Title: Linear Optics Quantum Process Tomography

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Abstract: The field of linear optics quantum computing (LOQC) allows the construction of conditional gates using only linear optics and measurement. This quantum computing paradigm bypasses a seemingly serious problem in optical quantum computing: it appears to be very hard to produce a meaningful interaction between two single photons. But what if this obstacle were instead an advantage? By assuming that none of the physical components that make up an LOQC gate produce a direct photon-photon interaction, we dramatically reduce the space of gates which are possible for a given number of input and output qubits. In fact, by parametrizing a gate according to it\'s action on single photons, instead of on multiple photons, it is possible to exponentially reduce the number of measurements necessary to fully characterize an LOQC gate. In addition, this approach to LOQC process tomography may have additional experimental advantages when non-ideal input states are used for this characterization.

Linear Optics
Quantum Process Tomography

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Department of Physics

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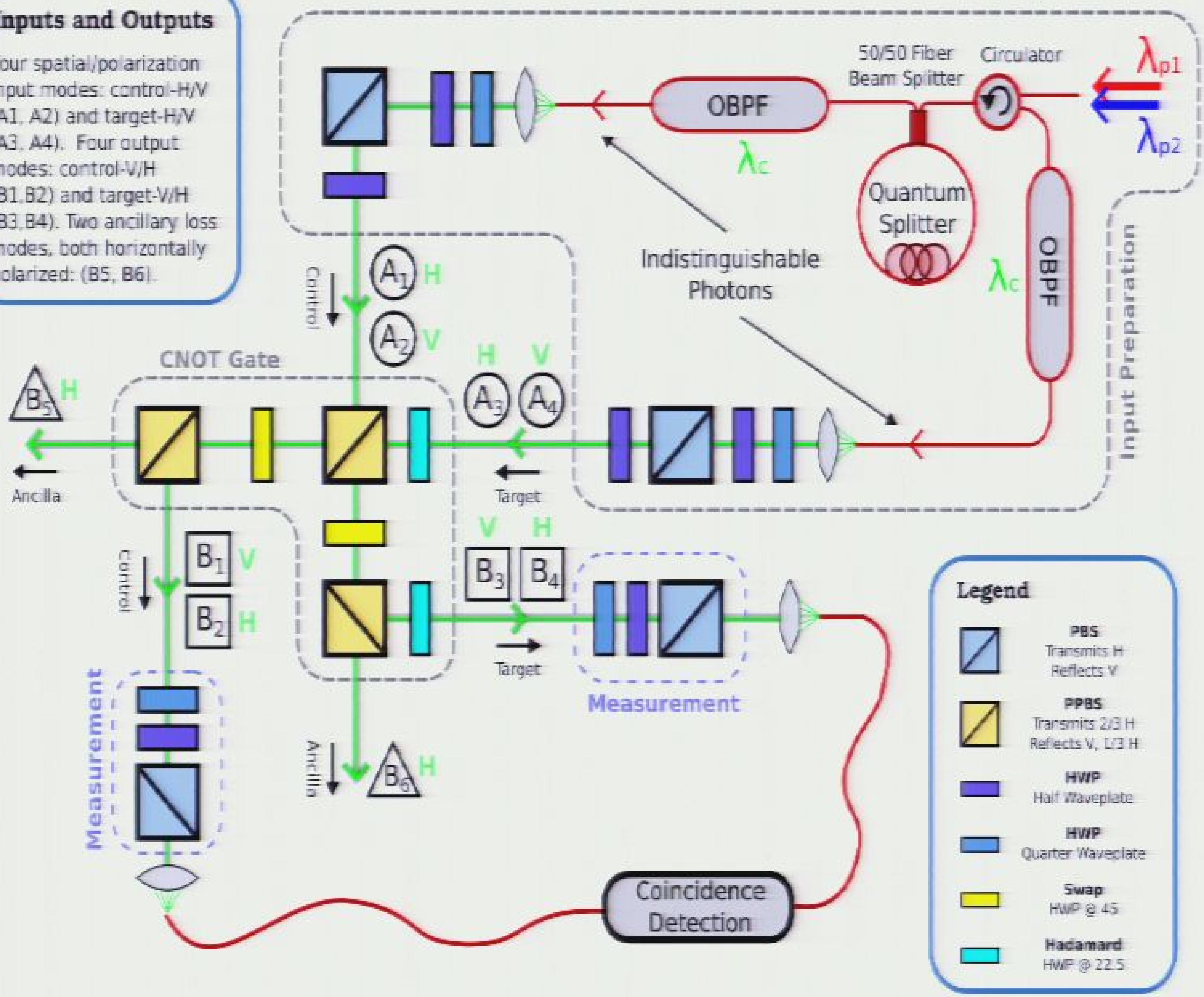
A Telecom-band CNOT Gate

