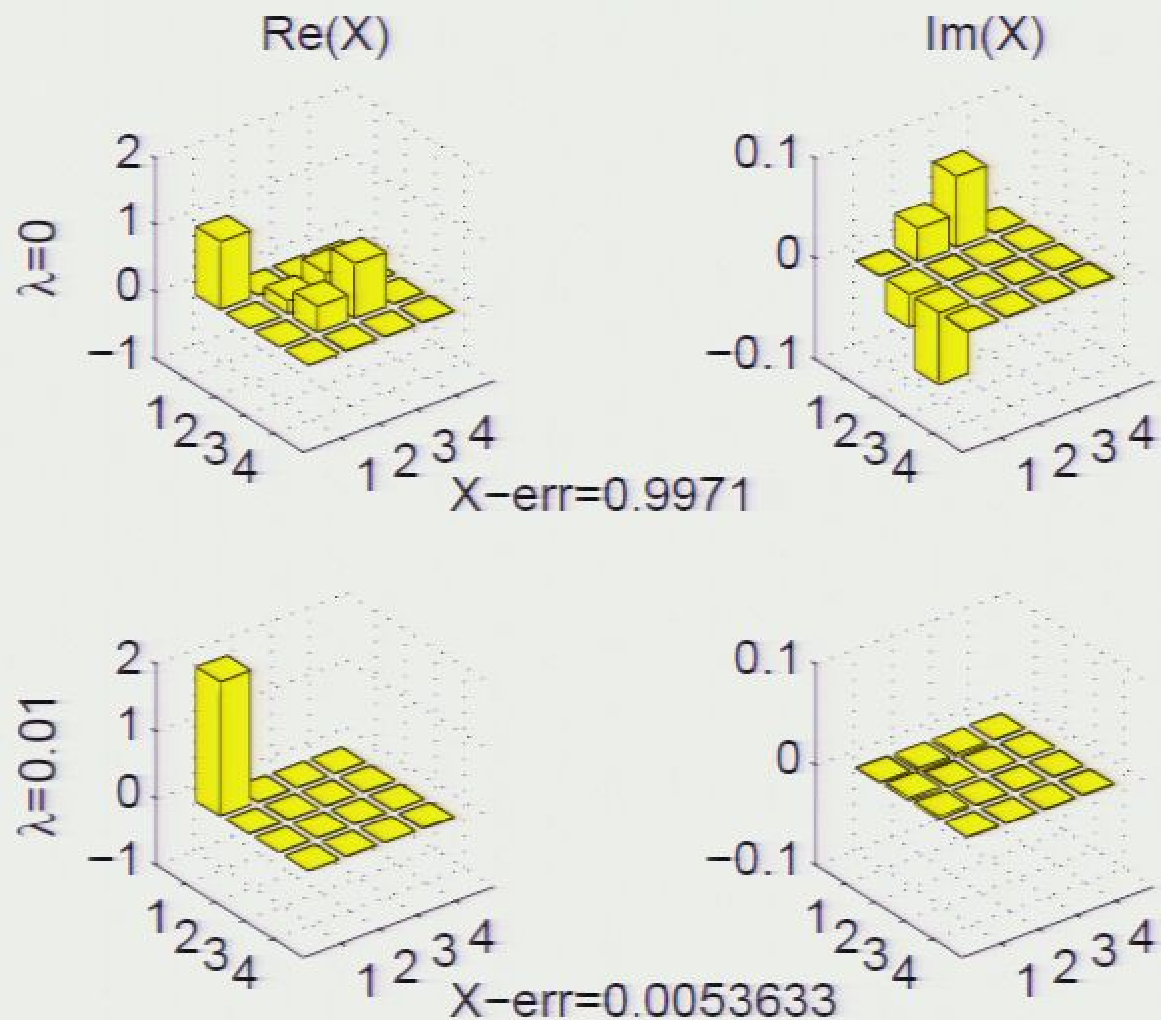


$q = 5,\ 30$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1 2 3 4
X−err=0.9971
X−err=0.0053633

