

Title: Relativity 5

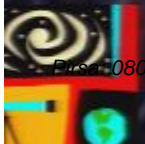
Date: Aug 15, 2008 02:00 PM

URL: <http://pirsa.org/08080020>

Abstract:



Black Holes



$$\phi(\vec{x}) = -G \int \frac{\rho(\vec{x}') d^3x'}{|\vec{x} - \vec{x}'|}$$

$$F_g = \frac{GMm}{r^2}$$

$$\nabla^2 \phi = 4\pi G \rho$$

$$S_r = S_e \left(1 + \frac{gH}{c^2} \right)$$
$$S_{\text{mass}} = \frac{GM}{c^2}$$

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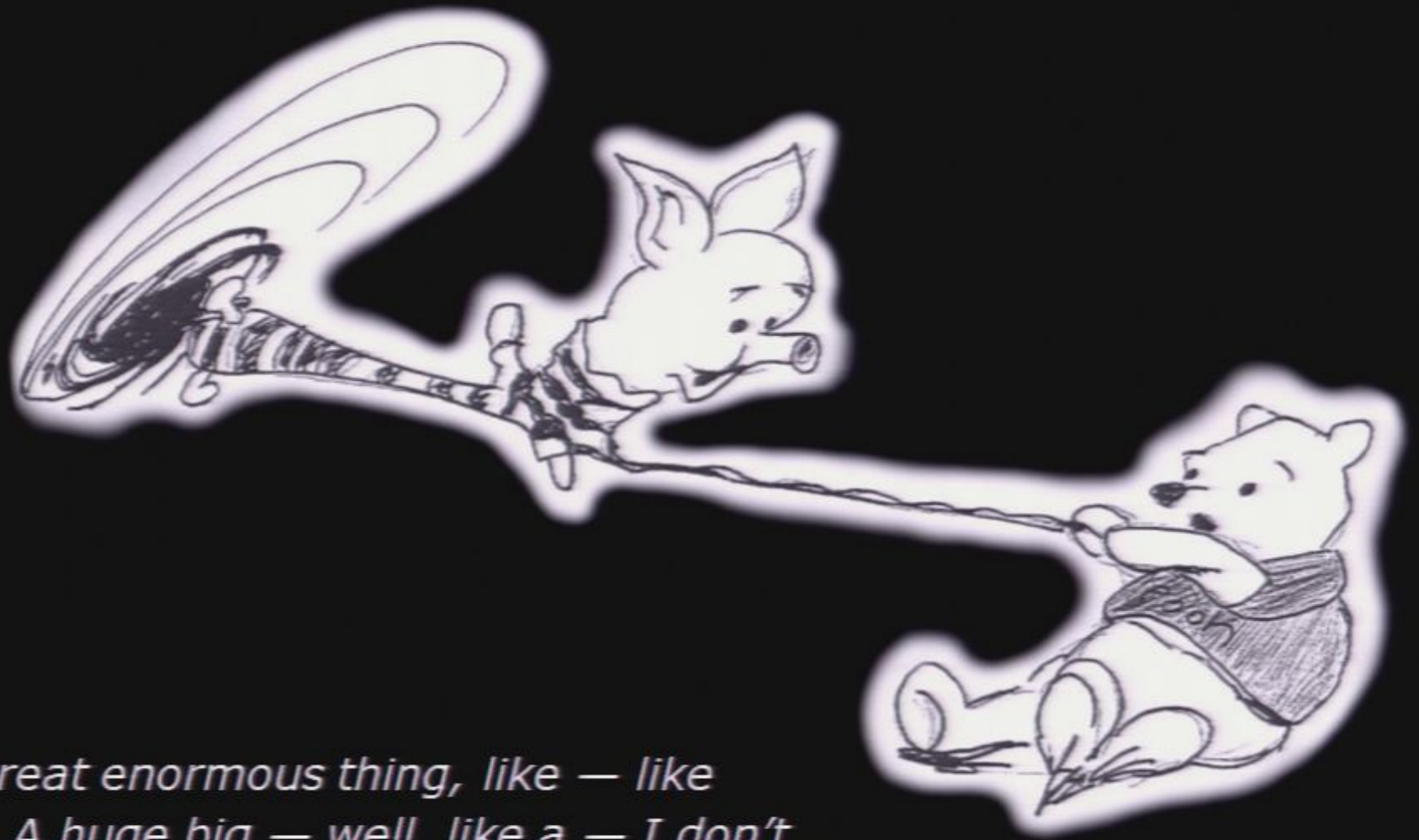
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A huge great enormous thing, like — like nothing. A huge big — well, like a — I don't know — like an enormous big nothing ...

**Piglet describes the Heffalump,
in *Winnie the Pooh* by A.A. Milne**



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Dark stars

- **Rev. John Michell (1783)**

A British born "natural philosopher" dared to combine the corpuscular description of light with Newton's gravitation laws to predict what large compact stars should look like.

- He showed that a star, that has the same density of the sun, but 500 time as big, would have such a gravity, that "All light emitted from such a body would be made to return towards it". He said we wouldn't be able to see such a body, but we sure will feel it's gravitational pull.
- We could fly close to this "Dark star" and look around and describe the features of the object.
- A novelty, world lost interest when light was shown to be waves in 1803 by Thomas Young.

Calculation of Escape Velocity for Earth

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

Calculation of Escape Velocity for Earth

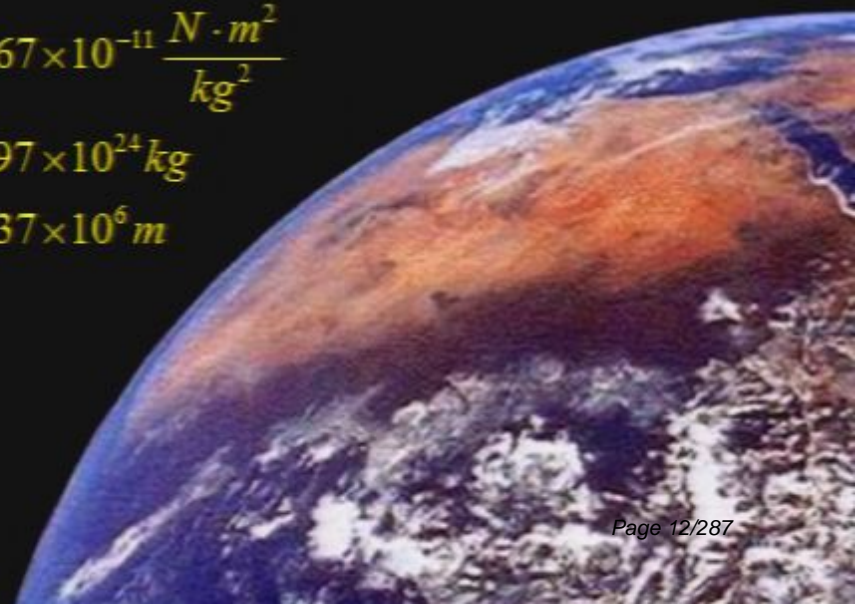
$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

$$M = 5.97 \times 10^{24} kg$$

$$r = 6.37 \times 10^6 m$$

Calculate Escape Velocity



Calculation of Escape Velocity for Earth

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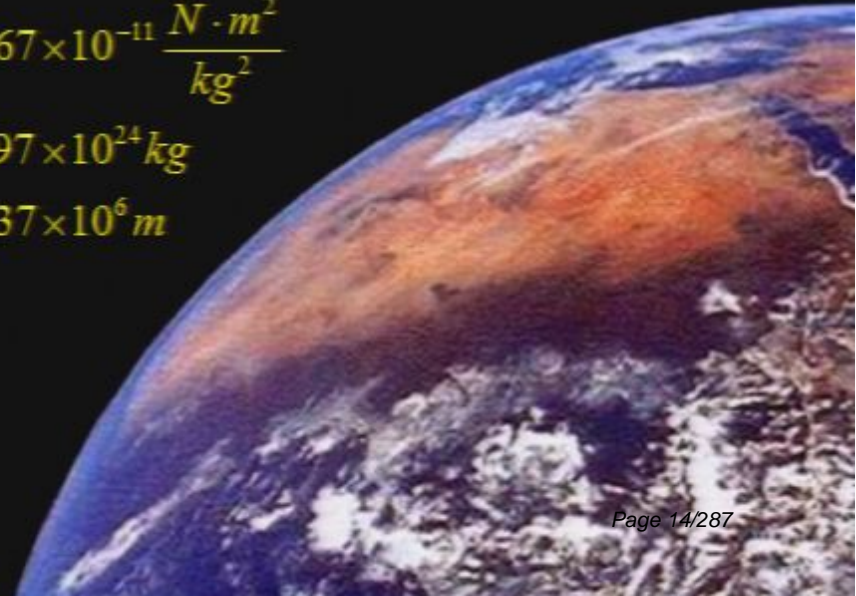
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11.2 km/s

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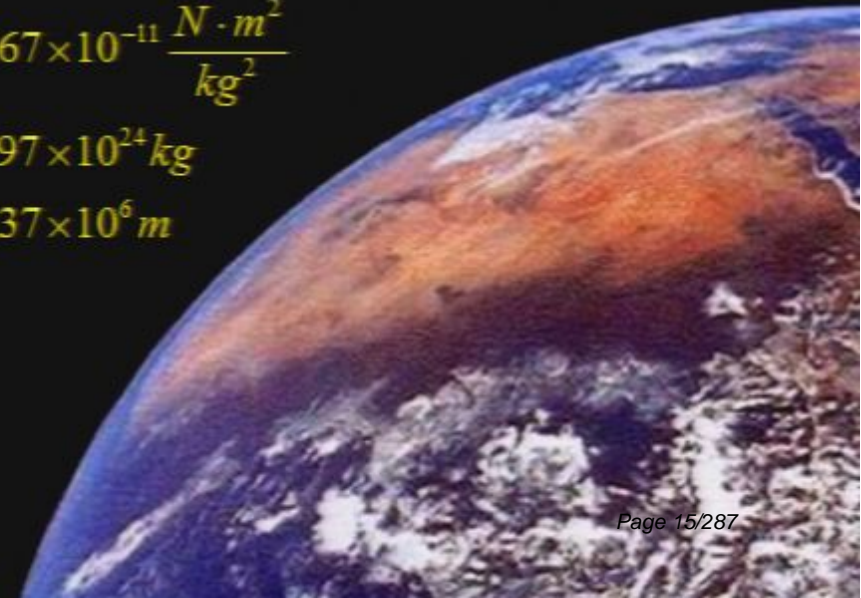
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~ 8.9 mm

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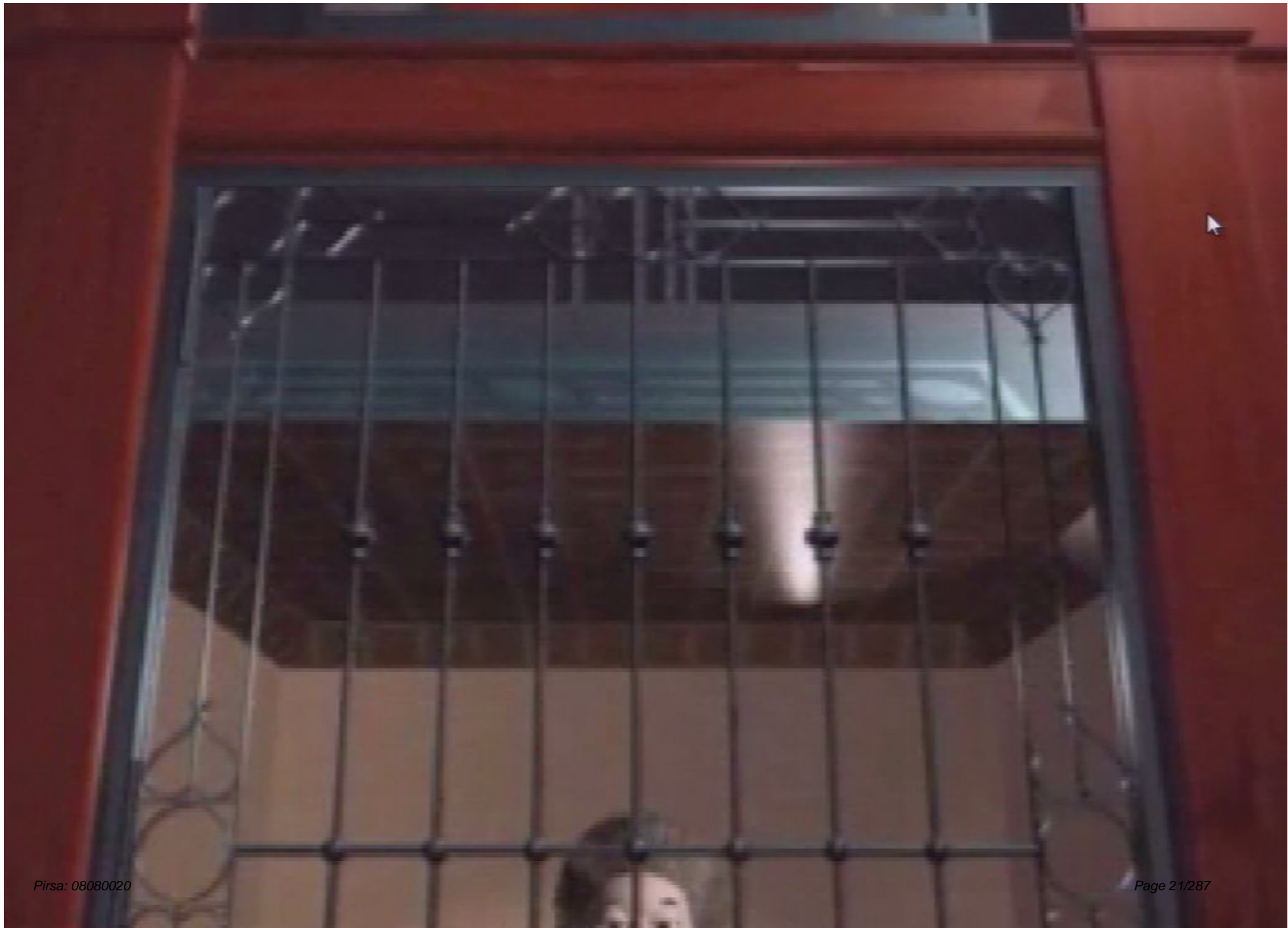


Einstein's Equivalence Principle

- There is no experiment that you can perform that will distinguish these two diagrams





















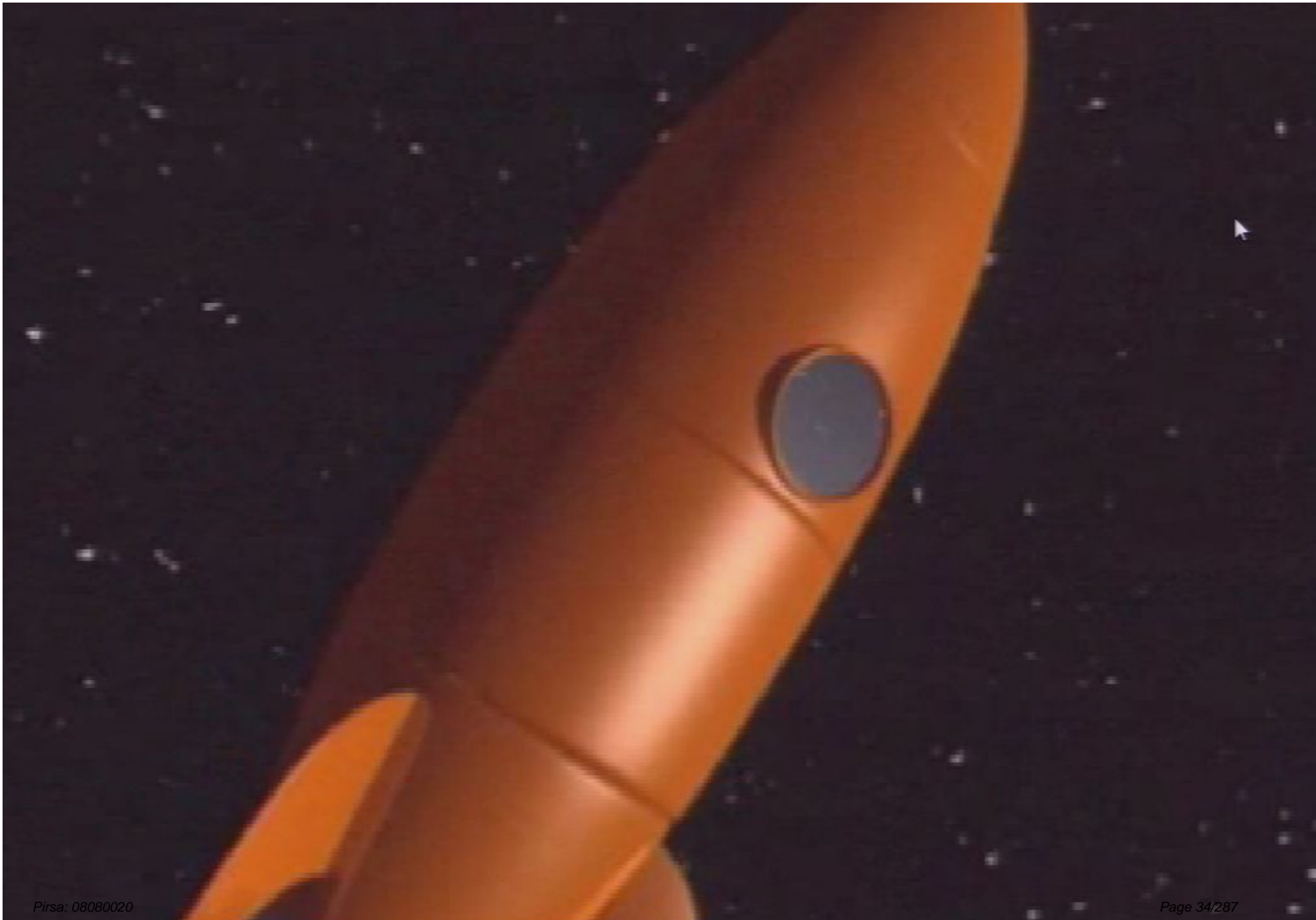


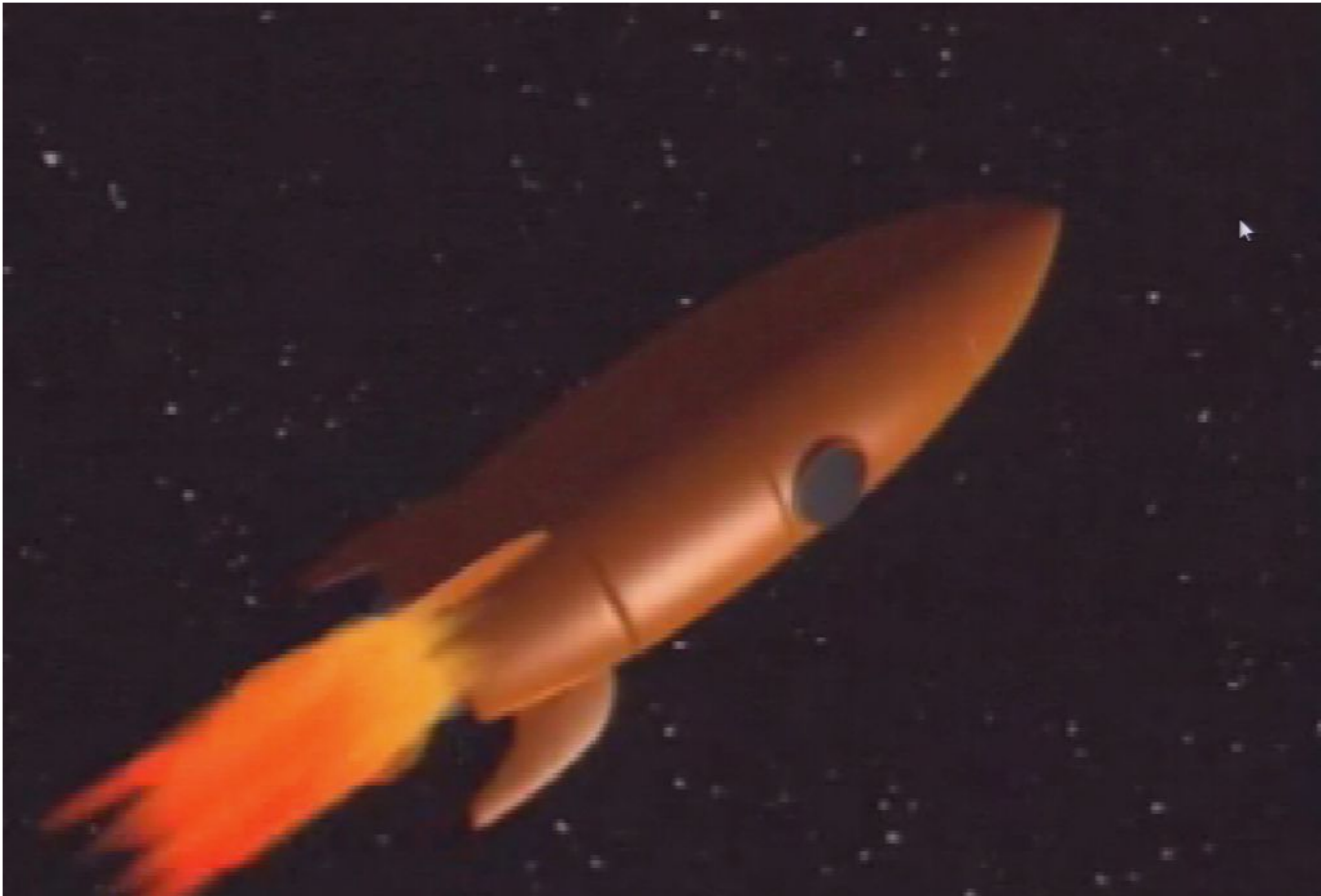




















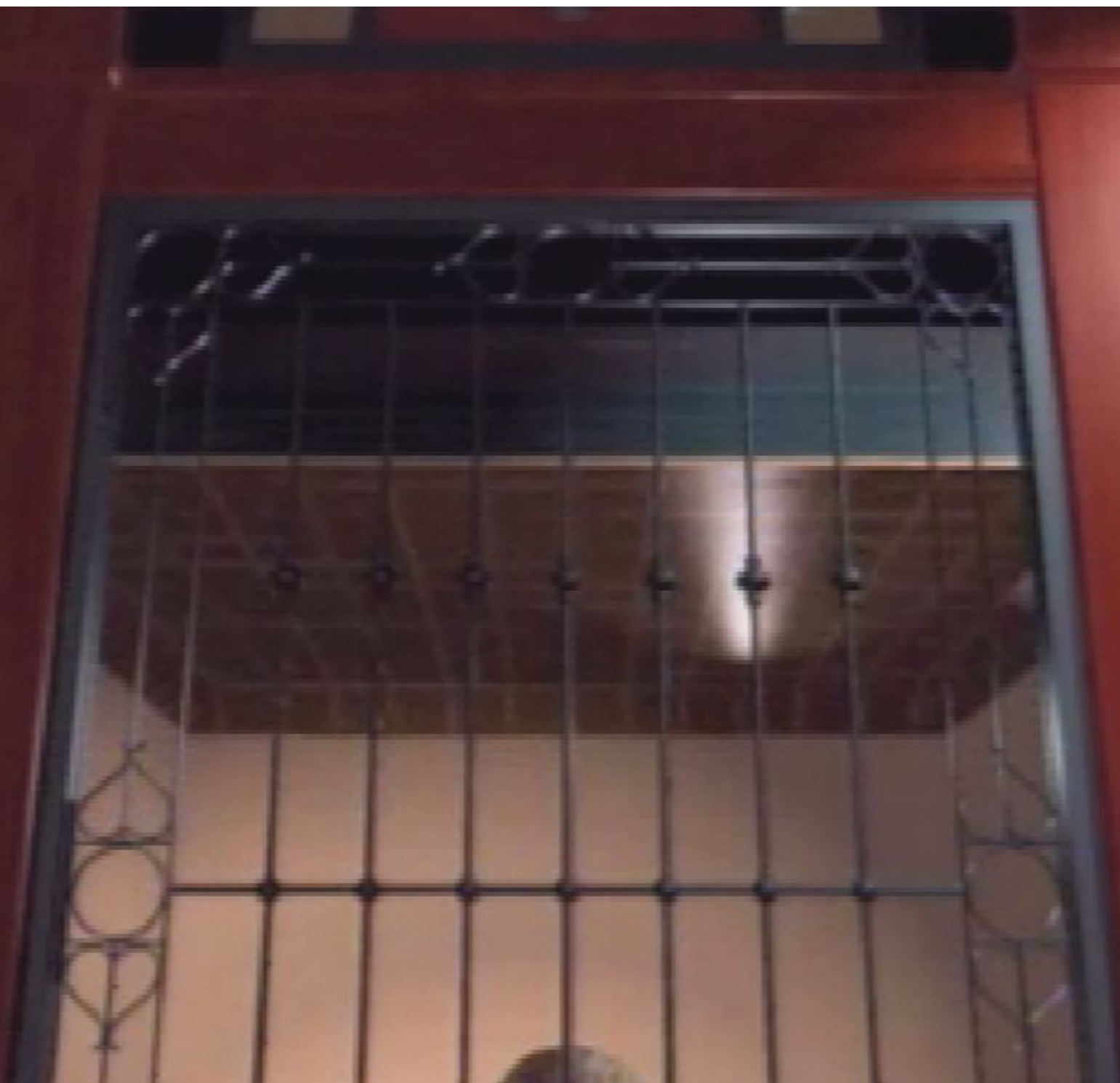














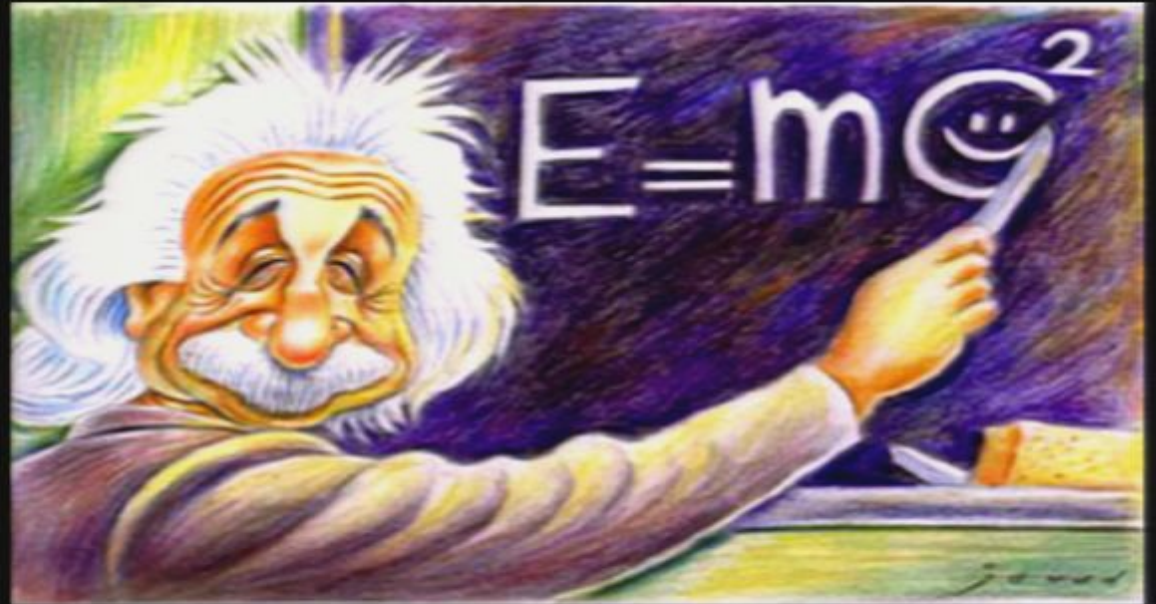




Gravity as a Curvature of Spacetime

The early 1900's changed the way gravity is looked at. Einstein didn't think of gravity as a force between objects, but as a curving of "straight lines" due to mass. Light always follows these straight lines.

Time also slows down near masses (space and time are different parts of "spacetime", which is what gets bent).

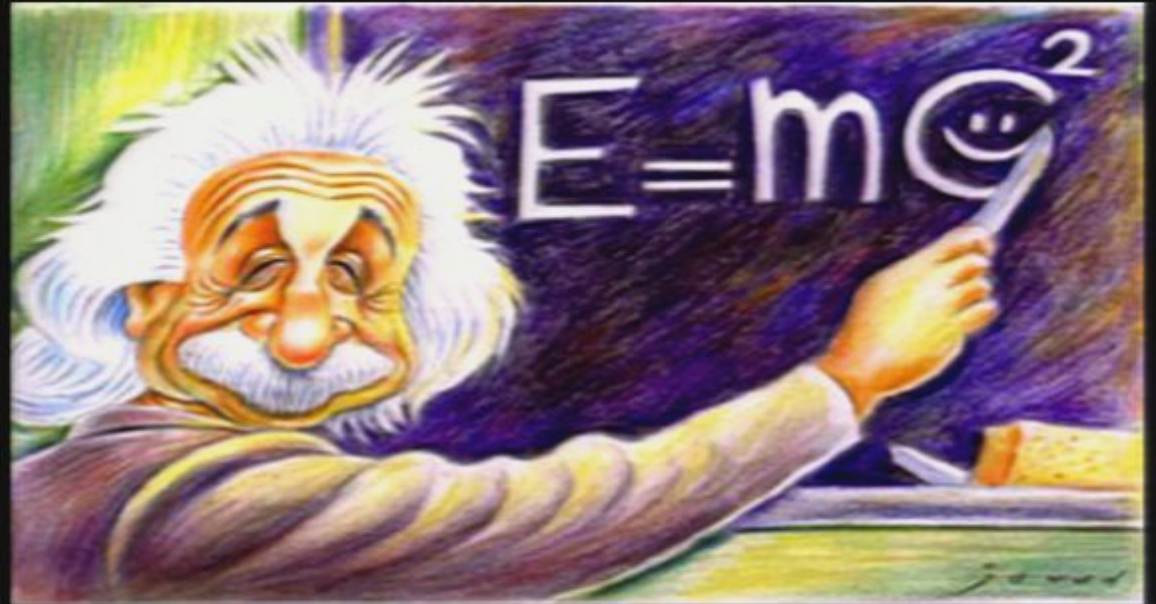


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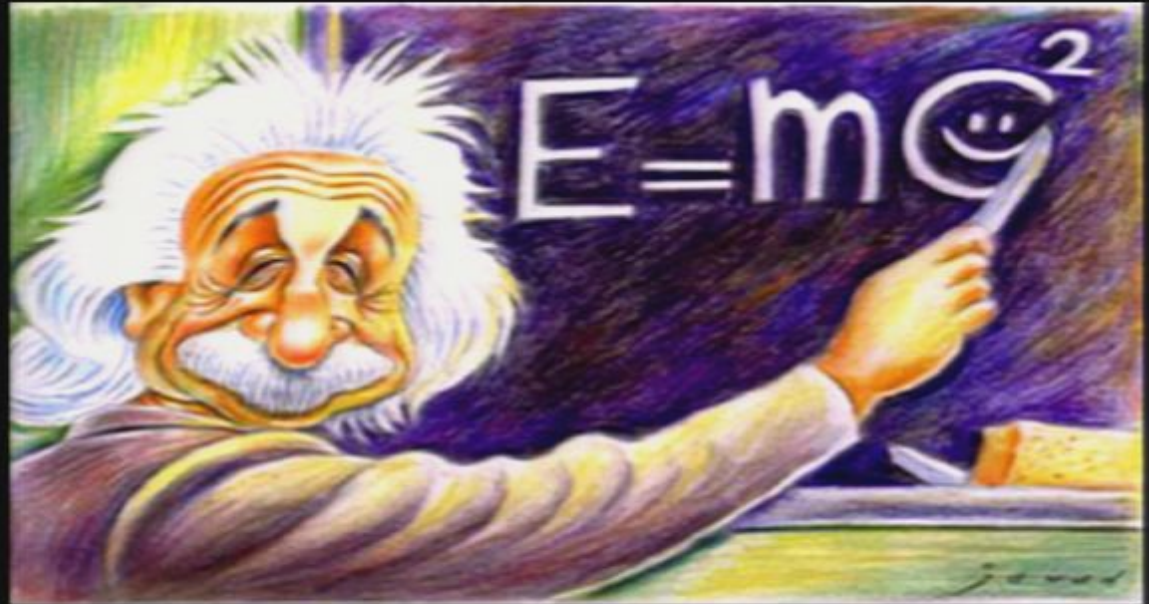


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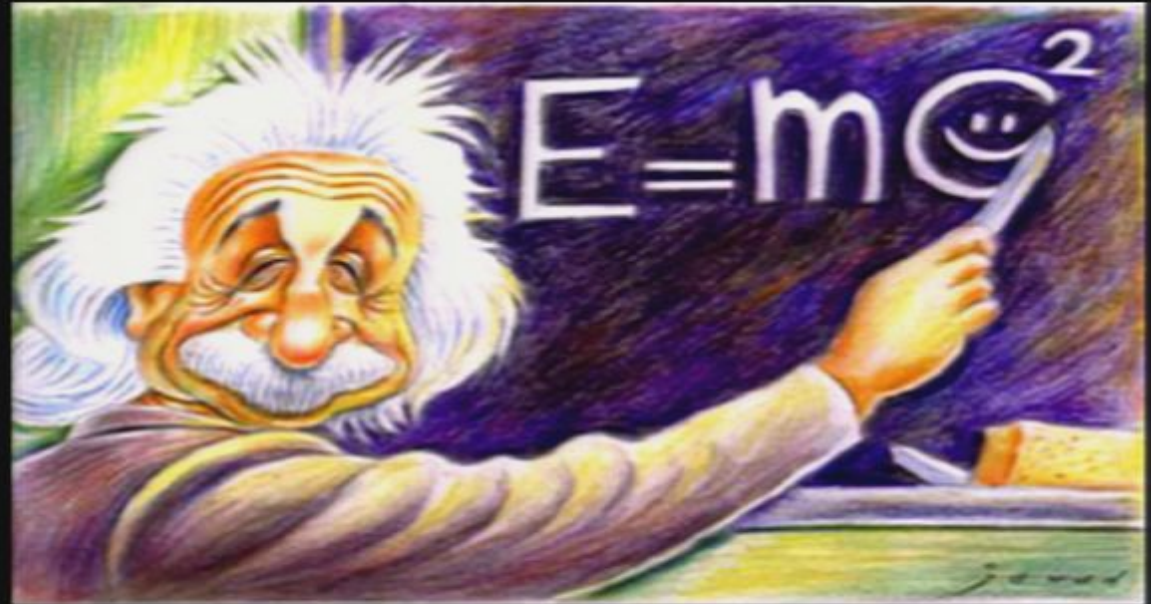


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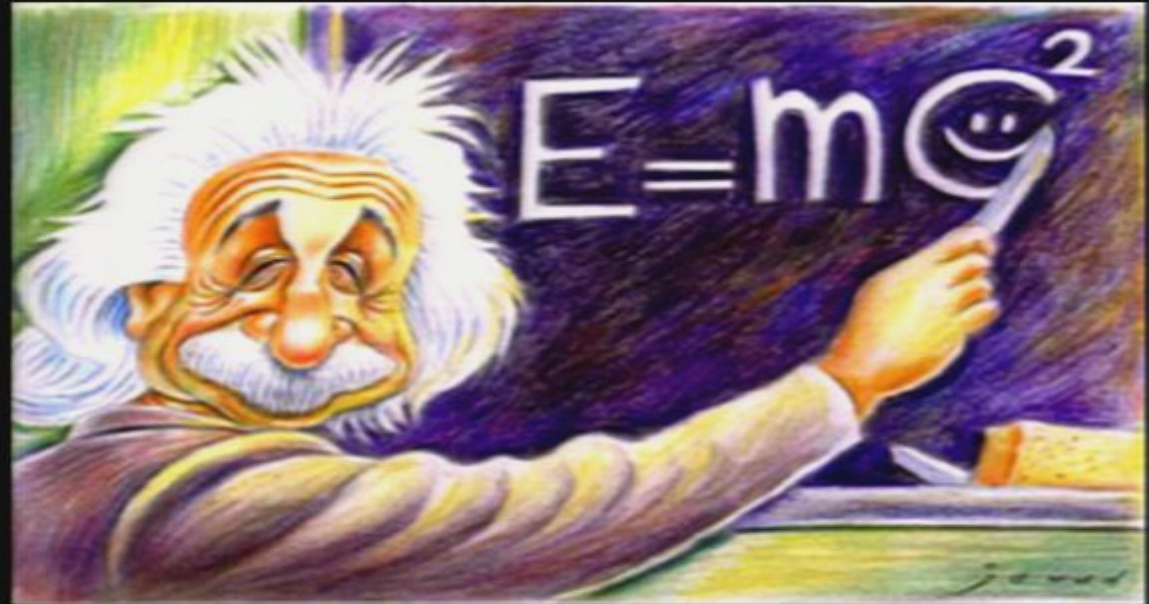


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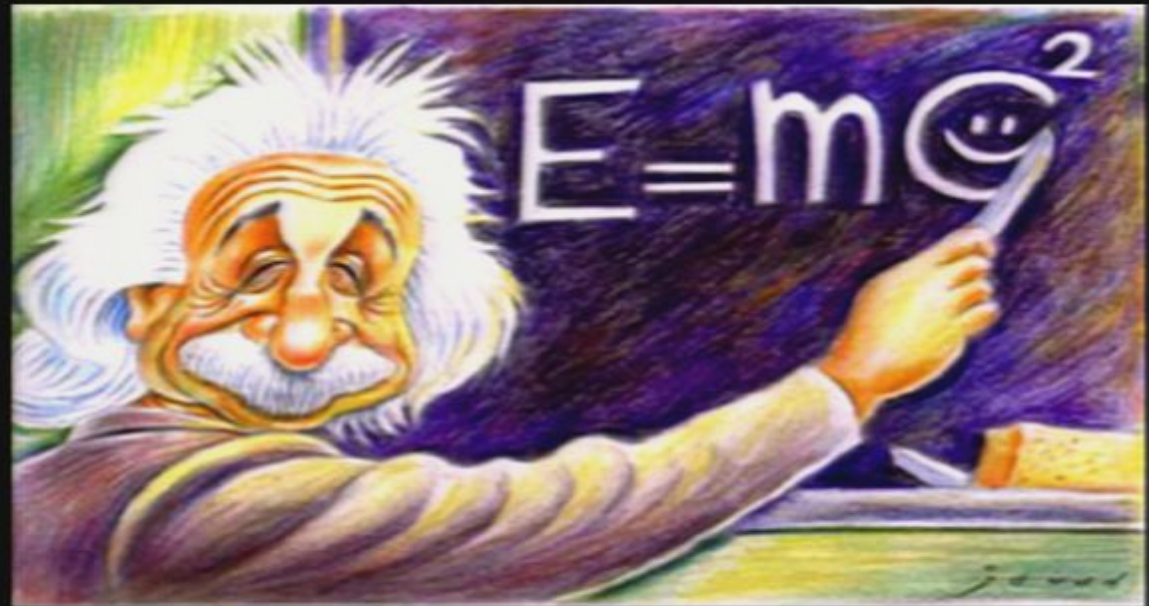
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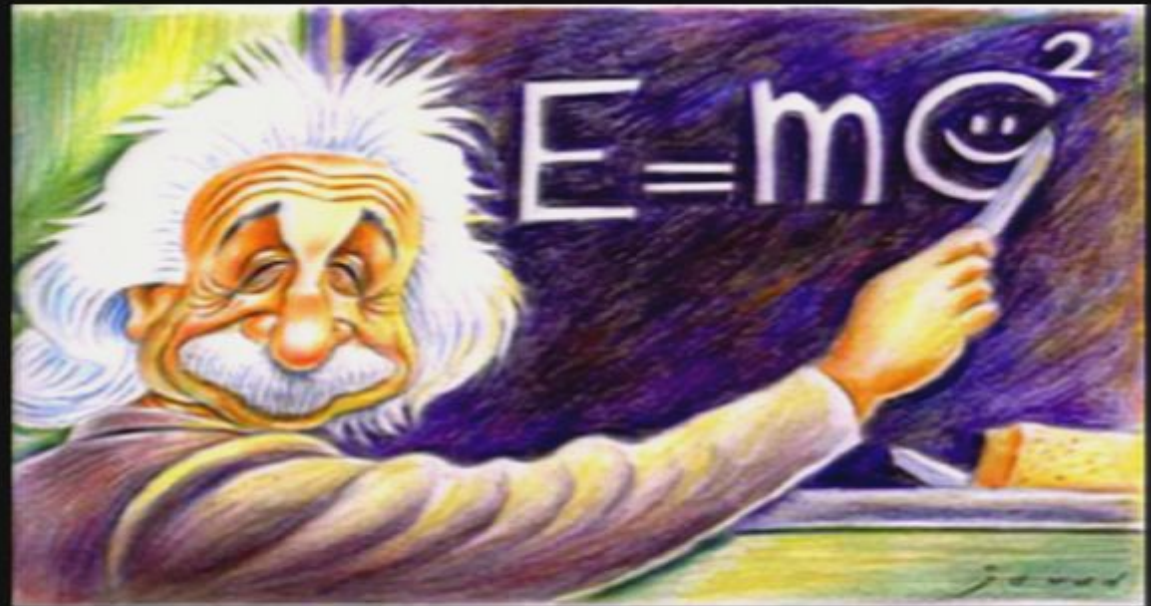
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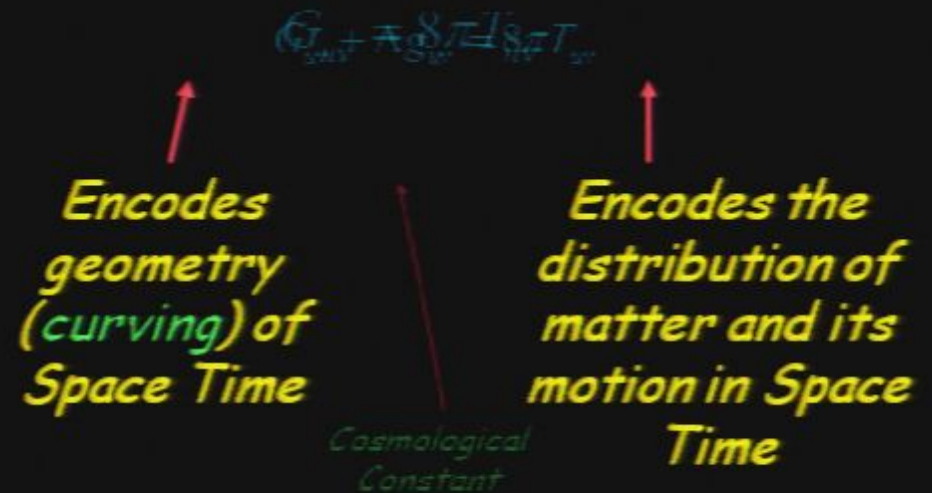
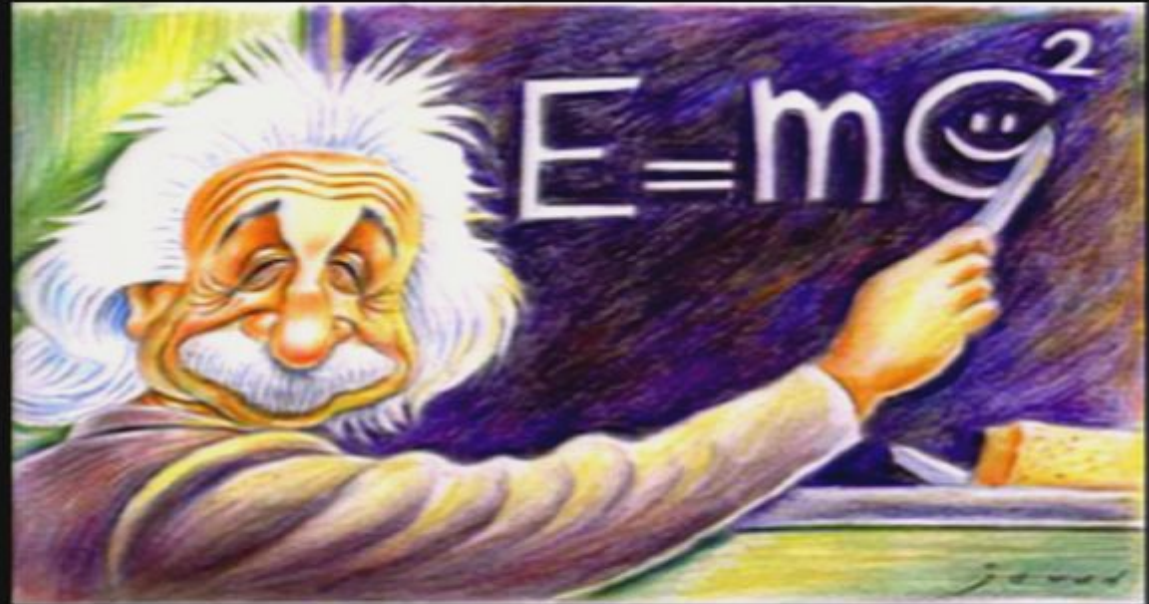
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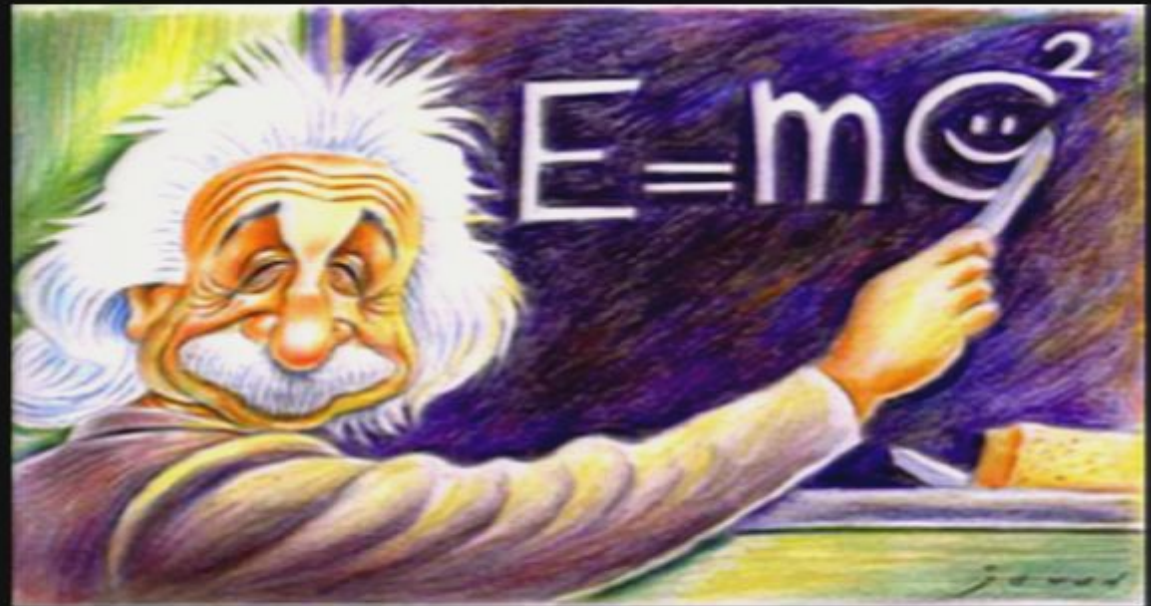
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Einstein Field Equation: another dissection

• *Generally speaking, Einstein field equation: $G_{\mu\nu} = 8\pi T_{\mu\nu}$*

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Let's Review

EMIT & CAPS

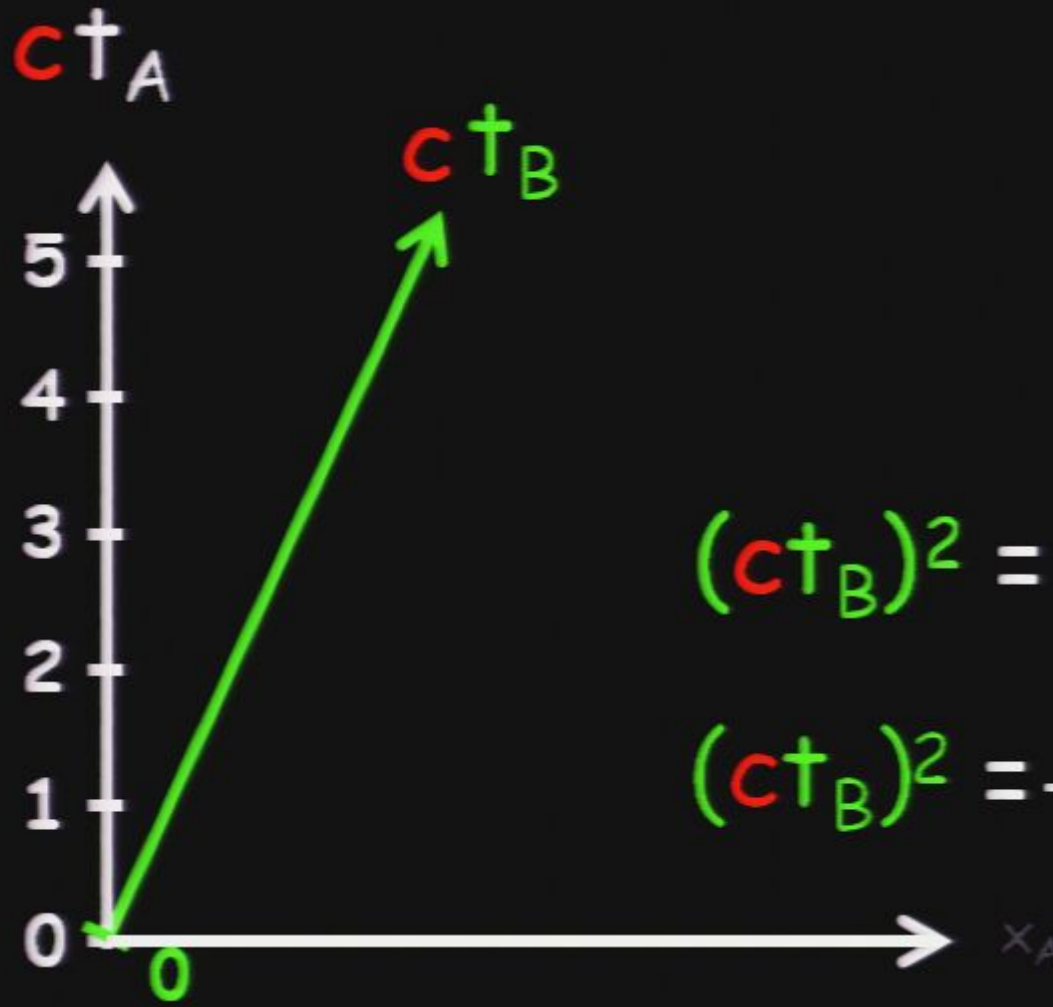
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SPACETIME

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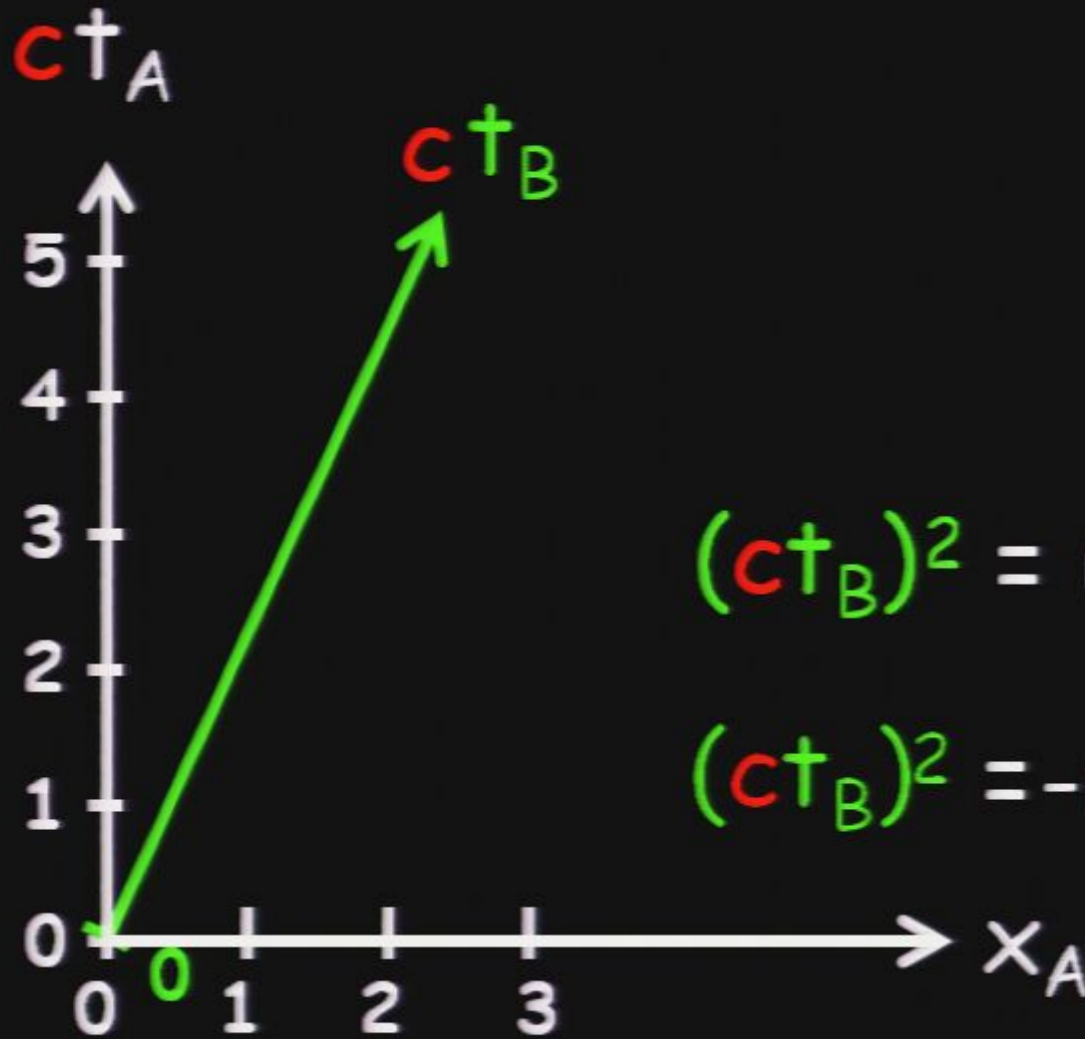


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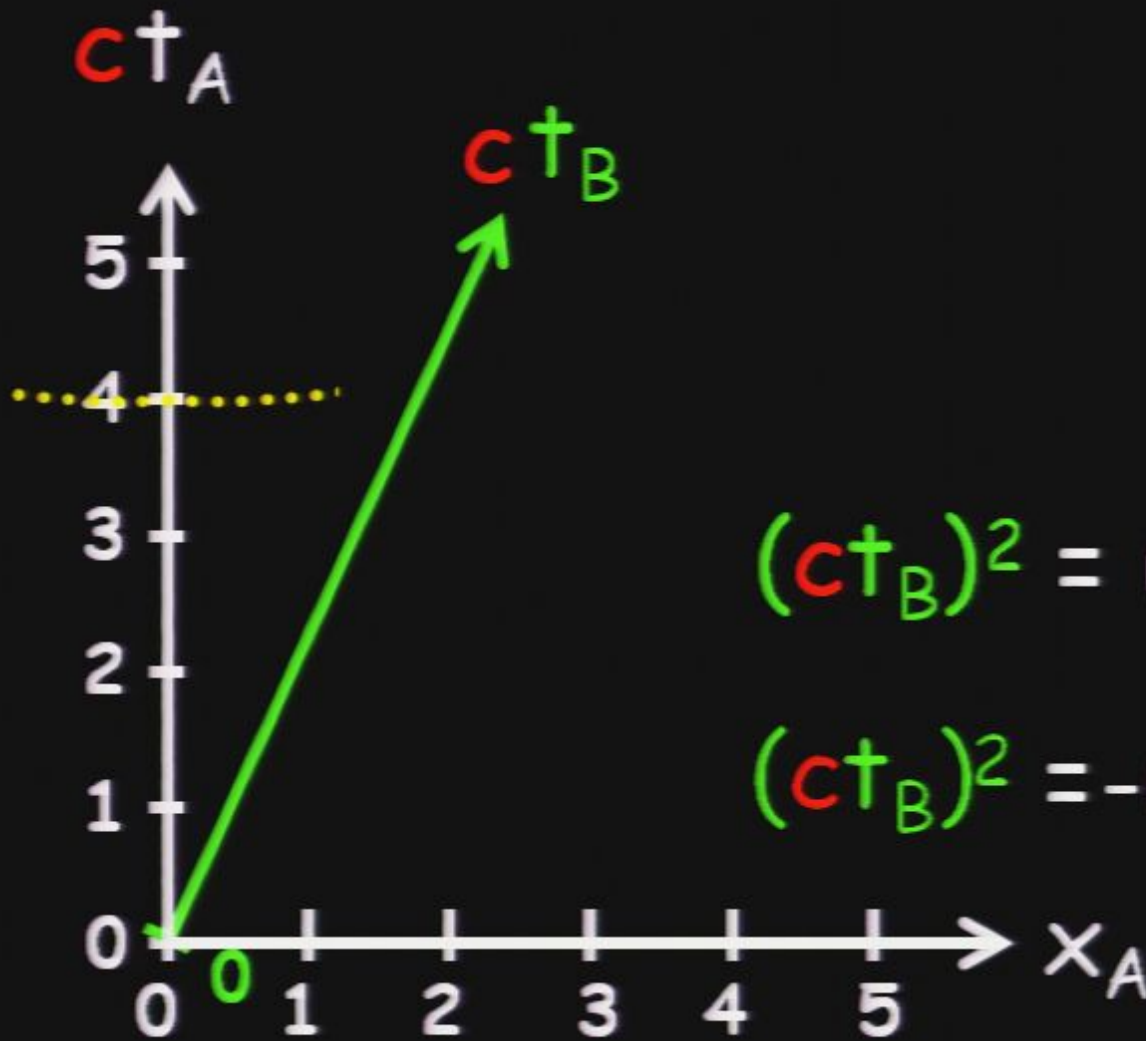


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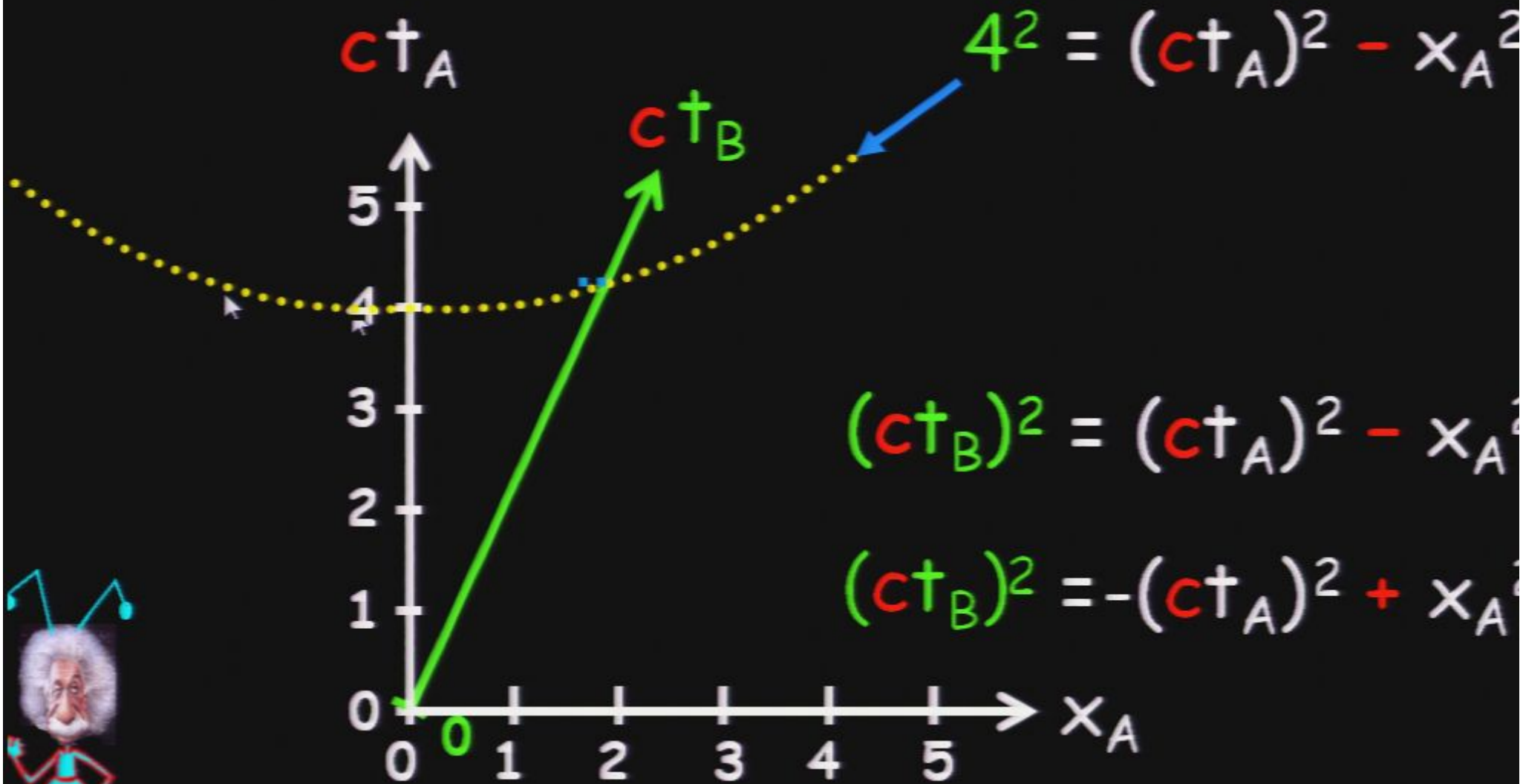


