

Title: Local quantum physics versus (relativistic) quantum mechanics: thermal- versus information theoretic- entanglement and the origin of the area law for "localization entropy".

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Abstract: The fundamentally different localization concepts of QT, i.e. the Born-(Newton-Wigner) localization of (relativistic) QM as compared with the causal localization (modular localization) of QFT, lead to significant differences in the nature of local observables and affiliated states. This in turn results in a rather sharp distinction between a tensor-factorization and information-theoretic entanglement in QM on the one hand, and a more radical "thermal entanglement" responsible for an area law for localization entropy. These surprising differences can be traced back to the very different nature of the localized operator algebras in QFT: they are all isomorphic (independent of the localization region) to one abstract "monad" (borrowing terminology from Leibniz) and the full reality of QFT (including its symmetries) is contained in the positioning of a finite rather small number (2 for chiral theories, 6 for $d=1+3, \dots$) within a joint Hilbert space. It is an important open question to what extent such positional characterizations (where the individual monads are void of any physical properties which reside fully in their relative placements) can be generalized to CST or QG.

Local Quantum Physics versus (relativistic) QM

(ref. B. Schroer, The interface between QM, QFT and QG, arXiv:0711.4600)

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Aim

- ① highlight structures of QFT (LQP) which are relevant (and have been ignored) in work on foundations of QT (e.g. thermal against information-theoretic entanglement). Facts which defy present foundational work on QT (apart from papers which Rob Clifton et al. wrote shortly before he died).
- ② mention some successes of the LQP approach in particle physics: area law for loc-entropy, rigorous construction of certain strictly ren. models (as special application of QFT encoded in positioning of finite number of "monads").

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- Fundamental differences in entanglement between QM/QFT
(information-theoretic versus thermal)

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 - The setting of QFT in terms of positioning of a finite number of "monads" in a joint H-space.
- **Conclusions, achievements**

Entanglement in QM

- QM(Fockspace notation)

$$H = H(\mathcal{O}) \otimes H(\mathcal{O}'), \quad H(\mathcal{O}) = P_{\text{Born}}(\mathcal{O})H, \quad \vec{x}_{op} = \int \vec{x} dP_{\text{Born}}(x)$$

$$B(H) = B(\mathcal{O}) \otimes B(\mathcal{O}'), \quad \mathcal{O} \subset \mathbb{R}^3, \quad B(\mathcal{O}') = B(\mathcal{O})'$$

$$B(\mathcal{O}) \equiv B(H(\mathcal{O}))$$

where $B(\mathcal{O}) \simeq B(\mathcal{O}) \otimes 1 \subset B(H)$ is Born-localized subalgebra and $B(\mathcal{O}') = B(\mathcal{O})'$ its commutant is Born-localized in the complement \mathcal{O}' .

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Convenient switch of terminology: state as positive lin. functional ω on an operator algebra \mathcal{A} . In this setting only intrinsic differences remain i.e. pure versus mixed (vector versus density matrix represents pure versus density matrix only if algebra is $B(H)$). States \rightarrow vectors in H via GNS construction. Behind lack of tensor factorization

Fact: states on $\mathcal{A}(\mathcal{O})$ are neither pure nor density operator ρ (help: (singular) KMS-like states), return to Gibbs density state is by creating a "attenuation-halo" (splitting \mathcal{O} and \mathcal{O}' by $\varepsilon =$ attenuation distance for vac. pol.). $\mathcal{O} \subset \mathcal{O}_{+\varepsilon} \curvearrowright \mathcal{O} \subset \mathcal{M}_{can} \subset \mathcal{O}_{+\varepsilon}$, \mathcal{M}_{can} is q.m. algebra (type I), vac. restr. to \mathcal{M}_{can} the is Gibbs state with $S_{entr}(\varepsilon) = \text{transv. area of } \text{Hor}(\mathcal{O}) \times s(\varepsilon)$

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- Factorizing and entangled vector states:

$$\Psi = \varphi \otimes \chi$$

entangled vector states and density matrix states

$$\Psi = \sum c_i \varphi_i \otimes \chi_i$$

$$\langle Op \rangle_\rho = \text{tr} \rho Op, \quad \rho > 0, \quad \text{tr} \rho = 1$$

