Title: Tomography on the Sphere

Date: Aug 28, 2008 03:15 PM

URL: http://pirsa.org/08080055

Abstract: a short presentation about work characterizing triphoton states via Wigner functions on the PoincarÃ© sphere.

Tomography on the Poincaré Sphere

Lynden K. Shalm, Robert B. Adamson, and Aephraim M. Steinberg

Center for Quantum Information and Quantum Control

Institute for Optical Sciences

Department of Physics

University of Toronto

August 28th, 2008

Triphoton States

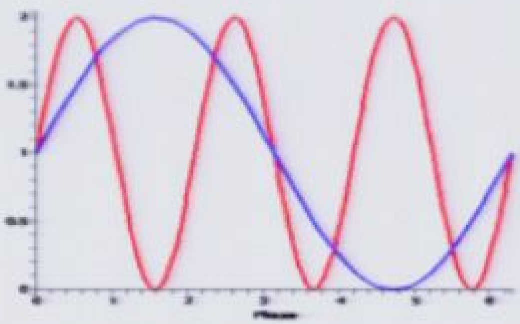
States made up of 3 indistinguishable photons (same spatial mode, frequency, chirp, etc...) with one degree of freedom (polarization).

$$|\psi\rangle = a|3,0\rangle_{H,V} + b|2,1\rangle_{H,V} + c|1,2\rangle_{H,V} + d|0,3\rangle_{H,V}$$

Most Famous Triphoton is the 3-N00N state

$$|3,0\rangle_{R,L} + e^{i3\phi}|0,3\rangle_{R,L}$$

Phase Super-Sensitivity, Phase Super-Resolution

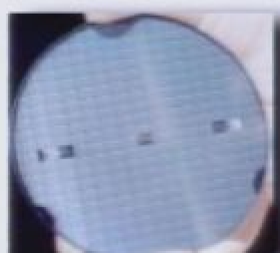


Shot-Noise Limit

$$\Delta\phi = \frac{1}{\sqrt{N}}$$

Heisenberg Limit

$$\Delta\phi = \frac{1}{N}$$



J.P. Dowling, Phys. Rev. A 57, 4736–4746 (1998)
M.W. Mitchell et al., Nature 429, 161 (2004)
Z. Y. Ou, Phys. Rev. A 55, 2598–26 (1997)
T. Na... e 316, 726 (2007)

Producing Triphoton States

Important Factorization!

$$c_1 a_H^{\dagger\,3} + c_2 a_H^{\dagger\,2} a_V^\dagger + c_3 a_H^\dagger a_V^{\dagger\,2} + c_4 a_V^{\dagger\,3} = (A_1 a_H^\dagger + B_1 a_V^\dagger)(A_2 a_H^\dagger + B_2 a_V^\dagger)(A_3 a_H^\dagger + B_3 a_V^\dagger)$$

NOON State

$$(a_R^{\dagger 3} + a_L^{\dagger 3}) = \underbrace{(a_R^\dagger + a_L^\dagger)}\underbrace{(a_R^\dagger + e^{2\pi i/3} a_L^\dagger)}\underbrace{(a_R^\dagger + e^{-2\pi i/3} a_L^\dagger)}$$

Linear Polarizations

$$|3,0\rangle_{R,L} + e^{i3\phi}|0,3\rangle_{R,L} =$$

Need to put three photons of different polarizations into the same mode.

H. Lee et al., Phys. Rev. A **65**, 030101 (2002)

J. Fiurášek, Phys. Rev. A **65**, 053818 (2002)

Triphoton States

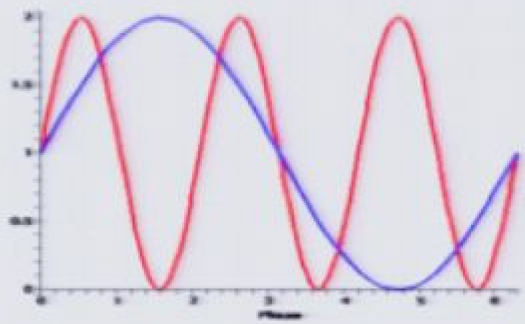
States made up of 3 indistinguishable photons (same spatial mode, frequency, chirp, etc...) with one degree of freedom (polarization).

$$|\psi\rangle = a|3,0\rangle_{H,V} + b|2,1\rangle_{H,V} + c|1,2\rangle_{H,V} + d|0,3\rangle_{H,V}$$

Most Famous Triphoton is the 3-N00N state

$$|3,0\rangle_{R,L} + e^{i3\phi}|0,3\rangle_{R,L}$$

Phase Super-Sensitivity, Phase Super-Resolution

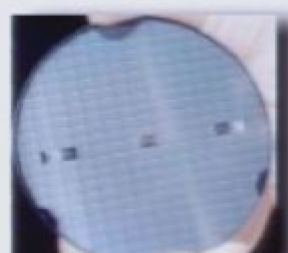


Shot-Noise Limit

$$\Delta\phi = \frac{1}{\sqrt{N}}$$

Heisenberg Limit

$$\Delta\phi = \frac{1}{N}$$



J.P. Dowling, Phys. Rev. A 57, 4736–4746 (1998)
M.W. Mitchell et al., Nature 429, 161 (2004)
Z. Y. Ou, Phys. Rev. A 55, 2598–26 (1997)
T Na... 316, 726 (2007)

Producing Triphoton States

Important Factorization!

$$c_1 a_H^{\dagger\,3} + c_2 a_H^{\dagger\,2} a_V^\dagger + c_3 a_H^\dagger a_V^{\dagger\,2} + c_4 a_V^{\dagger\,3} = (A_1 a_H^\dagger + B_1 a_V^\dagger)(A_2 a_H^\dagger + B_2 a_V^\dagger)(A_3 a_H^\dagger + B_3 a_V^\dagger)$$

NOON State

$$(a_R^{\dagger 3} + a_L^{\dagger 3}) = (\underbrace{a_R^\dagger + a_L^\dagger})(a_R^\dagger \underbrace{+ e^{2\pi i/3}} a_L^\dagger)(a_R^\dagger \underbrace{+ e^{-2\pi i/3}} a_L^\dagger)$$

Linear Polarizations

$$|3,0\rangle_{R,L} + e^{i3\phi}|0,3\rangle_{R,L} =$$ 

Need to put three photons of different polarizations into the same mode.

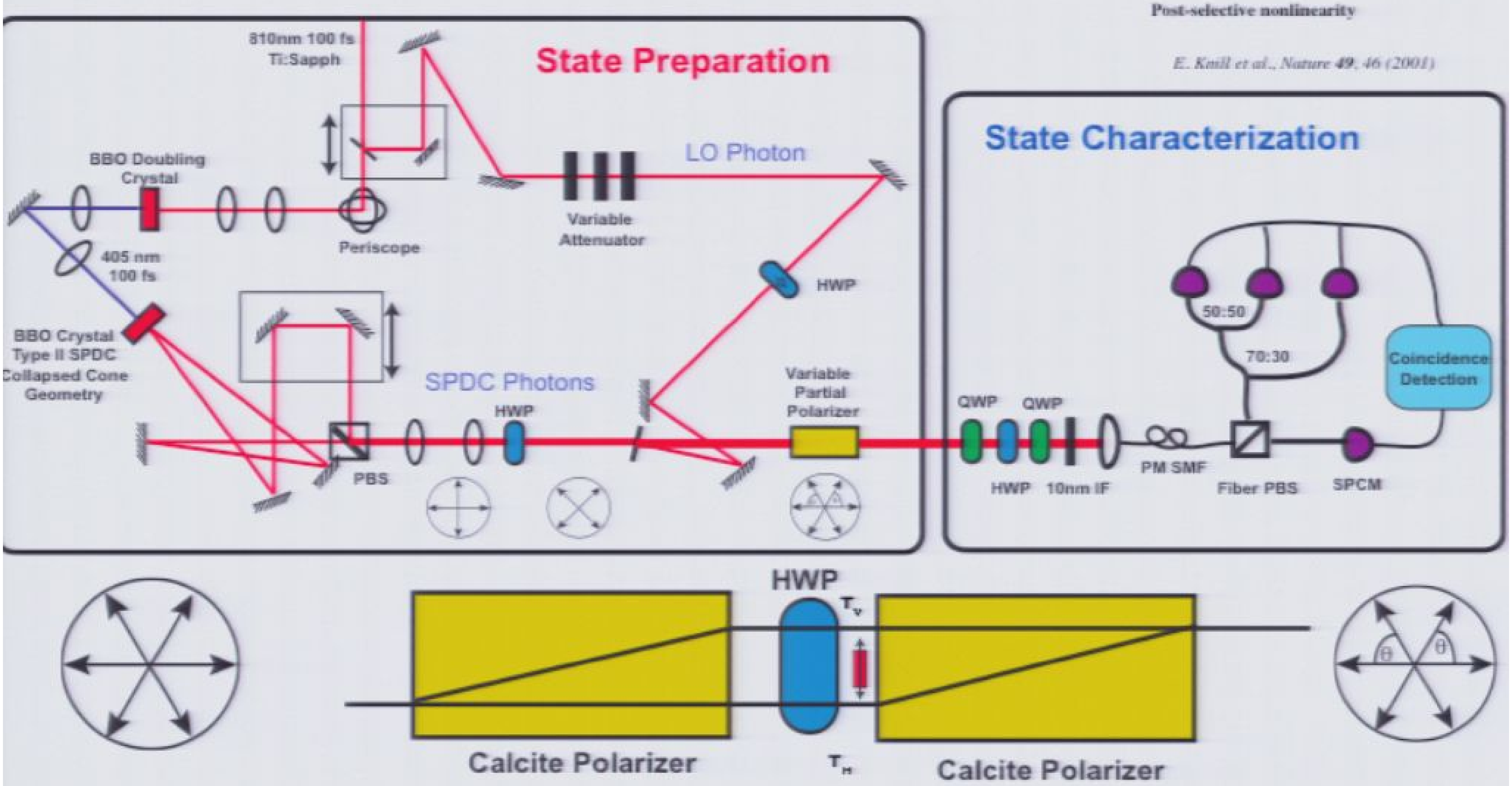
H. Lee et al., Phys. Rev. A **65**, *030101 (2002)*

J. Fiurášek, Phys. Rev. A **65**, *053818 (2002)*

Experimental Setup

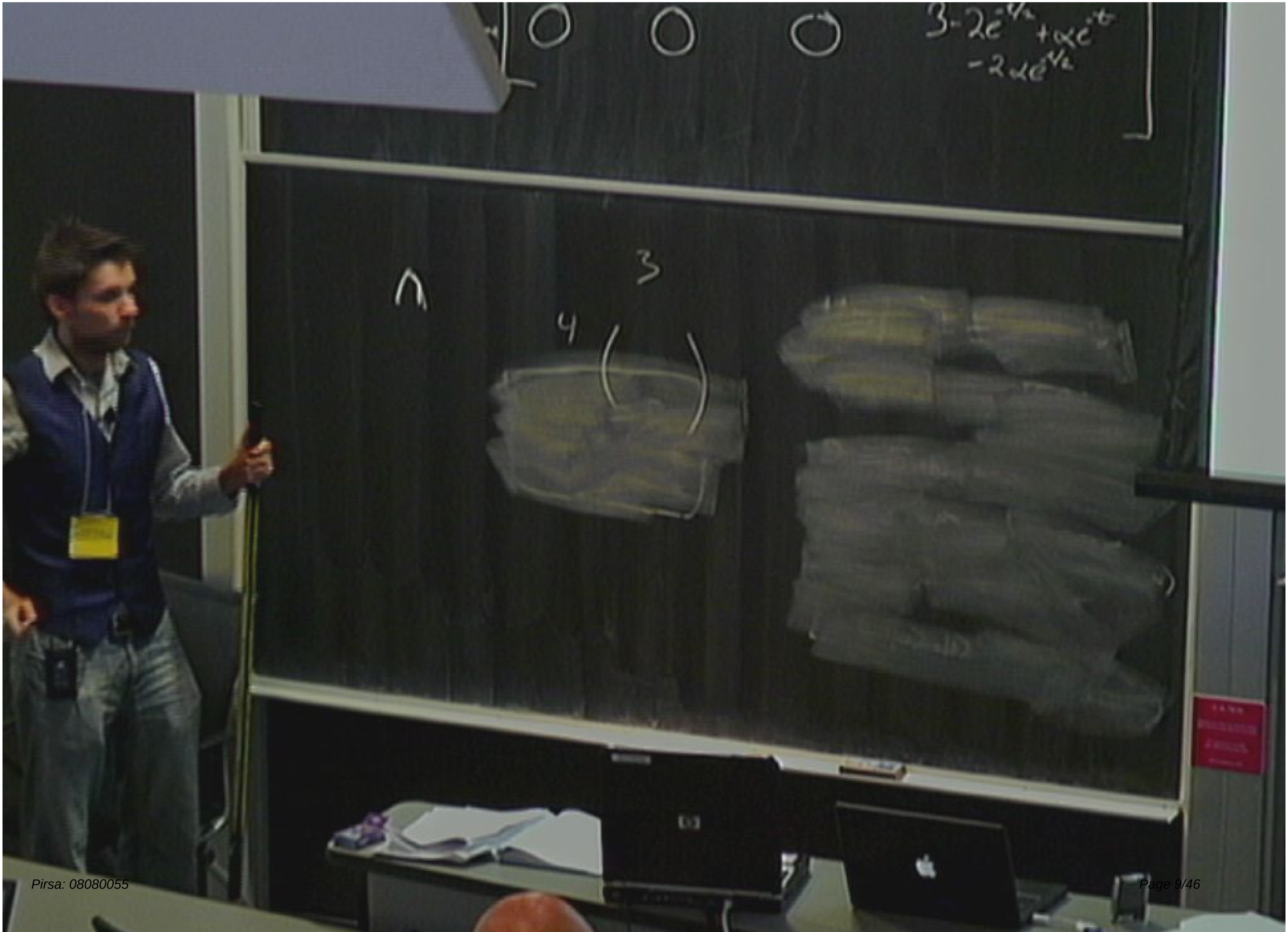
Post-selective nonlinearity

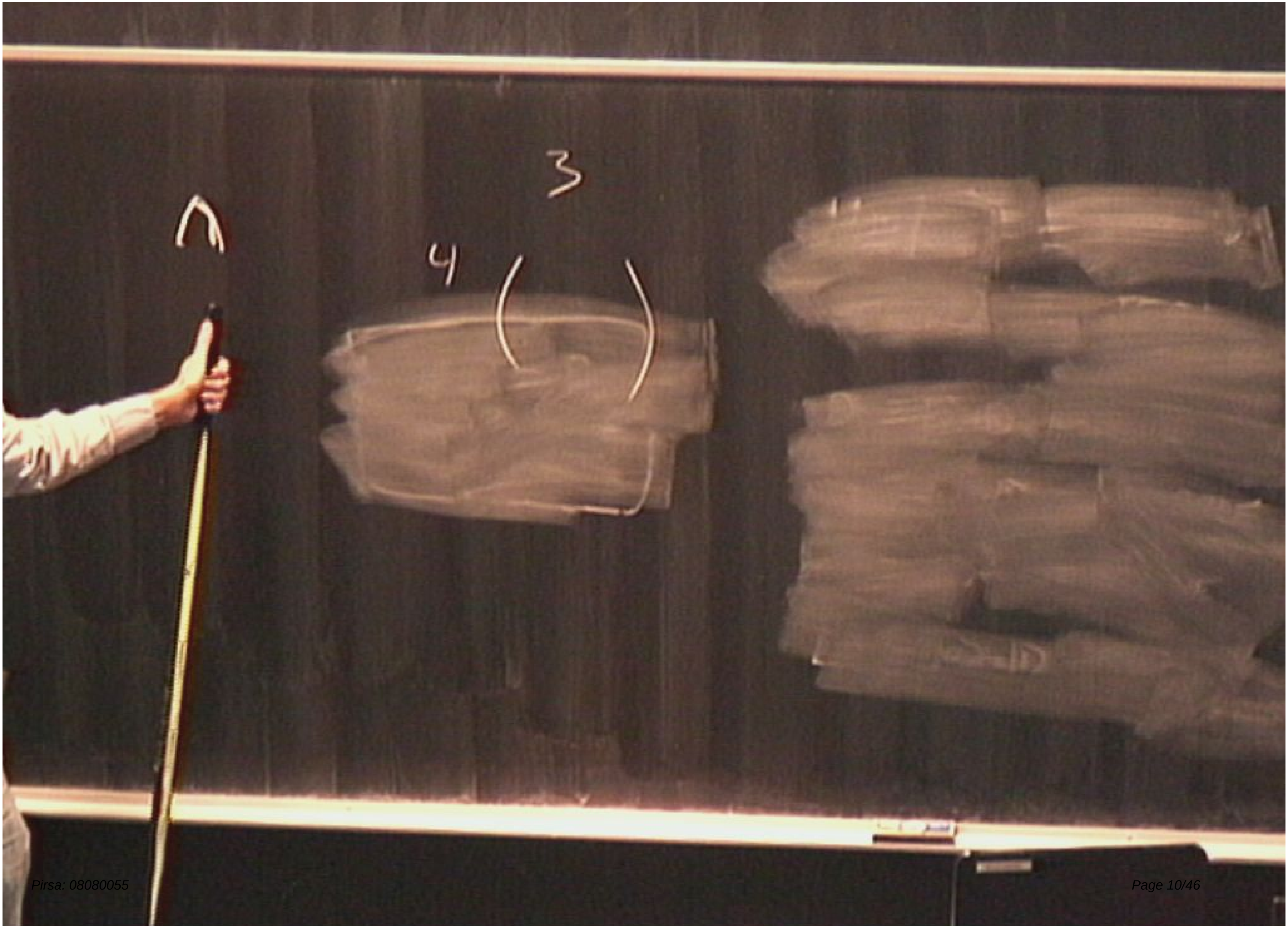
E. Knill et al., Nature 409, 46 (2001)