CPCC

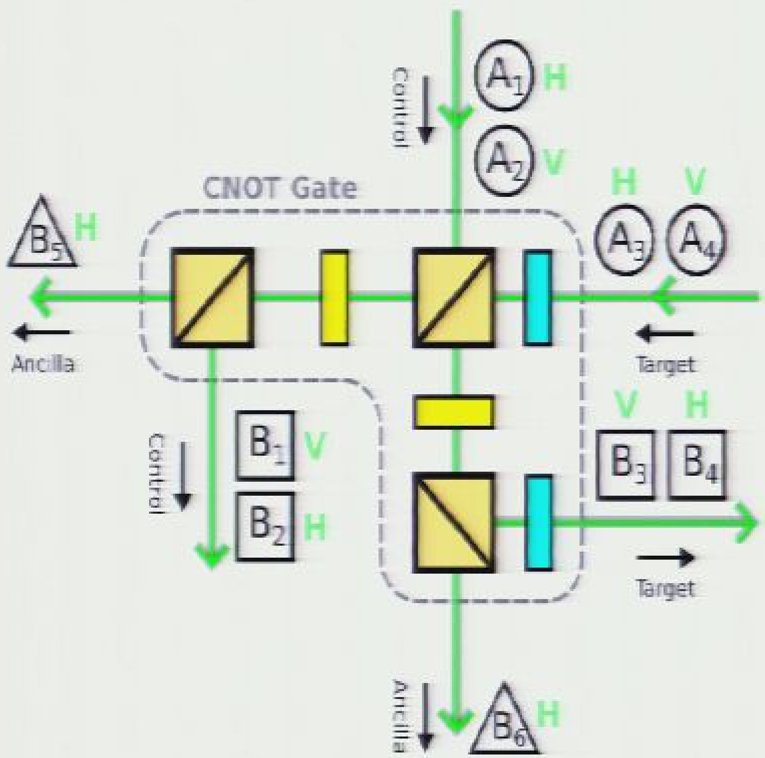
Inputs and Outputs

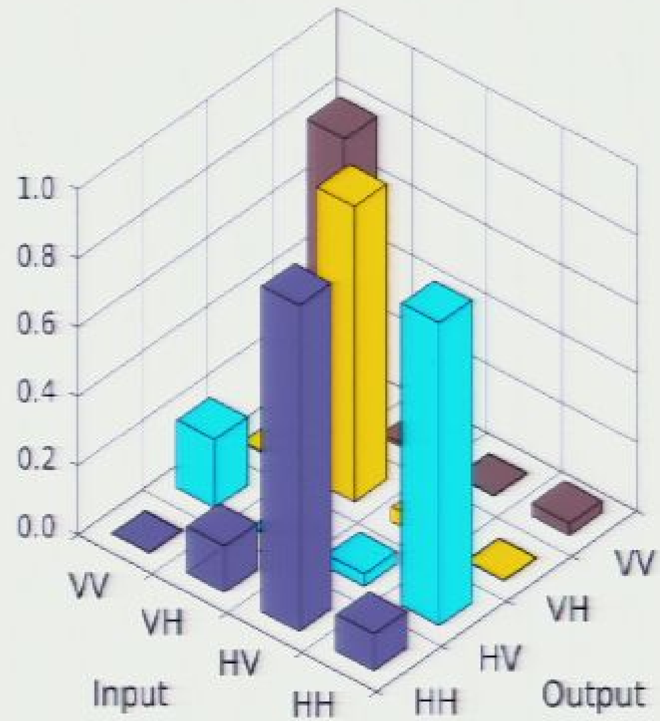
Four spatial/polarization input modes: control-H/V (A1, A2) and target-H/V (A3, A4). Four output modes: control-V/H (B1,B2) and target-V/H (B3,B4). Two ancillary loss modes, both horizontally polarized: (B5, B6).



First Experimental Results

- Errors not from mode mismatch.





State	F	T	S_L
HH+VV	0.80(2)	0.43(6)	0.42(4)
HH-VV	0.75(3)	0.30(5)	0.53(4)
HV+VH	0.75(3)	0.34(6)	0.53(4)
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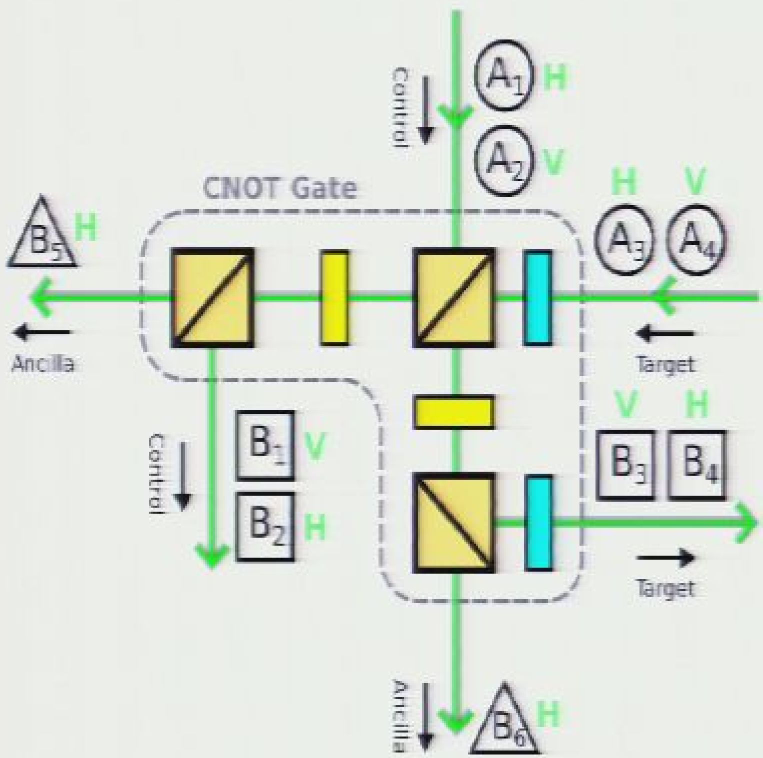

