

Title: Observational Constraints on Gravitational Degrees of Freedom

Date: Aug 12, 2008 02:00 PM

URL: <http://pirsa.org/08080009>

Abstract: TBA

# The Phenomenology of Gravity



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# A Case For / Introduction To “Phenomenology” (with examples)





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- Parsimony (*gratia* Alan Cooney, U. Arizona)



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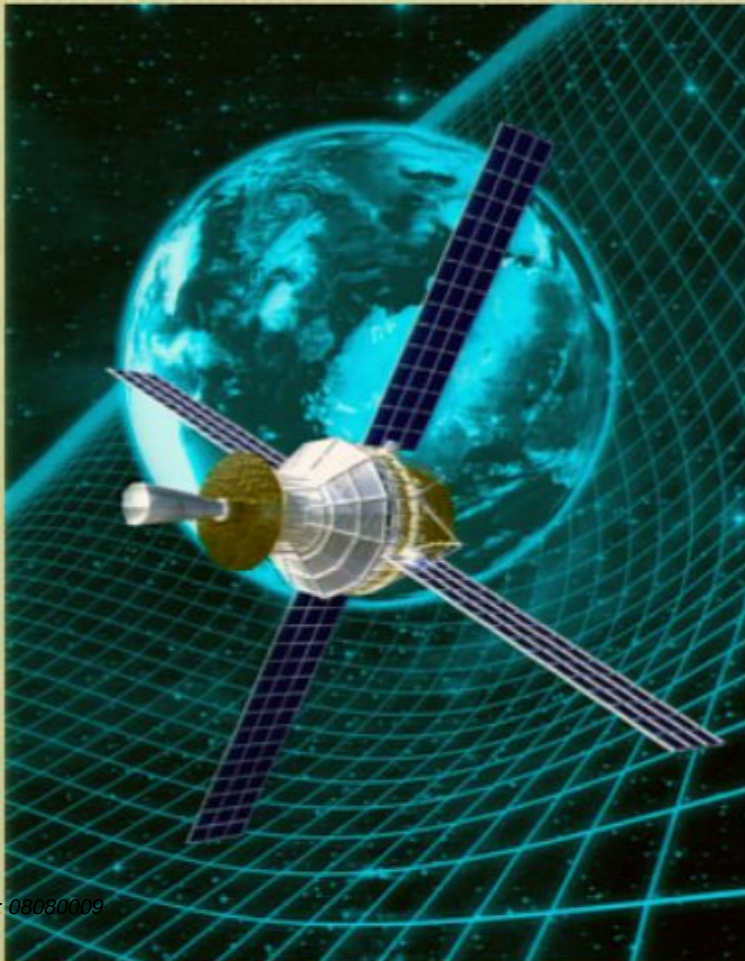


# Example I.

## Degrees of Freedom in Gravity

(work in collaboration with Alan Cooney & Dimitrios Psaltis @ U. Arizona)

# “Parametrized Post-Newtonian”



Pioneered by Will and Nordvedt in the 1970s —  
a general “phenomenology”  
of solar system gravity

more than just “interoperability”  
— a deep assertion of physical  
principles



Newtonian  
gravity

$$g_{00} = -1 + 2U$$

$$U = \int \frac{\rho(x')}{|x - x'|} d^3x'$$

“First order”  
post-Newtonian  
(in a particular  
gauge)

$$g_{00} = -1 + 2U + 2U^2$$

possible modifications . . .

$$g_{00} = -1 + 2U + 2\beta U^2 - 2\xi \Phi_W \dots$$

$$\Phi_W = \int \frac{\rho'(x')\rho''(x'')}{|x - x'|^3} \cdot \left( \frac{x' - x''}{|x - x''|} - \frac{x - x''}{|x' - x''|} \right) d^3x' d^3x''$$



# “Matter Dictatorship”

All possible modifications to the metric are functions of the matter/energy distribution and its velocity [& preferred frames]

$$\Phi_W = \int \frac{\rho' \rho''(x - x')}{|x - x'|^3} \cdot \left( \frac{x' - x''}{|x - x''|} - \frac{x - x''}{|x' - x''|} \right) d^3 x' d^3 x''$$

$$A = \int \frac{\rho' [v' \cdot (x - x')]^2}{|x - x'|^3} d^3 x'$$

&c., &c.

