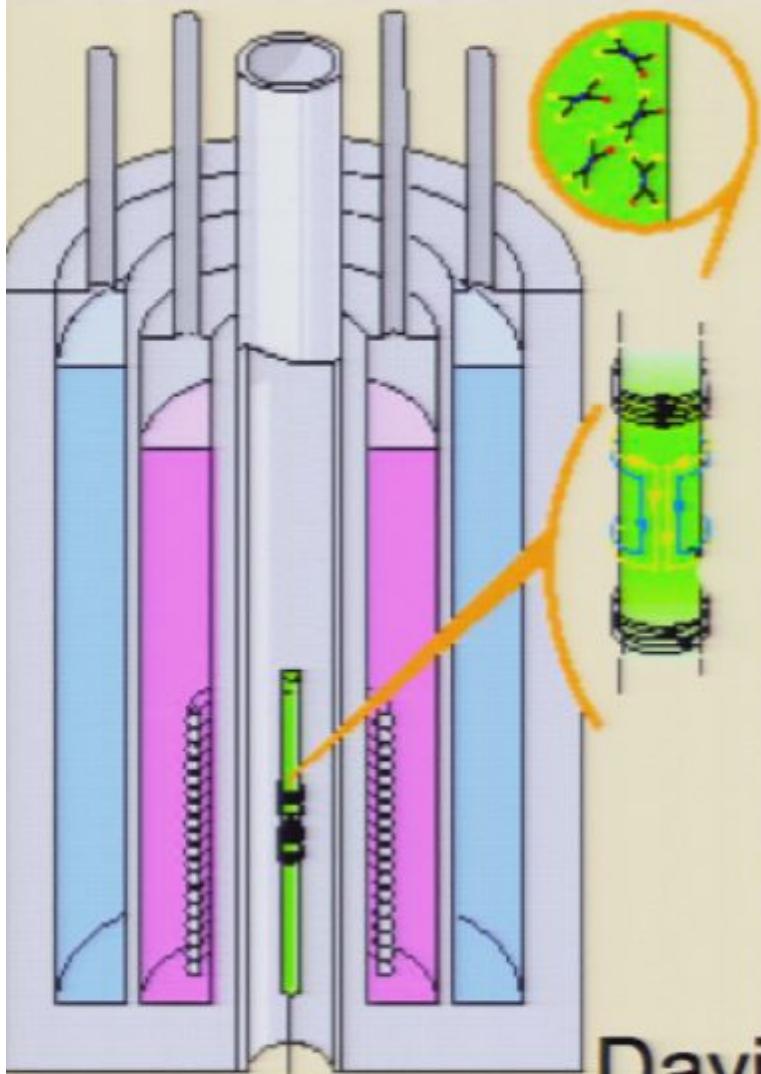


Title: Learning from experiments on coherent control

Date: Aug 28, 2008 04:45 PM

URL: <http://pirsa.org/08080039>

Abstract: I will discuss a few case studies of coherent control experiments and how we use quantum estimation to motivate improved experiments. Examples from NMR with physical and logical quits, electron/nuclear spin systems and persistent current flux qubits



# Learning from experiments on coherent control

David G. Cory  
Nuclear Science & Engineering  
Massachusetts Institute of Technology

We work to develop experimental measures that would inform discussions of fault tolerance.

- how quickly do 2-body terms fall off with distance?
- do they fall below a threshold?
- how important are n-body terms?
- what are the correlation (time/space) functions of noise?

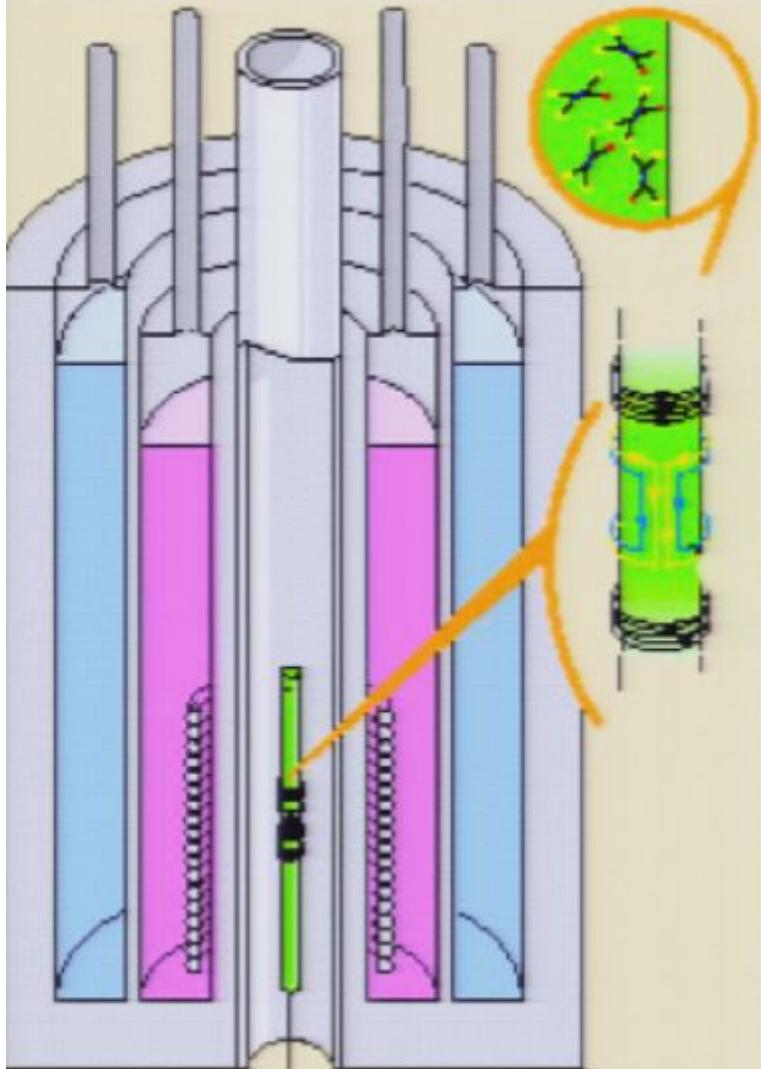
I will not discuss these today.

Bread and Butter of lab is finding means of bending coherent dynamics to our desires.

- to accomplish this we carryout a few measurements
- try to understand them in terms of a physically relevant model
- use this to motivate the next experiment

Today I will work through a few case studies with you

- liquid state NMR, physical qubits (nuclear spins)
- logical qubits
- superconducting, persistent current flux qubits



ARDA

Dr. Timothy Havel  
Professor Seth Lloyd  
Professor Raymond Laflamme (UW)  
Dr. Sekhar Ramanathan  
Professor Joseph Emerson (UW)  
Dr. Nicolas Boulant  
Dr. Paola Cappellaro  
Dr. Yaakov Weinstein  
Dr. Michael Henry  
Dr. Jonathan Hodges  
Dr. Jamie Yang  
Dr. Jonas Bylander  
Dr. Will Oliver



Pirsa: 08080039



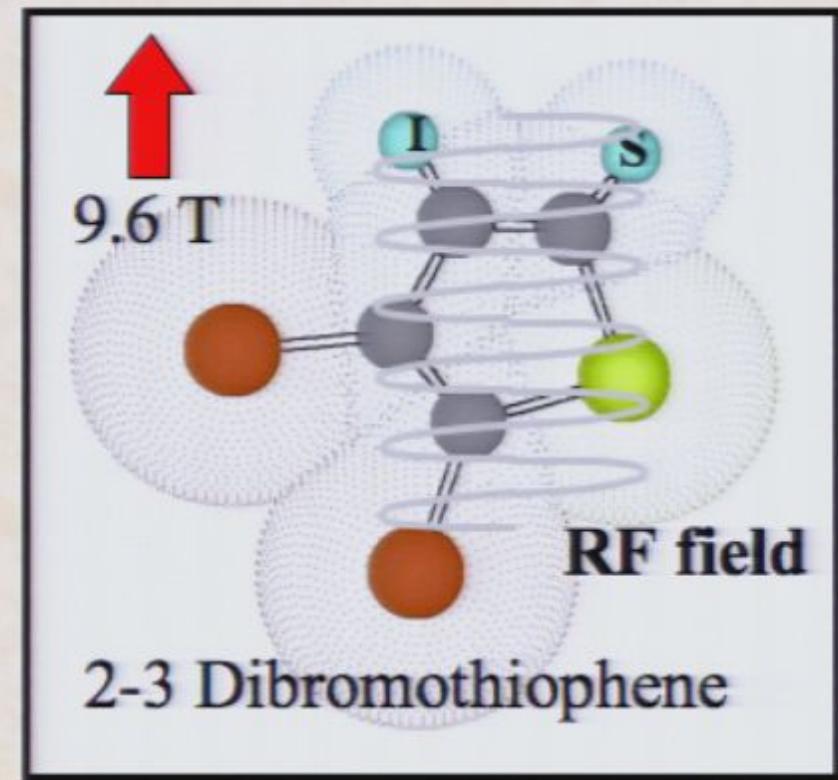
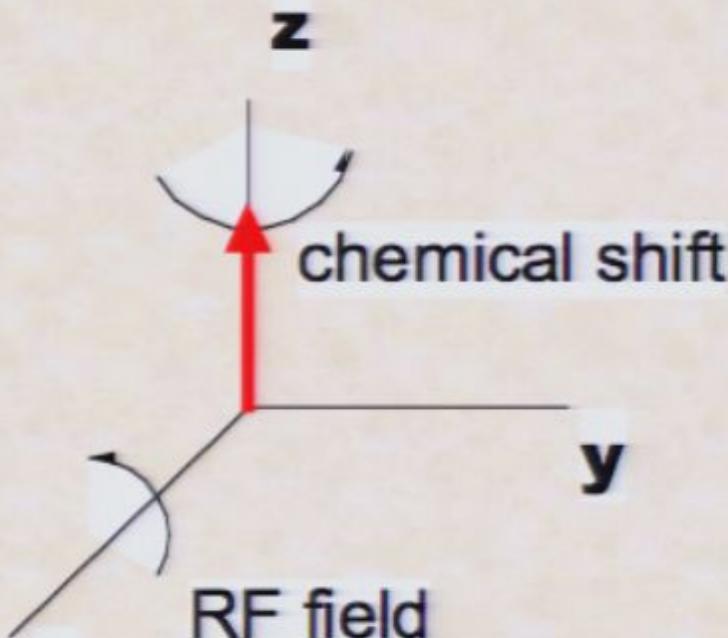
Page 5/48

# Control through time-dependent spin Hamiltonian

$$H_{int} = \underbrace{\omega_I I_z + \omega_S S_z}_{\text{interaction with B field}} + 2\pi J_{IS} I_z S_z$$

interaction with B field

spin-spin coupling



$$H_{ext} = \underbrace{\omega_{RF}(t) I_x}_{\text{I spin with RF field}} + \underbrace{\omega_{RF'}(t) S_x}_{\text{S spin with RF field}}$$

I spin with RF field

S spin with RF field

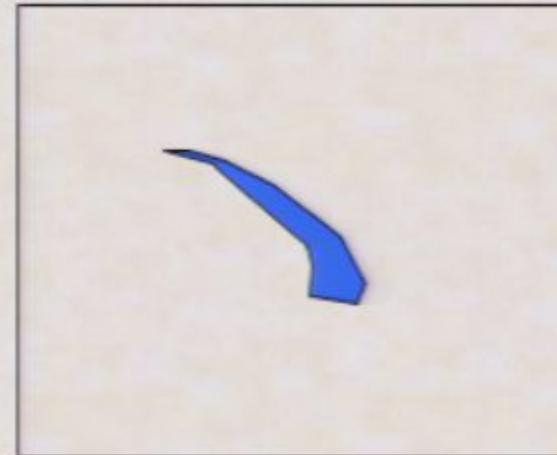
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interaction with B field

spin-spin coupling

RF



## Model

- chemical shifts
- coupling constants
- range of fields (correlations, distributions)
- other spins in the system
- model of RF transmitter (non-linearity, transients)
- relaxation times
- correlation times

$$H_{ext} = \underbrace{\omega_{RF}(t) I_x}_{\text{I spin with RF field}} + \underbrace{\omega_{RF'}(t) S_x}_{\text{S spin with RF field}}$$

I spin with RF field

S spin with RF field

# Superoperator - Liouvillian Space

Robust Method for Estimating the Lindblad Operators of a Dissipative Quantum Process from Measurements of the Density Operator at Multiple Time Points. N. Boulant, T. F. Havel, M. A. Pravia and D. G. Cory, *Physical Review A*, 2003, **67**, 042322-1-12.

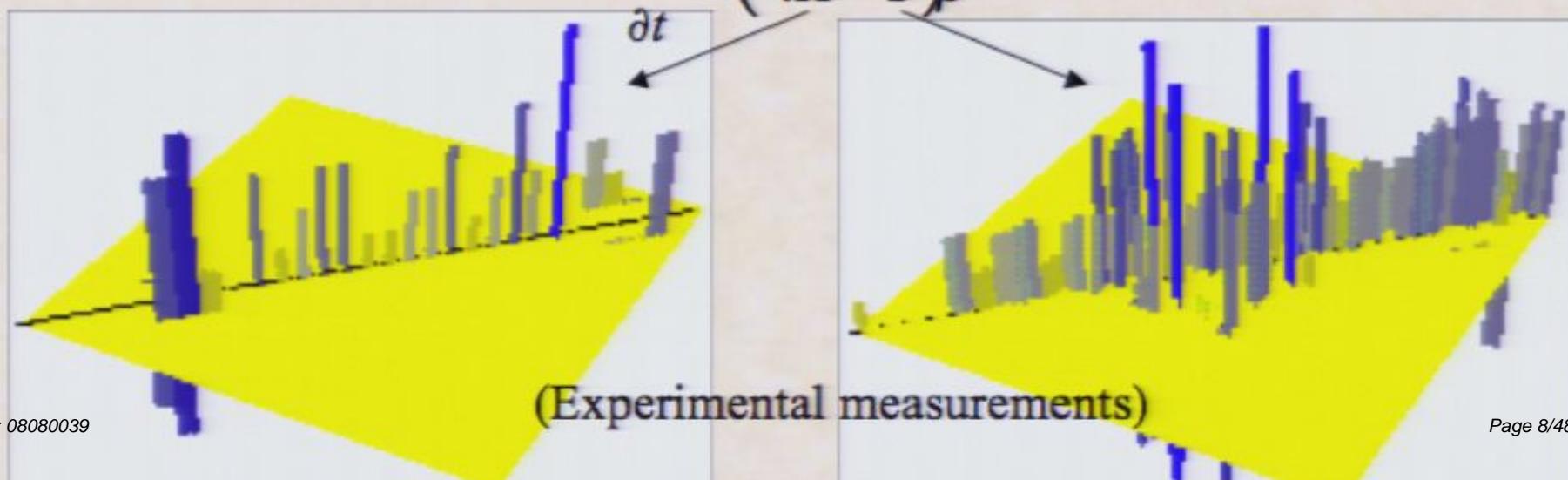
- Superoperator - a compact representation of the complete system evolution, including coherent, incoherent, and decoherent parts.

A closed quantum system evolves via the Liouville-von Neumann equation:

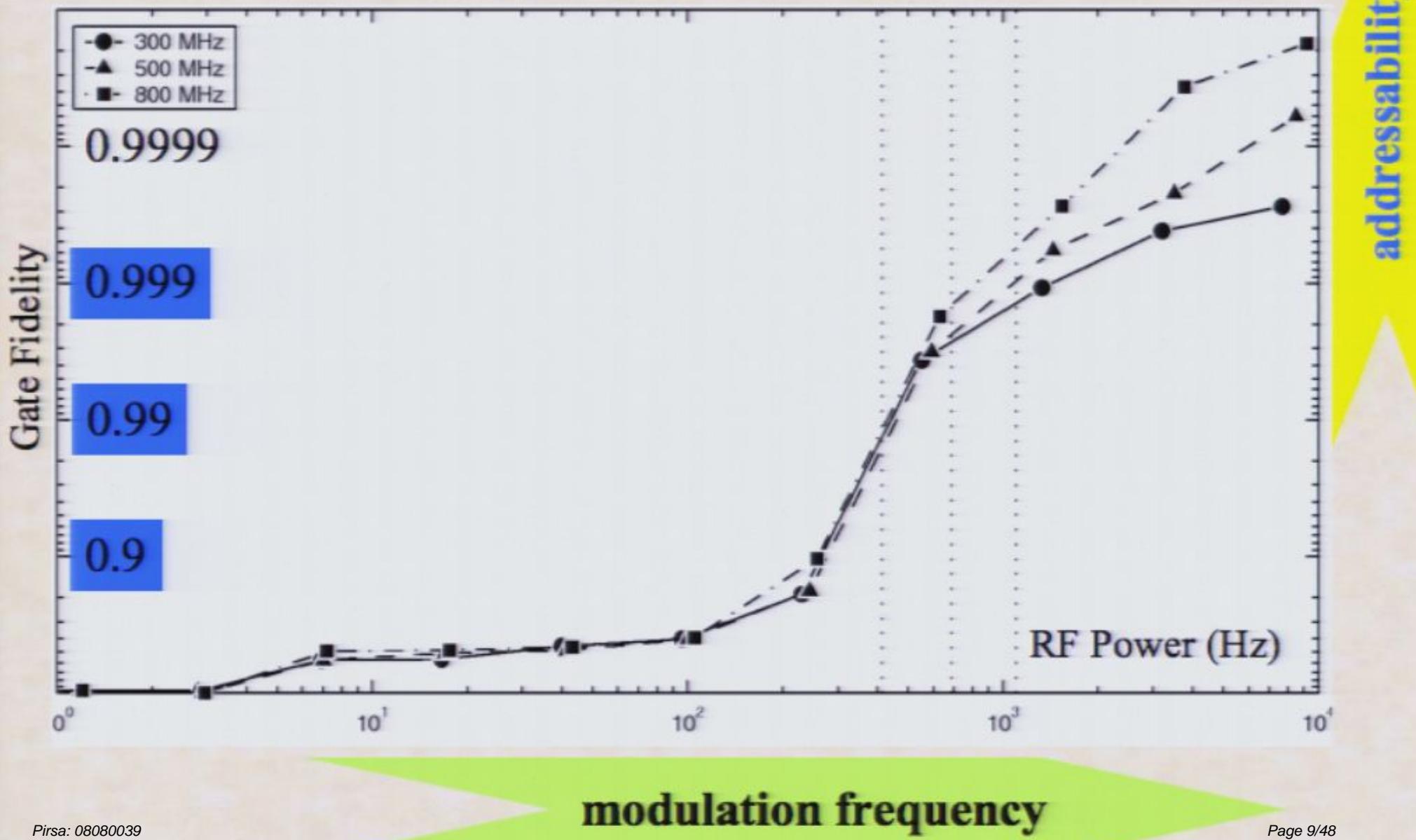
$$\frac{\partial \rho}{\partial t} = -i[H, \rho]$$

An open quantum system evolves via the Master Equation:

$$\frac{\partial \hat{\rho}}{\partial t} = (-i\tilde{H} - \tilde{\Gamma})\hat{\rho}$$



