Title: A new approach to Quantum Estimation: Theory and Applications

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Abstract: A new approach to Quantum Estimation Theory will be introduced, based on the novel notions of \'quantum comb\' and \'quantum tester\', which generalize the customary notions of \'channel\' and \'POVM\' [PRL 101 060401 (2008)]. The new approach opens completely new possibilities of optimization in Quantum Estimation, beyond the classic approach of Helstrom and Holevo. Using comb theory it is possible to optimize the input-output arrangement of the black boxes for estimation with many uses. In this way it is possible to prove equivalence of arrangements for optimal estimation of unitaries, and the need of memory assisted protocols for for optimal discrimination of memory channels [arXive:0806.1172]. This also leads to a new notion of distance for channels with memory. Using the theory of quantum testers the optimal tomography schemes are derived---both state and for channel tomography---for arbitrary prior ensemble and arbitrary representation [arXive:0803.3237]. Finally, using the method of generalized pseudo-inverse for optimal data-processing [PRL. 98 020403 (2007)], we derived two improved data-processing for quantum tomography: Adaptive Bayesian and Frequentist [arXive:0807.5058].

A new approach to Quantum Estimation: Theory and Applications

Giacomo Mauro D'Ariano

QUIT group, University of Pavia

Quantum Estimation: Theory and Practice

August 25-30, Perimeter Institute, Waterloo, Canada

QUIT Group in Pavia


Chiara Macchiavello


Lorenzo Maccone


Massimiliano Sacchi


Paolo Perinotti


Giulio Chiribella


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Stefano Facchini


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Theory of Quantum Combs in collaboration with

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Application to Optimal Quantum Tomography in collaboration with

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  - Optimal discrimination of unitary operators and quantum memory channels
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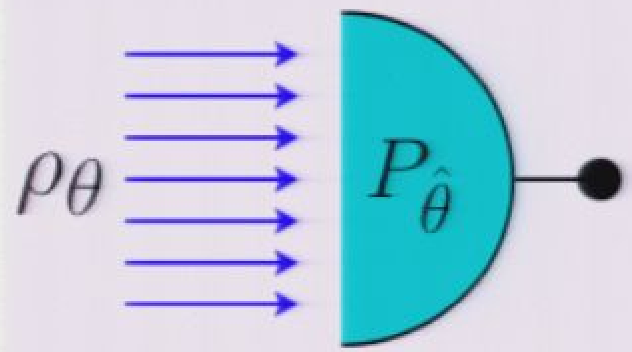
  - Cloning of processes, quantum learning, quantum strategies and algorithms, ...

Helstrom

Quantum Estimation Theory

Quantum state ρ_θ parameterized by θ

Problem: estimate θ optimally according to the cost function $C(\theta, \hat{\theta})$



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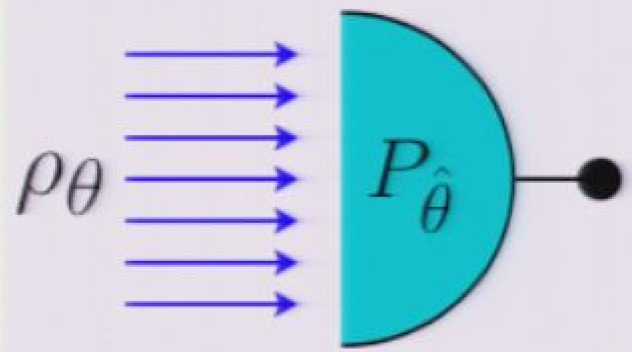

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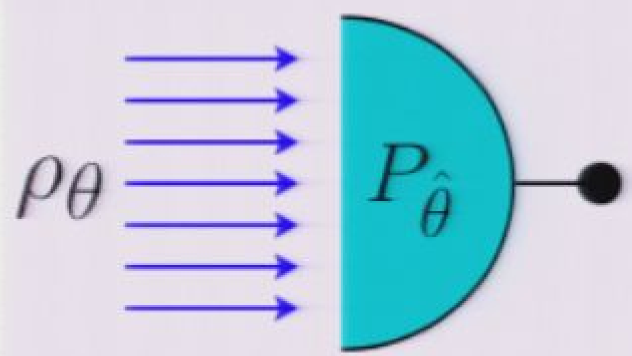

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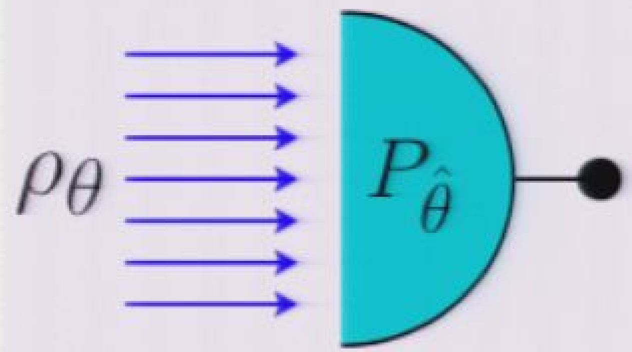
$$\theta \Longrightarrow \rho_\theta = U_\theta \rho U_\theta^\dagger$$



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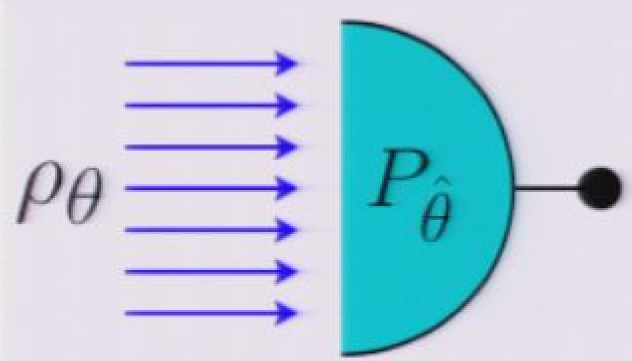

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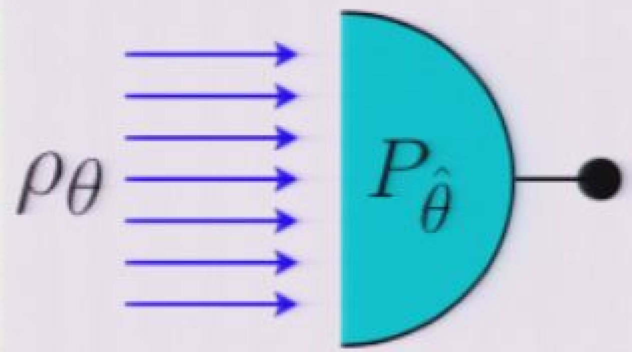

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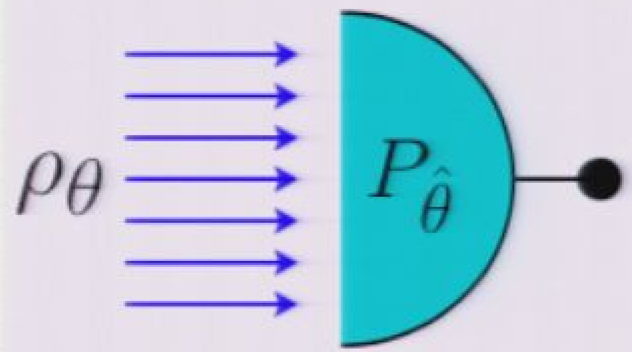

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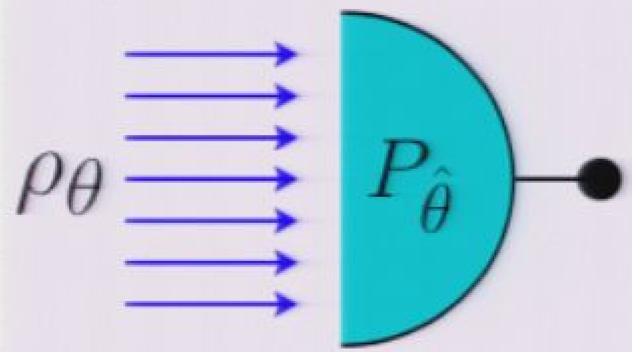

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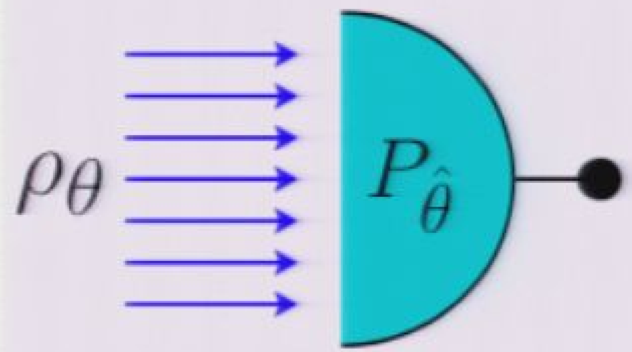

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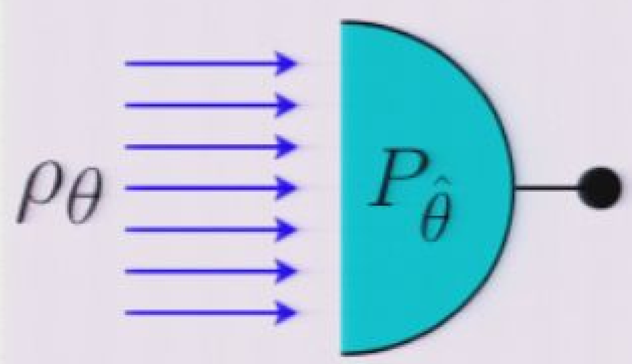

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Interesting situation: the parameter to be estimated is encoded on a transformation---not on the state!

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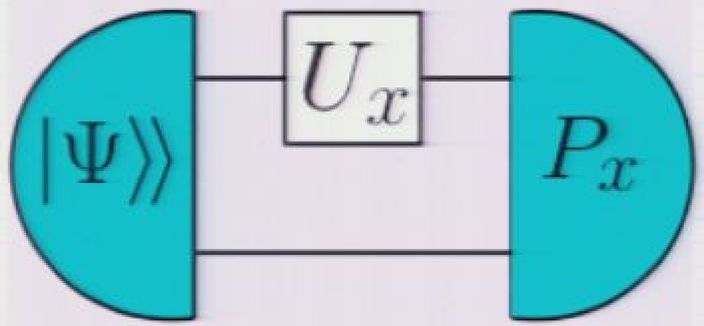

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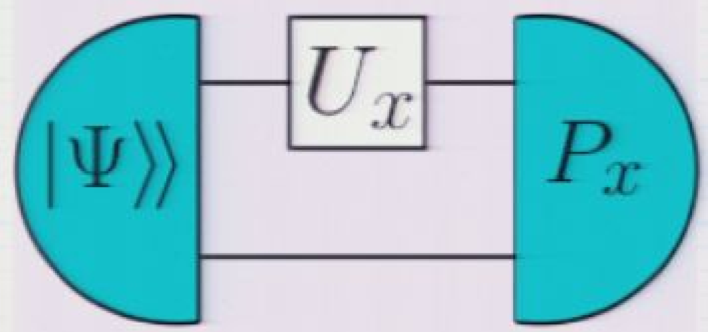

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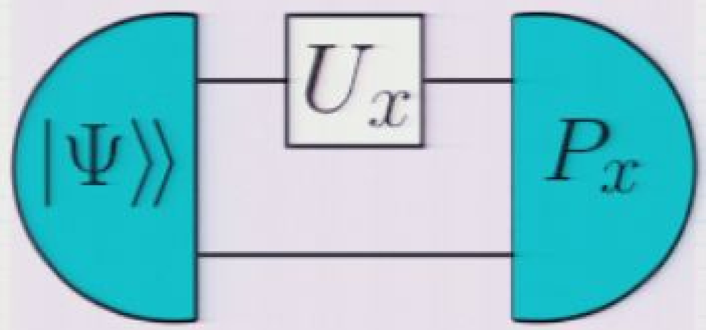
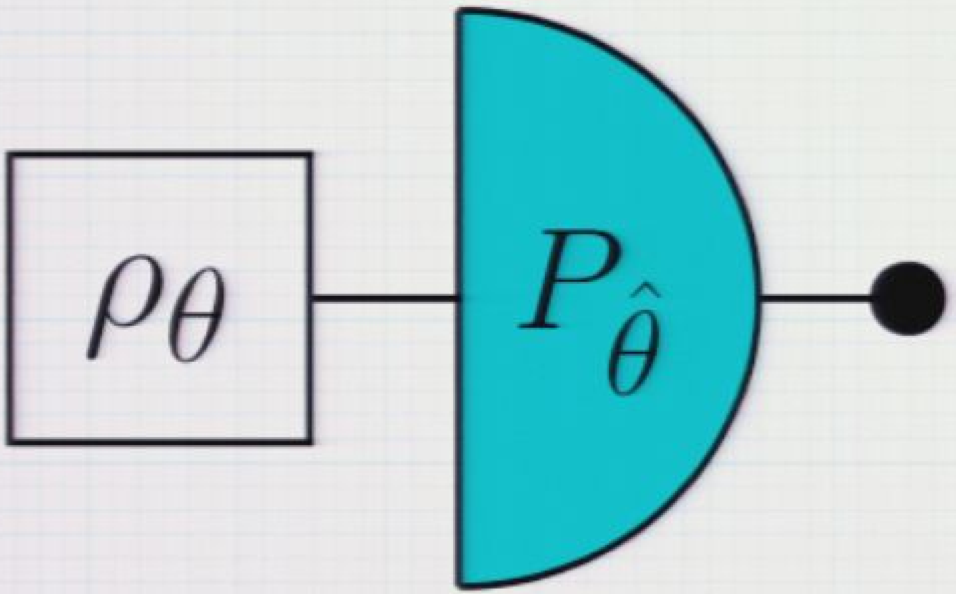


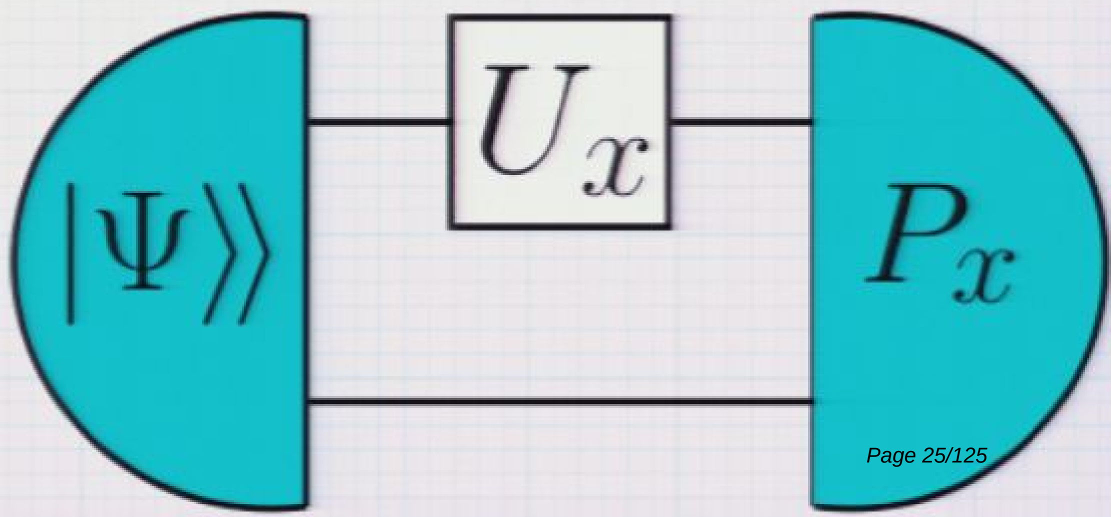
With the phase we were lucky!

Quantum Estimation Theory



Quantum Estimation Theory

New scheme



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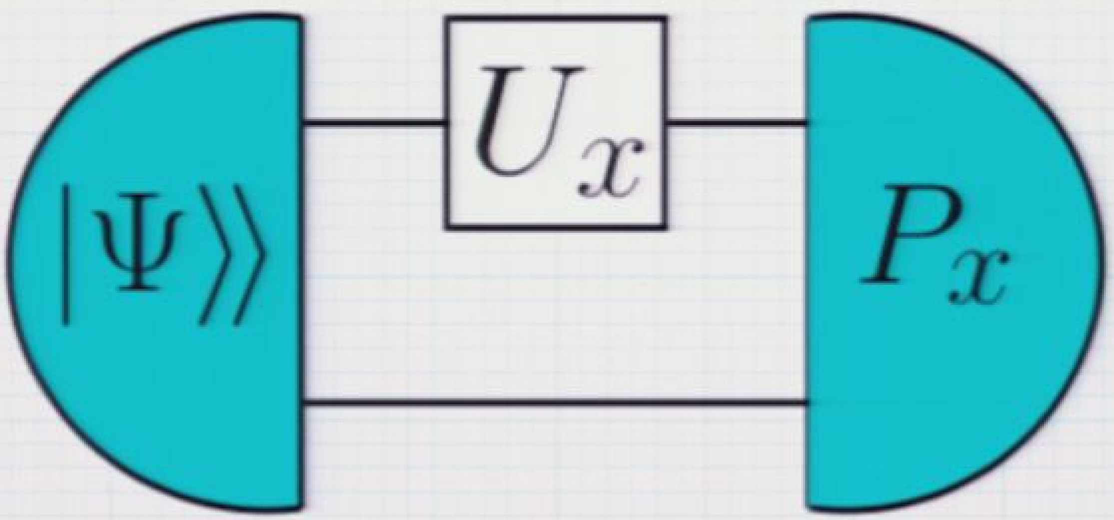
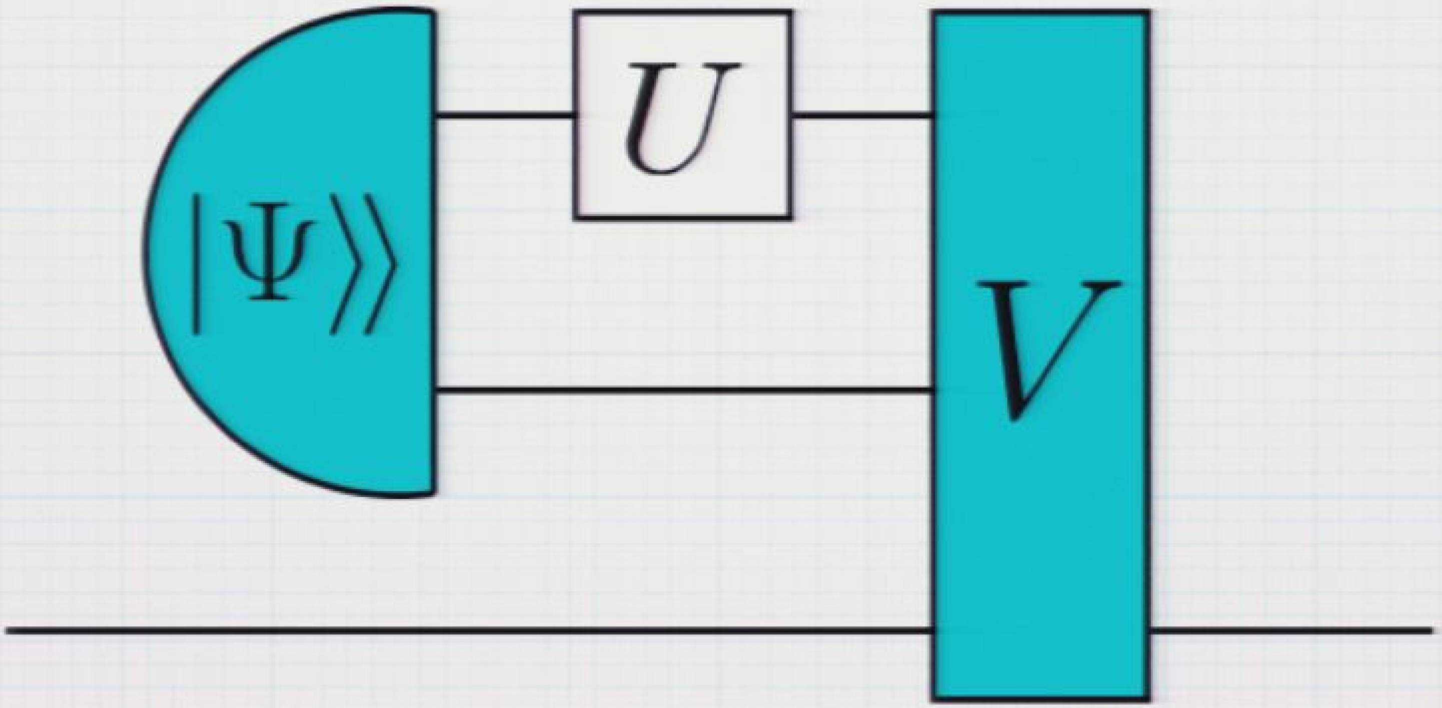


Quantum Feedback



- quantum feedback: perform a transformation $\mathscr{T}_U$ on a system which depends on an unknown unitary transformation U occurring on a (generally different) system (e.g. reference-frames realignment).

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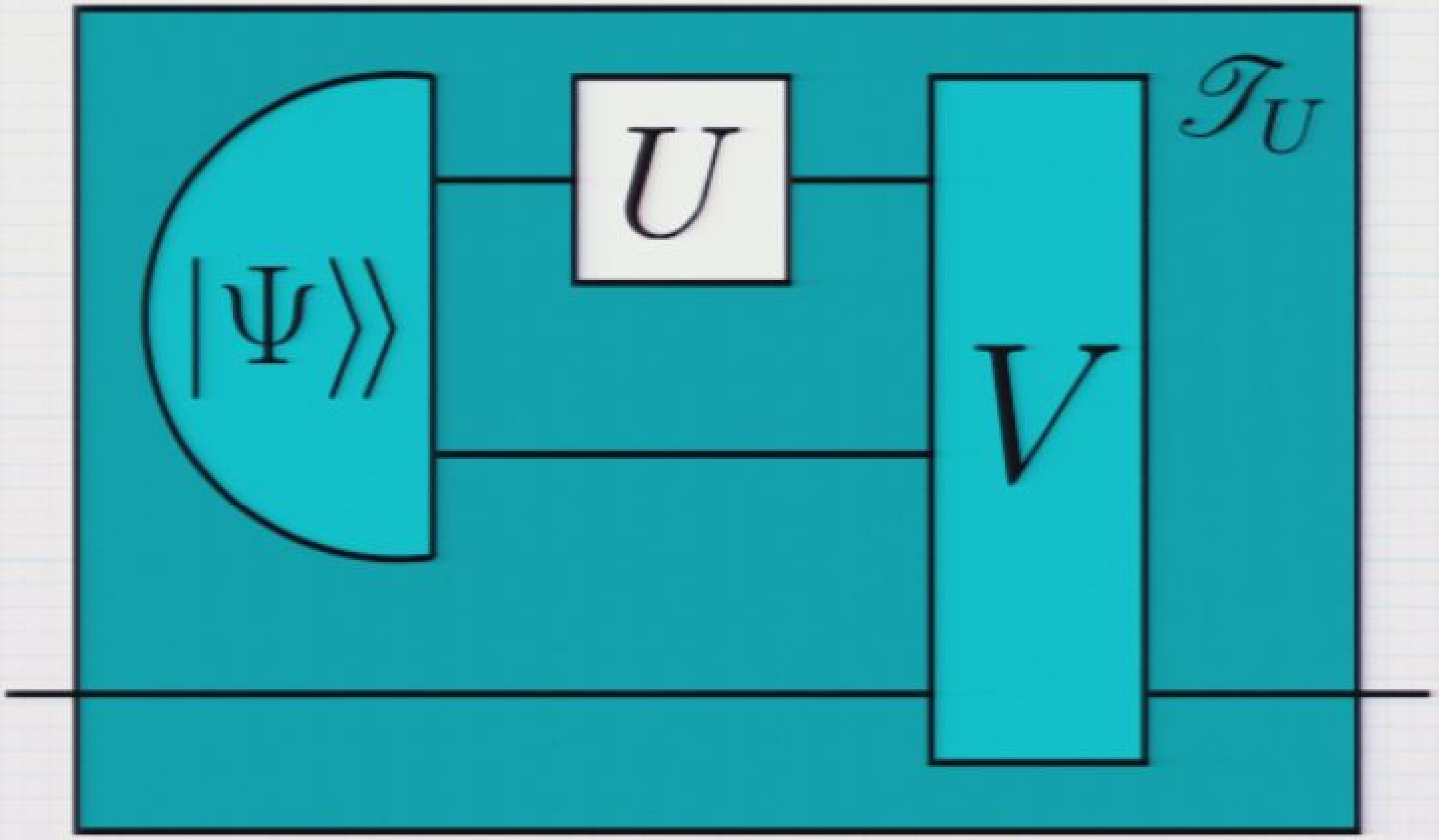

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QUit quantum information theory group

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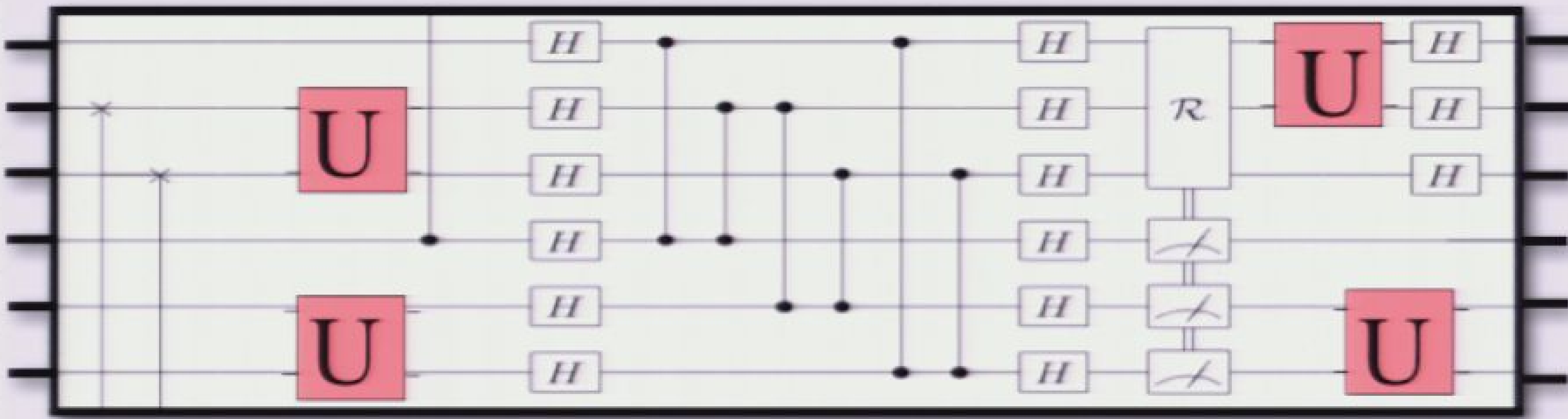
Use a Quantum Board!



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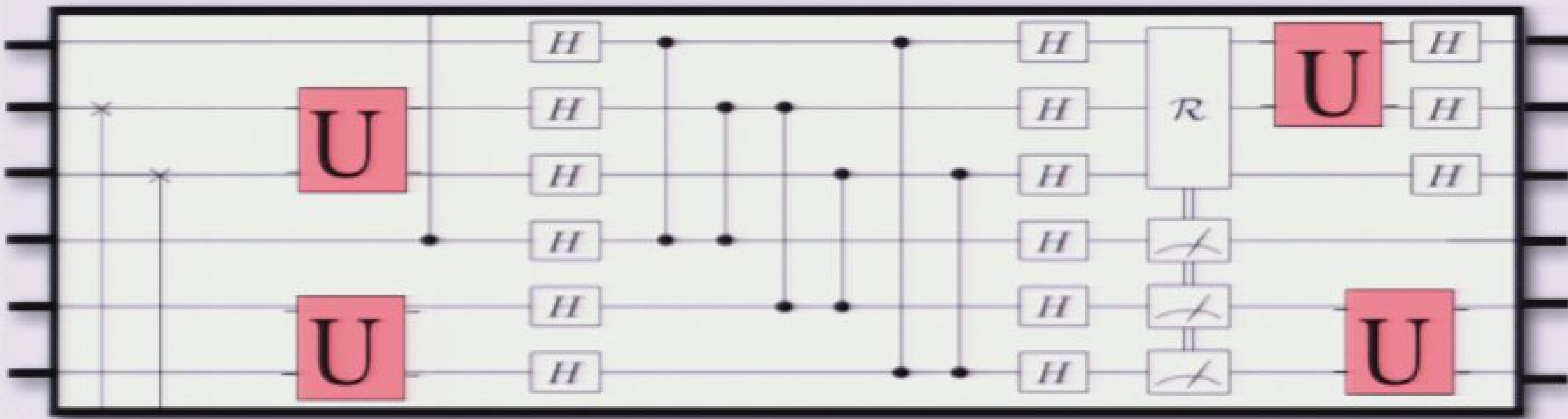

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Quantum circuit board: input and output are themselves circuits that are slotted into the board.

