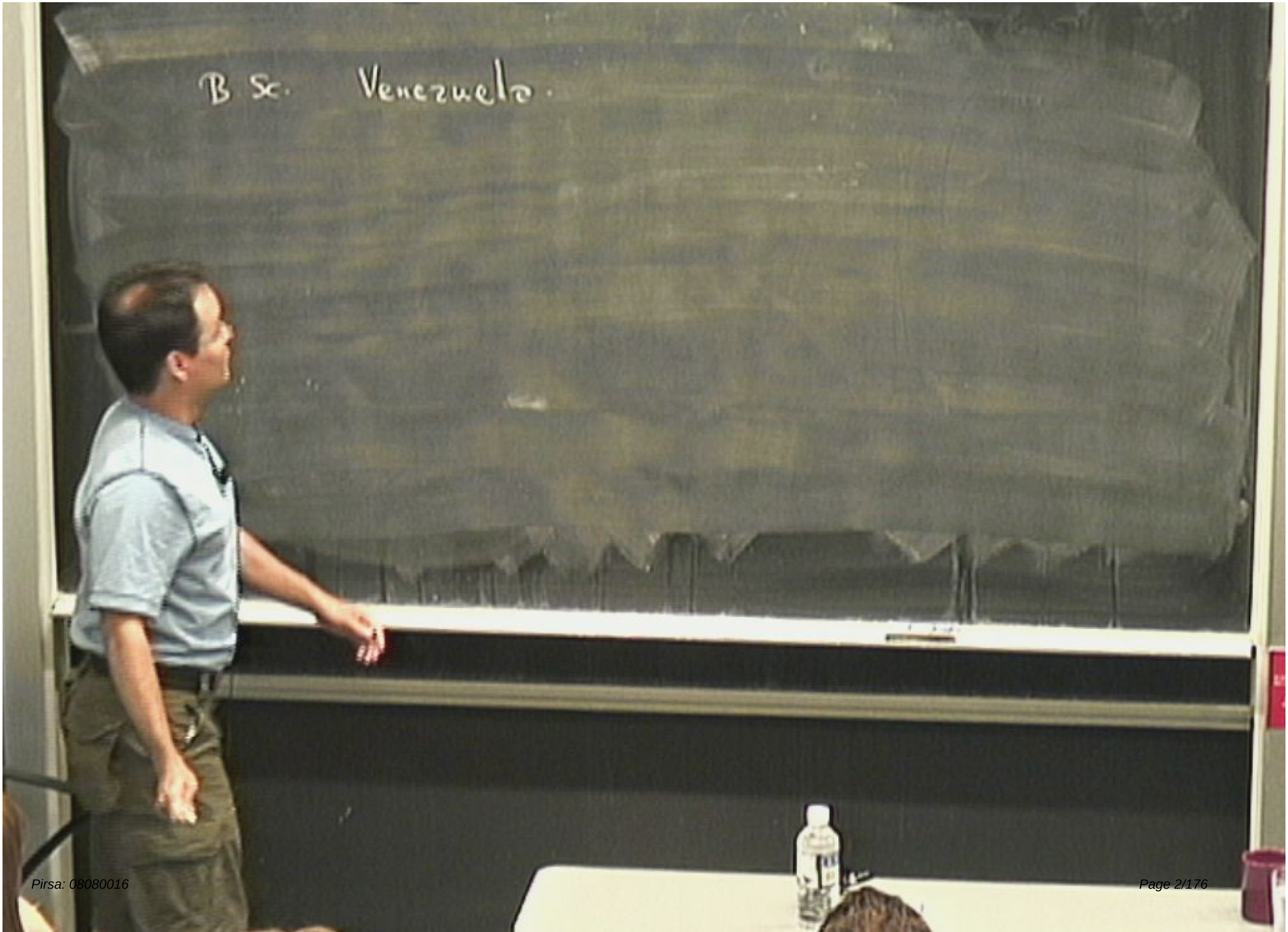
Title: From Symmetries to String Theory

Date: Aug 12, 2008 01:00 PM

URL: http://pirsa.org/08080016

Abstract: Symmetry principles in physics are a very powerful guiding principle. Sometimes they are so powerful that they can determine a theory completely. This talk will be a tour from the Standard Model of particle physics to string theory compactifications using mostly symmetry arguments.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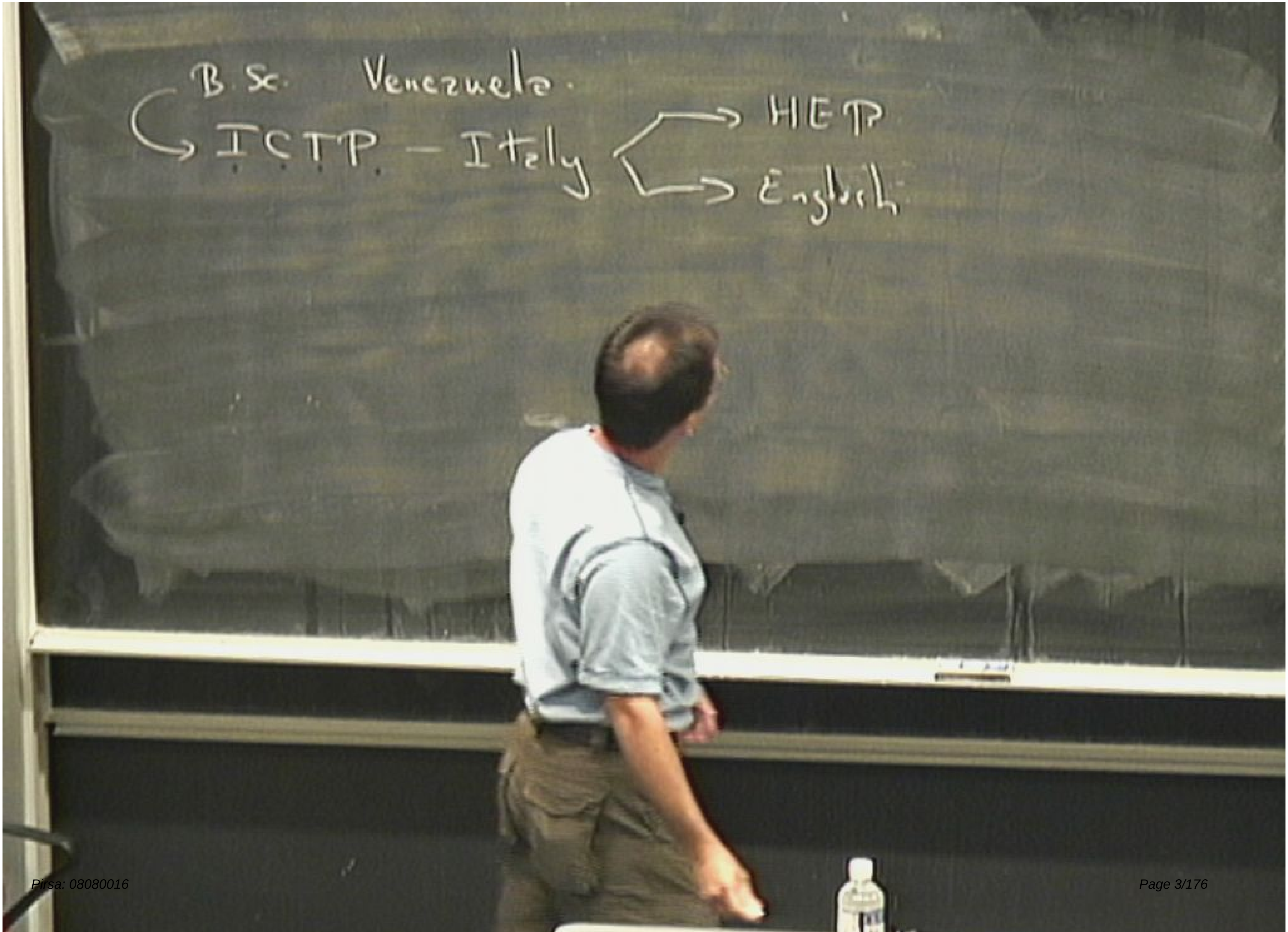
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B. Sc. Venezuela.

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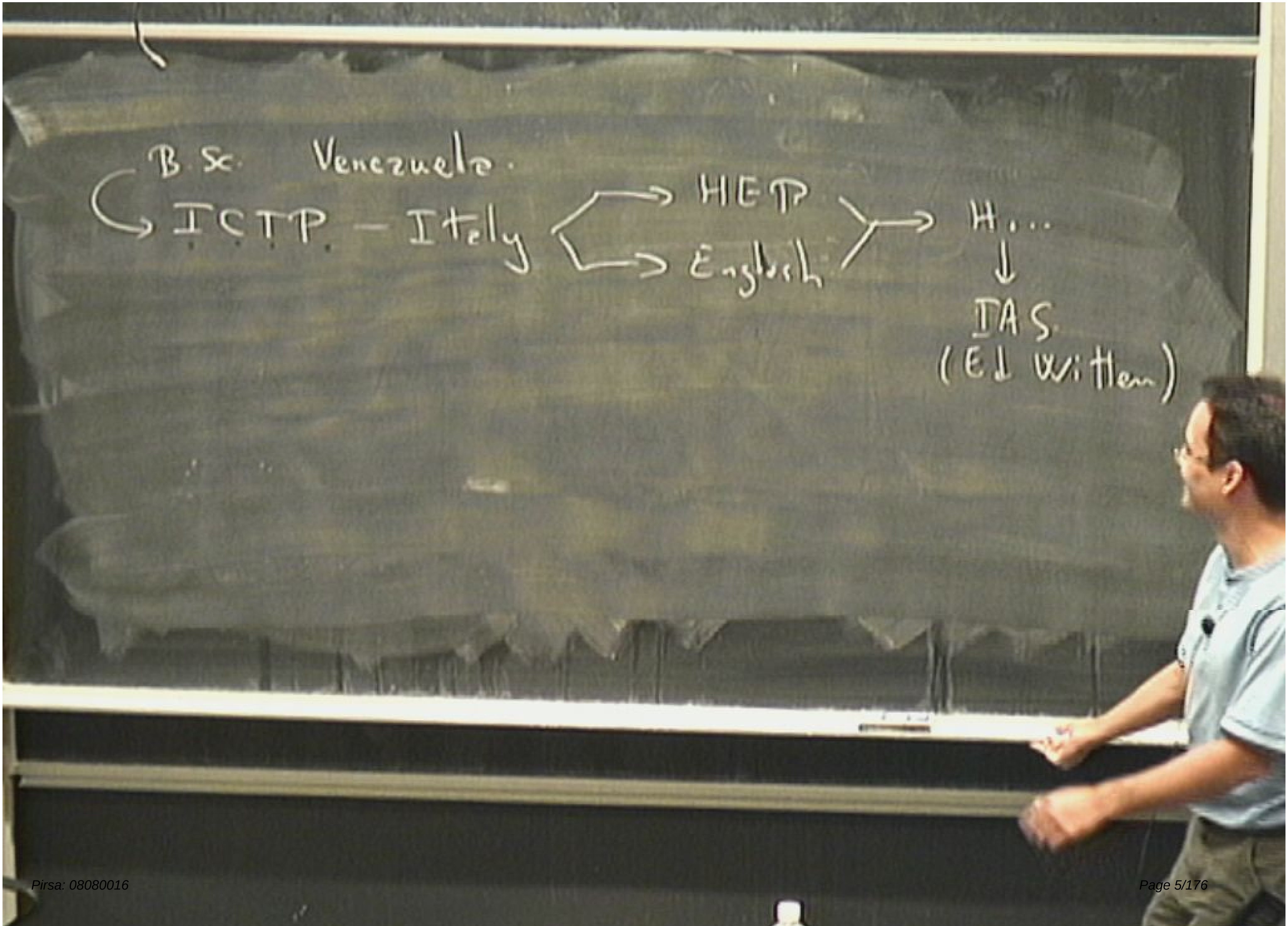
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B Sc. Venezuela.

ICTP - Italy → HEP → H...

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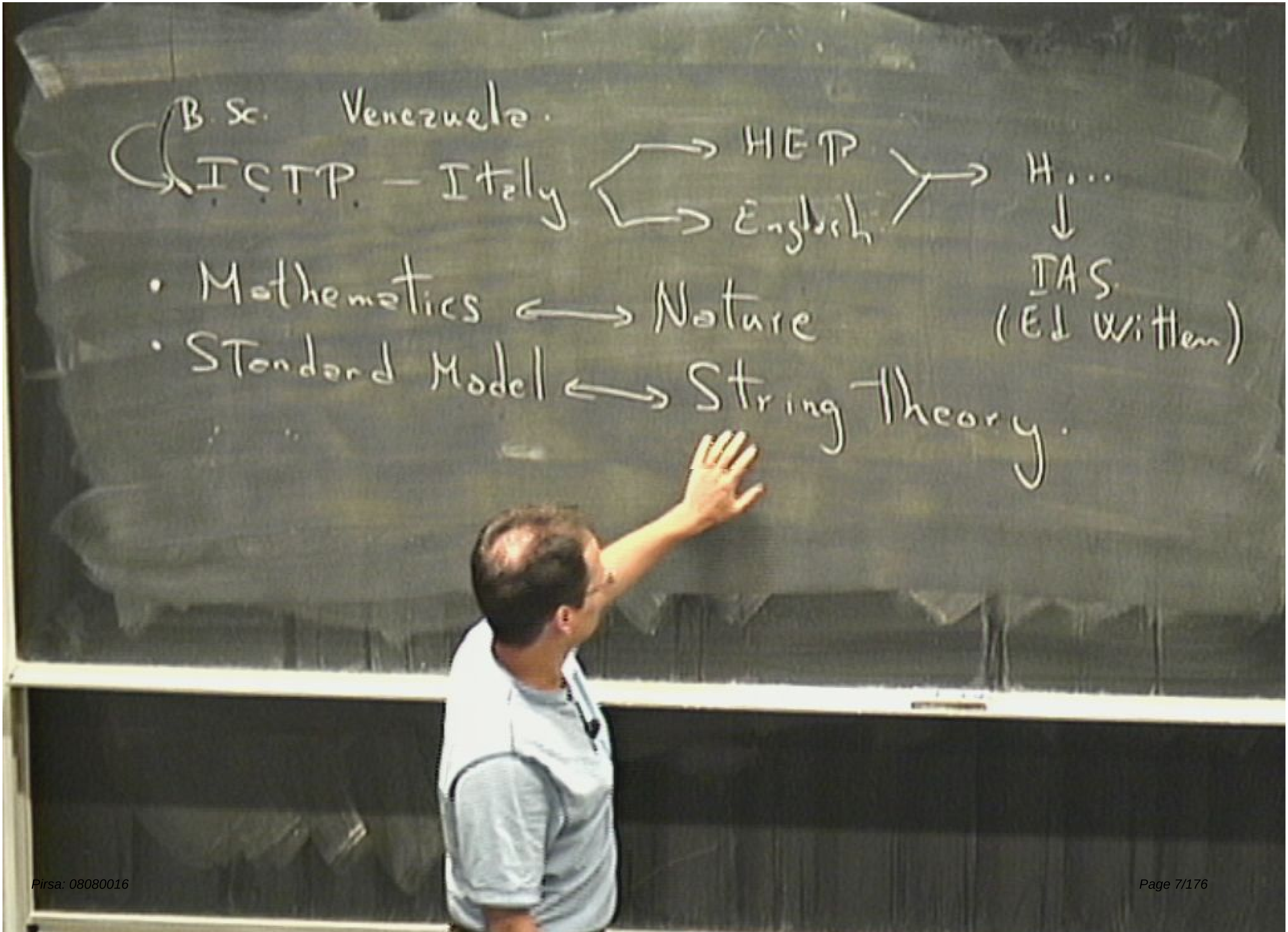
(Ed Witten)

• Mathematics ⟷ Nature


B. Sc. Venezuela.
ICTP - Italy
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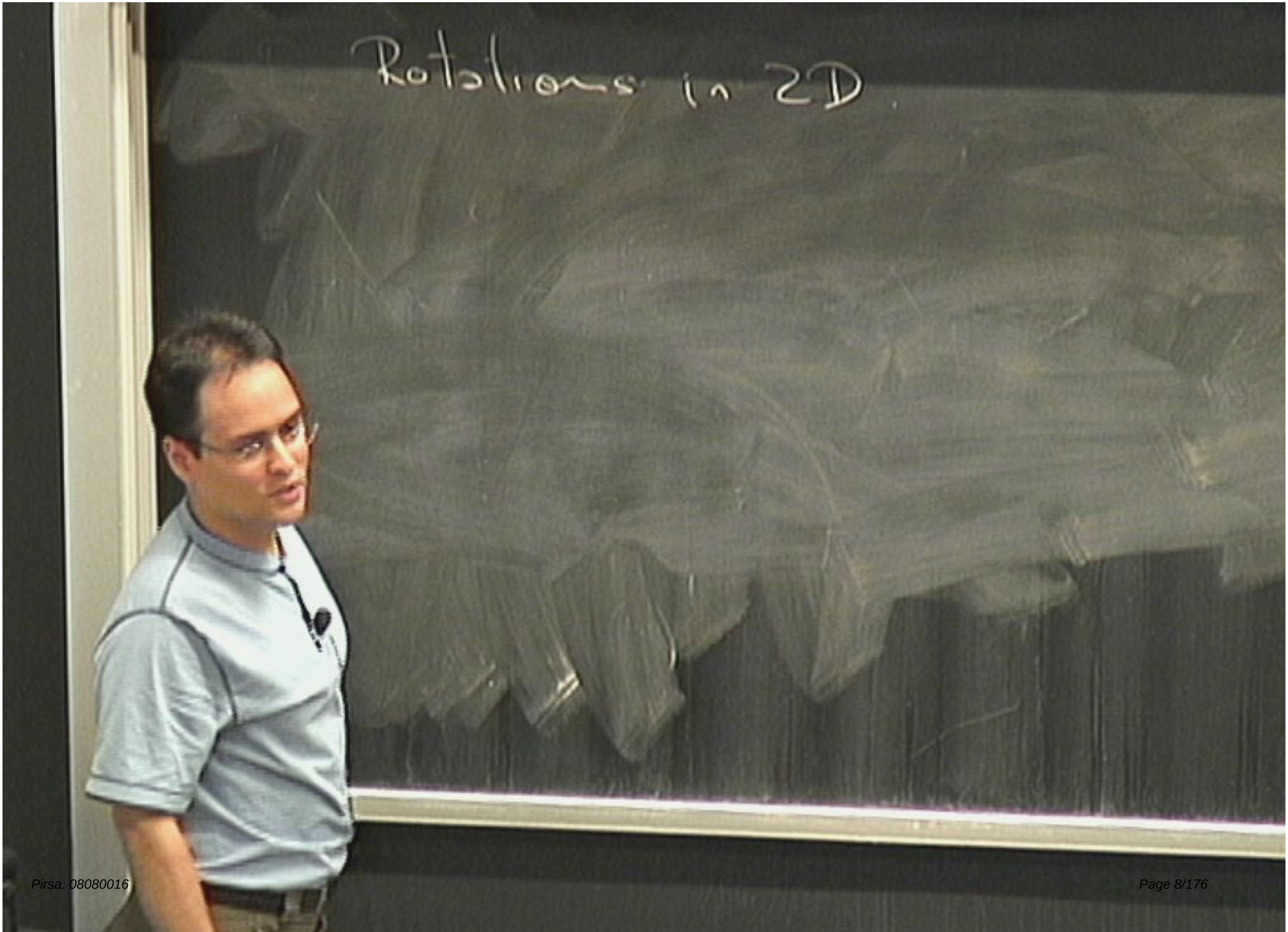
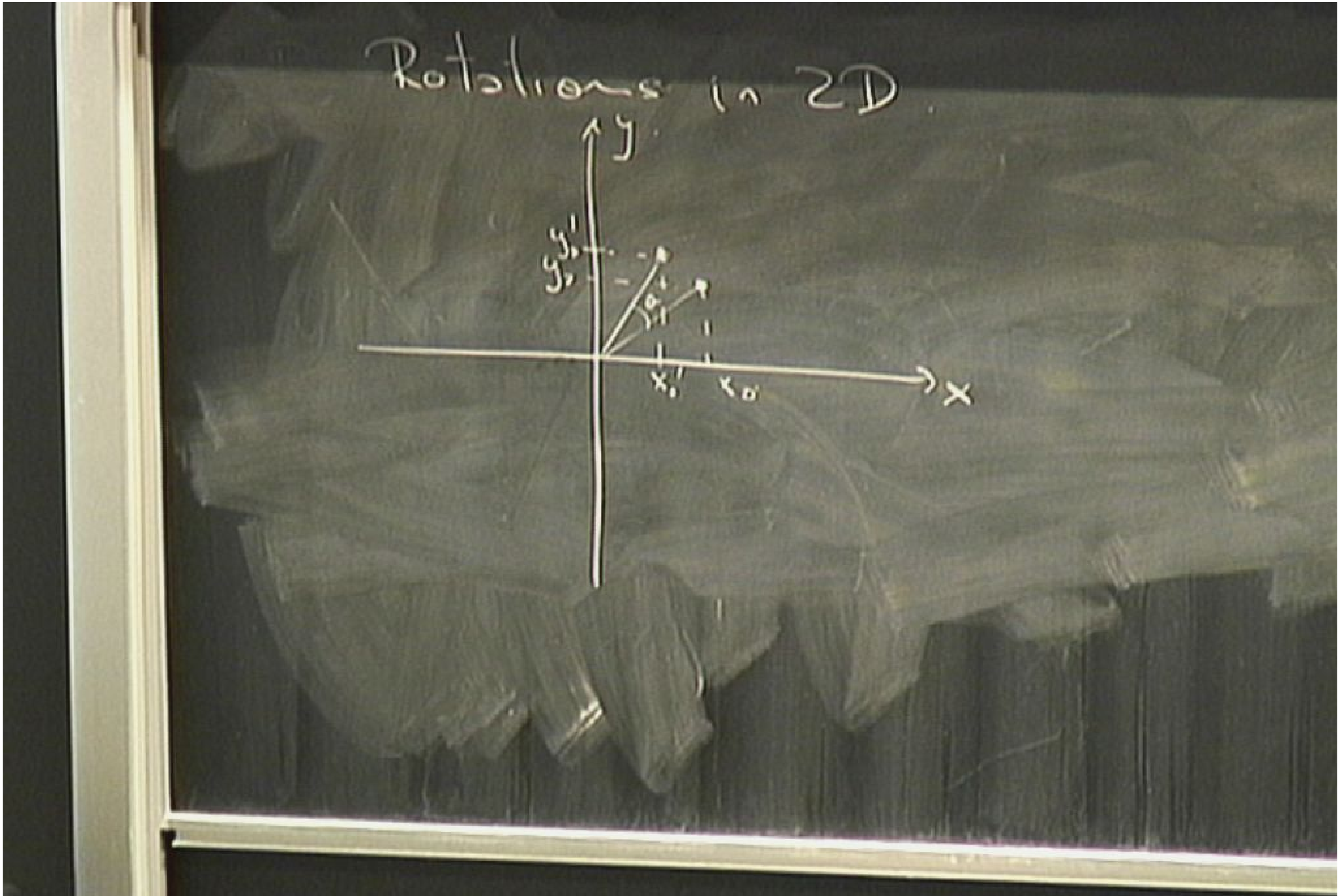
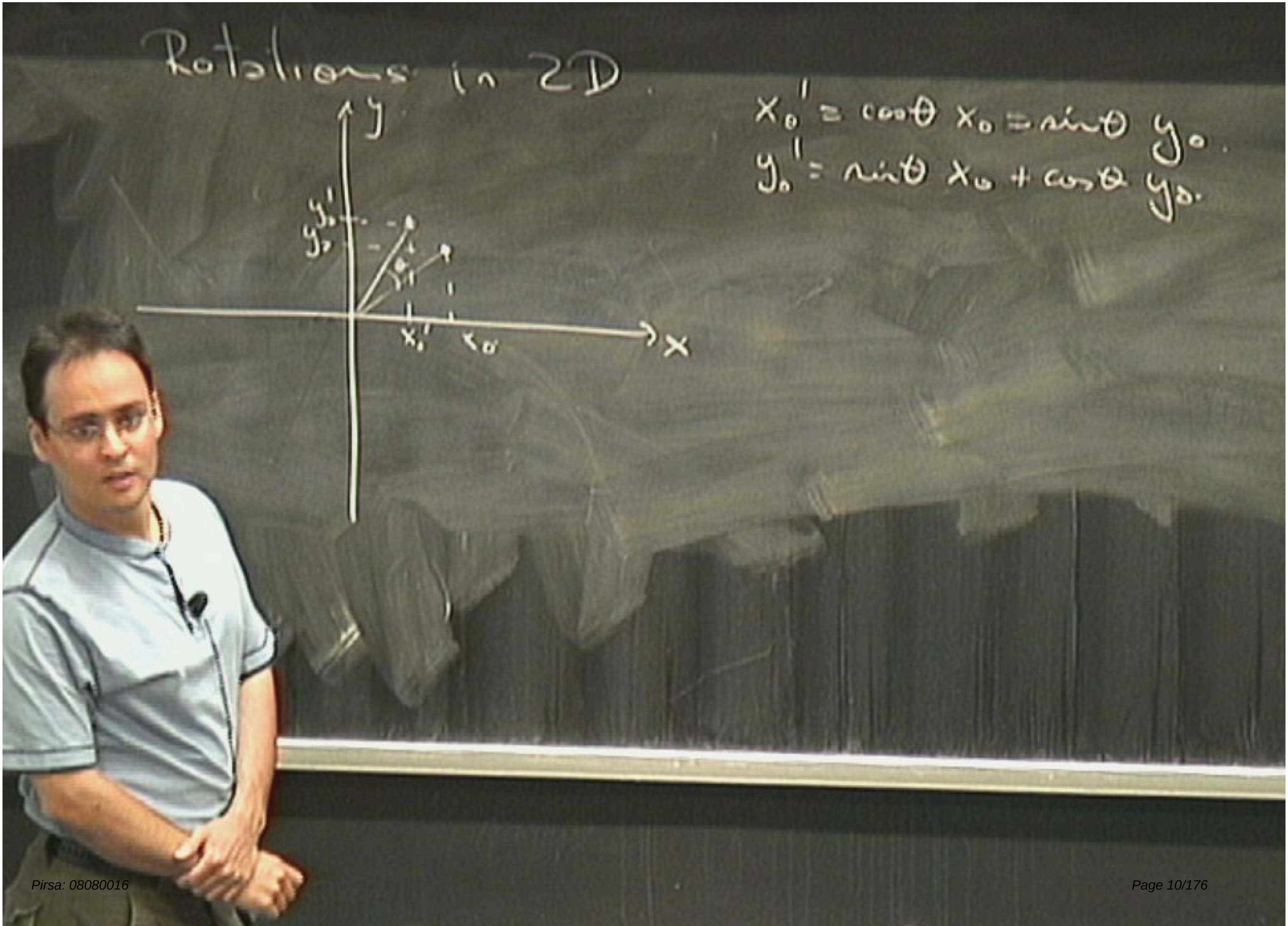
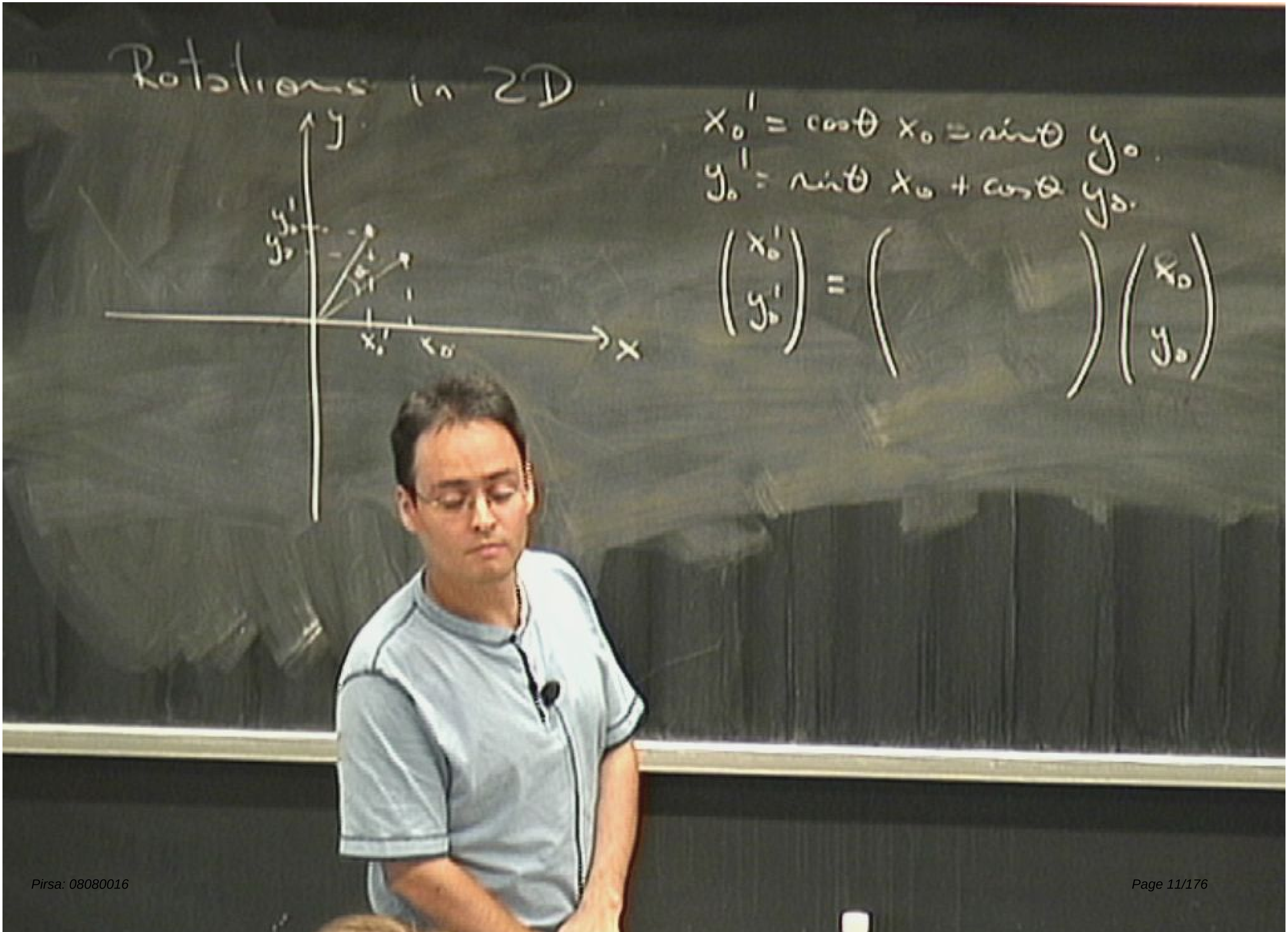
• Mathematics ⟷ Nature
• Standard Model ⟷ String Theory.


Rotations in 2D.


Rotations in 2D.
y
x


Rotations in 2D.
y
x
x0' = cosθ x0 − sinθ y0.
y0' = sinθ x0 + cosθ y0.


Rotations in 2D
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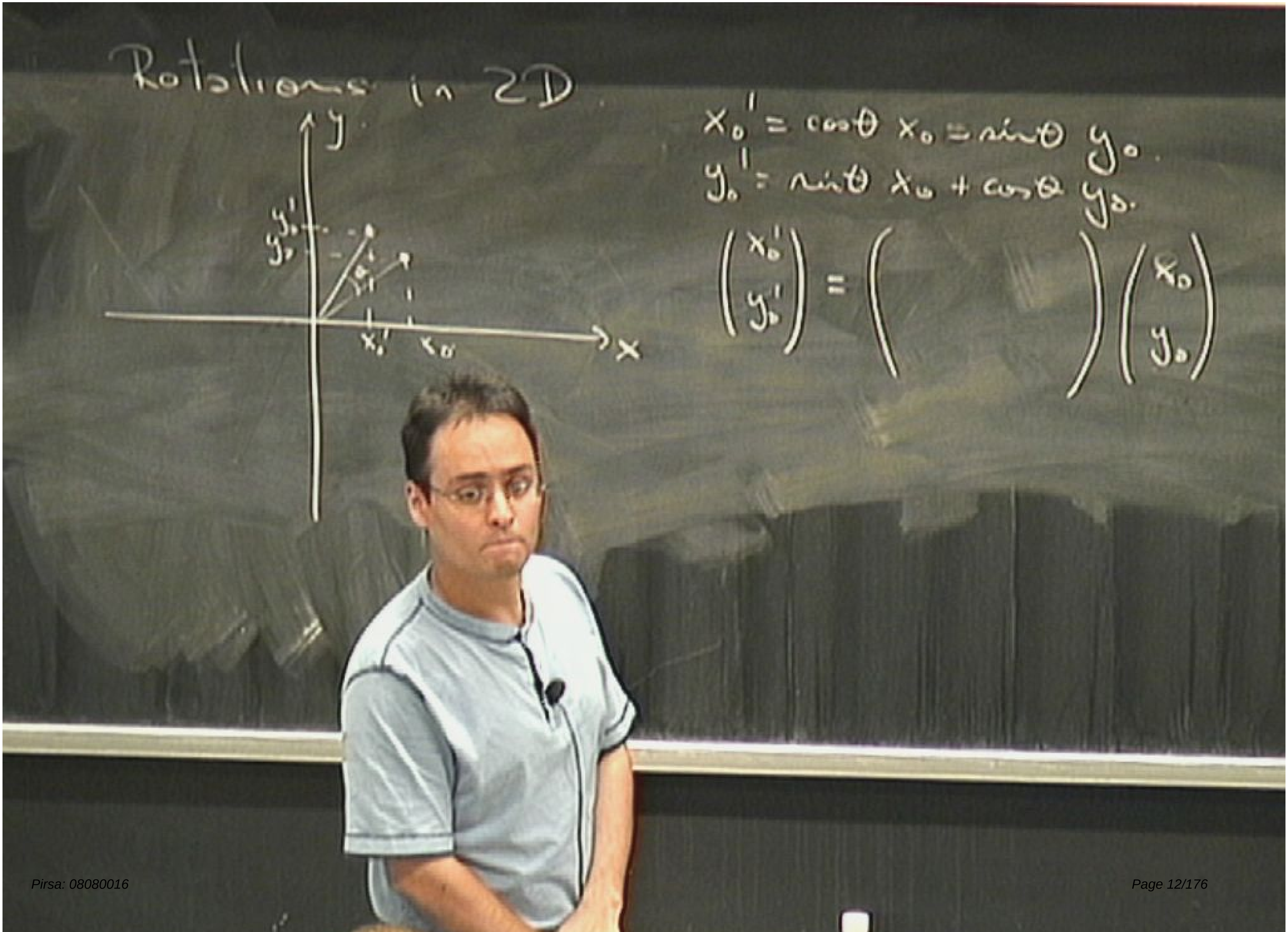
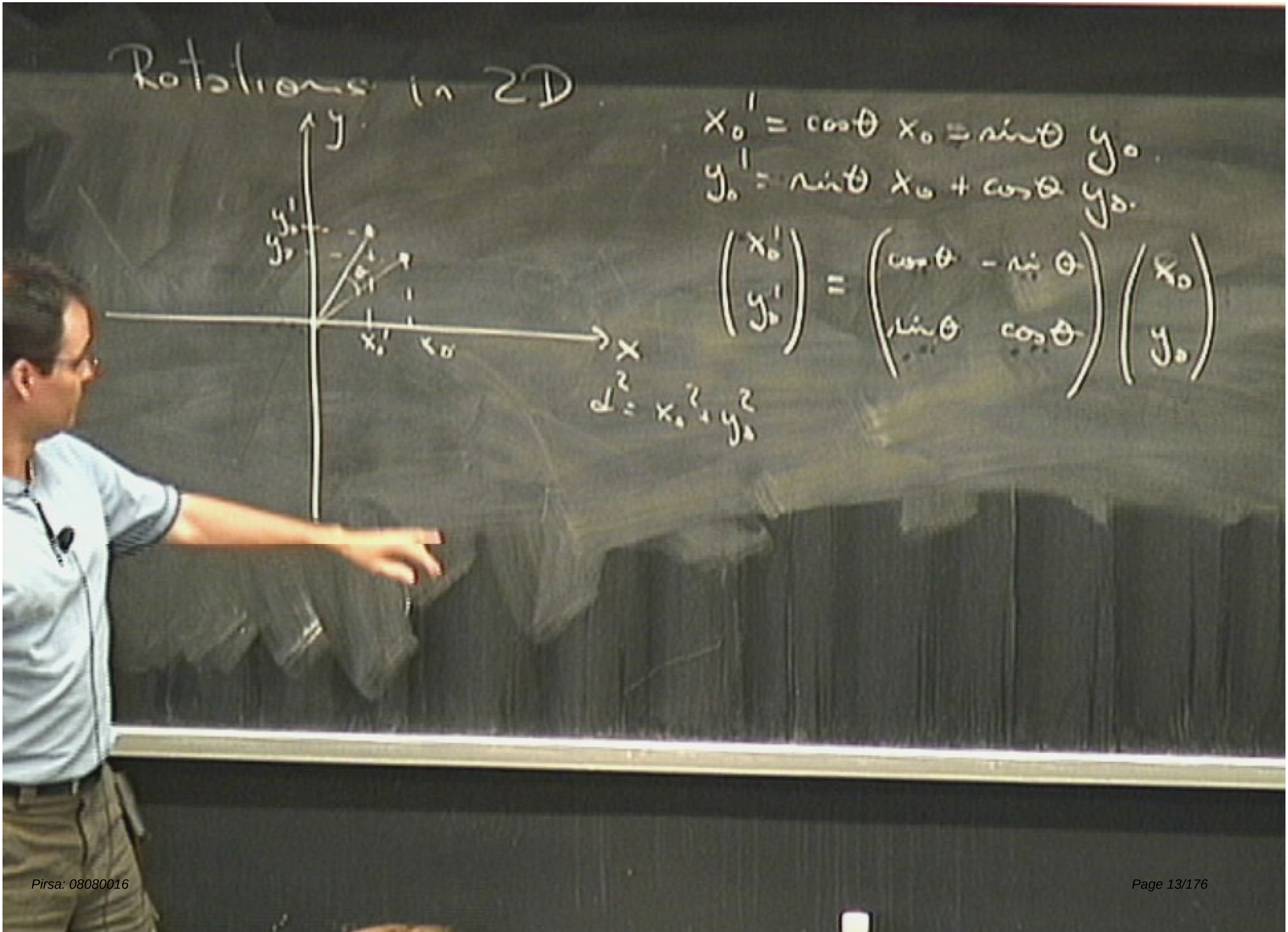
x
x0' = cosθ x0 − sinθ y0
y0' = sinθ x0 + cosθ y0


Rotations in 2D
$x_0' = \cos\theta\, x_0 - \sin\theta\, y_0$
$y_0' = \sin\theta\, x_0 + \cos\theta\, y_0$
$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$
y
x
y_0'
y_0
x_0'
x_0
θ


Rotations in 2D
$x_0' = \cos\theta \, x_0 - \sin\theta \, y_0$
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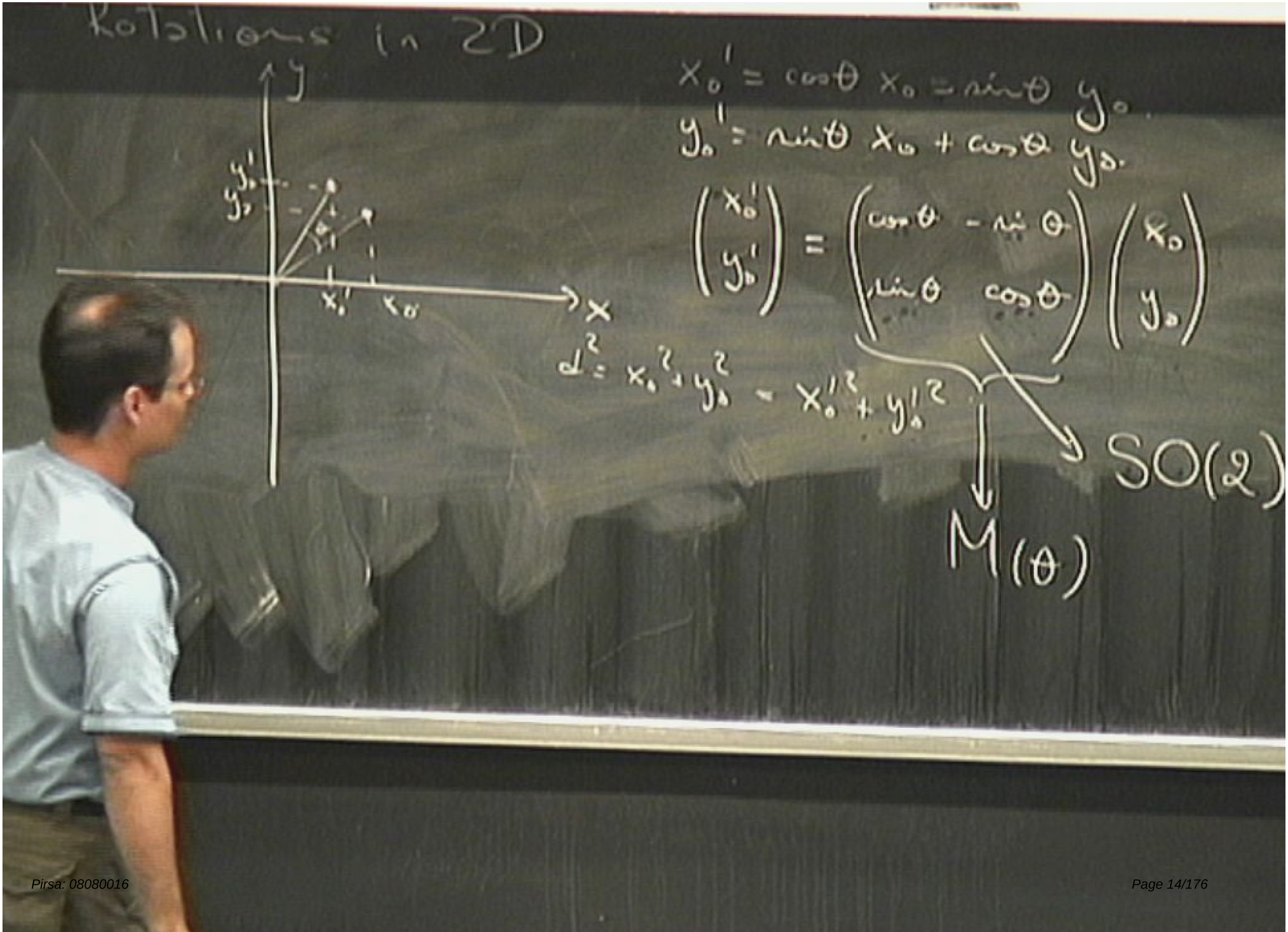
$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$
$d^2 = x_0^2 + y_0^2$
y
x
y_0'
y_0
x_0'
x_0
θ


Rotations in 2D
$x_0' = \cos\theta\, x_0 - \sin\theta\, y_0$
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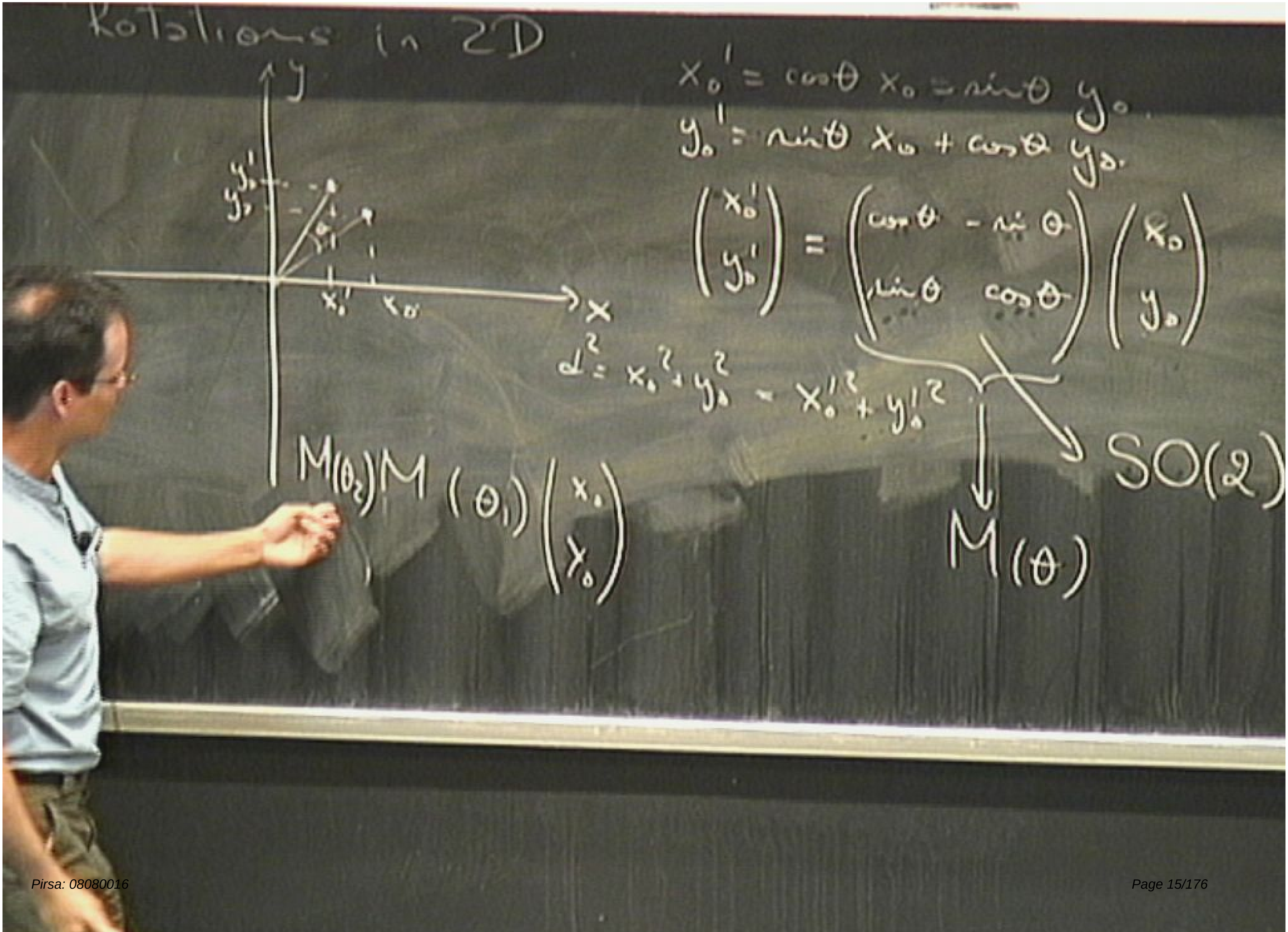
$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$
$d^2 = x_0^2 + y_0^2 = x_0'^2 + y_0'^2$
$M(\theta)$
$SO(2)$
y
x
y_0'
y_0
x_0'
x_0
θ


Rotations in 2D
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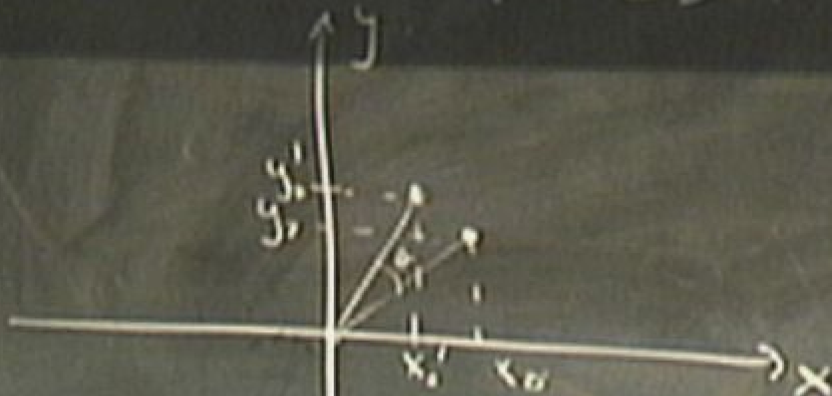
$d^2 = x_0^2 + y_0^2 = x_0'^2 + y_0'^2$
$M(\theta)$
$SO(2)$
$M(\theta_2) M(\theta_1) \begin{pmatrix} x_0 \\ x_0 \end{pmatrix}$

Rotations in 2D



$$x_0' = \cos\theta\, x_0 - \sin\theta\, y_0$$

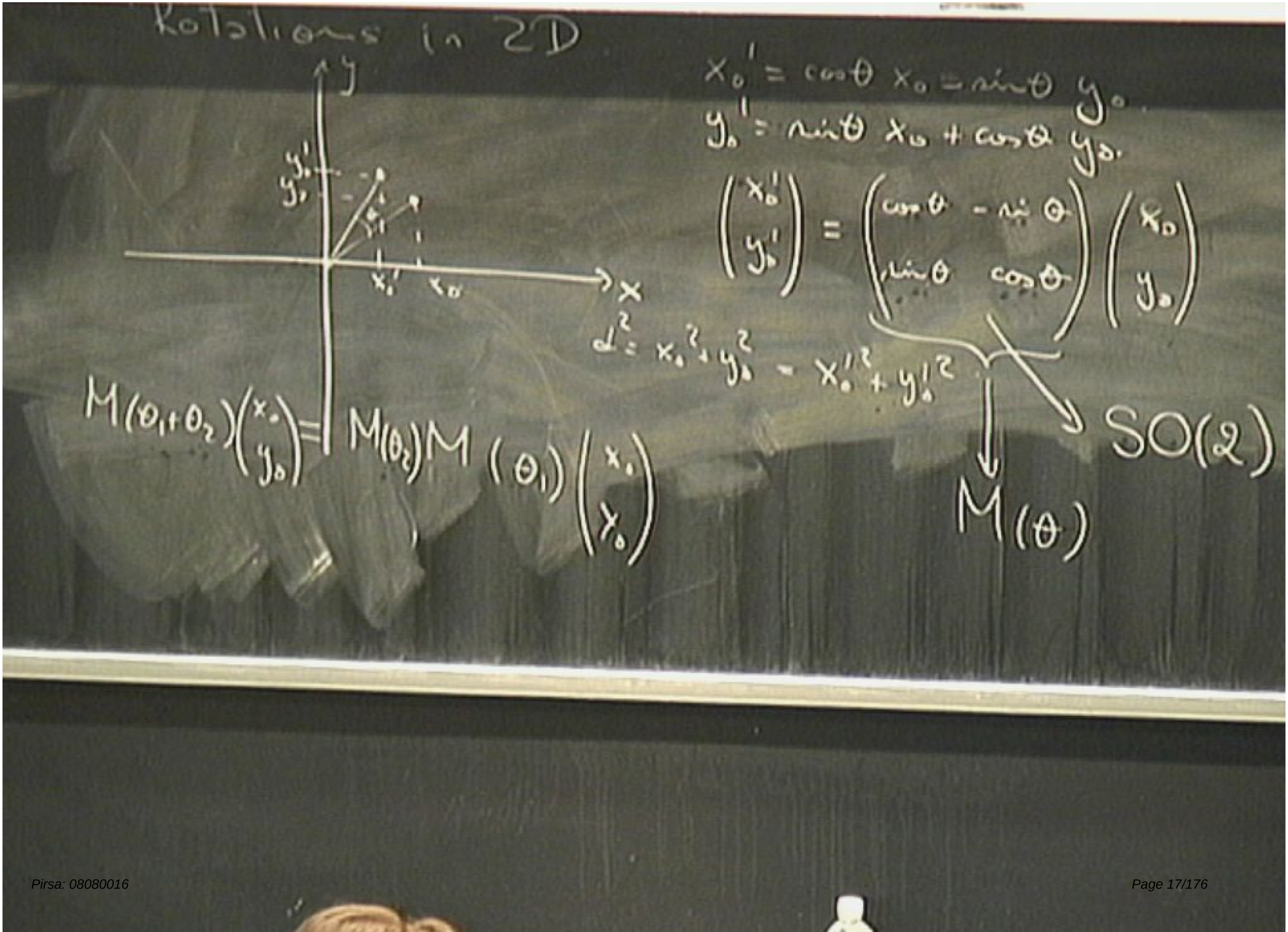
$$y_0' = \sin\theta\, x_0 + \cos\theta\, y_0$$

$$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{M(\theta)} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

SO(2)

$$d^2 = x_0^2 + y_0^2 = x_0'^2 + y_0'^2$$

$$M(\theta_1+\theta_2)\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = M(\theta_2)M(\theta_1)\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$


Rotations in 2D
x
y
$x_0' = \cos\theta\, x_0 - \sin\theta\, y_0$
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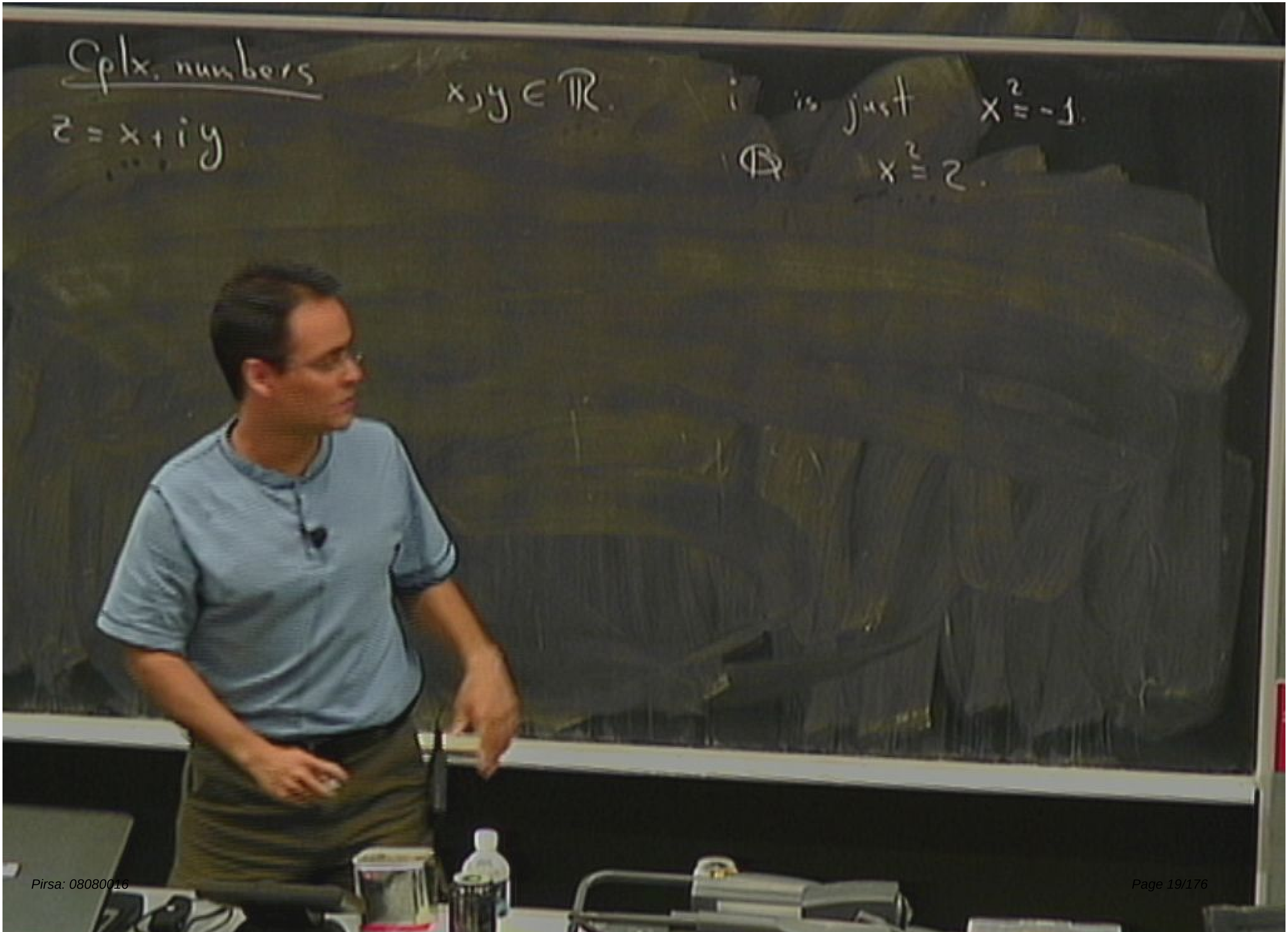
$d^2 = x_0^2 + y_0^2 = x_0'^2 + y_0'^2$
SO(2)
M(θ)
$M(\theta_1+\theta_2)\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = M(\theta_2) M(\theta_1) \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$

Cplx. numbers

$z = x + iy \qquad x, y \in \mathbb{R}. \qquad i$ is just $x^2 = -1$.