# Strongly Modulating Pulses



# Superoperator - Liouvillian Space

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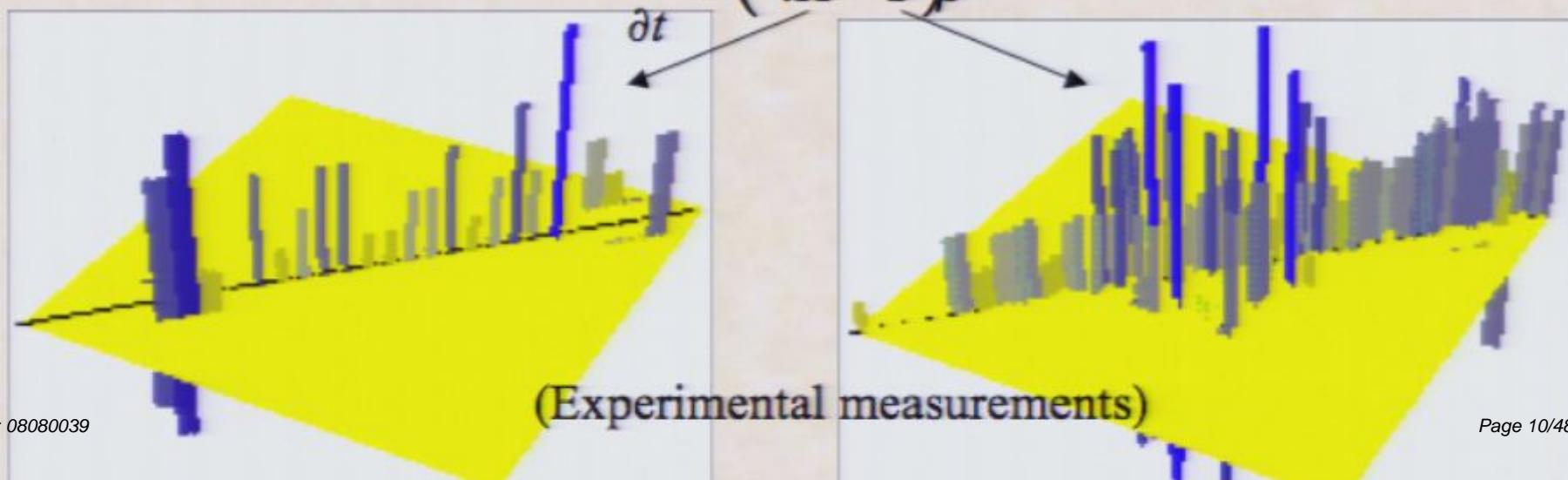
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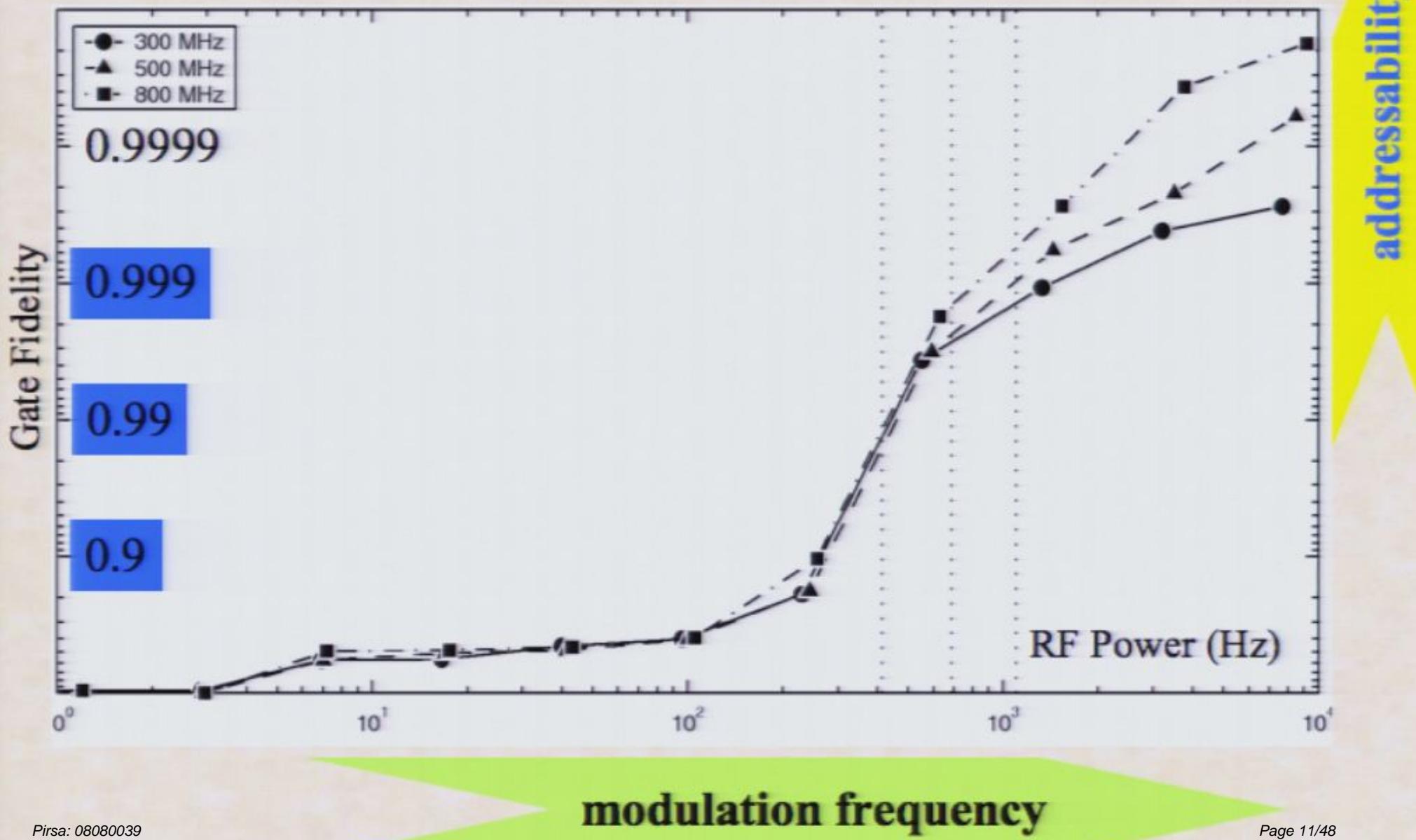
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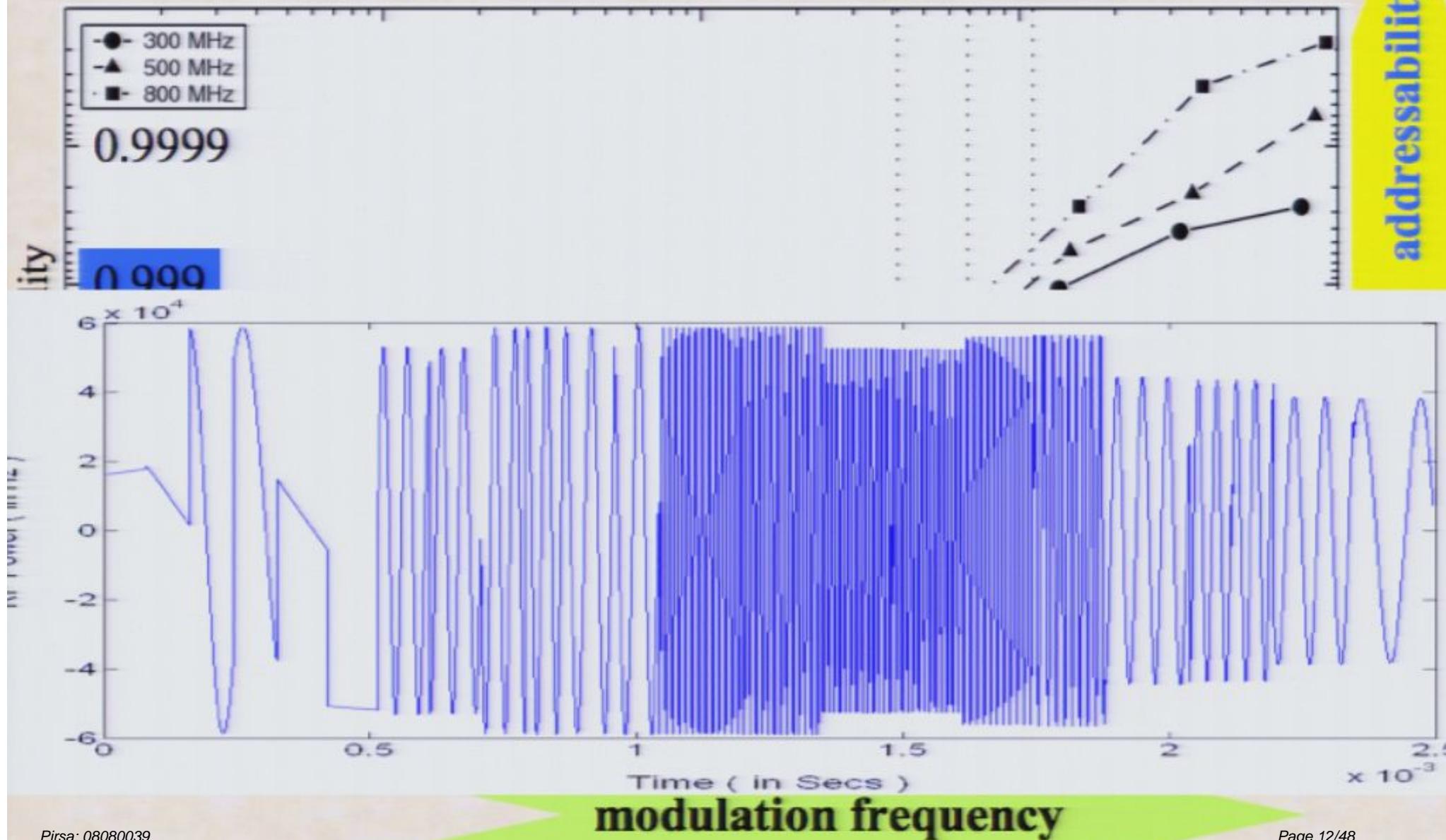
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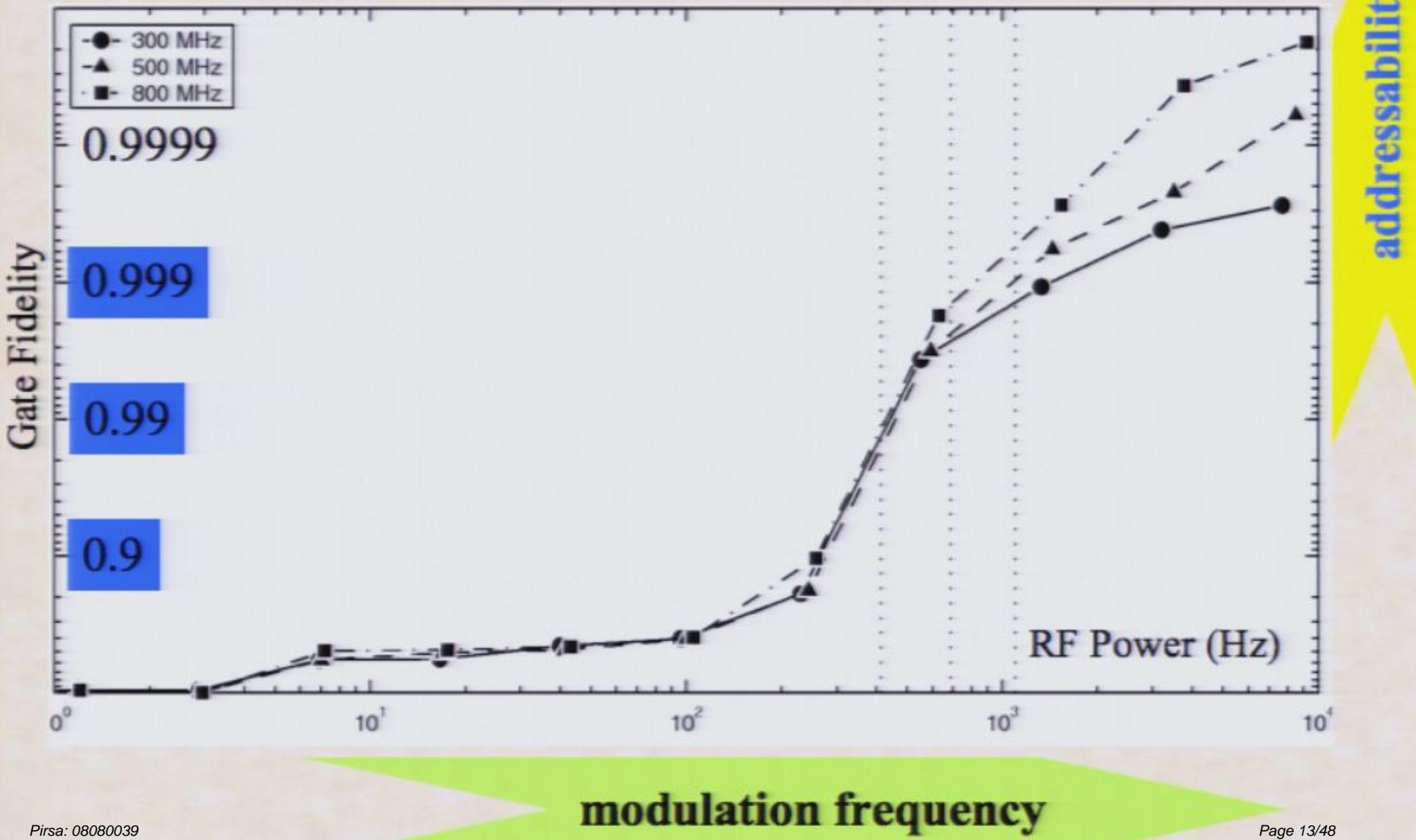
# Strongly Modulating Pulses



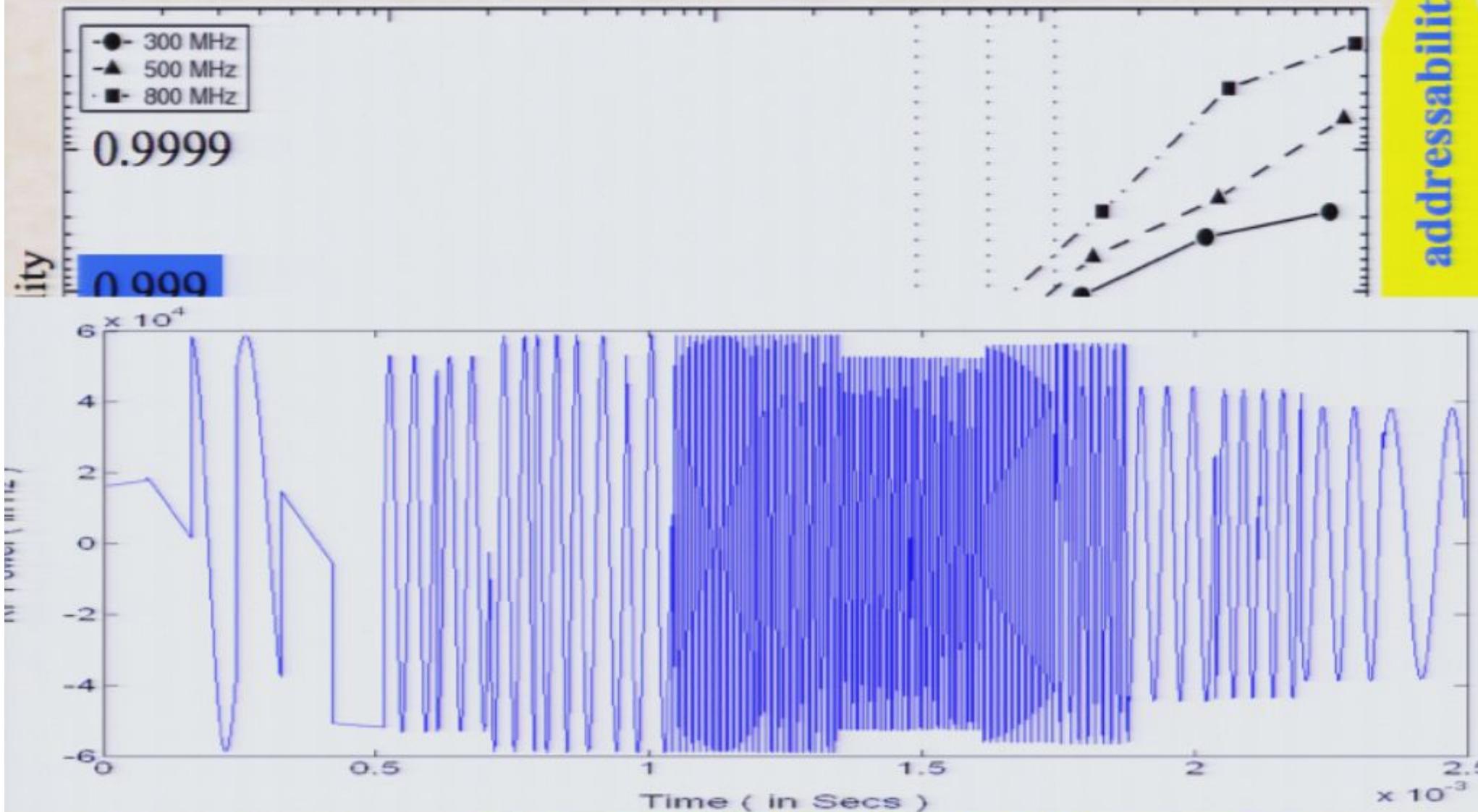
# Strongly Modulating Pulses



# Strongly Modulating Pulses



# Strongly Modulating Pulses



# QFT Superoperator

Theoretical QFT Superoperator



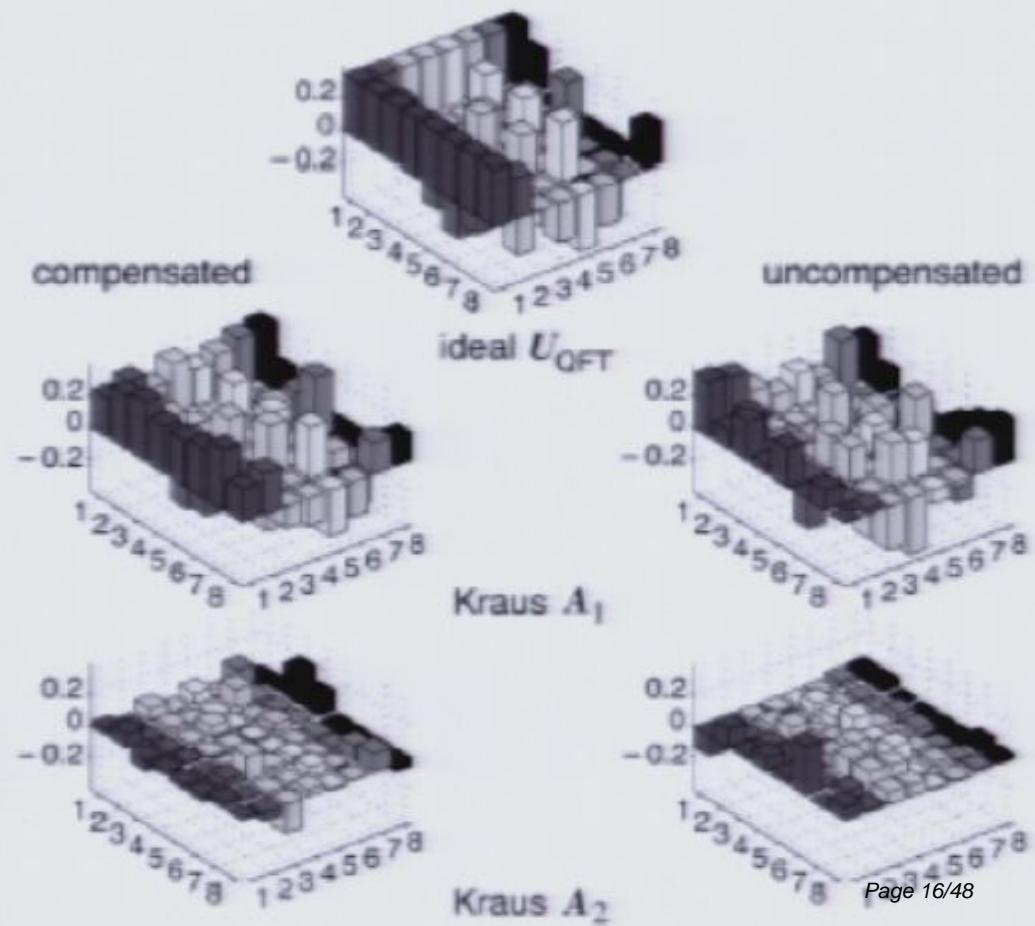
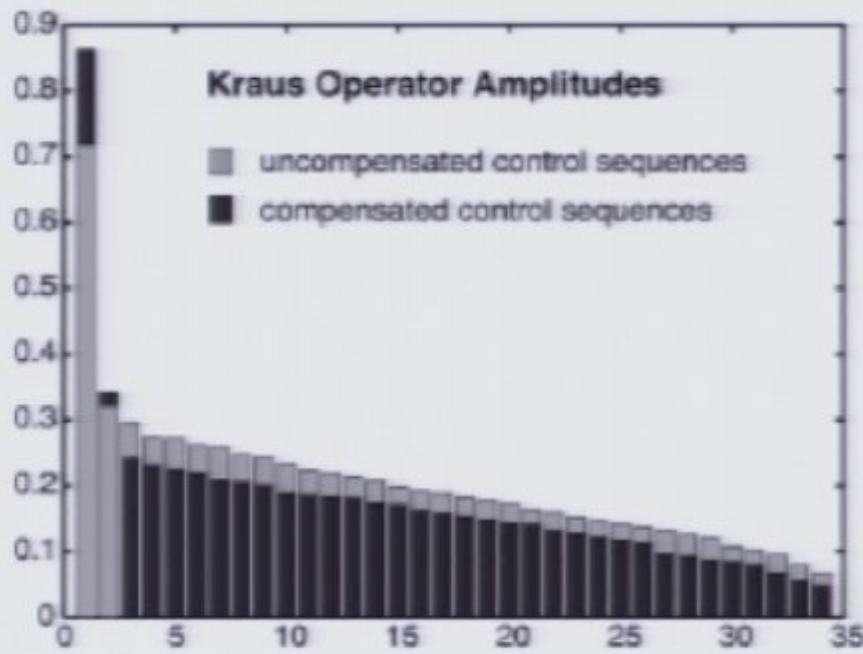
Experimental QFT Superoperator



