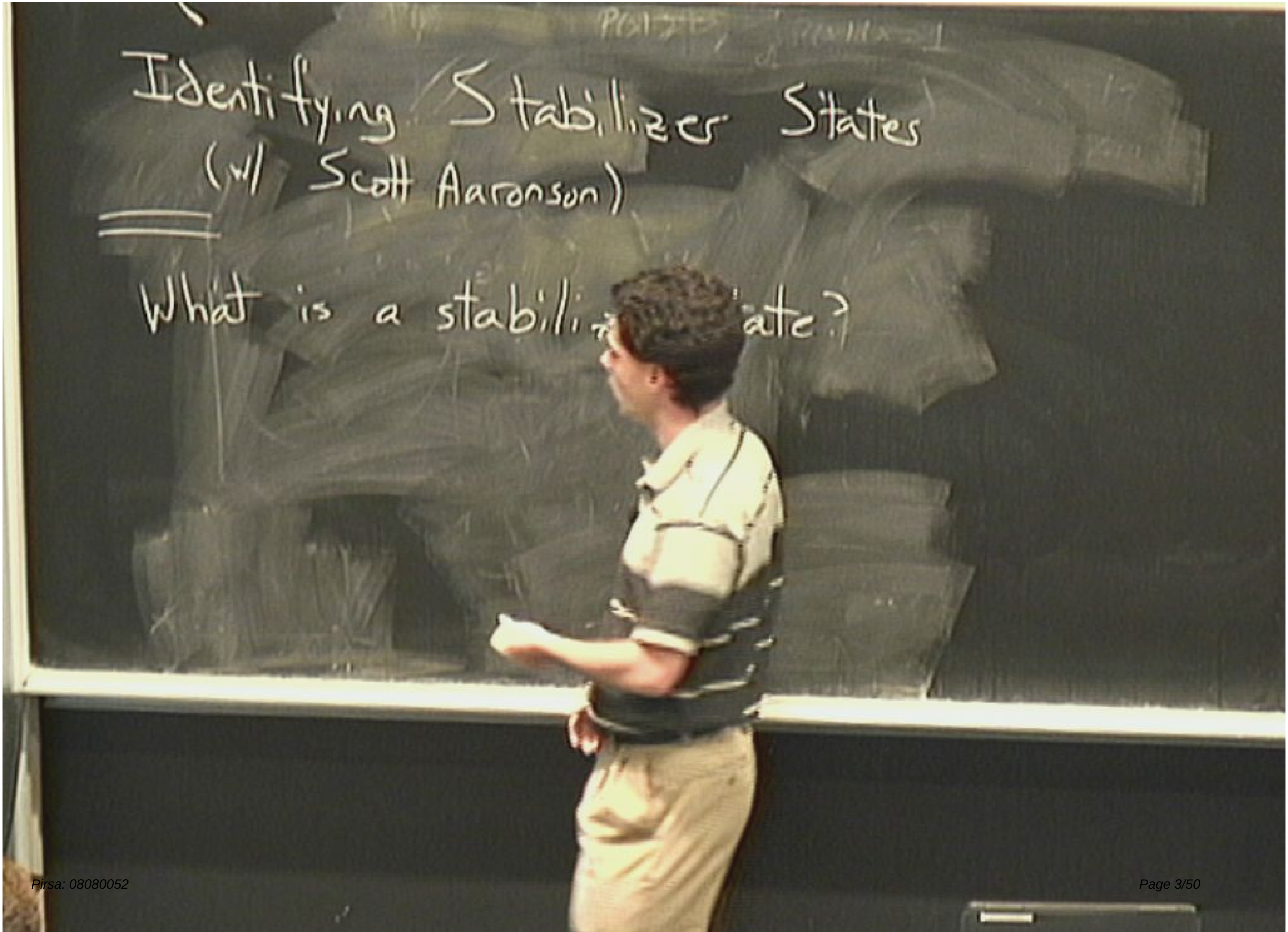
Title: Identifying Stabilizer States

Date: Aug 29, 2008 02:45 PM

URL: http://pirsa.org/08080052

Abstract: Suppose you are given m copies of an unknown n-qubit stabilizer state. How many copies do you need before you can figure out exactly what state it is? Just to specify the state requires about n^2/2 bits, so certainly m is at least n/2. Using only single-copy measurements, we show how to identify the state with high probability using m=O(n^2) copies. If one can make joint measurements, O(n) copies is sufficient.This is joint work with Scott Aaronson.

