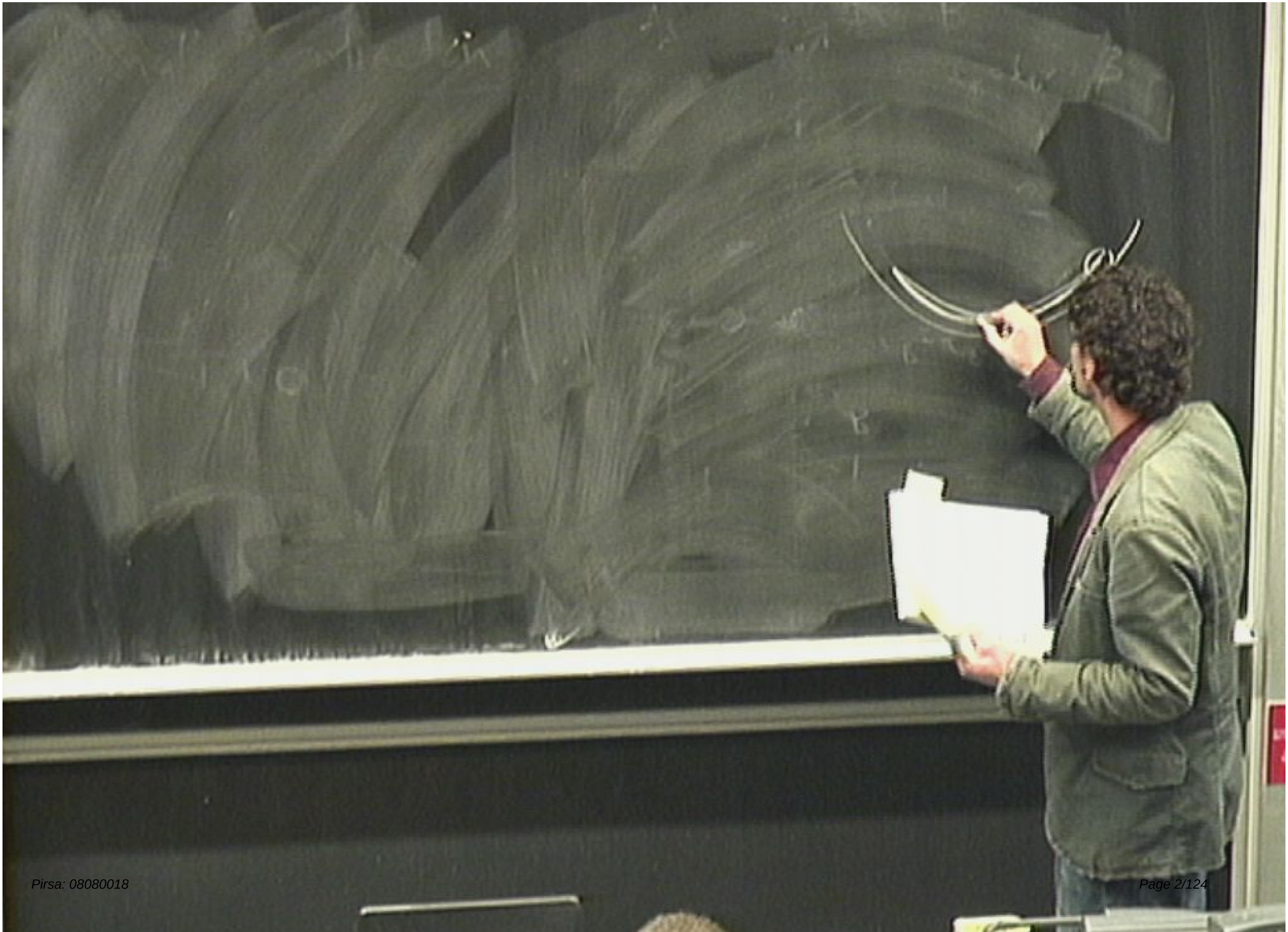
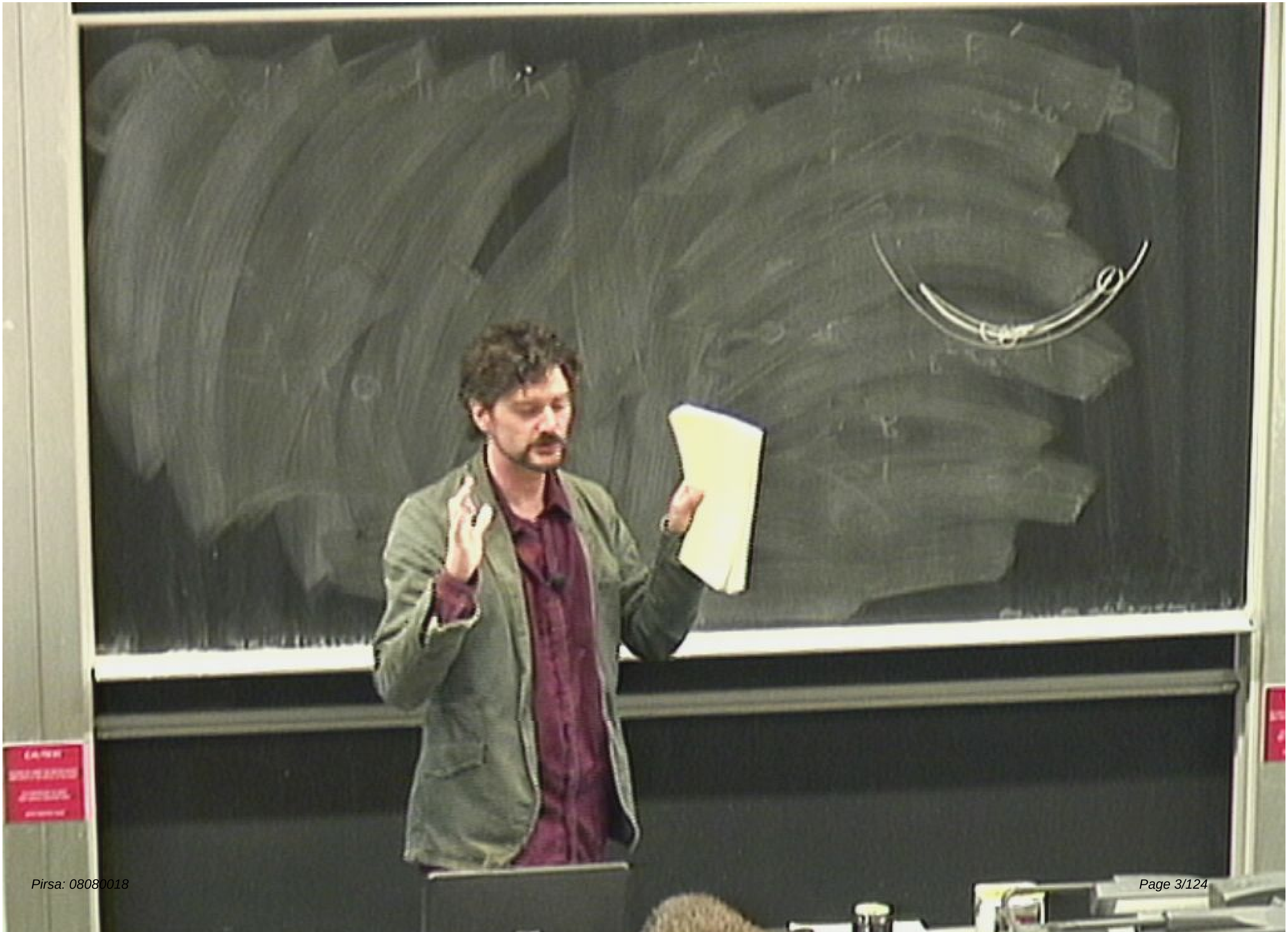
Title: Quantum 4

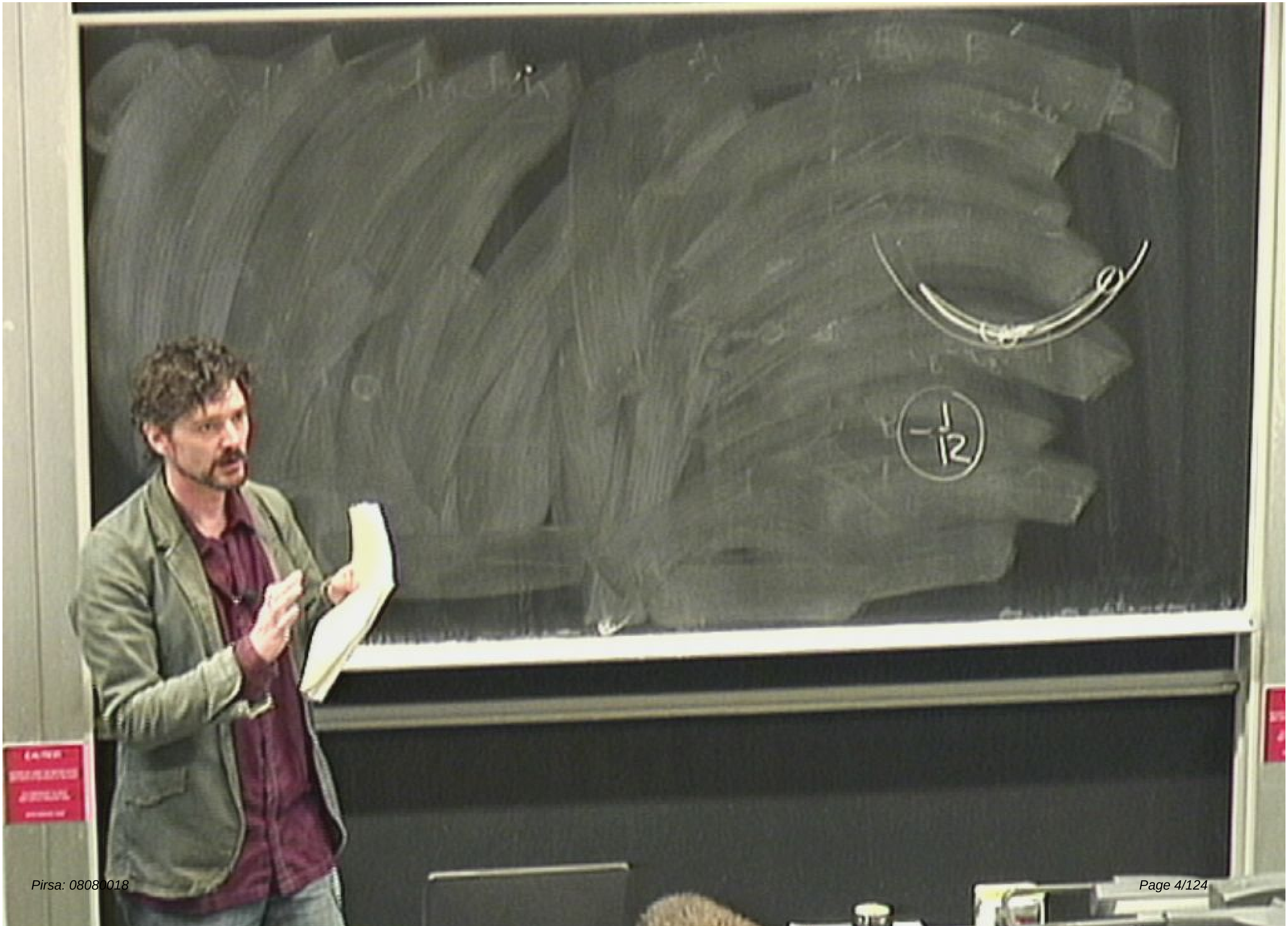
Date: Aug 13, 2008 10:30 AM

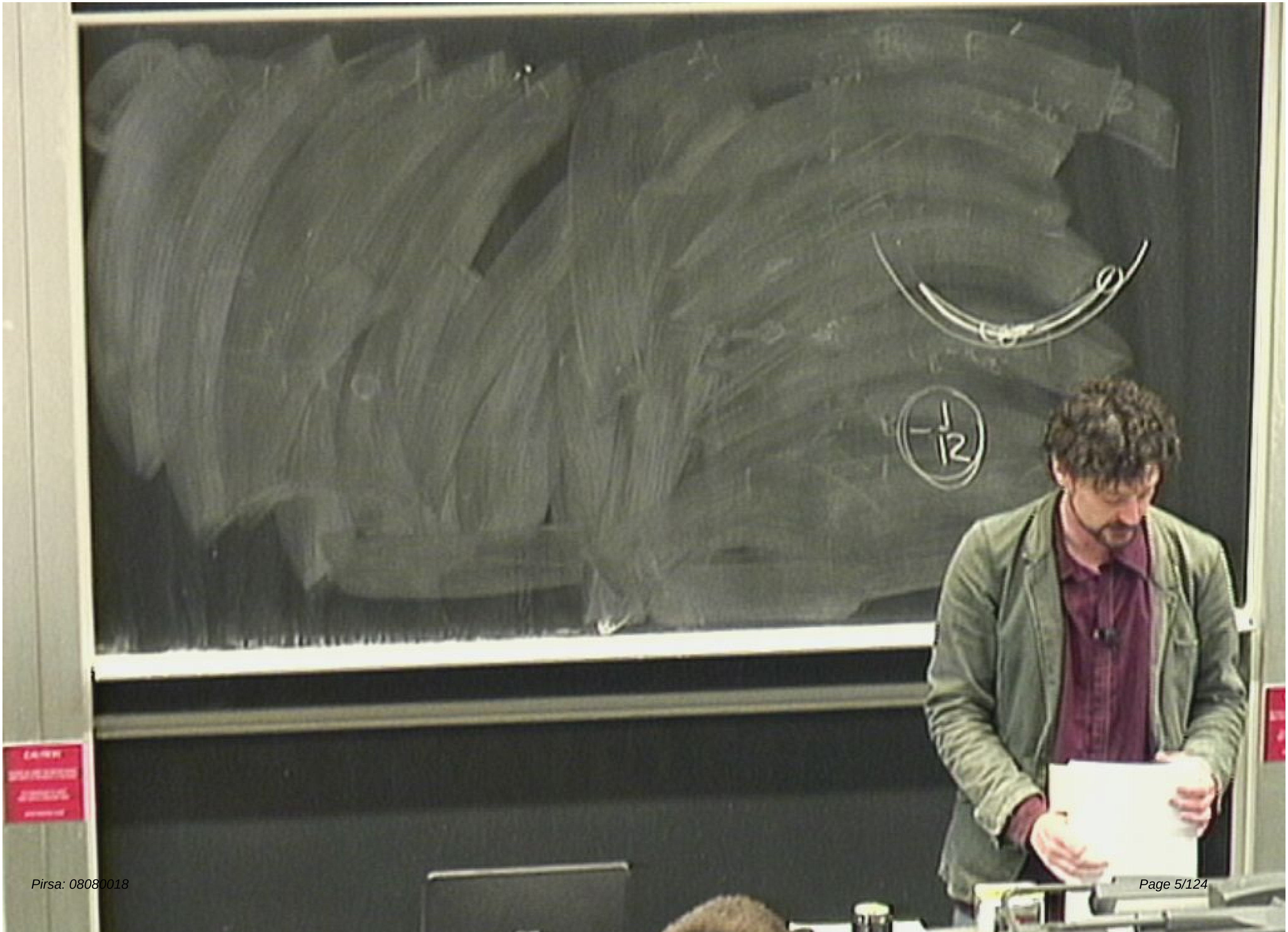
URL: http://pirsa.org/08080018

Abstract:

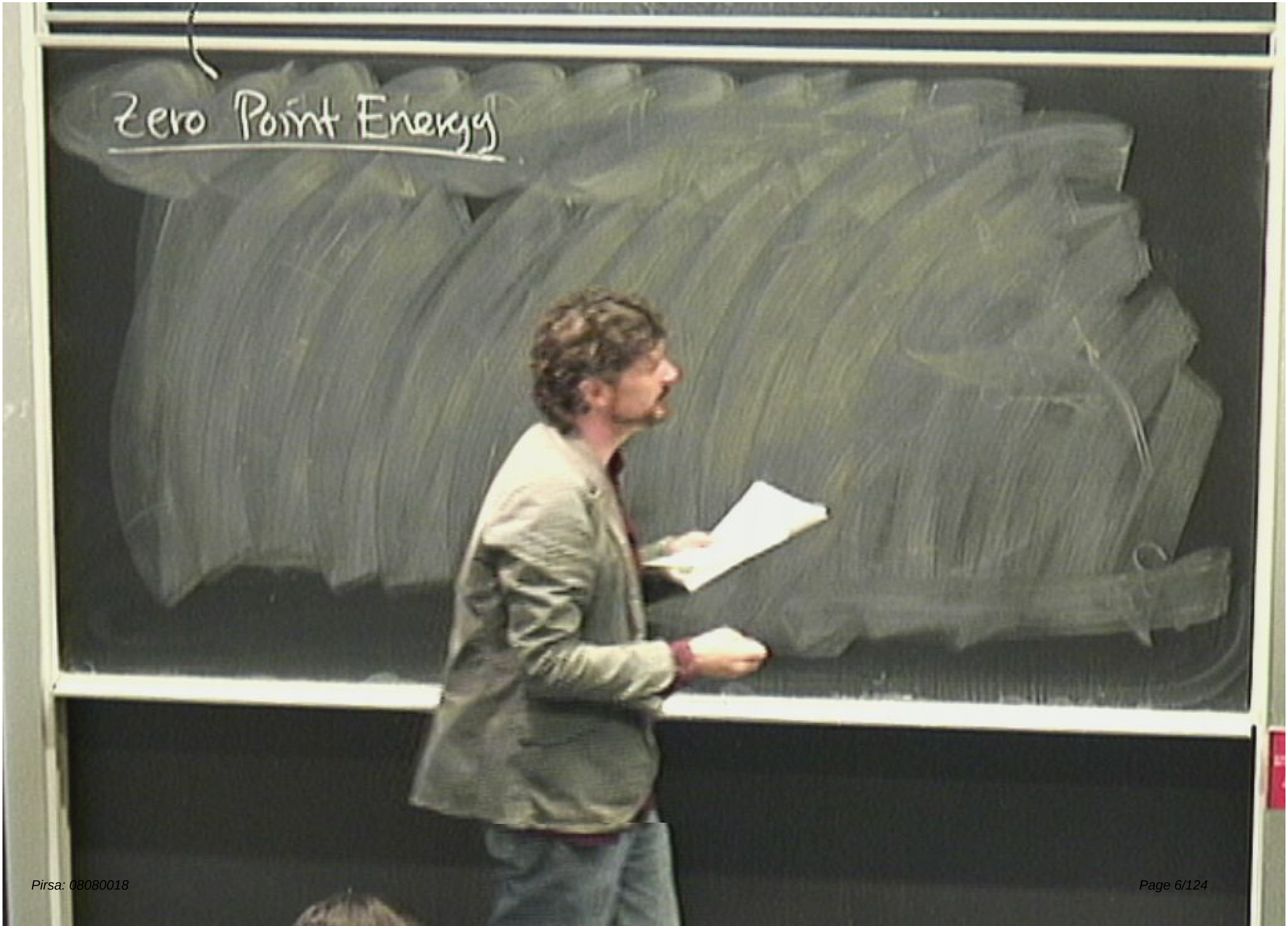


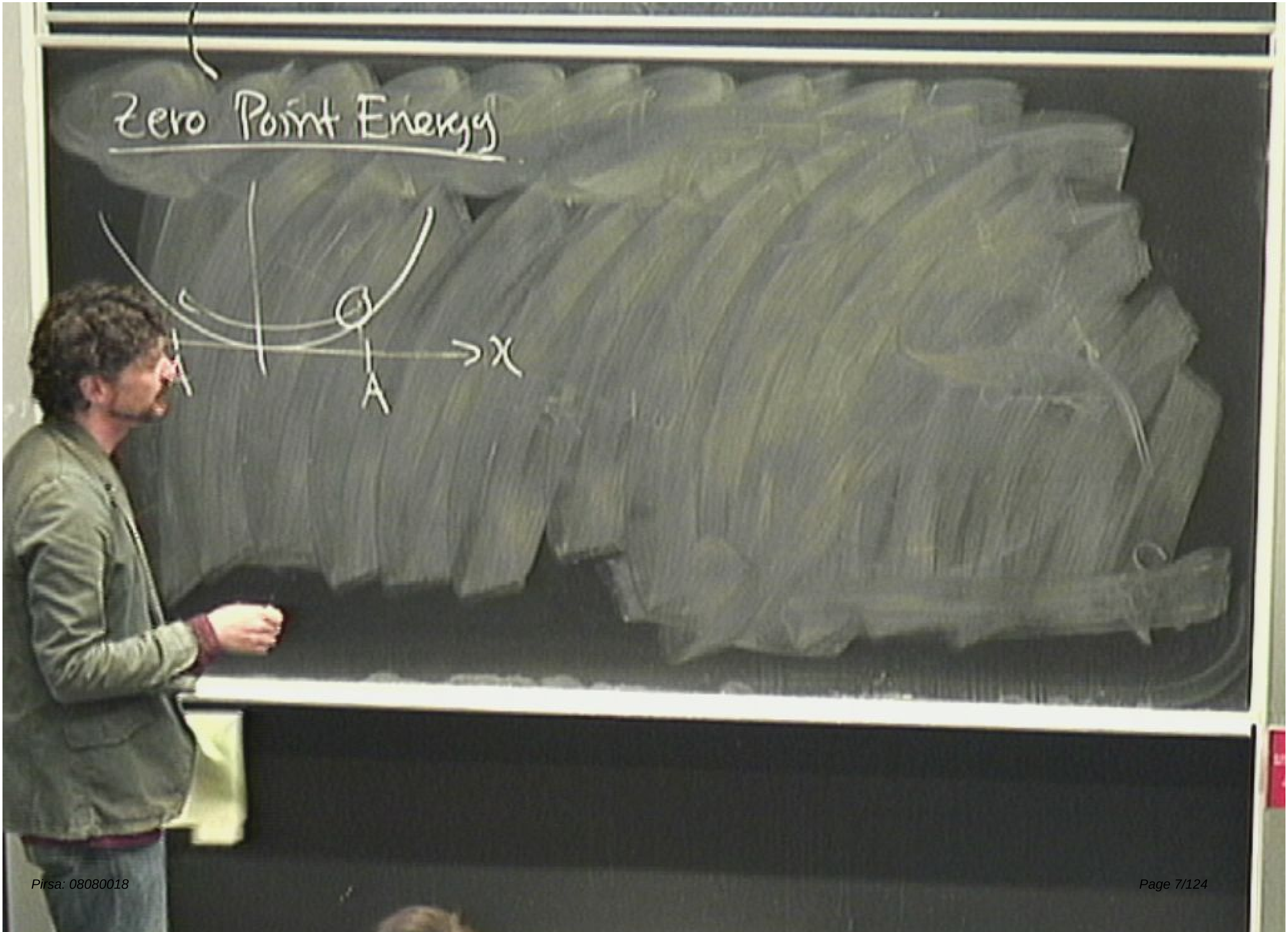


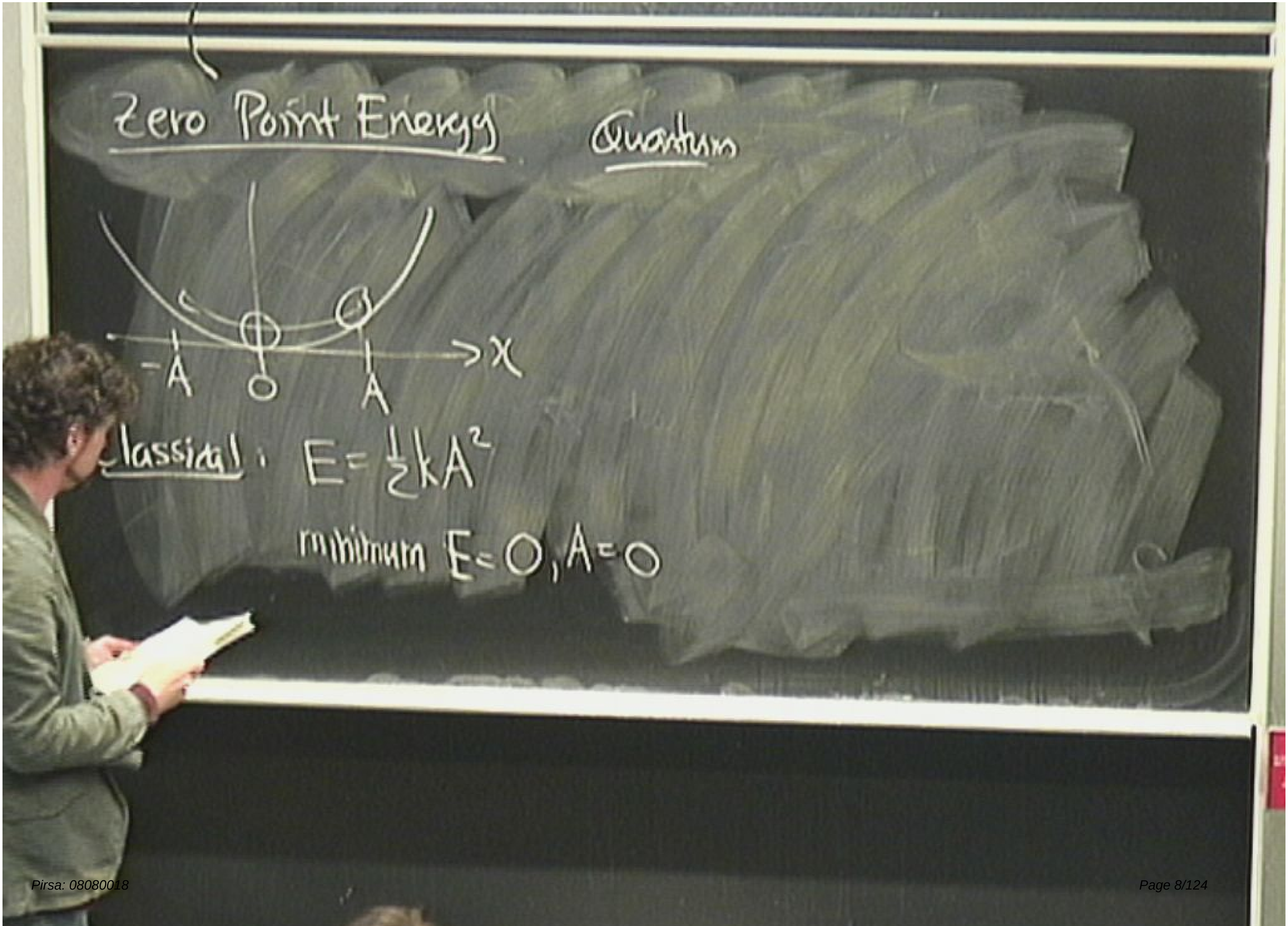

-1/12

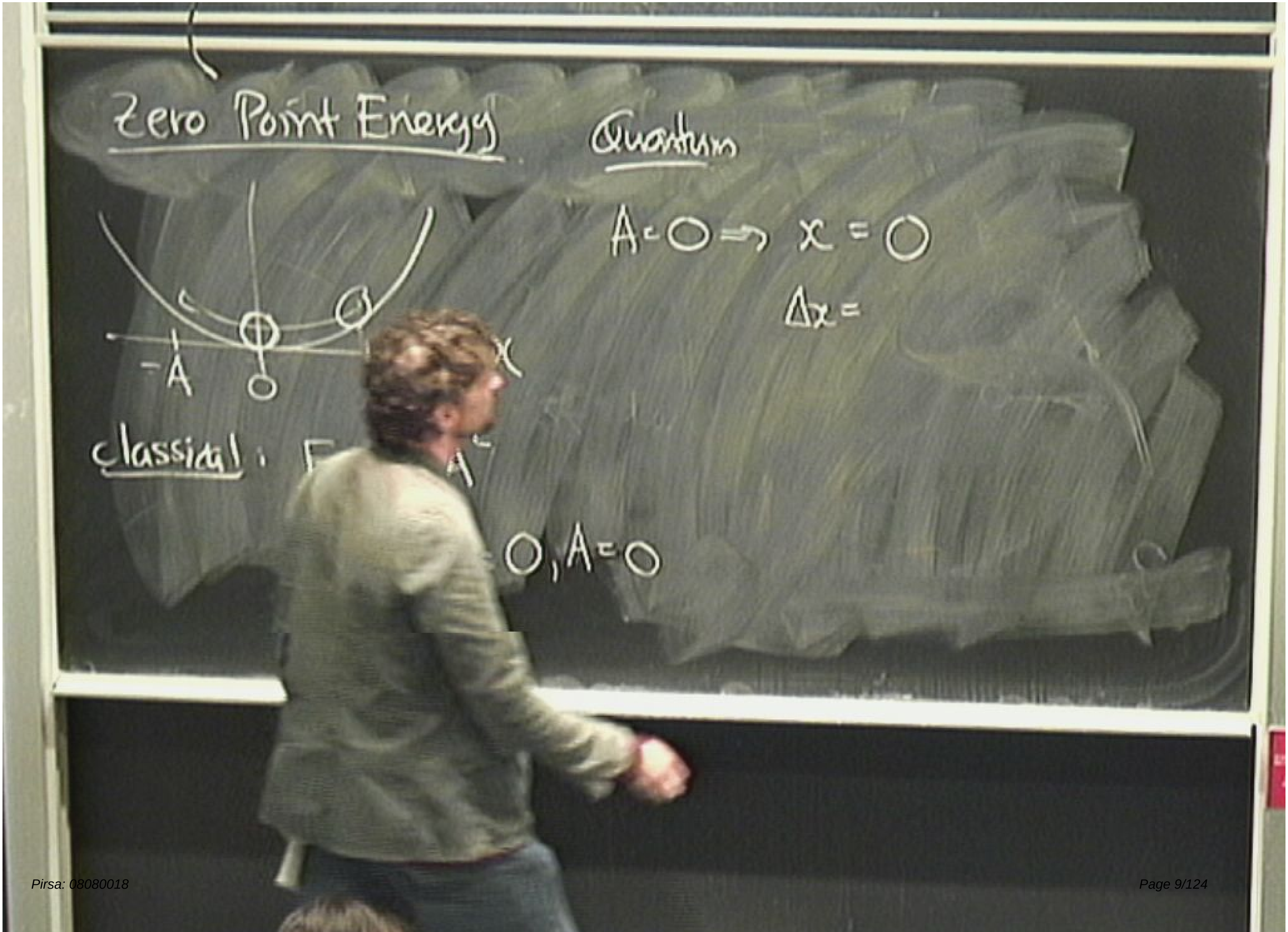


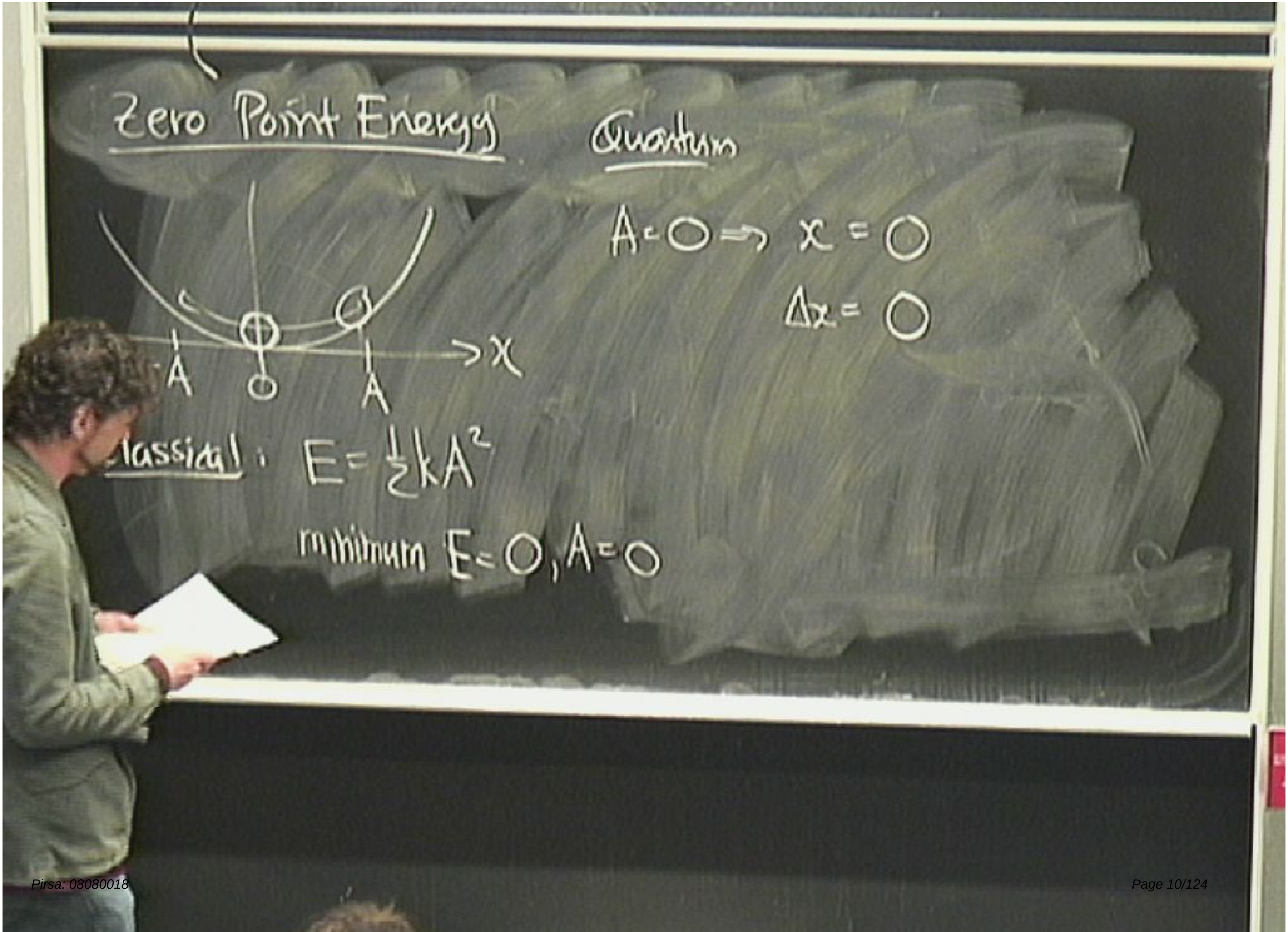
$-\frac{1}{12}$

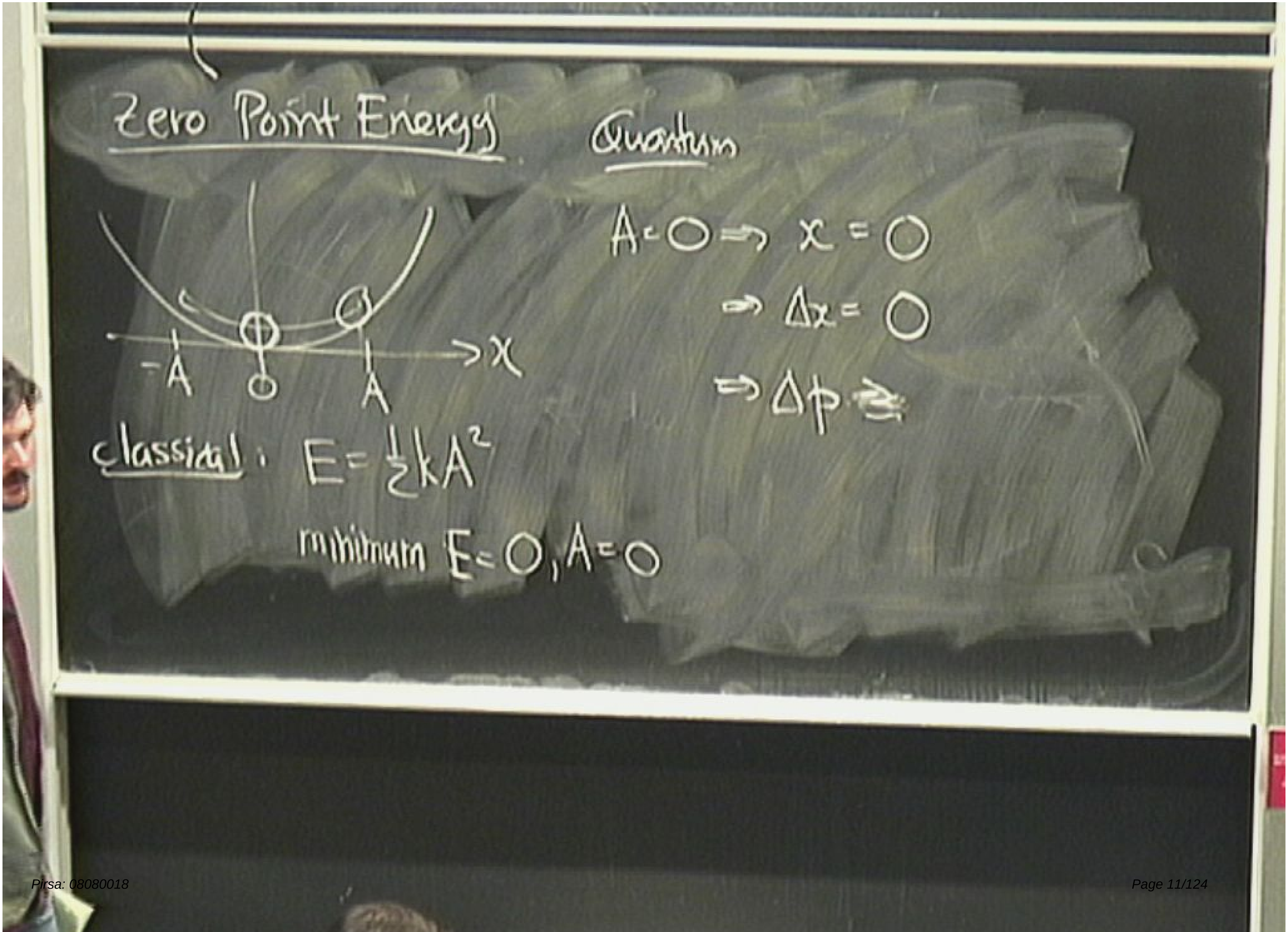

Zero Point Energy

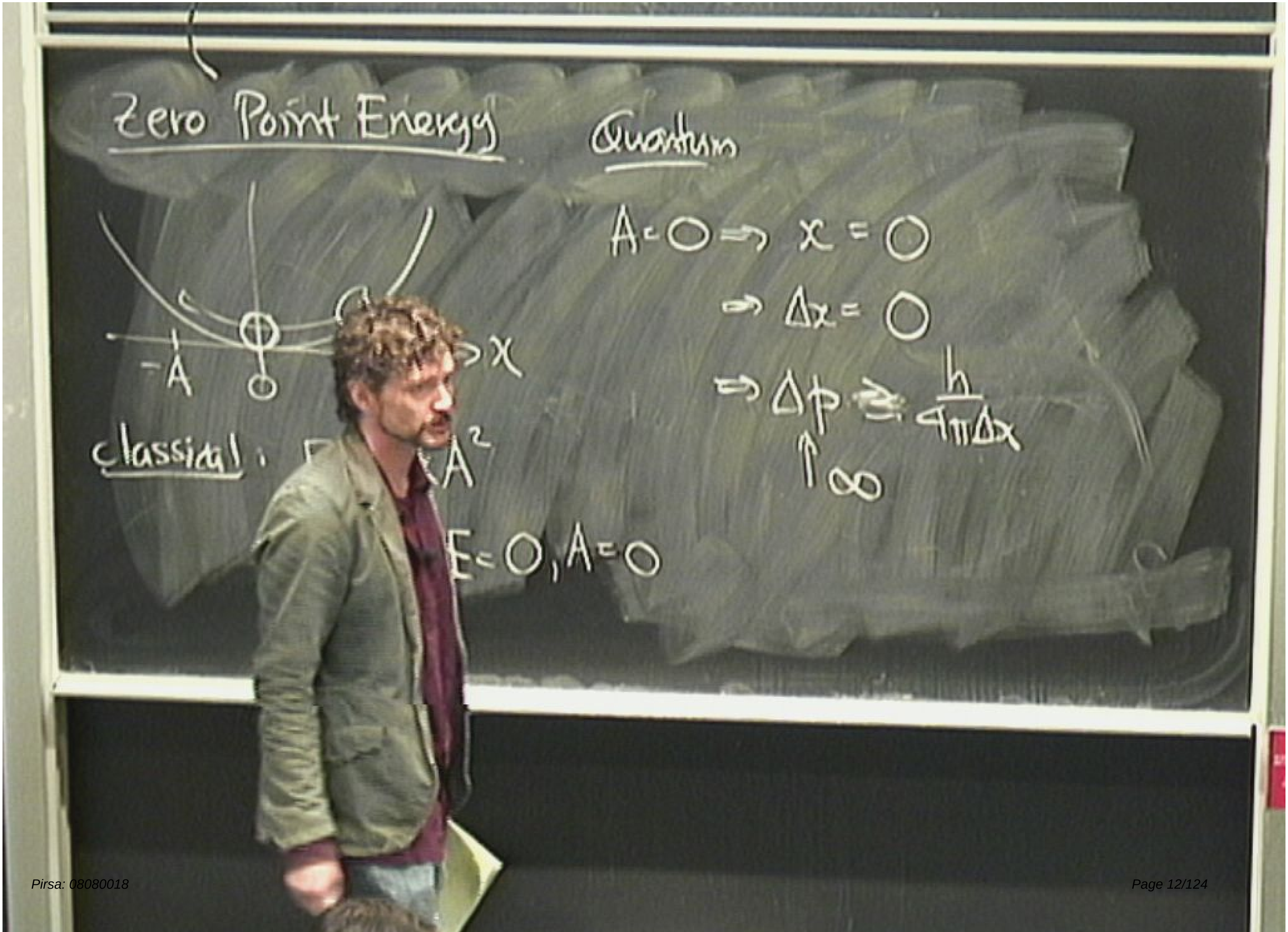

Zero Point Energy
X
A
A

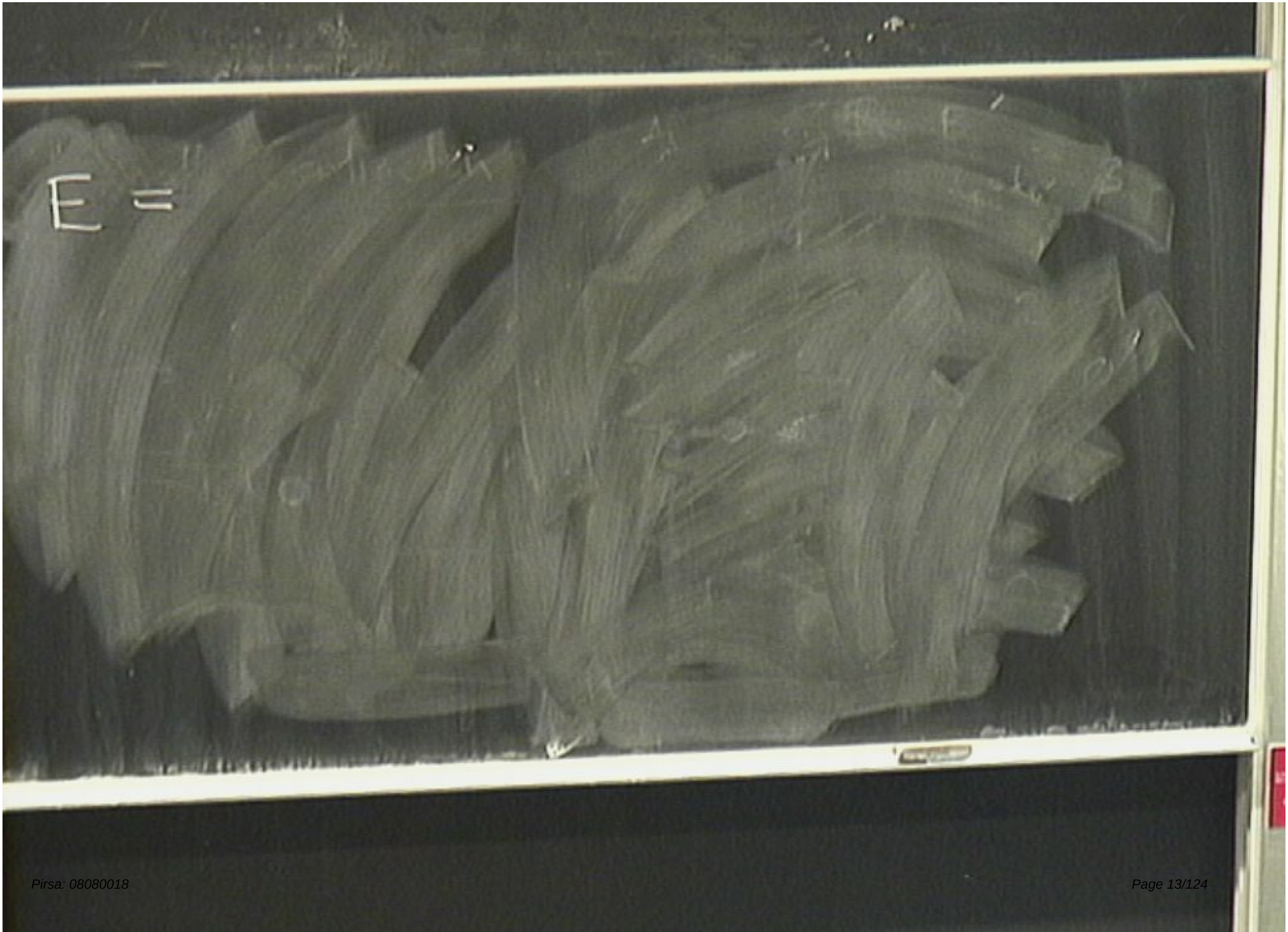

Zero Point Energy
Quantum
-A
0
A
x
Classical: E = ½kA²
minimum E=0, A=0

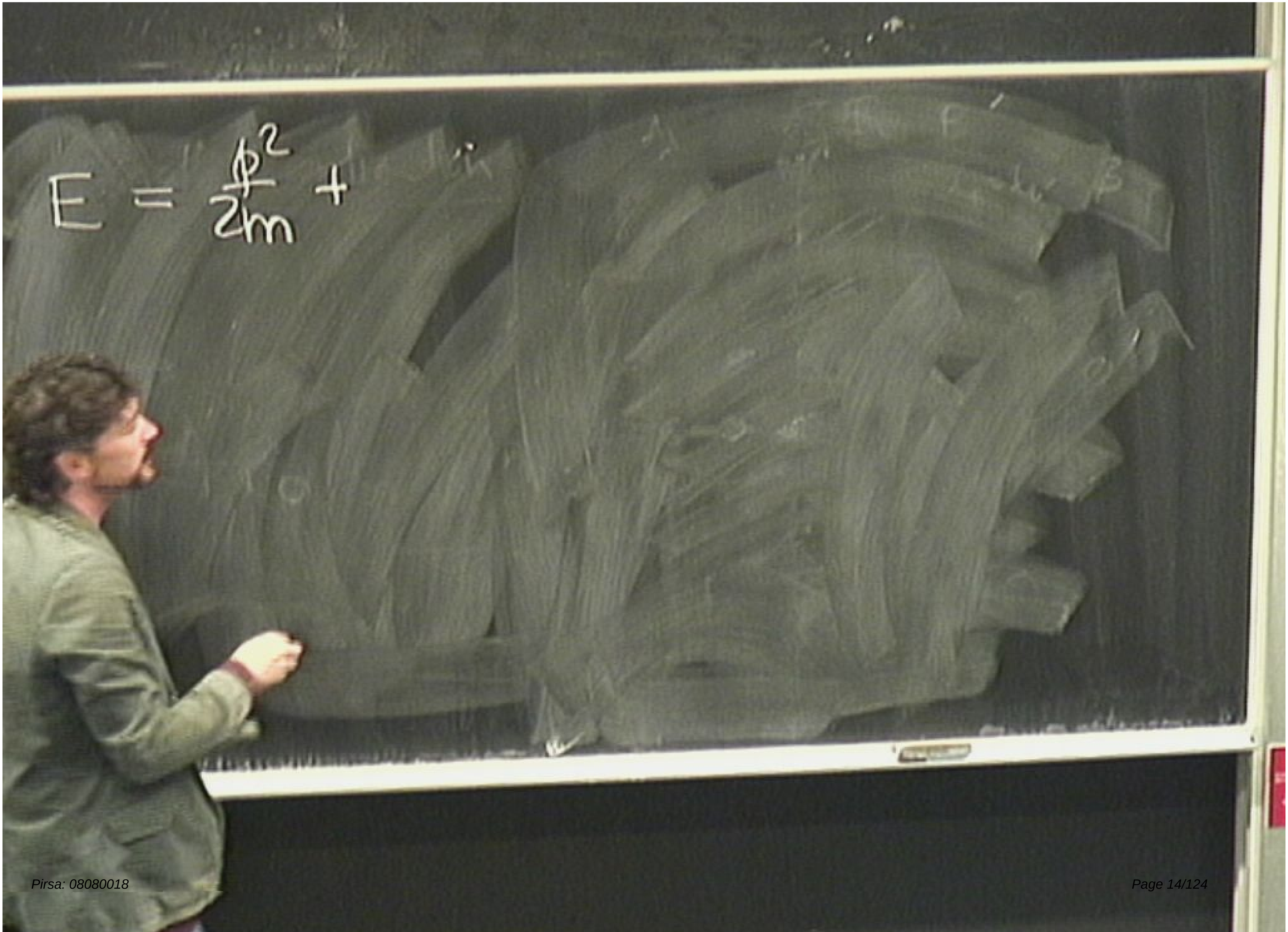

Zero Point Energy
Quantum
A=0 ⇒ x=0
Δx =
-A
0
x
classical:
0, A=0

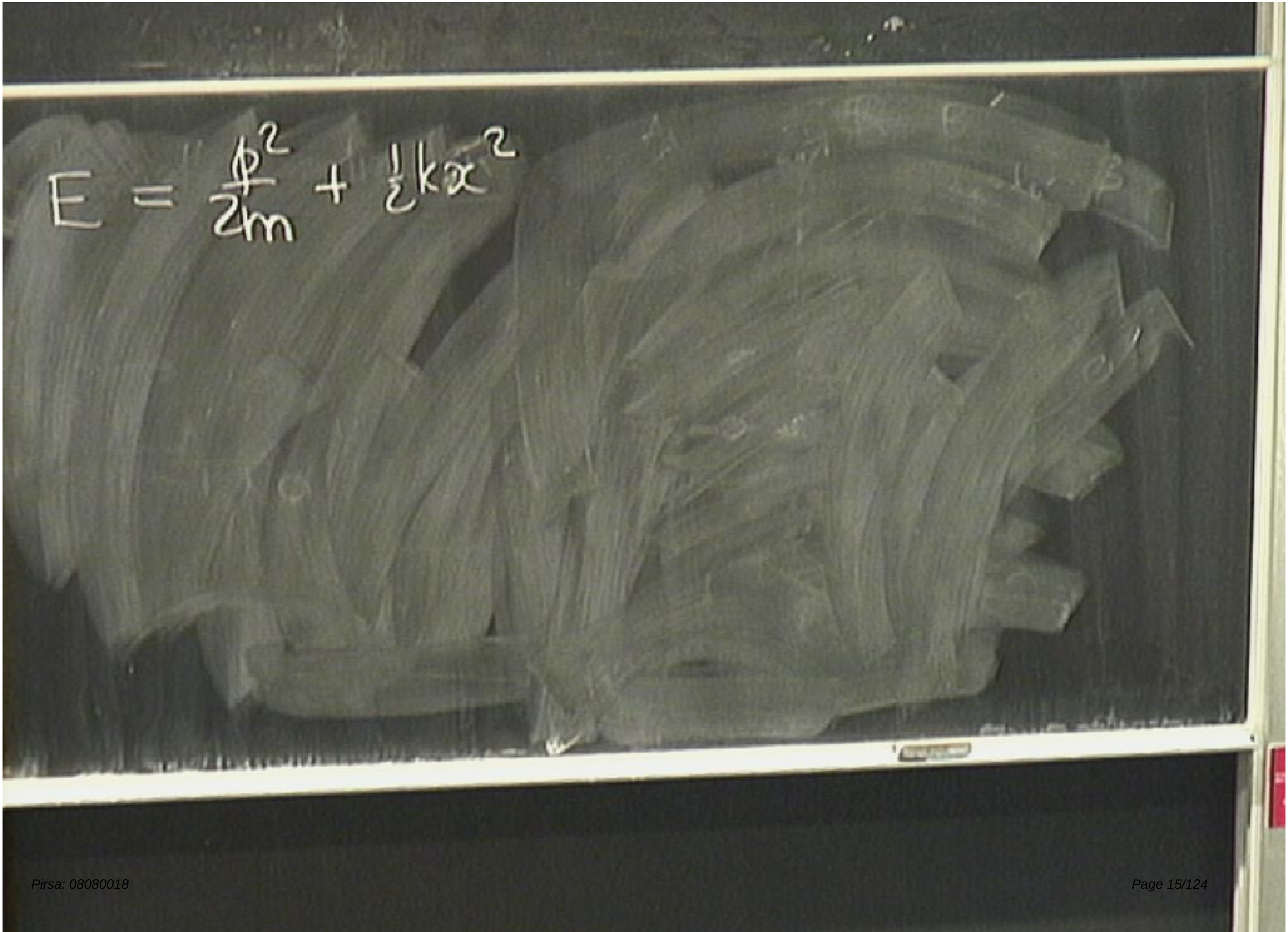

Zero Point Energy
Quantum
A=0 ⇒ x = 0
Δx = 0
x
-A 0 A
lassical : E = ½kA²
minimum E=0, A=0


