Title: Quantum state tomography in the real world: the search for rigor by an increasingly confused experimentalist

Date: Aug 25, 2008 02:45 PM

URL: http://pirsa.org/08080034

Abstract: The basic principles of quantum state tomography were first outlined by Stokes for the context of light polarisation more than 150 years ago. For an experimentalist the goal is clear: to use a series of measurement outcomes to make the best possible estimate of a system\'s quantum state, including phase information, with the least amount of measurement (and analysis) time and, if possible, also the least expensive and complicated apparatus. However, although the general ideas are straightforward, there is still much flexibility of choice in the practical details of how to implement tomography in the laboratory - details which may substantially influence the tomographic performance. For example, how do I make my measurements? How do I analyse my data? How confident can I be of my reconstruction anyway? Say, for example, I\'ve made my measurements and taken my data and I\'m now faced with two different ways, essentially equivalent at first glance, of analysing my data, where one way gives me \'better numbers\', e.g., a state with higher entanglement. Which do I choose? One may lead to a higher chance of a successful publication, but this is hardly a satisfactory way of making the decision. The perspective of an experimentalist motivates a fairly pragmatic approach to choosing the best technique for performing tomography. Does it work? How well? Can it work better? In this talk, I will use this approach and discuss issues which arise at each stage of the tomography process, using both numerical and real lab data to characterise the performance quality.

Quantum state tomography in the real world: the search for rigor by an increasingly confused experimentalist

Nathan K. Langford

THEN...

THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

quantum technology lab

Group leader: Andrew G. White

NOW...

universität wien

IQI

Group leader: Anton Zeilinger

Quantum state tomography in the real world: the search for rigor by an increasingly confused experimentalist

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OVERVIEW

- Motivation and introduction
- Taking data
- Analysing data
- Interpreting the results

COLLEAGUES…

- Alexei Gilchrist, Andrew White, Andrew Doherty, Aephraim Steinberg, Bill Munro, Robin Blume-Kohout, Mark de Burgh, Rohan Dalton, Geoff Gillett, Marco Barbieri, …

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- Motivation and introduction
- Taking data
- Analysing data
- Interpreting the results

"There may be a couple of little details remaining, but this tomography lark basically seems to be a solved problem…" (paraphrased) - anonymous colleague

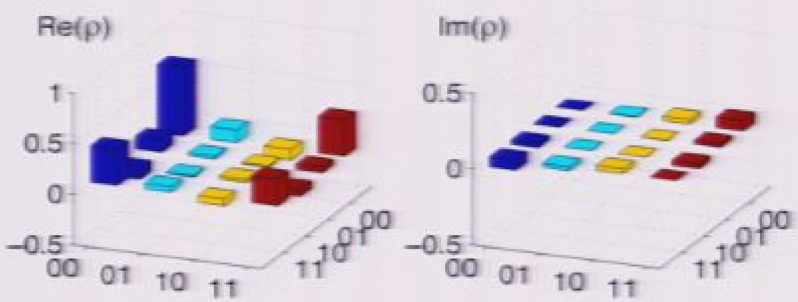
Motivation: the devil's in the details

Two-qubit data set

[PRL 93, 053601 (2004)]

$q_2 \backslash q_1$	0	1	0+1	0−1	0+i1	0−i1
0	9914	601	3784	6368	4943	6105
1	369	5615	3072	1355	1497	2572
0+1	7392	861	7570	2497	2154	4523
0−1	3416	2509	250	6652	3091	3544
0+i1	3918	1819	2894	4242	1014	7375
0−i1	6061	1778	3253	2808	6078	882

$$\Pi(\rho) = \sum_m \frac{(N_m - n_m(\rho))^2}{\Delta n_m^2(\rho)}$$



$S_L = 0.178 \pm 0.005$ $T = 0.567 \pm 0.007$

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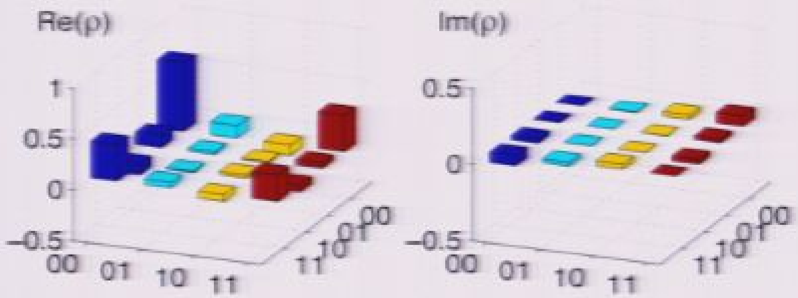
$$\Pi(\rho) = \sum_m \frac{(N_m - n_m(\rho))^2}{\Delta n_m^2(\rho)} \qquad \tilde{\Pi}(\rho) = \sum_m \frac{(N_m - n_m(\rho))^2}{\Delta N_m^2}$$



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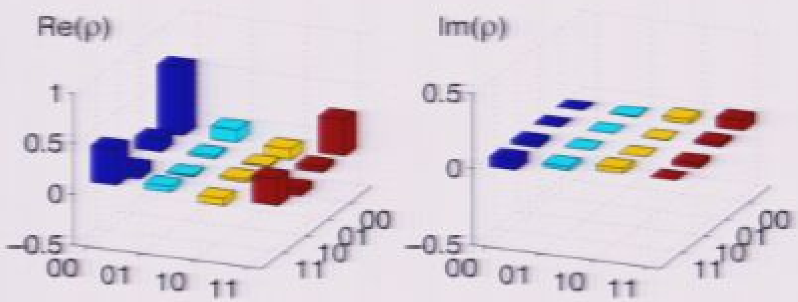

