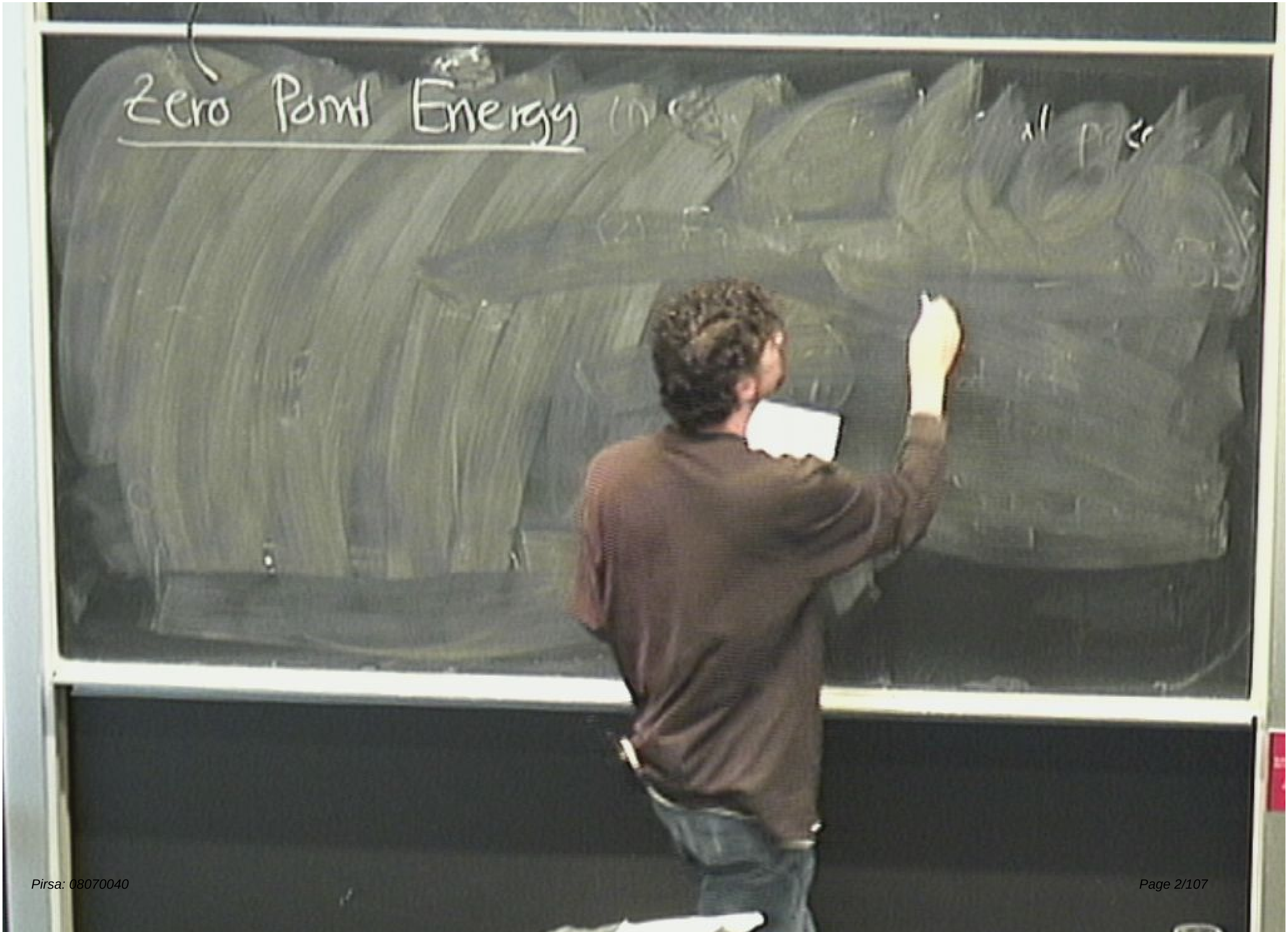
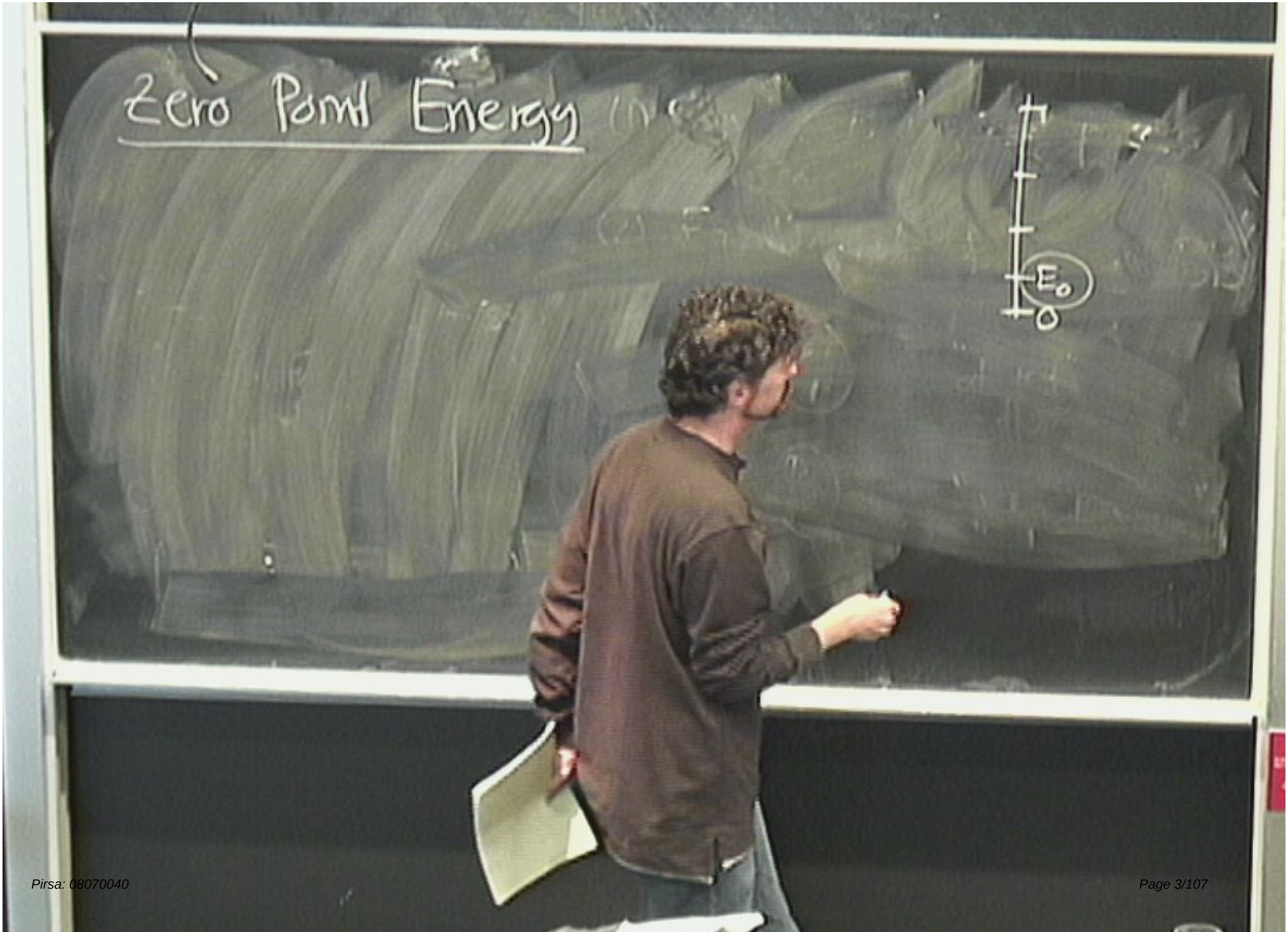
Title: Quantum 3