Zero Point Energy
Quantum
A=O ⇒ x = O
⇒ Δx = O
⇒ Δp
-A
O
A
x
classical : E = ½kA²
minimum E=O, A=O

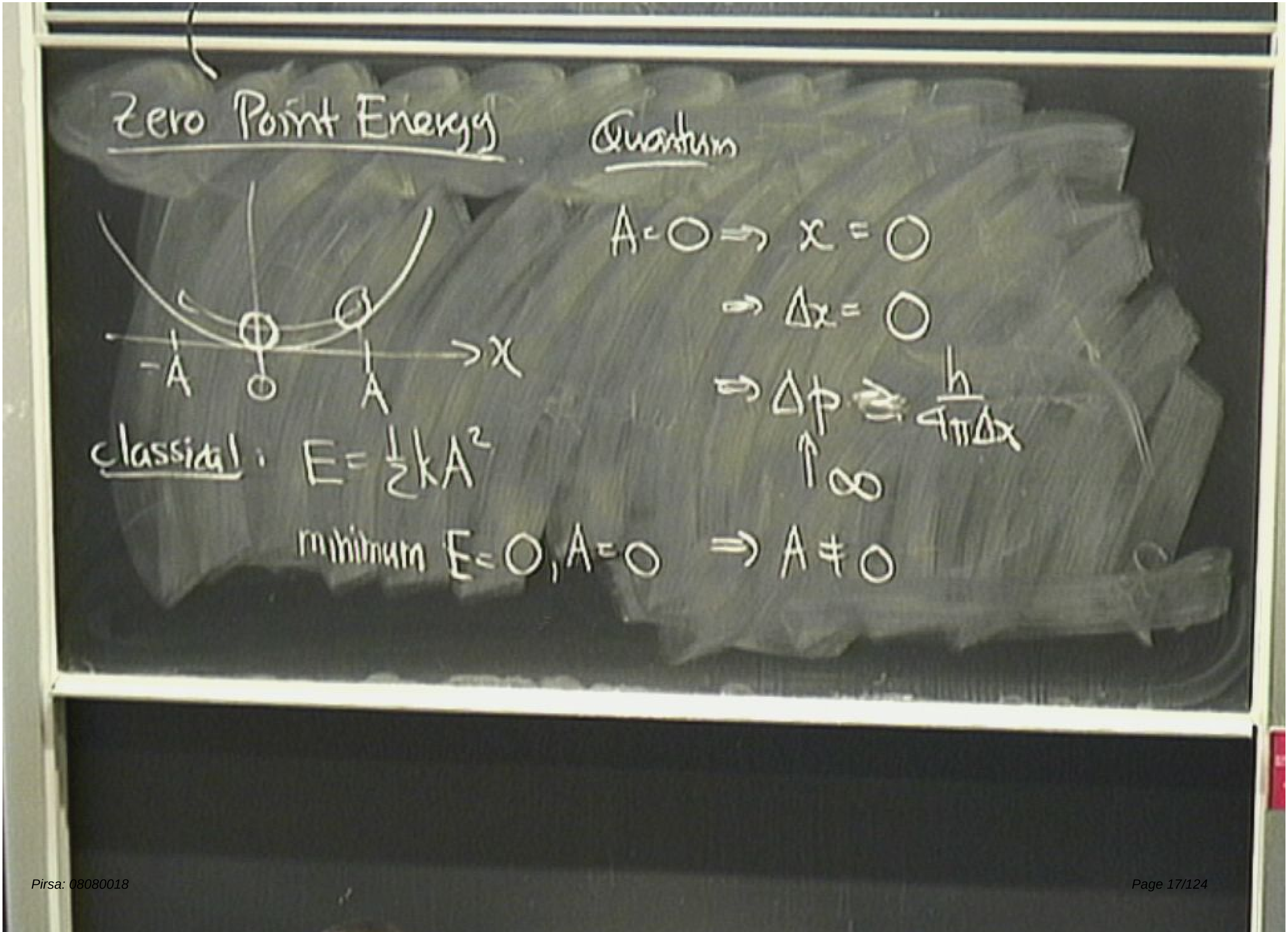

Zero Point Energy
Quantum
A=0 ⇒ x=0
⇒ Δx = 0
⇒ Δp ≥ h/4πΔx
↑ ∞
-A
0
x
Classical:
E=0, A=0

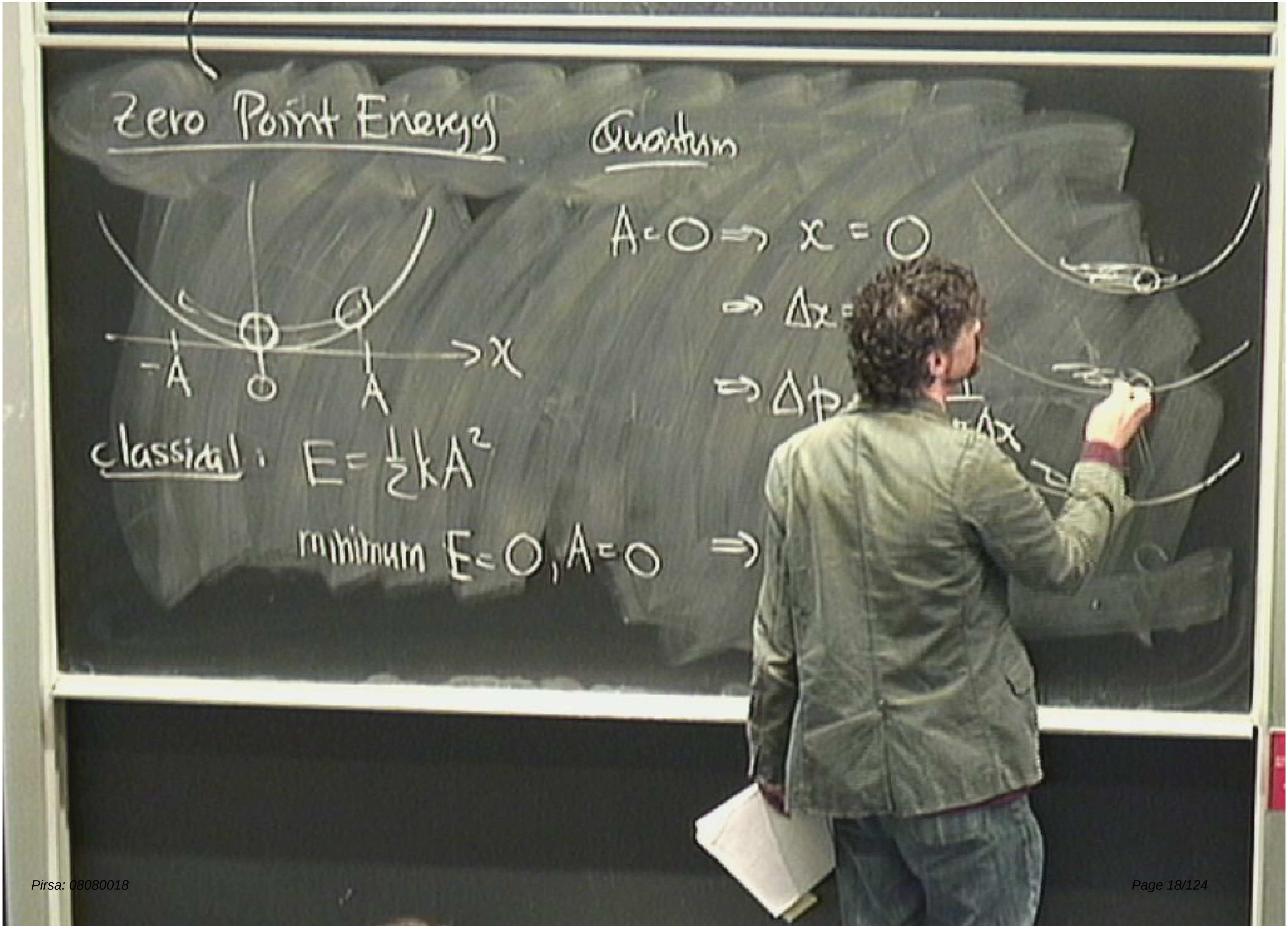

E =

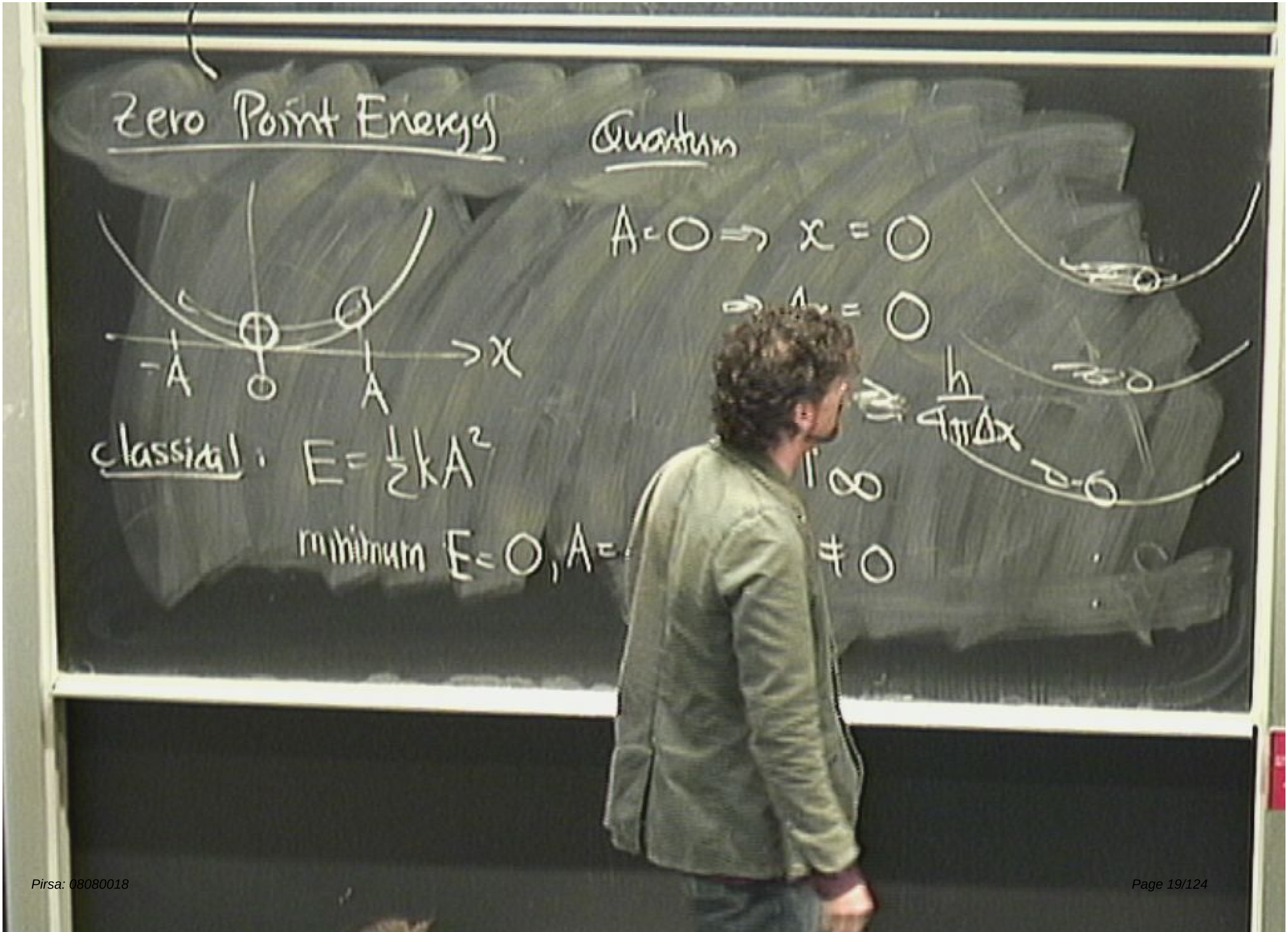

$E = \frac{p^2}{2m} +$


$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$

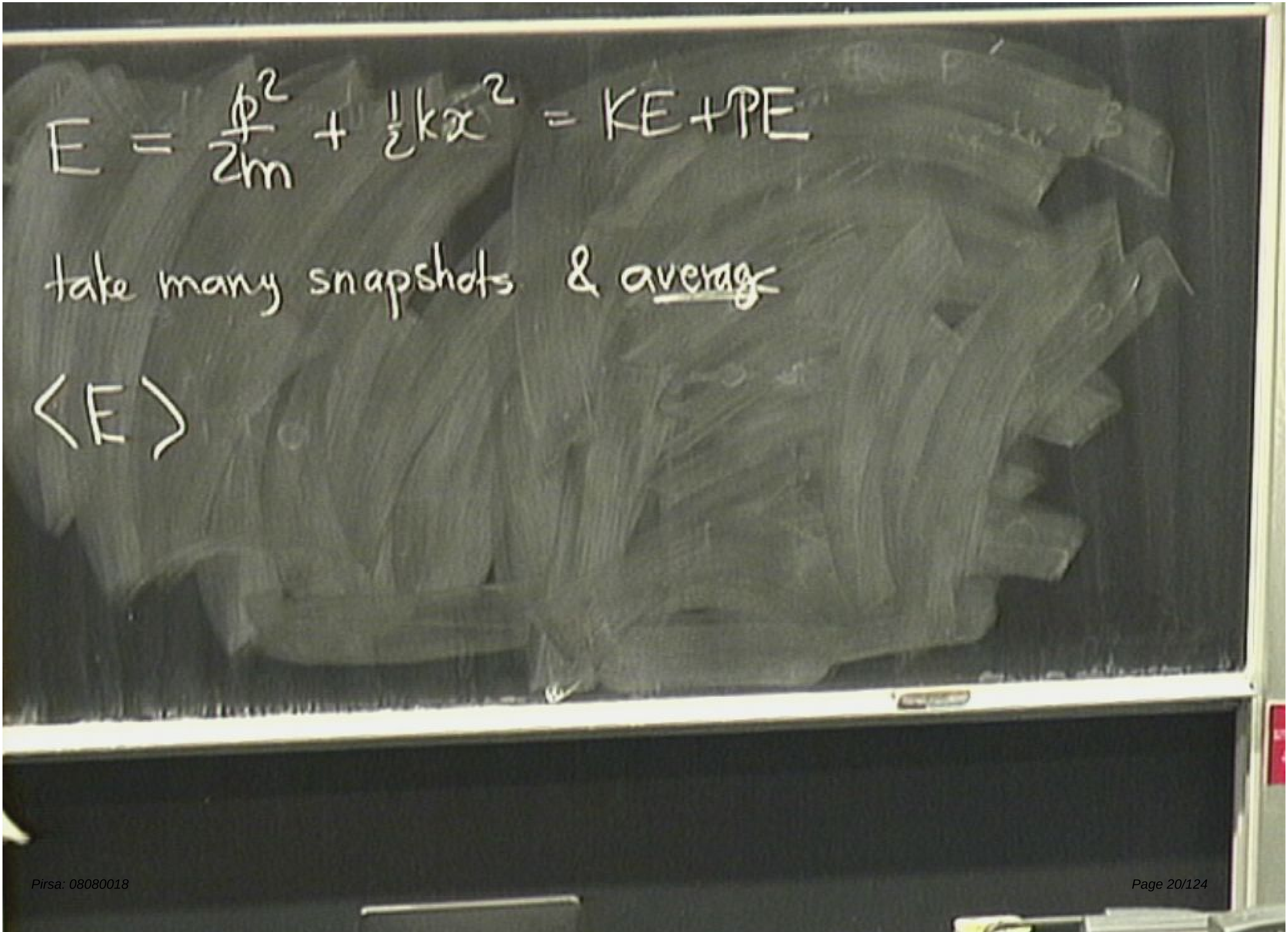
$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$


Zero Point Energy
Quantum
-A
0
A
x
classical: $E = \frac{1}{2}kA^2$
minimum $E = 0, A = 0$
$A = 0 \Rightarrow x = 0$
$\Rightarrow \Delta x = 0$
$\Rightarrow \Delta p \geq \frac{h}{4\pi\Delta x}$
$\uparrow \infty$
$\Rightarrow A \neq 0$


Zero Point Energy
Quantum
A=0 ⇒ x=0
⇒ Δx=
⇒ Δp
-A 0 A
x
classical: E = ½kA²
minimum E=0, A=0 ⇒



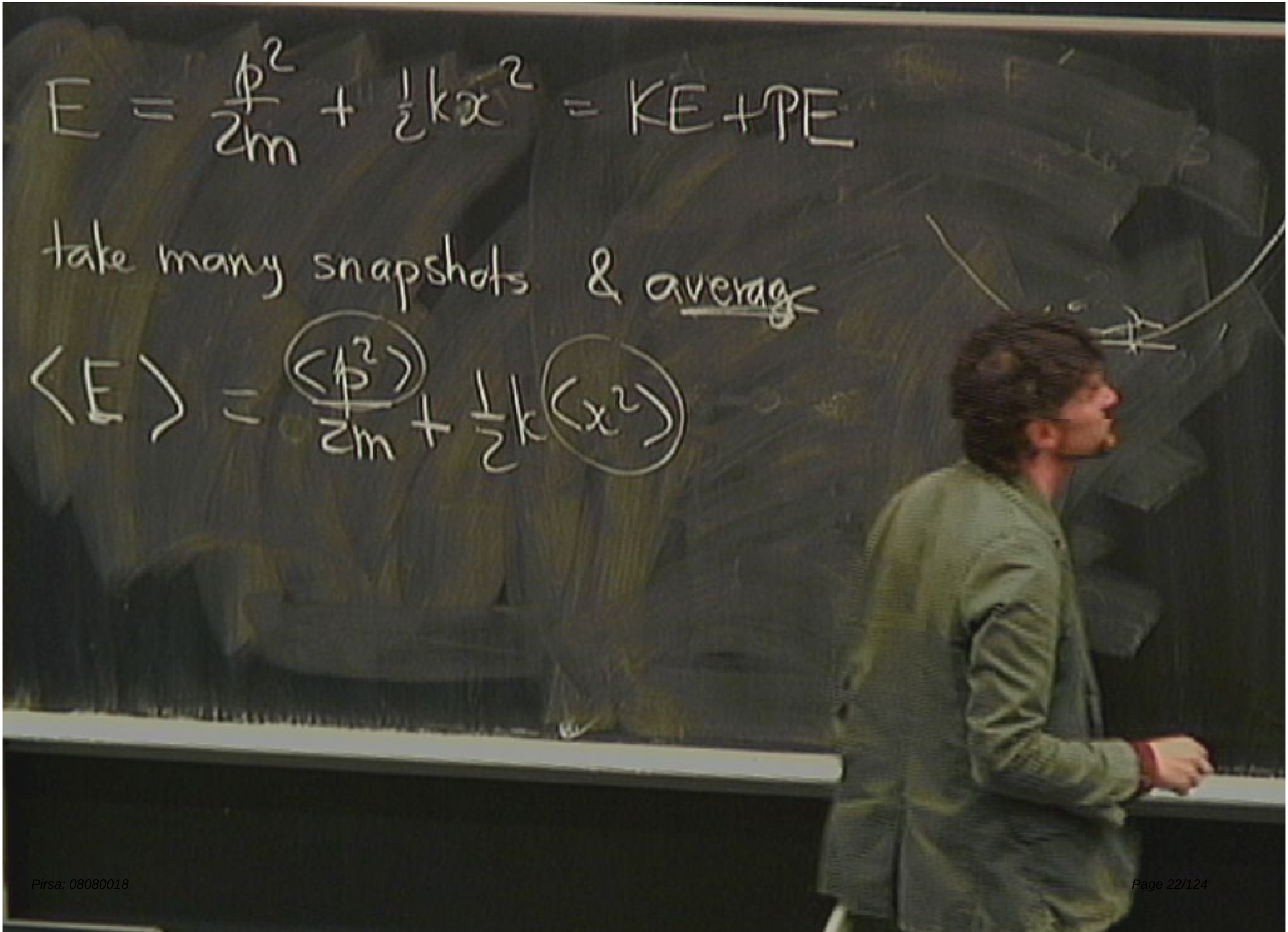
Zero Point Energy
Quantum
A=0 ⇒ x=0
-A
0
A
x
classical:
E = ½kA²
minimum E=0, A=
h
4πΔx
∞
≠0

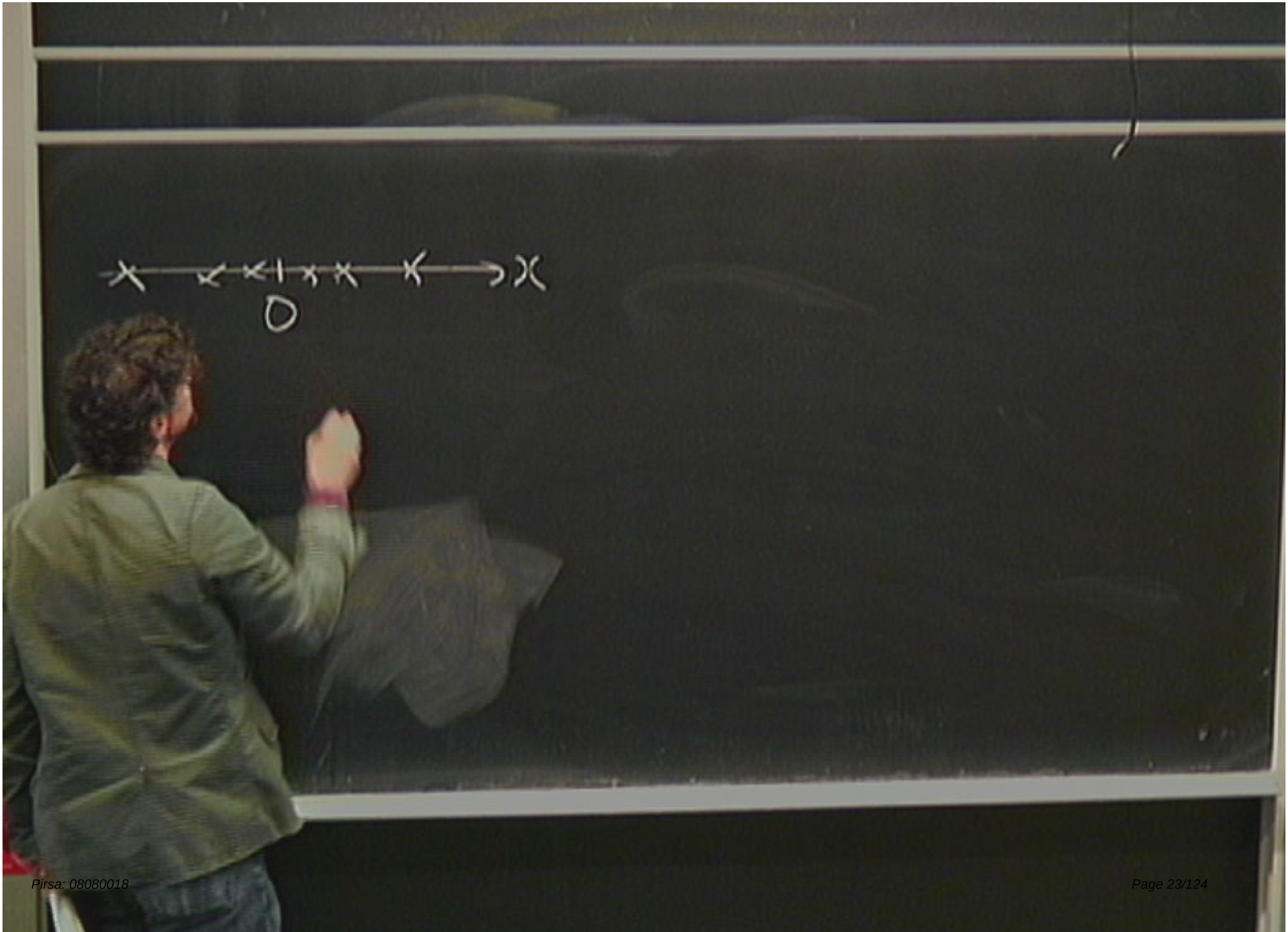

$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle$

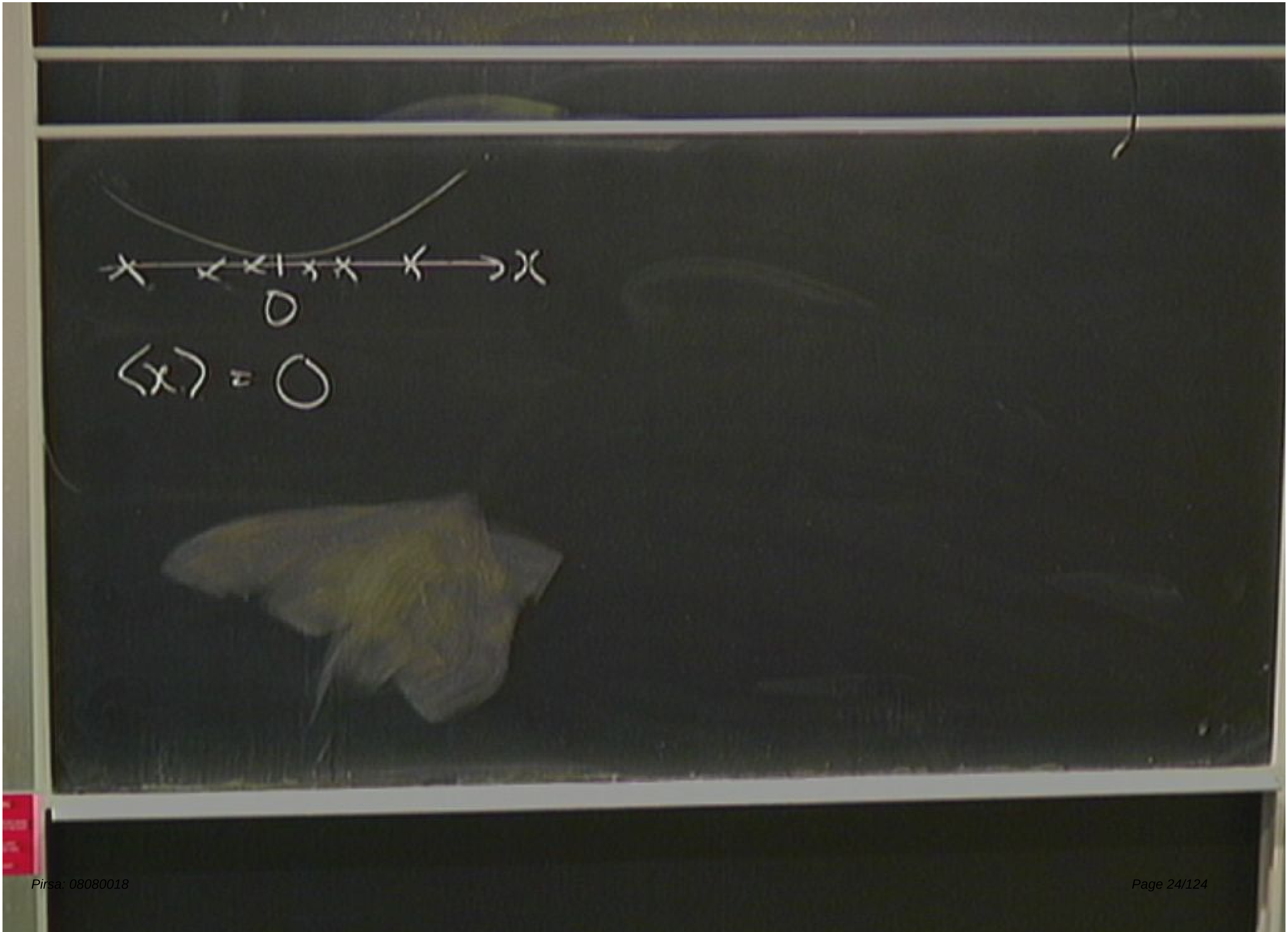
$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

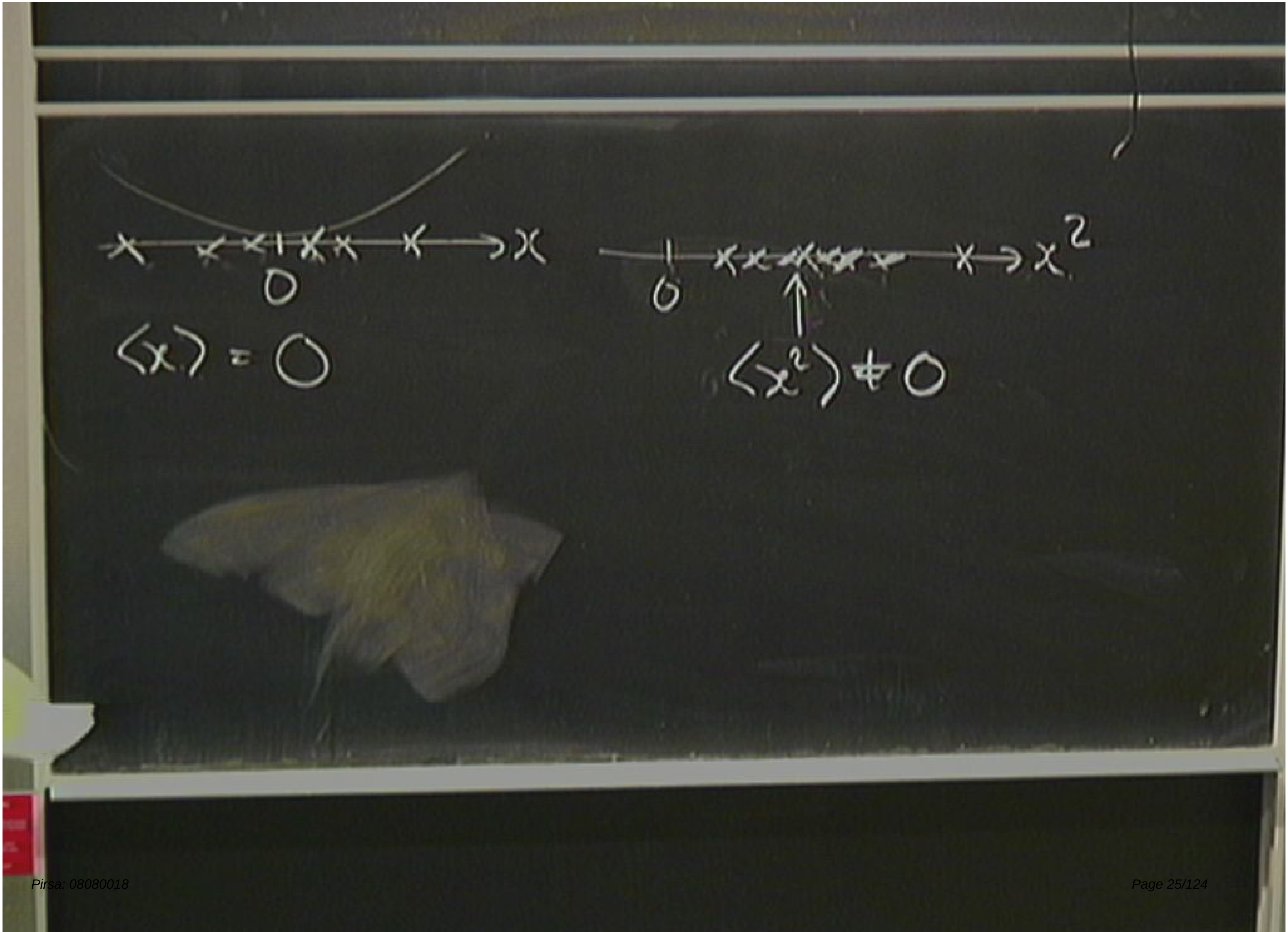
take many snapshots & average

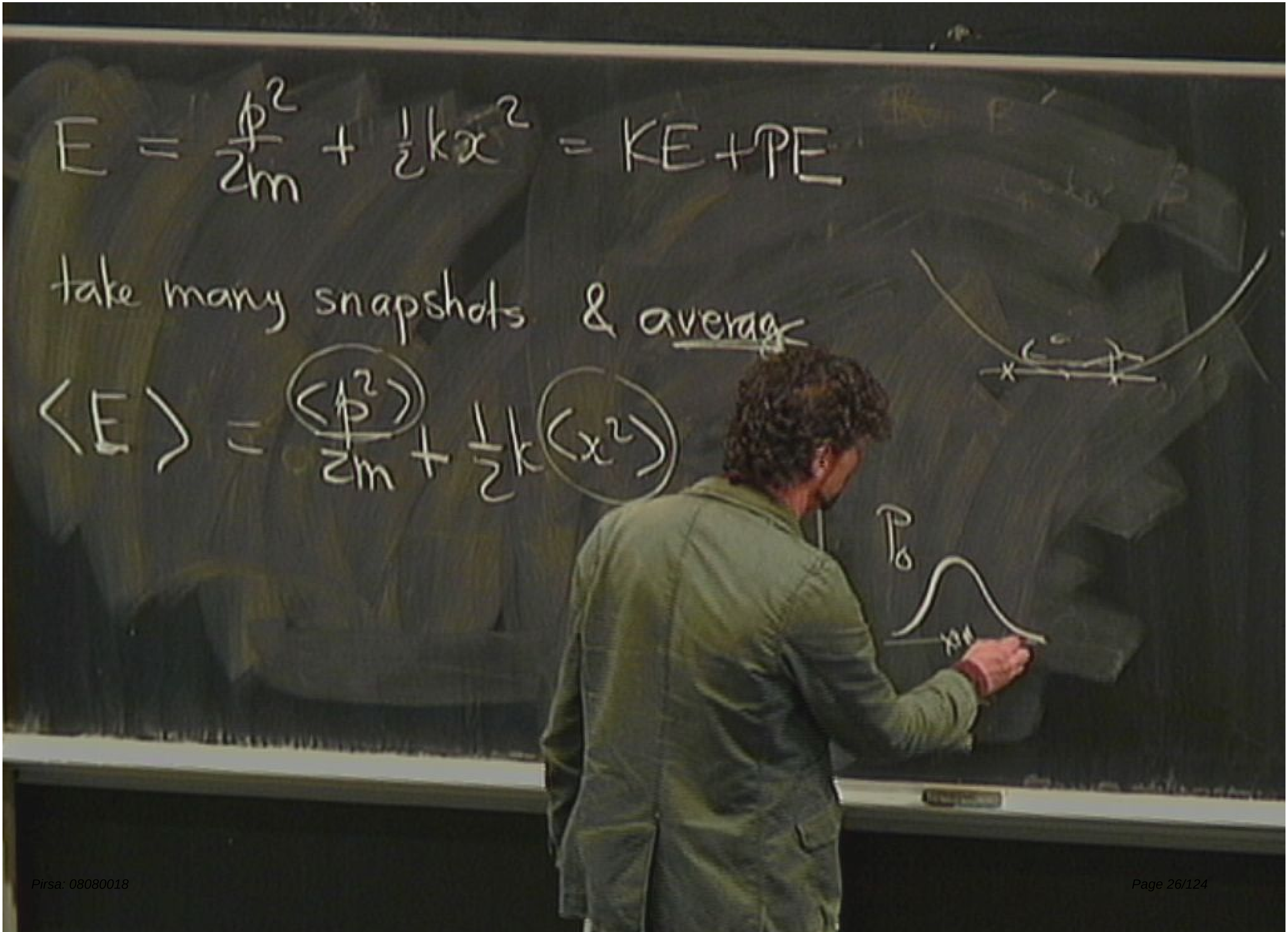
$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$$

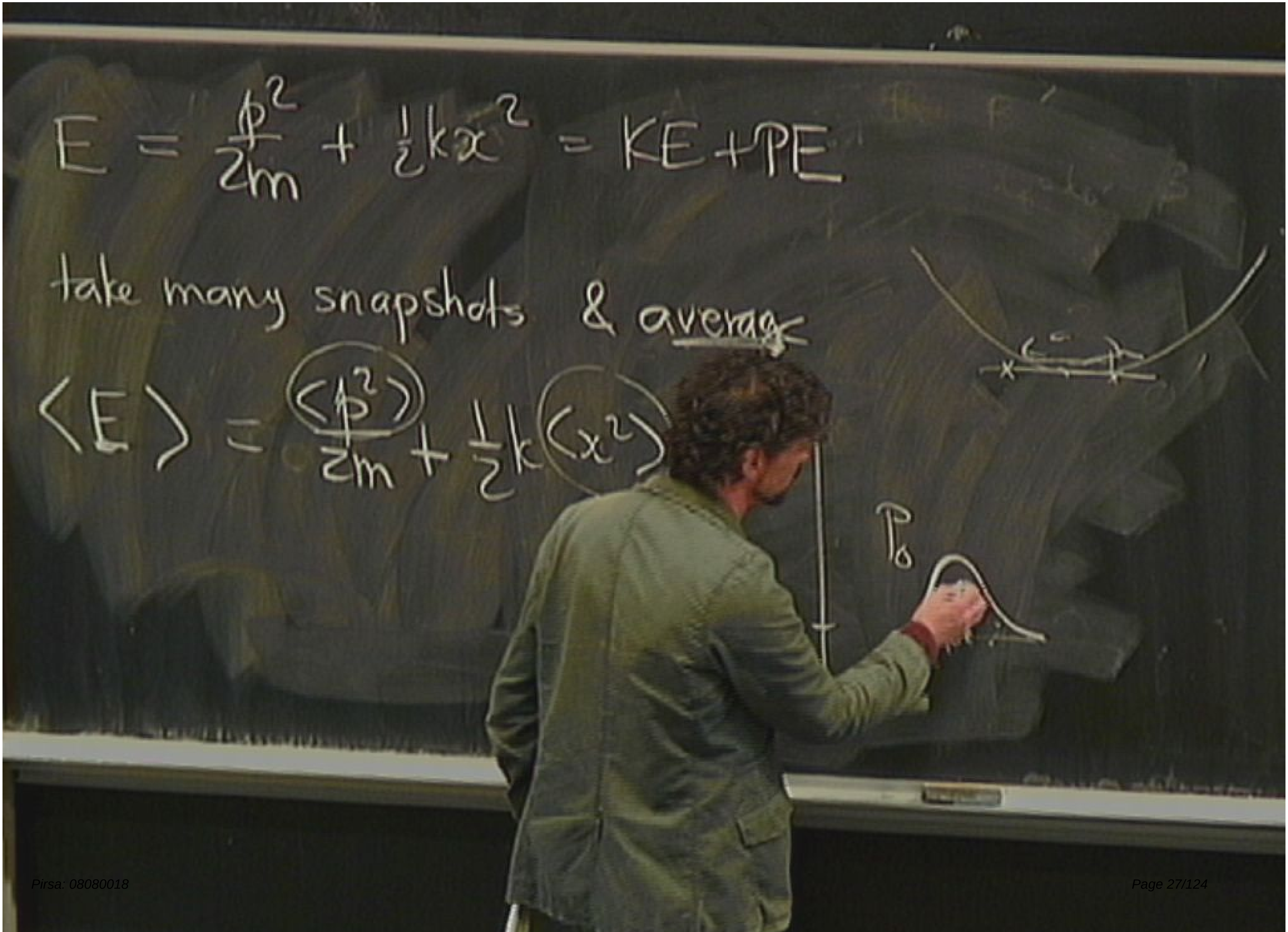

E = p²/2m + ½kx² = KE+PE
take many snapshots & average
⟨E⟩ = ⟨p²⟩/2m + ½k⟨x²⟩

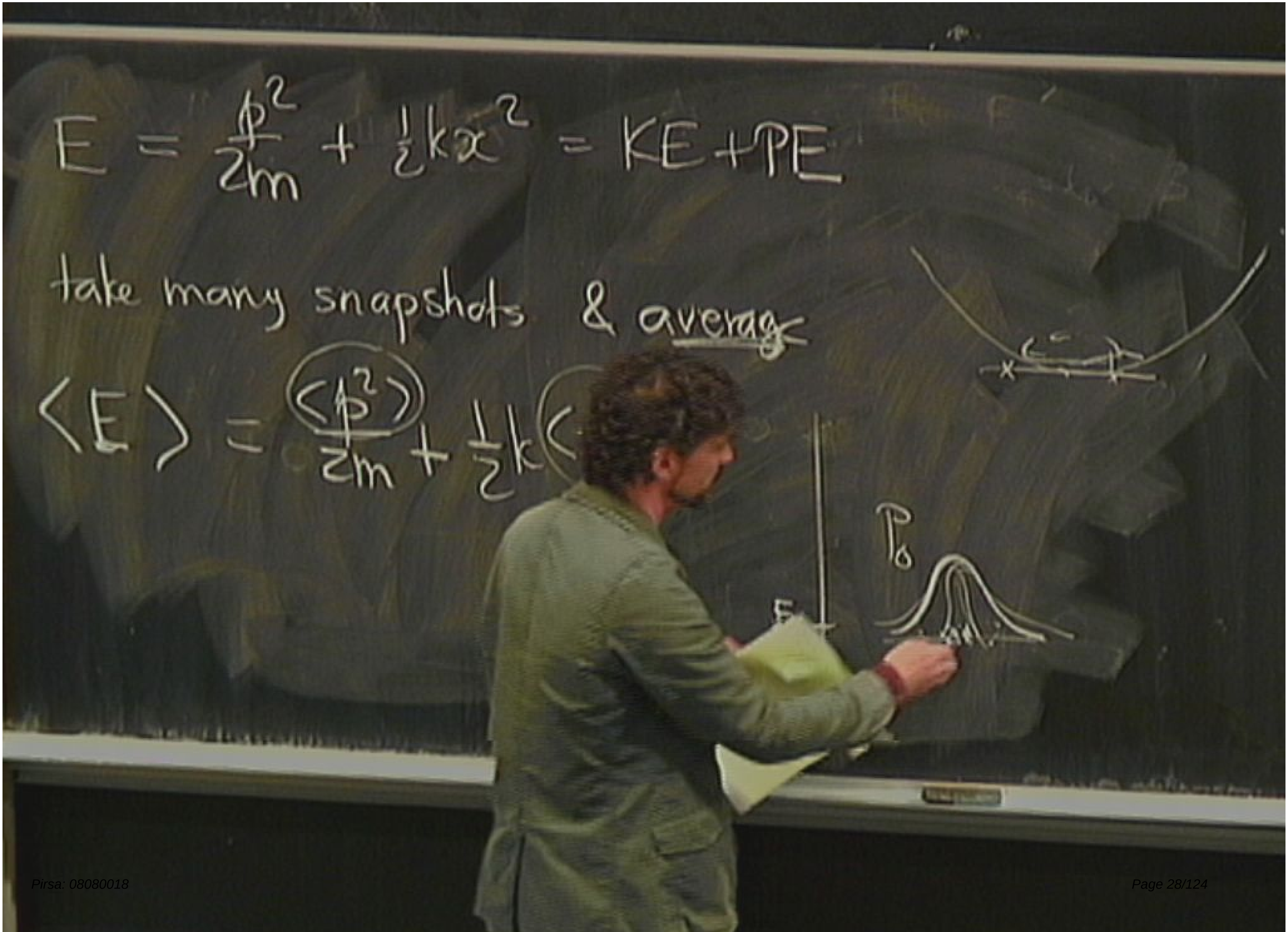

X
0

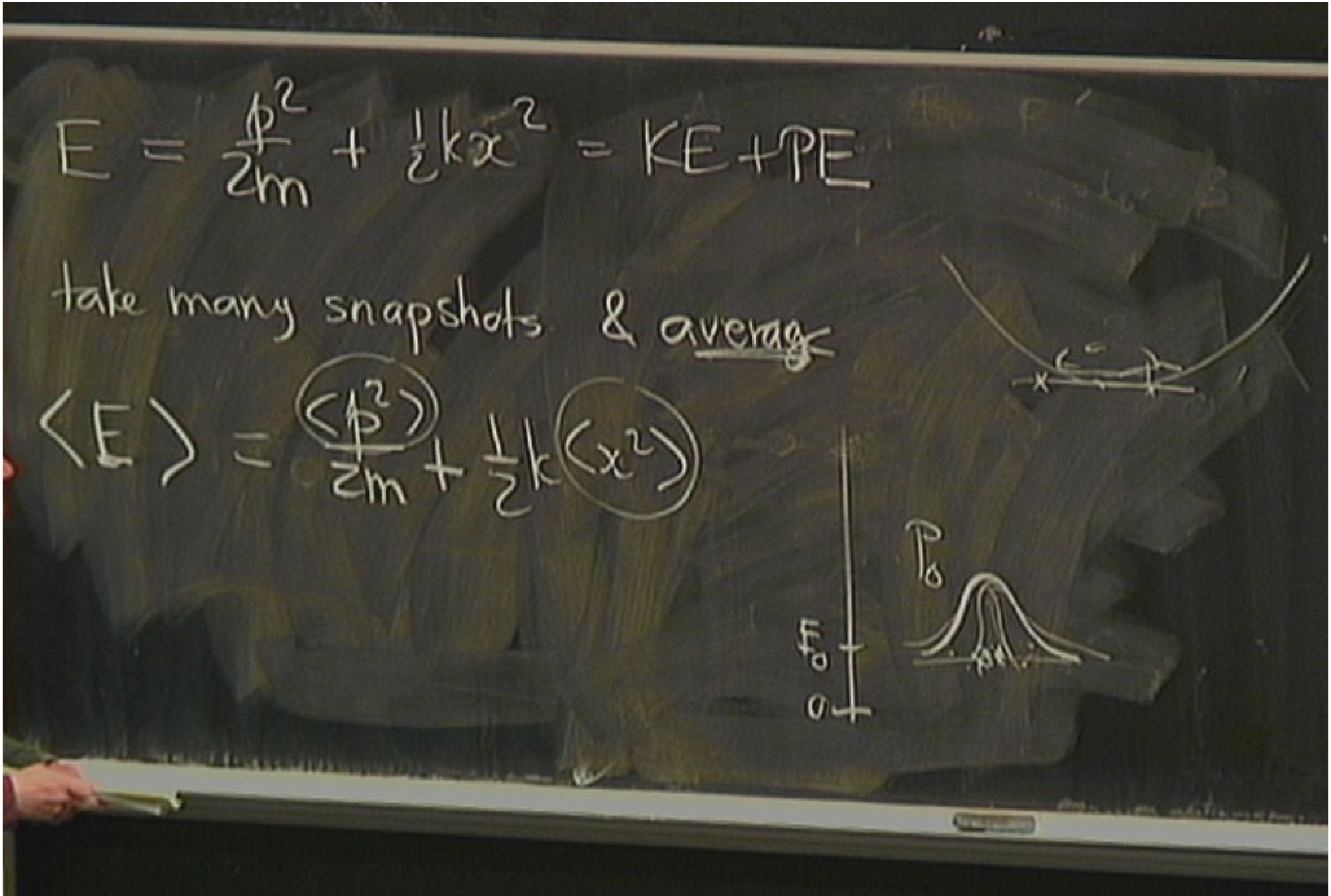

x
0
⟨x⟩ = 0

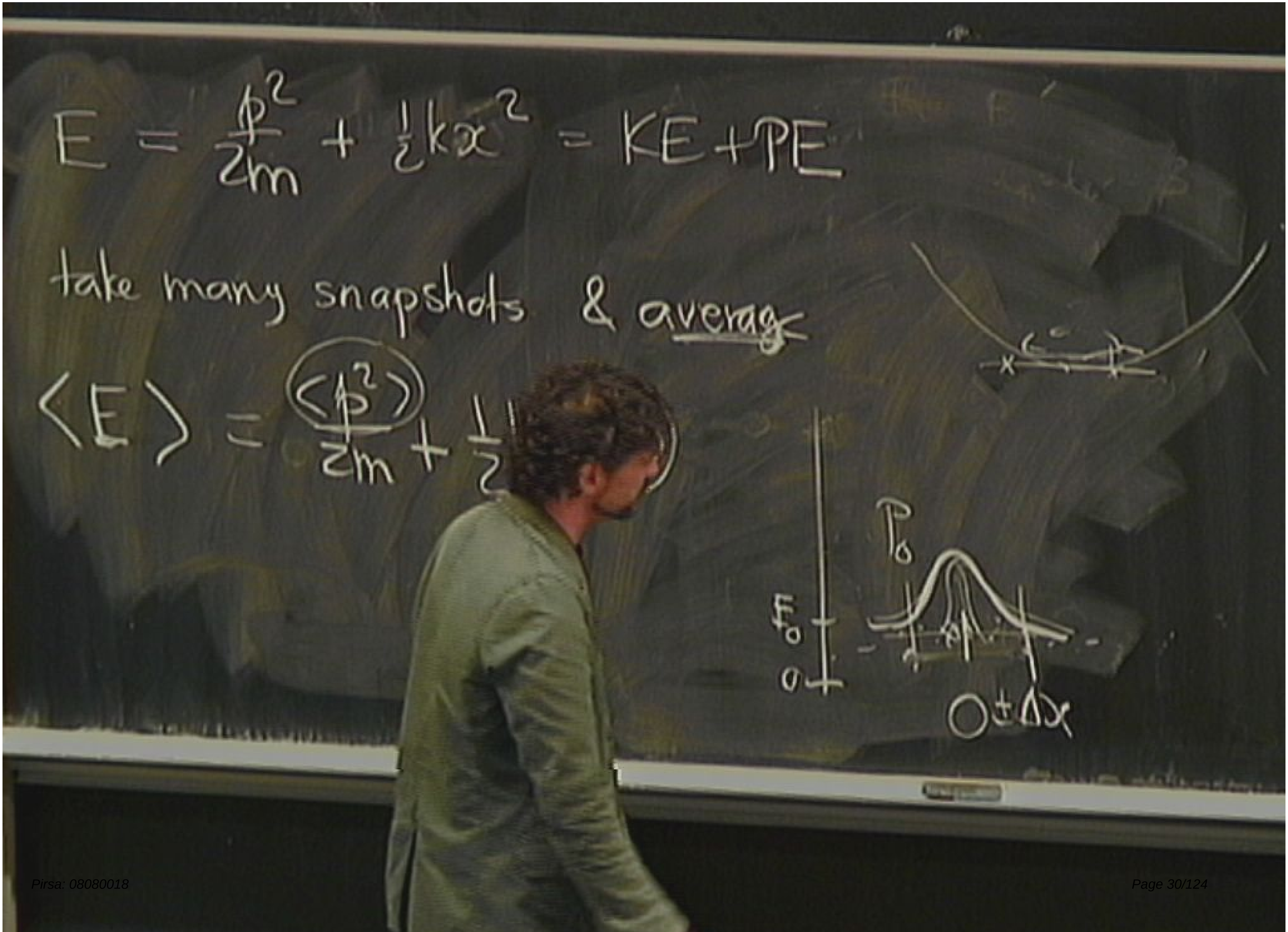

x
0
$\langle x \rangle = 0$
x^2
0
$\langle x^2 \rangle \neq 0$

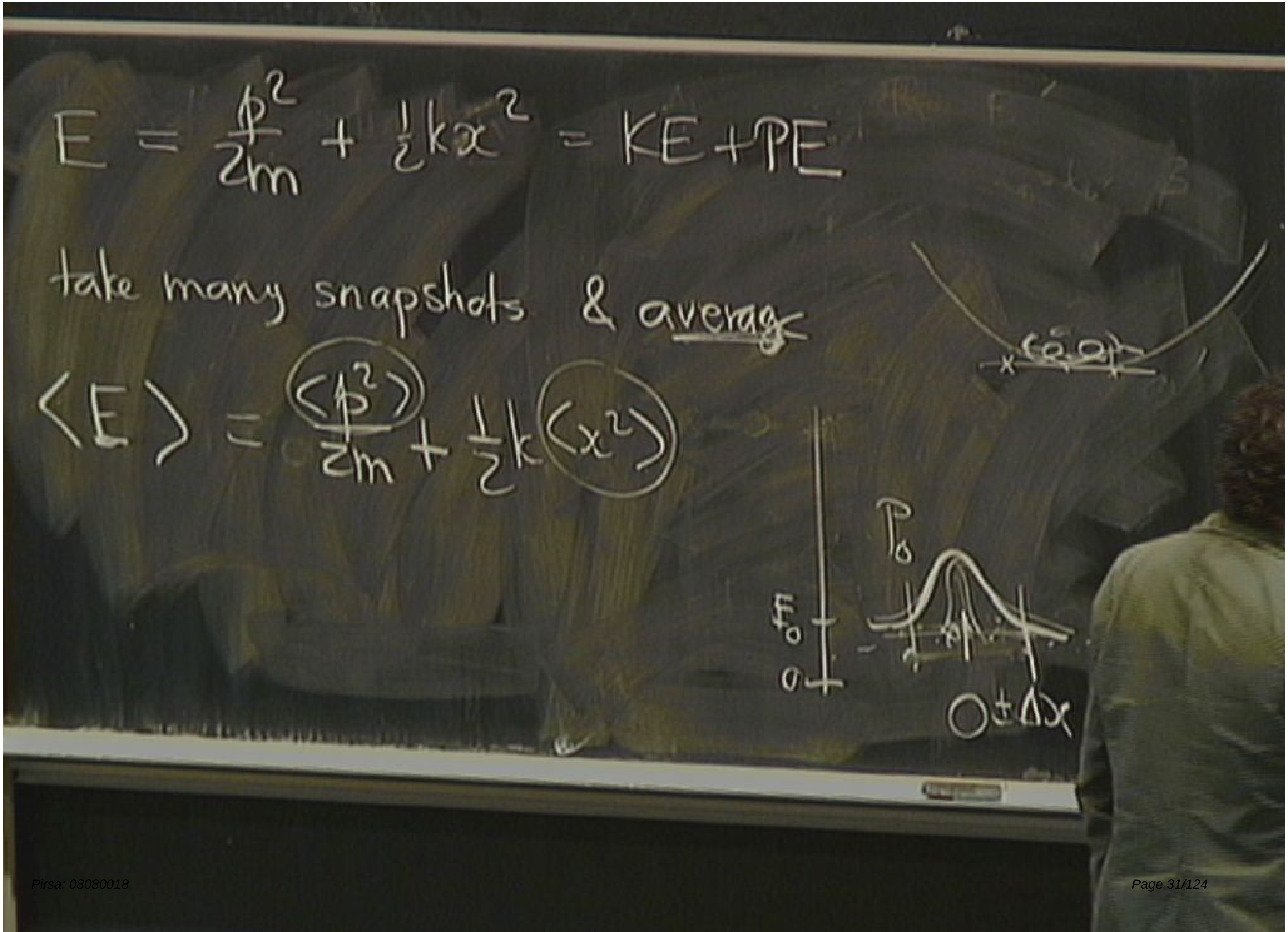

E = p²/2m + ½kx² = KE + PE
take many snapshots & average
⟨E⟩ = ⟨p²⟩/2m + ½k⟨x²⟩
P₀

