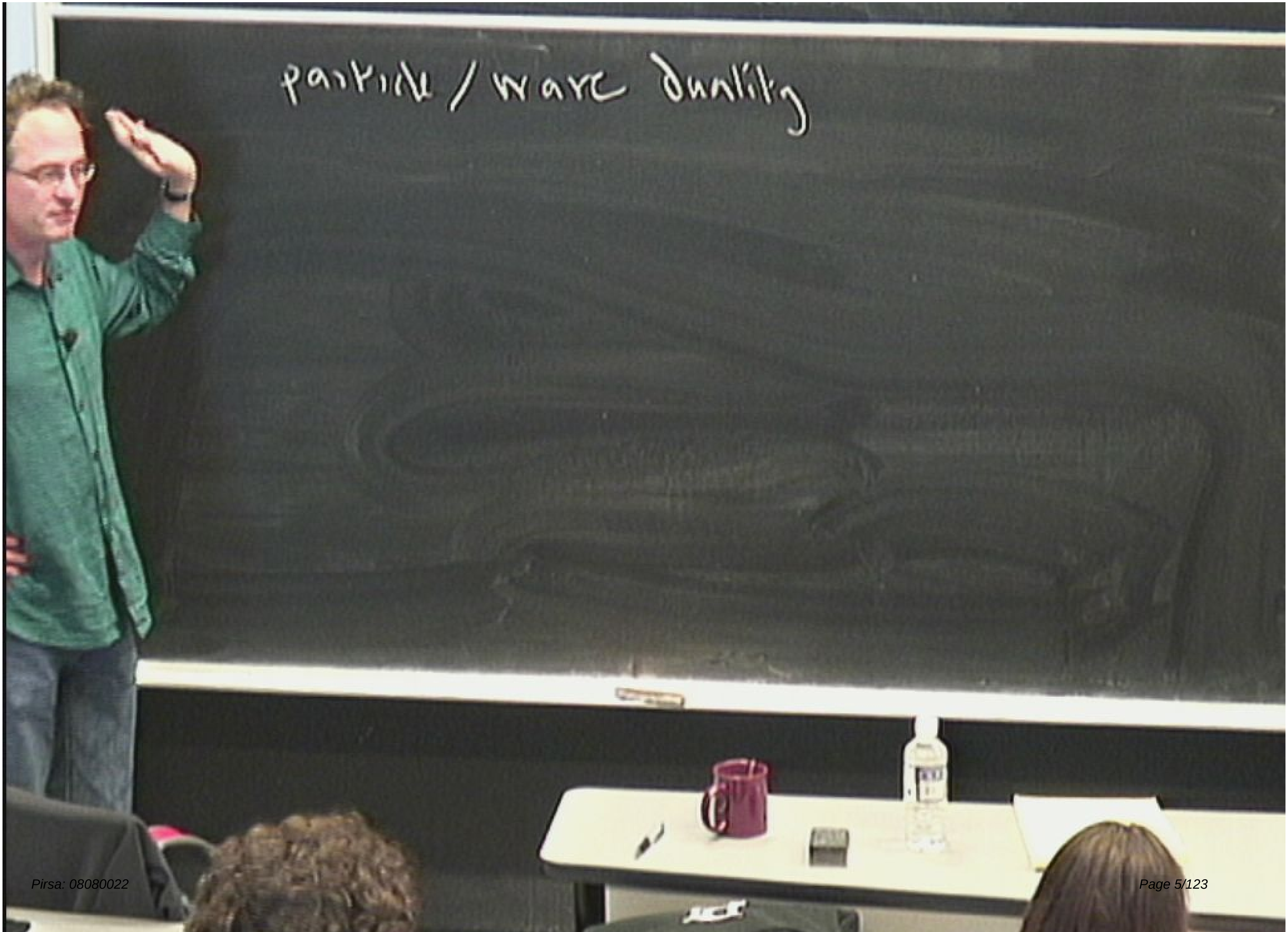
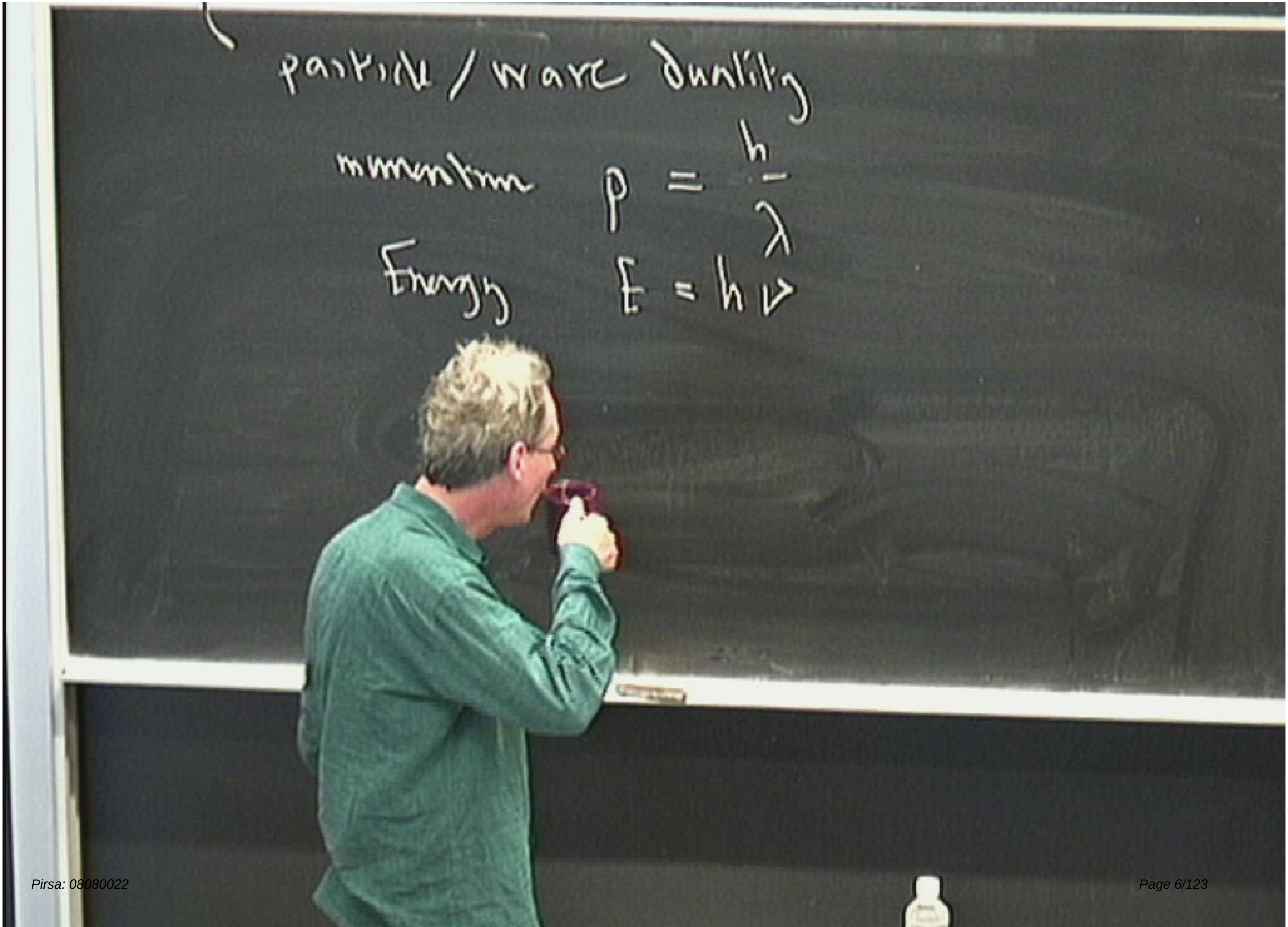
Title: Heisenberg's Uncertainty Principle

Date: Aug 16, 2008 09:00 AM

URL: http://pirsa.org/08080022

Abstract: We will review the uncertainty principle of quantum mechanics, first formulated by Werner Heisenberg in 1927, and the role they played in the famous debate between Einstein and Bohr on the meaning of quantum theory. Along the way we will focus on questions like: what do we mean by "uncertainty", and how do we express that in the theory? What, in fact, is a physical property? Does a theory like quantum mechanics provide a description of physical reality? Interestingly, some of these questions do not have a unique answer.


particle / wave duality


particle / wave duality
momentum $p = \frac{h}{\lambda}$
Energy $E = h\nu$

particle / wave duality

momentum $p = \frac{h}{\lambda}$

Energy $E = h\nu$

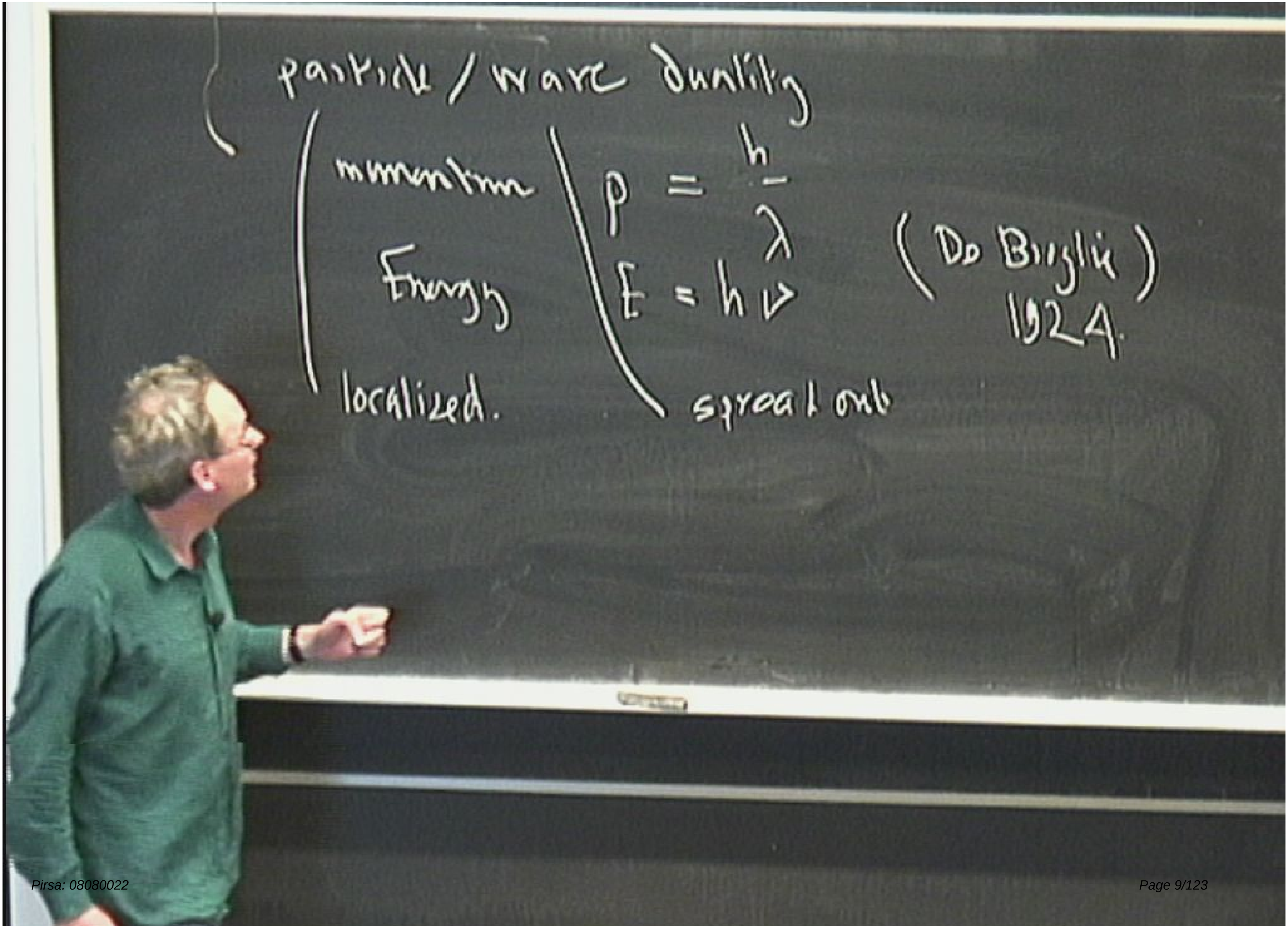
(De Broglie) 1924

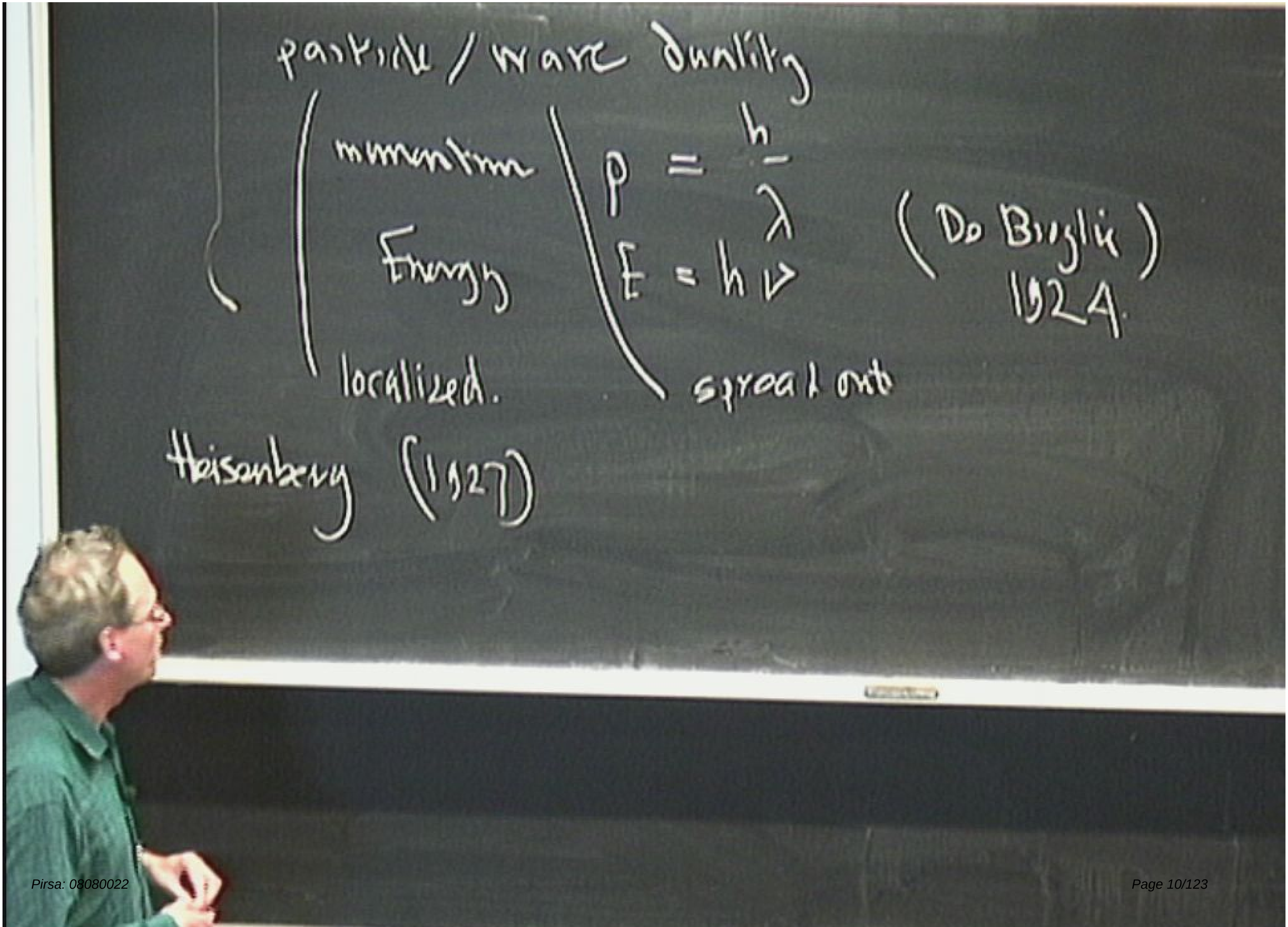
particle / wave duality

momentum $p = \frac{h}{\lambda}$

Energy $E = h\nu$ (De Broglie) 1924

localized.


particle / wave duality
momentum
$p = \frac{h}{\lambda}$
Energy
$E = h\nu$
(De Broglie)
1924
localized.
spread out


particle / wave duality
momentum
Energy
localized.
$p = \frac{h}{\lambda}$
$E = h\nu$
spread out
(De Broglie)
1924.
Heisenberg (1927)

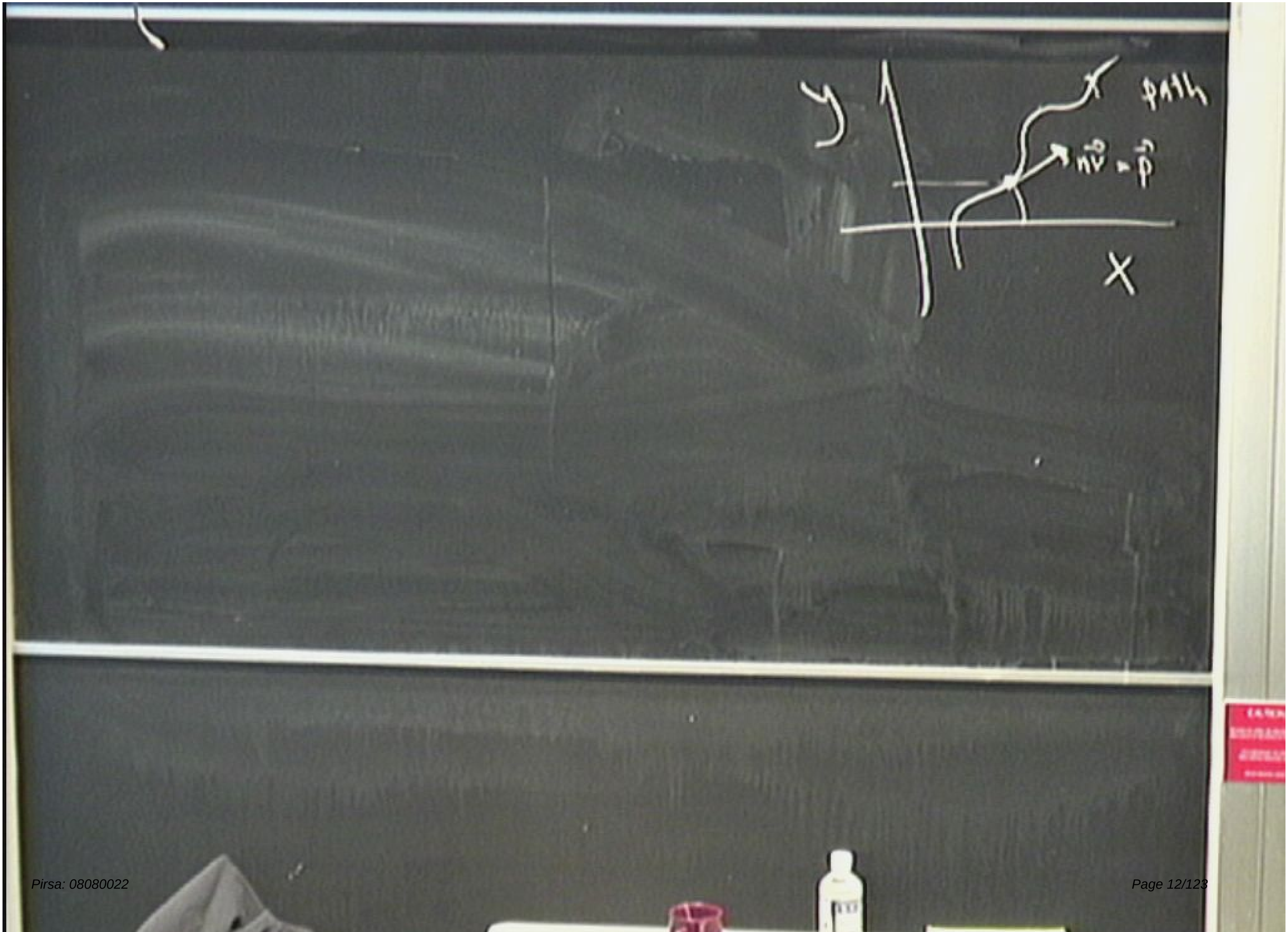
particle [illegible]

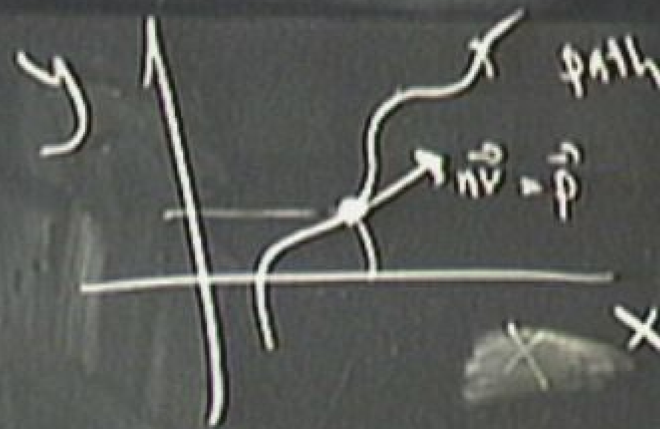
momentum $p = \frac{h}{\lambda}$ (De Broglie) 1924

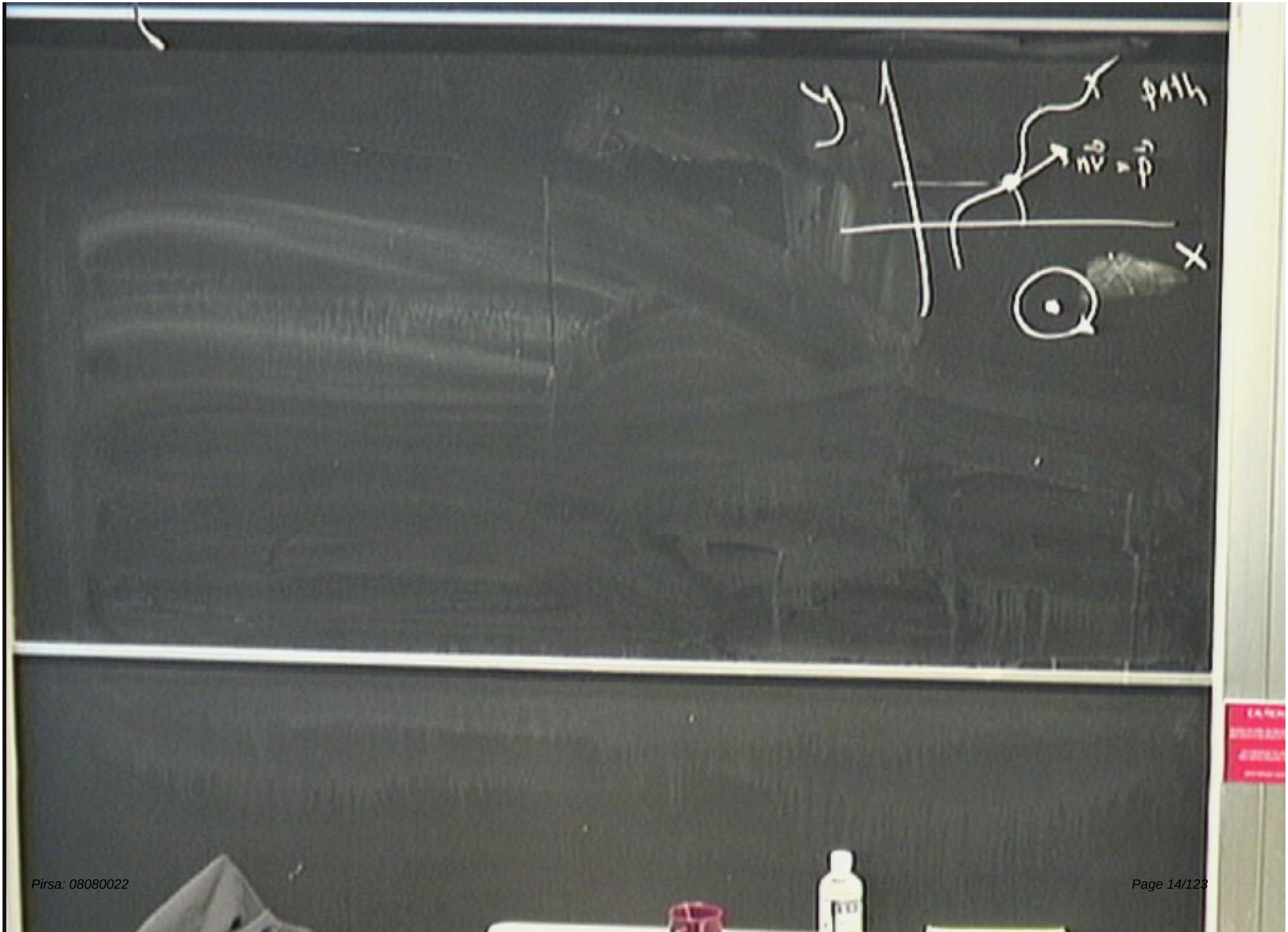
Energy $E = h\nu$

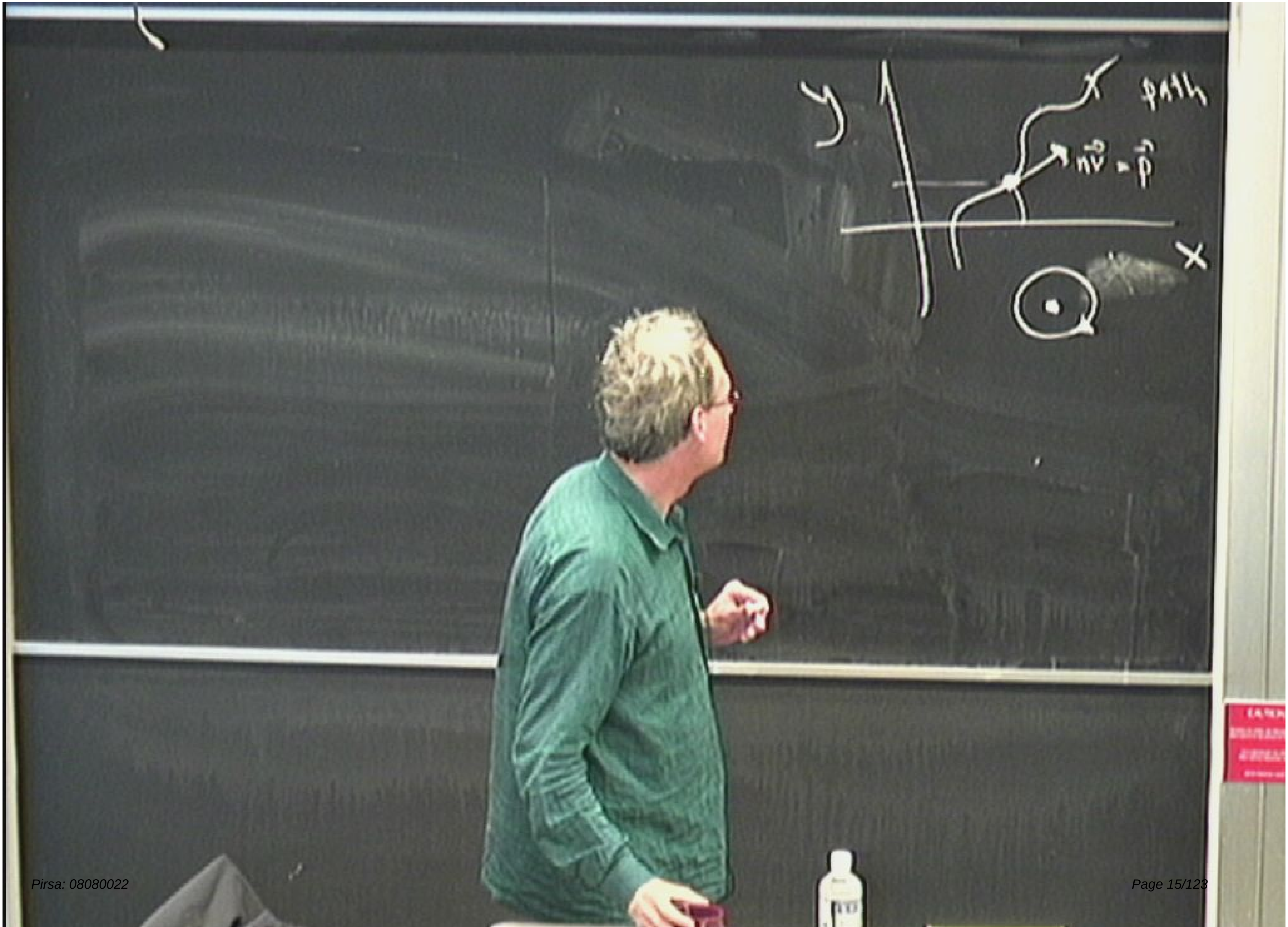
localized. spread out

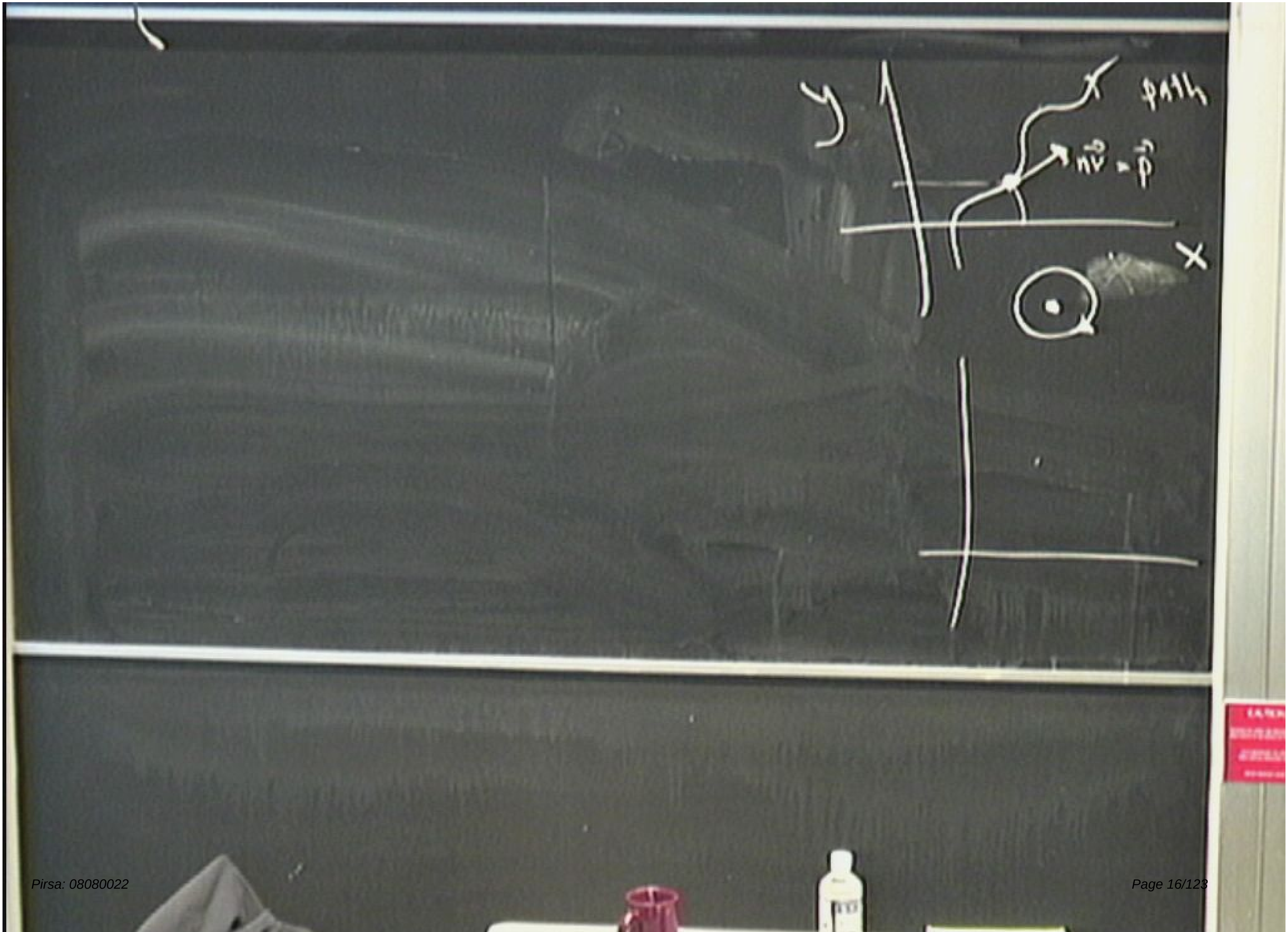
Heisenberg (1927) what is position of a particle?