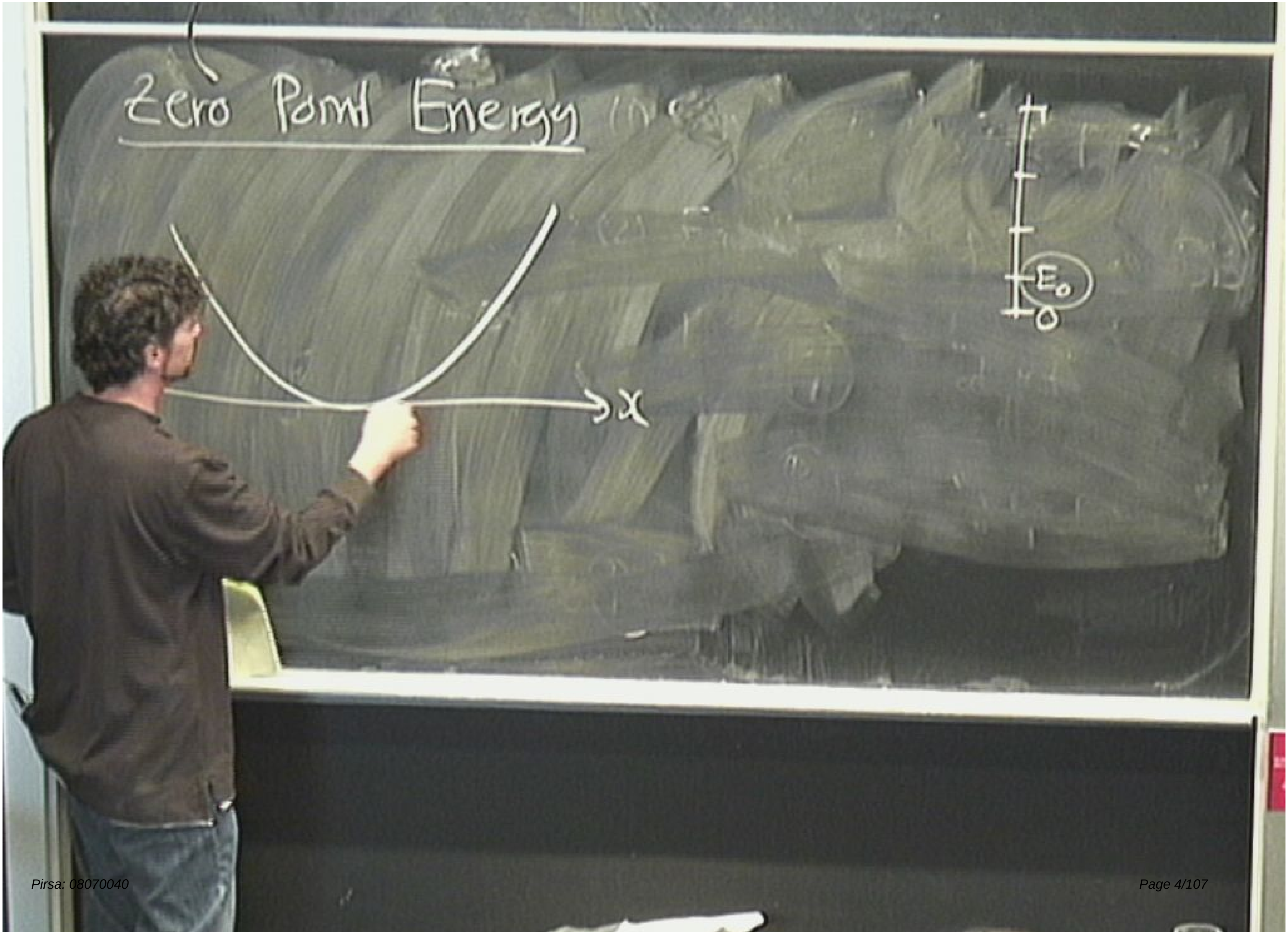
Date: Jul 26, 2008 10:30 AM

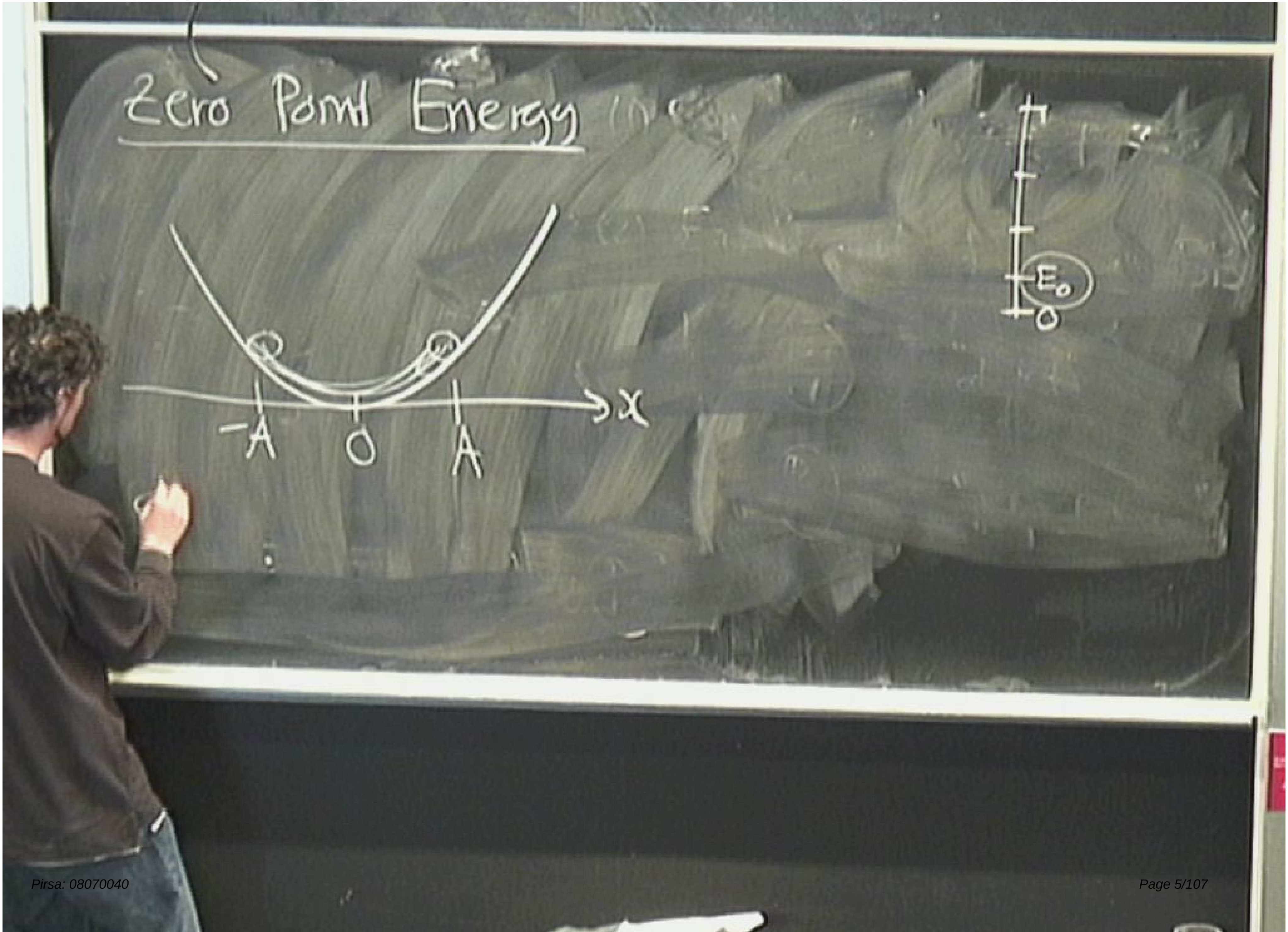
URL: http://pirsa.org/08070040

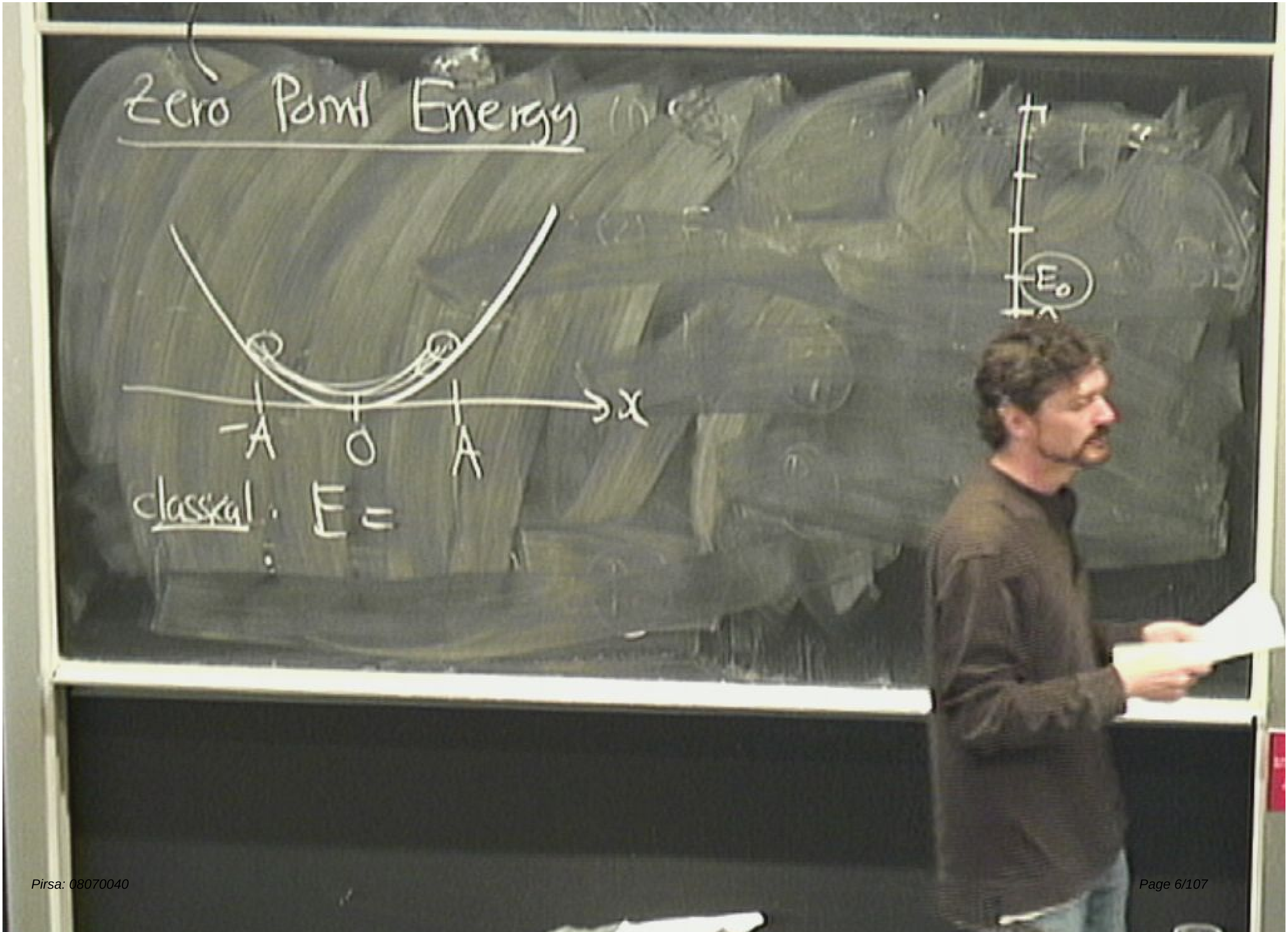
Abstract:

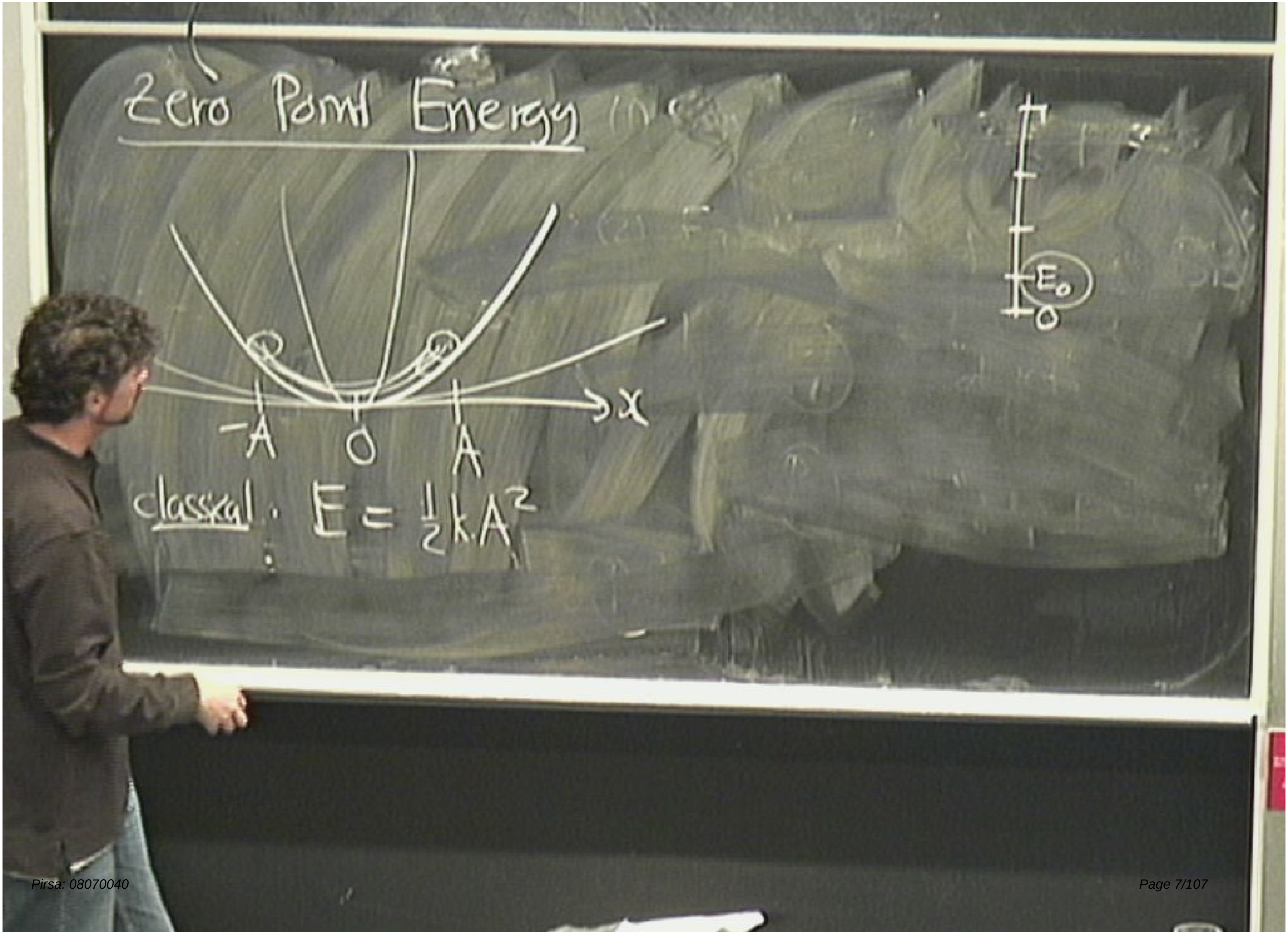

Zero Point Energy

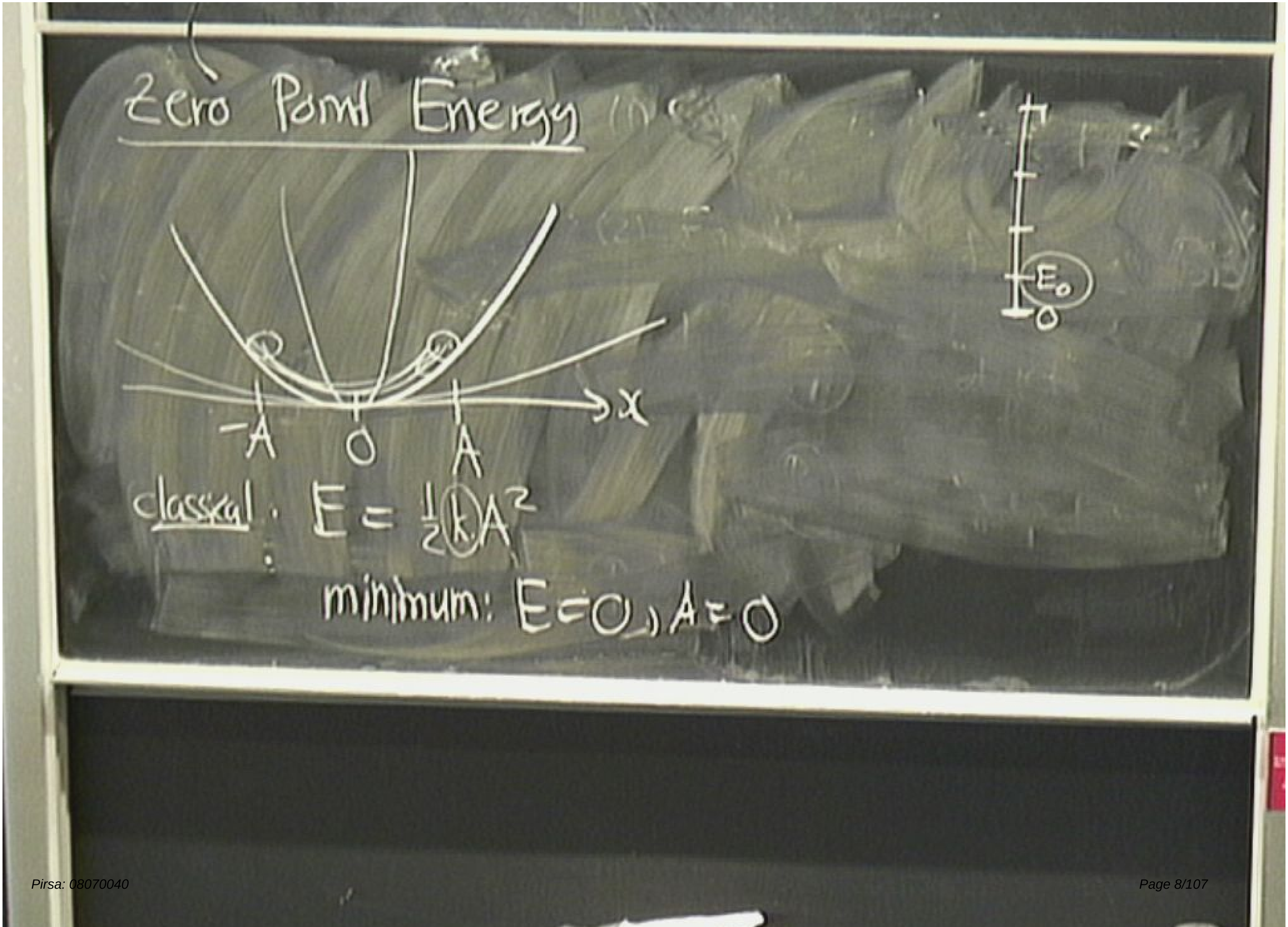

Zero Point Energy
E0
0


Zero Point Energy
x
E_0
0

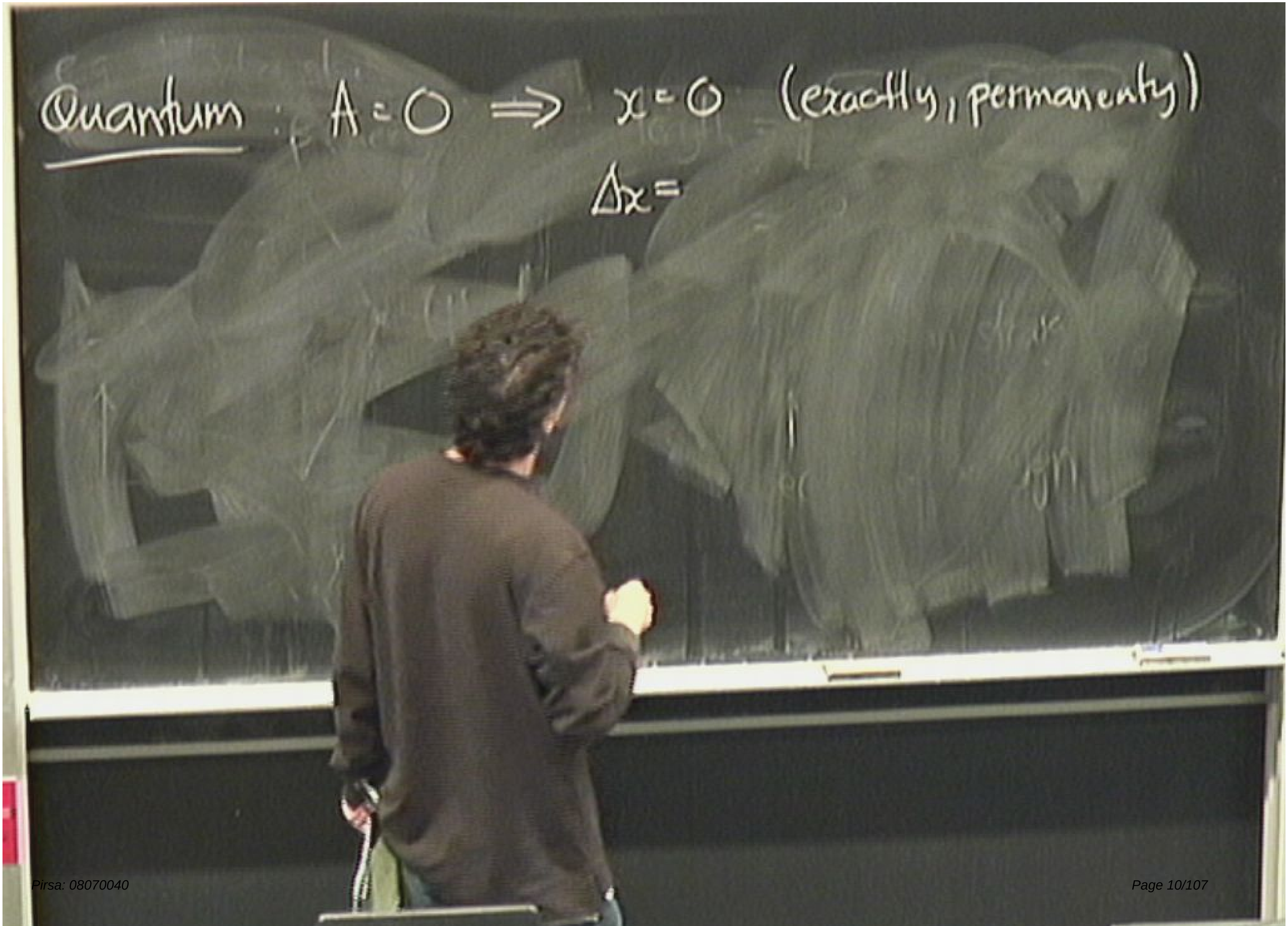

Zero Point Energy
x
-A
0
A
E₀
0

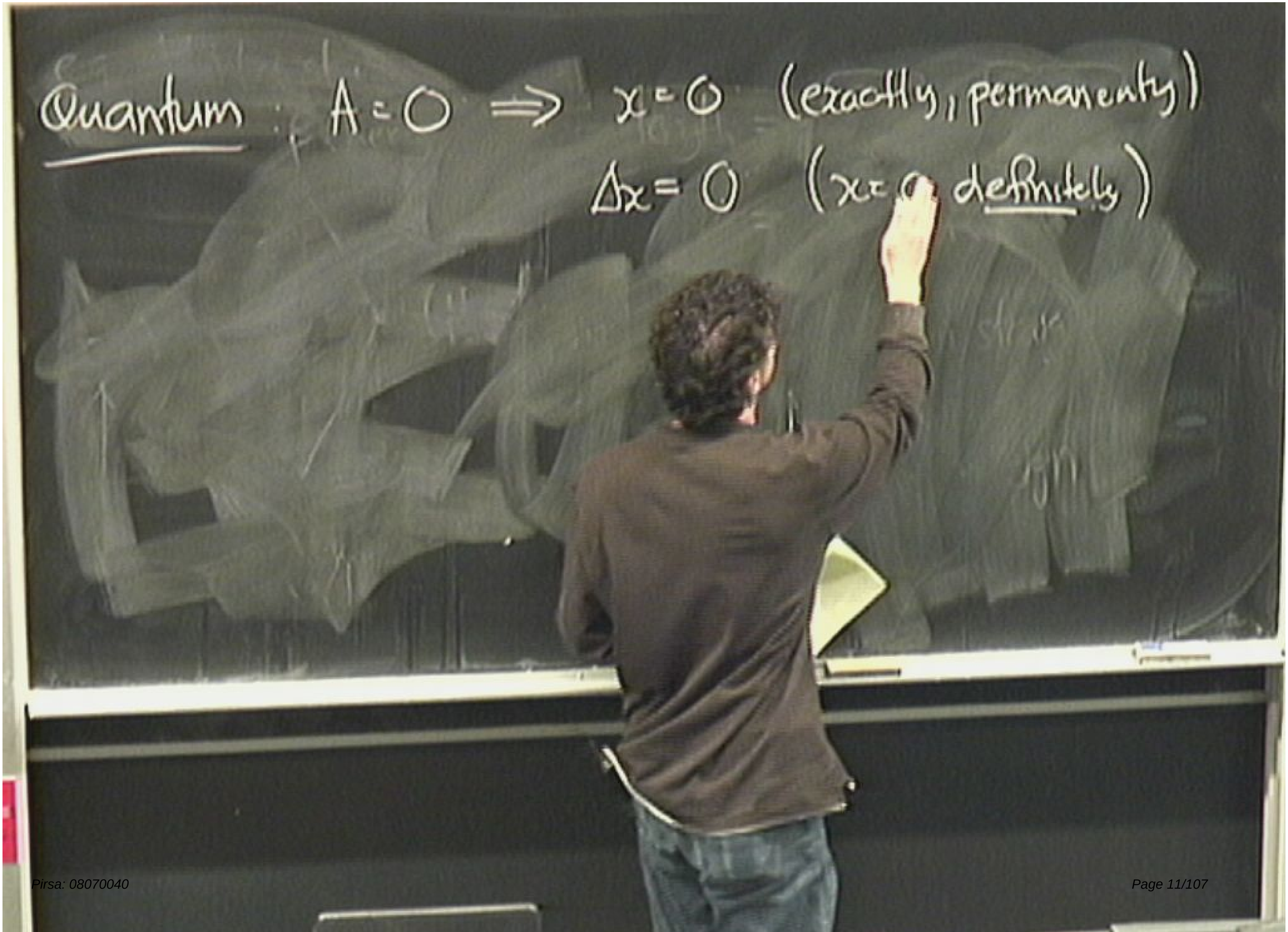

Zero Point Energy
x
-A
0
A
classical
E =
E0

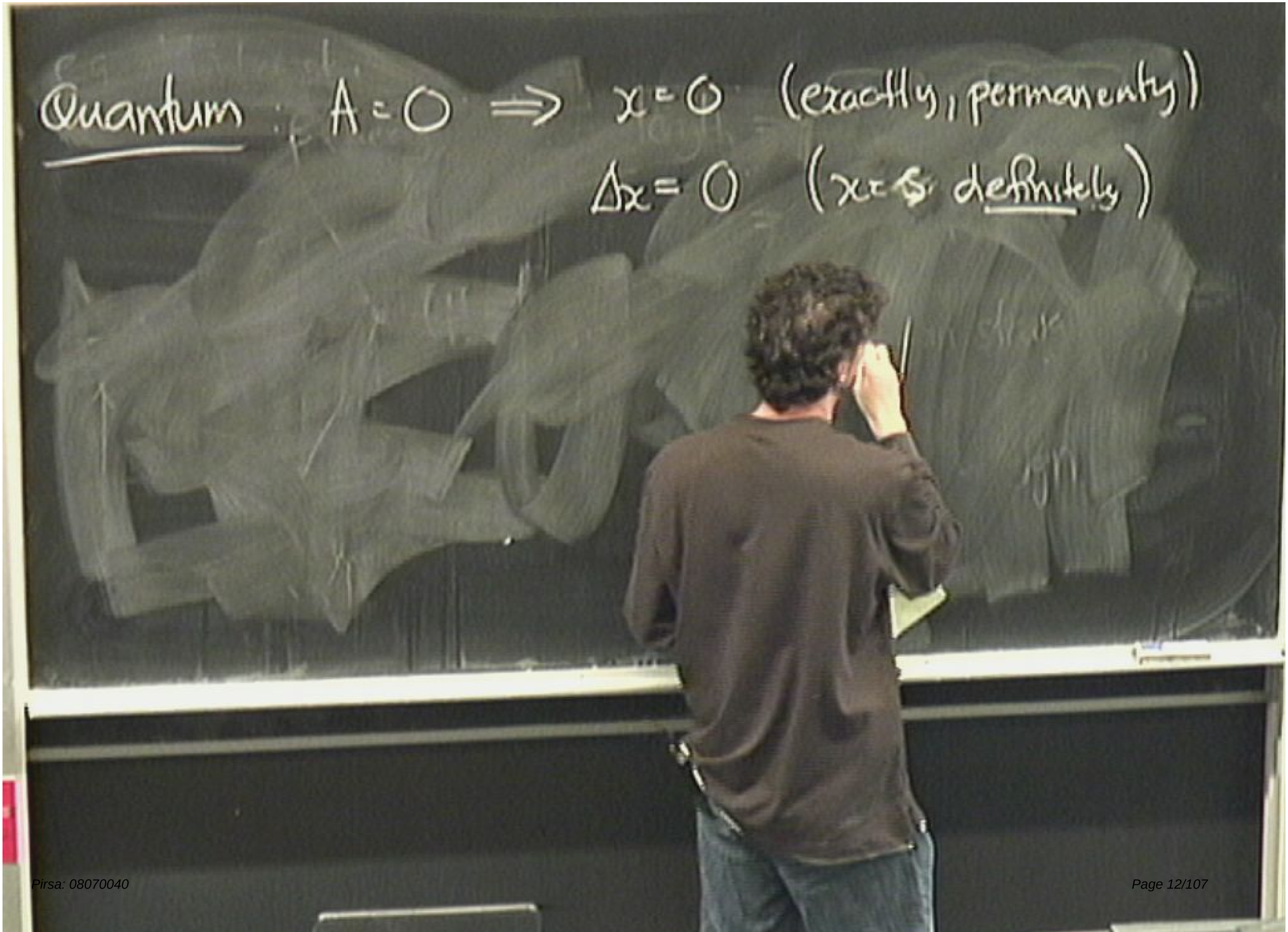

Zero Point Energy
x
-A
0
A
classical: $E = \frac{1}{2}kA^2$
E_0
0

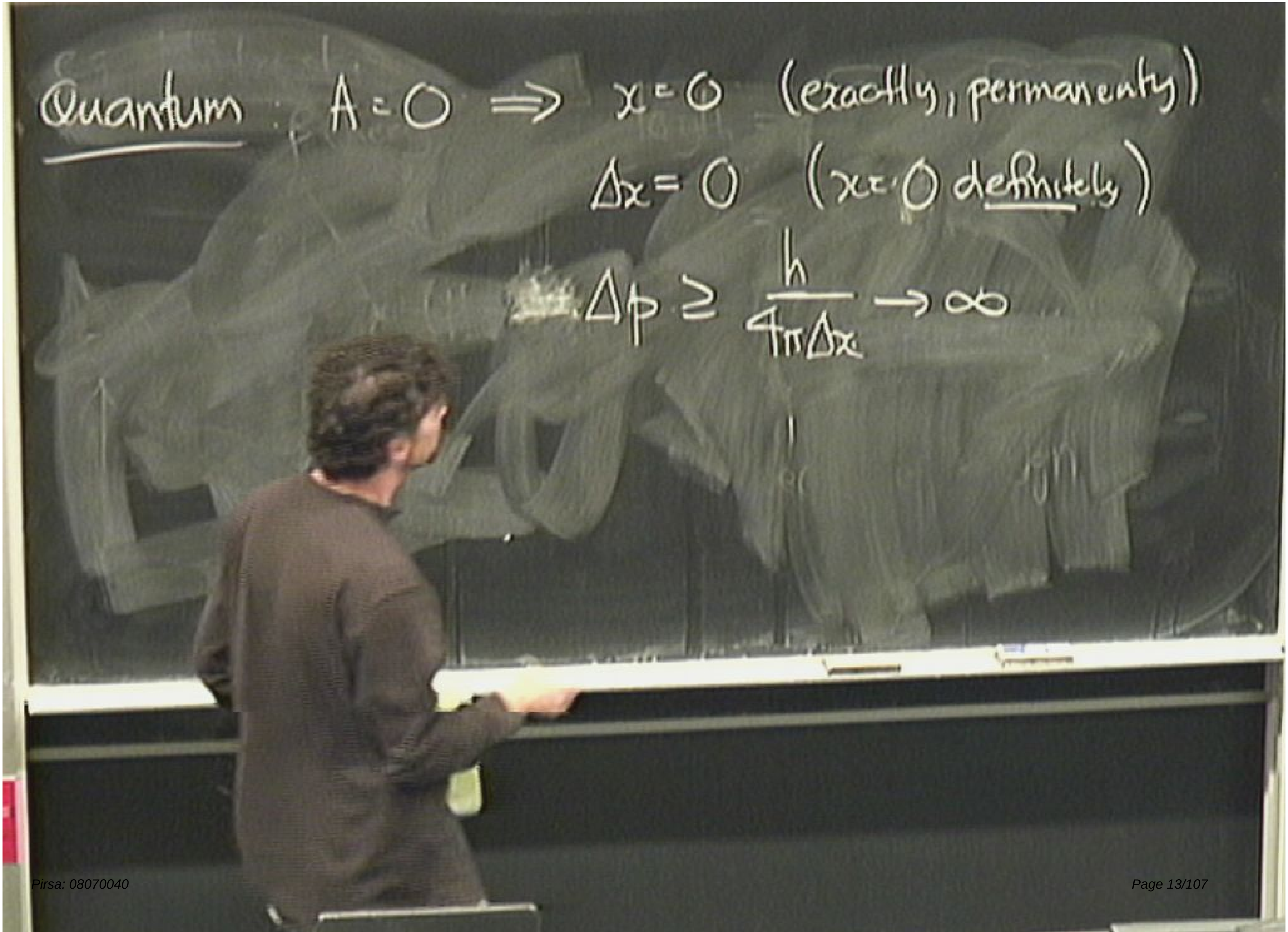

Zero Point Energy
x
-A
0
A
classical: $E = \frac{1}{2}kA^2$
minimum: $E=0$, $A=0$
E_0
0

<u>Quantum</u> : $A = 0 \Longrightarrow x = 0$ (exactly, permanently)


Quantum: $A=0 \Rightarrow x=0$ (exactly, permanently)
$\Delta x =$


Quantum : A = 0 ⟹ x = 0 (exactly, permanently)
Δx = 0 (x = 0 definitely)


Quantum : A = 0 ⟹ x = 0 (exactly, permanently)
Δx = 0 (x is definitely)


Quantum : $A = 0 \Rightarrow x = 0$ (exactly, permanently)
$\Delta x = 0$ ($x = 0$ definitely)
$\Delta p \geq \frac{h}{4\pi \Delta x} \rightarrow \infty$

Quantum: $A = 0 \implies x = 0$ (exactly, permanently)

$\implies \Delta x = 0$ ($x = 0$ definitely)

$\implies \Delta p \geq \dfrac{h}{4\pi\Delta x} \to \infty$

$\implies$ instantly "jump into motion"

