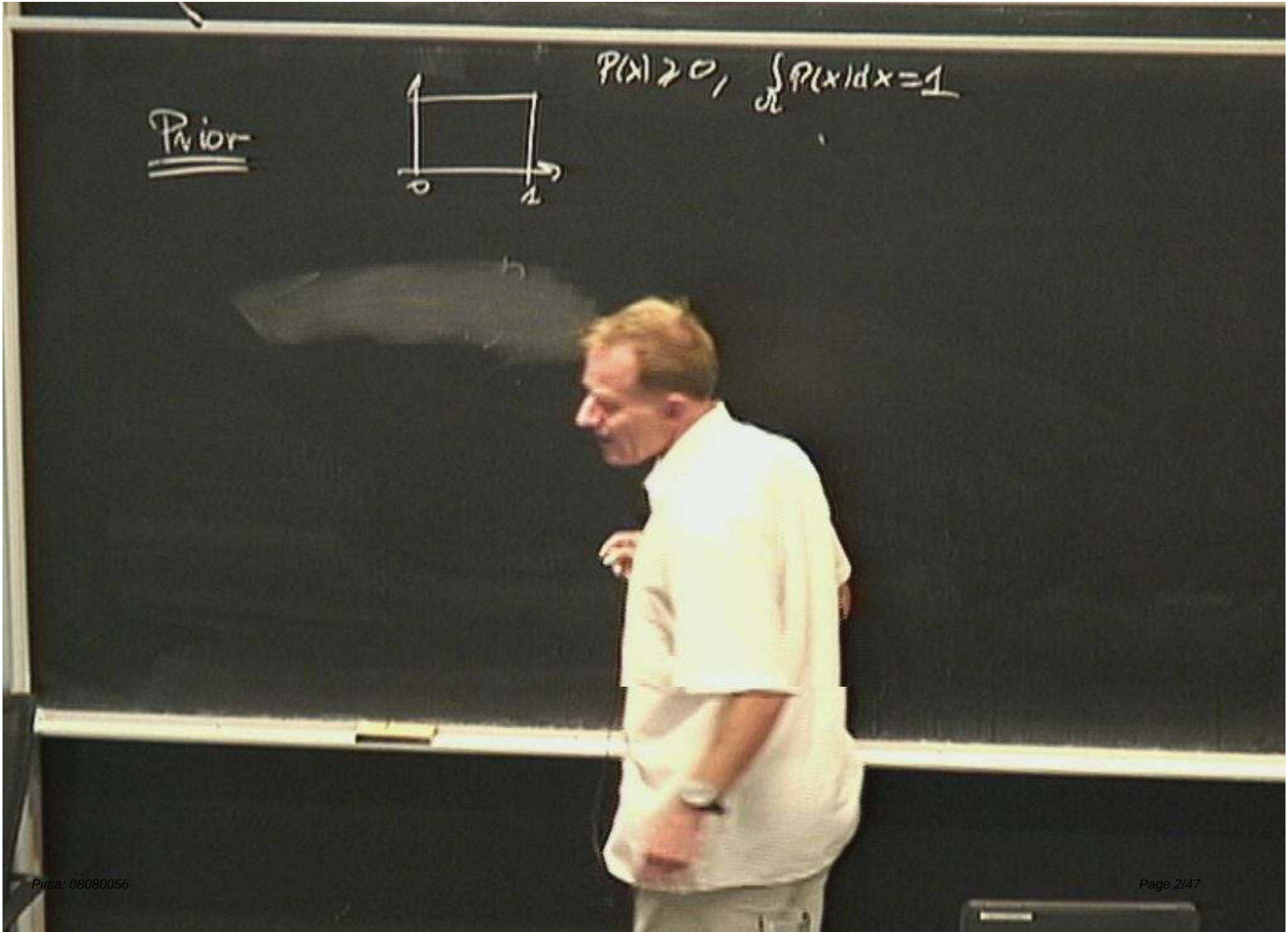
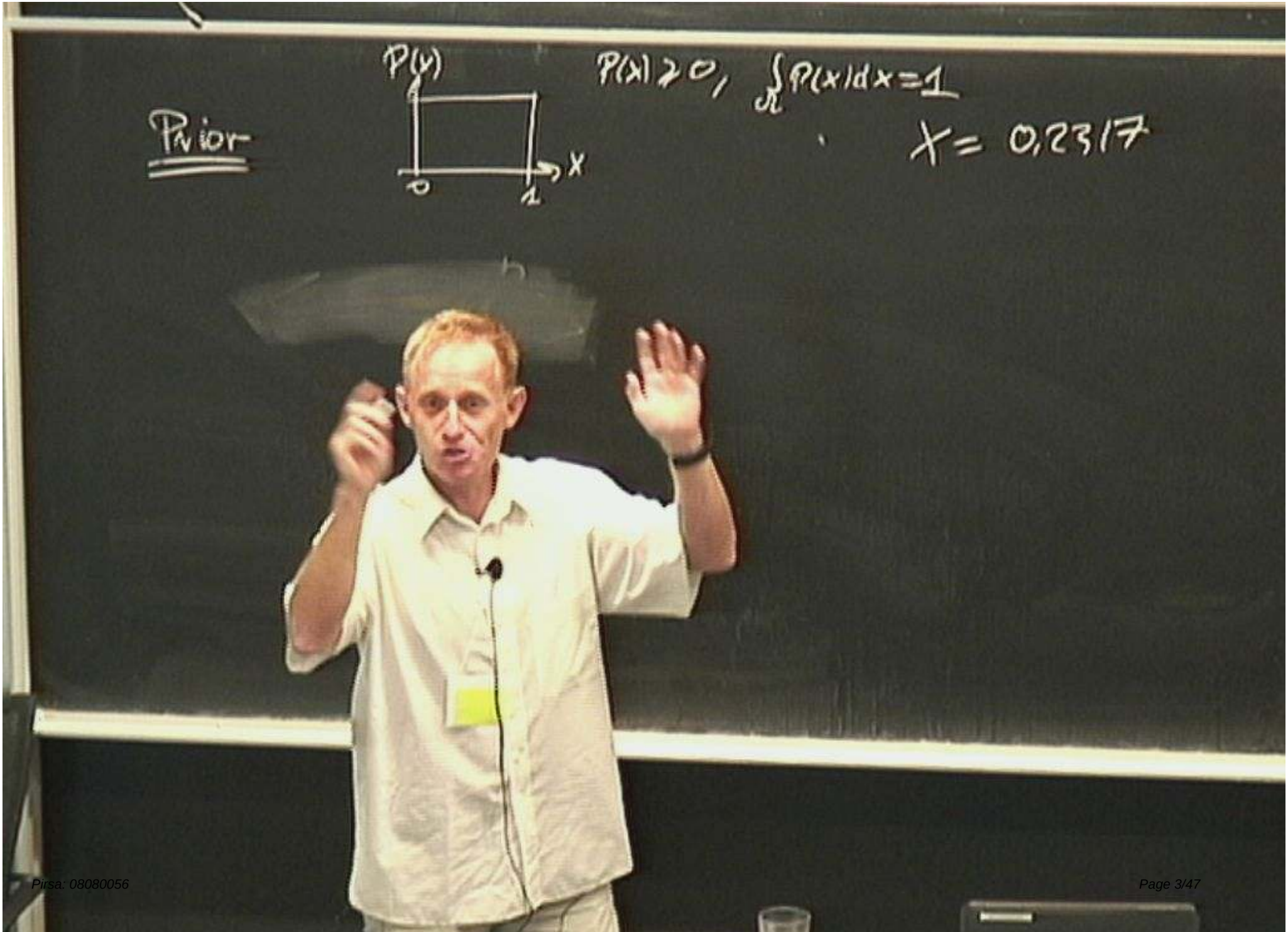
Title: Priors for quantum states & processes

Date: Aug 29, 2008 11:00 AM

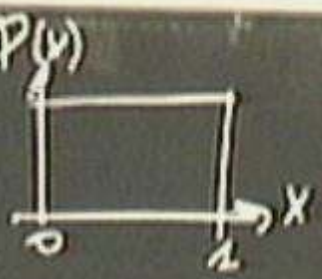
URL: http://pirsa.org/08080056

Abstract: As became apparent during Koenraad\'s talk, there are some important subleties to concepts like \'flat prior\' and \'uniform distribution\'... especially over probability simplices and quantum state spaces. This is a key problem for Bayesian approaches. Perhaps we\'re more interested in Jeffreys priors, Bures priors, or even something induced by the Chernoff bound! I\'d like to start a discussion of the known useful distributions over quantum states & processes, and I nominate Karol Zyckowski to lead it off.


Prior
P(x) ≥ 0, ∫ P(x) dx = 1
0
1


Prior
P(x)
x
0
1
P(x) ≥ 0, ∫ P(x) dx = 1
x = 0.2317

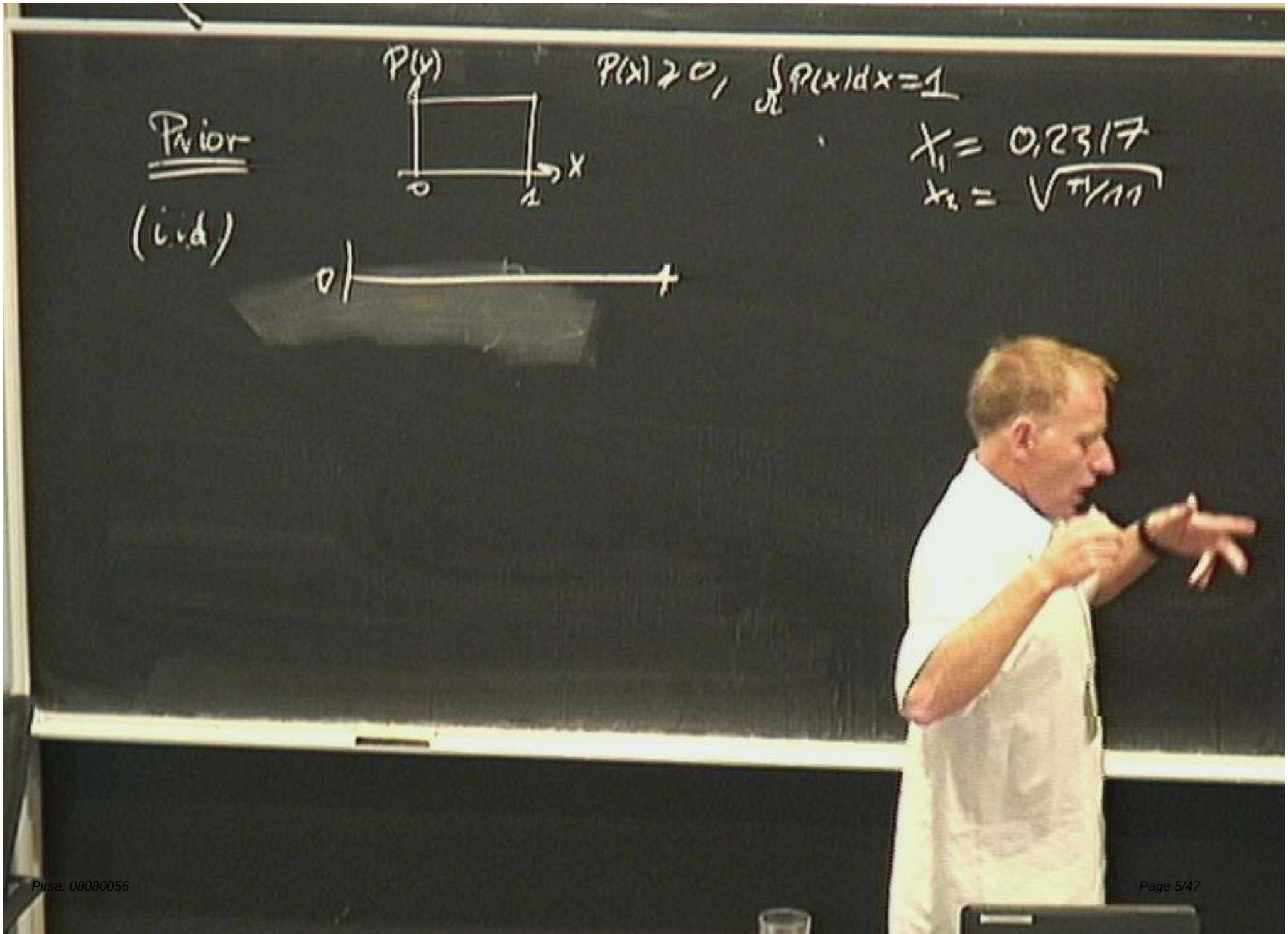
Prior

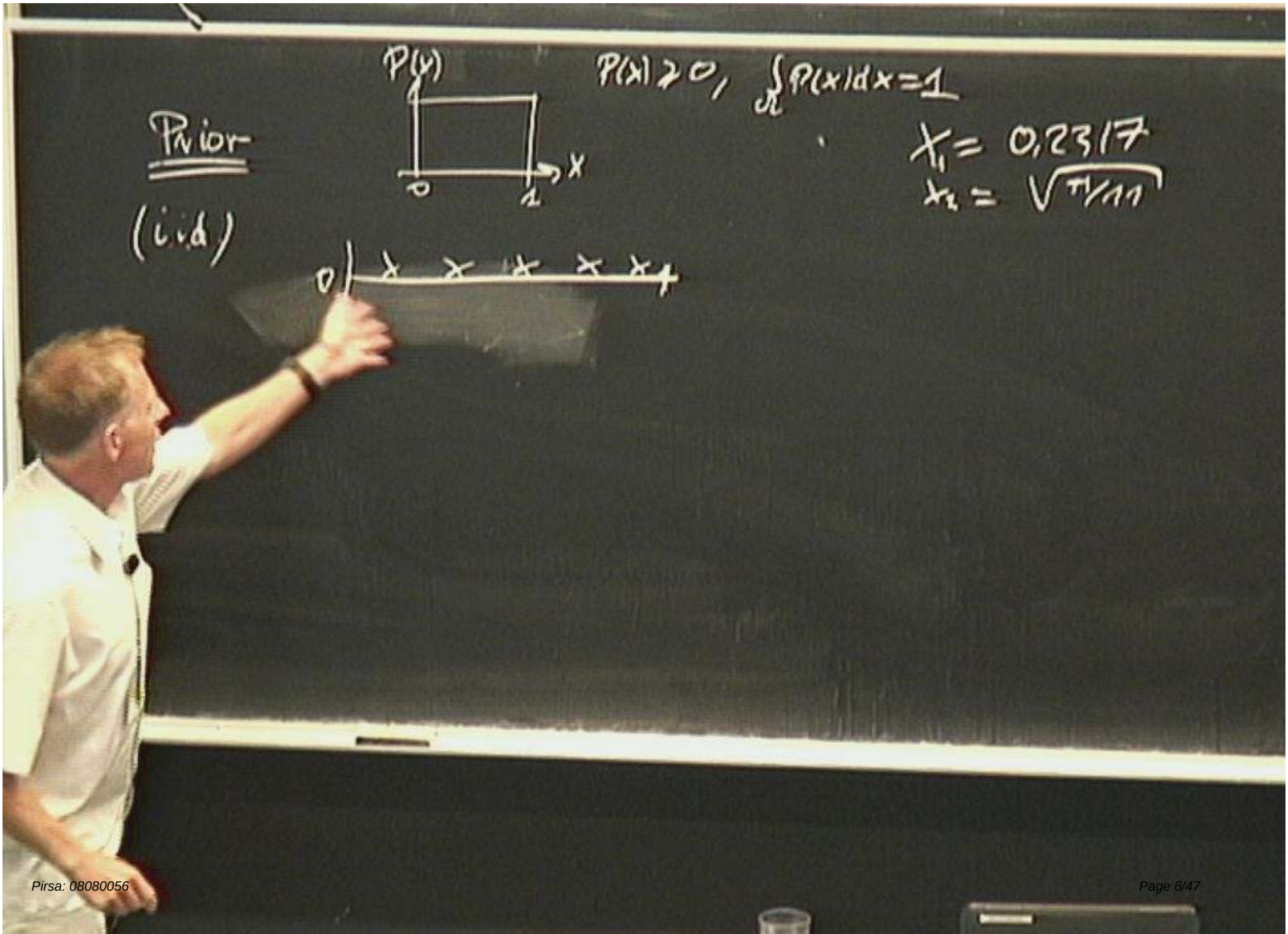


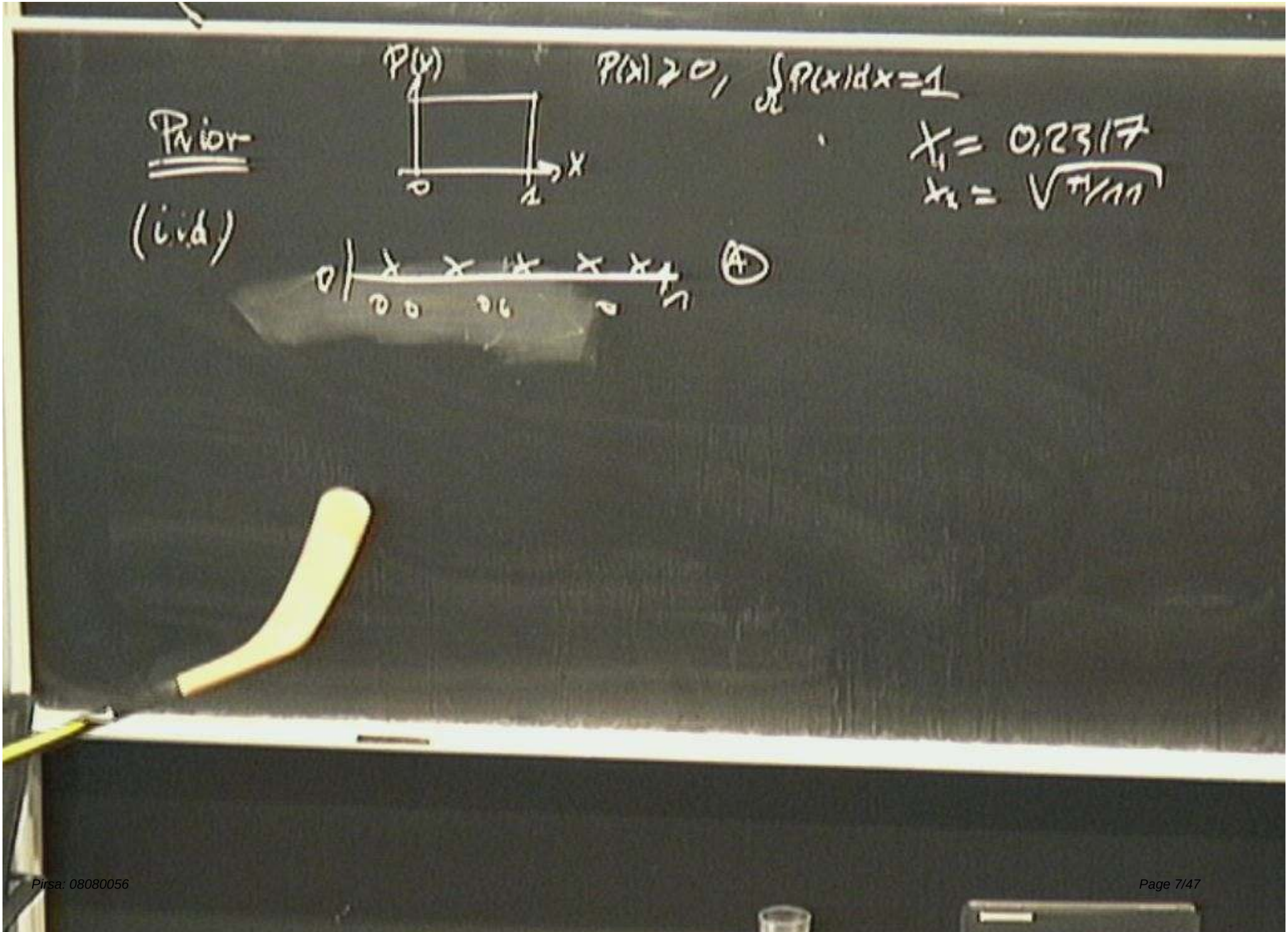
$P(x) \geq 0, \quad \int_{\Omega} P(x)\,dx = 1$