Identifying Stabilizer States
(w/ Scott Aaronson)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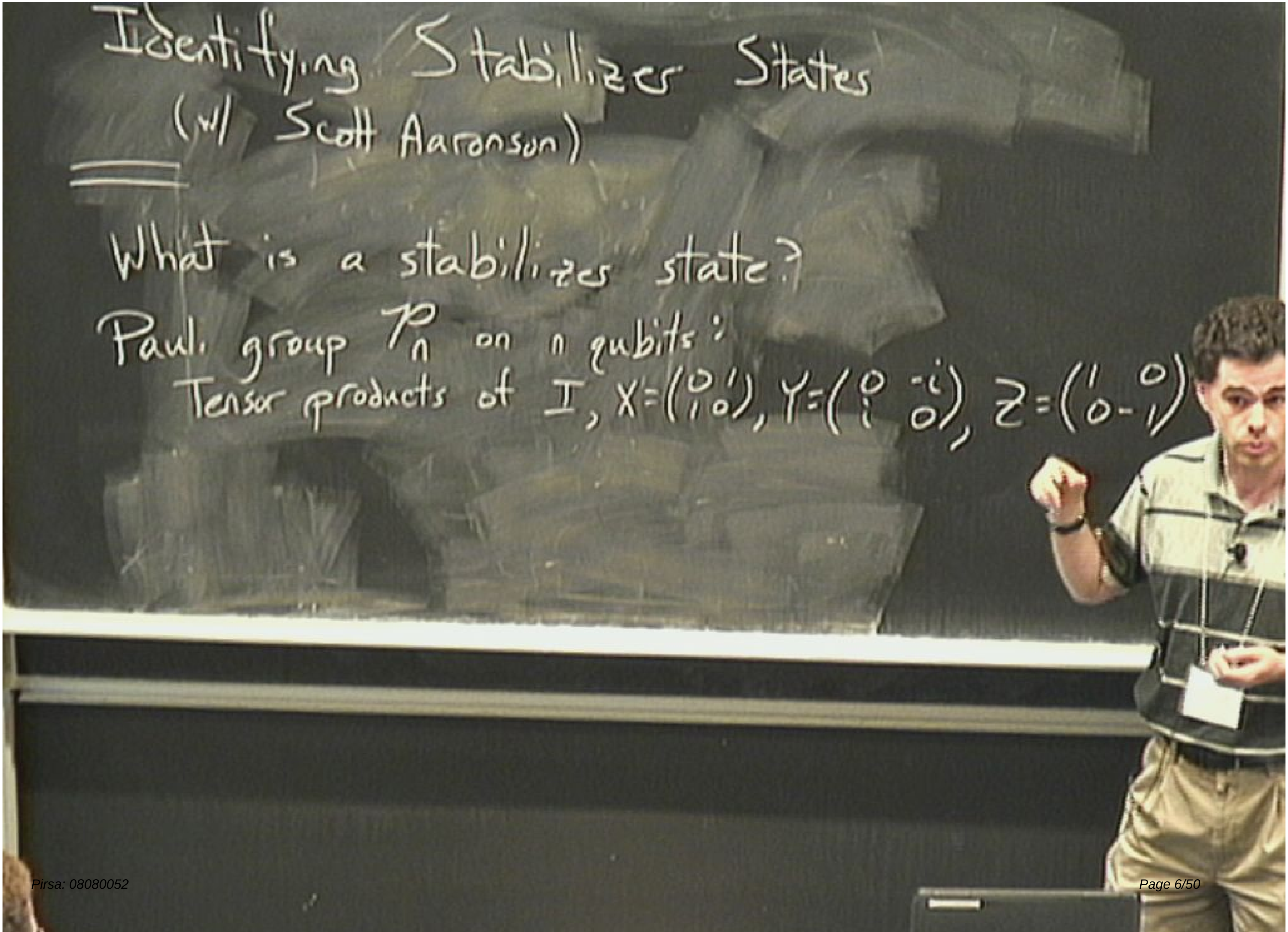
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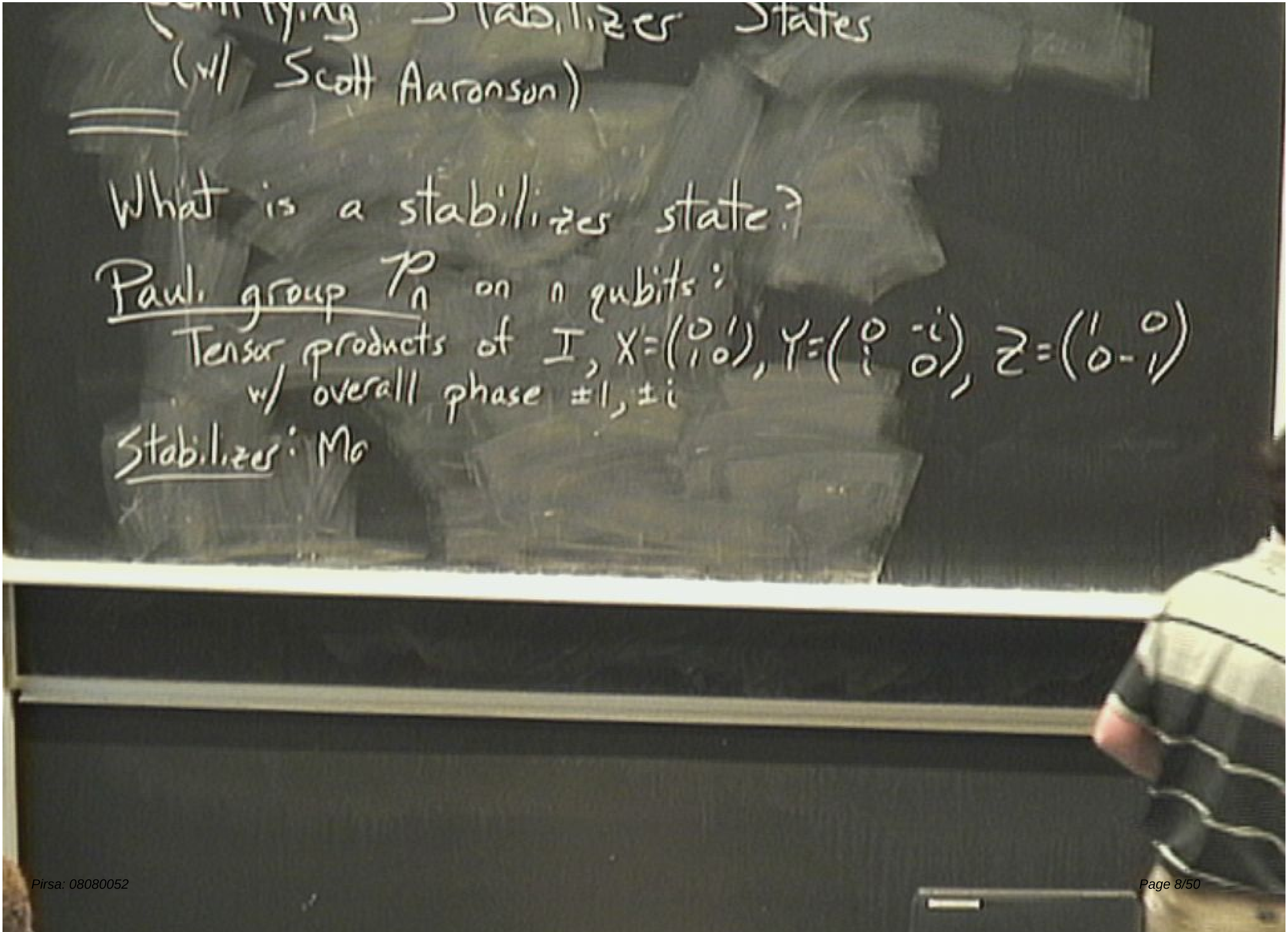
w/ overall phase $\pm 1, \pm i$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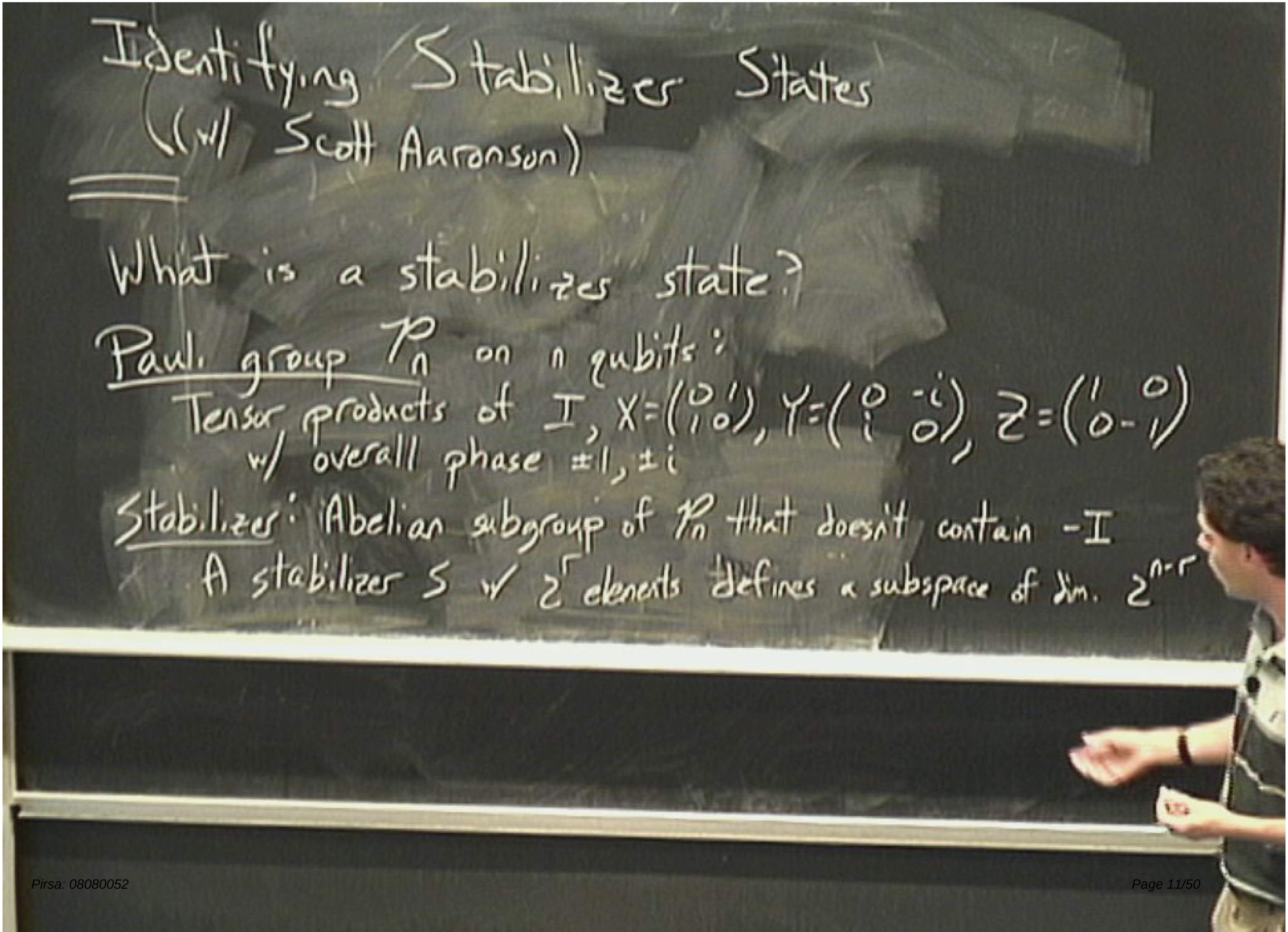
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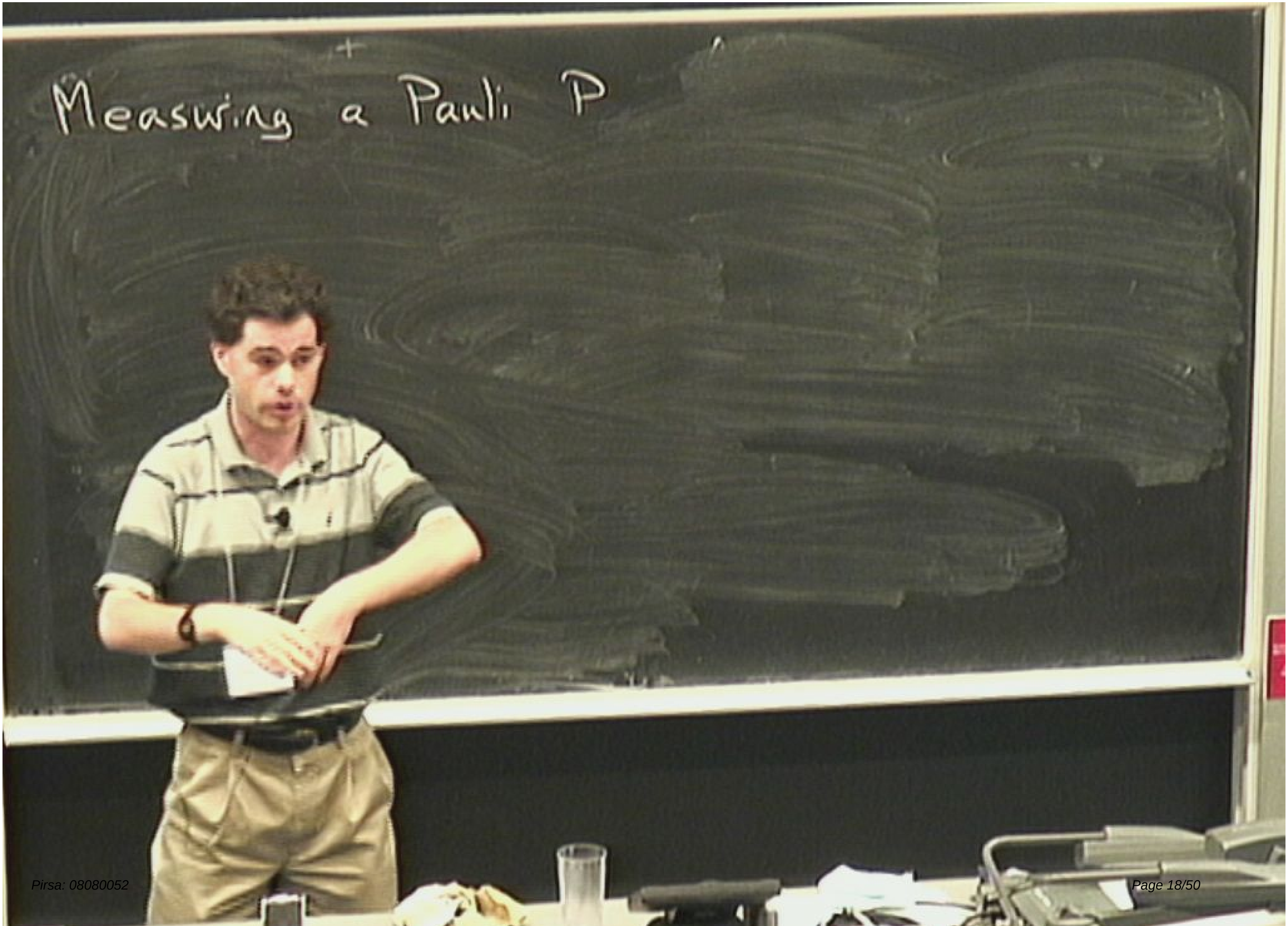
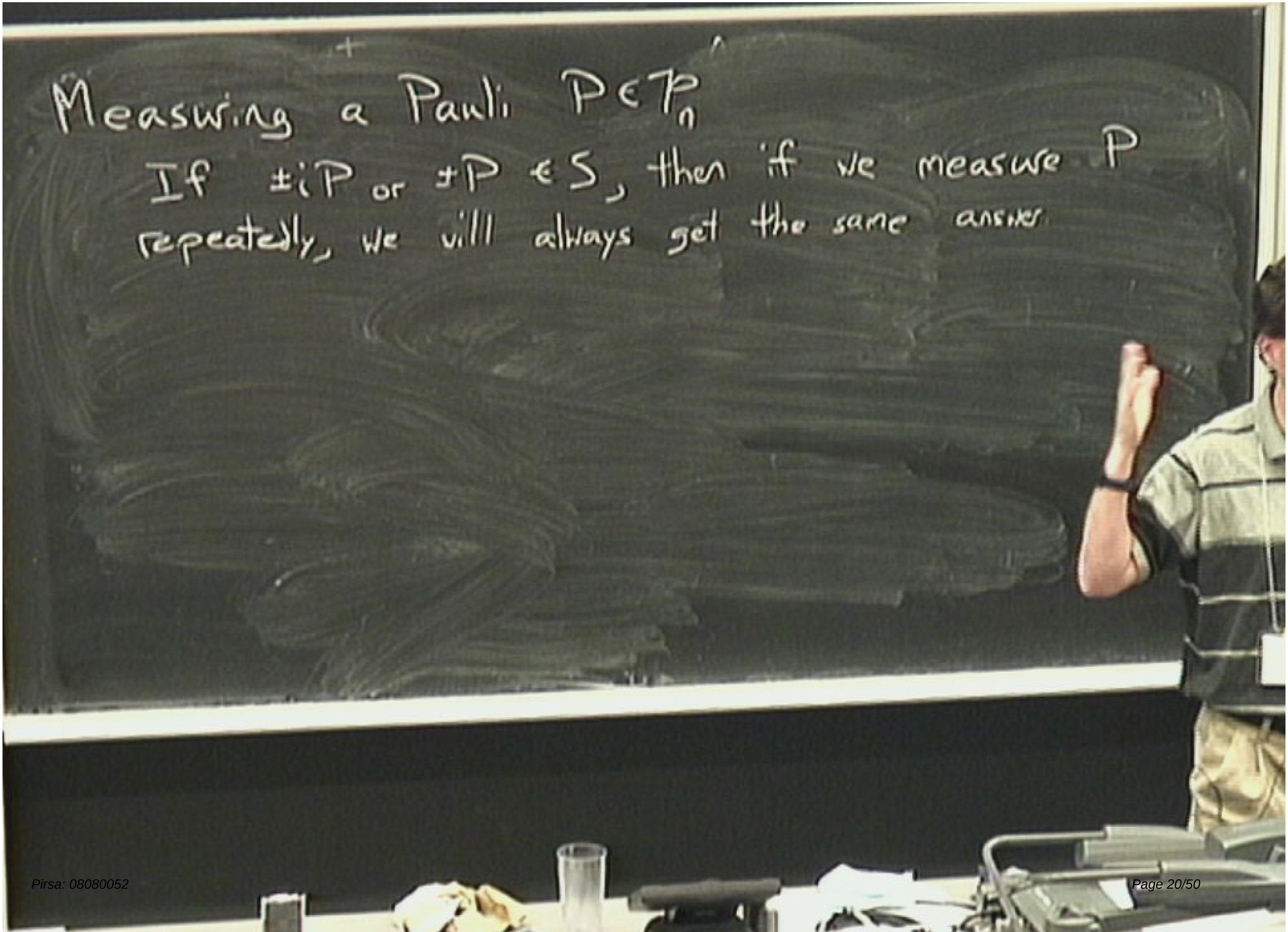