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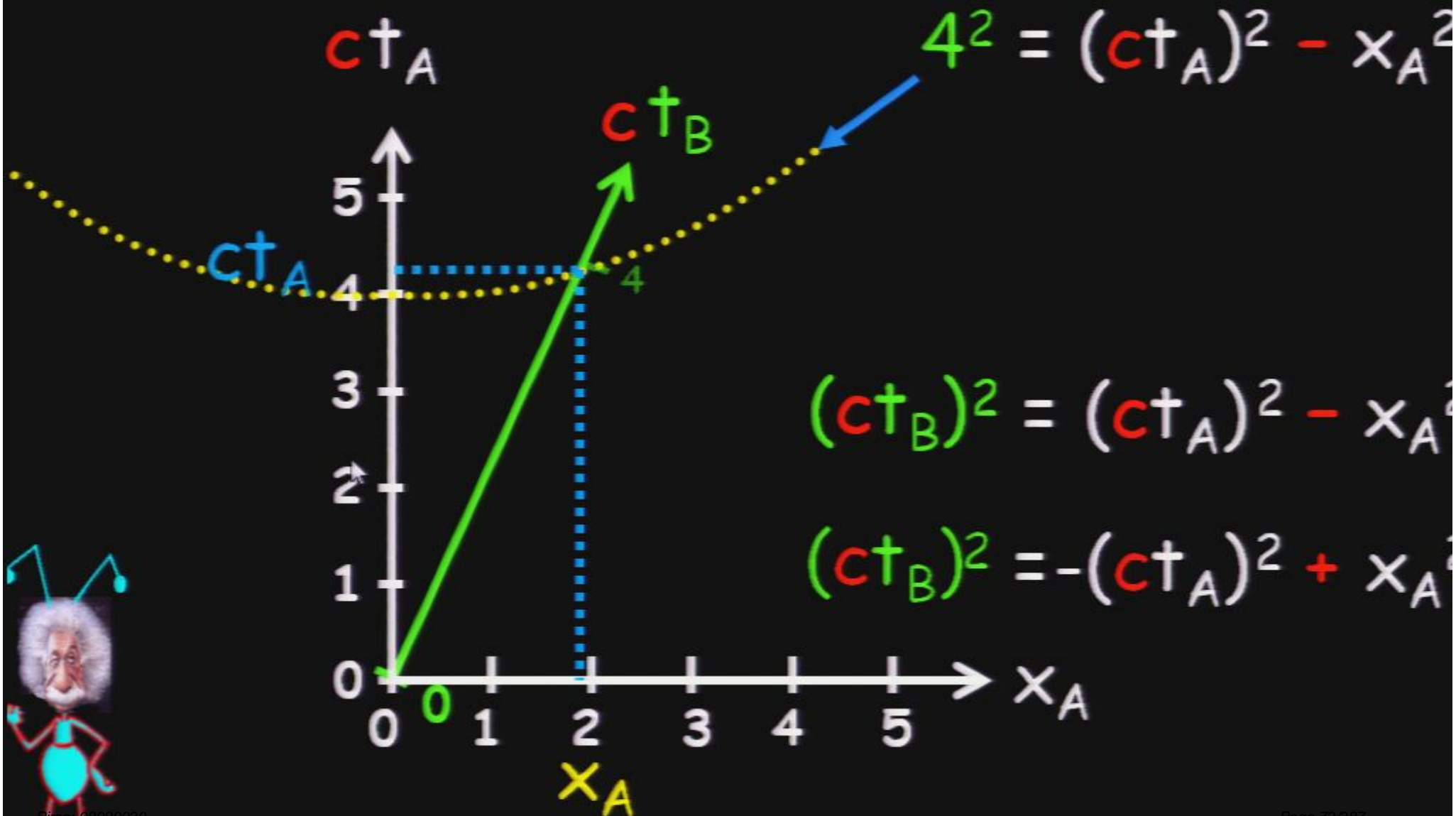
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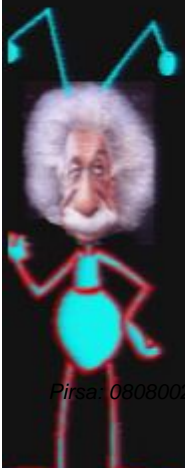
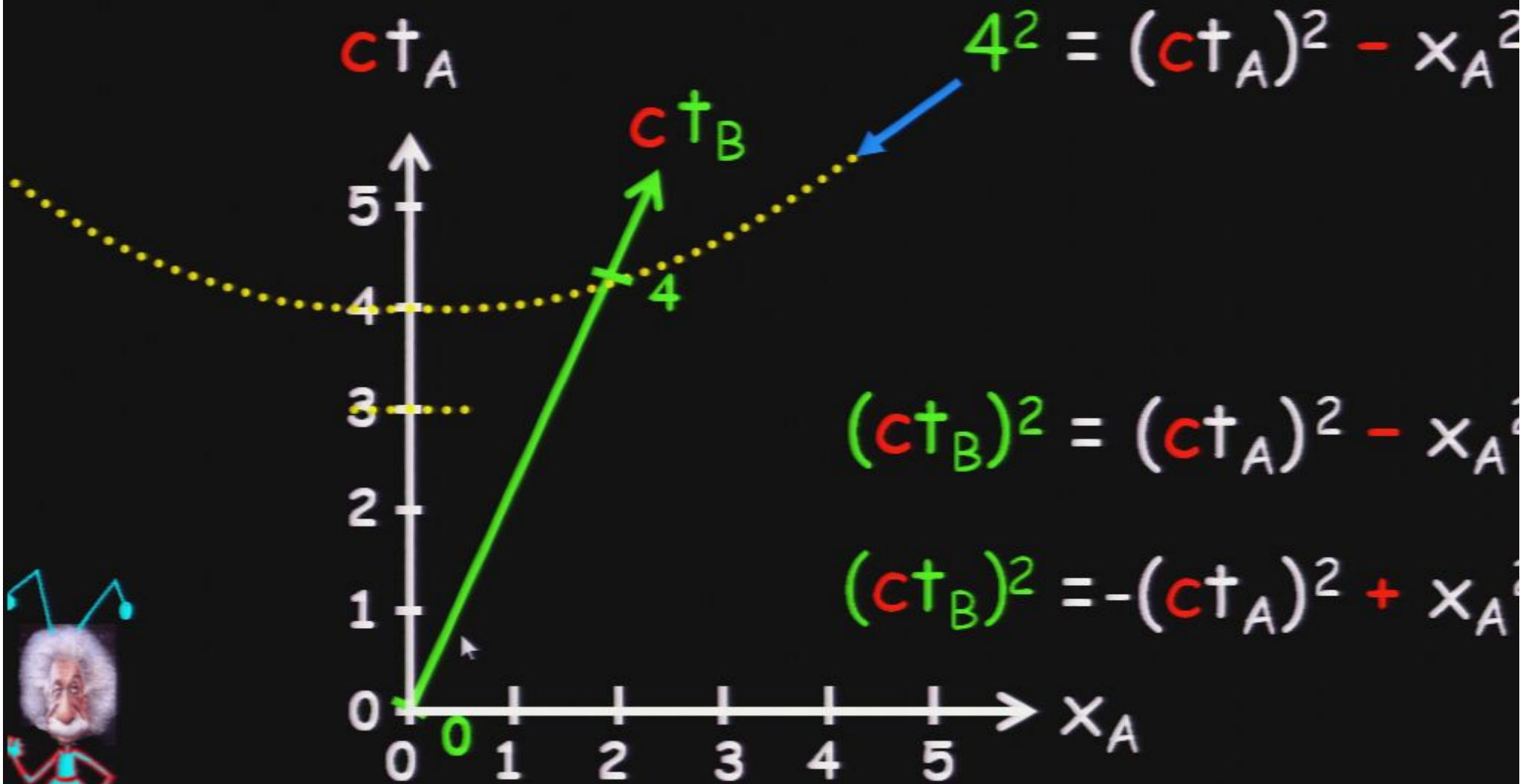
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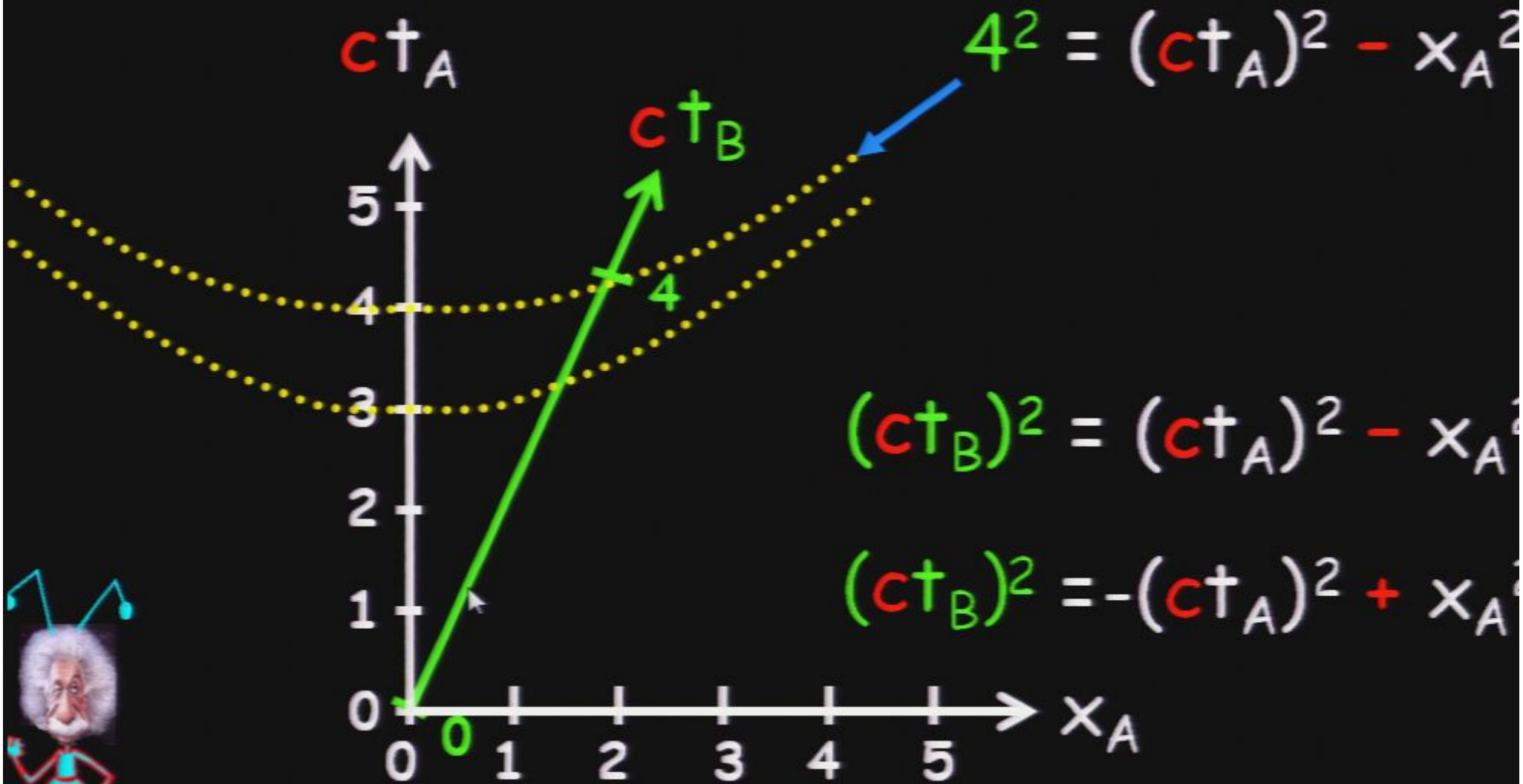
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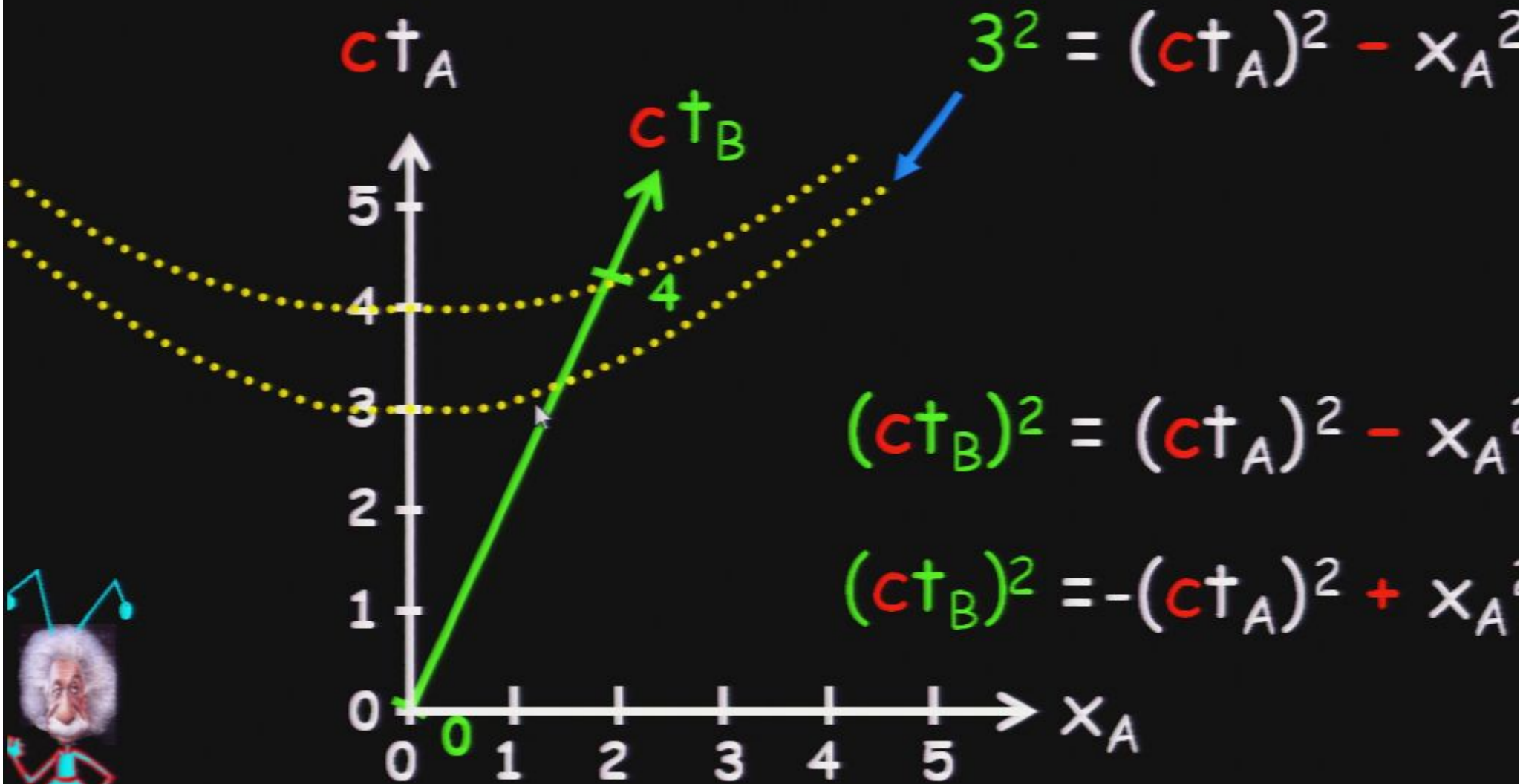
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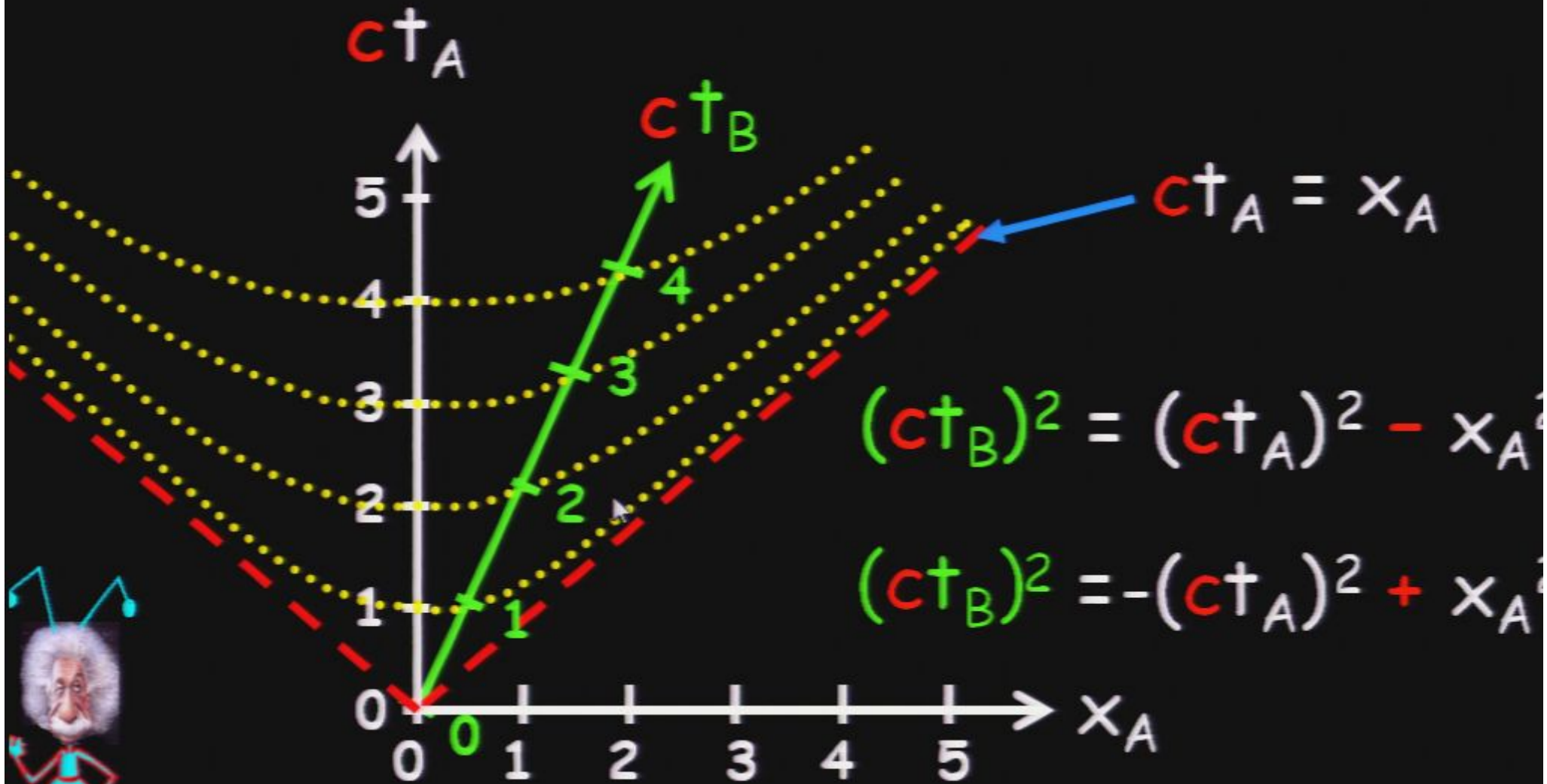
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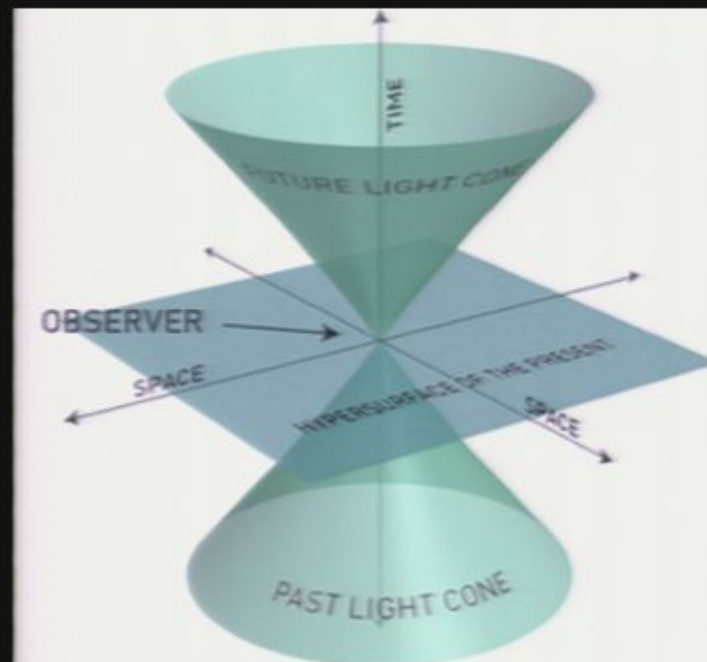
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Spacelike, Null, Timelike

Special relativity implies that all matter must move at less than or equal to the speed of light.



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- Example two dimensional Euclidean Space

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- Example of two dimensional Minkowski Space

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Usual representation

$$(\Delta s)^2 = -(\Delta t)^2 + t^2 (\Delta \phi)^2$$

Milne representation

The Geodesics

Einstein needed to modify Newton's First Law and he did it in
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Curves of shortest distance are known in relativistic jargon as geodesics.

You are familiar with the Geodesic



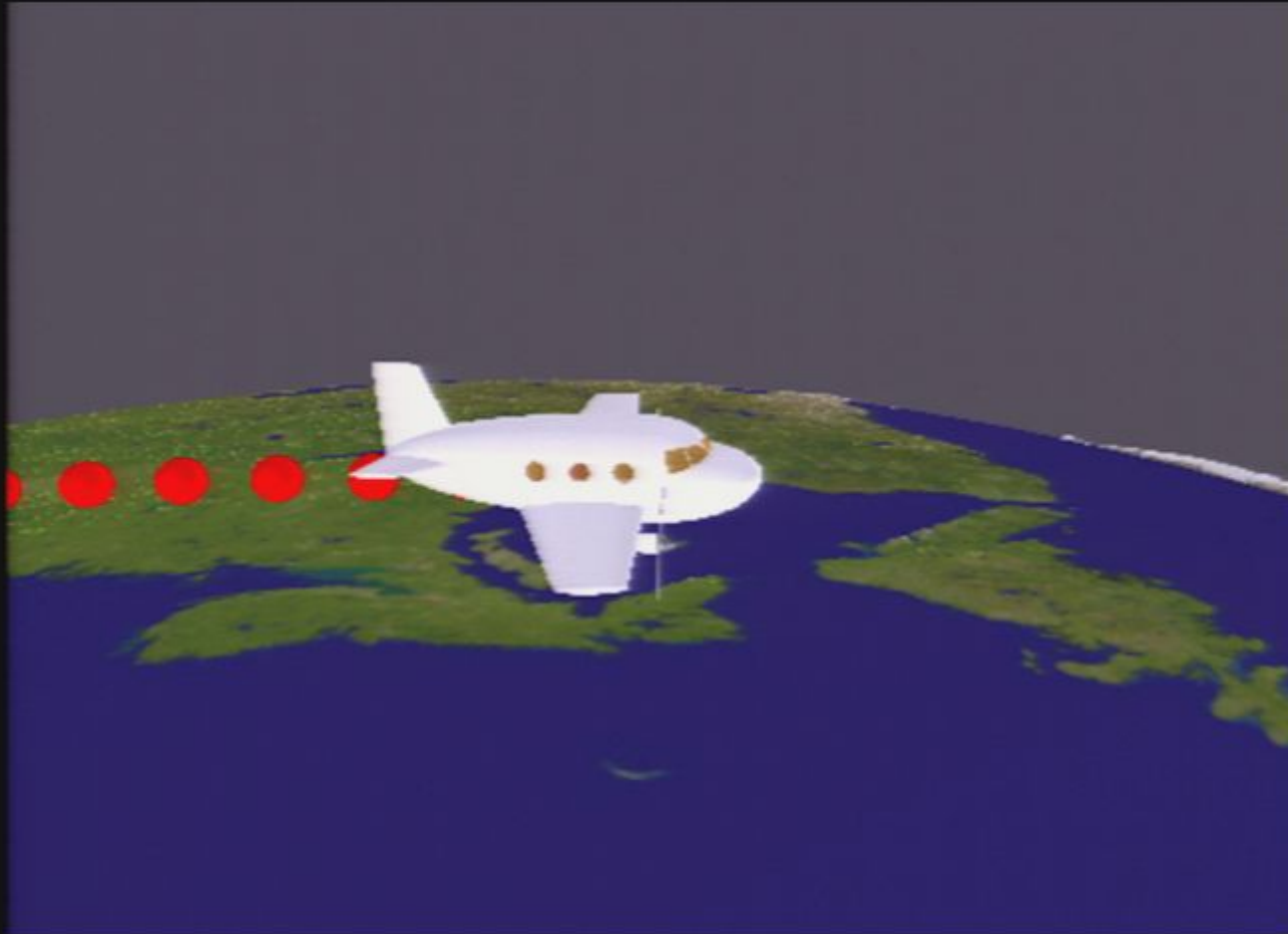
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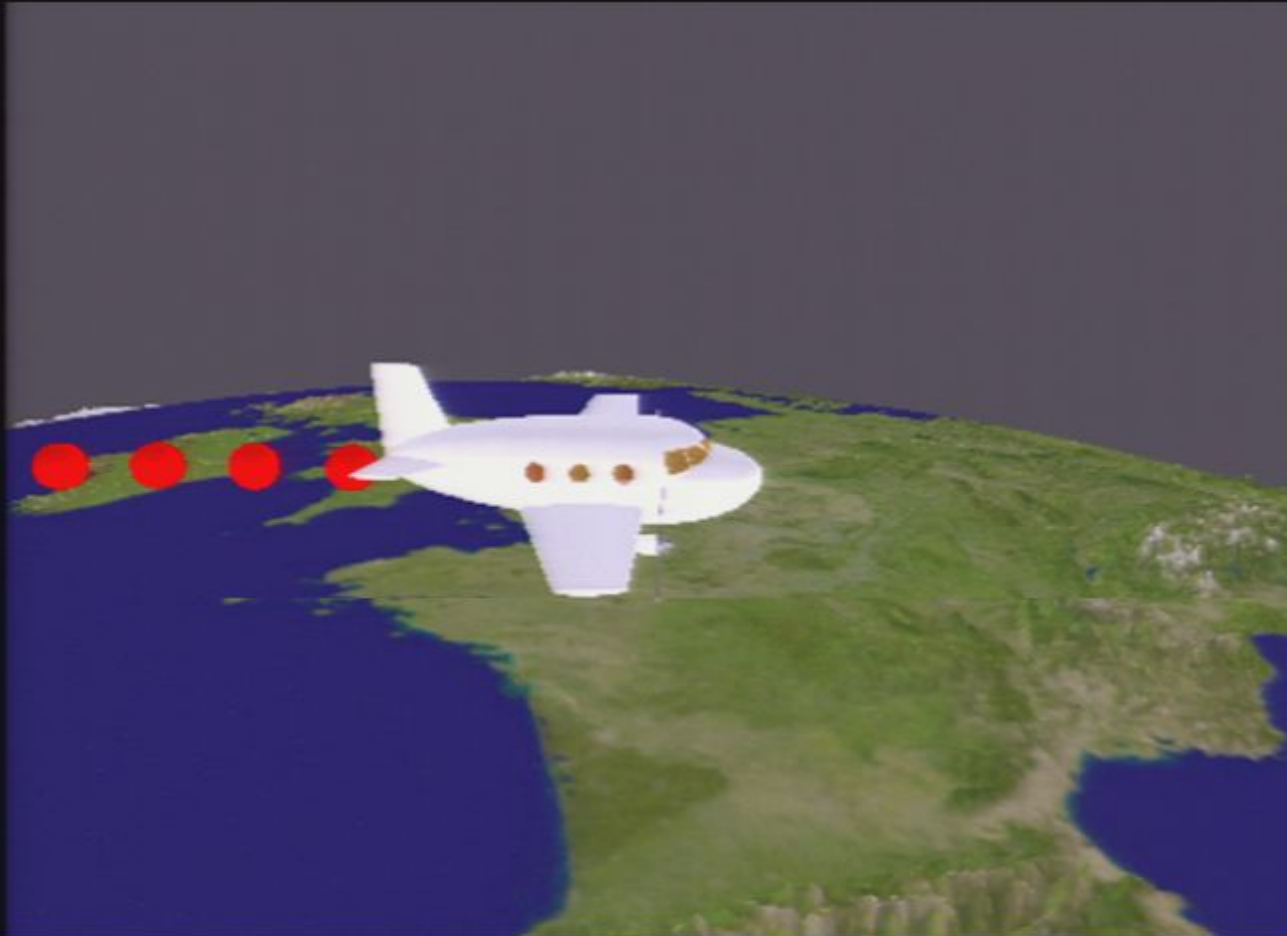
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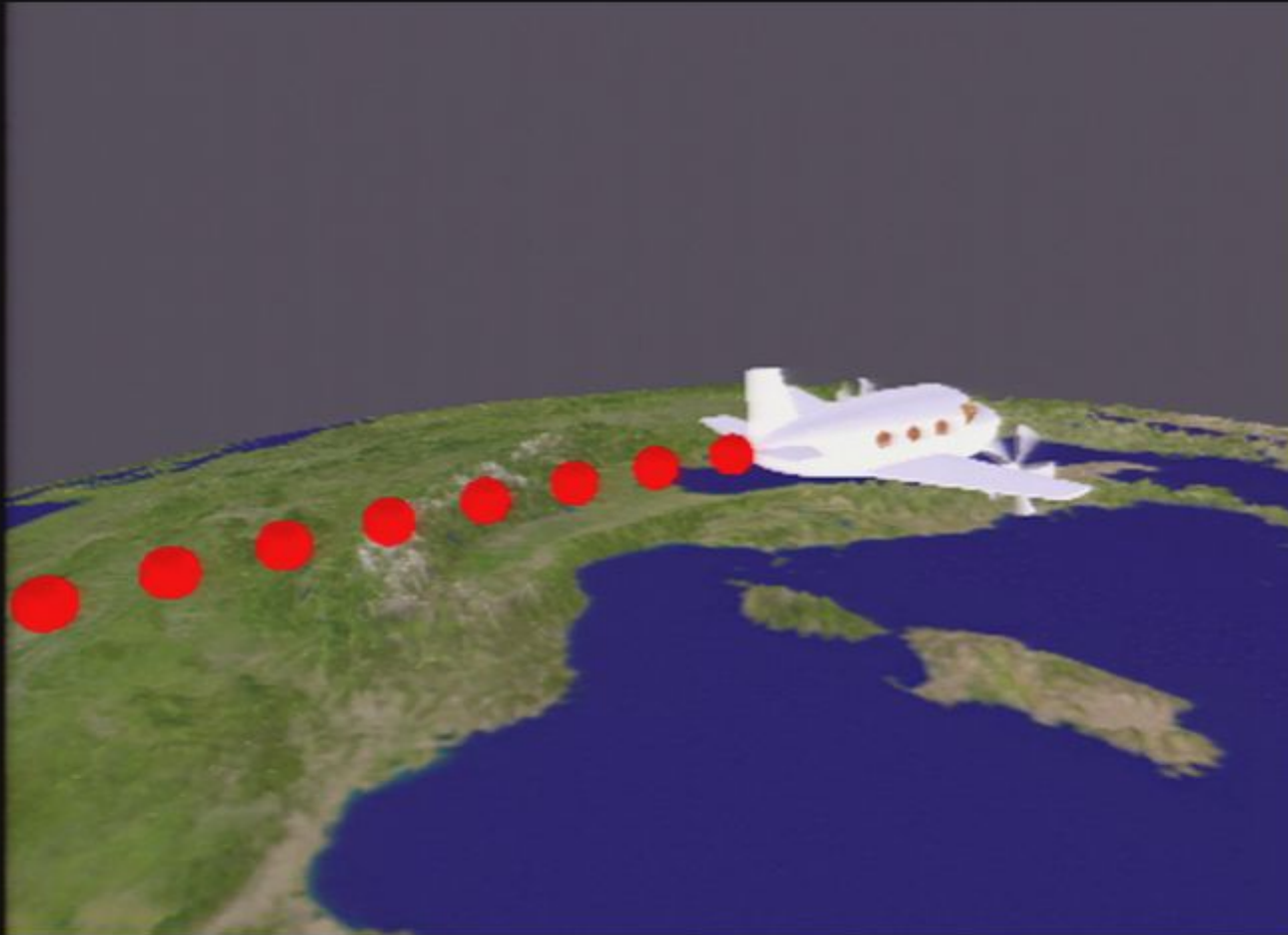
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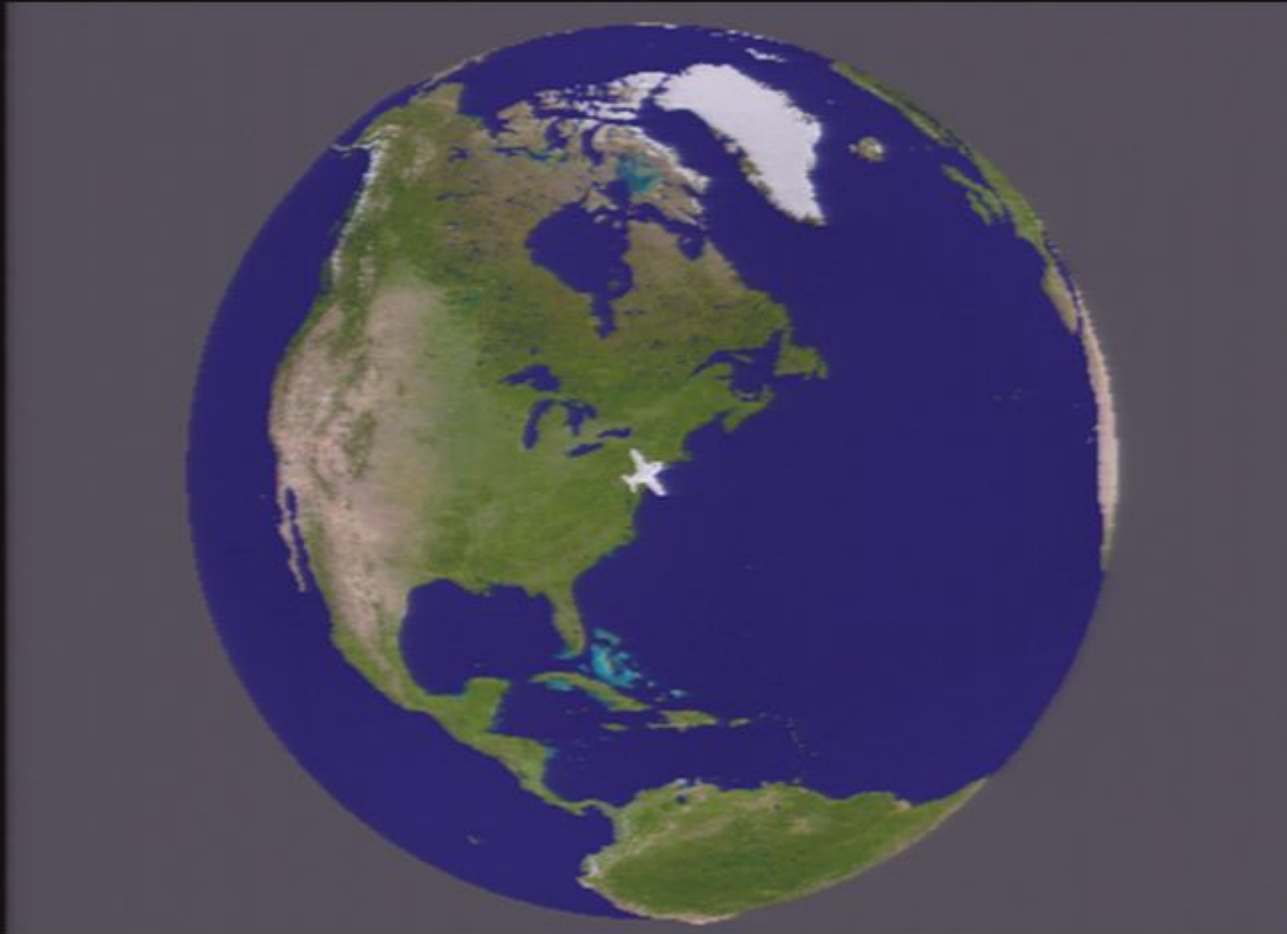
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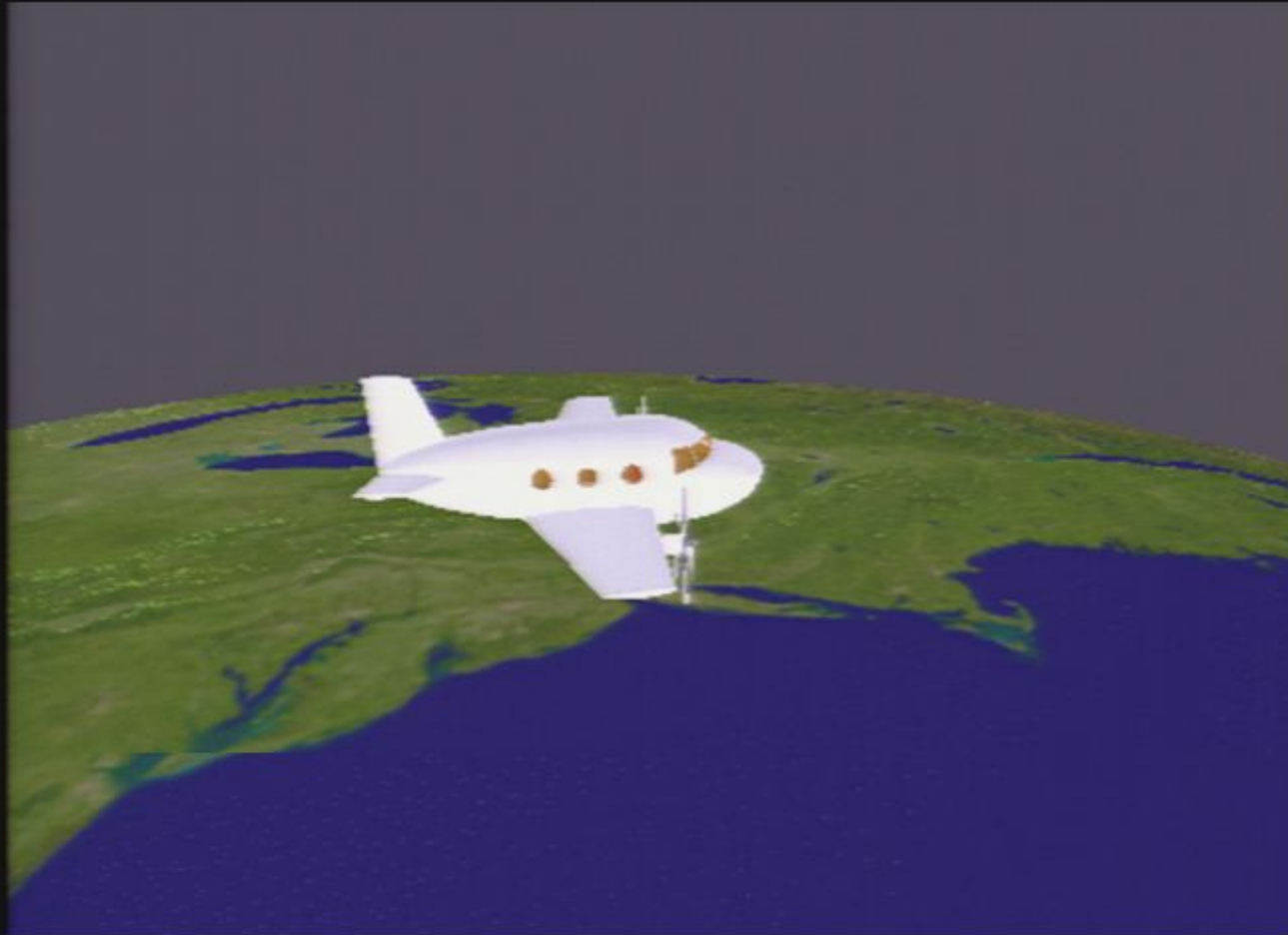
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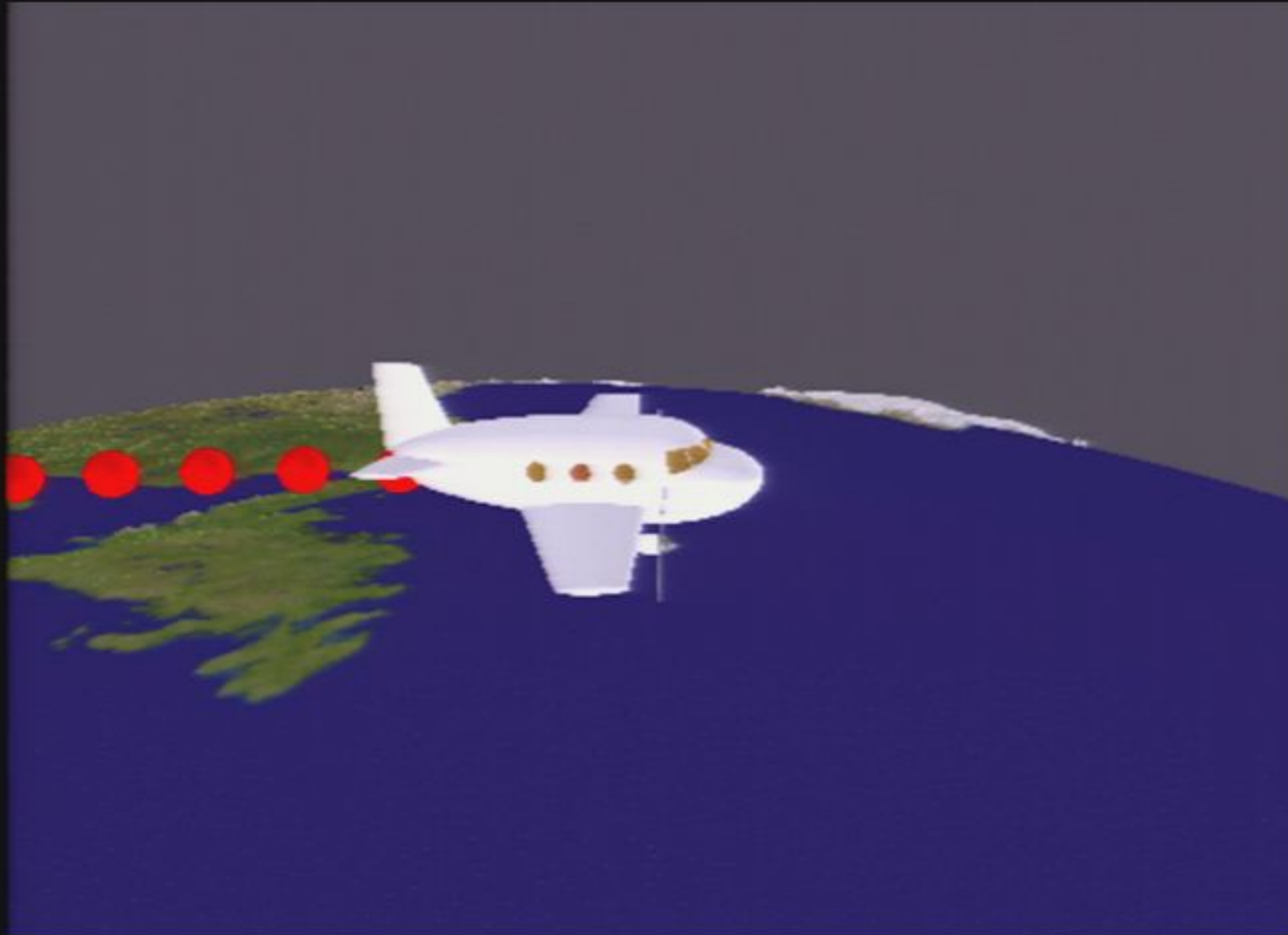
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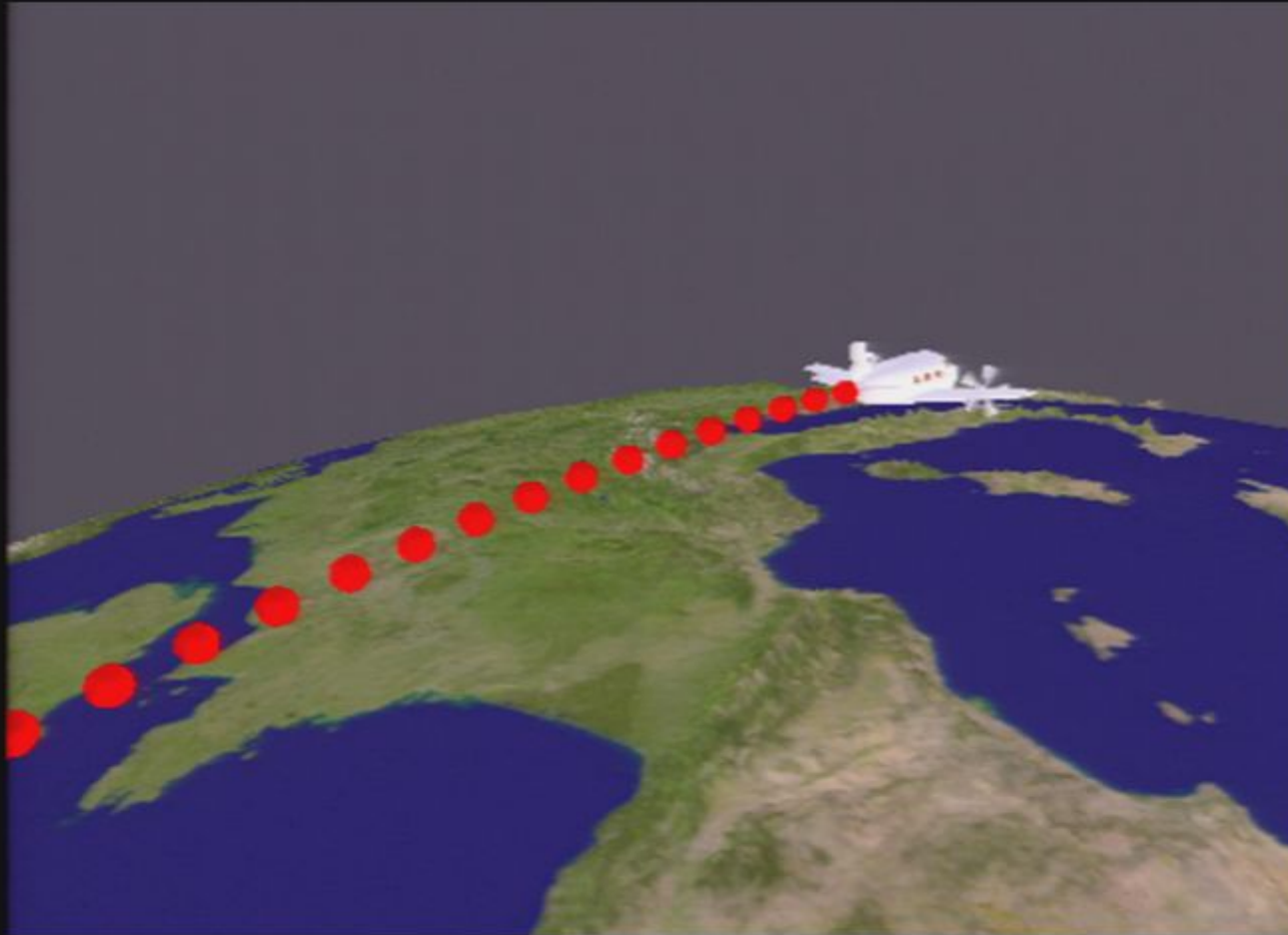
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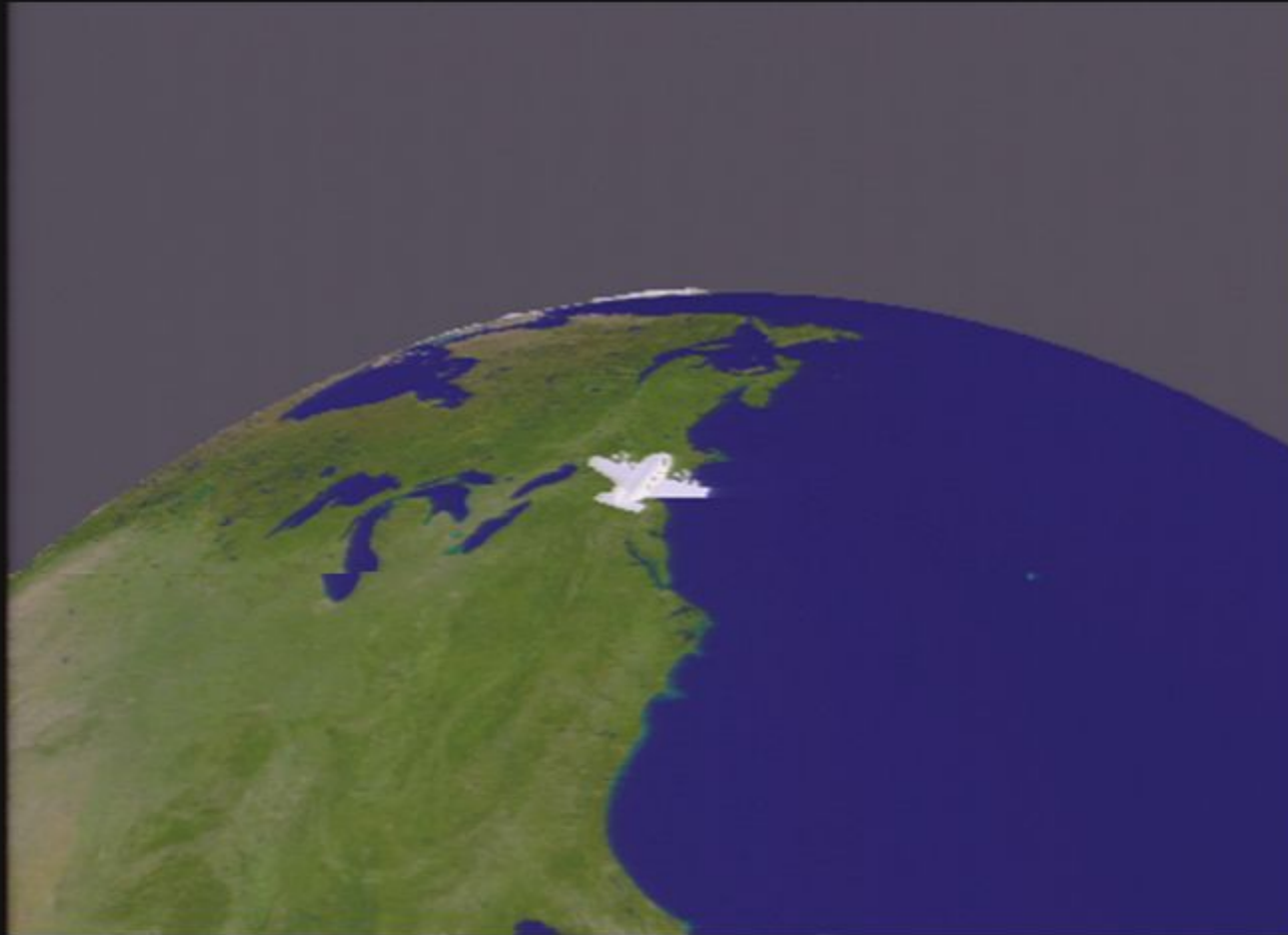
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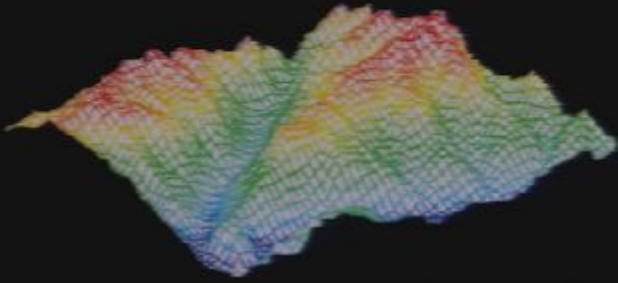
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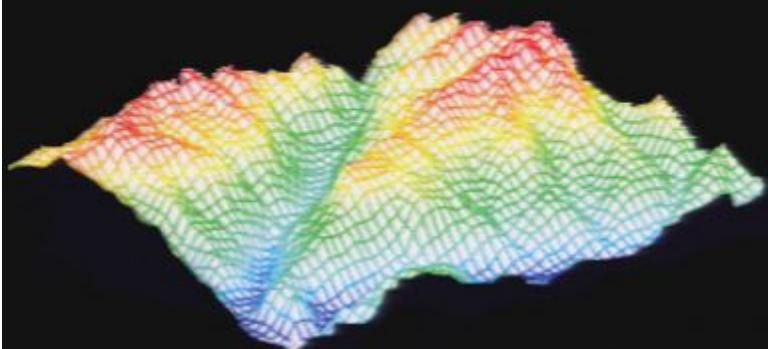
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Geodesics are very difficult to calculate in general. Imagine surveying a complex landscape with hills and valleys. How is one to calculate the shortest distance over this terrain?



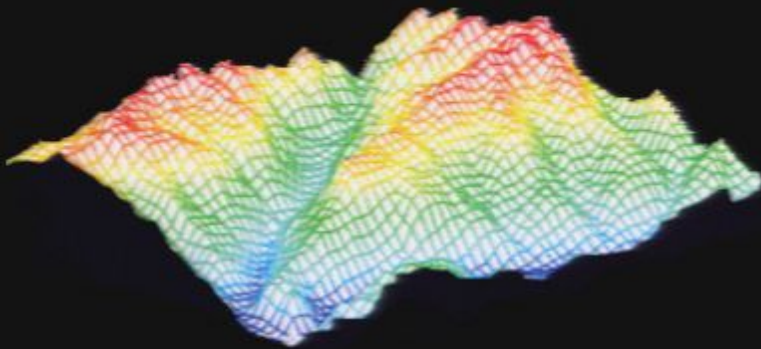


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...and I had to do it in four dimensions!, Yep, that time thing.

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...but I needed the distance in SPACETIME, so I used a mathematical quantity which converts the flat map distances into actual distances on our curved space. This is the METRIC of the space. Denoted by "g".



The Metric

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The "taxi metric" can depend on time, it can depend on location.



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The Metric, g

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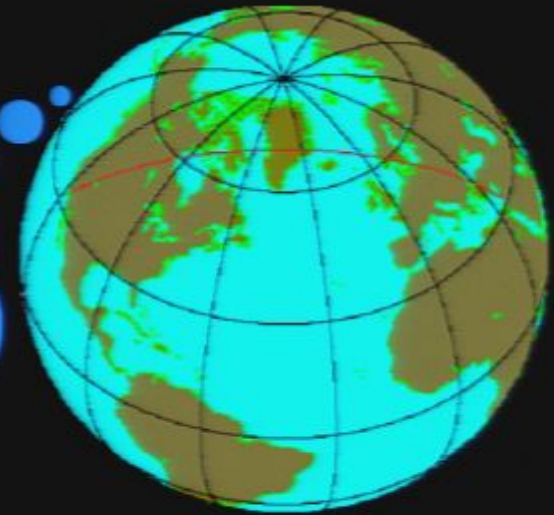
A metric is not a single number at each point in space, otherwise how could it tell us that the cylinder and sphere are curved differently



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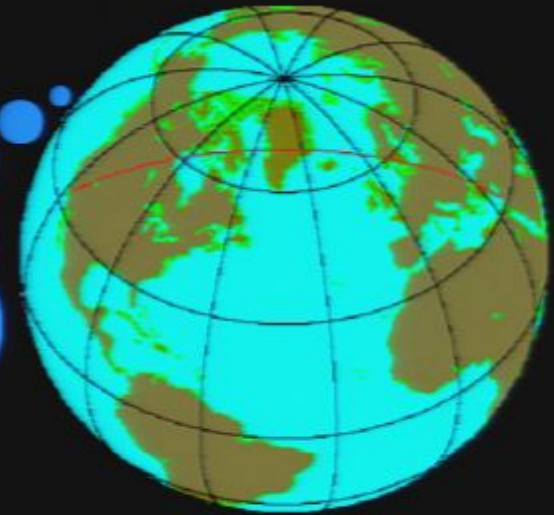
I curve in two directions (north-south, and east-west)



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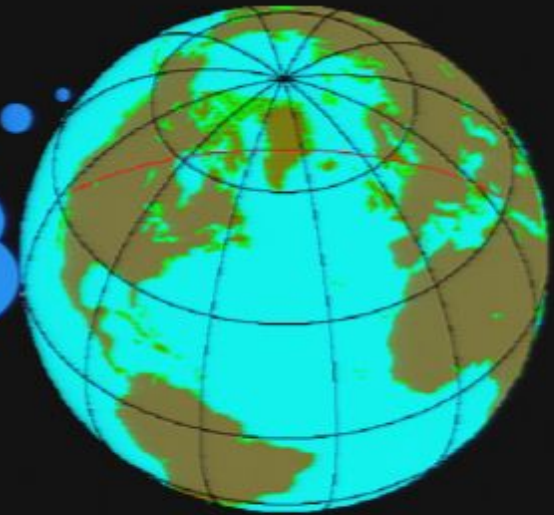
Me, I only bend in east-west.



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A metric is not a single number at each point in space, otherwise how could it tell us that the cylinder and sphere are curved differently

My mama calls me an intrinsic curvature



My mama calls me my little extrinsic curvature



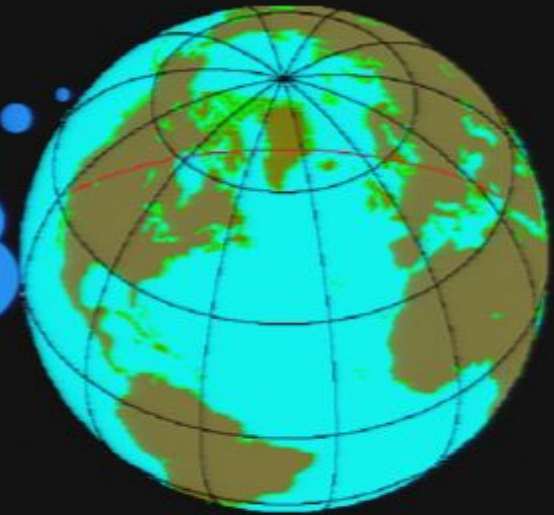
So, for these geometries we need 2 numbers to uniquely specify the curvature of the surface



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We will designate two numbers representing the metric, g_{xx} and g_{yy} , to show that they are associated with curvature in the x and y direction.

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2-tensor is a matrix of $4 \times 4 = 16$ numbers

$$B_{ij} = \begin{pmatrix} 3 & -1 & 17 & 2 \\ 7 & 99 & 0 & 34 \\ 1000 & 3 & 0 & -1 \\ 4 & -2.5 & 7 & -12.3 \end{pmatrix}$$



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With these tools, I can finally write down how the Geometry of Space Time is affected by mass or is it how mass is affected by the geometry of Space Time

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The Einstein field equation (EFE) is usually written in the form

$$G_{ij} = 8\pi T_{ij} + \Lambda g_{ij}$$

Einstein's
Tensor

Stress-Energy
Tensor

Metric
Tensor

$$\begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{pmatrix} = 8\pi \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{pmatrix} + \Lambda \begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{pmatrix}$$

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$$

Curvature of
Space Time

describes the density and
flux of energy and momentum
in spacetime

The EFE is a tensor equation relating a set of symmetric 4 x 4 tensors. Einstein's equations are actually 16 equations in the form: $G_{11} = 8\pi T_{11} + \Lambda g_{11}$

If you sit down and write down the Ricci tensor for a general case of a 2-dimensional space with axial symmetry, you would get something like this:

$$\begin{aligned}
R_{\psi\psi} = & -\frac{2a^2 \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2ac \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{a \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} - \frac{\frac{\partial a}{\partial \psi} c \cot \theta}{\delta \psi} - \frac{a \frac{\partial a}{\partial \psi} \cot \theta}{\delta \psi} - \frac{2a^2 \frac{\partial^2 \psi}{\partial \psi^2}}{\delta \psi^2} \\
& - \frac{2a^2 \left(\frac{\partial c}{\partial \psi}\right)^2}{\delta \psi^2} + \frac{4ac \frac{\partial c}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi^2} - \frac{a^2 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi} + \frac{a \frac{\partial c}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi^2} + \frac{2a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi^2} - \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{4\delta^2} + \frac{a \left(\frac{\partial a}{\partial \psi}\right)^2 \delta}{4\delta^2} \\
& + \frac{3a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{2a^2 c \frac{\partial c}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} + \frac{2a^2 \delta \frac{\partial c}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{a^2 \frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{\delta \psi^2} - \frac{a \frac{\partial a}{\partial \psi} \delta c}{\delta \psi^2} + \frac{a^3 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi^2} \\
R_{\psi d} = & \frac{2ac \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2ab \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} - \frac{2 \frac{\partial a}{\partial \psi} \cot \theta}{2d} - \frac{\frac{\partial a}{\partial \psi} c \cot \theta}{2\delta} + \frac{a \frac{\partial c}{\partial \psi} \cot \theta}{2\delta} \\
& + \frac{a^2 \frac{\partial a}{\partial \psi} \delta \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{2ab \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{2 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{4ac \frac{\partial c}{\partial \psi}}{\delta \psi^2} - \frac{2ab \left(\frac{\partial c}{\partial \psi}\right)^2}{\psi^2} + \frac{6 \left(\frac{\partial a}{\partial \psi}\right)^2}{\psi^2} \\
& + \frac{2ac \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{2ac \left(\frac{\partial c}{\partial \psi}\right)^2}{\delta \psi^2} + \frac{4ab \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi^2} + \frac{2 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\psi^2} - \frac{ac \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi} + \frac{ab \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi} \\
& + \frac{ac \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi} - \frac{ab \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi} - \frac{2c \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{\frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{2a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{\frac{\partial a}{\partial \psi} \delta \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{2a^2 b \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} \\
& + \frac{2a^2 b \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{2ab c \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} + \frac{a^2 \frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{\delta^2 \psi} + \frac{a \frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{\delta^2 \psi} + \frac{a^2 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{2c \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} \\
& + \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{2\delta d} - \frac{\frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{4\delta d} + \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{\left(\frac{\partial a}{\partial \psi}\right)^2}{\delta \psi^2} + \frac{c \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{2\delta d} - \frac{\frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{\delta c \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi} \\
& + \frac{\frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{4\delta d} + \frac{\frac{\partial a}{\partial \psi} \delta \frac{\partial c}{\partial \psi}}{2\delta d} - \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} - \frac{a \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{ac \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi} + \frac{2ab c \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{2ab \delta^2 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi}
\end{aligned}$$

... and just a little bit more.

$$\begin{aligned}
 R_{\psi\psi} &= -\frac{2a^2 \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2ac \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{a \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} - \frac{\frac{\partial a}{\partial \psi} c \cot \theta}{\delta \psi} - \frac{a \frac{\partial a}{\partial \psi} \cot \theta}{\delta \psi} - \frac{2a^2 \frac{\partial^2 \psi}{\partial \psi^2}}{\delta \psi^2} \\
 &\quad - \frac{2a^2 \left(\frac{\partial \psi}{\partial \psi}\right)^2}{\delta \psi^2} \\
 &\quad - \frac{3a \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} : \\
 R_{\psi\psi} &= -\frac{2ac \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2ab \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2 \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} - \frac{\frac{\partial a}{\partial \psi} c \cot \theta}{\delta \psi} - \frac{\frac{\partial a}{\partial \psi} c \cot \theta}{\delta \psi} + \frac{a \frac{\partial b}{\partial \psi} \cot \theta}{\delta \psi} \\
 &\quad + \frac{a^2 \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi} \\
 &\quad + \frac{2ac \frac{\partial^2 \psi}{\partial \psi^2}}{\delta \psi} - \frac{2ac \left(\frac{\partial \psi}{\partial \psi}\right)^2}{\delta \psi^2} + \frac{4ab \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{c \delta \psi} + \frac{2 \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} - \frac{ac \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} + \frac{ab \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} \\
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 &\quad + \frac{2abc \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi} + \frac{a^2 \frac{\partial b}{\partial \psi} c \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi} + \frac{a \frac{\partial a}{\partial \psi} b c \frac{\partial \psi}{\partial \psi}}{4\delta^2} + \frac{a^2 b \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{1\delta^2} + \frac{a \frac{\partial a}{\partial \psi} b^2 \frac{\partial \psi}{\partial \psi}}{4\delta^2} + \frac{2bc \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{4\delta^2} \\
 &\quad + \\
 &\quad + \frac{4ab \frac{\partial^3 \psi}{\partial \psi^3}}{\delta \psi} - \frac{6 \frac{\partial^3 \psi}{\partial \psi^3}}{2a \psi} \cot \theta + \frac{2bc \left(\frac{\partial \psi}{\partial \psi}\right)^2}{\delta \psi} - \frac{ab \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} - \frac{\frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} - \frac{bc \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} + \frac{\frac{\partial a}{\partial \psi} c \cot \theta}{2\delta} + \frac{a \frac{\partial a}{\partial \psi} \cot \theta}{2\delta} \\
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$$-\frac{2a^2 (\frac{\partial c}{\partial \psi})^2}{\delta \psi^2}$$

$$-\frac{3a \frac{\partial a}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta \psi} :$$

$$R_{\psi\psi} = -\frac{2ac \frac{\partial c}{\partial \psi}}{\delta}$$

$$+\frac{a^2 \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi}$$

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$$+\frac{ac \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi}$$

$$-\frac{\frac{\partial a}{\partial \psi} c}{4\delta}$$

$$+\frac{2a^2 b \frac{\partial c}{\partial \psi} \frac{\partial \psi}{\partial \psi}}{\delta^2 \psi}$$

$$+\frac{2abc \frac{\partial c}{\partial \psi}}{\delta^2 \psi}$$

$$+\frac{4a}{\delta}$$

$$+\frac{2b \frac{\partial c}{\partial \psi} c}{\delta \psi}$$

$$+\frac{ab \frac{\partial b}{\partial \psi} c \frac{\partial c}{\partial \psi}}{\delta^2 \psi} + \frac{\frac{\partial a}{\partial \psi} b^2 c \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{a^2 b \frac{\partial b}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2 \psi} - \frac{a \frac{\partial a}{\partial \psi} b^2 \frac{\partial c}{\partial \psi}}{\delta^2 \psi} + \frac{\frac{\partial a}{\partial \psi} \frac{\partial a}{\partial \psi}}{4d^2} - \frac{\frac{\partial a}{\partial \psi} c \frac{\partial c}{\partial \psi}}{4\delta d}$$

$$+\frac{a \frac{\partial b}{\partial \psi} \frac{\partial a}{\partial \psi}}{4\delta d} - \frac{\frac{\partial^2 a}{\partial \psi^2} d}{2d} - \frac{\frac{\partial b}{\partial \psi} c \frac{\partial a}{\partial \psi}}{4\delta d} + \frac{\frac{\partial a}{\partial \psi} b \frac{\partial a}{\partial \psi}}{4\delta d} - \frac{\frac{\partial c}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta} + \frac{\frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{2\delta}$$

$$+\frac{c \frac{\partial^2 c}{\partial \psi^2}}{\delta} + \frac{\frac{\partial b}{\partial \psi} \frac{\partial c}{\partial \psi}}{2\delta} - \frac{\frac{\partial^2 b}{\partial \psi^2} c}{2\delta} - \frac{\frac{\partial^2 a}{\partial \psi^2} c}{2\delta} - \frac{\frac{\partial a}{\partial \psi} \frac{\partial b}{\partial \psi}}{4\delta} + \frac{\frac{\partial b}{\partial \psi} \frac{\partial b}{\partial \psi}}{4\delta}$$

$$+\frac{ab \frac{\partial c}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta^2} - \frac{\frac{\partial a}{\partial \psi} b c \frac{\partial c}{\partial \psi}}{2\delta^2} - \frac{a \frac{\partial a}{\partial \psi} b \frac{\partial c}{\partial \psi}}{2\delta^2} - \frac{a \frac{\partial b}{\partial \psi} c \frac{\partial c}{\partial \psi}}{2\delta^2} - \frac{ab \frac{\partial b}{\partial \psi} \frac{\partial c}{\partial \psi}}{2\delta^2} + \frac{a \frac{\partial a}{\partial \psi} \frac{\partial b}{\partial \psi} c}{4\delta^2}$$

$$+\frac{a (\frac{\partial b}{\partial \psi})^2 c}{4\delta^2} + \frac{\frac{\partial a}{\partial \psi} b \frac{\partial b}{\partial \psi} c}{4\delta^2} + \frac{(\frac{\partial a}{\partial \psi})^2 b c}{4\delta^2} + \frac{a \frac{\partial a}{\partial \psi} b \frac{\partial b}{\partial \psi}}{4\delta^2} - \frac{a \frac{\partial a}{\partial \psi} b \frac{\partial b}{\partial \psi}}{4\delta^2}$$

$$R_{\psi\psi} = -\frac{2ab \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} + \frac{2bc \frac{\partial c}{\partial \psi} \cot \theta}{\delta \psi} - \frac{\frac{\partial a}{\partial \psi} \cot \theta}{d} - \frac{c \frac{\partial c}{\partial \psi} \cot \theta}{\delta} + \frac{\frac{\partial b}{\partial \psi} c \cot \theta}{2\delta} + \frac{a \frac{\partial a}{\partial \psi} \cot \theta}{2\delta}$$

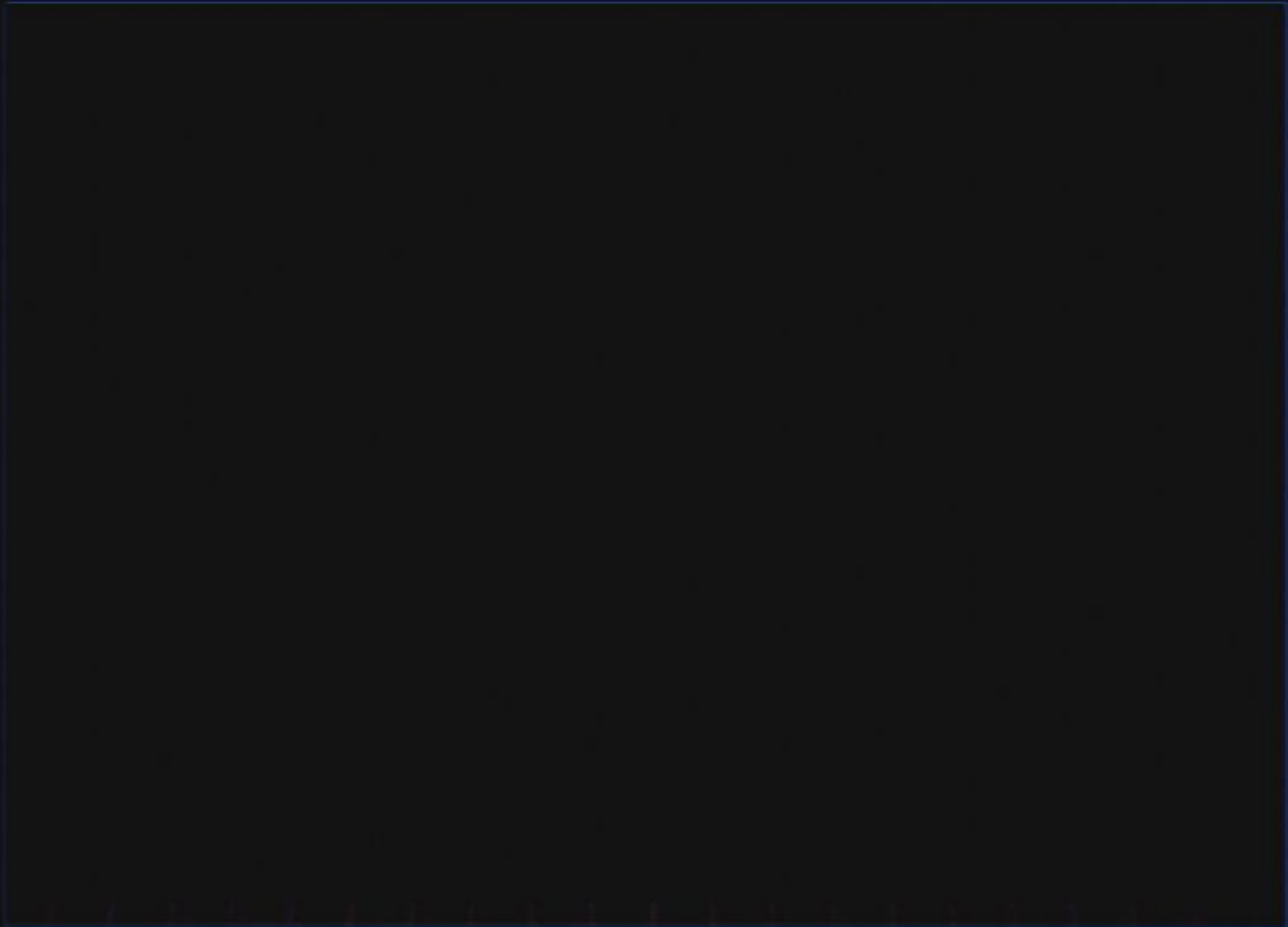
$$-\frac{2ab \frac{\partial^2 \psi}{\partial \psi^2}}{\delta \psi} - \frac{2 \frac{\partial^2 \psi}{\partial \psi^2}}{\psi} - \frac{2ab (\frac{\partial c}{\partial \psi})^2}{\delta \psi^2} + \frac{6(\frac{\partial c}{\partial \psi})^2}{\psi^2} + \frac{4bc \frac{\partial b}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta \psi^2} - \frac{ab \frac{\partial a}{\partial \psi} \frac{\partial c}{\partial \psi}}{\delta d \psi}$$

This is a general expression for Ricci tensor R_{mn} in only two dimensions, with axial symmetry.