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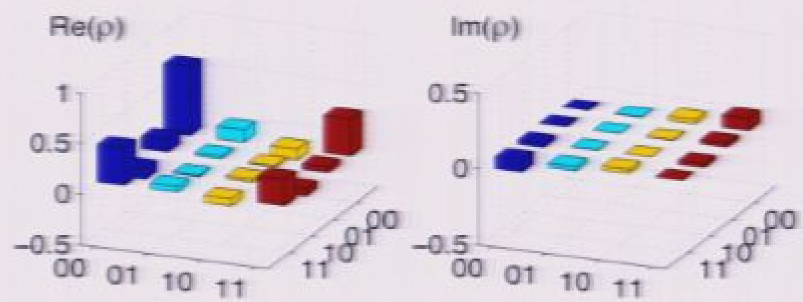
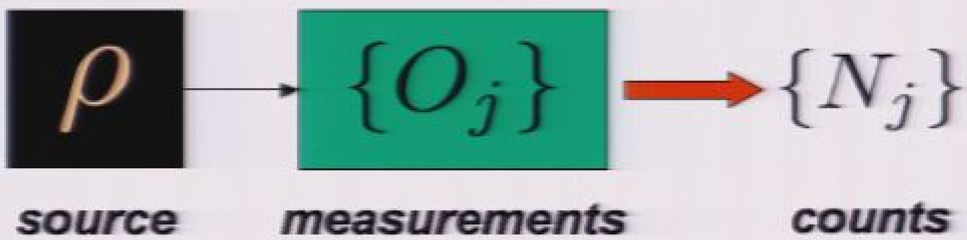


$S_L = 0.146 \pm 0.004$ $T = 0.605 \pm 0.006$

The Basic Framework

The irreducible components



- inversion procedure
  → physical state
- informationally complete measurements
  → complete density matrix

My framework - state tomography

- a measurement = counting for a single projector $\quad O_j$
- maximum likelihood reconstruction $\quad N_j \approx N \, \mathrm{Tr}\{\rho \, O_j\}$
- photons = poissonian counts statistics $\quad \Delta N_j^2 = N_j$

Does it work? Can it work better?

- pragmatic approach - no philosophical value judgements (I hope!)
- fundamental issues - intrinsic to theory
- practical issues - result from non-ideal behaviour in experiments

Taking Data - minimum vs over-complete

What measurements should I take?

standard qubit measurements $|\mathbf{0}\rangle, |\mathbf{1}\rangle, |\mathbf{0}\rangle+|\mathbf{1}\rangle, |\mathbf{0}\rangle+i|\mathbf{1}\rangle$

over-complete qubit measurements $|\mathbf{0}\rangle, |\mathbf{1}\rangle, |\mathbf{0}\rangle\pm|\mathbf{1}\rangle, |\mathbf{0}\rangle\pm i|\mathbf{1}\rangle$

Minimum measurement sets

- longer count times per measurement = better statistics
- stability required for entire data set

Over-complete measurement sets

- information redundancy = more insensitive to noise
- shorter stability times

Polarisation labels

$$|H\rangle = |0\rangle \qquad |D\rangle = |0\rangle+|1\rangle \qquad |R\rangle = |0\rangle+i|1\rangle$$
$$|V\rangle = |1\rangle \qquad |A\rangle = |0\rangle-|1\rangle \qquad |L\rangle = |0\rangle-i|1\rangle$$

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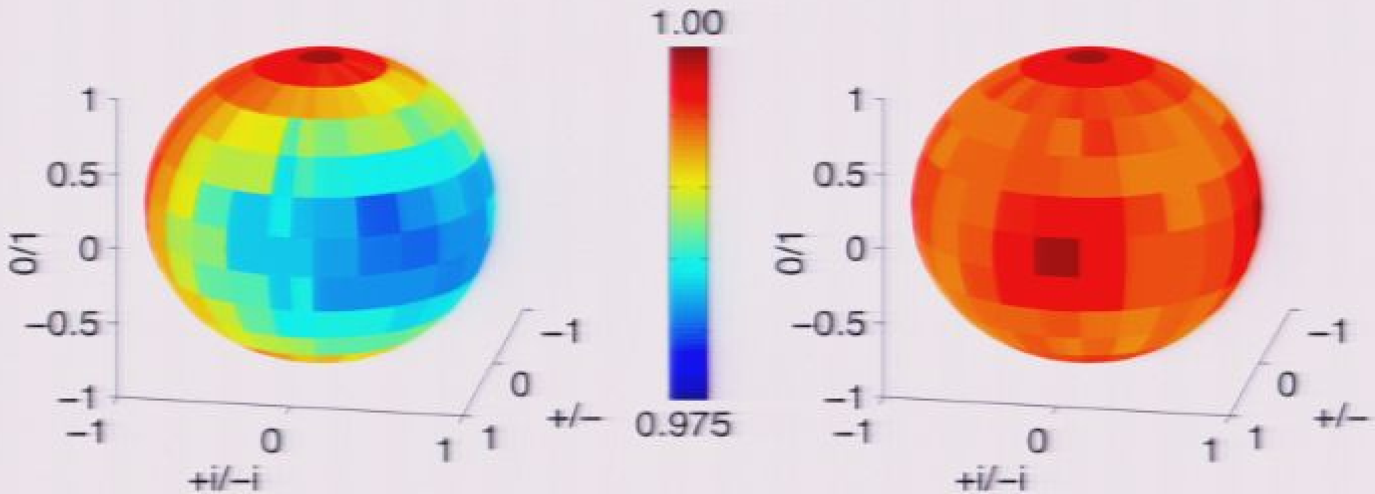
Single-qubit simulations