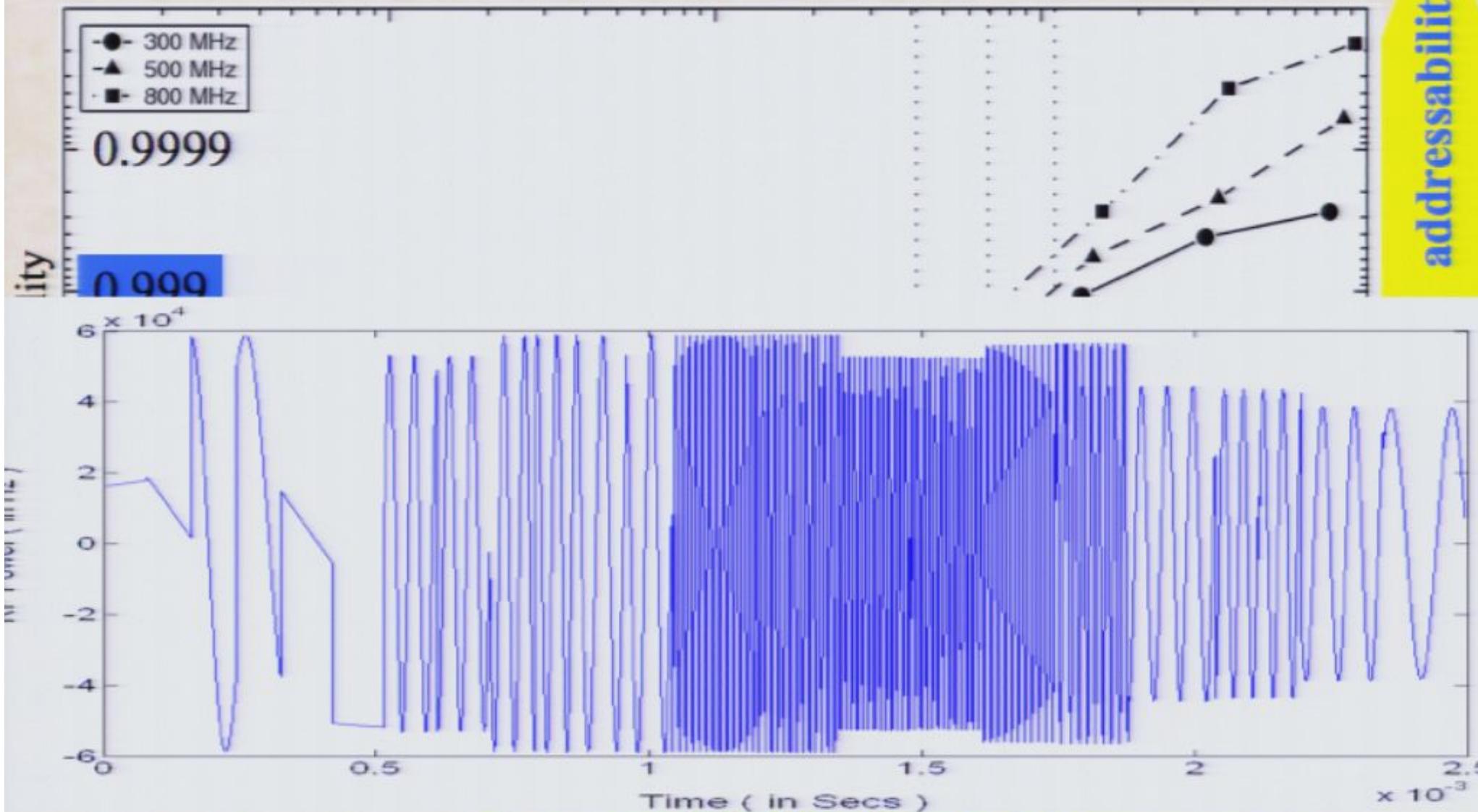
Quantum Process Tomography of the Quantum Fourier Transform, Y. Weinstein, T. Havel, J. Emerson, N. Boulant, M. Saraceno, S. Lloyd and D. G. Cory, *Journal of Chemical Physics*, 2004, 121, 6117

# Quantum Fourier Transform

A comparison of two different control schemes,  
the “compensated” data set removes  
most of the errors from RF inhomogenieties.

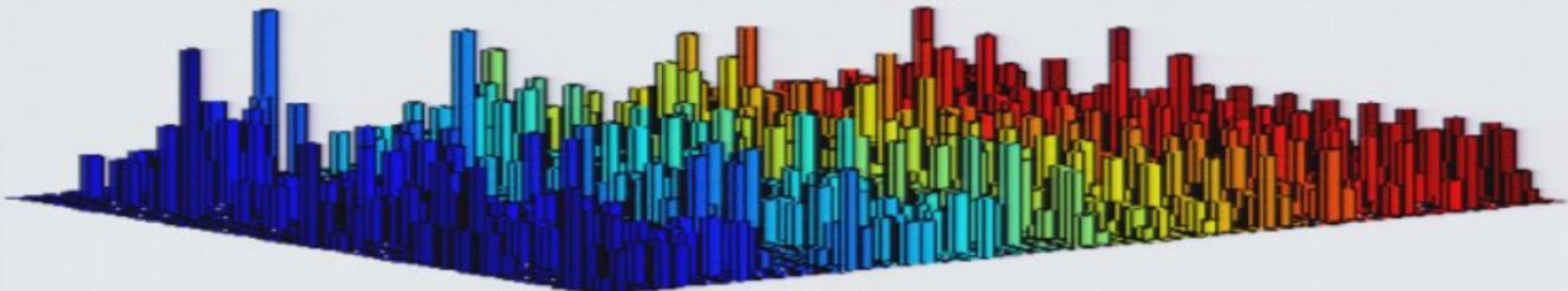


# Strongly Modulating Pulses



# QFT Superoperator

Theoretical QFT Superoperator



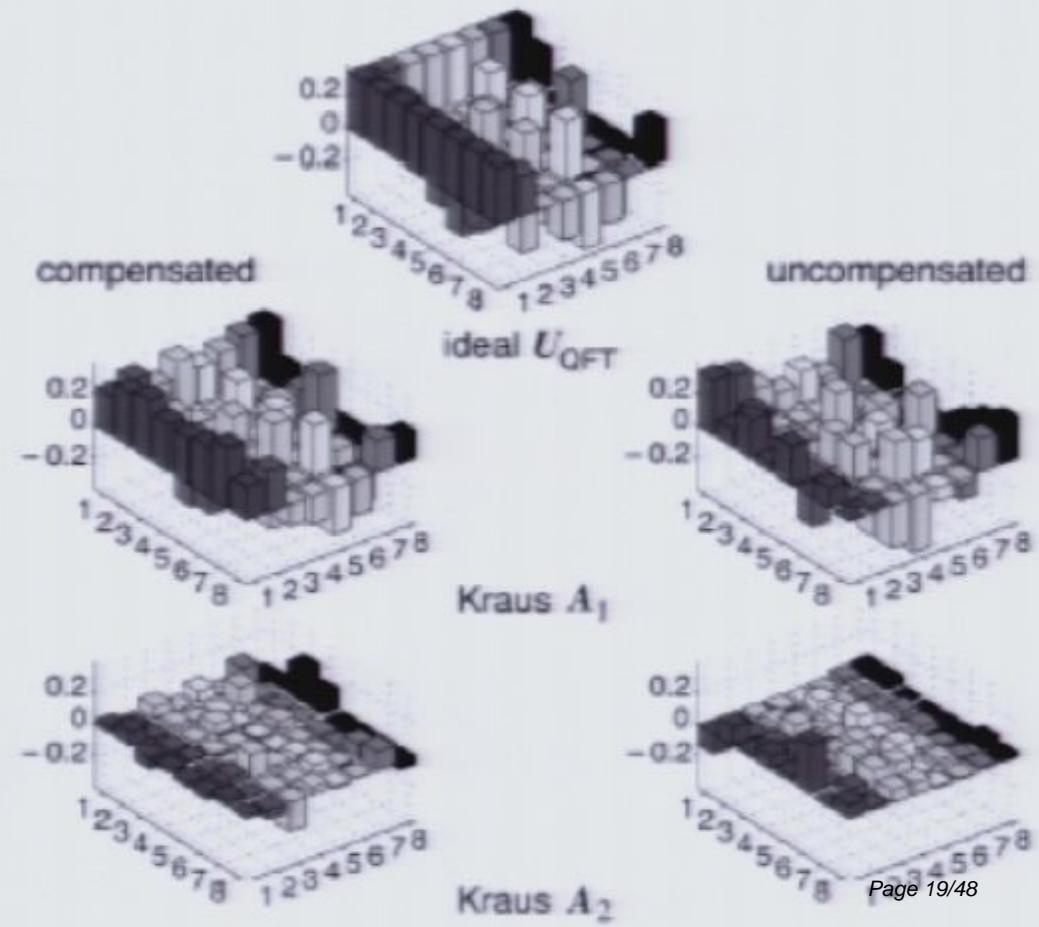
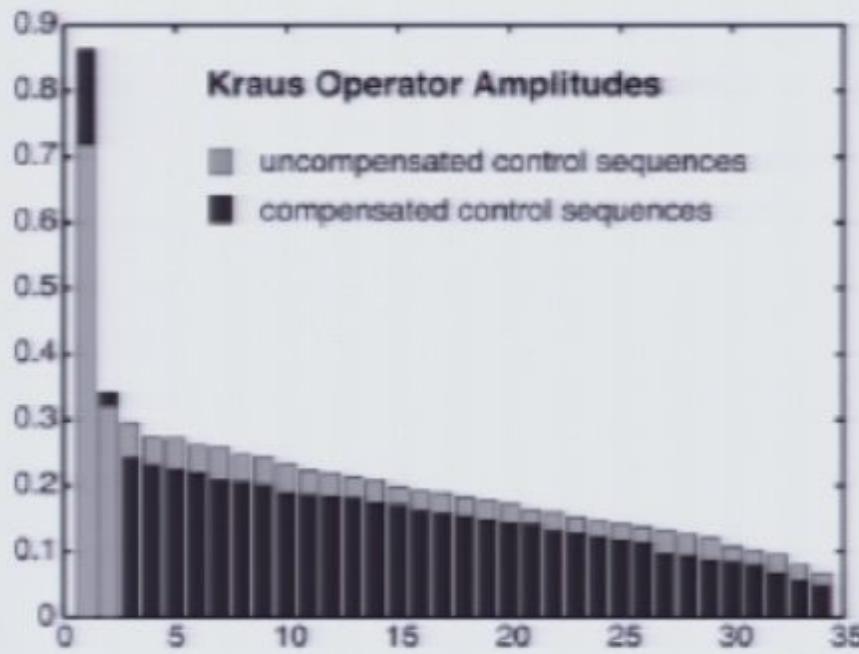
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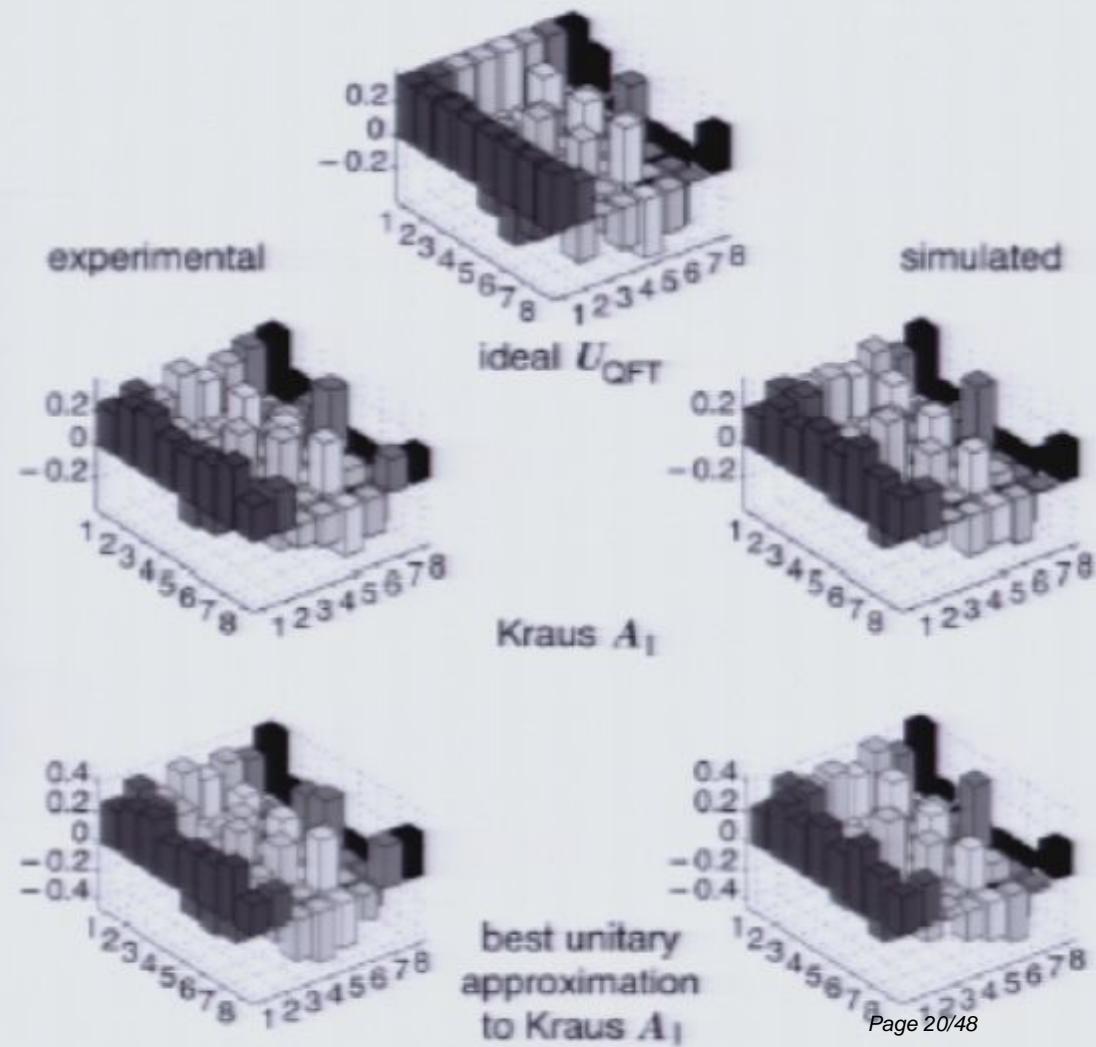
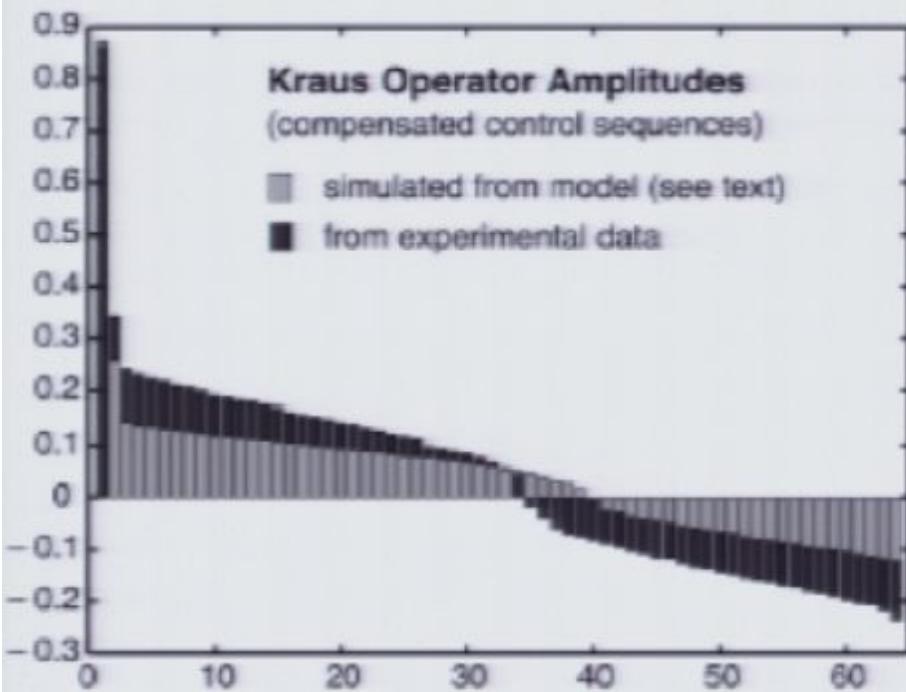
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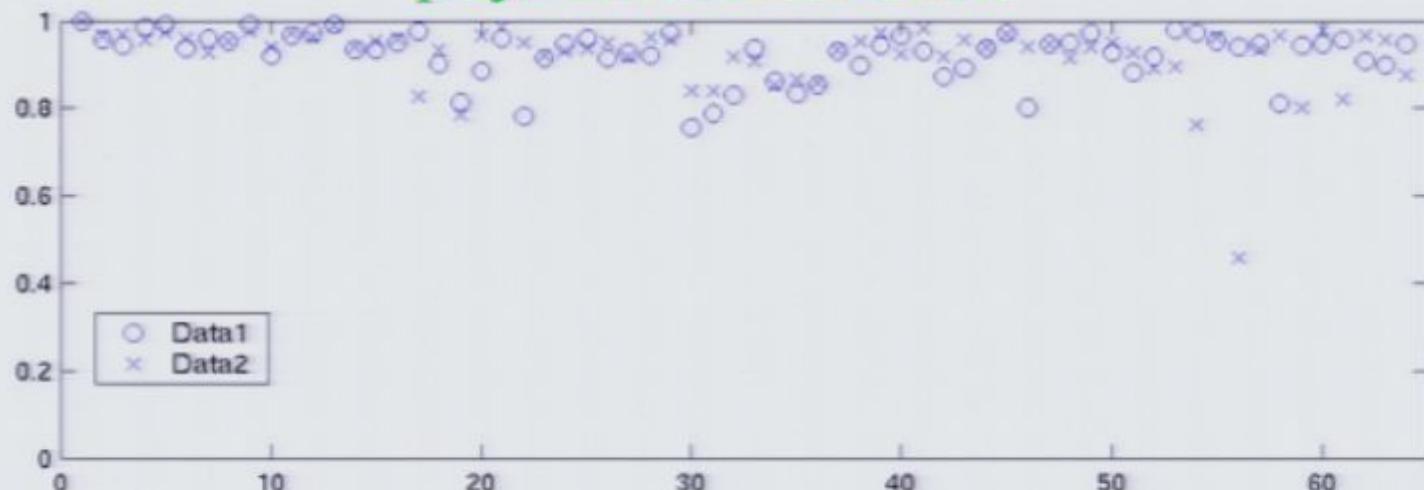
# Quantum Fourier Transform

