Title: Quantum state and process estimation in trapped ion experiments

Date: Aug 25, 2008 11:15 AM

URL: http://pirsa.org/08080032

Abstract: The experimental realization of entangled states requires tools for characterizing the produced states as well as the processes used for creating the entanglement. In my talk, I will present examples of quantum measurements occuring in trapped ion experiments aiming at creating high-fidelity quantum gates.

(Nonoptimal) Characterization of quantum states and gates in trapped ion experiments

Christian Roos

Institute for Quantum Optics and Quantum Information
Austrian Academy of Sciences
Innsbruck, Austria

Outline :

1. Quantum information processing with trapped ions
2. Tomography of entangled states and quantum gates

Waterloo, 25.8.2008

(Nonoptimal) Characterization of quantum states and gates in trapped ion experiments

Christian Roos

Institute for Quantum Optics and Quantum Information
Austrian Academy of Sciences
Innsbruck, Austria

Outline :

1. Quantum information processing with trapped ions
2. Tomography of entangled states and quantum gates

Waterloo, 25.8.2008

Experiments with single trapped ions

Precision spectroscopy / Optical frequency standards

Absolute frequency measurement of the $S_{1/2}$-$D_{5/2}$ transition in $^{40}Ca^+$

M. Chwalla *et al.*, arXiv:0806.1414

Quantum information processing

Entangled states of 4...8 ions

H. Häffner *et al*, Nature **438**, 643 (2005)

High-fidelity two-ion quantum gate

J. Benhelm *et al.*, Nat. Phys. **4**, 463 (2008)

Quantum metrology

'Designer atoms' for quantum metrology

C. F. Roos *et al*, Nature **443**, 316 (2006)

Experiments with single trapped ions

Precision spectroscopy / Optical frequency standards

Absolute frequency measurement of the $S_{1/2}$-$D_{5/2}$ transition in $^{40}Ca^+$

M. Chwalla *et al.*, arXiv:0806.1414

Quantum information processing

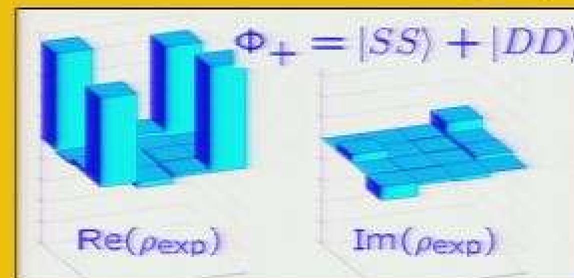
Enta

H. Häffr

High-

J. Benhel

Quantum state tomography

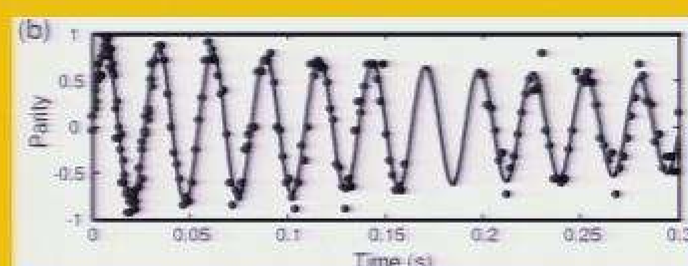


Quantum metrology

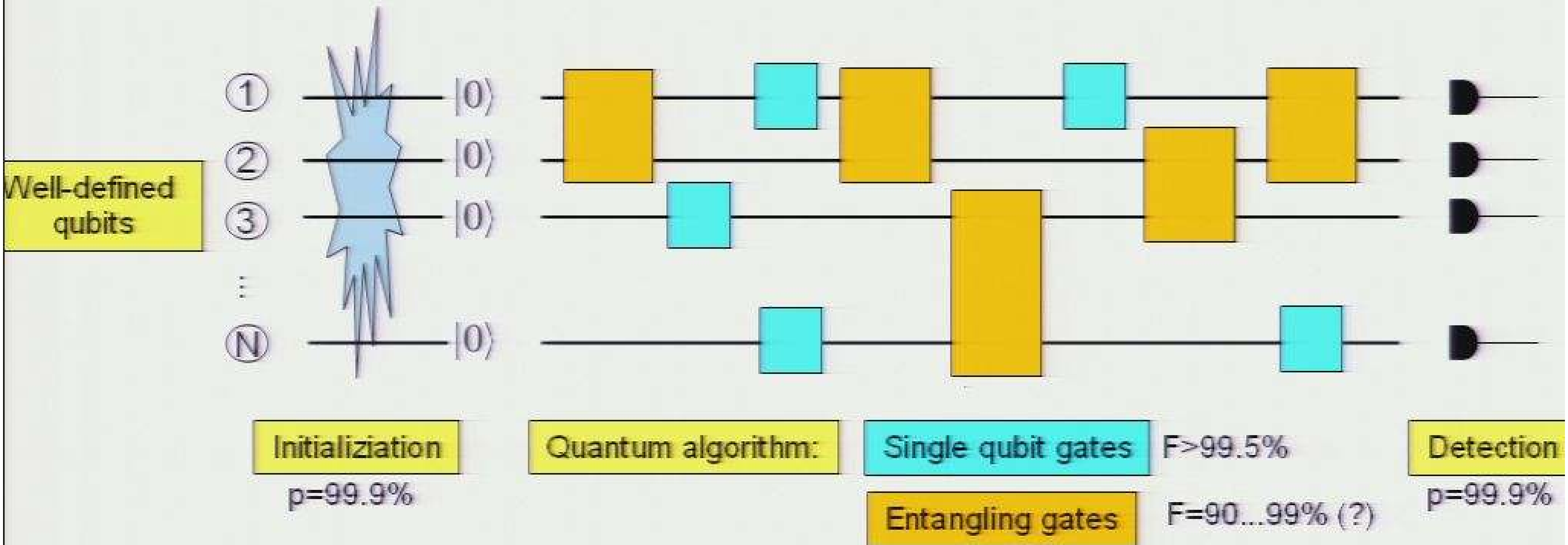
'Designer atom

C. F. Roos *et al*

Phase estimation



Quantum information processing (with trapped ions)



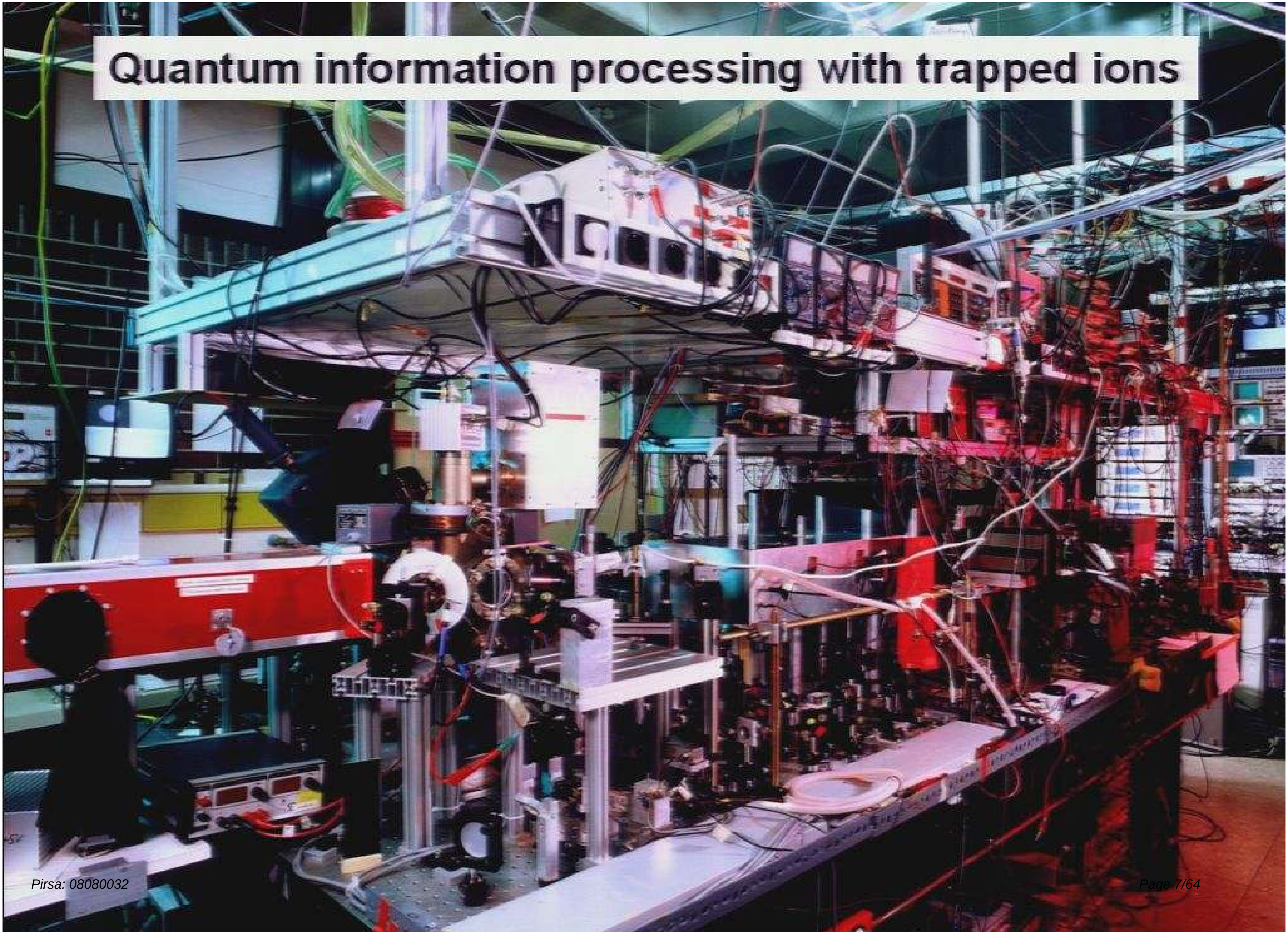
Current experiments:

2 - 4 ions, few entangling gates

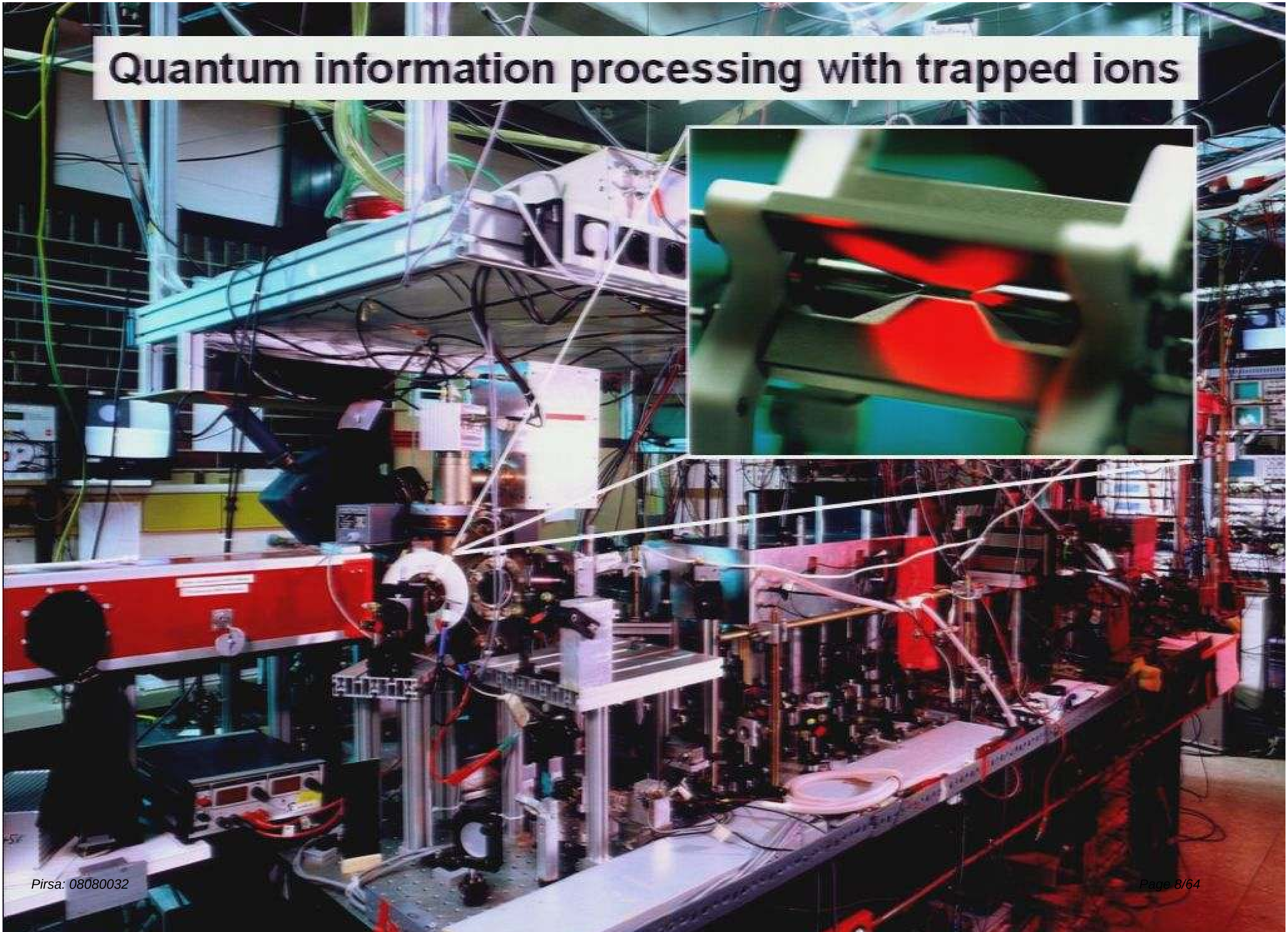
Challenges:

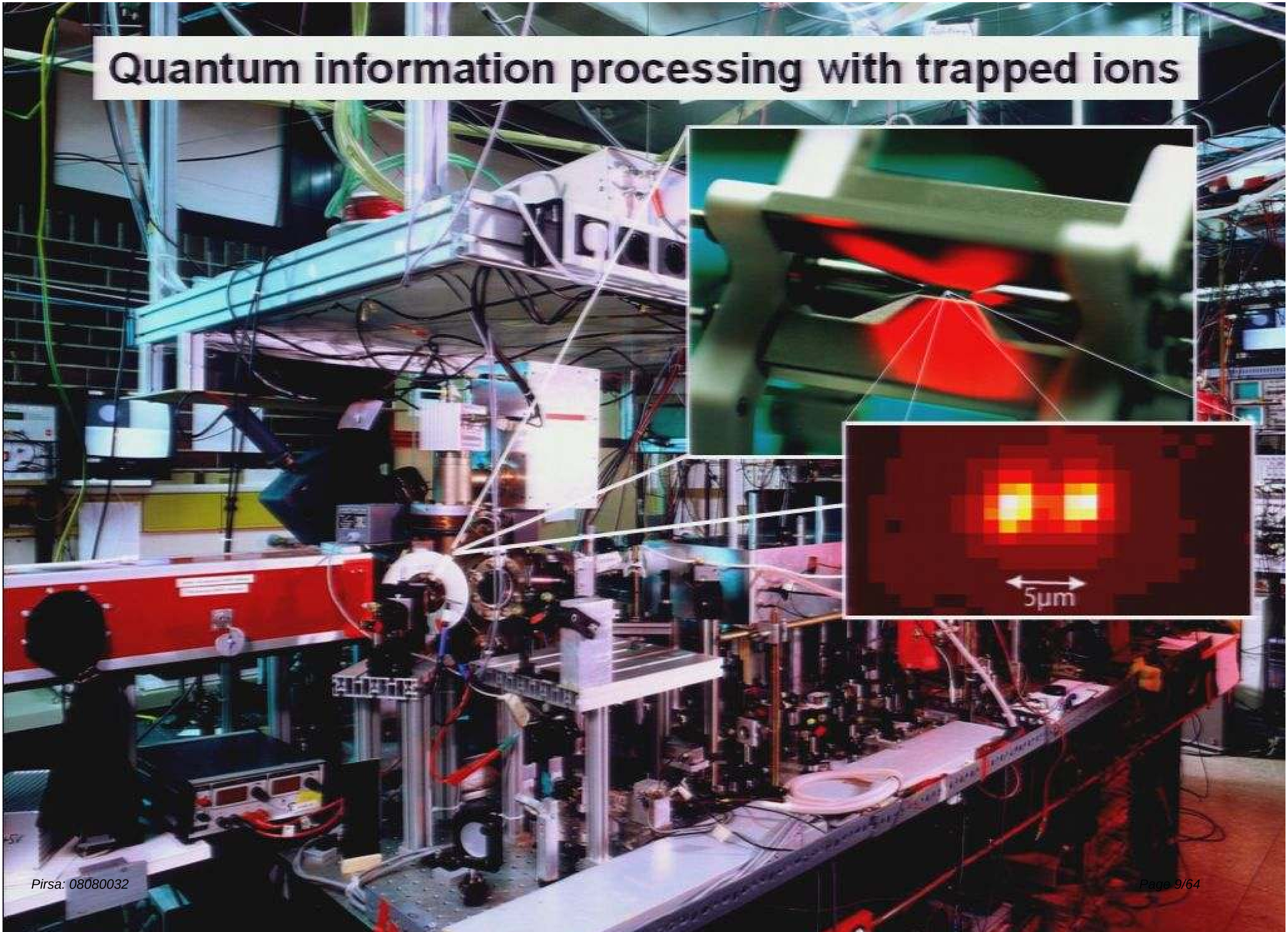
- Increase number of ions
- Reduce gate error rates