- single-qubit pure states (surface of qubit sphere)
- equal total no. of copies of quantum state
- Poissonian statistics



mean fidelity

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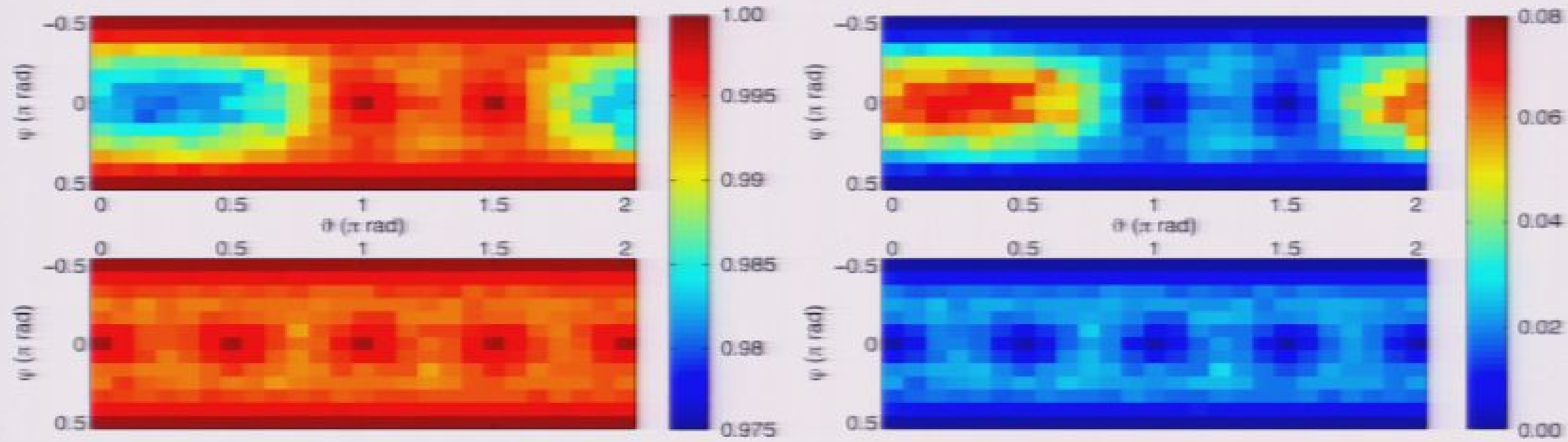
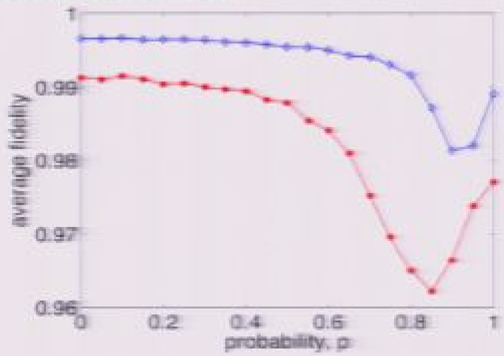


mean fidelity *mean linear entropy*

Taking Data - minimum vs over-complete

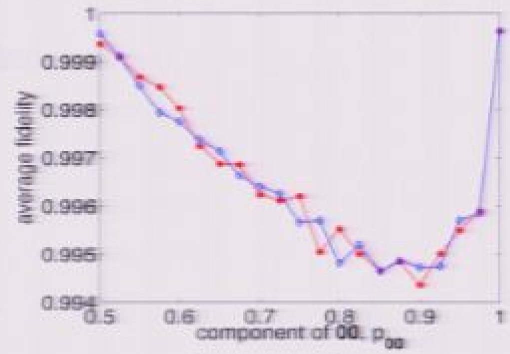
Two-qubit simulations *(red: standard, blue: over-complete)*

mixture



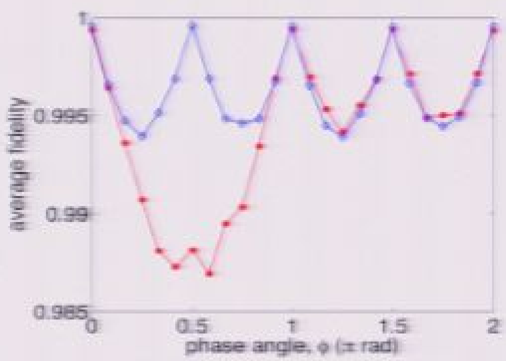
$$\rho = p|\psi_{\rm max}\rangle\langle\psi_{\rm max}| + (1-p)I_4/4$$

entanglement



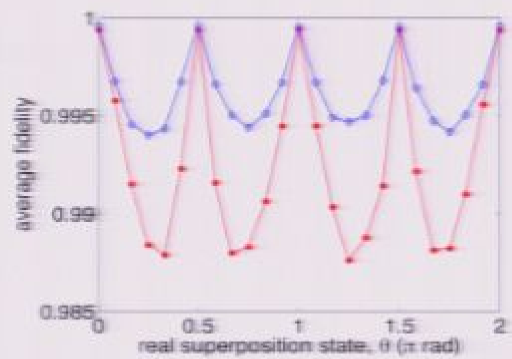
$$|\psi\rangle = \sqrt{p_{00}}|00\rangle + \sqrt{1-p_{00}}|11\rangle$$

phase



$$|\psi\rangle \sim |00\rangle + \exp(i\theta)|11\rangle$$

polarisation



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- stability required for entire data set

Over-complete measurement sets

- information redundancy = more insensitive to noise
- shorter stability times

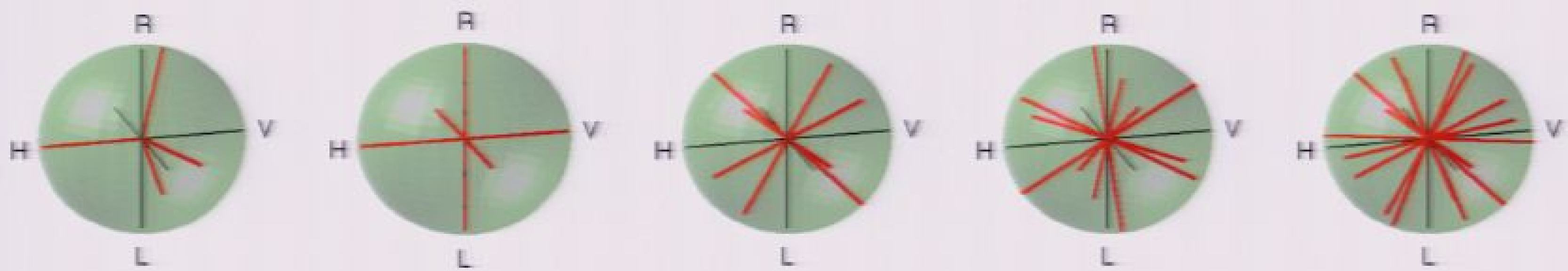
Results so far…

- results are strongly state sensitive, BUT…
- over-complete measurement set seems to perform better over different states and shot-to-shot for individual states
- is the standard minimum set the best minimum set?

