

Title: Undergraduate Summer Research Project Presentations

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Abstract: TBA

$2\frac{1}{2}$ Apps of BCFW

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$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \left(\frac{1}{2}, 0\right) \quad \left(0, \frac{1}{2}\right)$$

$[\cdot]$

$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \left(\frac{1}{2}, 0\right) \quad \left(0, \frac{1}{2}\right)$$

$[\cdot]$ γ^a $\tilde{\gamma}^a$ $a=1,2$

$$\sigma^\mu = \left(\sigma^0, \underbrace{\vec{\sigma}}_{\text{Pauli}} \right)$$

\mathbb{I}

$$P_\mu^f \rightarrow P_{aa} = P_\mu \sigma_{aa}^\mu$$

$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \begin{matrix} (\frac{1}{2}, 0) & (0, \frac{1}{2}) \\ \nearrow^a & \nearrow^a \\ & a=1,2 \end{matrix}$$

$$\sigma^\mu = \left(\underbrace{\sigma^0}_{\mathbb{I}}, \underbrace{\vec{\sigma}}_{\text{Pauli}} \right)$$

$$P_\mu \rightarrow P_{a\dot{a}} = P_\mu \sigma^\mu_{a\dot{a}}$$
$$P_\mu P^\mu = \det(P_{a\dot{a}})$$

$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \begin{matrix} (\frac{1}{2}, 0) \\ \gamma^a \end{matrix} \quad \begin{matrix} (0, \frac{1}{2}) \\ \tilde{\gamma}^a \end{matrix} \quad a=1,2$$

$$\sigma^\mu = \left(\underbrace{\sigma^0}_{I}, \underbrace{\vec{\sigma}}_{\text{Pauli}} \right)$$

$$P_\mu^{\nu} \Rightarrow P_{\alpha\dot{\alpha}} = P_\mu \sigma^\mu_{\alpha\dot{\alpha}}$$

$$P_\mu P^\mu = \det(P_{\alpha\dot{\alpha}})$$

$$P_{\alpha\dot{\alpha}} = \gamma^a_{\alpha\dot{\alpha}}$$

$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \left(\frac{1}{2}, 0\right) \quad \left(0, \frac{1}{2}\right)$$

$$e_{ab} \quad e_{12} = 1$$

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$\gamma^a$$

$$\tilde{\gamma}^a$$

$a = 1, 2$

$$\langle i j \rangle = \epsilon_{ab} \tilde{\gamma}_i^a \tilde{\gamma}_j^b$$

$$\sigma^\mu = (\sigma^0, \vec{\sigma})$$

Pauli

$$P_{\mu\nu} \rightarrow P_{\mu a} = P_{\mu} \sigma^{\mu}_{a\dot{a}}$$

$$P_{\mu} P^{\mu} = \det(P_{\mu a})$$

$$P_{\mu a} = \gamma_a \tilde{\gamma}_{\dot{a}}$$

$$[i j] = \epsilon_{ab} \gamma_i^a \gamma_j^b$$

$$A(1^-, 2^-, 3^+) = X \left(\frac{\langle 12 \rangle^2}{\langle 23 \rangle \langle 31 \rangle} \right)^5$$

$$A(1^+, 2^+, 3^-) = \text{" } (\langle \rangle \rightarrow [])$$

$$A(1^-, 2^-, 3^-) = X' (\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle)^5$$

$$A(1^+, 2^+, 3^+) = \text{" } (\langle \rangle \rightarrow [])$$

$$2p \cdot p' = \langle T, T' \rangle [\hat{T}, \hat{T}']$$

$$A(1^-, 2^-, 3^+) = \kappa \left(\frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle} \right)^5$$

$$A(1^+, 2^+, 3^-) = \text{" } (\langle \rangle \rightarrow [])$$

$$A(1^-, 2^-, 3^-) = \kappa' (\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle)^5$$

$$A(1^+, 2^+, 3^+) = \text{" } (\langle \rangle \cdot)$$

$$2p \cdot p' = \langle T, T' \rangle [\tilde{T}, \tilde{T}']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^\alpha \frac{\partial}{\partial \tilde{T}_i^\alpha} \right) | T_i$$

$$A(1^-, 2^-, 3^+) = \kappa \left(\frac{\langle 12 \rangle^2}{\langle 23 \rangle \langle 31 \rangle} \right)^5$$

$$A(1^+, 2^+, 3^-) = \text{" } (\langle \rangle \rightarrow [])$$

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$$A(1^+, 2^+, 3^+) = \text{" } (\langle \rangle \rightarrow [])$$

$$2p \cdot p' = \langle T, T' \rangle [\tilde{T}, \tilde{T}']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^{\dot{\alpha}} \frac{\partial}{\partial \tilde{T}_i^{\dot{\alpha}}} \right) |T_i, \tilde{T}_i, h\rangle = -2h_i |T_i, \tilde{T}_i, h\rangle$$

$$A(1^-, 2^-, 3^+) = \kappa \left(\frac{\langle 12 \rangle^2}{\langle 23 \rangle \langle 31 \rangle} \right)^5$$

$$A(1^+, 2^+, 3^-) = \text{" } (\langle \rangle \rightarrow [])$$

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$$2p \cdot p' = \langle T, T' \rangle [\tilde{T}, \tilde{T}']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^{\dot{\alpha}} \frac{\partial}{\partial \tilde{T}_i^{\dot{\alpha}}} \right) |T_i, \tilde{T}_i, h\rangle = -2h_i |T_i, \tilde{T}_i, h\rangle$$

$$A(1^-, 2^-, 3^+) = \kappa \left(\frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle} \right)^5$$

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$$A(1^-, 2^-, 3^-) = \kappa' (\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle)^5$$

$$A(1^+, 2^+, 3^+) = \text{" } (\langle \rangle \rightarrow [])$$

$$2p \cdot p' = \langle T, T' \rangle [\tilde{T}, \tilde{T}']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^{\dot{\alpha}} \frac{\partial}{\partial \tilde{T}_i^{\dot{\alpha}}} \right) |T_i, \tilde{T}_i, h\rangle = -2h_i |T_i, \tilde{T}_i, h\rangle$$

$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \left(\frac{1}{2}, 0\right) \quad \left(0, \frac{1}{2}\right)$$

$$\epsilon_{ab} \epsilon_{12} = 1 \quad \left[\begin{array}{c} \cdot \\ \cdot \end{array} \right] \quad \gamma^a \quad \tilde{\gamma}^a \quad a=1,2$$

$$\langle i, j \rangle \equiv \epsilon_{ab} \tilde{\gamma}_i^a \tilde{\gamma}_j^b \quad \sigma^\mu = \left(\sigma^0, \vec{\sigma} \right)$$

$$[i, j] \equiv \epsilon_{ab} \tilde{\gamma}_i^a \tilde{\gamma}_j^b \quad \mathbb{I} \quad \underbrace{\vec{\sigma}}_{\text{Pauli}}$$

$$P_{ai} = P_\mu \sigma^\mu_{ai}$$

$$\Delta = \det(P_{ai})$$

$$\tilde{\gamma}_a$$



$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \left(\frac{1}{2}, 0\right) \quad \left(0, \frac{1}{2}\right)$$

$$\epsilon_{ab} \epsilon_{12} = 1 \quad \left[\begin{array}{c} \cdot \\ \cdot \end{array} \right] \quad \gamma^a \quad \tilde{\gamma}^a \quad a=1,2$$

$$\langle i j \rangle \equiv \epsilon_{ab} T_i^a T_j^b \quad \sigma^\mu = \left(\sigma^0, \vec{\sigma} \right)$$

$$[i j] \equiv \epsilon_{ab} \tilde{T}_i^a \tilde{T}_j^b \quad \begin{array}{c} \mathbb{I} \\ \parallel \\ \text{Pauli} \end{array}$$

$$P_\mu \Rightarrow P_{a\dot{a}} = P_\mu \sigma^\mu_{a\dot{a}}$$

$$P_\mu P^\mu = \det(P_{a\dot{a}})$$

$$P_{a\dot{a}} = \gamma_a \tilde{\gamma}_{\dot{a}}$$

$2\frac{1}{2}$ Apps of BCFW

$$SO(3,1) \cong SL(2, \mathbb{C}) \quad \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \quad \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

$\gamma^a \quad \gamma^{\dot{a}} \quad a=1,2$

$$\epsilon_{ab} \epsilon_{\dot{a}\dot{b}} = 1$$

$$\langle i, j \rangle = \epsilon_{ab} \gamma_i^a \gamma_j^b \quad \sigma^\mu = \left(\sigma^0, \vec{\sigma} \right)$$

$\mathbb{I} \quad \text{Pauli}$

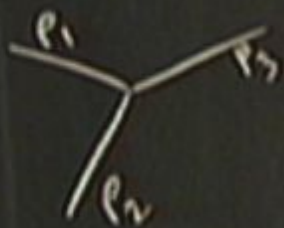
$$[i, j] = \epsilon_{ab} \tilde{\gamma}_i^a \tilde{\gamma}_j^b$$

$$P_\mu \rightarrow P_{aa} = P_\mu \sigma_{\dot{a}\dot{a}}^\mu$$

$$P_\mu P^\mu = \det(P_{aa})$$

$$P_{aa} = \gamma_a^\mu \tilde{\gamma}_{\dot{a}\dot{a}}^\mu$$

P_1, P_2, P_3



$$(P_1 + P_2 + P_3)^2 = 0$$

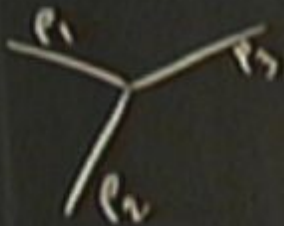
$$A(1^-, 2^-, 3^-) = \kappa' (\langle 1, 2 \rangle \langle 2, 3 \rangle \dots)$$

$$A(1^+, 2^+, 3^+) = \dots (\langle \rangle \rightarrow L \dots)$$

$$2p \cdot p' = \langle T, T' \rangle [\tilde{\gamma}, \tilde{\gamma}']$$

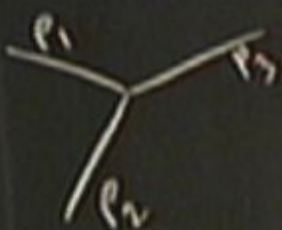
$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{\gamma}_i^\alpha \frac{\partial}{\partial \tilde{\gamma}_i^\alpha} \right) |T_i, \tilde{\gamma}_i, h\rangle = -L |T_i, \tilde{\gamma}_i, h\rangle$$

p_1, p_2, p_3



$$(p_1 + p_2 + p_3)^2 = 0 \quad p_i^2 = 0$$
$$\Rightarrow p_i \cdot p_j = 0$$
$$\langle i, j \rangle [i, j] = 0$$

P_1, P_2, P_3



$$(P_1 + P_2 + P_3)^2 = 0 \quad P_i^2 = 0$$

$$\Rightarrow P_i \cdot P_j = 0$$

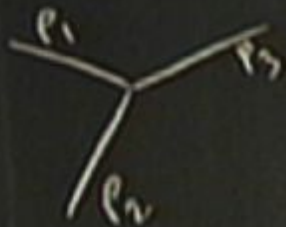
$$\langle i, j \rangle [i, j] = 0$$

$$\langle i, j \rangle = 0$$

$$\Rightarrow T^i \propto T^j$$

$$A = c_1 A_1 (\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle) + c_2 A_2 ([12] [23] [31])$$

p_1, p_2, p_3



$$(p_1 + p_2 + p_3)^2 = 0 \quad p_i^2 = 0$$

$$\Rightarrow p_i \cdot p_j = 0$$

$$\langle i | j \rangle [i | j] = 0$$

$$\langle i | j \rangle = 0$$

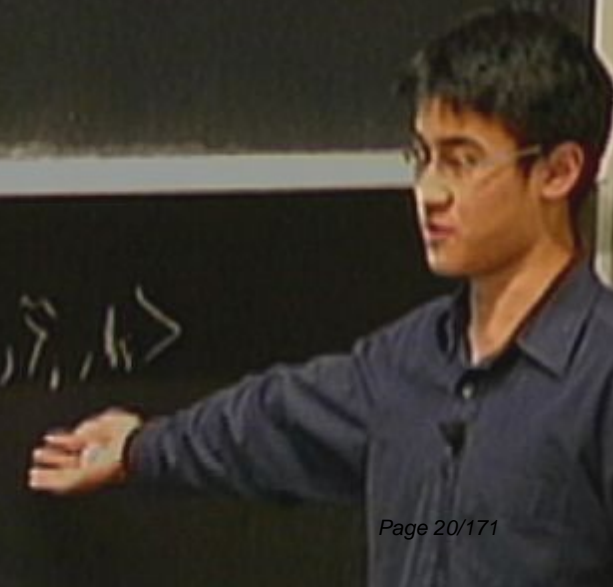
$$\Rightarrow T^i \propto T^j$$

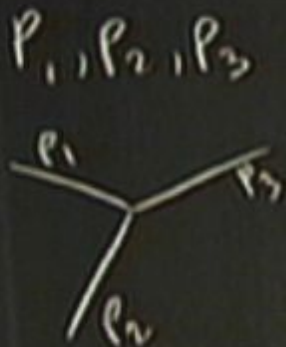
$$A = c_1 A_1 (\langle 12 | \langle 23 | \langle 31 |) + c_2 A_2 ([12] [23] [31])$$

$$\equiv \sum_{d_1, d_2, d_3}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^{d_1} \langle 23 \rangle^{d_2} \langle 13 \rangle^{d_3} \quad \langle out | S | in \rangle$$

$$2p \cdot p' = \langle T, T' \rangle [T, T']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^\alpha \frac{\partial}{\partial \tilde{T}_i^\alpha} \right) |T_i, \tilde{T}_i, h\rangle = -2h |T_i, \tilde{T}_i, h\rangle$$





$$(P_1 + P_2 + P_3)^2 = 0 \quad P_i^2 = 0$$

$$\Rightarrow P_i \cdot P_j = 0$$

$$\langle i | j \rangle [i | j] = 0$$

$$\langle i | j \rangle = 0$$

$$\Rightarrow T^i \propto T^j$$

$$\begin{aligned} d_3 &= h_3 - h_1 - h_2 \\ d_1 &= h_1 - h_3 - h_2 \\ d_2 &= h_2 - h_3 - h_1 \end{aligned}$$

