Title: A continuous-variable approach to process tomography

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Abstract: We propose and demonstrate experimentally a technique for estimating quantum-optical processes in the continuous-variable domain. The process data is determined by applying the process to a set of coherent states and measuring the output. The process output for an arbitrary input state can then be obtained from its Glauber-Sudarshan expansion. Although such expansion is generally singular, it can be arbitrarily well approximated with a regular function.

Quantum-process tomography in a continuous-variable setting

M. Lobino, E. Figueroa, D. Korystov, C. Kupchak,
B. C. Sanders, A. I. Lvovsky

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THE MOTIVATION

Quantum memory for light: state of the art

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The "holy grail"

- Store and retrieve arbitrary states of light for unlimited time
- State after retrieval must be identical to initial

Existing work

- L. Hau, 1999: slow light
- M. Fleischauer, M. Lukin, 2000: original theoretical idea for EIT light storage
- M. Lukin, D. Wadsworth *et al.*, 2001: storage and retrieval of a classical state
- A. Kuzmich *et al.*, M. Lukin *et al.*, 2005: storage and retrieval of single photons
- M. Kozuma *et al.*, A. Lvovsky *et al.*, 2008: memory for squeezed vacuum
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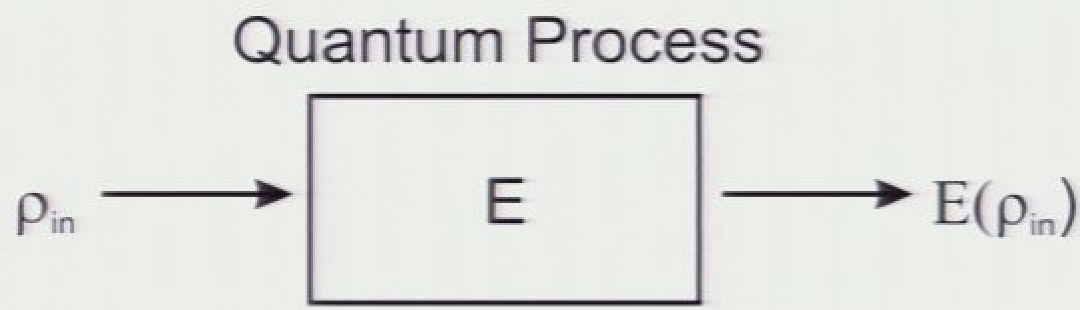
Memory characterization: the big problem

- **Predict how an arbitrary state of light will be preserved**

Quantum processes

- **General properties**
  - Positive mapping
  - Trace preserving or decreasing
  - Linear in density matrix space
    $E(\hat{\rho}_1 + \hat{\rho}_2) = E(\hat{\rho}_1) + E(\hat{\rho}_2)$



  - Not always linear in the quantum Hilbert space

    $E(|\psi_1\rangle + |\psi_2\rangle) \neq |E(\psi_1)\rangle + |E(\psi_2)\rangle$

- **The superoperator**
  - Matrix $\mathbf{E}_{lk}^{mn}$ such that for any input density matrix $(\rho_{\text{in}})_{mn}$ the output density matrix is $(\rho_{\text{out}})_{lk} = \mathbf{E}_{lk}^{nm}(\rho_{\text{in}})_{nm}$
  - Characterizing the process means finding the superoperator
  - Dimension = (Dimension of the Hilbert space)^4
    - Process on a single qubit → 16-dimensional matrix
    - Process on two qubits → 256-dimensional matrix

Quantum process tomography. Existing approaches

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- **The goal**
  - Fully characterize the process, i. e. find the rank 4 matrix of $\boldsymbol{E}$.
- **Direct approach** [Laflamme *et al.*, 1998; Steinberg *et al.*, 2005; etc.]
  - Prepare a set of states $\{\rho_{in}\}$ that form a full basis in the space of input density matrices (basis of the Hilbert space is insufficient!)
  - Subject each of them to the process
  - Characterize each output $\{\boldsymbol{E}(\rho_{in})\}$
- **Ancilla-assisted approach** [D'Ariano, Di Martini *et al.*, 2003; etc.]
  - Use two isomorphic ancilla Hilbert spaces
  - Prepare a maximally entangled state
  - Subject one of the spaces to the process
  - Characterize the output → obtain the superoperator
- **Chalenges**
  - Need to prepare multiple, complex quantum states of light

→ All work so far restricted to discrete Hilbert spaces of very low dimension

Tutorial: coherent states

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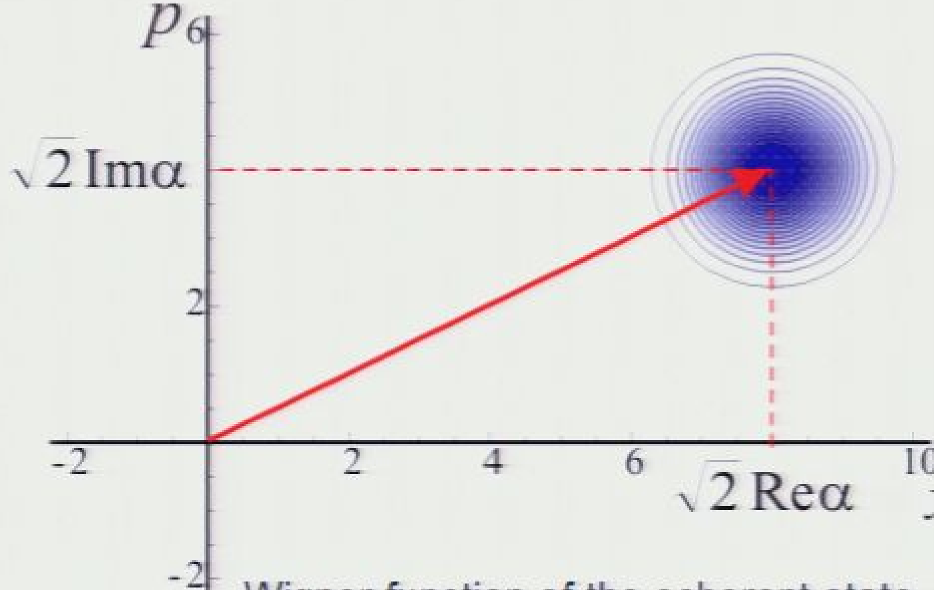
- Eigenstate of the annihilation operator

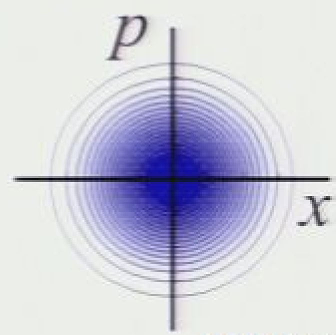
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

- Minimum-uncertainty state
- Uncertainty in position = uncertainty in momentum
- Produced by the laser (under some approximation)



Wigner function of the coherent state

- Important special case: vacuum state (no light)



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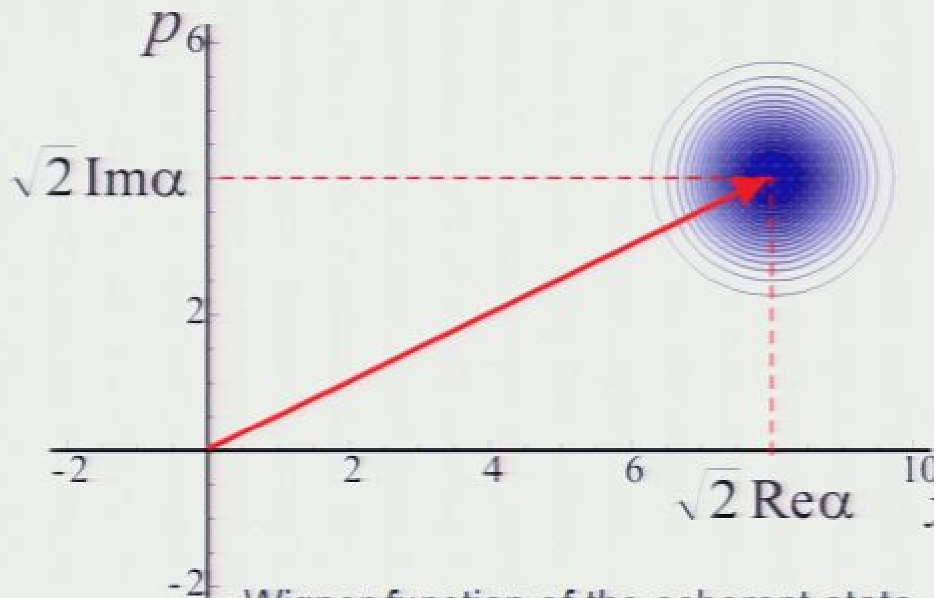

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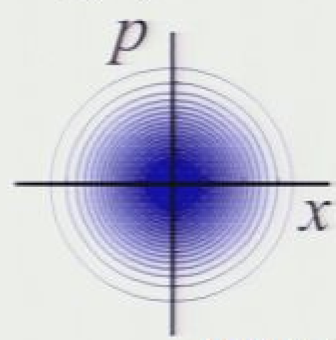


The main idea

- **Decomposition into coherent states**
  - Coherent states form a "basis" in the space of optical density matrices
  - Glauber-Sudarshan P-representation (Nobel Physics Prize 2005)

$$\hat{\rho}_{in} = \int_{\substack{\text{phase}\\ \text{space}}} P_{\hat{\rho}_{in}}(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$$

- **Application to process tomography**
  - Suppose we know the effect of the process $\boldsymbol{E}(|\alpha\rangle\langle\alpha|)$ on each coherent state
  - Then we can predict the effect on any other state

$$\boldsymbol{E}(\hat{\rho}_{in}) = \int_{\substack{\text{phase}\\ \text{space}}} P_{\hat{\rho}_{in}}(\alpha)\boldsymbol{E}(|\alpha\rangle\langle\alpha|)d^2\alpha$$

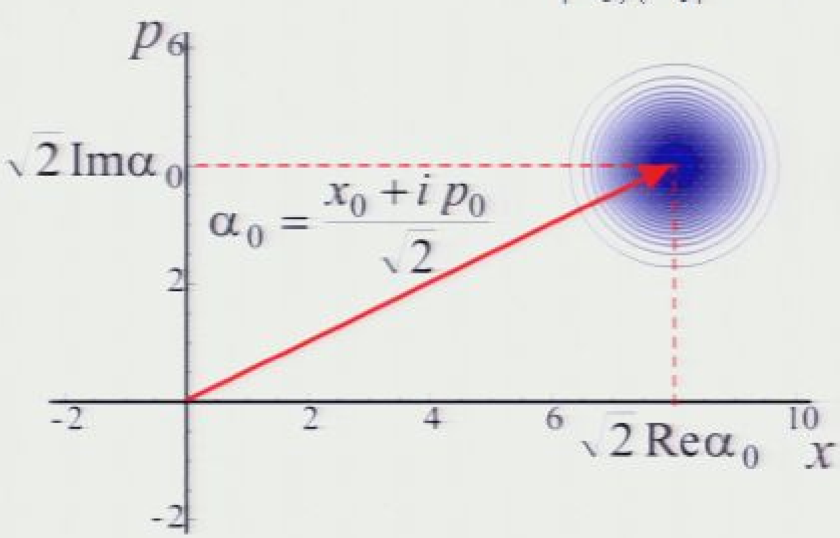
- **The good news**
  - Coherent states are readily available from a laser.
    No nonclassical light needed
  - **Works in infinite dimensions**