Cplx. numbers
x, y ∈ ℝ.
i is just x² = -1.
z = x + iy
ℂ x² = z.

Cplx. numbers $x, y \in \mathbb{R}$. i is just $x^2 = -1$.

$z = x + iy$

$\mathbb{Q}$ $x^2 = 2$. $x = \sqrt{2}$

Cplx. numbers $\quad x, y \in \mathbb{R}. \quad i$ is just $x^2 = -1.$

$z = x + i y. \quad \mathbb{Q} \quad x^2 = 2. \quad x = \sqrt{2}$

Cplx. numbers

$x, y \in \mathbb{R}$.

i is just $x^2 = -1$.

$\mathbb{Q}$ $x^2 = 2$. $x = \sqrt{2}$

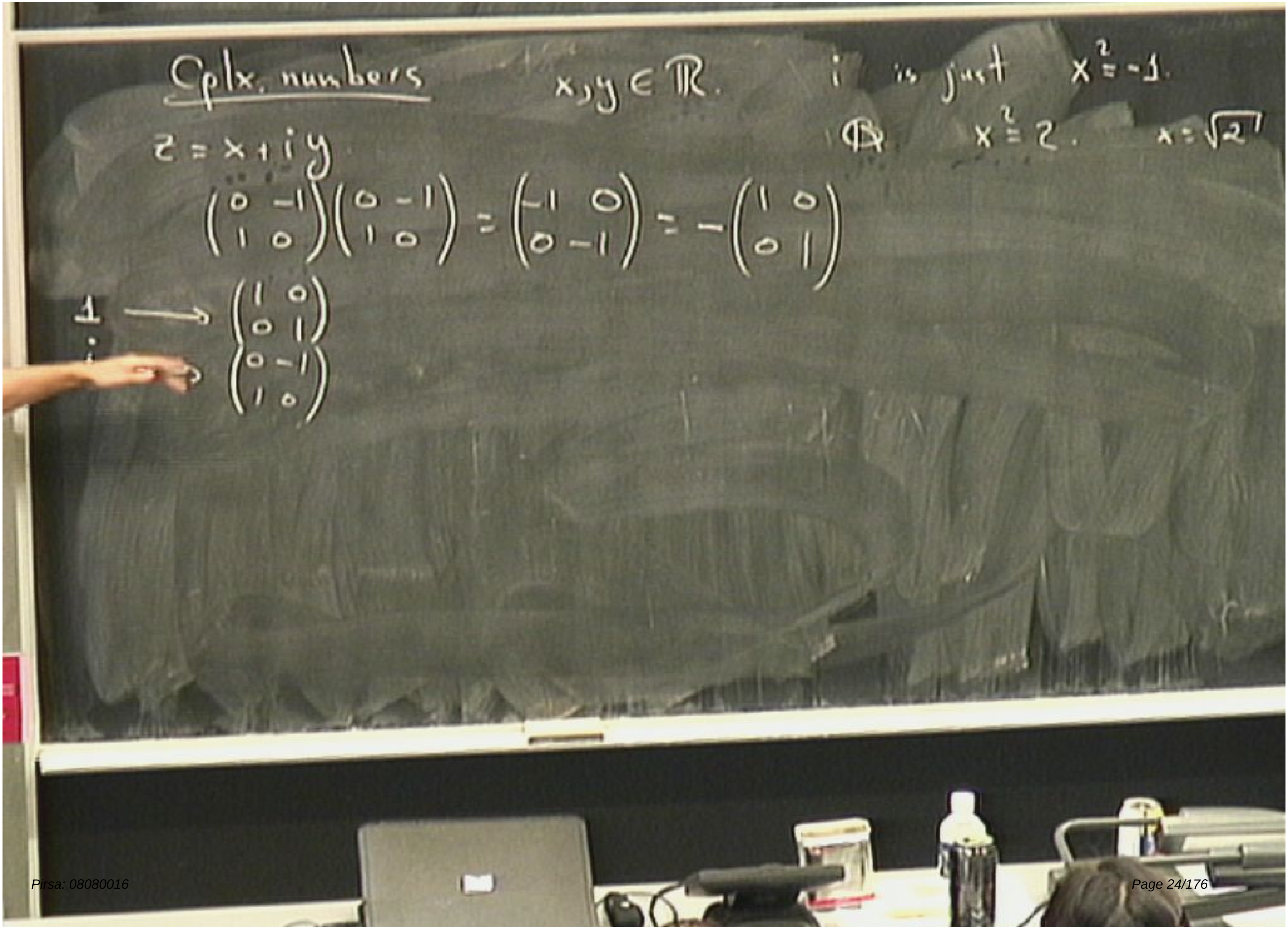
$z = x + iy$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Cplx. numbers $\quad x, y \in \mathbb{R}$. $\quad i$ is just $x^2 = -1$.

$z = x + iy$ $\quad \mathbb{Q} \quad x^2 = 2. \quad x = \sqrt{2}$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$


Cplx. numbers
$x, y \in \mathbb{R}$.
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$z = x + iy$
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$1 \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
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Special: $\qquad$ Euler.

Cplx. numbers $\quad x, y \in \mathbb{R}$. $\quad i$ is just $x^2 = -1$.

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Special: $\quad$ Euler. $\quad e^{i\pi} + 1 = 0$

z

Cplx. numbers

$x, y \in \mathbb{R}$. i is just $x^2 = -1$.

$z = x + iy$

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$$i \longrightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Special: Euler.

$$u = e^{i\theta} \qquad \theta \in \mathbb{R}. \qquad 0 < \theta < 2\pi$$

$$e^{i\pi} + 1 = 0$$

$$i \longrightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad z \longrightarrow \begin{pmatrix} x & -y \\ y & x \end{pmatrix}$$

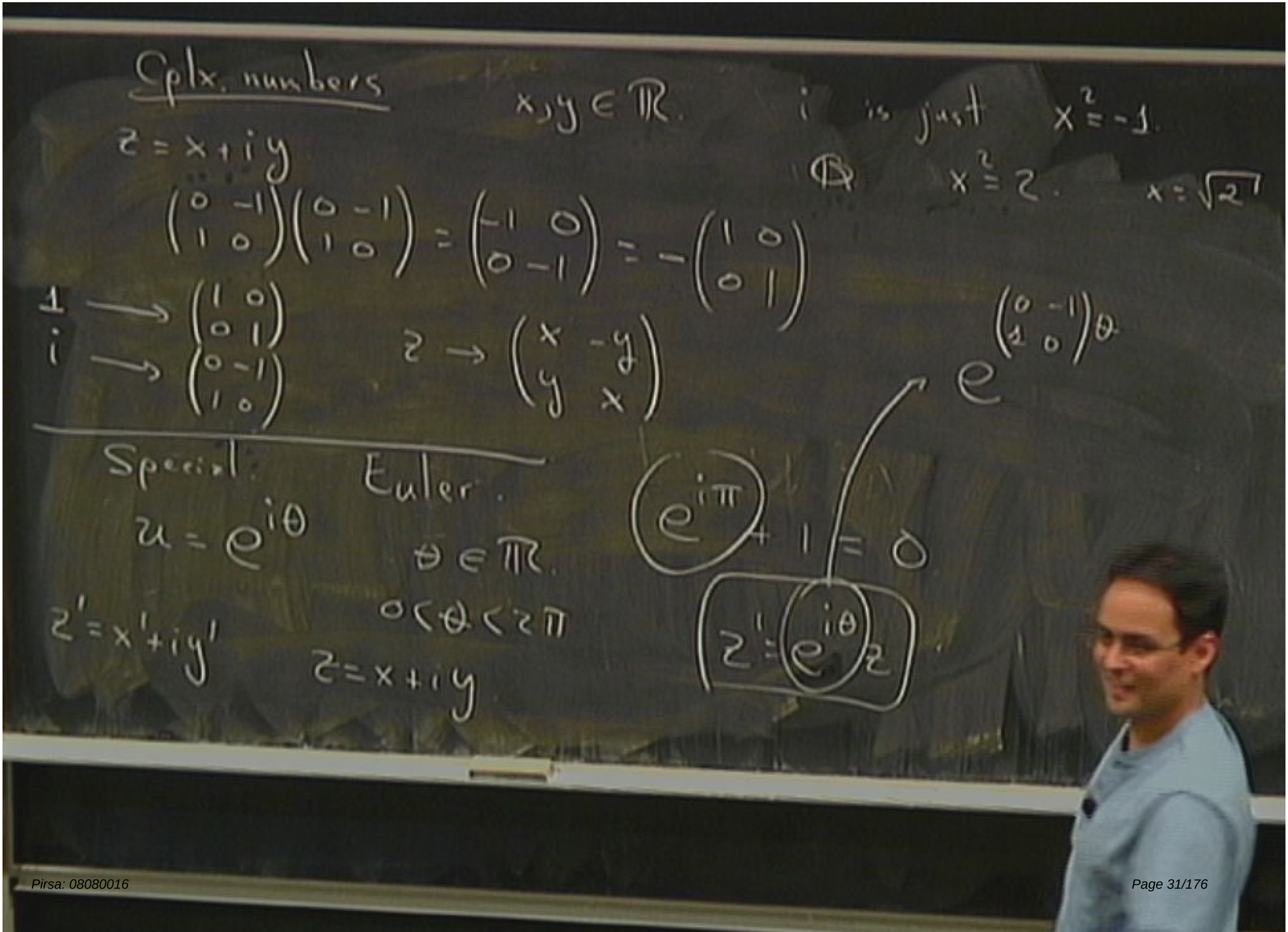
Special: Euler.

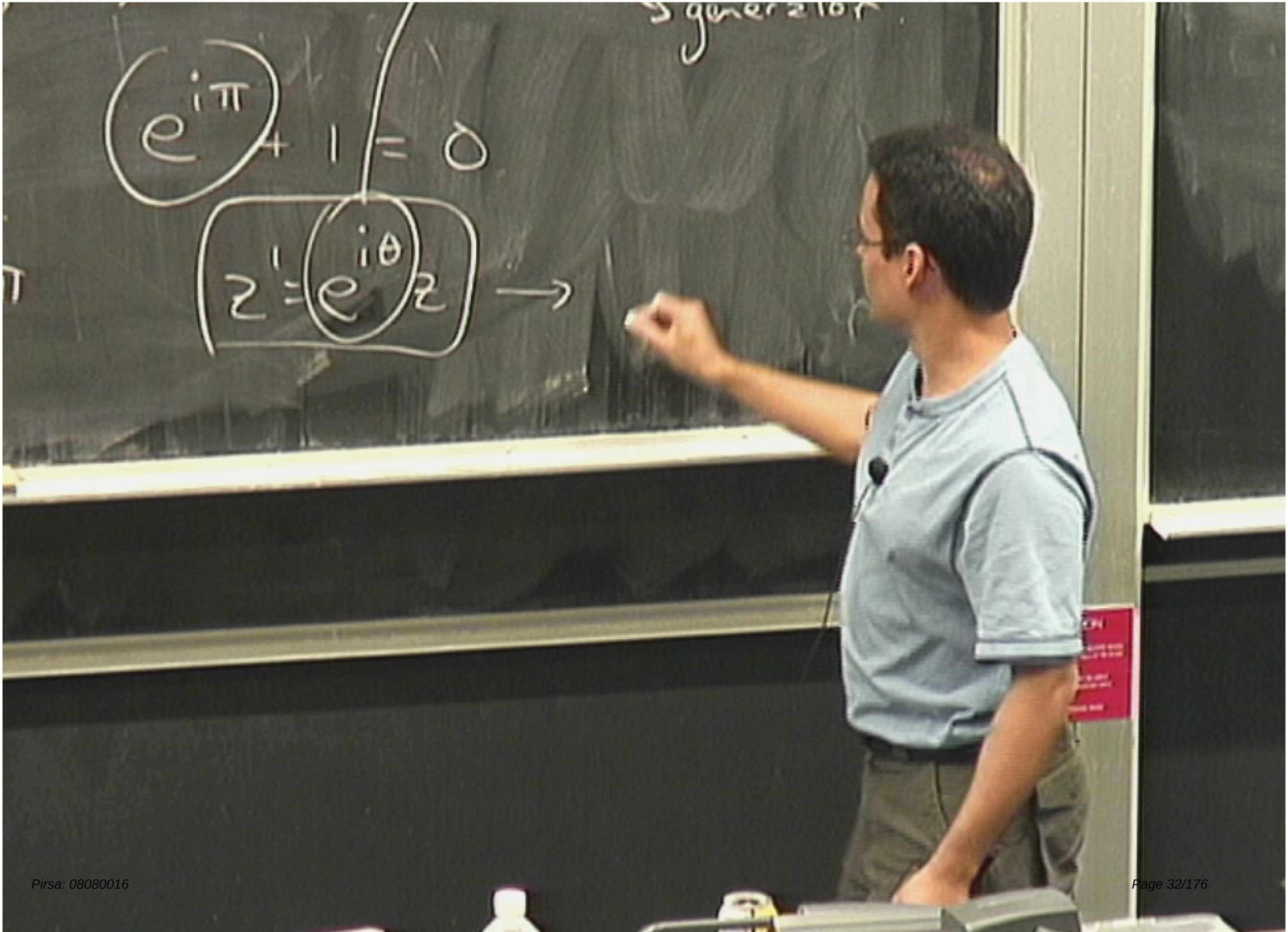
$$e^{i\pi} + 1 = 0$$

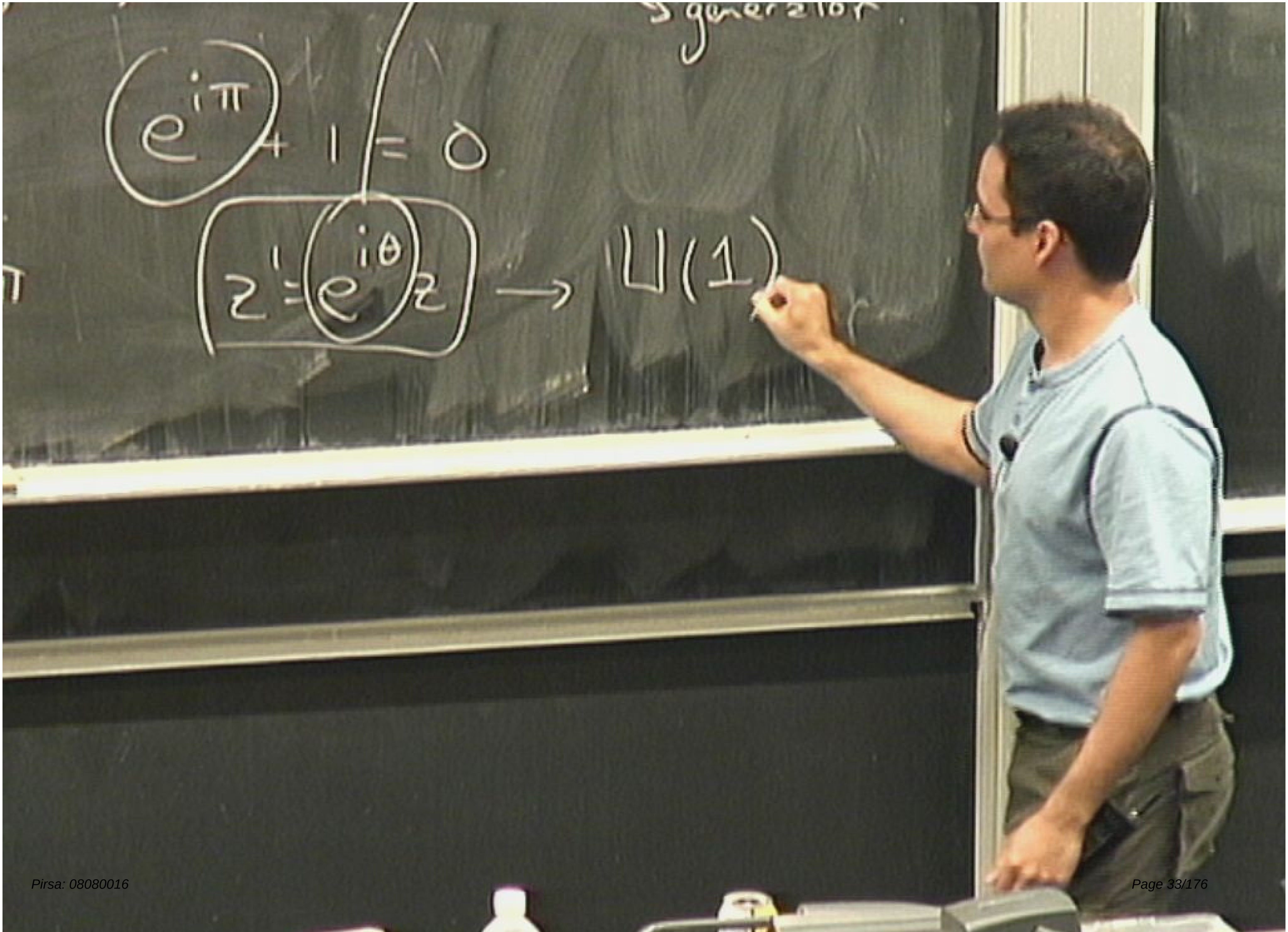
$$u = e^{i\theta} \qquad \theta \in \mathbb{R}$$

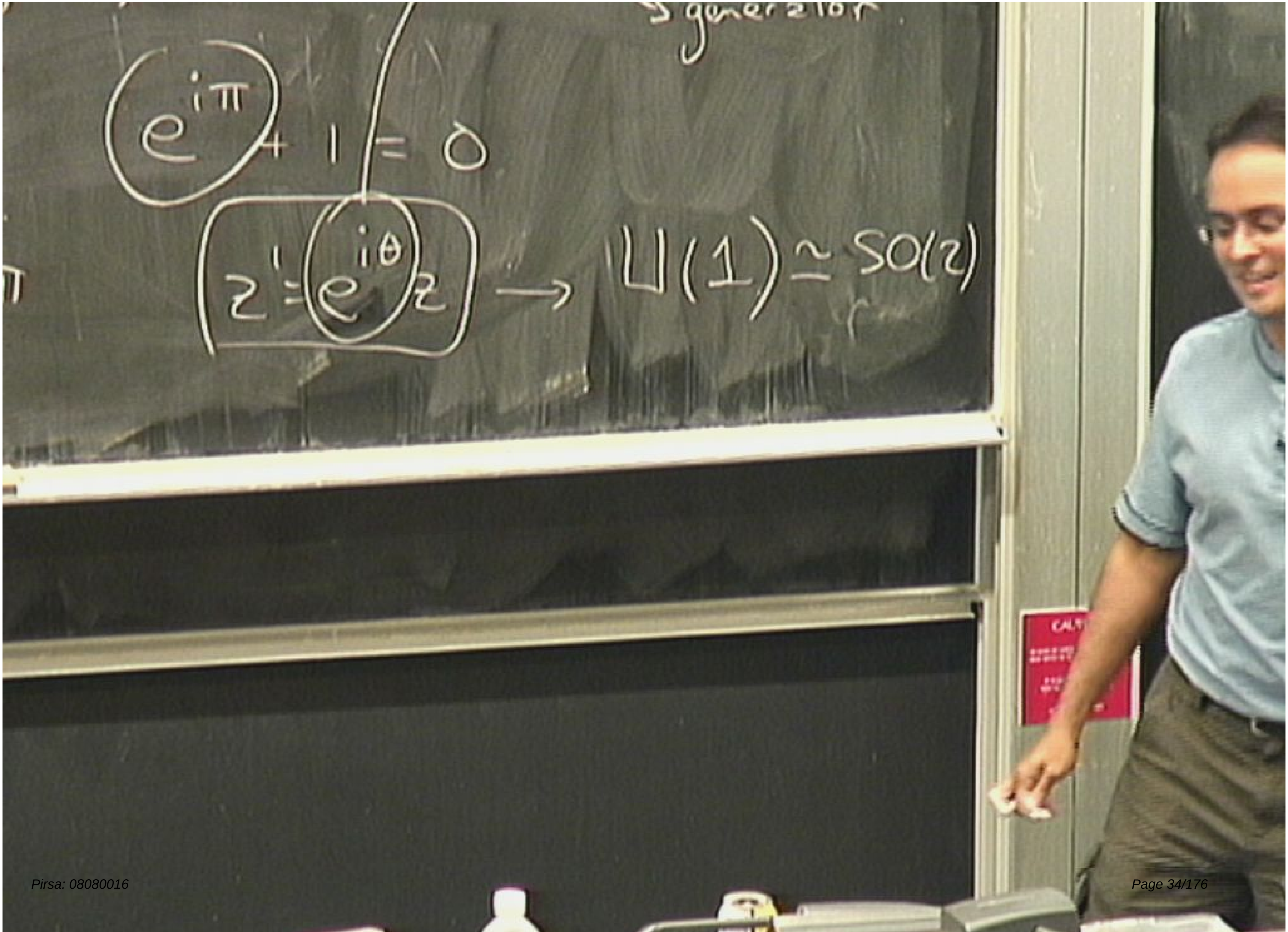
$$0 < \theta < 2\pi \qquad z' = e^{i\theta} z$$

$$z' = x' + iy' \qquad z = x + iy$$


Cplx. numbers
x, y ∈ ℝ
i is just x² = -1
z = x + iy
x² = 2
x = √2
(0 -1; 1 0)(0 -1; 1 0) = (-1 0; 0 -1) = -(1 0; 0 1)
1 → (1 0; 0 1)
i → (0 -1; 1 0)
z → (x -y; y x)
e^{(0 -1; 1 0)θ}
Special:
u = e^{iθ}
Euler:
θ ∈ ℝ
e^{iπ} + 1 = 0
0 < θ < 2π
z' = x' + iy'
z = x + iy
z' = e^{iθ} z


generator
e^{iπ} + 1 = 0
z' = e^{iθ} z →


generator
$e^{i\pi} + 1 = 0$
$z' = e^{i\theta} z \longrightarrow U(1)$


generator
e^{iπ} + 1 = 0
z' = e^{iθ} z → U(1) ≃ SO(2)

Cplx. numbers

$z = x + iy$ $\qquad x, y \in \mathbb{R}$. $\qquad i$ is just $x^2 = -1$.

$\mathbb{Q}$ $\qquad x^2 = 2$. $\qquad x = \sqrt{2}$

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$$1 \longrightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$i \longrightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$z \longrightarrow \begin{pmatrix} x & -y \\ y & x \end{pmatrix}$$

$$e^{\underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\theta}_{\text{generator}}}$$

Special. $\qquad$ Euler.

$u = e^{i\theta}$ $\qquad \theta \in \mathbb{R}$. $\qquad e^{i\pi} + 1 = 0$

$0 < \theta < 2\pi$

$z' = x' + iy'$ $\qquad z = x + iy$

$$\boxed{z' = e^{i\theta} z} \longrightarrow U(1) \simeq SO(2$$

Cplx. numbers $\quad x, y \in \mathbb{R} \quad$ i is just $x^2 = -1$

$z = x + iy \qquad \mathbb{Q} \quad x^2 = 2 \quad x = \sqrt{2}$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$e^{\underbrace{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}_{\text{generator}}\theta}$$

Special. $\qquad$ Euler. $\qquad e^{i\pi} + 1 = 0$

$u = e^{i\theta} \qquad \theta \in \mathbb{R}$

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$z' = x' + iy' \qquad z = x + iy$

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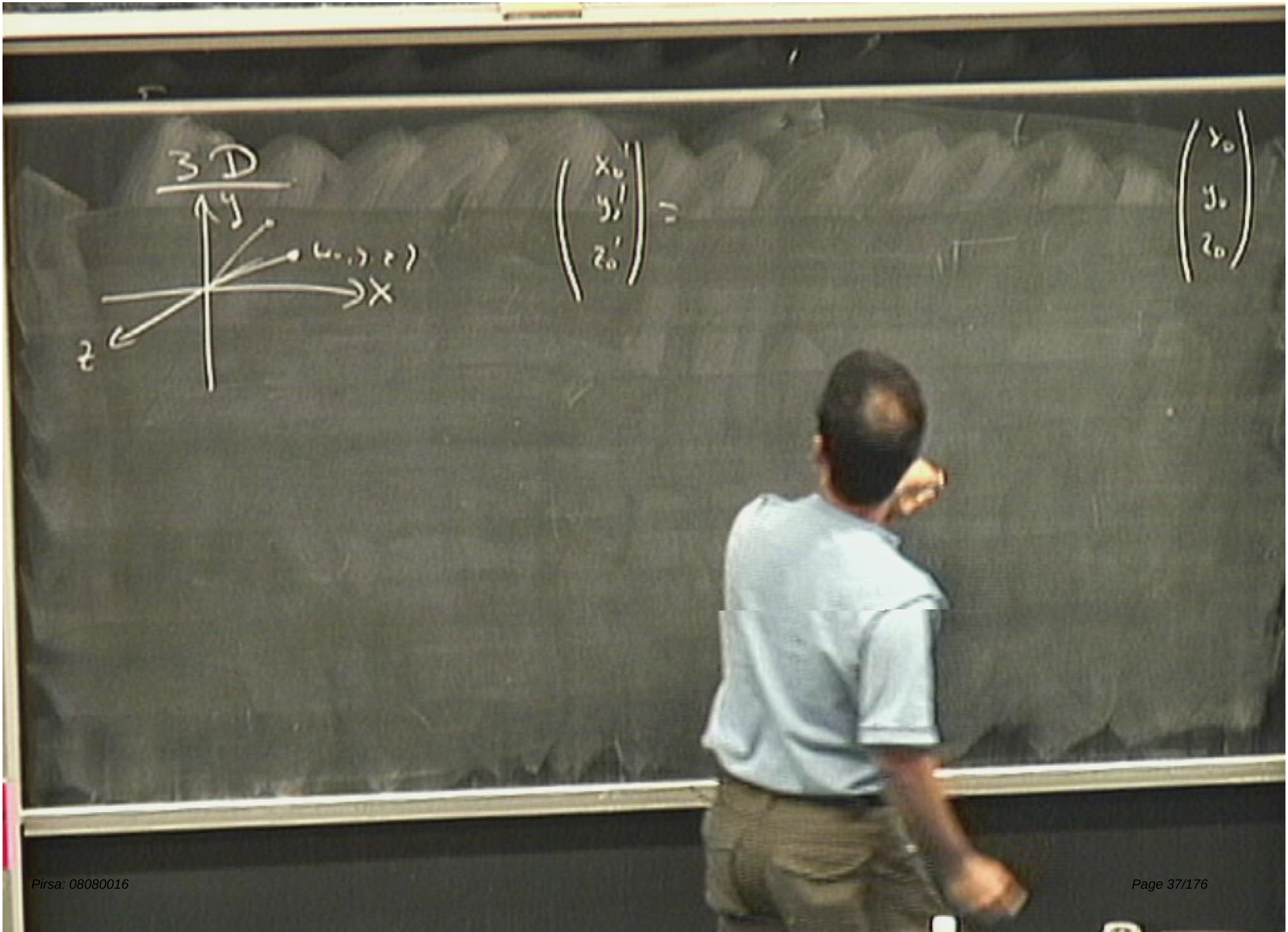
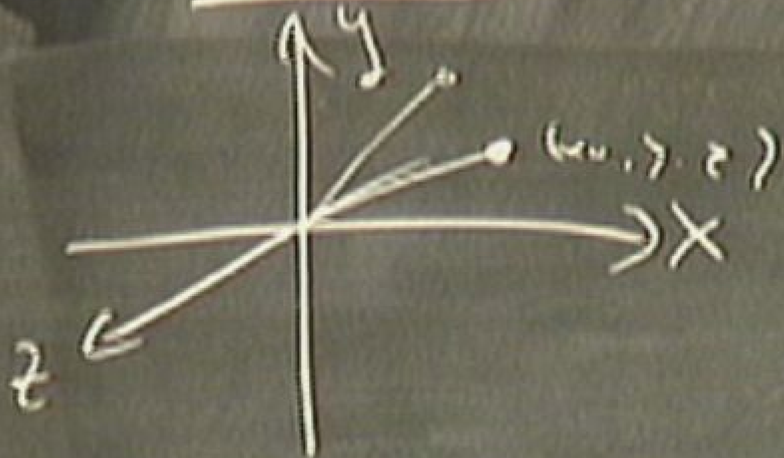
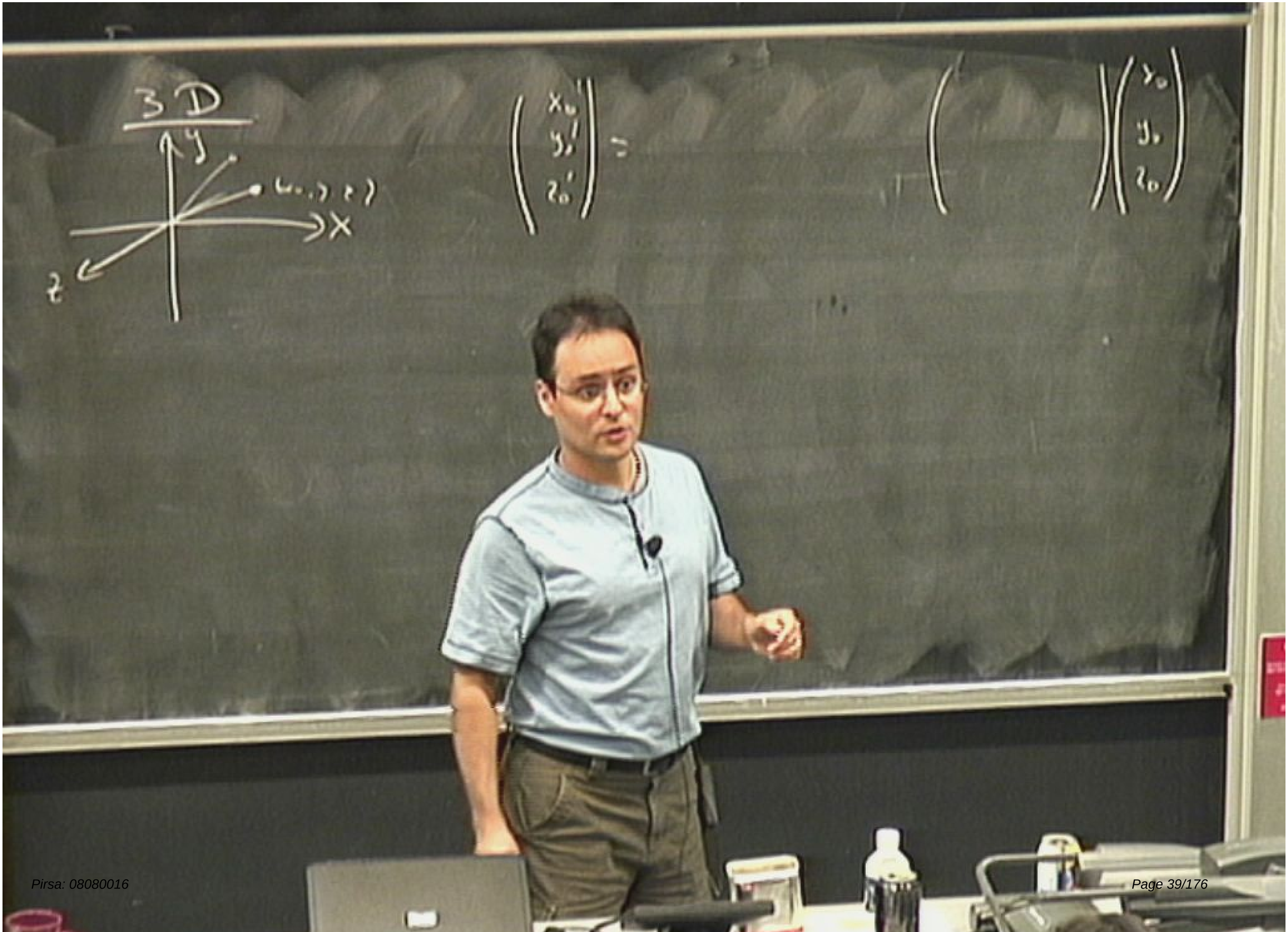

3 D
y
X
z
x₀'
y₀'
z₀'
=
x₀
y₀
z₀

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} =$$

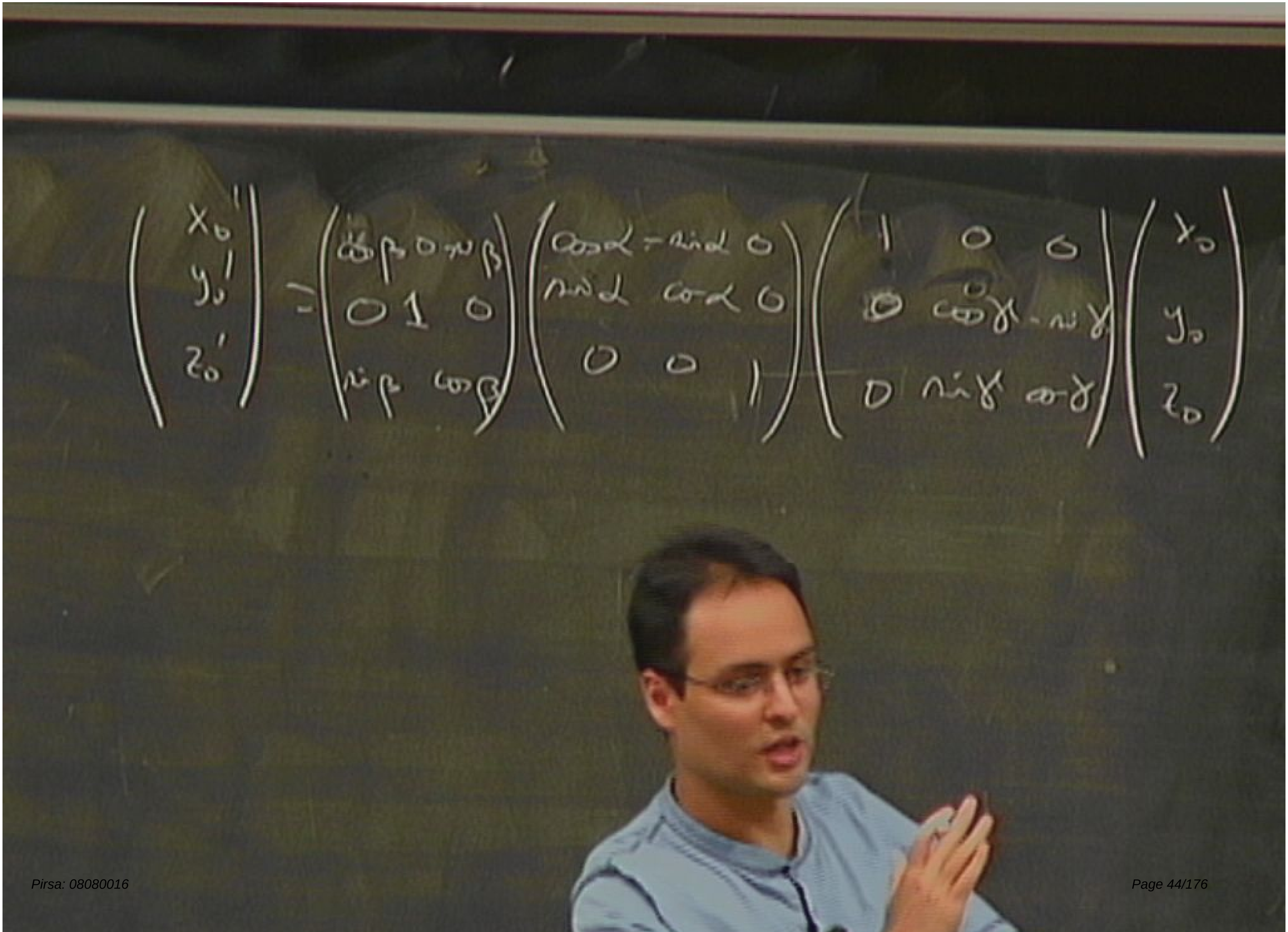

3 D
y
x
z

$$= \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

$$\begin{pmatrix} \\ \\ \\ \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

$$\begin{pmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\gamma & -\sin\gamma\\ 0 & \sin\gamma & \cos\gamma\end{pmatrix}\begin{pmatrix}x_0\\ y_0\\ z_0\end{pmatrix}$$

$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$


$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ \sin\beta & & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

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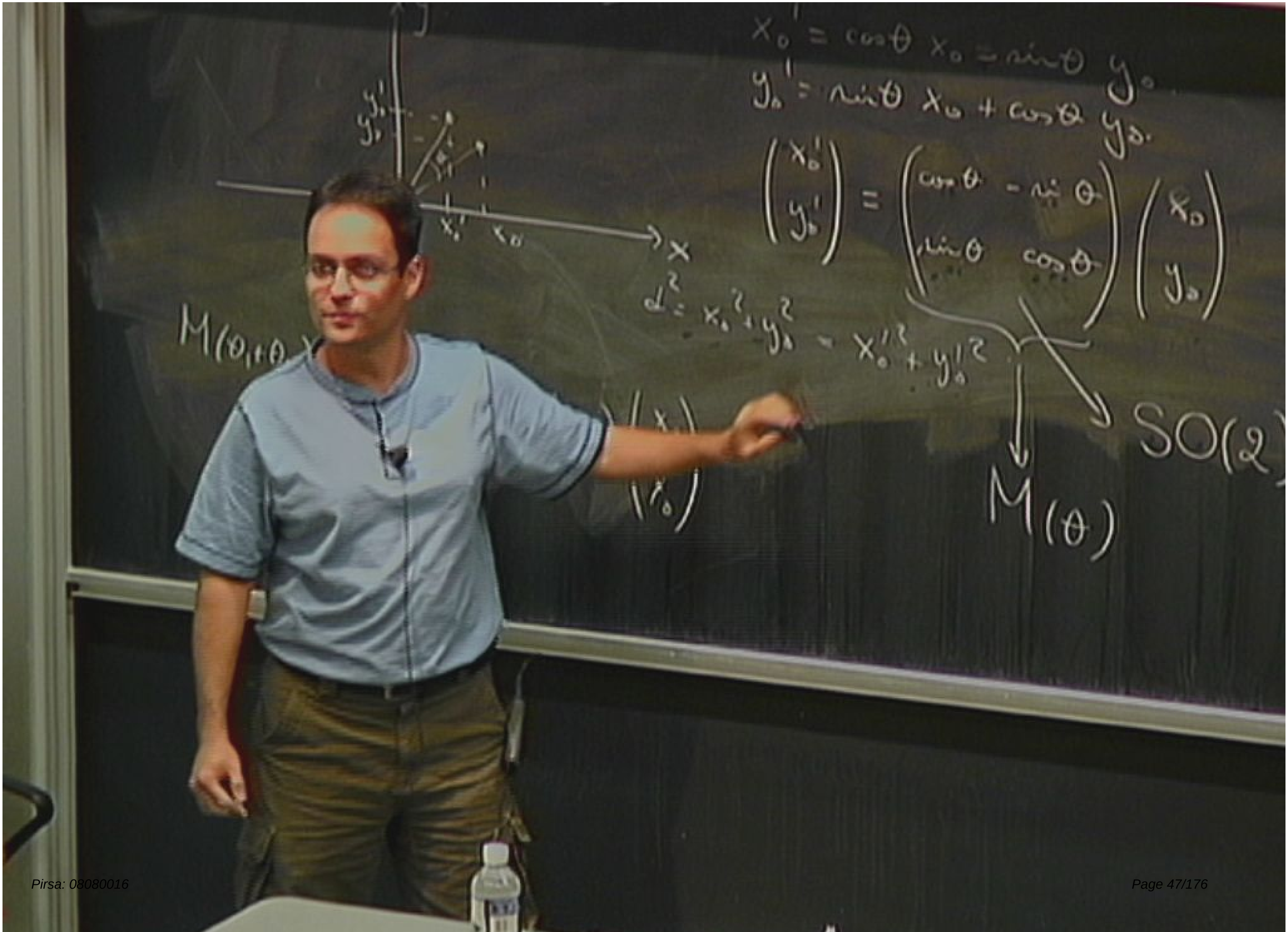
3 angles

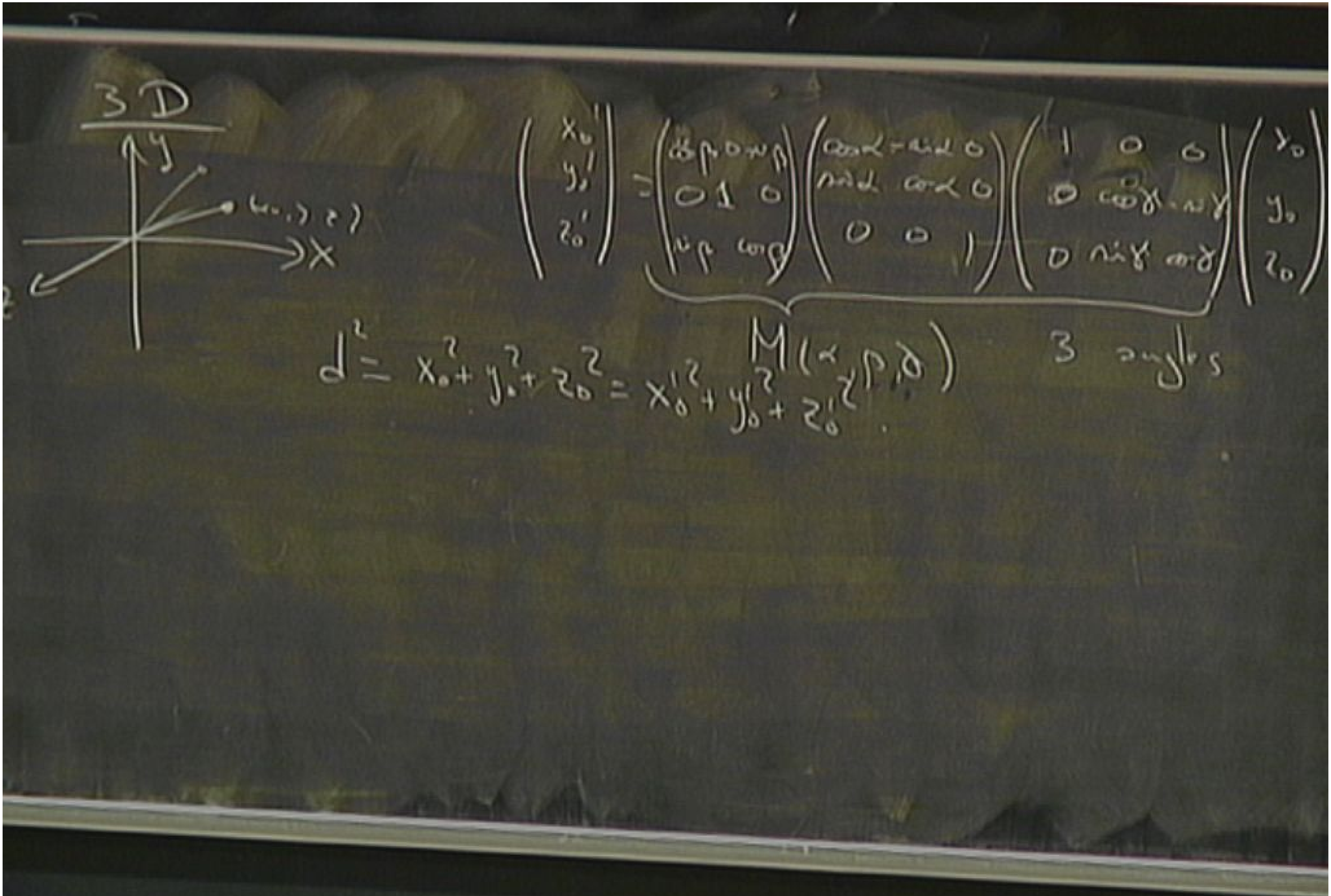
$(\ldots, z)$

$\rightarrow X$

$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ \sin\beta & & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles


x₀' = cosθ x₀ − sinθ y₀
y₀' = sinθ x₀ + cosθ y₀
d² = x₀² + y₀² = x₀'² + y₀'²
M(θ)
SO(2)


3D
y
x
z
$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$
3 angles
$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2$

$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

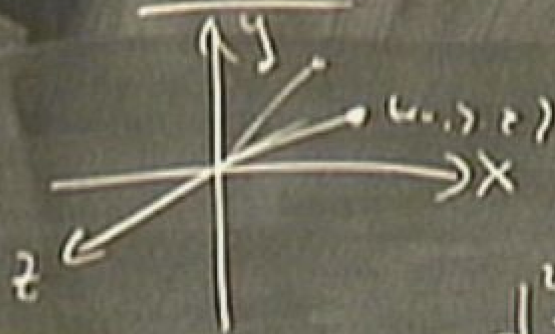
$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2$$

$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2.$$

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

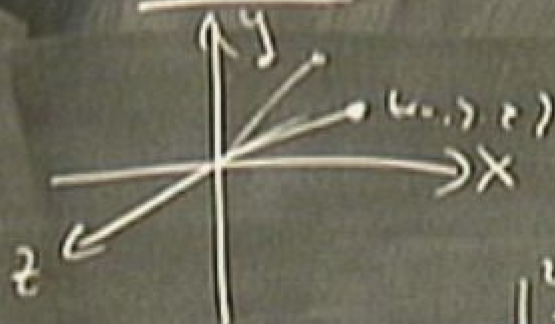
SO(3)

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

Q: Is there something like U(1)?