$$A = c_1 A_1 (\langle 12 | \langle 23 | \langle 31 |) + c_2 A_2 ([12][23][31])$$

$$= \sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^{d_1} \langle 23 \rangle^{d_2} \langle 13 \rangle^{d_3}$$

$$\langle \text{out} | S | \text{in} \rangle$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - T_i^\beta \frac{\partial}{\partial T_i^\beta} \right) |T_i, \tilde{T}_i, h\rangle = -2h_i |T_i, \tilde{T}_i, h\rangle$$

$$\sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^2 \langle 23 \rangle^2 \langle 13 \rangle \quad \langle \text{out} | S | \text{in} \rangle$$

$$A(1^-, 2^-, 3^+) = \kappa \left(\frac{\langle 12 \rangle^2}{\langle 23 \rangle \langle 31 \rangle} \right)^2$$

$$A(1^+, 2^+, 3^-) = \kappa \langle \dots \rangle$$

$$A(1^-, 2^-, 3^-) = \kappa' \langle 12 \rangle \langle 23 \rangle \langle 31 \rangle$$

$$A(1^+, 2^+, 3^+) = \kappa'' \langle \dots \rangle$$

$$2p \cdot p' = \langle T, T' \rangle [\tilde{T}, \tilde{T}']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^\alpha \frac{\partial}{\partial \tilde{T}_i^\alpha} \right) |T_i, \tilde{T}_i, h\rangle = -2h |T_i, \tilde{T}_i, h\rangle$$

$$\sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^{d_1} \langle 23 \rangle^{d_2} \langle 13 \rangle^{d_3}$$

$$\langle \text{out} | S | \text{in} \rangle$$

$$A(p_1, \dots, p_m)$$

$$p_i(z) = p_i$$



$$\sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^{d_1} \langle 23 \rangle^{d_2} \langle 13 \rangle^{d_3}$$

$$\langle \text{out} | S | \text{in} \rangle$$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$\sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^3 \langle 23 \rangle^1 \langle 13 \rangle$$

$\langle \text{out} | S | \text{in} \rangle$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z}$$

$$\sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^{d_1} \langle 23 \rangle^{d_2} \langle 13 \rangle^{d_3} \quad \langle \text{out} | S | \text{in} \rangle$$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint A$$

$$\sum_{-\infty}^{+\infty} C_{d_1 d_2 d_3} \langle 12 \rangle^{d_1} \langle 23 \rangle^{d_2} \langle 13 \rangle^{d_3} \quad \langle \text{out} | S | \text{in} \rangle$$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint_{z=z_i} \frac{A(z)}{z}$$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint_{z=z_i} \frac{A(z)}{z} = -\sum_{z_i \neq 0} \text{Res} \left(\frac{A(z)}{z} \right)$$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint_{z=z_i} \frac{A(z)}{z} = -\sum_{z_i \neq 0} \text{Res} \left(\frac{A(z)}{z} \right)$$

$$z_i \quad (p_1(z_i) + p_a + \dots + p_\ell)^2 = 0$$

$$A(p_1, \dots, p_m)$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint_{z=z_i} \frac{A(z)}{z} = -\sum_{z_i \neq 0} \text{Res} \left(\frac{A(z)}{z} \right)$$

$$z_i \quad (p_1(z_i) + p_a + \dots + p_\lambda)^2 = 0$$



$$A(p_1, \dots, p_m)$$

$$q^2 = q \cdot p_1 = q \cdot p_2 = 0$$

$$p_1(z) = p_1 + zq$$

$$p_2(z) = p_2 - zq$$

$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint_{z=z_i} \frac{A(z)}{z} = -\sum_{z_i \neq 0} \text{Res} \left(\frac{A(z)}{z} \right)$$

$$z_i \quad (p_1(z_i) + p_a + \dots + p_\lambda)^2 = 0$$



$$g = g \cdot p_1 = g \cdot p_2 = 0$$

$$p_1(z) = p_1 + zq$$

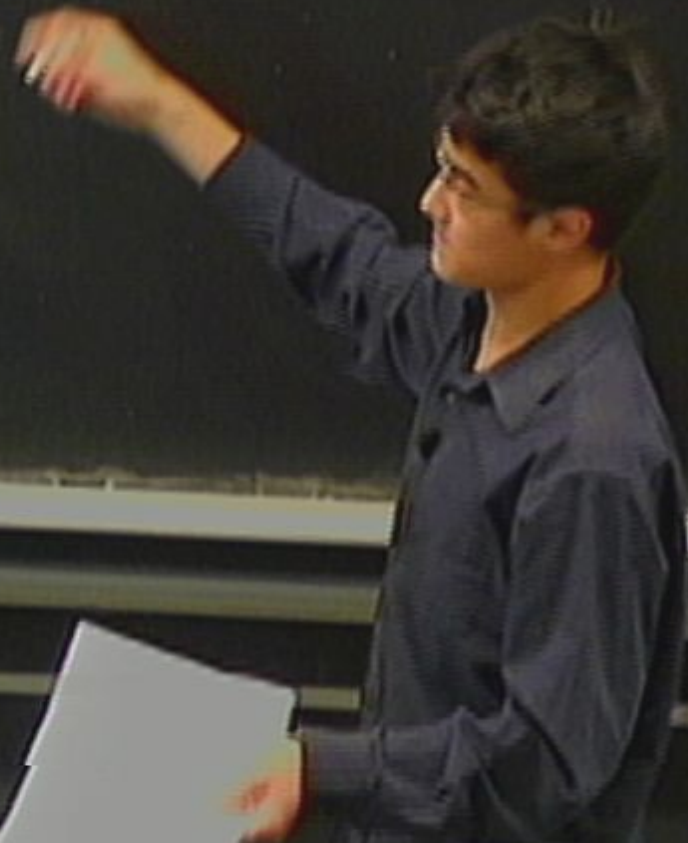
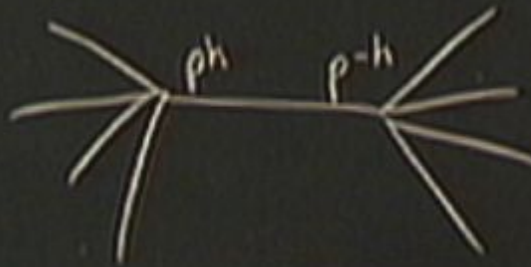
$$p_2(z) = p_2 - zq$$

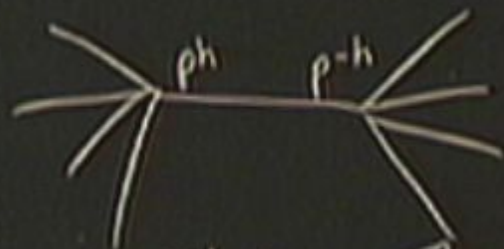
$$A = A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = -\frac{1}{2\pi i} \sum_{z_i \neq 0} \oint_{z=z_i} \frac{A(z)}{z} = -\sum_{z_i \neq 0} \text{Res} \left(\frac{A(z)}{z} \right)$$

$$z_i \quad (p_1(z_i) + p_a + \dots + p_\lambda)^2 = 0$$



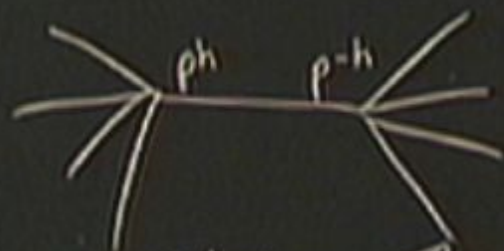
$A(s_1(z), p^h) A(s_2(z), p^h)$





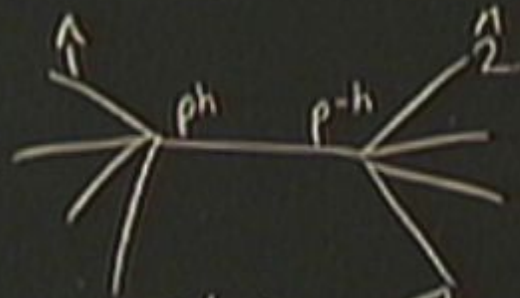
$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p^{-h})^2} A(S_2(z), p^{-h})$$

$$A = \sum_{\substack{\text{finite} \\ z_i}} \sum_h A(S_1(z_i), p^h) \frac{1}{\left(\sum (S_2(z_i), p^{-h})\right)}$$



$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_j t_j)^2} A(S_2(z), p^{-h})$$

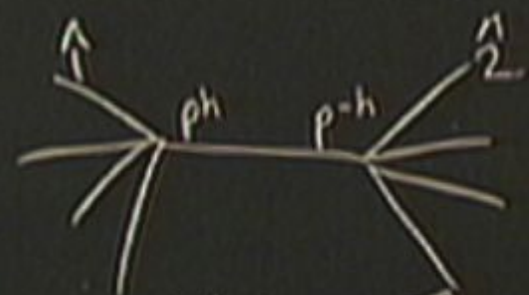
$$A = \sum_{\substack{\text{finite} \\ z_i}} \sum_h A(S_1(z_i), p^h) \frac{1}{(\sum_j p_j)^2} A(S_2(z_i), p^{-h})$$



$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_j t_j)^2} A(S_2(z), p^{-h})$$

$$A = \sum_{\substack{\text{finite} \\ z_i}} \sum_h A(S_1(z_i), p^h) \frac{1}{(\sum_j p_j)^2} A(S_2(z_i), p^{-h})$$

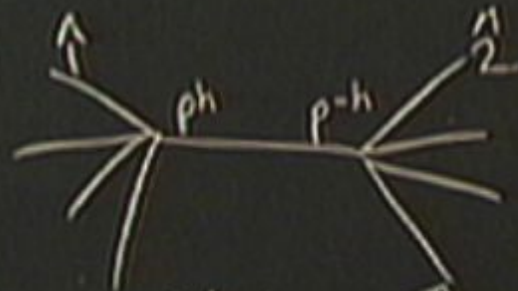
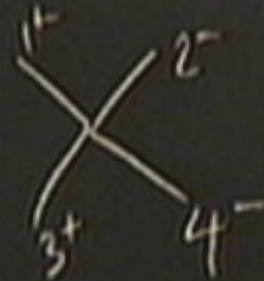
$$A(1^+, 2^-, 3^+, 4^-) = -K^2 \frac{([13]K_2 4^-)^4}{stu}$$



$$A(z) = \sum_h A(S_1(z), p^h) \cdot A(S_2(z), p^{-h})$$

$$A = \sum_{\substack{\text{finite} \\ z_i}} \sum_h A(S_1(z_i), p^h) \cdot A(S_2(z_i), p^{-h})$$

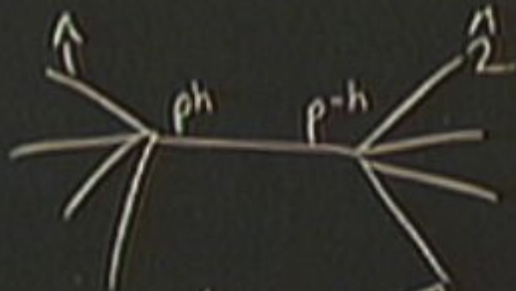
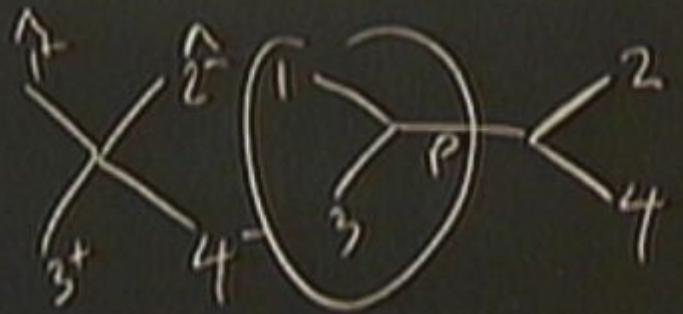
$$A(1^+, 2^-, 3^+, 4^-) = -K^2 \frac{([13]K[24])^4}{stu}$$



$$A(z) = \sum_h A(S_i(z), p^h) \frac{1}{(s(z) + \sum_i t_i)} A(z, p^{-1})$$

$$A = \sum_{\text{finite } z_i} \sum_h A(S_i(z_i), p^h) \frac{1}{(\sum_i t_i)^2} A(z, p^{-1})$$

$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{([13]k_2 4^-)^{\text{tr}}}{stu}$$



$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_i t_i)^2} A(S_2(z), p^{-h})$$

$$A = \sum_{\substack{\text{finite} \\ z_i}} \sum_h A(S_1(z_i), p^h) \frac{1}{(\sum_i p_i)^2} A(S_2(z_i), p^{-h})$$

$$A(1^-, 2^-, 3^+) = \kappa \left(\frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle} \right)^5$$

$$A(1^+, 2^+, 3^-) = \text{" } (\langle \rangle \rightarrow [])$$

$$A(1^-, 2^-, 3^-) = \kappa' (\langle 12 \rangle \langle 23 \rangle \langle 31 \rangle)^5$$

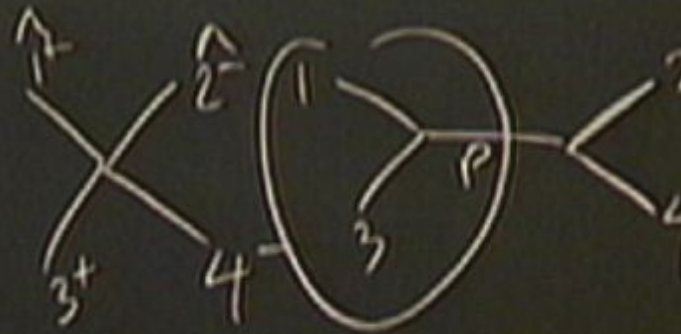
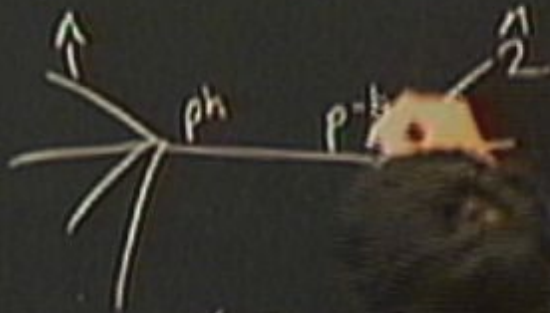
$$A(1^+, 2^+, 3^+) = \text{" } (\langle \rangle \rightarrow [])$$

$$L_P \cdot P' = \langle T, T' \rangle [\tilde{T}, \tilde{T}']$$

$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^{\alpha'} \frac{\partial}{\partial \tilde{T}_i^{\alpha'}} \right) |T_i, \tilde{T}_i, h\rangle = -2h_i |T_i, \tilde{T}_i, h\rangle$$

