Title: Towards a generally covariant averaging process for metrics in general relativity

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Abstract: The speculation that Dark Energy can be explained by the backreaction of present inhomogeneities on the evolution of the background cosmology has been increasingly debated in the recent literature. We demonstrate quantitively that the backreaction of linear perturbations on the Friedmann equations is small but is nevertheless non-vanishing. This indicates the need for an improved averaging procedure capable of averaging tensor quantities in a generally covariant way. We present an averaging process which decomposes the metric into Vielbeins selected employing a variational principle, and parallel-transports them to a single point at which they can be averaged. The functionality of the process is discussed in specific 2-d examples, and its application to 3-surfaces and metric recovery in cosmology is outlined.

Towards a generally covariant averaging process for metrics in general relativity

Juliane Behrend[1]

Iain A. Brown[2] Georg Robbers[2] Otto Nachtmann[2] Thomas Richter[2]

[1]University of Ulm, Germany
[2]University of Heidelberg, Germany

Perimeter Institute, Canada
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Introduction

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⇒ Use exact solution to Einstein equations (FLRW metric) to model the universe

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$\Rightarrow$ Modifications can in principle act as a dark enegy

$$G_{\mu\nu}(\langle g_{\mu\nu}\rangle) = 8\pi G \langle T_{\mu\nu}\rangle + 8\pi G T^{g}_{\mu\nu} + \Lambda \langle g_{\mu\nu}\rangle$$

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⇒ Hamiltonian constraint:

$$\mathcal{R} + K^2 - K^i_j K^j_i = 16\pi G\rho + 2\Lambda$$

⇒ Evolution equation:

$$\frac{1}{\alpha}\dot{K}_{ij} =$$

$$\mathcal{R}_{ij} - 2K^n_i K_{nj} + KK_{ij} - 8\pi G S_{ij} + 4\pi G h_{ij}(S - \rho) - \Lambda h_{ij} - \frac{1}{\alpha} D_i D_j \alpha$$

Buchert Equations

- Define average $\langle A\rangle = \frac{1}{V}\int_{\mathcal{D}} A\sqrt{h}d^3\mathbf{x}$
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$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3} \langle \alpha^2 \rho \rangle + \frac{\Lambda}{3} \langle \alpha^2 \rangle - \frac{1}{6} (\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}})$$

⇒ Extrinsic curvature evolution ⇒ Raychaudhuri equation:

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Modifications

- Kinematical backreaction:

$$\mathcal{Q}_{\mathcal{D}} = \left\langle \alpha^2 \left(K^2 - K^i_j K^j_i \right) \right\rangle - \frac{2}{3} \langle \alpha K \rangle^2$$

- Dynamical backreaction:

$$\mathcal{P}_{\mathcal{D}} = \langle \dot{\alpha} K \rangle + \langle \alpha D^i D_i \alpha \rangle$$

- Curvature contribution:

$$\mathcal{R}_{\mathcal{D}} = \langle \alpha^2 \mathcal{R} \rangle$$

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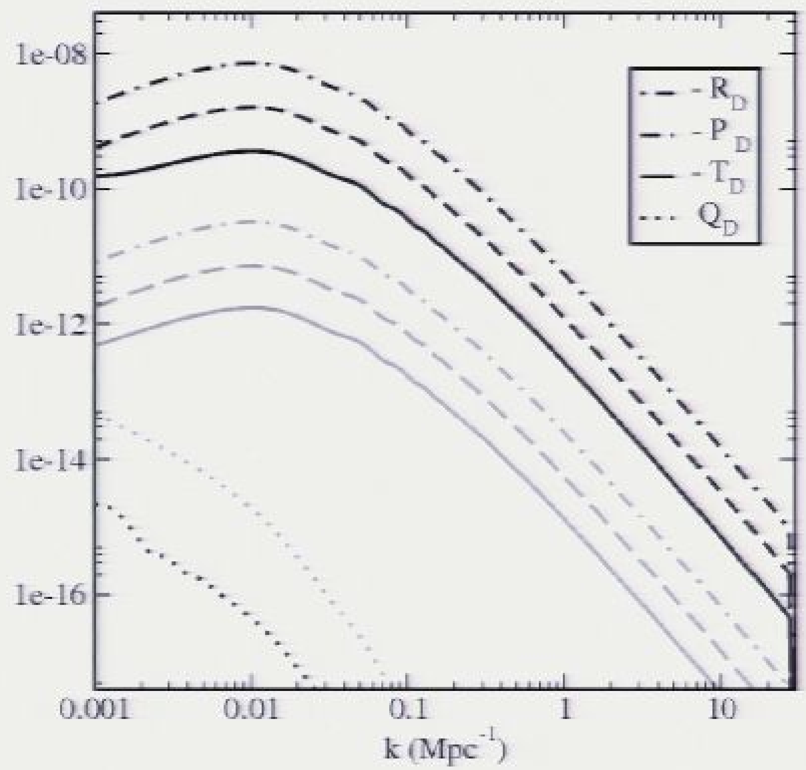
Modifications at $z = 10$ and $z = 0$



- Evaluate corrections with CMBEasy, e.g.

$$\mathcal{Q}_{\mathcal{D}} = 6 \int \mathcal{P}_{\psi}(k) \left| \dot{\phi}(t,k) \right|^2 \frac{dk}{k}$$

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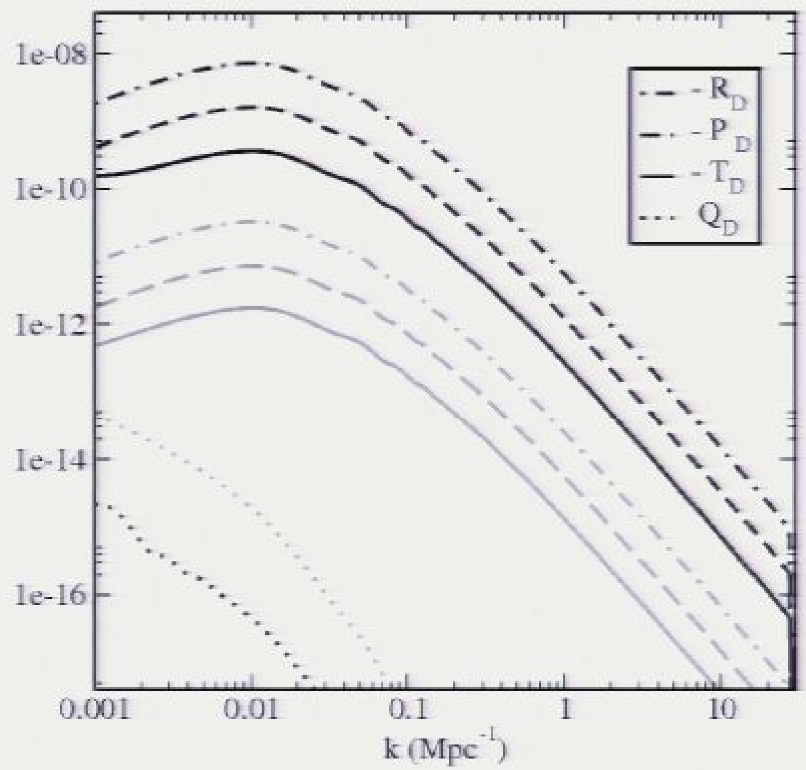