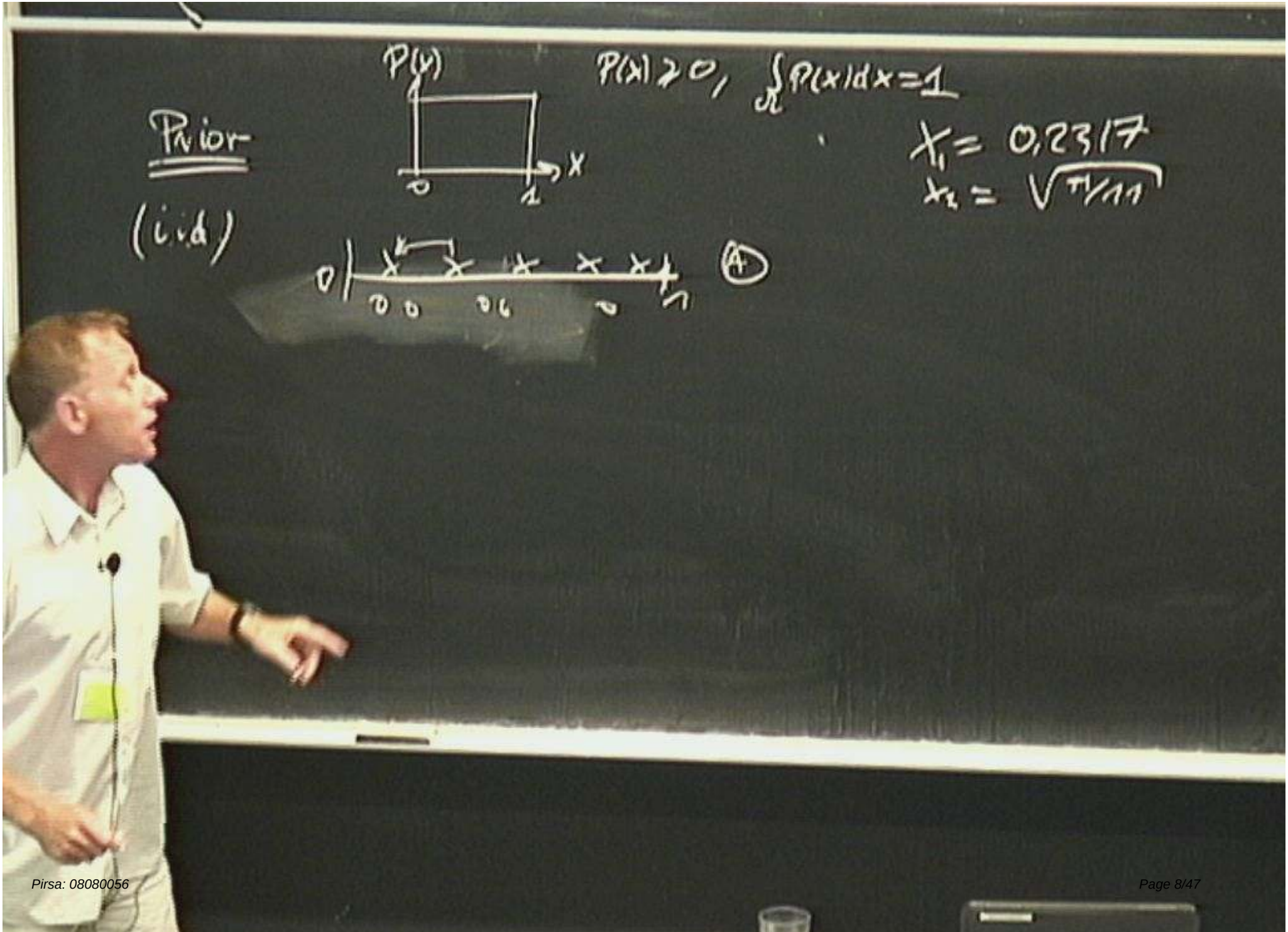
$\chi_1 = 0{,}2317$

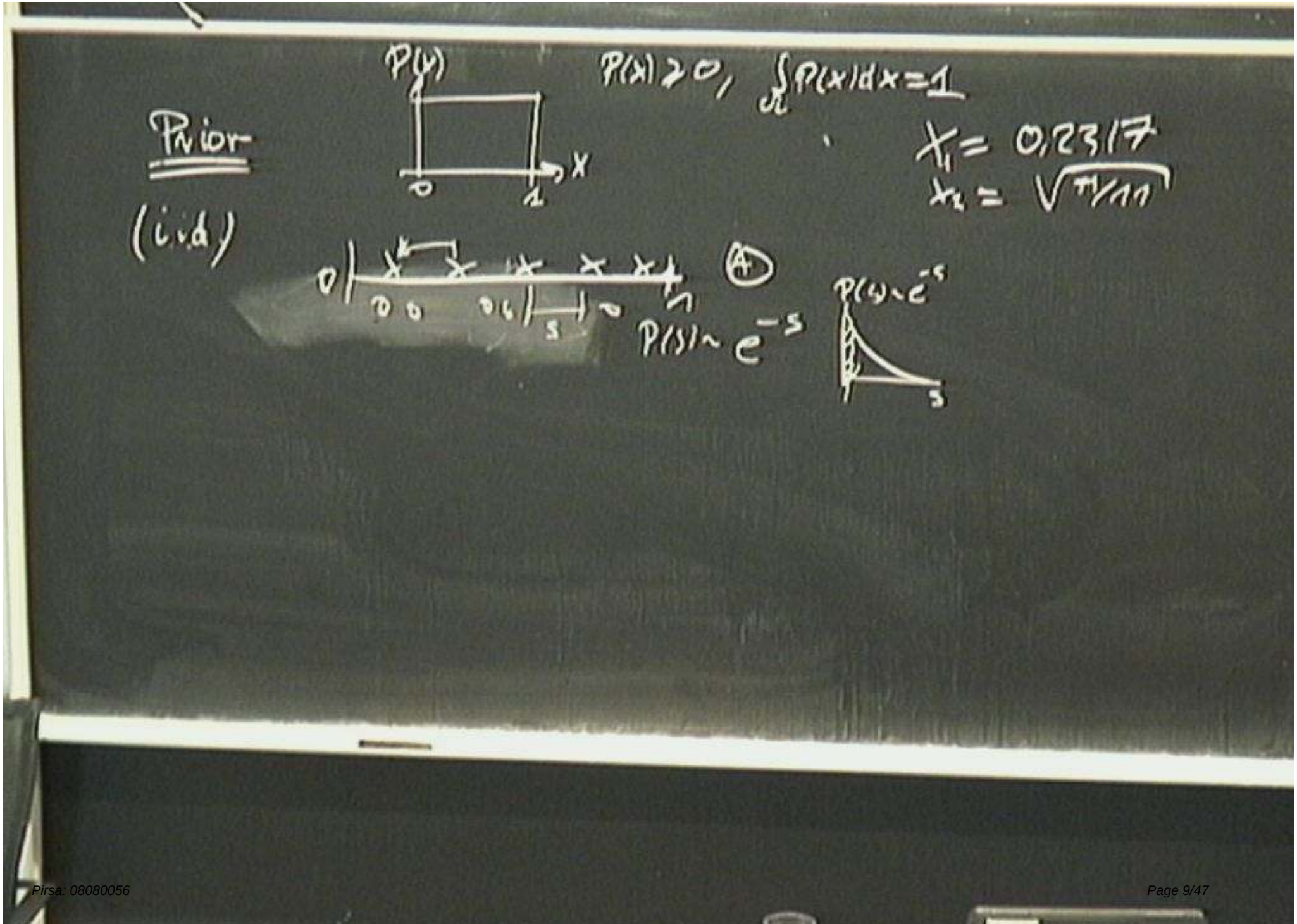
$\lambda_1 = \sqrt{\pi/11}$

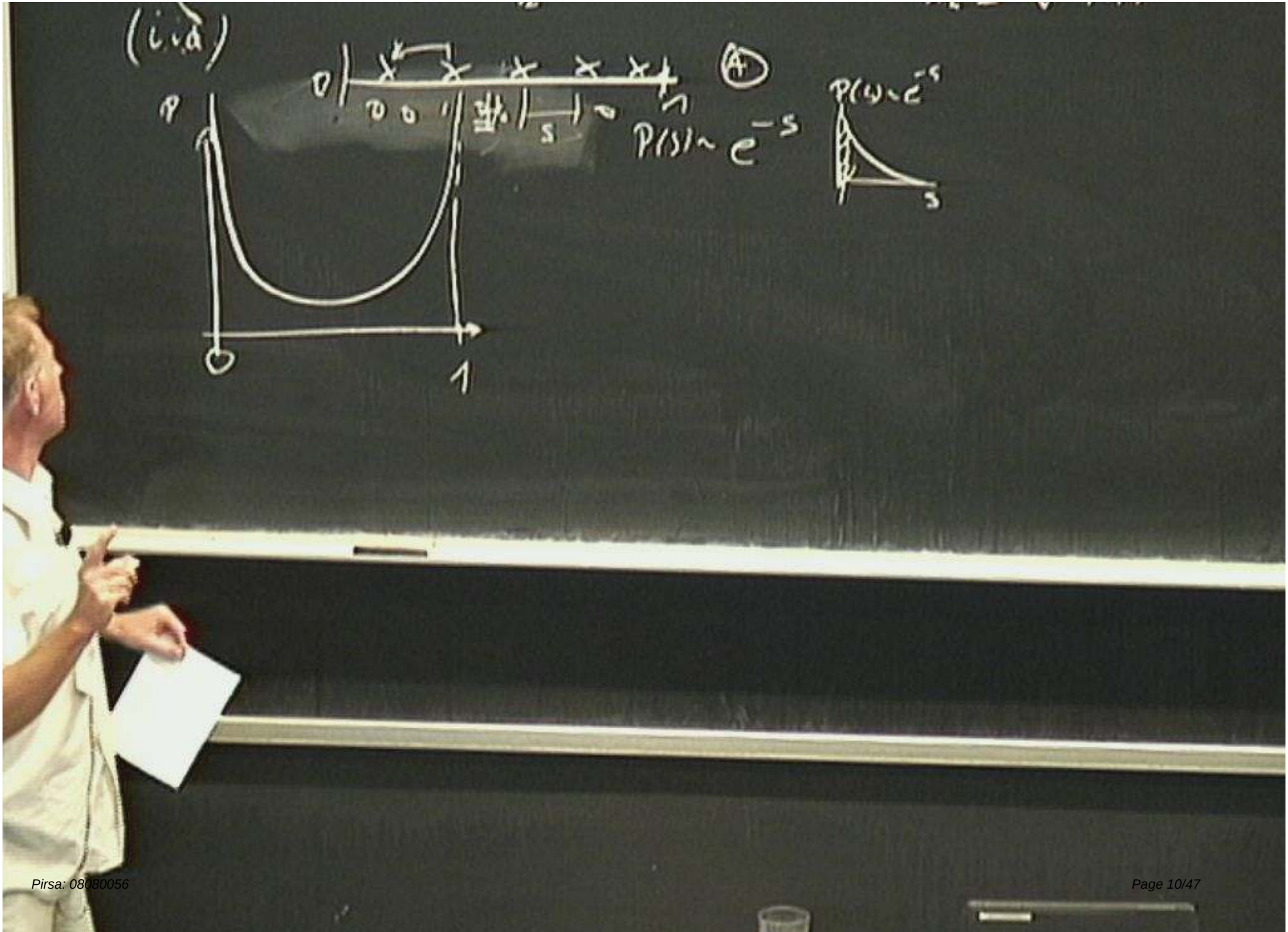

P(x)
x
0
1
P(x) ≥ 0, ∫ P(x) dx = 1
Prior
(i.i.d)
0
+
x1 = 0,2317
x2 = √11/11

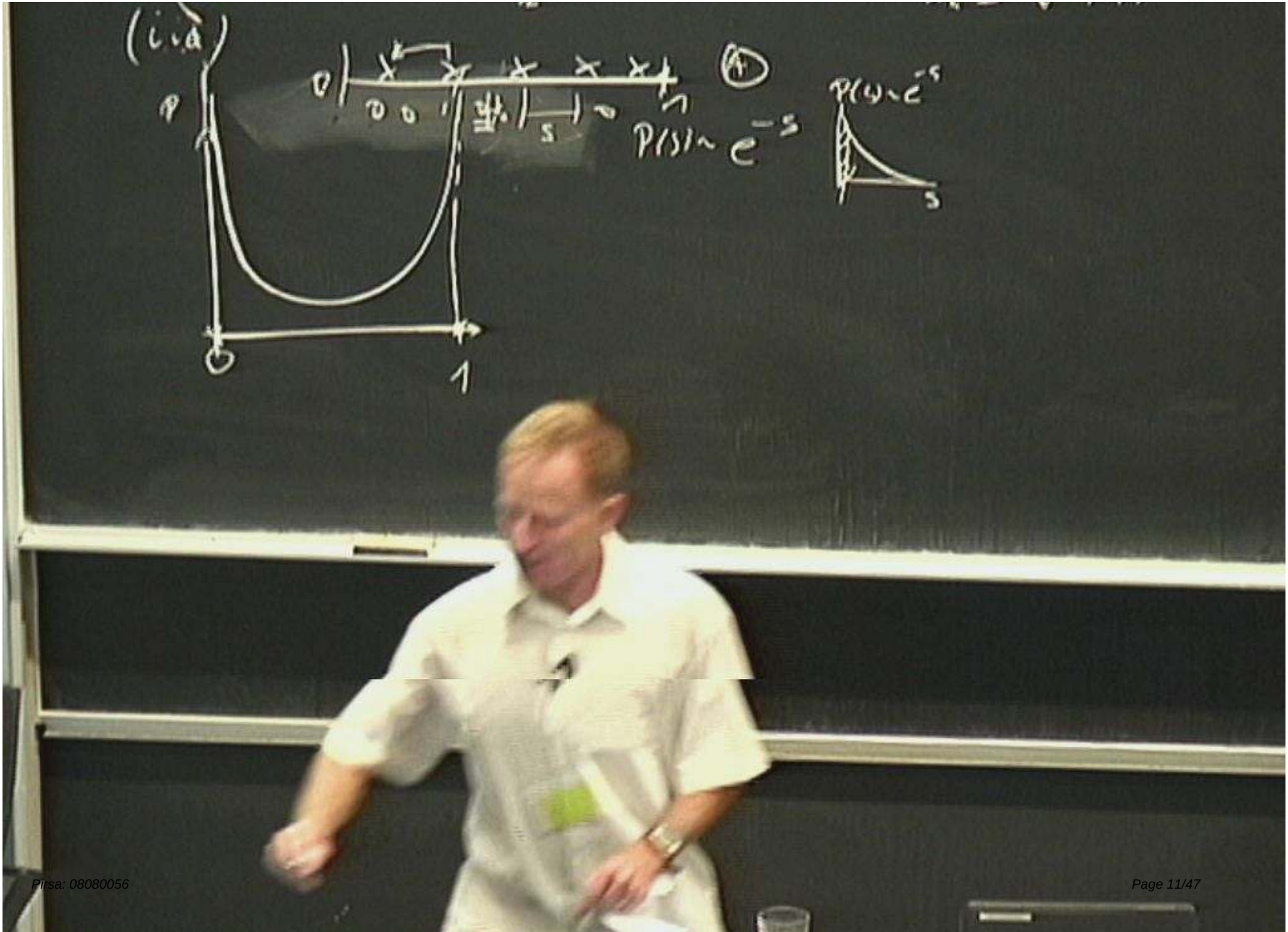

P(x)
P(x) ≥ 0, ∫ P(x) dx = 1
Prior
(i.i.d)
x
0
1
x1 = 0,2317
x2 = √π/11
0
x1 x2 x3 x4