THE P-FUNCTION

The P-function

[Glauber,1963; Sudarshan, 1963]

- **Relation to the Wigner function**
  - Associate a point in the phase space ($x = \sqrt{2}\,\mathrm{Re}\,\alpha$, $p = \sqrt{2}\,\mathrm{Im}\,\alpha$) with α
  - Wigner function is the convolution of the P-function with the Wigner function of the vacuum state $W_{\hat{\rho}}(\alpha) = P_{\hat{\rho}}(\alpha) * W_0(\alpha)$
- **Example**
  - P-function of a coherent state $|\alpha_0\rangle$: Dirac delta-function $P_{|\alpha_0\rangle\langle\alpha_0|}(\alpha) = \delta^2(\alpha - \alpha_0)$



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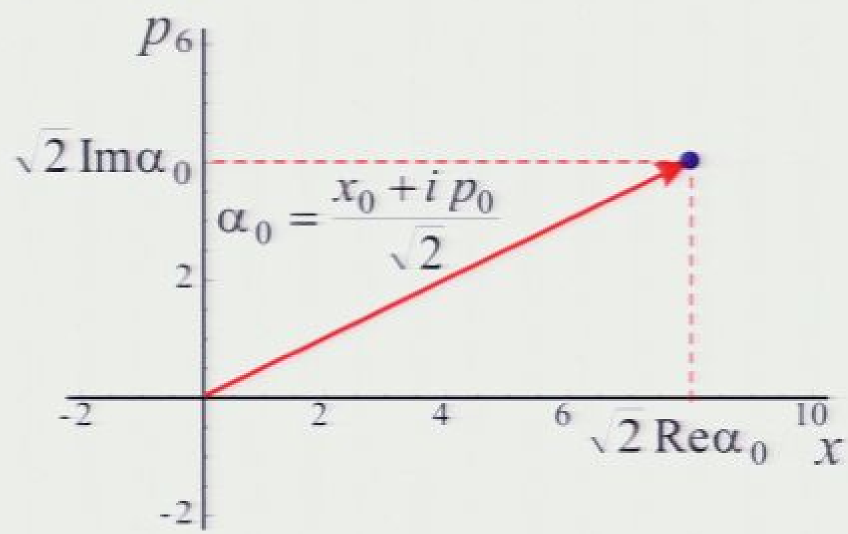

P-function of a coherent state

The P-function

[Glauber,1963; Sudarshan, 1963]

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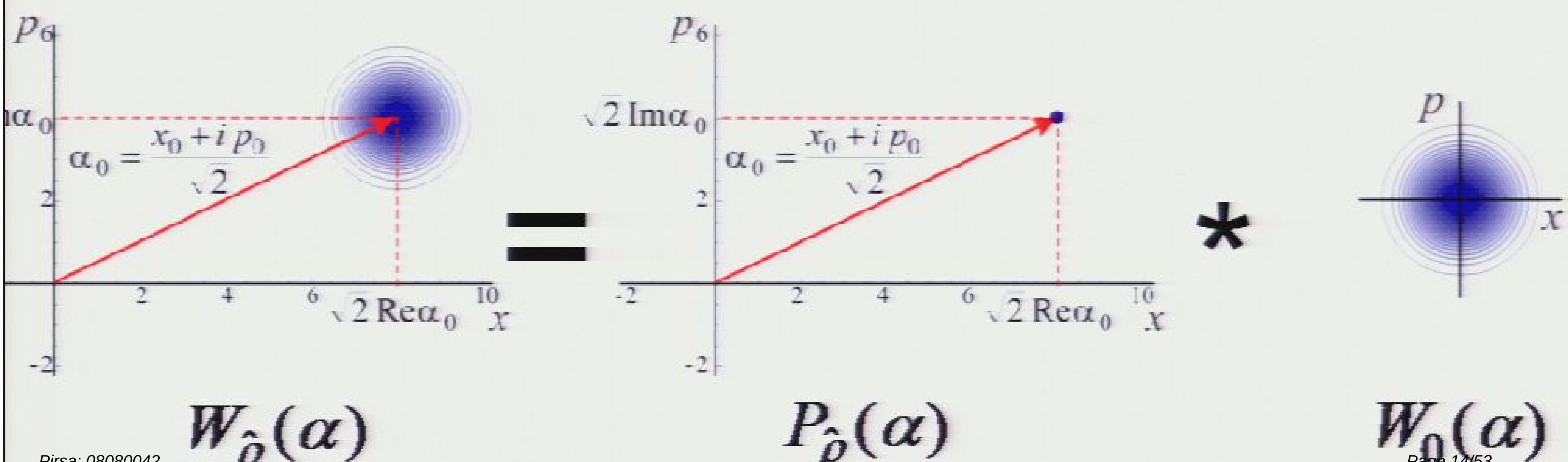

The P-function
(…continued)

- **More properties**
  - For classical states (statistical mixtures of coherent states): positive definite
  - For nonclassical states (photon-number, squeezed, etc.): extremely ill-behaved

Example: $P_{|n\rangle\langle m|}(\alpha) \propto \left(-\frac{\partial}{\partial|\alpha|} \right)^{n+m} \delta(|\alpha|)$

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Artist's view of P-function

The P-function
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    Example: $P_{|n\rangle\langle m|}(\alpha) \propto \left(-\frac{\partial}{\partial|\alpha|}\right)^{n+m} \delta(|\alpha|)$
  - Sounds like bad news: can't do process tomography for nonclassical states
  - Fortunately,…

Artist's view of P-function

Klauder's theorem

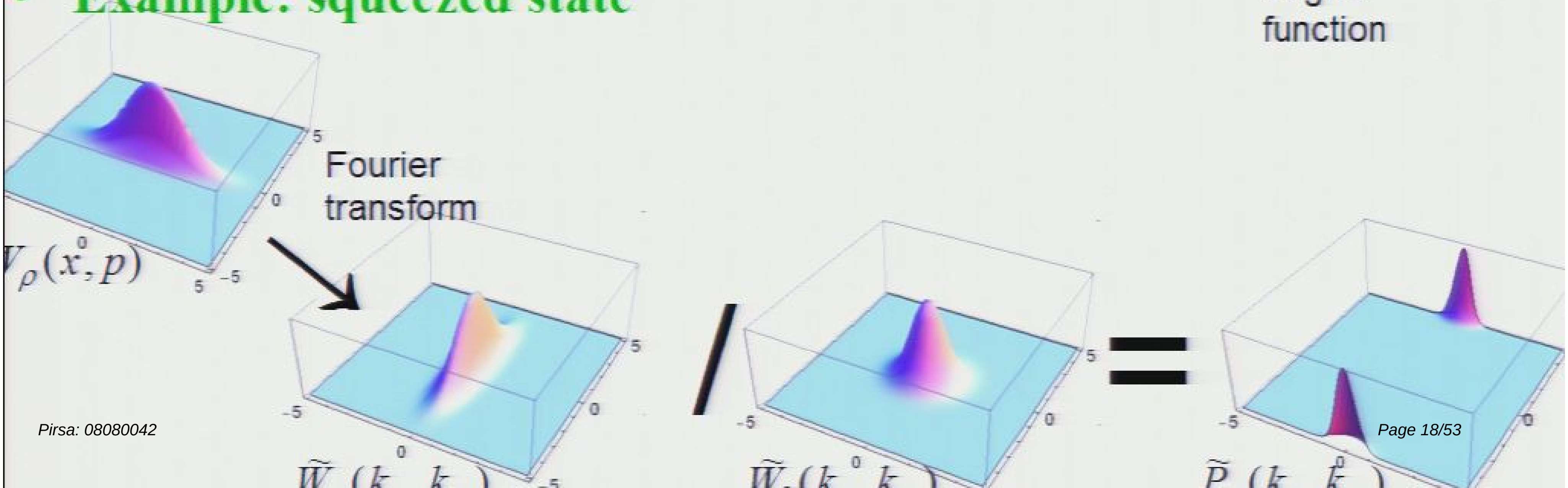
[Klauder, 1966]

Any state can be infinitely well approximated by a state with a P function in S-class (infinitely smooth, rapidly decaying)

- **Proof (sketch)**
  - For every "reasonable" state, the Wigner function $W_\rho(x,p)$ is "nice"
  - So is its Fourier transform $\tilde{W}_\rho(k_x, k_p)$
  - The Fourier transform of the P-function $\tilde{P}_\rho = \tilde{W}_\rho / \tilde{W}_0$ is infinitely smooth, but may be rapidly growing

