Title: Dynamical Evolution of Superoperator for Hamiltonian Identification and Quantum Dynamical Control

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Abstract: Estimation of quantum Hamiltonian systems is a pivotal challenge to modern quantum physics and especially plays a key role in quantum control. In the last decade, several methods have been developed for complete characterization of a \'superopertor\', which contains all information about a quantum dynamical process. However, it is not fully understood how the estimated elements of the superoperator could lead to a systematic reconstruction of many-body Hamiltonians parameters generating such dynamics. Moreover, it is often desirable to utilize the relevant information obtained from quantum process estimation experiments for optimal control of a quantum device. In this work, we introduce a general approach for monitoring and controlling evolution of open quantum systems. In contrast to the master equations describing time evolution of density operators, here, we develop a dynamical equation for the evolution of the superoperator acting on the system. This equation does not presume any Markovian or perturbative assumptions, hence it provides a broad framework for analysis of arbitrary quantum dynamics. As a result, we demonstrate that one can efficiently estimate certain classes of Hamiltonians via application of particular quantum process tomography schemes. We also show that, by appropriate modification in the data analysis techniques, the parameter estimation procedures can be implemented with calibrated faulty state generators and measurement devices. Furthermore, we propose an optimal control theoretic approach for manipulating quantum dynamics of Hamiltonian systems, specifically for the task of decoherence suppression.

Motivation

➢ The tasks of **characterizing** and **controlling** quantum Hamiltonian systems are fundamentally **complementary**

(1) Efficient characterization of the relevant properties of the system

How to control certain dynamical parameters of the system

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(2) [characterization observables, controllable observables] $\neq 0$

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(2) [characterization observables, controllable observables] $\neq 0$

□ Control

■ Characterization

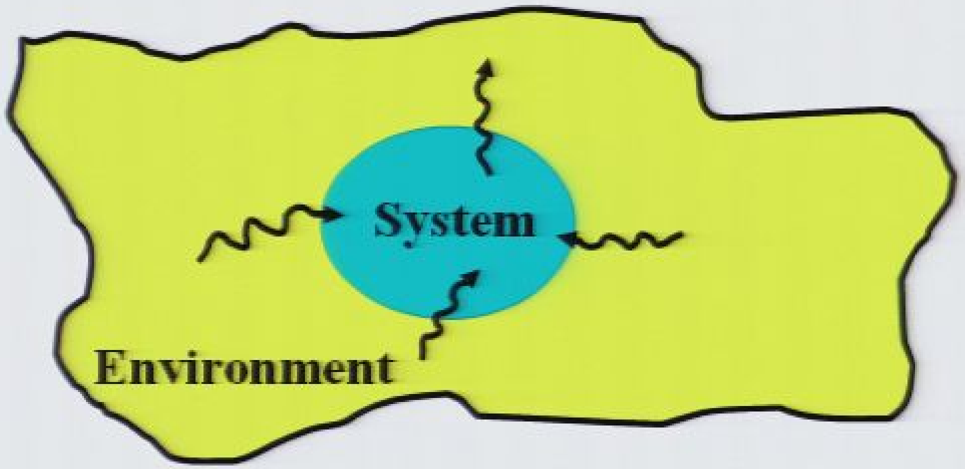
➢ Quantum process tomography is the general strategy to estimate the parameters of a "superoperator" or "process matrix", which contains all information about the dynamics.

➢ How the relevant information obtained from process estimation experiments can be utilized for optimal control of a quantum device?

➢ How to estimate Hamiltonians parameters via quantum process tomography?

Motivation

Equation of motion for open quantum systems



	State	Dynamics
	$\lvert\psi\rangle, \rho$	U, χ
Closed systems	$id\lvert\psi\rangle/dt = H\lvert\psi\rangle$	$idU/dt = HU$
Open systems	$id\rho/dt = [H, \rho] + \mathcal{L}(\rho)$	

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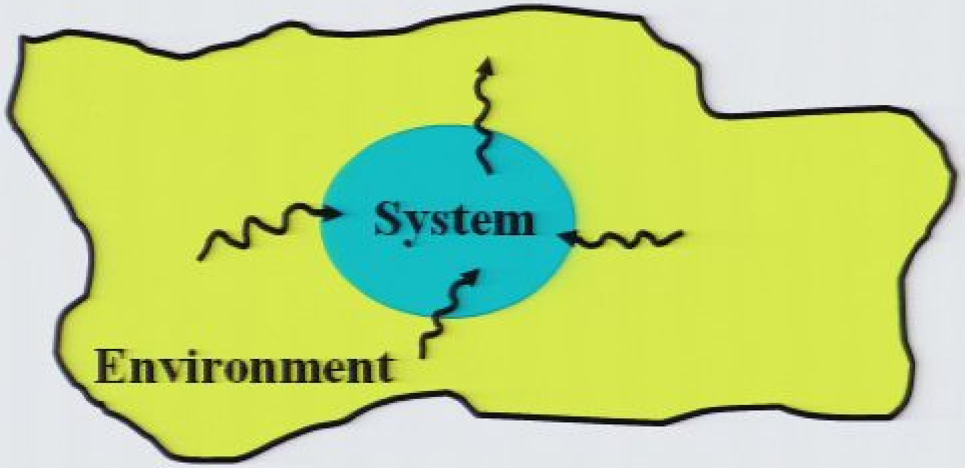

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Heisenberg picture for open quantum systems!

Is there a dynamical equation for the process matrix?

Does it lead us to new ways for monitoring and/or controlling open quantum systems?

Outline

Part I

- Equation of motion for quantum processes
- Using quantum process tomography for Hamiltonian identification
- Efficient estimation of sparse Hamiltonians in short-time behavior
- Dynamical control of quantum Hamiltonian systems: decoherence suppression

Part II

- Direct characterization of process matrix
- Efficient estimation of error probabilities
- Process estimation with faulty preparations and measurements

Quantum Dynamical Maps

$$\varepsilon(\rho) = \sum_i A_i \rho A_i^\dagger$$

$$A_i = \sum_{m=0}^{d^2-1} e_{im} E_m$$

$$\longrightarrow \quad \varepsilon(\rho) = \sum_{m,n=0}^{d^2-1} \chi_{mn} E_m \rho E_n^\dagger$$

χ is positive-Hermitian matrix and $Tr\chi \leq 1$

For a single qubit: $E_m \in \{I, \sigma_x, \sigma_y, \sigma_z\}$

Superoperator, χ, has
4x4 = 16 elements

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{01}^* & \rho_{11} \end{pmatrix} \longrightarrow \begin{pmatrix} \rho'_{00} & \rho'_{01} \\ \rho'^*_{01} & \rho'_{11} \end{pmatrix} = \varepsilon(\rho)$$

Coherence

Population

Quantum dynamical probabilities

$$\chi = \begin{pmatrix} \chi_{00} & \chi_{01} & \chi_{02} & \chi_{03} \\ \chi_{01}^* & \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{02}^* & \chi_{12}^* & \chi_{22} & \chi_{23} \\ \chi_{03}^* & \chi_{13}^* & \chi_{23}^* & \chi_{33} \end{pmatrix}$$

Quantum dynamical coherence

Quantum Process Tomography

Standard Quantum Process Tomography

I.L. Chuang and M.A. Nielsen, J. Mod. Opt. (97); J. F. Poyatos, J. I. Cirac, and P. Zoller, PRL (97)

Ancilla-assisted Process Tomography

D'Ariano, G. M. & Lo Presti, P. PRL.(01); D. W. Leung, PhD thesis (Stanford University, 2000) and J. Math. Phys. (03).

Direct Characterization of Quantum Dynamics

M. Mohseni and D. A. Lidar, PRL. (06).

Symmetrized Characterization of Noisy Quantum Processes

J. Emerson, et. Al., Science (07)

Selective and Efficient Quantum Process Tomography

A. Bendersky, F. Patawski, and J. P. Paz, PRL (08).

What can we do with them?

Dynamical Evolution of the Process Matrix: Closed Systems

$$idU/dt = HU$$

$$U(t) = \sum_m a_m(t) E_m$$

$$H(t) = \sum_k h_k(t) E_k$$

$$E_k E_l = \sum_m \alpha^{kl}{}_m E_m$$

$$\chi_{mn}(t) = a_m(t) \bar{a}_n(t)$$

$$[\widetilde{\boldsymbol{H}}]_{mn} = \sum_k \alpha^{kn}{}_m h_k(t)$$

M. M. and A. T. Rezakhani, arXiv:0805.3188.

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$$id\chi_{mn}/dt = \sum_l \tilde{h}_{ml}\chi_{ln}(t) - \chi_{ml}(t)\tilde{h}^*_{nl}$$

$$id\chi/dt = \widetilde{\boldsymbol{H}}\chi - \chi\widetilde{\boldsymbol{H}}^\dagger$$

$$id\chi/dt = [\widetilde{H}, \chi]^*$$

The dynamical equation for process matrix

A generalized commutator notation:

$$[A,B]^* = AB - B^\dagger A^\dagger$$

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Hamiltonian Identification

knowing $\chi(t)$ & $\chi(t+\delta t)$ via tomography $\Longrightarrow$ $H(t)$ (*Hamiltonian identification*)

(ensemble measurement)

M. M. and A. T. Rezakhani, arXiv:0805.3188.

Efficient estimation of sparse Hamiltonians in short time-scale

hort time limit: $\mathcal{O}(t^3)$

$$\chi \approx$$

$$\begin{pmatrix} 1-\frac{1}{2}t^2\sum_{ij\neq 0}\left(a_0^{ij}h_ih_j+\bar{a}_0^{ij}\bar{h}_i\bar{h}_j\right) & it\bar{h}_1-\frac{1}{2}t^2\sum_{ij\neq 0}\bar{a}_1^{ij}\bar{h}_i\bar{h}_j & \cdots & it\bar{h}_{d^2-1}-\frac{1}{2}t^2\sum_{ij\neq 0}\bar{a}_{d^2-1}^{ij}\bar{h}_i\bar{h}_j \\ -ith_1-\frac{1}{2}t^2\sum_{ij\neq 0}a_1^{ij}h_ih_j & & & \\ \vdots & & \chi_{mn}=t^2 h_m\bar{h}_n & \\ -ith_{d^2-1}-\frac{1}{2}t^2\sum_{ij\neq 0}a_{d^2-1}^{ij}h_ih_j & & & \end{pmatrix}$$

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if $H(t)$ is *sparse*, the number of the nonzero elements of the red block is *polylog(d)*

Generic *N*-**body** and *L-local* quantum systems (e.g., *N*-qubits, 2-local controllable Hamiltonian systems for quantum information processing):

$$H(t)=\sum_k h_\sigma \sigma \;\rightarrow\; \#(N,L)=\sum_{l=0}^{L}\binom{N}{l}3^l \sim \mathcal{O}(N^L)\sim \mathrm{polylog}(d)$$

E.g., exchange interaction:

$$H=-\sum_{i=1}^{N-1} J_X^{(i)}X^{(i)}X^{(i+1)}+J_Y^{(i)}Y^{(i)}Y^{(i+1)}+J_Z^{(i)}Z^{(i)}Z^{(i+1)}$$

$\Rightarrow$ any *direct* or *selective* process tomography schemes can *efficiently* identify $H(t)$
(Caveat: we should know *which* h_m coefficients are nonzero)

SQPT ✗
DCQD ✓
SEQPT ✓

Dynamical Evolution of Process Matrix for Open Quantum Systems

$$H(t) = H_S(t) + H_B(t) + H_{SB}(t)$$

$$H_{SB}(t) = \sum_k \lambda_k(t) E_k \otimes B_k$$

(system-bath coupling)

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In the *interaction picture*:

Known from the system free Hamiltonian of system

A property of operator basis

$$\begin{cases} [\widetilde{\mathbf{H}}]_{n\,\underbrace{imj}} := \sum_{pq} \lambda_p e_{qp} \alpha^{qp}{}_n \; {}_B\langle b_j | \widetilde{B}_m | b_i \rangle_B \\ [\mathbf{K}]_{\underbrace{imj}\,n} := a_{im} \bar{a}_{jn} \end{cases}$$