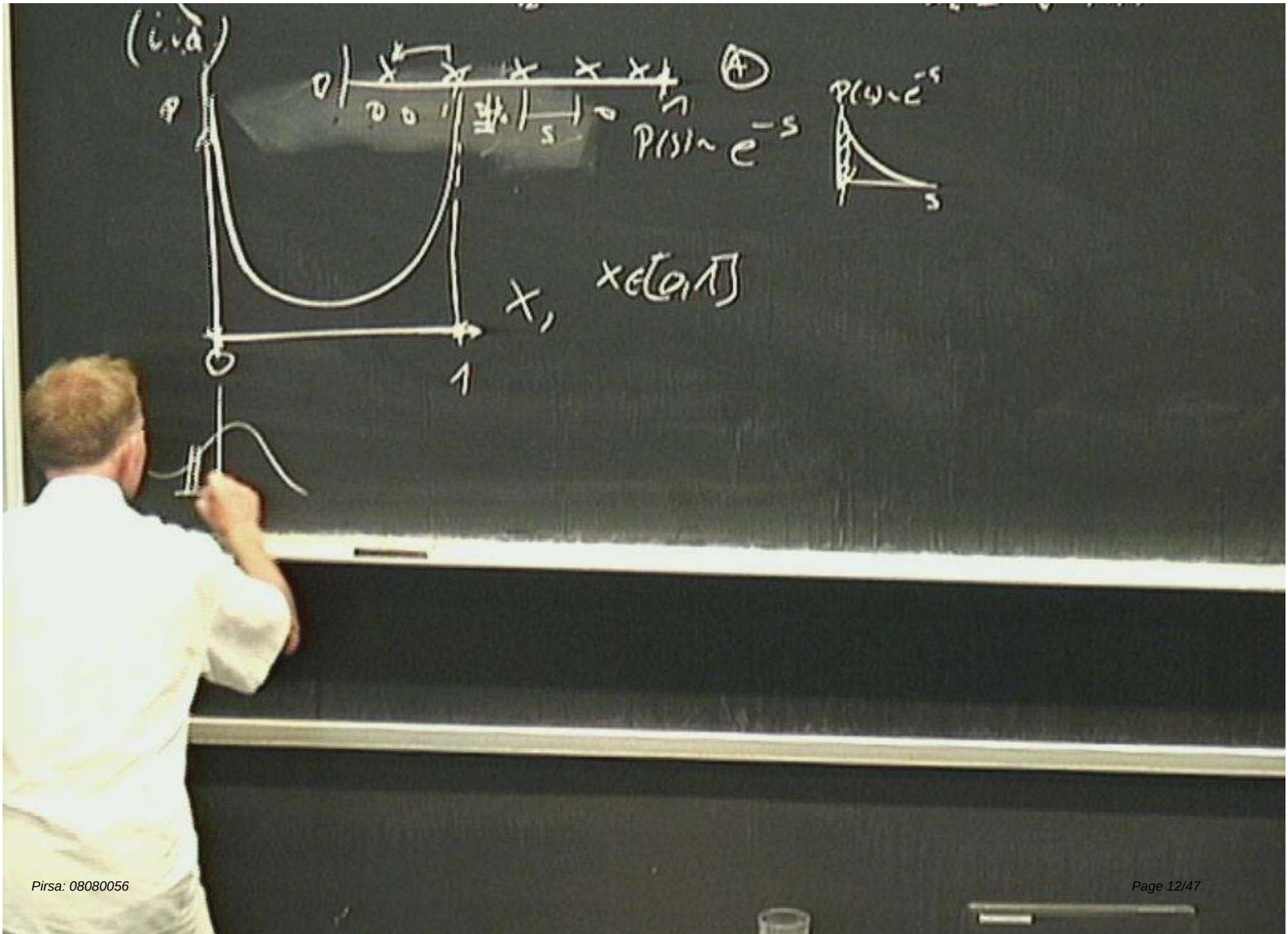

P(x)
P(x) ≥ 0, ∫ P(x) dx = 1
Prior
(i.i.d)
X
0
1
$x_1 = 0{,}2317$
$x_n = \sqrt{n/11}$
0
1
A

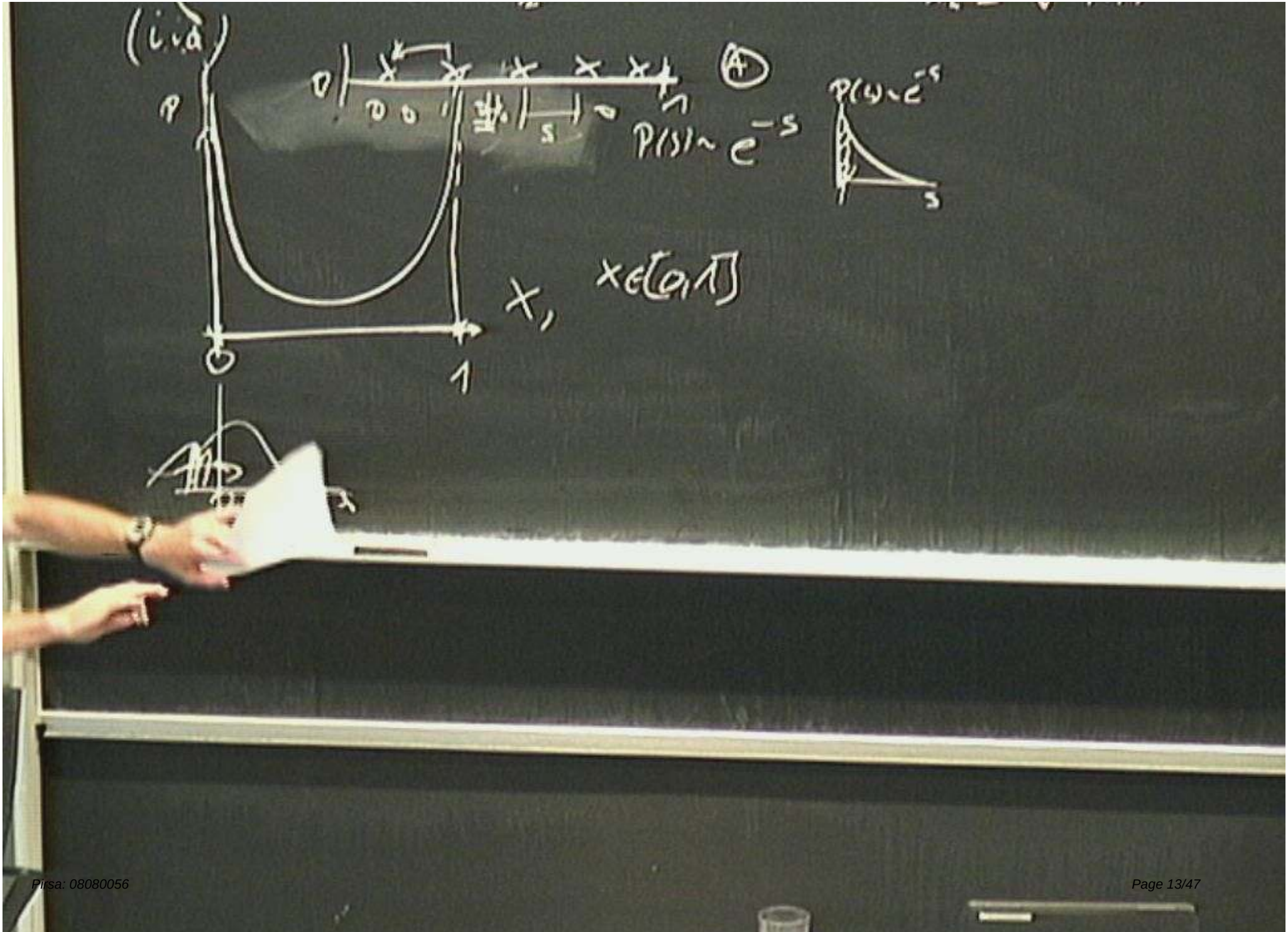

P(x)
P(x) ≥ 0, ∫ P(x) dx = 1
Prior
(i.i.d)
x
0
1
x1 = 0,2317
x1 = √11/11

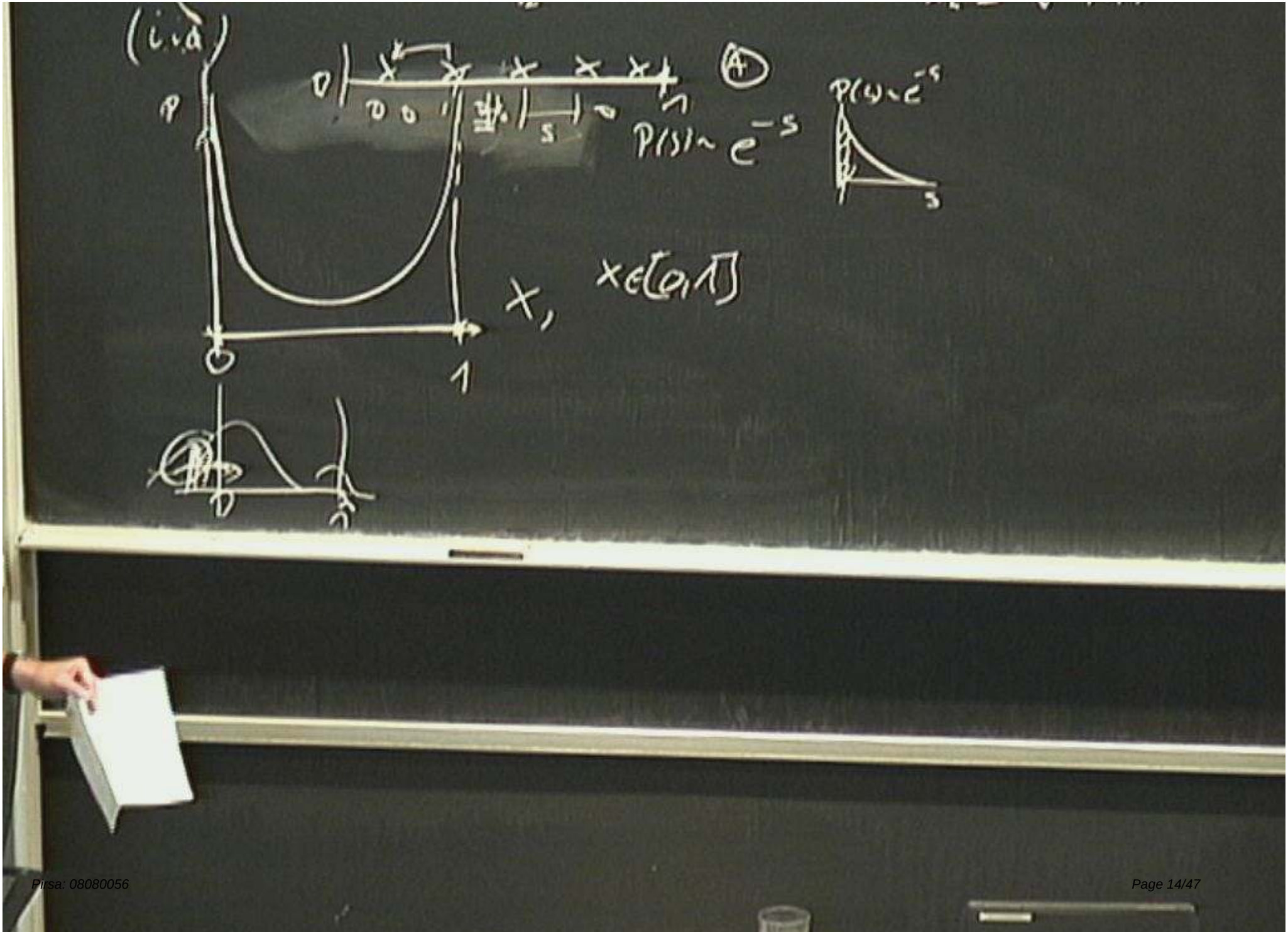

P(x)
P(x) ≥ 0, ∫ P(x) dx = 1
Prior
(i.i.d)
X
0
1
x1 = 0,2317
x2 = √11/11
0
s
P(s) ~ e^-s
P(s) ~ e^-s
s