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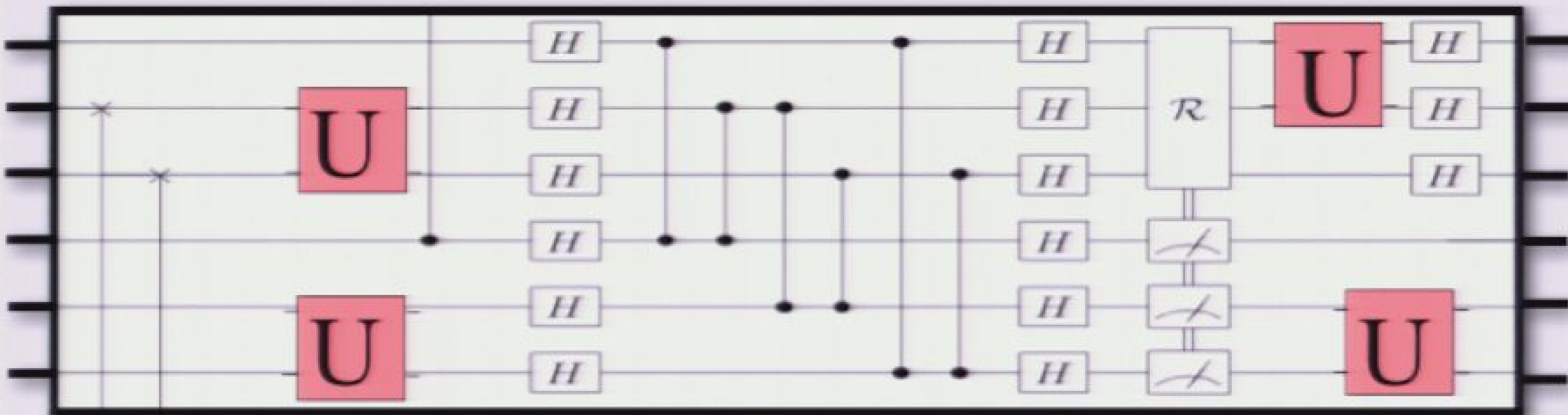

Formulation of the problem:

Optimize the quantum circuit board for all possible dispositions of the slots

It looks a difficult problem ...

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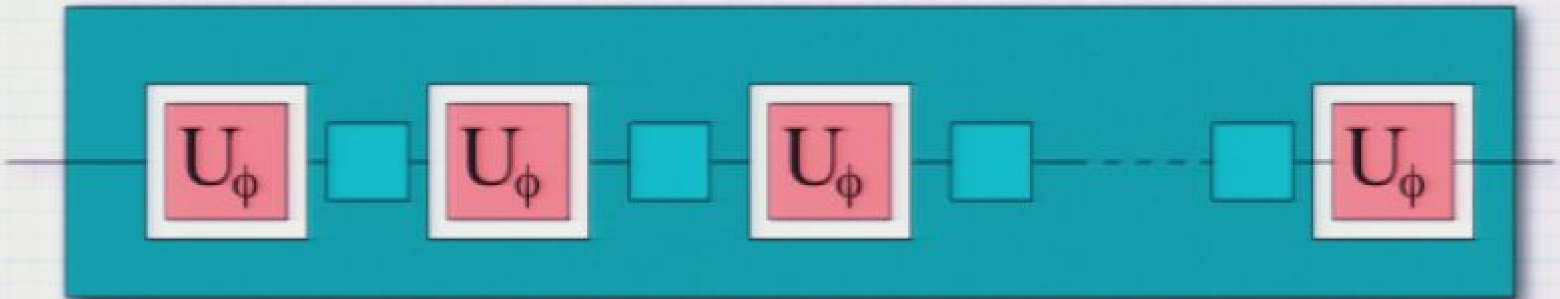
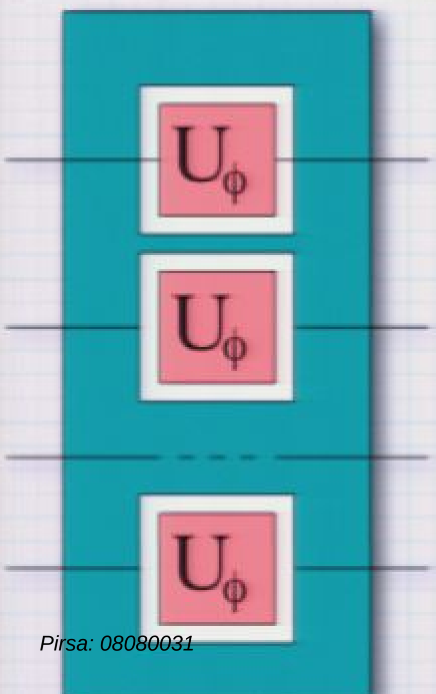


In sequence intercalated by some unitary?



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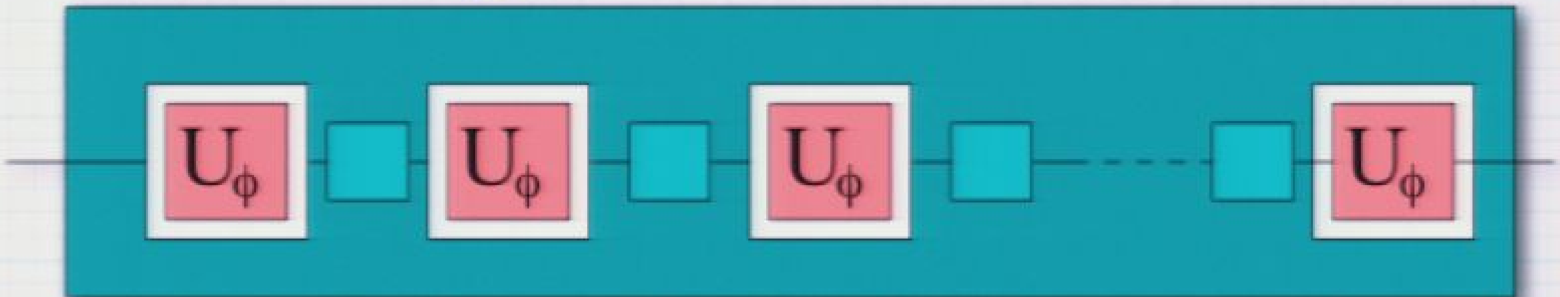

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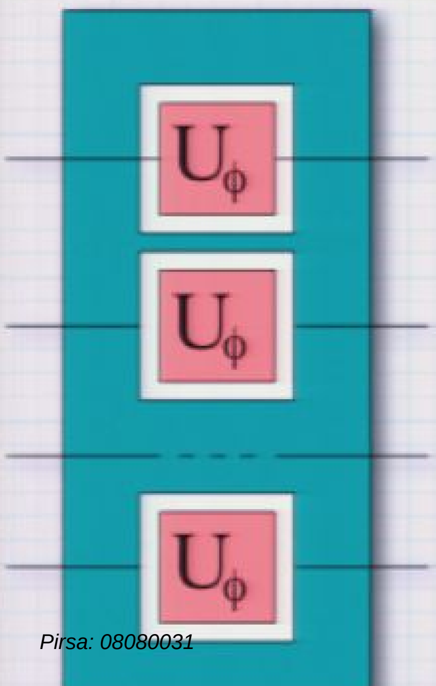

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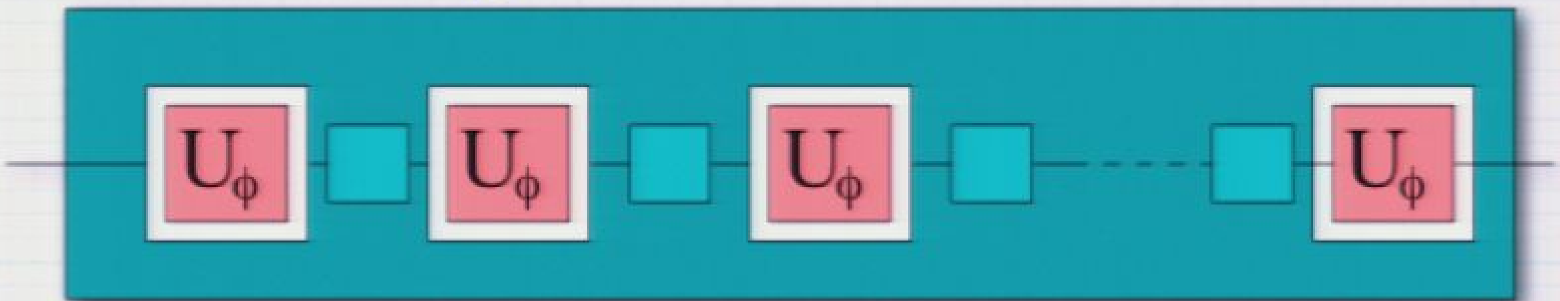

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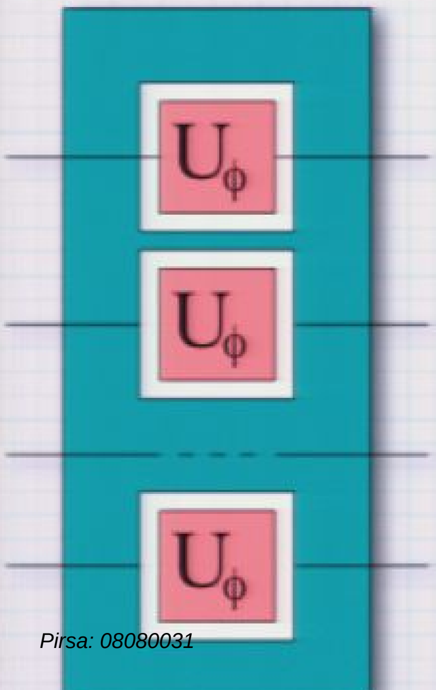

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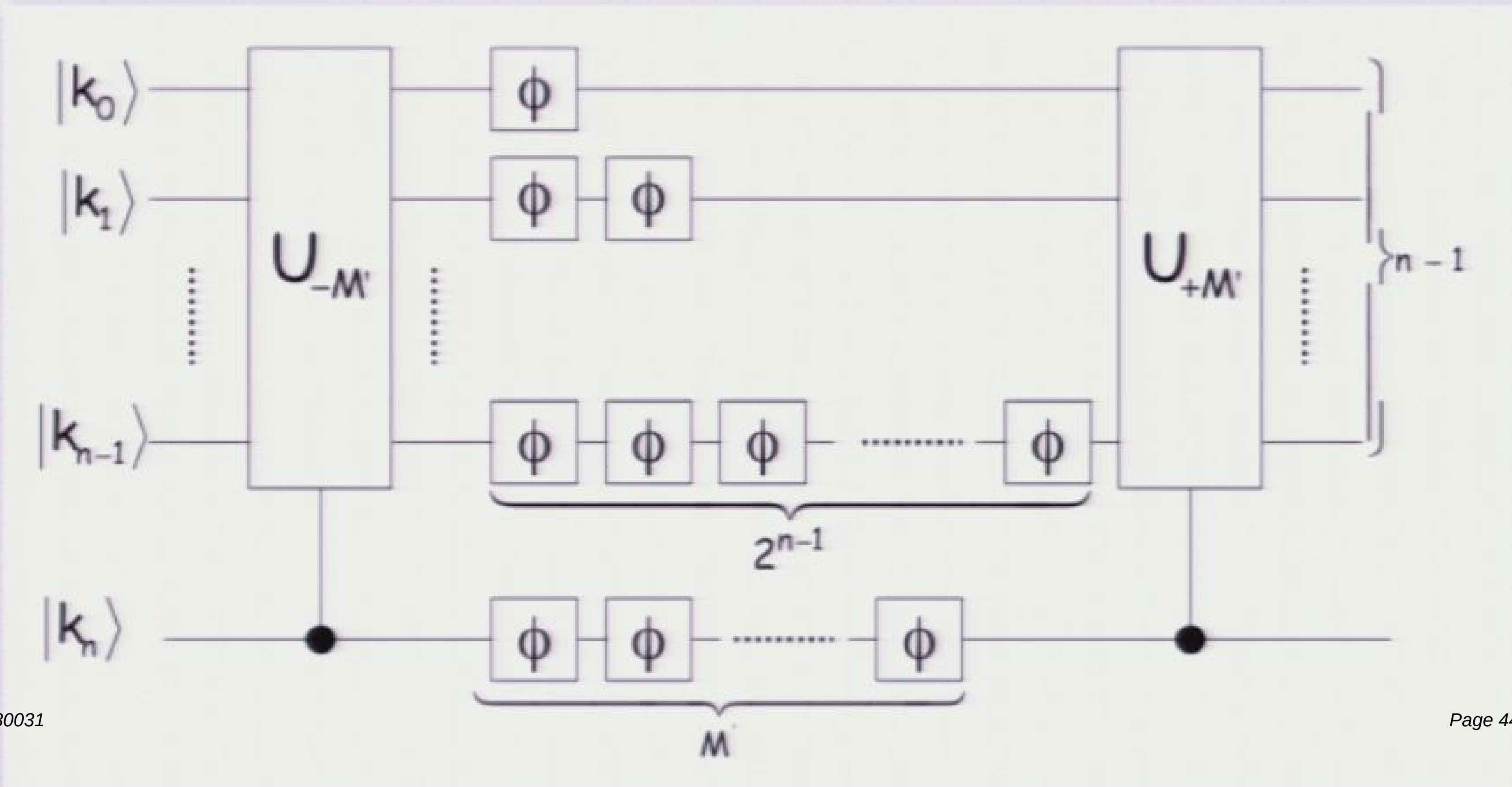

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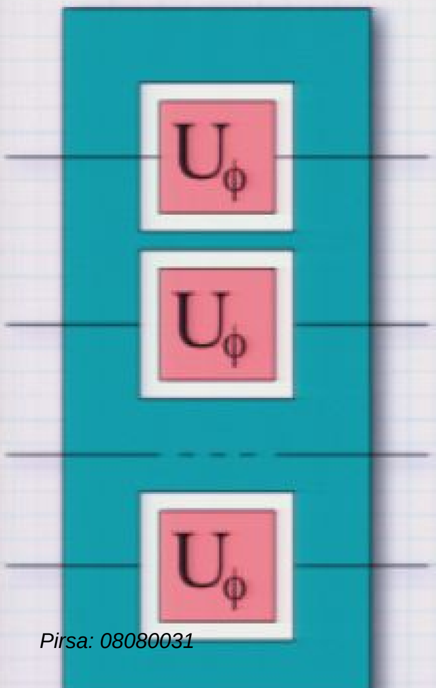

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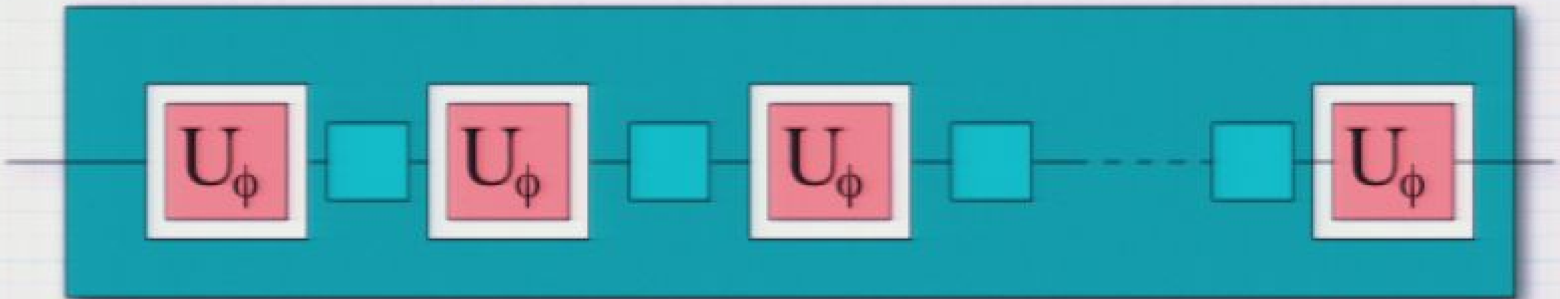

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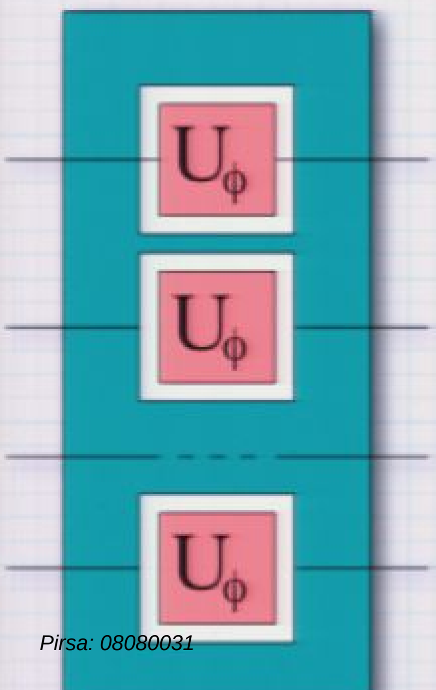

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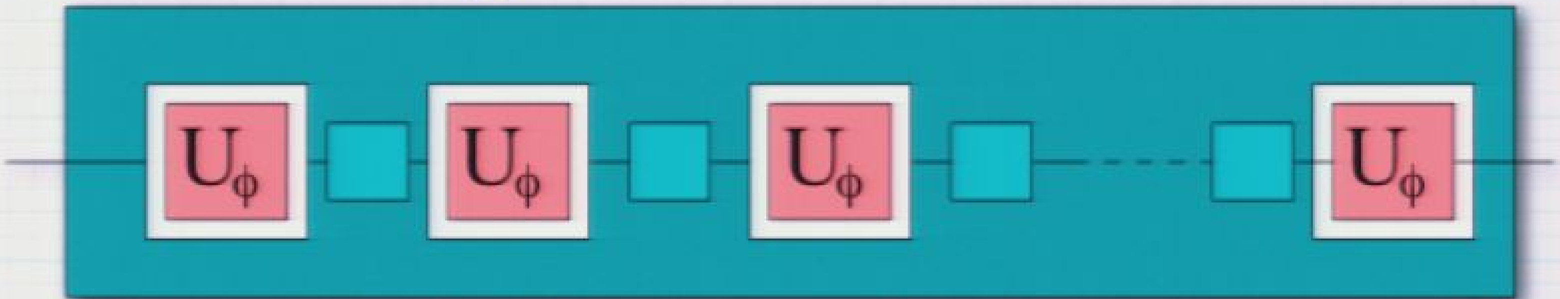

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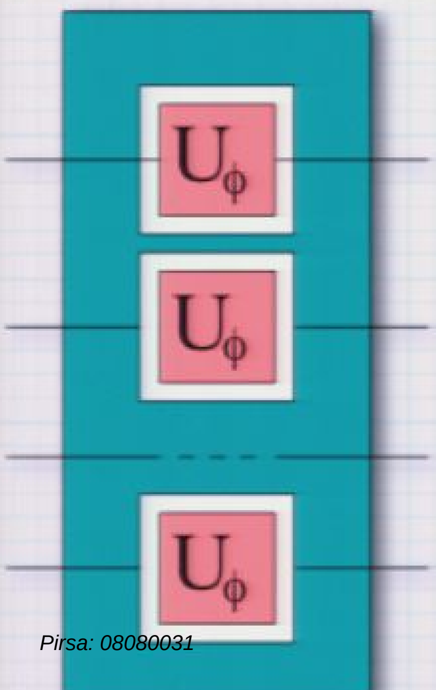

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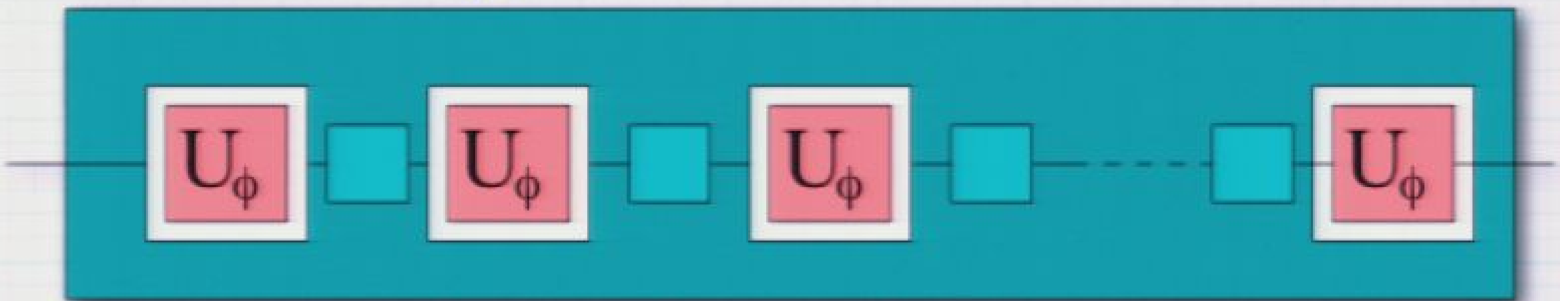

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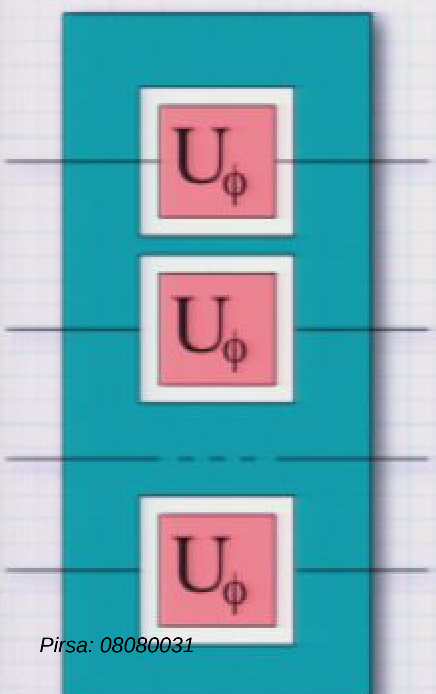

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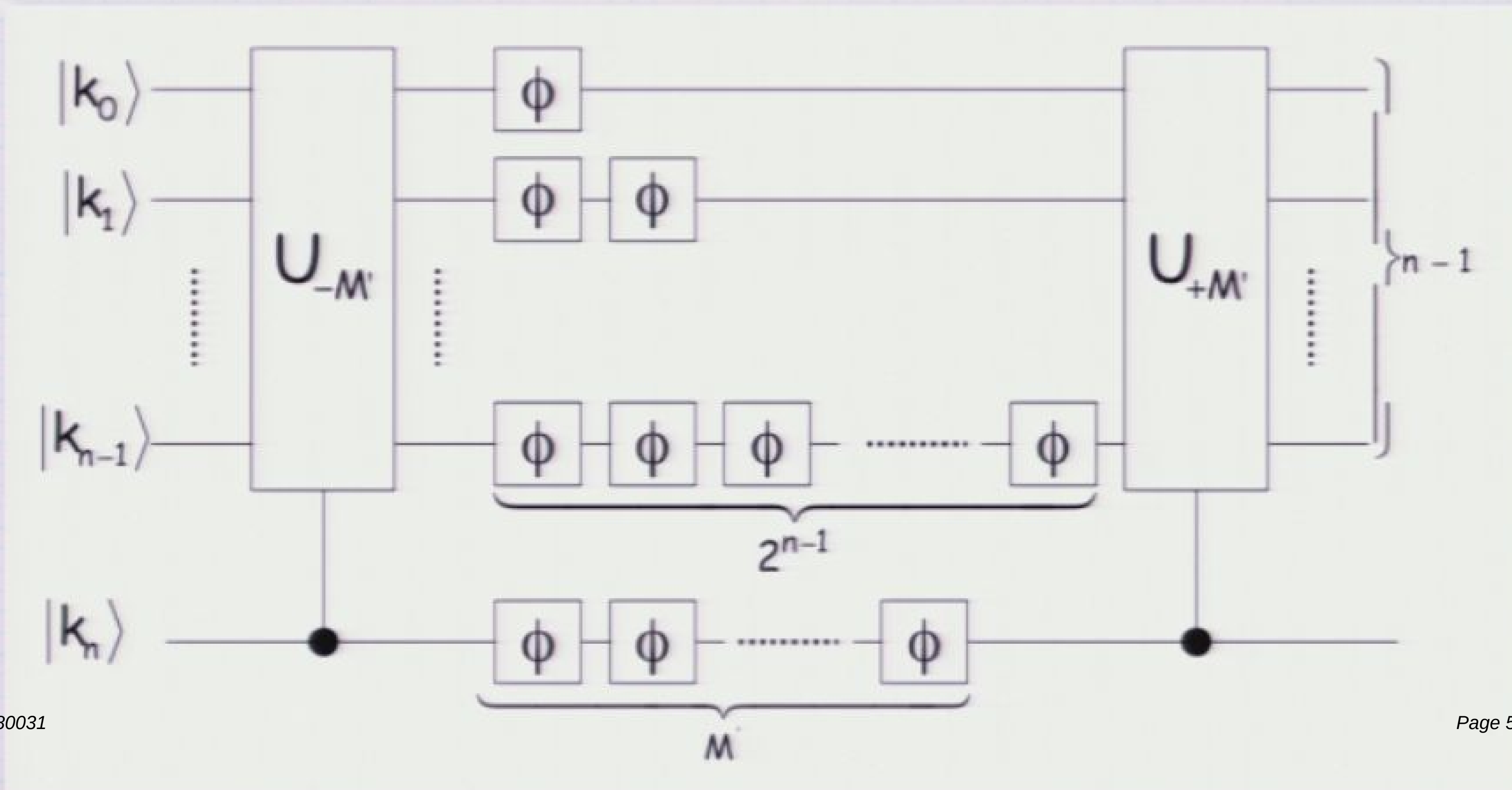