Yes

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

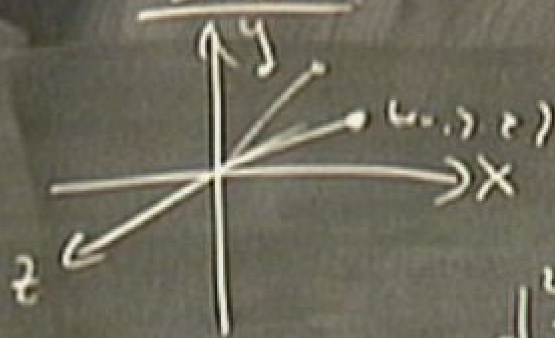
SO(3)

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

Q: Is there something like $U(1)$?

Yes?

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)}\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

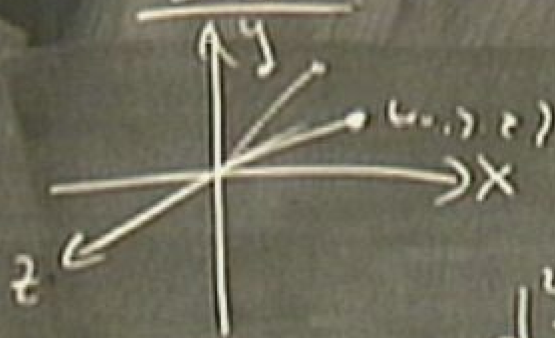
SO(3)

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

Q: Is there something like U(1)?

Yes $\overset{D}{o}$ $\begin{pmatrix} z' \\ \omega' \end{pmatrix}$

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

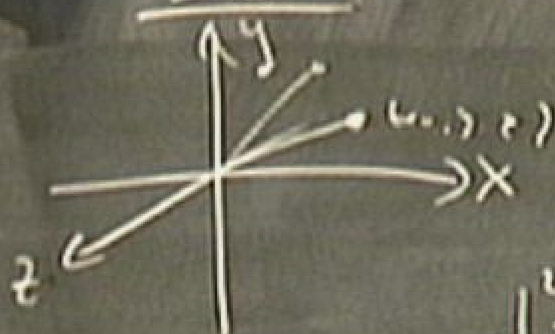
$SO(3)$

Q: Is there something like $U(1)$?

Yes!

$\mathbb{C}^2$ $\begin{pmatrix} z' \\ \omega' \end{pmatrix}$

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

SO(3)

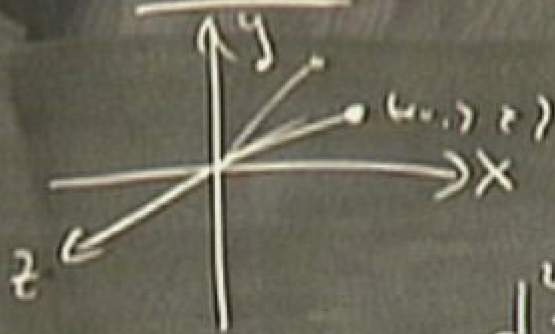
$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

Q: Is there something like U(1)?

Yes $\mathbb{C}^2$

$$\begin{pmatrix} z' \\ \omega' \end{pmatrix}$$

<u>3D</u>



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

$SO(3)$

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

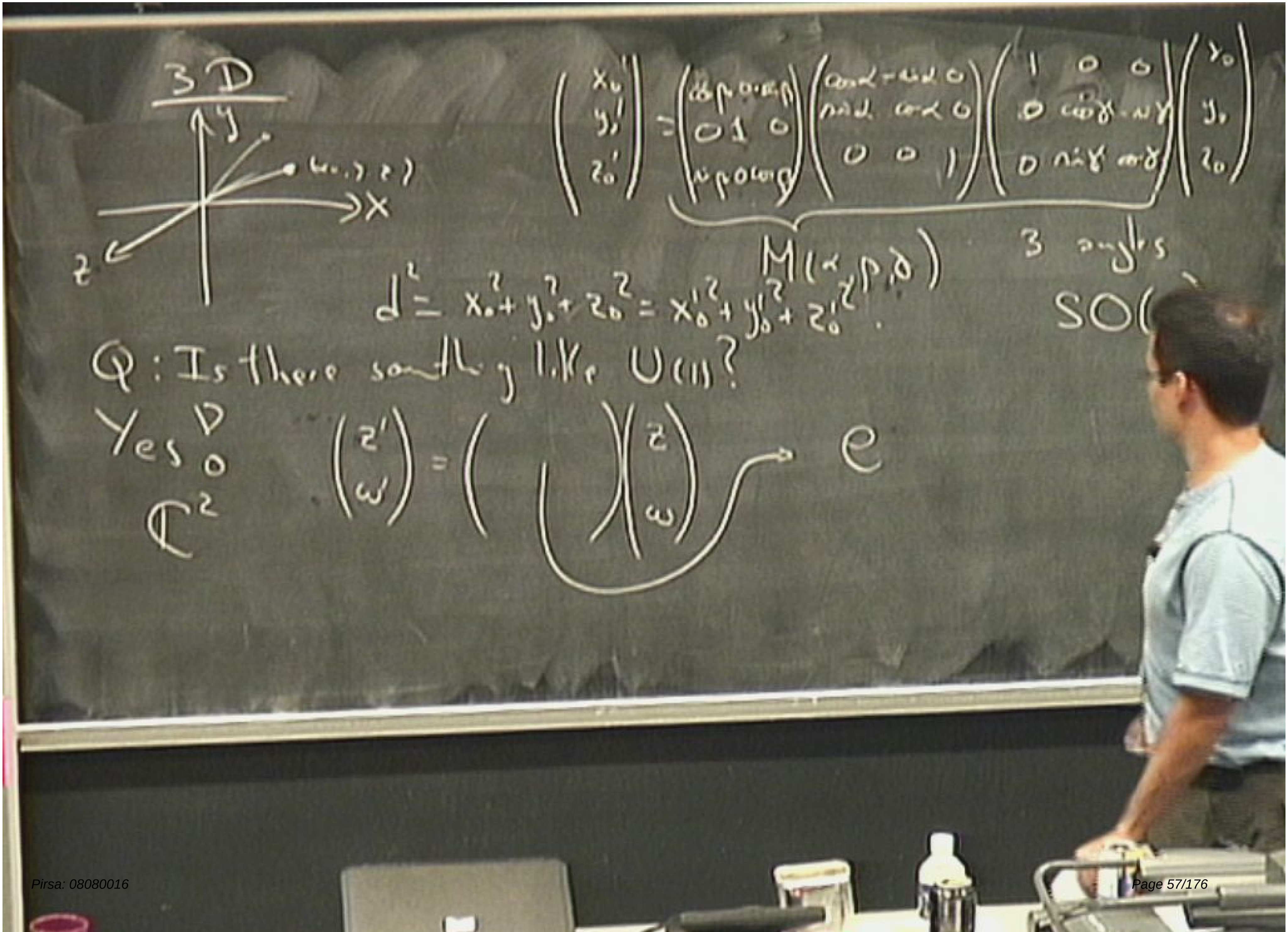
Q: Is there something like $U(1)$?

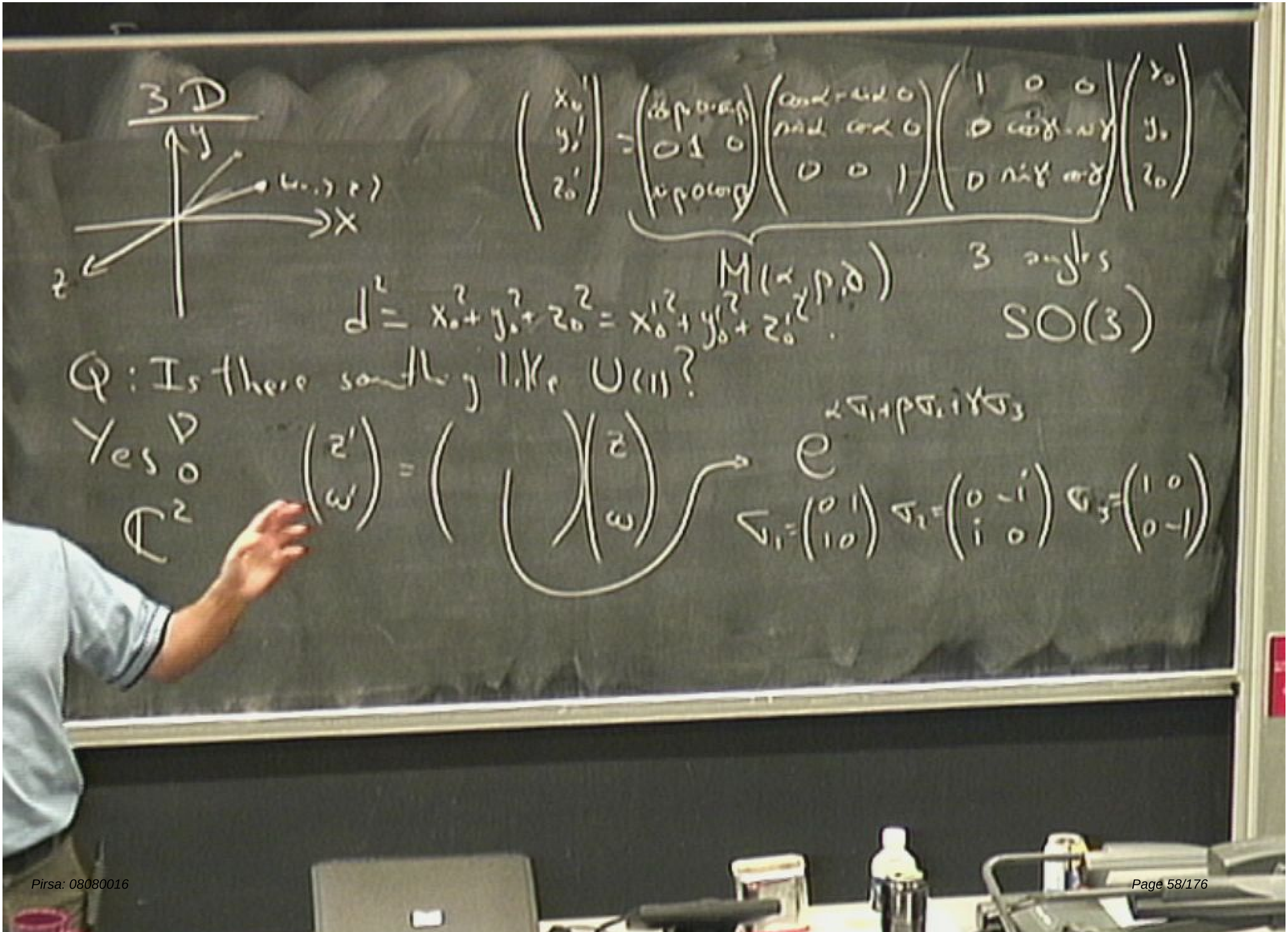
Yes

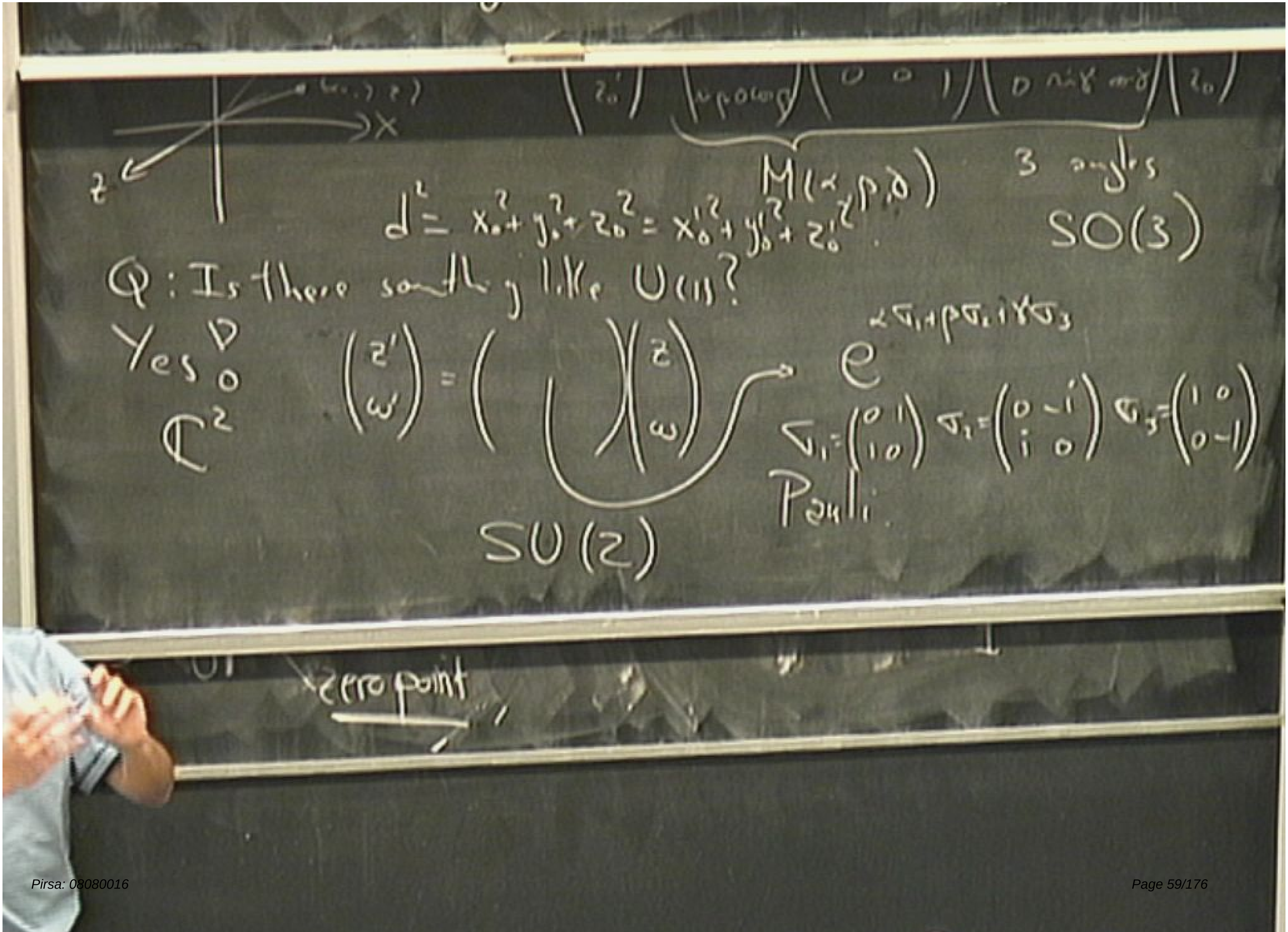
$\mathbb{C}^2$

$$\begin{pmatrix} z' \\ \omega' \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} z \\ \omega \end{pmatrix}$$

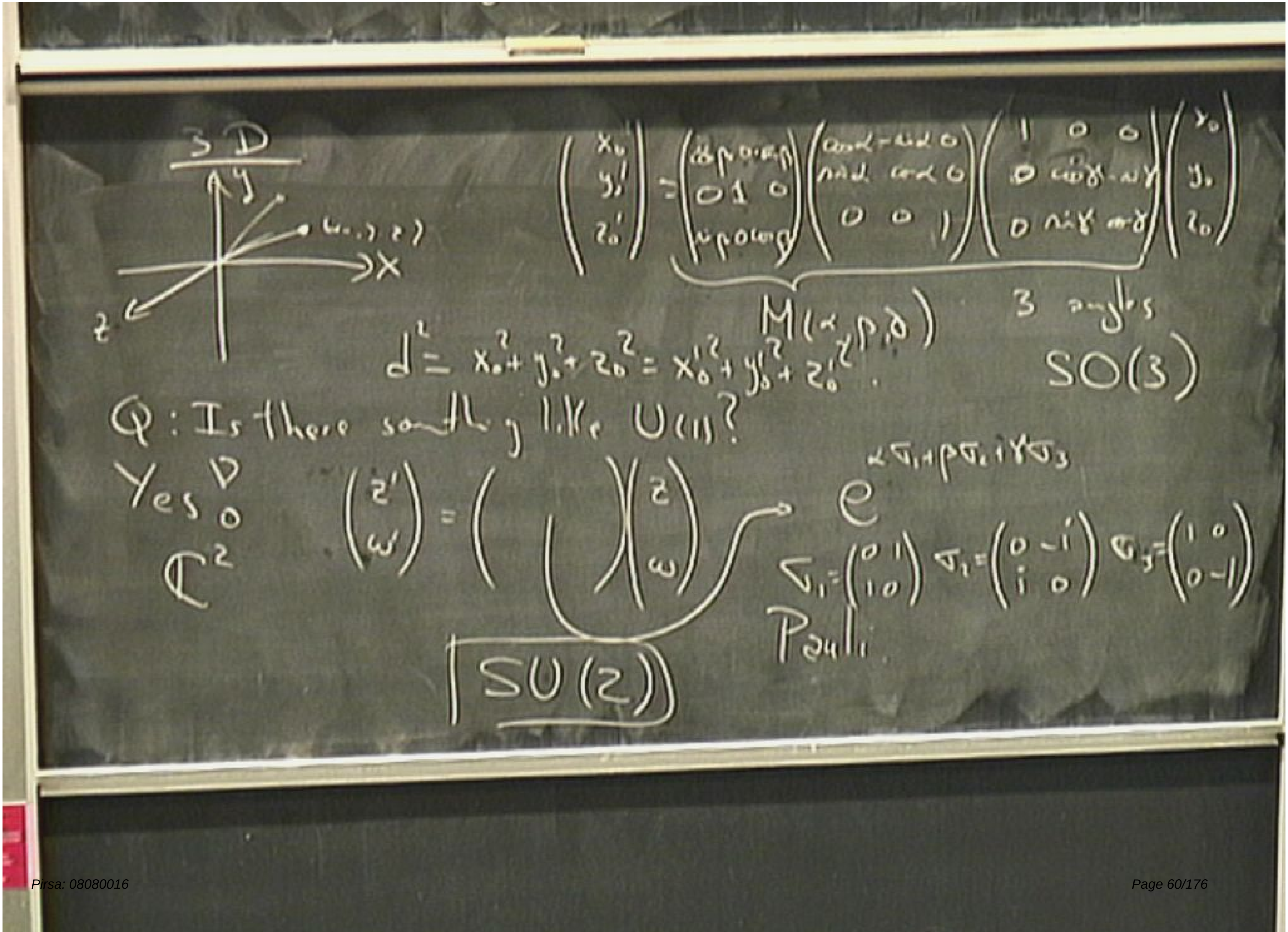
e


3D
y
X
z
3 angles
SO(
Q: Is there something like U(1)?
Yes
e


3D
y
x
z
(x₀, y₀, z₀)
M(α, β, γ)
3 angles
SO(3)
Q: Is there something like U(1)?
Yes
ℂ²
σ₁ = (0 1 / 1 0) σ₂ = (0 −i / i 0) σ₃ = (1 0 / 0 −1)

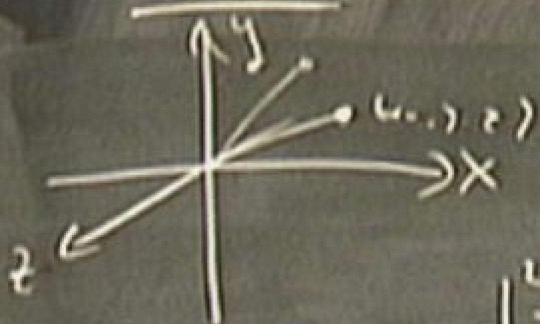


X
z
$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2$
M(α, β, γ)
3 angles
SO(3)
Q: Is there something like U(1)?
Yes
$\mathbb{C}^2$
$\begin{pmatrix} z' \\ \omega' \end{pmatrix} = \begin{pmatrix} & \end{pmatrix} \begin{pmatrix} z \\ \omega \end{pmatrix}$
$e^{\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3}$
$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Pauli
SU(2)
zero point



3 D
y
X
z
M(α, β, γ)
3 angles
SO(3)
Q: Is there something like U(1)?
Yes
C²
SU(2)
Pauli

3D



$$\begin{pmatrix} x_0' \\ y_0' \\ z_0' \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}}_{M(\alpha,\beta,\gamma)}\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

3 angles

SO(3)

$$d^2 = x_0^2 + y_0^2 + z_0^2 = x_0'^2 + y_0'^2 + z_0'^2 .$$

Q: Is there something like U(1)?

Yes

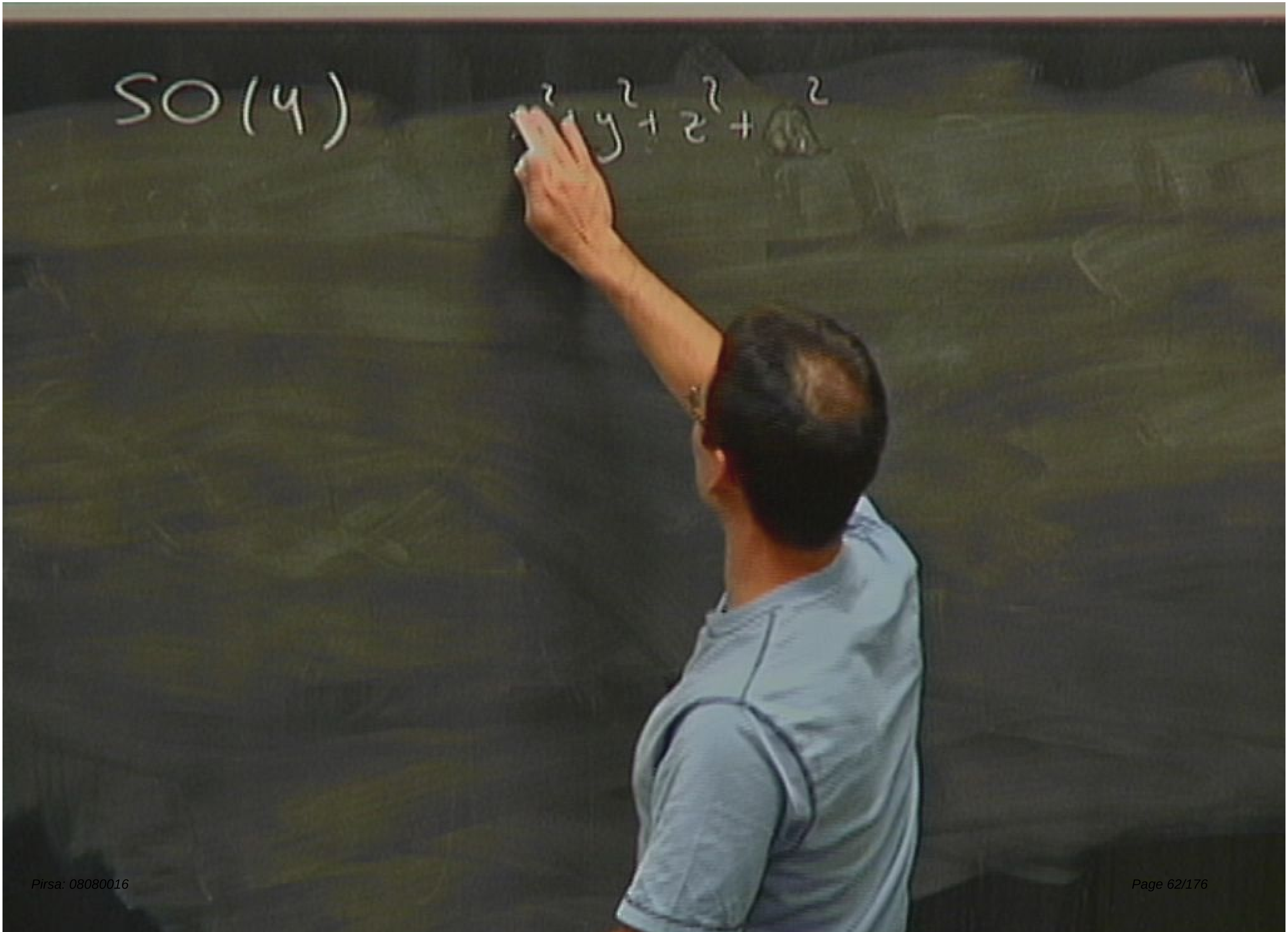
$\mathbb{C}^2$

$$\begin{pmatrix} z' \\ \omega' \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix} z \\ \omega \end{pmatrix} \longrightarrow e^{i\alpha\sigma_1 + i\beta\sigma_2 + i\gamma\sigma_3}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

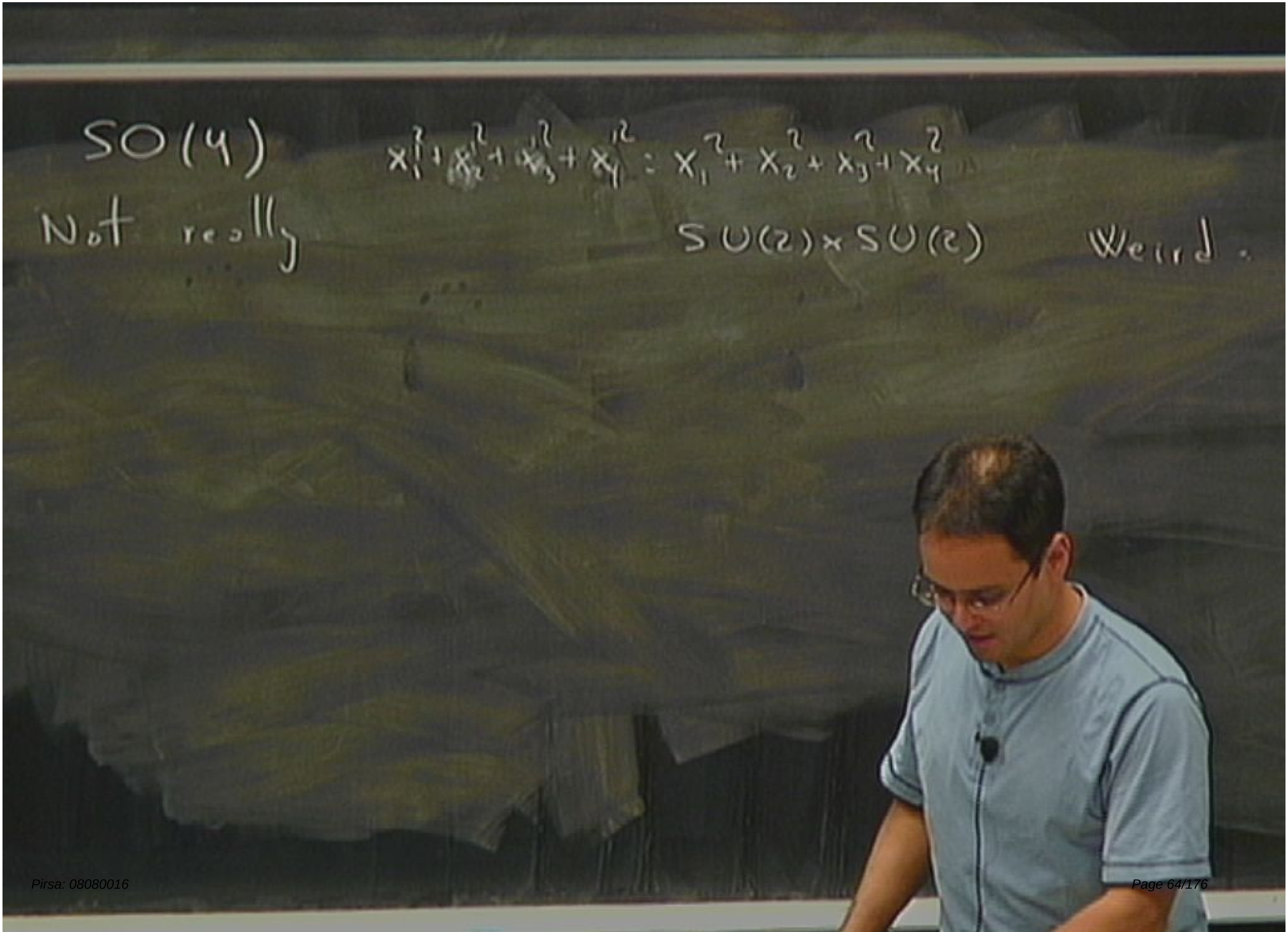
Pauli

$\boxed{SU(2)}$

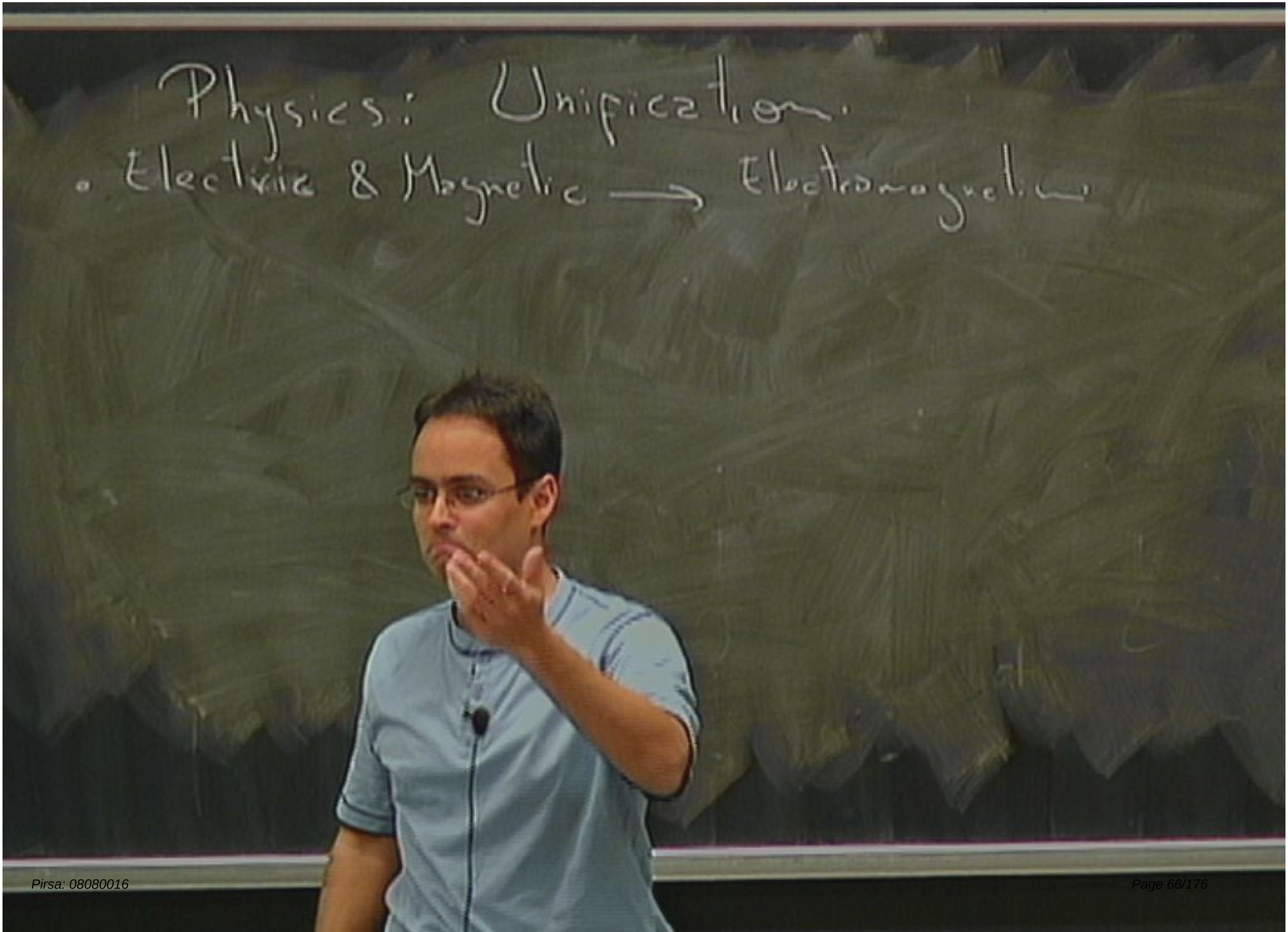

SO(4)

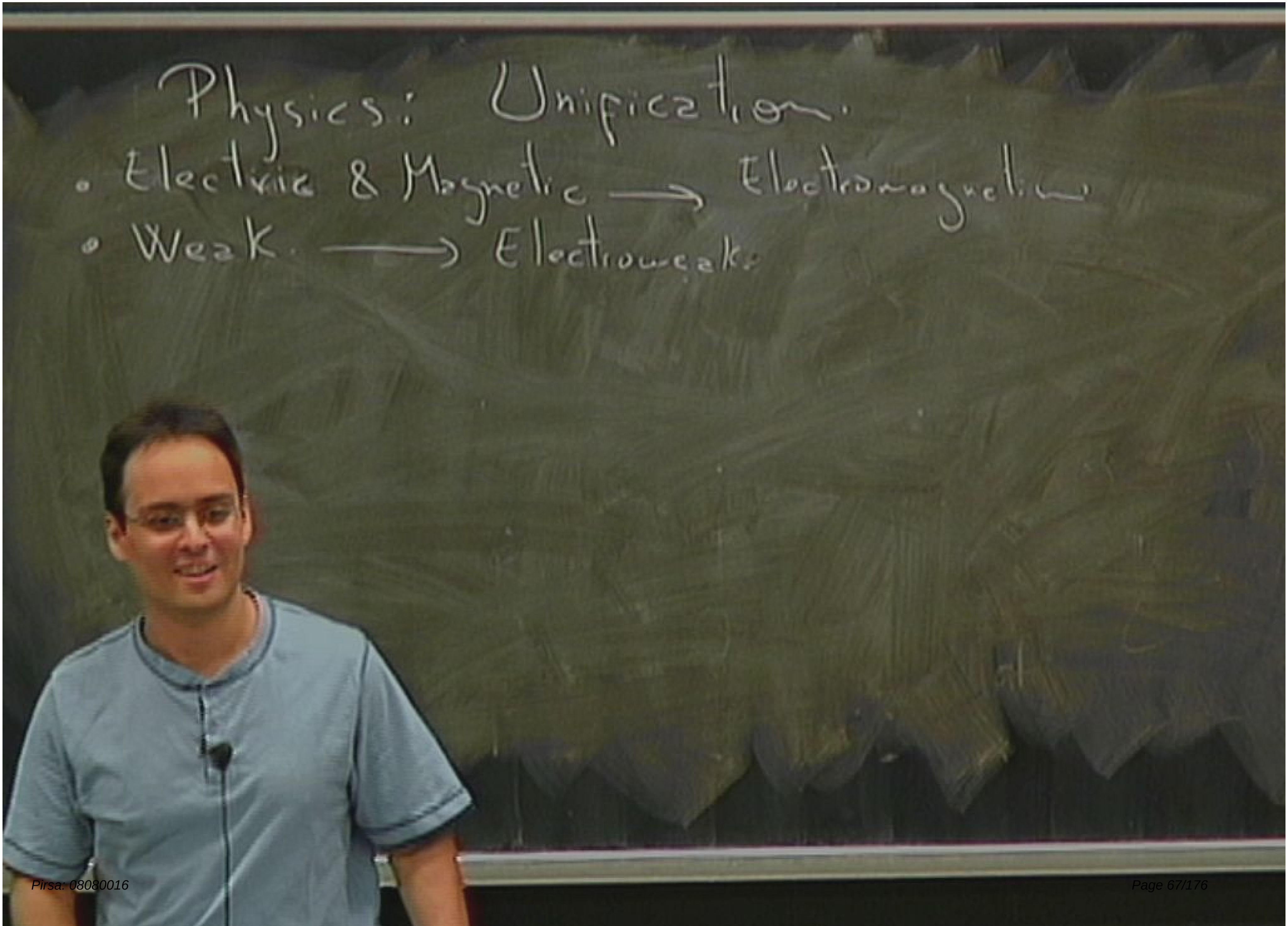
SO(4)

$$x_1'^2 + x_2'^2 + x_3'^2 + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$$


SO(4)
$x_1'^2 + x_2'^2 + x_3'^2 + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$
Not really
SU(2) × SU(2)
Weird.

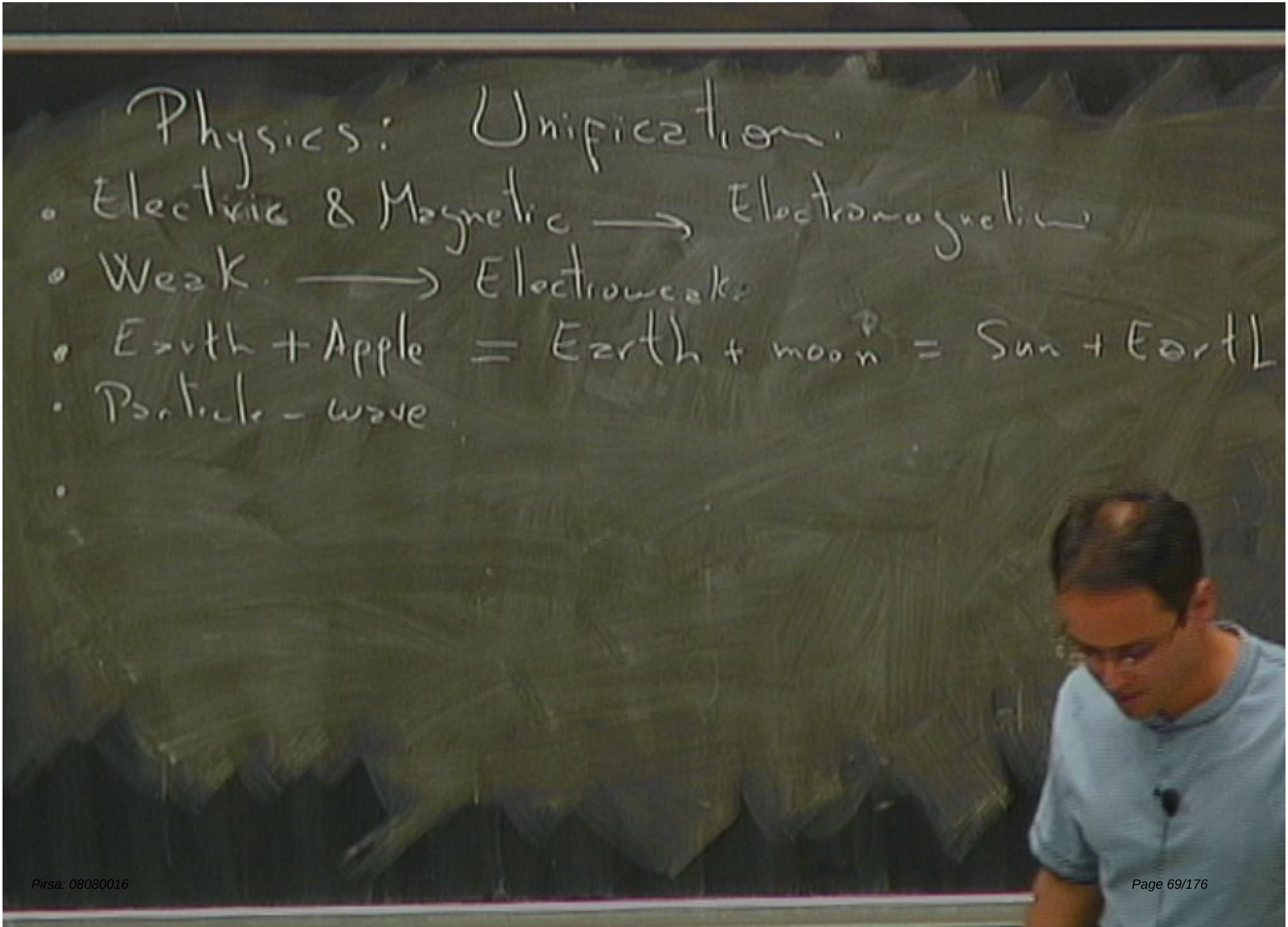
Physics: Unification.


Physics: Unification.
• Electric & Magnetic → Electromagnetism


Physics: Unification.
• Electric & Magnetic → Electromagnetism
• Weak → Electroweak.

Physics: Unification.

- Electric & Magnetic $\longrightarrow$ Electromagnetism
- Weak $\longrightarrow$ Electroweak.
- Earth + Apple = Earth + moon ? = Sun + Earth

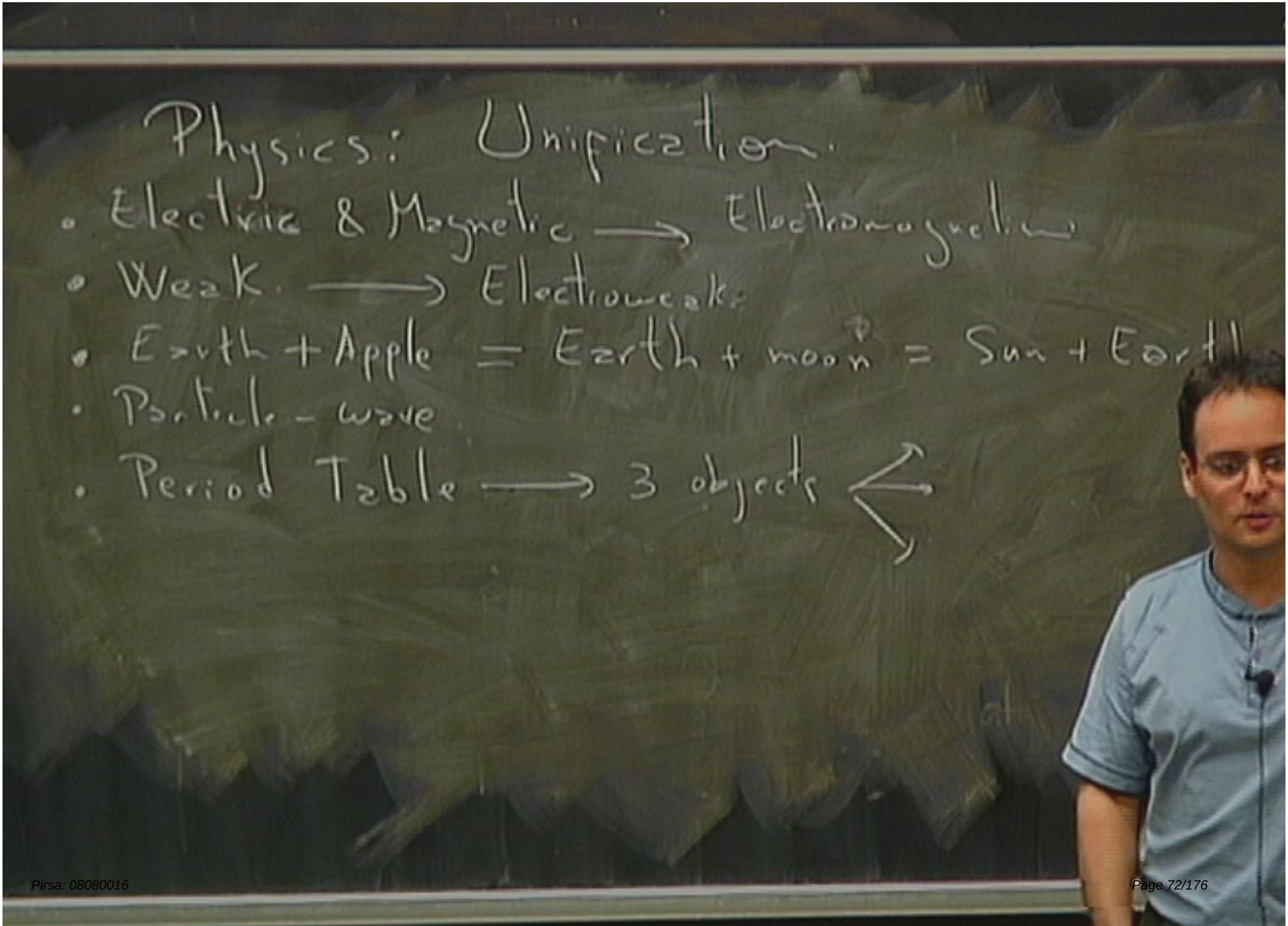

Physics: Unification.
• Electric & Magnetic → Electromagnetism
• Weak → Electroweak
• Earth + Apple = Earth + moon ? = Sun + Earth
• Particle - wave
•

Physics: Unification.

- Electric & Magnetic ⟶ Electromagnetism
- Weak ⟶ Electroweak.
- Earth + Apple = Earth + moon = Sun + Earth
- Particle – wave
- Period Table

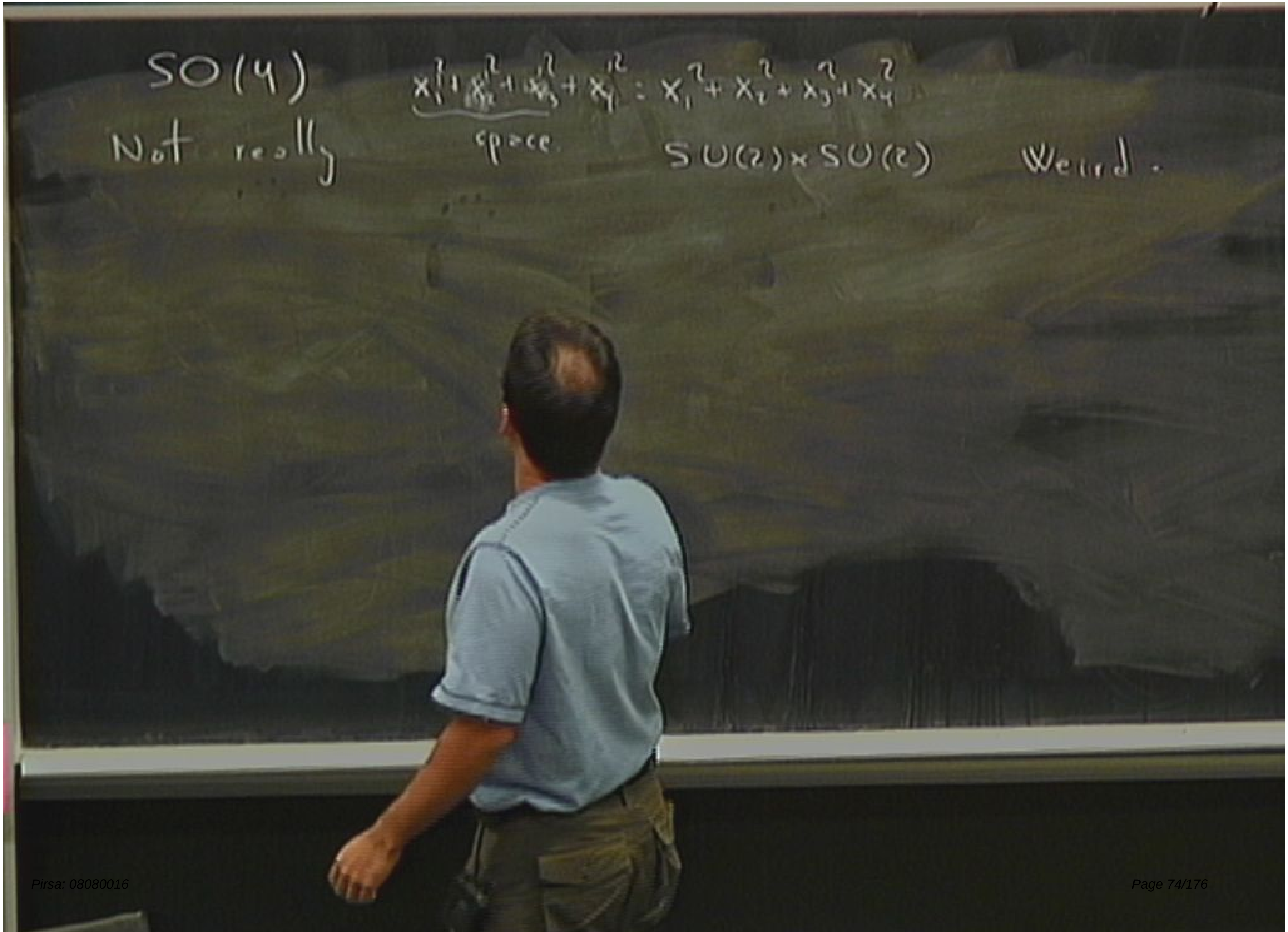
Physics: Unification.