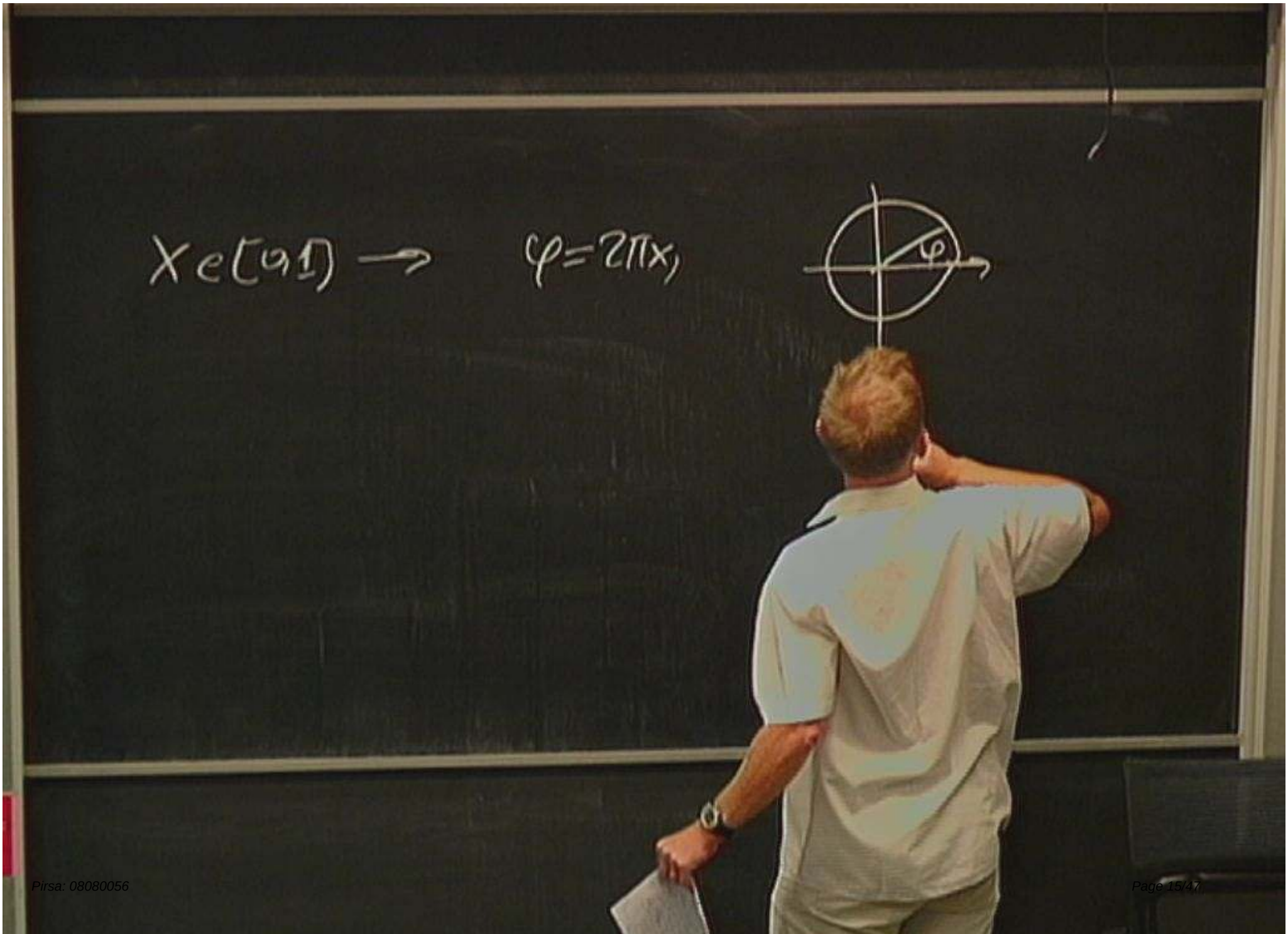

(i.i.d)
P
1
P(s) ~ e^{-s}
s
P(s) ~ e^{-s}
s


(i.i.d)
P
0
1
P(s) ~ e^{-s}
P(s) ~ e^{-s}
s


(i.i.d.)
P(s) ~ e^{-s}
P(s) ~ e^{-s}
s
x_i, x ∈ [0,1]
0
1

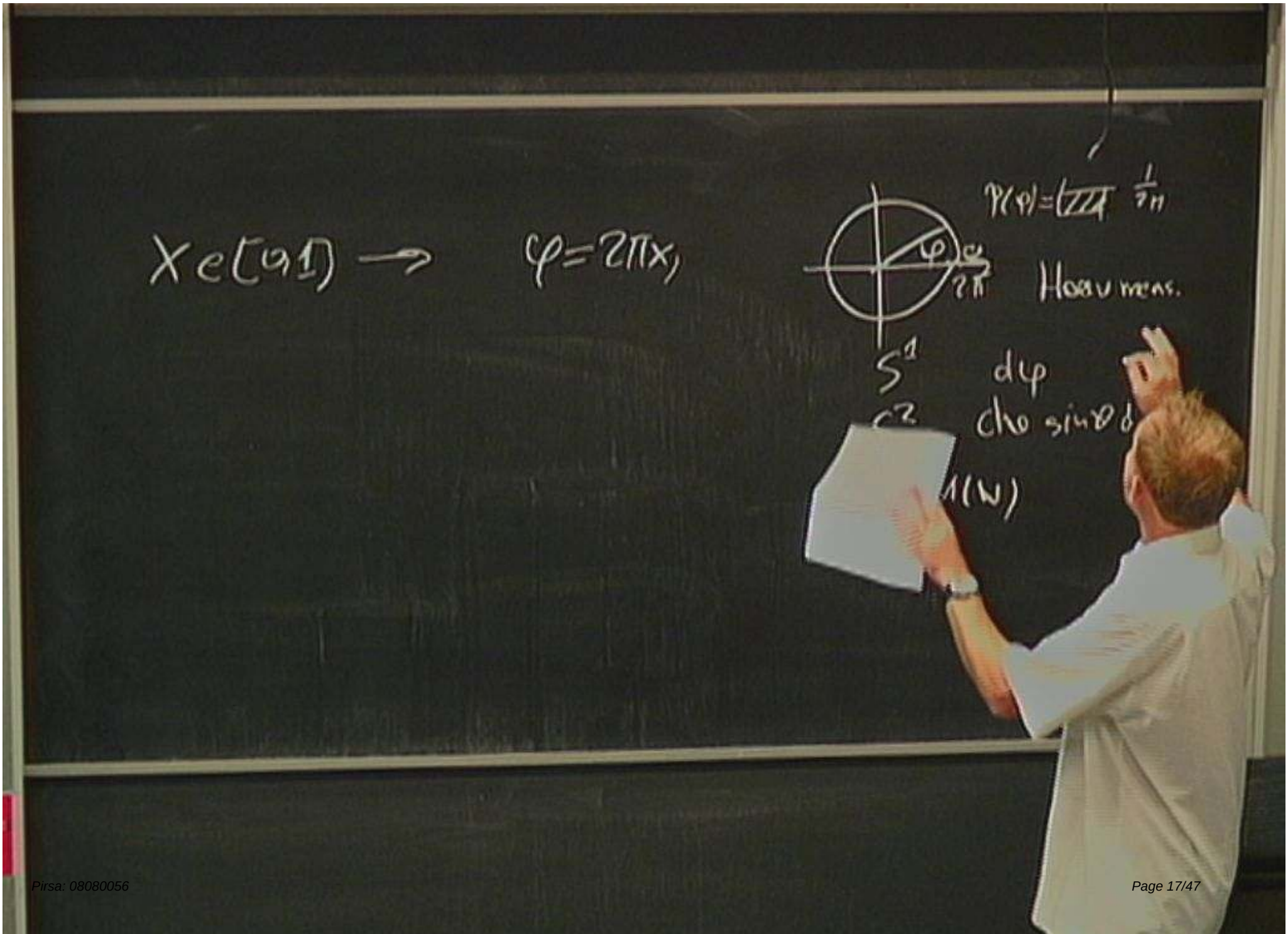

(i.i.d)
P
0
1
x_1
$x \in [0,1]$
$P(s) \sim e^{-s}$
$P(s) \sim e^{-s}$
s


(i.i.d.)
P
0
1
x_1
$x \in [0,1]$
$P(s) \sim e^{-s}$
s

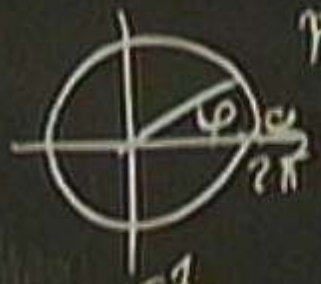

$x \in [0,1) \rightarrow$
$\varphi = 2\pi x$
φ

$x \in [0,1) \rightarrow \varphi = 2\pi x$

$P(\varphi) = \frac{1}{2\pi}$


$x \in [0,1) \rightarrow \varphi = 2\pi x$
2π
S^1
$d\varphi$
Haar meas.

$x \in [0,1) \rightarrow \varphi = 2\pi x$



$P(\varphi) = \frac{1}{2\pi}$

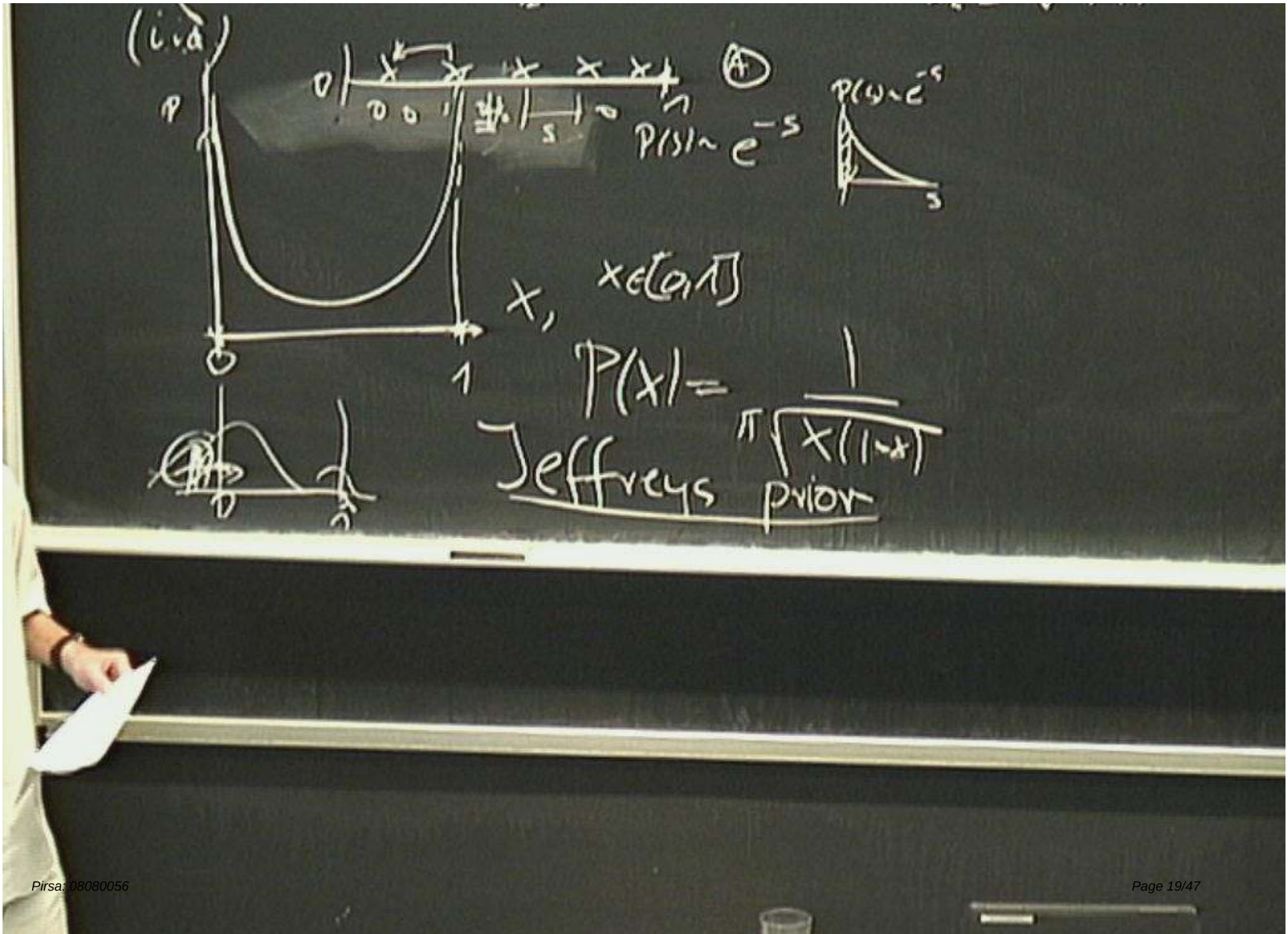
Haar meas.

