Title: Selective and efficient process tomography

Date: Aug 26, 2008 03:15 PM

URL: http://pirsa.org/08080044

Abstract:

χ representation

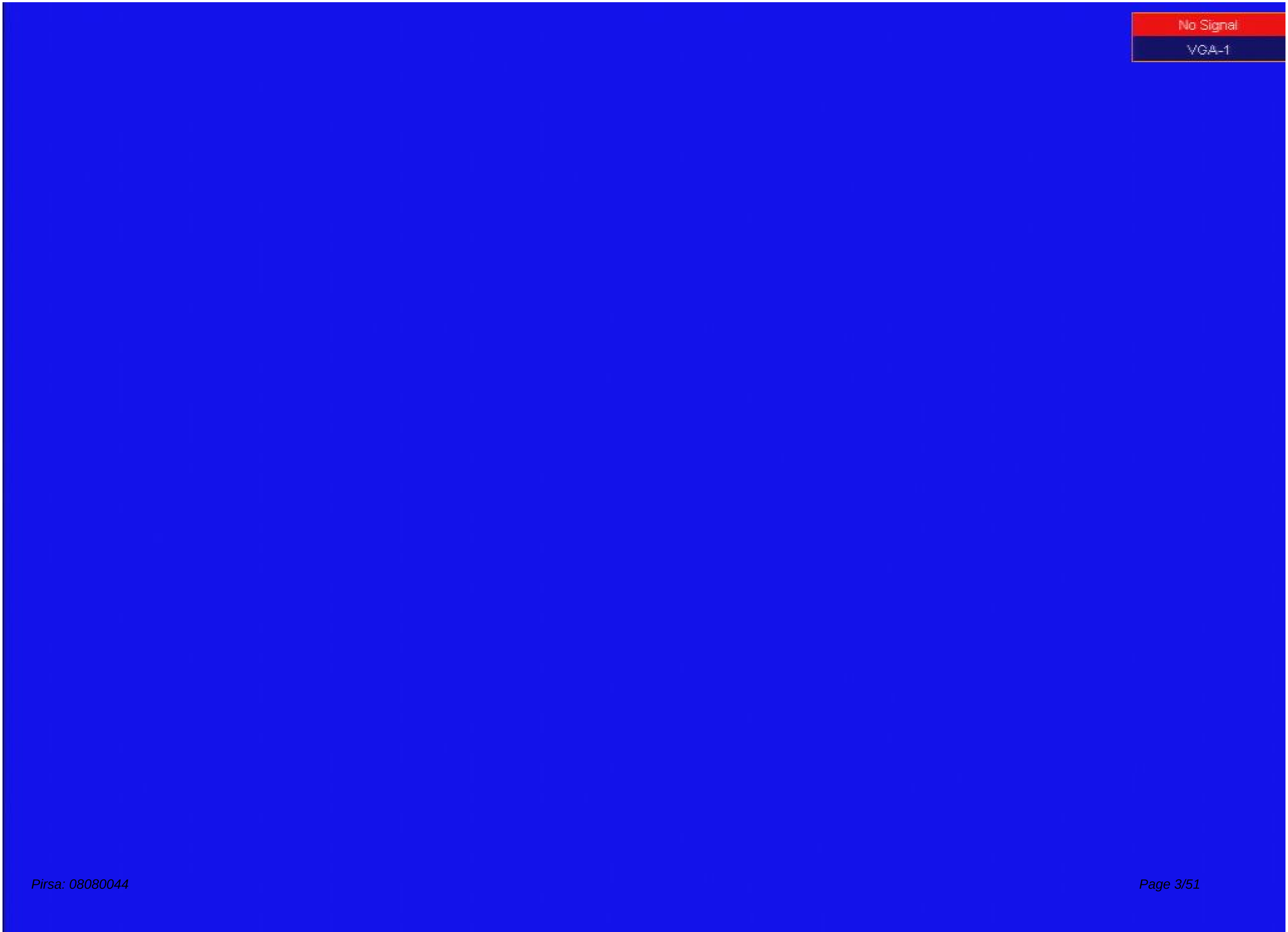
- Parametrized by operator base: $\{E_1, \ldots, E_{D^2}\}$

$$\Lambda(\rho) = \sum_{m,n} \chi_{m,n} E_m \rho E_n^\dagger$$

- χ is hermitian and positive defined.
- Trace preserving:

$$\sum_{m,n} \chi_{m,n} E_n^\dagger E_m = I$$

- Required for deciding on error correcting codes.
- Has large number of parameters $D^4 = 2^{4N}$.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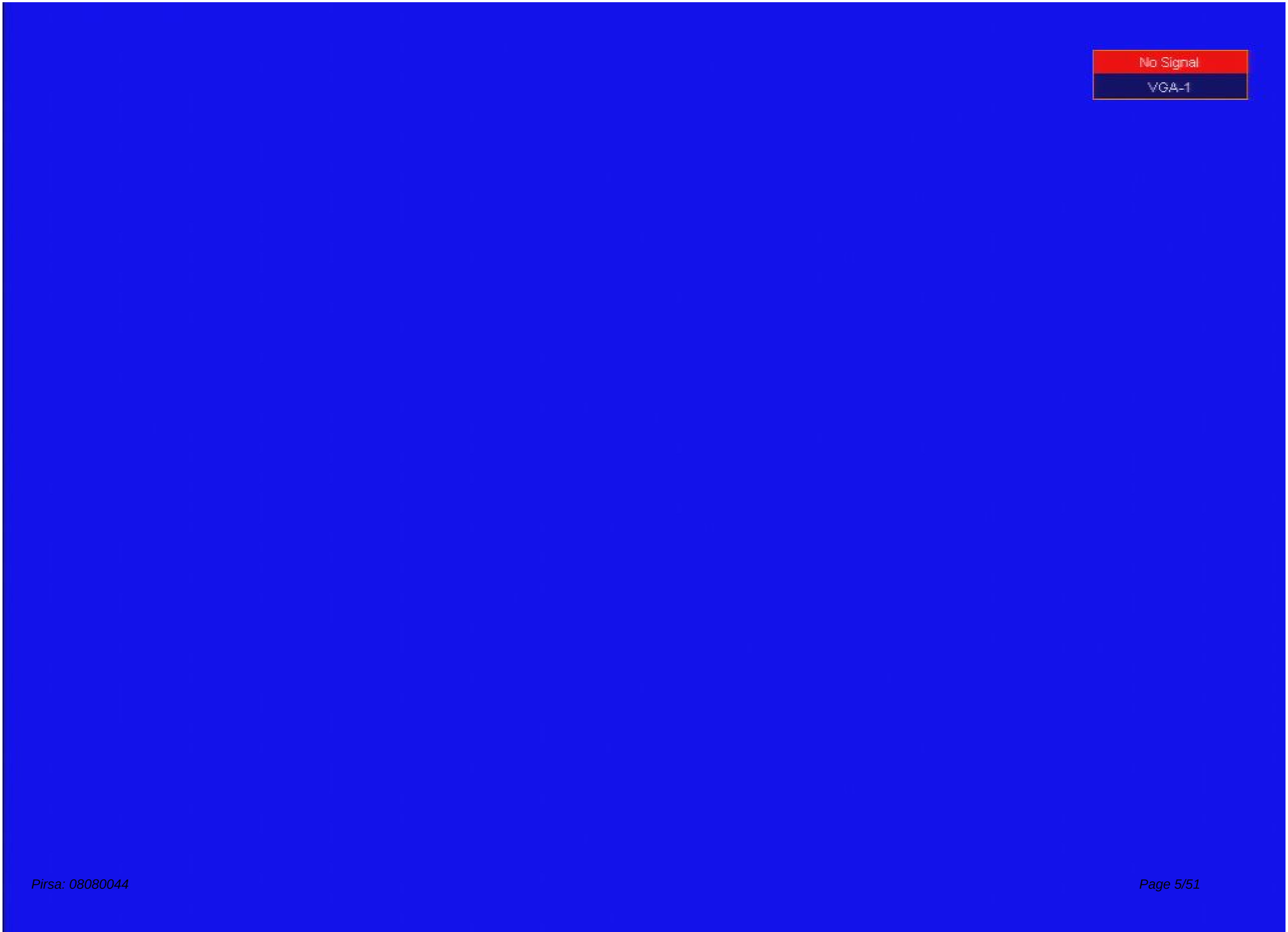
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Some interesting questions.