$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{([13][24])^4}{stu}$$

$$\kappa^2 = 0$$



$$A(z) = \frac{1}{(p_i(z) + \sum_j t_j)^2} A(S_2(z), p^h)$$

$$A = \sum_{\text{finite } z_i} \sum_h \frac{1}{(\sum_j p_j)^2} A(S_2(z), p^h)$$

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$$2p \cdot p' = \langle \gamma, \gamma' \rangle [\vec{\gamma}, \vec{\gamma}']$$

$$\left(\gamma_i \frac{\partial}{\partial x_i} - \gamma'_i \frac{\partial}{\partial x'_i} \right) \psi = -2h_i | \gamma_i, \vec{\gamma}_i, h_i \rangle$$

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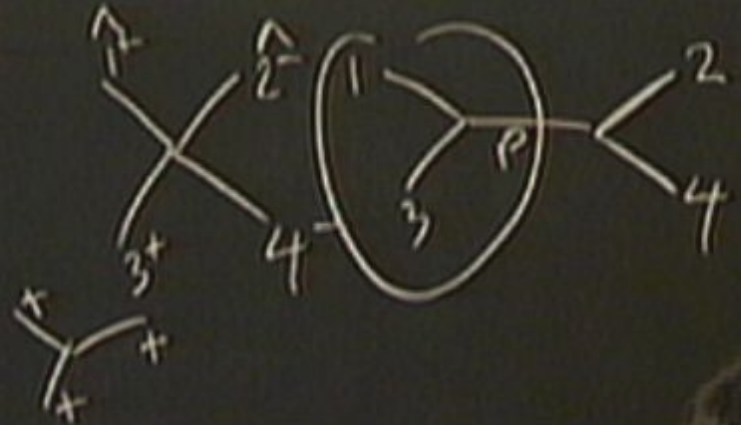
$$A(1^+, 2^+, 3^+) = \text{" } (\langle \rangle \rightarrow [])$$

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$$\left(T_i^\alpha \frac{\partial}{\partial T_i^\alpha} - \tilde{T}_i^\alpha \frac{\partial}{\partial \tilde{T}_i^\alpha} \right) | T_i, \tilde{T}_i, h \rangle = -2h_i | T_i, \tilde{T}_i, h_i \rangle$$

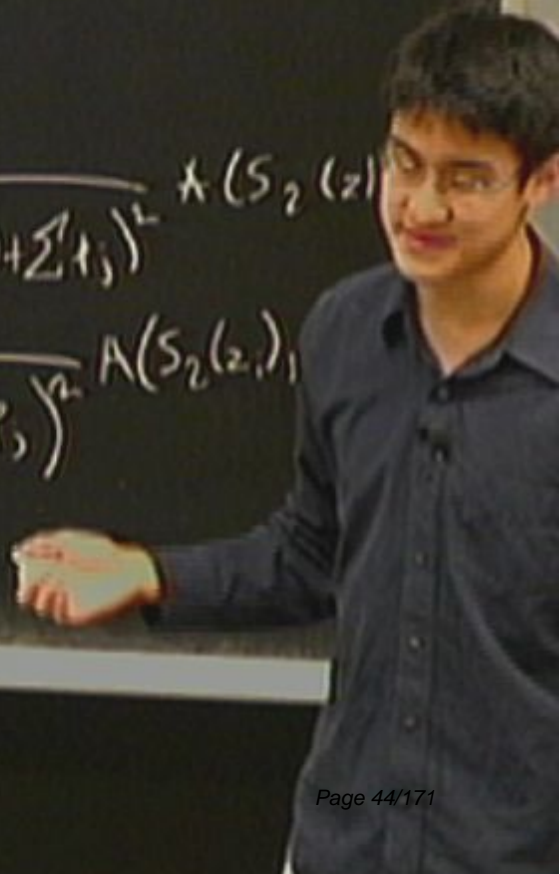
$$A(1^+, 2^-, 3^+, 4^-) = -K^2 \frac{([13][24])^4}{stu}$$

$$K^2 = 0$$



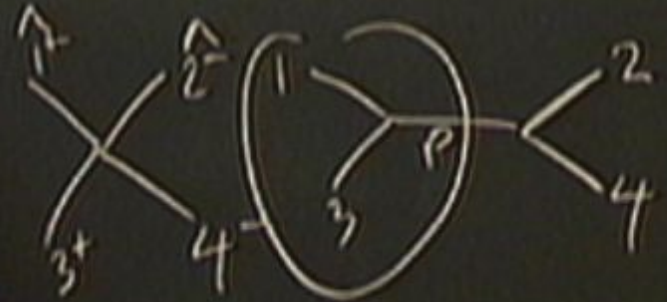
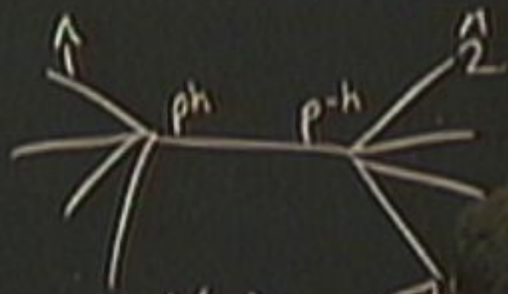
$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_i t_i)^2} * A(S_2(z))$$

$$A = \sum_{\text{finite } z_i} \sum_h A(S_1(z_i), p^h) \frac{1}{(\sum_i p_i)^2} A(S_2(z_i))$$



$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{(1234)^4}{stu}$$

$$\kappa^2 = 0$$

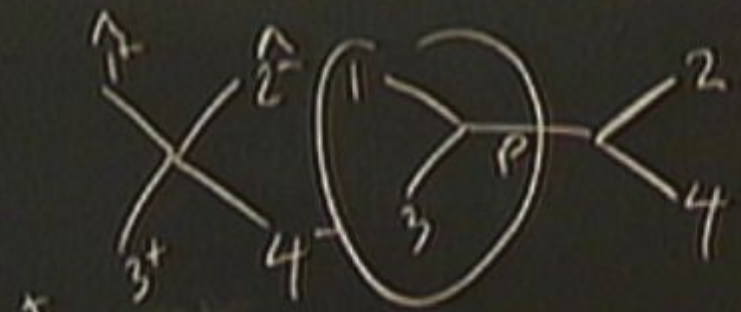
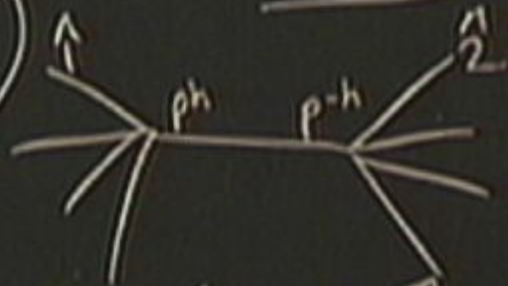


$$A(z) = \sum_h A(S_2(z, p^h)) \frac{1}{(p_1(z) + \sum_i t_i)^2} \kappa(S_2(z, p^{-h}))$$

$$A = \sum_{\text{finite } z_i} \sum_h A(S_2(z, p^h)) \frac{1}{(\sum_i p_i)^2} A(S_2(z, p^{-h}))$$

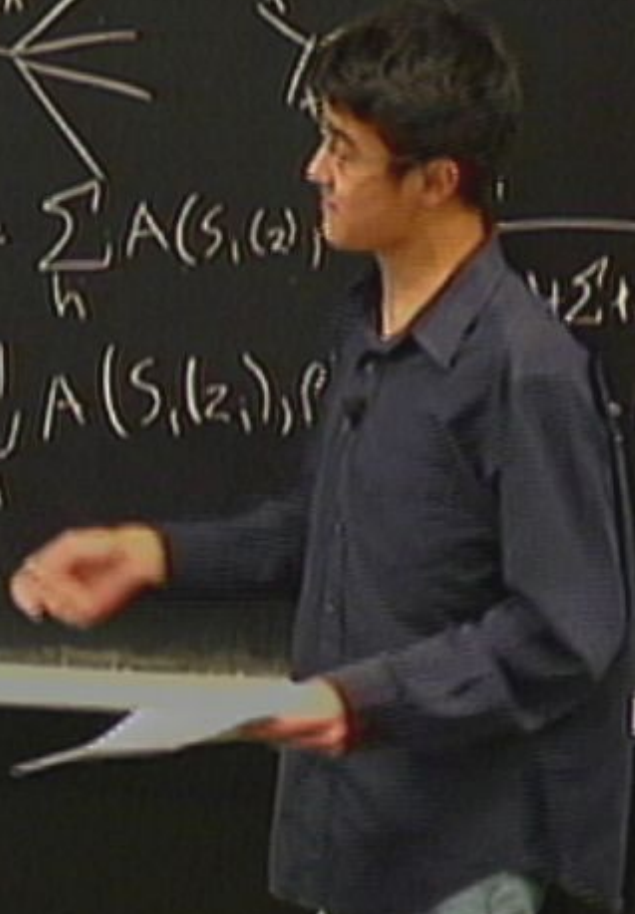
$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{(013k24^-)^4}{stu}$$

$$A(1^+, 2^-, 3^-, 4^-) \quad \kappa^2 = 0$$



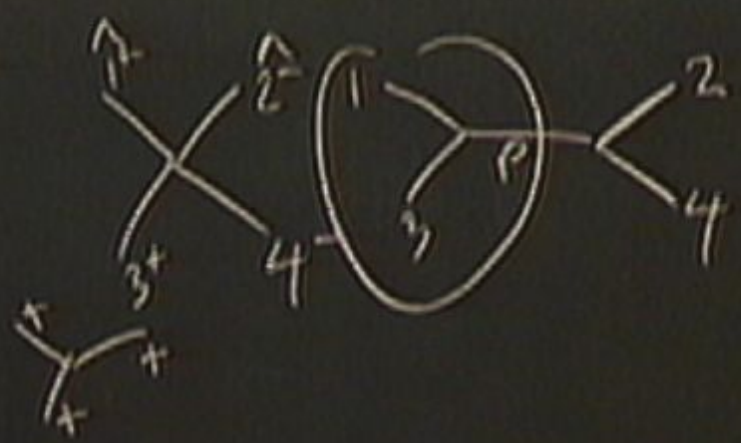
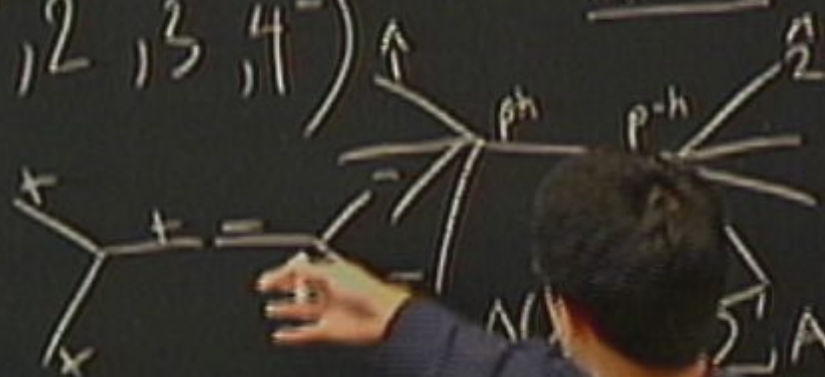
$$A(z) = \sum_h A(S_1(z), P^h) \cdot A(S_2(z), P^{-h})$$

$$A = \sum_{\text{finite } z_i} \sum_h A(S_1(z_i), P^h) \cdot A(S_2(z_i), P^{-h})$$



$$A(1^+, 2^-, 3^+, 4^-) = -K^2 \frac{(1234)^4}{stu}$$

$$A(1^+, 2^-, 3^-, 4^-) \quad \chi^2 = 0$$



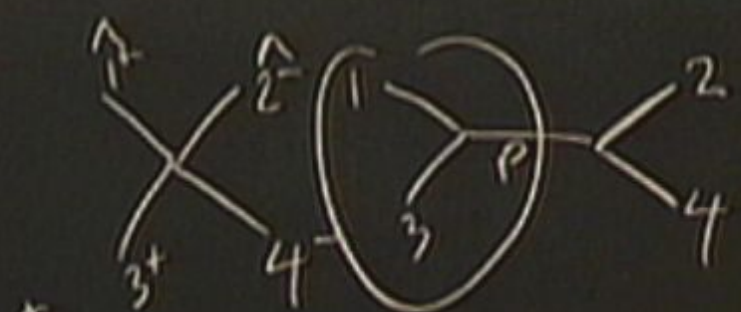
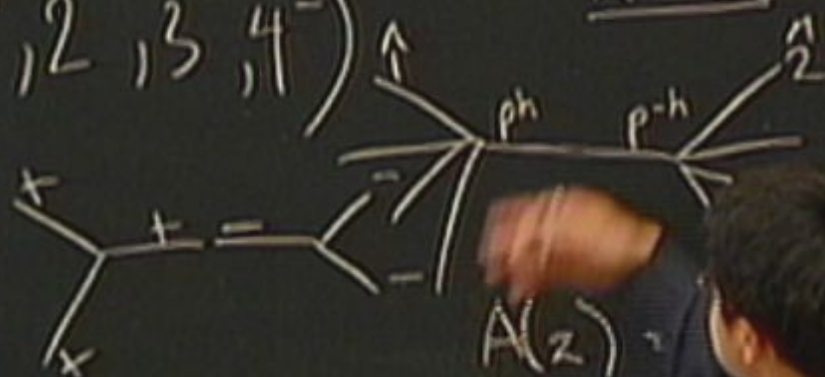
$$A = \sum_i A(S_1(z_i), p^h) \frac{1}{(p_1(z_i) + \sum_j t_j)^2} A(S_2(z_i), p^{-h})$$