Fourier transform of vacuum Wigner function

- **Example: squeezed state**



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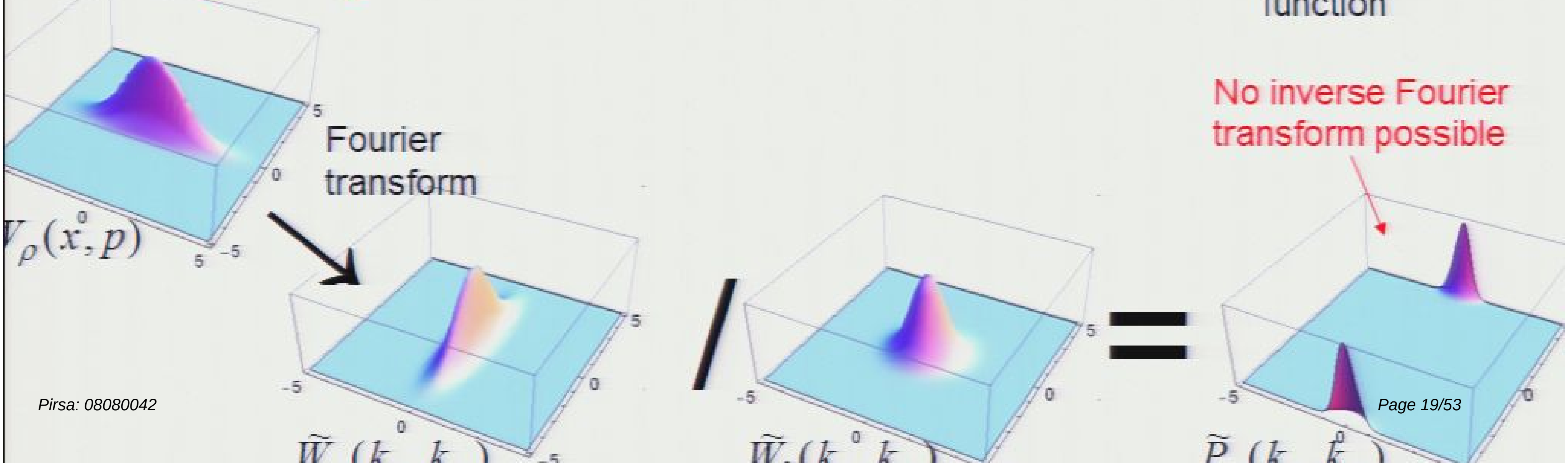
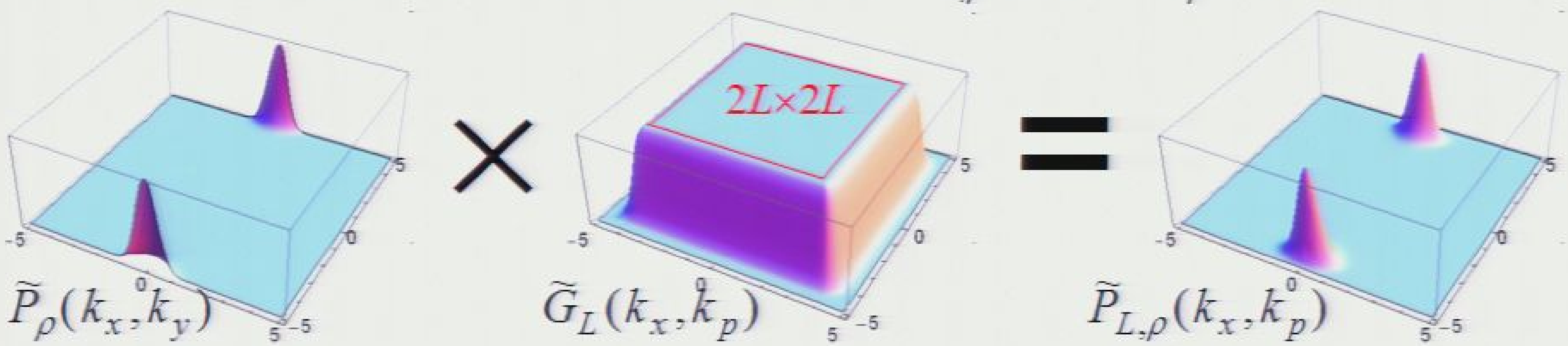


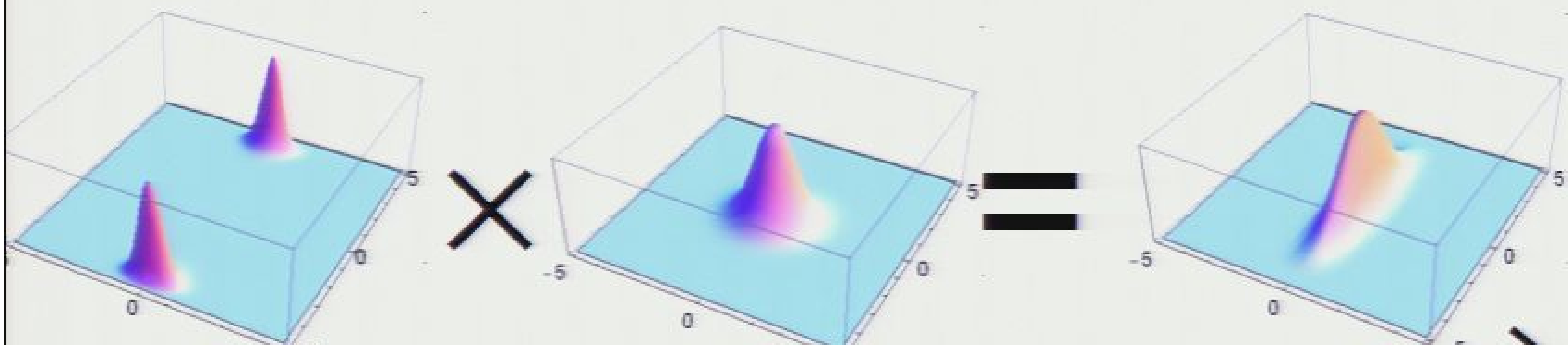
Klauder's theorem
(…continued)

- **Klauder's approximation**
  - Multiply $\widetilde{P}_\rho(k_x, k_p)$ by a smoothened top-hat function $\widetilde{G}_L(k_x, k_p)$
  - The result $\widetilde{P}_{L,\rho} = \widetilde{P}_\rho \widetilde{G}_L$ is a "nice" function
  - and so is its inverse Fourier transform $P_{L,\hat{\rho}}(x,p) = P_{\hat{\rho}}(x,p) * G_L(x,p)$

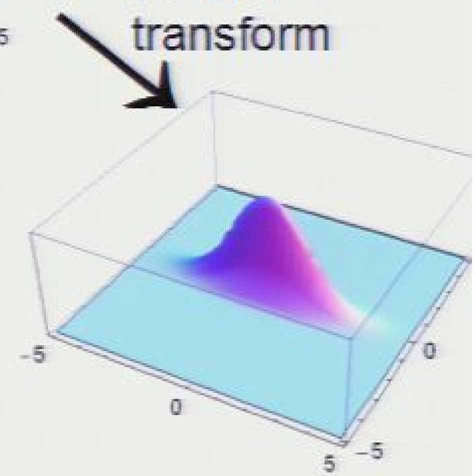


Klauder's theorem
(…continued)

- Let us construct a new quantum state based on this smoothened P-function



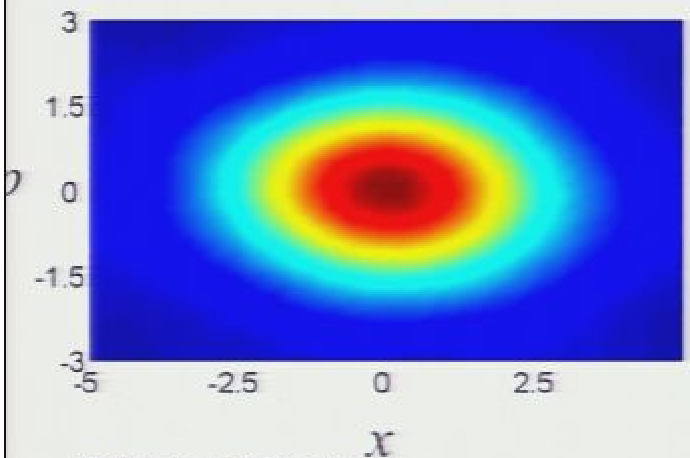
$\tilde{P}_{L,\rho}(k_x, k_p)$ $\quad \widetilde{W}_0(k_x, k_p) \quad \widetilde{W}_{L,\hat{\rho}}(k_x, k_p)$



- If we choose a sufficiently large L, the function $G_L(x,p) = \text{Fourier}^{-1}\left[\widetilde{G}_L(k_x,k_p)\right]$ is very narrow
  - Wigner function $W_{L,\rho}(x,p)$ can be made arbitrarily similar to $W_\rho(x,p)$
  - This implies similarity of states □

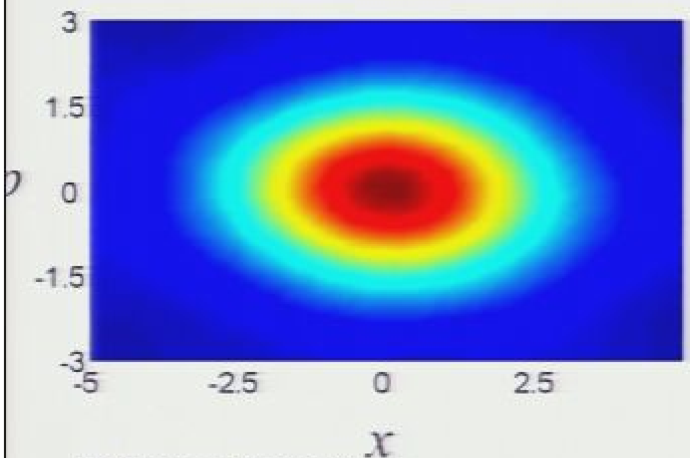
$$W_{L,\hat{\rho}}(x,p) = W_{\hat{\rho}}(x,p) * G_L(x,p)$$

Example: CW squeezed vacuum



Wigner function
from experimental data

Example: CW squeezed vacuum



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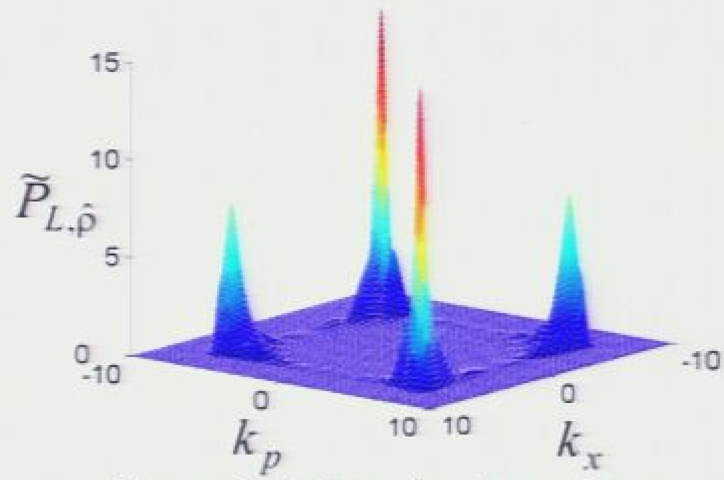

Bounded Fourier transform
of the P-function

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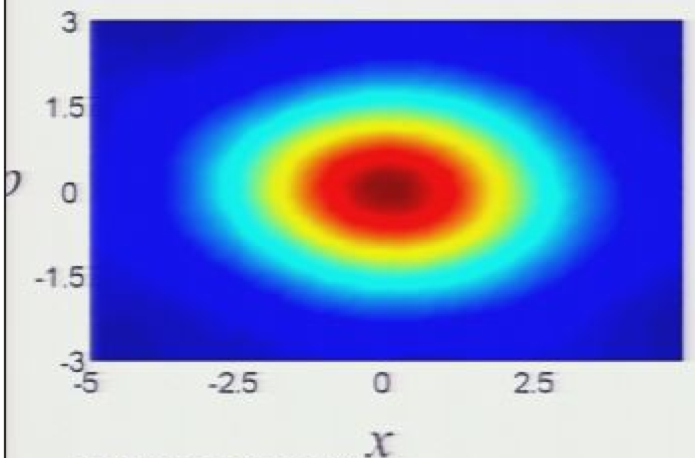