Measuring a Pauli P

Measuring a Pauli $P \in \mathcal{P}_n$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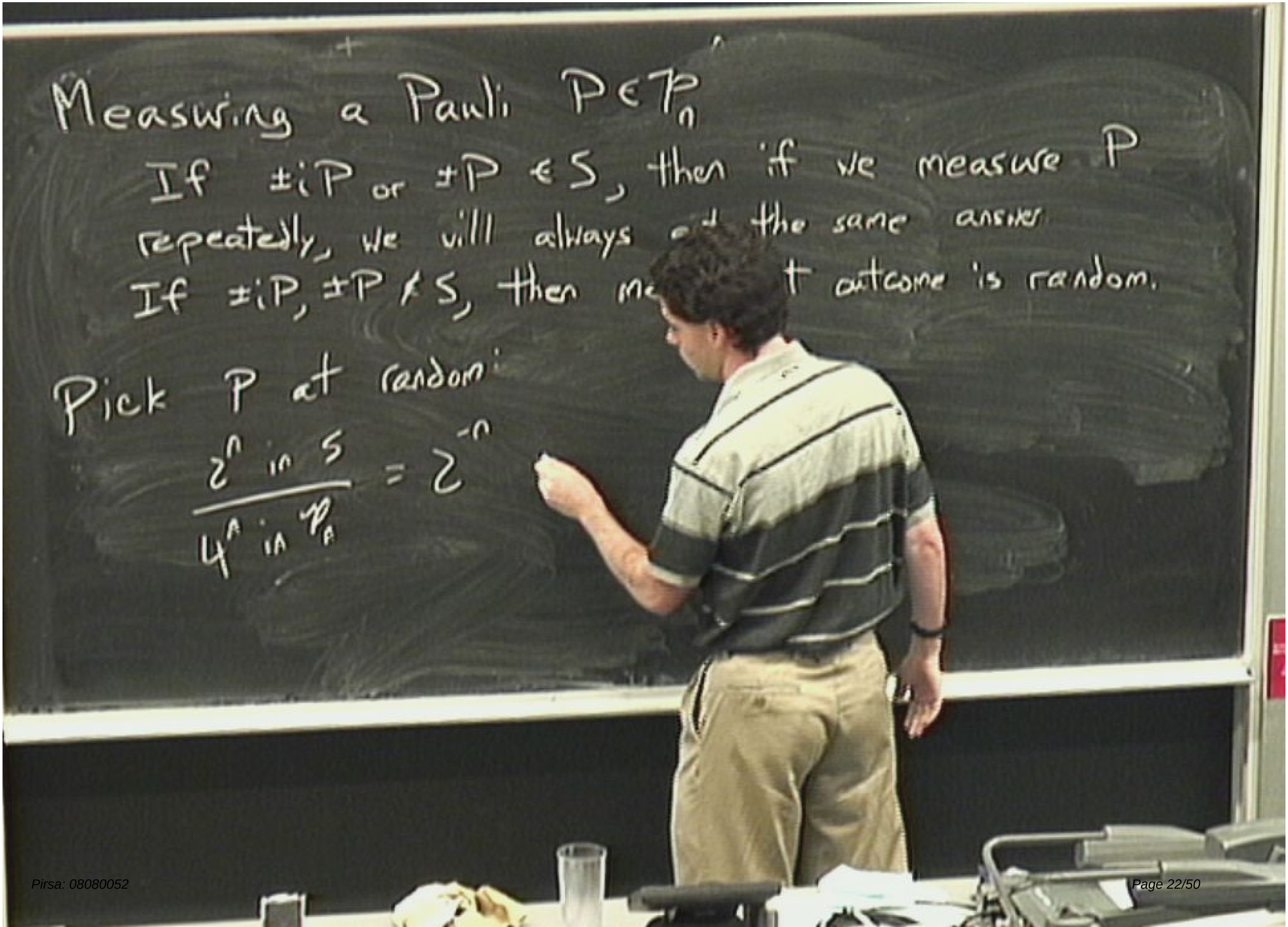
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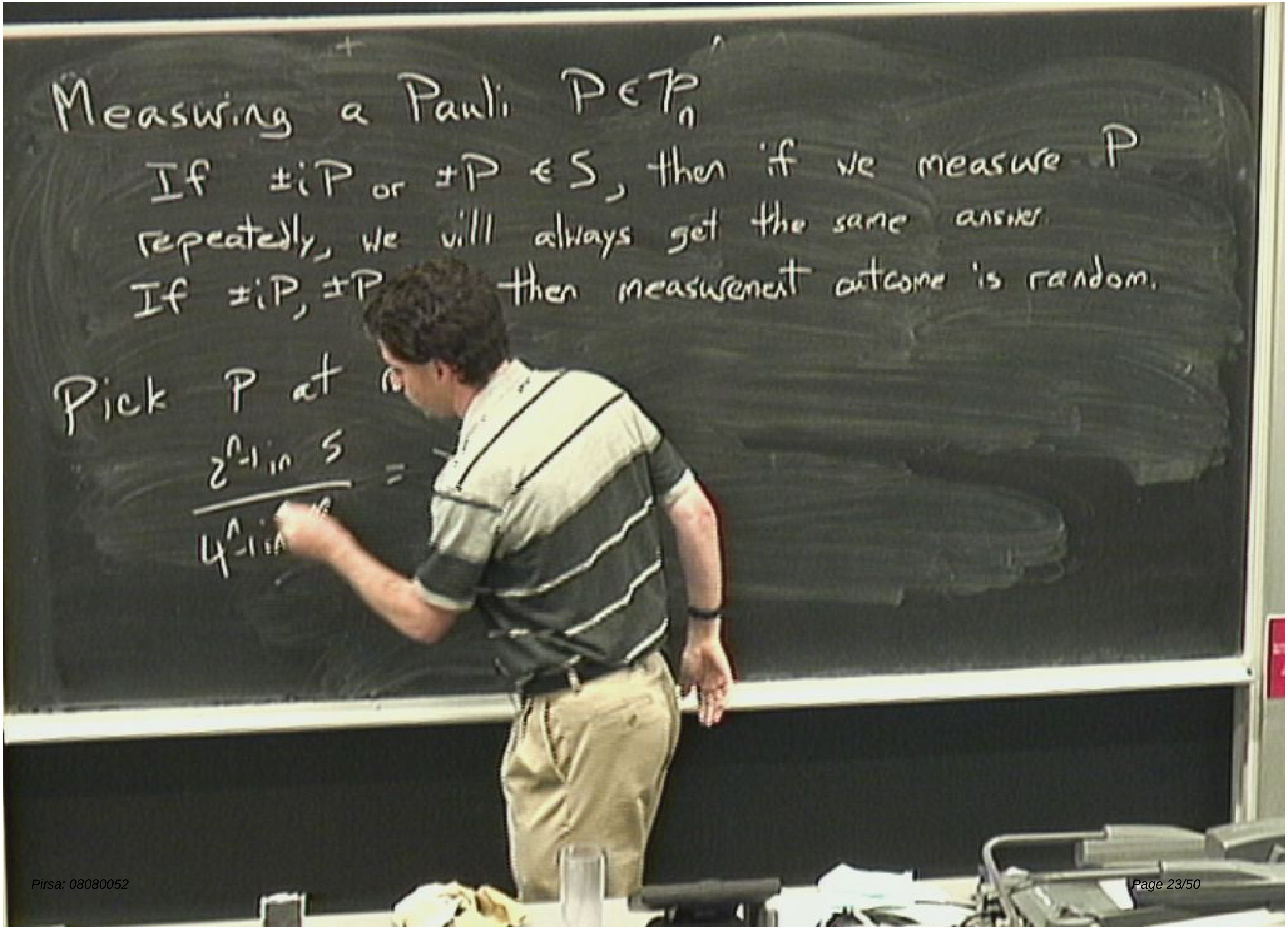
Pick P at random:
$\frac{2^n \text{ in } S}{4^n \text{ in } \mathcal{P}_n} = 2^{-n}$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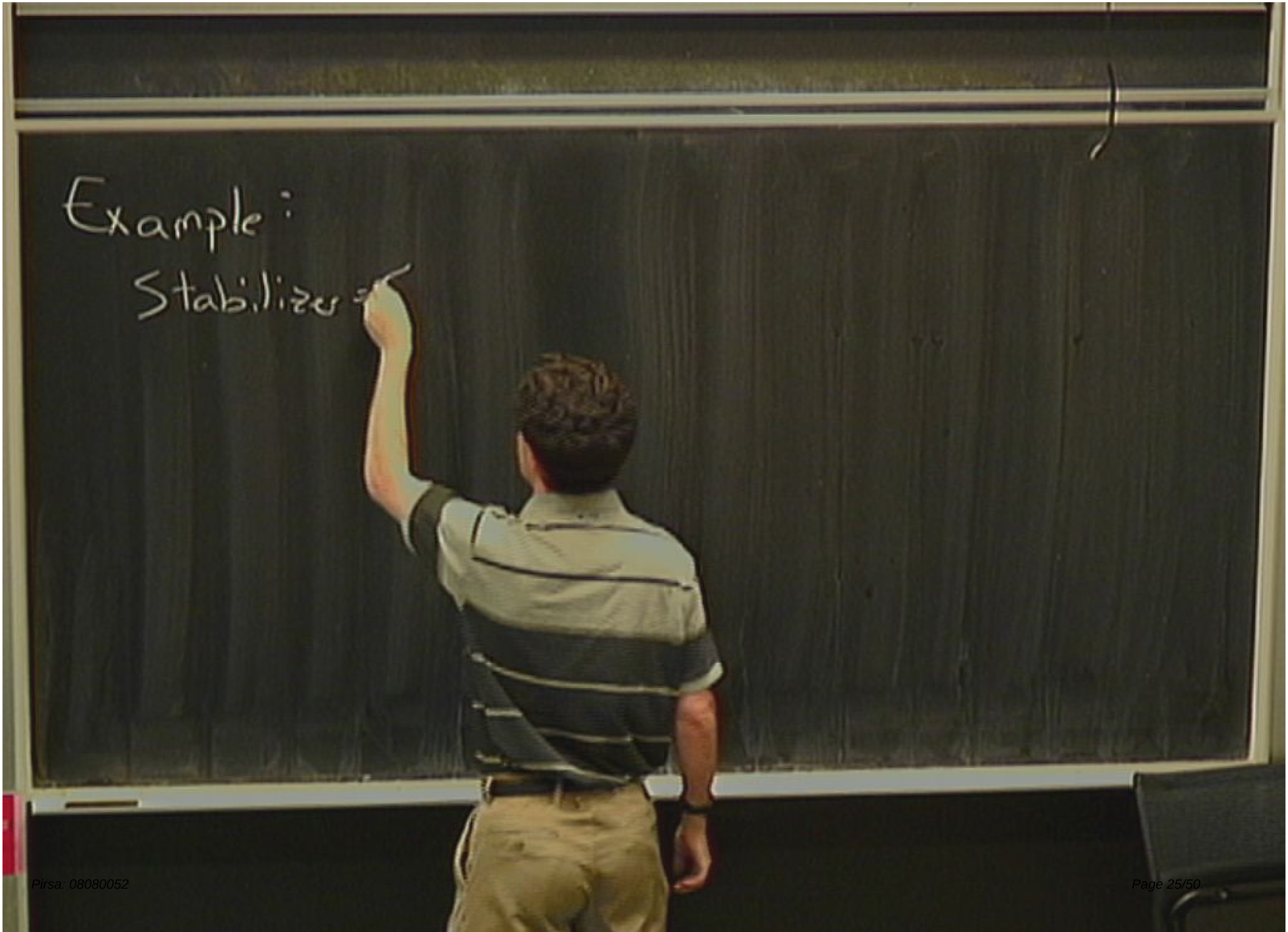
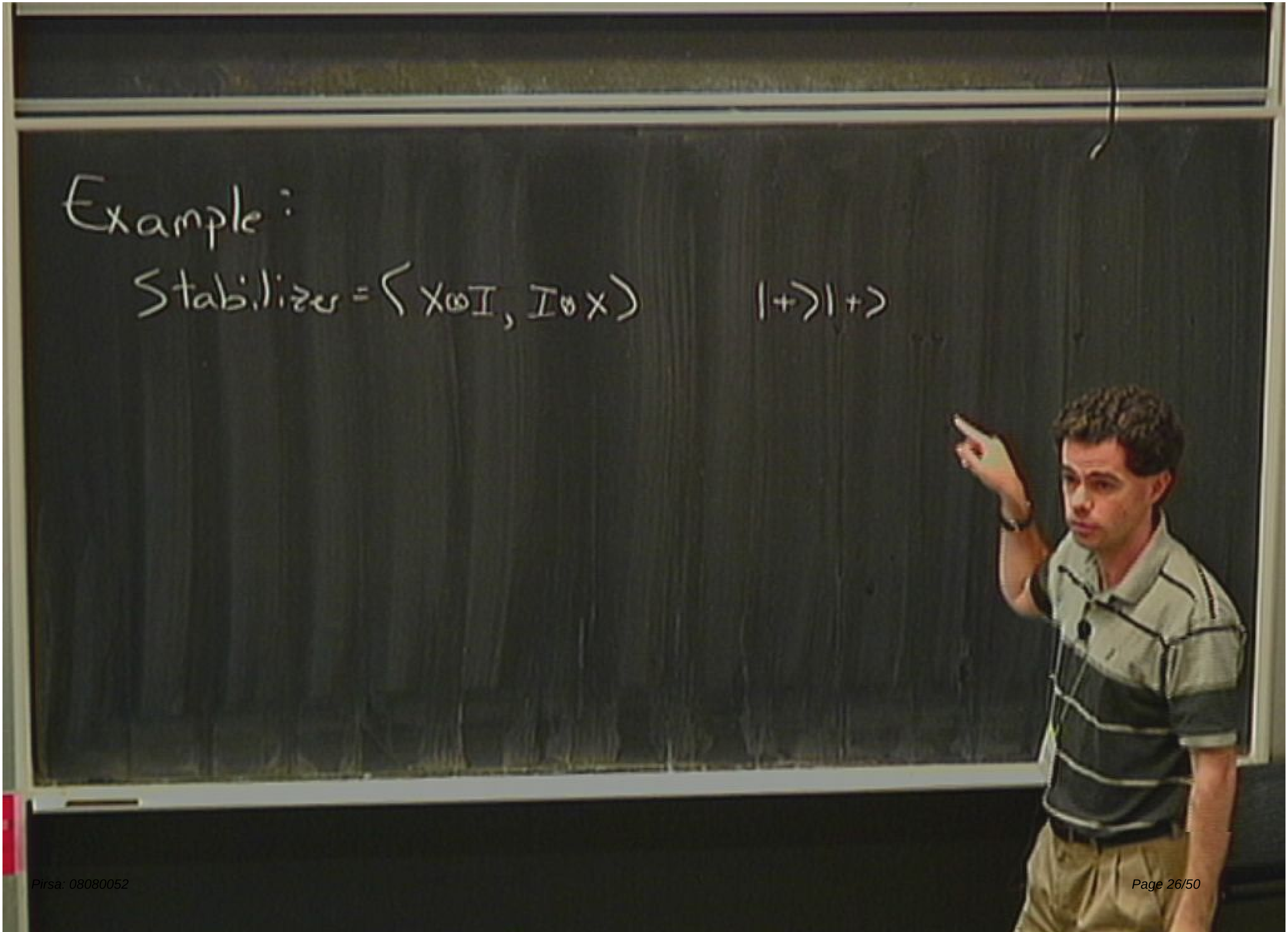
Pick P at random:

$$\frac{2^n - 1 \text{ in } S - \{I\}}{4^n - 1 \text{ in } \mathcal{P}_n - \{I\}} \approx 2^{-n} \quad (\text{Prob. of choosing } P \in S)$$