not quite “no new degrees of freedom”  
— *e.g.*, Brans-Dickie theories can fit in,  
but not  $f(R)$  (in general)



# PPN as Phenomenology

*plus sides*

$$A = \int \frac{\rho' [v' \cdot (x - x')]^2}{|x - x'|^3} d^3 x'$$

“matter dictatorship” : parsimony without restriction  
forms of dictatorship easy to cover (but see Alexander, 2008)

*downsides (for cosmology &c.)*

requires “post-Newtonian” conditions:  
shallow potentials that fade at large distance  
(but can adapt the gauge to cosmology)



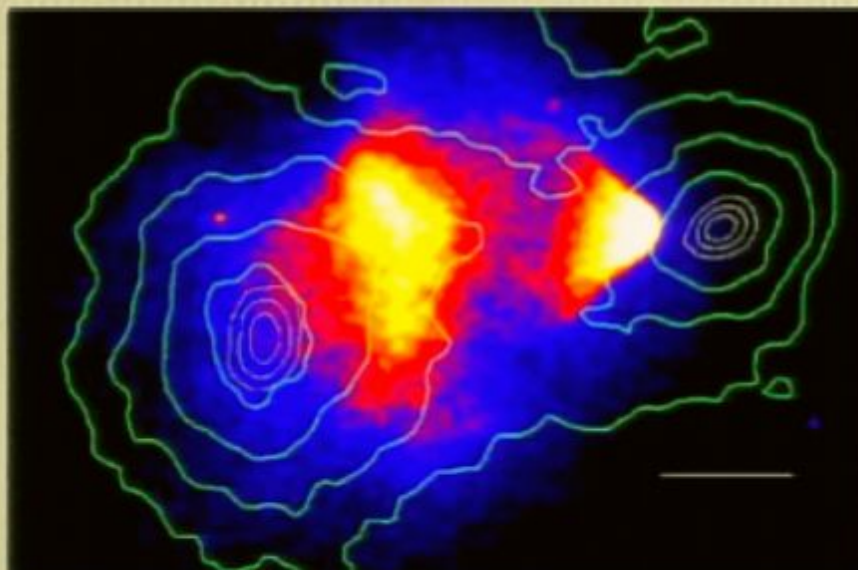
# MOND *vs.* Dark Matter

MOND has parsimony — one parameter “acceleration scale,”  
but not relativistic

*TeV*S has phenomenological parsimony — matter dictatorship[\*]  
but not fundamentally so (new vector and scalar fields)

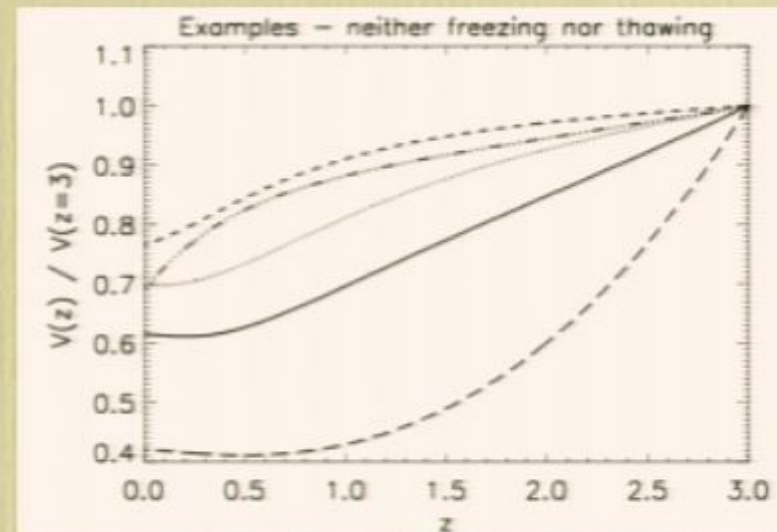
[\*]. *caveat*. — only sort of

observation ruled out (?) both — Clowe *et al.* 2006



# The Age of Quintessence

- “in for a penny, in for a pound” — a newly cavalier attitude towards new degrees of freedom (but a time of great hope.)
- pick your  $V(\varphi)$ , find your E.O.M., discover great fundamental physics!
- where we are today:





# Modified *Gravity*?

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*but!* — as an “exact” theory, can be rewritten:



$$\int \sqrt{-g} [R + (\partial\phi)^2 + \tilde{V}(\phi) + \mathcal{L}_{\text{matt}}(\phi)]$$

# $f(R)$ is the new $V(\varphi)$ ?

- Certainly new behaviors, with various clustering scales — looks in many ways like  $k$ -essence, which has non-trivial sound speeds. (See, *e.g.*, work by Sawicki & Hu.)
- One of the most interesting things in “classical cosmology” today
- But not *conceptually* distinct.
- Still new degrees of freedom!