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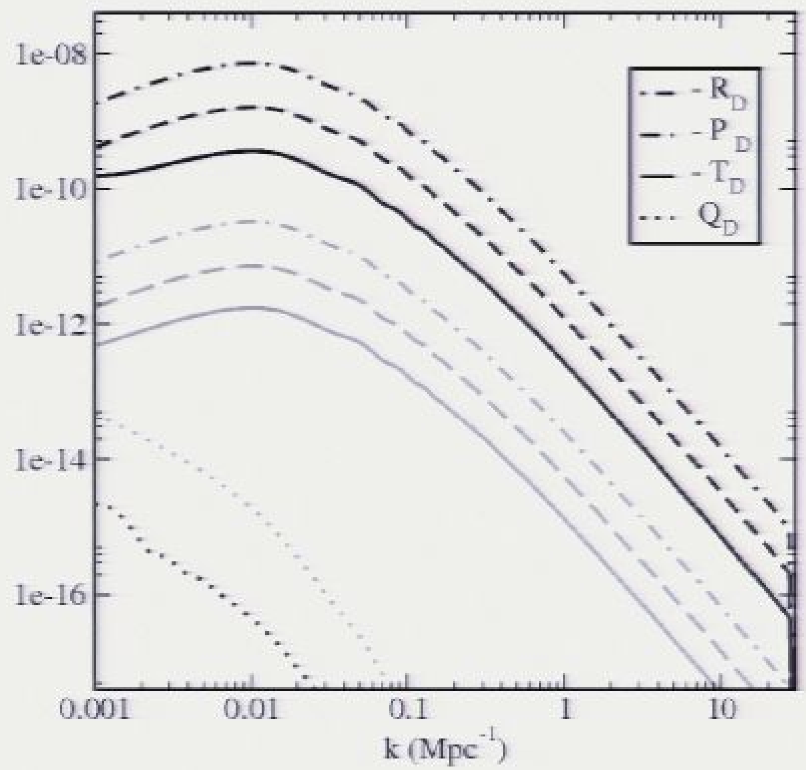

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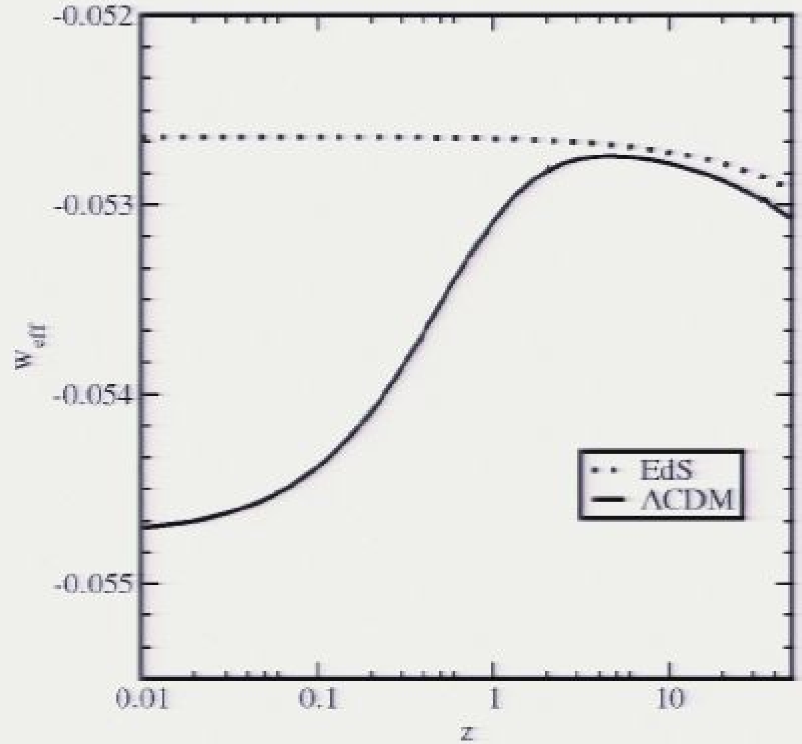
Effective Equation of State

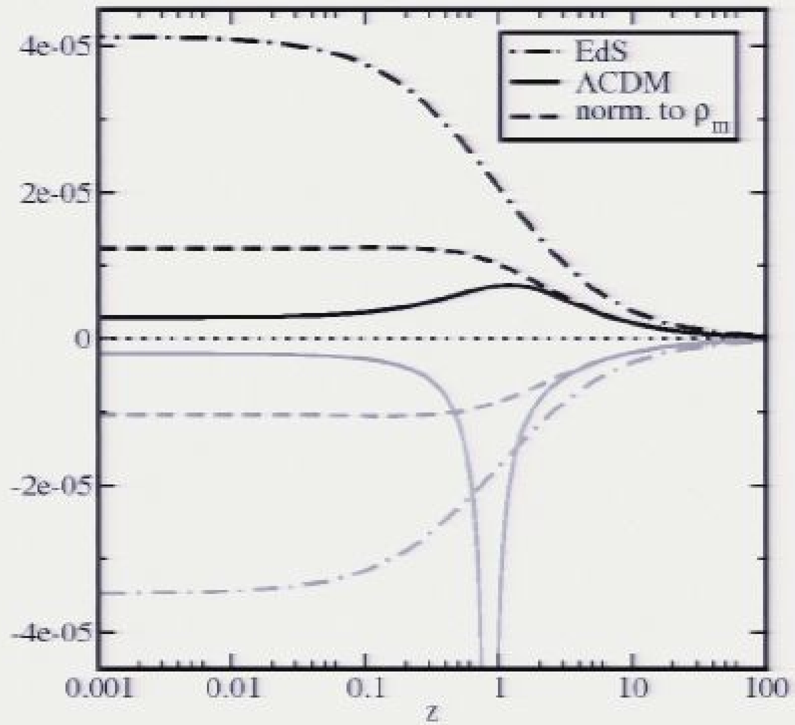


- Effective density: $(8\pi G/3)\rho_{\text{eff}} = \mathcal{T}_{\mathcal{D}} - (\mathcal{Q}_{\mathcal{D}} + \mathcal{R}_{\mathcal{D}})/6$
- Effective pressure: $16\pi G p_{\text{eff}} = \mathcal{R}_{\mathcal{D}}/3 - \mathcal{Q}_{\mathcal{D}} - 4\mathcal{P}_{\mathcal{D}}/3$

⇒ Effective equation of state:

$$w_{\text{eff}} = -(1/3)(\mathcal{R}_{\mathcal{D}} - 4\mathcal{P}_{\mathcal{D}} - 3\mathcal{Q}_{\mathcal{D}})/(\mathcal{R}_{\mathcal{D}} - 6\mathcal{T}_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}) \approx -1/19$$