Example:
Stabilizer =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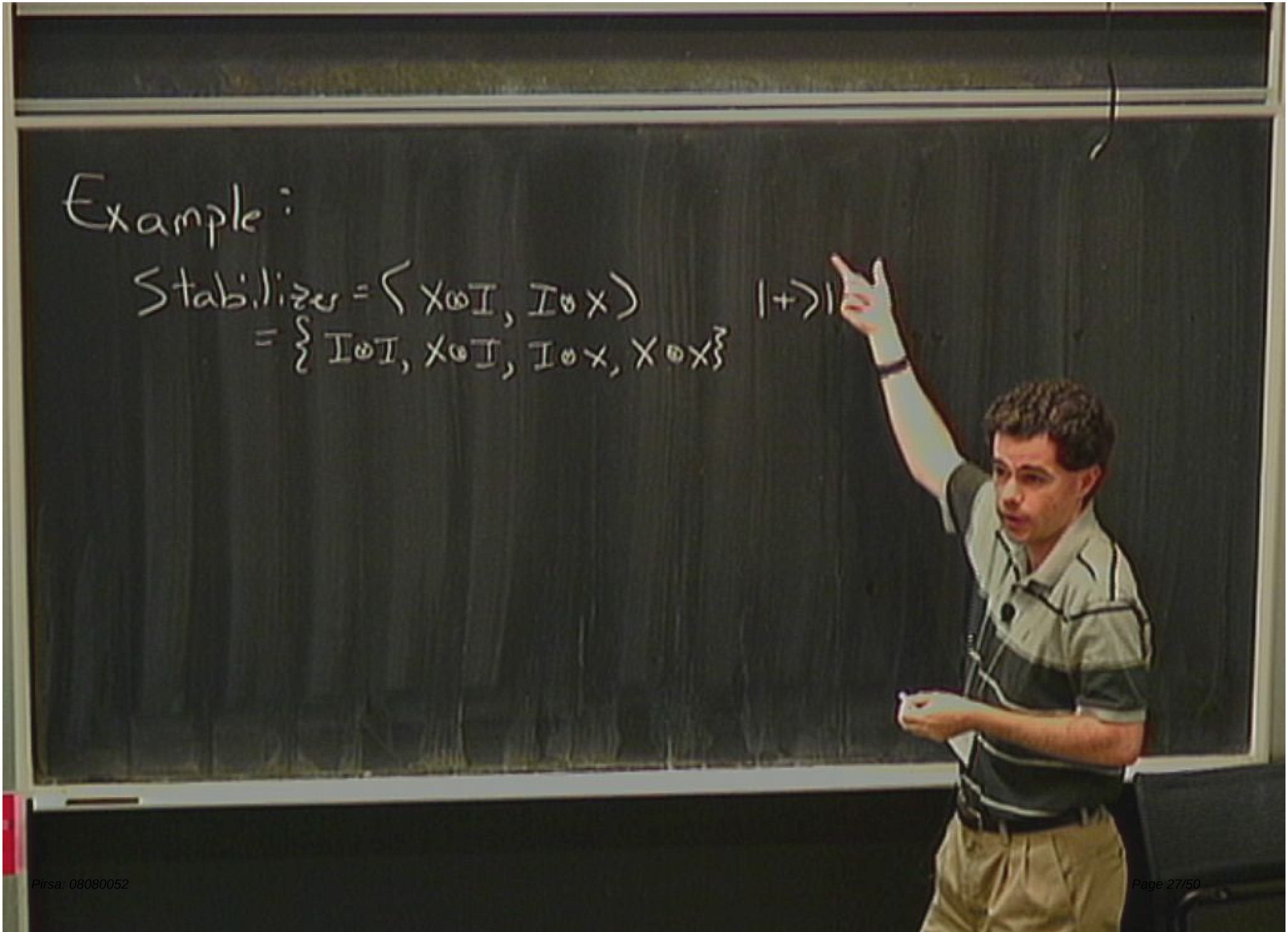
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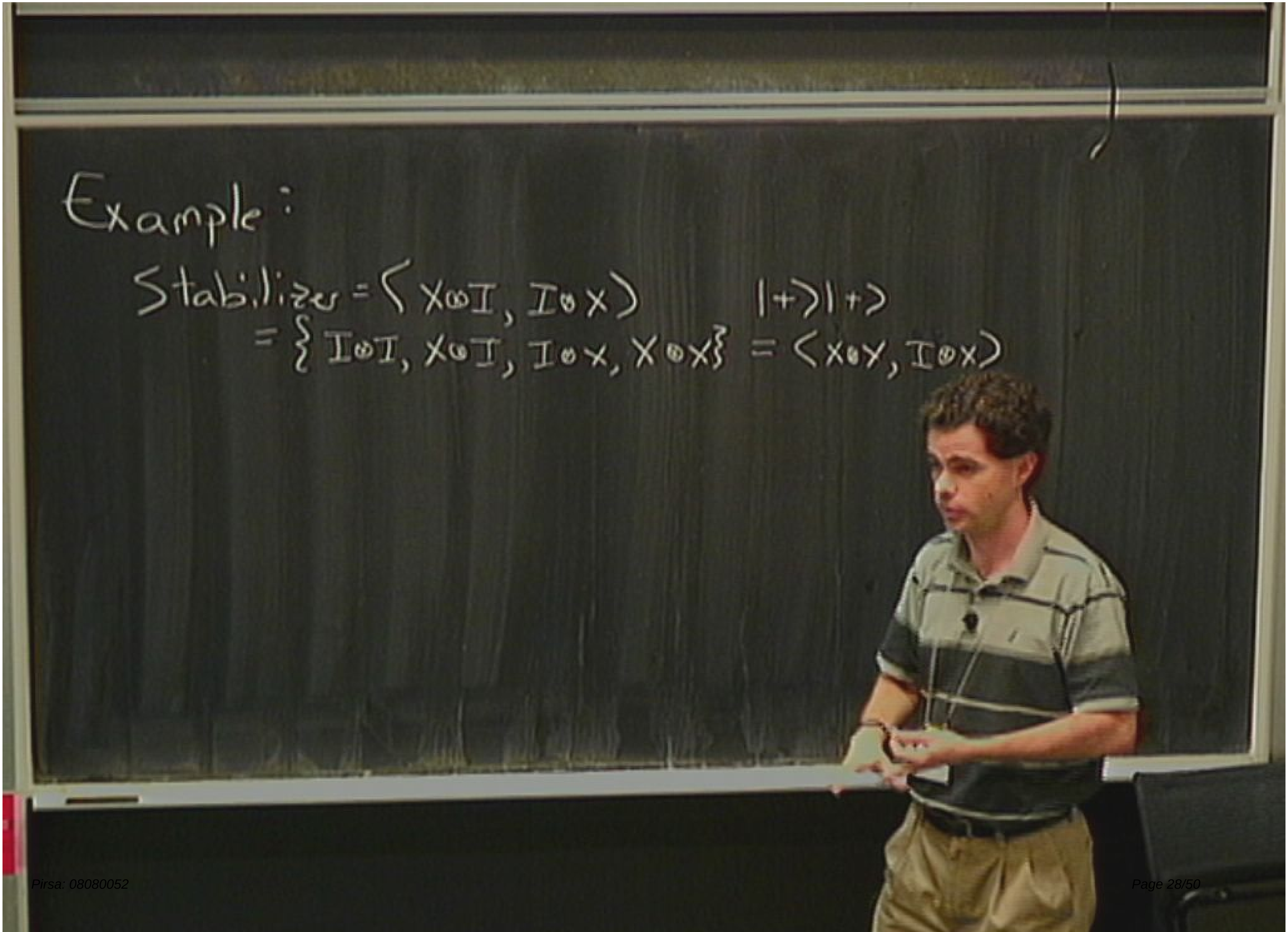
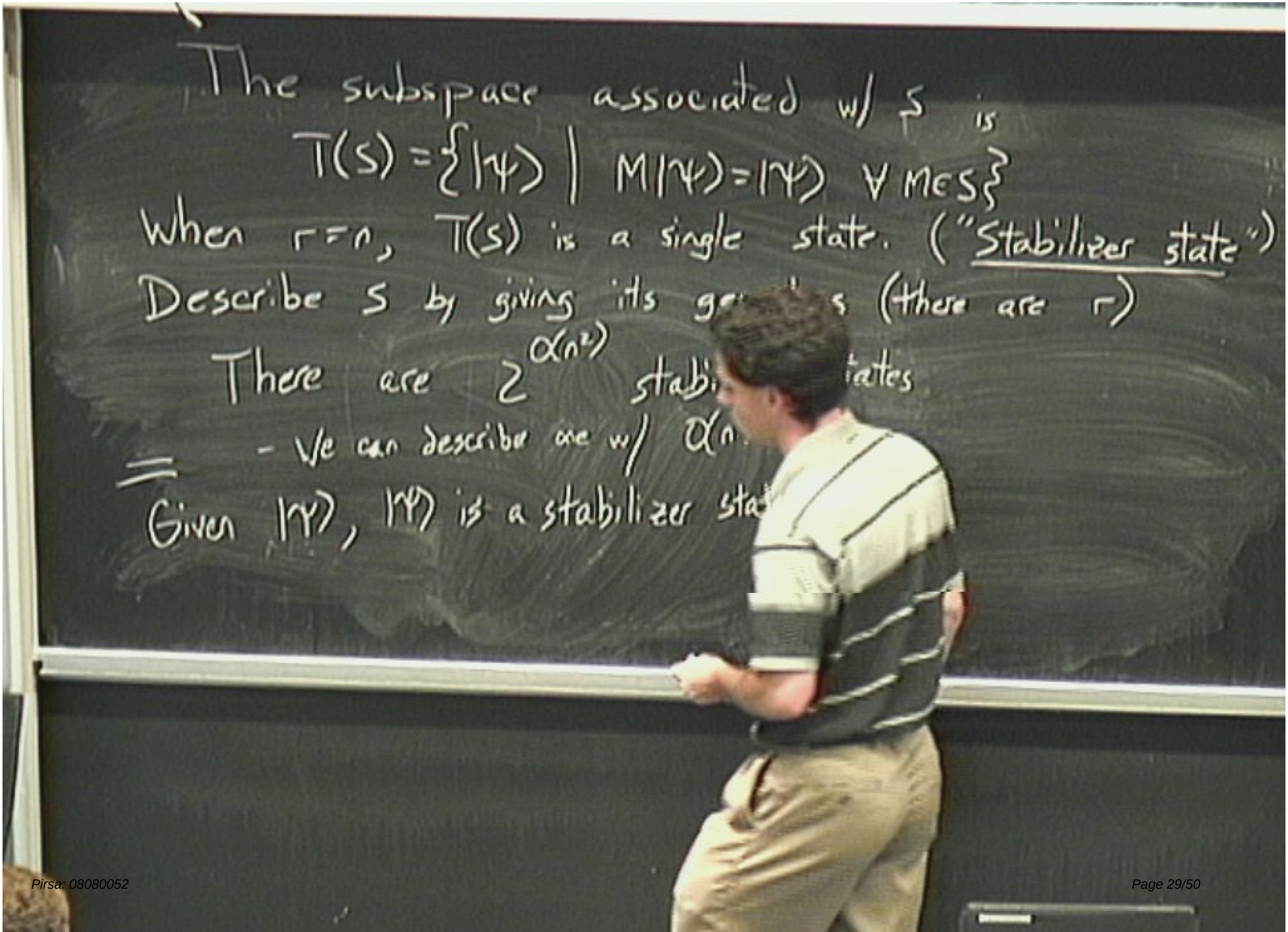
= {I⊗I, X⊗I, I⊗X, X⊗X}