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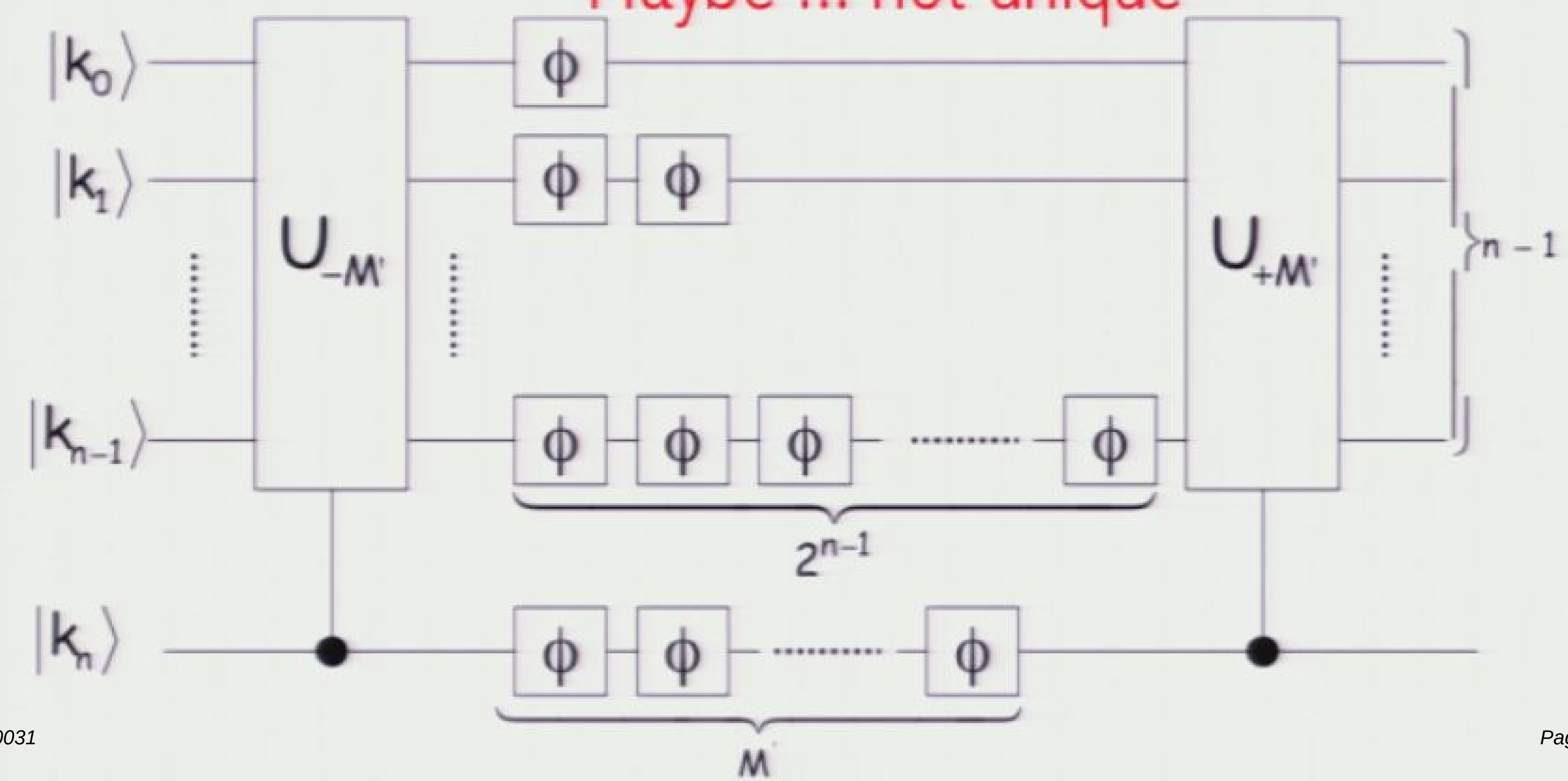

QUit quantum information theory group

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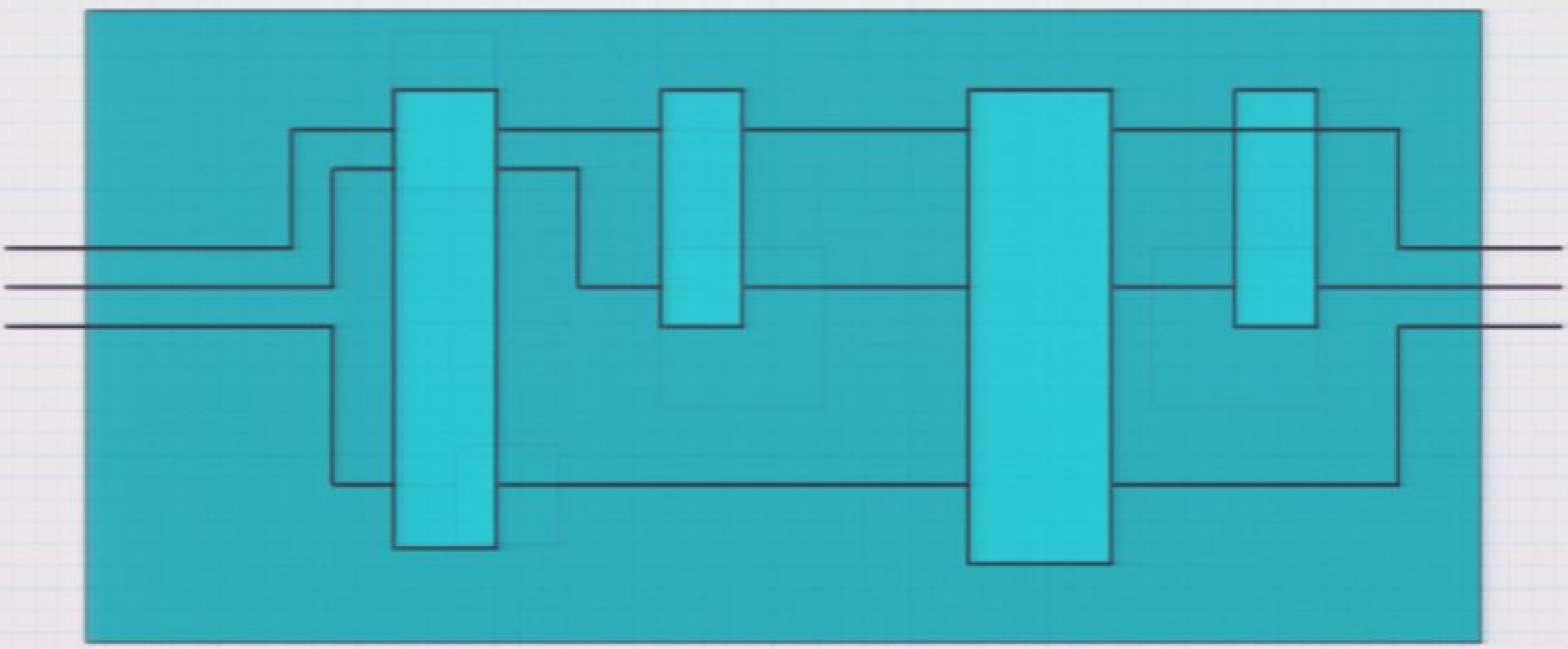

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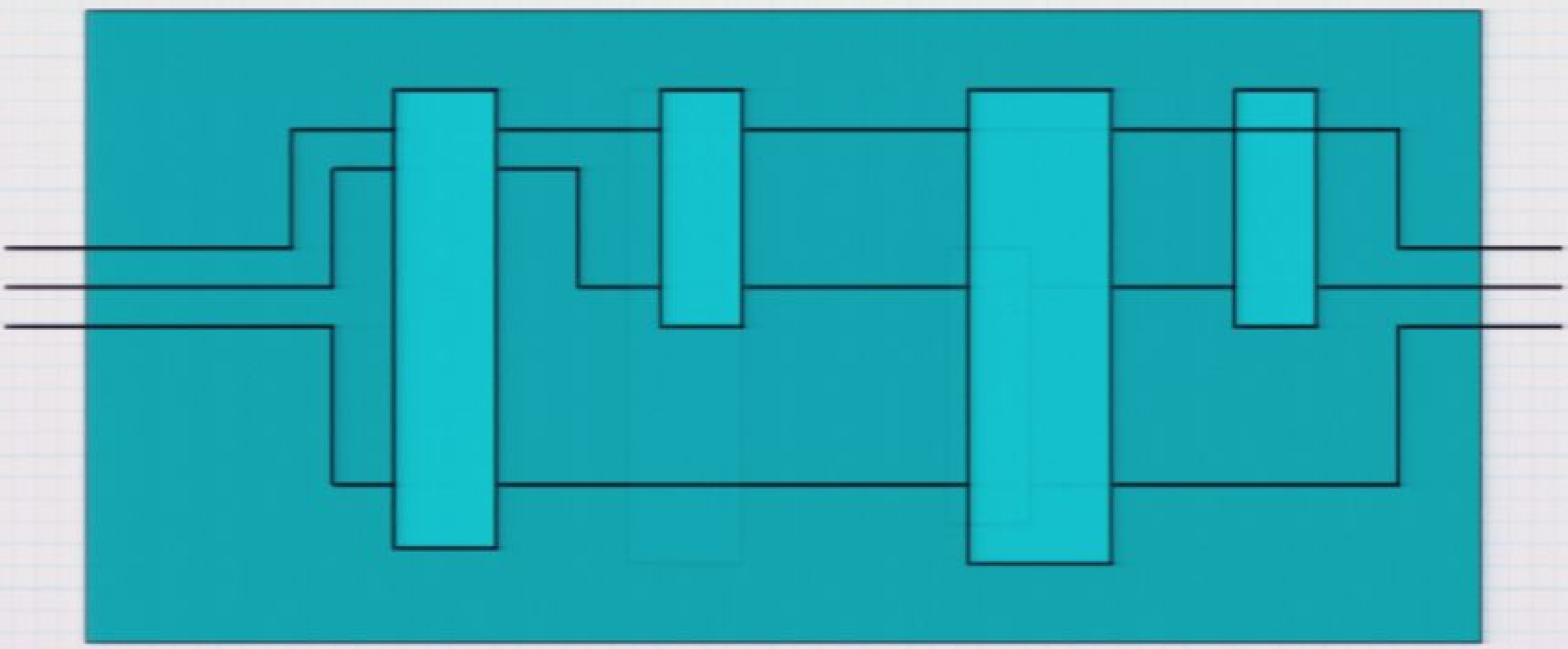

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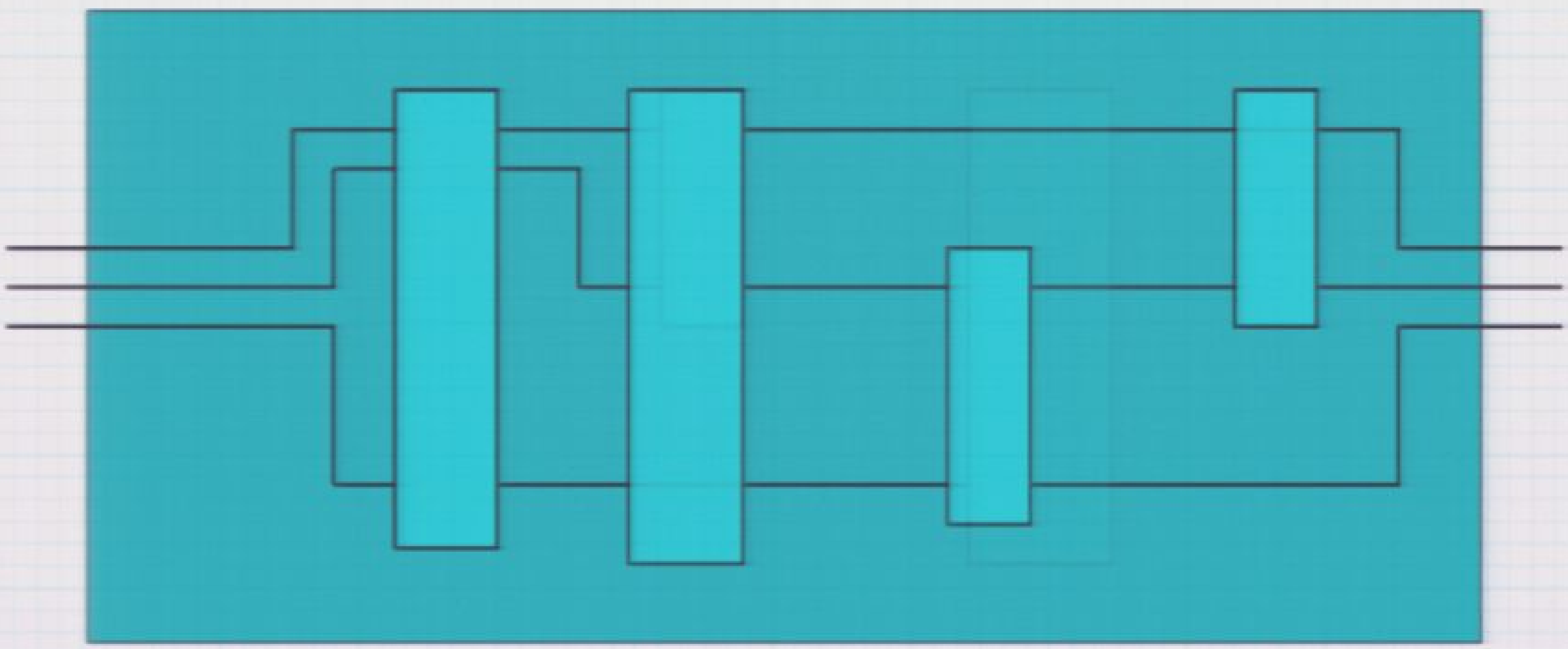

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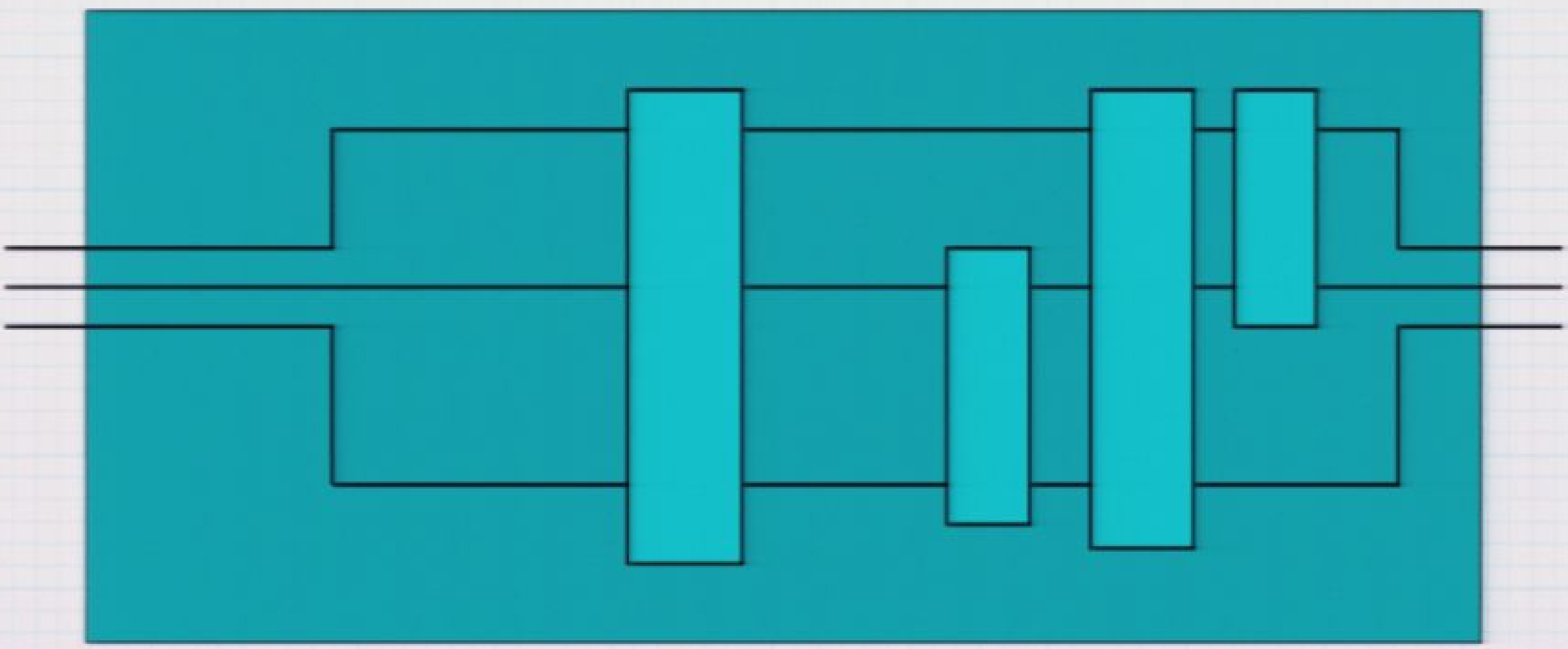

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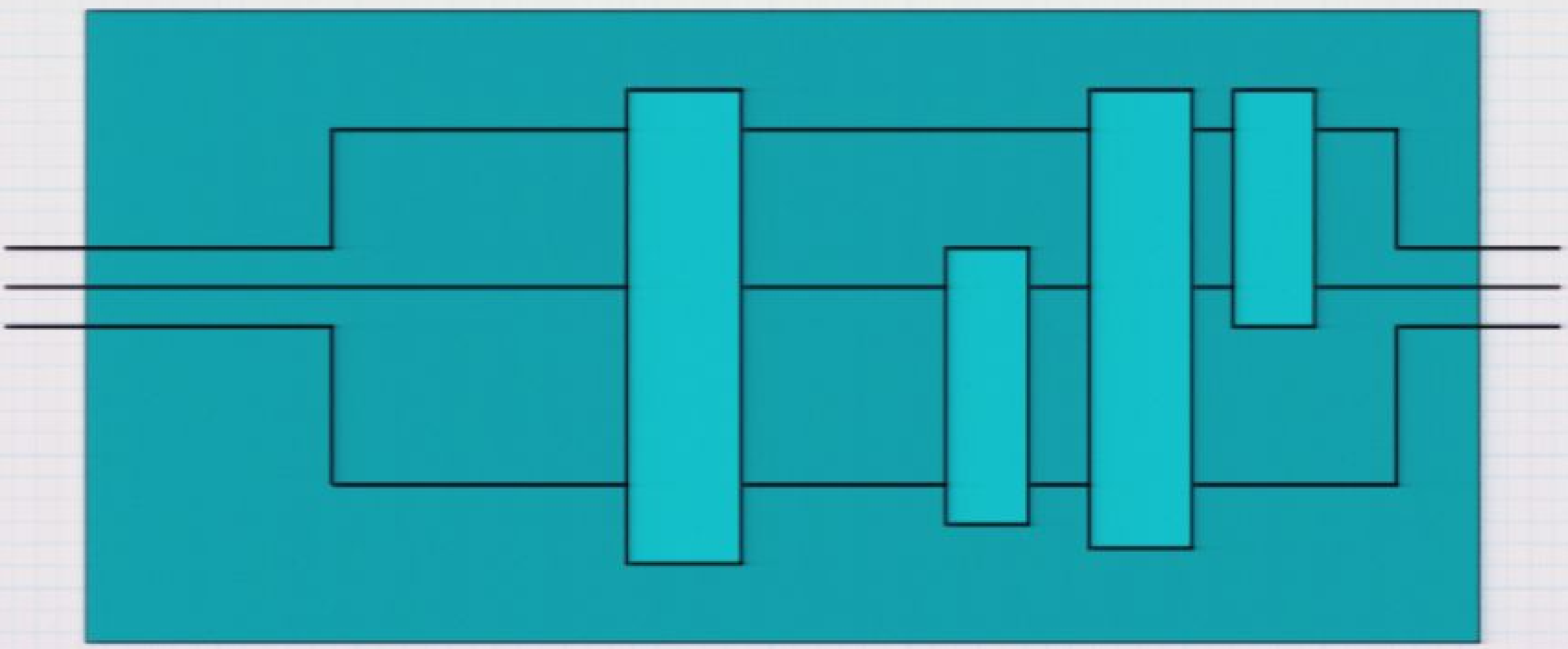
For a channel the input and the output are states



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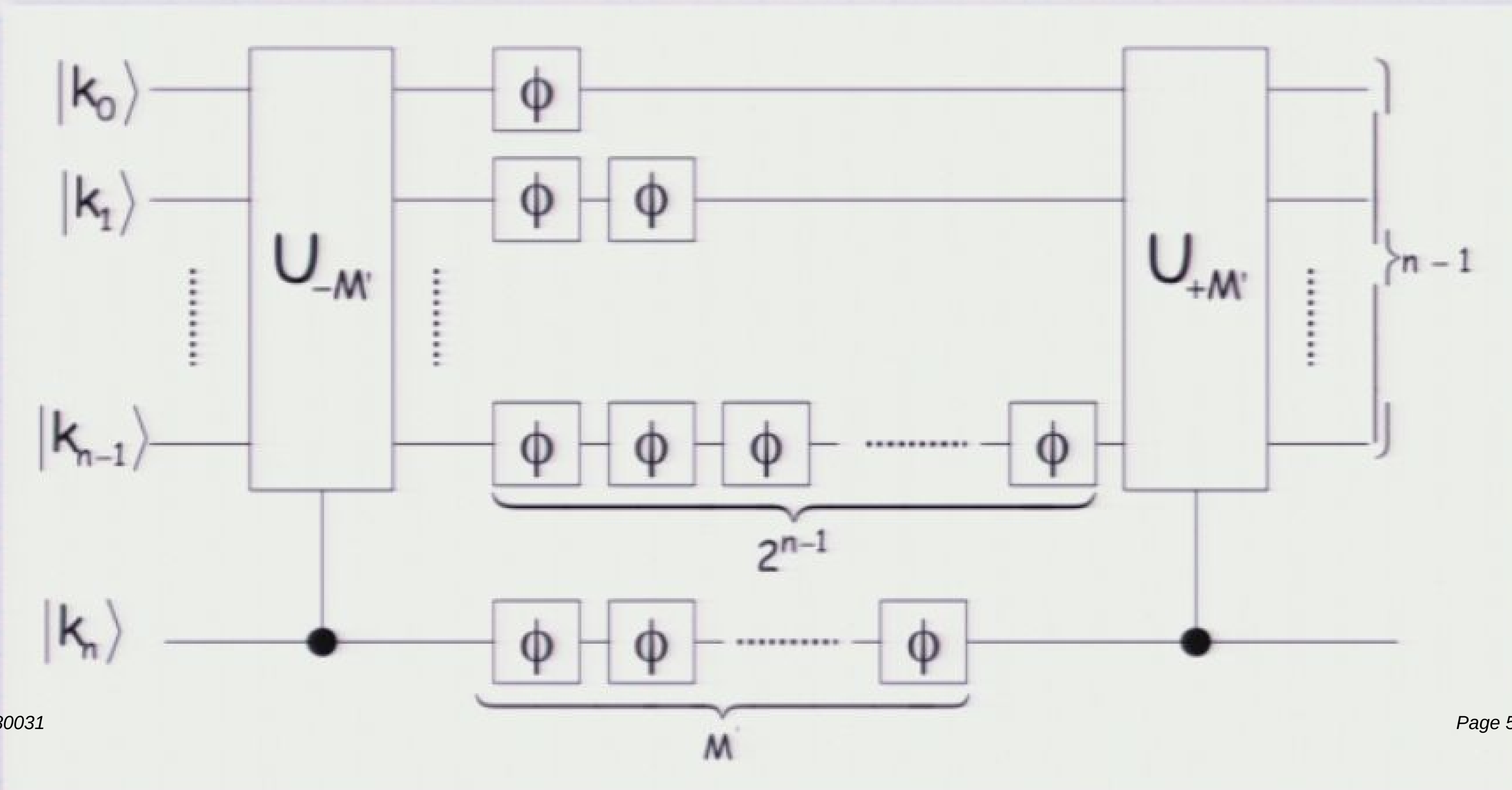

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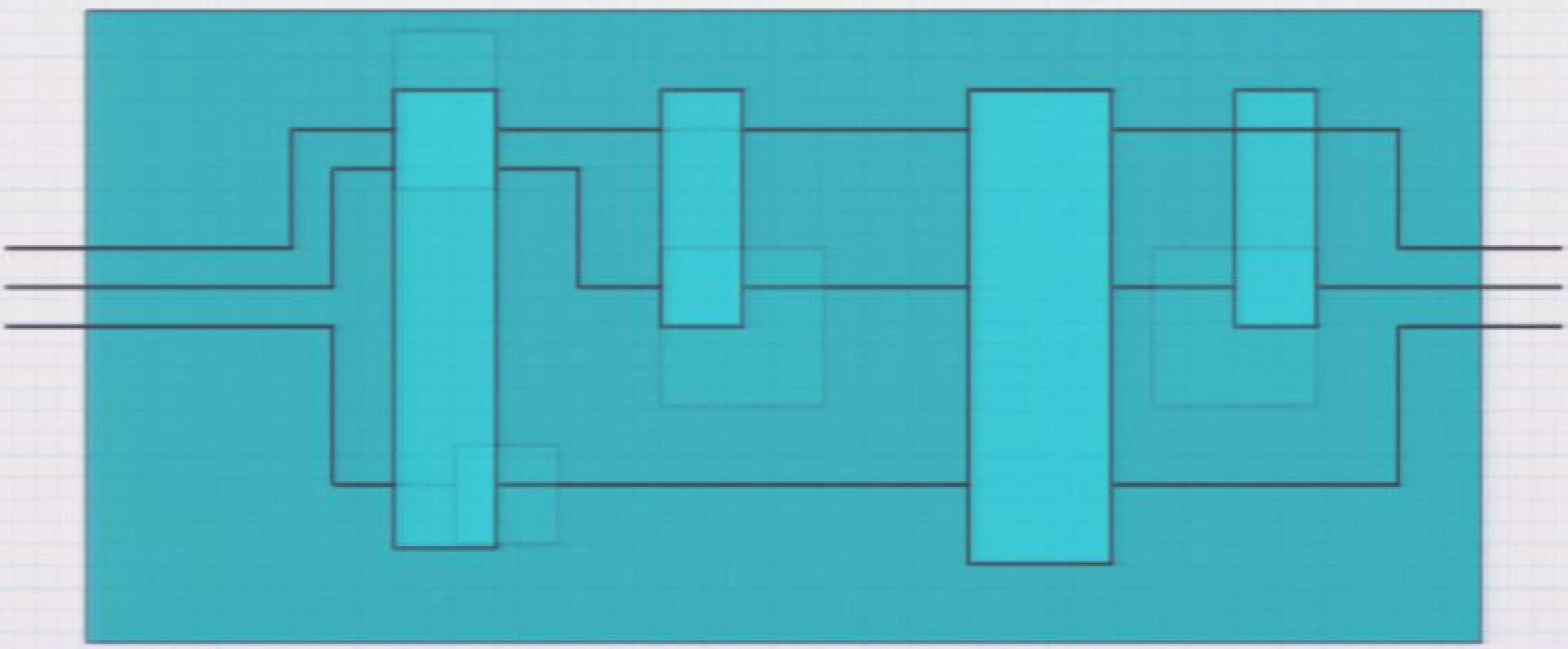

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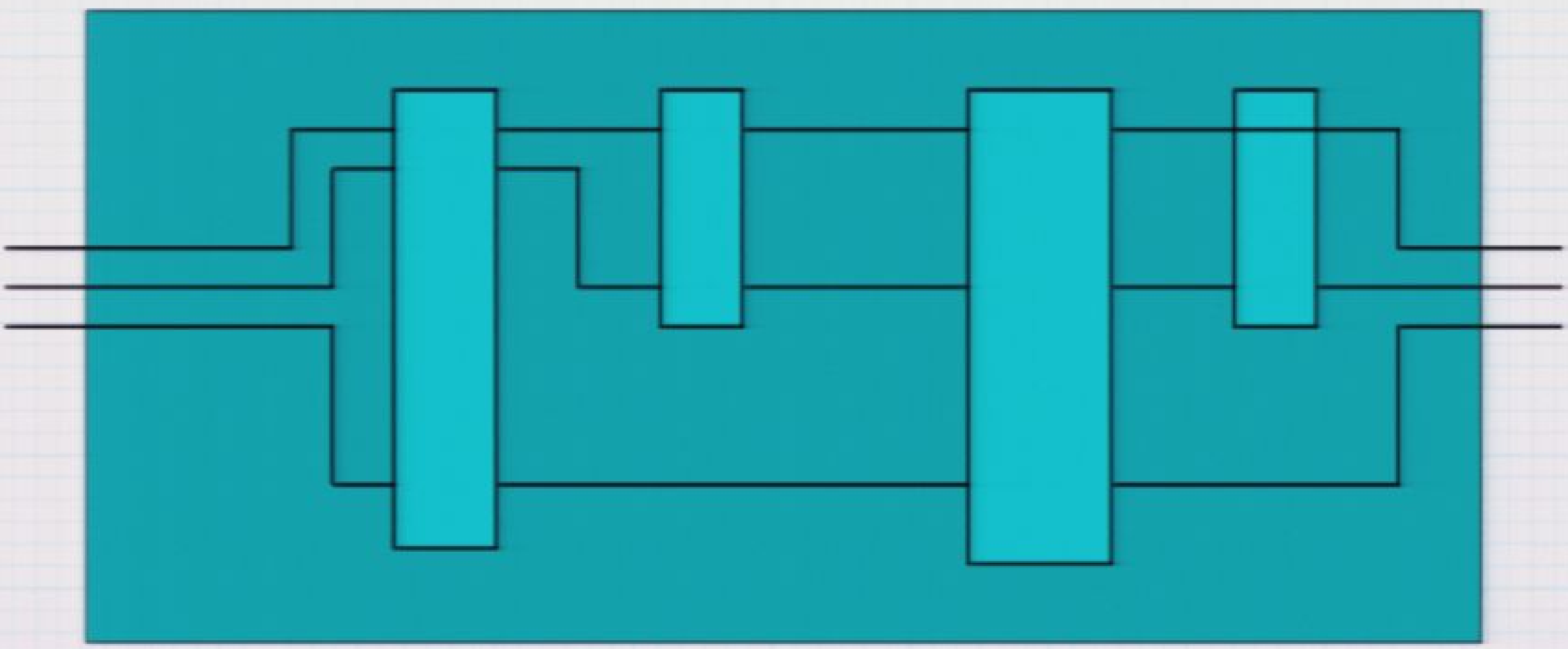