$$A = \sum_i A(S_1(z_i), p^h) \frac{1}{(\sum_j p_j)^2} A(S_2(z_i), p^{-h})$$

Spin 2

$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{([13]k_2 4^-)^4}{stu}$$

$$A(1^+, 2^-, 3^-, 4^-) \quad \kappa^2 = 0$$



$$A(z) = \dots$$

$$A = \sum_{\text{finite } z_i} \sum_h \dots$$

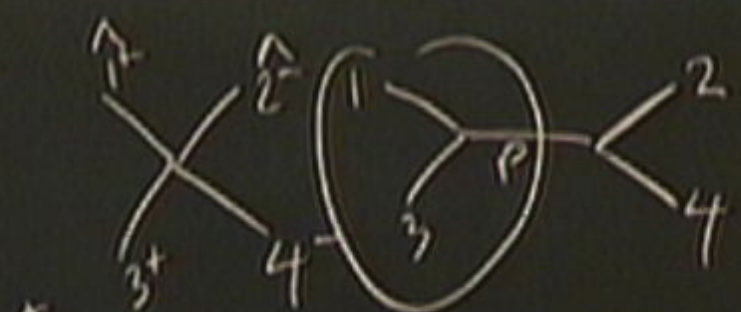
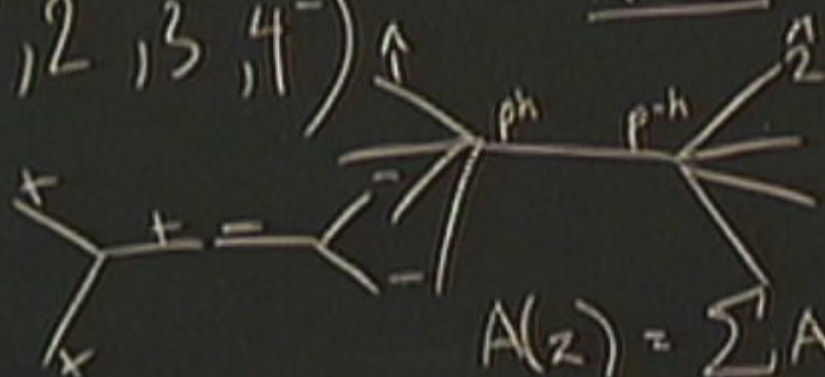
$$\dots \frac{1}{(p_i(z) + \sum_j t_j)^2} A(S_2(z), P^{-1})$$

$$\dots \frac{1}{(\sum_j p_j)^2} A(S_2(z), P^{-h})$$

Spin 2

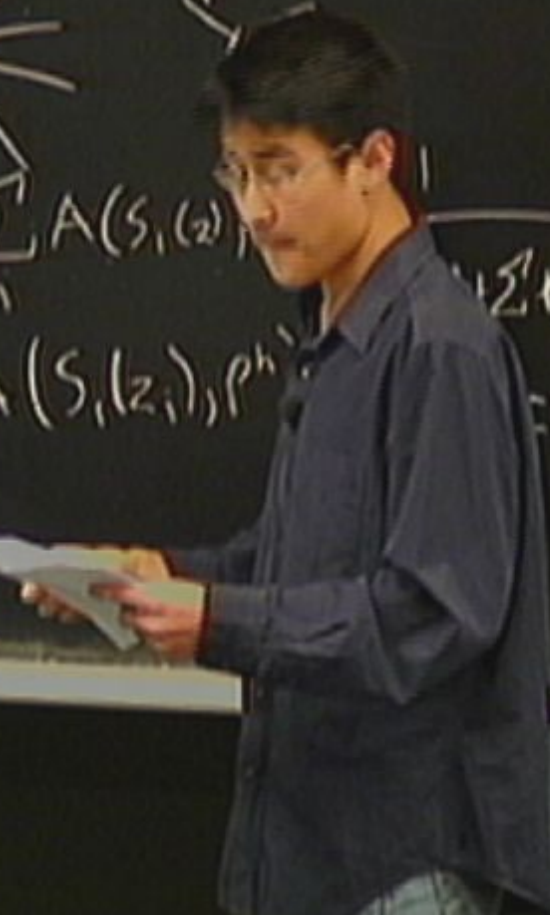
$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{(013k24^-)^4}{stu}$$

$$A(1^+, 2^-, 3^-, 4^-) \quad \kappa^2 = 0$$



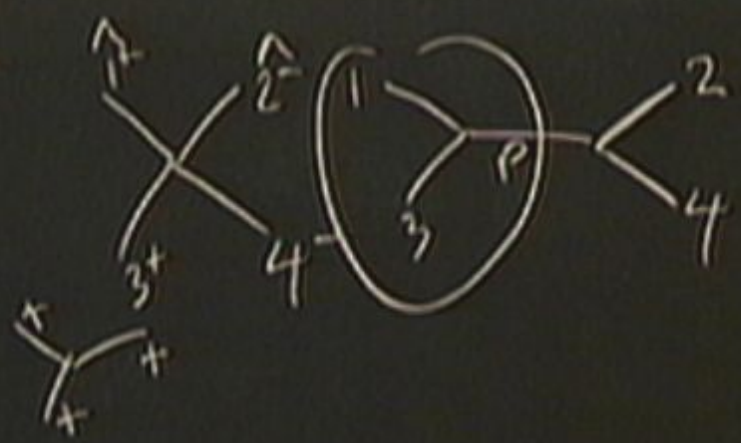
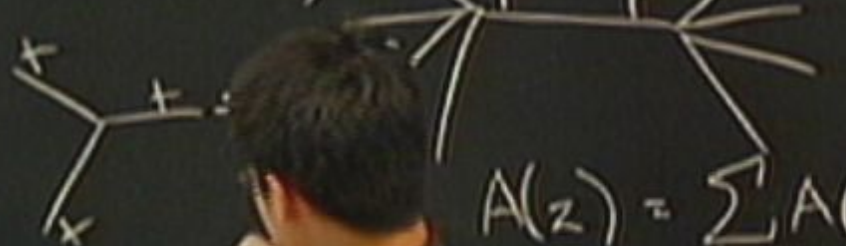
$$A(z) = \sum_h A(S_1(z), P^h) \rightarrow A(S_2(z), P^{-h})$$

$$A = \sum_{\text{finite } z_i} \sum_h A(S_1(z_i), P^h) = A(S_2(z_i), P^{-h})$$



$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{(\langle 13 | k_2 | 4 \rangle)^4}{stu}$$

$$A(1^+, 2^-, 3^-, 4^-) \quad \kappa^2 = 0$$



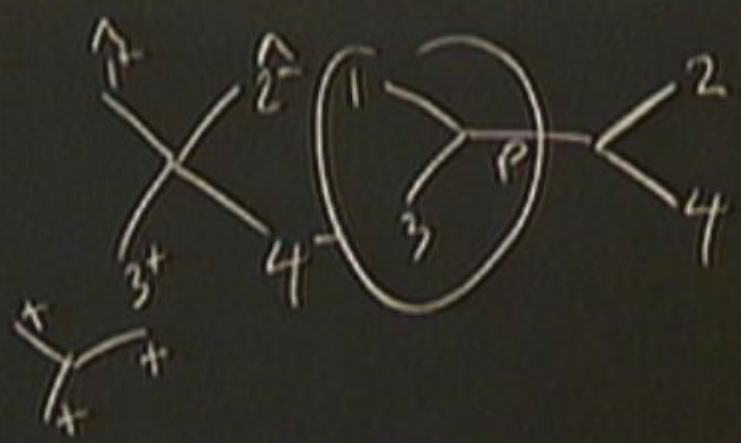
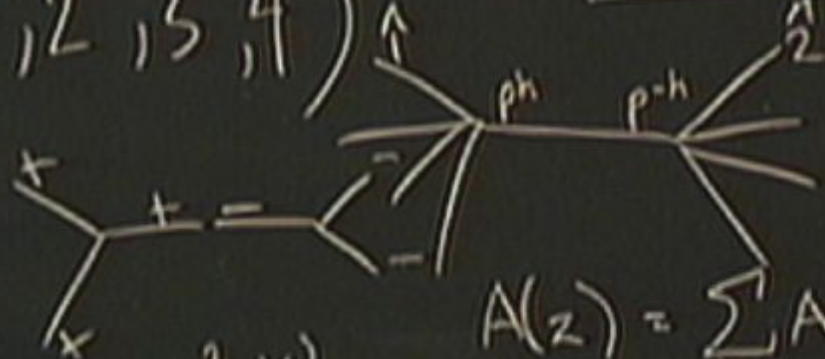
$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_j t_j)^2} A(S_2(z), p^{-h})$$

$$\sum_h A(S_1(z, i), p^h) \frac{1}{(\sum_j p_j)^2} A(S_2(z, i), p^{-h})$$

Spin 2

$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{(013k24^-)^4}{stu}$$

$$A(1^+, 2^-, 3^-, 4^-) \quad \kappa^2 = 0$$



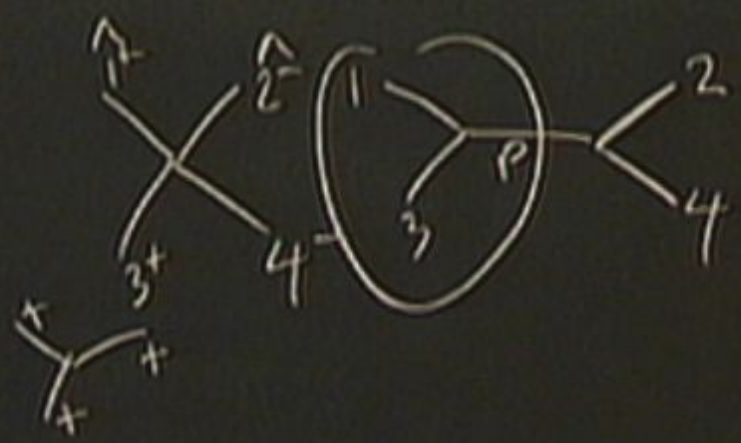
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$$A = \sum_{\text{finite } z_i} \sum_h A(S_1(z_i), p^h) \frac{1}{(\sum_i p_i)^2} A(S_2(z_i), p^h)$$

Spin 2

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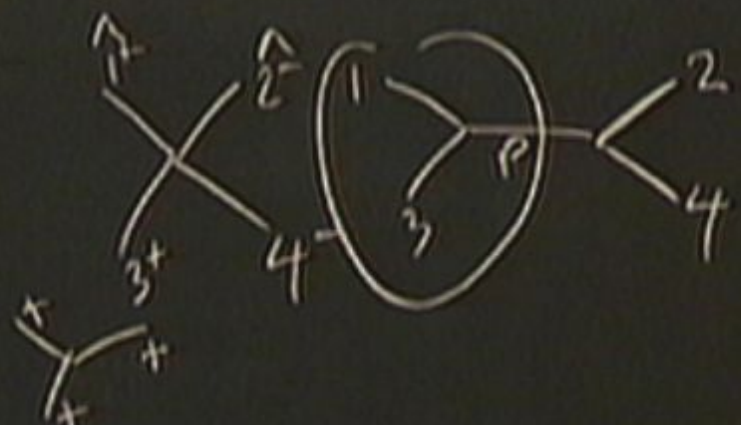
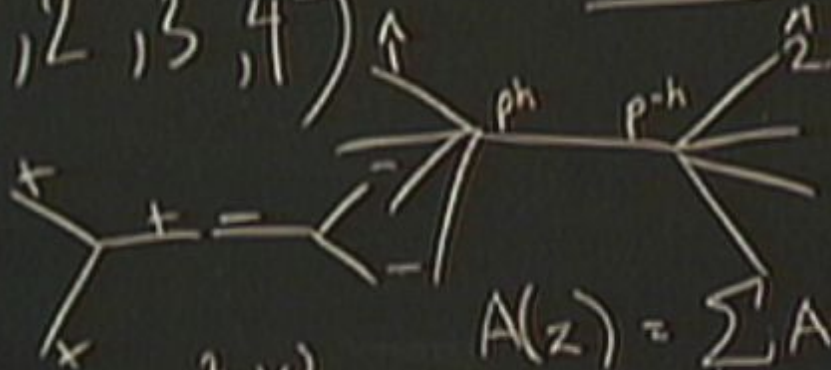
$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_j t_j)^2} A(S_2(z), p^{-h})$$

$$\sum_b \sum_h A(S_1(z_b), p^h) \frac{1}{(\sum_j p_j)^2} A(S_2(z_b), p^{-h})$$

$\kappa^2 \kappa^2$
 $\kappa^2 \kappa^2$
 $\kappa^2 \kappa^2$

$$A(1^+, 2^-, 3^+, 4^-) = -\kappa^2 \frac{([13]k_2 4^-)^4}{stu}$$

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$$A(z) = \sum_h A(S_1(z), p^h) \frac{1}{(p_1(z) + \sum_j t_j)^2} A(S_2(z), p^{-h})$$

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Schedule

4:00 Tim Hsieh

4:10 Francesca Vidotto

5 min break

5:05 Alexandru Dăfină

5:35 Robert Mooney

Schedule

- 4:00 Tim Hsieh
- 4:10 Francesca Vidotto
- 5 min break
- 5:05 Alexandru Dafinca
- 5:35 Robert Mooney

$$\gamma^i(z) = \gamma^i + z \langle X_{i,i} \rangle \eta$$

4:00 Tim Hsieh
4:10 Francesca Vidotto
5 min break
5:05 Alexandru Dăfini
5:35 Robert Mooney

$$T^1(z) = T^1 + z \langle X_{11} \rangle \eta$$

$$T^1(z) = T^1 + z T^2$$

$$\tilde{T}^2(z) = T^1 - z T^2$$

Schedule

- 4:00 Tim Hsieh
4:10 Francesca Vidotto
5 min break
5:05 Alexandru Dafinca
5:35 Robert Mooney

$$T^i(z) = T^i + z \langle X, i \rangle \eta$$

$$T^1(z) = T^1 + z T^2$$

$$\tilde{T}^2(z) = T^1 - z T^2$$

Schedule

- 4:00 Tim Hsieh
- 4:10 Francesca Vidotto
- 5 min break
- 5:05 Alexandru Dăfîncea
- 5:35 Robert Mooney

$$T^i(z) = T^i + z \langle X_{ii} \rangle \eta$$

$$T^1(z) = T^1 + z T^2$$

$$\hat{T}^2(z) = T^1 - z T^2$$

$$\sum$$

$$\langle \hat{i}, \hat{j} \rangle = \langle i, j \rangle (1 + z \langle \hat{i}, \hat{j} \rangle)$$