Impact on Large-Scale Evolution



- Einstein de-Sitter and ΛCDM (WMAPIII concordance)
- $\sim 10^{-5}$ impact as predicted, maxima at $z \approx 1.4$ and $z \approx 0.7$

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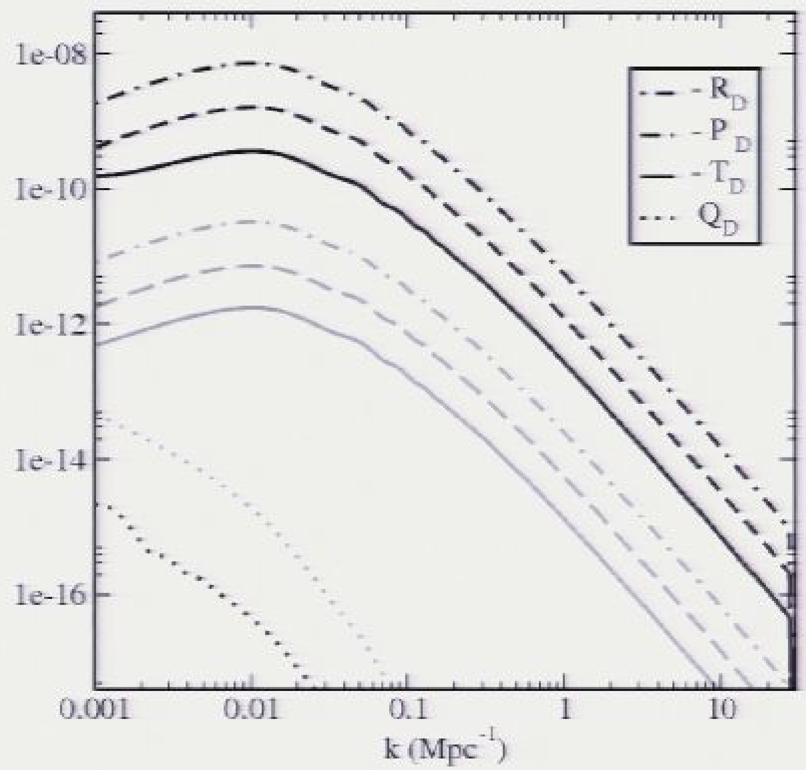

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⇒ Need a generally covariant averaging process

Generally Covariant Averaging Process for the Metric

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- Decompose metric into a right-handed orthochronous Minkowski tetrad

$$g_{\mu\nu}(x) = \eta_{\alpha\beta} E^{\alpha}{}_{\mu}(x) E^{\beta}{}_{\nu}(x)$$

Generally Covariant Averaging Process for the Metric

- Averaging Process must be independent of coordinate system

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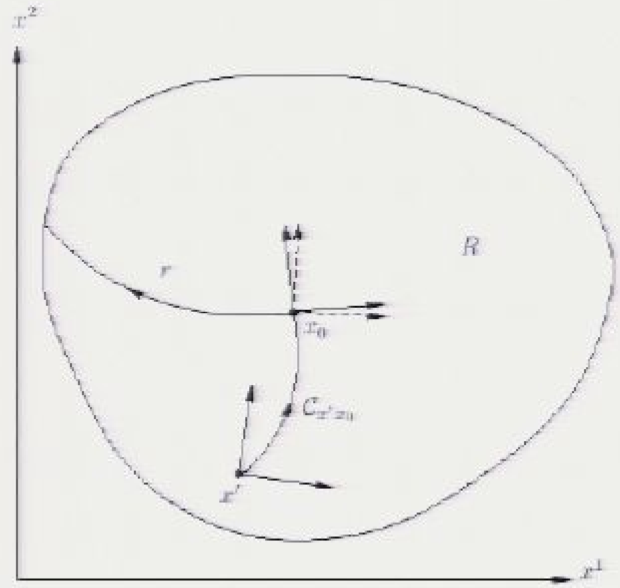
- Decompose metric into a right-handed orthochronous Minkowski tetrad

$$g_{\mu\nu}(x) = \eta_{\alpha\beta} E^{\alpha}{}_{\mu}(x) E^{\beta}{}_{\nu}(x)$$

- Find (up to global Lorentz-transformations) unique tetrad field, the maximally smooth tetrad field, by following Lagrangian

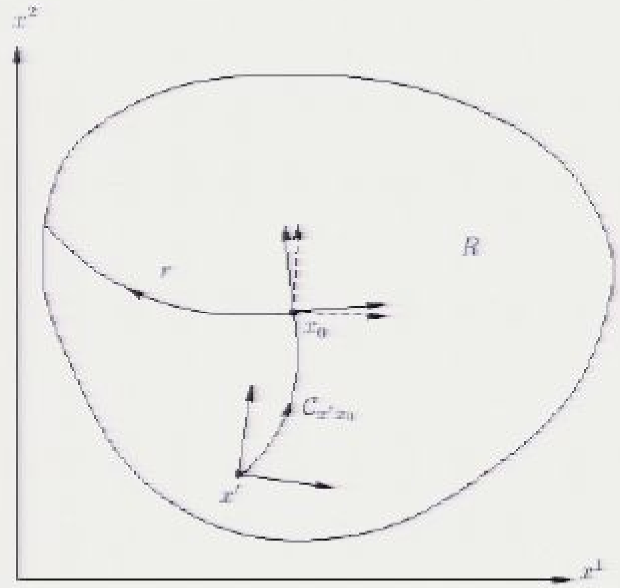
$$\mathcal{L}_{\mathrm{MS}} = (D_{\mu} E^{\alpha}{}_{\rho})(D_{\nu} E^{\beta}{}_{\lambda}) g^{\mu\nu} g^{\rho\lambda} \eta_{\alpha\beta}$$



Parallel transport along geodesics $\mathcal{C}_{x_0 x'}$ realized by Wegner-Wilson line operator

$$V(x', x_0; \mathcal{C}_{x_0 x'}) = \mathcal{P} \exp \left[- \int_{\mathcal{C}_{x_0 x'}} dz^\mu \, \Gamma_\mu(z) \right]$$

where $\Gamma_\mu(x)$ are four matrices with components $(\Gamma_\mu(x))^\lambda_\nu = \Gamma^\lambda_{\mu\nu}(x)$