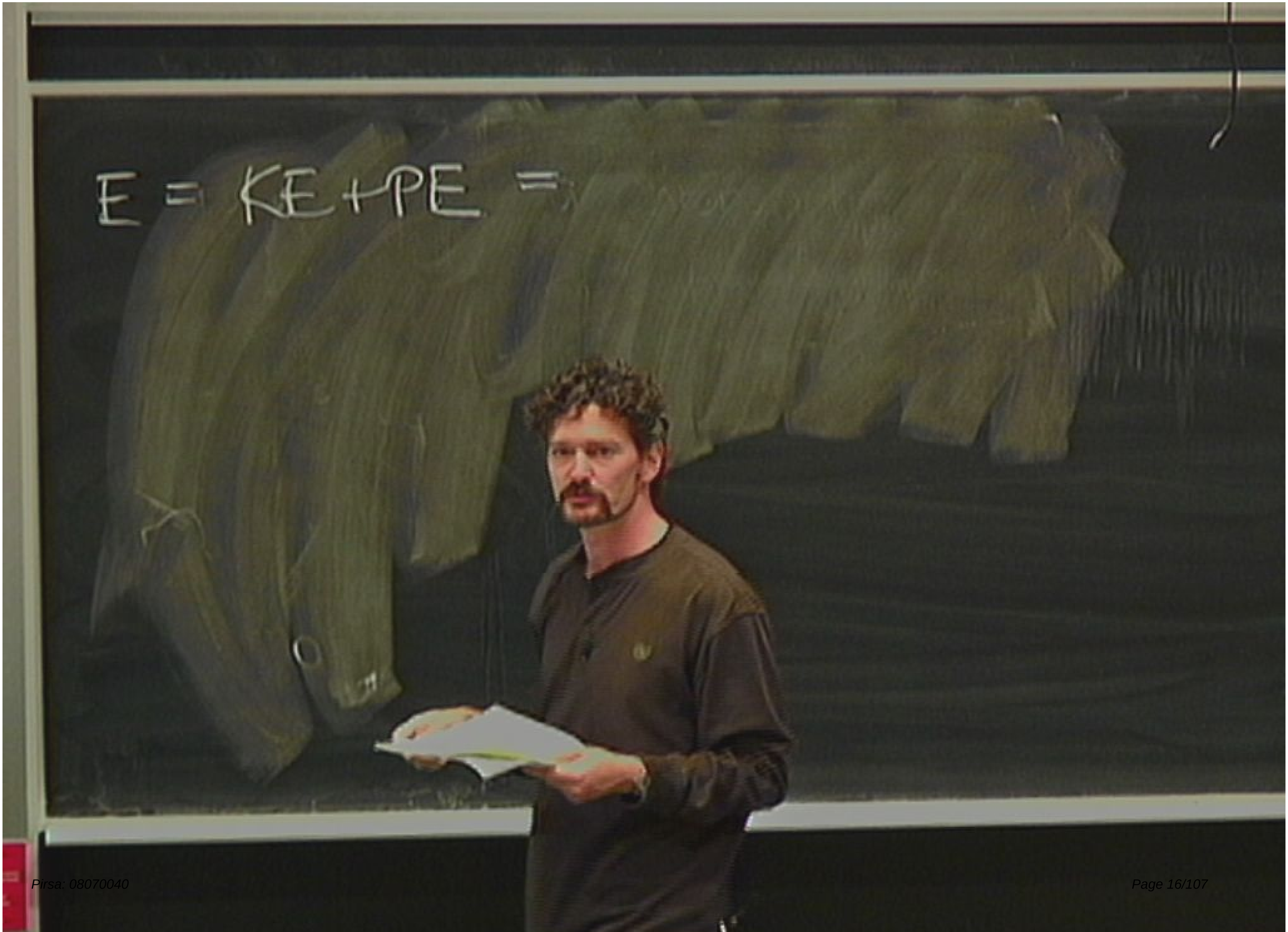
<u>Quantum</u>: $A = 0 \Rightarrow x = 0$ (exactly, permanently)

$\Rightarrow \Delta x = 0$ ($x = 0$ <u>definitely</u>)

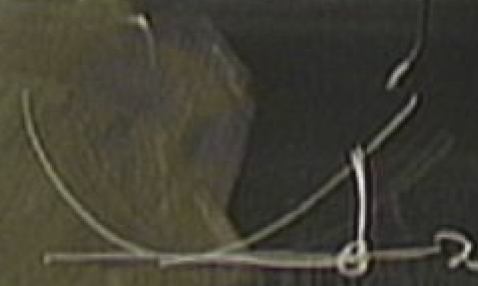
$\Rightarrow \Delta p \geq \dfrac{h}{4\pi \Delta x} \rightarrow \infty$

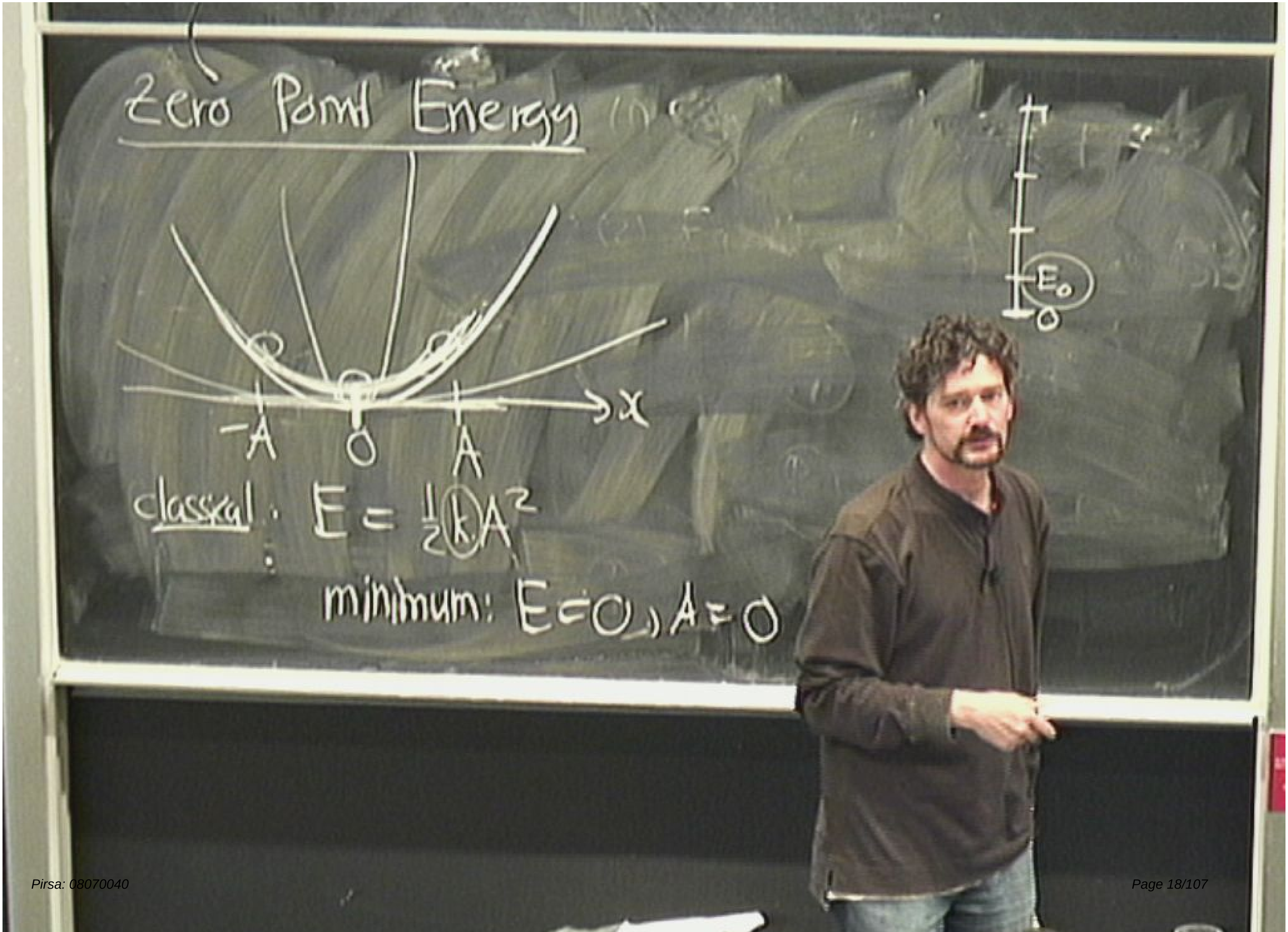
$\Rightarrow$ instantly "<u>jump into motion</u>"