- Can experiments directly access χ elements?
- Is it possible to estimate these elements efficiently? (good scaling with the number of qubits?)
- Can experiments efficiently determine which of all coefficients are 'important'? (or wether a subset is important?

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ANSWER: Yes (estimation with fixed, D-independent, precision)

Quantum process tomography

- SQPT: Direct access to λ coefficients ('transition matrix elements). Inefficient for accessing χ representation.
- DCQD: Efficient access to diagonal $\chi_{m,m}$ Is not selective for other coefficients. Requires N clean ancilla qubits.
- SCNQP: Based on Λ map symmetrization. Efficient access to limited number of important χ parameter groups

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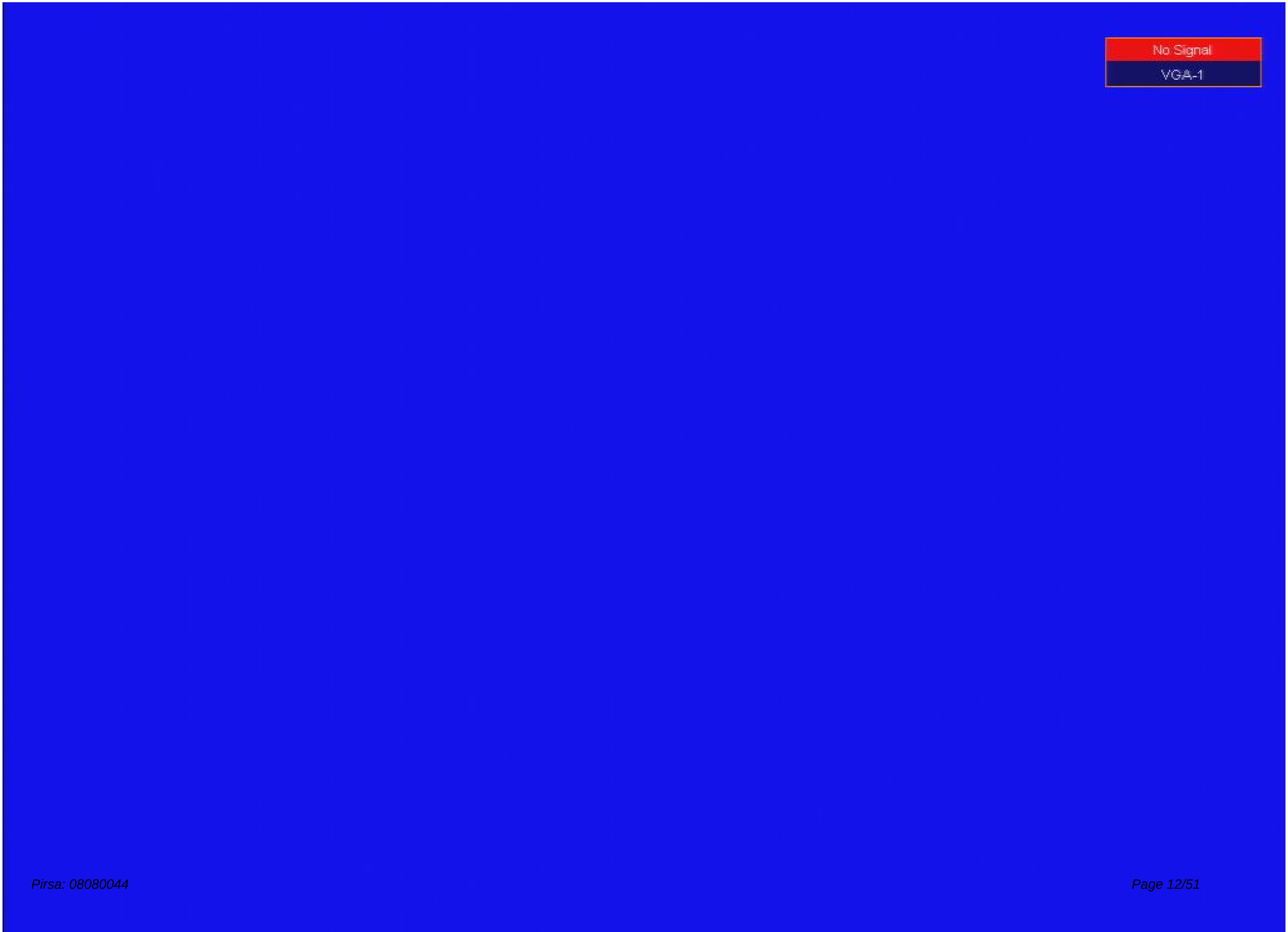
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(all acronyms are ARC-aproved)

Operator basis

- We could use Pauli operators (or any other basis)

