

Title: Quantum state and process estimation in trapped ion experiments

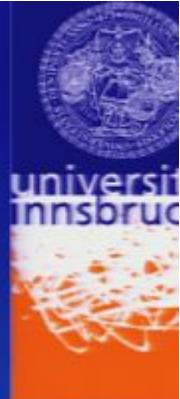
Date: Aug 25, 2008 11:15 AM

URL: <http://pirsa.org/08080032>

Abstract: The experimental realization of entangled states requires tools for characterizing the produced states as well as the processes used for creating the entanglement. In my talk, I will present examples of quantum measurements occurring in trapped ion experiments aiming at creating high-fidelity quantum gates.



# (Nonoptimal) Characterization of quantum states and gates in trapped ion experiments



Christian Roos

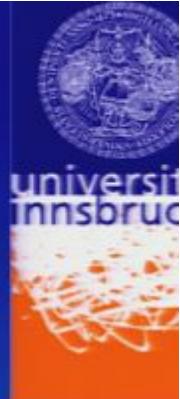
Institute for Quantum Optics and Quantum Information  
Austrian Academy of Sciences  
Innsbruck, Austria

Outline :

1. Quantum information processing with trapped ions
2. Tomography of entangled states and quantum gates



# (Nonoptimal) Characterization of quantum states and gates in trapped ion experiments



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Outline :

1. Quantum information processing with trapped ions
2. Tomography of entangled states and quantum gates

# Experiments with single trapped ions

Precision spectroscopy /  
Optical frequency standards

Absolute frequency measurement  
of the  $S_{1/2}$ - $D_{5/2}$  transition in  $^{40}\text{Ca}^+$

M. Chwalla *et al.*, arXiv:0806.1414

Quantum information processing

Entangled states of 4...8 ions  
H. Häffner *et al.*, Nature **438**, 643 (2005)

High-fidelity two-ion quantum gate  
J. Benhelm *et al.*, Nat. Phys. **4**, 463 (2008)

Quantum metrology

'Designer atoms' for quantum metrology  
C. F. Roos *et al.*, Nature **443**, 316 (2006)

# Experiments with single trapped ions

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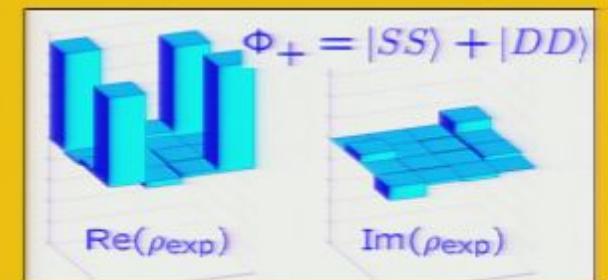
M. Chwalla *et al.*, arXiv:0806.1414

Quantum information processing

Entanglement  
H. Häffner

High-fidelity  
J. Benhelm

Quantum state tomography

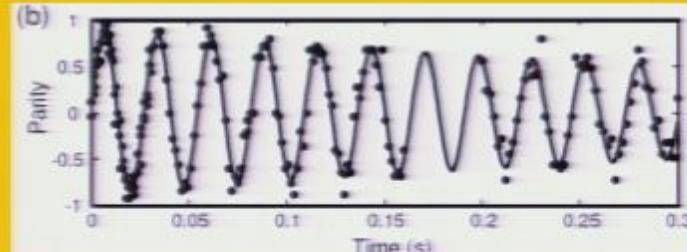


Quantum metrology

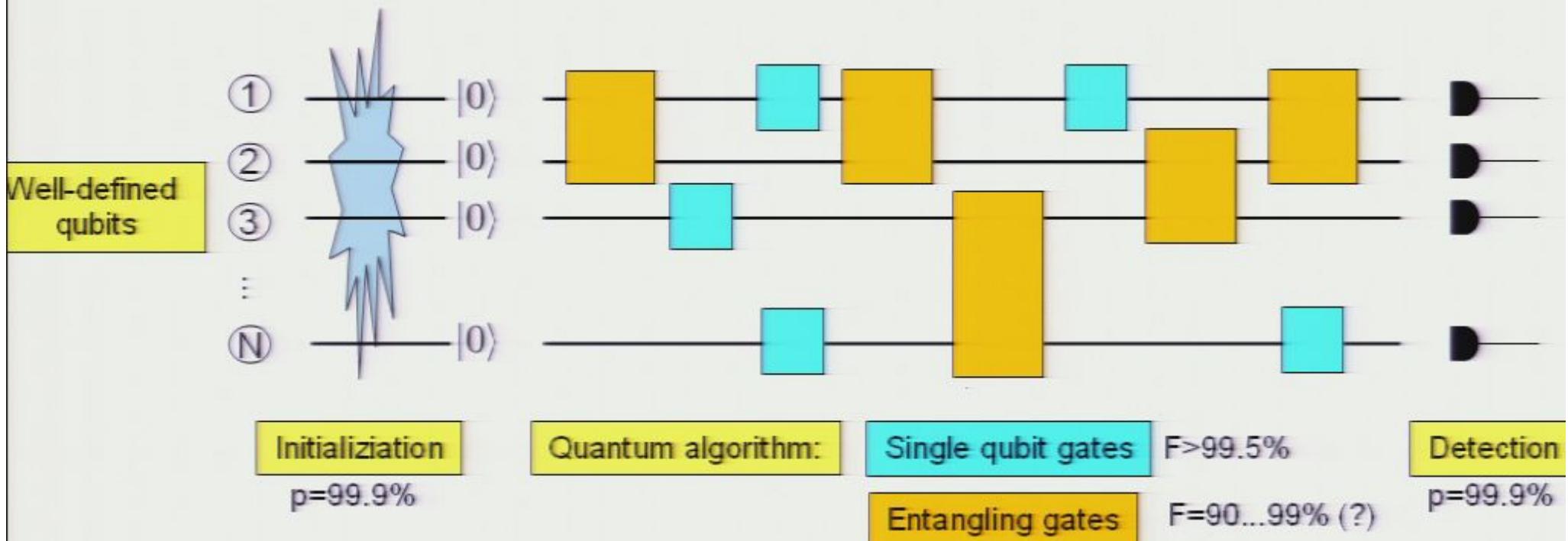
'Designer atom'

C. F. Roos *et al.*

Phase estimation



# Quantum information processing (with trapped ions)



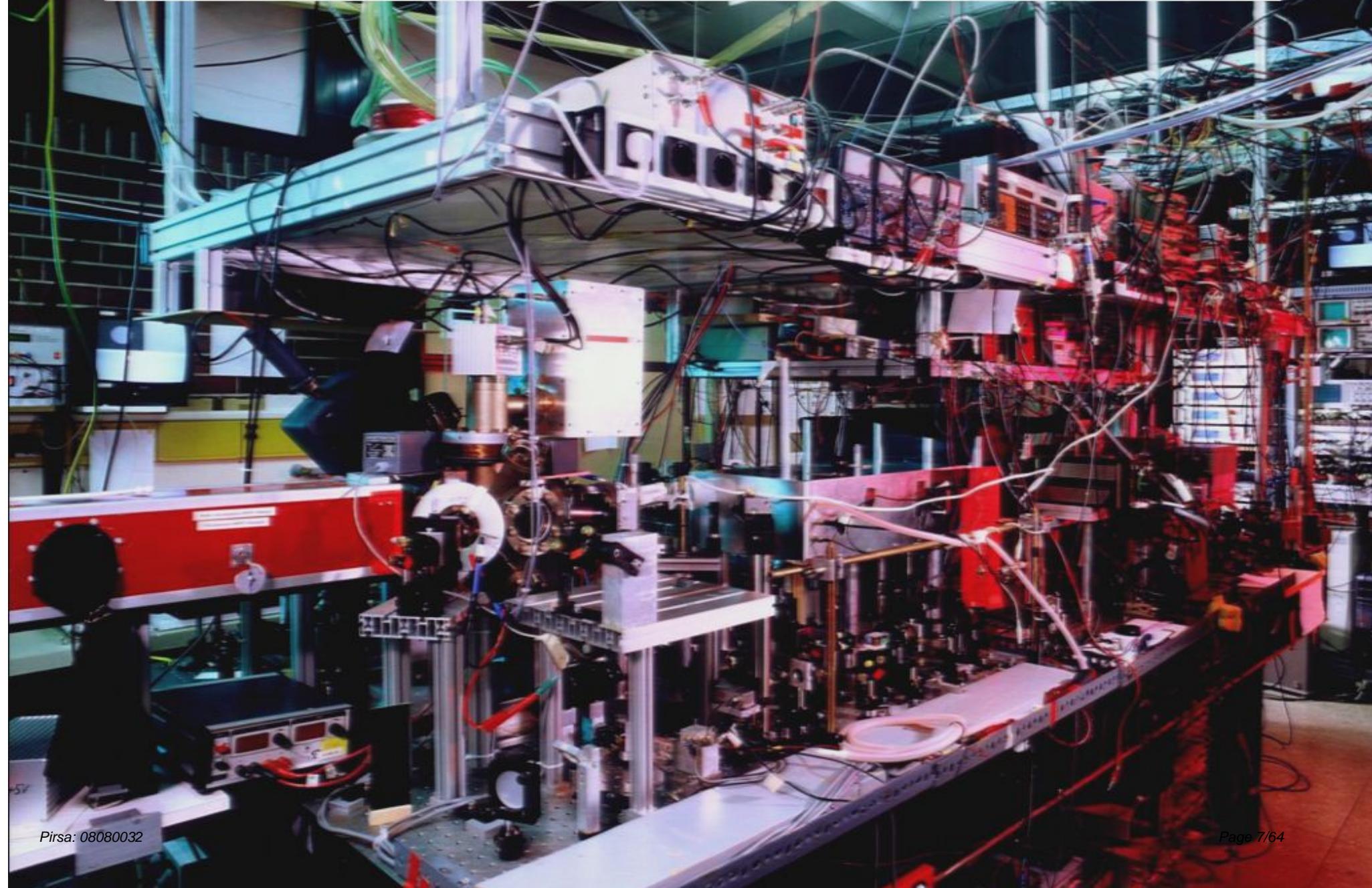
Current experiments:

2 - 4 ions, few entangling gates

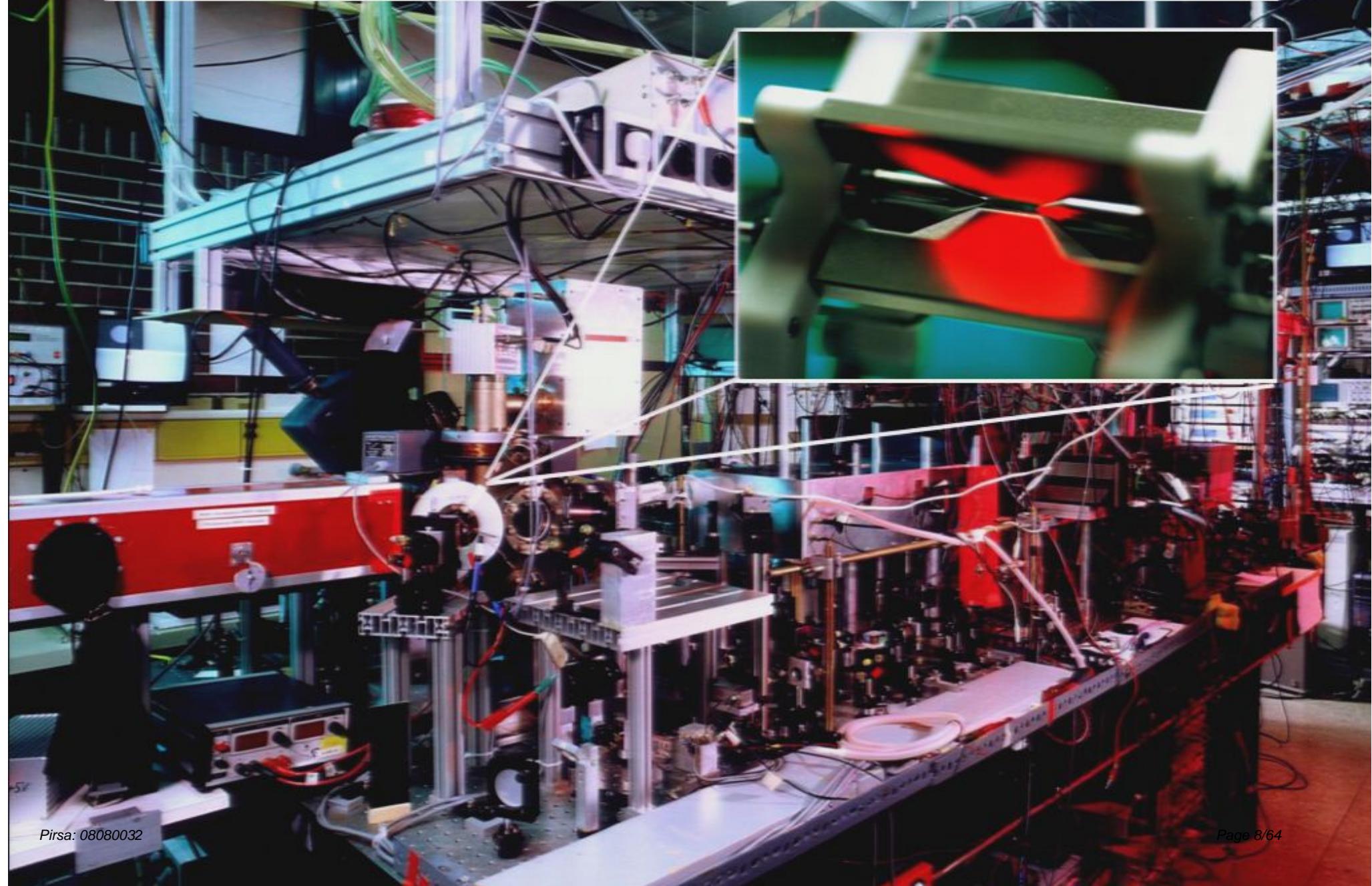
Challenges:

- Increase number of ions
- Reduce gate error rates

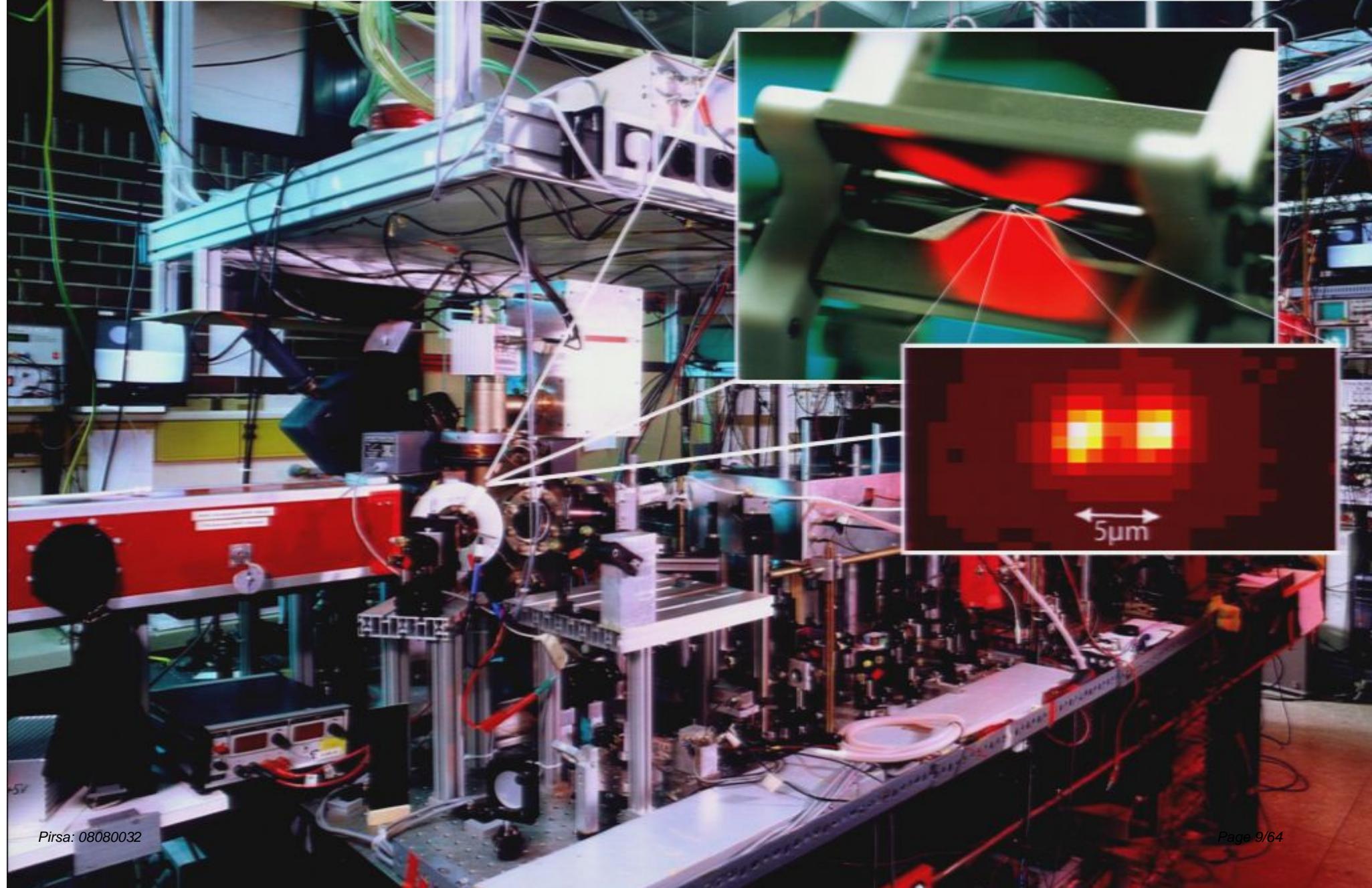
# Quantum information processing with trapped ions