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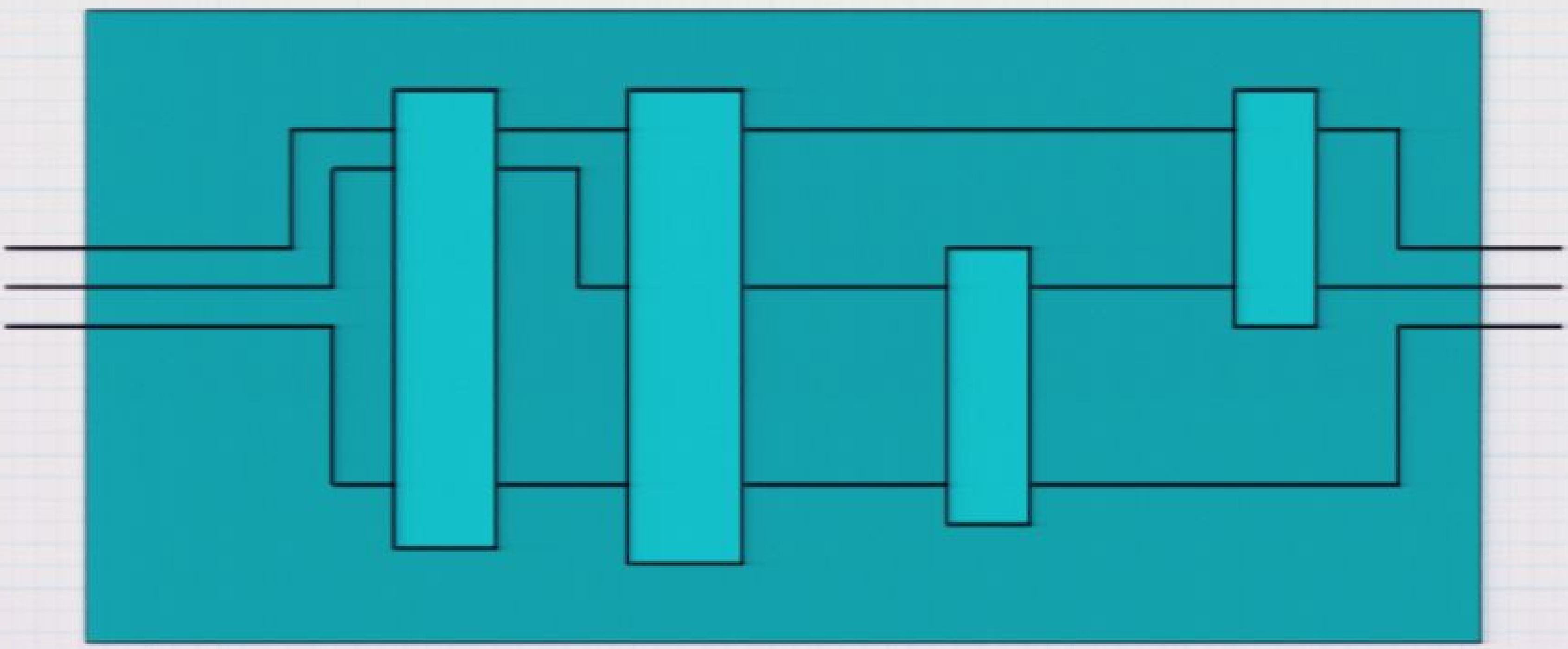

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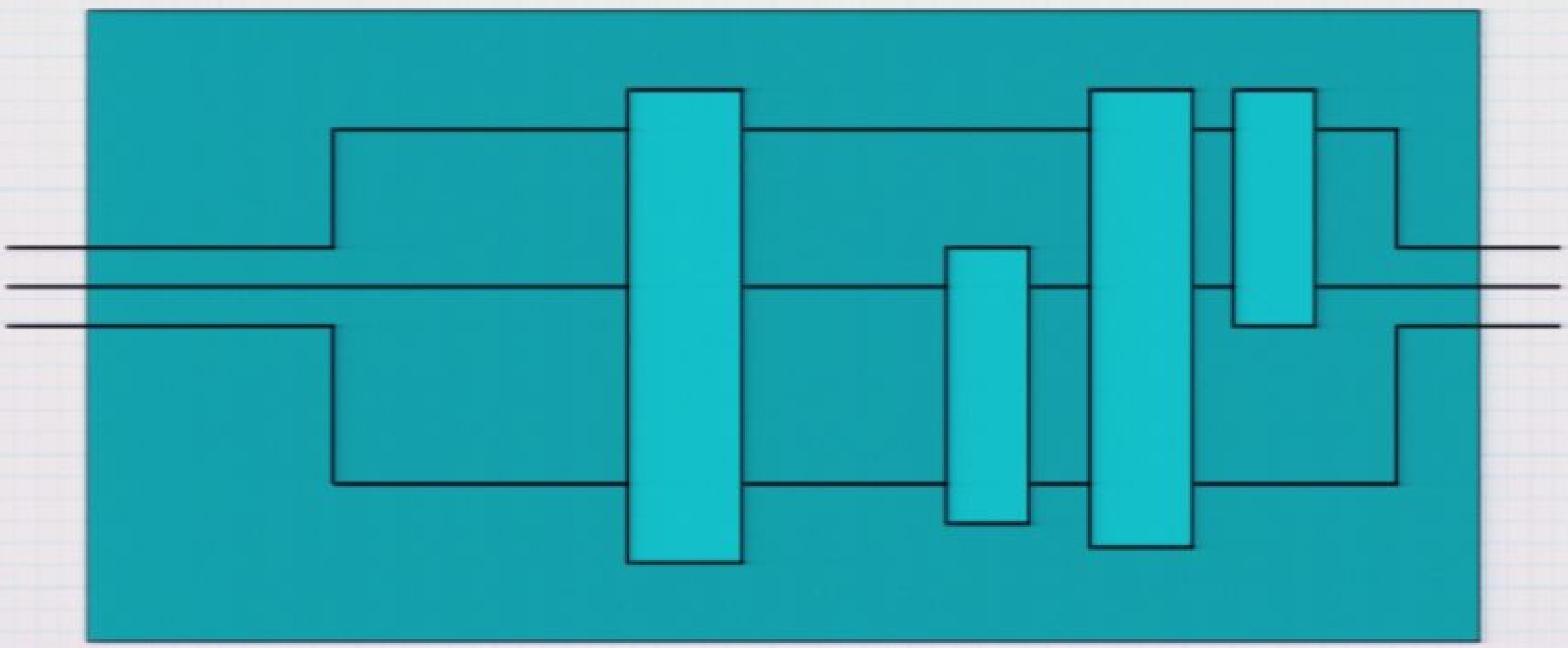

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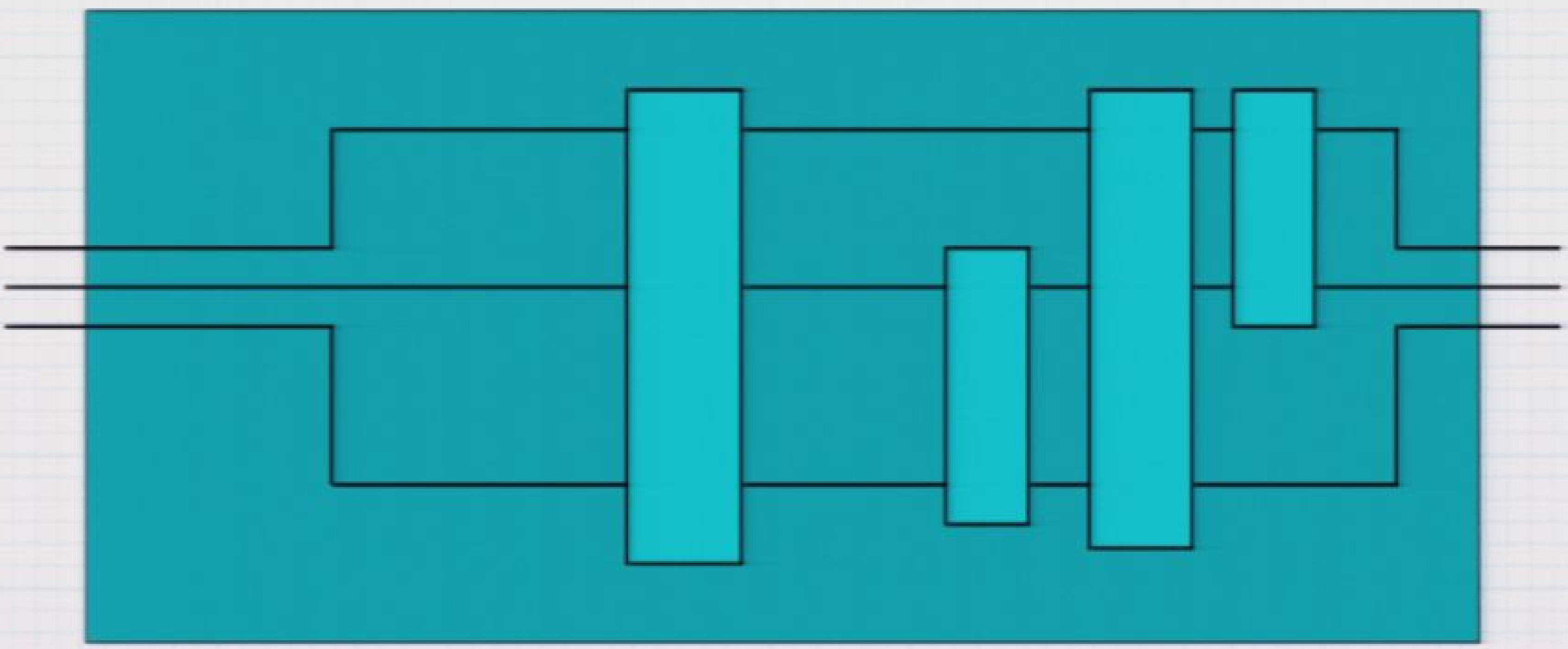
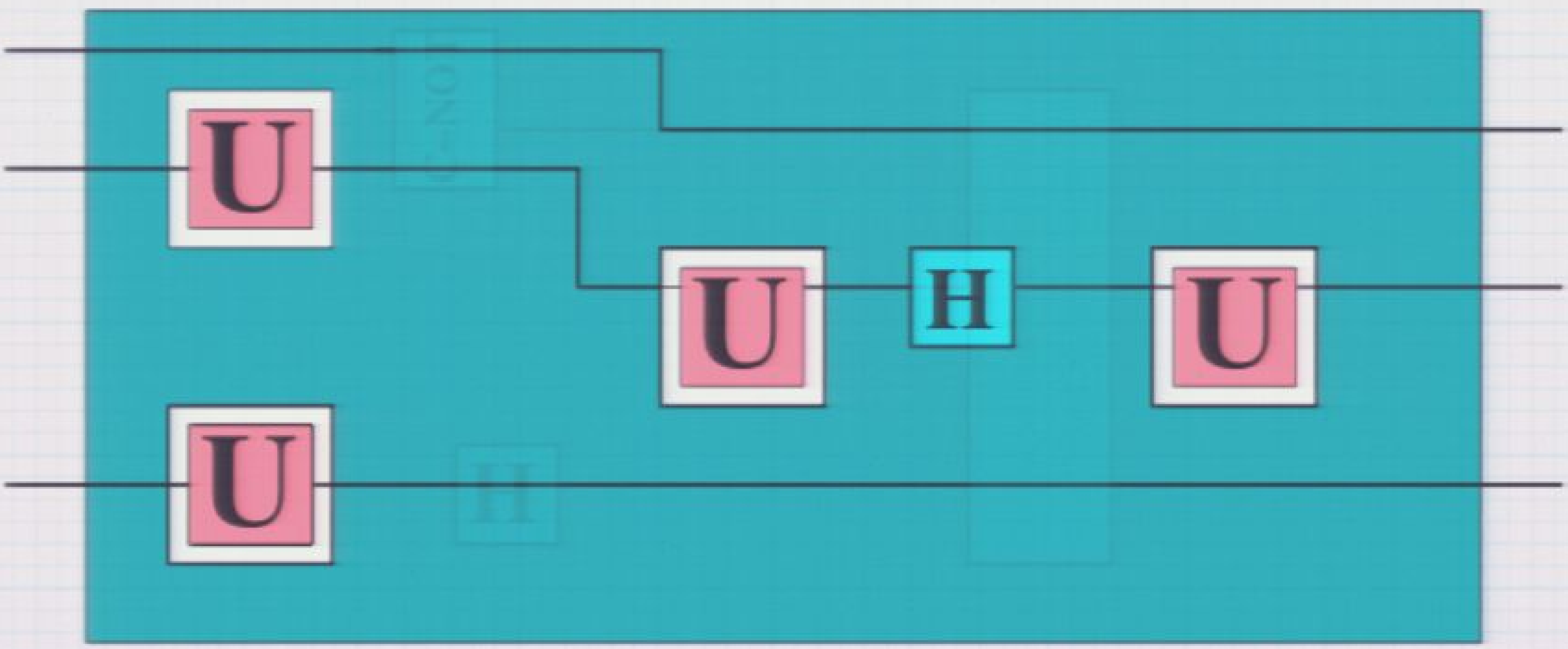
For a channel the input and the output are states



Quantum Board

Equivalence class of quantum circuits boards performing the same overall input-output transformation ...



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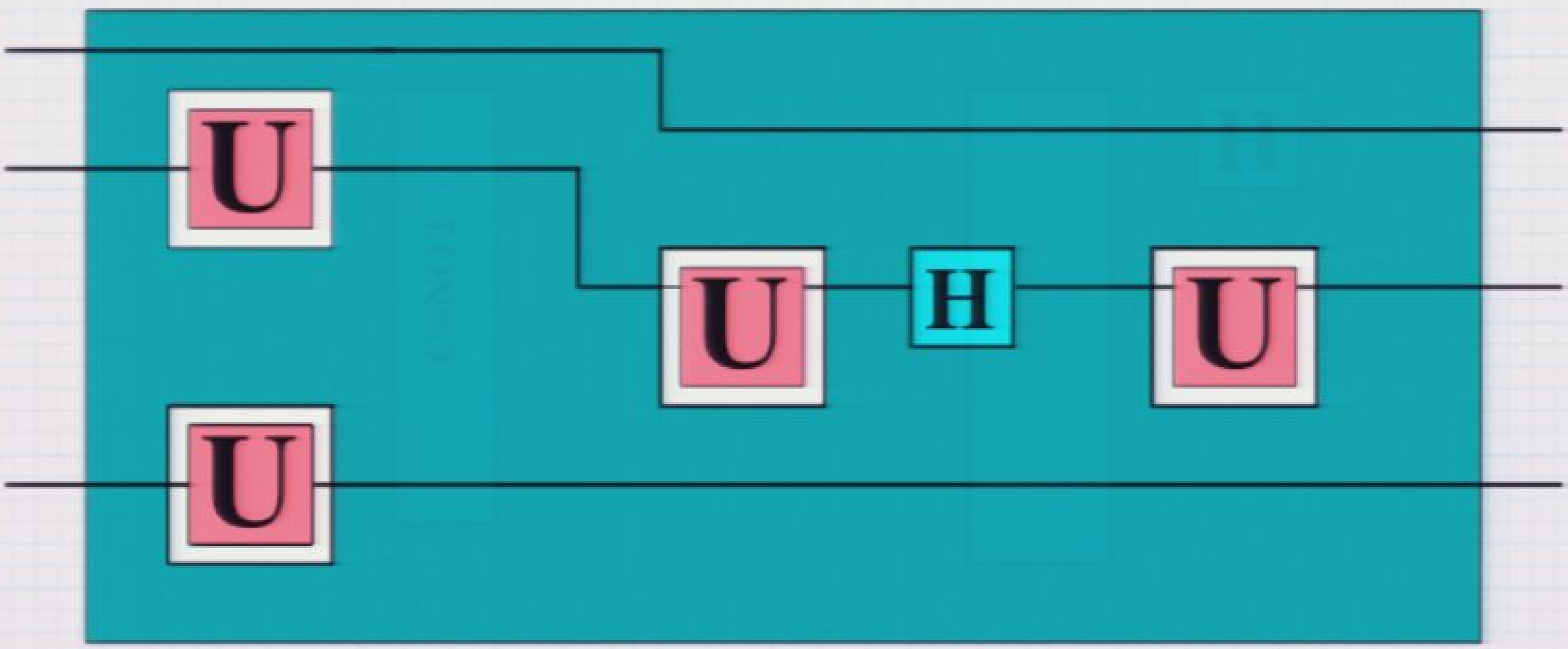

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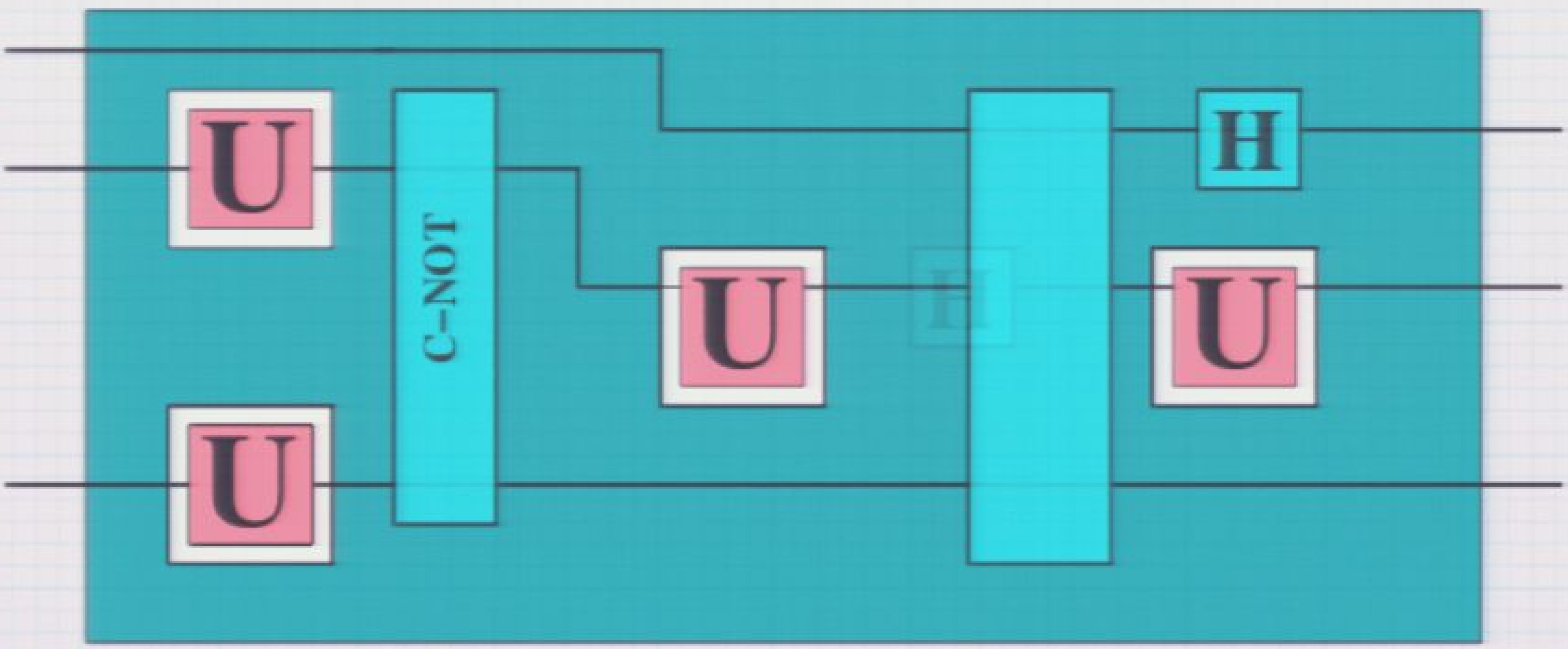

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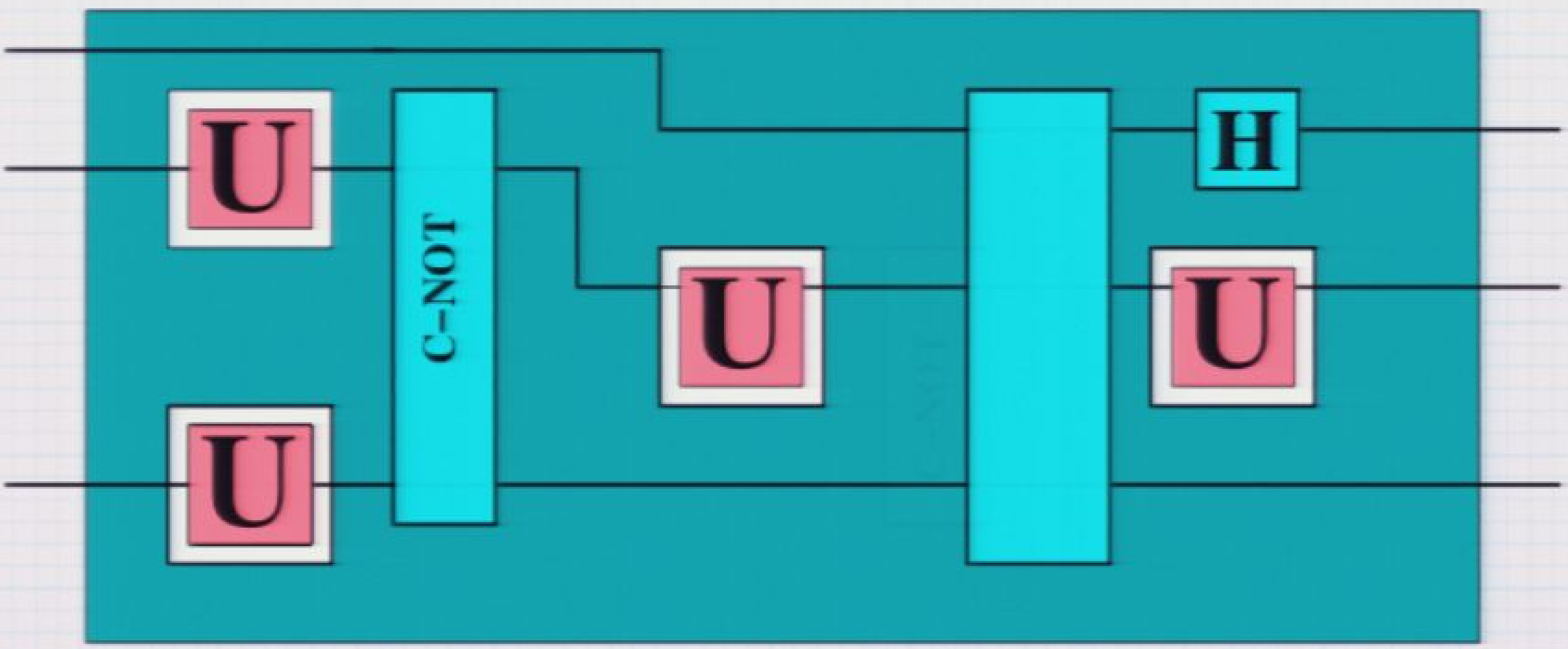

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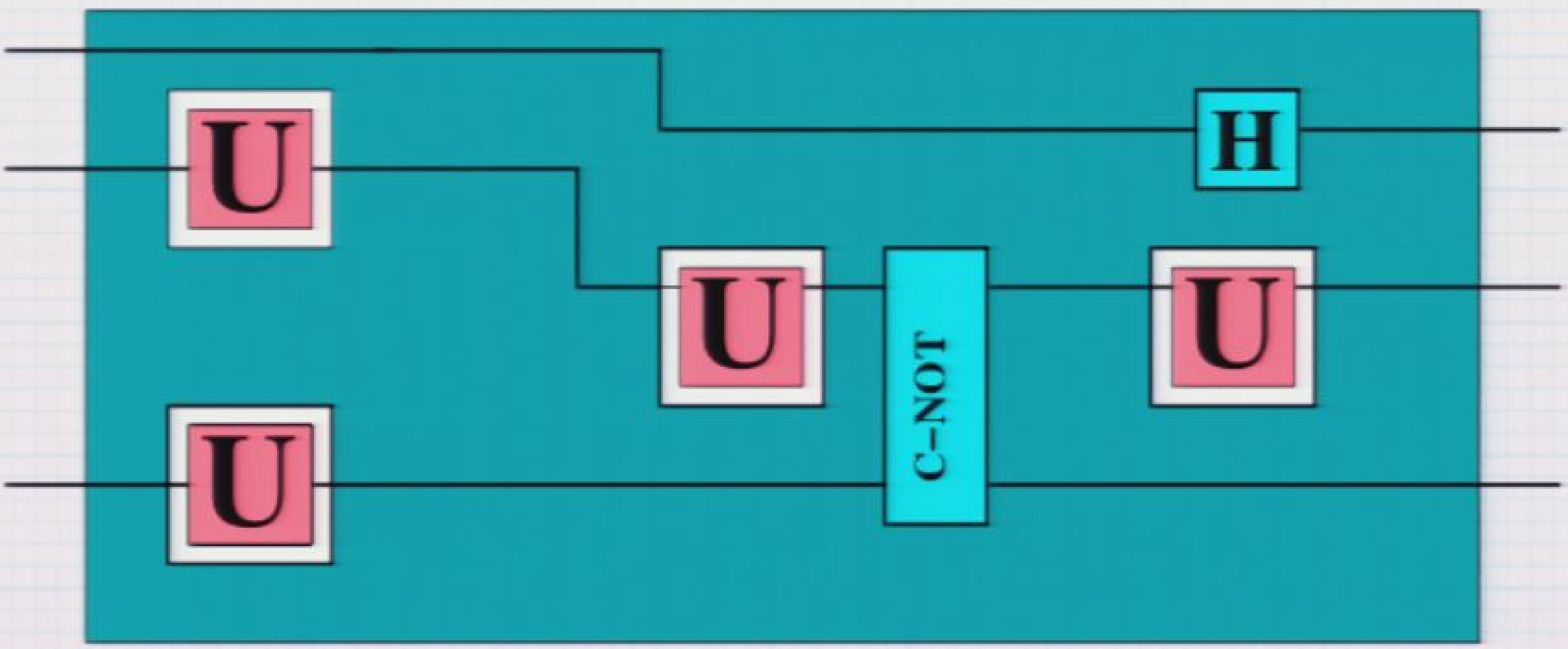