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# One “escape route”

Forget about it! — just parametrize  $\tau w$ , the E.O.S. :  
pick one you like, *e.g.*,

$$w(a) = w_0 + (1 - a)w_a$$

$$w(z) = w_0 + zw_1$$

nice for doing observational tests given the current data —  
but “so open-minded your brain falls out”? You are never  
sure what physics you are holding constant.

$\tau w_a \approx 0.1$  means what, exactly? is it inconsistent with  
structure formation (*e.g.*)?

Pirsa: 08080009 — you’ll never know without a “real” model behind it. Page 19/58

# Should we bite the bullet?

- Classical, local  $f(R)$  theories that are IR complete generically have new degrees of freedom, and are massless in cosmologically relevant regimes.
- Can we “do more with less”? Do the observations *require* new degrees? Is there a dark energy bullet cluster? [Not yet!]



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# Modified Gravity

- Stay with Lagrangian formalism — retain, for free, general covariance (so no nasty MOND  $\Rightarrow$  *TeV*S surprises.)
- But assume it is a *perturbative* approximation to a larger theory “we know not what” that fixes “we know not how” *fictitious* extra degrees that appear perturbatively.

$$\mathcal{L} = \int \frac{1}{2} \dot{q}^2 + \alpha q(x + \delta x) q(x + \delta x') d^3x \quad \text{second order UV non-local}$$

$$= \int \frac{1}{2} \dot{q}^2 + \alpha \left[ q + \frac{dq}{dx} \delta x + \frac{1}{2} \frac{d^2q}{dx^2} (\delta x)^2 + \dots \right] \left[ q + \frac{dq}{dx} \delta x' + \frac{1}{2} \frac{d^2q}{dx^2} (\delta x')^2 + \dots \right]$$

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# The Formalism

- Adapt the “perturbative localization” of Woodard, Simon *et al.* — developed for the classic case of (UV) non-local theories.

deceptive “formalism overlap”

$$\int \sqrt{-g} \left[ R + \frac{\mu^4}{R} + \dots \right]$$

*looks* like a “standard”  $f(R)$  theory — but appearances are deceptive; here we write it in terms of an action, but only because it keeps us honest in spacetime symmetries

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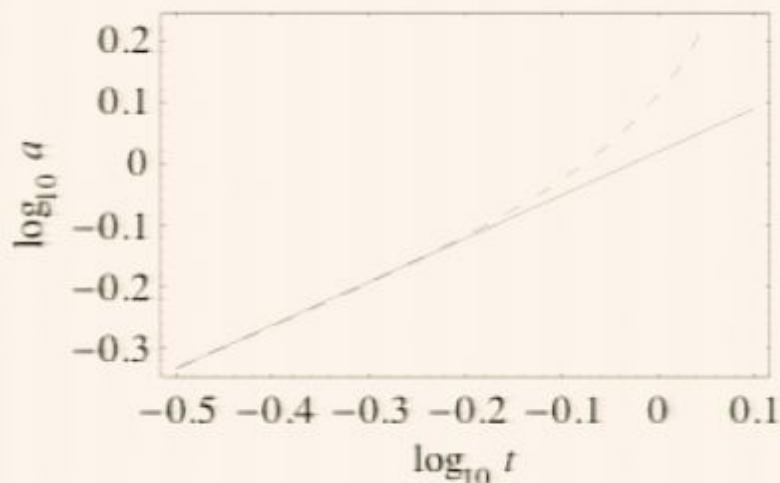
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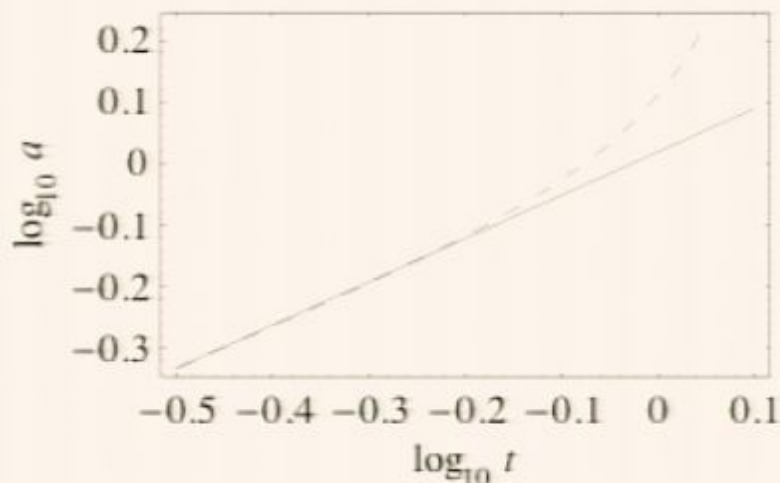