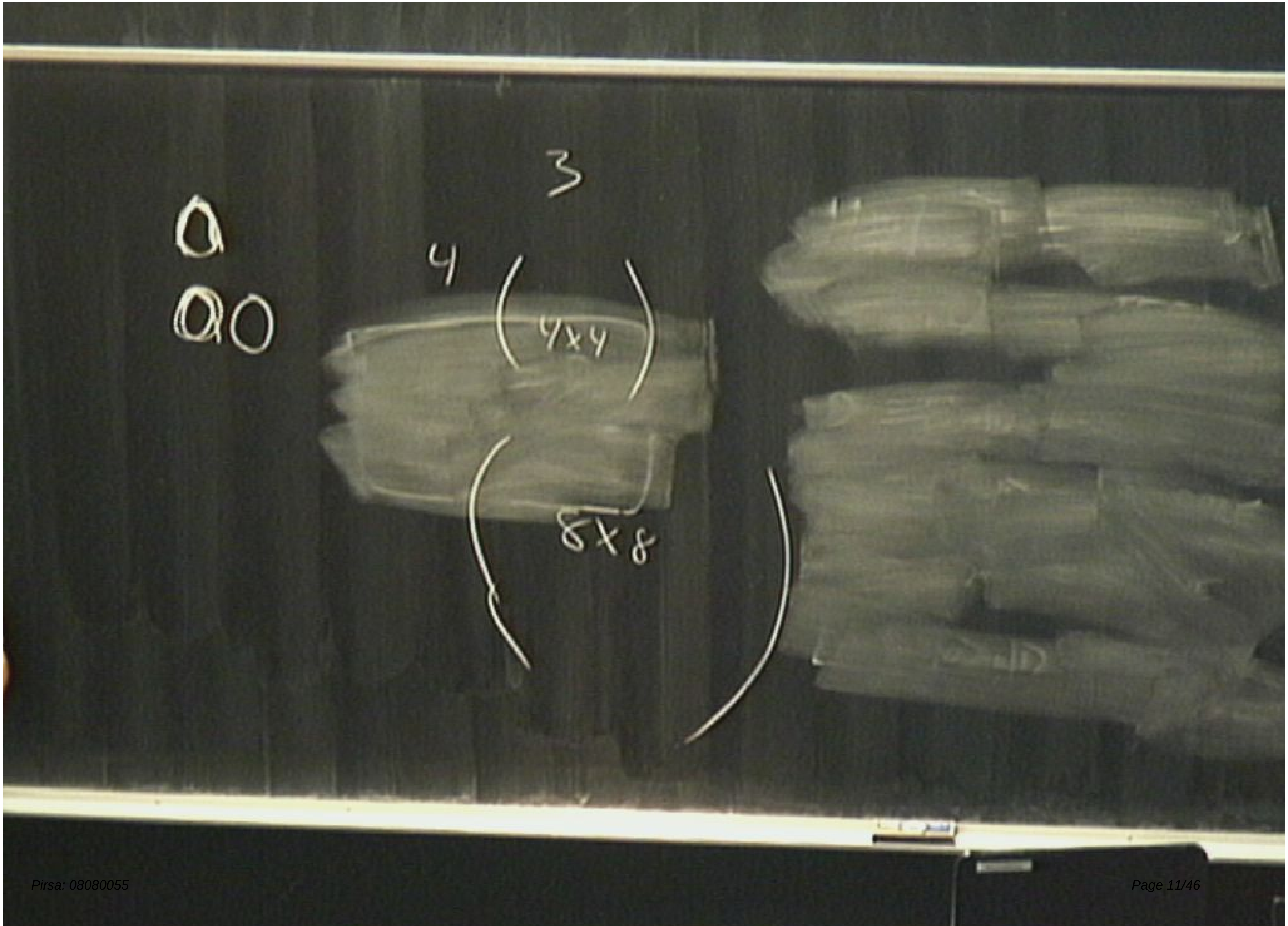


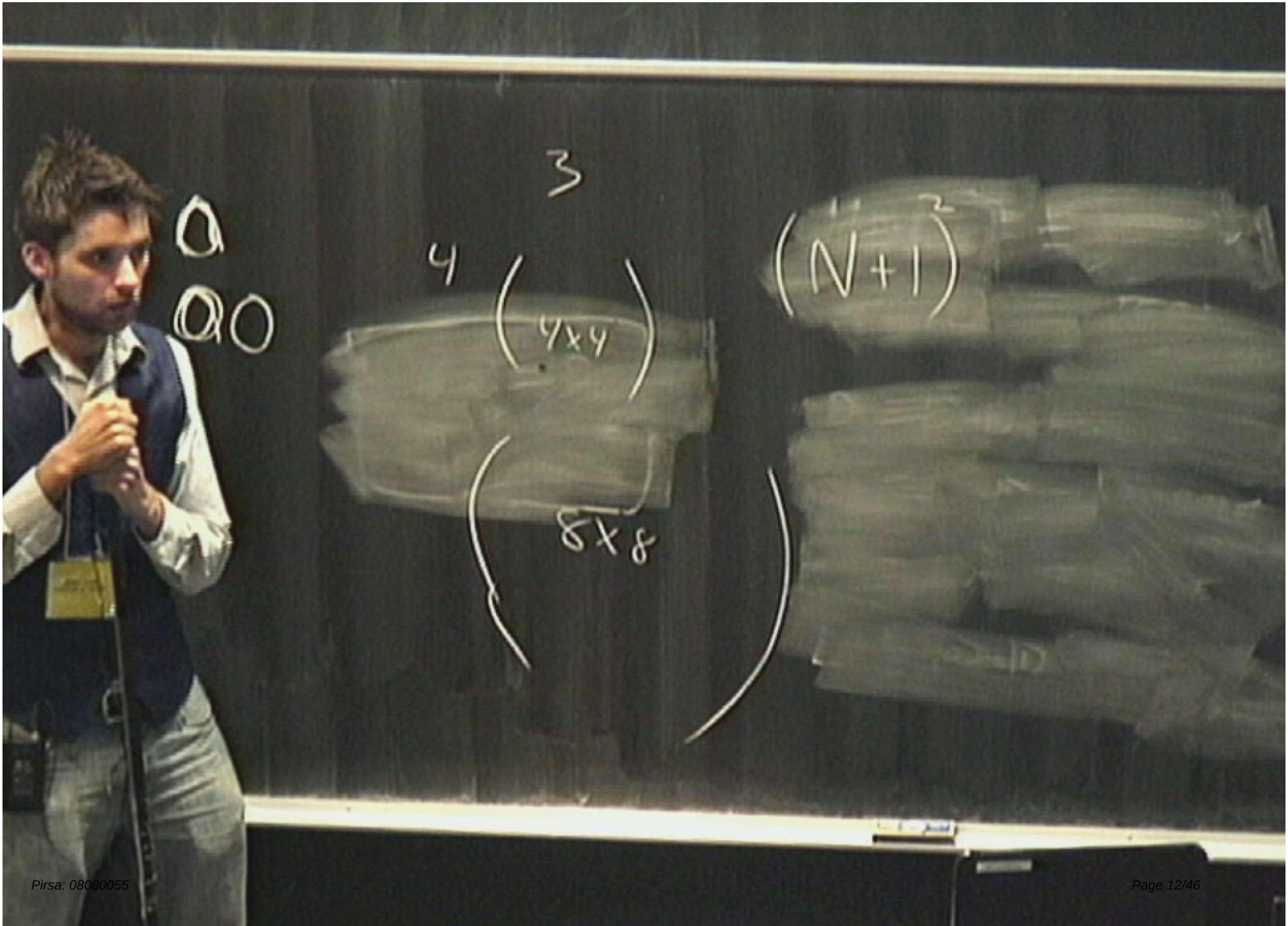
$$T = \frac{T_V}{T_H}$$

How do we do Tomography?



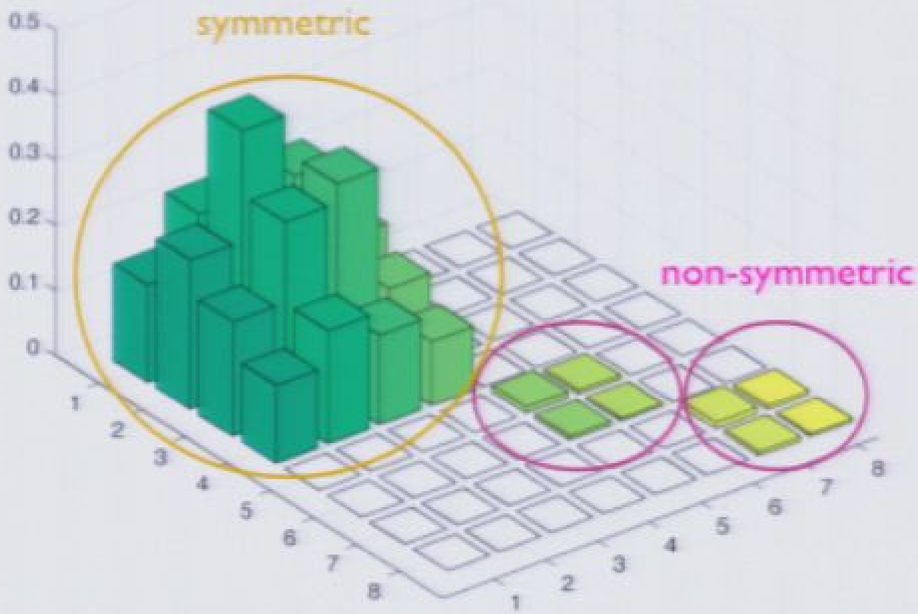

Λ
3
4 (   )


