

Title: Quantum 3

Date: Aug 12, 2008 10:30 AM

URL: <http://pirsa.org/08080015>

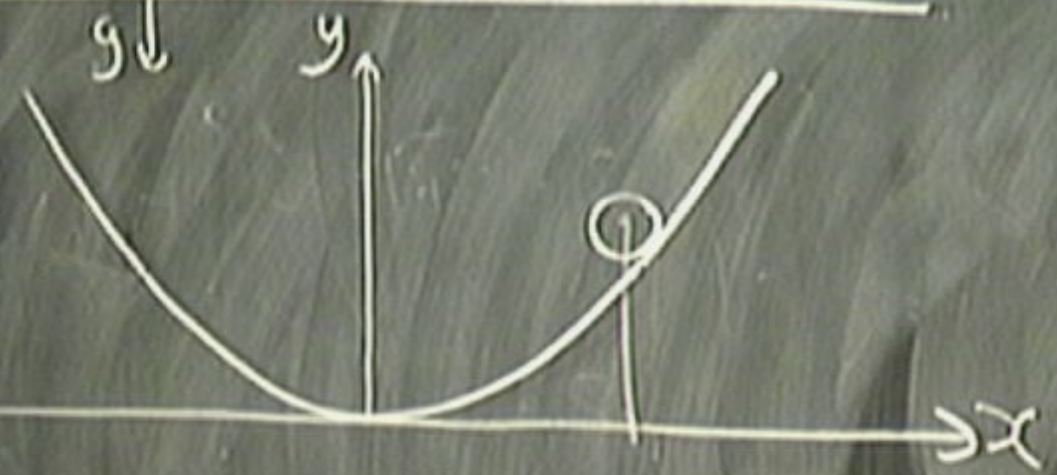
Abstract:

Bound Particles in General

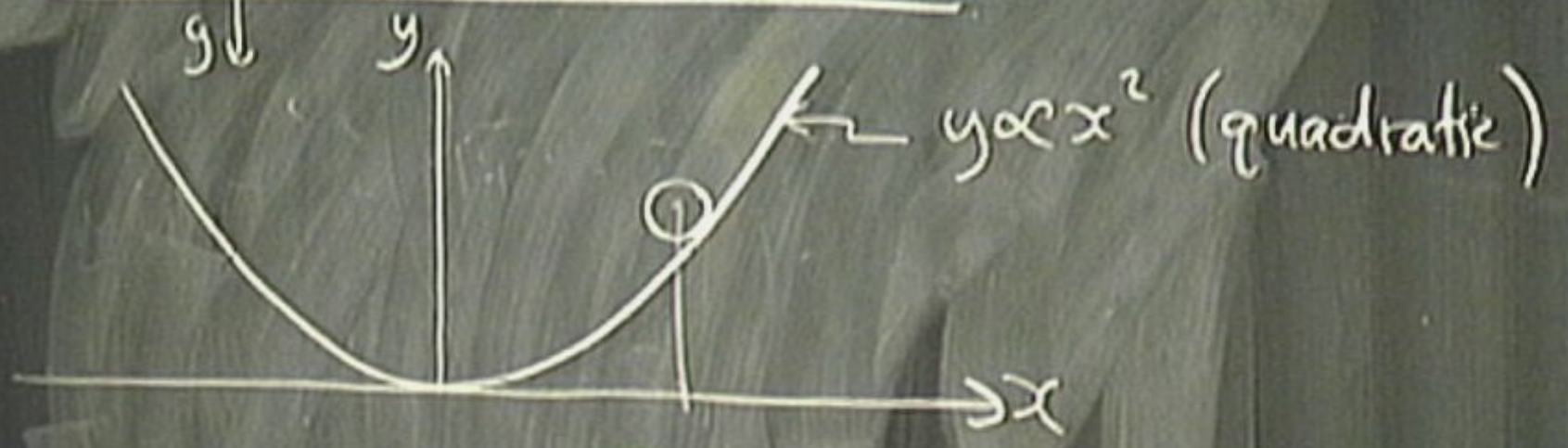
Bound Particles in General



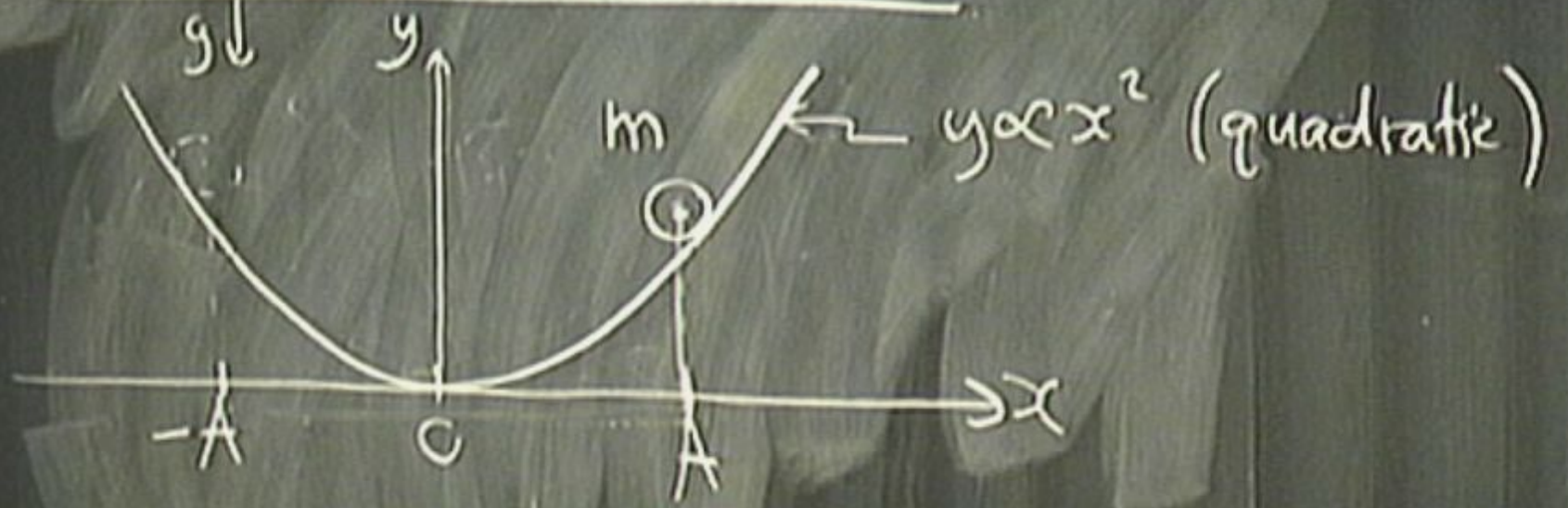
Bound Particles in General



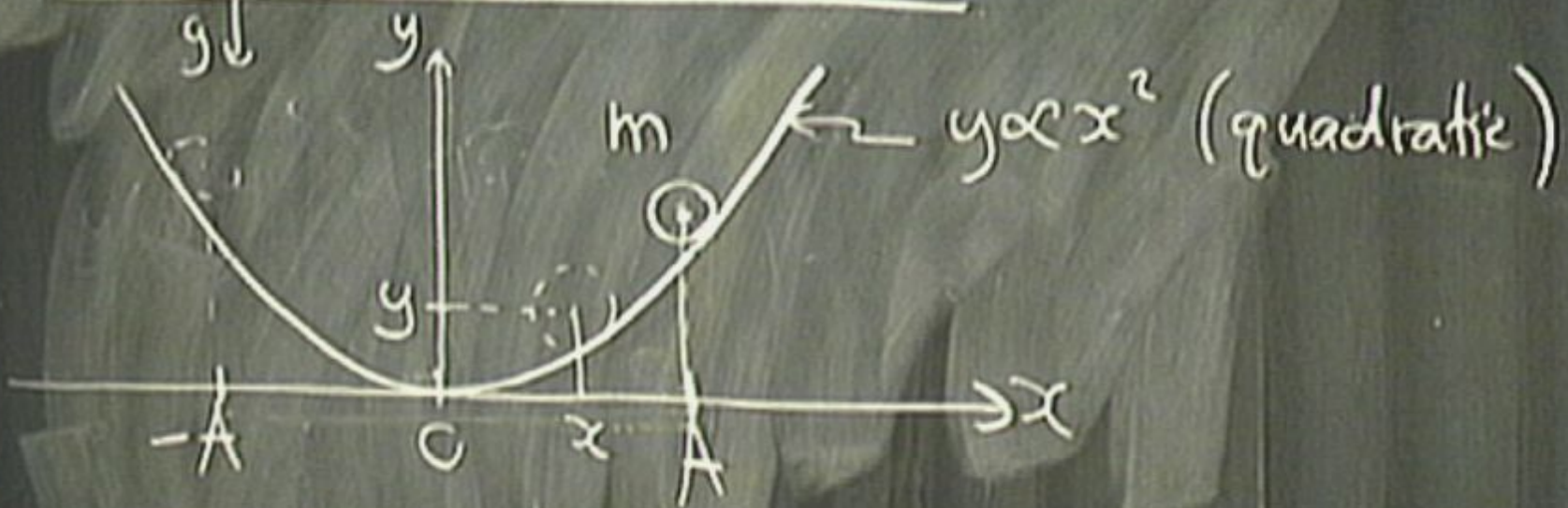
Bound Particles in General



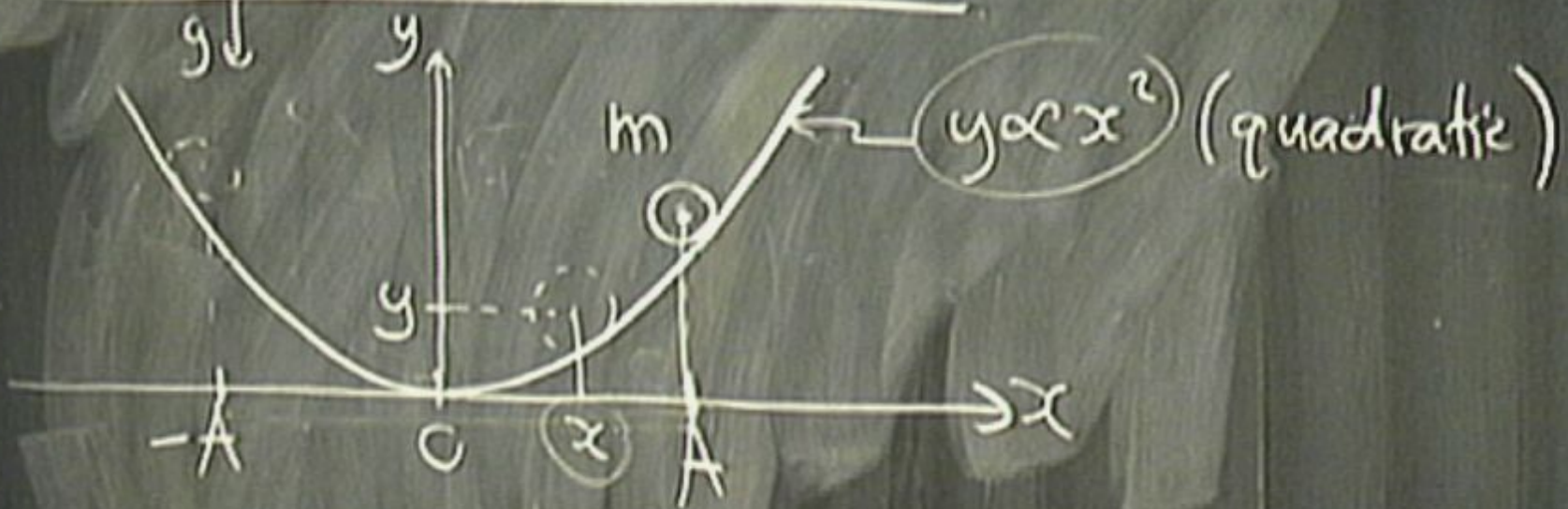
Bound Particles in General



Bound Particles in General

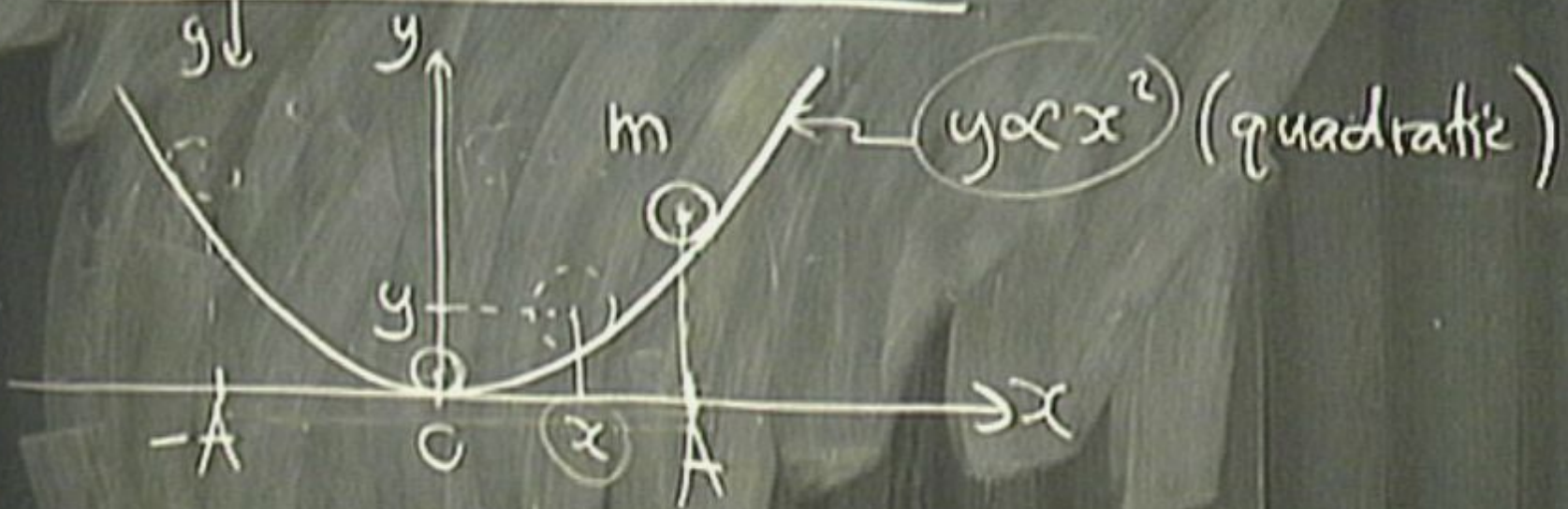


Bound Particles in General



$$PE = mgy = \frac{1}{2}kx^2$$

Bound Particles in General



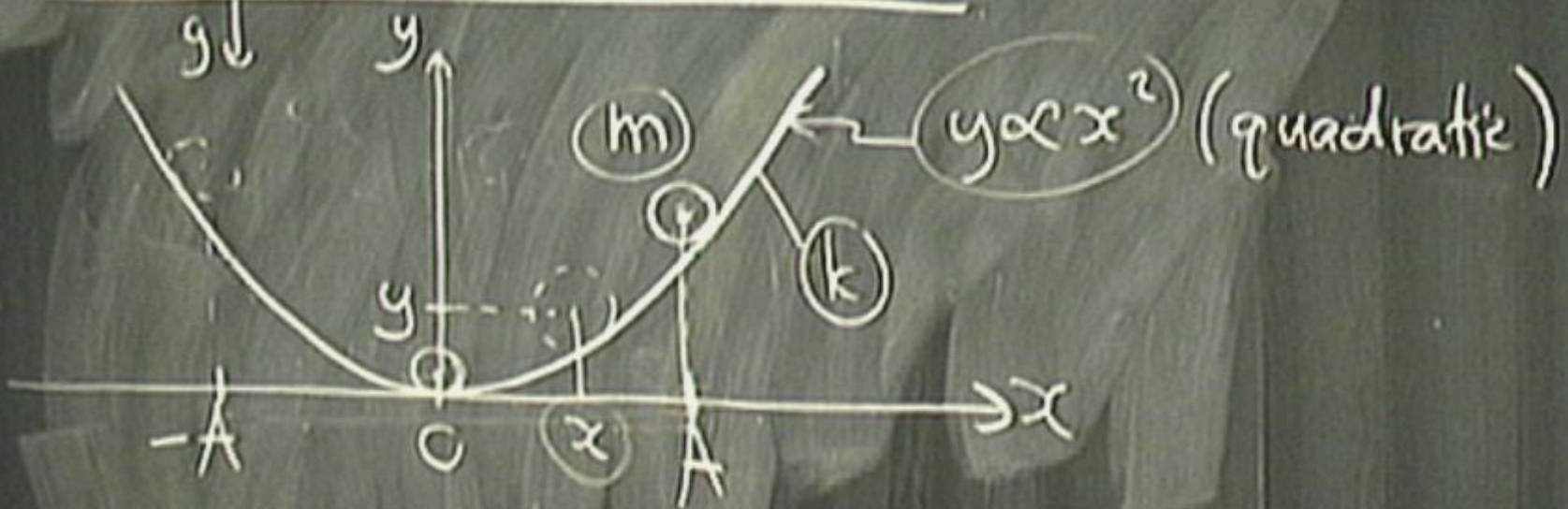
$$PE = mgy = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

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Bound Particles in General

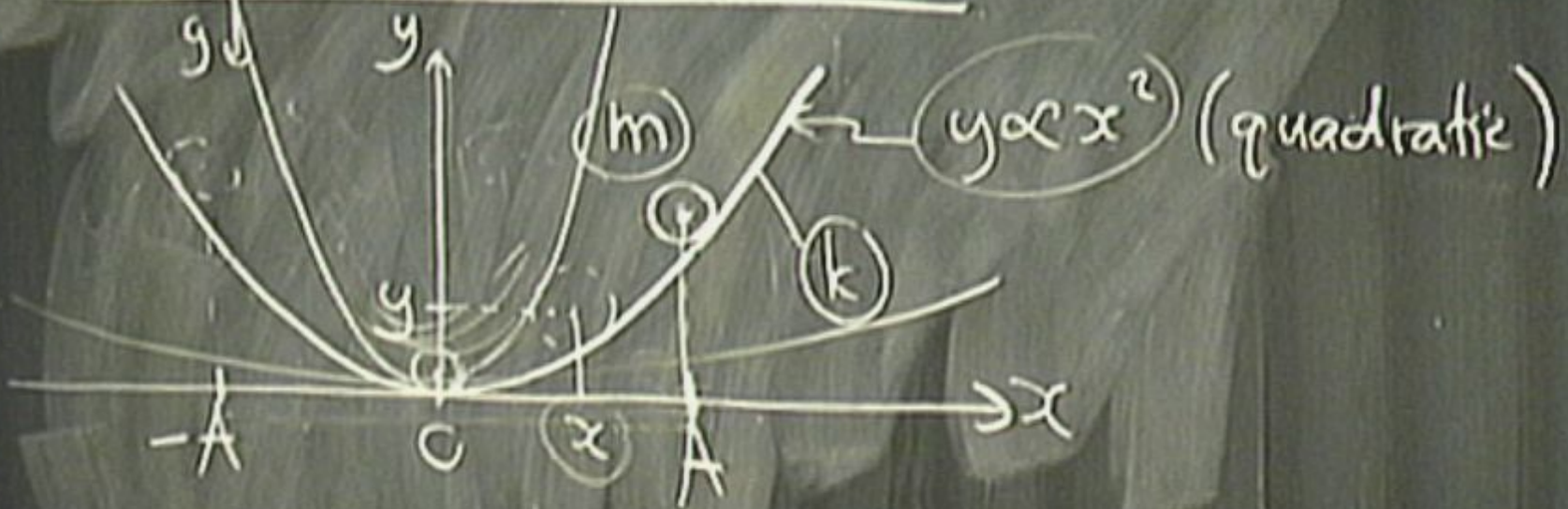


$$PE = mgy = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Bound Particles in General



$$PE = mgy \stackrel{y = \frac{1}{k}x^2}{=} \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

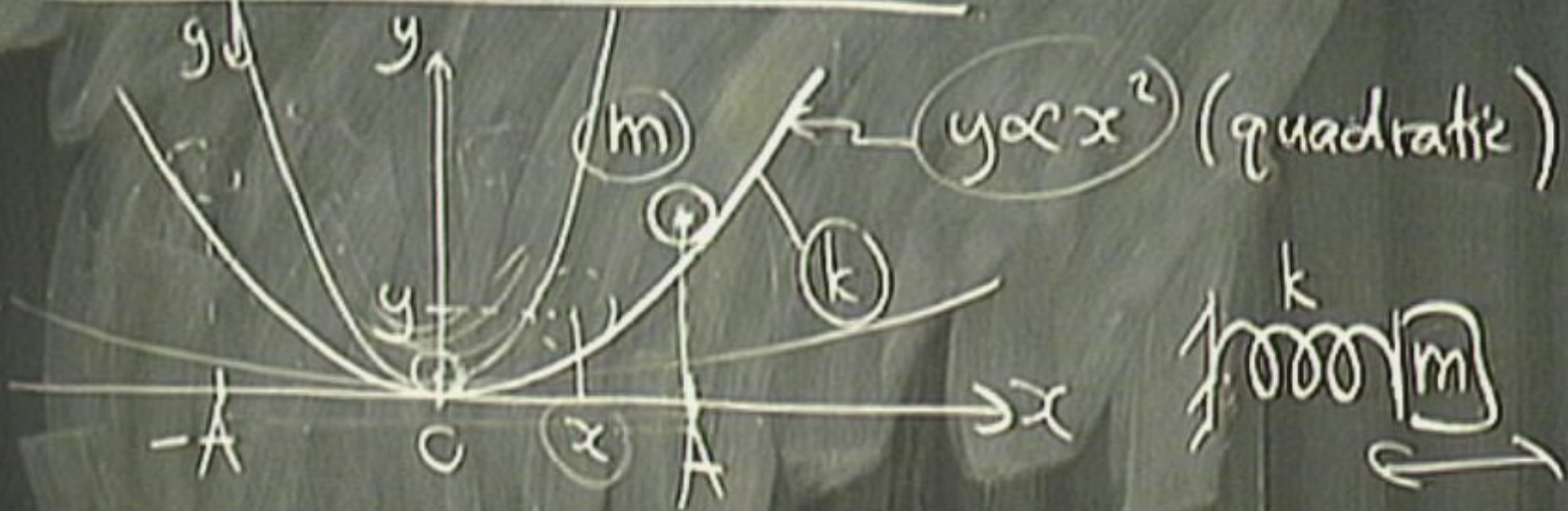
$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$PE = mgy \stackrel{y = \frac{1}{2k}x^2}{=} \frac{1}{2}kx^2$$

