

Title: Selective and efficient process tomography

Date: Aug 26, 2008 03:15 PM

URL: <http://pirsa.org/08080044>

Abstract:

χ representation

- Parametrized by operator base: $\{E_1, \dots, E_{D^2}\}$

$$\Lambda(\rho) = \sum_{m,n} \chi_{m,n} E_m \rho E_n^\dagger$$

- χ is hermitian and positive defined.

- Trace preserving:
$$\sum_{m,n} \chi_{m,n} E_n^\dagger E_m = I$$

- Required for deciding on error correcting codes.
- Has large number of parameters $D^4 = 2^{4N}$.

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Some interesting questions.

- Can experiments directly access χ elements?
- Is it possible to estimate these elements efficiently? (good scaling with the number of qubits?)
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ANSWER: Yes (estimation with fixed, D-independent, precision)

Quantum process tomography

- SQPT: Direct access to λ coefficients ('transition matrix elements). Inefficient for accessing χ representation.
- DCQD: Efficient access to diagonal $\chi_{m,m}$
Is not selective for other coefficients.
Requires N clean ancilla qubits.
- SCNQP: Based on Λ map symmetrization.
Efficient access to limited number of important χ parameter groups



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(all acronyms are ARC-approved)

Operator basis

- We could use Pauli operators (or any other basis)

$$P_0 = I, \quad P_1 = X, \quad P_2 = Y, \quad P_3 = Z$$

$$E_m = P_{m_1} \otimes P_{m_2} \otimes \dots \otimes P_{m_N}$$

- We take: $E_0 = I^{\otimes N}$

Average Fidelity

- The average fidelity for a map Λ : survival probability averaged over all pure states:

$$F(\Lambda) = \int \langle \psi | \Lambda(|\psi\rangle\langle\psi|) | \psi \rangle d|\psi\rangle$$

- Key: All $\chi_{m,m}$ coefficient can be related to average fidelity of a channel.

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How? Simple argument leads to $\chi_{0,0}$

- Useful identity

$$\int \langle \psi | A | \psi \rangle \langle \psi | B | \psi \rangle d | \psi \rangle = \frac{\text{tr}(A)\text{tr}(B) + \text{tr}(AB)}{D(D+1)}$$

J.M. Renes et. al. Journal of Mathematical Physics **45**, 2171 (2004)

- Combine it with the channel chi-representation

$$F(\Lambda) = \int \langle \psi | \Lambda(| \psi \rangle \langle \psi |) | \psi \rangle d | \psi \rangle$$

- Use that operator basis is orthonormal

$$F(\Lambda) = \frac{D\chi_{0,0} + 1}{D+1}$$

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Other diagonal $\chi_{m,m}$ coefficients

- Other $\chi_{m,m}$ are obtainable from the fidelity of modified maps Λ_m .

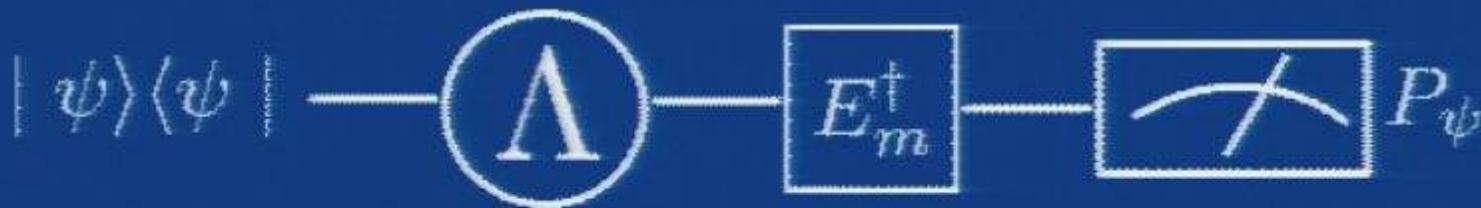
$$F(\Lambda_m) = \frac{D\chi_{m,m} + 1}{D + 1}$$

- Λ_m may be readily implemented through the successive application of Λ and E_m .

$$\Lambda_m(\rho) = E_m^\dagger \Lambda(\rho) E_m$$

Quantum circuit

- More formally, the quantum circuit for measuring fidelity is written as:



- To implement this we need to compute the average over all pure states ψ (use 2-design S...)