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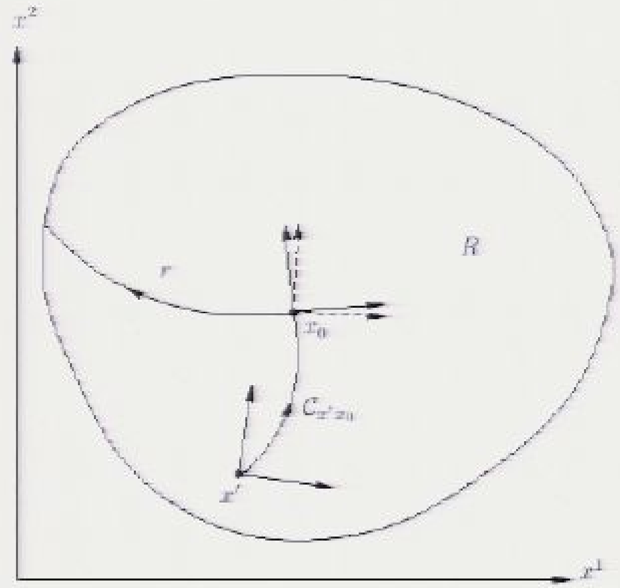
$$\langle E^\alpha{}_\mu(x_0) \rangle$$
$$= \int_R f(x_0, x'; \mathcal{C}_{x' x_0}) \widehat{V}_\mu{}^\nu(x_0, x'; \mathcal{C}_{x' x_0}) E^\alpha{}_\nu(x') \sqrt{-g(x')} \, d^4x'$$



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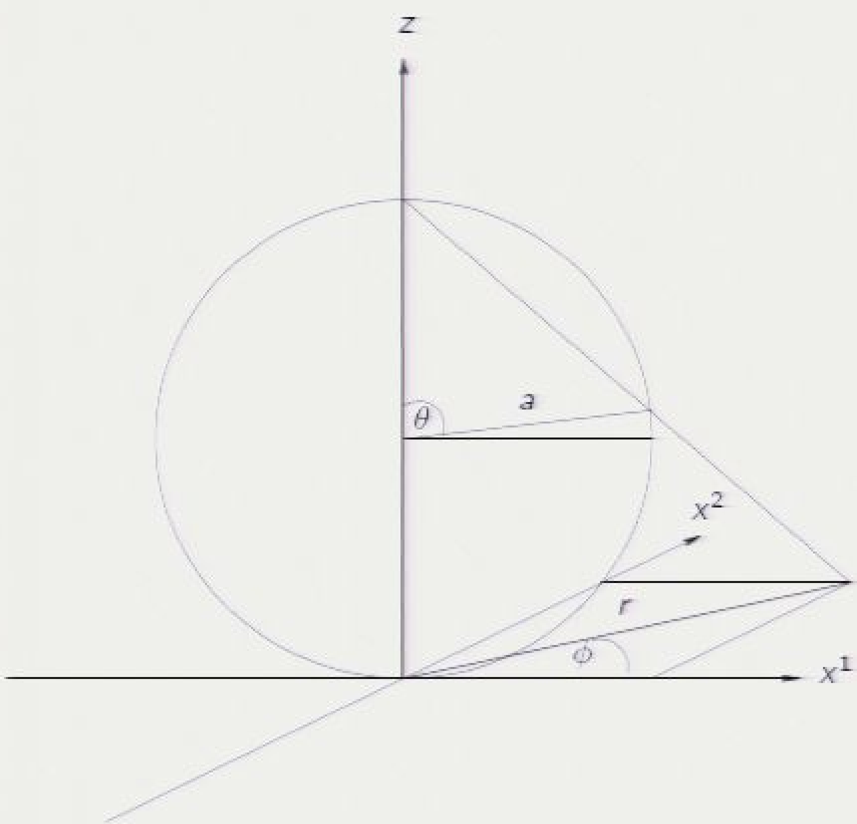
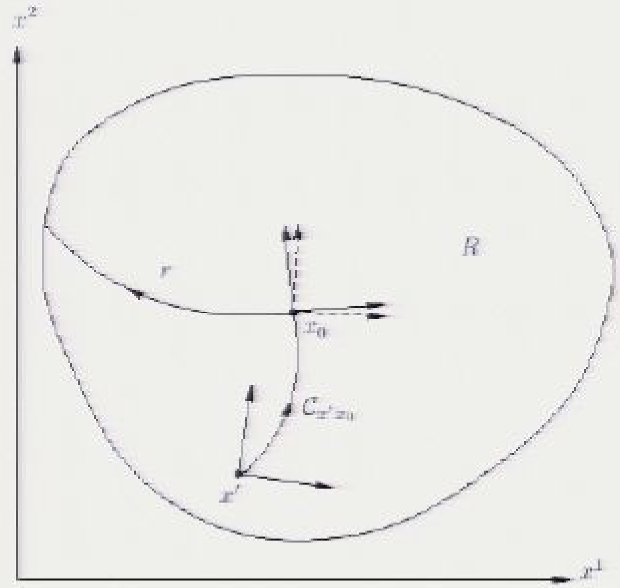
$$\langle g_{\mu\nu}(x) \rangle = \eta_{\alpha\beta} \langle E^\alpha{}_\mu(x) \rangle \langle E^\beta{}_\nu(x) \rangle$$

Averaging the Metric of a Two-Sphere

Stereographic Projection:



- Metric: $g_{ij} = (\frac{2a}{L})^4 \delta_{ij}$ where $L^2 = 4a^2 + (x^1)^2 + (x^2)^2$



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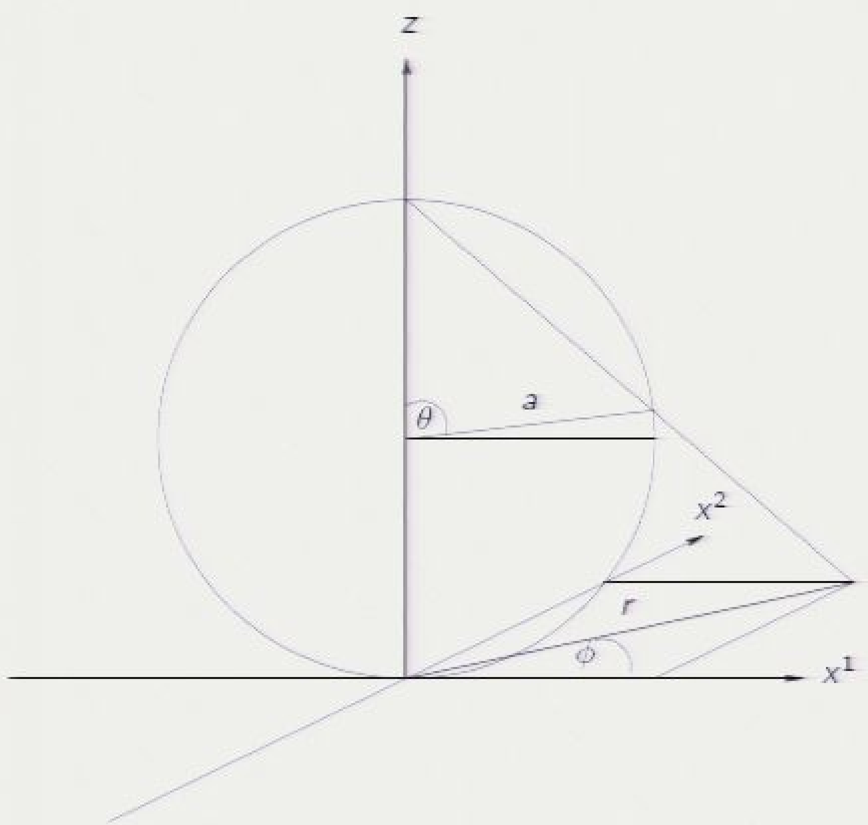