$$P_0 = I, \quad P_1 = X, \quad P_2 = Y, \quad P_3 = Z$$

$$E_m = P_{m_1} \otimes P_{m_2} \otimes \ldots \otimes P_{m_N}$$

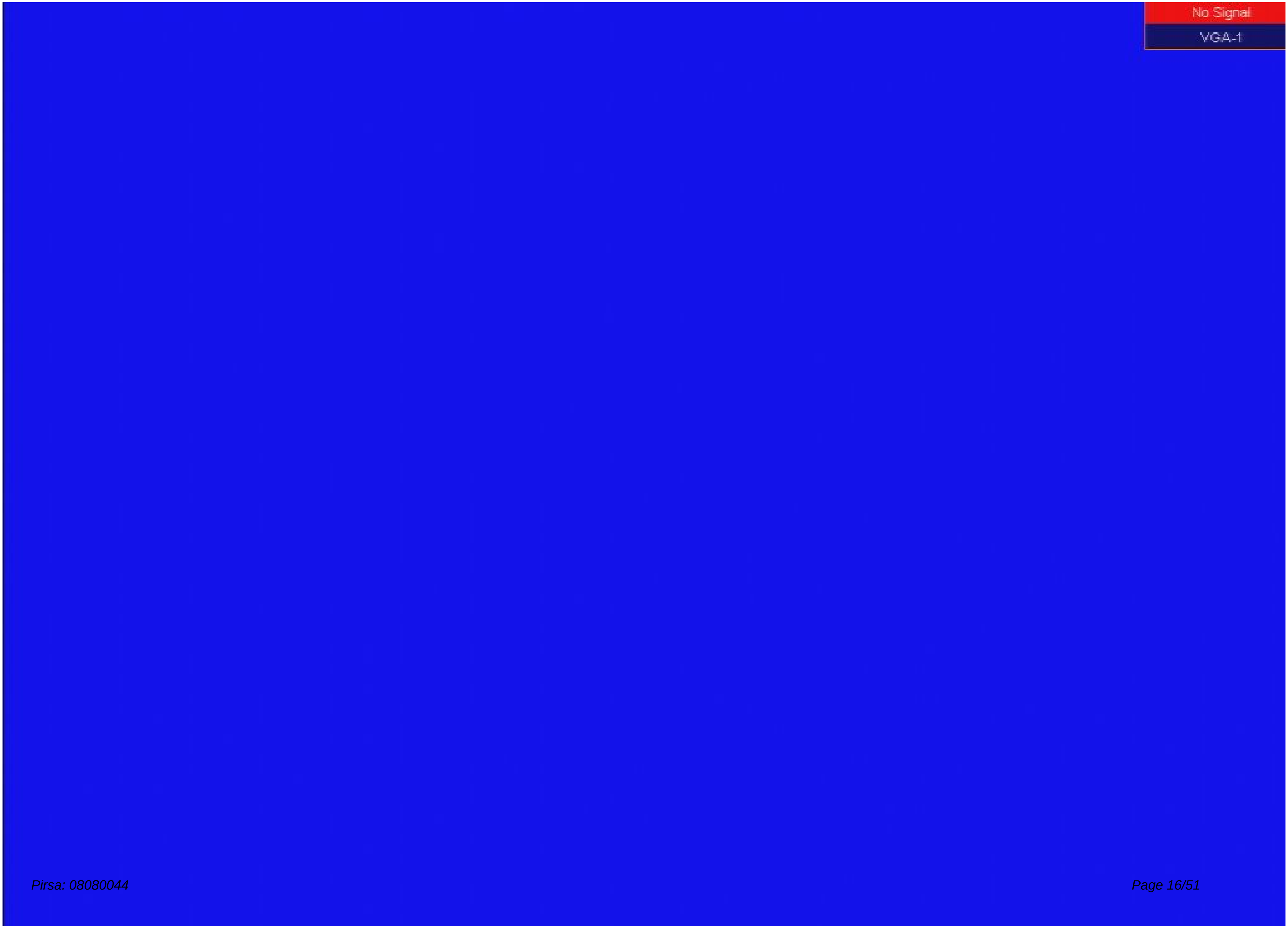
- We take: $E_0 = I^{\otimes N}$

Average Fidelity

- The average fidelity for a map Λ: survival probability averaged over all pure states:

$$F(\Lambda) = \int \langle\psi \mid \Lambda(\mid \psi\rangle\langle\psi \mid) \mid \psi\rangle d \mid \psi\rangle$$