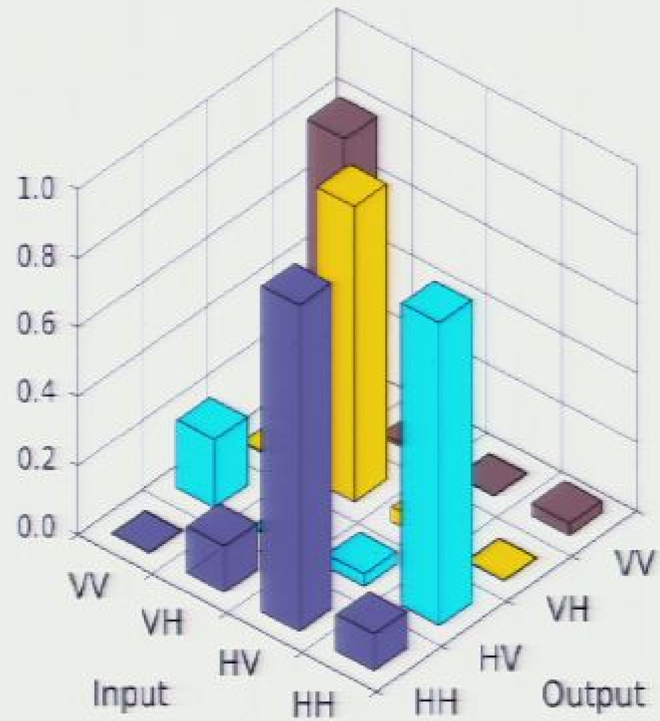

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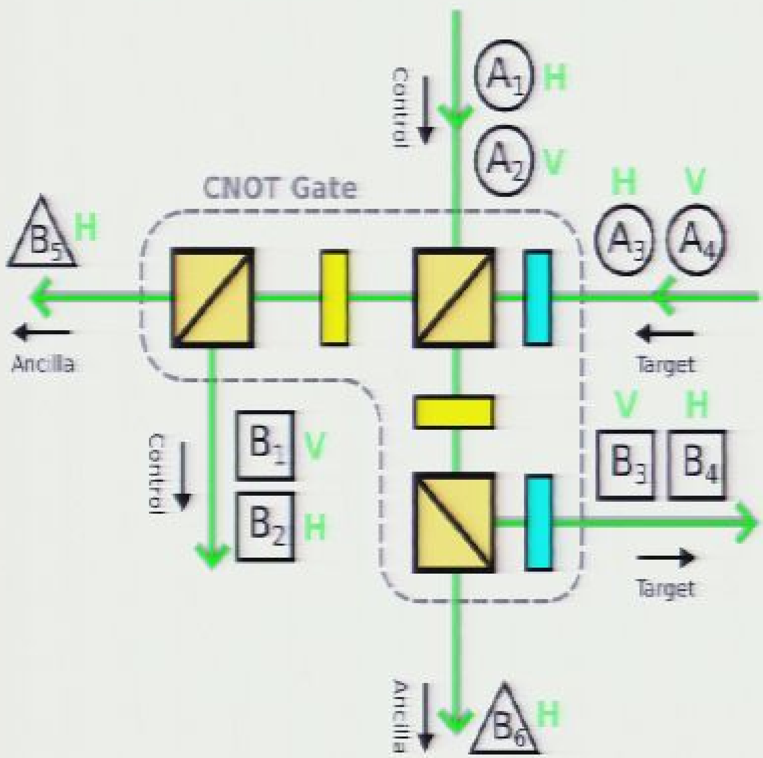
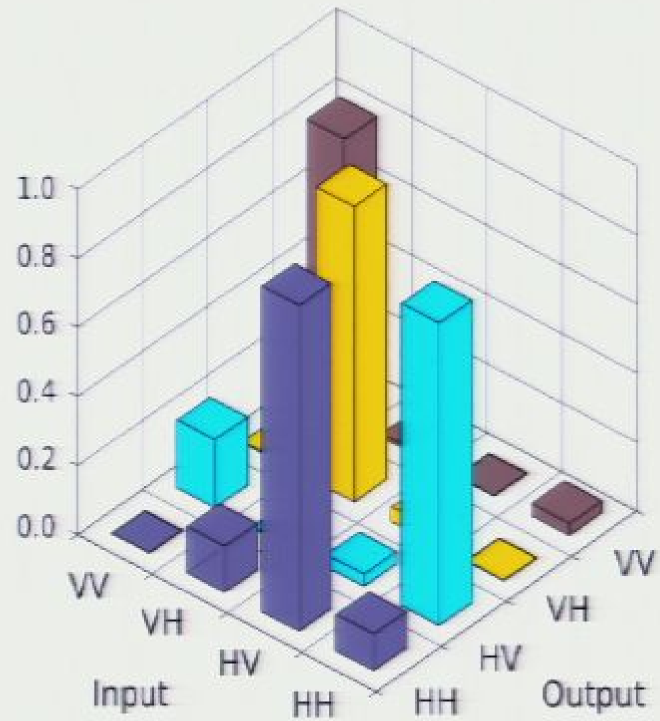
- What about multi-pairs?





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Explaining the Multipair Errors

How does a CNOT gate operate on two photons?