Quantum information processing with trapped ions

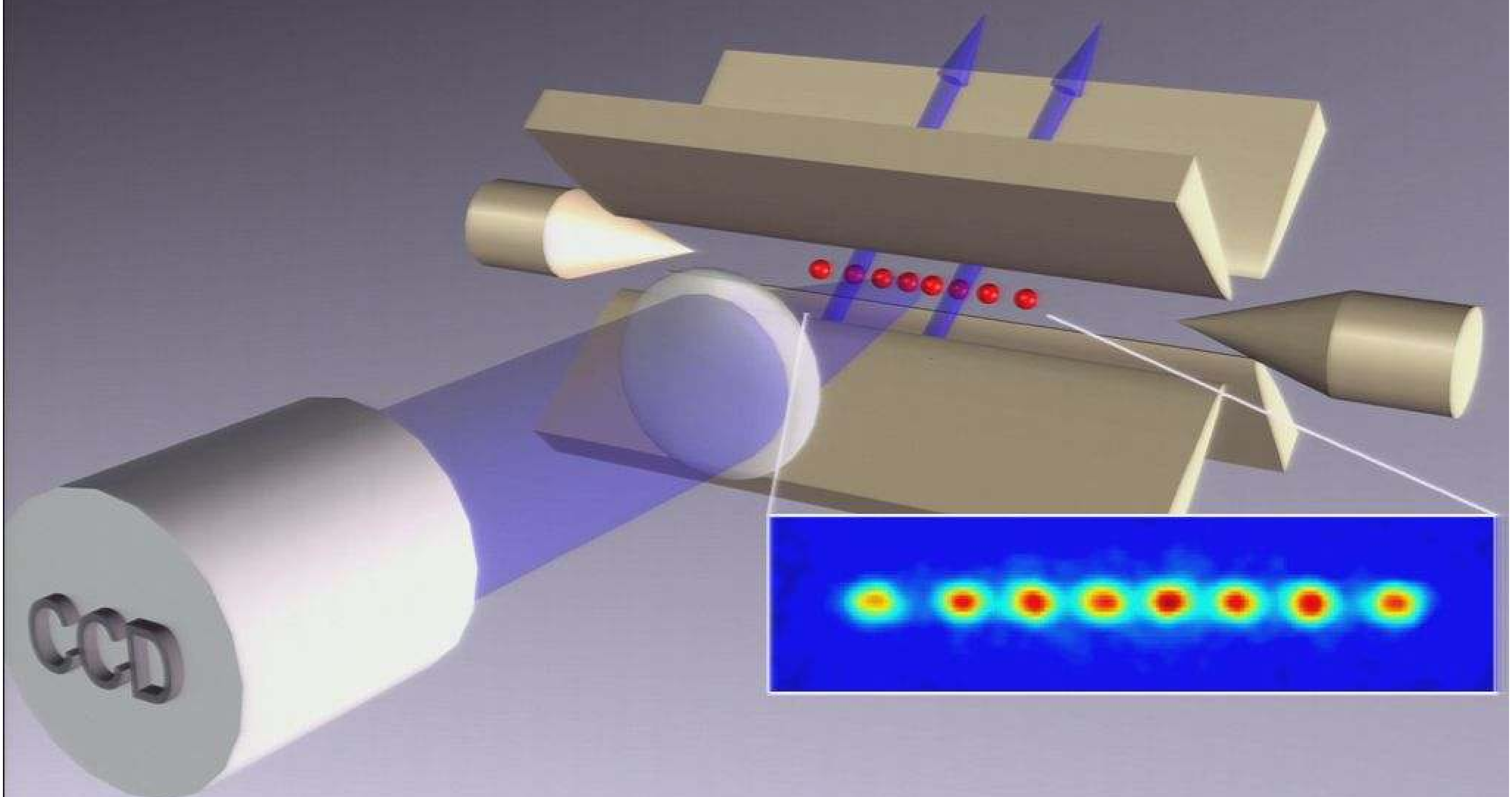


Quantum information processing with trapped ions

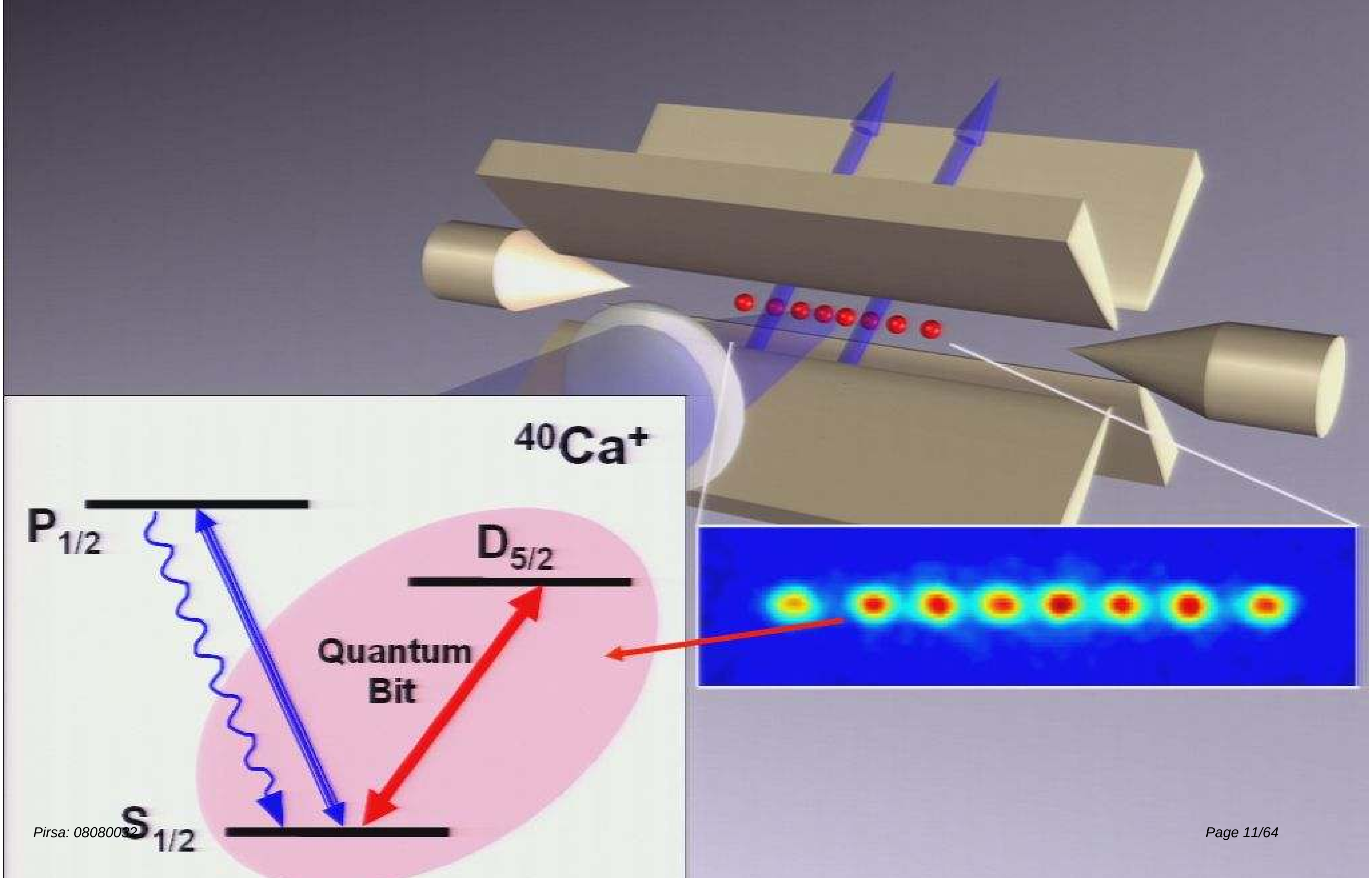



Quantum information processing with trapped ions
5μm

Quantum information processing with trapped ions



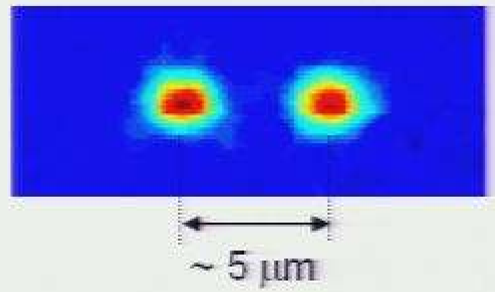
Quantum information processing with trapped ions



Coupling ions for quantum gate operations

Ion crystals:

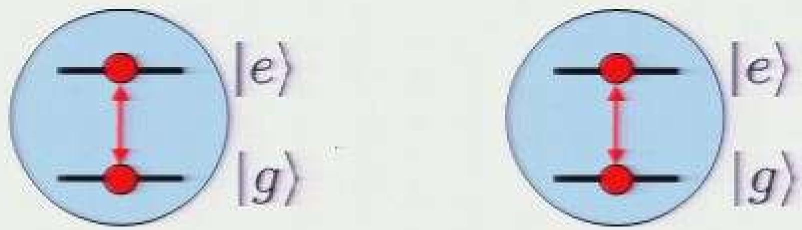
Carriers of quantum information



Internal degrees of freedom

Storage of quantum information

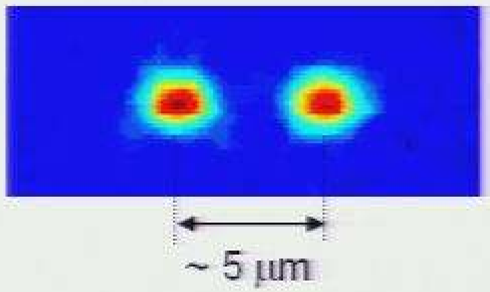
$$\Psi = \alpha|g\rangle + \beta|e\rangle$$



Coupling ions for quantum gate operations

Ion crystals:

Carriers of quantum information

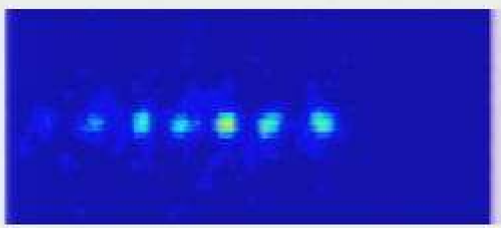


Internal degrees of freedom

Storage of quantum information

$$\Psi = \alpha|g\rangle + \beta|e\rangle$$

External degrees of freedom



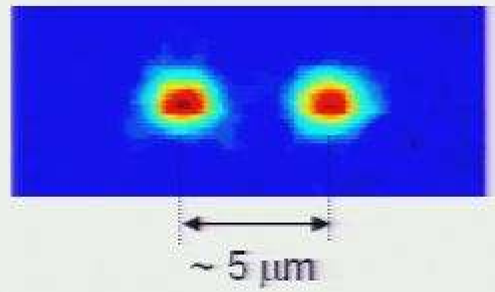
Coulomb coupling:

„center-of-mass mode"

Coupling ions for quantum gate operations

Ion crystals:

Carriers of quantum information

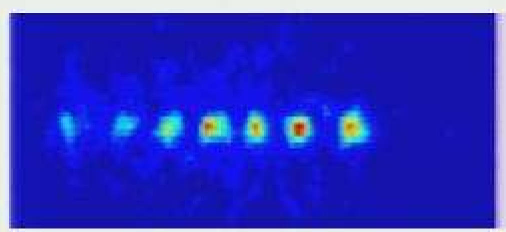


Internal degrees of freedom

Storage of quantum information

$$\Psi = \alpha|g\rangle + \beta|e\rangle$$

External degrees of freedom



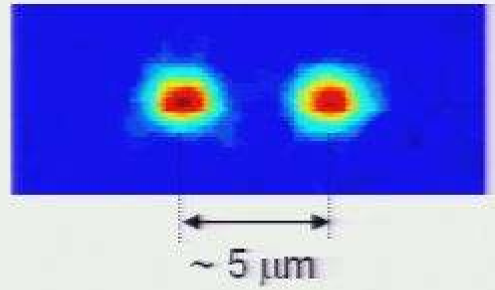
Coulomb coupling:

„center-of-mass mode"

Coupling ions for quantum gate operations

Ion crystals:

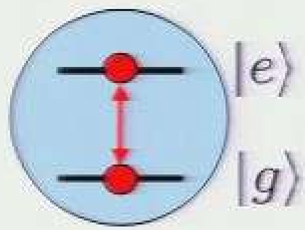
Carriers of quantum information

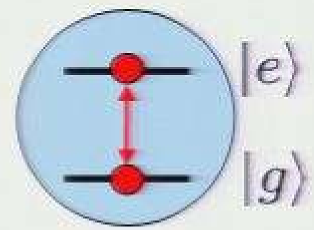


Internal degrees of freedom

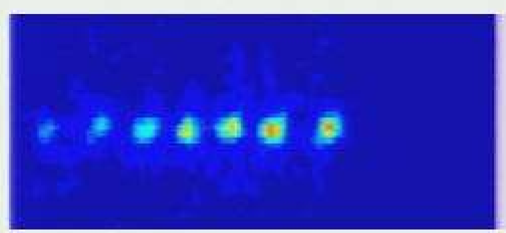
Storage of quantum information

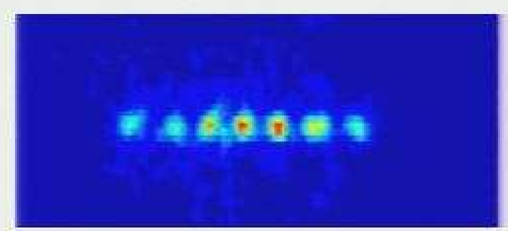
$$\Psi = \alpha|g\rangle + \beta|e\rangle$$





External degrees of freedom



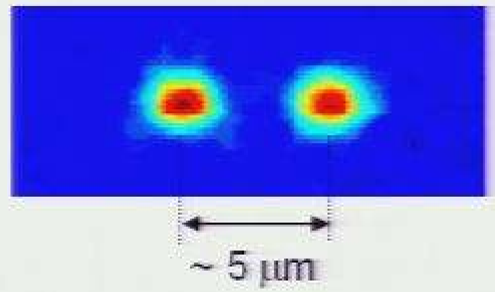


Coulomb coupling: „center-of-mass mode" „stretch mode"

Coupling ions for quantum gate operations

Ion crystals:

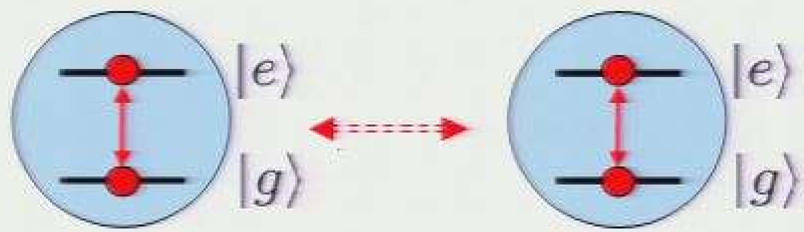
Carriers of quantum information



Internal degrees of freedom

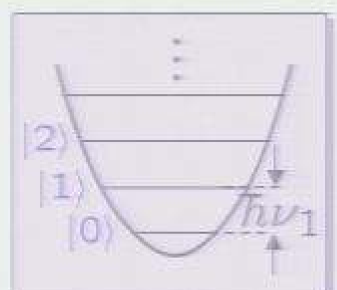
Storage of quantum information

$$\Psi = \alpha|g\rangle + \beta|e\rangle$$

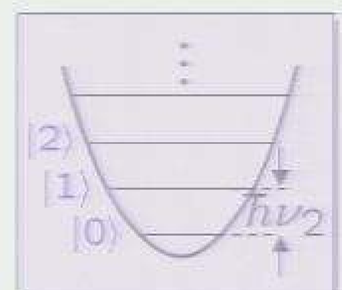


External degrees of freedom

Effective ion-ion interaction
y coupling internal and external
states with lasers

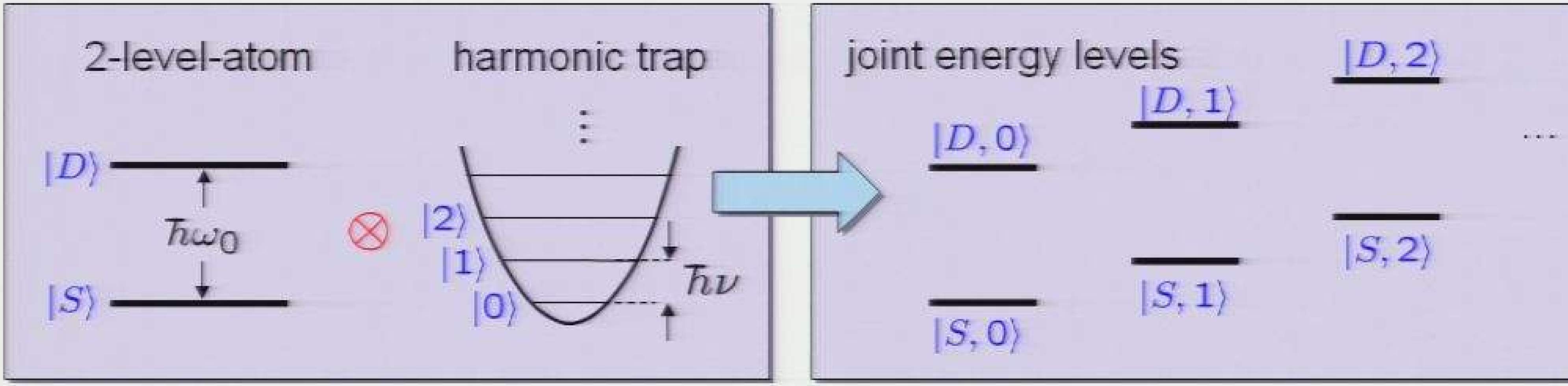


„center-of-mass mode"

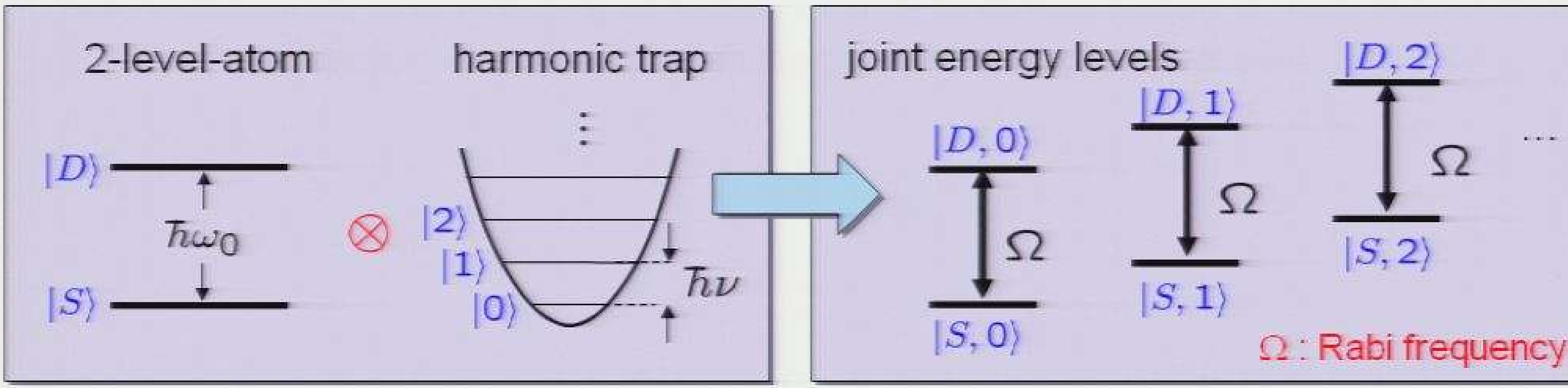


„stretch mode"