$q = 5$

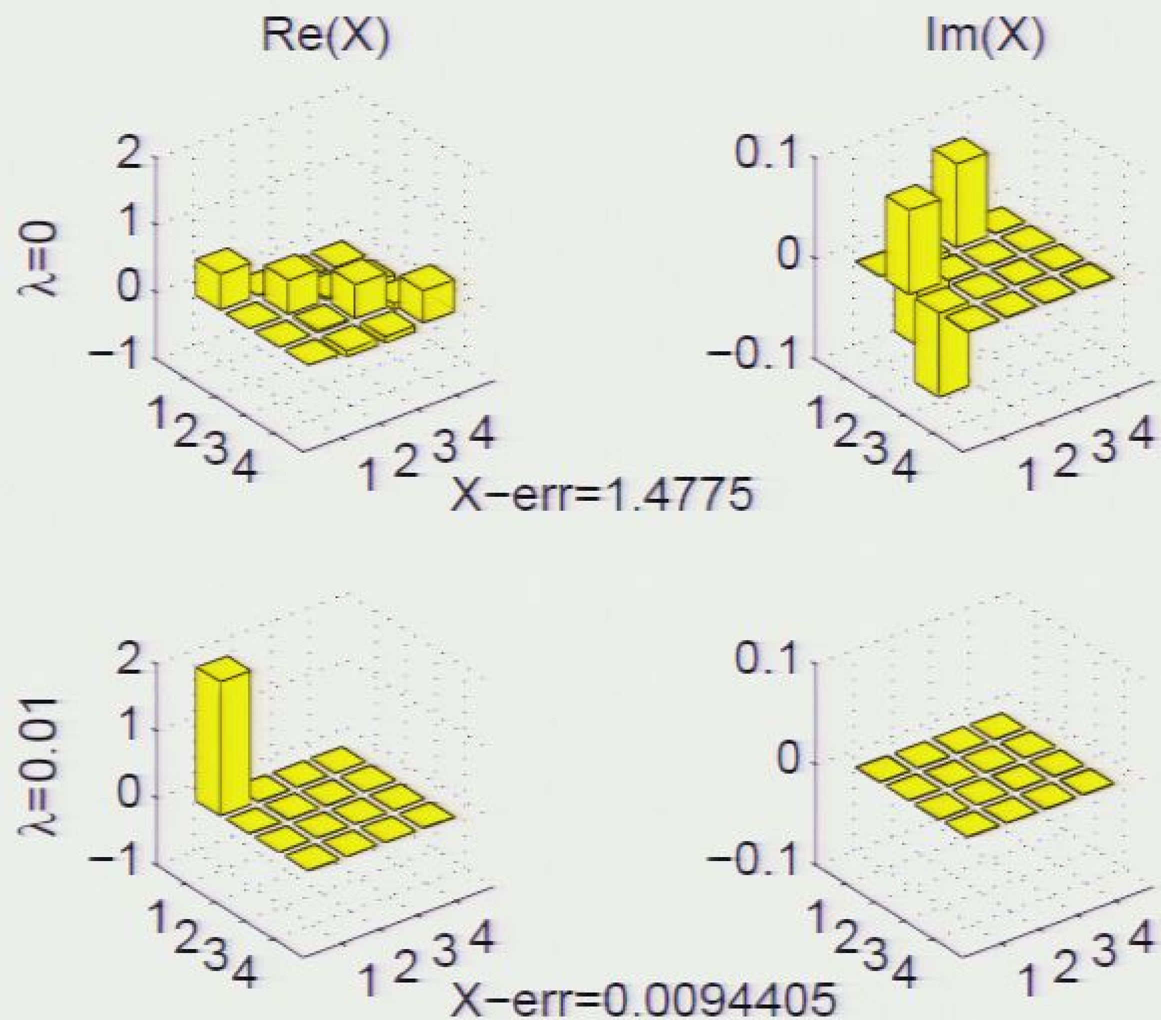

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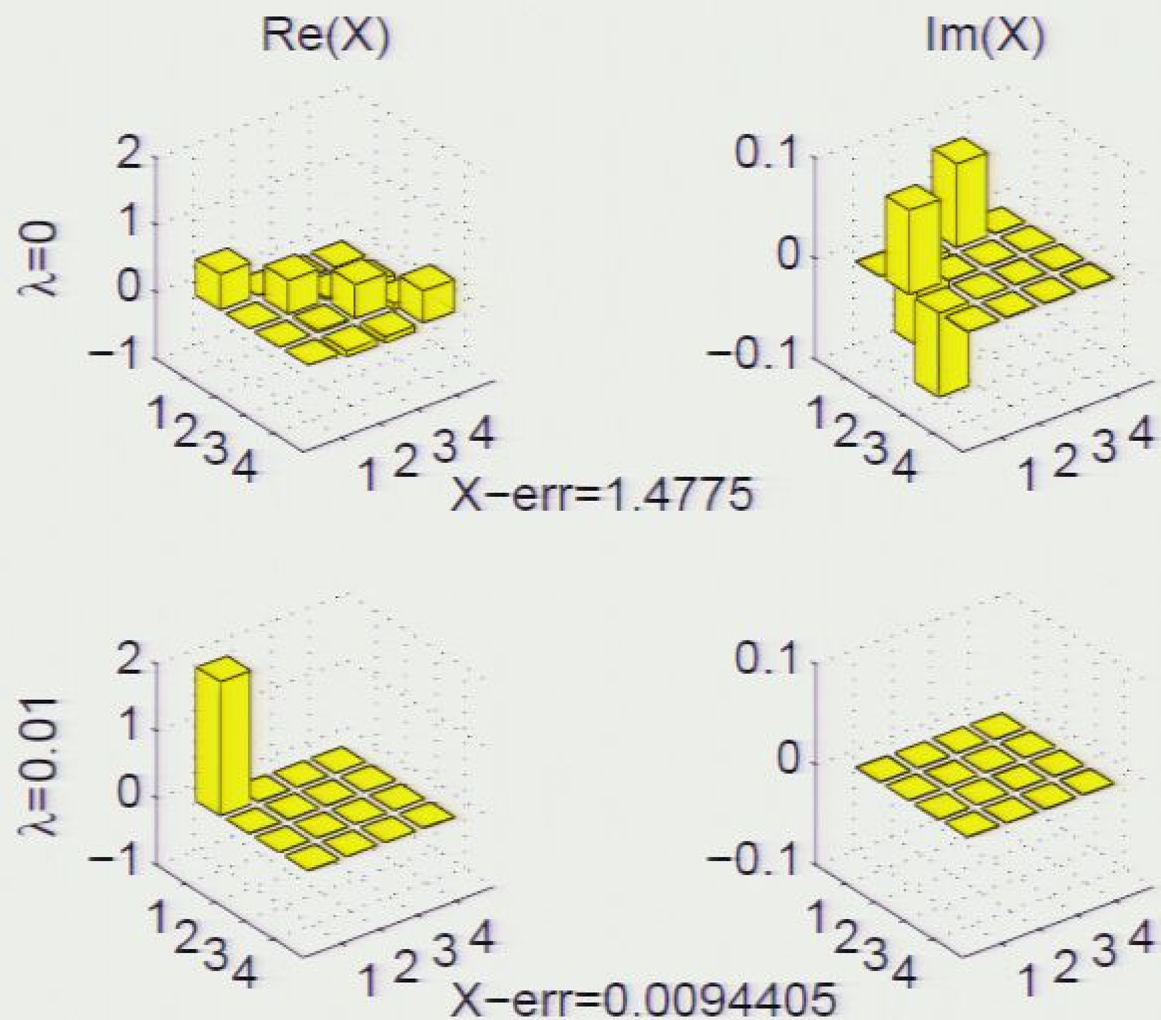
Table of $\|P(X_{\text{true}}) - P(X)\|_{\text{fro}}$

	$q = 5, 30, 60$	$q = 5, 30$	$q = 5$
$\lambda = 0$	0.0019	0.0031	0.0014
$\lambda = 0.01$	0.0019	0.0031	0.0014

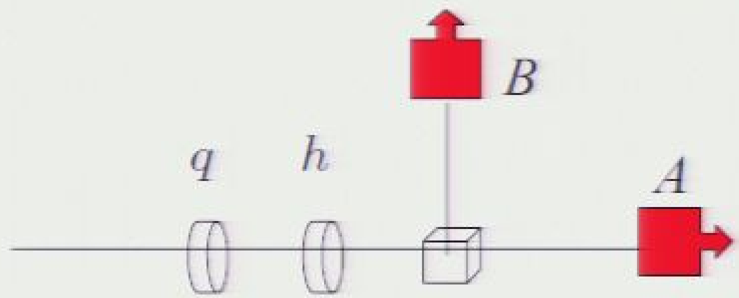
Table of $\|X_{\text{true}} - X\|$

	$q = 5, 30, 60$	$q = 5, 30$	$q = 5$
$\lambda = 0$	0.0029	0.9971	1.4775
$\lambda = 0.01$	0.0028	0.0054	0.0094

$q = 5$


Re(X)
Im(X)
λ=0
λ=0.01
X−err=1.4775
X−err=0.0094405
2
1
0
−1
0.1
0
−0.1
1 2 3 4
1 2 3 4

example



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