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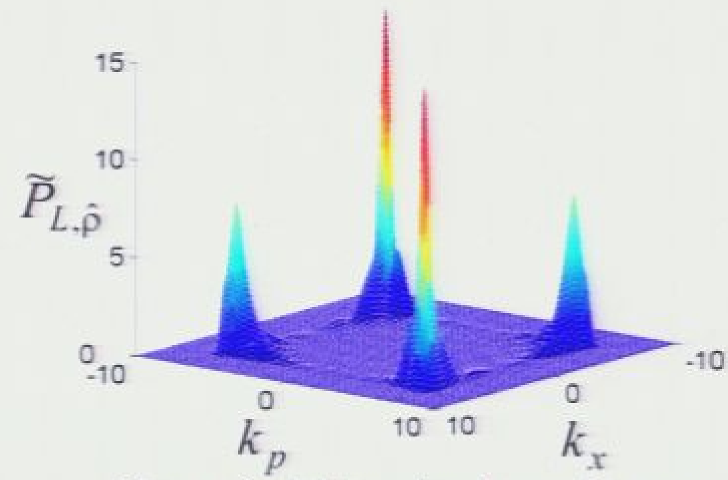

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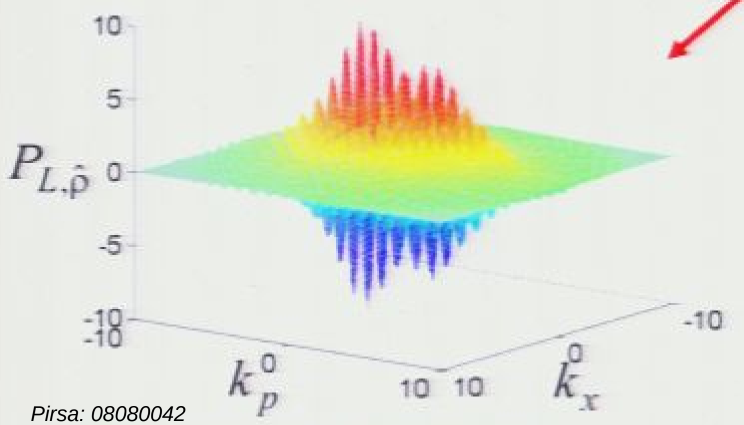

Regularized P-function

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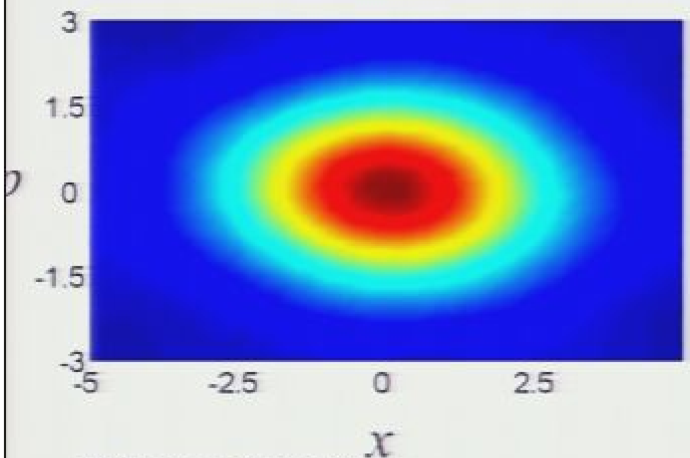
Example: CW squeezed vacuum



Wigner function
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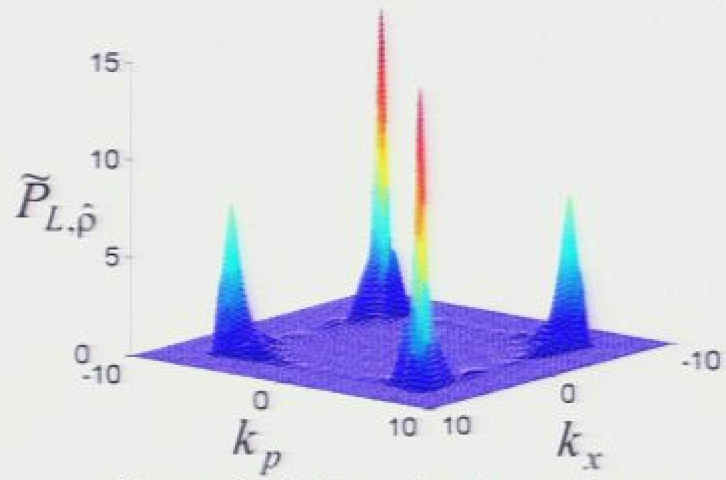

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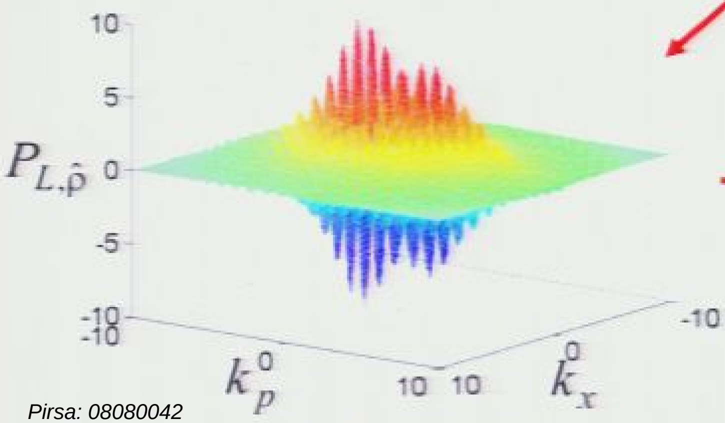

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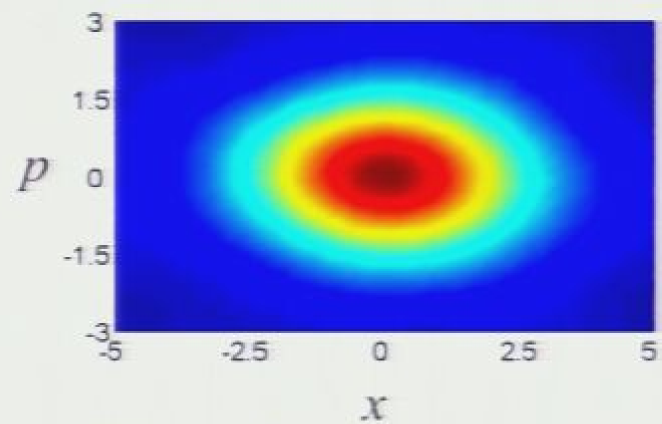

Wigner function
from approximated P-function

Wait a second…

- Does this mean that every nonclassical state of light can be approximated with a classical state??

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Wait a second…

- Does this mean that every nonclassical state of light can be approximated with a classical state??
- No, because for a classical state the P-function is positive definite. Here we don't have this restriction.

Finding the superoperator

- Assume we did the experiment and found $\mathbf{E}(|\alpha\rangle\langle\alpha|)$ for every α.
  Let us find the superoperator in the photon number basis.
- We are interested in the matrix $\mathbf{E}_{lk}^{nm} = \langle l|\,\mathbf{E}(|m\rangle\langle n|)\,|k\rangle$
  So we need to know the process outcome for every "density matrix" $|m\rangle\langle n|$
- Use smoothened P-decomposition!

$$|m\rangle\langle n| = \int_{\substack{\text{phase}\\\text{space}}} P_{L,|m\rangle\langle n|}(\alpha)|\alpha\rangle\langle\alpha|\,d^2\alpha$$

$$\mathbf{E}(|m\rangle\langle n|) = \int_{\substack{\text{phase}\\\text{space}}} P_{L,|m\rangle\langle n|}(\alpha)\mathbf{E}(|\alpha\rangle\langle\alpha|)\,d^2\alpha$$

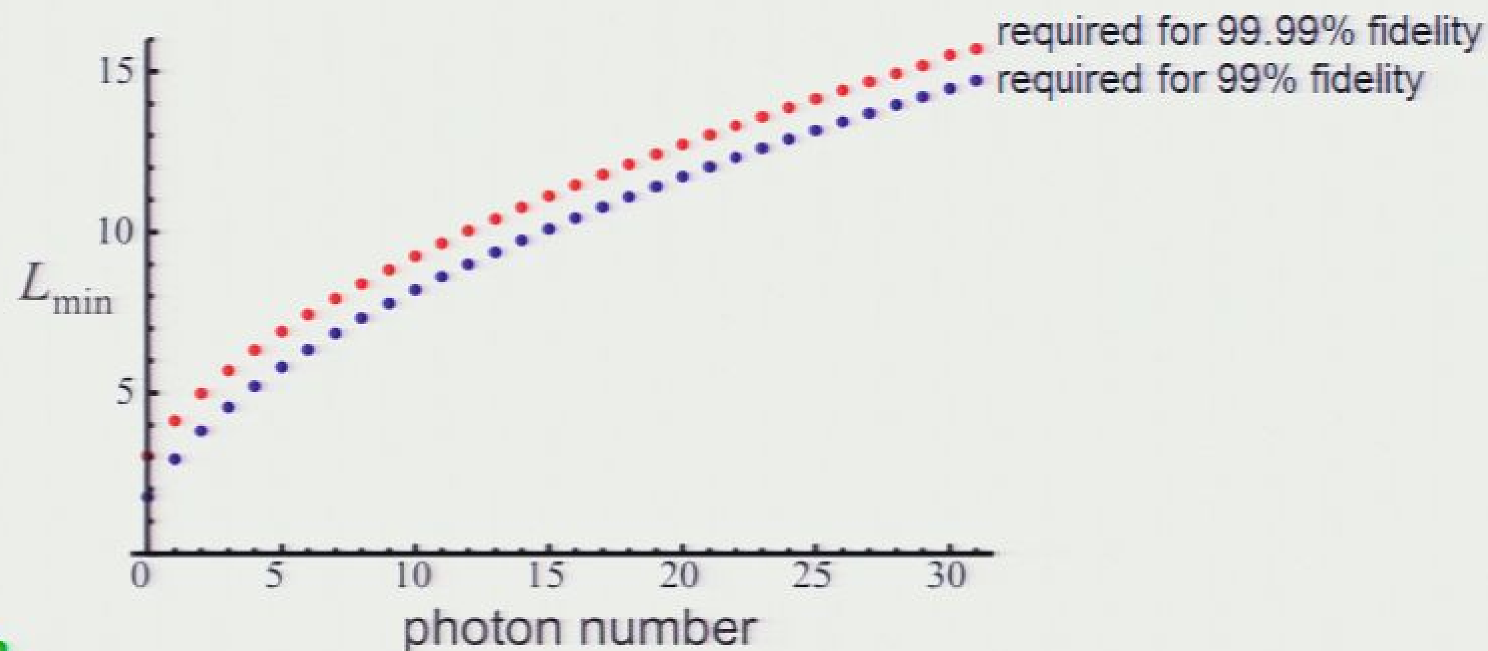
- So the superoperator is

$$\mathbf{E}_{lk}^{nm} = \int_{\substack{\text{phase}\\\text{space}}} P_{L,|nm\rangle\langle nm|}(\alpha)\mathbf{E}_{kl}(|\alpha\rangle\langle\alpha|)\,d^2\alpha$$

- Note: $P_{L,|nm\rangle\langle nm|}(\alpha)$ are process-independent. Can be pre-calculated and tabulated!