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Phase Space Descriptions of 4d Simplicial Geometries

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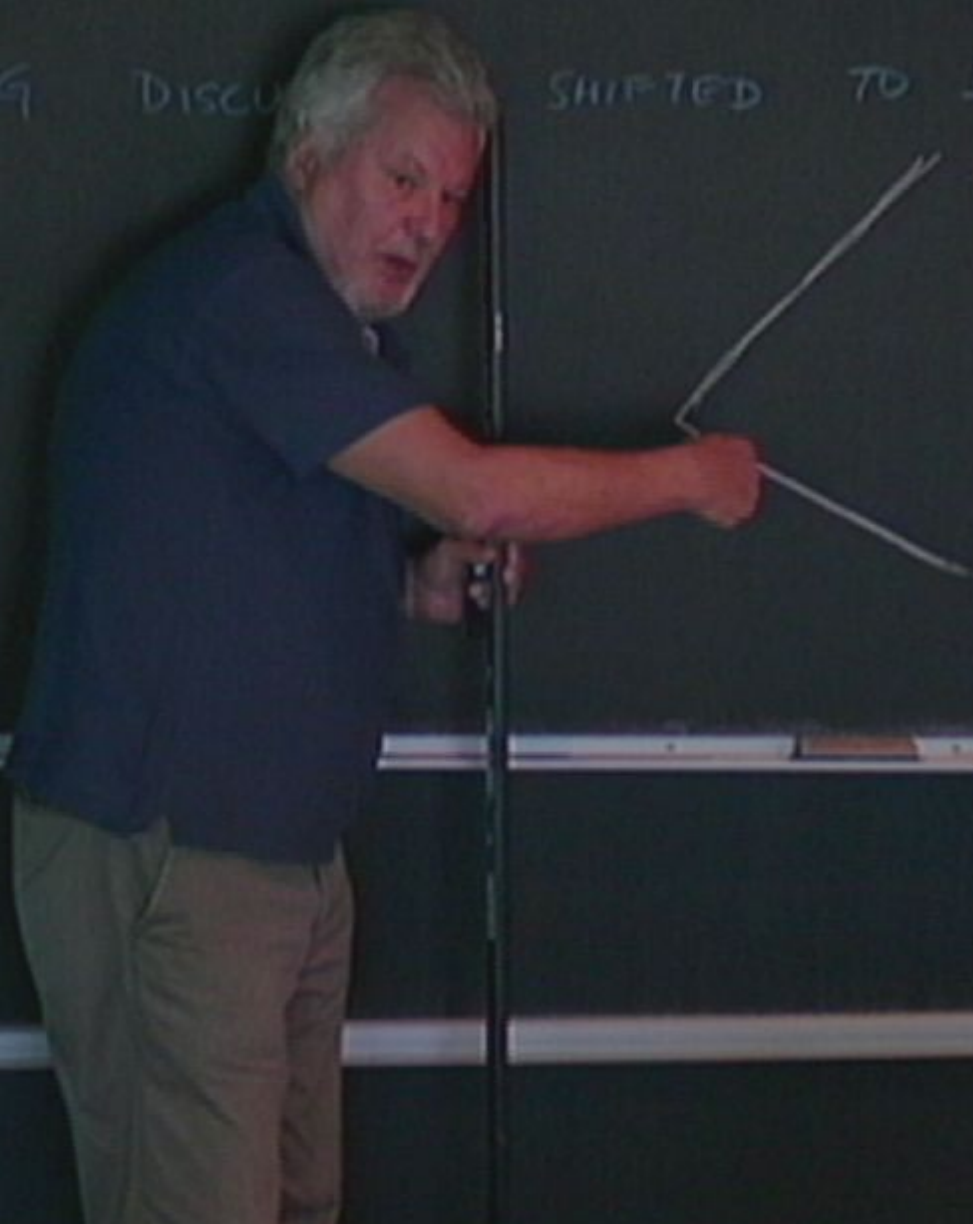
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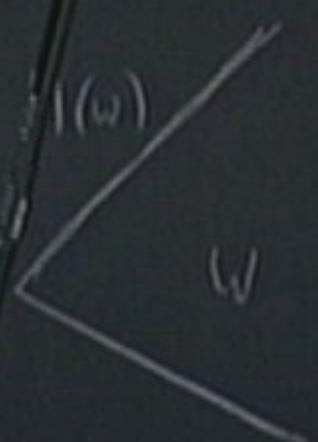
Phase Space Descriptions of 4d Simplicial Geometries