## A test of our system model

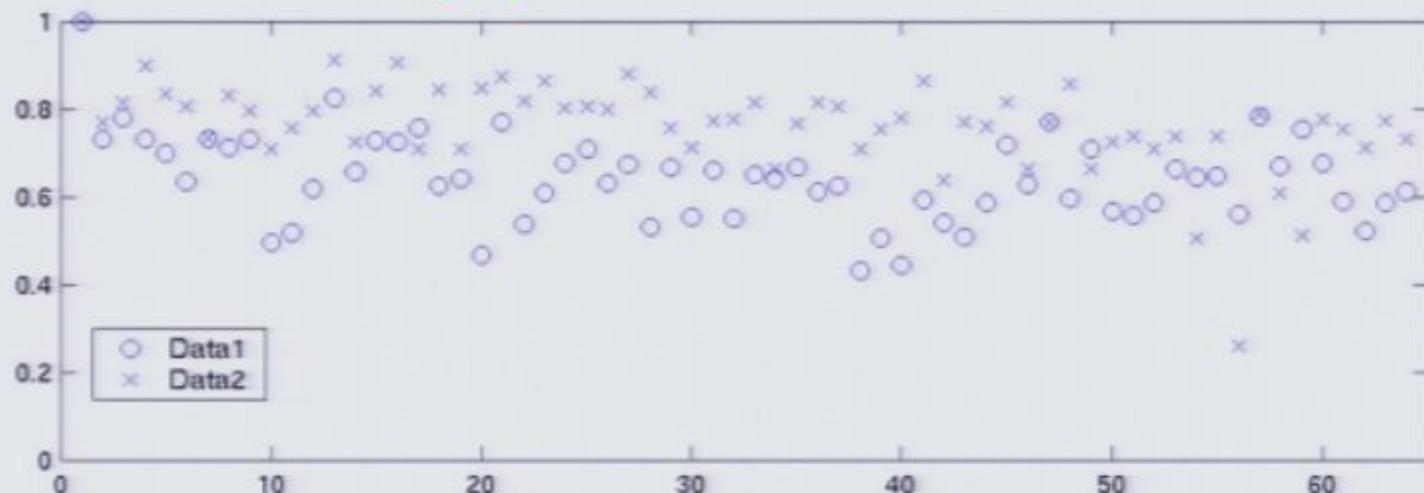


# Experimental Improvement

projection of initial states

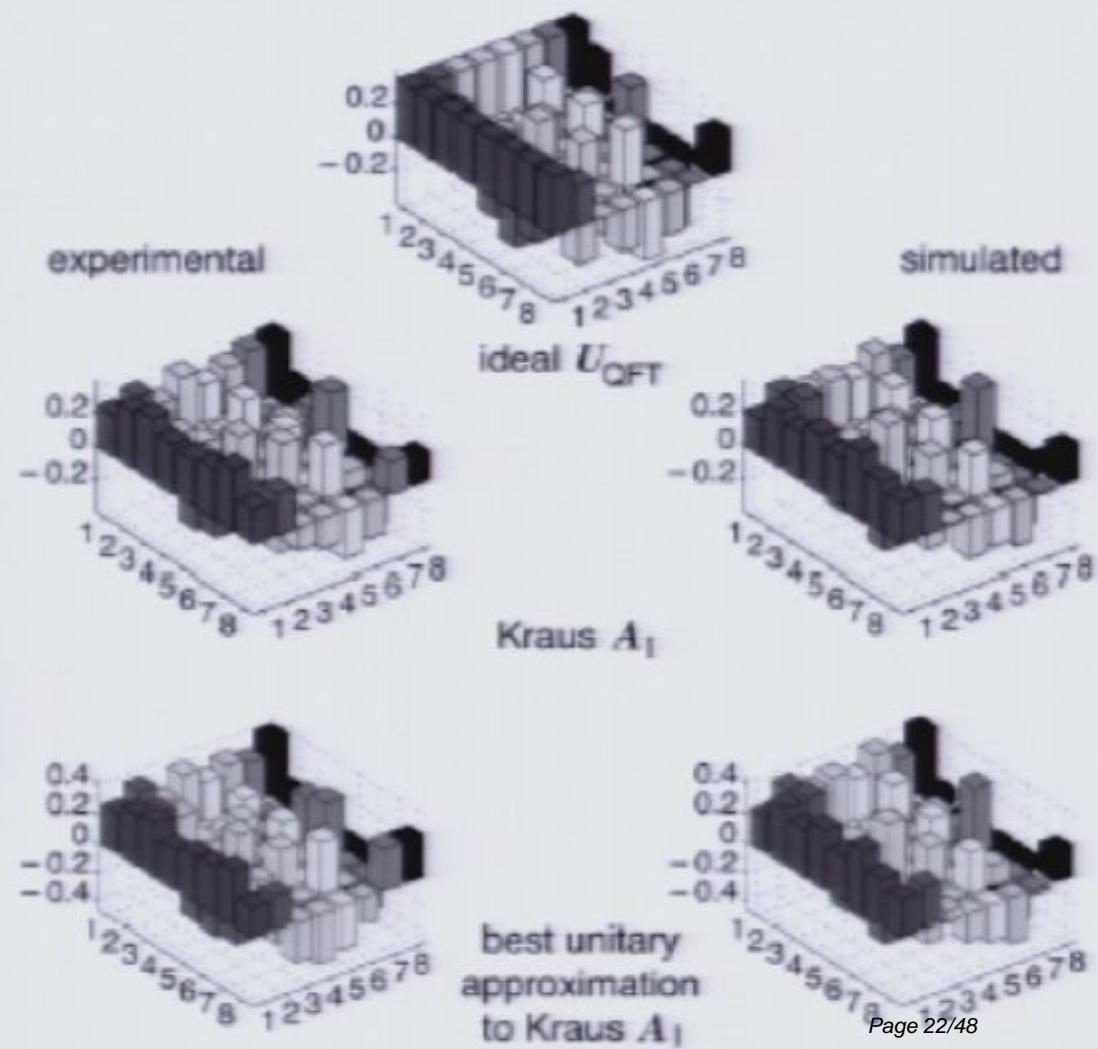
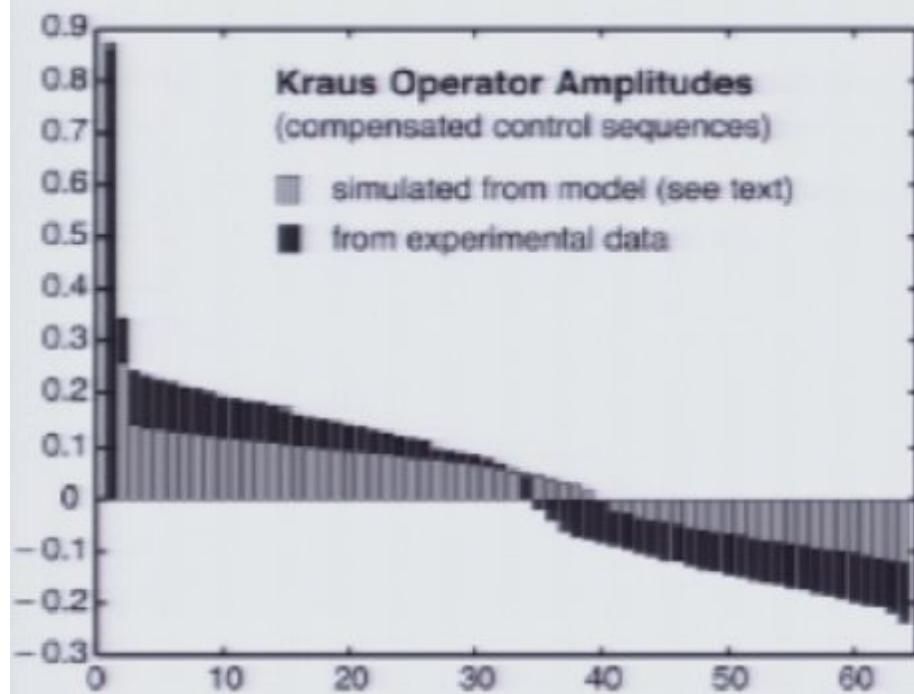


projection of final states



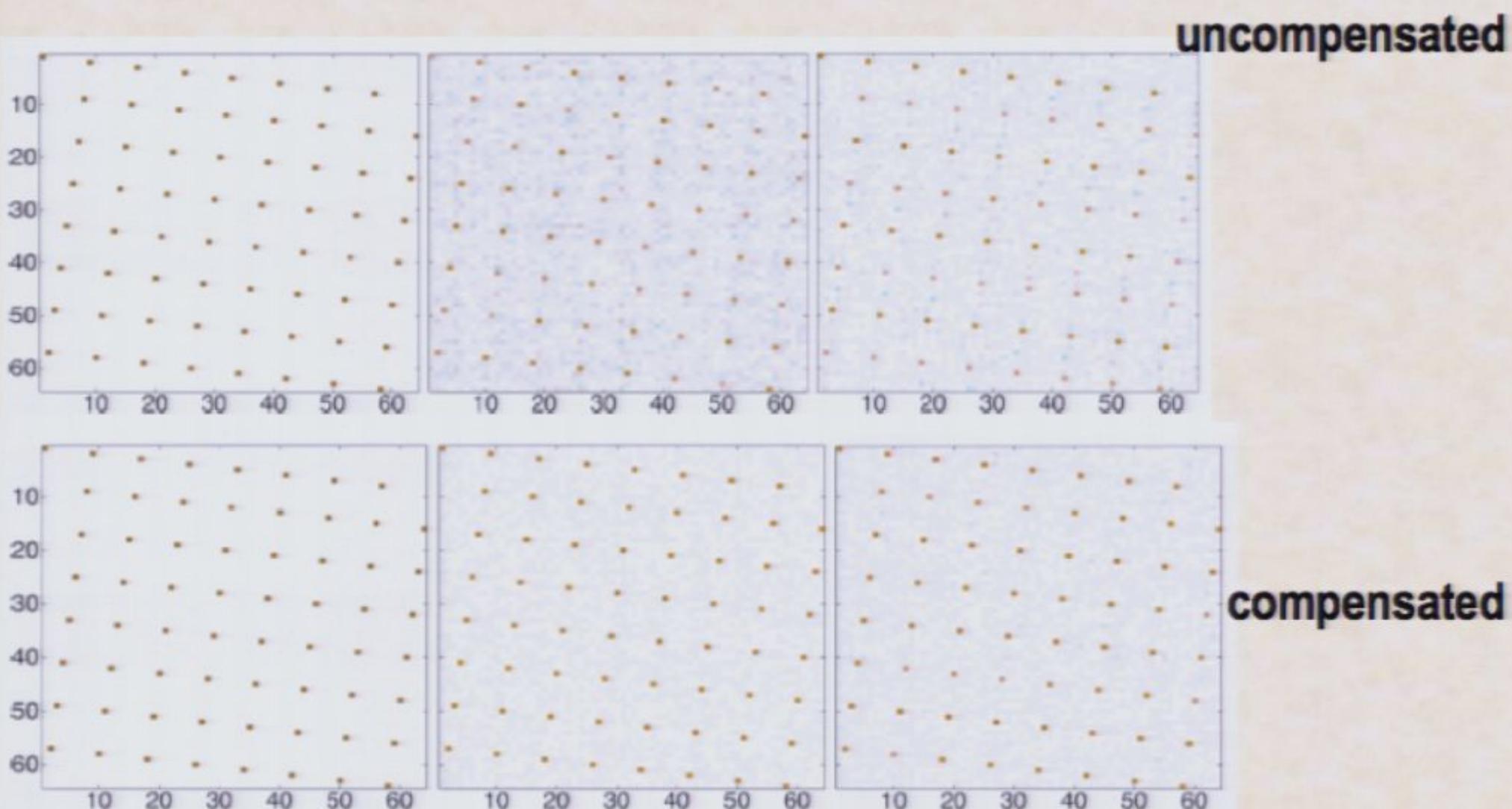
# Quantum Fourier Transform

## A test of our system model



# Quantum Fourier Transform

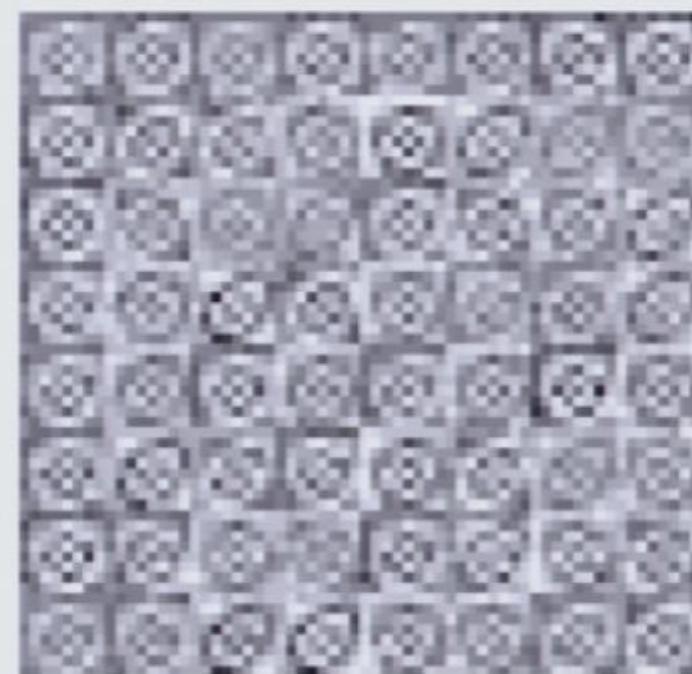
The Quantum Fourier transform is a permutation matrix in phase space.



# QFT Superoperator



Ideal QFT Superoperator



Experimental QFT Superoperator

Basis is Zeeman.

# Product Operator Basis

Largest error:  $\sigma_z^3$

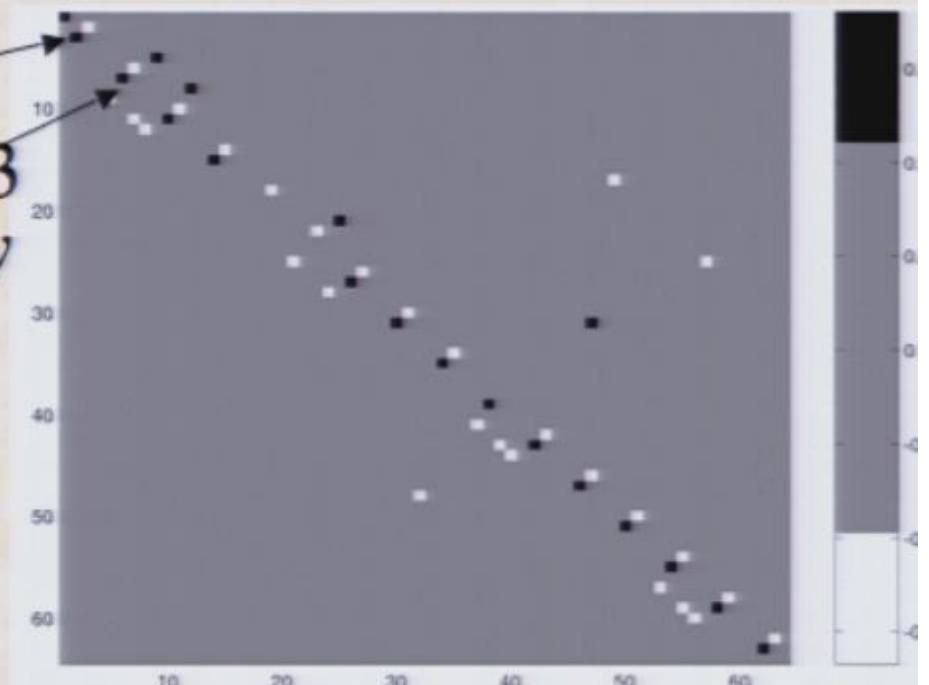
$$\sigma_x^3 \Rightarrow \sigma_y^3$$

$$\sigma_x^2 \sigma_x^3 \Rightarrow \sigma_x^2 \sigma_y^3$$

2nd largest error:  $\sigma_z^2$

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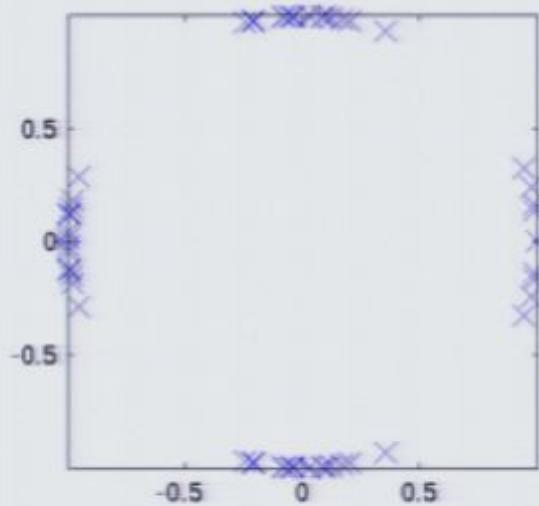


Errors show up clearly when the theoretical action of the QFT is removed and the remainder is transformed to the product operator basis.

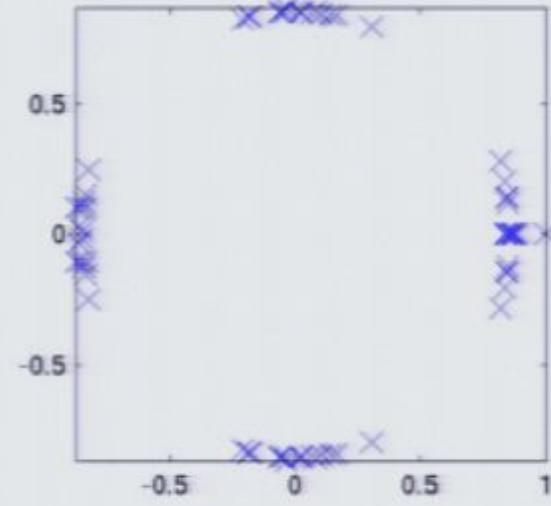
# Superoperator Eigenvalues

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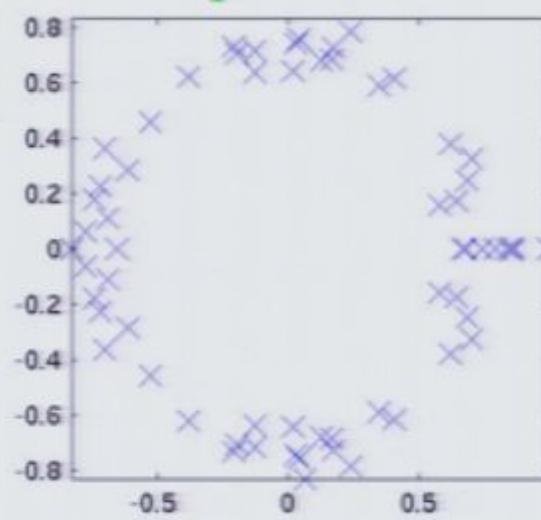
unitary simulation



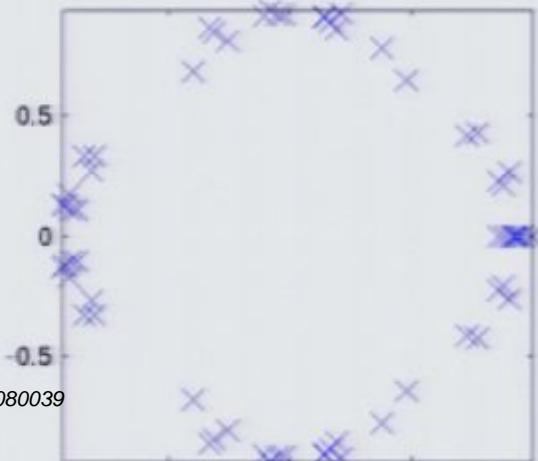
simulation with decoherence



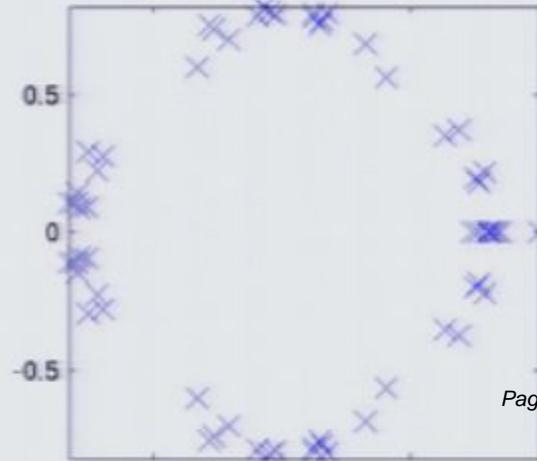
experimental



simulation with incoherence



simulation with decoherence and incoherence



# Quantum Process Tomography of a 3-Qubit Fourier Transform

## What we learned:

Gate fidelity with theoretical superoperator of 0.64

Correlation with superoperator simulated from full system model 0.79

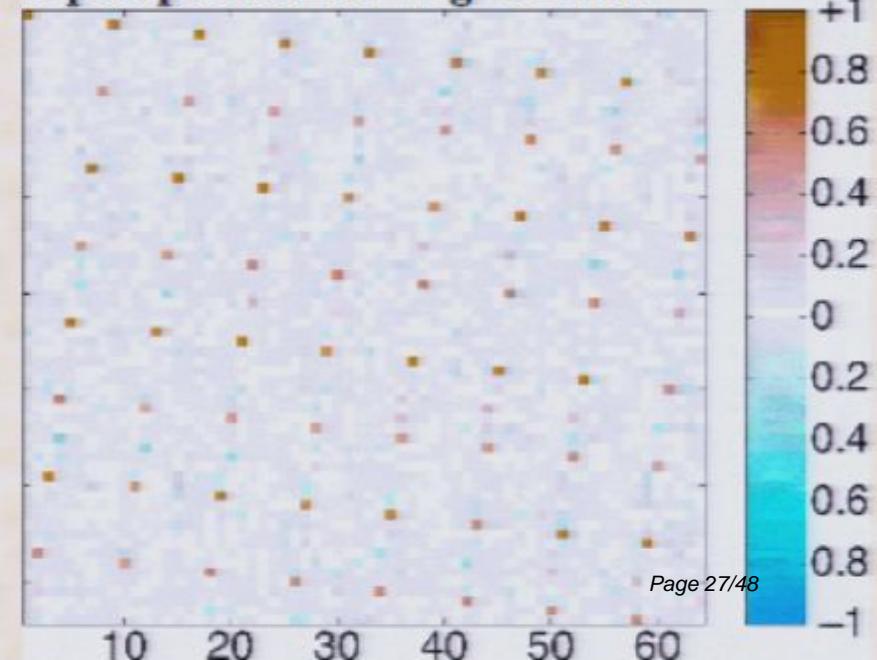
Lack of complete positivity of experimental due primarily to systematic variation of operations over the ensemble

Complete positivity could however be restored with only small changes to the superoperator (correlation 0.97 before & after)

Correlation with simulation could be increased to 0.97 by single qubit rotations

System model is very good, but could be still better. Know the statics, now we need to add transients.