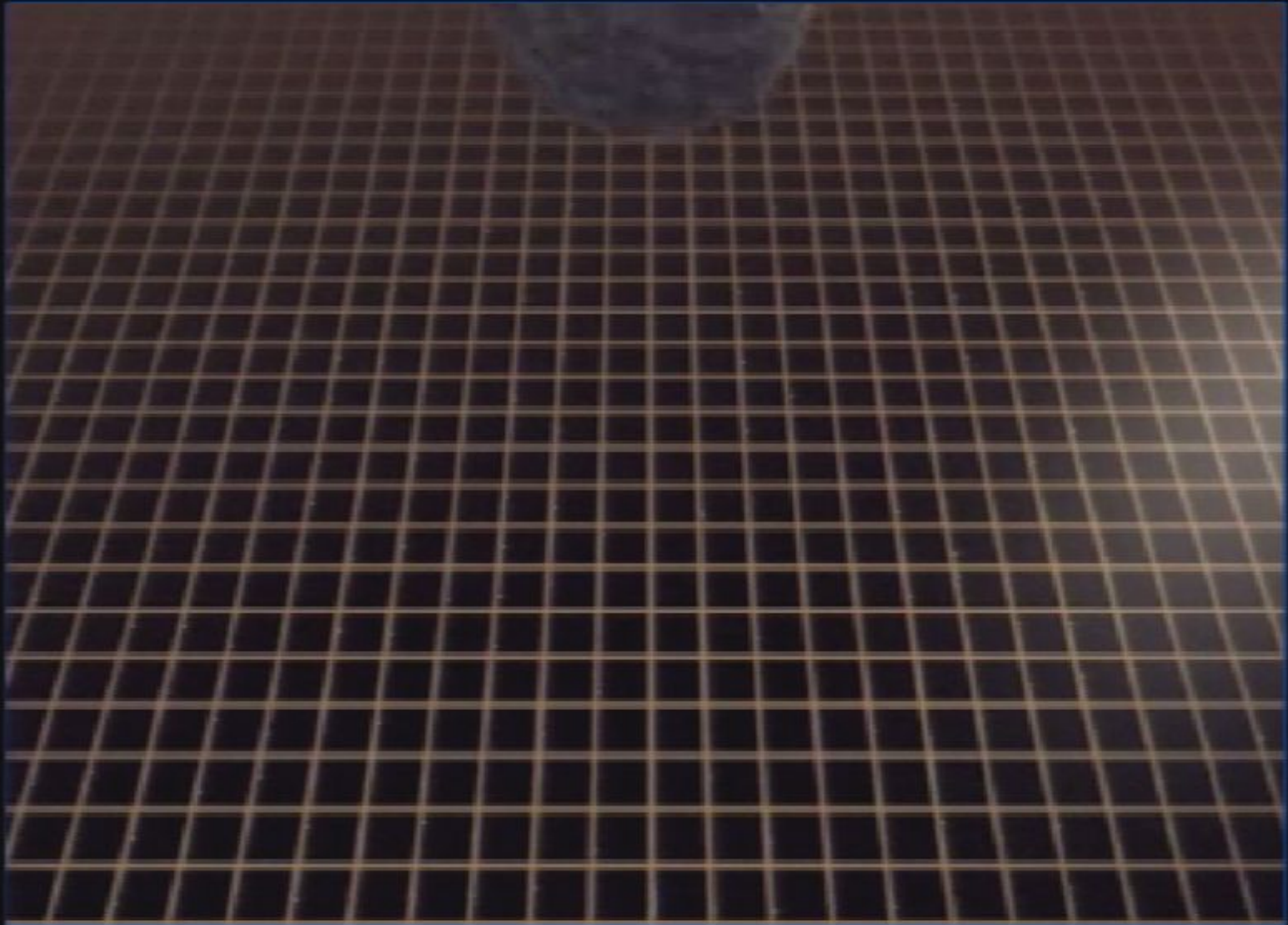
Just try to imagine all of three dimensions of space plus one of time!

What does all this say?

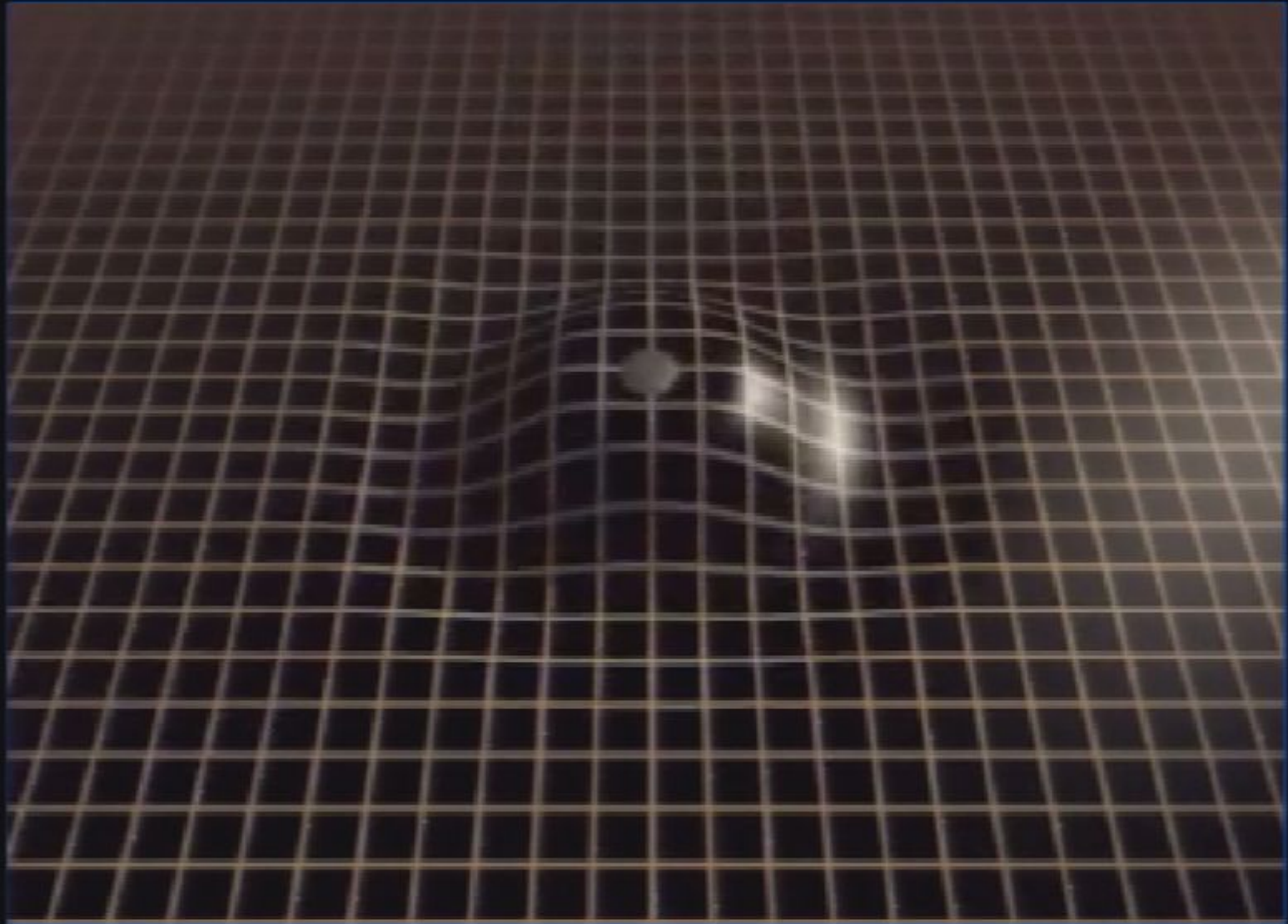
Curvature



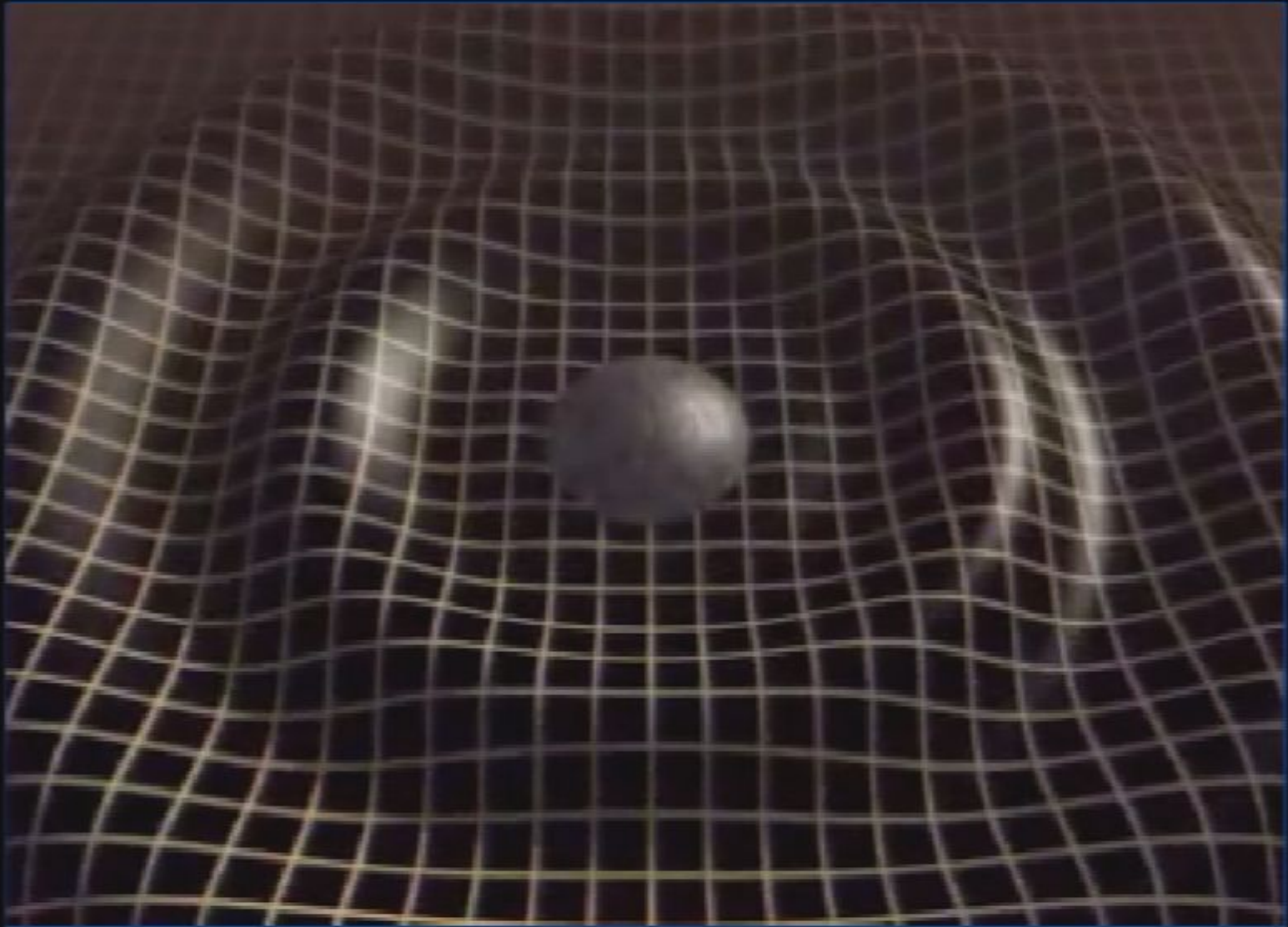
Curvature



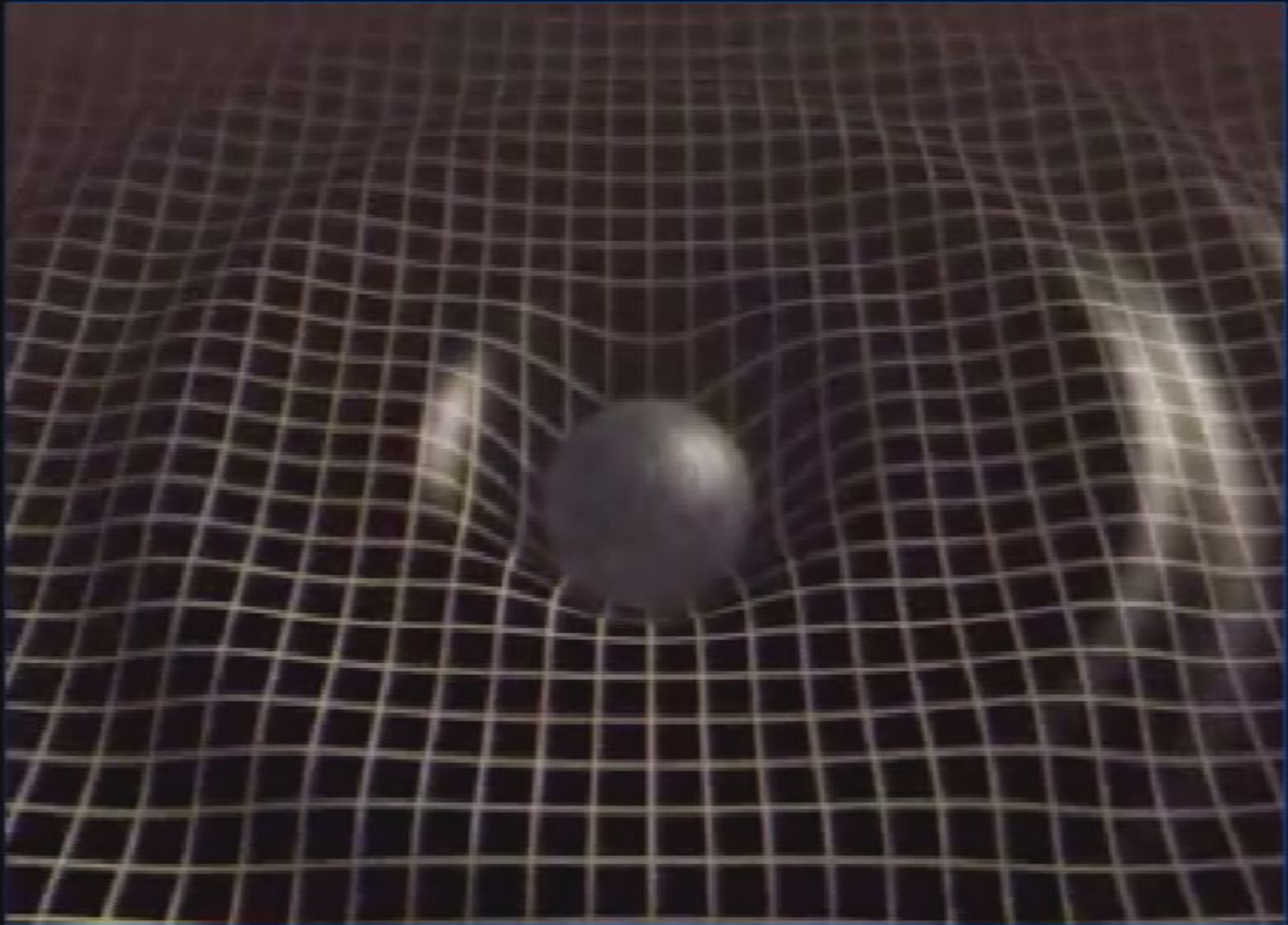
Curvature



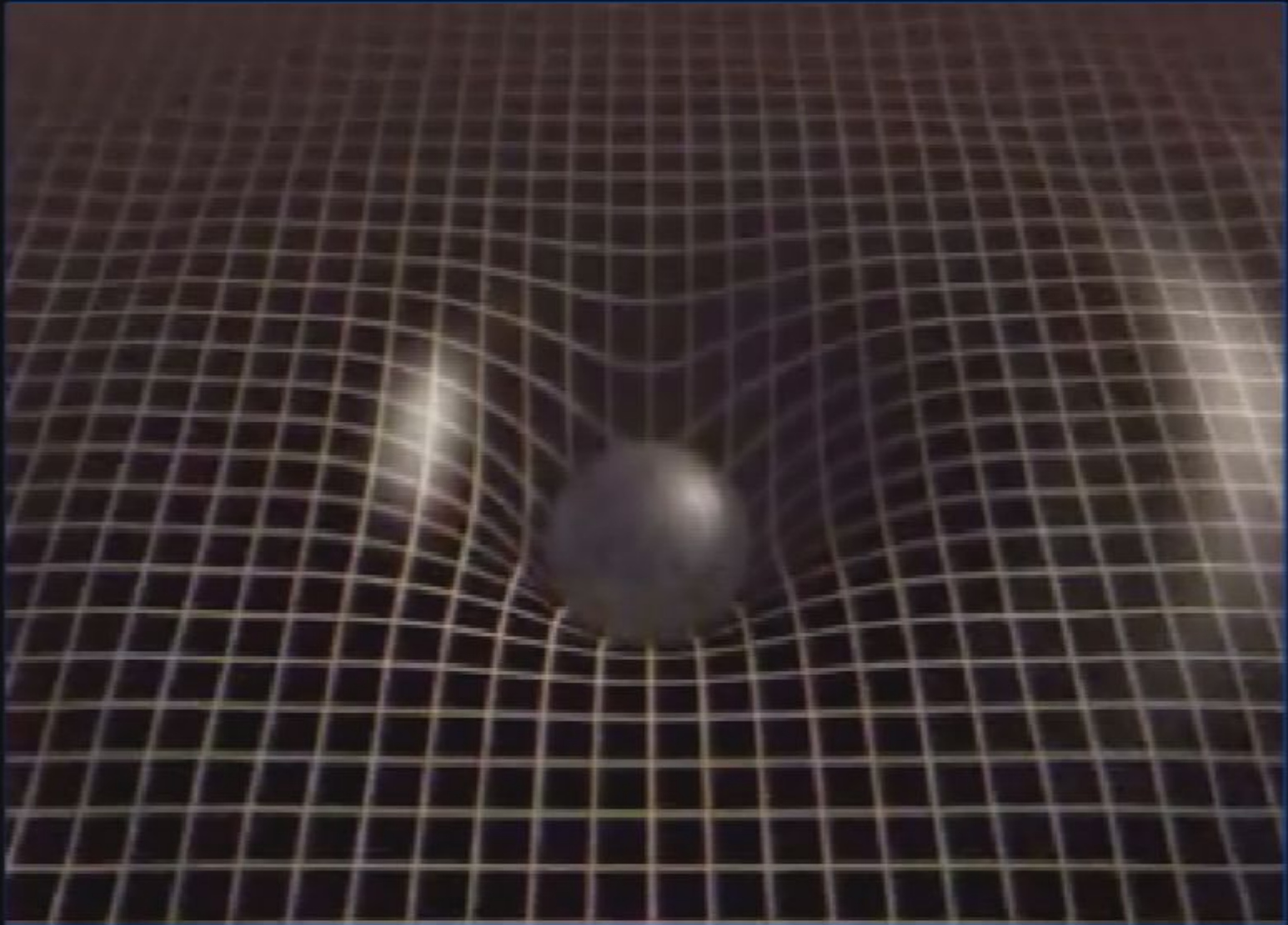
Curvature



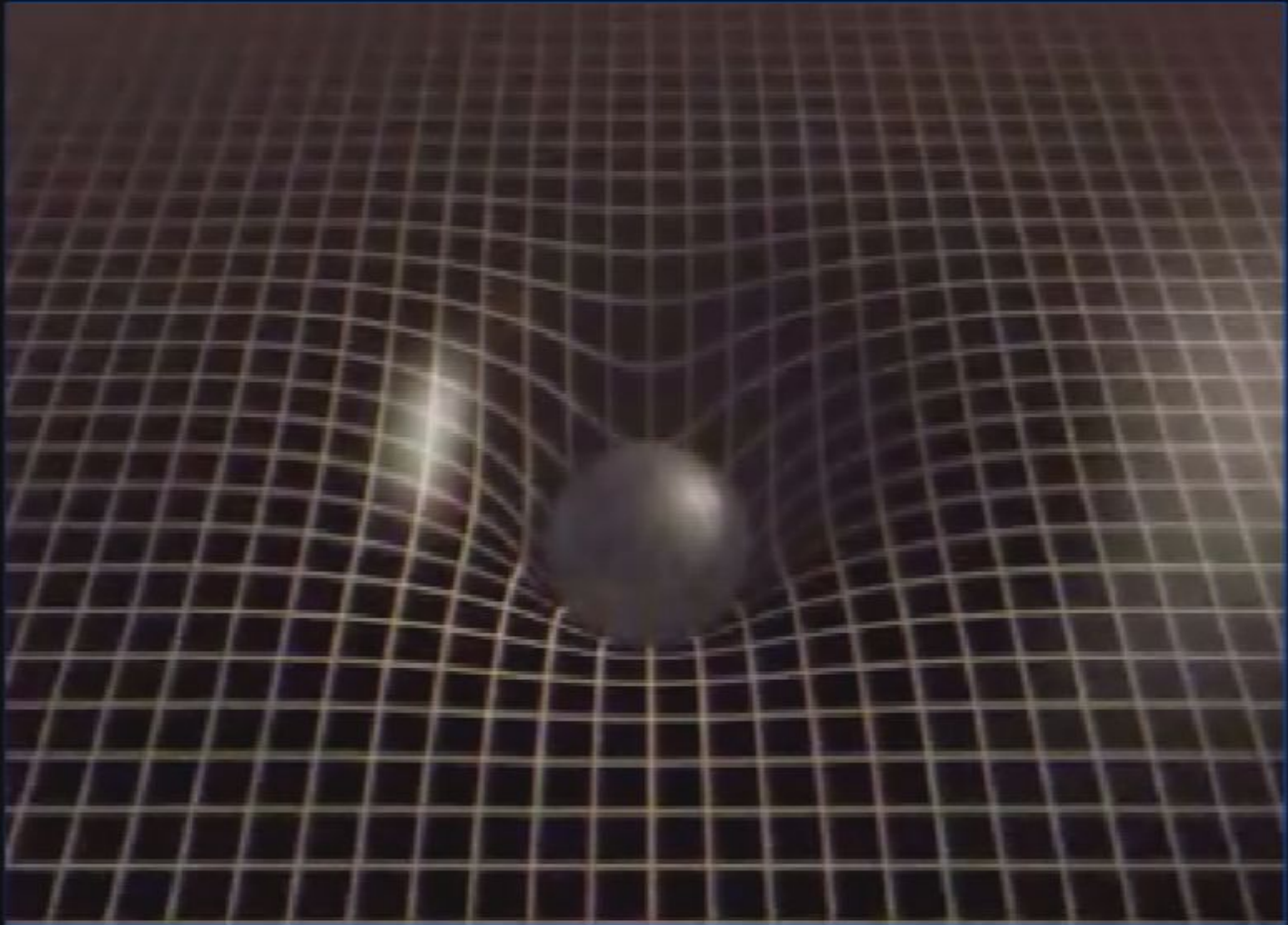
Curvature



Curvature



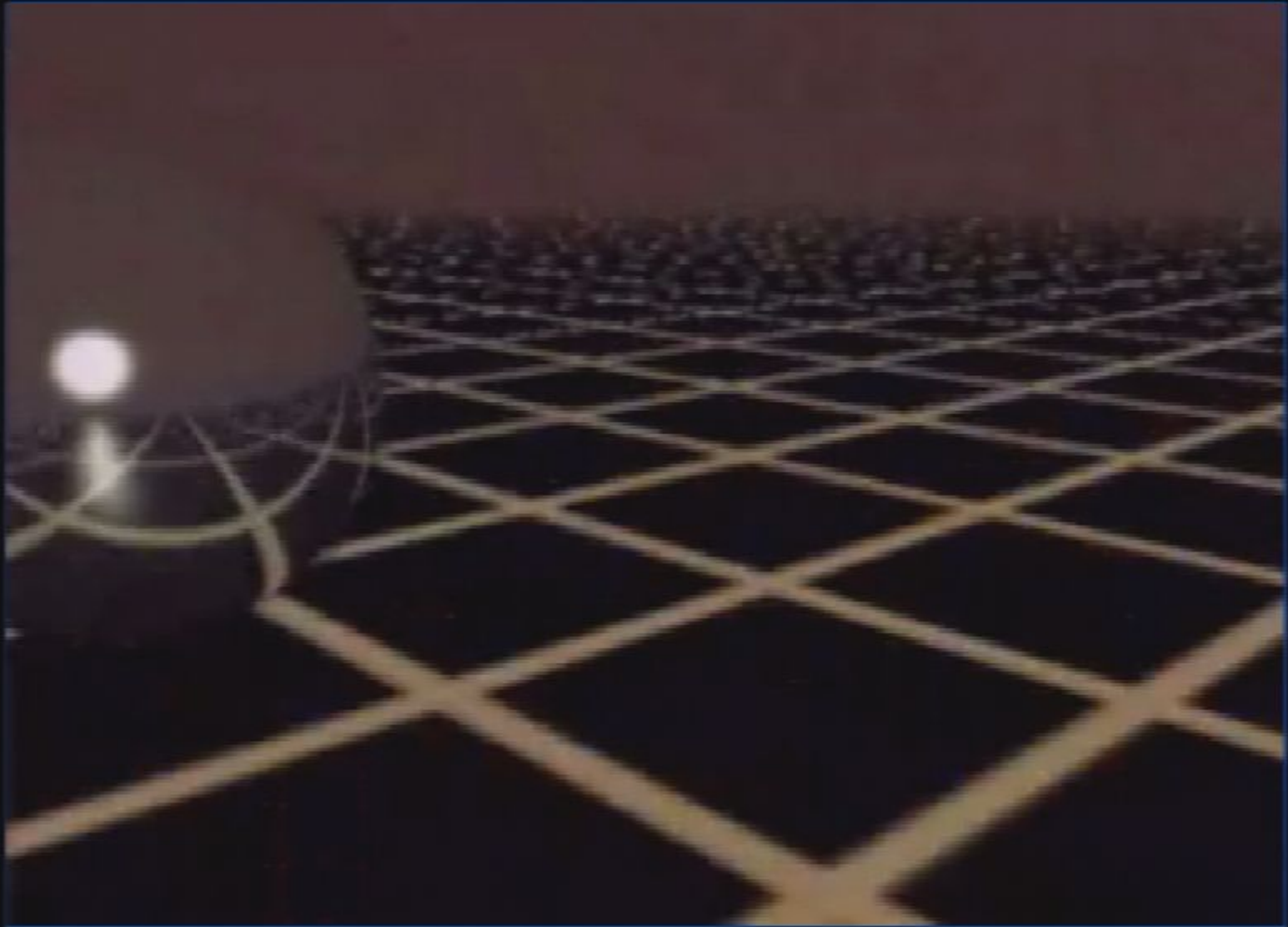
Curvature



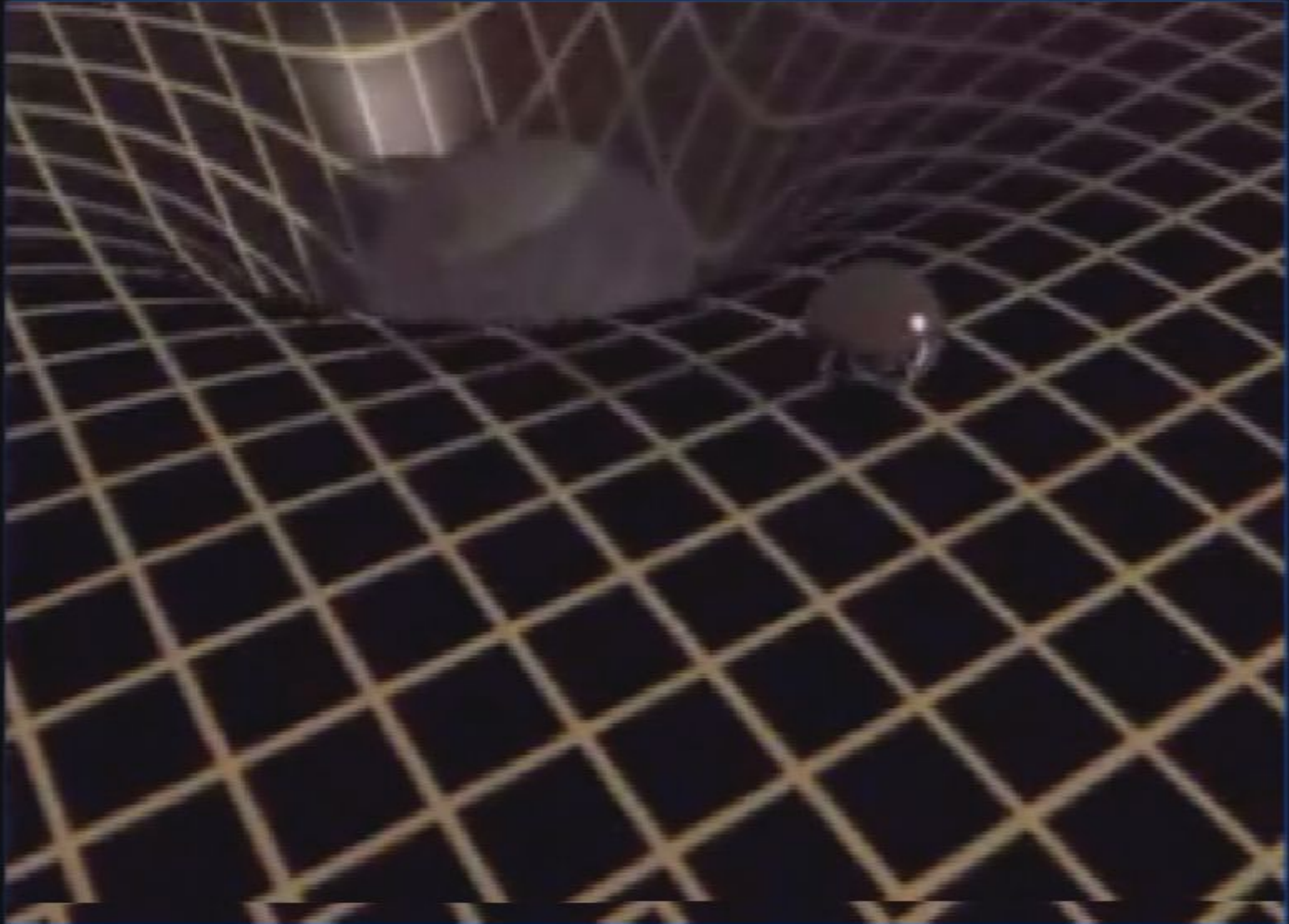
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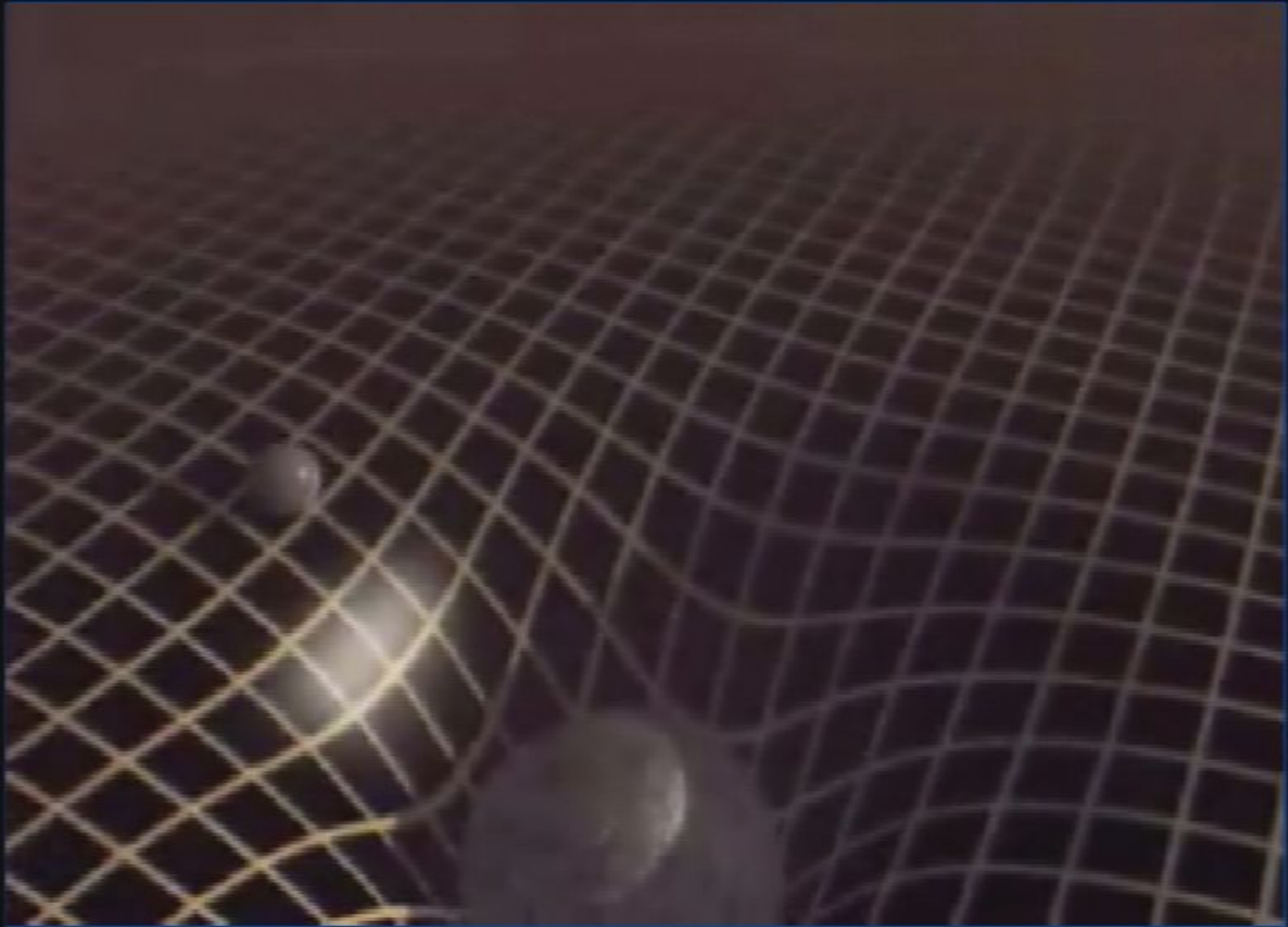
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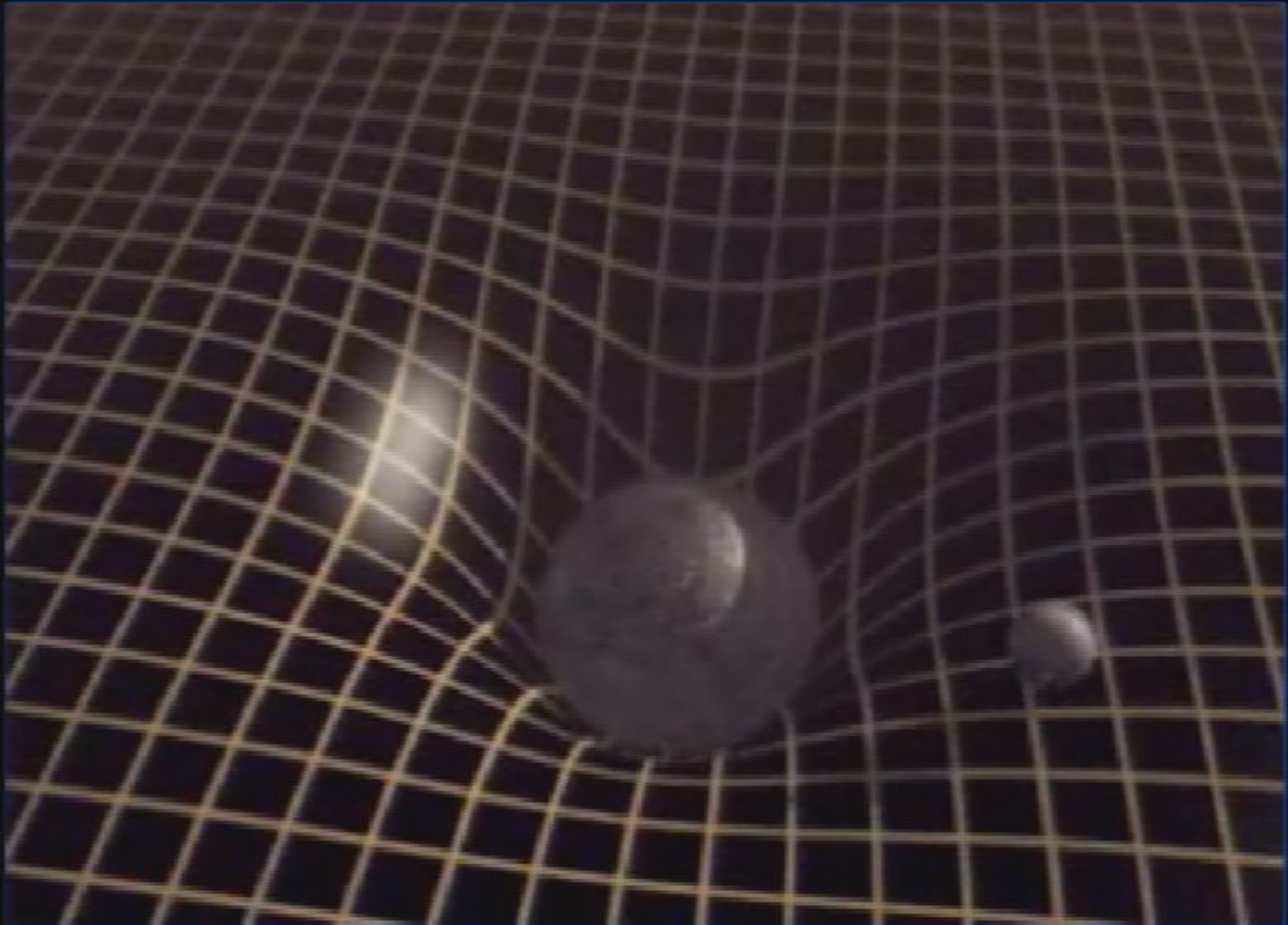
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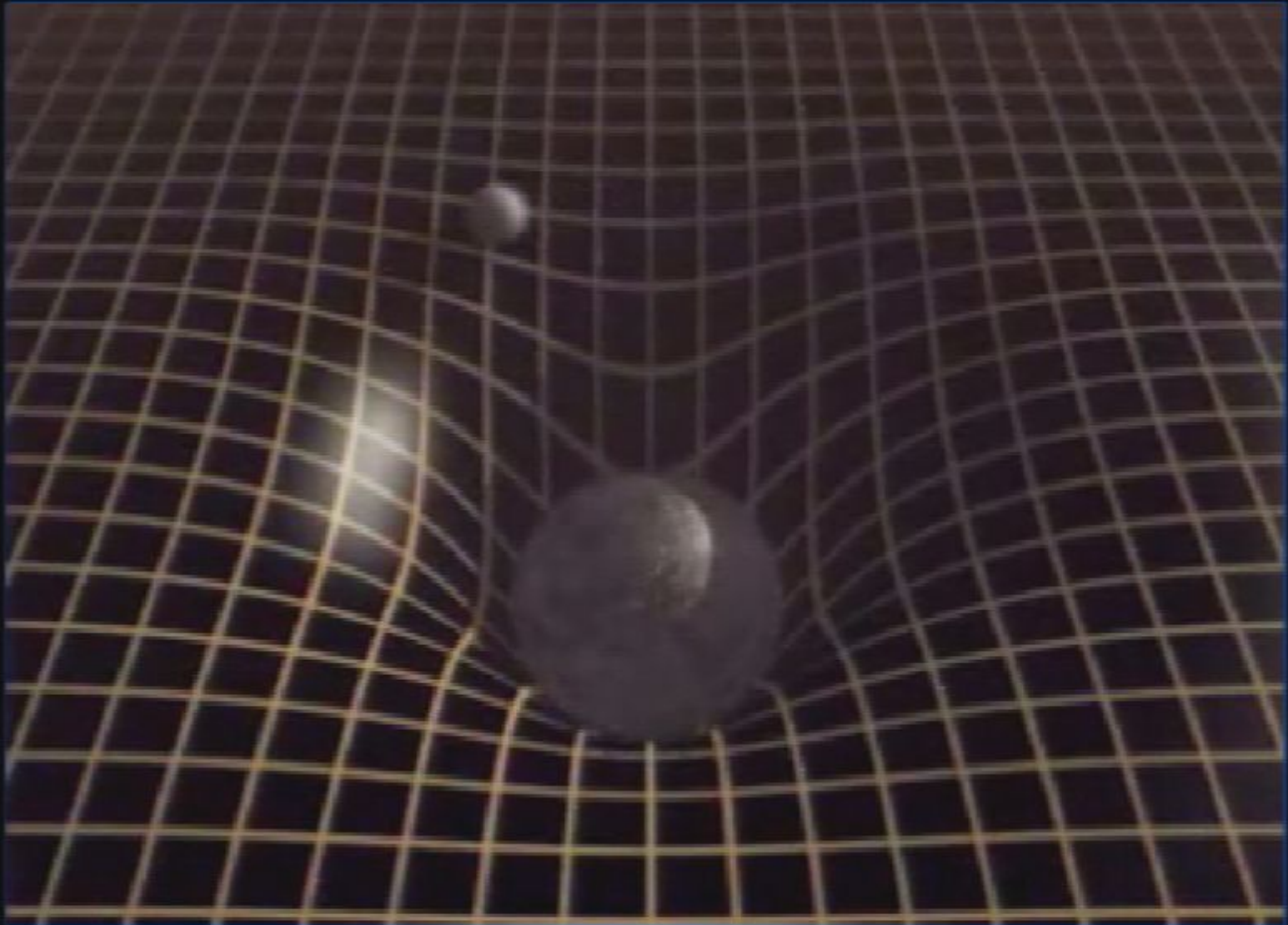
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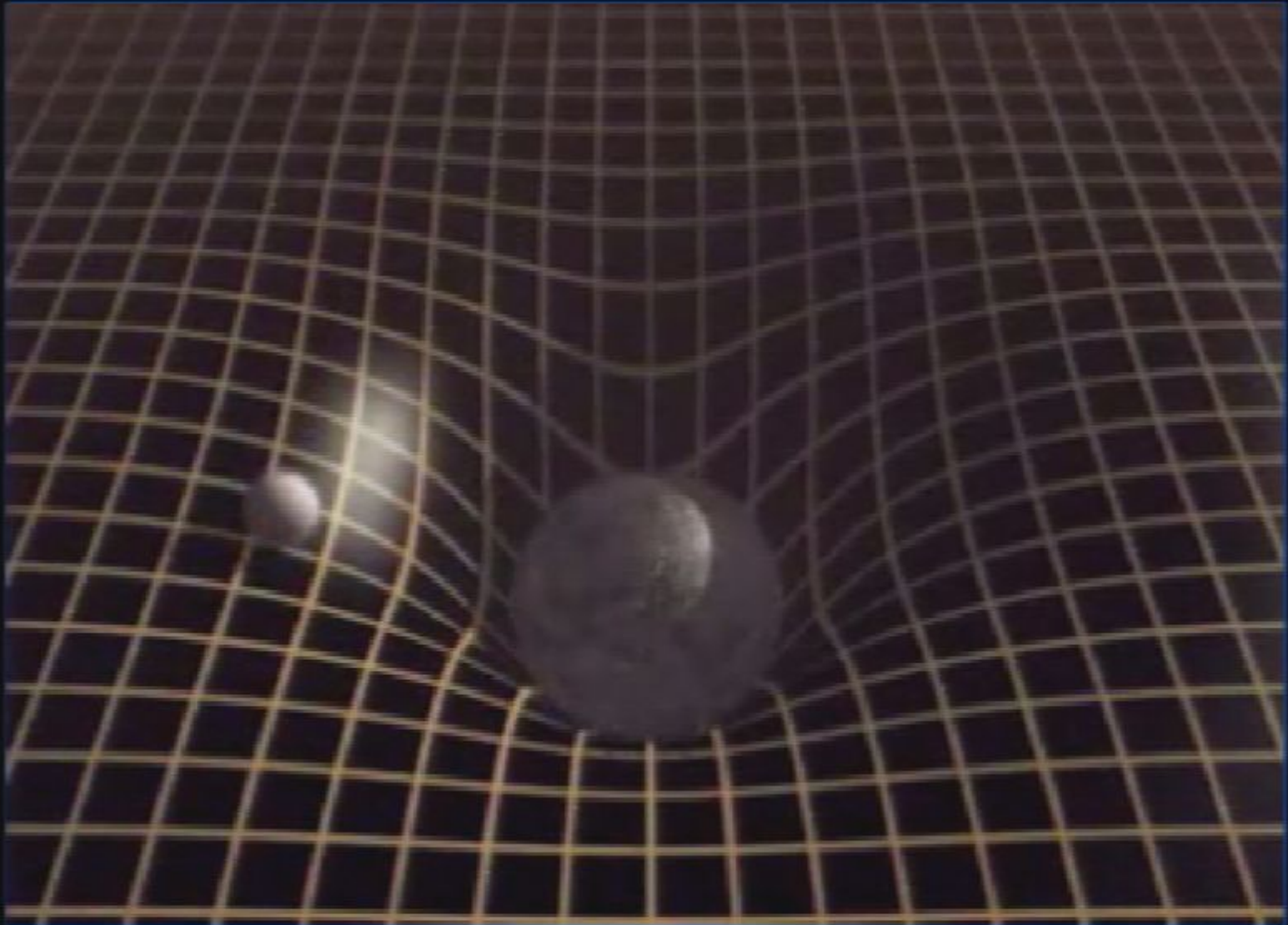
Curvature



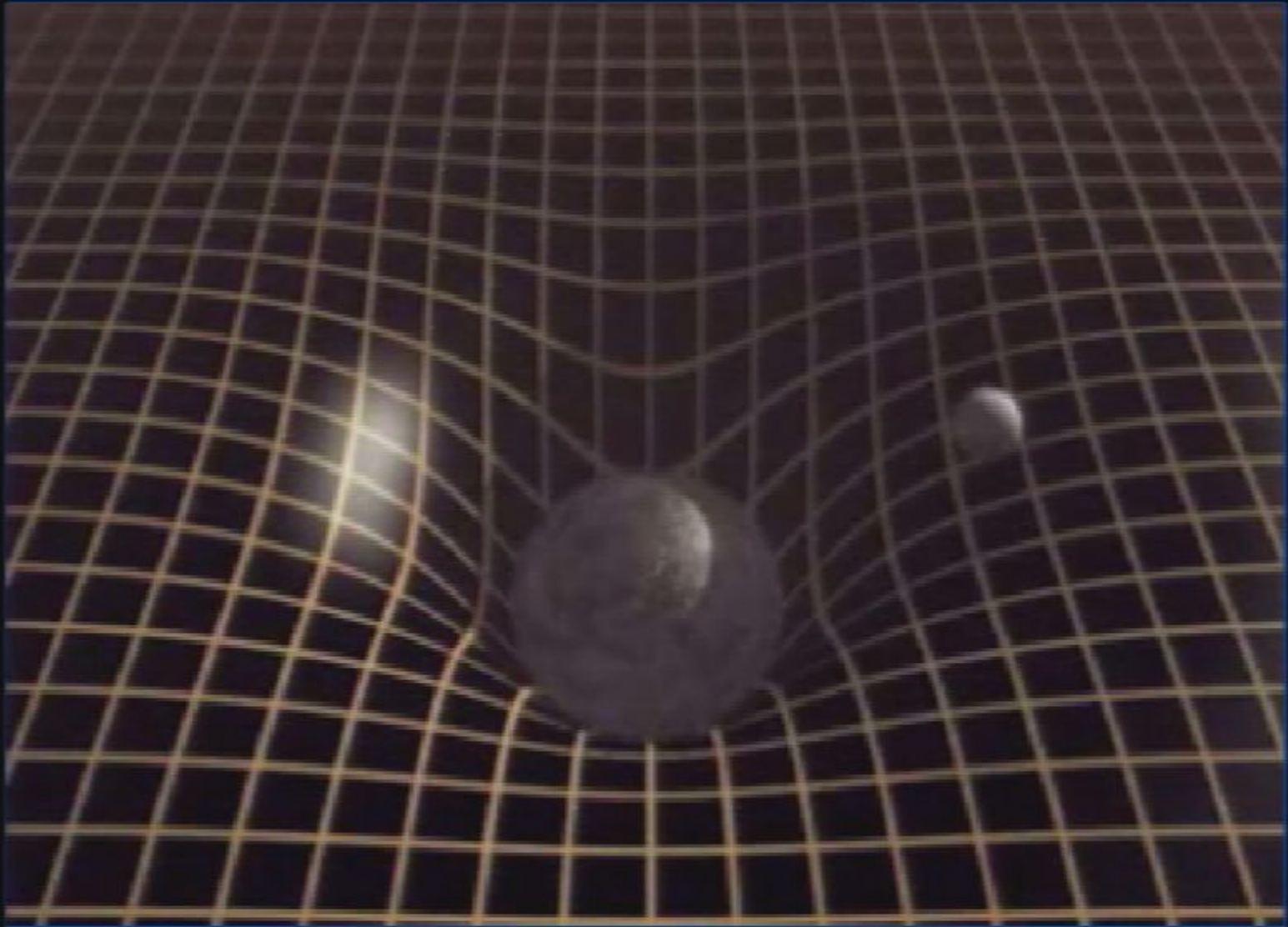
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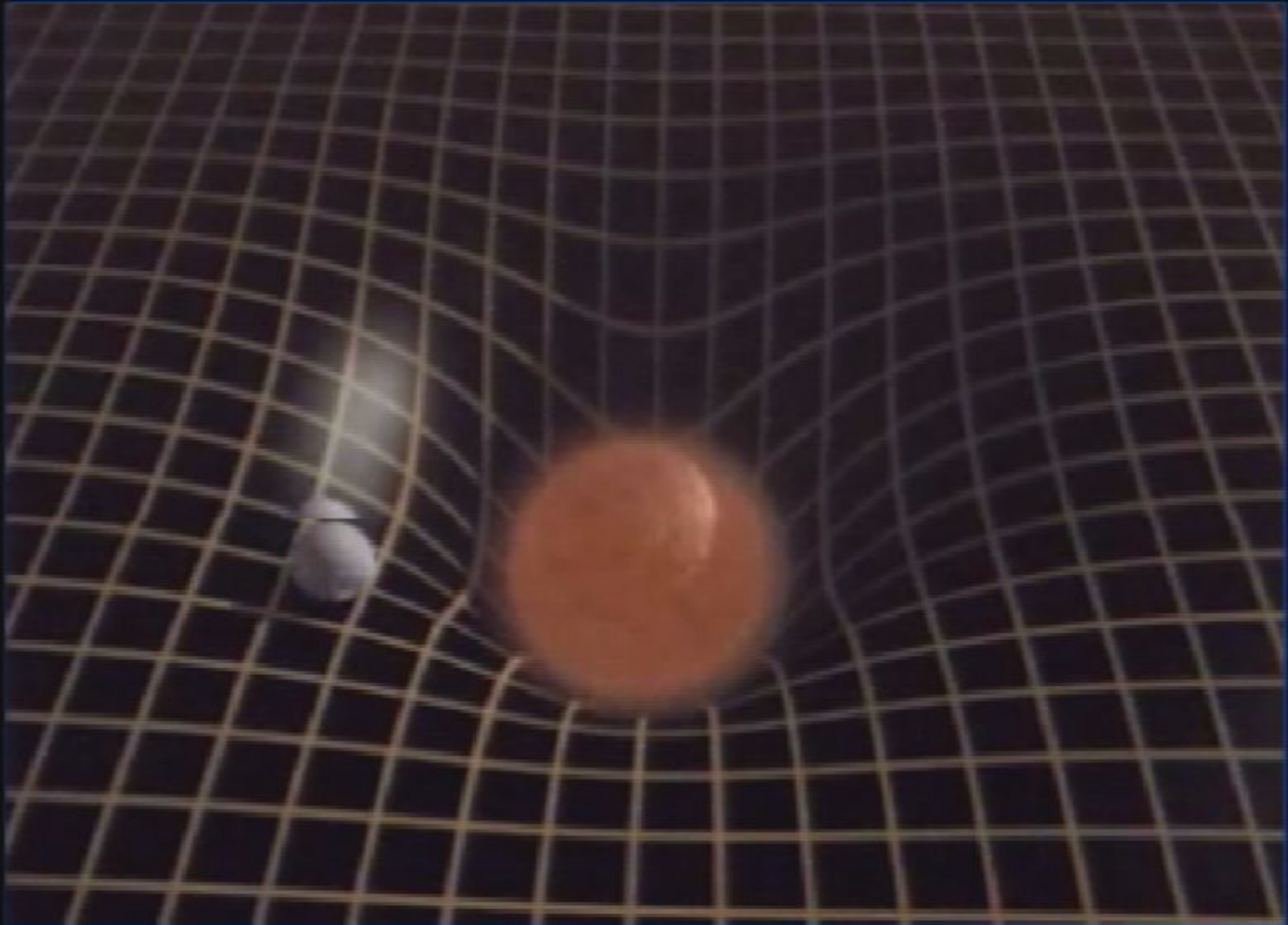
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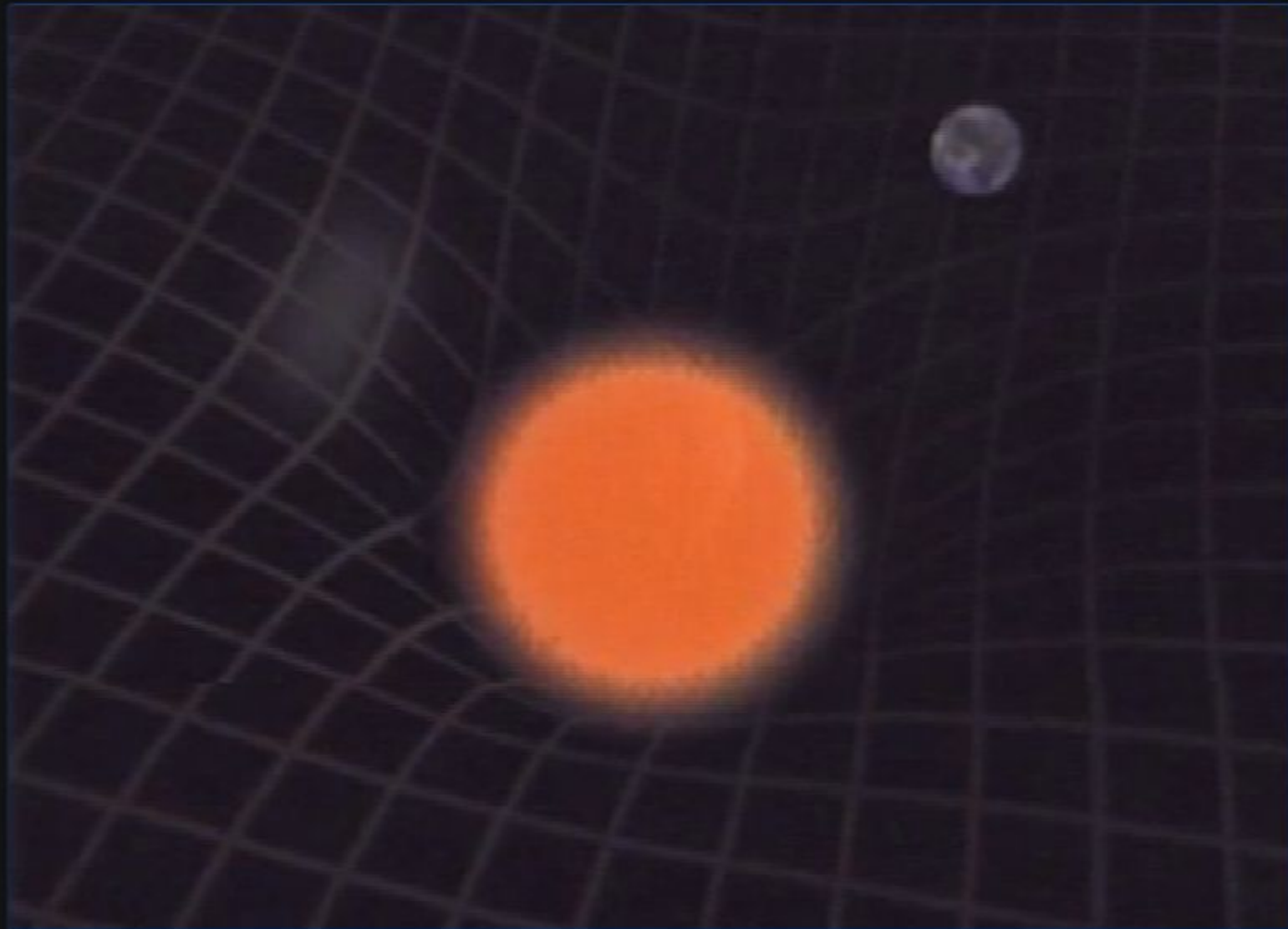
Curvature