Schedule

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5:35 Robert Mooney

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$$T^1(z) = T^1 + z T^2$$

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$$\sum \langle \uparrow, \uparrow \rangle = \langle i, j \rangle (1 + z \langle \uparrow, \uparrow \rangle)$$

Schedule

- 4:00 Tim Hsieh
- 4:10 Francesca Vidotto
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$$T^i(z) = T^i + z \langle X_{,i} \rangle \eta$$

$$T^1(z) = T^1 + z T^2$$

$$\hat{T}^2(z) = T^1 - z T^2$$

\sum

$$\langle \hat{i}, \hat{j} \rangle = \langle i, j \rangle \left(1 + z \frac{\langle X_{,i} \rangle \langle X_{,j} \rangle}{\langle X_{,i} \rangle} \right)$$

Schedule

- 7:00 Tim Hsieh
- 7:10 Francesca Vidotto
- 5 min break
- 5:05 Alexandru Dafinca
- 5:35 Robert Mooney

$$T^i(z) = T^i + z \langle X, i \rangle \eta$$

$$T^1(z) = T^1 + z T^2$$

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Loop Quantum Cosmology

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supervisor: Parampreet Singh

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Perimeter Institute, Summer Student Research Project 2008

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Plan of the talk

Loop Quantum Cosmology

Isotropic Flat Model

The Friedmann-Raichauduri equation

Isotropic Curved Models
to conclude...

Introduction

Motivations

Loop Quantum Gravity

Loop Quantum Cosmology

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LQC: Homogeneous and Isotropic setting

Spatial homogeneity and isotropy: fix a fiducial triad \mathfrak{e}_i^a and co-triad ω_a^i .

Symmetries $\Rightarrow A_a^i = cV_0^{-\frac{1}{3}}\omega_a^i$, $E_i^a = pV_0 - \frac{2}{3}(\det \omega)\mathfrak{e}_i^a$

where c and p satisfy $\{c, p\} = \frac{8}{3}\pi G\gamma$.

Elementary variables:

Holonomies: $h_k(\mu) = \cos(\mu \frac{c}{2})\mathbb{I} + 2\sin(\mu \frac{c}{2})\tau_k$, $\mu \in (-\infty, \infty)$.

Elements of form $\exp(i\mu \frac{c}{2})$ generate algebra of almost periodic functions.

Hilbert space: $\mathcal{H}_{kin} = L^2(\mathbb{R}, d\mu)$

Orthonormal basis: $N(\mu) = \exp(i\mu \frac{c}{2})$; $\langle N(\mu) | N(\mu') \rangle = \delta_{\mu, \mu'}$

Hamiltonian Constraint $C_{grav} = \int_{\mathcal{V}} d^3x \epsilon^{ijk} F_{ab}^i \frac{E^{aj} E^{bk}}{\sqrt{\det E}}$

Procedure:

- ▶ Express C_{grav} in terms of elementary variables and their Poisson brackets
- ▶ Classical identity of the phase space:

$$\epsilon^{ijk} \frac{E^{aj} E^{bk}}{\sqrt{\det E}} \rightarrow \text{Tr}(h^k(\mu) \{h^k(\mu)^{-1}, V\})$$

- ▶ Express field strength in terms of holonomies

Flat Theory

FRW with $k = 0$ is instructive because every classical solution is singular and provides a foundation for more complicated models.

Consider the conjugate variables (c, p) related to scale factor such that:

$p = a^2$ (with two possible orientations for the triad)

$c = \gamma \dot{a}$ (on the space of solutions of GR). where volume and laps are normalized to 1.

The fundamental Poisson bracket is given by:

$$\{c, p\} = \frac{8}{3}\pi G\gamma \quad (1)$$

where γ is the Barbero-Immirzi parameter. It's defined a *minial area gap* so that $\bar{\mu}^2 |p| = 2\sqrt{3}\pi G\hbar\gamma$. The effective Hamiltonian turns out to be

$$\mathcal{H}_{\text{eff}} = -\frac{3}{8\pi G\gamma^2} \frac{|p|^{\frac{1}{2}}}{\bar{\mu}^2} \sin^2(\bar{\mu}c) + \mathcal{H}_m \quad (2)$$

The classical equations

Classically we have the well known equations:

$$\text{Friendmann eq. } H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho \quad (3)$$

which is derived from the 00 component of Einstein field equations;

$$\text{Conservation law } \dot{\rho} + 3H(\rho + p) = 0 \quad (4)$$

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which is derived from the trace of Einstein's field equations, and can be easily be computed putting together 3 and 4.

The modified Friedmann equation can be obtained from the vanishing of the Hamiltonian constraint (2) and using the equations of motion. It turns out to be

$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}} \right), \quad \text{where} \quad \rho_{\text{crit}} = \left(\frac{8}{3} \pi G \gamma^2 \Delta \right)^{-1} \quad (6)$$

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where $\rho = \mathcal{H}_m p^{-\frac{3}{2}}$ and $P = -\frac{\partial \mathcal{H}_m}{\partial V} = -\frac{2}{3} \frac{1}{\sqrt{p}} \frac{\partial \mathcal{H}_m}{\partial p}$.

The classical equations

Classically we have the well known equations:

$$\text{Friendmann eq. } H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho \quad (3)$$

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Flat Theory

FRW with $k = 0$ is instructive because every classical solution is singular and provides a foundation for more complicated models.

Consider the conjugate variables (c, p) related to scale factor such that:

$p = a^2$ (with two possible orientations for the triad)

$c = \gamma \dot{a}$ (on the space of solutions of GR). where volume and laps are normalized to 1.

The fundamental Poisson bracket is given by:

$$\{c, p\} = \frac{8}{3}\pi G\gamma \quad (1)$$

where γ is the Barbero-Immirzi parameter. It's defined a *minial area gap* so that $\bar{\mu}^2 |p| = 2\sqrt{3}\pi G\hbar\gamma$. The effective Hamiltonian turns out to be

$$\mathcal{H}_{\text{eff}} = -\frac{3}{8\pi G\gamma^2} \frac{|p|^{\frac{1}{2}}}{\bar{\mu}^2} \sin^2(\bar{\mu}c) + \mathcal{H}_m \quad (2)$$

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$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}} \right), \quad \text{where} \quad \rho_{\text{crit}} = \left(\frac{8}{3} \pi G \gamma^2 \Delta \right)^{-1} \quad (6)$$

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$$\dot{\phi} \{p, H_{eff}\} = -\{c, p\} \frac{\partial H_{eff}}{\partial c}$$

$$H \rightarrow \frac{\dot{p}}{2p} \rightarrow \left(\frac{\dot{p}}{2p}\right)^2 = H^2$$

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Criteria for the physical viability

We shall ask that the resulting model be such:

1. That the predictions about physical entities, like the spacetime curvature and the energy density that do not depend on “auxiliary structures”, should be independent in the quantum theory, of any choice related to such structures.
2. That the quantization prescription gives a well defined notion of “Planck scale”, that is, the scale for which “quantum gravitational corrections” should become important;
3. That there exists a well defined classical limit that approximates general relativity when spacetime curvatures are small.

The “ $\bar{\mu}$ -quantization” satisfies all these requests.

Other Loop-Inspired Quantizations

▶ Old Quantization

Δ is treated as a *constant* and plays the role of affine parameter.

▷ dependence on the fiducial cell

▷ critical density depends on P_ϕ and appearance of bounce at low density

▶ Polymerized WDW Quantization

Wheeler-DeWitt model where the quantum constraint is written in terms of P_a .

▷ sensitive to change of fiducial structures

▷ critical density depends on P_ϕ and appearance of bounce at low density

▶ Lattice Refinement

Conjugate variables: $P_g = cp^m$ and $g = \frac{\rho^{1-m}}{1-m}$ with $-1 < m < 0$

▷ dependence on change of variables

▷ critical density depends on P_ϕ

$$\dot{p} \{p, H_{eff}\} = -\{c, p\} \frac{\partial H_{eff}}{\partial c}$$

$$H = \frac{\dot{p}}{2p} \Rightarrow \left(\frac{\dot{p}}{2p}\right)^2 = H^2$$

$$\rho \sim 0,8 \text{ per}$$

$$-\frac{1}{2} < m < 0$$

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All these quantization are ruled out by our criteria for physical viability. It has been shown in details for $k = 0$, but there is a lack of literature for $k \neq 0$. Note that when the spatial topology is compact there is no need to introduce an auxiliary structure such as a cell. Nonetheless there are more physically motivated conditions that need to be satisfied by any viable physical theory. In particular, apart from a well defined Planck scale, a “low curvature limit ” should also exist. These conditions turn out to be sufficient to rule out some quantizations.

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$k \neq 0$ models

$$|\rho| = a^2 \qquad c = \gamma \dot{a} - k$$

where the variable c is changed respect to the case $k = 0$. The fundamental Poisson bracket is still:

$$\{c, \rho\} = \frac{8\pi G\gamma}{3} \qquad (8)$$

$$\mathcal{H}_{\text{eff}}^k = -\frac{3}{8\pi G\gamma^2} \frac{\sqrt{\rho}}{\bar{\mu}^2} \left[\sin^2(\bar{\mu}(c-k)) - kM \right] + \mathcal{H}_m. \qquad (9)$$

where $\Delta = \bar{\mu}^2 |\rho|$ and $M := [\sin^2(\bar{\mu}) - \bar{\mu}^2(1 + \gamma^2)]$

We call $\bar{\mu}\sqrt{\rho} = \lambda$, that is the square root of the minimal area gap.

The modified Friedmann equation can be obtained from the vanishing of the Hamiltonian constraint (??):

$$\sin^2(\bar{\mu}(c-k)) = \frac{8}{3}\pi G\gamma^2 \Delta \rho + kM = \frac{\rho}{\rho_{\text{crit}}} + kM \qquad (10)$$

and using the equations of motion.

$$\dot{\rho} = \{\rho, \mathcal{H}_{\text{eff}}^k\} = \frac{2}{\gamma\mu} \sqrt{\rho} \sin(\bar{\mu}(c-k)) \cos(\bar{\mu}(c-k)) \Rightarrow H = \frac{\dot{\rho}}{2\rho} \qquad (11)$$

$k \neq 0$ for generalized matter

$$H^2 = \left(\frac{8}{3} \pi G \rho + \frac{kM}{\gamma^2 \Delta} \right) \left(1 - \frac{\rho}{\rho_{\text{crit}}} - kM \right) \quad (12)$$

$\rho = -M\rho_{\text{crit}}$	$a = a_{\text{max}}$	$\rho = \rho_{\text{min}}$	recollapse
$\rho = (1 - M)\rho_{\text{crit}}$	$a = a_{\text{min}}$	$\rho = \rho_{\text{max}}$	bounce

Recollapse: Agreement with the classical Friedmann formula to one part in 10^5 . For macroscopic universes, LQC prediction on recollapse indistinguishable from the classical Friedmann formula.

Bounces: ρ_{max} equals ρ_{crit} to within 2%. For large universes, the two are indistinguishable.

$$\frac{\ddot{a}}{a} = -\frac{4}{3} \pi G \rho \left[1 - 4 \left(\frac{\rho}{\rho_{\text{crit}}} + kM \right) \right] - 4\pi G P \left[1 - 2 \left(\frac{\rho}{\rho_{\text{crit}}} + kM \right) \right] + \frac{k\tilde{M}}{\gamma^2 \Delta} \left[1 - 2 \left(\frac{\rho}{\rho_{\text{crit}}} + kM \right) \right] + \frac{kM}{\gamma^2 \Delta} \left[1 - \frac{\rho}{\rho_{\text{crit}}} - kM \right] \quad (13)$$

$$\tilde{M} = -\sin(\bar{\mu}) \cos(\bar{\mu}) \bar{\mu} + \bar{\mu}^2 (1 + \gamma^2)$$

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$k \neq 0$ for different quantizations

$$\Delta = \bar{\mu}^2 \rho \quad \rightarrow \quad \Delta = \bar{\mu}^2 \rho^{2n} \quad n \in \frac{1}{2}\mathbb{N}$$

$$H^2 = \left(\frac{8}{3} \pi G \rho \rho^{1-2n} + \frac{kM_h}{\gamma^2 \Delta} \right) \left(1 - \frac{\rho}{\rho_{\text{crit}}} \rho^{1-2n} - kM_h \right) \quad (14)$$

$$\begin{aligned} \frac{\ddot{a}}{a} = & -\frac{4}{3} \pi G \rho \rho^{1-2n} \left[1 - 4 \left(\frac{\rho}{\rho_{\text{crit}}} \rho^{1-2n} + kM_h \right) \right] - 4\pi G \rho \rho^{1-2n} \left[1 - 2 \left(\frac{\rho}{\rho_{\text{crit}}} \rho^{1-2n} + kM_h \right) \right] \\ & + 2n \frac{k\tilde{M}_h}{\gamma^2 \Delta} \left[1 - 2 \left(\frac{\rho}{\rho_{\text{crit}}} \rho^{1-2n} + kM_h \right) \right] + \frac{kM_h}{\gamma^2 \Delta} \left[1 - \frac{\rho}{\rho_{\text{crit}}} \rho^{1-2n} - kM_h \right] \end{aligned} \quad (15)$$

\tilde{M}_h comes from the derivative of M respect to ρ , so it differs now from \tilde{M} .