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Bound Particles in General



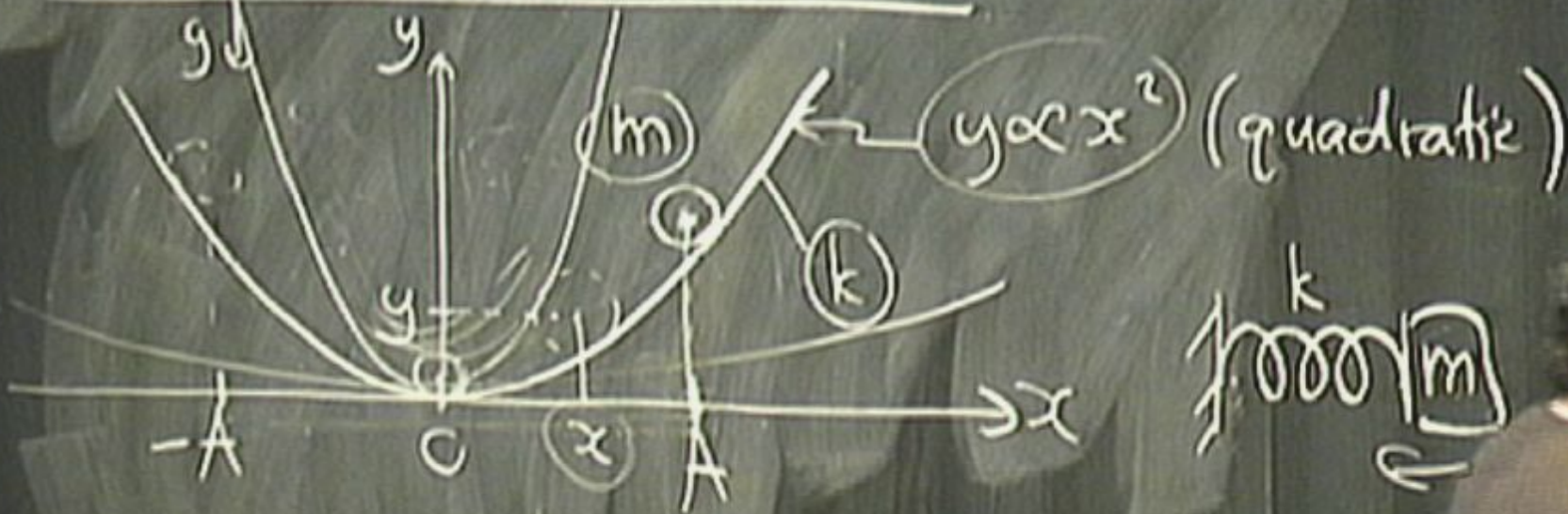
$$PE = mgy \stackrel{y = \frac{1}{k}x^2}{=} \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$E = KE + PE = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

Bound Particles in General



$$PE = mgy \quad \left. \begin{array}{l} y = kx^2 \\ \end{array} \right) = \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$E = KE(A) + PE(A) = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

constant \uparrow

$$PE = mgy \quad (y = kx^2)$$
$$= \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$E = KE(A) + PE(A) = \frac{p(A)^2}{2m} + \frac{1}{2}kx(A)^2$$

↑
constant

$$PE = mgy \quad (y = kx^2)$$

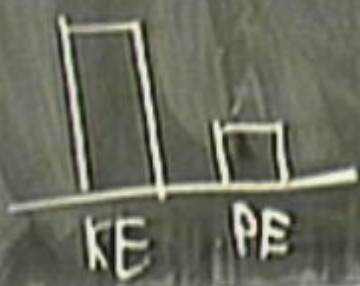
$$= \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$E = KE(A) + PE(A) = \frac{p(A)^2}{2m} + \frac{1}{2}kx(A)^2$$

constant



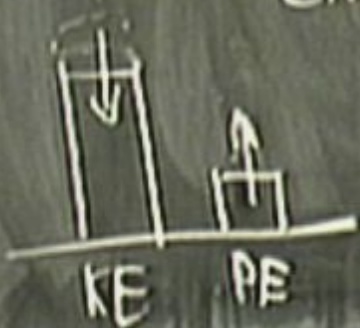
$$PE = mgy \stackrel{y = \frac{1}{2}kx^2}{=} \frac{1}{2}kx^2$$