Taking Data - platonic solids

Platonic solid qubit measurement sets

- varying amount of information redundancy: 4 measurements (min) to 20 meas.
- uniformly distributed over the Bloch sphere (t-designs)



tetra (4) *cube (6)* *octa (8)* *dodeca (12)* *icosa (20)*

(with M. de Burgh, A. Gilchrist and A. Doherty)

Taking Data - minimum vs over-complete

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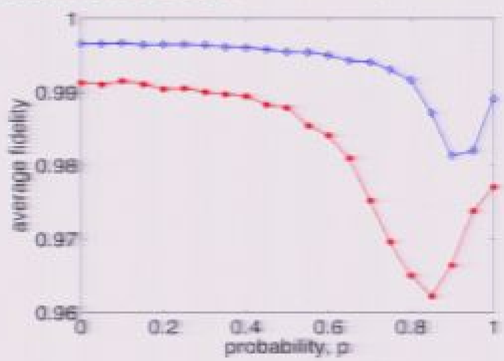

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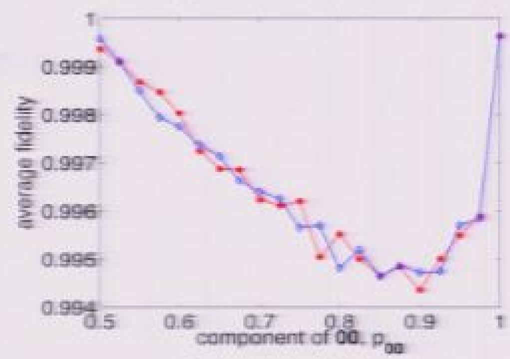

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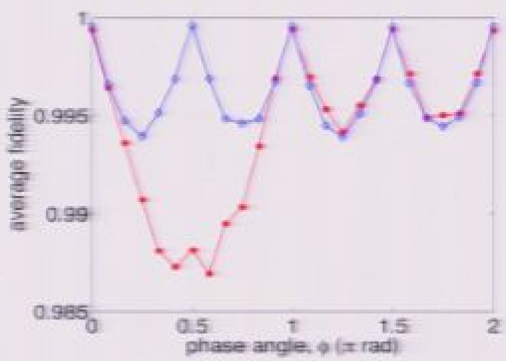

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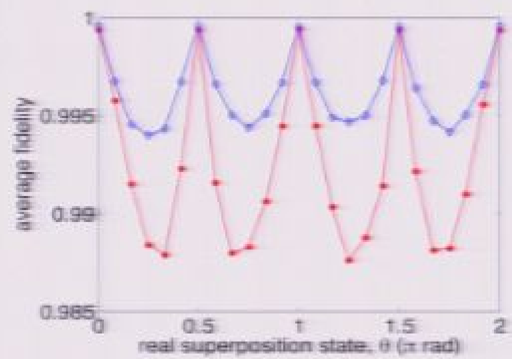

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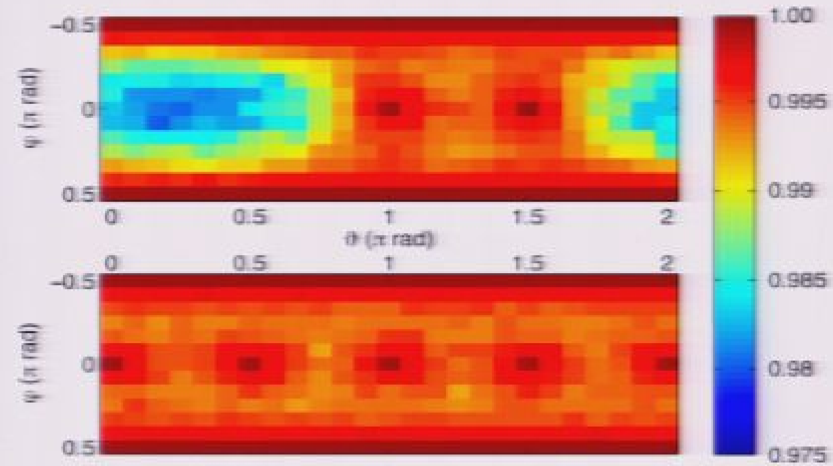
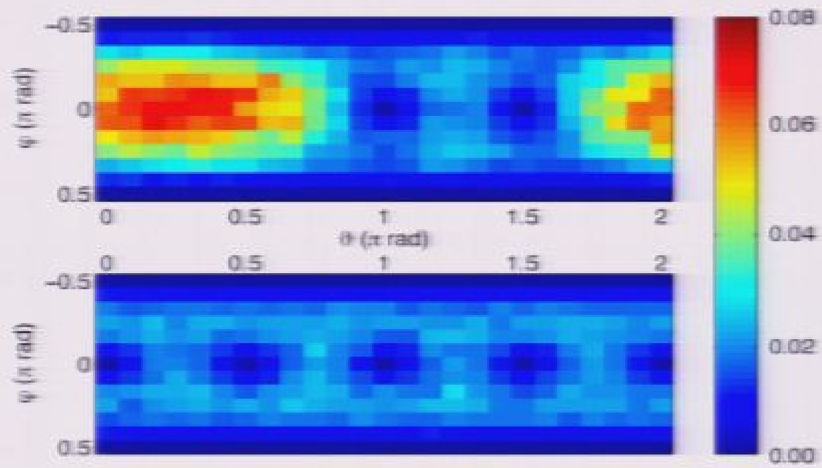