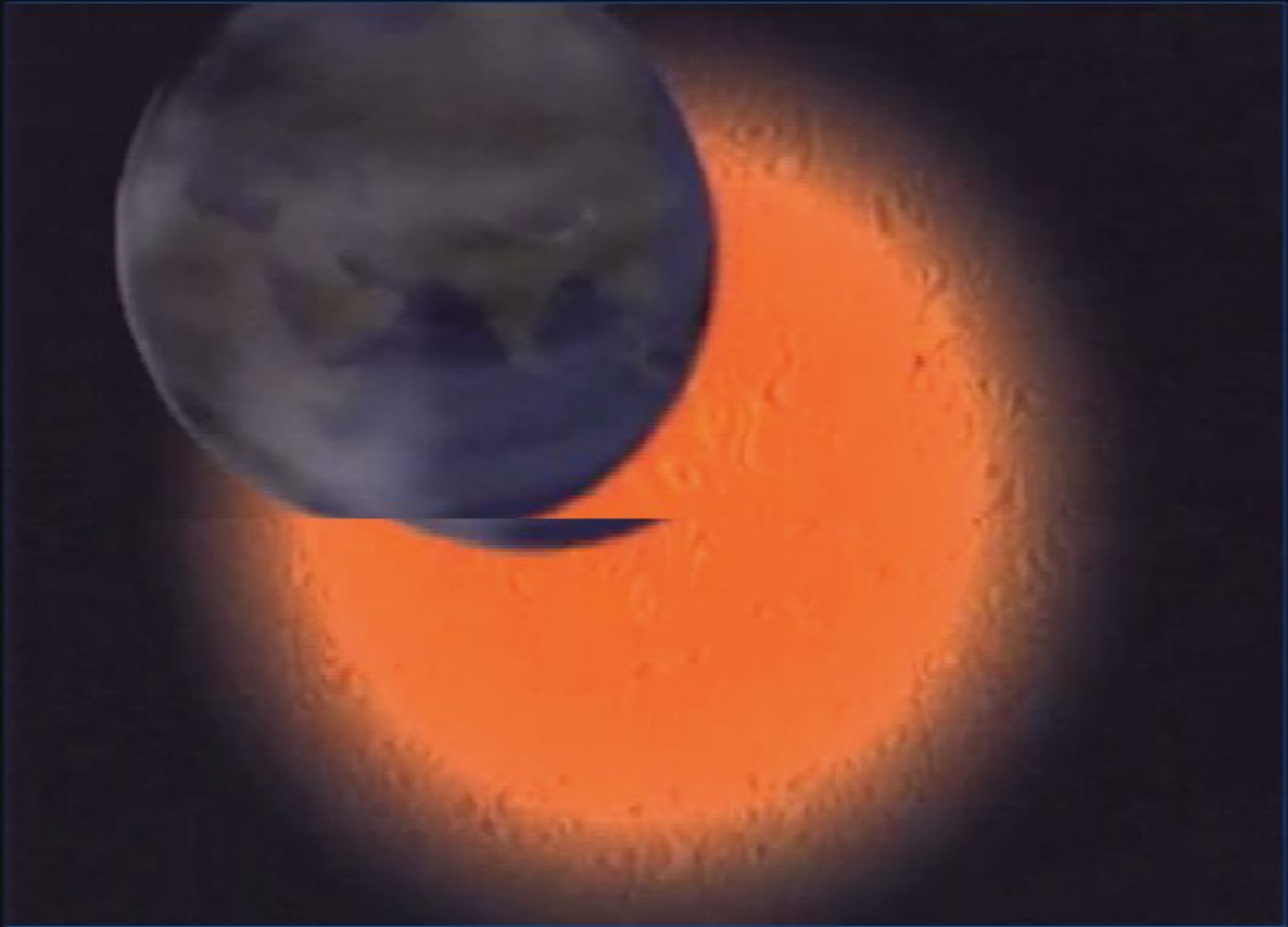
Curvature



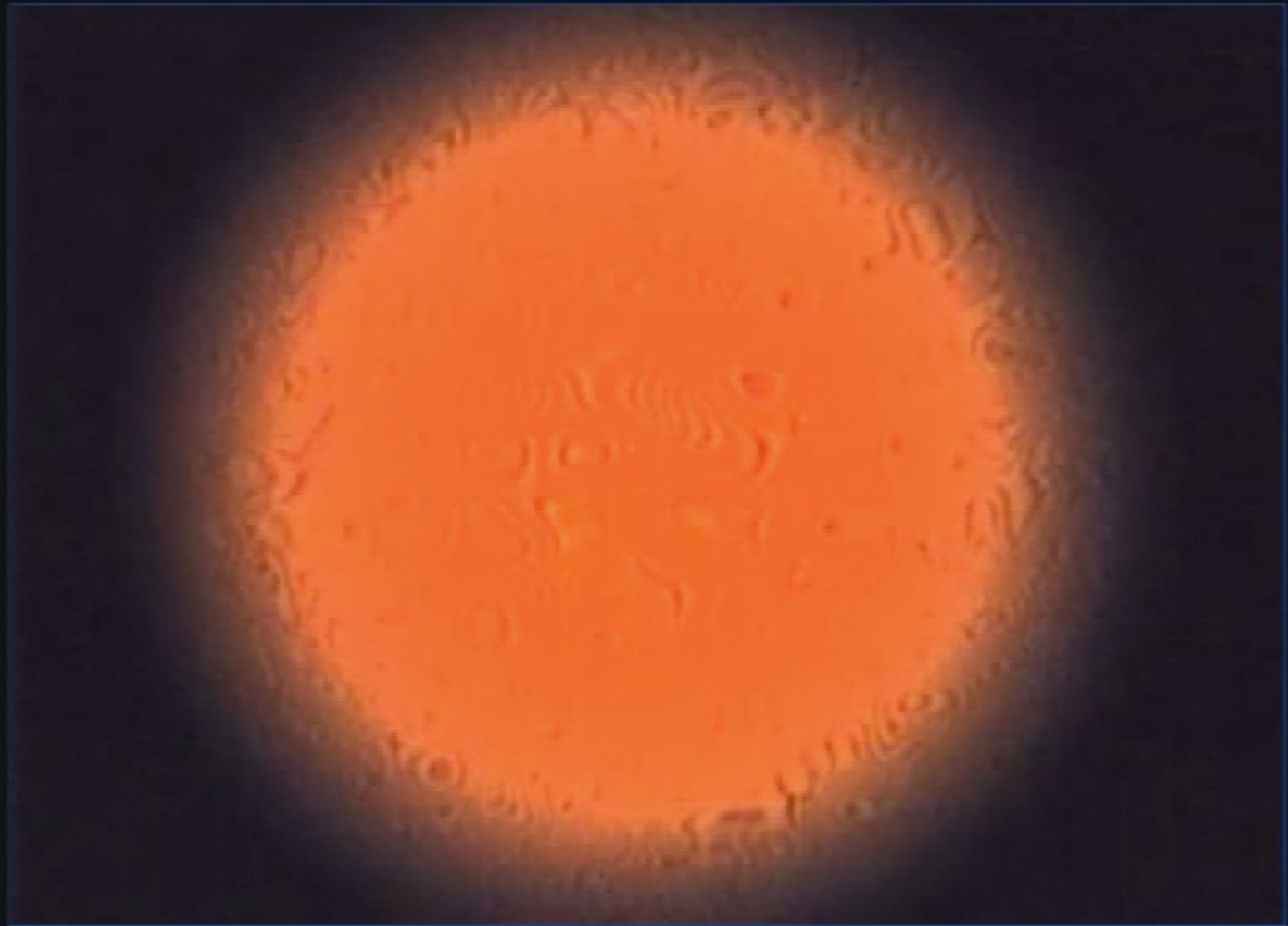
Curvature



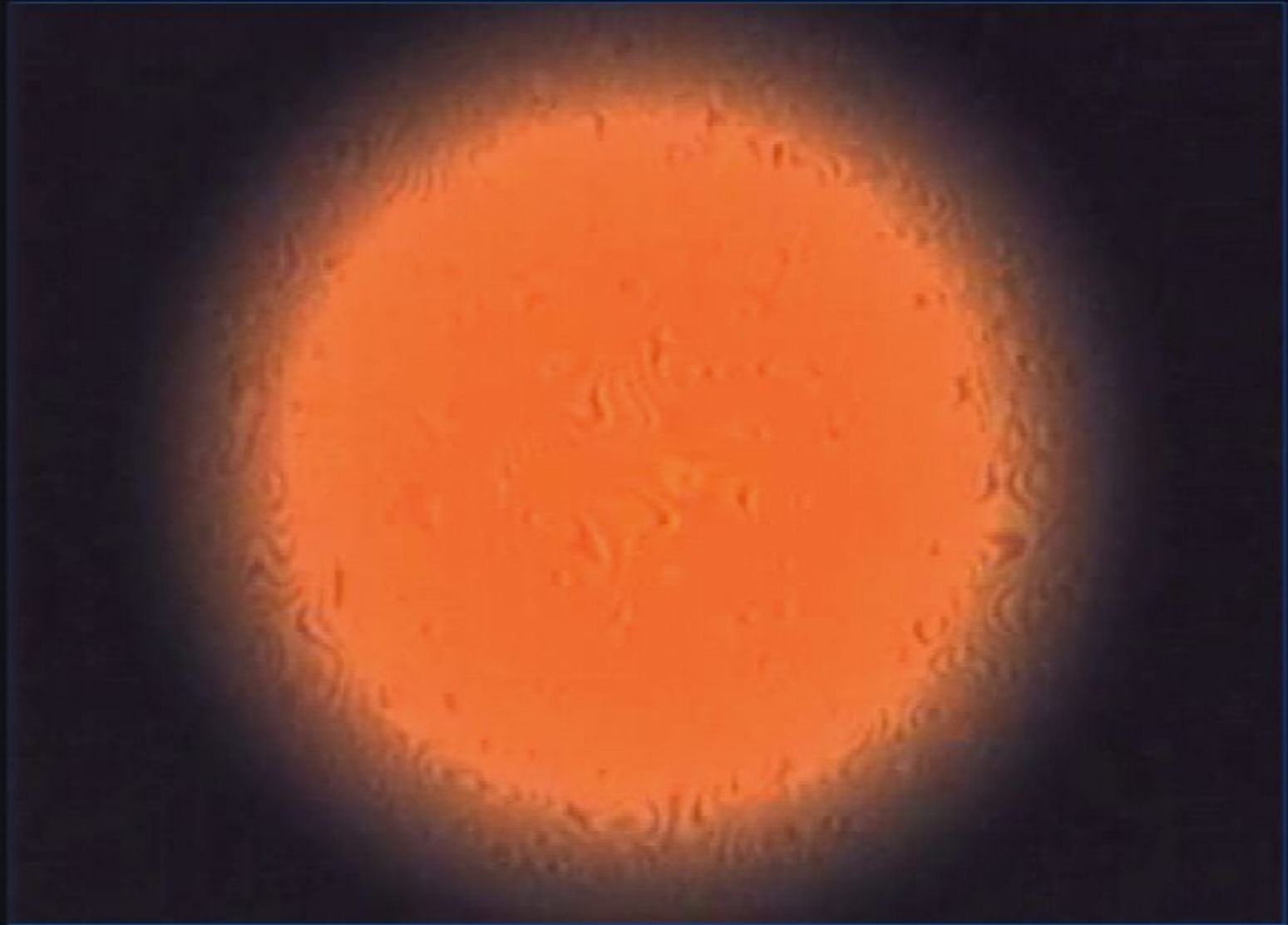
Curvature



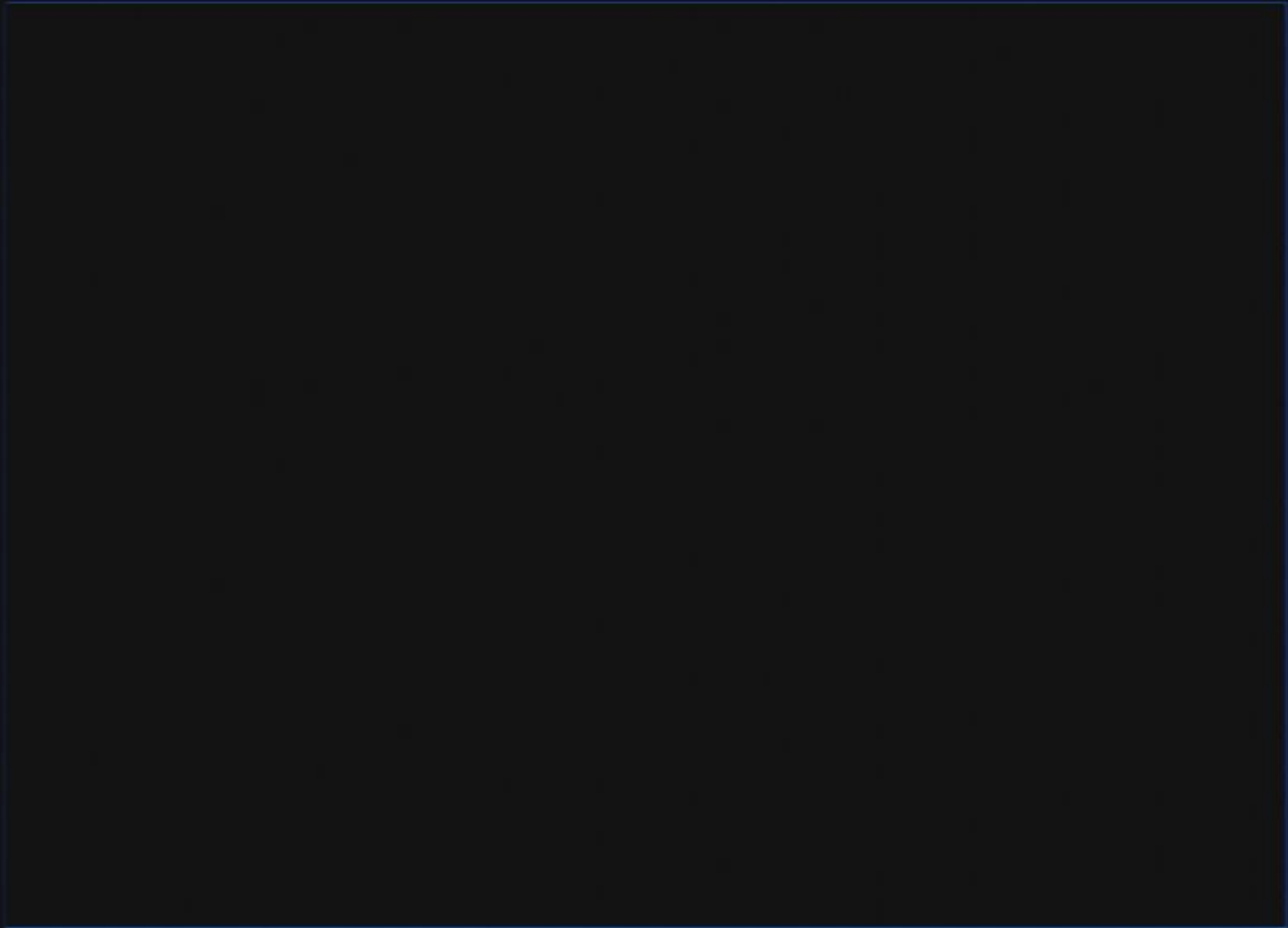
Curvature

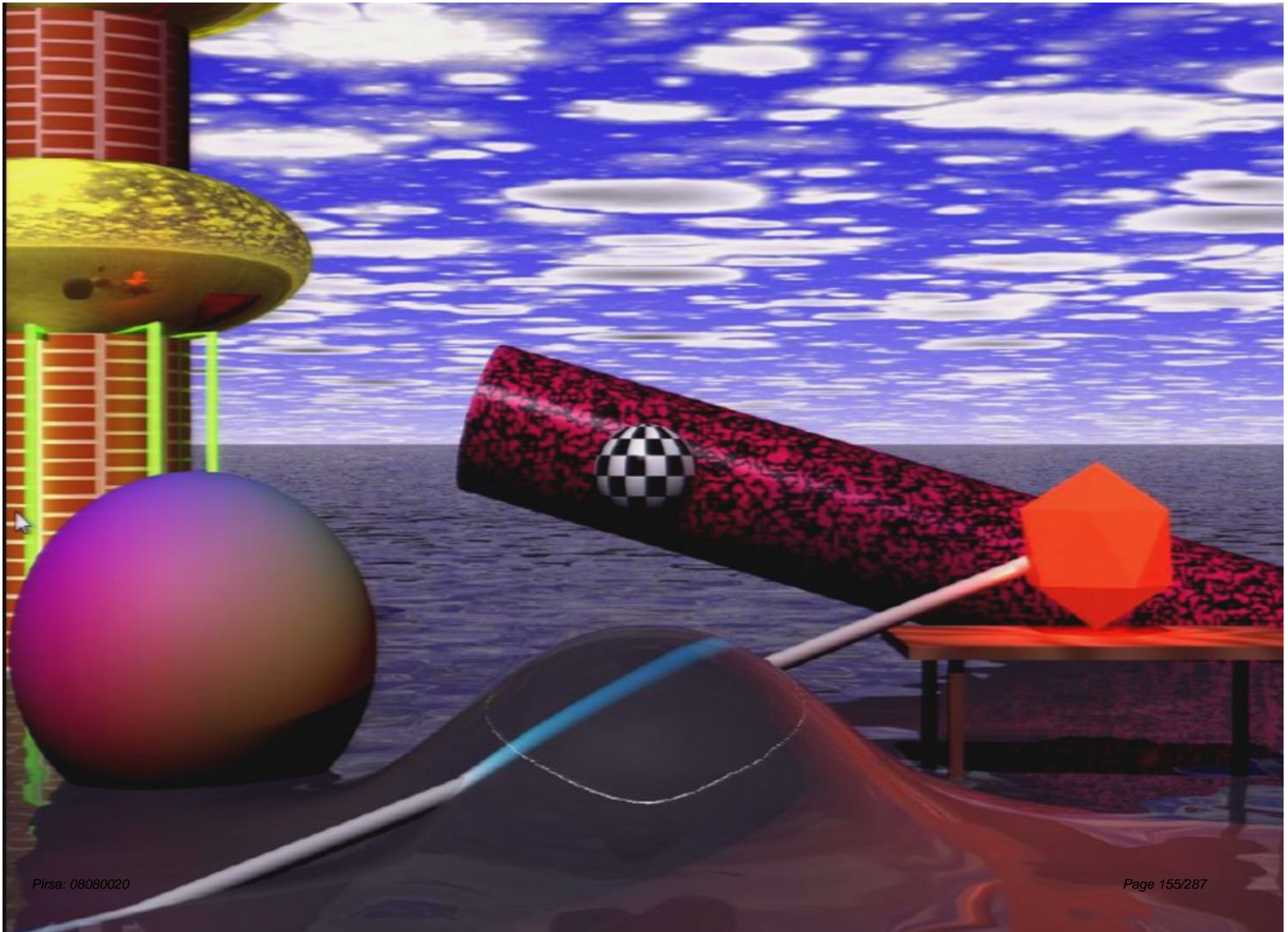


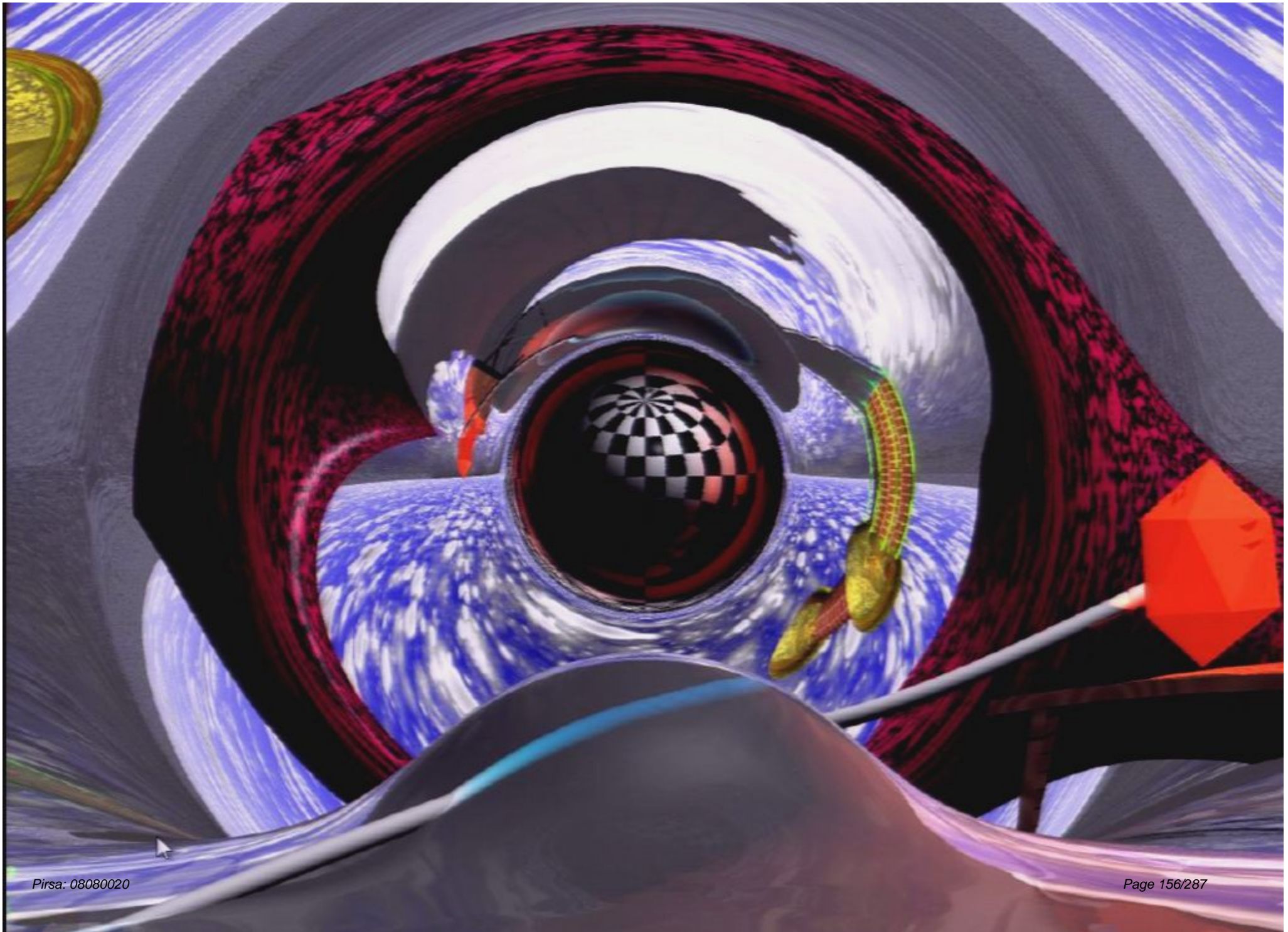
Curvature



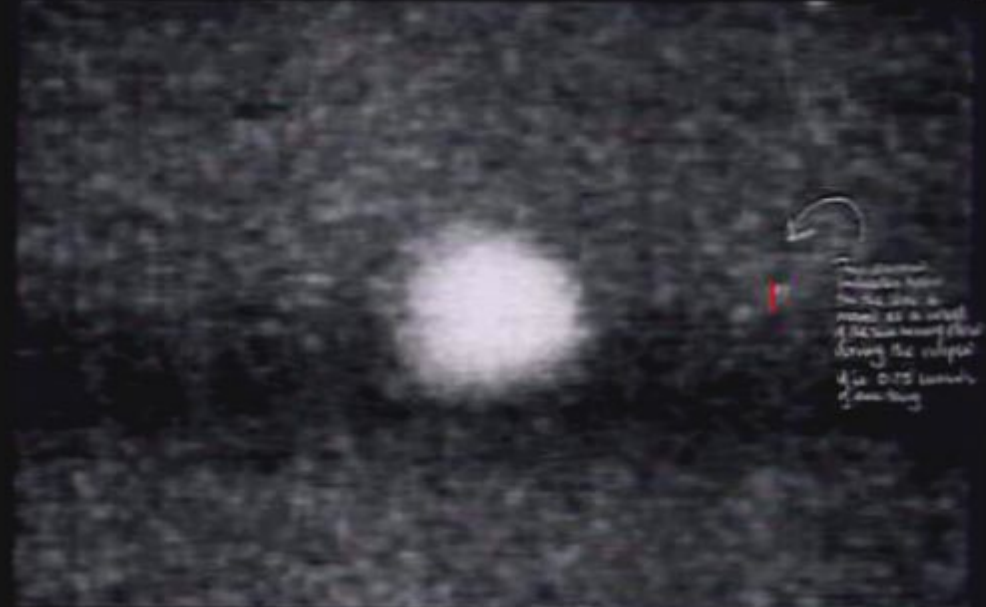
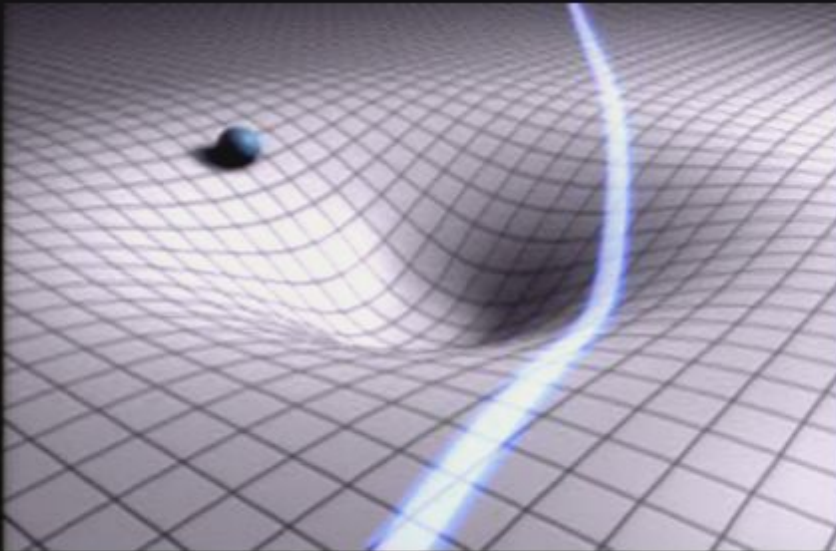
Curvature







1919 Verification



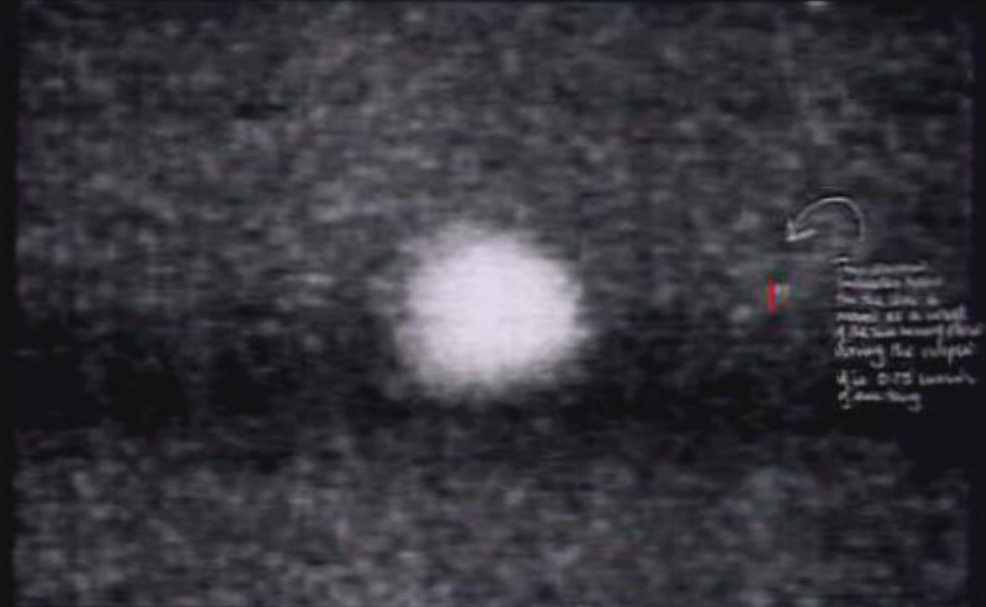
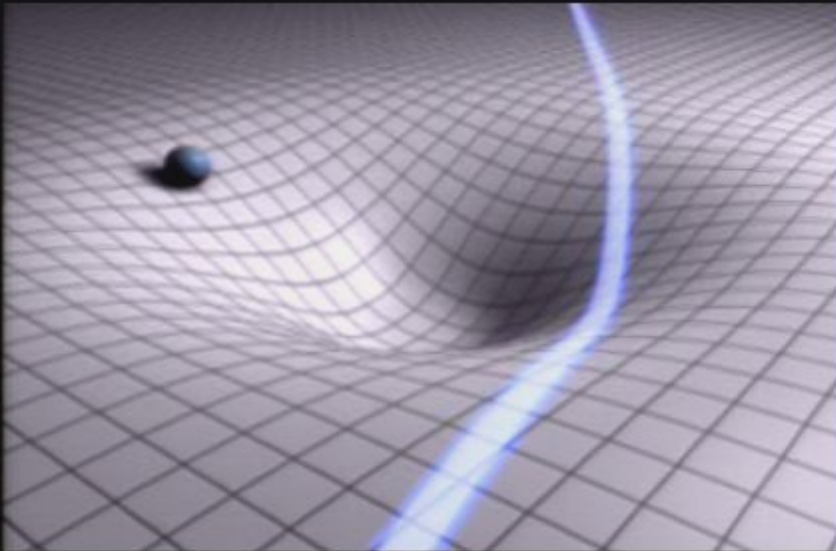
This image is magnified 251 times, compared with glass plate.



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



This image is superposed 291 hours, compared with glass plate.

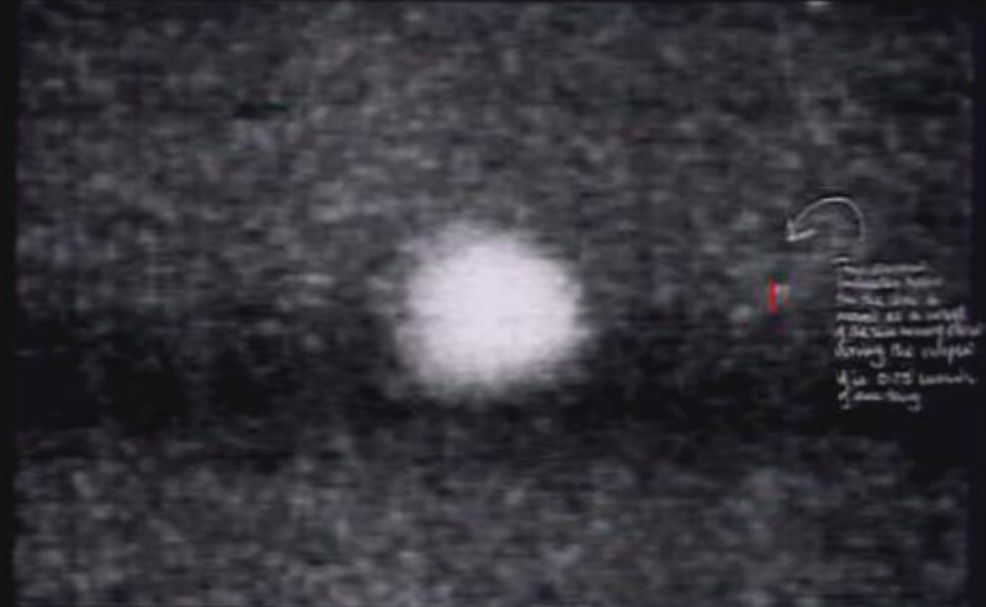
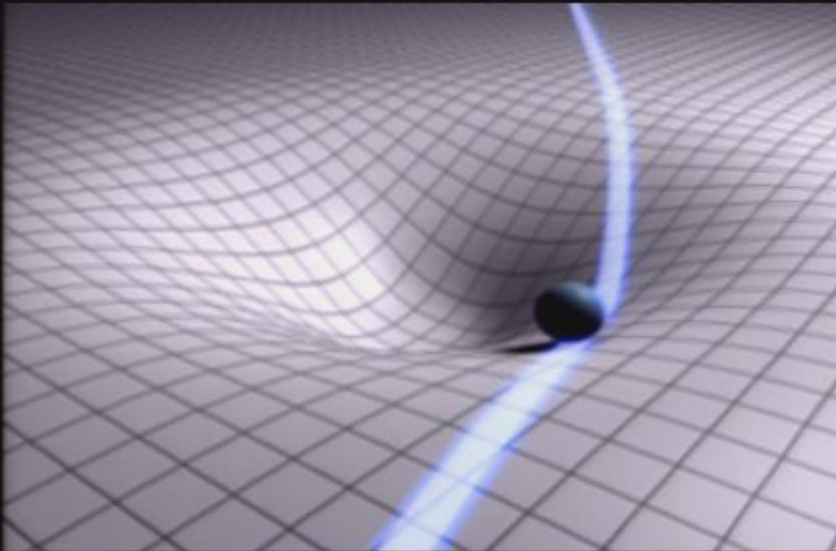


Pirsa: 08080020



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.

1919 Verification



This image is superimposed 291 times, compared with other plates.

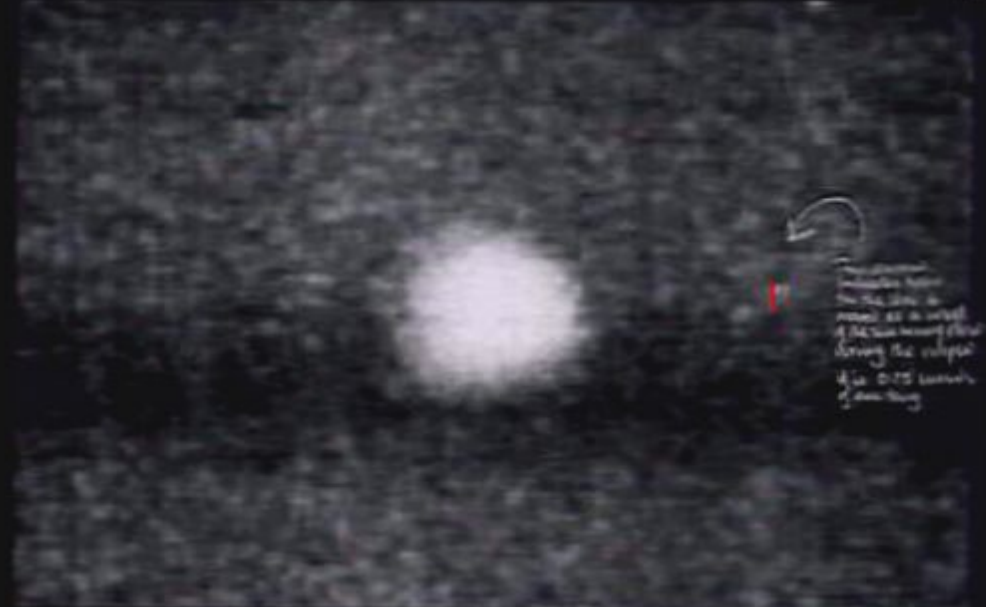
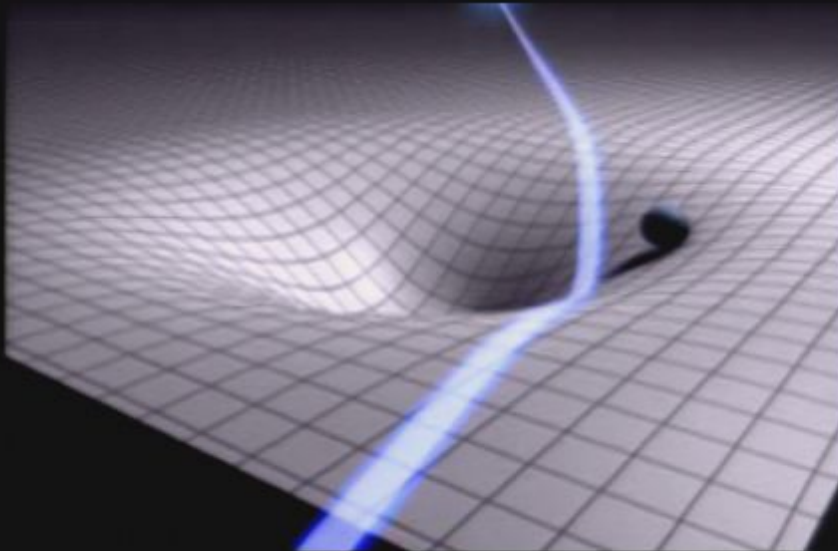


Pirsa: 08080020



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.

1919 Verification



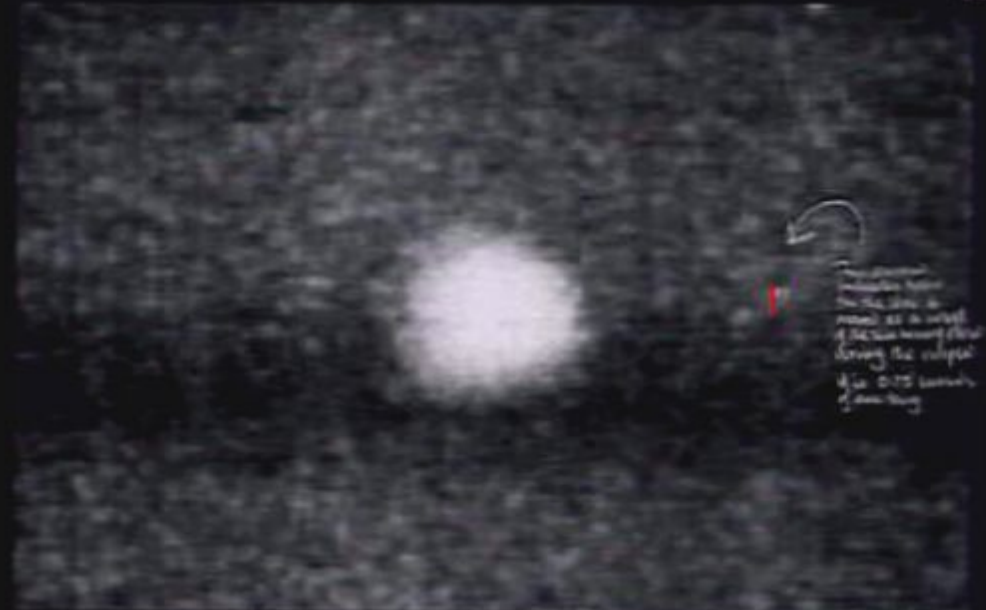
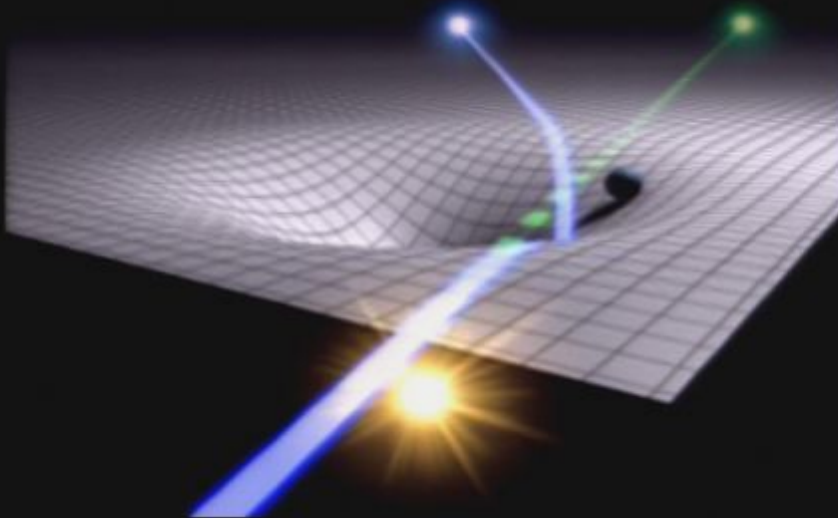
This image is superimposed 291 times, compared with glass plate



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



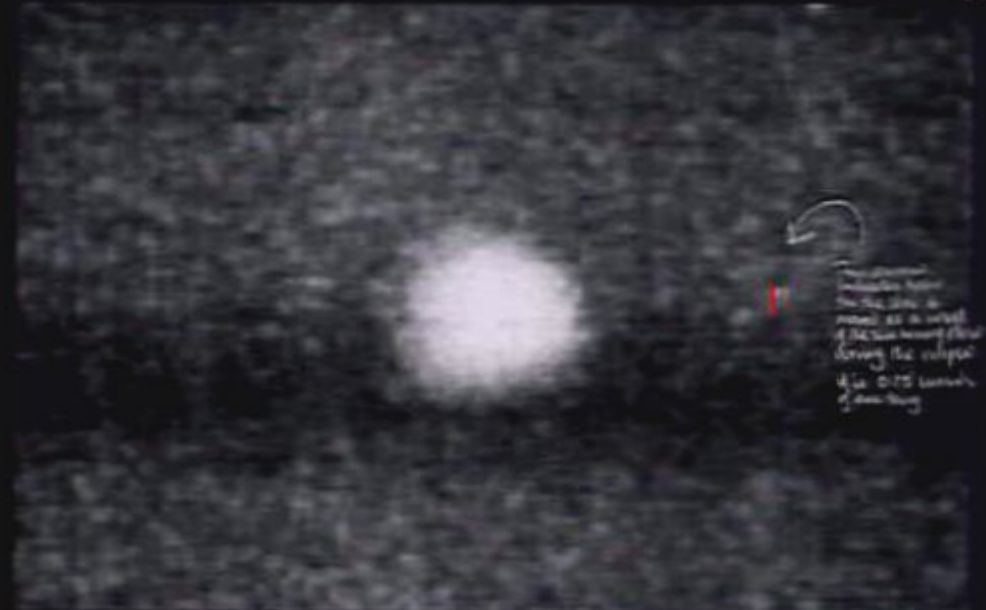
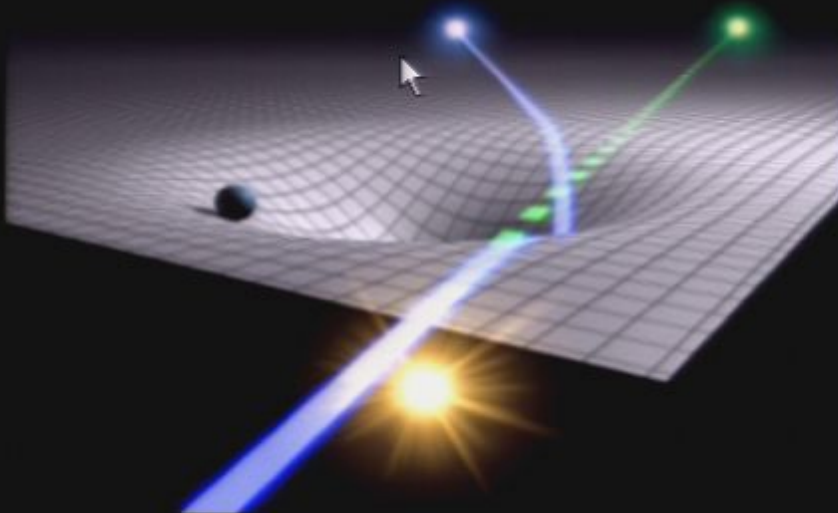
This image is superposed 291 times, compared with glass plate



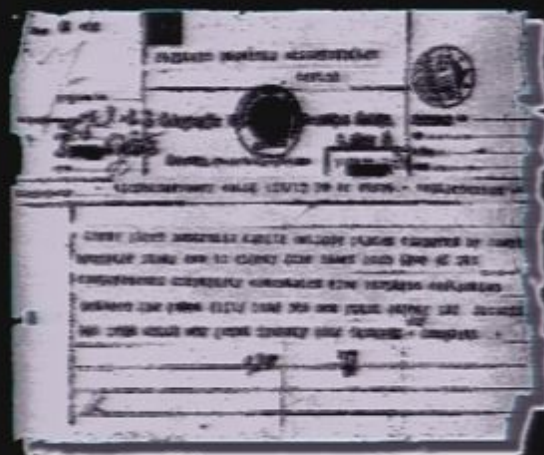
The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



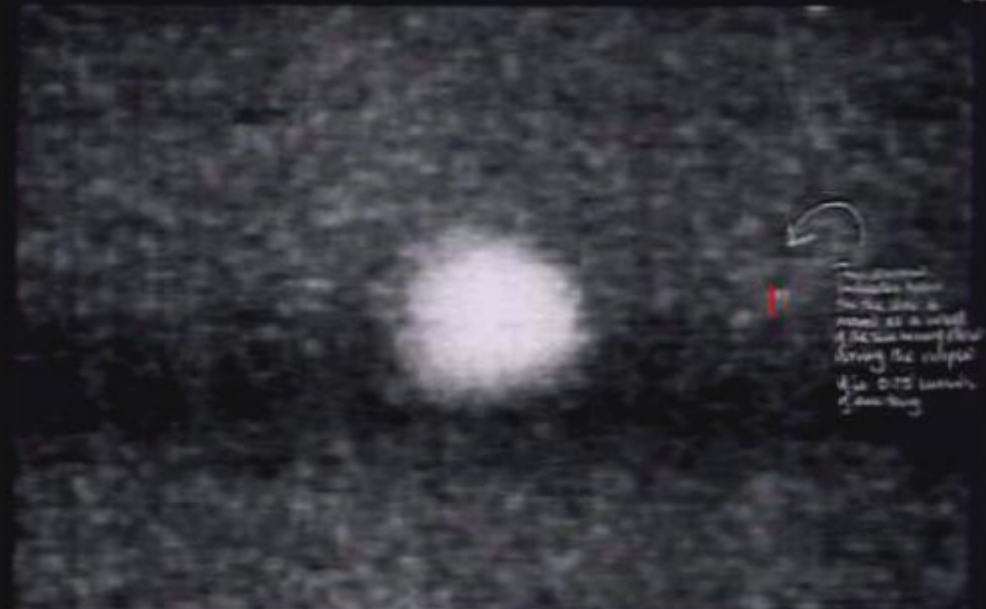
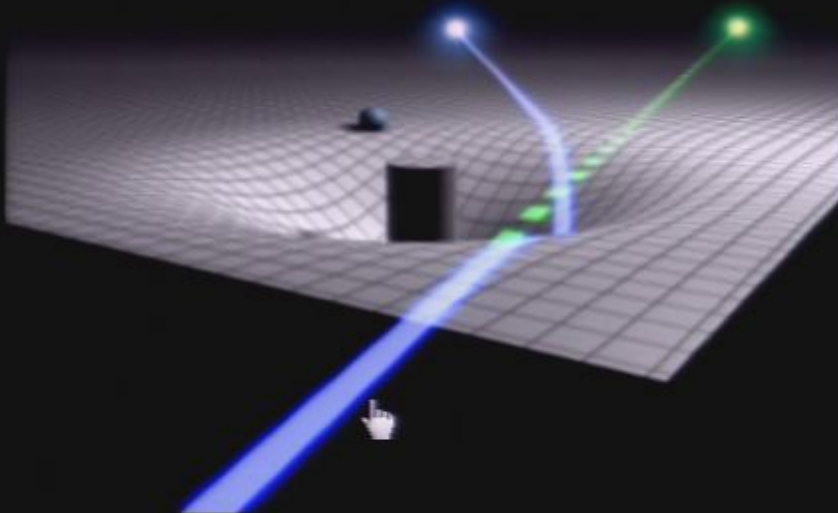
This image is superposed 281 times, compared with glass plate



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



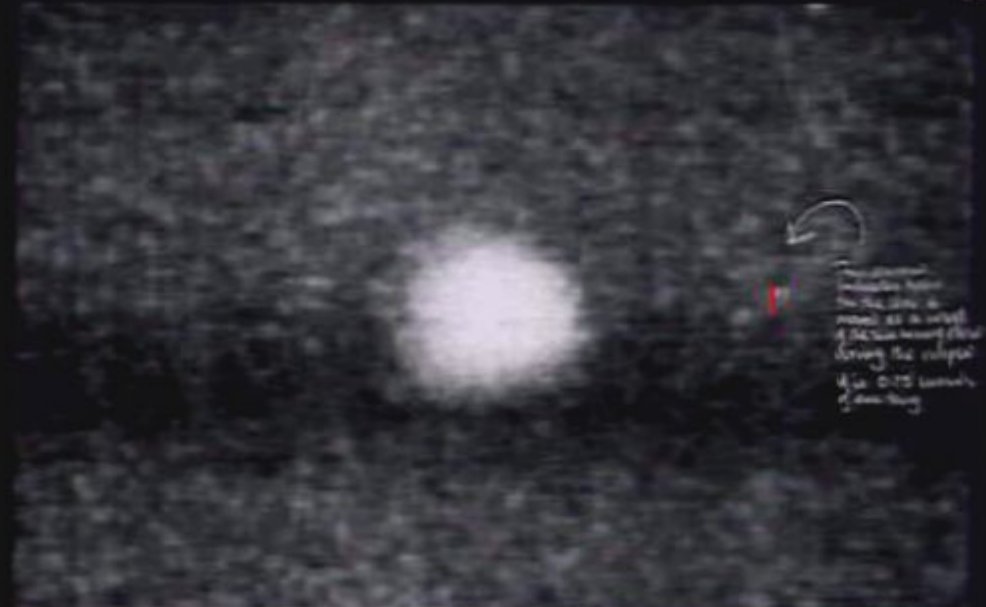
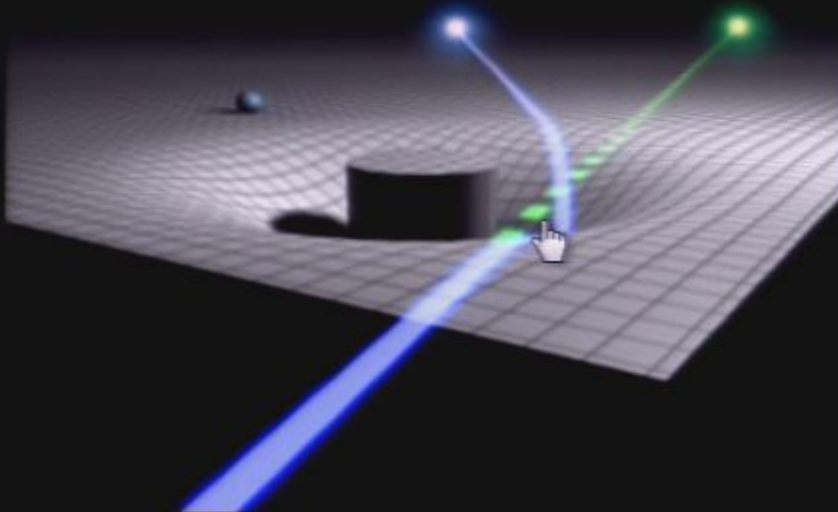
This image is superimposed 291 times, compared with a normal plate.



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



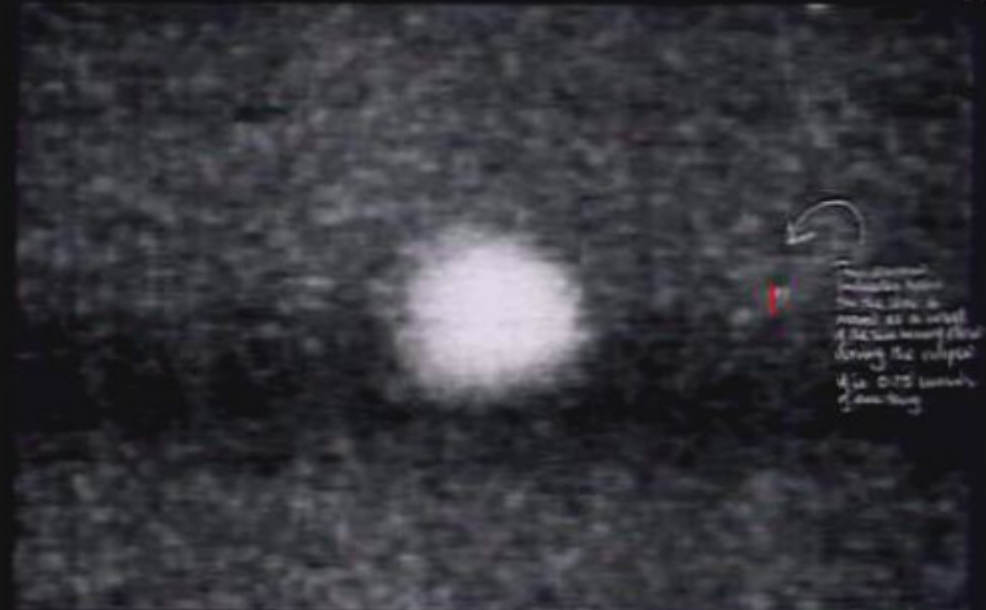
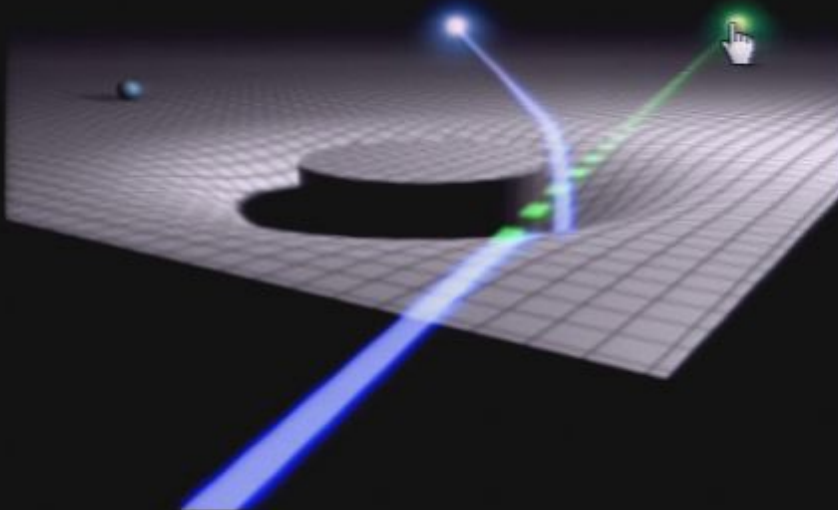
This image is superposed 281 times, compared with glass plate



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



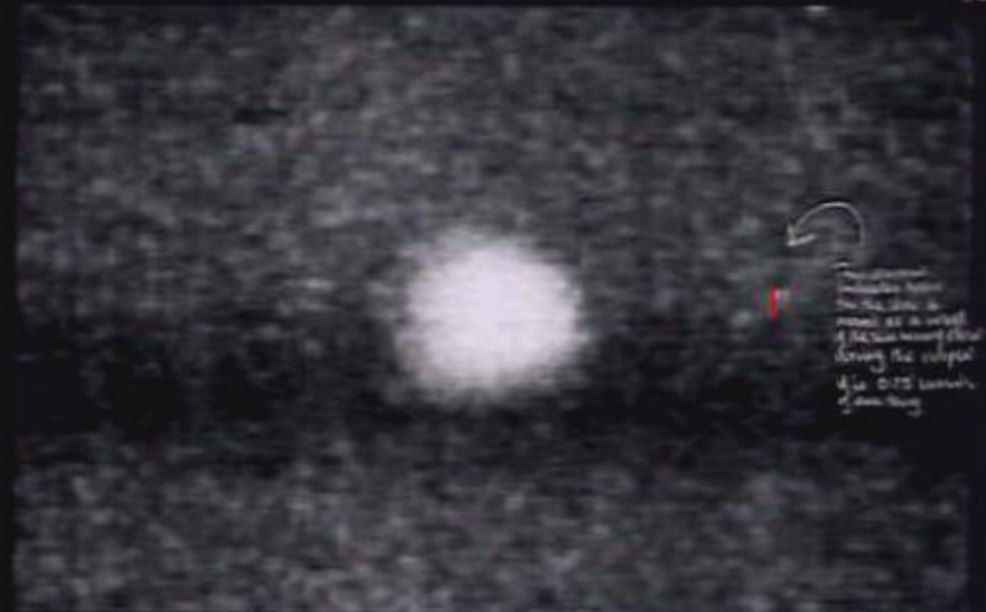
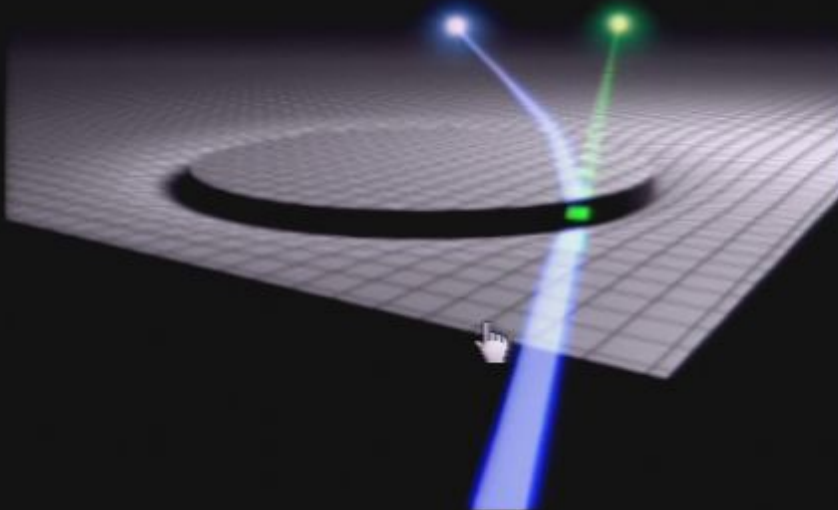
This image is superimposed 281 times, compared with glass plate.



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



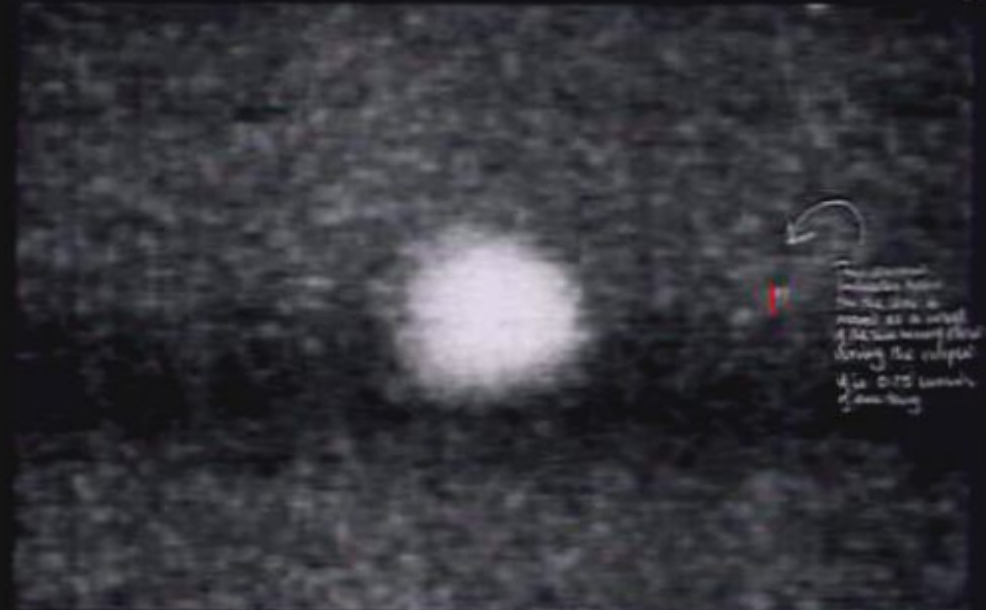
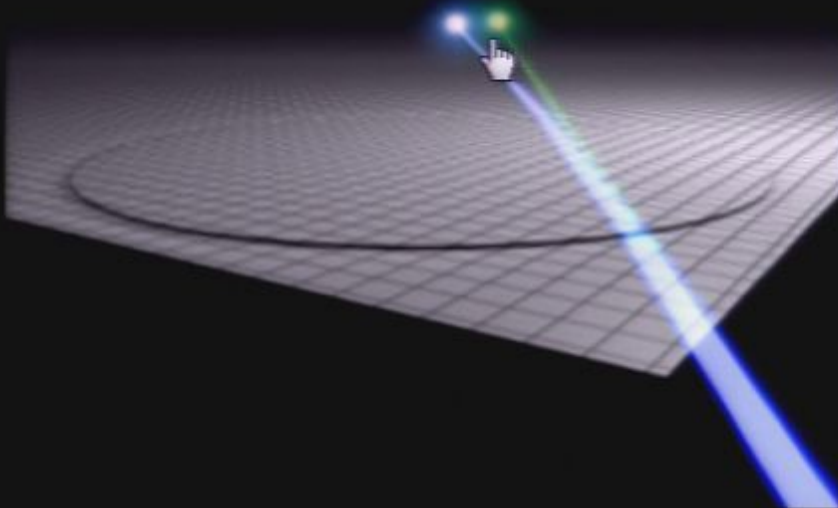
This image is superimposed 291 times, measured with glass plate.



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



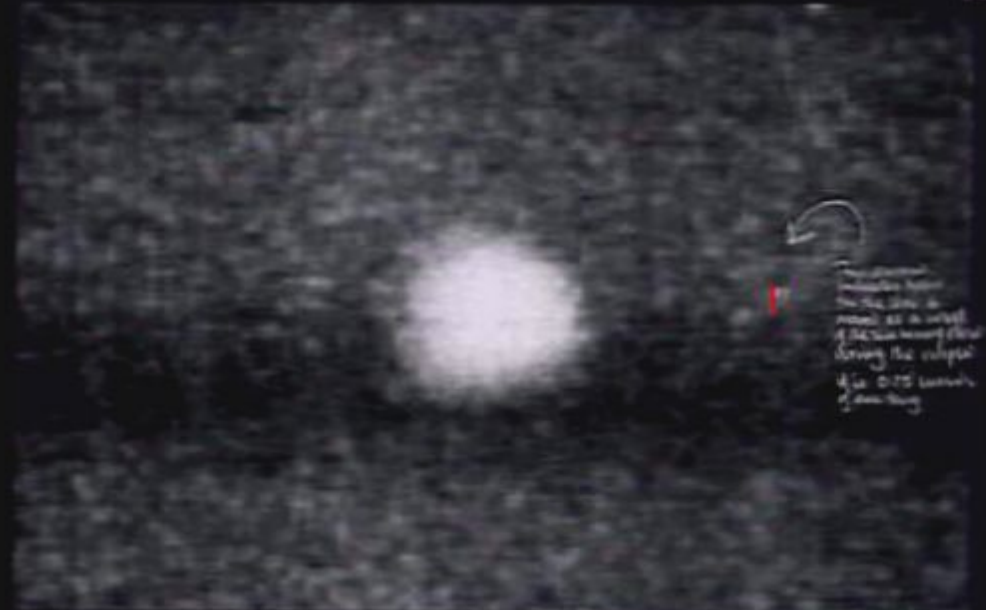
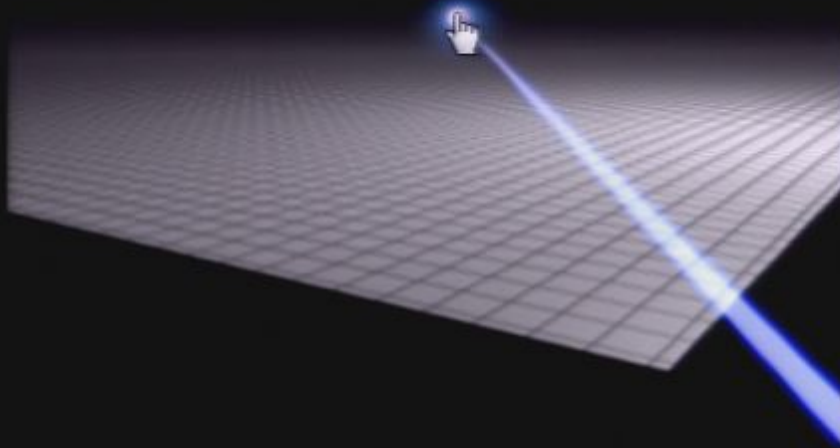
This image is superposed 281 times, compared with a glass plate.



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



1919 Verification



This image is magnified 251 times, compared with glass plate.



The final proof: the small red line shows how far the position of the star has been shifted by the Sun's gravity.