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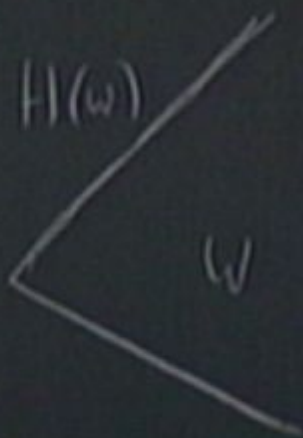
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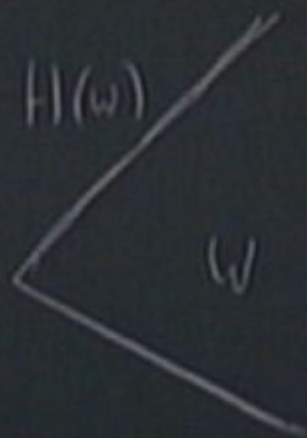
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Local QFT algebras $\mathcal{A}(\mathcal{O})$ are radically different from algebras in QM, they are copies of a "monad" with preempted thermal aspect: the vacuum state (and all other global finite energy states) restricted to $\mathcal{A}(\mathcal{O})$ becomes thermal (KMS with respect to a (modular) "Hamiltonian" K which depends on \mathcal{O})

Localization-caused entanglement in QFT is not information-theoretical but rather thermal (KMS not describable by density operator). KMS $\omega(AB) = \omega(Be^{-K}A)$, $\text{spec}(K)$ cont., density matrix iff $\text{tr}e^{-K} < \infty$

All (finite energy) states of particle physics behave similar to the vacuum i.e. KMS where the "modular Hamiltonian" K depends on $\mathcal{A}(\mathcal{O})$ and the chosen state. Example: for $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$, $\mathcal{O} = W$ the modular Hamiltonian is $K = K_{boost}(W)$

In general (in particular for compact regions) $K(\mathcal{O})$ acts "fuzzy" i.e. not as a diffeomorphism in \mathcal{O} . These thermodynamic equilibrium states are limits of Gibbs density matrix states. Later: this parallelism between localization thermality and heat bath thermality is much more than an analogy: with the help of LF holography it becomes an isomorphism.

$\mathcal{A}(\mathcal{O})$ unique hyperfinite Type III₁ factor operator algebra (studied by Connes) shorter name: *monad*

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Existence of DPI theory (Coester, Coester-Polyzou) shows: fundamental conceptual differences between QM/QFT are not explained in terms of P-representations, QFT requires causal propagation.

- Relativistic interaction in a Wigner two-particle space

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Eigenfunctions of \vec{x}_{op}^{NW} not delta functions but expon. fall-off with Compton radius.

Theorem (Malament): No covariant system of loc. projectors $P(\mathcal{O})$ in pos. energy rpts. (in fact no loc. $P(\mathcal{O})$ which annihilate Ω_{vac}).

Good news: relation between large timelike separated events is covariant (origin of *physical* energy-momenta p). essential for L-invariance of S_{scat} . but its illegitimate use for finite distances feighns superluminal propagation.

The localization carried by cov. fields is for all distances consistent with a maximal velocity and the relevant causal localization does not lead to projectors and probabilities (see modular localization) but becomes part of modular theory (discovered by Tomita-Takesaki on the math. side and by Haag Hugenholtz and Winnink on the physical side around the middle of the 60s)

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- N particles: inductive clustering in appropriate reference frames. For N=3

$$M = M_0 + V_{12} + V_{13} + V_{23} (+V_{123})$$

$$V_{12} = M(12, 3) - M_0(12, 3), \quad V_{13} = \dots$$

$$M(12, 3) = \sqrt{\vec{p}_{12,3}^2 + M_{12}^2} + \sqrt{\vec{p}_{12,3}^2 + m^2}$$

For $N > 3$ use *scattering equivalences*, $(N-1)$ -body forces \rightarrow N -body M
Weinberg's conjecture: relativistic S +cluster factorization \leadsto QFT is
disproved by DPI construction.