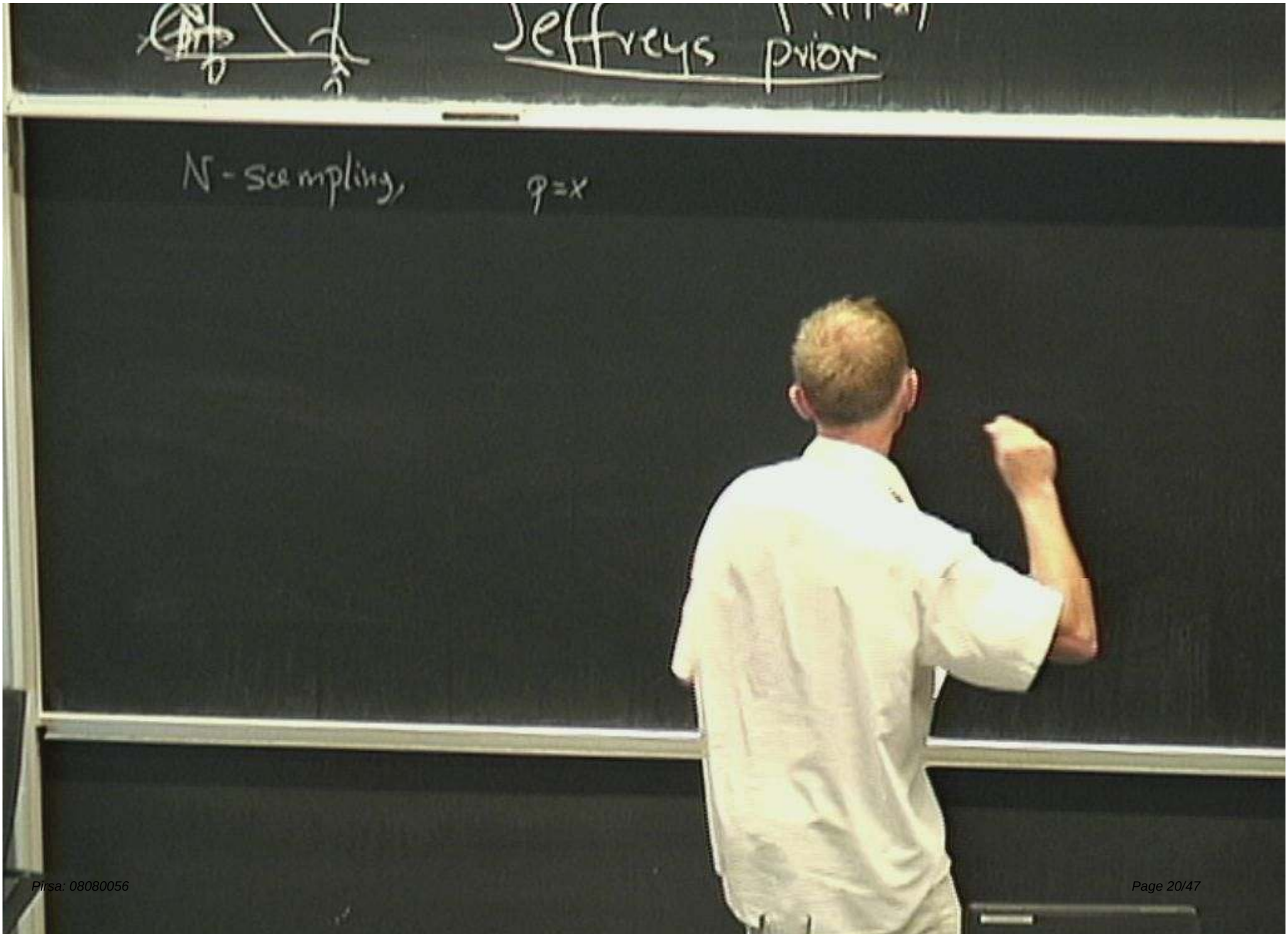
S^1 $d\varphi$

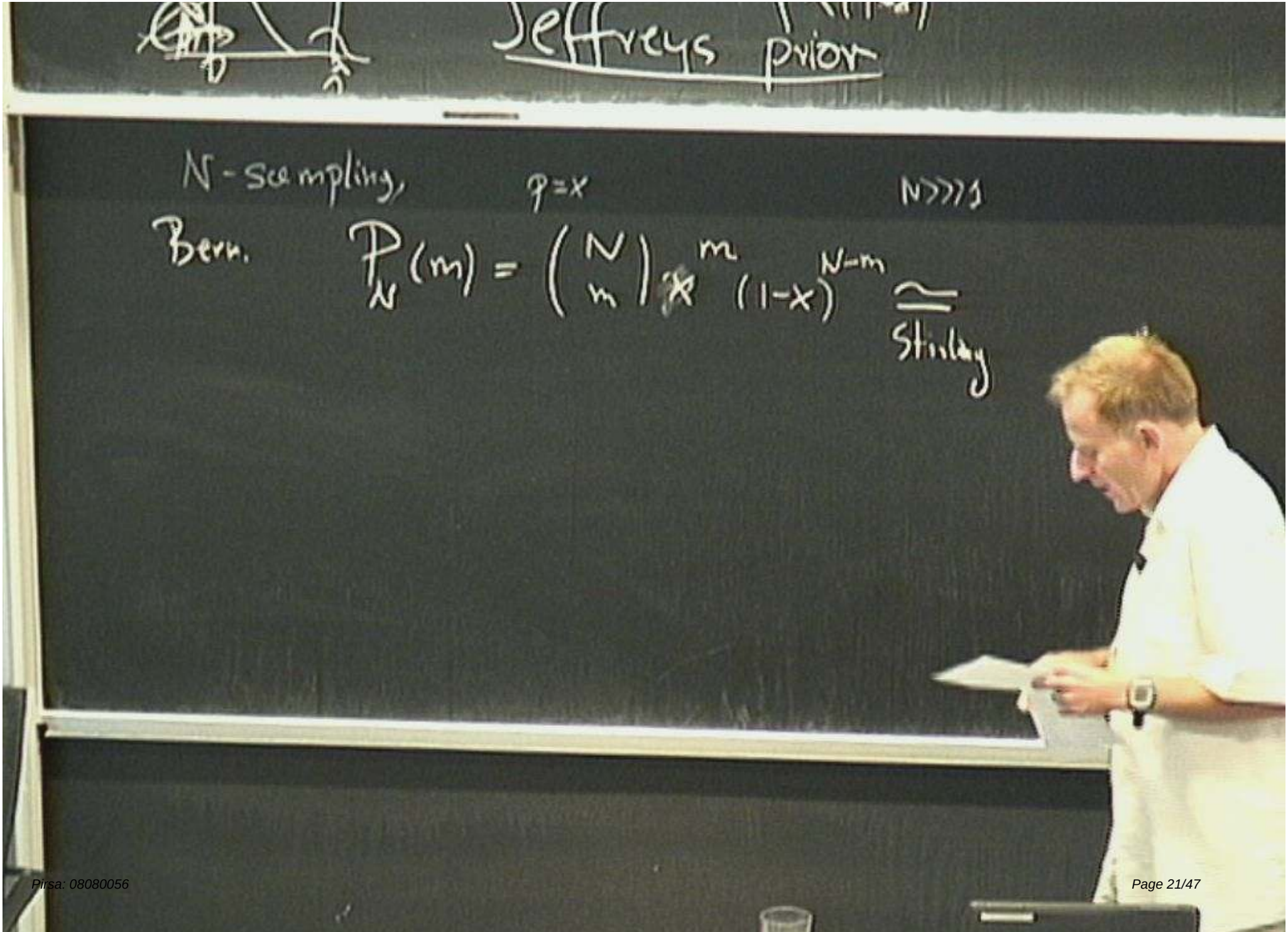
S^2 $\sin\theta\, d\theta\, d\varphi$

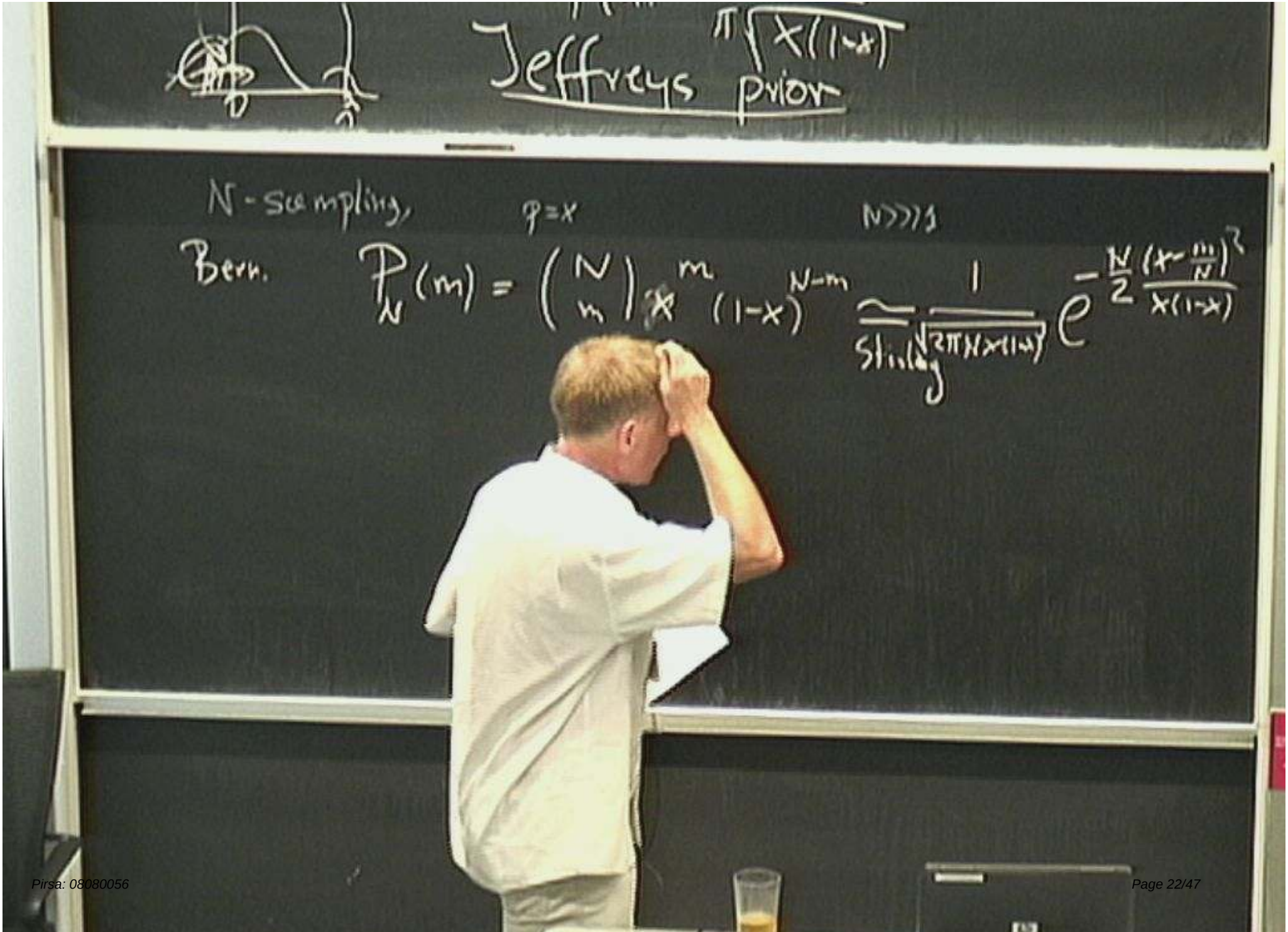
$\vdots$

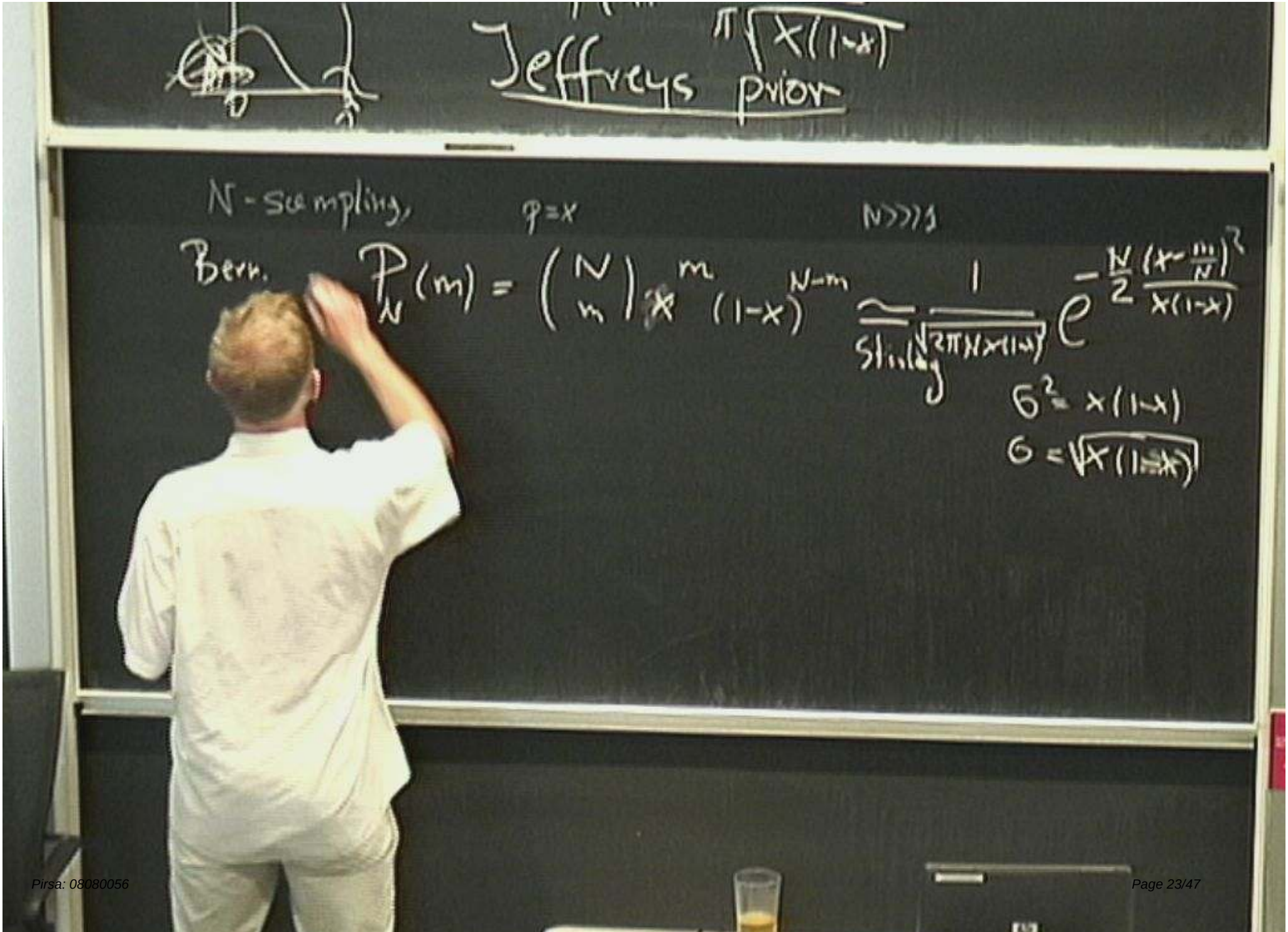
$U(N)$

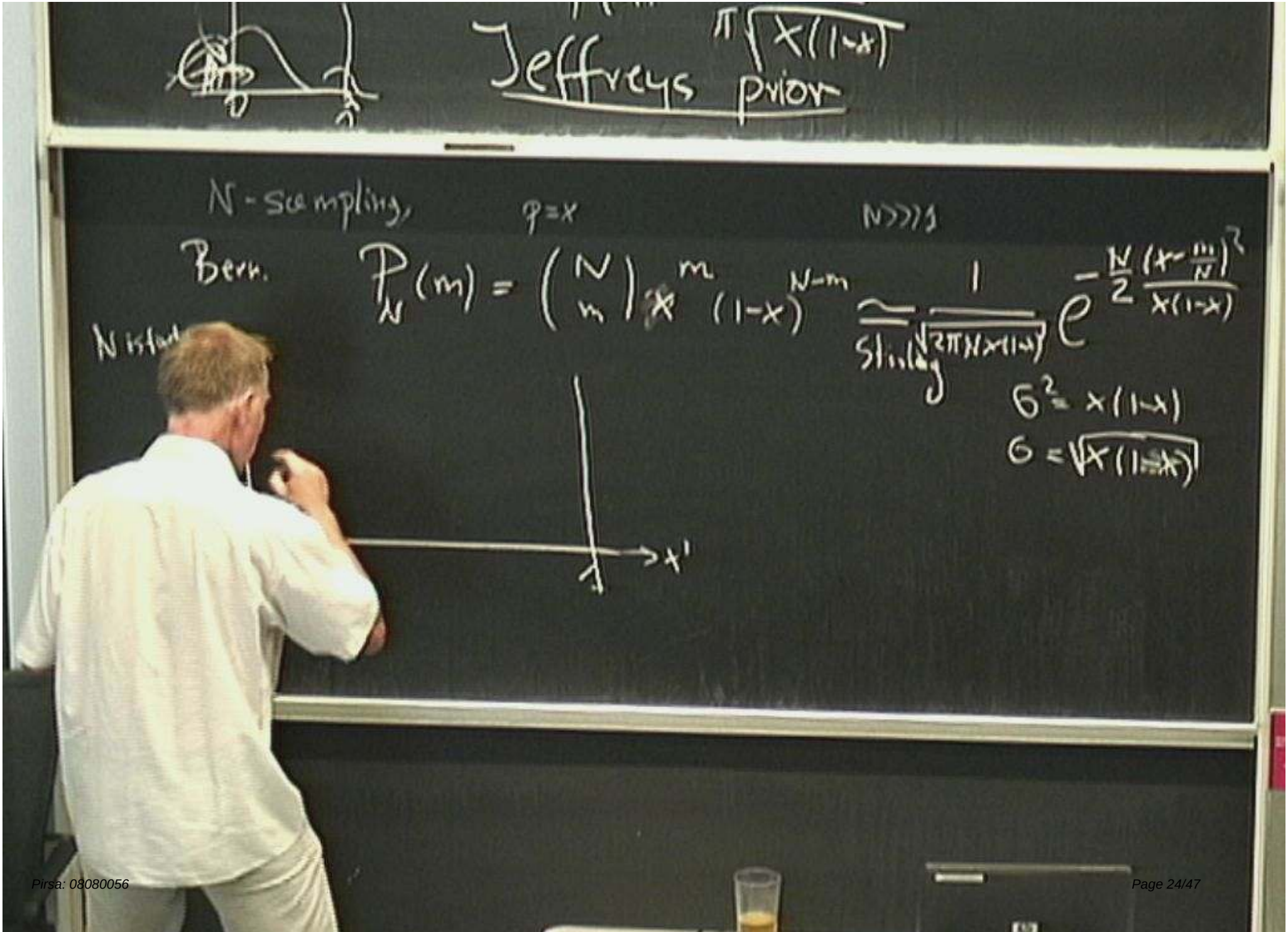

(i.i.d)
P
0
1
x, x∈[0,1]
P(x) = 1 / π√(x(1-x))
Jeffreys prior
P(s|~e^-s
P(s)~e^-s
s


Jeffreys prior
N - sampling,
q = x


Jeffreys prior
N - sampling, p=x N>>1
Bern. $P_N(m) = \binom{N}{m} x^m (1-x)^{N-m} \simeq$
Stirling


Jeffreys prior
N - sampling, p=x
N>>1
Bern.
Stirling


Jeffreys prior
N - sampling,
p = x
N >>> 1
Bern.
Stirling
$\pi\sqrt{x(1-x)}$
$P_N(m) = \binom{N}{m} x^m (1-x)^{N-m} \simeq \frac{1}{\sqrt{2\pi N x(1-x)}} e^{-\frac{N}{2}\frac{(x-\frac{m}{N})^2}{x(1-x)}}$
$\sigma^2 = x(1-x)$
$\sigma = \sqrt{x(1-x)}$


Jeffreys prior
N - sampling, p = x
N >>> 1
Bern.
$P_N(m) = \binom{N}{m} x^m (1-x)^{N-m}$
Stirling
$\sigma^2 = x(1-x)$
$\sigma = \sqrt{x(1-x)}$
x'

Jeffreys prior

$\pi \propto \sqrt{x(1-x)}$

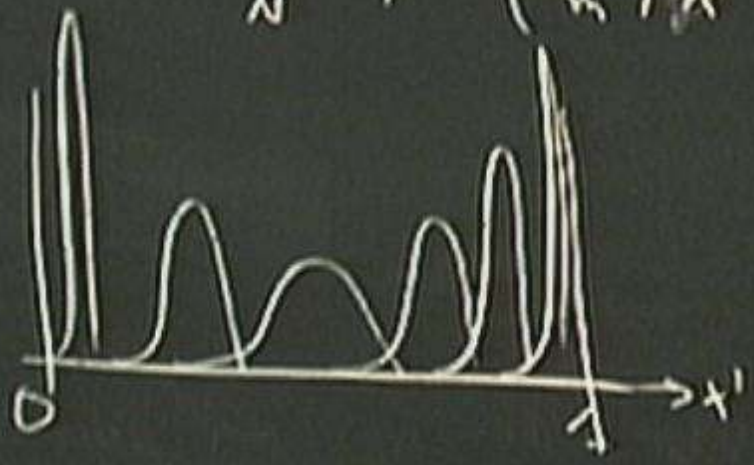
N - sampling, $p = x$, $N \gg 1$

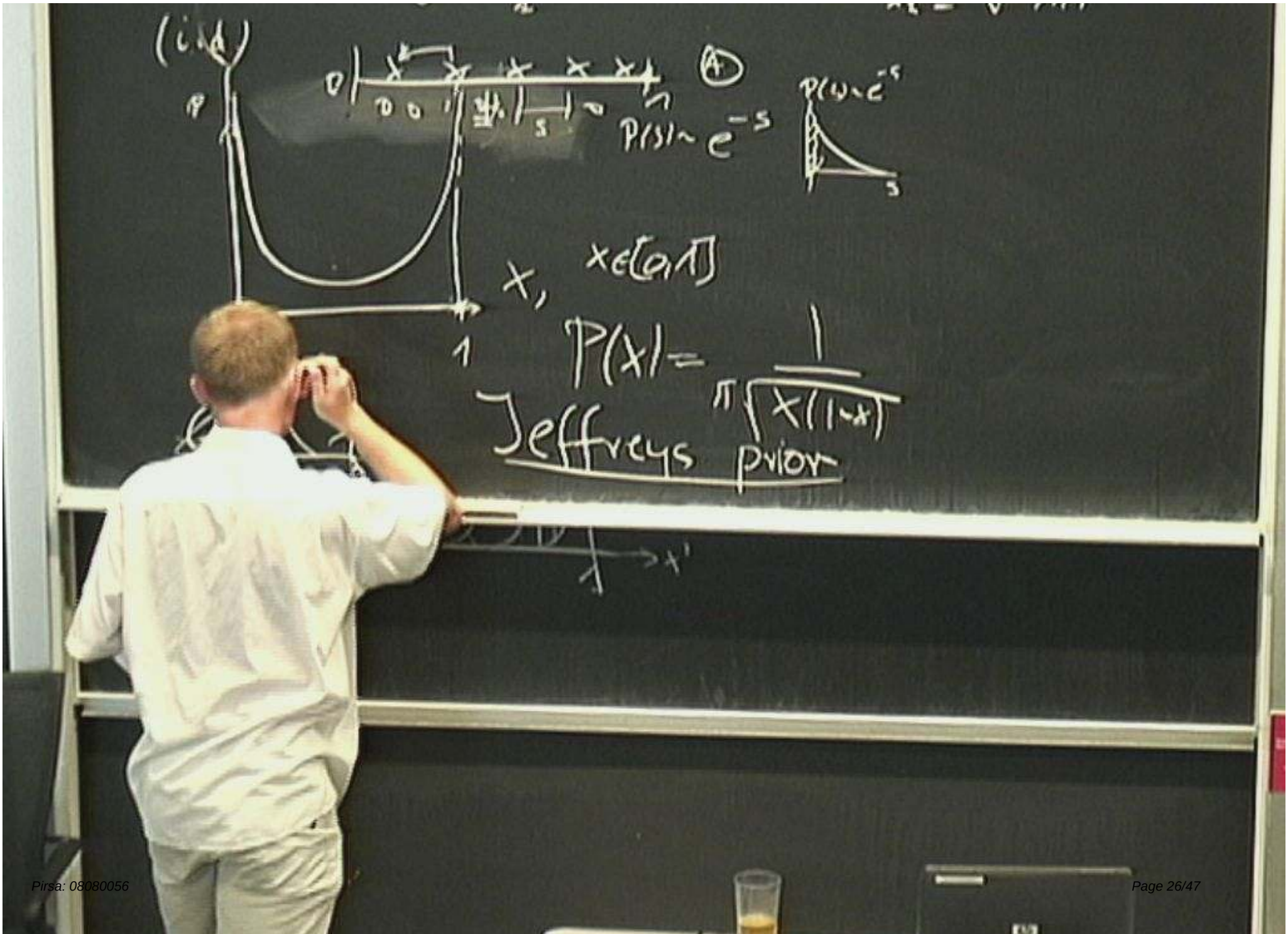
Bern. $P_N(m) = \binom{N}{m} x^m (1-x)^{N-m} \simeq \frac{1}{\text{Stirling}\sqrt{2\pi N x(1-x)}} e^{-\frac{N}{2}\frac{(x-\frac{m}{N})^2}{x(1-x)}}$