mean fidelity



mean linear entropy

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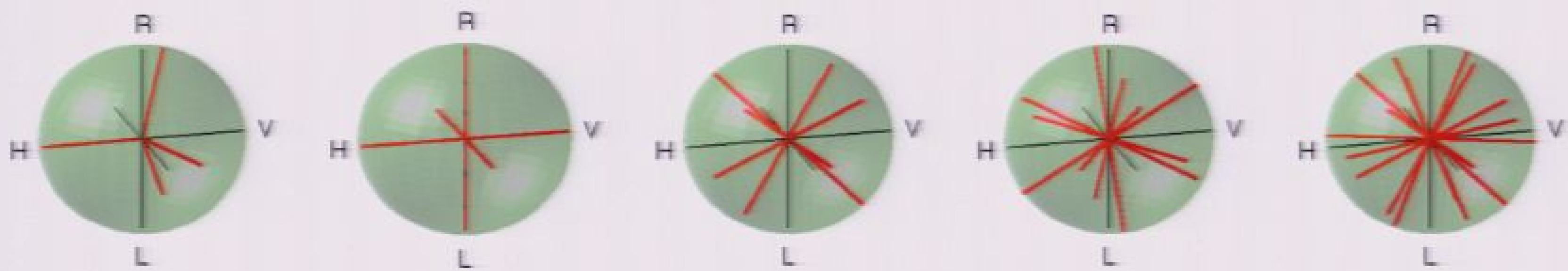

tetra (4) *cube (6)* *octa (8)* *dodeca (12)* *icosa (20)*

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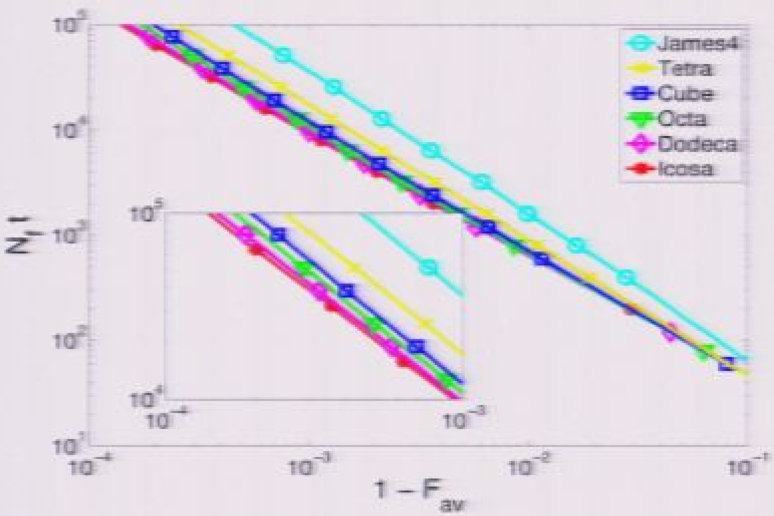
Taking Data - platonic solids

Average reconstruction performance - simulations

- average reconstruction fidelity - calculated numerically
- randomly selected mixed states uniformly distributed w.r.t. rotations (Haar measure) and w.r.t. mixture (Bures metric, fidelity based)



one photonic qubit

- all platonic solids out-perform standard minimum set
- cube out-performs tetrahedron (over-complete is good)
- higher-order platonic solids get better

(with M. de Burgh, A. Gilchrist and A. Doherty)

Taking Data - platonic solids

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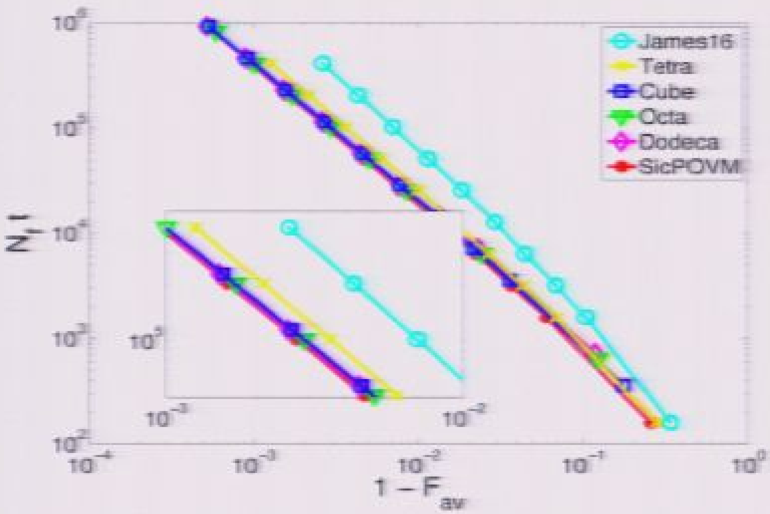


two photonic qubits

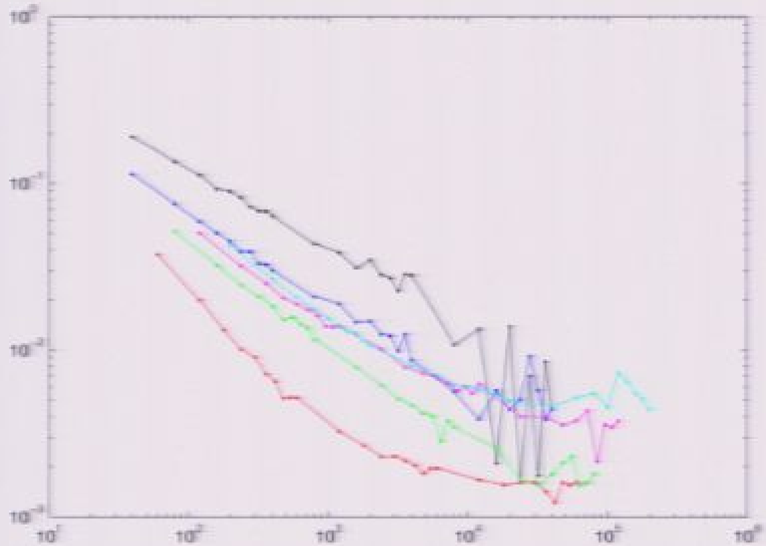
- all platonic solids out-perform standard minimum set
- cube still out-performs tetrahedron (over-complete is good)
- negligible improvement with higher-order solids