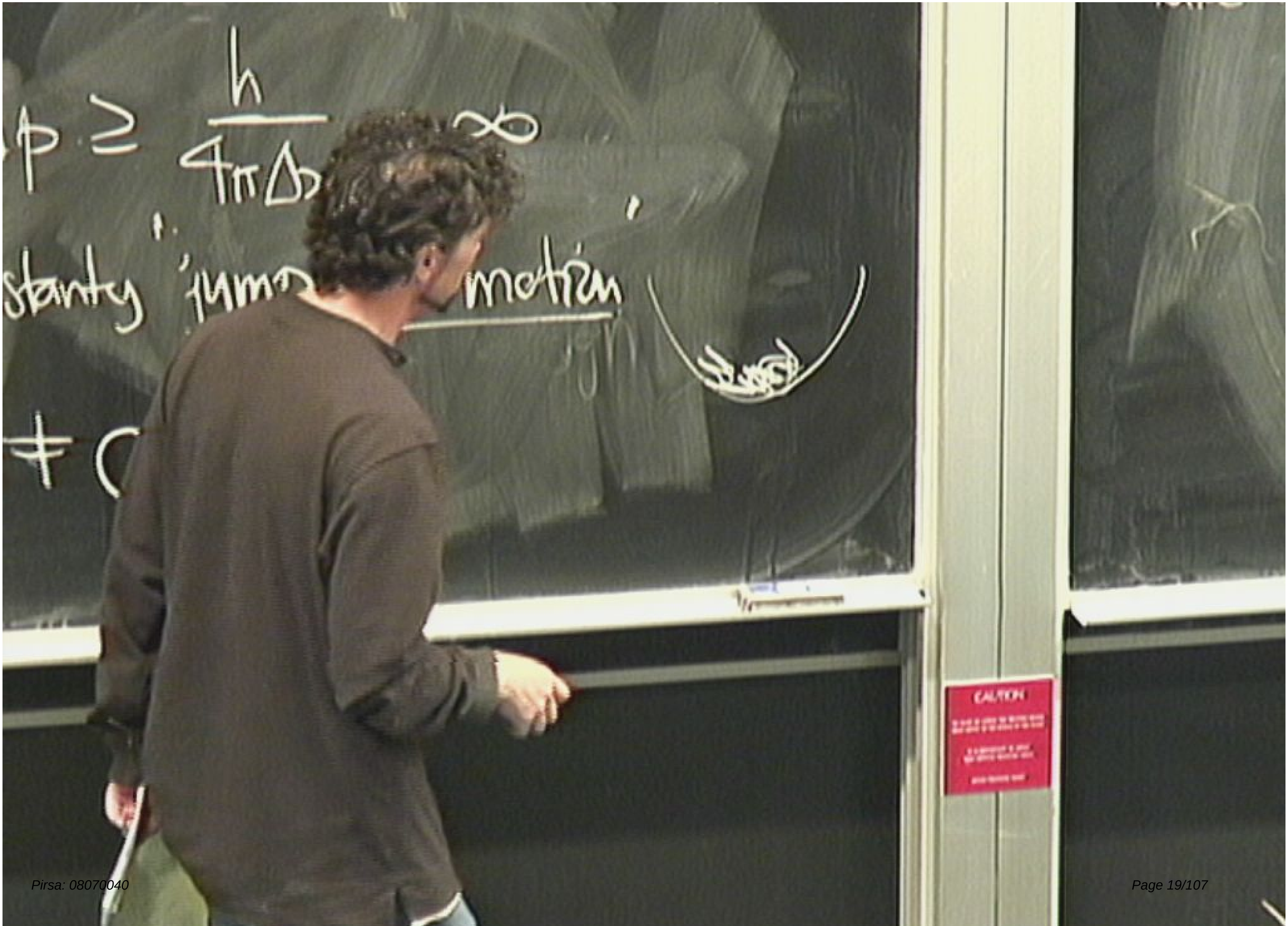
$\Rightarrow A \neq 0$

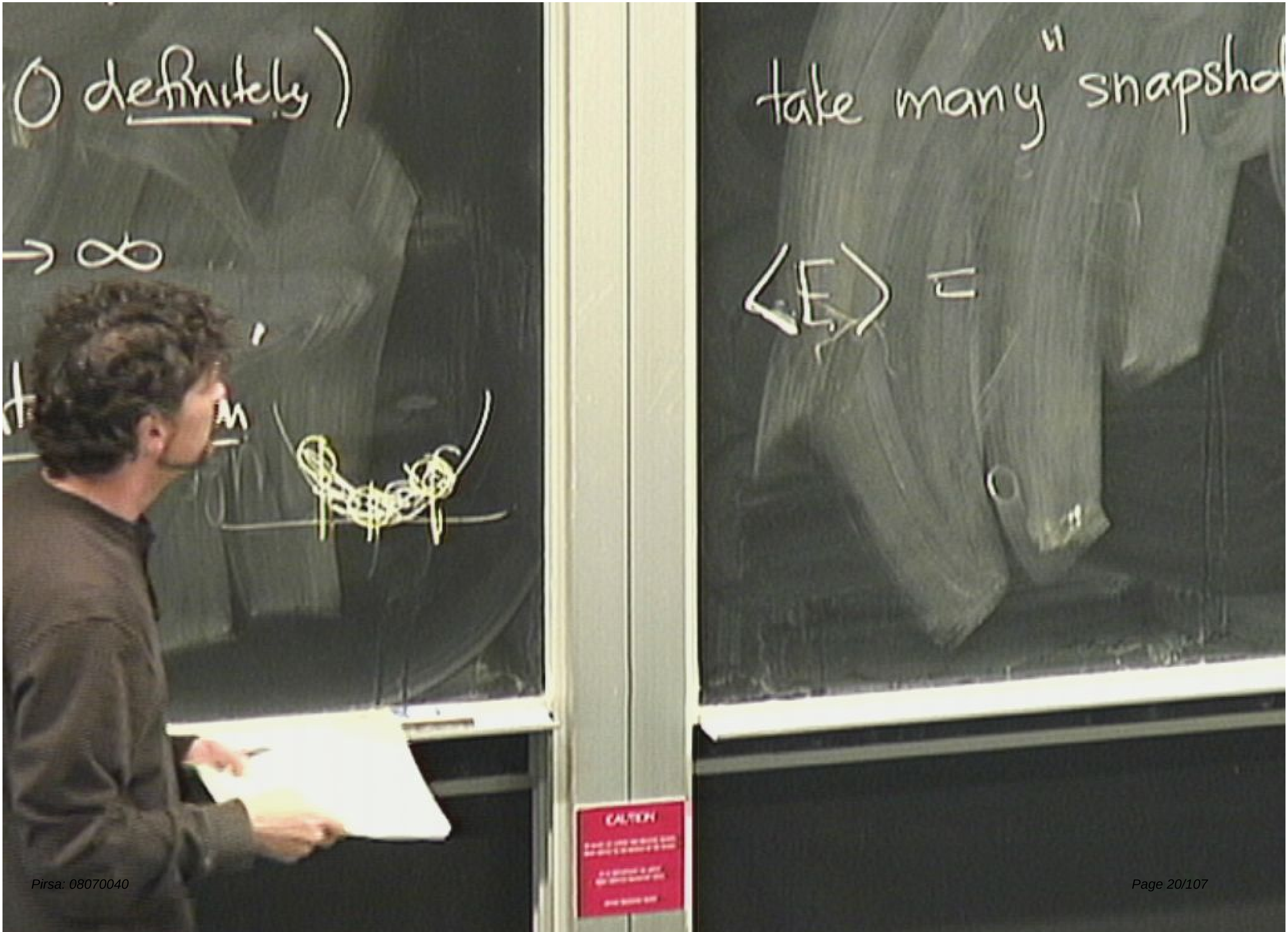

E = KE+PE =

$$E = KE + PE = \frac{p^2}{2m} + \frac{1}{2}kx^2$$




Zero Point Energy
-A
0
A
x
classical: E = 1/2 k A^2
minimum: E=0, A=0
E_0
0




(0 definitely)
→ ∞
take many "snapsho
⟨E⟩ =
CAUTION

$$E = KE + PE = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

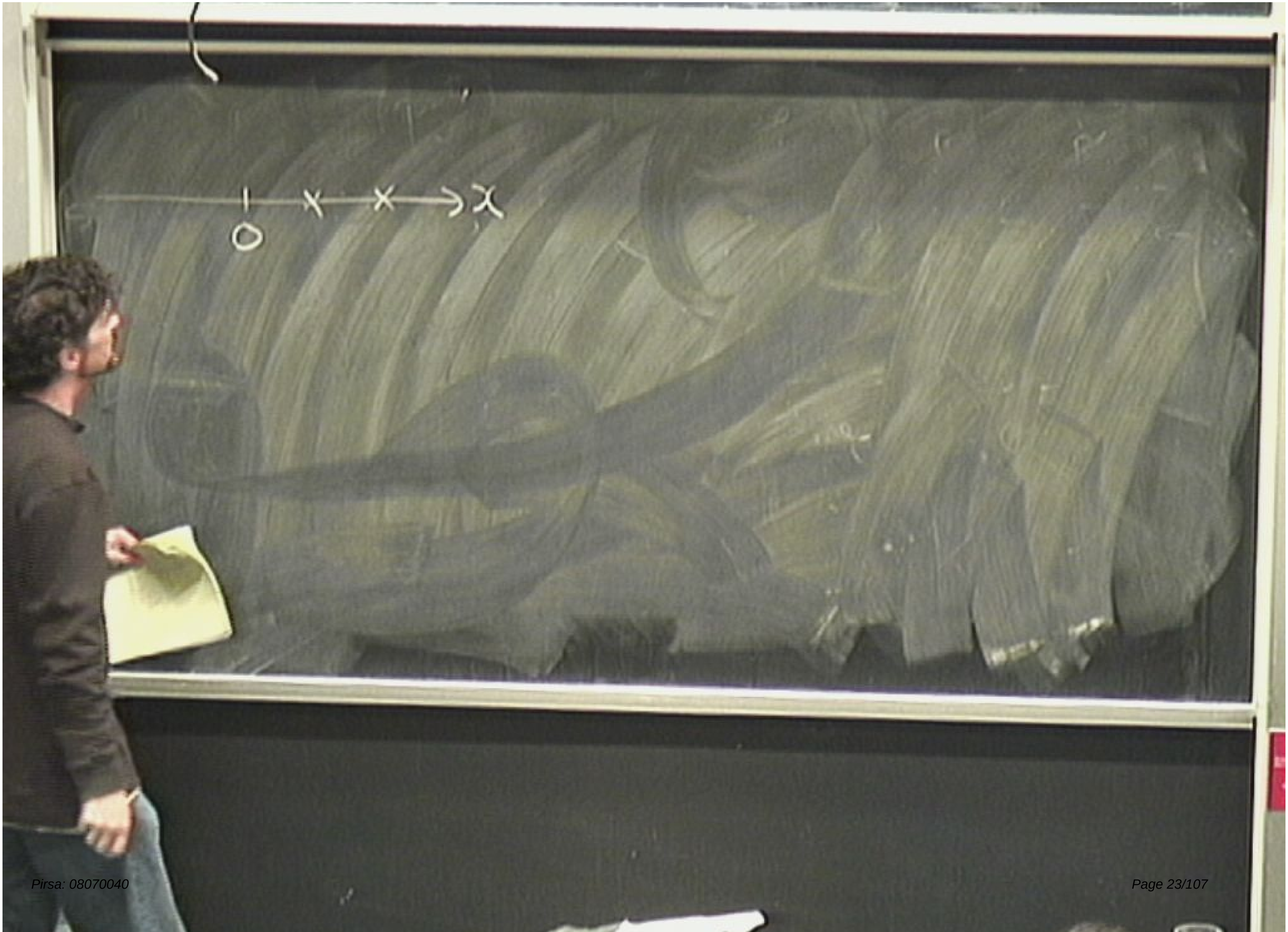
take many "snapshots" and <u>average</u>