S -matrix from DPI fulfills all requirements which can be formulated in
terms of particles as: unitarity, P -invariance, macrocausality (cluster
factorization, timelike corrections from "causal rescattering").

missing "crossing" property: all n -particle formfactors from one
masterfunction

DPI setting can be generalized to coupling of creation channels (finite
number of π 's produced in nucleon collisions), see work by
Coester-Polyzou (nuclear phenomenology within principles of particle
physics).

DPI setting is a quantum mechanical construction of relativistic S -matrix;
representation of \mathcal{P} , but no off-shell covariant objects. Fulfillment of old
(pre crossing) S -matrix dreams (Heisenberg/Stueckelberg).

Irony: in his pursuit of macrocausality Stueckelberg discovered
"accidentally" Feynman rules by using the asymptotic (macro-causal)
one-particle structure for finite distances (together with pointlike
interaction vertices).

Spatial modular localization

In QFT the intrinsic objects are the local algebras whereas the fields are local coordinatizations

$$\mathcal{A} = \{ \mathcal{A}(\mathcal{O}) \}_{\mathcal{O} \subset \mathbb{R}^4}, \Phi(f)_{\text{supp} f \subset \mathcal{O}} \text{ gen. } \mathcal{A}(\mathcal{O})$$

infinitely many field objects Φ generate same $\mathcal{A}(\mathcal{O})$. Liberate localization from the infinitely many field coordinatization: modular localization directly related to algebras. Simplest road: Wigner representation theory (m,s=0)

$$H_{Wig} = \left\{ \psi(p) \mid \int |\psi(p)|^2 d\mu(p) < \infty \right\}$$

$$(u(\Lambda, a)\psi)(p) = e^{ipa} \psi(\Lambda^{-1}p),$$

$$(u(j_{W_0})\psi)(p) = \overline{\psi(-j_{W_0}p)}$$

two commuting operators in H_{Wig} associated to the $t-x$ wedge

$W_0 = \{x \mid x_1 > |x_0|\}$: the wedge-preserving boost $u(\Lambda_{W_0}(\chi))$ commutes

with the antiunitary reflection $j_{W_0} = u(j_{W_0})$ along edge of wedge.

Define unbounded closed antilinear involutive operator in H_{Wig} ("Tomita S-operator")

$$\mathfrak{s}(W_0) := j_{W_0} \delta_{W_0}^{\frac{1}{2}}, \quad \delta_{W_0}^{it} := u_{W_0}(\chi = -2\pi t), \quad \mathfrak{s}^2(W_0) \subset \mathbf{1}$$

$$(\mathfrak{s}(W_0)\psi)(p) = \overline{\psi(-p)}, \quad \text{dom } \mathfrak{s}(W_0) = \text{dom } \delta_{W_0}^{\frac{1}{2}}$$

more detailed characterization of the $\text{dom } \mathfrak{s}(W_0)$ in terms of real subspaces of H_{Wig}

$$\mathfrak{K}(W_0) = \{\psi \mid \mathfrak{s}(W_0)\psi = \psi\}, \quad \mathfrak{s}(W_0)i\psi = -i\psi$$

$$\text{dom } \mathfrak{s}(W_0) = \mathfrak{K}(W_0) + i\mathfrak{K}(W_0)$$

$$\mathfrak{s}(W_0)(+i\varphi) = -i\varphi$$

In order to construct an abstract modular setting need "standard"