3
4
4x4
8x8

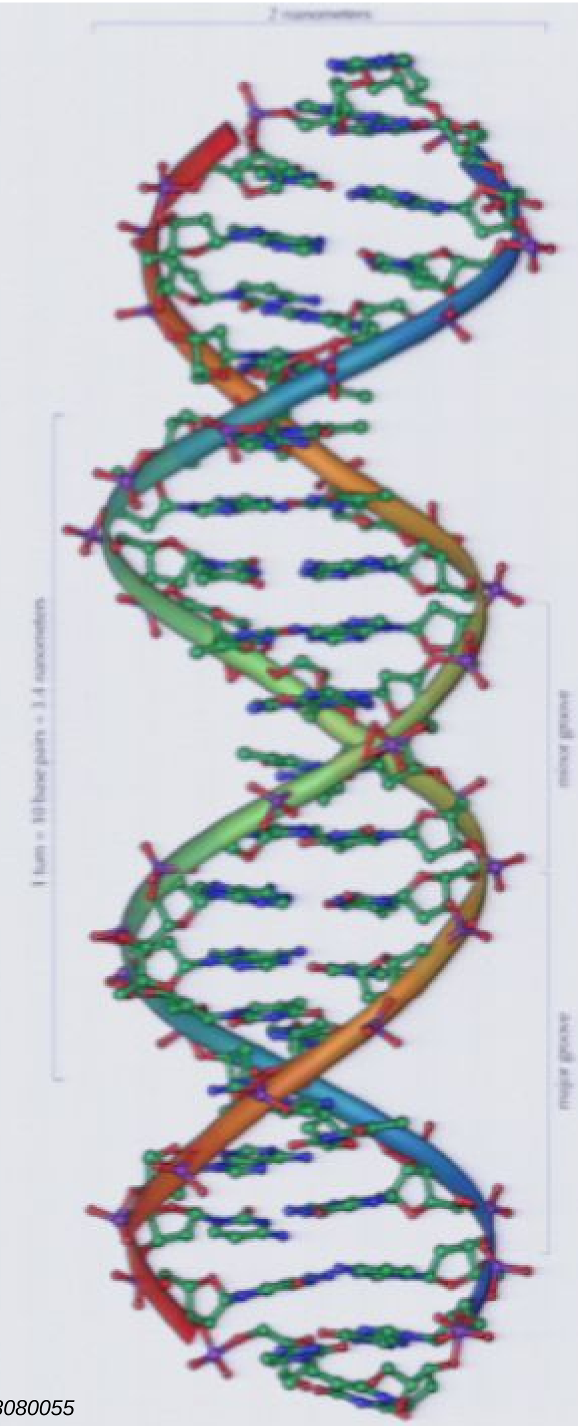

3
4
4x4
8x8
(N+1)
2

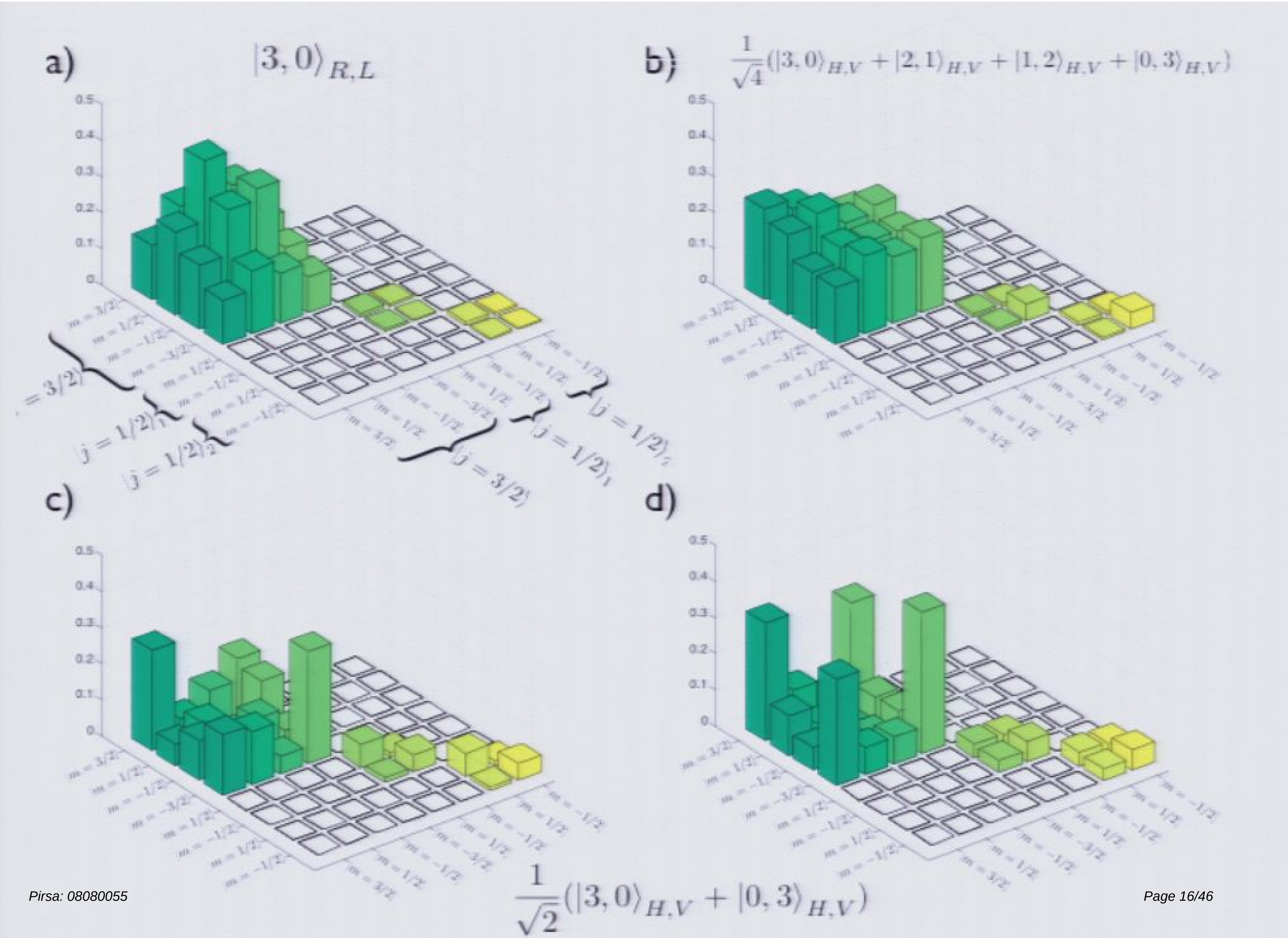

j = 3/2

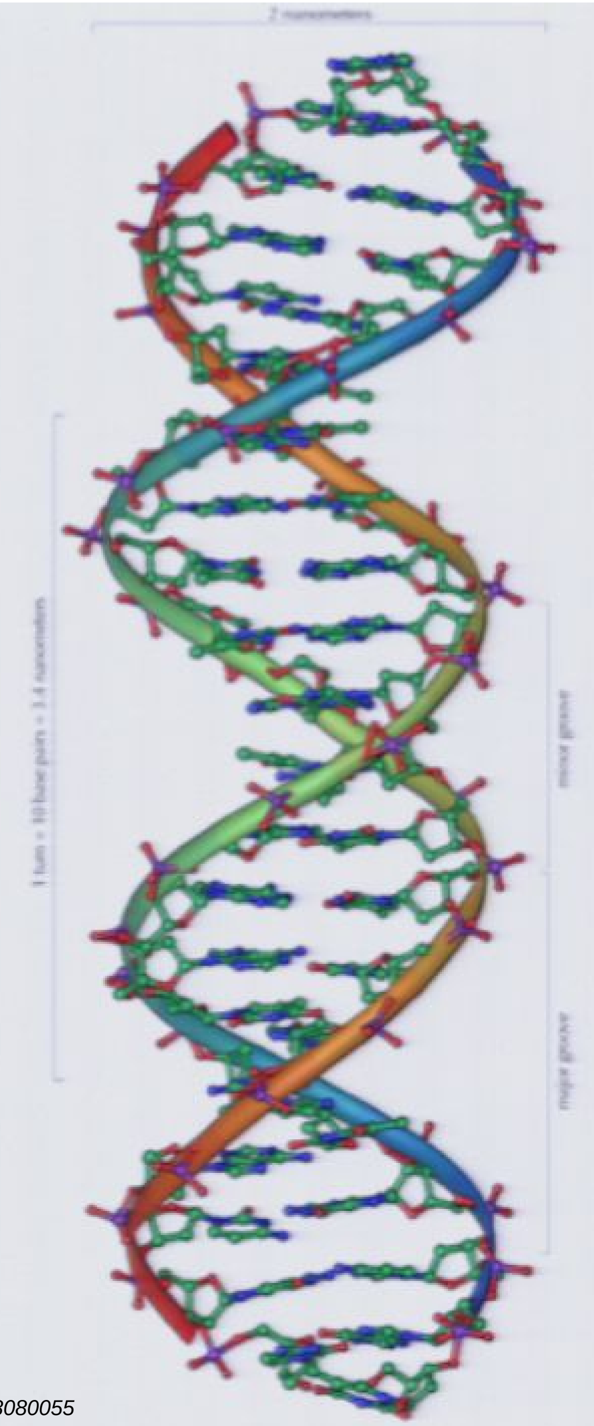
Density Matrix Representation

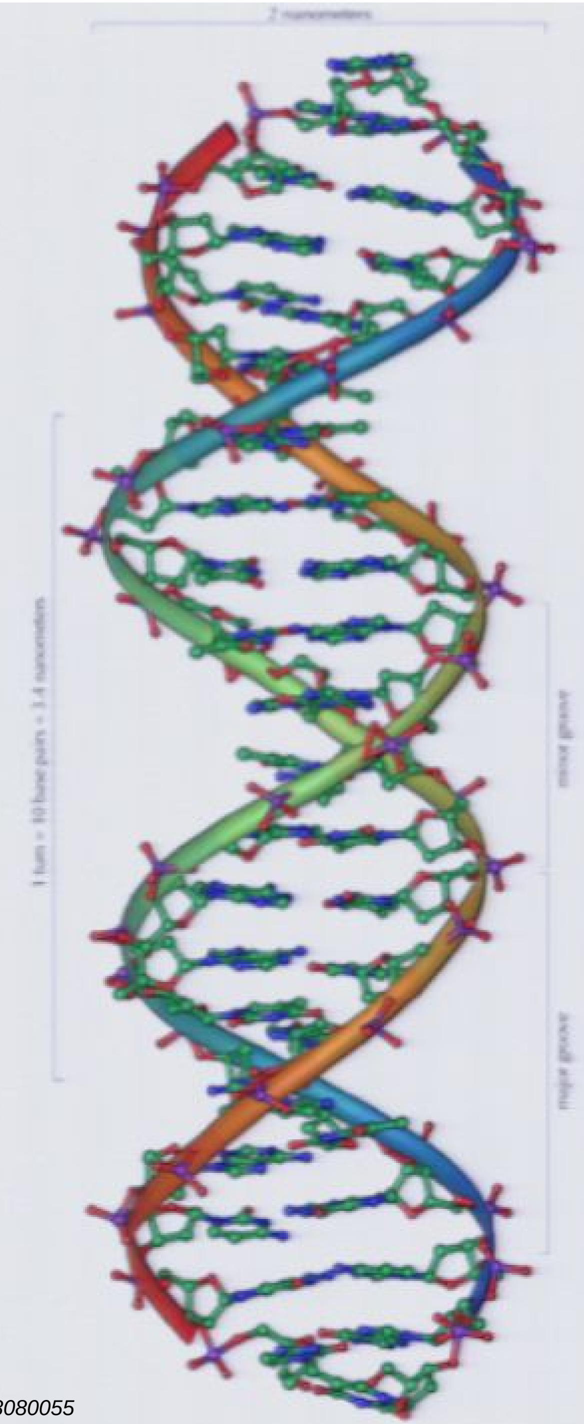


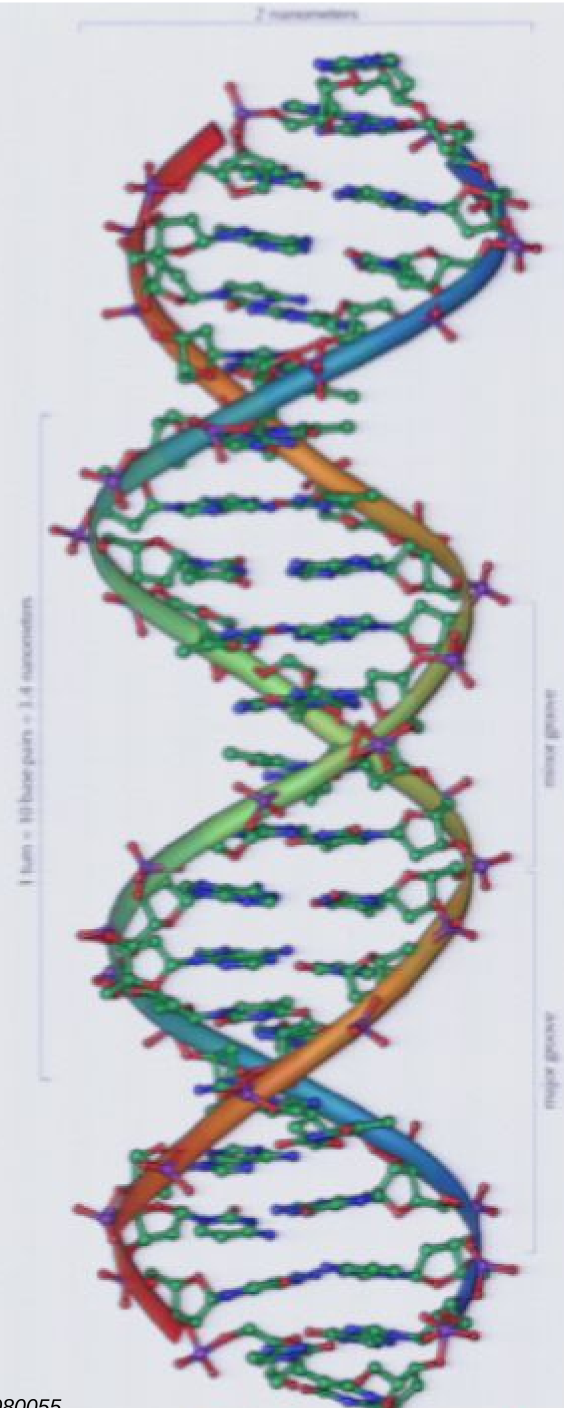
Adamson, Turner, Mitchell, Steinberg quant-ph/0612081
Adamson, Shalm, Mitchell, Steinberg, PRL 98, 043601 (2007)




a)
|3, 0⟩_{R,L}
b)
1/√4 (|3, 0⟩_{H,V} + |2, 1⟩_{H,V} + |1, 2⟩_{H,V} + |0, 3⟩_{H,V})
|j = 3/2⟩
|j = 1/2⟩₁
|j = 1/2⟩₂
c)
d)
1/√2 (|3, 0⟩_{H,V} + |0, 3⟩_{H,V})