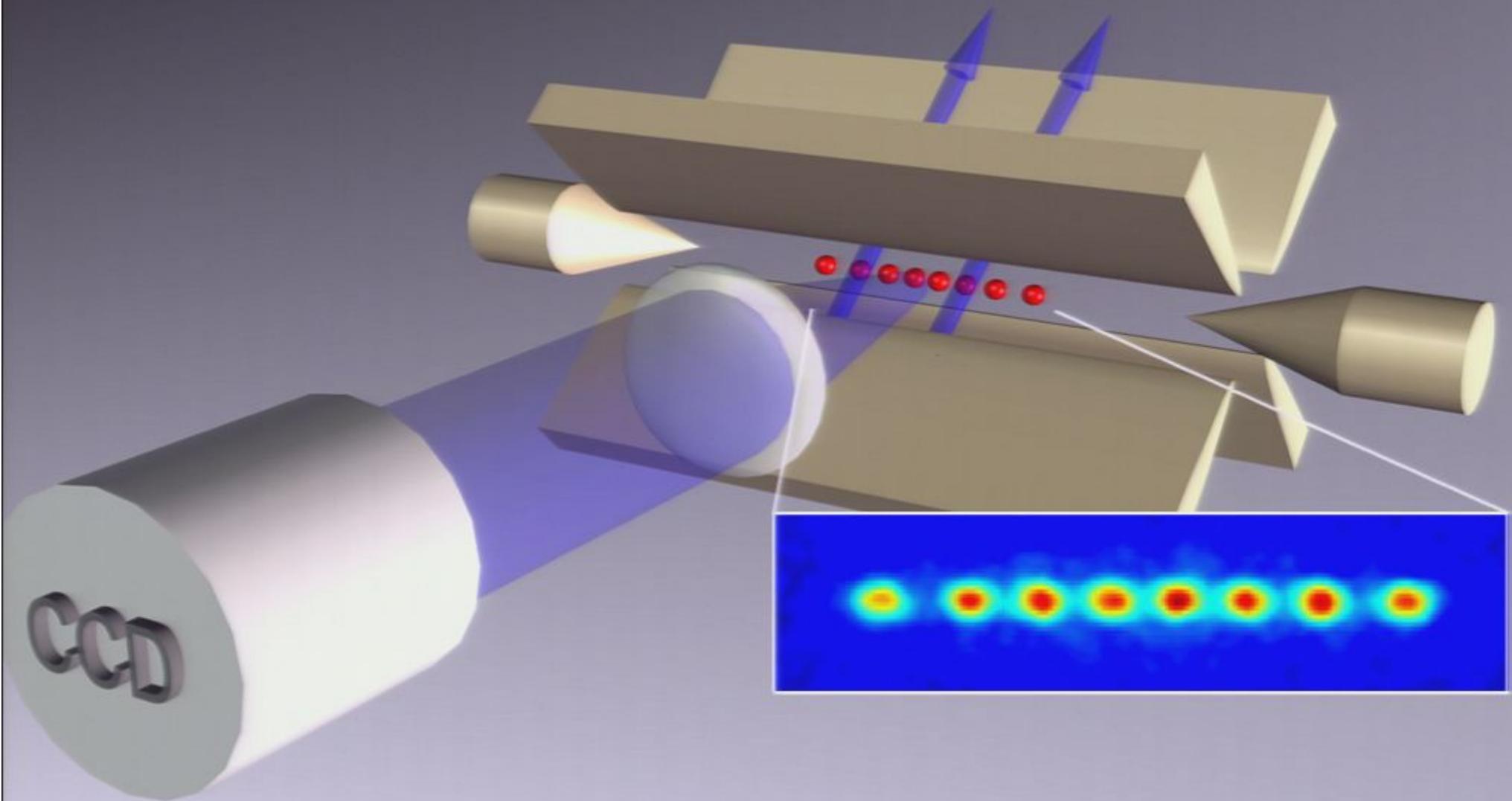
# Quantum information processing with trapped ions



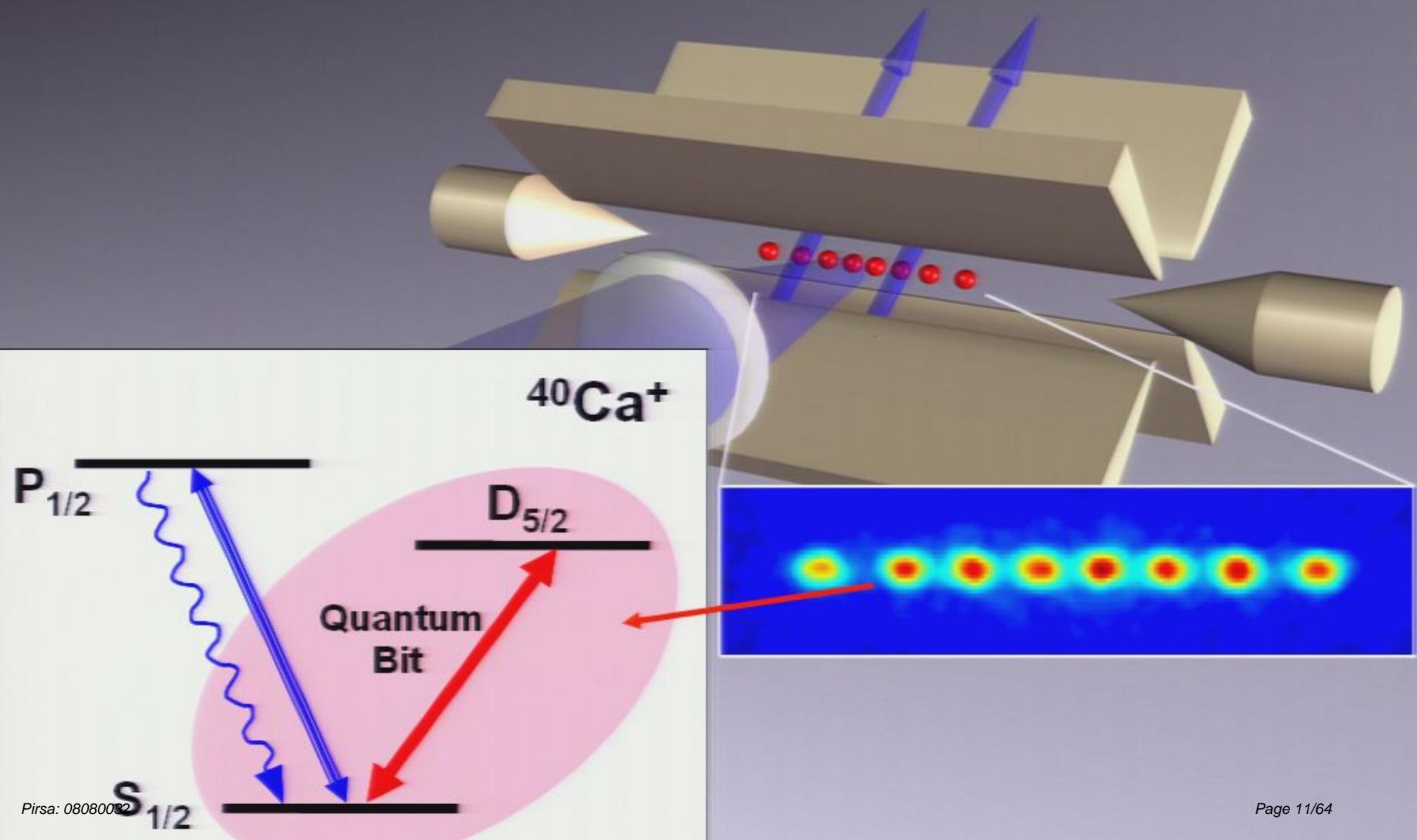
# Quantum information processing with trapped ions



# Quantum information processing with trapped ions



# Quantum information processing with trapped ions



# Coupling ions for quantum gate operations

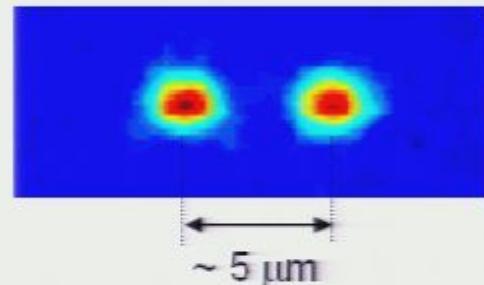
Ion crystals:

Carriers of quantum information

Internal  
degrees of freedom

Storage of quantum information

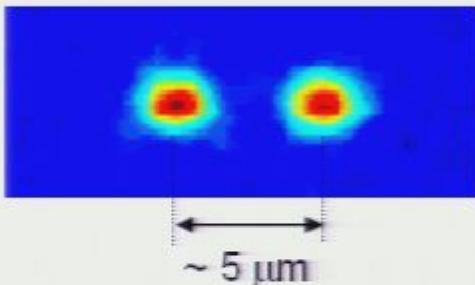
$$\Psi = \alpha|g\rangle + \beta|e\rangle$$



# Coupling ions for quantum gate operations

Ion crystals:

Carriers of quantum information



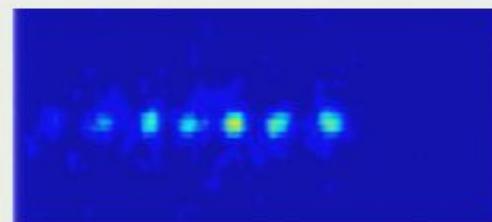
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External  
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Coulomb coupling:

„center-of-mass mode“

# Coupling ions for quantum gate operations

Ion crystals:

Carriers of quantum information

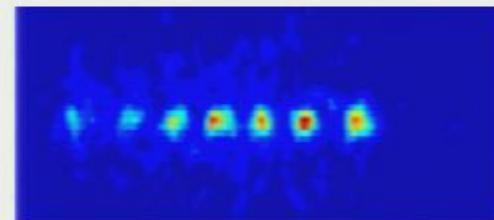
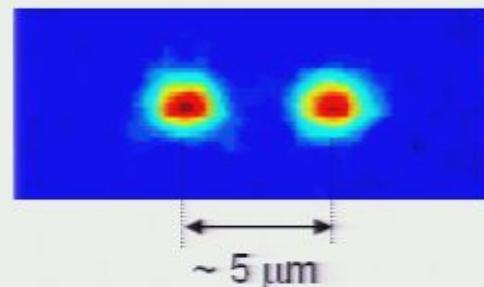
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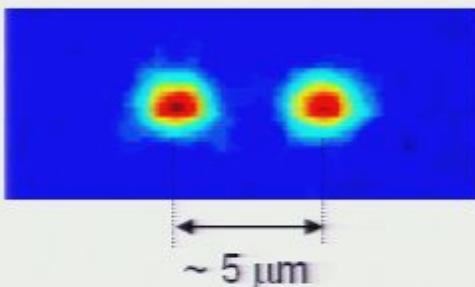


„center-of-mass mode“

# Coupling ions for quantum gate operations

Ion crystals:

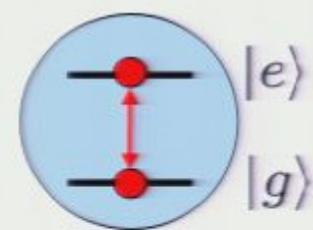
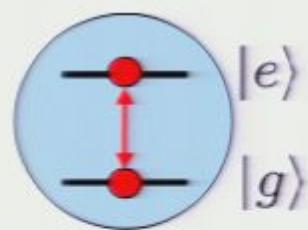
Carriers of quantum information



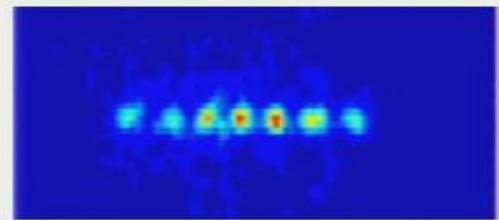
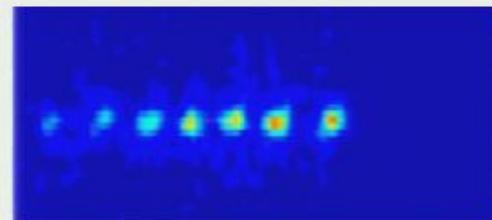
Internal  
degrees of freedom

Storage of quantum information

$$\Psi = \alpha|g\rangle + \beta|e\rangle$$



External  
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Coulomb coupling:

„center-of-mass mode“

„stretch mode“

# Coupling ions for quantum gate operations

Ion crystals:

Carriers of quantum information

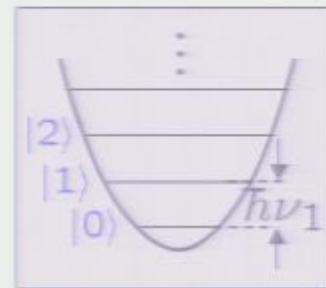
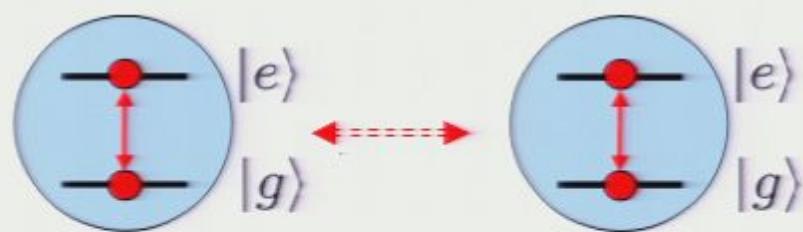
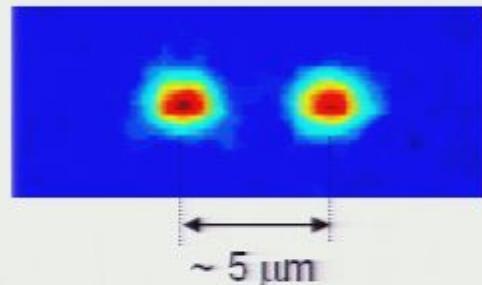
Internal  
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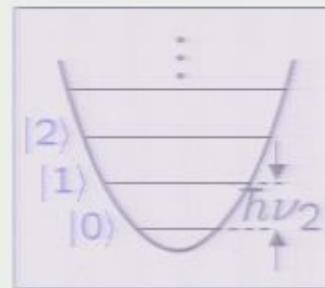
$$\Psi = \alpha|g\rangle + \beta|e\rangle$$

External  
degrees of freedom

Effective ion-ion interaction  
by coupling internal and external  
states with lasers