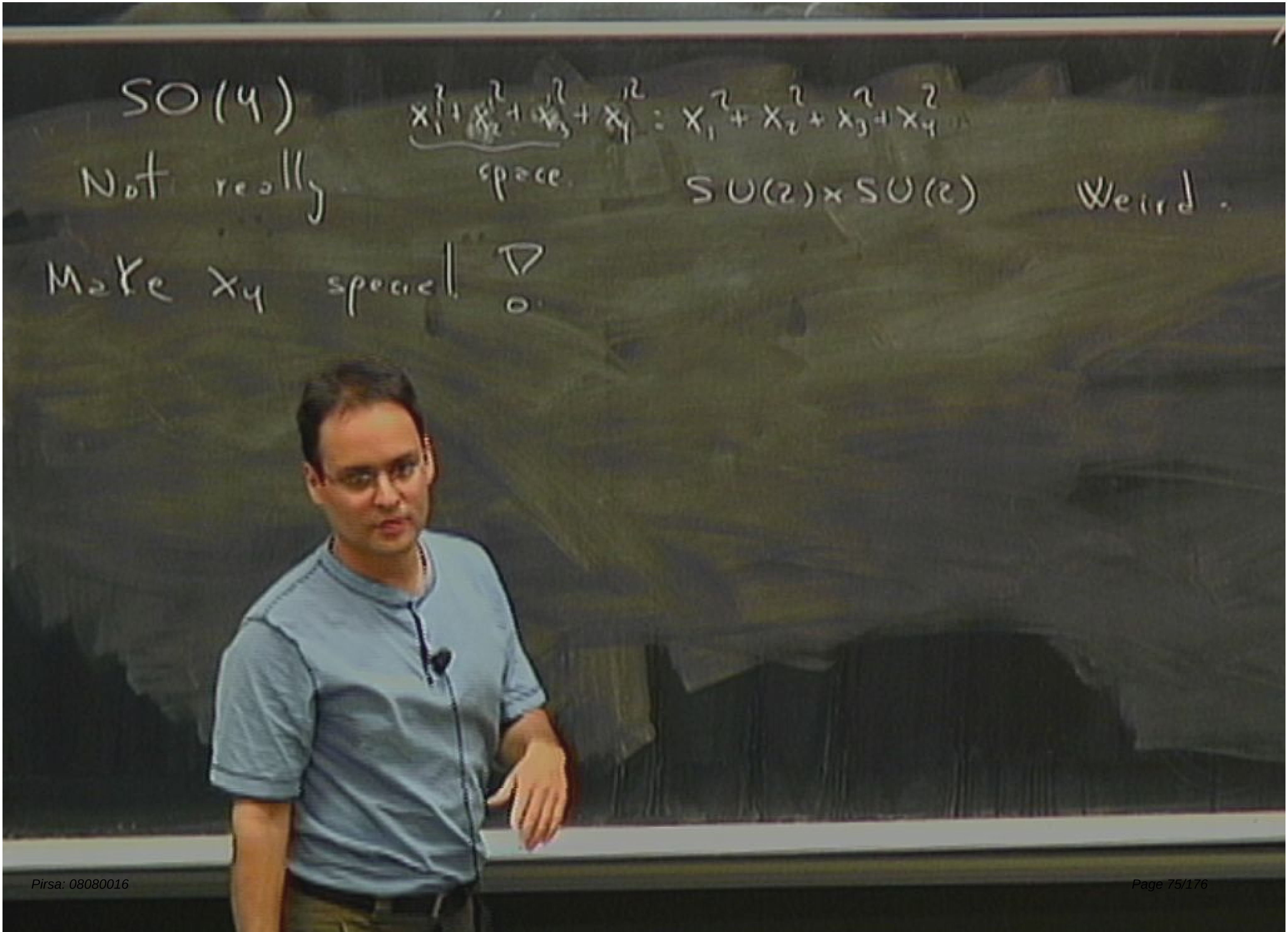
- Electric & Magnetic $\longrightarrow$ Electromagnetism
- Weak $\longrightarrow$ Electroweak
- Earth + Apple = Earth + moon ? = Sun + Earth
- Particle - wave
- Period Table

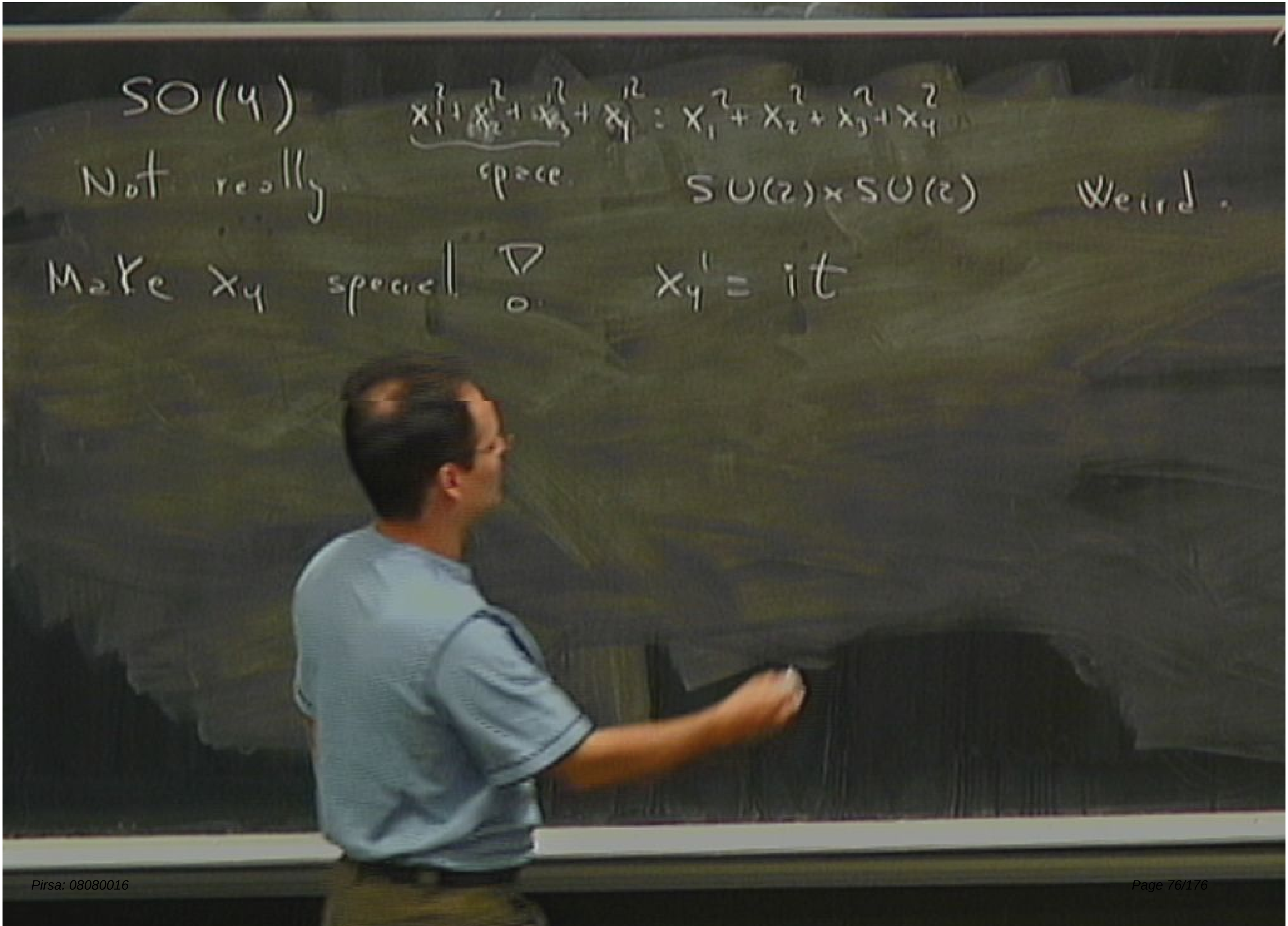

Physics: Unification.
• Electric & Magnetic ⟶ Electromagnetism
• Weak ⟶ Electroweak
• Earth + Apple = Earth + moon = Sun + Earth
• Particle - wave
• Period Table ⟶ 3 objects

Physics: Unification.

- Electric & Magnetic $\longrightarrow$ Electromagnetism
- Weak $\longrightarrow$ Electroweak.
- Earth + Apple = Earth + moon = Sun + Earth
- Particle - wave
- Period Table $\longrightarrow$ 3 objects $\begin{cases} \to N \\ \to P \\ \to e \end{cases}$


SO(4)
$x_1'^2 + x_2'^2 + x_3'^2 + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$
Not really
space
SU(2)×SU(2)
Weird.

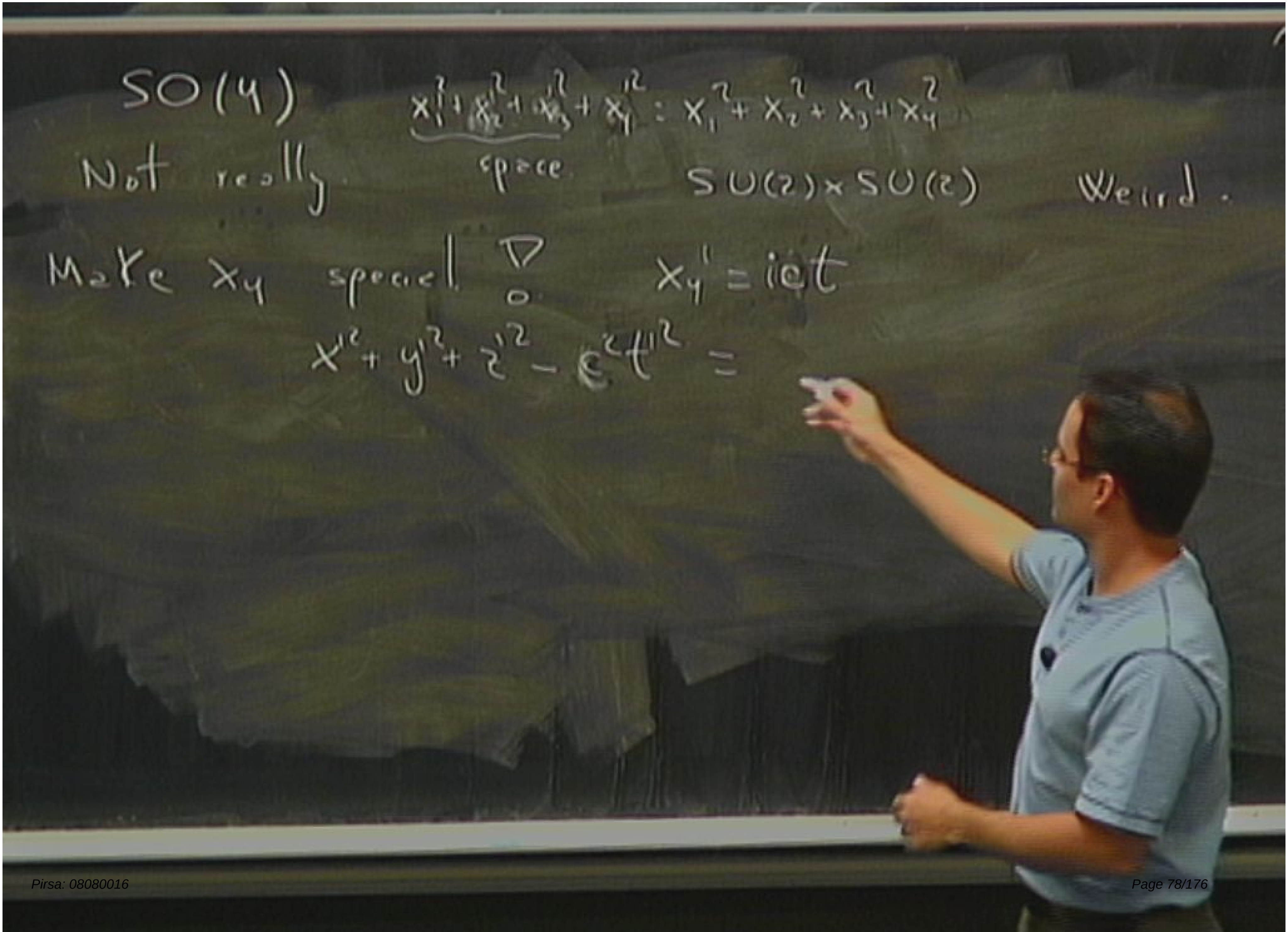

SO(4)
x1'2 + x2'2 + x3'2 + x4'2 = x1 2 + x2 2 + x3 2 + x4 2
space
Not really
SU(2) × SU(2)
Weird.
Make x4 special


SO(4)
$\underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$
Not really
SU(2)×SU(2)
Weird.
Make x_4 special
$x_4' = it$

SO(4) $\qquad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\qquad SU(2) \times SU(2) \qquad$ Weird.

Make x_4 special! $\qquad x_4' = ict$


SO(4)
$\underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$
Not really
SU(2) × SU(2)
Weird.
Make x_4 special
$x_4' = i\epsilon t$
$x'^2 + y'^2 + z'^2 - \epsilon^2 t'^2 =$

SO(4) $\quad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\quad$ $SU(2) \times SU(2)$ $\quad$ Weird.

Make x_4 special $\quad$ $x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(4)$ $\quad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\qquad SU(2) \times SU(2) \qquad$ Weird.

Make x_4 special! $\quad x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(3,1)$

$SO(4)$ $\underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $SU(2) \times SU(2)$ Weird.

Make x_4 special ∇ $x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(3,1)$

$SO(4)$ $\underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $SU(2) \times SU(2)$ Weird.

Make x_4 special ∇ $x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2$$

$SO(3,1) \longrightarrow SL(2,\mathbb{C})$

$SO(4)$ $\qquad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\qquad SU(2) \times SU(2)$ $\qquad$ Weird.

Make x_4 special $\qquad x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$$SO(3,1) \longrightarrow SL(2,\mathbb{C})$$

$$\hookrightarrow e^{\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3}$$

SO(4) $\quad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

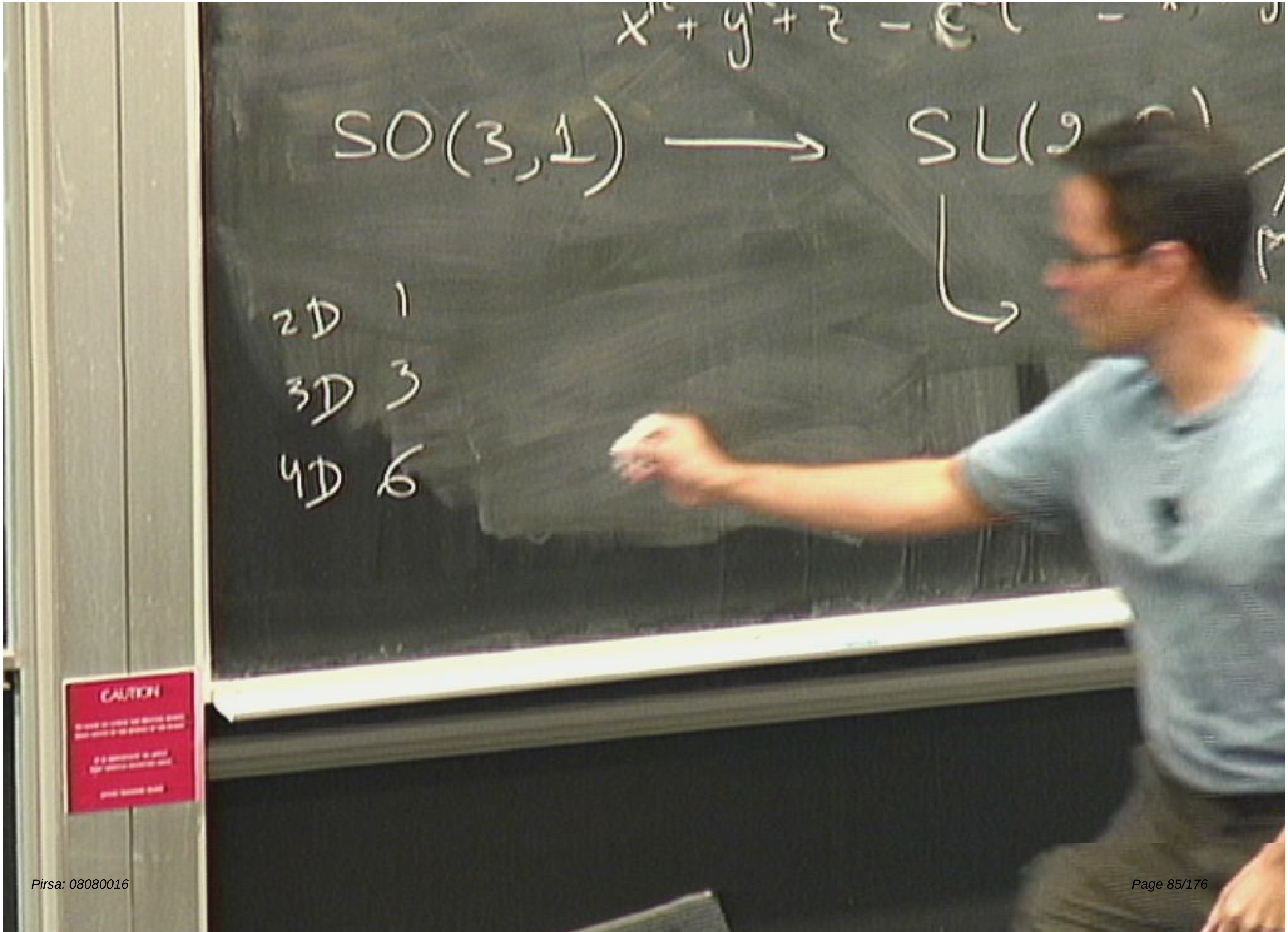
Not really $\quad$ SU(2) × SU(2) $\quad$ Weird.

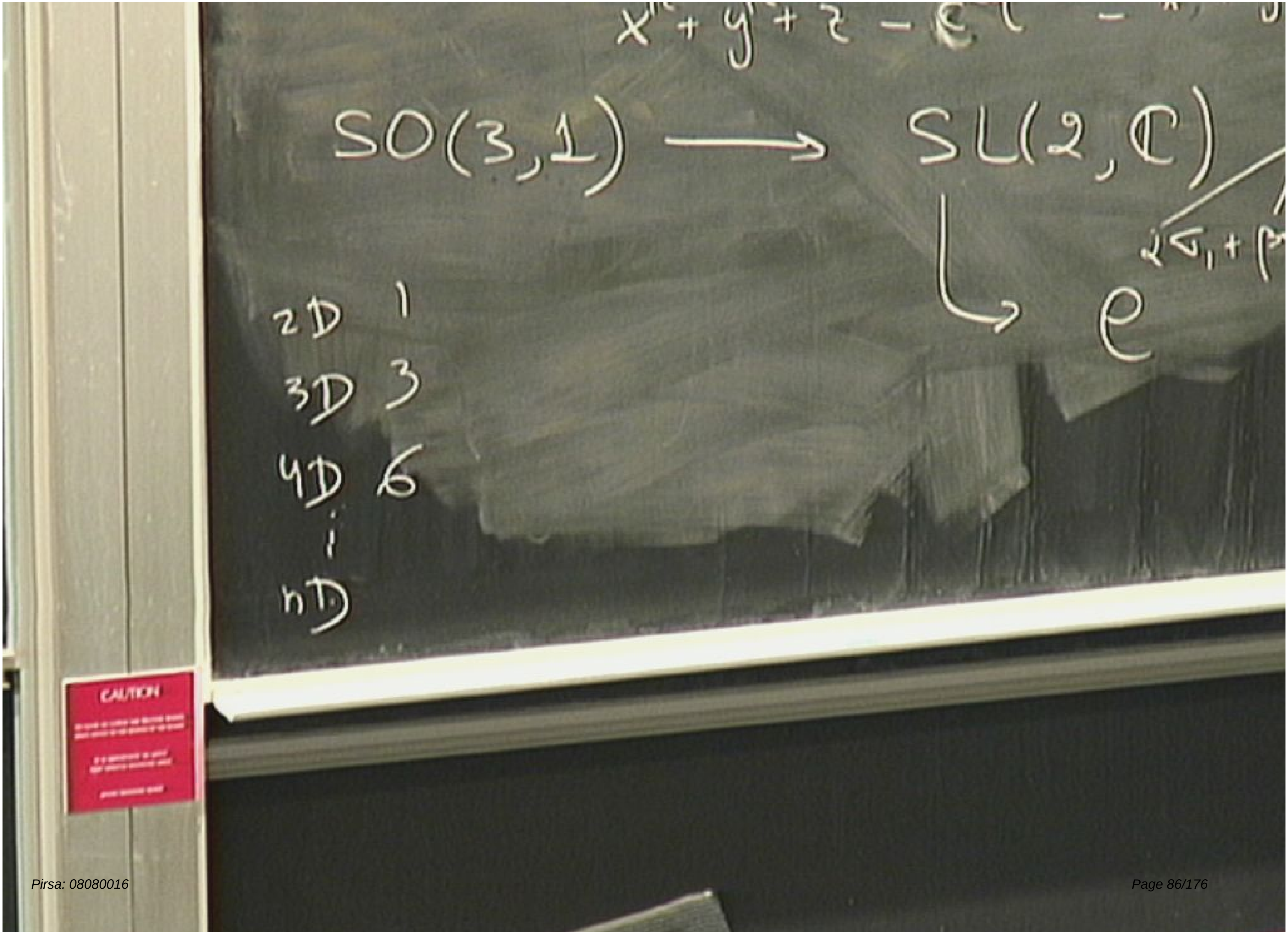
Make x_4 special ∇ $\quad x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$$SO(3,1) \longrightarrow SL(2,\mathbb{C})$$

$$\hookrightarrow e^{\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3}$$


SO(3,1) → SL(2
2D 1
3D 3
4D 6
CAUTION



SO(3,1) → SL(2,ℂ)
e
2D 1
3D 3
4D 6
⋮
nD
CAUTION

SO(4) $\qquad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\qquad SU(2) \times SU(2) \qquad$ Weird.

Make x_4 special ∇ $\qquad x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(3,1) \longrightarrow SL(2,\mathbb{C})$

$\hookrightarrow e^{\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3}$ $\longrightarrow$ cplx numbers.

$\alpha = \alpha_1 + i\alpha_2$

2D 1

3D 3

4D 6

⋮

nD

SO(4) $\underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $SU(2) \times SU(2)$ Weird.

Make x_4 special! $x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(3,1) \longrightarrow SL(2,\mathbb{C})$ → cplx numbers.

$\hookrightarrow e^{\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3}$

$\alpha = \alpha_1 + i\alpha_2$

$\beta = \beta_1 + i\beta_2$

$\gamma = \gamma_1 + i\gamma_2$

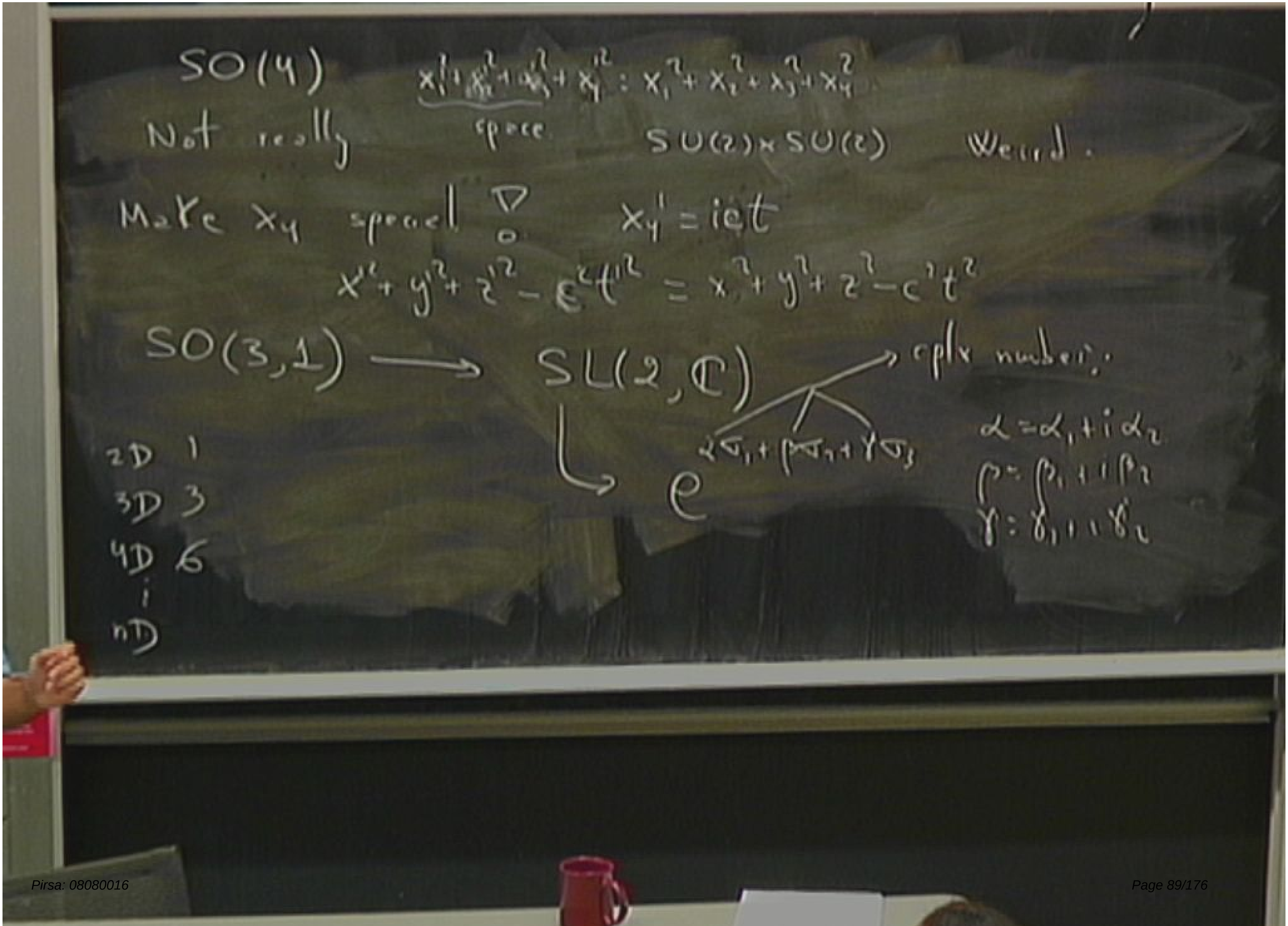
2D 1

3D 3

4D 6

⋮

nD


SO(4) x'₁² + x'₂² + x'₃² + x'₄² = x₁² + x₂² + x₃² + x₄²
Not really
space
SU(2)×SU(2)
Weird.
Make x₄ special
x'₄ = ict
x'² + y'² + z'² − c²t'² = x² + y² + z² − c²t²
SO(3,1) → SL(2,C)
cplx numbers.
e^(ασ₁ + βσ₂ + γσ₃)
α = α₁ + iα₂
β = β₁ + iβ₂
γ = γ₁ + iγ₂
2D 1
3D 3
4D 6
nD

$SO(4)$ $\quad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\quad SU(2) \times SU(2)$ Weird.

Make x_4 special ∇ $\quad x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(3,1) \longrightarrow SL(2,\mathbb{C})$ $\quad$ cplx numbers.

2D 1

3D 3

4D 6

⋮

nD

$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ 4-vectors

$e^{\alpha \sigma_1 + \beta \sigma_2 + \gamma \sigma_3}$

$\alpha = \alpha_1 + i\alpha_2$

$\beta = \beta_1 + i\beta_2$

$\gamma = \gamma_1 + i\gamma_2$

$\begin{pmatrix} z \\ w \end{pmatrix}$ Spinors.

$SO(4)$ $\quad \underbrace{x_1'^2 + x_2'^2 + x_3'^2}_{\text{space}} + x_4'^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2$

Not really $\quad$ $SU(2) \times SU(2)$ $\quad$ Weird.

Make x_4 special! $\quad x_4' = ict$

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

$SO(3,1) \longrightarrow SL(2,\mathbb{C})$ $\quad$ cplx numbers.

$\alpha = \alpha_1 + i\alpha_2$

$\beta = \beta_1 + i\beta_2$

$\gamma = \gamma_1 + i\gamma_2$

2D 1

3D 3

4D 6

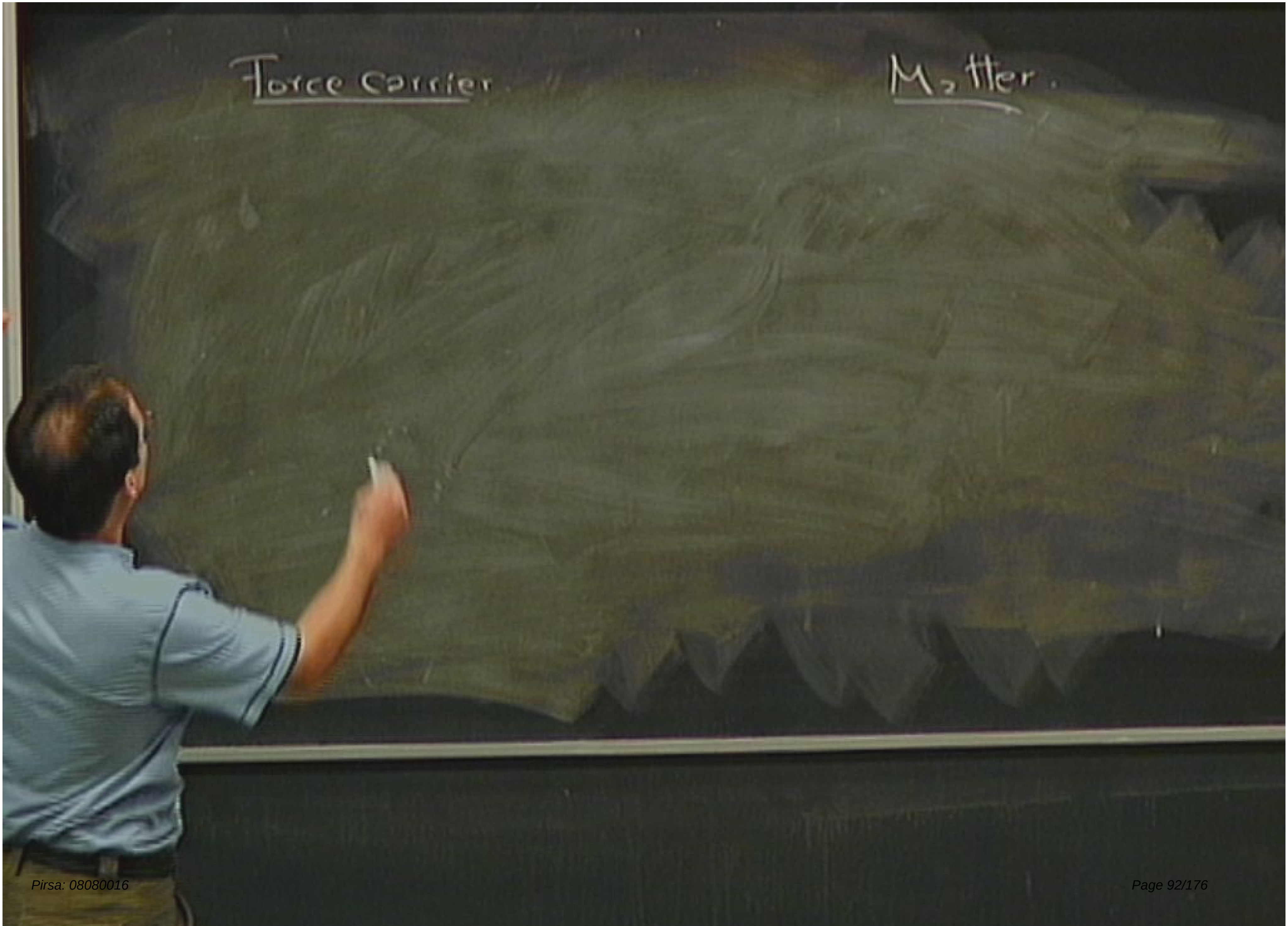
$\vdots$

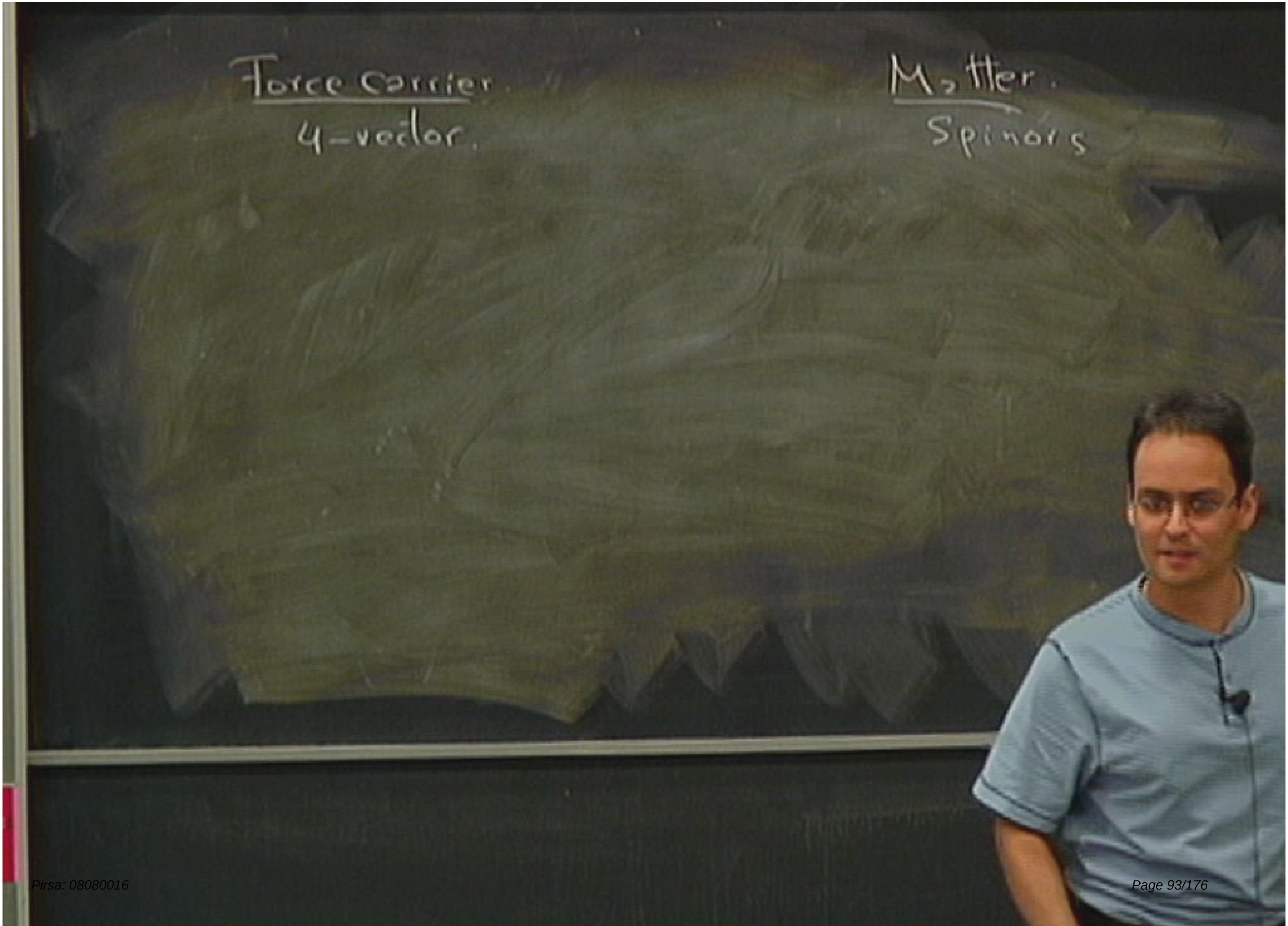
nD

$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ 4-vectors

$e^{\alpha\sigma_1 + \beta\sigma_2 + \gamma\sigma_3}$

$\begin{pmatrix} z \\ w \end{pmatrix}$ Spinors.


Force carrier
Matter.

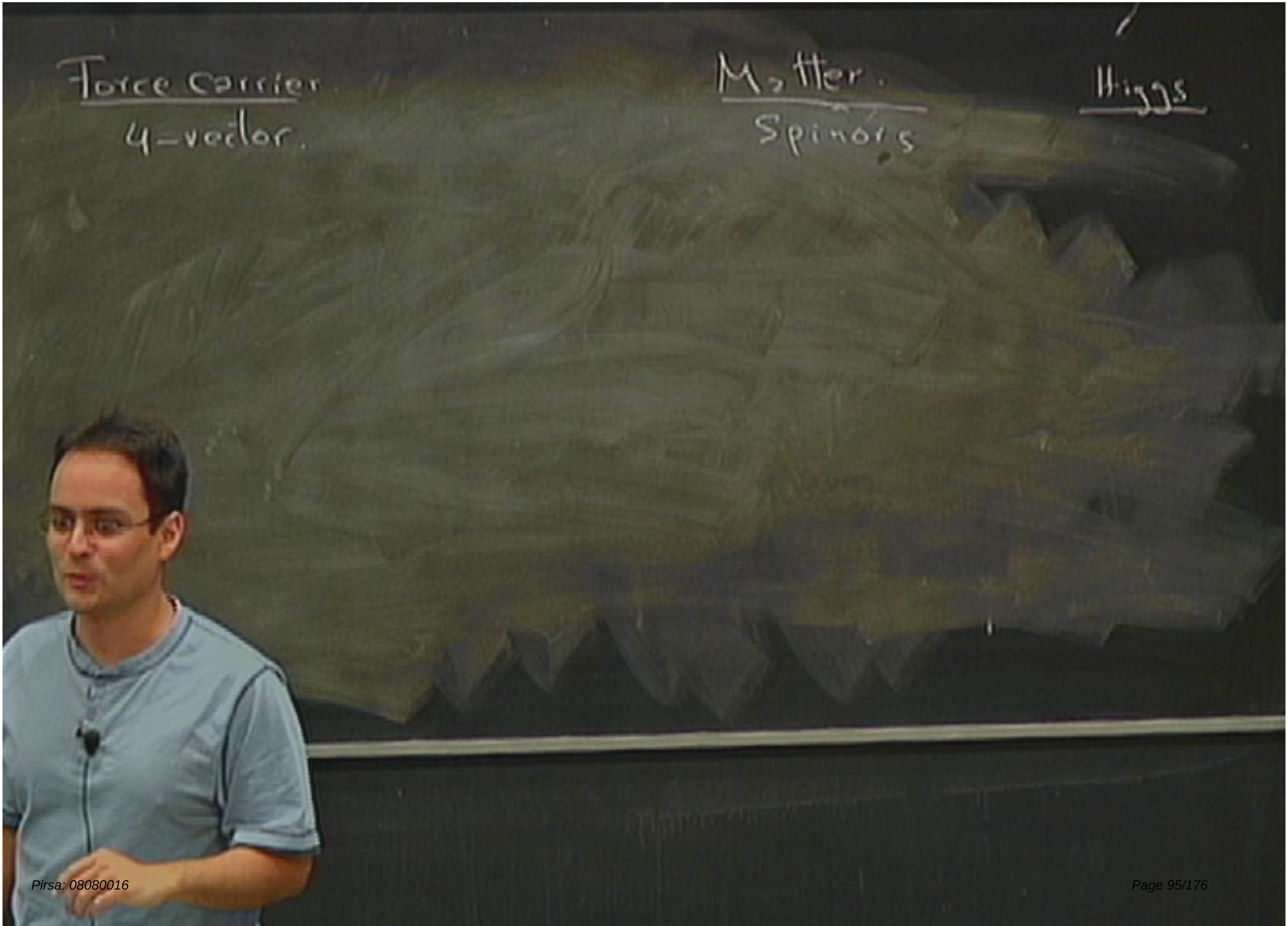

Force carrier
4-vector.
Matter.
Spinors

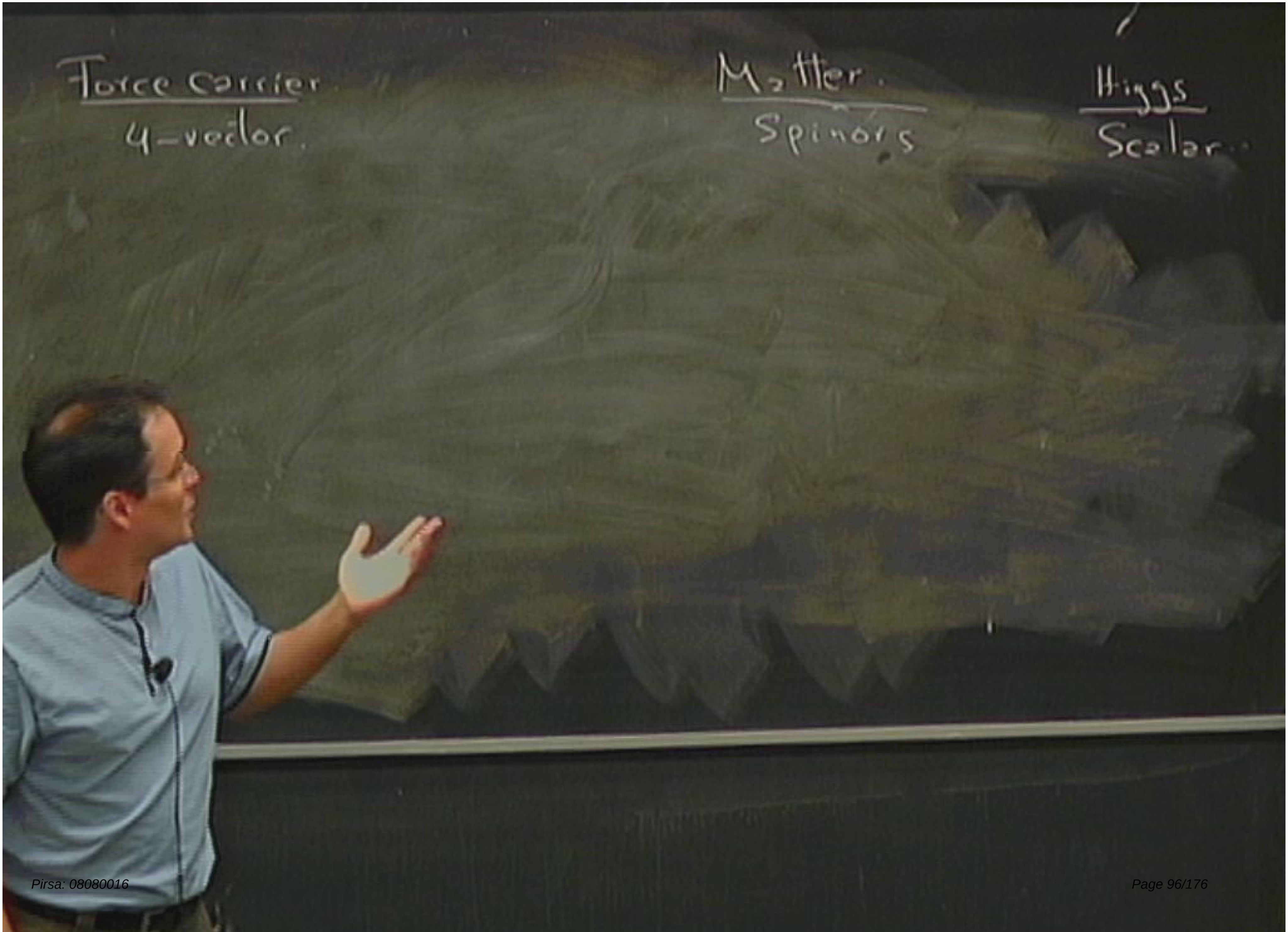
Force carrier

4-vector.

Matter.

Spinors

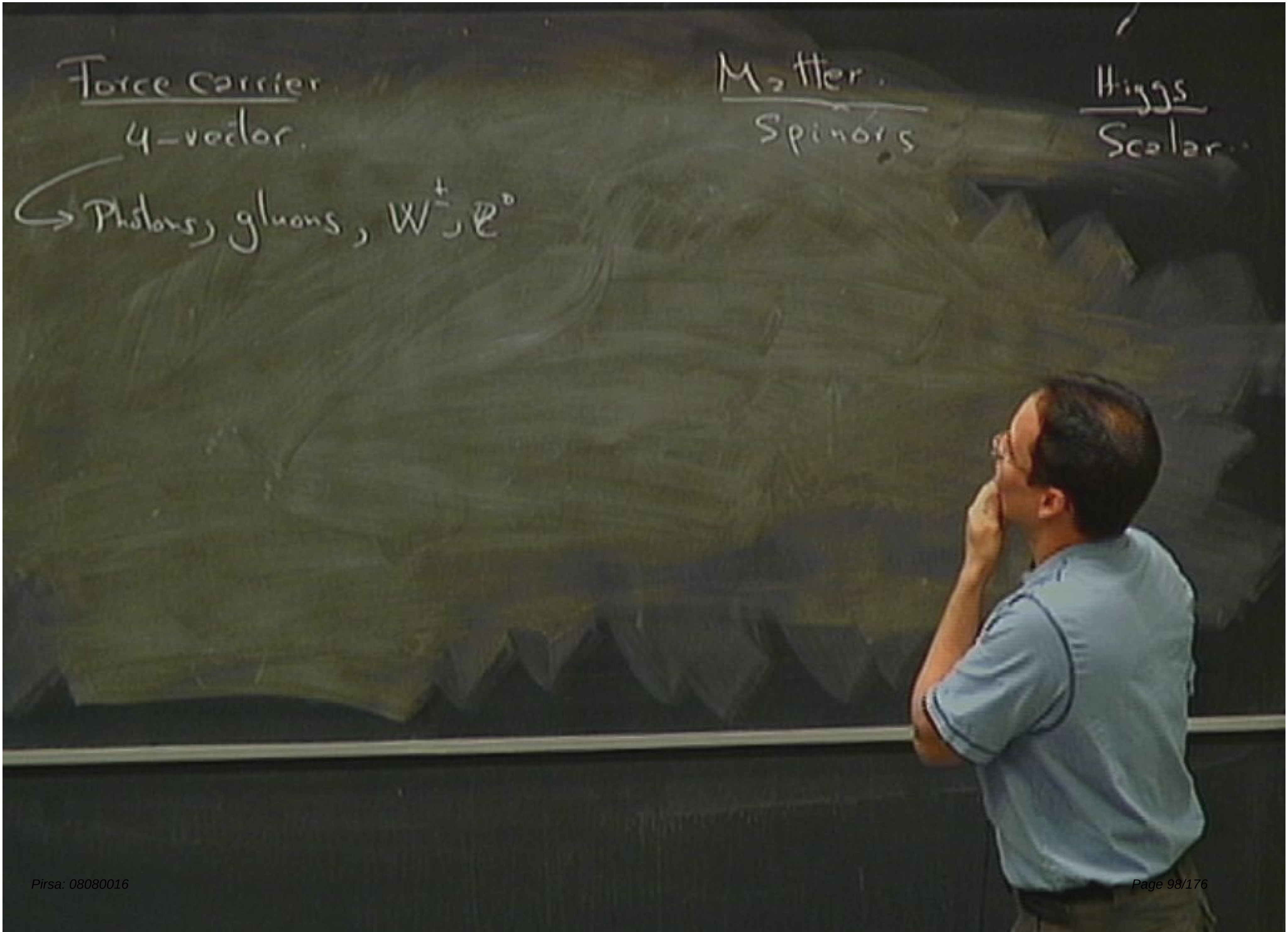

Force carrier
4-vector.
Matter.
Spinors
Higgs

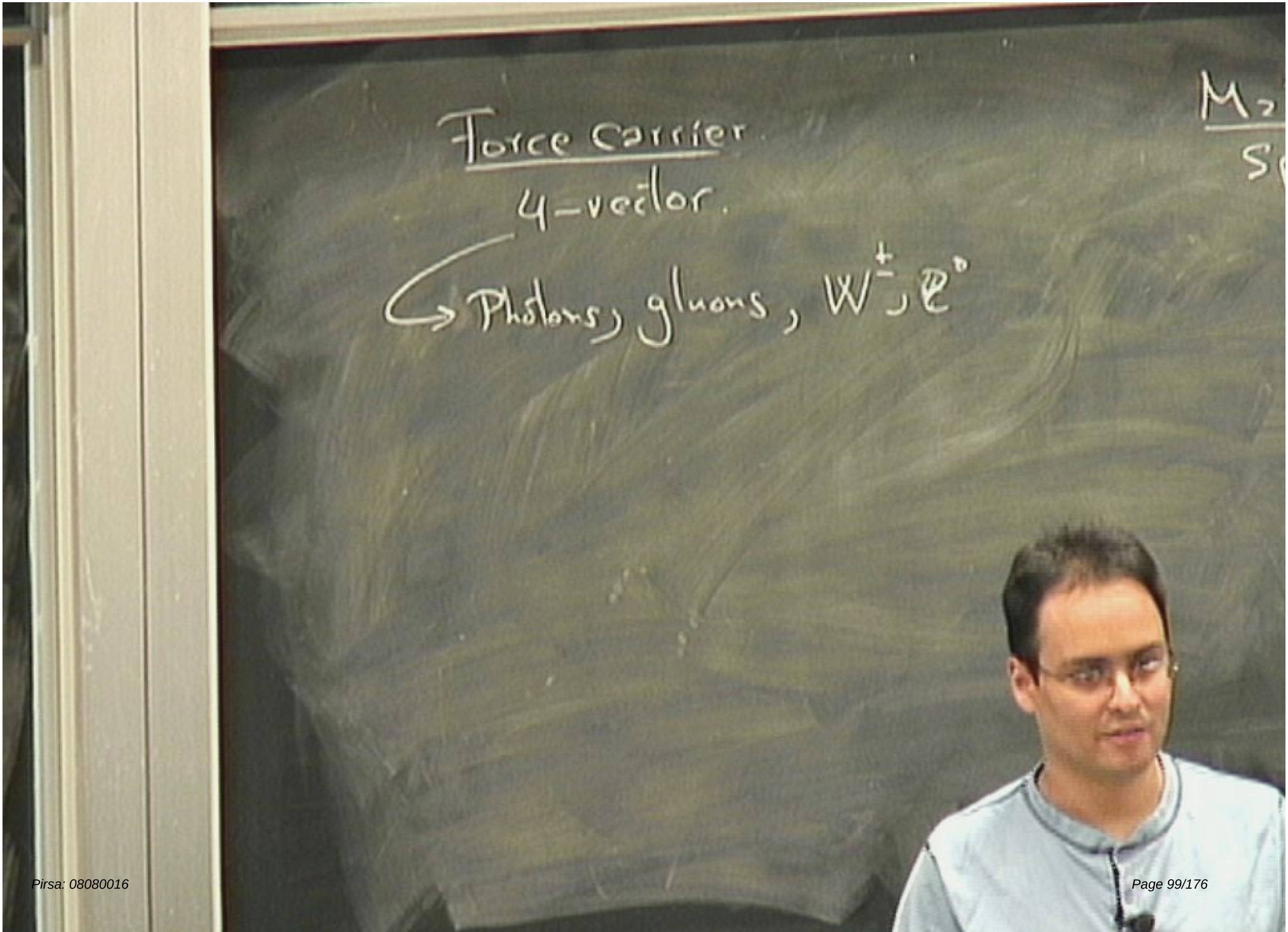

Force carrier
4-vector.
Matter.
Spinors
Higgs
Scalar

Force carrier
4-vector.
Photons, gluons,

Matter.
Spinors

Higgs
Scalar.