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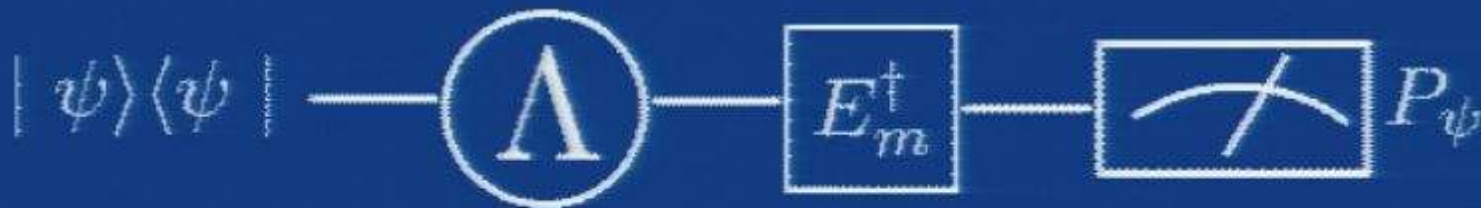
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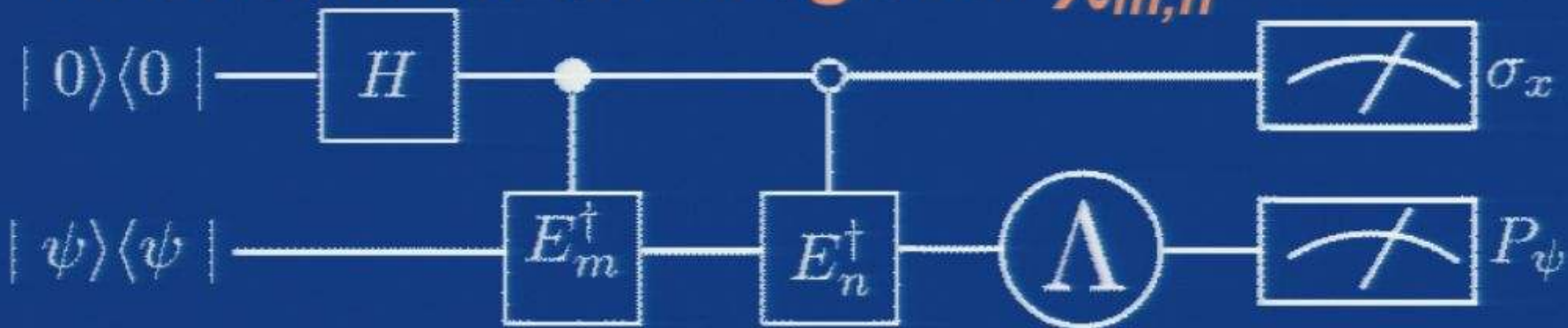
What about off diagonal $\chi_{m,n}$

- They are more. Many more ($\sim D^4$).
- How could we measure them?