$\mathfrak{K} : \overline{\mathfrak{K}} + i\mathfrak{K} = H, \quad \mathfrak{K} \cap i\mathfrak{K} = 0$. Vice versa a standard $\mathfrak{K} \Leftrightarrow \mathfrak{s}$

$$\mathfrak{K}(\mathcal{O}) = \bigcap_{W \supset \mathcal{O}} \mathfrak{K}(W)$$

$$\mathfrak{K}(\mathcal{O}') = \mathfrak{K}(\mathcal{O})' \text{ sympl.compl., Haag duality}$$

4. From spatial to algebraic modular localization

functorial ascend to the net o.a. $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O} \subset \mathbb{R}^4}$ acting in H_{WF} , $H_{Wig} = P_1 H_{WF}$

$$\text{Weyl}(\psi) = \exp i(a(\psi) + a^*(\psi)), \quad \psi \in \mathfrak{K}(W)$$

$$\mathcal{A}(W) := \text{alg} \{ \text{Weyl}(\psi) \mid \psi \in \mathfrak{K}(W) \}, \quad \mathcal{A} := \cup_{\mathcal{O}} \mathcal{A}(W)$$

$$K = \overline{\{(A + A^*) \Omega \mid A \in \mathcal{A}(W)\}} \subset H_{W-F}, \quad \mathfrak{K}(W) = P_1 K(W)$$

$$\Delta^{it}, J \text{ from } \delta^{it}, j \rightarrow \text{Ad} \Delta^{it} \mathcal{A}(W) = \mathcal{A}(W), \quad J \mathcal{A}(W) J = \mathcal{A}(W')$$

$\mathfrak{K}(\mathcal{O}) \rightarrow \mathcal{A}(\mathcal{O})$ unique, indep. of multitude of generating fields.

$$\begin{array}{ccc} \{\mathfrak{K}(W)\}_{W \subset \mathcal{W}} & \longrightarrow & \{\mathcal{A}(W)\}_{W \subset \mathcal{W}} \\ \downarrow \cap & & \downarrow \cap \\ \mathfrak{K}(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array}$$

algebraic modular theory: $S_{Tom} A \Omega = A^* \Omega$, $A \in \mathcal{A}$, $S_{Tom} = J \Delta^{1/2}$

Mathematical interlude on abstract modular theory

Let \mathcal{A} be a (v. Neumann) operator algebra in standard position with respect to Ω

(i.e. Ω is cyclic and separating)

Theorem (Tomita-Takesaki): the polar decomposition of the Tomita S -operator

$$S_{Tom}A\Omega = A^*\Omega, \quad A \in \mathcal{A}$$

$$S = J\Delta^{\frac{1}{2}}$$

leads to a modular automorphism of \mathcal{A} as well as a antiunitary morphism with its commutant

$$\Delta^{it}\mathcal{A}\Delta^{-it} = \mathcal{A}$$

$$J\mathcal{A}J = \mathcal{A}'$$

In QFT $\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$ i.e. the commensurable observables have known localization

Modifications from interactions

The particle-field relation of the commuting square breaks down; no vacuum-polarization-free generators (PFG) for compact localized $\mathcal{A}(\mathcal{O})$.

Def. let $A \in \mathcal{A}(\mathcal{O})$ (affiliated), then A is PFG if $A\Omega = \Psi_{1p}$.

Theorem: a model with \mathcal{O} -localized PFG is a free field model, interactions have infinite v.p. "cloud"

This change is related to change of modular operators

$$S(W) = J\Delta^{\frac{1}{2}}, J = J_0 S_{scat}$$

The "smallest" localization for which PFGs exist in presence of interactions is the wedge: with $\mathcal{A}(\mathcal{O})$ for $\mathcal{O} \subset W$ only "field states", in $\mathcal{A}(W)$ also operators generate particle states: i.e. W is the best compromise between particles and fields in terms of localization.

Def. tempered PFG A as above, but in addition "good" domain properties

Theorem: tempered PFGs exist only in $d=1+1$ and lead to the factorizing models (BBS). Modular theory together with a related property "modular nuclearity" has led to an existence proof of many factorizing models

(Sinh-Gordon,....).

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Lightfront Holography, entropic area law

LFH is a rather radical change of spacetime ordering of an algebraic substrate. Start from net of wedge algebras in bulk with upper horizon on same LF. Obtain localization structure on LF by forming relative commutants of such wedge algebras.