Force carrier
4-vector.
Photons, gluons, W±, Z0
Matter
Spinors
Higgs
Scalar


Force carrier
4-vector.
Photons, gluons, $W^{\pm}$, Z^0

Force carrier.

4-vector.

→ Photons, gluons, $W^{\pm}$, Z^0

U(1)	SU(2)	

Matter.

Spinors

Force carrier

4-vector.

→ Photons, gluons, $W^{\pm}$, Z^0

Matter.

Spinors

U(1)	SU(2)	SU(3)	SO(3,1)

Force carrier.

4-vector.

Photons, gluons, $W^{\pm}$, Z^0

Matter.

Spinors

U(1)	SU(2)	SU(3)	SO(3,1)	

Force carrier

4-vector.

→ Photons, gluons, $W^{\pm}$, Z^0

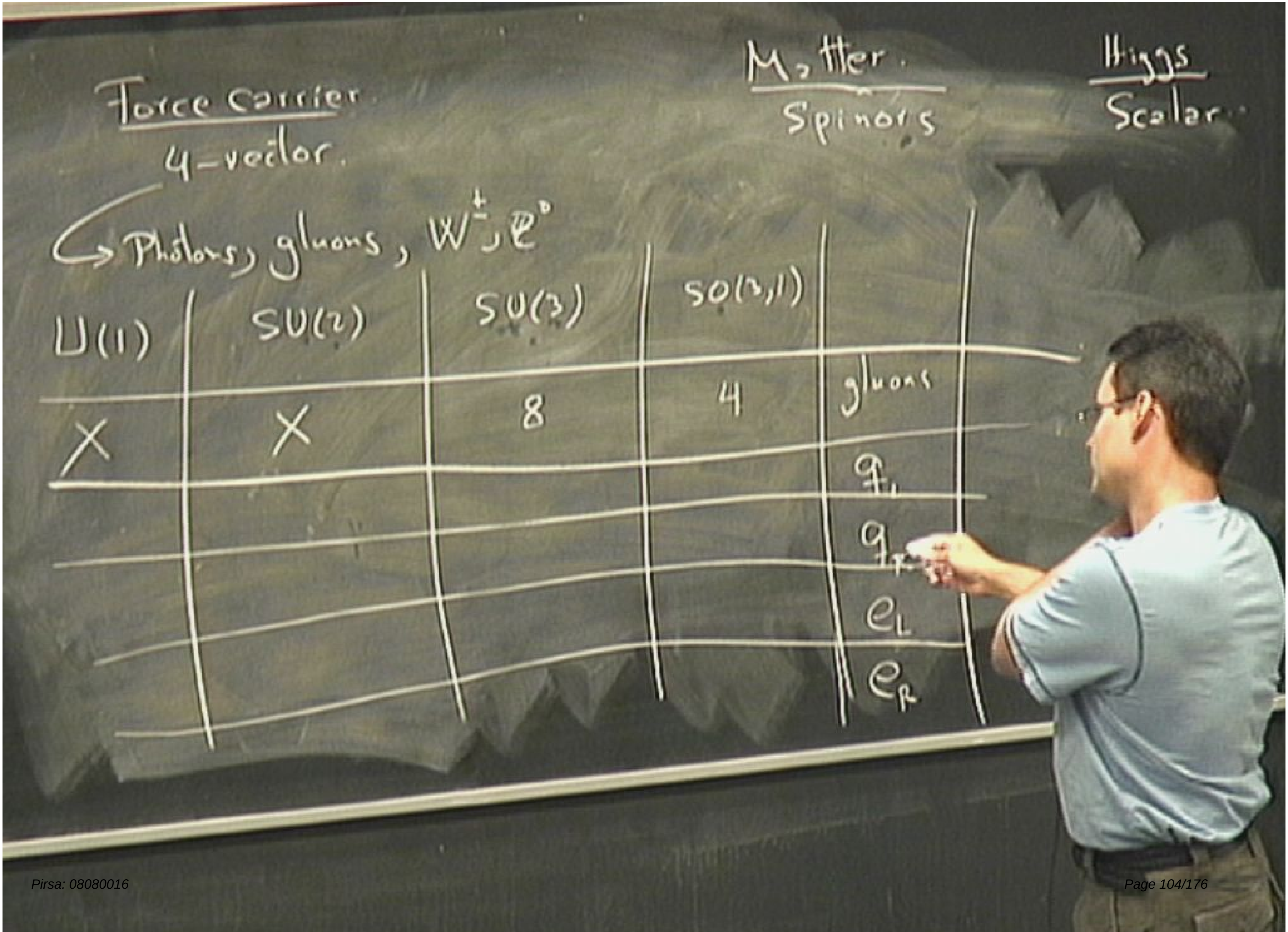
Matter.

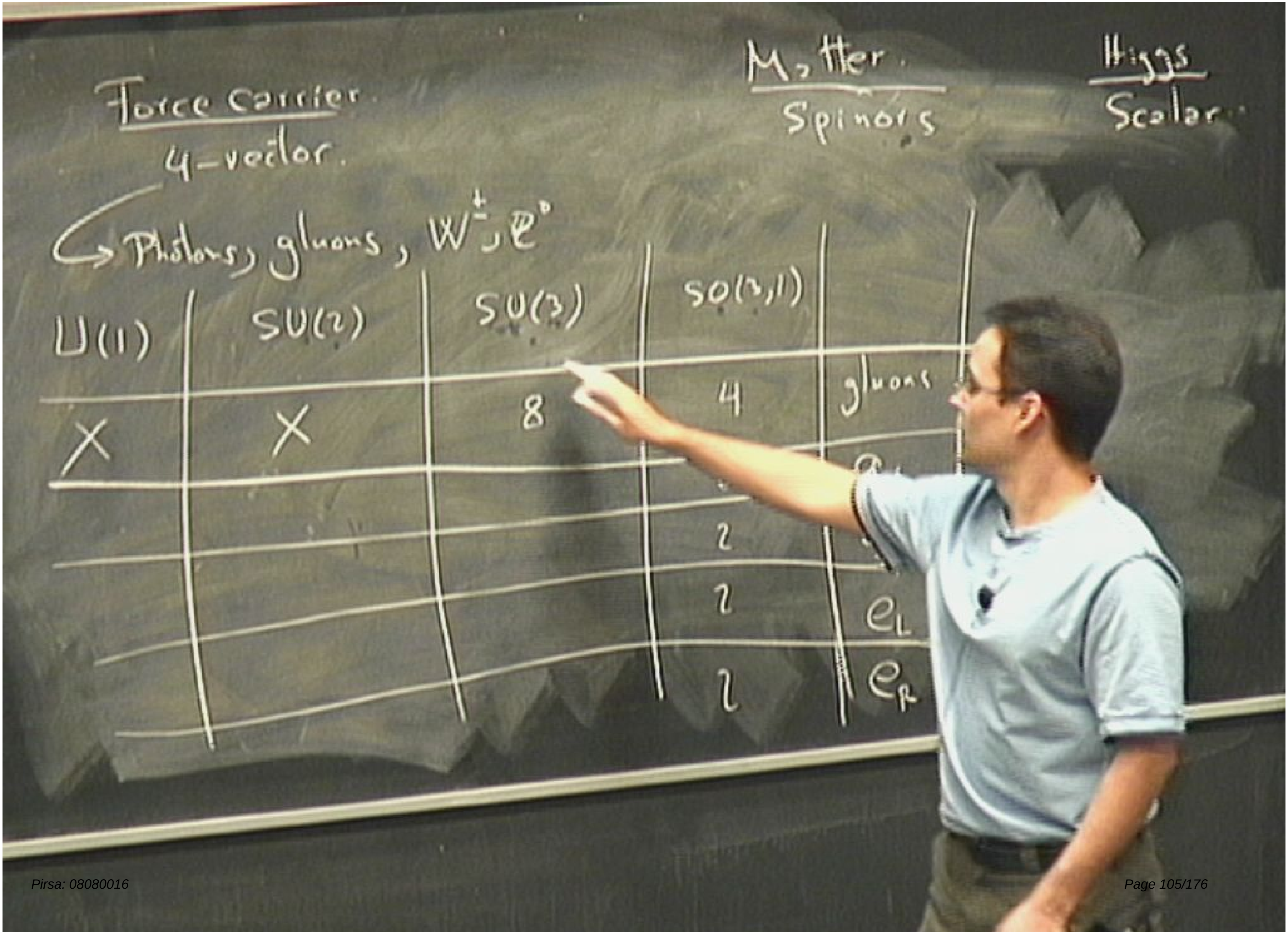
Spinors

Higgs

Scal

U(1)	SU(2)	SU(3)	SO(3,1)		
X	X	8	4	gluons	
				q	


Force carrier
4-vector
Photons, gluons, W±, Z⁰
Matter
Spinors
Higgs
Scalar
U(1)
SU(2)
SU(3)
SO(3,1)
gluons
X
X
8
4
q_L
q_R
e_L
e_R


Force carrier
4-vector
Photons, gluons, W±, Z0
Matter
Spinors
Higgs
Scalar
U(1)
SU(2)
SU(3)
SO(3,1)
X
X
8
4
gluons
2
2
2
eL
eR

Force carrier.
4-vector.
Photons, gluons, $W^{\pm}$, Z^0

Matter.
Spinors

Higgs
Scalar.

U(1)	SU(2)	SU(3)	SO(3,1)	
X	X	8	4	gluons
		3	2	q_L
		3	2	q_R
		X	2	e_L
		X	2	e_R

Force carrier
4-vector.

Photons, gluons, $W^{\pm}$, Z^0

Matter.
Spinors

Higgs
Scalar.

$U(1)$	$SU(2)$	$SU(3)$	$SO(3,1)$	
X	X	8	4	gluons
✓	2	3	2	q_L
✓	X	3	2	q_R
✓	2	X	2	e_L
✓	X	X	2	e_R

Force carrier
4-vector.

→ Photons, gluons, $W^{\pm}$, Z^0

Matter.
Spinors

Higgs
Scalar.

$U(1)$	$SU(2)$	$SU(3)$	$SO(3,1)$		
X	X	8	4	gluons	
✓	2	3	2	q_L	quarks
✓	X	3	2	q_R	
✓	2	X	2	e_L	Leptons.
✓	X	X	2	e_R	

Force carrier

4-vector.

↳ Photons, gluons, $W^{\pm}$, Z^0

Matter.

Spinors

Higgs

Scalar

U(1)	SU(2)	SU(3)	SO(3,1)		
X	X	8	4	gluons	
✓	2	3	2	q_L	quarks
✓	X	3	2	q_R	
✓	2	X	2	e_L	Leptons
✓	X	X	2	e_R	μ, τ

Force carrier

4-vector.

→ Photons, gluons, $W^{\pm}$, Z^{0}

Matter.

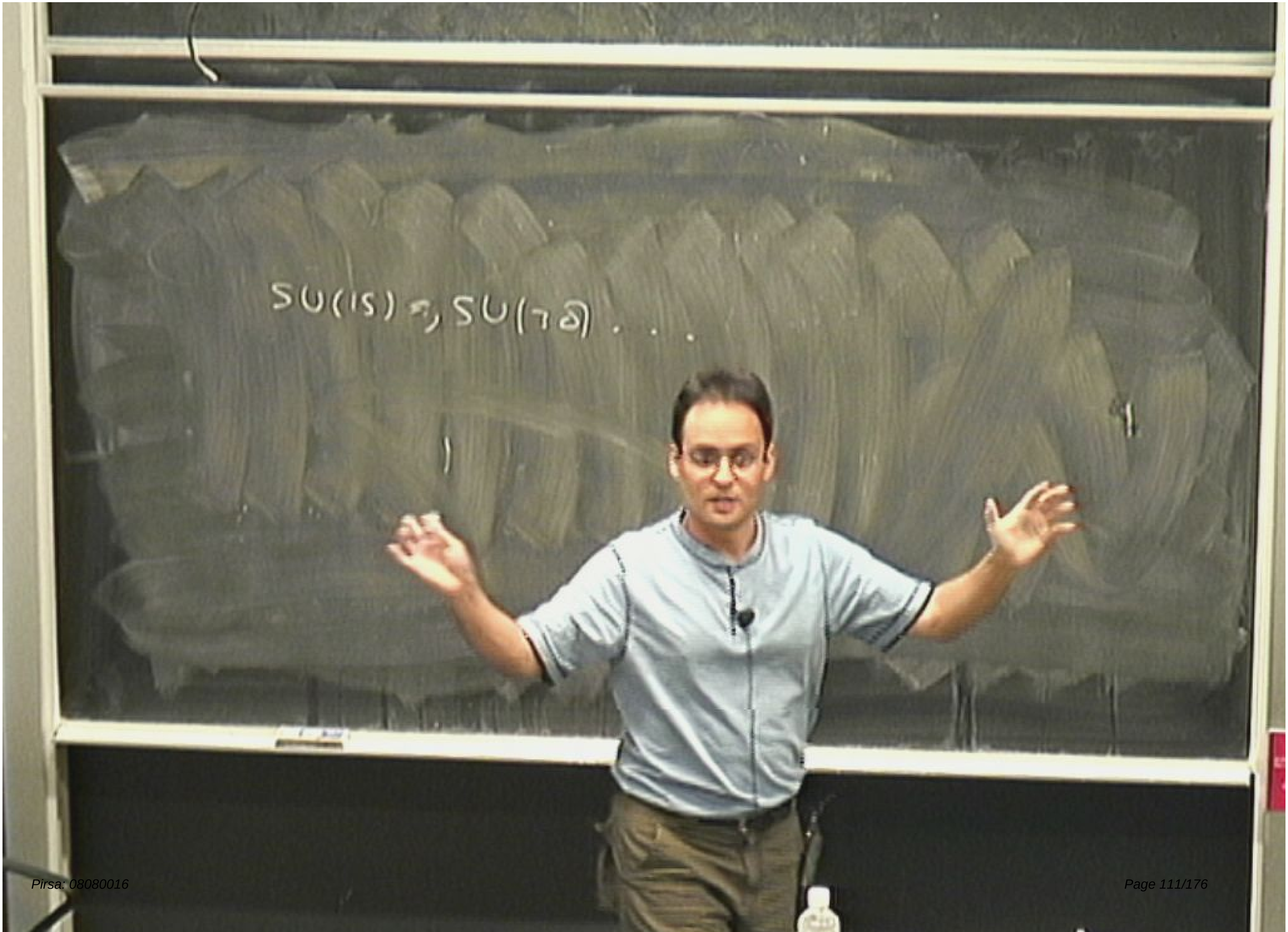
Spinors

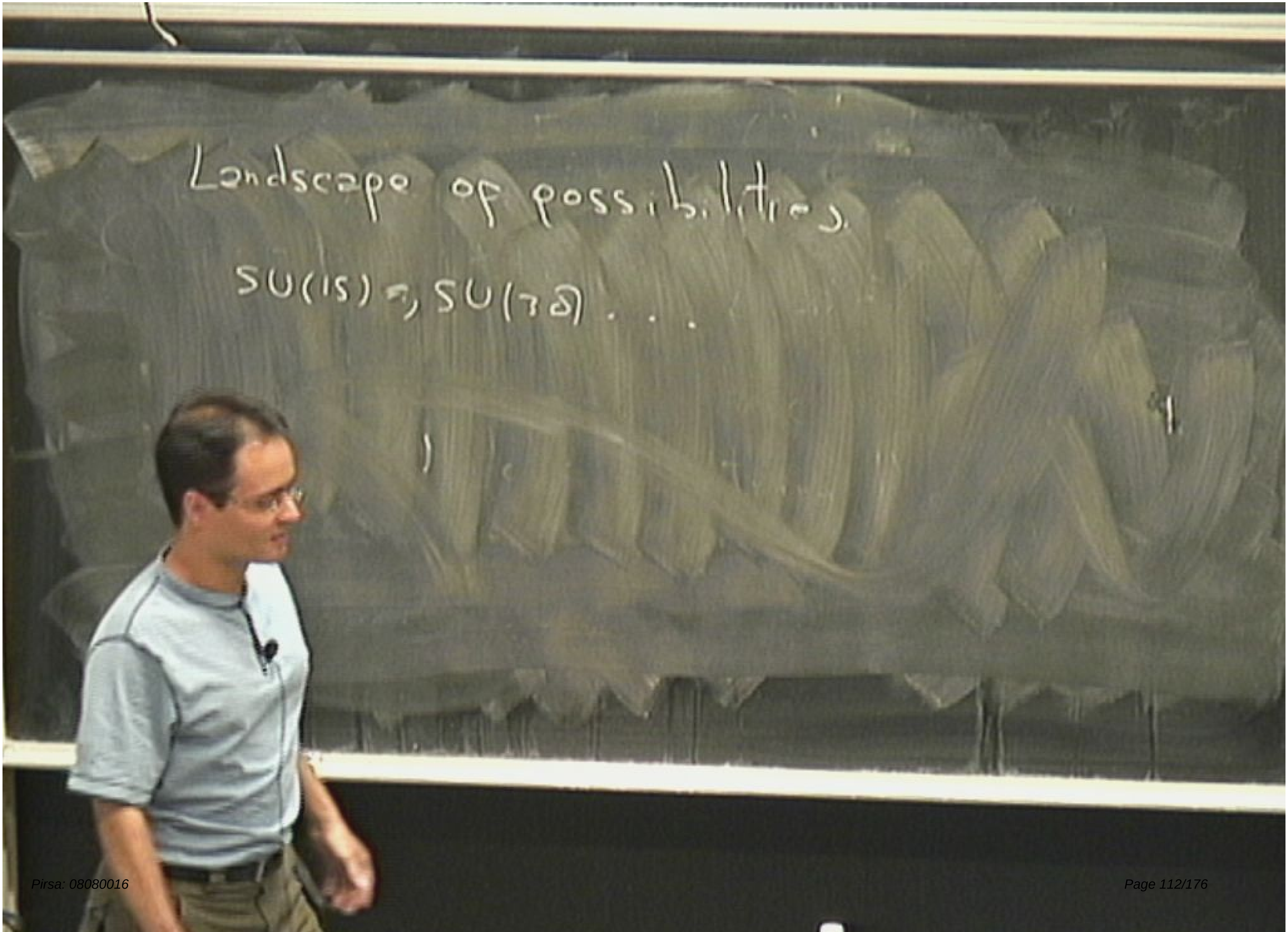
Higgs

Scalar

$$\left.\begin{array}{ll} 4 & A_{1,2,3} \\ 4 & B \end{array}\right\} \rightarrow \begin{array}{l} \text{Photon } \gamma \\ W^{\pm}, Z^{0}. \end{array}$$

U(1)	SU(2)	SU(3)	SO(3,1)		
X	X	8	4	gluons	
✓	2	3	2	q_L	quarks
✓	X	3	2	q_R	
✓	2	X	2	e_L	Leptons.
✓	X	X	2	e_R	μ, τ
✓	X				

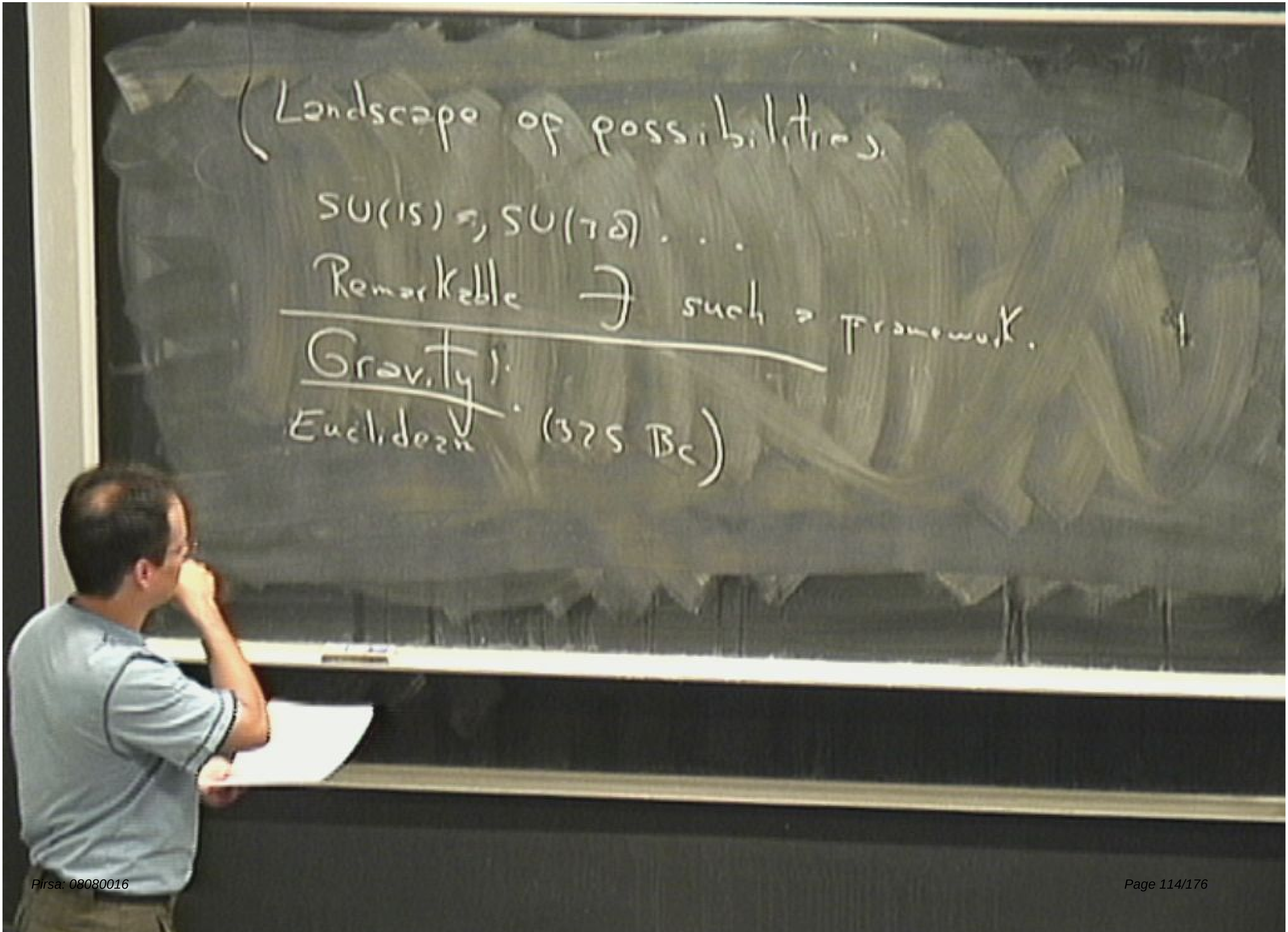

SU(15)


Landscape of possibilities
SU(15), SU(78) . . .

Landscape of possibilities.

$SU(15) \Rightarrow, SU(78) \ldots$

Remarkable $\exists$ such a framework.


(Landscape of possibilities.
SU(15), SU(78) . . .
Remarkeble ∃ such a framework.
Gravity:
Euclidean (375 BC)

(Landscape of possibilities.

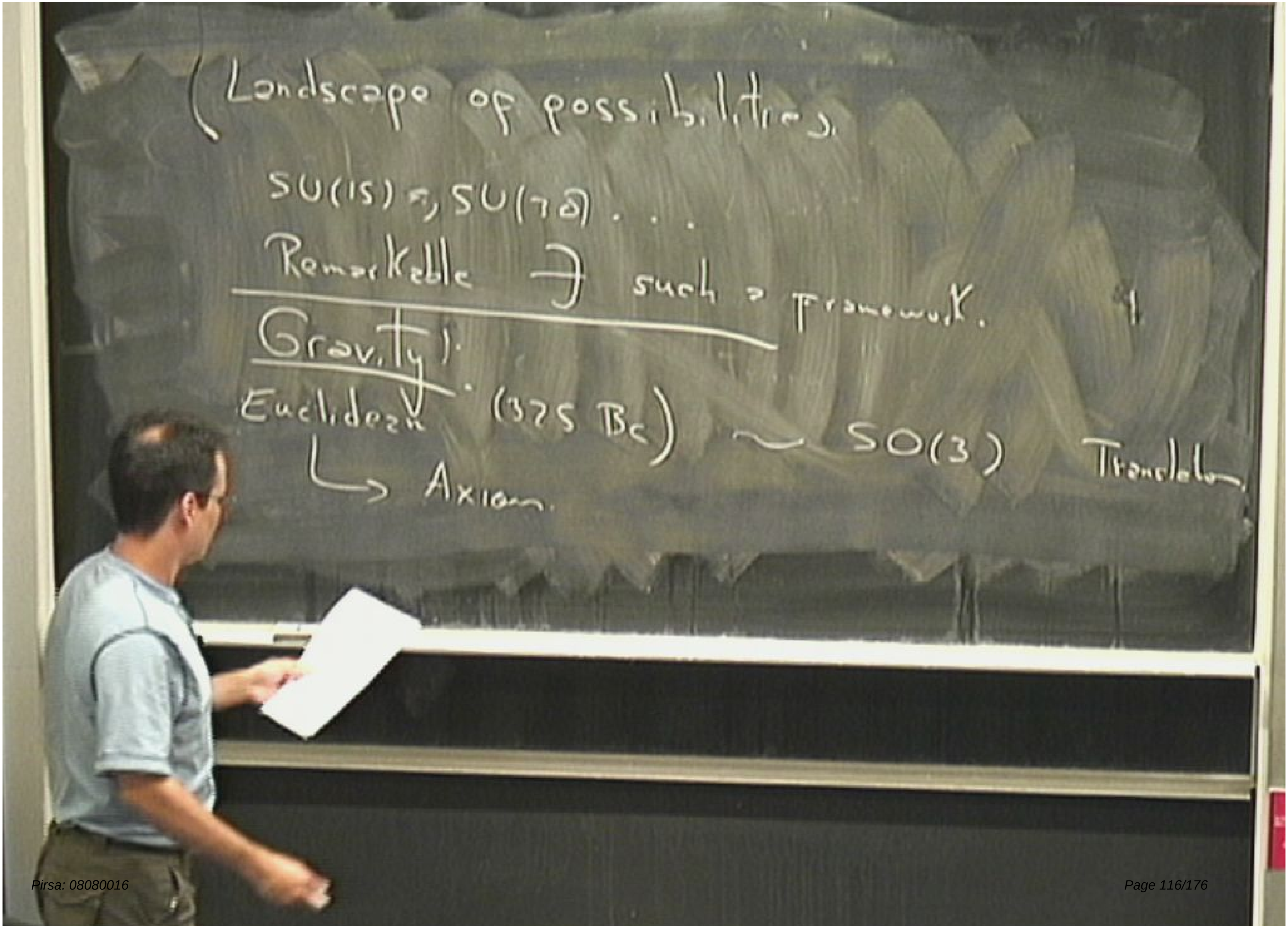
SU(15), SU(78) . . .

Remarkable $\exists$ such a framework.

Gravity:

Euclidean (325 BC)

↳ Axiom.


(Landscape of possibilities.
SU(15), SU(78) . . .
Remarkable ∃ such a framework.
Gravity).
Euclidean (325 BC) ~ SO(3) Translation
Axiom.

(Landscape of possibilities.

SU(15) ~, SU(78) . . .

Remarkable $\exists$ such a framework.

Gravity:

Euclidean (325 BC) ~ SO(3) Translation

↳ Axiom.

Newton (1643)

(Landscape of possibilities.

SU(15), SU(78) . . .

Remarkable ∃ such a framework.

Gravity!

Euclidean (325 Bc) ~ SO(3) Translation

↳ Axiom.

Newton (1643) ~ Move in a straight line in S-T

Landscape of possibilities.

SU(15) ?, SU(78) . . .

Remarkable $\exists$ such a framework.

Gravity:

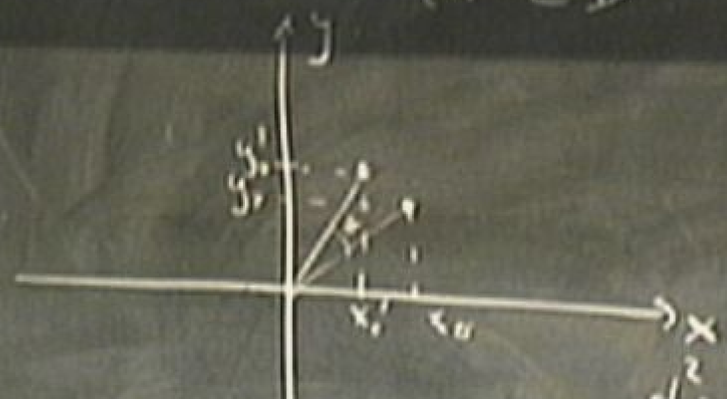
Euclidean (325 BC) ~ SO(3) Translation

↳ Axiom.

(Newton (1643) ~ Move in a straight line in S-T

SO(3,1)

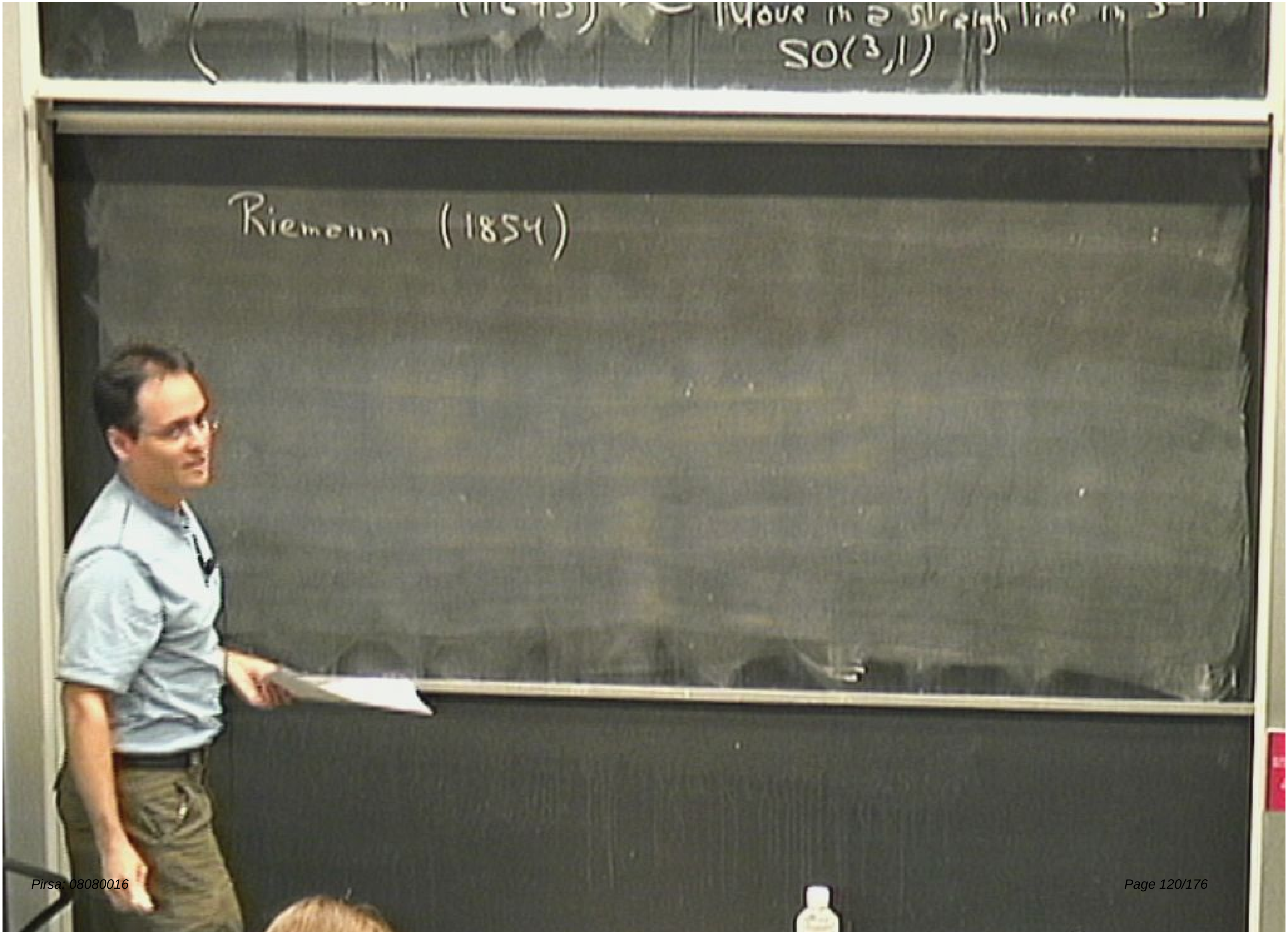
Rotations in 2D

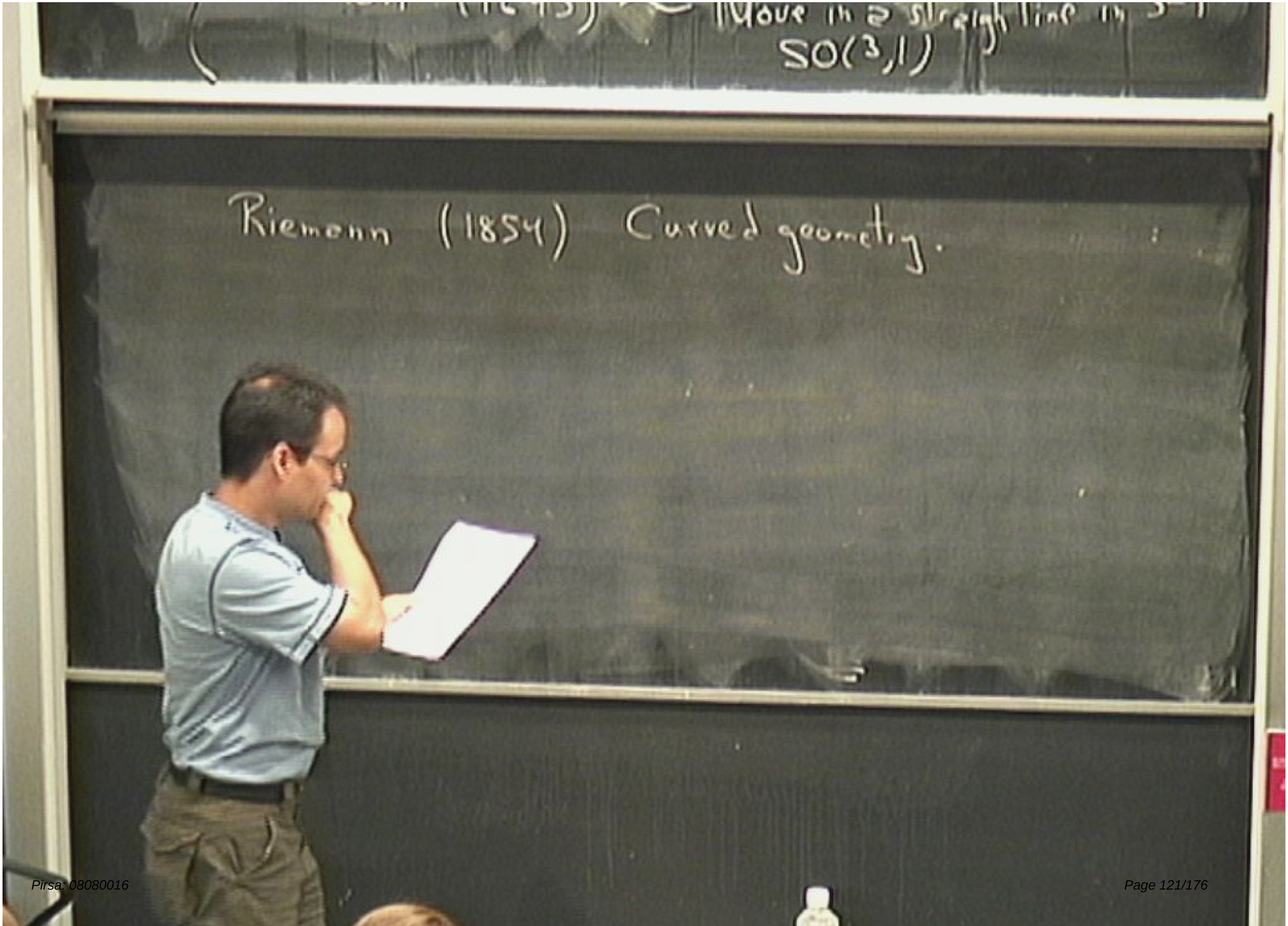


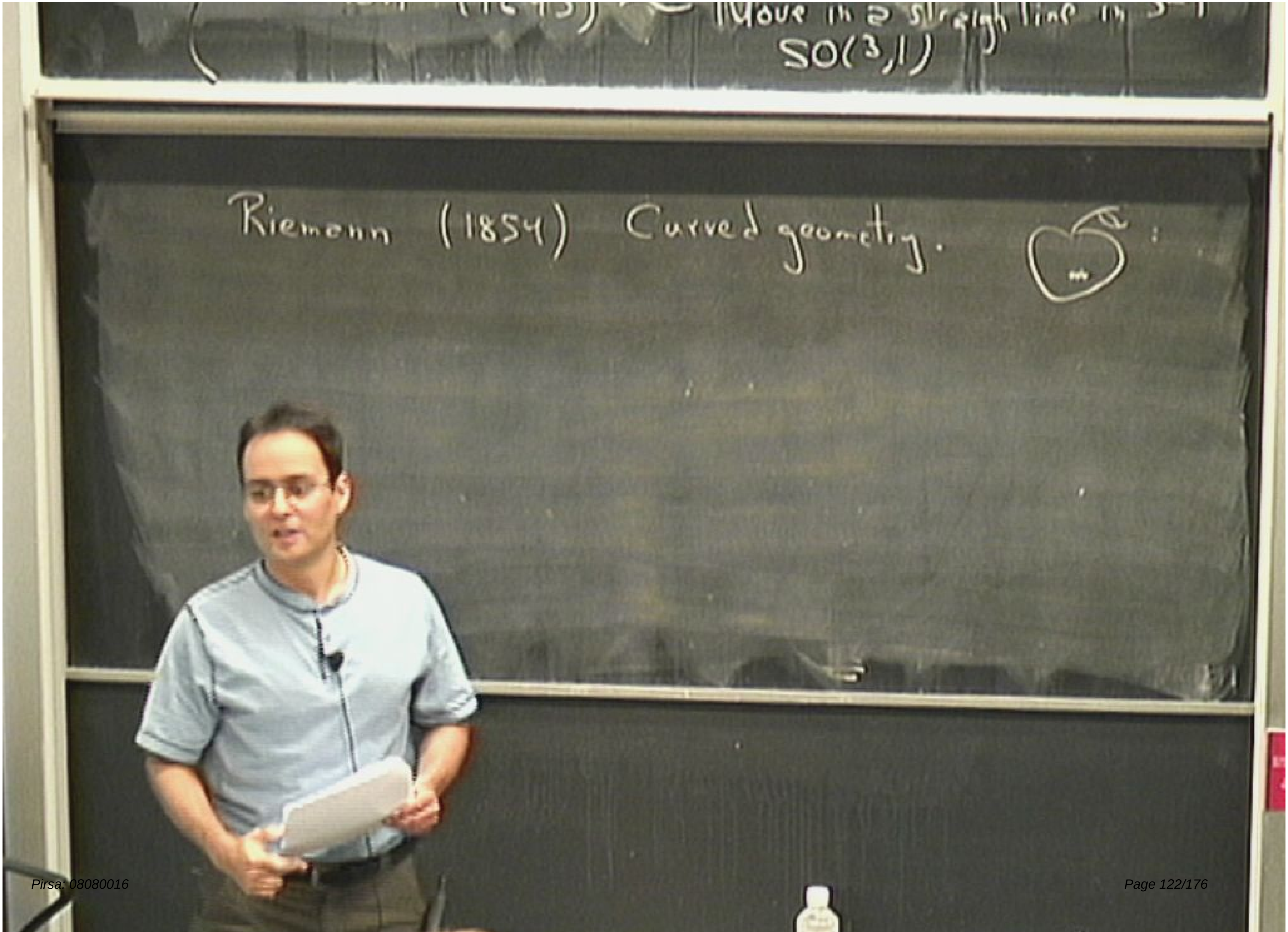
$$x_0' = \cos\theta\, x_0 - \sin\theta\, y_0$$

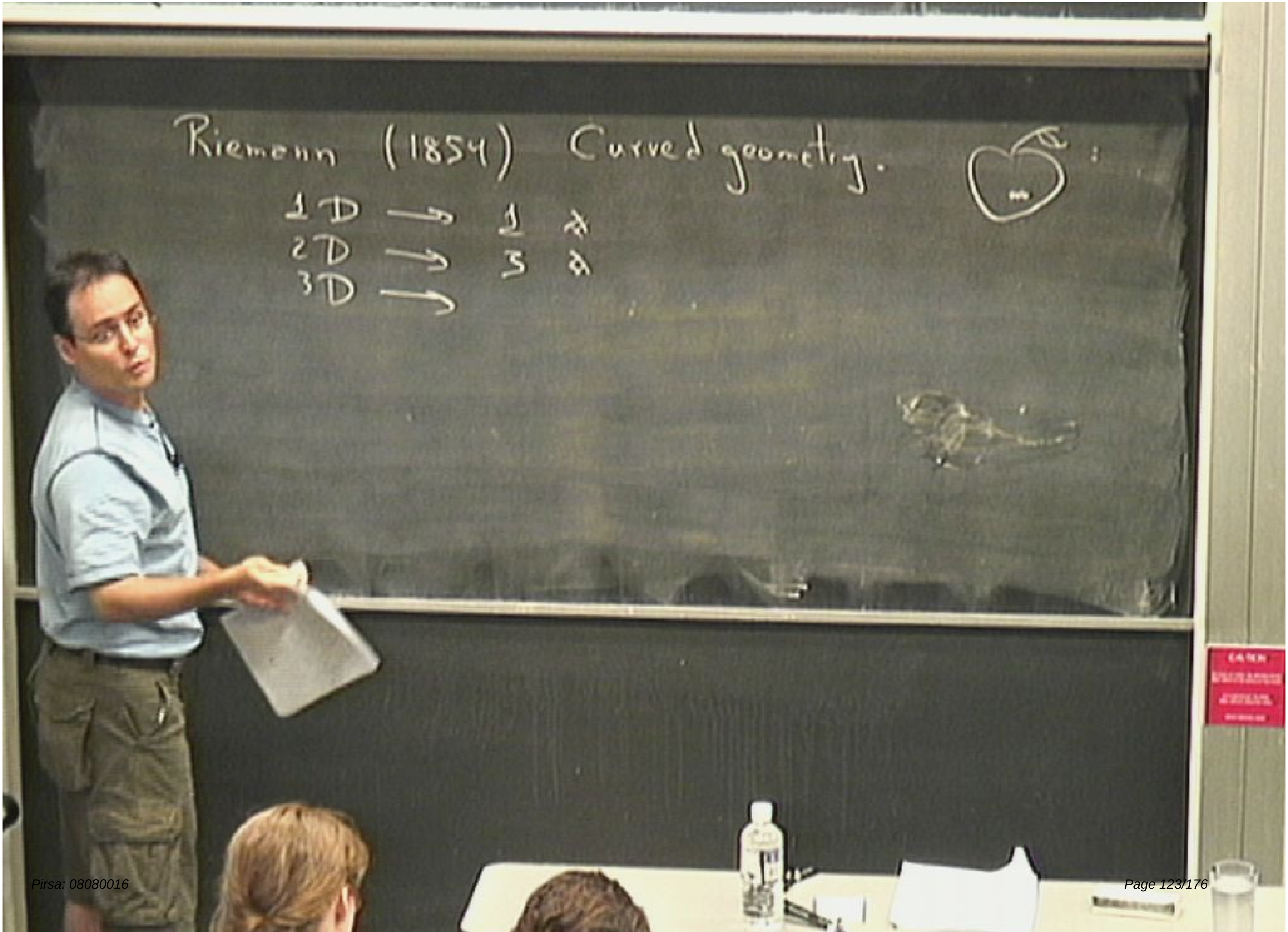
$$y_0' = \sin\theta\, x_0 + \cos\theta\, y_0$$

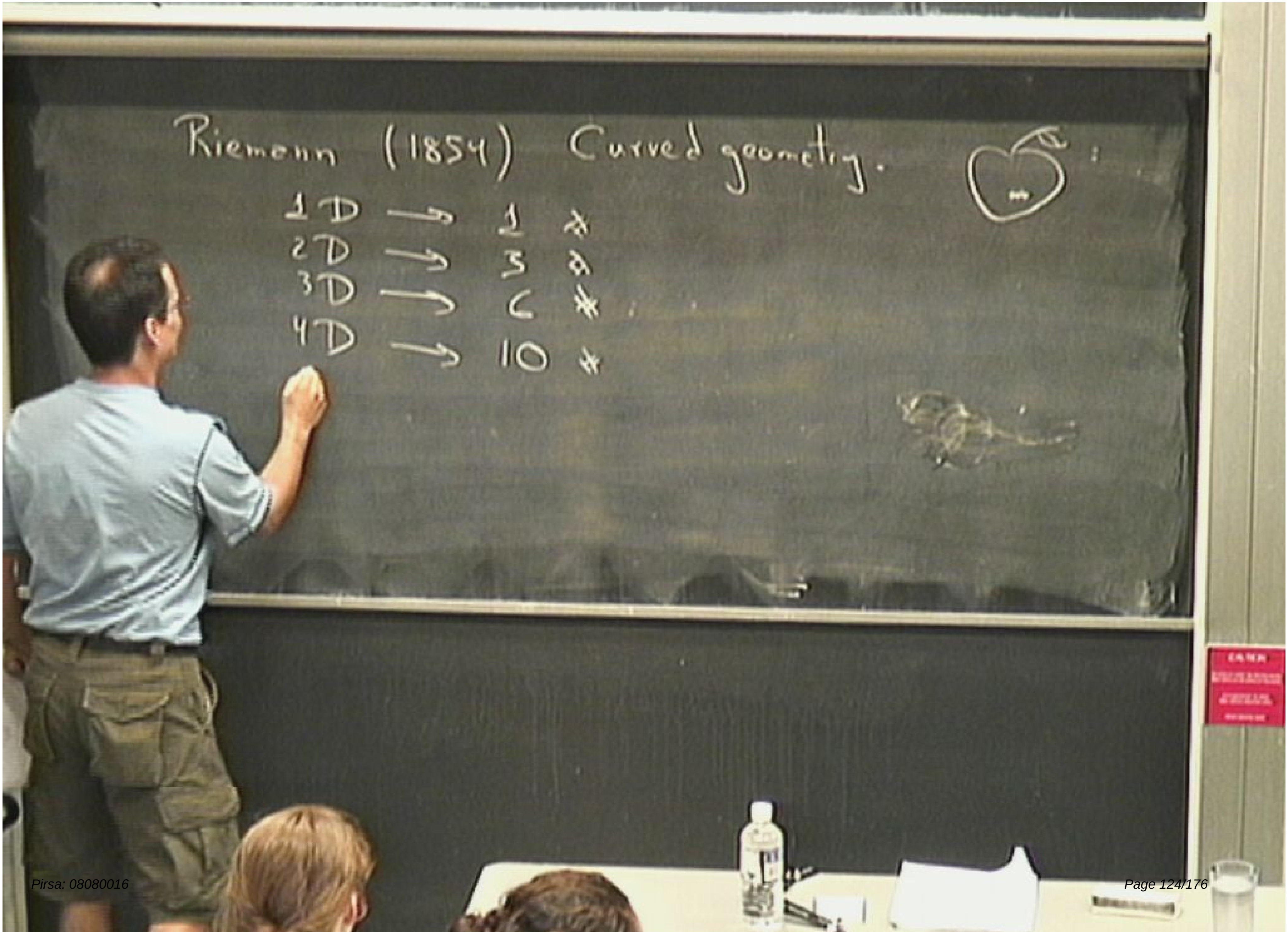
$$\begin{pmatrix} x_0' \\ y_0' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

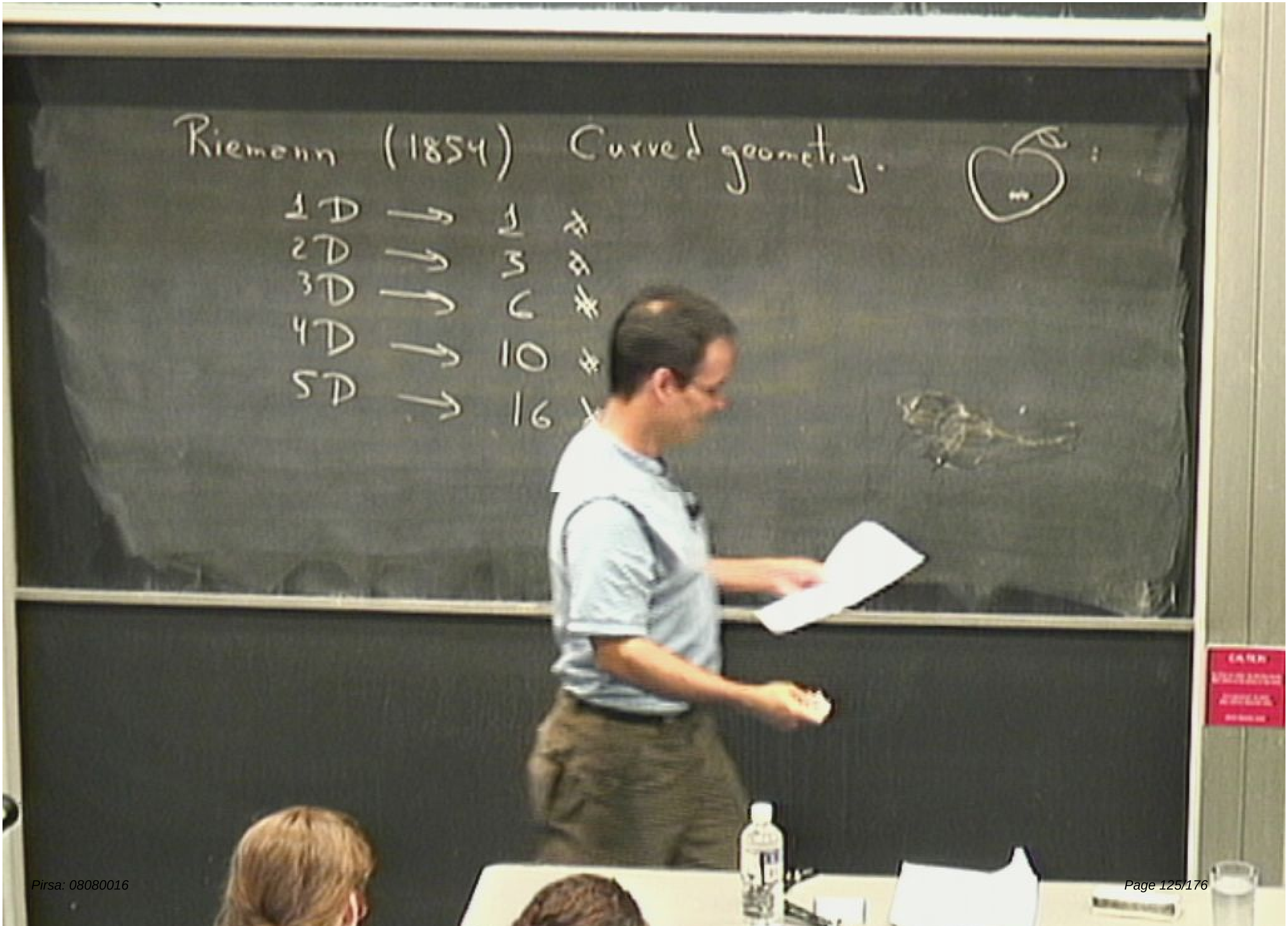

Move in a straight line in
SO(3,1)
Riemann (1854)

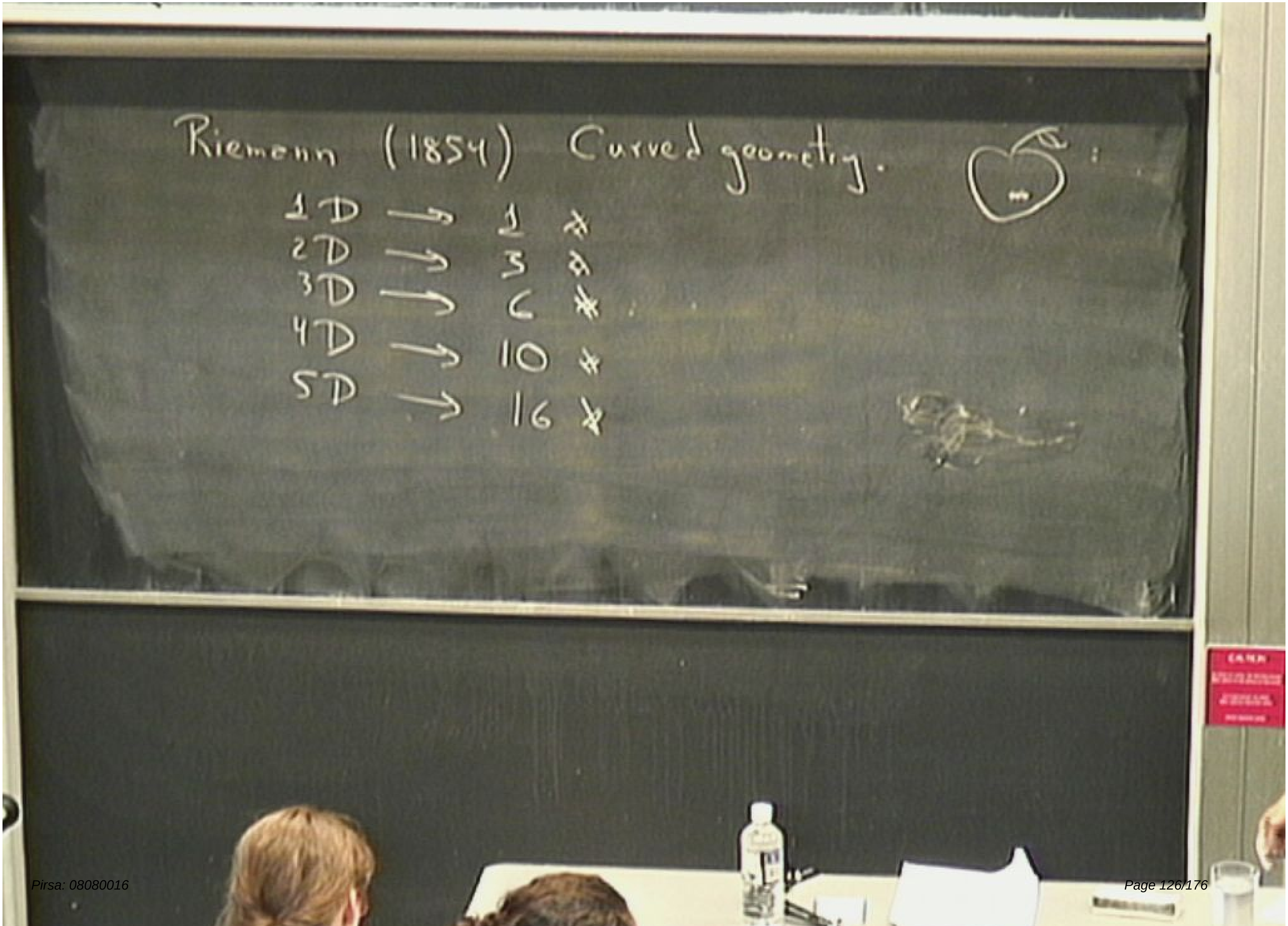

Move in a straight line
SO(3,1)
Riemann (1854) Curved geometry.