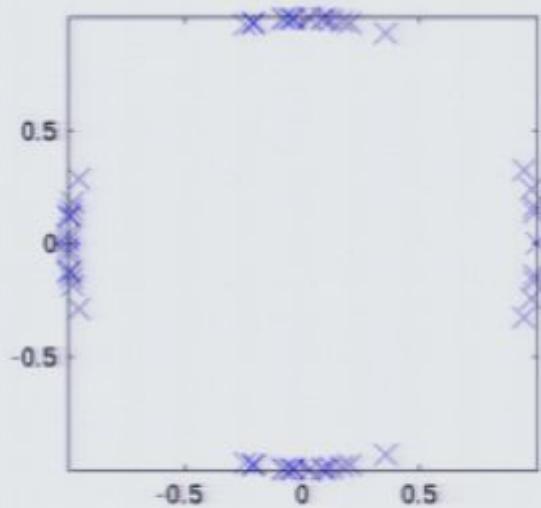
Superoperator vs. Wigner basis



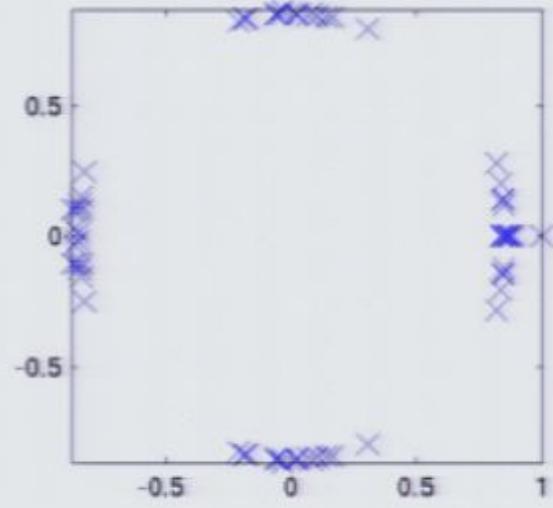
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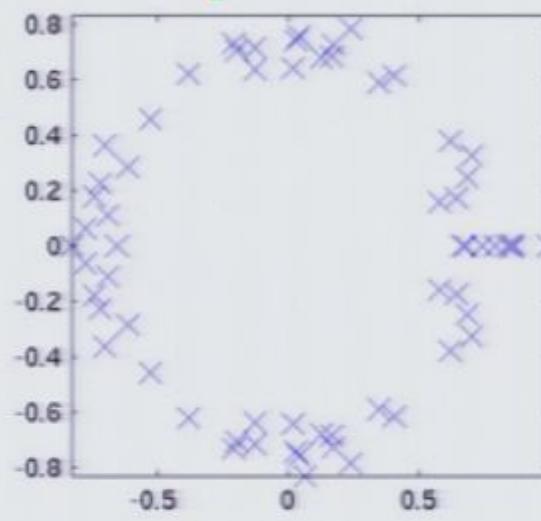
unitary simulation



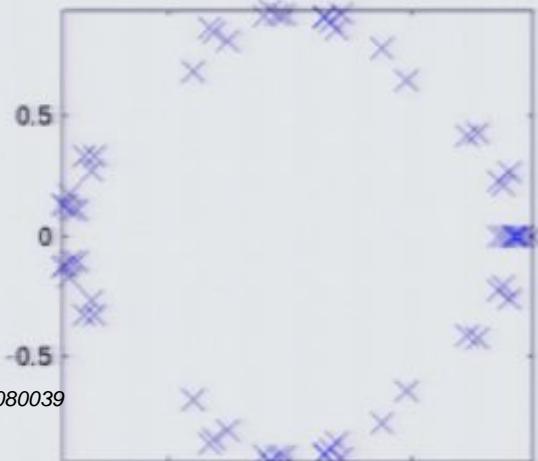
simulation with decoherence



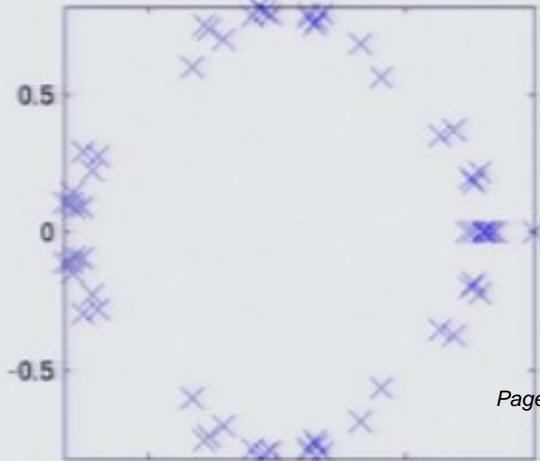
experimental



simulation with incoherence



simulation with decoherence and incoherence



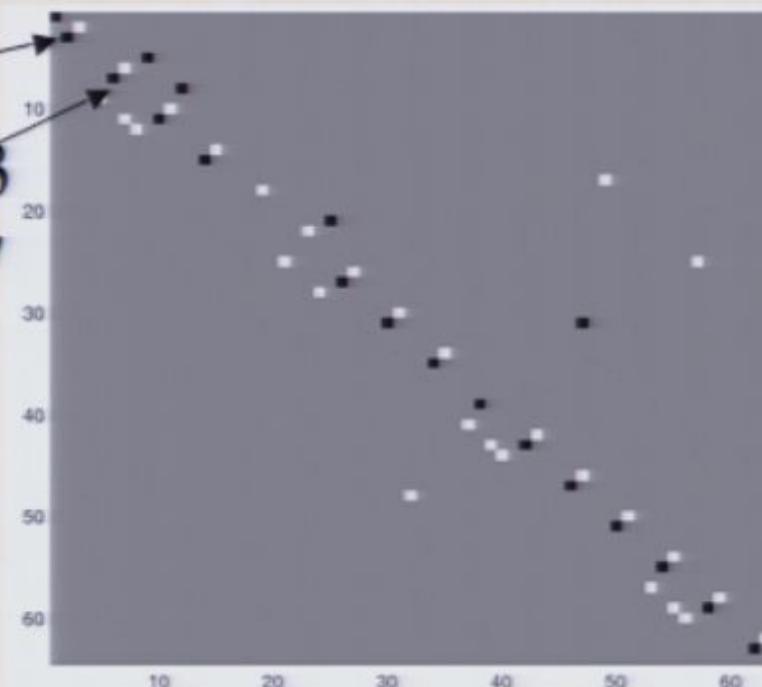
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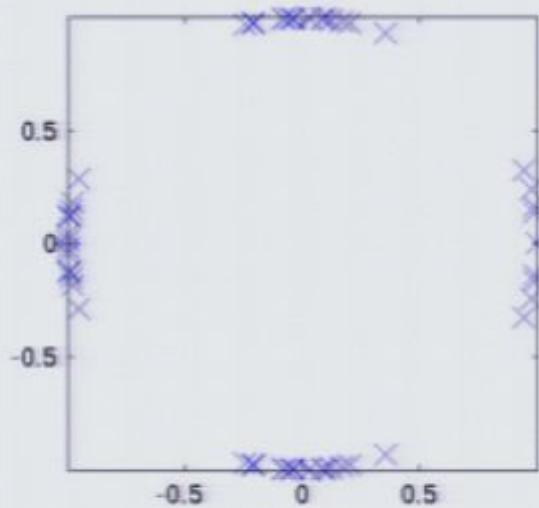


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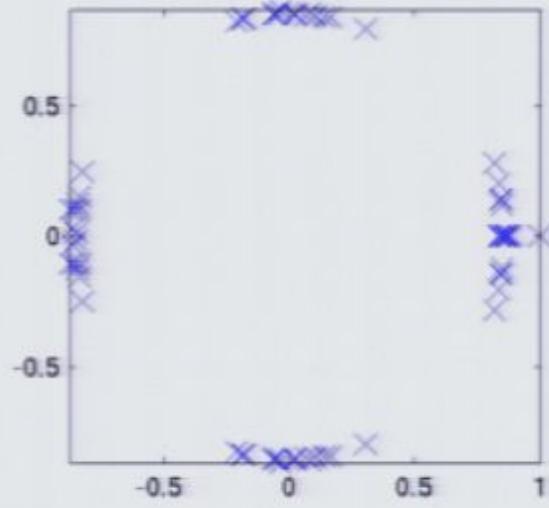
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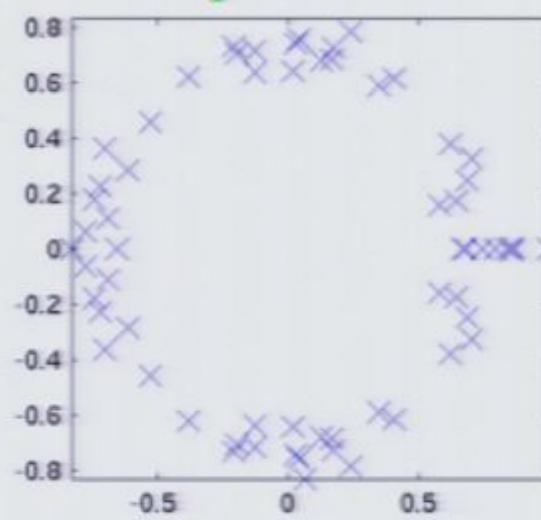
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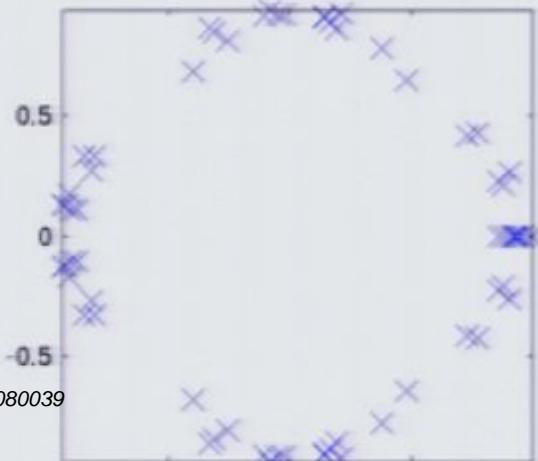
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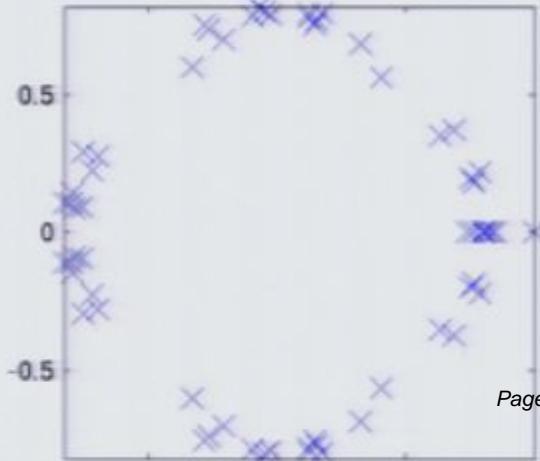
experimental



simulation with incoherence



simulation with decoherence and incoherence



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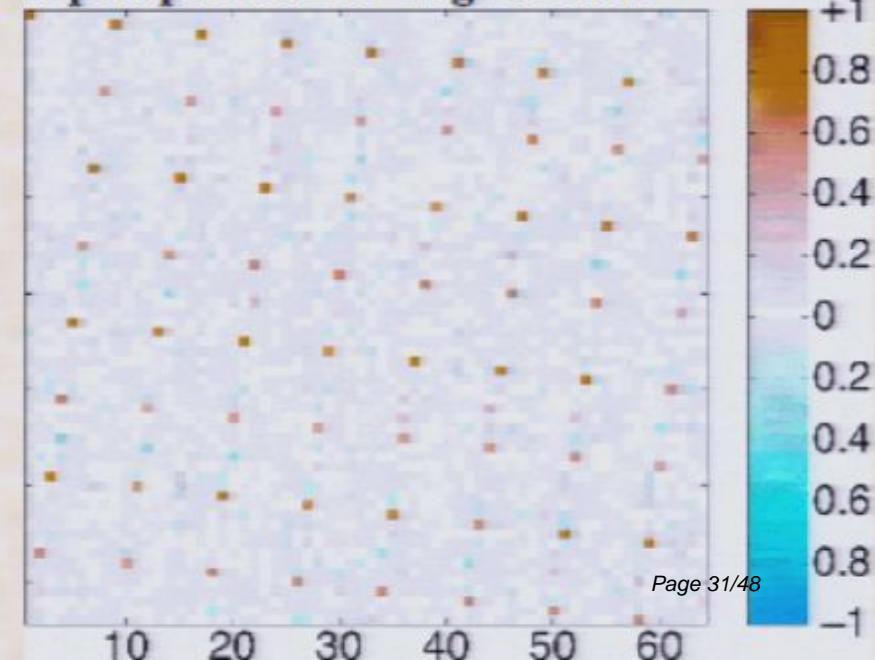
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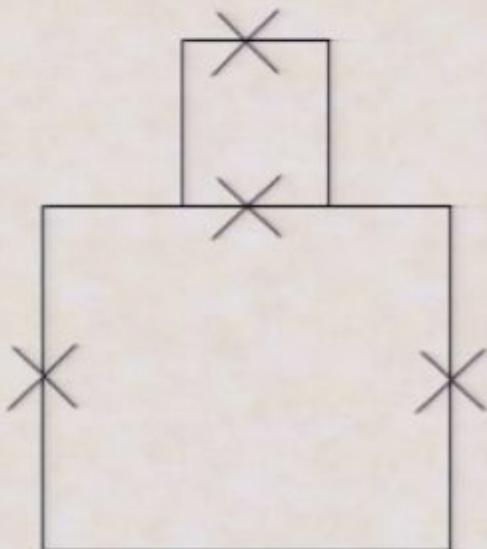
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Superoperator vs. Wigner basis





$$H_{\text{int}} = \omega_0 \sigma_x$$

$$H_{\text{bias}}(t) = \omega_B(t)(\sigma_z + \epsilon \sigma_x)$$

$$H_{\text{couple}}(t) = \omega_C(t)(\sigma_x + \epsilon' \sigma_z)$$

$$H_{\mu w}(t) = \omega_A(t) \cos[\omega_T t + \phi](\sigma_z + \epsilon \sigma_x)$$