- Key: All $\chi_{m,m}$ coefficient can be related to average fidelity of a channel.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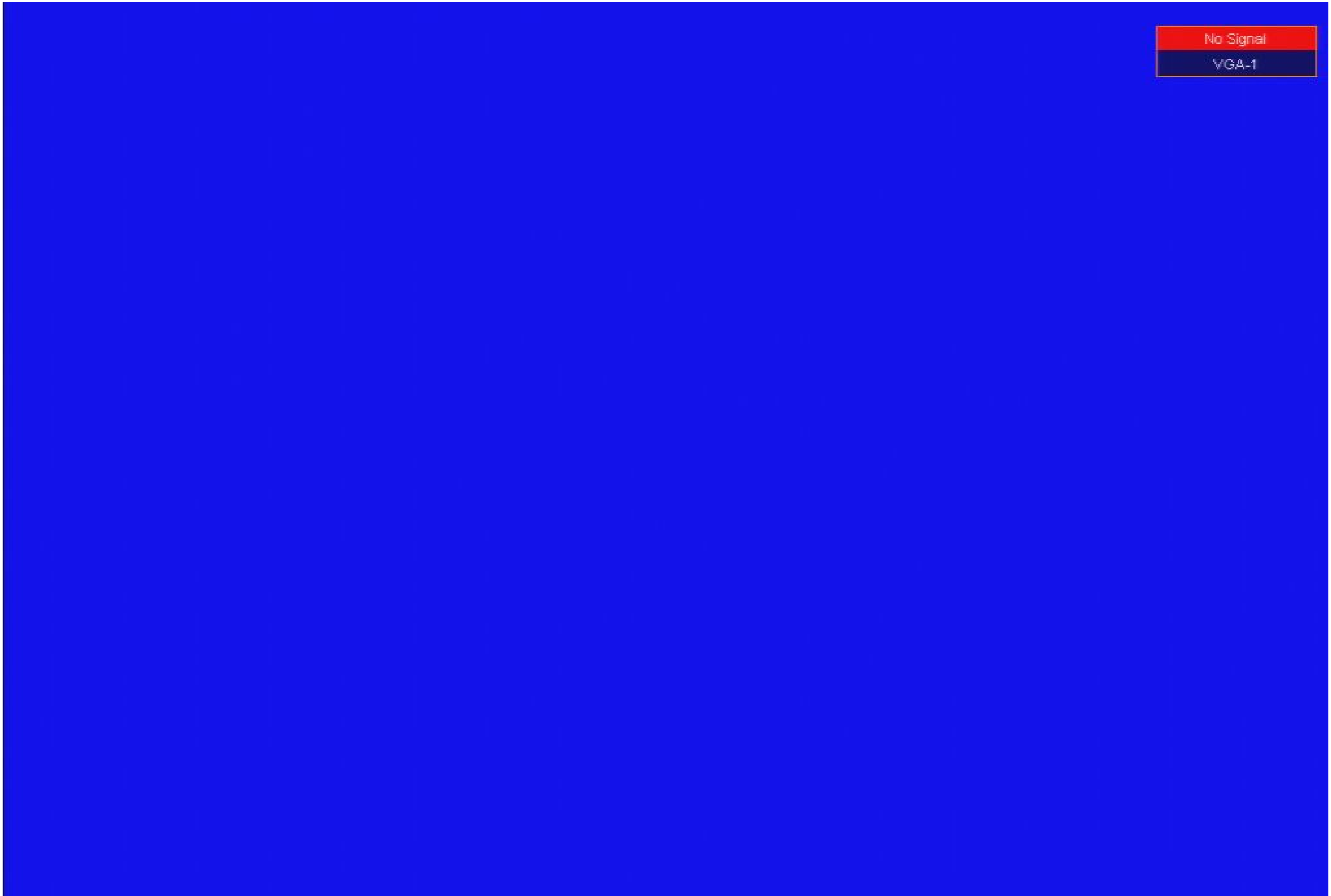
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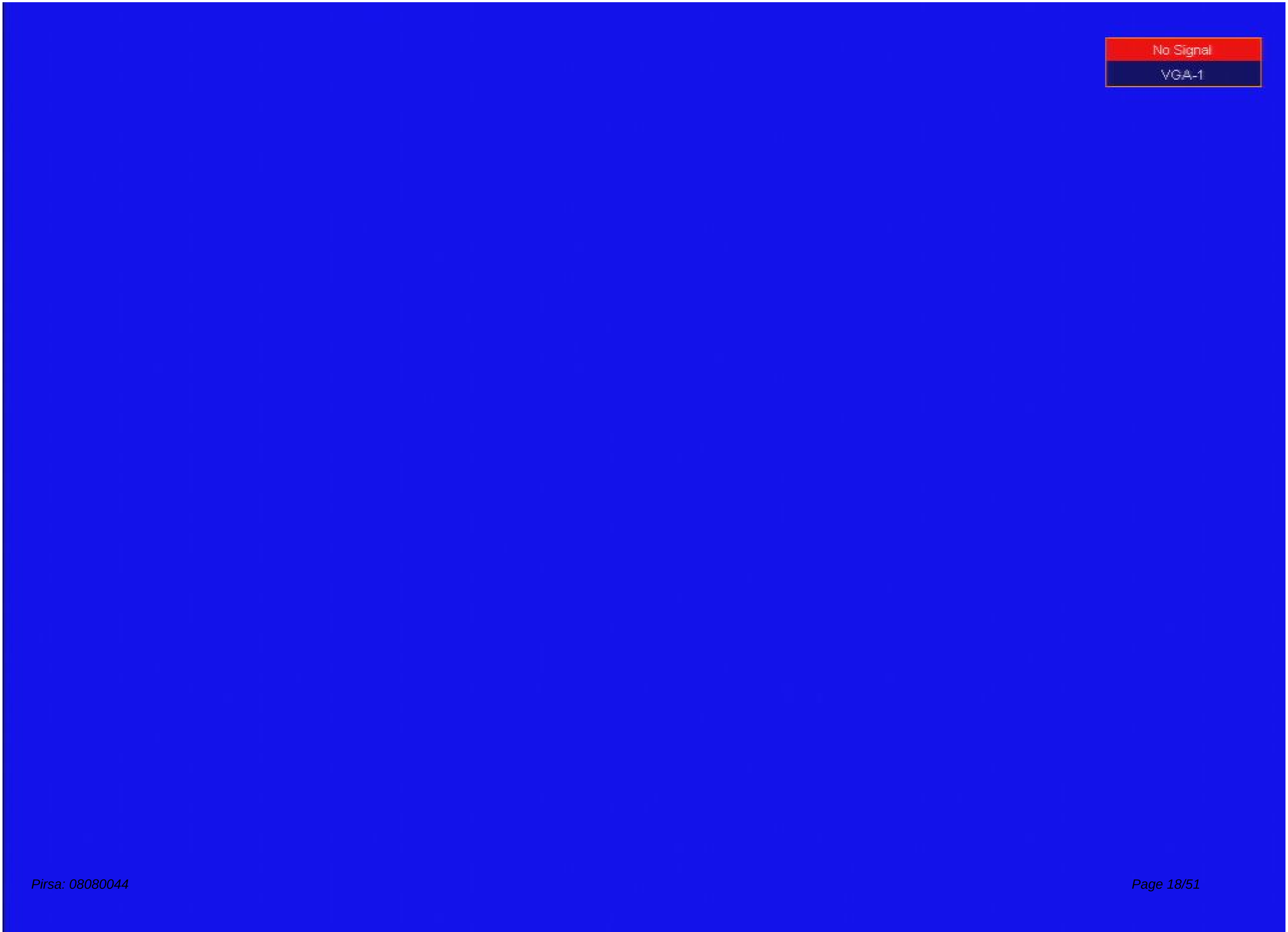
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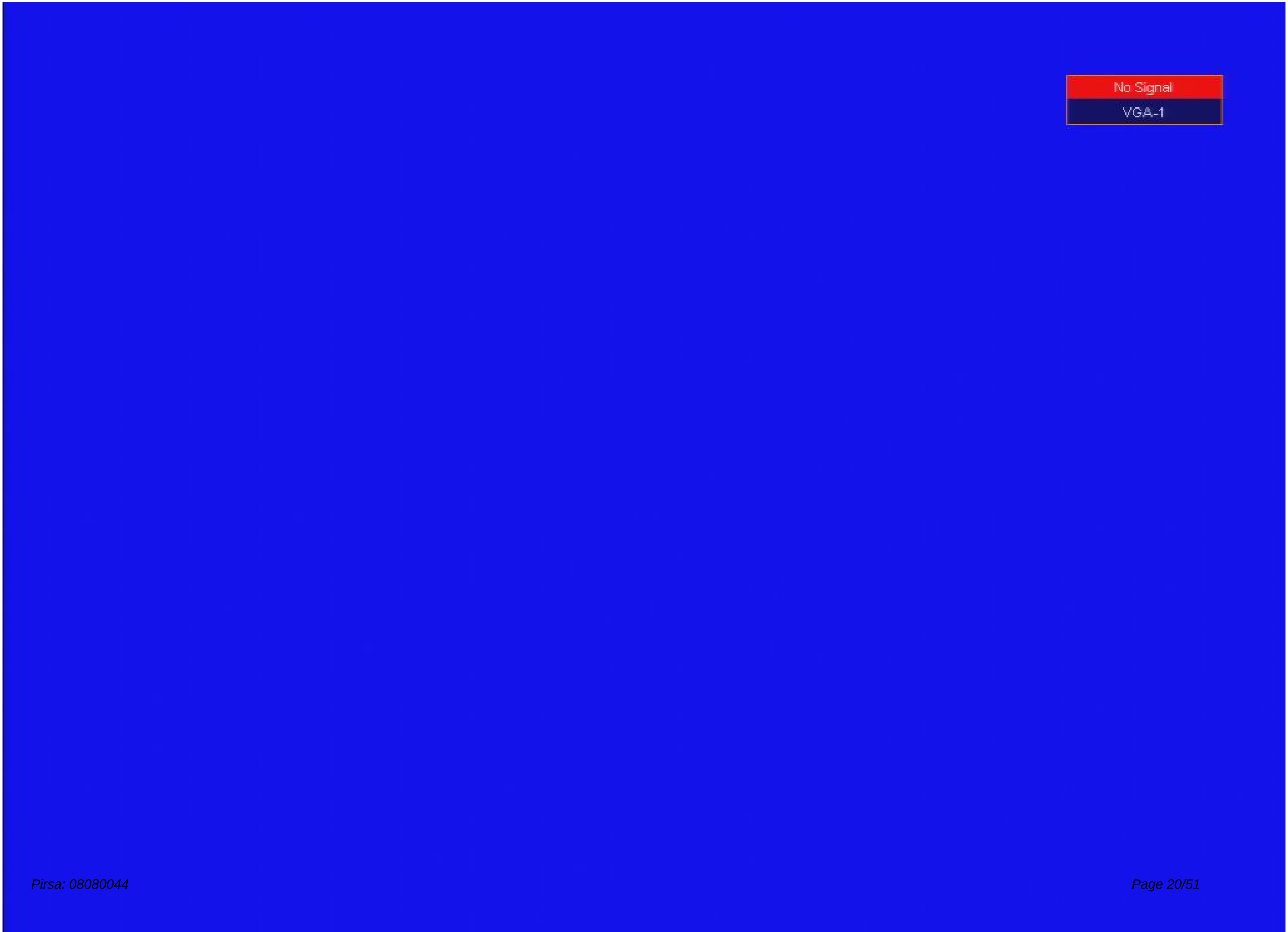
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How? Simple argument leads to $\chi_{0,0}$

- Useful identity

$$\int \langle \psi \mid A \mid \psi \rangle \langle \psi \mid B \mid \psi \rangle d \mid \psi \rangle = \frac{\mathrm{tr}(A)\mathrm{tr}(B) + \mathrm{tr}(AB)}{D(D+1)}$$

J.M. Renes et. al. Journal of Mathematical Physics **45**, 2171 (2004)

- Combine it with the channel chi-representation

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Other diagonal $\chi_{m,m}$ coefficients

- Other $\chi_{m,m}$ are obtainable from the fidelity of modified maps Λ_m.