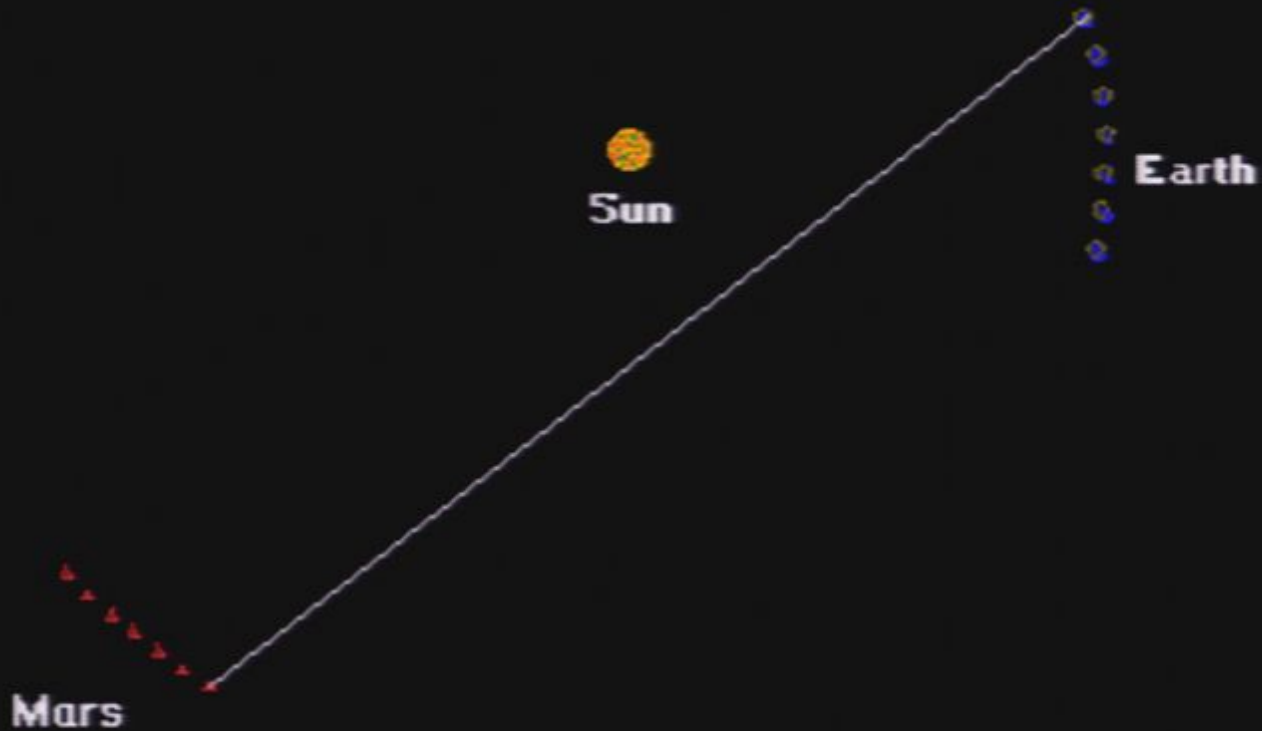
General Relativity Test (1976)

60
Excess Time Delay,
Microseconds



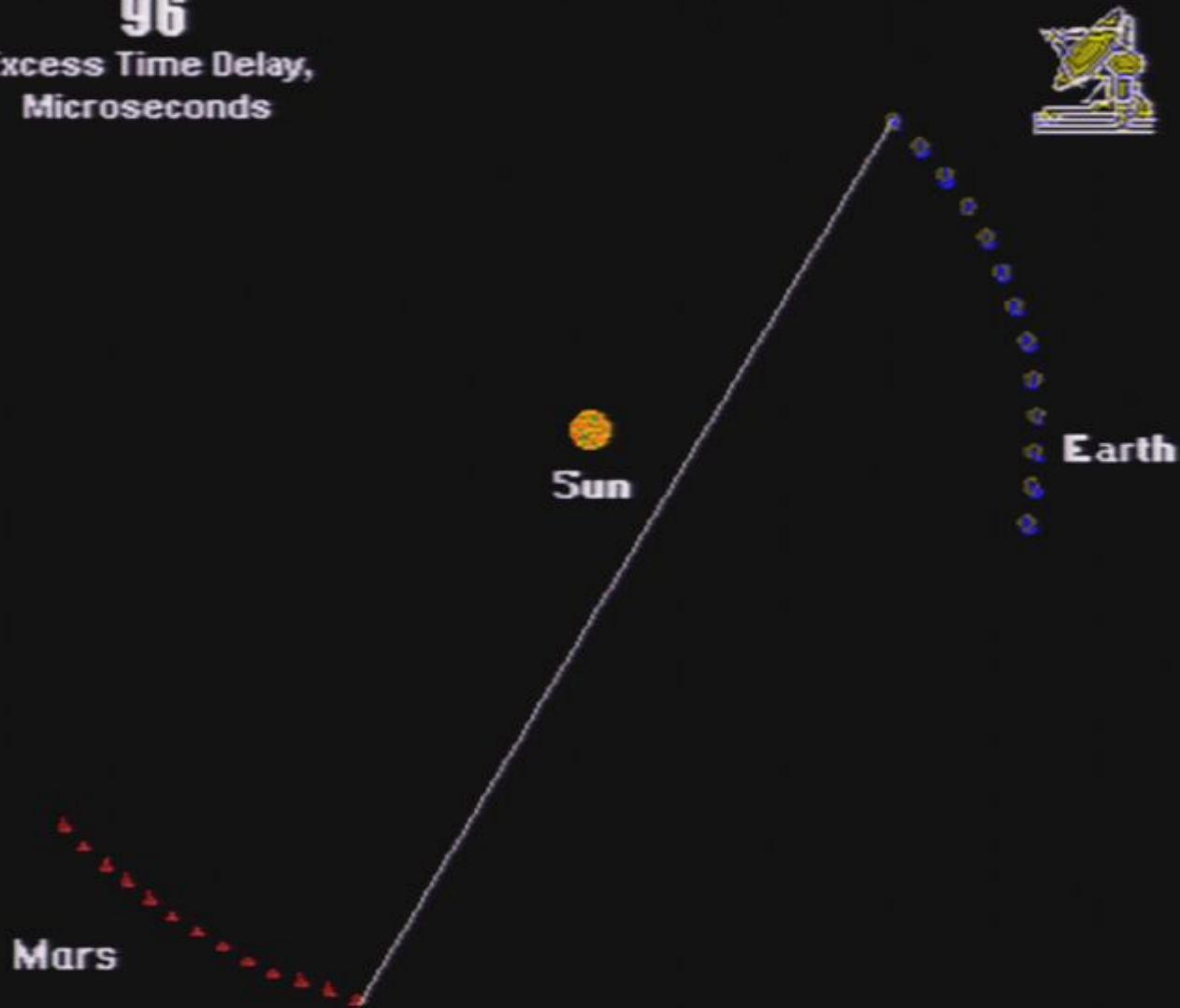
General Relativity Test (1976)

74
Excess Time Delay,
Microseconds



General Relativity Test (1976)

96
Excess Time Delay,
Microseconds



General Relativity Test (1976)

124
Excess Time Delay,
Microseconds



General Relativity Test (1976)

190
Excess Time Delay,
Microseconds



General Relativity Test (1976)

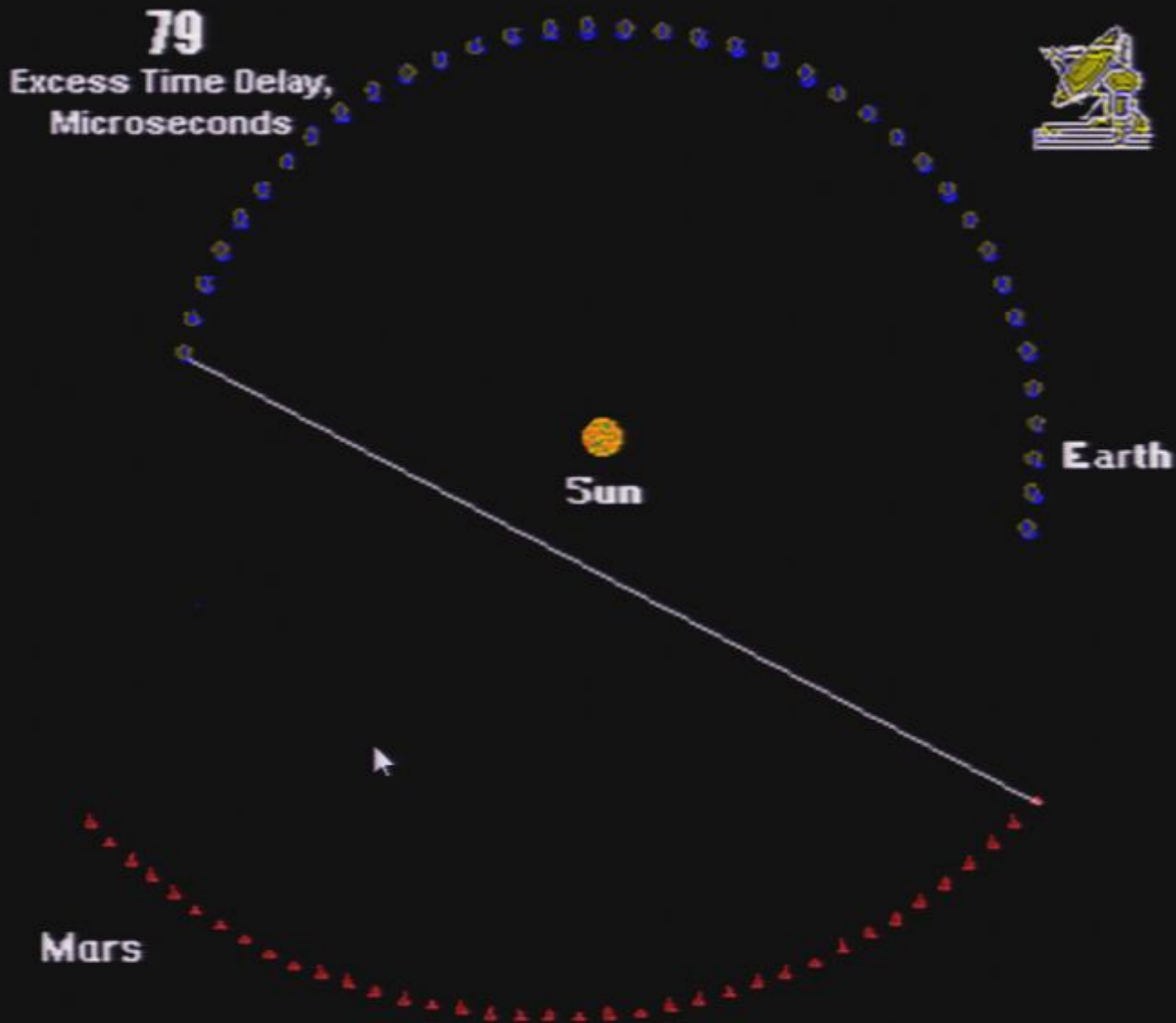
190
Excess Time Delay,
Microseconds



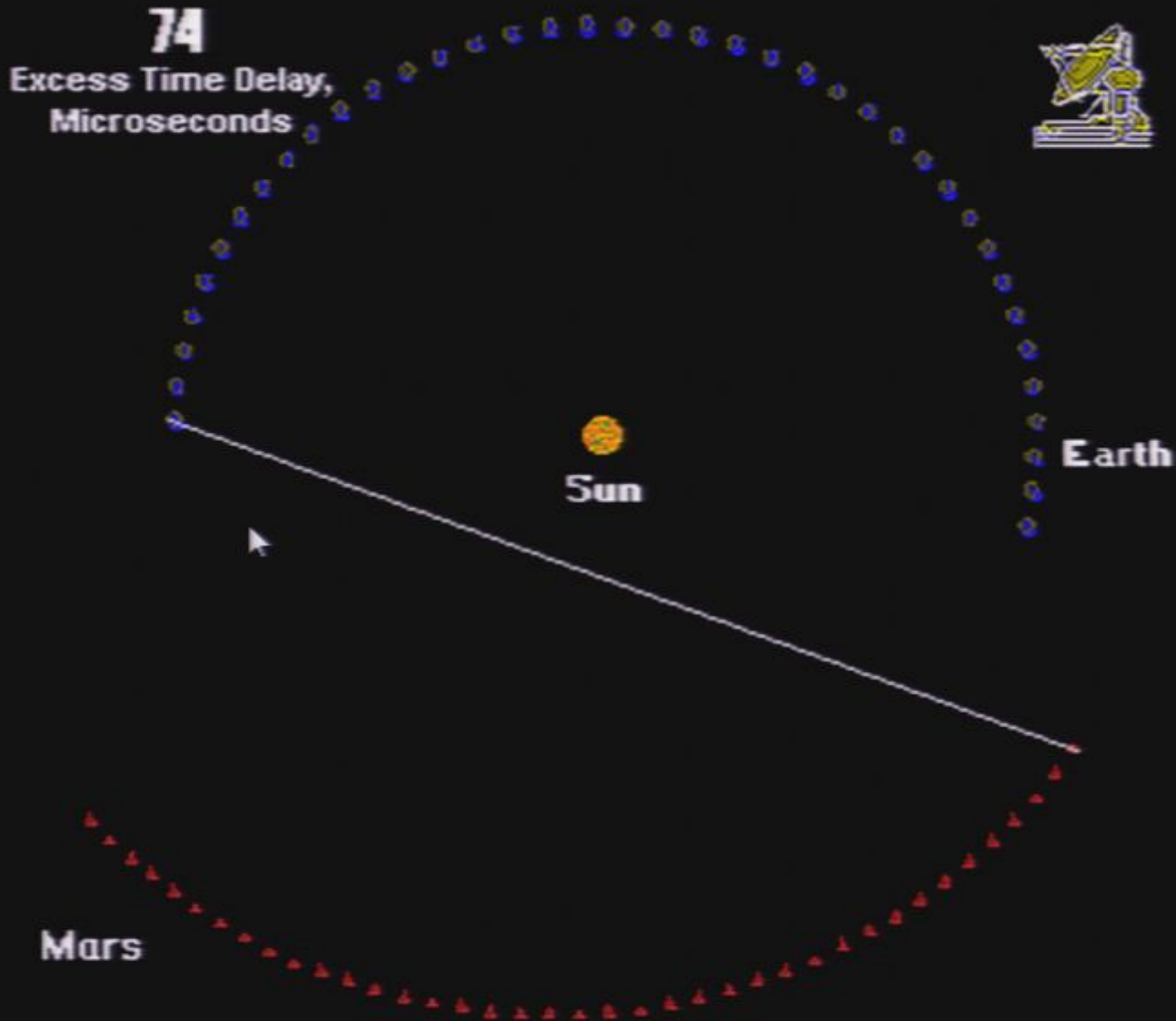
General Relativity Test (1976)



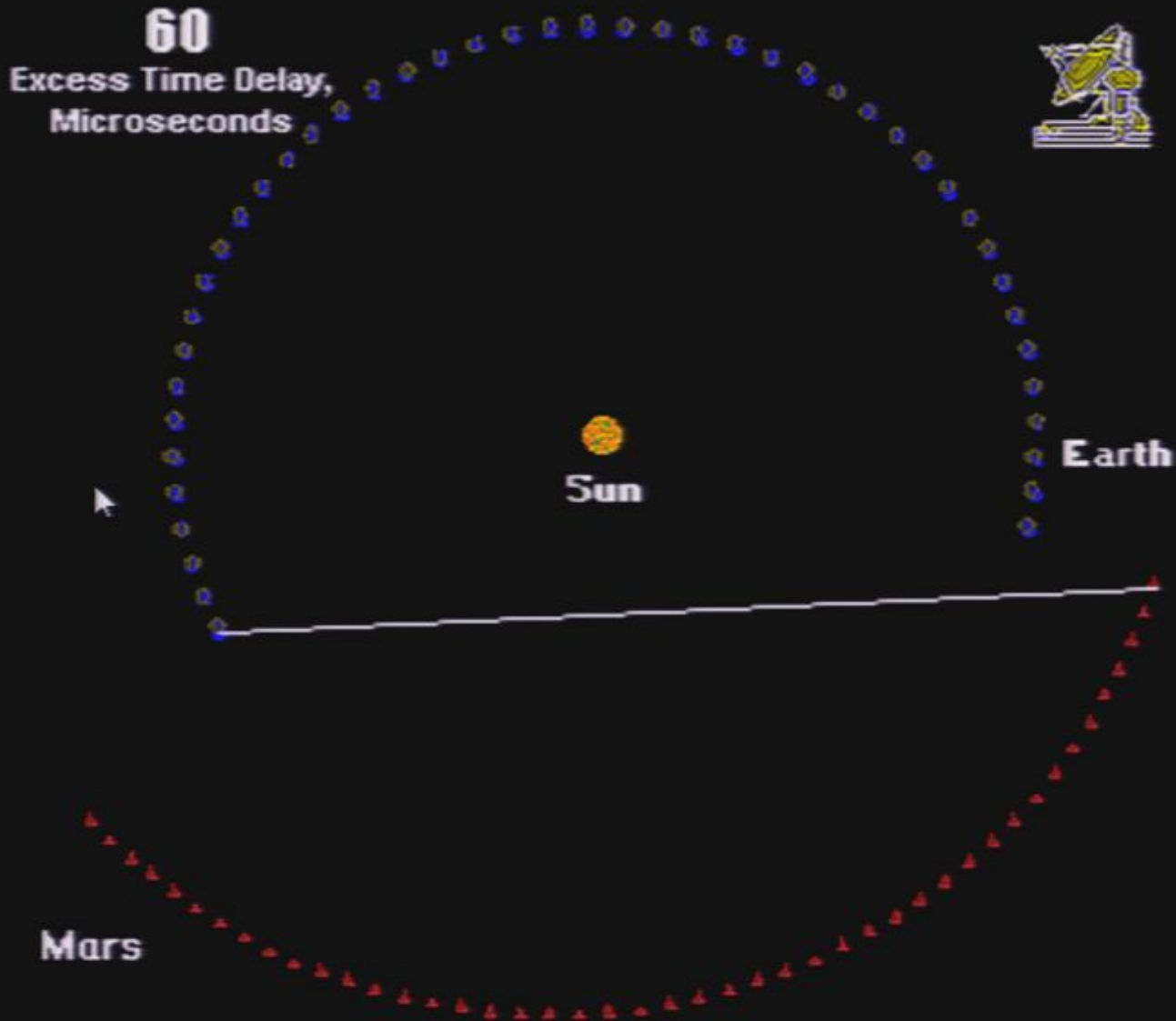
General Relativity Test (1976)



General Relativity Test (1976)



General Relativity Test (1976)



General Relativity Test (1976)

65
Excess Time Delay,
Microseconds



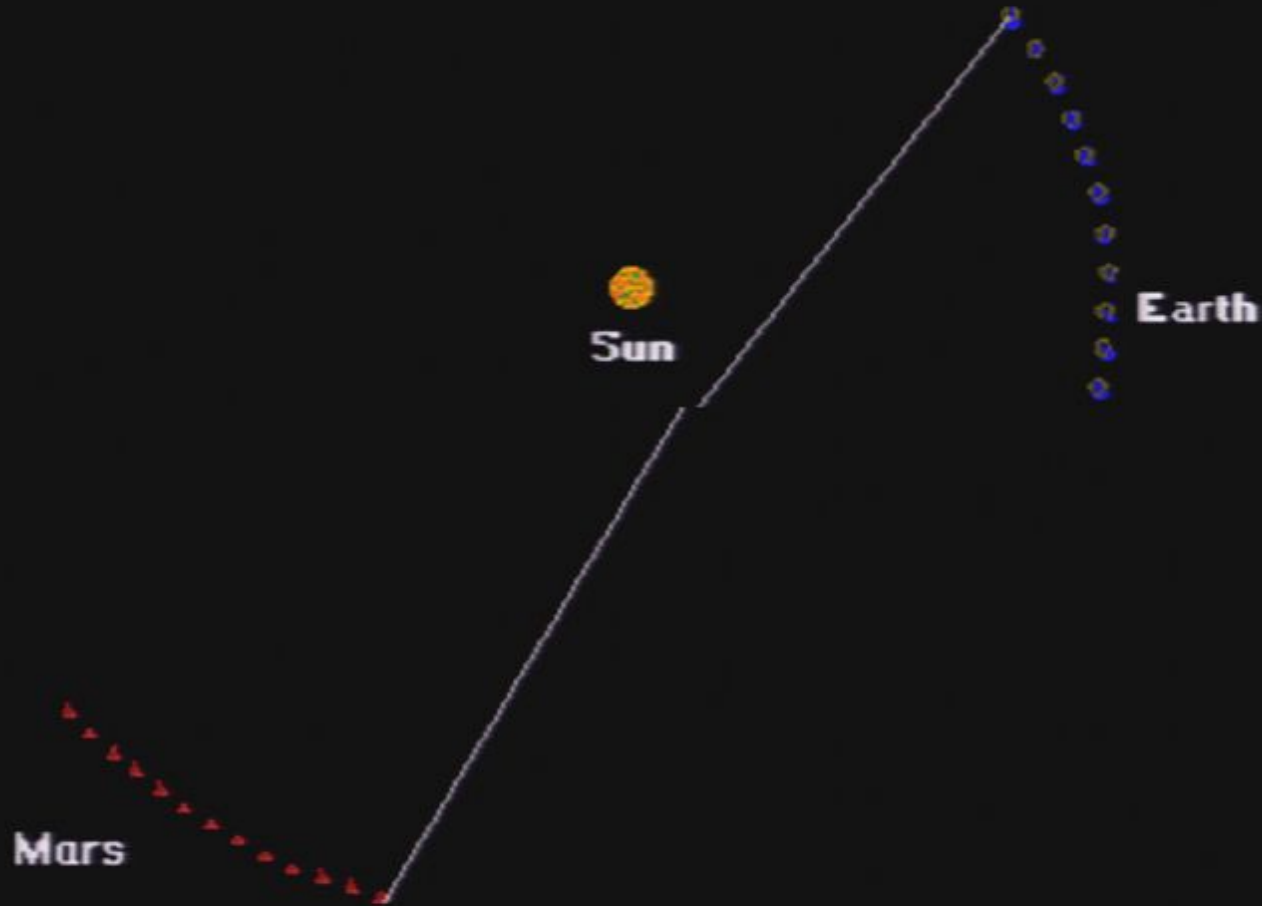
General Relativity Test (1976)

79
Excess Time Delay,
Microseconds



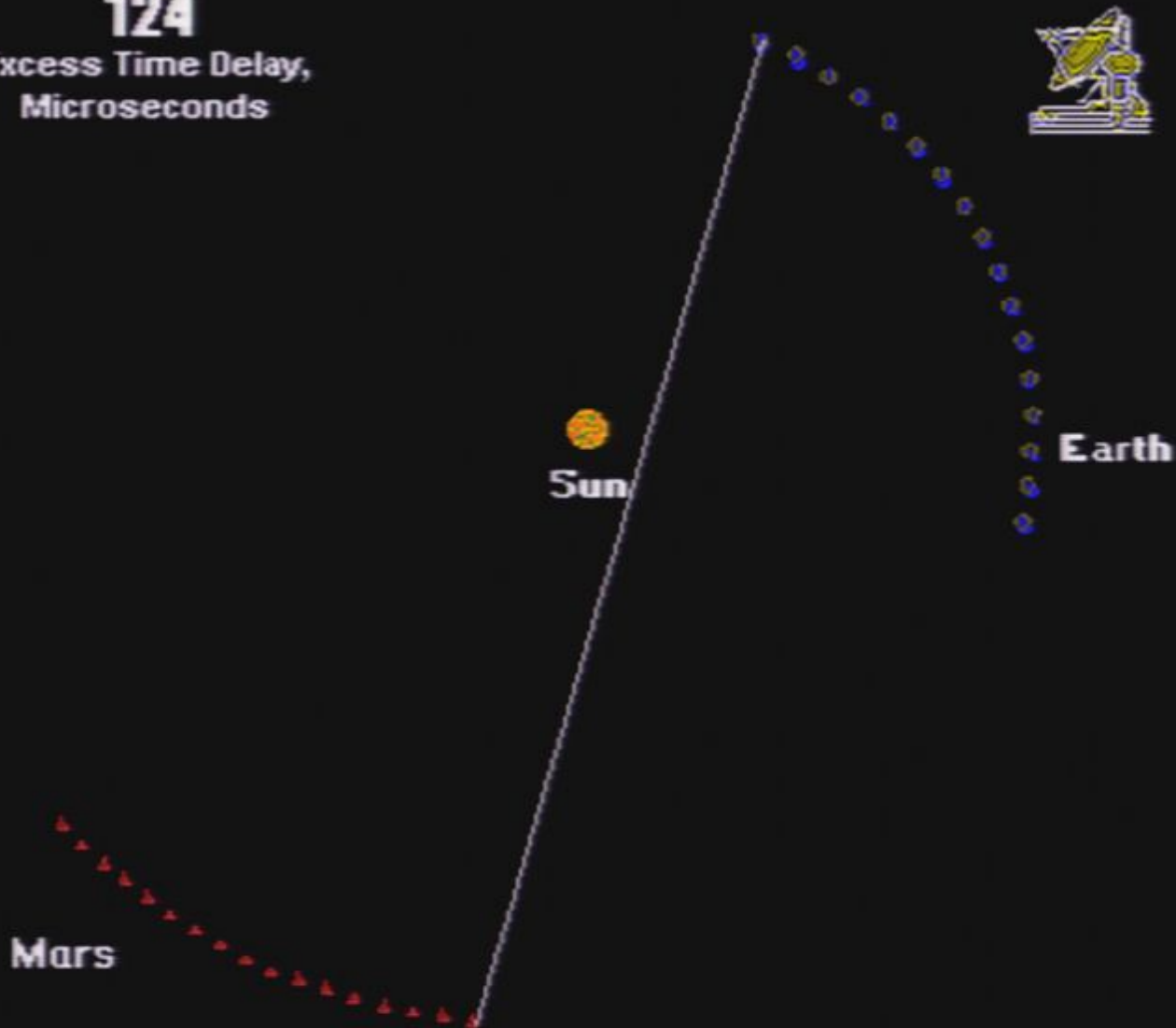
General Relativity Test (1976)

85
Excess Time Delay,
Microseconds



General Relativity Test (1976)

124
Excess Time Delay,
Microseconds



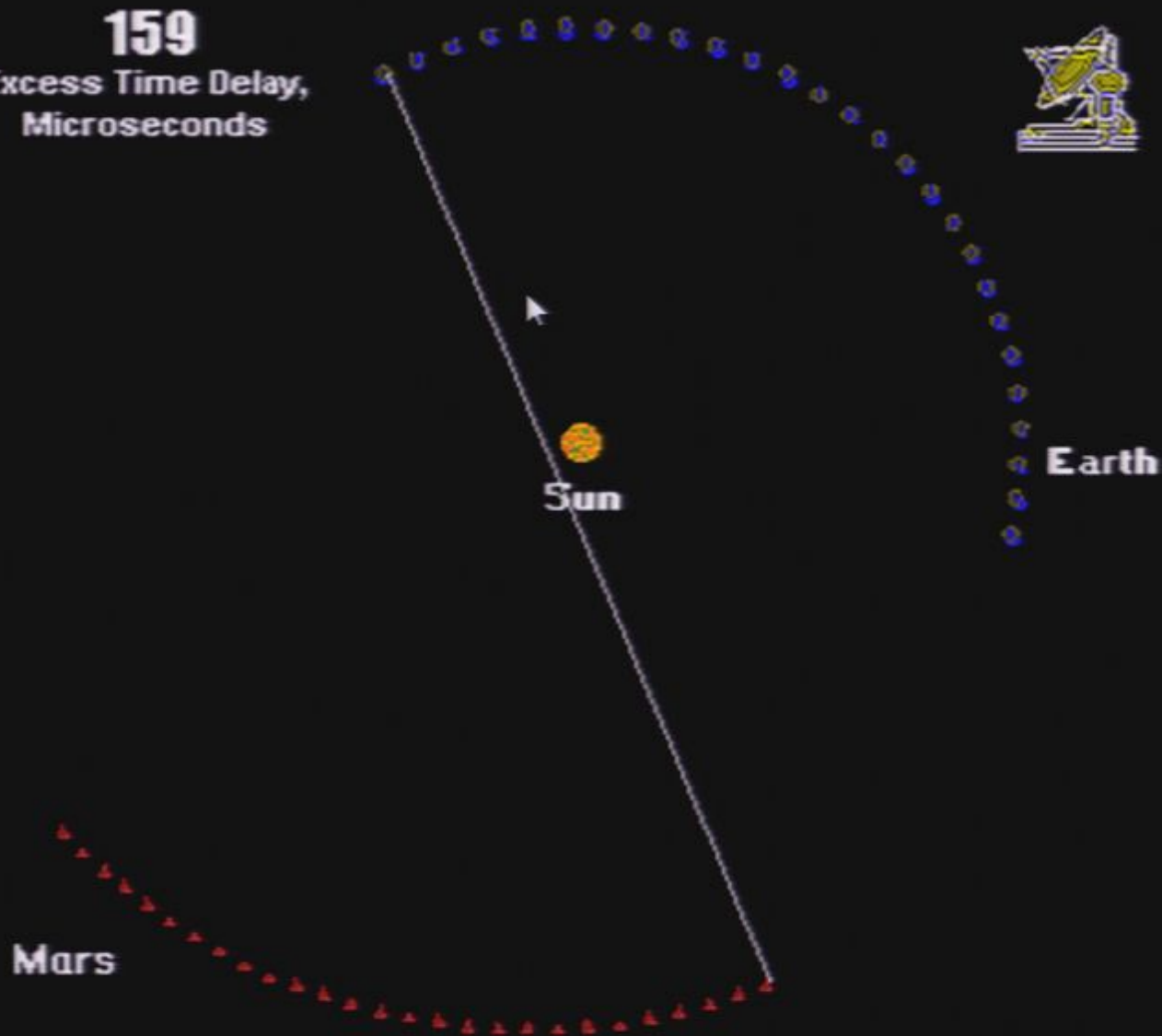
General Relativity Test (1976)

190
Excess Time Delay,
Microseconds

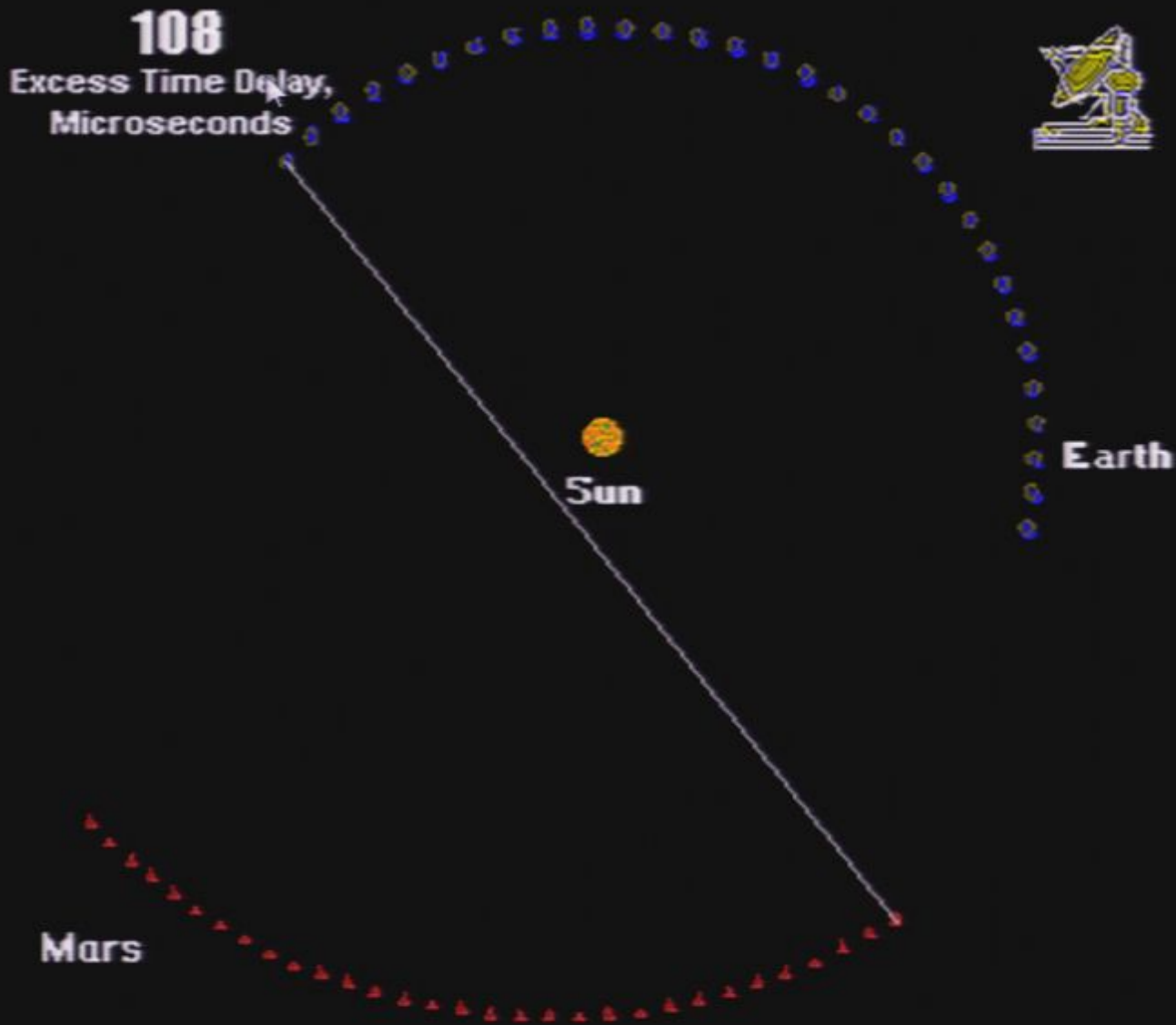


General Relativity Test (1976)

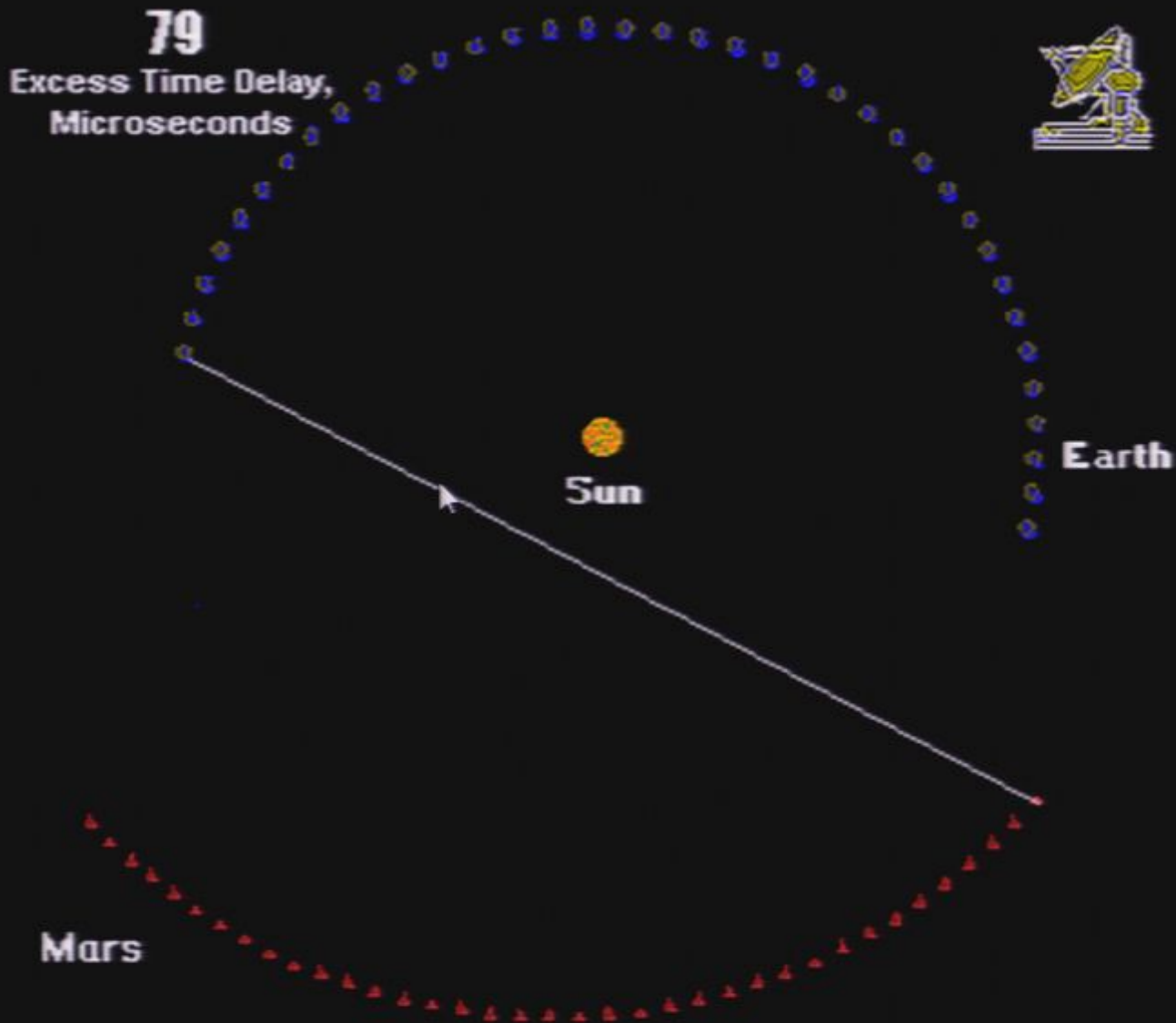
159
Excess Time Delay,
Microseconds



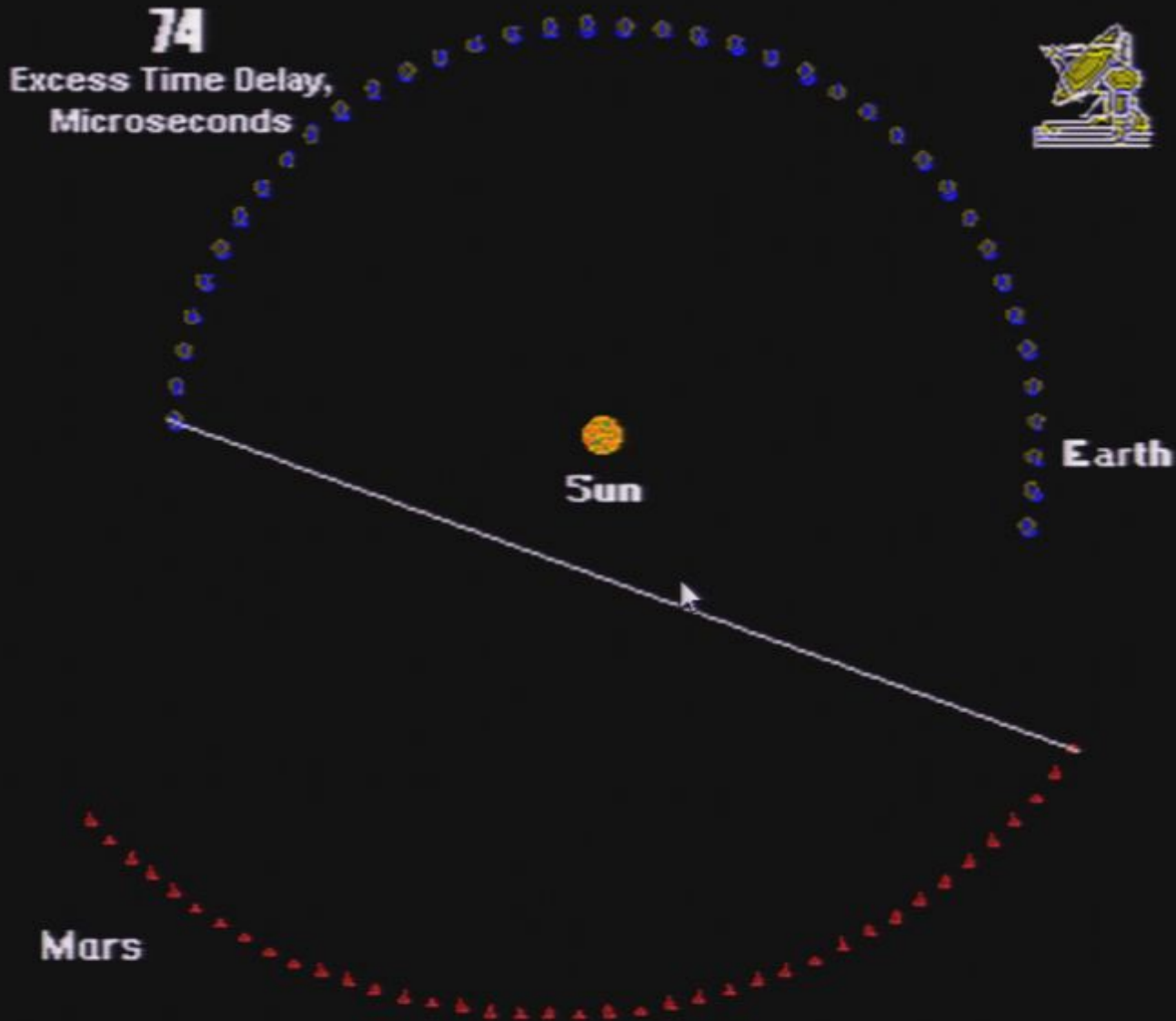
General Relativity Test (1976)



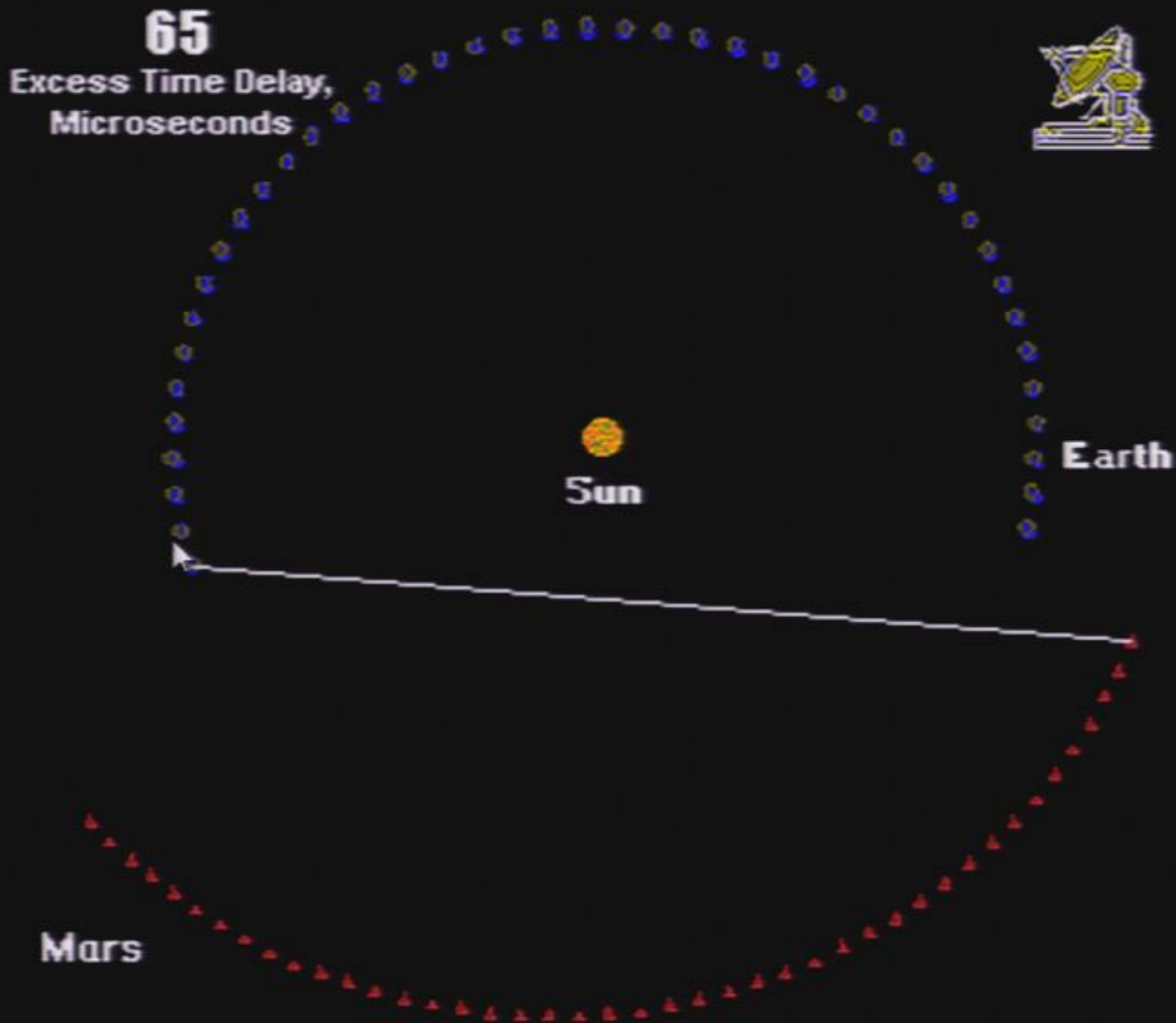
General Relativity Test (1976)



General Relativity Test (1976)



General Relativity Test (1976)



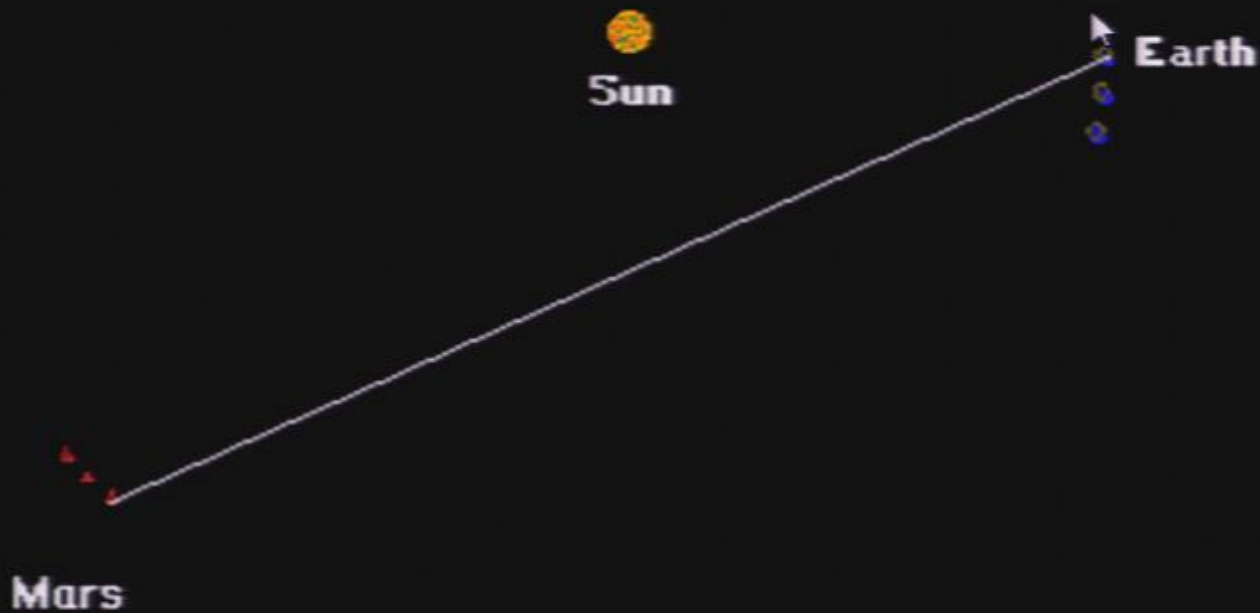
General Relativity Test (1976)

60
Excess Time Delay,
Microseconds



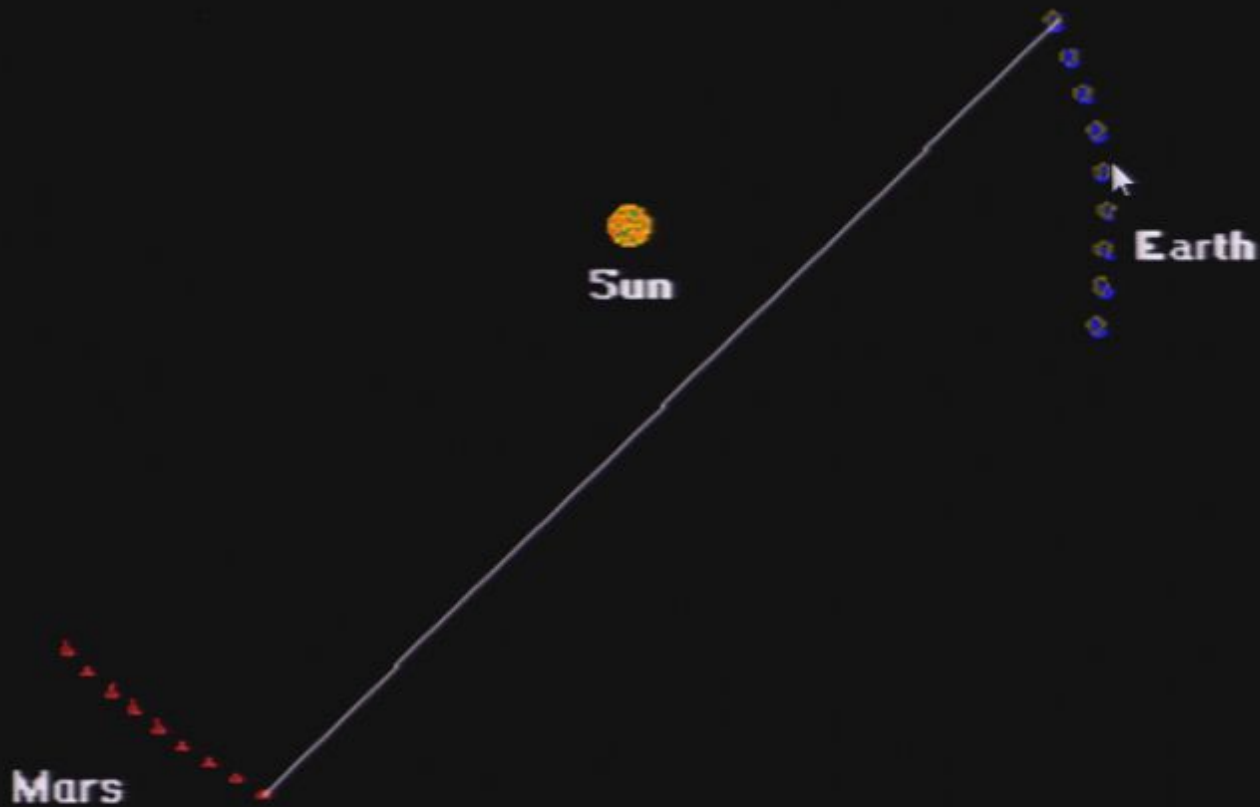
General Relativity Test (1976)

65
Excess Time Delay,
Microseconds



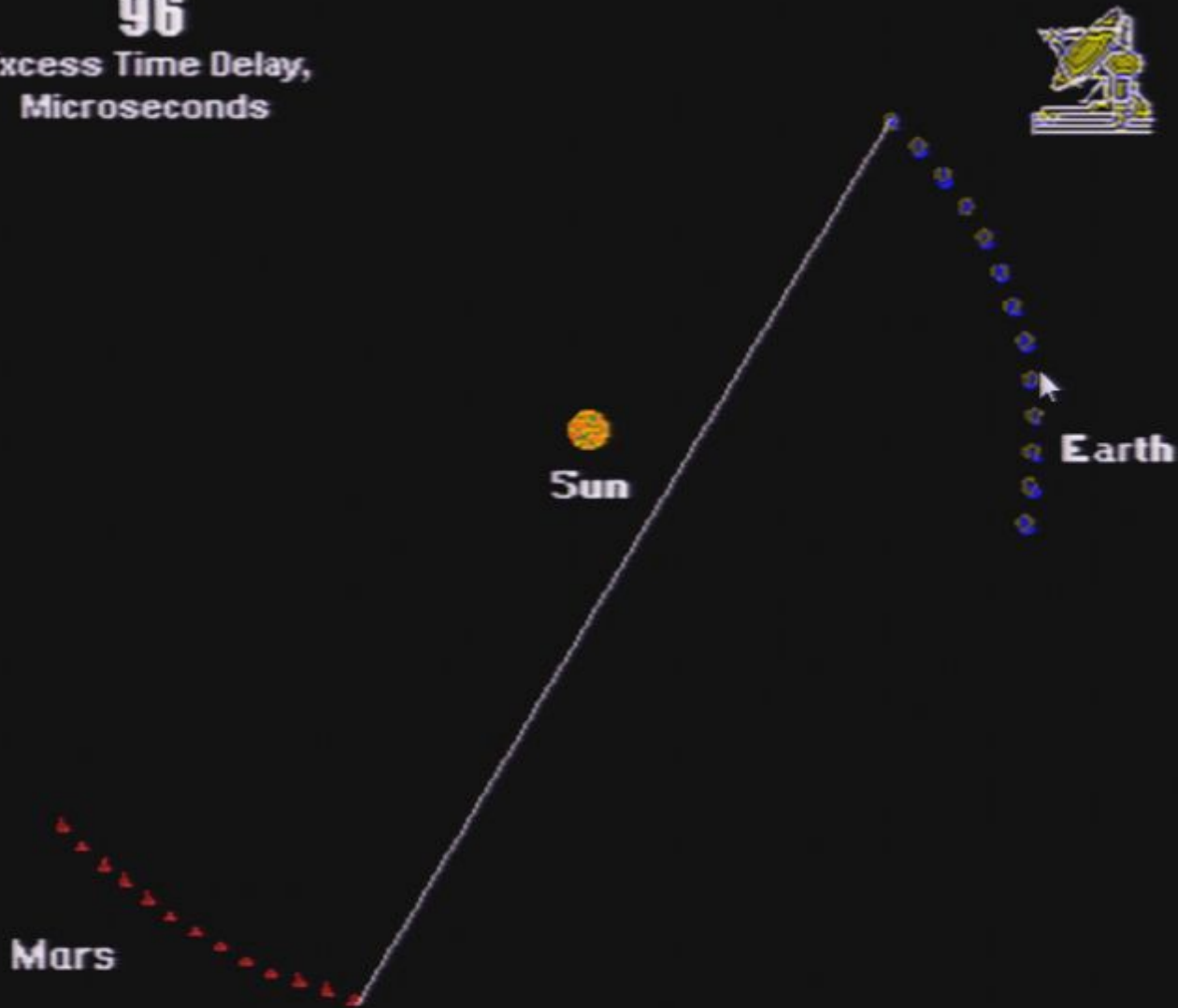
General Relativity Test (1976)

79
Excess Time Delay,
Microseconds



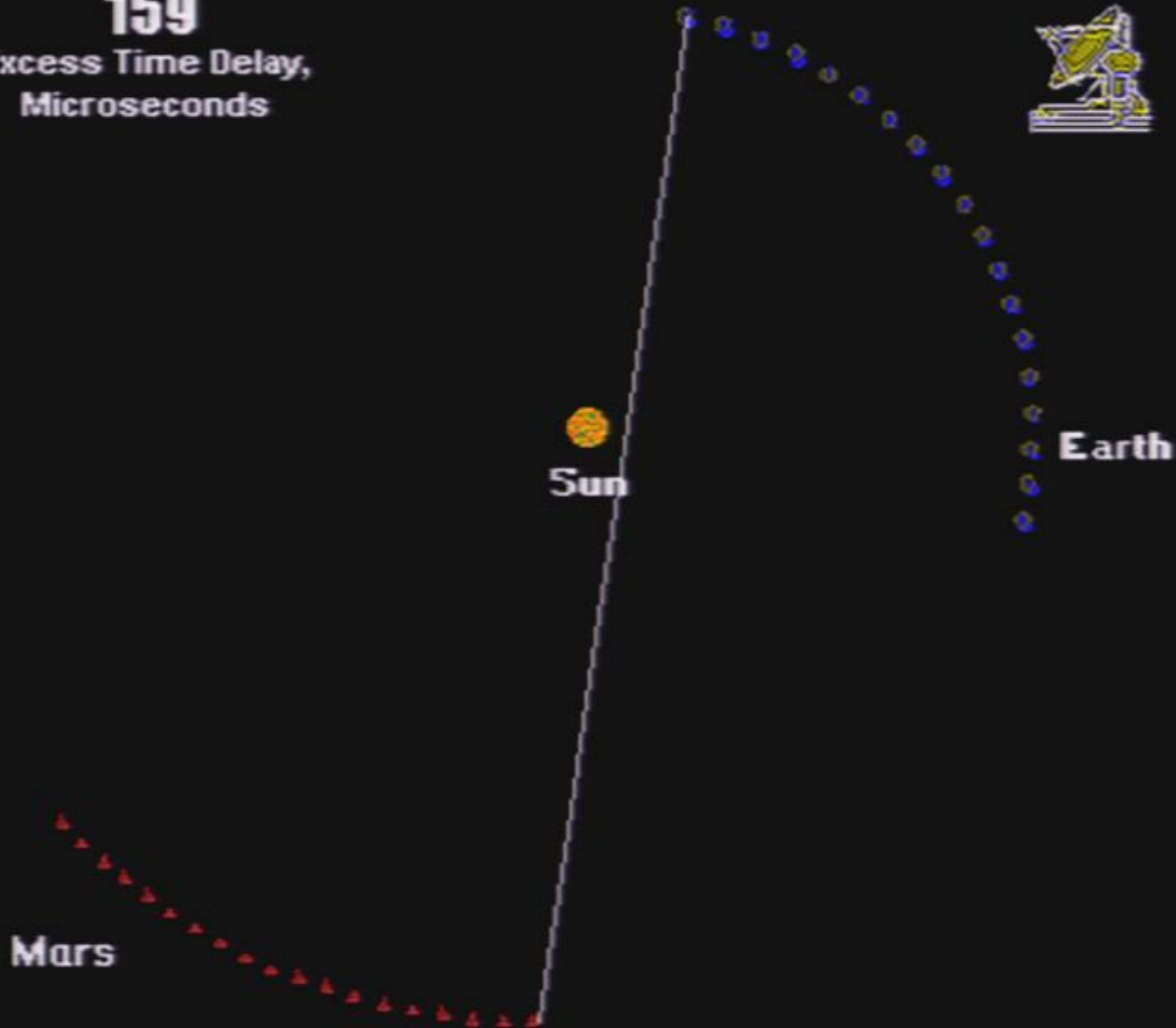
General Relativity Test (1976)

96
Excess Time Delay,
Microseconds



General Relativity Test (1976)

159
Excess Time Delay,
Microseconds



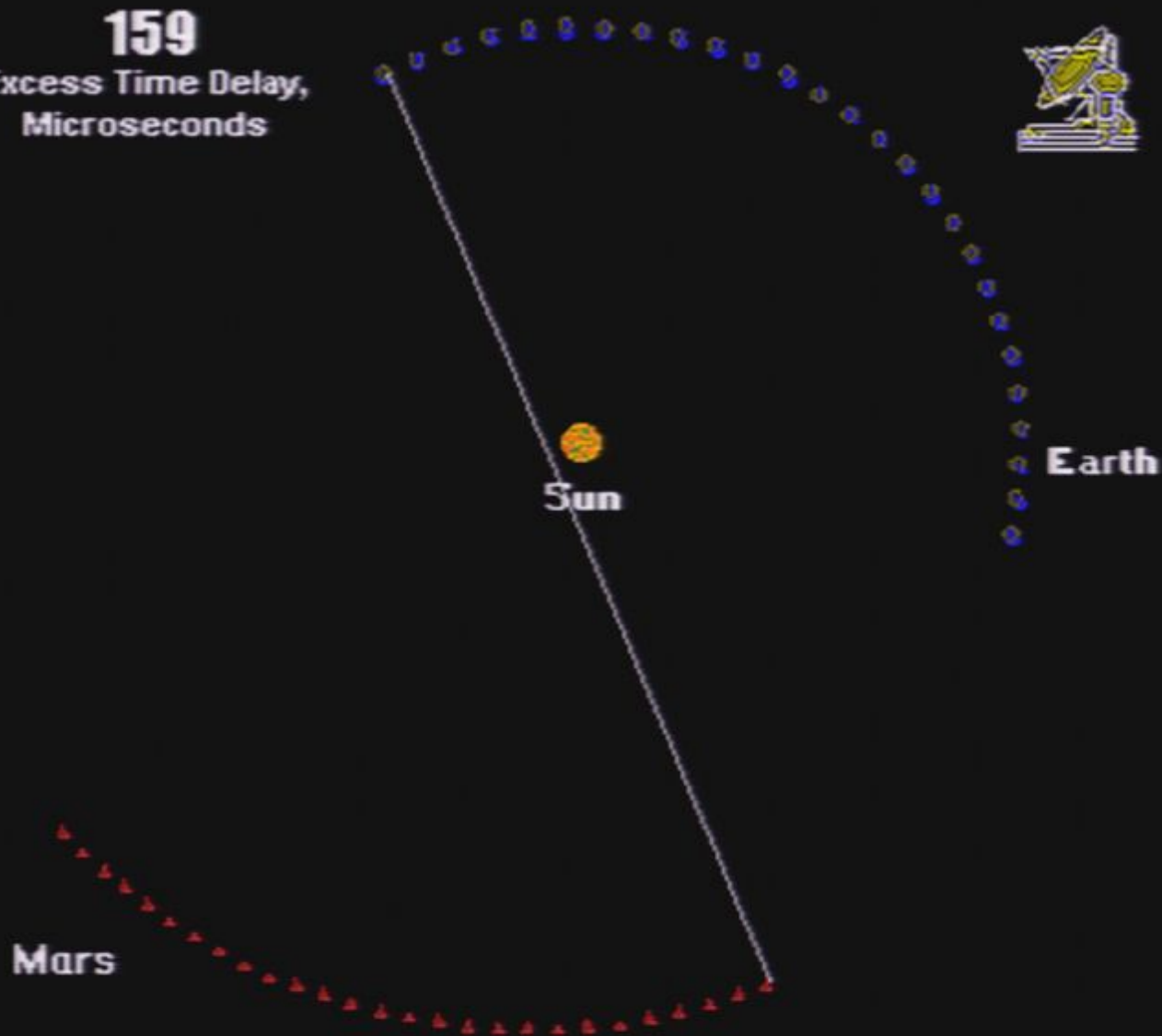
General Relativity Test (1976)

Excess Time Delay,
Microseconds

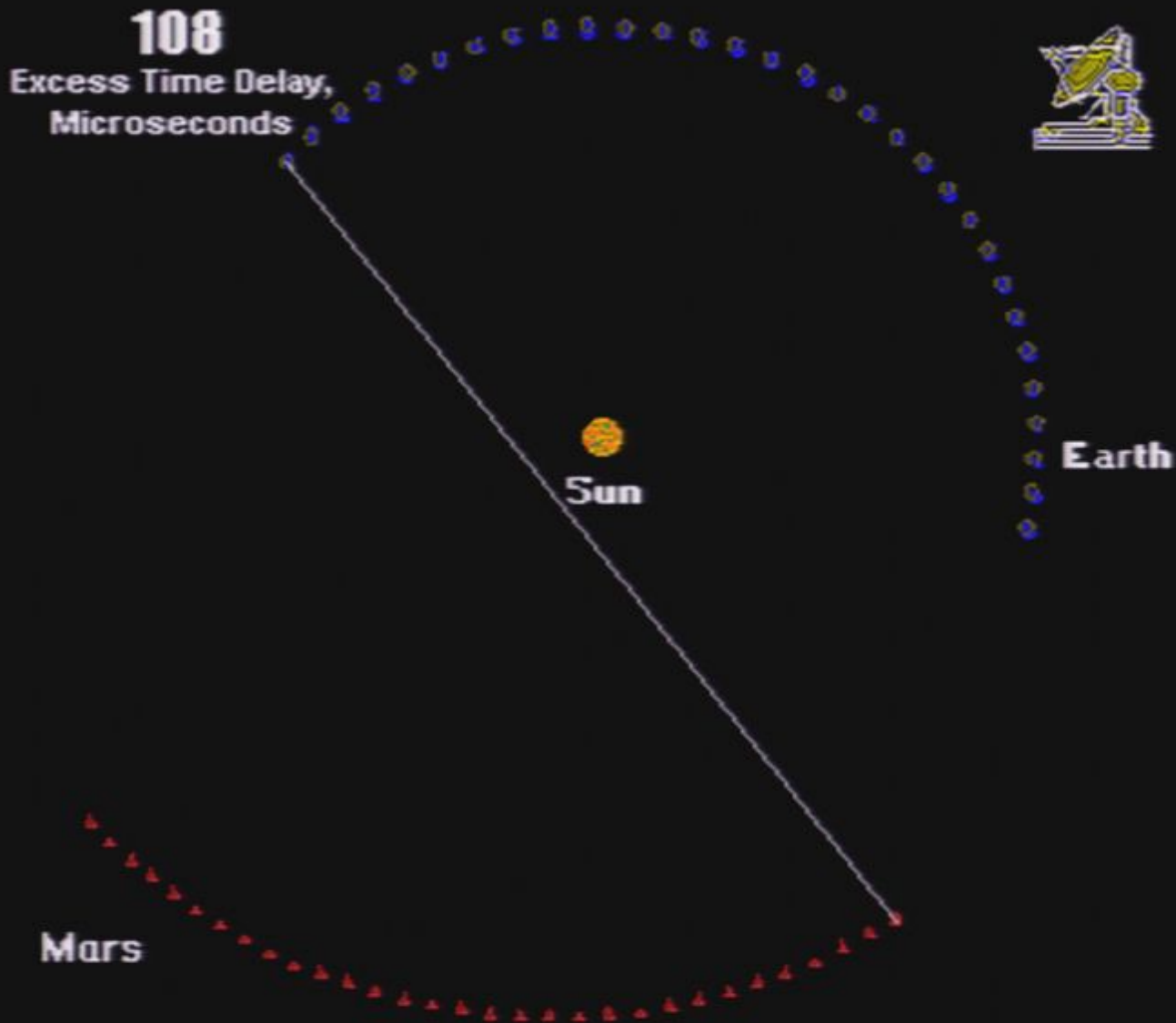


General Relativity Test (1976)