(with M. de Burgh, A. Gilchrist and A. Doherty)

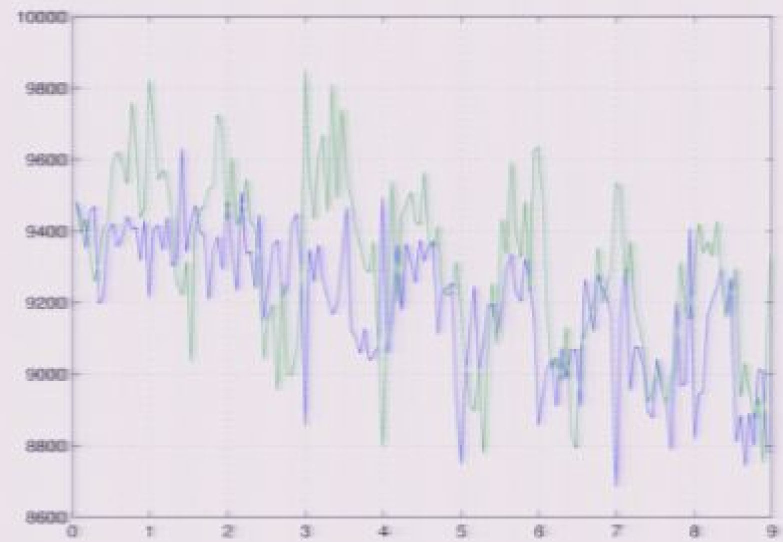
Taking Data - platonic solids

Point-wise reconstruction performance - experiment

- each measurement - 100s, 0.1s samples (1000 samples)



input: D



stability (9 input states)

J4 4 6 8 12 20

(with R. Dalton, G. Gillett, M. Barbieri and others)

Taking Data - platonic solids

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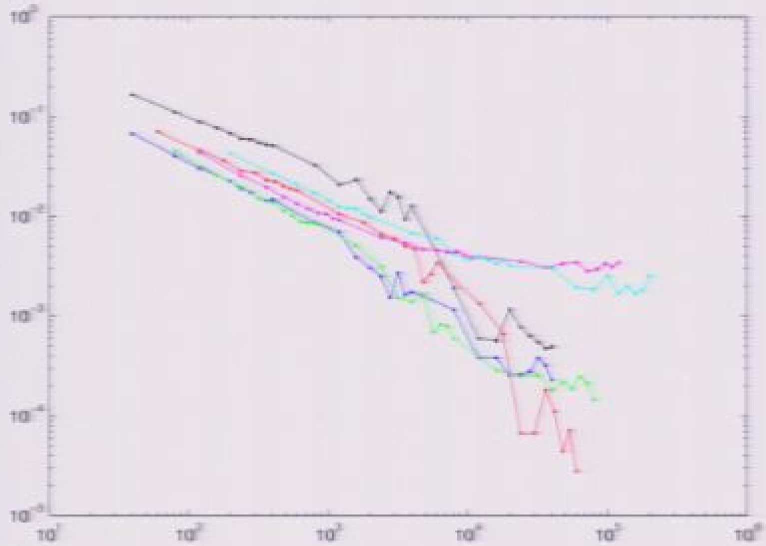

input: cube worst case

- cube still performs well overall
- flattened tail - stability? slightly inaccurate wave plates?
- not seeing the expected improvement for higher-order solids - probably stability

TO DO

- improve source stability
- improve high-count statistics
- take data for full sphere average

J4 4 6 8 12 20

(with R. Dalton, G. Gillett, M. Barbieri and others)

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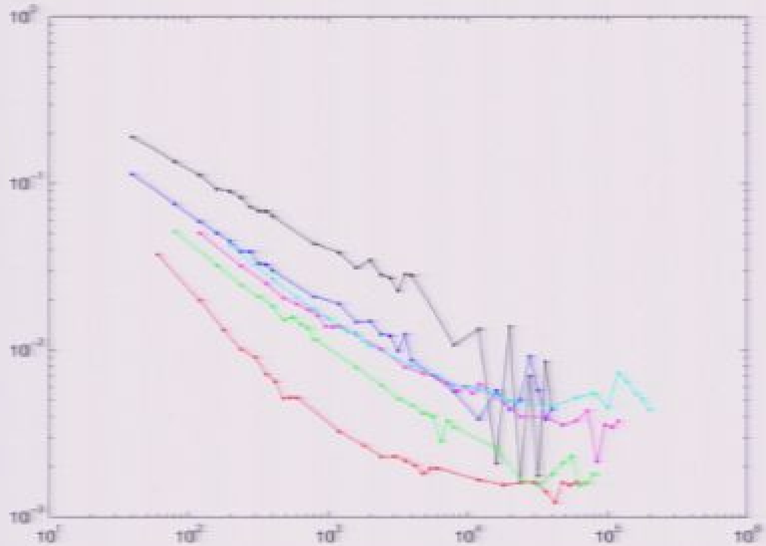
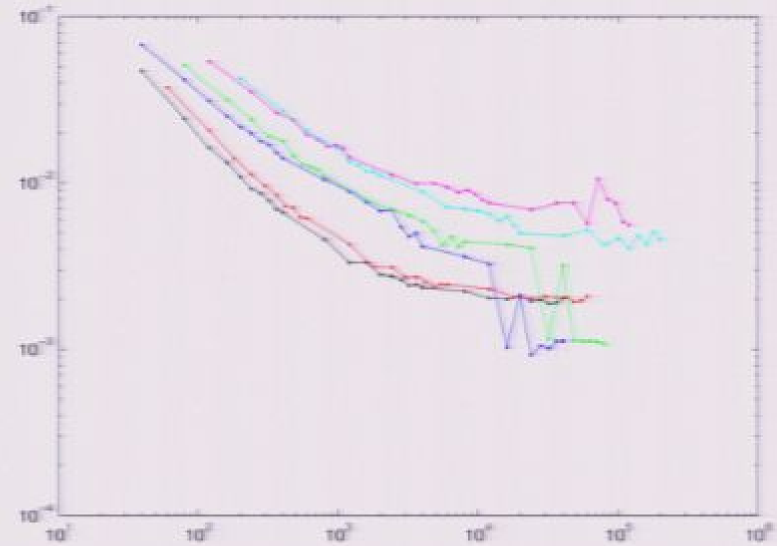
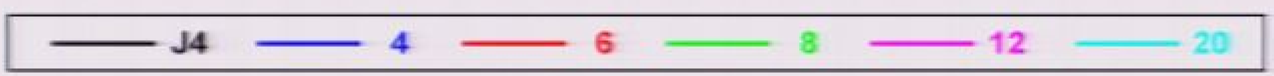