Coherent manipulation by laser pulses

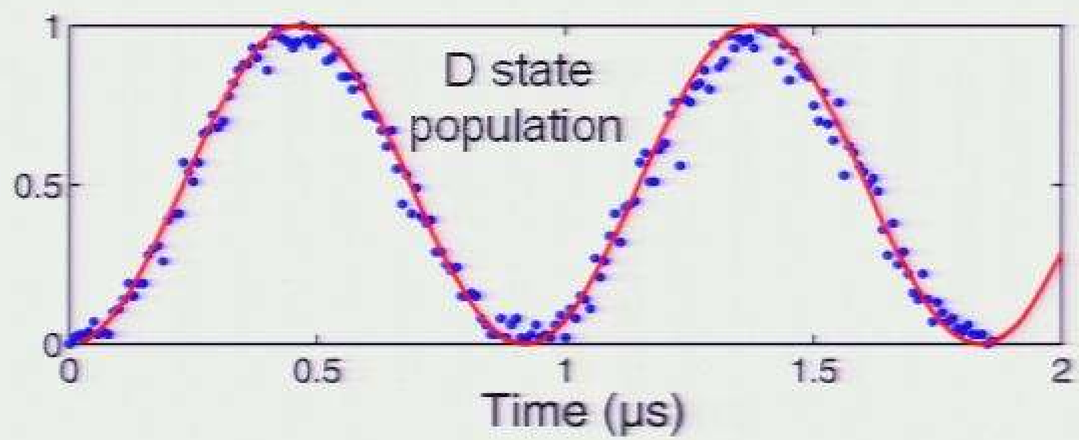


Coherent manipulation by laser pulses

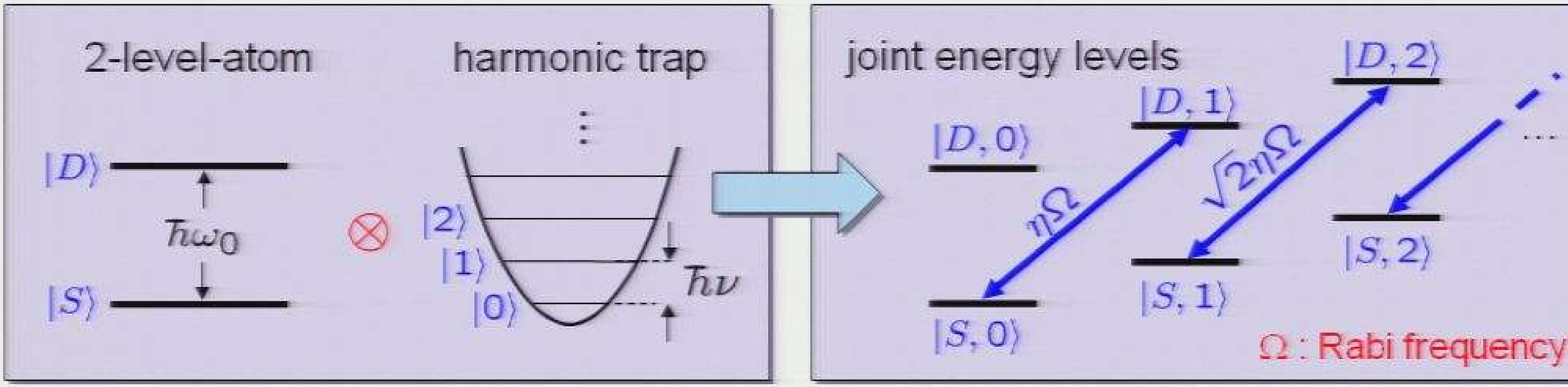


Carrier excitation:
Manipulation of the internal state

$$|S\rangle \longleftrightarrow |D\rangle$$

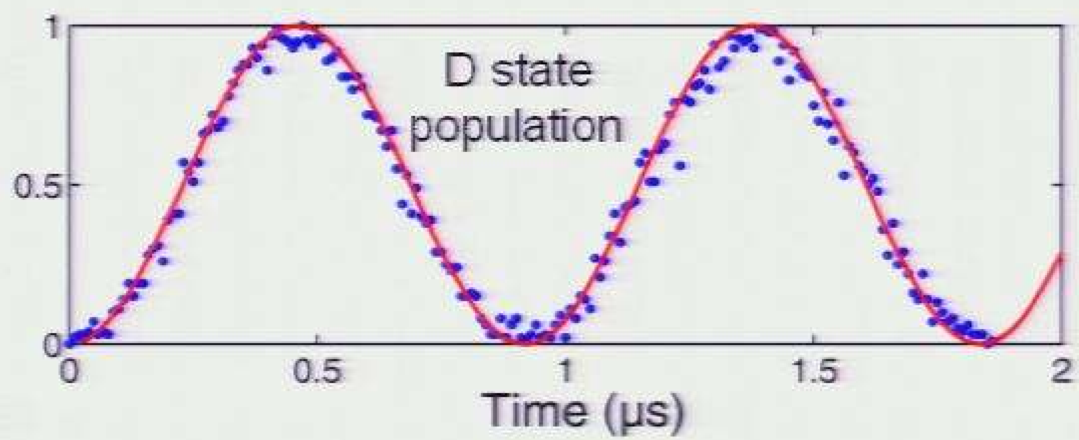


Coherent manipulation by laser pulses



Carrier excitation:
Manipulation of the internal state

$$|S\rangle \longleftrightarrow |D\rangle$$



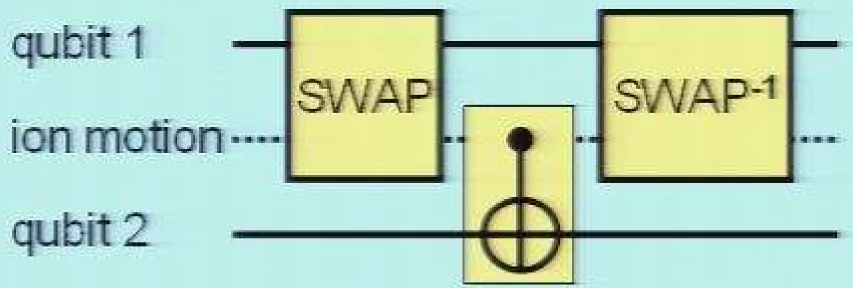
Blue sideband excitation:
Entangling internal and motional state

$$|S\rangle|n\rangle \longleftrightarrow |D\rangle|n+1\rangle$$

Quantum algorithms with trapped ions

Elementary building blocks: entangling quantum gates

Controlled-NOT gate



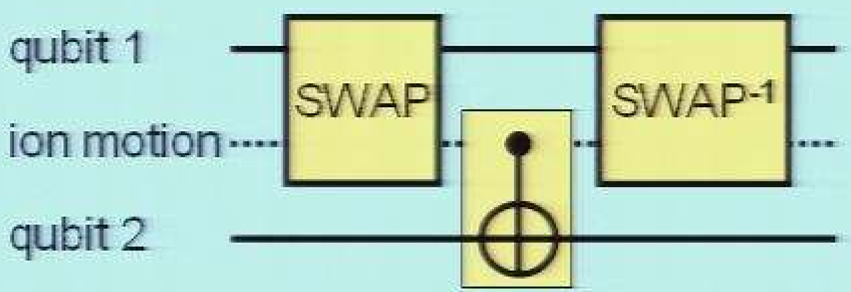
Bell state fidelity: F≈ 95%

M. Riebe *et al.*, PRL **97**, 220407 (2006)

Quantum algorithms with trapped ions

Elementary building blocks: entangling quantum gates

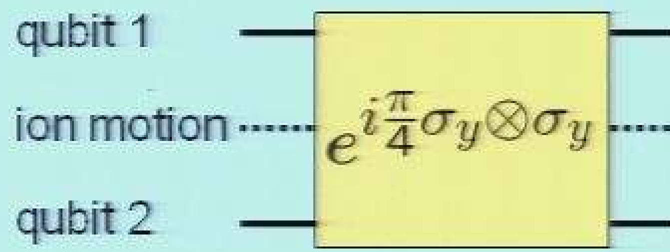
Controlled-NOT gate



Bell state fidelity: F≈ 95%

M. Riebe *et al.*, PRL **97**, 220407 (2006)

Sørensen-Mølmer gate



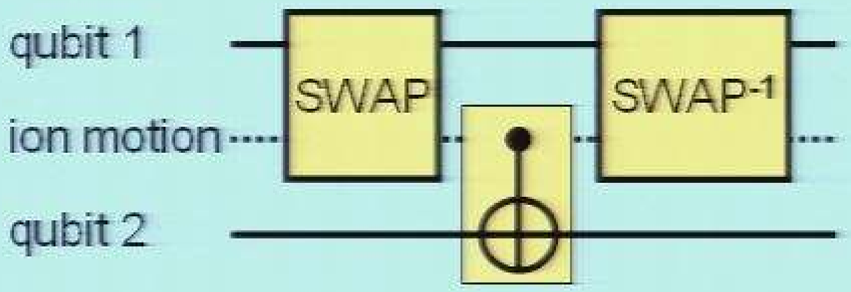
Bell state fidelity: F = 99.3(1)%

J. Benhelm *et al.*, Nat. Phys. **4**, 463 (2008)

Quantum algorithms with trapped ions

Elementary building blocks: entangling quantum gates

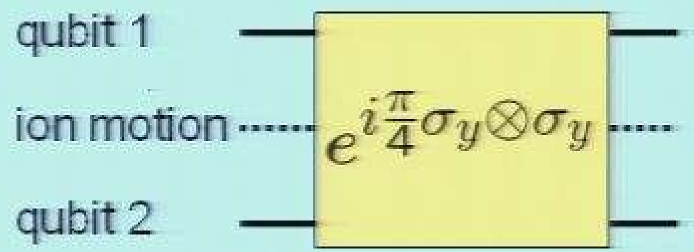
Controlled-NOT gate



Bell state fidelity: F≈ 95%

M. Riebe *et al.*, PRL **97**, 220407 (2006)

Sørensen-Mølmer gate

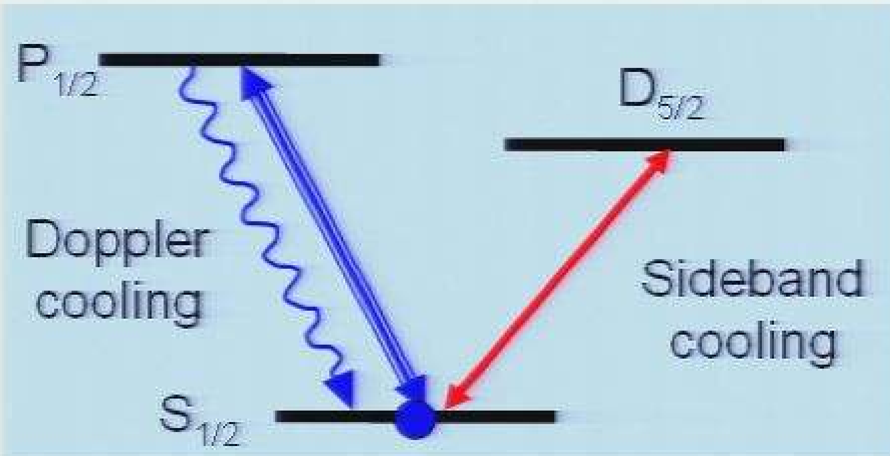


Bell state fidelity: F = 99.3(1)%

J. Benhelm *et al.*, Nat. Phys. **4**, 463 (2008)

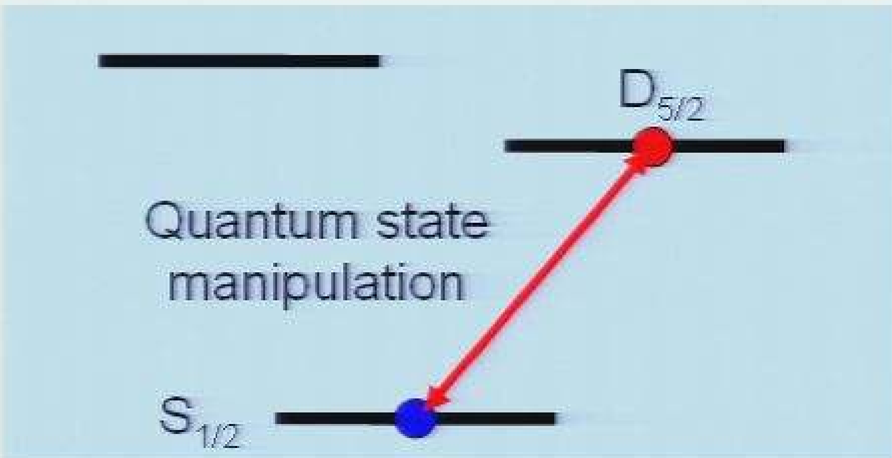
→ Characterize the quantum gates by tomography of its output states

Experiments with $^{40}Ca^{+}$: Experimental procedure



1. Initialization

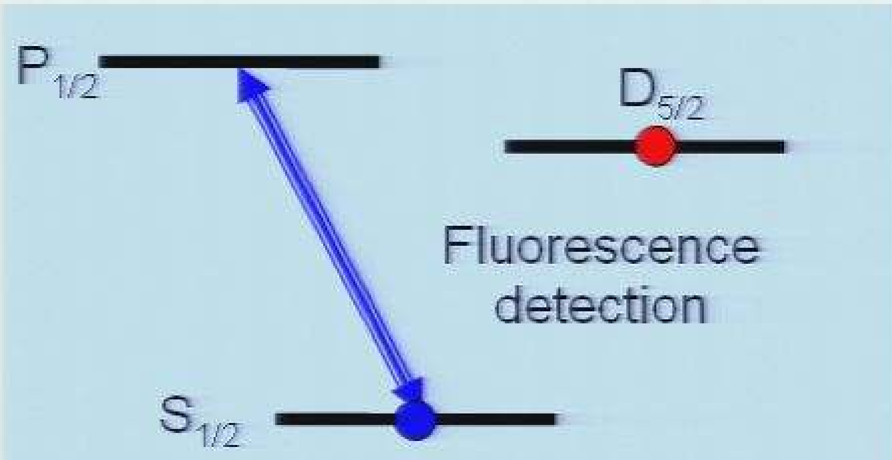
Experiments with $^{40}Ca^+$: Experimental procedure



1. Initialization

2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition

Experiments with $^{40}Ca^+$: Experimental procedure

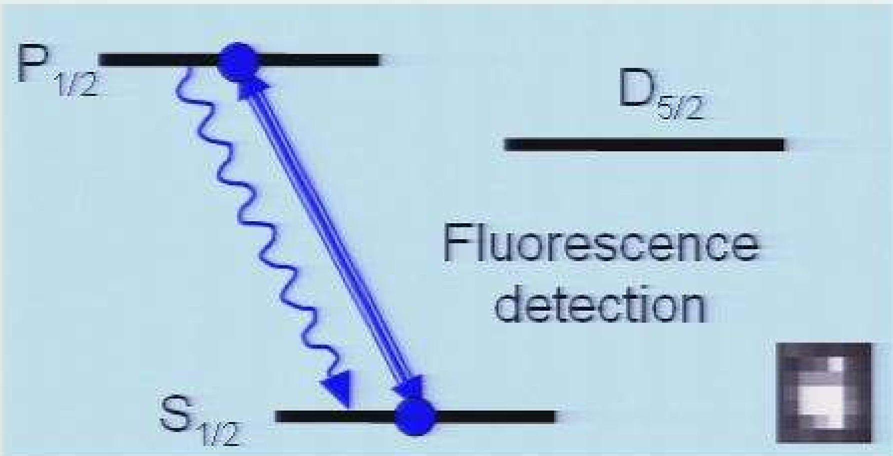


1. Initialization

2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

Experiments with $^{40}Ca^+$: Experimental procedure

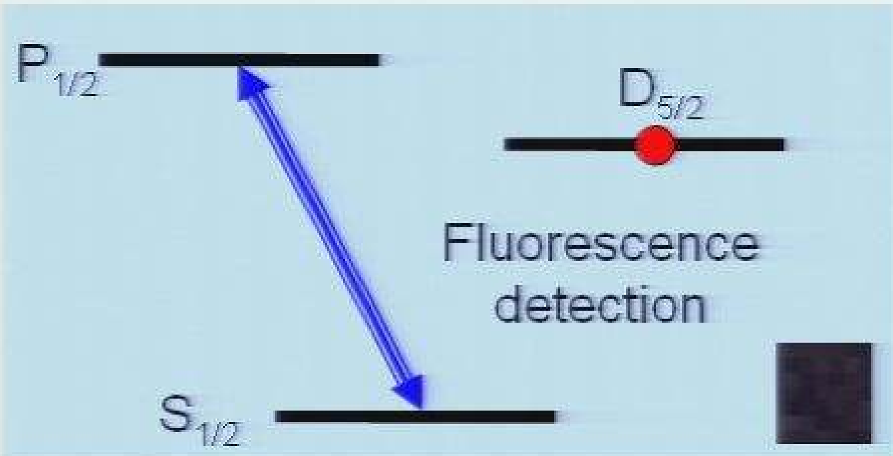


1. Initialization

2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition

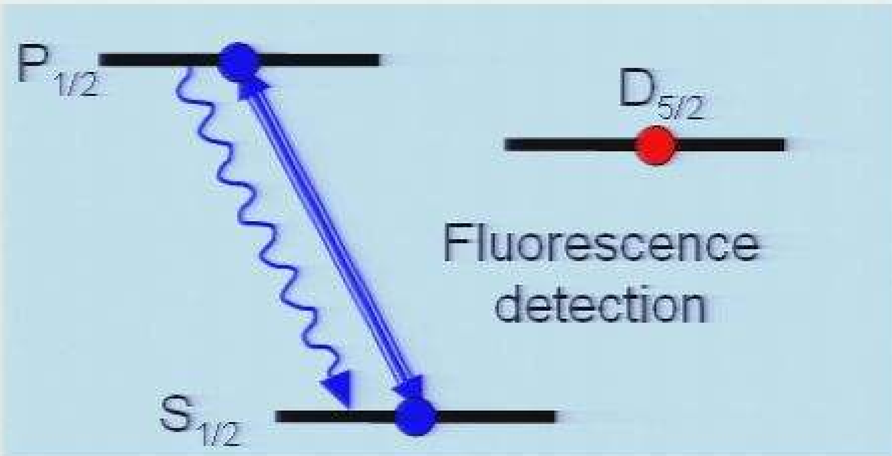
3. Quantum state measurement by fluorescence detection

Experiments with $^{40}Ca^{+}$: Experimental procedure



1. Initialization
2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Experiments with $^{40}Ca^{+}$: Experimental procedure



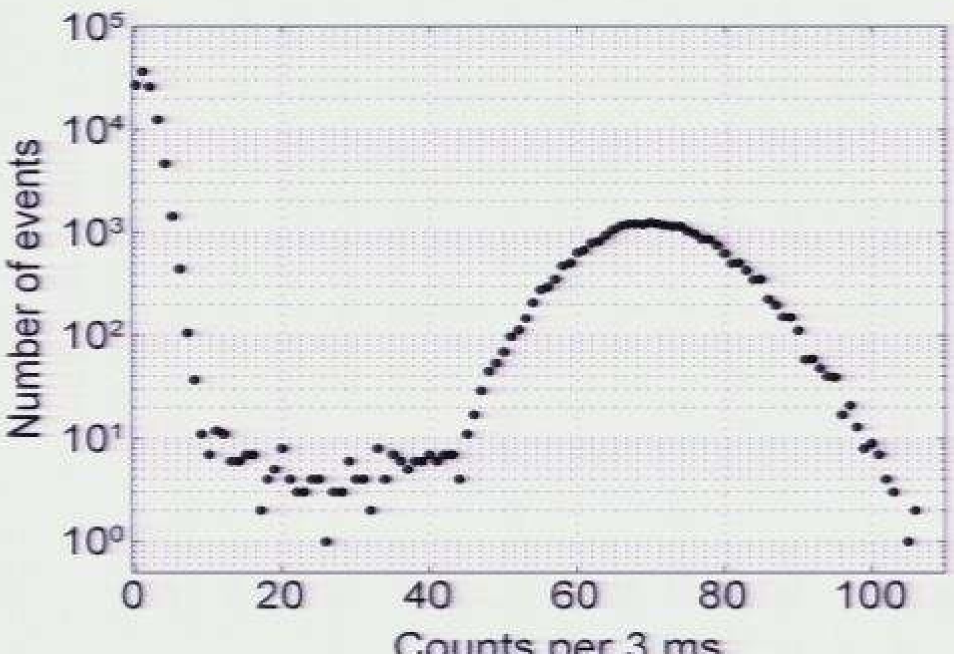
1. Initialization

2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition

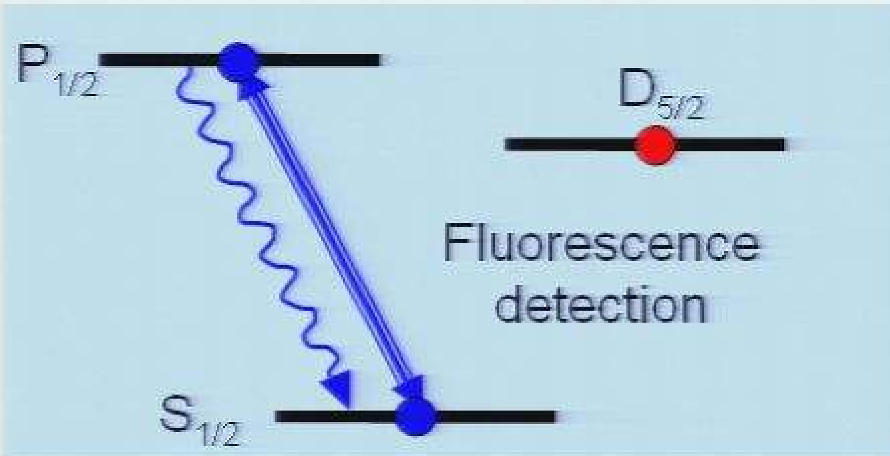
3. Quantum state measurement by fluorescence detection