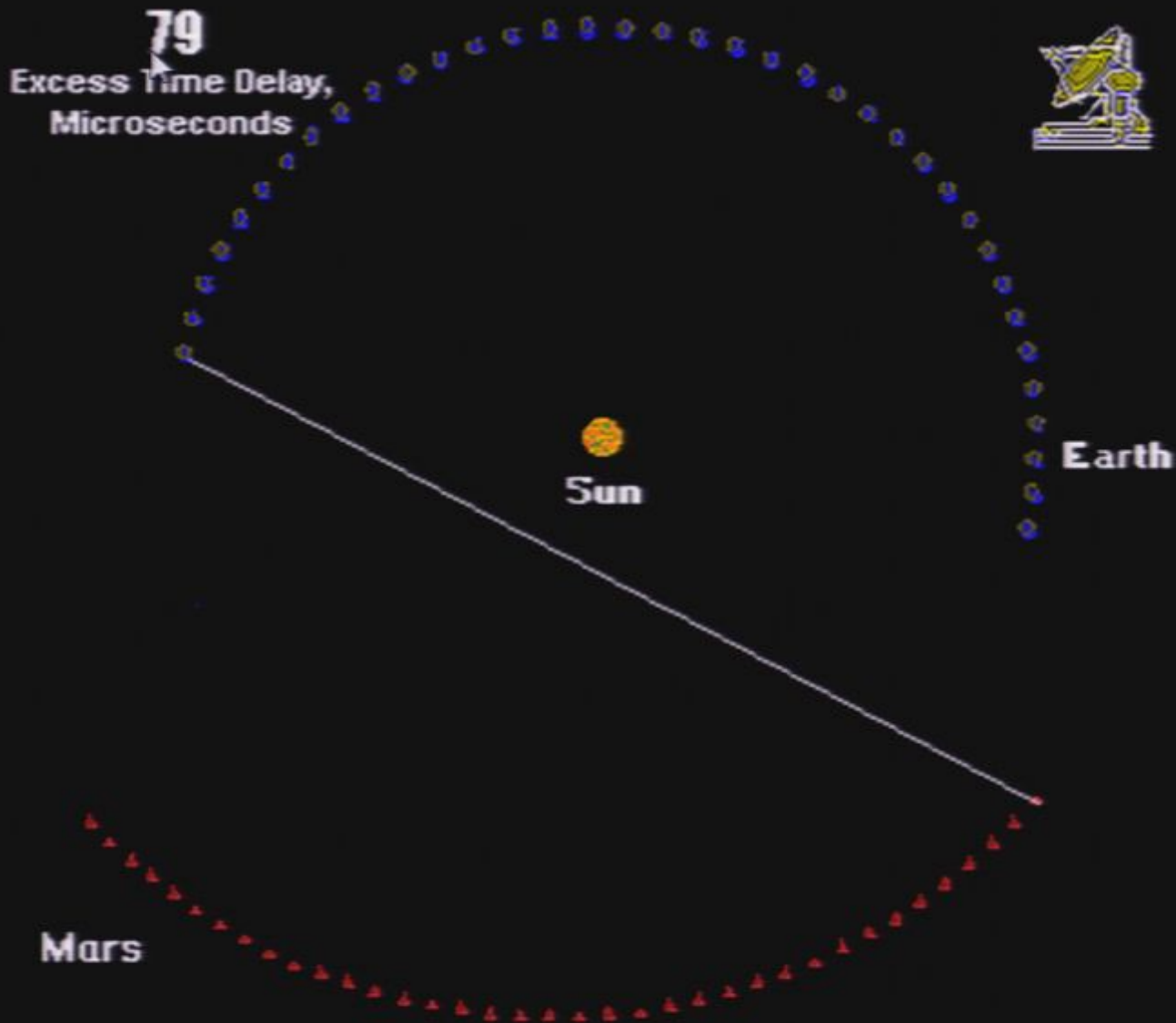
159
Excess Time Delay,
Microseconds



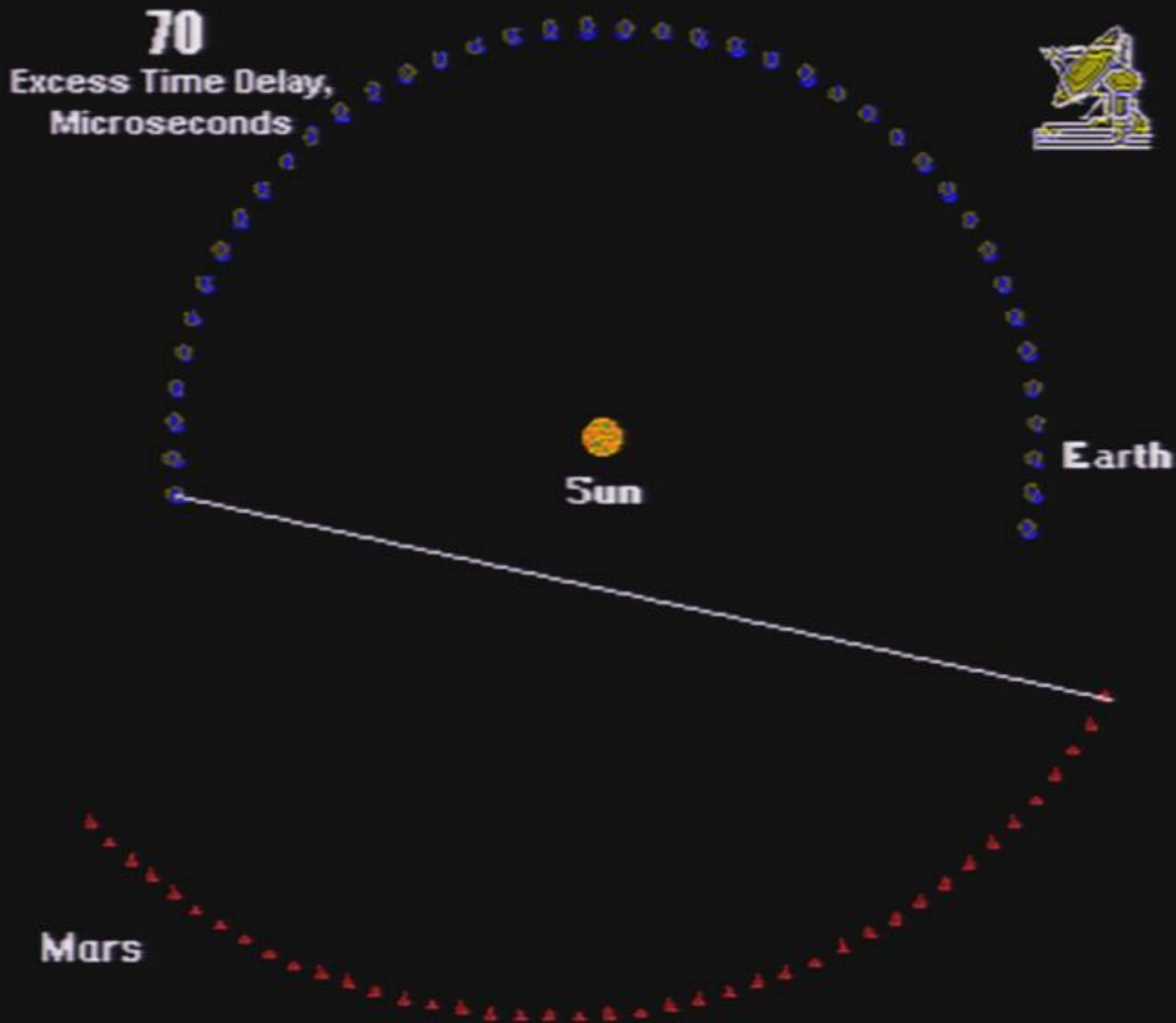
General Relativity Test (1976)



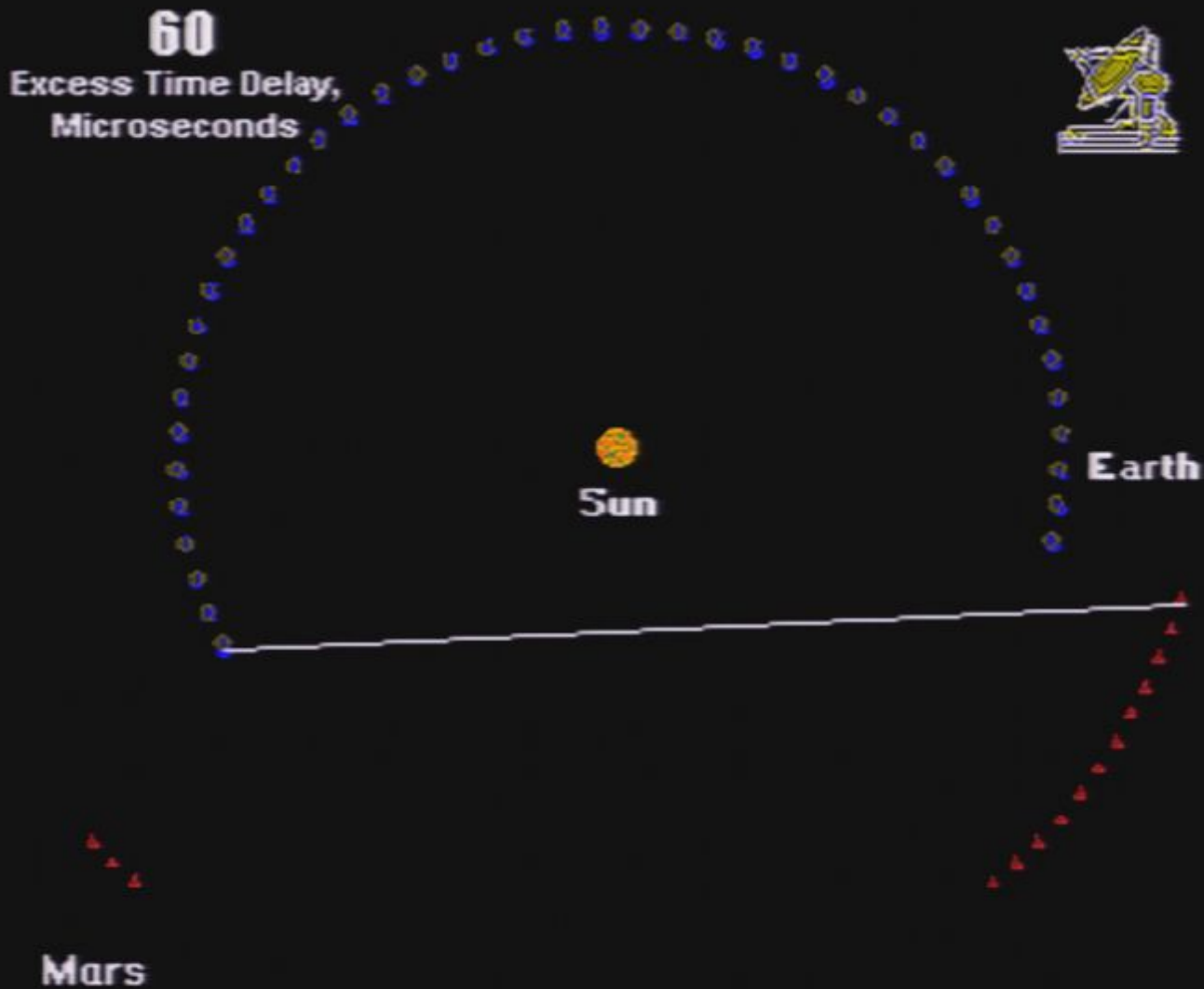
General Relativity Test (1976)



General Relativity Test (1976)

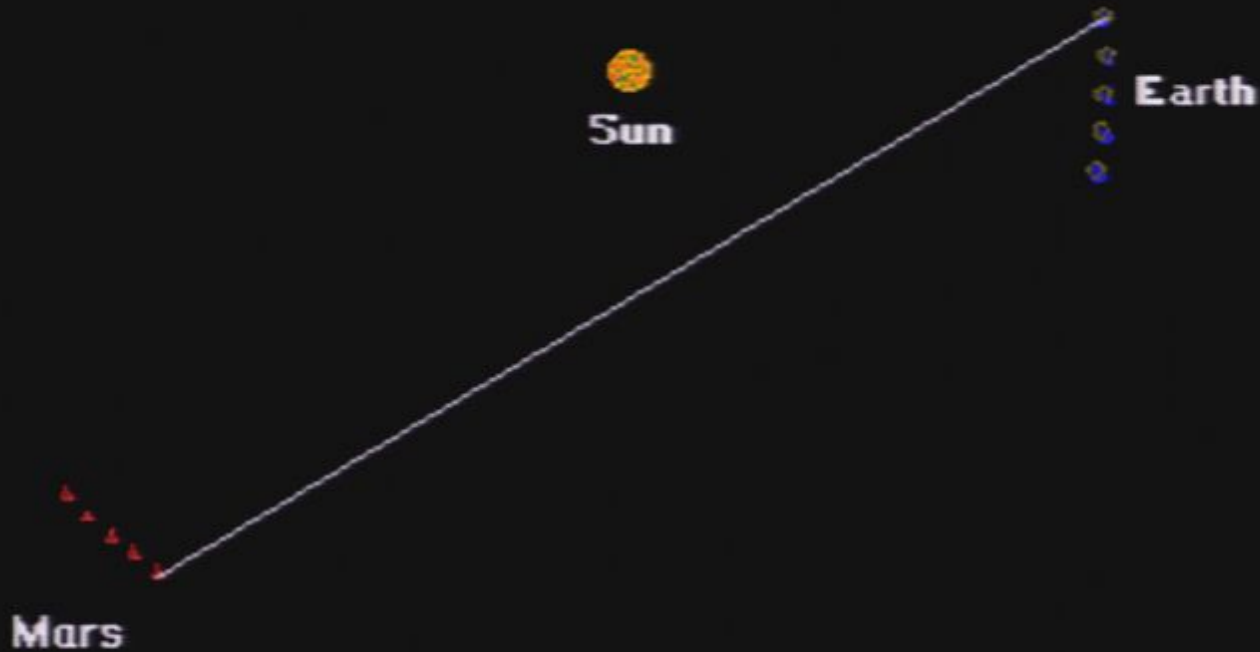


General Relativity Test (1976)



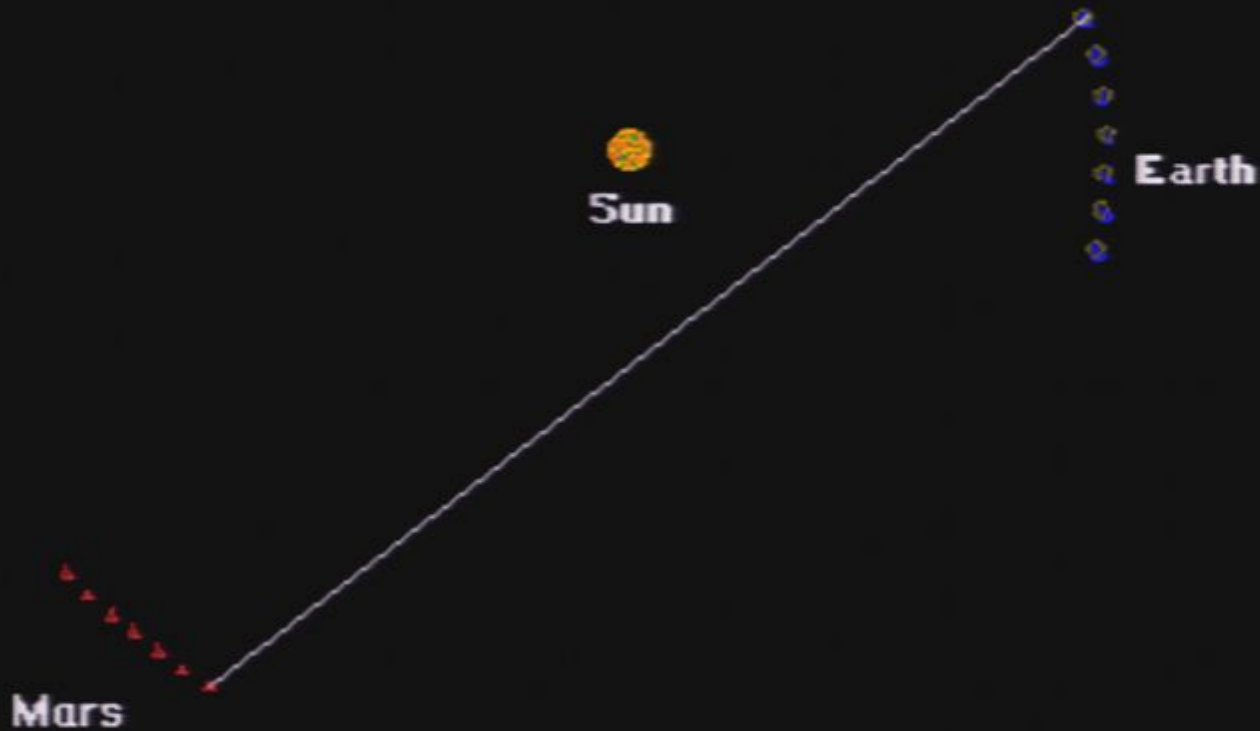
General Relativity Test (1976)

70
Excess Time Delay,
Microseconds



General Relativity Test (1976)

74
Excess Time Delay,
Microseconds



General Relativity Test (1976)

85
Excess Time Delay,
Microseconds



General Relativity Test (1976)

108

Excess Time Delay,
Microseconds



General Relativity Test (1976)

159
Excess Time Delay,
Microseconds



General Relativity Test (1976)

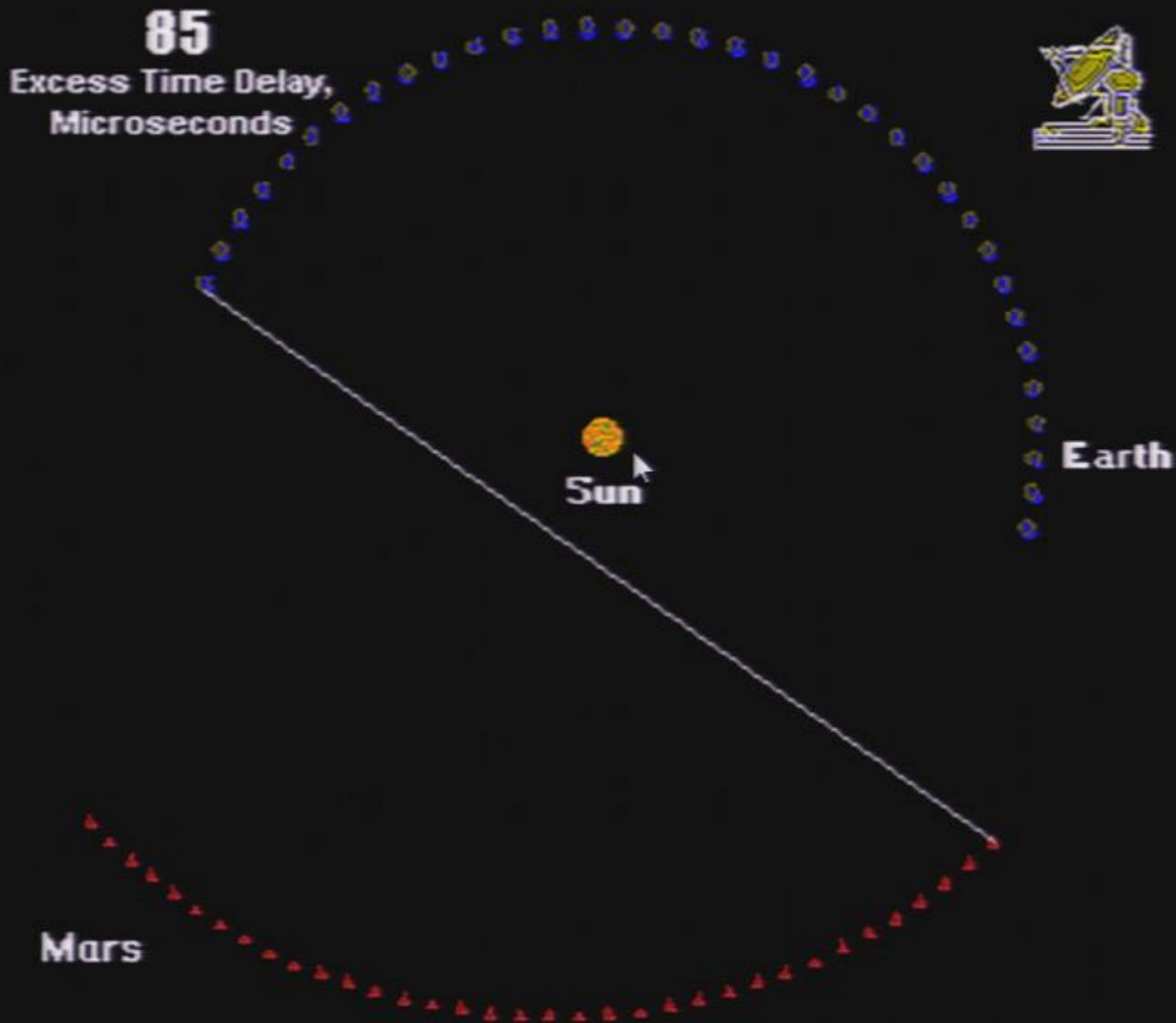
Excess Time Delay,
Microseconds



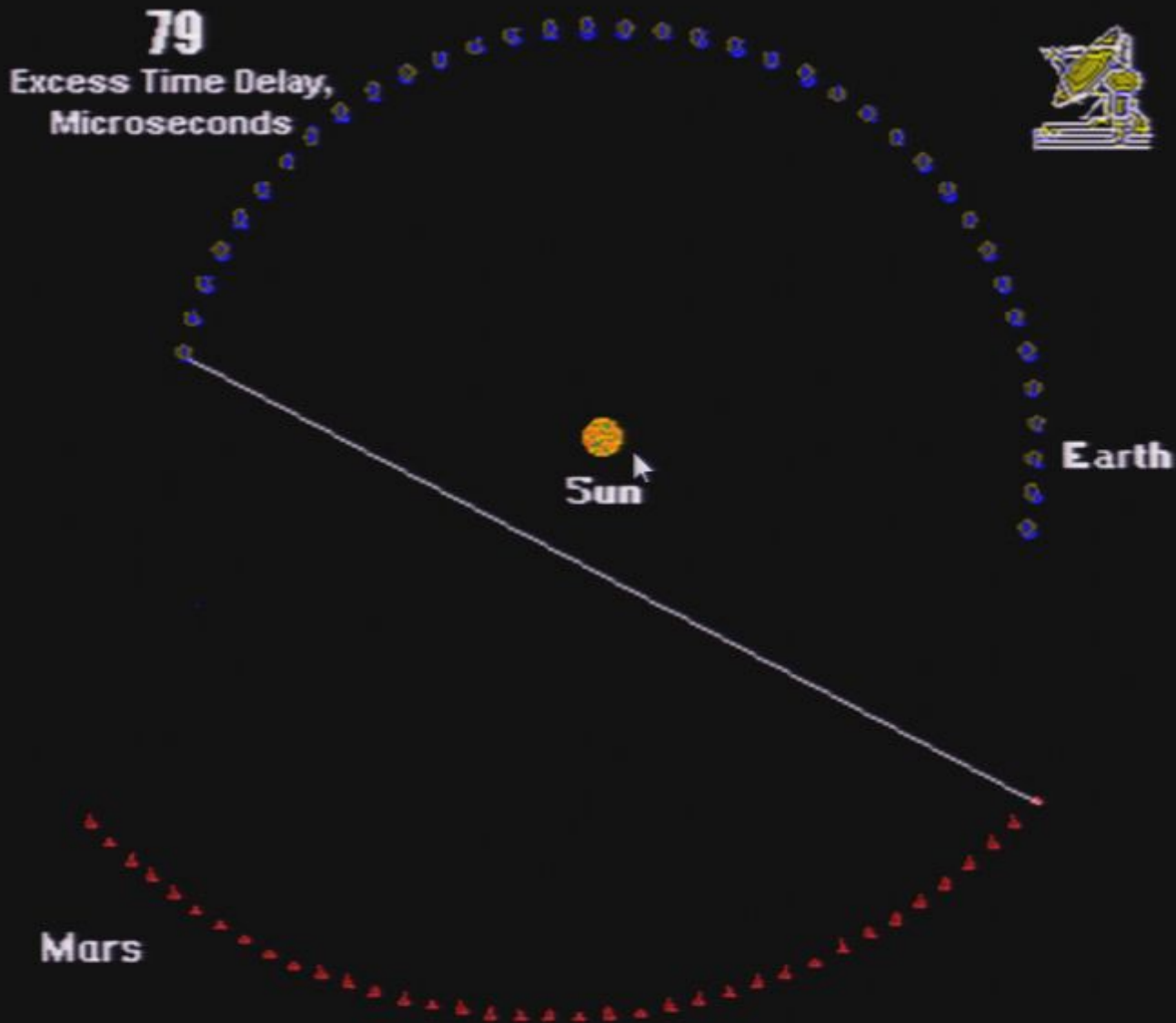
General Relativity Test (1976)



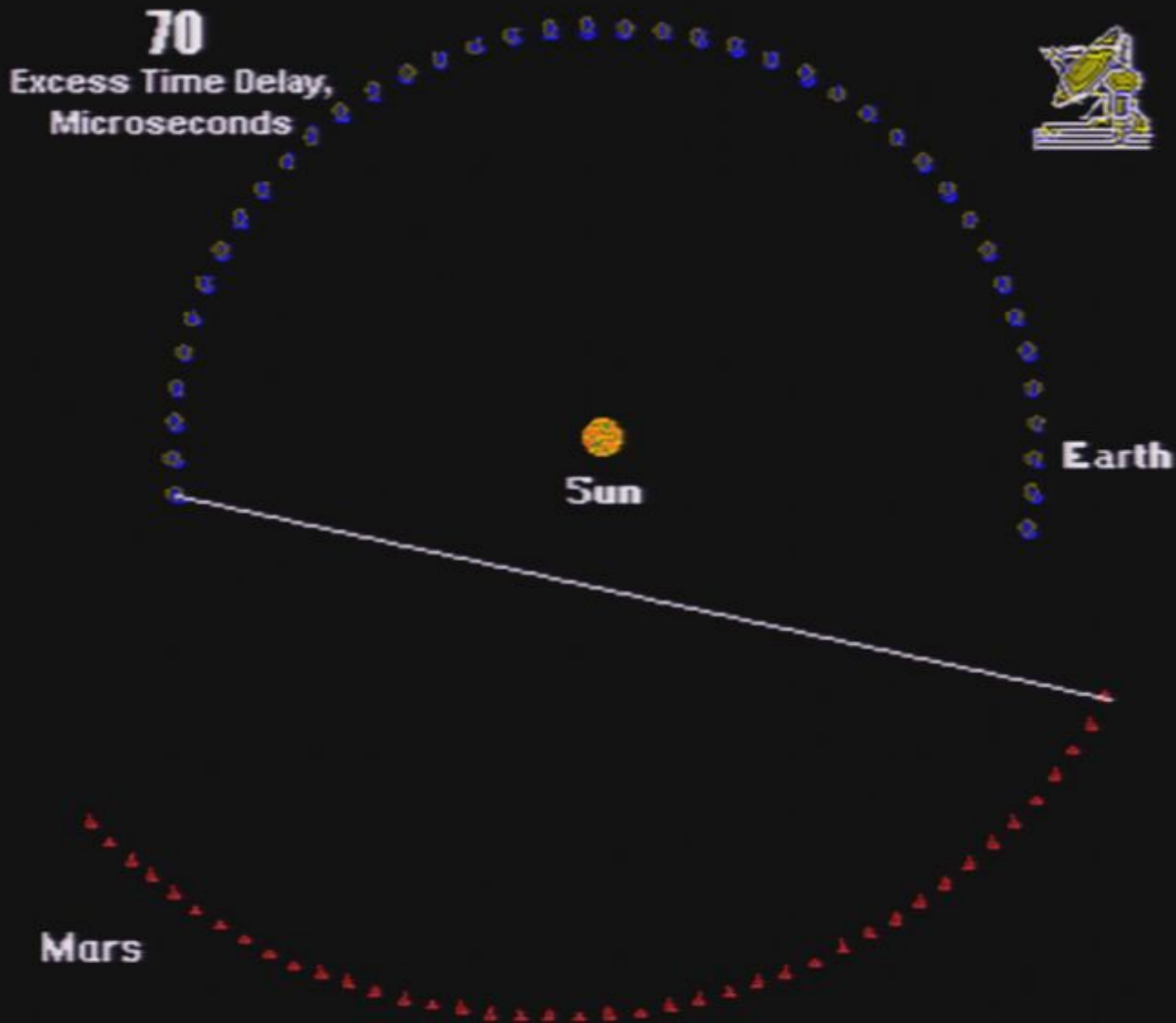
General Relativity Test (1976)



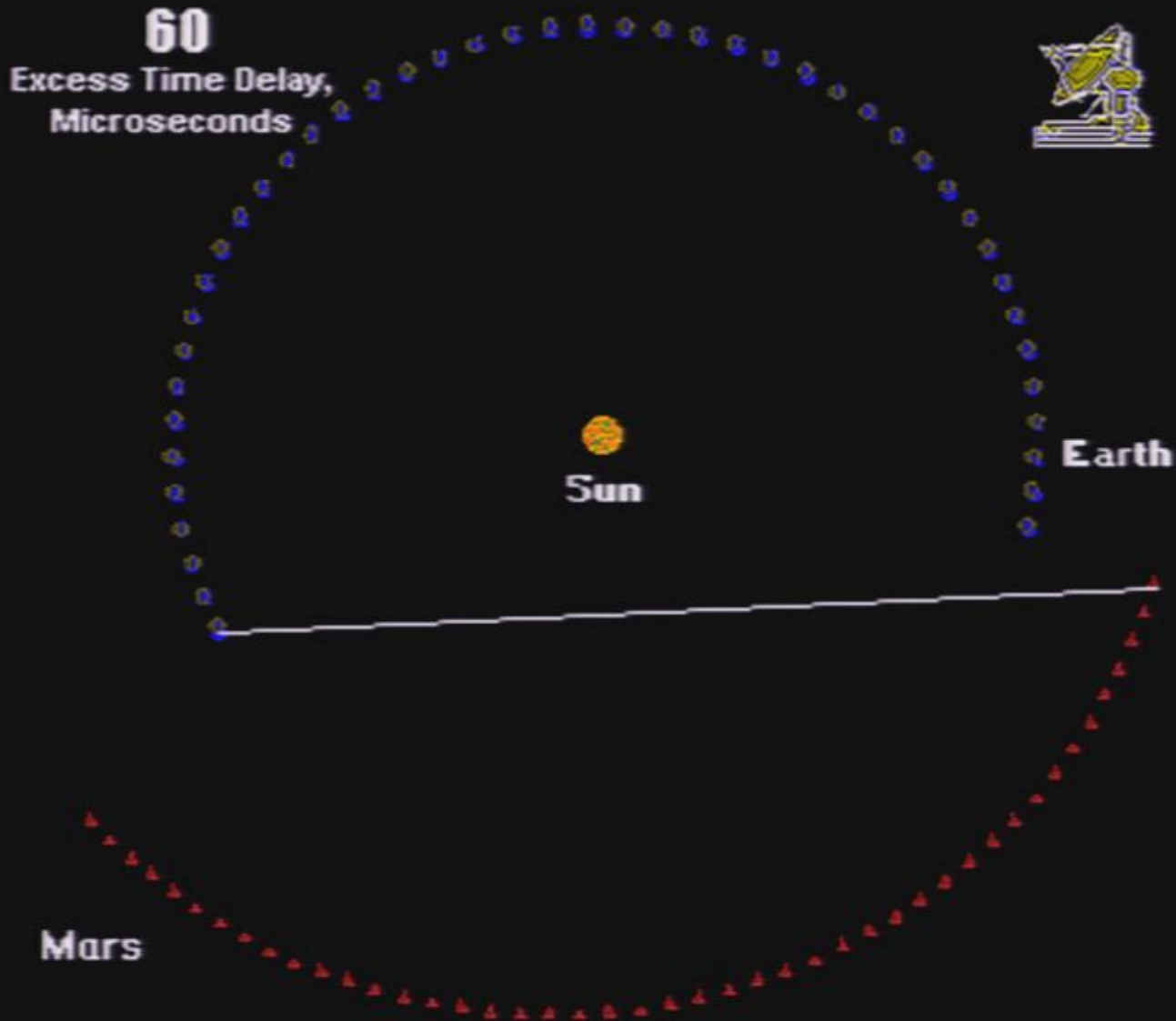
General Relativity Test (1976)



General Relativity Test (1976)



General Relativity Test (1976)



General Relativity Test (1976)

60
Excess Time Delay,
Microseconds




Sun

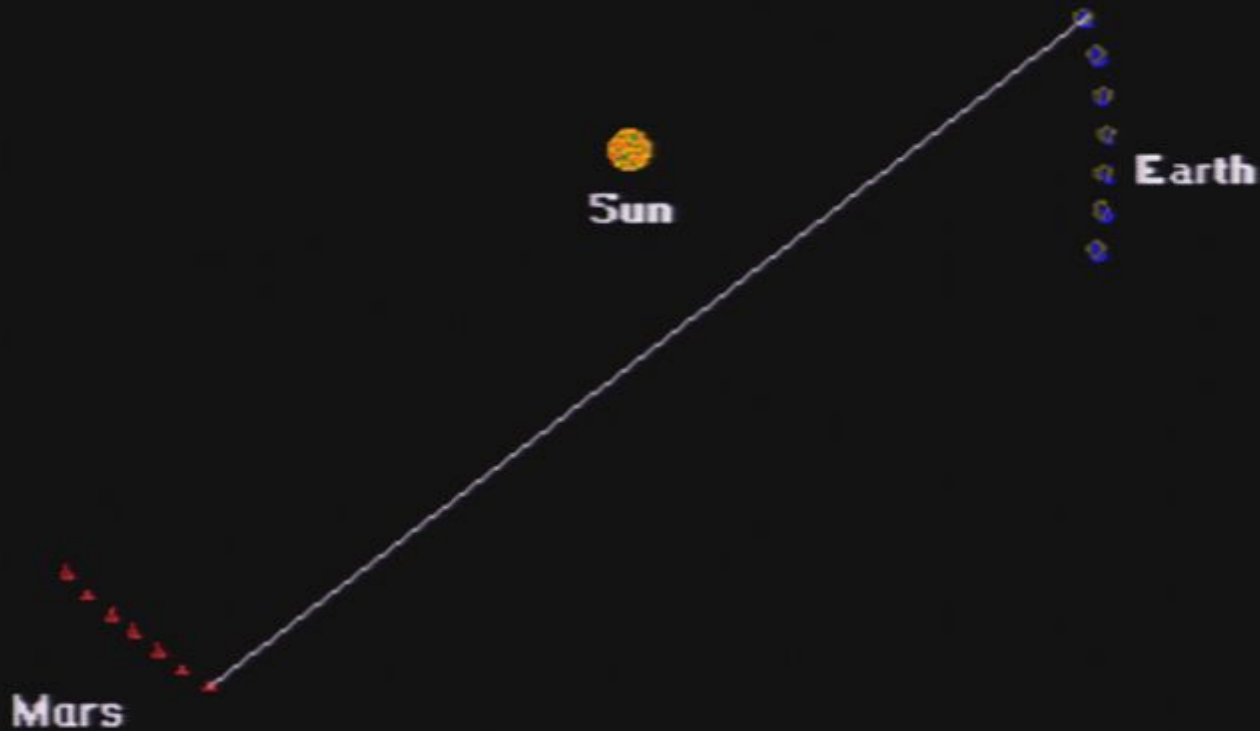
Earth

Mars



General Relativity Test (1976)

74
Excess Time Delay,
Microseconds



General Relativity Test (1976)

85
Excess Time Delay,
Microseconds



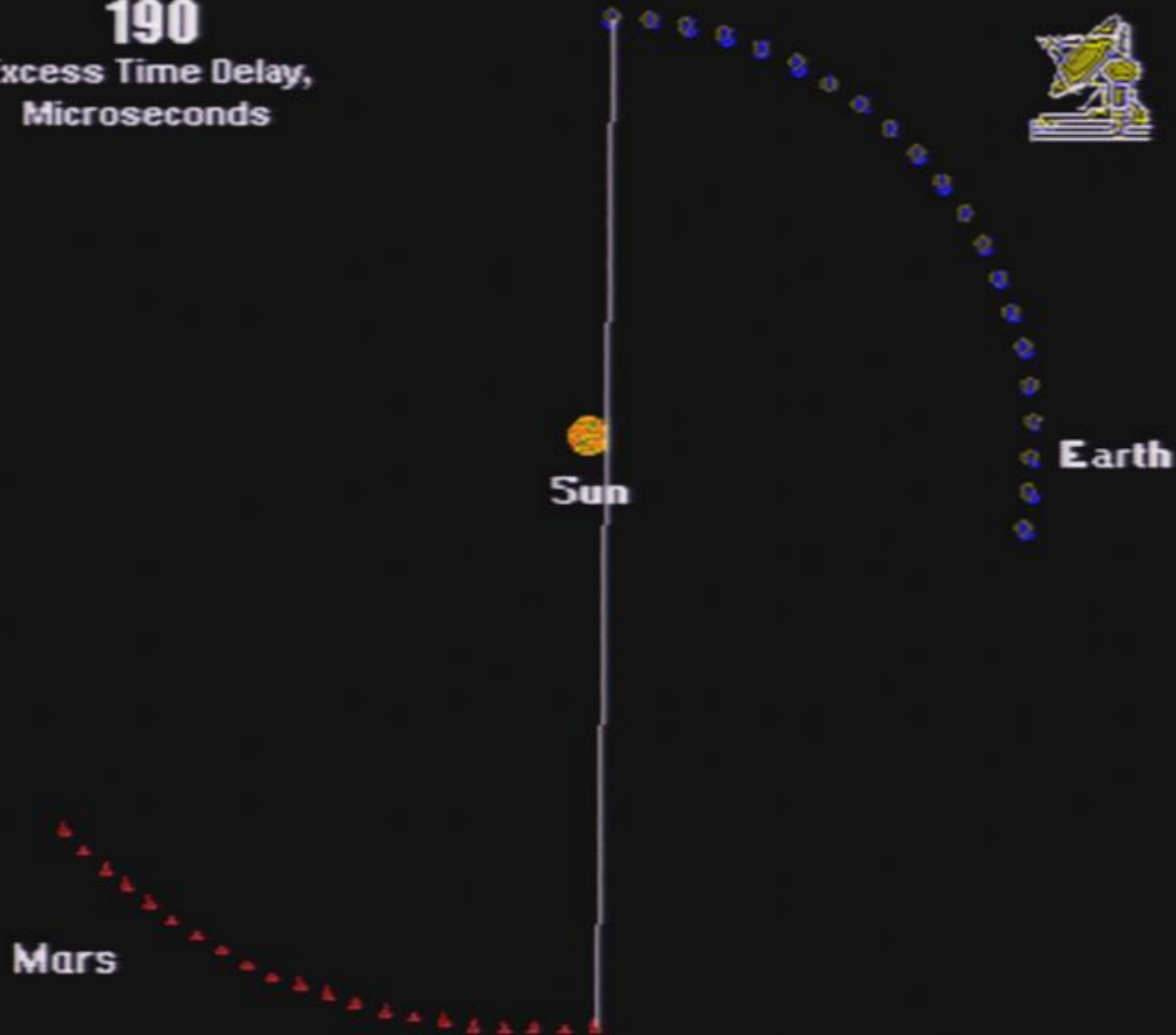
General Relativity Test (1976)

108
Excess Time Delay,
Microseconds



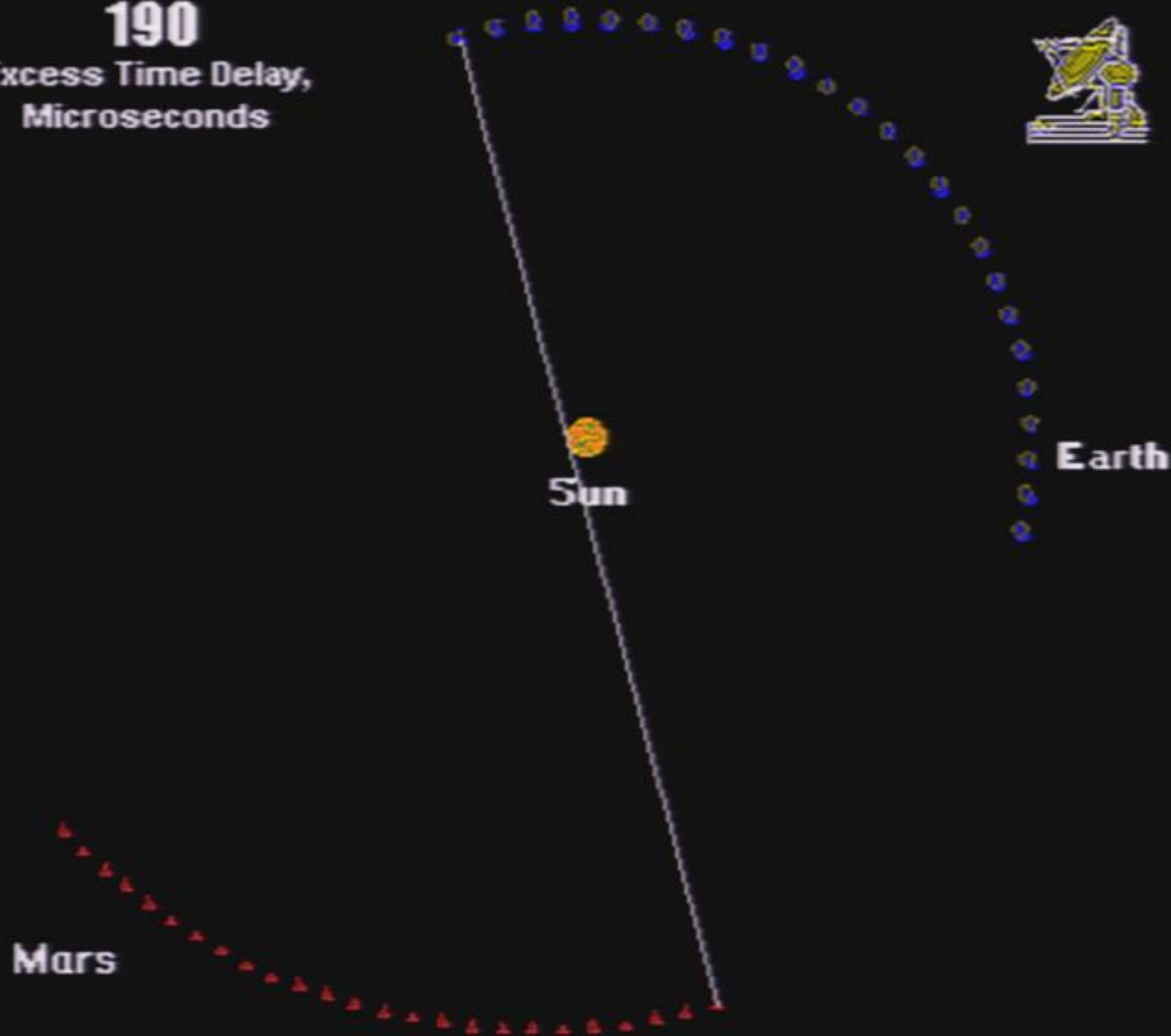
General Relativity Test (1976)

190
Excess Time Delay,
Microseconds



General Relativity Test (1976)

190
Excess Time Delay,
Microseconds



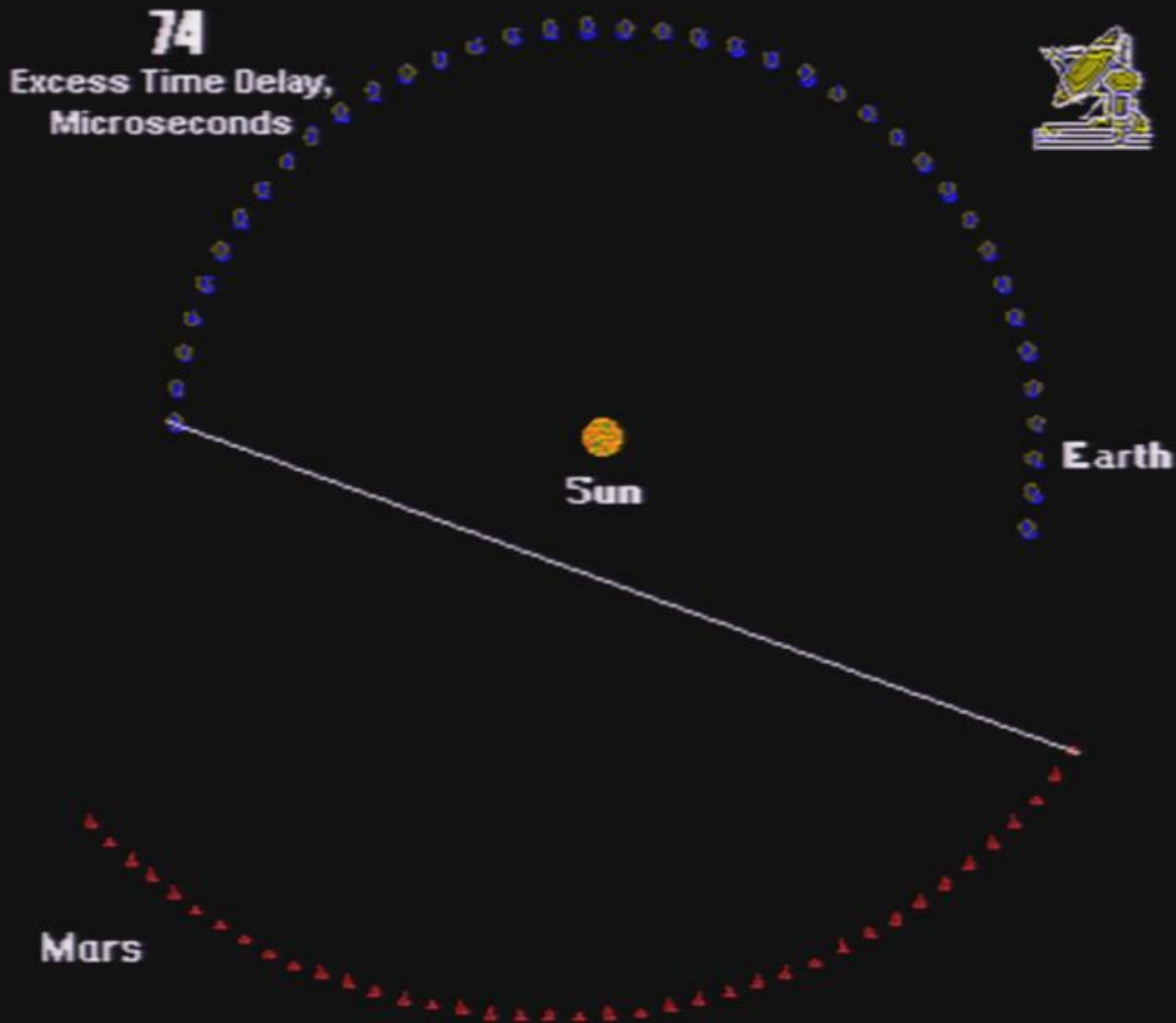
General Relativity Test (1976)



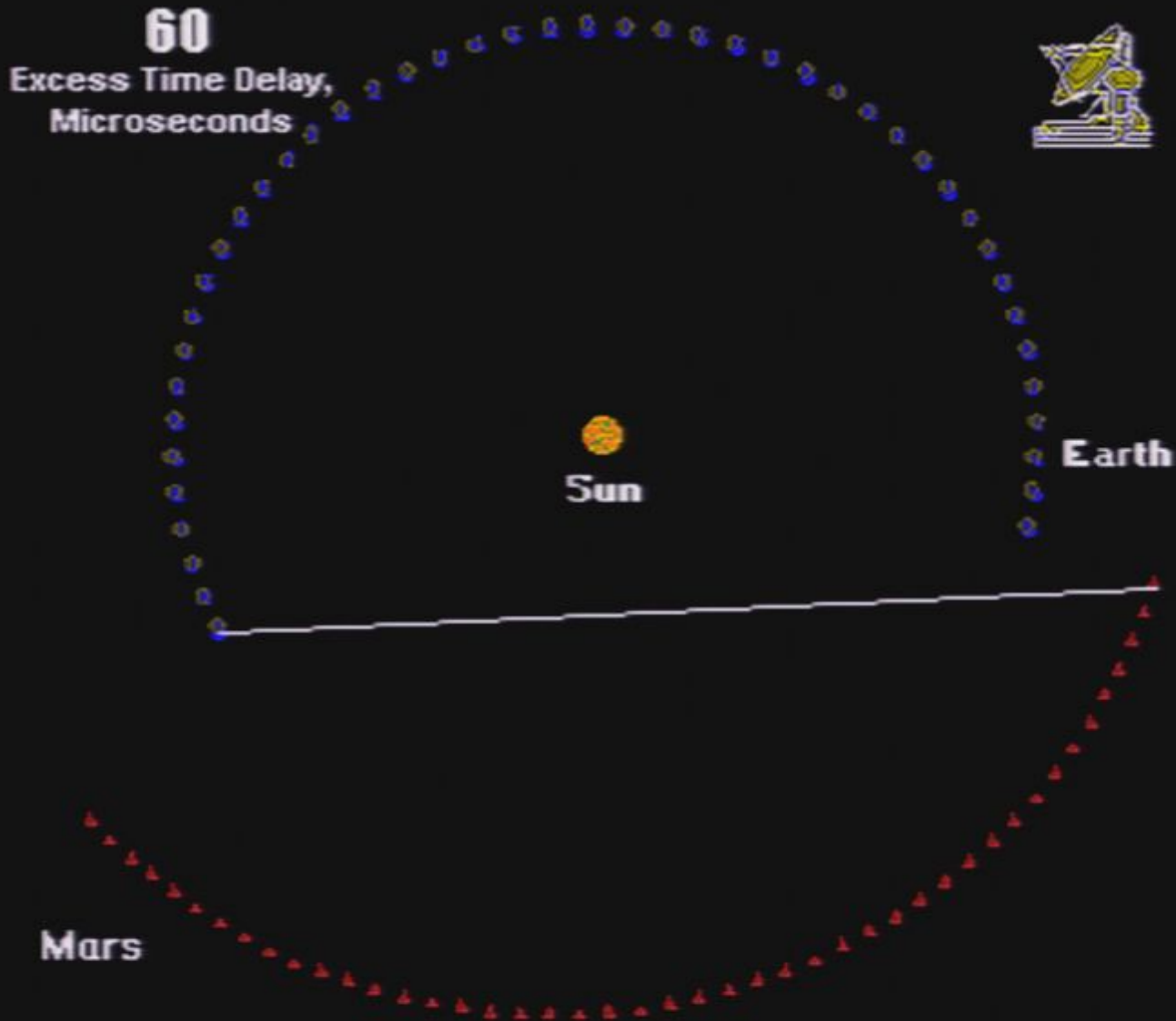
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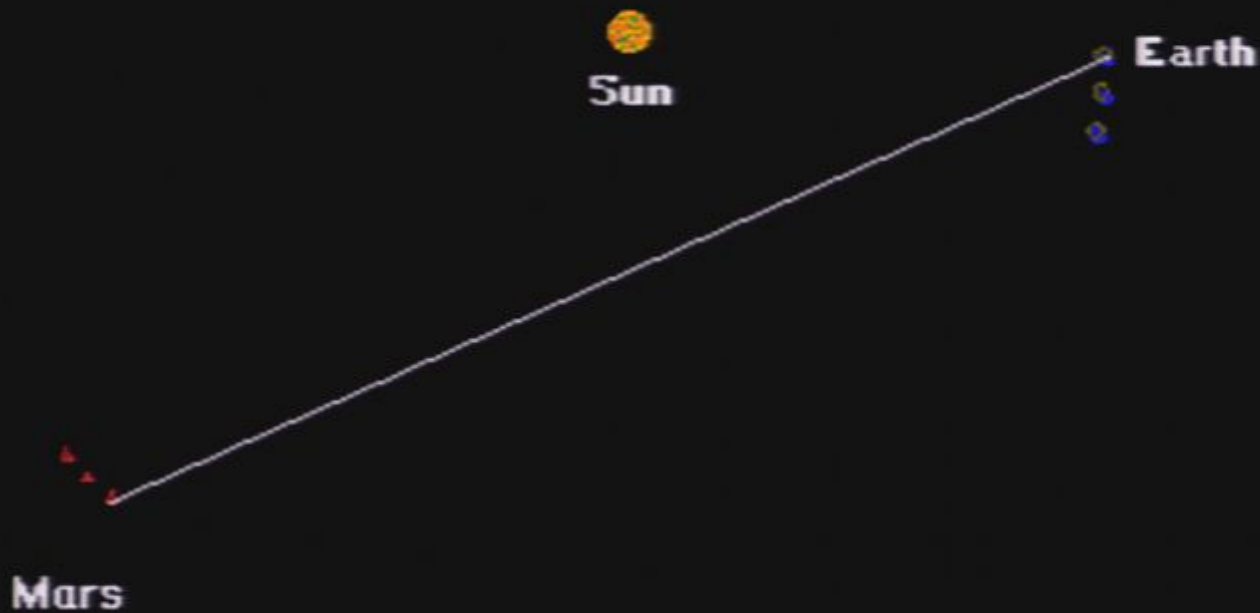


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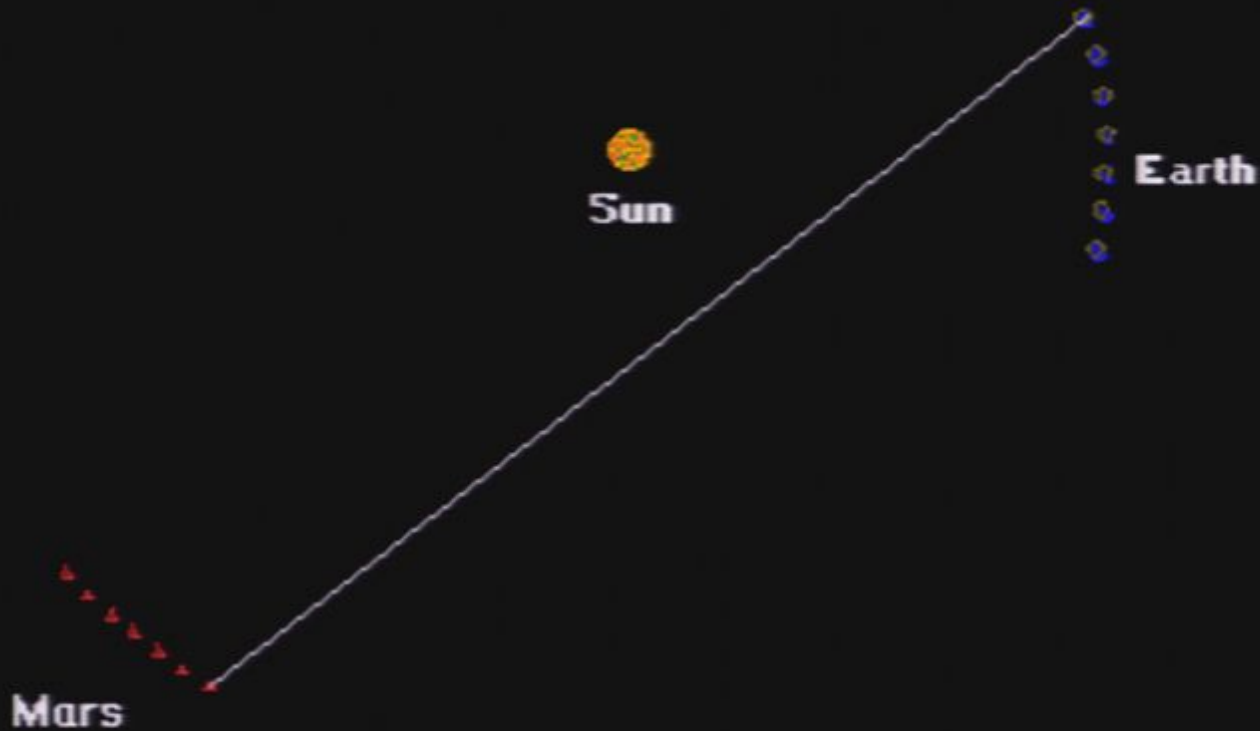
General Relativity Test (1976)

65
Excess Time Delay,
Microseconds



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74
Excess Time Delay,
Microseconds



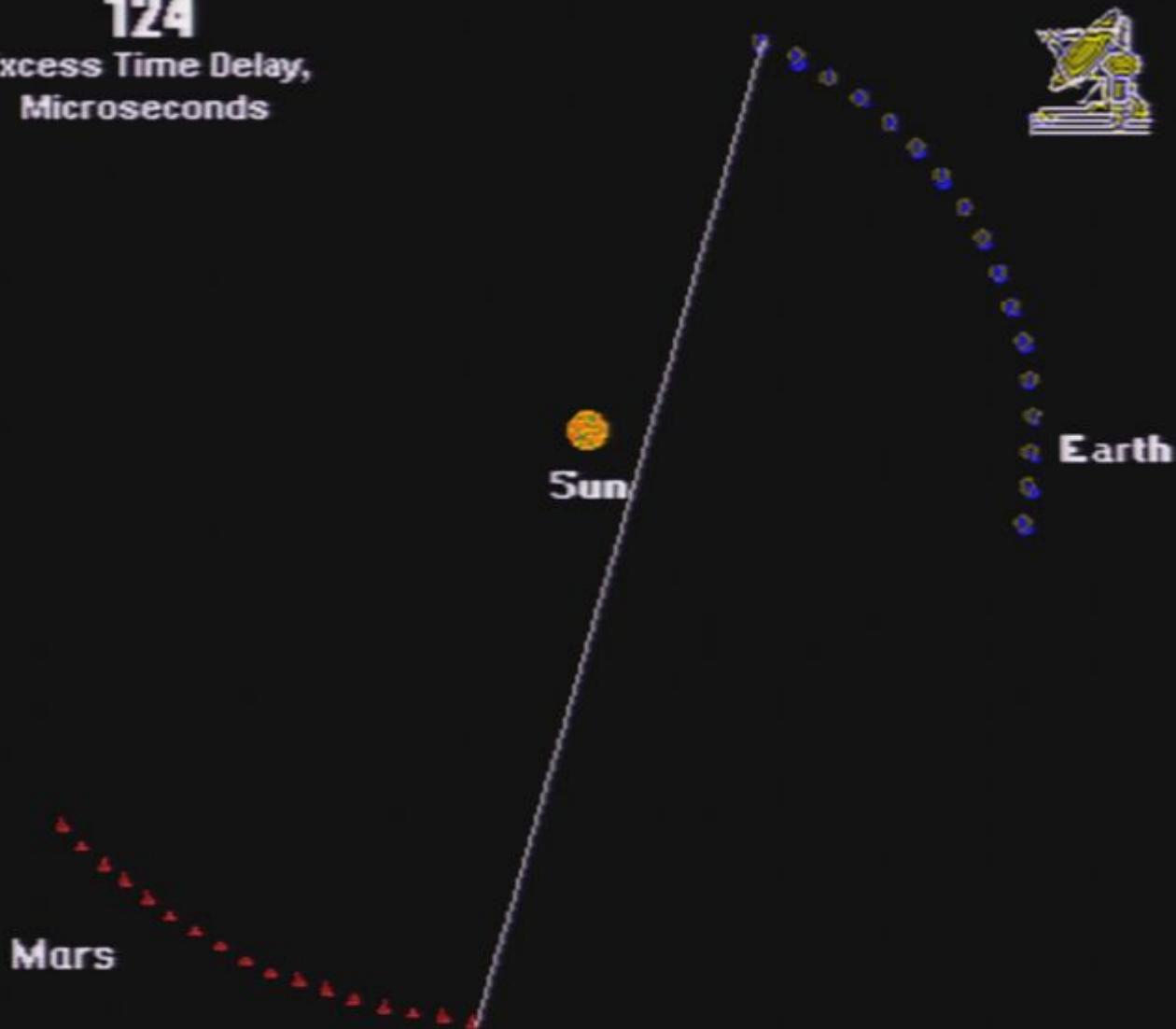
General Relativity Test (1976)

85
Excess Time Delay,
Microseconds



General Relativity Test (1976)

124
Excess Time Delay,
Microseconds



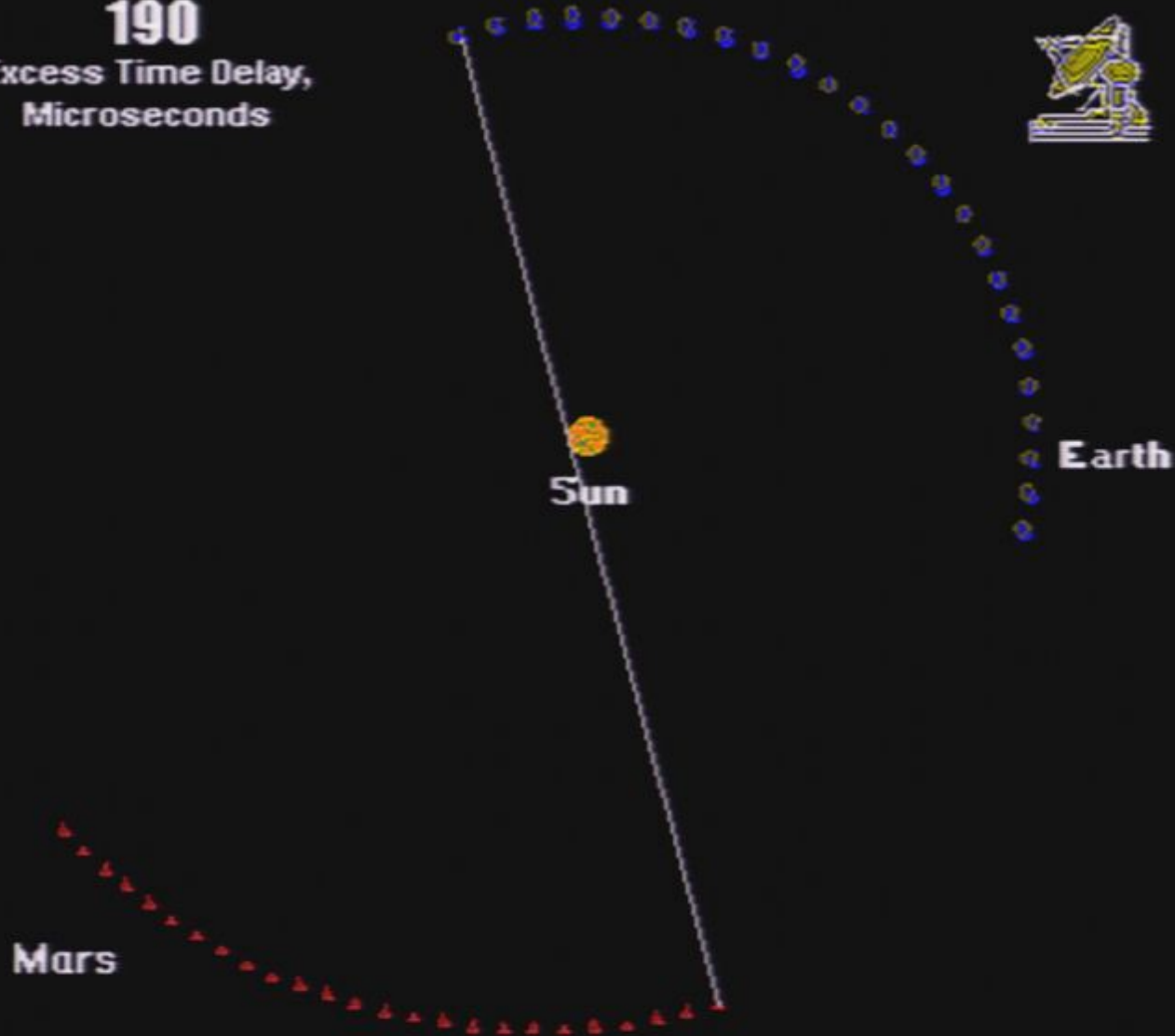
General Relativity Test (1976)

Excess Time Delay,
Microseconds



General Relativity Test (1976)