Surprising structural property: the transverse part of the LF theory is QM with a tensor factorizing vacuum, all vacuum polarization is compressed into lightlike direction. The LF theory is an extended chiral theory i.e. its generating fields $A_{LF}(x_+, \mathbf{x}_{tr})$ are of the form

$$[A_{LF}(x_+, \mathbf{x}_{tr}), B(x'_+, \mathbf{x}'_{tr})] = \delta(\mathbf{x}_{tr} - \mathbf{x}'_{tr}) \sum_{i \text{ odd}} C_i(x_+, \mathbf{x}_{tr}) \delta^{(i)}(x_+ - x'_+)$$

where the derivative chiral delta function are typical for the presence of vacuum fluctuation

LF holography reduces degrees of freedom, LF \rightarrow bulk only with additional informations (e.g. action of additional P-transformations to the 7-parametric LF invariance subgroup). Hol. rig. structure in QFT, different from 't Hooft's idea in gravity context.

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Localization entropy from LF holography

No transverse localization-entropy since Ω tensor factorizes; \curvearrowright area proportionality on structural grounds (as TCP, spin&statistics); remaining problem calculate localization entropy in chiral models.

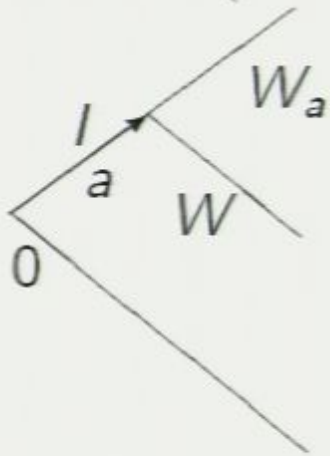
The *inverse chiral Unruh effect* relates the KMS state (which results from reducing the global chiral vacuum state to an interval) to a global heat bath thermal state at a fixed temperature (proportional to ϵ and S). Here the "volume" factor L in *chiral localization entropy* of a region in the light-ray direction this means $h.b.entr. = V \times s$ passes

$$\begin{aligned} \mathcal{A}(\mathcal{D}) &= \mathcal{A}(I \times \mathbb{R}^2) \cap \\ &\text{tilting done with } E(2) \\ \underline{loc.entr.} &= A |\ln \epsilon| \times s \end{aligned}$$

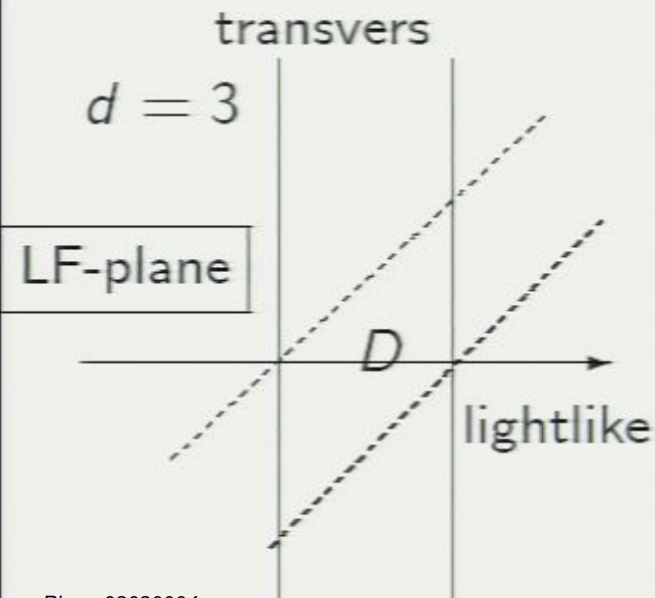
The remaining task to compute c is achieved by the approximation of the translative by the rotational conformal Hamiltonian ("relativistic box"). A more detailed investigation reveals that the split parameter ϵ is really the conformally invariant cross ratio between 4 points which characterizes the split between an interval and its complement i.e. $\underline{loc.entr.} = f(\underline{crossratio})$.