The superoperator: practical issues 1

- *L* cannot be infinite
  - The higher *L*, the uglier the approximated P-function
  - A lower *L* will compromise the approximation fidelity
  - Higher photon numbers require higher *L*'s



Solution

- Restrict to a subspace with a certain maximum photon number n_{max}
- State $|n_{max}\rangle\langle n_{max}|$ provides the worst case scenario for the subspace
- Pick the appropriate value of *L*

The superoperator: practical issues 2

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- α **cannot be infinite**
  - Can't send infinitely strong coherent states as inputs
  - Restricting to finite α's will compromise the fidelity

$$\hat{\rho}_{\text{in}} = \int_{\substack{\text{phase}\\\text{space}}} P_{L,\hat{\rho}_{\text{in}}}(\alpha)|\alpha\rangle\langle\alpha|\, d^2\alpha$$

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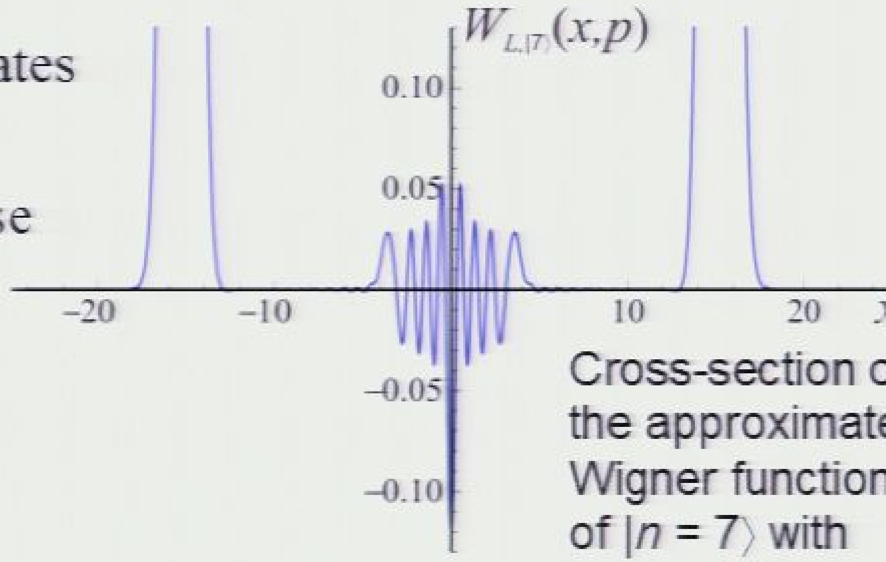

Cross-section of the approximated Wigner function of $|n=7\rangle$ with $\alpha_{\max} = 11$

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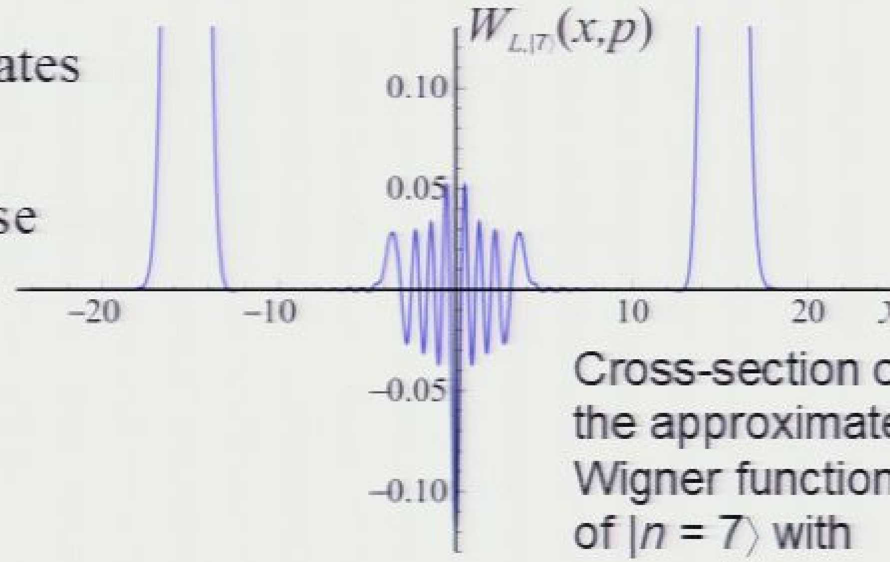

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  - To avoid this, we need to choose a very large α_{max} (much larger than $\sqrt{n_{max}}$)

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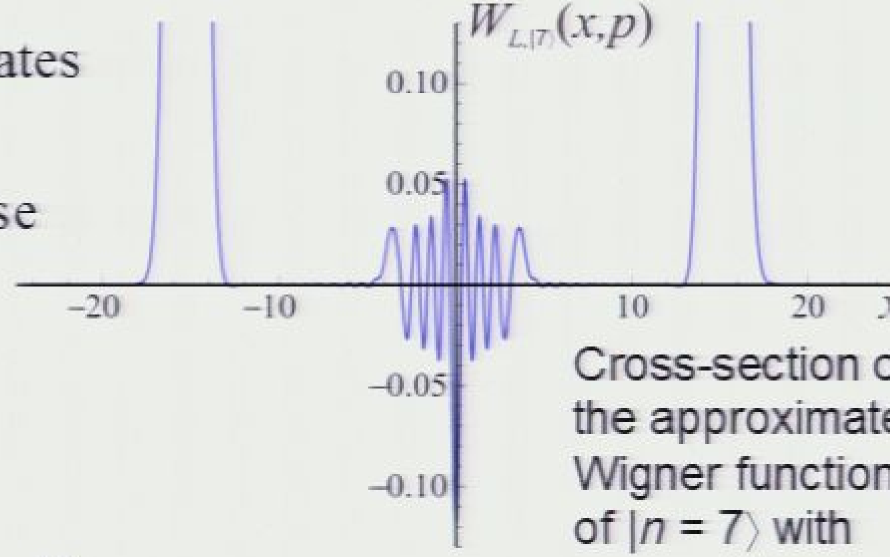

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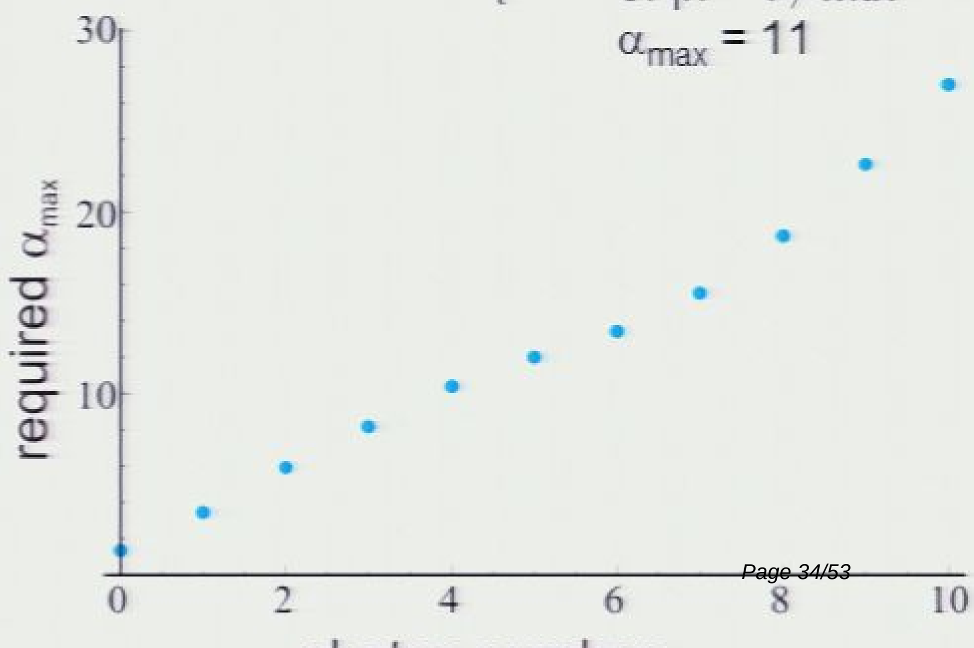

PROCESS TOMOGRAPHY

Experiment

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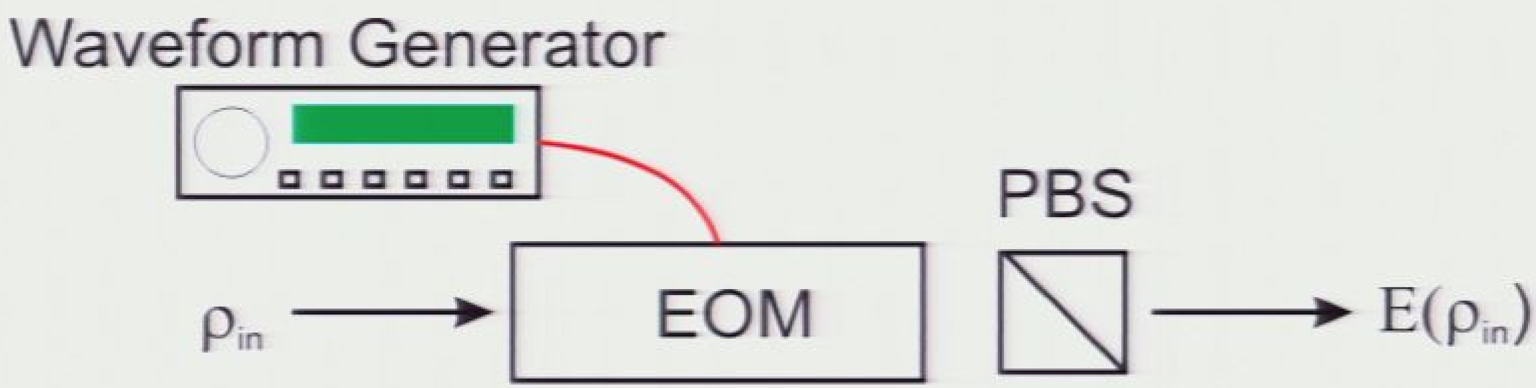
Quantum Process: electro-optical modulation