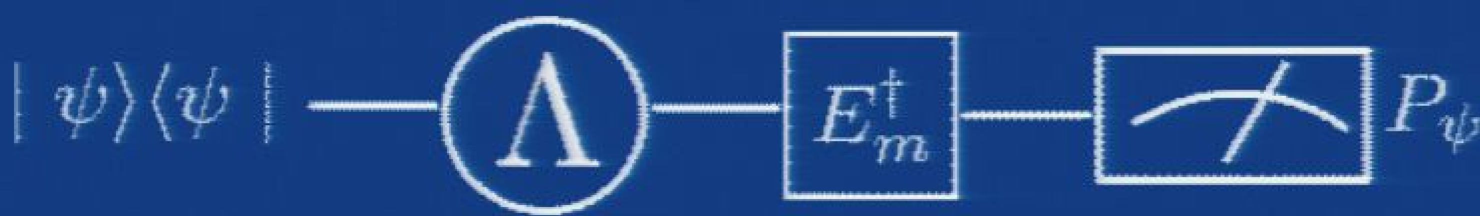
$$F(\Lambda_m) = \frac{D\chi_{m,m} + 1}{D + 1}$$

- Λ_m may be readily implemented through the successive application of Λ and E_m.

$$\Lambda_m(\rho) = E_m^\dagger \Lambda(\rho) E_m$$

Quantum circuit

- More formally, the quantum circuit for measuring fidelity is written as:



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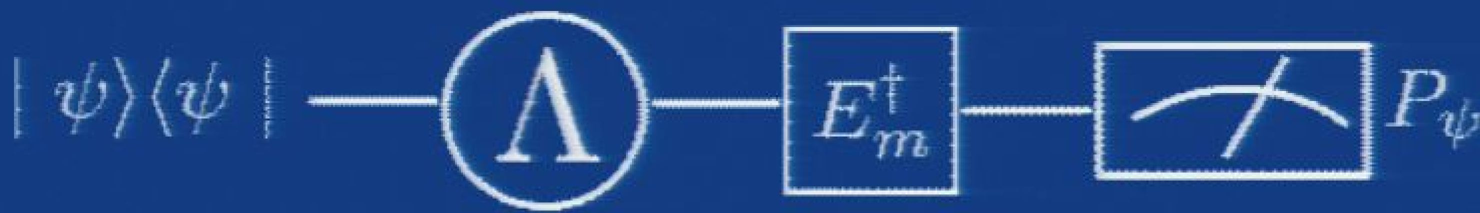

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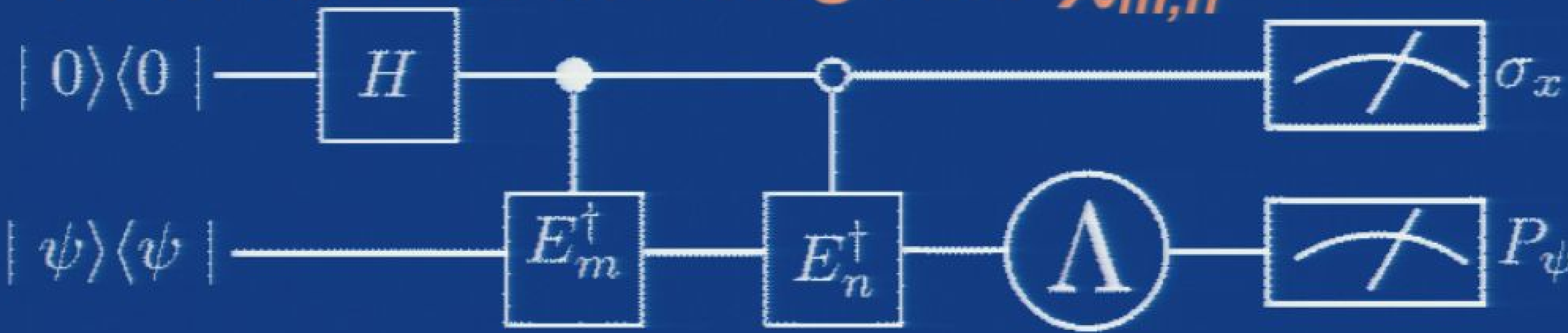
What about off diagonal $\chi_{m,n}$

- They are more. Many more ($\sim D^4$).
- How could we measure them?

$$\int \langle \psi \mid \Lambda(E_m^\dagger P_\psi E_n) \mid \psi \rangle = \frac{D\chi_{m,n} + \delta_{m,n}}{D+1}$$

- Need a different strategy.
- Need an extra resource (one clean qubit)

A circuit for off-diagonal $\chi_{m,n}$



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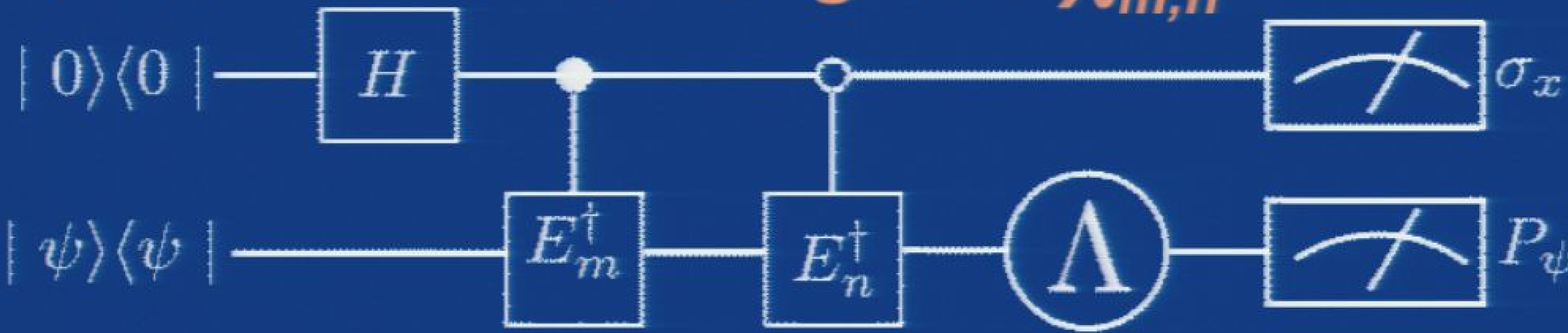