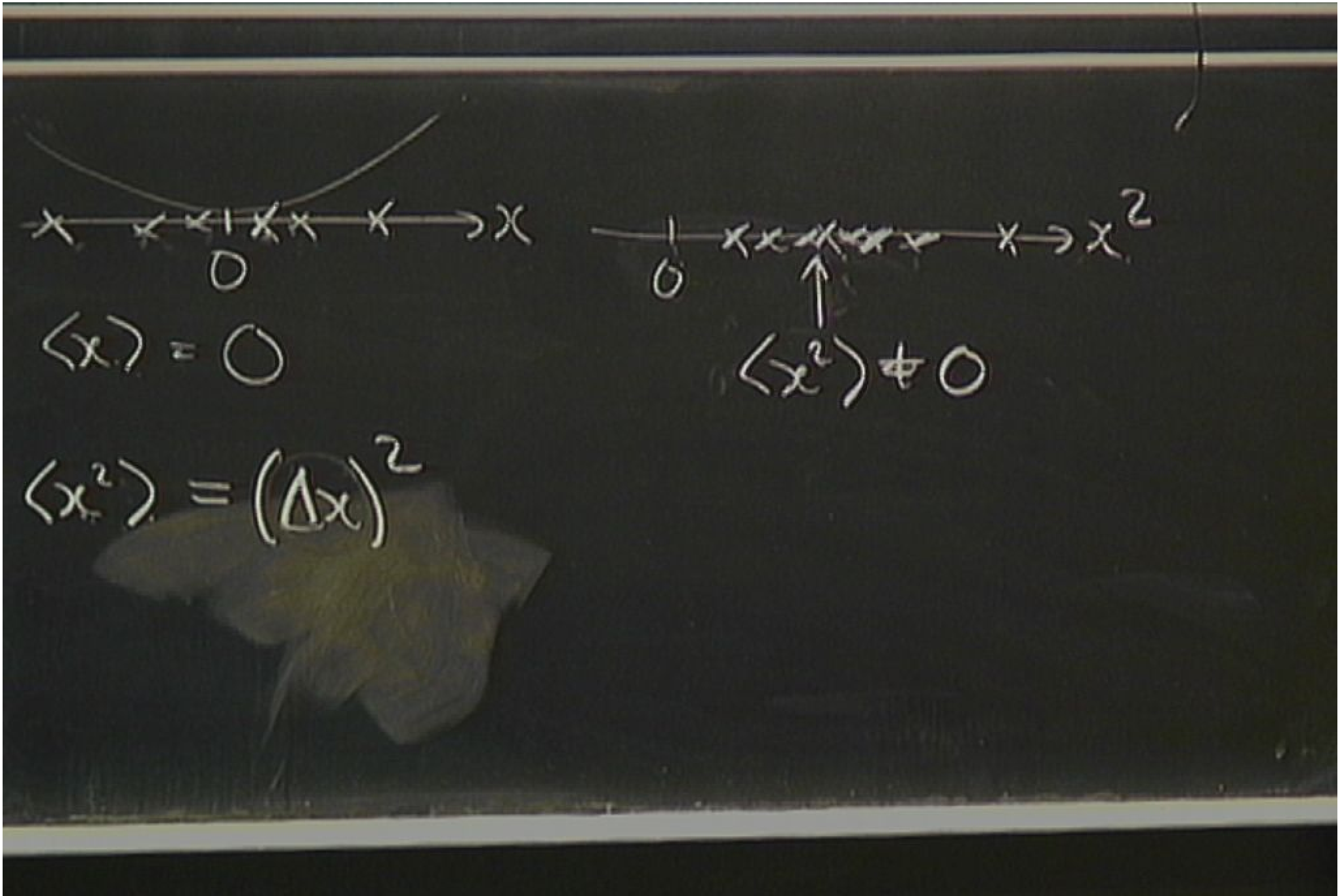

E = p²/2m + ½kx² = KE + PE
take many snapshots & average
⟨E⟩ = ⟨p²⟩/2m + ½k⟨x²⟩
P₀

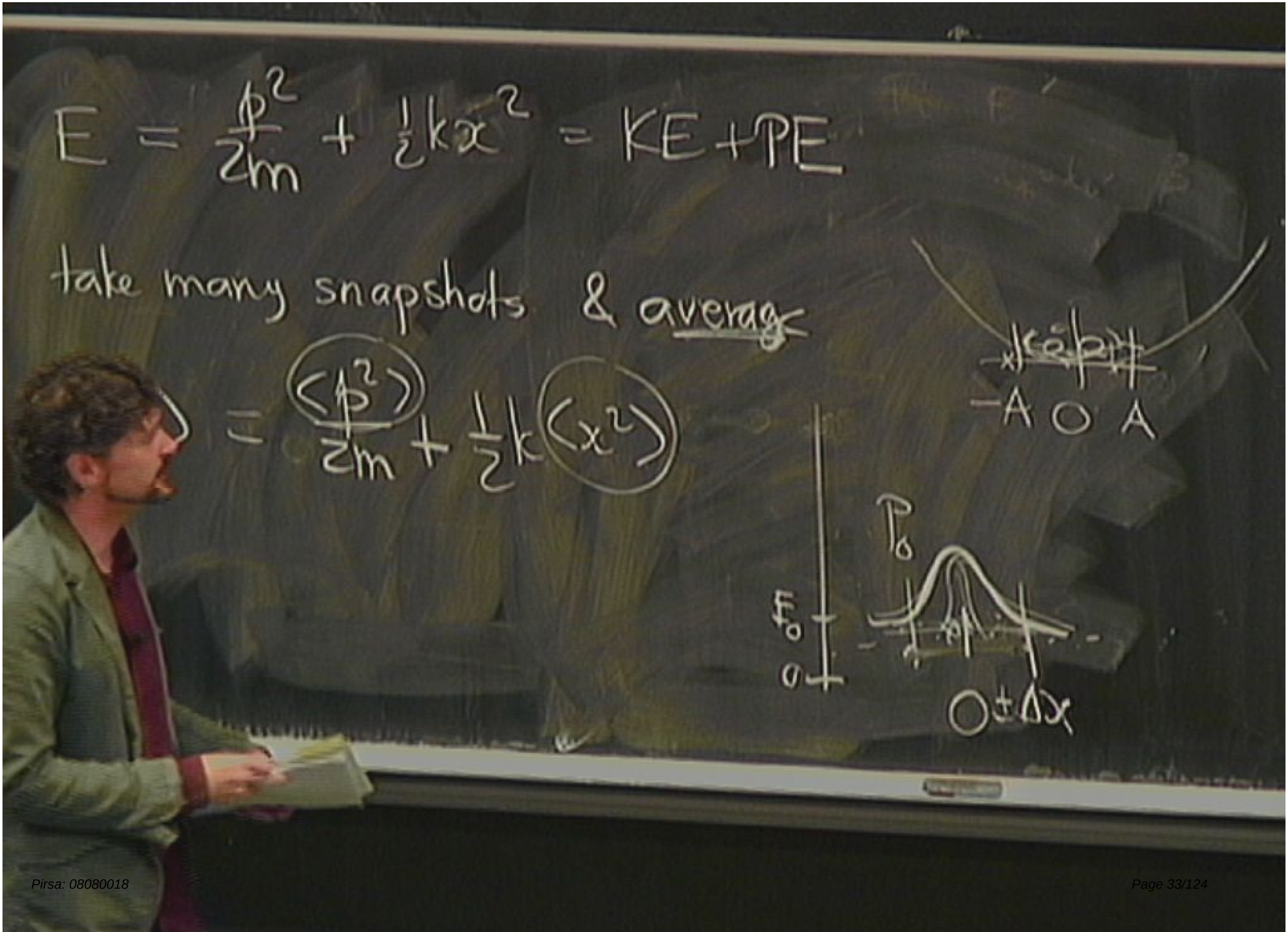

E = p^2/2m + 1/2 kx^2 = KE+PE
take many snapshots & average
<E> = <p^2>/2m + 1/2 k<
p_0


$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$
P_0
E_0
0

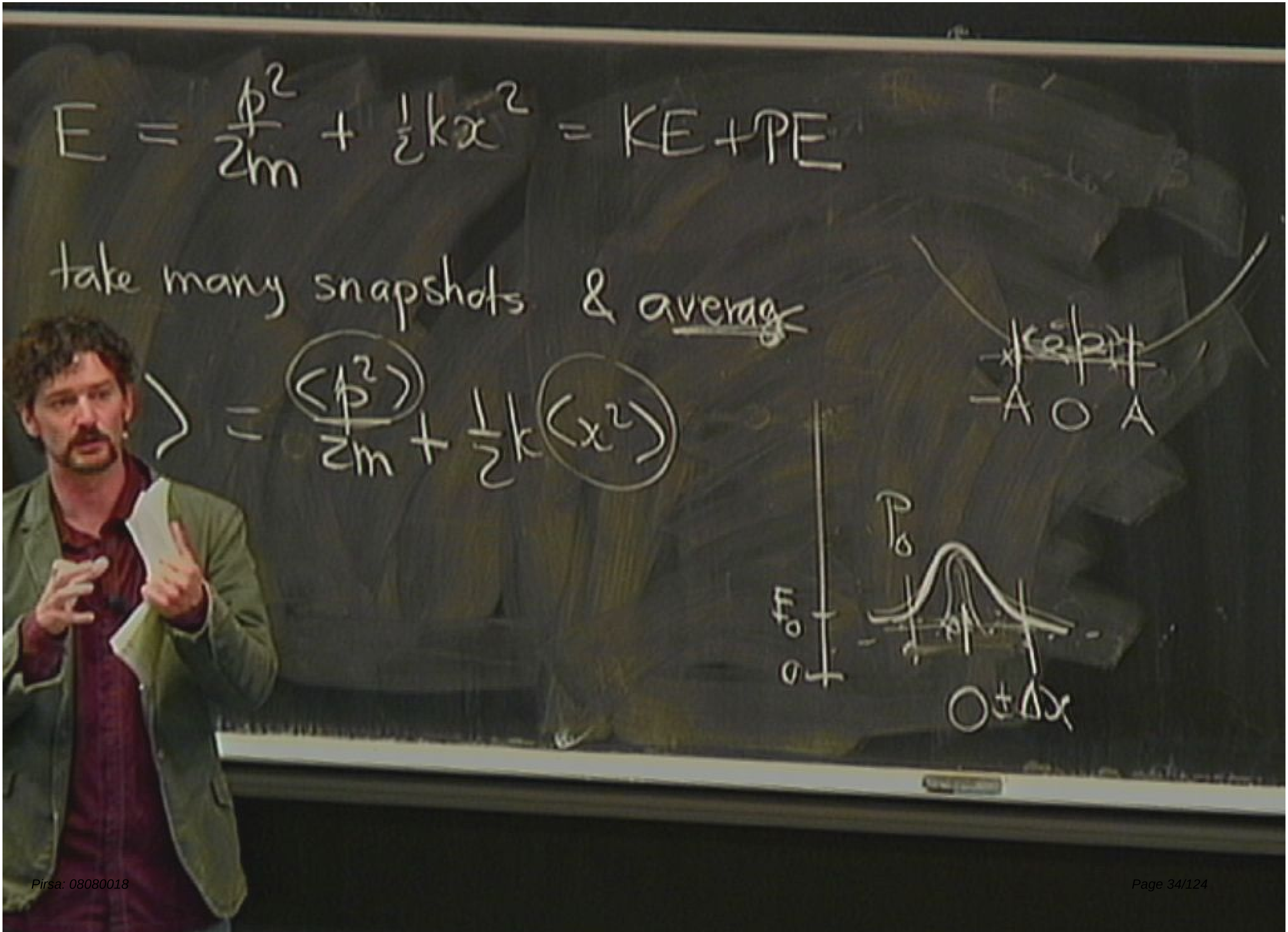

E = p^2/2m + ½kx^2 = KE+PE
take many snapshots & average
<E> = <p^2>/2m + ½
E_0
0
P_0
0 ± Δx

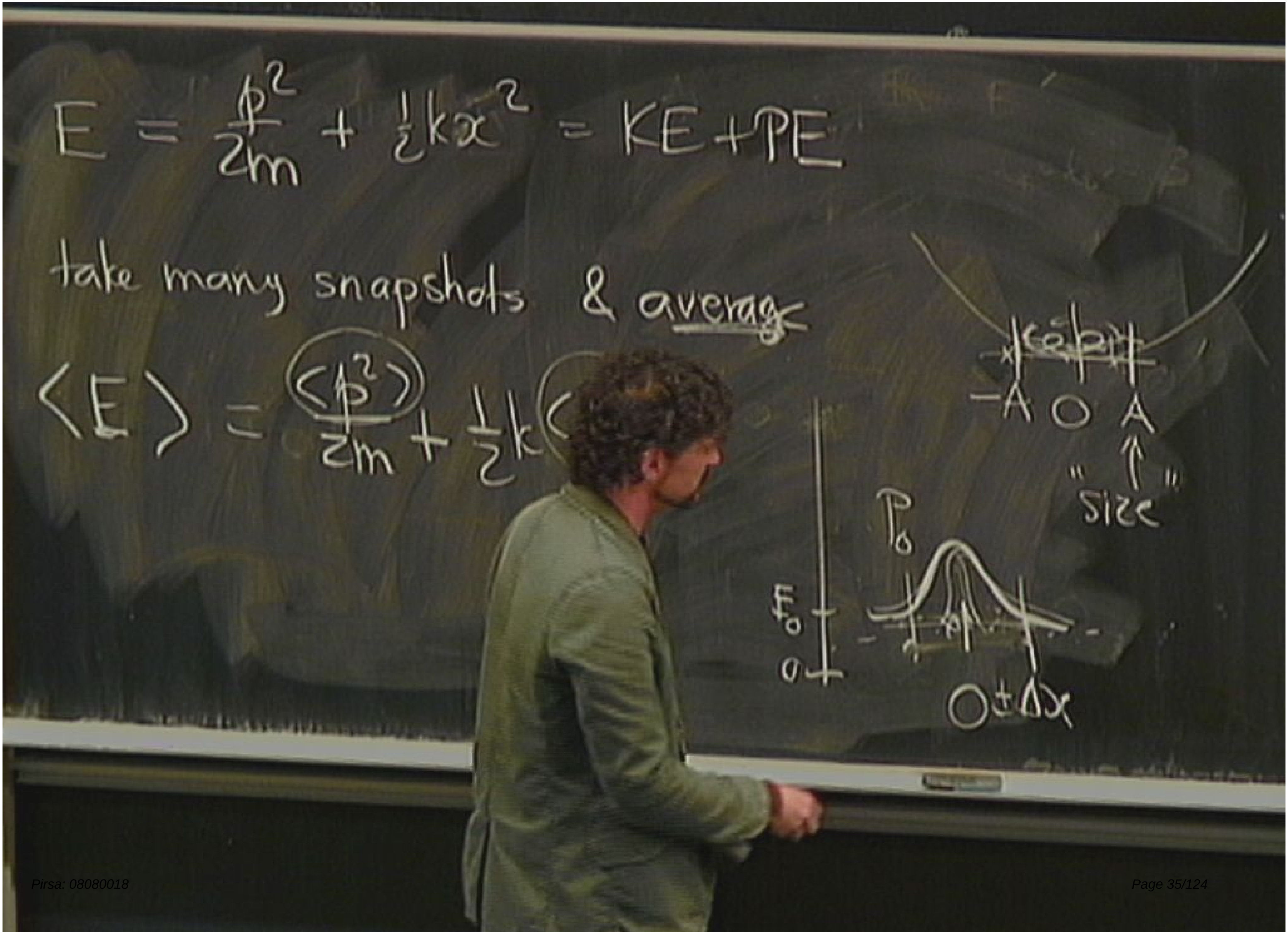

$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$
E_0
0
p_0
$0 \pm \Delta x$

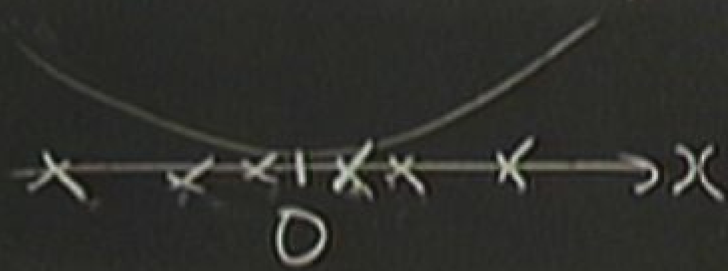

x
0
x²
0
⟨x⟩ = 0
⟨x²⟩ ≠ 0
⟨x²⟩ = (Δx)²



$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$= \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$
$-A \quad 0 \quad A$
P_0
E_0
0
$0 \pm \Delta x$


E = p²/2m + ½kx² = KE + PE
take many snapshots & average
⟩ = ⟨p²⟩/2m + ½k⟨x²⟩
A O A
P₀
E₀
0
0 ± Δx


$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$
take many snapshots & average
$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle$
−A 0 A
"size"
P_0
F_0
0
$0 \pm \Delta x$



$$\langle x \rangle = 0$$

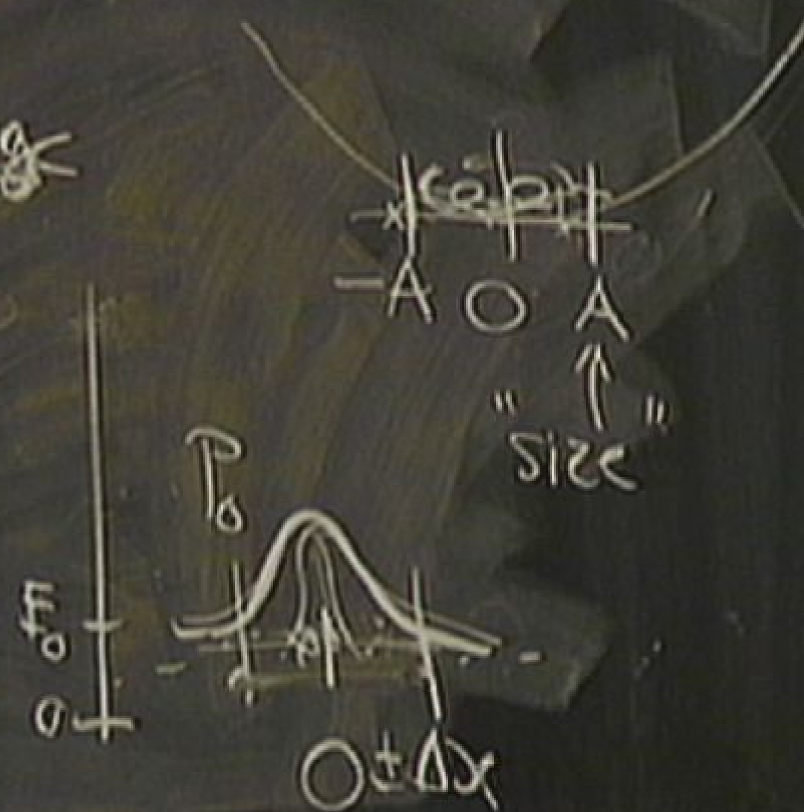
$$\langle x^2 \rangle \neq 0$$

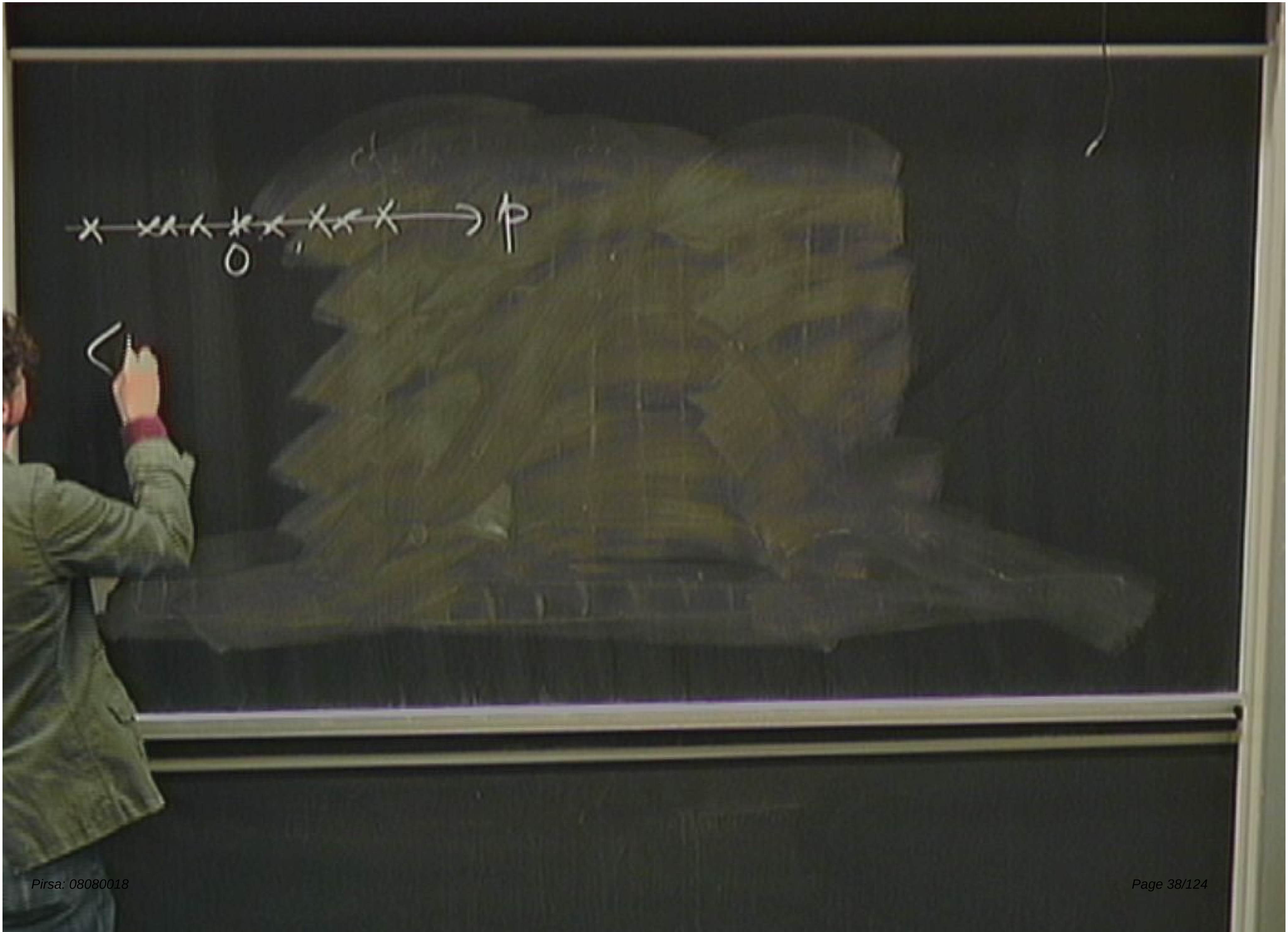
$$\langle x^2 \rangle = (\Delta x)^2 = A^2$$

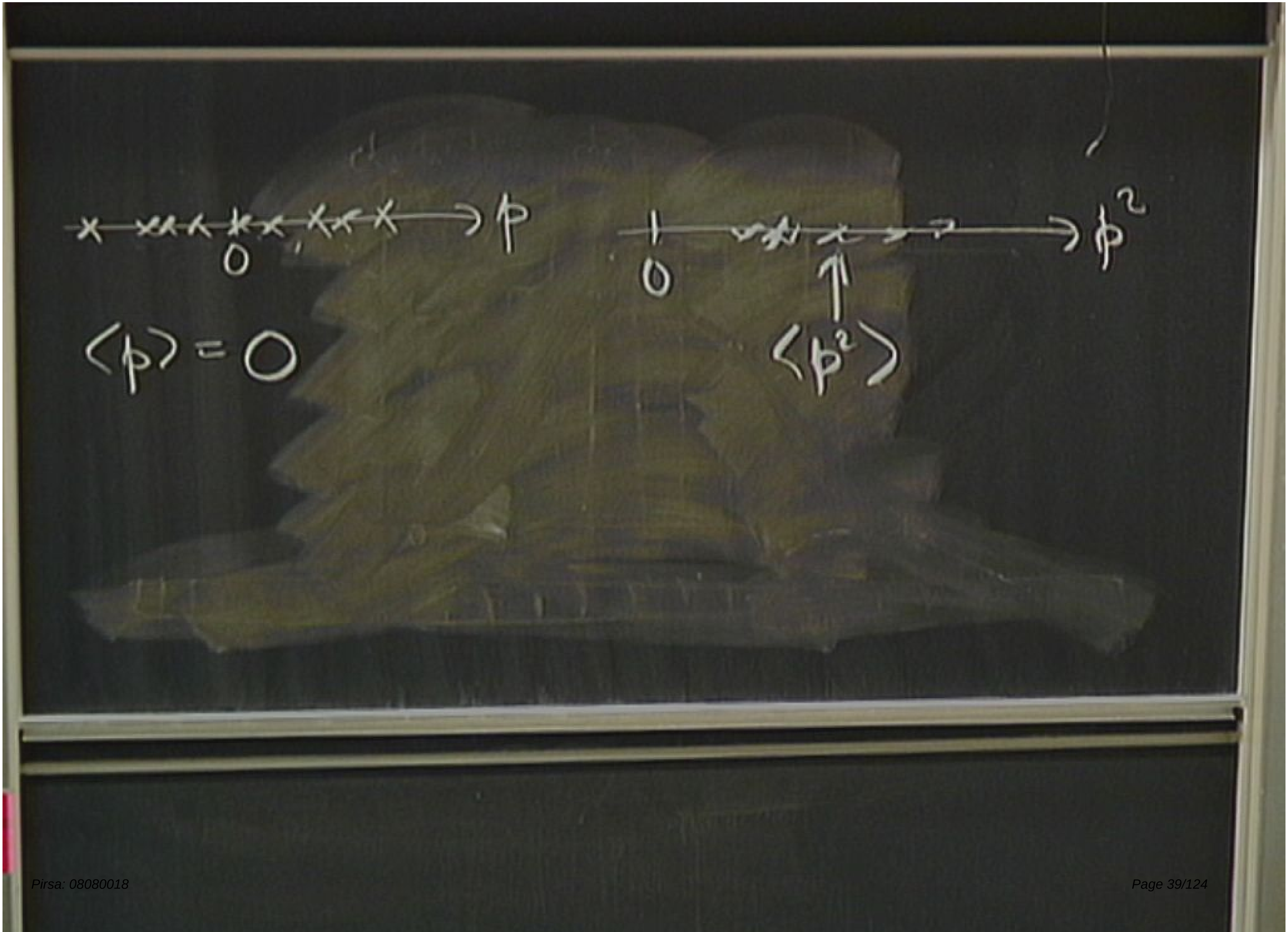
$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2 = KE + PE$$

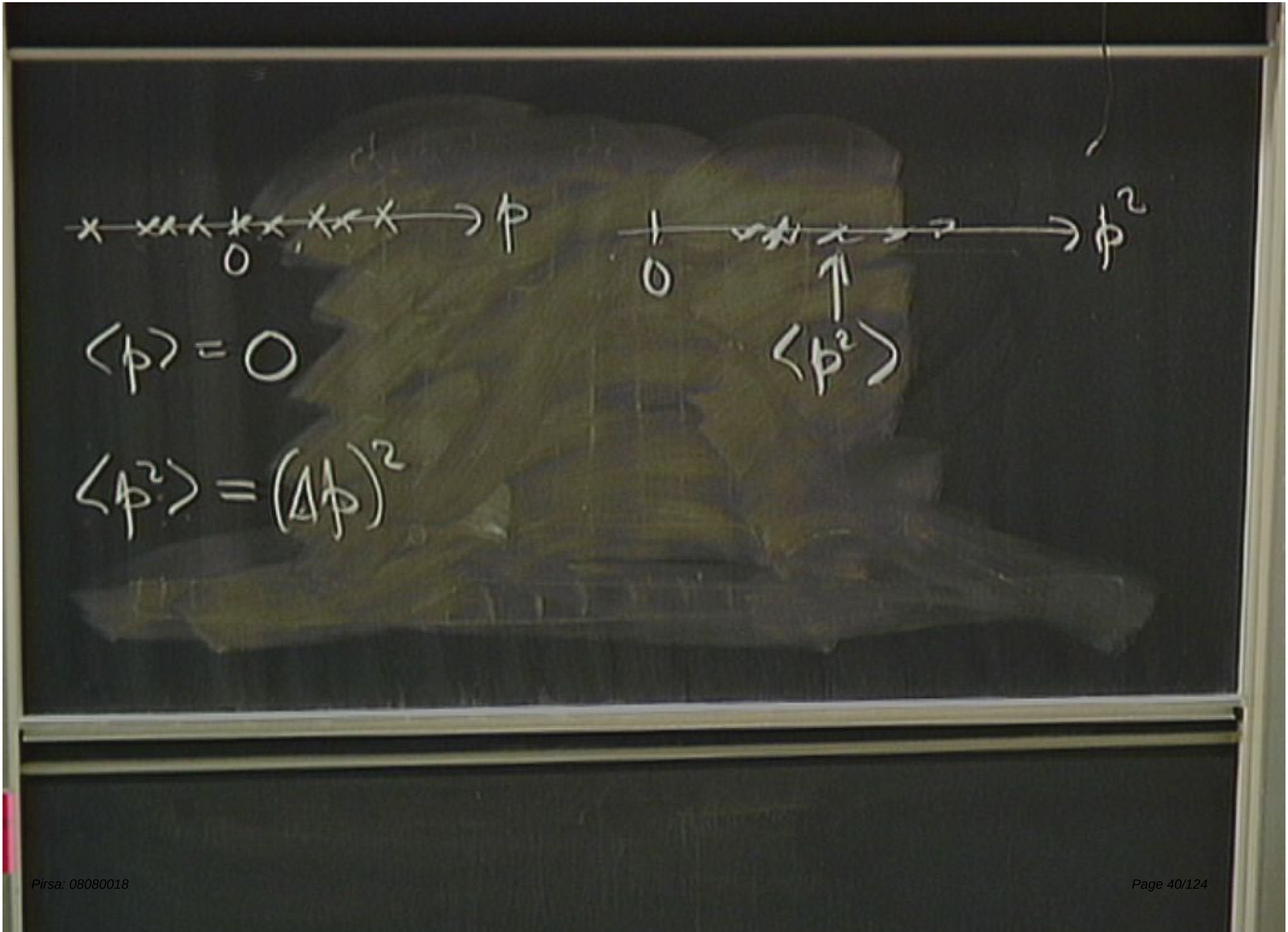
take many snapshots & average

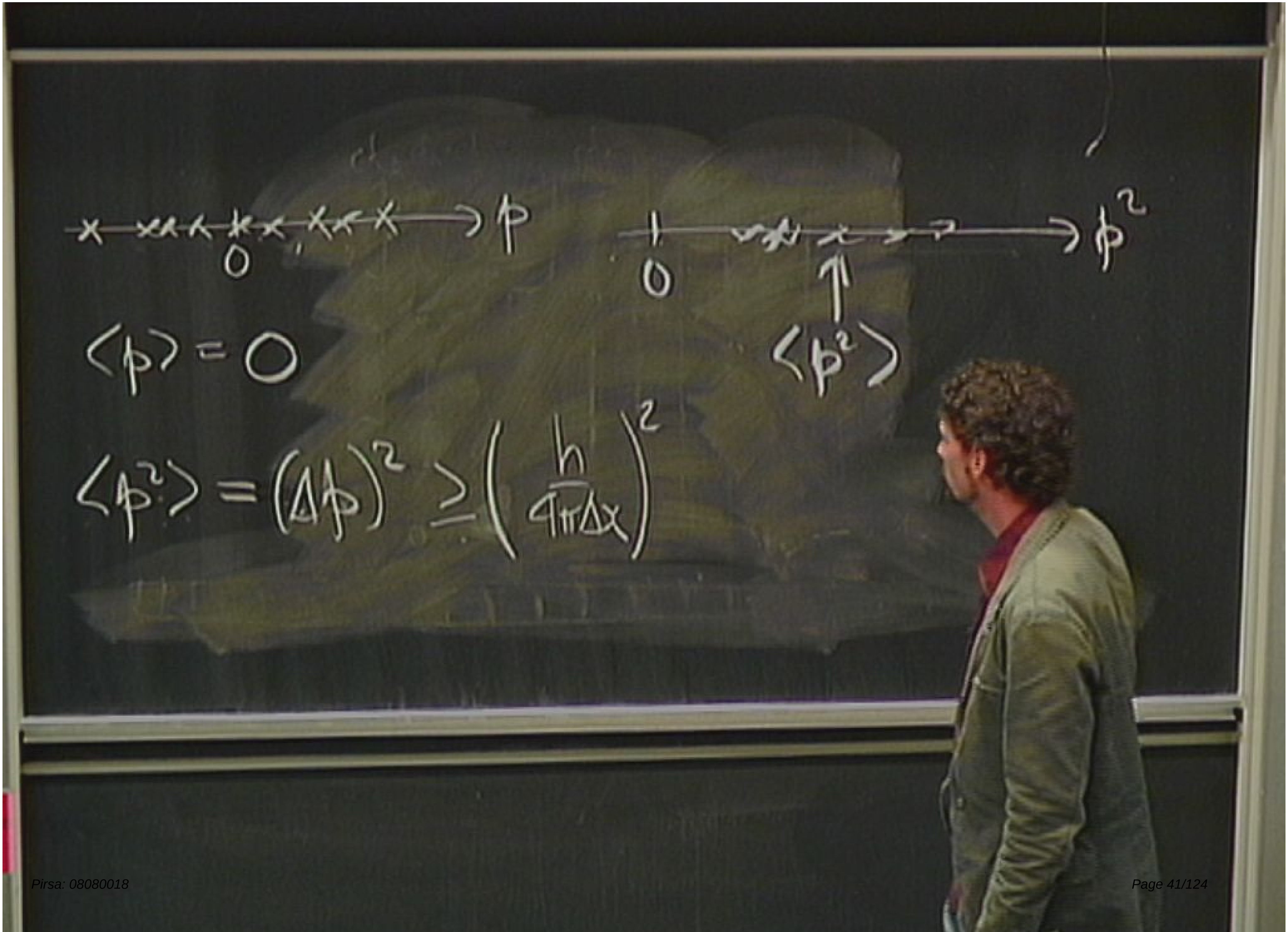
$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \frac{1}{2}k\langle x^2 \rangle$$





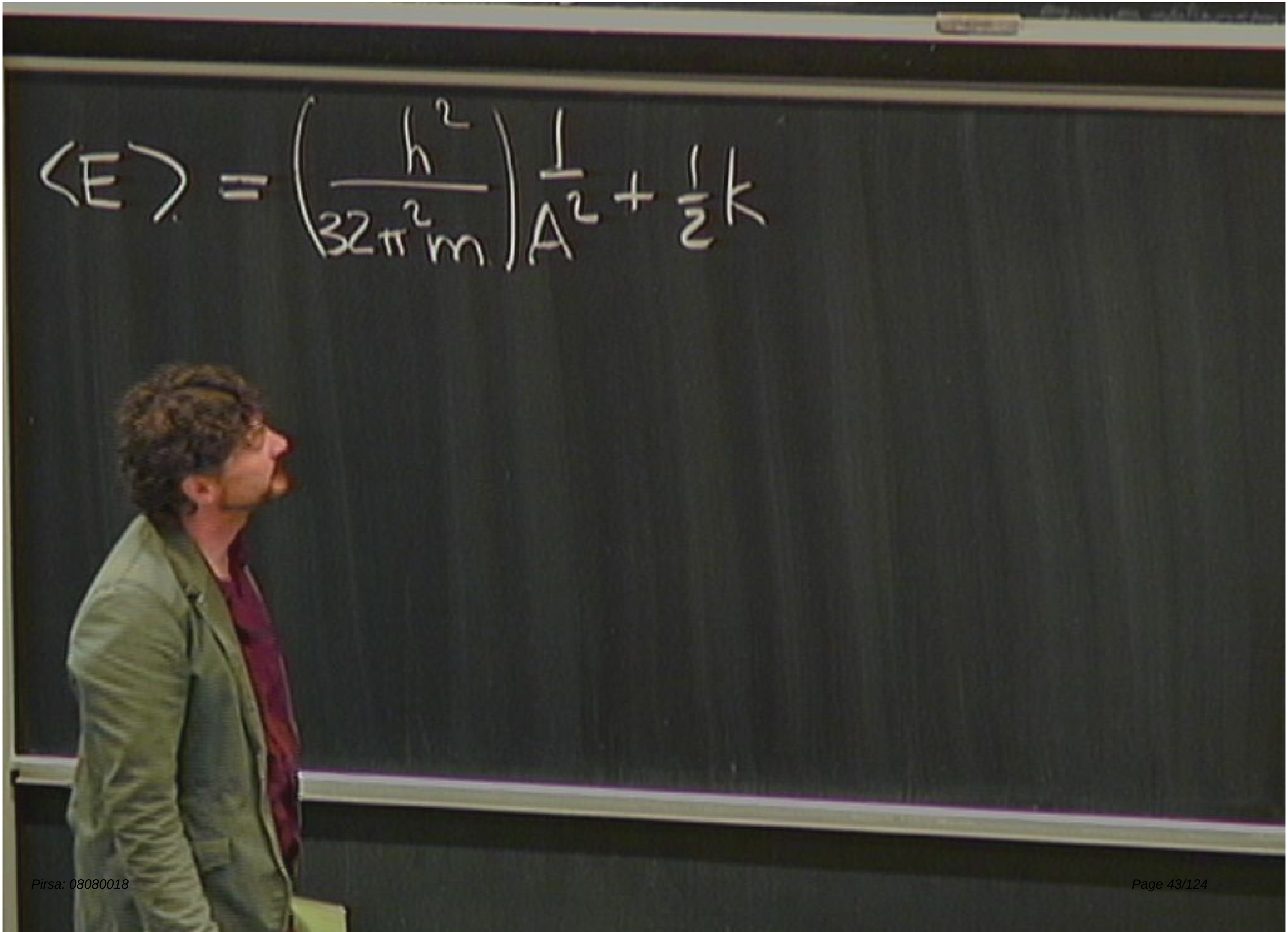

p
0
⟨p⟩ = 0
p²
0
⟨p²⟩

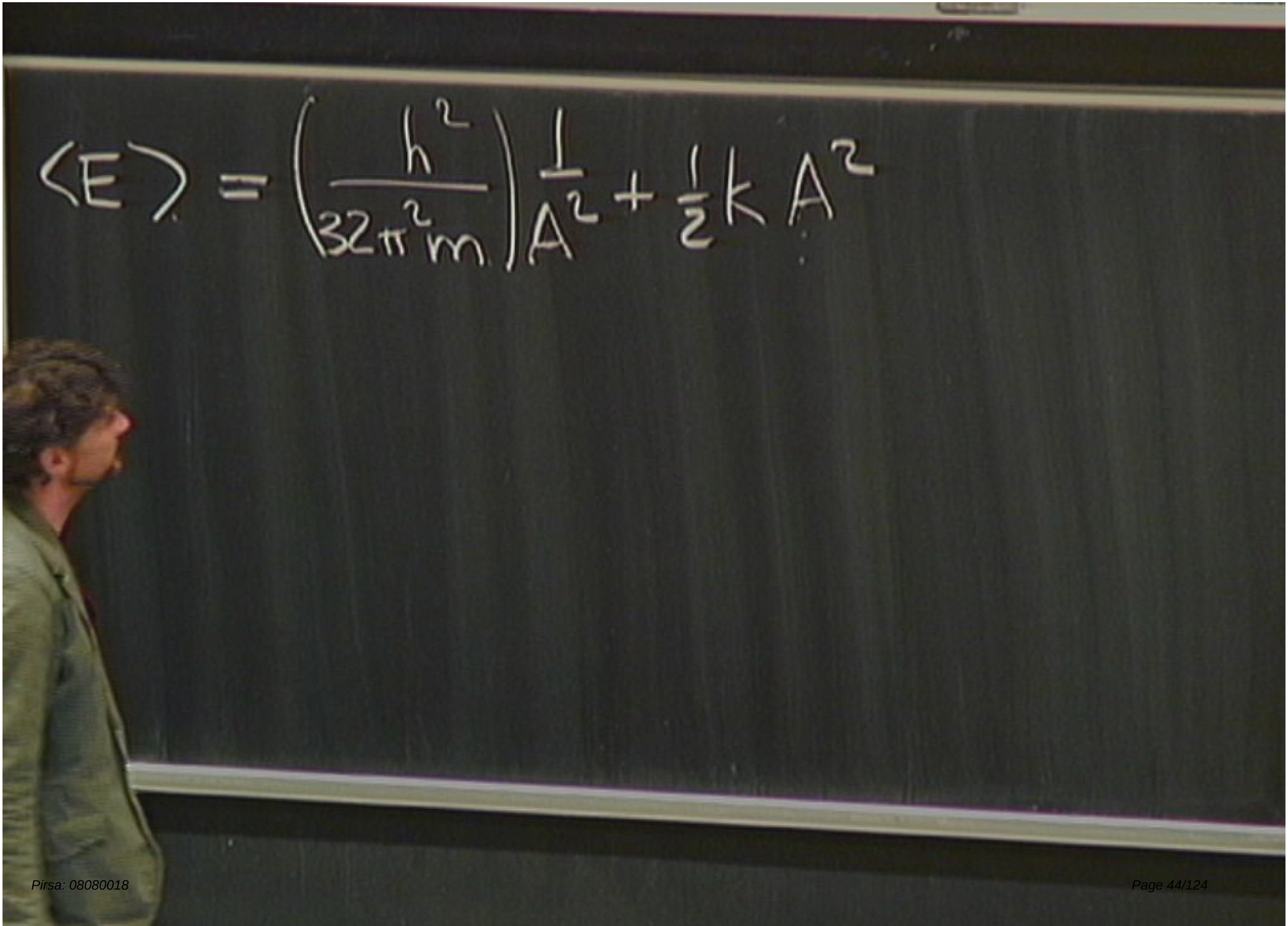

p
0
p^2
0
$\langle p^2 \rangle$
$\langle p \rangle = 0$
$\langle p^2 \rangle = (\Delta p)^2$