Process description

- Voltage applied to the EOM
  → Birefringence of crystal
  → Polarization rotation
- Transmission through a polarizer
  → Selection of polarization
- Resulting process: loss + phase shift

Superoperator reconstruction: the procedure

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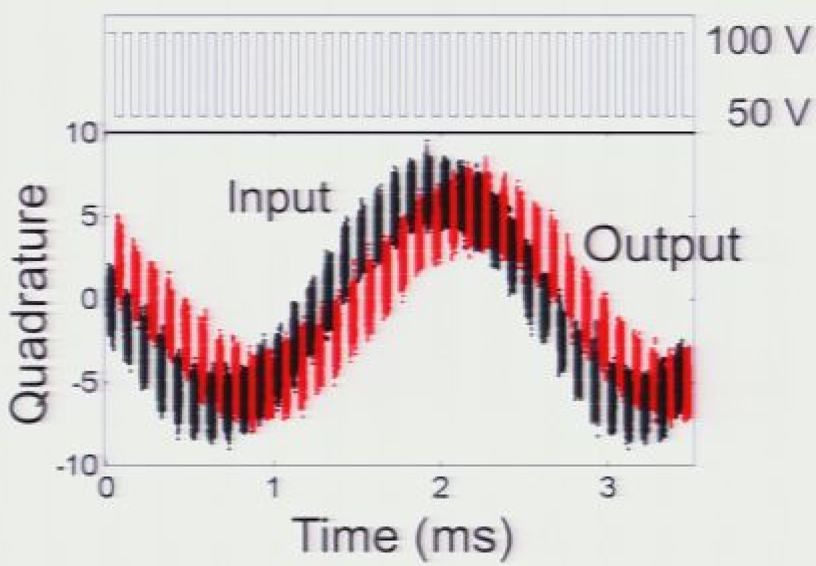
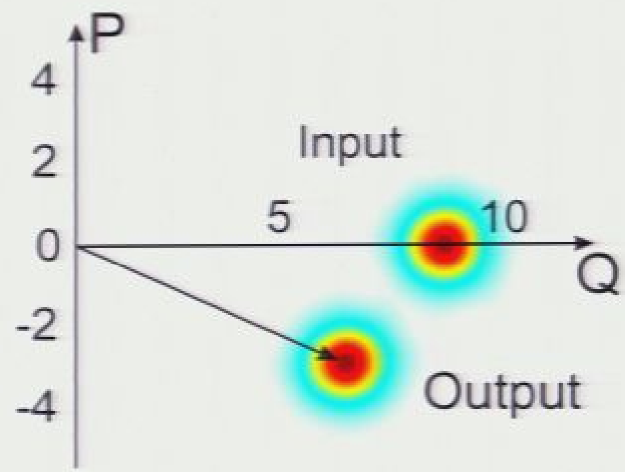
- Input: coherent states up to $\alpha_{max}=11$, 11 different amplitudes
- Output quantum state reconstruction by maximum likelihood
- Process assumed phase invariant
- Interpolation

Voltage waveform



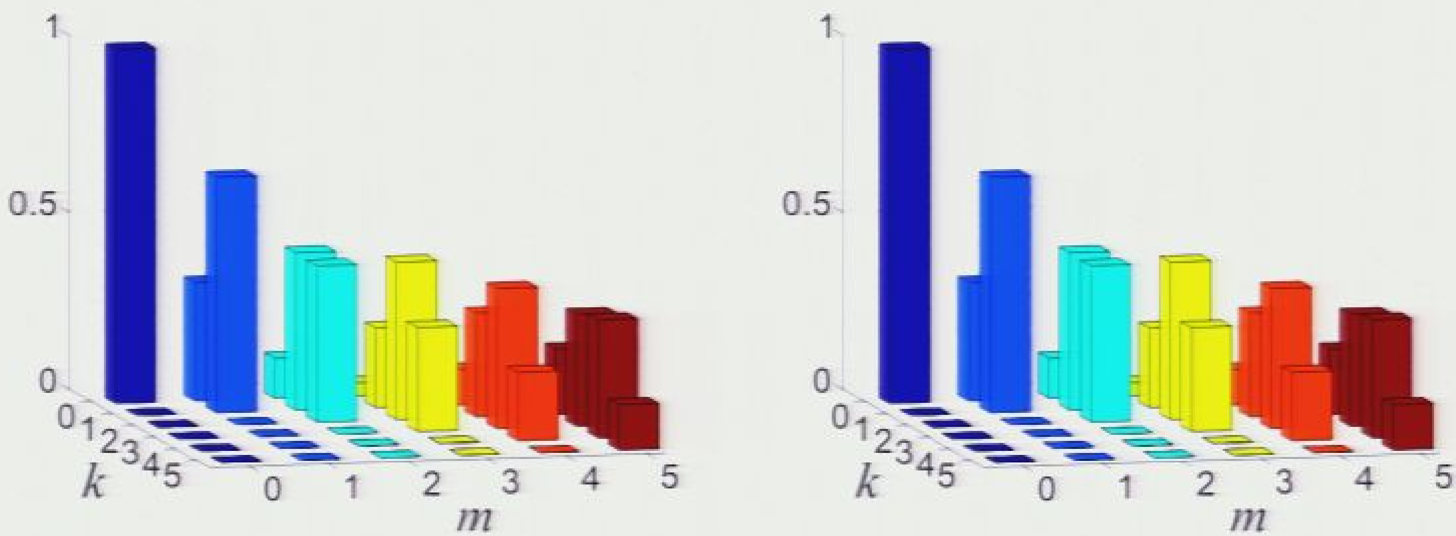
Oscilloscope trace



Wigner representation of coherent state input and output

Superoperator reconstruction: the result

- Shown: diagonal elements $\mathbf{E}_{kk}^{mm}$ of the process superoperator
- Each color: diagonal elements of the output density matrix for input $|m\rangle$

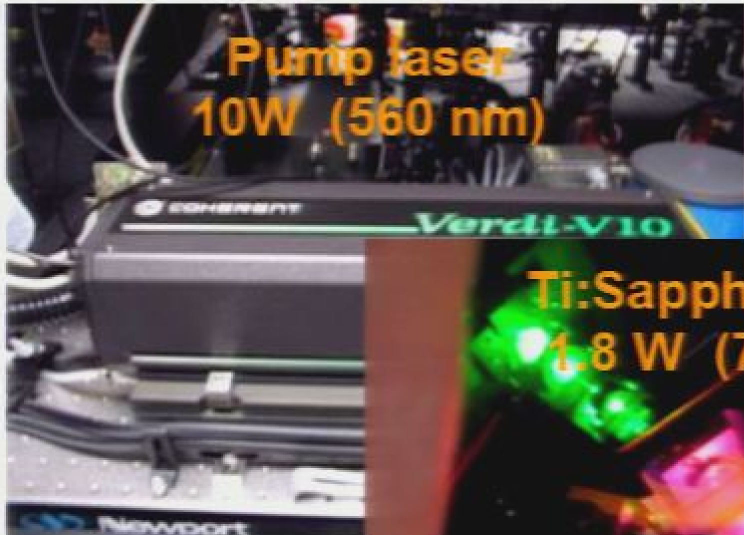


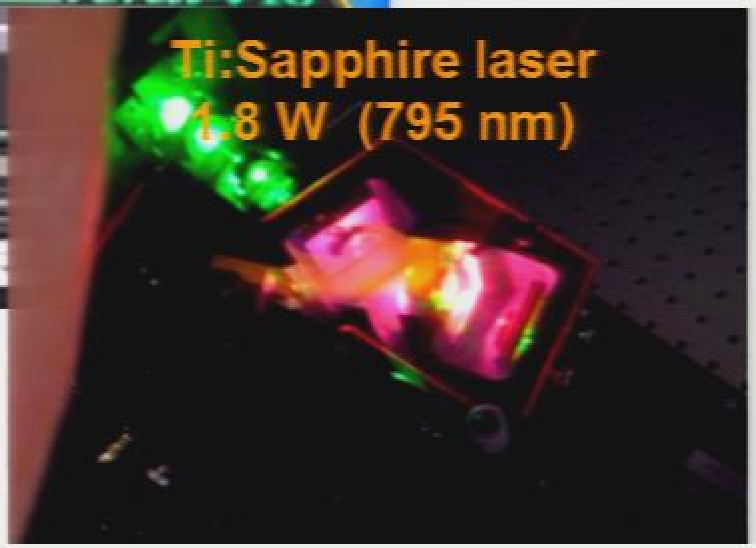
Prediction (35% loss) | Experimental result