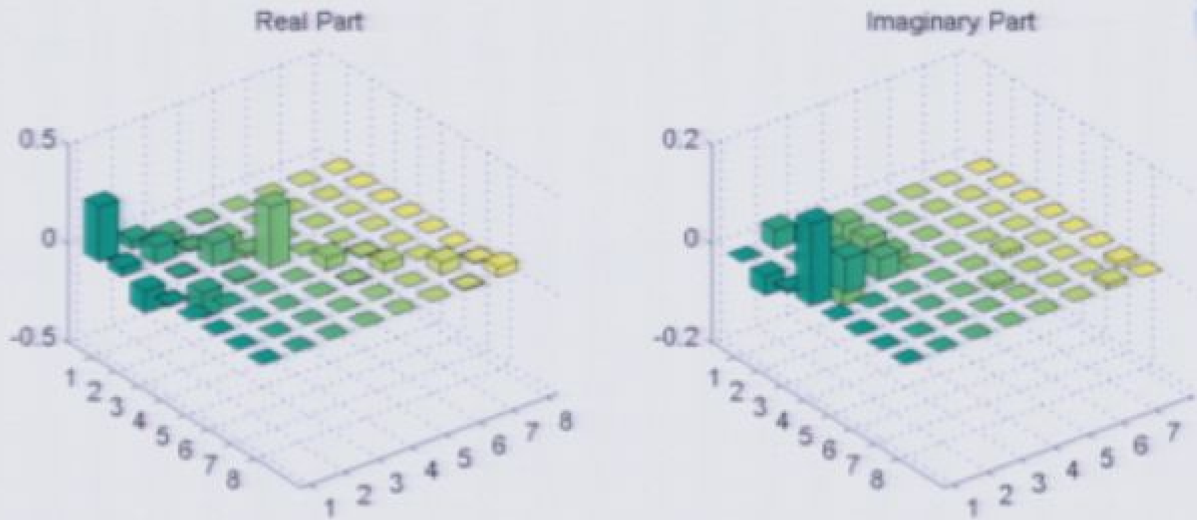




A picture is worth 10^3 DNA strands

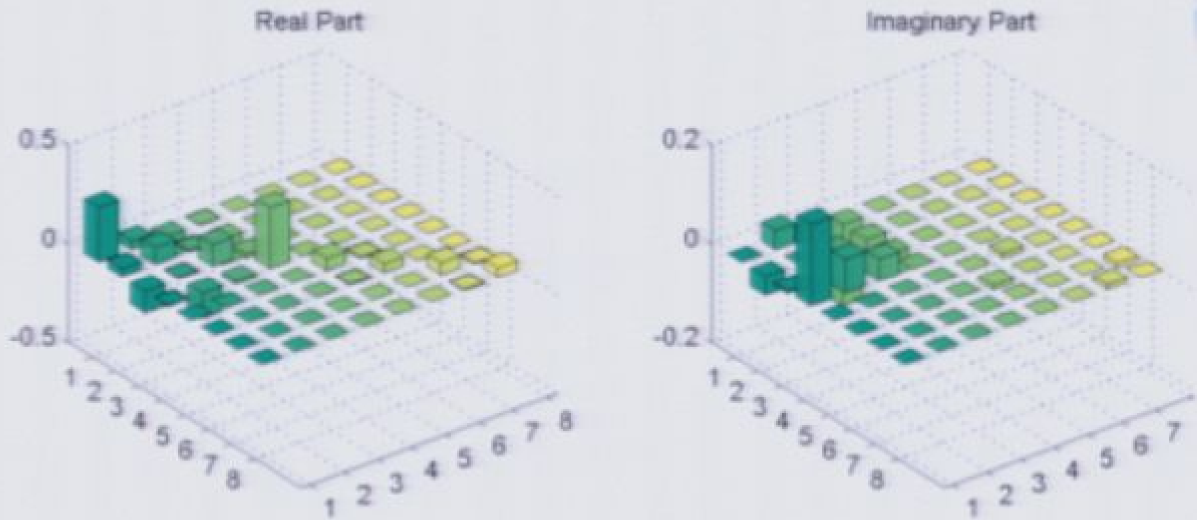
Density Matrix Representation

=



Density Matrix Representation

=

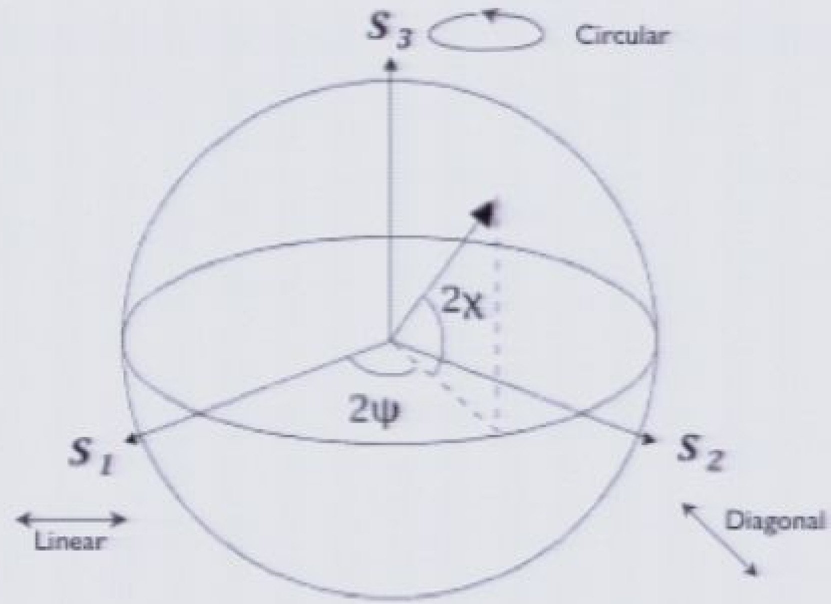


Poincaré Sphere

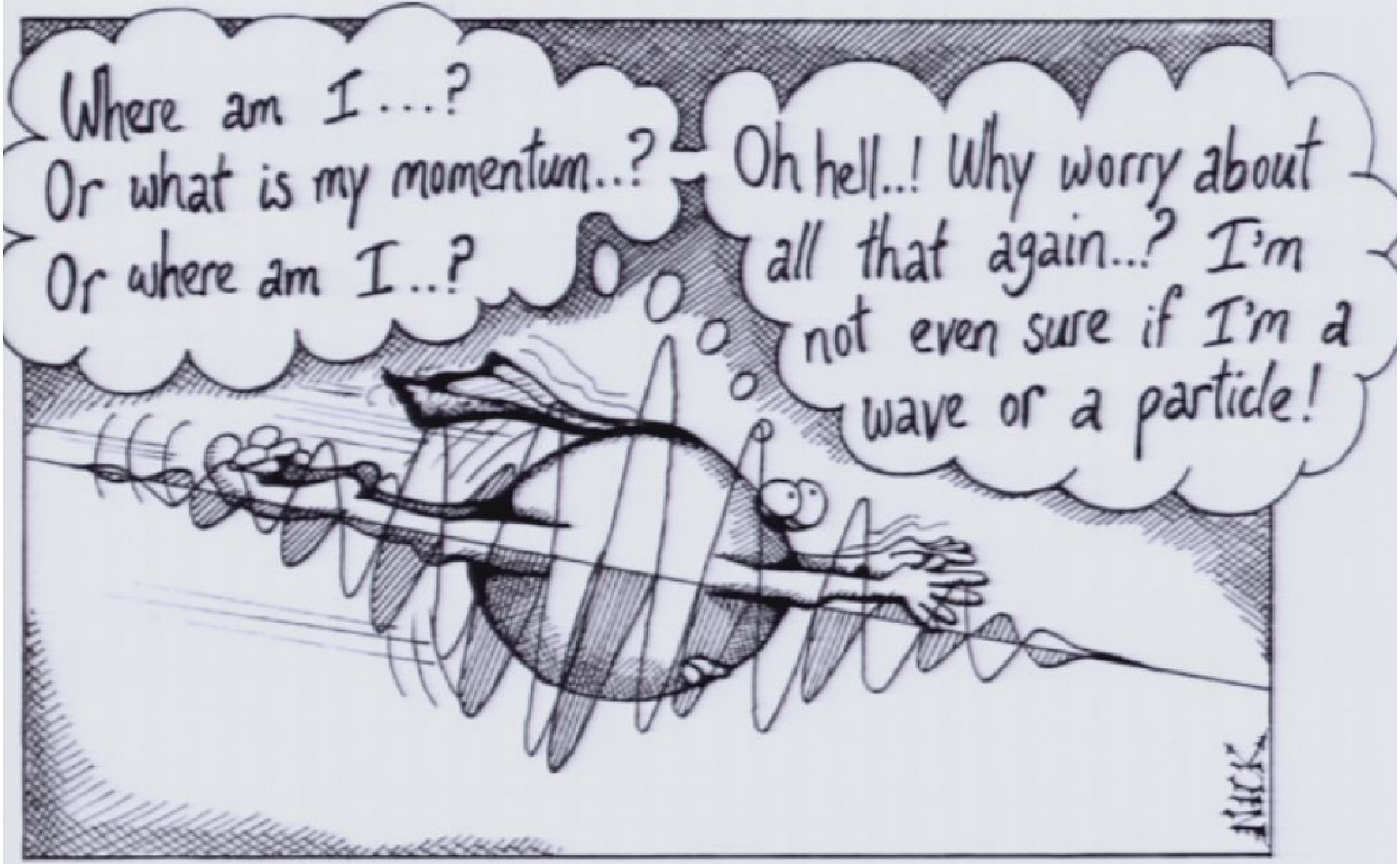
To characterise the polarisation of a beam of light, use the Stokes parameters. A pure polarization beam is represented by a vector terminating at a point on the Poincaré Sphere.

Analogous to the Bloch sphere for a Qubit.

$$S_0 = I$$
$$S_1 = Ip\cos 2\psi \cos 2\chi$$
$$S_2 = Ip\sin 2\psi \cos 2\chi$$
$$S_3 = Ip\sin 2\chi$$



What does a polarisation entangled quantum state look like on the Poincaré Sphere?



Photon self-identity problems.

Quasi-Probability Distributions

Treat polarization as a spin 1/2 particle. A collection of N photons represents a composite particle with spin J=N/2.

ntroduce the Quantum Poincaré Sphere with Stokes Operators that replace the Stokes 'arameters.

$$\hat{S}_0 = a_H^\dagger a_H + a_V^\dagger a_V = \hat{n}_H + \hat{n}_V = N,$$

$$\hat{S}_1 = a_H^\dagger a_H - a_V^\dagger a_V = \hat{n}_H - \hat{n}_V,$$

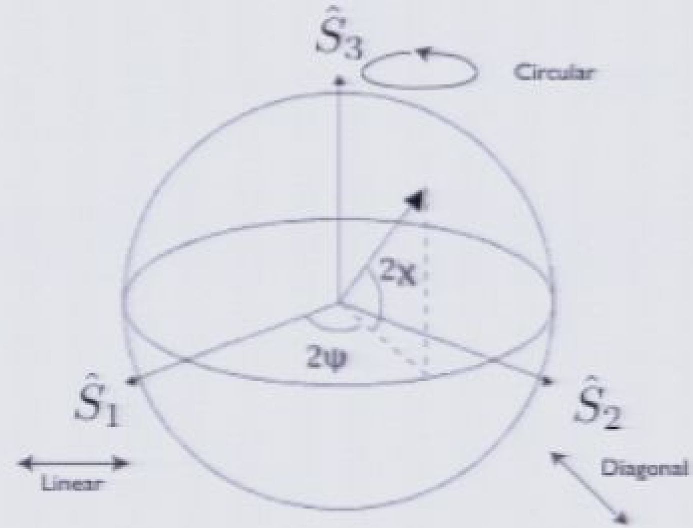
$$\hat{S}_2 = a_H^\dagger a_V + a_V^\dagger a_H = \hat{n}_D - \hat{n}_A,$$

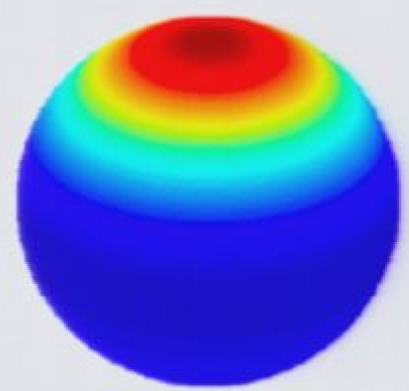
$$\hat{S}_3 = i(a_V^\dagger a_H - a_H^\dagger a_V) = \hat{n}_R - \hat{n}_L$$

$$V_1 V_2 \geqslant |\langle \hat{S}_3 \rangle|^2, \quad V_3 V_1 \geqslant |\langle \hat{S}_2 \rangle|^2, \quad V_2 V_3 \geqslant |\langle \hat{S}_1 \rangle|^2.$$

J. Hald et. al, Phys. Rev. Lett, 83, 1319 (1999)

Quantum Poincaré Sphere





'olarization state is represented by a Wigner quasi-probability distribution over the **surface** of the sphere. Mixed states are also represented by a broad distribution on the surface.

Dowling et. al, Phys. Rev. A, **49**, 4101 (1994)

R.L. Stratonovich, Sov. Phys. JETP, V. **31**, p. 1012 (1956)

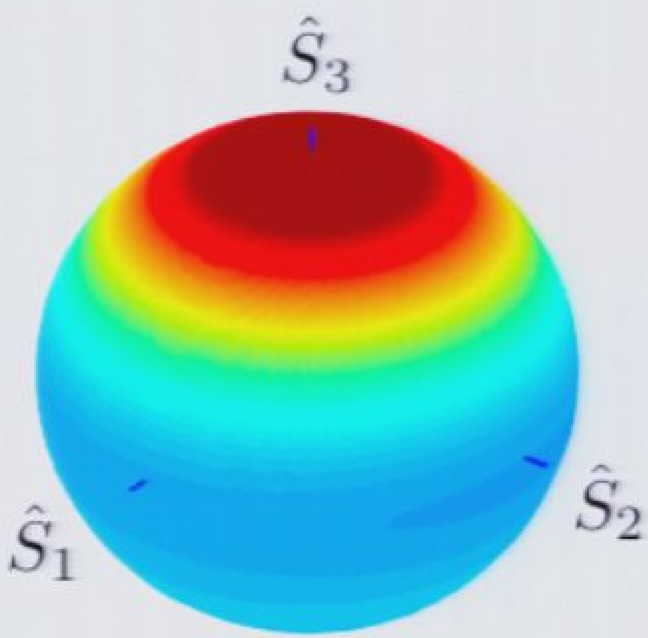
$$W(\theta,\phi) = \sum_{k=0}^{2j} \sum_{q=-k}^{+k} Y_{kq}(\theta,\phi) \mathrm{Tr}\left[\rho \hat{T}_{kq}\right]$$

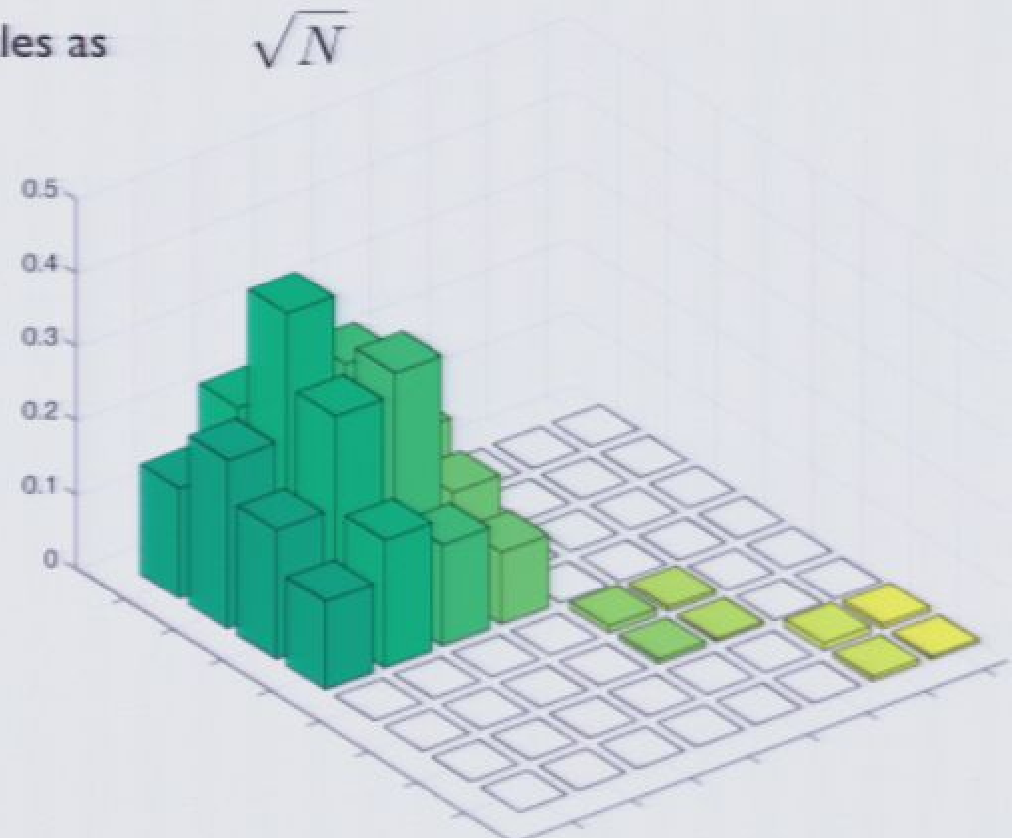
Quasi-Probability Distributions

- *Spin coherent state* is analogous to a coherent state in a harmonic oscillator
- Formed by all *N* photons being polarized in the same direction.
- Equivalent to a binomial distribution in any other basis.

$$|3,0\rangle_{R,L} = \frac{1}{\sqrt{8}}(|3,0\rangle_{H,V} + \sqrt{3}\,|2,1\rangle_{H,V} + \sqrt{3}\,|1,2\rangle_{H,V} + |0,3\rangle_{H,V})$$

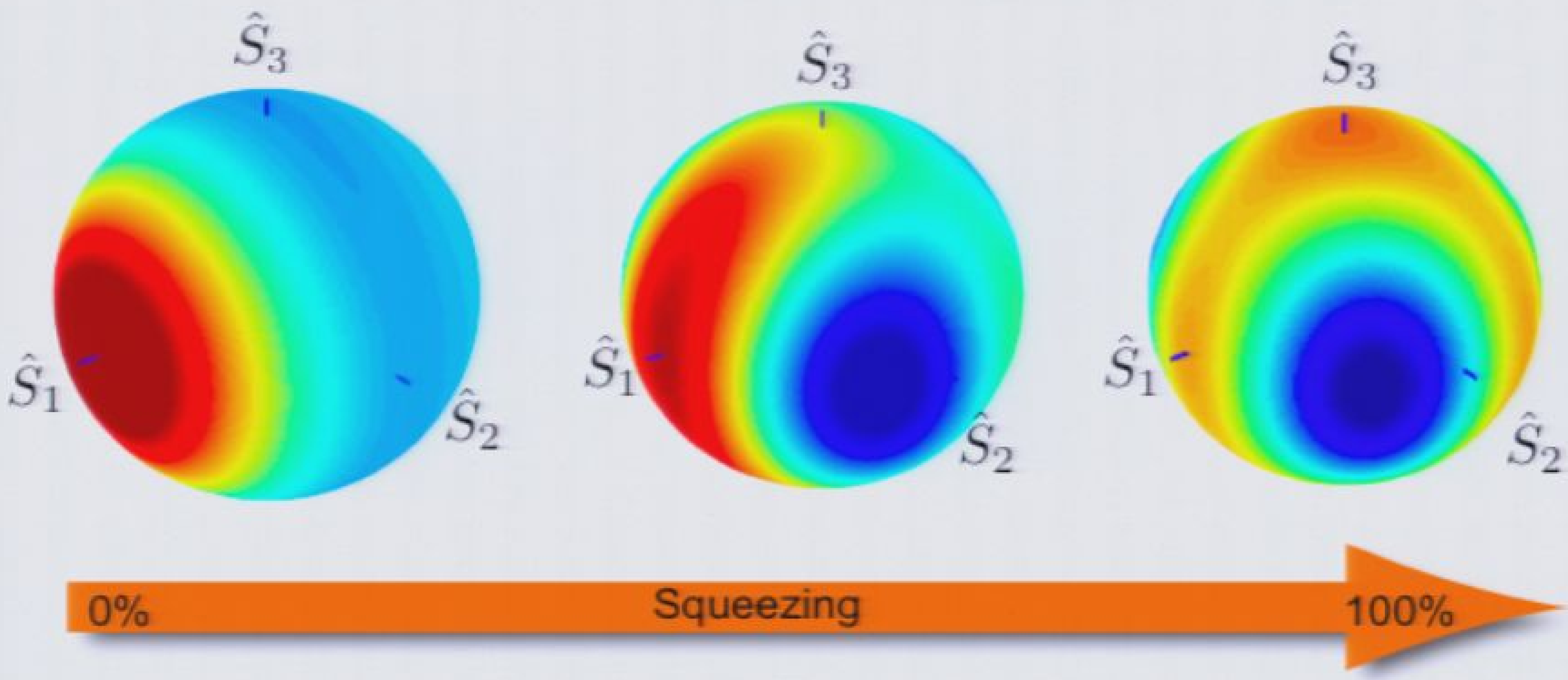
- Uncertainty in other Stokes operators scales as $\sqrt{N}$