Example:
Stabilizer = $\langle X\otimes I, I\otimes X\rangle$
$|+\rangle|+\rangle$
$= \{I\otimes I, X\otimes I, I\otimes X, X\otimes X\} = \langle X\otimes X, I\otimes X\rangle$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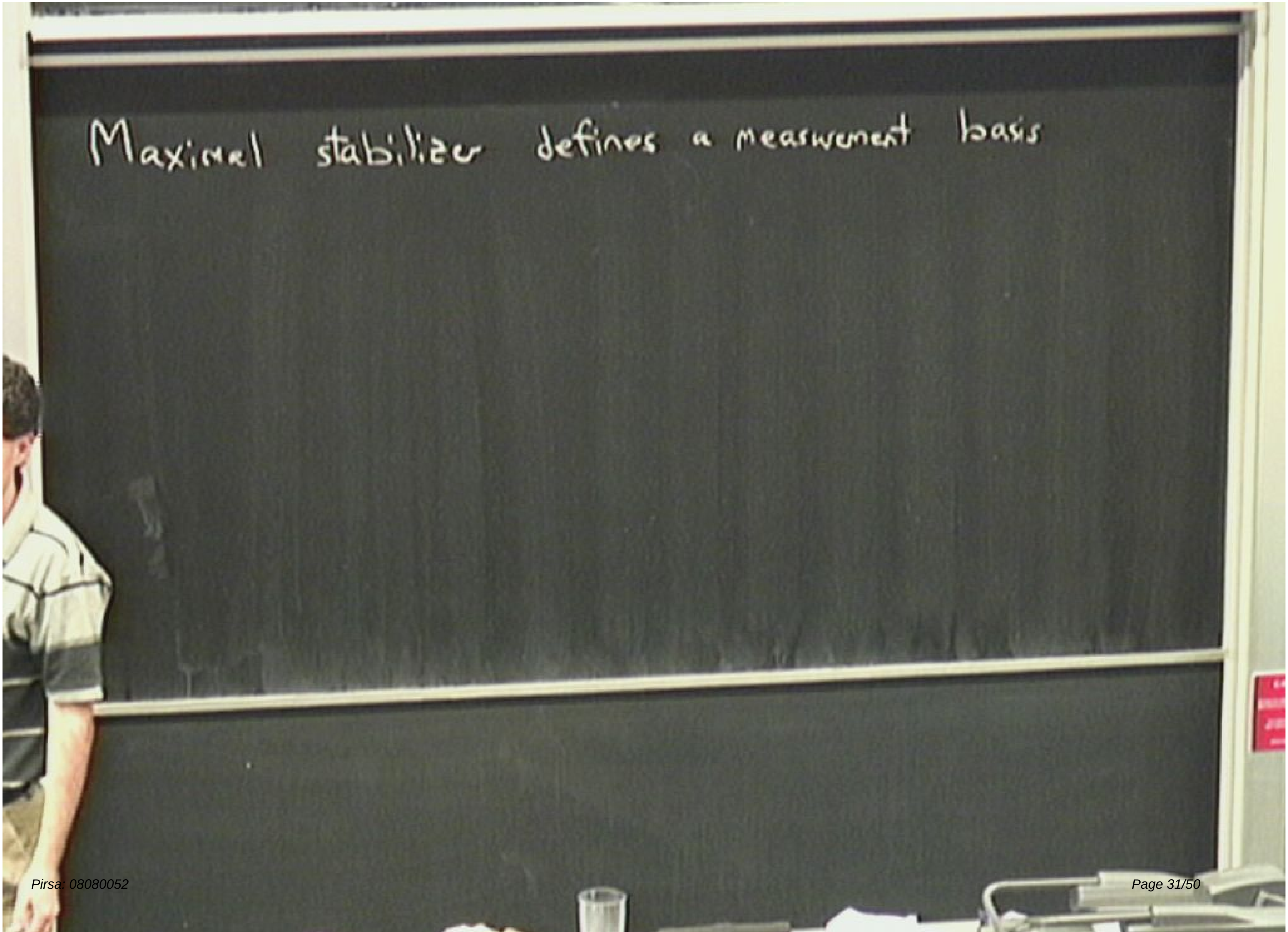
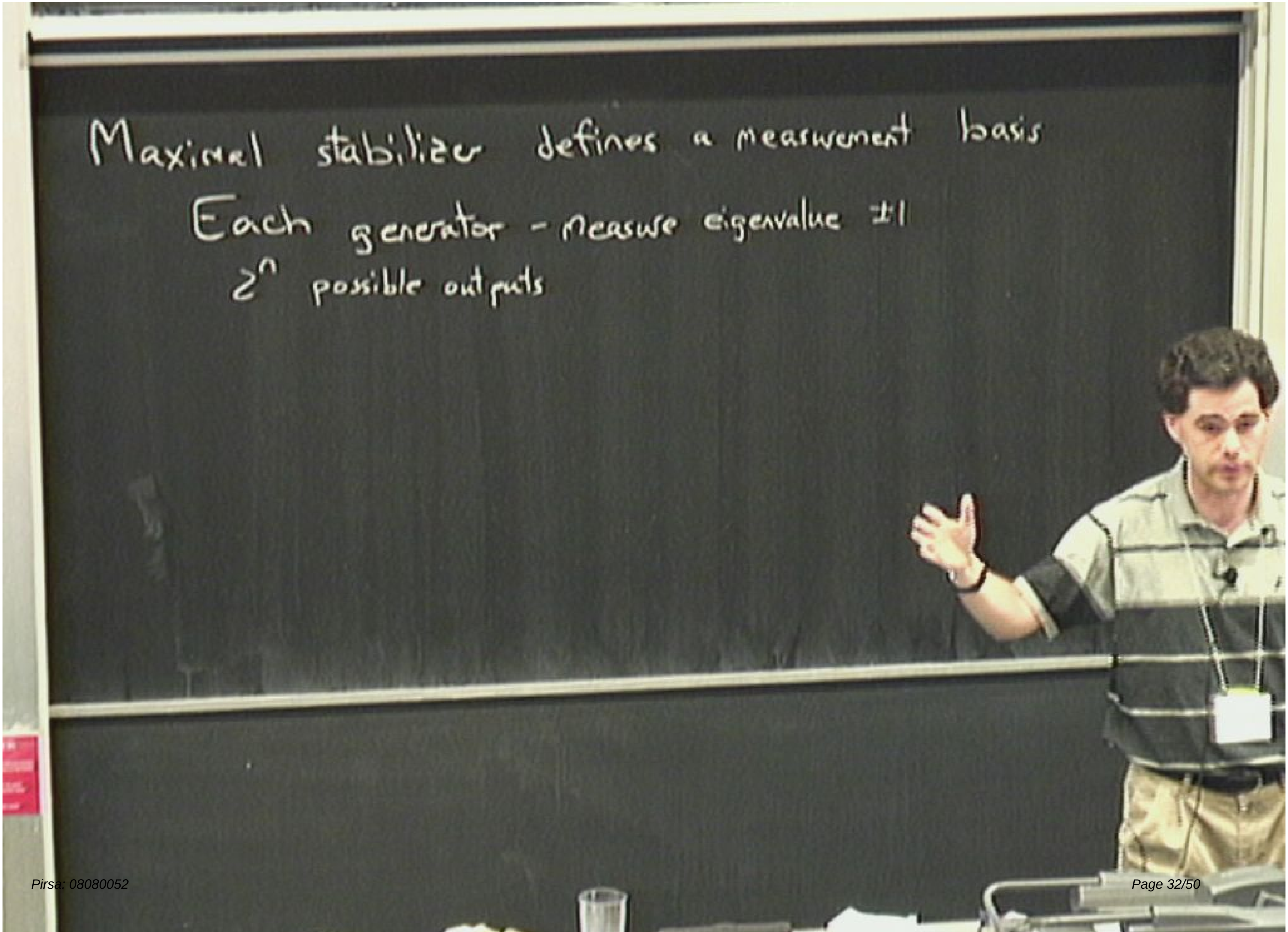
Given $|\psi\rangle^{\otimes M}$, $|\psi\rangle$ is a stabilizer state