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constant



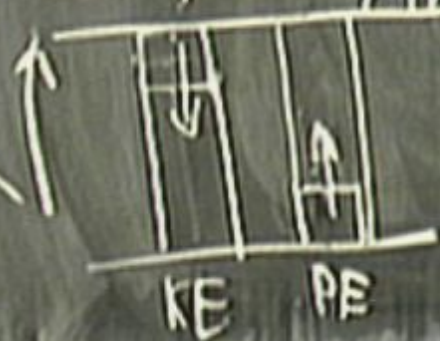
$$PE = mgy \stackrel{y=kx^2}{=} \frac{1}{2}kx^2$$

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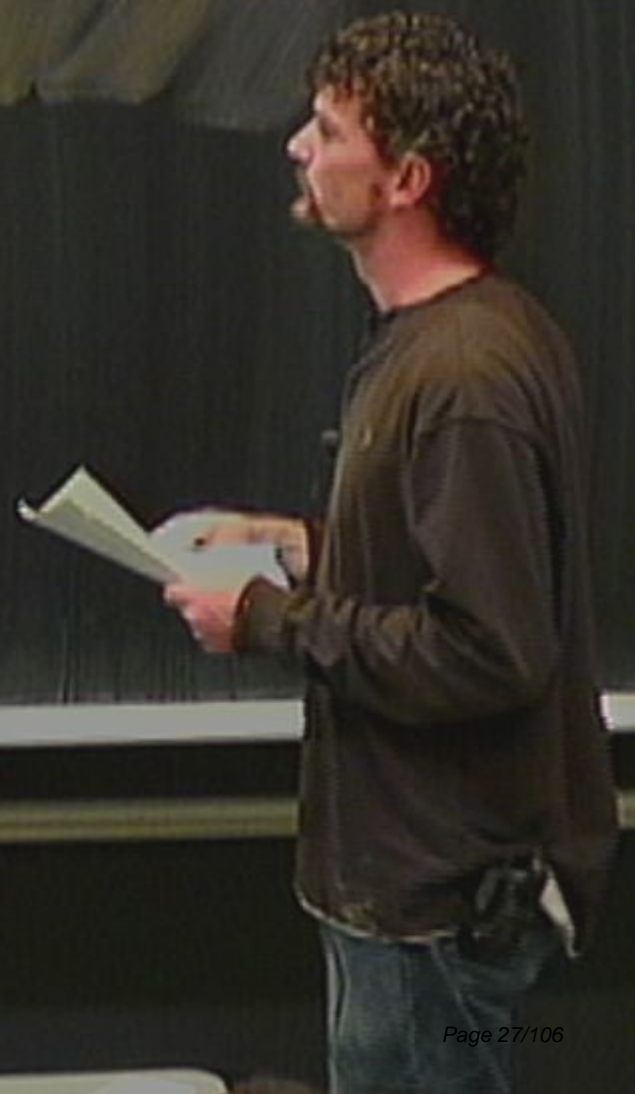
↑
constant



"turning points":



"turning points" : $\phi = 0, x = \pm A$



"turning points" : $\phi = 0, x = \pm A$



$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$A =$$

"turning points" : $\phi = 0, x = \pm A$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$



$$A = \sqrt{\frac{2E}{k}}$$

"turning points" : $\phi = 0, x = \pm A$



$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$A = \sqrt{\frac{2E}{k}}$$



qualitatively similar to $\frac{1}{\epsilon} \left(\frac{1}{\epsilon} \right)$

qualitatively similar to $\mathbb{R} \rightarrow \mathbb{C} \rightarrow \mathbb{R}$

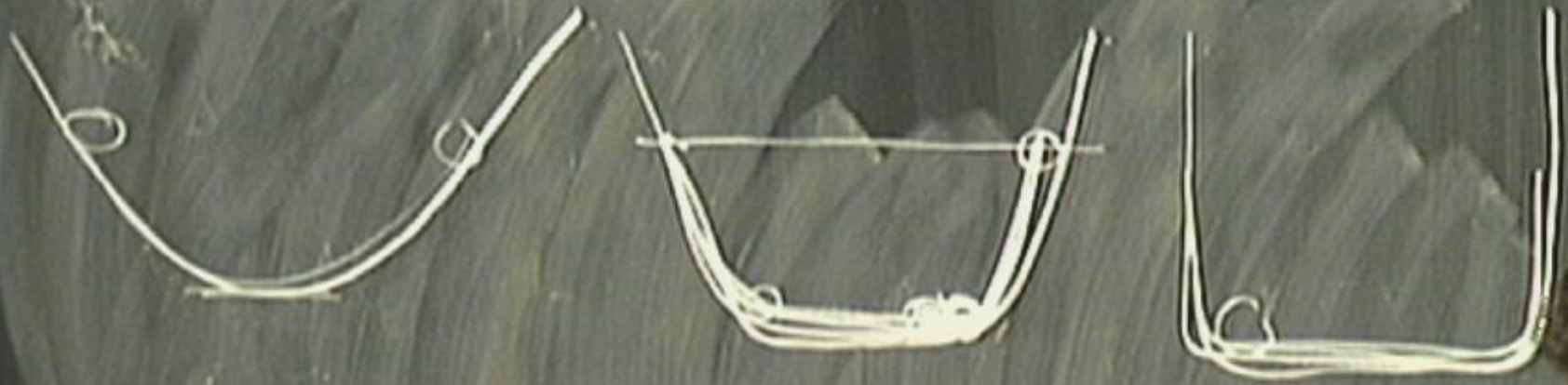
qualitatively similar to $\mathbb{R} \times \mathbb{S}^1$



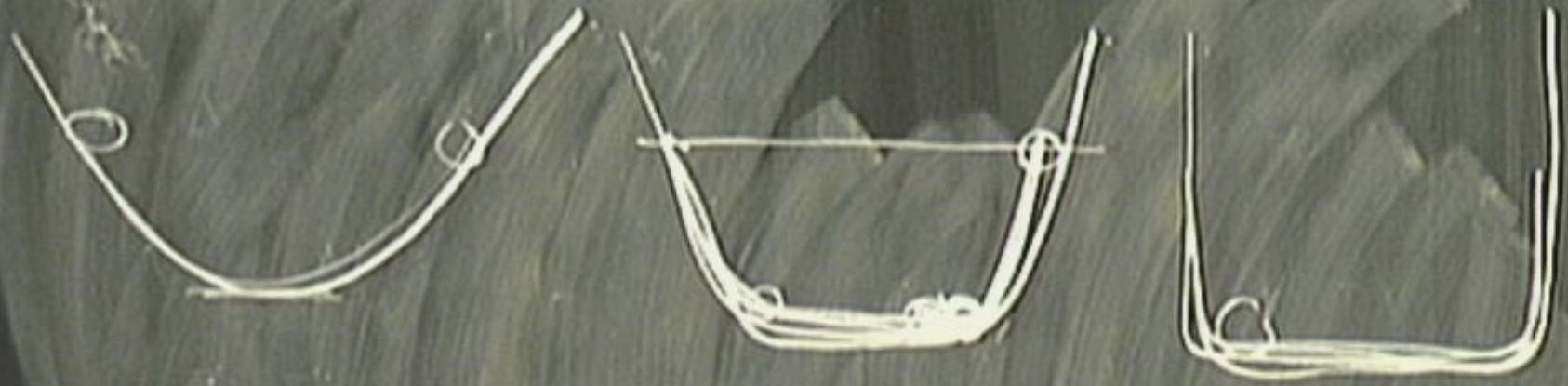
qualitatively similar to $\mathbb{R} \times \mathbb{R}^2$



qualitatively similar to $\mathbb{A}^1 \times \mathbb{A}^1$



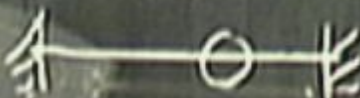
qualitatively similar to $\mathbb{R} \times \mathbb{S}^1$