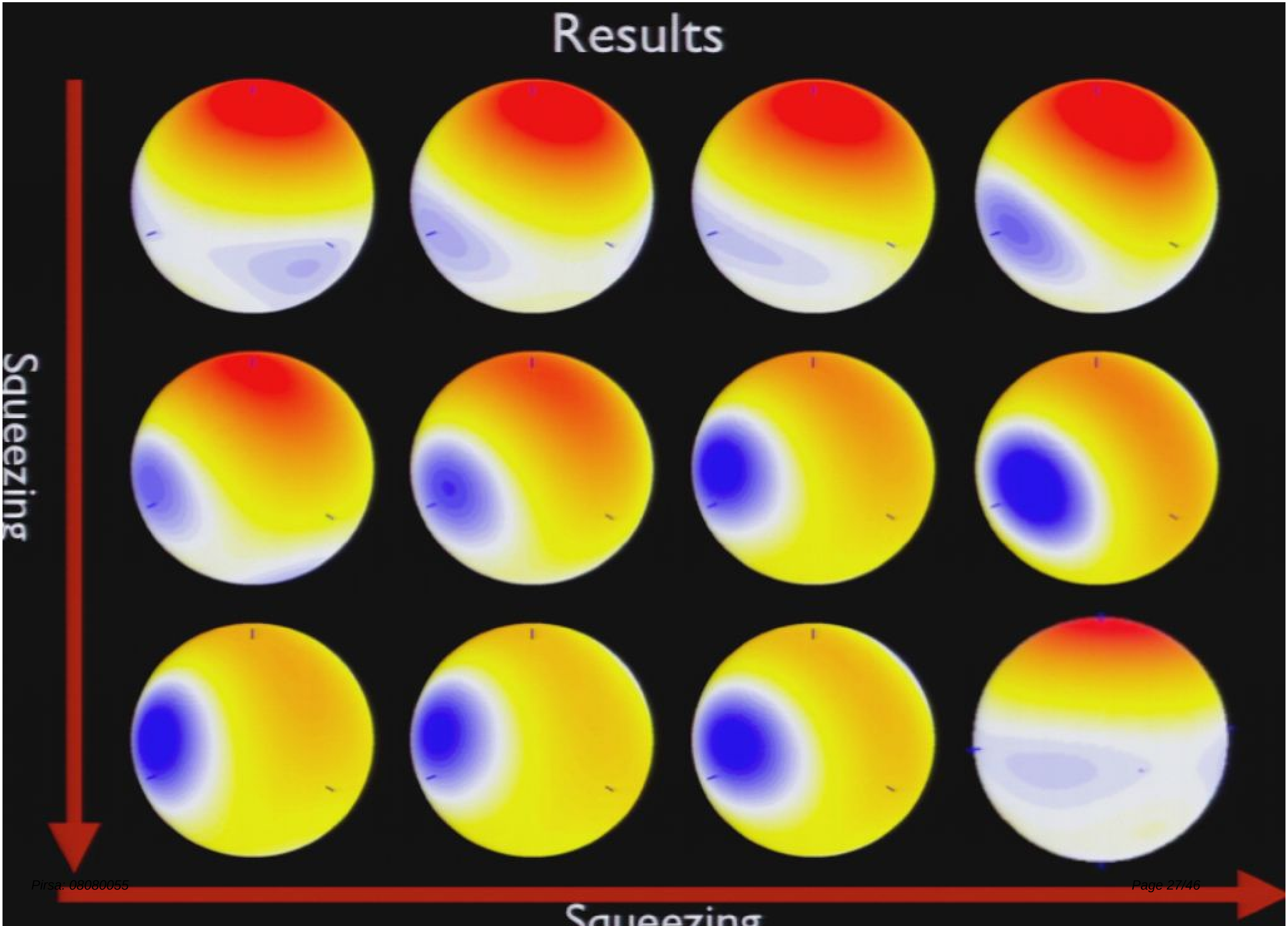




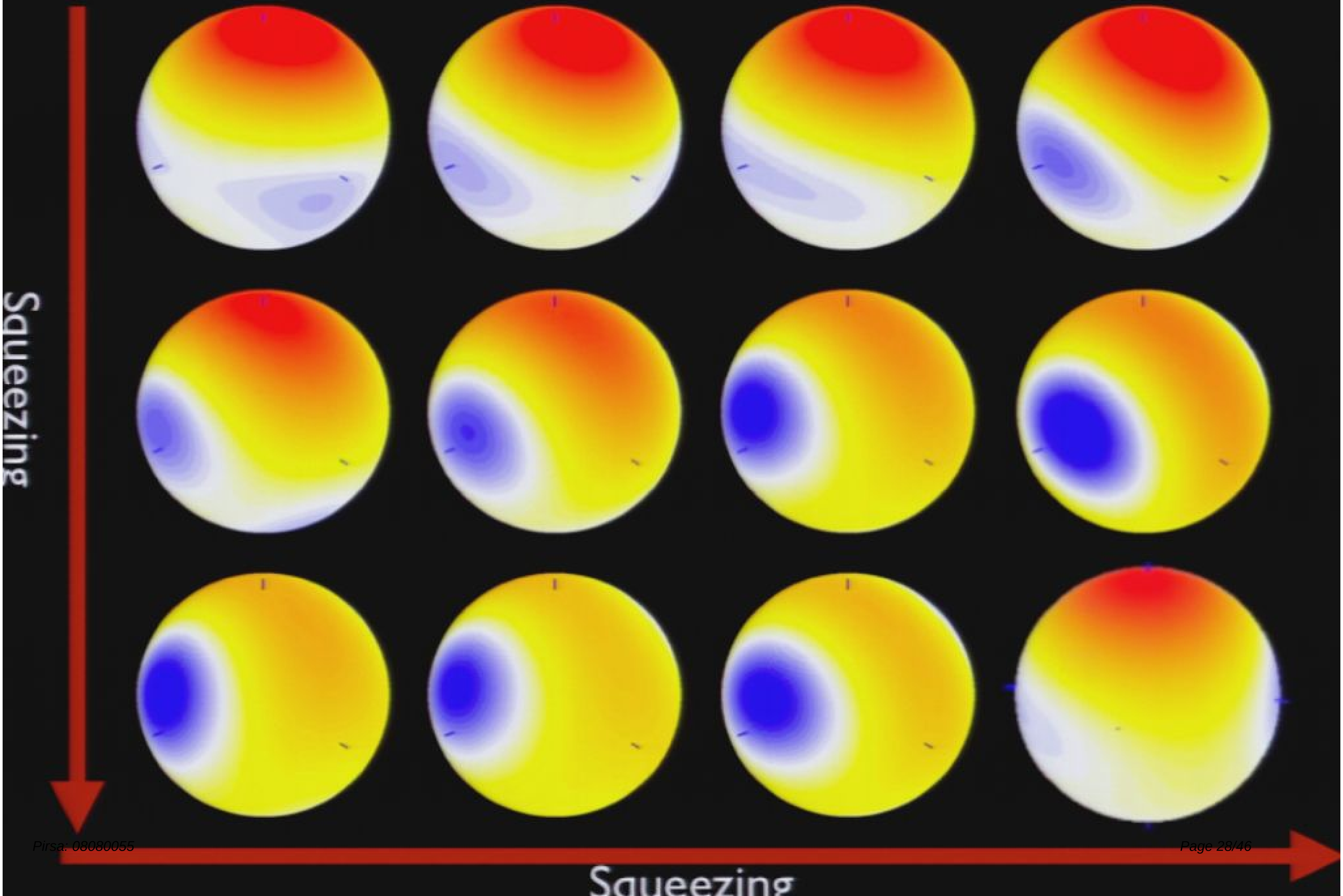
Quasi-Probability Distributions

•Possible to achieve squeezing at the few Photon level!