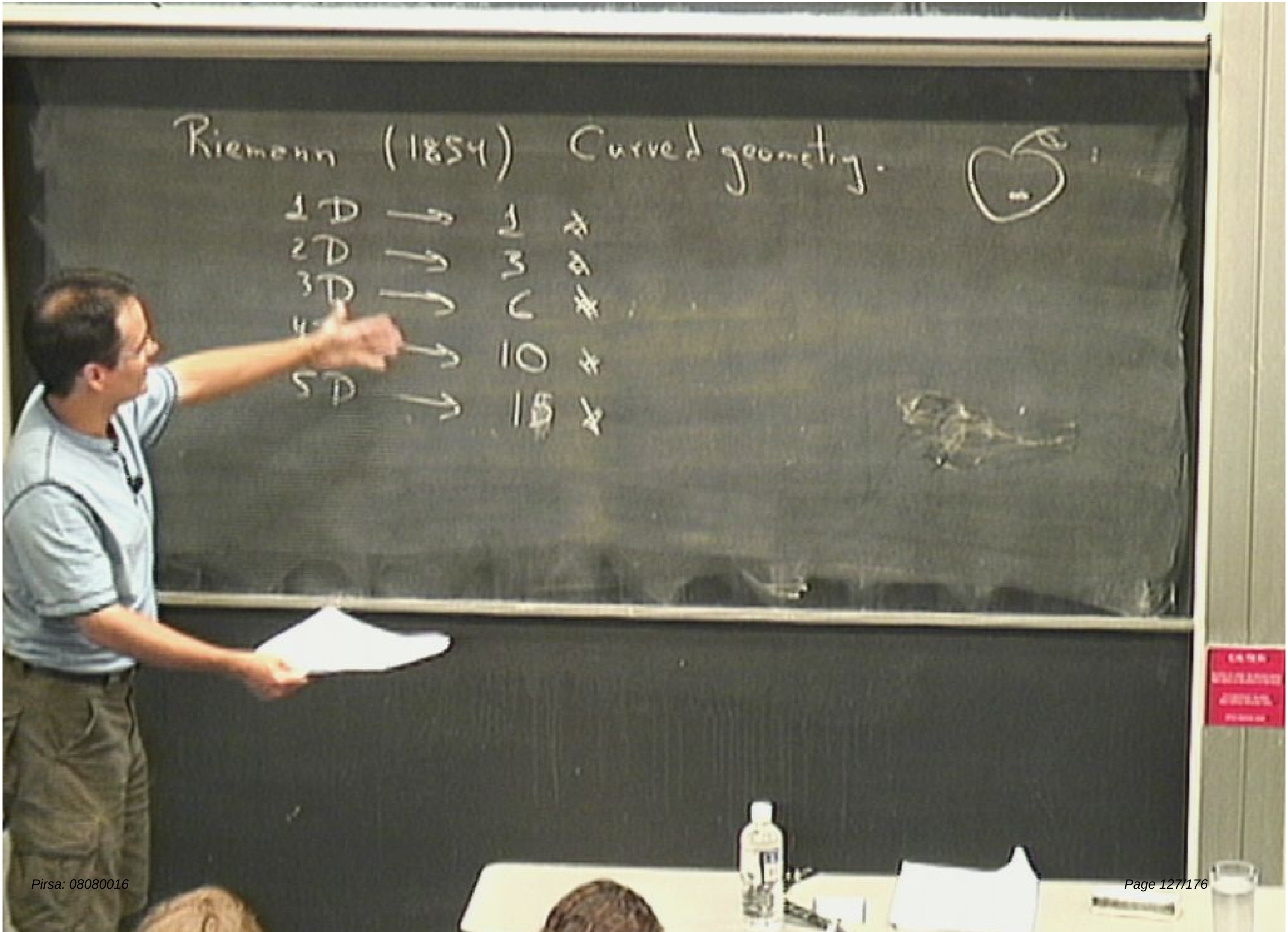

Move in a straight line in
SO(3,1)
Riemann (1854) Curved geometry.


Riemann (1854) Curved geometry.
1D → 1
2D → 3
3D →


Riemann (1854) Curved geometry.
1D → 1
2D → 3
3D → 6
4D → 10

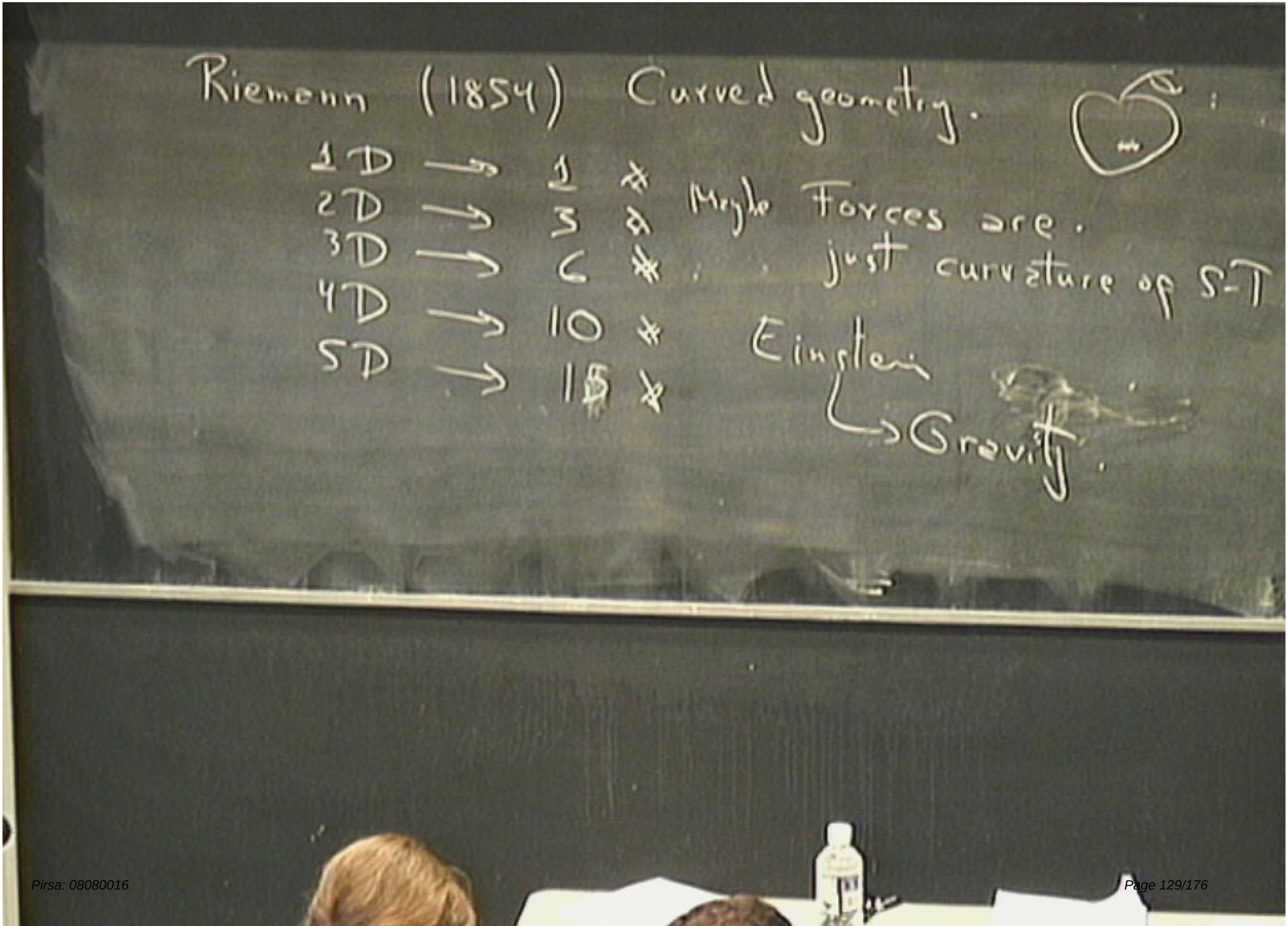

Riemann (1854) Curved geometry.
1D → 1 #
2D → 3 #
3D → 6 #
4D → 10 #
5D → 16


Riemann (1854) Curved geometry.
1D → 1 *
2D → 3 *
3D → 6 *
4D → 10 *
5D → 16 *

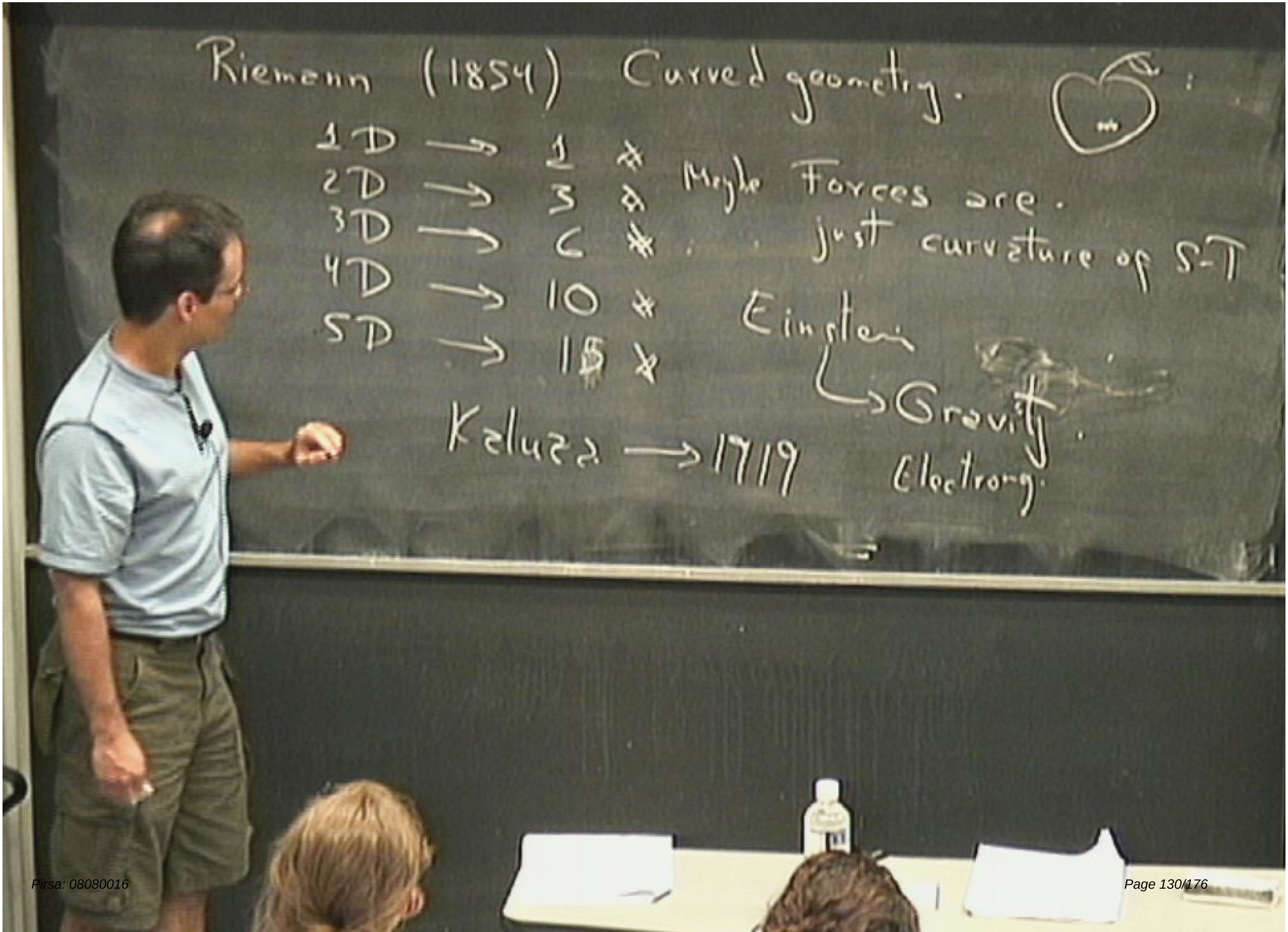

Riemann (1854) Curved geometry.
1D → 1
2D → 3
3D → 6
→ 10
5D → 15

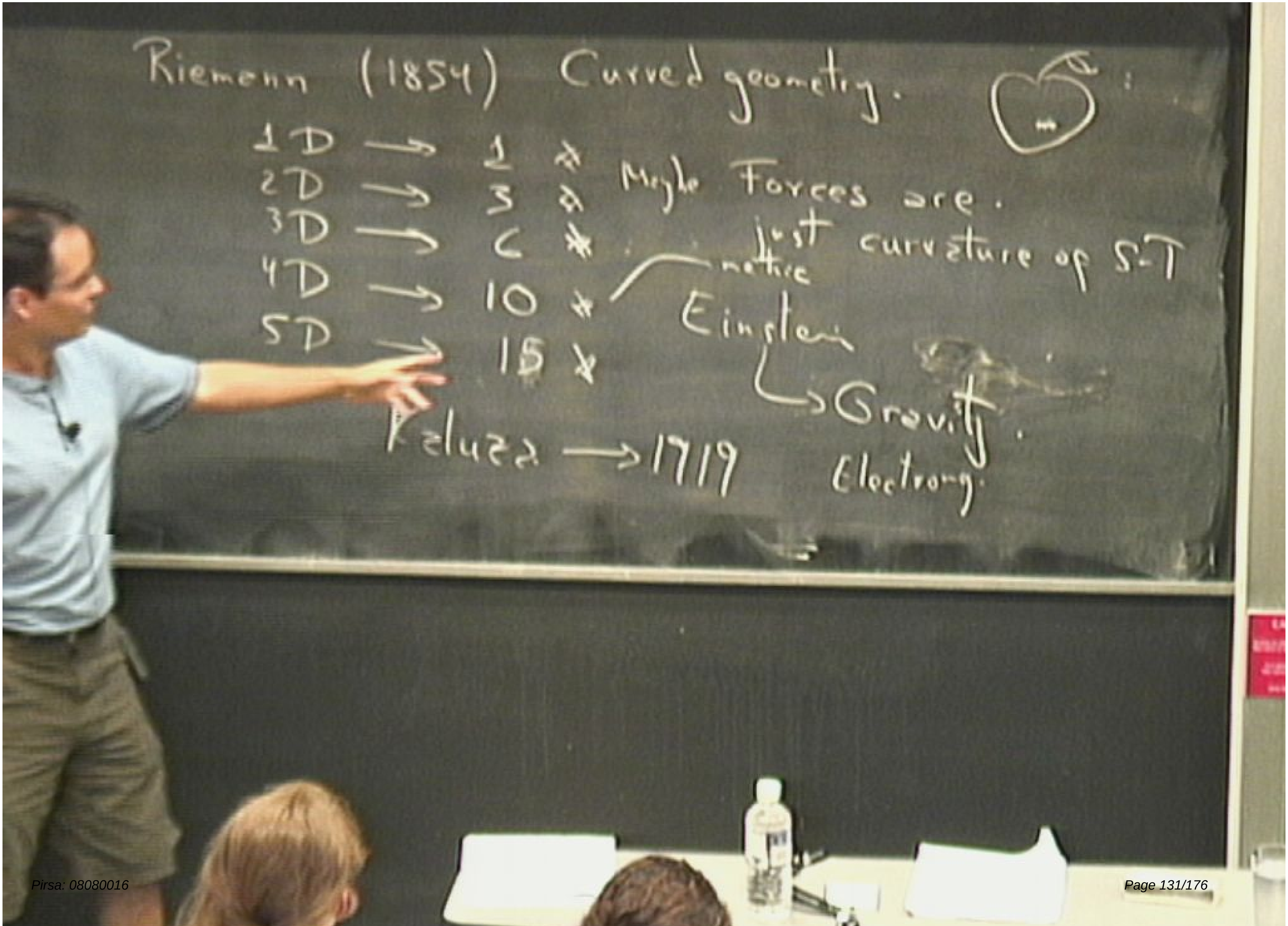
Riemann (1854) Curved geometry.

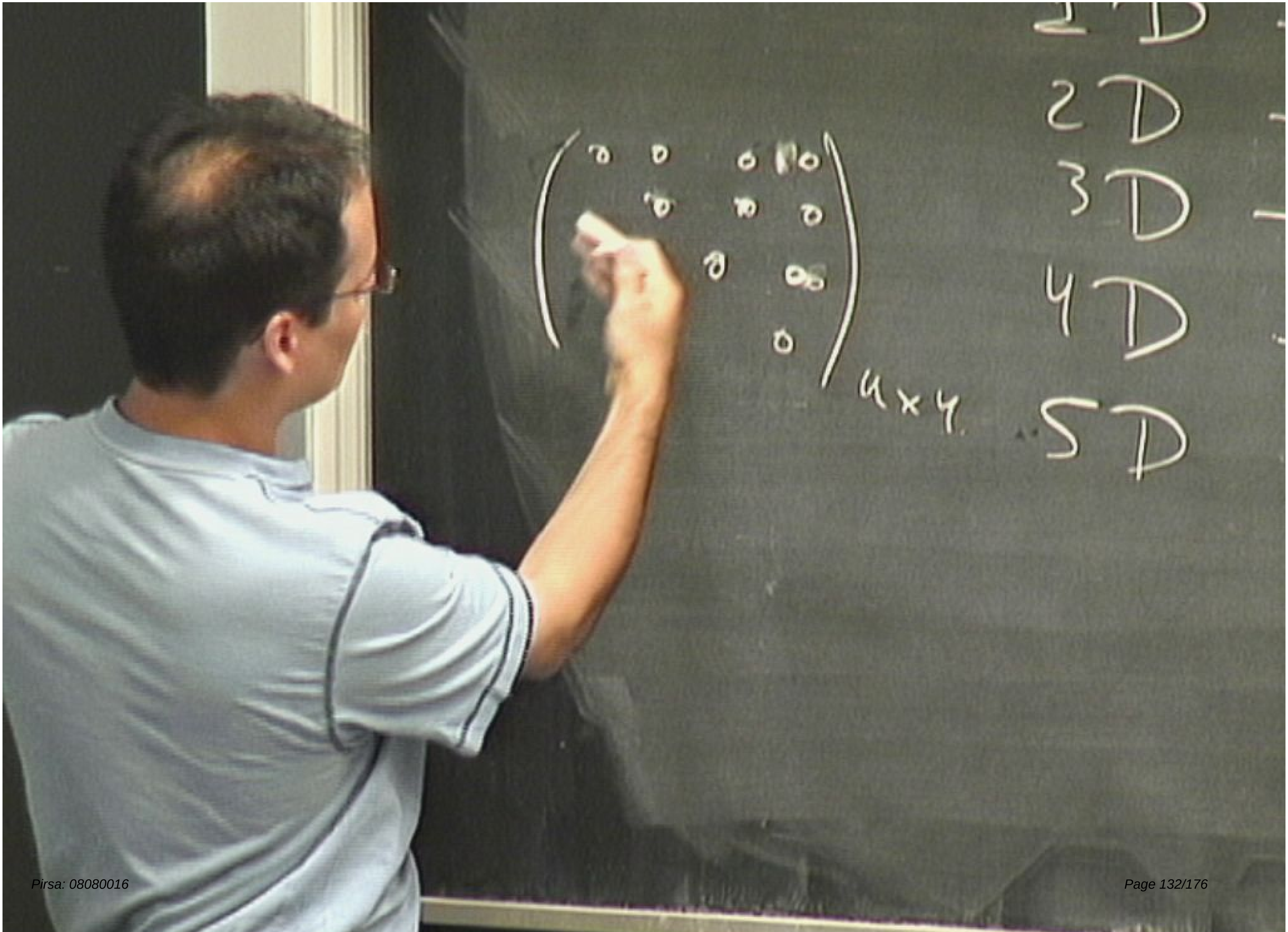
1D → 1 #
2D → 3 #
3D → 6 #
4D → 10 #
5D → 15 #

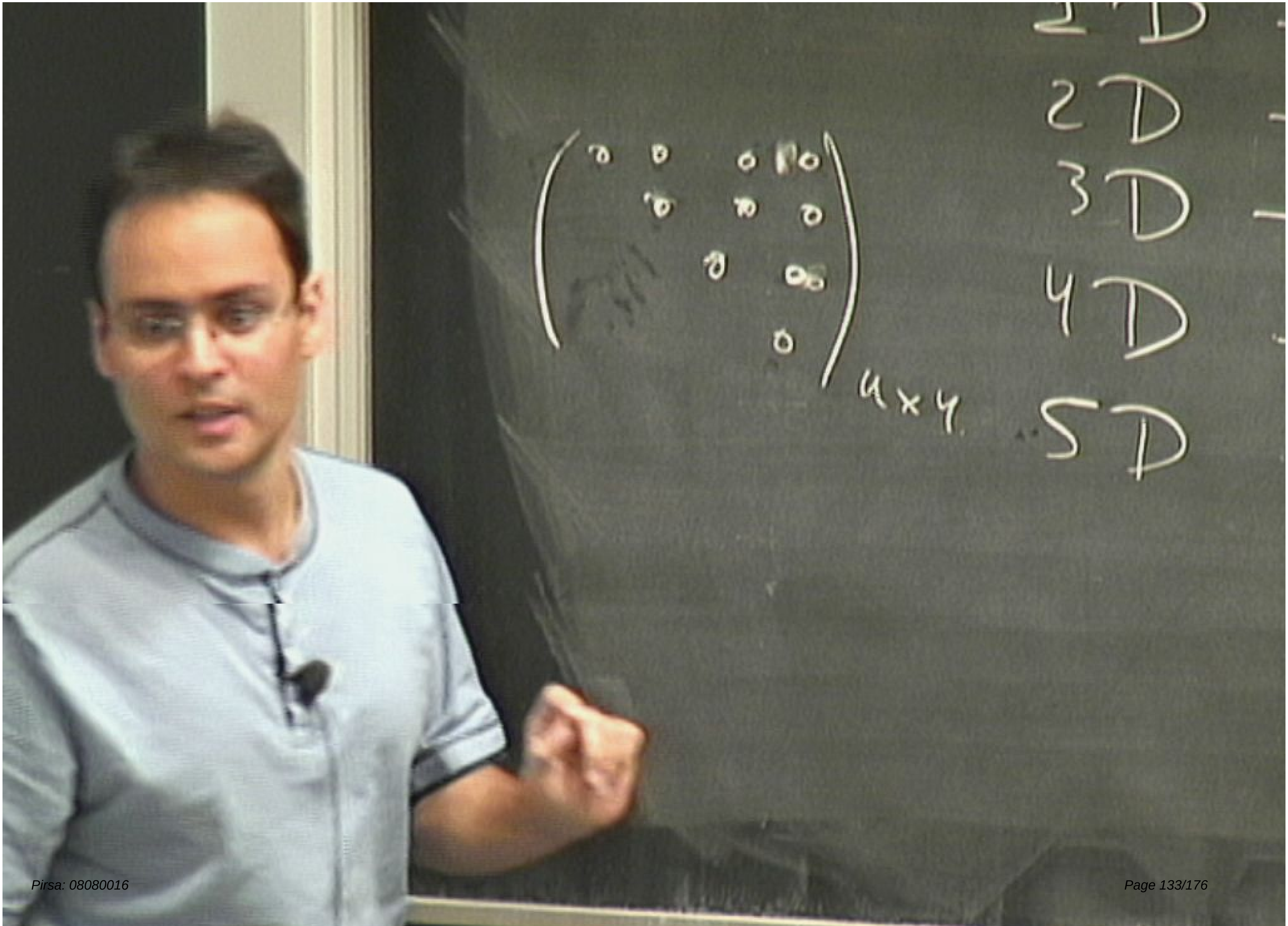


Riemann (1854) Curved geometry.
1D → 1 *
2D → 3 * Maybe Forces are.
3D → 6 * just curvature of S-T
4D → 10 * Einstein
5D → 15 *
Gravity.


Riemann (1854) Curved geometry.
1D → 1 *
2D → 3 * Maybe Forces are.
3D → 6 * just curvature of S-T
4D → 10 *
5D → 15 *
Einstein
↳ Gravity.
Kaluza → 1919 Electromag.


Riemann (1854) Curved geometry.
1D → 1
2D → 3
3D → 6
4D → 10
5D → 15
Maybe Forces are
just curvature of S-T
metric
Einstein
Gravity.
Kaluza → 1719
Electromag.


1D
2D
3D
4D
5D
4×4


2D
3D
4D
5D
4×4

Force carrier

4-vector.

Photons, gluons, $W^{\pm}$, Z^0

Matter.

Spinors

4 $A_{1,2,3}$

4 B

U(1)	SU(2)	SU(3)	SO(3,1)	
X	X	8	4	gluons
	2	3	2	q_L
✓	X	3	2	q_R
✓	2	X	2	e_L
✓	X	X	2	e_R

Riemann (1854) Curved geometry.

$1D \longrightarrow 1$ *

$2D \longrightarrow 3$ * Maybe Forces are.

$3D \longrightarrow 6$ * just curvature of S-T

$4D \longrightarrow 10$ * metric

$5D \longrightarrow 15$ *

Einstein
↳ Gravity.

4×4

5×5

Kaluza $\longrightarrow$ 1919 Electromag.

Riemann (1854) Curved geometry.

1D → 1 #

2D → 3 # Maybe Forces are.

3D → 6 # just curvature of S-T

4D → 10 # metric

Einstein

→ Gravity.

5D → 15 #

4×4

5×5

Kaluza → 1919 Electromag.

SO(3,1)

Riemann (1854) Curved geometry.

1D → 1 #

2D → 3 # Maybe Forces are

3D → 6 # just curvature of S-T

4D → 10 # metric

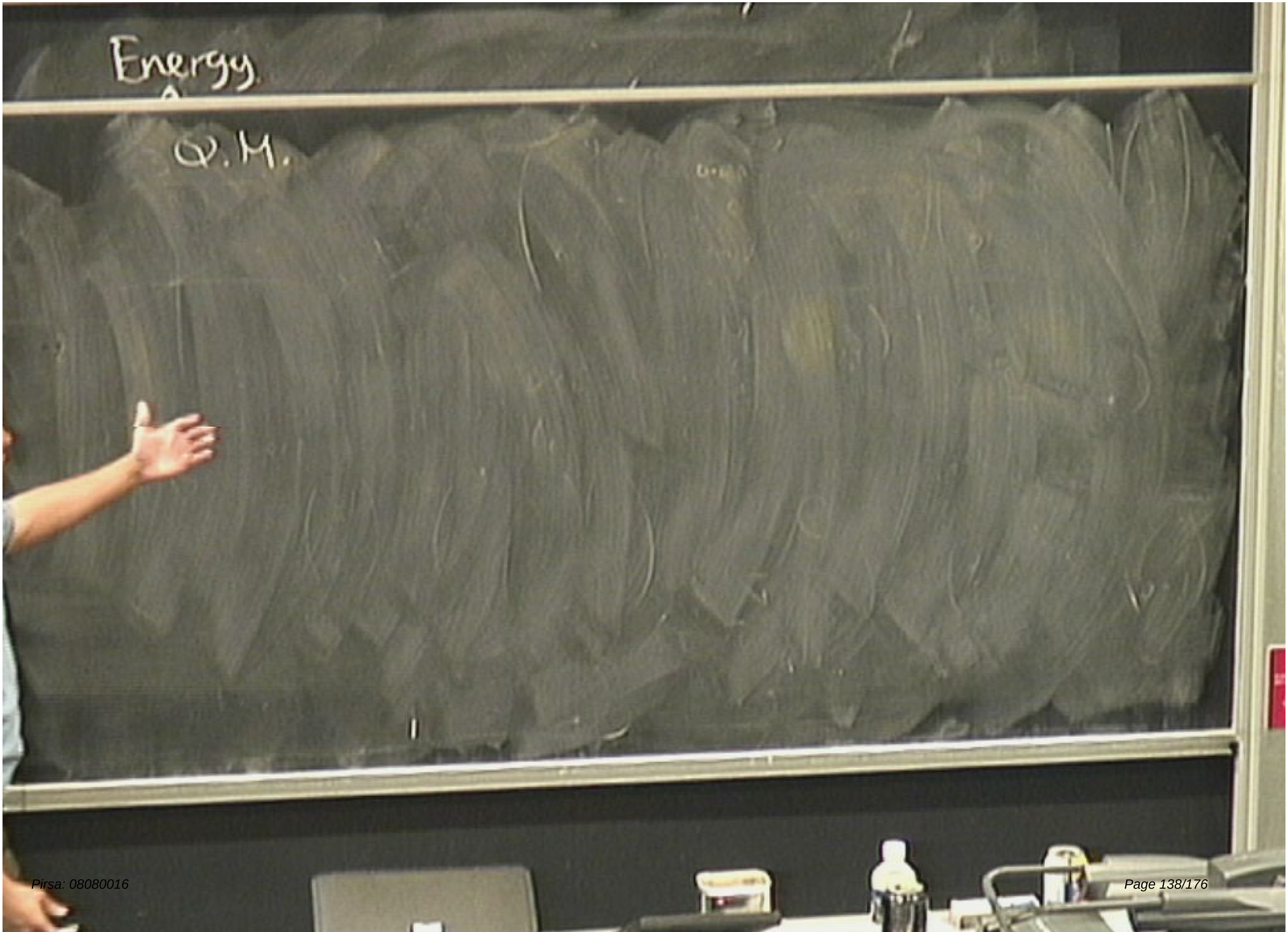
5D → 15 # Einstein

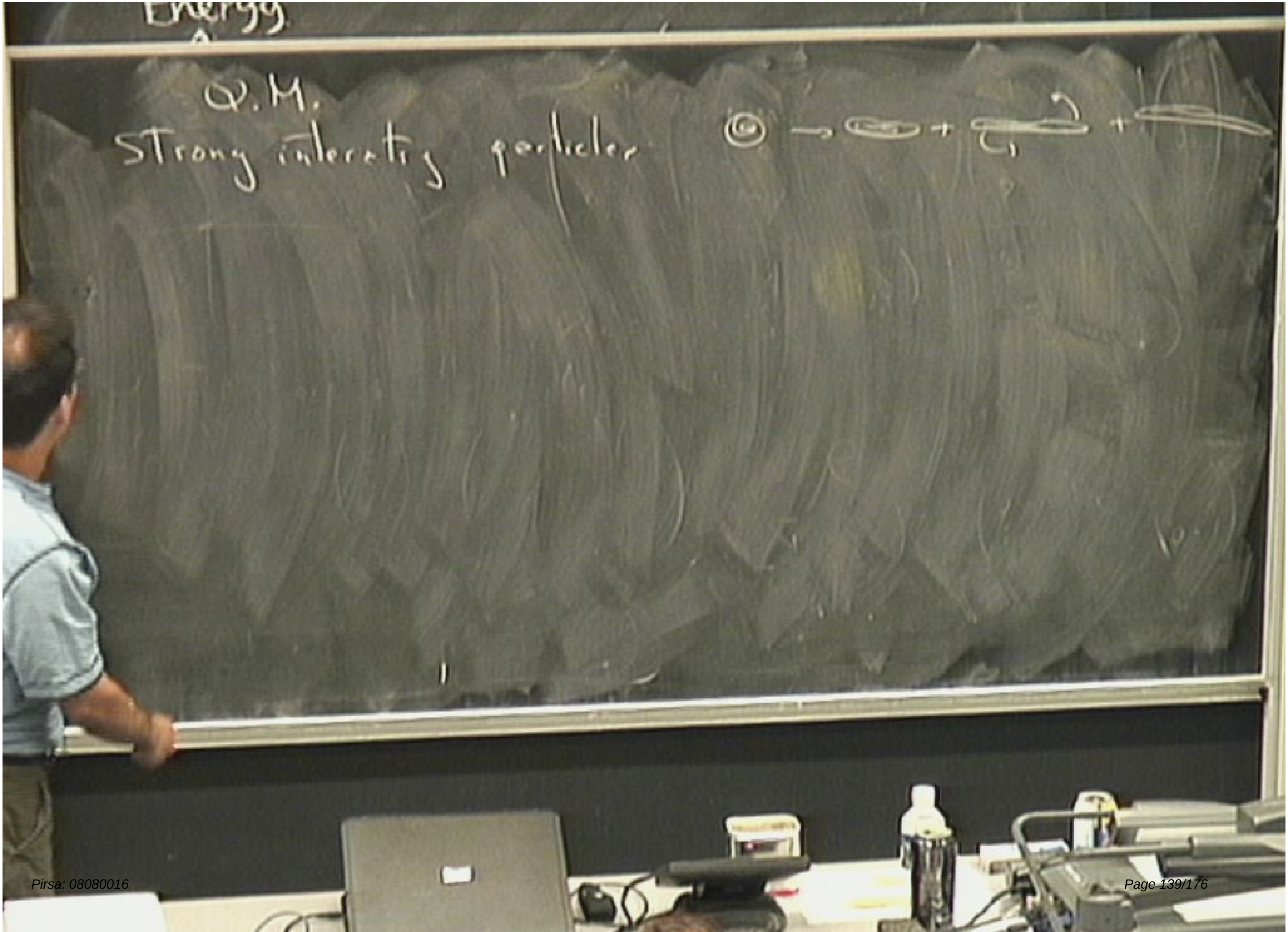
↳ Gravity.

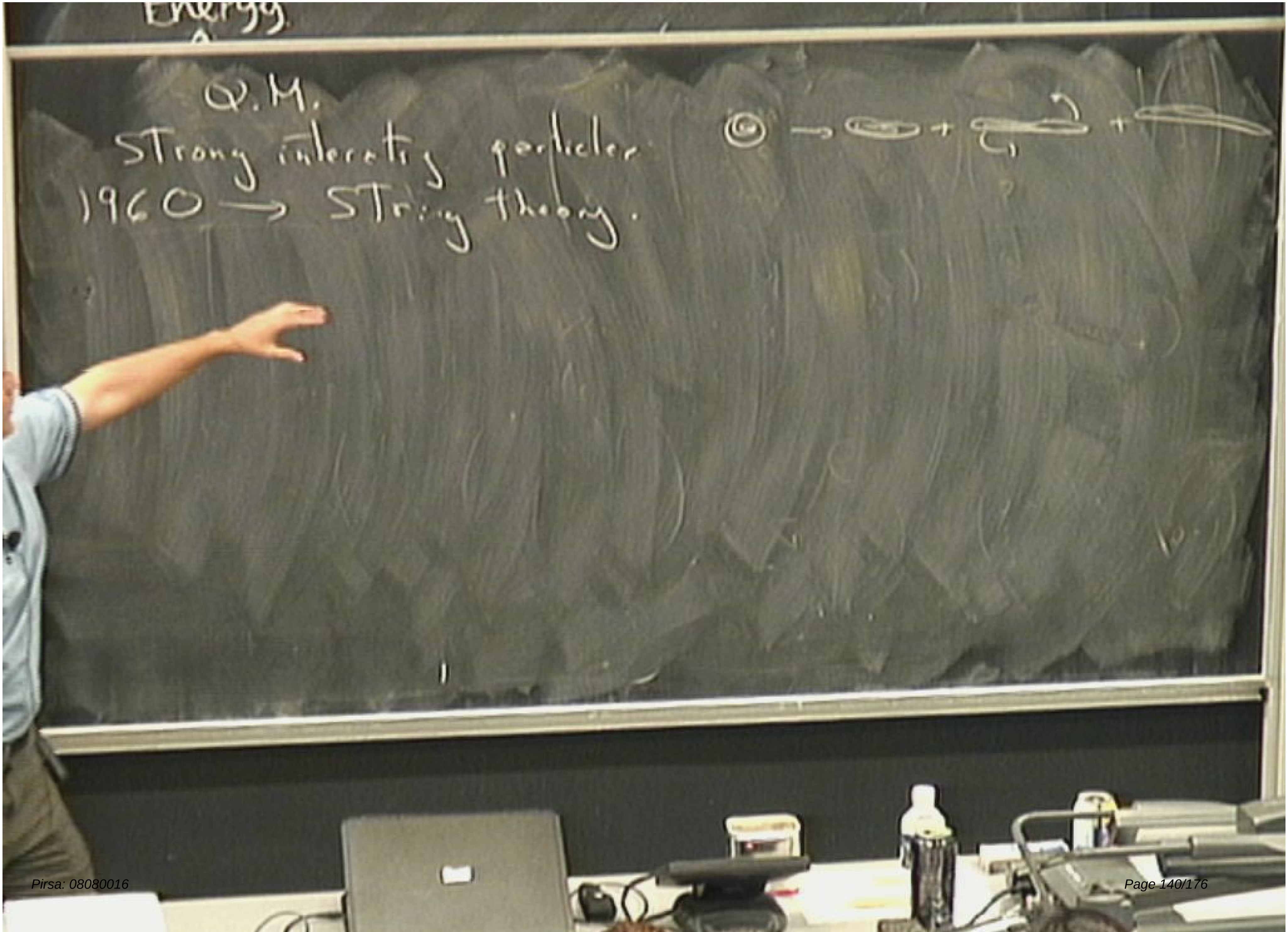
4×4

5×5

Kaluza → 1919 Electromag.


Energy
Q.M.


Energy
Q.M.
Strong interacting particles


Q.M.
Strong interacting particles
1960 → String theory.

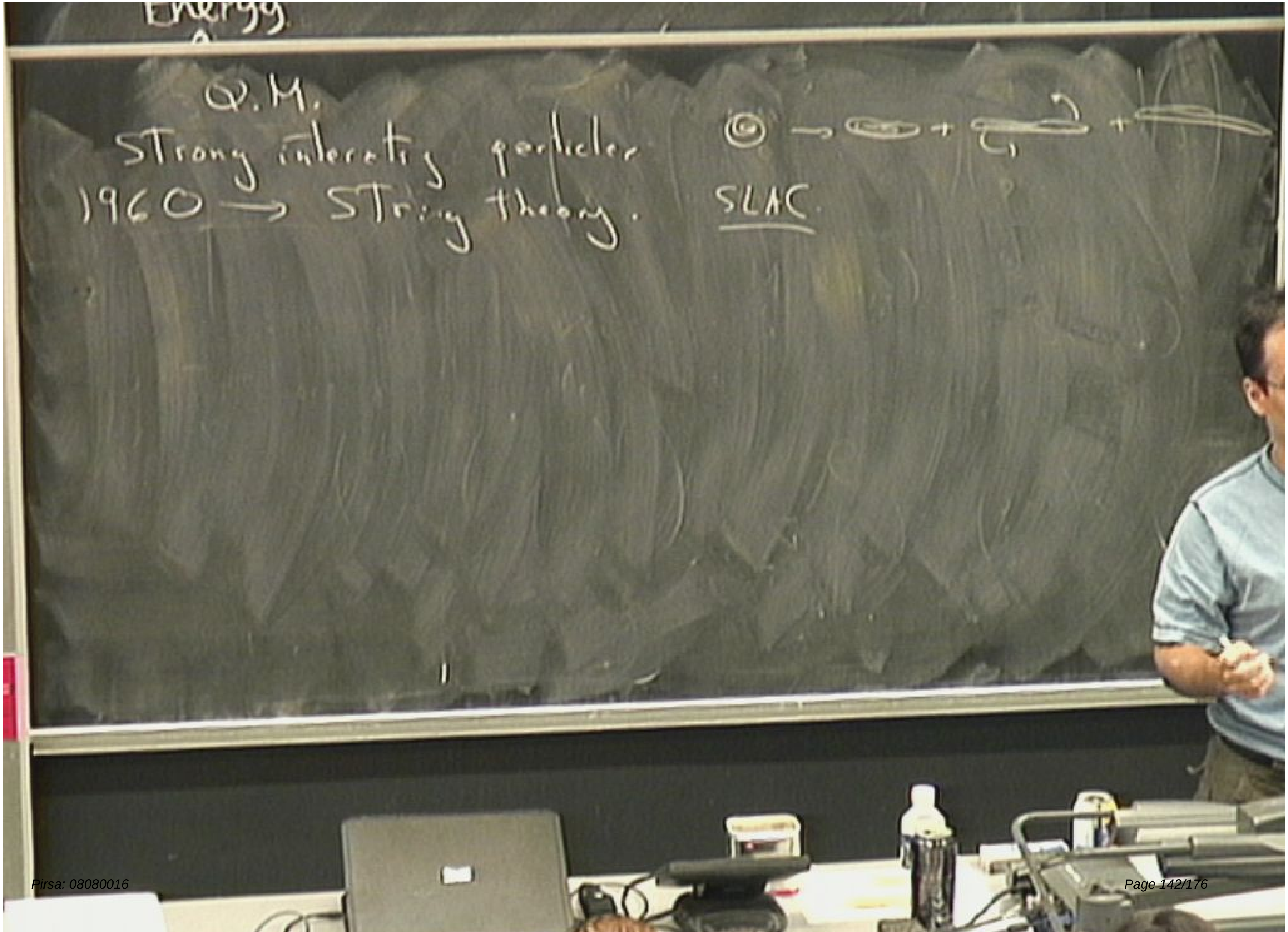
Energy

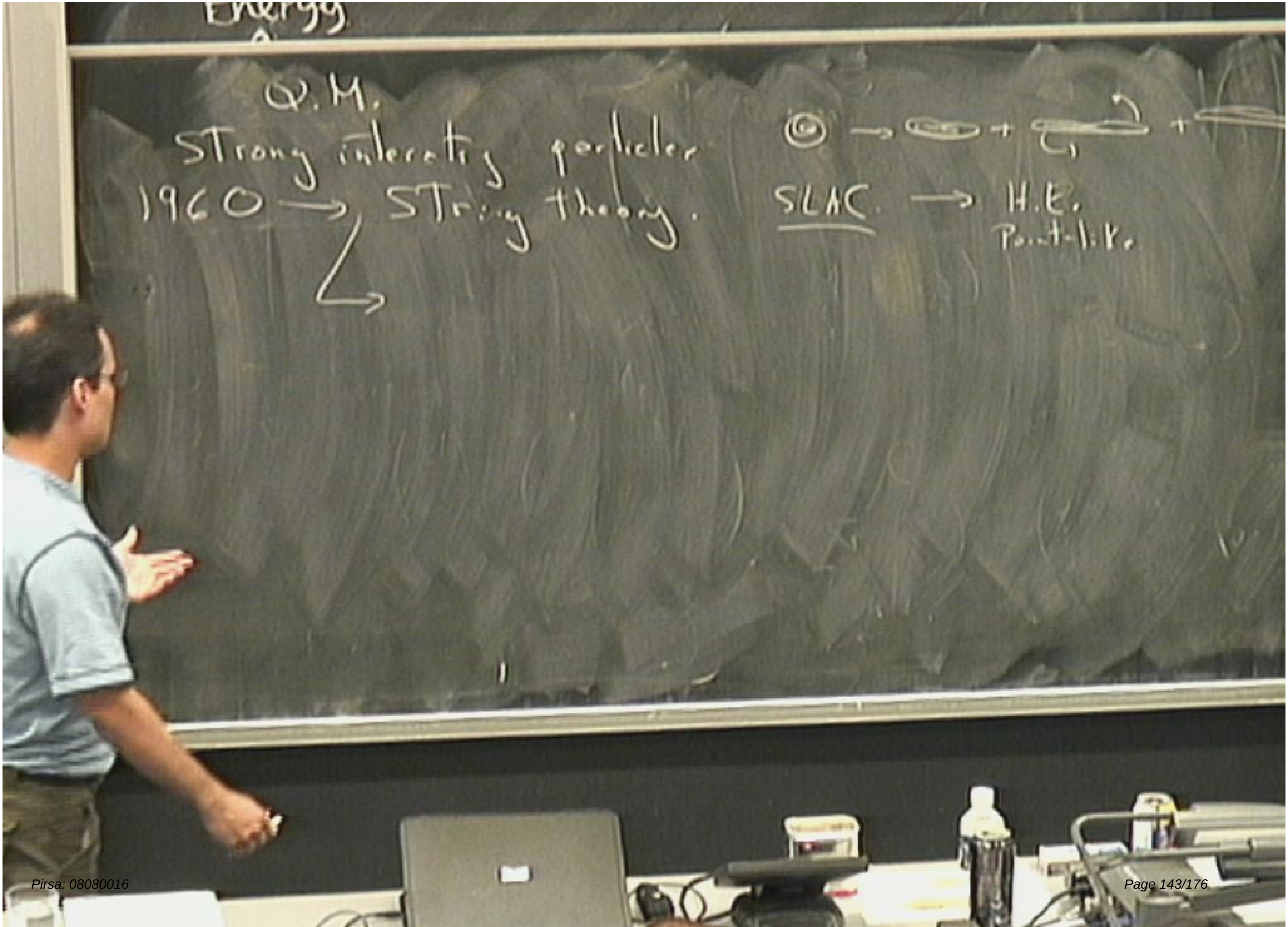
Q.M.