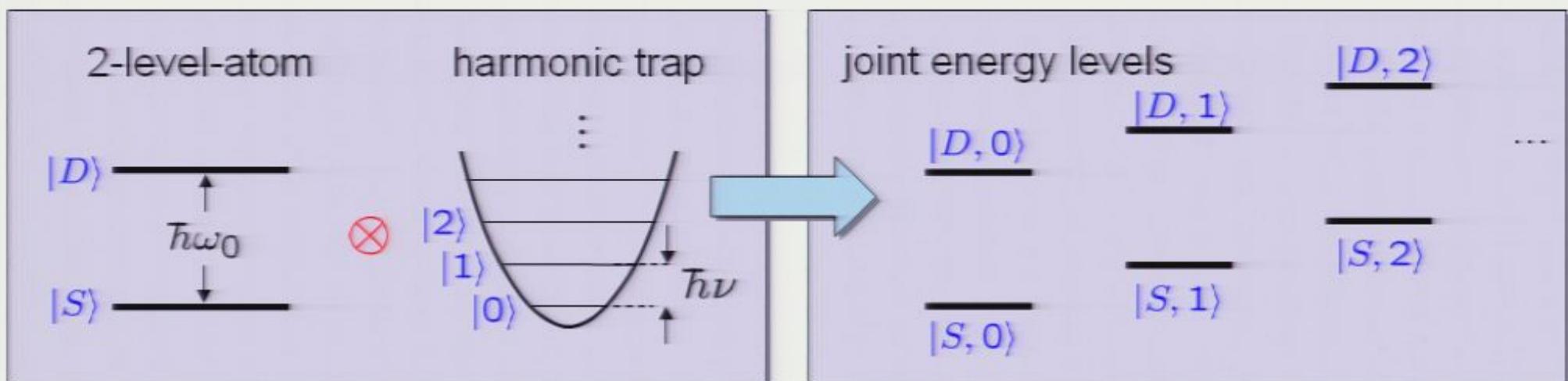


„center-of-mass mode“

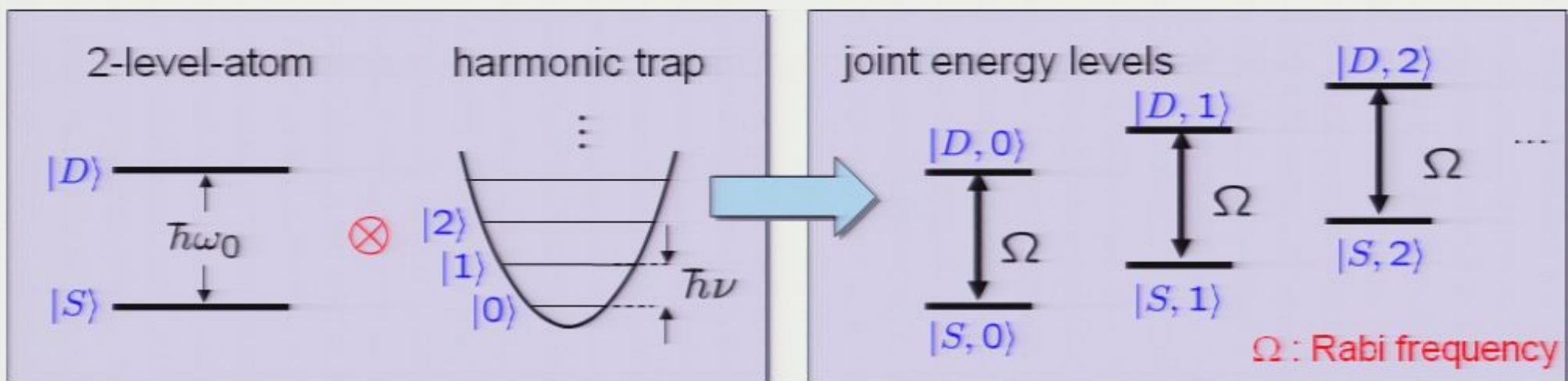


„stretch mode“

# Coherent manipulation by laser pulses

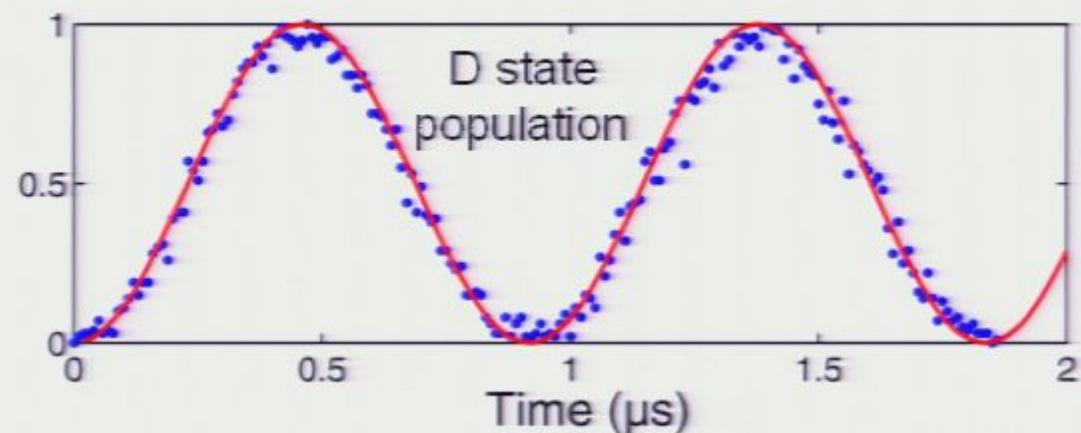


# Coherent manipulation by laser pulses

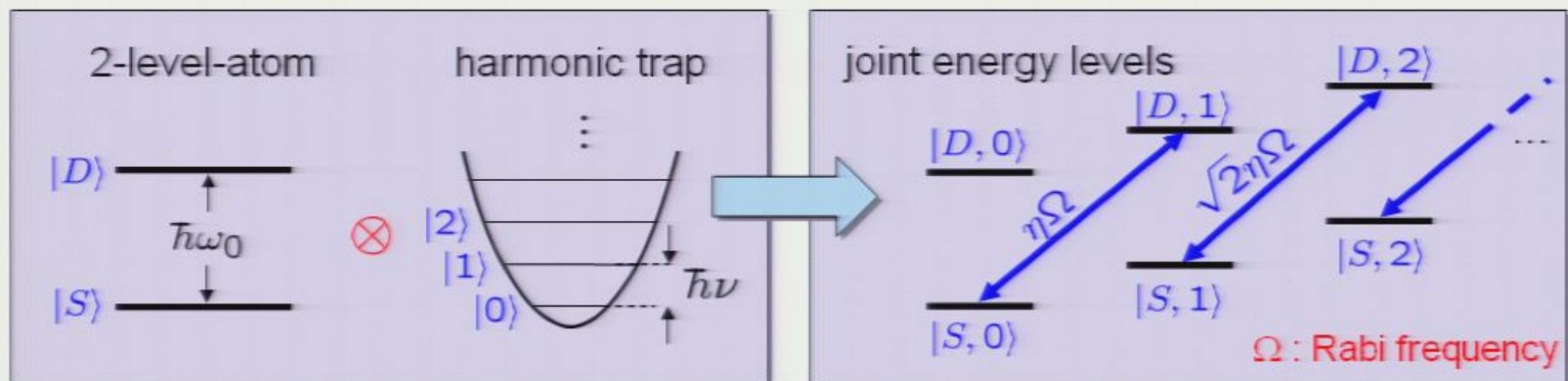


Carrier excitation:  
Manipulation of the internal state

$$|S\rangle \longleftrightarrow |D\rangle$$



# Coherent manipulation by laser pulses

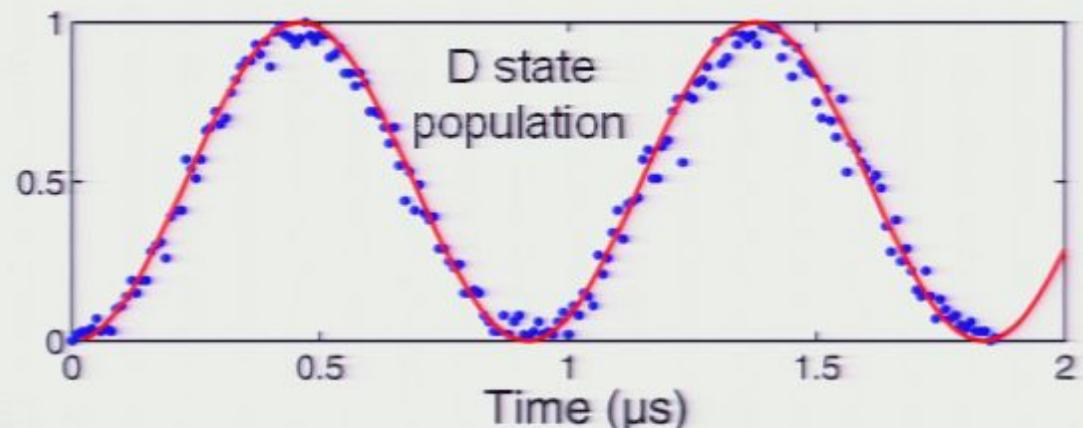


Carrier excitation:  
Manipulation of the internal state

$$|S\rangle \longleftrightarrow |D\rangle$$

Blue sideband excitation:  
Entangling internal and motional state

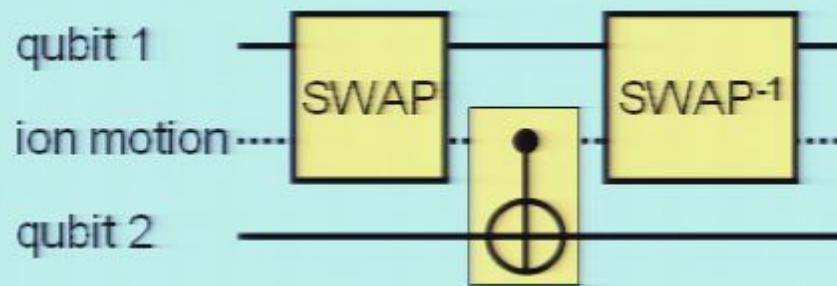
$$|S\rangle|n\rangle \longleftrightarrow |D\rangle|n+1\rangle$$



# Quantum algorithms with trapped ions

Elementary building blocks: entangling quantum gates

Controlled-NOT gate



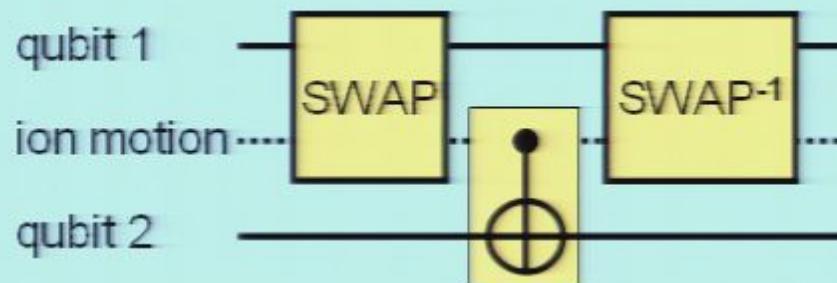
Bell state fidelity:  $F \approx 95\%$

M. Riebe et al., PRL 97, 220407 (2006)

# Quantum algorithms with trapped ions

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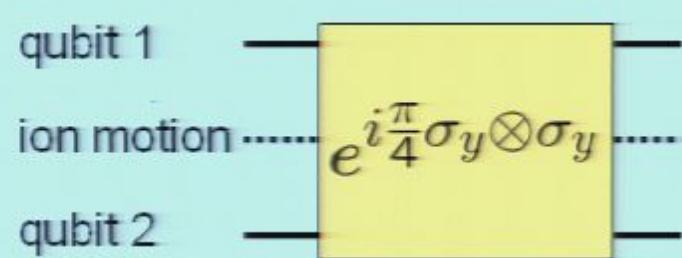
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Sørensen-Mølmer gate



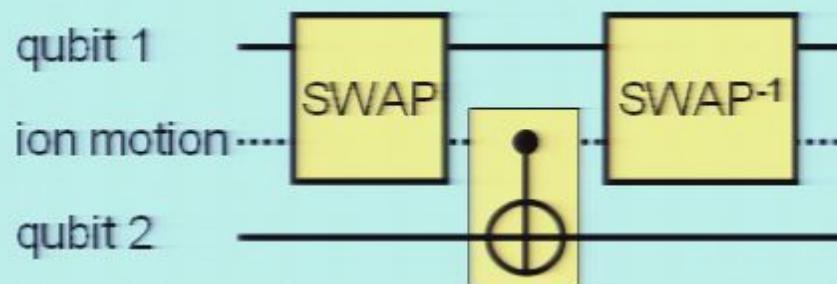
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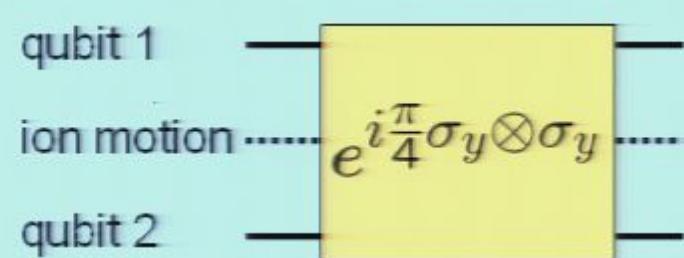
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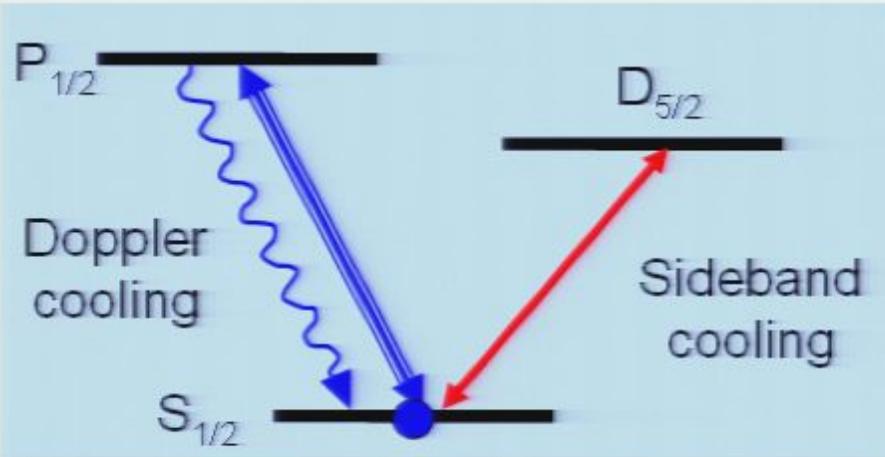
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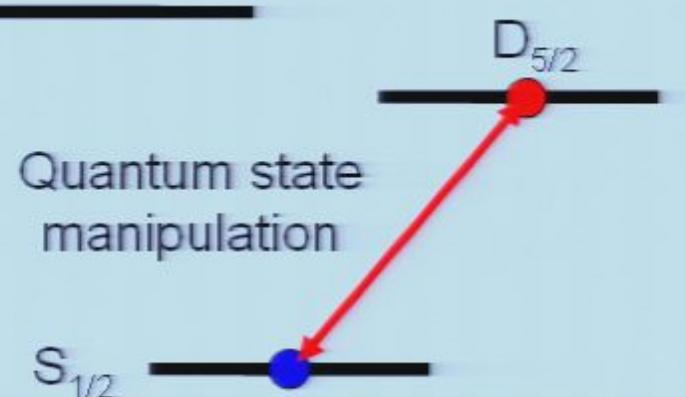
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# Experiments with $^{40}\text{Ca}^+$ : Experimental procedure



## 1. Initialization

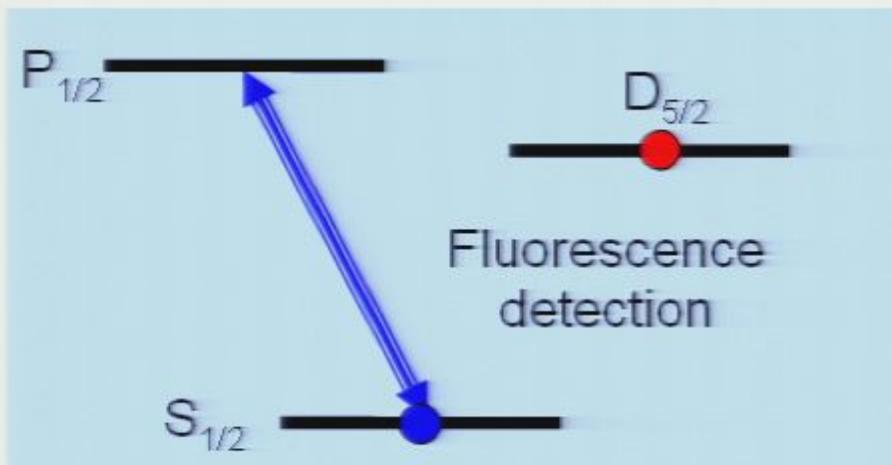
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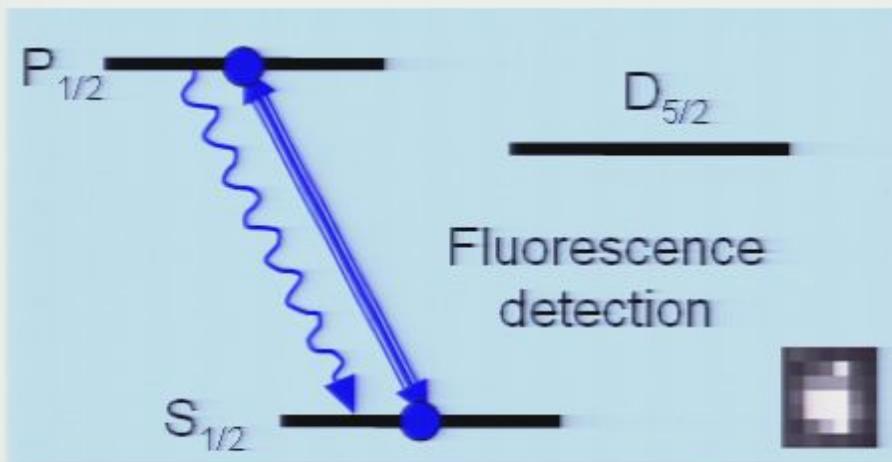
2. Coherent excitation of  
 $S_{1/2} - D_{5/2}$  transition

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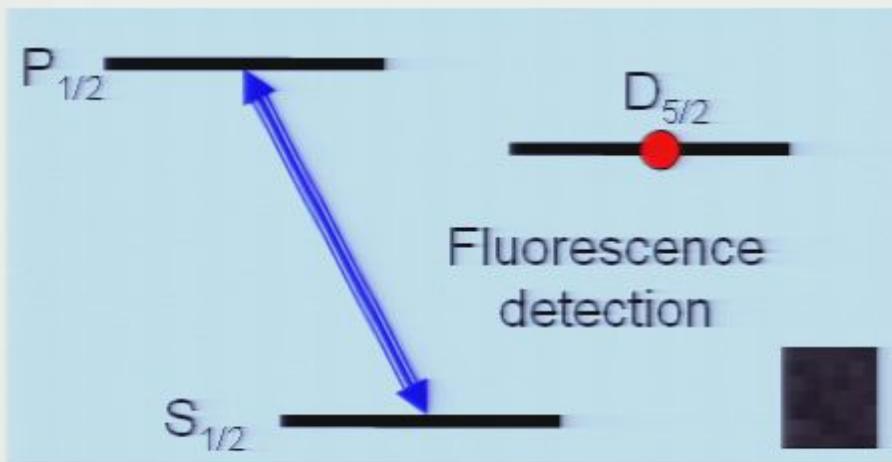
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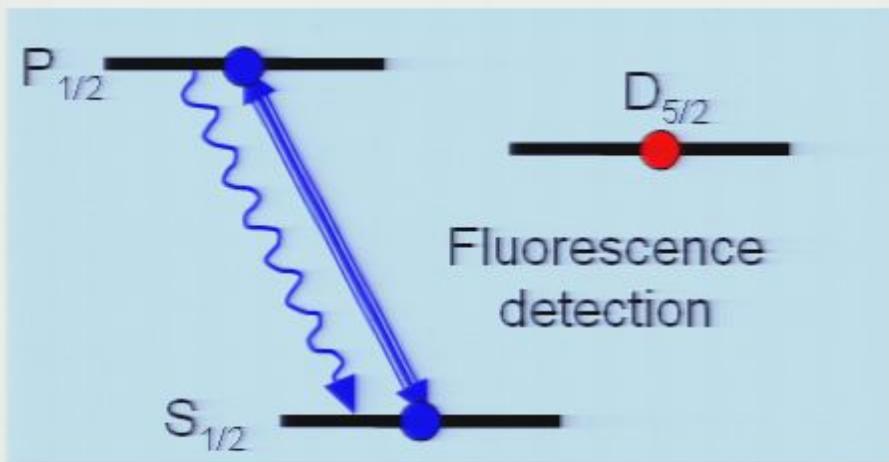
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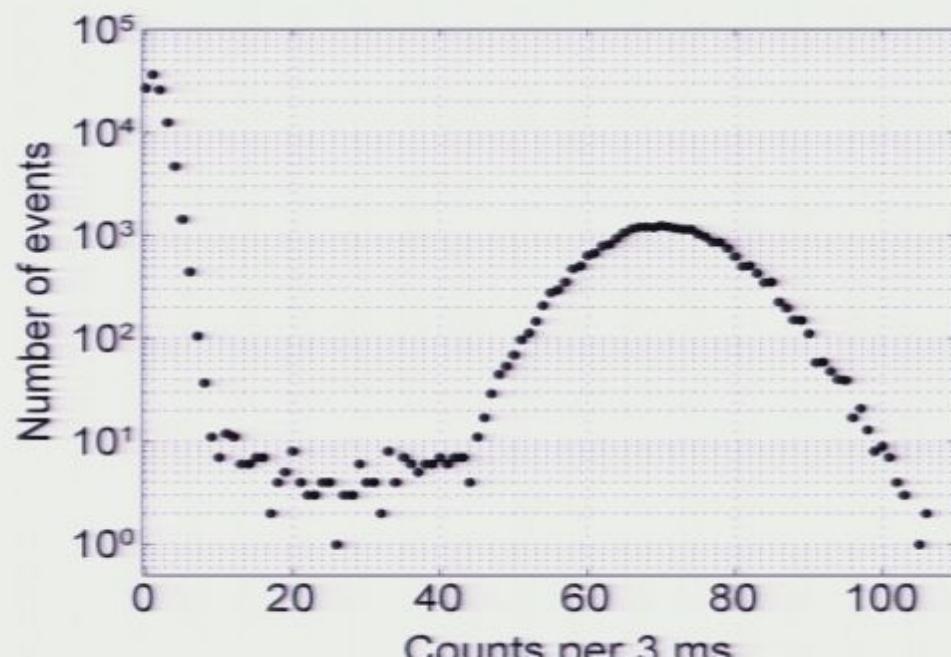
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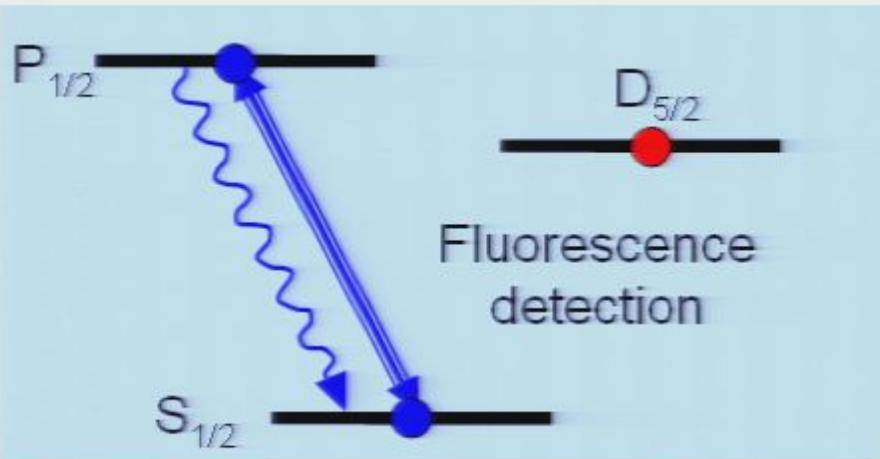
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One ion:

Photon count histogram



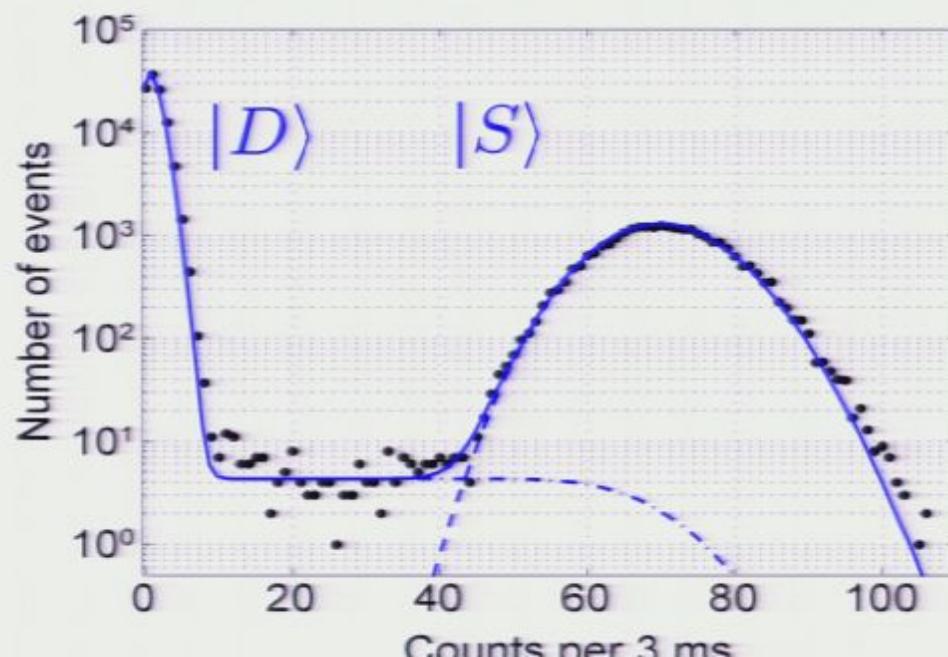
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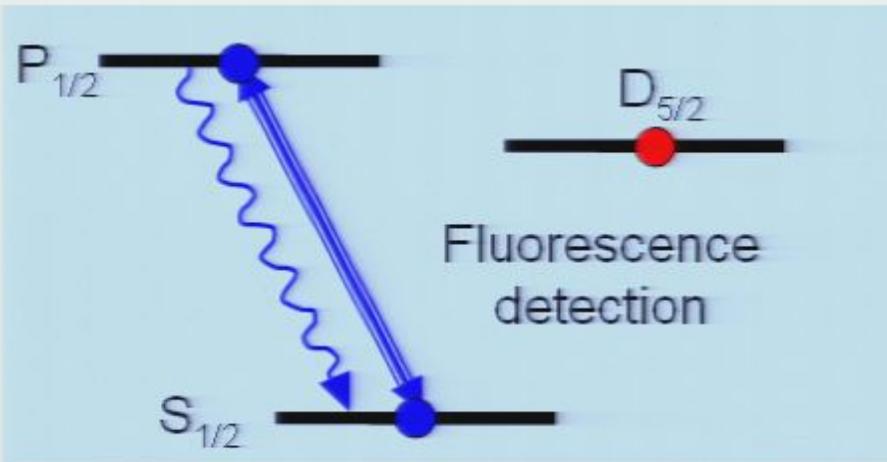
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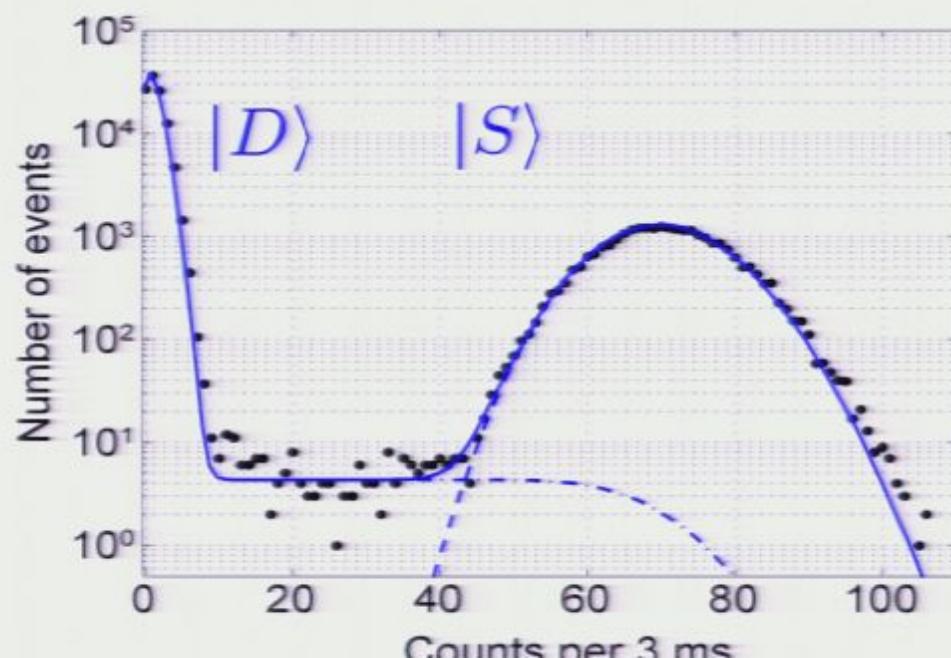
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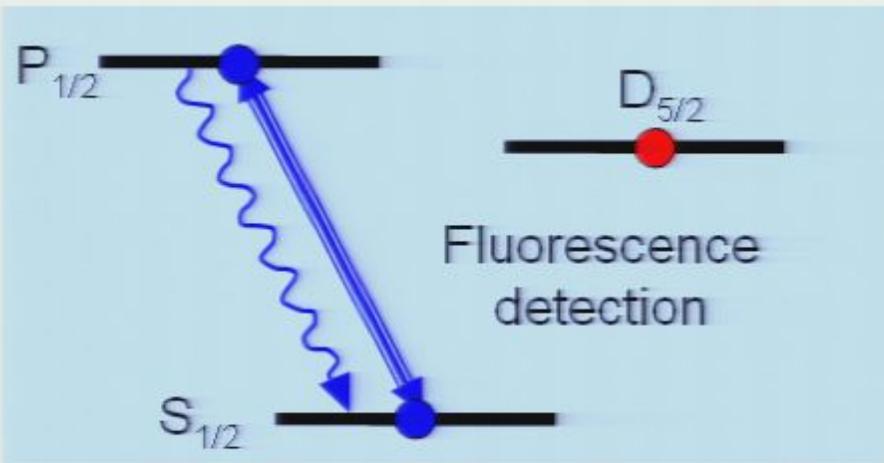
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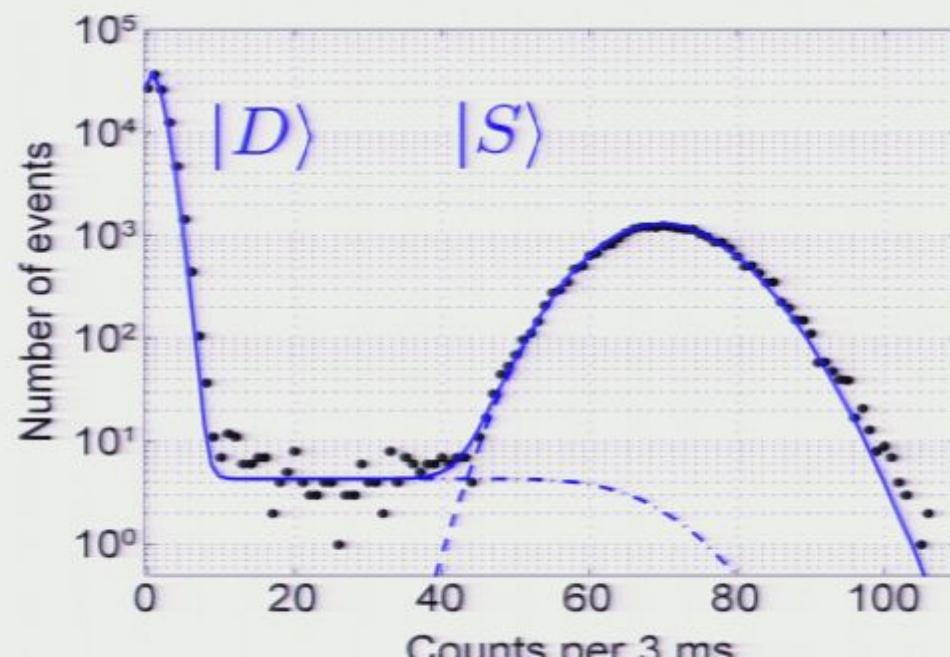
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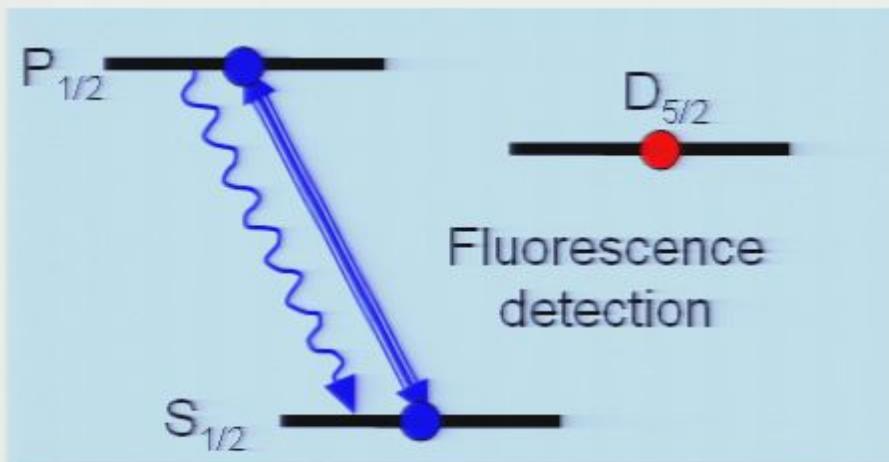
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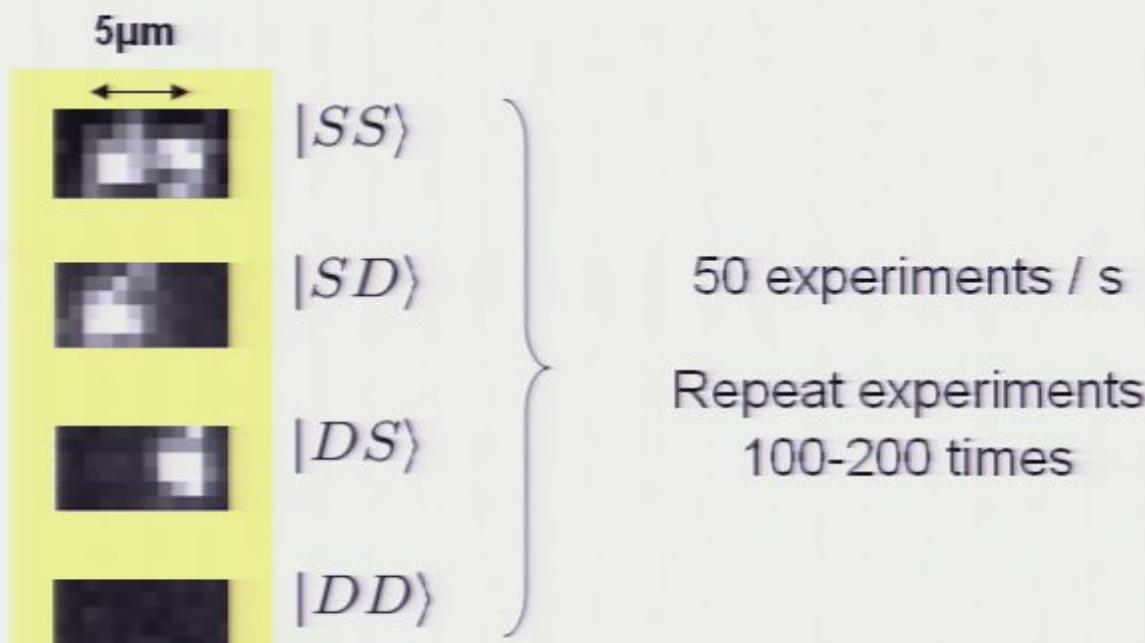
# Experiments with $^{40}\text{Ca}^+$ : Experimental procedure



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Two ions:

Spatially resolved  
detection with  
CCD camera

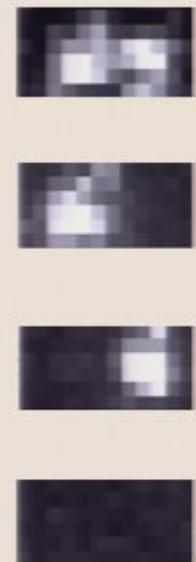


## Bell state analysis

$$|SD\rangle + |DS\rangle$$

Fluorescence  
detection with  
CCD camera:

$$\begin{cases} |SS\rangle \\ |SD\rangle \\ |DS\rangle \\ |DD\rangle \end{cases}$$



## Bell state analysis

$$|SD\rangle + |DS\rangle$$

Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

Fluorescence  
detection with  
CCD camera:

$ SS\rangle$	
$ SD\rangle$	
$ DS\rangle$	
$ DD\rangle$	

# Bell state analysis

$$|SD\rangle + |DS\rangle$$

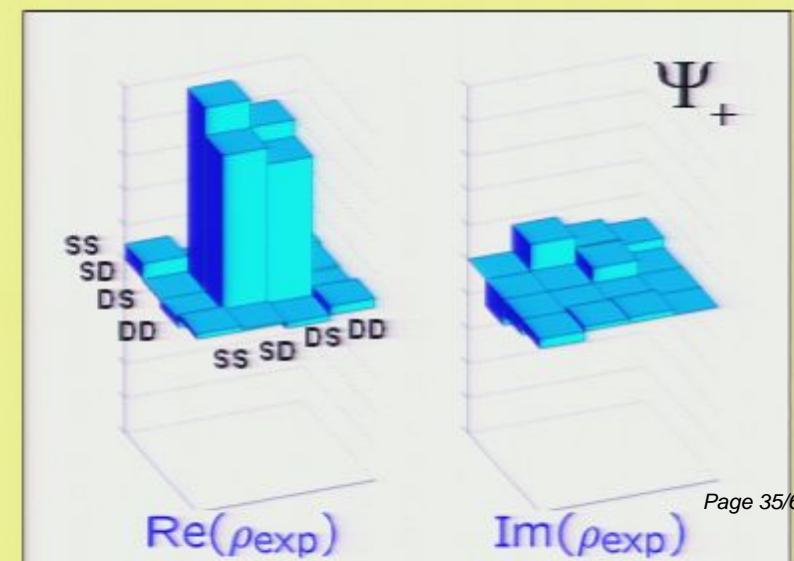
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Fluorescence  
detection with  
CCD camera:



→ Measurement of the density matrix:



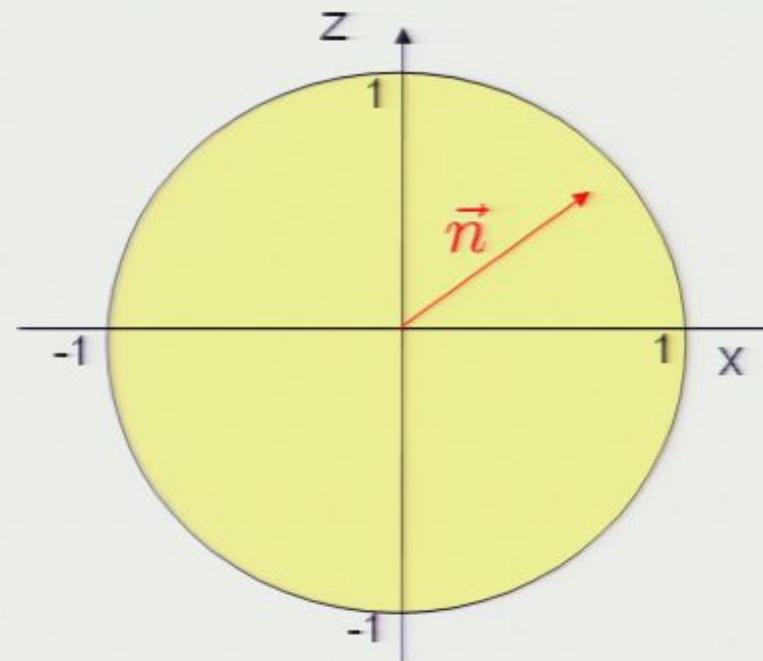
# Reconstruction of the density matrix

One qubit:

$$\rho = \frac{1}{2} (I + n_x \sigma_x + n_y \sigma_y + n_z \sigma_z)$$

Observables: Pauli spin matrices  $I, \sigma_x, \sigma_y, \sigma_z$

Expectation values:  $n_i = \langle \sigma_i \rangle$



# Reconstruction of the density matrix

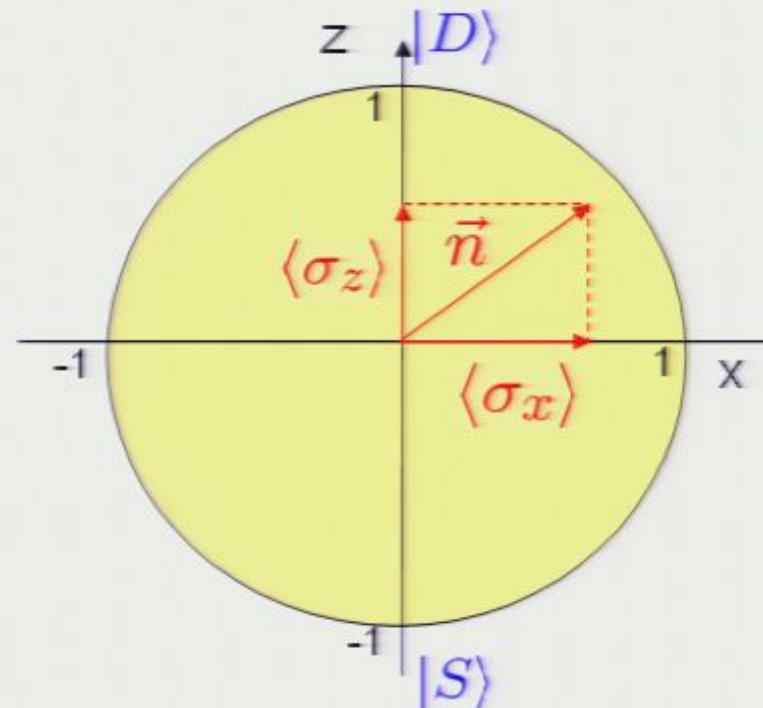
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Expectation values:  $n_i = \langle \sigma_i \rangle$

Natural measurement basis:  $\begin{array}{l} |+\rangle_z \leftrightarrow |D\rangle \\ |-\rangle_z \leftrightarrow |S\rangle \end{array} \quad \left. \right\}$  Fluorescence measurement