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$$A_1^\dagger A_3^\dagger \rightarrow B_1^\dagger B_3^\dagger$$

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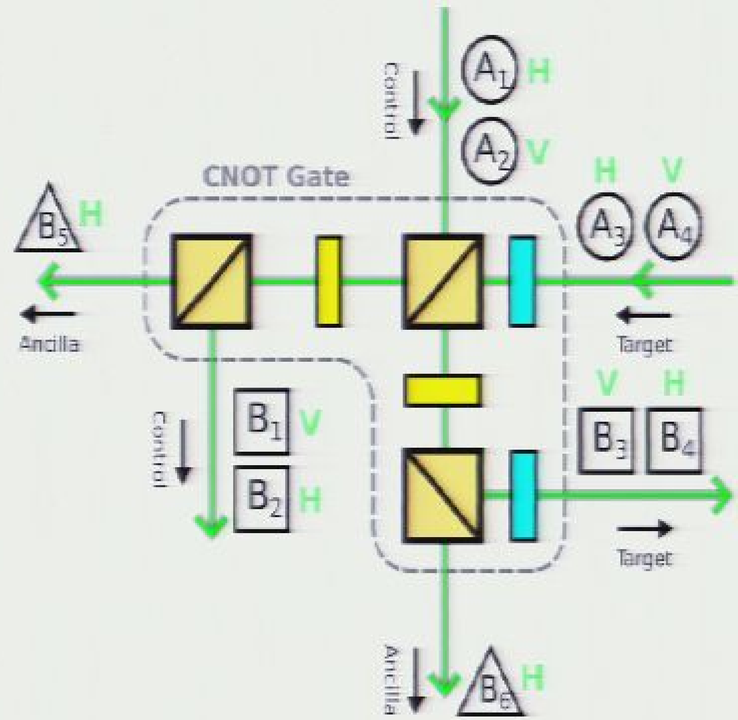

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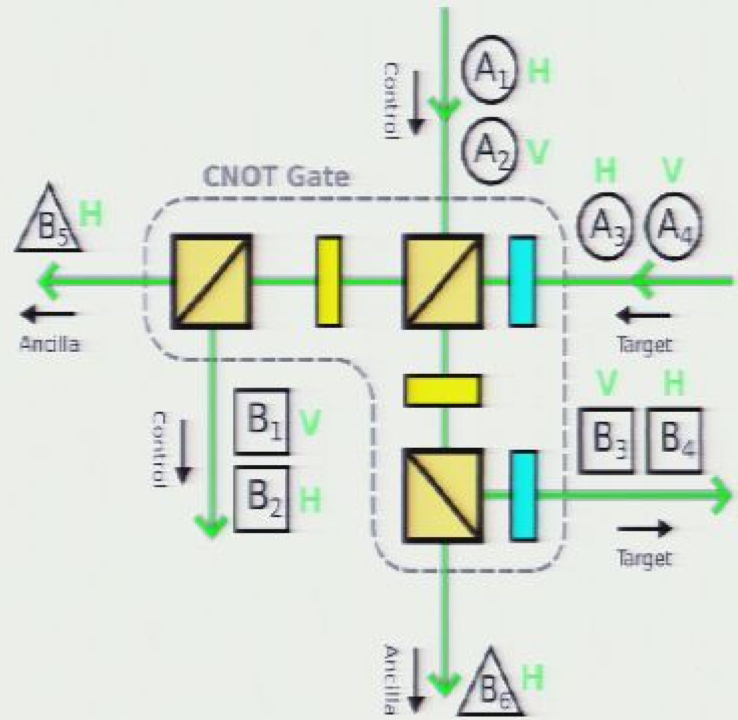
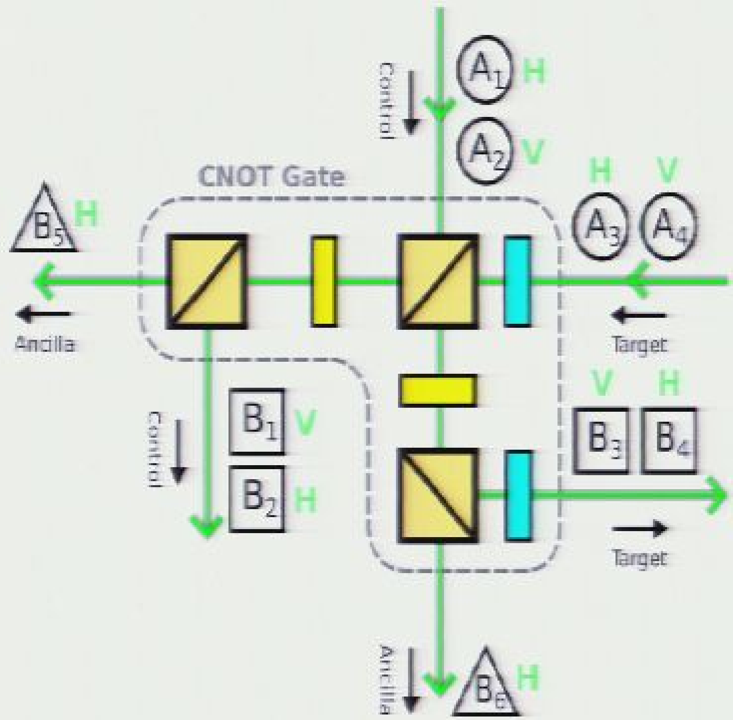


How do we use this to predict the CNOT's effect on four input photons? Seems like a hard problem...

Back to the Drawing Board

What about a single photon model?