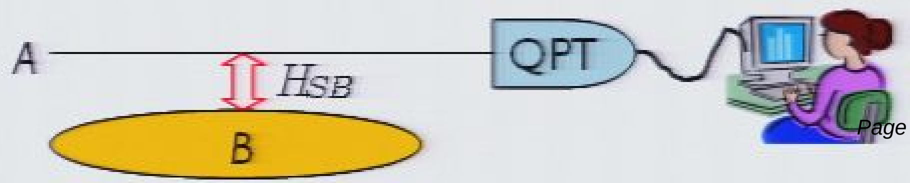
$$i d\chi / dt = \widetilde{\mathbf{H}}\mathbf{K} - \mathbf{K}^\dagger \widetilde{\mathbf{H}}^\dagger$$

The evolution does not depend on state of the system

$$i d\chi / dt = [\widetilde{H}, K]^*$$

Dynamical equation for open quantum systems, applicable to non-Markovian and strong-coupling regimes

Identification of system-bath Hamiltonian



Optimal Quantum Control

main idea of OCT for *states*:

$|\psi_i\rangle \rightsquigarrow |\psi_f\rangle$ + the equation of motion : *control field?* (e.g., a shaped laser pulse $\epsilon(t)$)

control field *maximizing/minimizing* a *yield/cost function* (e.g., $Y = |\langle\psi_i|U^\dagger(0,T;\epsilon)|\psi_f\rangle|^2$)

optimization by variational calculus $(\widetilde{Y}[\epsilon] \equiv Y[0,T,\epsilon] + \int_0^T C(\psi,\epsilon)dt \longrightarrow \frac{\delta\widetilde{Y}}{\delta\epsilon} = 0)$
(constraint)

solve by Krotov/Rabitz/... methods

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⟶ generalization to unitary *dynamics* [Palao & Kosloff, PRL (02)]

$$H(t) = H_0 - \mu\epsilon \;\rightarrow\; idU(t)/dt = H(t)U(t)$$

(free) (control)

Goal: find optimal $\epsilon(t)$ such that a *given* unitary dynamics U^* is achieved in time T

$$U(T) = U^* \quad Y = \mathrm{Re}(\mathrm{Tr}[U^{*\dagger}U(T)])$$

$$\widetilde{Y} = \mathrm{Re}\left(Y - \int_0^T \mathrm{Tr}[(dU(t)/dt + iH(t)U(t))\boldsymbol{B}(t)]dt\right) - \eta\int_0^T |\epsilon(t)|^2/s(t)dt$$

Decoherence control

Ideally, we wish to have no decoherence: $\widetilde{H}(t)=0\ \ ,\chi^{I}_{mn}=\delta_{m0}\delta_{n0}=[E_{00}]_{mn}$

Natural system-bath Hamiltonian

External control field

Effective decoherence

$$\widetilde{H}(t)=\widetilde{H}_0-\mu\pi(t)$$

The yield function is the fidelity: $Y=\mathrm{Re}[Tr[\chi^{I\dagger}(T)E_{00}]]$

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The control strategy is to find the optimal $\pi(t)$ such that the constrained fidelity become minimal

$$\widetilde{Y}=\mathrm{Re}[Y-\int_0^T dtTr\{d\chi^I/dt+i[\widetilde{H}(t),K(t)]^*)\Lambda(t)\}]-\eta\int_0^T dt|\pi(t)|^2/s(t)$$

We set $\delta\widetilde{Y}=0$ and vary Lagrange multipliers to get:

quantum process tomography

$$\pi(t)=-\frac{f(t)}{2\eta}\mathrm{Im}[Tr([\mu,K(t)]^*\Lambda(t)]]$$

$$-i[K\frac{d\Lambda}{dt}]_{imim}=\sum_{njl}\Lambda_{ln}\widetilde{H}_{nimj}K_{imjn}-\Lambda_{nm}\widetilde{H}_{mjli}\overline{K}_{jlim}$$

Part II

Direct Characterization of the Process Matrix

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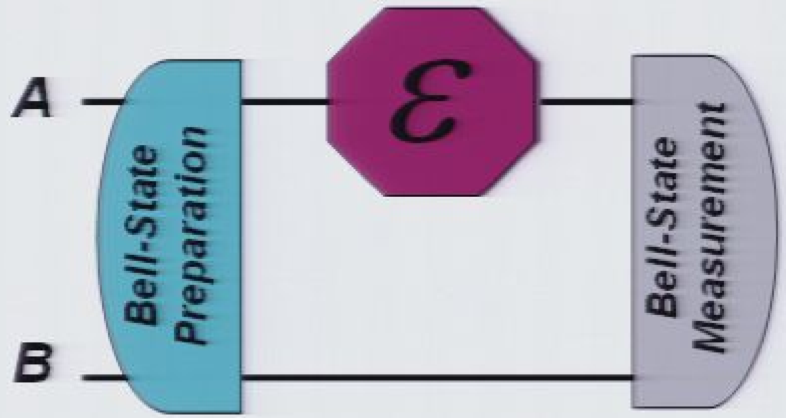

Input state	Measurement: Stabilizer	Measurement: Normalizer	Output
$(\|0\rangle\|0\rangle+\|1\rangle\|1\rangle)/\sqrt{2}$	ZZ, XX	N/A	χ_{00}, χ_{11}, χ_{22}, χ_{33}
$\alpha\|0\rangle\|0\rangle+\beta\|1\rangle\|1\rangle$	ZZ	XX	χ_{03}, χ_{12}
$\alpha\|+\rangle_x\|+\rangle_x+\beta\|-\rangle_x\|-\rangle_x$	XX	ZZ	χ_{01}, χ_{23}
$\alpha\|+\rangle_y\|+\rangle_y+\beta\|-\rangle_y\|-\rangle_y$	YY	ZZ	χ_{02}, χ_{13}

$$|\alpha| \neq |\beta| \neq 0; \mathrm{Im}(\alpha^* \beta)$$

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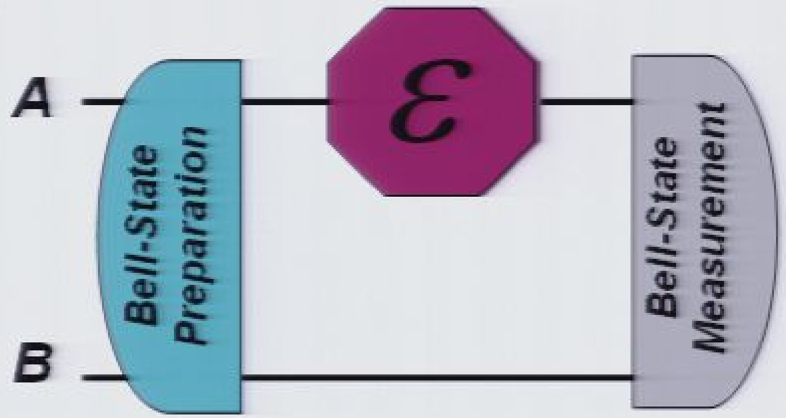

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$$|\alpha| \neq |\beta| \neq 0; \mathrm{Im}(\alpha^* \beta)$$

Four independent real parameters of superoperator are determined in each measurement

Quantum Error Detection

The information is encoded in a larger Hilbert space (with some redundancy) such that any arbitrary error that may occur on the data can be unambiguously detected.

Stabilizer code:

It is a subspace of the Hilbert space of n qubits, V_C , that has eigenvalue +1 under the action of a given Abelian subgroup of the n-qubit Pauli group.

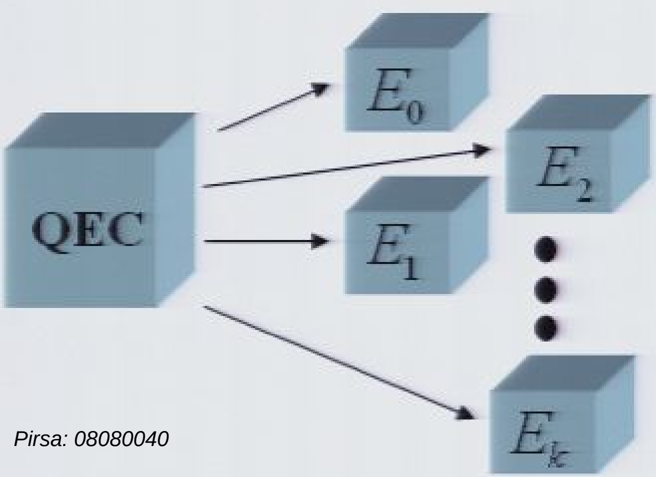
$$S_i|\psi_C\rangle = |\psi_C\rangle \; ; \; |\psi_C\rangle \in V_C \quad ; \; S_i \in S \subset G_n$$

Error detection:

$$S_i|\psi_C\rangle = |\psi_C\rangle \quad ; \quad \{S_i, E\} = 0$$

$$S_i(E|\psi_C\rangle) = -ES_i|\psi_C\rangle = -(E|\psi_C\rangle)$$

Non-degenerate codes:



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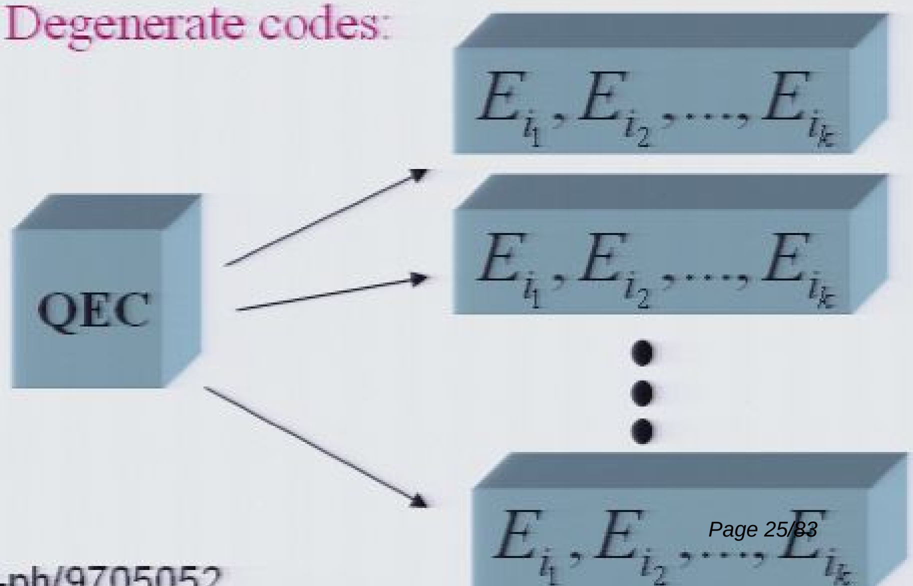

D. Gottesman, quant-ph/9705052

Characterization of Quantum Dynamical Population

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3- Perform the Bell-state measurement $P_k \in \{P_0, P_1, P_2, P_3\}$ as:

$$\varepsilon(\rho) \longrightarrow P_k \varepsilon(\rho) P_k$$

$$P_0 = |\phi^+\rangle\langle\phi^+| \quad P_2 = |\psi^-\rangle\langle\psi^-|$$

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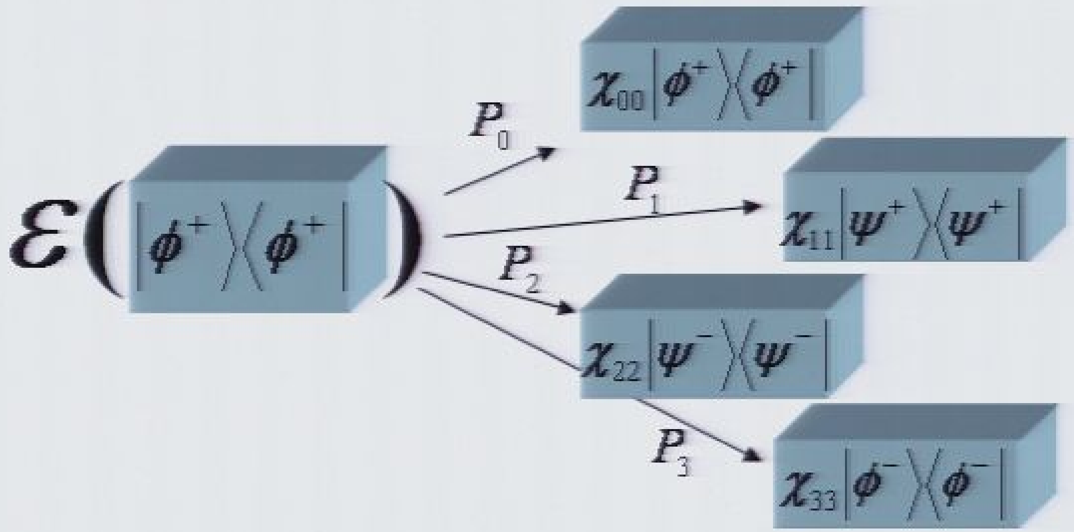