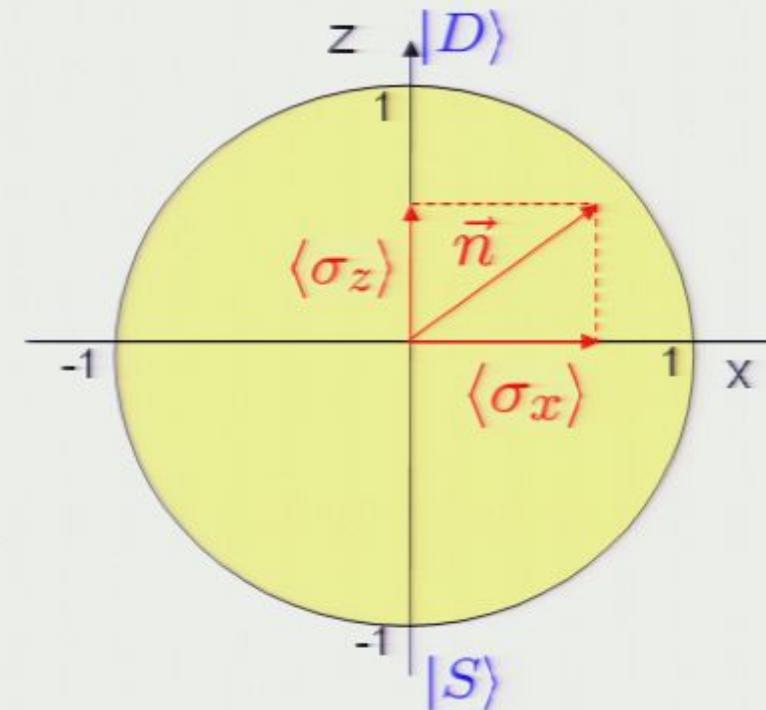
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Natural measurement basis:  $\left. \begin{array}{l} |+\rangle_z \leftrightarrow |D\rangle \\ |-\rangle_z \leftrightarrow |S\rangle \end{array} \right\}$  Fluorescence measurement

Other measurement bases:

$\left. \begin{array}{l} |+\rangle_{x,y} \\ |-\rangle_{x,y} \end{array} \right\}$   $\pi/2$  pulse  
+  
fluorescence measurement

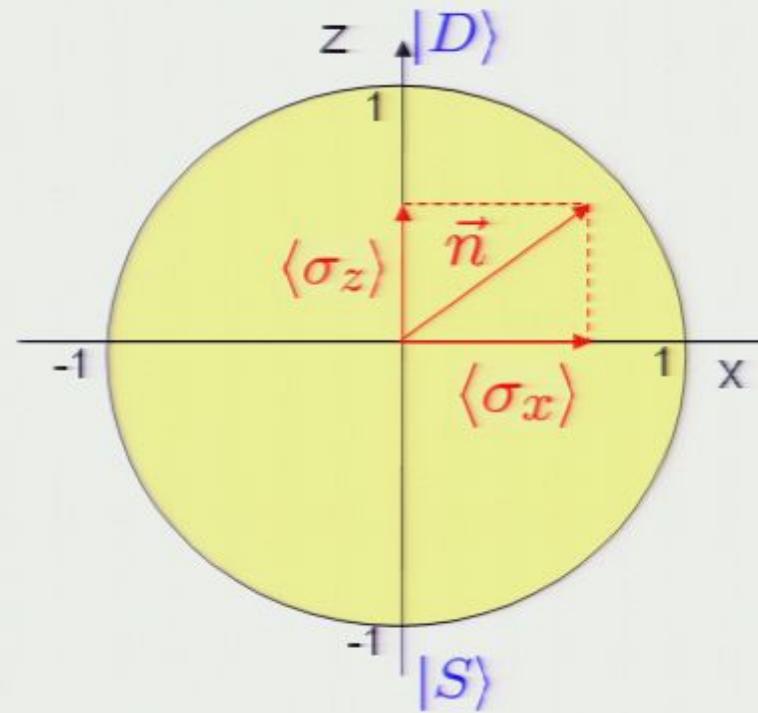
# Reconstruction of the density matrix

One qubit:

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Observables: Pauli spin matrices  $I, \sigma_x, \sigma_y, \sigma_z$

Expectation values:  $n_i = \langle \sigma_i \rangle$



N qubits:

Representation of  $\rho$  as a sum of orthogonal observables  $A_i$ :

$$\rho = \sum_i \langle A_i \rangle A_i$$

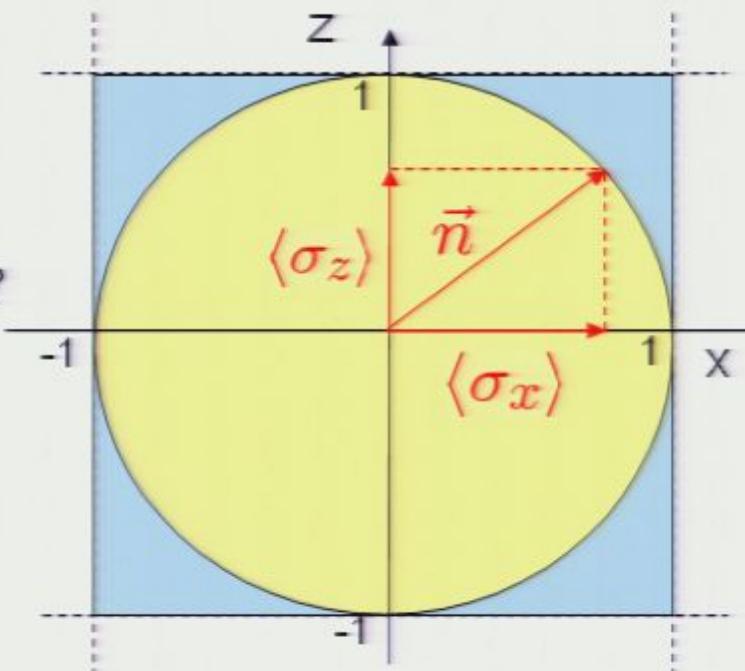
$$A_i = \sigma_m^{(1)} \otimes \sigma_n^{(2)} \otimes \dots$$

# Measurement uncertainties

Direct reconstruction:

Is  $\rho_R = \sum_i \langle A_i \rangle A_i$  positive semidefinite ?

... not necessarily:

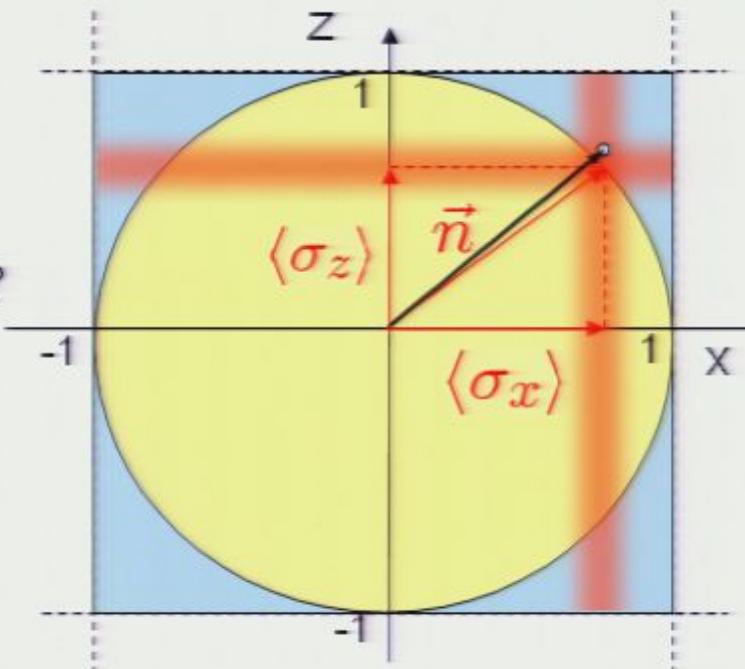


# Measurement uncertainties

## Direct reconstruction:

Is  $\rho_R = \sum_i \langle A_i \rangle A_i$  positive semidefinite ?

... not necessarily:



Shot noise in the measurement  
might give rise to  
unphysical density matrices

## Maximum likelihood estimation:

Determine density matrix that is most likely to reproduce the  
experimentally observed results.

# Maximum likelihood estimation

Maximum likelihood estimation: (Hradil '97, Banaszek '99)

Find the density matrix that is most likely to reproduce the experimentally observed results.

In  $N$  experiments, the quantum state is projected onto the outcomes  $|y_j\rangle$ .

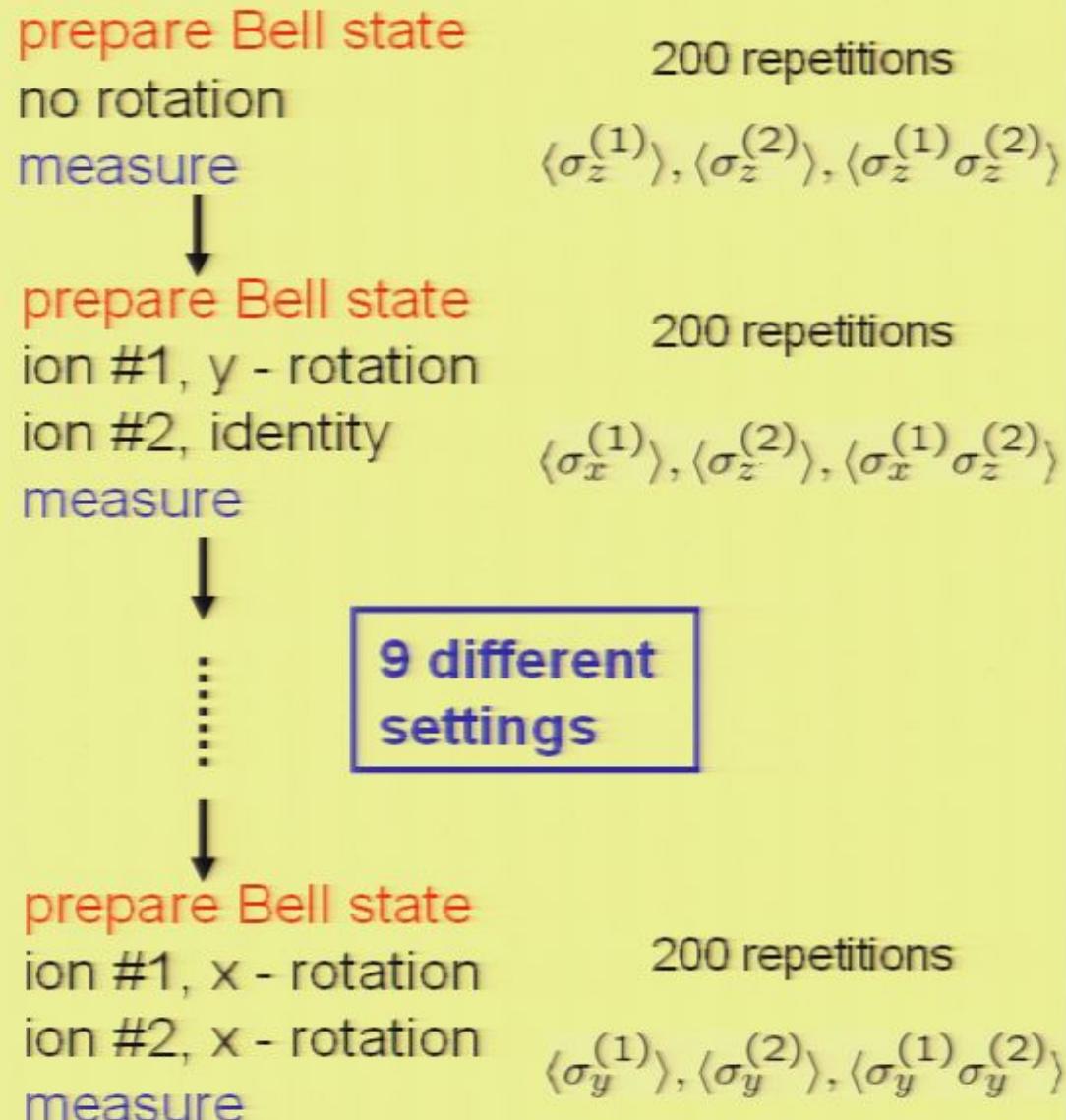
$f_j$ : relative frequency of the outcome  $|y_j\rangle$

On the set of density matrices  $\rho$ , look for the matrix that maximizes

$$\mathcal{L}(\rho) = \prod_j \langle y_j | \rho | y_j \rangle^{N f_j}$$

$$\text{Maximize } L(\rho) = \log \mathcal{L}(\rho) = N \sum_j f_j \log \langle y_j | \rho | y_j \rangle$$

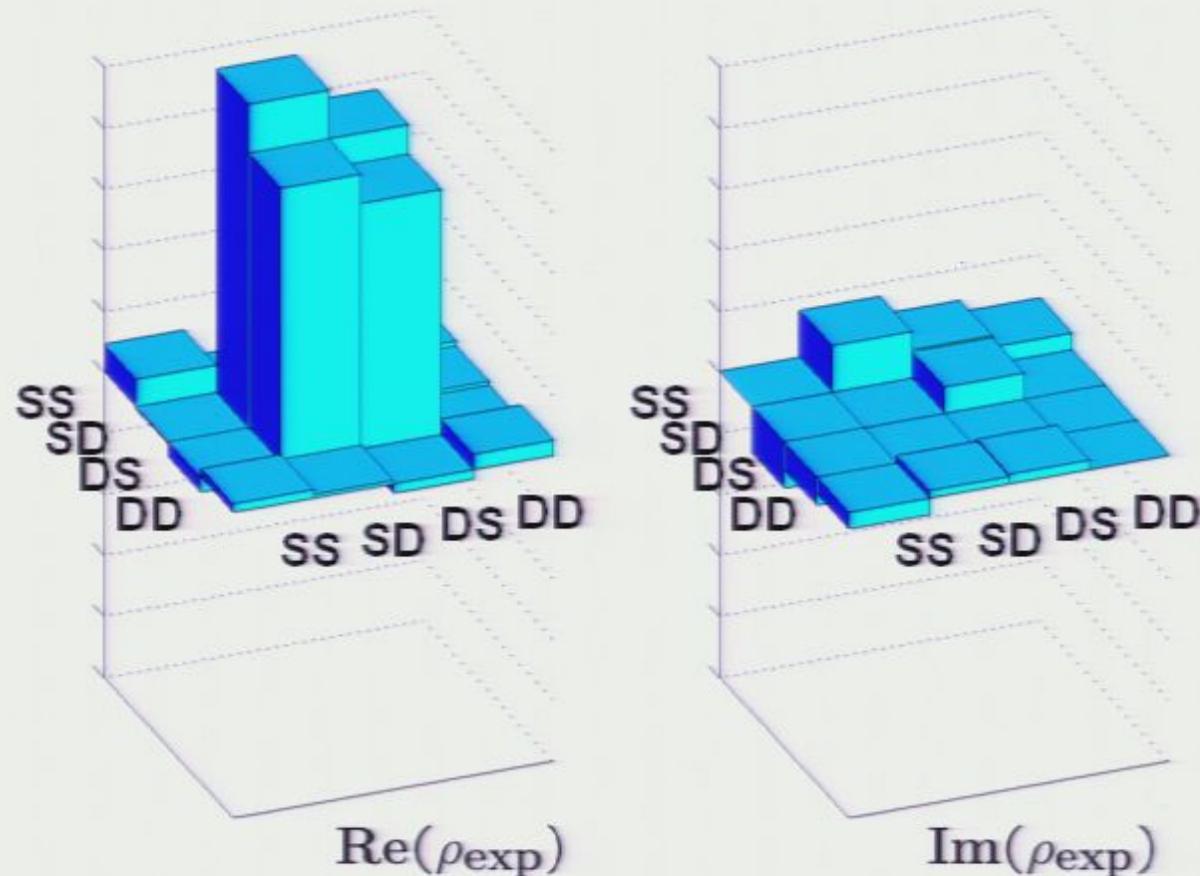
# Analysis of two-qubit states



# Bell state reconstruction

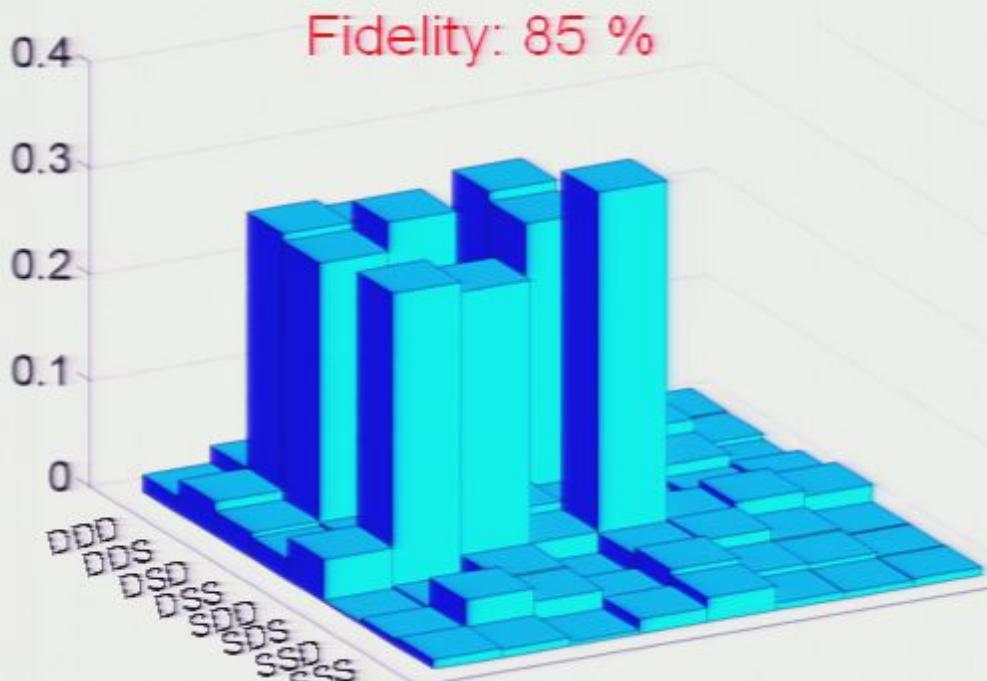
Measurement of 16 observables  
9 different settings, measurement time  $\approx 40$  s