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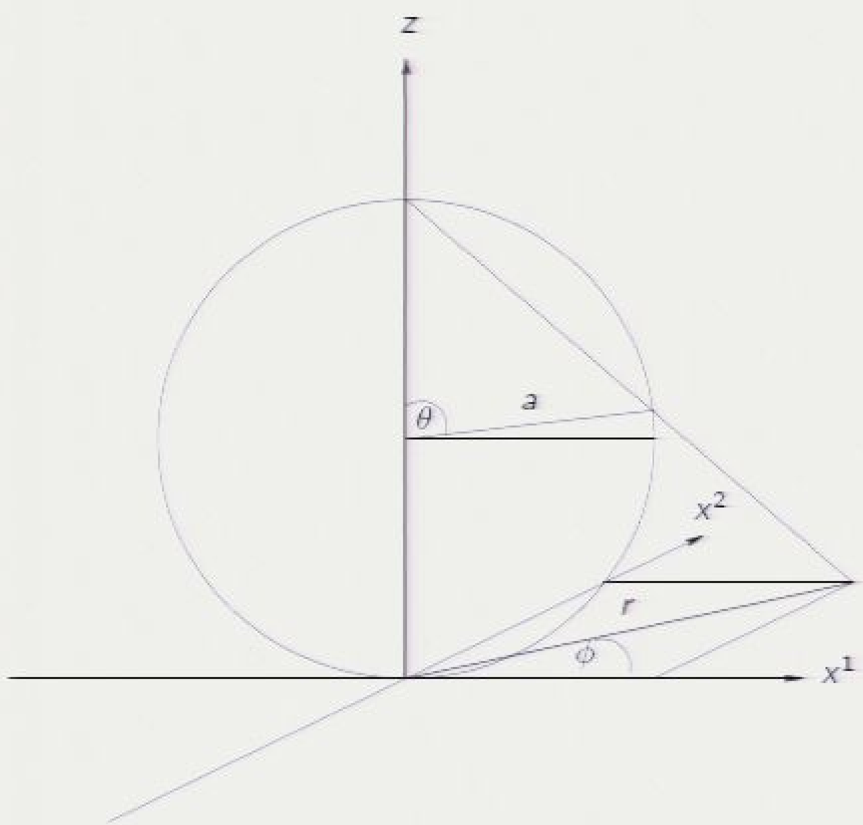

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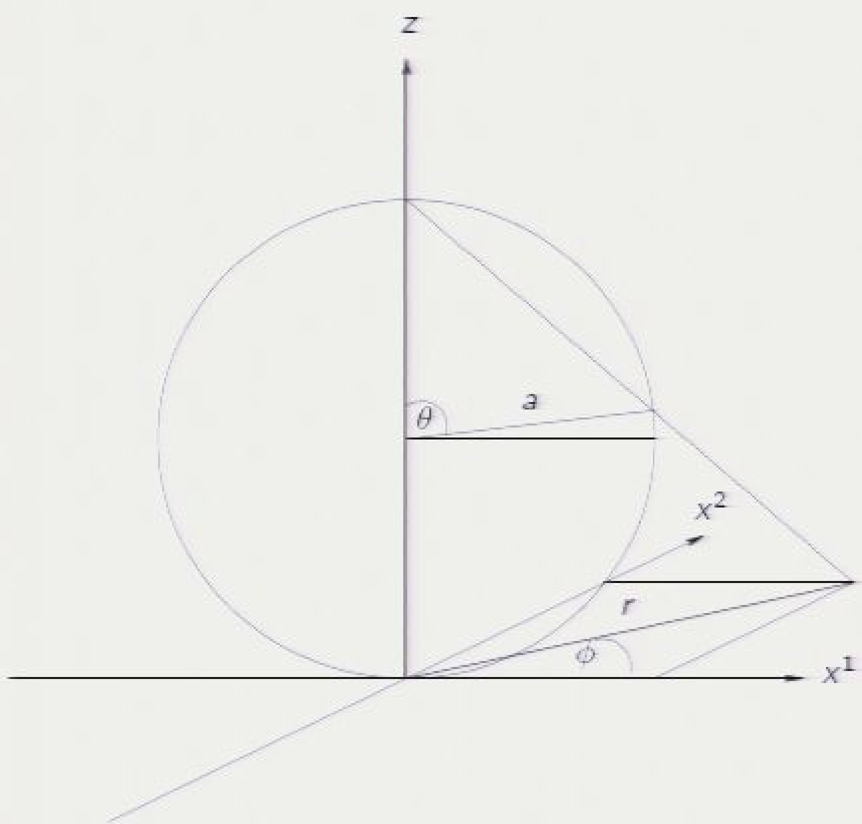

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Averaging the Metric of a Two-Sphere

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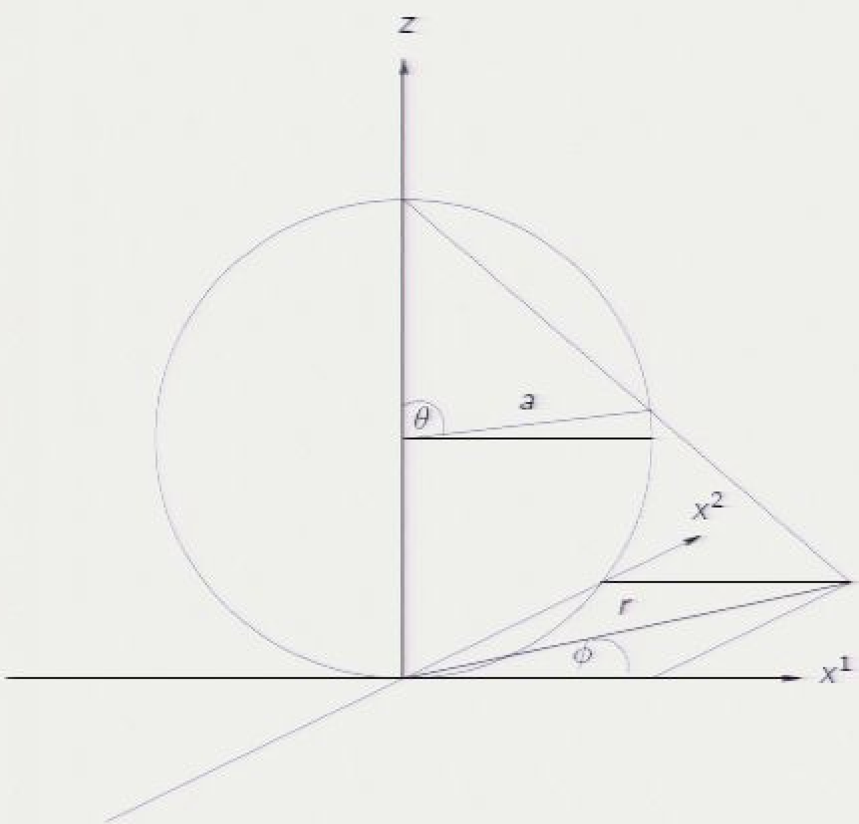