- Now let us verify the result for a squeezed state input

Squeezing in our experiment

Squeezing in our experiment





Squeezing in our experiment

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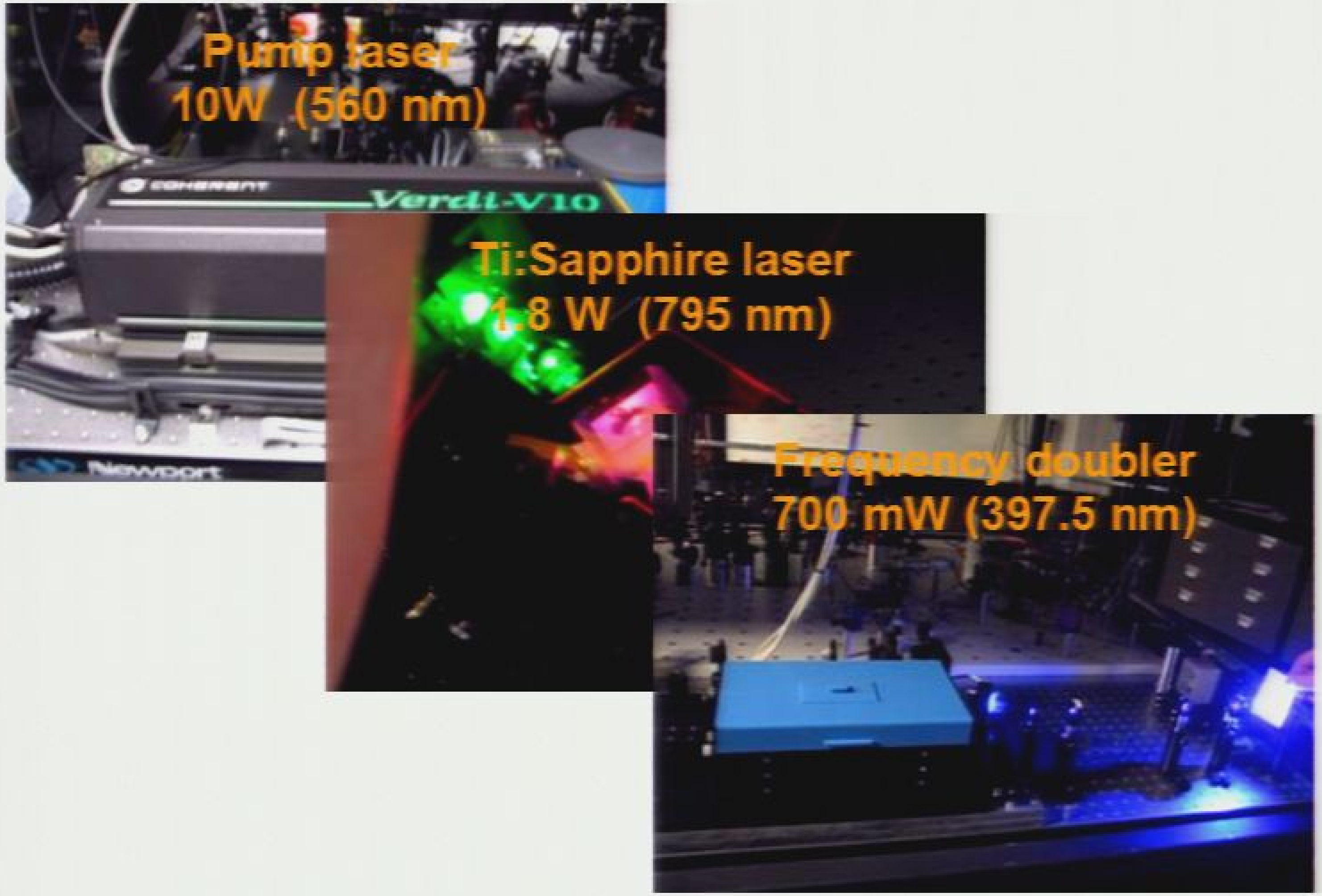
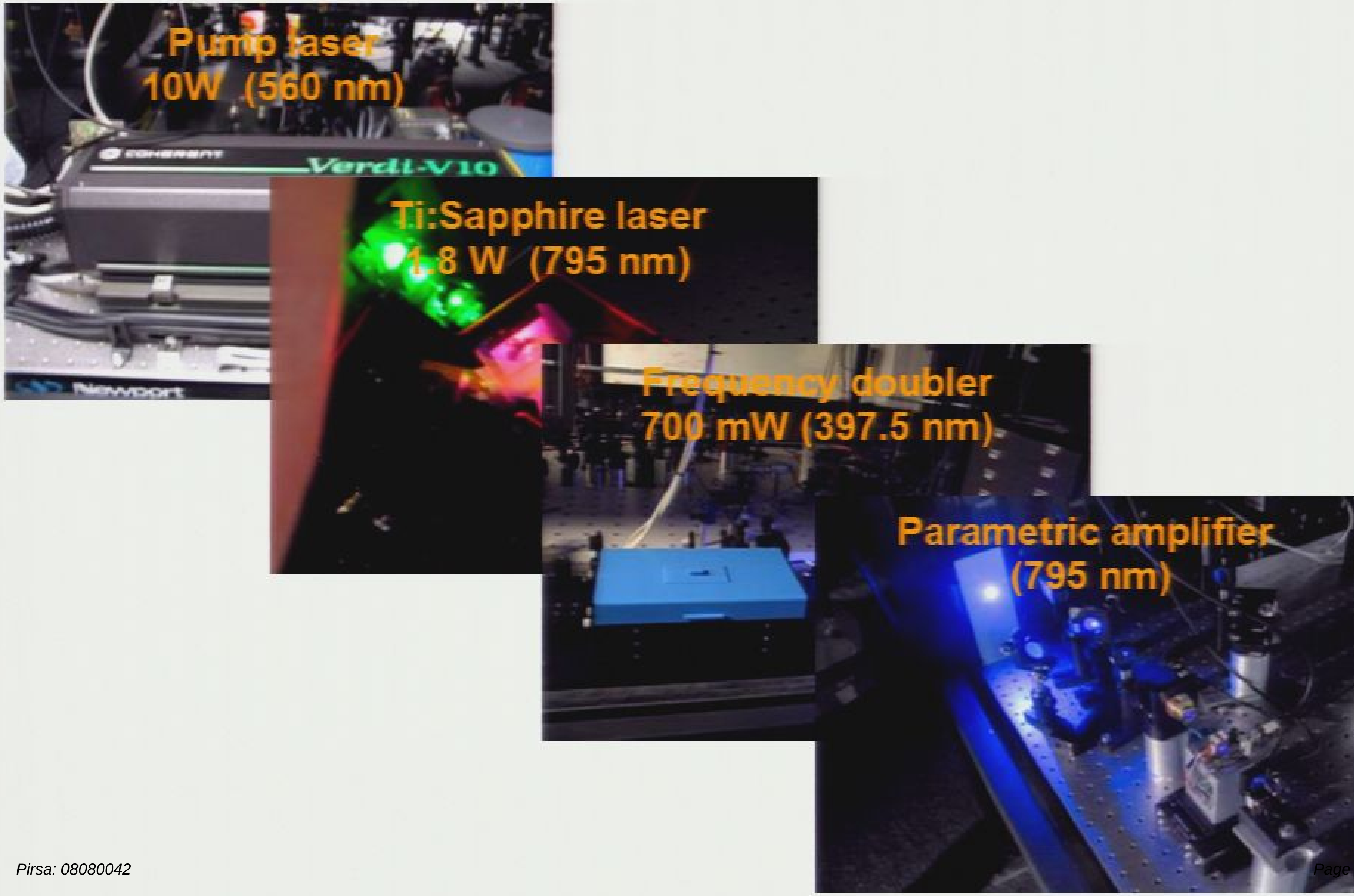


Squeezing in our experiment



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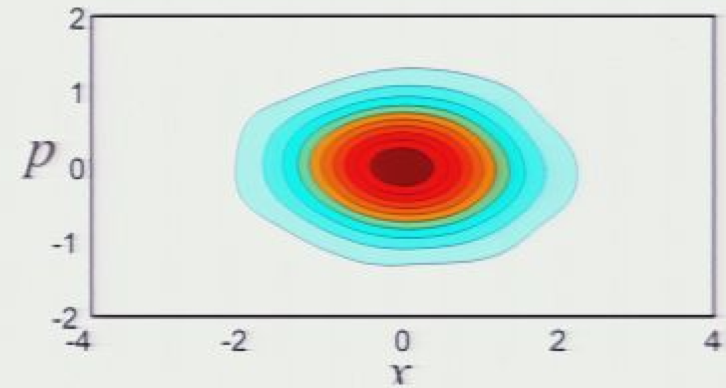
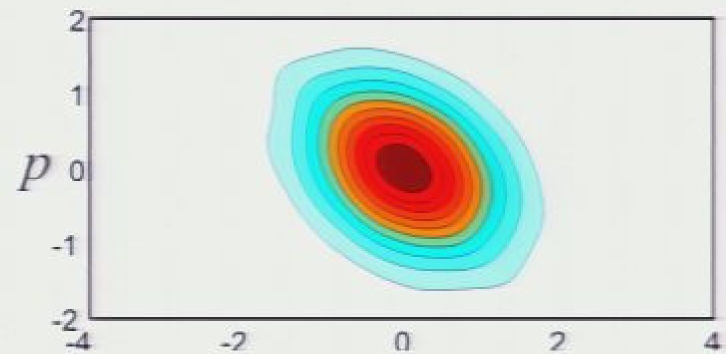
Test with the squeezed state

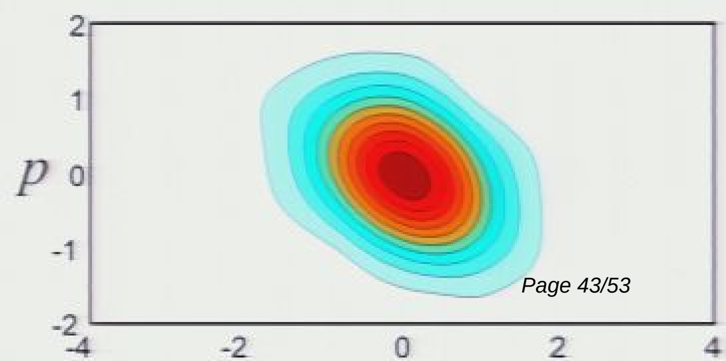
- **Input state**
  - 1.58 dB squeezing
  - 2.91 dB antisqueezing



- **Prediction with calculated superoperator**
  - 1.15 dB squeezing
  - 2.12 dB antisqueezing



- **Measured**
  - 1.07 dB squeezing
  - 2.19 dB antisqueezing



Fidelity = 0.997

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Summary

- **A new technique for characterizing quantum-optical processes**
  - Based on homodyne tomography
  - Works in the continuous-variable domain, but can be applied to discrete variables
  - Does not require any nonclassical input
- **Experiment**
  - Reconstruction of a simple process
  - Superoperator applied to a squeezed state → prediction for output made
  - Prediction verified

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The superoperator: practical issues 1

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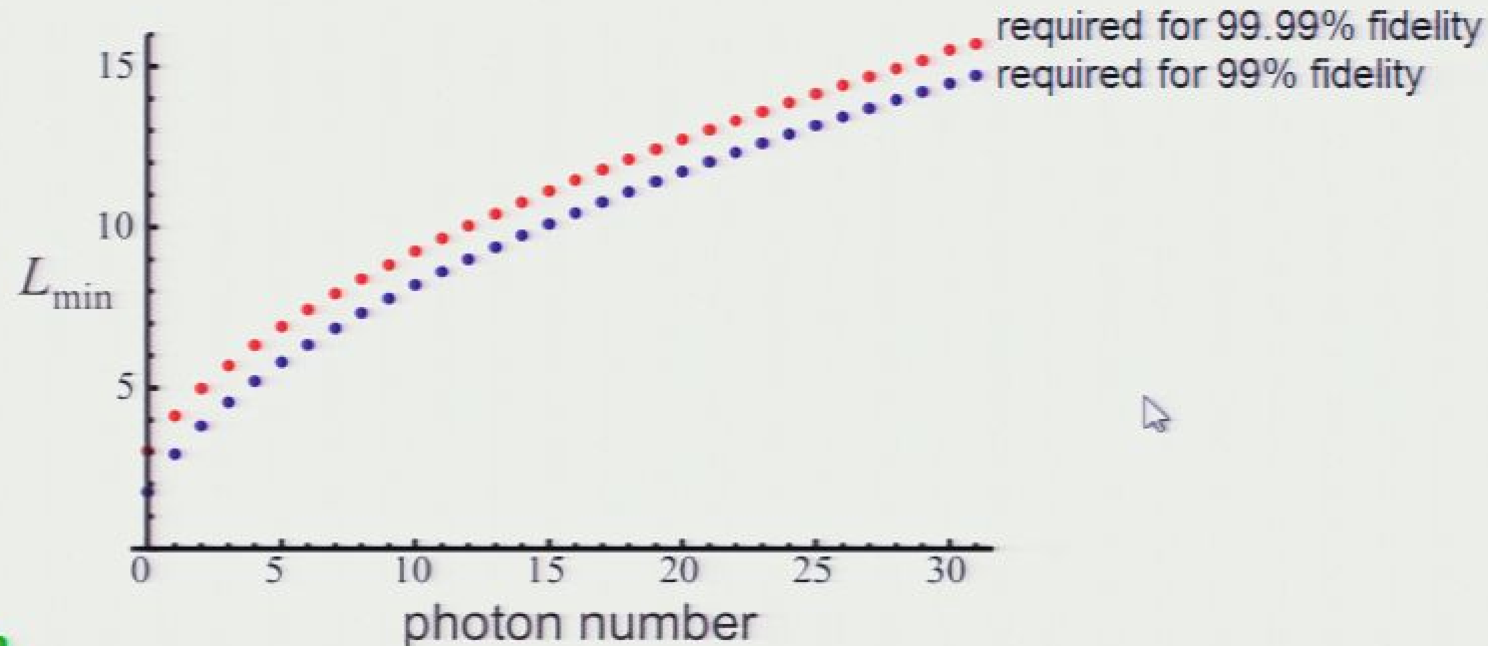
- *L* cannot be infinite
  - The higher *L*, the uglier the approximated P-function
  - A lower *L* will compromise the approximation fidelity
  - Higher photon numbers require higher *L*'s