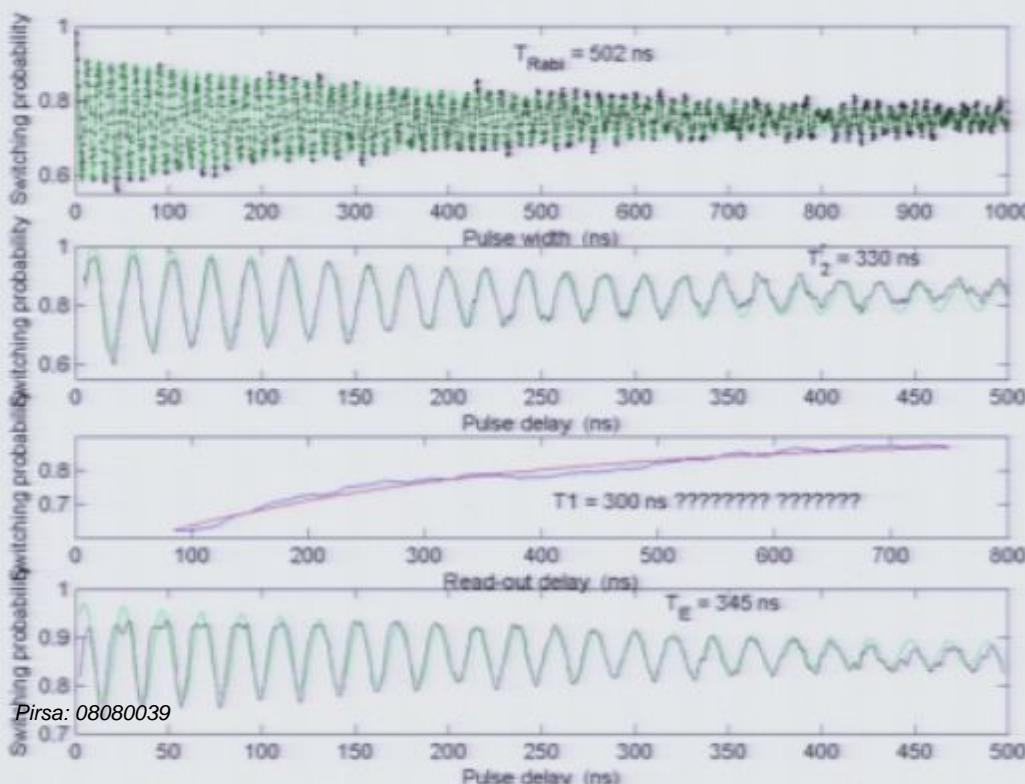
**At anti-crossing**

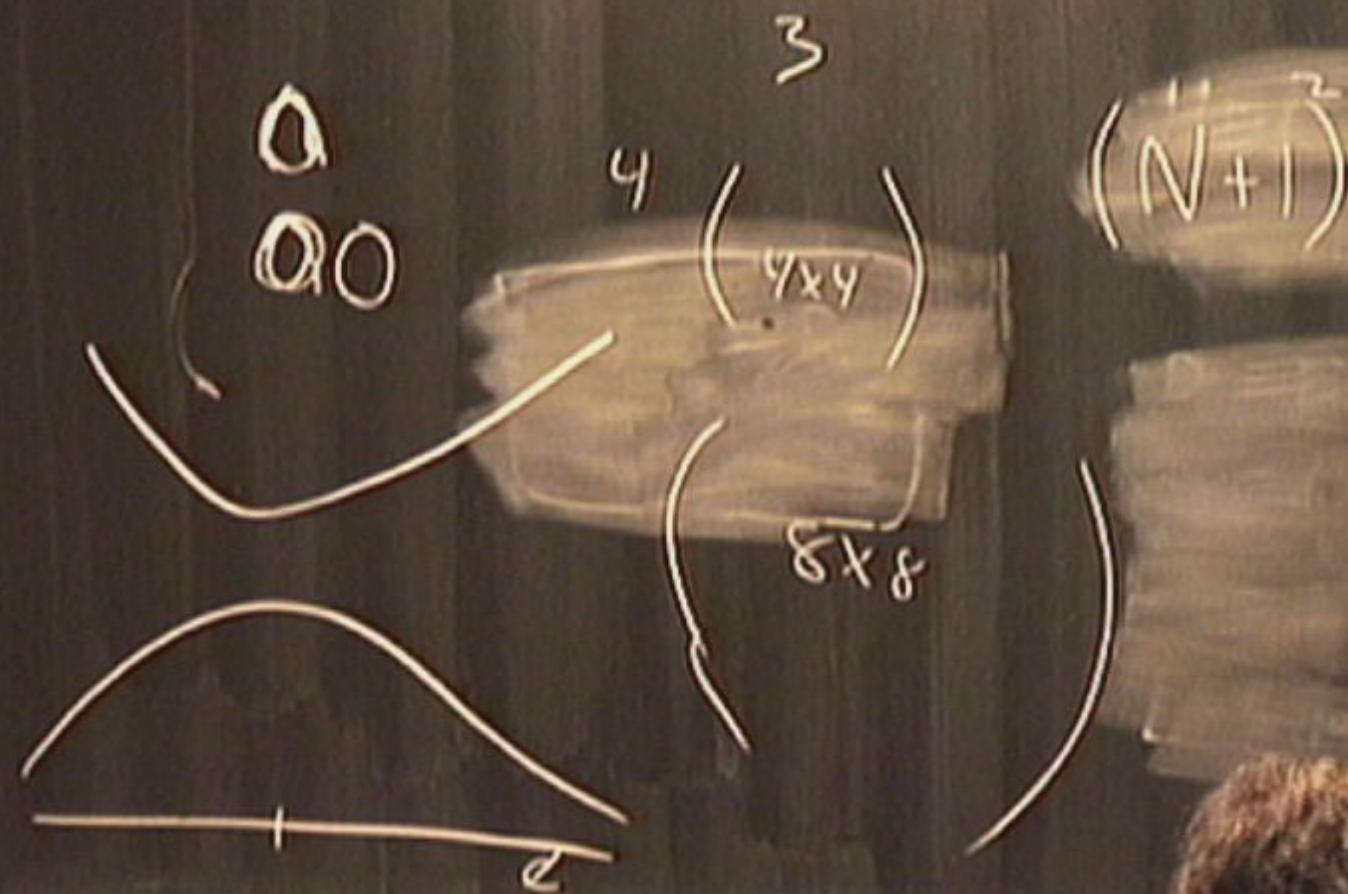
$$T_1 = 300 \text{ ns}$$

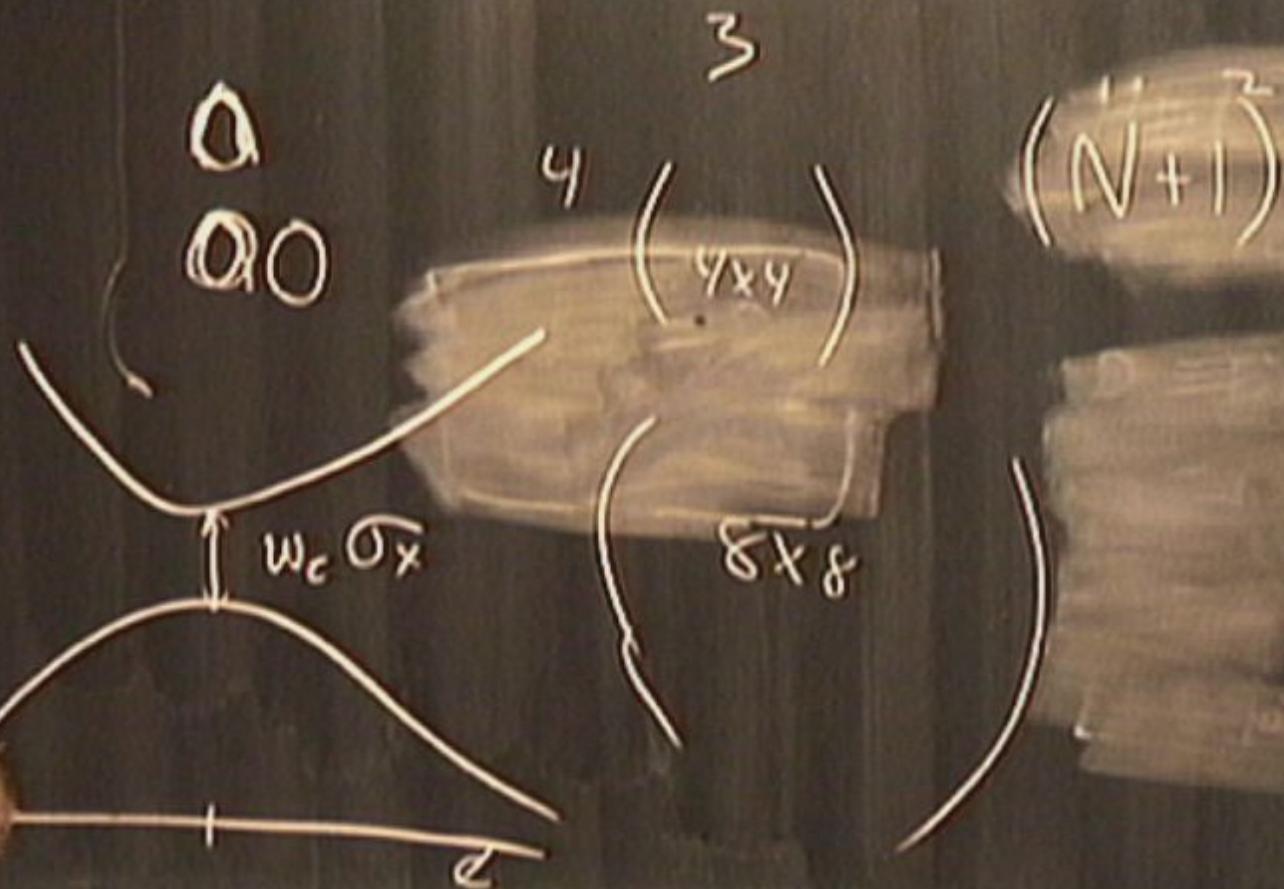
$$T_2^* = 330 \text{ ns}$$

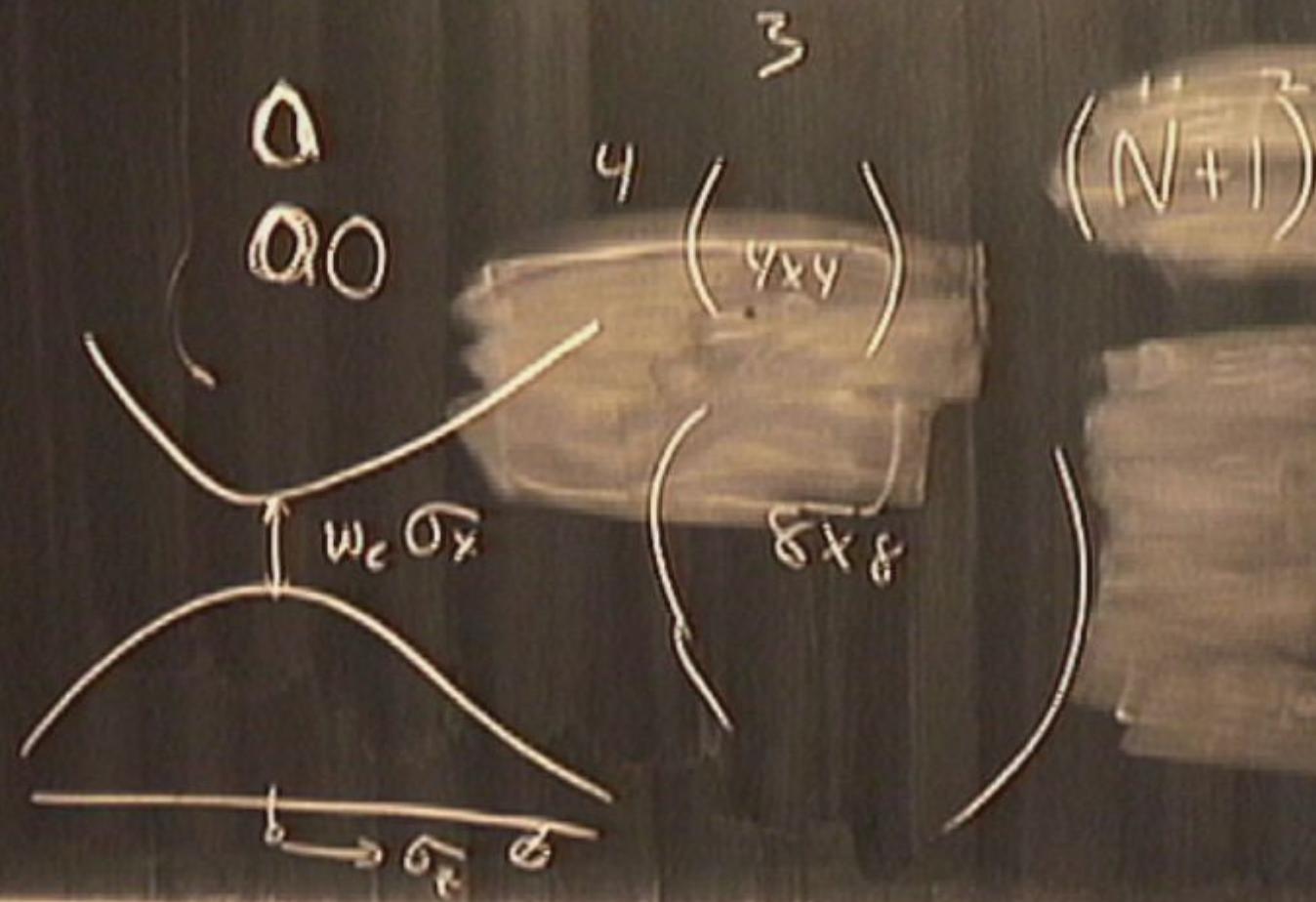
$$T_{\text{echo}} = 345 \text{ ns}$$

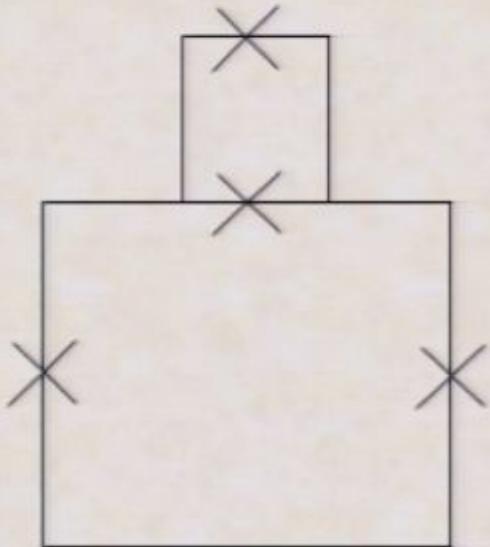
$$T_{\text{Rabi}} = 500 \text{ ns}$$











$$H_{\text{int}} = \omega_0 \sigma_x$$

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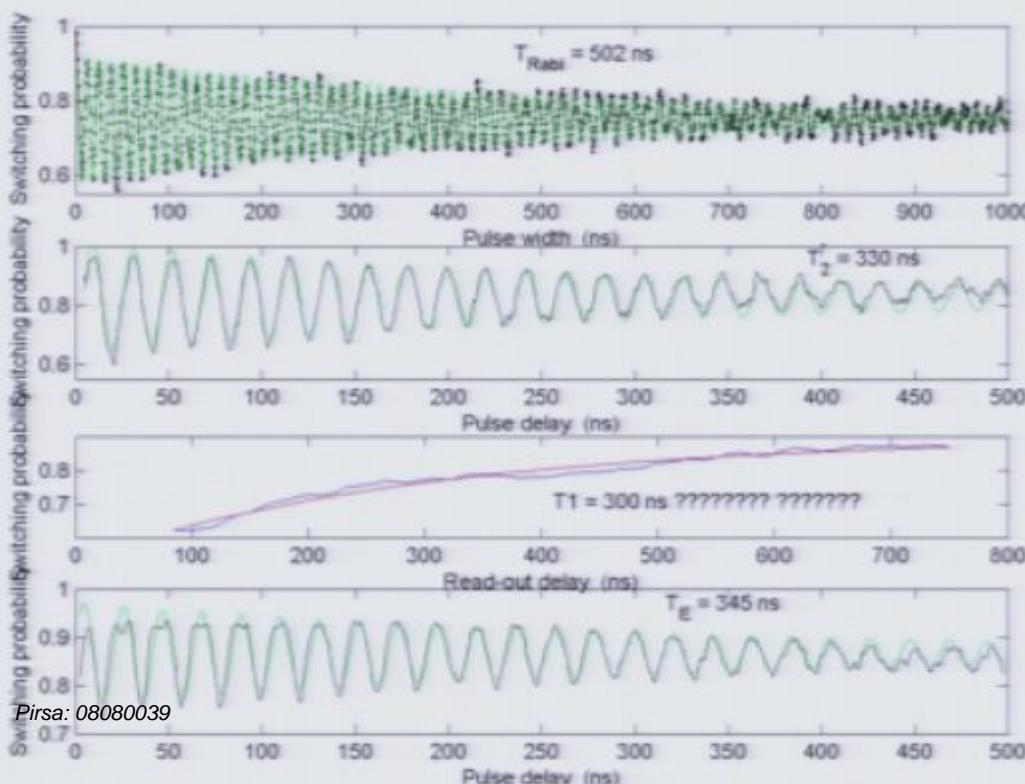
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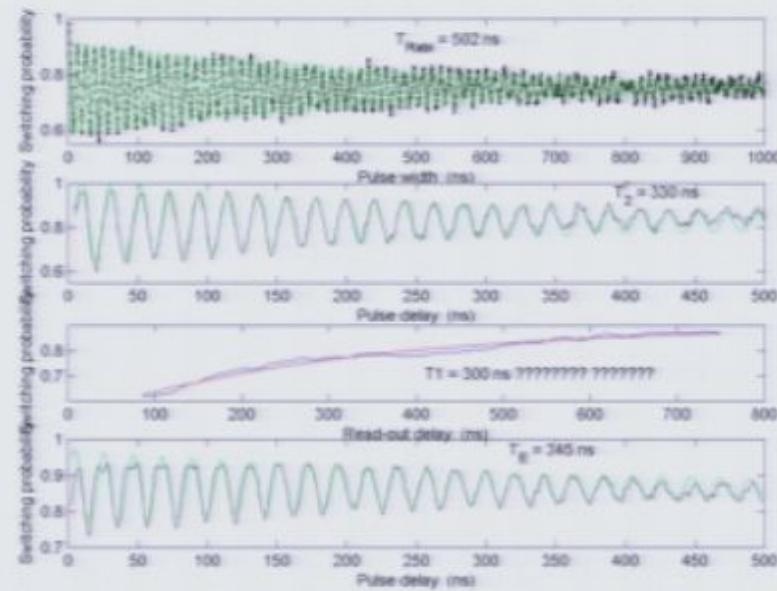
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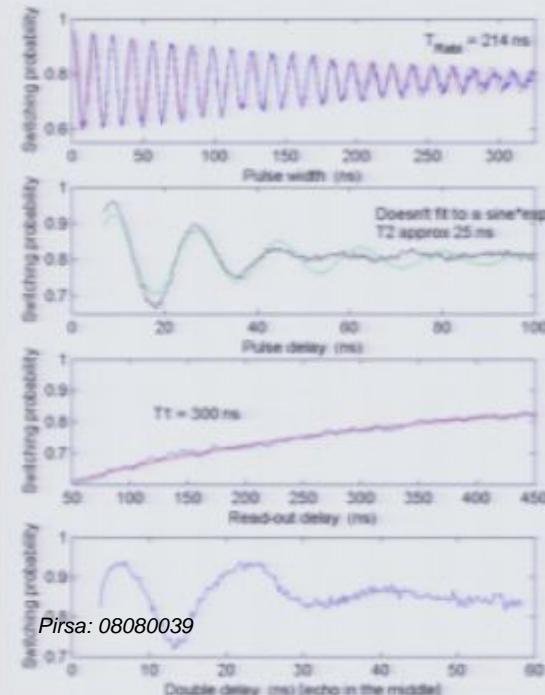


# Coherence dies fast when off degeneracy

**A**

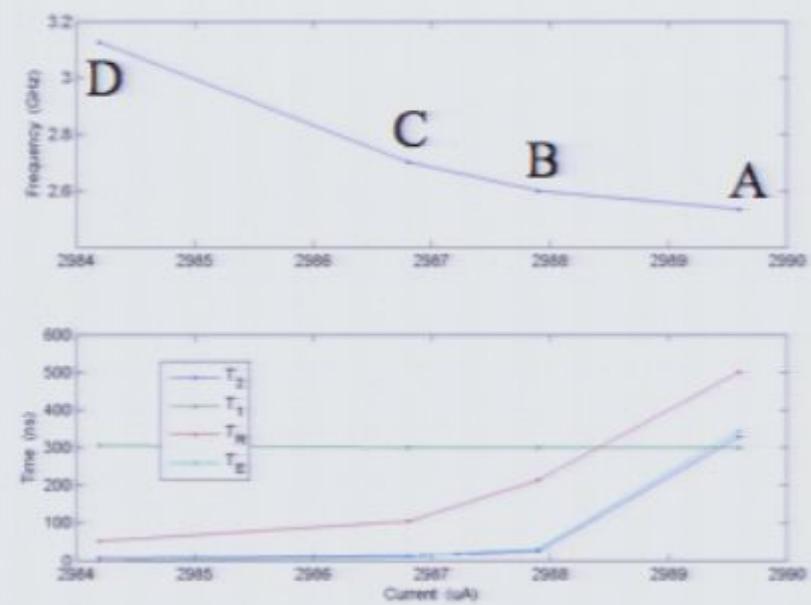
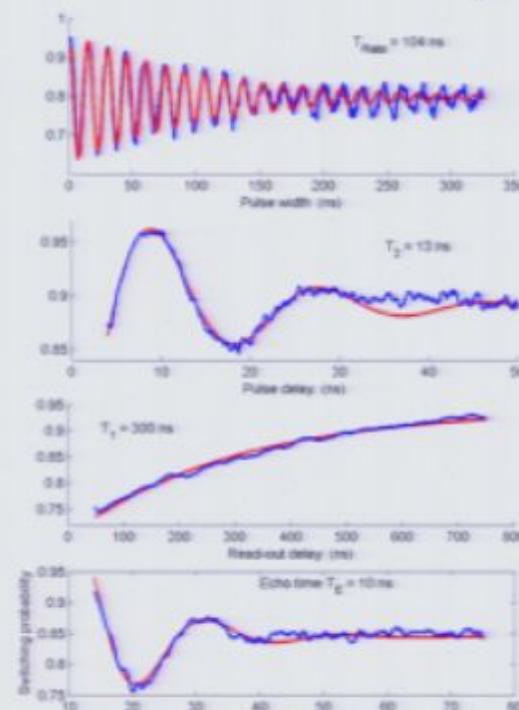


**B**

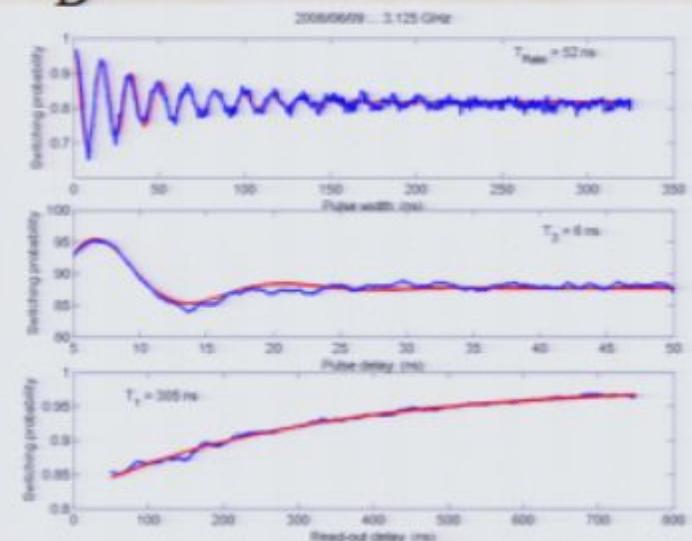


Pirsa: 08080039

**C**



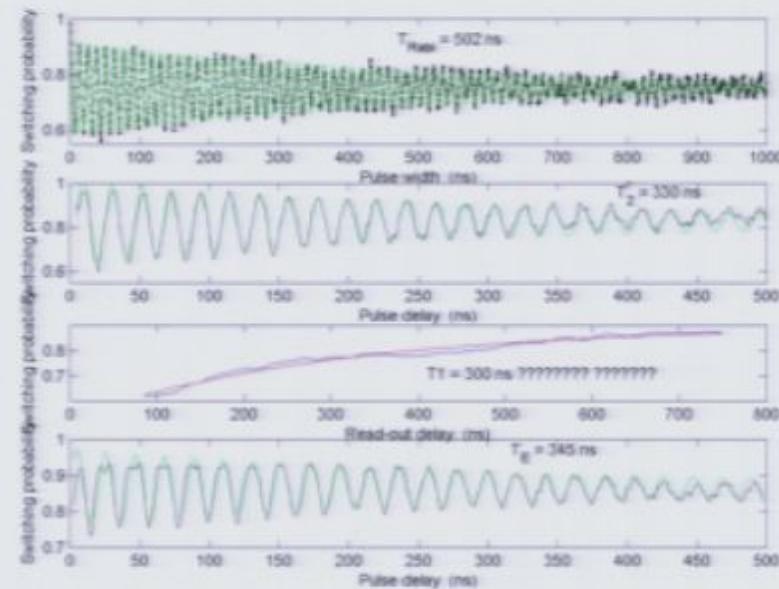
**D**



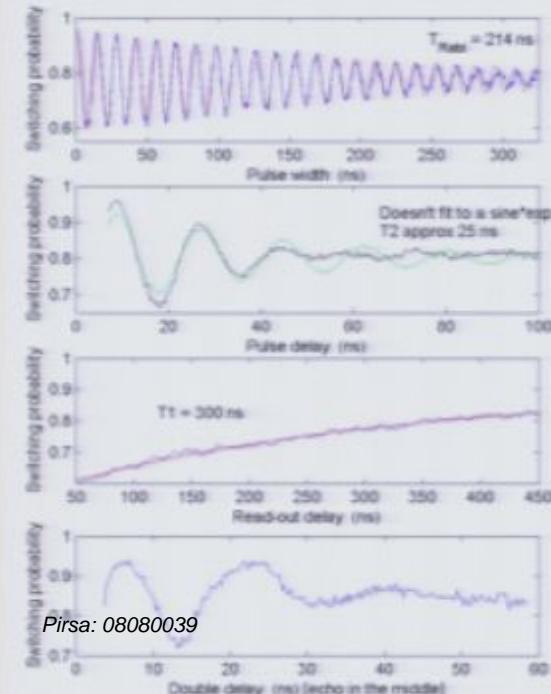
Page 37/48

# Coherence dies fast when off degeneracy

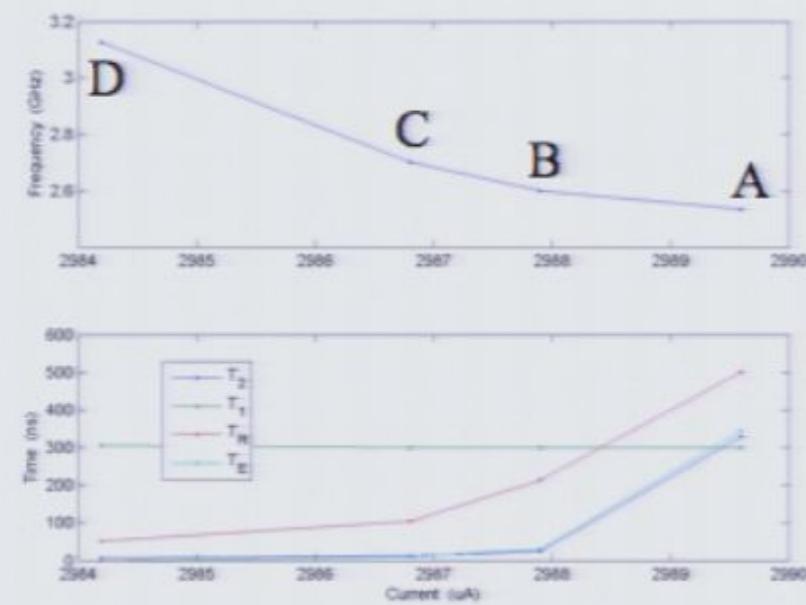
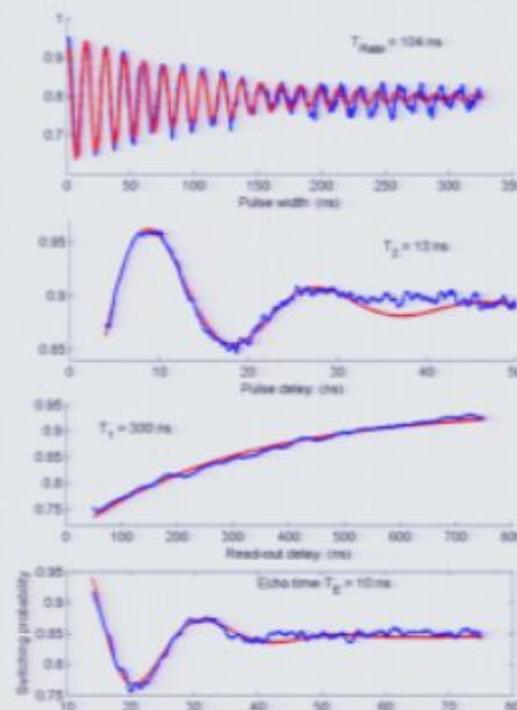
**A**