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tilting done with $E(2) \subset \mathcal{L} \subset \mathcal{P}$

result: local net structure of $\mathcal{A}(LF)$, $G(7) \subset \mathcal{P}$

transverse tensor factorization as in QM

$$\mathcal{A}((\mathcal{O}_1 \cup \mathcal{O}_2) \times I) = \mathcal{A}(\mathcal{O}_1 \times I) \otimes \mathcal{A}(\mathcal{O}_2 \times I)$$

even if the transverse regions touch,

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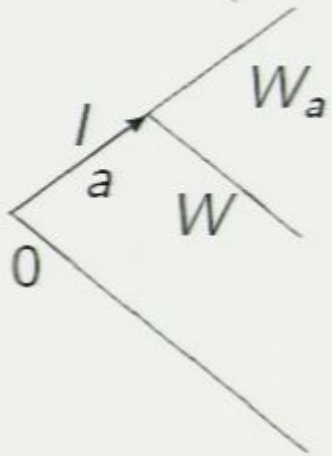
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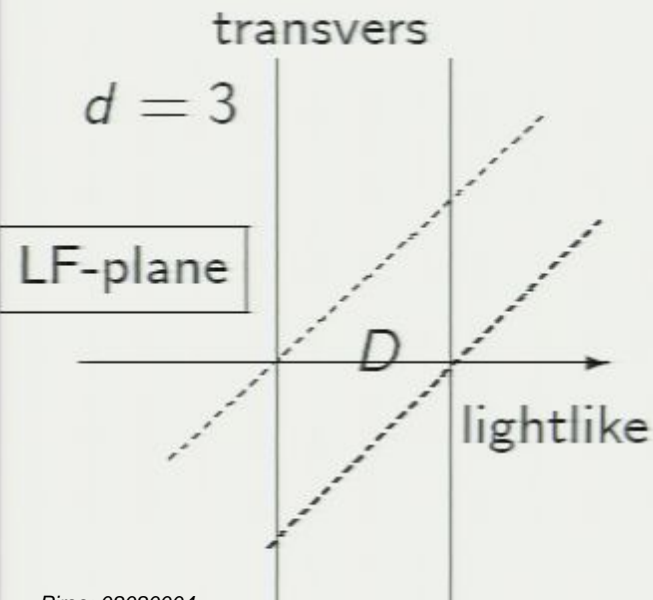
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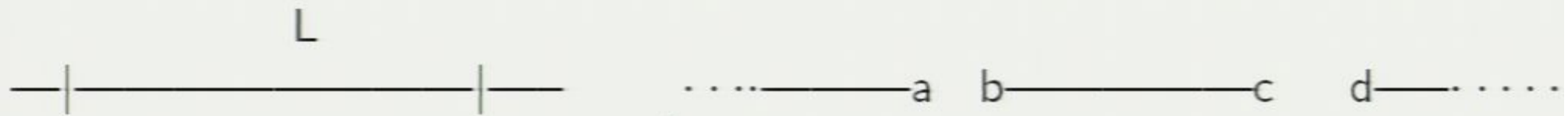
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inverse Unruh effect (WS)



$$S_{hb}(\beta) = \lim_{L \rightarrow \infty} L \times s_{hb}(\beta) \xrightarrow{\beta=2\pi} S_{loc} = \lim_{\varepsilon \rightarrow 0} |\ln \varepsilon| \times s_{loc}, \quad \varepsilon = e^{-L}, \quad s_{loc} = S_{hb}$$

using more refined techniques of chiral theories obtain:

$$\varepsilon = \frac{(d-a)(c-b)}{(b-a)(d-c)}, \quad s_{loc} = \frac{c_{Vir}}{6}, \text{ leading term of a canonically def. conformal entropy function}$$

$\curvearrowright S_{loc}(Hor) = \text{transverse Area} \times |\ln \varepsilon| \frac{\varepsilon}{6}$ dependence on hol.proj. matter

Important: ε not cutoff but rather attenuation length in fixed model, conf. equivalent to lightlike contribution to volume factor on lightfront. What is erroneously referred to cutoff is in reality the unavoidable attenuation factor whose conformal inverse becomes the length factor which together with the area makes up the volume factor. Both the thermal

$(\mathcal{A}_{glob}, \Omega_{KMS})$ and $(\mathcal{A}(\mathcal{O}), \Omega_{vac})$ represent the same monad and therefore localization and thermal aspects are two sides of the same coin. Supports

Jacobson's: thermodyn. \rightarrow G.R.