Find S, with $T(S) = \{|\psi\rangle\}$


Maximal stabilizer defines a measurement basis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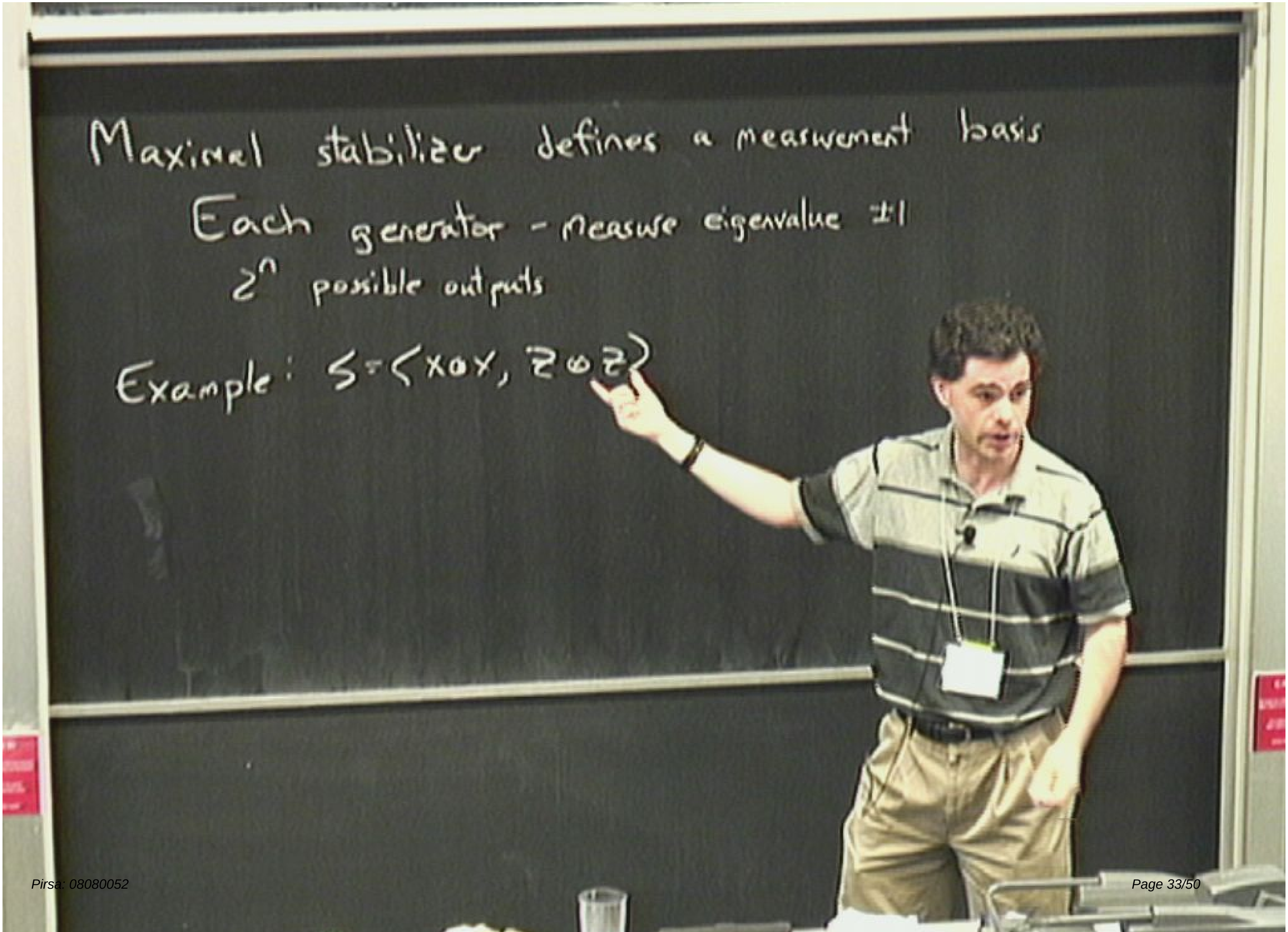
Each generator - measure eigenvalue ±1
2^n possible outputs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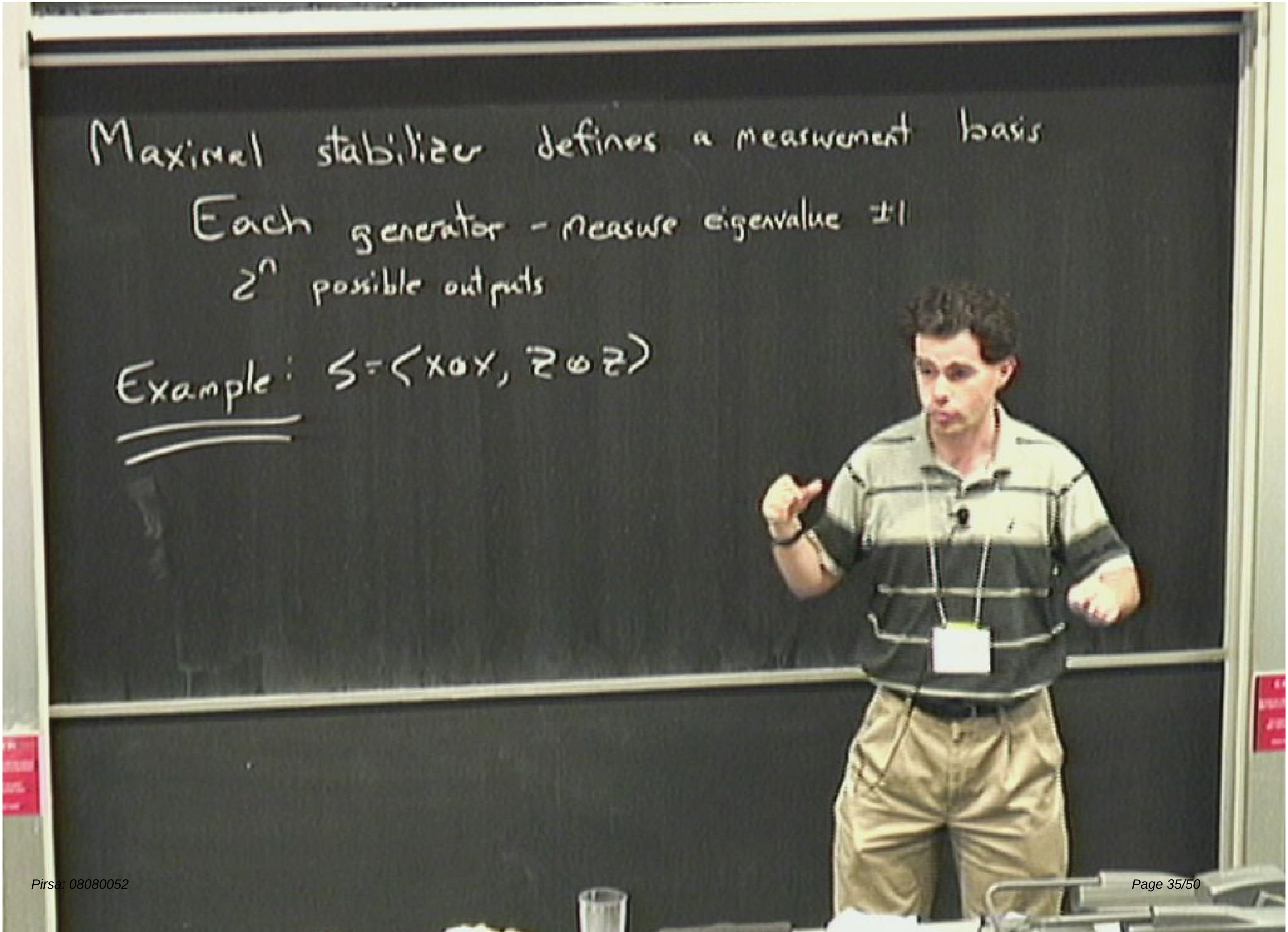
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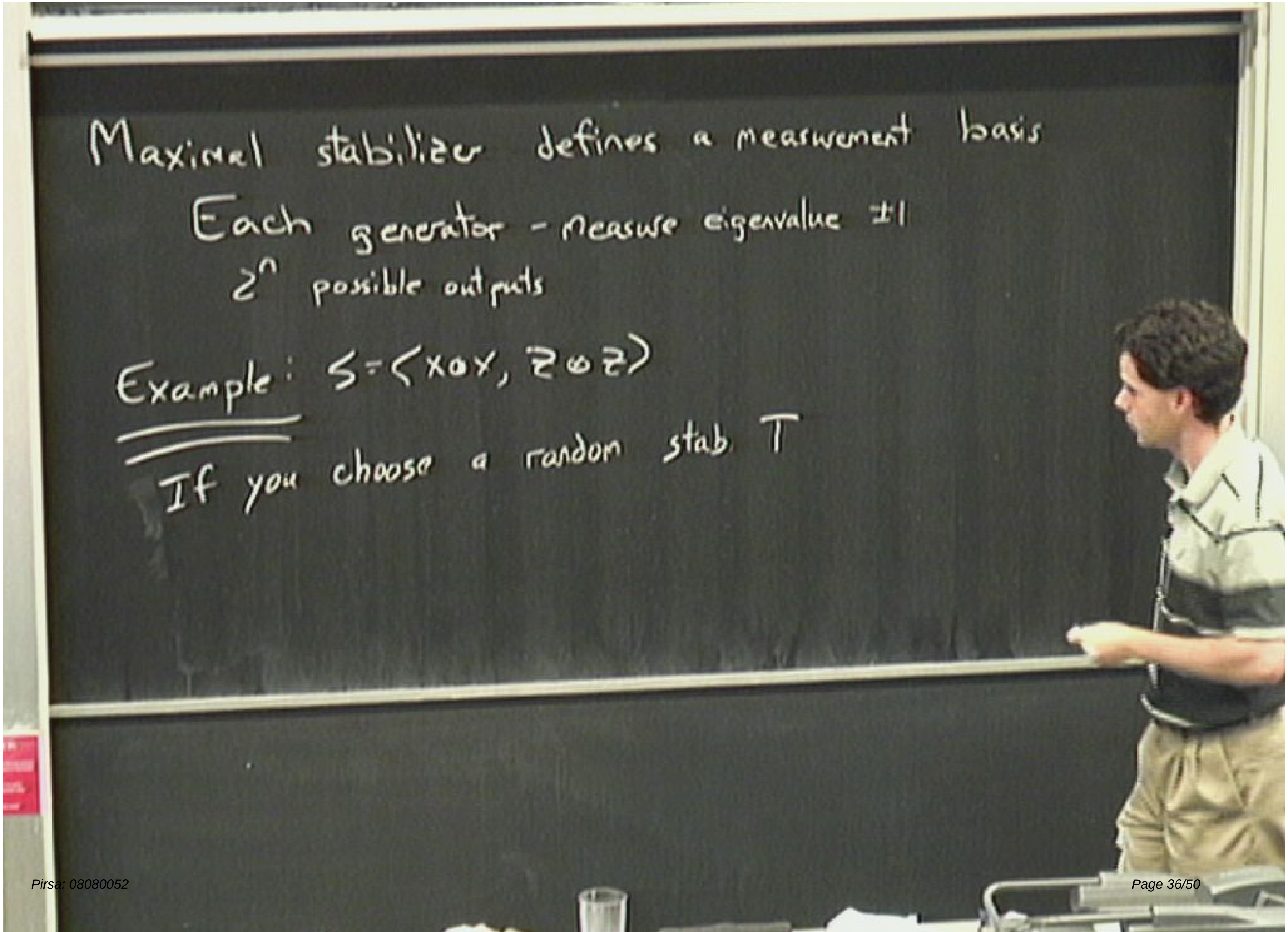
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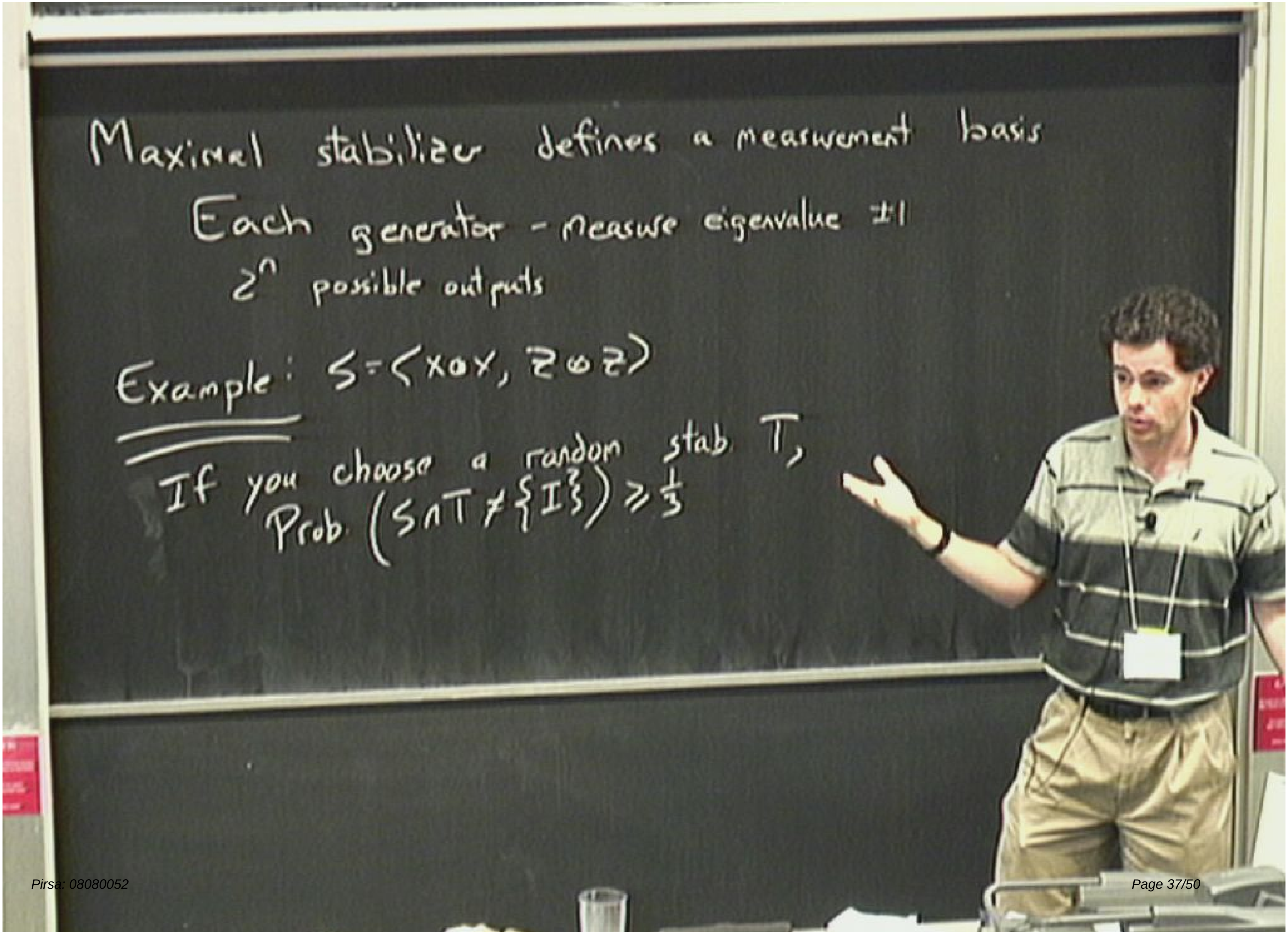
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Prob $(S \cap T \neq \{I\}) \geq \frac{1}{3}$

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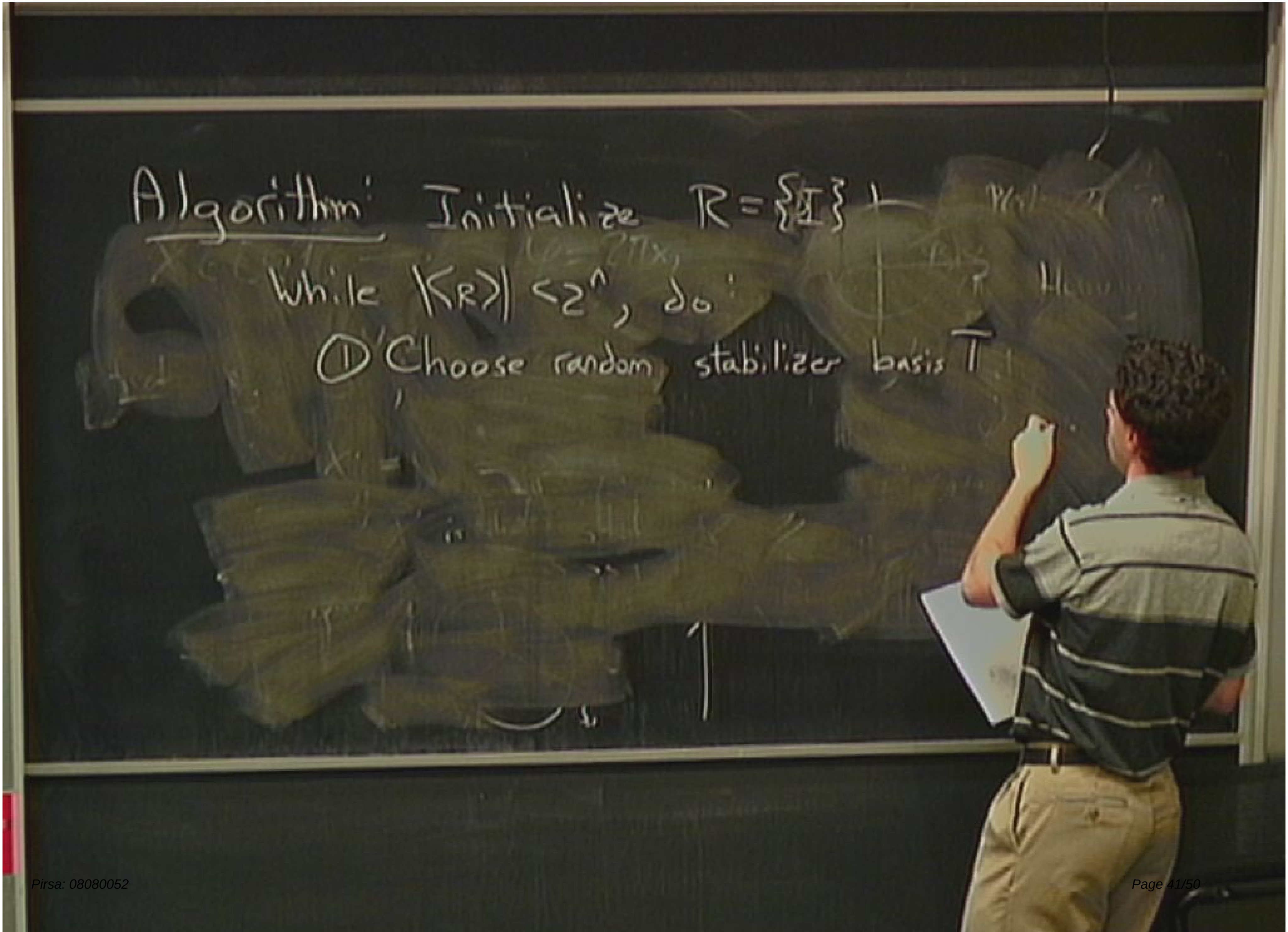
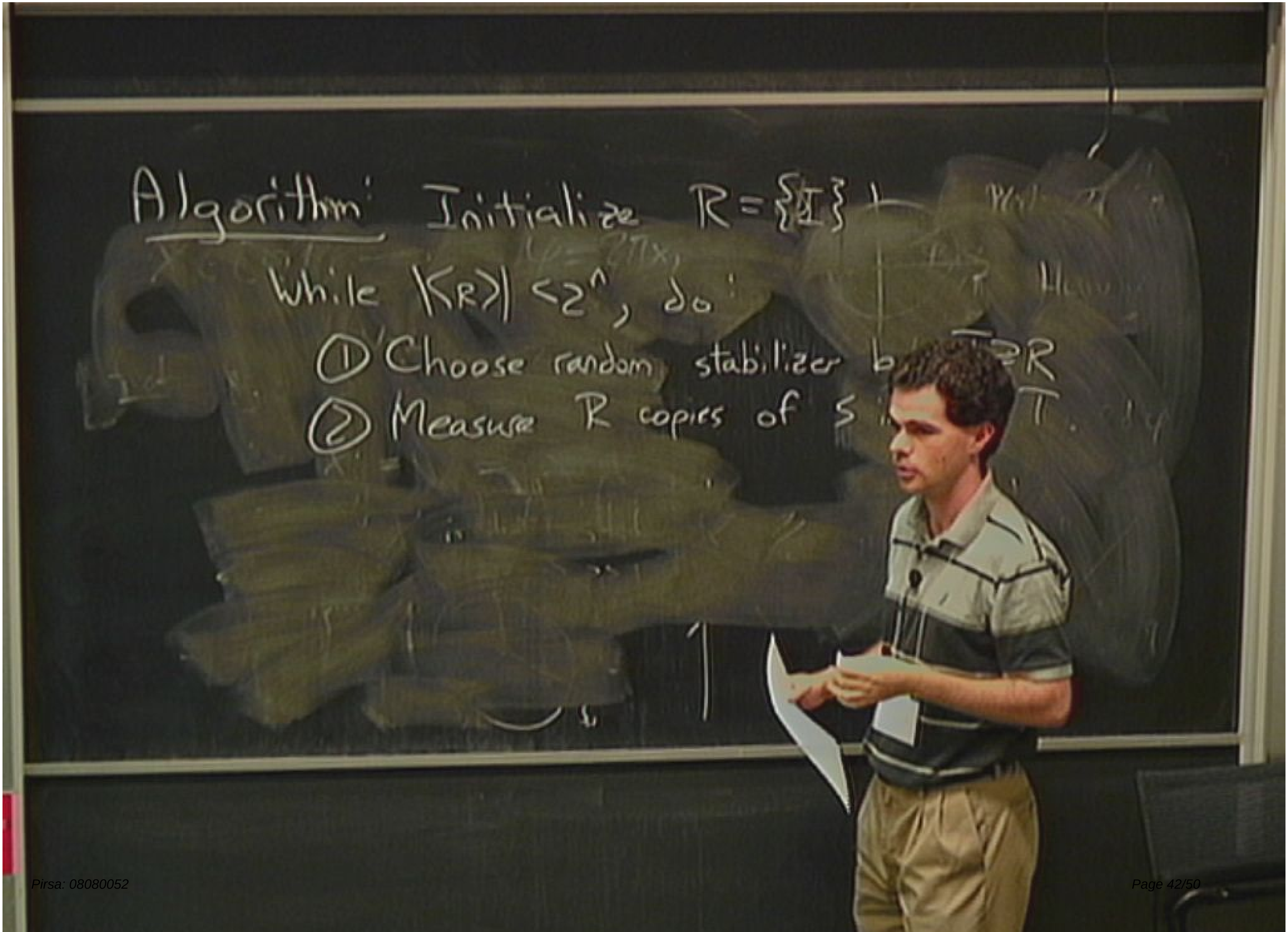