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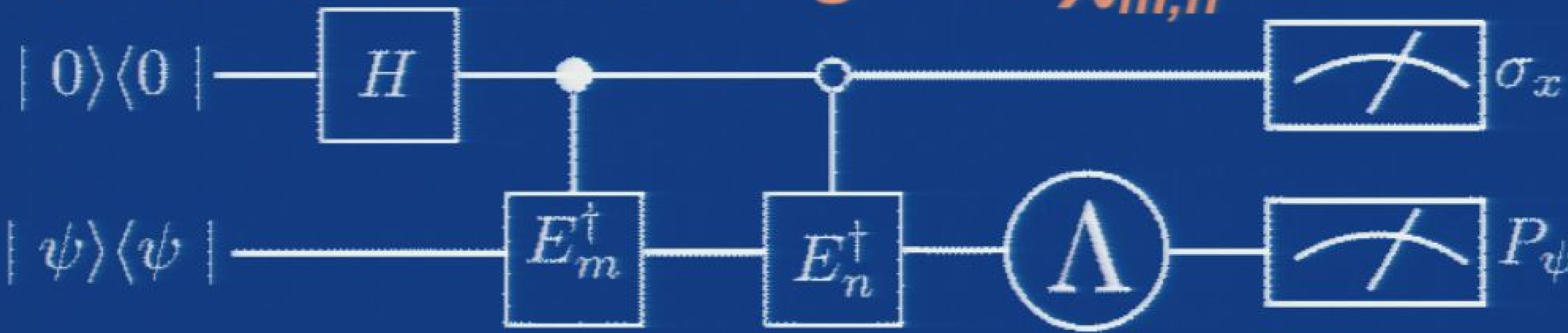

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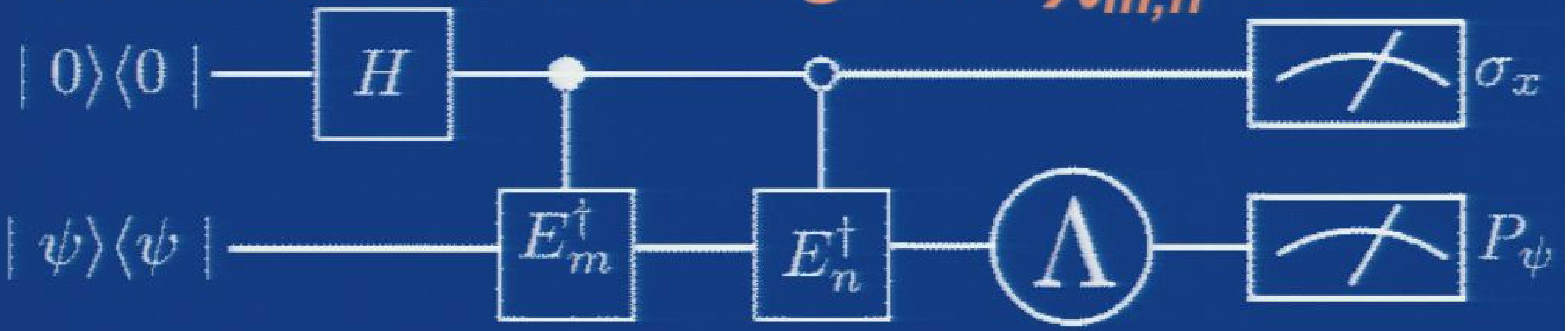

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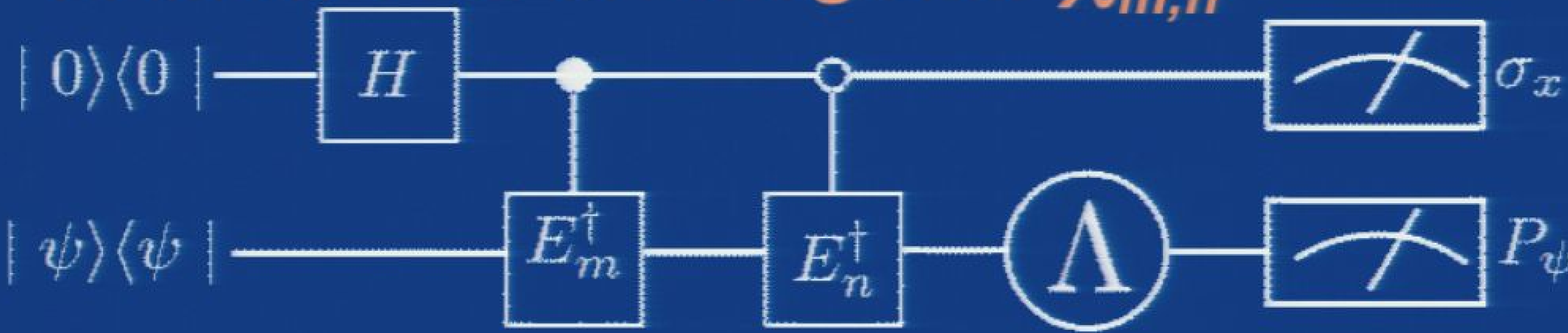

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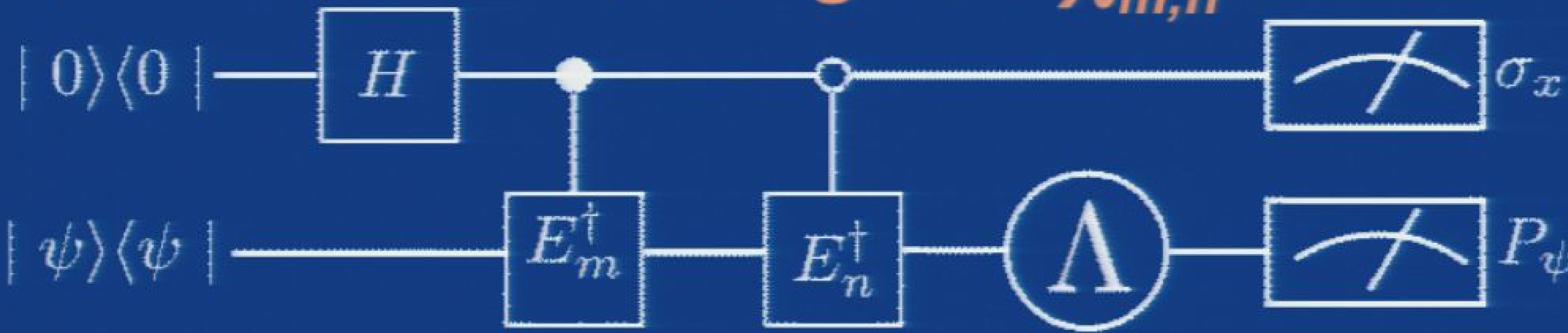