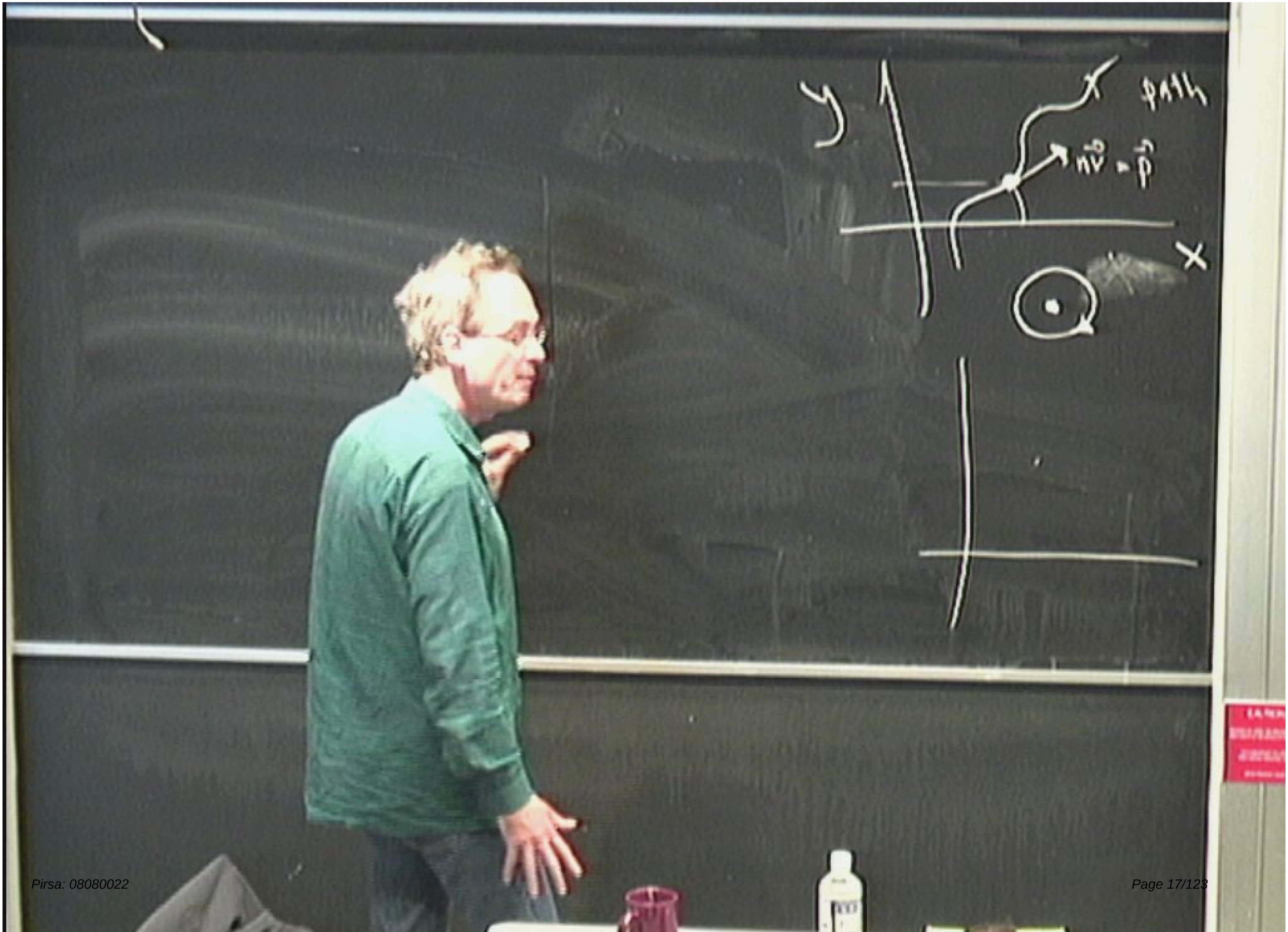

y
path
$m\vec{v} = \vec{p}$
x

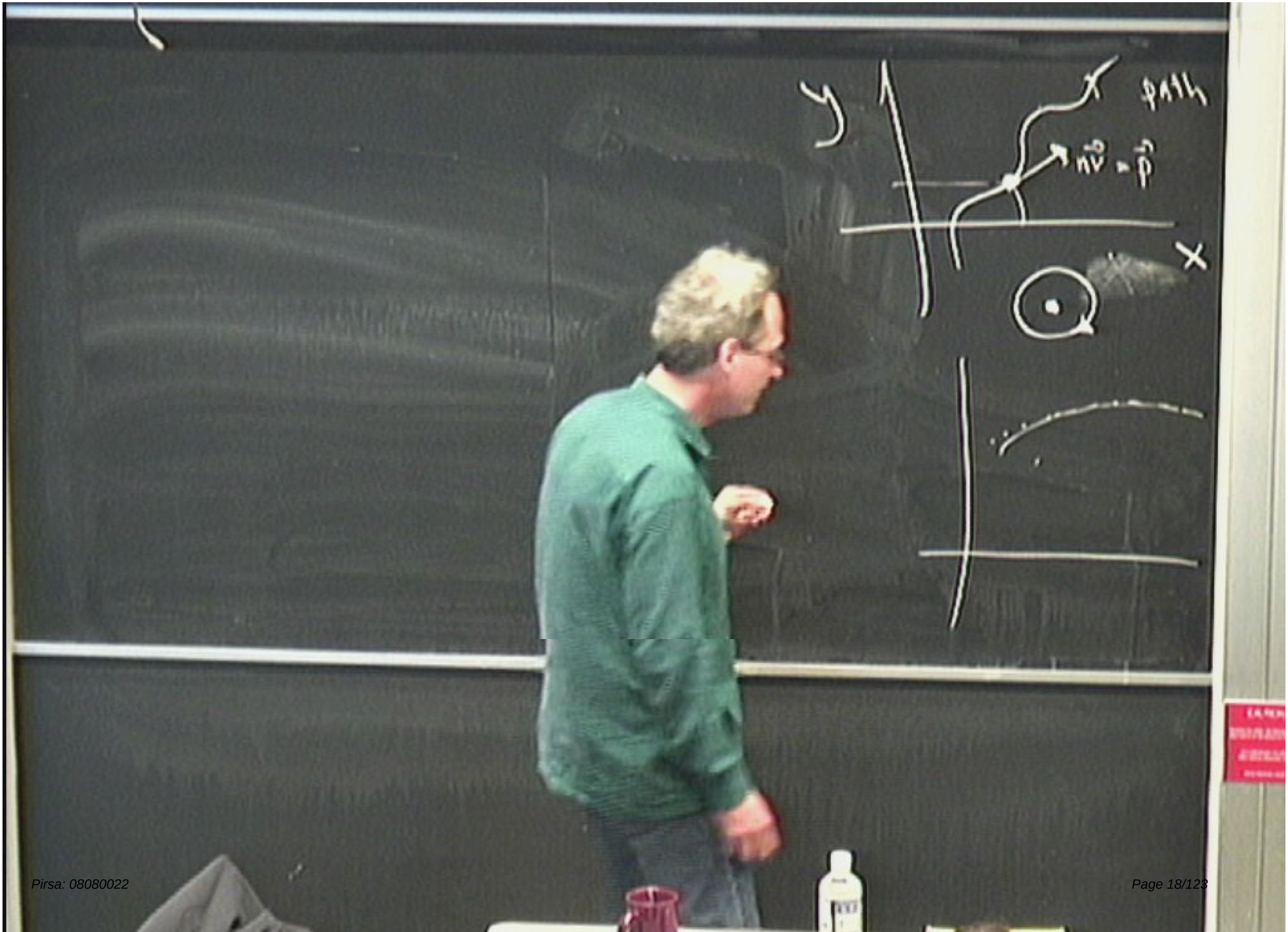

y
path
$m\vec{v} = \vec{p}$
x

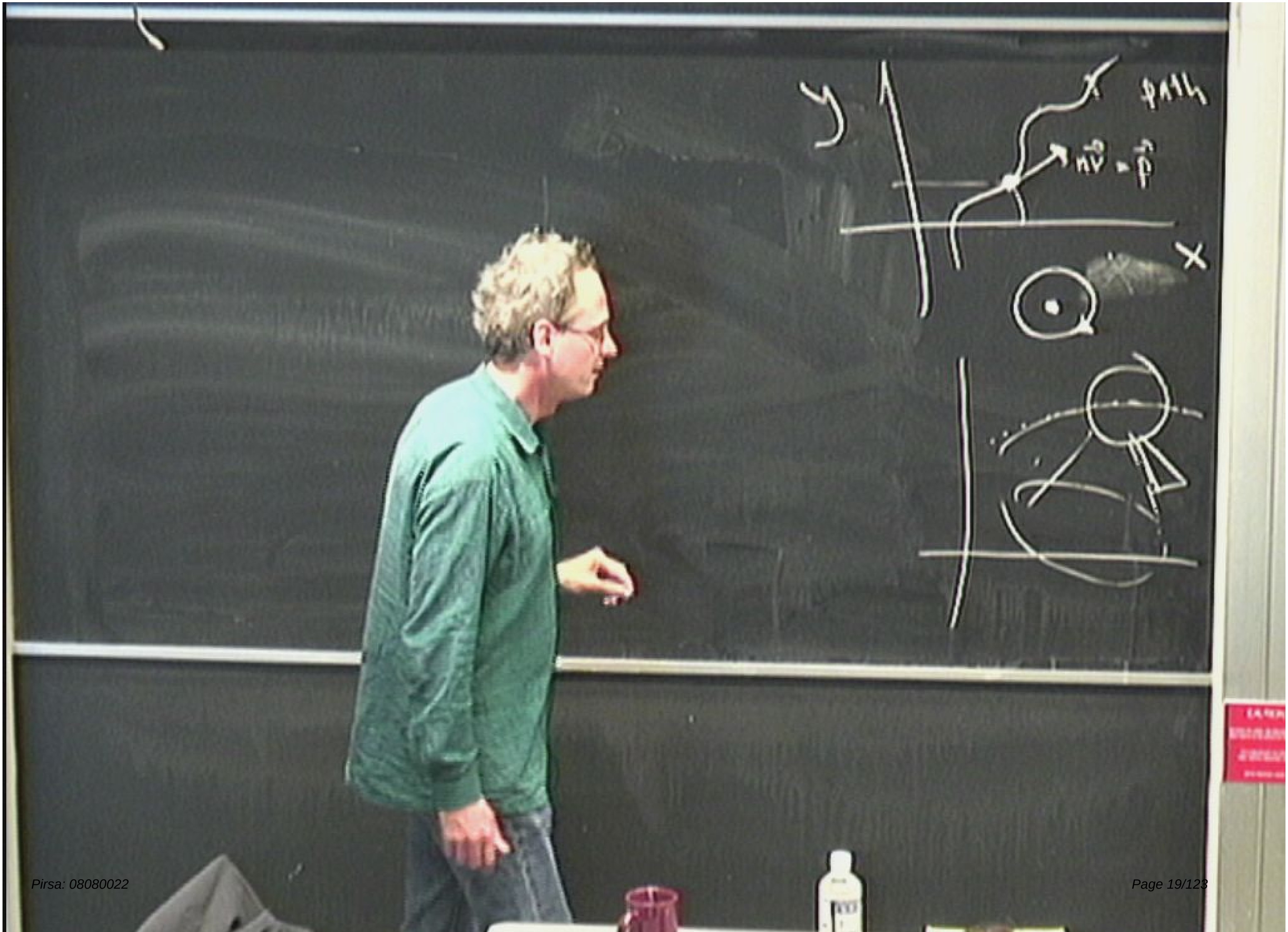

y
path
$m\vec{v} = \vec{p}$
x

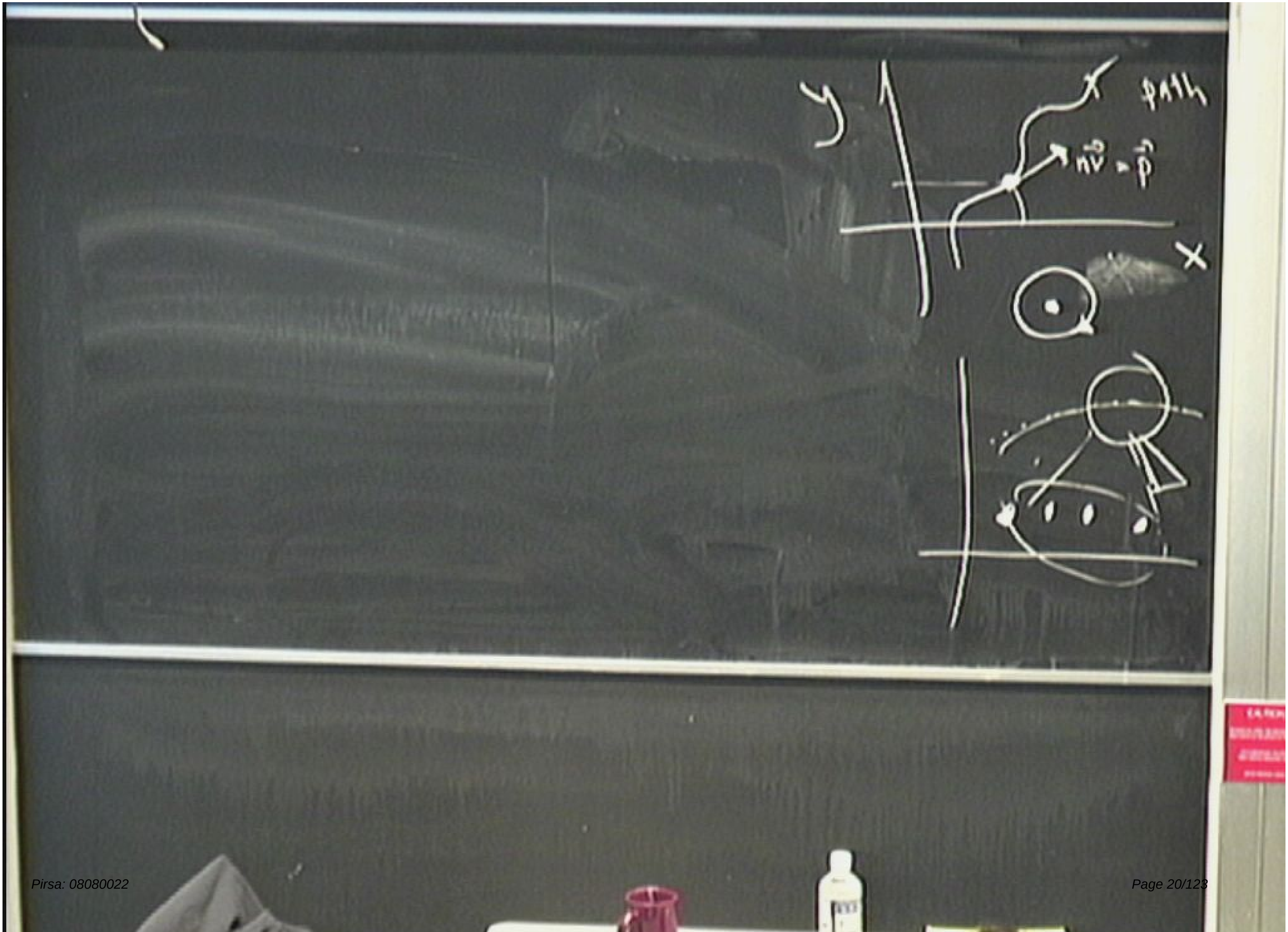

y
path
$m\vec{v} = \vec{p}$
x

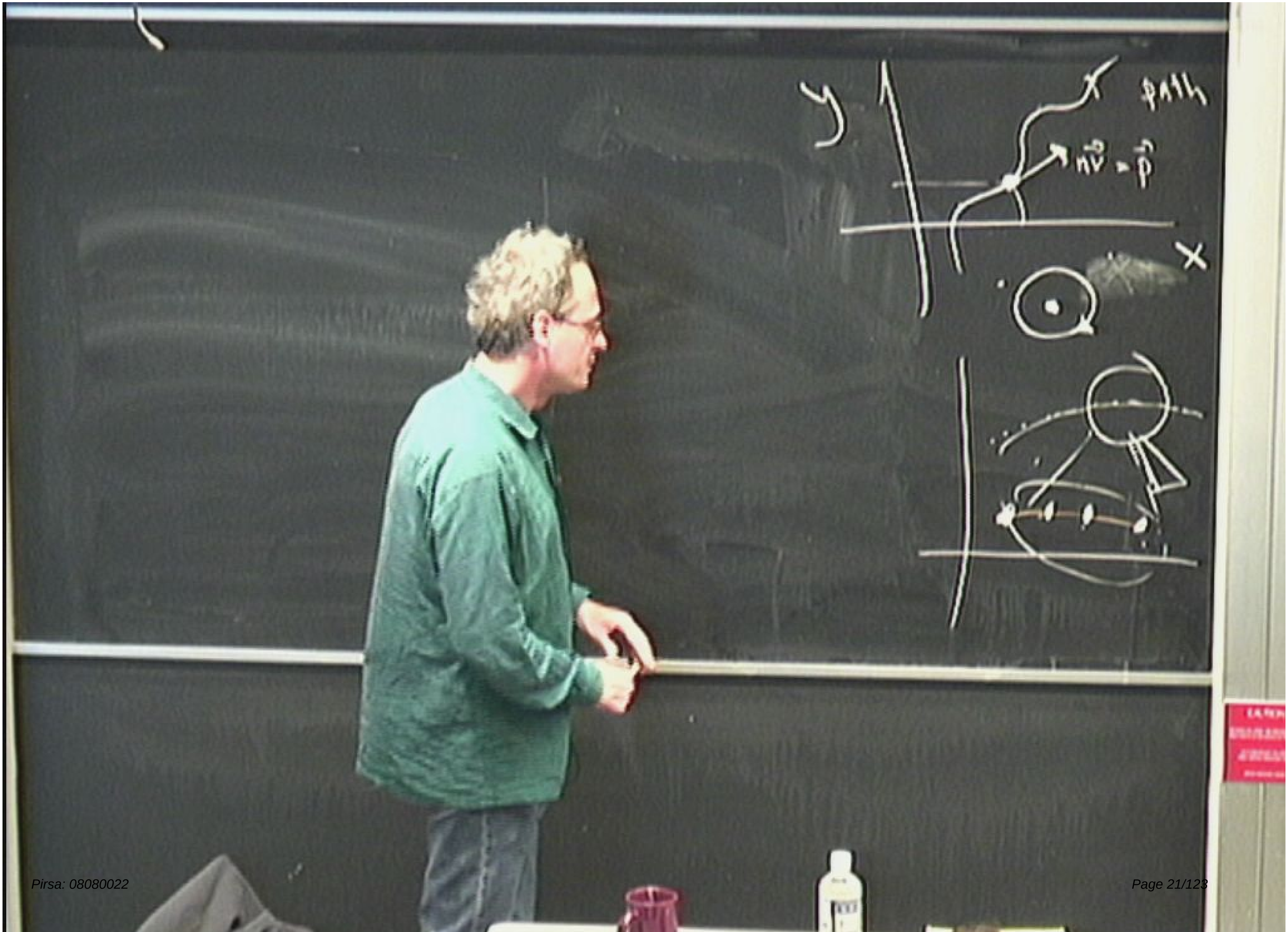

y
path
$m\vec{v} = \vec{p}$
x

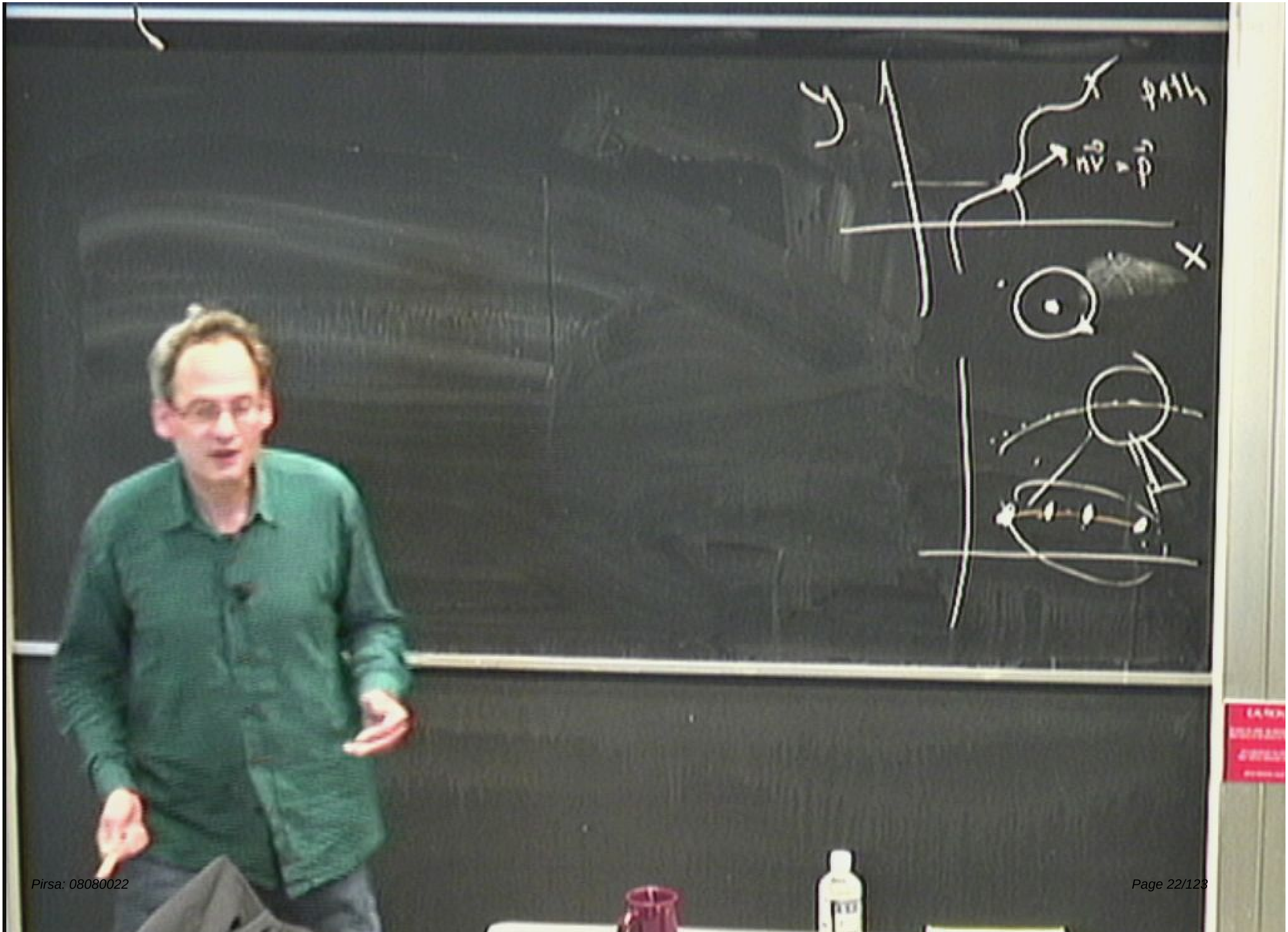

y
path
$m\vec{v} = \vec{p}$
x

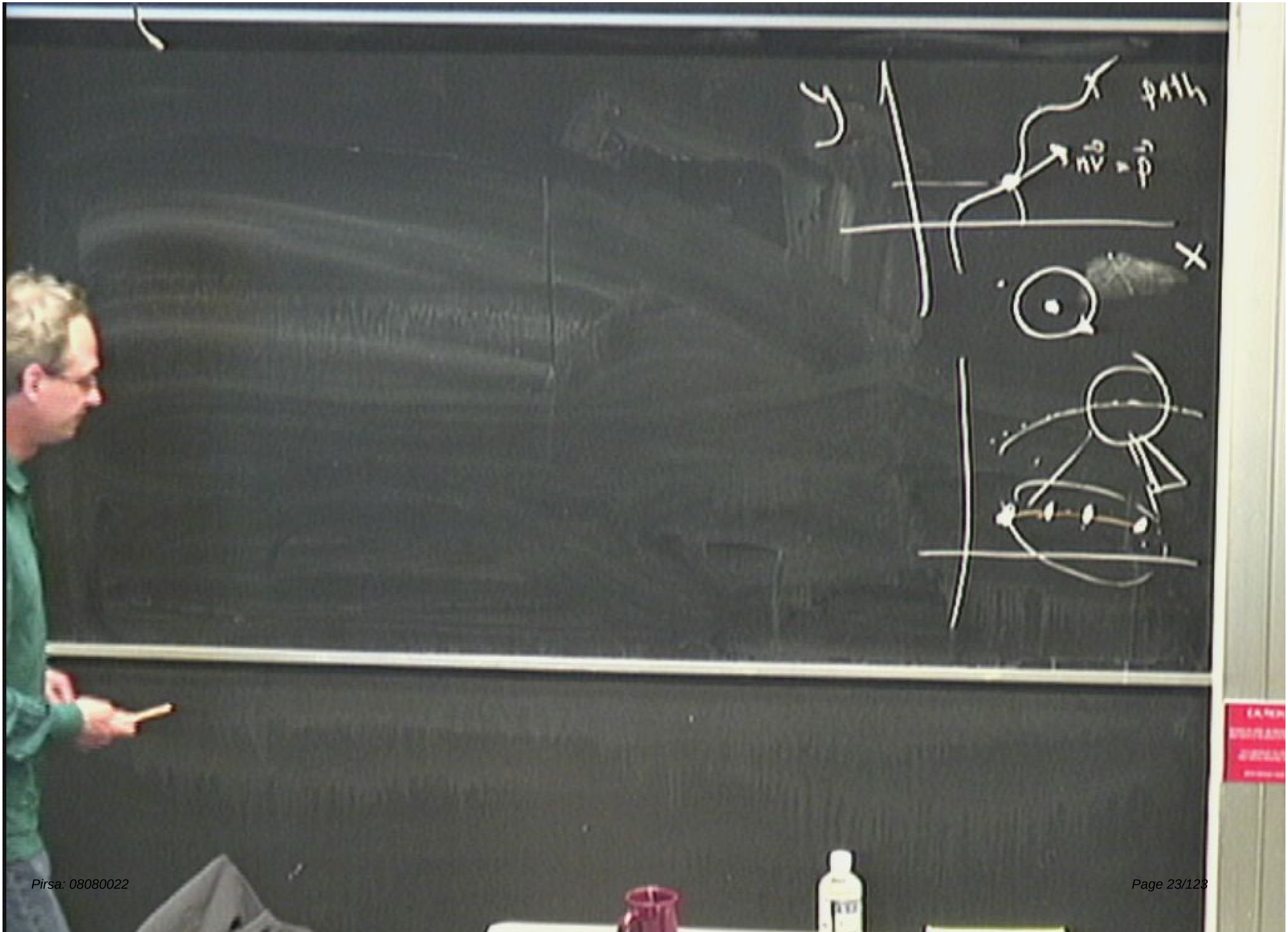

y
path
$m\vec{v} = \vec{p}$
x

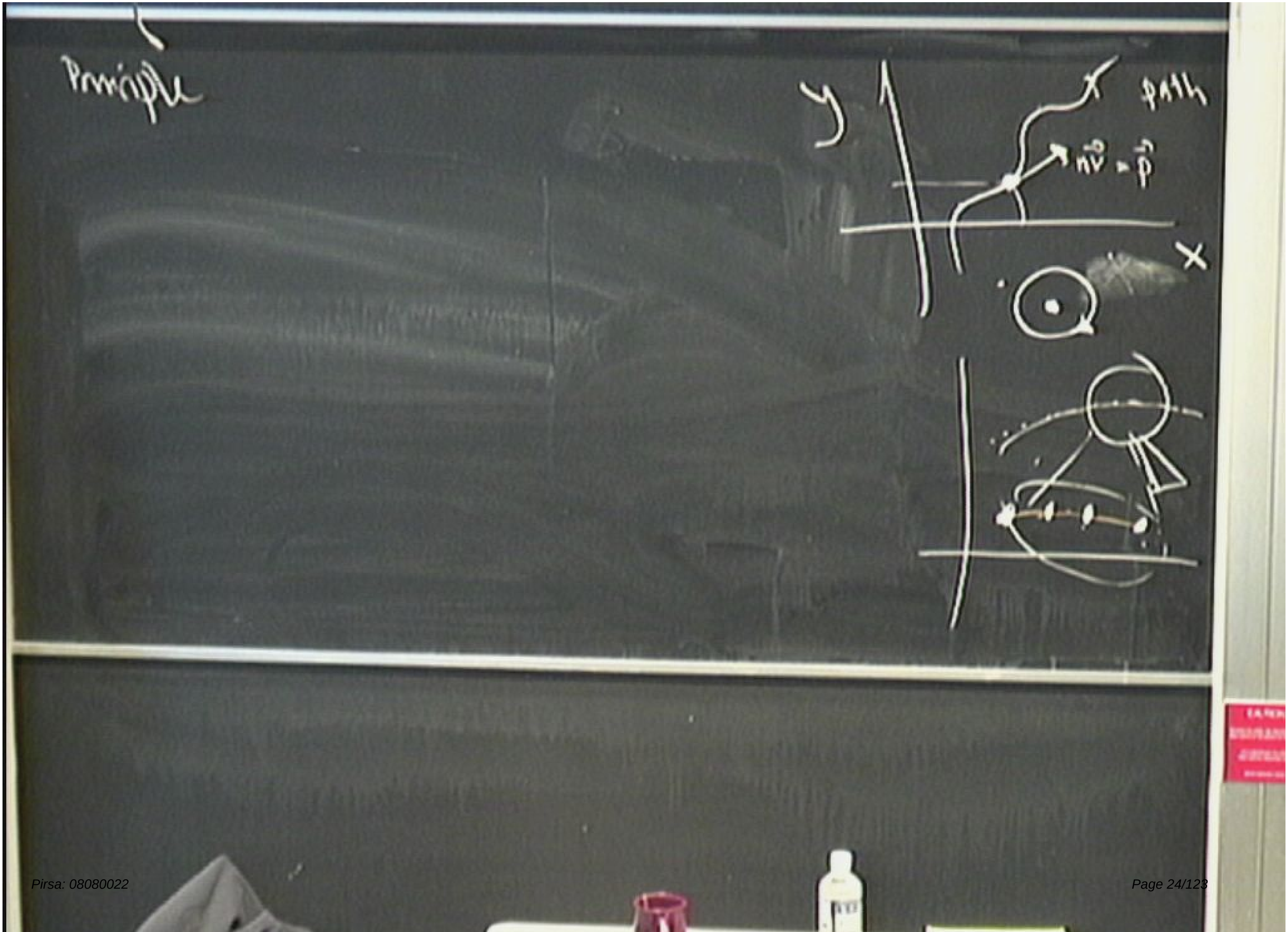

y
path
$m\vec{v} = \vec{p}$
x

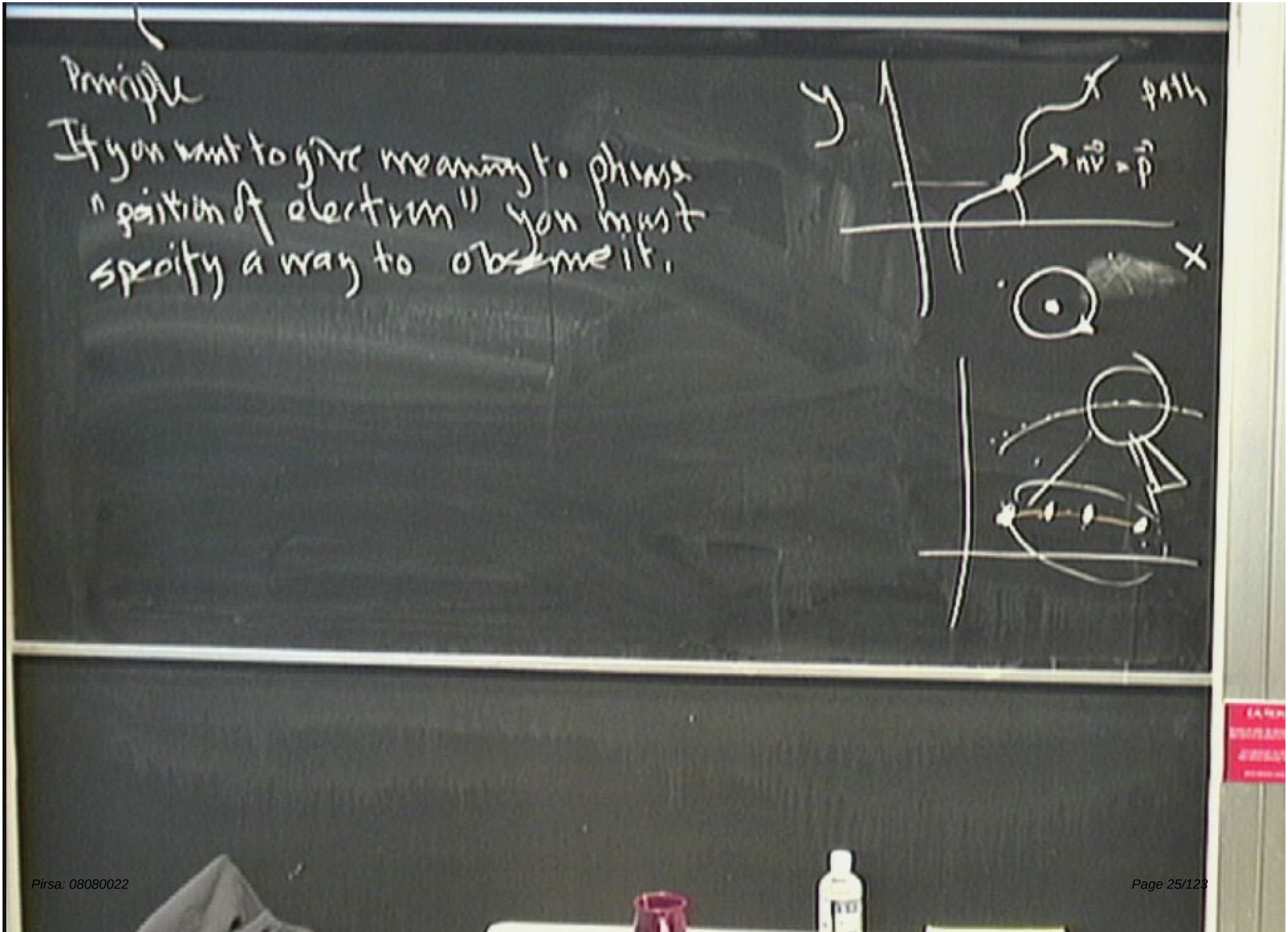

y
path
mv = p
x


y
path
mv = p
x


y
path
$m\vec{v} = \vec{p}$
x

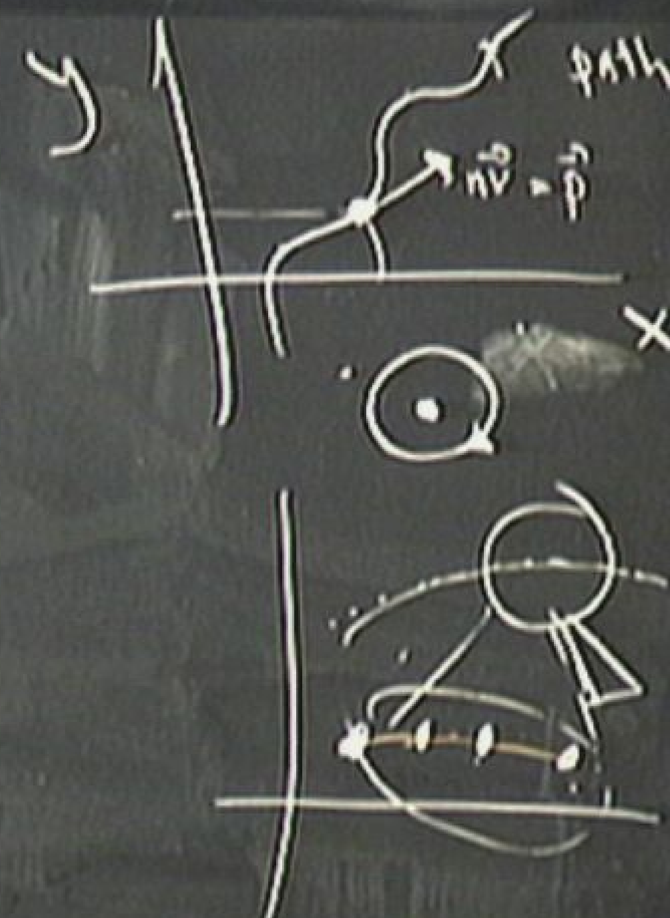

y
path
$m\vec{v} = \vec{p}$
x


Principle
y
path
$m\vec{v} = \vec{p}$
x


Principle
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.
y
path
$m\vec{v} = \vec{p}$
x

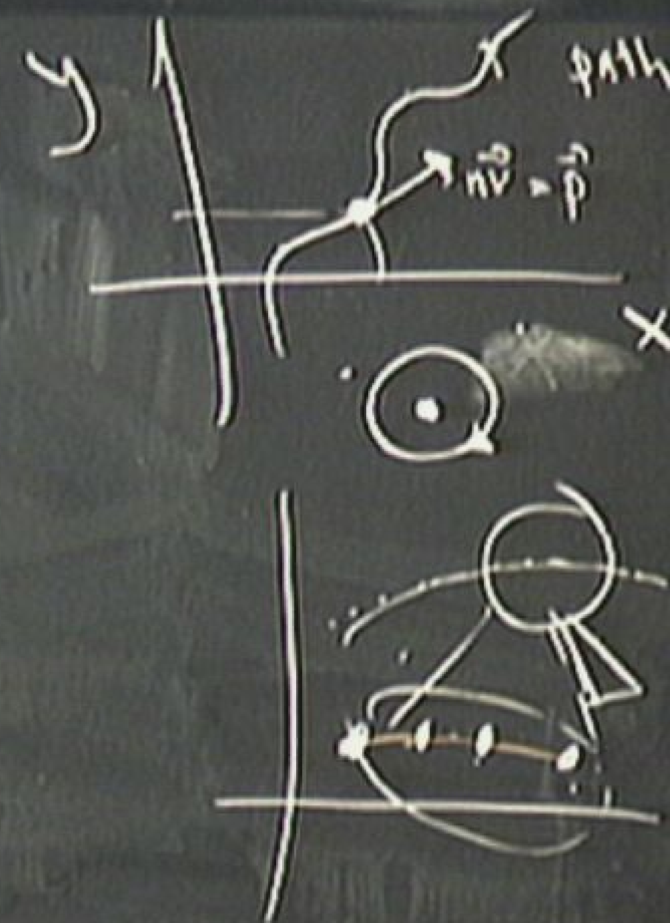
Principle

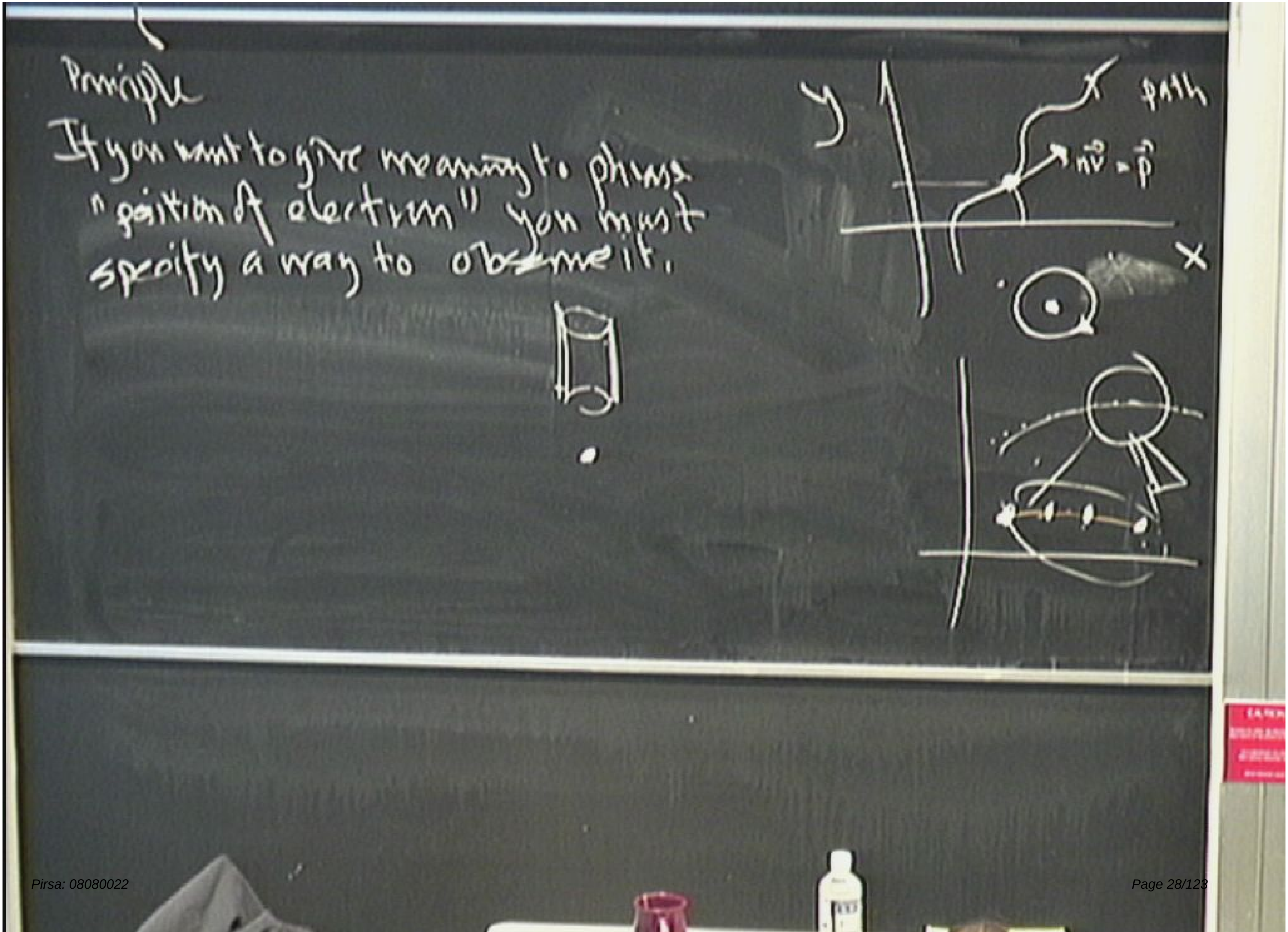
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.