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$$A_1^\dagger \rightarrow \sqrt{\frac{1}{3}}\left(B_1^\dagger + B_4^\dagger - B_3^\dagger\right)$$

$$A_2^\dagger \rightarrow \sqrt{\frac{1}{3}}\left(-\sqrt{2}\,B_5^\dagger - B_2^\dagger\right)$$

$$A_3^\dagger \rightarrow \sqrt{\frac{1}{3}}\left(B_1^\dagger - B_4^\dagger - B_6^\dagger\right)$$

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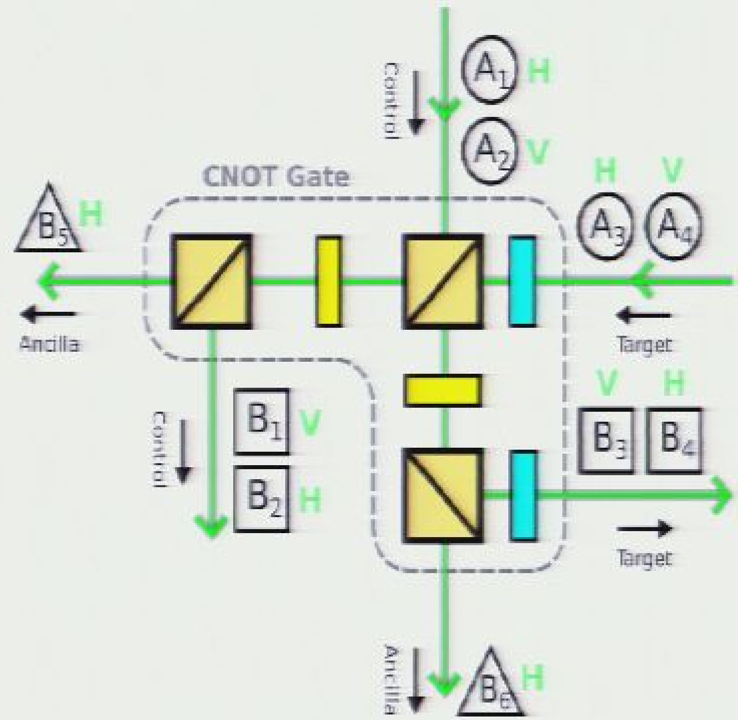
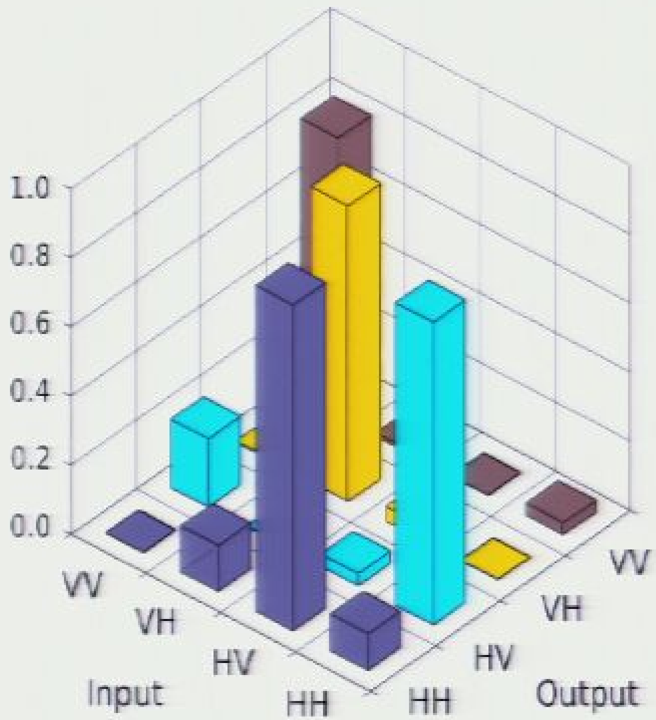


Now we can calculate how any number of photons will be affected by the gate.

Corrected Experimental Results

Original Results



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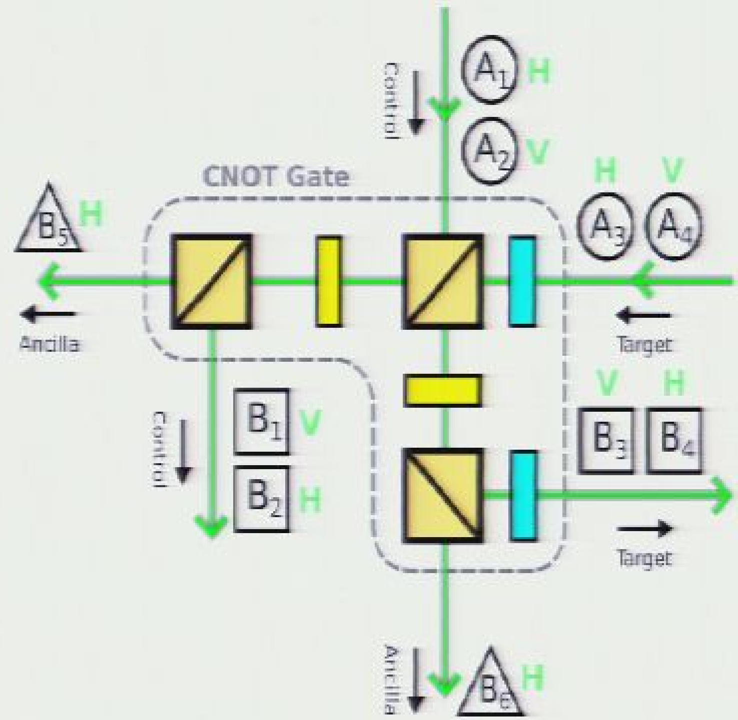