Summary

- ▶ We have studied in detail equations of LQC for the case of $k \neq 0$, going through an explicit calculation of equations of motion.
- ▶ We have stated from the case of a massless scalar field and we have generalized our analysis for every kind of matter.
- ▶ We have considered quantizations other than respect to $\bar{\mu}$ and we have ruled out them.

$$\dot{p} \{p, H_{eff}\} = -\{c, p\} \frac{\partial H_{eff}}{\partial c}$$

$$\frac{\partial}{\partial \alpha} \psi(a, \phi) = \frac{\partial}{\partial \phi} \psi(a, \phi)$$

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Schedule

- 4:00 Tim Hsieh
- 4:10 Francesca Vidotto
- 5 min break
- 5:05 Alessandro Delfino
- 5:35 Robert Mooney

Thoughts about

Information theory

I Quantum Foundations

→ reconstruct QM

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II State 2 principles

of the preparation of the system.

Summarizations

- (S1) In any experiment we have +ve information gain (we learn sth. about nature)
- (S2) In n trials eliminate the same no. of hypothesis about the system studied, regardless of the outcome $\hat{=}$ information gain independent of the preparation of the system.

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$$p = 1/2$$

$$p = 1/10$$



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2 1 a c b c 1 1

$$P_H = 1/2$$

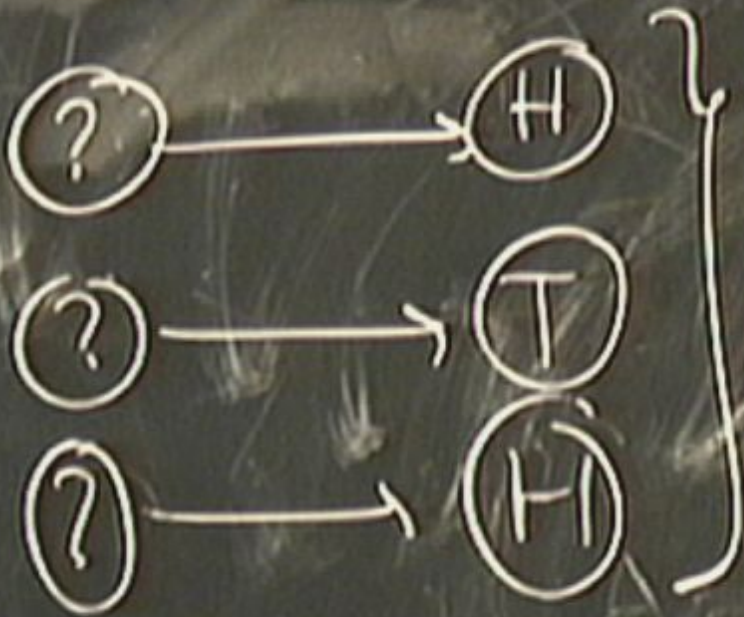
$$P_H = 1/10$$



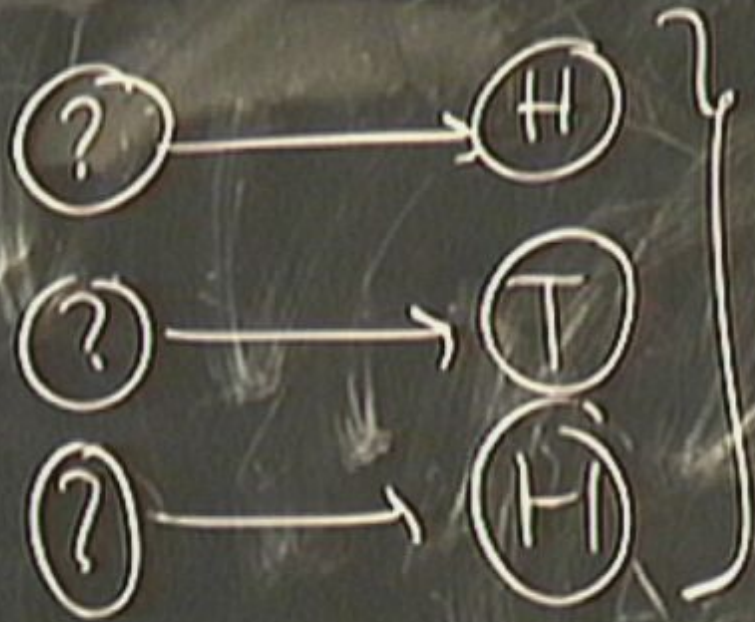
$$f_i = (p_H, 1 - p_H)$$



$$P = (p_H, 1 - p_H)$$



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n tosses $\rightarrow f \times n$ heads
 $(1-f) \times n$ tails

$$0 < f < 1$$

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$$P_r(A, B | C) = P_r(A) \times P_r(B | A, C) \\ = P_r(B) \times P_r(A | B, C)$$



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$$\propto p^f \times (1-p)^{m-f}$$

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$$H(p_1, p_2, \dots, p_n) = -\lambda \sum_i p_i \log p_i$$

$$H[p_1, p_2, \dots, p_n] = -\lambda \sum_i p_i \log p_i$$

$$H[P_n(p|\mathcal{I}, n, \mathbb{I})] = -\int P_n(p|\mathcal{I}, n, \mathbb{I}) \times \log P_n(p|\mathcal{I}, n, \mathbb{I})$$

$$H[p_1, p_2, \dots, p_n] = -\lambda \sum_i p_i \log p_i$$

$$H[P_n(p|\mathcal{I}, n, T)] = -\int P_n(p|\mathcal{I}, n, T) \times \log[P_n(p|\mathcal{I}, n, T)]$$

$$H[p_1, p_2, \dots, p_n] = -\lambda \sum_i p_i \ln p_i$$

$$H[P_n(p|I, n, I)] = - \int P_n(p|I, n, I) \times \log \left[\frac{P_n(p|I, n, I)}{P_n(p|I, n, I)} \right]$$

$$K_3 = H[R(p|m, I)] - H[R(p|f, m, I)]$$

$$= \int R(p|f, m, I) \cdot \log \left[\frac{R(p|f, m, I)}{P_{\text{prior}}} \right]$$



$$H[R(p|f,m)] = - \int R(p|f,m,I) \times \log \left[\frac{R(p|f,m,I)}{R(p|m,I)} \right]$$

$$K_3 = H[R(p|m,I)] - H[R(p|f,m,I)]$$

$$= \int R(p|f,m,I) \times \log \left[\frac{R(p|f,m,I)}{R(p|m,I)} \right]$$



$$K_m = H[R(p|m, I)] - H[R(p|f, m, I)]$$

$$= \int R(p|f, m, I) \times \log \left[\frac{R(p|f, m, I)}{P_{\text{prior}}} \right]$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log P_{\text{prior}}$$

$$= \frac{1}{m} \log_2 m + C + \log_2 \frac{1}{m} - \log_2 P_{prior}$$

$$P_r(A, B | C) = P_r(A) \cdot P_r(B | A, C)$$

$$= P_r(B) \cdot P_r(A | B, C)$$

$$P_r(p | f, m, I) = \frac{P_r(f | p, m, I) \times P_r(p | m, I)}{P_r(f | m, I)}$$

$$\propto p^f \times (1-p)^{m-f}$$

$$\rightarrow \frac{d}{dp} \left(\frac{p^f (1-p)^{m-f}}{2 \cdot 2^m} \right)$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}}$$

$$P_r(p|m, I) \propto \frac{1}{\sqrt{p(1-p)}}$$

Jeffrey's prior

$$P_r(A, B|C) = P_r(A) \times P_r(B|A, C) \\ = P_r(B) \times P_r(A|B, C)$$

$$P_r(I|p, m, I) \times P_r(p|m, I)$$

$$= \frac{1}{2} \log n + C + \log \frac{1}{\sqrt{p(1-p)}}$$

$$P_r(p|n, I) \propto \frac{1}{\sqrt{p(1-p)}}$$

Jeffrey's p

$$P_r(A, B|C) = P_r(A) \times P_r(B|A, C)$$

$$= P_r(B) \times P_r(A|B, C)$$

$$P_r(A|p, n, I) \times P_r(p|n, I)$$

$$H[\ln(p|f, m, I)] = - \int \ln(p|f, m, I) \times \log \left[\frac{Pr(p|m, I)}{Pr(p|m, I)} \right]$$

$$K_3 = H[R(p|m, I)] - H[R(p|f, m, I)]$$

$$= \int R(p|f, m, I) \times \log \left[\frac{Pr(p|f, m, I)}{Pr(p|m, I)} \right]$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log Pr(p)$$

$$Pr(A, B|C) = Pr(A) \times Pr(B|A, C)$$

$$= Pr(B) \times Pr(A|B, C)$$

$$Pr(f|p, m, I) \times Pr(p|m, I)$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log \text{Prior}$$

$$P_r(p|m, I) \propto \frac{1}{\sqrt{p(1-p)}}$$

Jeffrey's prior

$$\text{Prior} \propto [p(1-p)]^{\nu-1}$$

$$S_2 \Rightarrow S_1$$

$$P_r(A, B|C) = P_r(A) \times P_r(B|A, C)$$

$$= P_r(B) \times P_r(A|B, C)$$

$$P_r(I|p, m, I) \times P_r(p|m, I)$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log \text{Prior}$$

$$P_n(p|m, I) \propto \frac{1}{\sqrt{p(1-p)}}$$

Jeffrey's prior

$$\text{Prior} \propto [p(1-p)]^{\nu-1}$$

$$S_2 \Rightarrow S_1$$

$$\Delta K_{mn} = K_{mn} - 1$$

$$P_n(A, B|C) = P_n(A|C) P_n(B|C)$$

$$= P_n(A) P_n(B)$$

$$P_n(p|m, I)$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log \text{Prior}$$

$$P_n(p|m, I) \propto \frac{1}{\sqrt{p(1-p)}} \quad \text{Jeffrey's prior}$$

$$\text{Prior} \propto [p(1-p)]^{\alpha-1}$$

$$S_2 \Rightarrow S_1$$

$$\Delta K_{m+1} = K_{m+1} - K_m > 0 \rightarrow \alpha \in (0, 1/2]$$

$$P_r(A, B|C) = P_r(A) \times P_r(B|A, C)$$

$$= P_r(B) \times P_r(A|B, C)$$

$$P_r(I|p, m, I) \times P_r(p|m, I)$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log \text{Prior}$$

$$P_n(p|m, I) \propto \frac{1}{\sqrt{p(1-p)}}$$

Jeffrey's prior

$$\text{Prior} \propto [p(1-p)]^{\alpha-1}$$

$$S_2 \Rightarrow S_1$$

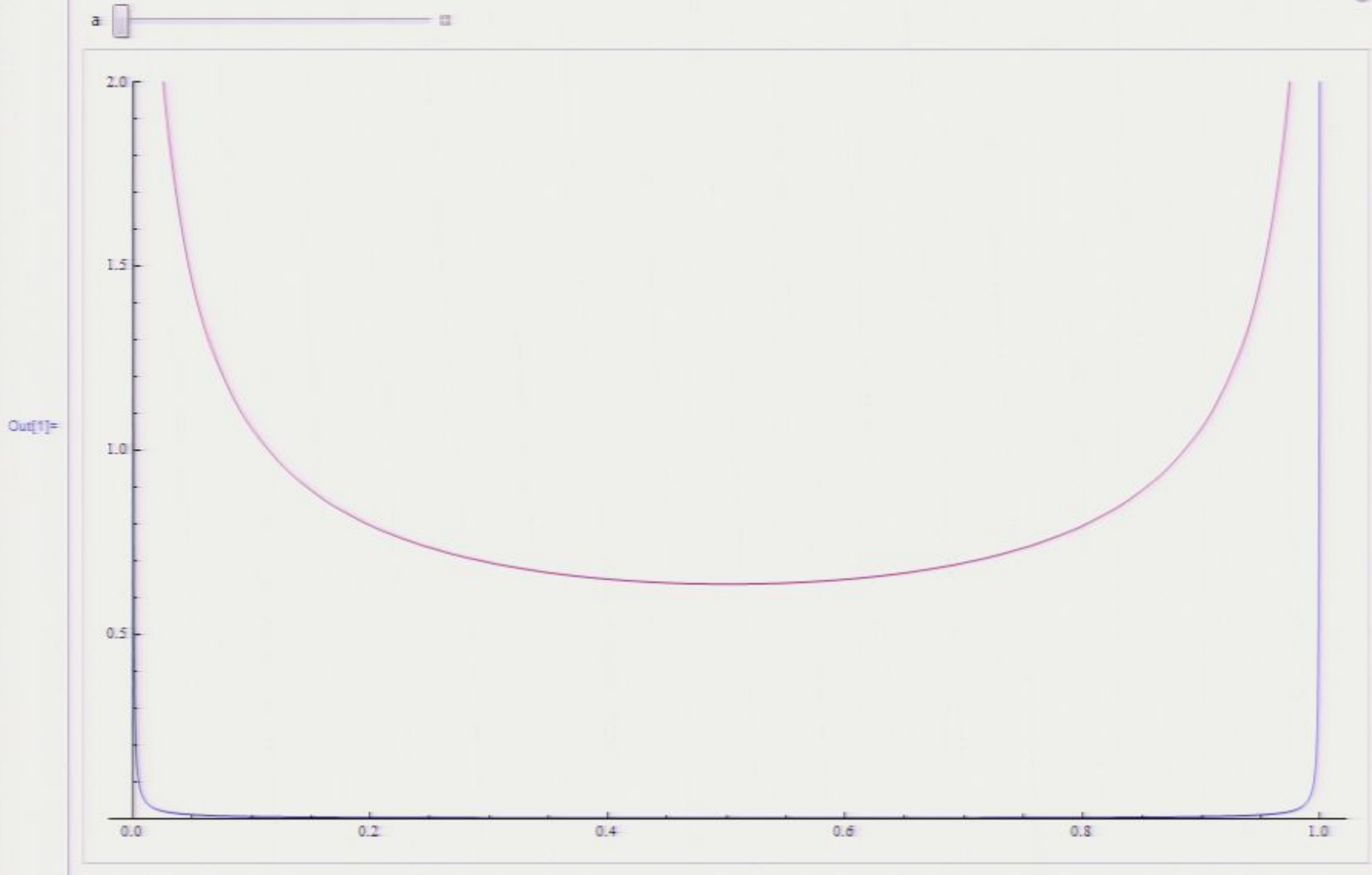
$m \rightarrow \infty$?