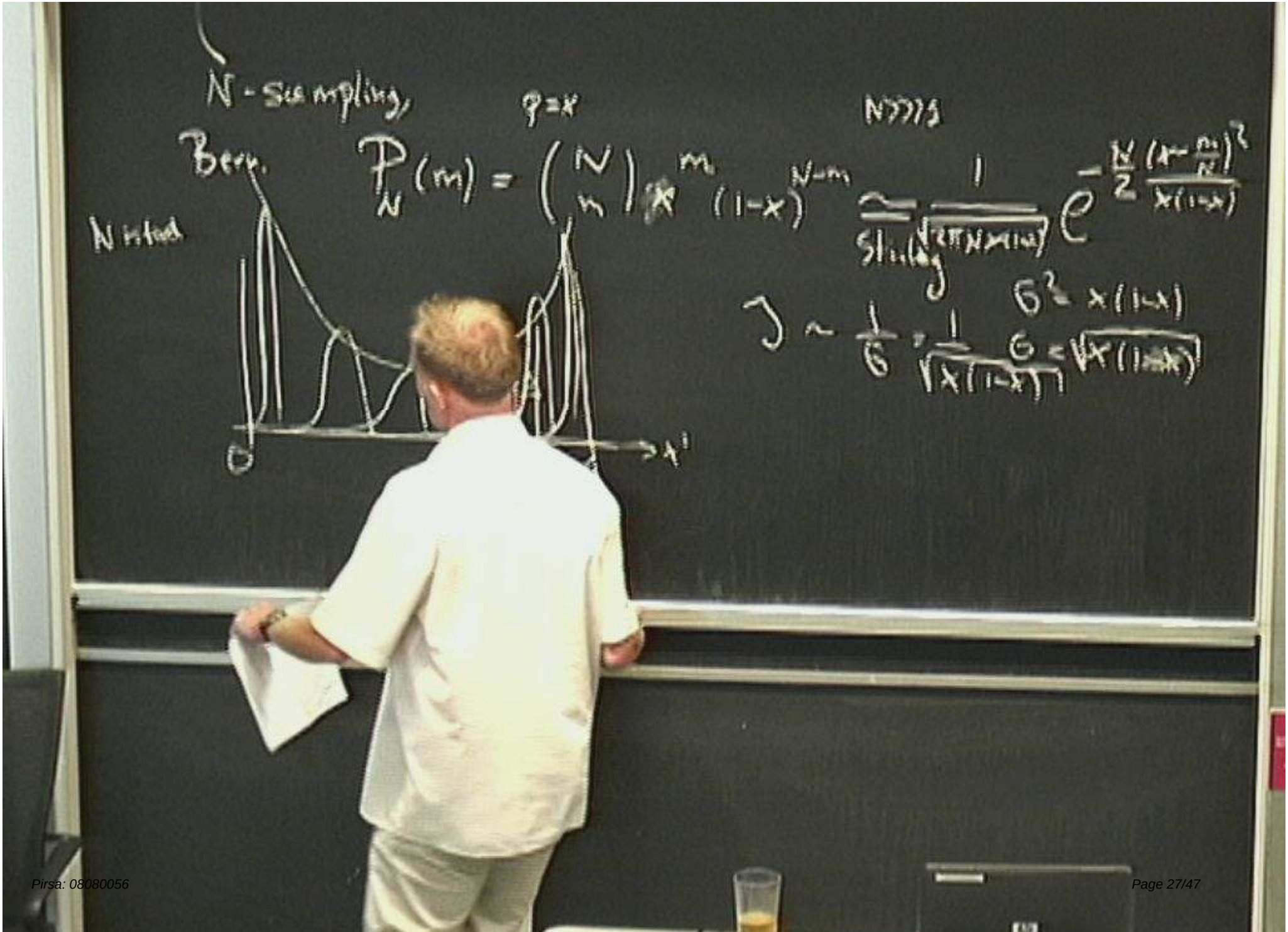
$\sigma^2 = x(1-x)$

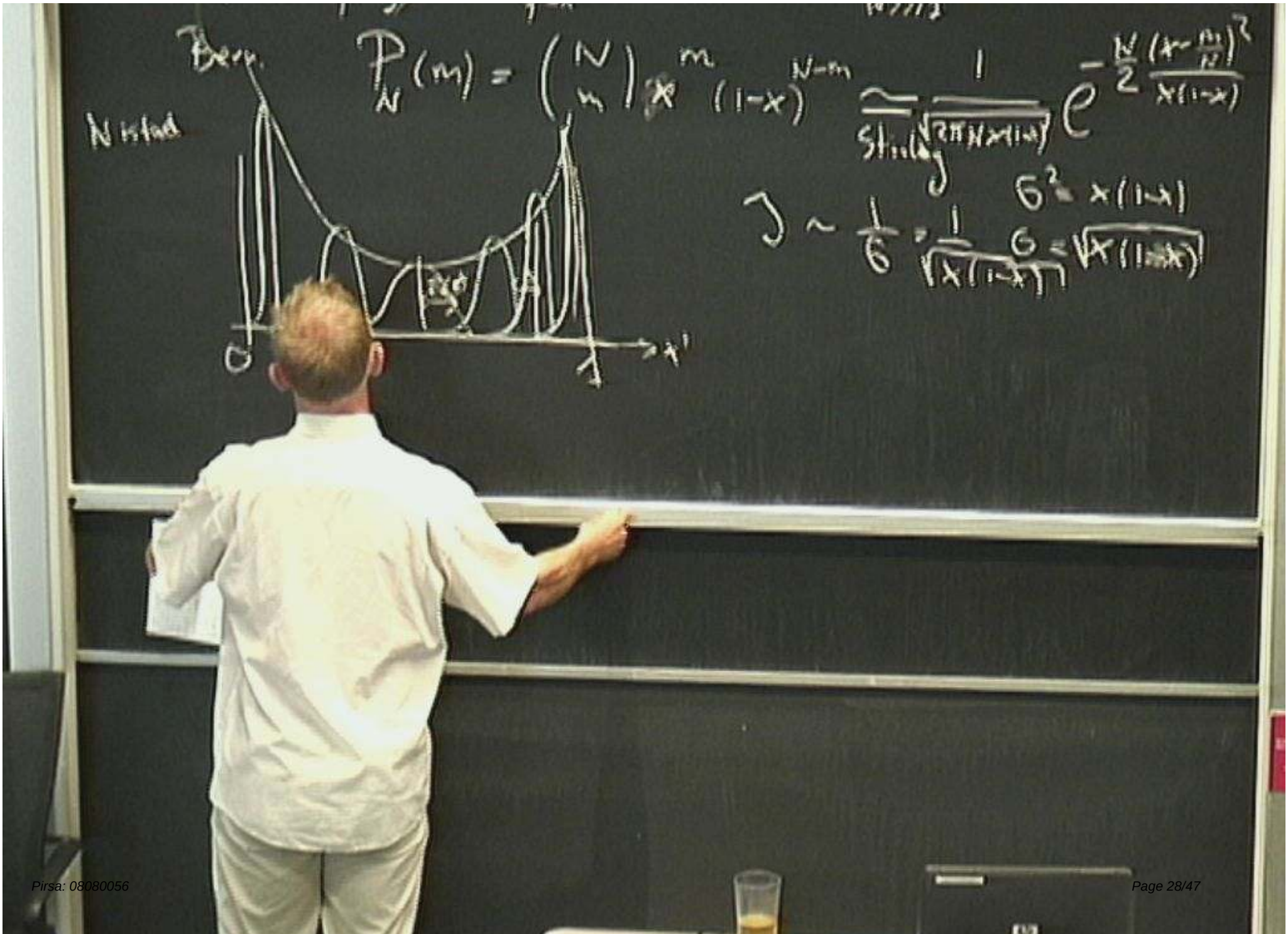
$\sigma = \sqrt{x(1-x)}$

N istod




P(s)~e^-s
s
x, x∈[0,1]
P(x)=
1
π√(x(1-x))
Jeffreys prior
x'

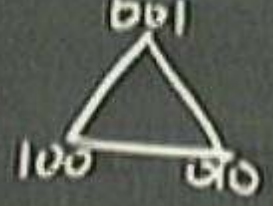

N - Sampling
p=x
N>>1
Bern.
$P_N(m) = \binom{N}{m} x^m (1-x)^{N-m}$
$\simeq \frac{1}{\sqrt{2\pi N x(1-x)}} e^{-\frac{N}{2}\frac{(x-\frac{m}{N})^2}{x(1-x)}}$
N fixed
$J \sim \frac{1}{\sigma}$
$\sigma^2 = x(1-x)$
$\sigma = \sqrt{x(1-x)}$

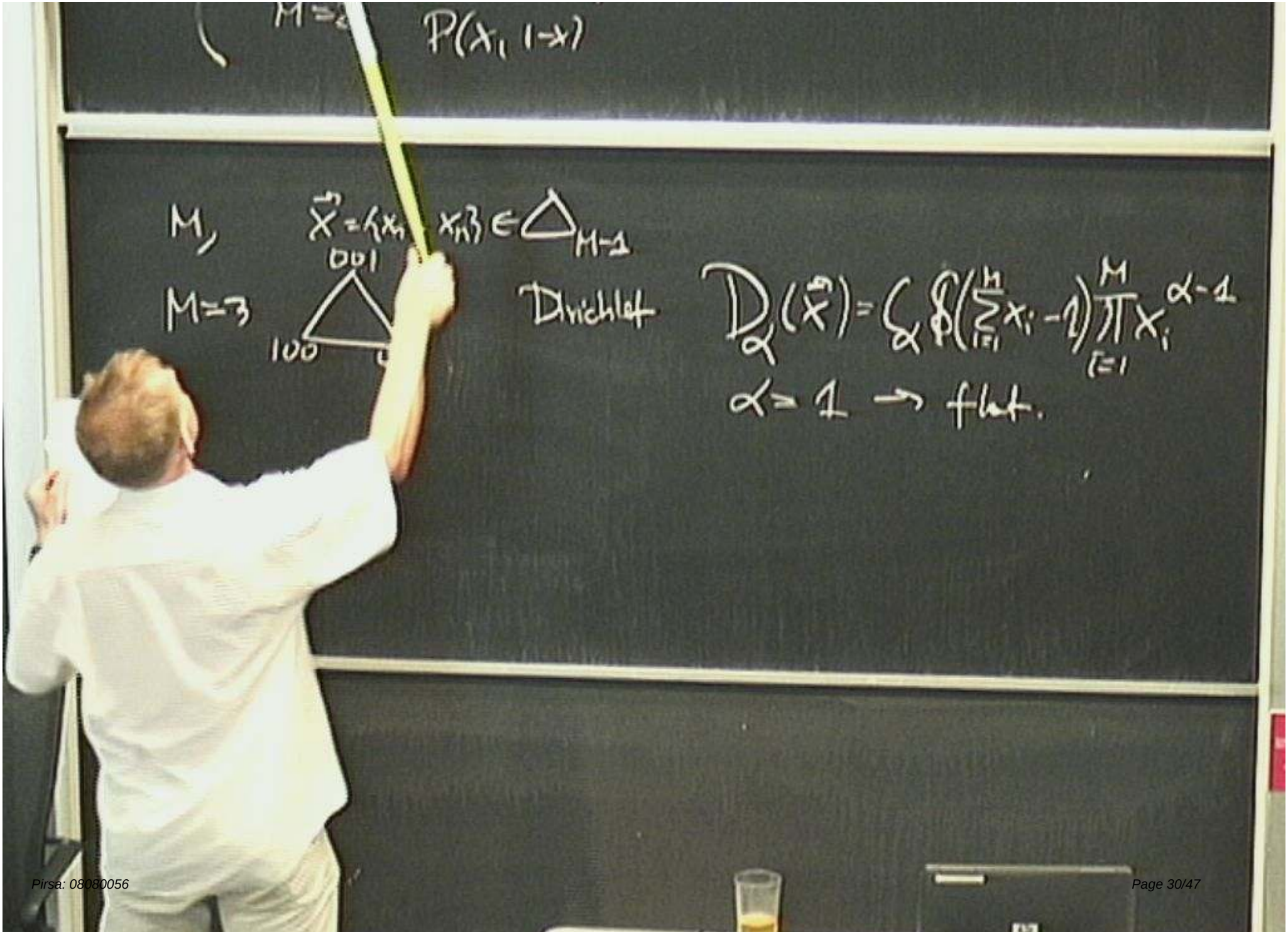

Bern.
$P_N(m) = \binom{N}{m} x^m (1-x)^{N-m}$
$\simeq \frac{1}{\sqrt{2\pi N x(1-x)}} e^{-\frac{N}{2}\frac{(x-\frac{m}{N})^2}{x(1-x)}}$
Stirling
$J \sim \frac{1}{6} \cdot \frac{1}{\sqrt{x(1-x)}}$
$\sigma^2 = x(1-x)$
$\sigma = \sqrt{x(1-x)}$
N fixed
0
x'

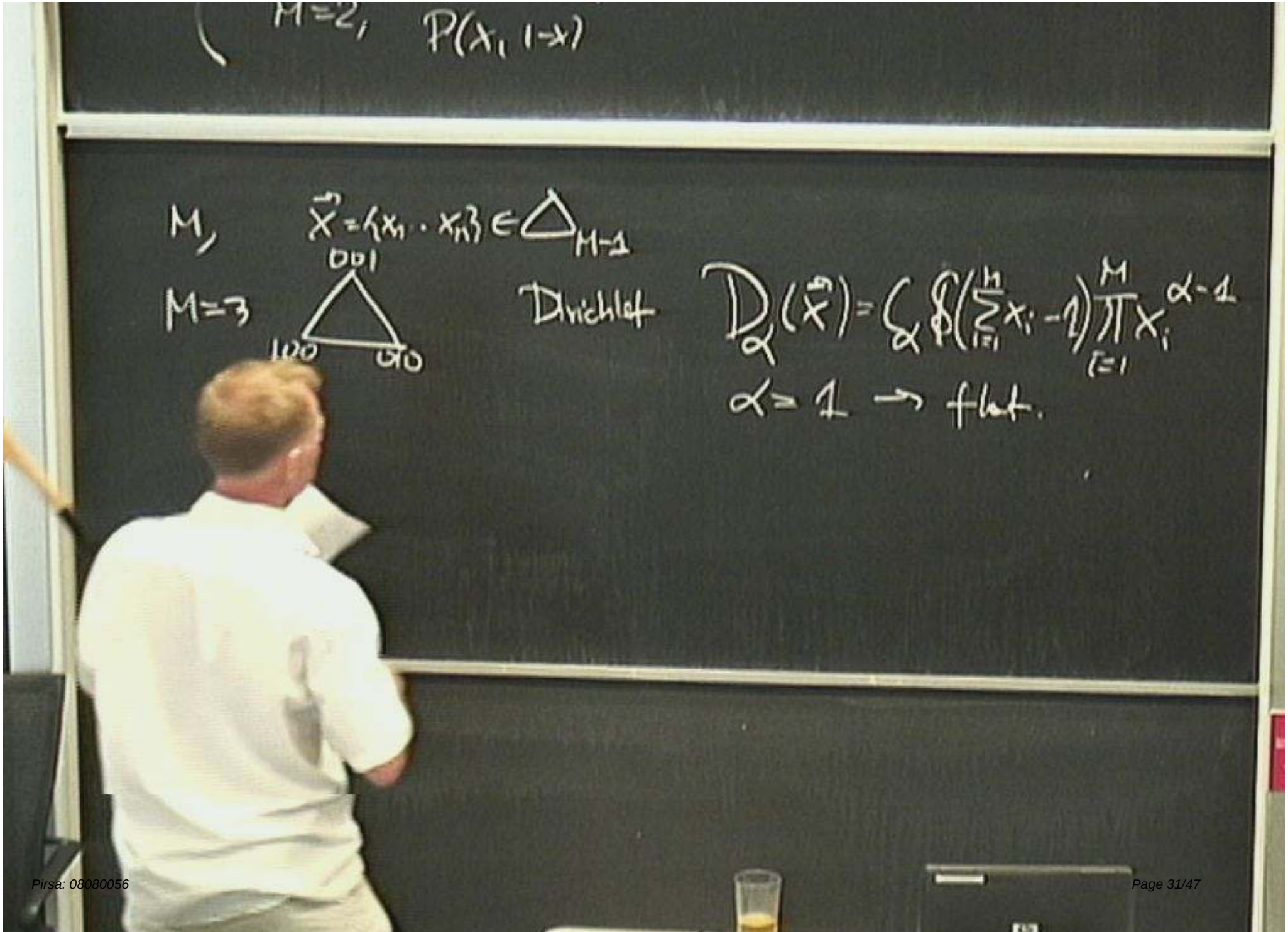
$M=2$, $P(x, 1-x)$

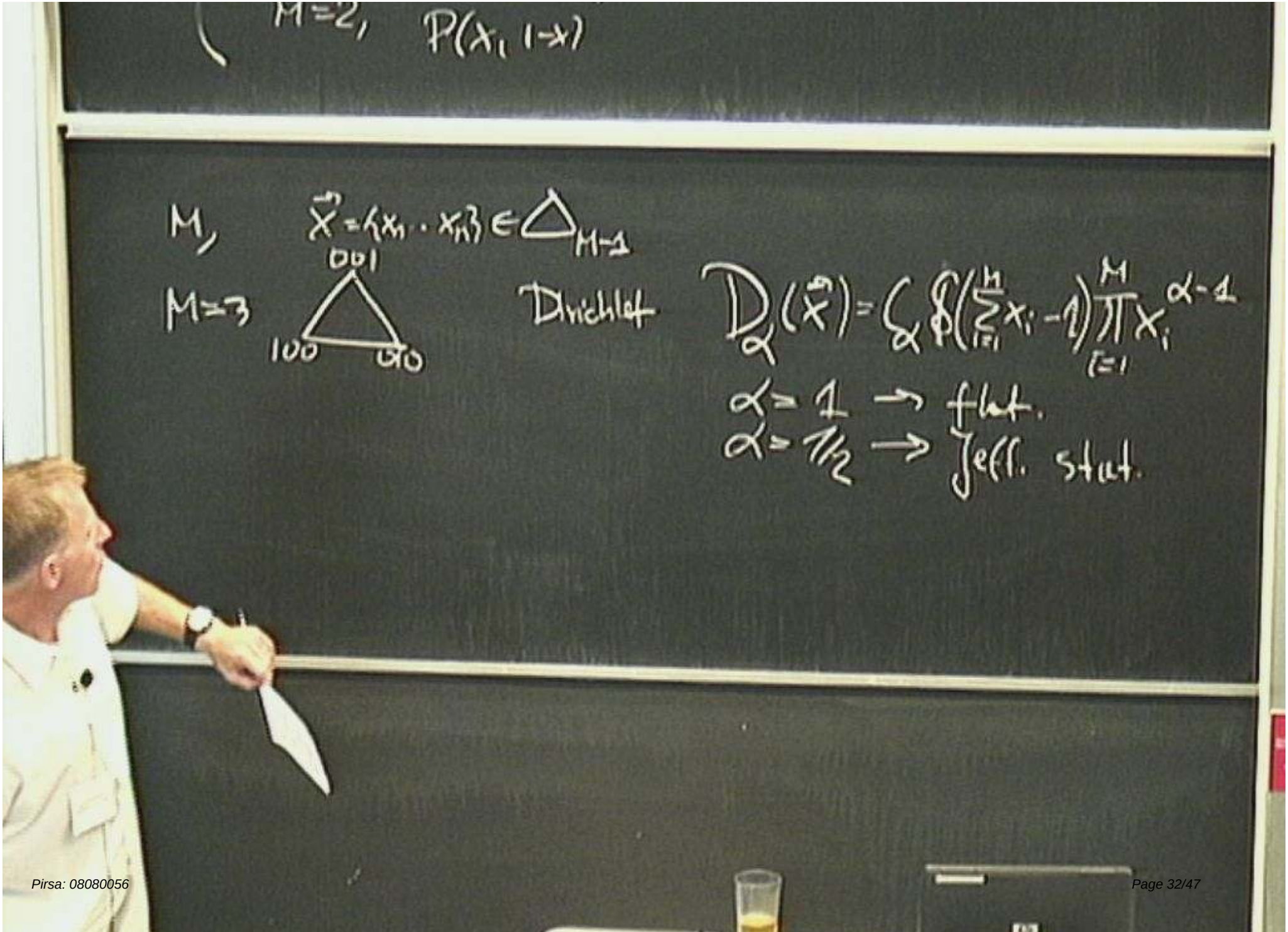
M, $\vec{x} = (x_1, \ldots, x_M) \in \Delta_{M-1}$

$M=3$



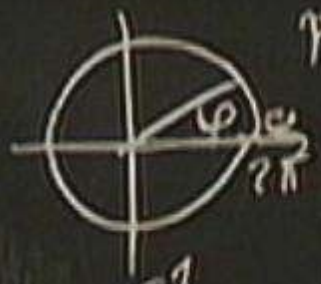

P(x, 1→)
M=3
001
100
Dirichlet
α=1 → flat.