One ion:

Photon count histogram



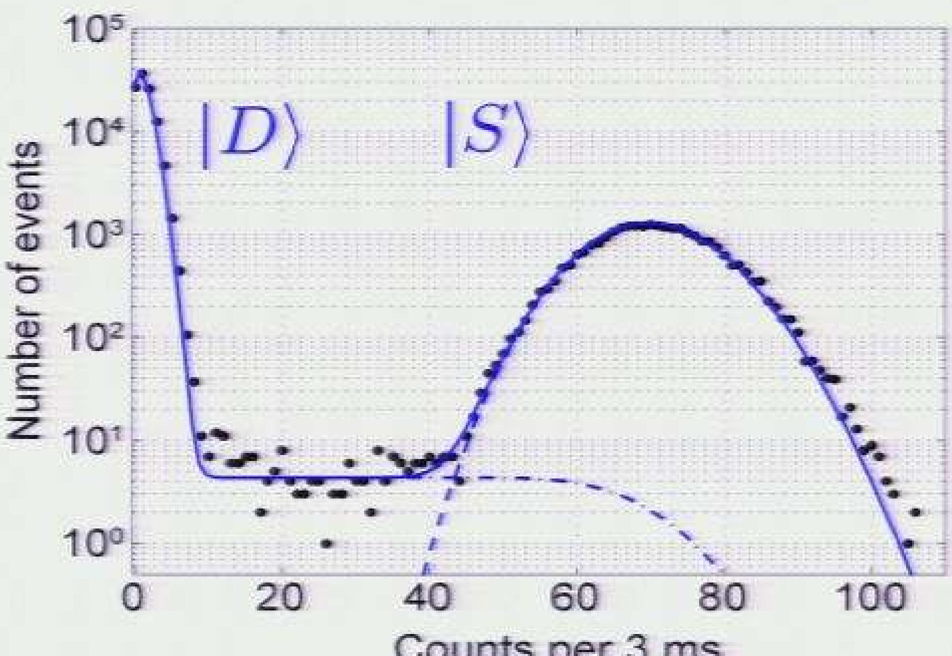
Experiments with $^{40}Ca^+$: Experimental procedure



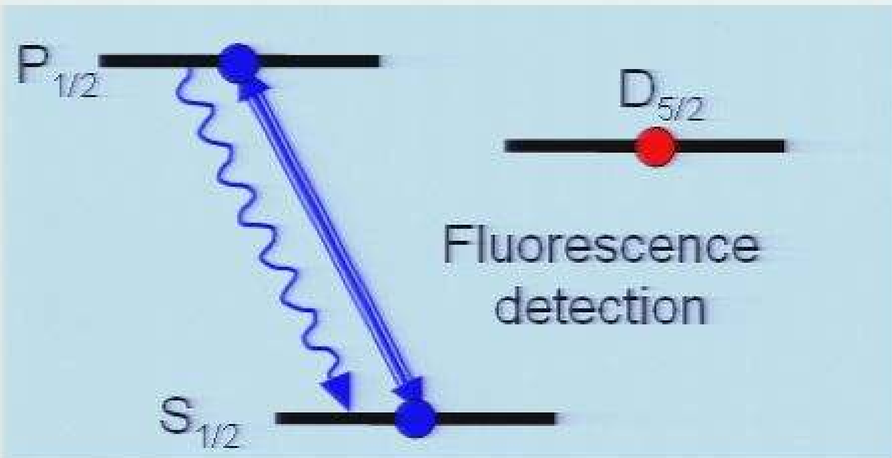
1. Initialization
2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

One ion:

Photon count histogram



Experiments with $^{40}Ca^{+}$: Experimental procedure



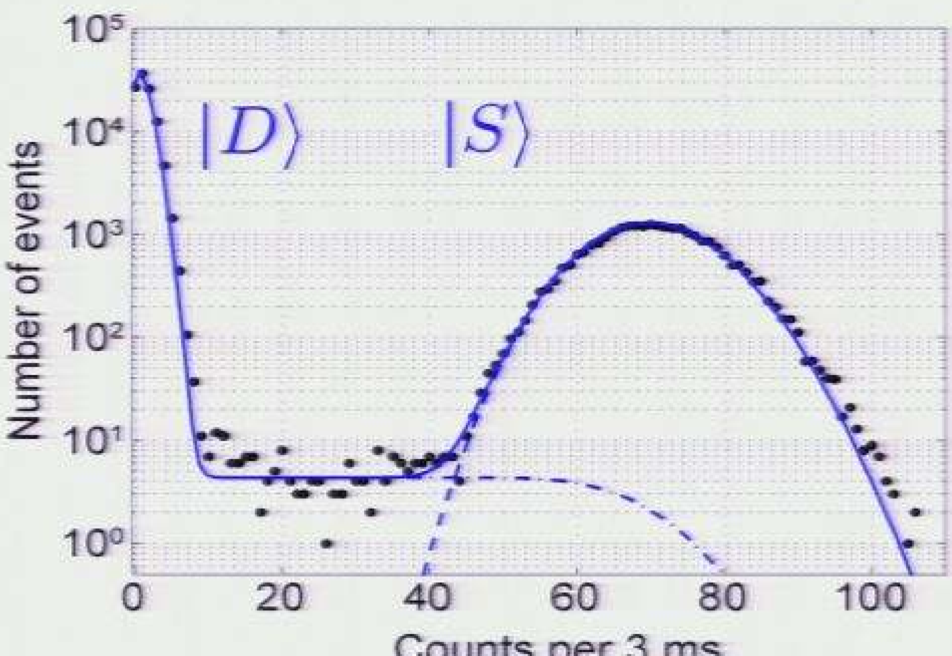
1. Initialization

2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

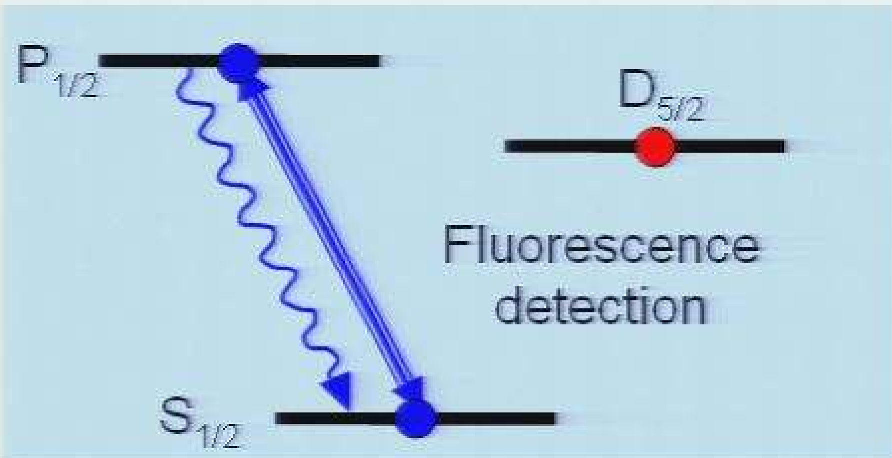
One ion:

Photon count histogram



ESET NOD32 Antivirus
Server nicht gefunden.

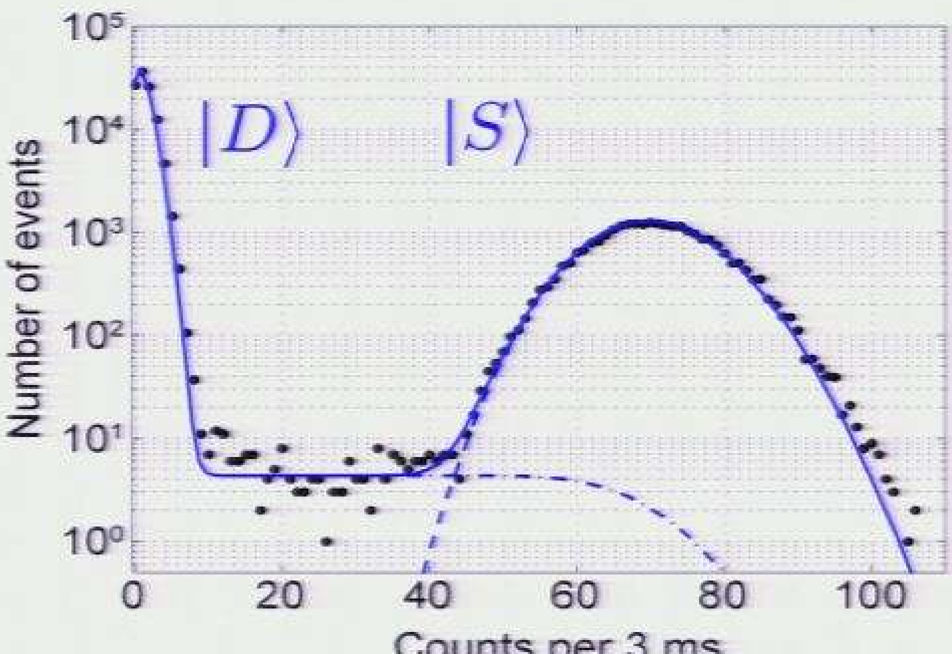
Experiments with $^{40}Ca^{+}$: Experimental procedure



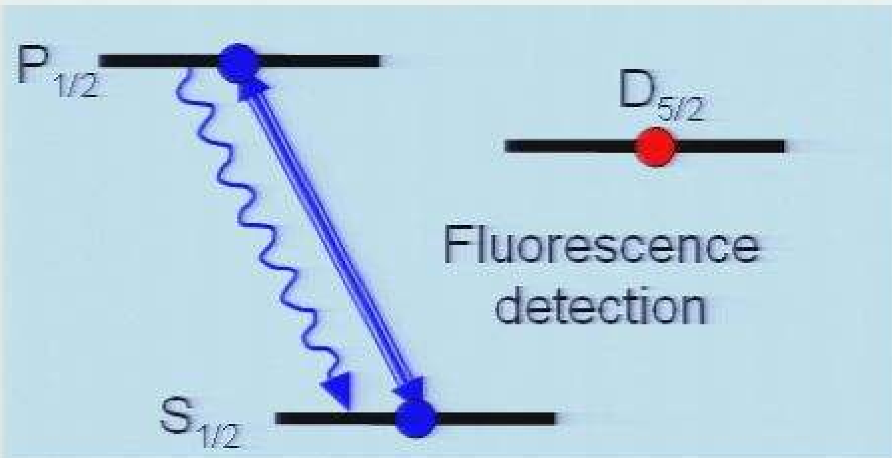
1. Initialization
2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

One ion:

Photon count histogram



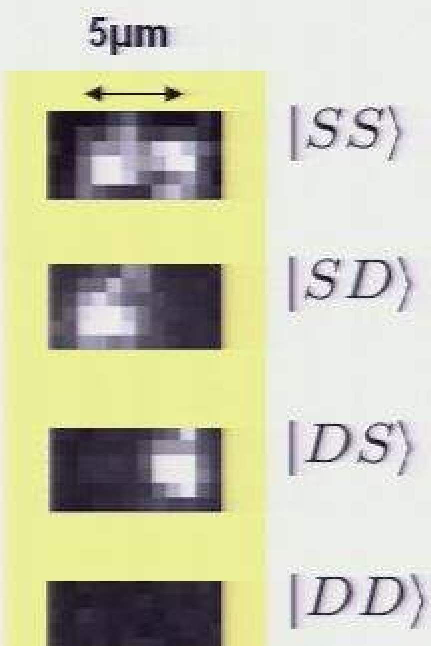
Experiments with $^{40}Ca^{+}$: Experimental procedure



1. Initialization
2. Coherent excitation of $S_{1/2} - D_{5/2}$ transition
3. Quantum state measurement by fluorescence detection

Two ions:

Spatially resolved detection with CCD camera



50 experiments / s

Repeat experiments 100-200 times

Bell state analysis

$$|SD\rangle + |DS\rangle$$

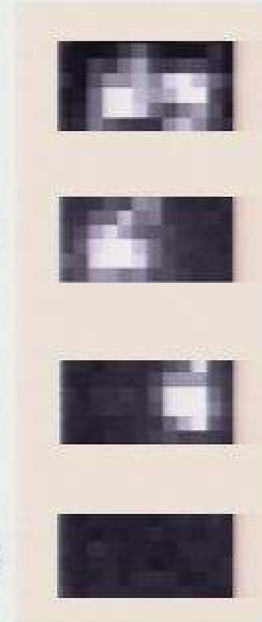
Fluorescence detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

$|DD\rangle$



Bell state analysis

$$|SD\rangle + |DS\rangle$$

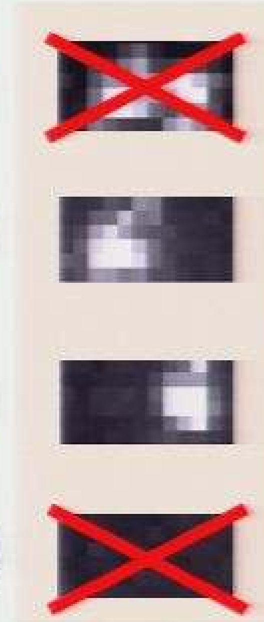
Fluorescence detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

$|DD\rangle$



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

Bell state analysis

$$|SD\rangle + |DS\rangle$$

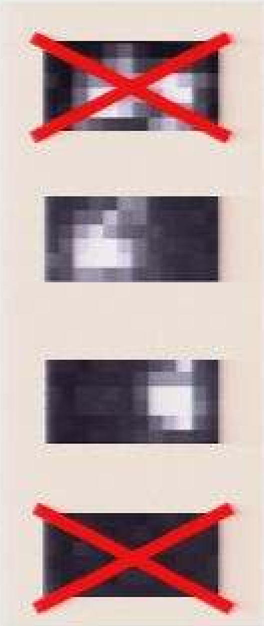
Fluorescence detection with CCD camera:

$|SS\rangle$

$|SD\rangle$

$|DS\rangle$

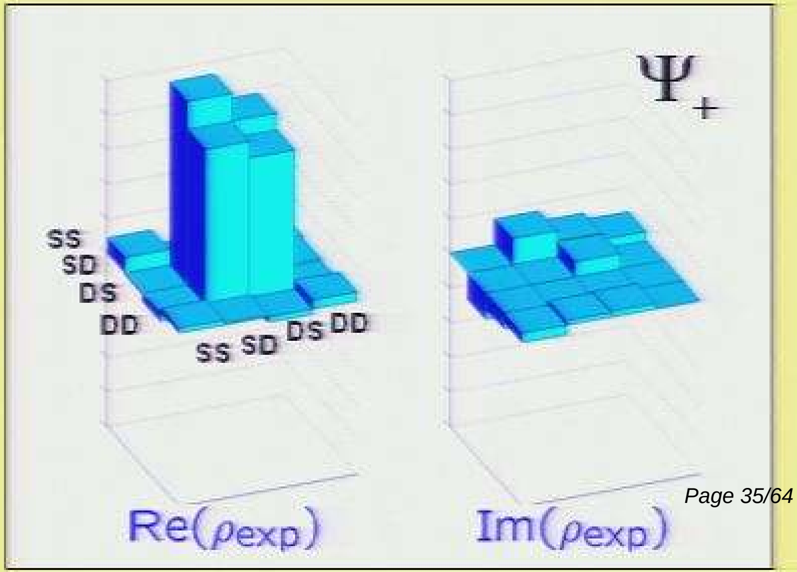
$|DD\rangle$



Coherent superposition or incoherent mixture ?

What is the relative phase of the superposition ?