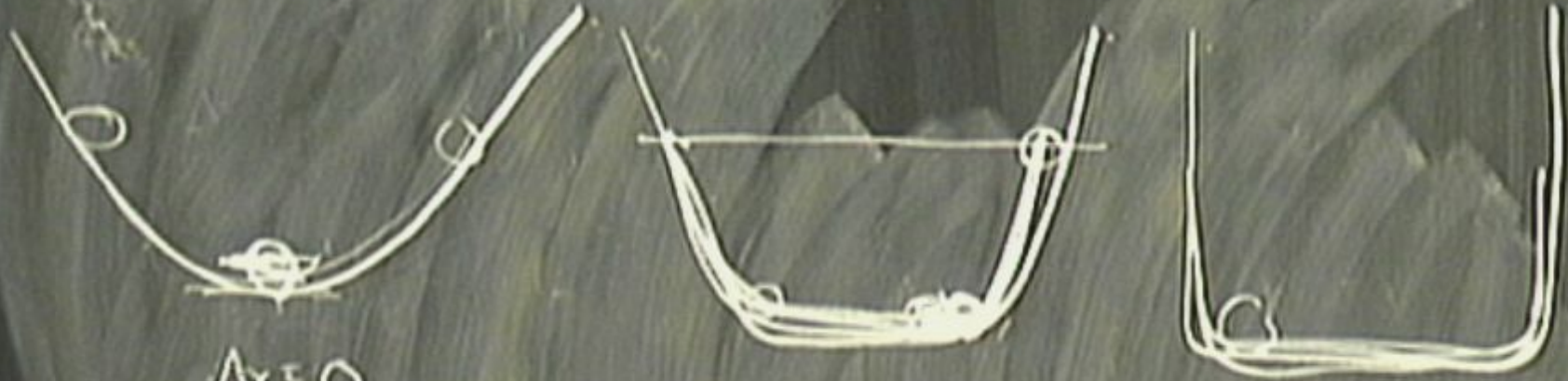


Energy



O

qualitatively similar to 



$$\Delta x = 0$$
$$\Delta p = 0$$

Energy



0 + E_0

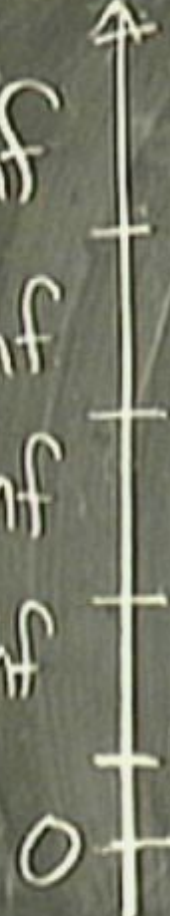
zero point



CAUTION
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MICHIGAN
ANN ARBOR

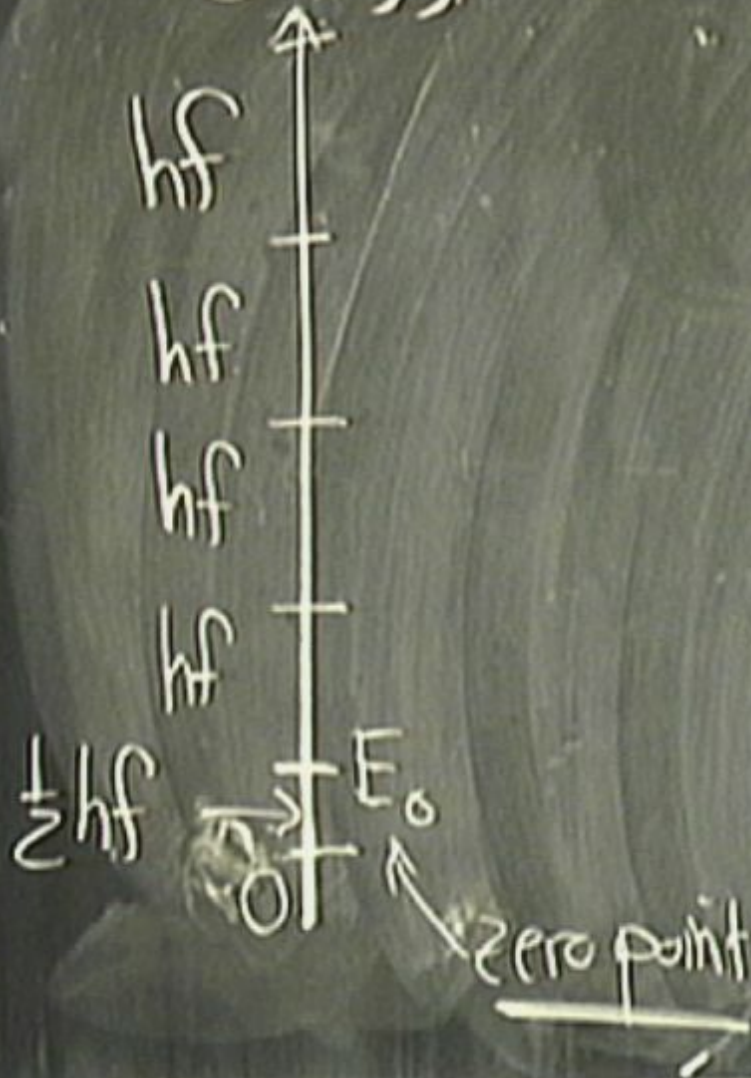
Energy

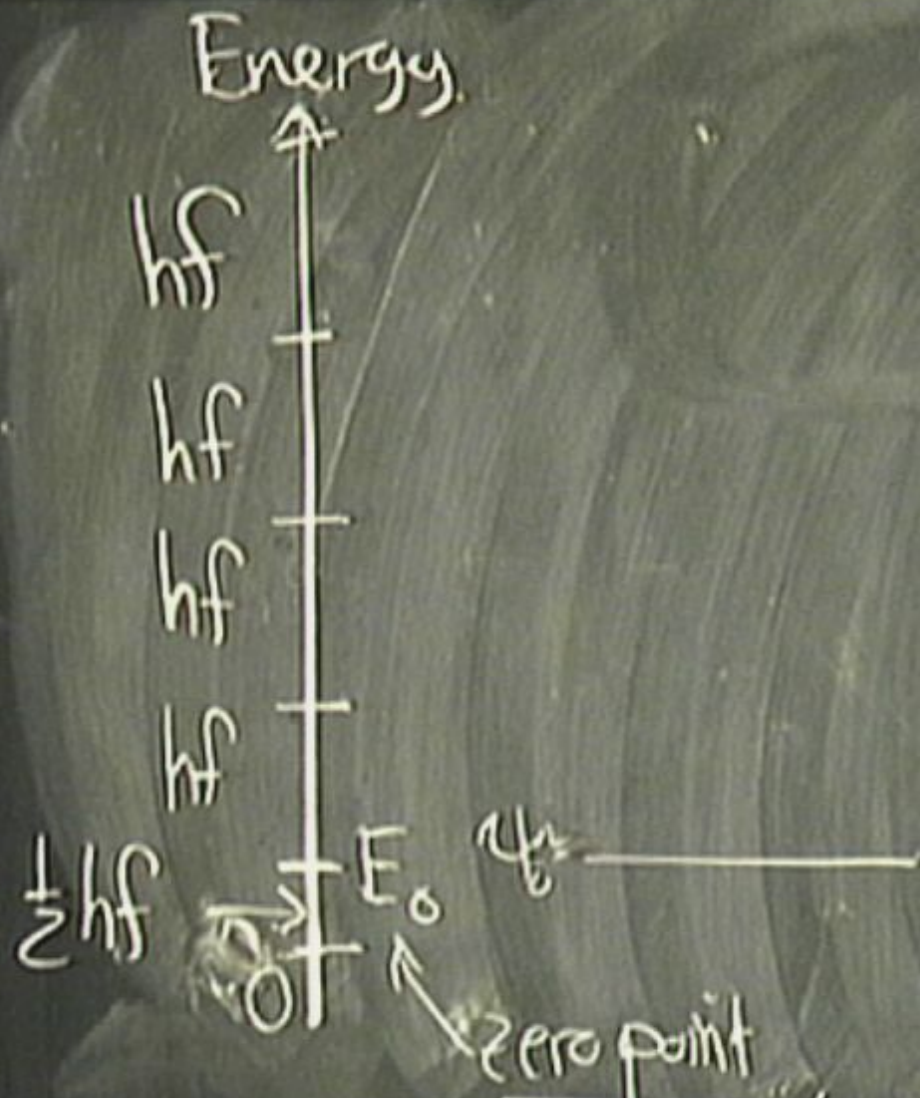
ψ_5
 ψ_4
 ψ_3
 ψ_2
 ψ_1

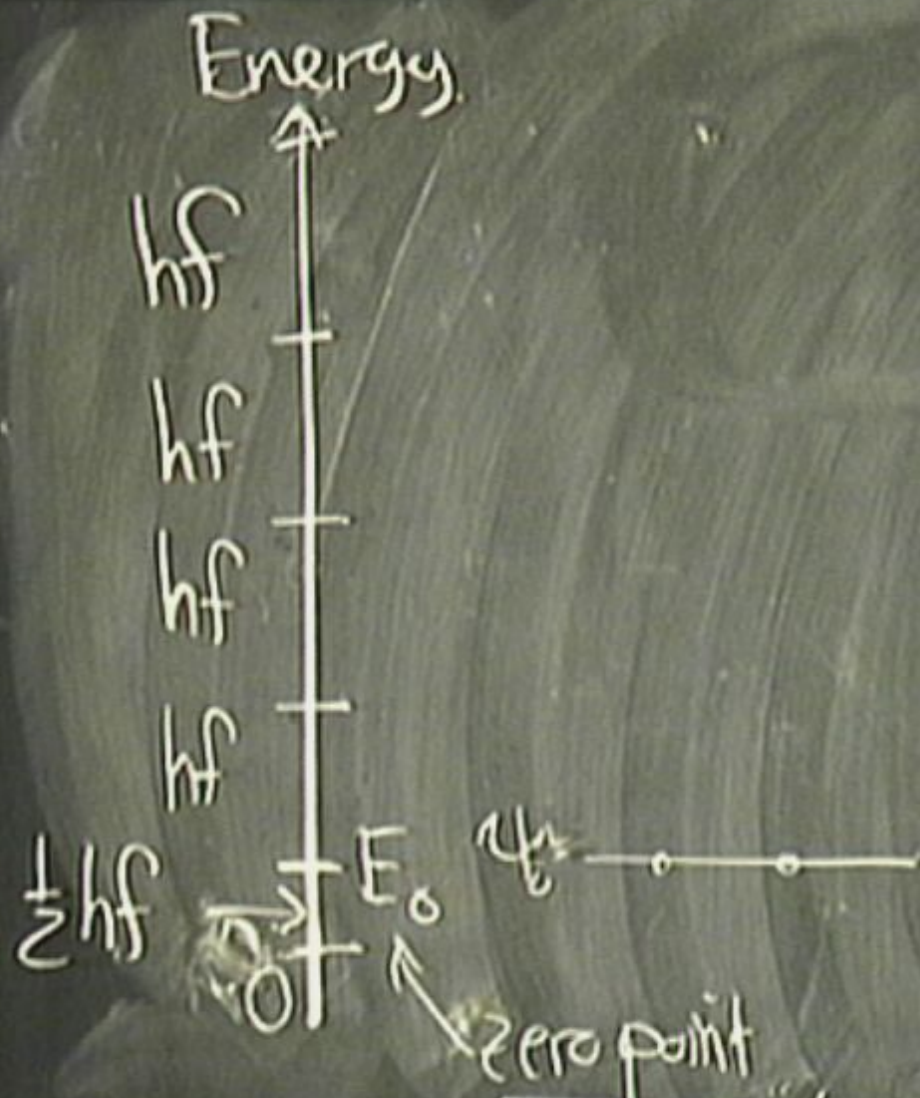


zero point

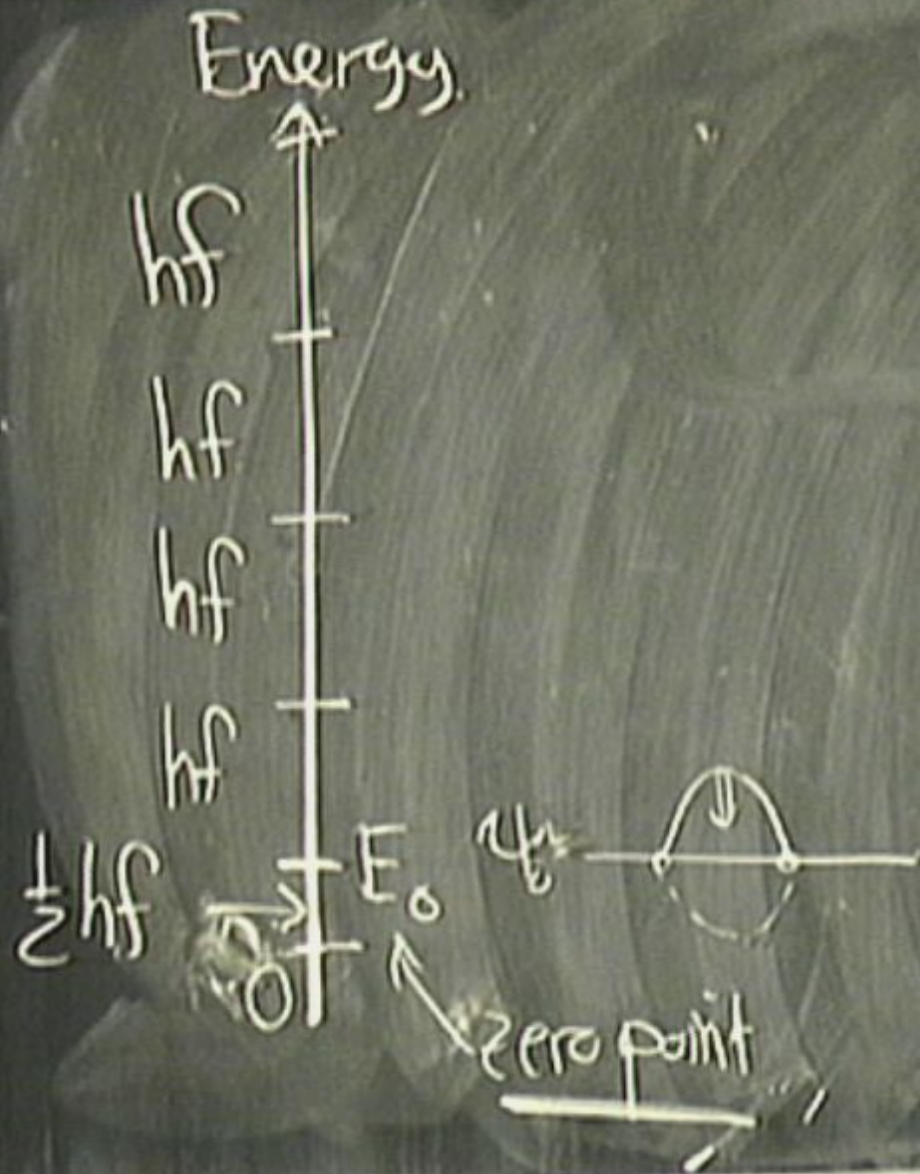
Energy



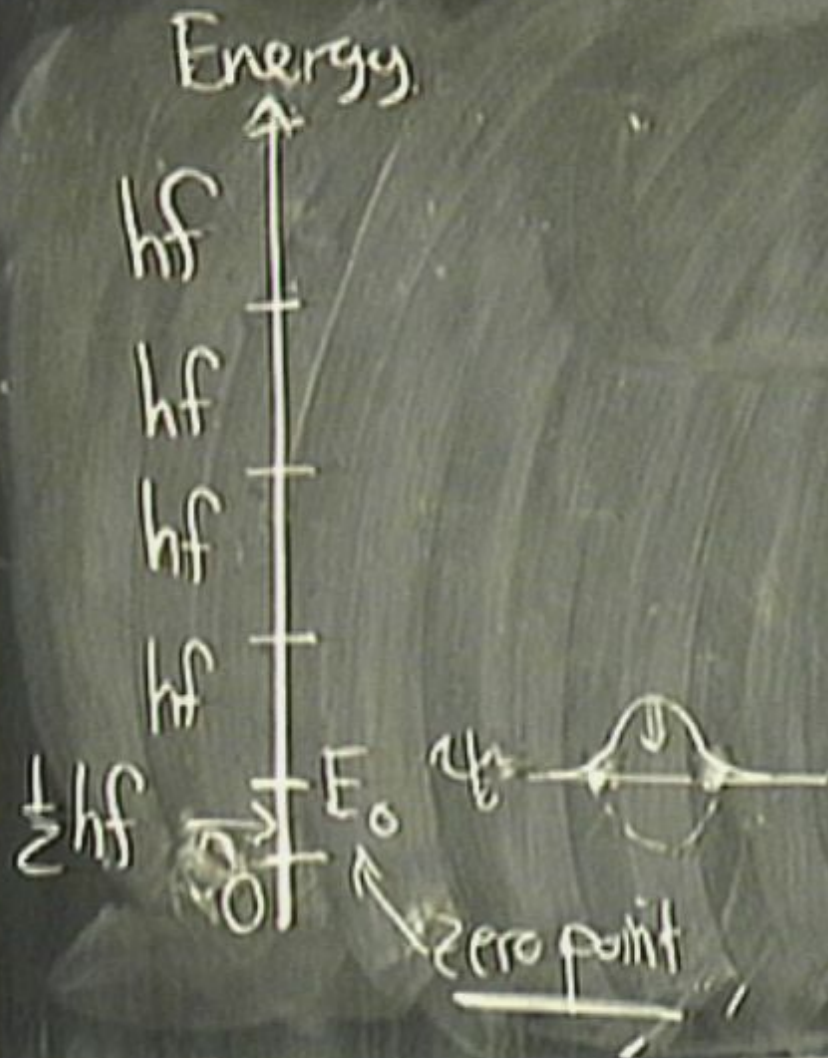




CAUTION
DO NOT TOUCH
THE SURFACE
OR THE CONTENTS
OF THIS CONTAINER



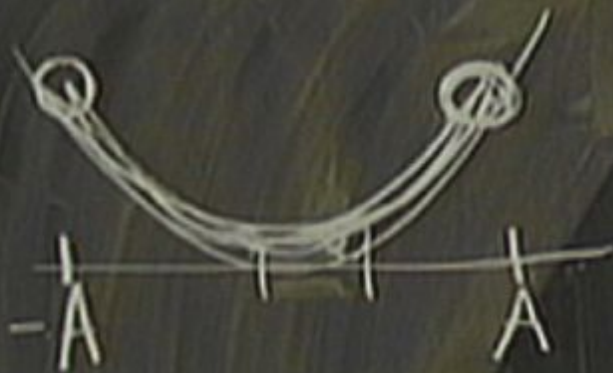
CAUTION
 DO NOT TOUCH
 THE SURFACE
 OF THE BOARD



CA 701
 UNIVERSITY
 OF CALIFORNIA
 BERKELEY

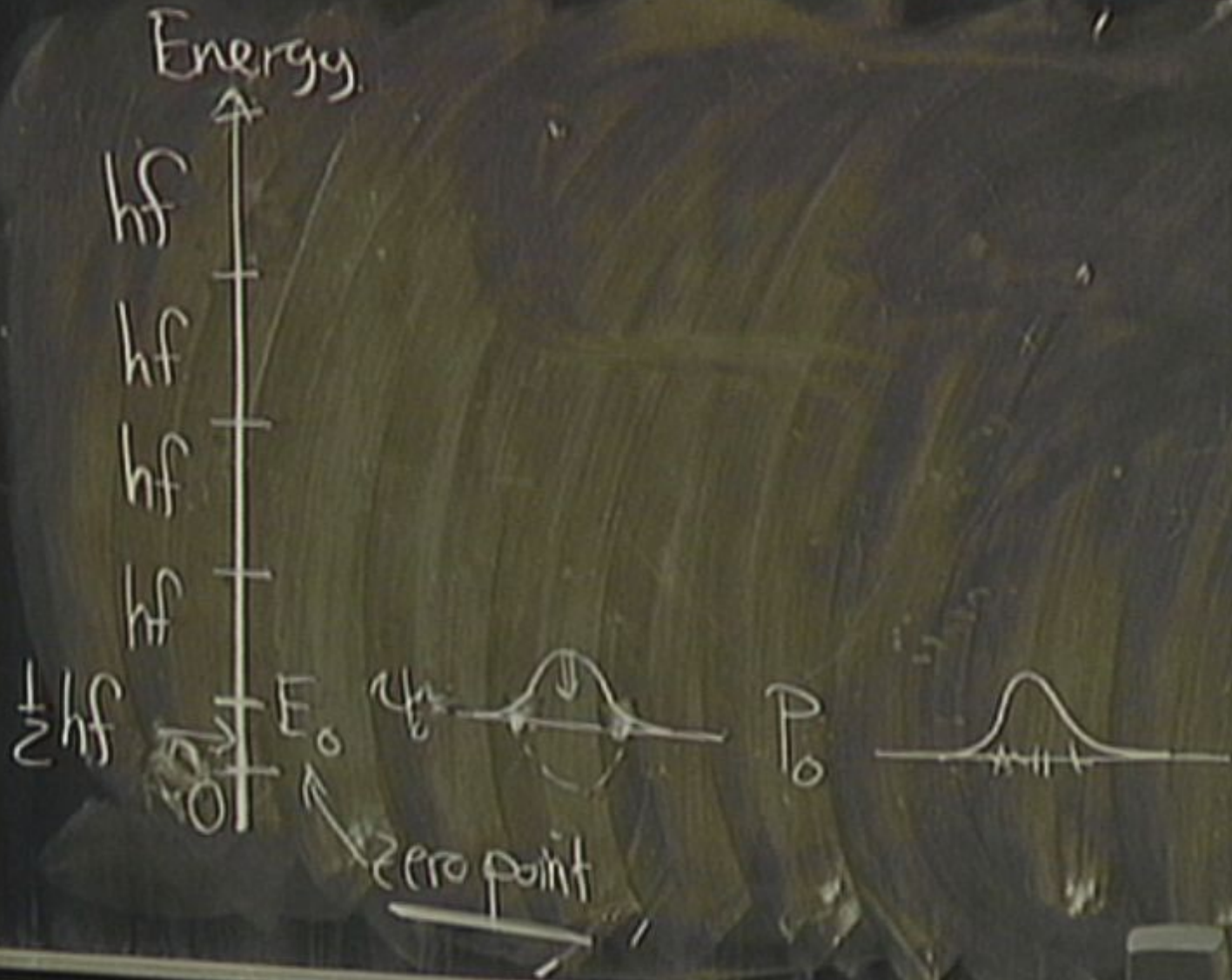
"turning points" : $\phi = 0, x = \pm A$