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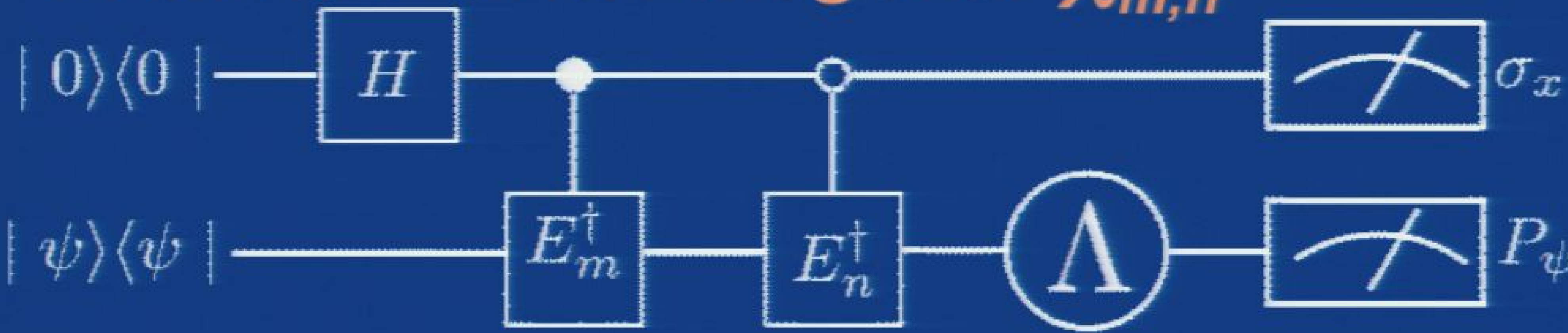

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Average over states: use 2-design

- S is a 2-design iff

$$\int \langle\psi|A|\psi\rangle\langle\psi|B|\psi\rangle d|\psi\rangle = \frac{1}{|S|}\sum_{|\psi\rangle\in S}\langle\psi|A|\psi\rangle\langle\psi|B|\psi\rangle$$

- Any degree 2 expression in bras and kets may be evaluated as an average over S (finite set).
- In particular, fidelities are averages over S.

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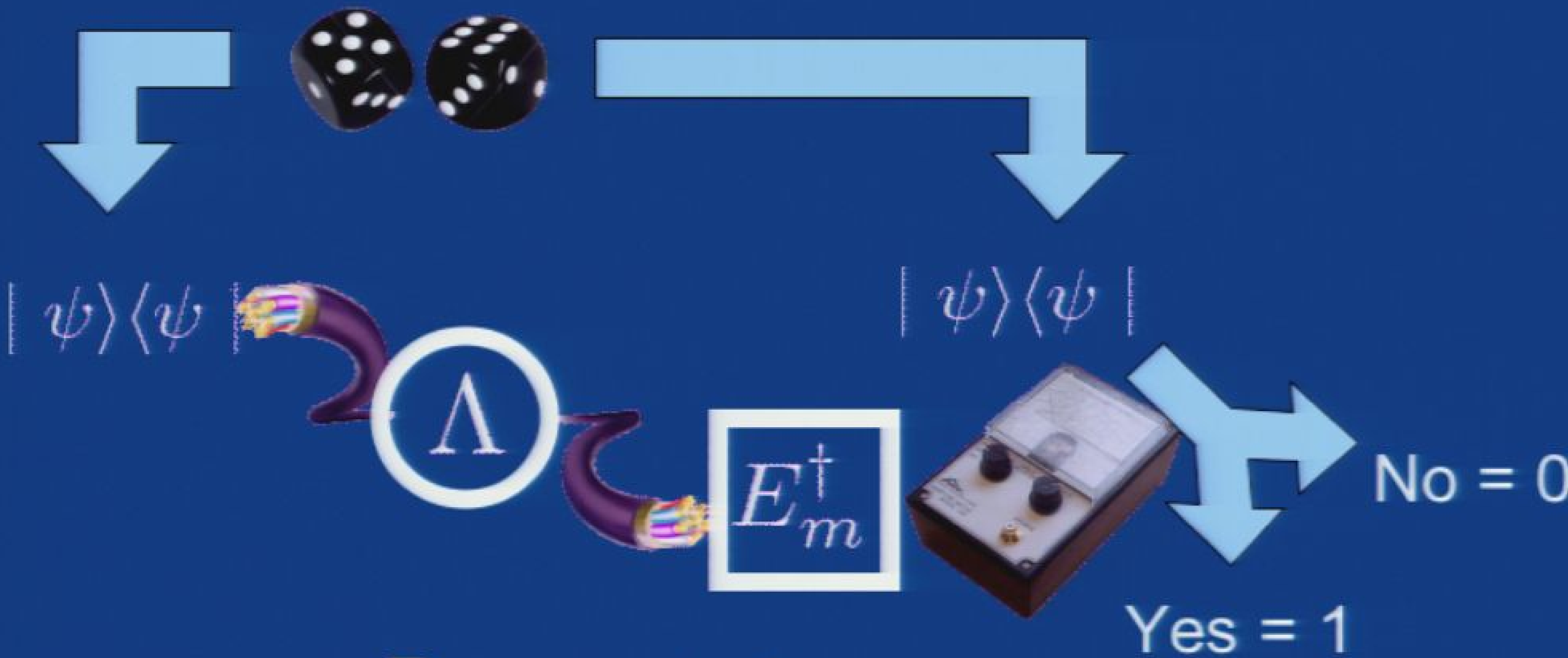
Nice things about 2-designs?

- Continuous to discrete.
- Efficient $O(N^2)$ constructions exist 2-design states.
- Approx state 2-designs (via approx unitary 2-d)

Measuring fidelity



$$F(\Lambda_m) = \frac{D\chi_{m,m} + 1}{D+1} =$$ Average results over M realizations.

Selective Efficient Quantum Process Tomography

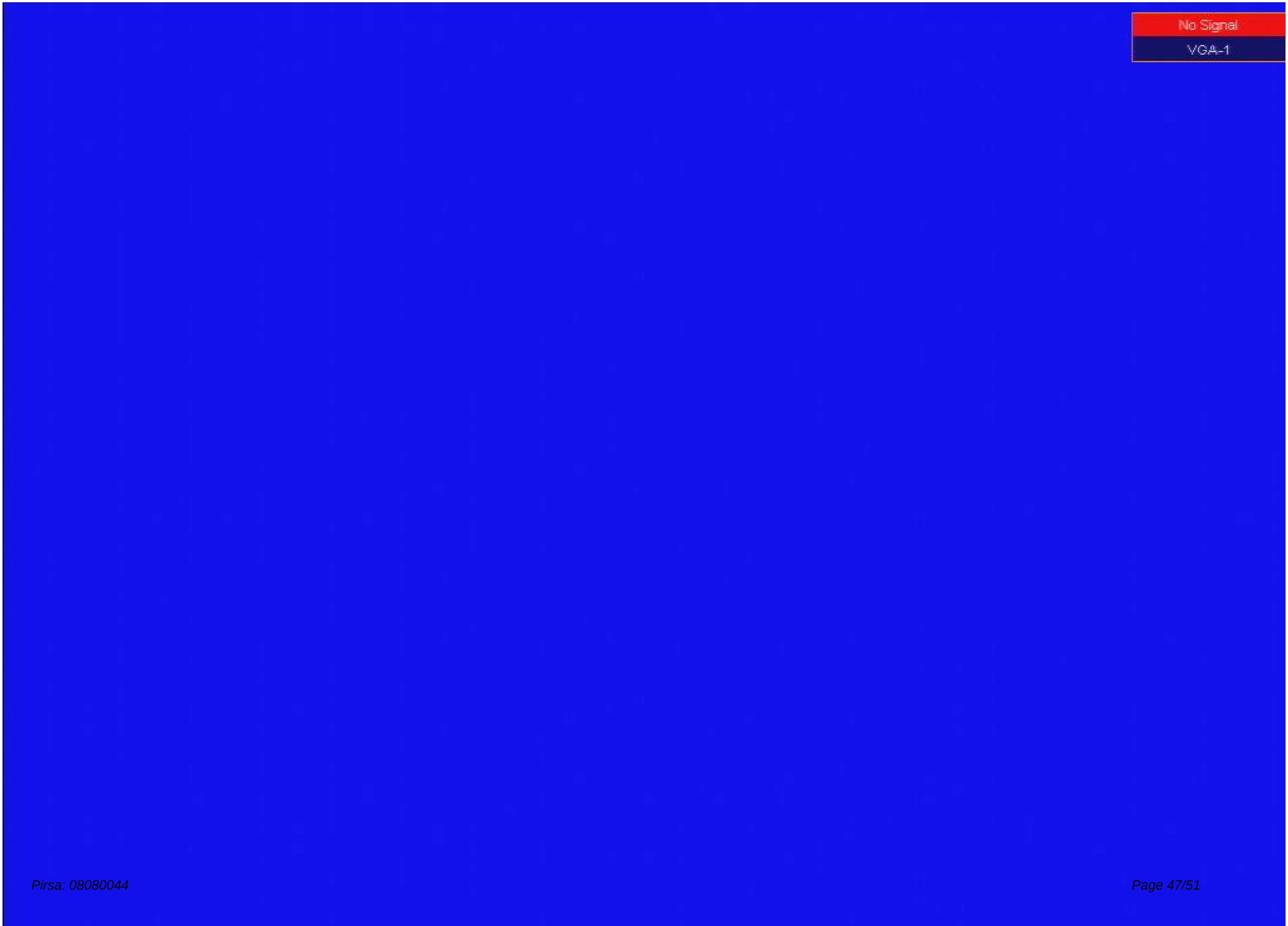
- M: number of experiments to determine parameter with precision ε and probability p is such that $M \geq \frac{1}{2\varepsilon^2}\log\left(\frac{2}{1-p}\right)$
- $O(N^2)$ Quantum gates required.
- $O(N^3)$ Classical processing required.