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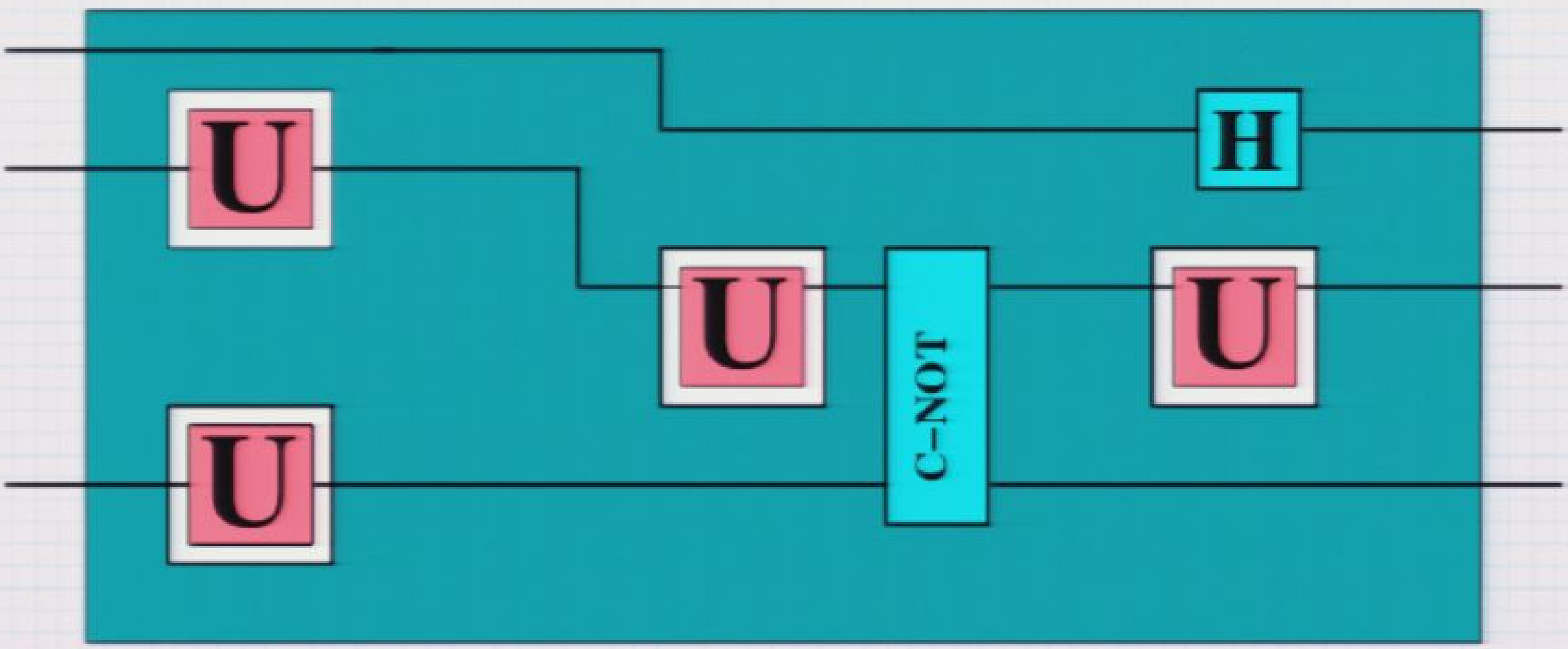

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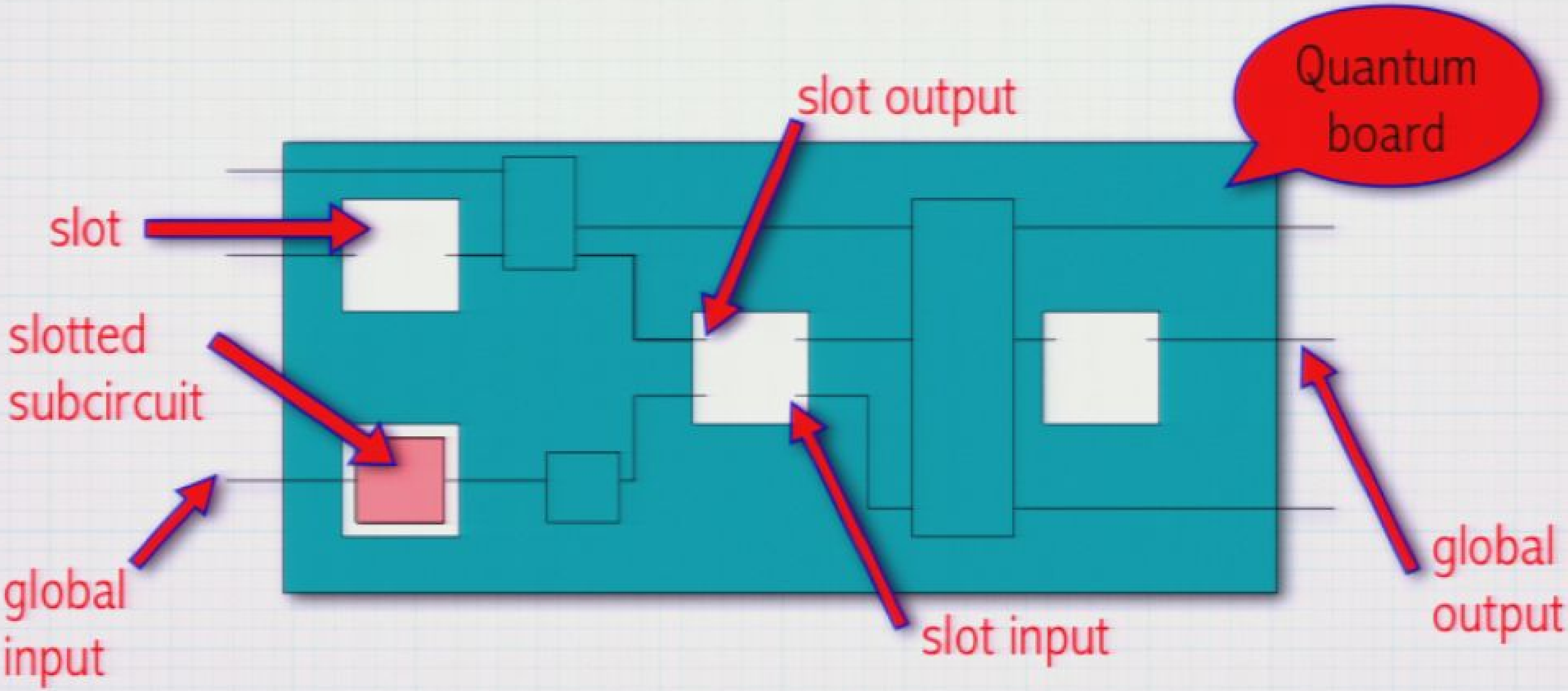
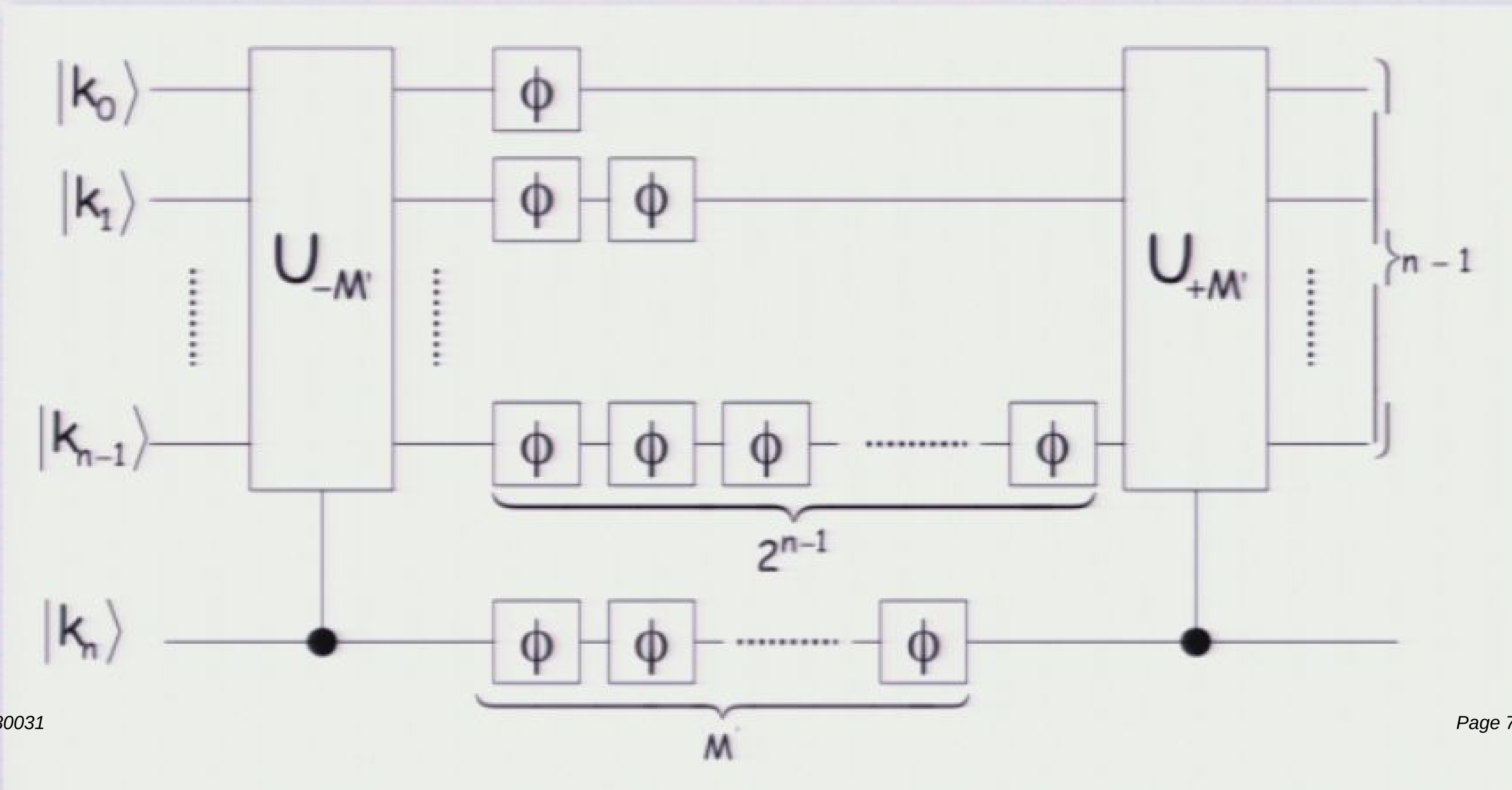


Problem: what is the optimal board for given slots achieving a global input/output transformation optimally according to a given cost function?

Quantum Board

[van Dam, D'Ariano, Ekert, Macchiavello, Mosca, PRL 98, 090501 (2007)]



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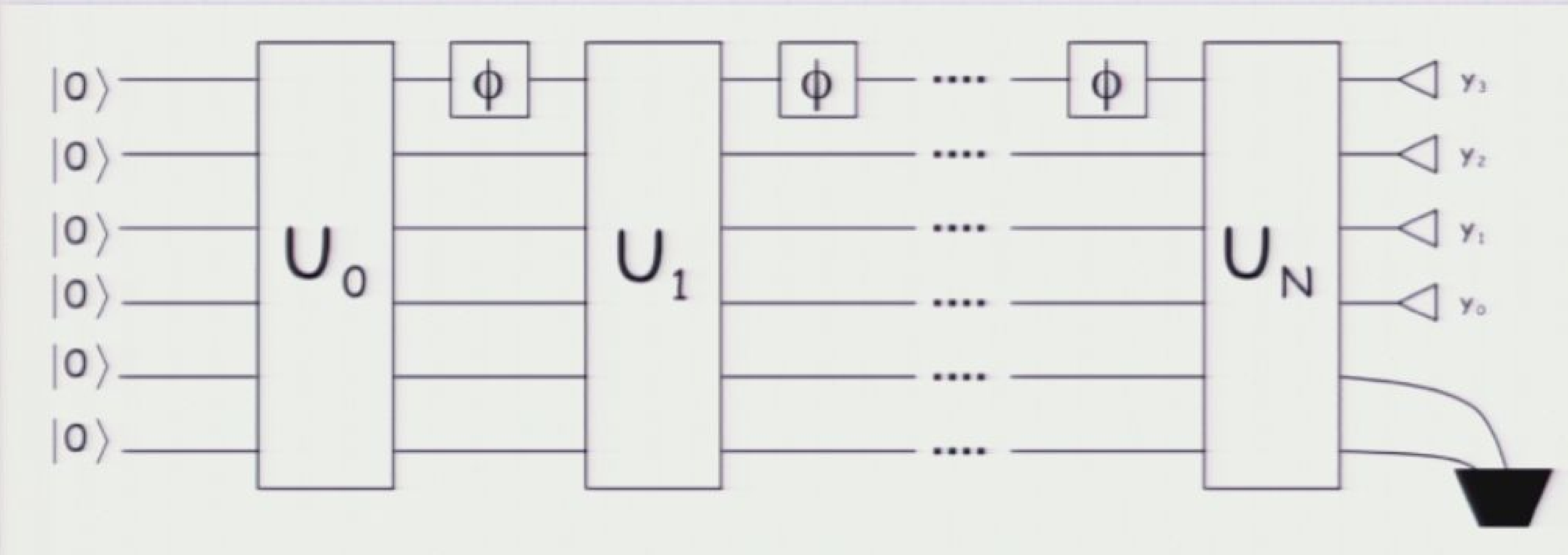

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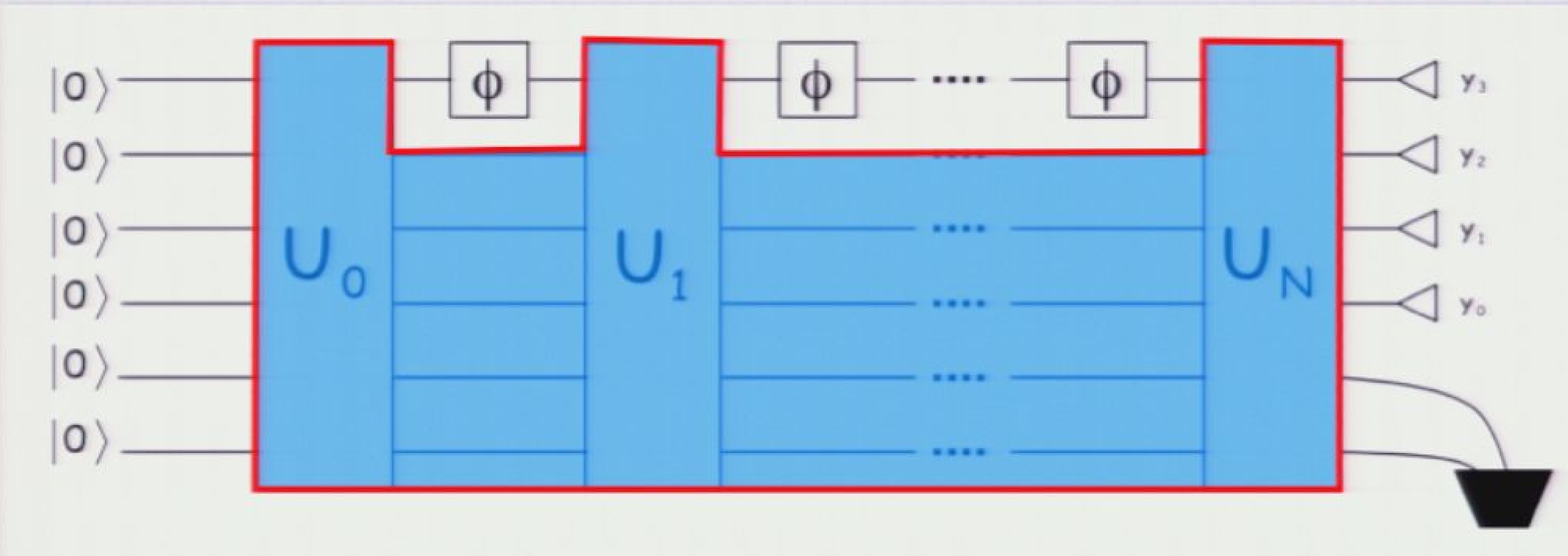
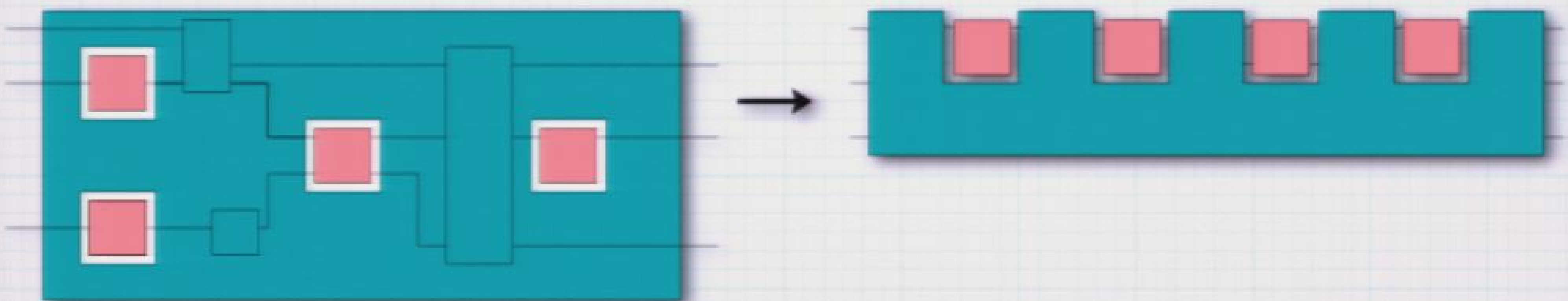


Quantum Combs

G.Chiribella, G.M.D'Ariano, P.Perinotti, PRL 101 060401 (2008)

All circuits-boards can be reshaped in form of "combs", with an ordered sequence of slots, each between two successive teeth



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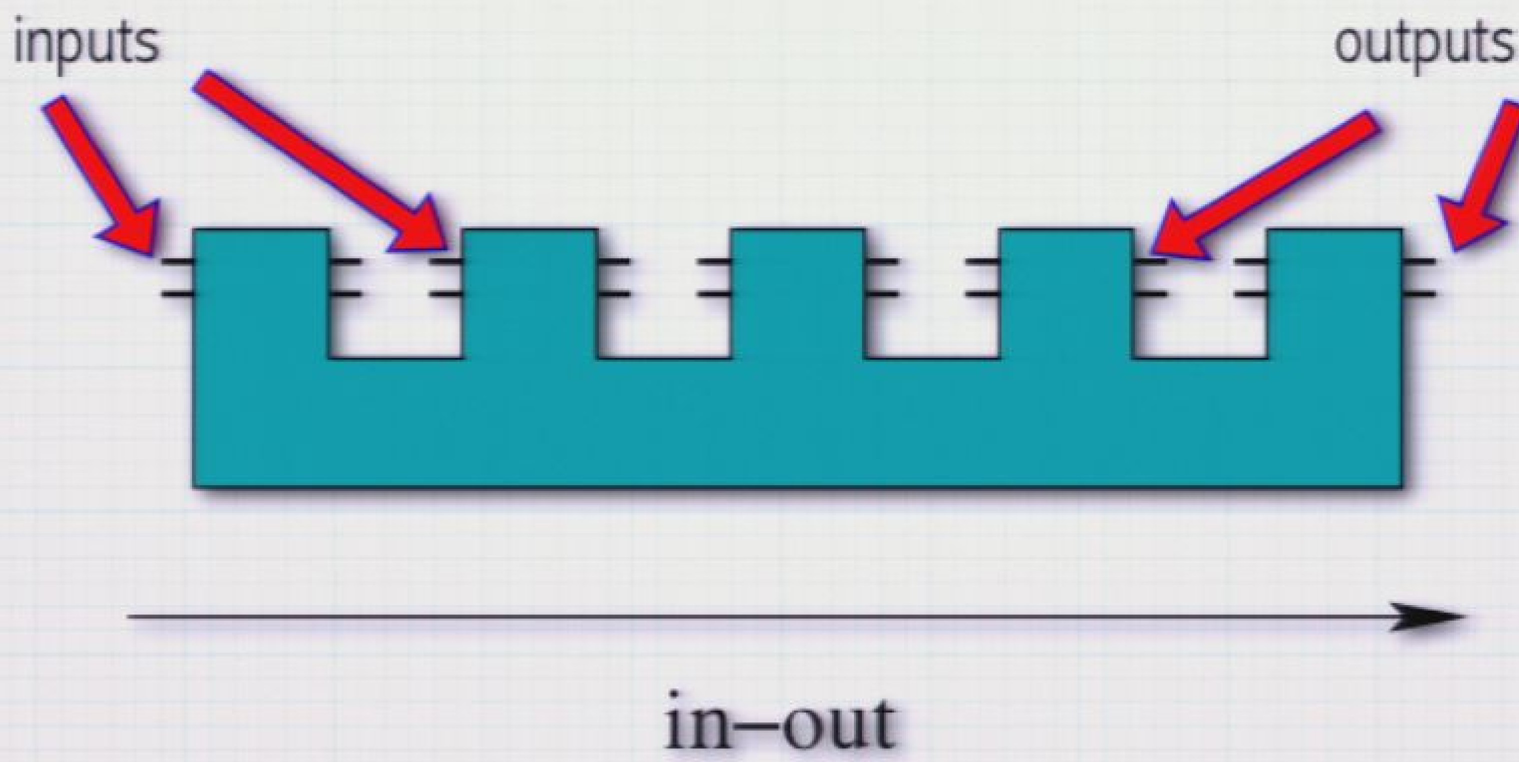

Pins = quantum systems with generally variable dimensions

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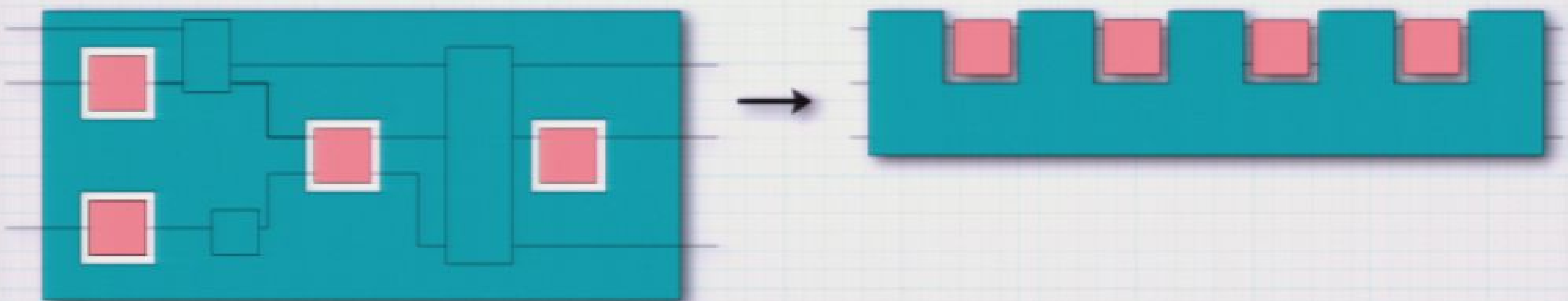

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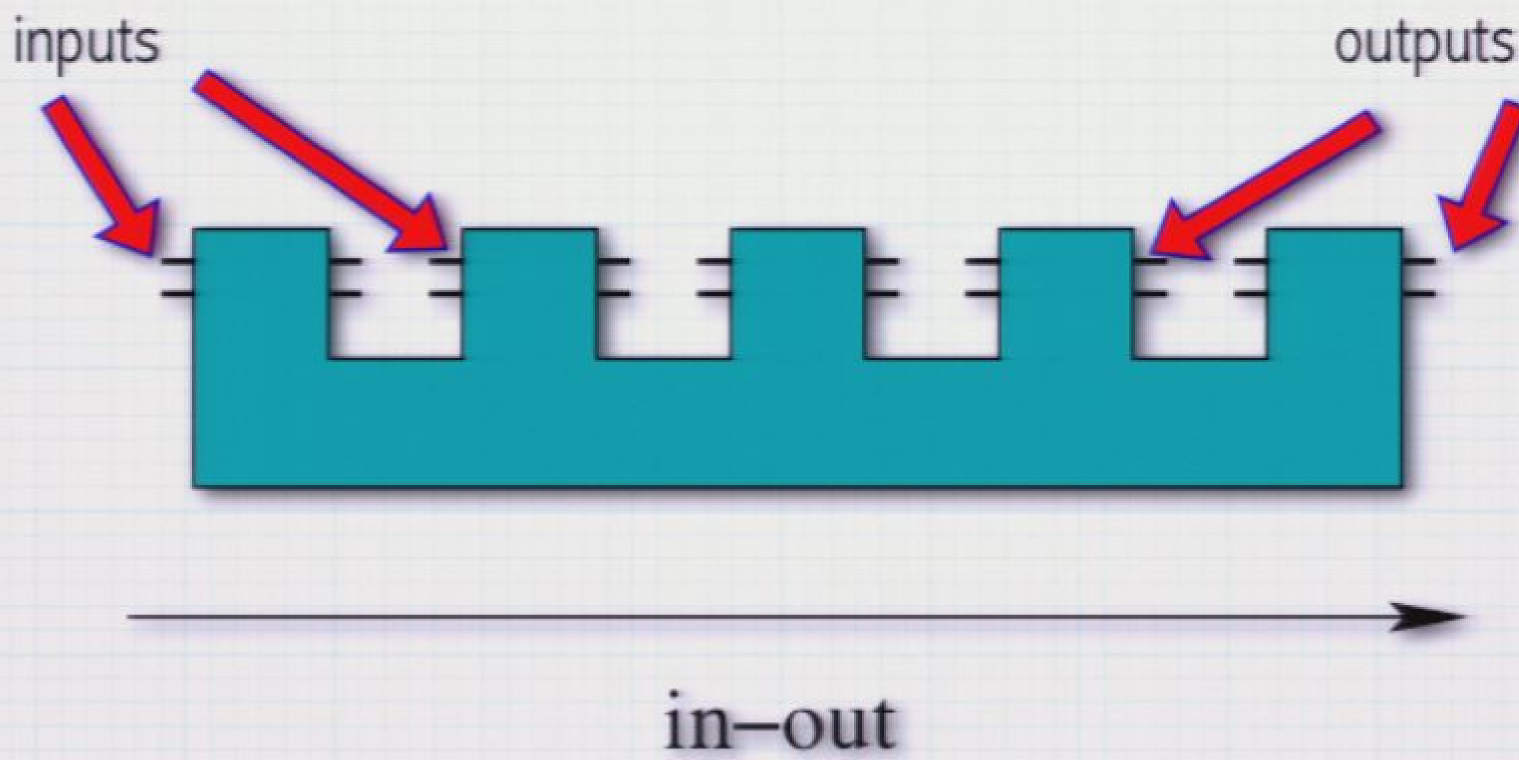
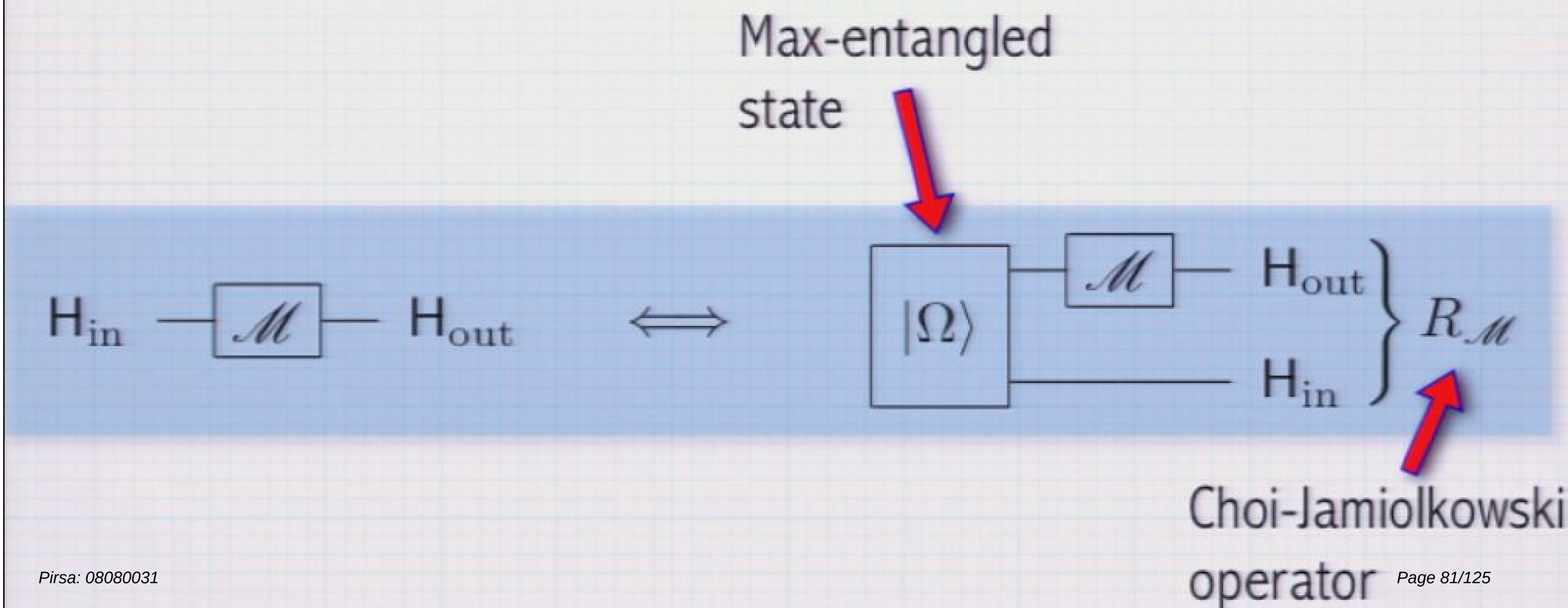


Pins = quantum systems with generally variable dimensions

How do we describe a quantum comb mathematically?