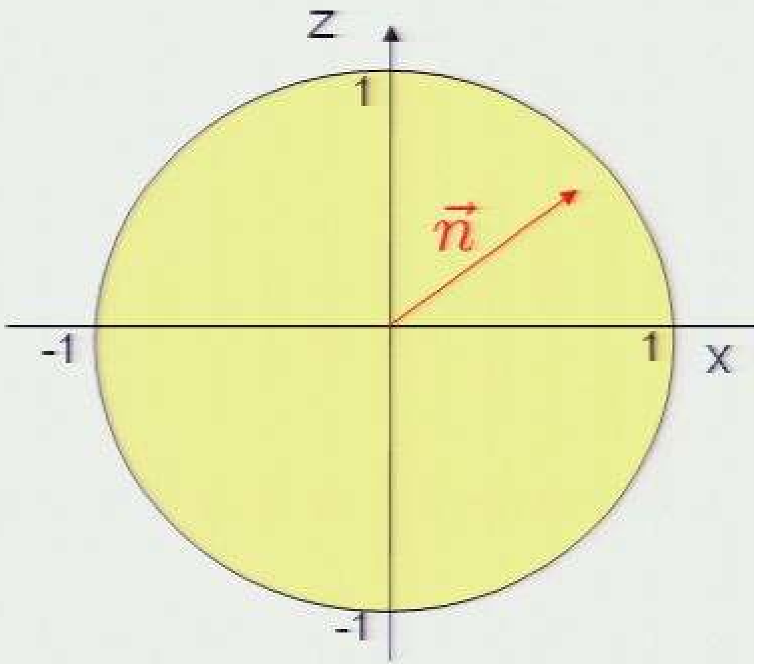
→ Measurement of the density matrix:



Reconstruction of the density matrix

One qubit:

$$\rho = \frac{1}{2}(I + n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$



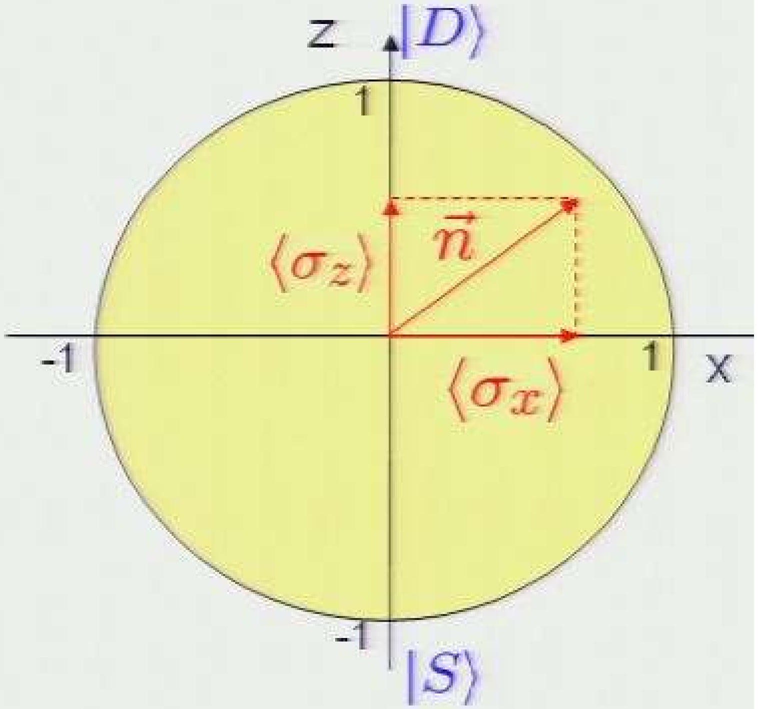
Observables: Pauli spin matrices $I, \sigma_x, \sigma_y, \sigma_z$

Expectation values: $n_i = \langle\sigma_i\rangle$

Reconstruction of the density matrix

One qubit:

$$\rho = \frac{1}{2}(I + n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$



Observables: Pauli spin matrices $I, \sigma_x, \sigma_y, \sigma_z$

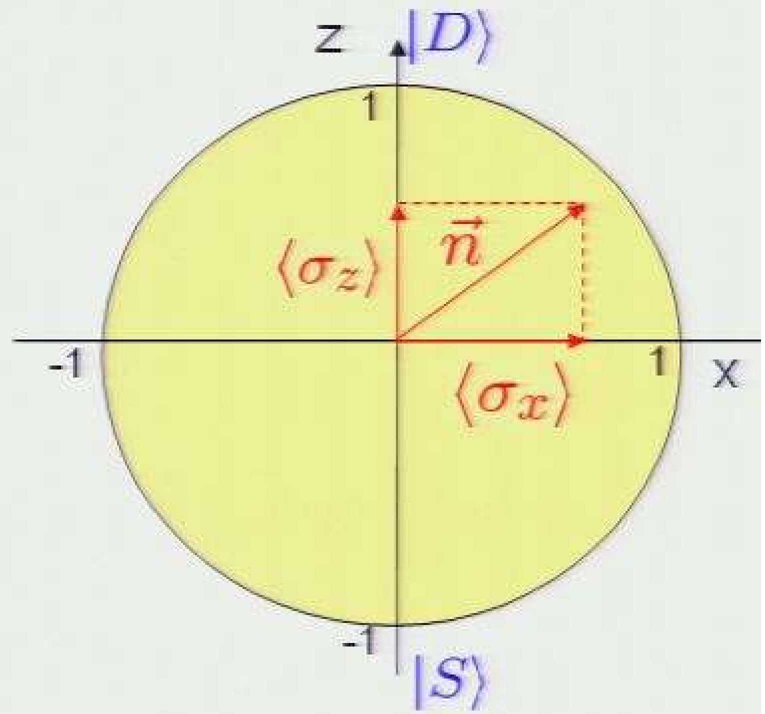
Expectation values: $n_i = \langle\sigma_i\rangle$

Natural measurement basis: $\left.\begin{array}{l} |+\rangle_z \leftrightarrow |D\rangle \\ |-\rangle_z \leftrightarrow |S\rangle \end{array}\right\}$ Fluorescence measurement

Reconstruction of the density matrix

One qubit:

$$\rho = \frac{1}{2}(I + n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$



Observables: Pauli spin matrices $I, \sigma_x, \sigma_y, \sigma_z$

Expectation values: $n_i = \langle\sigma_i\rangle$

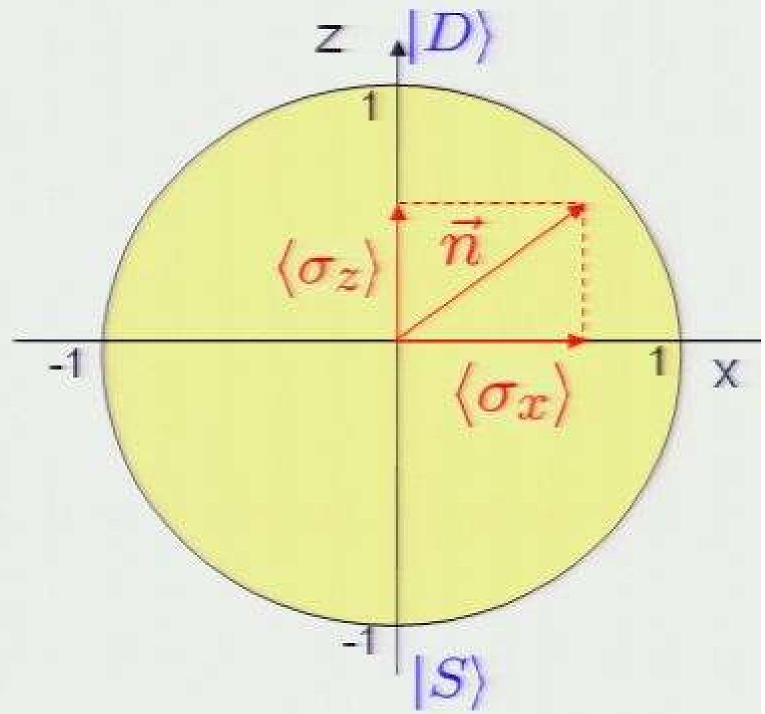
Natural measurement basis: $|+\rangle_z \leftrightarrow |D\rangle$, $|-\rangle_z \leftrightarrow |S\rangle$ } Fluorescence measurement

Other measurement bases: $|+\rangle_{x,y}$, $|-\rangle_{x,y}$ } π/2 pulse + fluorescence measurement

Reconstruction of the density matrix

One qubit:

$$\rho = \frac{1}{2}(I + n_x\sigma_x + n_y\sigma_y + n_z\sigma_z)$$



Observables: Pauli spin matrices $I, \sigma_x, \sigma_y, \sigma_z$

Expectation values: $n_i = \langle\sigma_i\rangle$

N qubits:

Representation of ρ as a sum of orthogonal observables A_i :

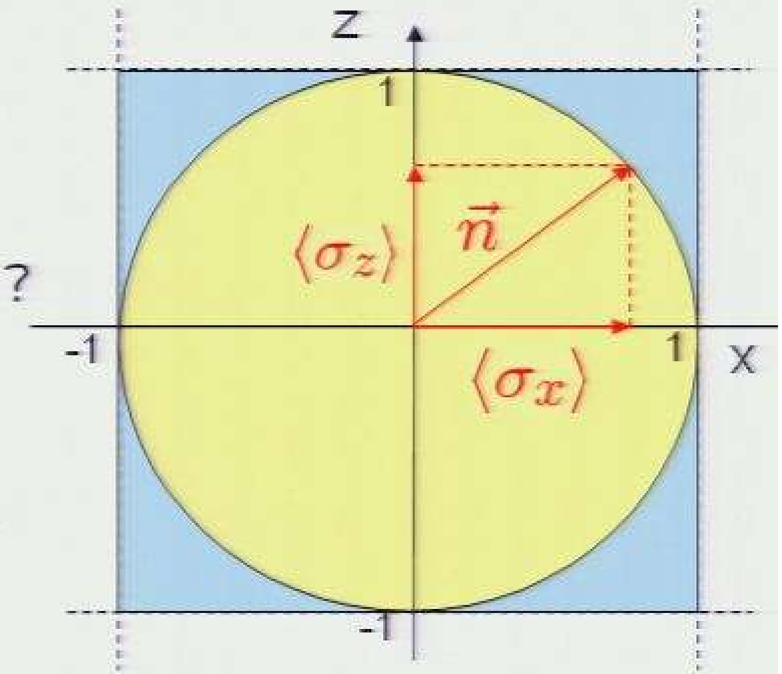
$$\rho = \sum_i \langle A_i\rangle\, A_i$$

$$A_i = \sigma_m^{(1)} \otimes \sigma_n^{(2)} \otimes \ldots$$

Measurement uncertainties

Direct reconstruction:

Is $\rho_R = \sum_i \langle A_i \rangle \, A_i$ positive semidefinite ?



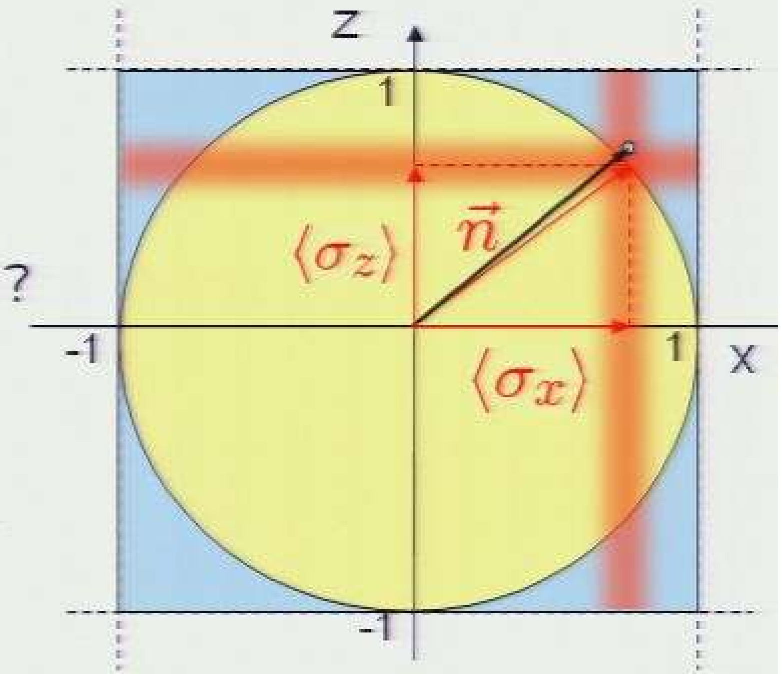
... not necessarily:

Measurement uncertainties

Direct reconstruction:

Is $\rho_R = \sum_i \langle A_i \rangle \, A_i$ positive semidefinite ?

... not necessarily:



Shot noise in the measuremen
might give rise to
unphysical density matrices

Maximum likelihood estimation:

Determine density matrix that is most likely to reproduce the experimentally observed results.

Maximum likelihood estimation

Maximum likelihood estimation: (Hradil '97, Banaszek '99)

Find the density matrix that is most likely to reproduce the experimentally observed results.

In N experiments, the quantum state is projected onto the outcomes $|y_j\rangle$.

f_j : relative frequency of the outcome $|y_j\rangle$

On the set of density matrices ρ, look for the matrix that maximizes

$$\mathcal{L}(\rho) = \prod_j \langle y_j|\rho|y_j\rangle^{N f_j}$$

Maximize $L(\rho) = \log \mathcal{L}(\rho) = N \sum_j f_j \log\langle y_j|\rho|y_j\rangle$

Analysis of two-qubit states

prepare Bell state
no rotation
measure

200 repetitions

$\langle\sigma_z^{(1)}\rangle, \langle\sigma_z^{(2)}\rangle, \langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle$

↓

prepare Bell state
ion #1, y - rotation
ion #2, identity
measure

200 repetitions

$\langle\sigma_x^{(1)}\rangle, \langle\sigma_z^{(2)}\rangle, \langle\sigma_x^{(1)}\sigma_z^{(2)}\rangle$

↓

⋮

9 different settings

↓

prepare Bell state
ion #1, x - rotation
ion #2, x - rotation
measure

200 repetitions

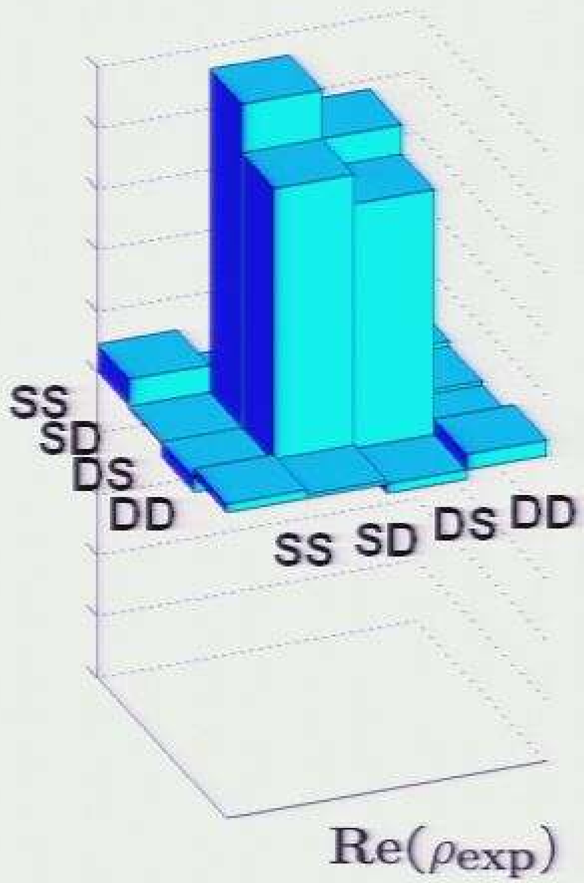
$\langle\sigma_y^{(1)}\rangle, \langle\sigma_y^{(2)}\rangle, \langle\sigma_y^{(1)}\sigma_y^{(2)}\rangle$

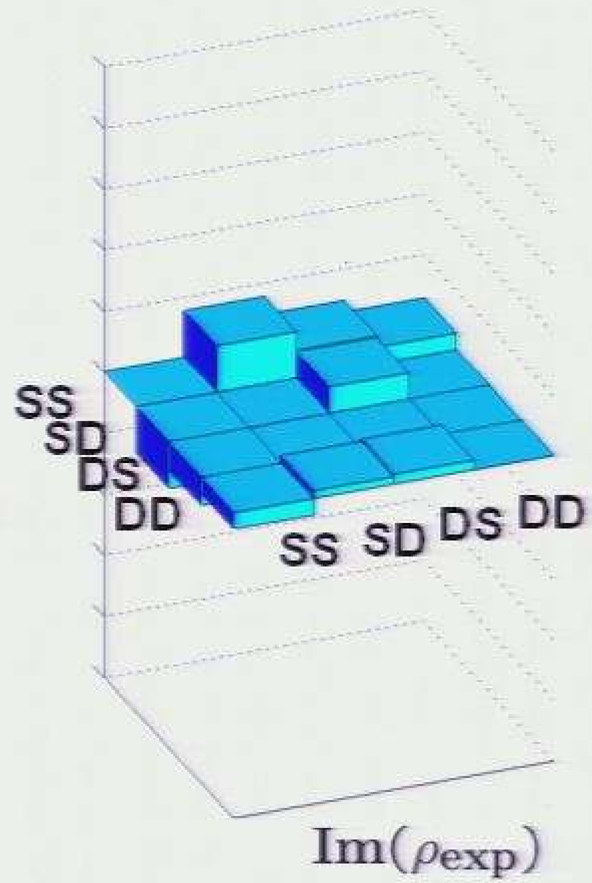
Bell state reconstruction

Measurement of 16 observables
9 different settings, measurement time ≈ 40 s

$|SD\rangle + |DS\rangle$

F=0.91

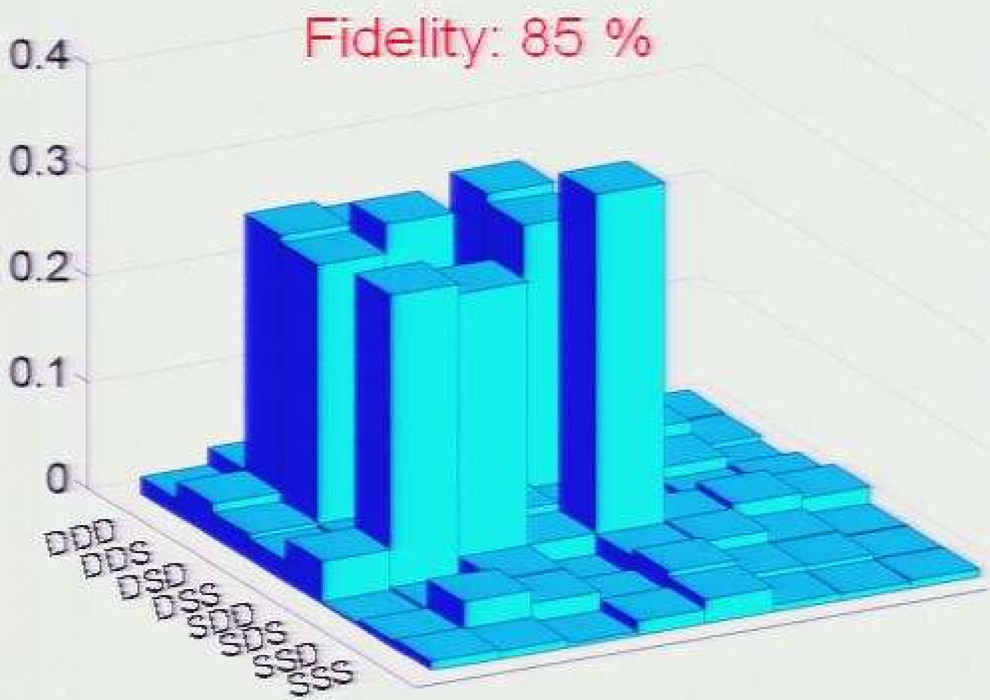




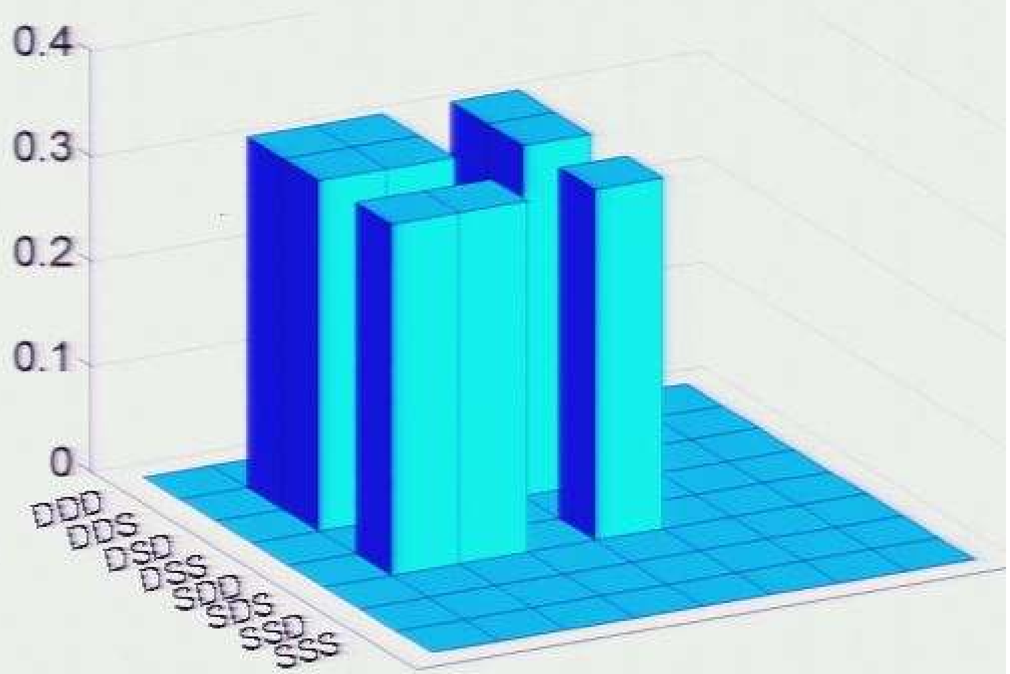
C. F. Roos *et al.*, Phys. Rev. Lett. **92**, 220402 (2004)