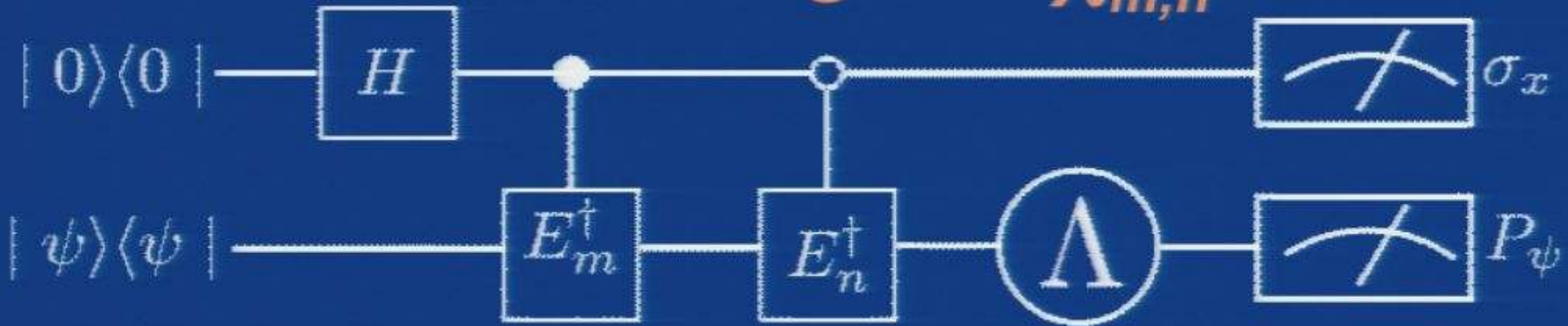
$$\int \langle \psi | \Lambda(E_m^\dagger P_\psi E_n) | \psi \rangle = \frac{D\chi_{m,n} + \delta_{m,n}}{D+1}$$

- Need a different strategy.
- Need an extra resource (one clean qubit)