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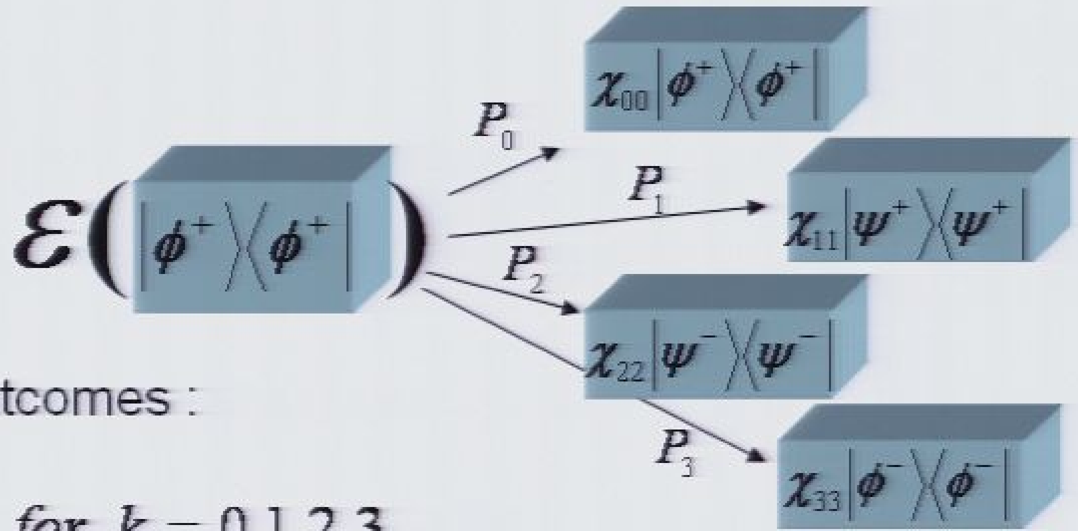

4-Calculate the probabilities of these outcomes :

$$Tr[P_k \varepsilon(\rho)] = \chi_{kk} \quad for\ k = 0,1,2,3$$

Outputs: $\chi_{00}, \chi_{11}, \chi_{22}, \chi_{33}$

We obtain four independent parameters of superoperator in a single measurement

Characterization of Quantum Dynamical Coherence

1- Prepare the Input state: $|\psi_C\rangle = \alpha|0\rangle_A|0\rangle_B + \beta|1\rangle_A|1\rangle_B$ $\quad |\alpha| \neq |\beta| \neq 0$

Sole stabilizer generator: $\sigma_z^A \sigma_z^B |\psi_C\rangle = |\psi_C\rangle$ ⟵ Degenerate stabilizer code

$(I^A, \sigma_z^A);(\sigma_x^A, \sigma_y^A)$

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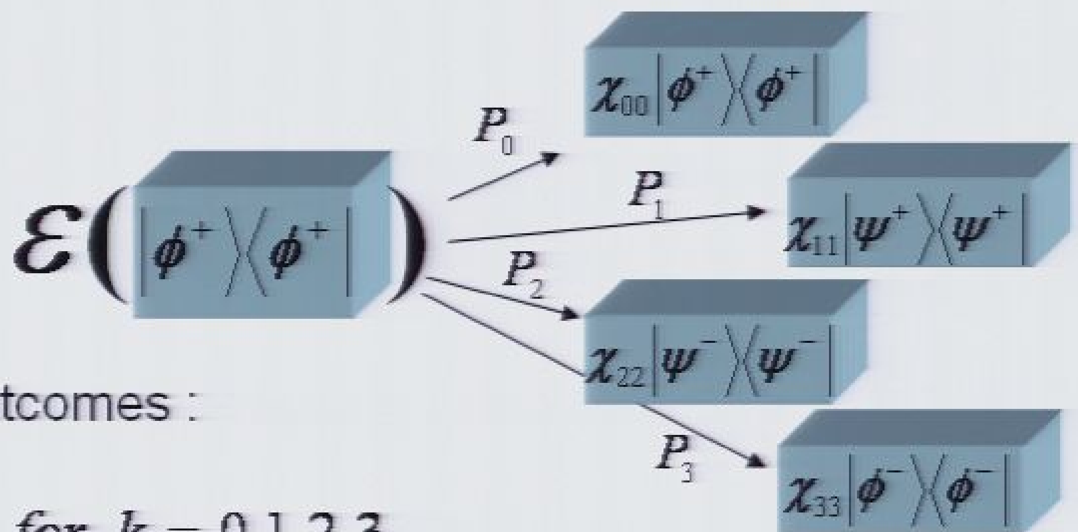

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Sole stabilizer generator: $\sigma_z^A\sigma_z^B|\psi_C\rangle = |\psi_C\rangle$ ⟵ Degenerate stabilizer code

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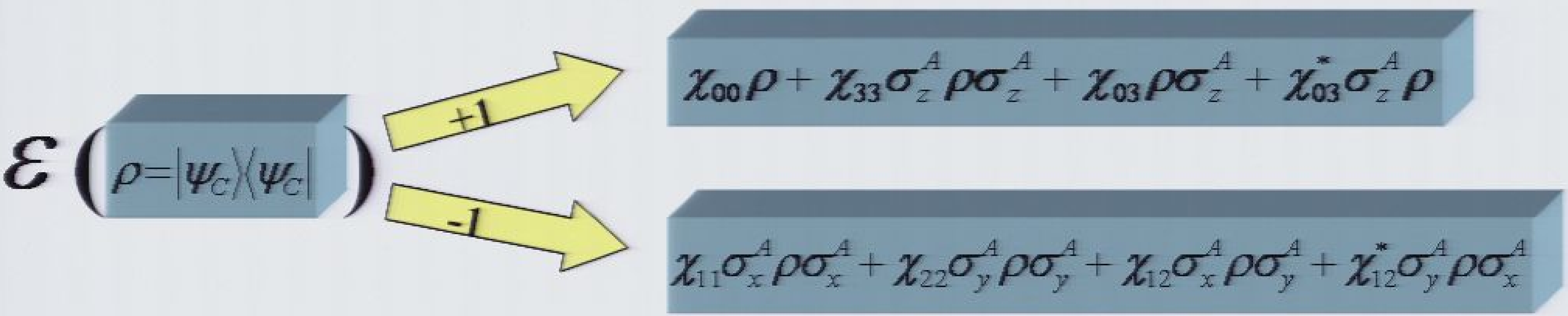

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$\mathcal{E}\left(\rho = |\psi_C\rangle\langle\psi_C|\right)$

+1: $\chi_{00}\rho + \chi_{33}\sigma_z^A \rho \sigma_z^A + \chi_{03}\rho\sigma_z^A + \chi_{03}^* \sigma_z^A \rho$

-1: $\chi_{11}\sigma_x^A \rho \sigma_x^A + \chi_{22}\sigma_y^A \rho \sigma_y^A + \chi_{12}\sigma_x^A \rho \sigma_y^A + \chi_{12}^* \sigma_y^A \rho \sigma_x^A$

4-Calculate the probability of these outcomes:

$$Tr[P_+ \varepsilon(\rho)] = \chi_{00} + \chi_{33} + 2\mathrm{Re}(\chi_{03}) Tr(\sigma_z \rho)$$

$$Tr[P_- \varepsilon(\rho)] = \chi_{11} + \chi_{22} + 2\,\mathrm{Im}(\chi_{12}) Tr(\sigma_z \rho)$$

5- Calculate the expectation values of a normalizer N (e.g., $\sigma_x^A \sigma_x^B$):

$$Tr[NP_+\varepsilon(\rho)] \rightarrow \mathrm{Im}(\chi_{03})$$
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M. M and D. A. Lidar, *Phys. Rev. Lett.* 97, 170501 (2006).

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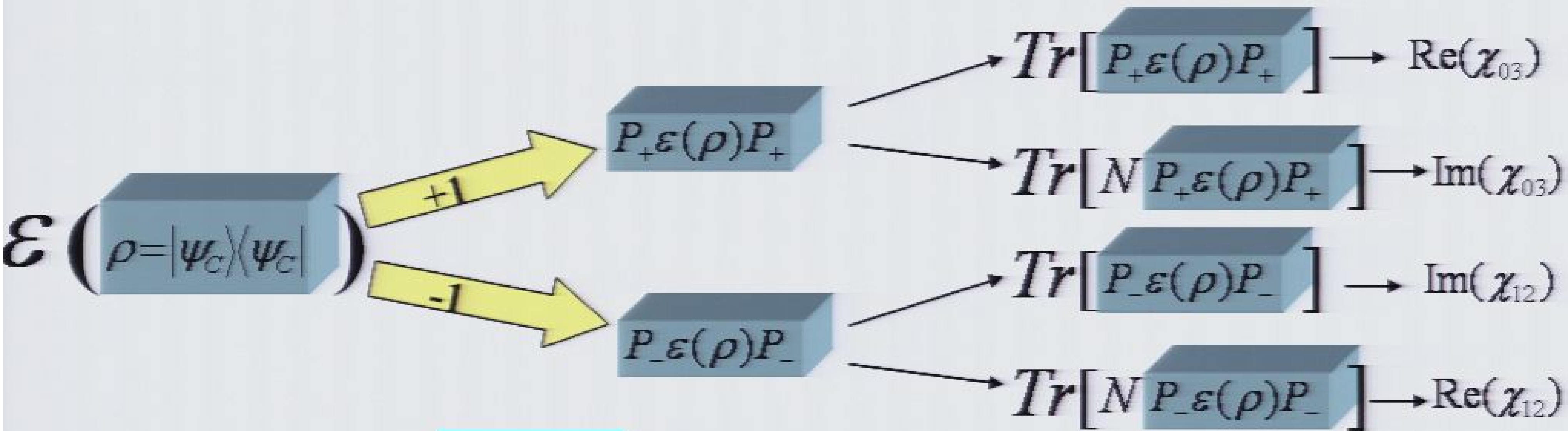
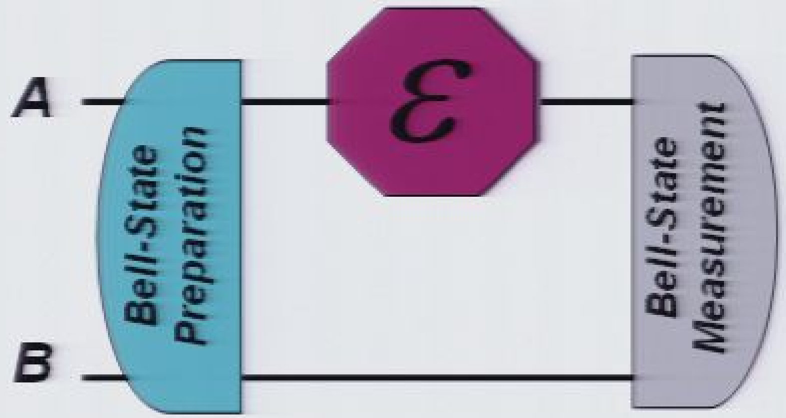