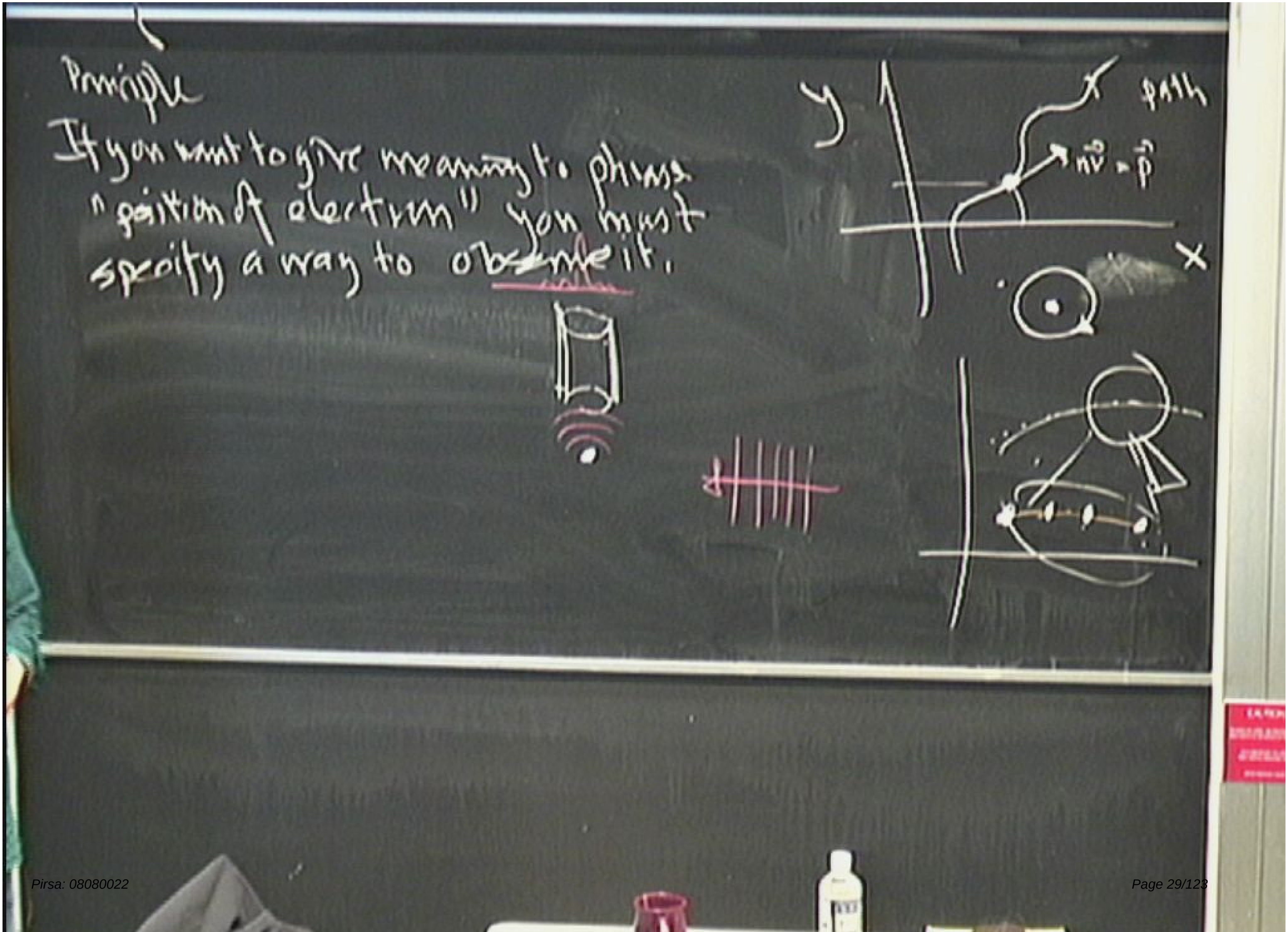


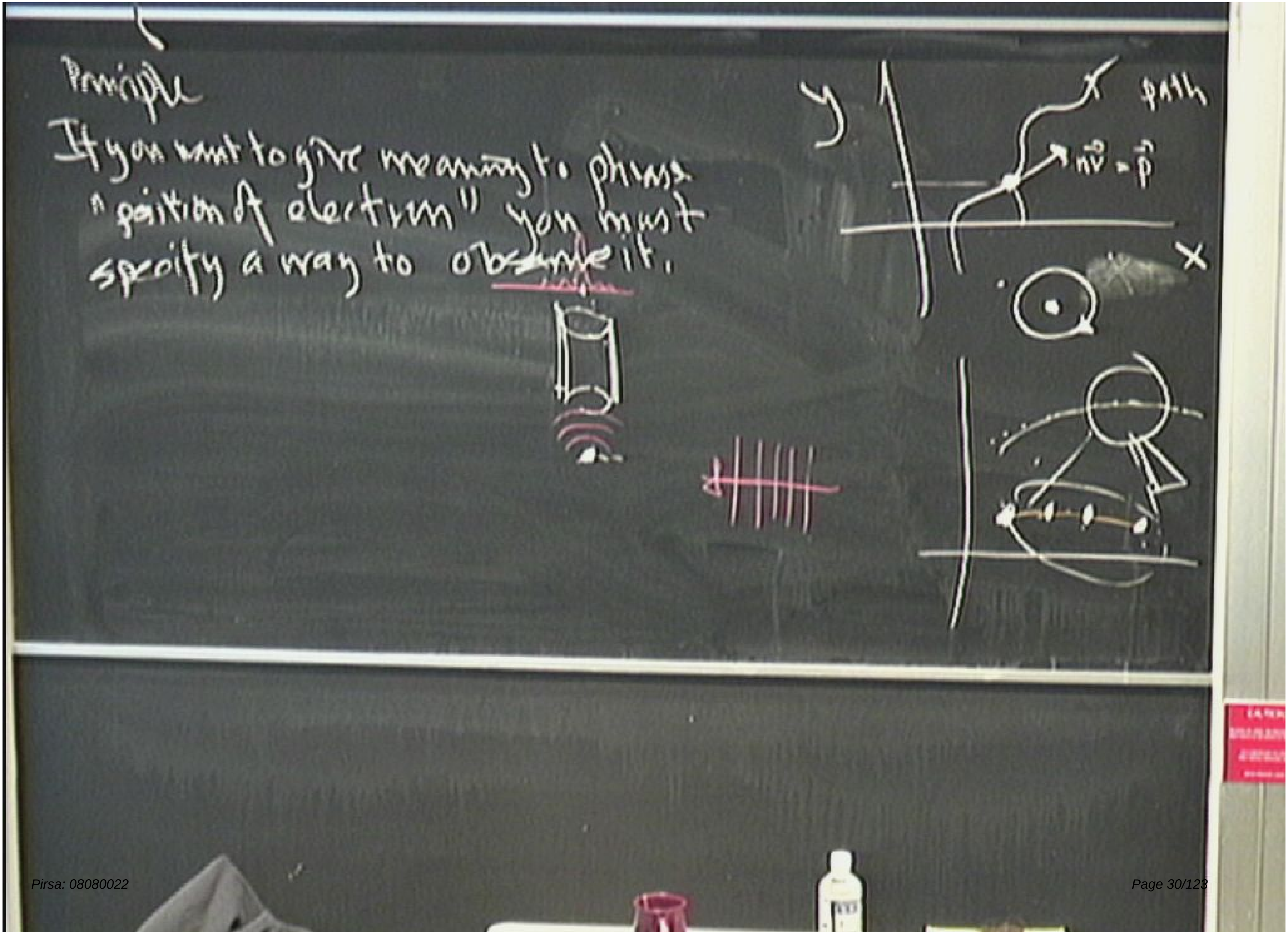
Principle

If you want to give meaning to phrase "position of electron" you must specify a way to observe it.



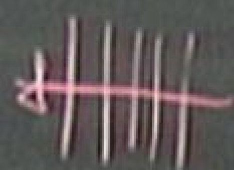

Principle
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.
y
path
m v = p
x

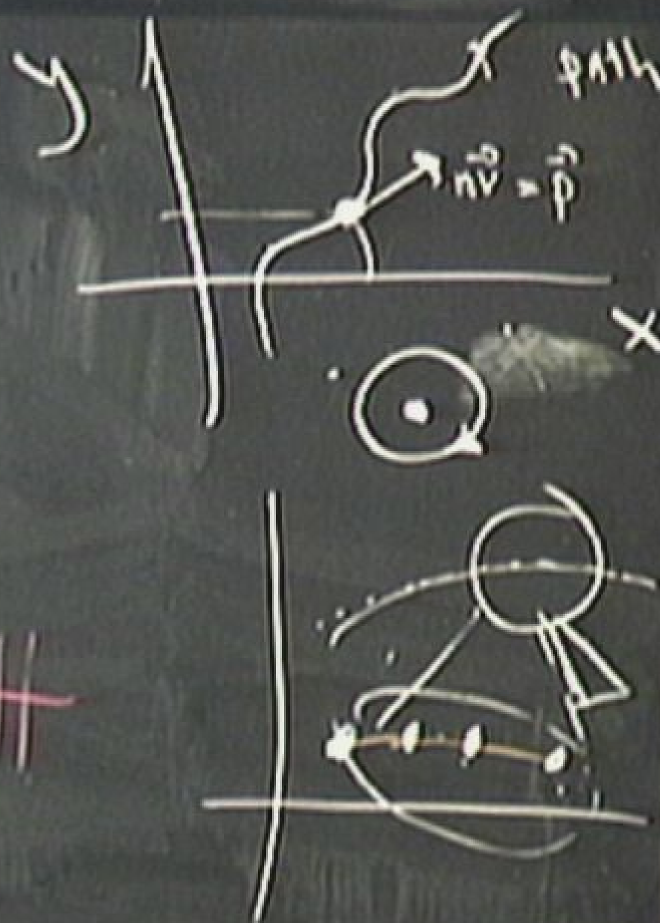

Principle
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.
y
path
$m\vec{v} = \vec{p}$
x


Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
y
path
$m\vec{v} = \vec{p}$
x

Principle

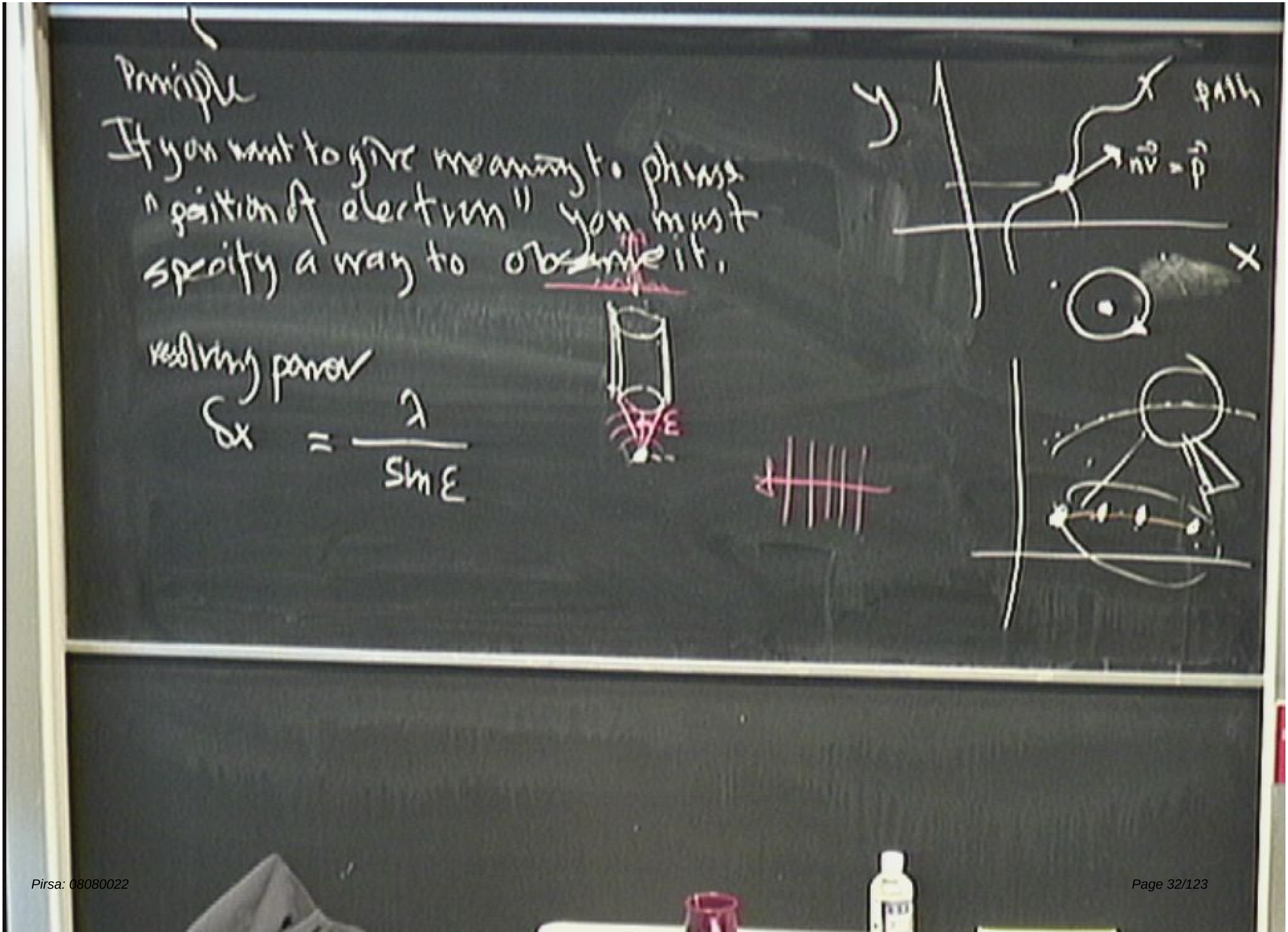
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.

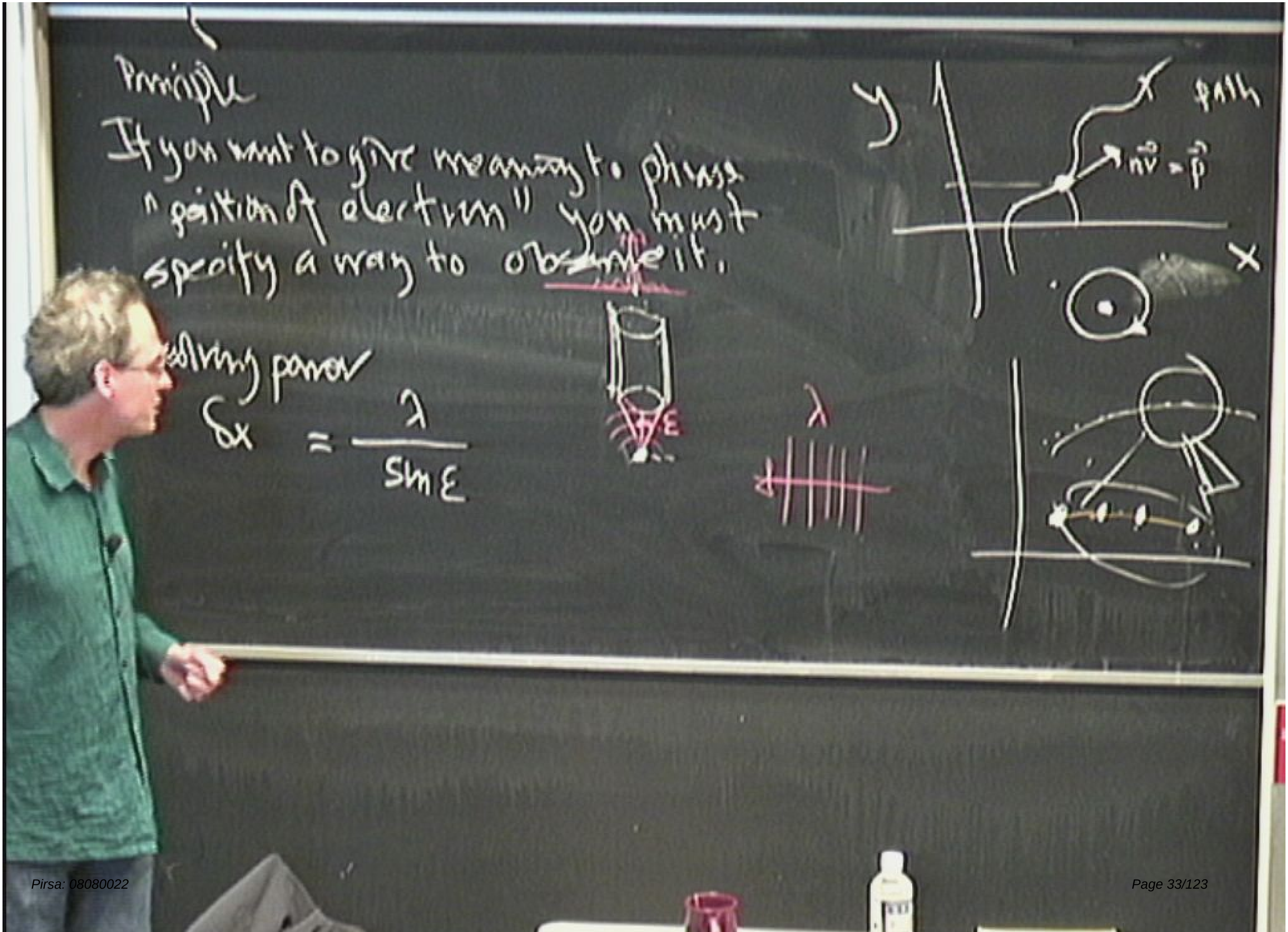


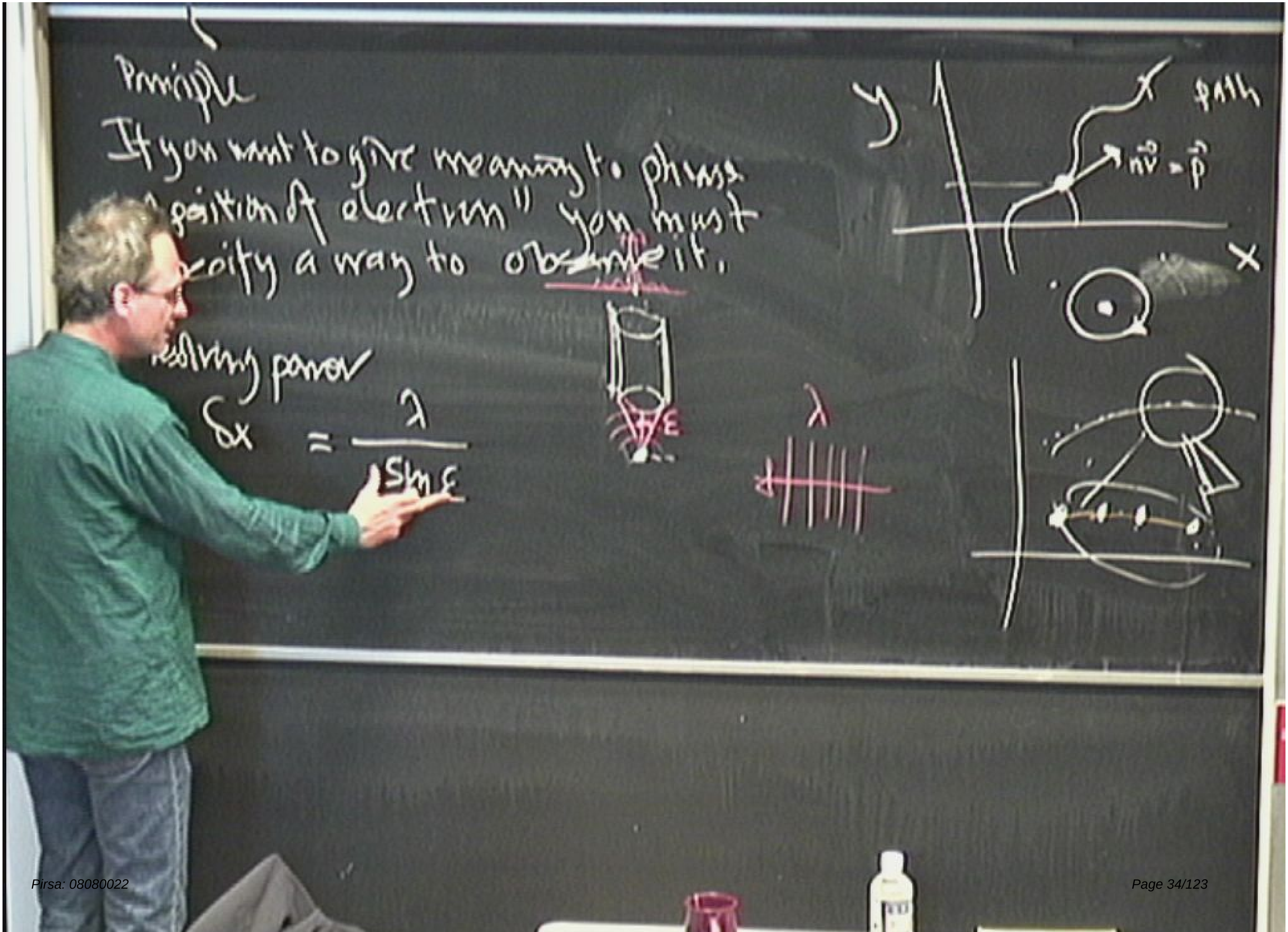


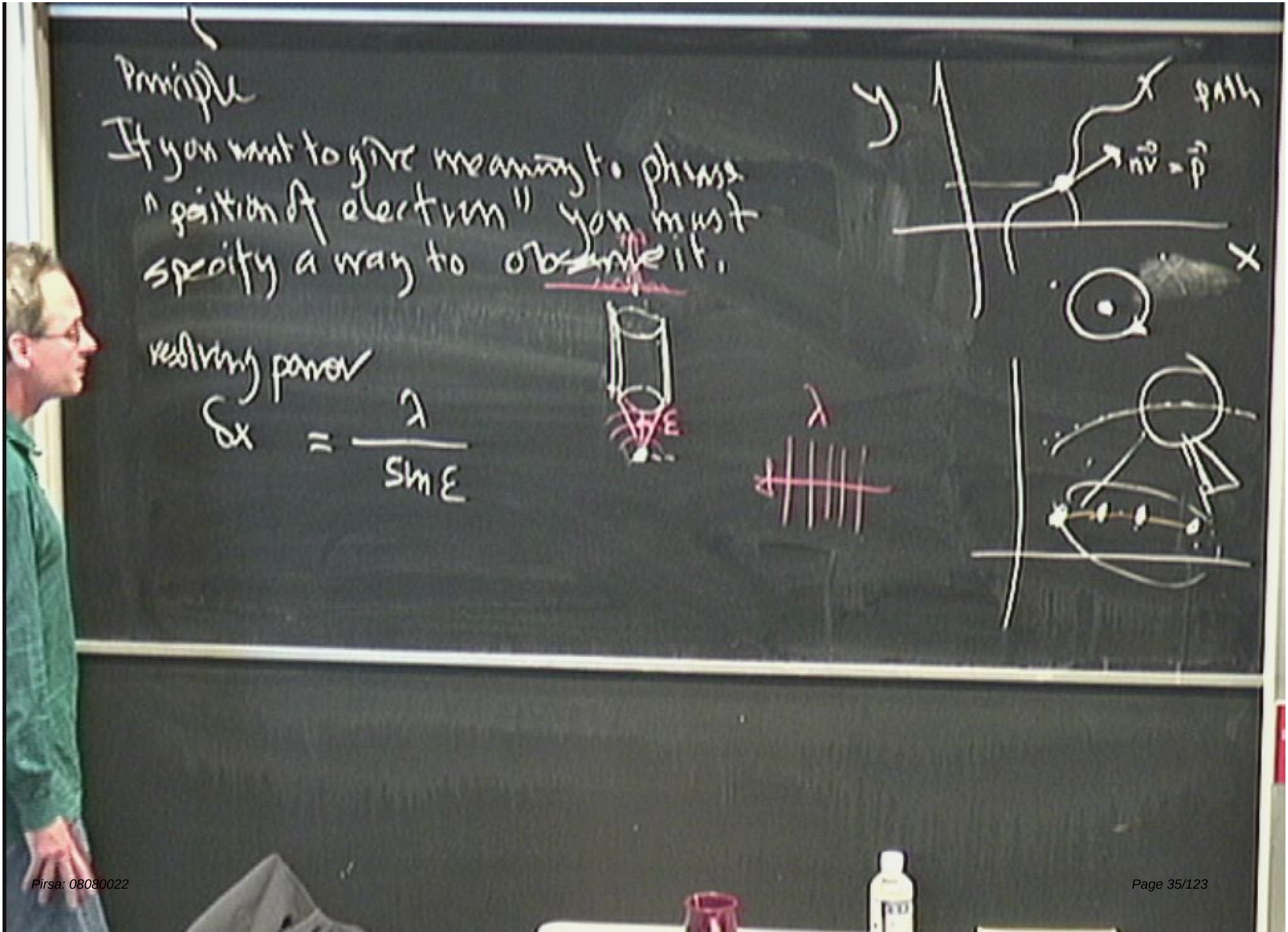
resolving power

δx


Principle
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.
resolving power
$\delta x = \frac{\lambda}{\sin \varepsilon}$
ε
y
x
path
$m\vec{v} = \vec{p}$

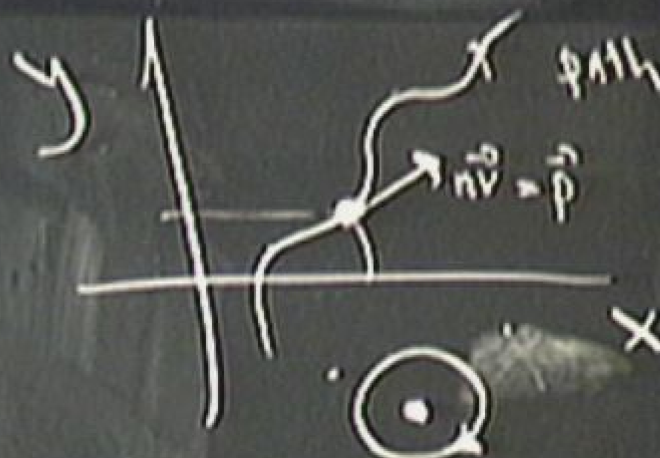

Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
esolving power
δx = λ / sin ε
ε
λ
y
x
path
mv = p


Principle
If you want to give meaning to phrase
position of electron" you must
cify a way to observe it.
solving power
δx = λ / sin ε
y
path
mv = p
x
ε
λ


Principle
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.
resolving power
$\delta x = \frac{\lambda}{\sin \varepsilon}$
ε
λ
y
path
$m\vec{v} = \vec{p}$
x

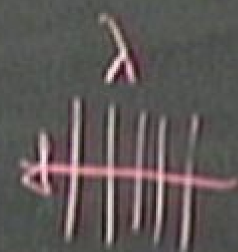
Principle

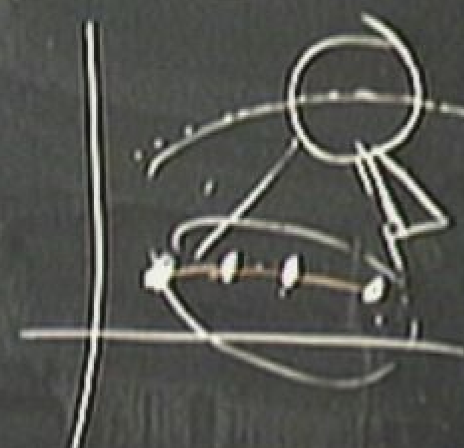
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.



resolving power

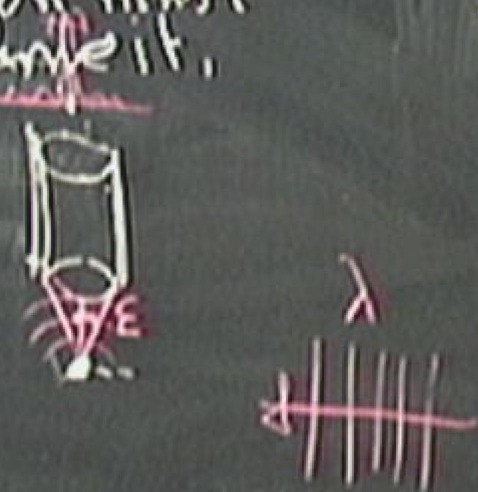
$$\delta x = \frac{\lambda}{\sin \varepsilon}$$





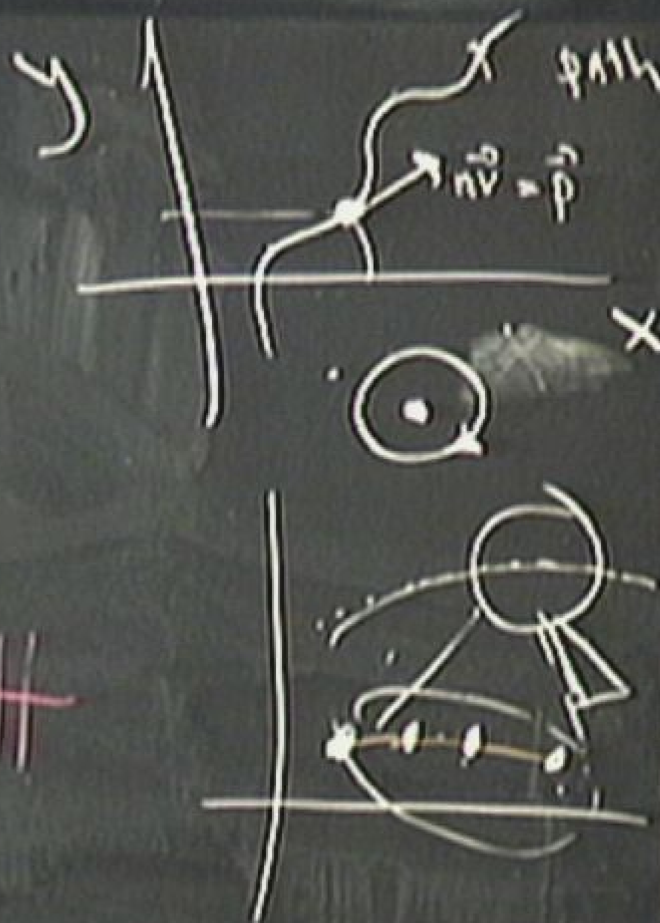
Principle

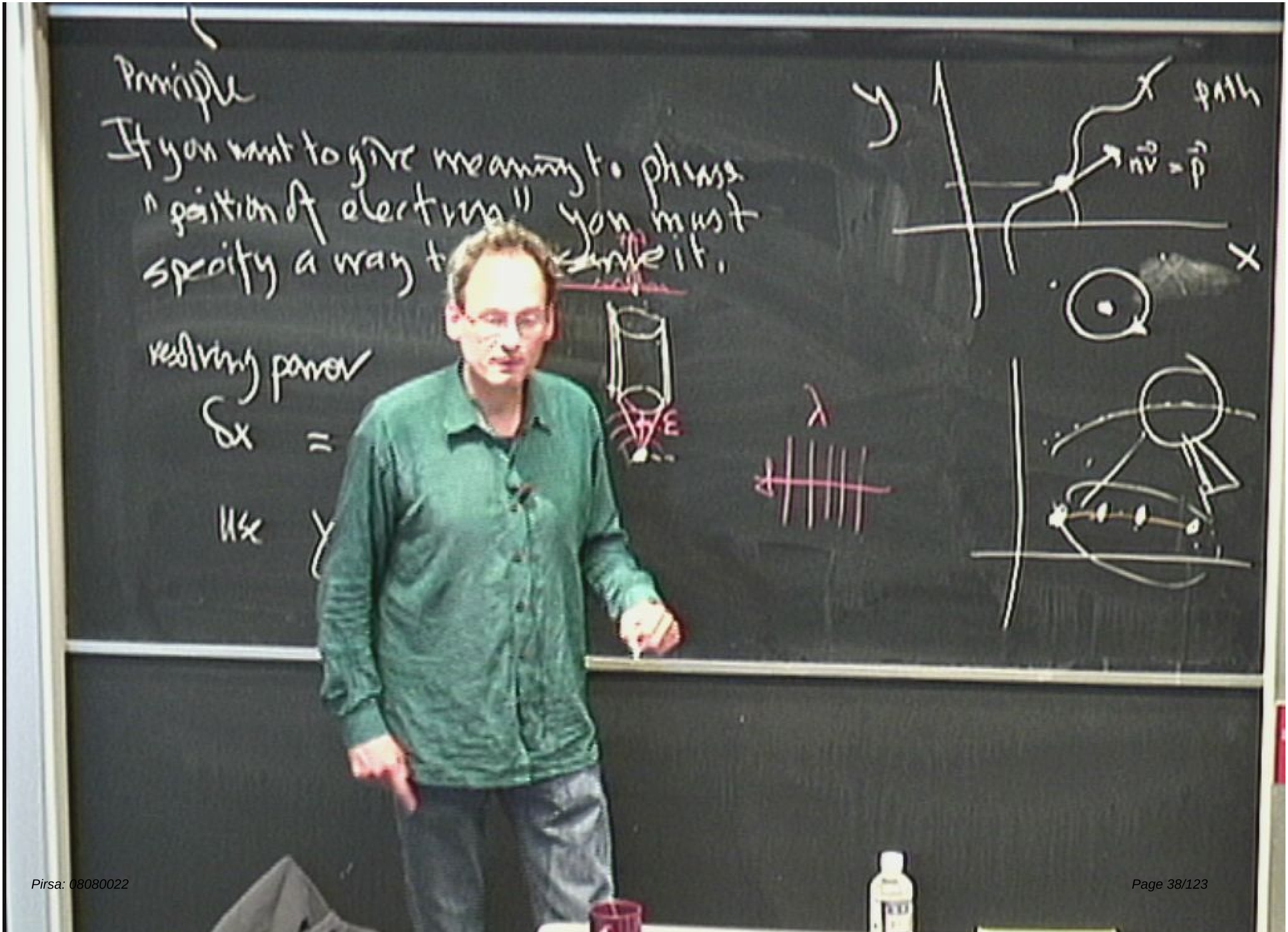
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.



resolving power

$$\delta x = \frac{\lambda}{\sin \varepsilon}$$




Principle
If you want to give meaning to phrase "position of electron" you must specify a way t
e it.
resolving power
δx =
Use
E
λ
y
x
path
mv = p

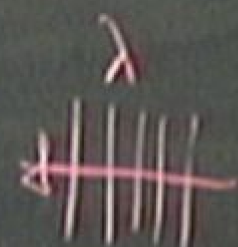
Principle

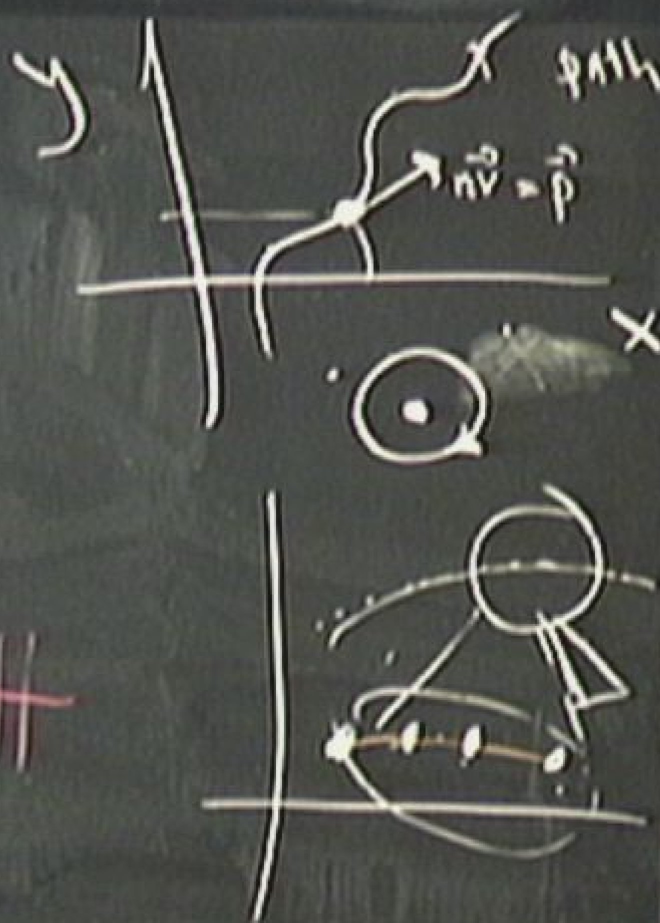
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.