Channel: Choi representation

Mathematically the input-output transformation operated by a quantum circuit is a CP map, and is in one-to-one correspondence with a positive operator called "Choi-Jamiolkowski operator", which is nothing but the output state of the map applied locally to a maximally entangled state.



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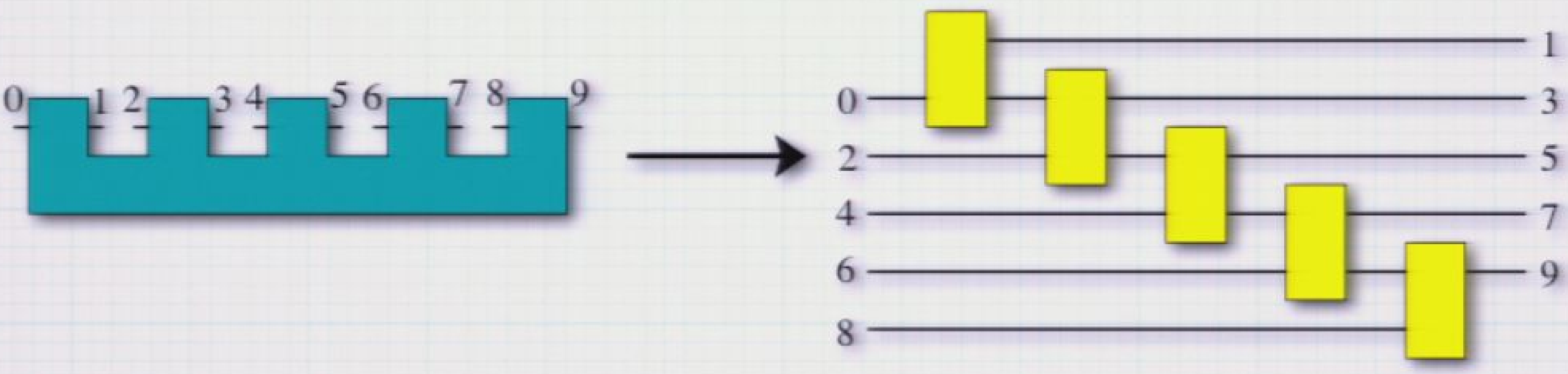
Causal networks

The quantum comb is equivalent to a causal network with all inputs on the left and all outputs on the right



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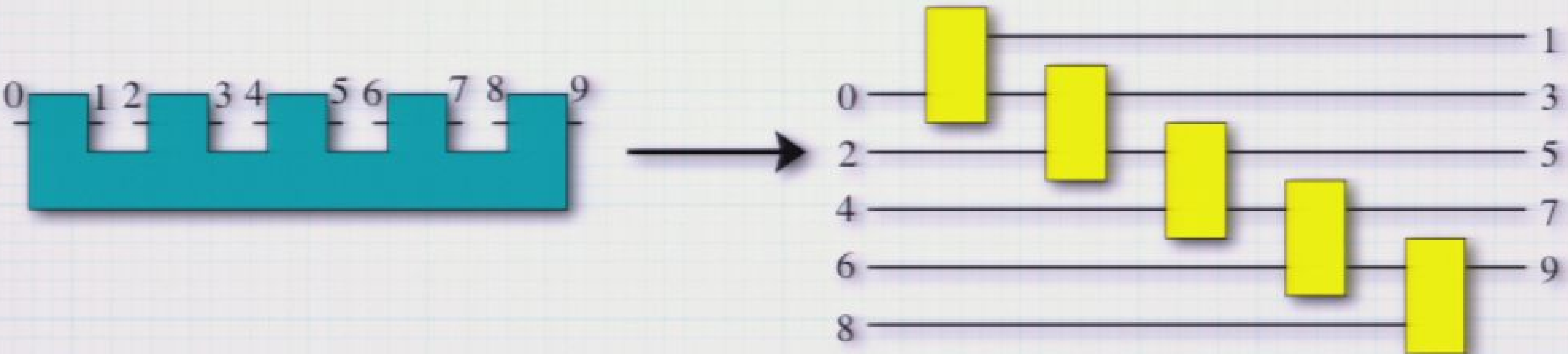
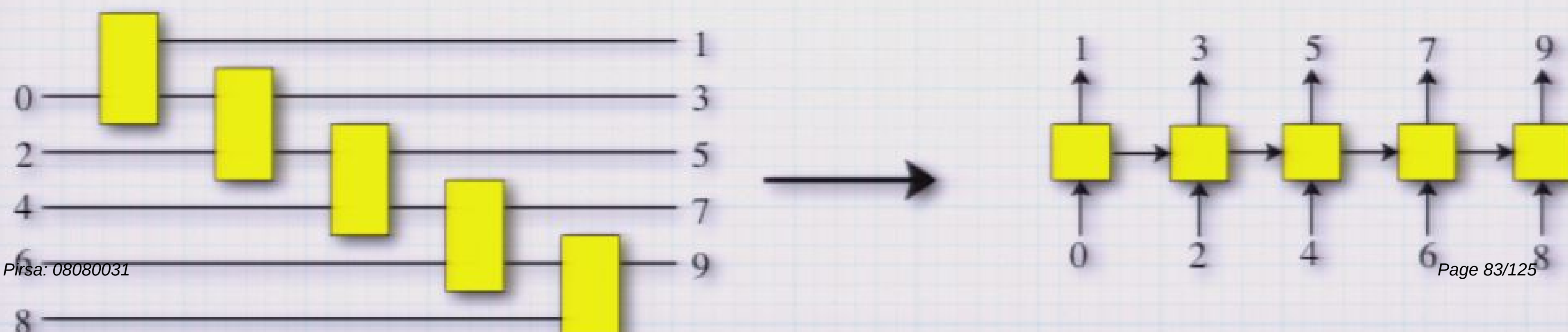


The causal network is also equivalent to the stack of memory channels



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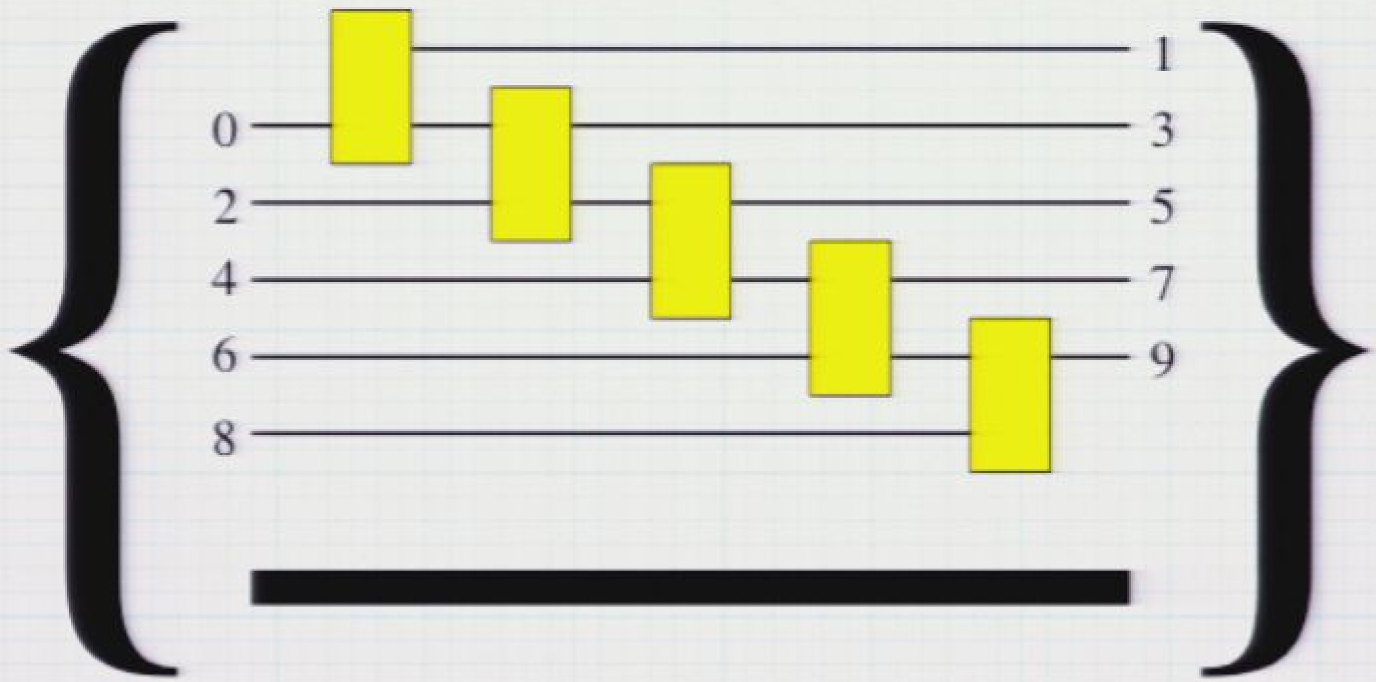
Choi representation

max entangled state



Choi–Jamiolkowski operator

R

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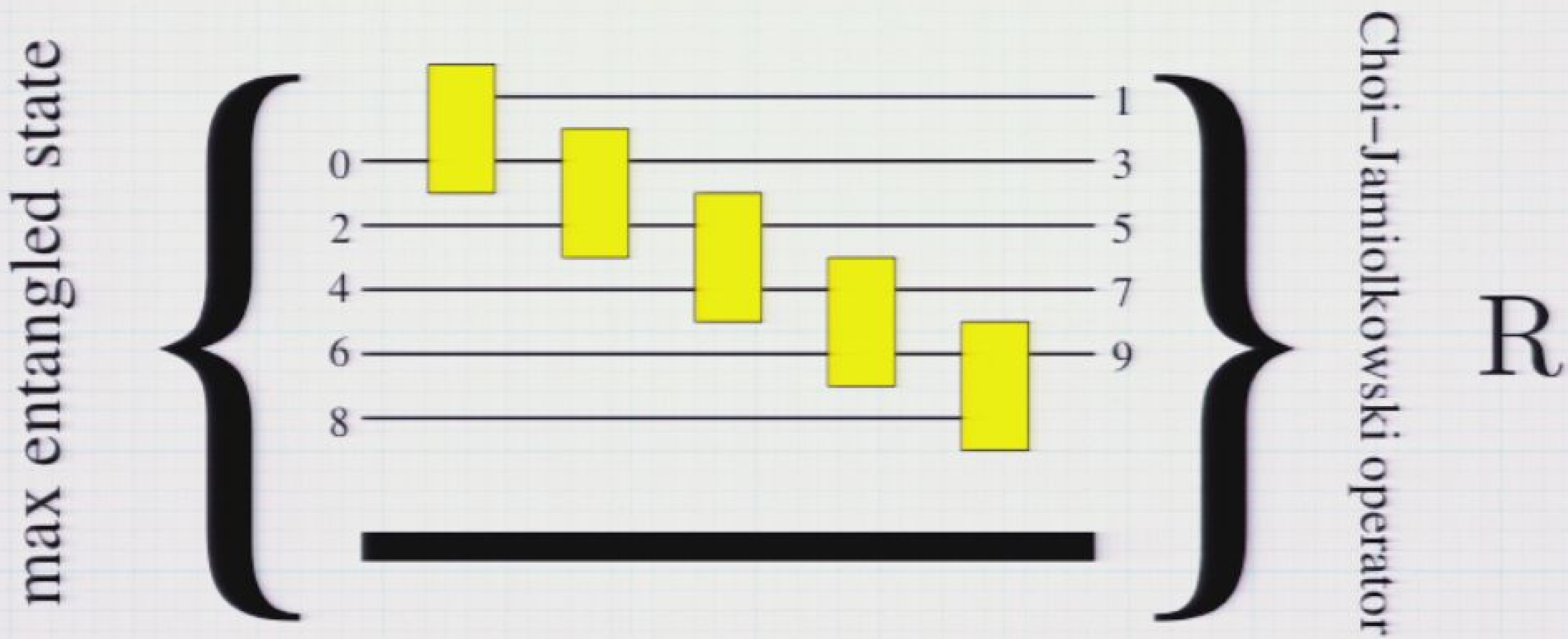
Choi representation



Causality constraints: (N+1 inputs/outputs)

$$\mathrm{Tr}_{2n+1}\left[R^{(n)}\right] = I_{2n} \otimes R^{(n-1)}, \quad n = 0, 1, N,$$

$$R^{(N)} \equiv R,\ R^{(-1)} = 1$$

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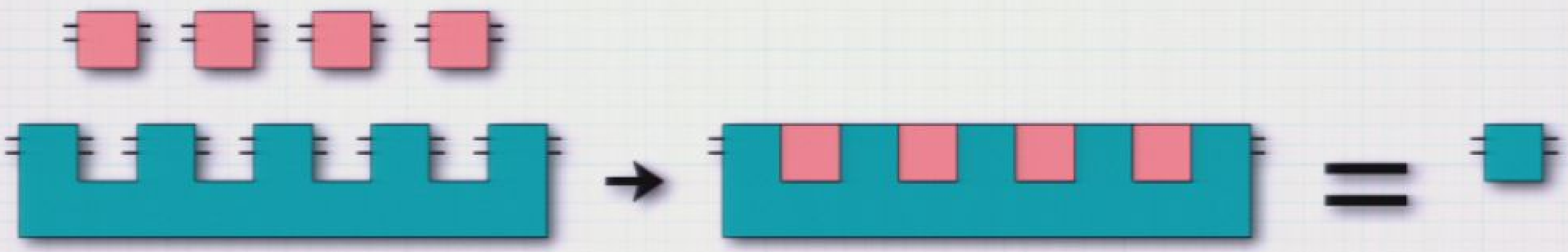
Supermaps

A quantum comb performs a transformation that is a generalization of the quantum operation: the so called "supermap"



A supermap sends a series of N channels to one channel.

Mathematically it is represented by a CP N-linear map which sends N Choi operators to one Choi operator, and with his own Choi operator satisfying the causality constraints.

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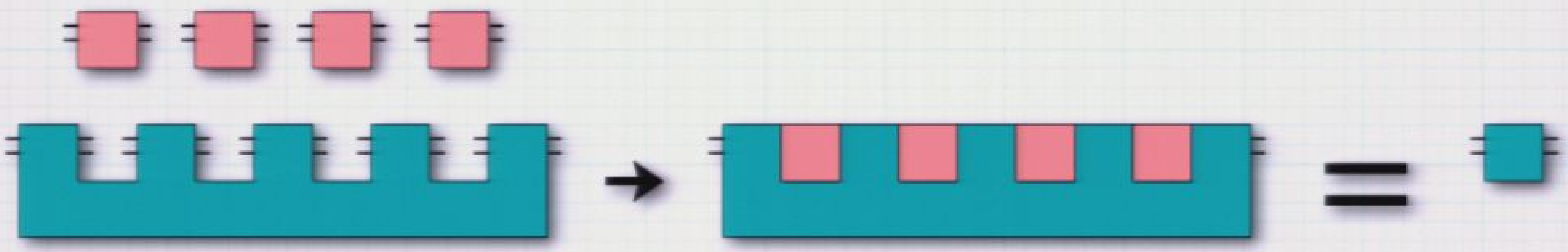

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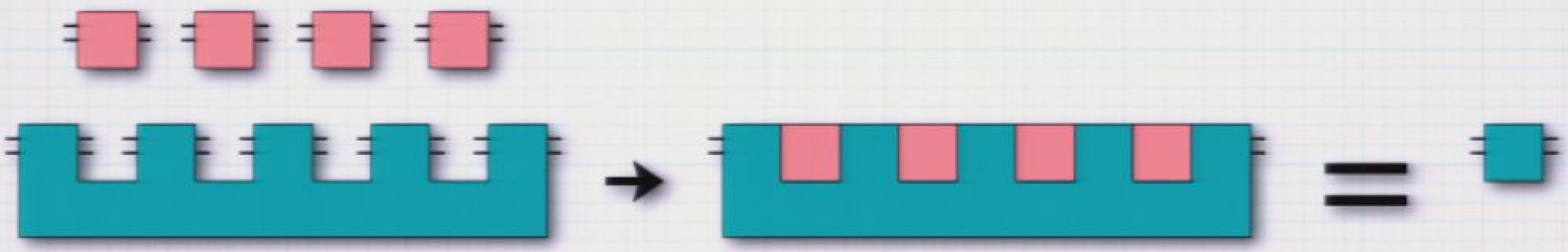

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(we can likewise consider probabilistic supermaps).

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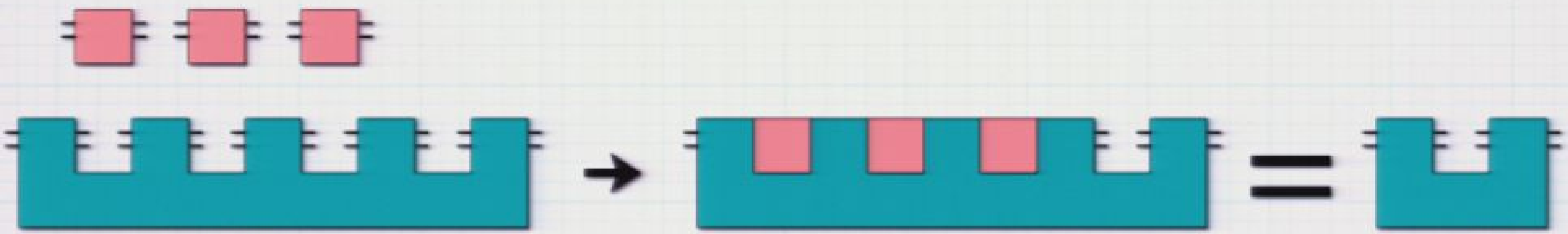
Supermaps

More generally, a quantum comb maps a series of channels into a comb



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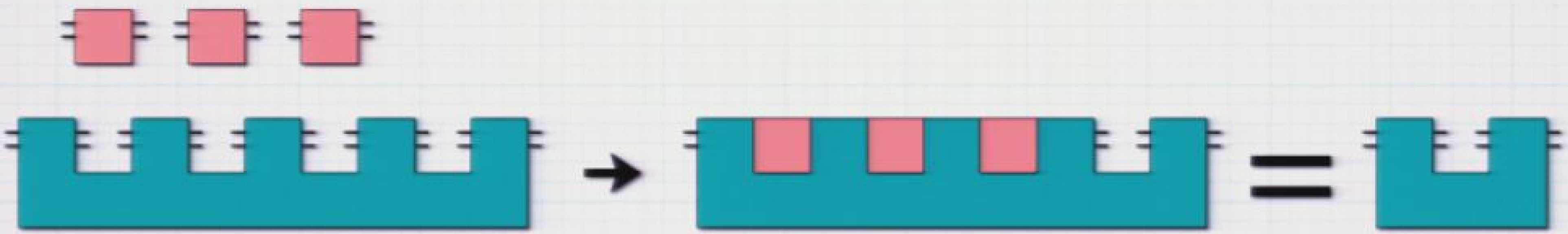
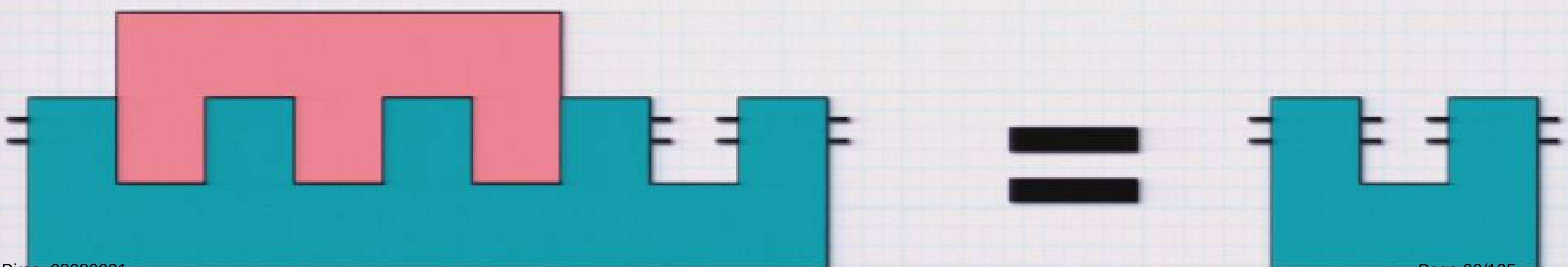


or, even more generally, a comb to a comb



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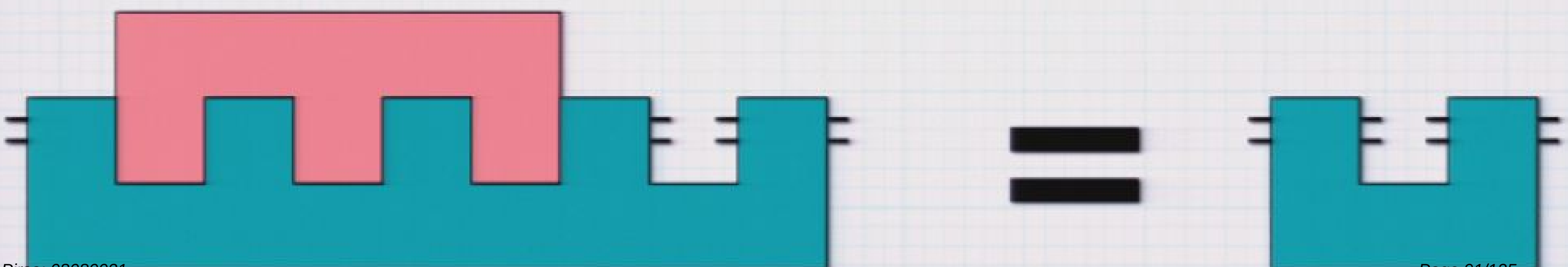
Supermaps

The notion of supermap is the last level of generalization, i.e. “super-supermaps” (mapping supermaps to supermaps) are still supermaps = quantum combs.



Link product

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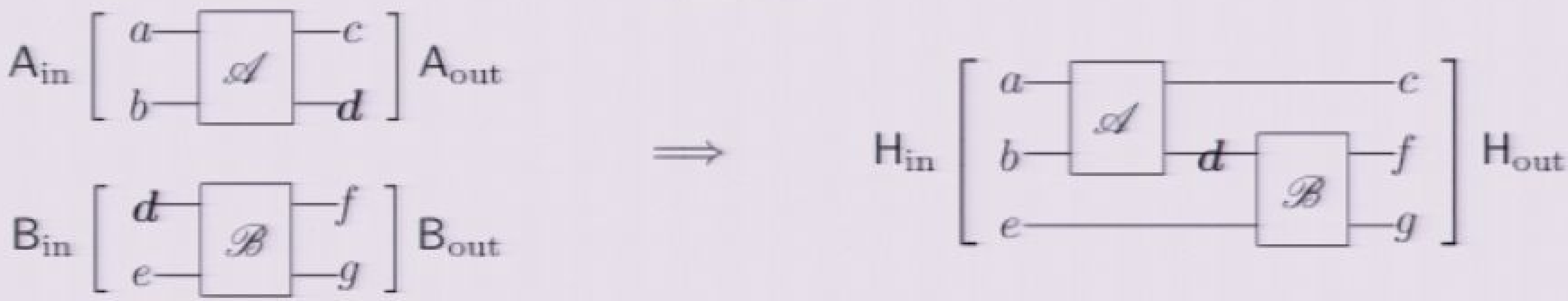

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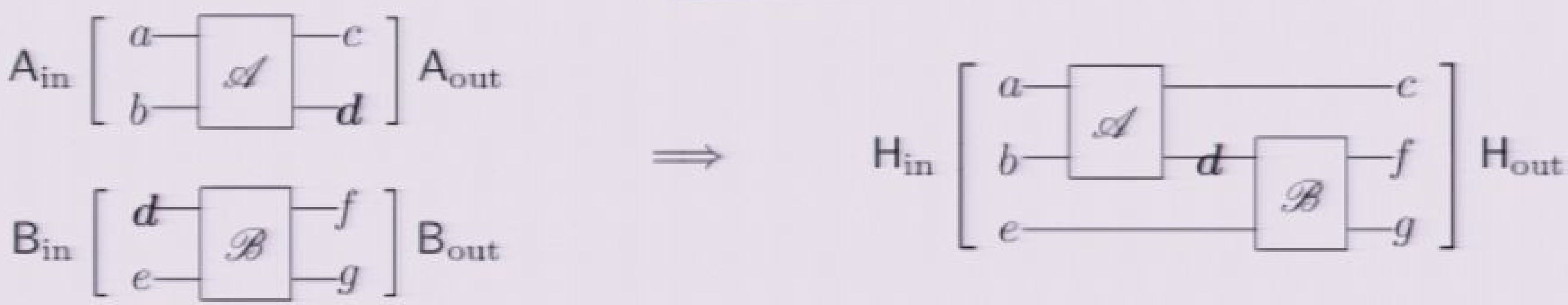

Choi-operator calculus

$$A \in \mathsf{B}(\mathsf{A}_{\rm out} \otimes \mathsf{A}_{\rm in}) = \mathsf{B}(\mathsf{H}_a \otimes \mathsf{H}_b \otimes \mathsf{H}_c \otimes \mathsf{H}_{\boldsymbol{d}}), \qquad \mathsf{J} \equiv \mathsf{H}_{\boldsymbol{d}}$$

$$B \in \mathsf{B}(\mathsf{B}_{\rm out} \otimes \mathsf{B}_{\rm in}) = \mathsf{B}(\mathsf{H}_{\boldsymbol{d}} \otimes \mathsf{H}_e \otimes \mathsf{H}_f \otimes \mathsf{H}_g)$$

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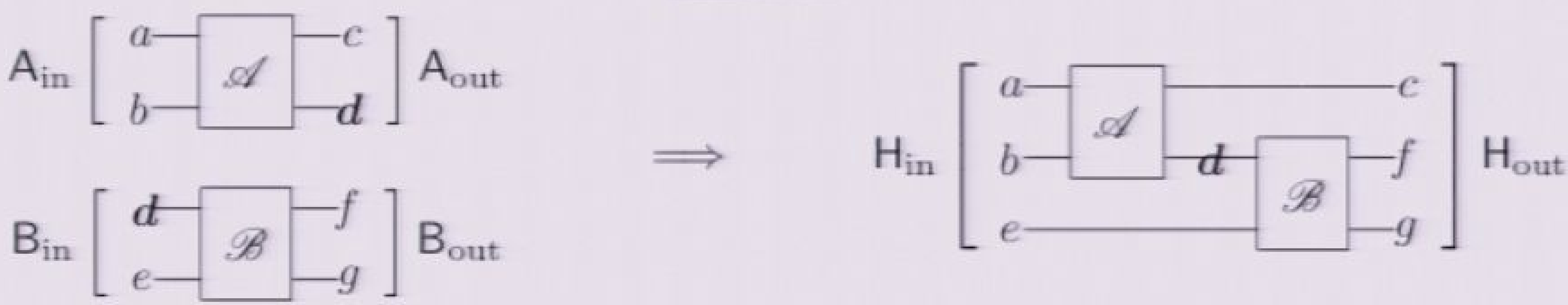

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$$AB := (A \otimes I_{e,f,g})(I_{a,b,c} \otimes B)$$

Link product

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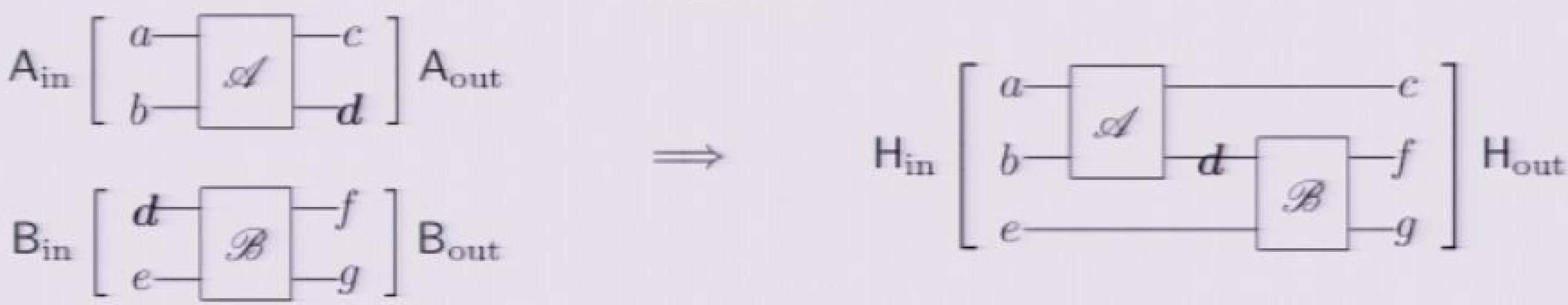

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$$A * B = \mathrm{Tr}_{\mathsf{J}}[A^{\theta_{\mathsf{J}}} B] \in \mathsf{B}(\mathsf{H}_{\text{out}} \otimes \mathsf{H}_{\text{in}})$$

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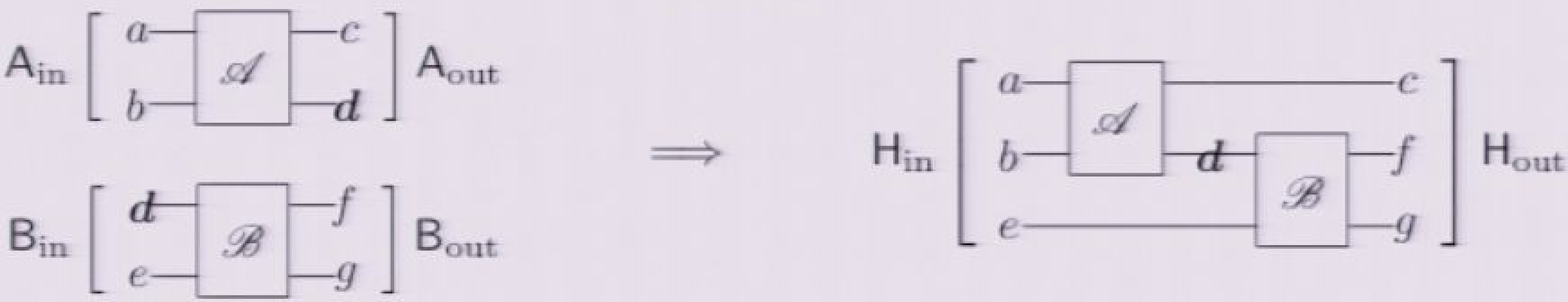
Link product



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$$A * B = \mathrm{Tr}_{\mathsf{J}}[A^{\theta_{\mathsf{J}}} B] \in \mathsf{B}(\mathsf{H}_{\rm out} \otimes \mathsf{H}_{\rm in})$$

The link-product is commutative!