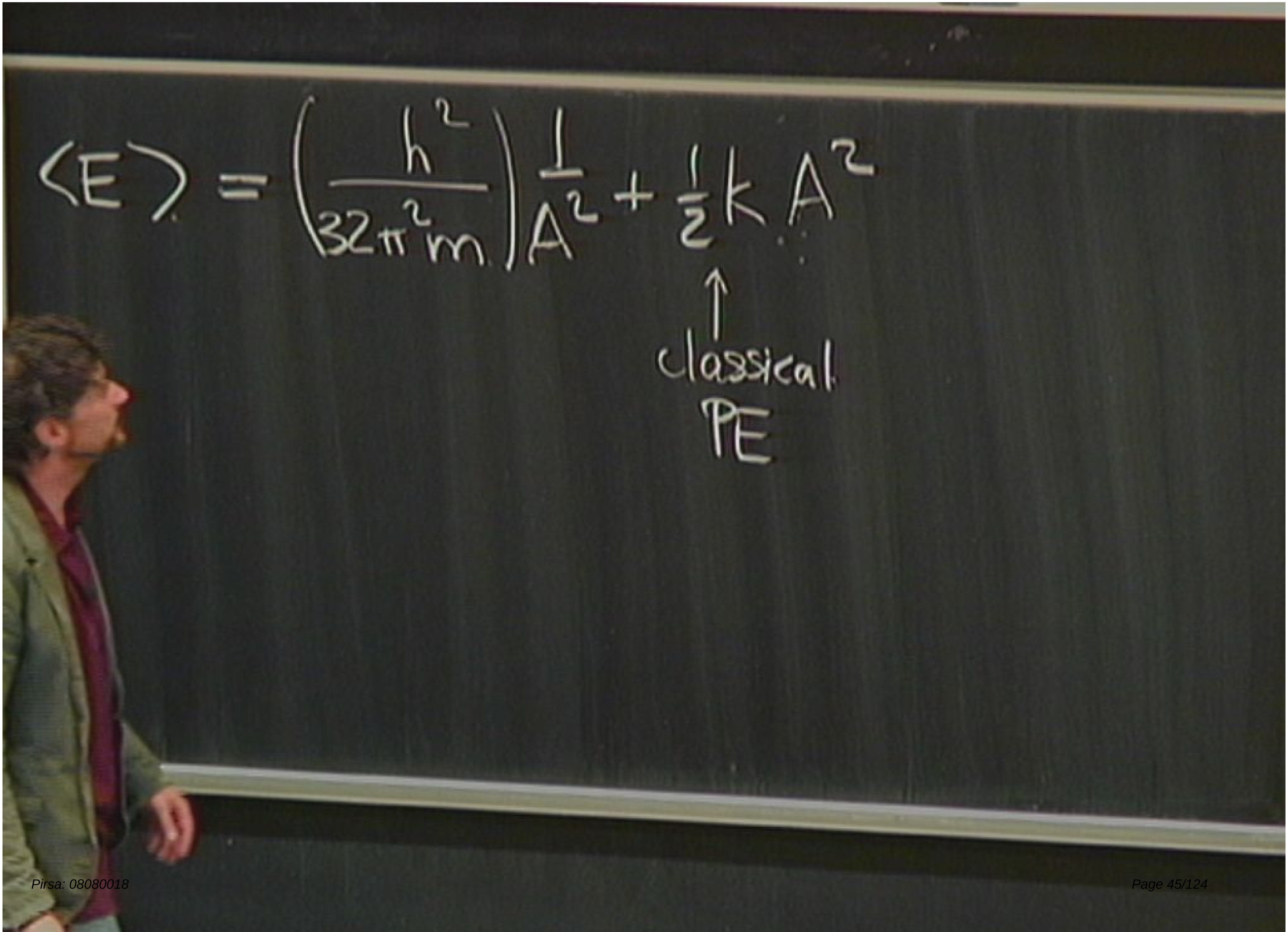

p
0
p²
0
⟨p⟩ = 0
⟨p²⟩
⟨p²⟩ = (Δp)² ≥ (h / 4πΔx)²

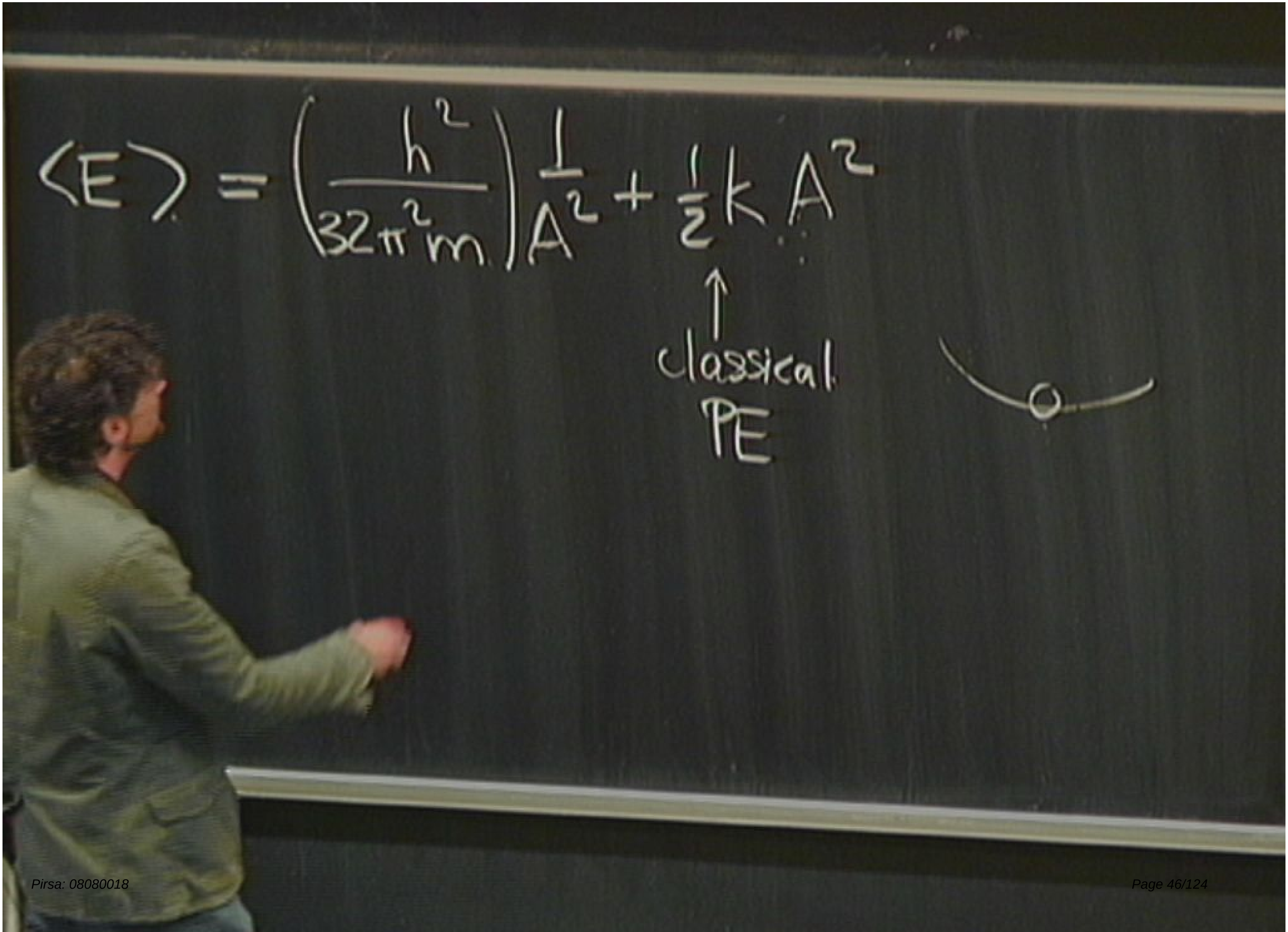
$$\langle p \rangle = 0 \qquad\qquad \langle p^2 \rangle$$

$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x} \right)^2 = \left(\frac{h}{4\pi A} \right)^2$$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}k$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m} \right) \frac{1}{A^2} + \frac{1}{2} k A^2$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$
classical
PE


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$
classical PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

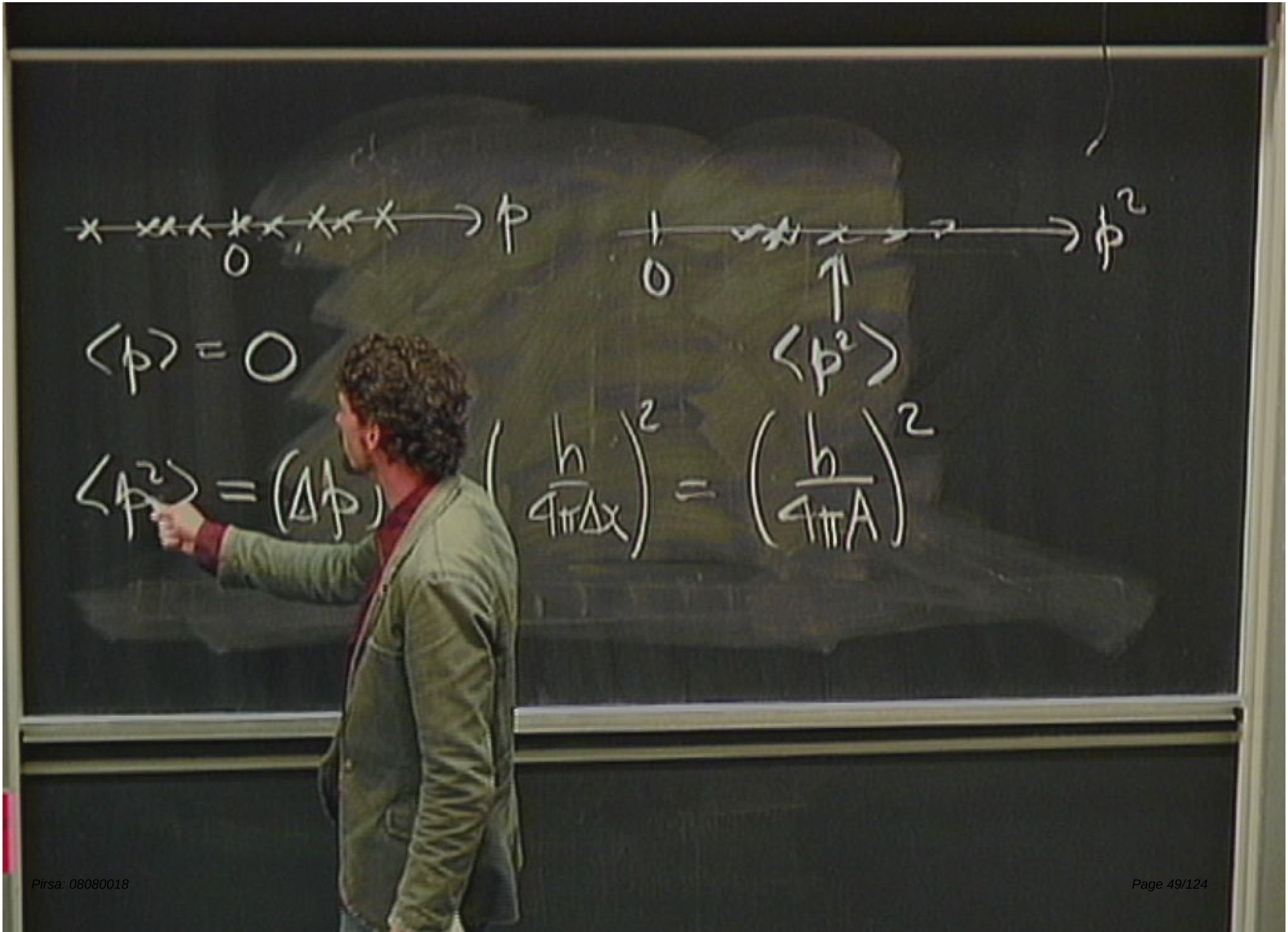
↑ quantum

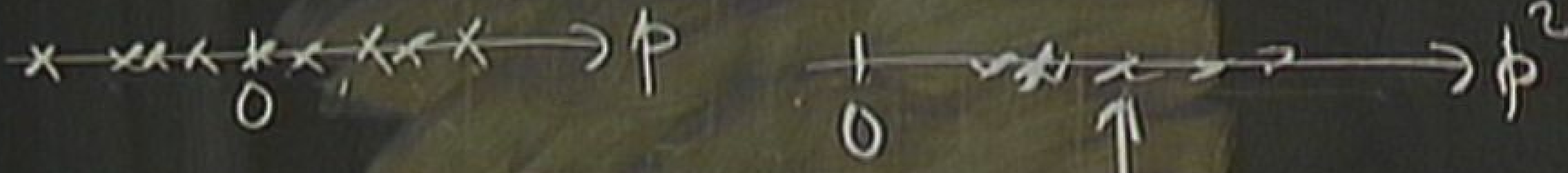
↑ classical PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE

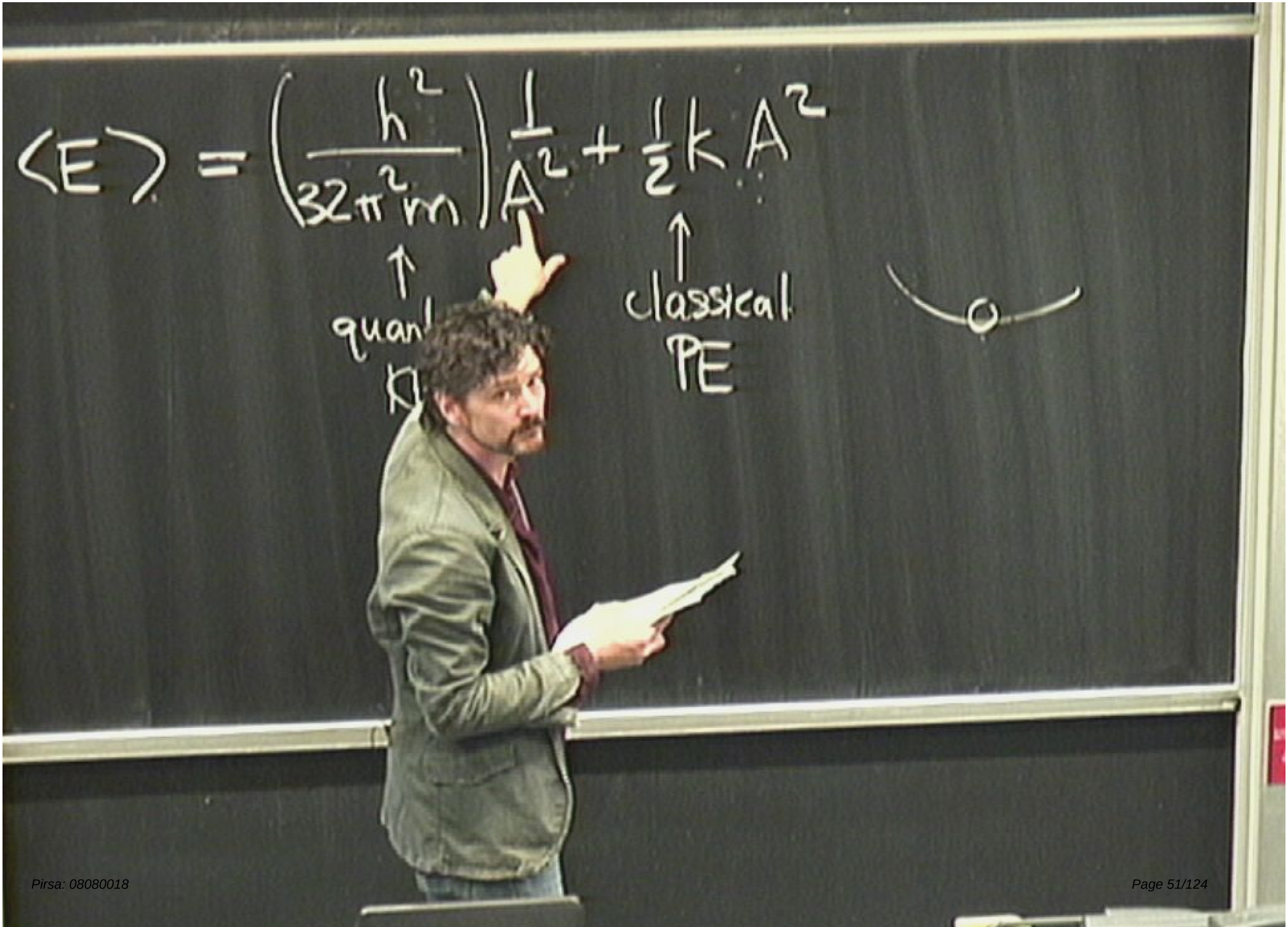

φ
0
φ²
0
⟨φ²⟩
⟨φ⟩ = 0
⟨φ²⟩ = (Δφ)² = (h/4πΔx)² = (h/4πA)²



$\langle p^2 \rangle$

$$\langle p \rangle = 0$$

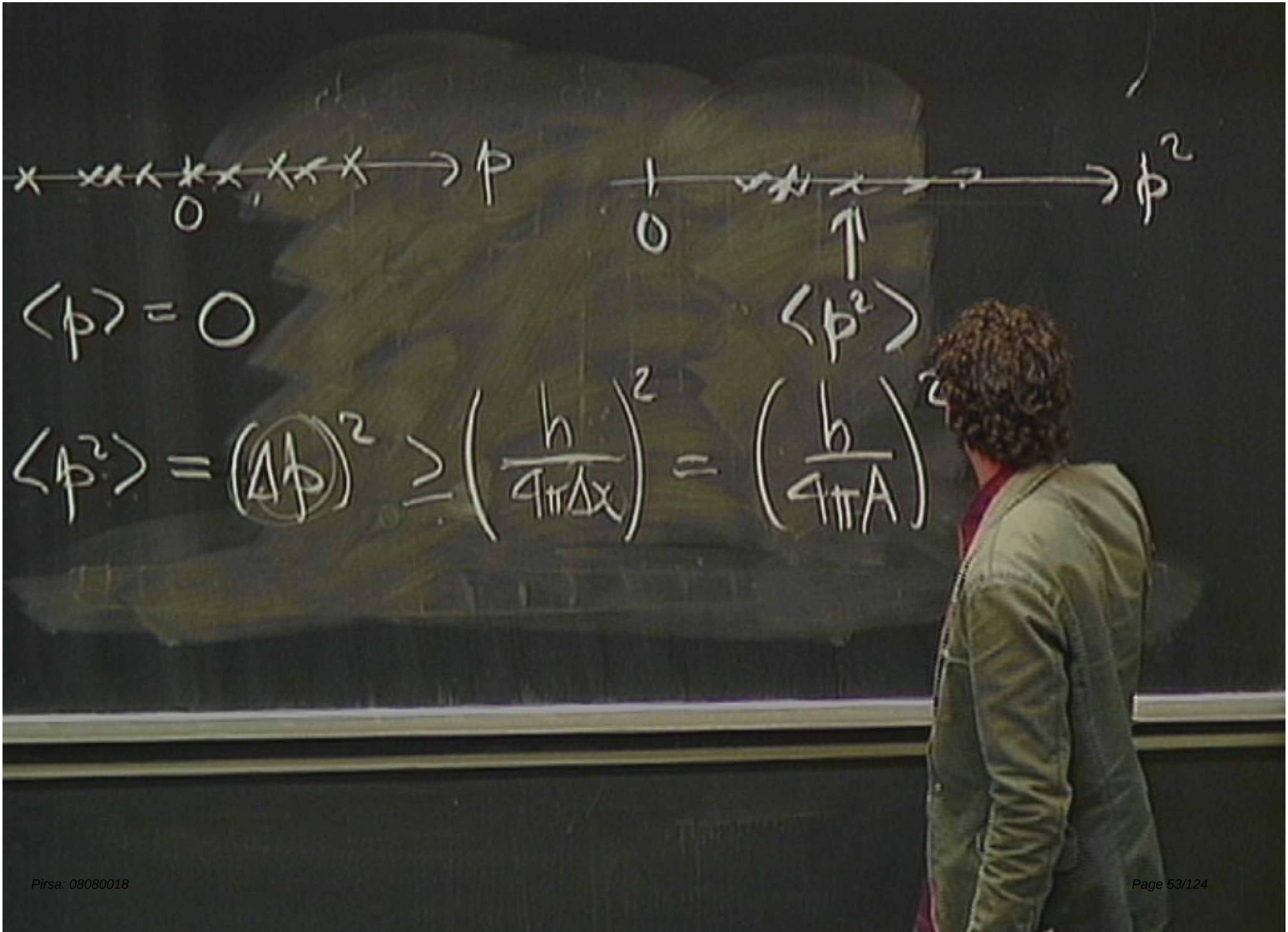
$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x} \right)^2 = \left(\frac{h}{4\pi A} \right)^2$$


$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$
classical
PE

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE

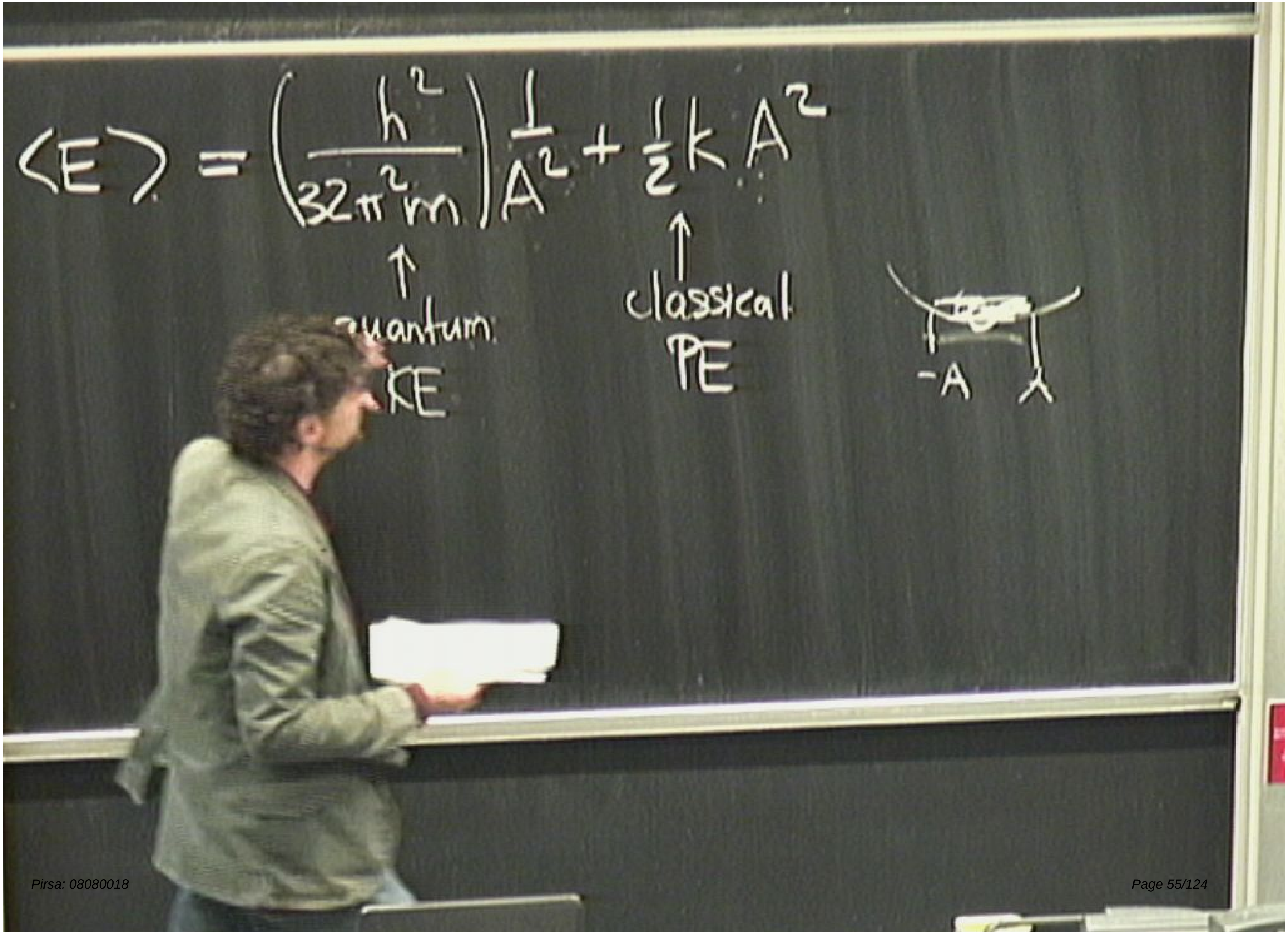

p
0
p²
0
⟨p⟩ = 0
⟨p²⟩
⟨p²⟩ = (Δp)² ≥ (h / 4πΔx)² = (h / 4πA)²

$$\langle p \rangle = 0$$

$$\langle p^2 \rangle$$

$$\Delta x = A$$

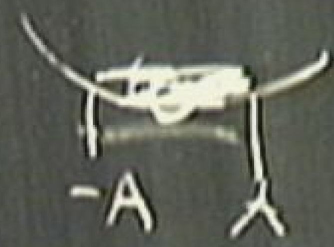
$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x}\right)^2 = \left(\frac{h}{4\pi A}\right)^2$$

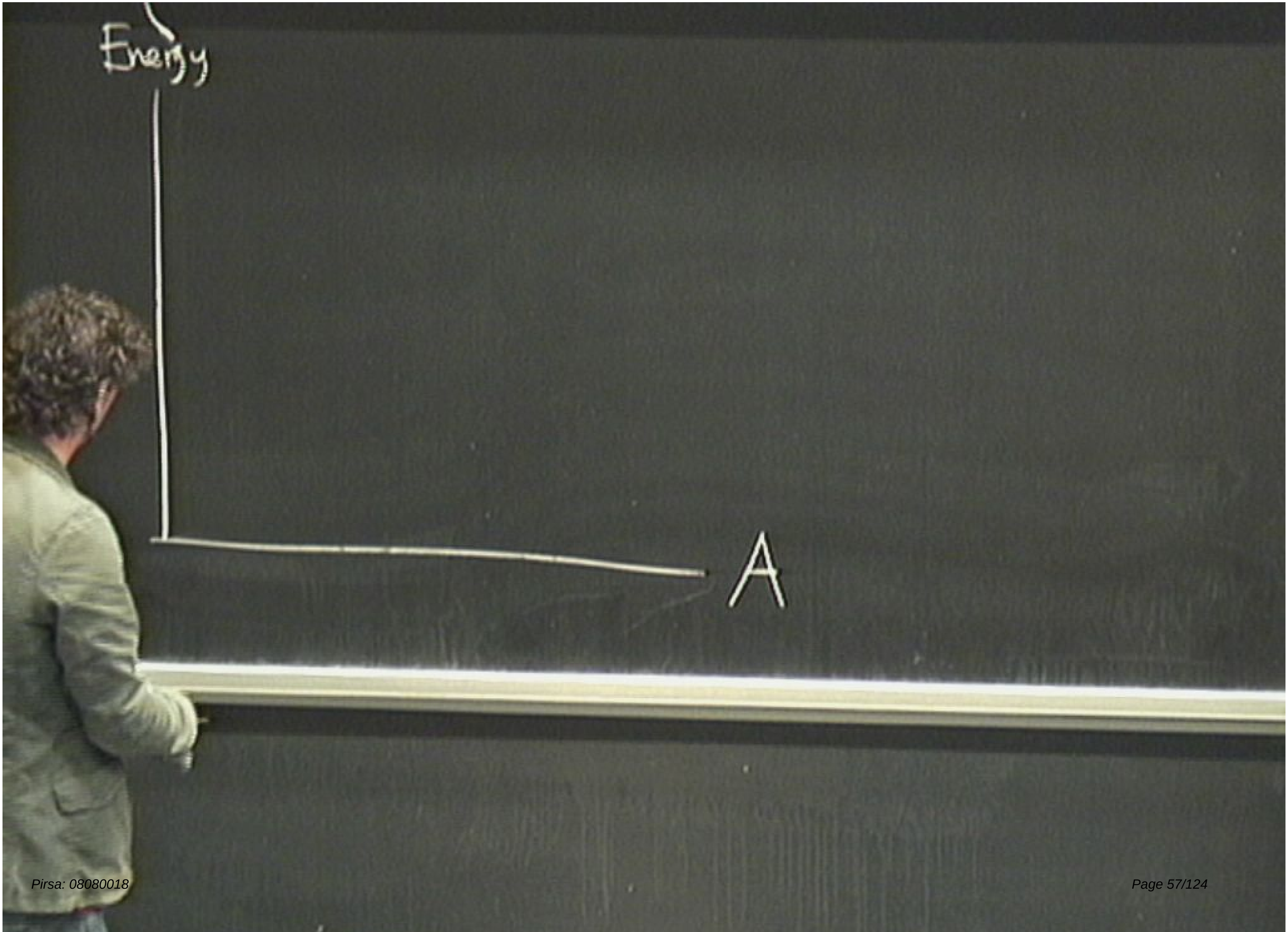

$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$
uantum
KE
classical
PE
-A
A

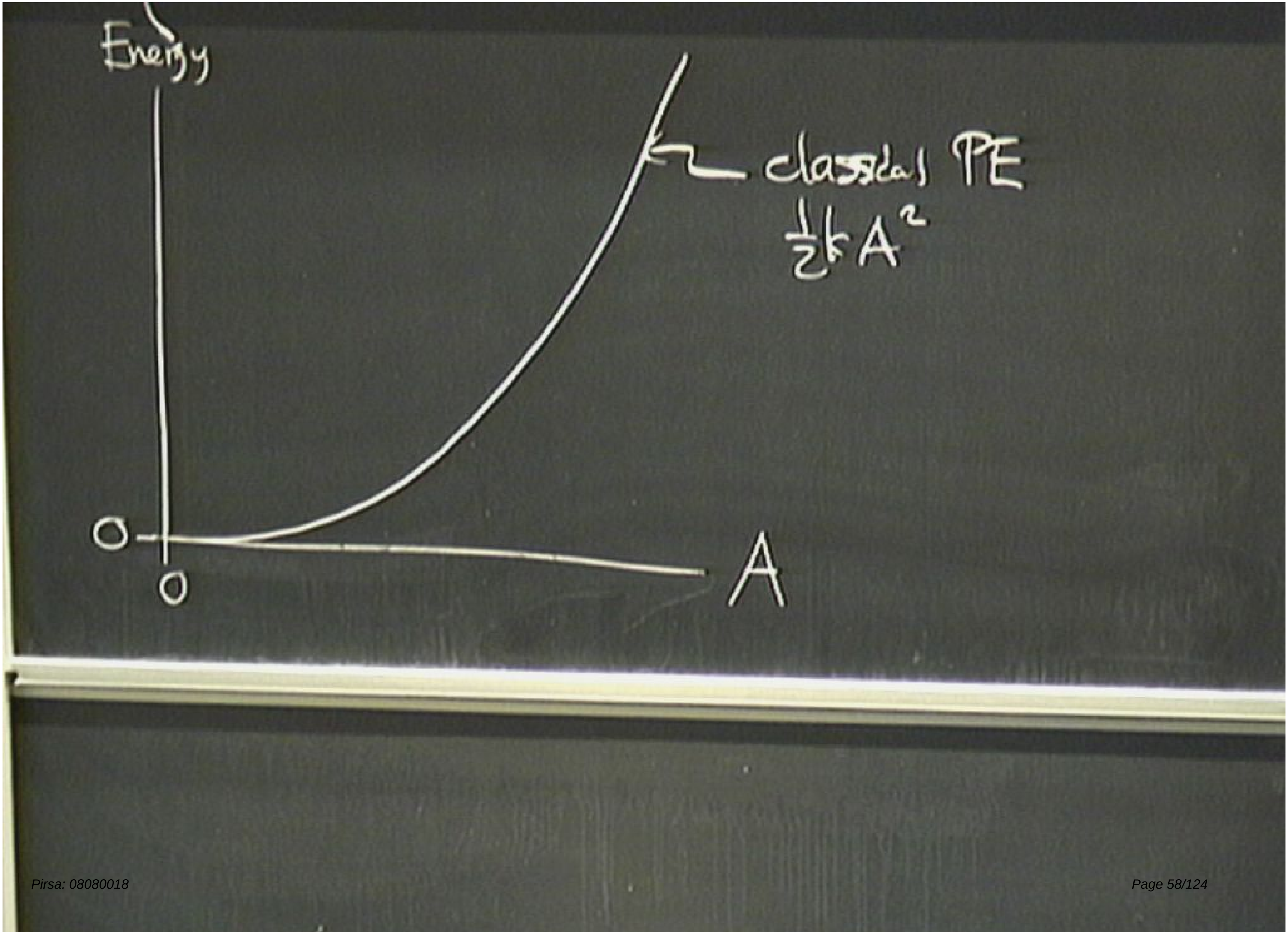
$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE



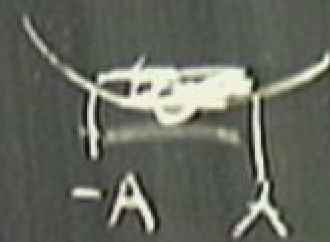

Energy
A

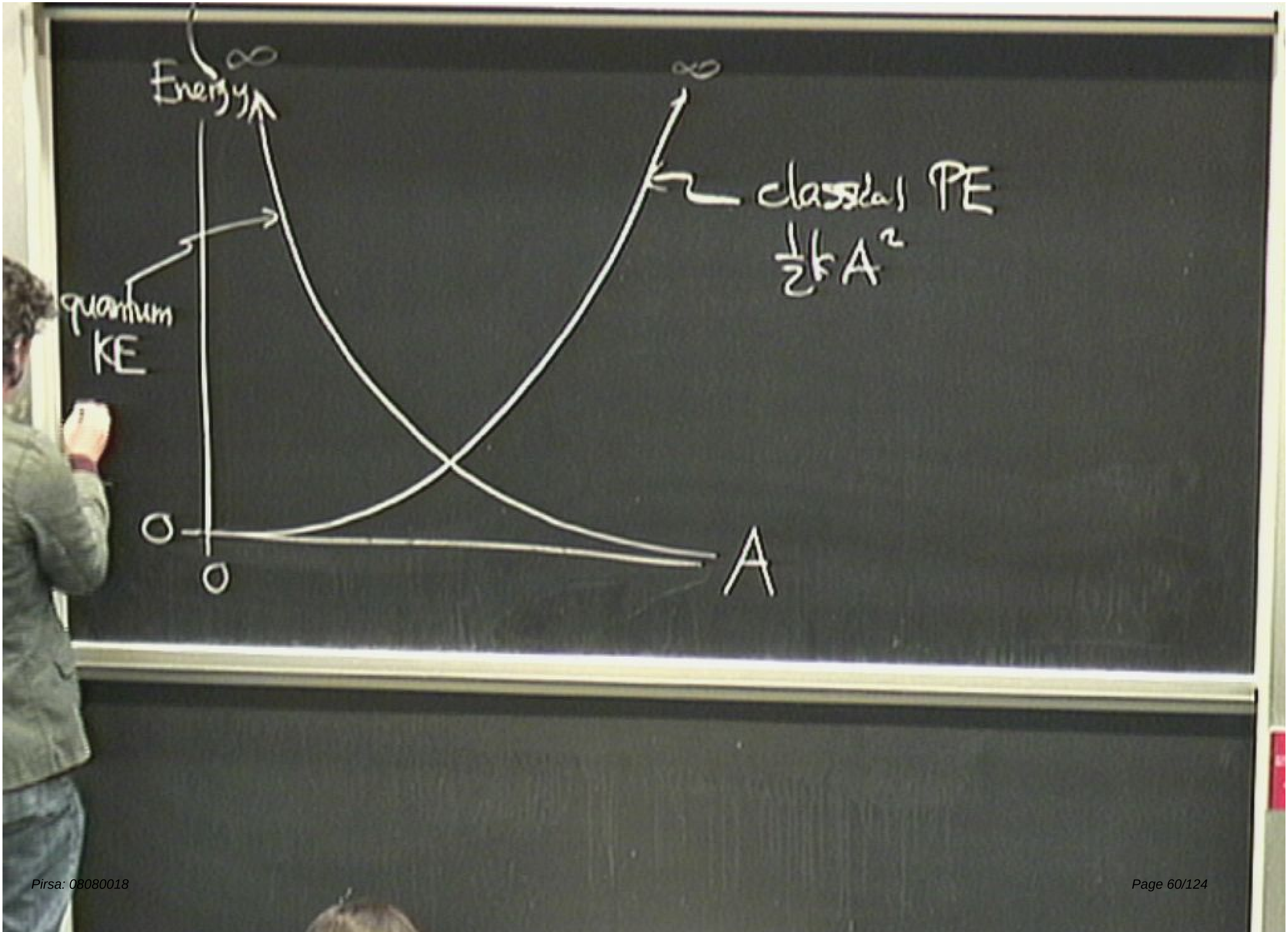

Energy
classical PE
$\frac{1}{2}kA^2$
0
0
A

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2} k A^2$$

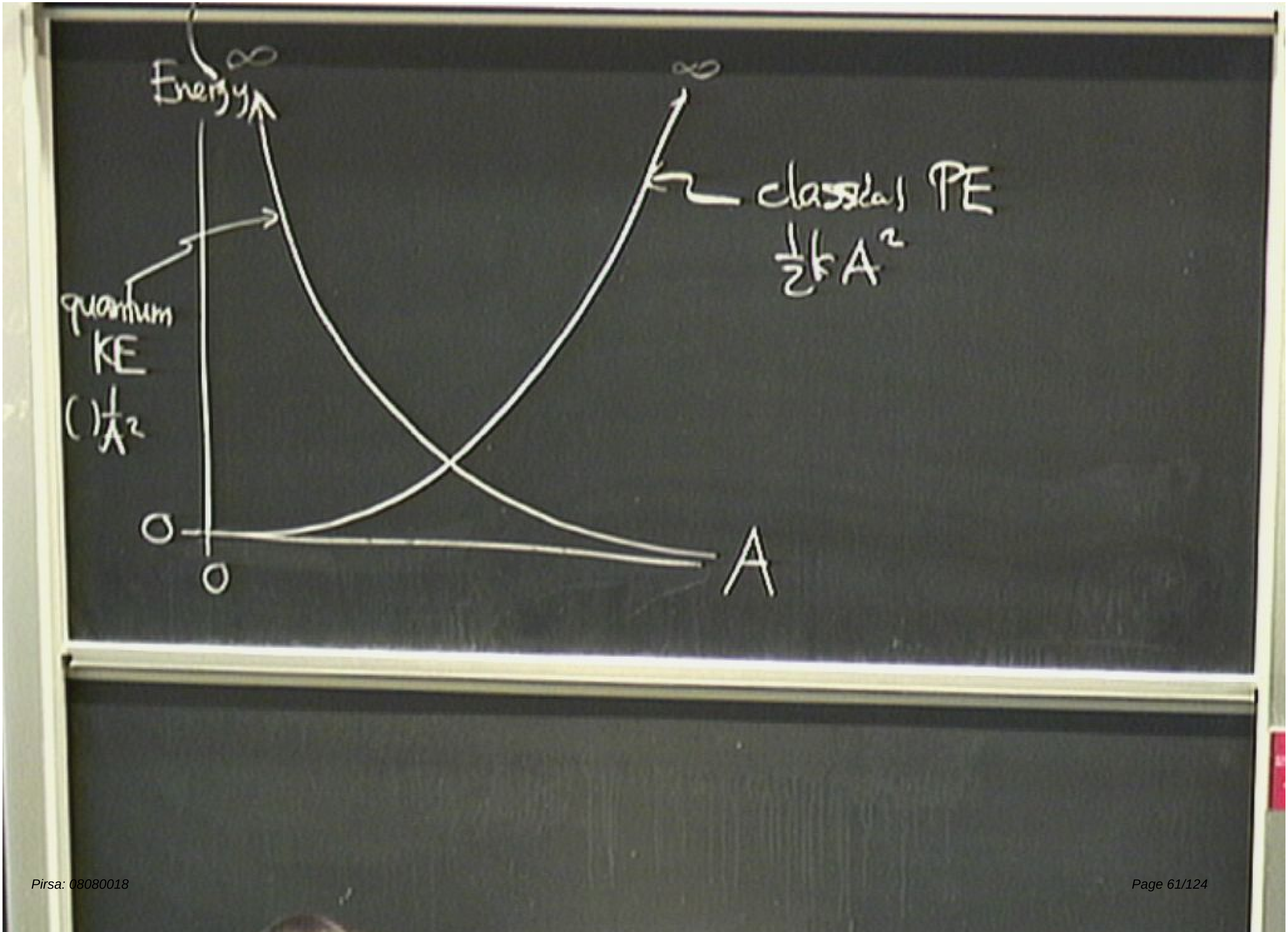
↑ quantum KE

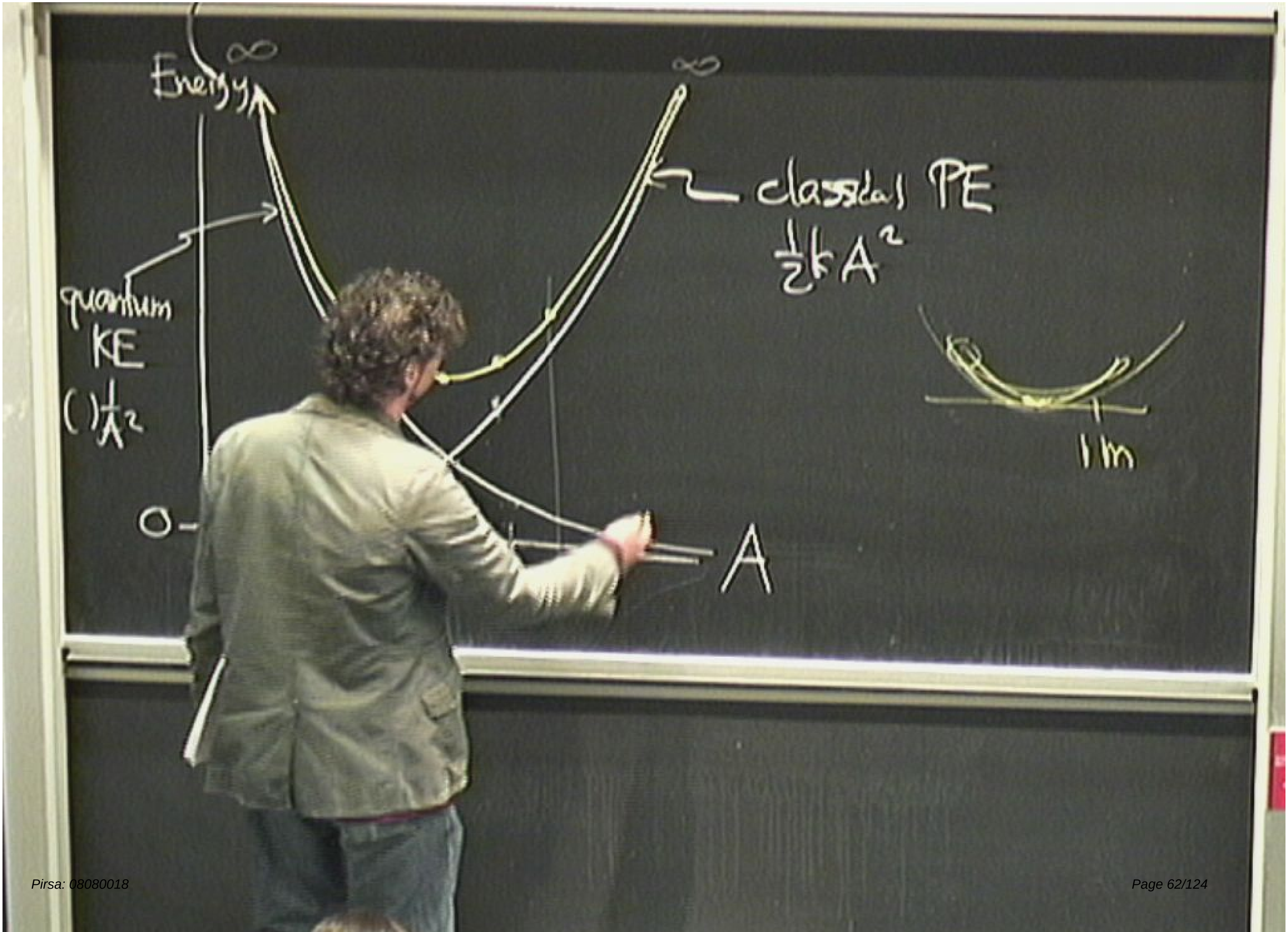
↑ classical PE

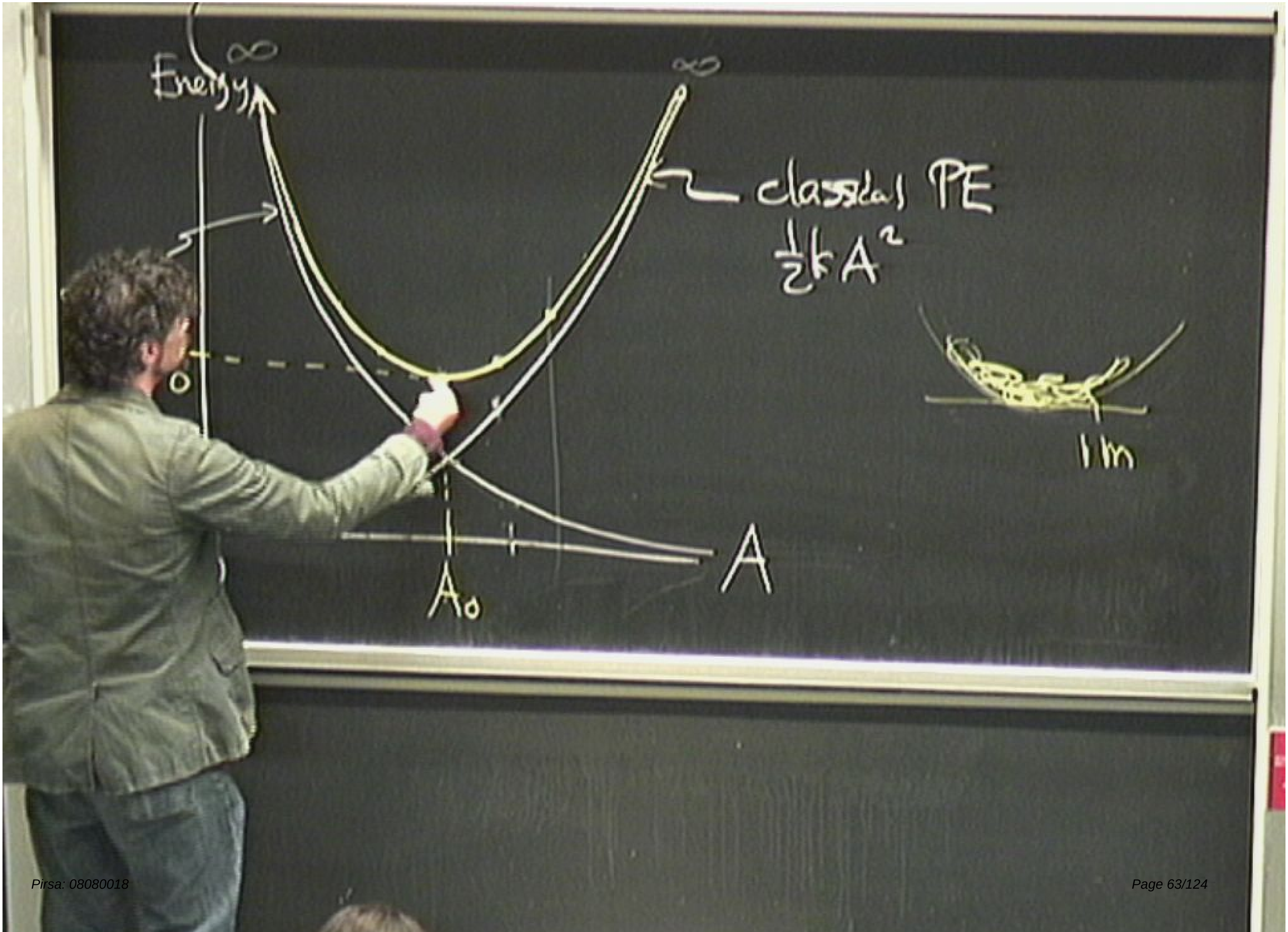




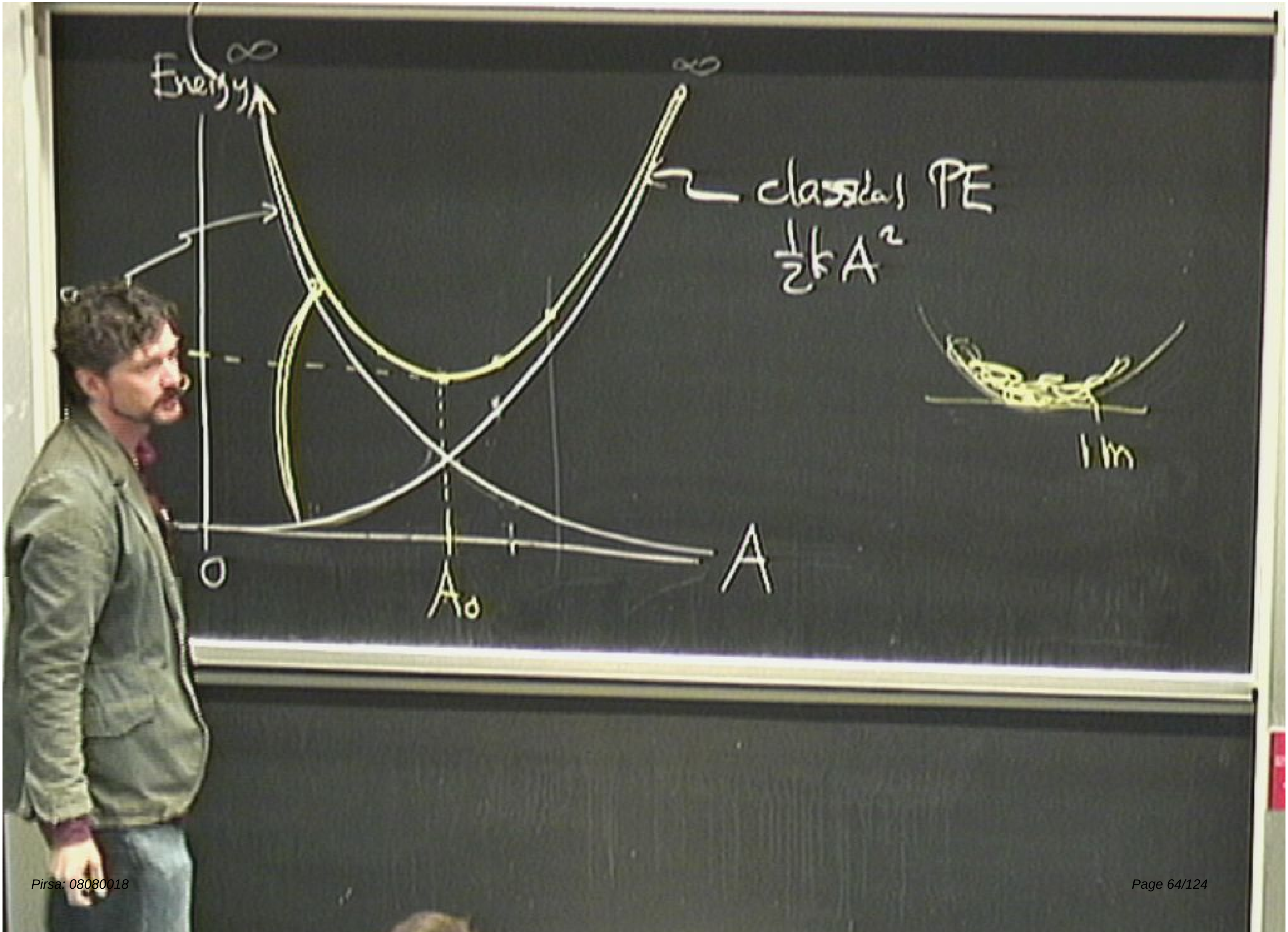
∞
Energy
∞
classical PE
$\frac{1}{2}kA^2$
quantum
KE
0
0
A