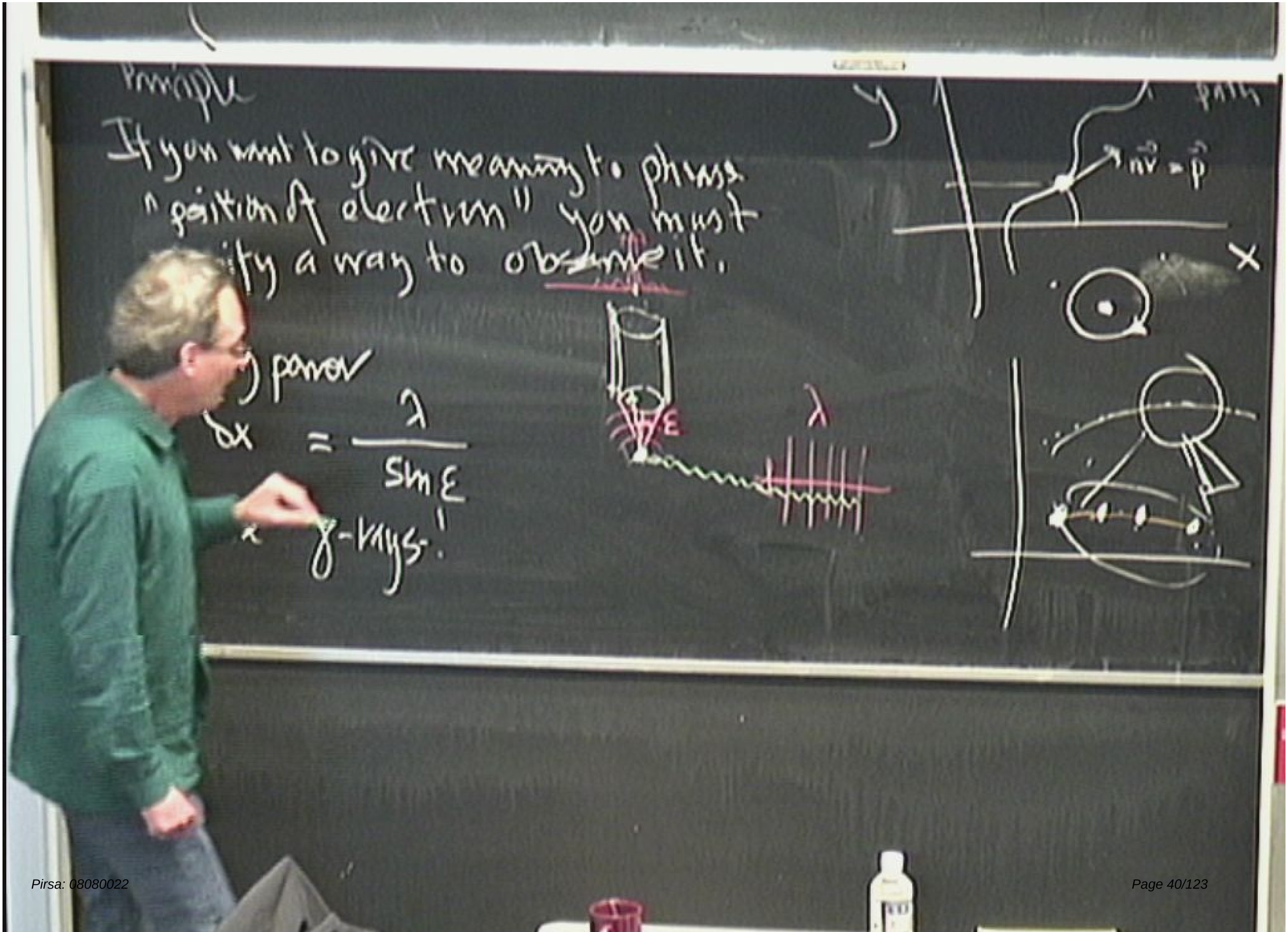
resolving power

$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

use γ-rays.







principle
If you want to give meaning to phrase
"position of electron" you must
ity a way to observe it.
power
δx = λ / sin ε
γ-rays!
ε
λ
mv = p
path
x

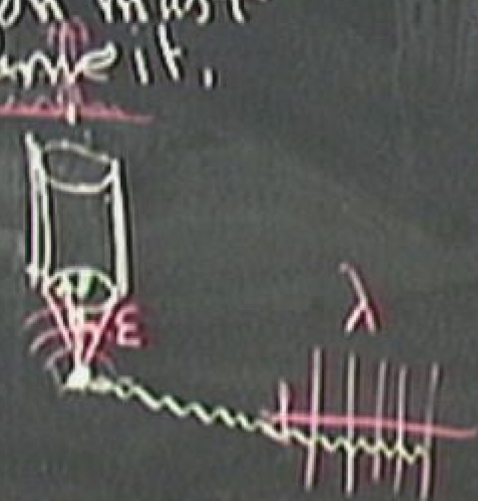
Principle

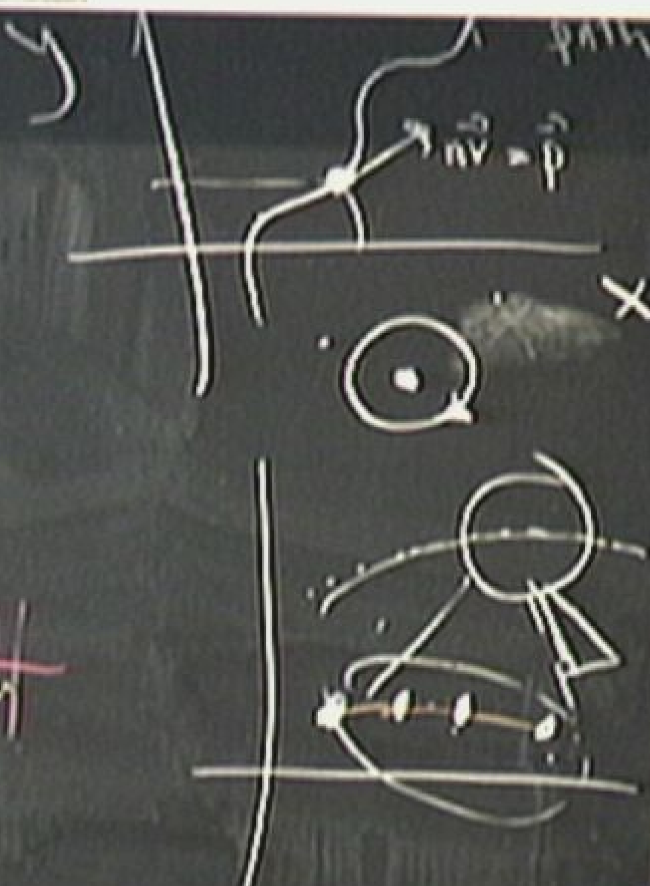
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.

resolving power

$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

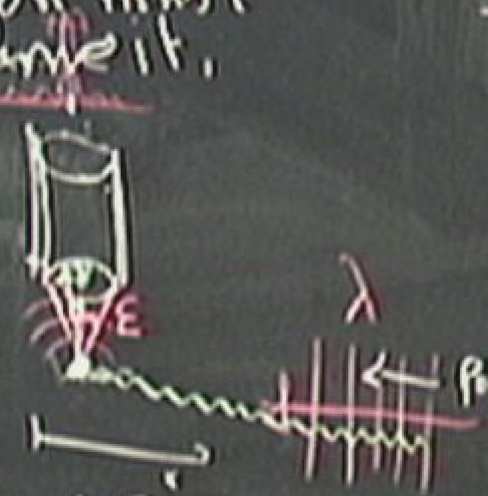
Use γ-rays!





Principle

If you want to give meaning to phrase "position of electron" you must specify a way to observe it.

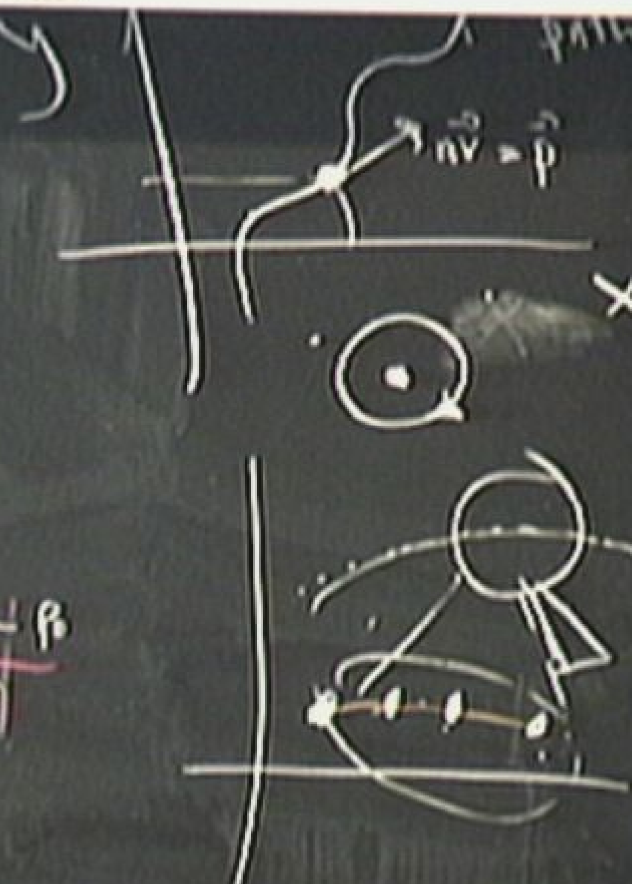


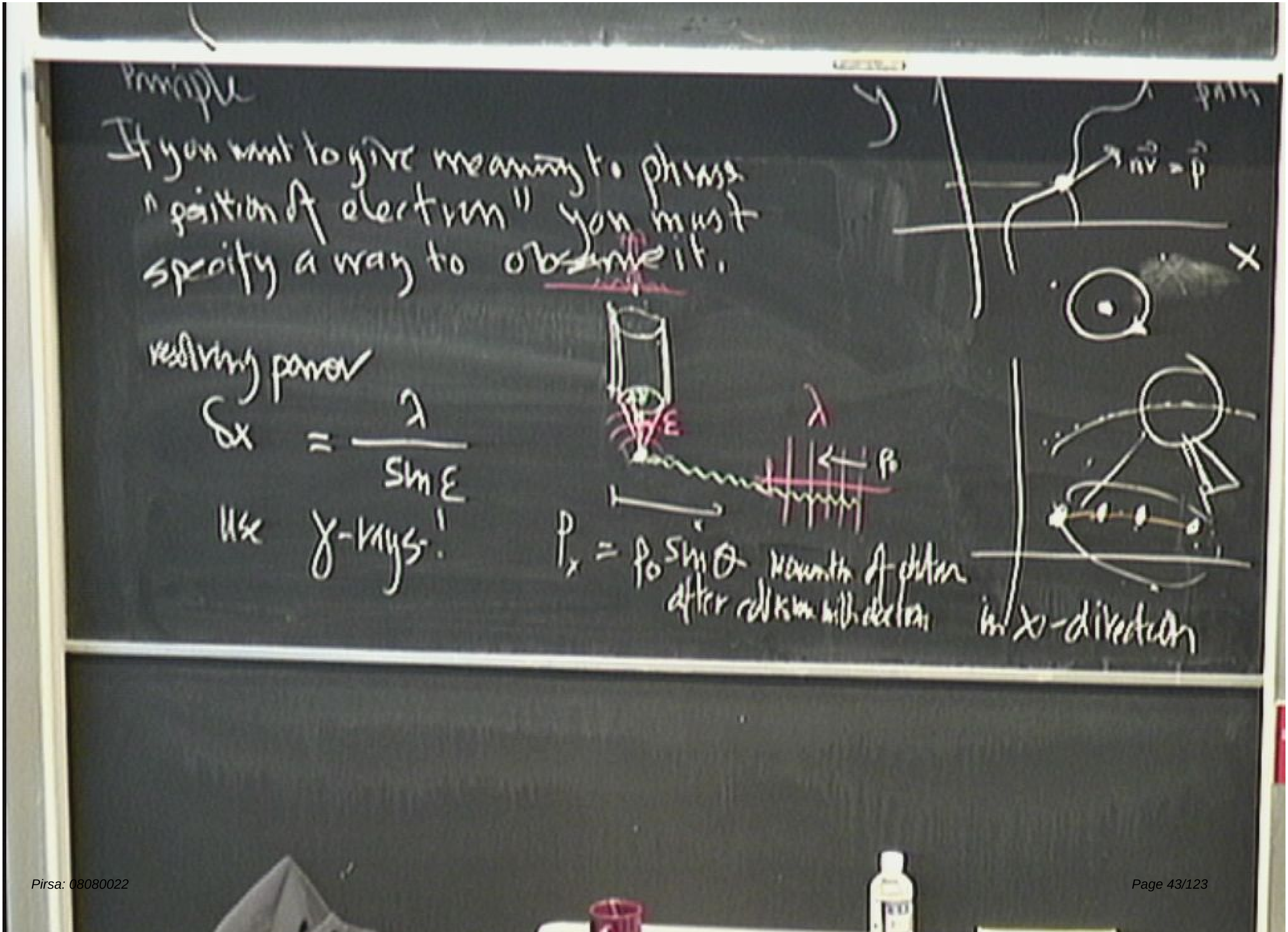
resolving power

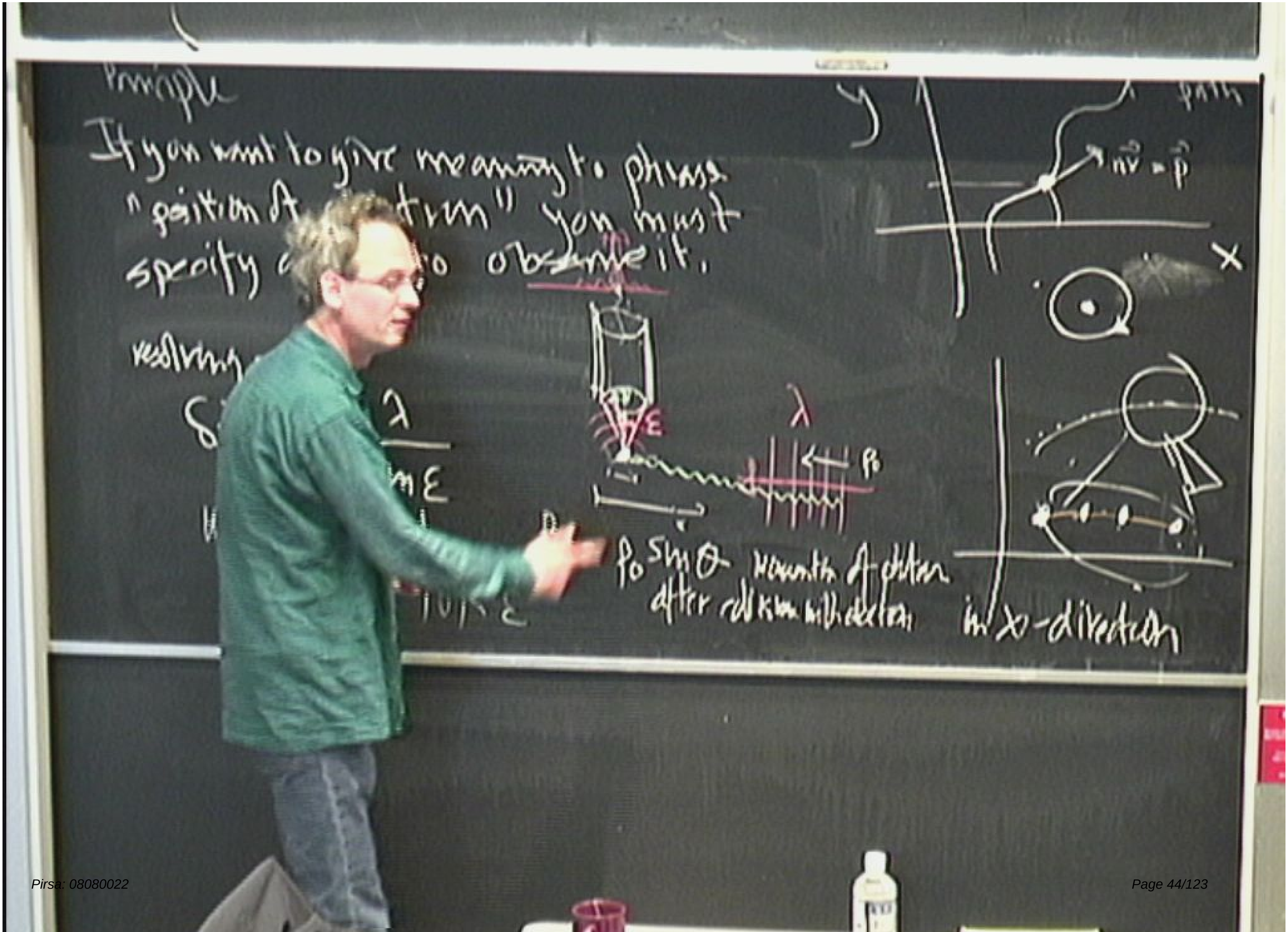
$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

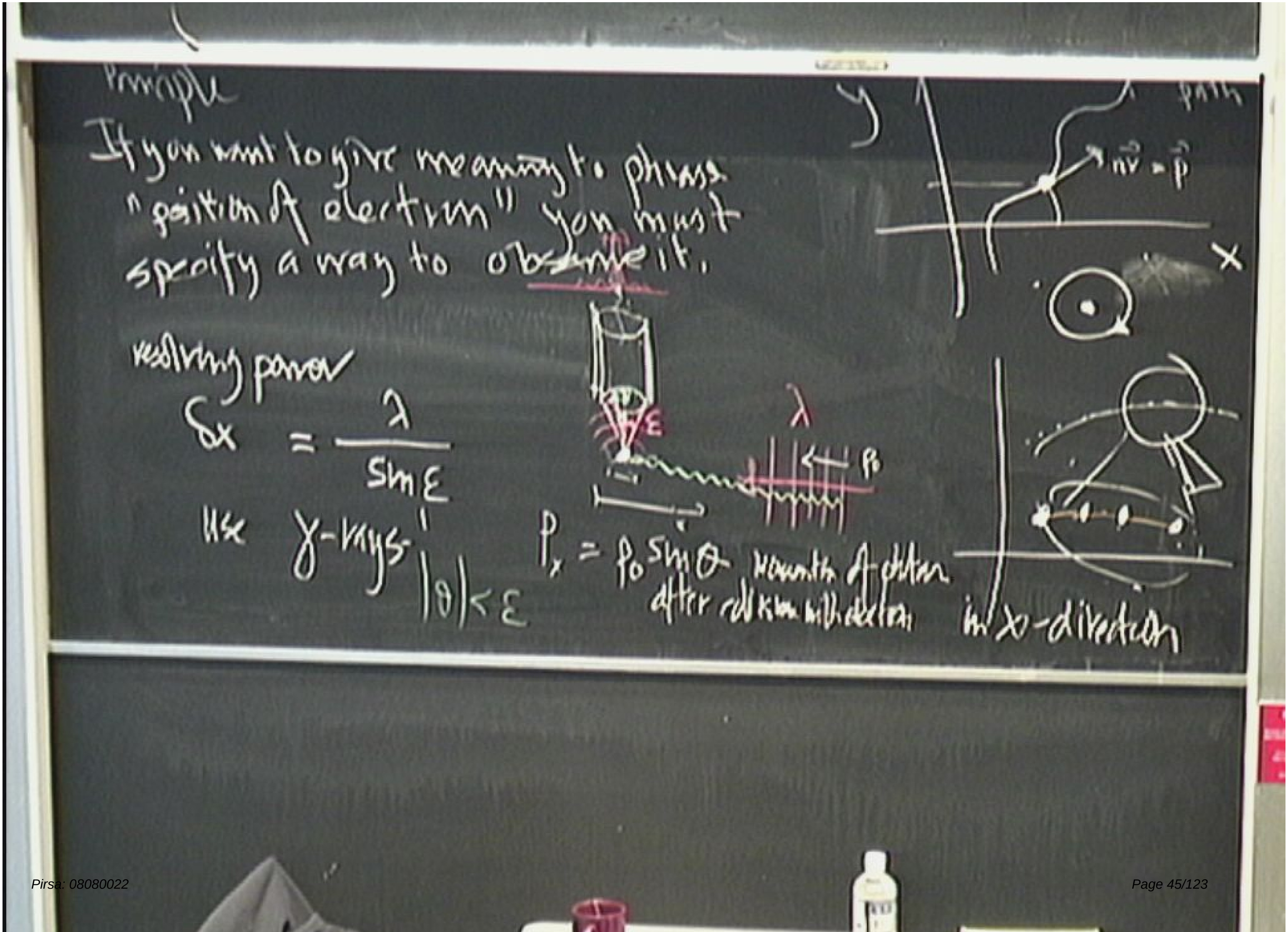
Use γ-rays!

$$p_x = p_0 \sin\theta$$




Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
resolving power
δx = λ / sin ε
use γ-rays!
ε
λ
p₀
pₓ = p₀ sin θ momentum of photon
after collision with electron in x-direction
path
mv = p
x

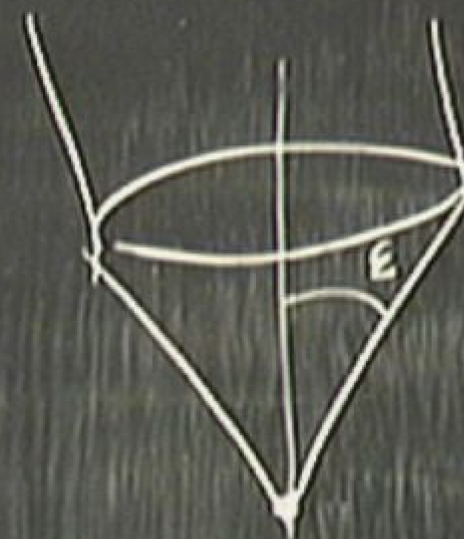

If you want to give meaning to phrase
"position of
tron" you must
specify
observe it.
resolving
path
$m\vec{v} = \vec{p}$
x
λ
p_0
$p_0 \sin\theta$ momentum of photon
after collision with electron
in x-direction


Principle
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.
resolving power
δx = λ / sin ε
use γ-rays
|θ| < ε
λ
p₀
p_x = p₀ sin θ momentum of photon after collision with electron in x-direction
mv = p
path
x

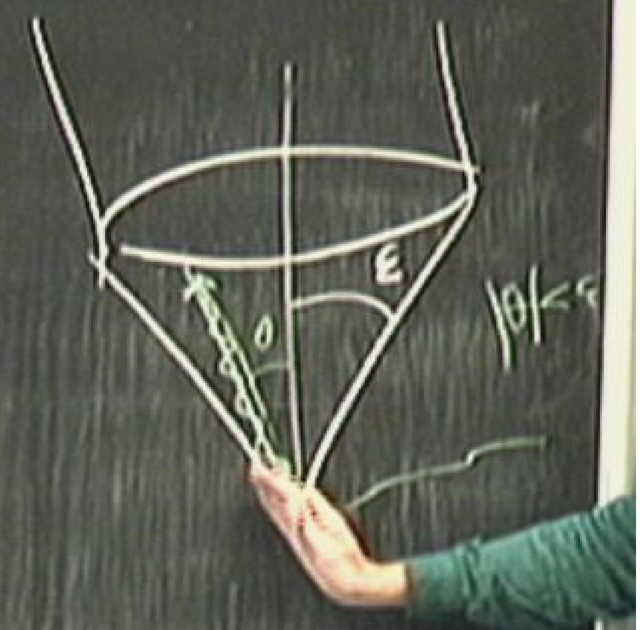
Uncertainty in momentum of photon $\delta p_x^{(ph)}$

uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.

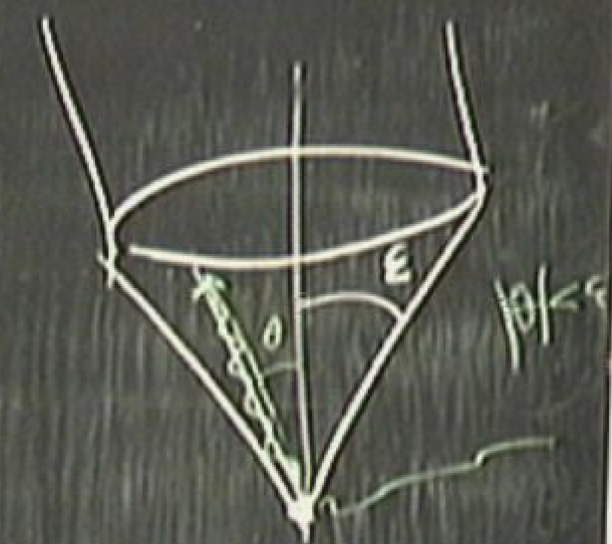
uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.



Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.



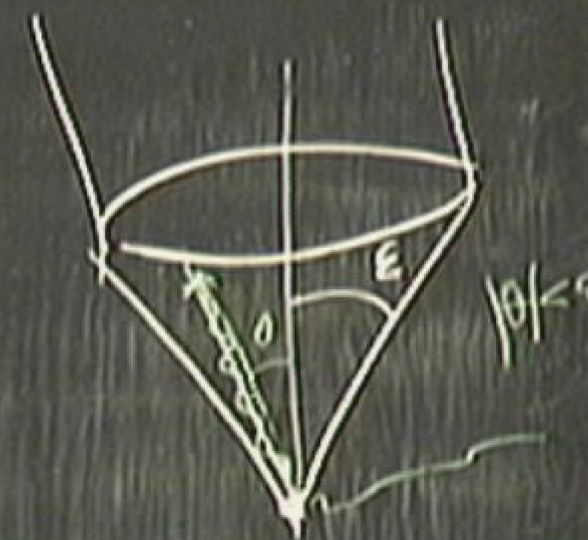
Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.

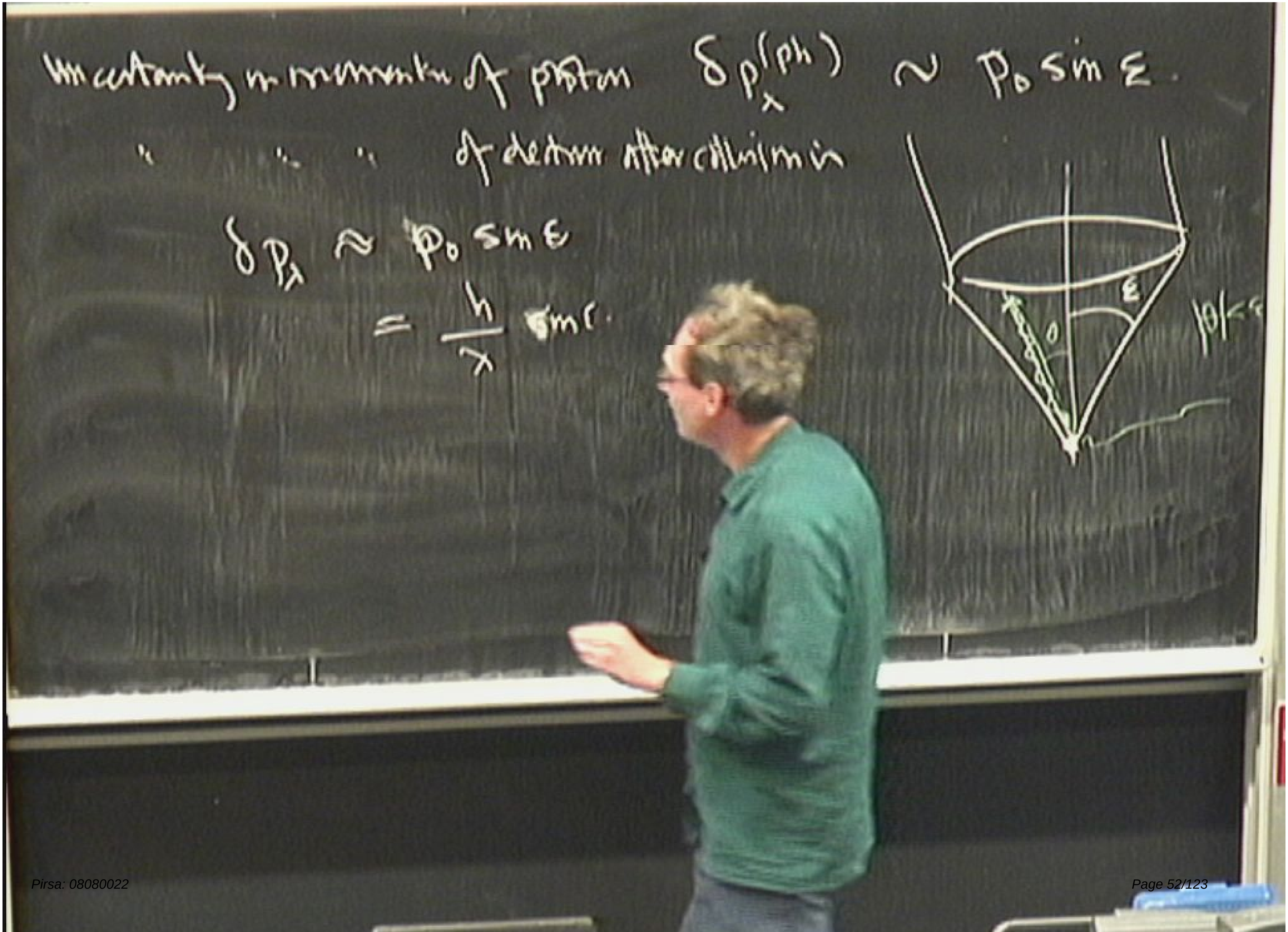


Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.

" " " of electron after collision in

$\delta p_x \approx p_0 \sin \varepsilon$




Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.
" " " of electron after collision is
$\delta p_x \approx p_0 \sin \varepsilon$
$= \frac{h}{\lambda} \sin \varepsilon$
ε
θ

Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin\epsilon$.

" " " of electron after collision is

$$\delta p_x \approx p_0 \sin\epsilon$$

$$= \frac{h}{\lambda} \sin\epsilon$$

$$\delta x = \frac{\lambda}{\sin\epsilon}$$

Principle

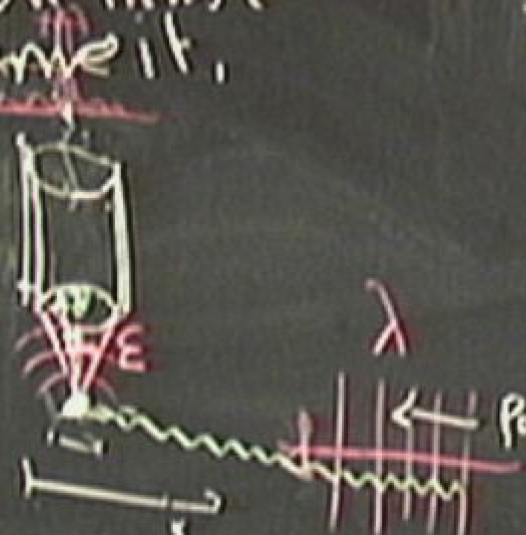
If you want to give meaning to phrase "position of electron" you must specify a way to observe it.

resolving power

$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

use γ-rays

$|\theta| < \varepsilon$

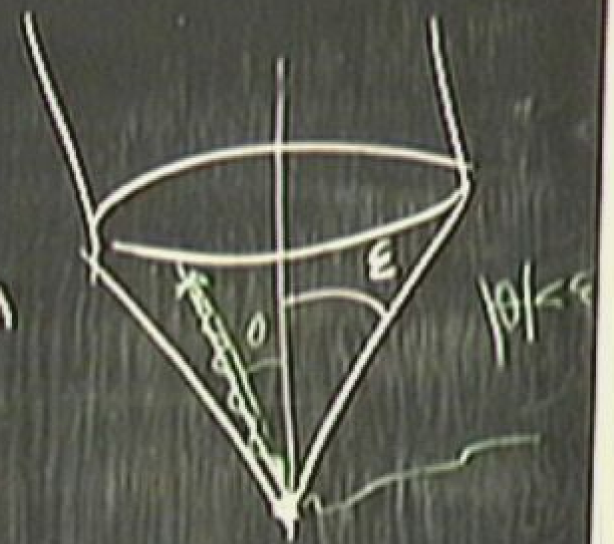


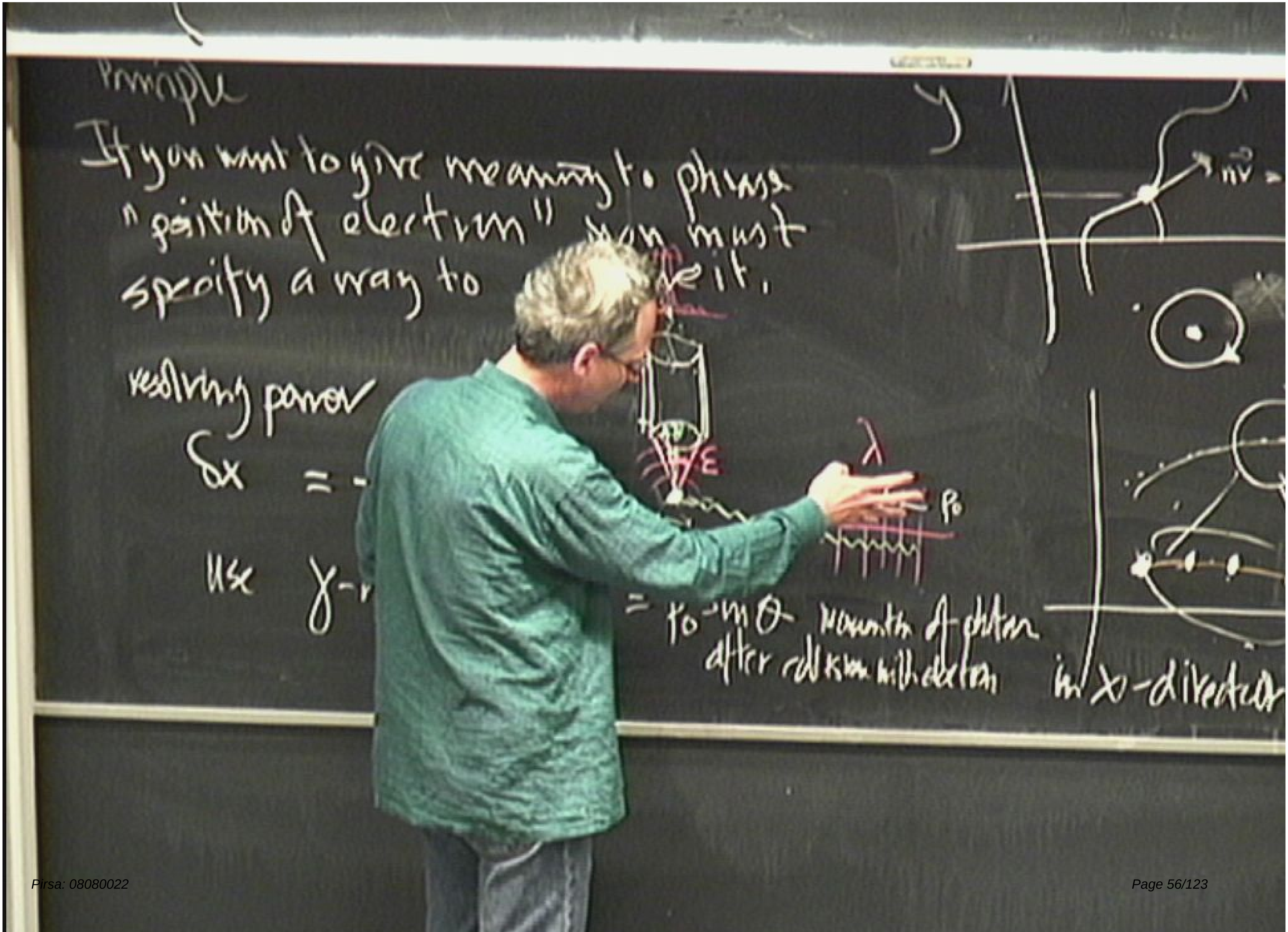
$p_x = p_0 \sin\theta$ momentum of photon after collision with electron in x-direction

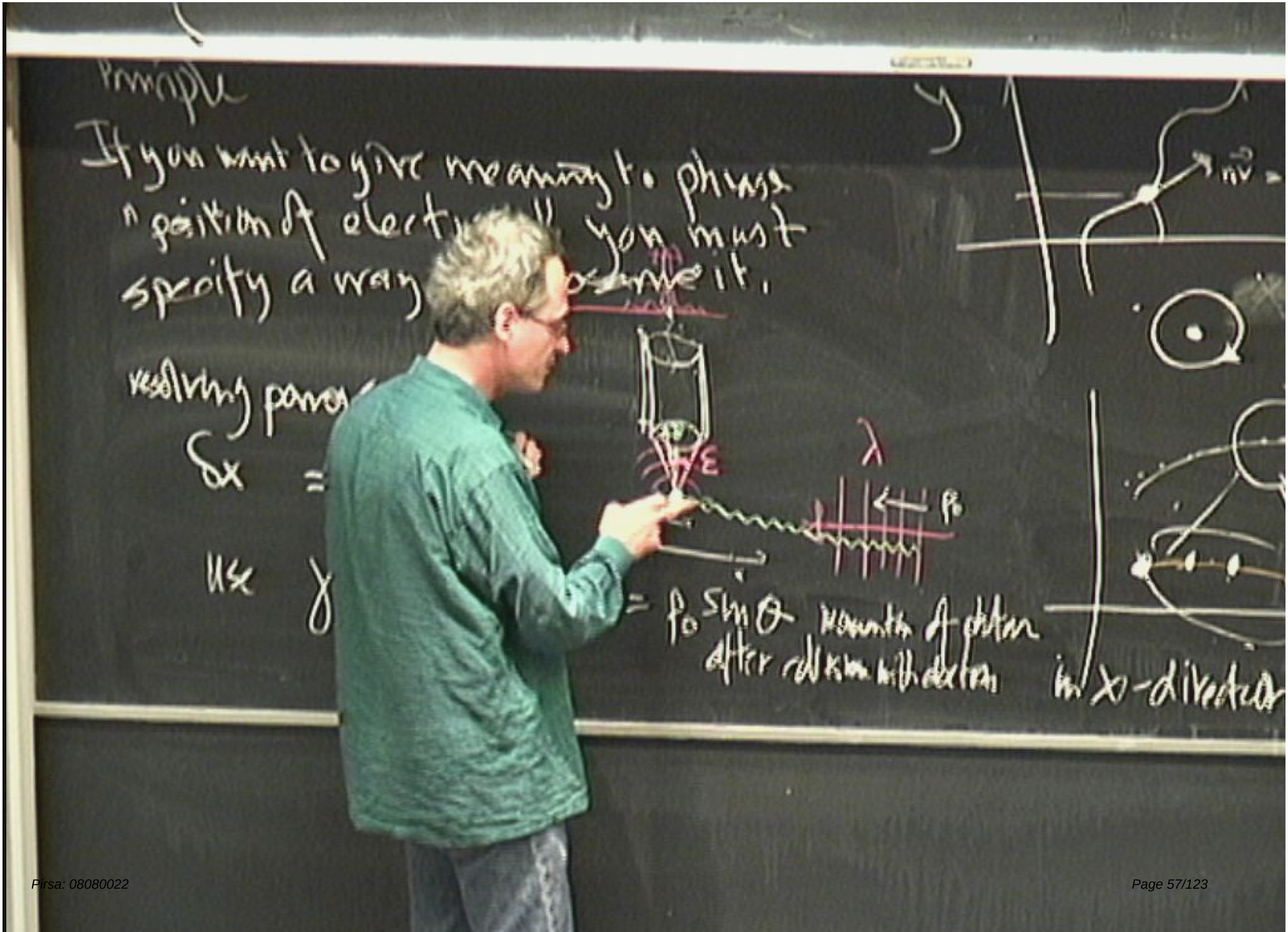
Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin\varepsilon$.