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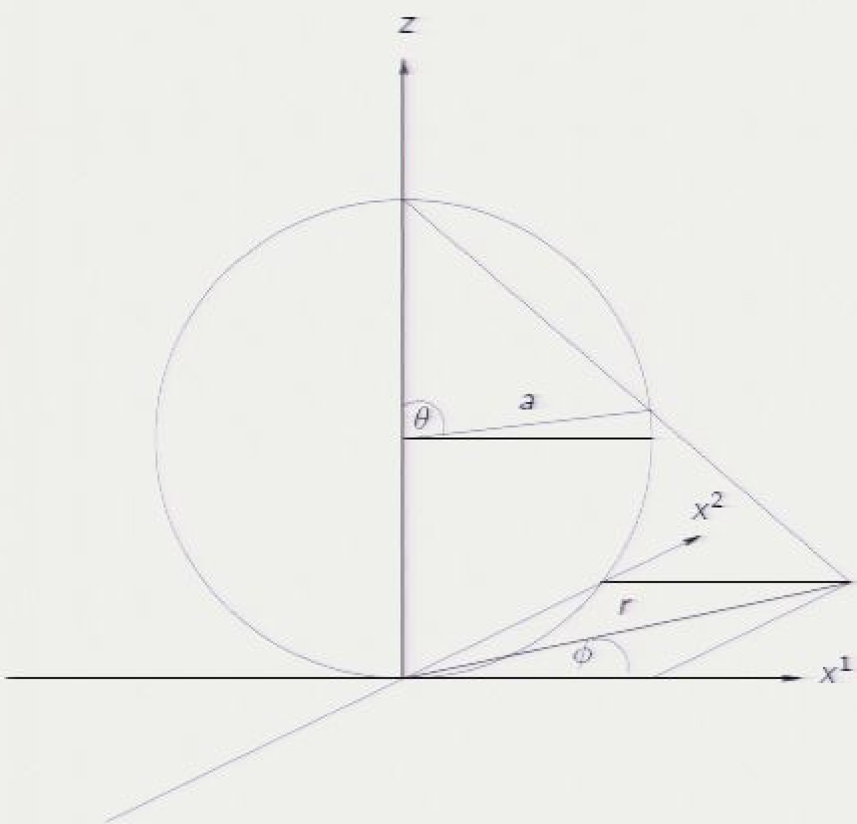

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- Averaged metric: $\langle g_{ij}(x) \rangle = g_{ij}(x)$

Averaging the Metric of a Perturbed Two-Sphere

- Perturb spherical coordinates with function $f(x, y, z)$
- Stereographic projection leads to perturbed metric:

$$(g_P)_{ij} = (1 + 2\eta f)(\tfrac{2a}{L})^4 \delta_{ij}$$

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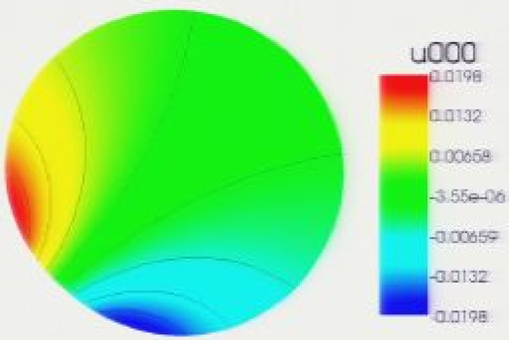
$\Rightarrow$ $$g^{ik}(\partial_i \partial_k \phi) = -\tfrac{1}{2} D_k u^k \text{ on } R$$

$\Rightarrow$ $$\frac{\partial \phi}{\partial n} = -\tfrac{1}{2} n_k u^k \text{ on } \partial R$$

The Gaussian Shaped Perturbation

Gascoigne3D



$$\left(\frac{\partial^2}{(\partial x^1)^2} + \frac{\partial^2}{(\partial x^2)^2}\right)\phi(x^1, x^2) = 0 \text{ on } R$$

Neumann boundary conditions on ∂R

$$\frac{\partial \phi}{\partial n} = \eta \cos^{-2}\left(\frac{r}{2a}\right)\left(h(r,\gamma) + \frac{1}{2a^2}\partial_\gamma \int_0^r f(s',\gamma)ds'\right)$$

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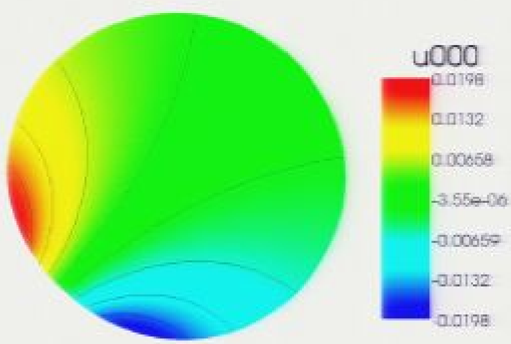

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R is the area inside ∂R given by

$$\alpha^1(\gamma)=2a\tan\left(\frac{r}{2a}\right)\cos\gamma+\eta\frac{v(r,\gamma)}{\cos^2(\frac{r}{2a})}\sin\gamma -\eta\frac{\cos\gamma}{\cos^2(\frac{r}{2a})}\int_0^r f(s',\gamma)ds'$$

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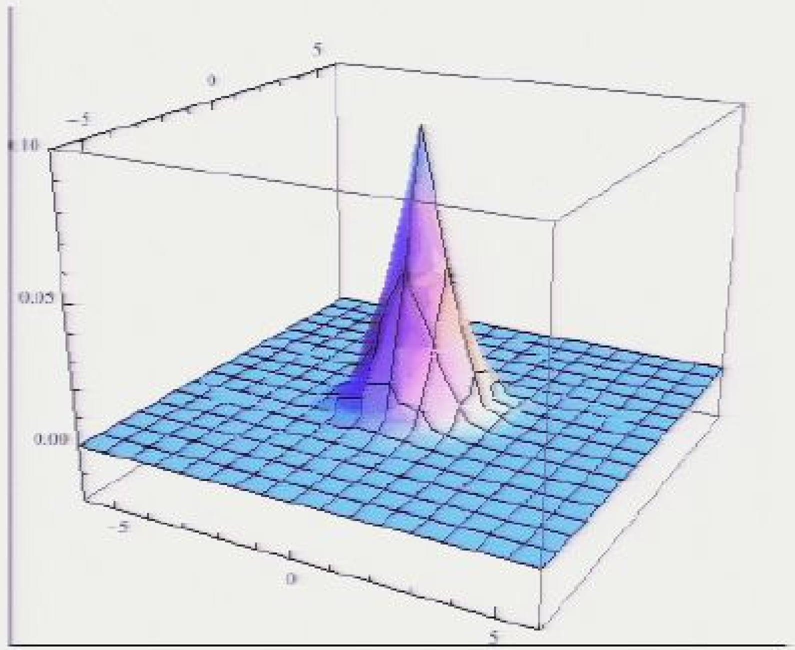
where $h(\tau)=\frac{\partial f}{\partial x^2}\Big|_{(x^1,x^2)=(z^1(\tau),z^2(\tau))}\frac{dz^1}{d\tau}(0)-\frac{\partial f}{\partial x^1}\Big|_{(x^1,x^2)=(z^1(\tau),z^2(\tau))}\frac{dz^2}{d\tau}(0)$