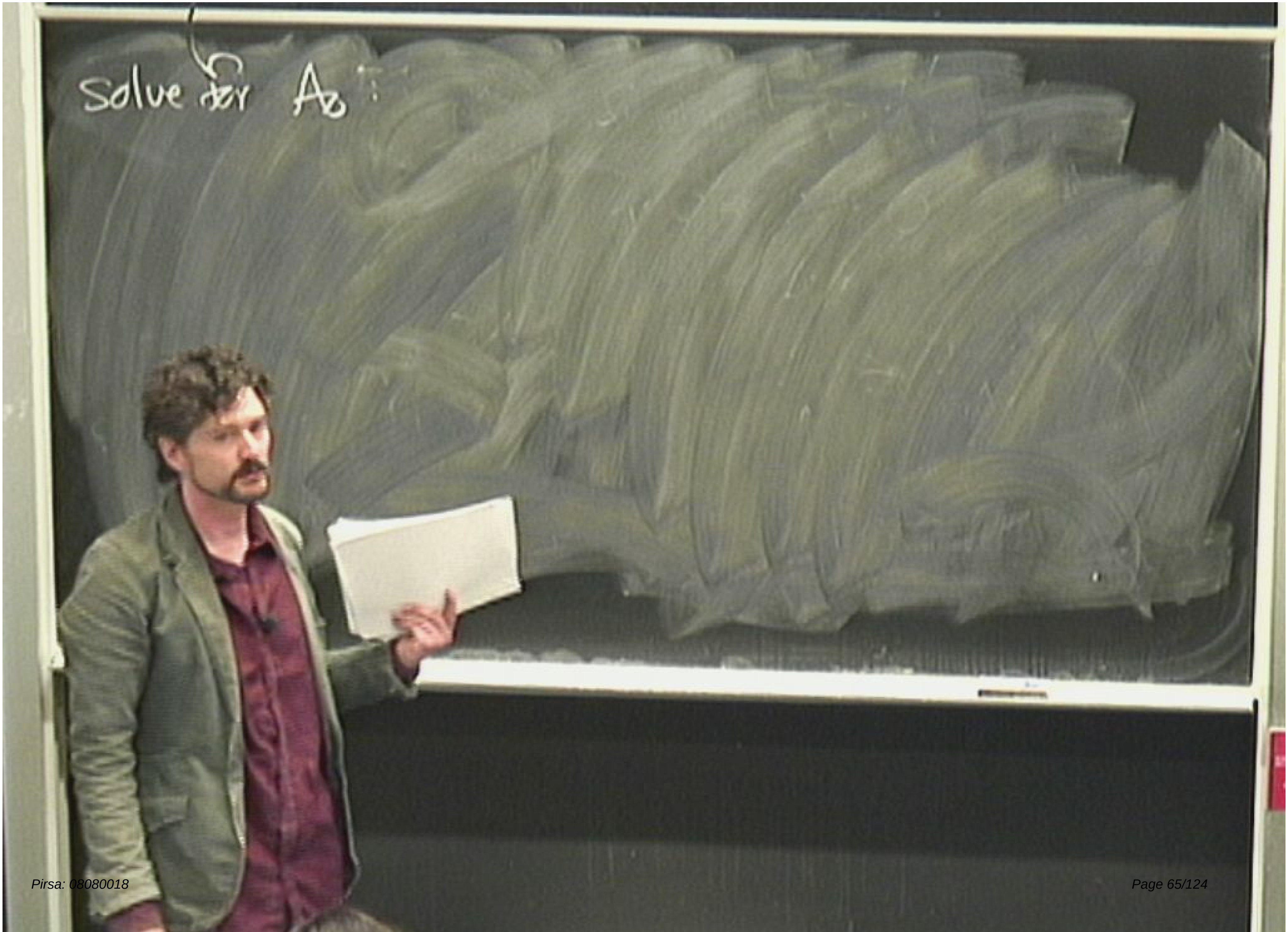

∞
Energy
∞
classical PE
$\frac{1}{2}kA^2$
quantum
KE
$(\ )\frac{1}{A^2}$
0
0
A

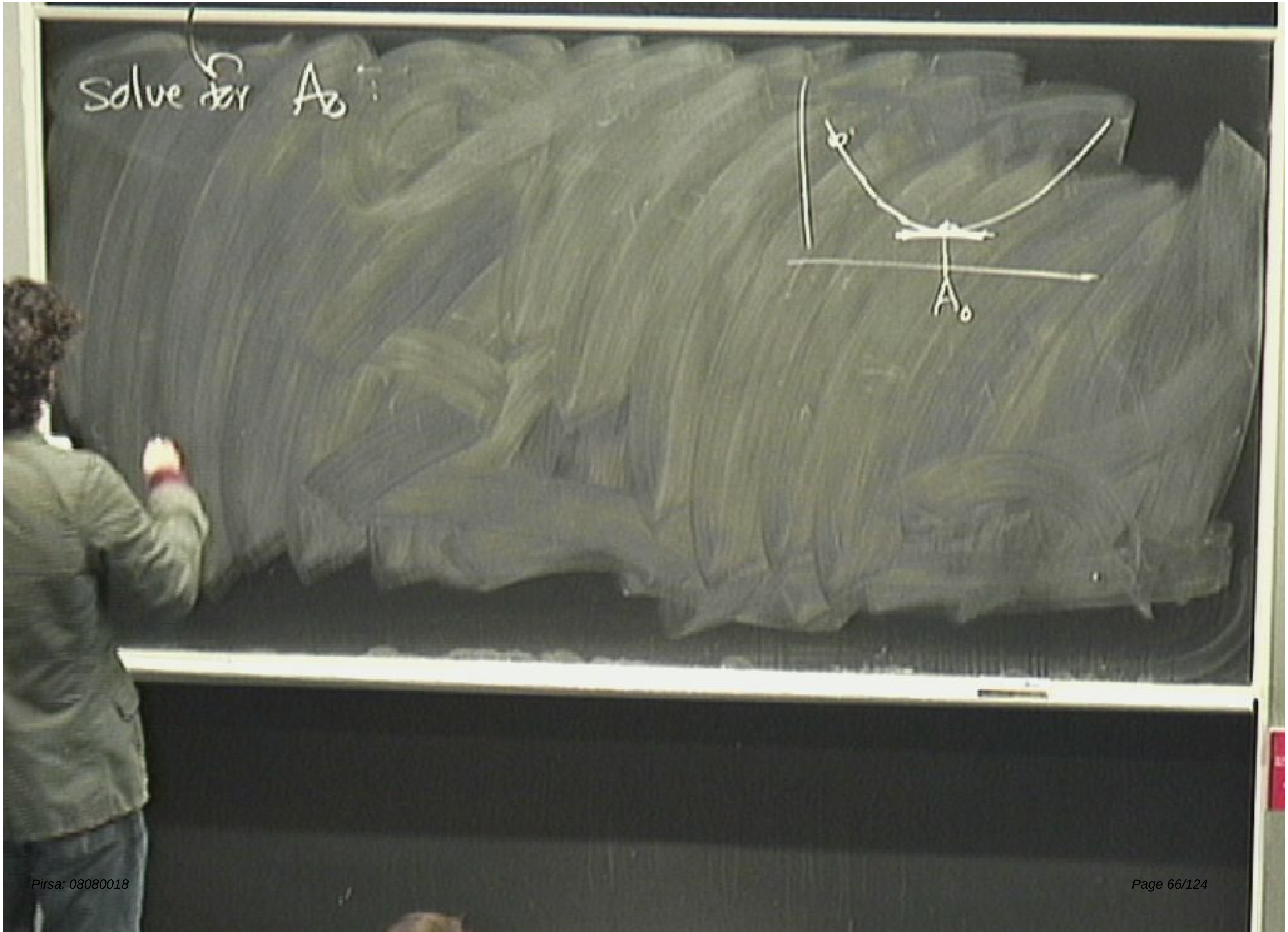

Energy
∞
∞
classical PE
$\frac{1}{2}kA^2$
quantum
KE
$()\frac{1}{A^2}$
0
A
1 m



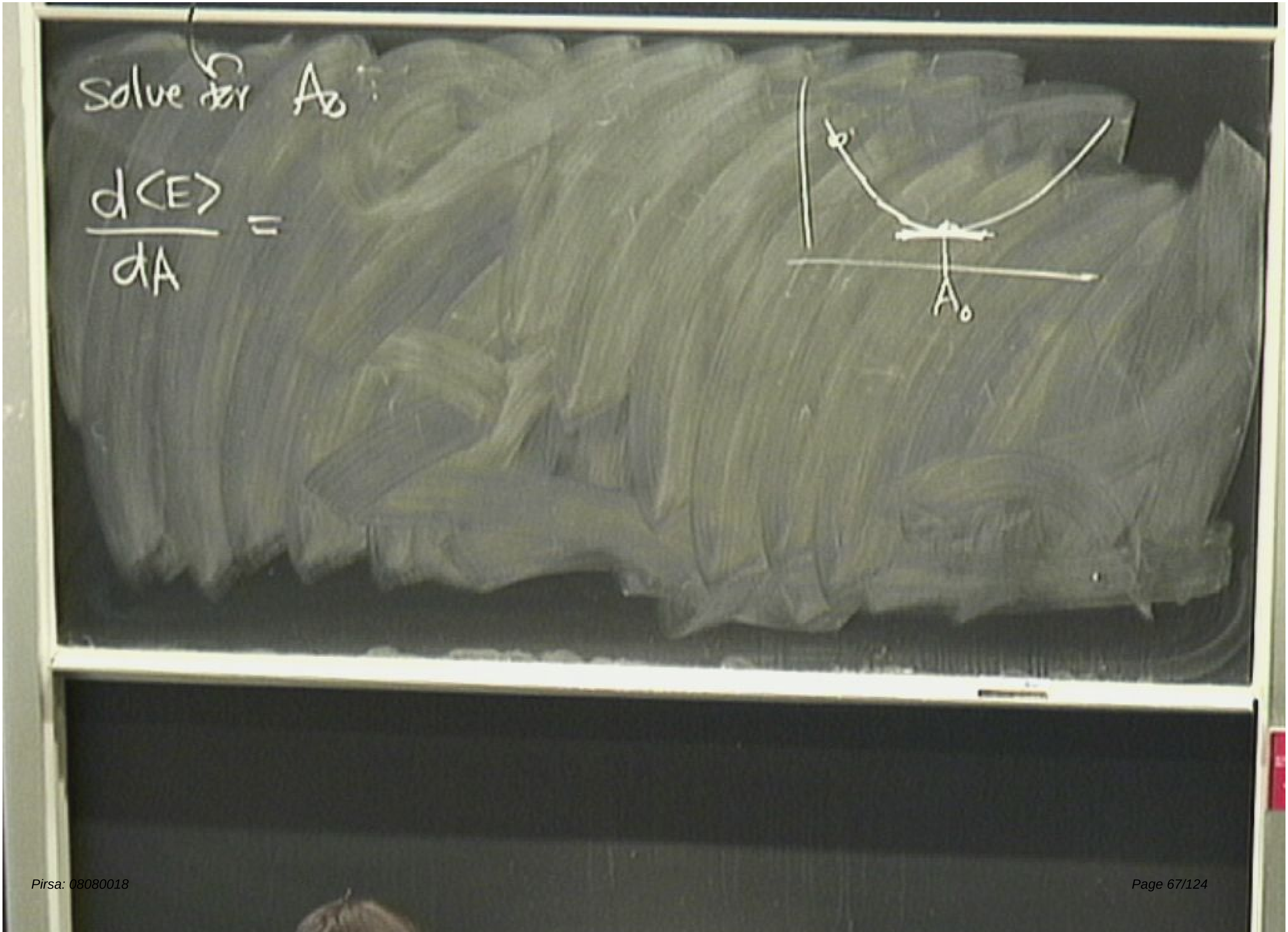
Energy
∞
∞
classical PE
$\frac{1}{2}kA^2$
0
A_0
A
1 m


Energy
∞
∞
classical PE
$\frac{1}{2}kA^2$
0
A_0
A
1m


Solve for A₀



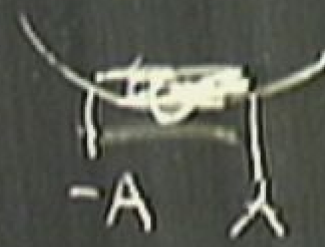
Solve for A_0
A_0


Solve for A_0:
$\frac{d\langle E\rangle}{dA} =$
A_0

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

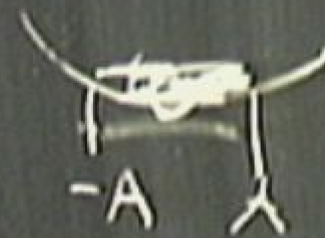
↑ classical PE



$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE

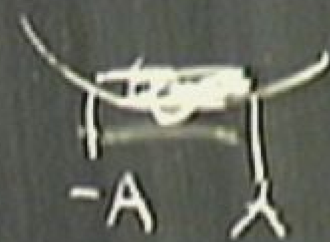


$$\frac{d}{dx}x^n =$$

$$\langle E \rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE

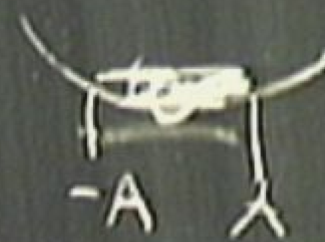


$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dA}(A^2) =$$

$$\langle E\rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

↑ classical PE



$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dA}(A^2) = 2A$$

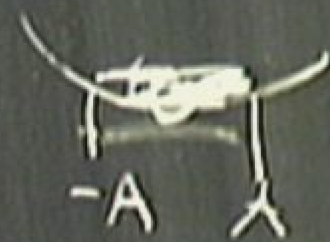
$$\frac{d}{dA}\left(\frac{1}{A^2}\right) =$$

$$\langle E\rangle = \left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^2} + \frac{1}{2}kA^2$$

↑ quantum KE

$\frac{1}{A^2} = A^{-2}$

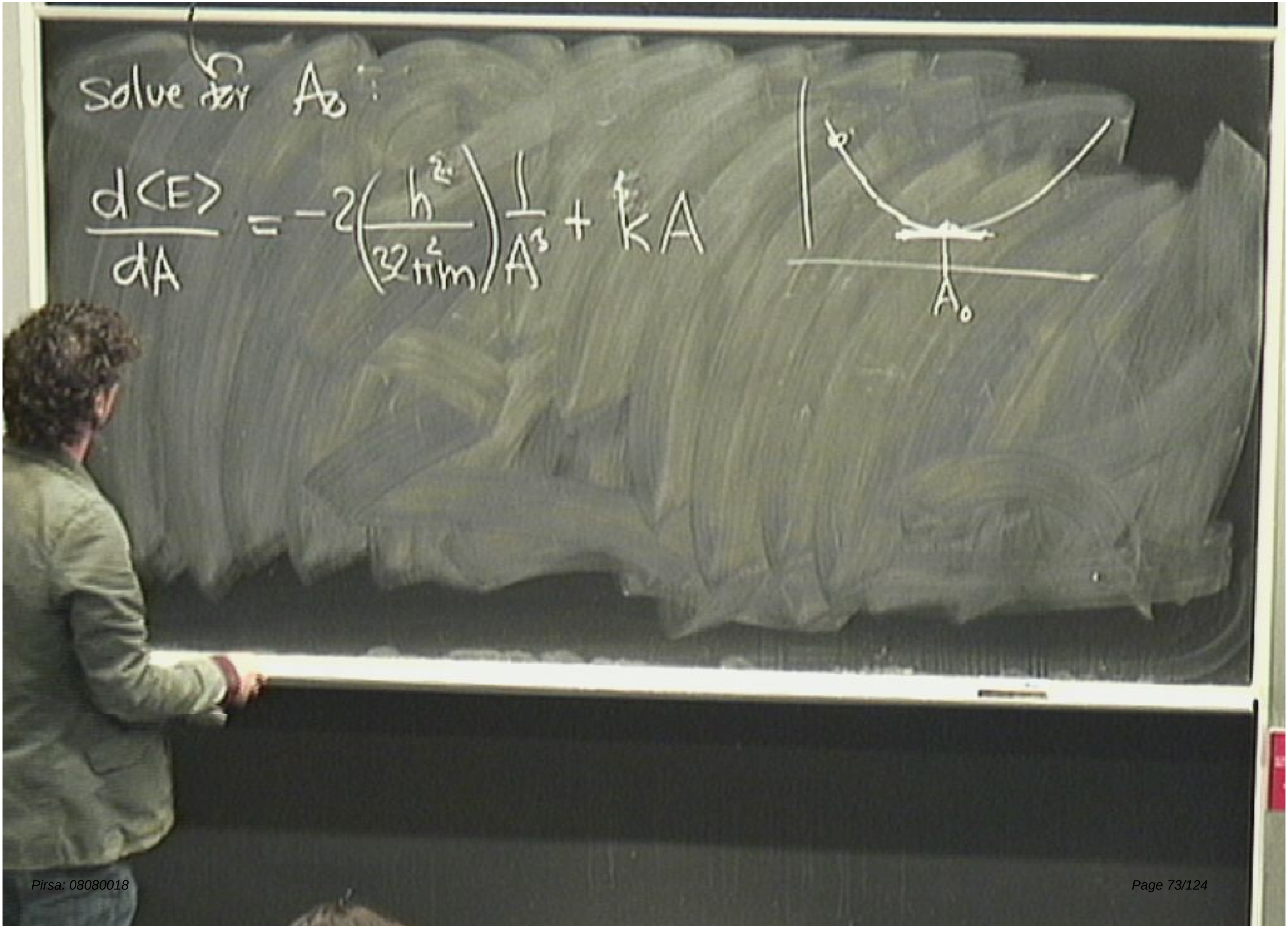
↑ classical PE

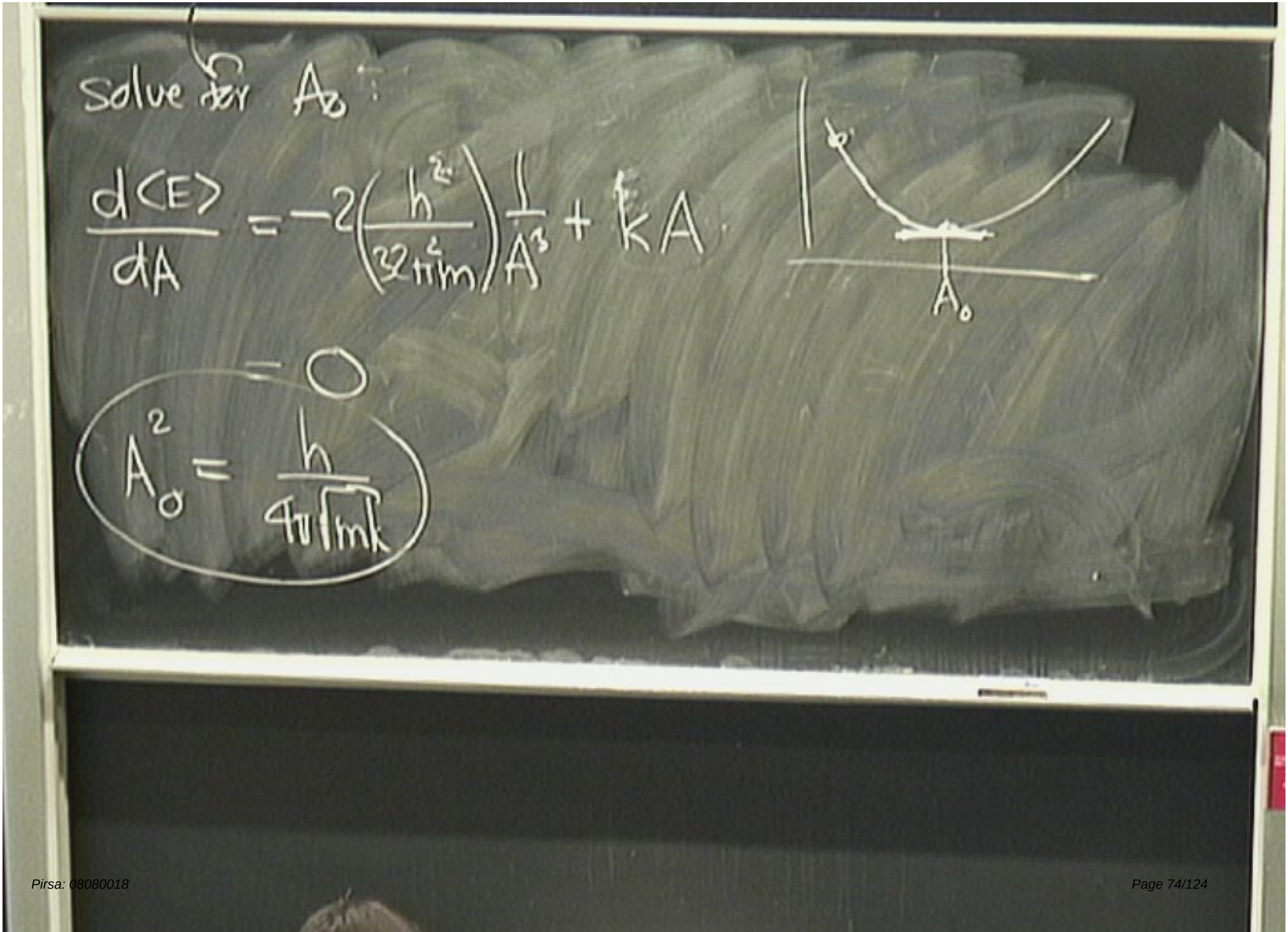


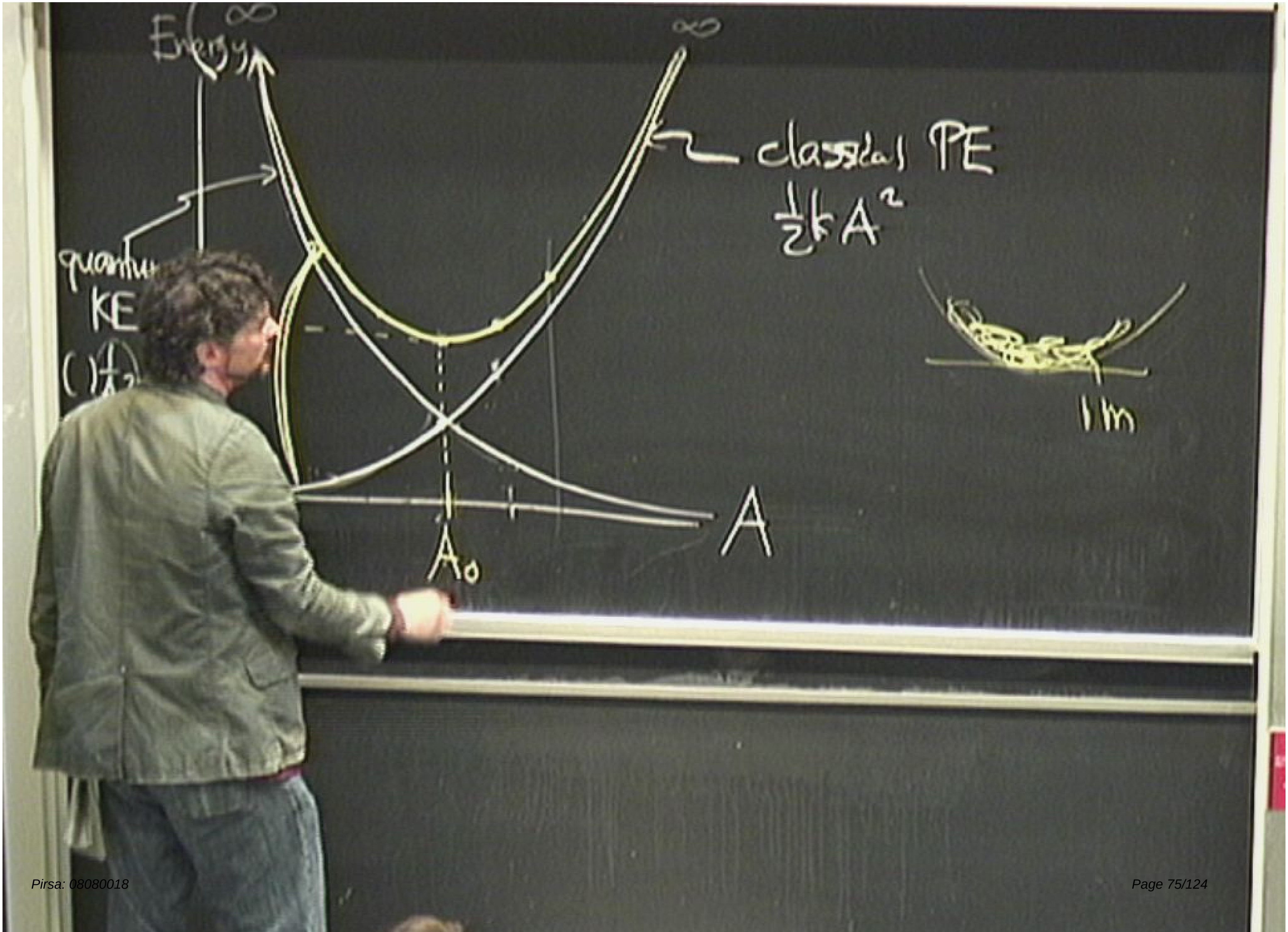
$$\frac{d}{dx}\left(x^{n}\right) = nx^{n-1}$$

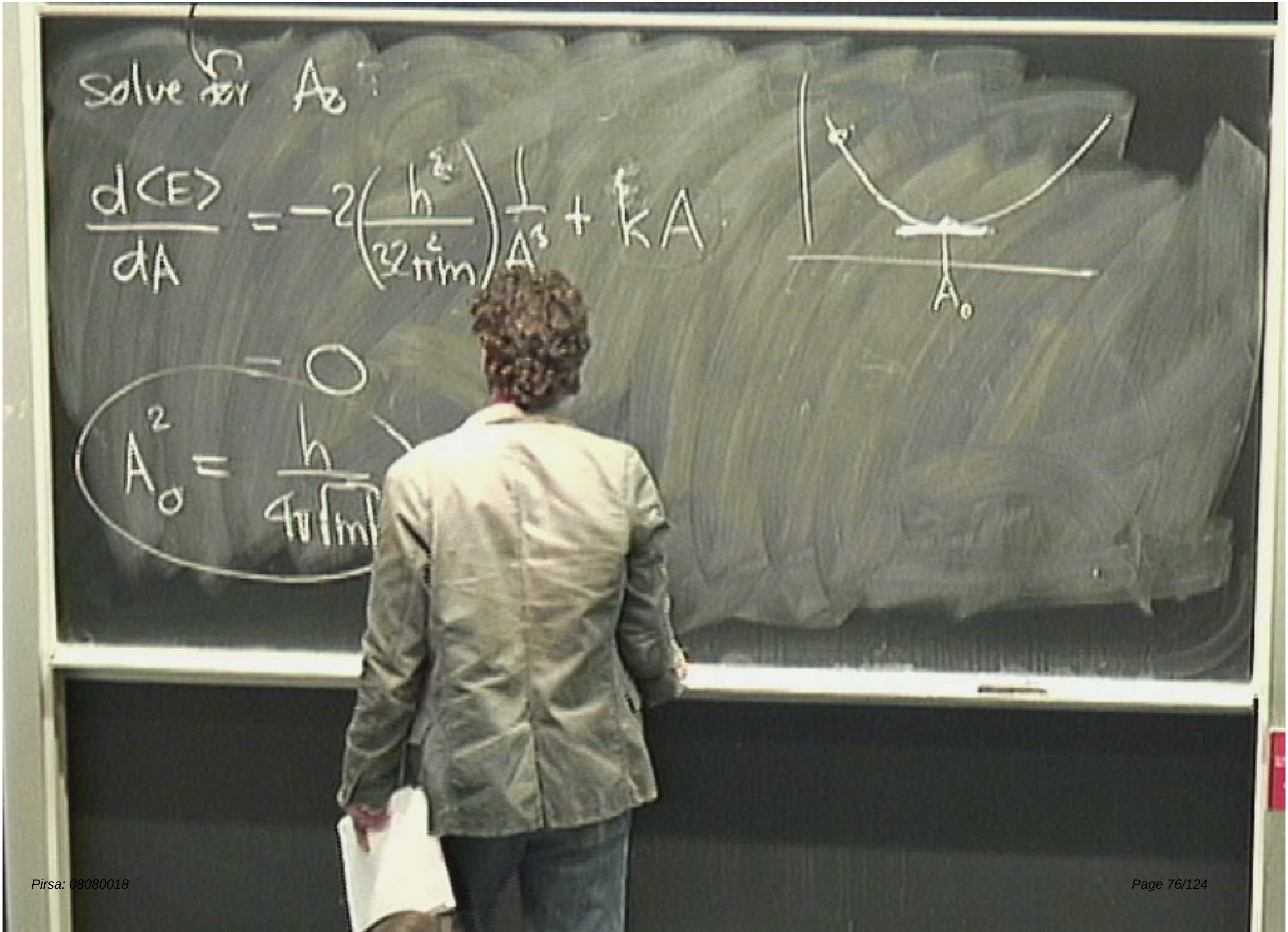
$$\frac{d}{dA}\left(A^2\right) = 2A$$

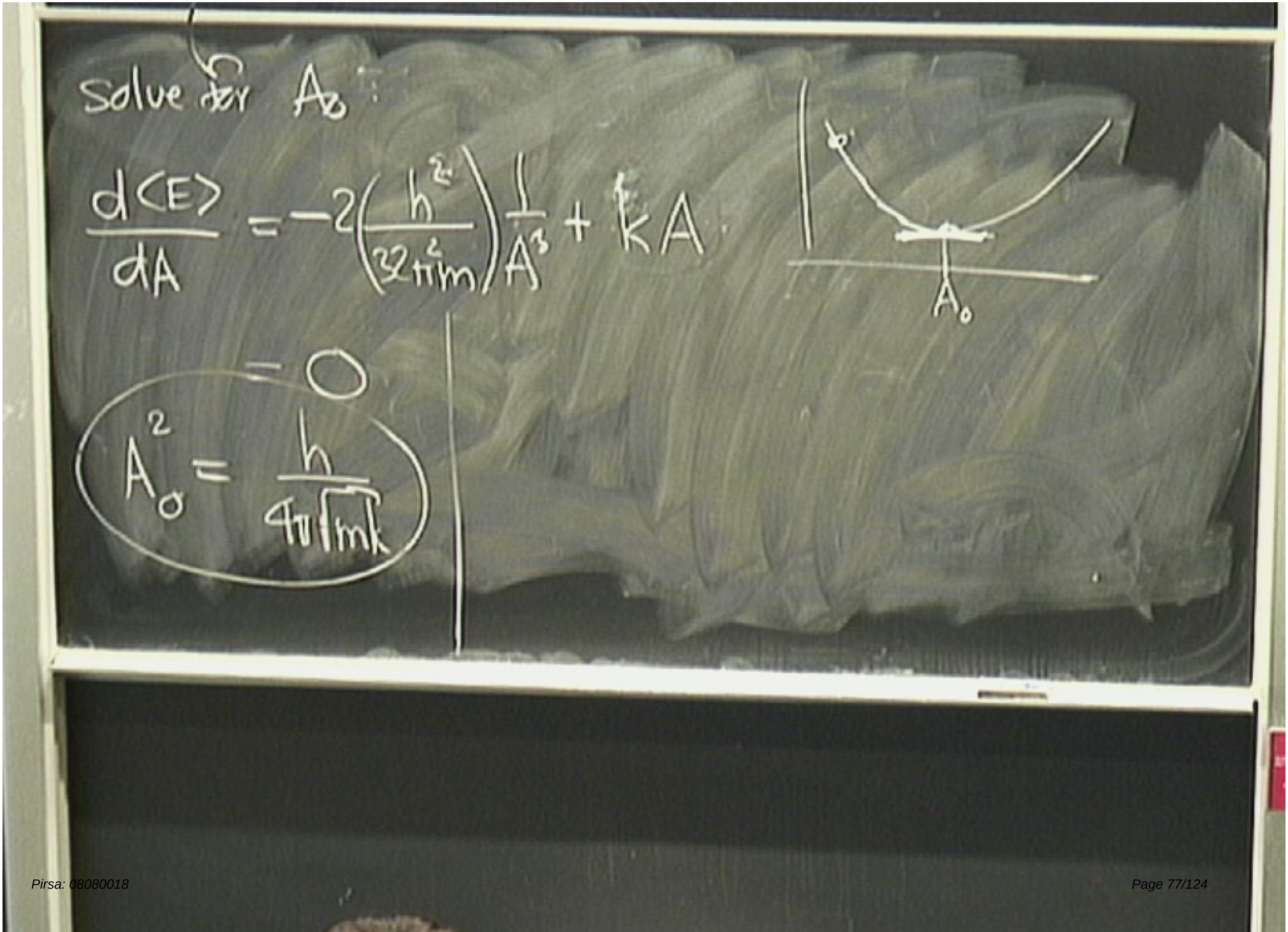
$$\frac{d}{dA}\left(\frac{1}{A^2}\right) = -2\frac{1}{A^3}$$

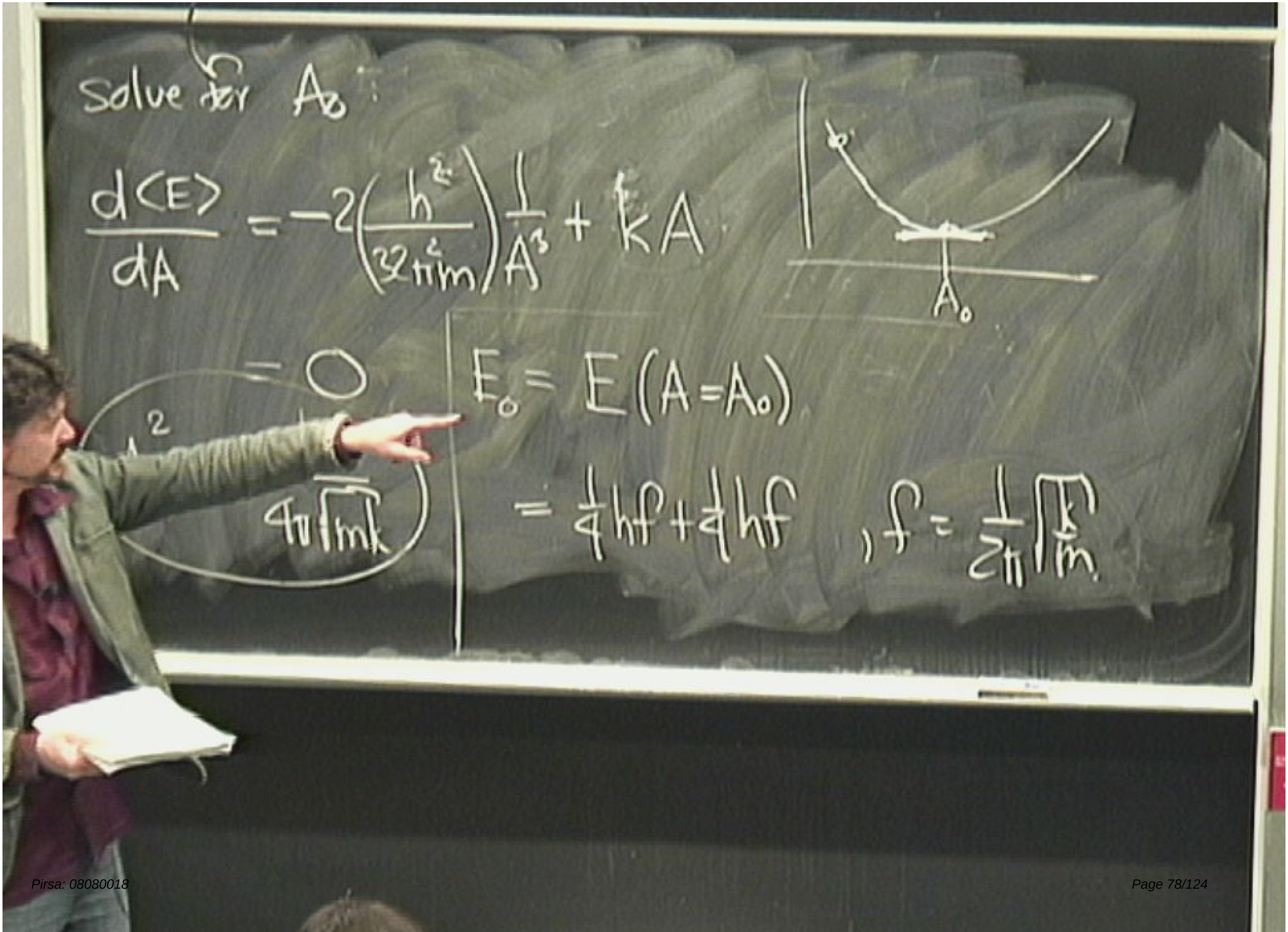

Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
A_0

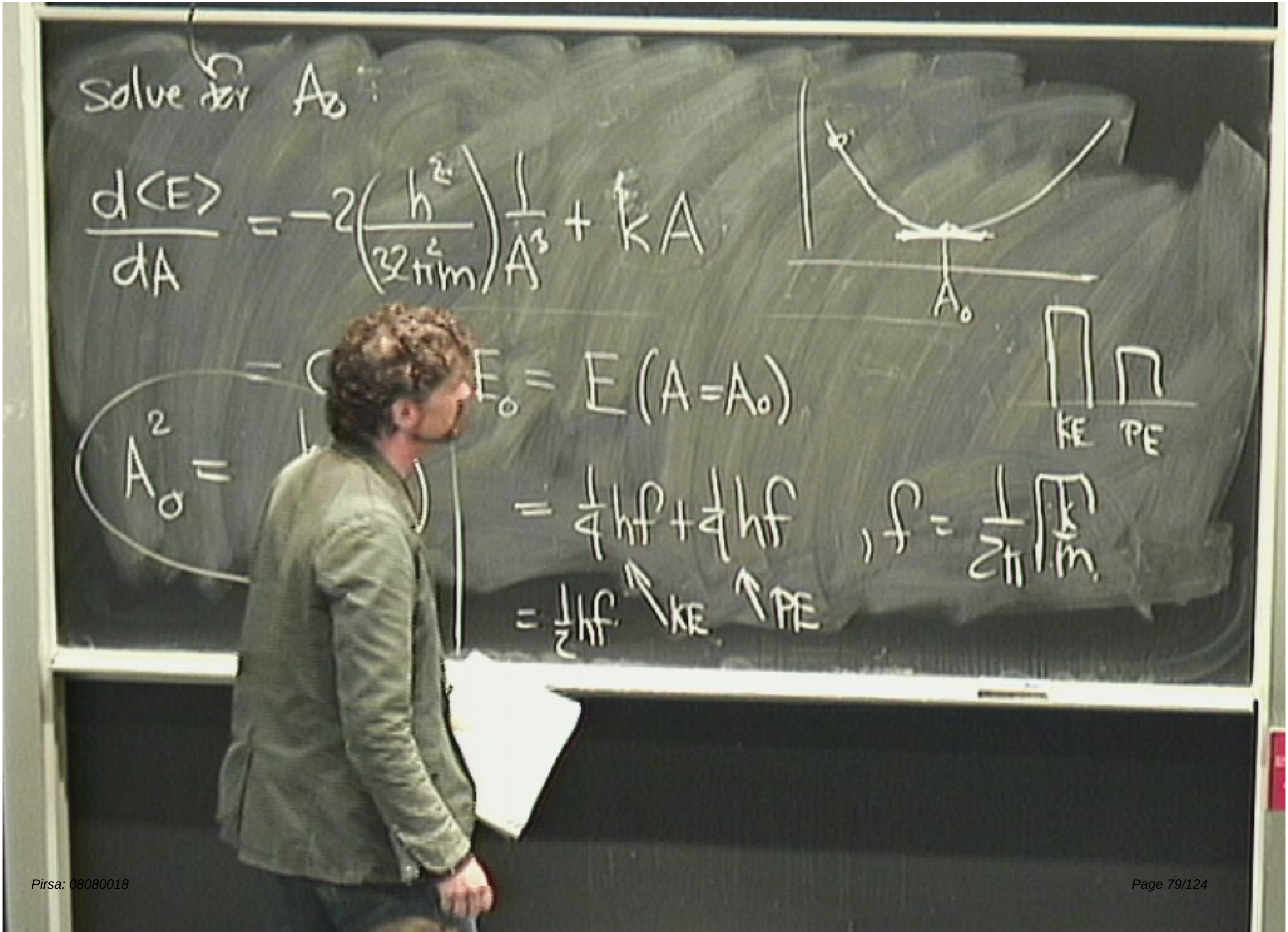

Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
$= 0$
$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$
A_0


Energy
∞
∞
classical PE
$\frac{1}{2}kA^2$
quantum
KE
A
A_0
1 m


Solve for A₀:
d⟨E⟩/dA = −2(h²/(32π²m)) 1/A³ + kA = 0
A₀² = h/(4π√m
A₀


Solve for A_0:
$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$
$= 0$
$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$
A_0


Solve for A0 :
d<E> / dA = −2 (h² / 32π²m) 1/A³ + kA
= 0
2
4π √mk
A0
E0 = E(A=A0)
= ¼ hf + ¼ hf , f = 1/2π √(k/m)


Solve for A₀:
d⟨E⟩/dA = −2(h²/(32π²m)) 1/A³ + kA
A₀
E₀ = E(A=A₀)
KE PE
= ¼hf + ¼hf , f = (1/2π)√(k/m)
KE PE
= ½hf

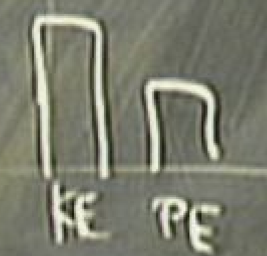
Solve for A_0:

$$\frac{d\langle E\rangle}{dA} = -2\left(\frac{h^2}{32\pi^2 m}\right)\frac{1}{A^3} + kA$$

$$= 0$$

$$A_0^2 = \frac{h}{4\pi\sqrt{mk}}$$

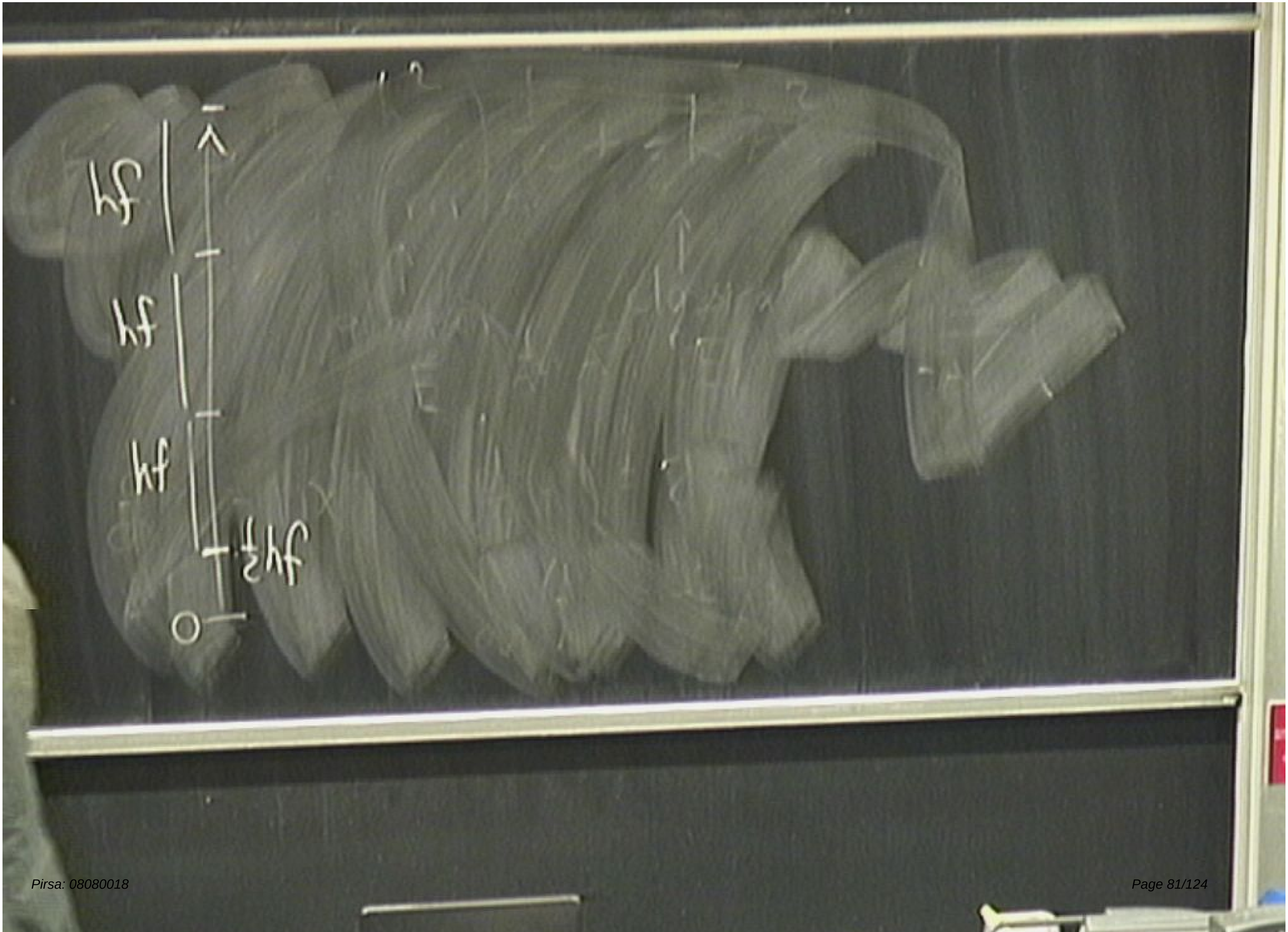
$$E_0 = E(A=A_0)$$

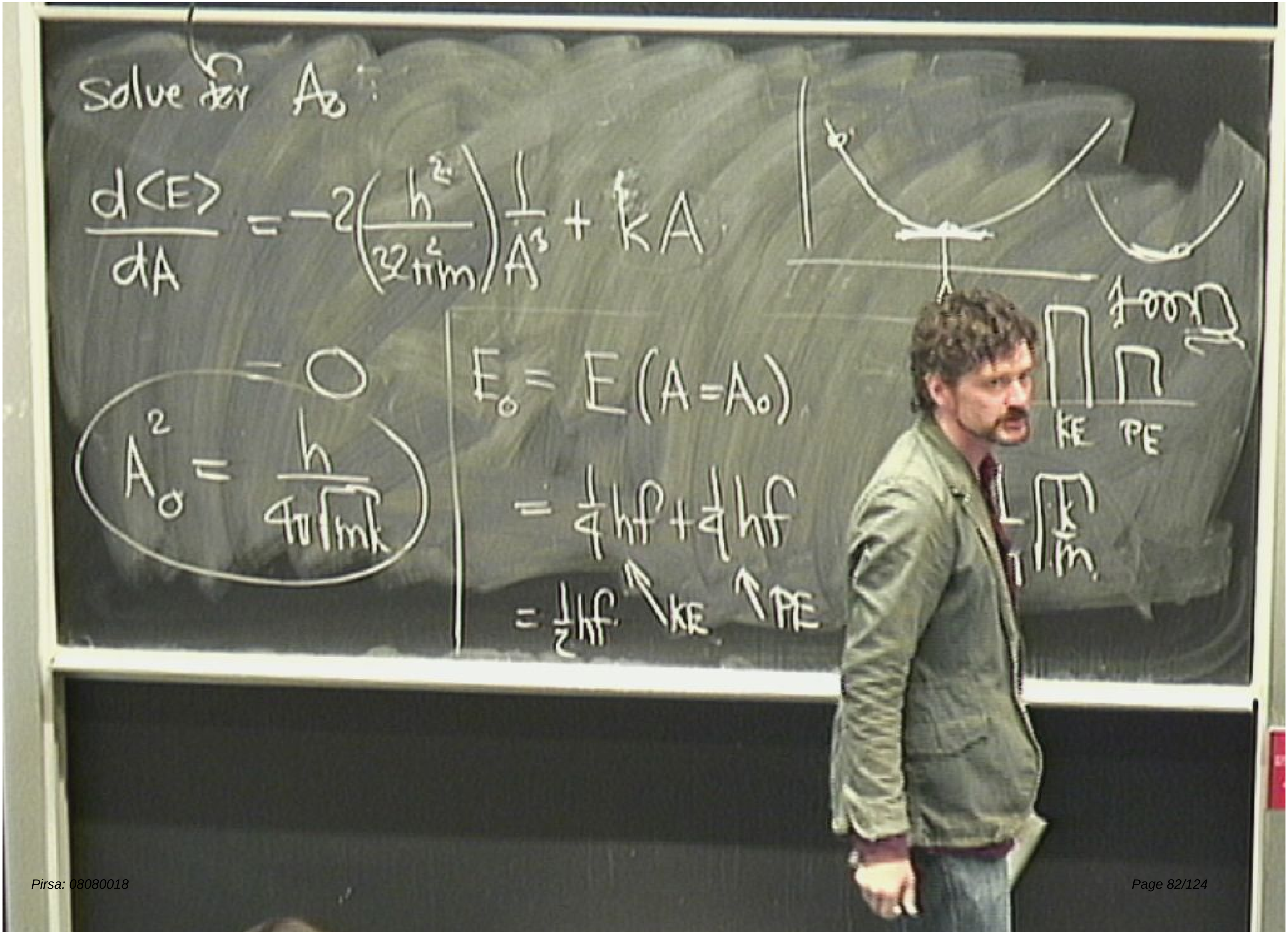


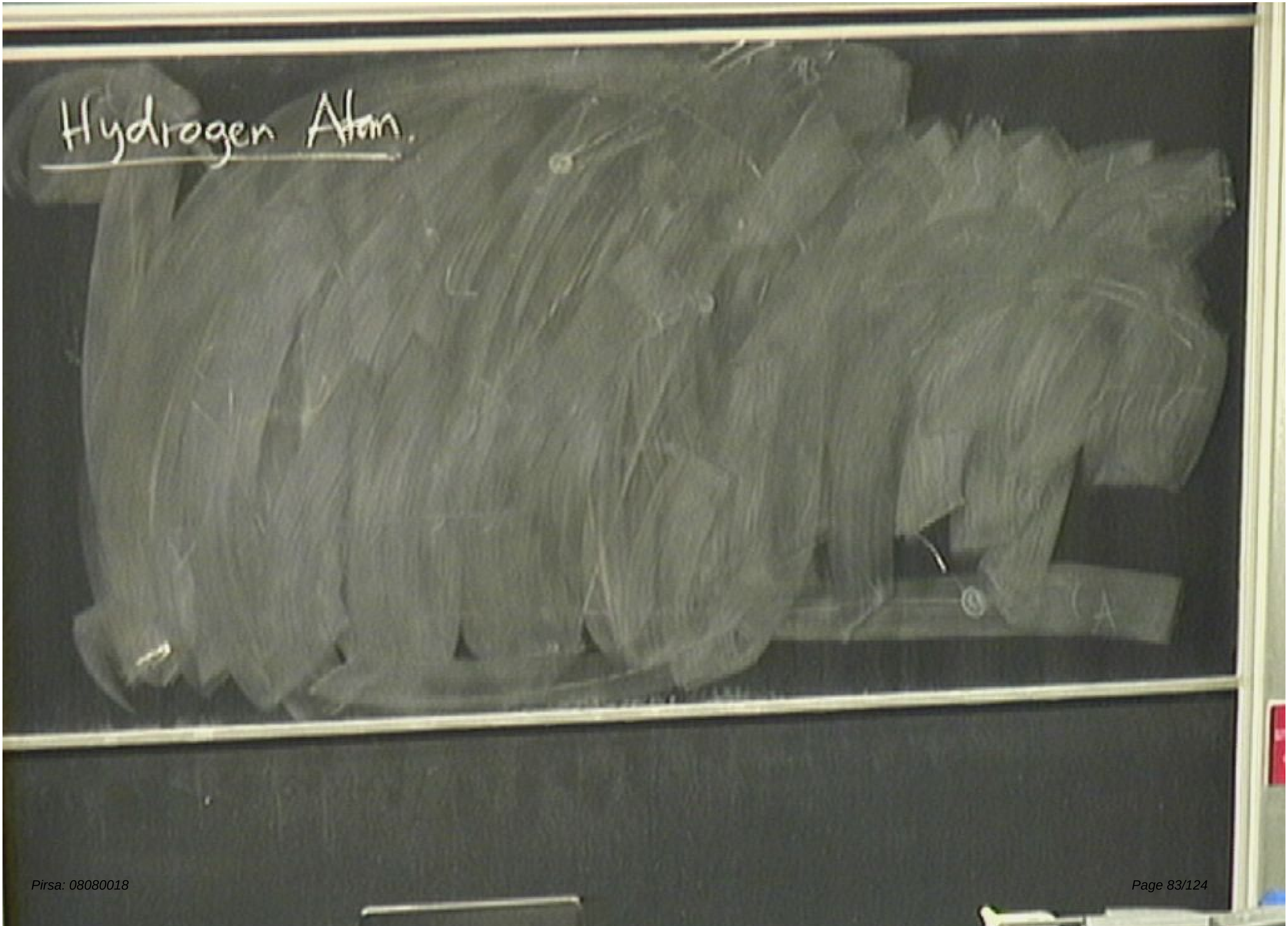
$$= \frac{1}{4}hf + \frac{1}{4}hf \quad , \; f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