$|SD\rangle + |DS\rangle$   
 $F=0.91$

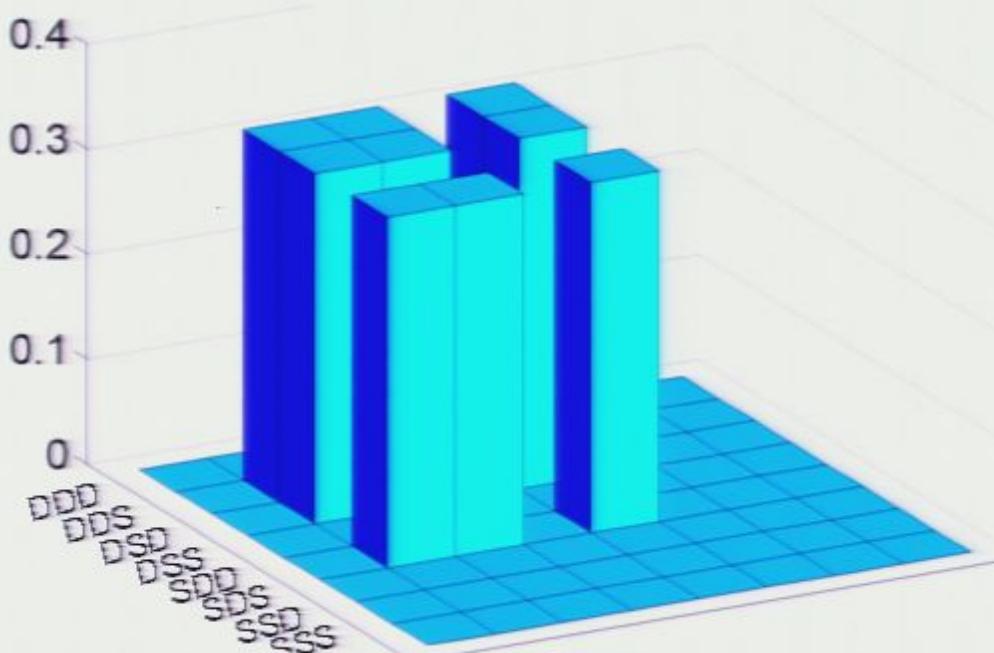


## Three-ion entangled states

$$|\Psi\rangle = \frac{1}{\sqrt{3}} (|SDD\rangle + |DSD\rangle + |DDS\rangle)$$



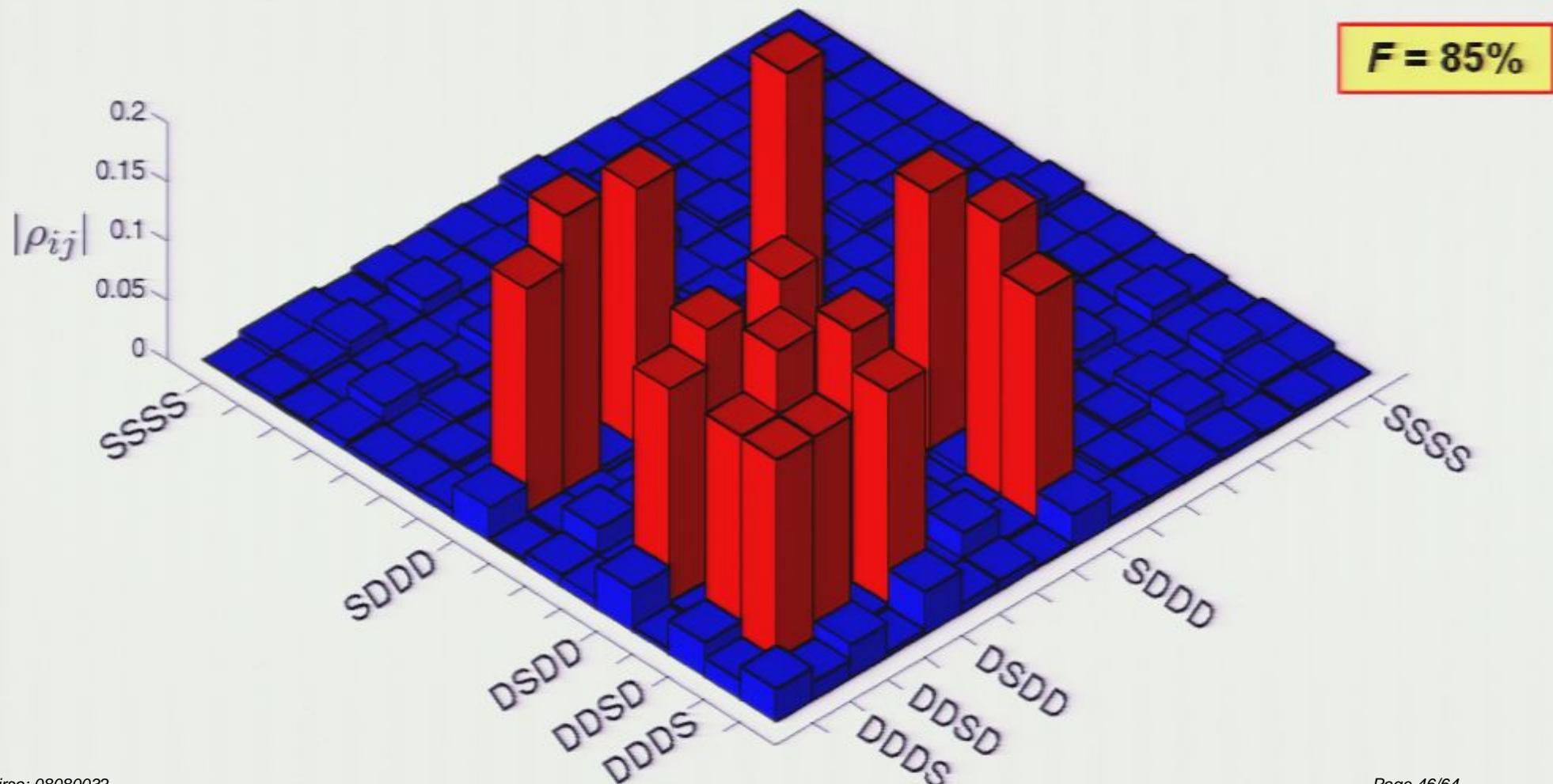
experimental result



theoretical expectation

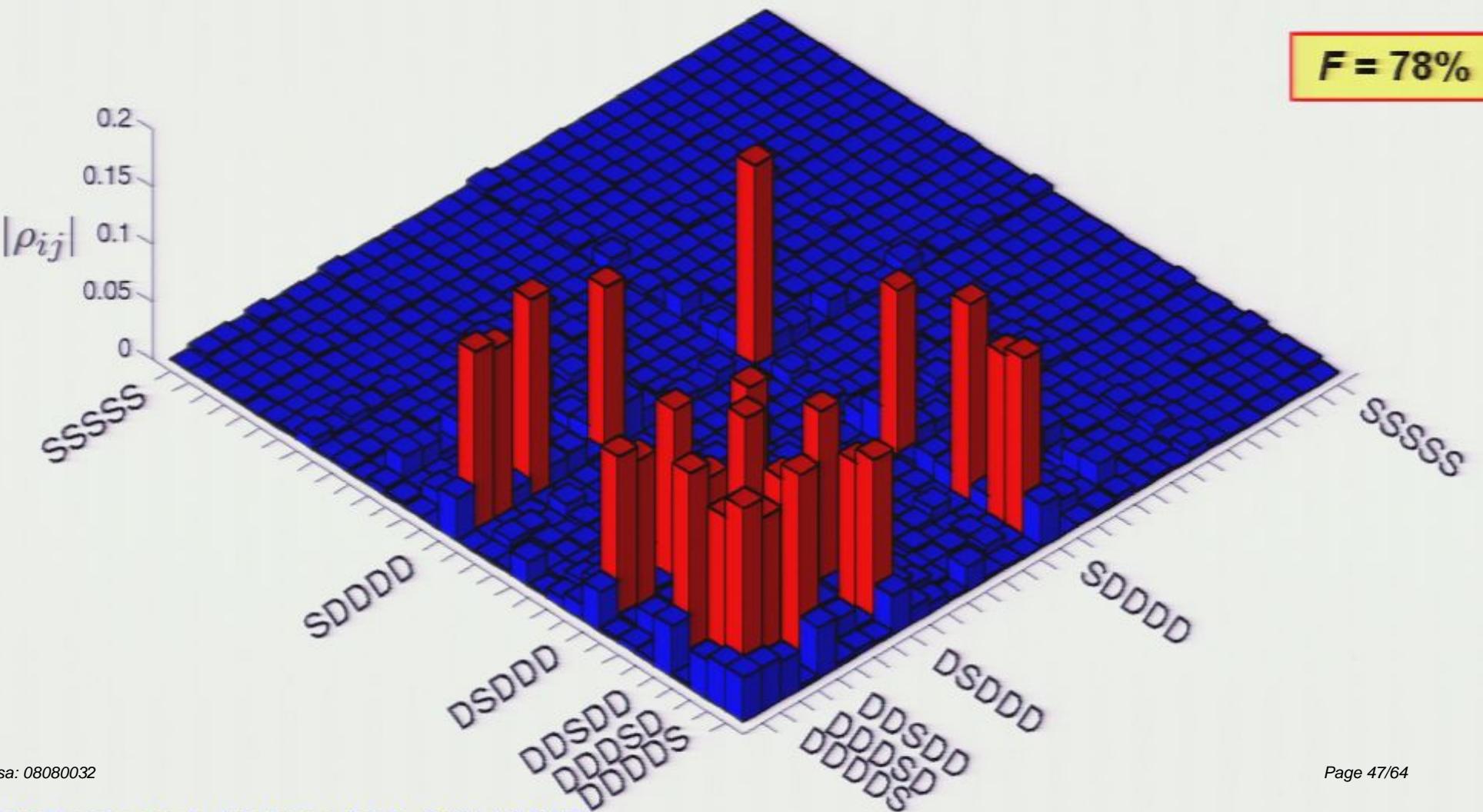
## Four-ion W-states

$$\Psi_4 = \frac{1}{\sqrt{4}}(|DDDS\rangle + |DDSD\rangle + |DSDD\rangle + |SDDD\rangle)$$



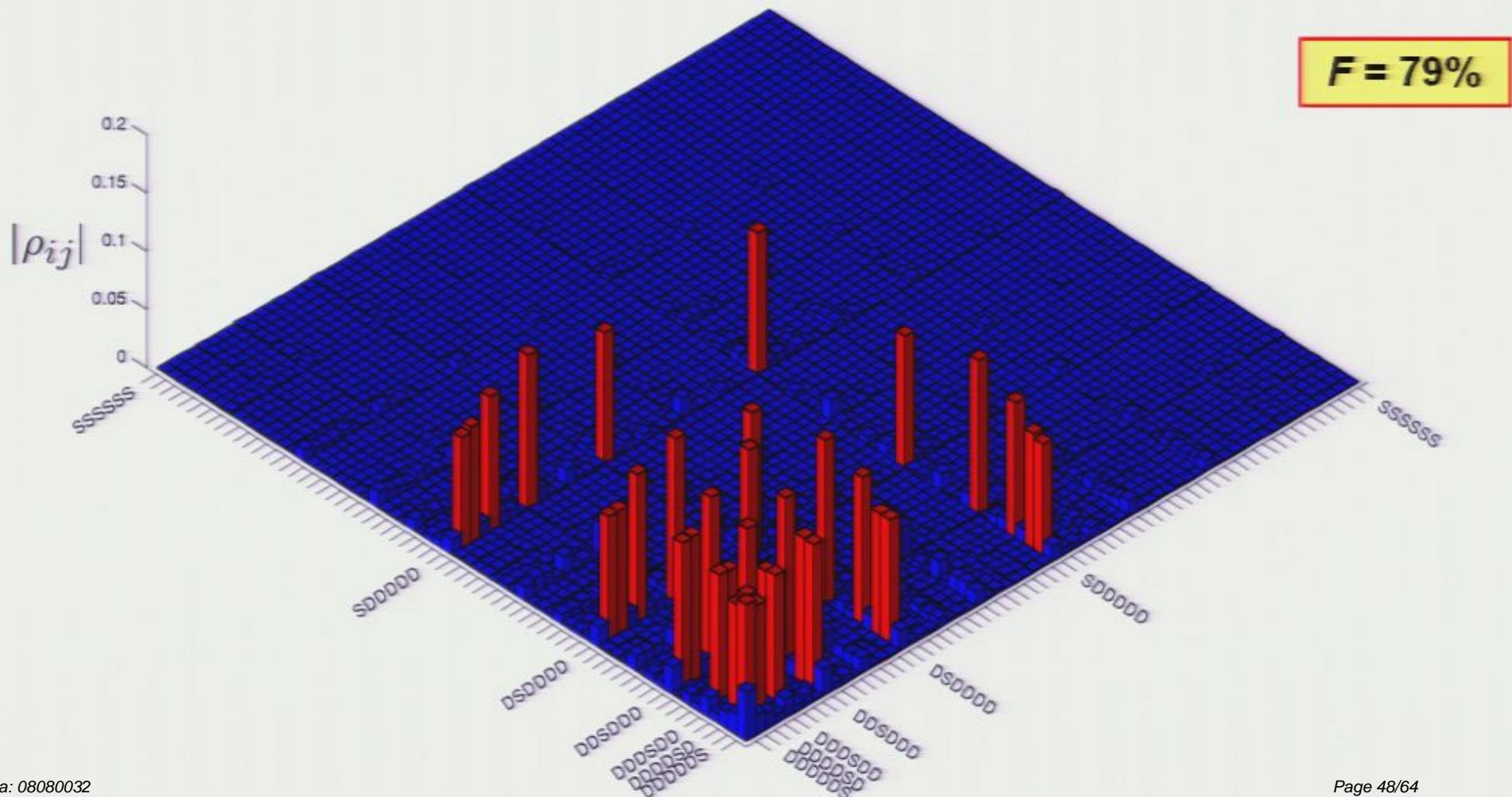
## Five-ion W-states

$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |DSDDD\rangle + |SDDDD\rangle)$$

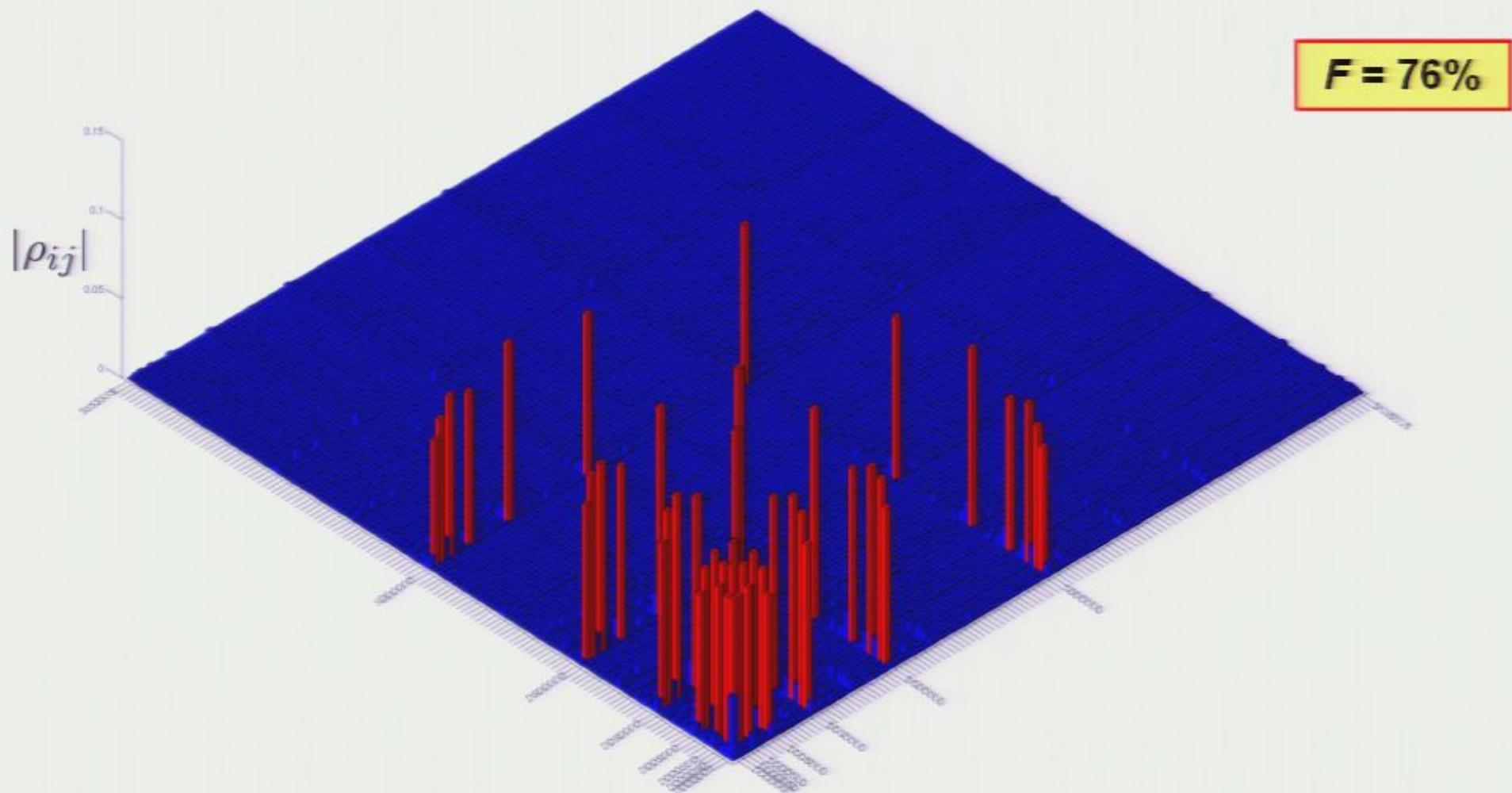


## Six-ion W-states

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSAAAA\rangle + |SAAAAA\rangle)$$