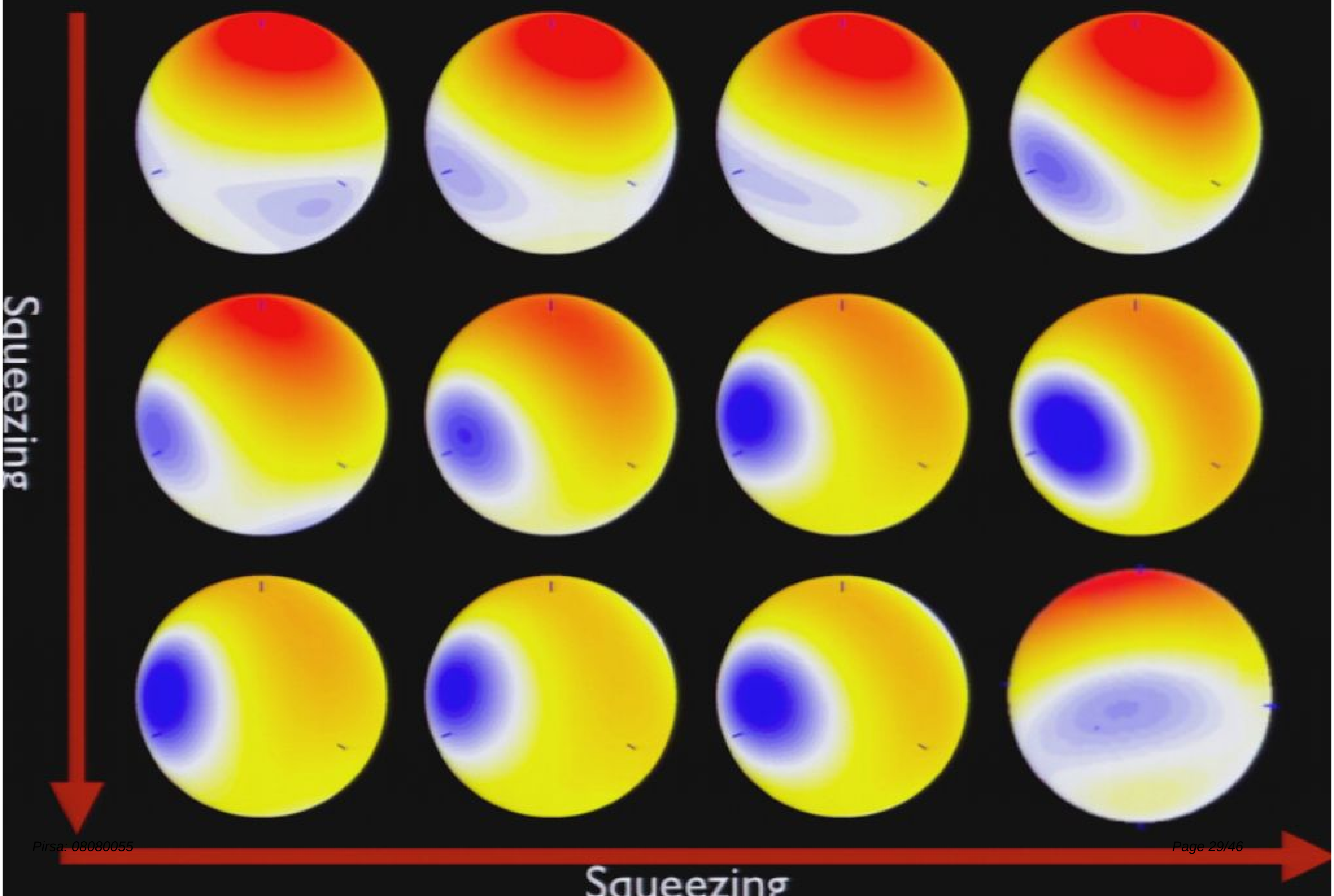



Results
Squeezing
Squeezing

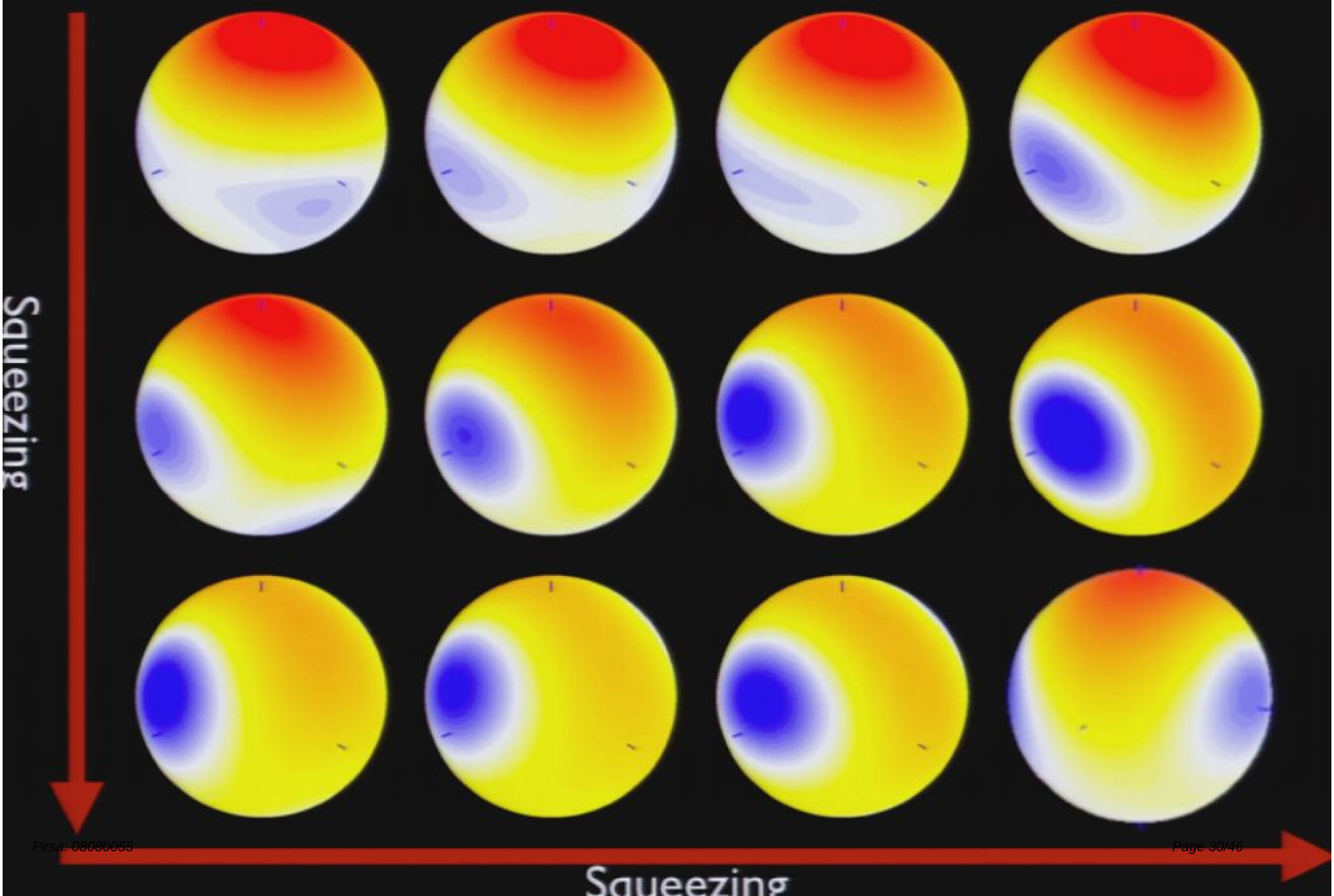
Results

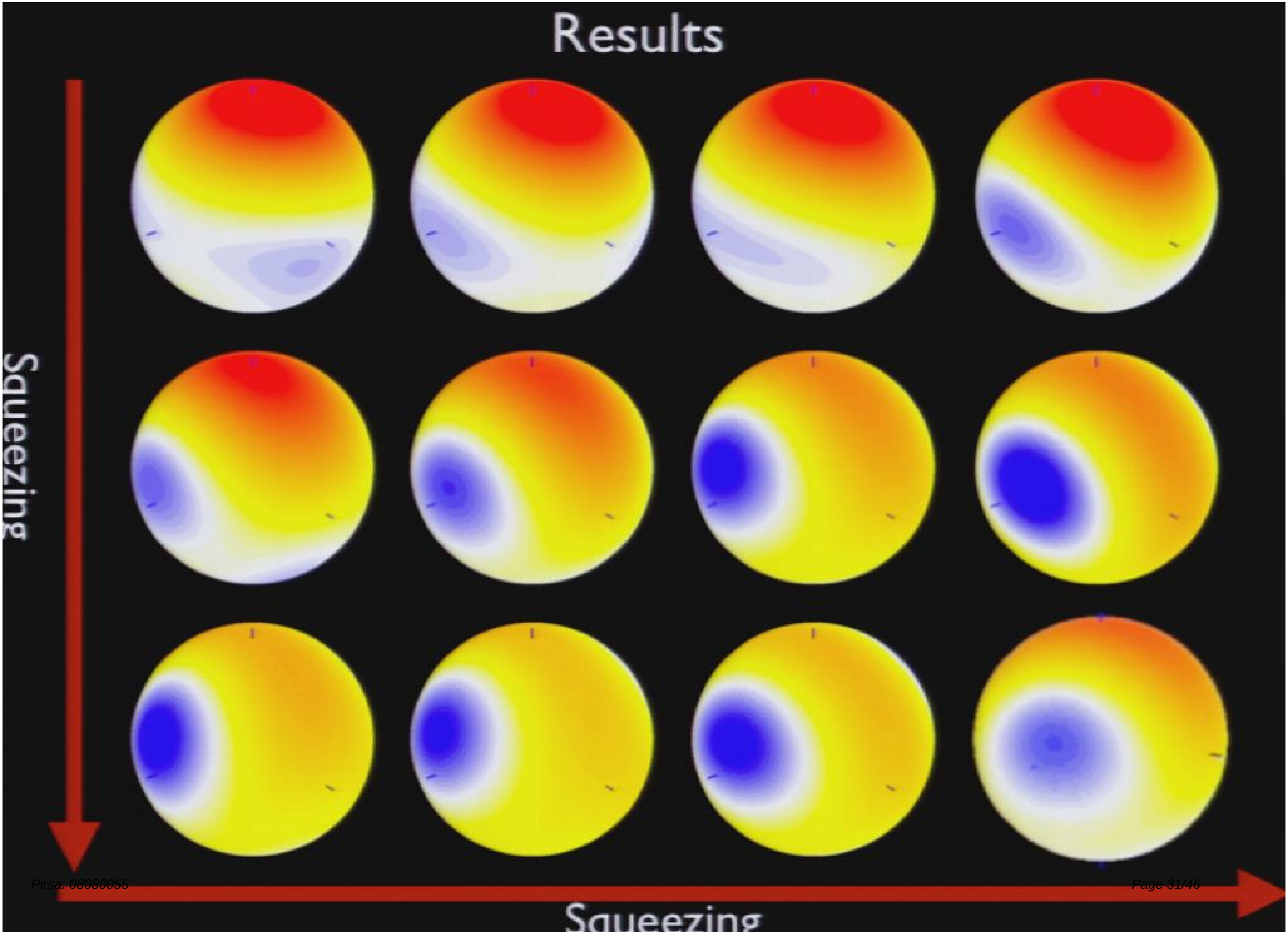


Results

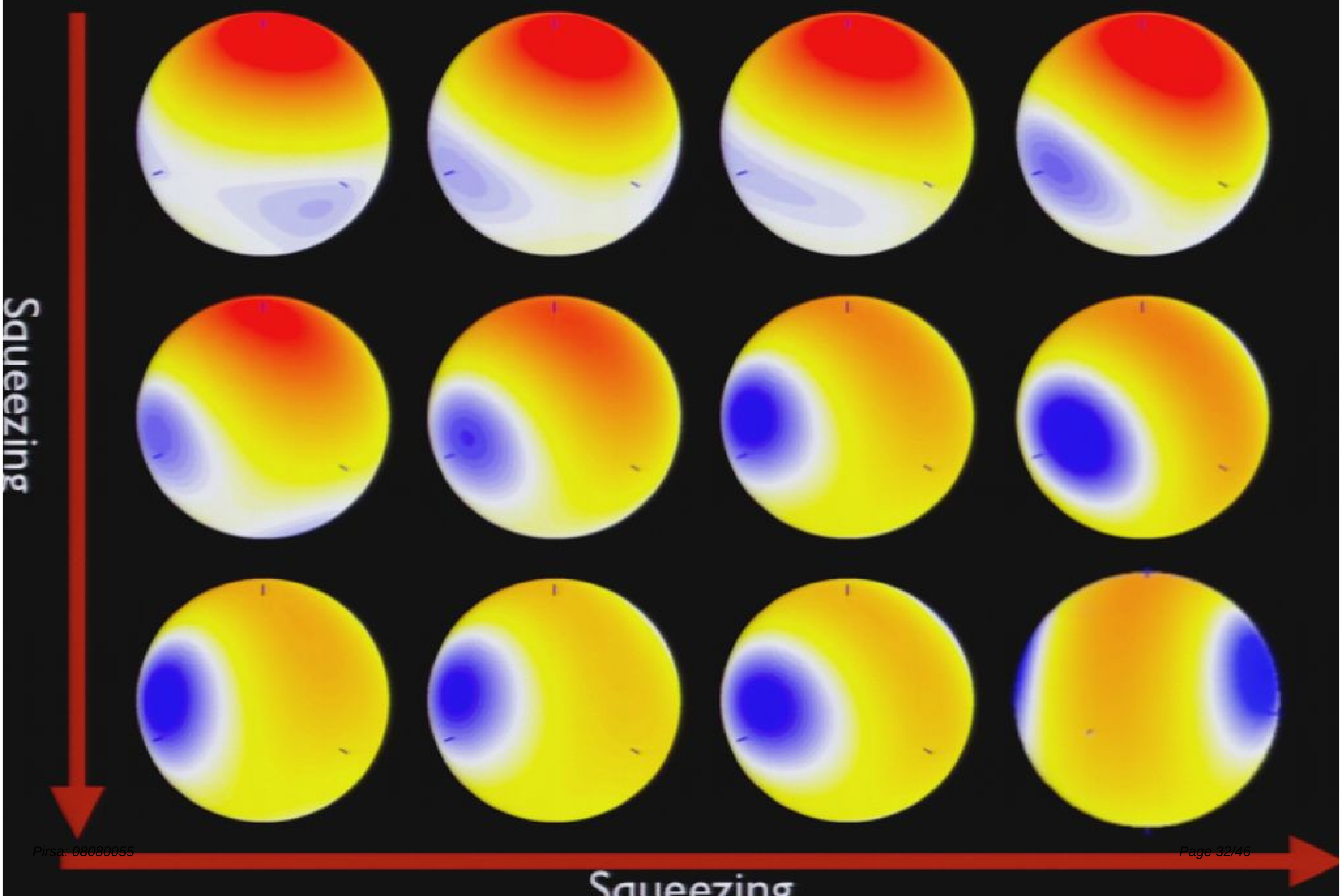


Results

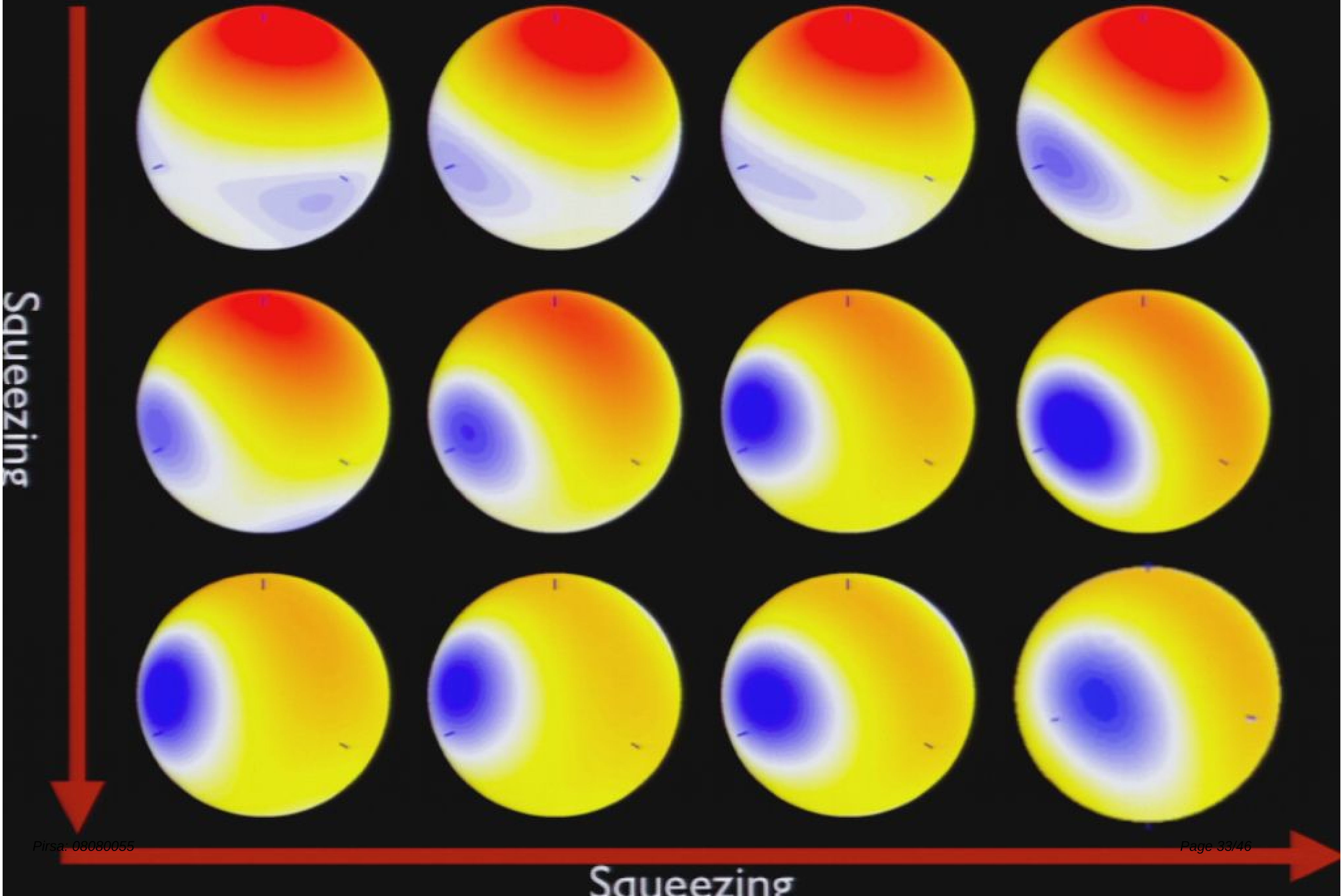



Results
Squeezing
Squeezing

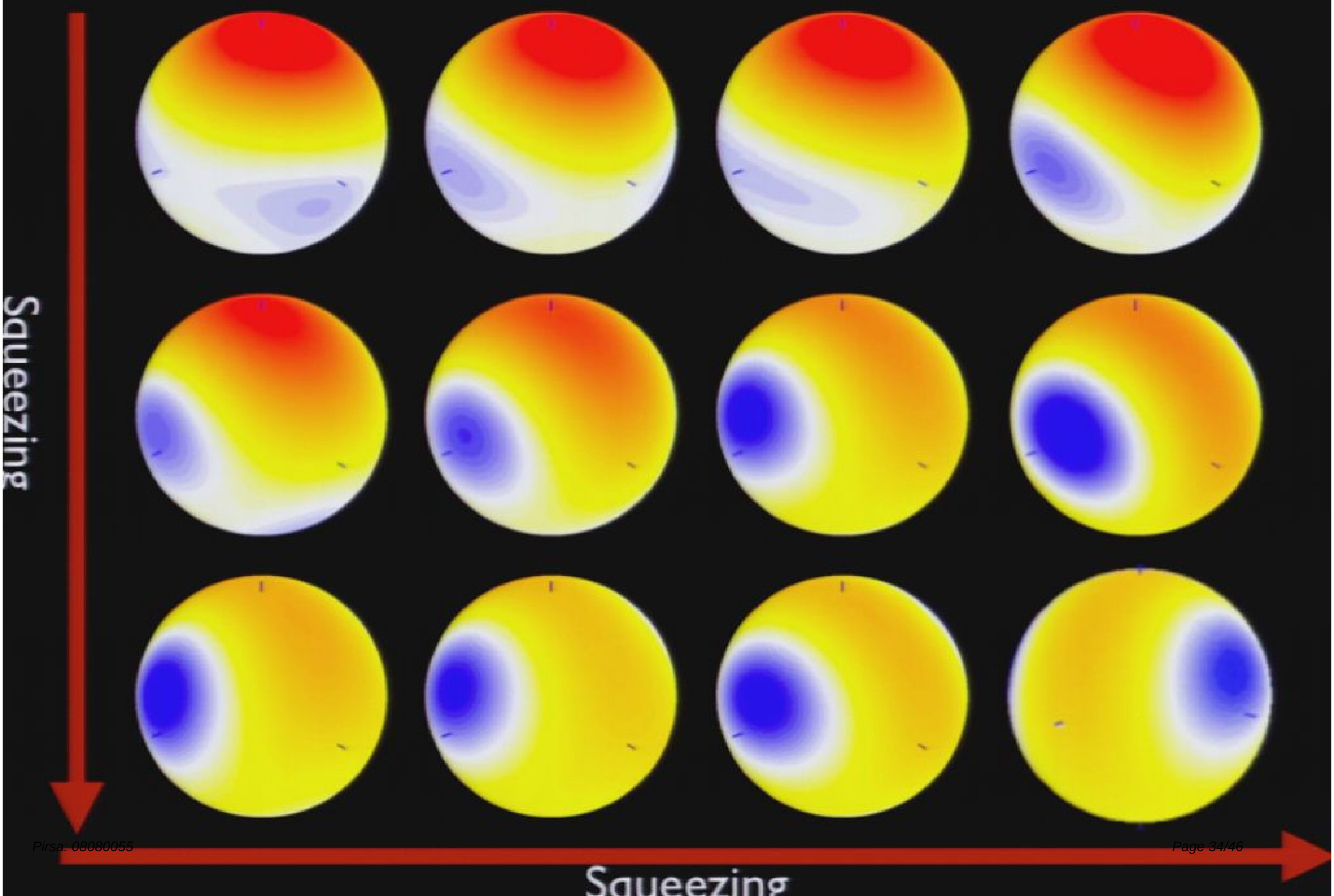
Results



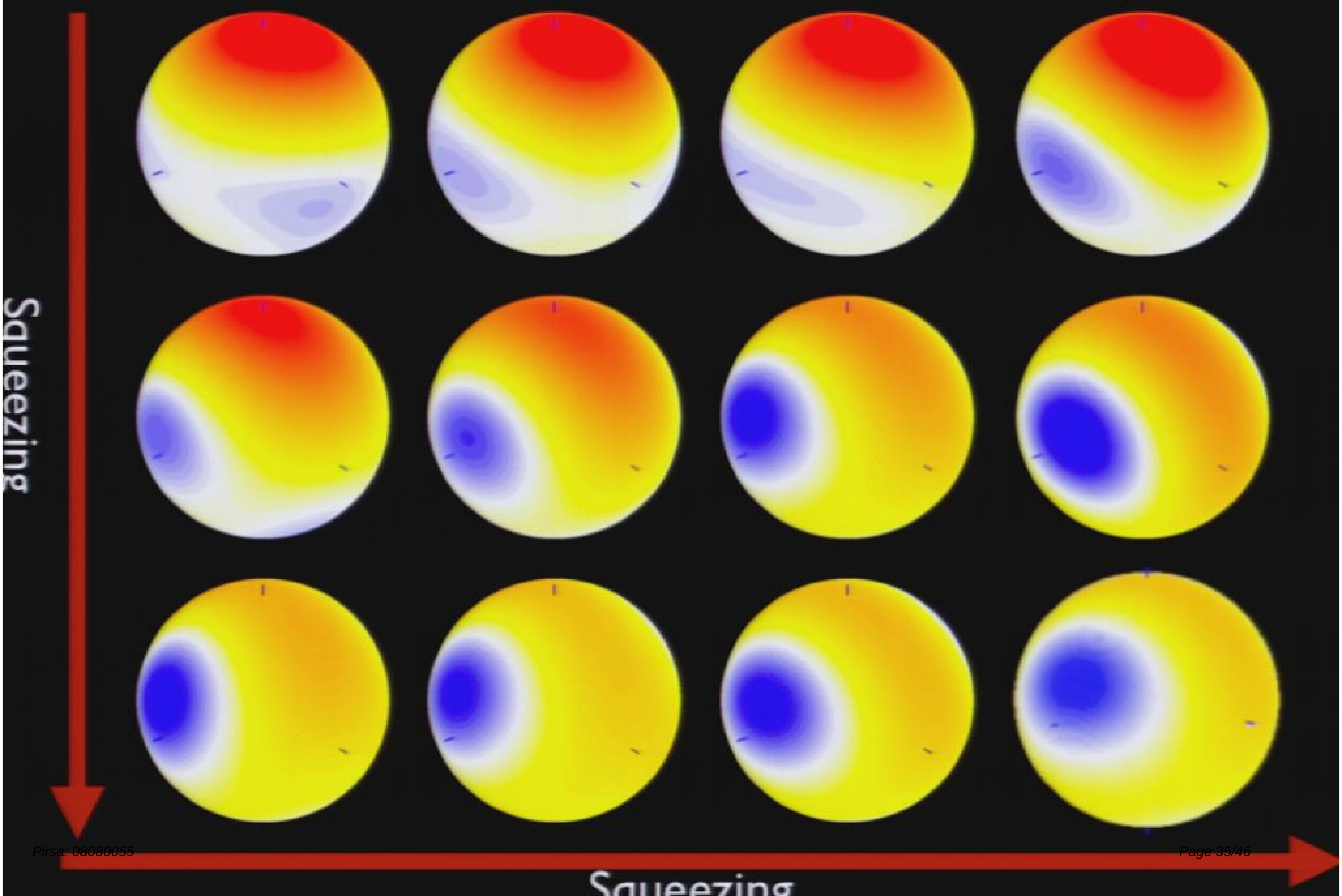
Results



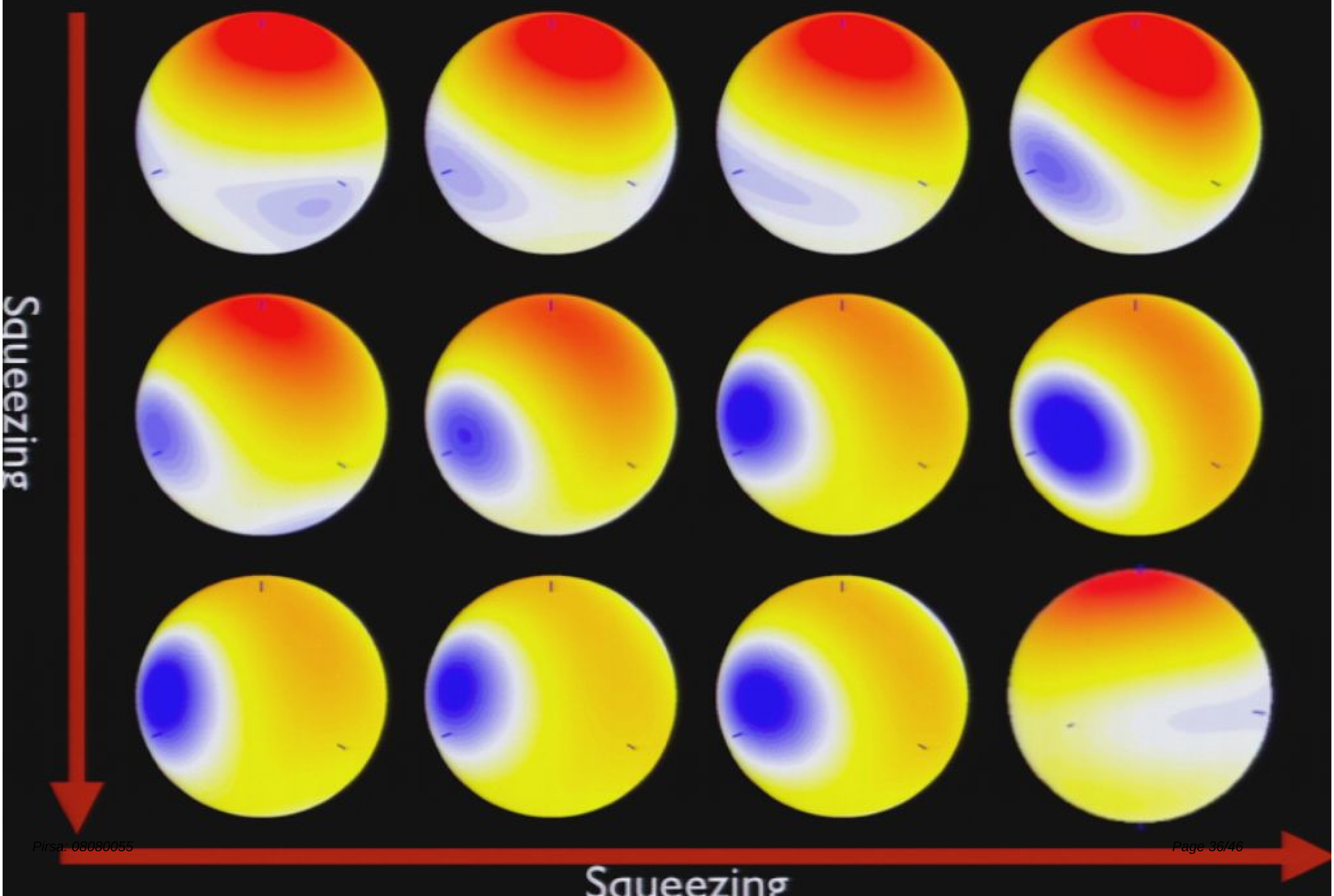
Results



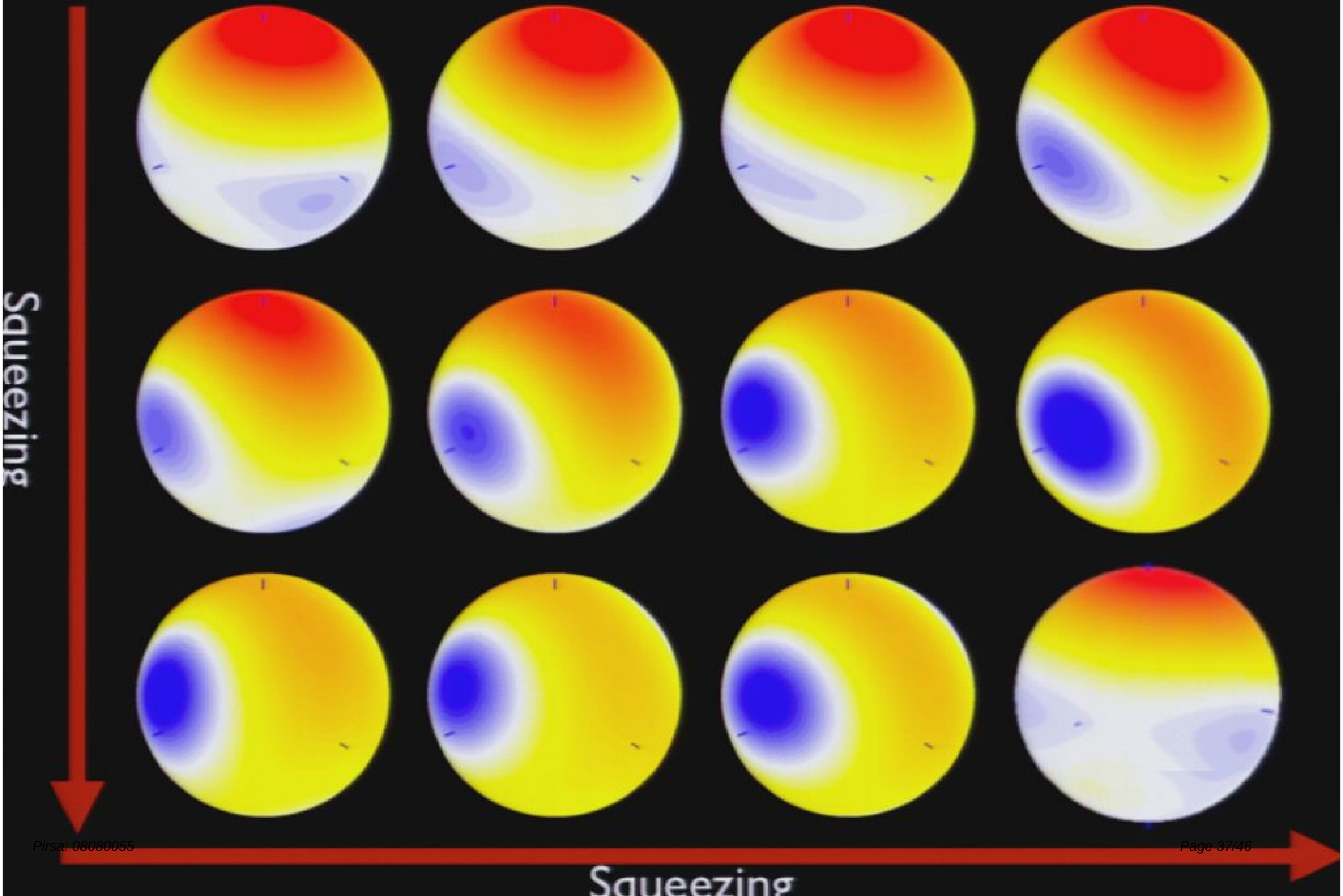
Results



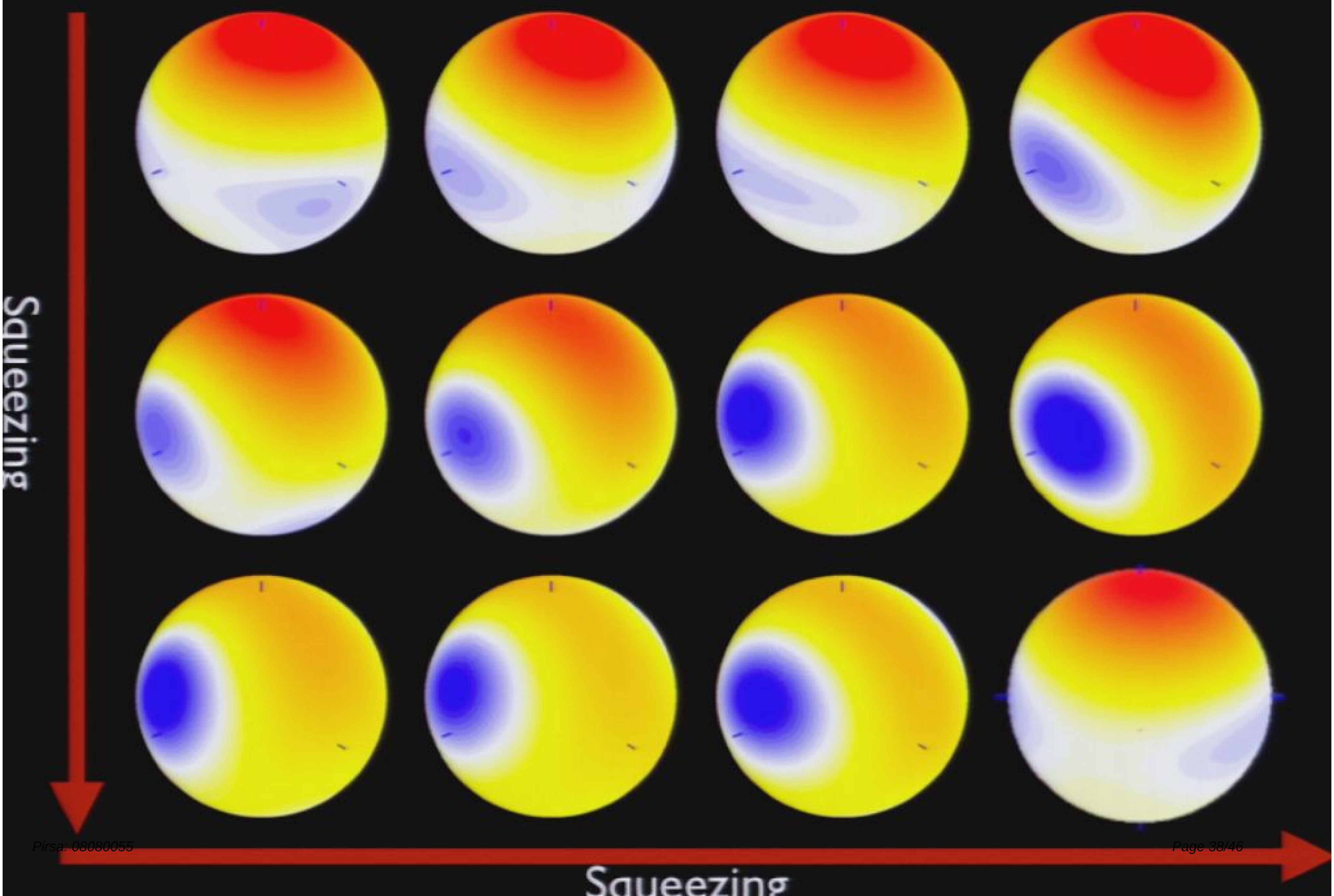
Results

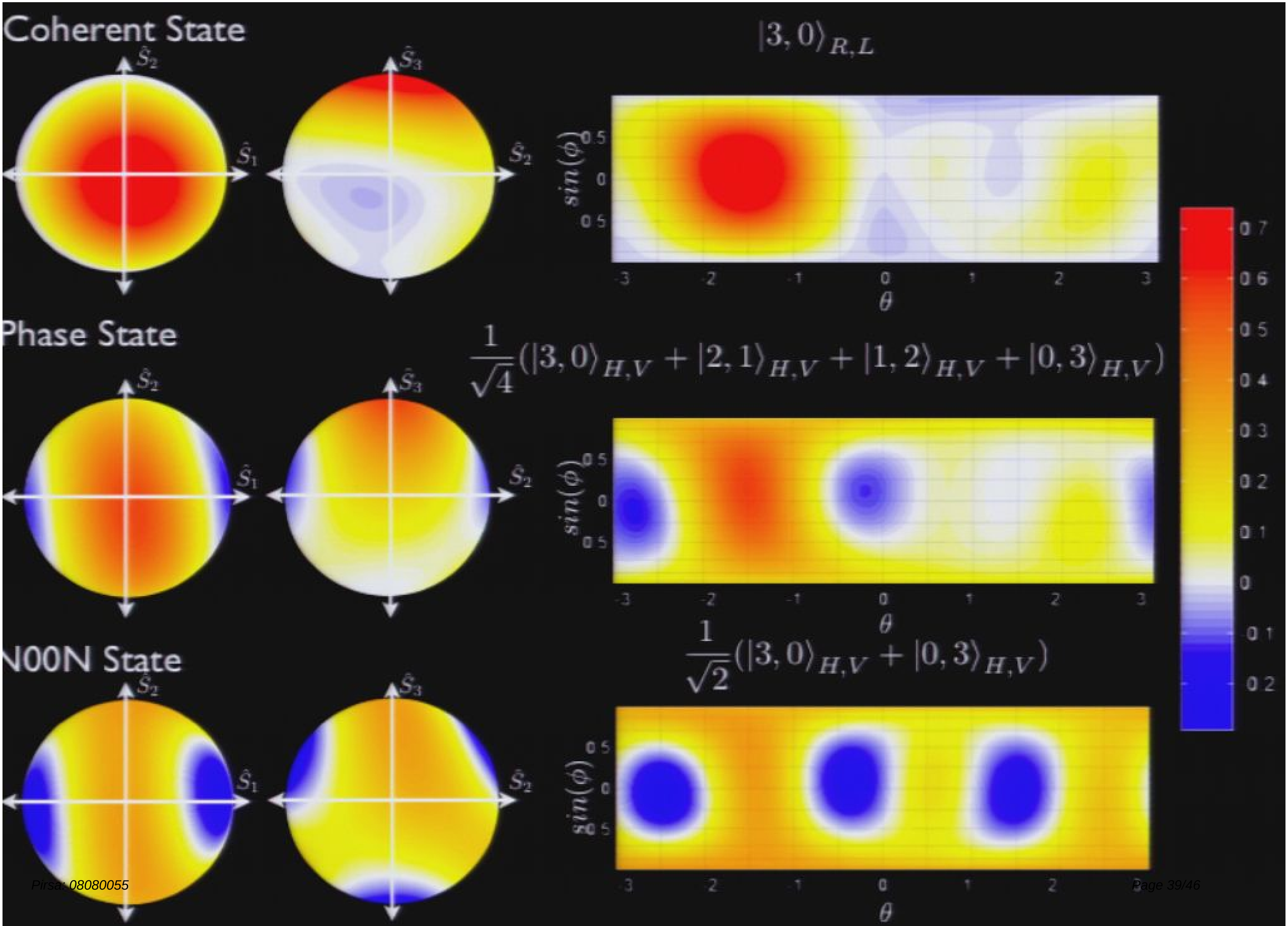


Results

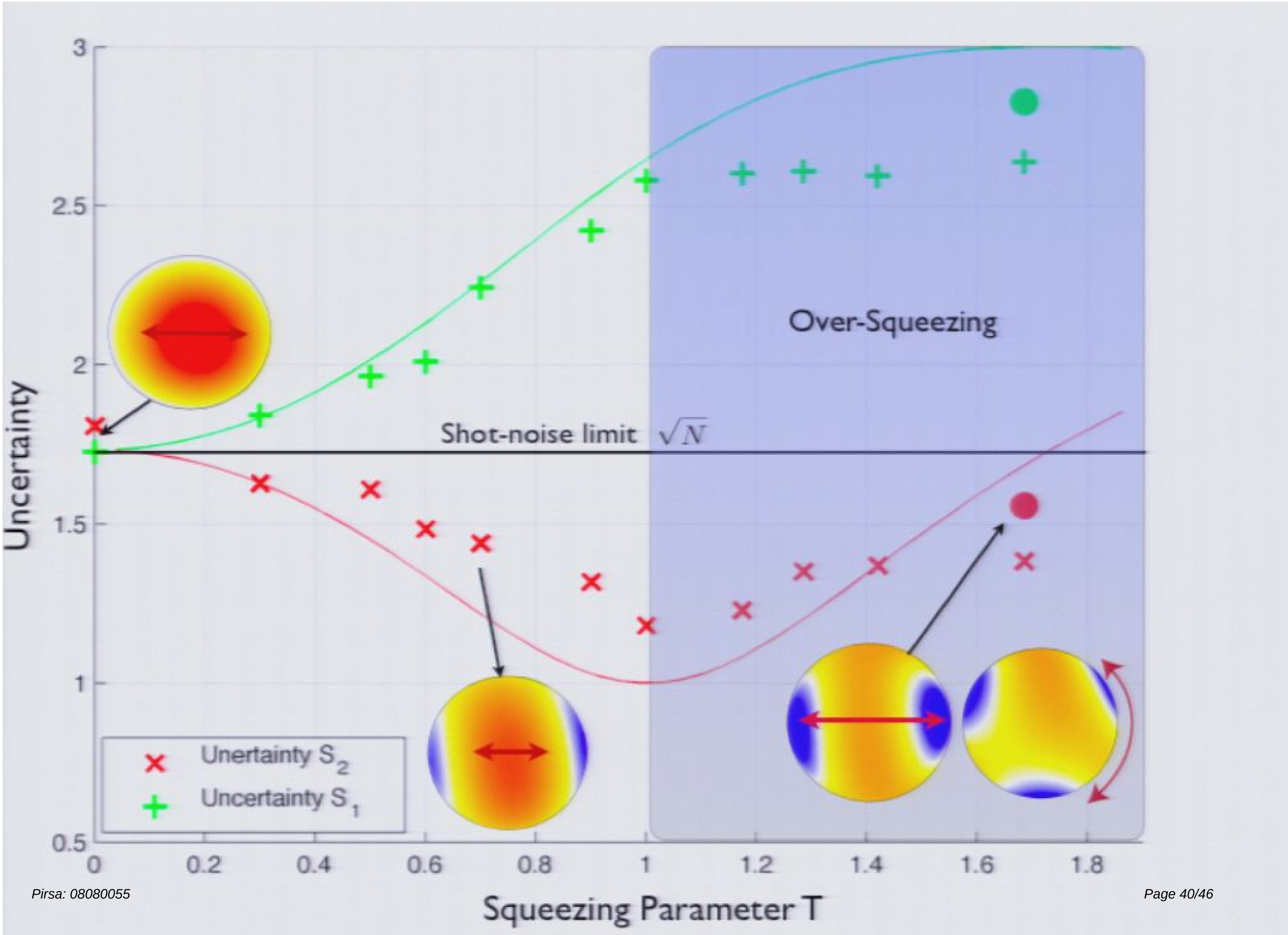


Results





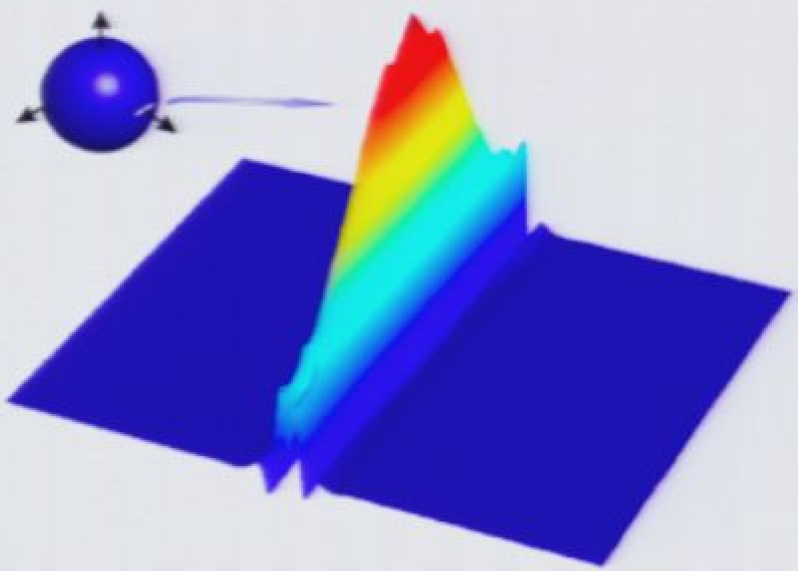
Coherent State
Phase State
N00N State
$|3,0\rangle_{R,L}$
$\frac{1}{\sqrt{4}}(|3,0\rangle_{H,V} + |2,1\rangle_{H,V} + |1,2\rangle_{H,V} + |0,3\rangle_{H,V})$
$\frac{1}{\sqrt{2}}(|3,0\rangle_{H,V} + |0,3\rangle_{H,V})$
$\hat{S}_1$
$\hat{S}_2$