$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

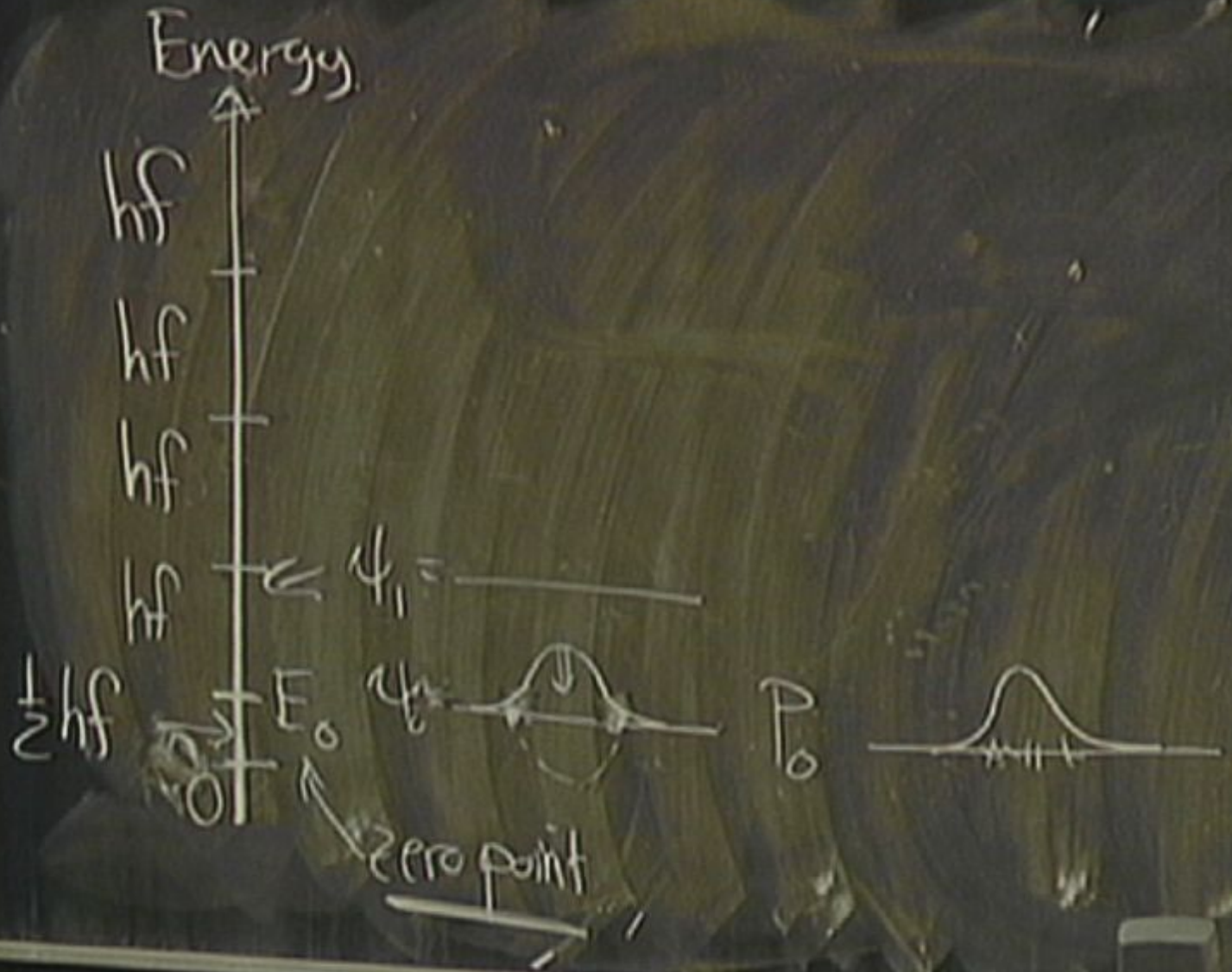


$$A = \sqrt{\frac{2E}{k}}$$

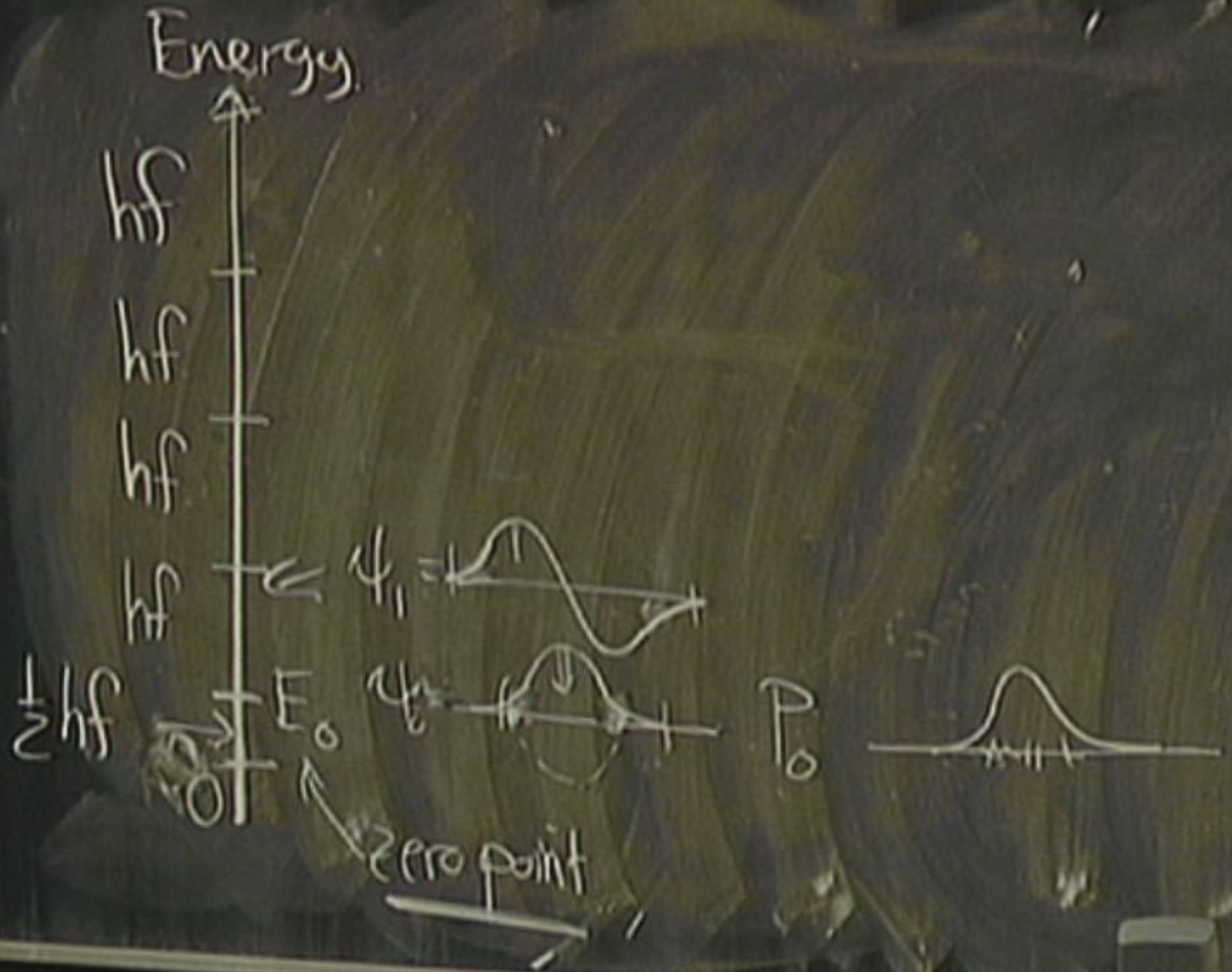




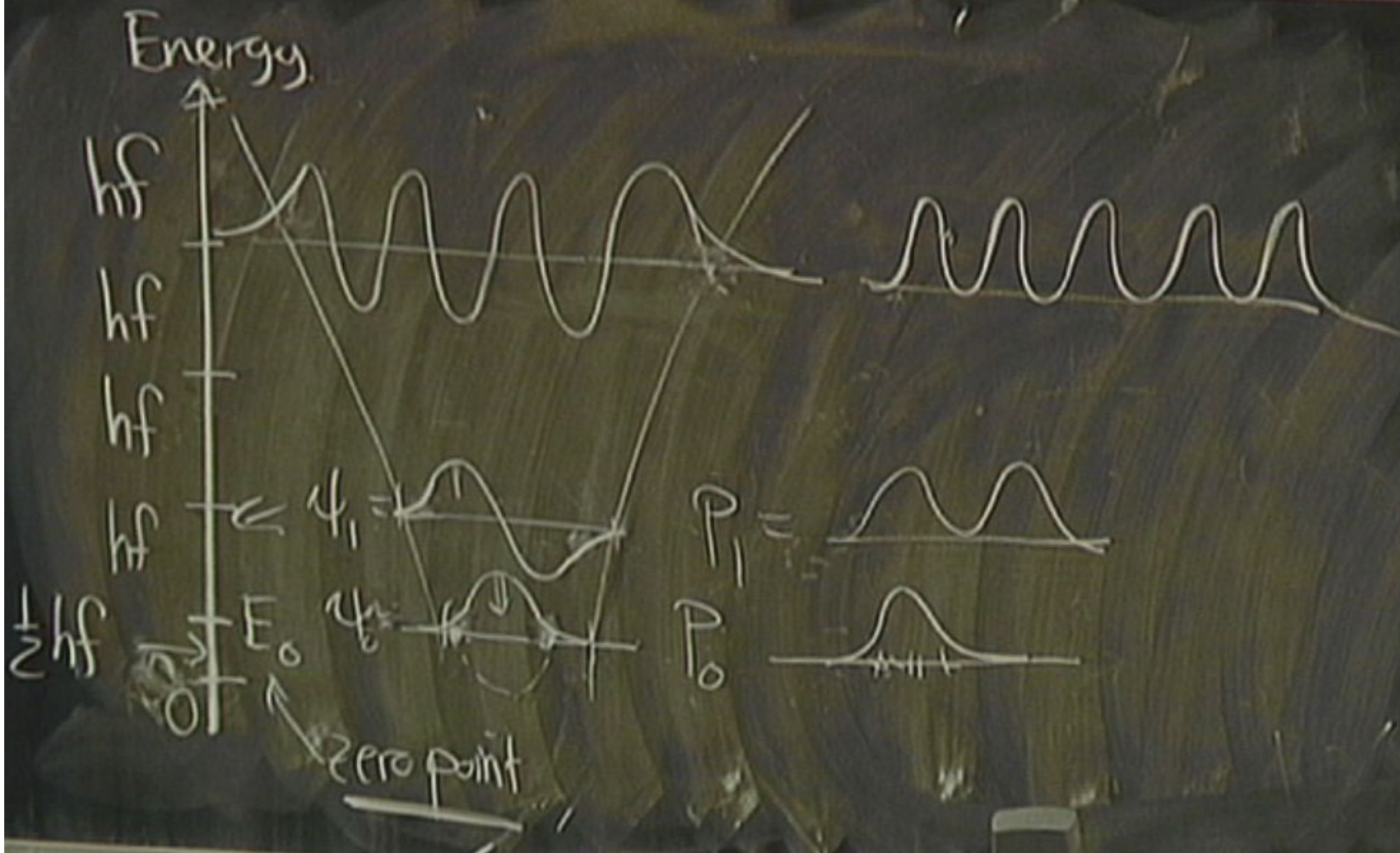
CAUTION



CAUTION



EATEN



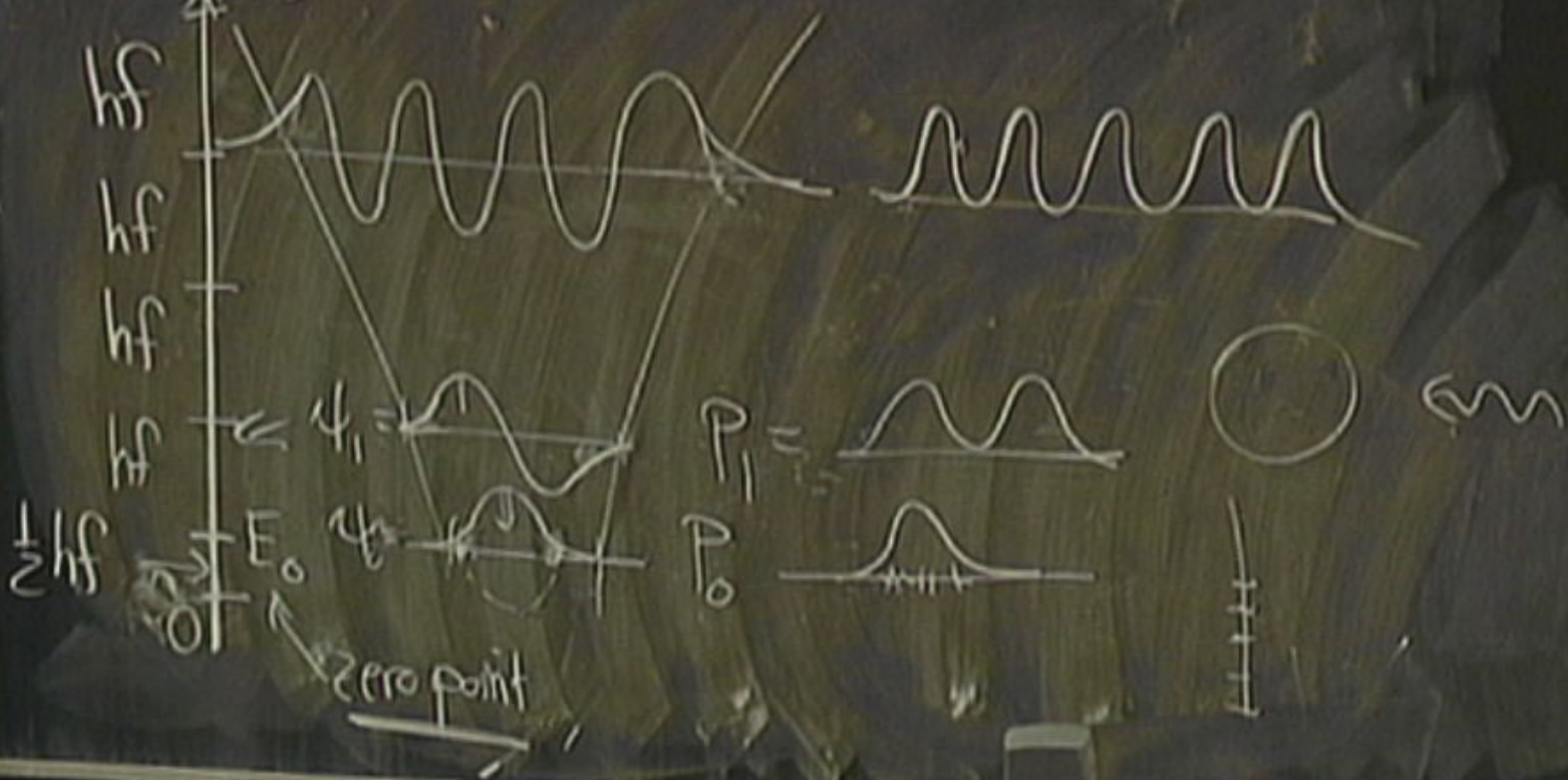
$$= \frac{1}{2} \frac{z^2}{m} + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$A = \sqrt{\frac{2E}{k}}$$

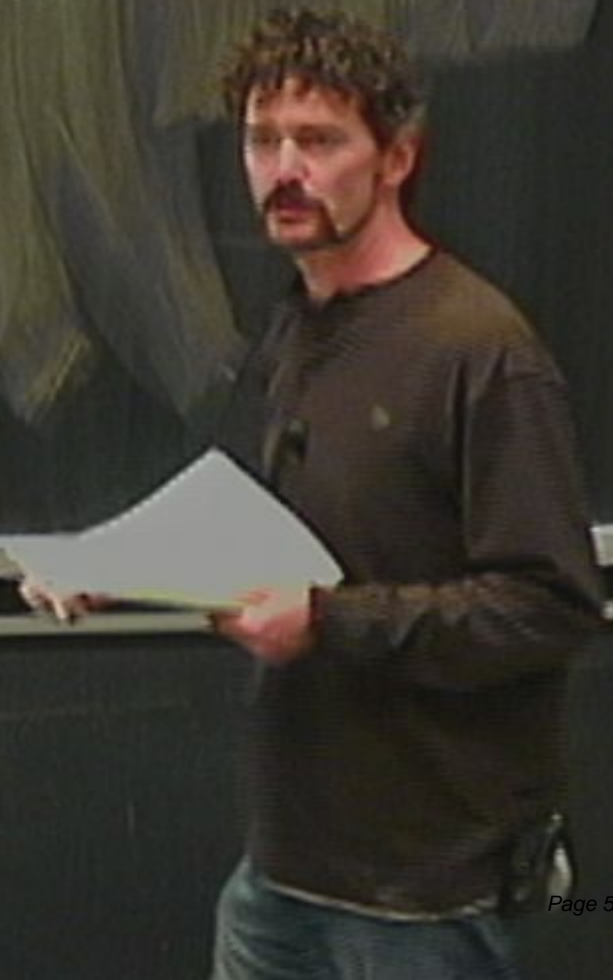
A diagram of a harmonic oscillator. A horizontal line represents the equilibrium position. A mass is shown as a small circle on the left, displaced by a distance x from the equilibrium. A vertical double-headed arrow indicates the displacement x . A larger vertical double-headed arrow on the right indicates the amplitude A . A curved line below the equilibrium position represents the potential energy well.



Energy



evenly spaced : ΔE



evenly spaced : ΔE

linear
geometric

evenly spaced: $\Delta E = hf$

$$E_{\text{photon}} = hf$$

evenly spaced: $\Delta E = hf$

$$E_{\text{photon}} = hf$$

photon \rightarrow !