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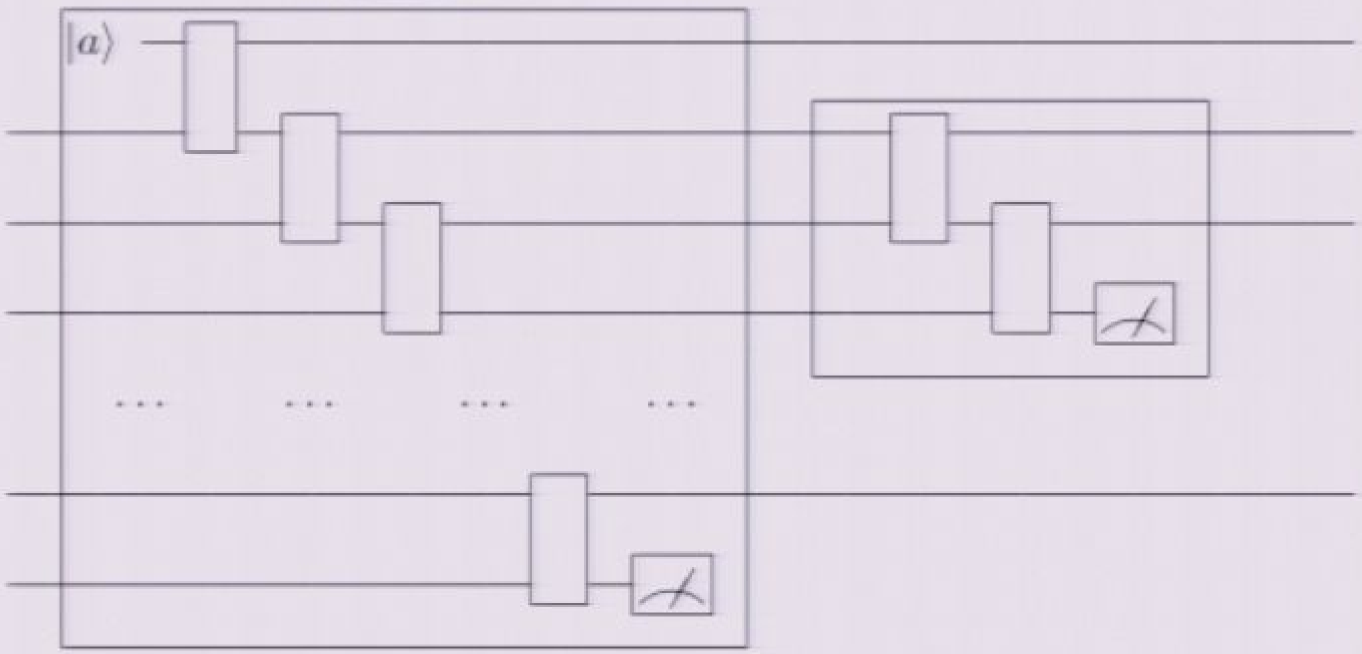
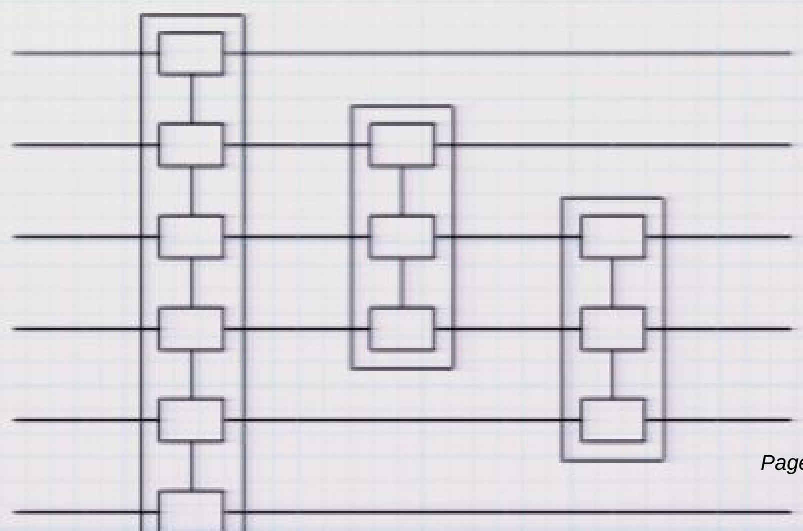
Link product

$R_1 * R_2 \iff$



$$R_1 * R_2 * R_3 \iff$$



Link product

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Special cases:

$$\mathscr{M}(\rho) = R_{\mathscr{M}} * \rho$$ quantum operation

$$\mathrm{Tr}[P^* \rho] = P * \rho$$ POVM

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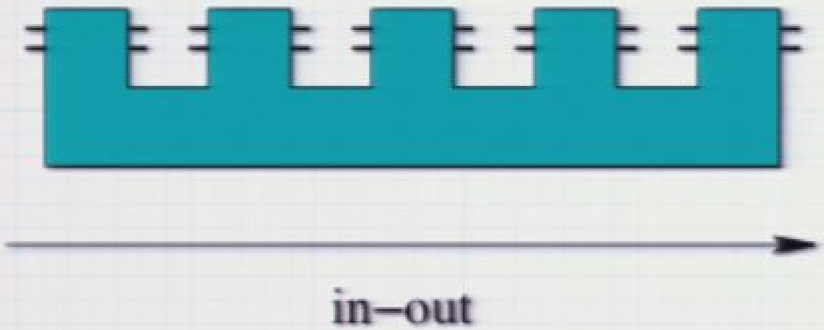
Circuits Architecture Optimization



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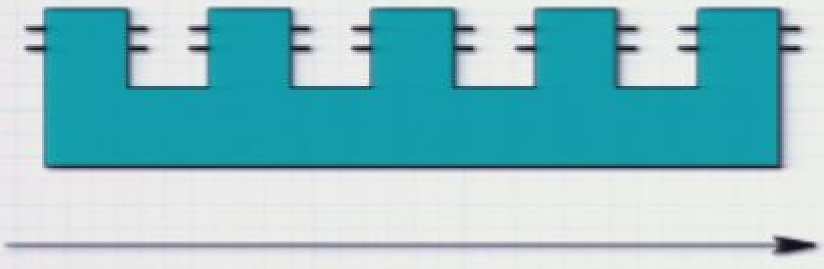
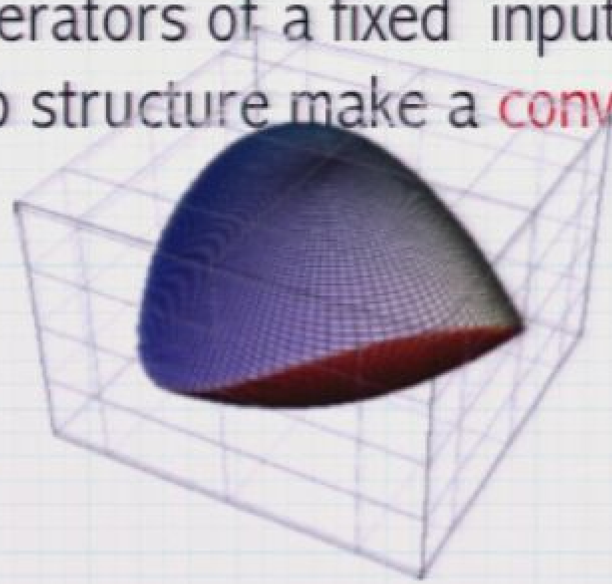


- The Choi operators of a fixed input-output comb structure make a convex set



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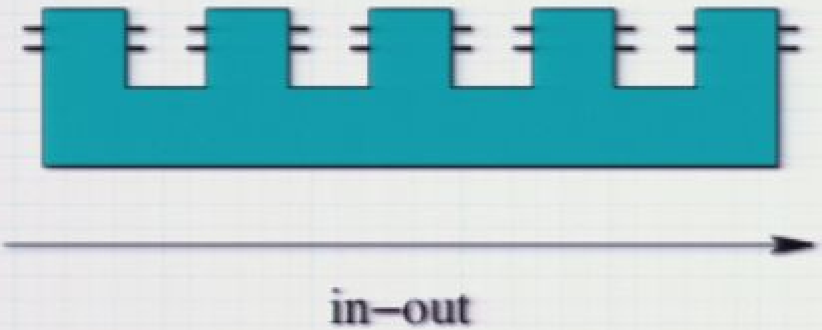

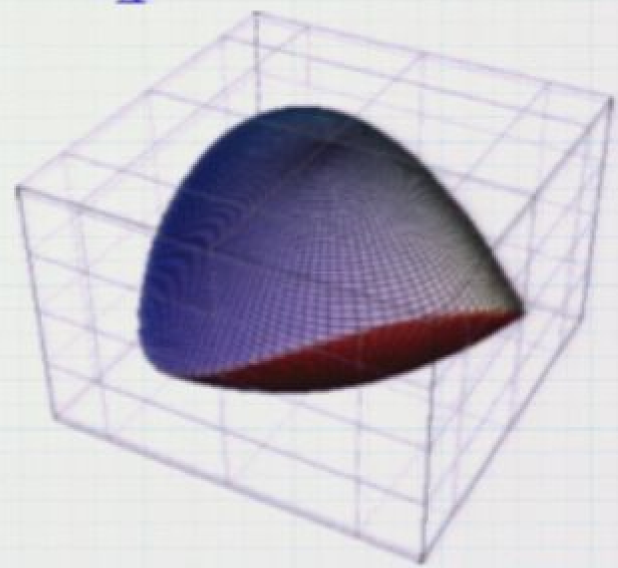

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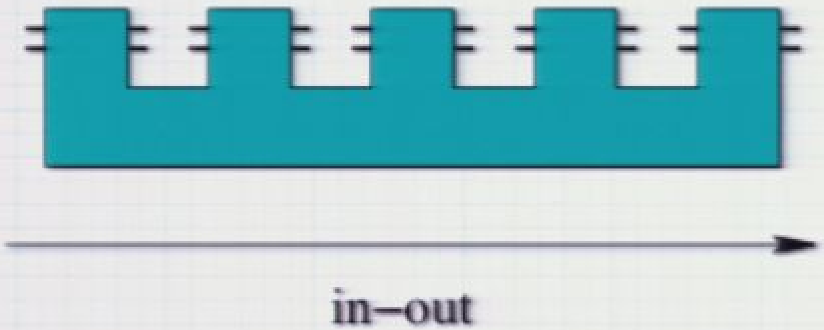

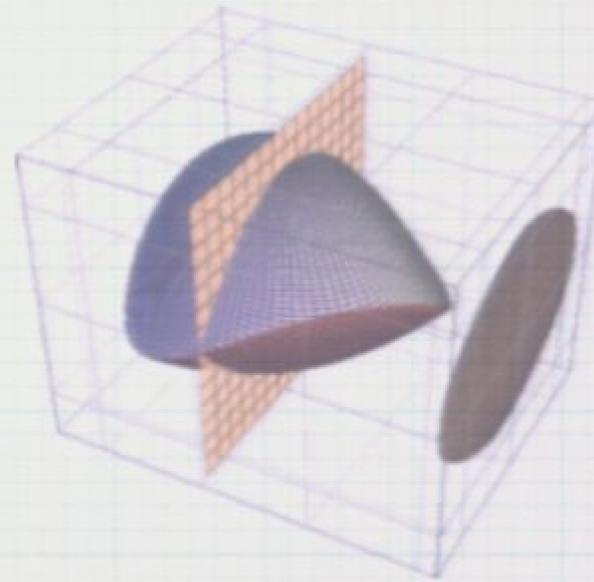

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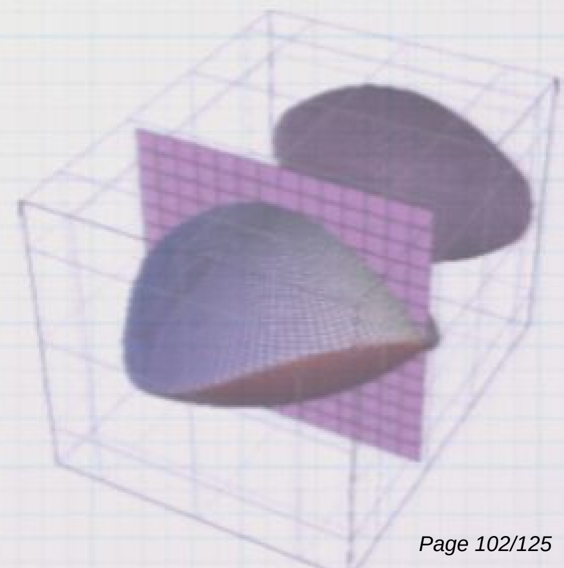

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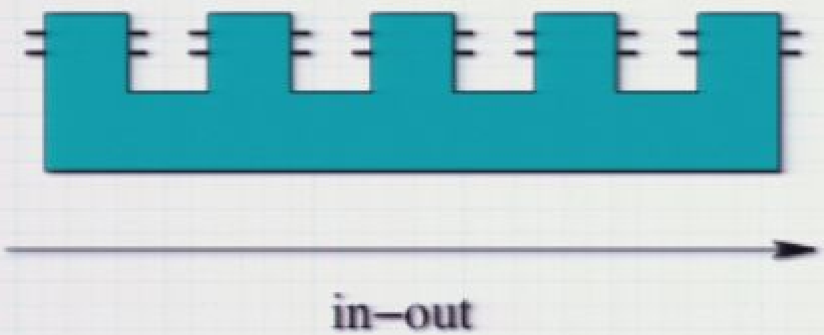

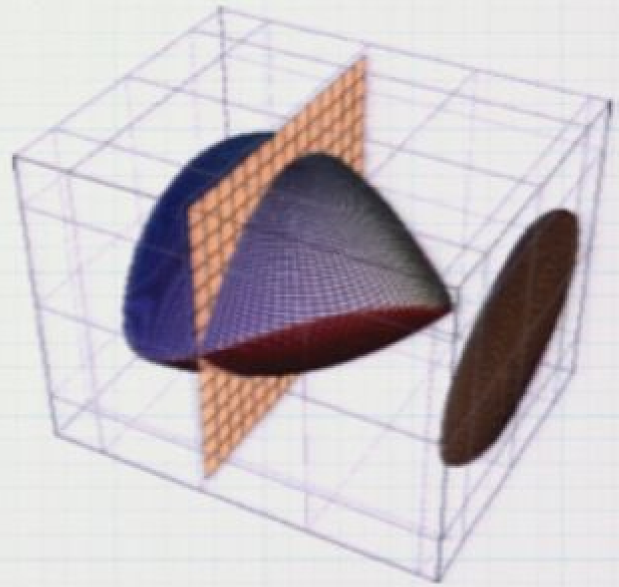

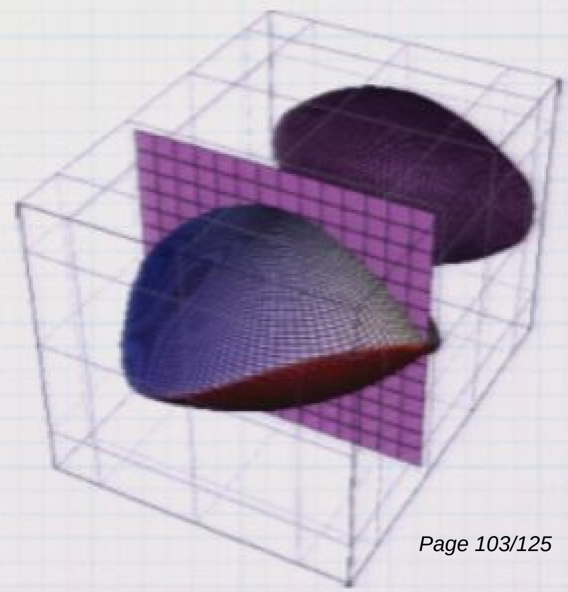

- The Choi operators of a fixed input-output comb structure make a convex set
- Causality constraints correspond to a hyperplane section of the convex
- Group-covariance gives ano

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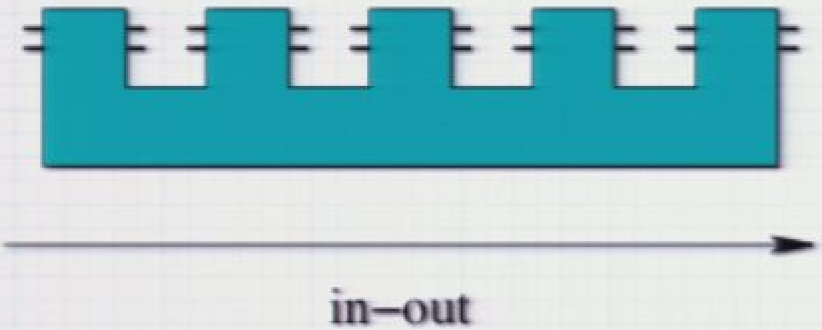
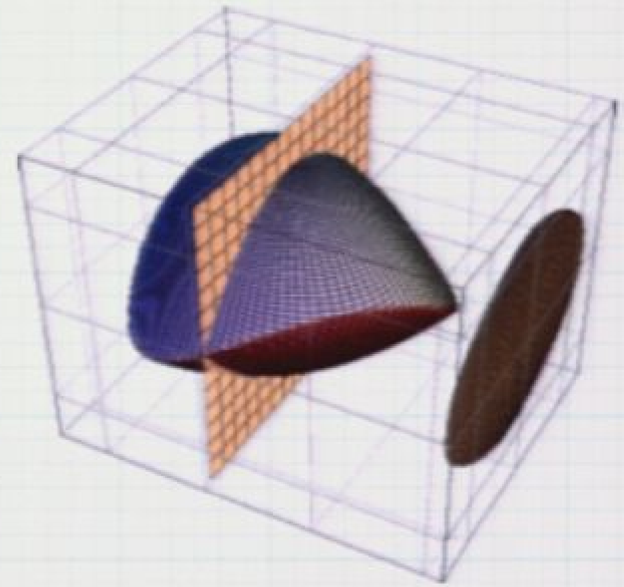
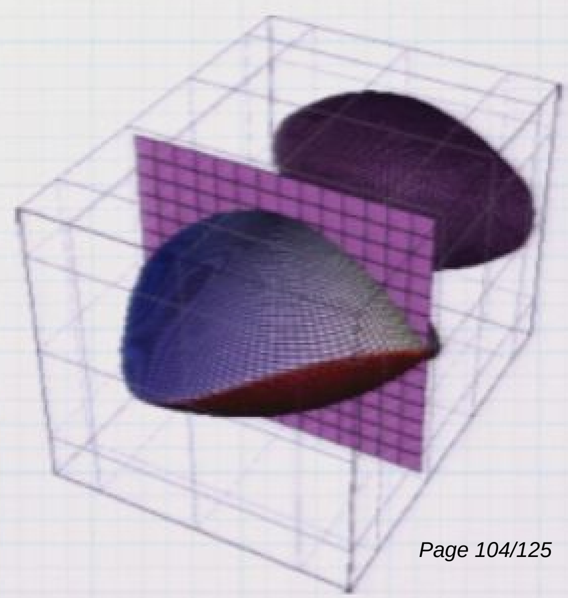
Circuits Architecture Optimization





- The Choi operators of a fixed input-output comb structure make a convex set
- Causality constraints correspond to a hyperplane section of the convex
- Group-covariance gives another linear constraint:



$$[R, V_g] = 0 \implies R = \bigoplus_j R_j \otimes \mathbb{1}_{m_j}$$

QUit quantum information theory group

The mathematical formulation is reduced to a convex problem!

QUit quantum information theory group

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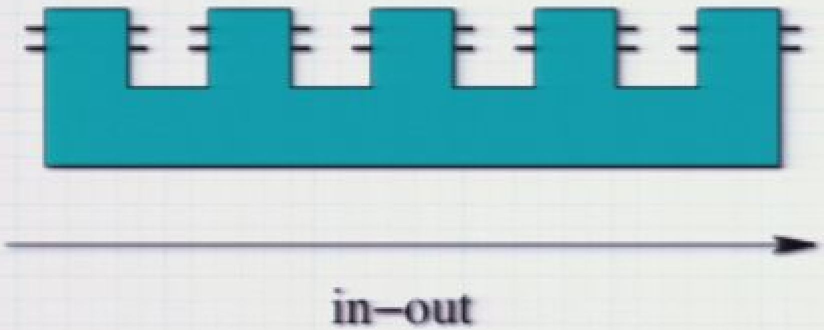
Circuits Architecture Optimization



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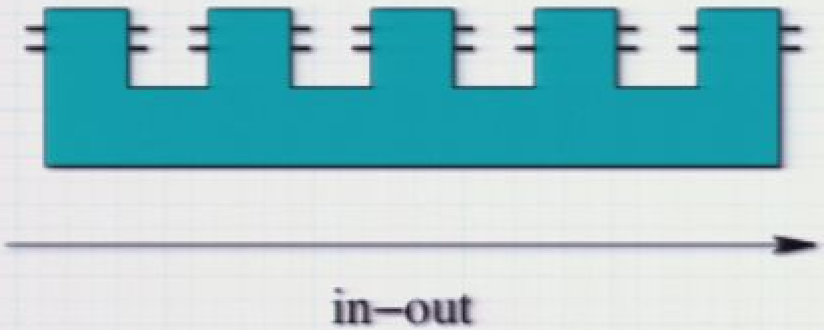
Circuits Architecture Optimization



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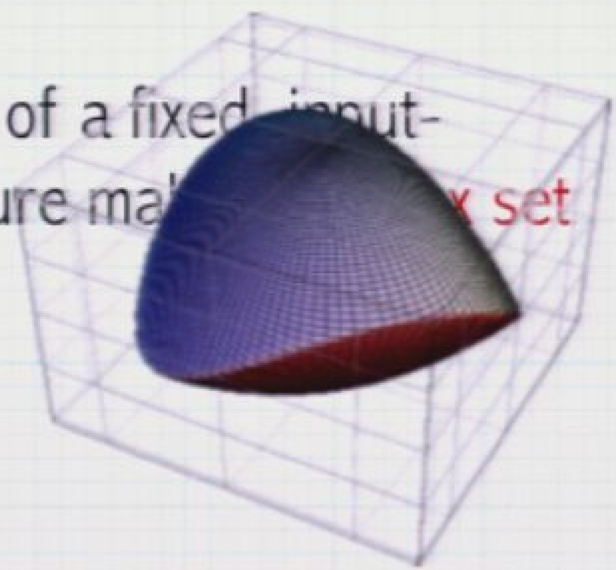

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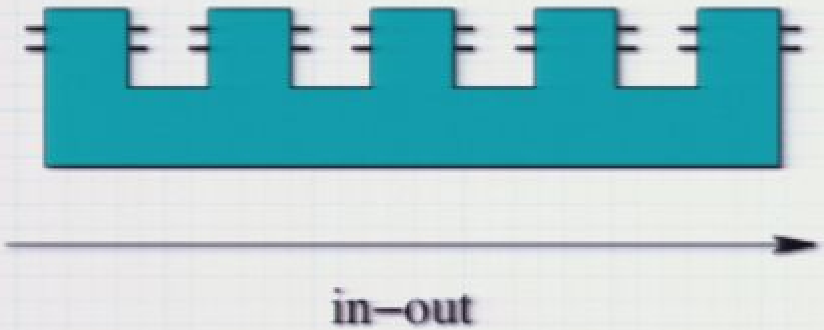
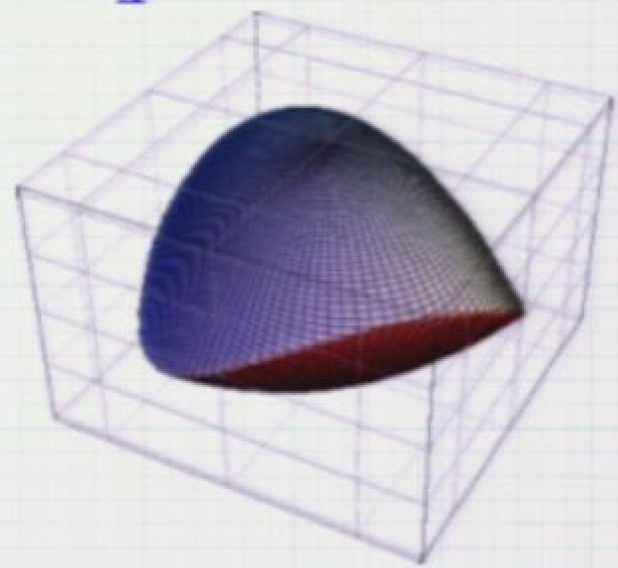
Circuits Architecture Optimization