Same scheme allowing efficient estimation of ANY $\chi_{m,n}$ coefficient.

A related strategy: measure not only survival probabilities (also transitions)

- Suppose we can generate the 2-design corresponding to the D+1 MUBs associated with the operator basis E_n (Pauli's).
- Measure $|\psi_{B,k}\rangle\langle\psi_{B,k}|$  $B\ ?k'$

- Each result contributes to the computation of D diagonal chi-matrix elements.
- Efficiently find if a diagonal chi is above some threshold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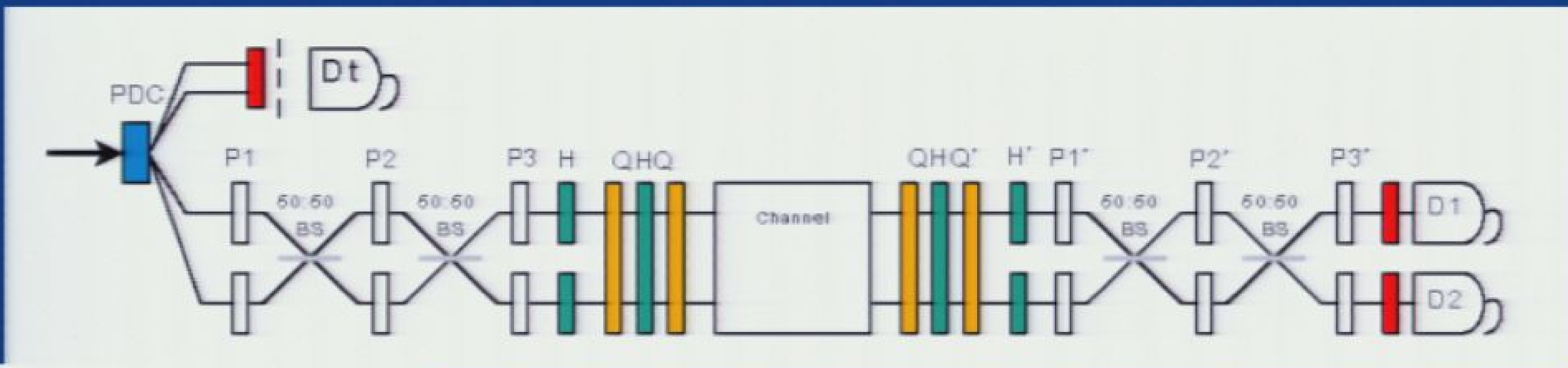
A quantum optical implementation

(C.Shmiegelow and J.P.P unpublished)



Base	$P1, P2, P3$	CNOT	QHQ
XX	$0, -\frac{\pi}{2}, 0$	✗	$\frac{\pi}{4}, \frac{\pi}{4}, 0$
YY	$\frac{\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{2}$	✗	$0, \frac{\pi}{4}, 0$
ZZ	0,0,0	✗	0,0,0
belle	$0, -\frac{\pi}{2}, \pi$	✓	$\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$
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TABLE II: Configurations for state preparation

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ZZ	0,0,0	✗	0,0,0
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beau	$\pi, \frac{\pi}{2}, 0$	✓	$\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$

TABLE III: Configurations for state readout

- Two qbits in one photon (hyper-entanglement).
- Preparation and readout in D+1=5 MUBs is simple.
- Channel characterization "efficient".

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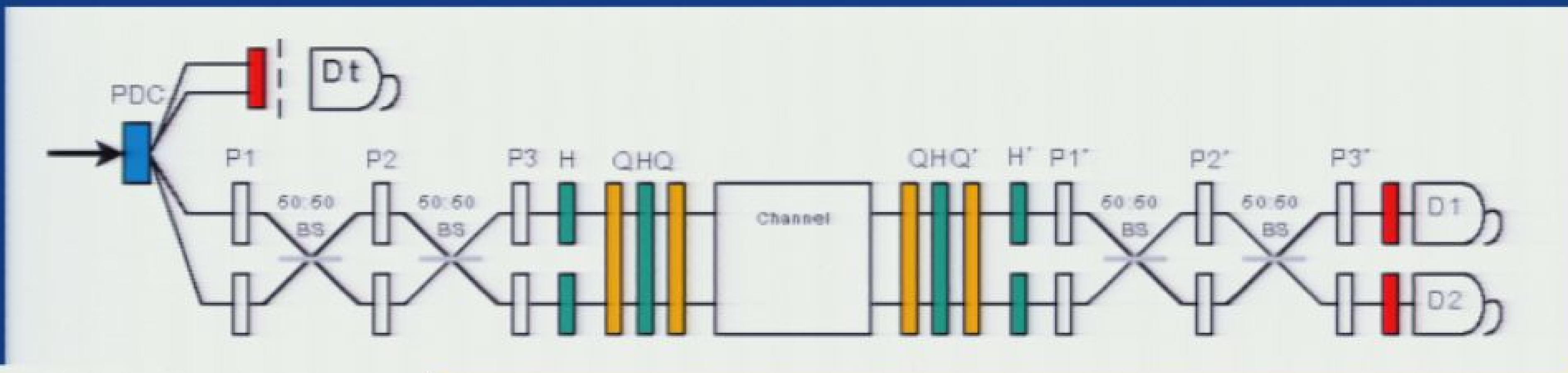

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