Outputs: χ_{03}, χ_{12}

M. M and D. A. Lidar, *Phys. Rev. Lett.* 97, 170501 (2006).

Direct Characterization of Process Matrix for a Single Qubit



Input state	Measurement: Stabilizer	Measurement: Normalizer	Output
$(\lvert 0\rangle\lvert 0\rangle+\lvert 1\rangle\lvert 1\rangle)/\sqrt{2}$	ZZ, XX	N/A	$\chi_{00}, \chi_{11}, \chi_{22}, \chi_{33}$
$\alpha\lvert 0\rangle\lvert 0\rangle+\beta\lvert 1\rangle\lvert 1\rangle$	ZZ	XX	χ_{03}, χ_{12}

$$|\alpha| \neq |\beta| \neq 0$$

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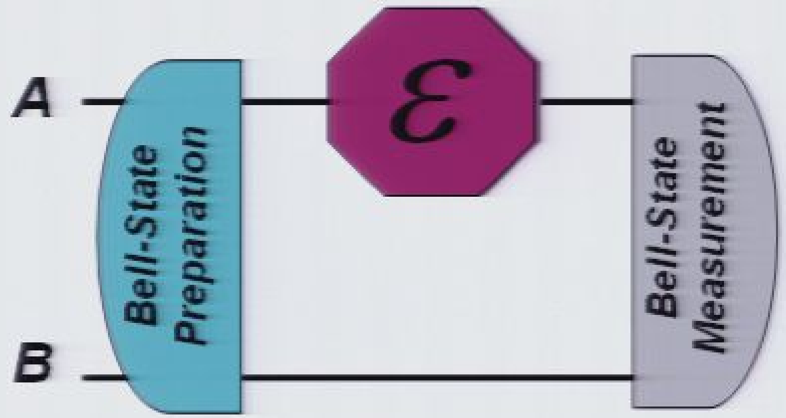

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$\alpha\|+\rangle_x\|+\rangle_x + \beta\|-\rangle_x\|-\rangle_x$	XX	ZZ	χ_{01}, χ_{23}
$\alpha\|+\rangle_y\|+\rangle_y + \beta\|-\rangle_y\|-\rangle_y$	YY	ZZ	χ_{02}, χ_{13}

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Physical Realization

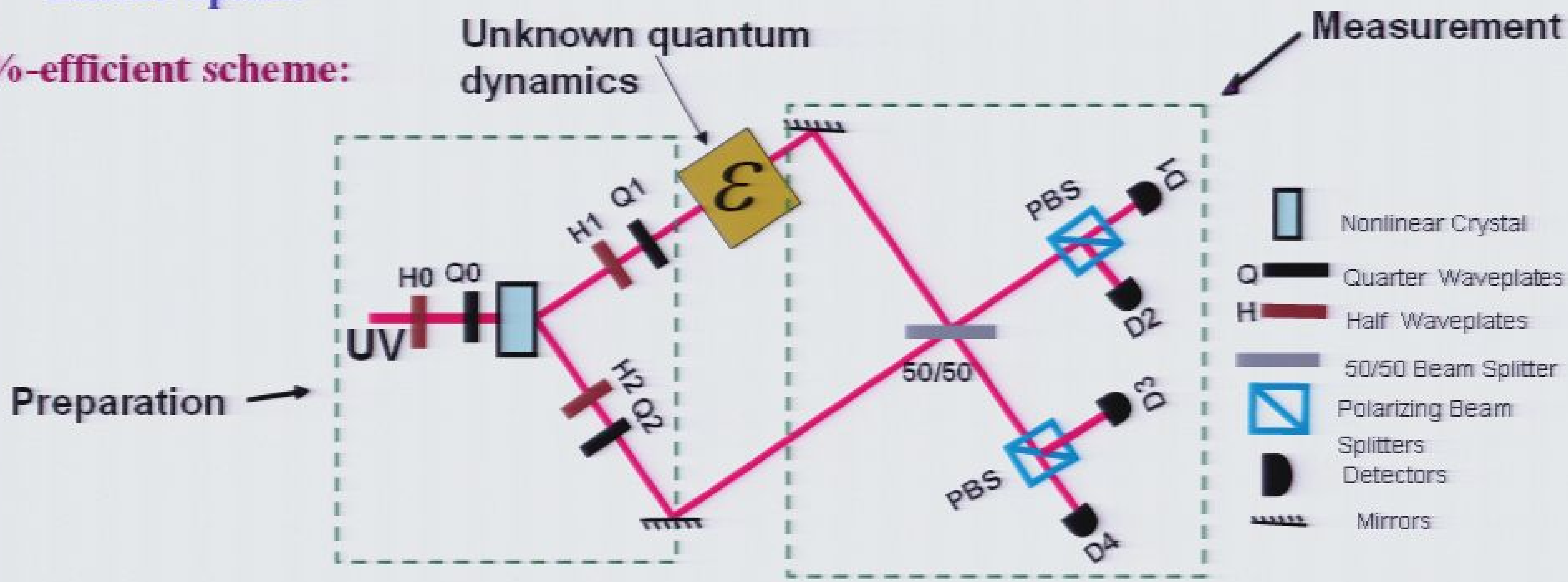
Required physical resources are:

Bell-state generation, Single-qubit rotations, Bell-state measurement

E.g., Liquid-state NMR, Trapped ions: experimentally demonstrated

Linear optics:

50%-efficient scheme:



Z. Wang *et al.*, PRA (07); R. Adamson, A. Steinberg (07); W. T. Liu *et al.*, PRA (08)

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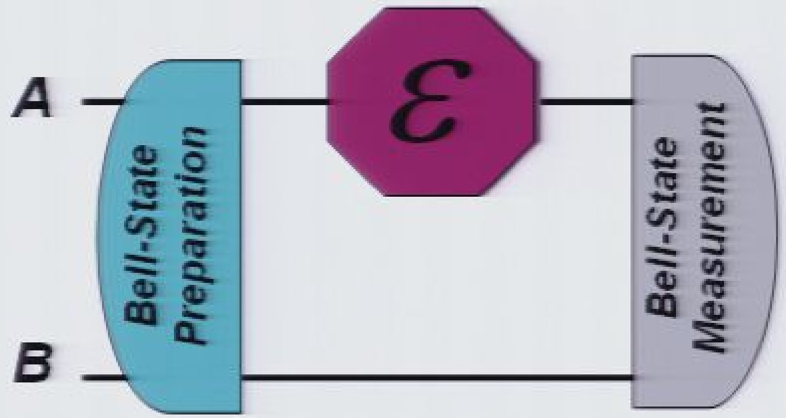

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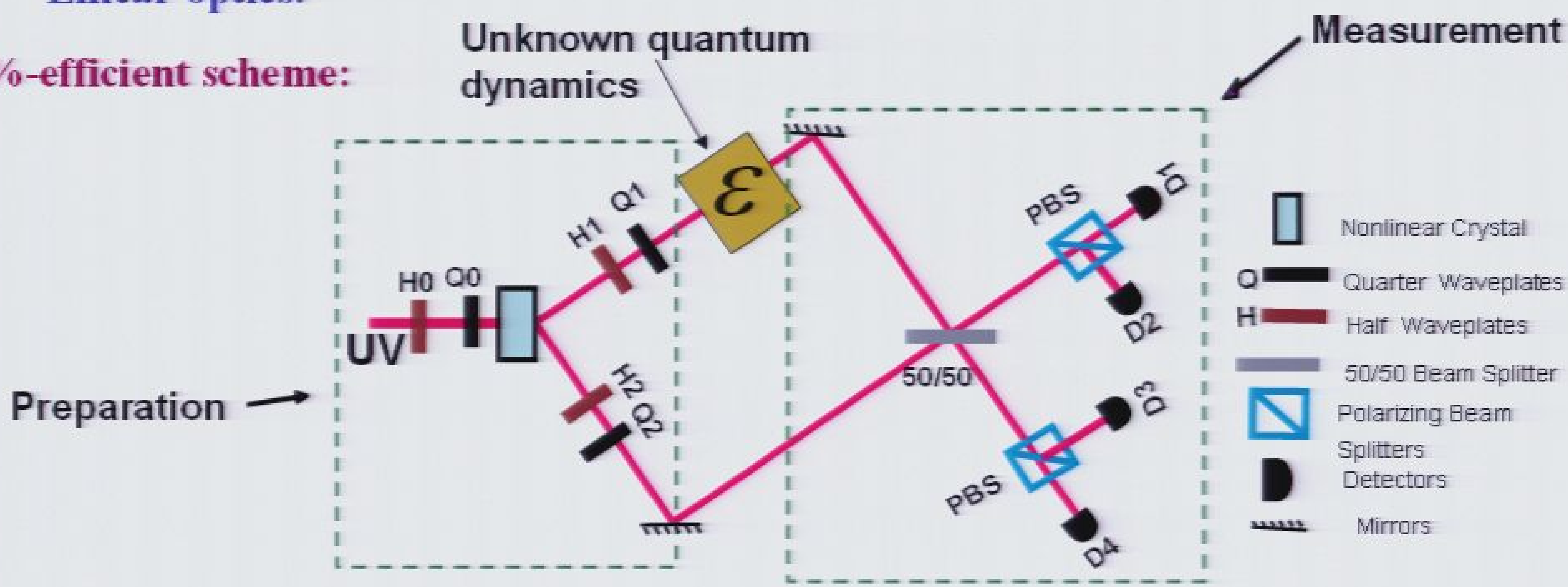

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Direct Characterization of Process Matrix for Higher Dimensional Systems

Unitary operator basis:

$$A_i = \sum_{i=0}^{d^2-1} e_{mi} E_i$$

$$E_i \in \left\{E_0, E_1, \ldots, E_{d^2-1}\right\};\ Tr\left(E_i^+ E_j\right) = d\delta_{ij}$$

$$E_{i=\{p,q\}} = \omega^a X_d^p Z_d^q\ ;\ \ \omega = e^{i2\pi/d}$$

Generalized Pauli operators:

$$Z_d\left|k\right\rangle = \omega^k\left|k\right\rangle;\quad X_d\left|k\right\rangle = \left|k+1\right\rangle$$

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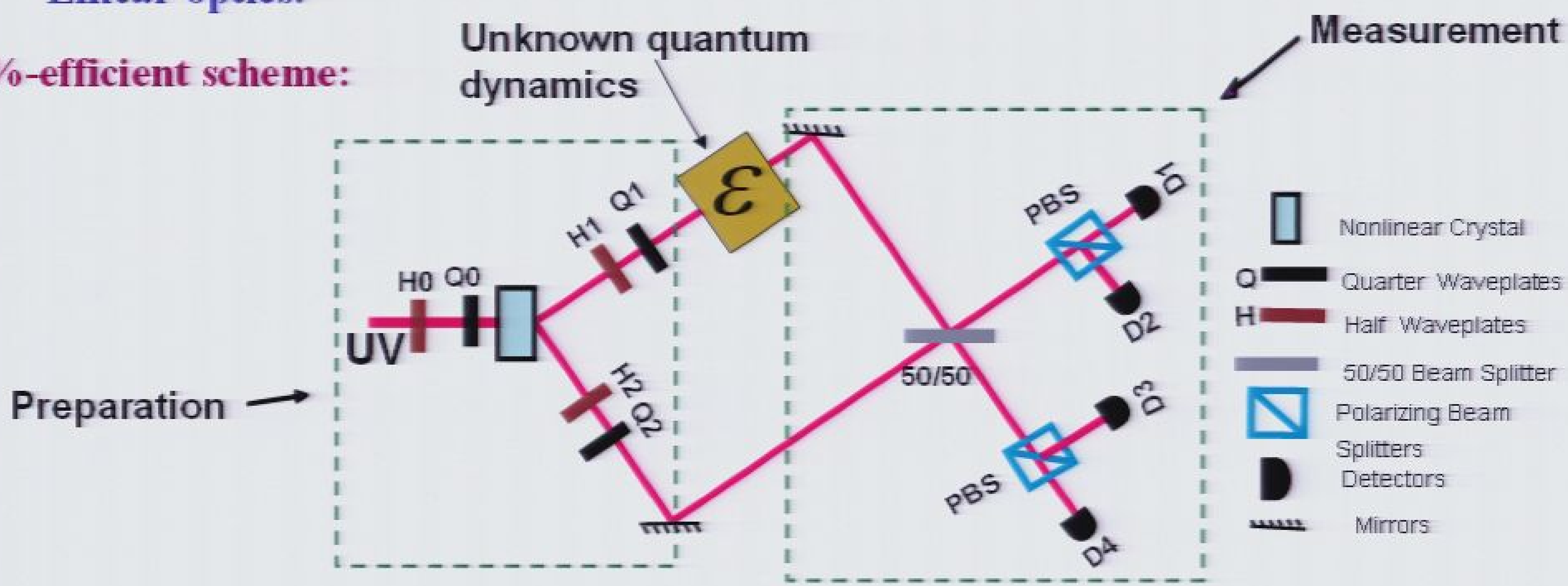