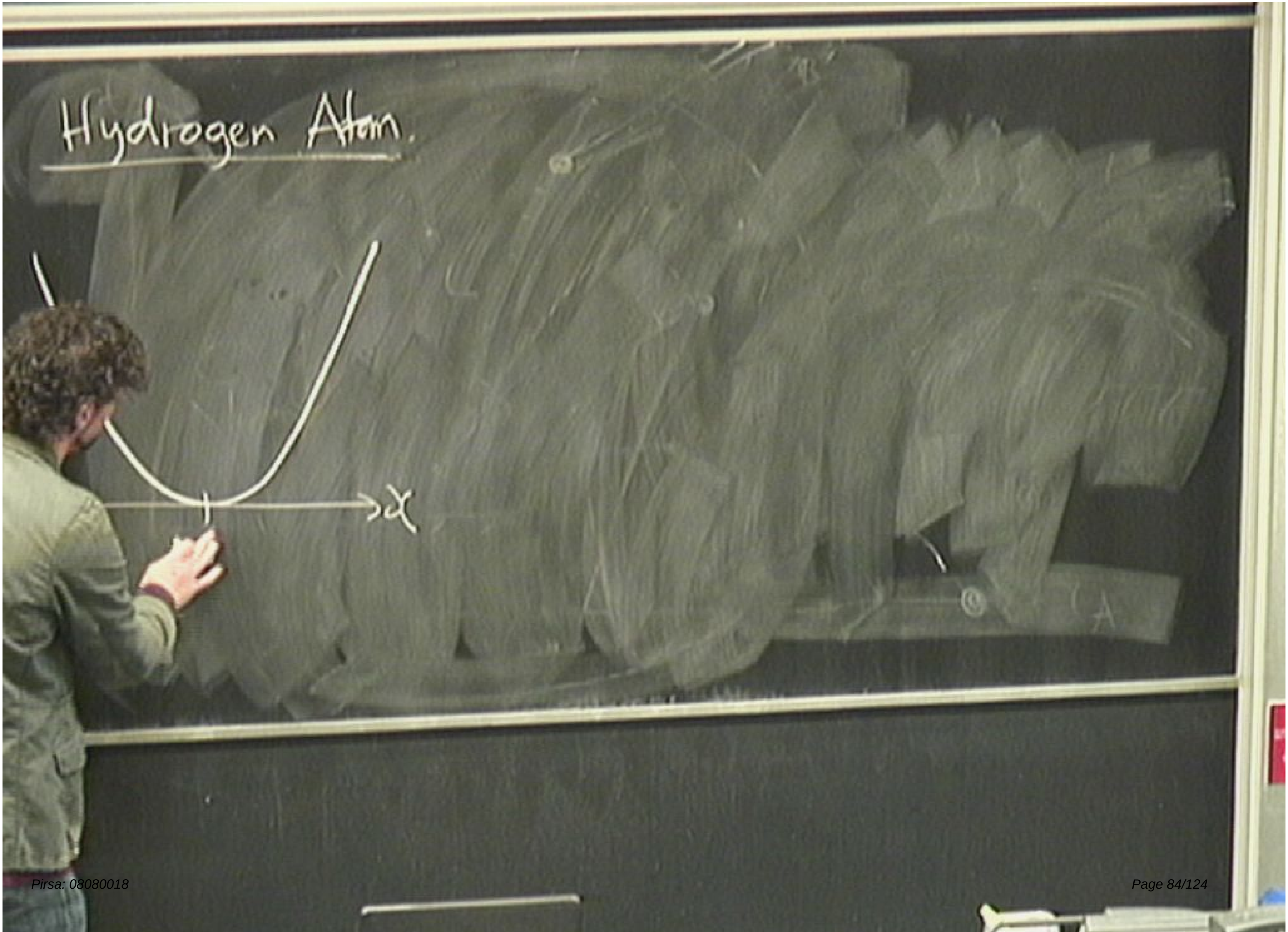
↑ KE ↑ PE

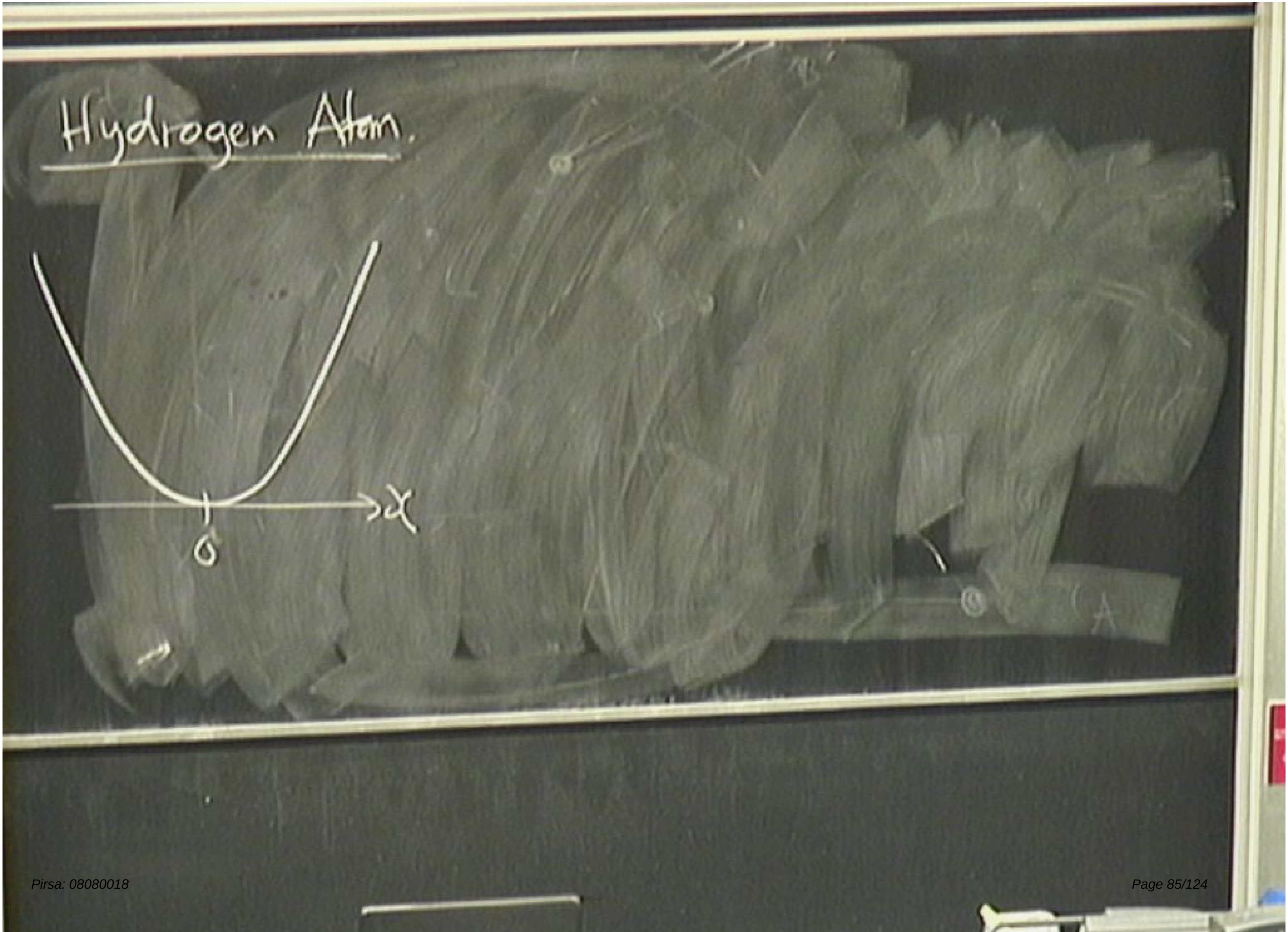
$$= \frac{1}{2}hf$$

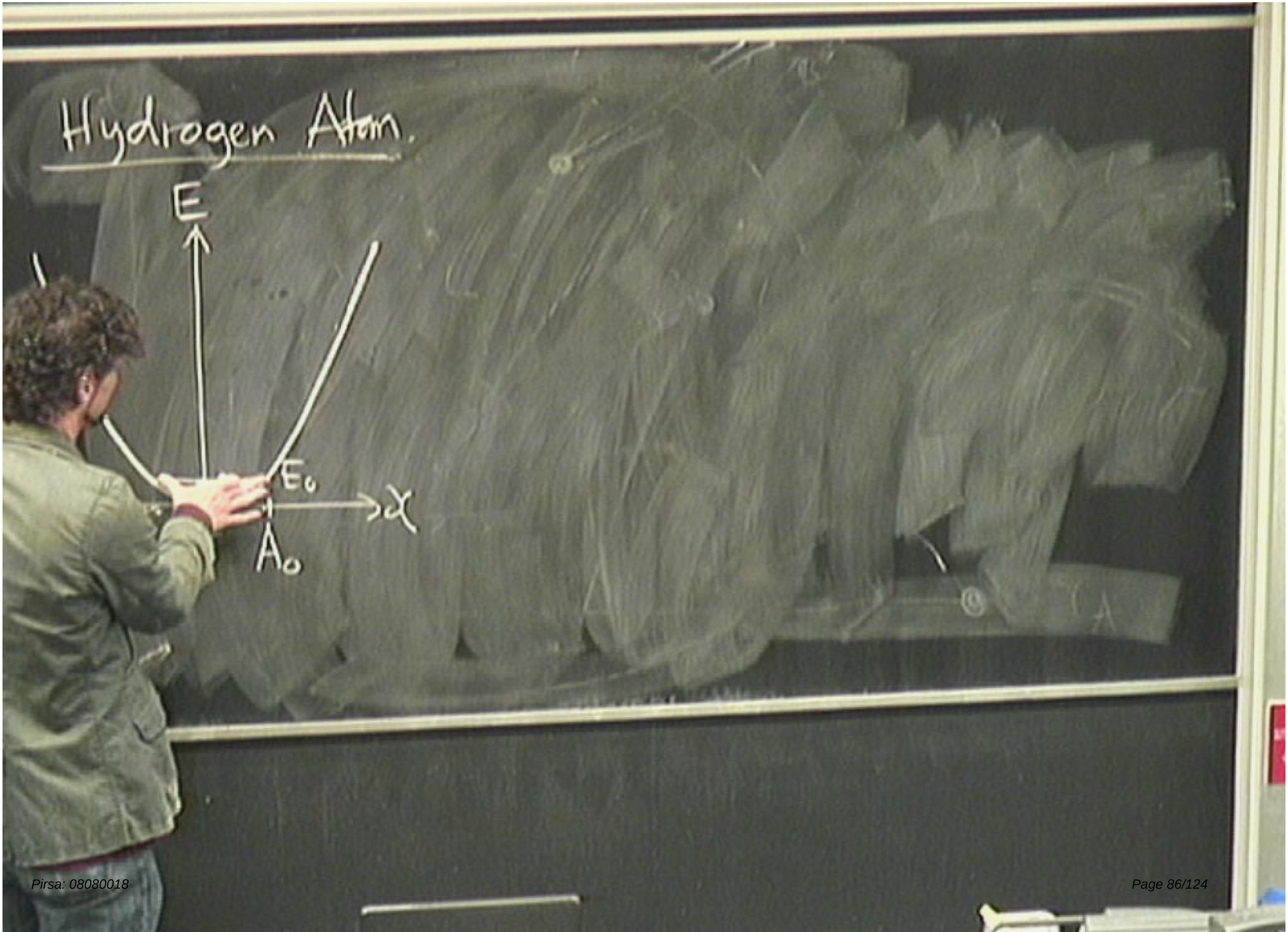

hf
hf
hf
½hf
0

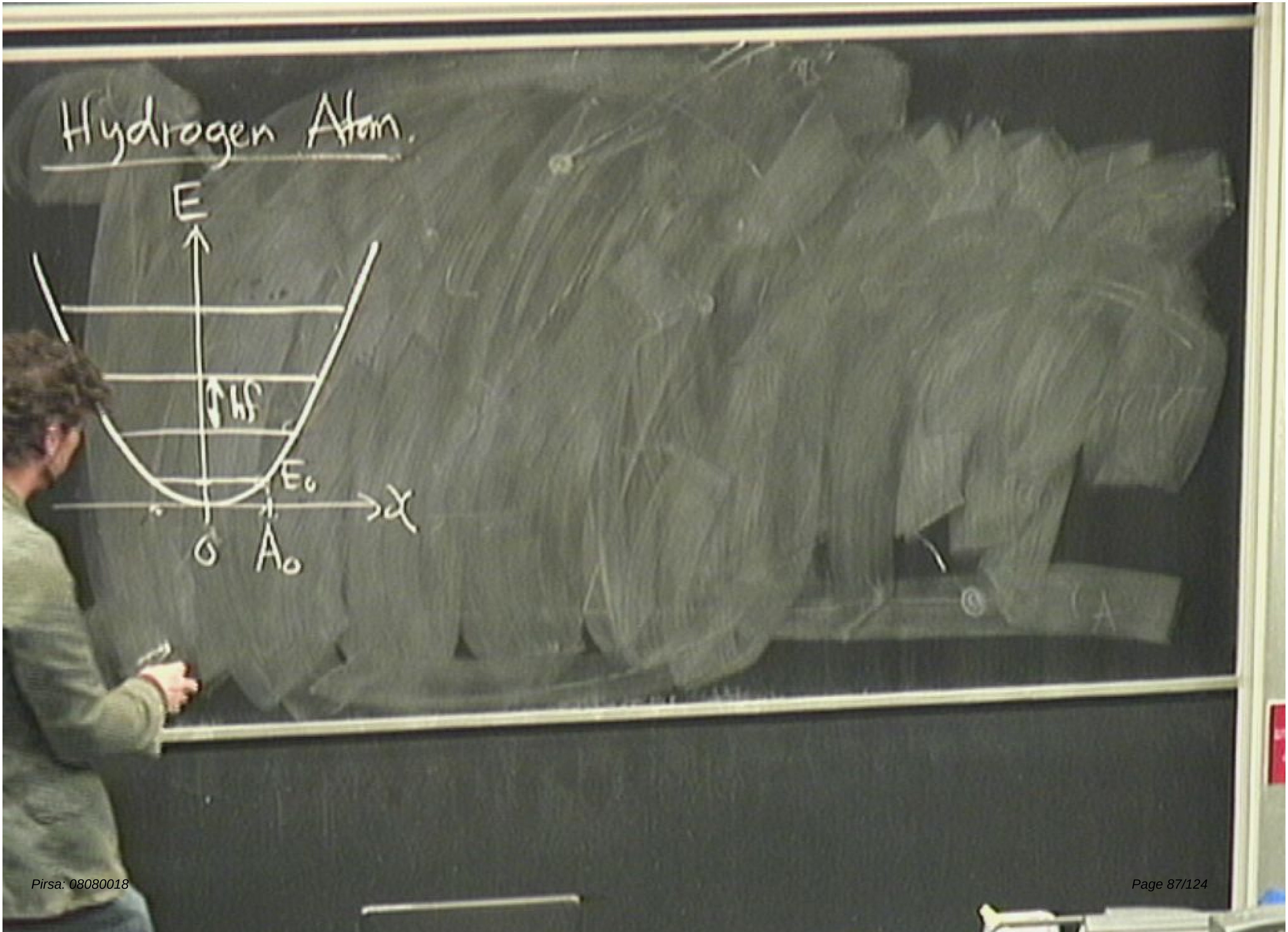

Solve for A0:
d<E>/dA = −2(h²/(32π²m)) 1/A³ + kA
= 0
A0² = h/(4π√(mk))
E0 = E(A=A0)
= ¼hf + ¼hf
↖KE ↖PE
= ½hf
KE PE

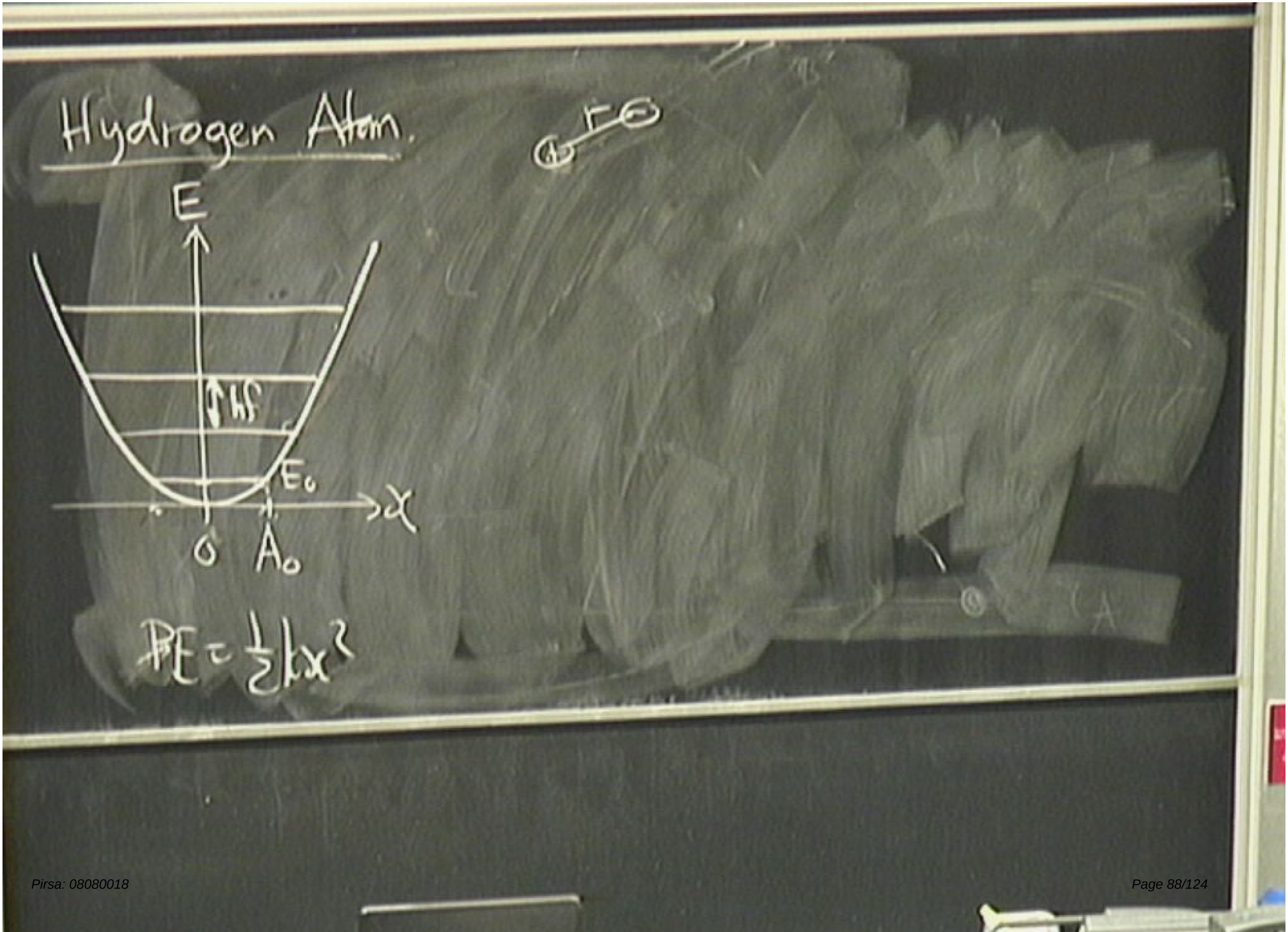

Hydrogen Atom.