QFT from positioning of monads

Definition

$(\mathcal{N} \subset \mathcal{M}, \Omega)$ is \pm modular inclusion if $\Delta_{\mathcal{M}}^{it} \mathcal{N} \Delta_{\mathcal{M}}^{-it} \subset \mathcal{N} \quad t \gtrsim 0$

Monad (hyperfinite type III₁ factor algebra) is appropriate short name for subalgebras $\mathcal{A}(\mathcal{O}) \subset B(H)$, model and loc.-independent, no individuality (seeing one = knowing them all). Richness of LQP from positioning.

Two ways of characterising a QFT in terms of positioning of monads

1. given $\mathcal{A}(W) \subset B(H)$ and action of P on H . By P -transforming $\mathcal{A}(W)$ obtain net $\{\mathcal{A}(W)\}_{W \subset \mathcal{W}}$, the associated compact net $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O} \subset \mathbb{R}^4}$ is obtained by intersecting $\mathcal{A}(W)$'s.

Practical method: know system of generating operators of $\mathcal{A}(W)$ and the action of P (situation for $d=1+1$ factorizing models). Construction of the LQP would fail if intersections are trivial (multiple of $\mathbf{1}$). Other possibility ($d \geq 3$) some intersections nontrivial, but algebras of compact regions are trivial, obtain nonlocal QFT.

QFT via positioning of monads

2. Encode everything in terms of positioning of a finite number of monads. Chiral models encoded in positioning of 2 monads in form of "standard" modular inclusion (in addition $((\mathcal{N}' \cap \mathcal{M}, \Omega)$ is standard). For 2-dim QFT need 3 algebras, \curvearrowright pos.energy unitary $P(2)$, $\{\mathcal{A}(\mathcal{O})\}_{\mathcal{O} \subset \mathbb{R}^2}$, for 3-dim. need 4 monads, for 4-dim. need 7 monads.

The observable algebra of every QFT permits such a representation simple Illustration QFT(2): $\mathcal{A}(W), \mathcal{A}(W_1), \mathcal{A}(W_2)$ such that

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Theorem: under those conditions $\Delta^{i\tau}, \Delta^{is}, \Delta^{it}$ gen. pos. energy rep. of $P(2)$. Fig.

Simplest monad description for chiral observable algebras in terms of 2 algebras in "standard" modular inclusion position. For $d \geq 3$ need concept of "modular inclusion". Positioning for $d=3$ in terms of 4 monads, for $d=4$ in terms of 7 monads. Impossible to find a counterpart in QM. **Holistic**

Conclusions, Achievements

- 1. Contrary to popular opinion QM and QFT are quite different QTs. Saying QFT results from replacing Galilei-covariance by P -covariance is insufficient. To explain the difference in terms of localization is more appropriate.

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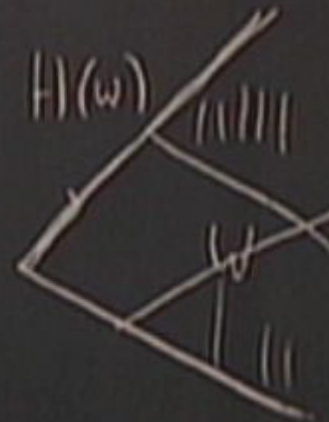
Phase Space Descriptions of 4d Simplicial Geometries

Q9. DIMENSION SHIFTED TO IInd floor TATIO



Phase Space Descriptions of 4d Simplicial Geometries

DISCUSSION SHIFTED TO IInd floor THTO



Phase Space Descriptions of 4d Simplicial Geometries

DISCUSSION SHIFTED TO IInd floor THT10

entropy $\left(\frac{(b-a)(d-c)}{(d-a)(c-b)} \right)$