M=2, P(x, 1→x)
M,
$\vec{x} = (x_1, \ldots, x_M) \in \Delta_{M-1}$
001
M=3
100
010
Dirichlet
$D_\alpha(\vec{x}) = \frac{\Gamma(\sum_{i=1}^{M} \alpha_i)}{\prod_{i=1}^{M} \Gamma(\alpha_i)} \prod_{i=1}^{M} x_i^{\alpha_i - 1}$
$\alpha = 1 \rightarrow$ flat.


M=2, P(x, 1->)
M,
$\vec{x} = (x_1 \cdots x_M) \in \Delta_{M-1}$
M=3
001
100
010
Dirichlet
$D_\alpha(\vec{x}) = \frac{\Gamma(\sum_{i=1}^{M} \alpha_i)}{\prod_{i=1}^{M} \Gamma(\alpha_i)} \prod_{i=1}^{M} x_i^{\alpha_i - 1}$
$\alpha = 1 \rightarrow$ flat.
$\alpha = 1/2 \rightarrow$ Jeffr. stat.

$x \in [0,1) \rightarrow \varphi = 2\pi x$

$P(\varphi) = \frac{1}{2\pi}$



Haar measure.

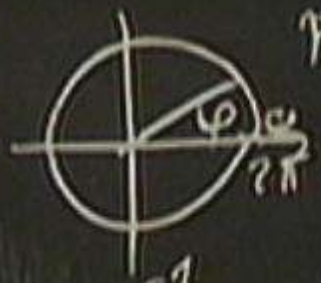
fact $\vec{y} \in S^{N-1}$, $\sum y_i^2 = 1$

S^1 $d\varphi$

S^2 $d\cos\theta\, d\varphi$

$U(N)$

$x \in [0,1) \rightarrow \varphi = 2\pi x$



$P(\varphi) = \frac{1}{2\pi}$

Haar mens.

fat $y \in S^{N-1}$, $\sum_i y_i^2 = 1$

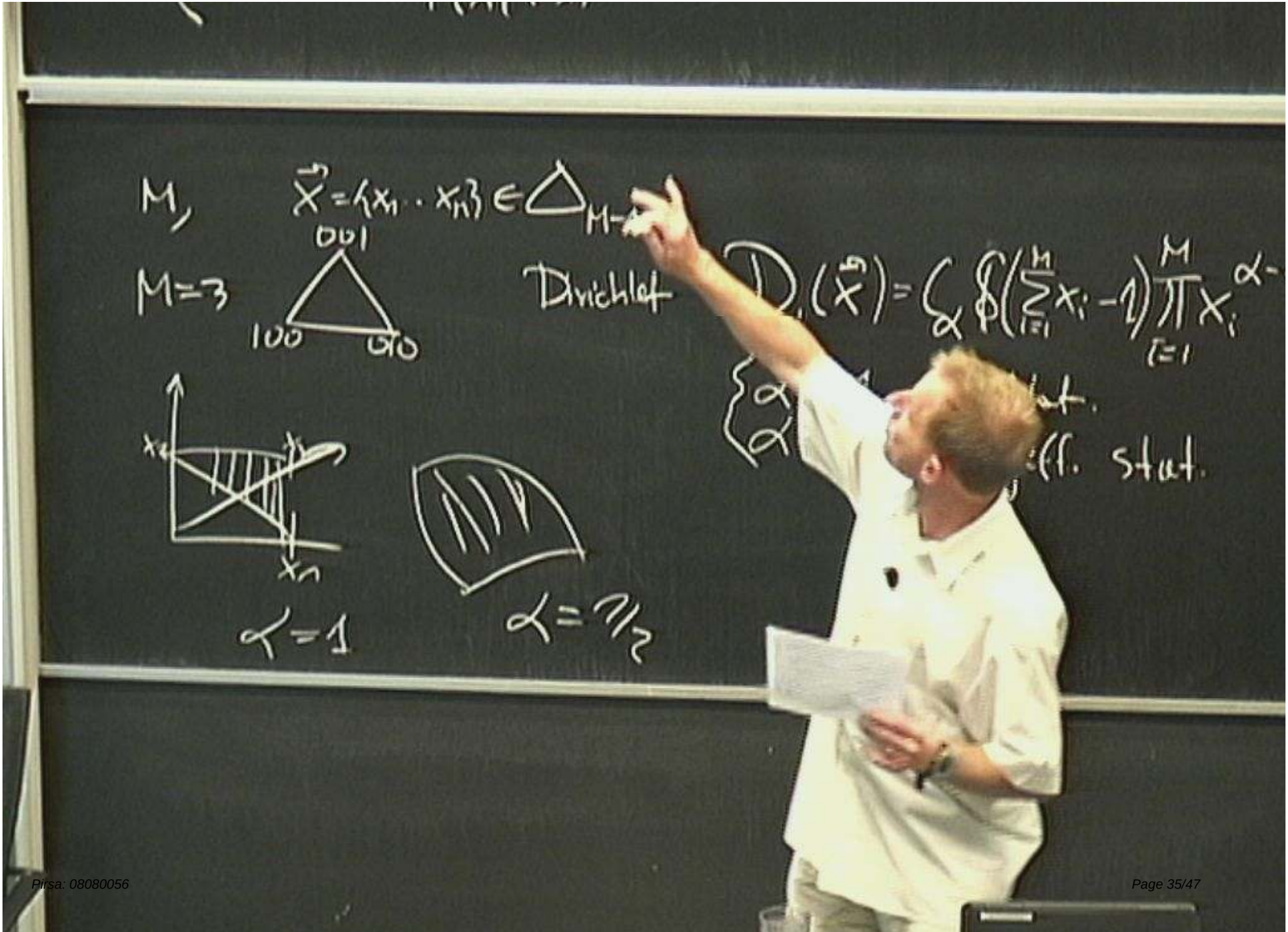
$x_i = y_i^2 \Rightarrow \vec{x} \in \Delta_{N-1}$

$\vec{x}$ is dist. stat. $\alpha = 1/2$

S^1 $d\varphi$

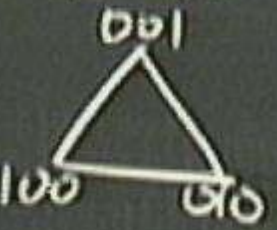
S^2 $d\cos\theta\, d\varphi$

$U(N)$


M,
M=3
001
100
010
Dirichlet
stat.
x_1
x_n
$\alpha = 1$
$\alpha = 1/2$

M, $\vec{x} = (x_1 \cdots x_M) \in \Delta_{M-1}$

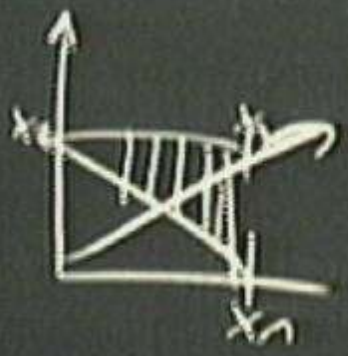
$M=3$



Dirichlet

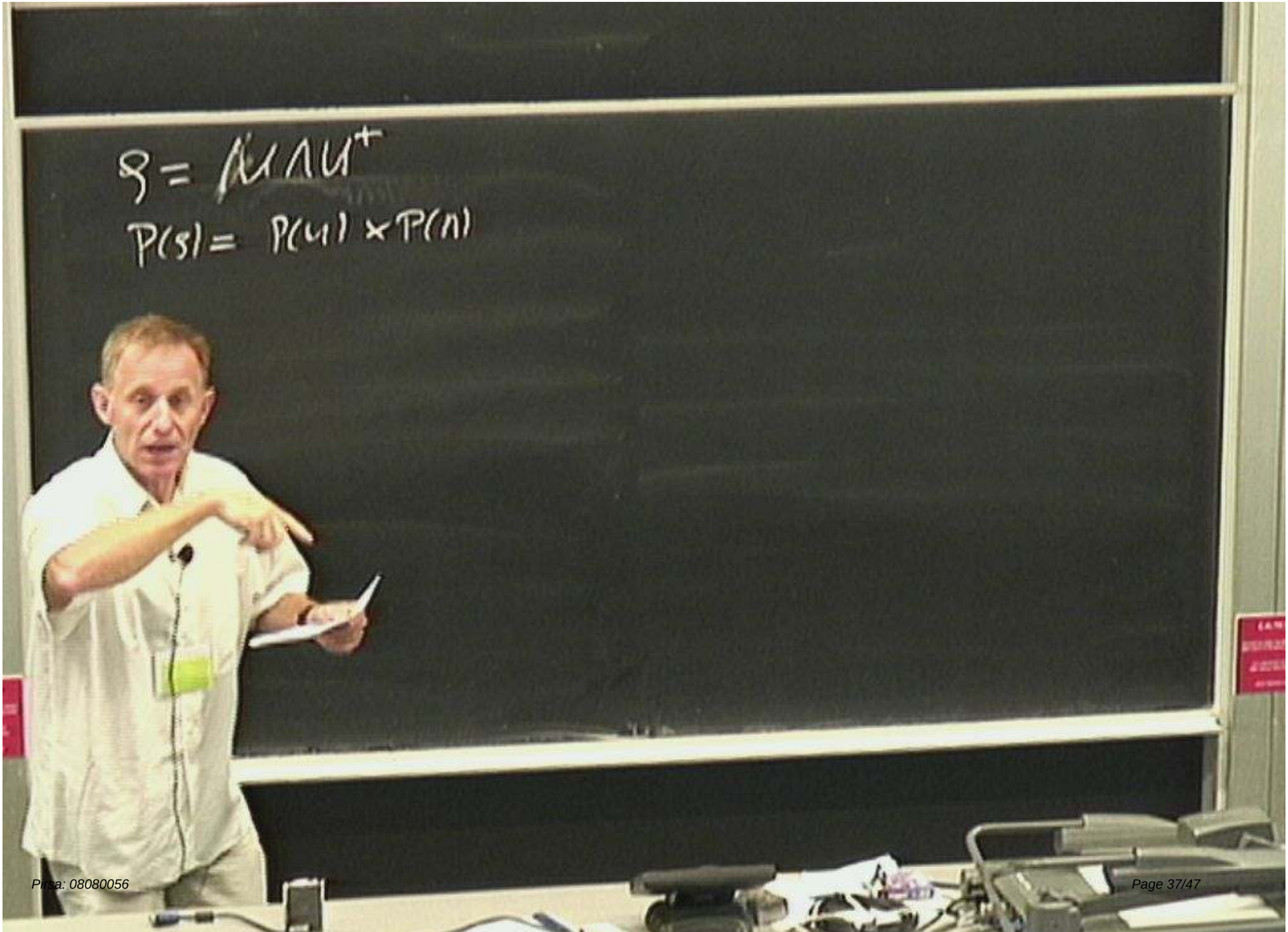
$$D_\alpha(\vec{x}) = \frac{\Gamma(\sum_{i=1}^{M} x_i)}{\prod_{i=1}^{M} \Gamma(x_i)} \prod_{i=1}^{M} x_i^{\alpha-1}$$

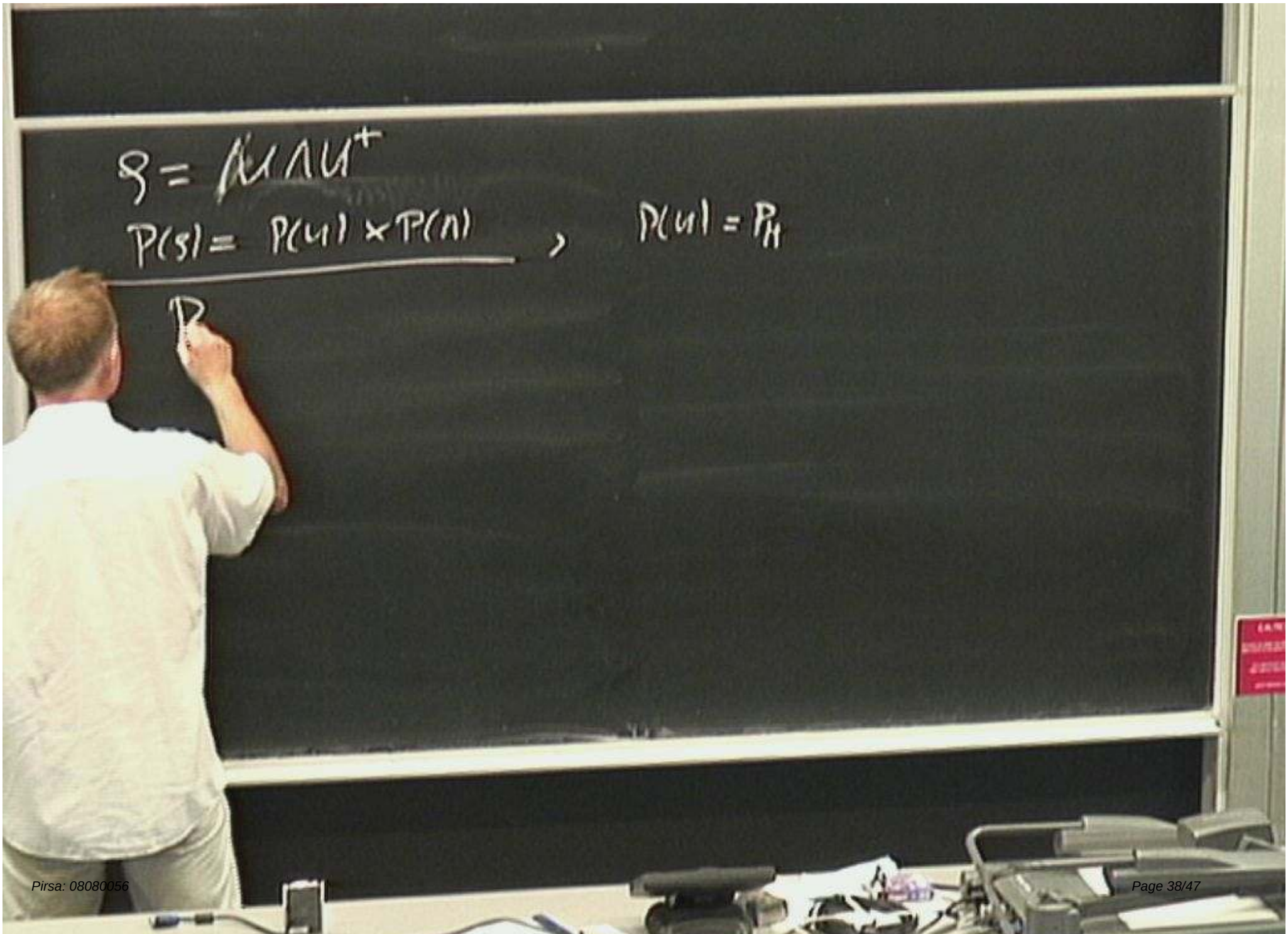
$\begin{cases} \alpha = 1 \rightarrow \text{flat.} \\ \alpha = 1/2 \rightarrow \text{Jeff. stat.} \end{cases}$



$\alpha = 1$

$\alpha = 1/2$


S = ΛU ΛU+
P(S) = P(U) × P(Λ)

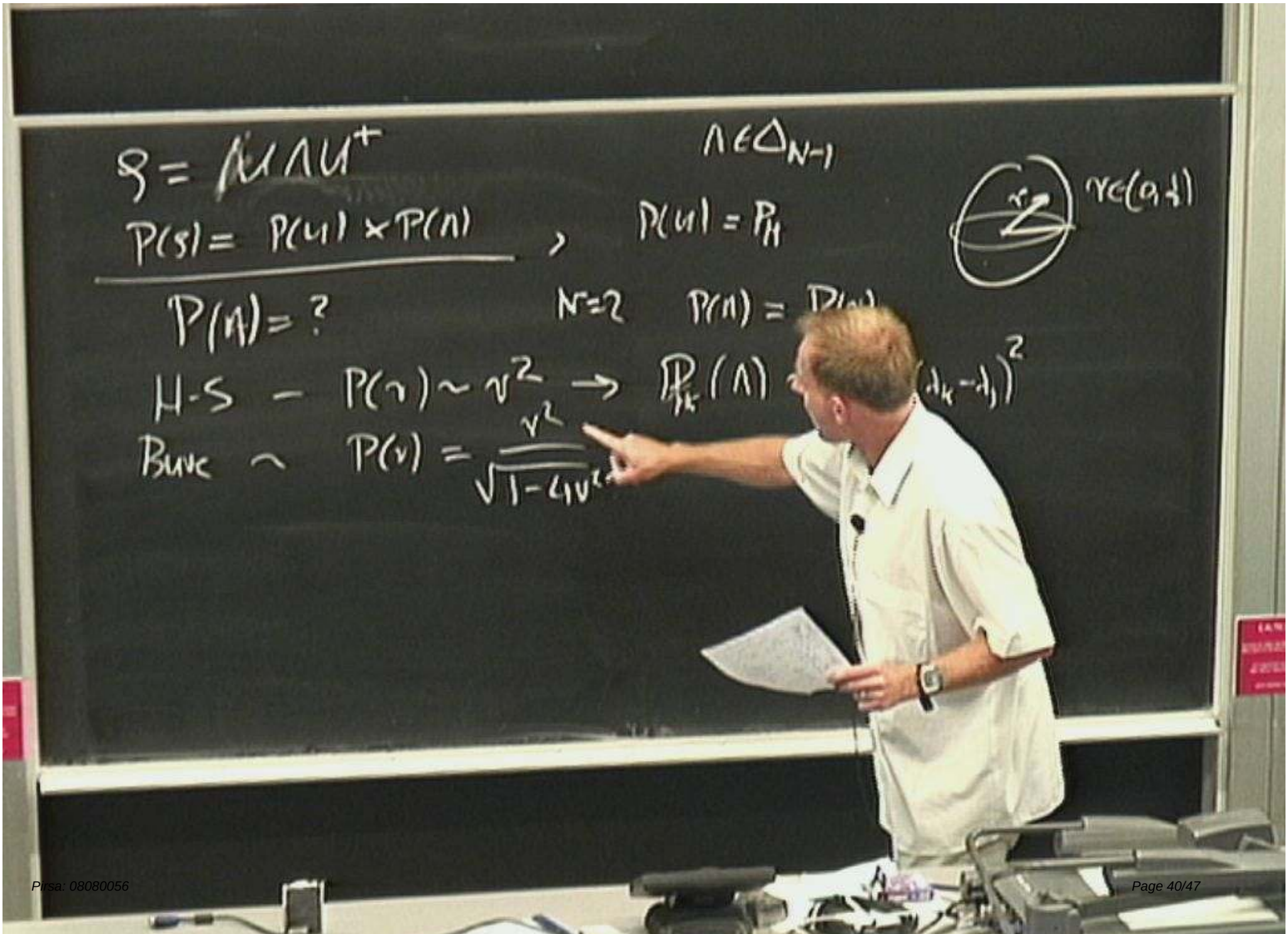

$S = U \Lambda U^{+}$
$P(s) = P(u) \times P(\Lambda)$
$P(u) = P_H$

$S = U \Lambda U^+$ $\quad \Lambda \in \Delta_{N-1}$

$P(S) = P(U) \times P(\Lambda)$, $\quad P(U) = P_H$

$P(\Lambda) = ?$ $\quad N=2 \quad P(\Lambda) \equiv P(\nu)$

$H\text{-}S - P(\nu) \sim \nu^2 \rightarrow R_k(\Lambda) \sim \prod_{j<k} (\lambda_k - \lambda_j)^2$