Three-ion entangled states

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|SDD\rangle + |DSD\rangle + |DDS\rangle)$$



experimental result



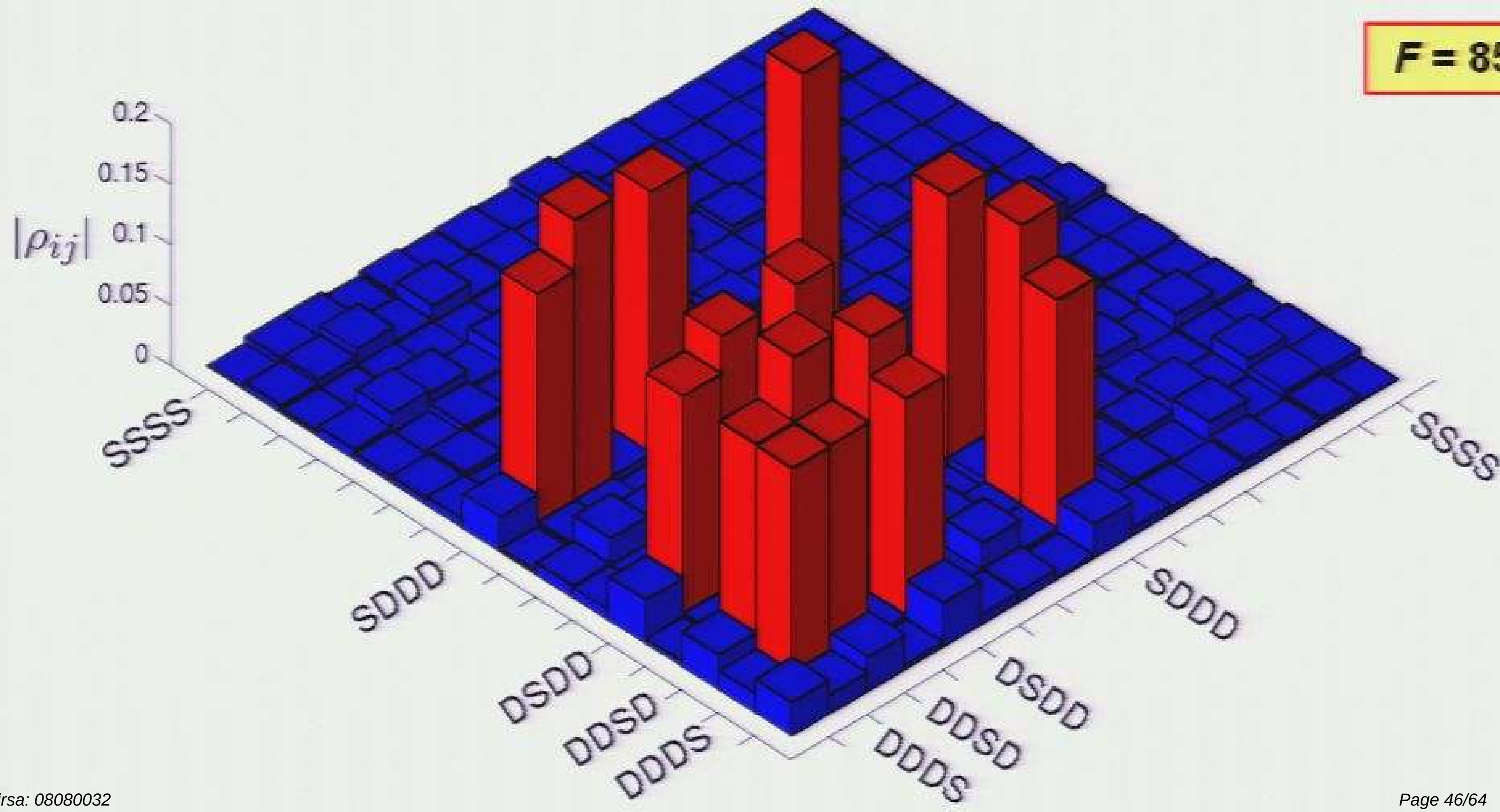
theoretical expectation

C. F. Roos *et al.*, Science **304**, 1478 (2004)

Four-ion W-states

$$\Psi_4 = \frac{1}{\sqrt{4}}(|DDDS\rangle + |DDSD\rangle + |DSDD\rangle + |SDDD\rangle)$$

F = 85%

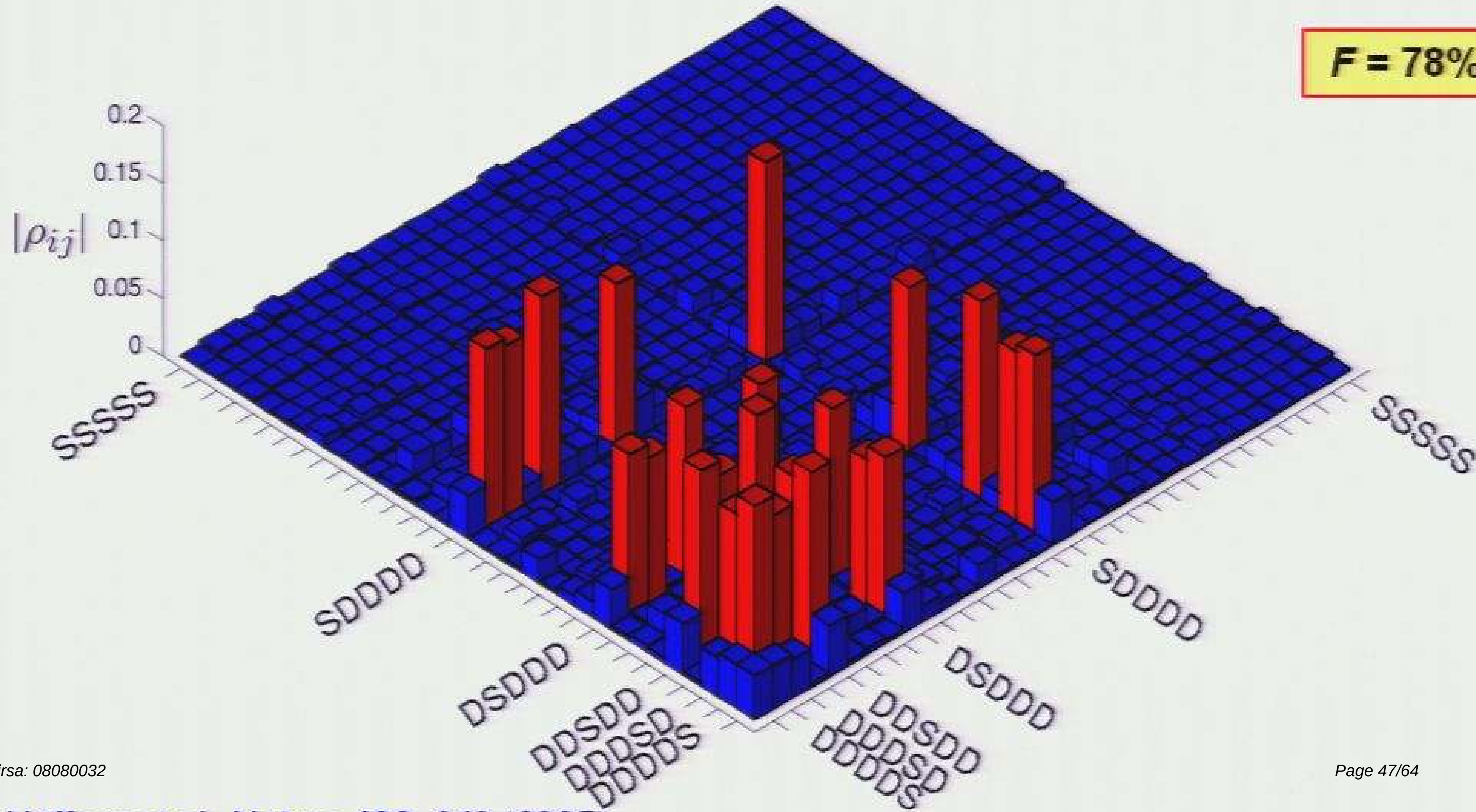


H. Häffner *et al.*, Nature **438**, 643 (2005)

Five-ion W-states

$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |DSDDD\rangle + |SDDDD\rangle$$