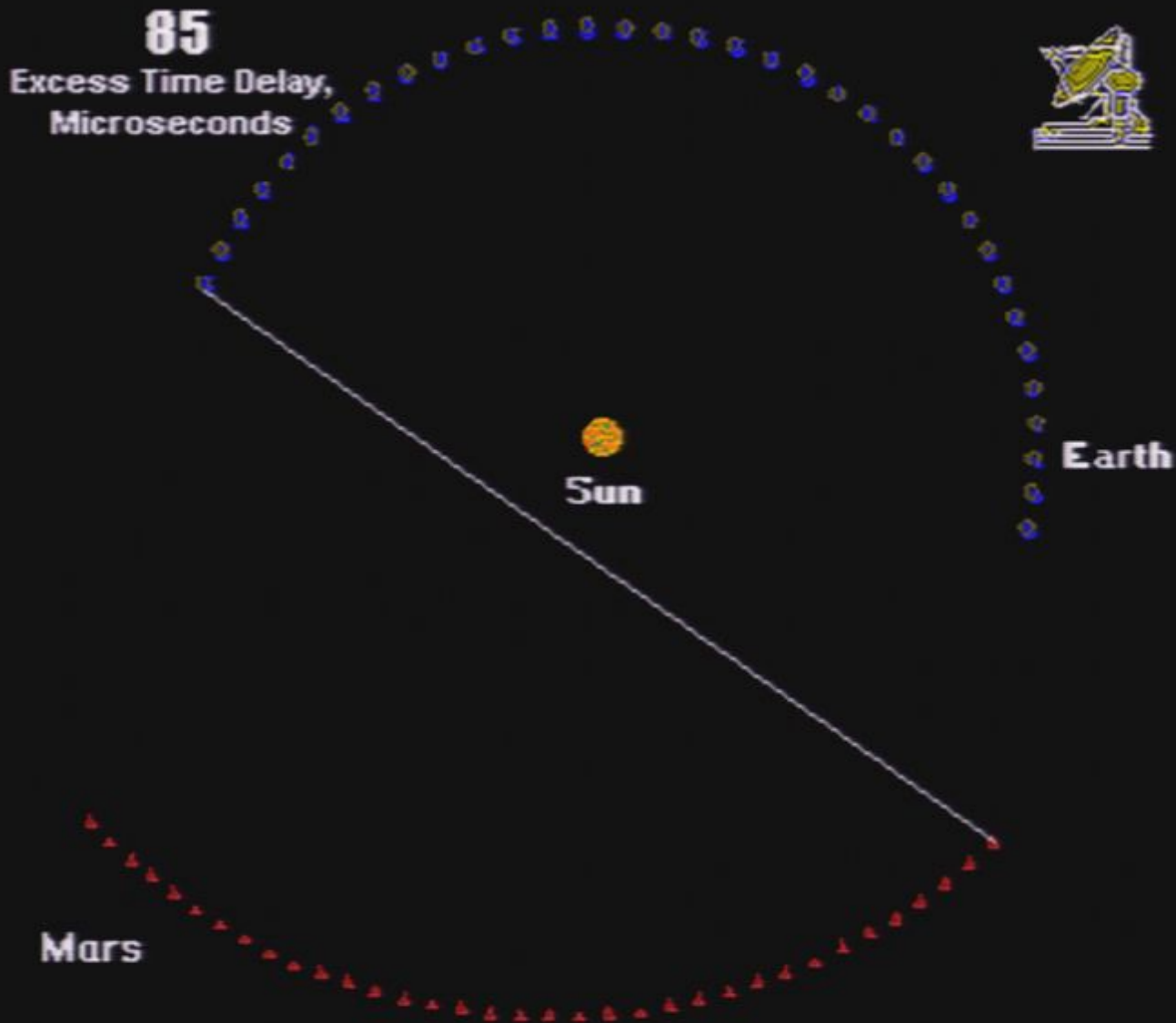
190
Excess Time Delay,
Microseconds



General Relativity Test (1976)



General Relativity Test (1976)



Calculation of Schwarzschild radius

- **In 1916 Karl Schwarzschild discovers a solution of the Einstein field equation, which describes a nonspinning, uncharged spherical body.**
- **Did this when serving in the German Army on the Russian front of **World War I****
- **Only required a few days to solve equation and describe spacetime curvature.**
- **Einstein presented solution on behalf of Schwarzschild to the Academy of Sciences.**
- **Schwarzschild died on the front 4 months later.**



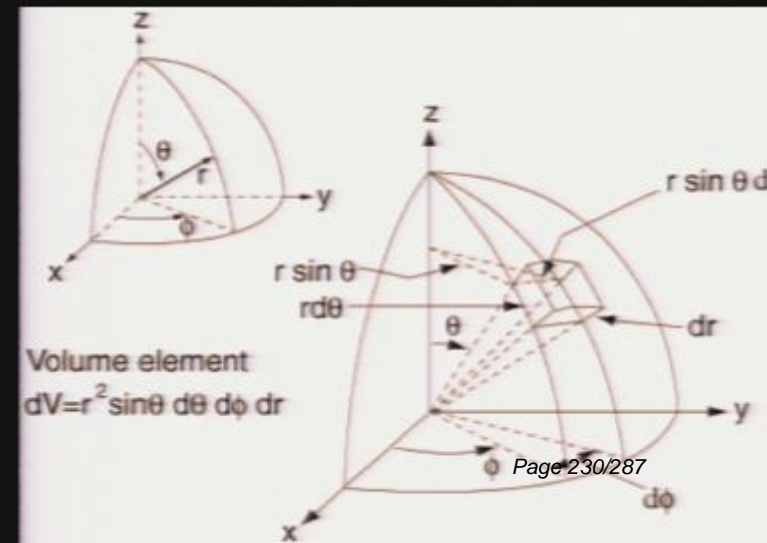
The Schwarzschild Radius

$$d\sigma^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2)$$

Curvature factor

$$r_s = \frac{2GM}{c^2}$$

r, θ, ϕ are the polar coordinates



The Schwarzschild Radius

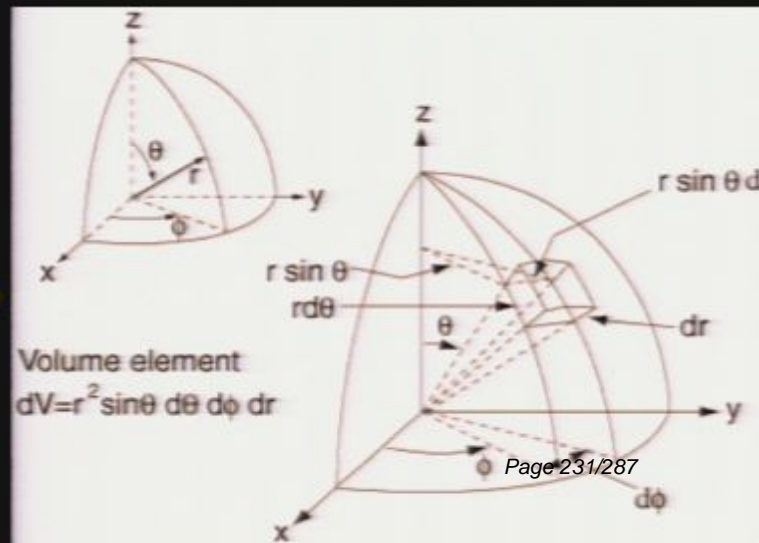
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$$r_s = \frac{2GM}{c^2}$$

r, θ, ϕ are the polar coordinates

$$g_{\mu\nu} = \begin{pmatrix} -\left(1 - \frac{2MG}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1 - \frac{2MG}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin(\theta) \end{pmatrix} \left. \begin{array}{l} \text{time} \\ \text{space} \end{array} \right\}$$



Schwarzschild Metric

$$d\sigma^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2 \right)$$

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2 \right)$$

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- σ is the proper distance (distance measured by a clock moving along path)
- t is the time coordinate (measured by a far away stationary observer)
- r is the radial coordinate (circumference of a circle centered on star divided by 2π)
- r_s is the Schwarzschild radius

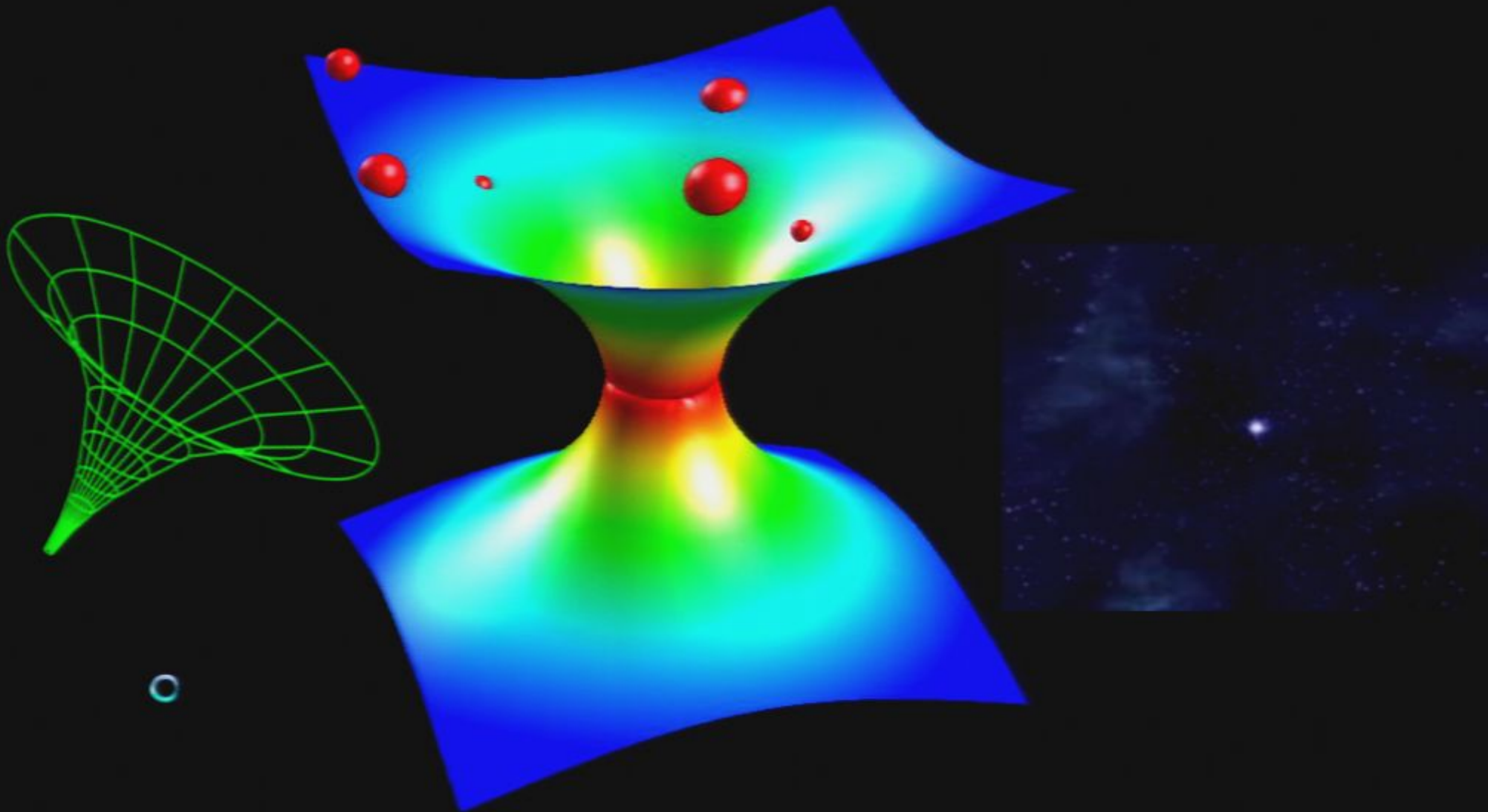
Schwarzschild radii for different objects

Object	Mass	R_S
Atom	10^{-26} kg	10^{-51} cm
Human Being	70 kg	10^{-23} cm
Earth	6.0×10^{24} kg	0.89 cm
Sun	2.0×10^{30} kg	3.0 km
Galaxy	$10^{11} M_S$	10^{-2} l.y.
Universe (if closed)	$10^{23} M_S$	10^{10} l.y.

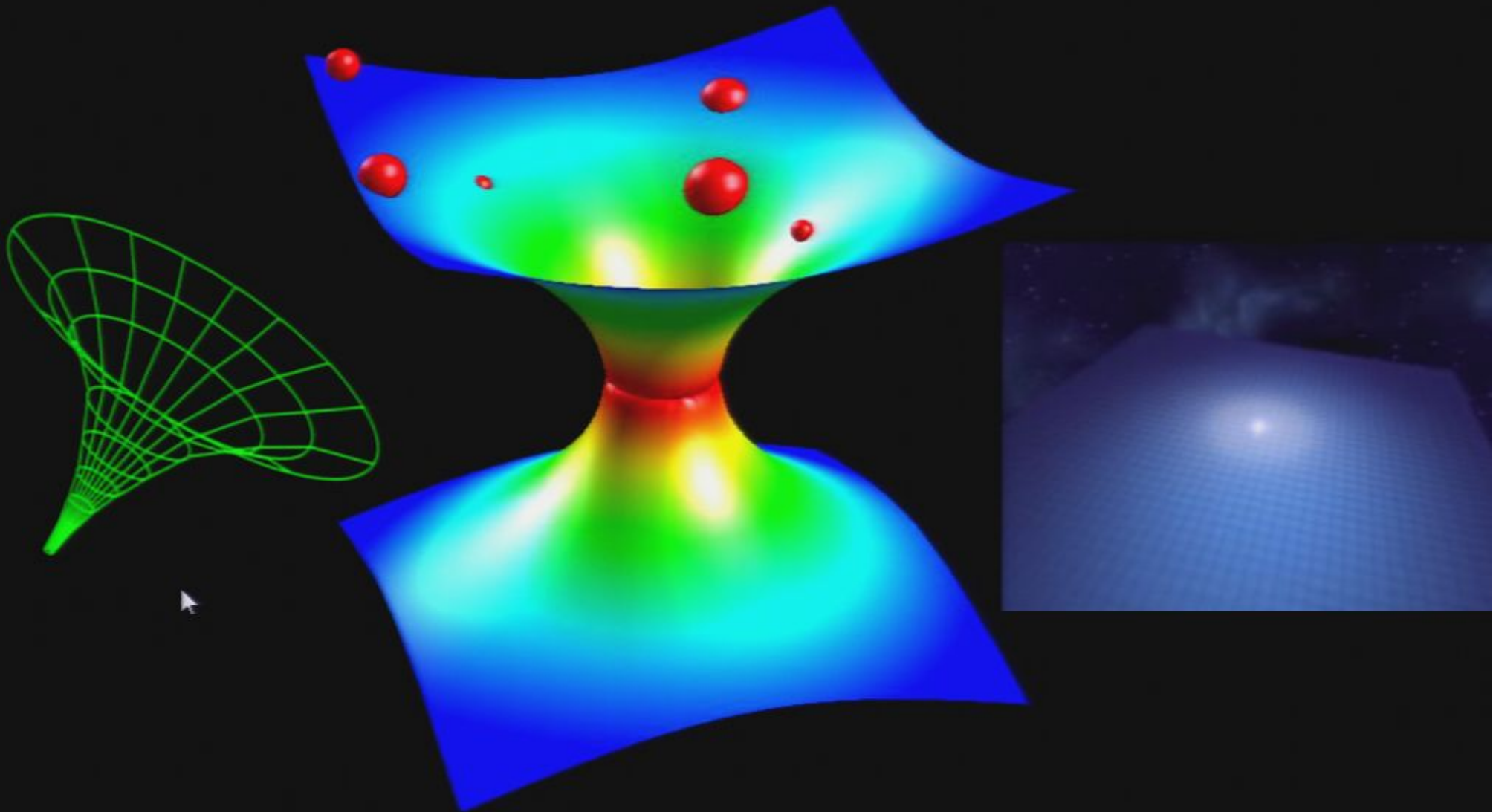
$$r_s = \frac{2GM}{c^2}$$



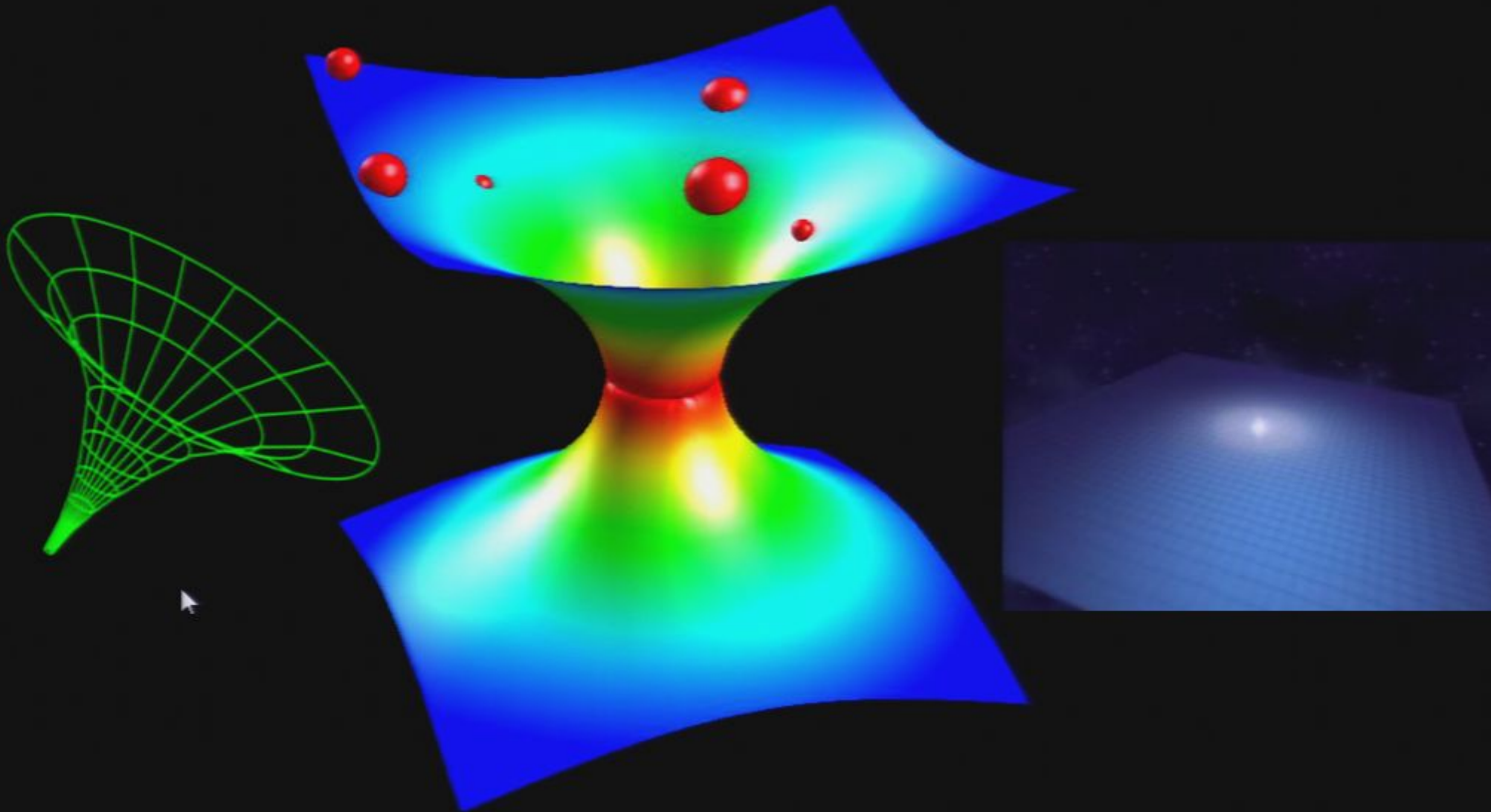
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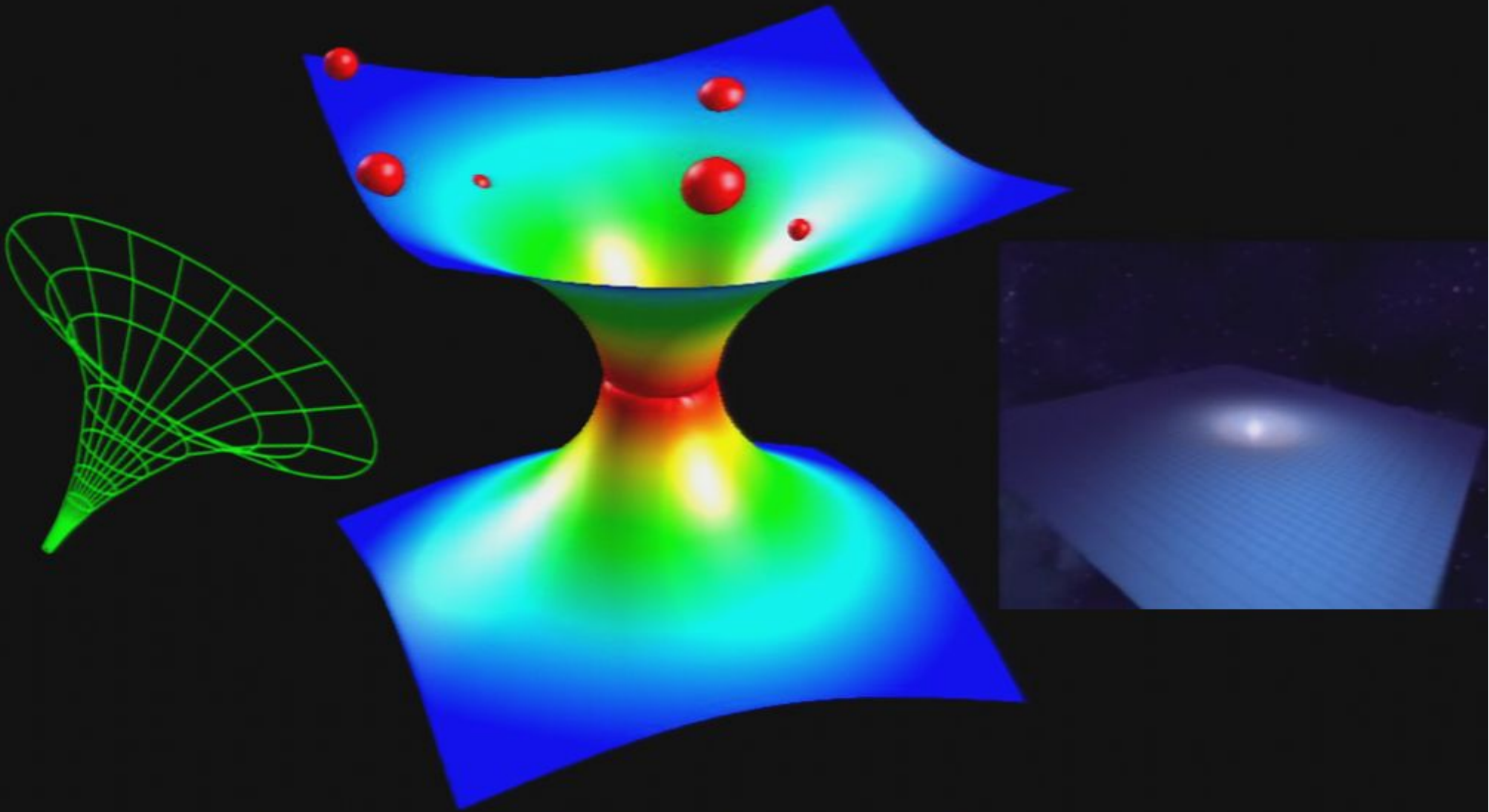
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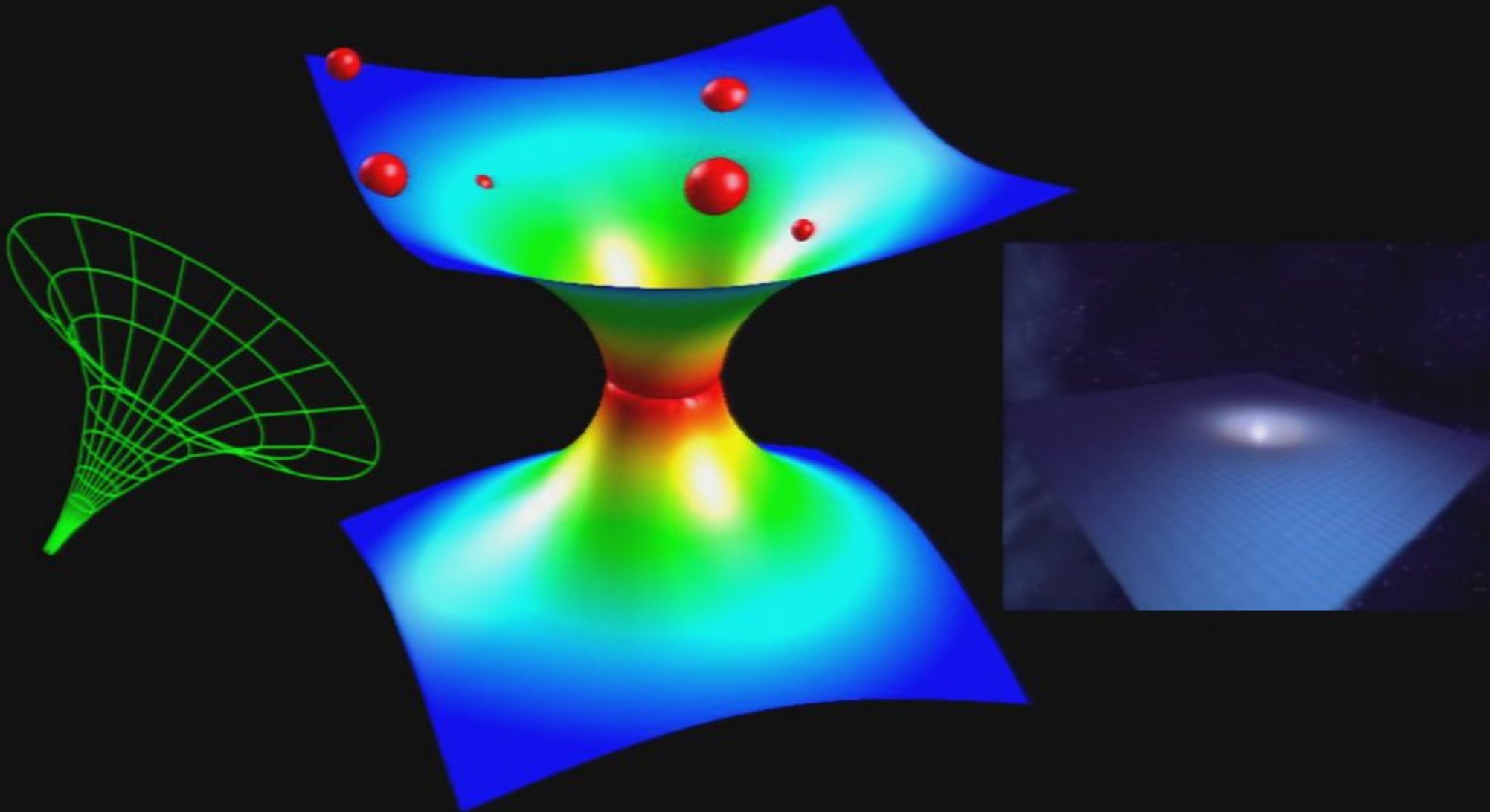
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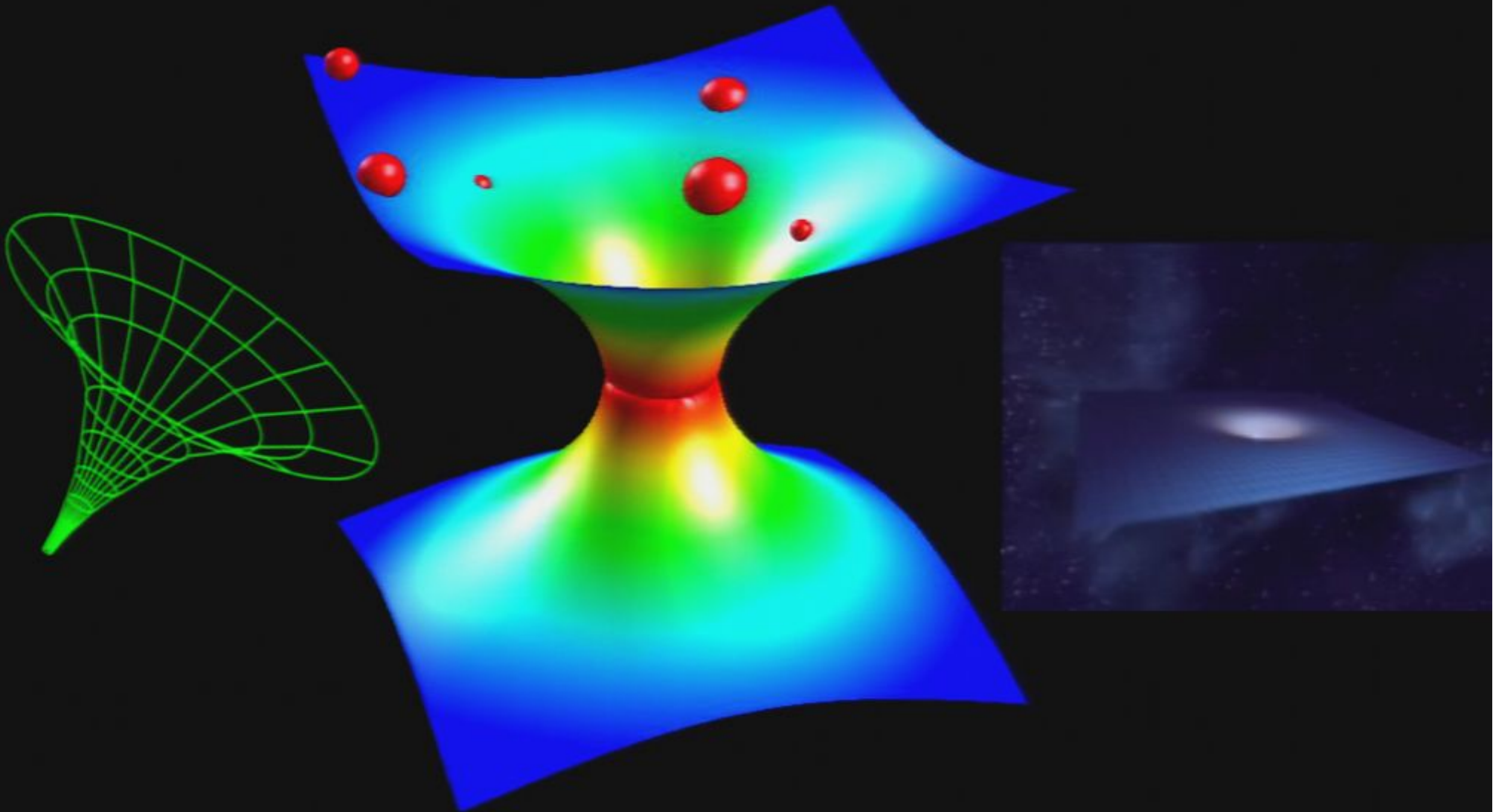
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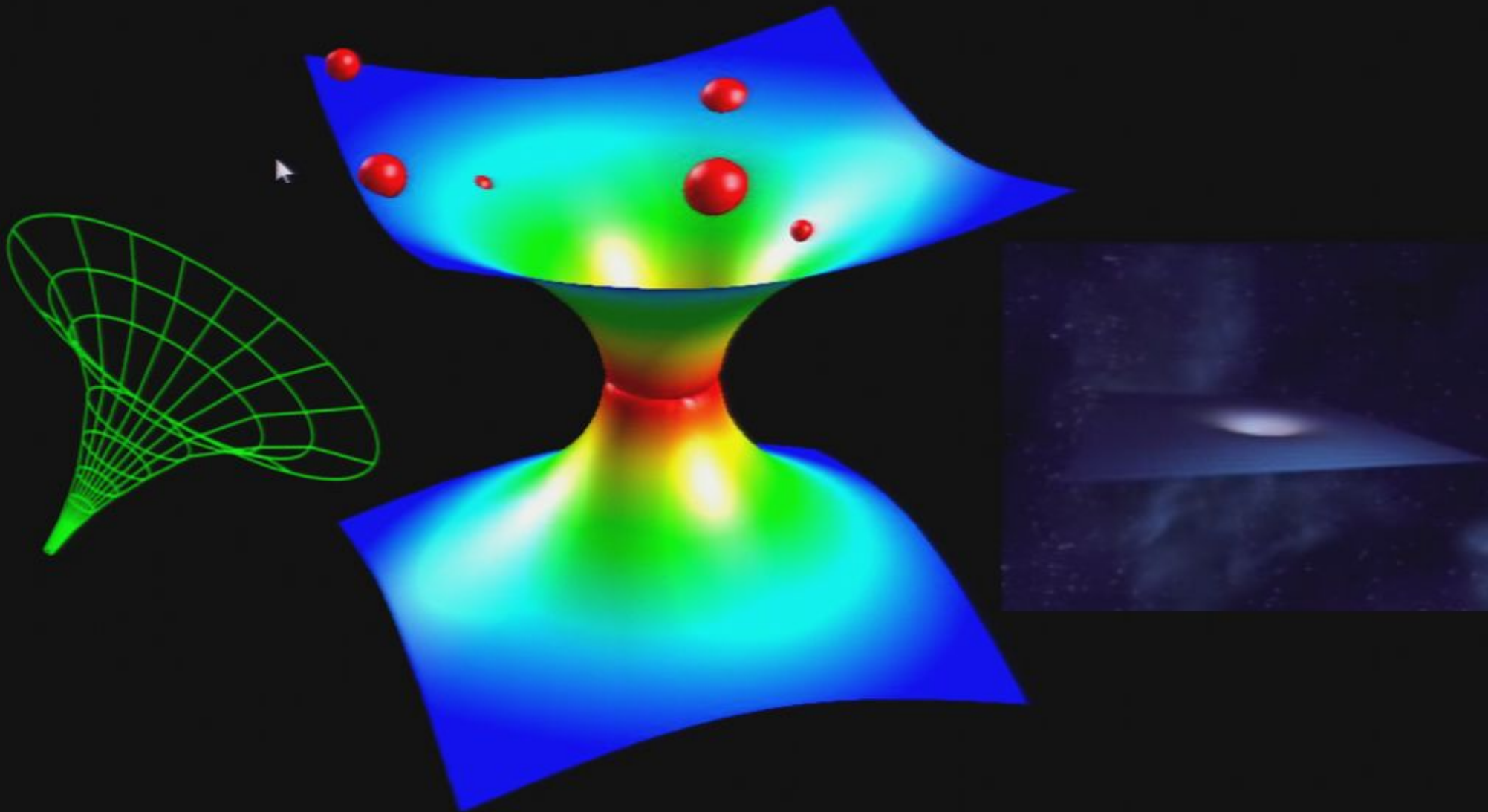
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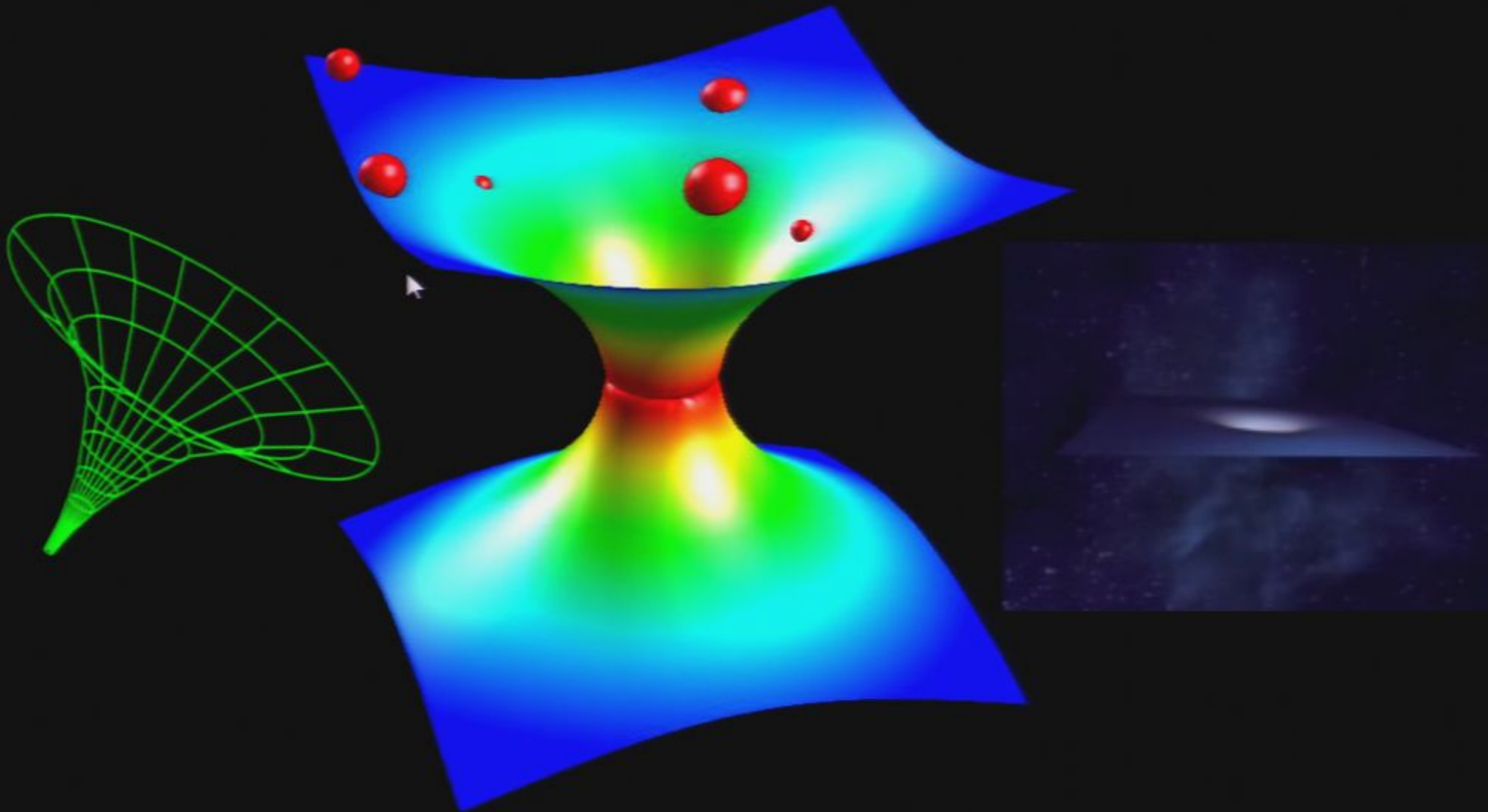
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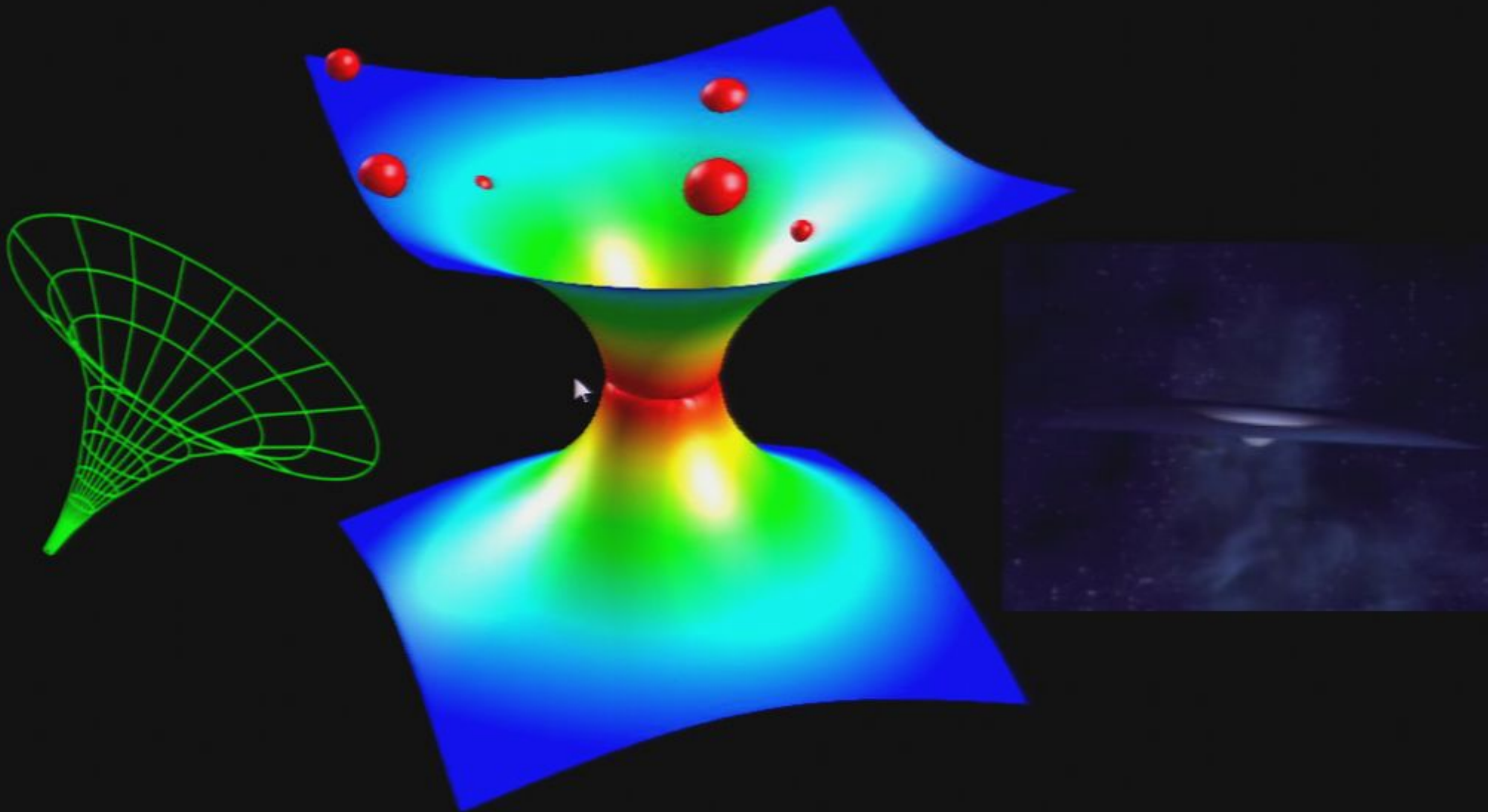
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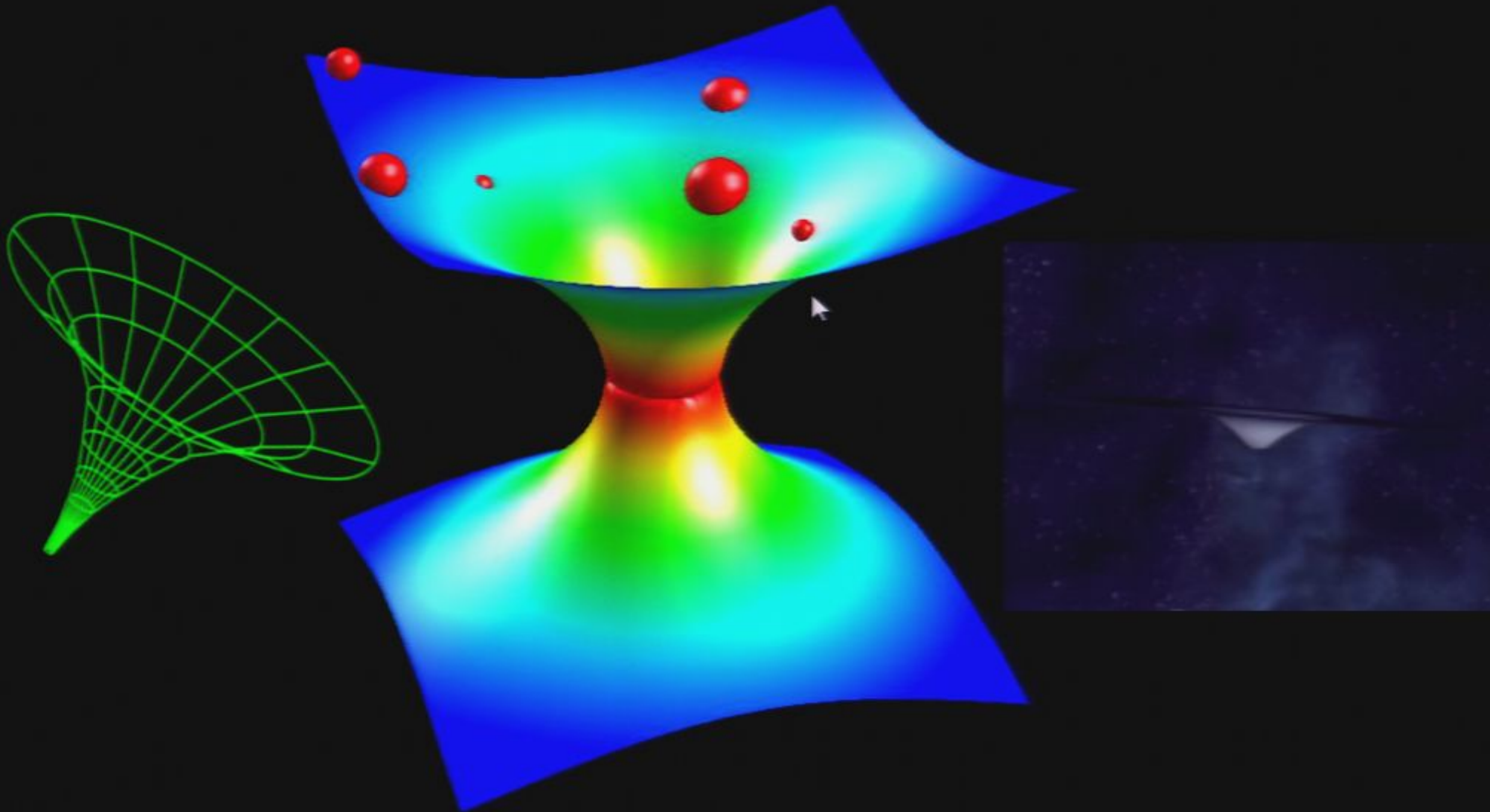
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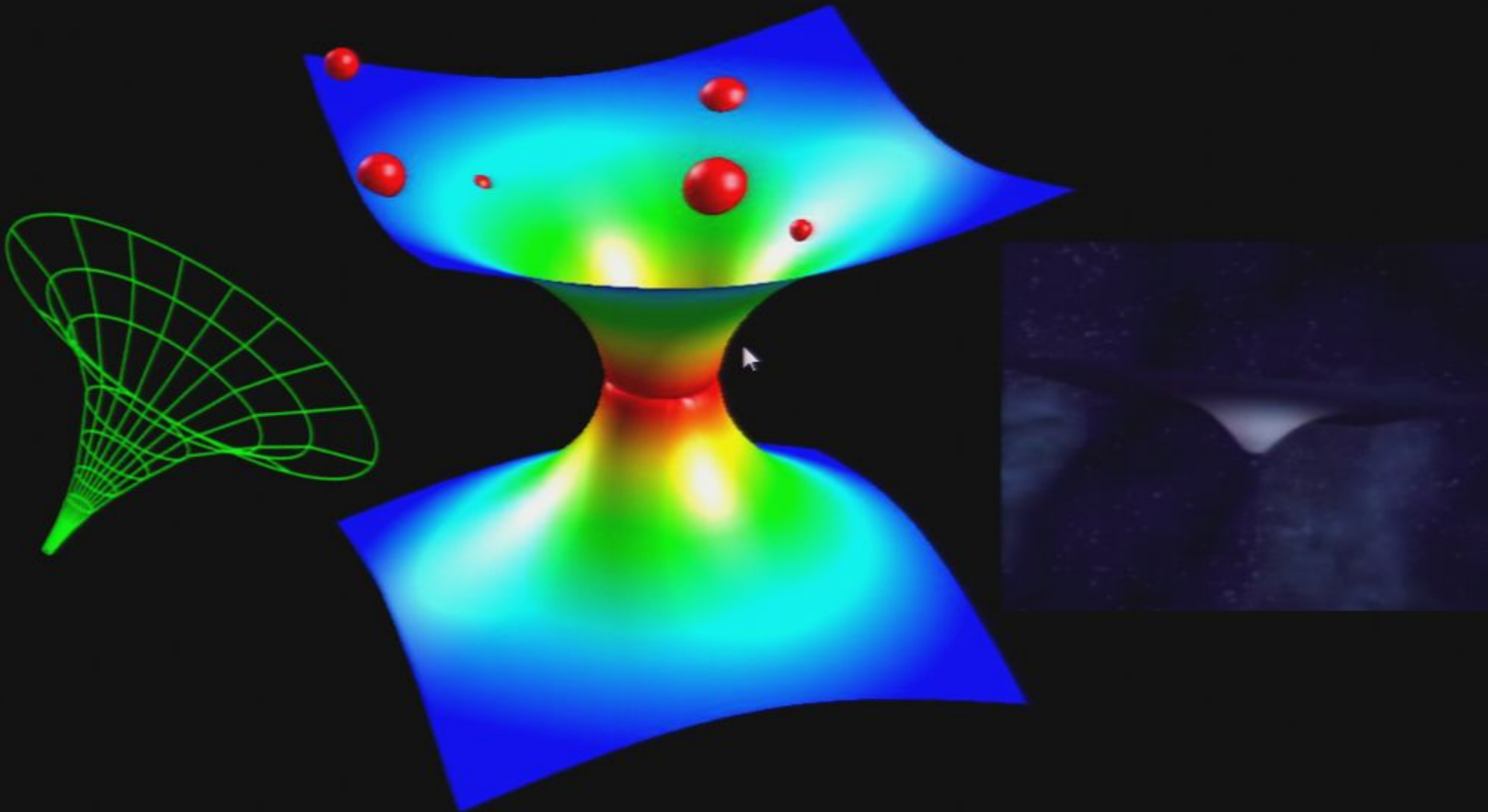
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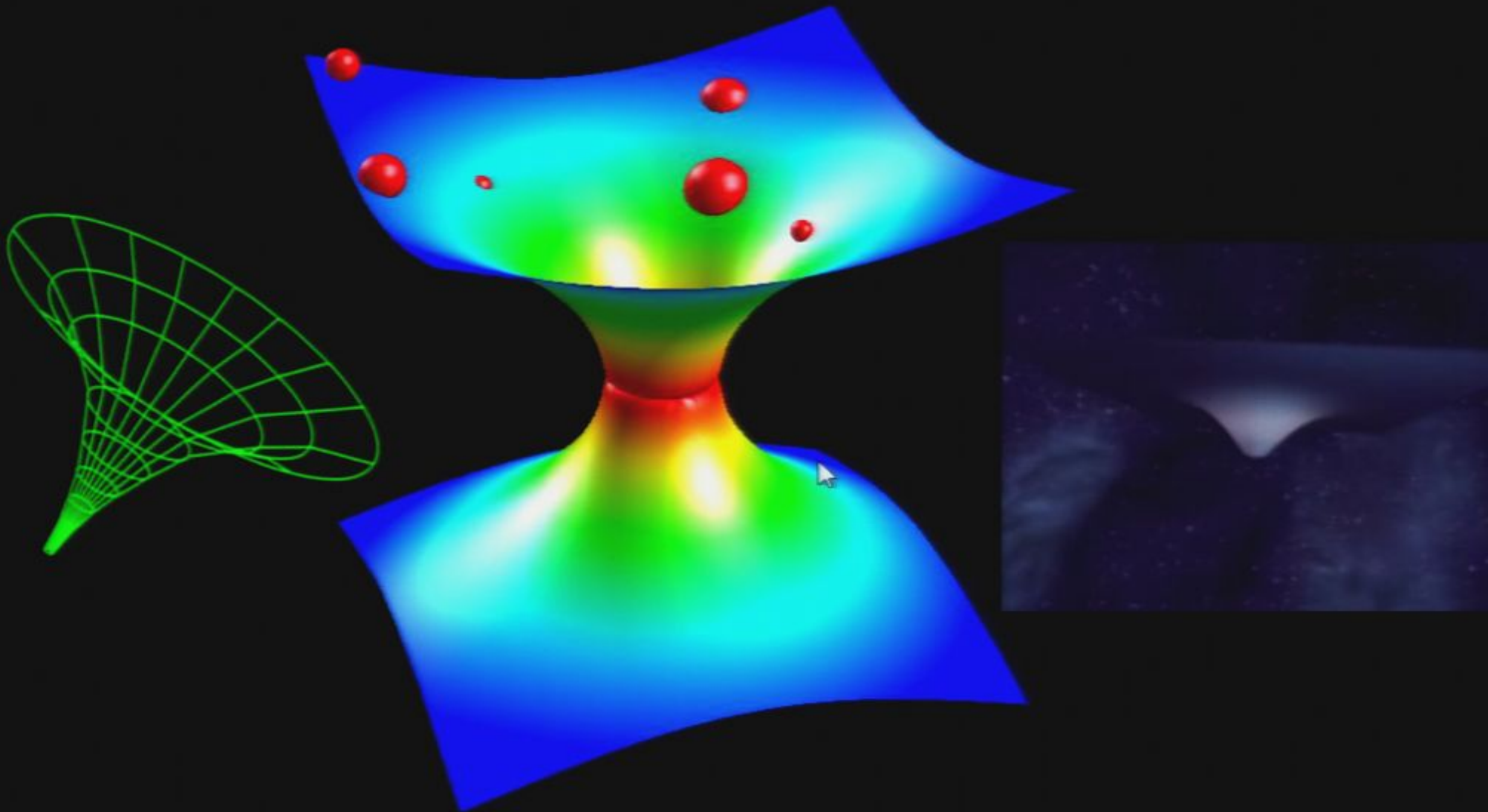
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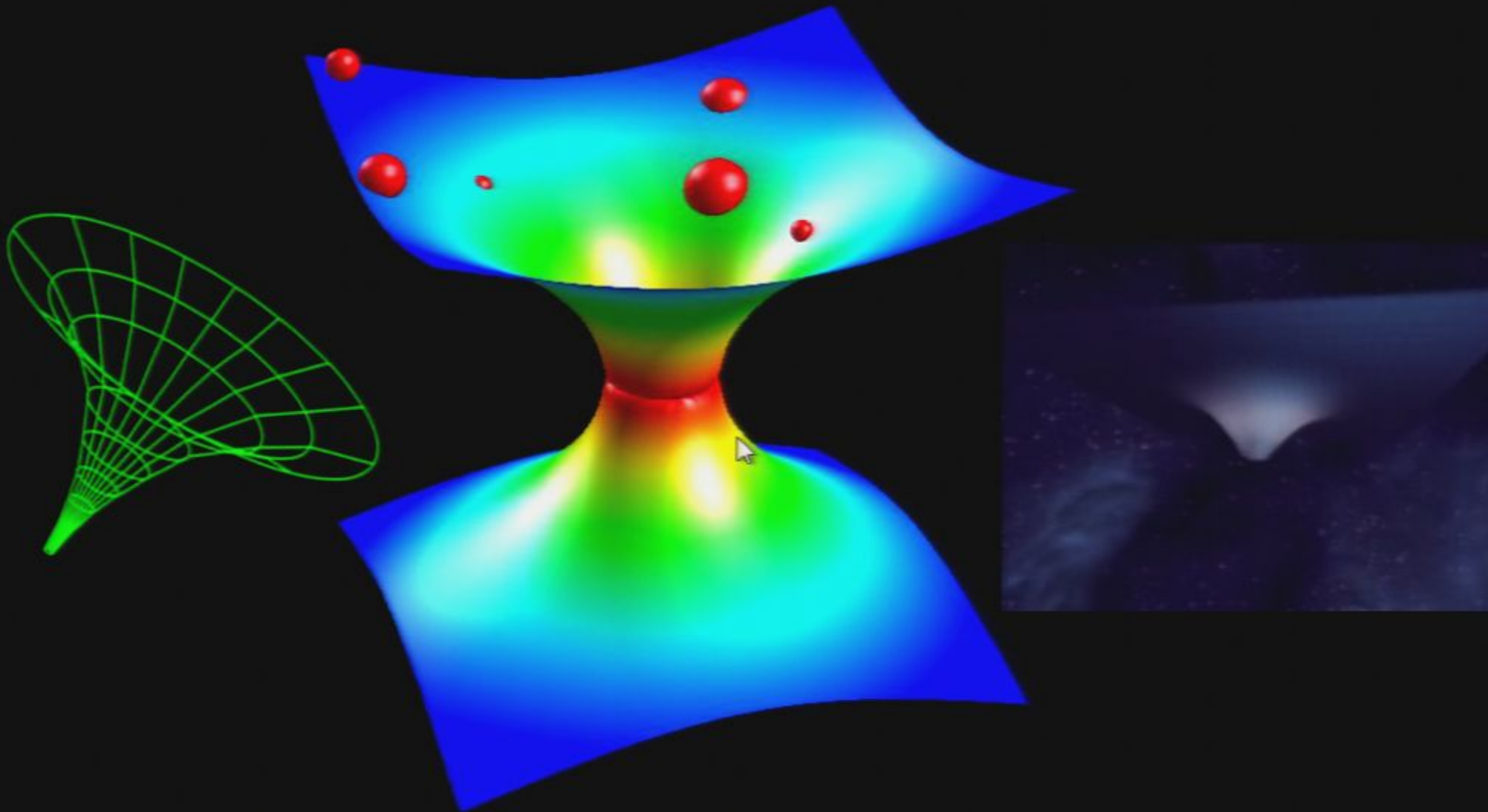
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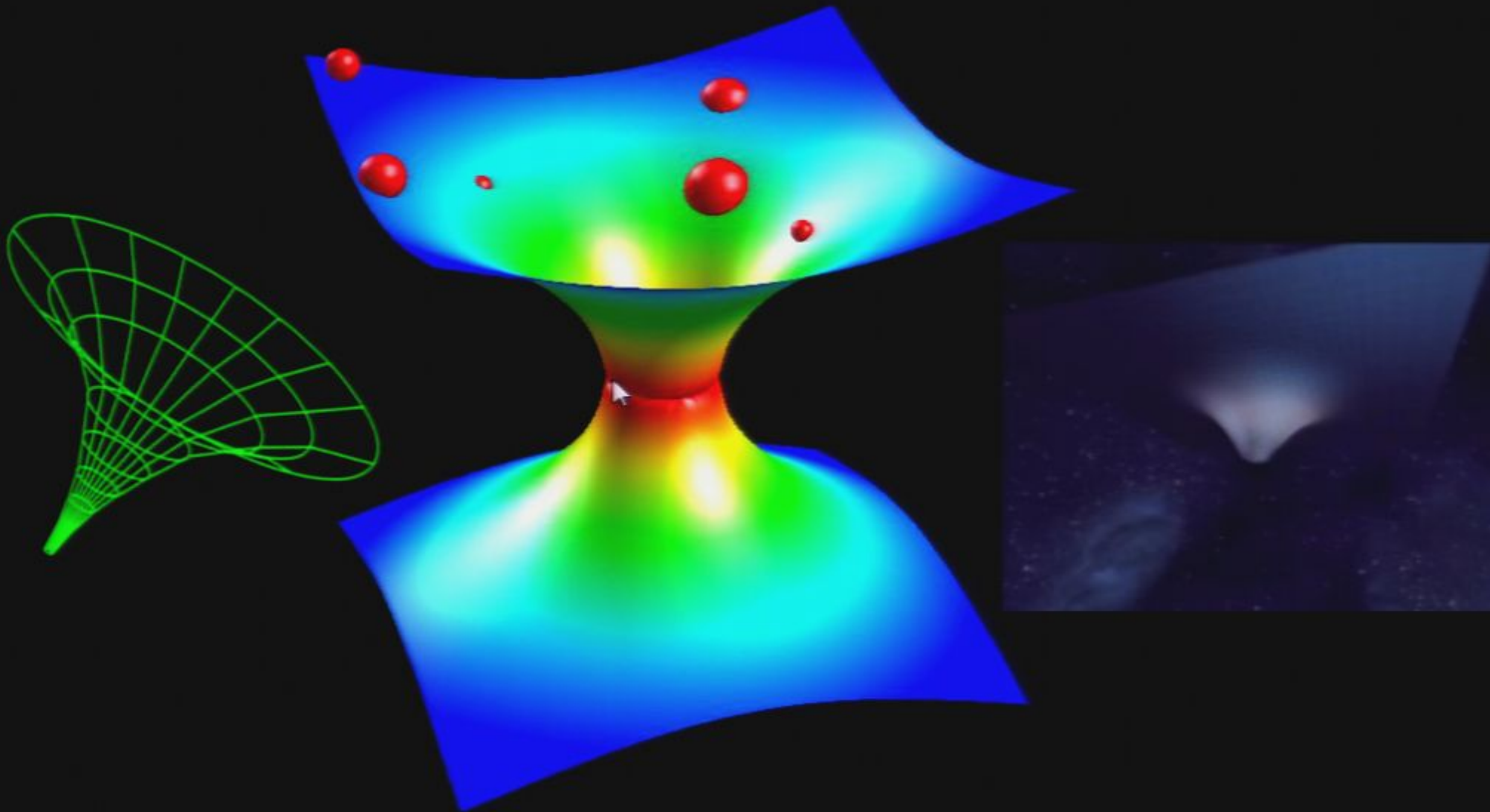
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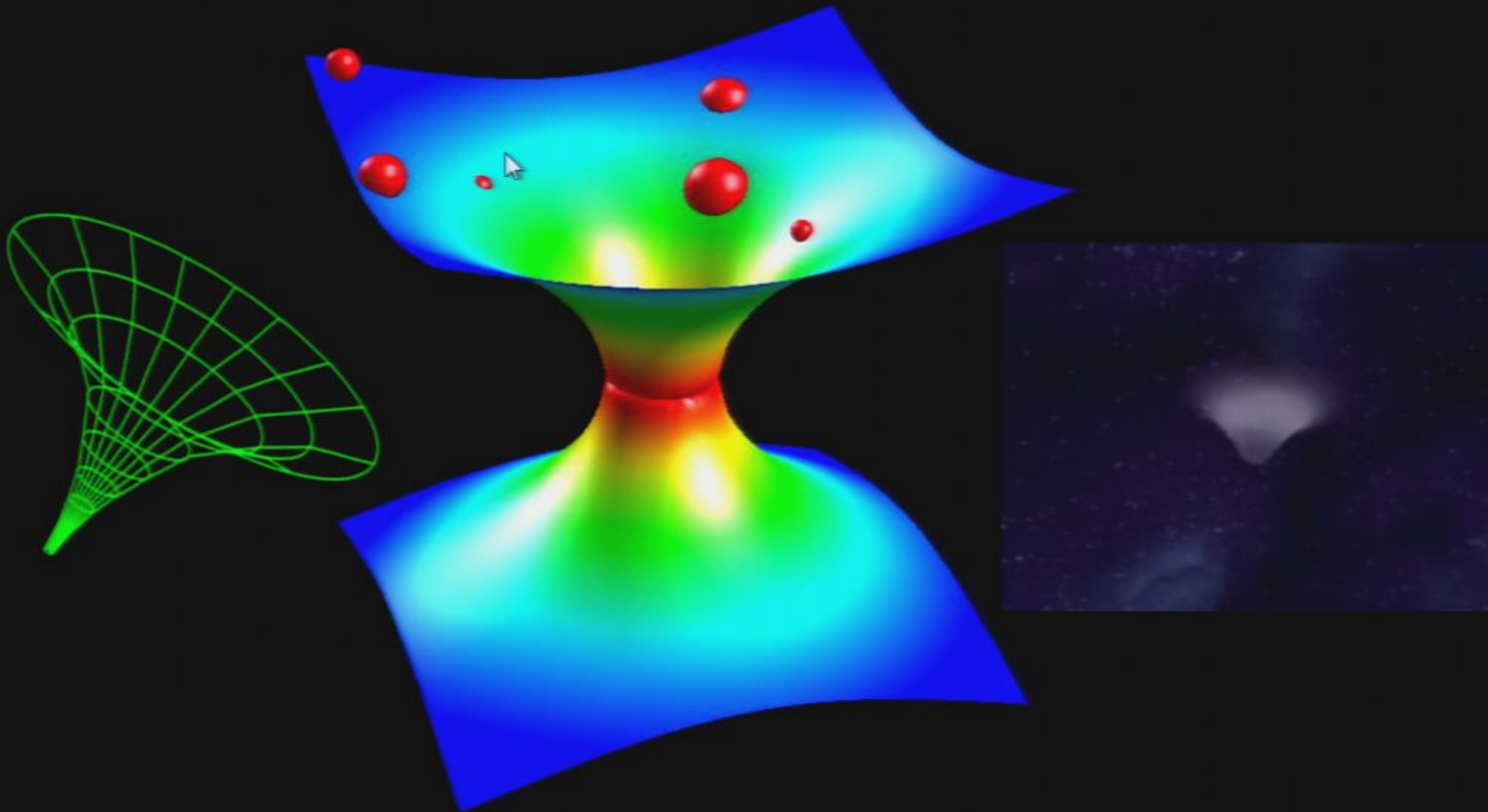
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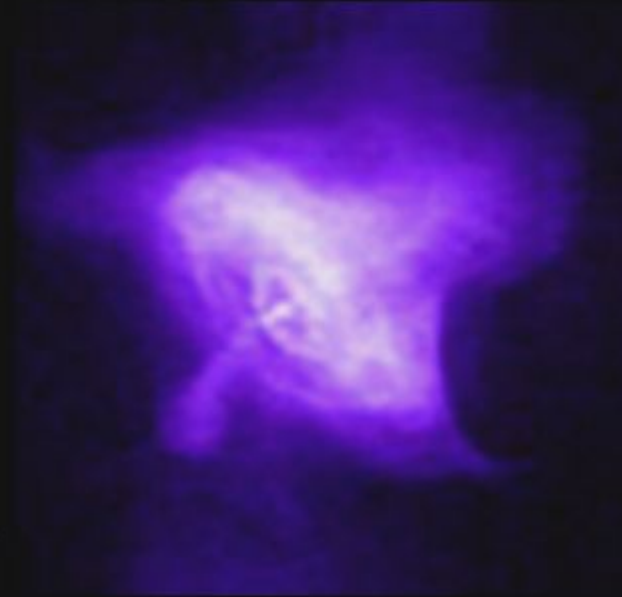


Sir Arthur Eddington



- *1926 Book - The internal constitution of the Stars*
- *Early proponent of Einstein's Theory of General Relativity (next to Einstein best expert on General Relativity)*
- *Poses the mystery of white dwarfs and attacks the reality of black holes predicted by Schwarzschild.*
- *Believed White Dwarf was last state in a stars life (rock Star)*
- *Paradox with White Dwarf*

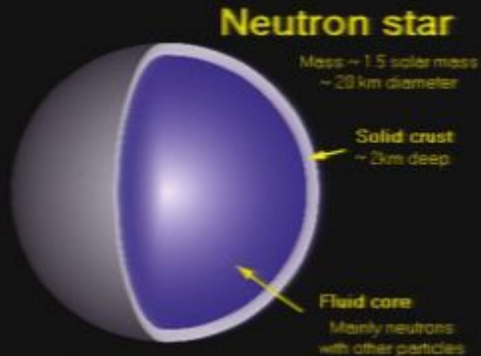
Subrahmanyan Chandrasekhar



- *Idolized Eddington, resolved Eddington's paradox*
- *In 1930 he showed that there is a maximum mass for White Dwarfs*
- *1935 Eddington attacks his work. "Chandra" left the field of Blackholes until 1970's*
- *Nobel Prize in Physics 1983*



Walter Baade and Fritz Zwicky

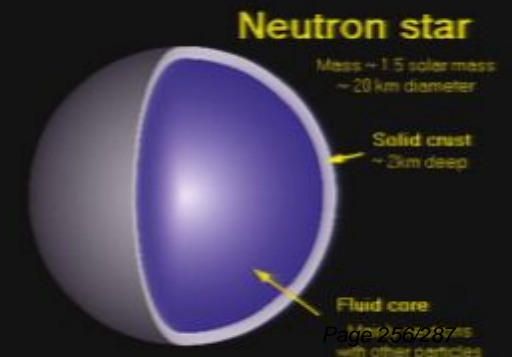


• *Identifies the process of a supernovae, predicted that this collapse strips the atoms of their electrons, packing the nuclei together as a neutron star.*

• *Neutron stars would not be verified observably until 1968.*

• *Identified the galaxies associated with cosmic radio sources.*

• *Still something was missing that took a star from fusion to supernovae.*



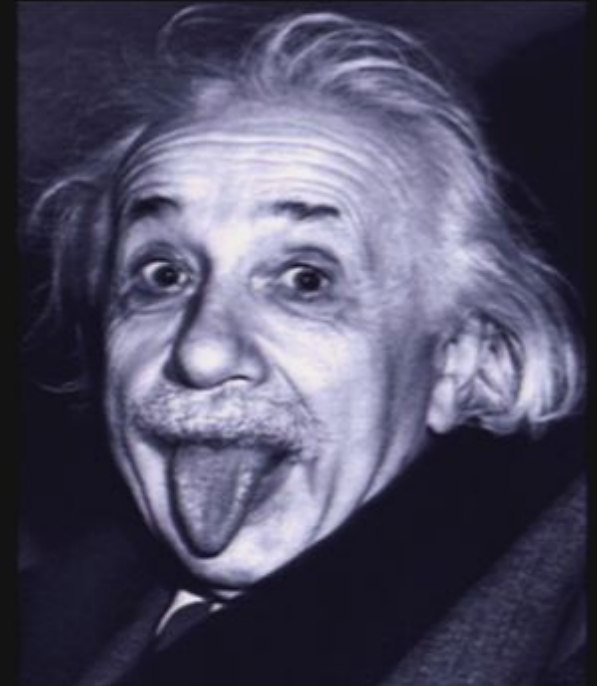
Robert J. Oppenheimer

- *Showed that there is a maximum mass for a neutron star from 1.5 to about 3 solar masses (1938).*
- *In a highly idealized calculation, showed that an imploding star forms a black hole.*
- *Led the American atomic bomb project.*
- *Which provided the opportunity to experimentally verify and test theories (too expensive for the universities) and the development of the atomic bombs which mimic the power source for the sun to come up with the mathematics and understanding of stellar mechanics*
- *Major battle with Wheeler.*



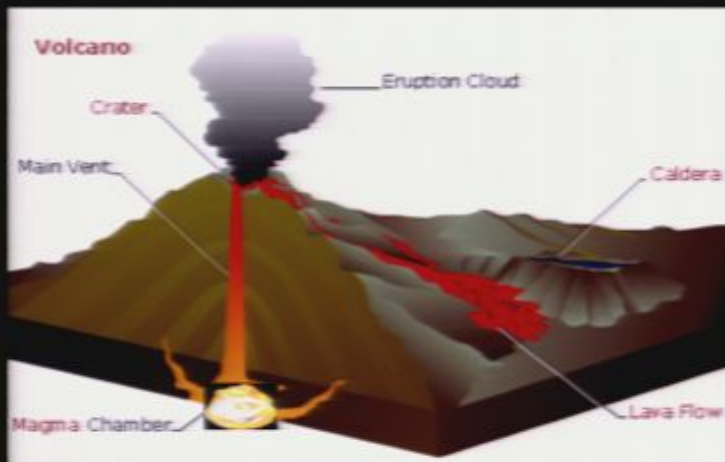
Robert J. Oppenheimer

In 1939 Einstein wrote a paper about his concerns about Oppenheimer's paper and the Schwarzschild radius and states "Schwarzschild singularities do not exist in physical reality". He demonstrated that a collapsing star is unstable when it reaches the Schwarzschild radius, which ended up being mute since that star collapses into a singularity there anyway.