$\hat{S}_3$
$sin(\phi)$
θ

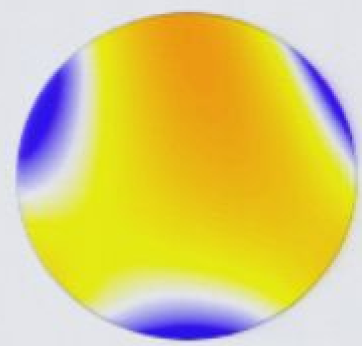


Over-Squeezing
Shot-noise limit $\sqrt{N}$
Uncertainty S_2
Uncertainty S_1
Uncertainty
Squeezing Parameter T
3
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

Squeezing & Over-Squeezing

First to see this phenomenon of over-squeezing. First experiment where the spherical topology of the Polarization space becomes important.

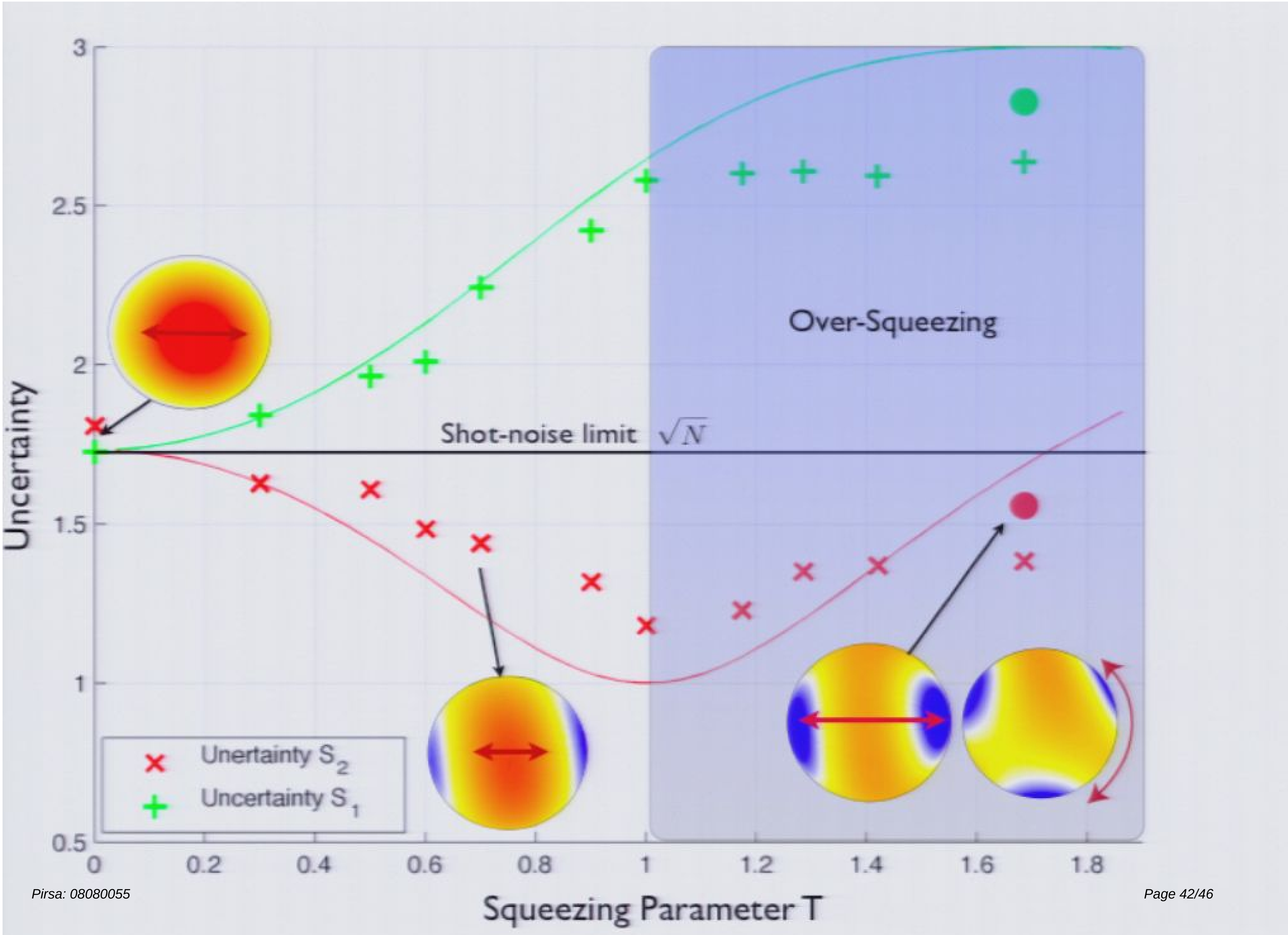




PRL 99, 220401 (2007) PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2007

Quantum Reconstruction of an Intense Polarization Squeezed Optical State

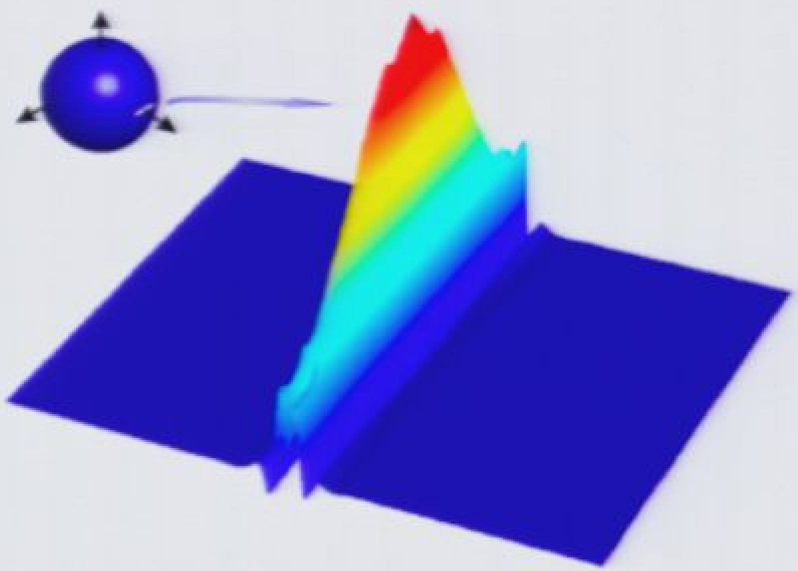
Ch. Marquardt,[1] J. Heersink,[1] R. Dong,[1] M. V. Chekhova,[2] A. B. Klimov,[3] L. L. Sánchez-Soto,[1]
U. L. Andersen,[1,4] and G. Leuchs[1]

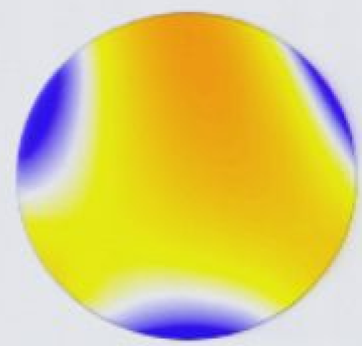


Over-Squeezing
Shot-noise limit $\sqrt{N}$
Uncertainty
Squeezing Parameter T
Unertainty S_2
Uncertainty S_1
0.5
1
1.5
2