and $v(\tau,\gamma)$ fulfills the differential equation $\frac{d^2v(\tau,\gamma)}{d\tau^2}+\frac{v(\tau,\gamma)}{a^2}=\frac{h(\tau,\gamma)}{\cos^2(\frac{\tau}{2a})}$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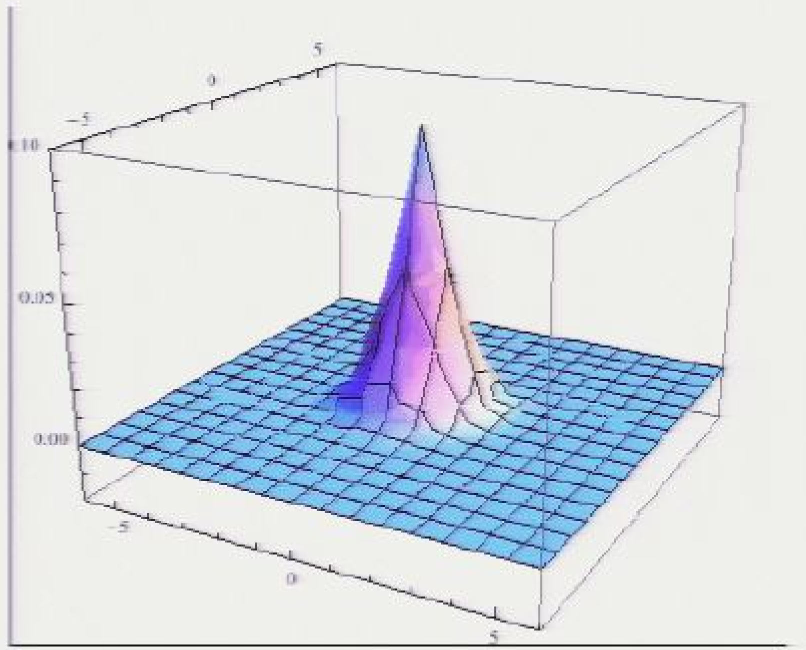
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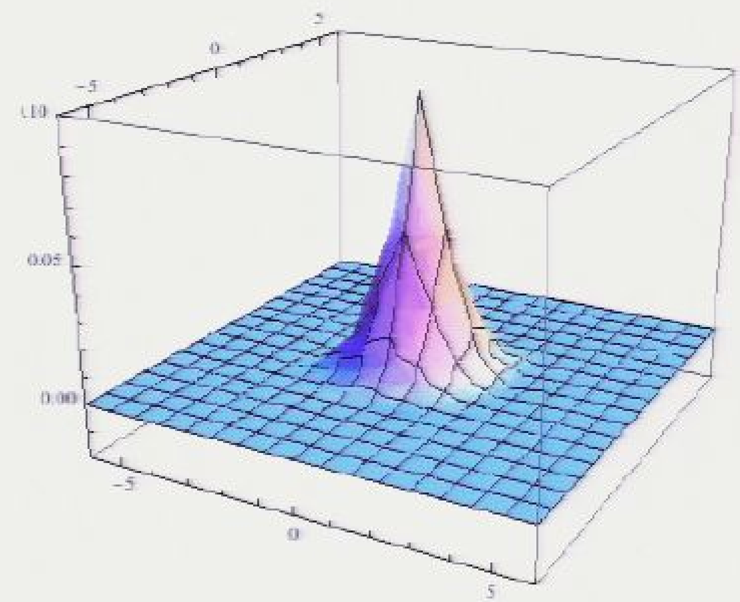
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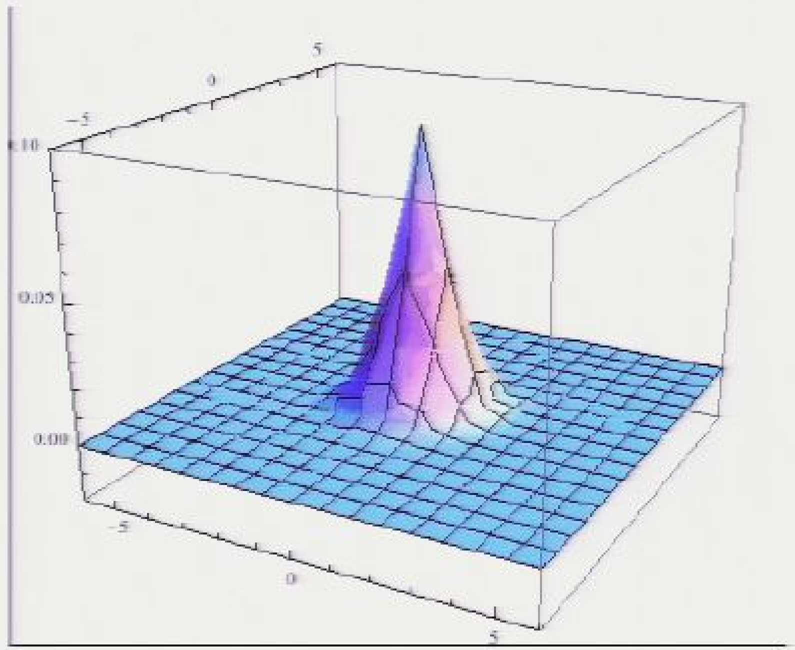