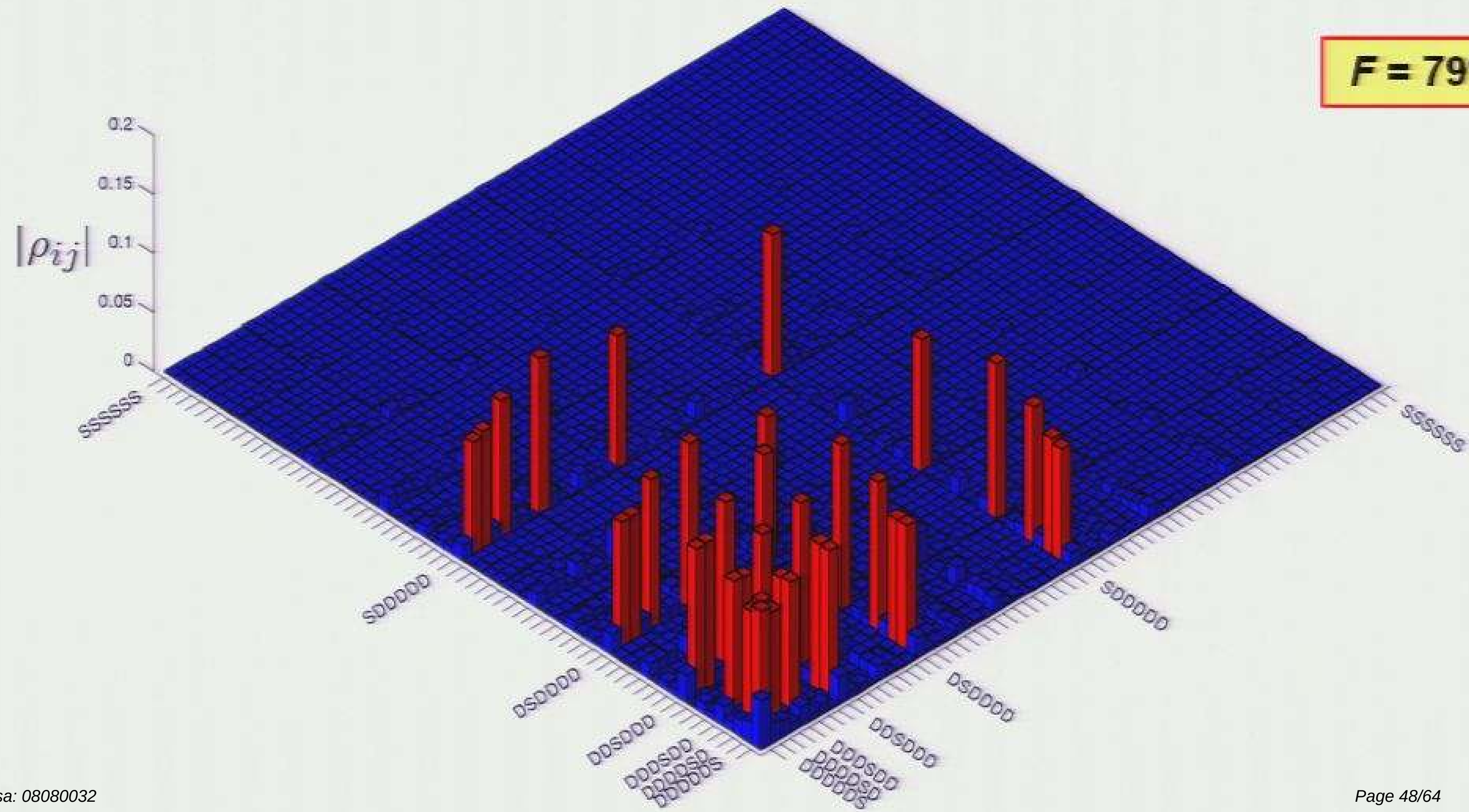
F = 78%



H. Häffner *et al.*, Nature **438**, 643 (2005)

Six-ion W-states

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$

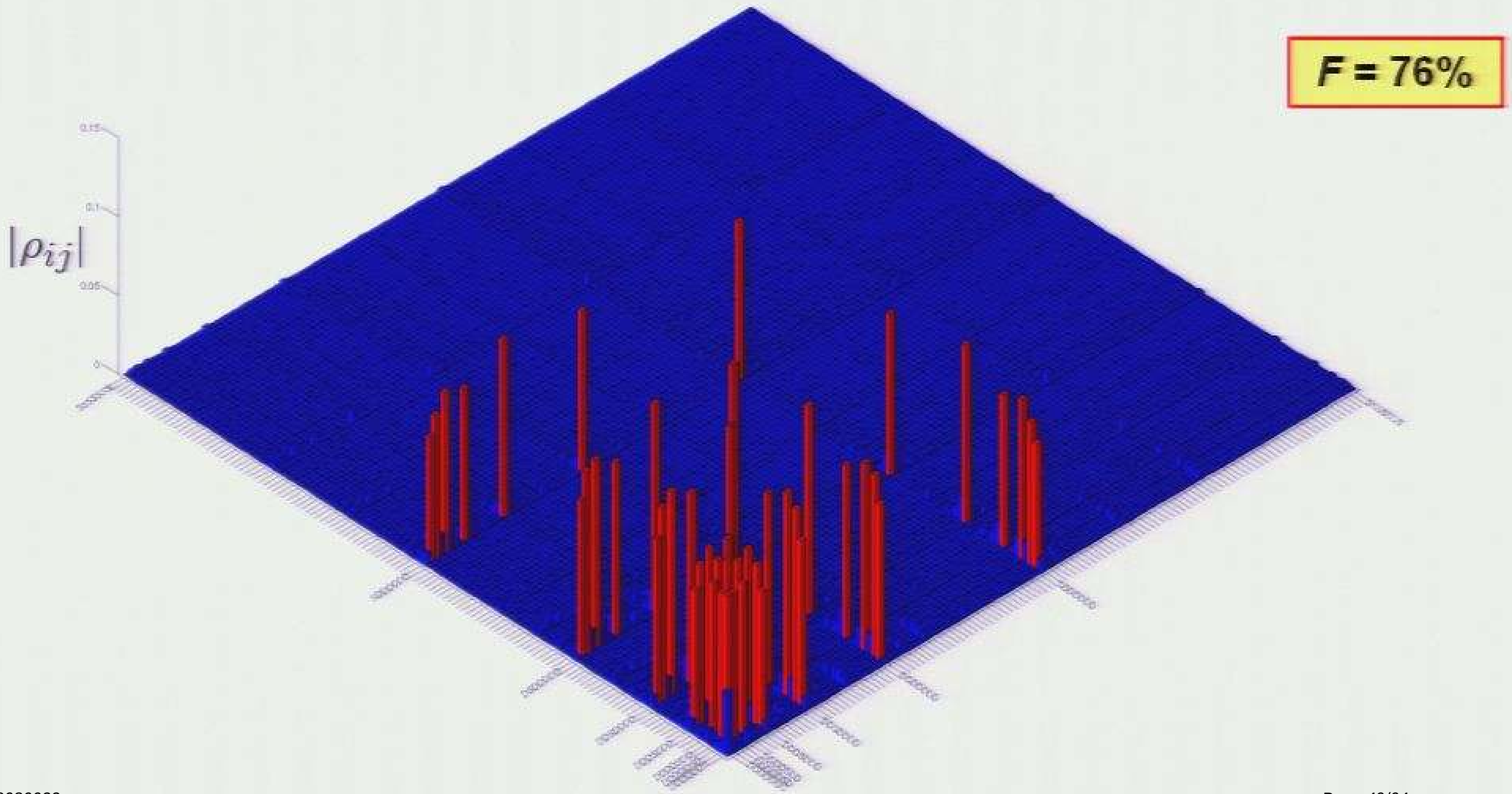


F = 79%

H. Häffner et al., Nature **438**, 643 (2005)

729 settings, measurement time: 40 minutes

Seven-ion W-states

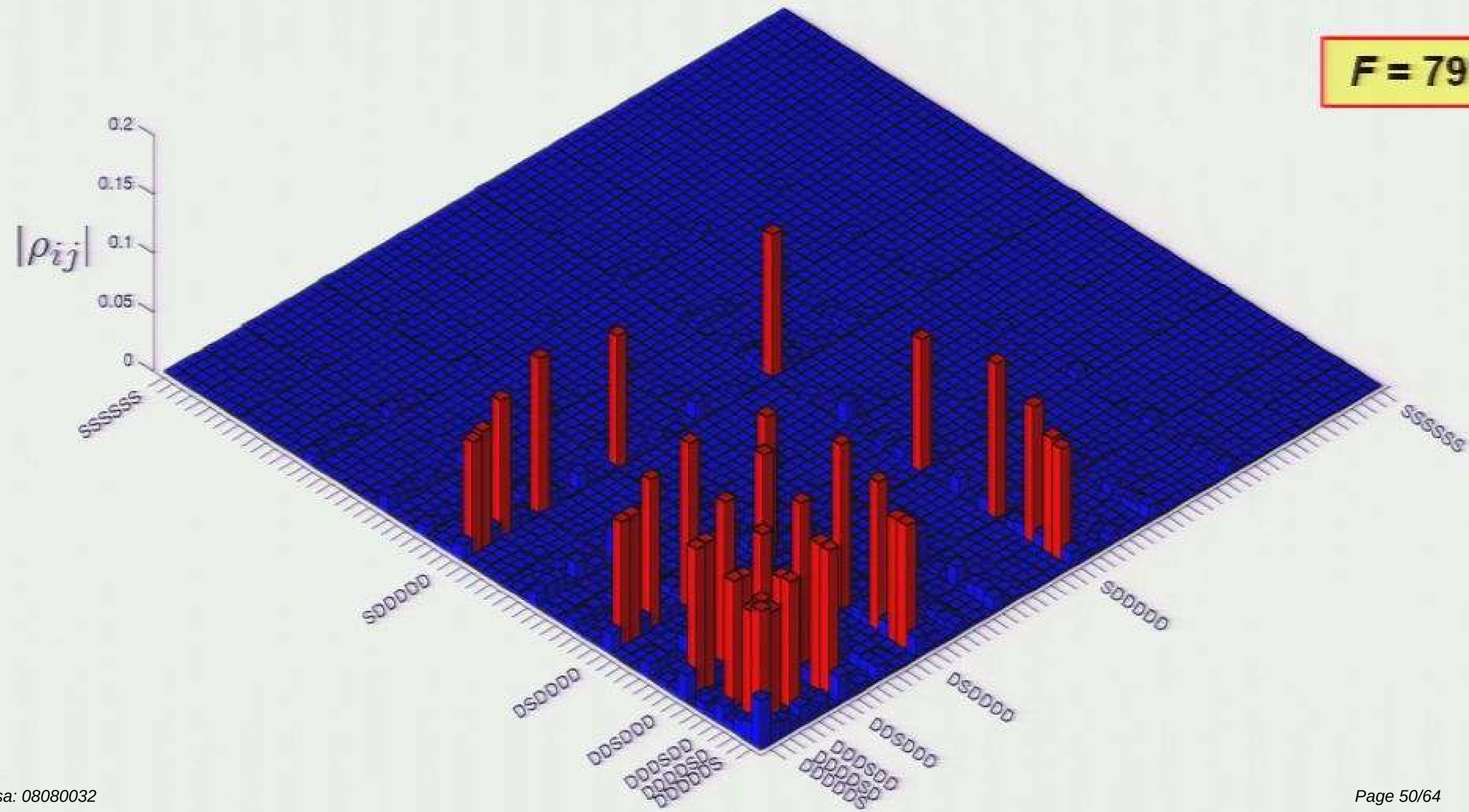


H. Häffner *et al.*, Nature **438**, 643 (2005) 2187 settings, measurement time: 2 hours

Six-ion W-states

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$

F = 79%

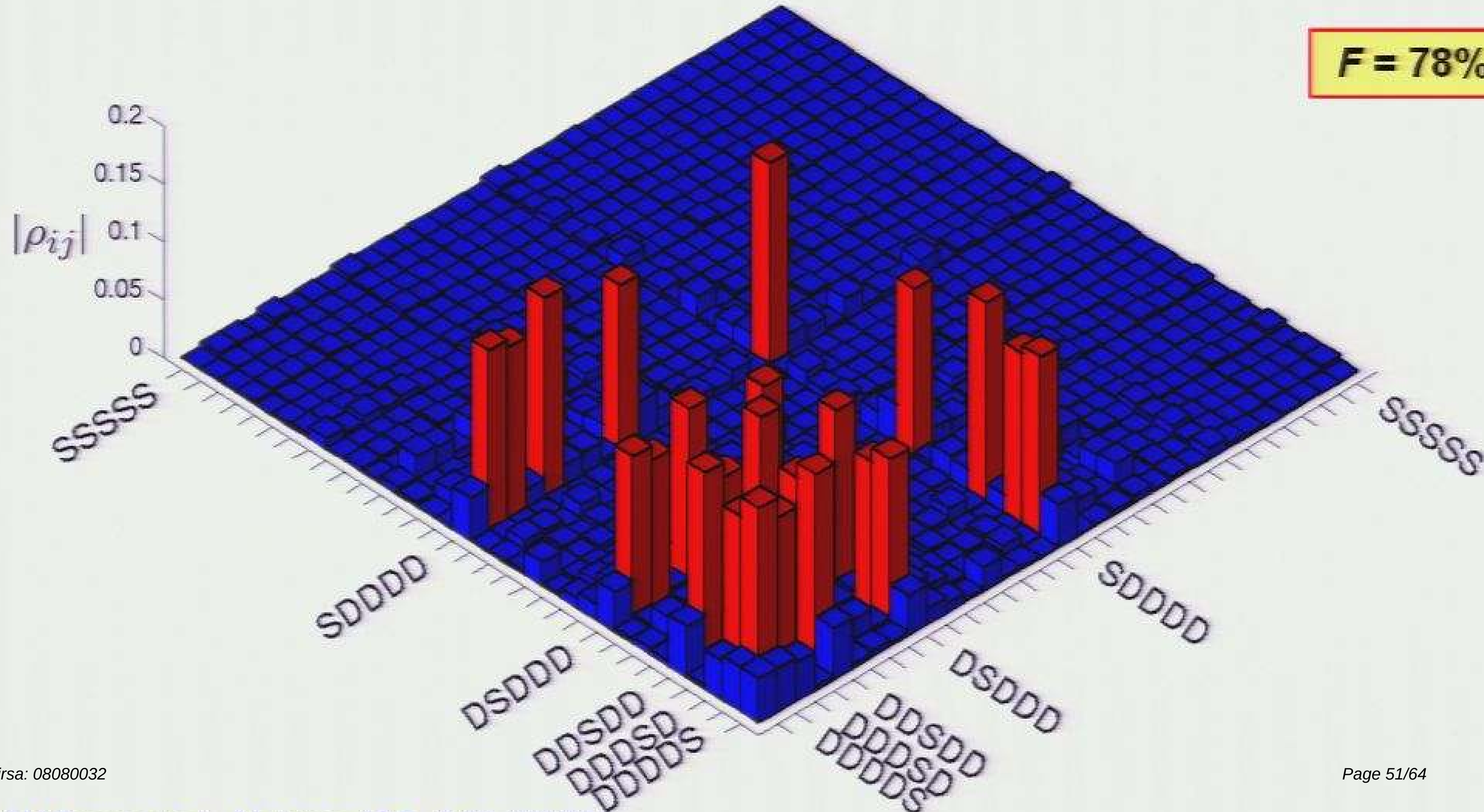


H. Häffner *et al.*, Nature **438**, 643 (2005) 729 settings, measurement time: 40 minutes

Five-ion W-states

$$\Psi_5 = \frac{1}{\sqrt{5}}(|DDDDS\rangle + |DDDSD\rangle + |DDSDD\rangle + |DSDDD\rangle + |SDDDD\rangle$$

F = 78%

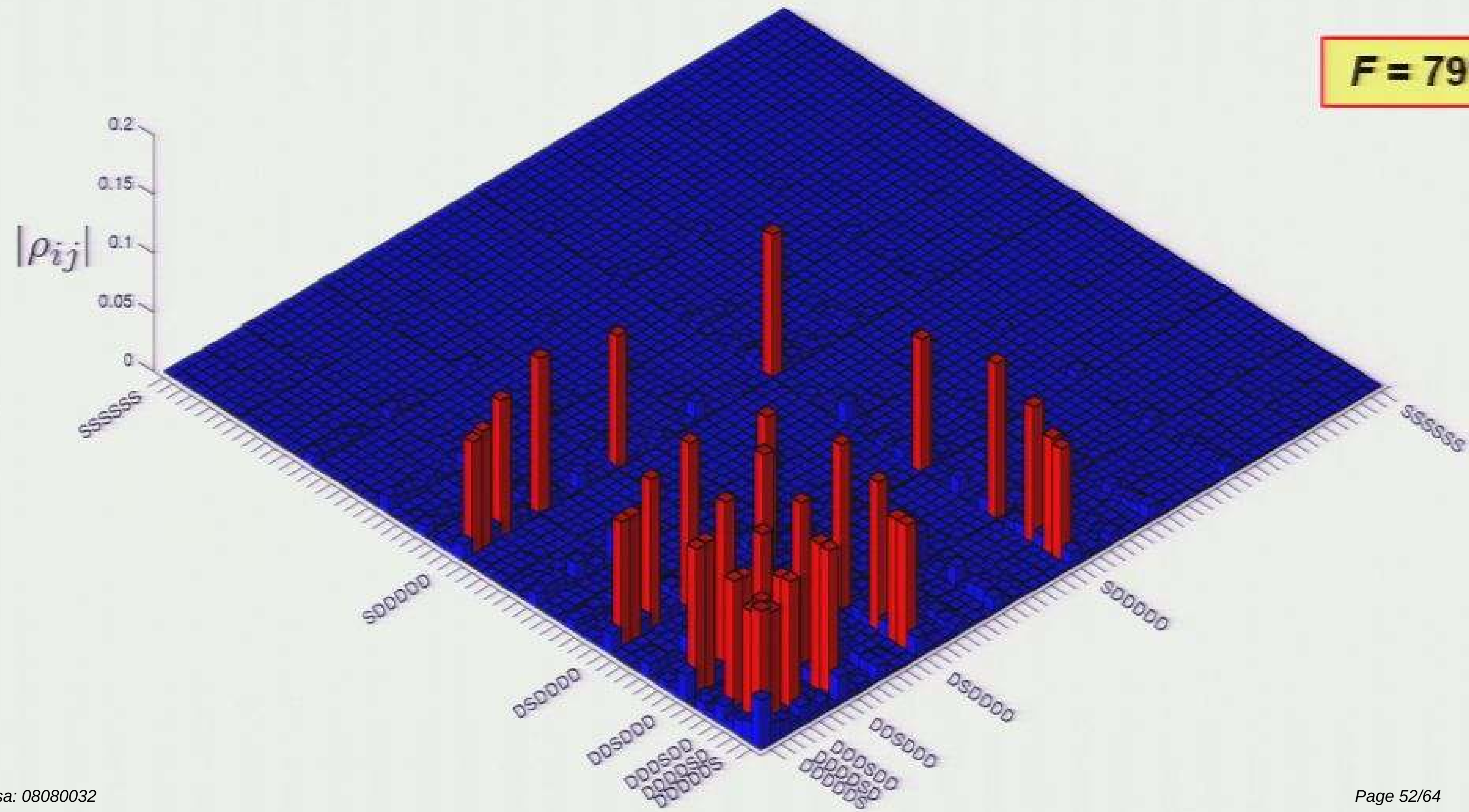


H. Häffner et al., Nature 438, 643 (2005)

Six-ion W-states

$$\Psi_6 = \frac{1}{\sqrt{6}}(|DDDDDS\rangle + |DDDDSD\rangle + |DDDSDD\rangle + |DDSDDD\rangle + |DSDDDD\rangle + |SDDDDD\rangle)$$

F = 79%

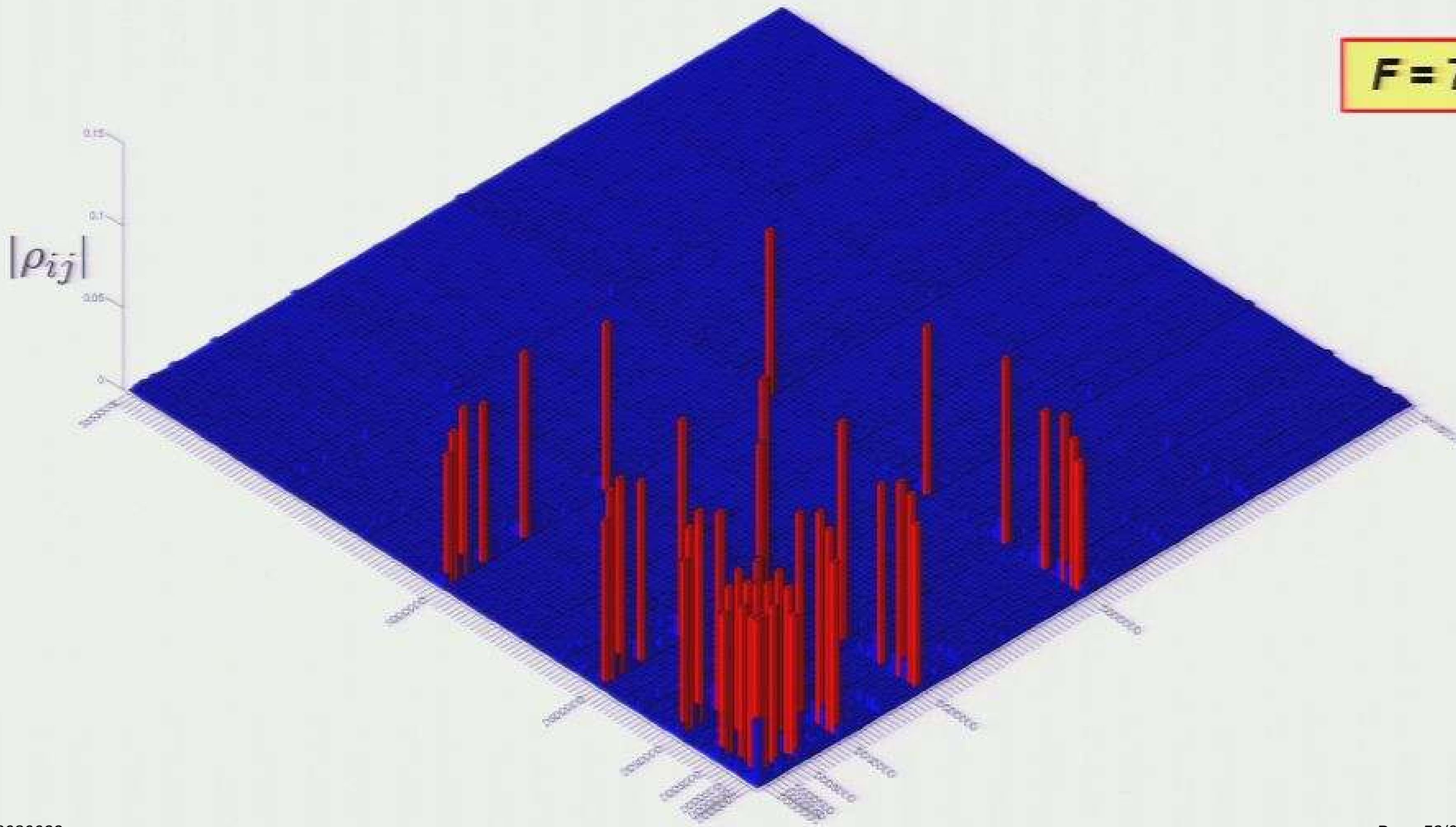


H. Häffner et al., Nature 438, 643 (2005)

729 settings, measurement time: 40 minutes

Seven-ion W-states

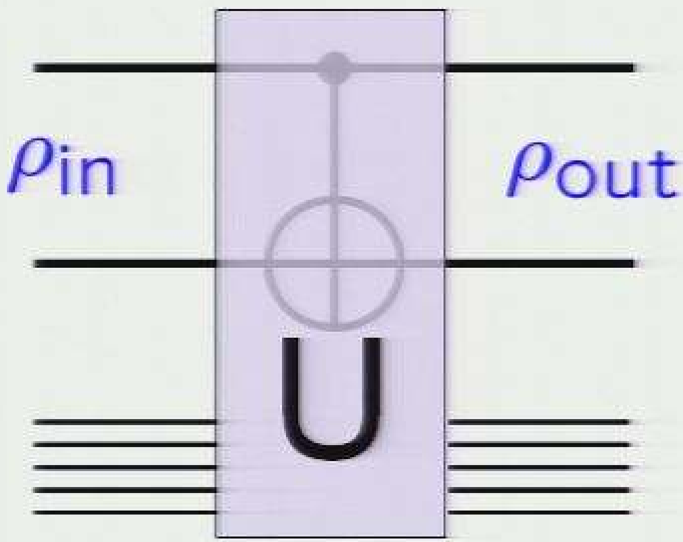
F = 76%



H. Häffner *et al.*, Nature **438**, 643 (2005)

2187 settings, measurement time: 2 hours

Quantum process tomography



Interaction with the environment

$$\rho_{\text{out}} = \mathcal{E}(\rho_{\text{in}})$$

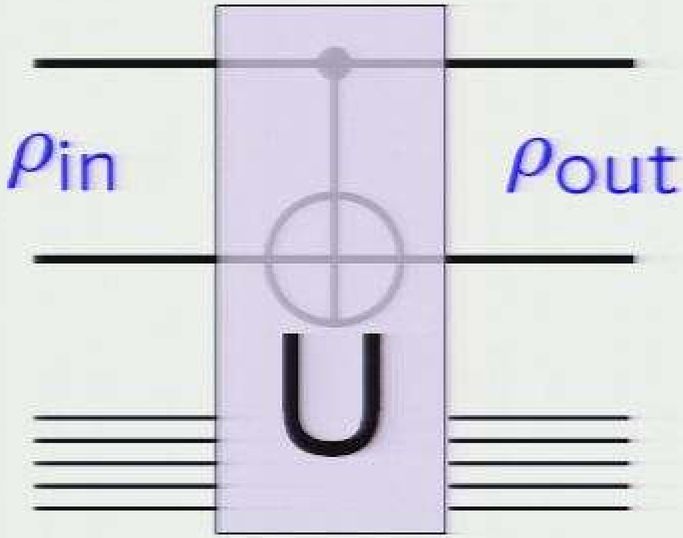
$$= \sum \chi_{ij} E_i \rho_{\text{in}} E_j^\dagger$$

with $\sum_i E_i^\dagger E_i = I$

χ_{ij} characterizes gate operation completely

Convenient choice: $E_i = A_i \otimes A_j, \quad A_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}$

Quantum process tomography



Interaction with the environment

$$\rho_{out} = \mathcal{E}(\rho_{in})$$

Experimentally applied to:

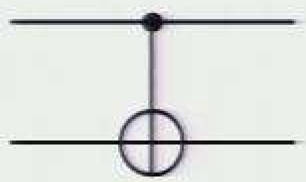
- Controlled-NOT gate operation
- Deterministic quantum teleportation

χ_{ij} characterizes gate operation completely

Convenient choice: $E_i = A_i \otimes A_j, \quad A_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}$

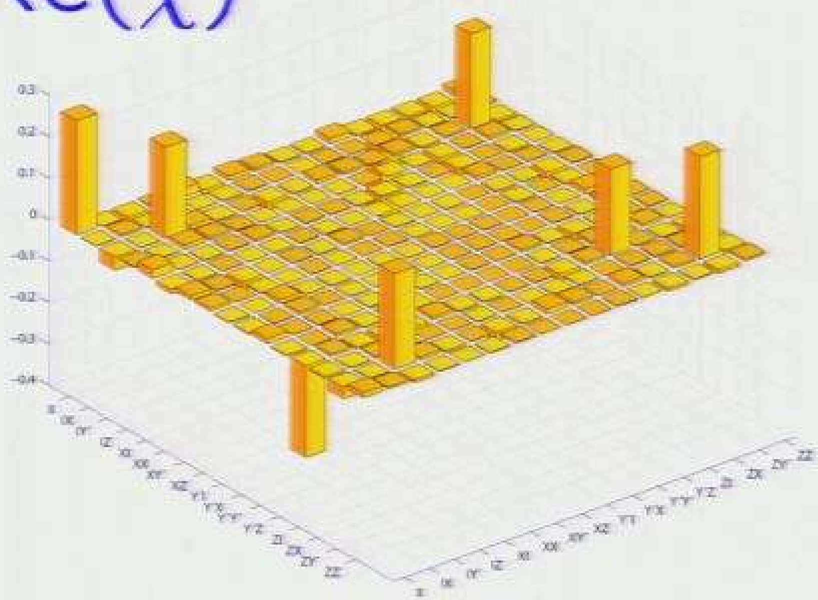
Process tomography of a CNOT gate

$$U^{12}_{\mathrm{CNOT}} = -\frac{1}{2}(I \otimes I + iI \otimes Y - Z \otimes I + iZ \otimes Y)$$

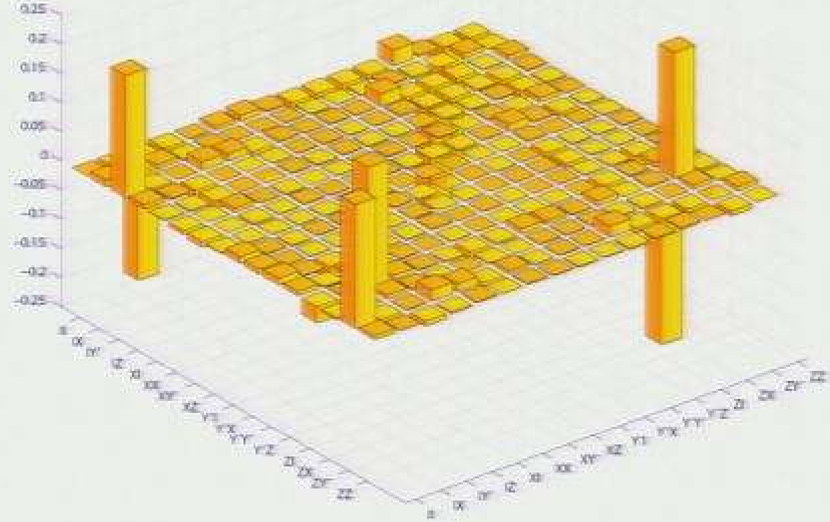


$$= \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Re(χ)



Im(χ)



M. Riebe et al., Phys Rev. Lett. **97**, 220407 (2006)

Gate performance

Measures of interest which quantify the perfomance of the gate operation:

Process fidelity: $$F_{proc} = tr(\chi_{id} \cdot \chi_{exp})$$

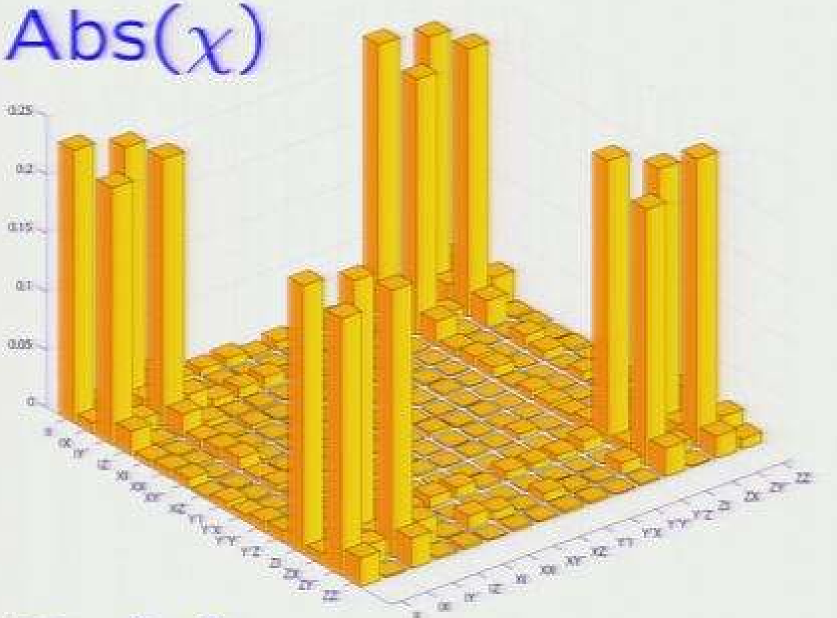
How well do the experimentally obtained and the ideal process matrix overlap ?