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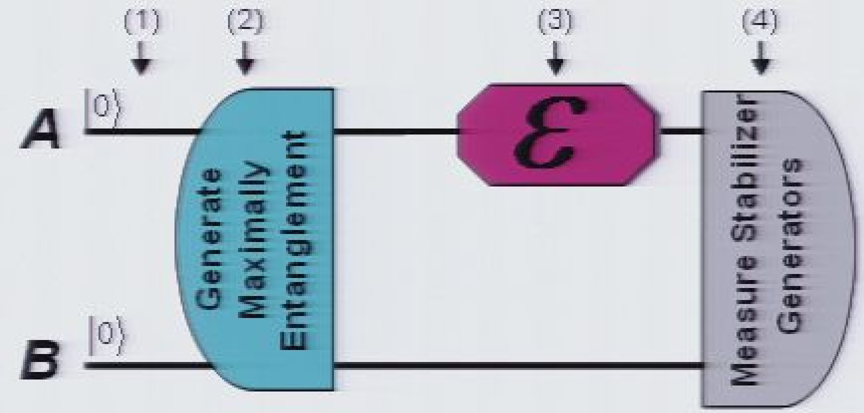
(a) Characterization of Quantum Dynamical Population (diagonal elements χ_{mm})



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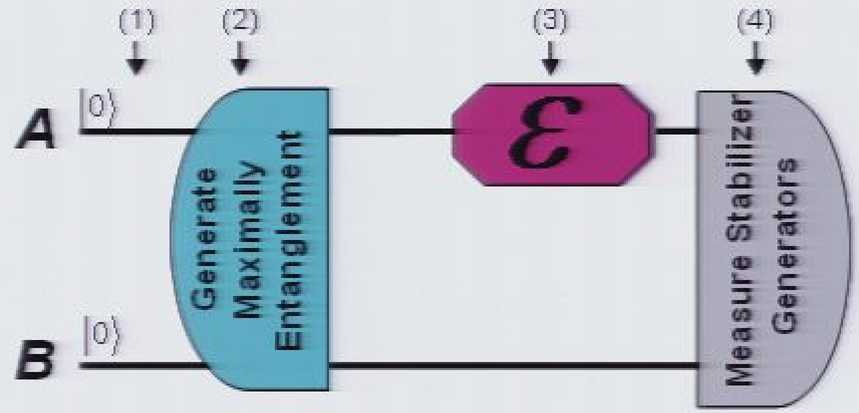

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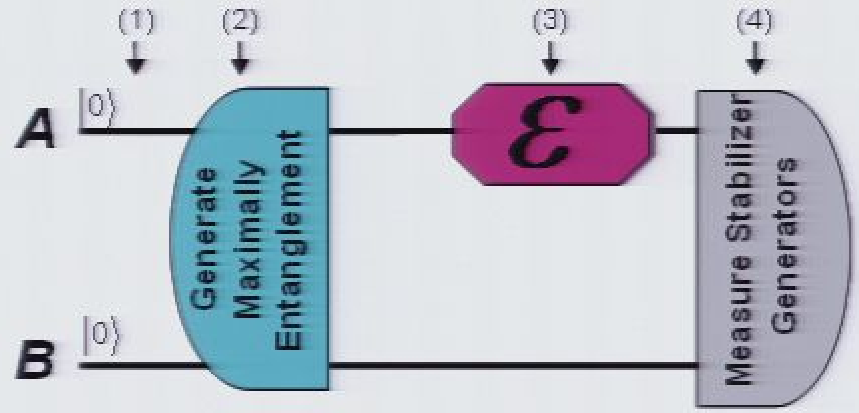

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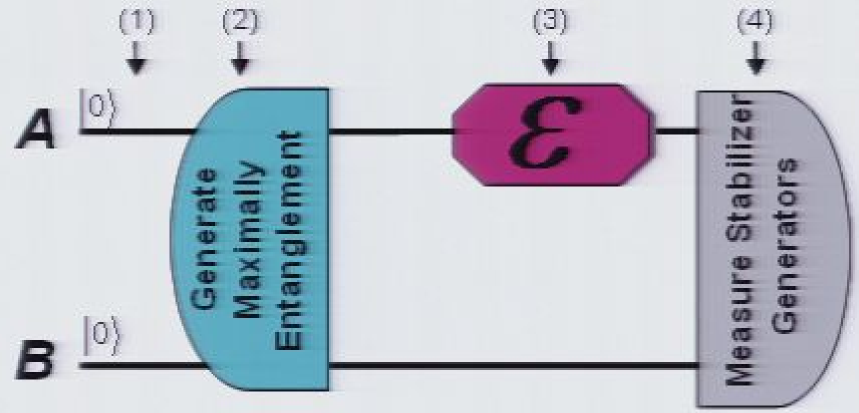

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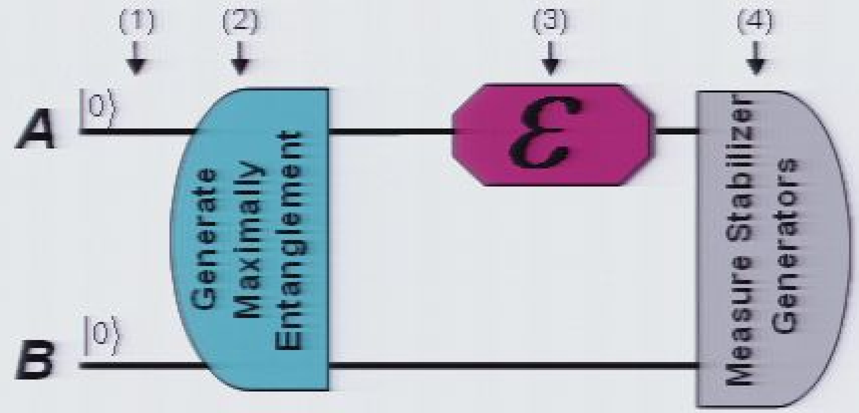

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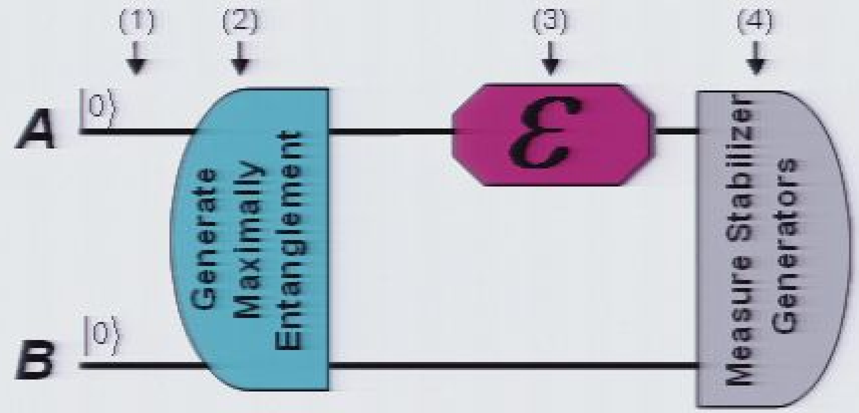

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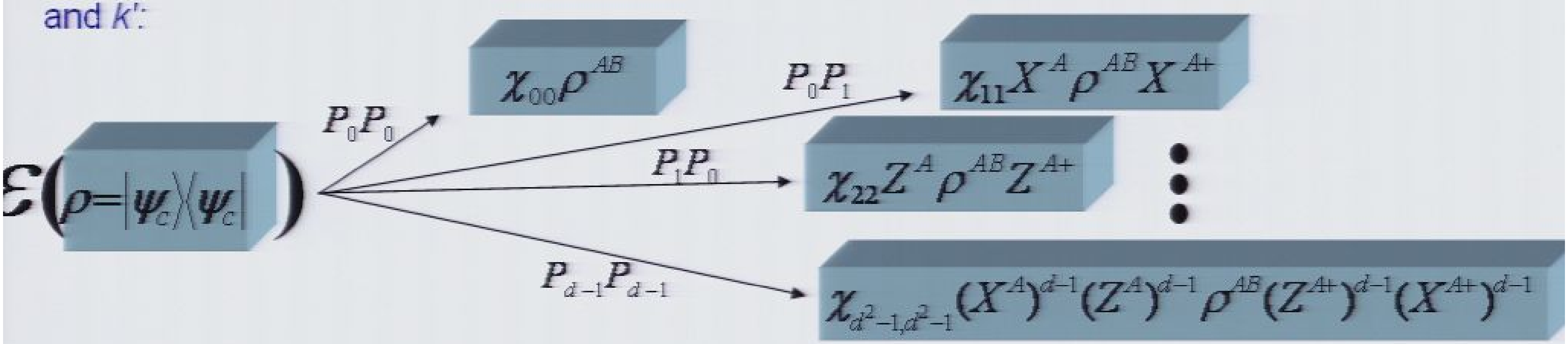

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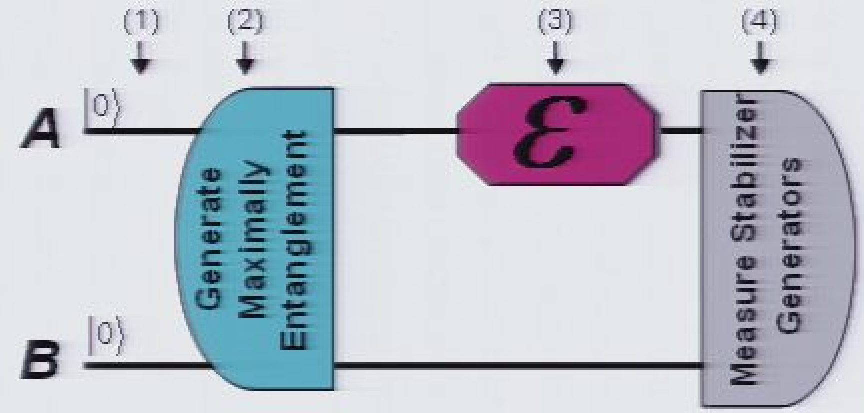