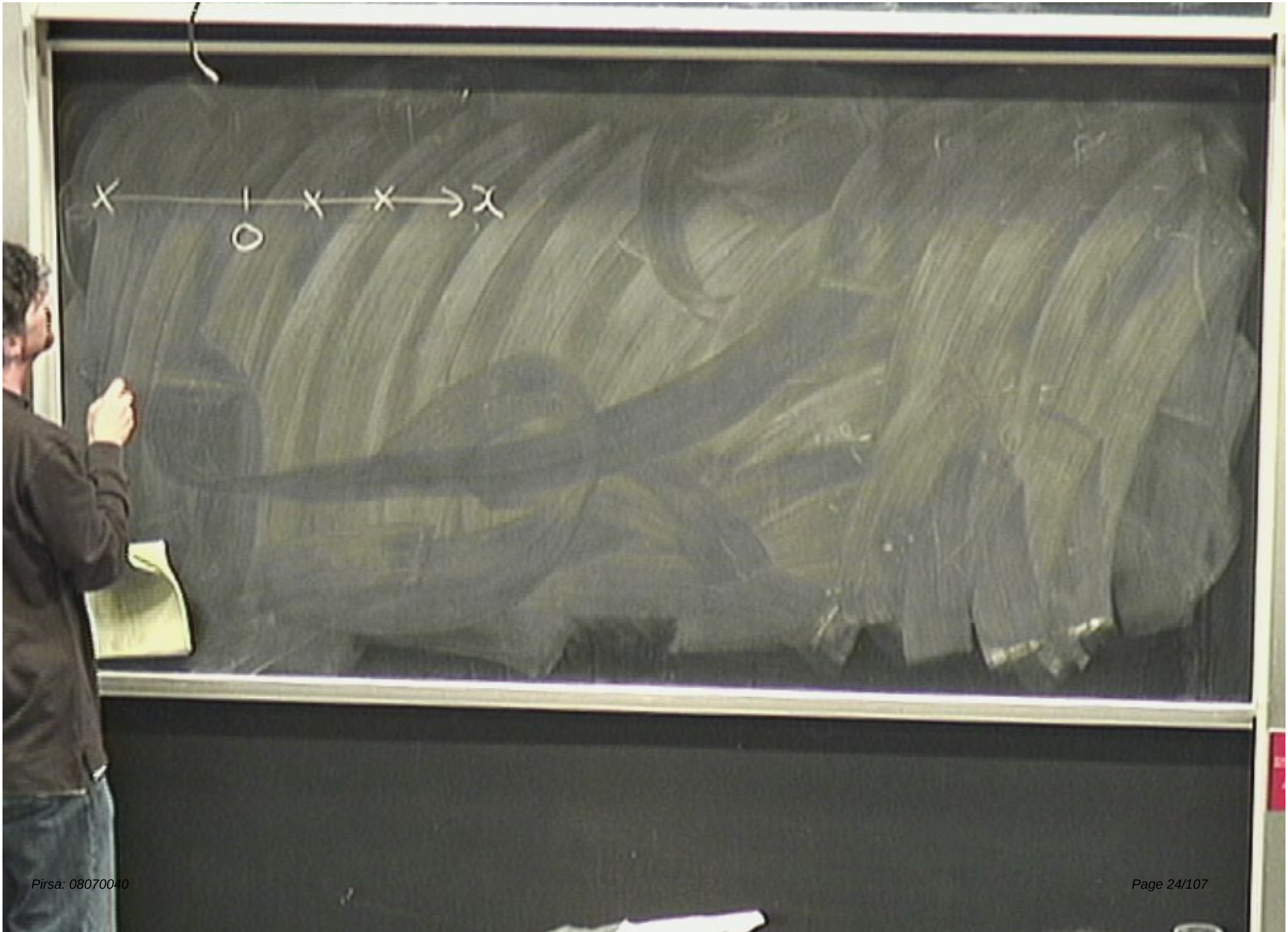
$$\langle E \rangle = \frac{1}{2m}\langle p^2 \rangle + \frac{1}{2}k\langle x^2 \rangle$$

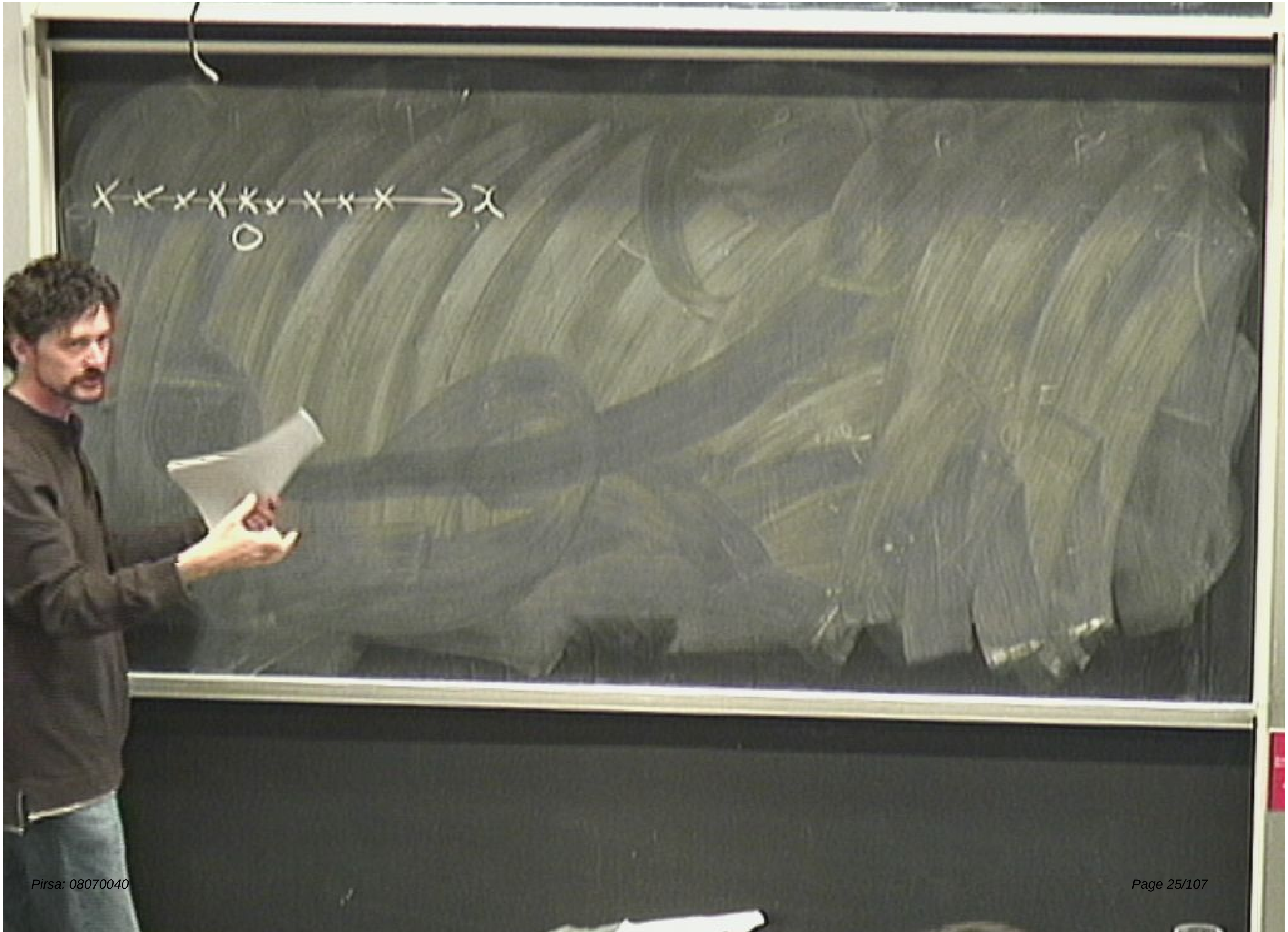
$$E = KE + PE = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

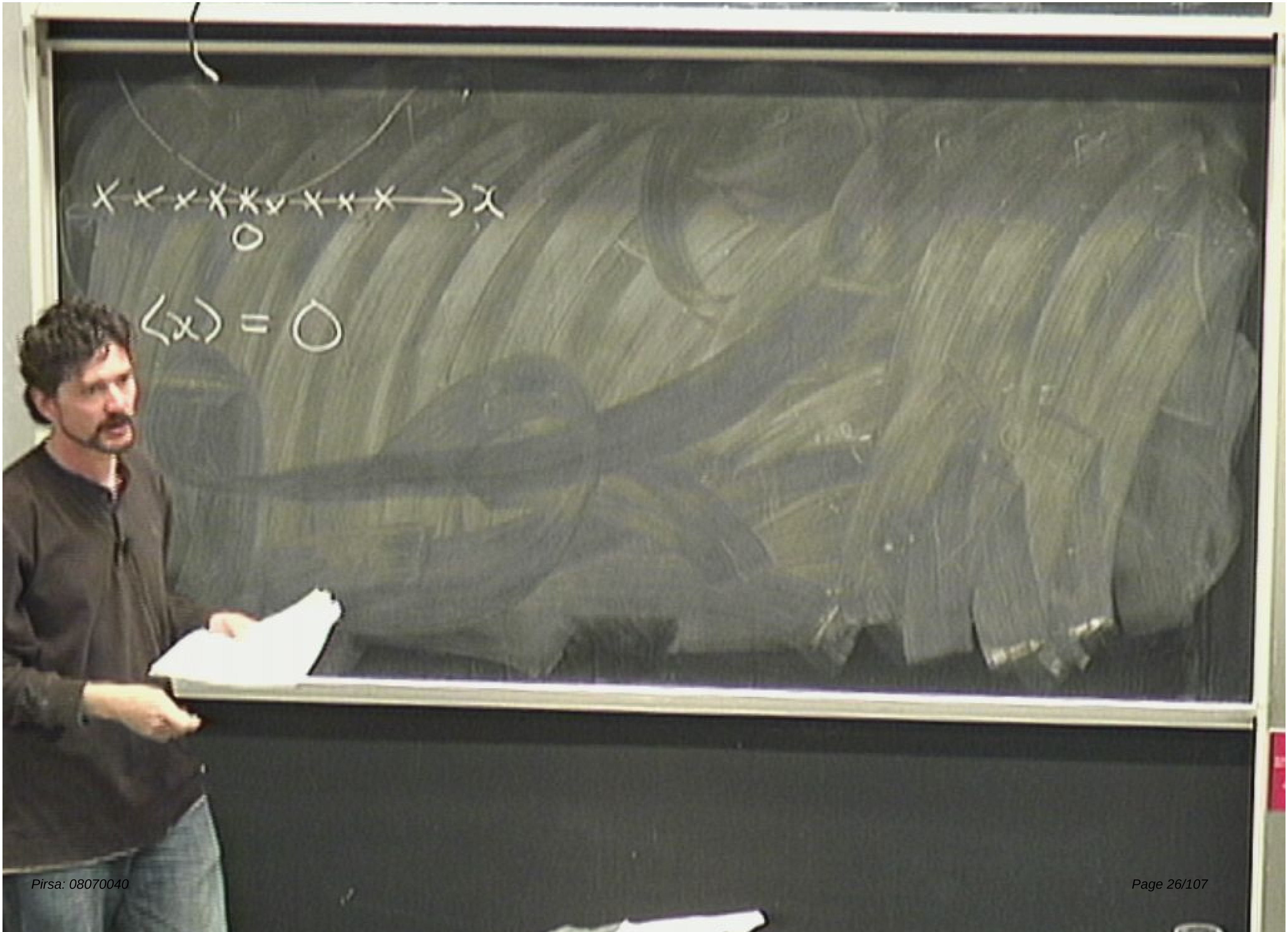
take many "snapshots" and <u>average</u>