A circuit for off-diagonal $\chi_{m,n}$

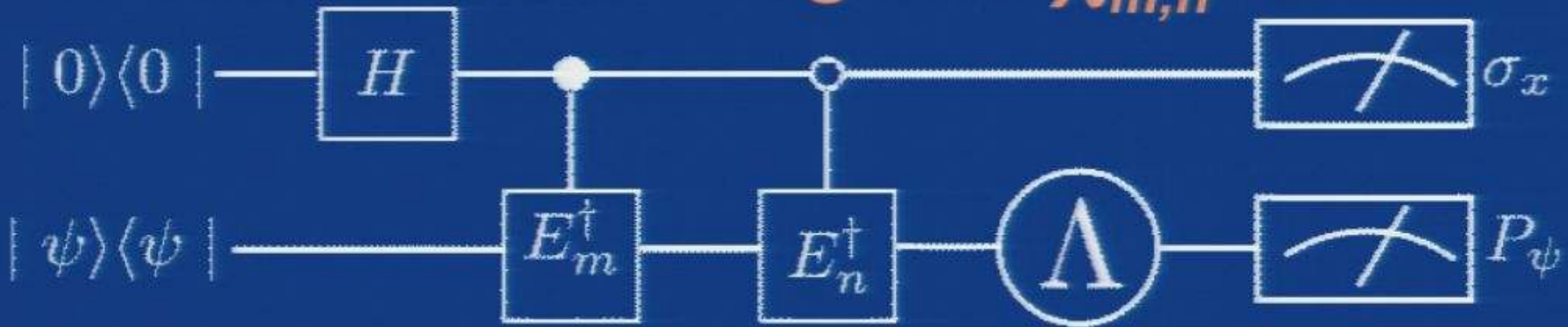


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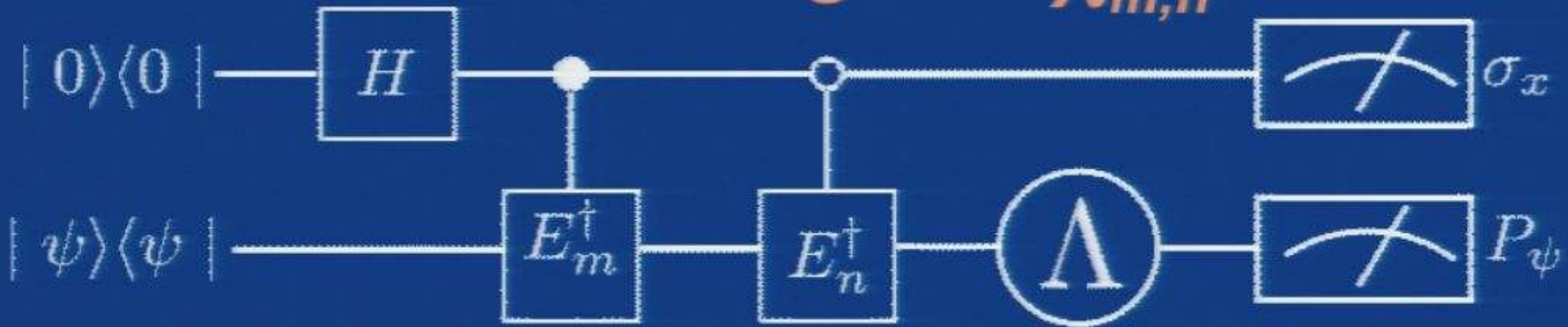
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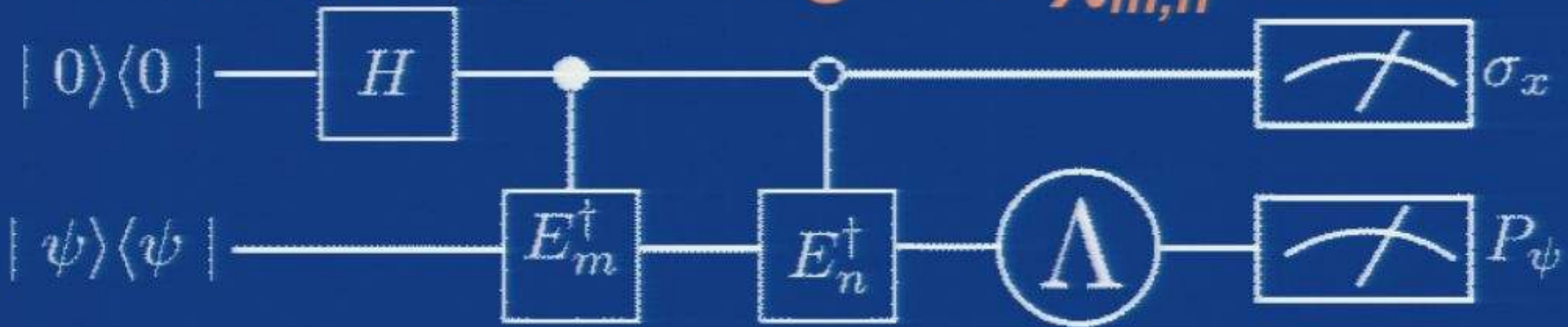
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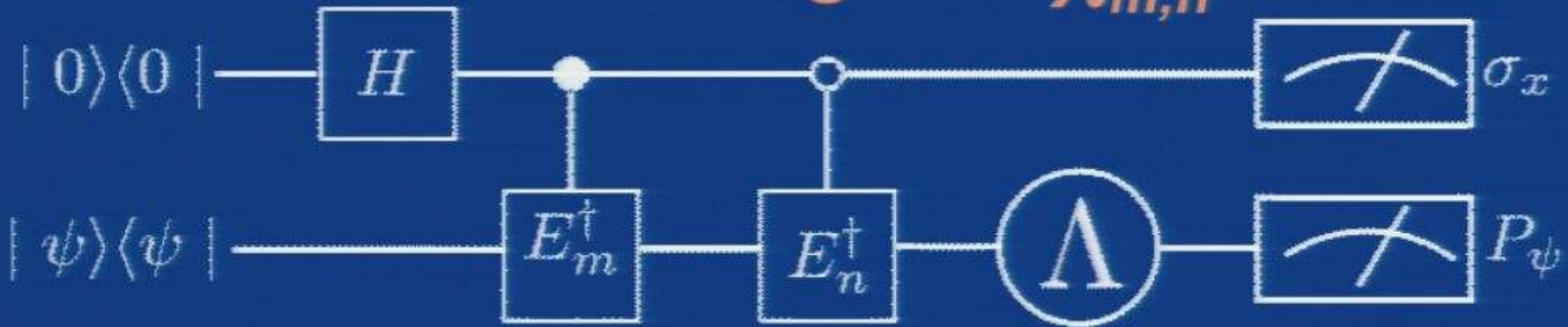
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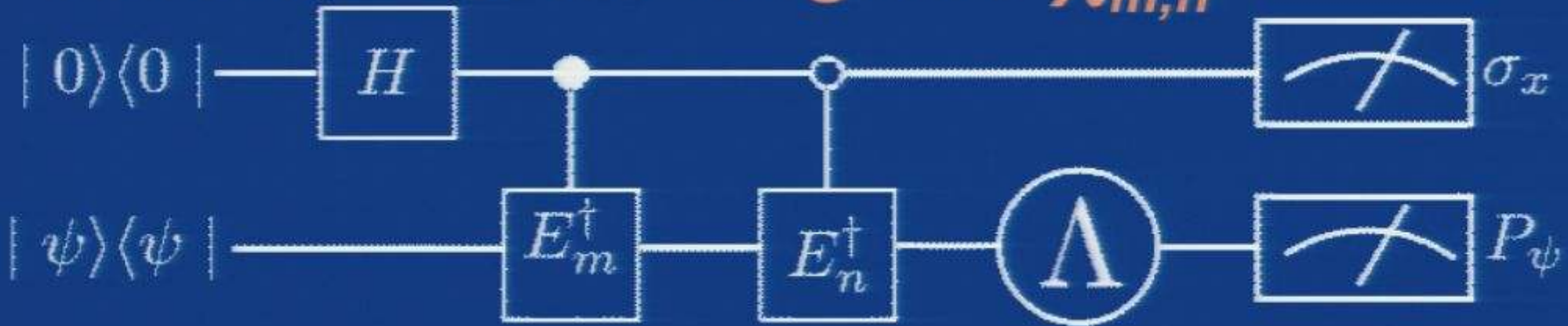
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Average over states: use 2-design