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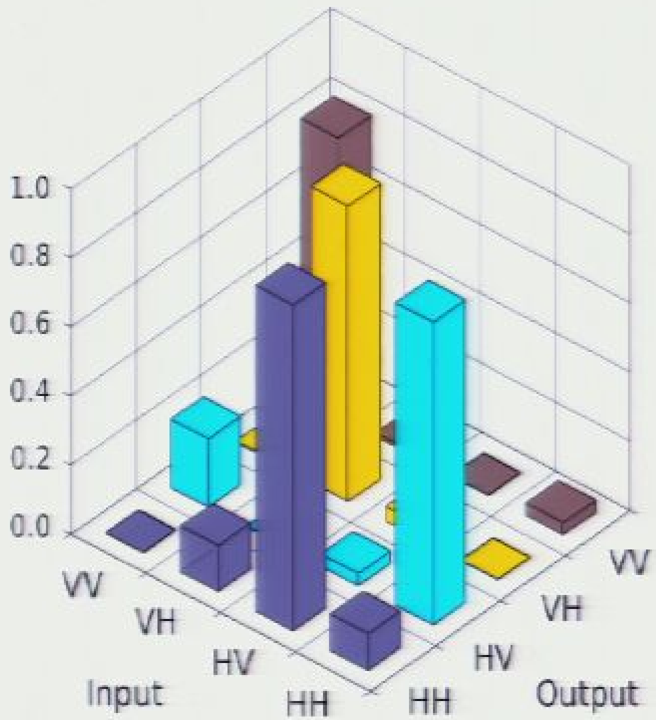

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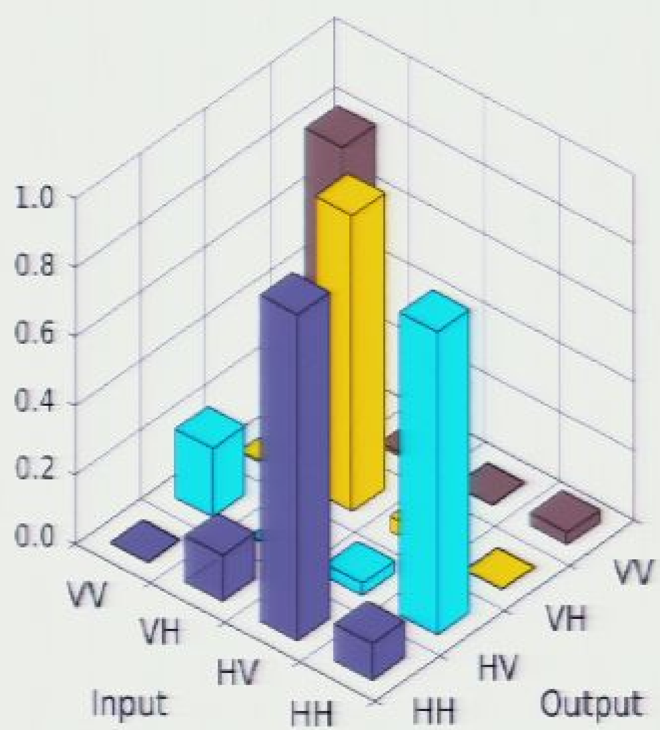

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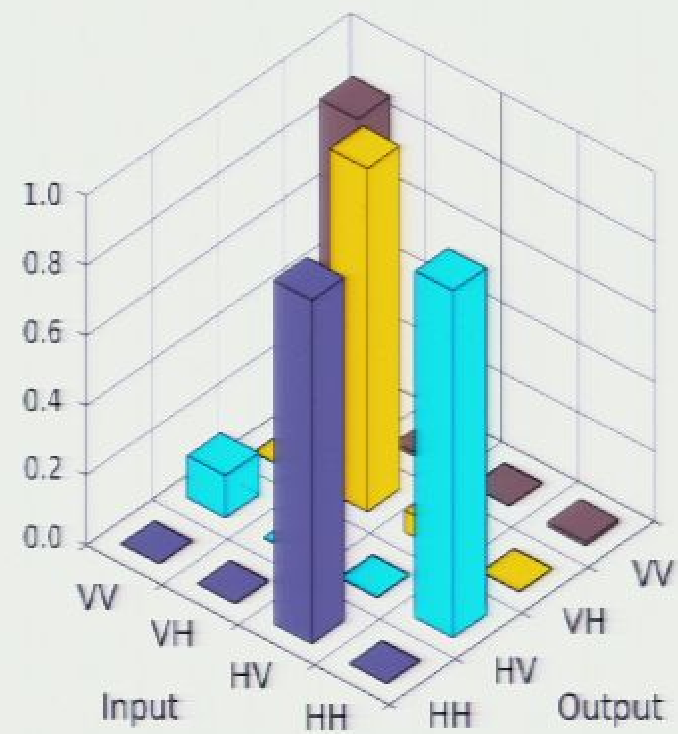
Multi-pairs Subtracted



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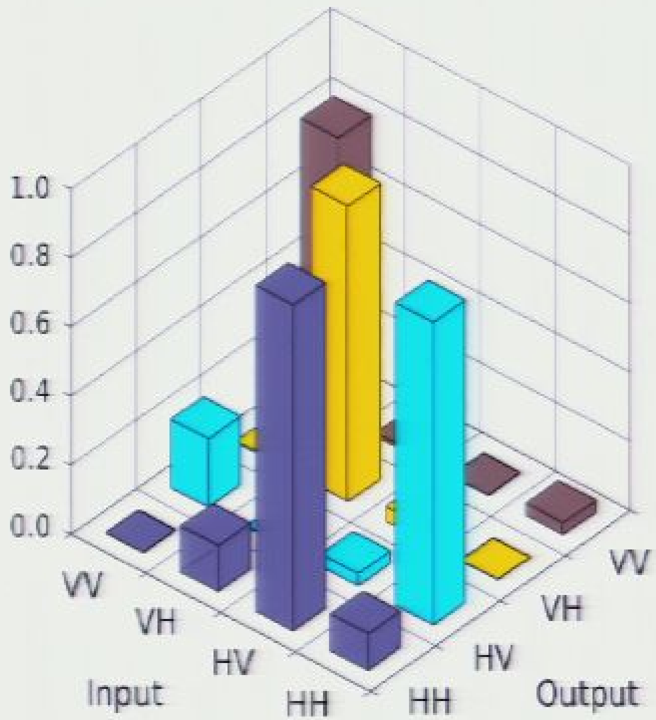