$$\Delta K_{m+1} = K_{m+1} - K_m > 0 \rightarrow \alpha \in (0, 1/2]$$

$$P_r(A, B|C) = P_r(A) \times P_r(B|A, C)$$

$$= P_r(B) \times P_r(A|B, C)$$

$$P_r(I|p, m, I) \times P_r(p|m, I)$$



$$P_2(p_1, p_2, \dots, p_n) \propto \frac{1}{\sqrt{p_1 p_2 \dots p_n}}$$

$$P_2(p_1, p_2, \dots, p_n) \propto \frac{1}{\sqrt{p_1 p_2 \dots p_n}}$$

$$g_i = p_i^{1/2}$$

e

$$P_n(p_1, p_2, \dots, p_n) \propto \frac{1}{\sqrt{p_1 p_2 \dots p_n}}$$

$$q_i = p_i^{1/2}$$

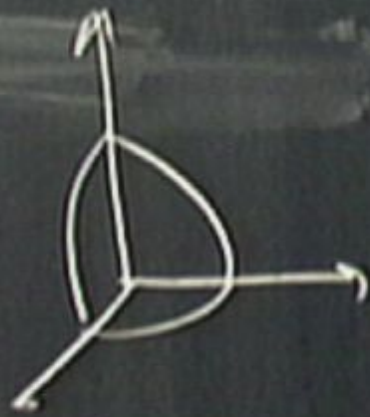
$$P_n(q_1, q_2, \dots, q_n) = \text{constant}$$

$$P_r(p_1, p_2, \dots, p_n) \propto \frac{1}{\sqrt{p_1 p_2 \dots p_n}}$$

$$q_i = p_i^{1/2}$$

$(p_1, p_2, \dots, p_n) = \text{constant}$
 $p_1 + p_2 + p_3 = 1$

$$q_1^2 + q_2^2 + q_3^2 = 1$$

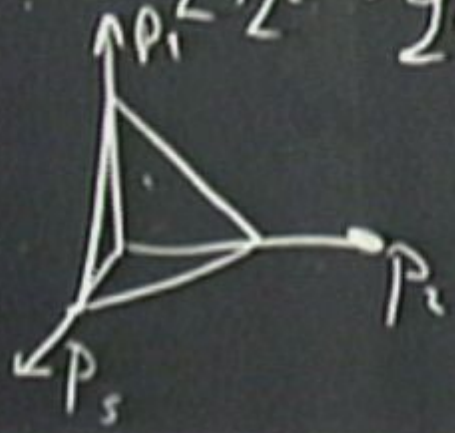


$$P_n(p_1, p_2, \dots, p_n) \propto \frac{1}{\sqrt{p_1 p_2 \dots p_n}}$$

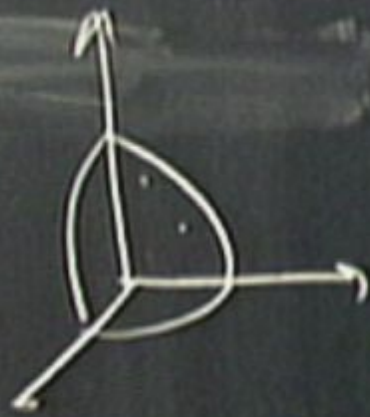
$$q_i = p_i^{1/2}$$

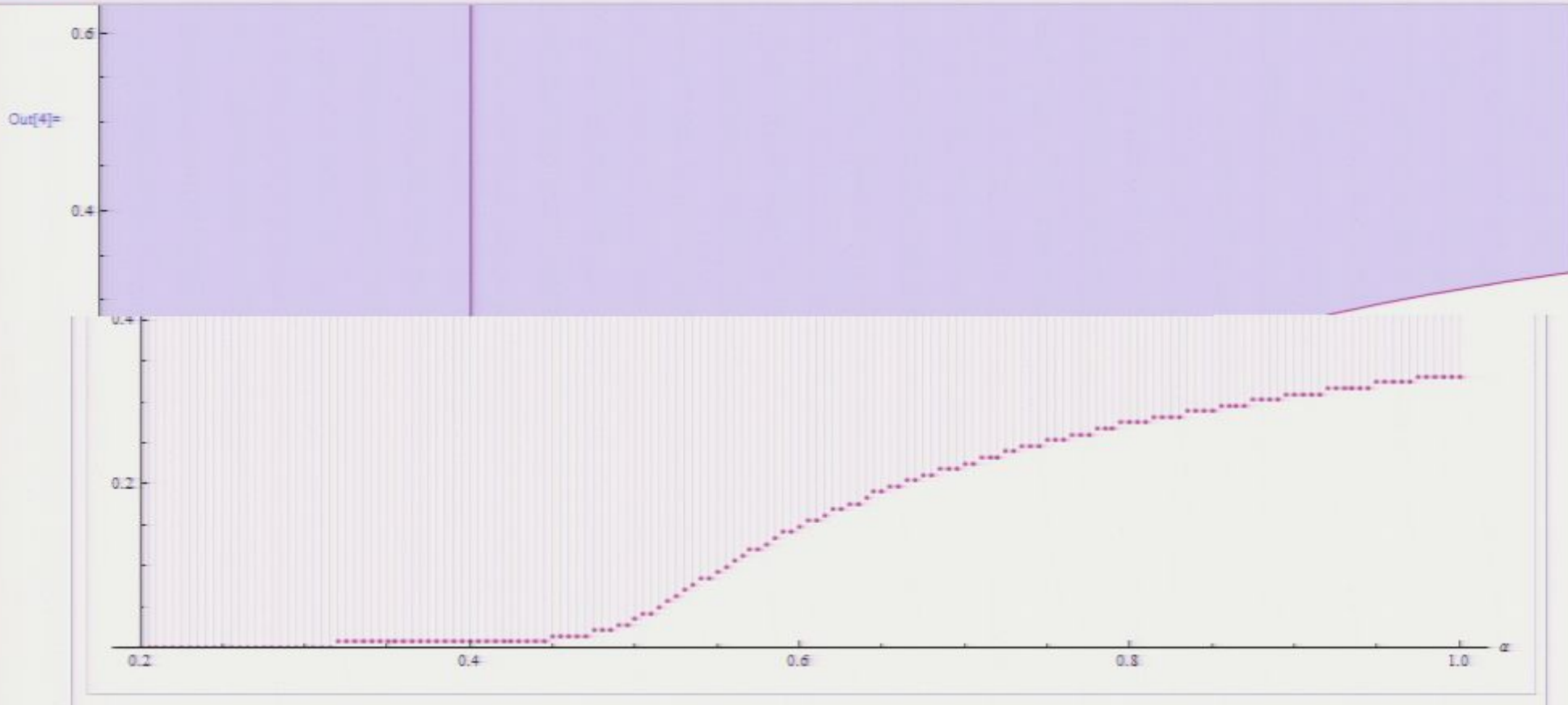
$$P_n(q_1, q_2, \dots, q_n) = \text{constant}$$