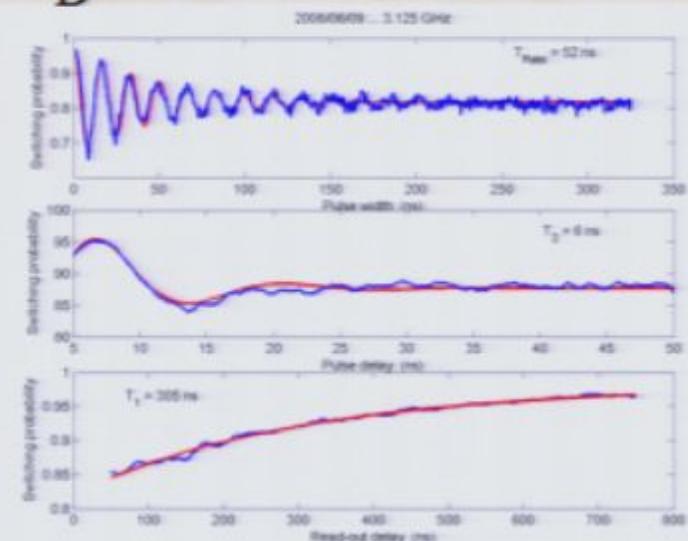
**B**



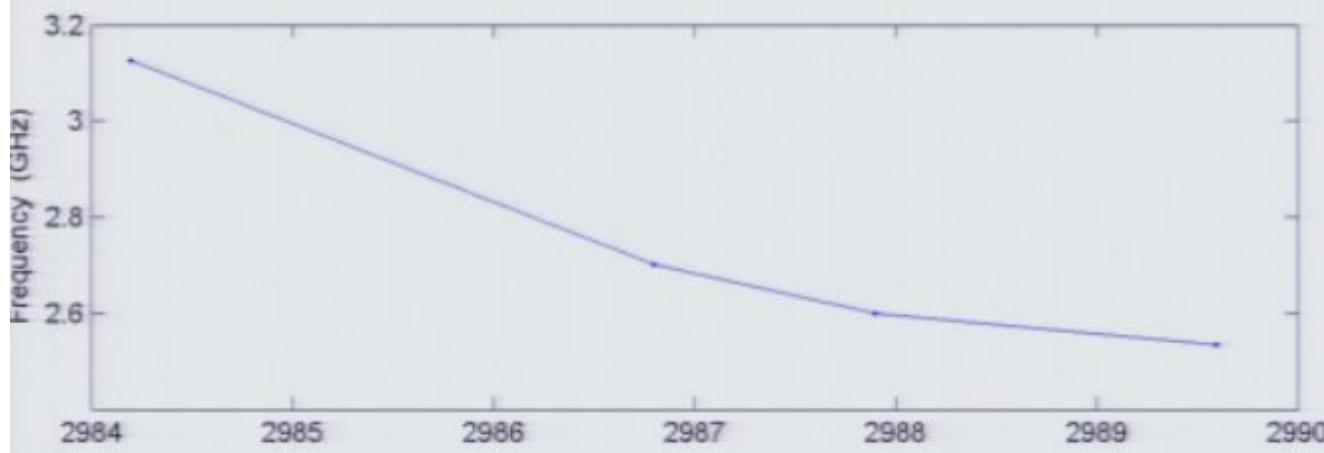
**C**



**D**

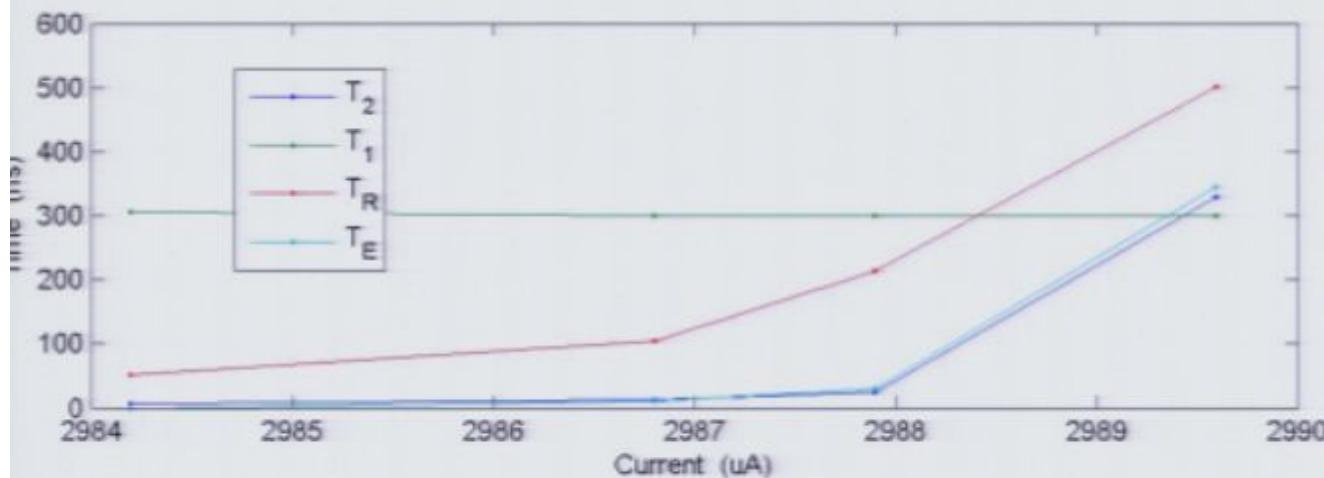


(No echo observed)



$T_1$  is constant  
-high frequency noise isotropic

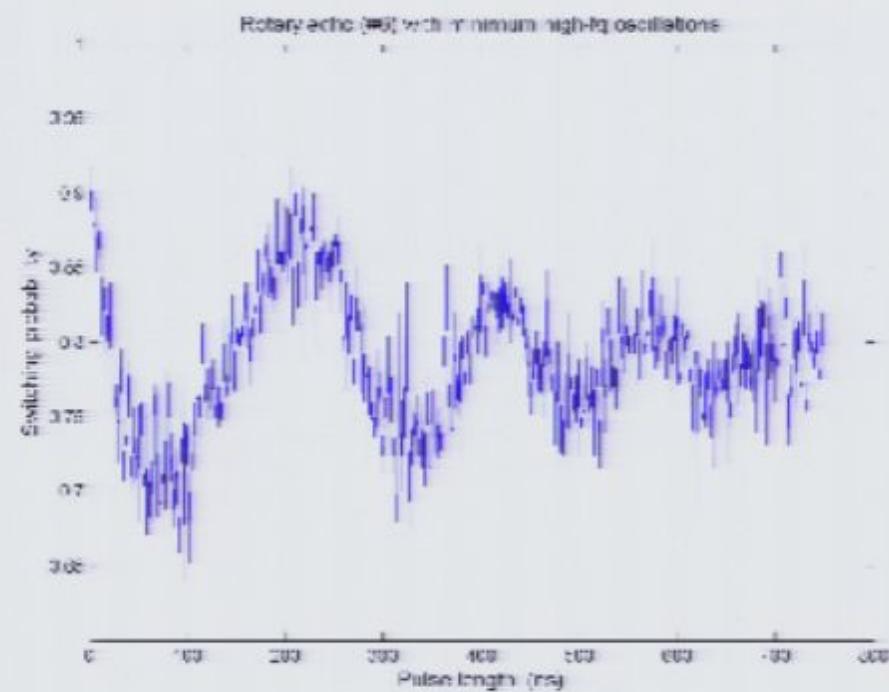
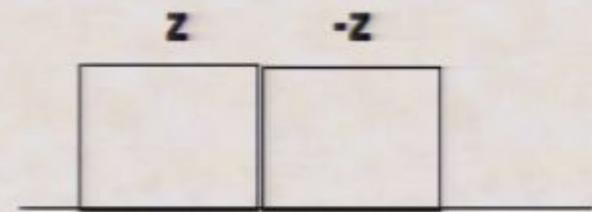
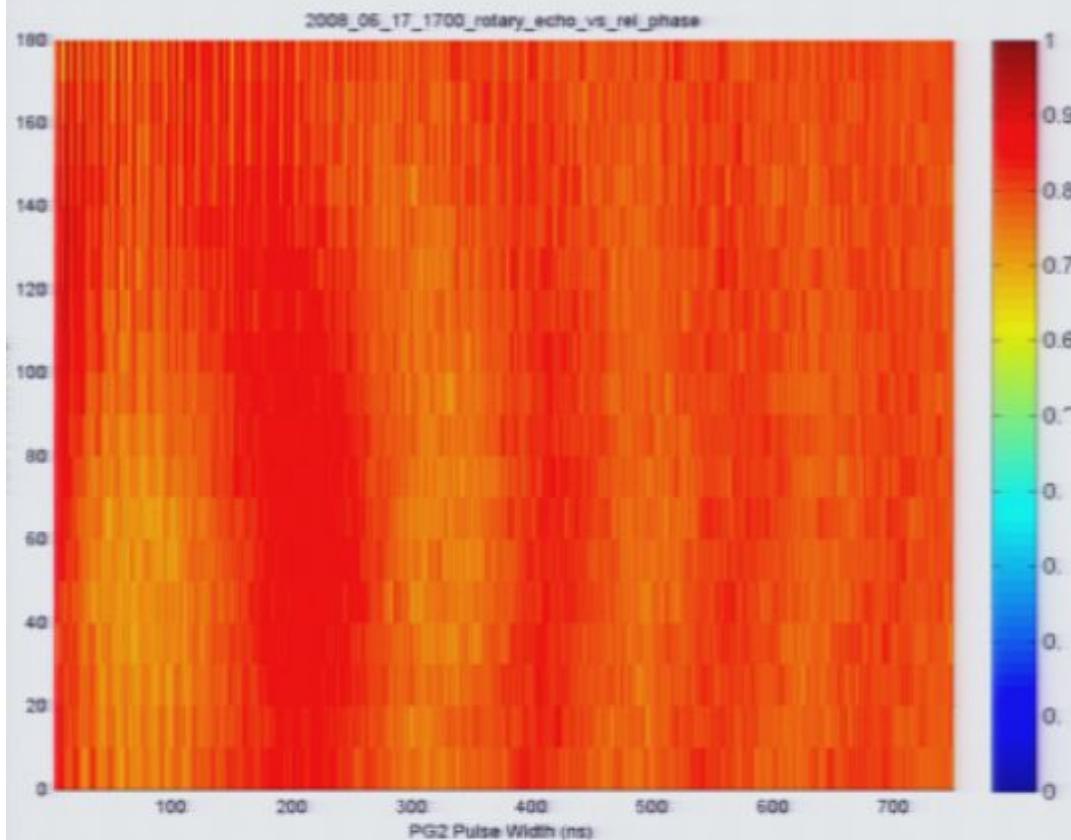
$T_2^* \sim T_{\text{echo}}$   
-noise along the quantization axis is not entirely static



$T_{\text{Rabi}} > T_2^*$   
-can refocus noise with modulation perpendicular to quantization axis

## Rotary echo

What limits  $T_{\text{Rabi}}$ ?



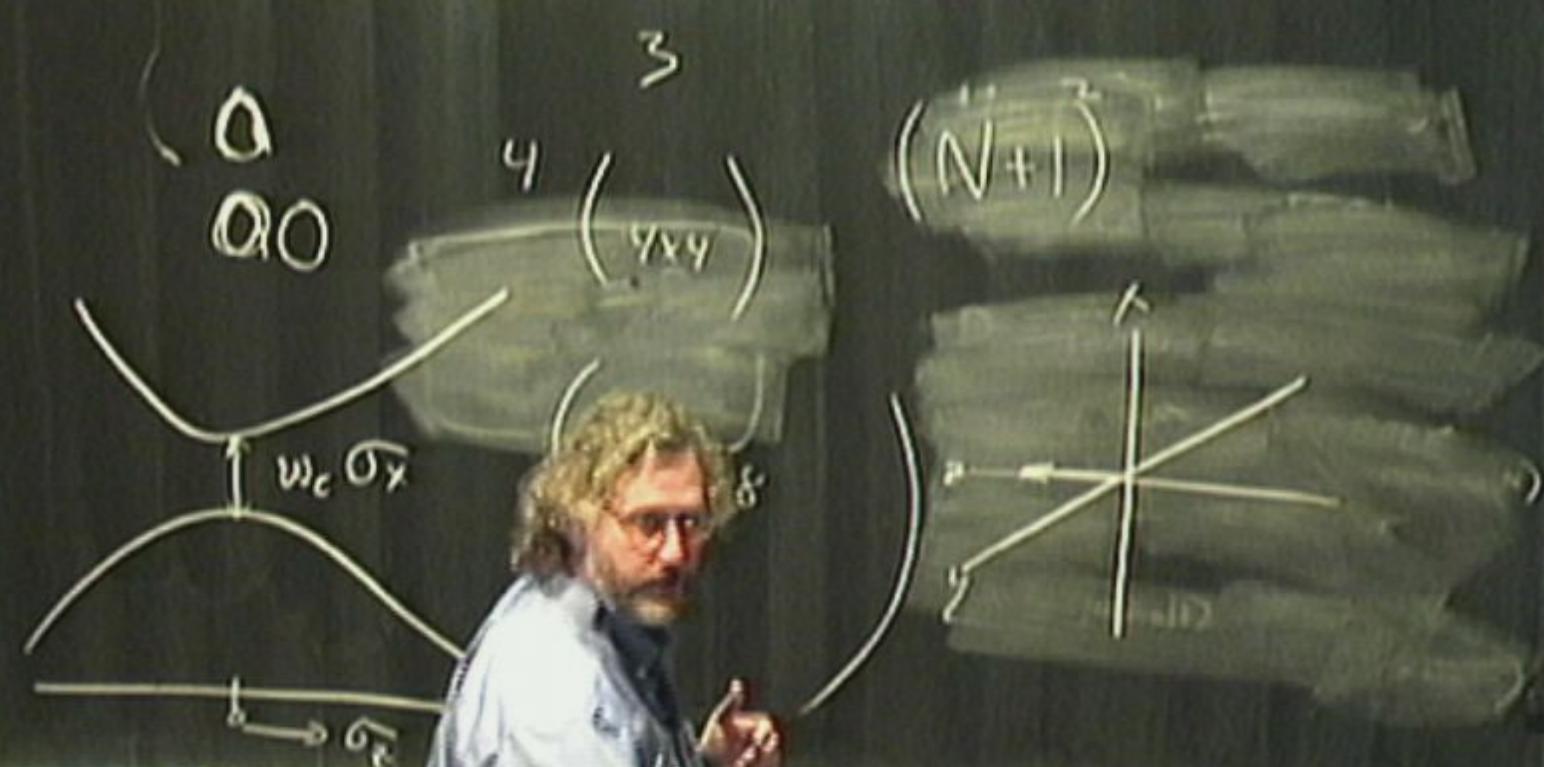
$T_{\text{Rotary}} > T_{\text{Rabi}}$  - noise in drive field