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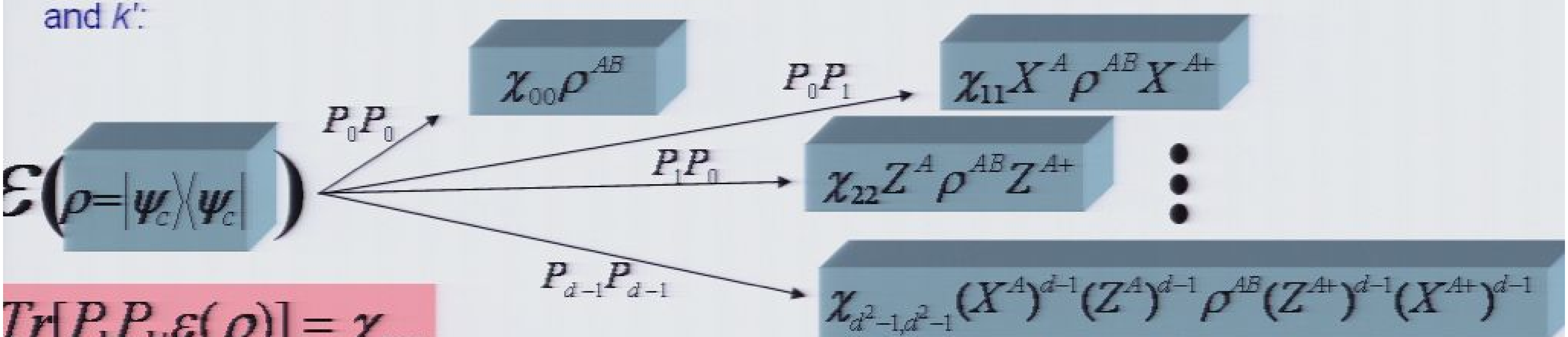
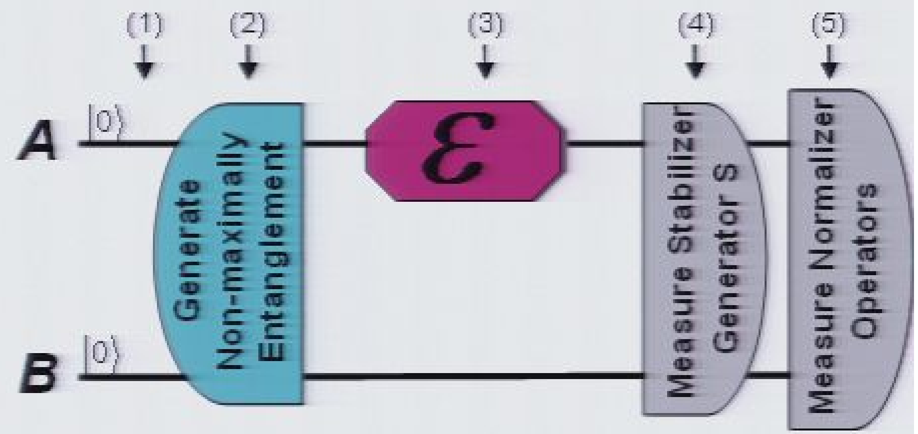


$$Tr[P_k P_{k'} \varepsilon(\rho)] = \chi_{mm}$$

$$m = 0, 1, \ldots, d^2 - 1$$

Number of ensemble measurements for (a): 1

b) Characterization of Quantum Dynamical Coherence (off-diagonal elements χ_{mn}



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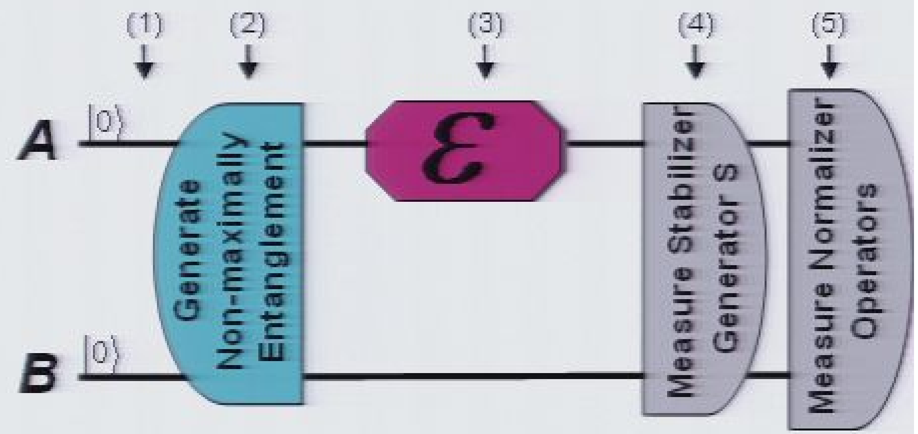

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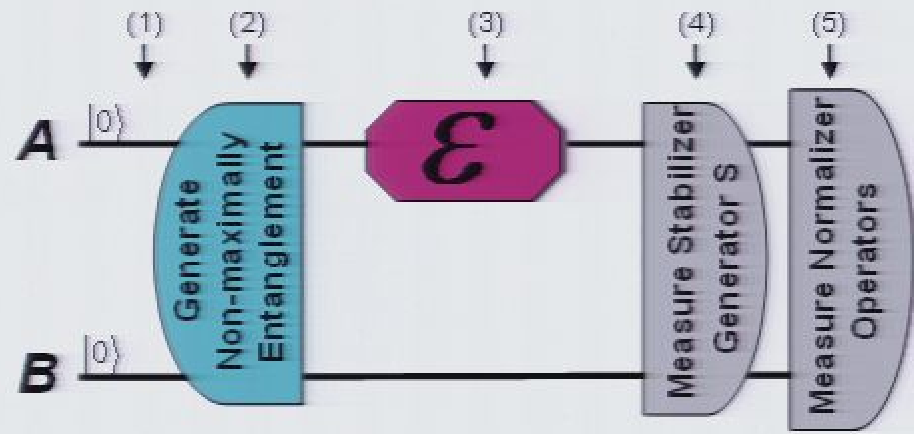

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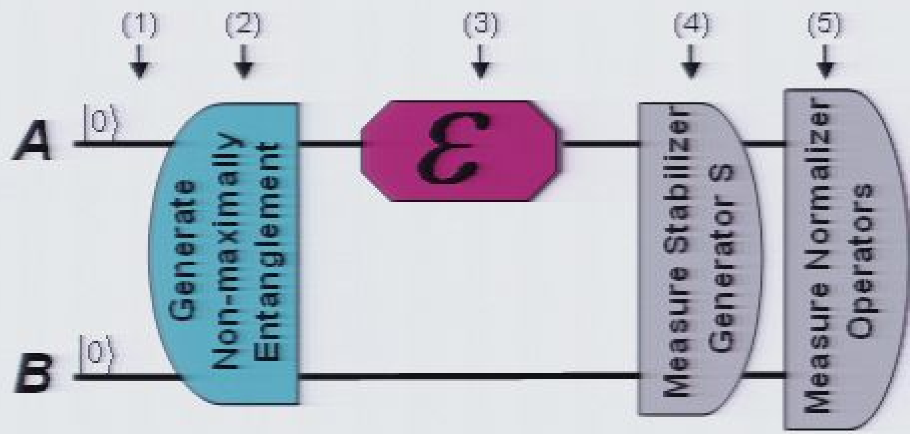

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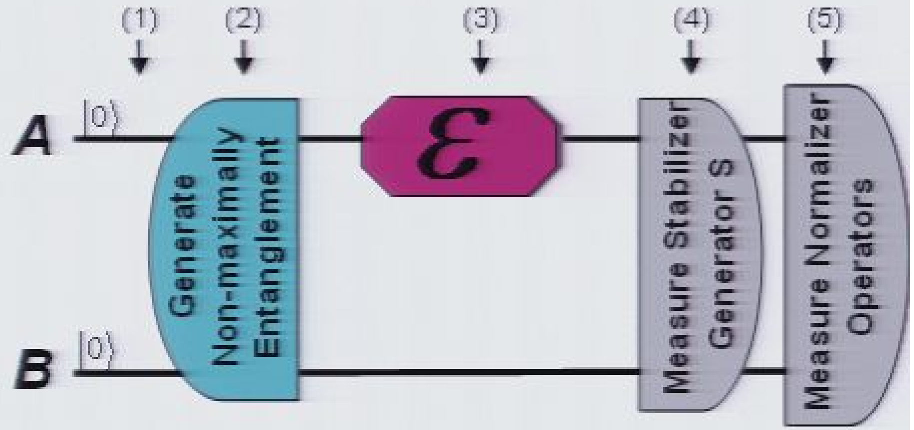

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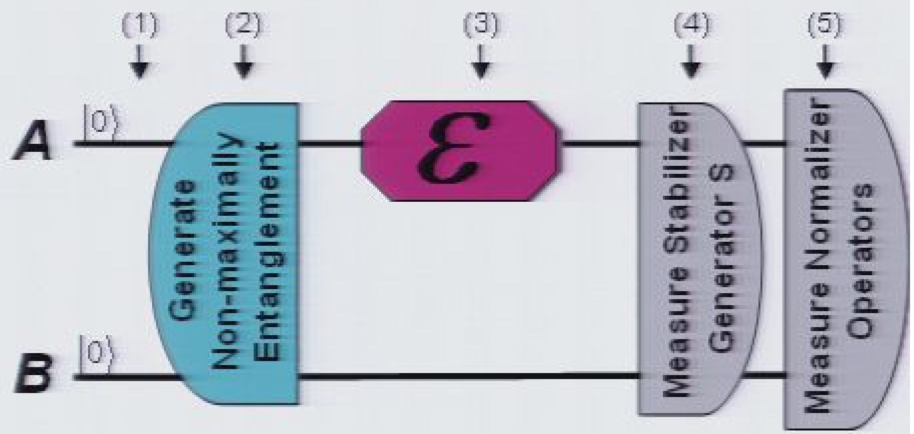

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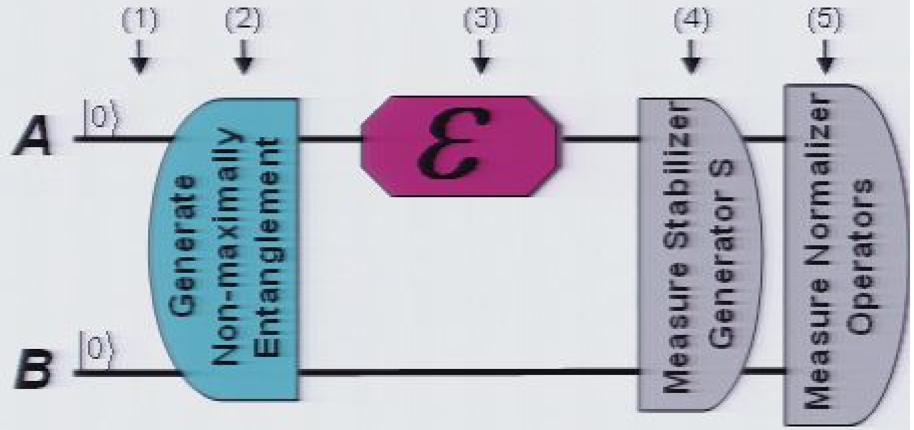

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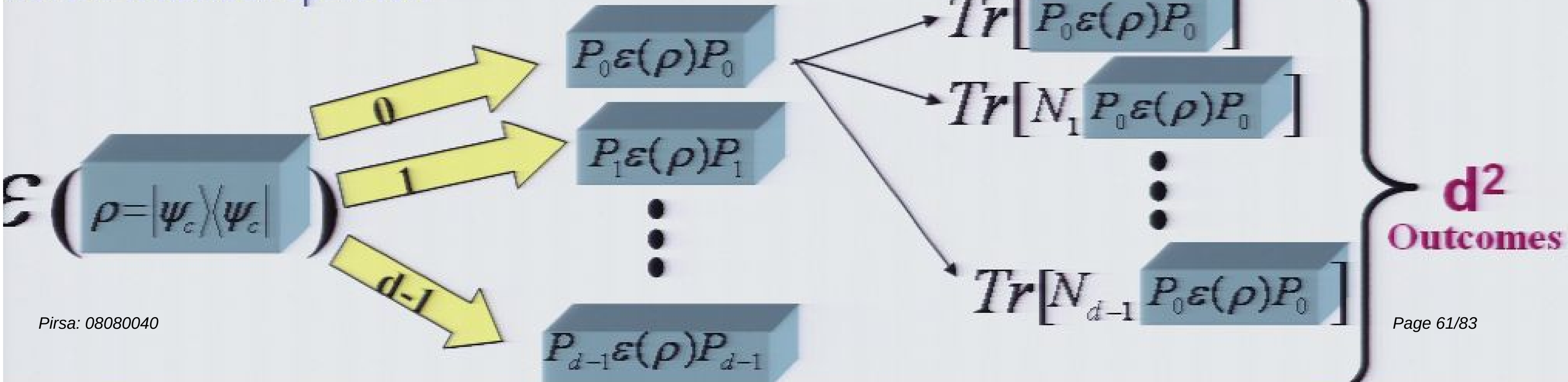

Example: Efficient estimation of k-qubit errors

$\chi_{00} I_1 \otimes I_2 \otimes \cdots \otimes I_N \rho I_1 \otimes I_2 \otimes \cdots \otimes I_N$