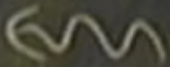
ground state

excited state

evenly spaced : $\Delta E = hf$

QM SR

$$E_{\text{photon}} = hf$$

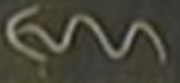


evenly spaced : $\Delta E = hf$

QM SR



$$E_{\text{photon}} = hf$$



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SCHOOL OF ENGINEERING
AND APPLIED SCIENCES
LIBRARY

evenly spaced: $\Delta E = hf$

QM SR

QFT

$$E_{\text{photon}} = hf$$



evenly spaced: $\Delta E = hf$

QM SR

QFT

$$E_{\text{photon}} = hf$$



evenly spaced: $\Delta E = hf$

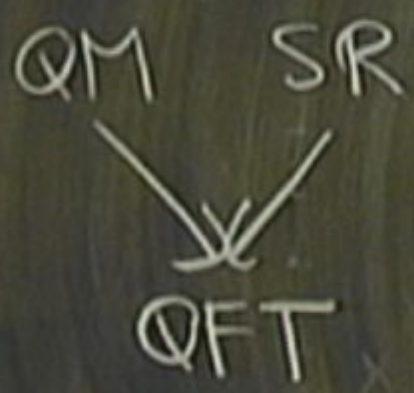
QM SR



$$E_{\text{photon}} = hf$$



evenly spaced: $\Delta E = hf$



$$E_{\text{photon}} = hf$$



each e/m standing wave
(mode) of freq f

evenly spaced: $\Delta E = hf$

quantization!
ground state

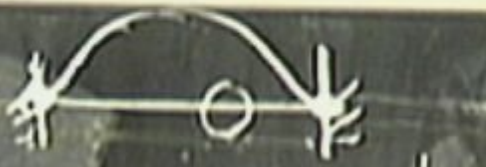
QM SR

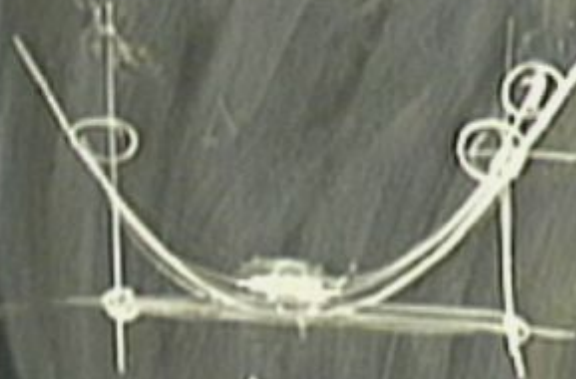
$$E_{\text{photon}} = hf$$

QFT



each e/m standing wave
(mode) of freq f
oscillates harmonically

qualitatively similar to 



$$\Delta x = 0$$
$$\Delta p = 0$$



energy in mode

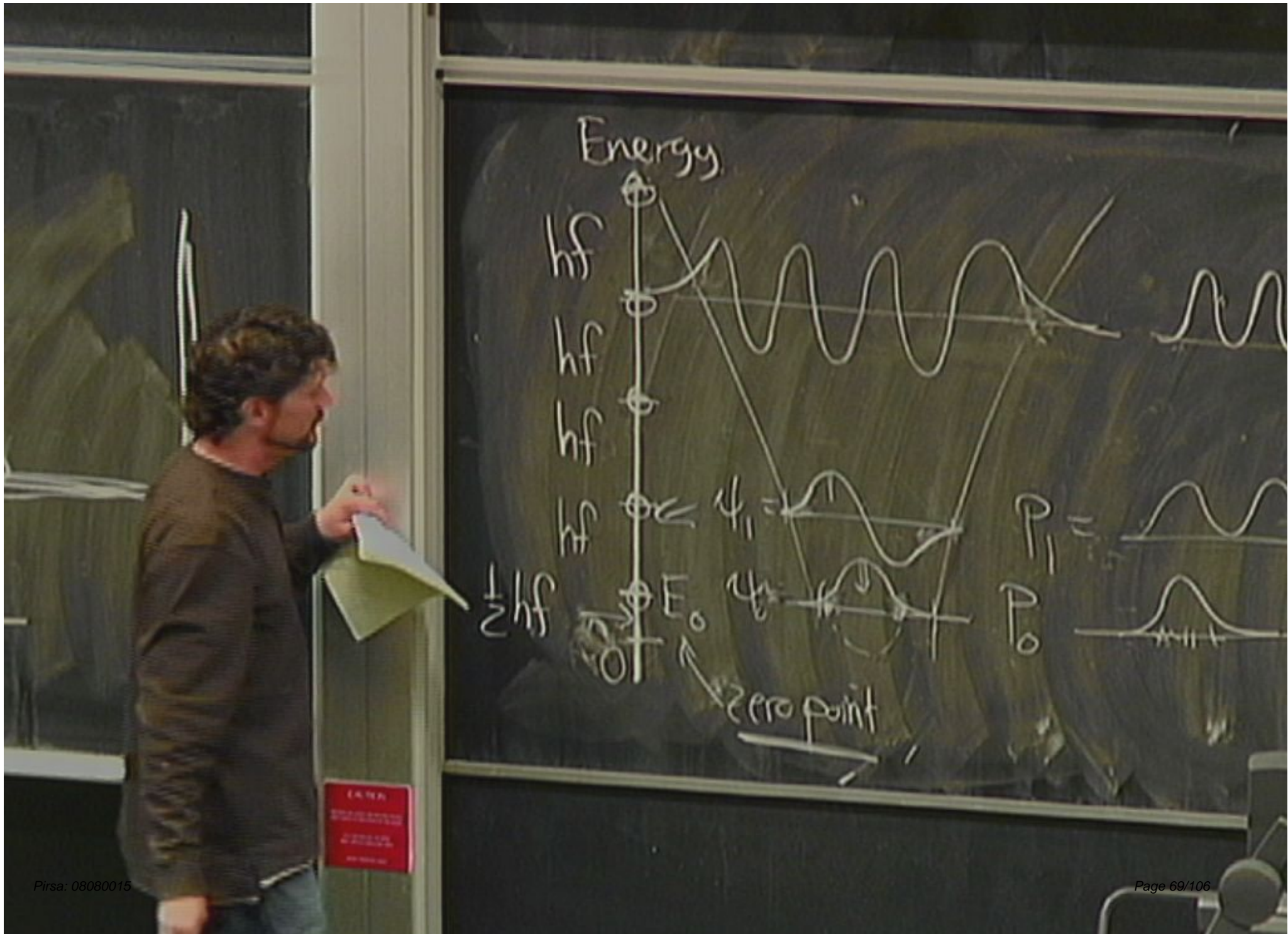
energy in matter is discrete

energy in mode is discrete

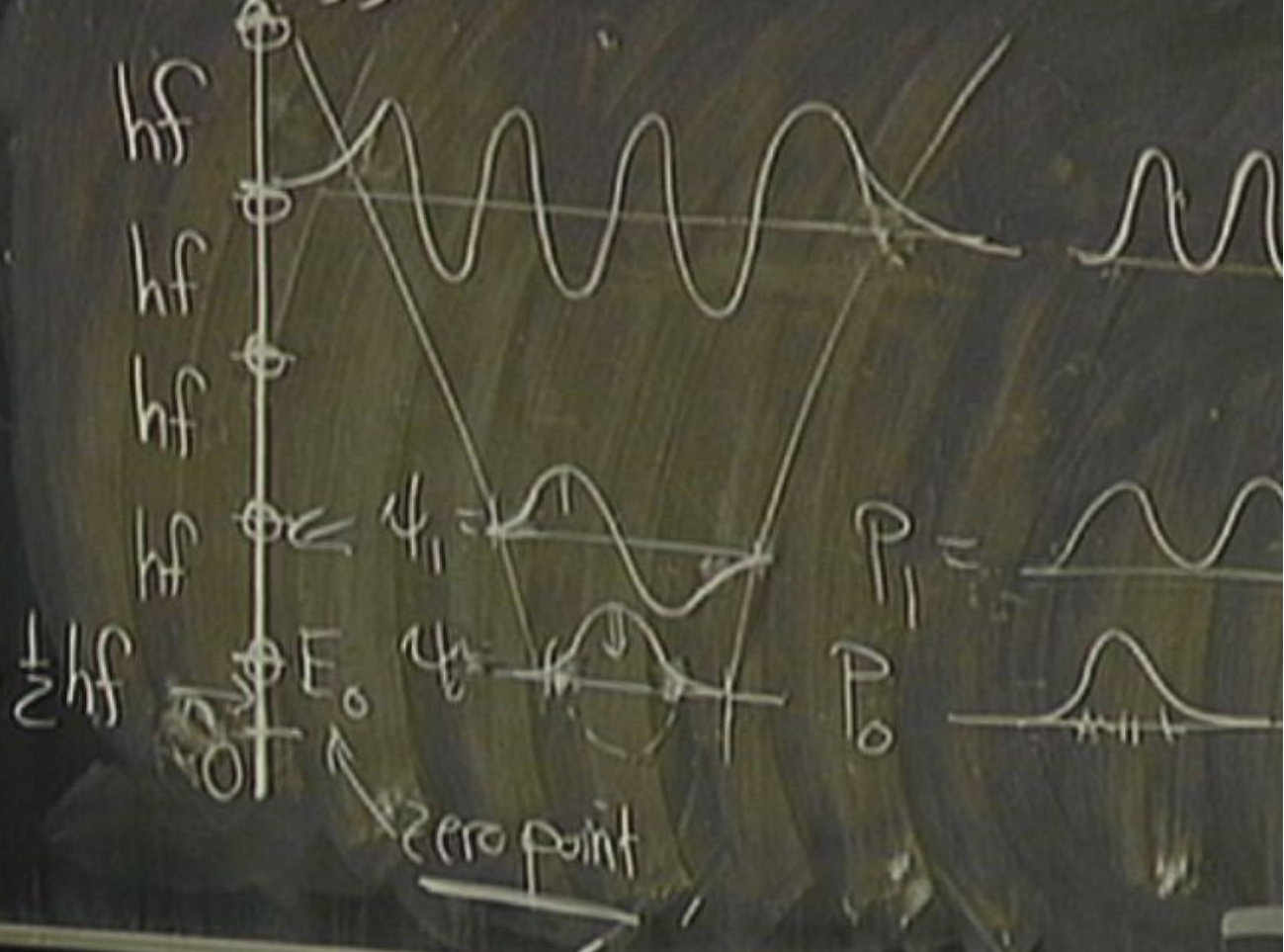
$$E_{\text{mode}}(f)$$

energy in mode is discrete

$$E_{\text{mode}(f)} = nhf + \frac{1}{2}hf$$



Energy



CAUTION

energy in mode is discrete

$$E_{\text{mode}(f)} = nhf + \frac{1}{2}hf, n=0,1,2,3, \dots$$

QFT



(mode) of freq f
oscillates harmonically

$$E_{\text{mode}(f)} = nhf + \frac{1}{2}hf, n=0,1,2,3, \dots$$

oscillates harmonically

$$E_{\text{mode}(f)} = n \hbar f + \frac{1}{2} \hbar f, \quad n=0,1,2,3, \dots$$

oscillates harmonically

$$E_{\text{mode}}(f) = n \underset{5}{hf} + \frac{1}{2} hf, \quad n = 0, 1, 2, 3, \dots$$

of photons.



oscillates harmonically

$$E_{\text{mode}(f)} = n \hbar f + \frac{1}{2} \hbar f, \quad n = 0, 1, 2, 3, \dots$$

$$\Delta x \Delta p \geq \frac{\hbar}{4\pi}$$

5

↖ # of photons.

Casimir



Casimir



$$\frac{1}{2}hf$$

Casimir



$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \dots$$

Casimir



$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \frac{1}{2}$$

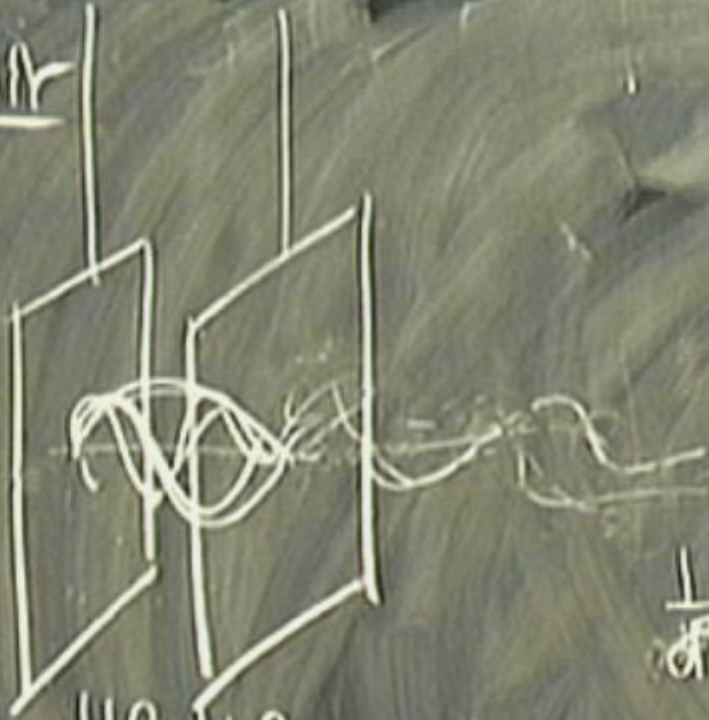
Casimir



$$\frac{1}{2}hf_1 + \frac{1}{2}hf_2 + \dots = \infty$$

$\frac{1}{df}$

Casimir



$$\frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{k^2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{k^2} = \infty$$

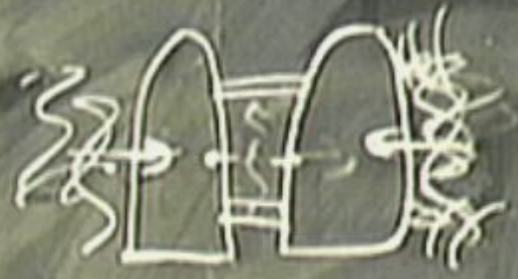
$\frac{1}{2}$

Casimir



$$\frac{1}{2}hf_1 + \frac{1}{2}hf_2 + \dots = \infty$$

Casimir



$$\frac{1}{2}hf_1 + \frac{1}{2}hf_2 + \dots = \infty$$

Casimir



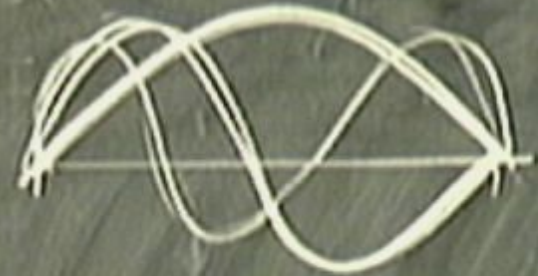
$$\frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{k^2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{k^2} = 8$$



$\infty - \infty = \text{anything}$

$\infty - \infty = \text{anything}$

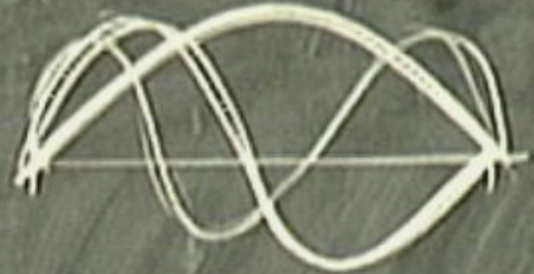
$$E_{2.P.} = \frac{1}{2} h f_1 + \frac{1}{2} h f_2 + \dots$$



1
2
3
4
...

$\infty - \infty = \text{anything}$

$1 \times 2 \times 3 \dots$



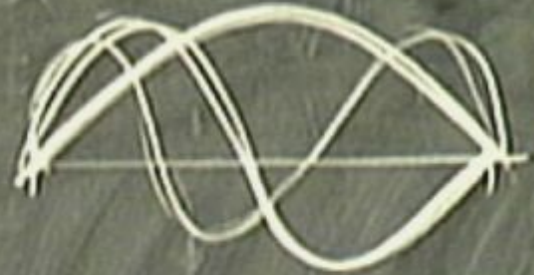
$$E_{2P} = \frac{1}{2} h f_1 + \frac{1}{2} h f_2 + \dots$$

$$= () (1 + 2 + 3 + 4 + \dots)$$

1
2
3
4
⋮

$\infty - \infty = \text{anything}$

$1 \times 2 \times 3 \dots$



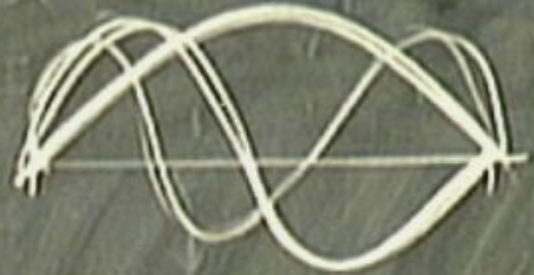
$$E_{2.P.} = \frac{1}{2} h f_1 + \frac{1}{2} h f_2 + \dots$$

$$= () (1 + 2 + 3 + 4 + \dots) = \infty$$

- 1
- 2
- 3
- 4
- ...