$$0 \leq \alpha \leq \frac{1}{3} :$$

Turbulence limit

$$e^{t/2} (\alpha + \delta e^{t/2}) e^{-\delta t}$$

3



$$0 \leq \alpha \leq \frac{1}{3} :$$

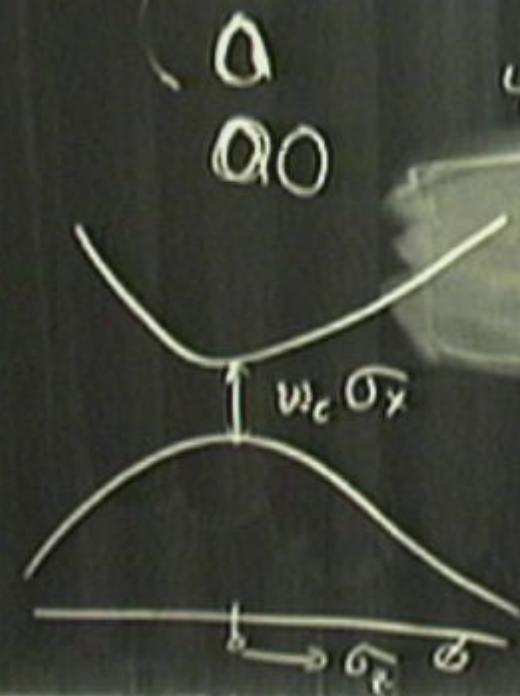
T depends on  $\alpha$

$$\tau^{\alpha}$$

$$e^{t/2}$$

$$(1+\alpha)e^{\alpha t}$$

$$\beta$$



$$3$$

$$4 \left( \gamma_{xy} \right)$$

$$5 \left( \gamma_{xx} \right)$$

$$6 \left( \gamma_{yy} \right)$$

$$(N+1)$$

$$(N+2)$$

$$(N+3)$$

$$(N+4)$$

$$(N+5)$$

$$(N+6)$$

$$(N+7)$$

$$(N+8)$$

$$(N+9)$$

$$(N+10)$$

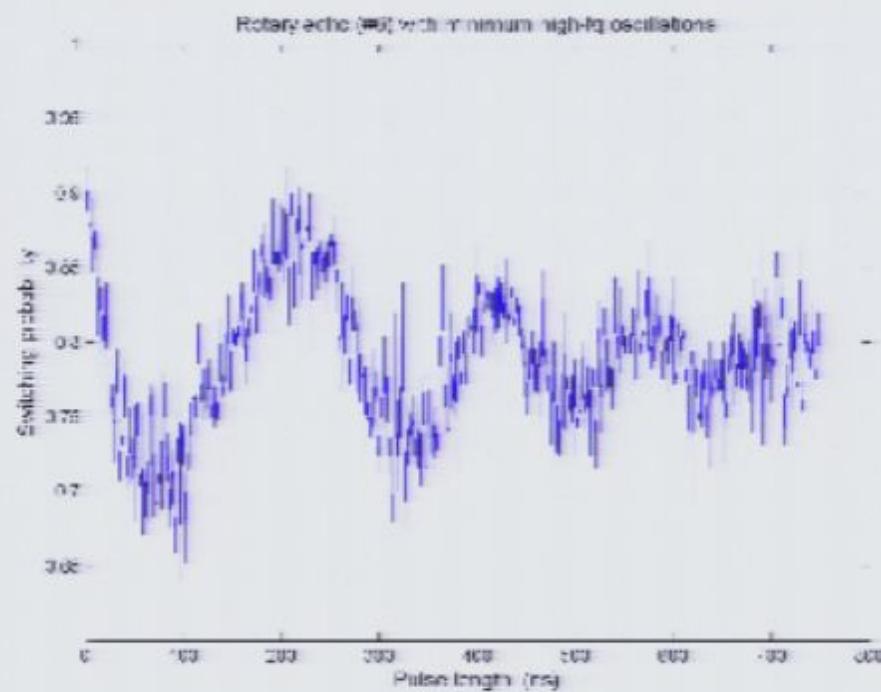
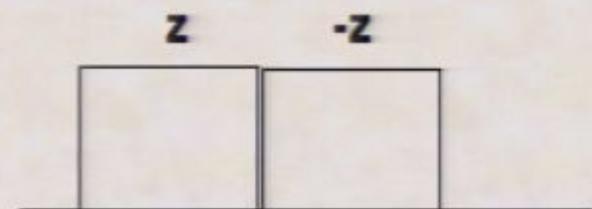
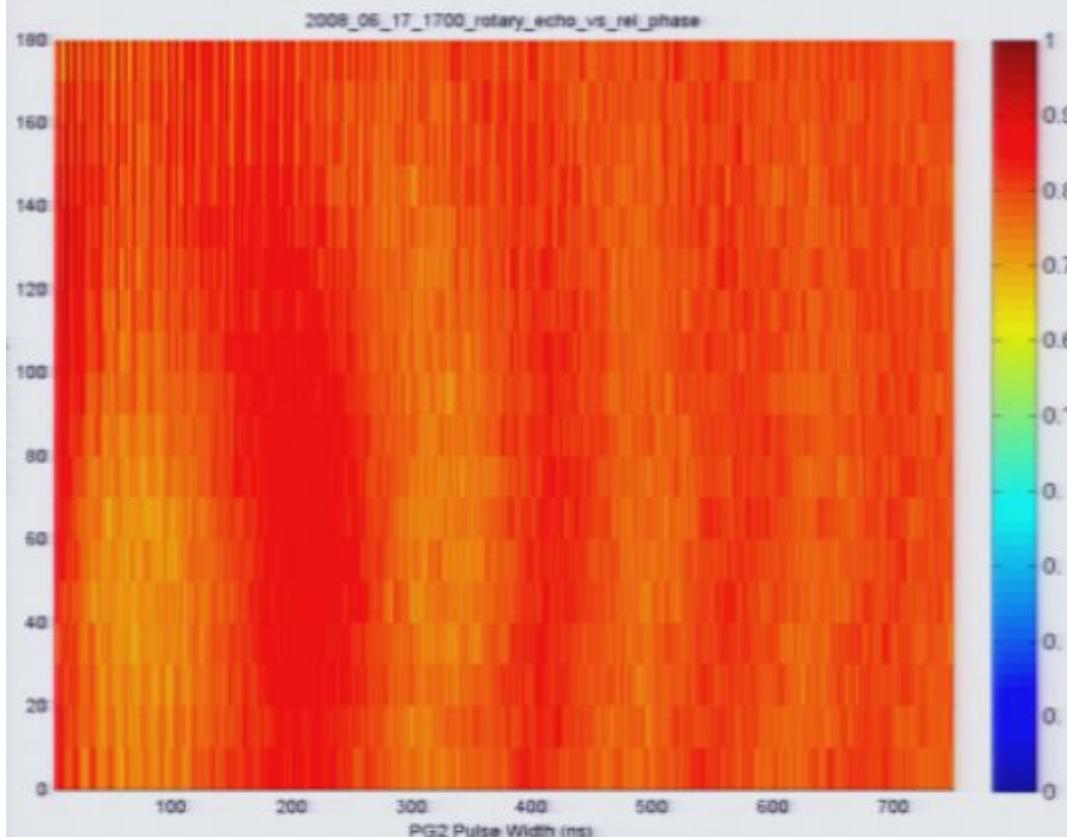
$$(N+11)$$

$$(N+12)$$



## Rotary echo

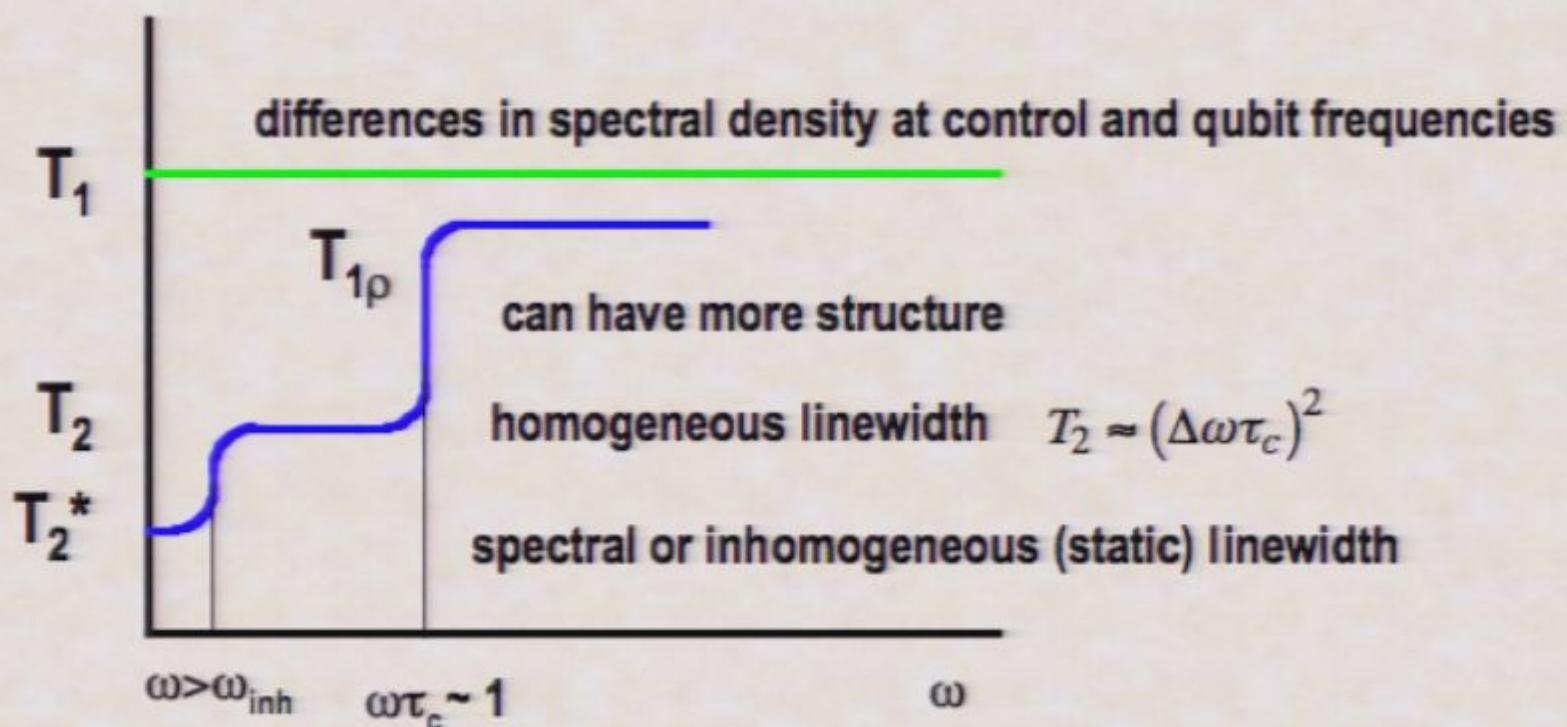
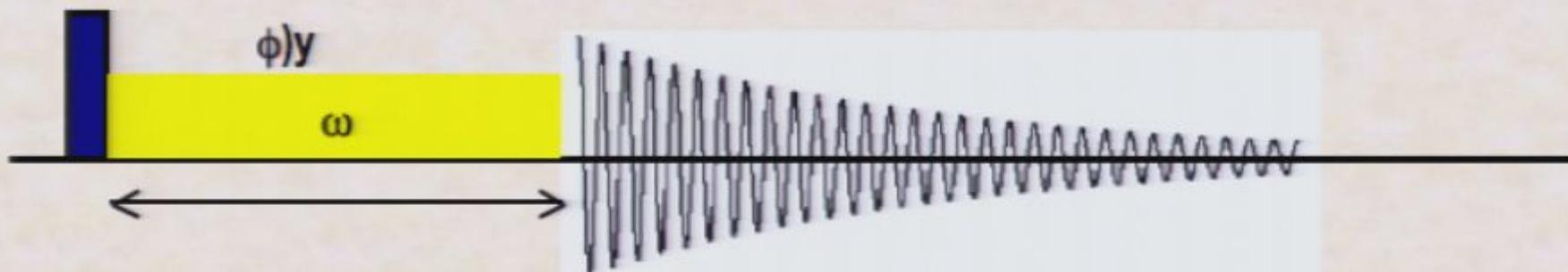
What limits  $T_{\text{Rabi}}$ ?



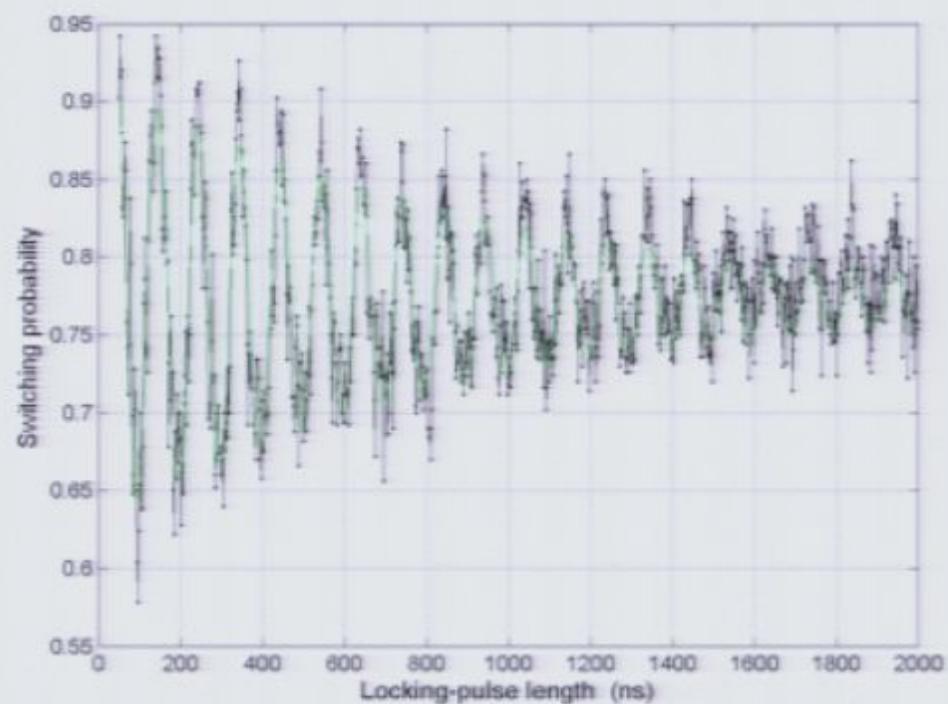
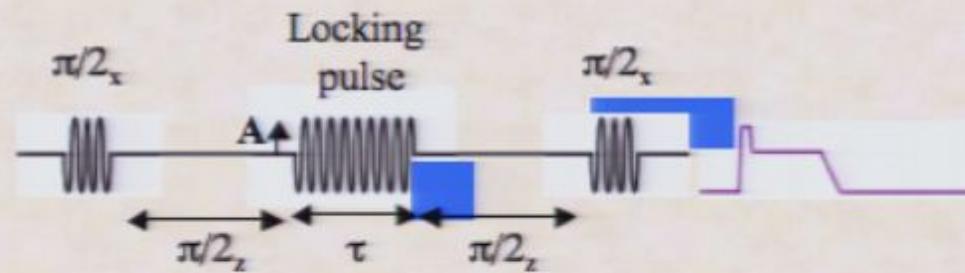
$T_{\text{Rotary}} > T_{\text{Rabi}}$  - noise in drive field

# Spin locking

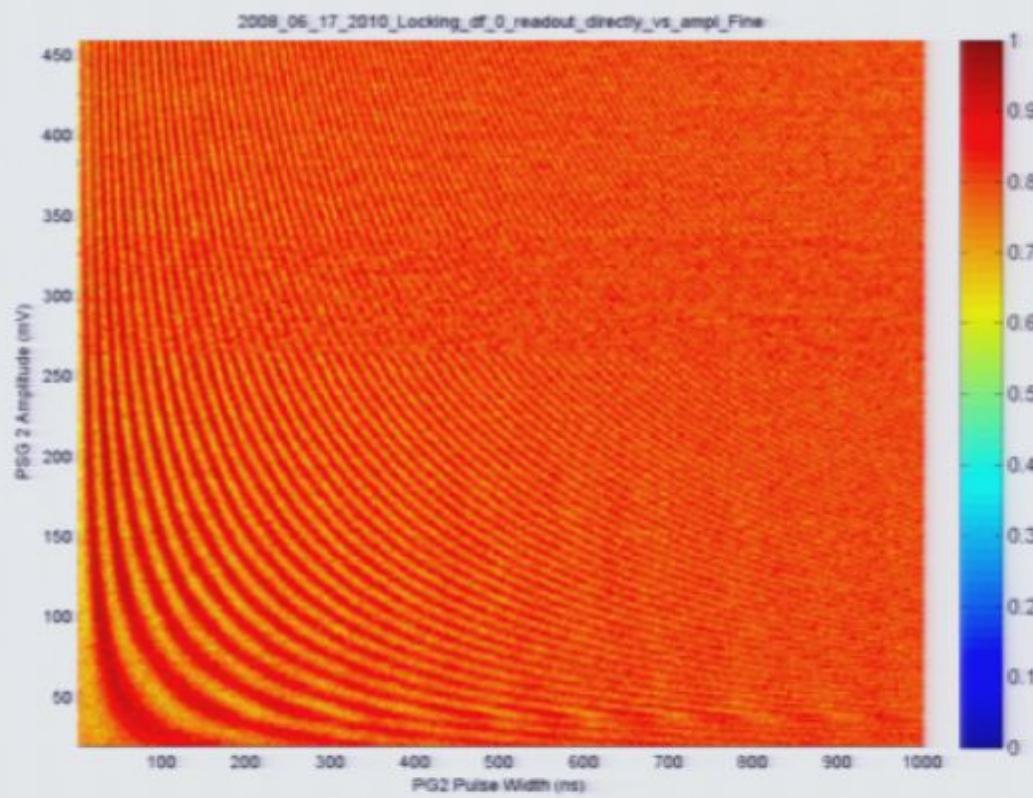
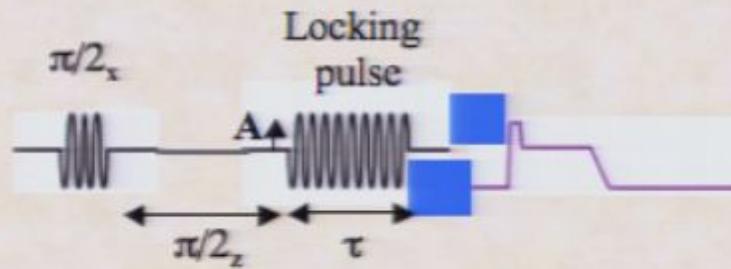
$\pi/2)x$

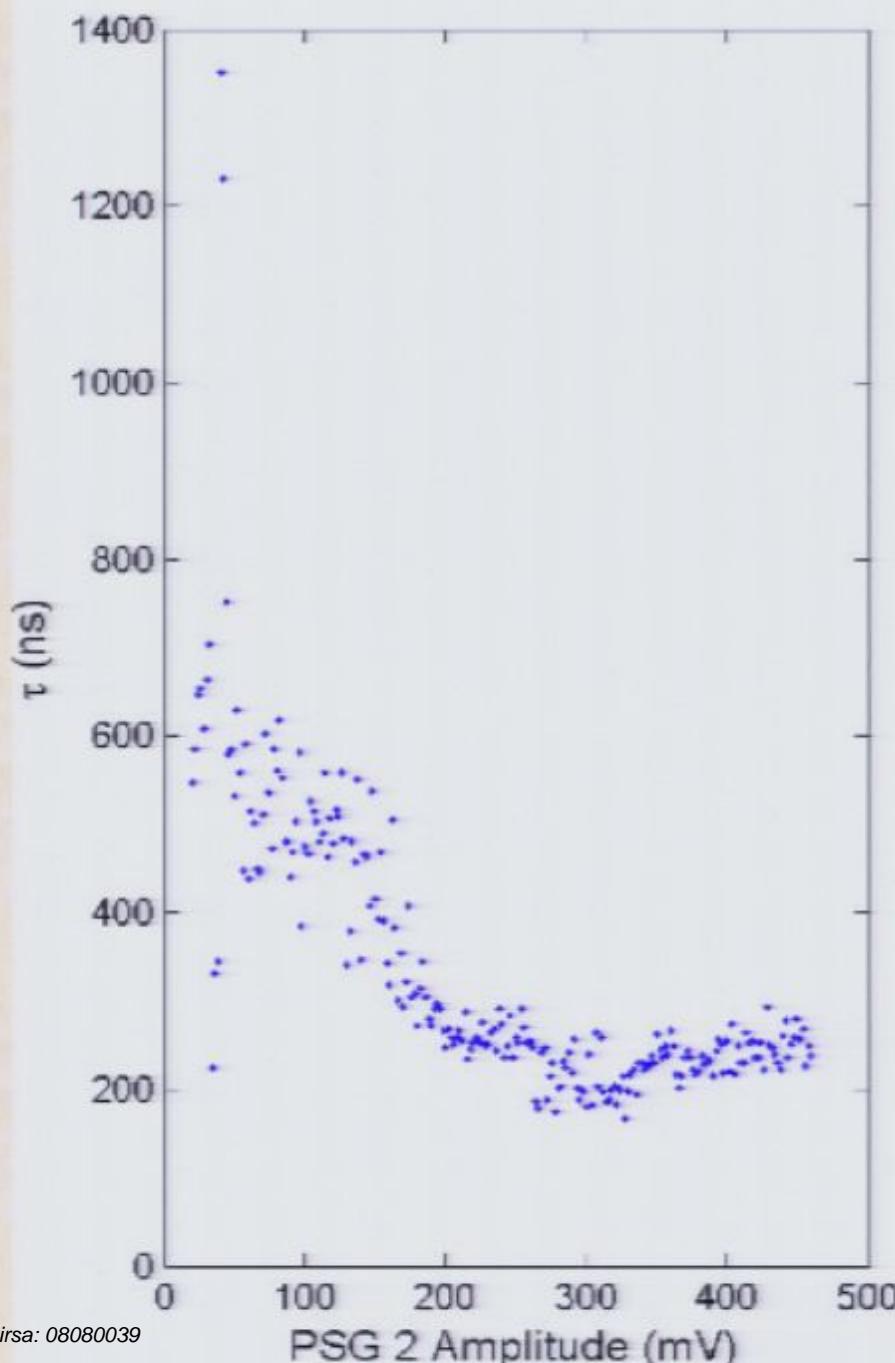


## Spin locking



## Spin locking





Here we plot the time constants of the  $T_{2\rho}$

The overall decay is probably the same physics as was seen in the rotary echo

What is happening at lower frequencies where we measure values up to  $> 4 T_1$  ?

## Bloch picture

define quantization axis

rotate into tilted reference frame

move into interaction frame of the transmitter frequency

suppress high frequency, transverse field

$$\frac{d\Phi_x}{dt} = \omega_A (\cos\theta - \varepsilon \sin\theta) (\cos(\phi)\Phi_y - \sin(\phi)\Phi_z) - \frac{\Phi_x}{T_1}$$

$$\frac{d\Phi_y}{dt} = -(\Delta\omega + \omega_A (\sin\theta + \varepsilon \cos\theta) \cos(\omega_T t + \phi))\Phi_z - \omega_A (\cos\theta - \varepsilon \sin\theta) \cos(\phi)\Phi_x - \frac{\Phi_y}{T_2}$$

$$\frac{d\Phi_z}{dt} = (\Delta\omega + \omega_A (\sin\theta + \varepsilon \cos\theta) \cos(\omega_T t + \phi))\Phi_y + \omega_A (\cos\theta - \varepsilon \sin\theta) \sin(\phi)\Phi_x - \frac{\Phi_z}{T_2}$$

$$\theta = \tan^{-1} \left( \frac{\omega_{bias}}{\omega_0 + \varepsilon \omega_{bias}} \right)$$

$$\Delta\omega = \sqrt{(\omega_0 + \varepsilon \omega_{bias})^2 + \omega_{bias}^2} - \omega_T$$