$\chi_{11}(\sigma_1^x \otimes I_2 \otimes \cdots \otimes I_N)\rho(\sigma_1^x \otimes I_2 \otimes \cdots \otimes I_N)$

$$\begin{pmatrix} \chi_{00} & & & \\ & \chi_{11} & & \\ & & \ddots & \\ & & & \chi_{NN} \end{pmatrix}$$

$\chi_{NN}(\sigma_1^z \otimes \sigma_2^z \otimes \cdots \otimes \sigma_N^z)\rho(\sigma_1^Z \otimes \sigma_1^Z \otimes \cdots \otimes \sigma_1^z)$

The number of possible k-qubit errors or less:

$$\sum_k \binom{N}{k} 3^k \sim O(N^k) \sim ploy \log(d)$$

Efficient estimation of certain non-sparse Hamiltonians in short time-scale

Let us consider a non-sparse Hamiltonian in a given basis $\{E_j\}$.

$$H = \sum_m h_m E_m$$

If :

$$E_m = E_j^k E_i^l,\ k = 0, \cdots, d-1,\ and\ l = 0, \cdots, \sqrt{d} + poly \log(d)$$

The estimation is possible with a single experimental configuration and a large number of copies

Required stabilizer state: $S = E_i^A E_{i'}^B$

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If : $E_m = E_j^k E_i^l,\ k = 0, \cdots, d-1,\ and\ fixed\ l$

Selective and efficient identification of such class of Hamiltonians is possible for polynomial subset of E_j^k

Simultaneous determination of T1 and T2

$$\rho_i = \begin{pmatrix} a & b \\ b^* & 1-a \end{pmatrix} \longrightarrow \boxed{\text{Quantum homogenization process}} \longrightarrow \varepsilon(\rho_i) = \begin{pmatrix} (a-a_0)\exp\left(-\frac{t}{T_1}\right)+a_0 & b\exp\left(-\frac{t}{T_2}\right) \\ b^*\exp\left(-\frac{t}{T_2}\right) & (a_0-a)\exp\left(-\frac{t}{T_1}\right)+1-a_0 \end{pmatrix}$$

M. M, A. T. Rezakhani, A. Aspuru-Guzik, Phys. Rev. A 77 , 042320 (2008).

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Preparation:

$$\rho_i = |\psi_C\rangle\langle\psi_C|; \quad |\psi_C\rangle = |0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B$$

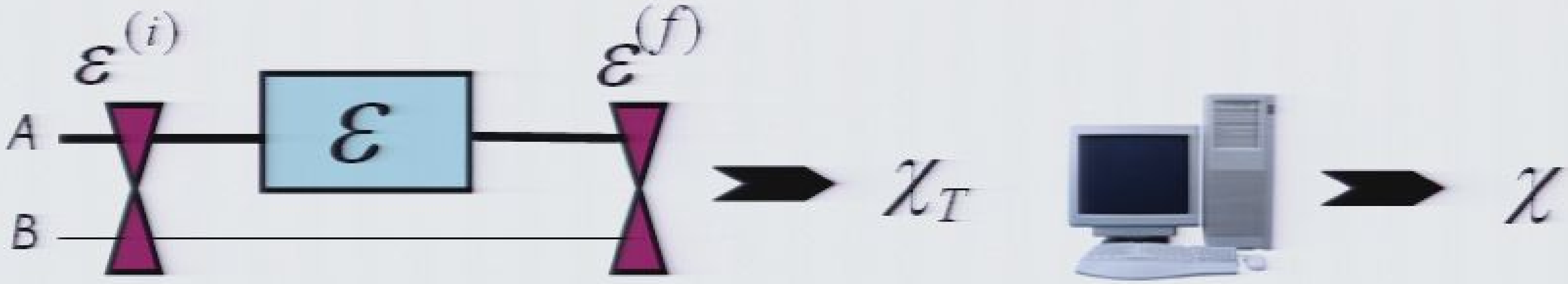
$$\frac{t}{T_1} = -\ln\{2tr[P_{\psi^+}\varepsilon(\rho_i)] + 2tr[P_{\psi^-}\varepsilon(\rho_i)] - 1\}$$

$$\frac{t}{T_2} = -\ln\{tr[P_{\varphi^+}\varepsilon(\rho_i)] - tr[P_{\varphi^-}\varepsilon(\rho_i)]\}$$

Both T1 and T2 can be measured via a single bell state measurement

M. M, A. T. Rezakhani, A. Aspuru-Guzik, Phys. Rev. A 77 , 042320 (2008).

Estimation of quantum dynamical processes via imperfect Bell-state analyzer



correlated input state (e.g., faulty BSP)

correlated measurement (e.g., faulty BSM)

S

Estimation of quantum dynamical processes via imperfect Bell-state analyzer

$$p_l(k) = \sum_{mnjj'} \lambda_{mn}^{jj'}(\rho^k)\mathrm{Tr}[P^{ll}\sigma_m^{(S)}P^{jj'}\sigma_n^{(S)}]\chi_{mn}$$

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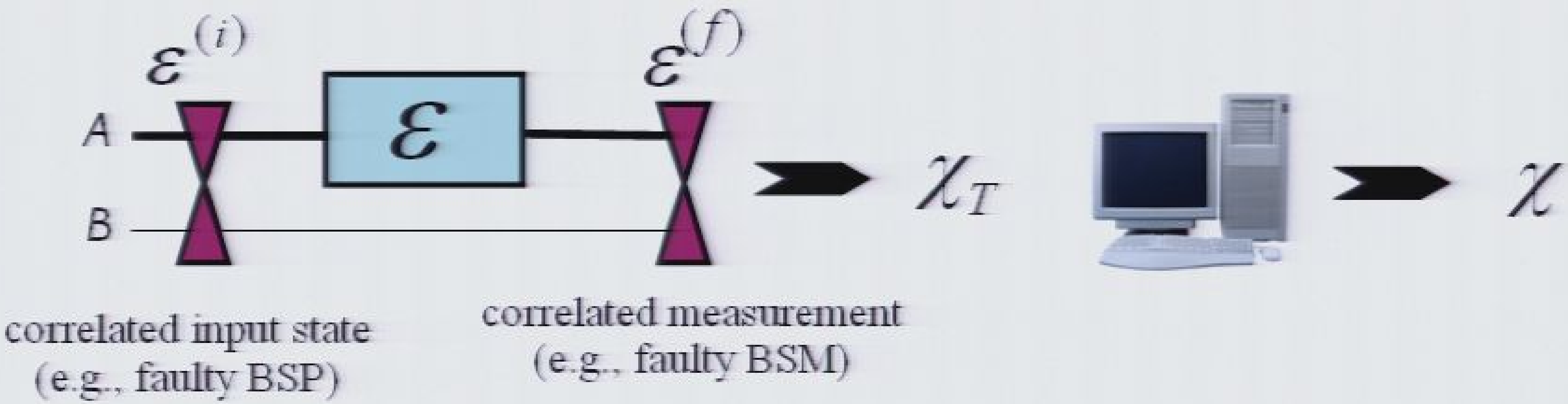

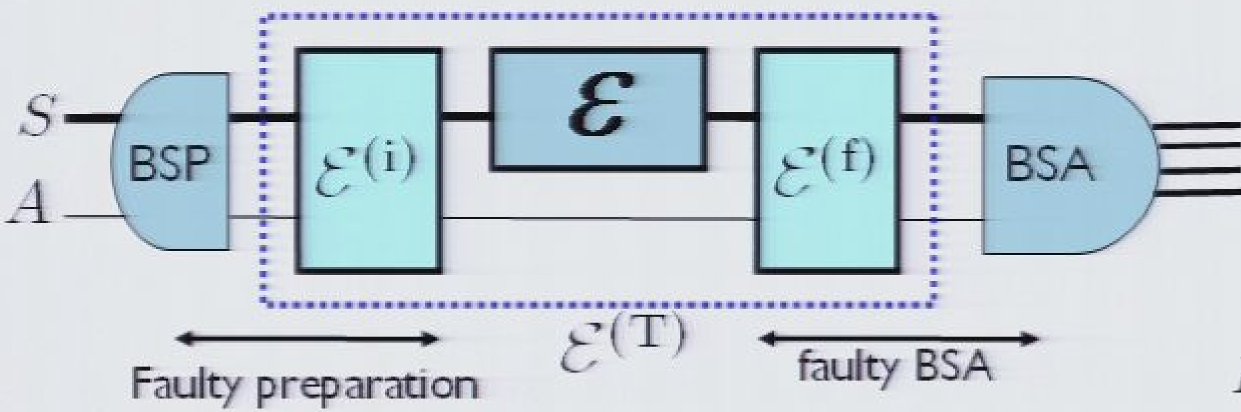

$$|\mathcal{B}_l\rangle : (\Phi^+, \Psi^+, \Psi^-, \Phi^-)$$

$$p_l(k) = \mathrm{Tr}[P^{ll}\mathcal{E}^{(\mathrm{T})}(\rho^k)]$$

$$P^{ll'} = |\mathcal{B}_l\rangle\langle\mathcal{B}_{l'}|, \rho^k = |\Phi_k^+\rangle\langle\Phi_k^+|$$

$$\begin{cases} \mathcal{E}^{(\mathrm{T})}(\rho) = \sum_{mn} \chi_{mn}\sigma_m^{(S)}\widetilde{\rho}_{mn}\sigma_n^{(S)} \\ \widetilde{\rho}_{mn} = \sum_{kk'} \lambda_{mn}^{kk'}(\rho)P^{kk'} \end{cases}$$

[collaboration with Paul Kwiat group @UIUC]

Estimation of quantum dynamical processes via imperfect Bell-state analyzer

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$$\left|\chi_T\right\rangle = \Lambda\left|\chi\right\rangle$$

$$\Lambda(\chi^{(i)},\chi^{(f)},\{\rho_i\})$$

$$\left|\chi_T\right\rangle = \begin{pmatrix} Tr[P_{\phi^+}\varepsilon^T(\rho_1)] \\ Tr[P_{\phi^-}\varepsilon^T(\rho_1)] \\ Tr[P_{\psi^+}\varepsilon^T(\rho_1)] \\ Tr[P_{\psi^-}\varepsilon^T(\rho_1)] \\ Tr[P_{\phi^+}\varepsilon^T(\rho_2)] \\ \cdots \\ \cdots \\ Tr[P_{\psi^-}\varepsilon^T(\rho_4)] \end{pmatrix} \qquad \left|\chi\right\rangle = \left.\begin{pmatrix} \chi_{00} \\ \chi_{01} \\ \chi_{02} \\ \chi_{03} \\ \chi_{10} \\ \cdots \\ \cdots \\ \chi_{33} \end{pmatrix}\right\} 16\times 1$$

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$$|\chi\rangle = \Lambda^{-1}(\chi^{(i)}, \chi^{(f)}, \{\rho_i\})|\chi_T\rangle$$

(extra data processing)

Optimal input states

Maximize $\left|\det \Lambda\right|$

$$\left|\psi_C\right\rangle = \cos\theta \left|++\right\rangle_{AB} + e^{i\varphi}\sin\theta\left|--\right\rangle_{AB}$$