$$p_1 + p_2 + p_3 = 1$$



$$q_1^2 + q_2^2 + q_3^2 = 1$$

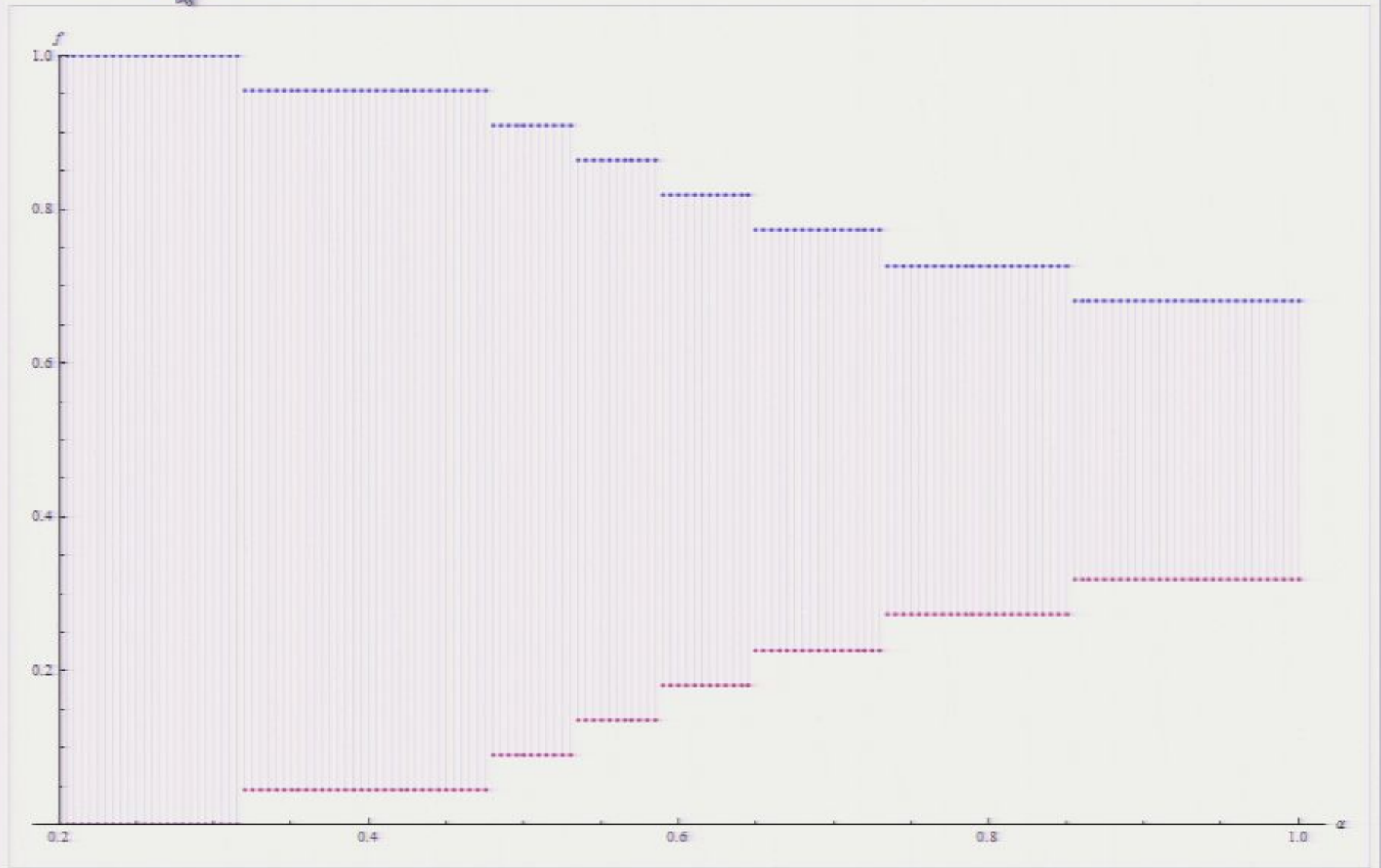




```
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0.332, 0.334, 0.335, 0.336, 0.338, 0.339, 0.34, 0.341, 0.343, 0.3440000000000000003,  
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0.3460000000000000003, 0.3470000000000000003, 0.3480000000000000003, 0.3490000000000000003, 0.3500000000000000003,  
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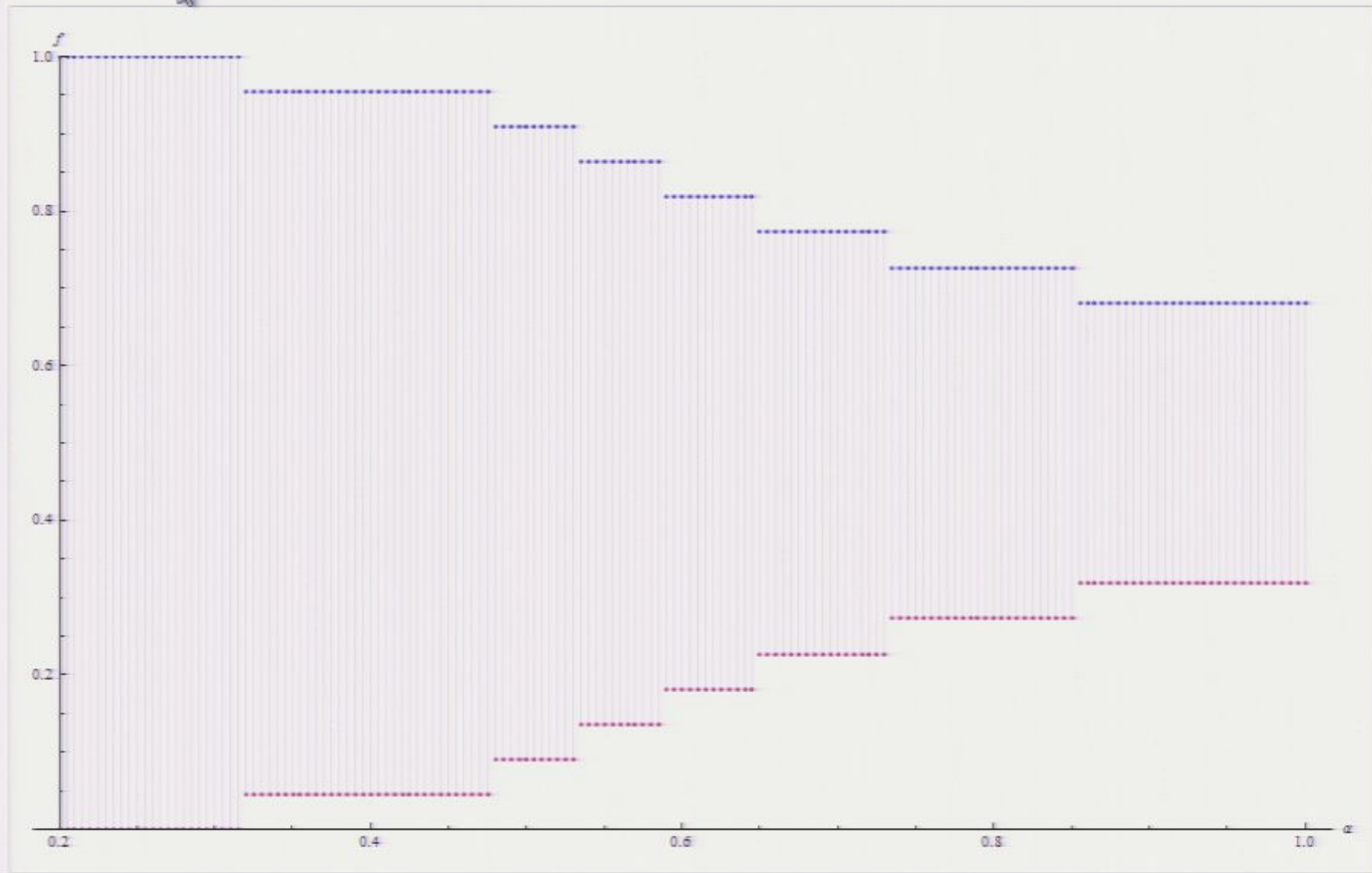
3 [Navigation icons]

Out[7]=



3 [Navigation icons]

Out[7]=



paradoxes

- (S1) In any experiment we have +ve information gain (we learn sth. about nature)
- (S2) n trials eliminate the same no. of hypothesis about the system studied, regardless of the outcome $\hat{=}$ information gain independent of the preparation of the system.

$$H[P_n(p|f, m, I)] = - \int P_n(p|f, m, I) \times \log \left[\frac{P_n(p|f, m, I)}{P_n(p|m, I)} \right]$$

$$K_m = H[P_n(p|m, I)] - H[P_n(p|f, m, I)]$$

$$= \int P_n(p|f, m, I) \times \log \left[\frac{P_n(p|f, m, I)}{P_{\text{prior}}(p)} \right]$$

$$= \frac{1}{2} \log m + C + \log \frac{1}{\sqrt{p(1-p)}} - \log P_{\text{prior}}(p)$$

$$P_n(A, B|C) = P_n(A) \times P_n(B|A, C)$$

$$= P_n(B) \times P_n(A|B, C)$$

$$P_n(p|f, m, I) = \frac{P_n(f|p, m, I) \times P_n(p|m, I)}{P_n(f|m, I)}$$

Summarizations

- (S1) In any experiment we have +ve information gain (we learn sth. about nature)
- (S2) In n trials eliminate the same no. of hypothesis about the system studied, regardless of the outcome $\hat{=}$ information gain independent of the preparation of the system.

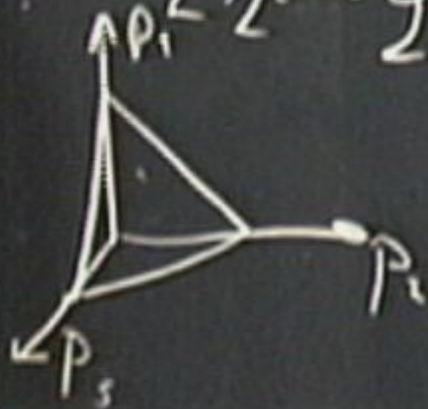
of the probability of the system.

$$P_r(p_1, p_2, \dots, p_n) \propto \frac{1}{\sqrt{p_1 p_2 \dots p_n}}$$

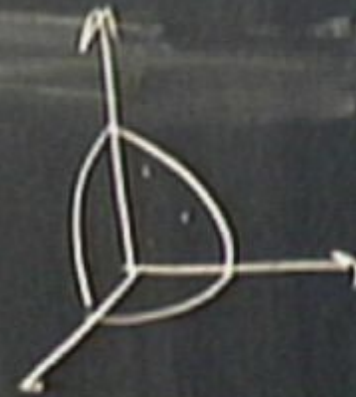
$$g_i = p_i^{1/2}$$

$$P_r(g_1, g_2, \dots, g_n) = \text{constant}$$

$$p_1 + p_2 + p_3 = 1$$



$$g_1^2 + g_2^2 + g_3^2 = 1$$



Fermions & CPT

4:00 Tim

4:30

5 min break

5:05

Alexander Dafinca

5:35

Robert Mooney

4:00 Tim

4:20

5 min break

5:05 Alessandro Daffina

5:35 Robert Mooney

Fermions & CPT in κ -Poincaré

$2\frac{1}{2}$ Apps of BCFW

Standard

$$\Delta(P_i) = \mathbb{1} \otimes P_i + P_i \otimes \mathbb{1}$$

2¹/₂ Apps of BCFW

Standard
 $\Delta(P_i) = \mathbb{1} \otimes P_i + P_i \otimes \mathbb{1}$
K:



2.1 Apps of BCFW

Standard

$$\Delta(P_j) = \mathbb{1} \otimes P_j + P_j \otimes \mathbb{1}$$

\mathcal{K} :

$$\Delta(P_j) = P_j \otimes \mathbb{1} + e^{-P_j} \otimes P_j$$

2¹/₂ Apps of BCFW

Standard

$$\Delta(P_j) = \mathbb{1} \otimes P_j + P_j \otimes \mathbb{1}$$

κ :

$$\Delta(P_j) = P_j \otimes \mathbb{1} + e^{-P_j/\kappa} \otimes P_j$$

$\tilde{P}_j = \tilde{P}_j$

Standard

$$\Delta(P_j) = \mathbb{1} \otimes P_j + P_j \otimes \mathbb{1}$$

κ_j :

$$\Delta(P_j) = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j$$
$$e^{-P_0/\kappa} P_j = \tilde{P}_j$$

standard

$$\Delta(P_j) = \mathbb{1} \otimes P_j + P_j \otimes \mathbb{1}$$

κ :

$$\Delta(P_j) = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j$$

$$e^{-P_0/\kappa} P_j = \tilde{P}_j$$

$[x_0]$

standard

$$\Delta(P_j) = 1 \otimes P_j + P_j \otimes 1$$

κ_j :

$$\Delta(P_j) = P_j \otimes 1 + e^{-P_0/\kappa} \otimes P_j$$

$$e^{-P_0/\kappa} P_j = \tilde{P}_j$$

$$[\kappa_0, \kappa_1] =$$

$$[\kappa_1, \kappa_2] = 0$$



standard

$$\Delta(P_j) = 1 \otimes P_j + P_j \otimes 1$$

κ_i :

$$\Delta(P_j) = P_j \otimes 1 + e^{-P_0/\kappa} \otimes P_j$$

$$e^{-P_0/\kappa} P_j = \tilde{P}_j$$

$$[\chi_0, \chi_1] = \frac{i}{\kappa} \chi_1$$

$$[\chi_1, \chi_2] = 0$$

Standard

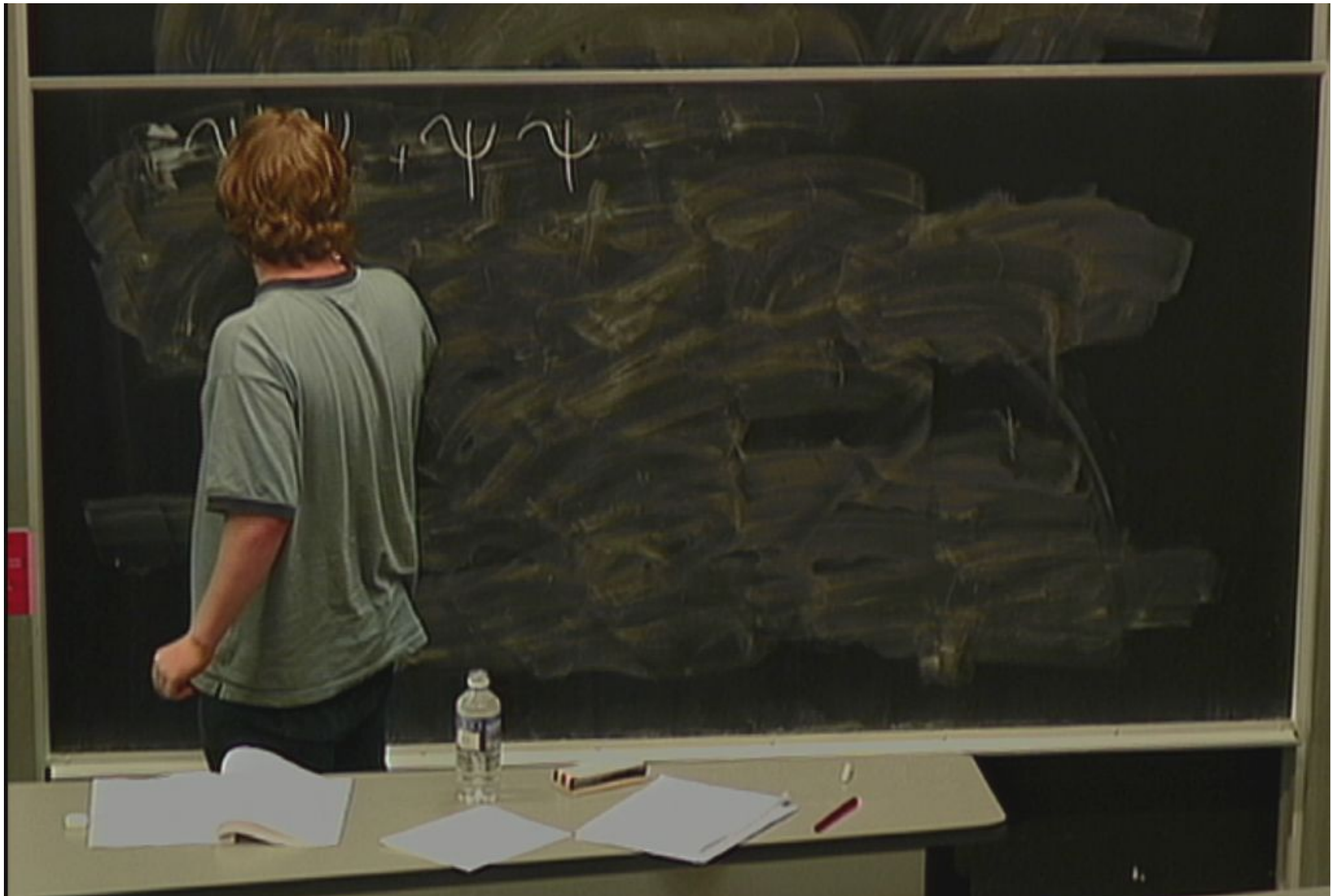
$$\Delta(P_j) = 1 \otimes P_j + P_j \otimes 1$$

$\kappa:$

$$\Delta(P_j) = P_j \otimes 1 + e^{-P_0/\kappa} \otimes P_j$$
$$e^{-P_0/\kappa} P_j = \tilde{P}_j$$

$$[\chi_0, \chi_1] = \frac{i}{\kappa} \chi_1$$

$$[\chi_1, \chi_2] = 0$$



$$\psi_{P_1} + \psi_{P_2} + \psi_{P_3} + \psi_{P_4}$$



$$\psi_{P_1} + \psi_{P_2} + \psi_{P_3} + \psi_{P_4}$$

$$\psi_{P_1} \psi_{P_2} + \psi_{P_3} \psi_{P_4}$$



$$\psi_{P_1} \psi_{P_2} + \psi_{P_3} \psi_{P_4}$$

$$\begin{matrix} \circ & \xrightarrow{\quad} & \circ \\ \text{P}_1 & & \text{P}_2 \end{matrix} \quad \begin{matrix} \text{e}^{-P_1 i X} \\ \text{e}^{-P_2 i X} \end{matrix}$$



$$\psi_{P_1} \psi_{P_2} + \psi_{P_3} \psi_{P_4}$$

$$\begin{pmatrix} 0 \\ P_1 \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} 0 \\ P_1 e^{-P_1 i x} \end{pmatrix}$$

$$\begin{array}{c}
 \psi_{P_1} \quad \psi_{P_2} \quad + \quad \psi_{P_3} \quad \psi_{P_4} \\
 \downarrow \quad \downarrow \quad \quad \quad \downarrow \quad \downarrow \\
 \left(\begin{array}{c} 0 \\ P_1 \end{array} \right) \quad \left(\begin{array}{c} 0 \\ P_2 \end{array} \right) \quad \left(\begin{array}{c} 0 \\ P_3 \end{array} \right) \quad \left(\begin{array}{c} 0 \\ P_4 \end{array} \right) \\
 \left(P_1 e^{-P_1 \cdot iK} \right) \quad \left(P_2 e^{-P_2 \cdot iK} \right) \quad \left(P_3 e^{-P_3 \cdot iK} \right) \quad \left(P_4 e^{-P_4 \cdot iK} \right)
 \end{array}$$



$$\psi_{P_1} \psi_{P_2} + \psi_{P_3} \psi_{P_4}$$

$$\begin{pmatrix} 0 & 1 \\ P_1 e^{-P_1 i x} & P_2 e^{-P_2 i x} \end{pmatrix}$$

$$\left(2\kappa \operatorname{unh} \frac{P_0}{2\kappa} \right)^2$$



$$\psi_{P_1} + \psi_{P_2} + \psi_{P_3} + \psi_{P_4}$$

$$\begin{pmatrix} 0 \\ P_0 \end{pmatrix} \begin{pmatrix} e^{-P_0 \cdot i x} \\ e^{P_0 \cdot i x} \end{pmatrix}$$

$$\left(2\kappa \sinh \frac{P_0}{2\kappa} \right)^2 - \vec{P}^2 e^{\frac{P_0}{\kappa}} = C \kappa$$

$$\psi_{p_1} \psi_{p_2} + \psi_{p_3} \psi_{p_4}$$

$$\begin{pmatrix} 0 \\ p_1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ p_2 e^{-p_1 i x} \end{pmatrix}$$

$$\begin{pmatrix} p_1 e^{-p_1 i x} \\ p_1 \end{pmatrix}, p_1 e^{p_1 i x}$$

$$\left(2\hbar \sin \frac{p_0}{2\hbar} \right)^2 - \vec{p}^2 e^{\frac{p_0}{\hbar}} = C \hbar$$

$$\partial \rightarrow \mathbb{D}^1$$



$$\theta \neq 0$$