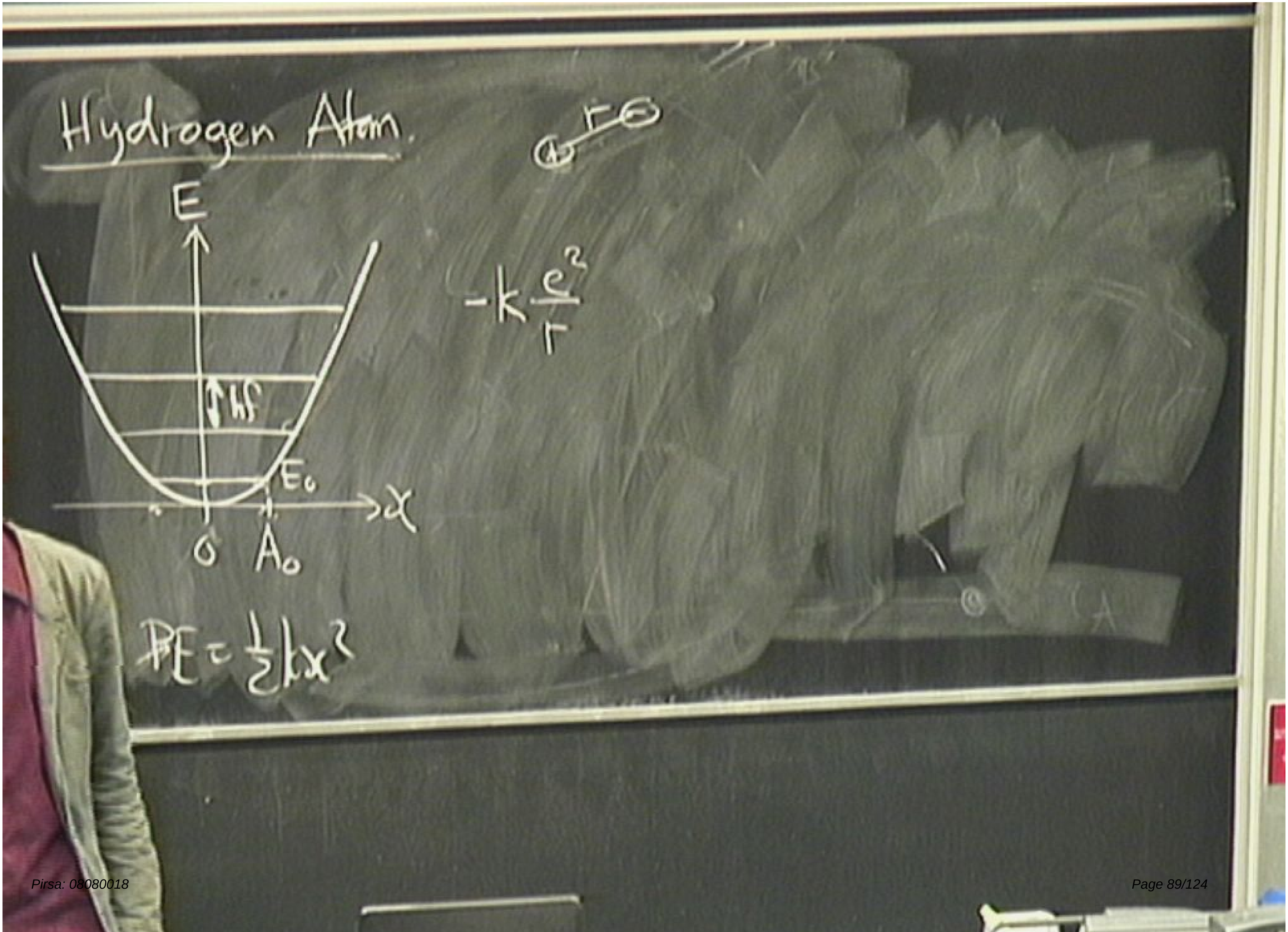

Hydrogen Atom.
x

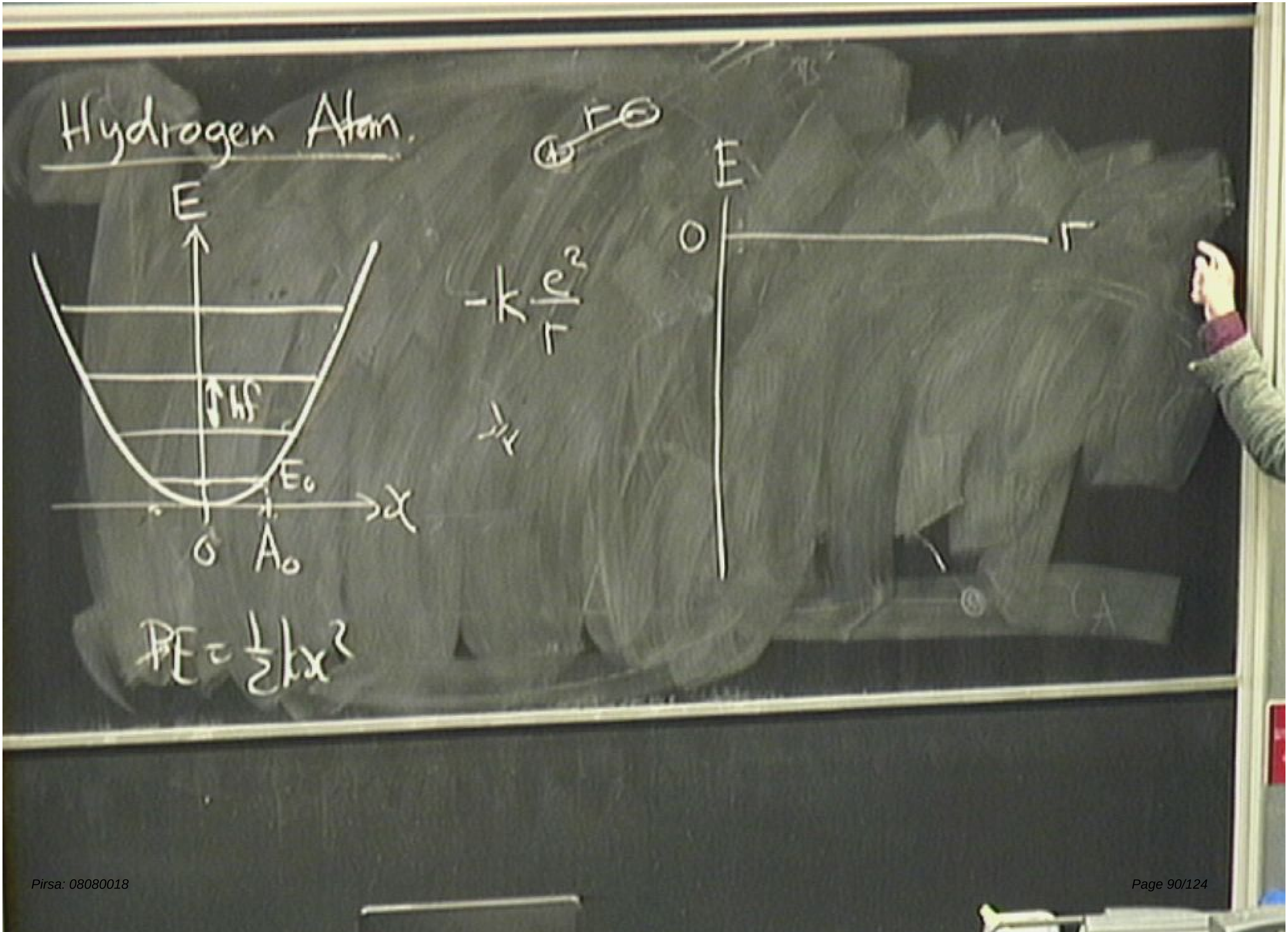

Hydrogen Atom.
x
0

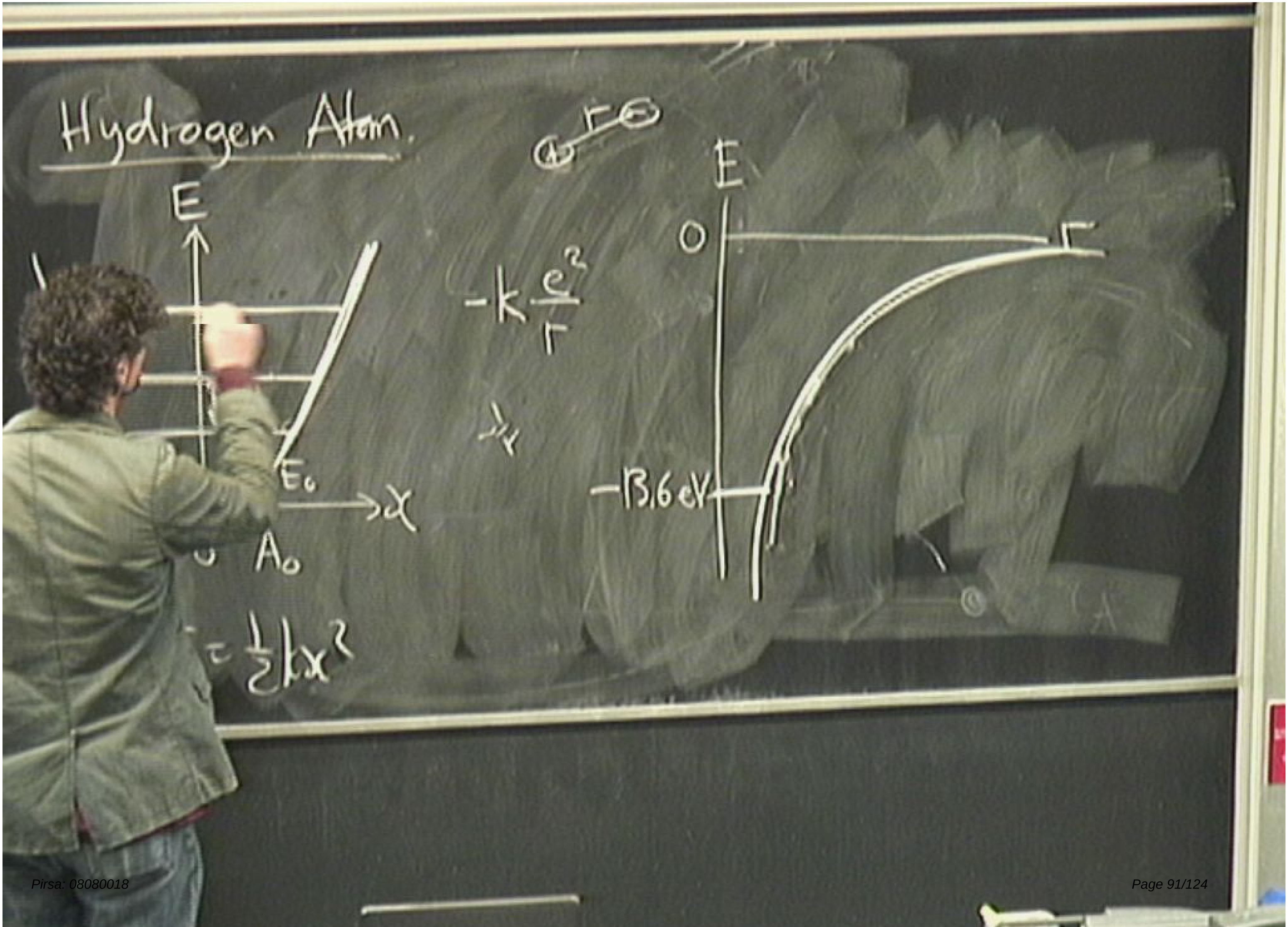

Hydrogen Atom.
E
E₀
x
A₀

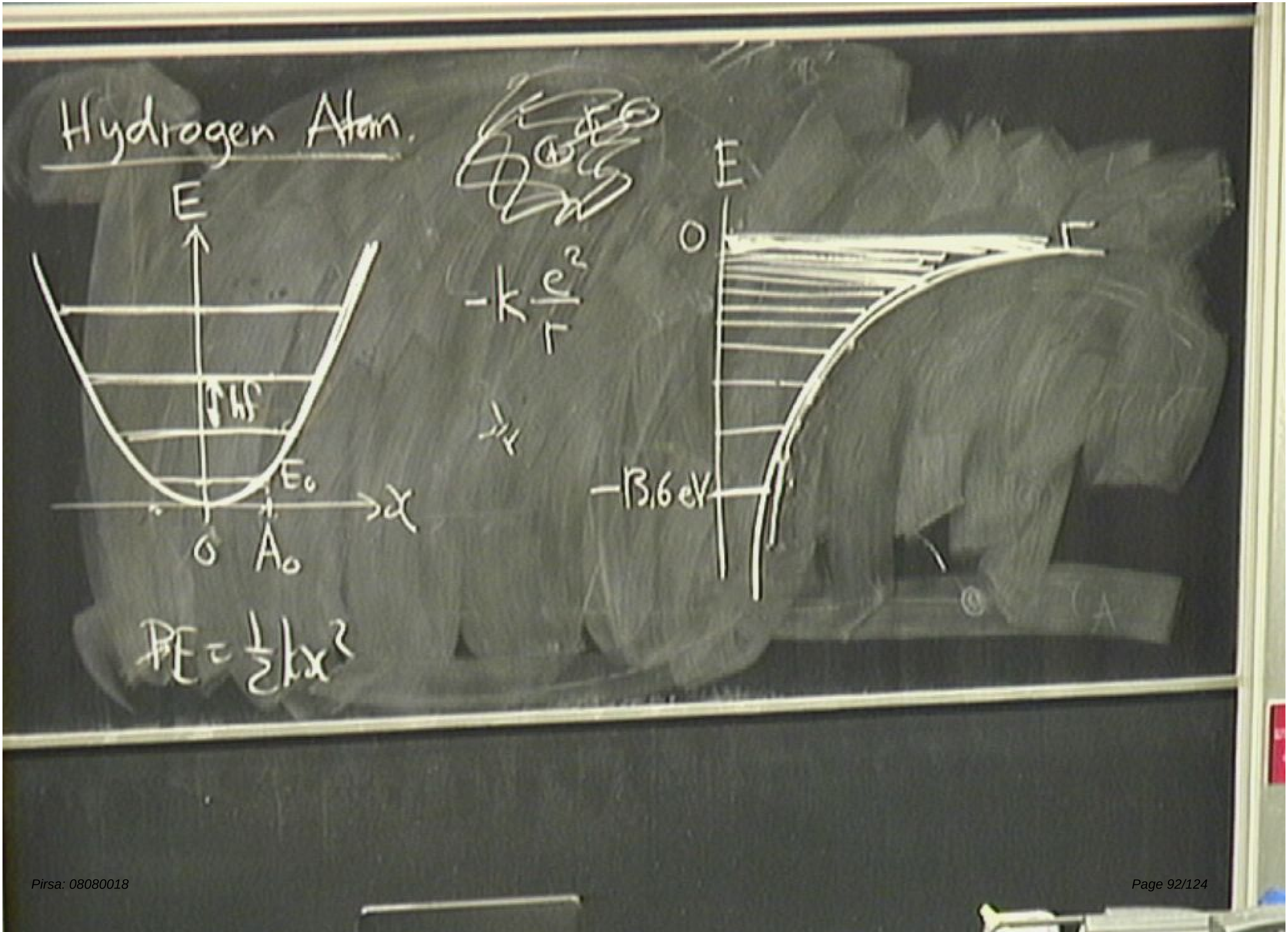

Hydrogen Atom.
E
hf
E0
X
0
A0

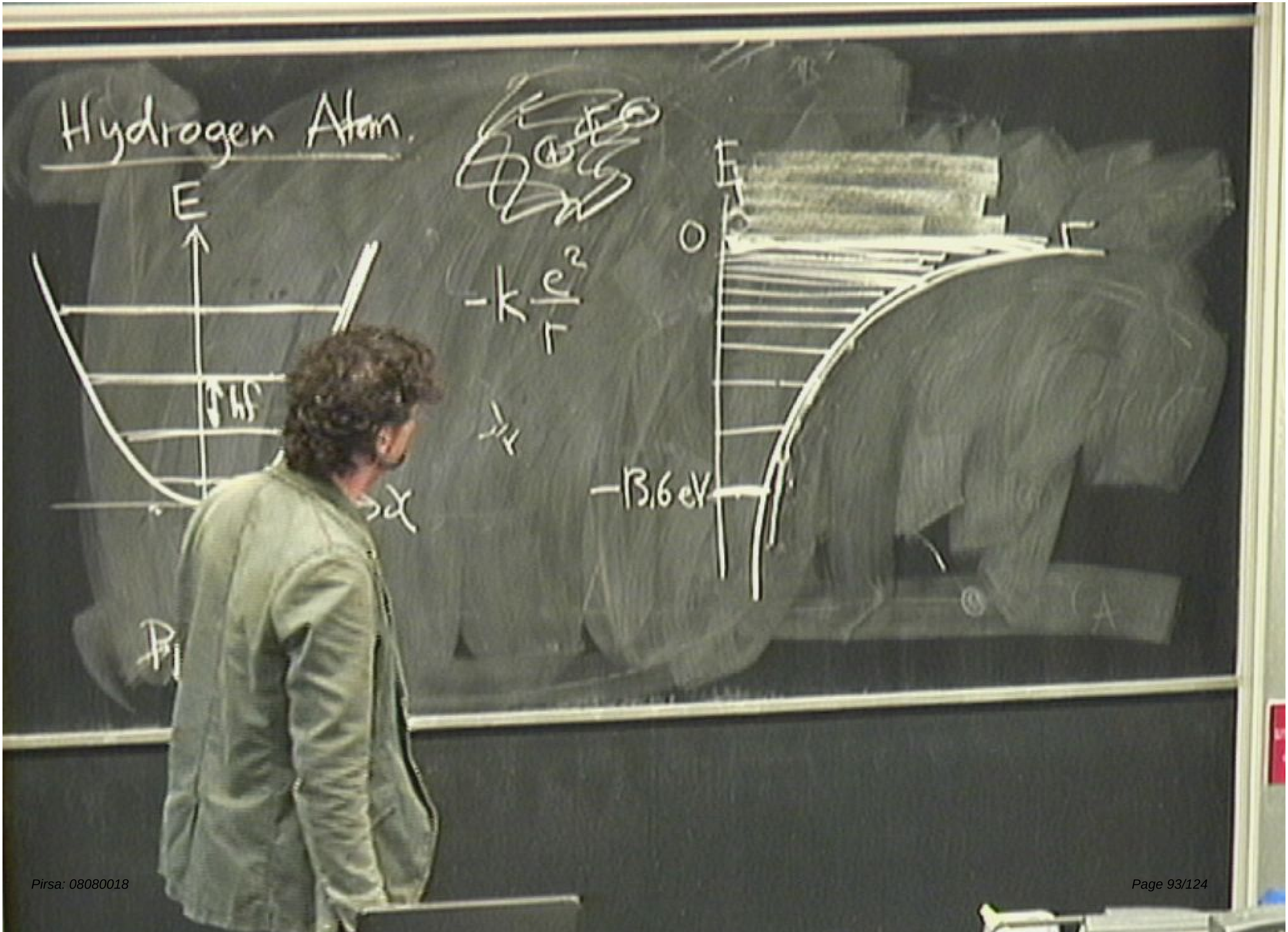

Hydrogen Atom.
E
hf
E₀
x
0
A₀
PE = ½kx²


Hydrogen Atom
E
hf
E0
x
0
A0
PE = ½kx²
r
-k e²/r

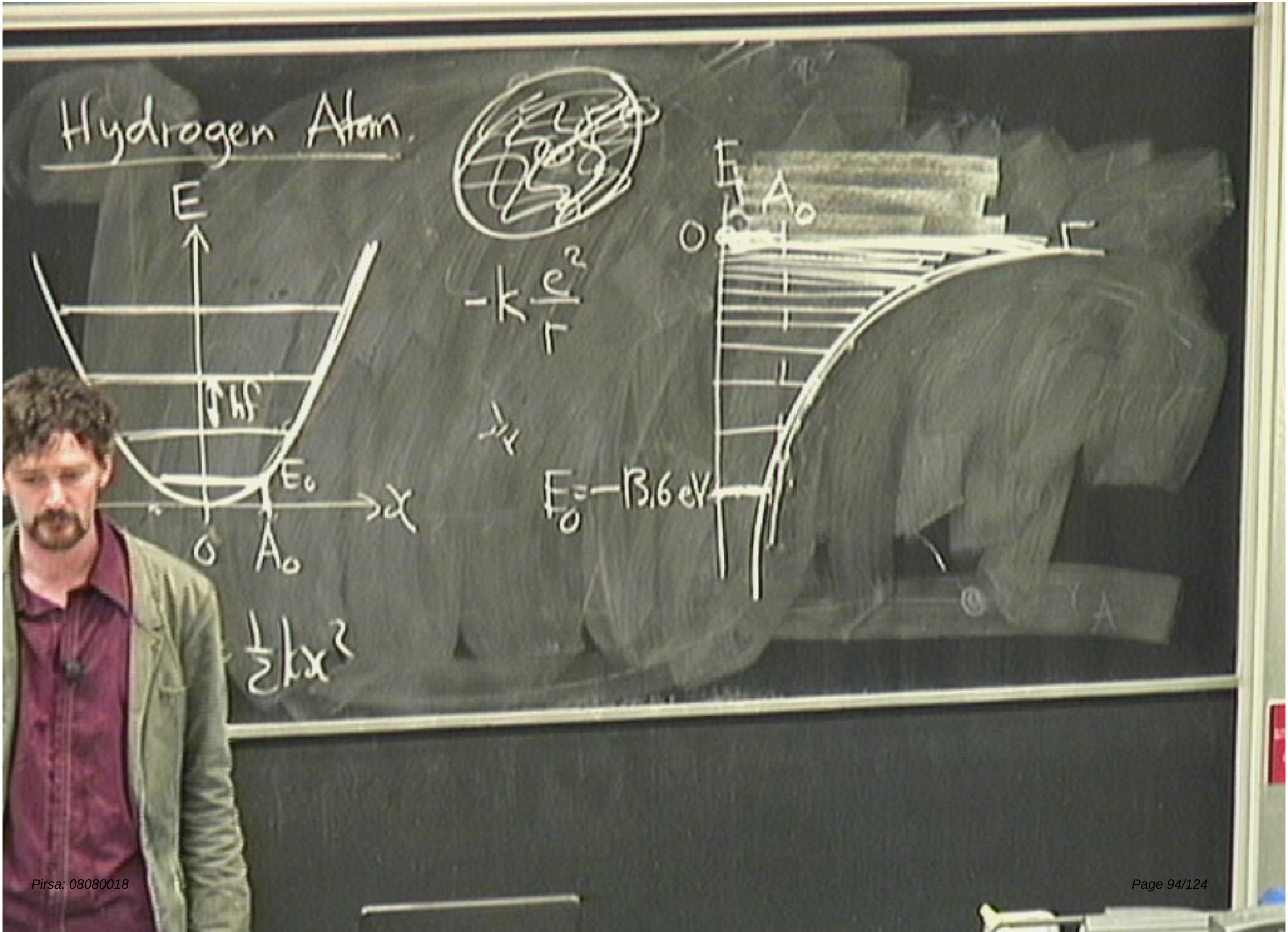

Hydrogen Atom
E
hf
E0
x
0
A0
PE = 1/2 kx2
-k e2/r
E
0
r

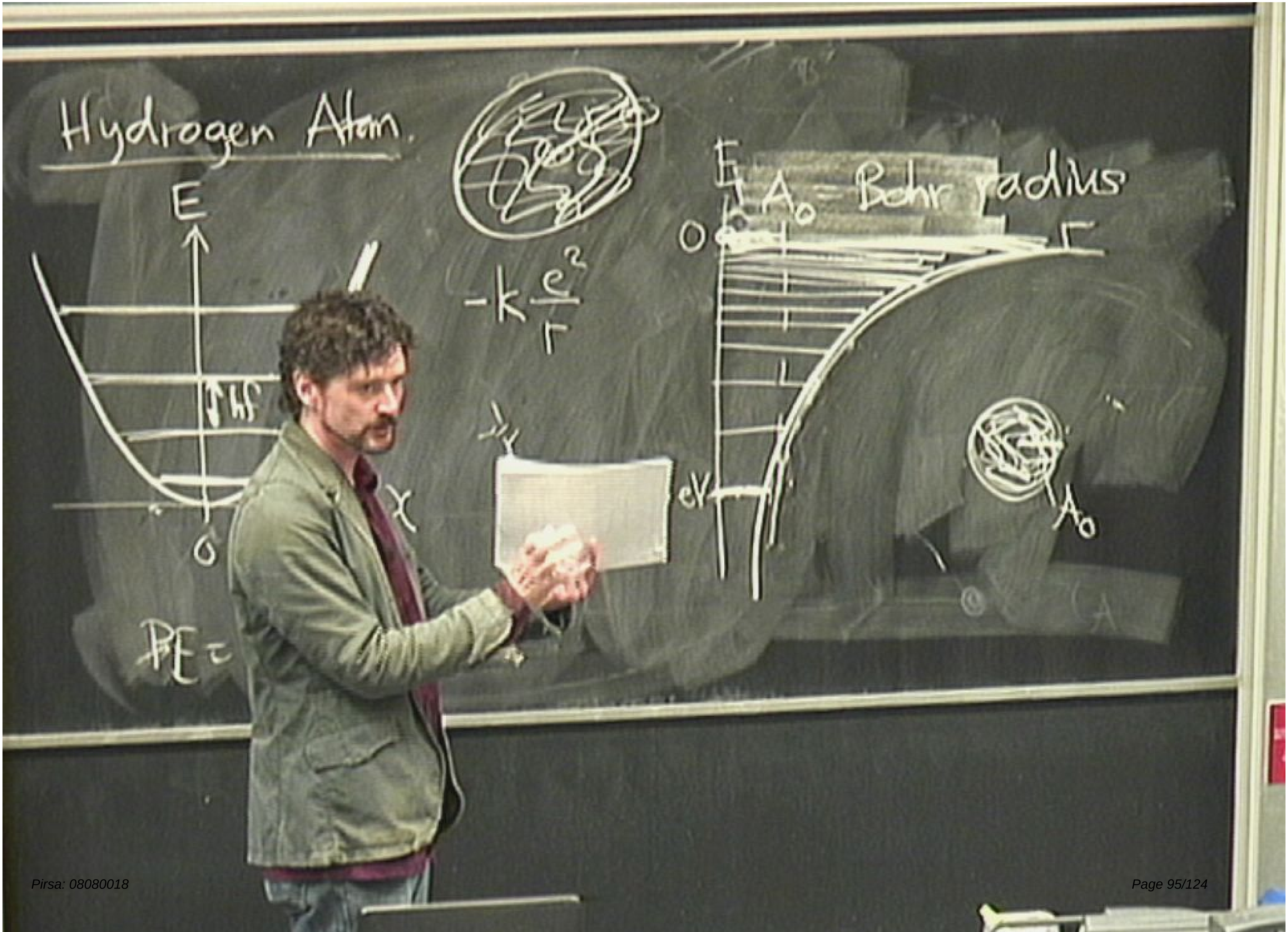

Hydrogen Atom
E
E_0
x
A_0
$\frac{1}{2}kx^2$
$-k\frac{e^2}{r}$
E
0
r
-13.6 eV

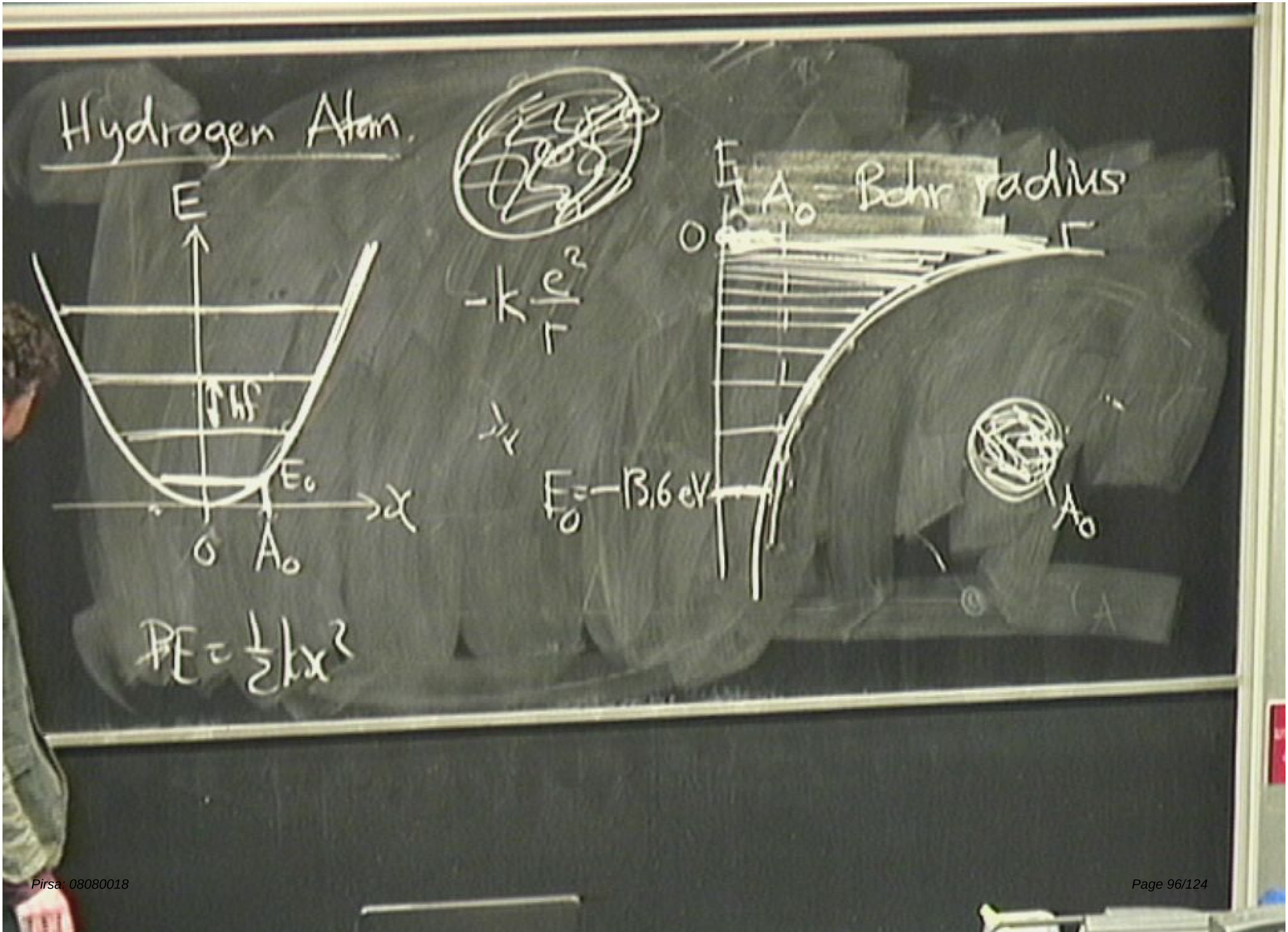

Hydrogen Atom.
E
hf
E0
x
0
A0
PE = ½kx²
-k e²/r
E
0
r
-13.6 eV



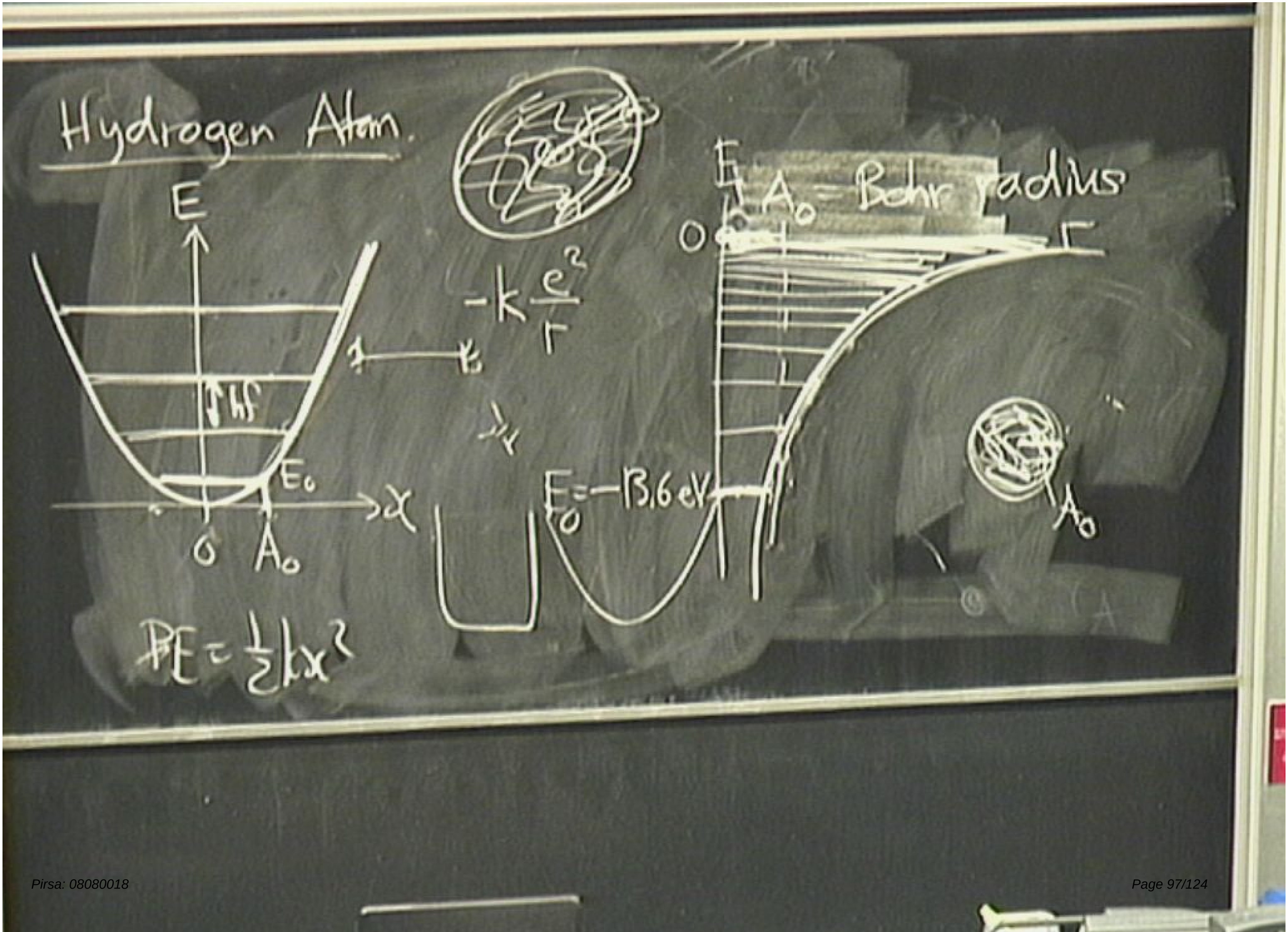
Hydrogen Atom.
E
ħf
x
$-k\frac{e^2}{r}$
E
0
r
-13.6 eV

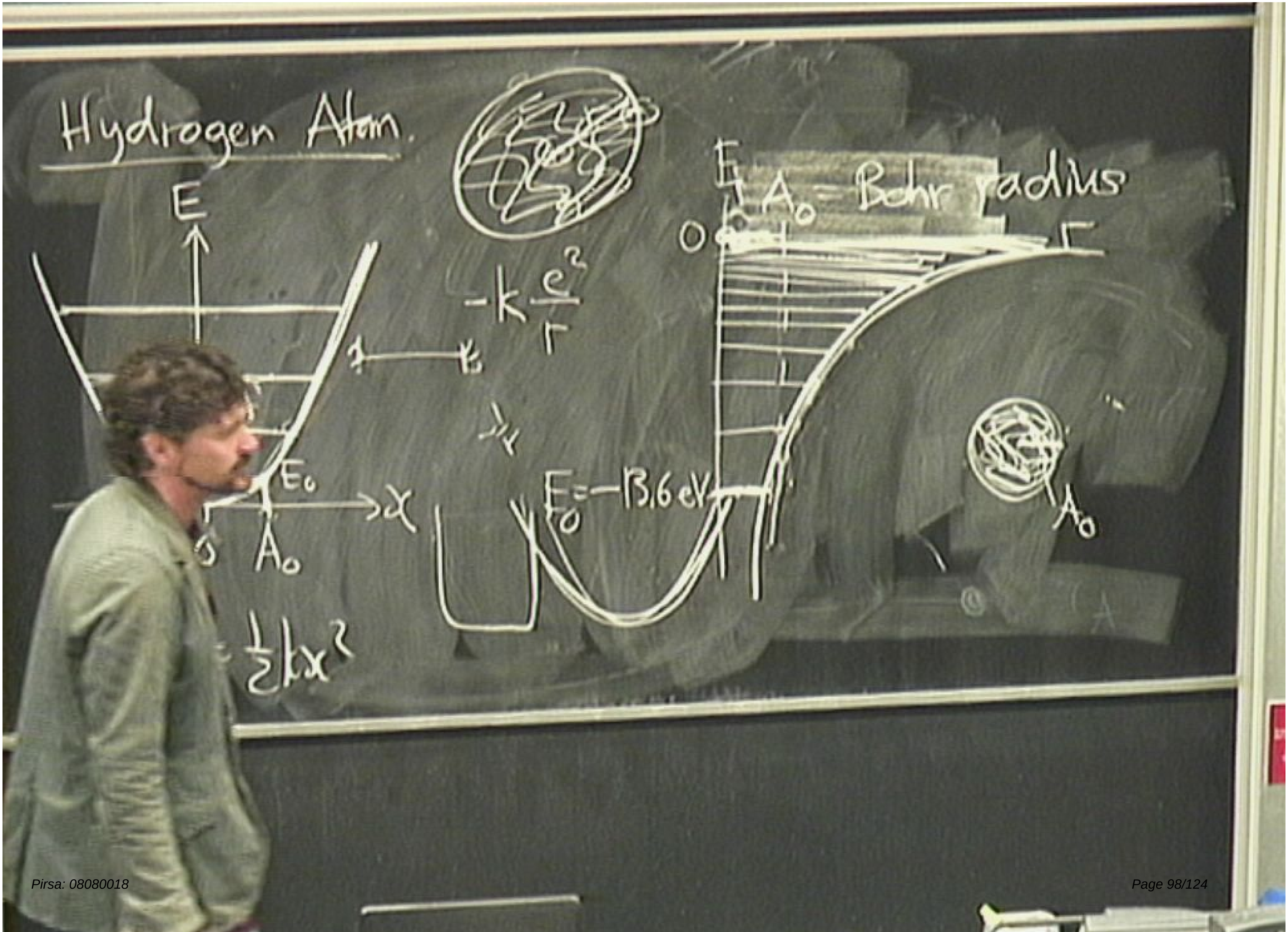

Hydrogen Atom
E
hf
E0
x
0
A0
1/2 kx^2
-k e^2/r
E
A0
0
r
E0 = -13.6 eV

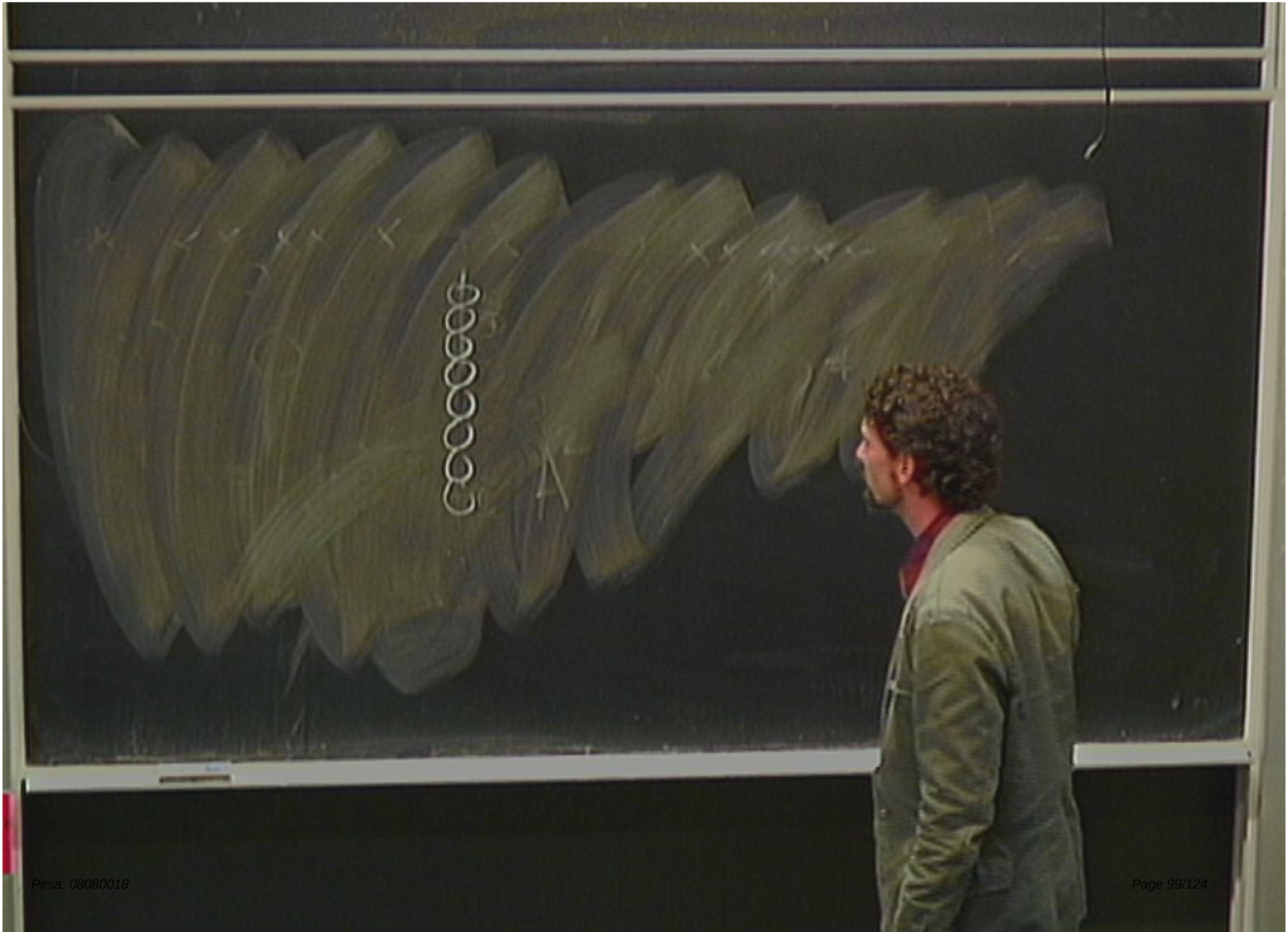

Hydrogen Atom
E
hf
0
x
-k e²/r
E
A₀ = Bohr radius
0
r
eV
A₀
PE =

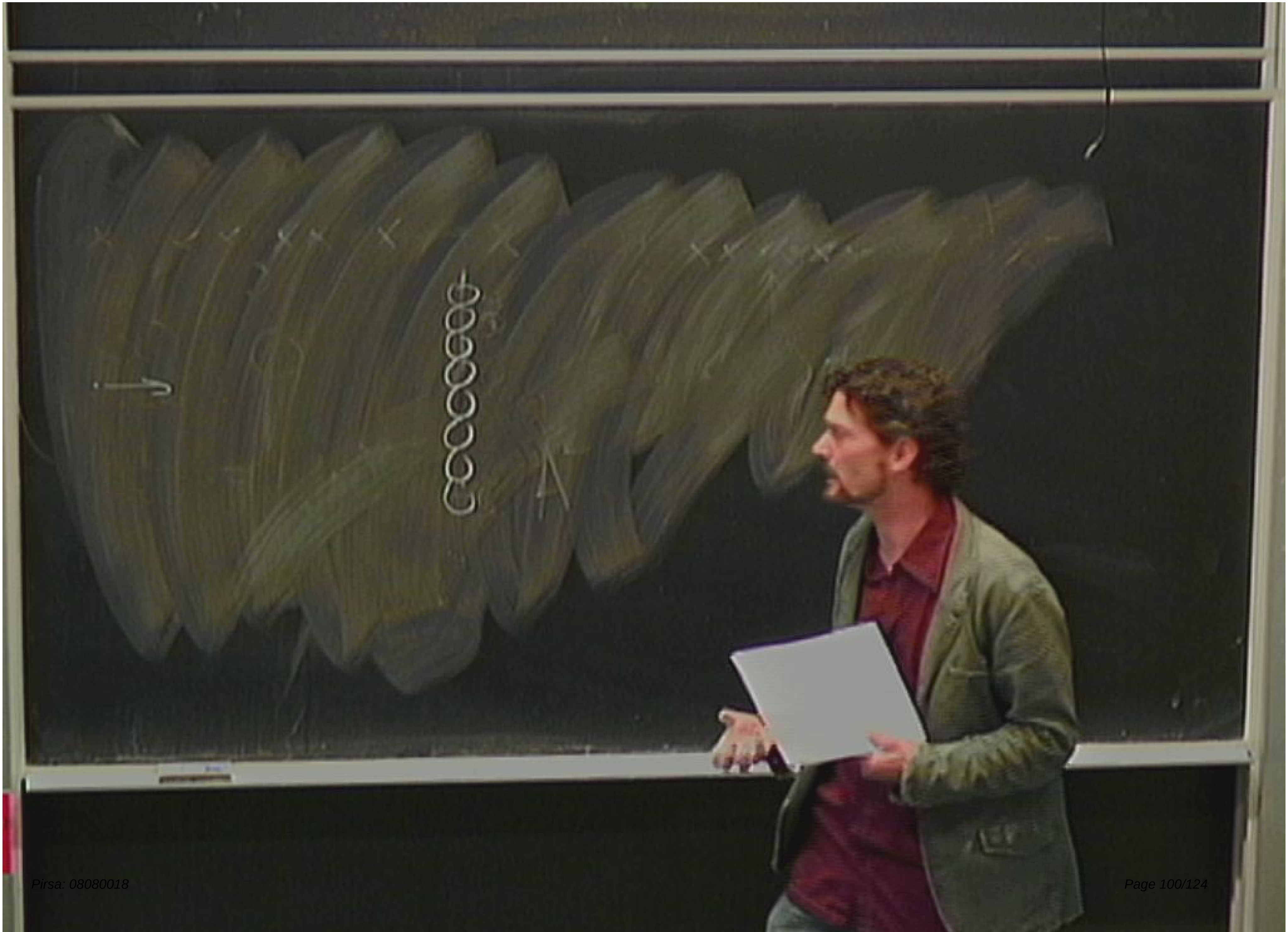


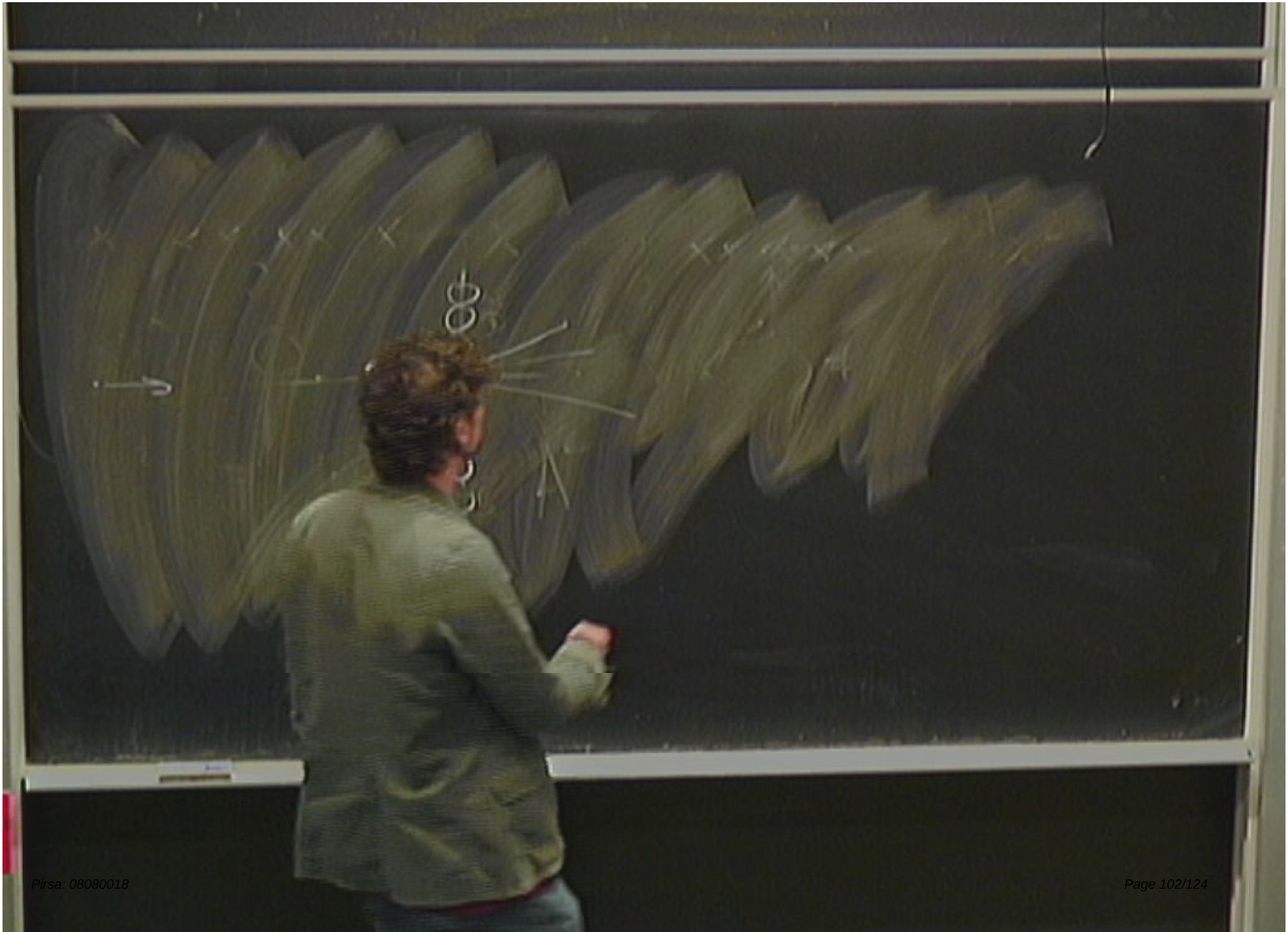
Hydrogen Atom
E
hf
E0
x
0 A0
PE = 1/2 kx²
-k e²/r
E0 = -13.6 eV
E
A0 = Bohr radius
0
r
A0

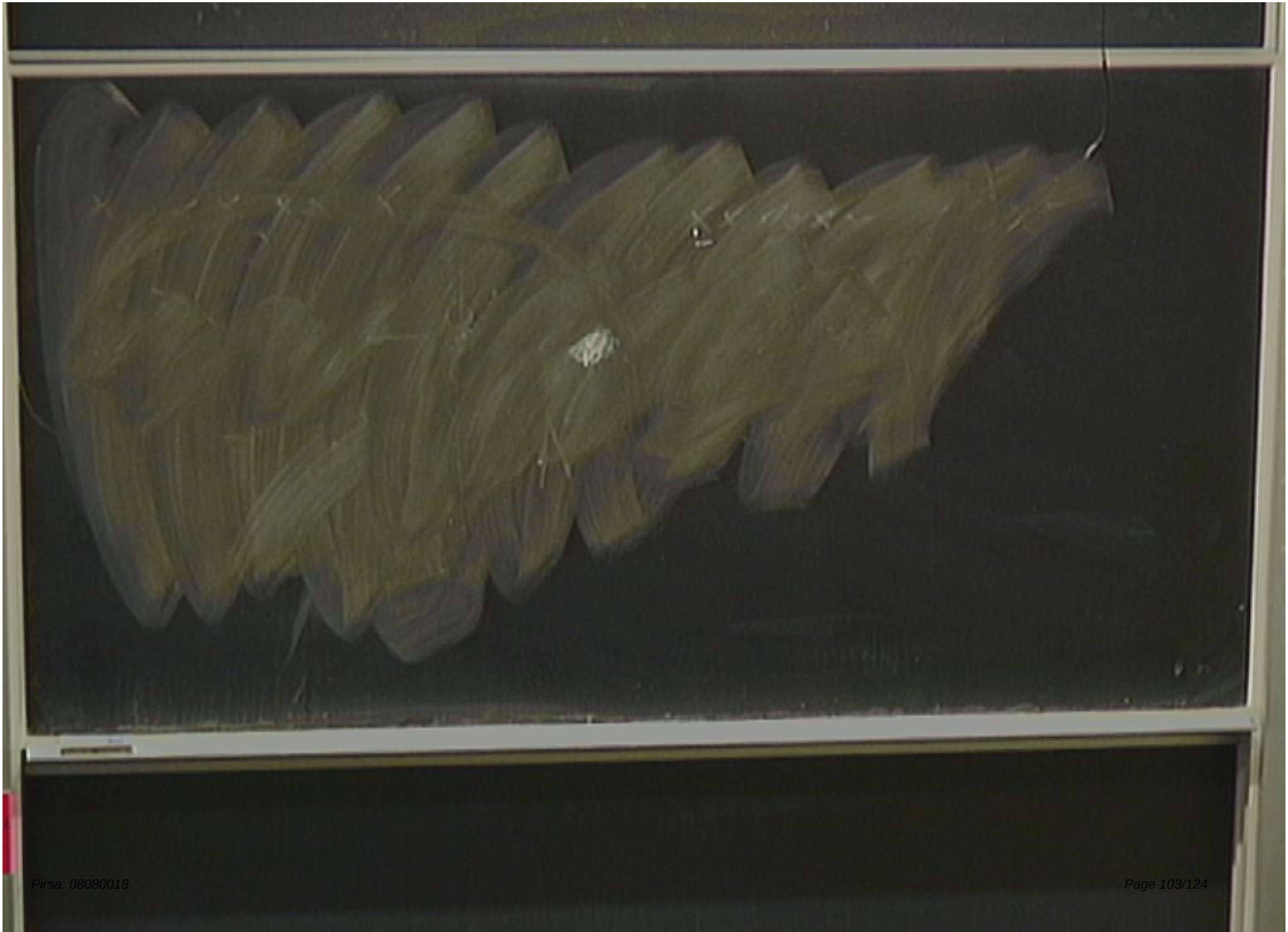

Hydrogen Atom
E
hf
E0
x
0
A0
PE = 1/2 kx^2
-k e^2/r
E
A0 = Bohr radius
0
r
E0 = -13.6 eV
A0

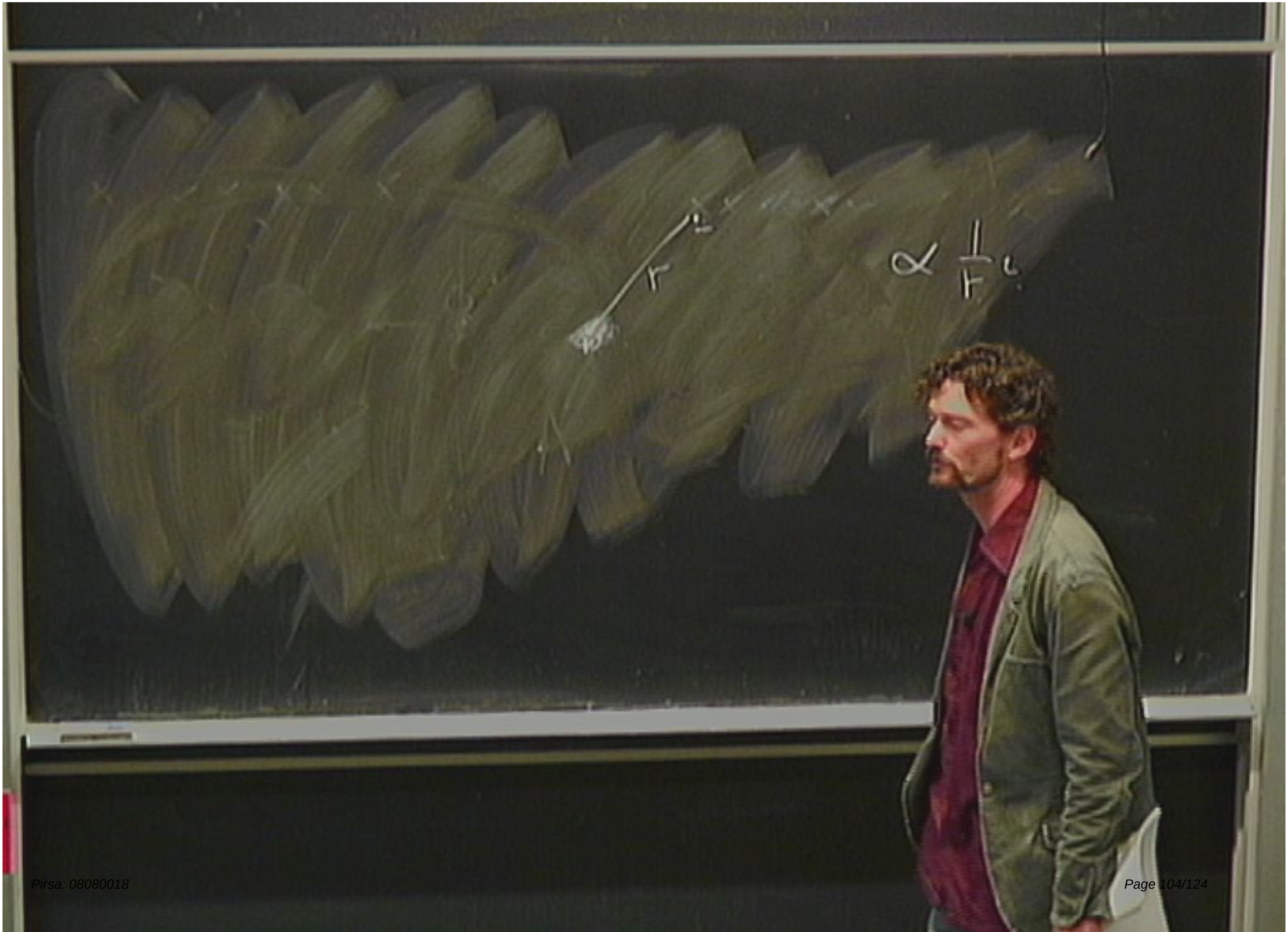

Hydrogen Atom
E
E_0
x
A_0
$\frac{1}{2}kx^2$
$-k\frac{e^2}{r}$
$E_0 = -13.6\,eV$
E
A_0 = Bohr radius
0
r
A_0

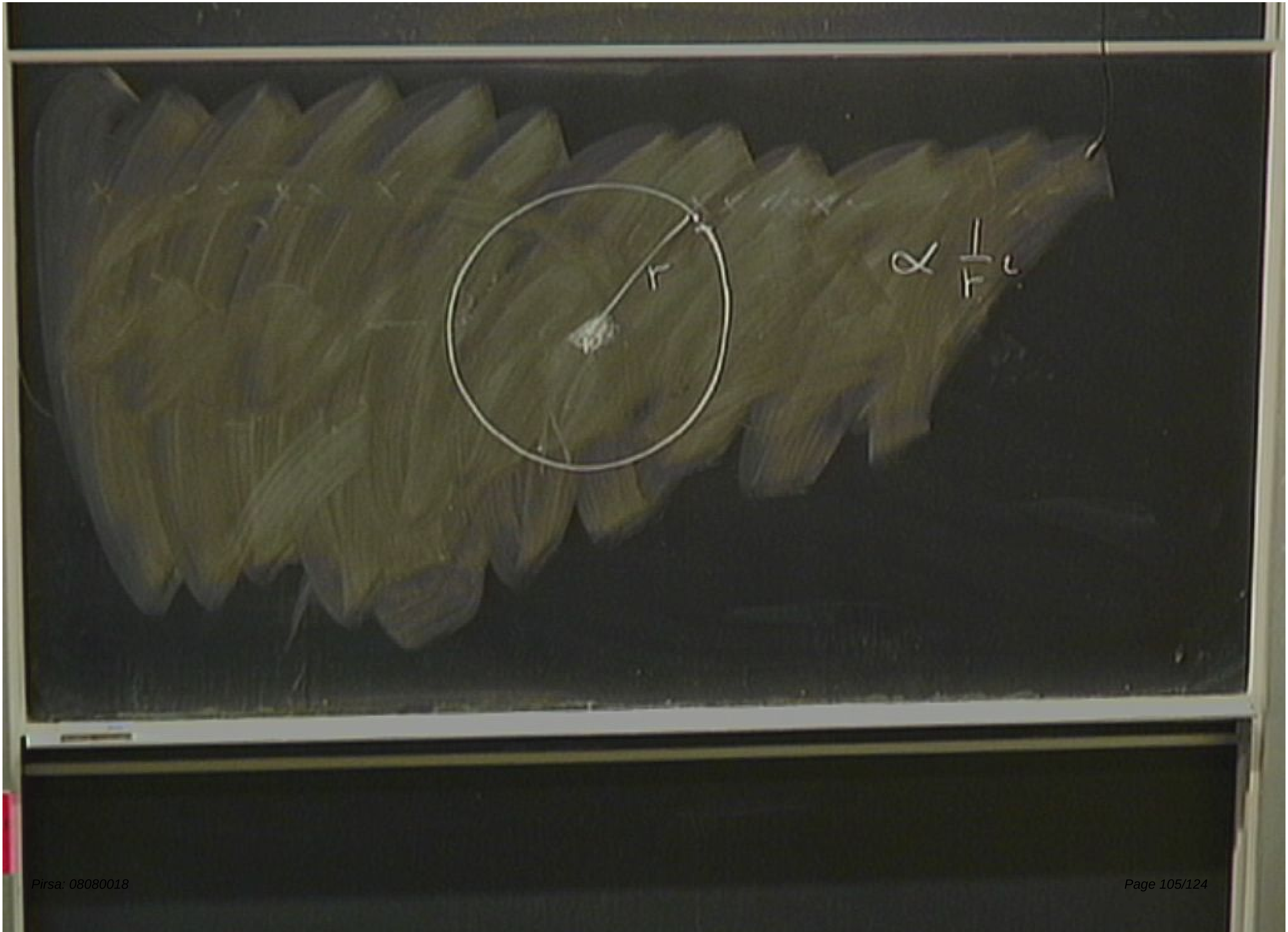


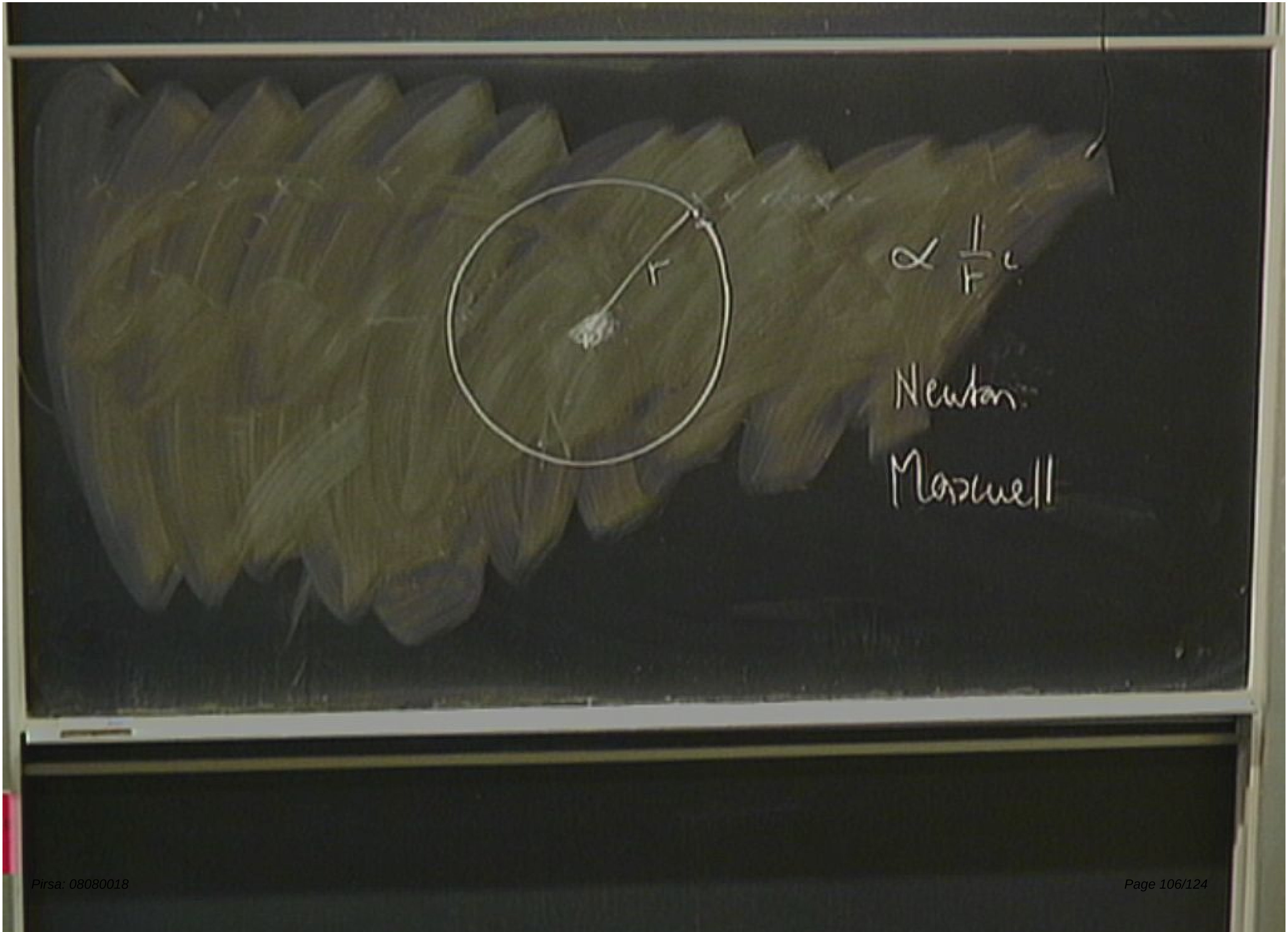


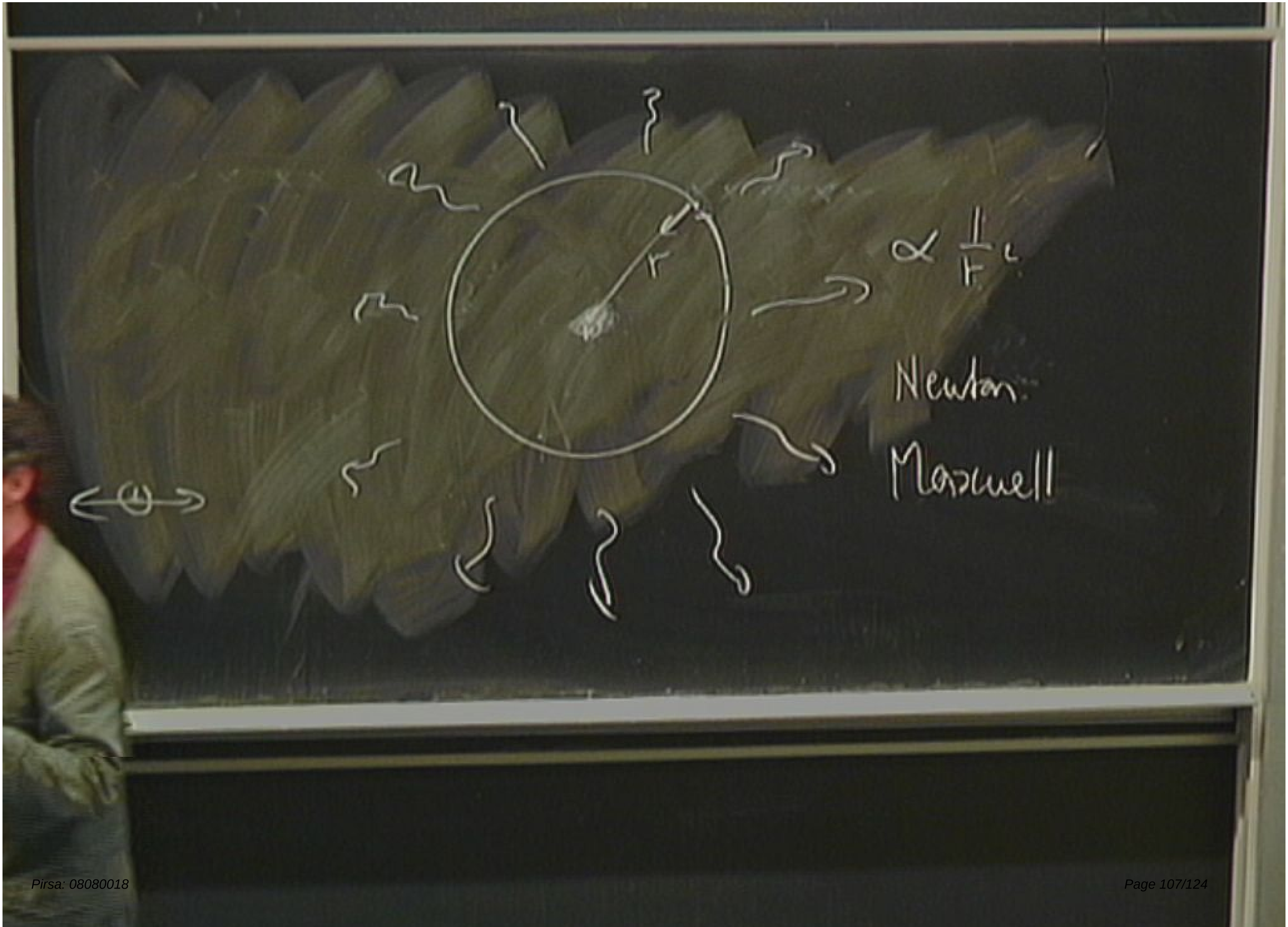




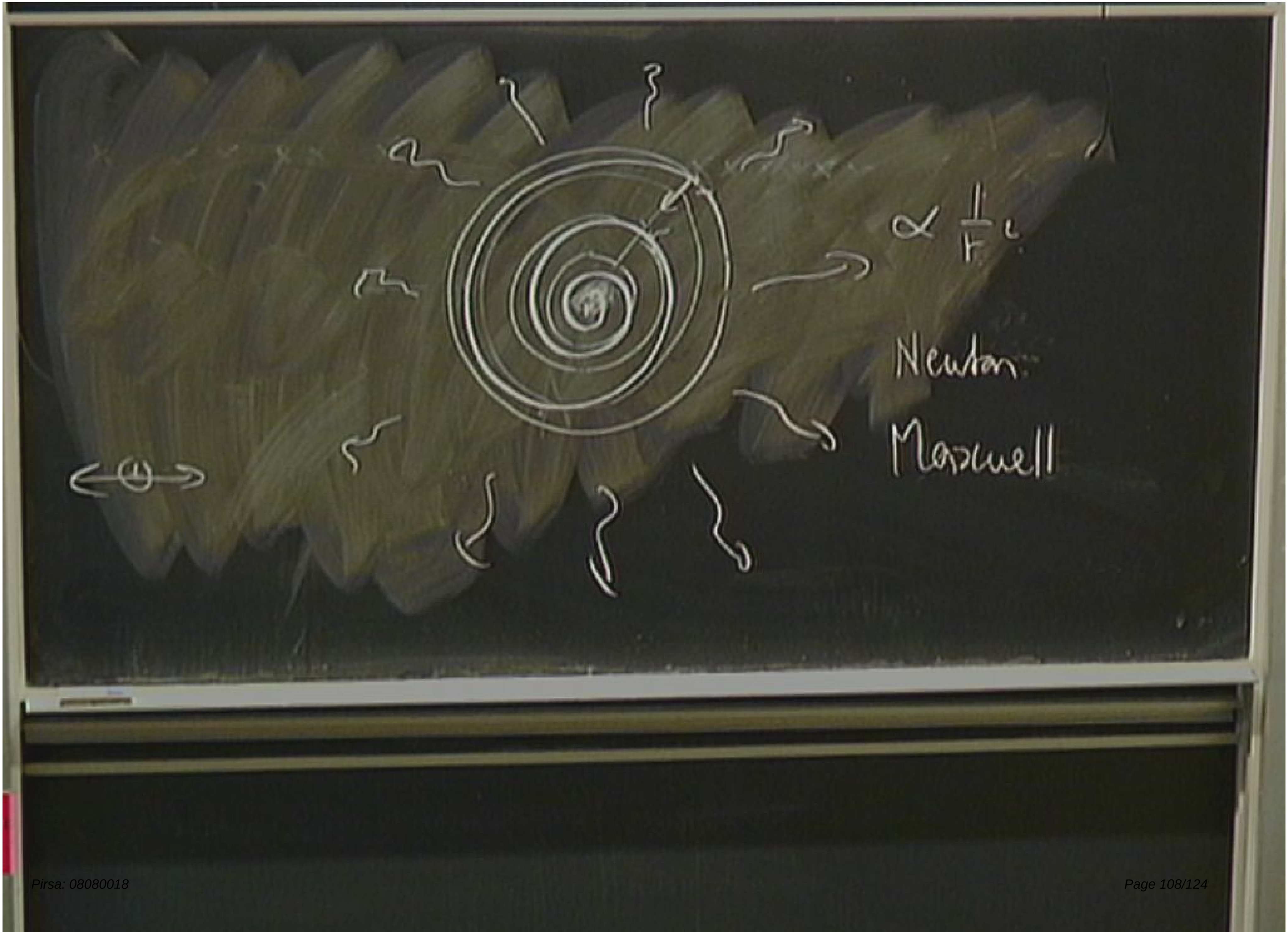



r

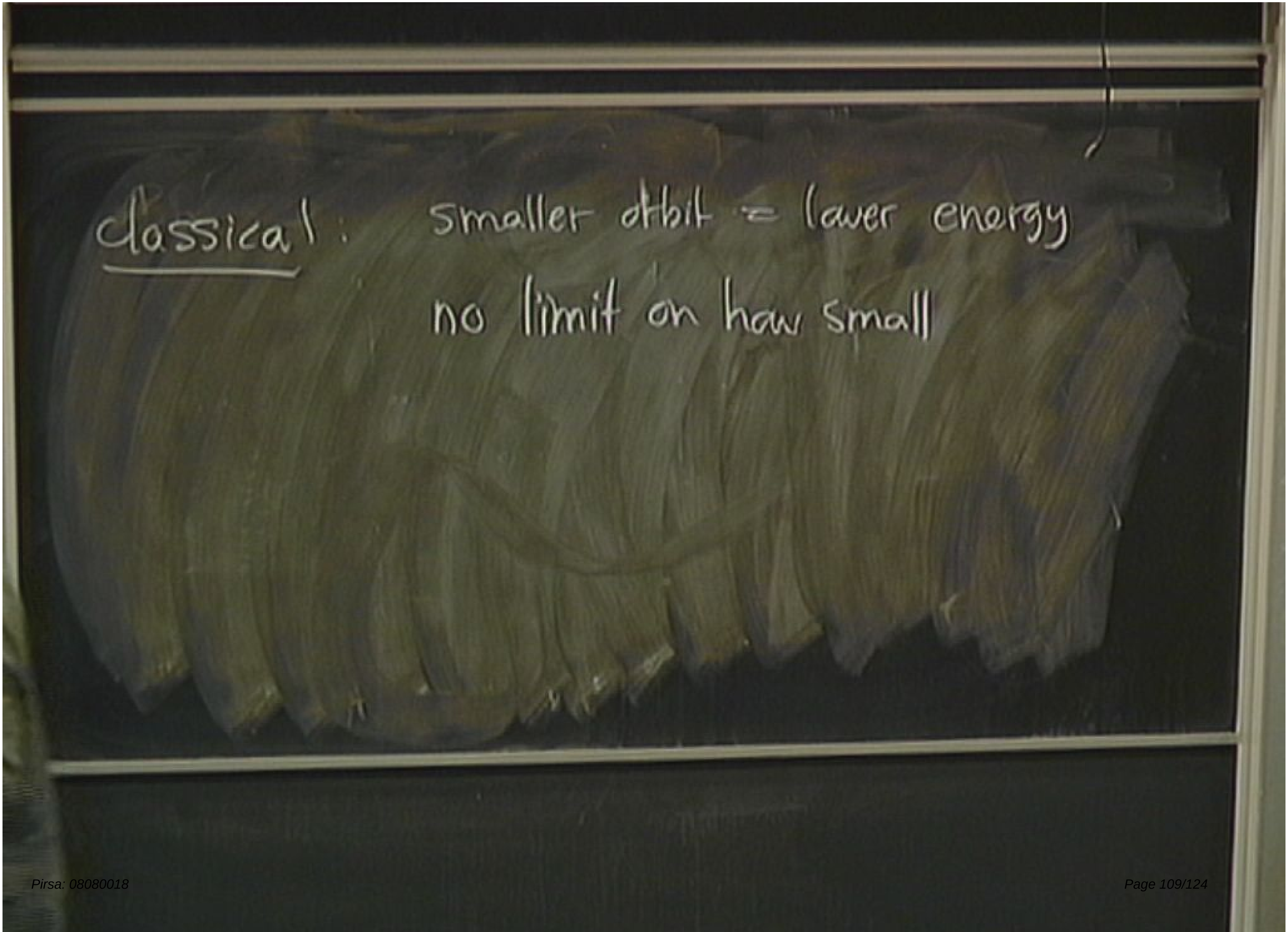

$\propto \frac{1}{r^2}$
r
Newton
Maxwell



r
$\propto \frac{1}{r^2}$
Newton
Maxwell



Newton
Maxwell

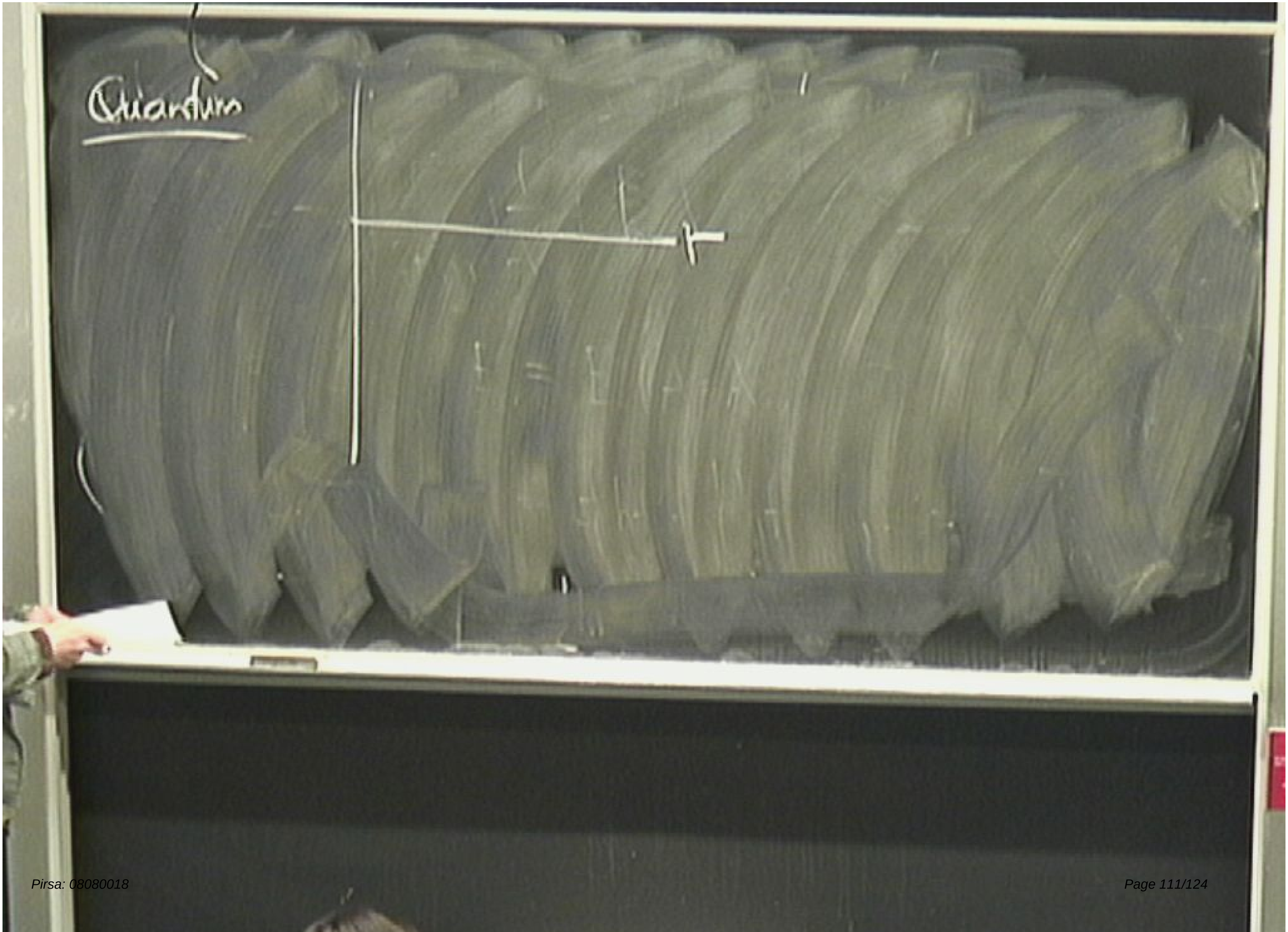

classical: smaller orbit = lower energy
no limit on how small

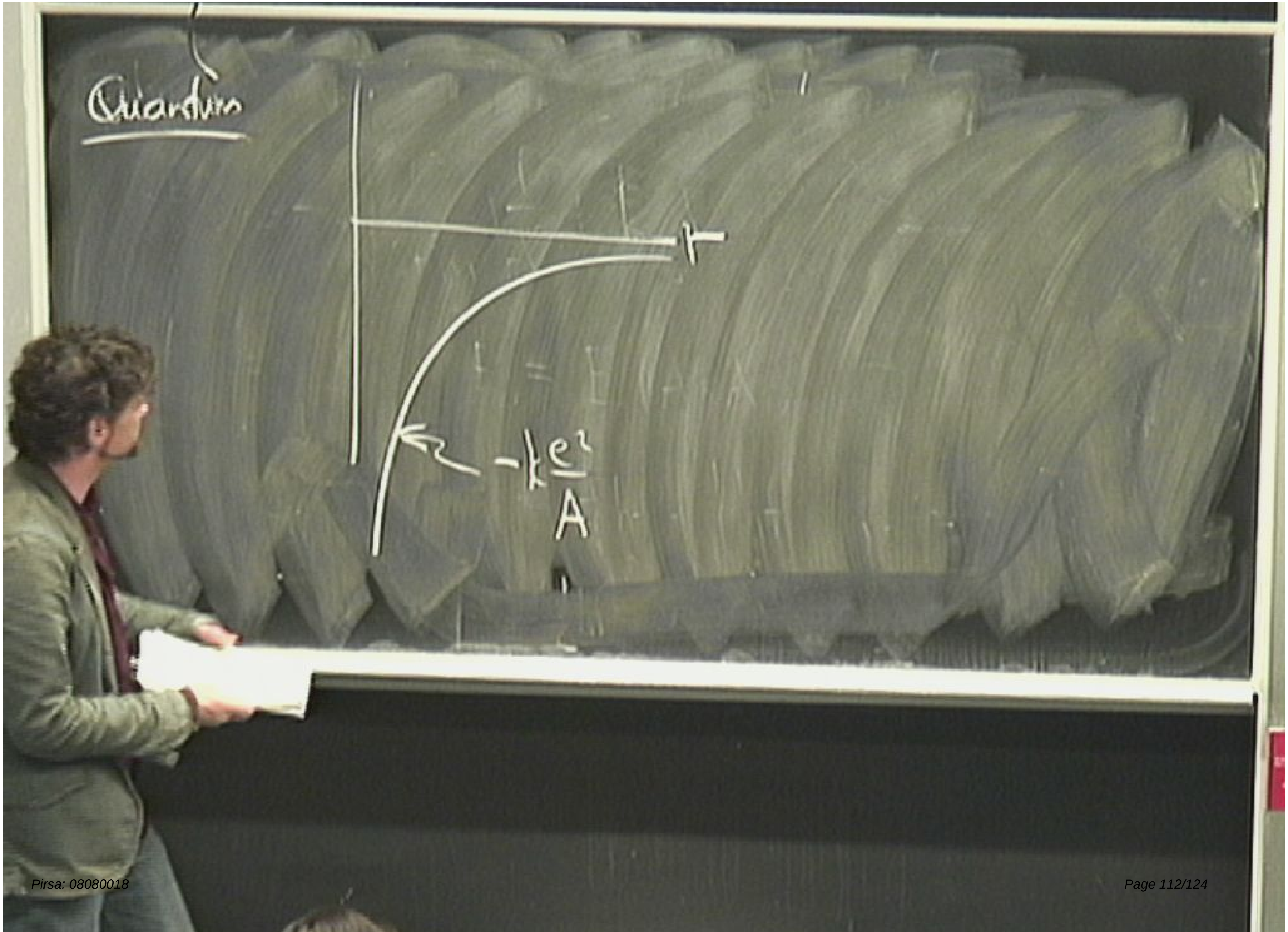
classical:

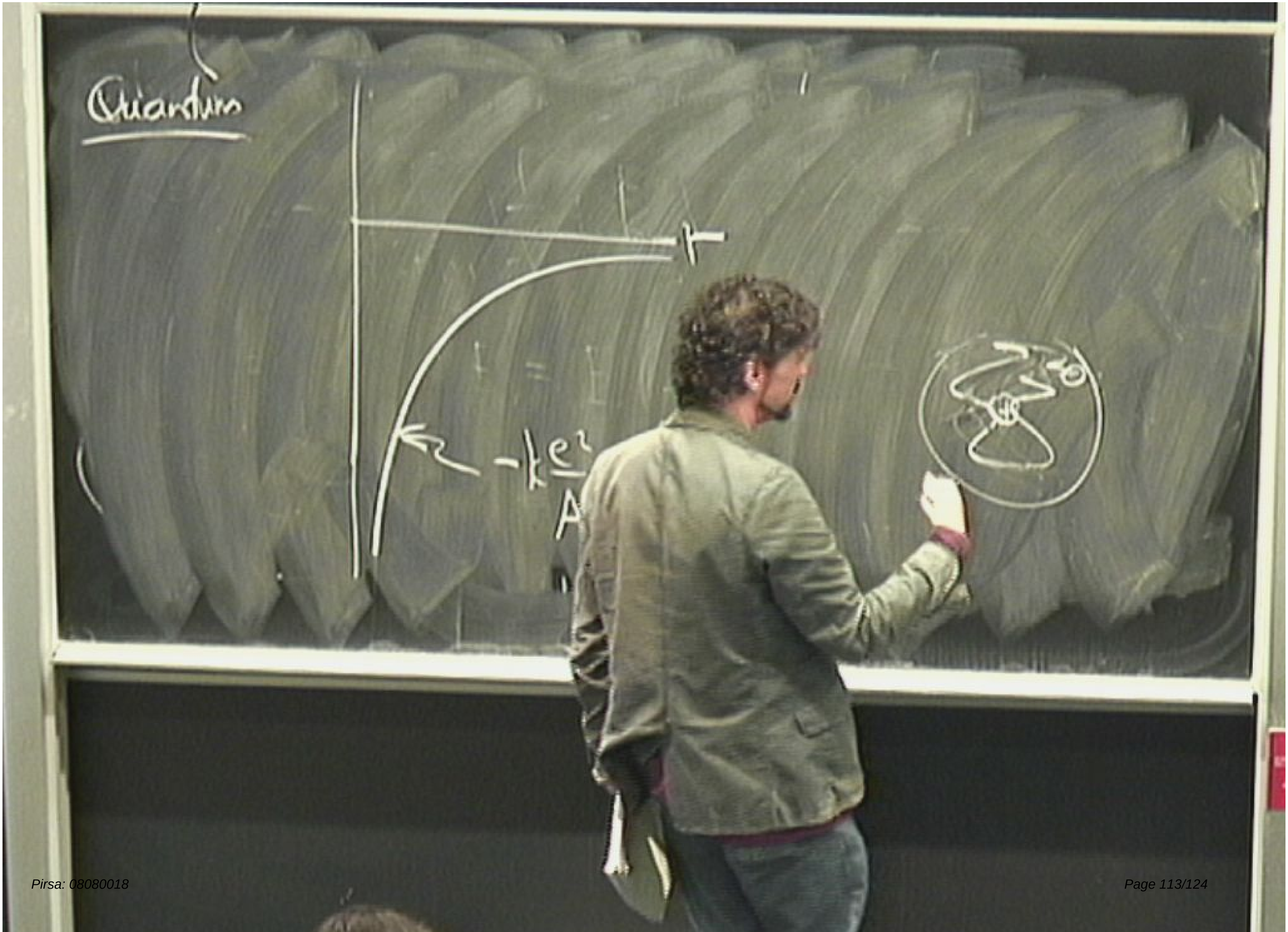
smaller orbit = lower energy

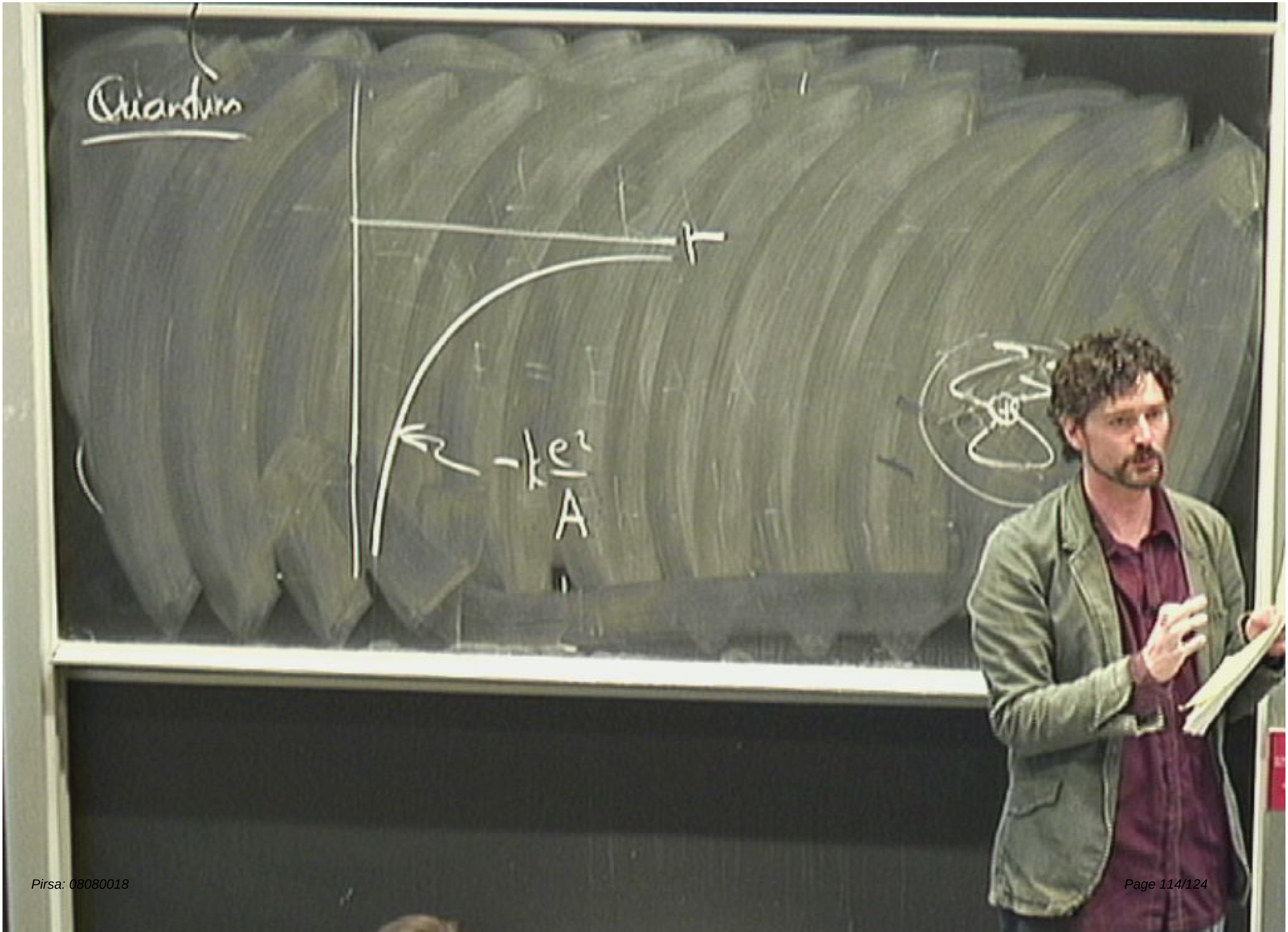
no limit on how small

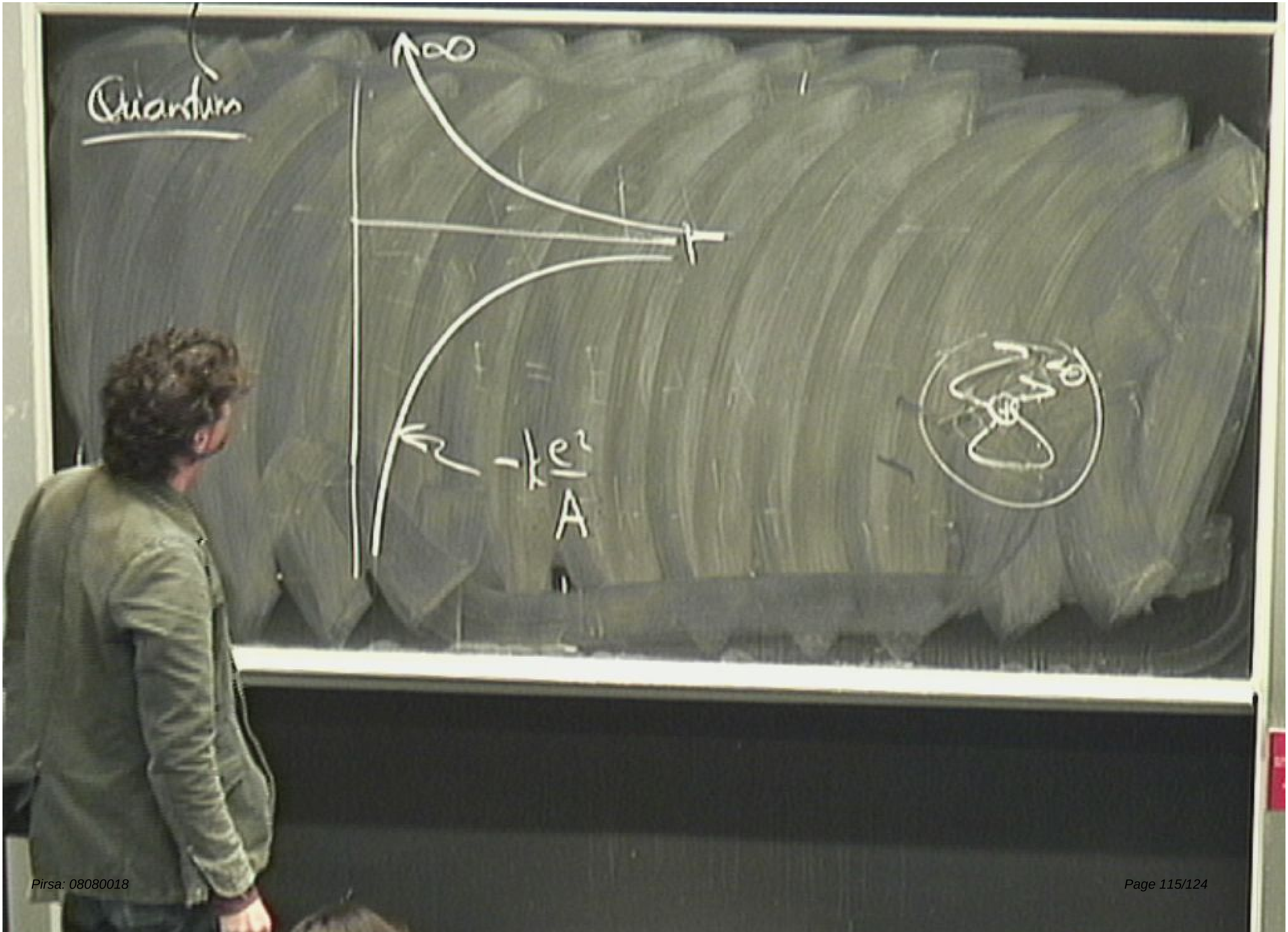
$\Rightarrow \infty$ amount of energy can be emitted

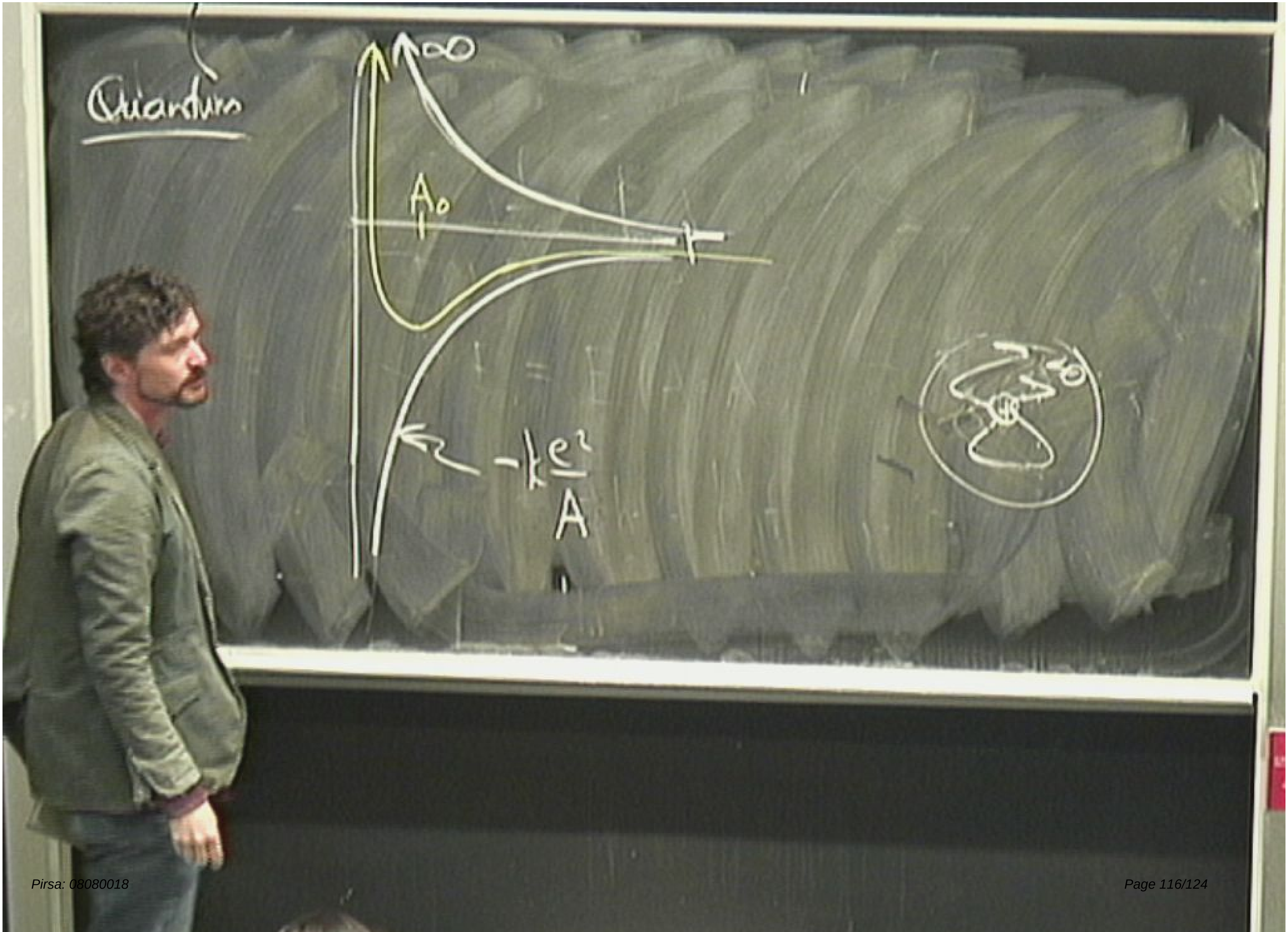

Quantum

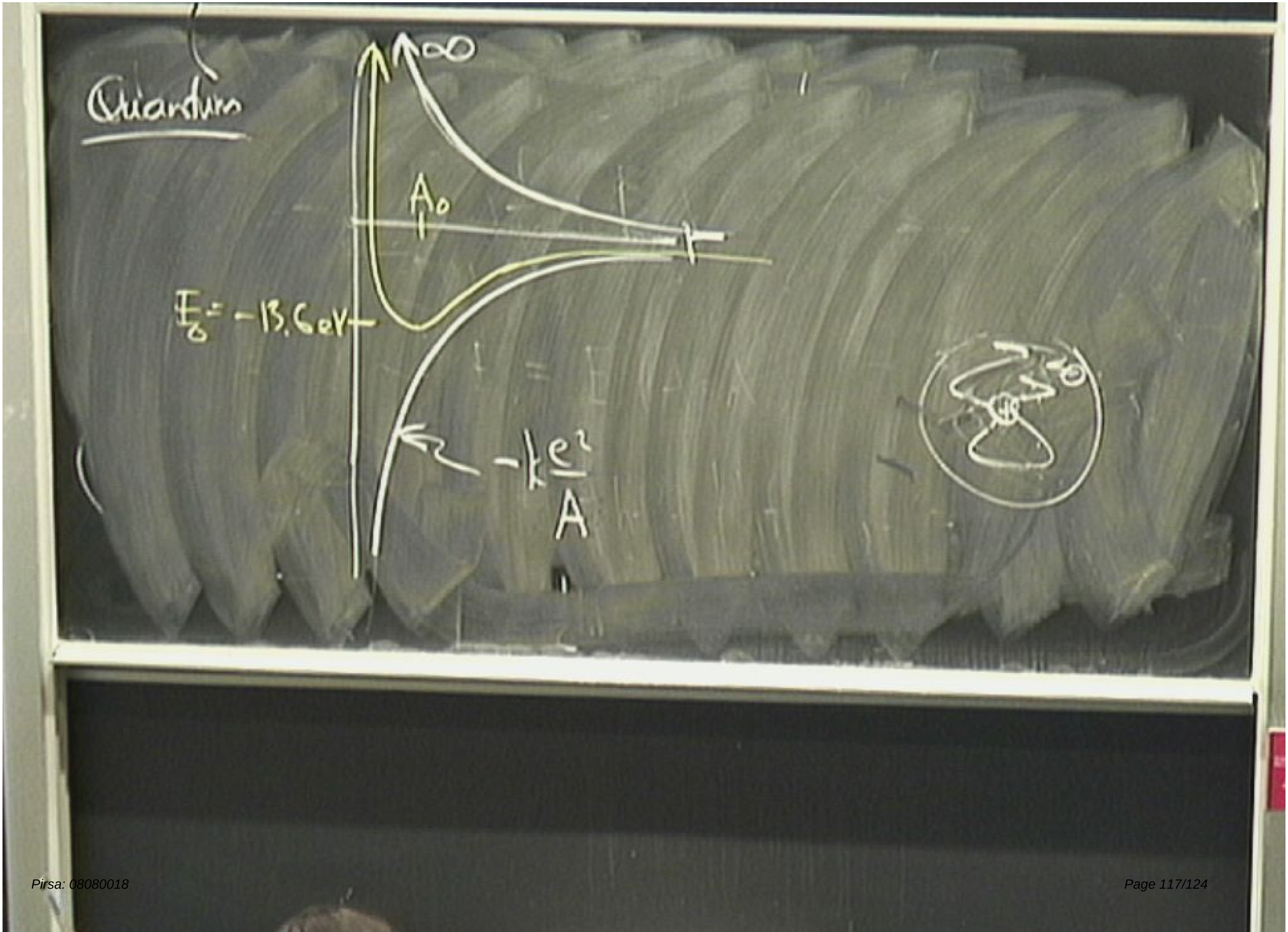

Quantum
r
$-\frac{ke^2}{A}$

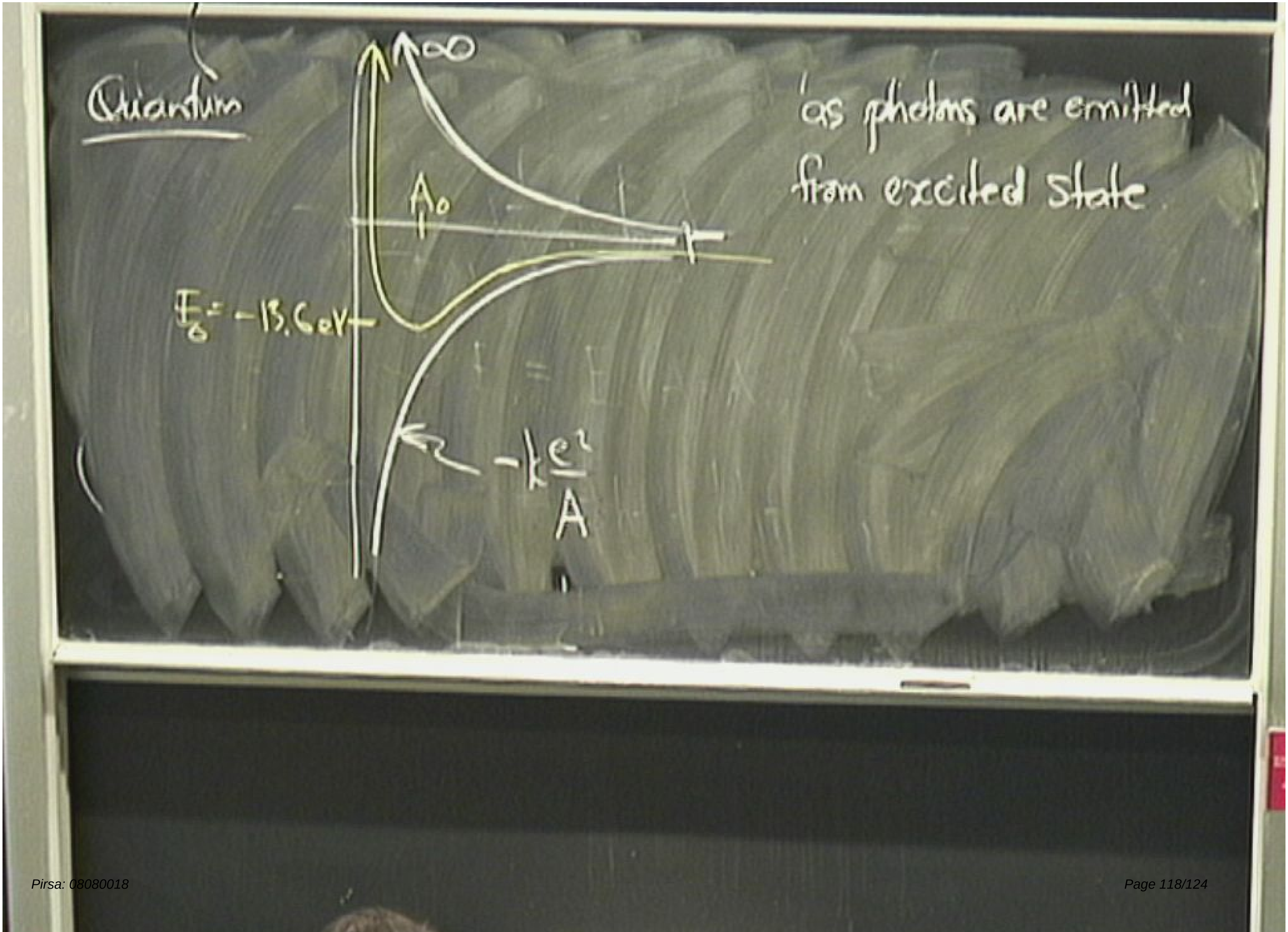

Quantum

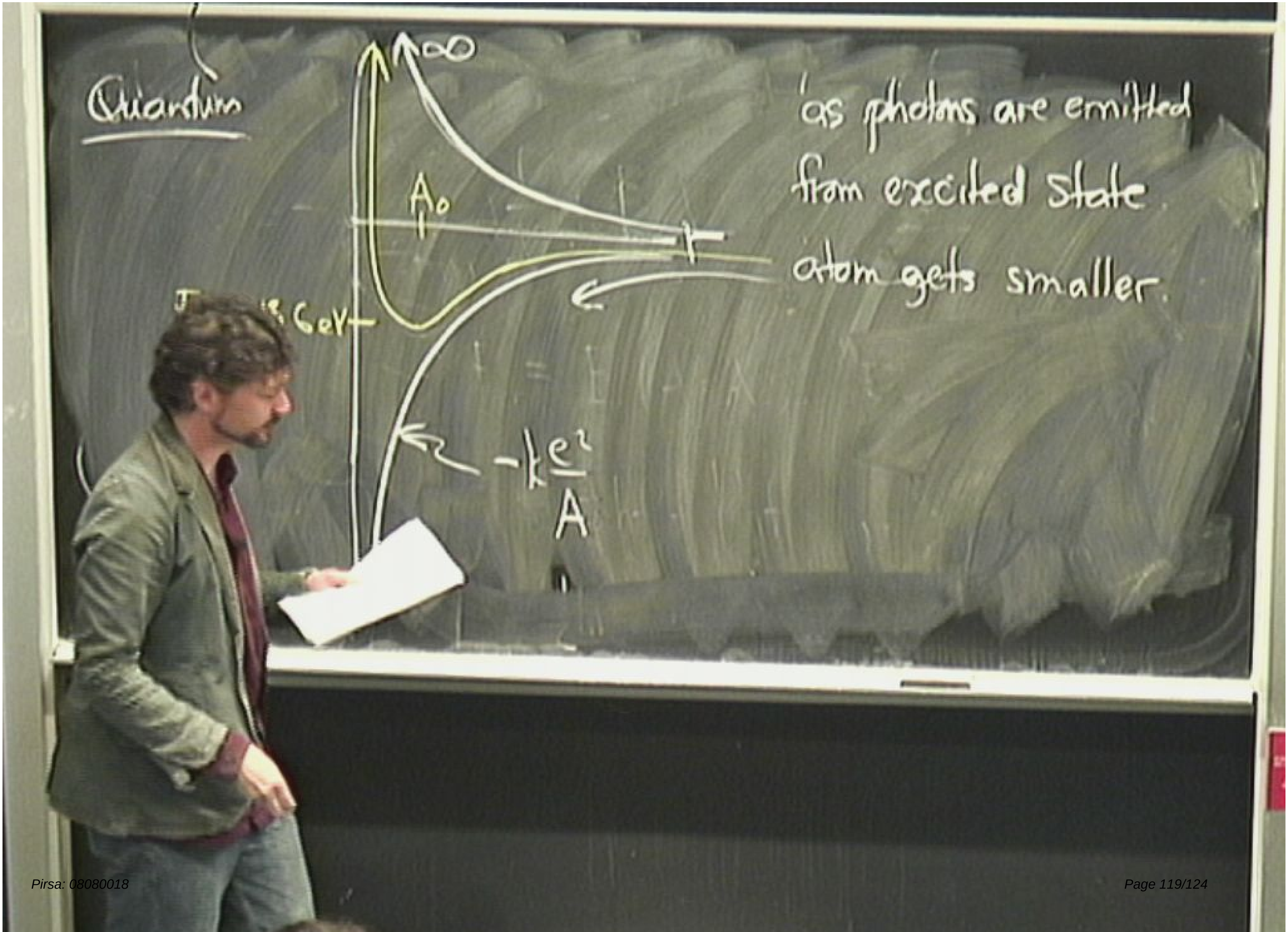

Quantum
r
$-\frac{ke^2}{r}$
A

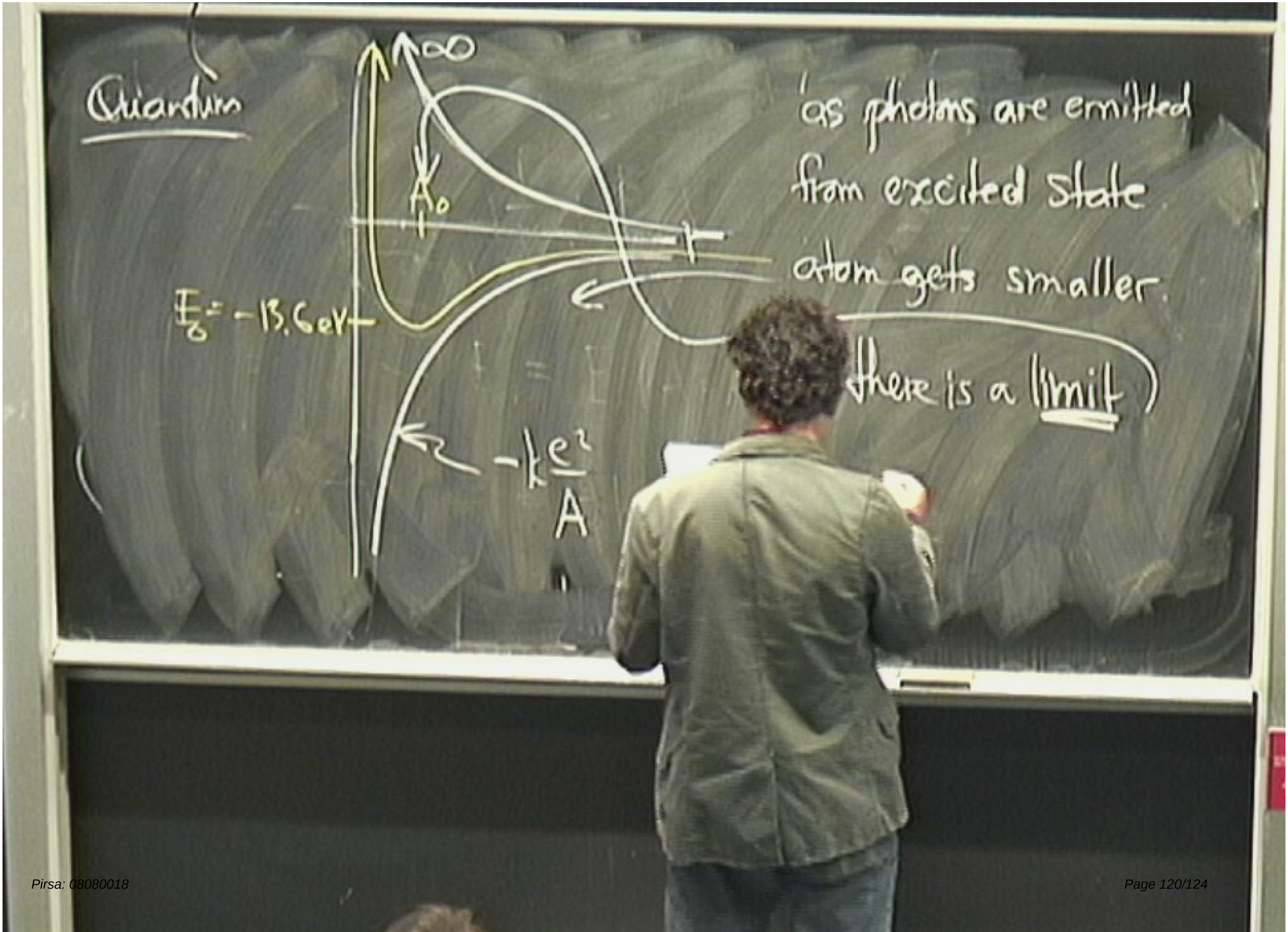

Quantum
∞
$-k\frac{e^2}{A}$

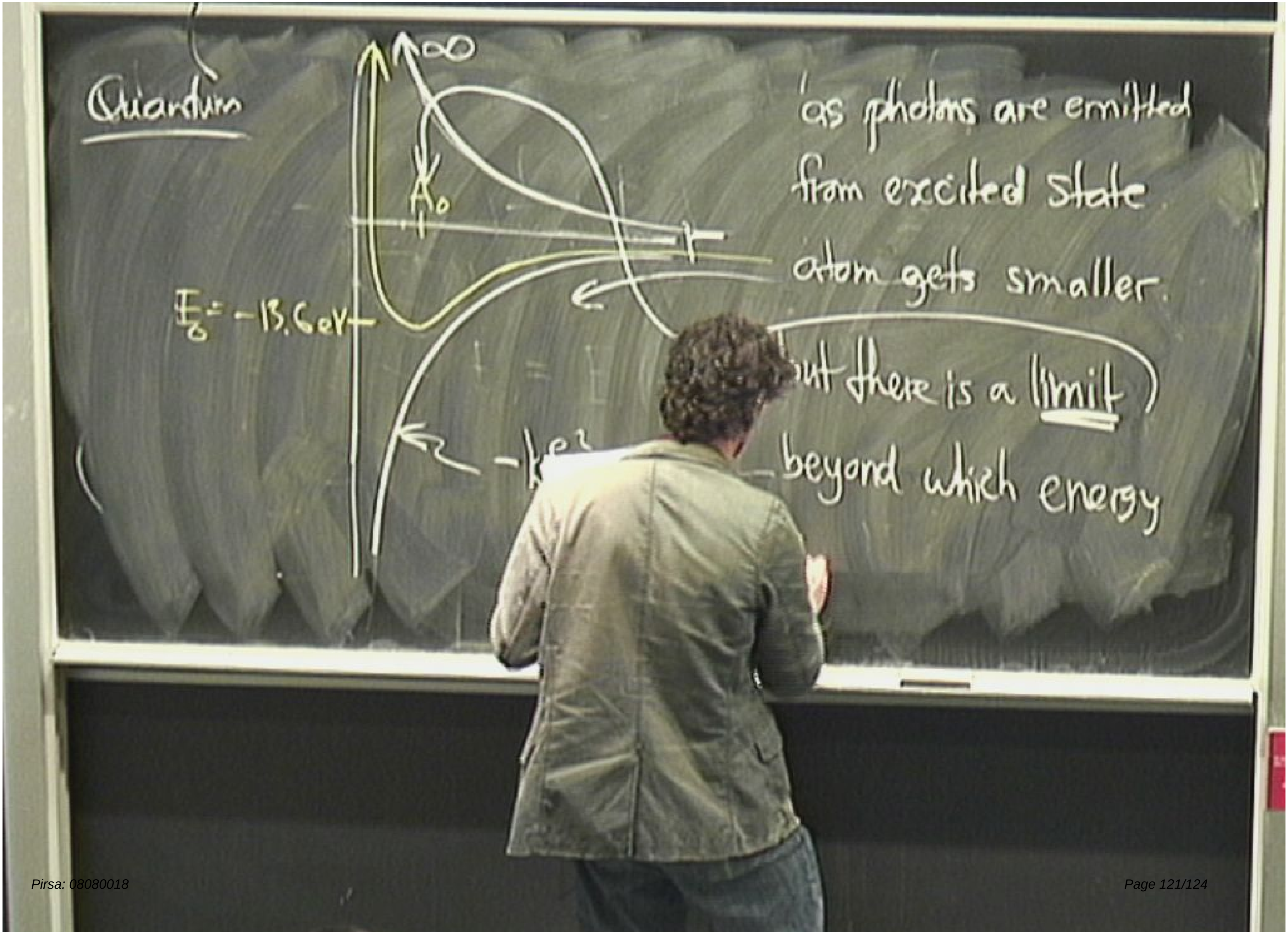

Quantum
∞
A_0
$-\frac{ke^2}{A}$
∞

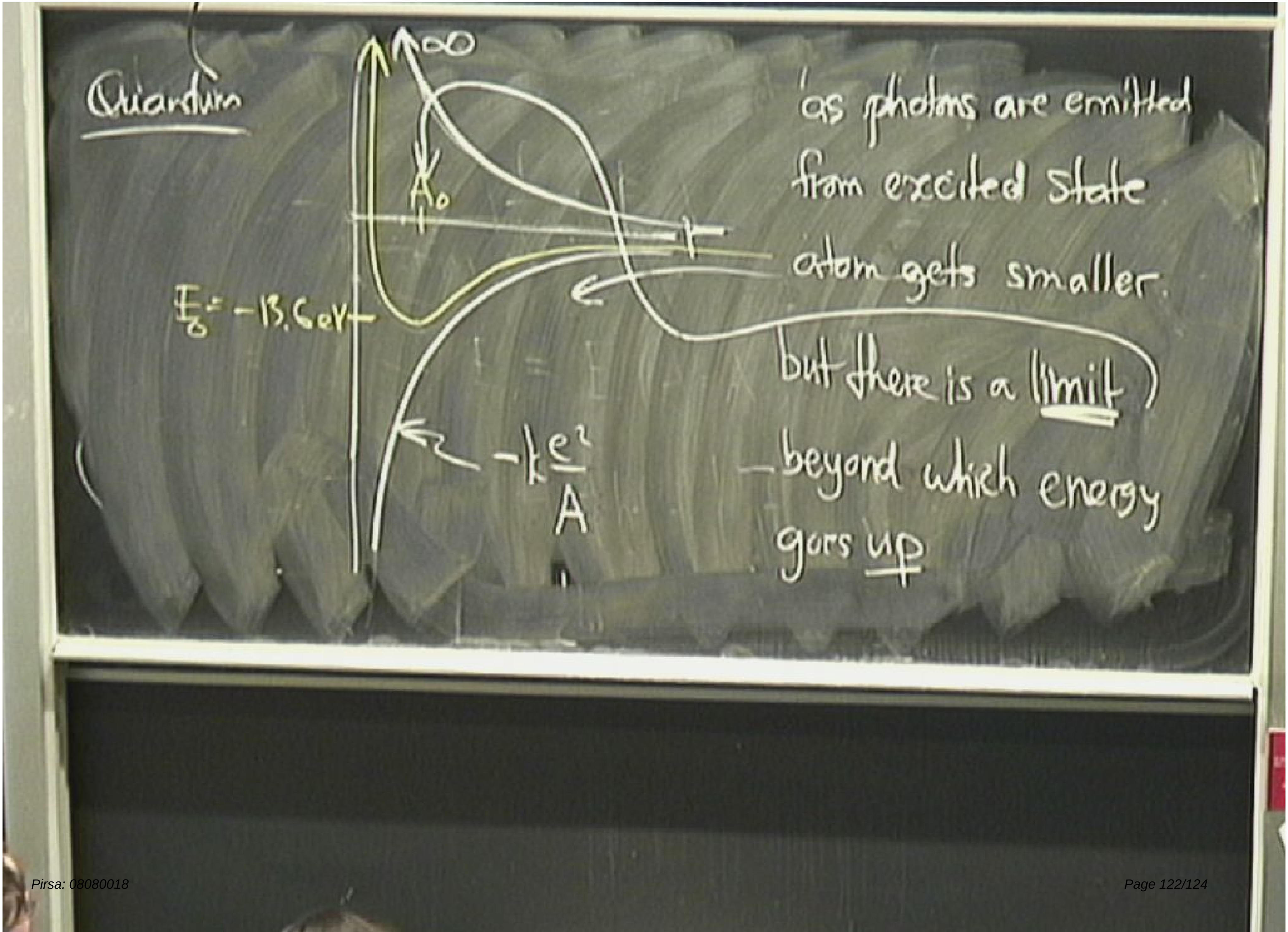

Quantum
∞
A₀
E₀ = -13.6 eV
-ke²/A
A

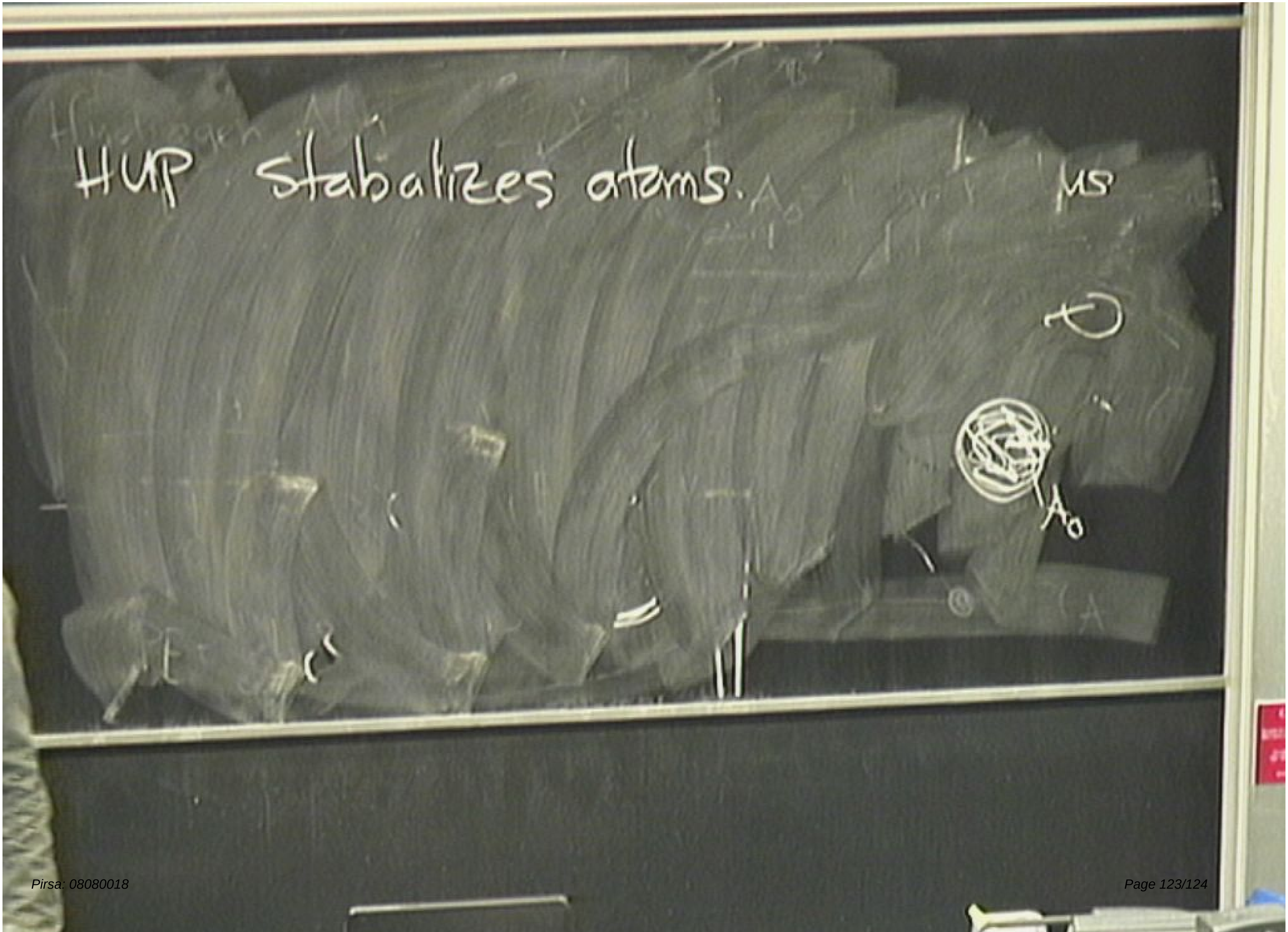

Quantum
∞
A_0
as photons are emitted
from excited State
$E_0 = -13.6\,eV$
$-k\frac{e^2}{A}$
A


Quantum
∞
A_0
as photons are emitted
from excited state
atom gets smaller.
$-k\frac{e^2}{A}$


Quantum
∞
A₀
r
E₀ = -13.6eV
-ke²/A
as photons are emitted
from excited State
atom gets smaller
there is a limit


Quantum
∞
r_0
r
$E_0 = -13.6\,eV$
as photons are emitted
from excited state
atom gets smaller.
but there is a limit
beyond which energy


Quantum
∞
A₀
r
E₀ = −13.6eV
−ke²/A
as photons are emitted
from excited state
atom gets smaller.
but there is a limit
beyond which energy
goes up


HUP stabalizes atoms.

HUP stabalizes atoms.