Optimal input values: $\theta = (2k+1)\dfrac{\pi}{8}$

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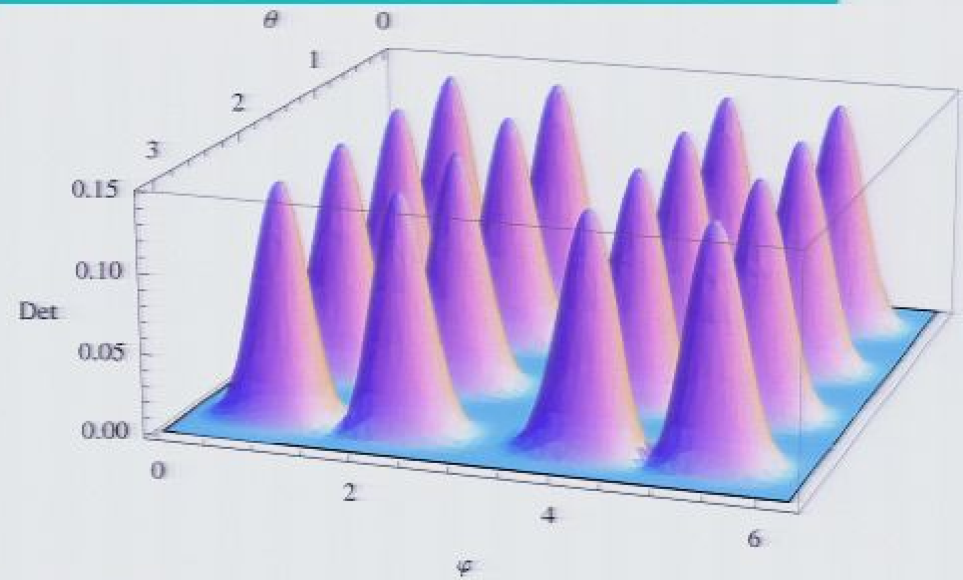

$$\Lambda(\theta,\varphi)=\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&0&x&0&0&0&0&0&0&0&0&x&0&0&1\\
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z&0&0&y&0&0&0&0&0&0&0&0&-iy&0&0&-z\\
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1&x&0&0&x&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&-ix&0&0&ix&1\\
z&z&0&0&-z&-z&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-z&iz&0&0&iz&z\\
1&0&x&0&0&0&0&0&x&0&1&0&0&0&0&0\\
0&0&0&0&0&1&0&x&0&0&0&0&0&-ix&0&1\\
z&0&iy&0&0&0&0&0&-iy&0&z&0&0&0&0&0\\
0&0&0&0&0&-z&0&y&0&0&0&0&0&y&0&z
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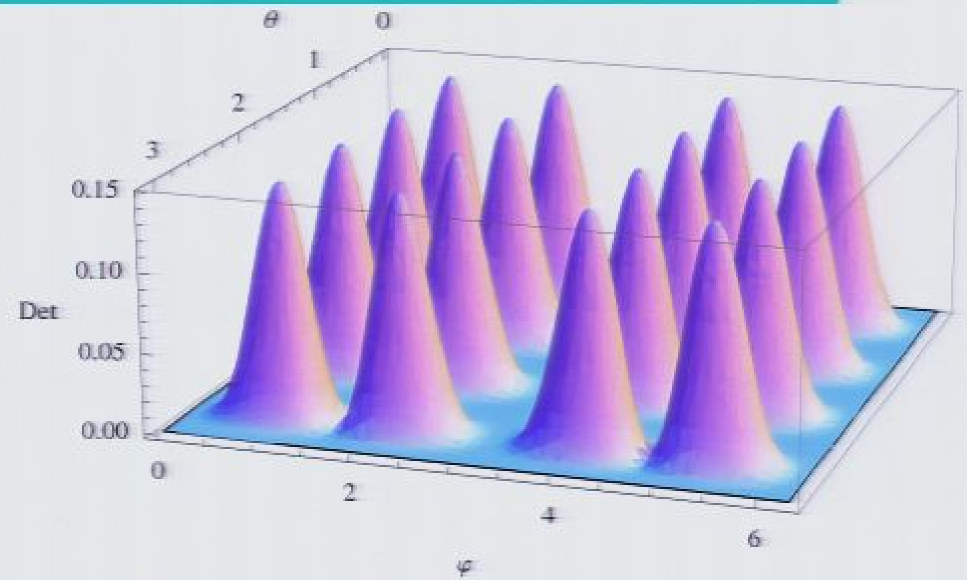

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0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & -ix & 0 & 0 & ix & 1 & 0 & 0 & 0 & 0 & 0 \\
z & 0 & 0 & y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -iy & 0 & 0 & -z \\
0 & 0 & 0 & 0 & 0 & z & y & 0 & 0 & y & -z & 0 & 0 & 0 & 0 & 0 \\
1 & x & 0 & 0 & x & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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z & z & 0 & 0 & -z & -z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z & iz & 0 & 0 & iz & z \\
1 & 0 & x & 0 & 0 & 0 & 0 & 0 & x & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
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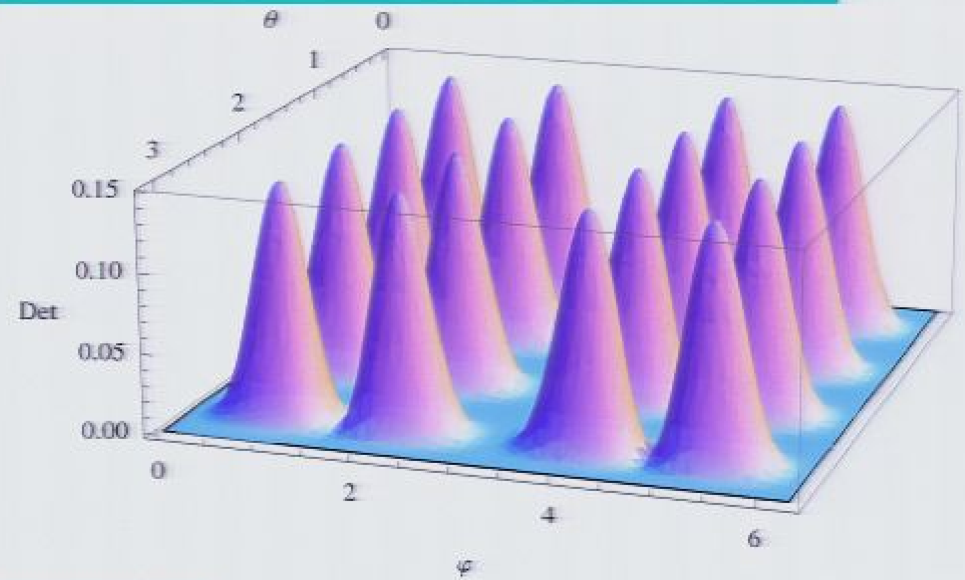

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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
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1 & 0 & x & 0 & 0 & 0 & 0 & 0 & x & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
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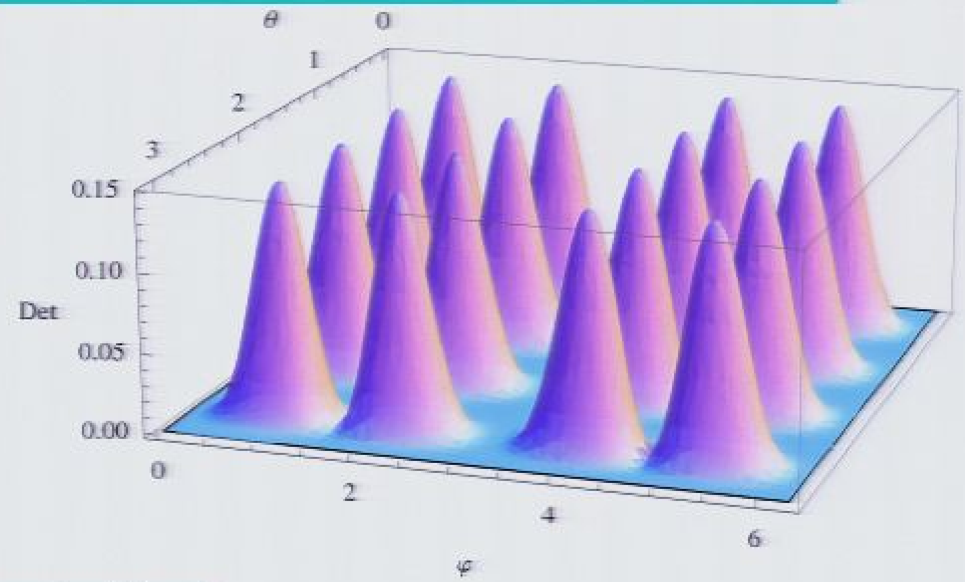

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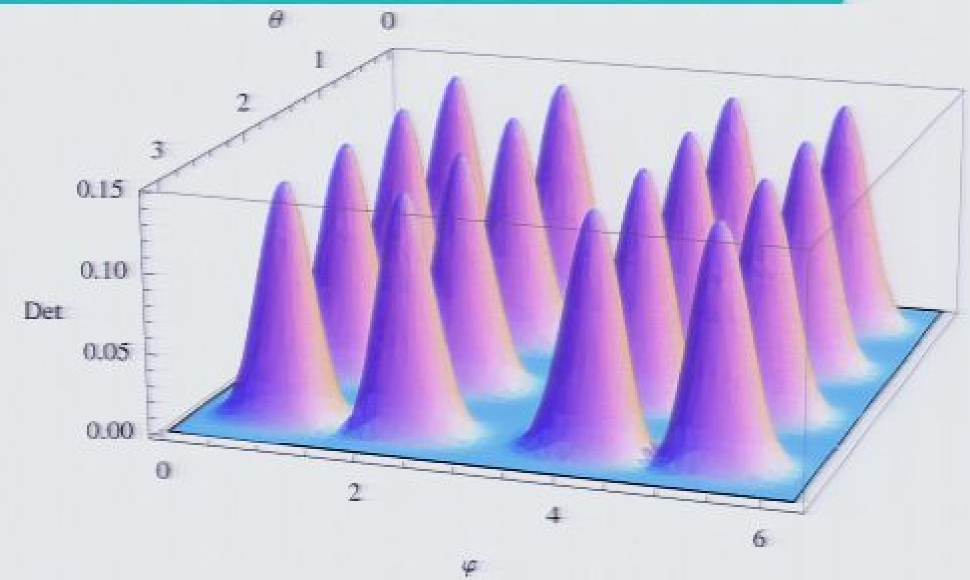

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0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -ix & 0 & 0 & ix & 1 \\
z & z & 0 & 0 & -z & -z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -z & iz & 0 & 0 & iz & z \\
1 & 0 & x & 0 & 0 & 0 & 0 & 0 & x & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & x & 0 & 0 & 0 & 0 & 0 & -ix & 0 & 1 \\
z & 0 & iy & 0 & 0 & 0 & 0 & 0 & -iy & 0 & z & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -z & 0 & y & 0 & 0 & 0 & 0 & 0 & y & 0 & z
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Correlated depolarizing noise

Example 1:

Effective input state: $$\rho^{(i)} \rightarrow \frac{1-\lambda^{(i)}}{4} I \otimes I + \lambda^{(i)} \rho^{(i)}$$

Effective measurement: $$P^{(f)} \rightarrow \frac{1-\lambda^{(f)}}{4} I \otimes I + \lambda^{(f)} P^{(f)}$$

Modification in data analysis:

$$tr[\varepsilon(\rho^{(i)})P^{(f)}] \rightarrow \lambda^{(i)}\lambda^{(f)} tr[\varepsilon(\rho^{(i)})P^{(f)}] + (1-\lambda^{(i)}\lambda^{(f)})/4$$

Example 2: Generalized depolarizing noise

Effective input state: $$\rho^{(i)} \rightarrow \frac{1-\lambda^{(i)}}{4} I \otimes I + \lambda^{(i)} U \rho^{(i)} U^{\dagger}$$

$$tr[\varepsilon(\rho^{(i)})P^{(f)}] \rightarrow \lambda^{(i)}\lambda^{(f)} tr[\varepsilon(U\rho^{(i)}U^{\dagger})P^{(f)}] + (1-\lambda^{(i)}\lambda^{(f)})/4$$

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Applications and Future works

➢ Dynamical open-loop/learning control of quantum Hamiltonian Systems.

➢ Utilizing weak measurements for real time quantum dynamical control.

➢ Efficient verification of certain correlated errors for quantum computers and quantum communication networks.

➢ Exploring the existence of the symmetries in the system-bath couplings which would lead to noiseless subspaces and subsystems.

➢ Studying energy transport in the multichoromophoric complexes in the non-Markovian and/or strong interaction regimes.

References:

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M. Mohseni, A. T. Rezakhani, A. Aspuru-Guzik, Phys. Rev. A 77 , 042320 (2008).
M. Mohseni, and A. T. Rezakhani, arXiv:0805.3188.

Thanks for Your Attention