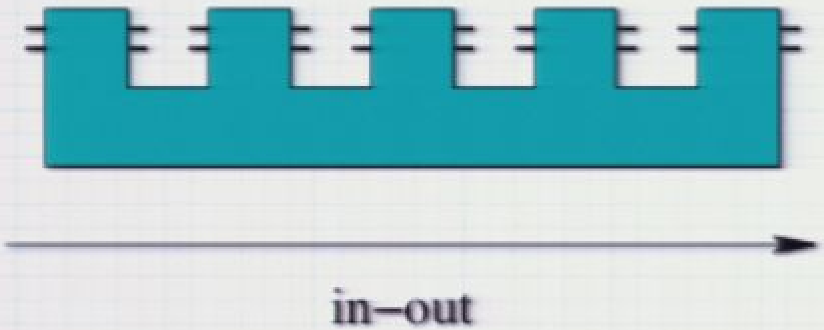


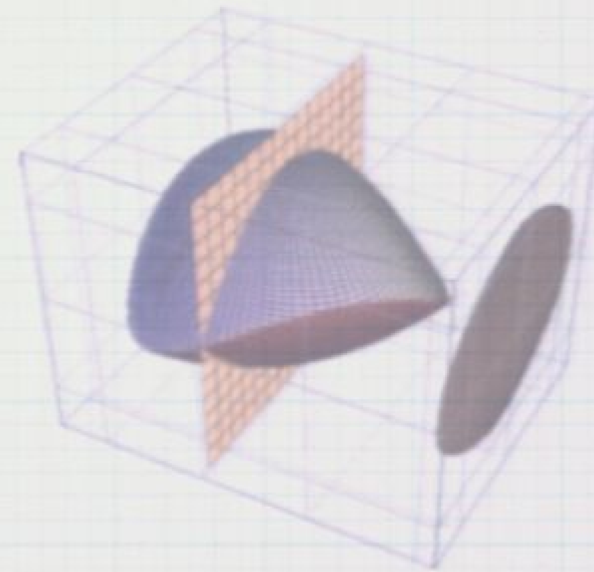


- The Choi operators of a fixed input-output comb structure make a convex set

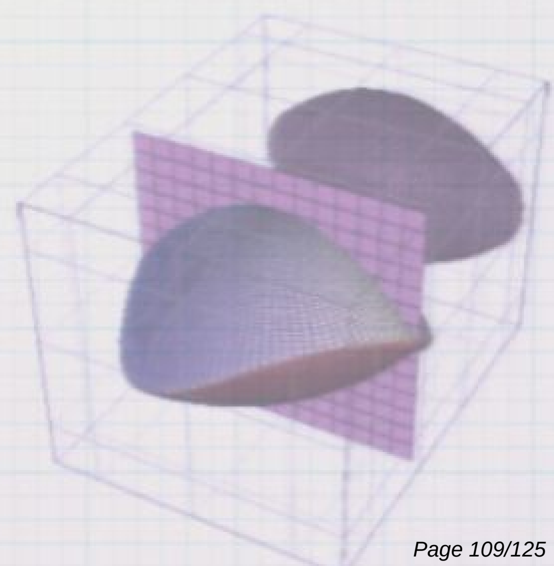
PRL 101 060401 (2008)

Circuits Architecture Optimization





- The Choi operators of a fixed input-output comb structure make a convex set



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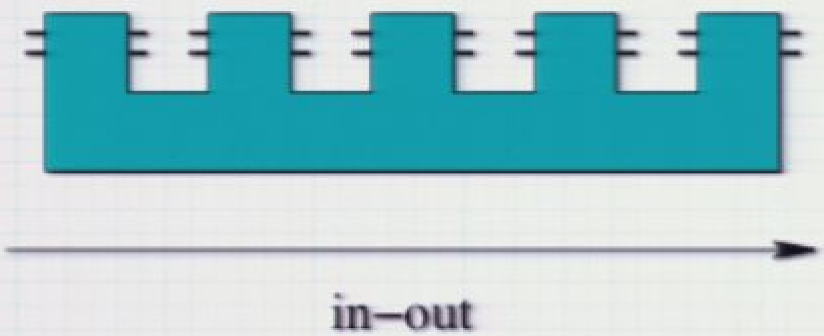
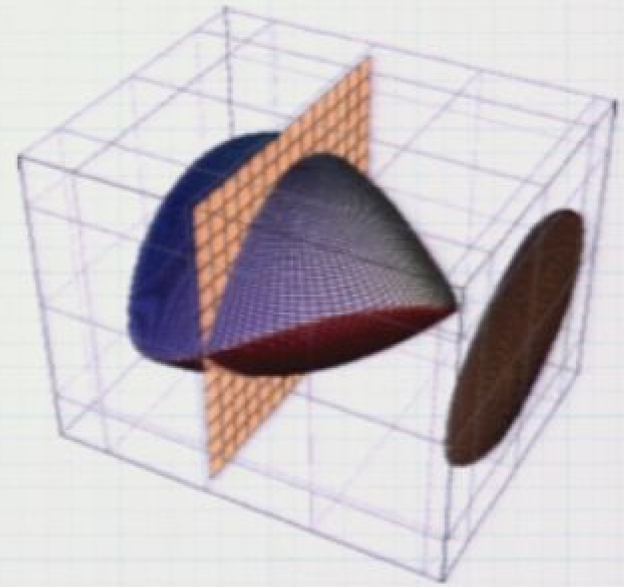
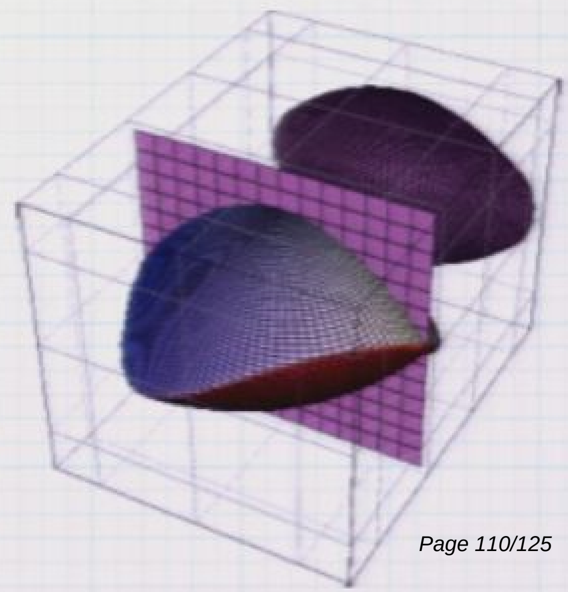
Circuits Architecture Optimization





- The Choi operators of a fixed input-output comb structure make a convex set
- Causality constraints correspond to a hyperplane section of the convex
- Group-covariance gives a



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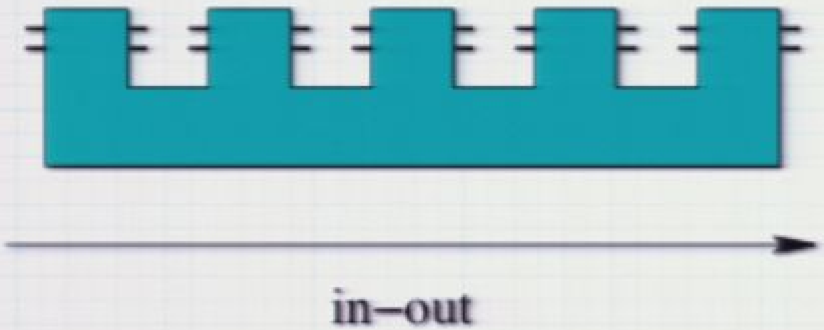
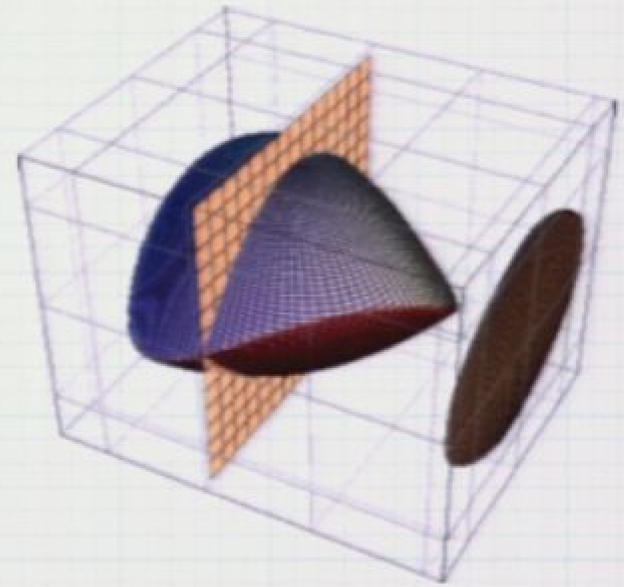
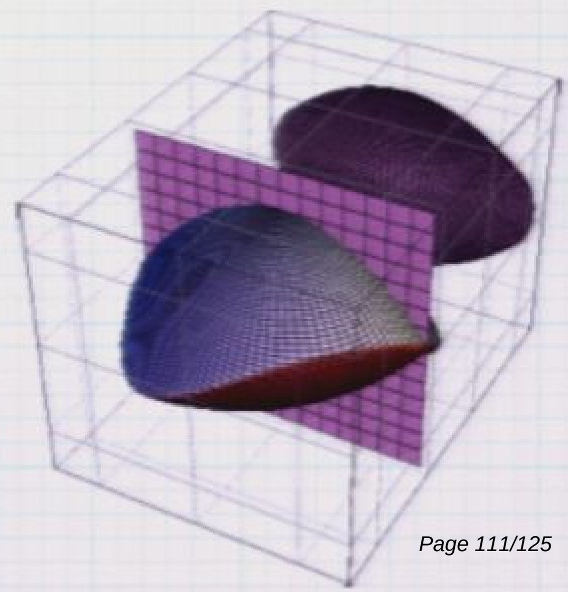
Circuits Architecture Optimization





- The Choi operators of a fixed input-output comb structure make a convex set
- Causality constraints correspond to a hyperplane section of the convex
- Group-covariance gives another linear constraint:

$$[R, V_g] = 0 \implies R = \bigoplus_j R_j \otimes \mathbb{1}_{m_j}$$



The mathematical formulation is reduced to a convex problem!

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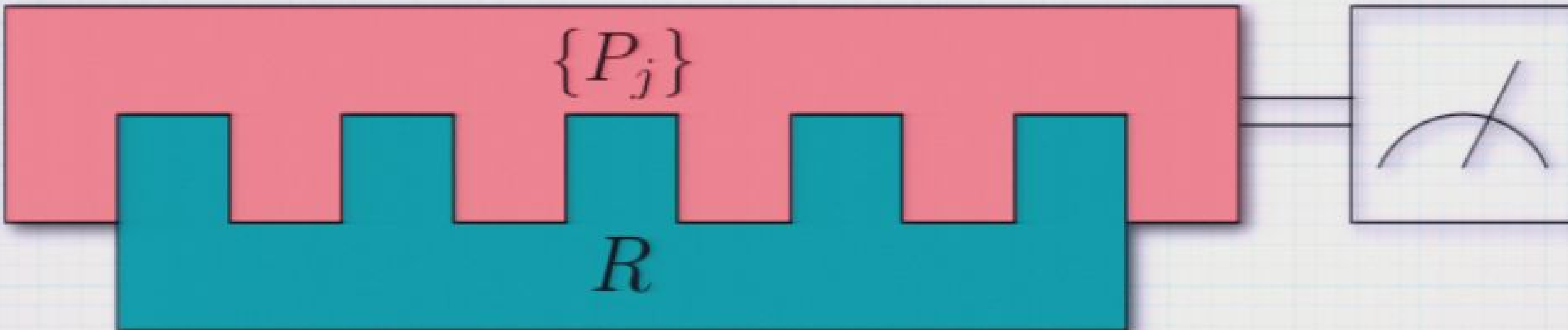
Quantum board testers



Tester Born rule:

$$\mathrm{Tr}[P_j R] = p_j, \qquad \sum_j P_j = \Xi$$

causality constraints:

$$\mathrm{Tr}_{2n+1}[\Xi^{(n)}] = I_{2n} \otimes \Xi^{(n-1)},\ n = 0, 1, \ldots, N$$
$$\Xi^{(N)} \equiv \Xi,\ \mathrm{Tr}_1[\Xi^{(0)}] = 1$$

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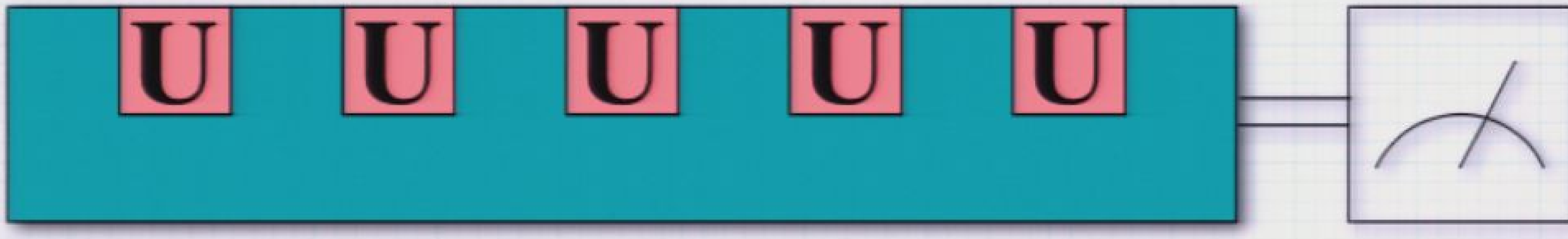
Estimating tester



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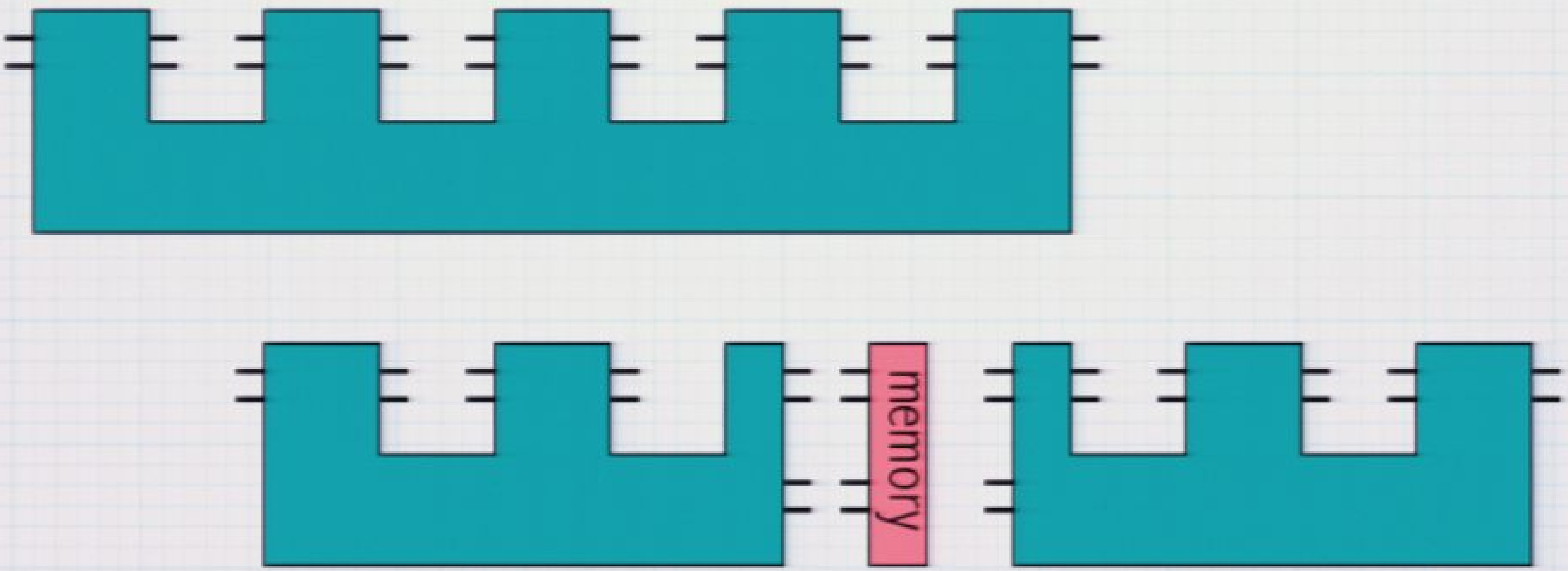
$$\Xi^{(N)} \equiv \Xi, \; \mathrm{Tr}_1[\Xi^{(0)}] = 1$$

Using quantum memory

delay the use of subcircuits by breaking the comb into subcombs + quantum memory




Applic
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Discrimination of unitaries
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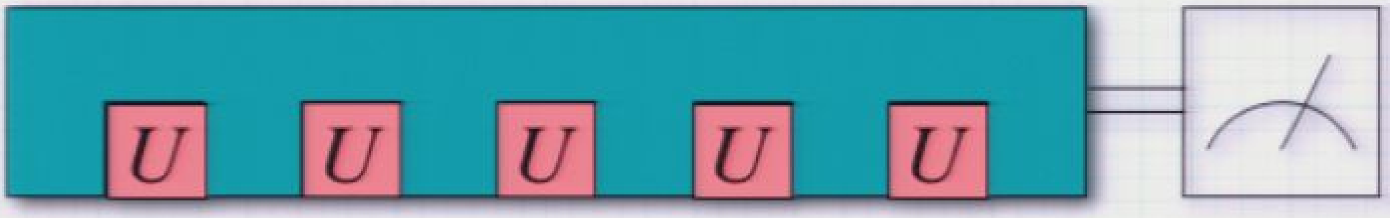
G.Chiribella, G.M.D'Ariano, P.Perinotti

arXiv:0803.3237

Discrimination of unitaries

Optimal discrimination between two possible unitary operators U_1 U_2



G.Chiribella, G.M.D'Ariano, P.Perinotti arXiv:0803.3237

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Discrimination of unitaries

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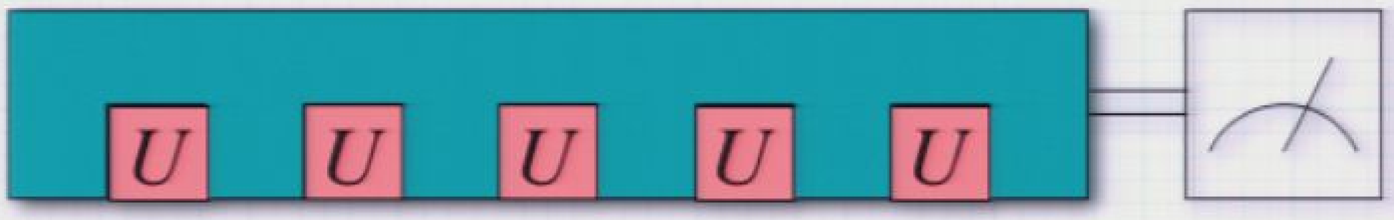


The parallel strategy is already optimal!

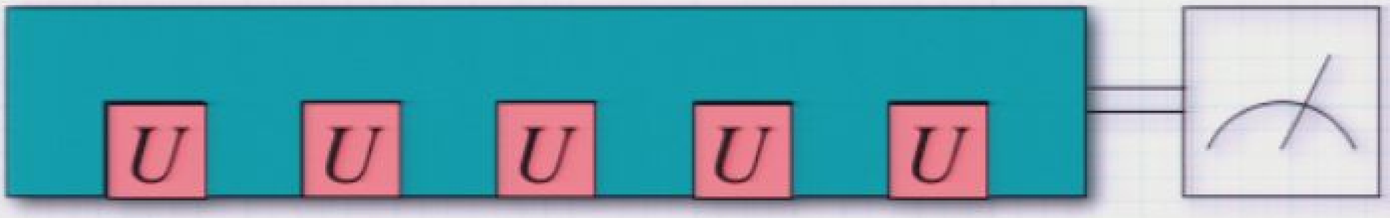
G.Chiribella, G.M.D'Ariano, P.Perinotti arXiv:0803.3237

QUit quantum information theory group

Discrimination of unitaries

arXiv:0803.3237

Optimal discrimination between two possible unitary operators U_1 U_2



The parallel strategy is already optimal!

Optimal discrimination of channels

Optimal estimation of unitaries

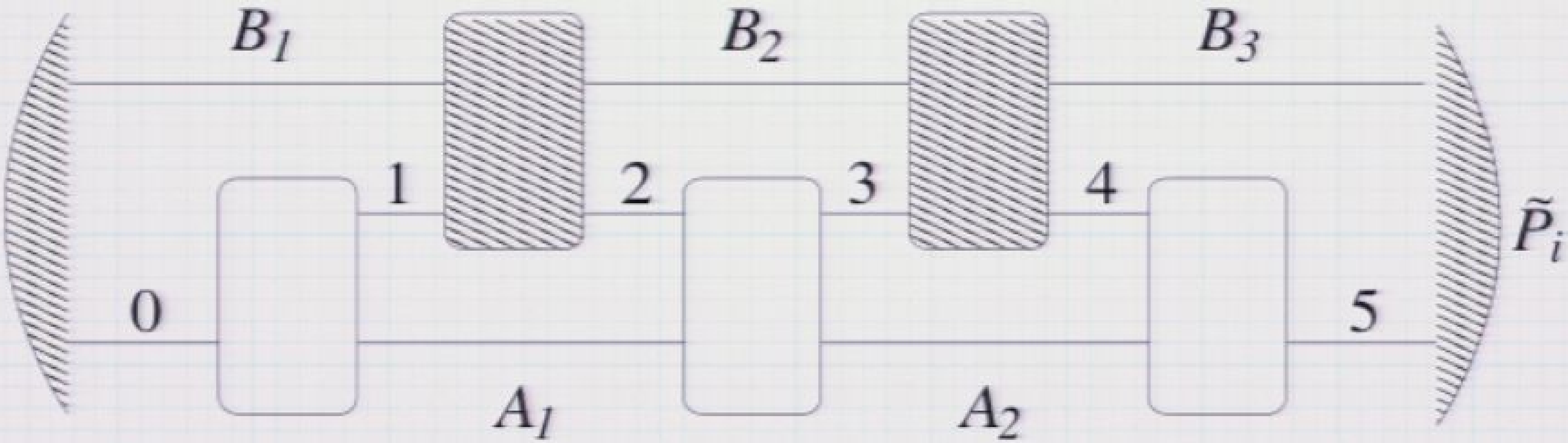
STILL OPEN PROBLEMS

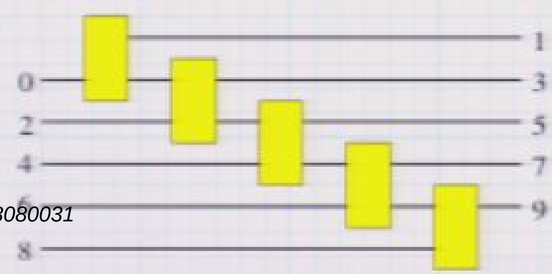
G.Chiribella, G.M.D'Ariano, P.Perinotti arXiv:0803.3237

Discrimination of memory channels

arXiv:0803.3237

There are memory channels that can be discriminated perfectly with a single use by a quantum tester, and not conventionally





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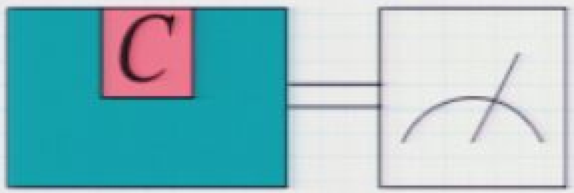
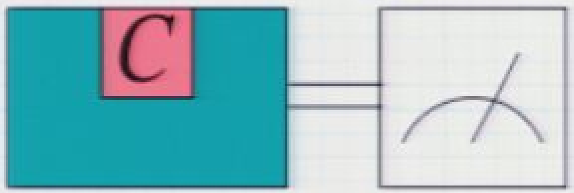
Optimal tomographers



(d^4 outcomes)

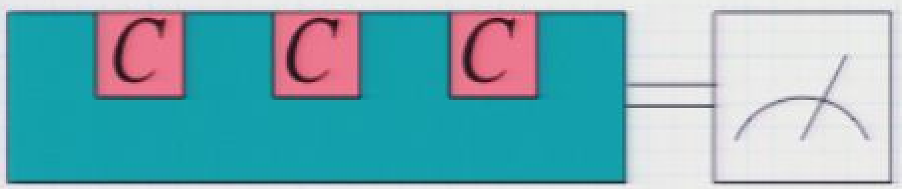
Informationally complete tester

Optimal tomographers



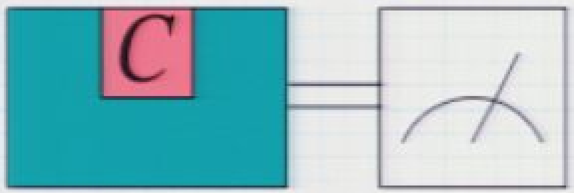
(d^4 outcomes)

Informationally complete tester



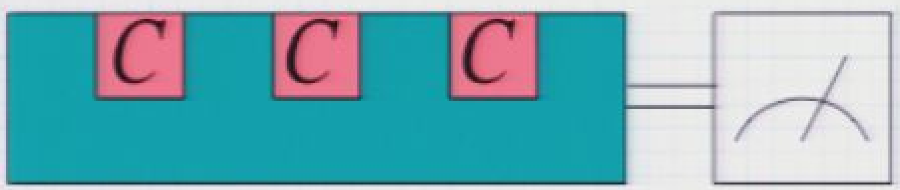
multiple uses

Optimal tomographers

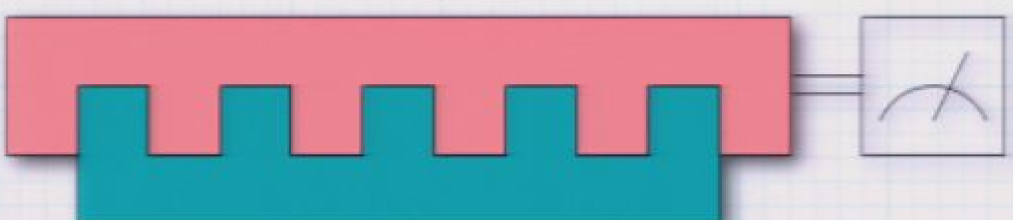


(d⁴ outcomes)

Informationally complete tester



multiple uses

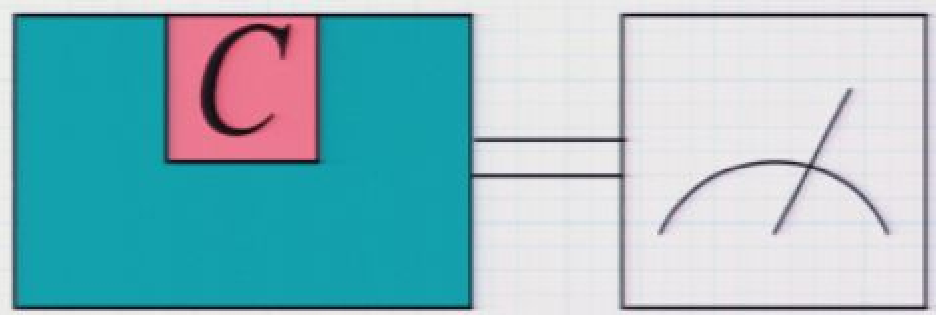


circuit board tomographer

arXiv: 0806.1172

Optimal tomography

Use different in and out dimensions to unify: states, channels, and POVMs

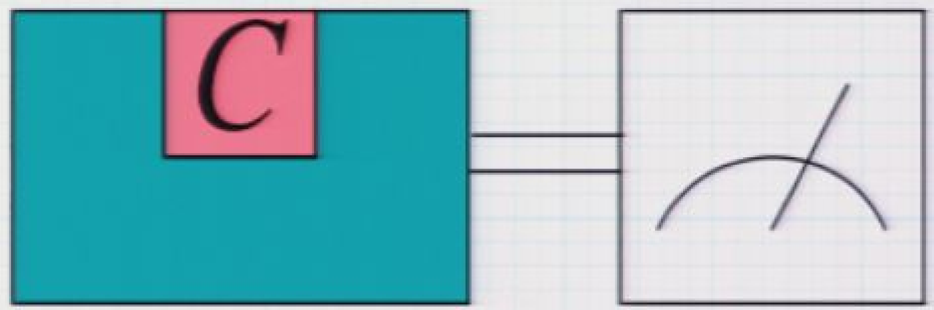


A. Bisio, G. Chiribella, G. M. D'Ariano, S. Facchini, P. Perinotti arXiv: 0806.1172 arXiv: 0806.1172

Optimal tomography

- Prior distribution of channels corresponding to the depolarizing average channel
- Cost function = representation, (equally weighted orthonormal set of operators)
- Further selection:
  1) quantum operations,
  2) channels,
  3) unital channels

Use different in and out dimensions to unify: states, channels, and POVMs



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