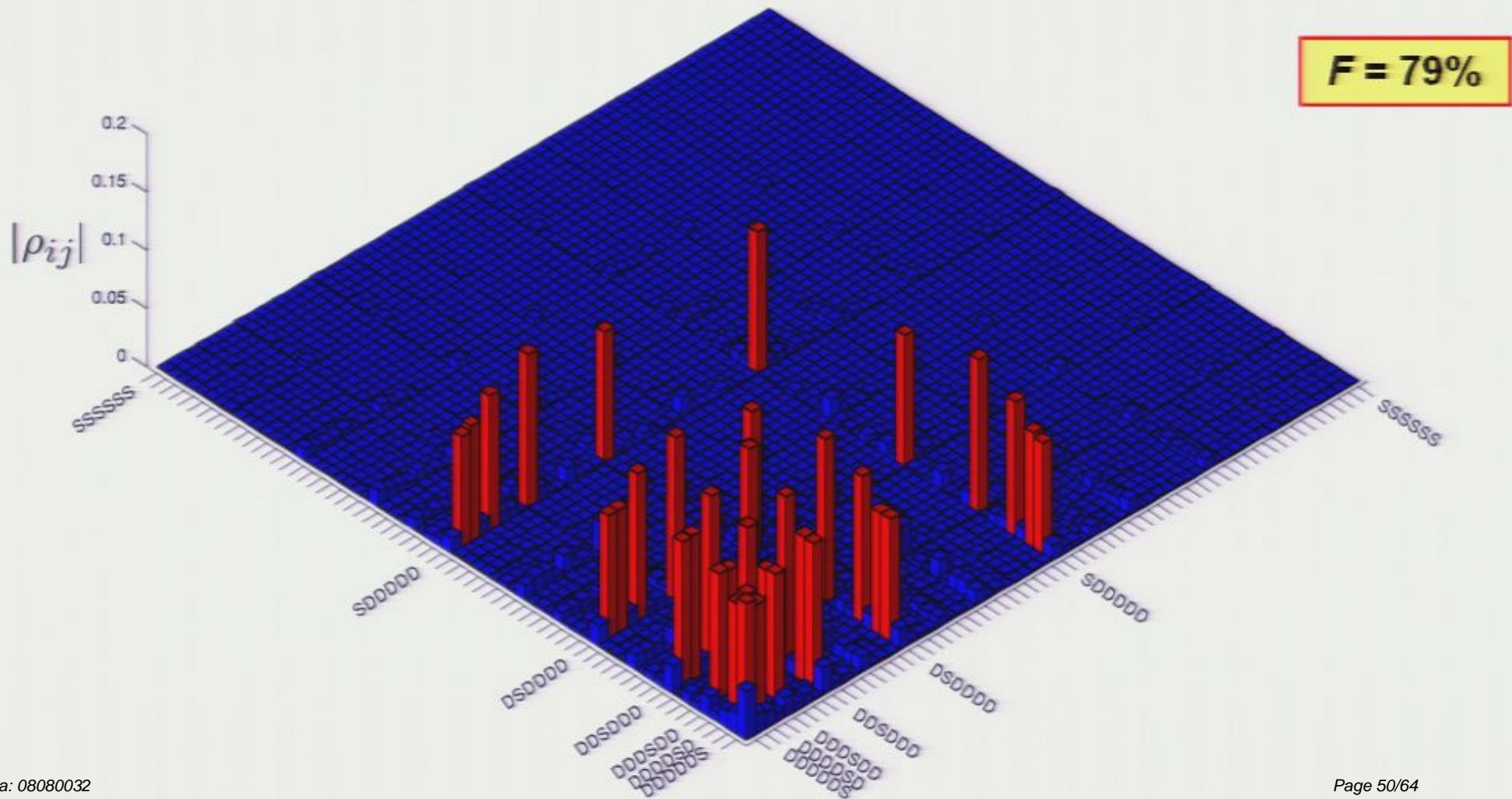


## Seven-ion W-states



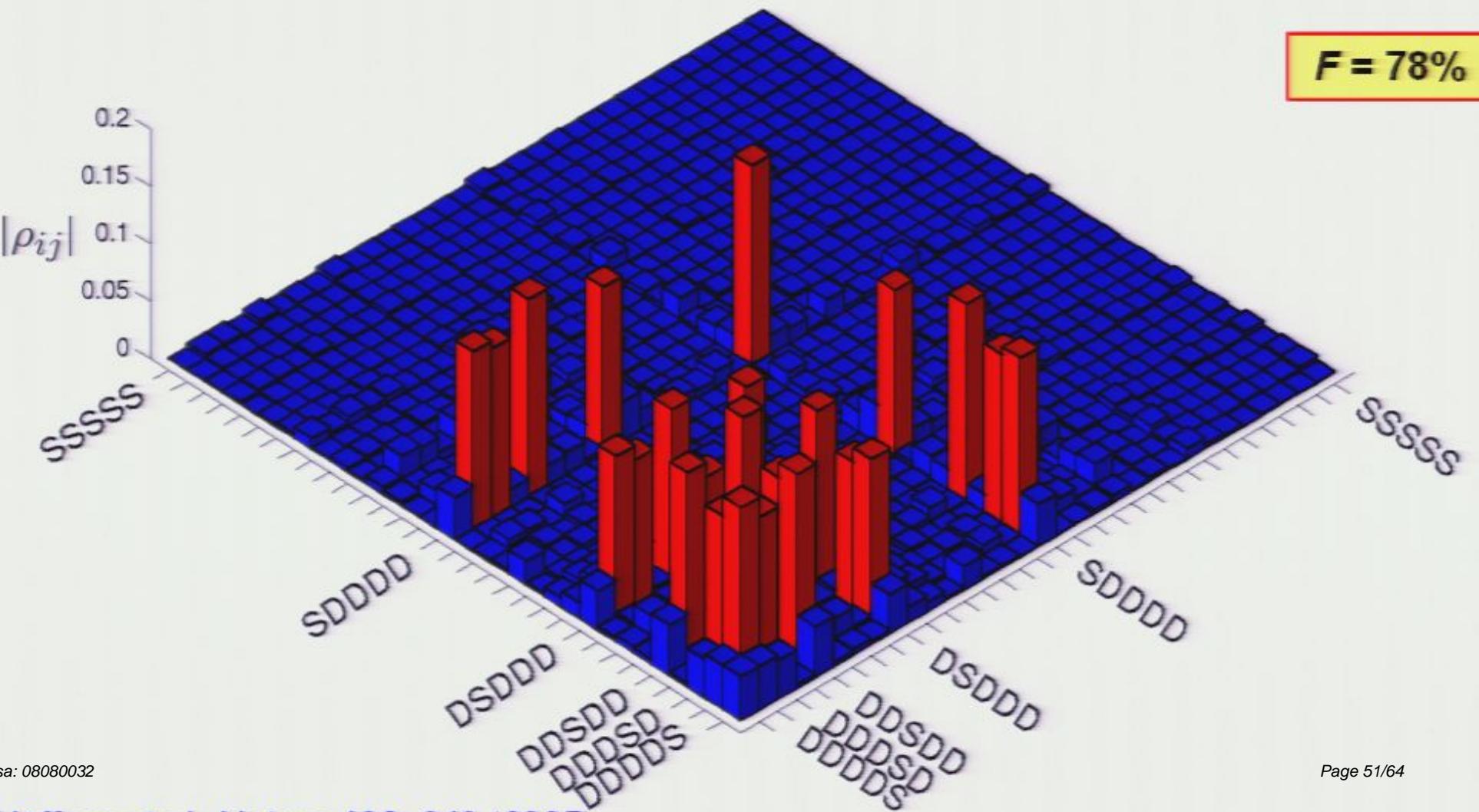
## Six-ion W-states

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDDD\rangle)$$



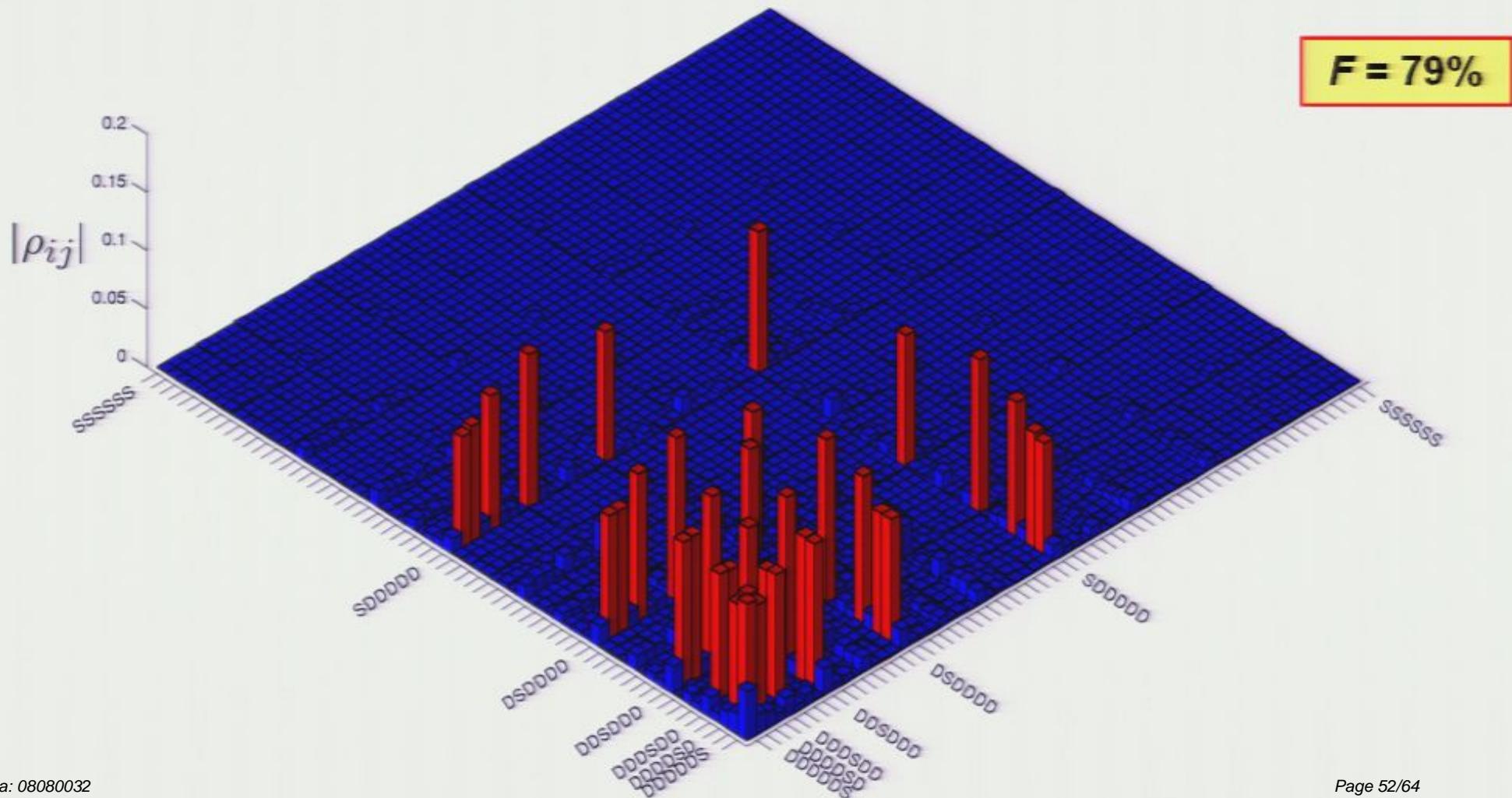
## Five-ion W-states

$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |DSDDD\rangle + |SDDDD\rangle)$$

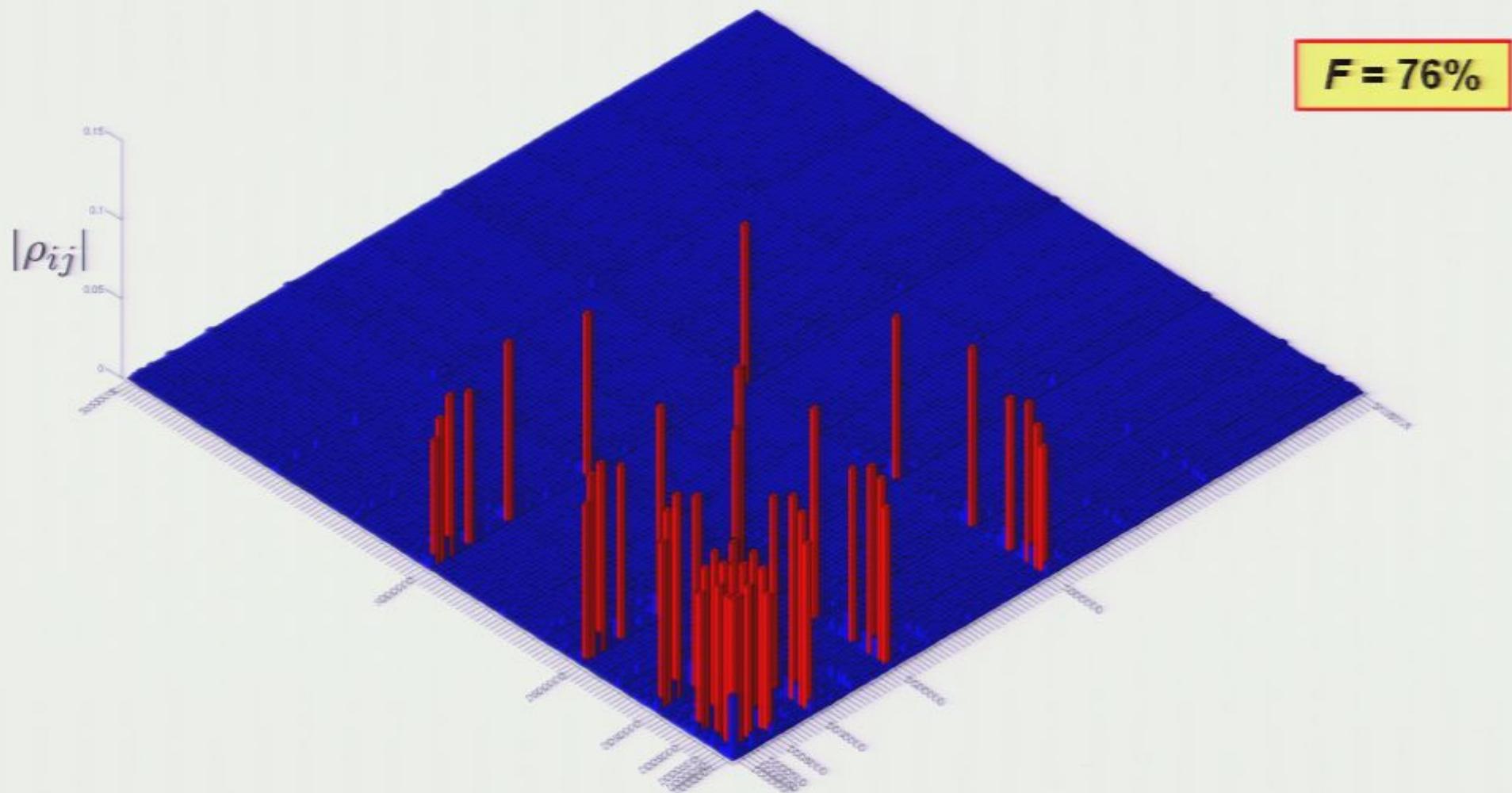


## Six-ion W-states

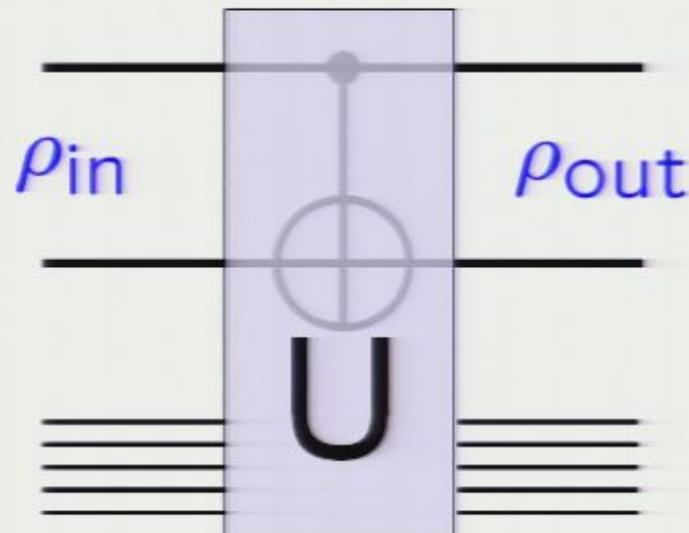
$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$



## Seven-ion W-states



## Quantum process tomography



Interaction with the environment

$$\rho_{\text{out}} = \mathcal{E}(\rho_{\text{in}})$$

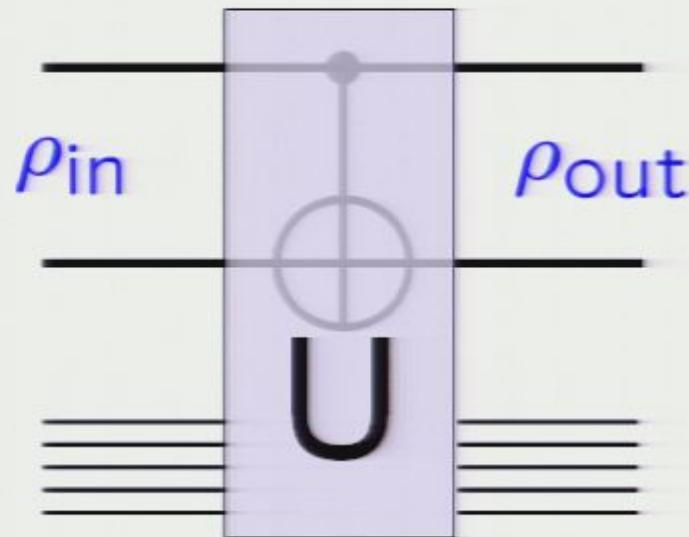
$$= \sum \chi_{ij} E_i \rho_{\text{in}} E_j^\dagger$$

with  $\sum_i E_i^\dagger E_i = I$

$\chi_{ij}$

characterizes gate operation completely

## Quantum process tomography



Interaction with the environment

$$\rho_{out} = \mathcal{E}(\rho_{in})$$

Experimentally applied to:

- Controlled-NOT gate operation
- Deterministic quantum teleportation

$\chi_{ij}$

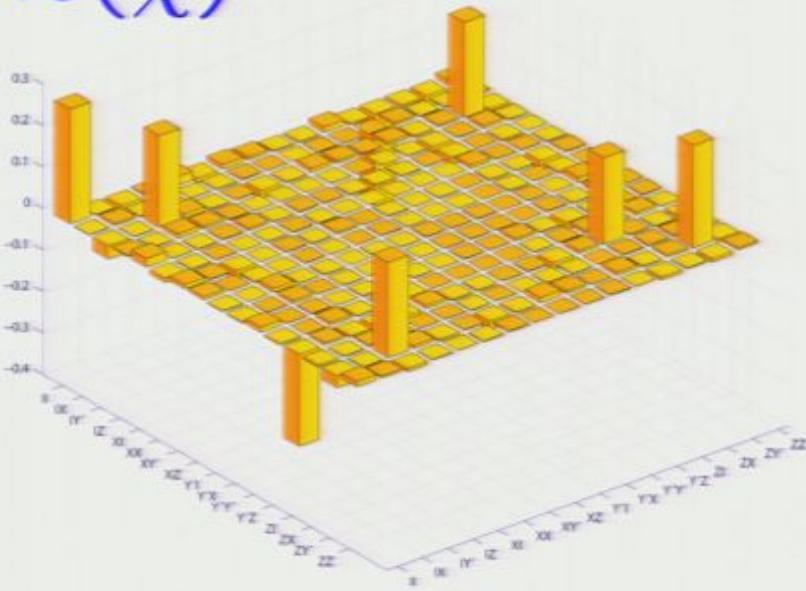
characterizes gate operation completely

## Process tomography of a CNOT gate

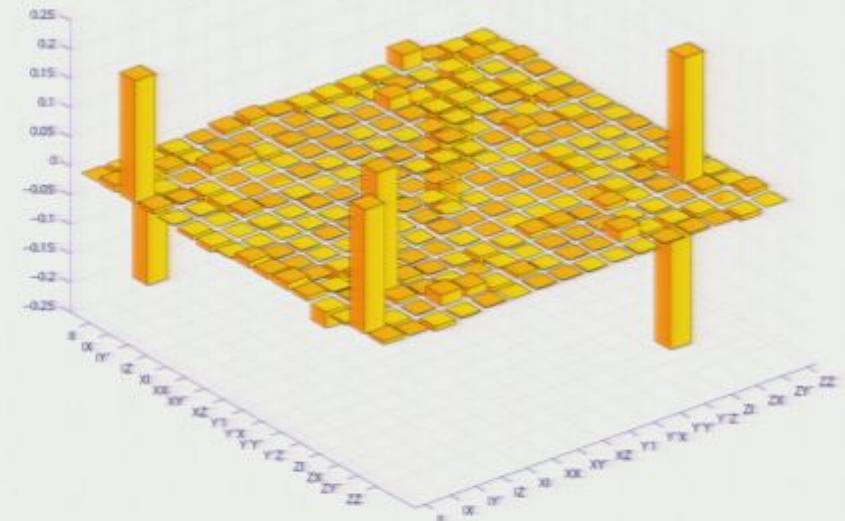
$$U_{\text{CNOT}}^{12} = -\frac{1}{2}(I \otimes I + iI \otimes Y - Z \otimes I + iZ \otimes Y)$$

$$\begin{array}{c} \text{Quantum Circuit Diagram} \\ \text{Control Line} \quad \text{Target Line} \end{array} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$\text{Re}(\chi)$