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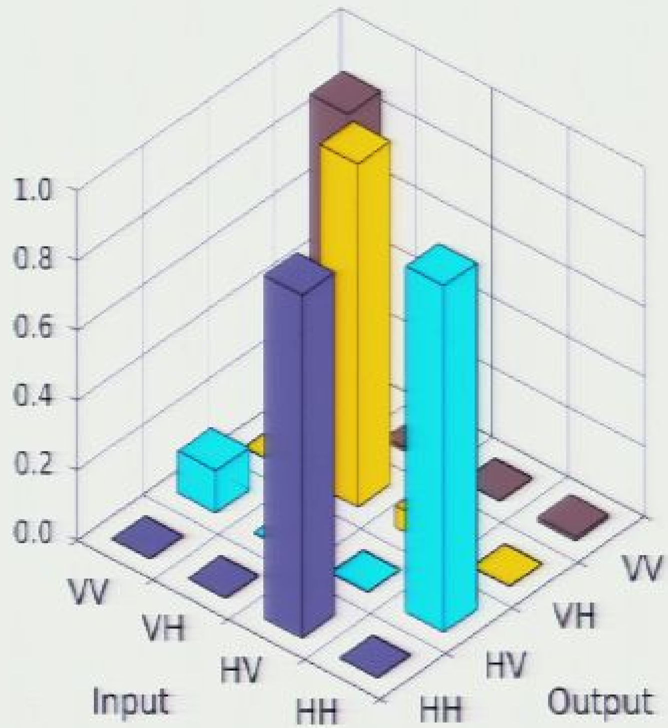

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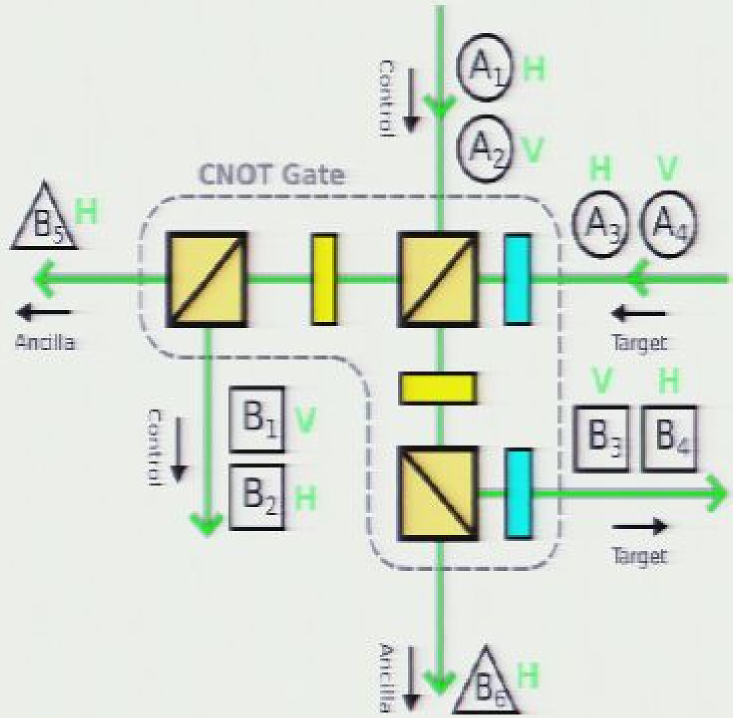
Is there more to this technique?

The Assumption:

$A^\dagger \rightarrow B^\dagger$



Center for Photonic Communication and Computing

McCormick School of Engineering and Applied Science

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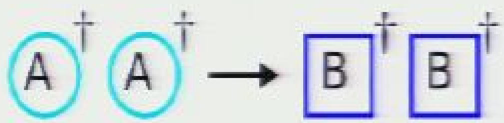
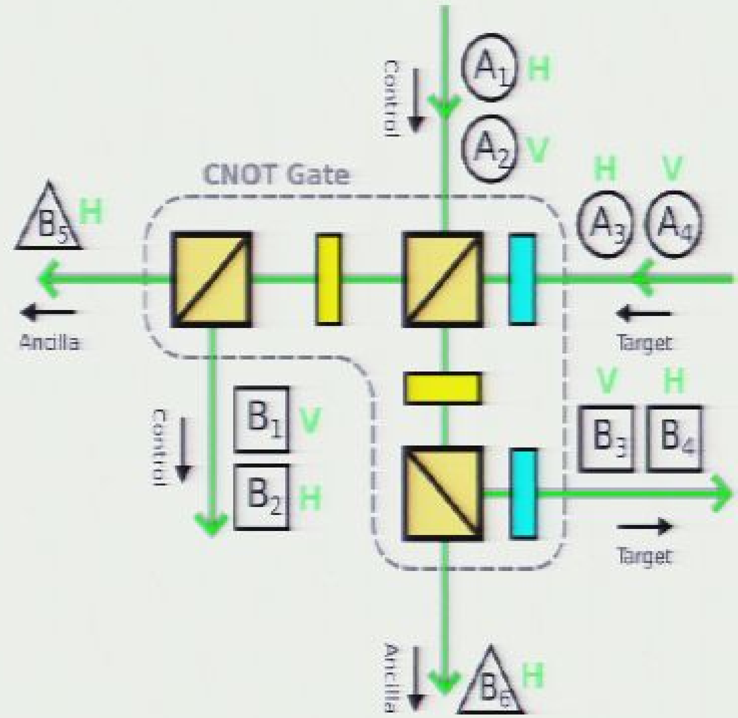
The Advantage:





No Loss, No Decoherence

The output is a coherent superposition of outputs:

$$A_1^\dagger \rightarrow \beta_{11} \boxed{B_1}^\dagger + \beta_{12} \boxed{B_2}^\dagger + \beta_{13} \boxed{B_3}^\dagger + \beta_{14} \boxed{B_4}^\dagger$$

$$A_2^\dagger \rightarrow \beta_{21} \boxed{B_1}^\dagger + \beta_{22} \boxed{B_2}^\dagger + \beta_{23} \boxed{B_3}^\dagger + \beta_{24} \boxed{B_4}^\dagger$$

$$A_3^\dagger \rightarrow \beta_{31} \boxed{B_1}^\dagger + \beta_{32} \boxed{B_2}^\dagger + \beta_{33} \boxed{B_3}^\dagger + \beta_{34} \boxed{B_4}^\dagger$$

$$A_4^\dagger \rightarrow \beta_{41} \boxed{B_1}^\dagger + \beta_{42} \boxed{B_2}^\dagger + \beta_{43} \boxed{B_3}^\dagger + \beta_{44} \boxed{B_4}^\dagger$$

In other words, it's a unitary operation.

Modelling Loss

General loss:

$$\textcircled{A}^\dagger \rightarrow \beta_B \boxed{B}^\dagger + \beta_L \triangle\!L^\dagger$$

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CPCC

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Let's make a half of a bigger unitary:

$$A_1^\dagger \rightarrow \beta_{11} B_1^\dagger + \beta_{12} B_2^\dagger + \beta_{13} B_3^\dagger + \beta_{14} B_4^\dagger + \beta_{15} B_5^\dagger + \beta_{16} B_6^\dagger + \beta_{17} B_7^\dagger + \beta_{18} B_8^\dagger$$

$$A_2^\dagger \rightarrow \beta_{21} B_1^\dagger + \beta_{22} B_2^\dagger + \beta_{23} B_3^\dagger + \beta_{24} B_4^\dagger + \beta_{25} B_5^\dagger + \beta_{26} B_6^\dagger + \beta_{27} B_7^\dagger + \beta_{28} B_8^\dagger$$

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$$A_4^\dagger \rightarrow \beta_{41} B_1^\dagger + \beta_{42} B_2^\dagger + \beta_{43} B_3^\dagger + \beta_{44} B_4^\dagger + \beta_{45} B_5^\dagger + \beta_{46} B_6^\dagger + \beta_{47} B_7^\dagger + \beta_{48} B_8^\dagger$$

Some Loss, No Decoherence

What about loss here?

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Explicit loss coherences are needed when they will coherently interfere. In other words, for low-order coincidences and singles.

Modelling Decoherence

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$$+$$

$$\left(\beta_{11}^{(2)} B_1^\dagger + \beta_{12}^{(2)} B_2^\dagger + \beta_{13}^{(2)} B_3^\dagger + \beta_{14}^{(2)} B_4^\dagger\right) \otimes D_2$$

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$$+$$

- In general, there can be any number of these decohering modes, leading to any number of matrices.... hmm....

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$$+ \ldots$$

A Brief Detour: Reviewing the χ-Matrix

$$\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

$$E_k = \sum_m e_{km} \tilde{E}_m$$

$$\chi_{mn} = \sum_k e_{km} e_{kn}^*$$

$$\mathcal{E}(\rho) = \sum_{mn} \tilde{E}_m \rho \tilde{E}_n^\dagger \chi_{mn}$$

Looks a lot Like Kraus Operators...

$$A_1^\dagger \rightarrow \left(\beta_{11}^{(1)} \boxed{B_1}^\dagger + \beta_{12}^{(1)} \boxed{B_2}^\dagger + \beta_{13}^{(1)} \boxed{B_3}^\dagger + \beta_{14}^{(1)} \boxed{B_4}^\dagger\right) \otimes D_1$$

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$$+ \ldots$$

The Point

Consider an N-qubit gate wth 2N single-photon input modes. The 2N-element vector α_i defines an arbitrary input creation operator:

$$A^\dagger \equiv \sum_i \alpha_i A_i^\dagger \qquad \rho' \equiv |\alpha\rangle\langle\alpha|$$

For any given LOQC process $\mathcal{E}$ (where primed objects are in the single-photon basis):

$$\mathcal{E}(\rho') = \sum_k \beta_{ij}^{(k)} \rho' \beta_{ij}^{(k)\dagger}$$

We can rewrite with the Hermitian χ_{mn}:

$$\mathcal{E}(\rho') = \sum_{mn} \tilde{E}'_m \rho' \tilde{E}'^\dagger_n \chi'_{mn}$$

By diagonalizing χ_{mn} we recreate the beta map, and can predict the output state for any input.

In Conclusion

Scaling for N qubit gates

Standard Process

2^{4N}

LOQC Process

$(2N)^4$

LOQC Process with Loss

$(4N)^4$

In Conclusion

Scaling looks good, but don't forget feed-forward.

Experimental Axioms:

- Errors are clues.
- Use all your data.

Ideas from smart theorists are always welcome.

Scaling for N qubit gates

Standard Process

2^{4N}

LOQC Process

$(2N)^4$

LOQC Process with Loss

$(4N)^4$