2.5
3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

Squeezing & Over-Squeezing

First to see this phenomenon of over-squeezing. First experiment where the spherical topology of the Polarization space becomes important.

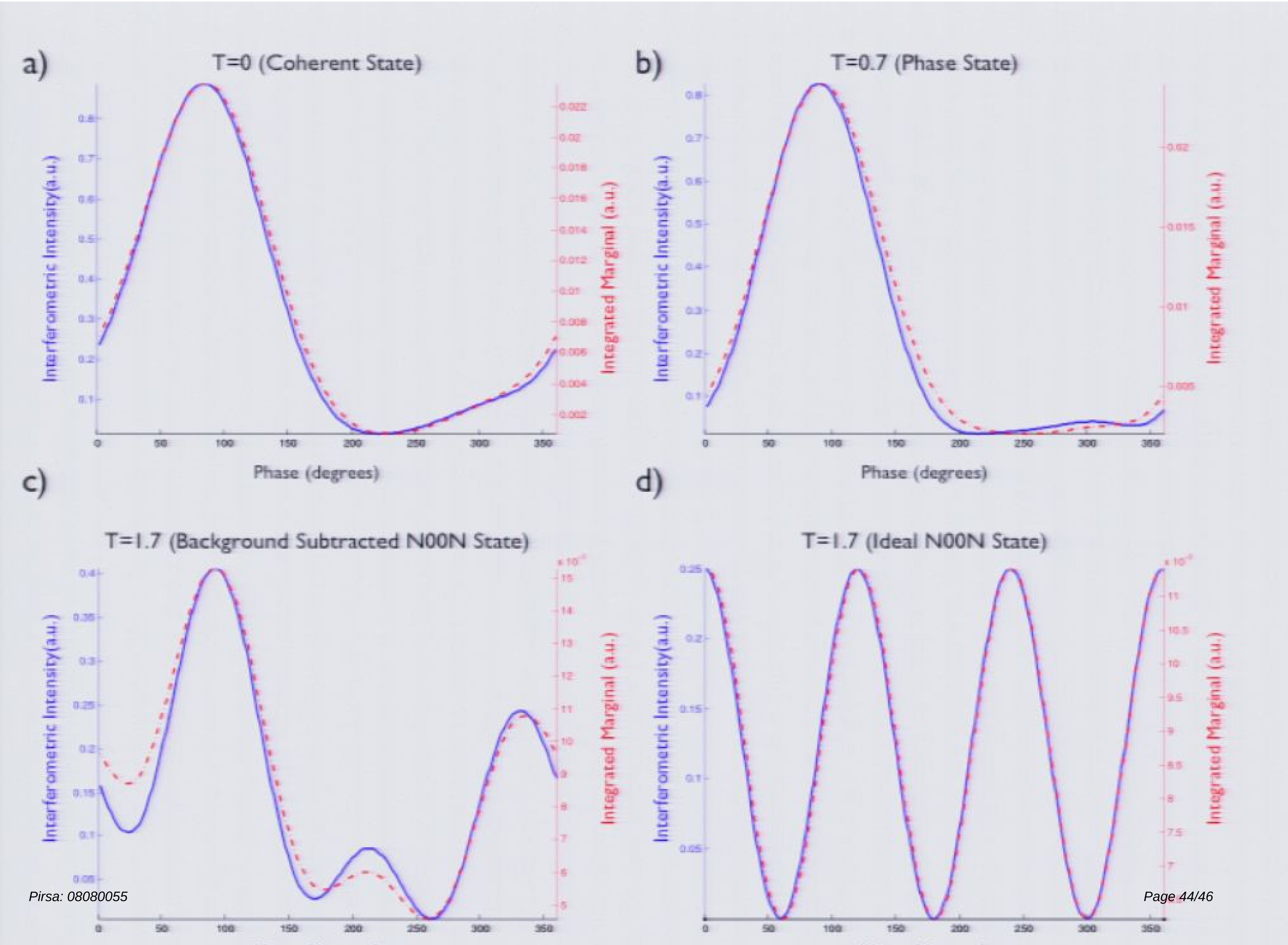




PRL 99, 220401 (2007) PHYSICAL REVIEW LETTERS week ending 30 NOVEMBER 2007

Quantum Reconstruction of an Intense Polarization Squeezed Optical State

Ch. Marquardt,[1] J. Heersink,[1] R. Dong,[1] M. V. Chekhova,[2] A. B. Klimov,[3] L. L. Sánchez-Soto,[1]
U. L. Andersen,[1,4] and G. Leuchs[1]


a)
T=0 (Coherent State)
Interferometric Intensity(a.u.)
Integrated Marginal (a.u.)
Phase (degrees)
b)
T=0.7 (Phase State)
Interferometric Intensity(a.u.)
Integrated Marginal (a.u.)
Phase (degrees)
c)
T=1.7 (Background Subtracted N00N State)
Interferometric Intensity(a.u.)
Integrated Marginal (a.u.)
d)
T=1.7 (Ideal N00N State)
Interferometric Intensity(a.u.)
Integrated Marginal (a.u.)