Mean fidelity: $$\bar{F} = \frac{1}{N} \sum_{i=1}^{N} \langle \psi_{id} | \rho_{out,i} | \psi_{id} \rangle$$

How good is the overlap between the calculated and the ideal output states ?

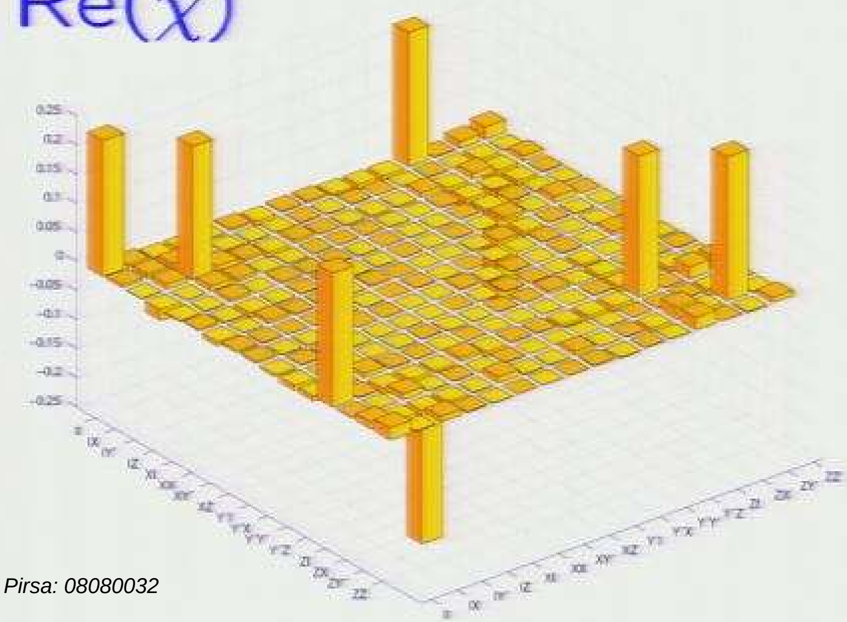
Results: Gate performance

Abs(χ)

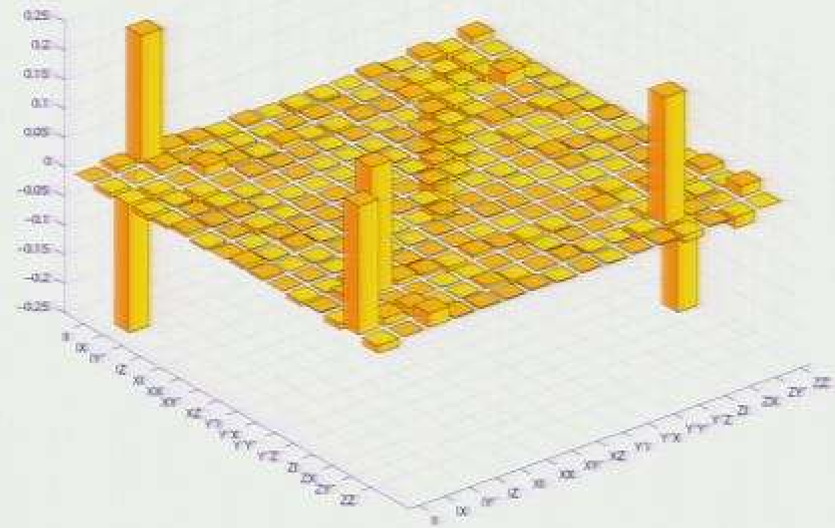


Measure	
Process fidelity F_p	90.8(6)%
Mean fidelity $\bar{F}$	92.6(6)%

Re(χ)



Im(χ)



M. Riebe et al., Phys. Rev. Lett. 97, 220407 (2006)

Double gate operation

$$U_{CNOT} \cdot U_{CNOT} = I$$

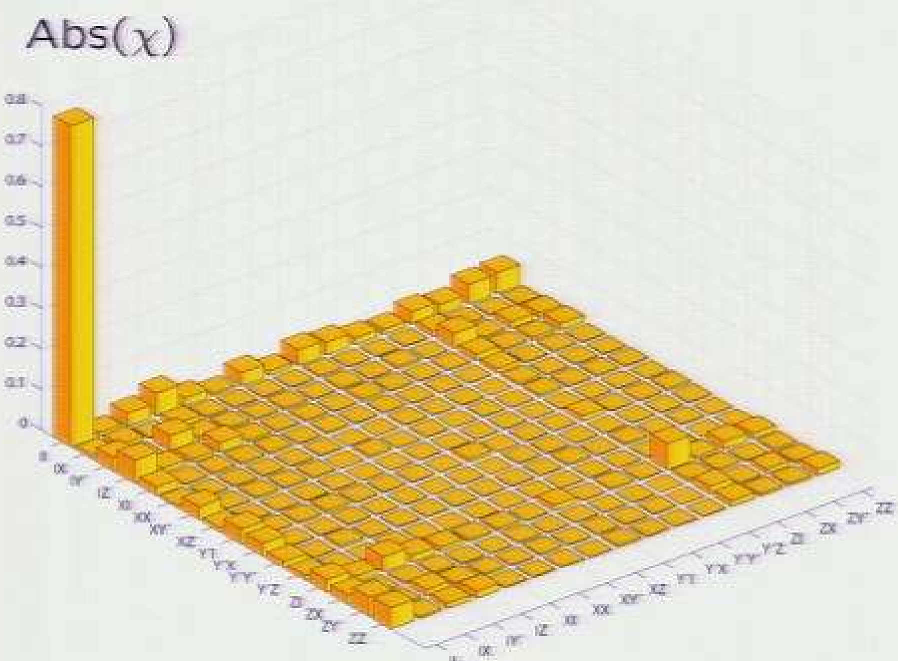
Can we infer the performance of combined gate operation from QPT of a single gate operation ?

Double gate operation

$$U_{CNOT} \cdot U_{CNOT} = I$$

Can we infer the performance of combined gate operation from QPT of a single gate operation ?

Experiment: QPT of two subsequently applied CNOT gates



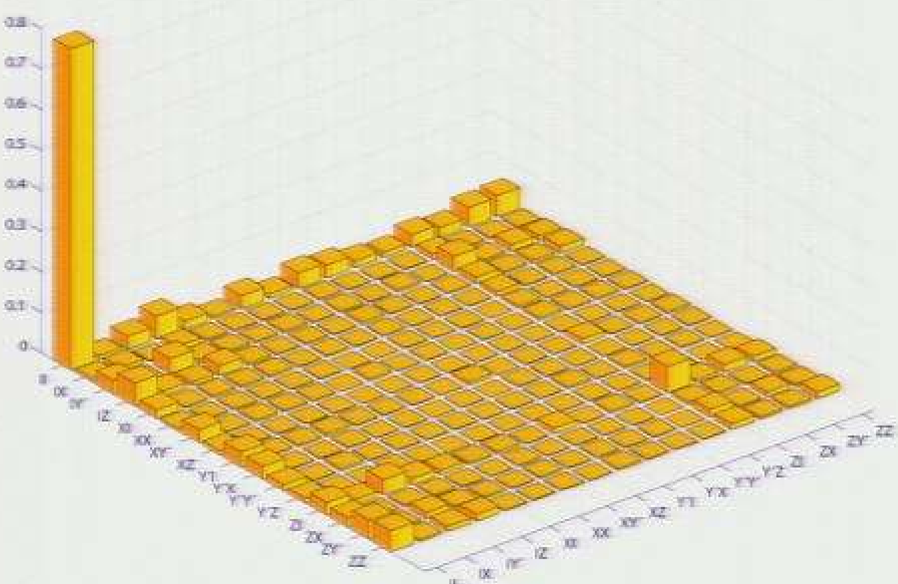
Double gate operation

$$U_{CNOT} \cdot U_{CNOT} = I$$

Can we infer the performance of combined gate operation from QPT of a single gate operation ?

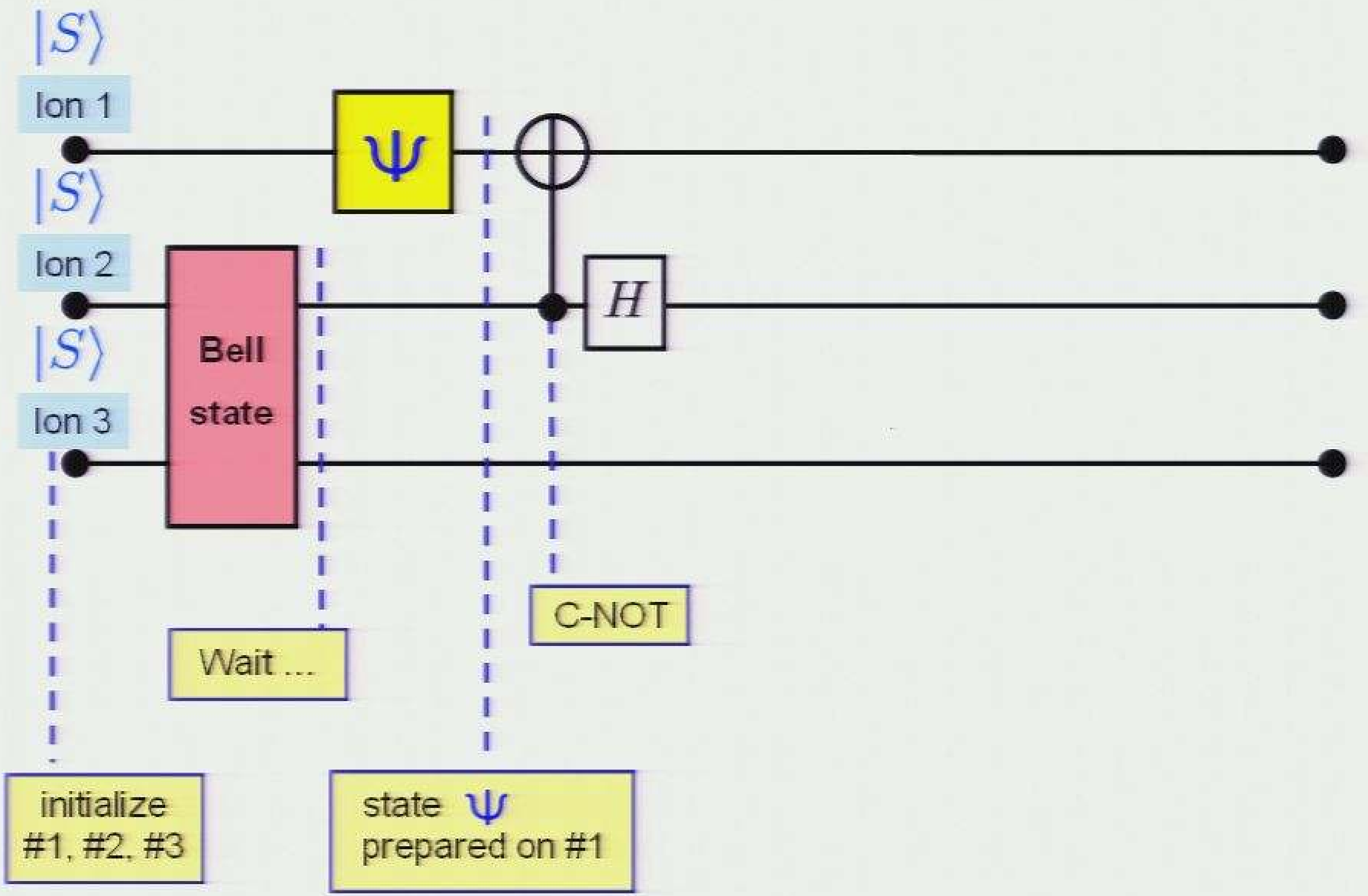
Experiment: QPT of two subsequently applied CNOT gates

Abs(χ)



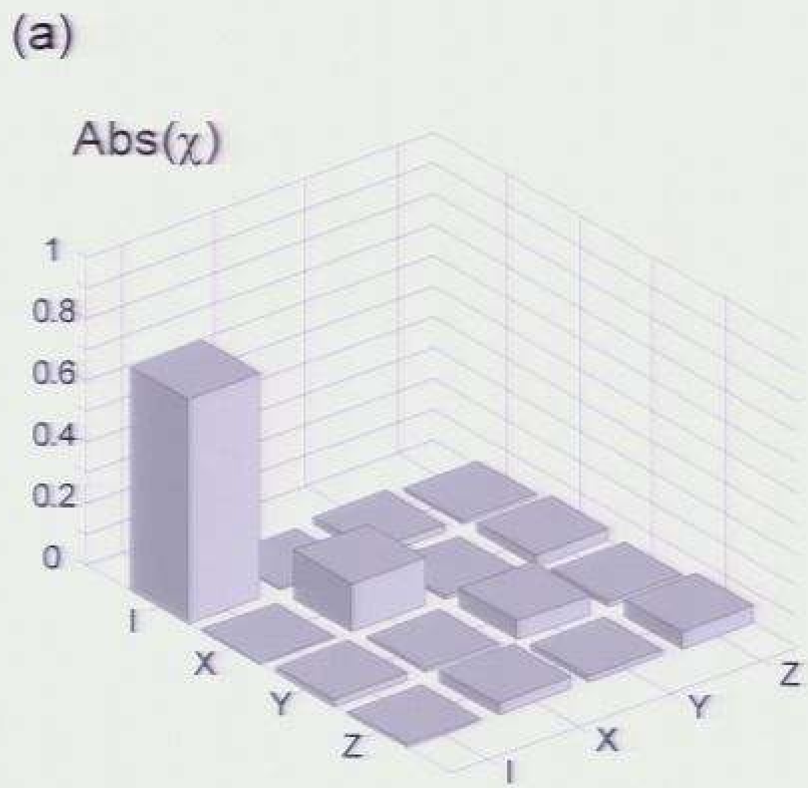
Experimental result	2x single gate result
$F_p = 79(1)\%$	$F_p = 82.8\%$
$\bar{F} = 83.4(8)\%$	$\bar{F} = 86.2\%$

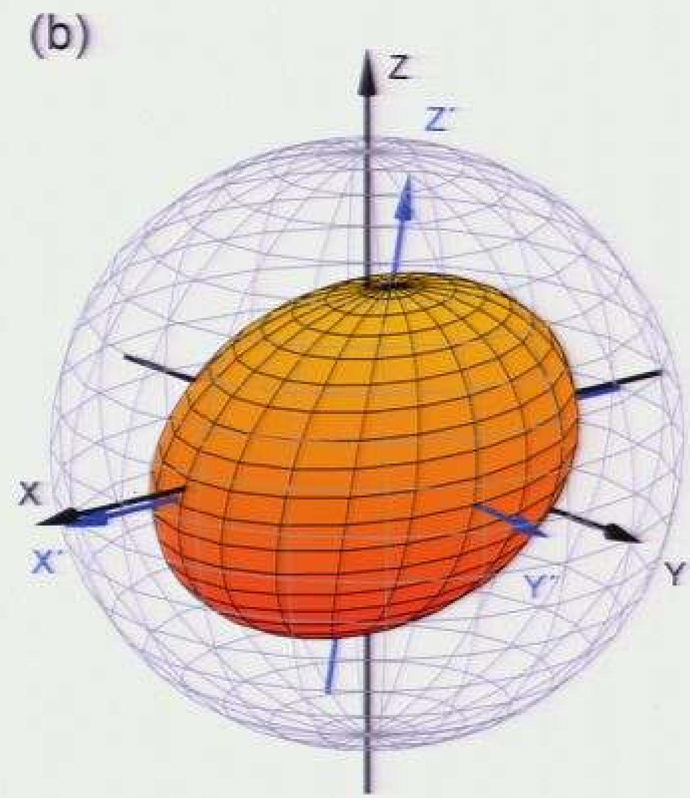
Deterministic quantum teleportation with ions



M. Riebe et al., Nature **429**, 734 (2004)

Process tomography of quantum teleportation





$$F_{proc} = \mathrm{Tr}(\chi_{id}\chi_{tele}) = 73\% \longrightarrow \overline{F} = \frac{2F_{proc}+1}{3} = 83\%$$

M. Riebe et al, New. J. Phys. 9, 211 (2007)

My questions for this workshop:

Process tomography

Does process tomography of gates provide a practical way to predict the fidelity of a quantum algorithm composed of these gates ?

How shall we deal with slowly fluctuating parameters ?

Quantum state tomography

What are reliable algorithms for reconstructing quantum states ?

Are there faster numerical reconstruction algorithms ?

Which approach should be taken when the state leaks out of the Hilbert space of interest ?