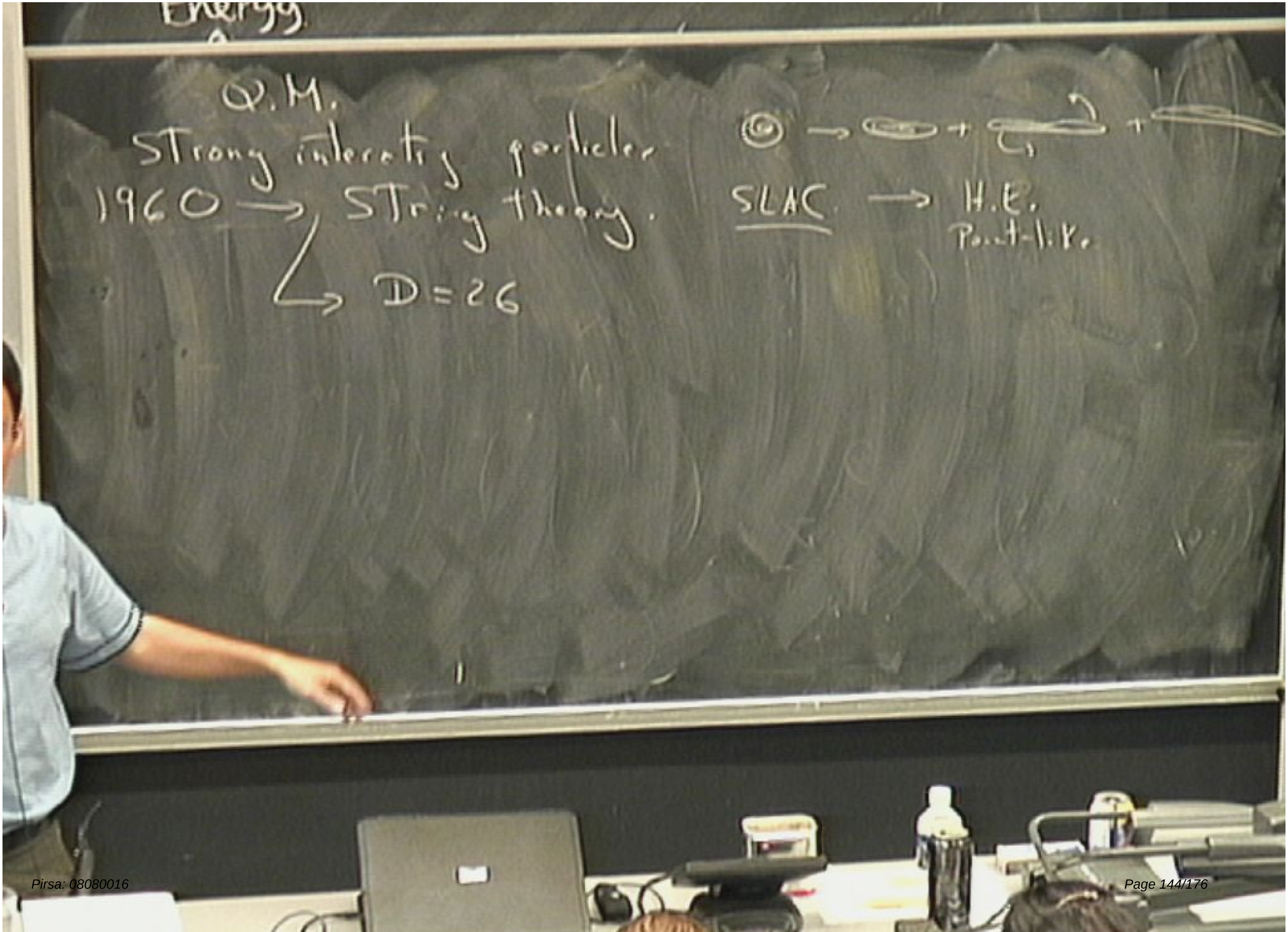
Strong interacting particles

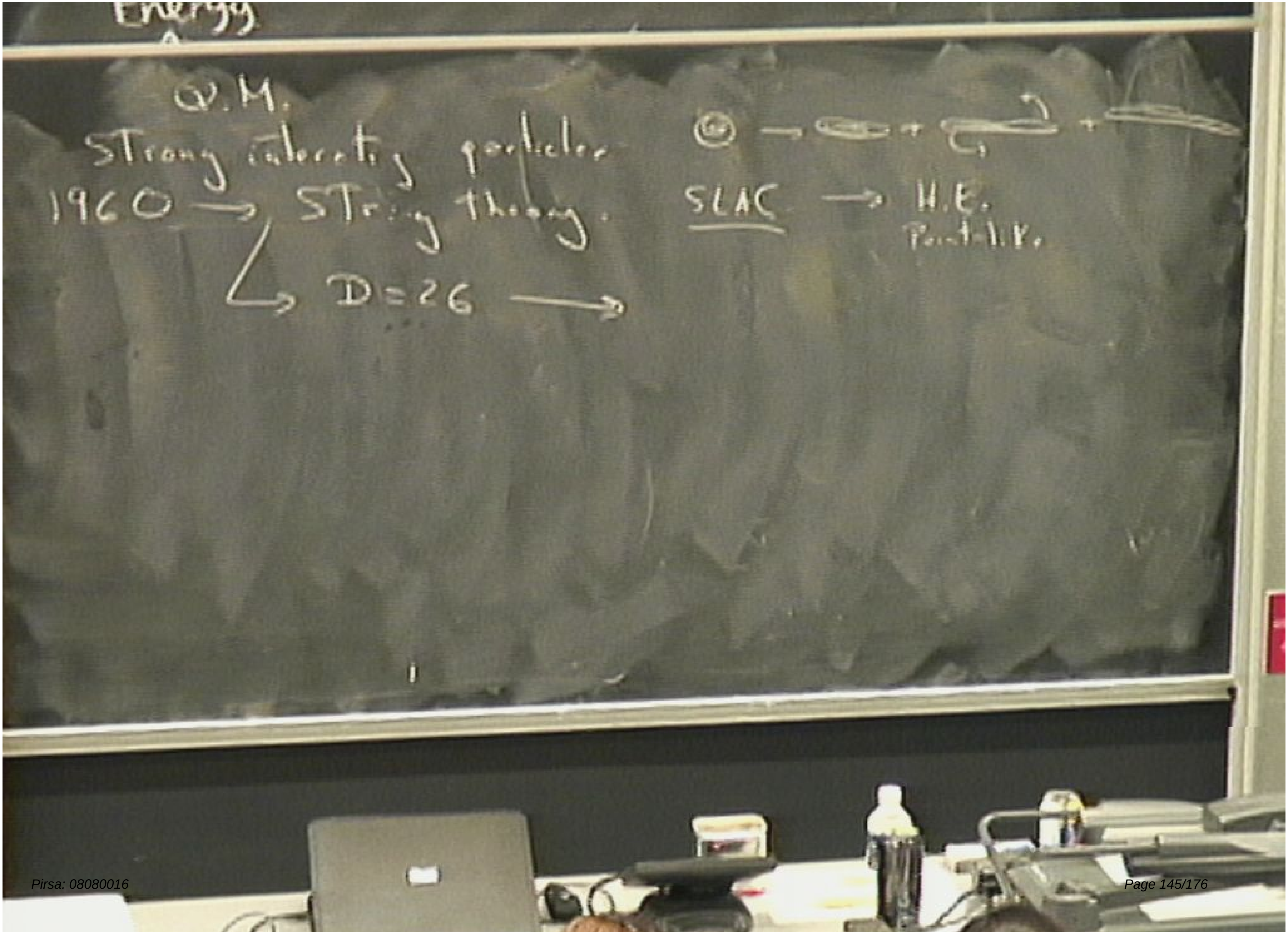
1960 $\longrightarrow$ String theory.

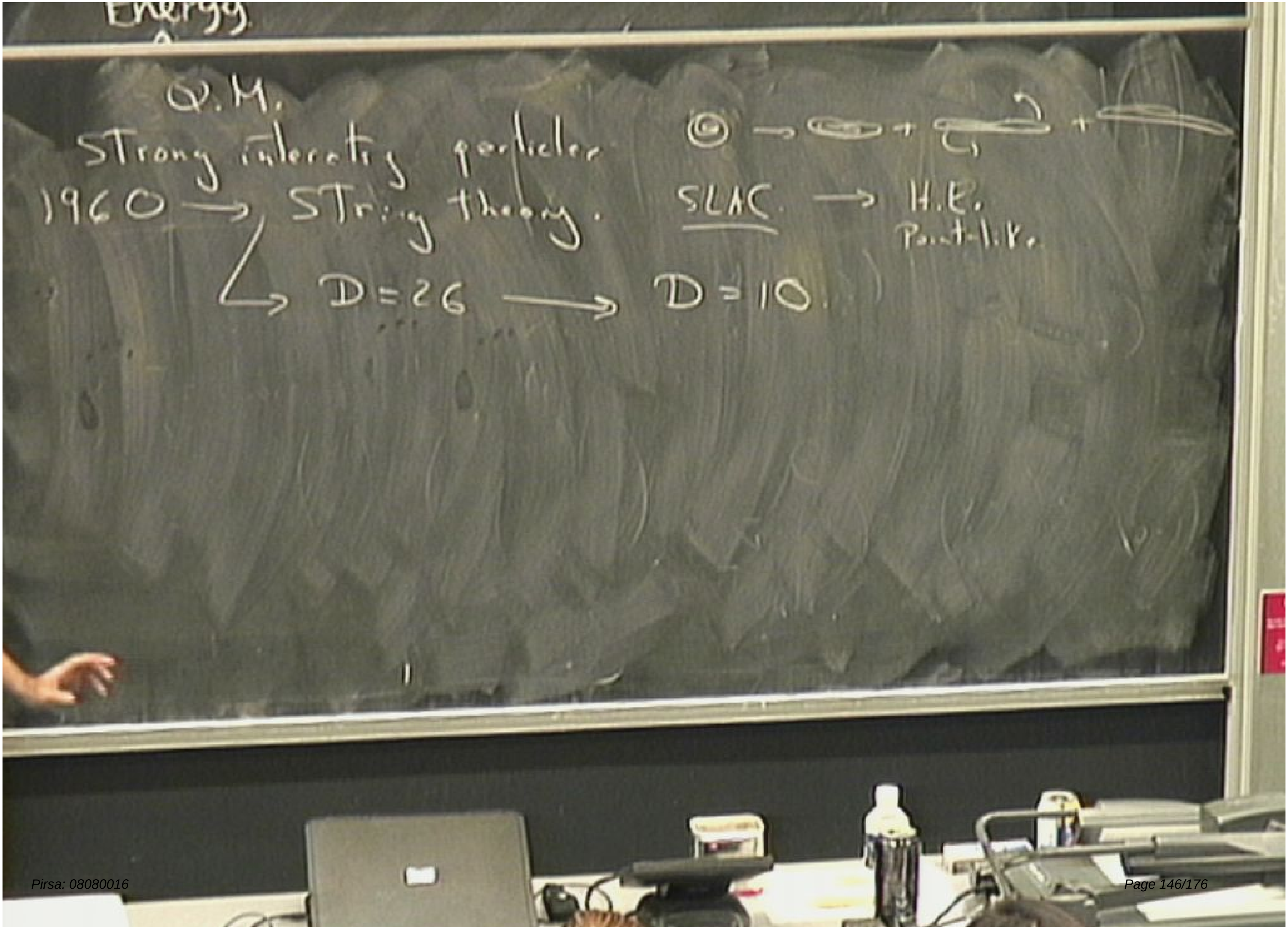
SLAC

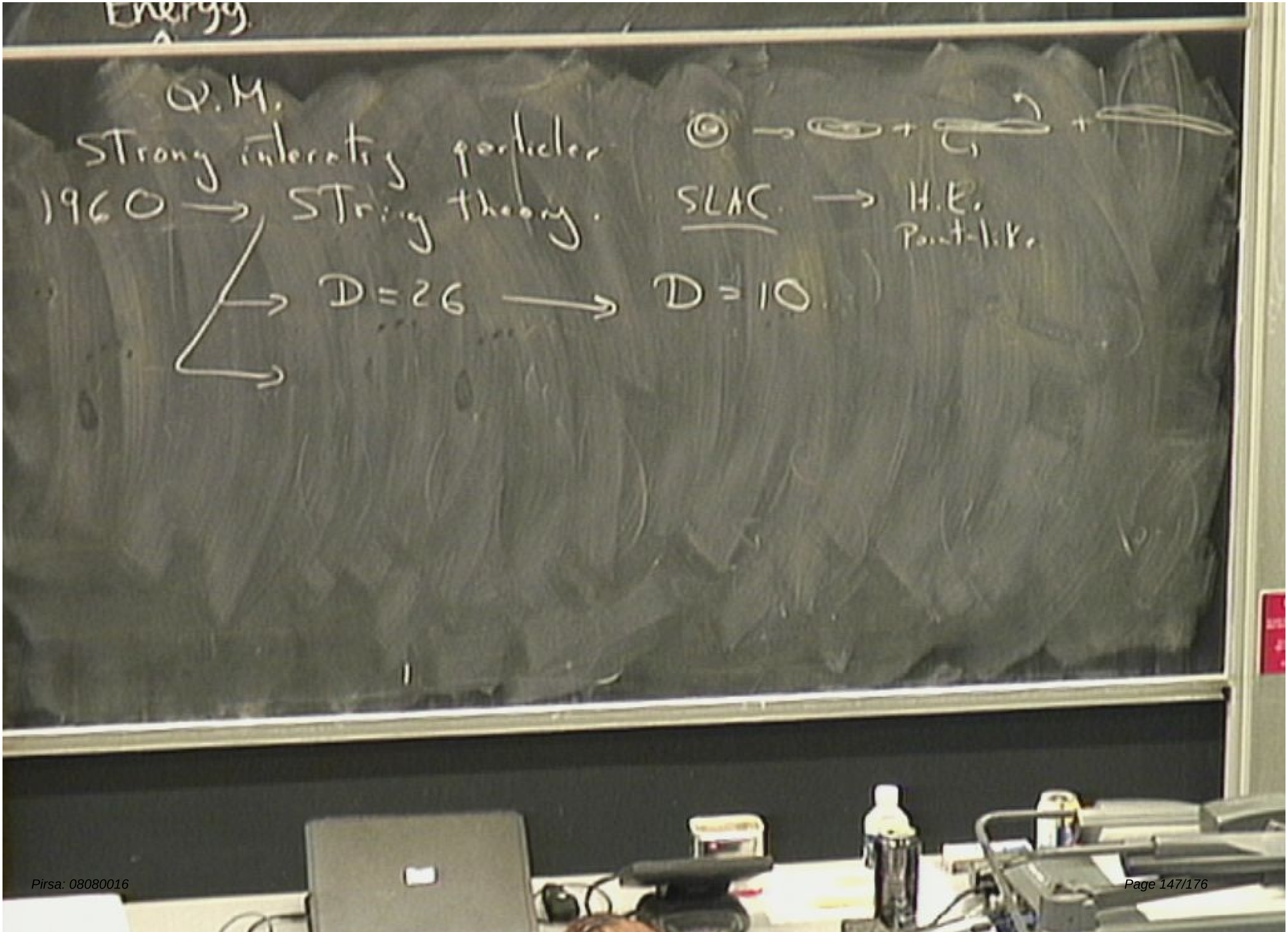

Energy
Q.M.
Strong interacting particles
1960 → String theory.
SLAC

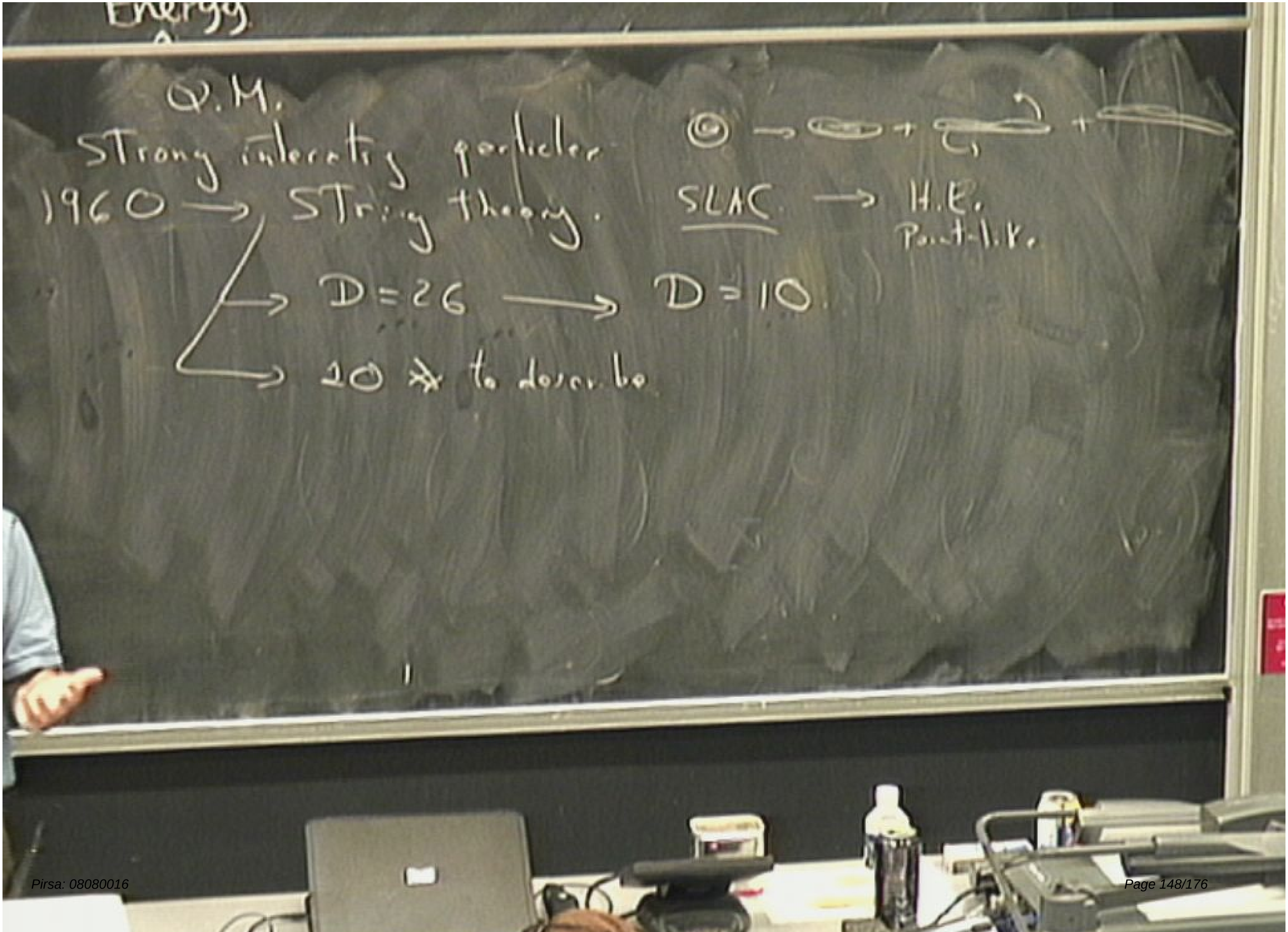

Energy
Q.M.
Strong interacting particles
1960 → String theory.
SLAC → H.E.
Point-like

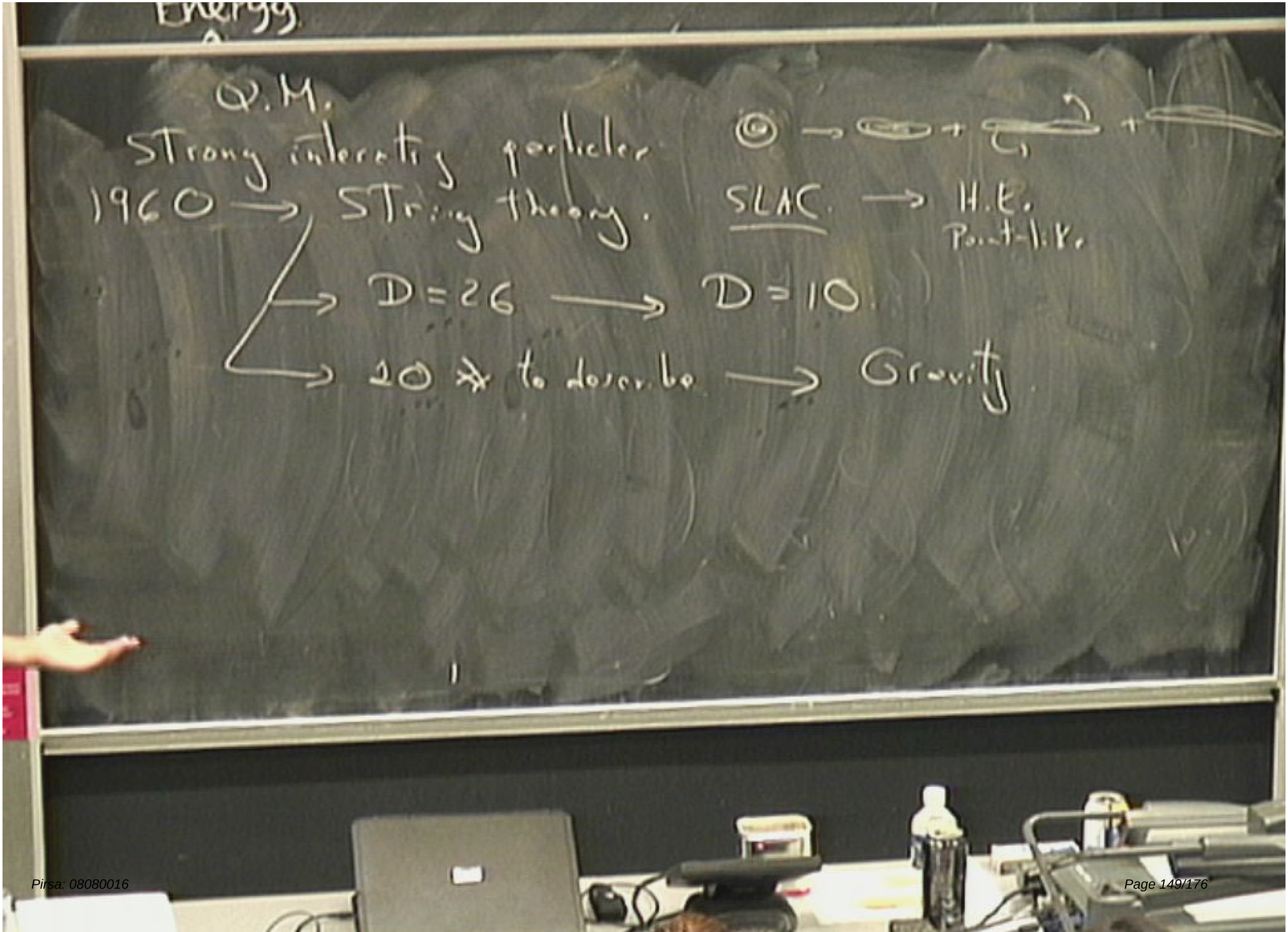

Energy
Q.M.
Strong interacting particles
1960 → String theory.
D=26
SLAC → H.E.
Point-like


Energy
Q.M.
Strong interacting particles
1960 → String theory.
D=26 →
SLAC → H.E.
Point-like


Energy
Q.M.
Strong interacting particles
1960 → String theory.
D=26 → D=10.
SLAC. → H.E.
Point-like.


Energy
Q.M.
Strong interacting particles
1960 → String theory.
SLAC → H.E.
Point-like
D=26 → D=10


Energy
Q.M.
Strong interacting particles
1960 → String theory.
SLAC → H.E.
Point-like
D=26 → D=10
10 to describe


Energy
Q.M.
Strong interacting particles
1960 → String theory.
SLAC → H.E.
Point-like
→ D=26 → D=10
→ 10 to describe → Gravity

Newton (1643) ~ Move in a straight line in S-T

SO(3,1)

Riemann (1854) Curved geometry.

1D → 1 *

2D → 3 * Maybe Forces are.

3D → 6 * just curvature of S-T

4D → 10 * metric

5D → 15 *

Einstein

↳ Gravity.

Kaluza → 1919 Electromag.

Klein.

4×4

5×5

Q.M.

Strong interacting particles

1960 → String theory. SLAC → H.E. Point-like

→ D = 26 → D = 10

→ 10 ... to describe → Gravity.

5-theories consistent.

IIA IIB I

Heterotic SO(32) Heterotic $E_8 \times E_8$

Q.M.

Strong interacting particles

1960 → String theory. SLAC → H.E. Point-like

→ D = 26 → D = 10

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Heterotic SO(32) Heterotic $E_8 \times E_8$

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Strong interacting particles

1960 $\longrightarrow$ String theory.

SLAC $\longrightarrow$ H.E.
Point-like

$\longrightarrow$ $D = 26$ $\longrightarrow$ $D = 10$

$\longrightarrow$ 10 to describe $\longrightarrow$ Gravity

5-theories consistent.

IIA

IIB

1995

I

Heterotic SO(32)

Heterotic $E_8 \times E_8$.

Q.M.

Strong interacting particles

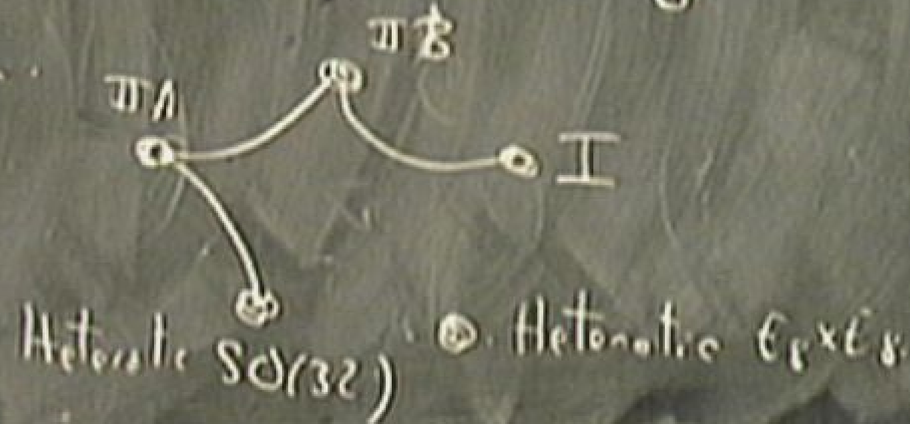
1960 $\longrightarrow$ String theory. SLAC $\longrightarrow$ H.E. Point-like

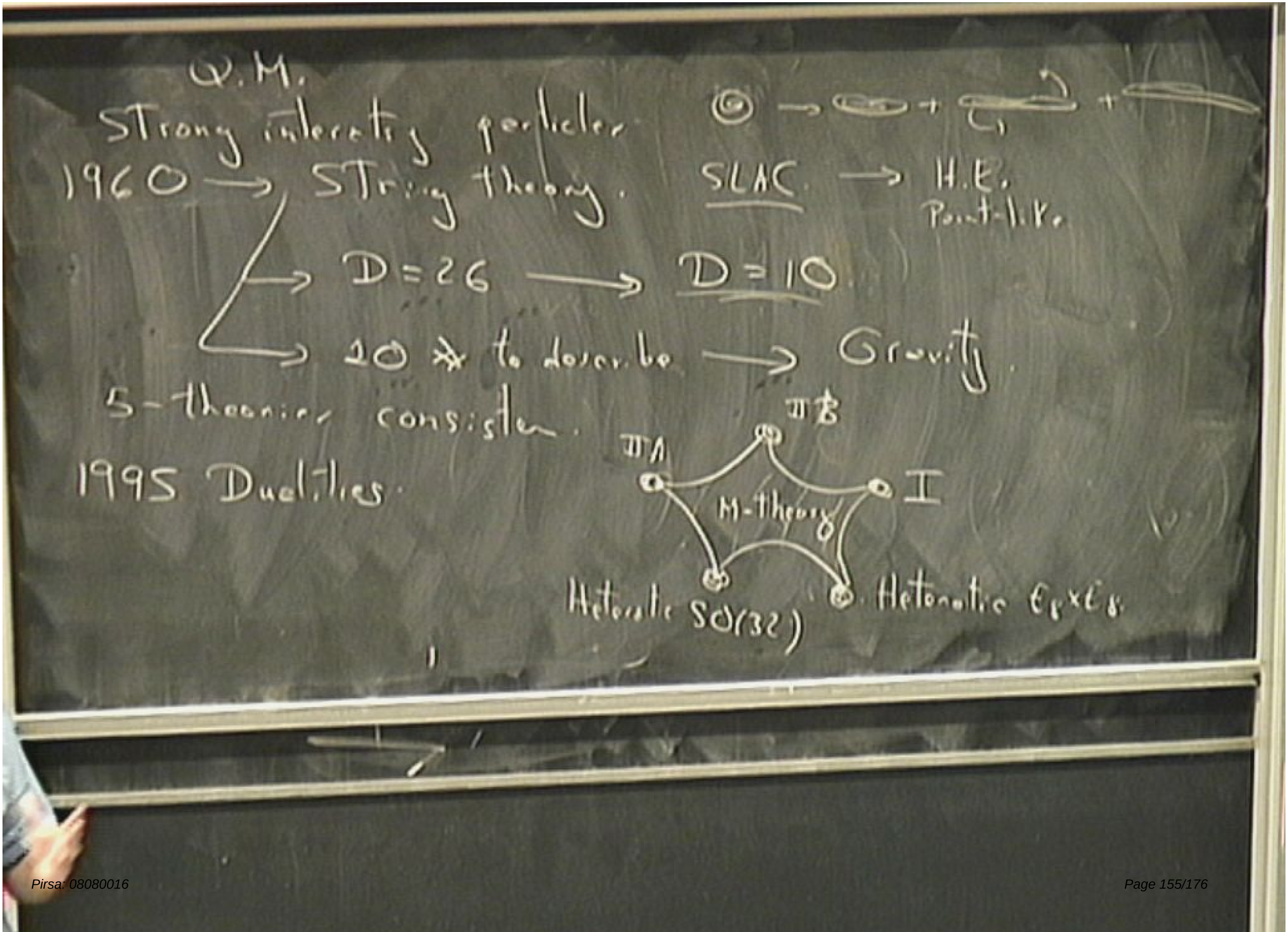
$\longrightarrow$ $D = 26$ $\longrightarrow$ $D = 10$

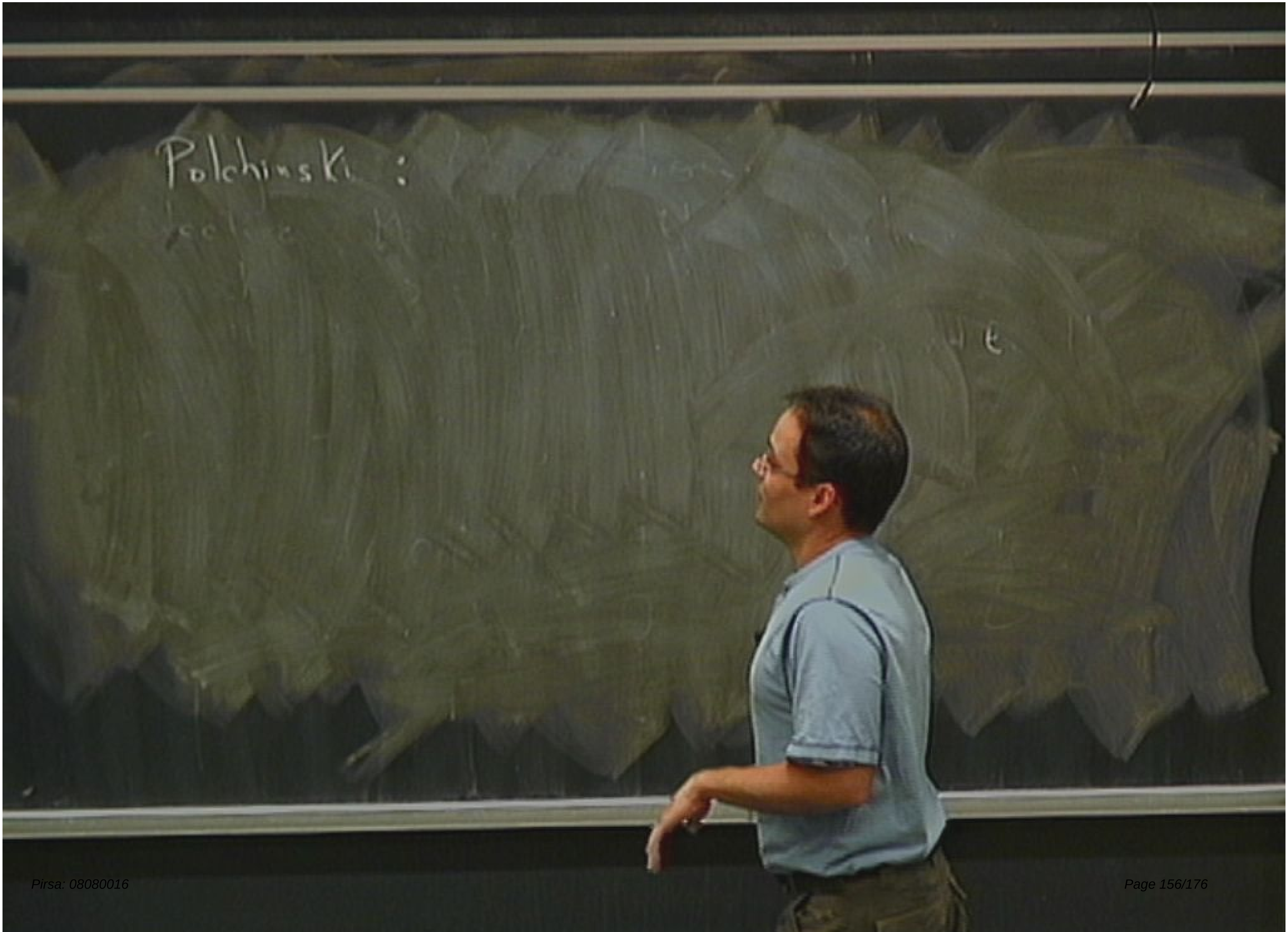
$\longrightarrow$ 10 ✱ to describe $\longrightarrow$ Gravity

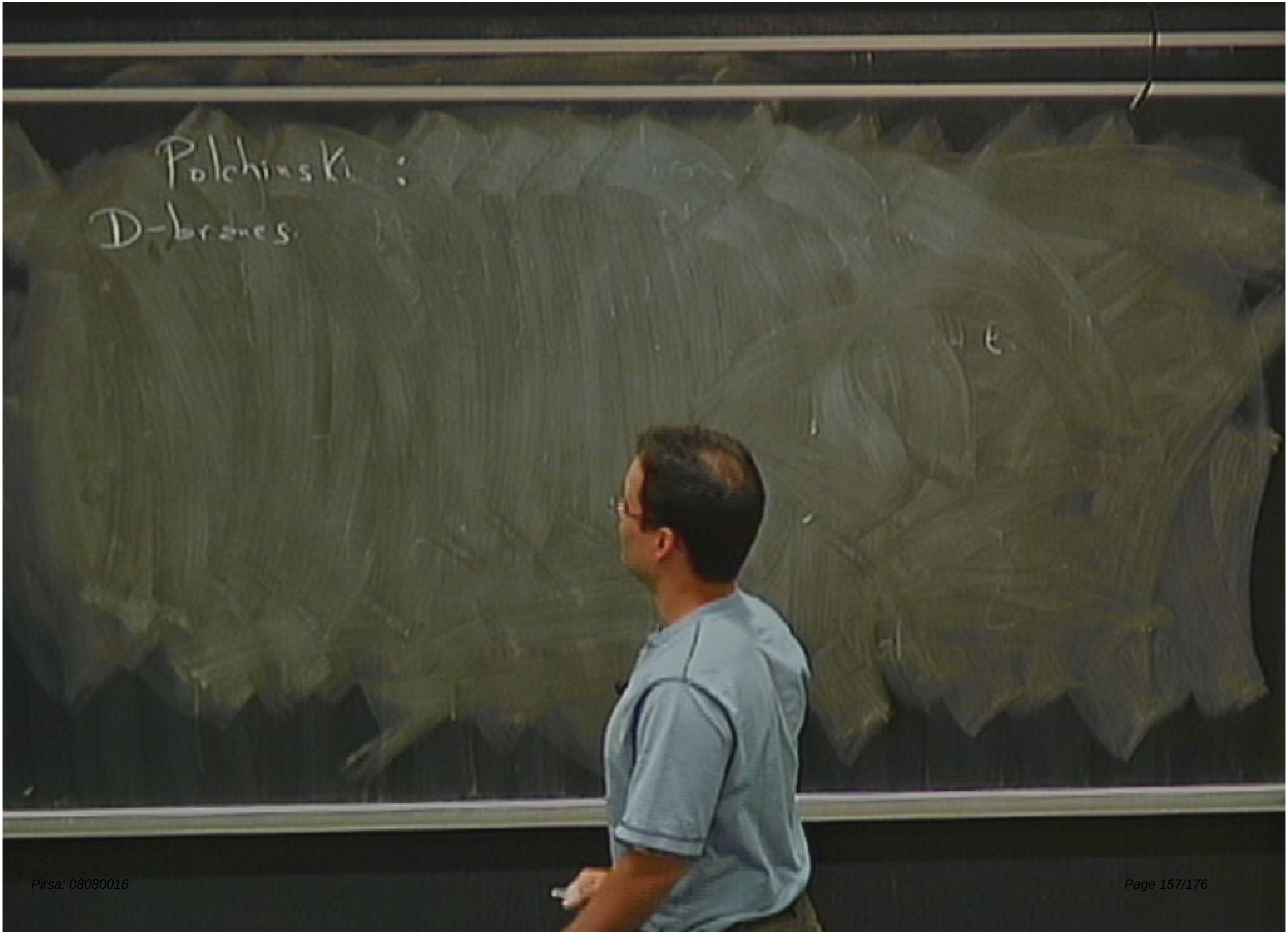
5-theories consistent.

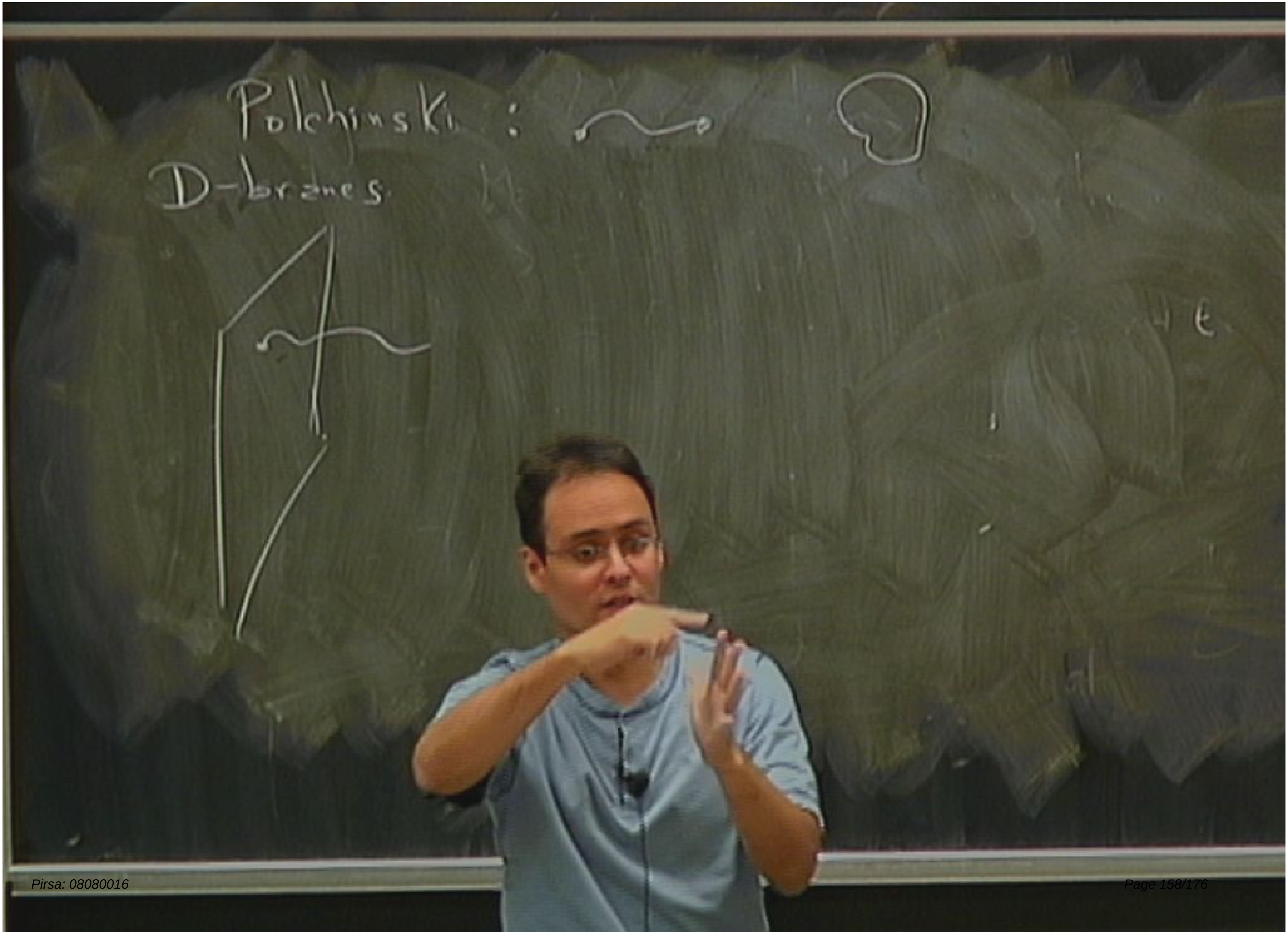
1995 Dualities.




Q.M.
Strong interacting particles
1960 → String theory.
SLAC → H.E.
Point-like
D=26 → D=10
10 to describe → Gravity
5-theories consistent.
1995 Dualities
IIB
IIA
I
M-theory
Heterotic SO(32)
Heterotic E8×E8


Polchinski :


Polchinski :
D-branes.


Polchinski :
D-branes

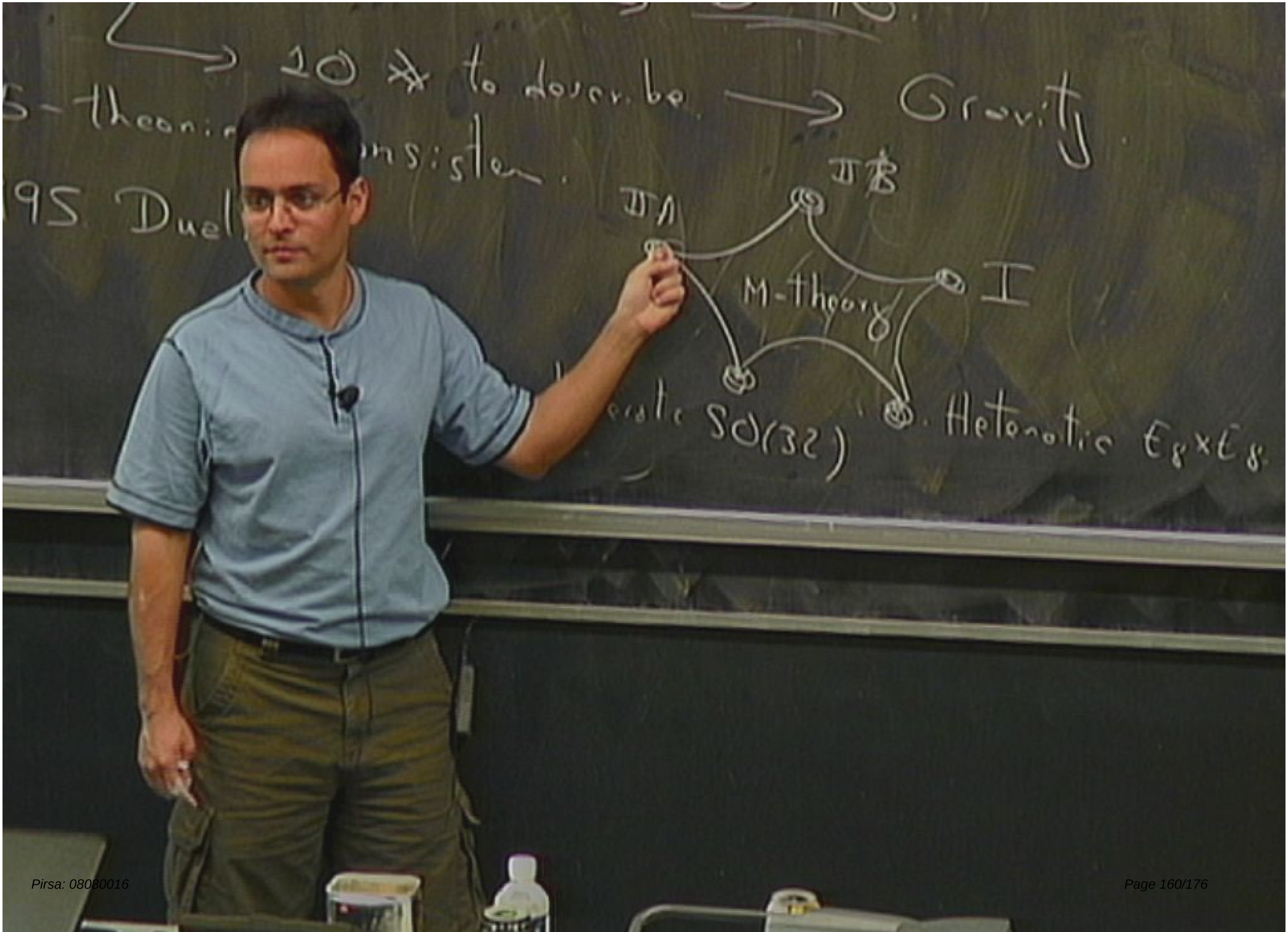
Polchinski : 

D-branes.

p - space dim'd

1 - 1mm

P-branes

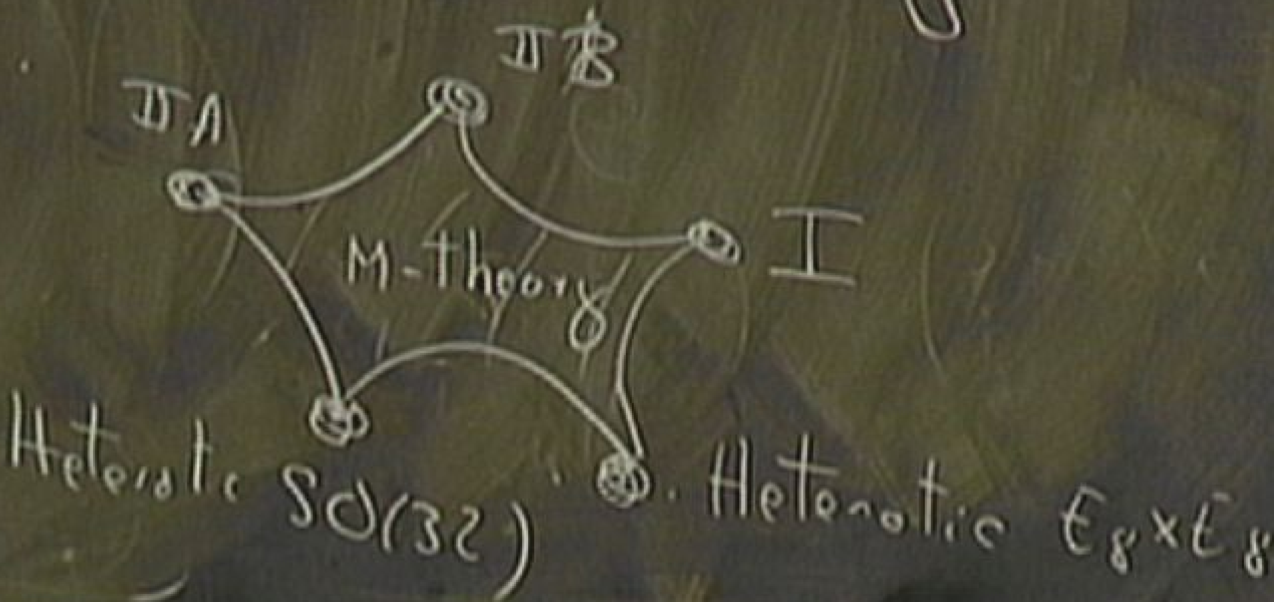

10 # to describe
Gravity
IIA
IIB
I
M-theory
SO(32)
Heterotic $E_8 \times E_8$

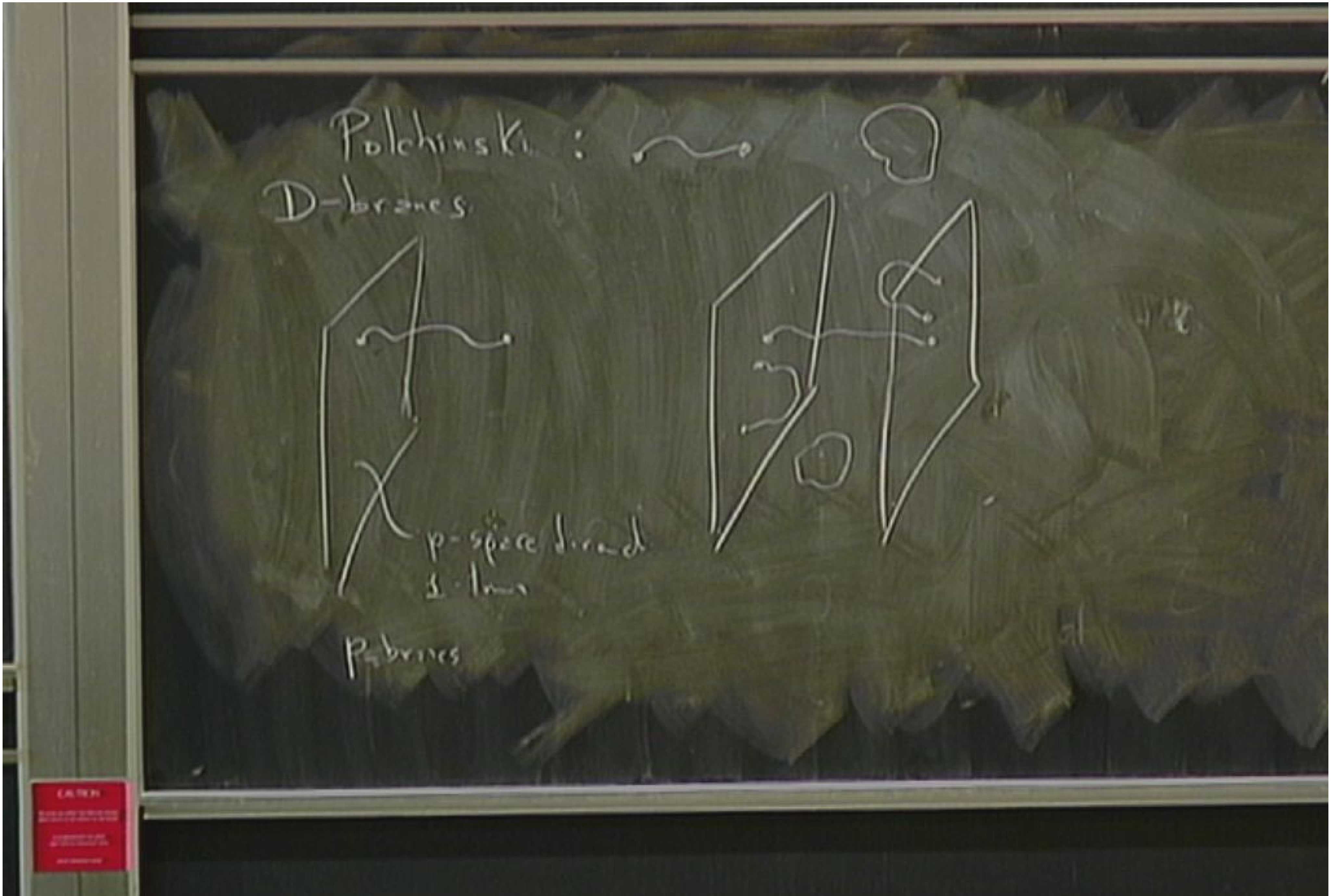
D=10.