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# Perturbative Validity?

control parameter for knowing when perturbative approximations break down :  $R^2 \gg \mu^4$ , or

$$H^4 \gg \frac{\mu^4}{36(1-q)^2}$$

*i.e.* — our solutions are valid even for “large” accelerations; (comes from the fact that  $H$  is not a scalar but rather gauge-dependent — Minkowski space can have large  $H$ !)




# This Kind of Modified Gravity . . .

- has “interesting” behavior (*e.g.*, can produce reliable cosmic acceleration.)
- may imply a “bizarre” fundamental theory but will respect many important *empirical* features such as general covariance, gauge independence, &c.
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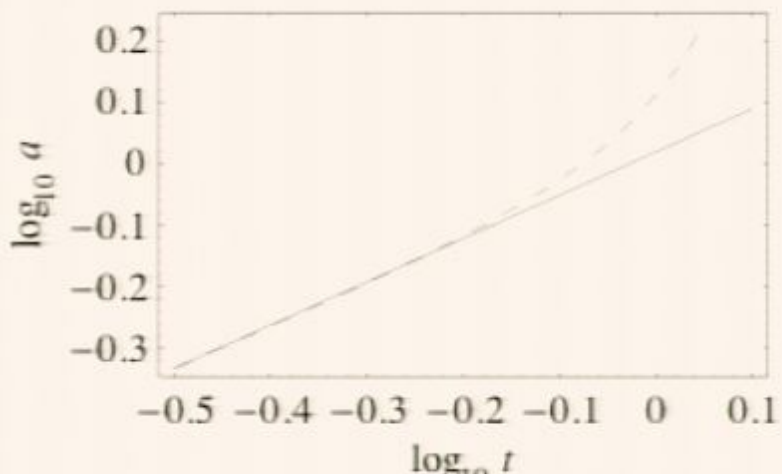

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# Constraining Modified Gravity *simpliciter*

$$\int \sqrt{-g} [R + f(R, G_B) + \dots]$$

(GB is the “Gauss-Bonnet” term — integral related to spacetime topology)

if  $f$  is an polynomial without a constant term,

$$w \leq -2$$

...ruled out by WMAP + Supernovae!



# Taking the theory further

- homogenous expansion already a way to rule out entire classes of modified gravities quickly and powerfully.

$$\int \sqrt{-g} [R + f(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) + \dots]$$

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# The Future of Modified Gravity

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- the simplicity of the mathematics means that large classes can be ruled out quickly; homogenous expansion + structure formation constraints may well rule out such a massive class that we can answer (to a string theorist's delight)



# Open Questions

- what fundamental theories lie beneath our perturbative approximations? Non-local theories are one obvious place to look, but the relationship is non-trivial because of our UV/IR exchange.

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- Niayesh & Ghazal's work on Cuscutons & *Gravitational Aethers* — are their d.o.f.'s “frozen in the Cooney-DeDeo-Psaltis way”?

*logically (but not physically?) independent ways to “freeze” apparent degrees of freedom:*

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# Example II.

## The Case of the Covariant Aether

(work in collaboration with Chanda Prescod-Weinstein @ P.I. & U. Waterloo)

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