$\rho = U \Lambda U^+$
$\Lambda \in \Delta_{N-1}$
$P(s) = P(u) \times P(\Lambda)$
$P(u) = P_H$
$v \in (0,1)$
$P(\Lambda) = ?$
$N=2$
H-S – $P(\eta) \sim v^2 \rightarrow$
Bures $\sim P(v) = \frac{v^2}{\sqrt{1-4v^2}}$

$\rho = U \Lambda U^+$ $\quad \Lambda \in \Delta_{N-1}$

$P(\rho) = P(U) \times P(\Lambda)$, $\quad P(U) = P_H$

$V \in (\mathbb{C}^N)^1$

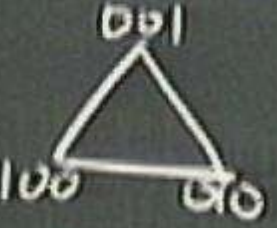
$P(\Lambda) = ?$ $\quad N=2 \quad P(\lambda) = P(\nu)$

H-S – $P(\nu) \sim \nu^2 \rightarrow P_k(\Lambda) \sim \prod_{j<k} (\lambda_k - \lambda_j)^2$

Bures $\sim P(\nu) = \frac{\nu^2}{\sqrt{1-4\nu^2}}$

M, $\vec{x} = \{x_1 \dots x_n\} \in \Delta_{M-1}$

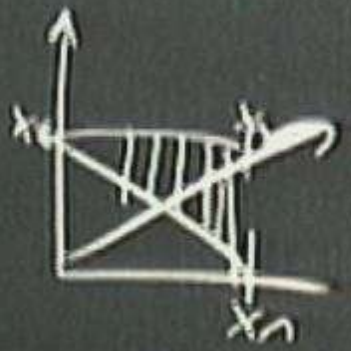
$M=3$



Dirichlet

$$D_\alpha(\vec{x}) = \frac{\Gamma(\sum_{i=1}^{M} \alpha_i)}{\prod_{i=1}^{M} \Gamma(\alpha_i)} \prod_{i=1}^{M} x_i^{\alpha_i - 1}$$

$\begin{cases} \alpha = 1 \rightarrow \text{flat.} \\ \alpha = 1/2 \rightarrow \text{Jeff. stat.} \end{cases}$



$\alpha = 1$

$\alpha = 1/2$

$\rho = U \Lambda U^+$ $\quad \Lambda \in \Delta_{N-1}$

$P(\rho) = P(U) \times P(\Lambda)$, $\quad P(U) = P_H$

$r \in (0, \frac{1}{2})$

$P(\Lambda) = ?$ $\quad N=2 \quad P(\Lambda) = P(r)$

H-S – $P(r) \sim r^2 \rightarrow P_{jk}(\Lambda) \sim \prod_{j<k} (\lambda_k - \lambda_j)^2 \rightarrow \alpha = 1$ (FWL)

Bures $\sim P(r) = \frac{r^2}{\sqrt{1-4r^2}} \longrightarrow \alpha = \frac{1}{2}$ (Stch.)

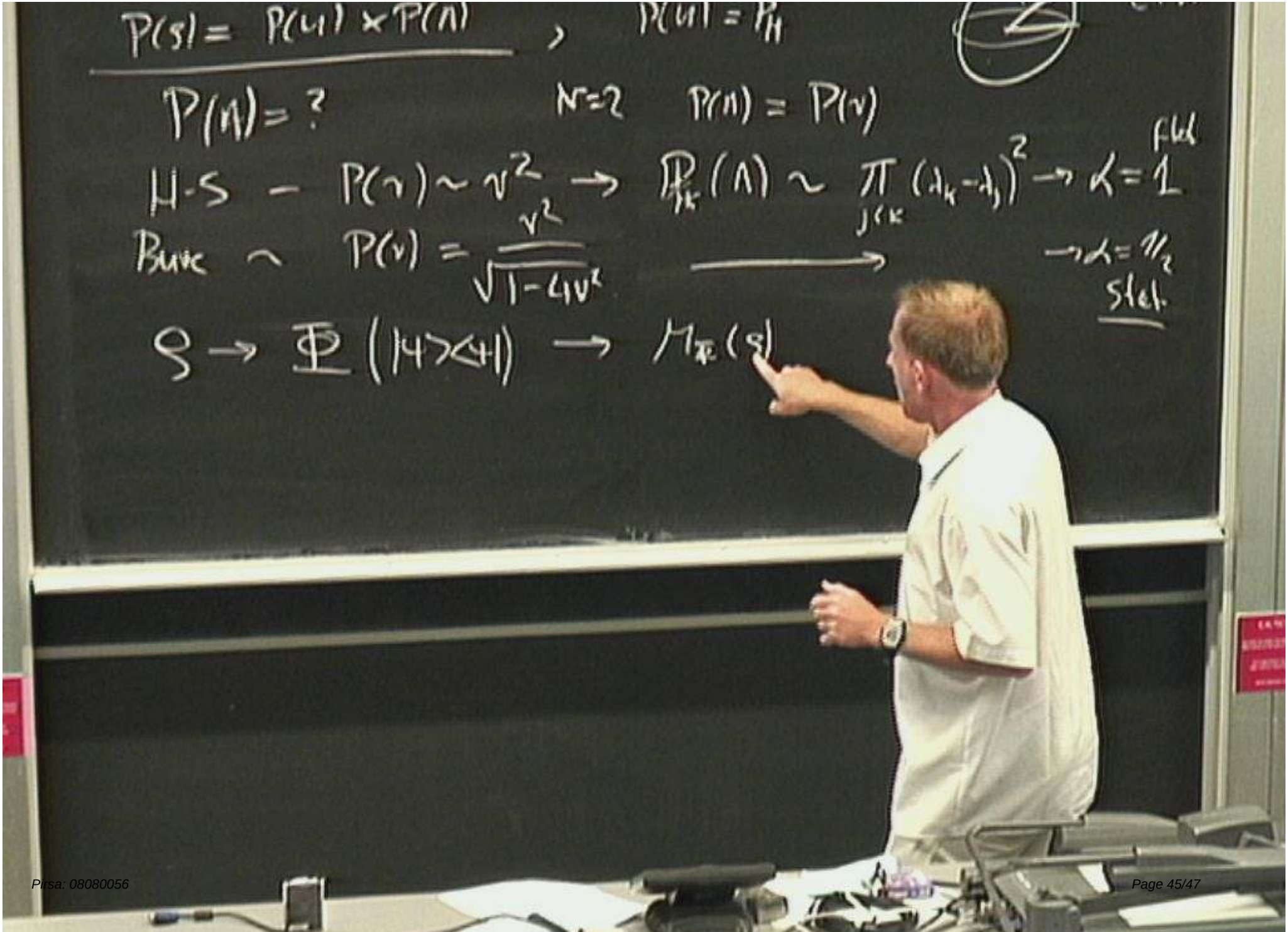
$P(s) = P(u) \times P(n)$, $P(u) = P_H$

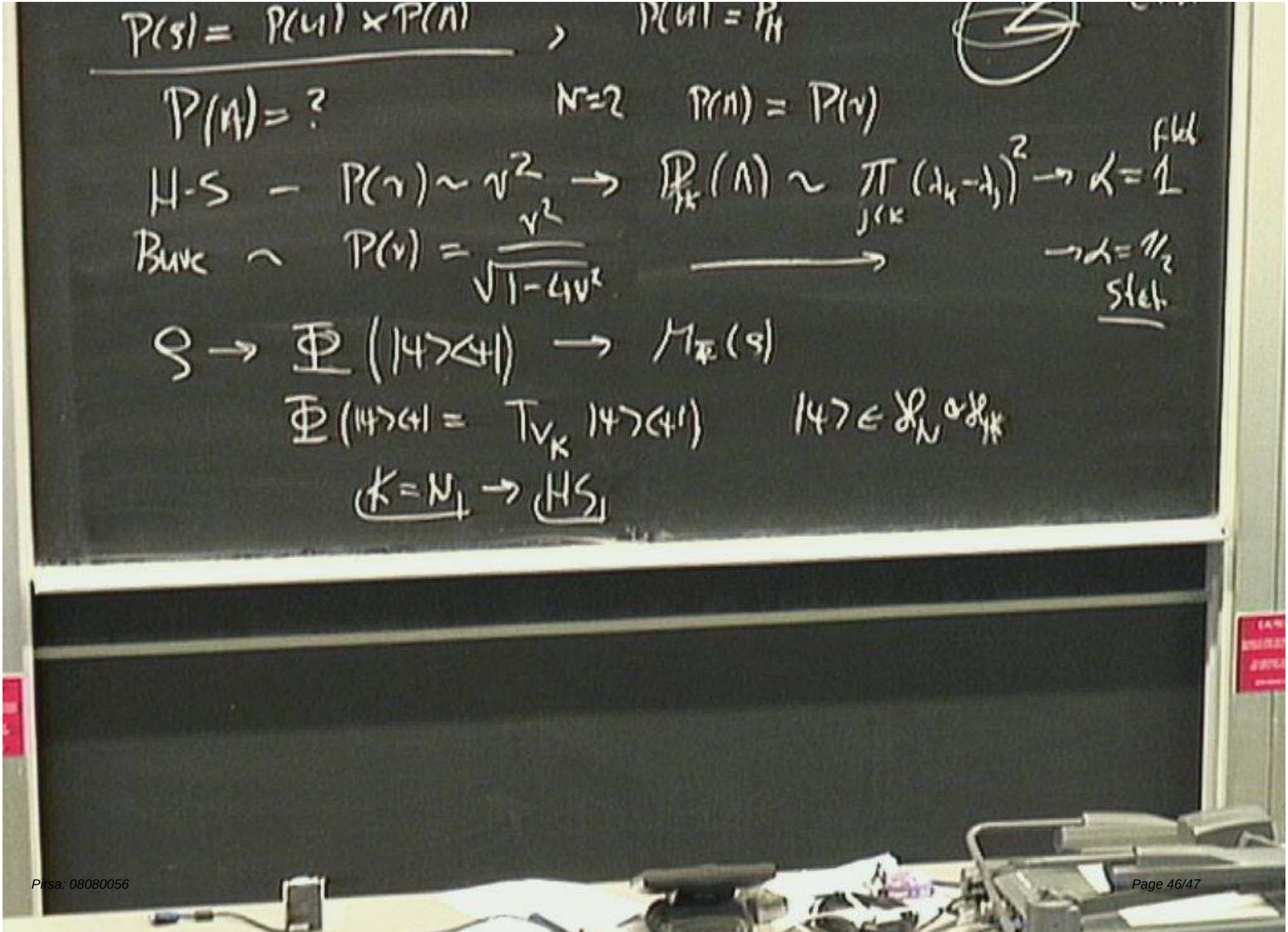
$P(n) = ?$ $N=2$ $P(n) = P(v)$

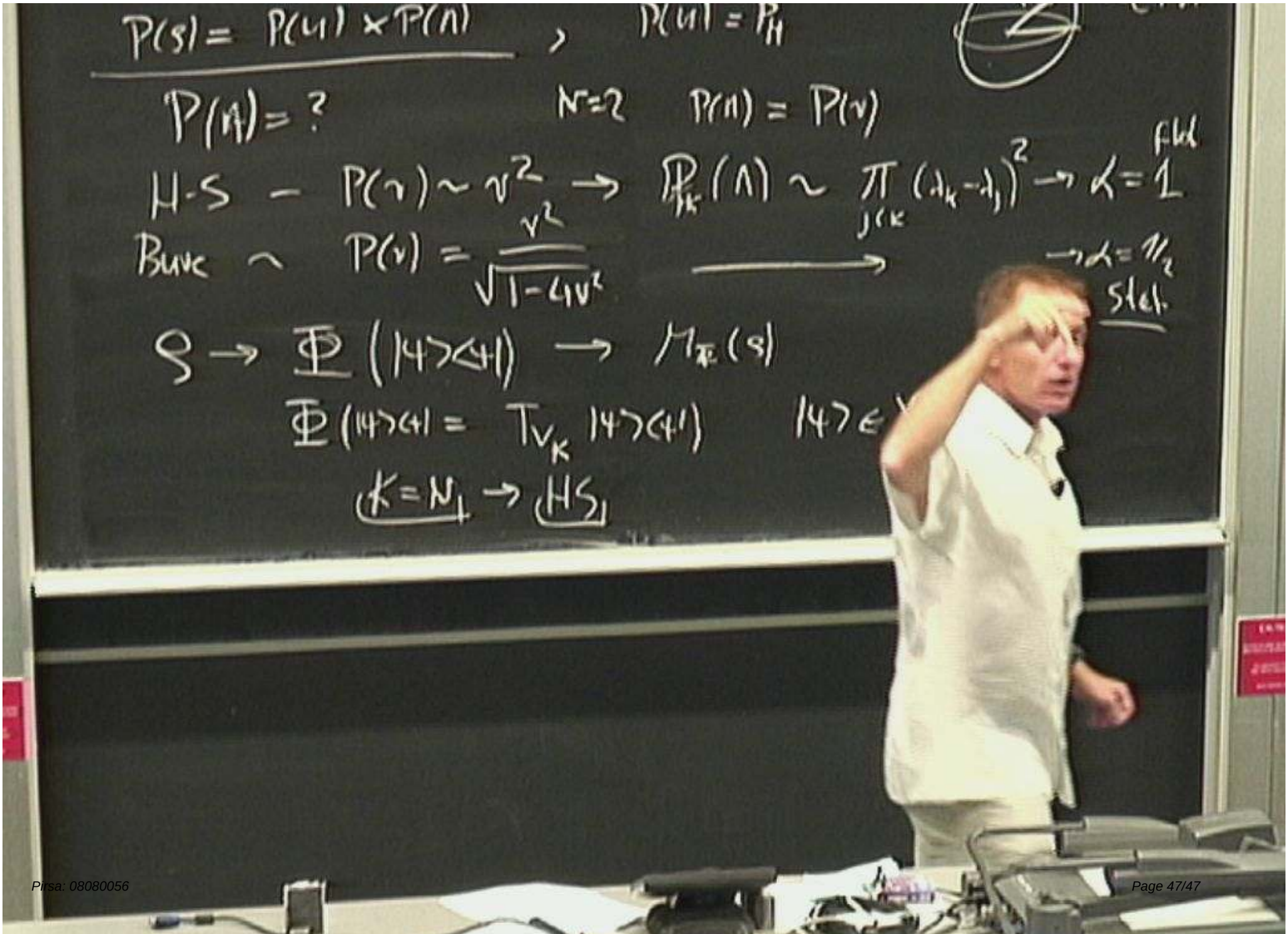
H-S – $P(v) \sim v^2 \rightarrow P_{H_k}(n) \sim \prod_{j<k} (\lambda_k - \lambda_j)^2 \rightarrow \alpha = 1$ flat

Buw $\sim P(v) = \frac{v^2}{\sqrt{1-4v^2}} \longrightarrow \alpha = 1/2$ Stat.

$\rho \rightarrow \Phi(|\psi\rangle\langle\psi|)$


P(s) = P(+) × P(-)
P(+) = ?
N=2
P(+) = P(-)
H-S
Born
Stat.


P(s) = P(u) × P(n)
P(n) = ?
N=2 P(n) = P(v)
H-S – P(r) ~ v² → P_k(Λ) ~ Π_{j<k} (λ_k − λ_j)² → α = 1
full
B_sing ~ P(v) = v² / √(1−4v²) → α = π/2
Stat
ρ → Φ(|ψ⟩⟨ψ|) → M_k(ρ)
Φ(|ψ⟩⟨ψ|) = T_{V_k} |ψ⟩⟨ψ|
|ψ⟩ ∈ H_N ⊗ H_k
k = N → HS


P(s) = P(u) × P(n)
P(n) = ?
N=2 P(n) = P(ν)
H-S – P(ν) ~ ν²
Bure ~ P(ν) = ν² / √(1-4ν²)
ρ → Φ (|ψ><ψ|) → M_R(s)
Φ(|ψ><ψ| = T_V_K |ψ><ψ|)
K = N_L → HS