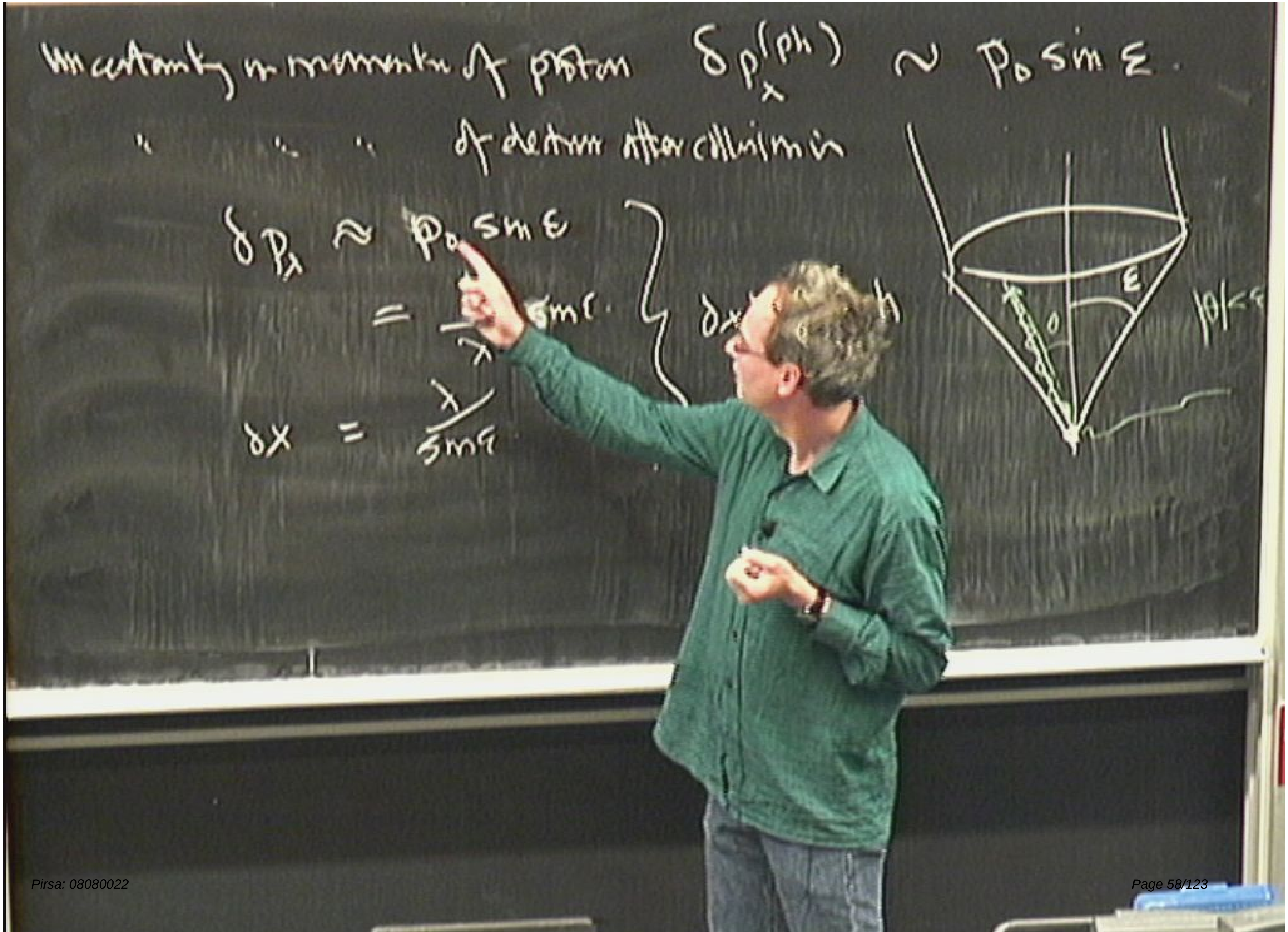
" " " of electron after collision in

$$\left.\begin{aligned} \delta p_x &\approx p_0 \sin\varepsilon \\ &= \frac{h}{\lambda}\sin\varepsilon \\ \delta x &= \frac{\lambda}{\sin\varepsilon} \end{aligned}\right\} \delta x\, \delta p_x \sim h$$




Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to
resolving power
δx =
Use
p_0
λ
$= p_0 - m\theta$ momentum of photon
after collision with electron
in x-direction

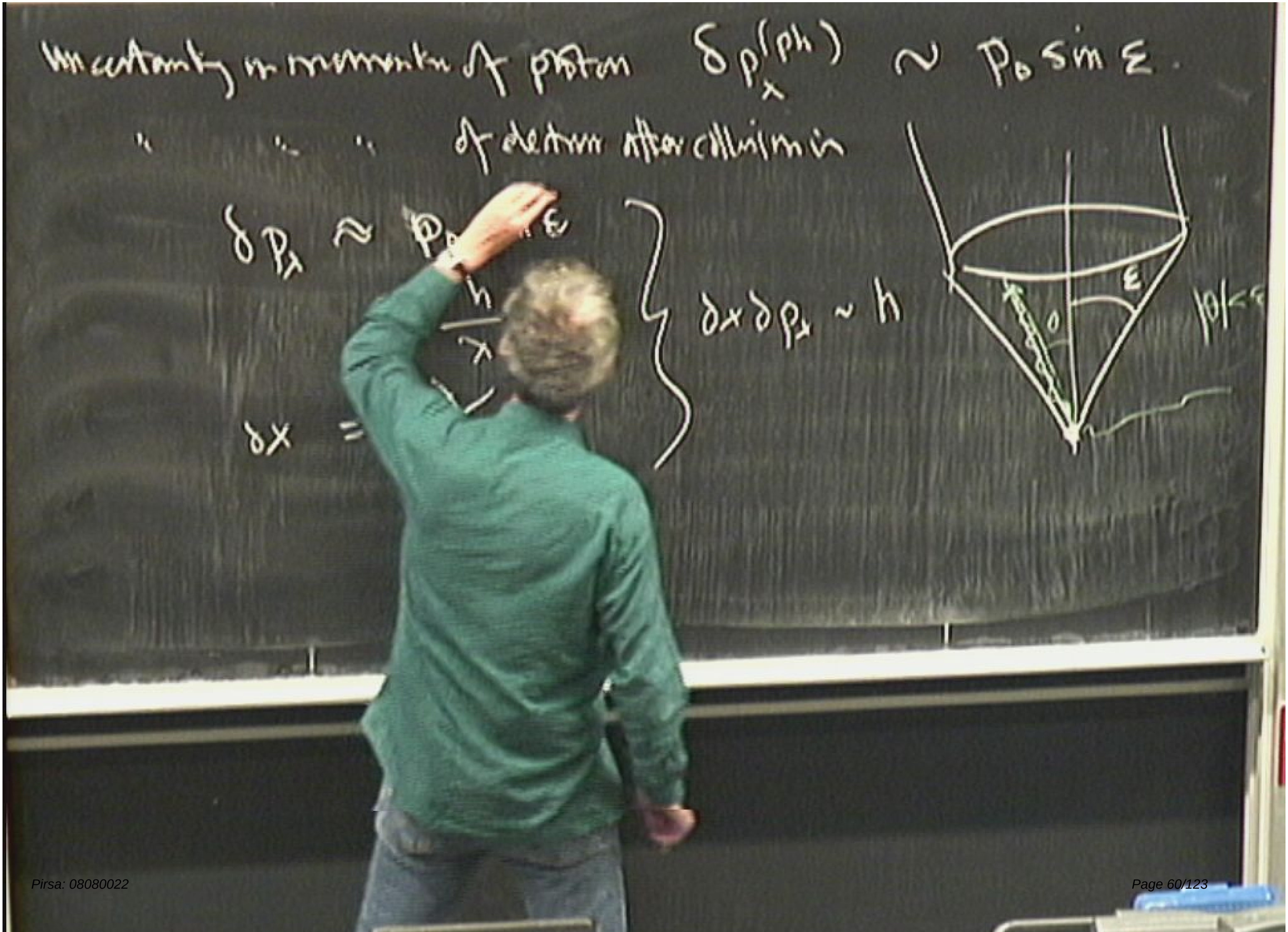

Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to measure it.
resolving power
δx =
use γ
λ
p₀
= p₀ sin θ momentum of photon
after collision with electron
in x-direction


Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.
" " " of electron after collision is
$\delta p_x \approx p_0 \sin \varepsilon$
$= \frac{h}{\lambda} \sin \varepsilon$
$\delta x = \frac{\lambda}{\sin \varepsilon}$
ε
θ
$|\theta| < \varepsilon$

Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \epsilon$.

" " " of electron after collision is

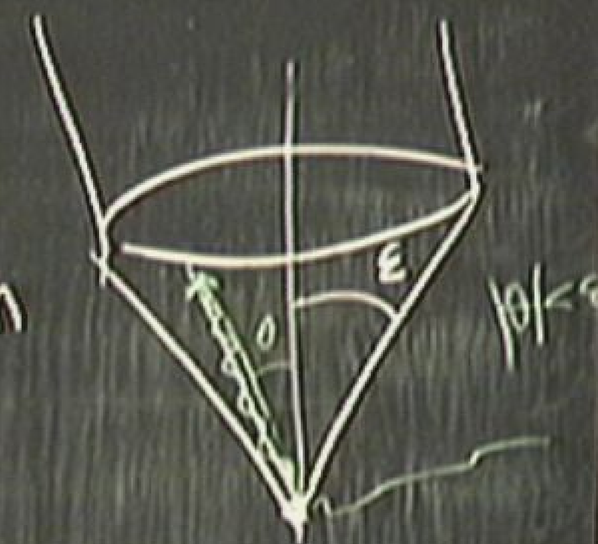
$$\left.\begin{aligned} \delta p_x &\approx p_0 \sin \epsilon \\ &= \frac{h}{\lambda} \sin \epsilon \\ \delta x &= \frac{\lambda}{\sin \epsilon} \end{aligned}\right\} \delta x \, \delta p_x \sim h$$


Uncertainty in momentum of photon δp_x^(ph) ~ p_0 sin ε.
" " " of electron after collision in
δp_x ≈ p_0 sin ε
h/λ
δx =
δx δp_x ~ h
ε
θ
|θ|<ε

Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.

" " " of electron after collision is

$$\left.\begin{aligned} \delta p_x &\approx p_0 \sin \varepsilon \\ &= \frac{h}{\lambda} \sin \varepsilon \\ \delta x &= \frac{\lambda}{\sin \varepsilon} \end{aligned}\right\} \delta x\, \delta p_x \sim h$$



Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin\varepsilon$.

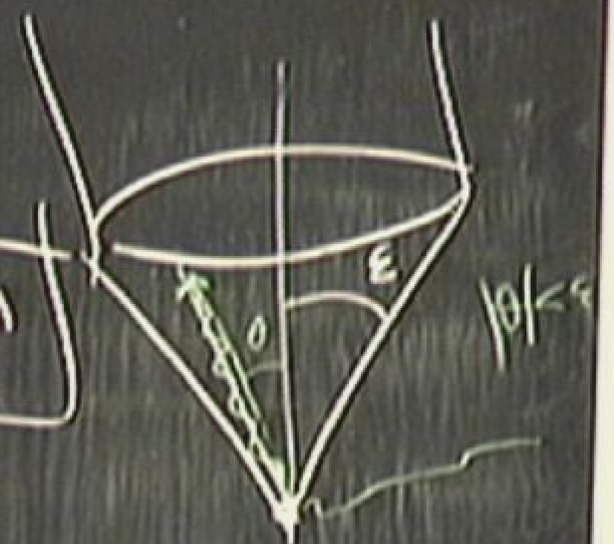
" " " of electron after collision in

$$\delta p_x^{el} \approx p_0 \sin\varepsilon$$

$$= \frac{h}{\lambda} \sin\varepsilon$$

$$\delta x = \frac{\lambda}{\sin\varepsilon}$$

$$\boxed{\delta x \, \delta p_x \sim h}$$



Uncertainty in momentum of photon $\quad \delta p_x^{(ph)} \sim p_0 \sin\varepsilon$.

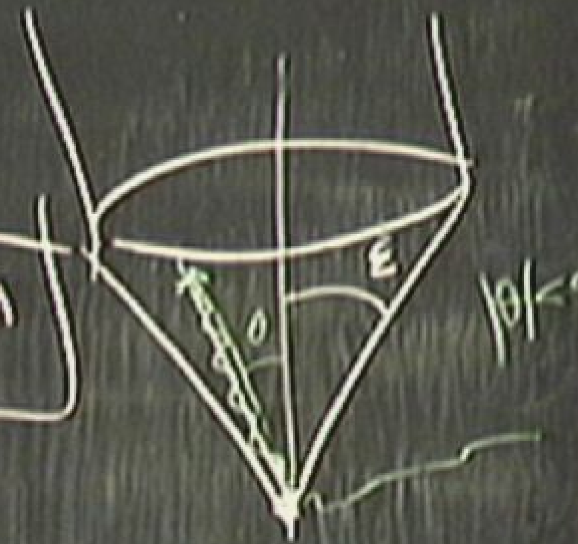
" " " of electron after collision is

$$\delta p_x^{el} \approx p_0 \sin\varepsilon$$

$$= \frac{h}{\lambda}\sin\varepsilon$$

$$\delta x = \frac{\lambda}{\sin\varepsilon}$$

$$\boxed{\delta x\, \delta p_x \sim h}$$



Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.

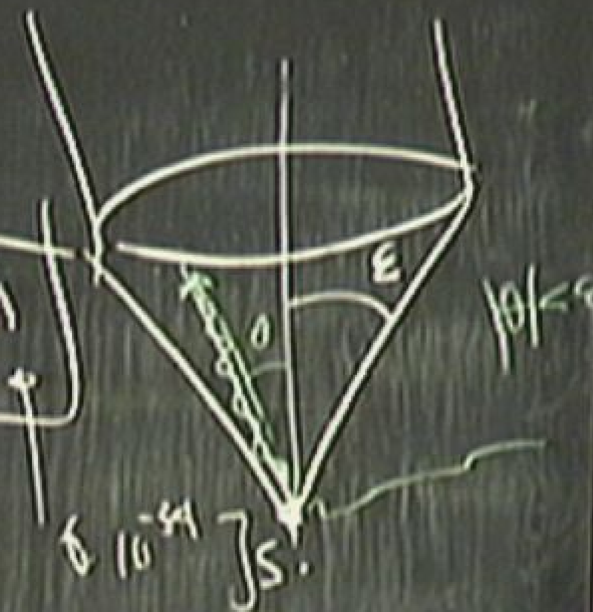
" " " of electron after collision is

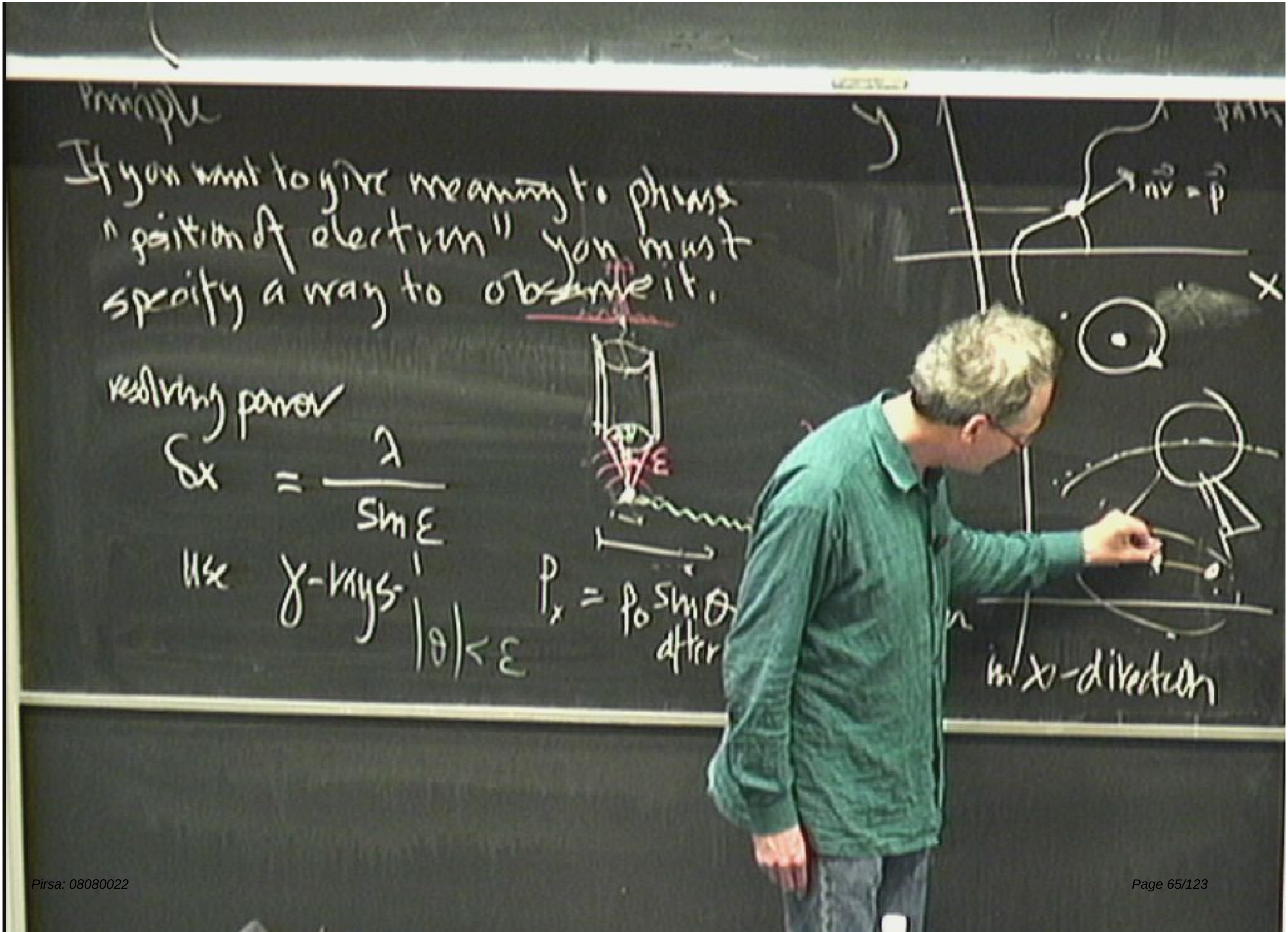
$$\delta p_x^{el} \approx p_0 \sin \varepsilon = \frac{h}{\lambda} \sin \varepsilon$$

$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

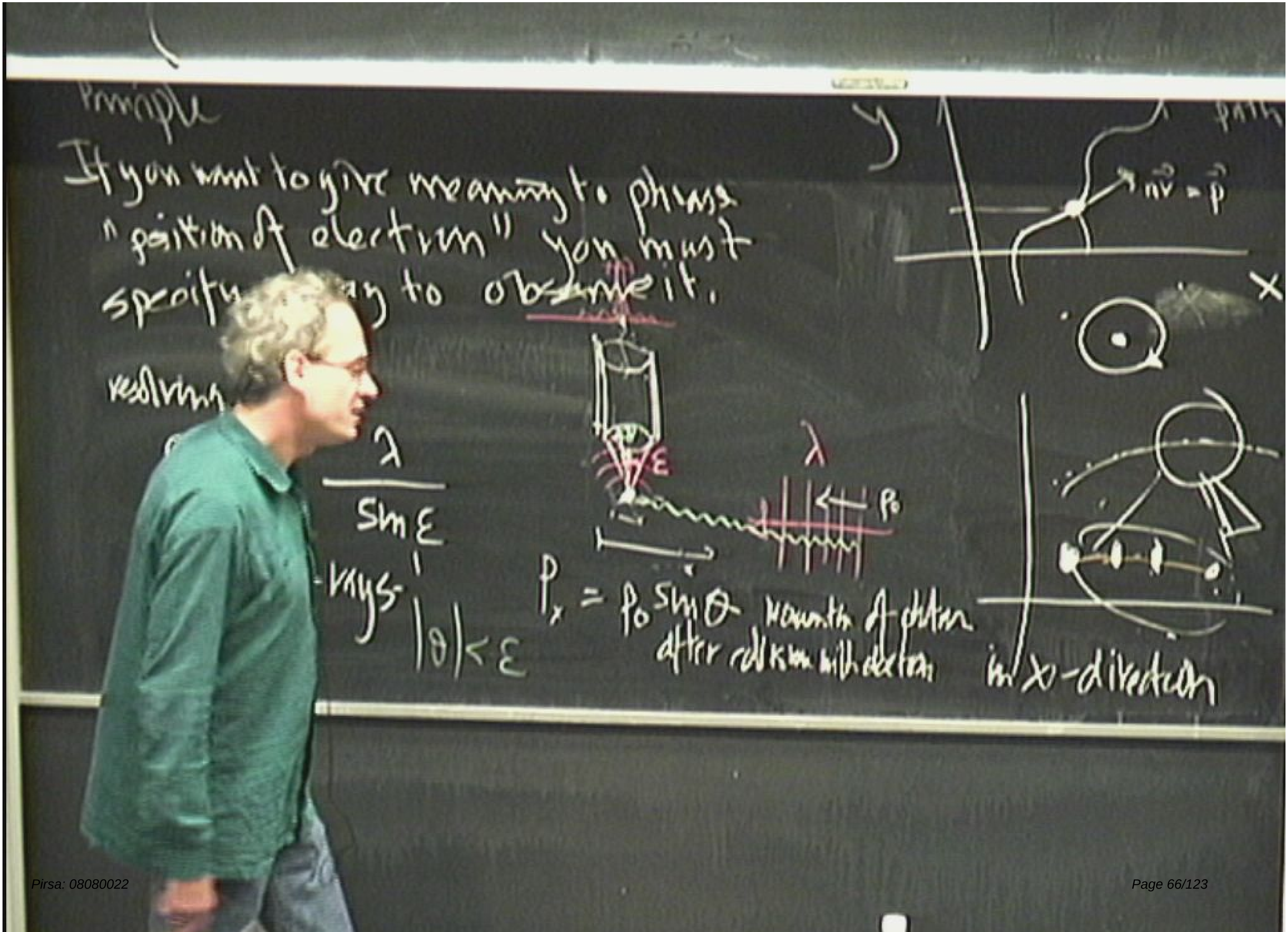
$$\boxed{\delta x \, \delta p_x \sim h}$$

$\approx 10^{-34}$ Js



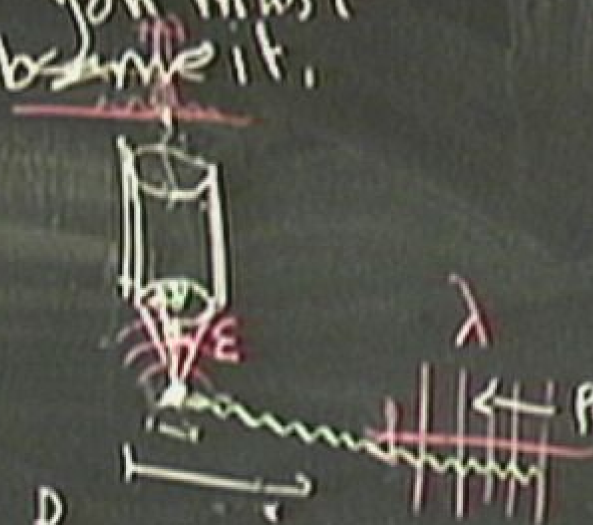


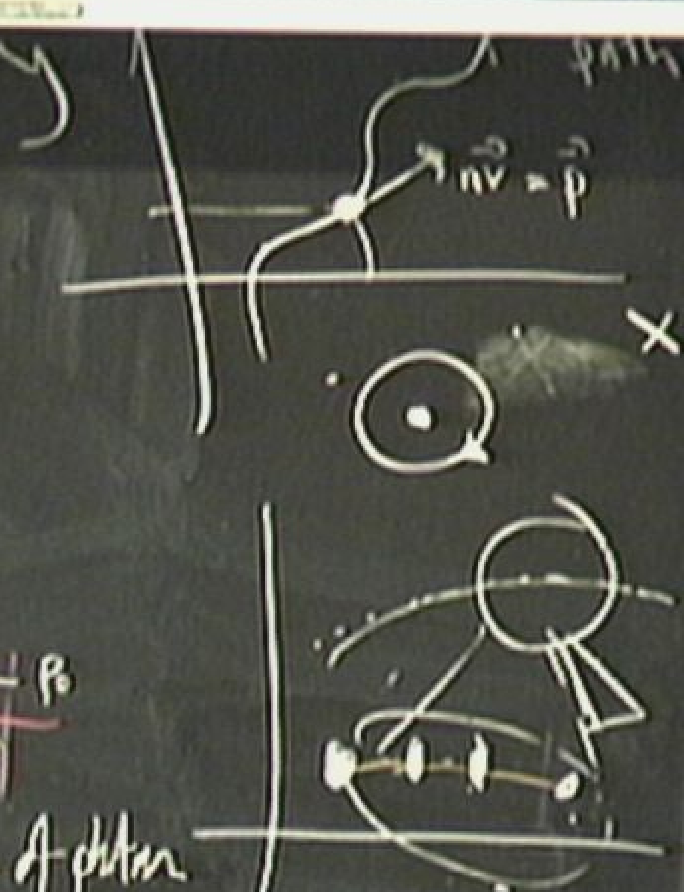
Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
resolving power
δx = λ / sin ε
use γ-rays
|θ| < ε
p_x = p_0 sin θ
after
ε
mv = p
x
in x-direction


Principle
If you want to give meaning to phrase "position of electron" you must specify way to observe it.
resolving
λ
sin ε
rays
|θ| < ε
ε
λ
p₀
Pₓ = p₀ sin θ momentum of photon after collision with electron in x-direction
mv = p
x
y

Principle

If you want to give meaning to phrase "position of electron" you must specify a way to observe it.





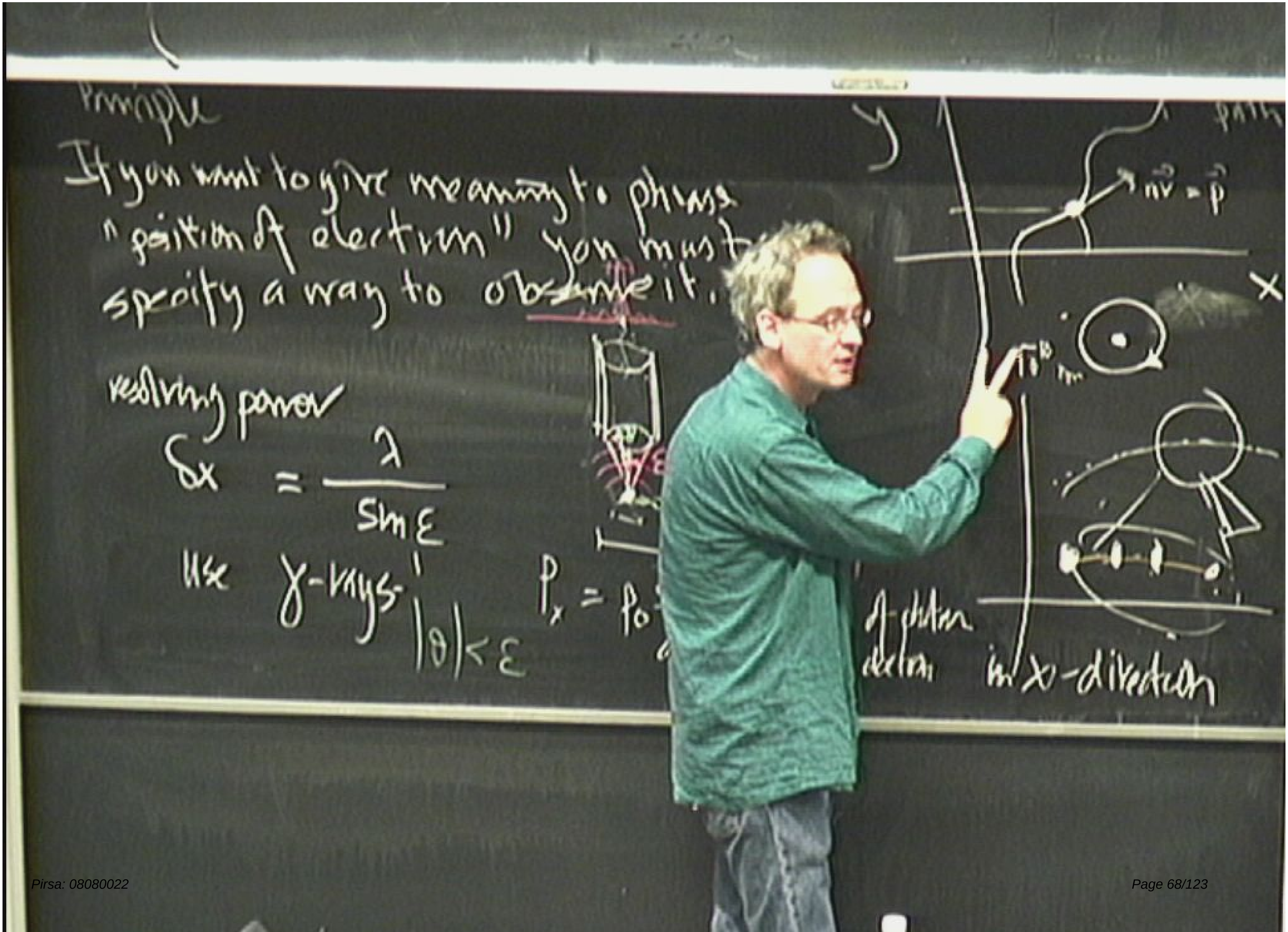
resolving power

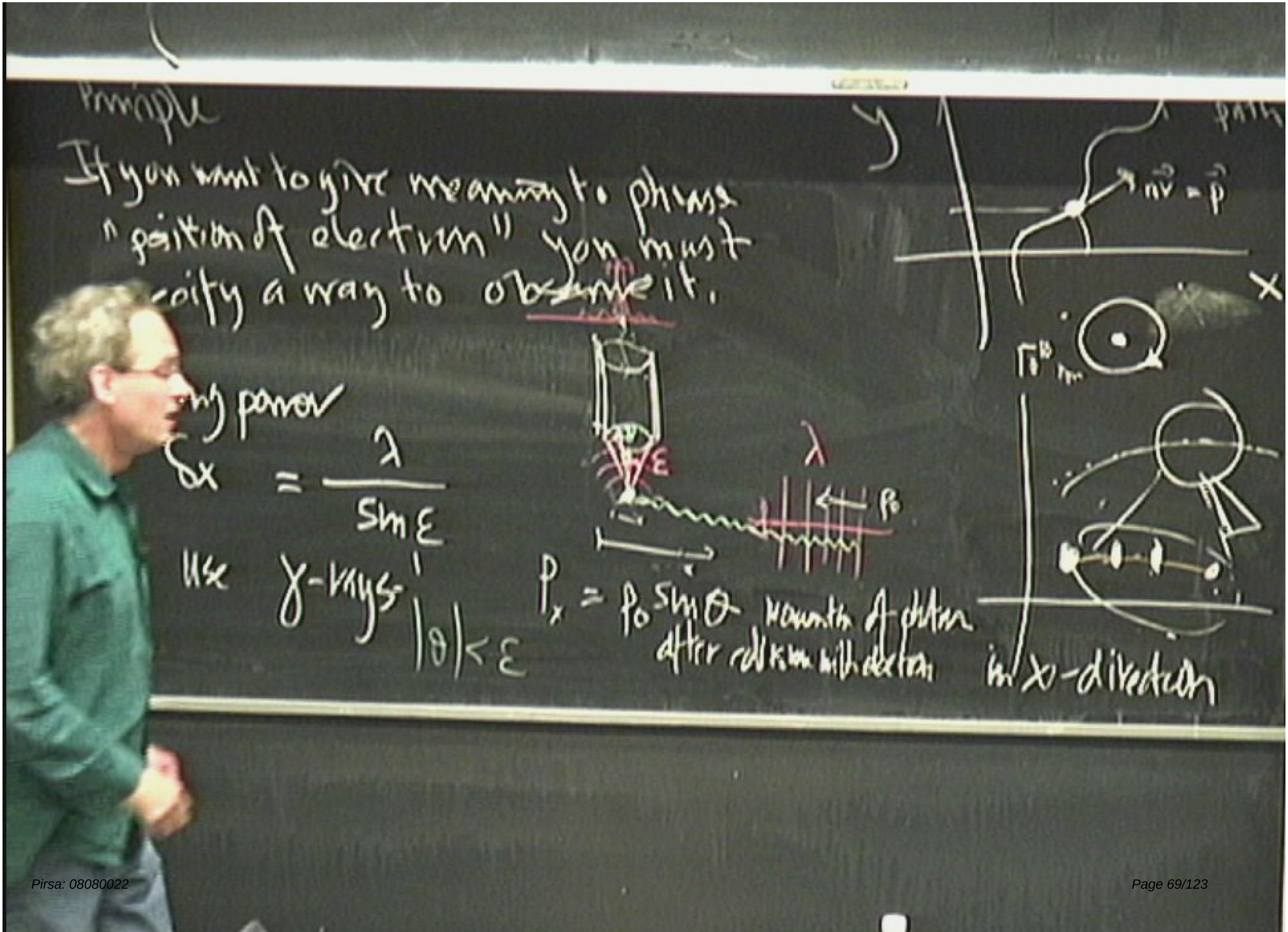
$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

use γ-rays!

$$|\theta| < \varepsilon$$

$p_x = p_0 \sin\theta$ momentum of photon after collision with electron in x-direction


Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
resolving power
δx = λ / sin ε
use γ-rays
|θ| < ε
p_x = p_0
in x-direction
mv = p


Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
power
δx = λ / sin ε
Use γ-rays
|θ| < ε
λ
p₀
ε
pₓ = p₀ sin θ momentum of photon
after collision with electron
in x-direction
mv = p
x

Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin \varepsilon$.

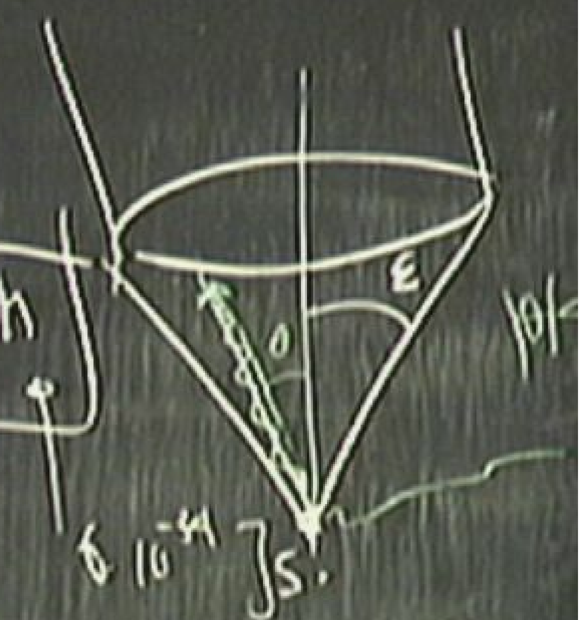
" " " of electron after collision in

$$\delta p_x^{el} \approx p_0 \sin \varepsilon$$

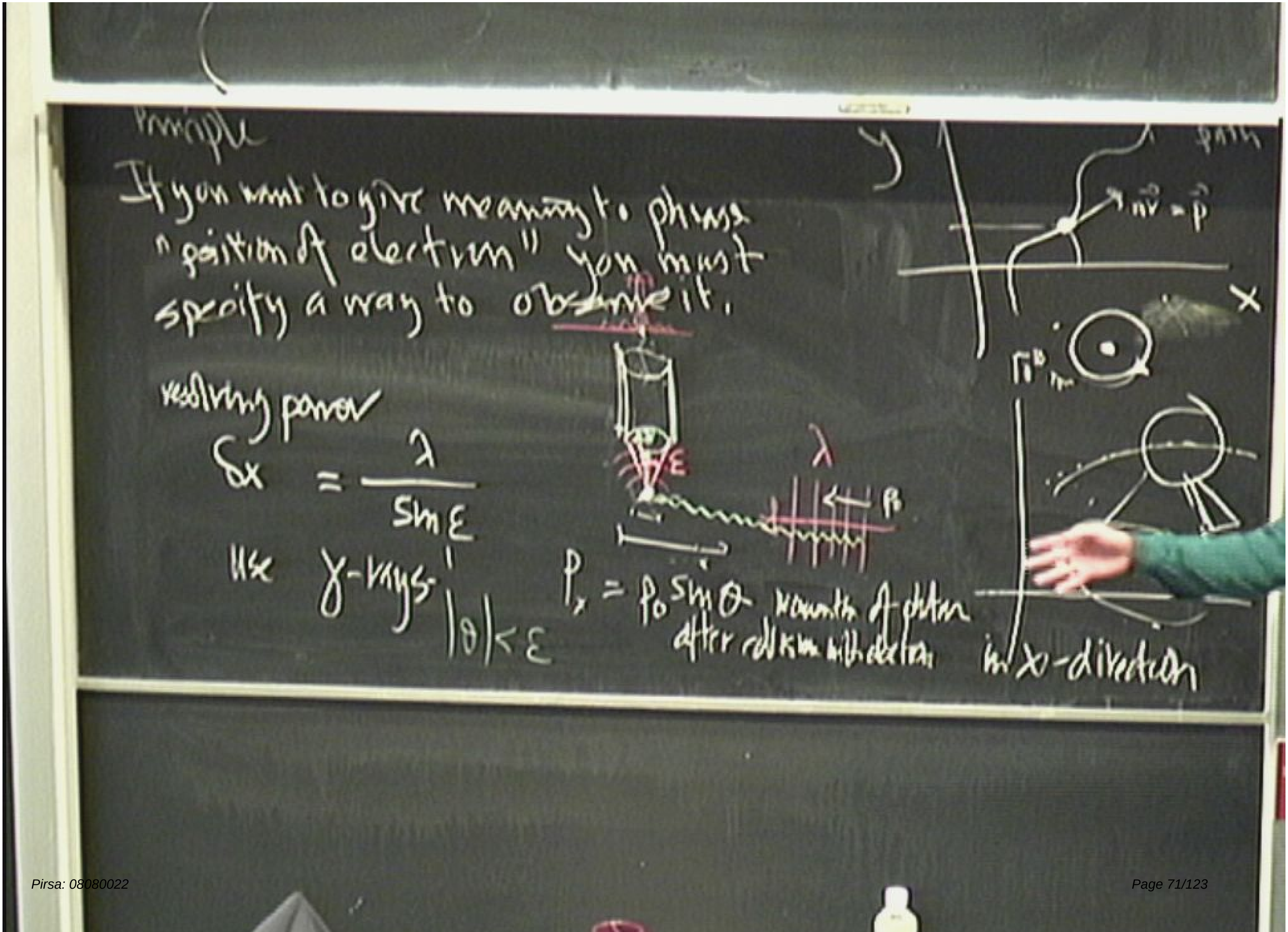
$$= \frac{h}{\lambda} \sin \varepsilon$$

$$\delta x = \frac{\lambda}{\sin \varepsilon}$$

$$\boxed{\delta x \, \delta p_x \sim h}$$



NB.



principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
resolving power
δx = λ / sin ε
use γ-rays!
|θ| < ε
p_x = p_0 sin θ momentum of photon
after collision with electron in x-direction
λ
p_0
ε
y
x
path
mv = p

Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin\epsilon$.