input: D



input: A



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Taking Data - instability

Types of instability

- state drift (all systems) - shot-to-shot or time-varying instability in the state
- flux drift (photonic systems) - limitation of current single-photon sources

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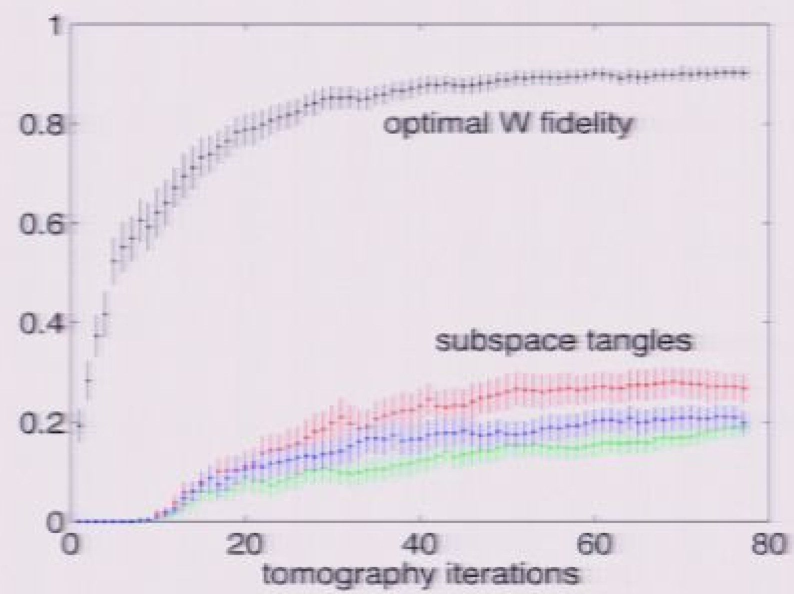
Iterative tomography - sequential complete measurement sets

- partially prevents long-term drift (longer than one iteration) from skewing reconstructed state - instead produces mixture
- when is the reconstruction limited by experimental properties and not by bad counting statistics?
- particularly useful for long data runs (low count rates)



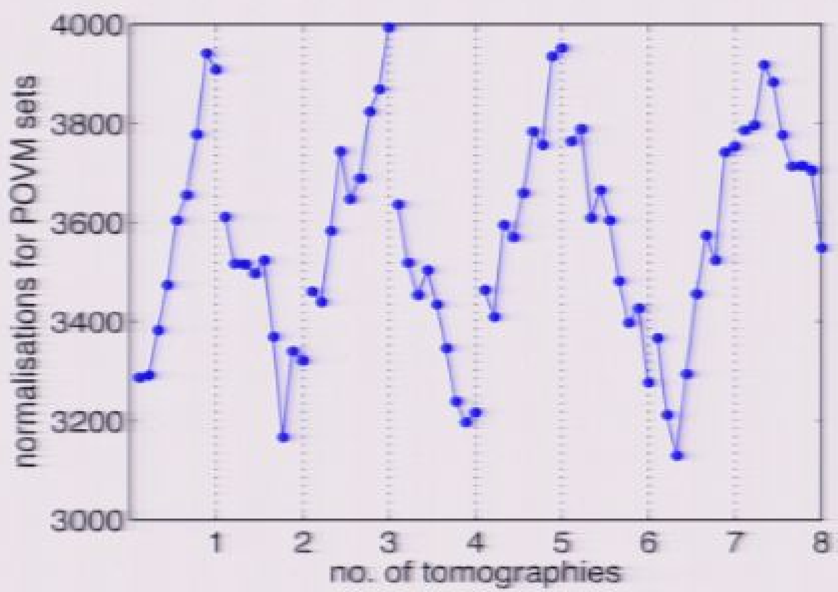
Taking Data - instability

Types of instability

- state drift (all systems) - shot-to-shot or time-varying instability in the state
- flux drift (photonic systems) - limitation of current single-photon sources