- S is a 2-design iff

$$\int \langle \psi | A | \psi \rangle \langle \psi | B | \psi \rangle d|\psi\rangle = \frac{1}{|S|} \sum_{|\psi\rangle \in S} \langle \psi | A | \psi \rangle \langle \psi | B | \psi \rangle$$

- Any degree 2 expression in bras and kets may be evaluated as an average over S (finite set).
- In particular, fidelities are averages over S .

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Nice things about 2-designs?

- Continuous to discrete.
- Efficient $O(N^2)$ constructions exist 2-design states.
- Approx state 2-designs (via approx unitary 2-d)

Measuring fidelity



$|\psi\rangle\langle\psi|$



$|\psi\rangle\langle\psi|$



No = 0

Yes = 1

$$F(\Lambda_m) = \frac{D\chi_{m,m} + 1}{D + 1} =$$

Average results
over M realizations.

Selective Efficient Quantum Process Tomography

- M : number of experiments to determine parameter with precision ϵ and probability p is such that
- $O(N^2)$ Quantum gates required.
- $O(N^3)$ Classical processing required.

$$M \geq \frac{1}{2\epsilon^2} \log\left(\frac{2}{1-p}\right)$$

**Same scheme allowing efficient estimation
of ANY $\chi_{m,n}$ coefficient.**

A related strategy: measure not only survival probabilities (also transitions)

- Suppose we can generate the 2-design corresponding to the $D+1$ MUBs associated with the operator basis E_n (Pauli's).

- Measure $|\psi_{B,k}\rangle\langle\psi_{B,k}|$ —  $B ?k'$

- Each result contributes to the computation of D diagonal chi-matrix elements.
- Efficiently find if a diagonal chi is above some threshold