" " " of electron after collision in

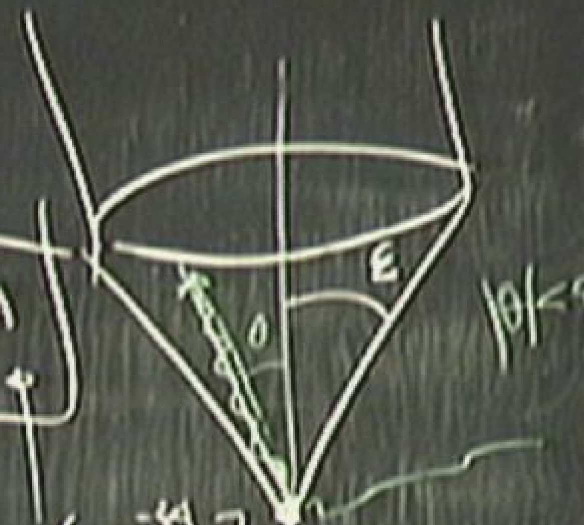
$$\delta p_x^{el} \approx p_0 \sin\epsilon$$

$$= \frac{h}{\lambda} \sin\epsilon$$

$$\delta x = \frac{\lambda}{\sin\epsilon}$$

$$\boxed{\delta x\, \delta p_x \sim h}$$

6×10^{-34} Js.



NB. Heisenberg's principle is a philosophical one and hence debatable.

Uncertainty in momentum of photon $\delta p_x^{(ph)} \sim p_0 \sin\varepsilon$

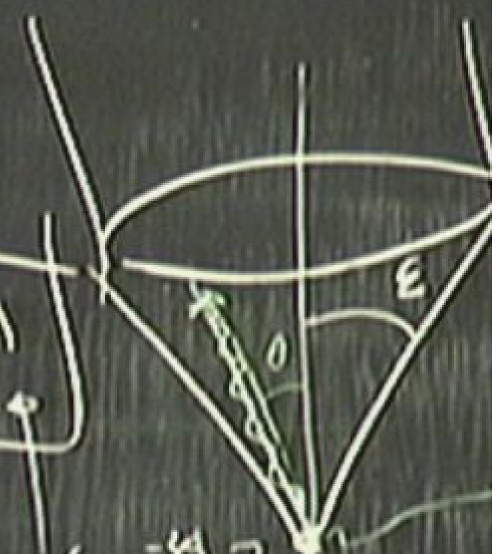
" " " of electron after collision is

$$\delta p_x^{el} \approx p_0 \sin\varepsilon = \frac{h}{\lambda}\sin\varepsilon$$

$$\delta x = \frac{\lambda}{\sin\varepsilon}$$

$$\boxed{\delta x\, \delta p_x \sim h}$$

$6 \cdot 10^{-34}$ Js



N.B. Heisenberg's principle is a philosophical one and hence debatable.

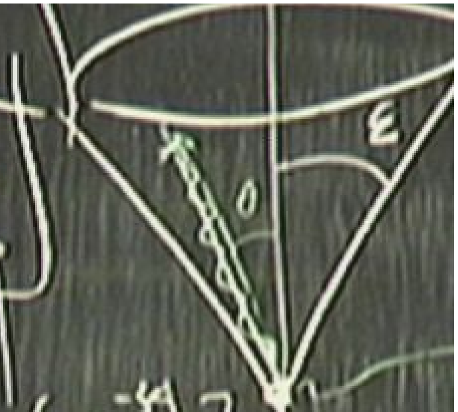
$$\delta p_x \approx p_0 \sin\epsilon$$

$$= \frac{h}{\lambda}\sin\epsilon$$

$$\delta x = \frac{\lambda}{\sin\epsilon}$$

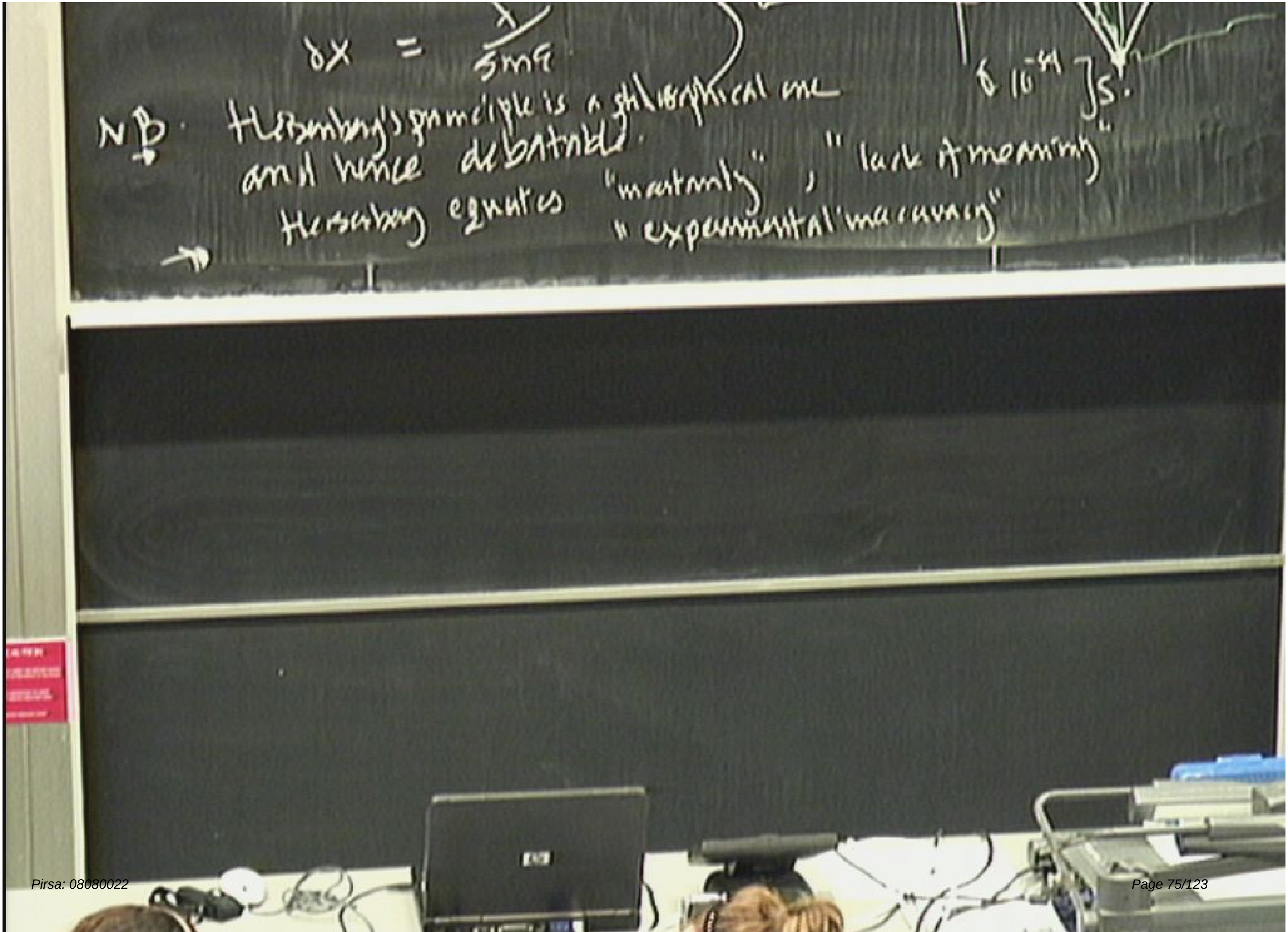
$$\boxed{\delta x\, \delta p_x \sim h}$$

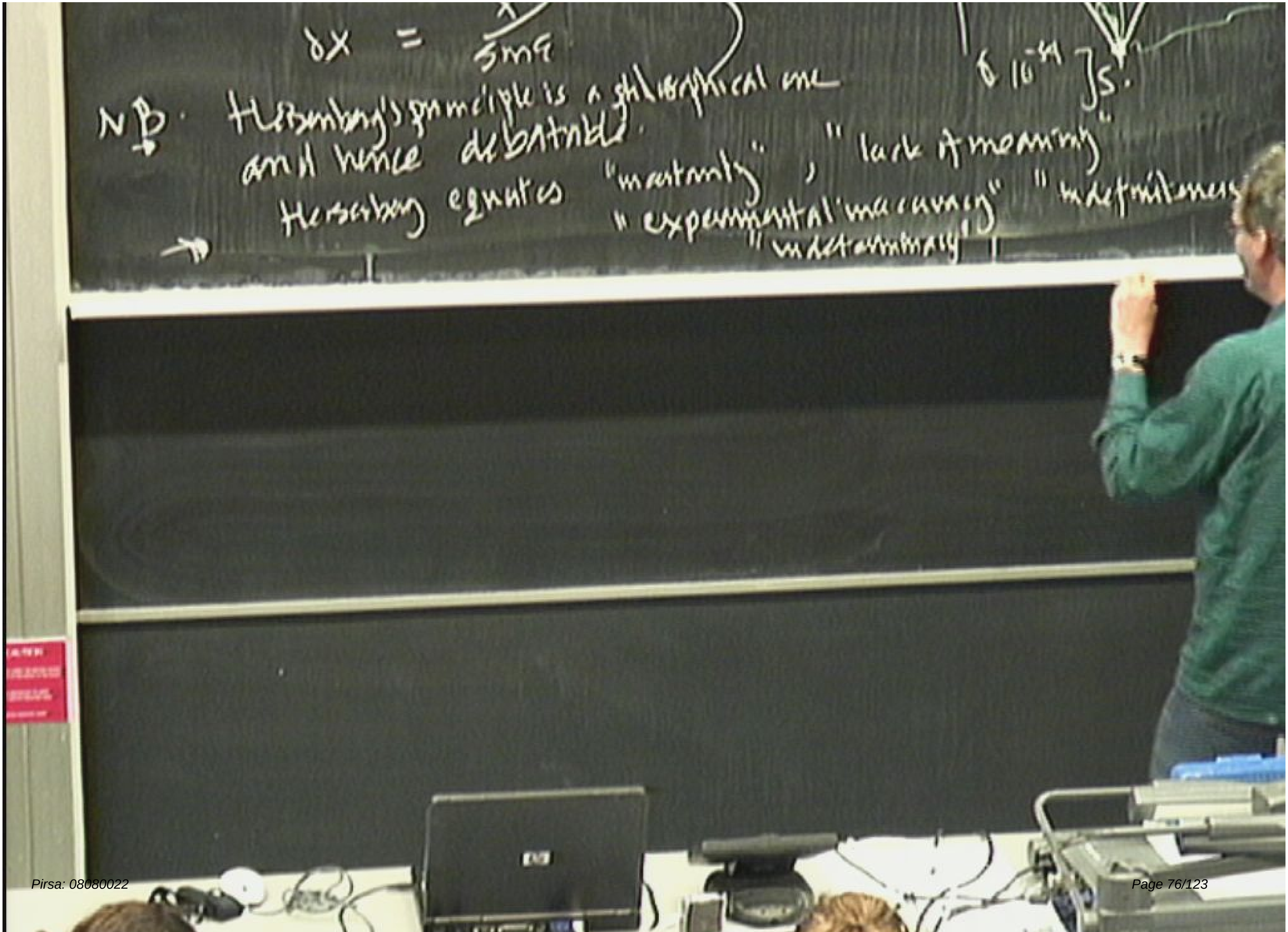
$6 \cdot 10^{-34}$ Js!



N.B. Heisenberg's principle is a philosophical one and hence debatable.

Heisenberg equates "uncertainty", "lack of meaning"


δx =
N.B. Heisenberg's principle is a philosophical one
and hence debatable.
Heisenberg equates "uncertainty", "lack of meaning"
"experimental inaccuracy"
Js.

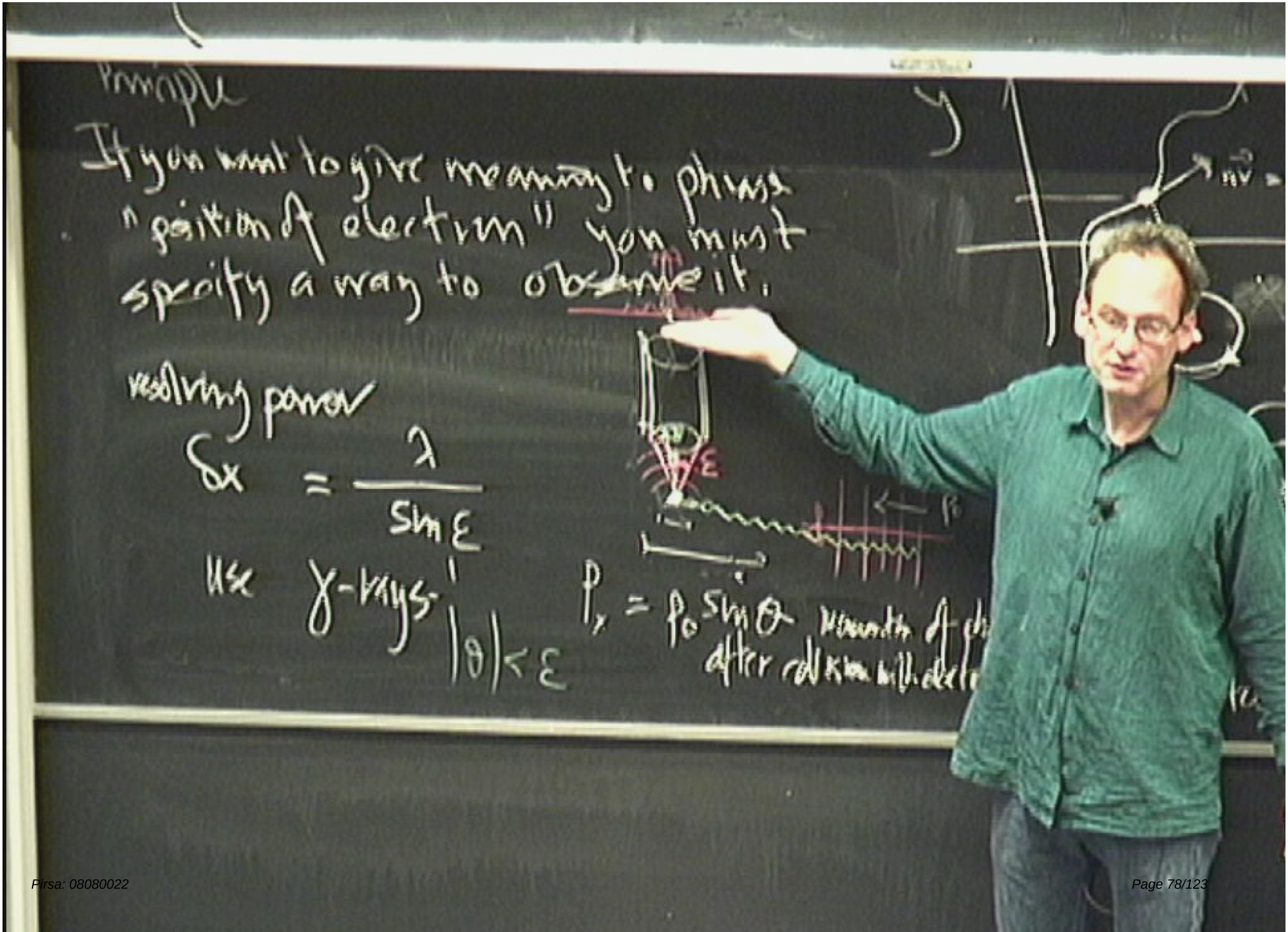

δx =
NB. Heisenberg's principle is a philosophical one
and hence debatable.
Heisenberg equates "uncertainty", "lack of meaning"
"experimental measuring" "indefiniteness"
"indeterminacy"

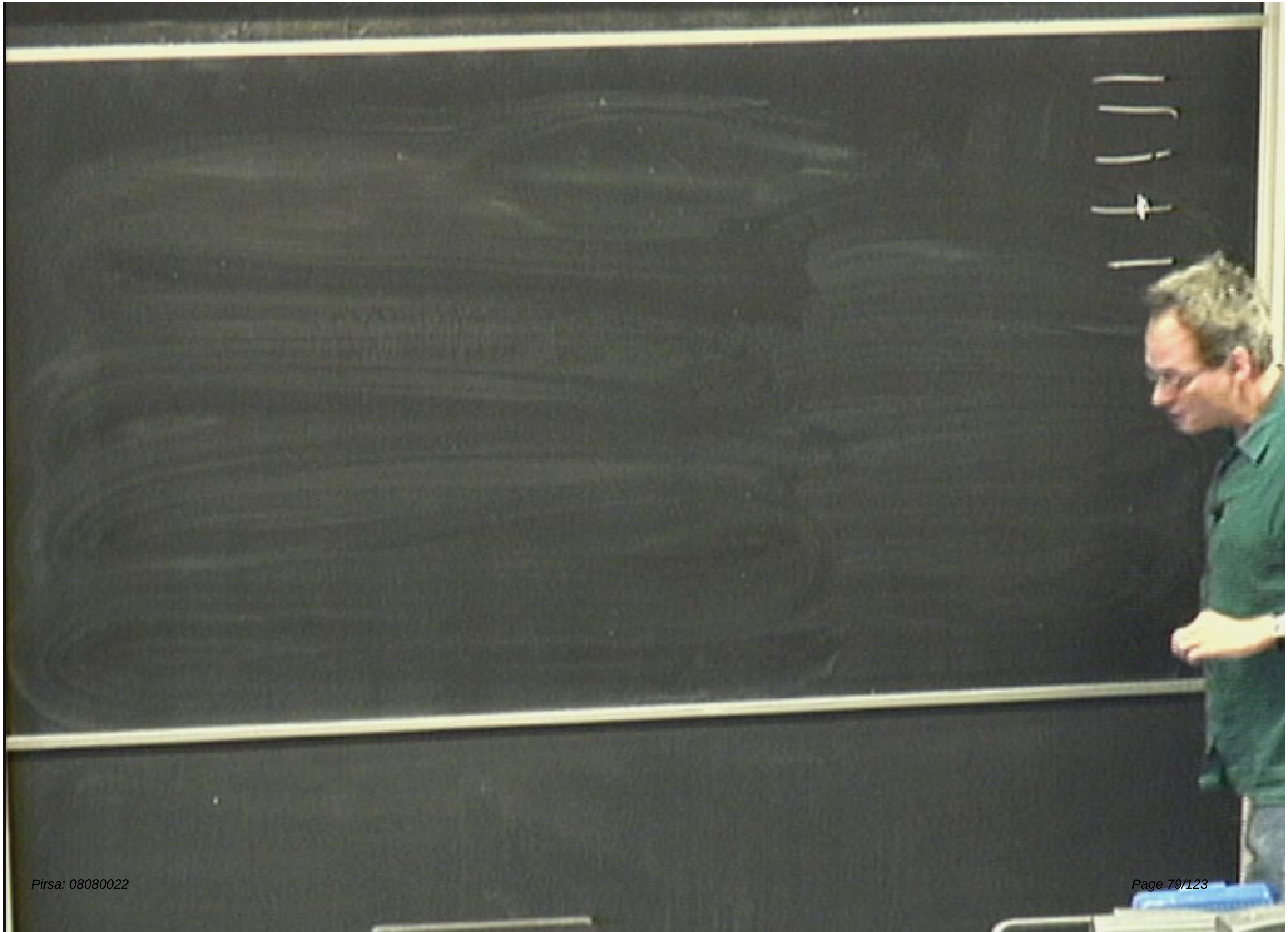
$$\delta x = \frac{\lambda}{5m\epsilon}$$

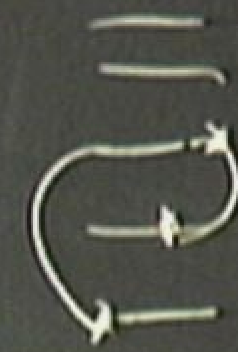
$\delta\ 10^{-34}\ Js$

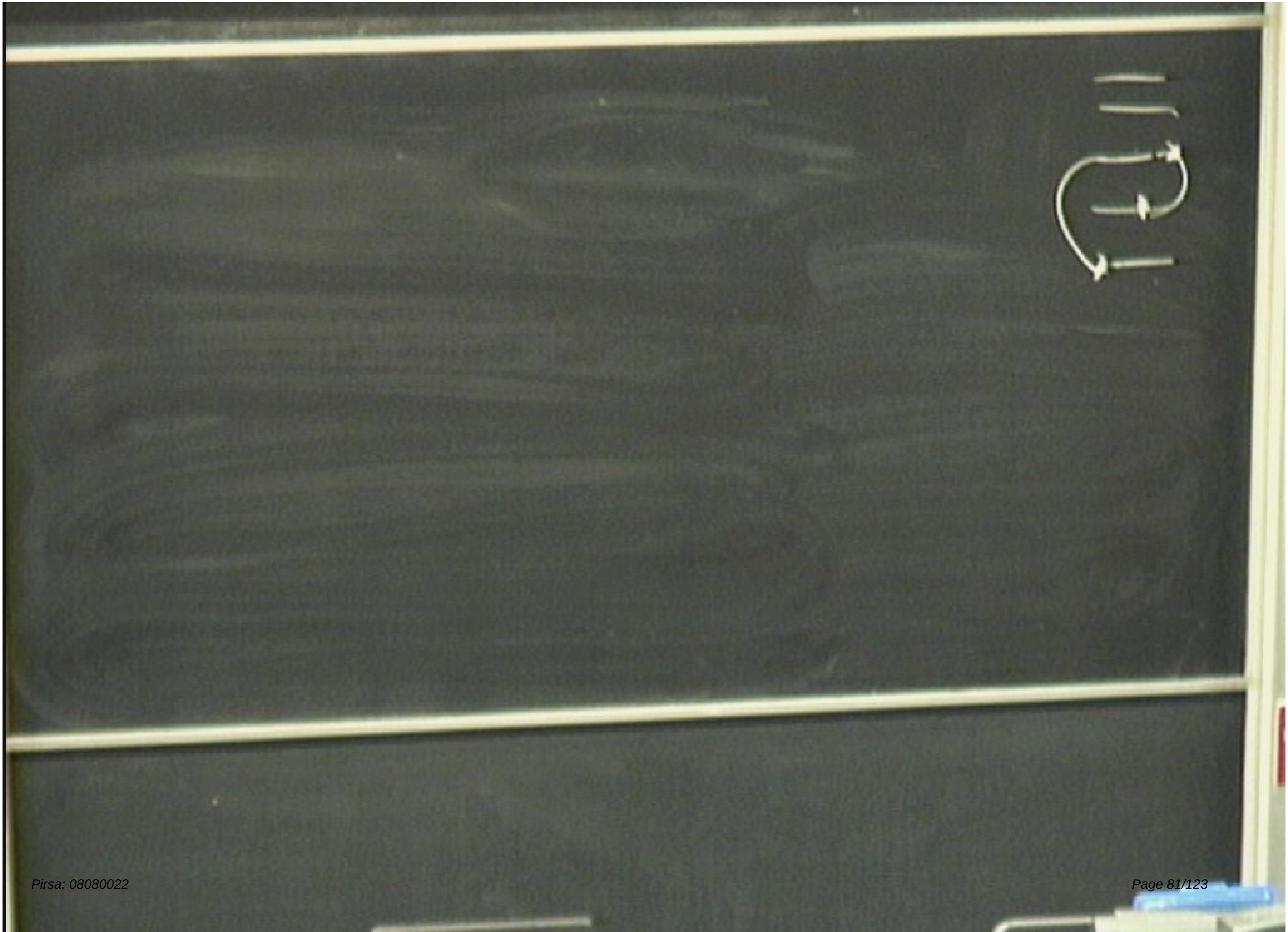
N.B. Heisenberg's principle is a philosophical one and hence debatable.

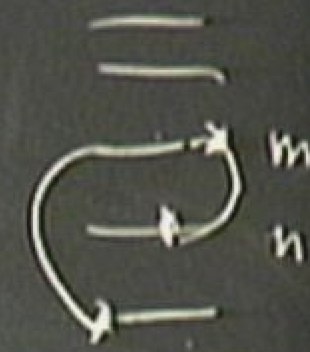
Heisenberg equates "uncertainty", "lack of meaning", "experimental inaccuracy", "indefiniteness", "indeterminacy" etc.


Principle
If you want to give meaning to phrase
"position of electron" you must
specify a way to observe it.
resolving power
$\delta x = \dfrac{\lambda}{\sin \varepsilon}$
use γ-rays!
$|\theta| < \varepsilon$
$p_x = p_0 \sin\theta$

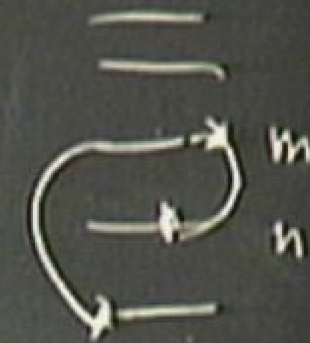




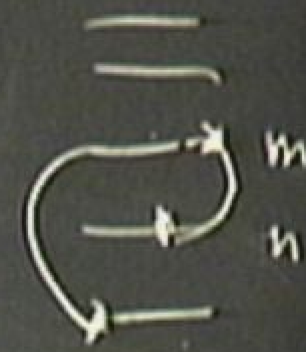




$E_{n,m}$



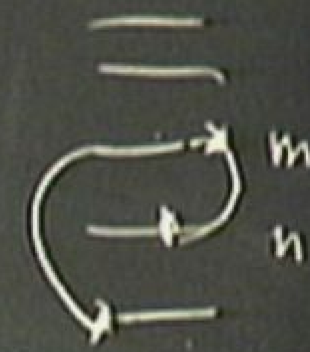
$$E_{n,m} \qquad \begin{array}{cc} & n \rightarrow \\ m \downarrow & \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \end{array}$$



$E_{n,m}$ $n \rightarrow$

$m \downarrow$

matrices

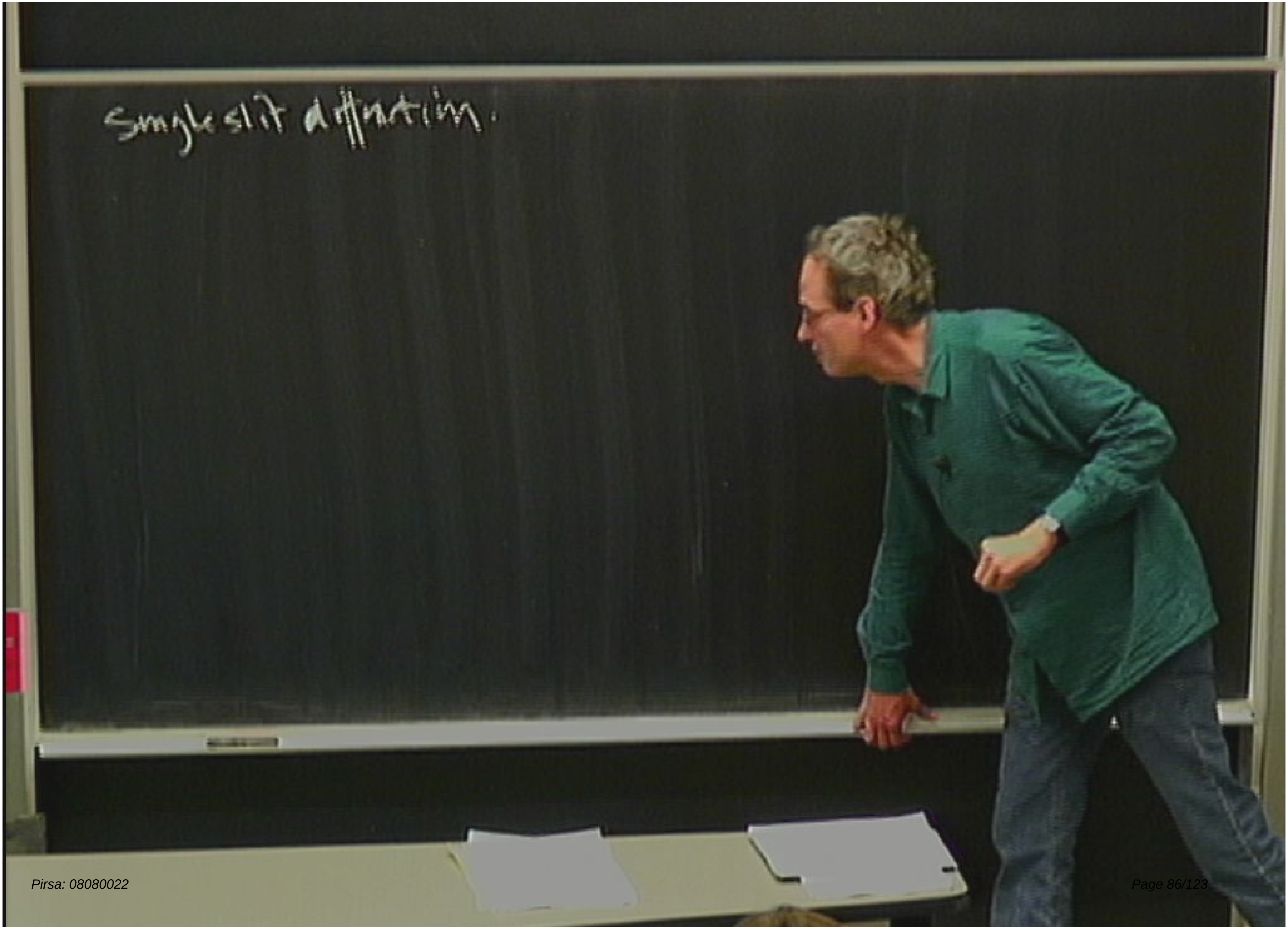


$E_{n,m}$

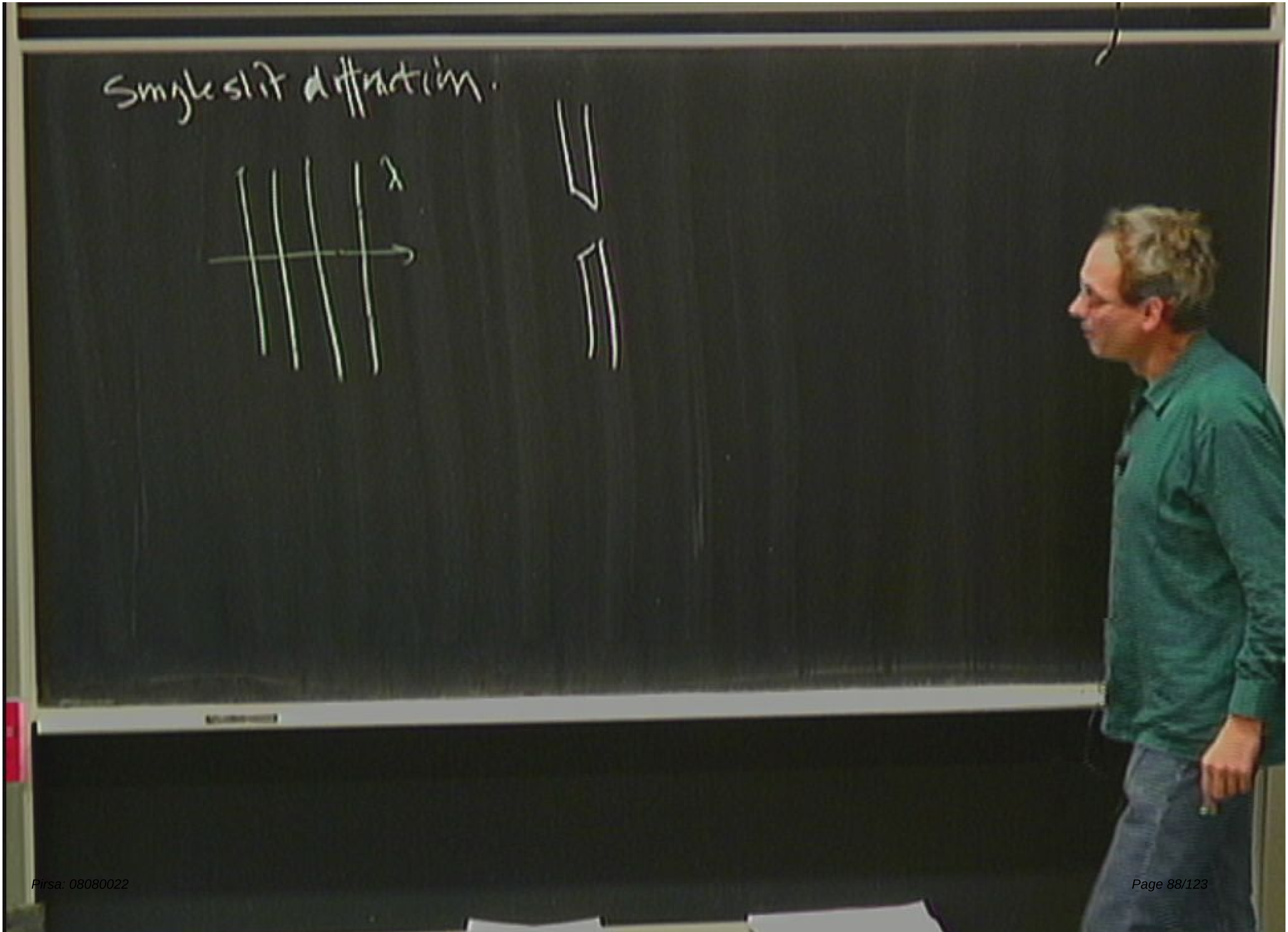
$$\begin{array}{cc} & n \rightarrow \\ m \downarrow & \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \end{array}$$

matrices

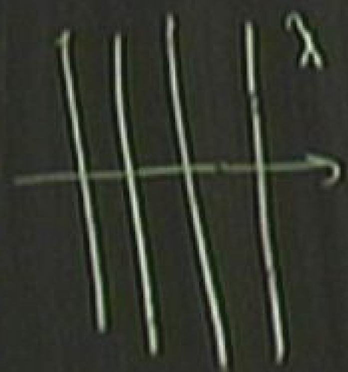
Heisenberg 1925 · matrix mechanics
Schrödinger 1926 wave mechanics.

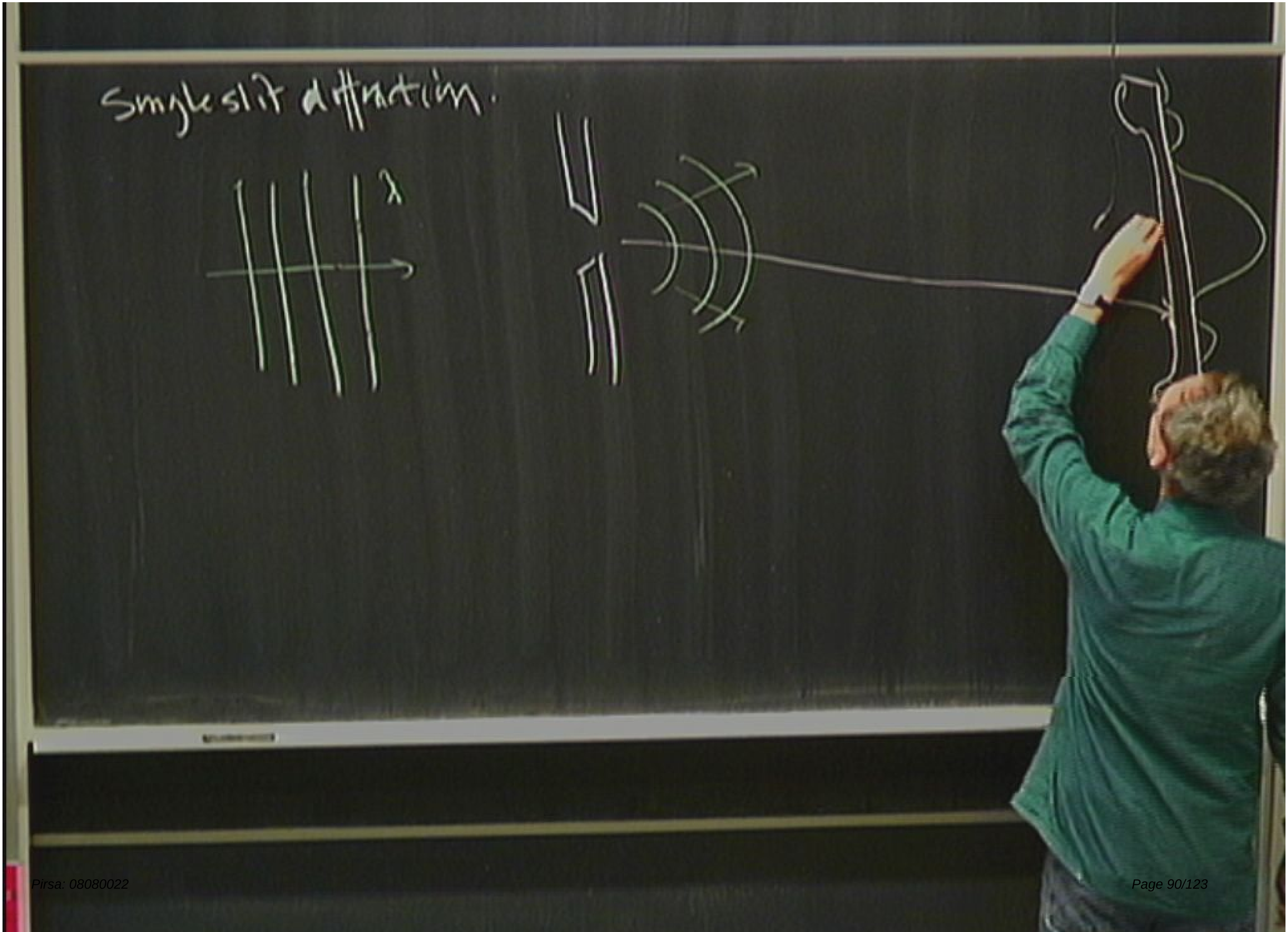

Single slit diffraction.

Single slit diffraction.