Algorithm: Initialize R = {I}
While |⟨R⟩| < 2^n, do:
① Choose random stabilizer basis T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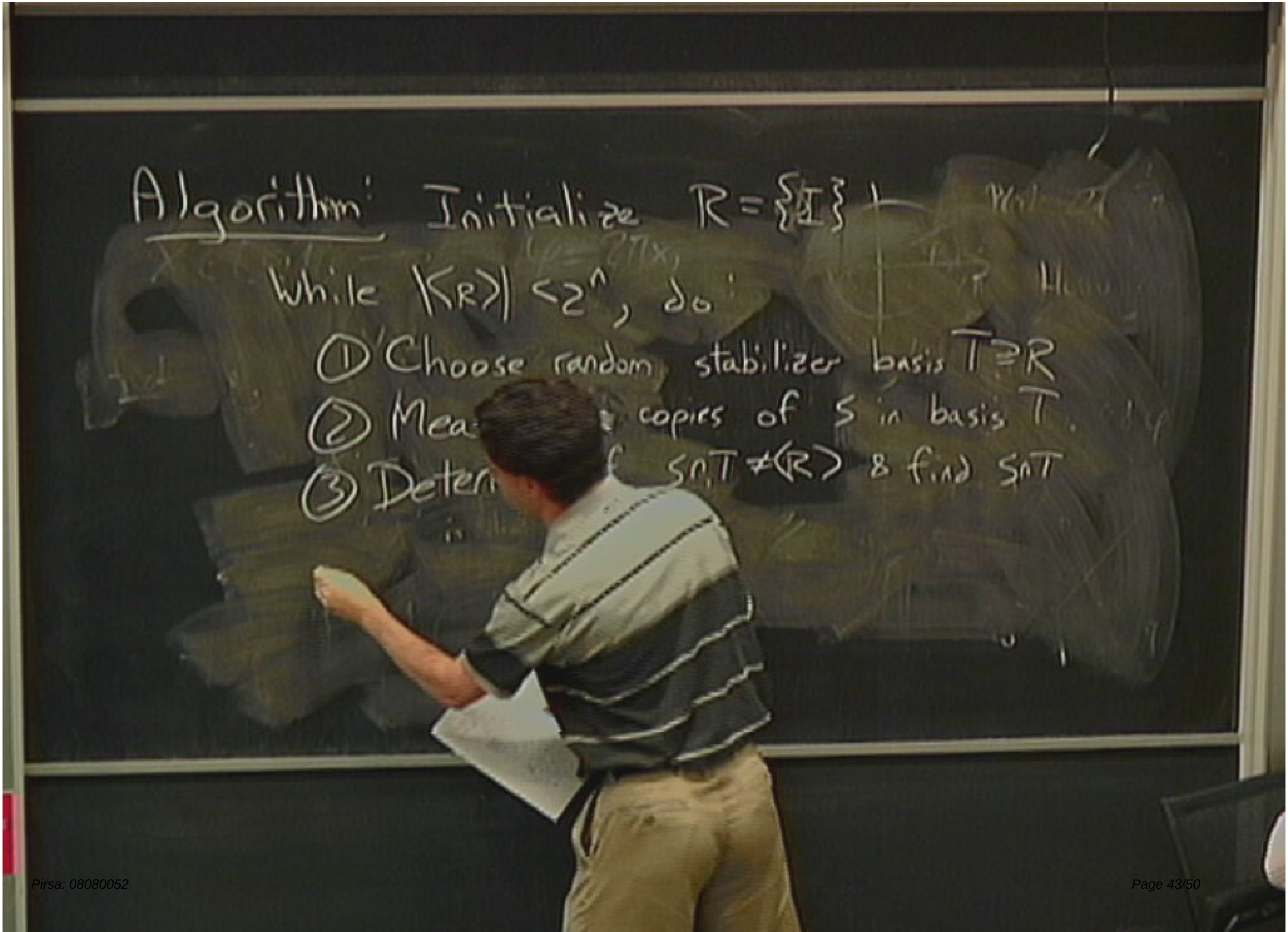
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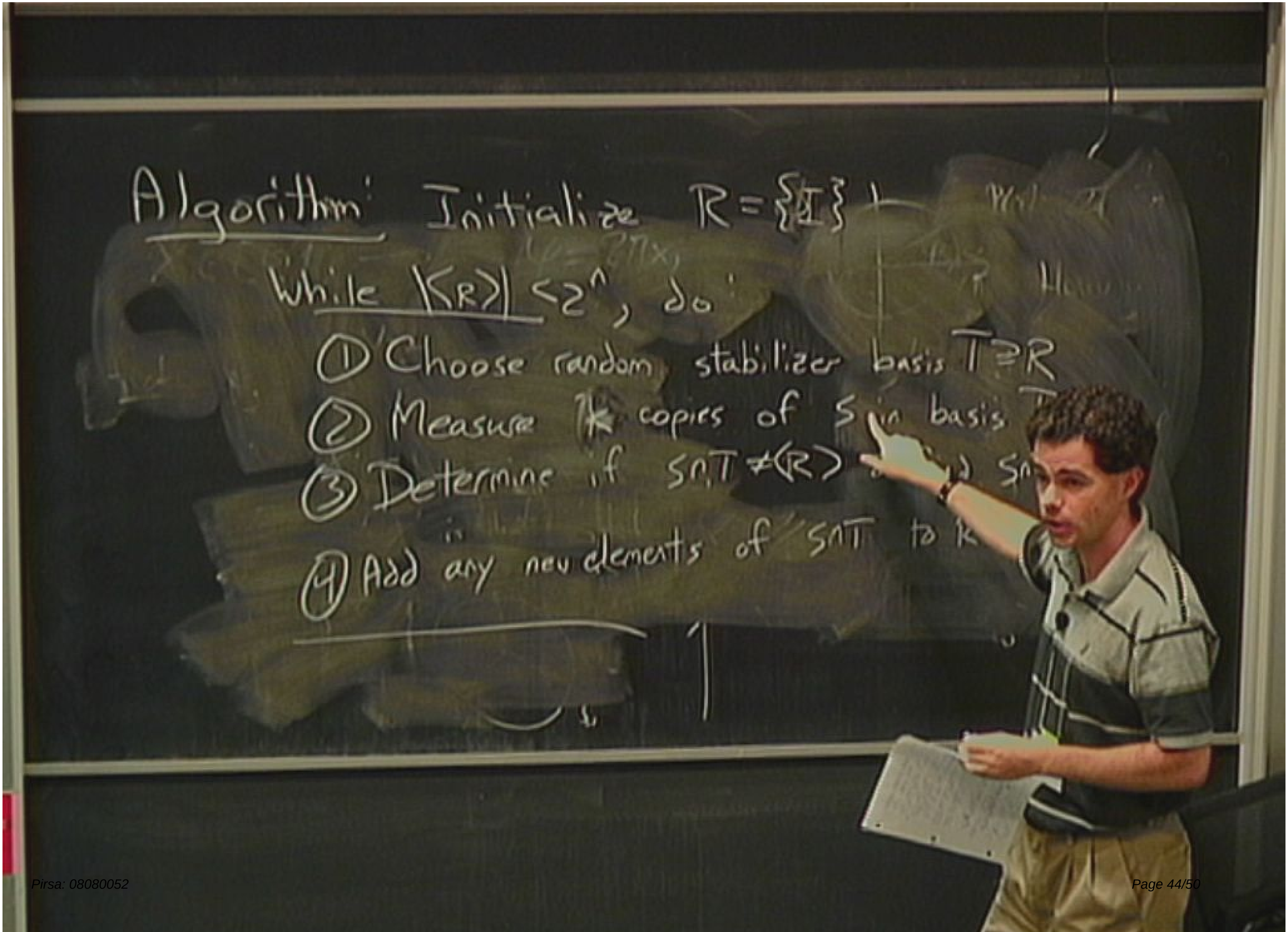
Algorithm: Initialize R = {I}
While |⟨R⟩| < 2^n, do:
① Choose random stabilizer basis T ⊇ R
② Measure k copies of S in basis
③ Determine if S∩T ≠ ⟨R⟩
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Algorithm: Initialize $R = \{I\}$

While $|\langle R \rangle| < 2^n$, do:

① Choose random stabilizer basis $T \supseteq R$

② Measure $k = O(n)$ copies of S in basis T.