$\infty - \infty = \text{anything}$

$1 \times 2 \times 3 \dots$



$$E_{2.P} = \frac{1}{2} h f_1 + \frac{1}{2} h f_2 + \dots$$

$$= \left(\frac{1}{12} \right) (1 + 2 + 3 + 4 + \dots) = \infty$$

$$-\frac{1}{12}$$

1
2
3
4
...

$$\frac{1}{1-x} =$$

$$1-x \sqrt{1}$$

Arithmetic series

Geometric

$$\frac{1}{1-x} = \frac{1+x+x^2+x^3}{1+x+x^2+x^3}$$

$$\frac{1+x+x^2}{1+x+x^2}$$

$$\begin{array}{r} 1-x \overline{) 1} \\ \underline{1-x} \\ x \\ \underline{x-x^2} \\ x^2 \end{array}$$

$$\frac{1}{1-x} = \frac{1}{1+x+x^2+x^3}$$

$$x = \frac{1}{2}$$

$$\frac{1+x+x^2}{1-x}$$

$$\begin{array}{r} 1-x \overline{) 1} \\ \underline{1-x} \\ x \\ \underline{x-x^2} \\ x^2 \end{array}$$

$$\lim_{x \rightarrow 2} \frac{1}{1-x} =$$

$$x=2$$

$$\sum_{x=2}^{\infty} \frac{1}{1-x} = -1$$

$$\sum_{x=2}^{\infty} x =$$

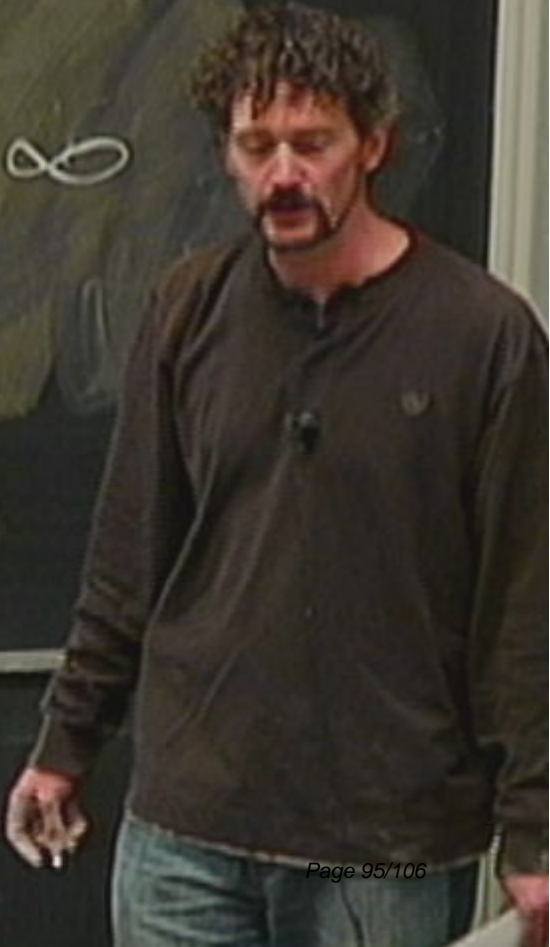
$$\sum_{x=2}^{\infty} (1+x) = 1+2+4+8+16+ \dots$$

$$\sum_{x=2}^{\infty} \frac{1}{1-x} = -1$$

$$\sum_{x=2}^{\infty} (1+x) = 1+2+4+8+16+\dots$$

$$\lim_{x \rightarrow 2} \frac{1}{1-x} = -\infty$$

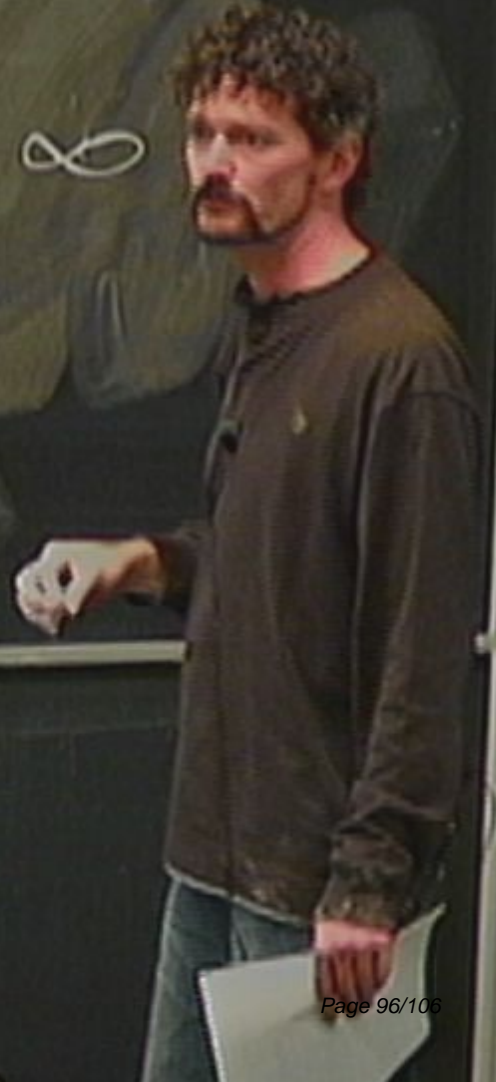
$$\sum_{x=2}^{\infty} (1+x) = 1+2+4+8+16+\dots = \infty$$



hence $\left| \frac{1}{1-x} \right|_{x=2} = -1$

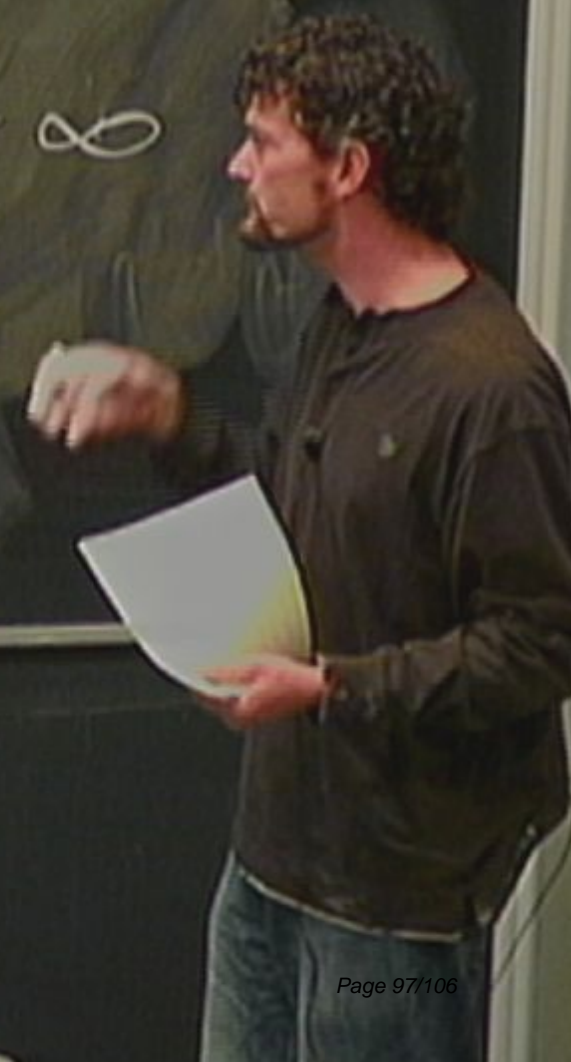
A horizontal number line is drawn with an arrow pointing to the right, labeled 'x'. A vertical tick mark is labeled '0'. To the right of 0, another vertical tick mark is labeled '1'. To the right of 1, a third vertical tick mark is labeled with an asterisk '*'. The asterisk is positioned at the value 2 on the number line.

$\left| 1+x+\dots \right|_{x=2} = \boxed{1+2+4+8+16+\dots} = \infty$

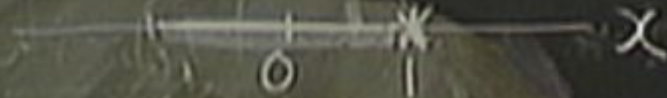


$\lim_{x \rightarrow 2} \frac{1}{1-x} = -1$

$\lim_{x \rightarrow 2} (1+x)^{-1} = 1+2+4+8+16+\dots = \infty$



$$\text{Hence } \frac{1}{1-x} \Big|_{x=2} = -1$$



$$\left. \frac{1}{1-x} \right|_{x=2} = \boxed{1+2+4+8+16+\dots} = \infty$$

Riemann Zeta function.

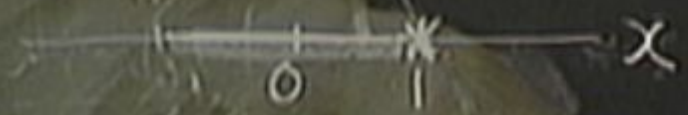
$\lim_{x \rightarrow 1} \frac{1}{1-x} = -1$

$\lim_{x \rightarrow 2} (1+x)$

Riemann Zeta function.



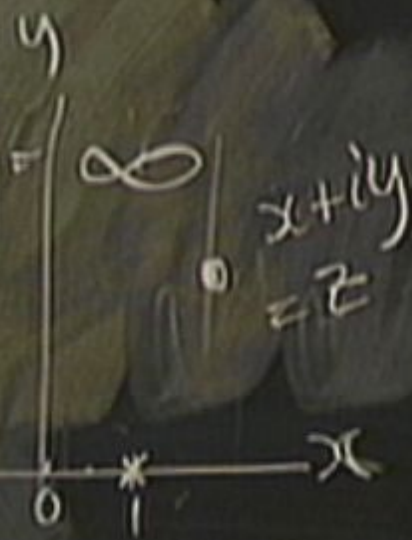
hence $\frac{1}{1-x} = \sum_{x=2}^{\infty} x^{-2}$



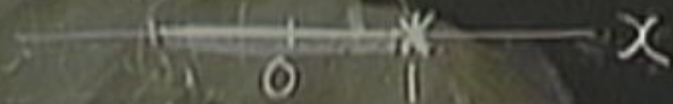
$\sum_{x=2}^{\infty} (1+x^{-2}) = 1 + 2^{-2} + 4^{-2} + 8^{-2} + 16^{-2} + \dots$

Riemann Zeta function.

$\zeta(z)$



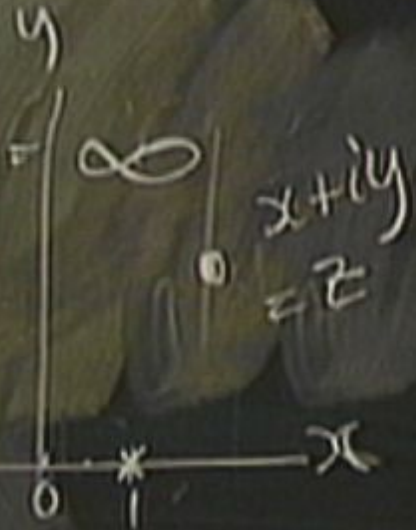
hence $\frac{1}{1-x} \Big|_{x=2} = -1$



$\left. \frac{1}{1-x} \right|_{x=2} = 1+2+4+8+16+ \dots$

Riemann Zeta function.

$\zeta(z)$

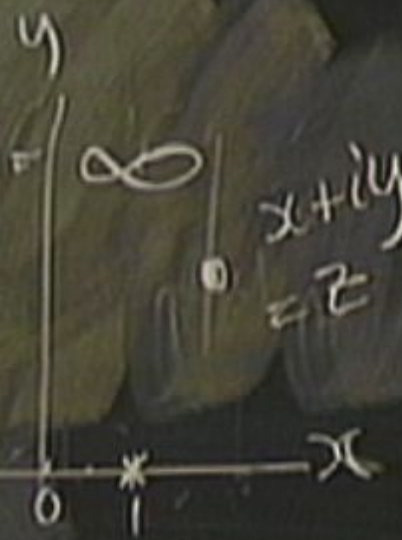


$\frac{1}{1-x} = 1 + x + x^2 + \dots$



A horizontal number line labeled 'x' with a tick mark at 0 and an asterisk at 1, representing a pole.

$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$



A diagram showing the real axis (x) and imaginary axis (y) of the complex plane. The real axis has a tick mark at 0 and an asterisk at 1. The imaginary axis has a tick mark at 1 and an infinity symbol. A point is marked on the real axis between 0 and 1, labeled 'x+iy = z'.

Riemann Zeta function.

$\zeta(z) =$

$\infty - \infty = \text{anything}$

$1 \times 2 \times 3 \dots$



$$E_{2R} = \frac{1}{2} h f_1 + \frac{1}{2} h f_2 + \dots$$

$$= \left(\frac{1}{2} h \right) (1 + 2 + 3 + 4 + \dots) = \infty$$

$$\frac{1}{12}$$

- 1
- 2
- 3
- 4
- ...

Casimir

$$E_{2P} = - \left(\frac{1}{4\pi} \right)$$

$$dE = F \cdot da$$



$$\sum_{f_2} \dots = \infty$$

Casimir



$$E_{2P} = - \left(\frac{1}{d^3} \right)$$

$$dE = E_c F d$$



$$\frac{1}{2} h f_1 + \frac{1}{2} h f_2 + \dots = \infty$$

Casimir



$$E_{2P} = - \left(\frac{1}{4\pi} \right)$$

$$dE = F d$$



$$\frac{1}{2}hf_1 + \frac{1}{2}hf_2 + \dots = \infty$$