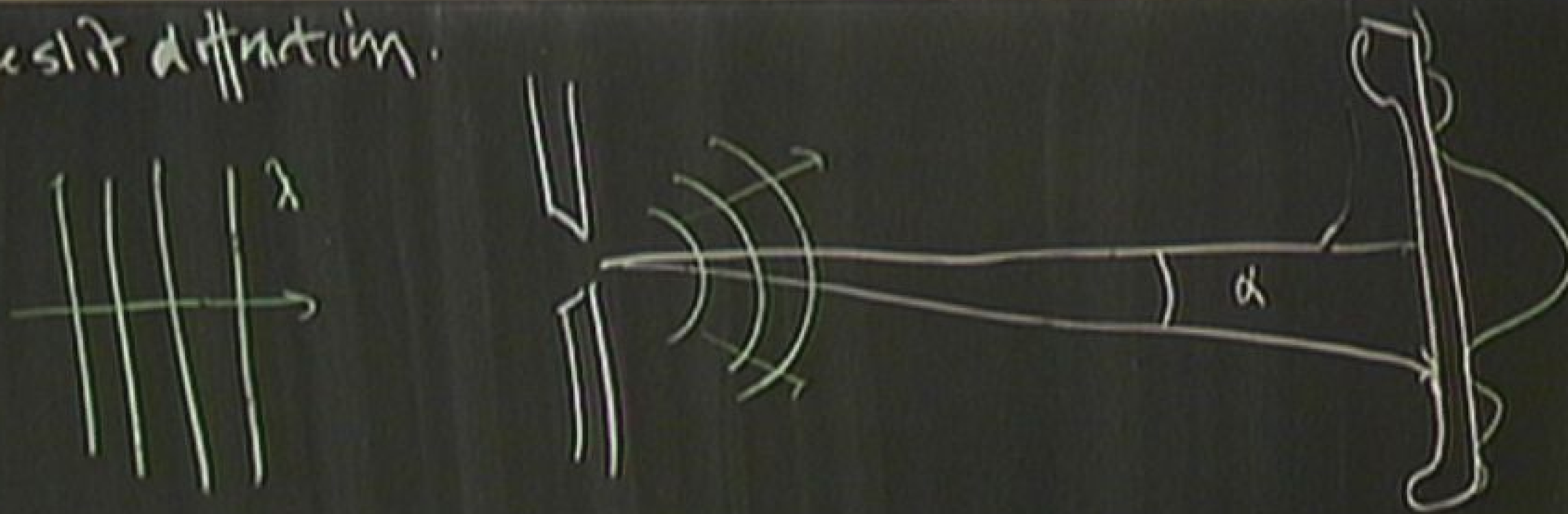

Single slit diffraction.
λ

Single slit diffraction.

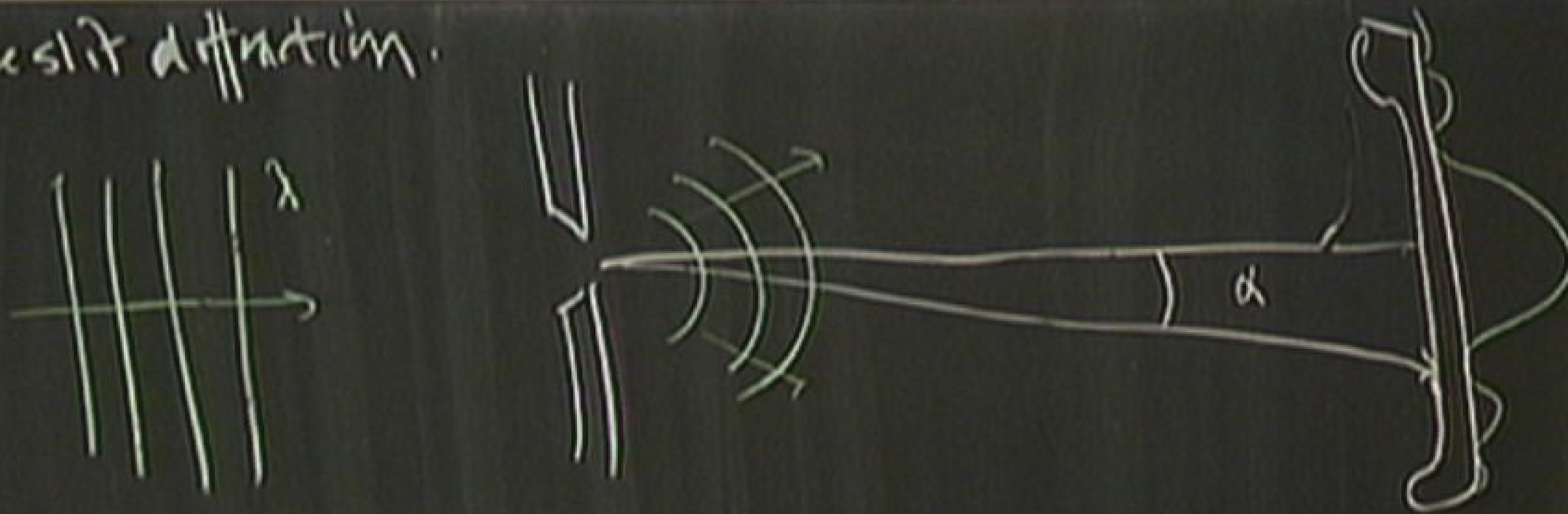


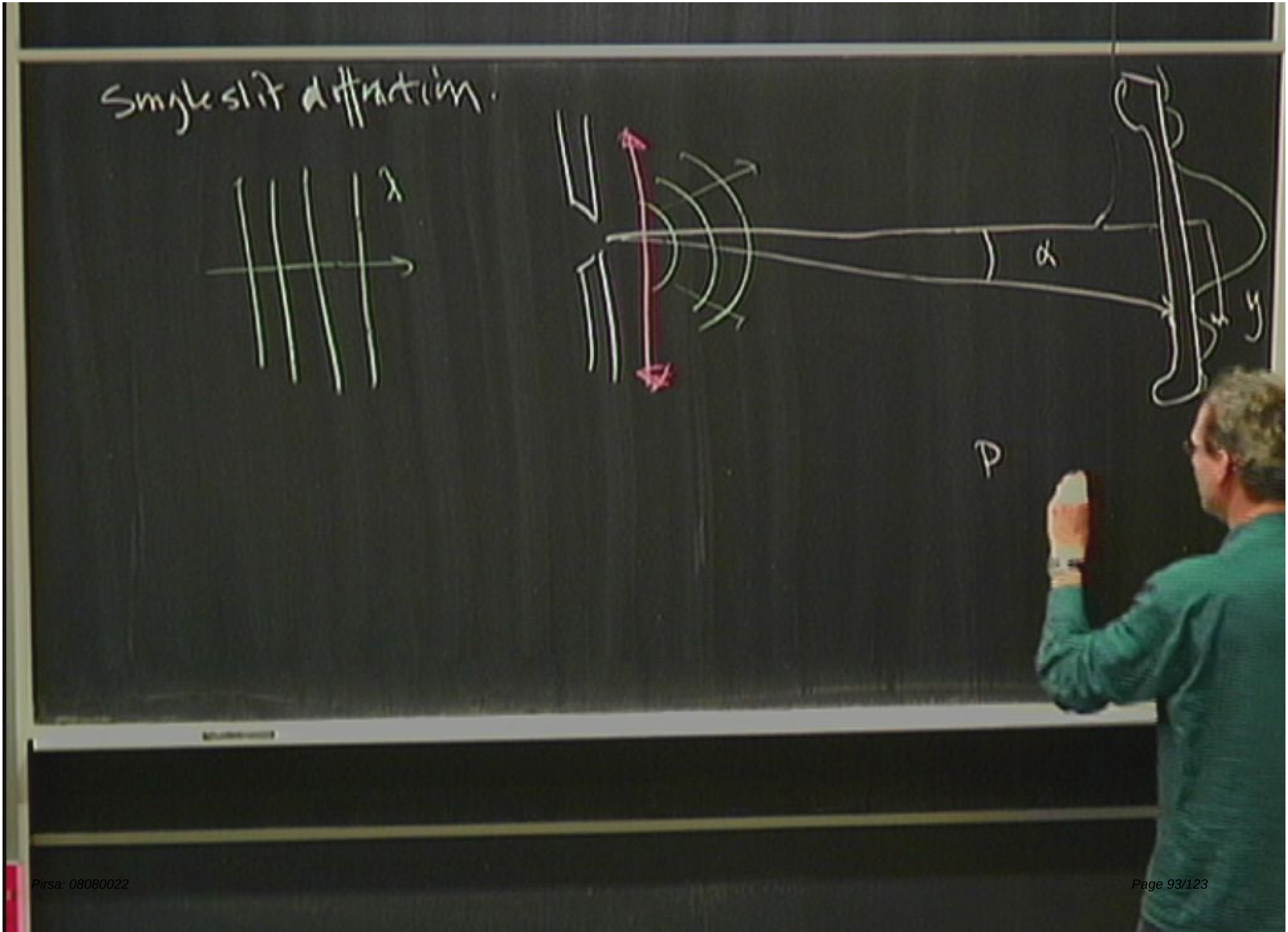

Single slit diffraction.
λ

Single slit diffraction.

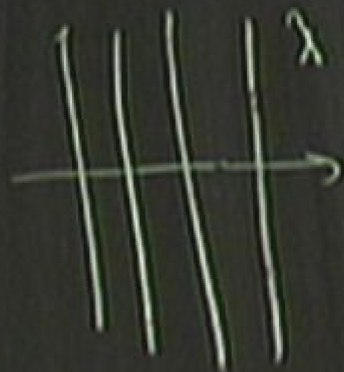


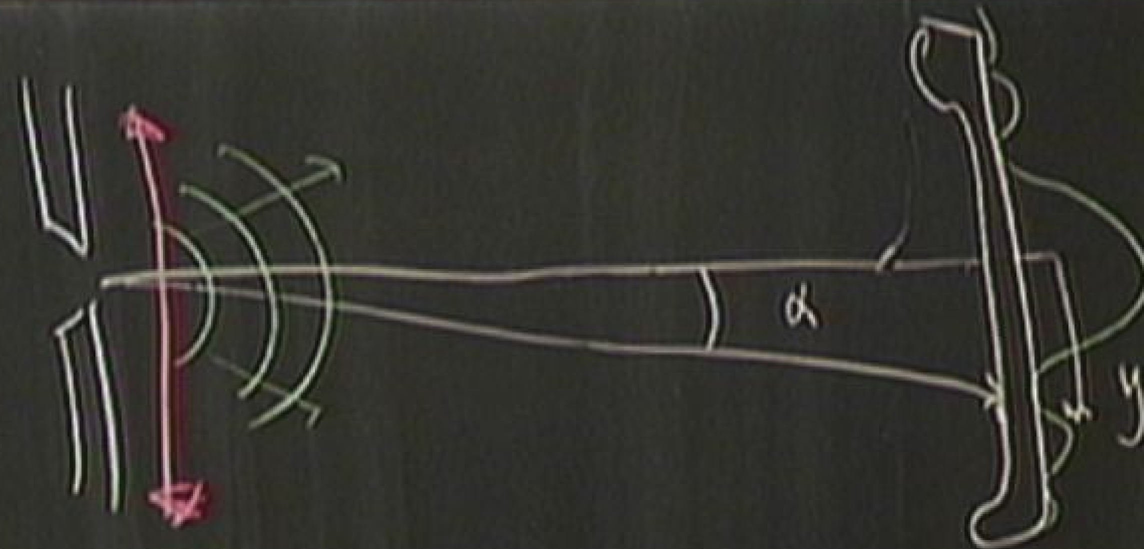
Single slit diffraction.




Single slit diffraction.
λ
α
y
P

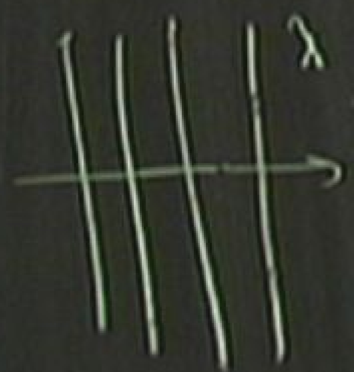
Single slit diffraction.

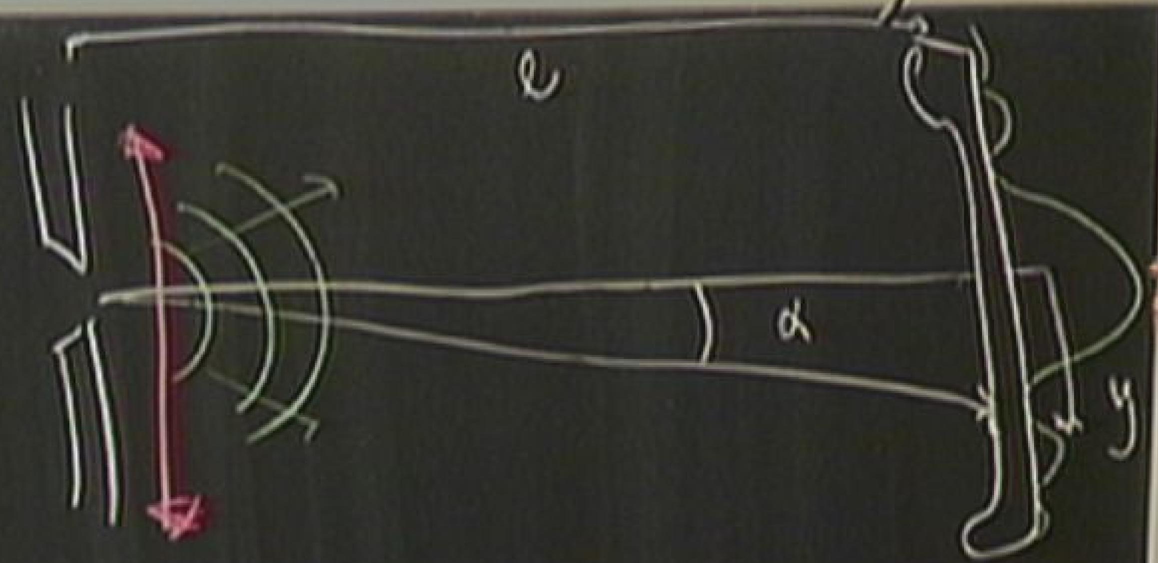




$$p = p_0 \sin\alpha$$
$$\approx p_0 \, y/L$$

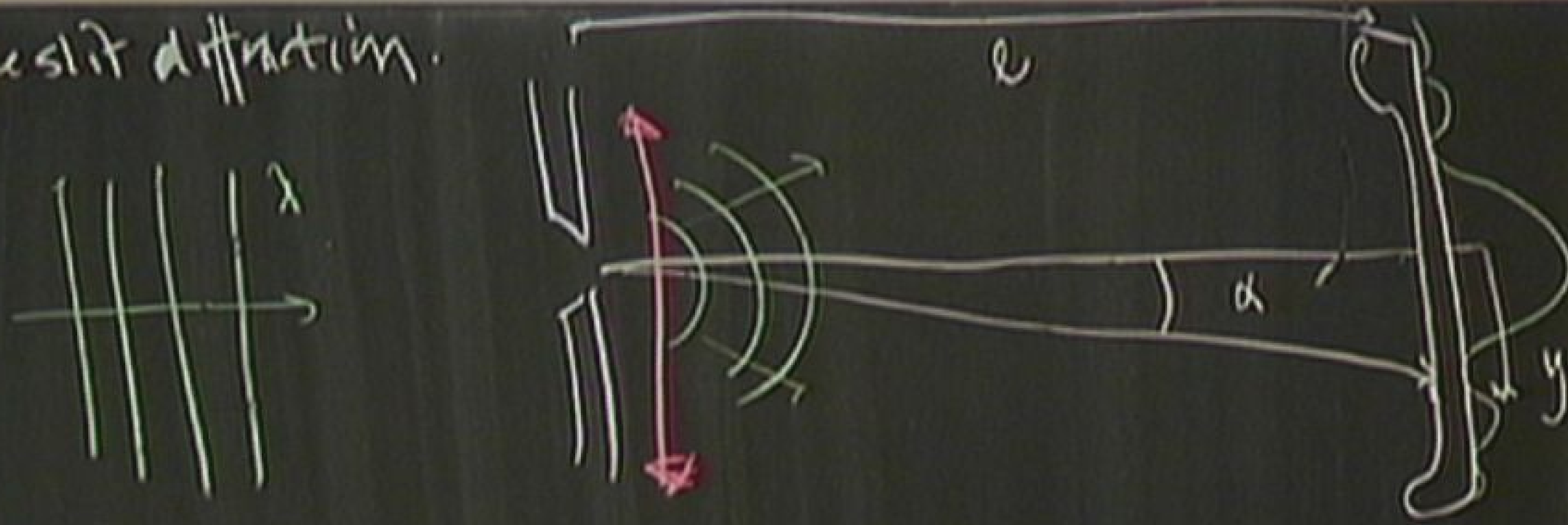
Single slit diffraction.



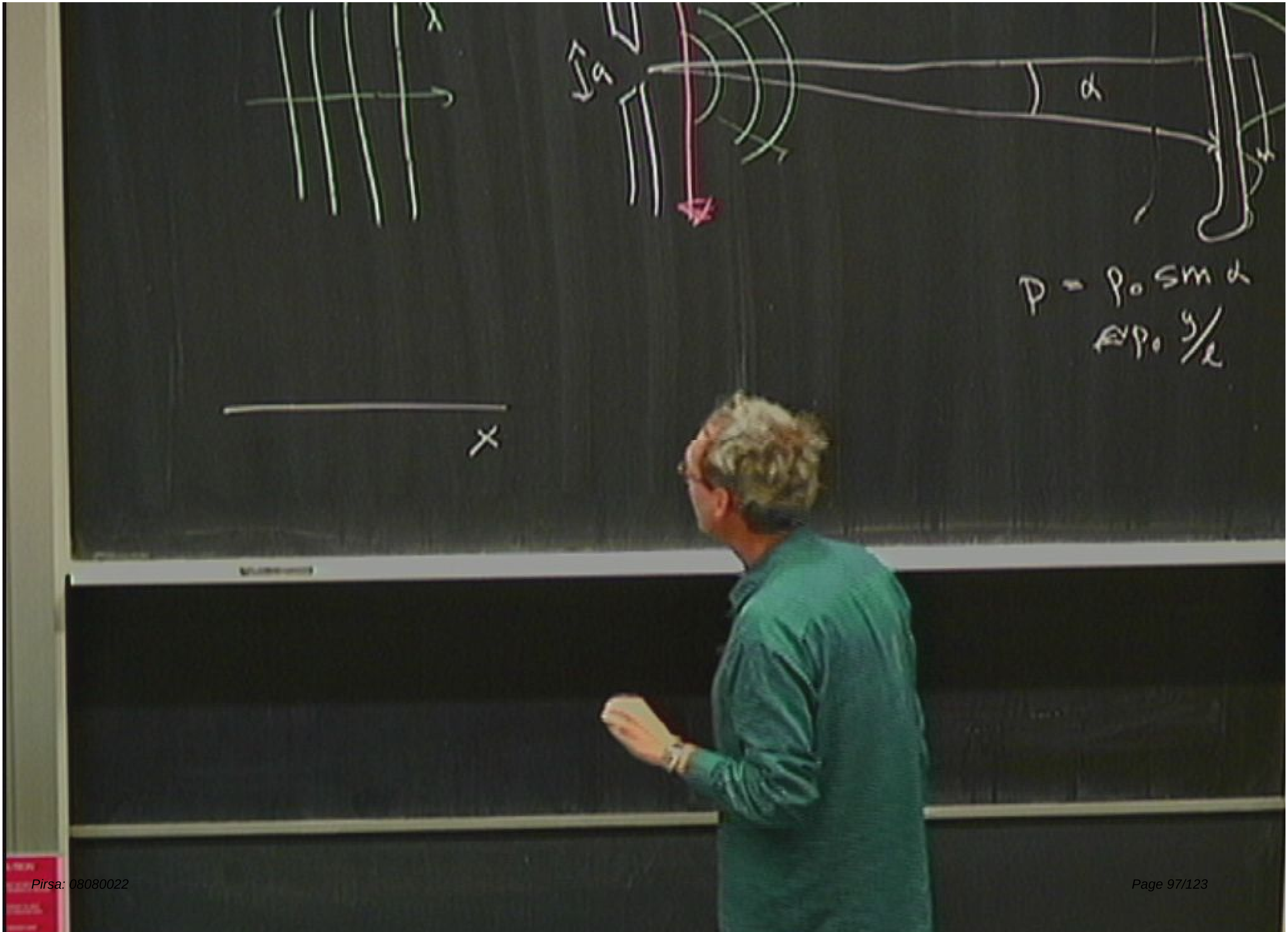


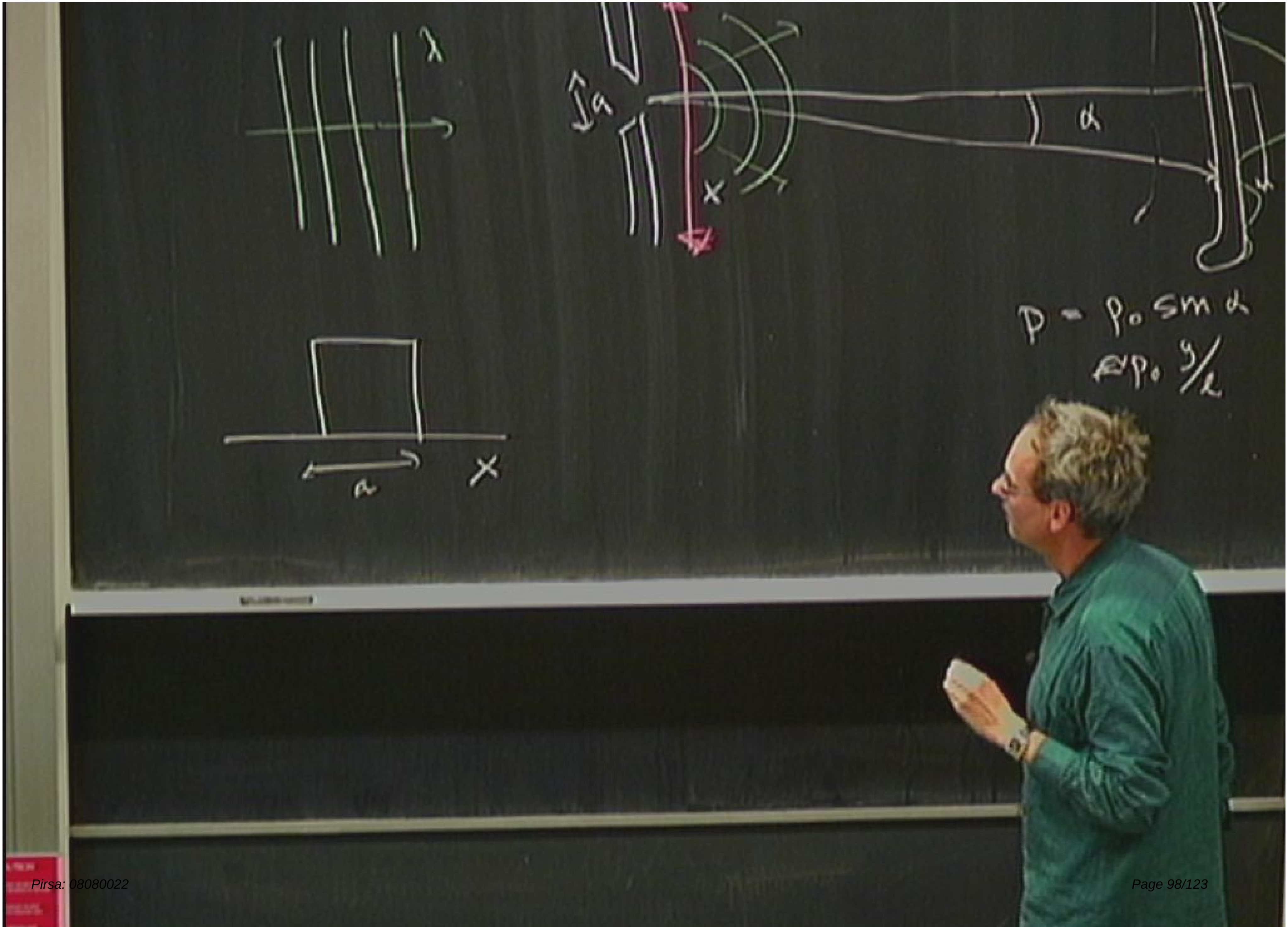
$$p = p_0 \sin \alpha$$
$$\approx p_0 \, y/\ell$$

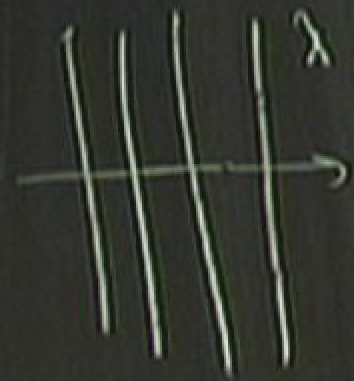
Single slit diffraction.

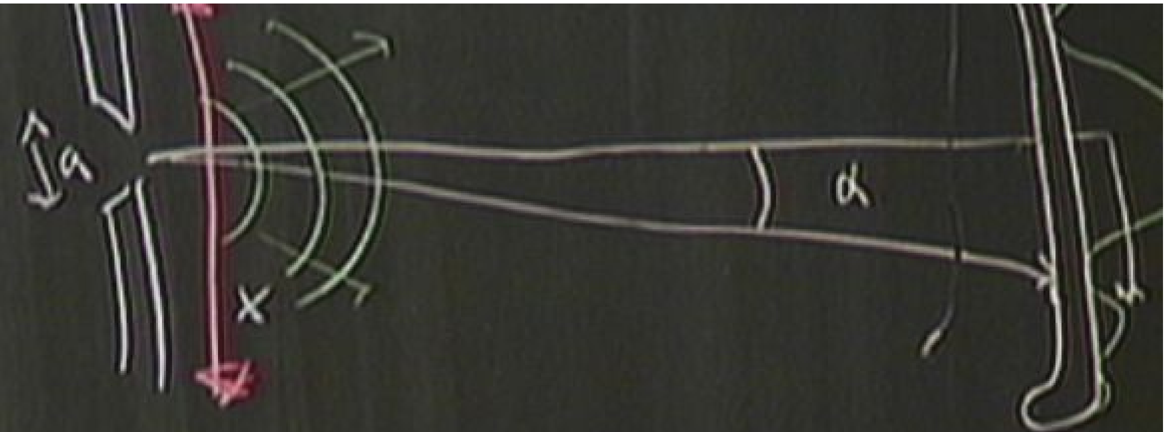


$$p = p_0 \sin\alpha$$
$$\approx p_0 \, y/\ell$$

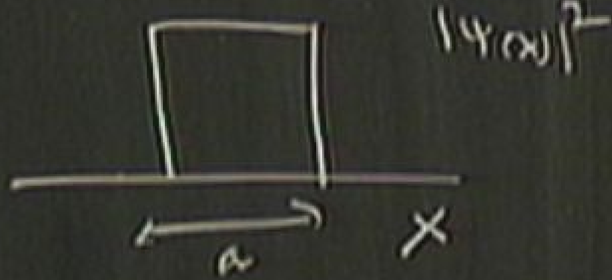

a
α
p = p₀ sin α
≈ p₀ y/ℓ
x

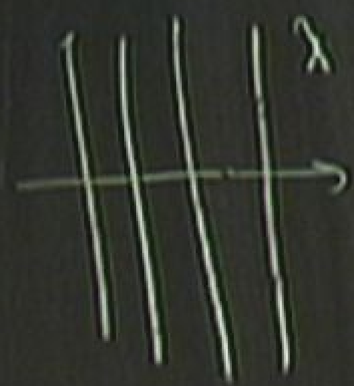

λ
a
α
x
p = p₀ sin α
≈ p₀ y/ℓ
a
x

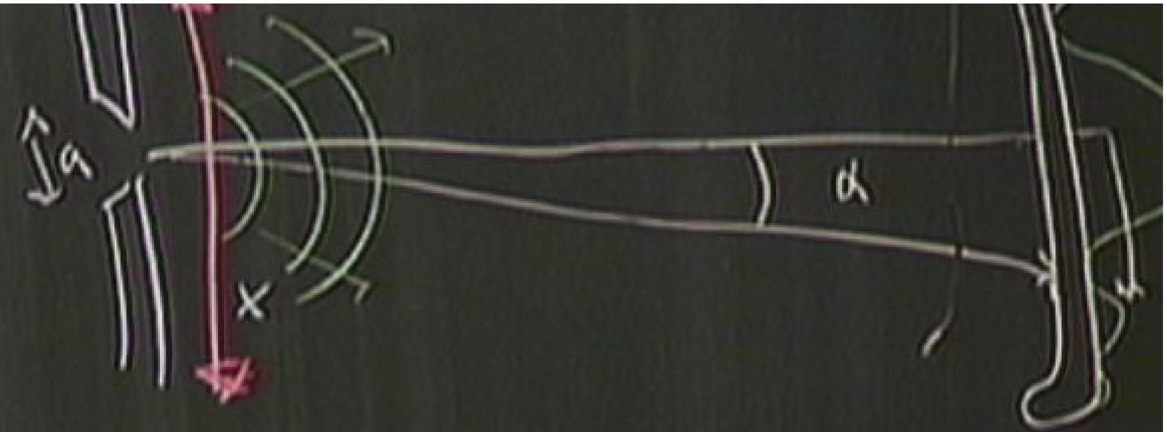




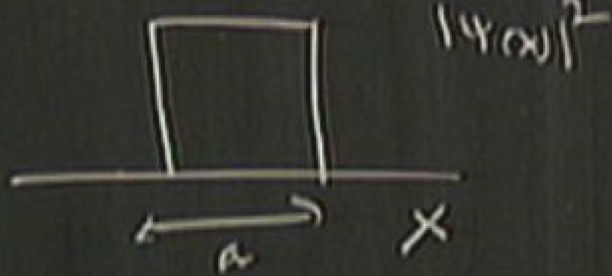
$$p = p_0 \sin\alpha$$
$$\approx p_0\, y/\ell$$



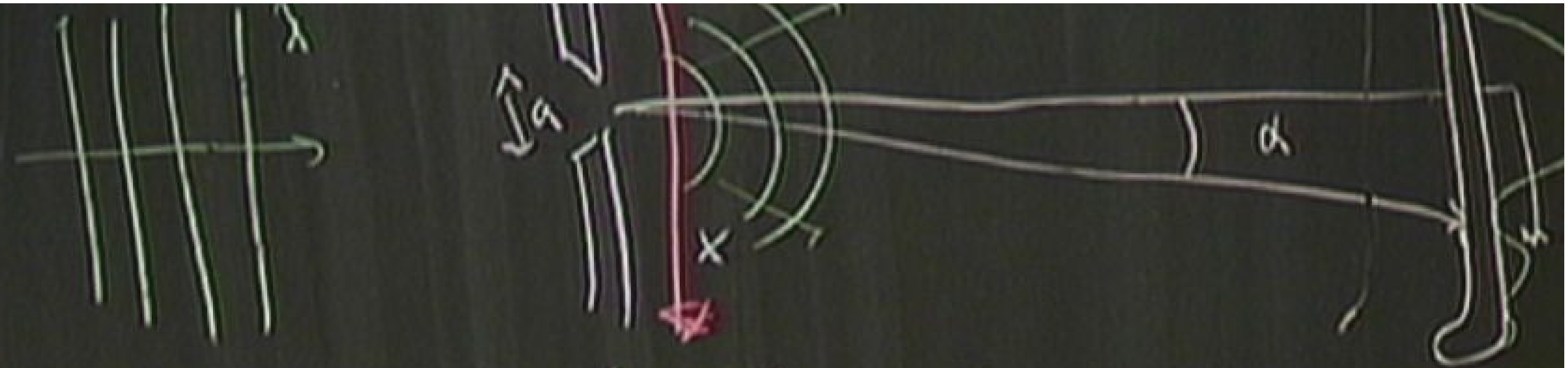




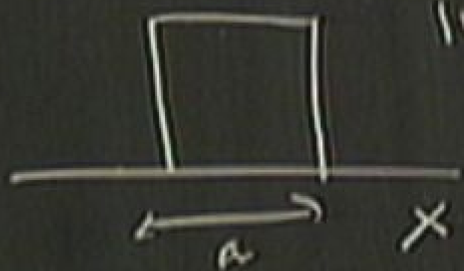
$\psi(x)$



$$p = p_0 \sin\alpha \approx p_0\, y/\ell$$



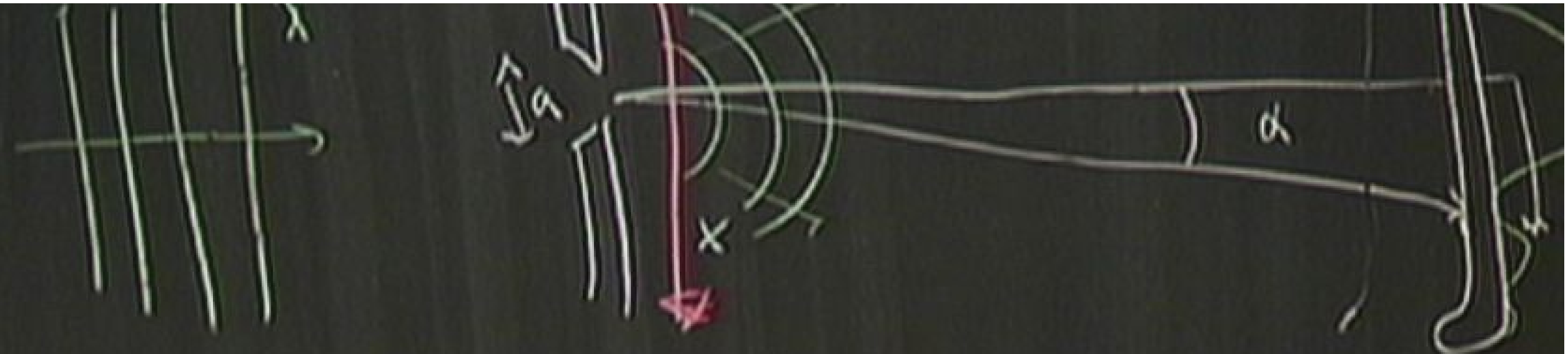
$\psi(x)$



$|\psi(x)|^2$ $\quad |\varphi(p)|^2$

$p = p_0 \sin\alpha$

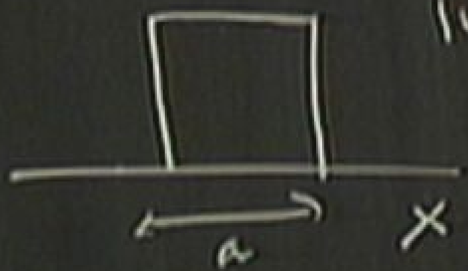
$\approx p_0\, y/\ell$

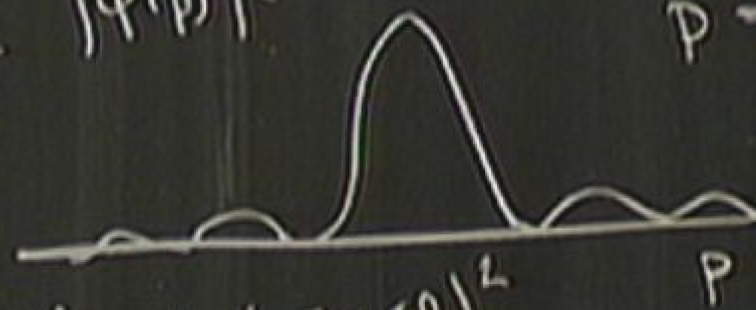


$\psi(x)$

$|\varphi(p)|^2$

$|\psi(x)|^2$

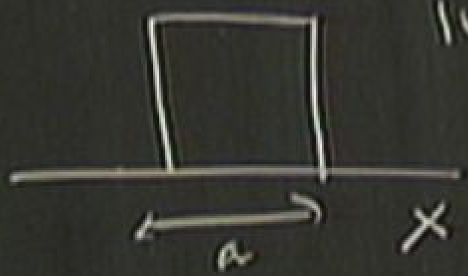




$$p = p_0 \sin\alpha \approx p_0\, y/\ell$$

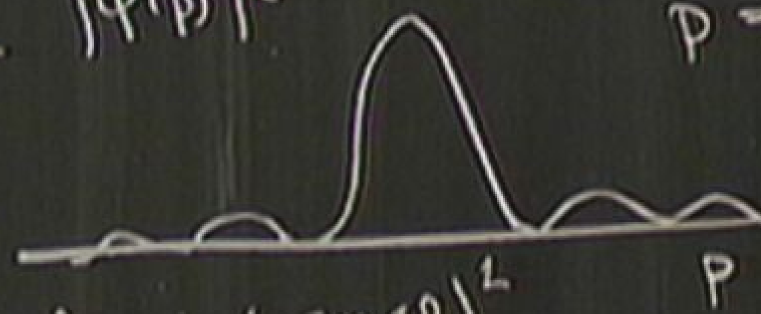
$$|\varphi(p)|^2 \quad \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$$