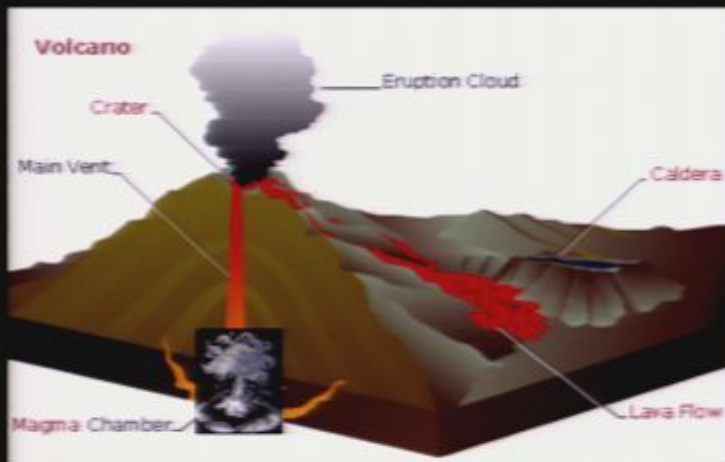
Yakov Zel'dovich

- *Soviet counterpart to Oppenheimer.*
- *Developed the theory of nuclear chain reactions. (1939)*
- *Lead theorist on USSR atomic bomb (1945)*
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- *Super massive black holes power Quasars (1960's).*



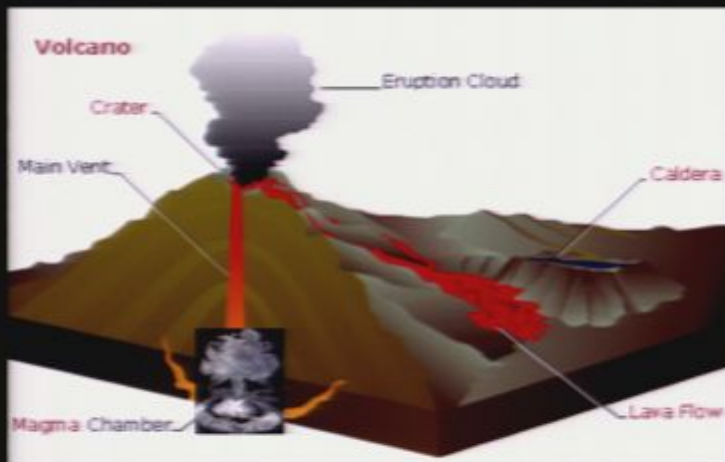
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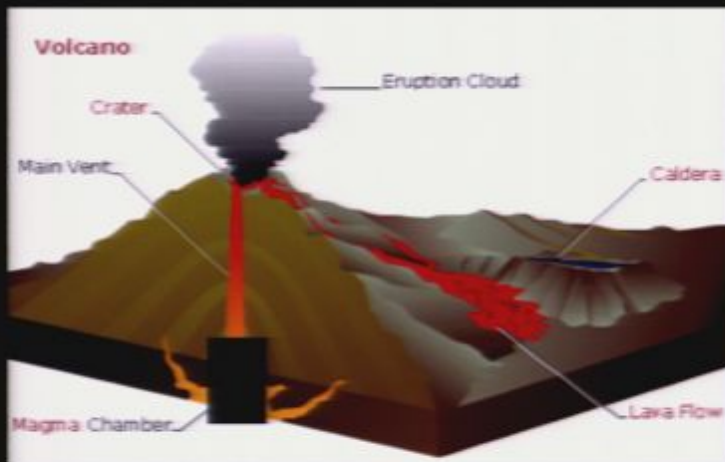
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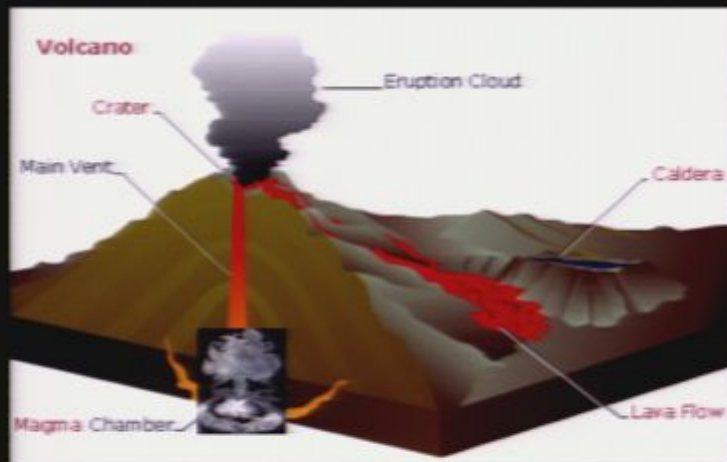
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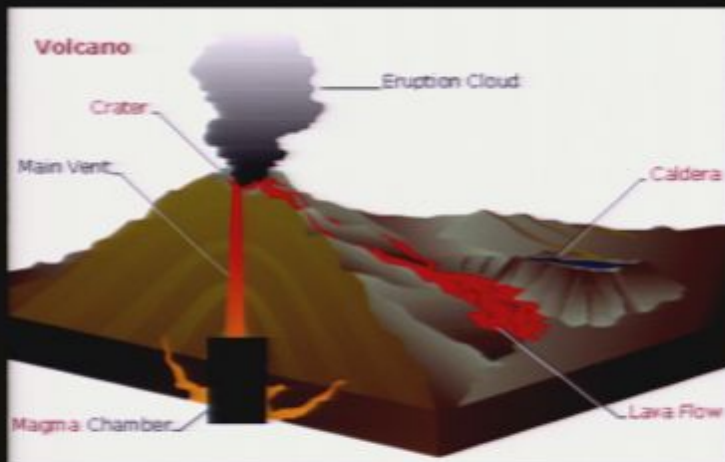
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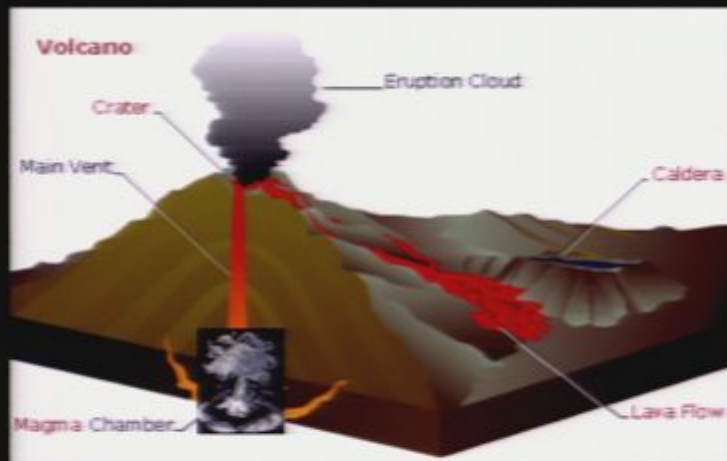
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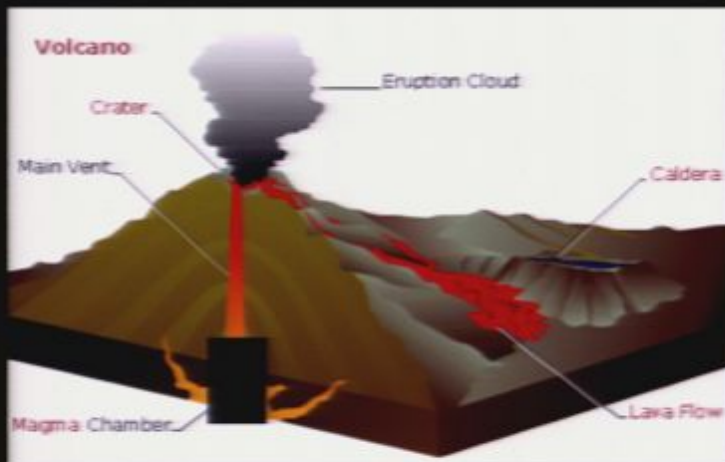
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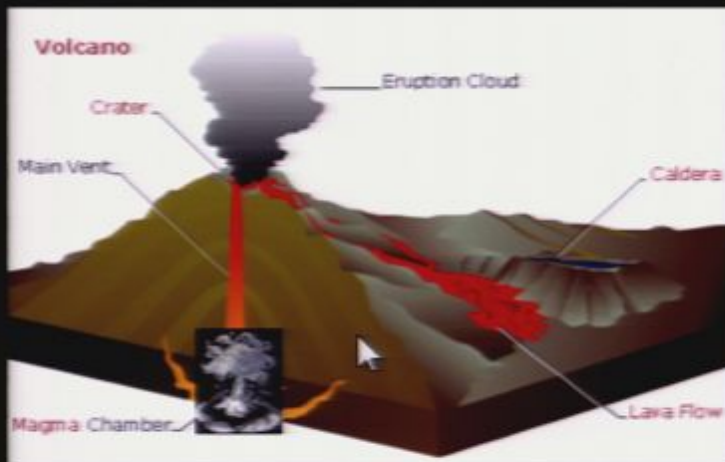
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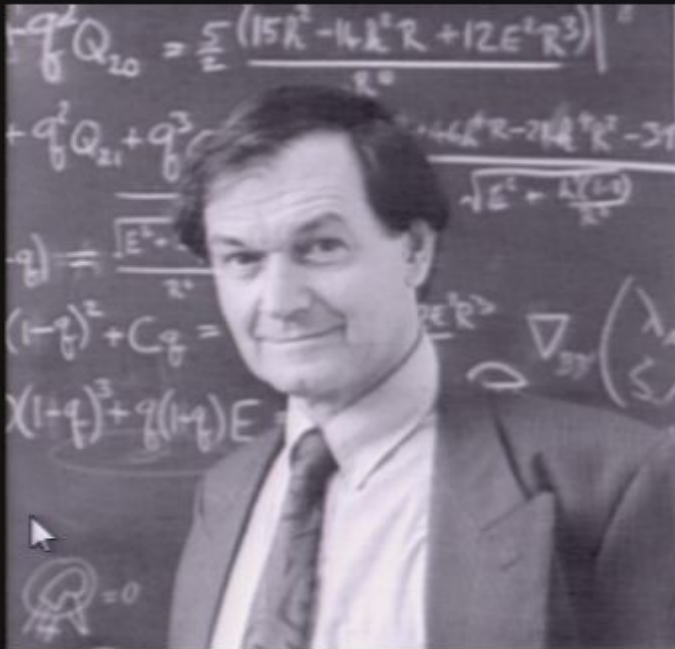


John Wheeler

- *With Bohr develops the theory of nuclear fission.*
- *Completes a catalog cold, dead stars firming up evidence of destiny of dead stars. (1957)*
- *Major battle with Oppenheimer about existence of black holes. (1957)*
- *Retracted argument and became the leading proponent of black hole. (1960)*
- *Coined the phrase "Black Hole".*
- *Coined the phrase "a Black Hole has no hair" (1968).*



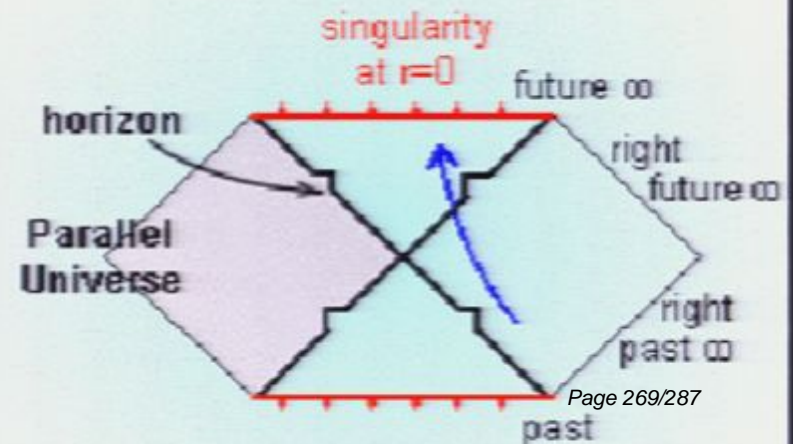
Roger Penrose



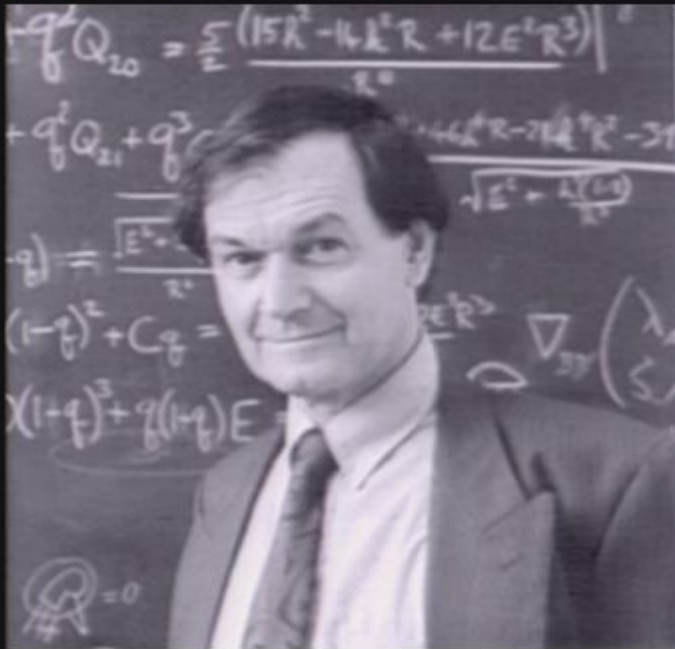
- Speculated black holes lose their hair by radiating it away.
- Discovered that spinning black holes store energy in space outside their horizon (1969).
- Discovered surface area of black holes must increase.
- Proved that black holes must have singularities at their core (1964).
- Proposed cosmic censorship conjecture (1969).



Topology



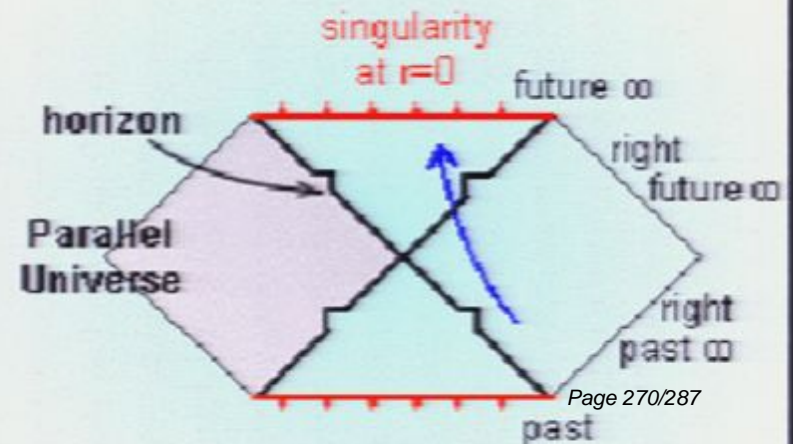
Roger Penrose



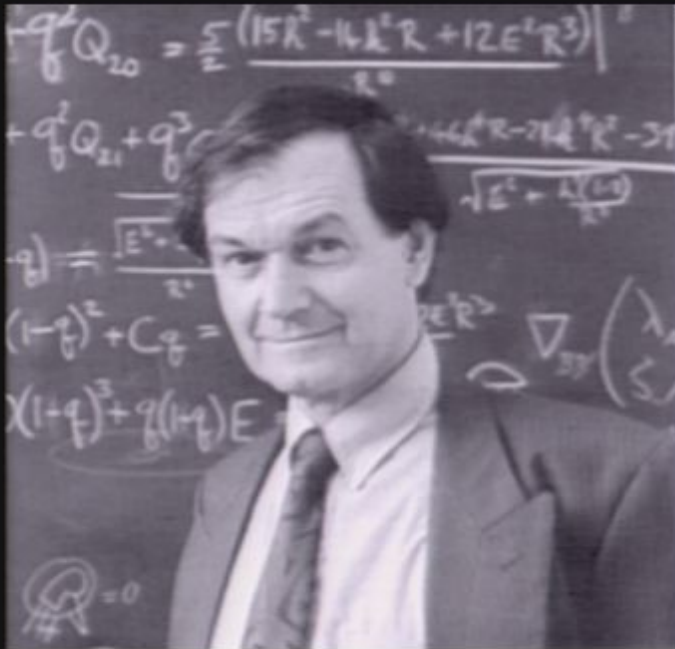
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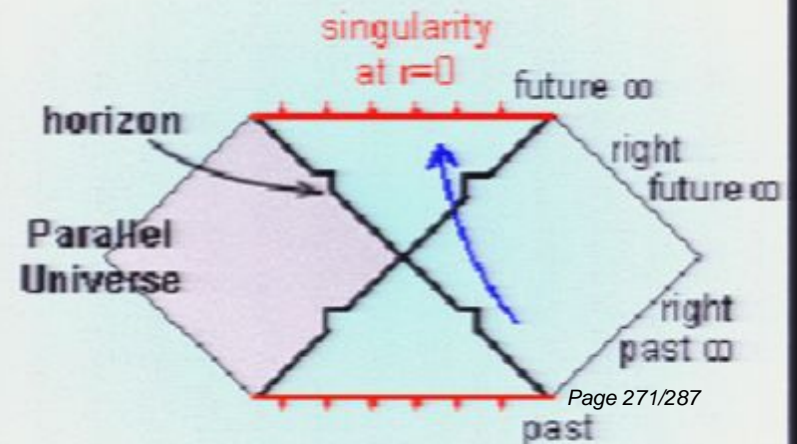
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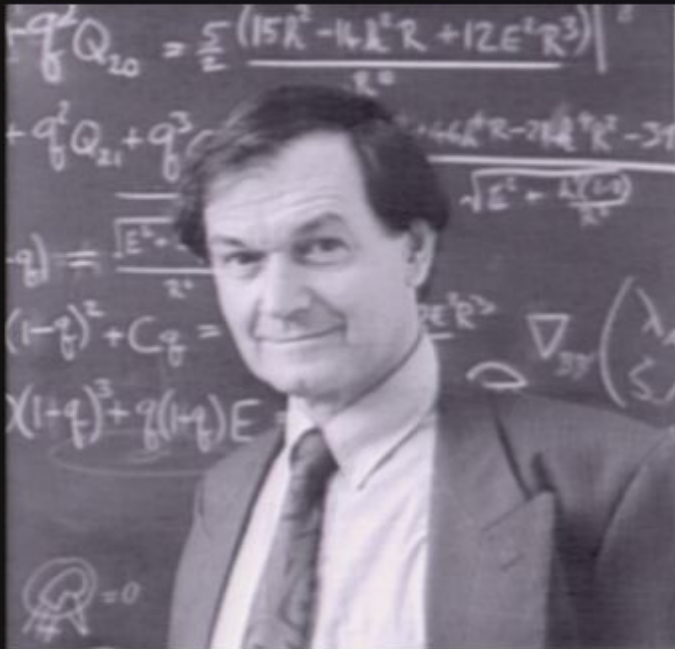
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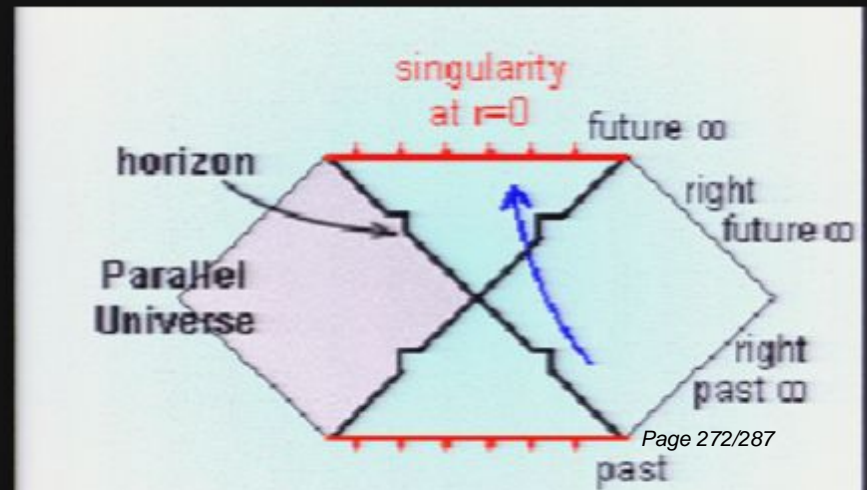
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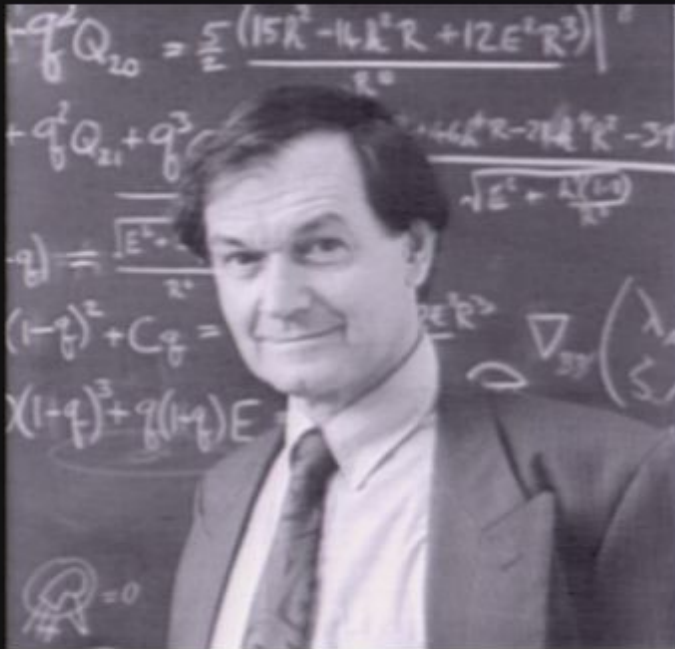
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Topology



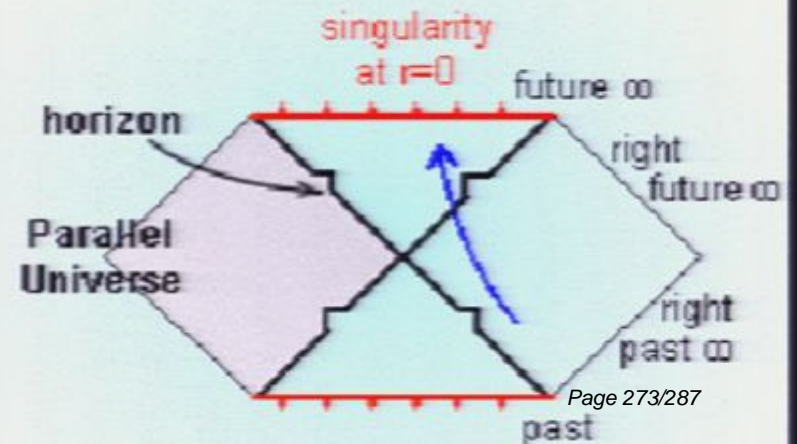
Roger Penrose



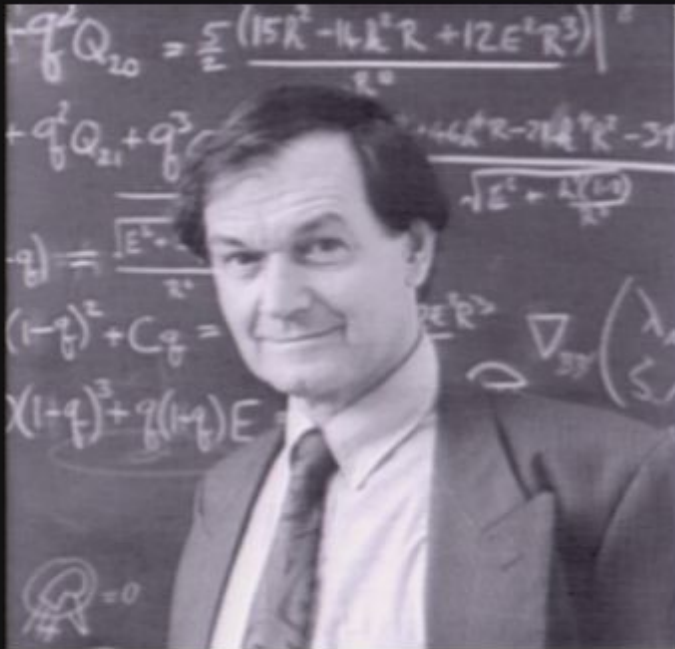
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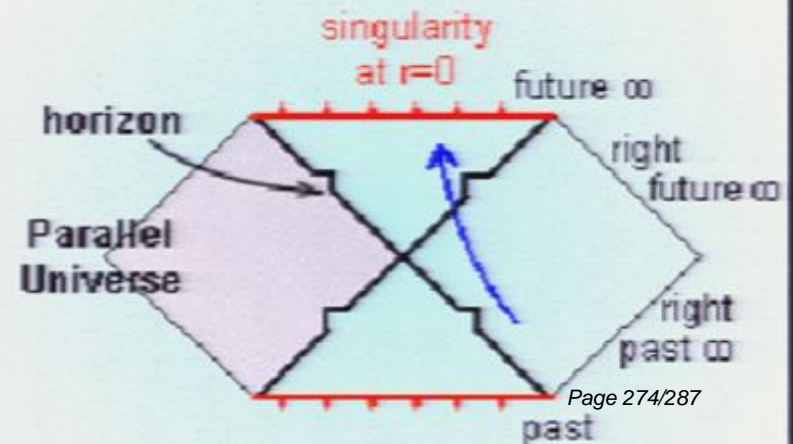
Roger Penrose



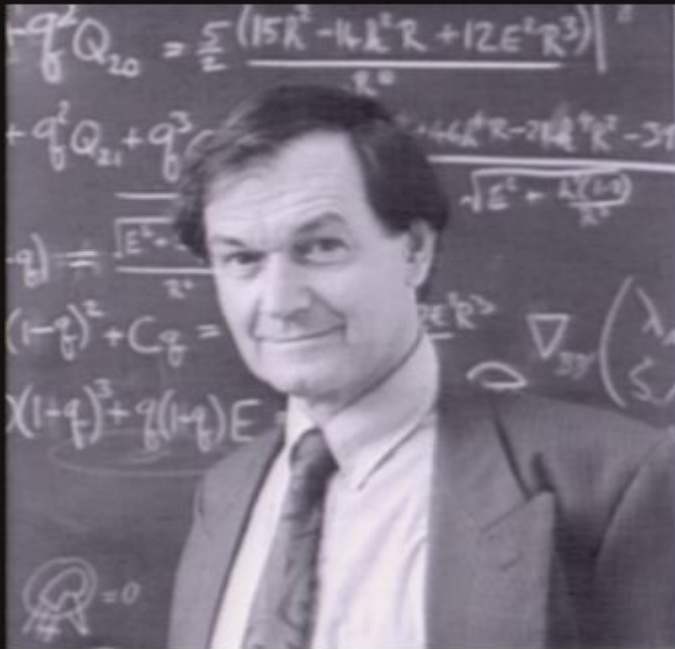
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Topology



Roger Penrose



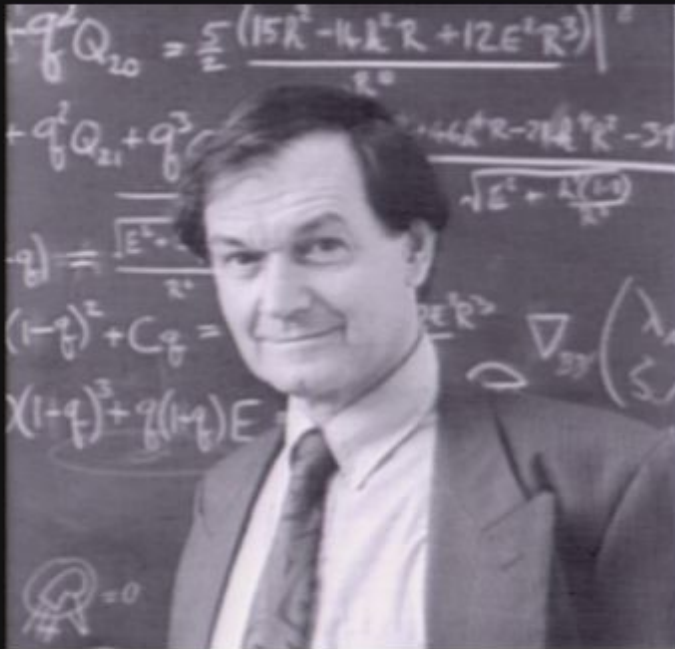
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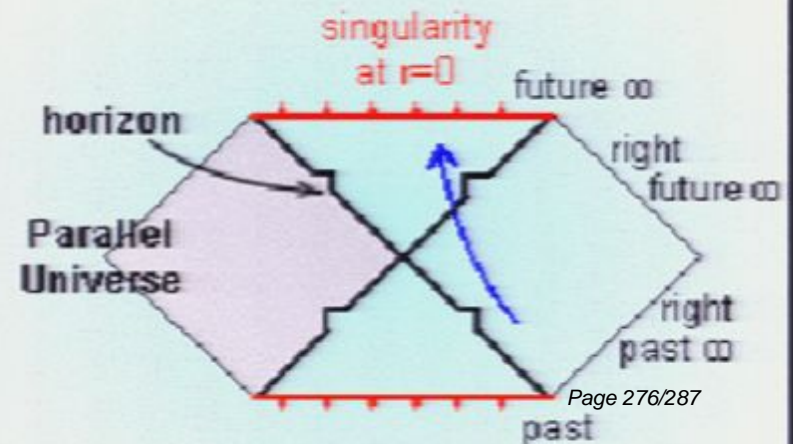
Roger Penrose



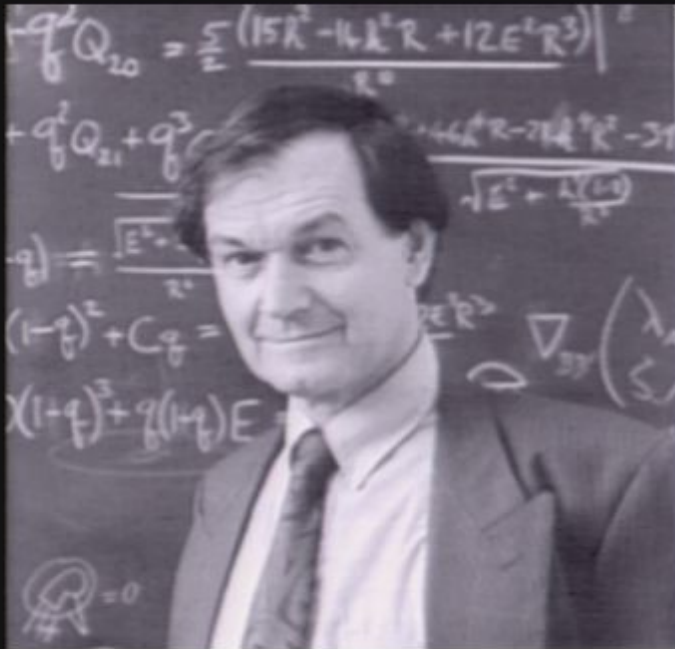
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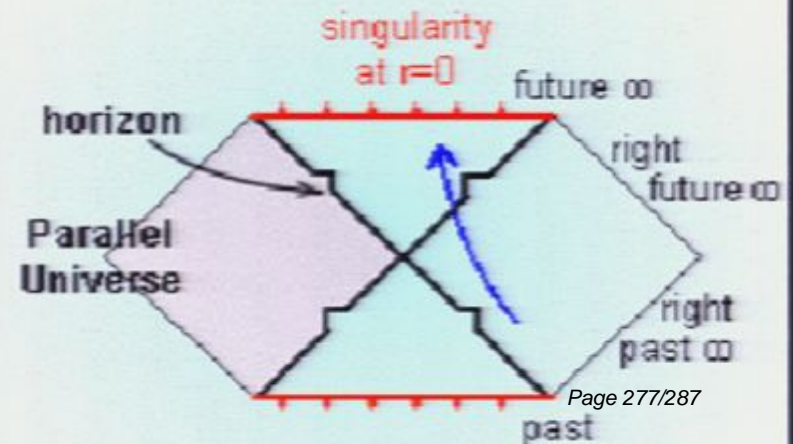
Roger Penrose



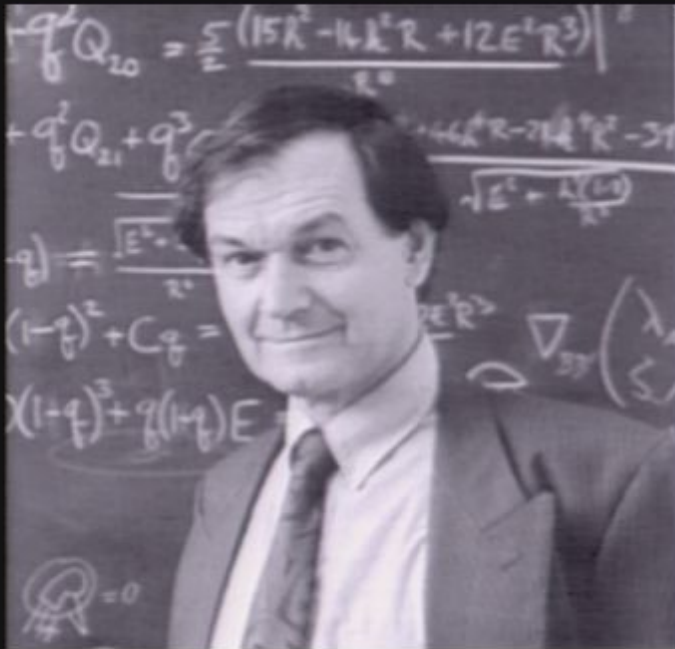
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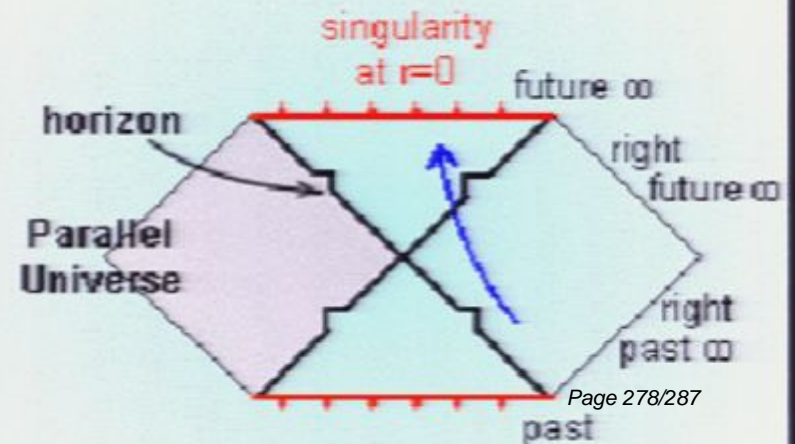
Roger Penrose



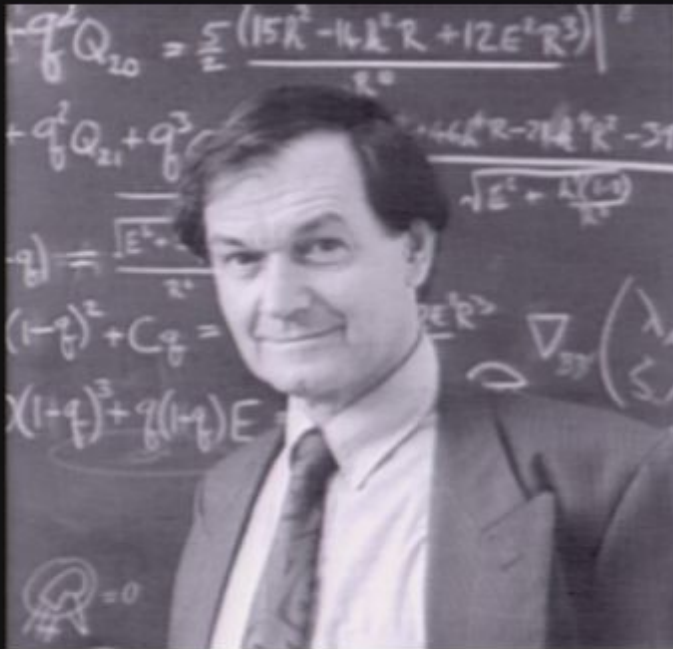
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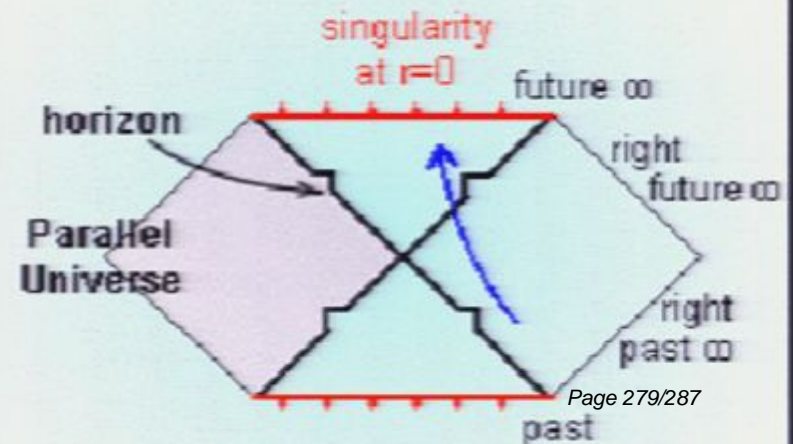
Roger Penrose



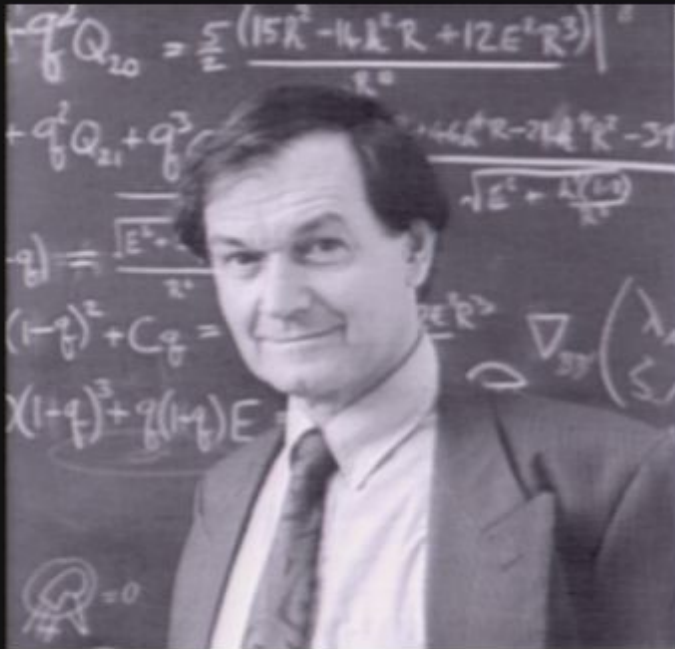
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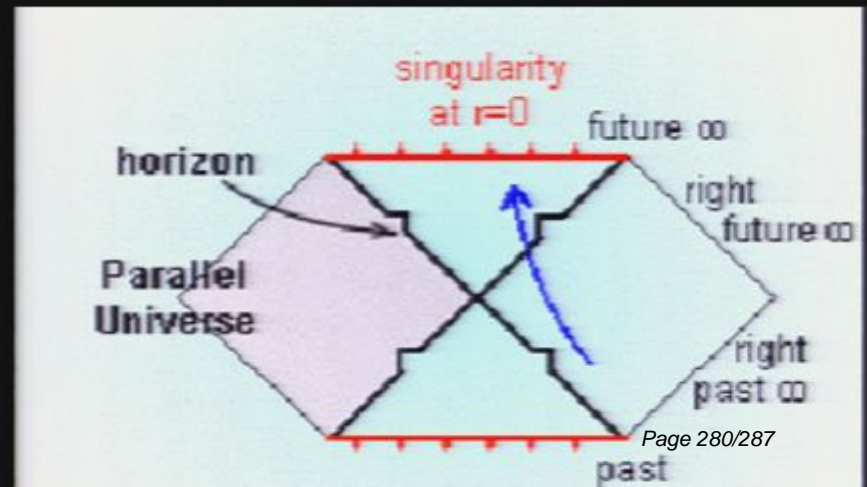
Roger Penrose



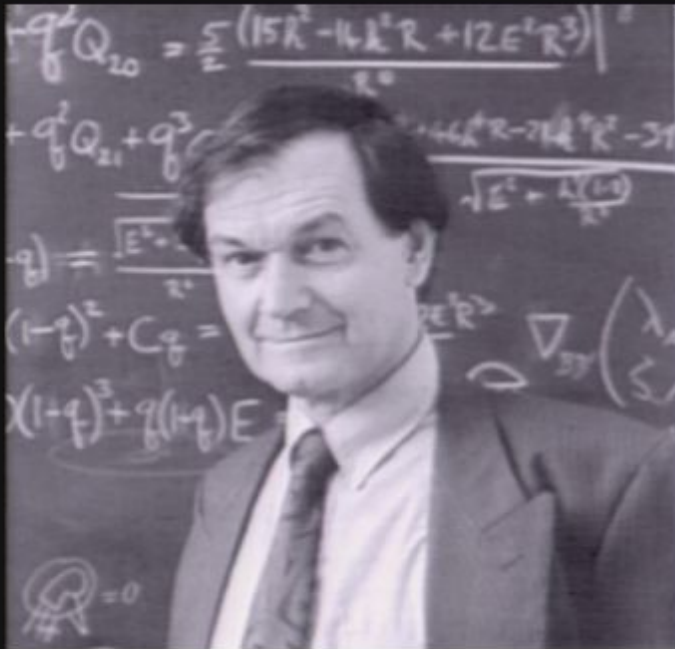
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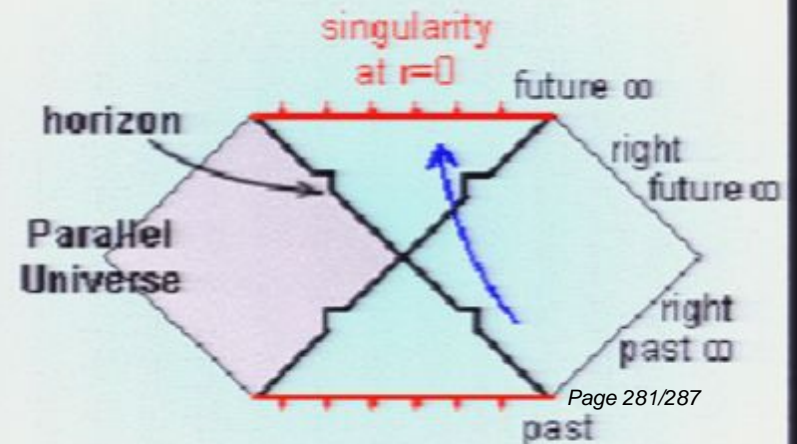
Roger Penrose



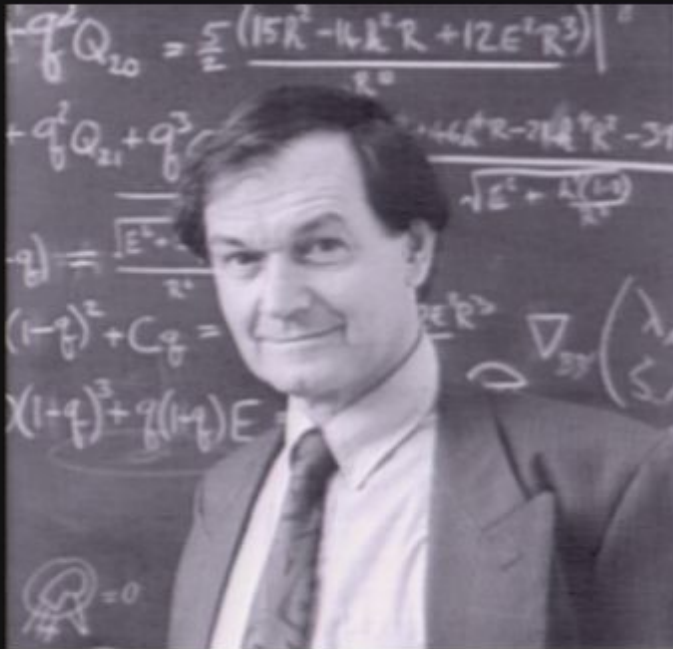
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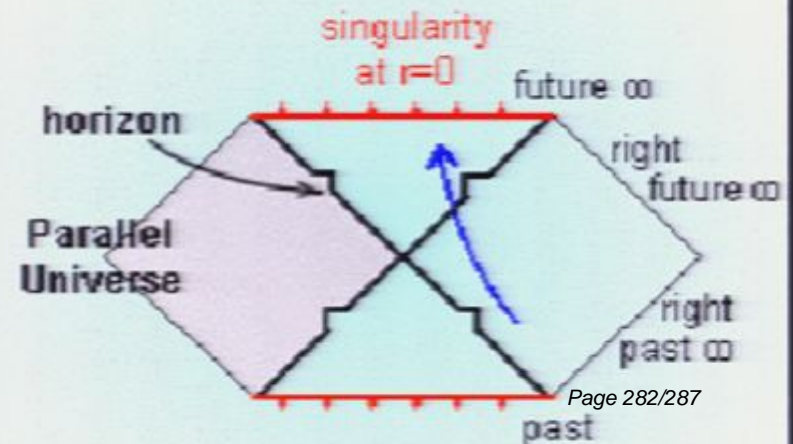
Roger Penrose



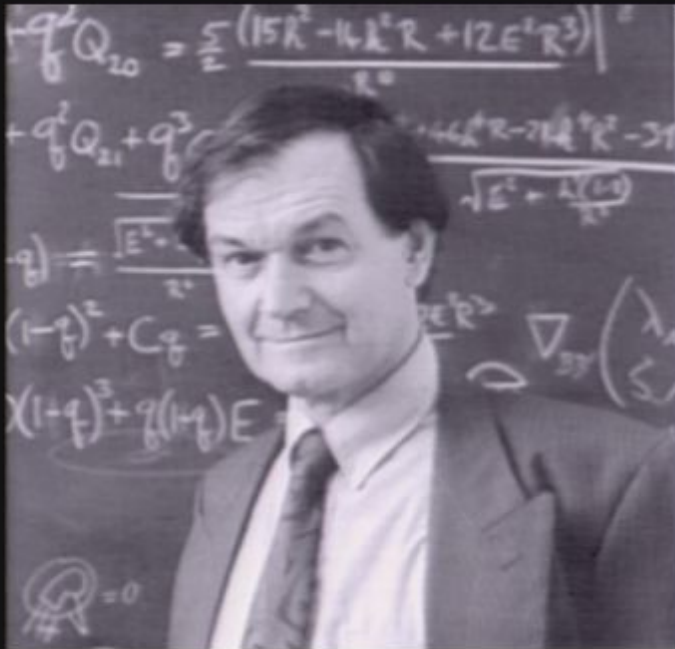
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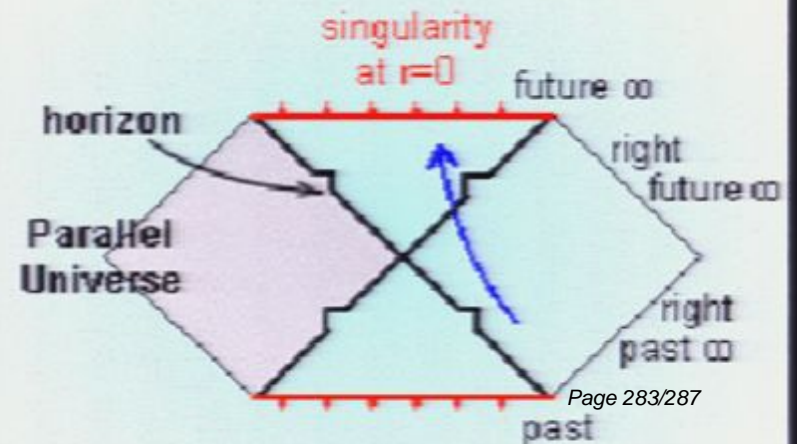
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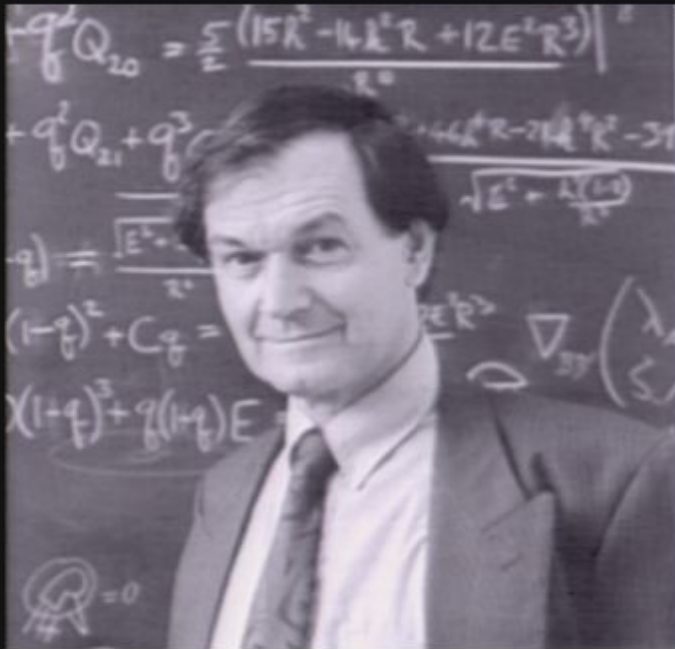
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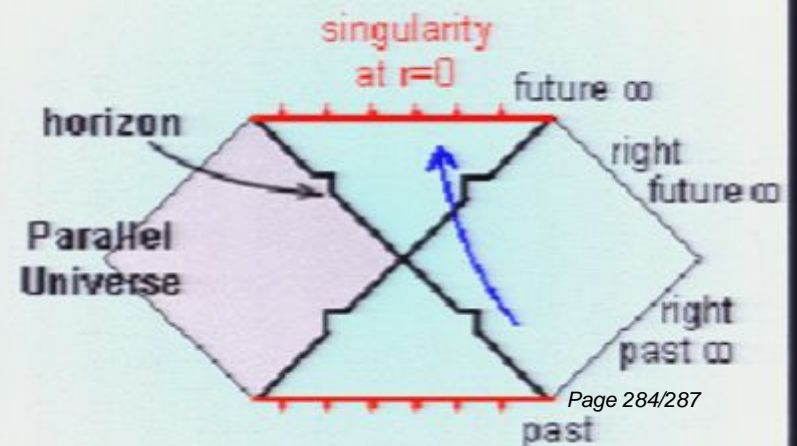
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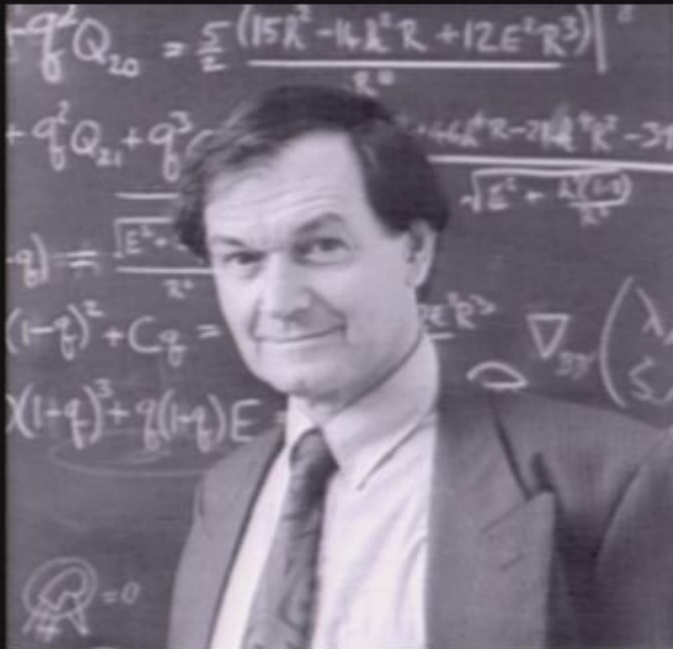
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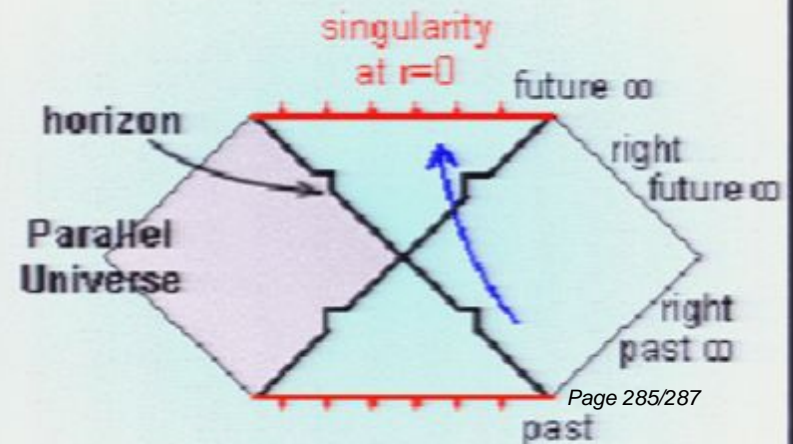
Roger Penrose



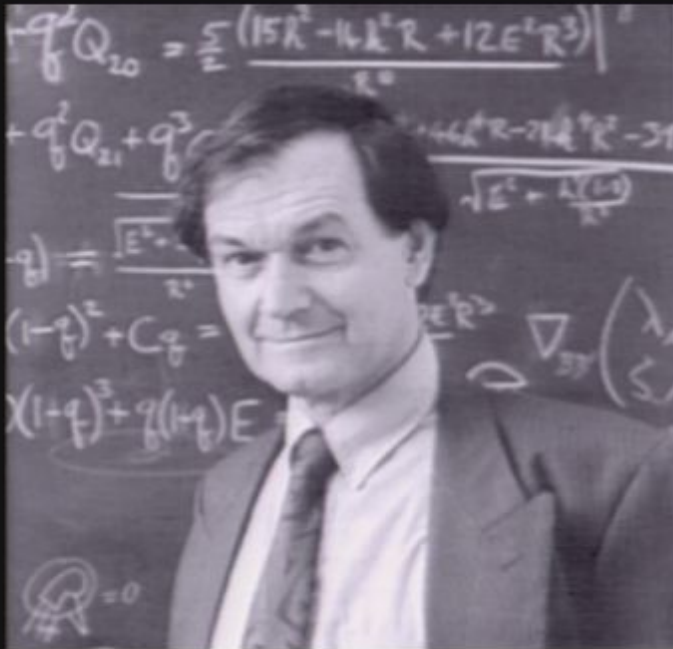
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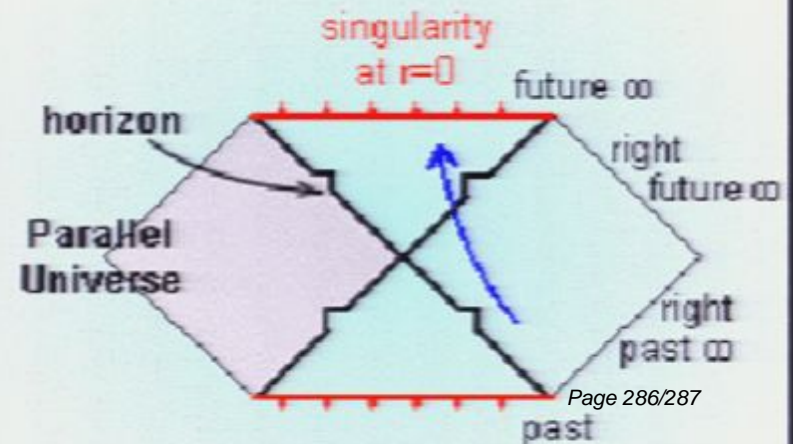
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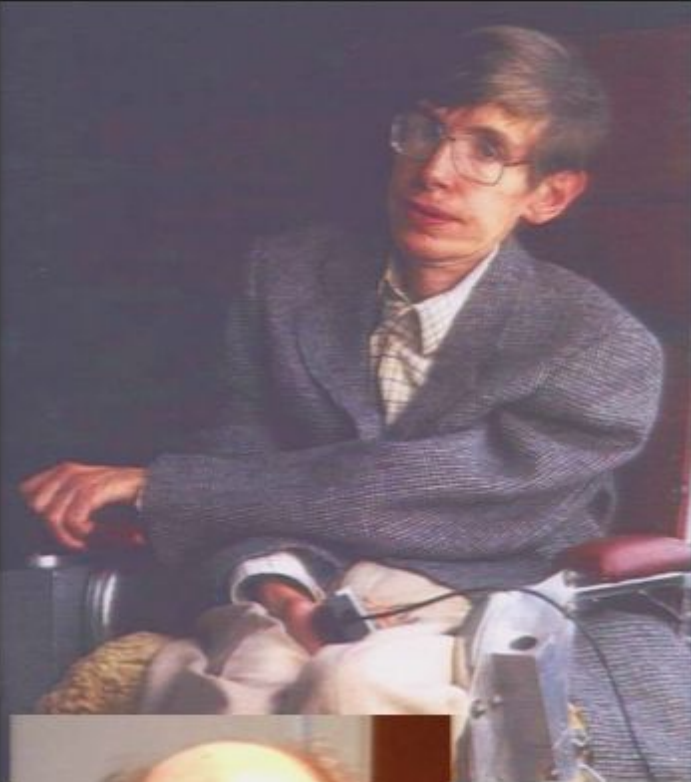
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Topology



The Blackhole Stars Today



Hawking



Bekenstein



Thorne



Susskind



Robert Wald

✦ *Werner Israel*