$\text{Im}(\chi)$



## Gate performance

Measures of interest which quantify the performance of the gate operation:

Process fidelity:  $F_{proc} = \text{tr}(\chi_{id} \cdot \chi_{exp})$

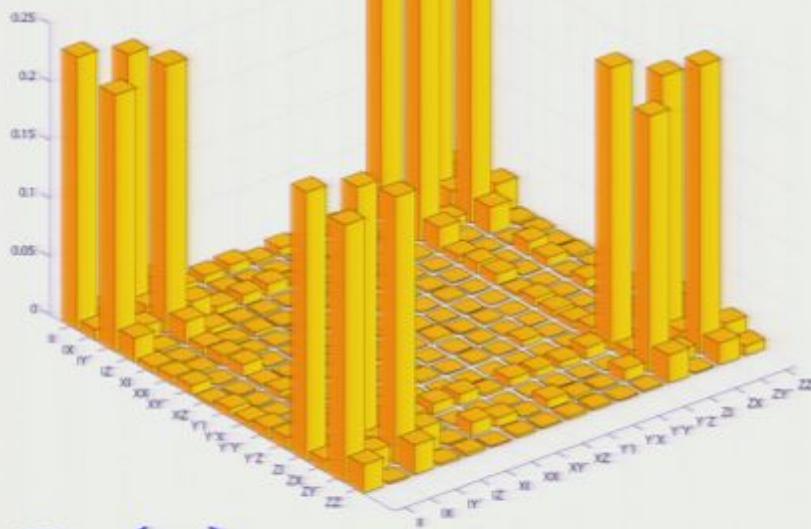
How well do the experimentally obtained and the ideal process matrix overlap ?

Mean fidelity:  $\bar{F} = \frac{1}{N} \sum_{i=1}^N \langle \psi_{id} | \rho_{out,i} | \psi_{id} \rangle$

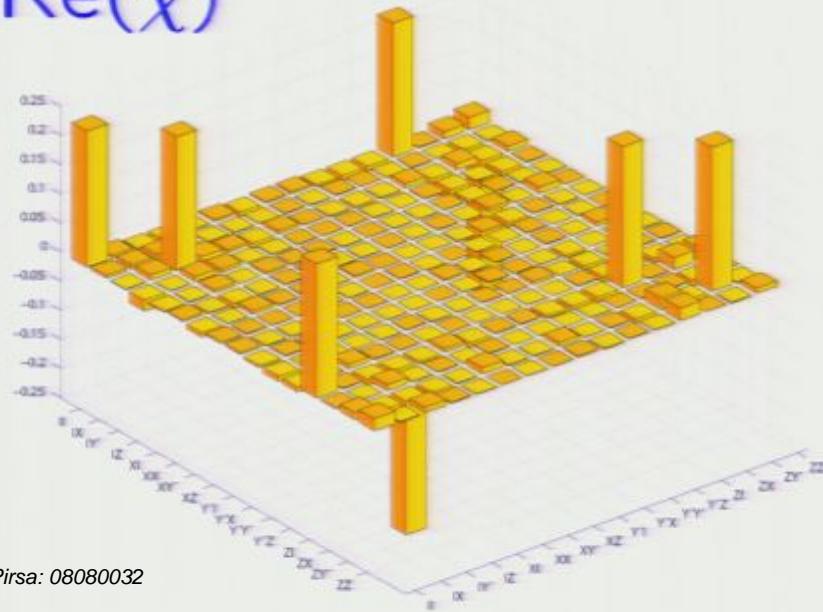
How good is the overlap between the calculated and the ideal output states ?

## Results: Gate performance

Abs( $\chi$ )



Re( $\chi$ )



Measure

Process fidelity

$$F_p$$

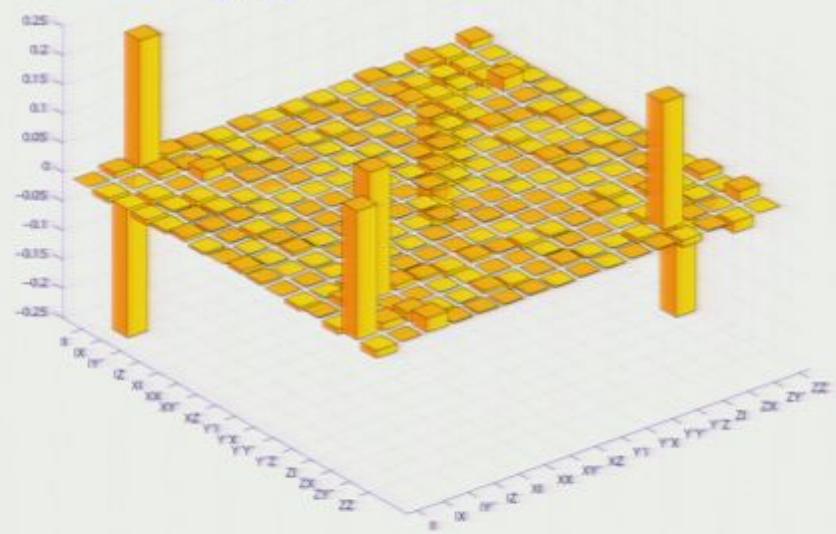
Mean fidelity

$$\bar{F}$$

90.8(6)%

92.6(6)%

Im( $\chi$ )



## Double gate operation

$$U_{CNOT} \cdot U_{CNOT} = I$$

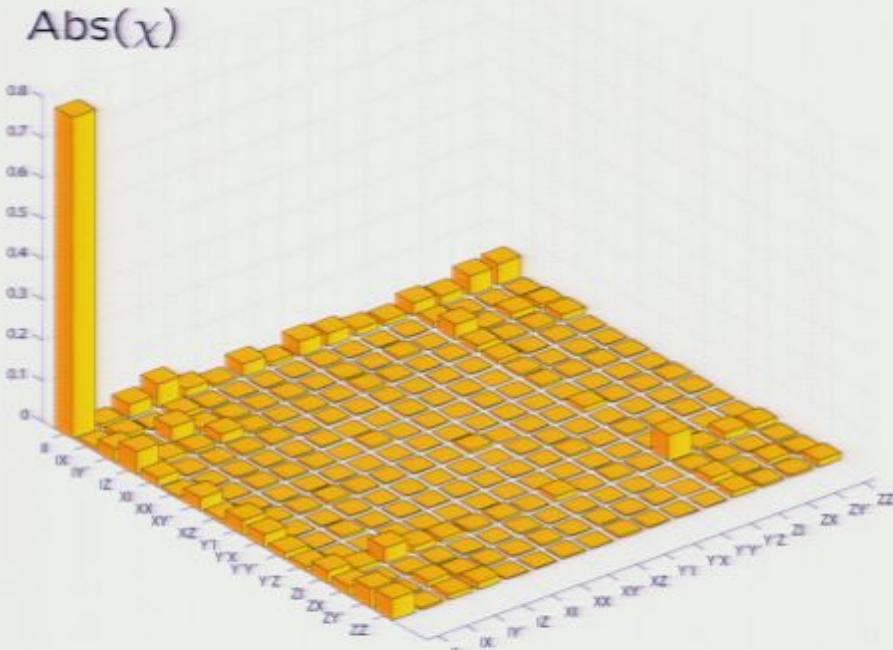
Can we infer the performance of combined gate operation from QPT of a single gate operation ?

## Double gate operation

$$U_{CNOT} \cdot U_{CNOT} = I$$

Can we infer the performance of combined gate operation from QPT of a single gate operation ?

**Experiment:** QPT of two subsequently applied CNOT gates

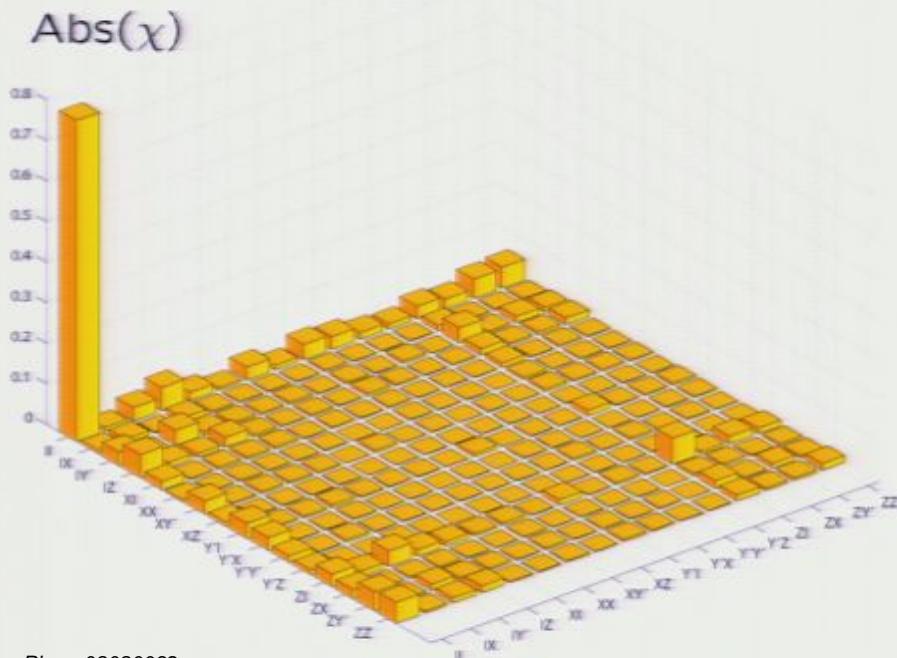


## Double gate operation

$$U_{CNOT} \cdot U_{CNOT} = I$$

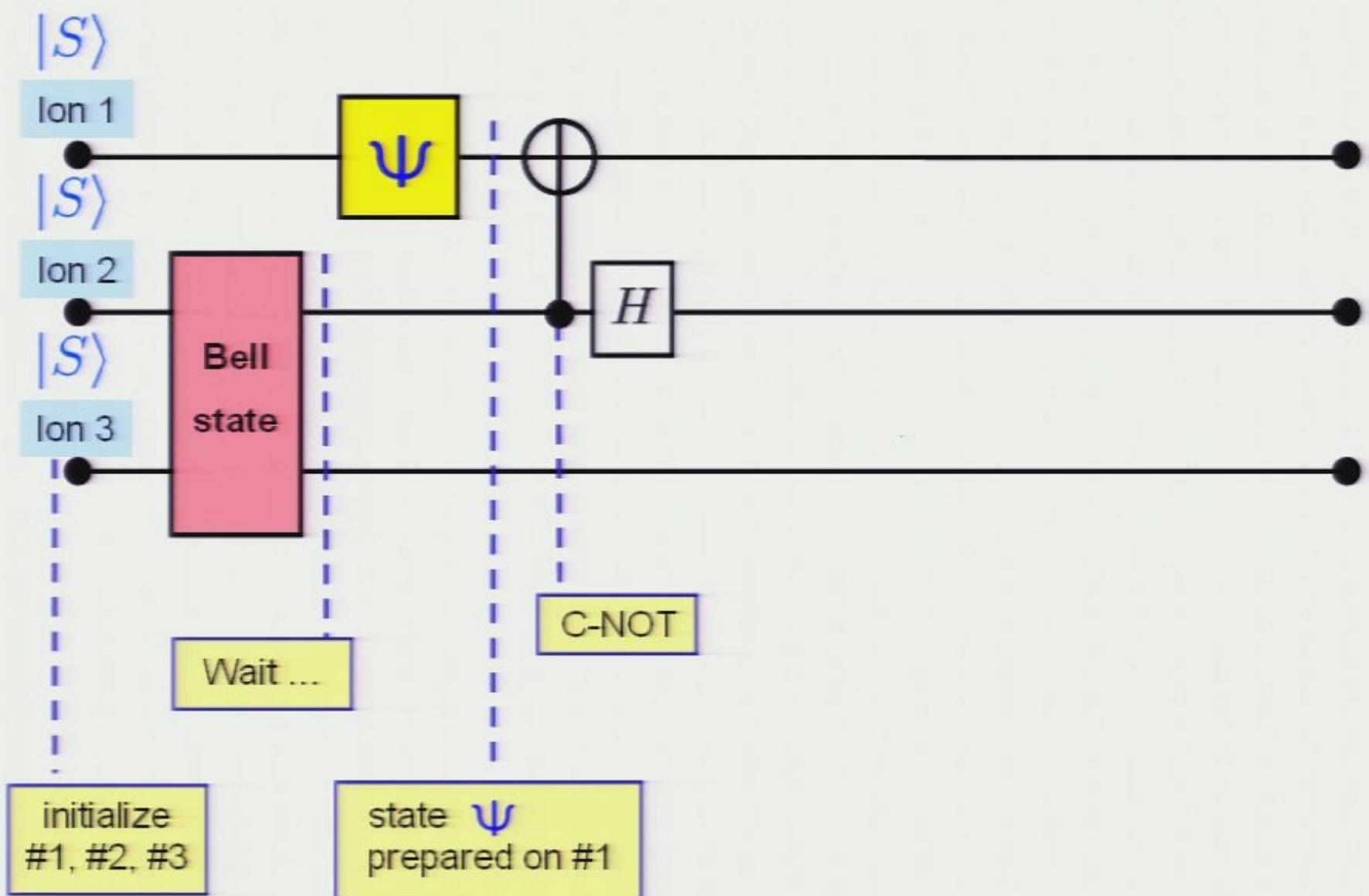
Can we infer the performance of combined gate operation from QPT of a single gate operation ?

**Experiment:** QPT of two subsequently applied CNOT gates



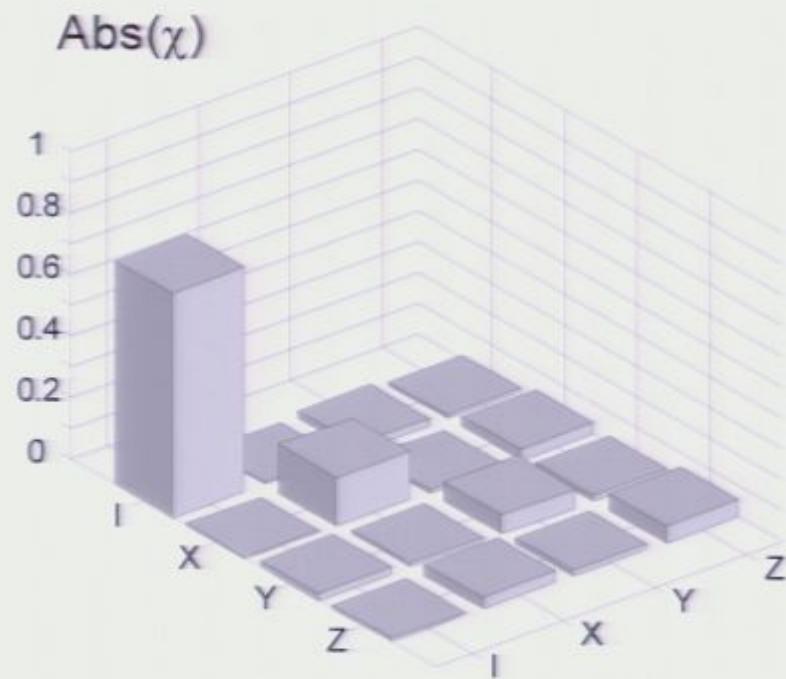
Experimental result	2x single gate result
$F_p = 79(1)\%$	$F_p = 82.8\%$
$\bar{F} = 83.4(8)\%$	$\bar{F} = 86.2\%$

## Deterministic quantum teleportation with ions

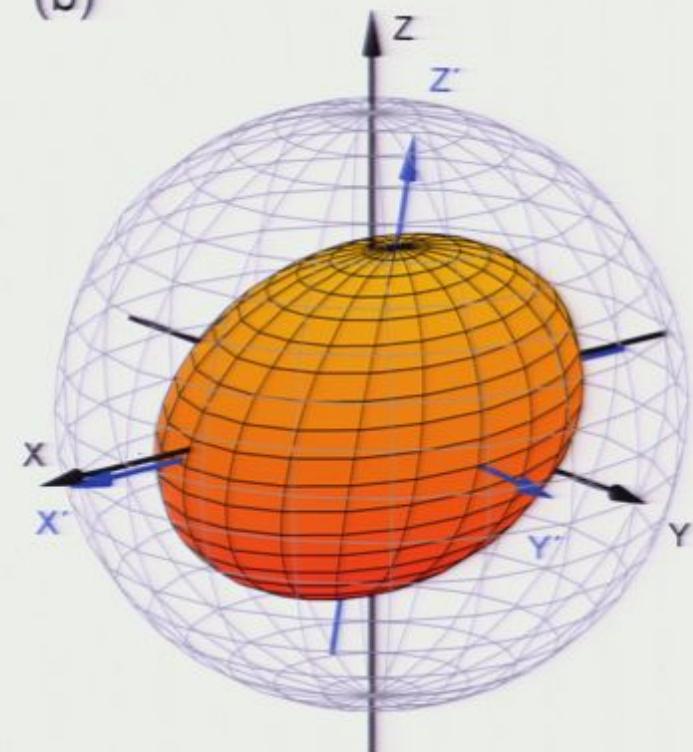


# Process tomography of quantum teleportation

(a)



(b)



$$F_{proc} = \text{Tr}(\chi_{id}\chi_{tele}) = 73\% \quad \rightarrow \quad \bar{F} = \frac{2F_{proc} + 1}{3} = 83\%$$

## **My questions for this workshop:**

### **Process tomography**

Does process tomography of gates provide a practical way to predict the fidelity of a quantum algorithm composed of these gates ?

How shall we deal with slowly fluctuating parameters ?

### **Quantum state tomography**

What are reliable algorithms for reconstructing quantum states ?

Are there faster numerical reconstruction algorithms ?

Which approach should be taken when the state leaks out of the Hilbert space of interest ?