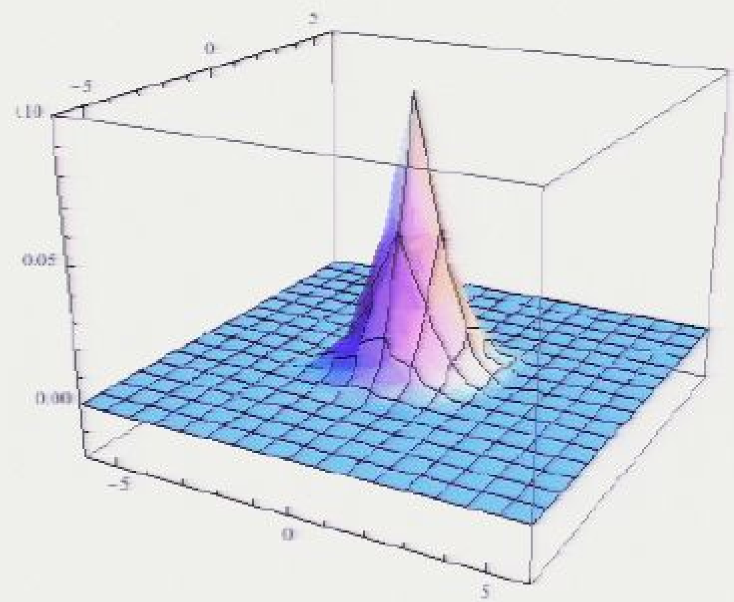

- Averaging effect in investigated example is too small

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

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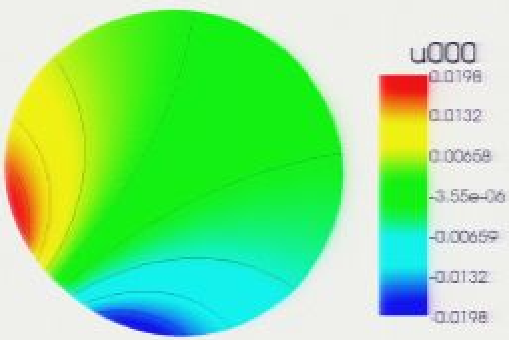

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Averaging the Metric of a Two-Sphere

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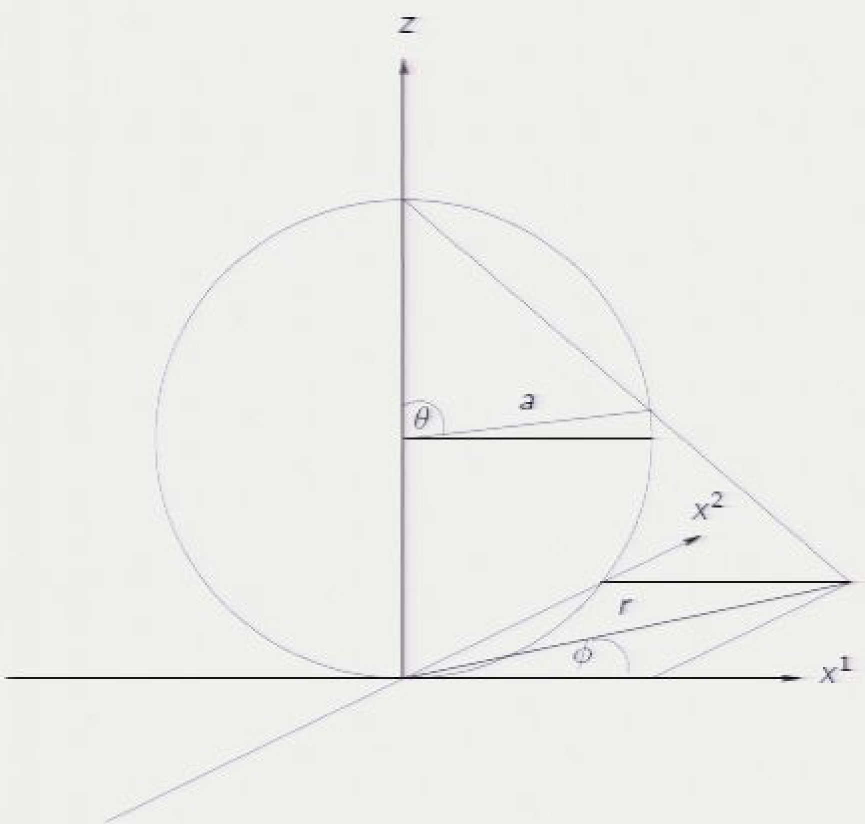

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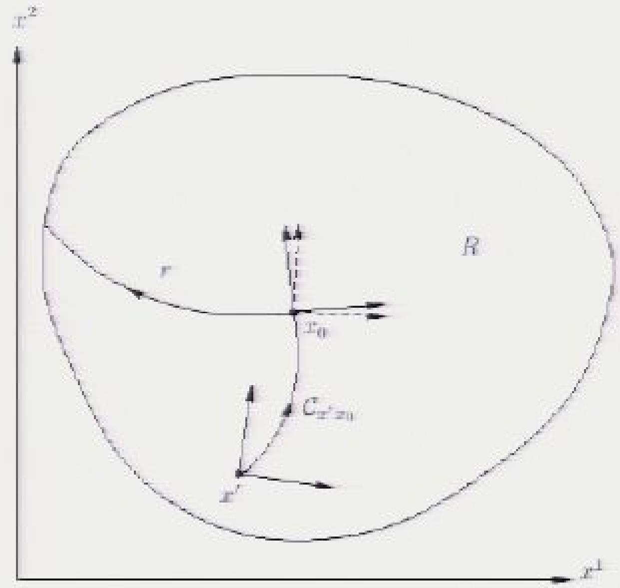

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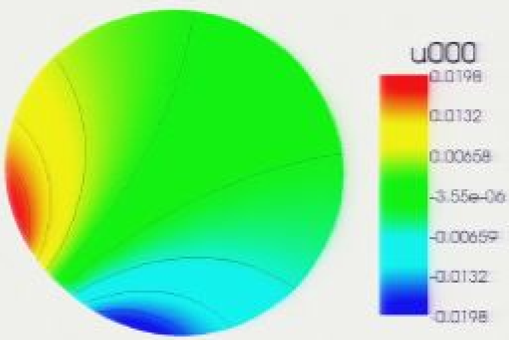

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

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- $\mathcal{L} = \left(L_n E^a{}_i\right)\left(L_n E^b{}_j\right)\delta_{ab} n^i n^j$
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- ...

⇒ They all fail to define a dyad field that can be used to average the perturbed plane in the desired way

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR

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⇒ Use different Lagrangian to define initial tetrad field:

- $\mathcal{L} = \left(L_t E^a{}_i\right)\left(L_t E^b{}_j\right)\delta_{ab}\left(t^i t^j + g^{ij} R s^2\right)$
- $\mathcal{L} = \left(L_n E^a{}_i\right)\left(L_n E^b{}_j\right)\delta_{ab} n^i n^j$
- $\mathcal{L} = \left(L_\xi E^a{}_i\right)\left(L_\xi E^b{}_j\right)\delta_{ab} \xi^i \xi^j$
- ...

- Problem: Used Lagrangian is similar to the Lagrangian which defines the geodetic induced parallel field

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- ...

⇒ They all fail to define a dyad field that can be used to average the perturbed plane in the desired way

Conclusions and Outlook

- Once we have a suitable Lagrangian we have a generally covariant averaging process which can be used to smooth metrics in the framework of GR
- Apply it to different perturbation functions to study their interaction with each other and with the background sphere
- Apply it to three-sphere and three-plane corresponding to hypersurfaces of closed and flat FLRW models

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$\Rightarrow$ Apply averaging process to Cosmology and combine the two lines of research