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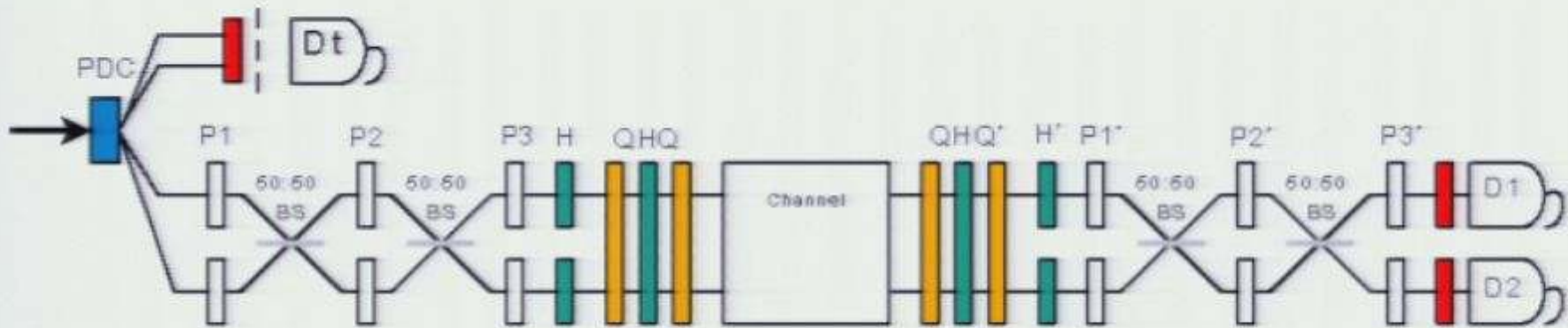
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A quantum optical implementation

(C. Shmiegelow and J.P.P unpublished)



Base	P_1, P_2, P_3	CNOT	QHQ
XX	$0, -\frac{\pi}{2}, 0$	✗	$\frac{\pi}{2}, \frac{\pi}{2}, 0$
YY	$\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2}$	✗	$0, \frac{\pi}{2}, 0$
ZZ	$0, 0, 0$	✗	$0, 0, 0$
belle	$0, -\frac{\pi}{2}, \pi$	✓	$\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}$
beau	$0, -\frac{\pi}{2}, \pi$	✓	$-\frac{\pi}{2}, -\frac{\pi}{2}, 0$

TABLE II: Configurations for state preparation

Base	P_1, P_2, P_3	CNOT	QHQ
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ZZ	$0, 0, 0$	✗	$0, 0, 0$
belle	$\pi, \frac{\pi}{2}, 0$	✓	$-\frac{\pi}{2}, -\frac{\pi}{2}, 0$
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TABLE III: Configurations for state readout

- Two qbits in one photon (hyper-entanglement).
- Preparation and readout in $D+1=5$ MUBs is simple.
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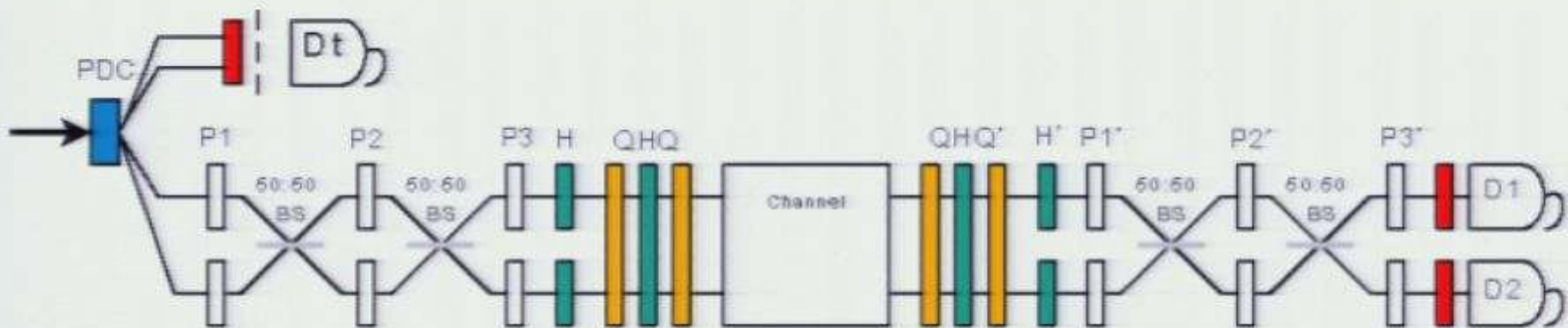
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