$\psi(x)$



$|\psi(x)|^2$

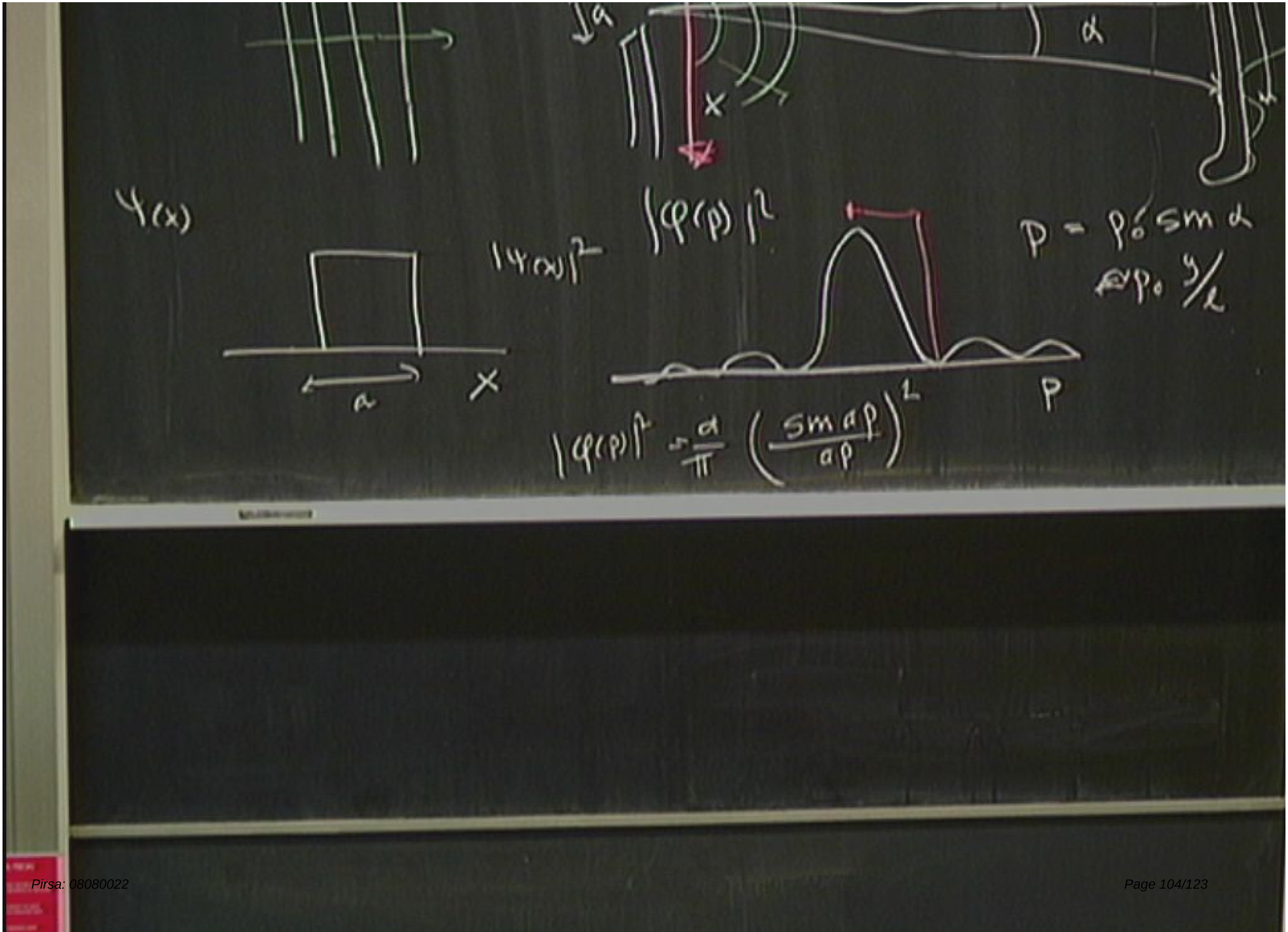
$|\varphi(p)|^2$

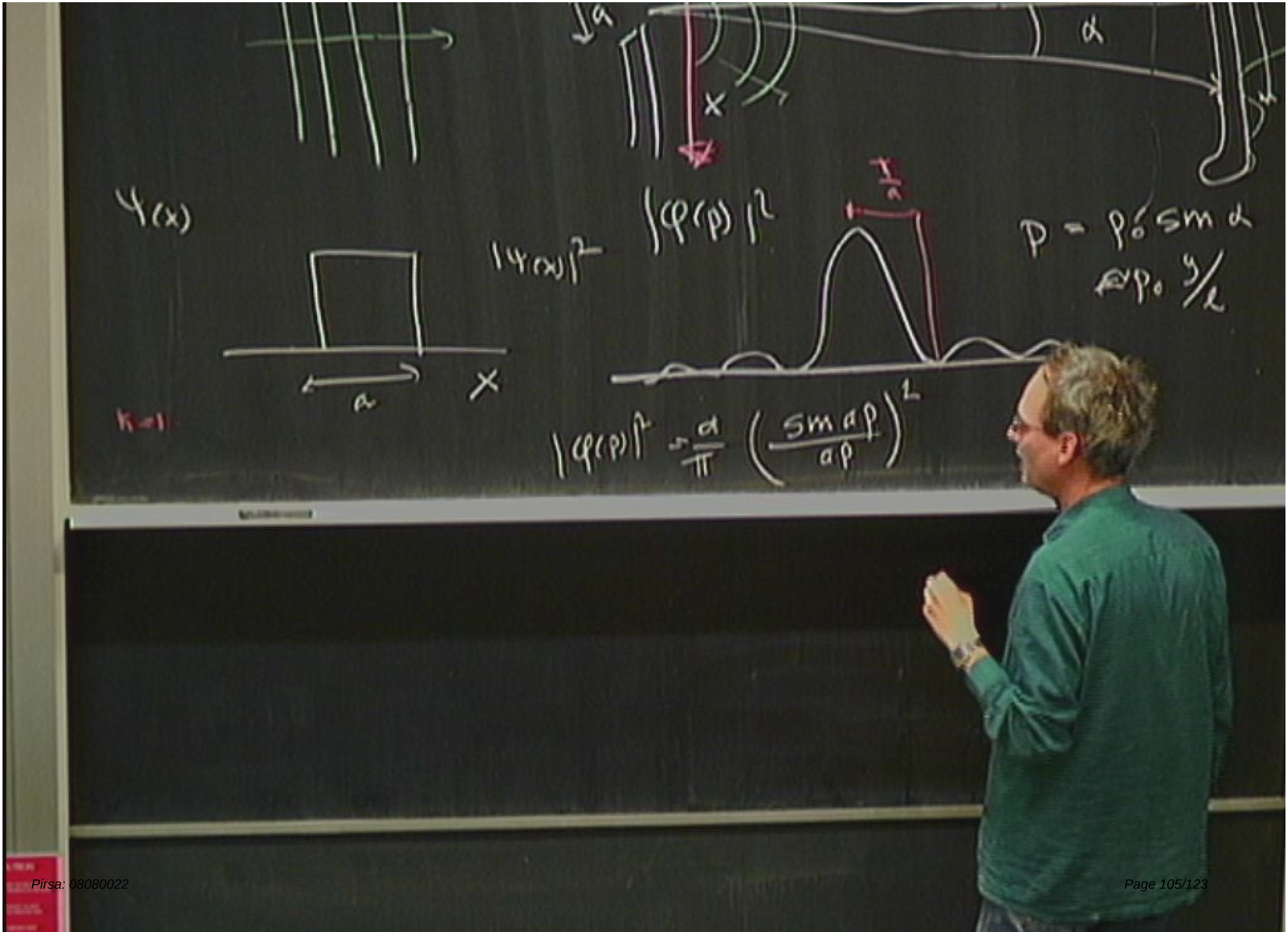


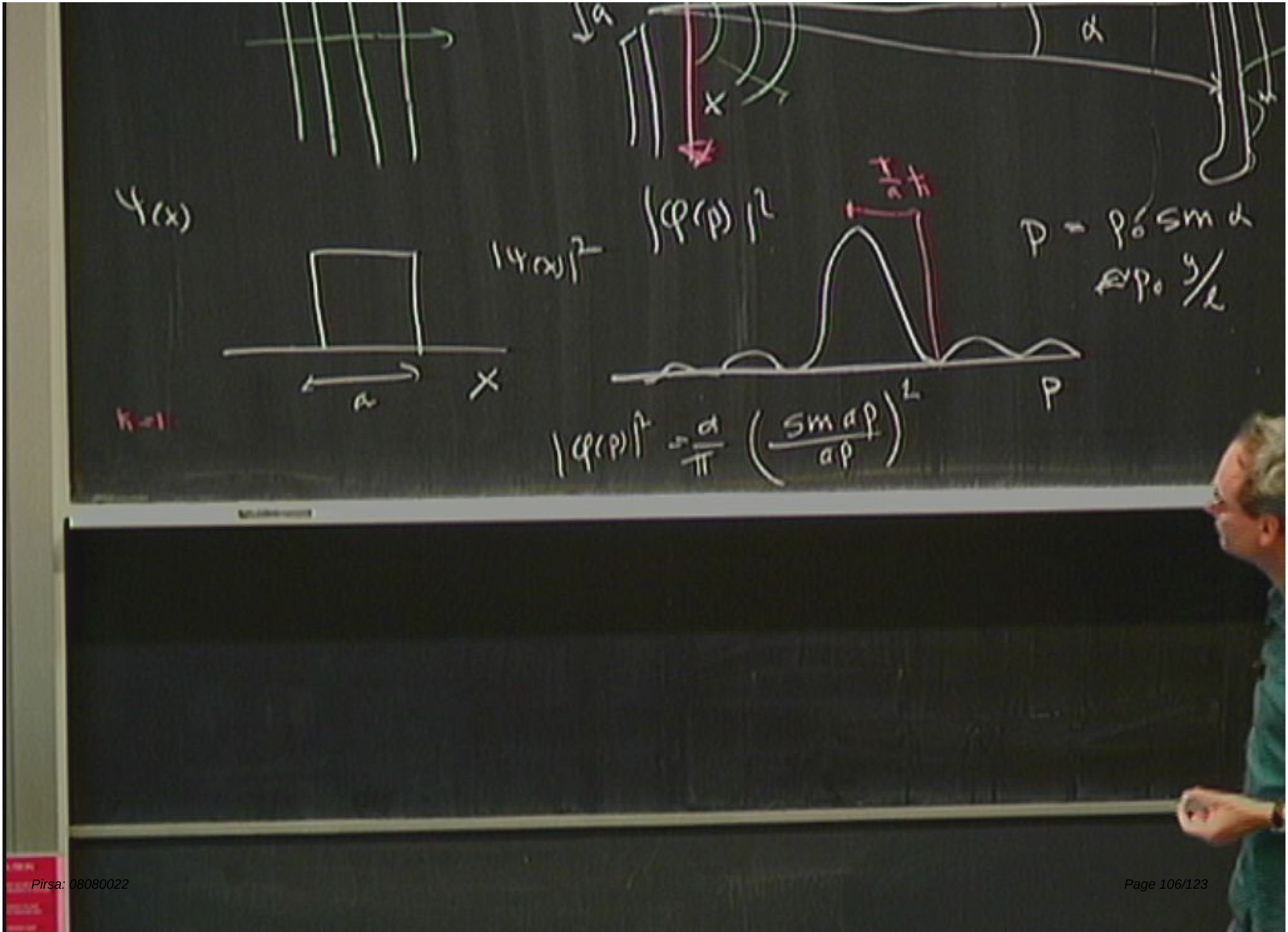
$p = p_0 \sin\alpha$

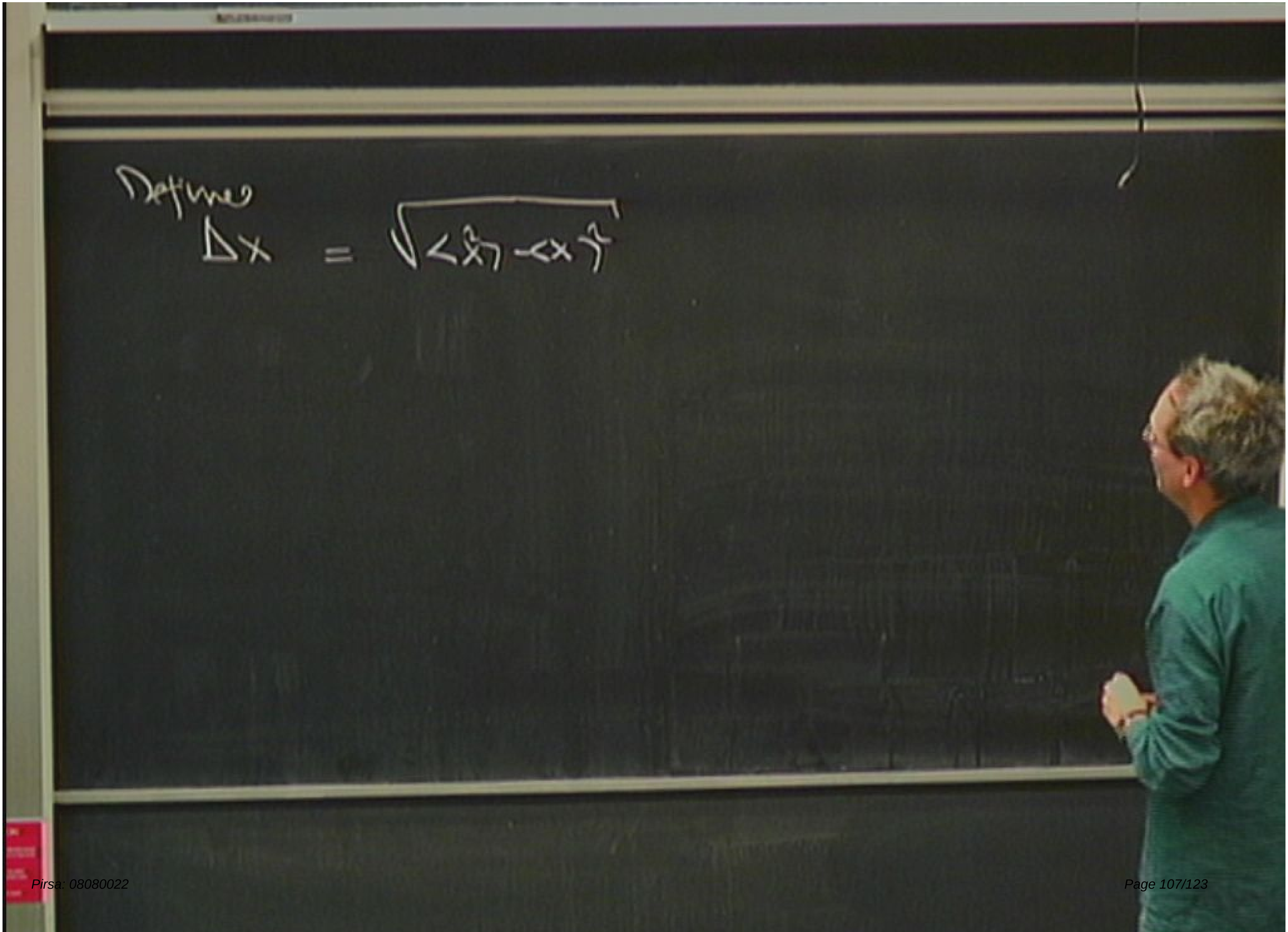
$\approx p_0\, y/L$

$$|\varphi(p)|^2 = \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$$


$\psi(x)$
$|\psi(x)|^2$
$|\varphi(p)|^2$
$p = p_0 \sin\alpha$
$\approx p_0\, y/L$
a
x
p
$|\varphi(p)|^2 = \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$


ψ(x)
|ψ(x)|²
|φ(p)|²
p = p₀ sin α
≈ p₀ y/L
|φ(p)|² = a/π (sin ap / ap)²


ψ(x)
|ψ(x)|²
a
x
|φ(p)|²
p
p = p₀ sin α
|φ(p)|² = a/π (sin ap / ap)²
α


Define
$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$

$\langle x \rangle = \int x |\psi(x)|^2 d$

etc.

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$

$\langle x \rangle = \int x |\psi(x)|^2 dx$

etc.

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 \, dx$

$\langle x \rangle = \int x \, |\psi(x)|^2 \, dx$

etc.

$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}}$$

([illegible]onnard 1927.)

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

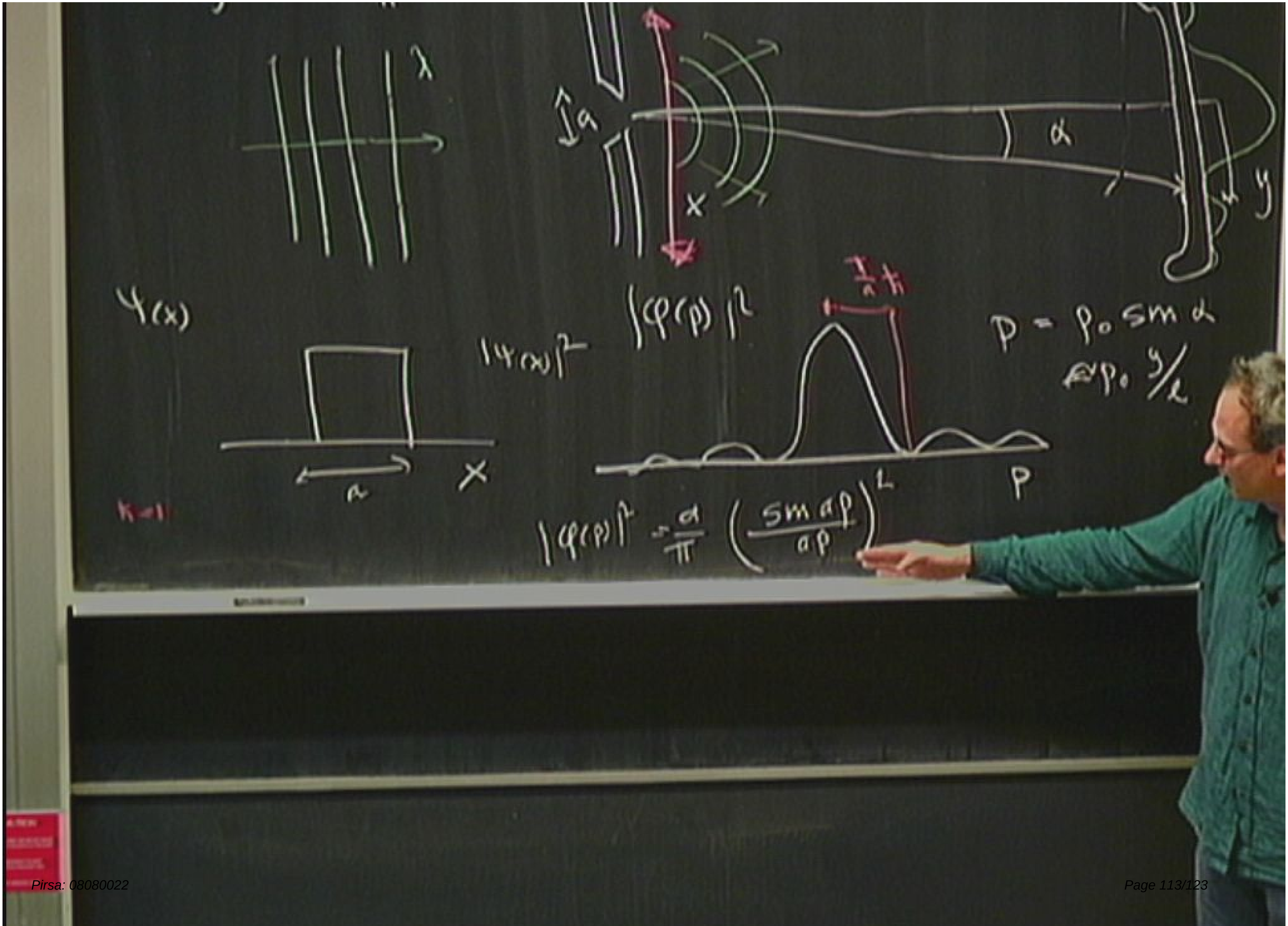
where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$

$\langle x \rangle = \int x |\psi(x)|^2 dx$

etc.

$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}}$$

(Kennard 1927)



λ
a
α
x
y
Ψ(x)
|Ψ(x)|²
|φ(p)|²
p = p₀ sin α
≈ p₀ y/L
a
x
p
|φ(p)|² = a/π (sin ap / ap)²

Define
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$
$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}}$$

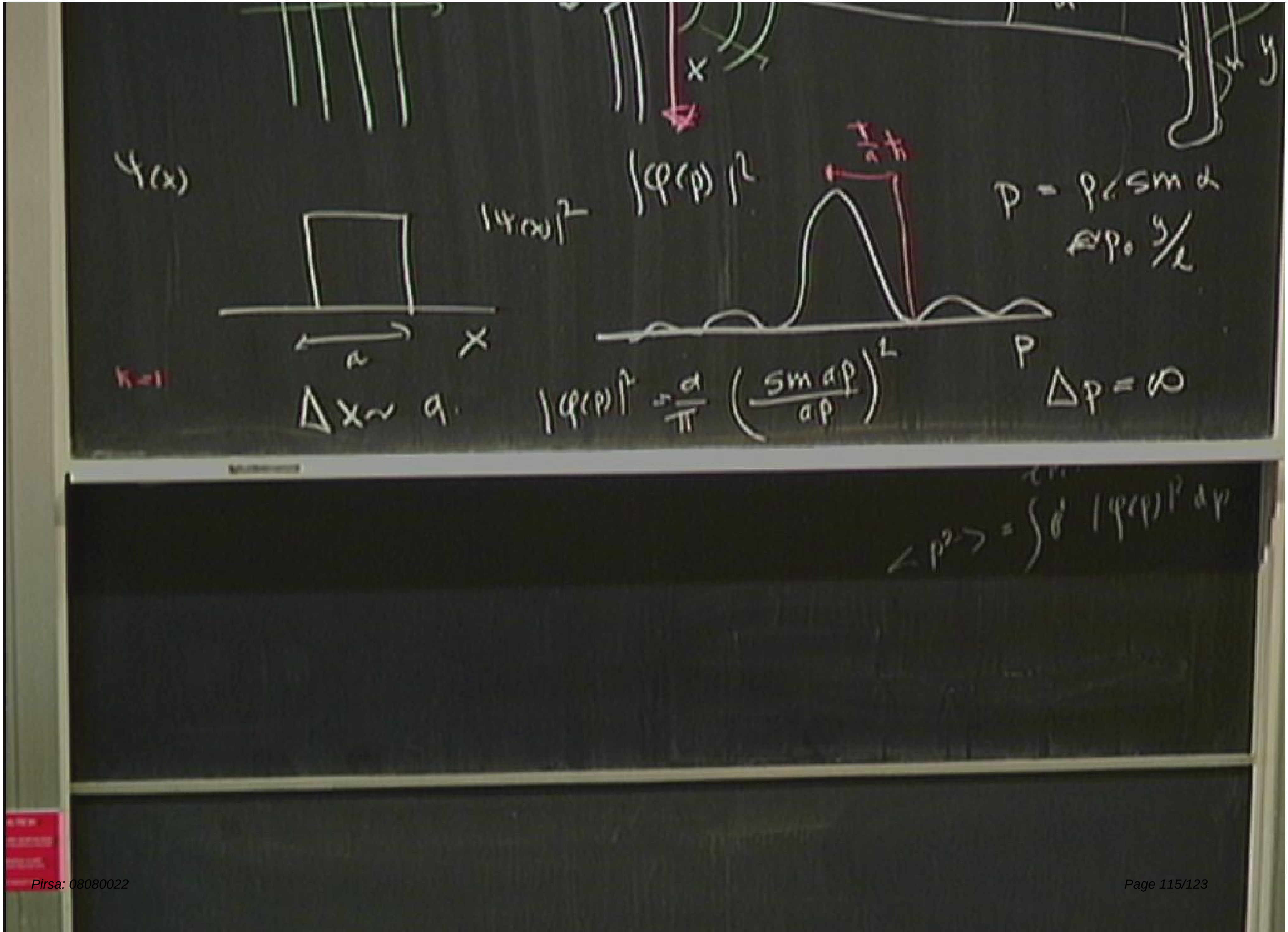
(Kennard 1927.)

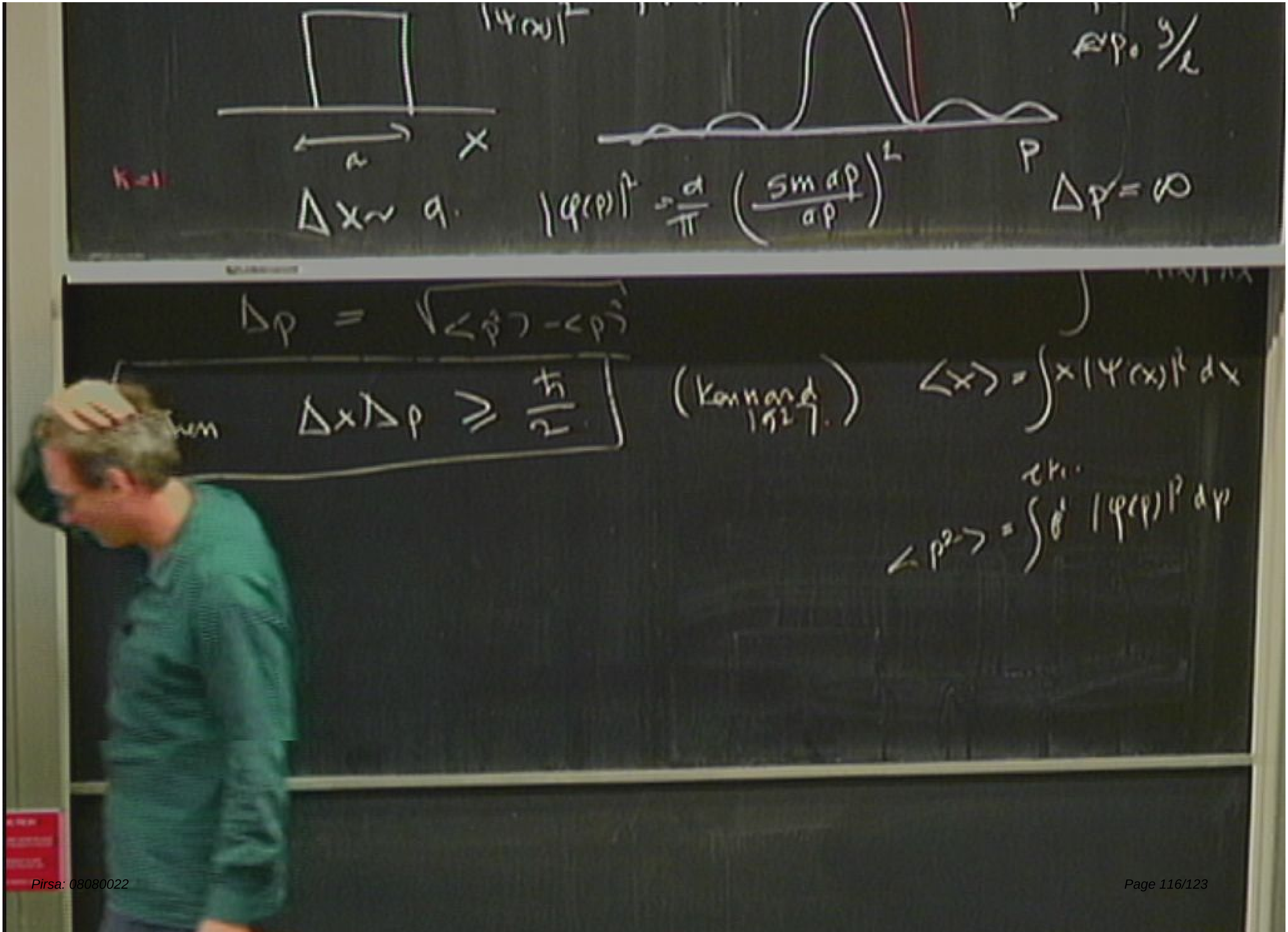
where $\langle x^2\rangle = \int x^2 |\psi(x)|^2 dx$

$\langle x\rangle = \int x |\psi(x)|^2 dx$

etc.

$\langle p^2\rangle = \int p^2 |\varphi(p)|^2 dp$


$\psi(x)$
$|\psi(x)|^2$
$|\varphi(p)|^2$
$\frac{1}{a}\hbar$
$p = p_0 \sin\alpha$
$\approx p_0\, y/L$
x
a
p
$\hbar = 1$
$\Delta x \sim a$
$|\varphi(p)|^2 = \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$
$\Delta p = \infty$
$\langle p^2 \rangle = \int p^2\, |\varphi(p)|^2\, dp$


|ψ(x)|²
a
x
Δx ~ a.
|φ(p)|² = a/π (sin ap / ap)²
p
Δp = ∞
Δp = √(<p²> − <p>²)
Δx Δp ≥ ħ/2
(Kennard 1927)
<x> = ∫ x |ψ(x)|² dx
<p²> = ∫ p² |φ(p)|² dp

$\hbar = 1$

$\Delta x \sim a.$ $\quad |\varphi(p)|^2 = \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$ $\quad \Delta p = \infty$

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

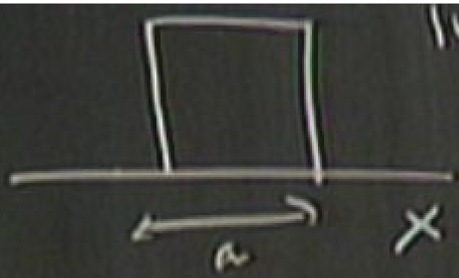
$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}.}$$

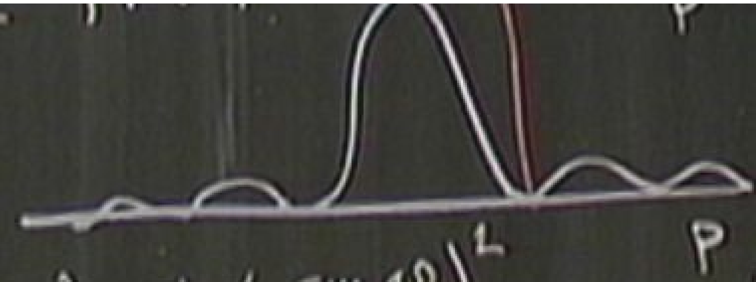
(Kennard 1927.)

$\langle x\rangle = \int x\,|\psi(x)|^2\,dx$

etc.

$\langle p^2\rangle = \int p^2\,|\varphi(p)|^2\,dp$





$\hbar = 1$

$\Delta x \sim a.$ $\quad |\varphi(p)|^2 = \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$ $\quad \Delta p = \infty$

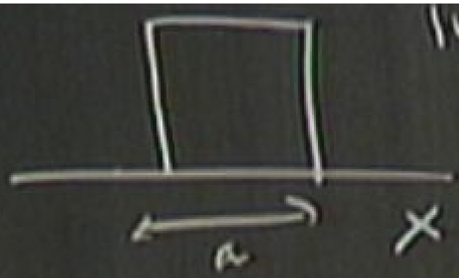
$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}}$$

(Kennard 1927.)

$$\langle x\rangle = \int x\,|\psi(x)|^2\,dx$$

$$\langle p^2\rangle = \int p^2\,|\varphi(p)|^2\,dp$$



$\hbar = 1$

$\Delta x \sim a.$ $\quad |\varphi(p)|^2 = \frac{a}{\pi}\left(\frac{\sin ap}{ap}\right)^2$ $\quad \Delta p = \infty$

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

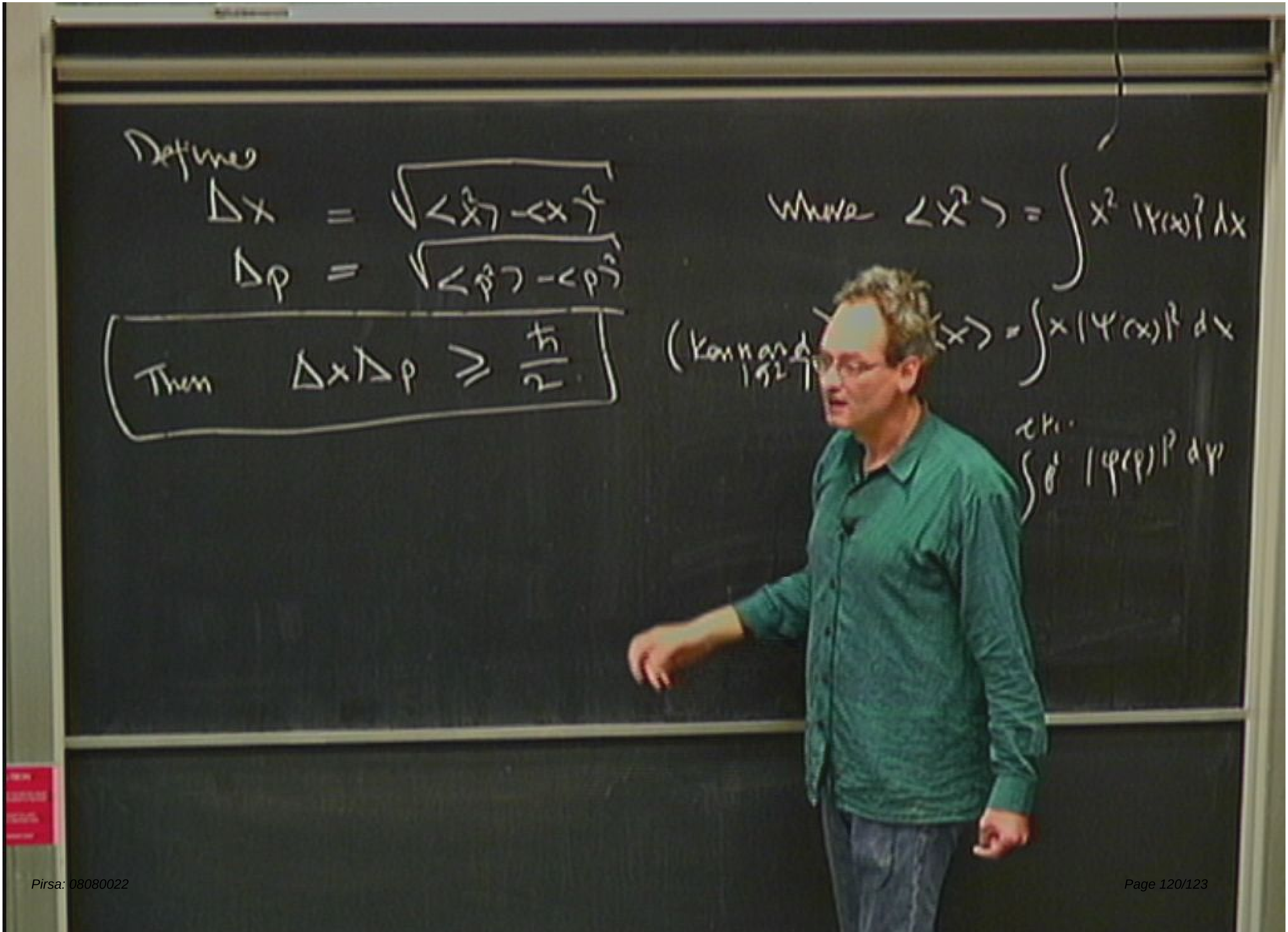
$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}.}$$

(Kennard 1927.)

$$\langle x\rangle = \int x\,|\psi(x)|^2\,dx$$

etc.

$$\langle p^2\rangle = \int p^2\,|\varphi(p)|^2\,dp$$


Define
$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$
$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$
Then $\Delta x \Delta p \geq \frac{\hbar}{2}$
Where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$
(Kennard 1927)
$\langle x \rangle = \int x |\psi(x)|^2 dx$
etc.
$\int p^2 |\varphi(p)|^2 dp$

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}}$$

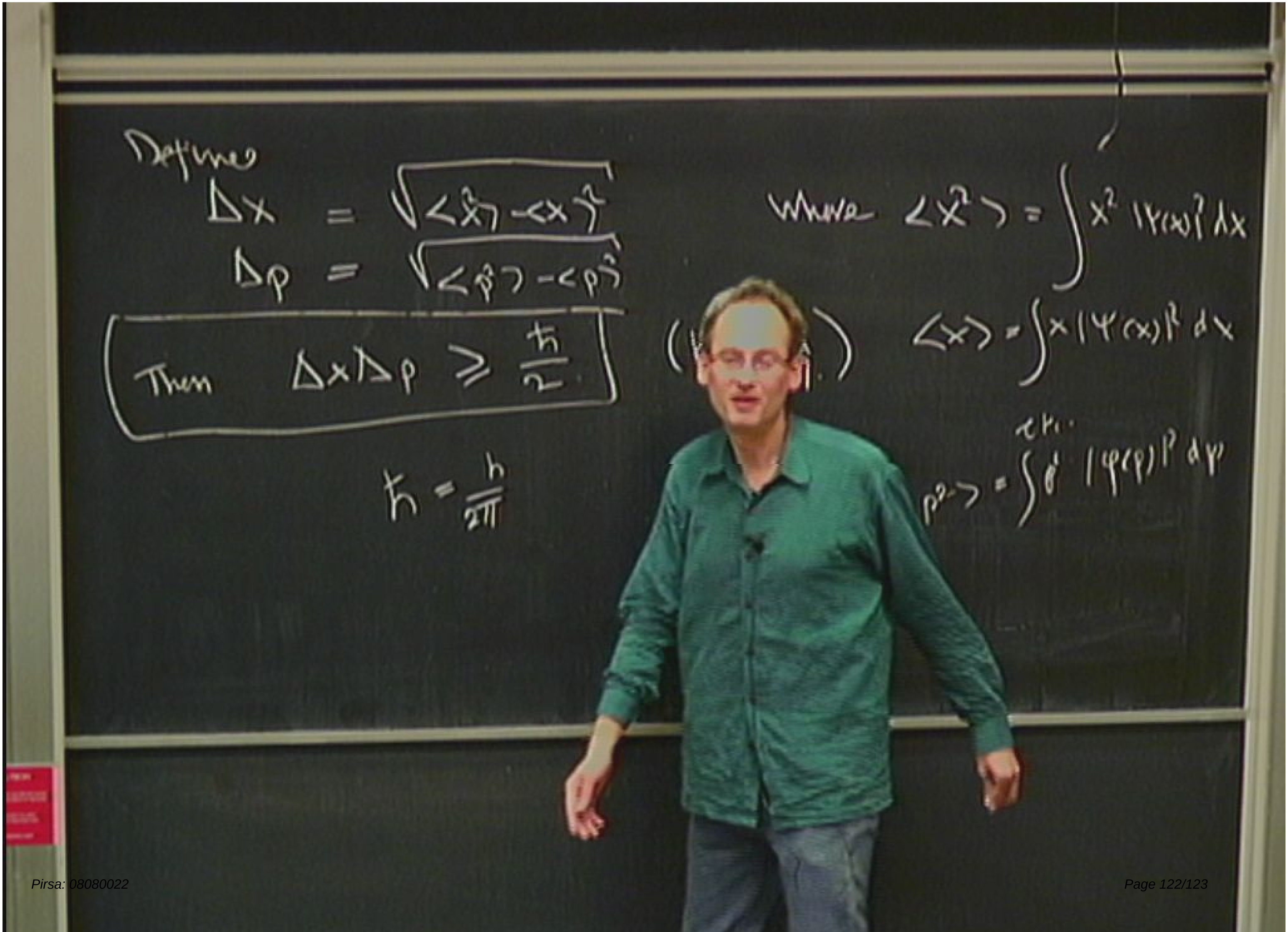
(Kennard 1927)

Where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$

$\langle x \rangle = \int x |\psi(x)|^2 dx$

etc.

$\langle p^2 \rangle = \int p^2 |\varphi(p)|^2 dp$


Define
$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$
$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$
Then $\Delta x \Delta p \geq \frac{\hbar}{2}$
$\hbar = \frac{h}{2\pi}$
Where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$
$\langle x \rangle = \int x |\psi(x)|^2 dx$
$\langle p^2 \rangle = \int p^2 |\varphi(p)|^2 dp$

Define

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle x \rangle^2}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2}$$

$$\boxed{\text{Then} \quad \Delta x \Delta p \geq \frac{\hbar}{2}}$$

(Kennard 1927)

$$\hbar = \frac{h}{2\pi}$$

Where $\langle x^2 \rangle = \int x^2 |\psi(x)|^2 dx$

$$\langle x \rangle = \int x |\psi(x)|^2 dx$$

etc.

$$\langle p^2 \rangle = \int p^2 |\varphi(p)|^2 dp$$