Solution

- Restrict to a subspace with a certain maximum photon number n_{max}
- State $|n_{max}\rangle\langle n_{max}|$ provides the worst case scenario for the subspace
- Pick the appropriate value of *L*

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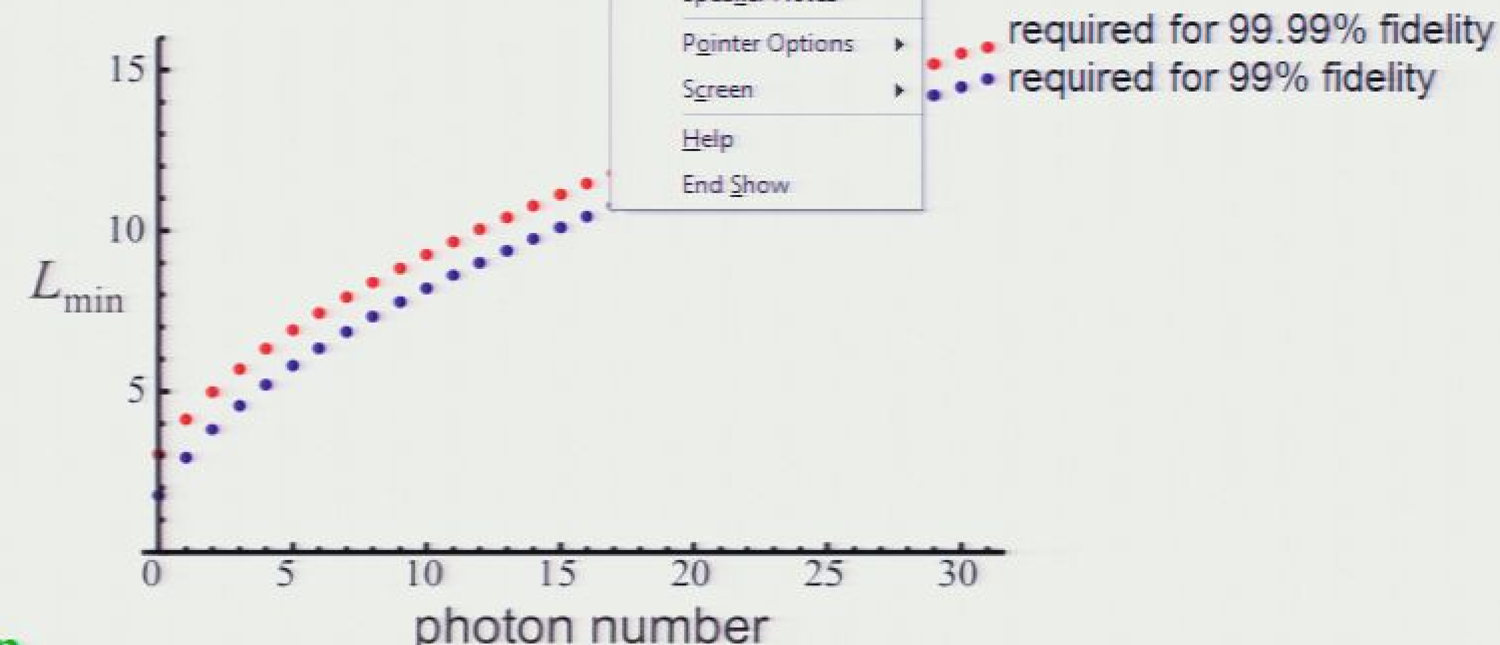

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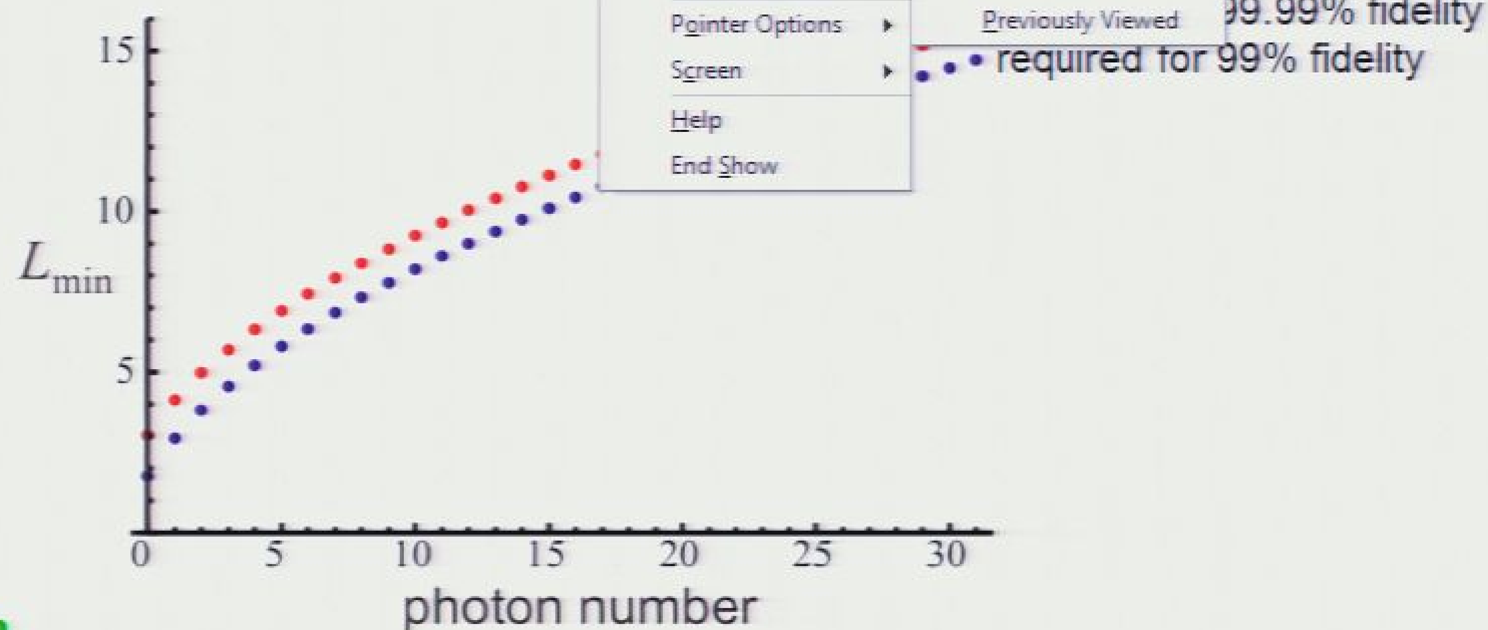

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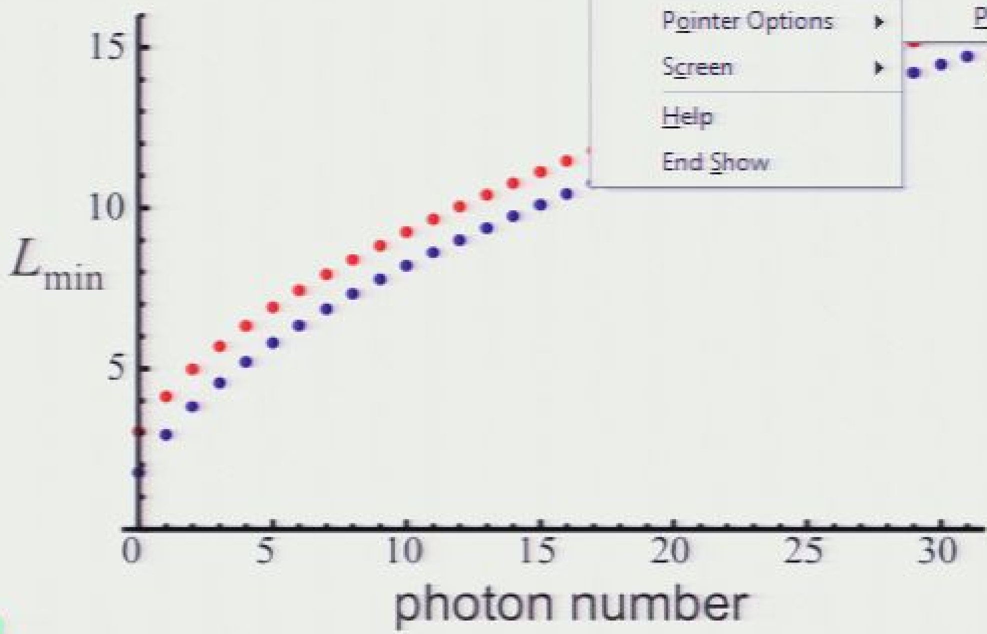

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THE P-FUNCTION

The main idea

- **Decomposition into coherent states**
  - Coherent states form a "basis" in the space of optical density matrices
  - Glauber-Sudarshan P-representation (Nobel Physics Prize 2005)

$$\hat{\rho}_{in} = \int_{\text{phase space}} P_{\hat{\rho}_{in}}(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$$

- **Application to process tomography**
  - Suppose we know the effect of the process $\boldsymbol{E}(|\alpha\rangle\langle\alpha|)$ on each coherent state
  - Then we can predict the effect on any other state

$$\boldsymbol{E}(\hat{\rho}_{in}) = \int_{\text{phase space}} P_{\hat{\rho}_{in}}(\alpha)\boldsymbol{E}(|\alpha\rangle\langle\alpha|)d^2\alpha$$

- **The good news**
  - Coherent states are readily available from a laser.
    No nonclassical light needed
  - **Works in infinite dimensions**