Internal normalisation - normalising data within individual POVM sets

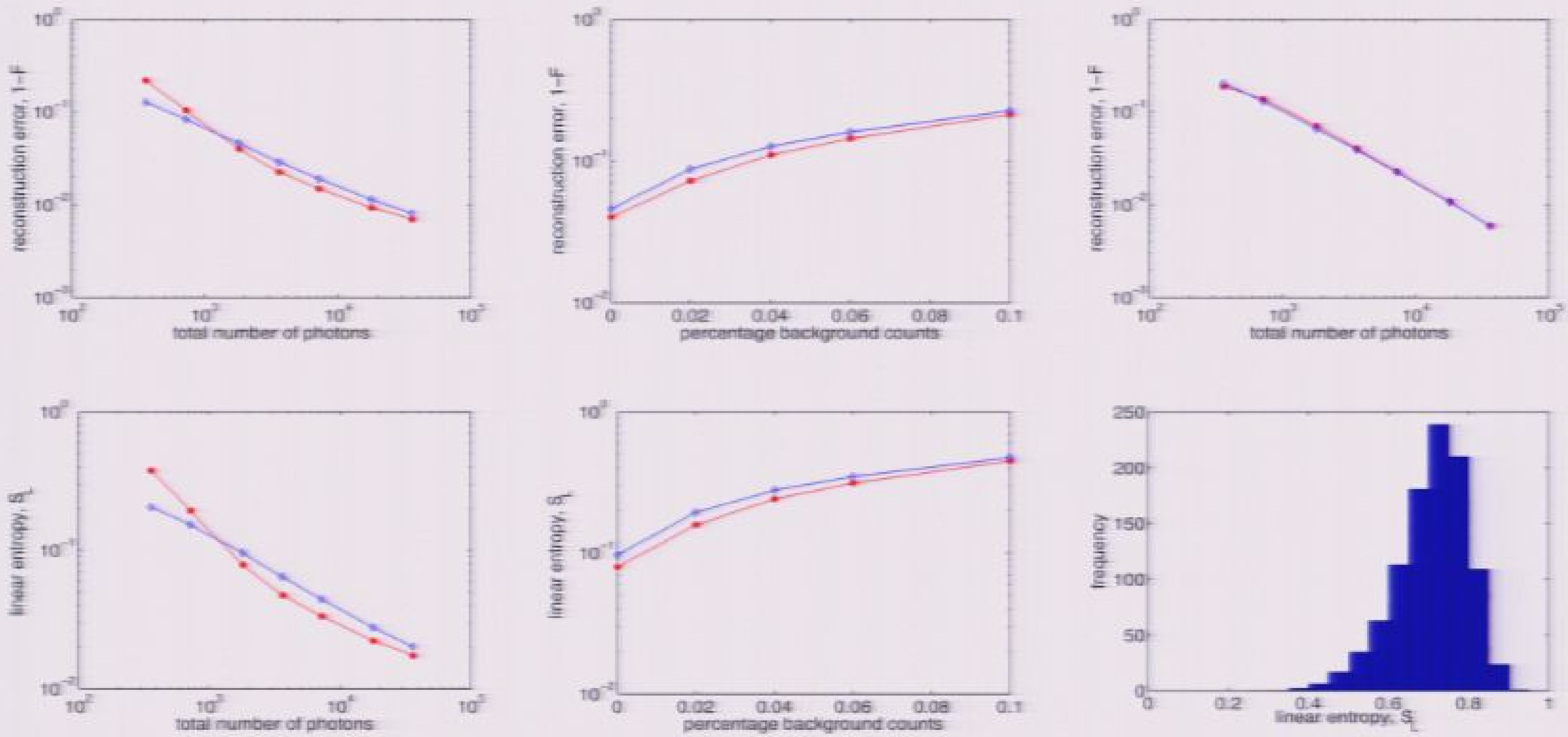
- standard measurement order: (HH, HV, HD, HA, HR, HL), (VH, VV, ...
- POVM measurement order: (HH, HV, VH, VV), (HD, HA, VD, VA), (HR, HL, ...
- requires shorter stability times ($d^n << d^{2n}$)
- particularly good for short data runs to compensate for shorter-term instability



what things sometimes really look like…

Analysing Data - the penalty function

Some initial investigation - random two-qubit states (1000 per point)



pure states *with background* *mixed states*

Calculating Errors

Errors in physical quantities

$$\{N_j\} \longrightarrow \tilde{\rho}(N_j) \longrightarrow f(\tilde{\rho}(N_j))$$

Analytical calculations based on linear tomography

$$\underset{\text{error model}}{\{\Delta N_j\}} \xrightarrow[\text{analytical error propagation}]{} \Delta f(N_j, \Delta N_j)$$

Problems

- analytical calculation very difficult & repeated for every physical quantity

Calculating Errors

Errors in physical quantities

$$\{N_j\} \longrightarrow \tilde{\rho}(N_j) \longrightarrow f(\tilde{\rho}(N_j))$$

Monte-Carlo simulations

$$\underbrace{\{\Delta N_j\}}_{\text{error model}} \rightarrow \{N_j^s\} \xrightarrow[\text{repeated optimisation}]{} \tilde{\rho}^s(N_j^s) \rightarrow f^s(\tilde{\rho}^s(N_j^s)) \xrightarrow{\text{simple statistics}} \Delta f(N_j^s)$$

Advantages
- better noise estimate
- requires only repeated calculations of quantity
- already incorporates effects of optimisation

The Simple Fit Quality Parameter

$$Q(\rho) = \sqrt{\frac{1}{M}\sum_m \frac{(N_m - n_m(\rho))^2}{\Delta n_m^2(\rho)}}$$

Ideal Properties

- Range: $0 \leq Q(\rho) < \infty$
- Expected mean: $\langle Q(\rho)\rangle = 1$
- Expected standard deviation: $\langle \Delta Q(\rho)\rangle = 1/\sqrt{2M}$

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Poissonian correction $\langle \Delta Q(\rho)\rangle = \frac{1}{\sqrt{2M}}\sqrt{1 + \frac{1}{2M}\sum \frac{1}{n_m}}$

The problem with the simple fit quality…

- reconstructed density matrix is an imperfect estimate of the "true" state
- skewed slightly away from "true" predicted data towards measured data
- expected mean in reality: $\langle Q(\tilde{\rho})\rangle \lesssim 1$

A Better Fit Quality Parameter

$$Q(\rho) = \sqrt{\frac{1}{\cancel{M}\ M-d^2}\sum_m \frac{(N_m - n_m(\rho))^2}{\Delta n_m^2(\rho)}}$$

Ideal Properties

- Range: $0 \leq Q(\rho) < \infty$
- Expected mean = ???
- Inside the surface of the Bloch sphere: $\langle Q(\rho)\rangle = 1$
- When near the surface: $\langle Q(\rho)\rangle > 1$
- Q > 1 means: (a) there are zero eigenvalues

  OR (b) the error model is no good