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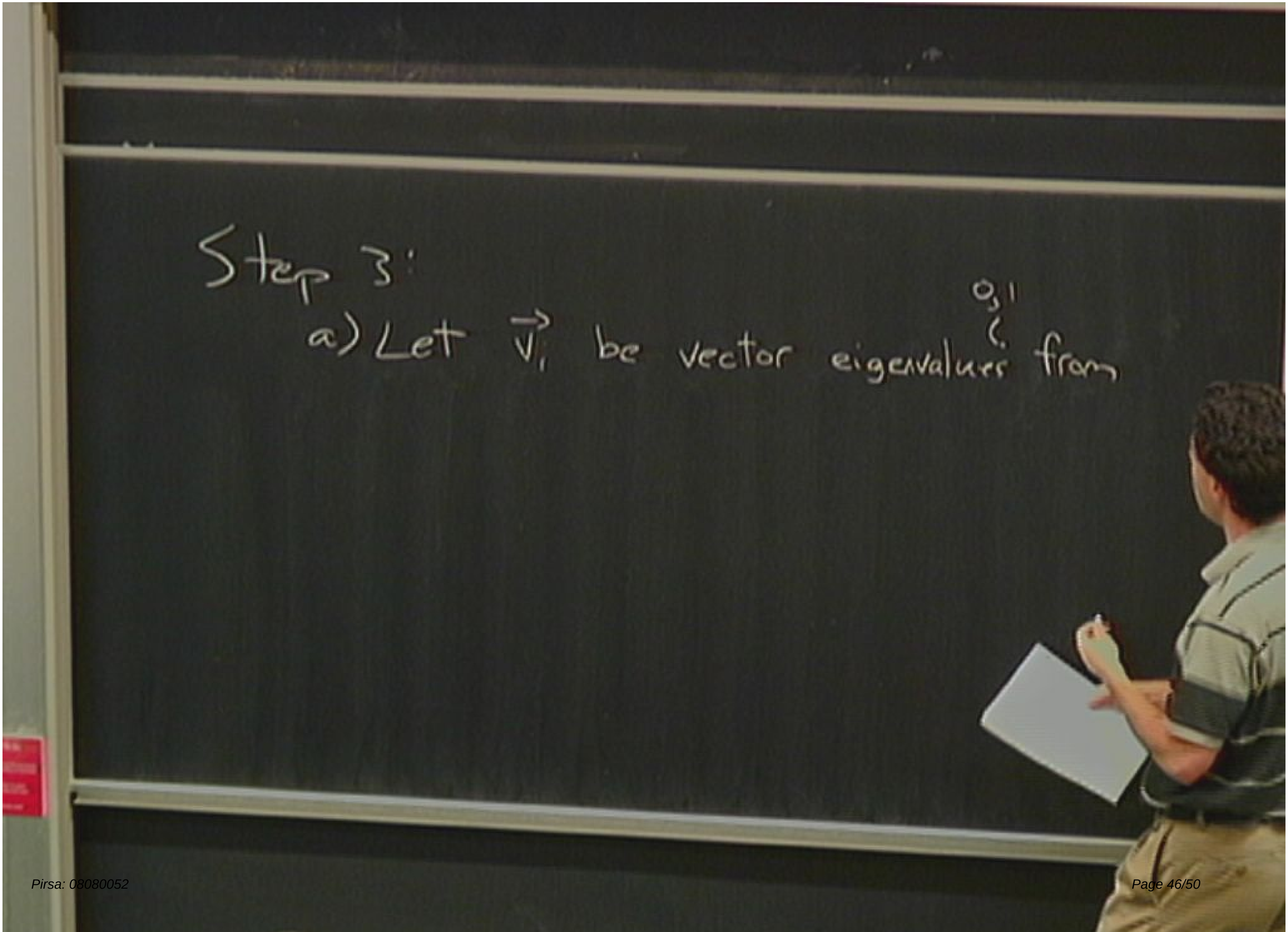
④ Add any new elements of $S \cap T$ to R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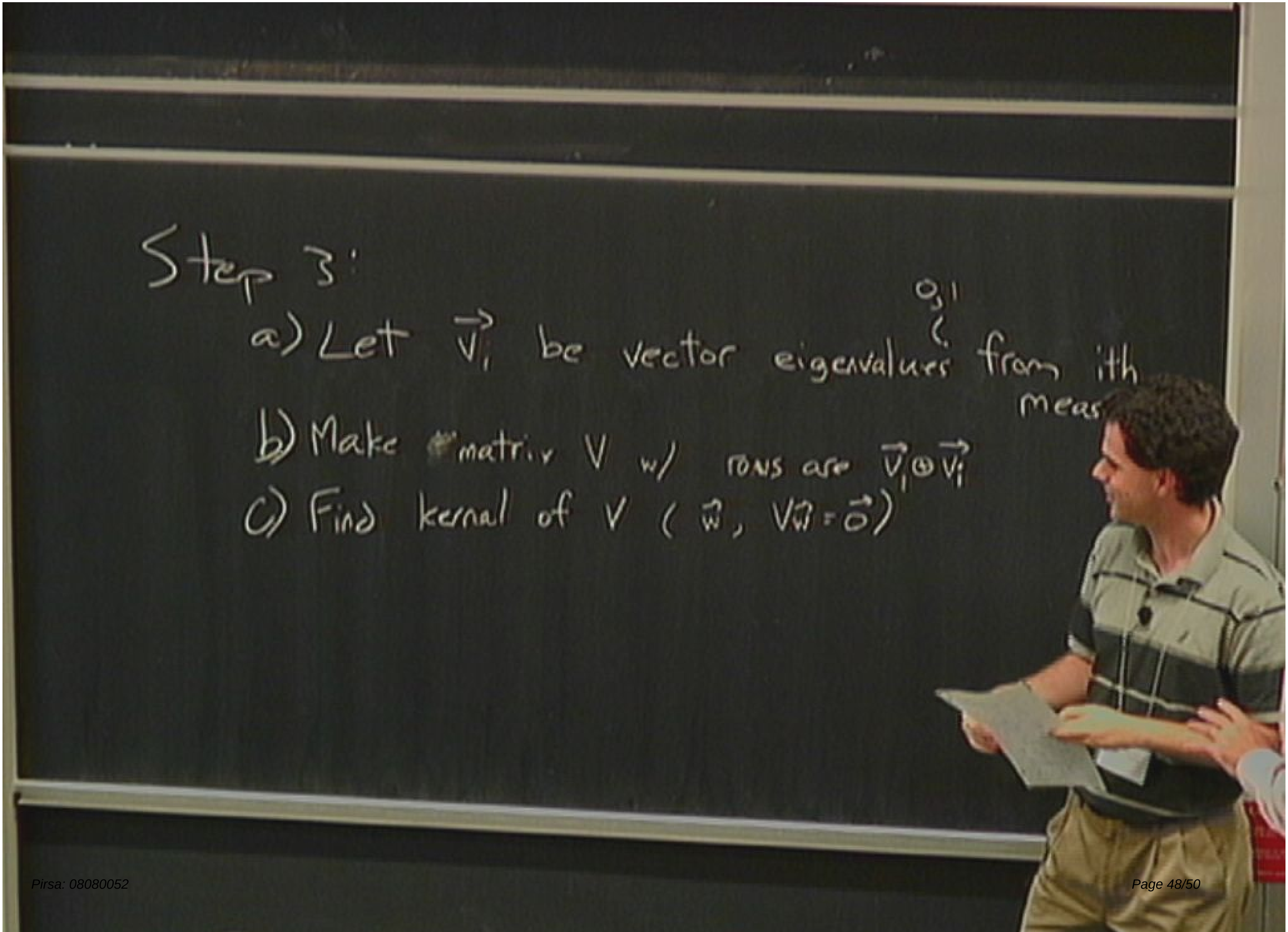
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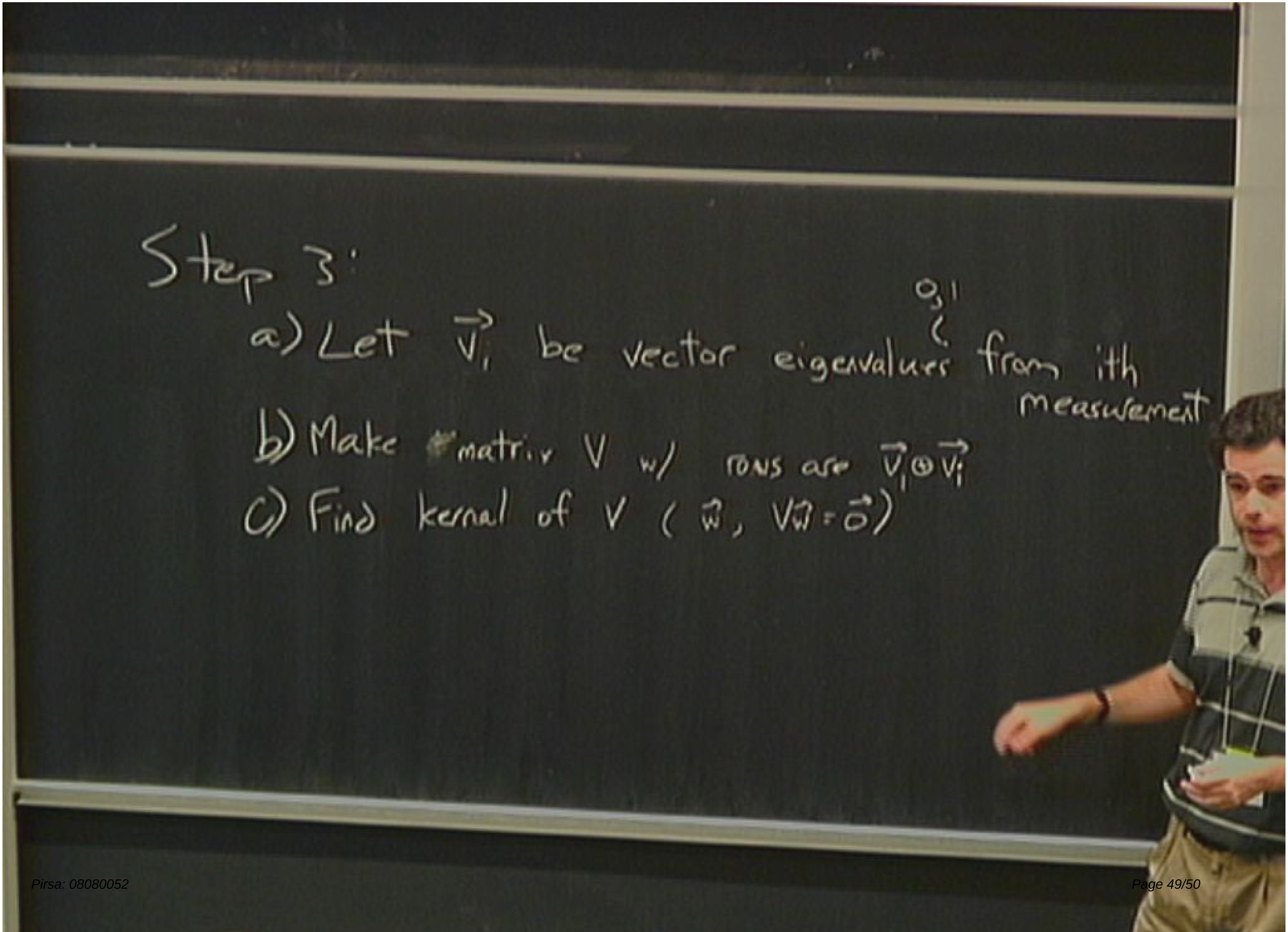
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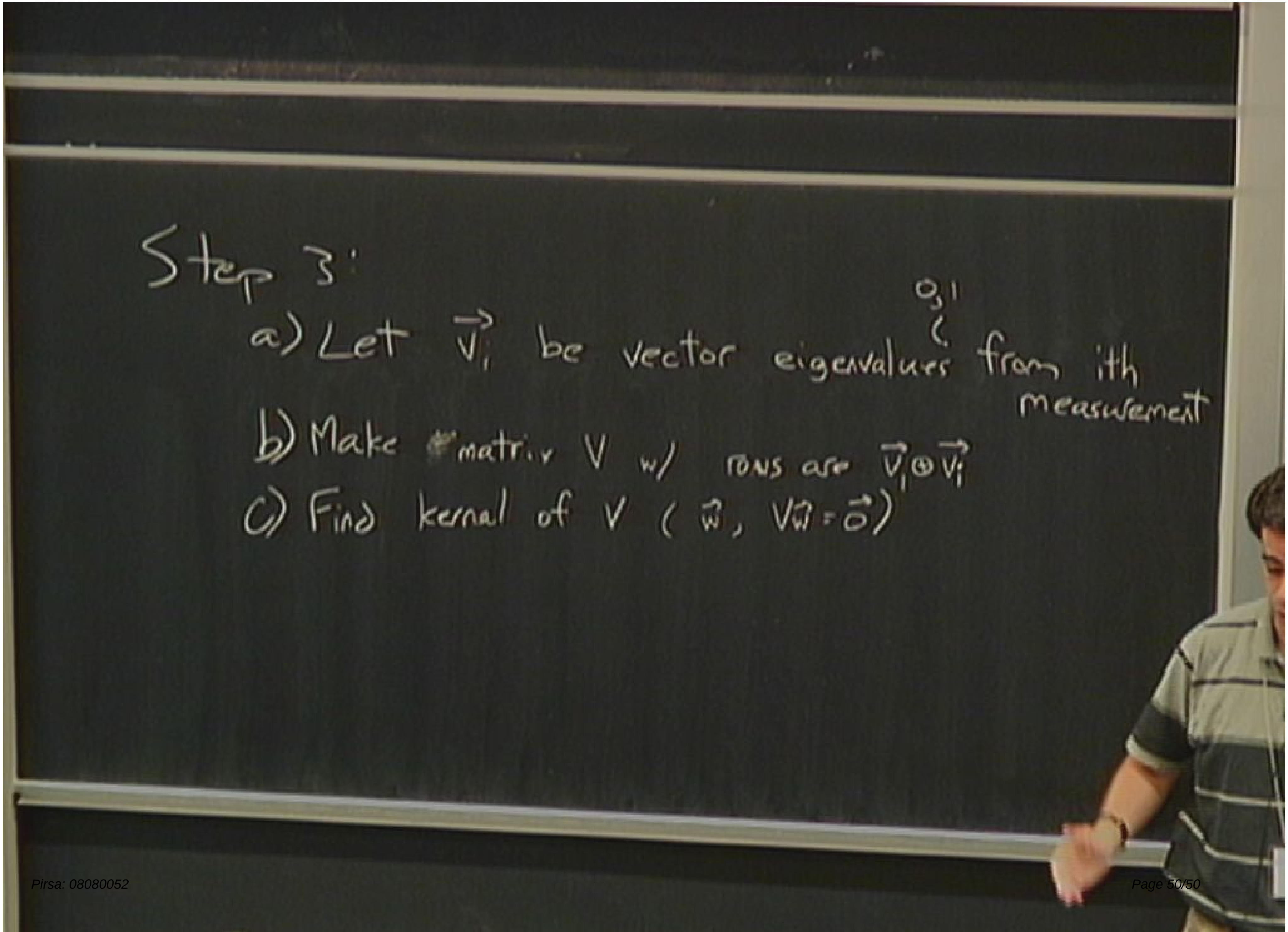
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