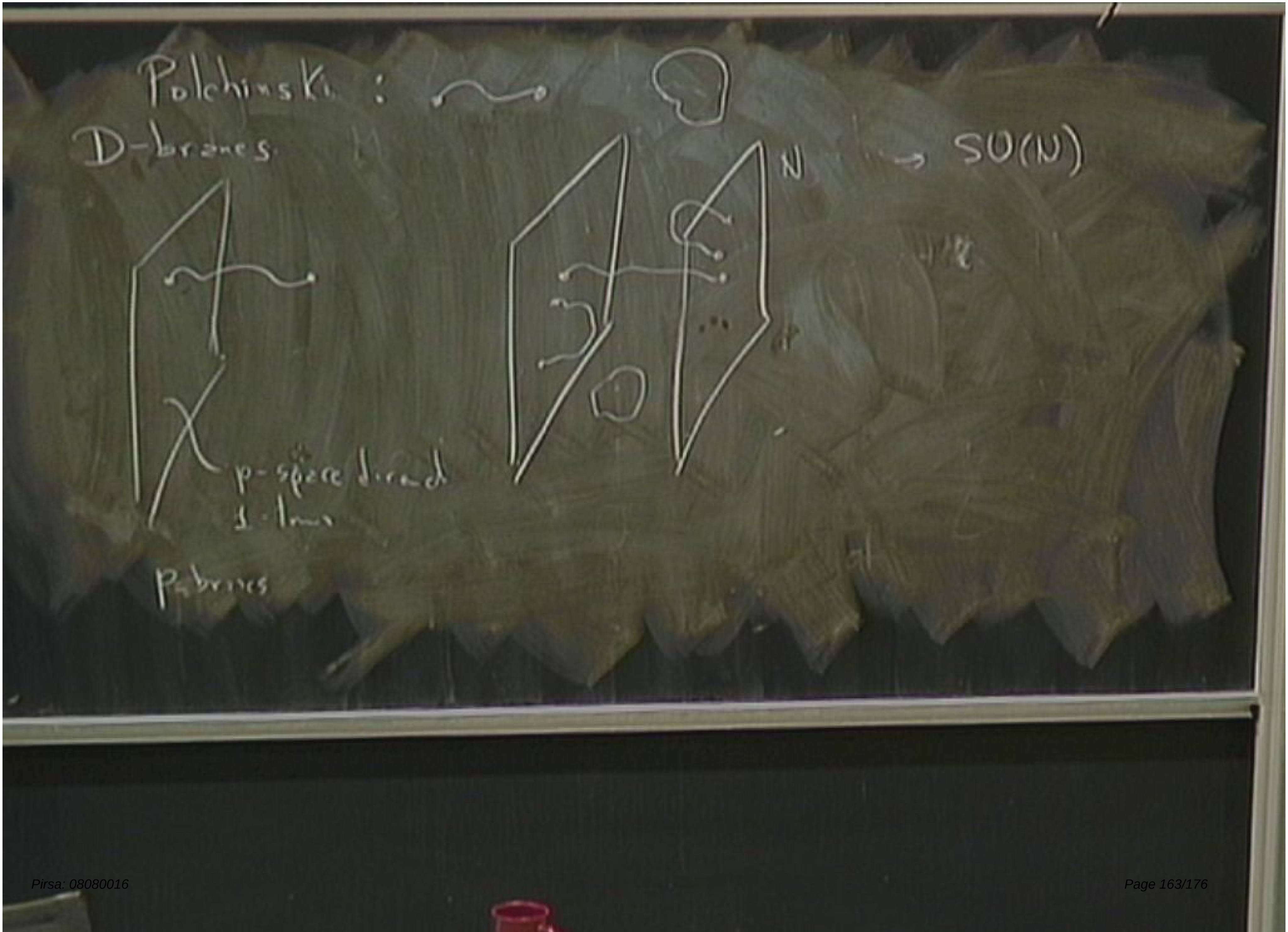
...0 # to describe → Gravity

consisten.

...es.

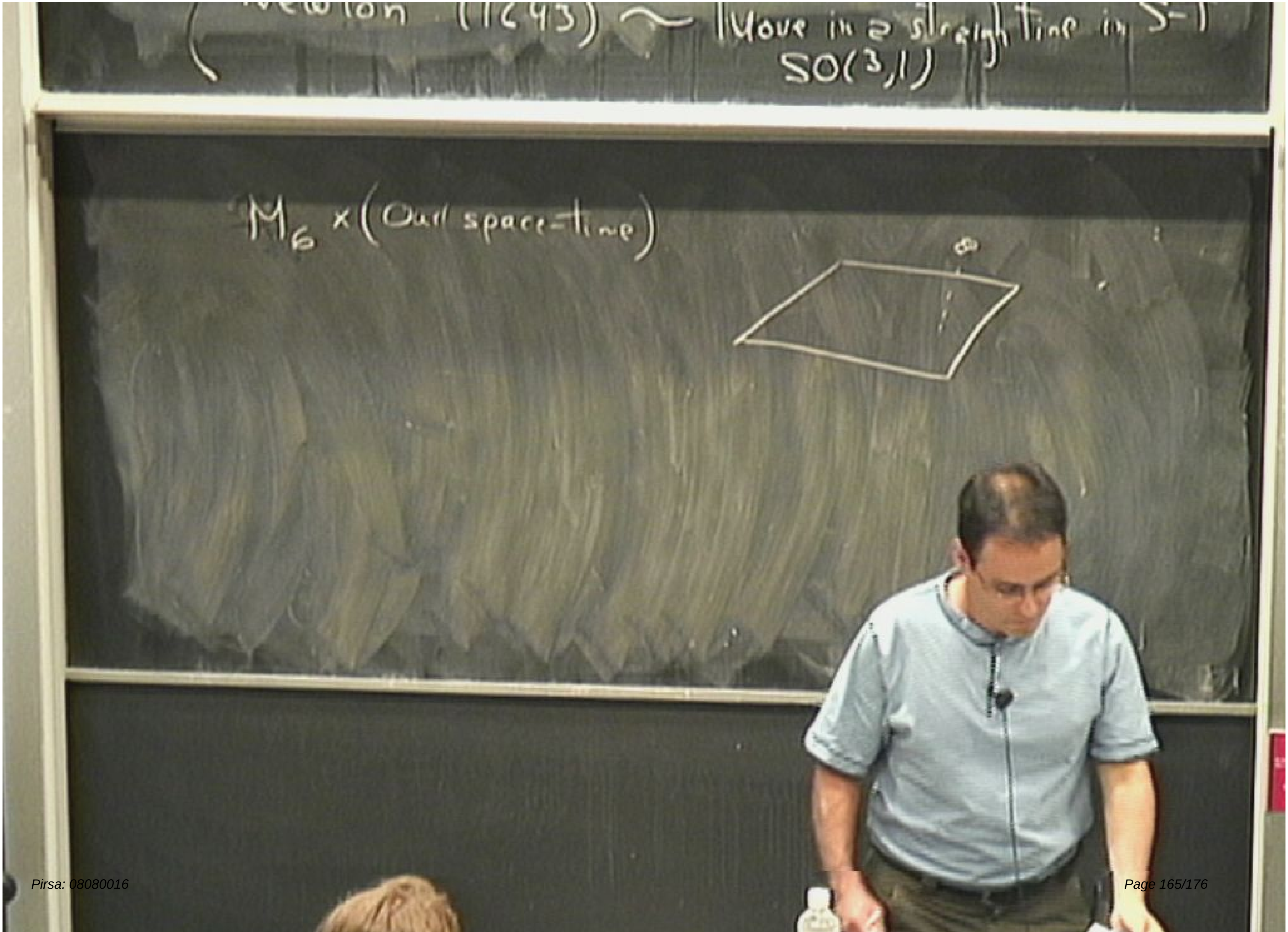


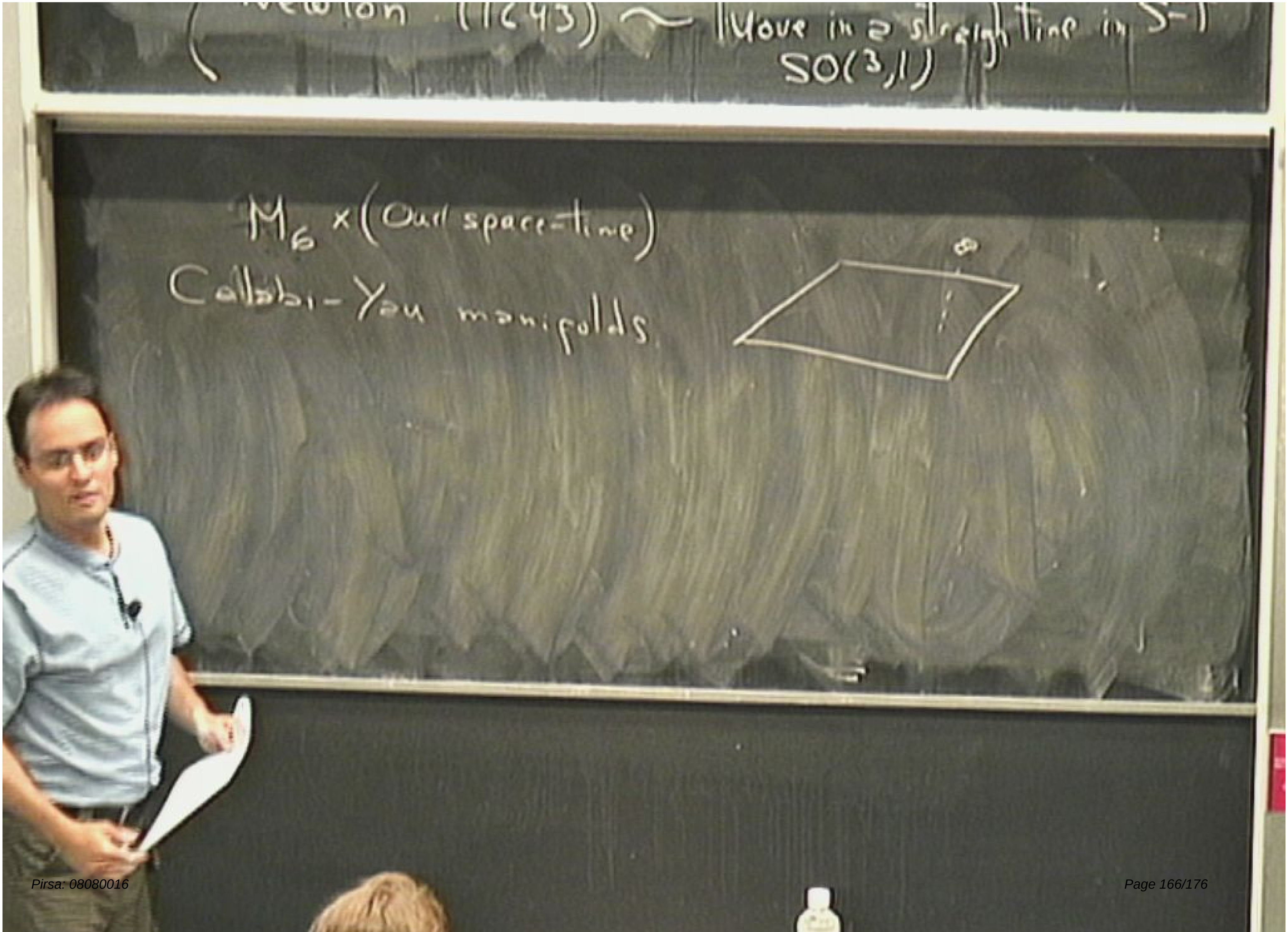

Polchinski :
D-branes

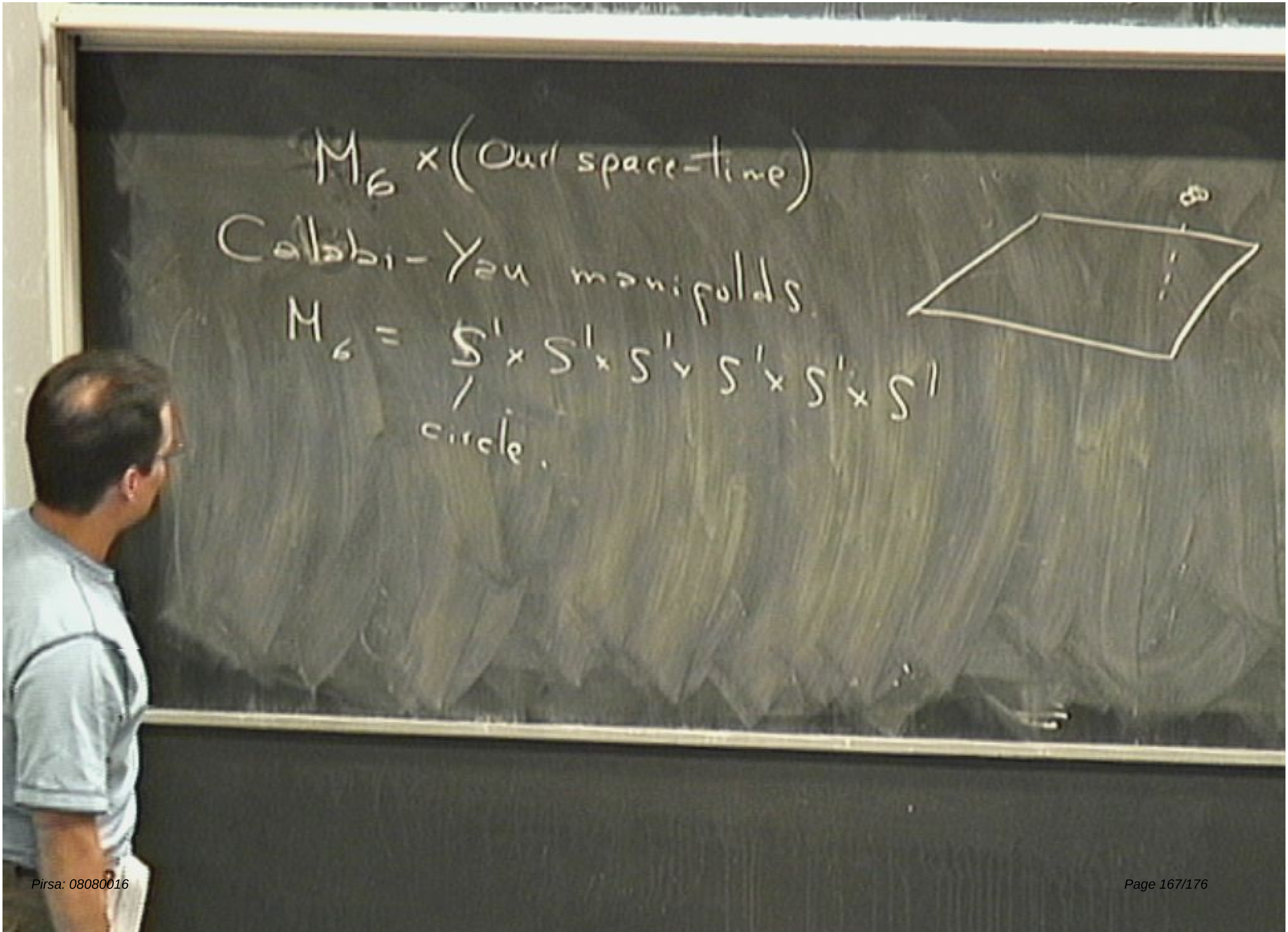

Polchinski :
D-branes
N
SU(N)
p-branes

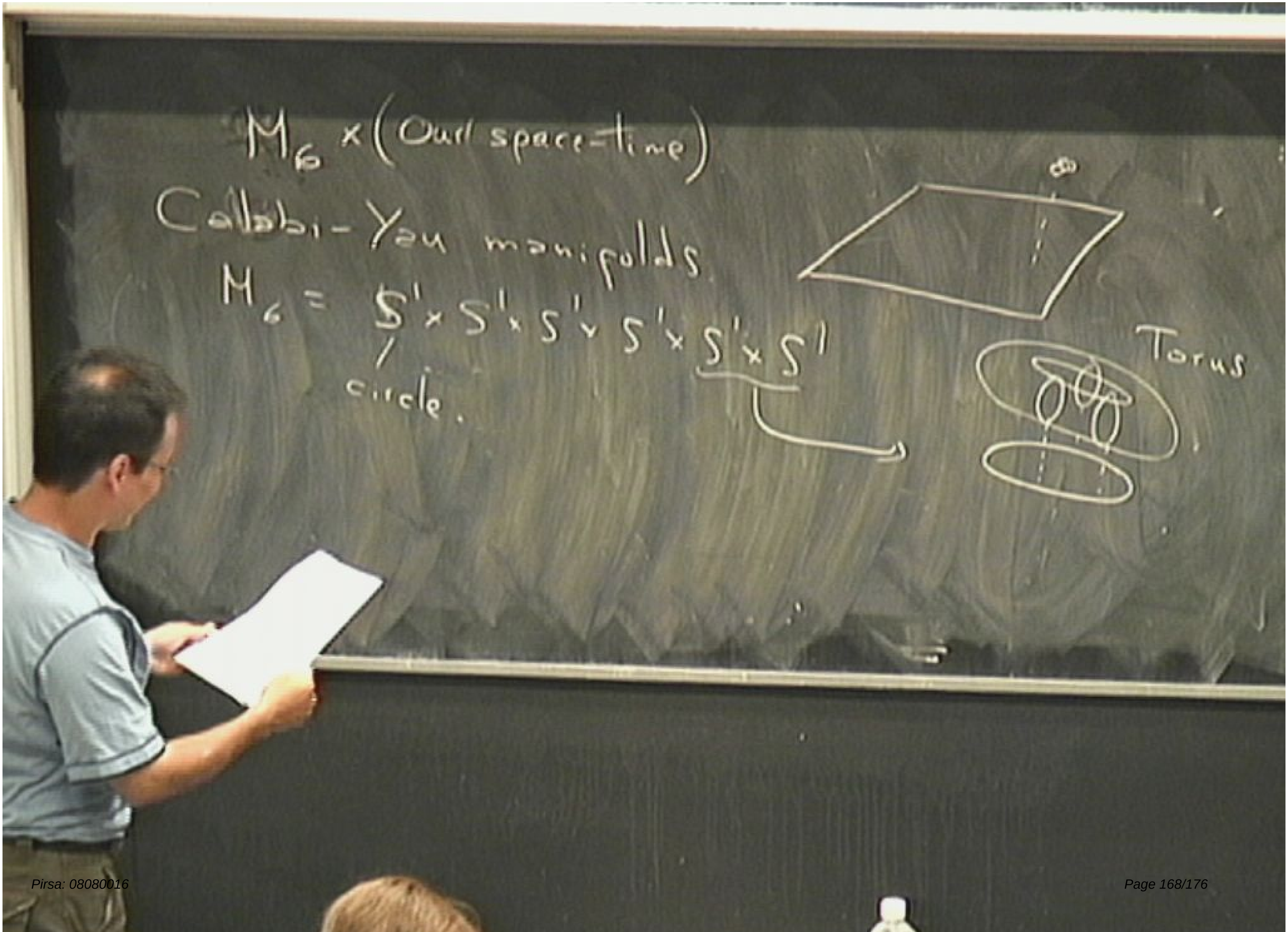
(Newton (1643) ~ (Move in a straight line in S-)
SO(3,1)

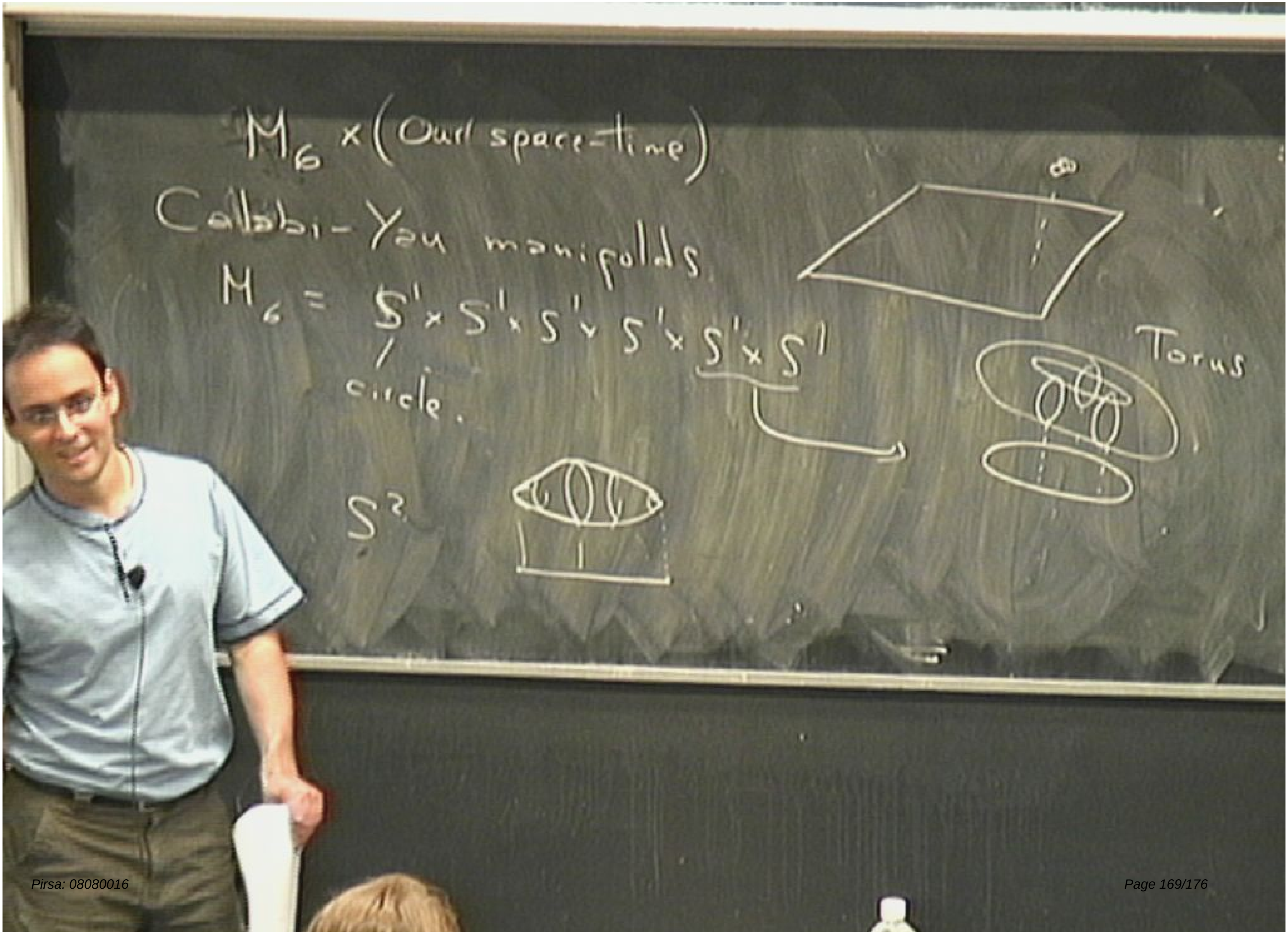
$M_6 \times$ (Our space-time)

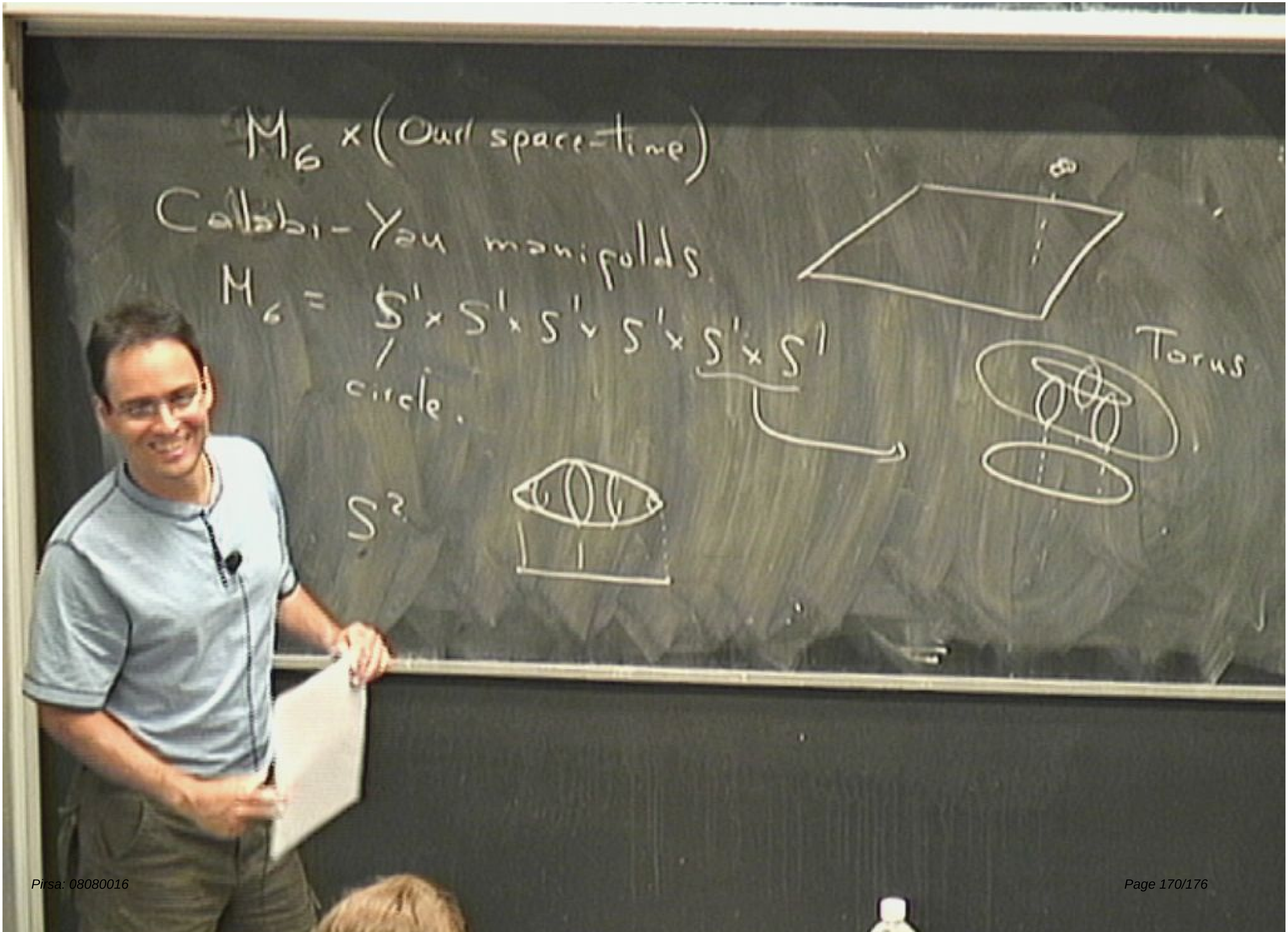

Move in a straight line in
SO(3,1)
M6 × (Our space-time)

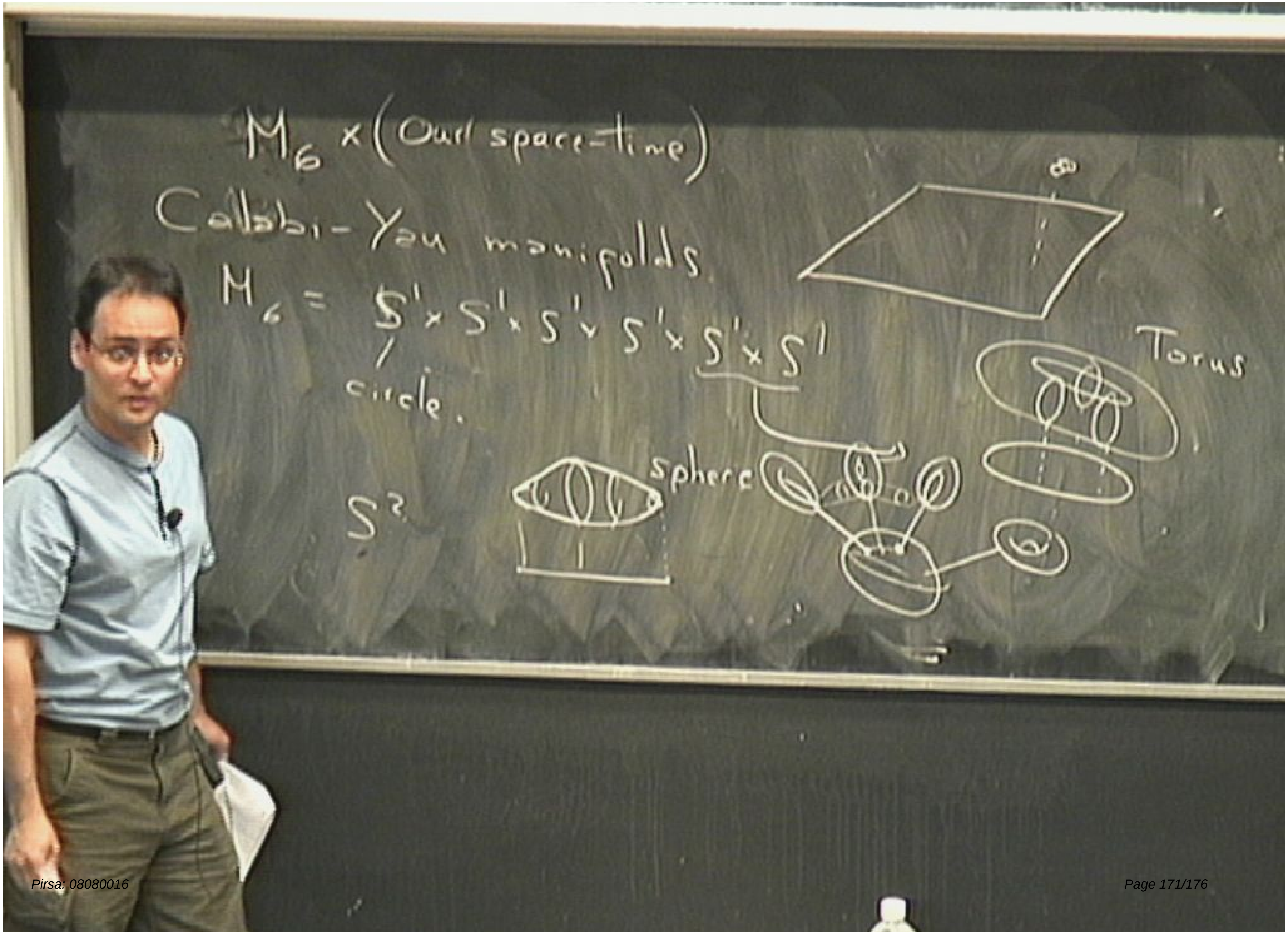

Move in a straight line in
SO(3,1)
M6 × (Our space-time)
Calabi-Yau manifolds


M6 x (Our space-time)
Calabi-Yau manifolds
M6 = S1 x S1 x S1 x S1 x S1 x S1
circle.


M_6 × (Our space-time)
Calabi-Yau manifolds
M_6 = S^1 × S^1 × S^1 × S^1 × S^1 × S^1
circle.
Torus


M6 x (Our space-time)
Calabi-Yau manifolds
M6 = S1 x S1 x S1 x S1 x S1 x S1
circle.
Torus
S2


M6 × (Our space-time)
Calabi-Yau manifolds
M6 = S1 × S1 × S1 × S1 × S1 × S1
circle.
Torus
S2



$M_6 \times$ (Our space-time)
Calabi-Yau manifolds
$M_6 = S^1 \times S^1 \times S^1 \times S^1 \times S^1 \times S^1$
circle.
S^2
sphere
Torus

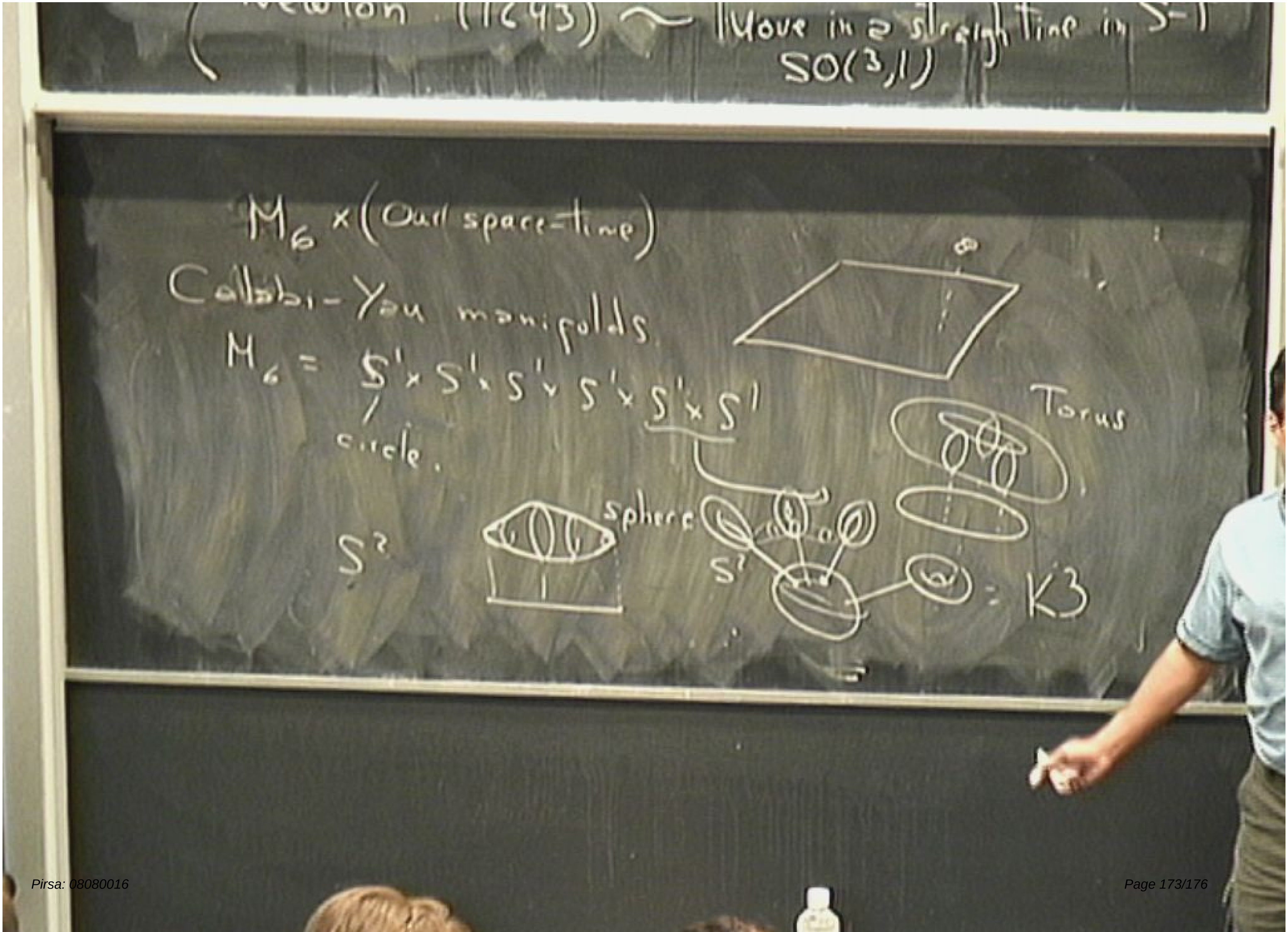
$M_6 \times$ (Our space-time)

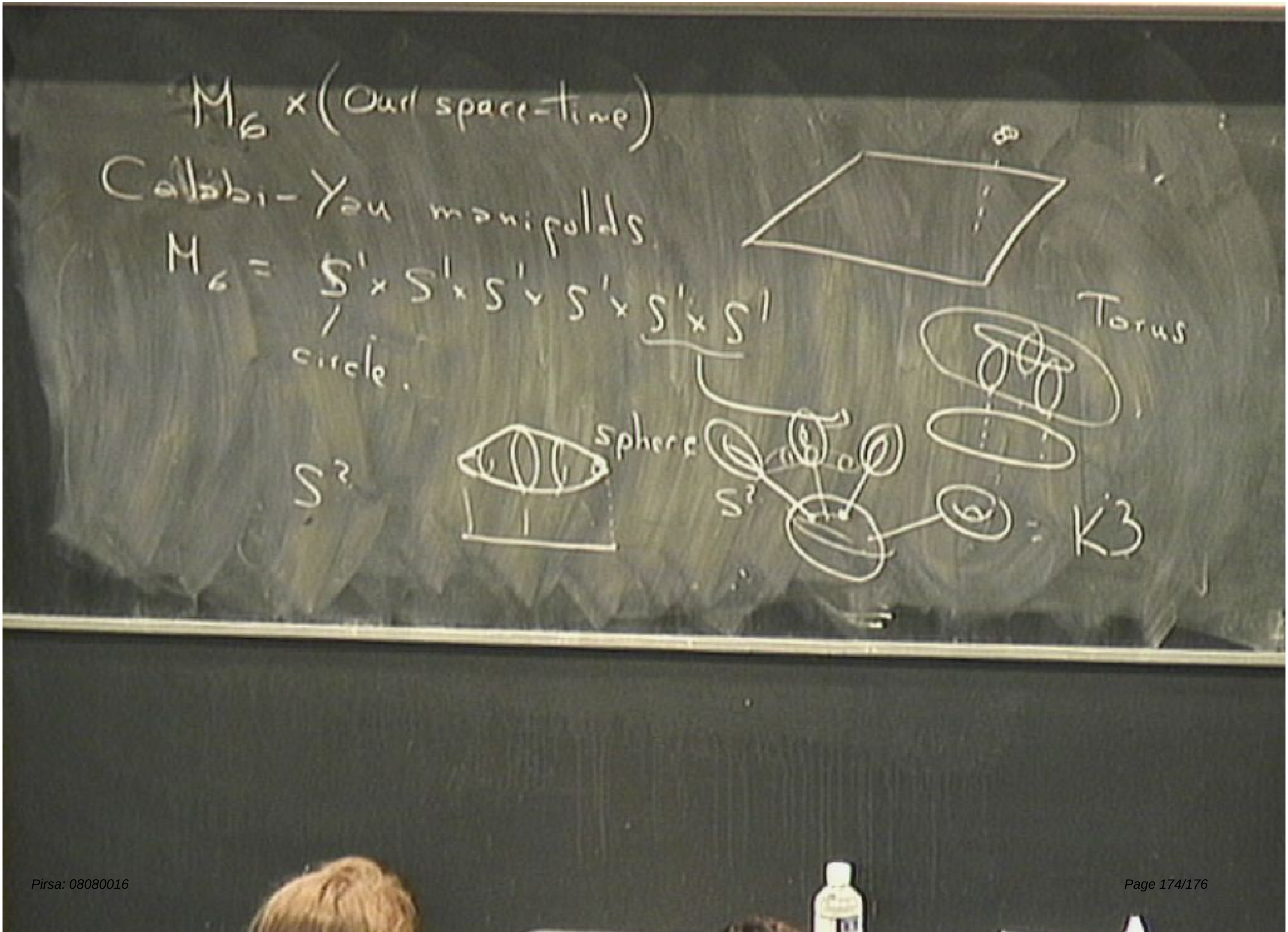
Calabi-Yau manifolds

$M_6 = S^1 \times S^1 \times S^1 \times S^1 \times S^1 \times \underline{S^1 \times S^1}$

circle.

S^2


Move in a straight line in
SO(3,1)
M6 × (Our space-time)
Calabi-Yau manifolds
M6 = S1 × S1 × S1 × S1 × S1 × S1
circle.
Torus
sphere
S2
S1
K3


M6 × (Our space-time)
Calabi-Yau manifolds
M6 = S1 × S1 × S1 × S1 × S1 × S1
circle.
S2
sphere
S2
Torus
K3

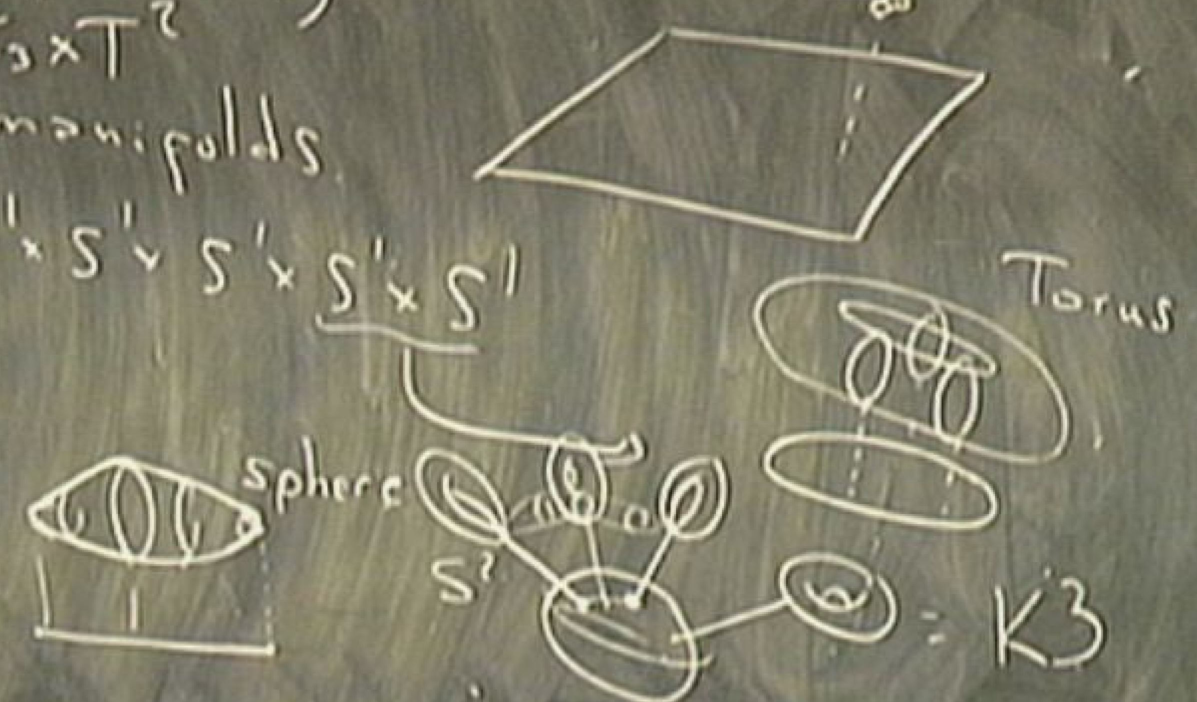
$M_6 \times$ (Our space-time)

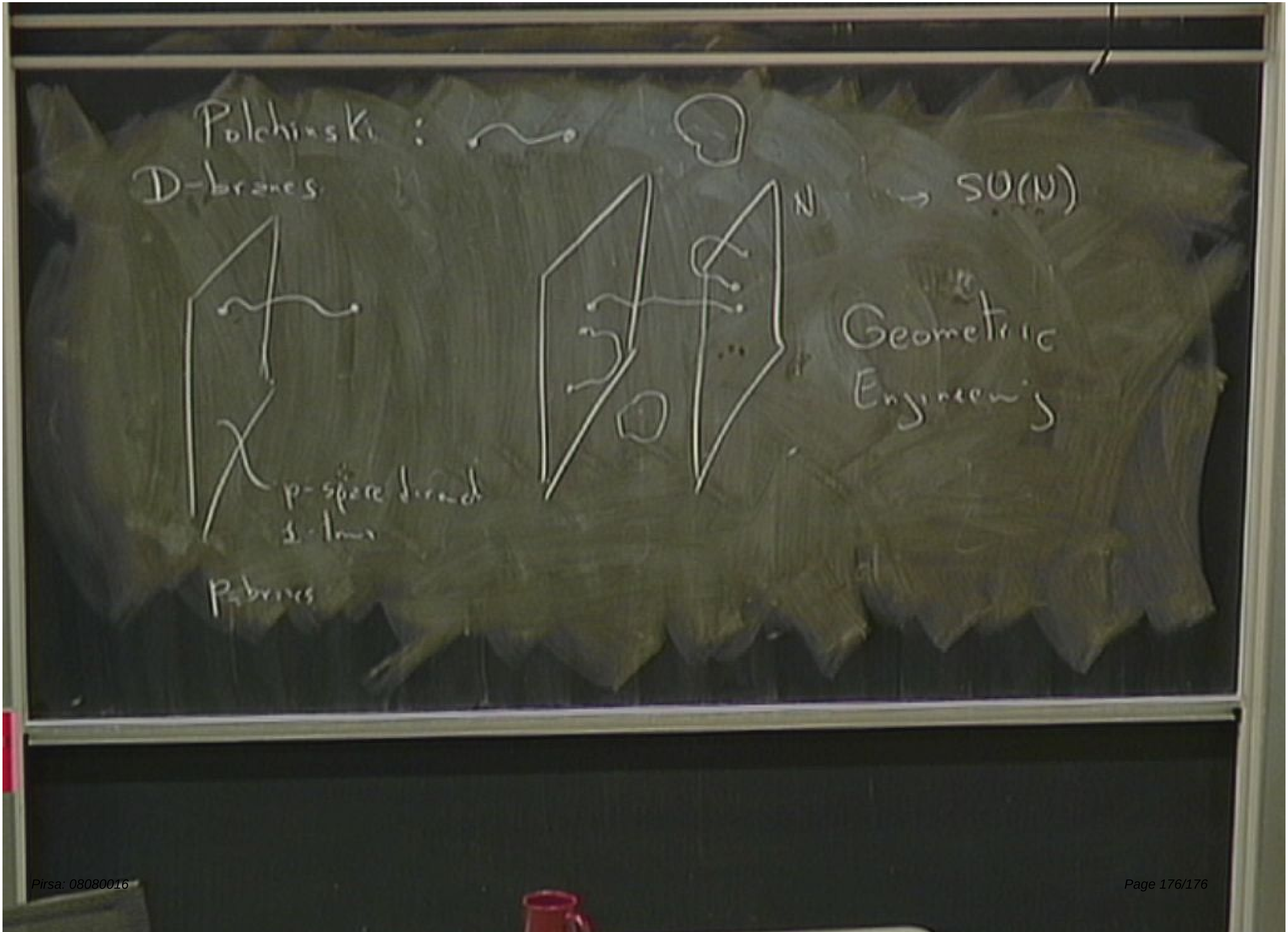
$M_6 \longrightarrow K_3 \times T^2$

Calabi-Yau manifolds

$$M_6 = \underset{\text{circle}}{S^1} \times S^1 \times S^1 \times S^1 \times \underbrace{S^1 \times S^1}$$

S^2




Polchinski :
D-branes
N
SU(N)
Geometric
Engineering
P-branes