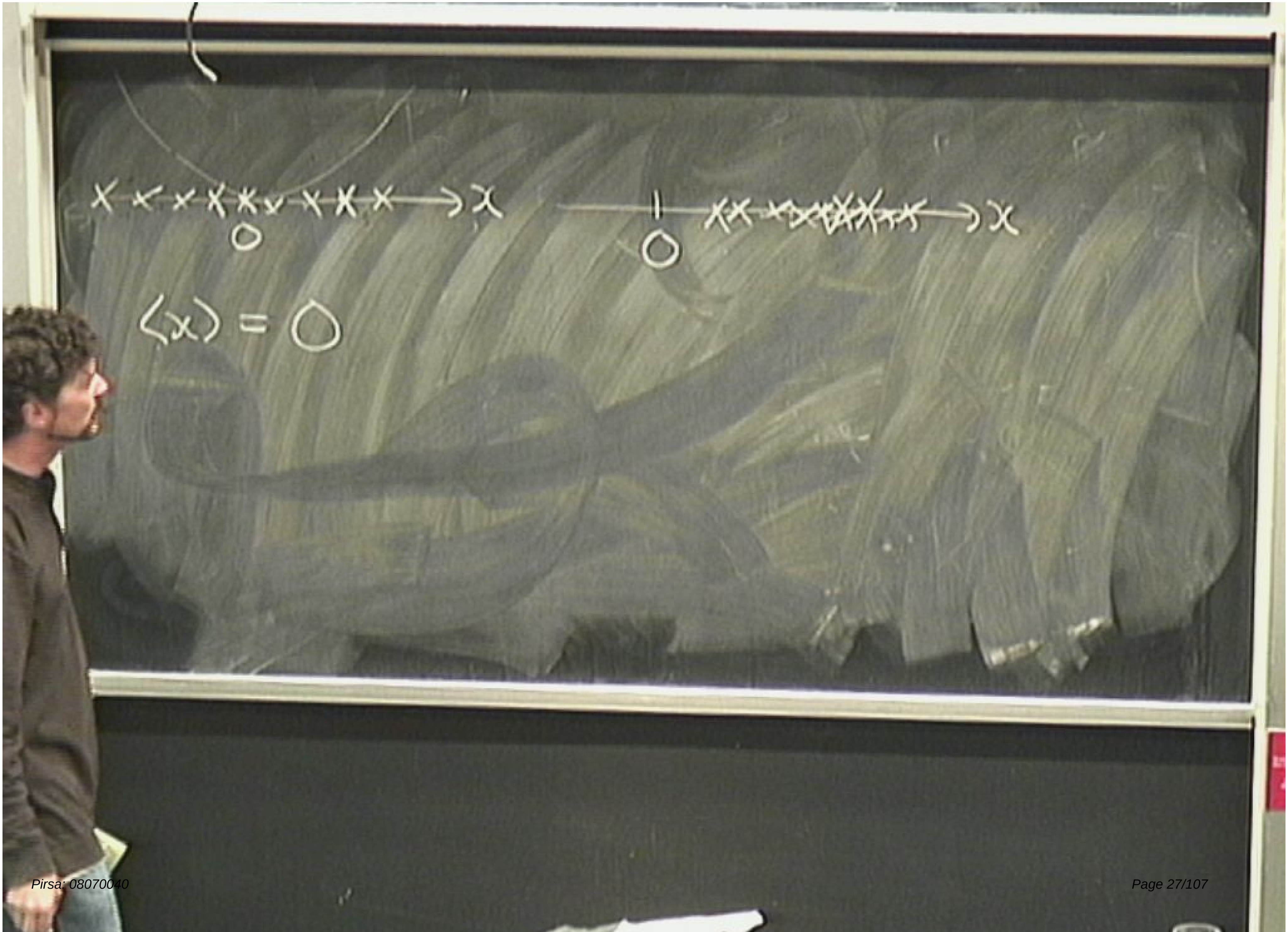
$$\langle E \rangle = \frac{1}{2m}\langle p^2 \rangle + \frac{1}{2}k\langle x^2 \rangle$$

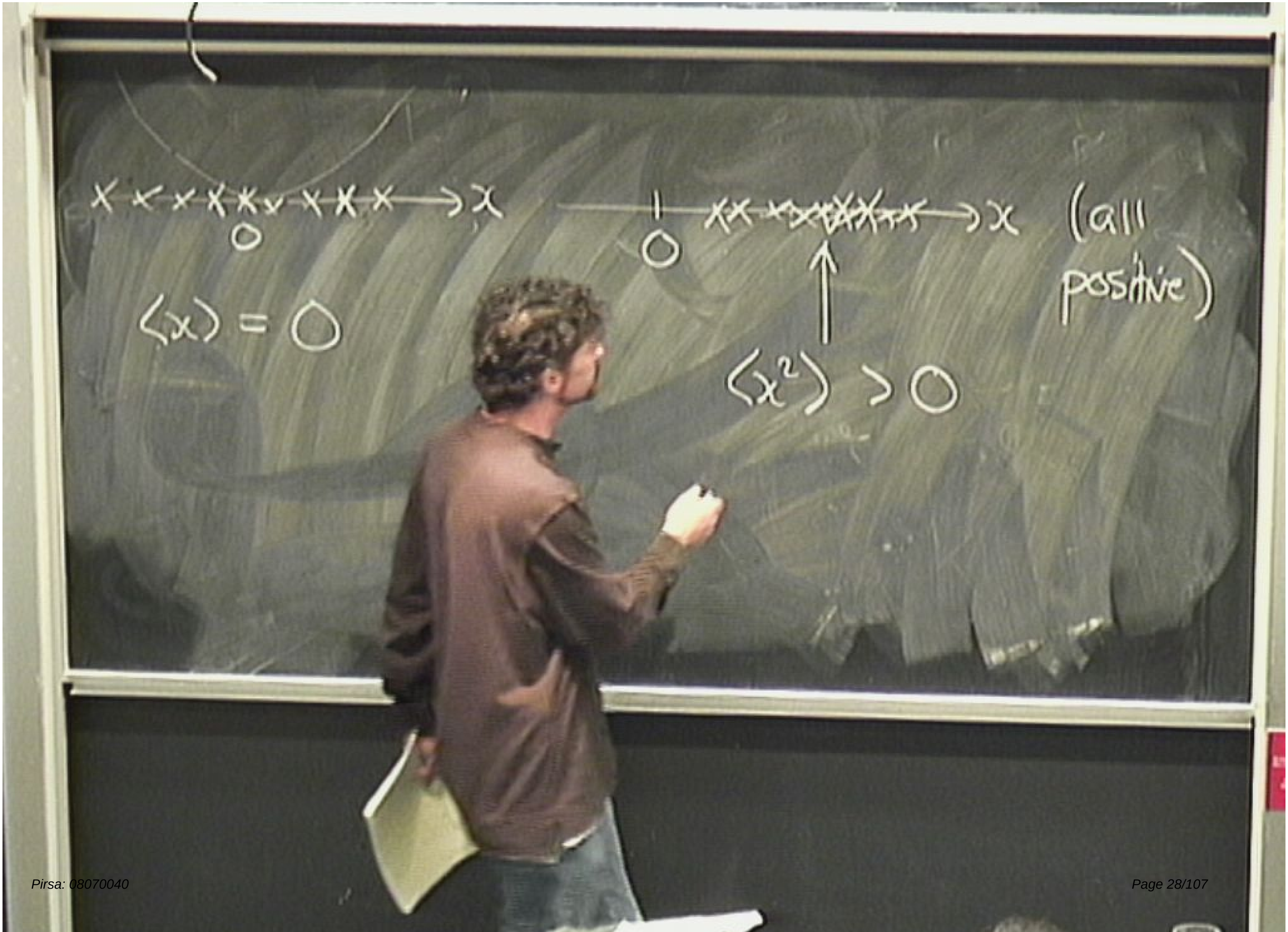


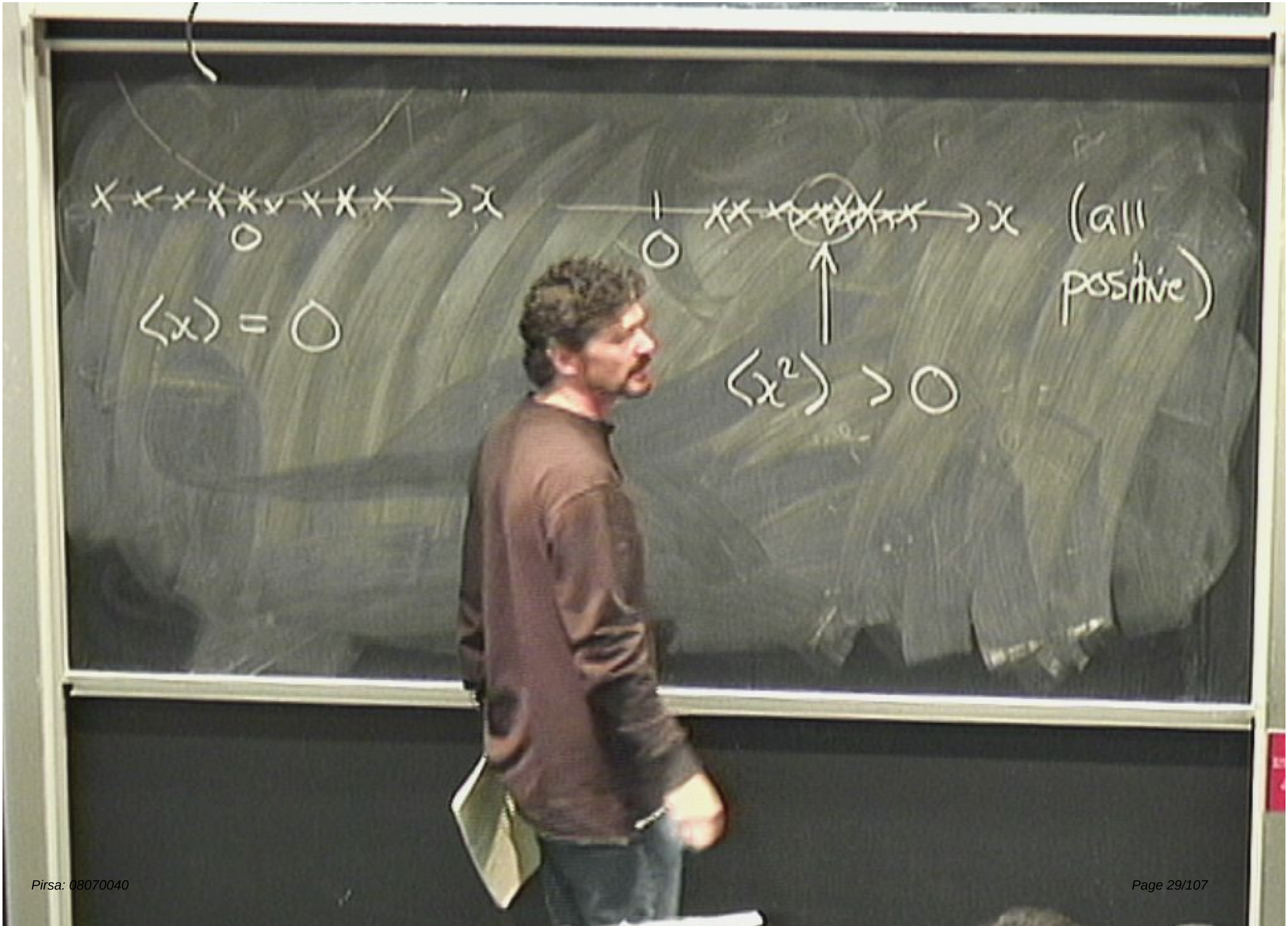

0
λ


λ
0


λ
0
⟨x⟩ = 0


x
0
x
0
⟨x⟩ = 0


x
0
⟨x⟩ = 0
0
x
(all positive)
⟨x²⟩ > 0


x
0
⟨x⟩ = 0
0
x
(all positive)
⟨x²⟩ > 0

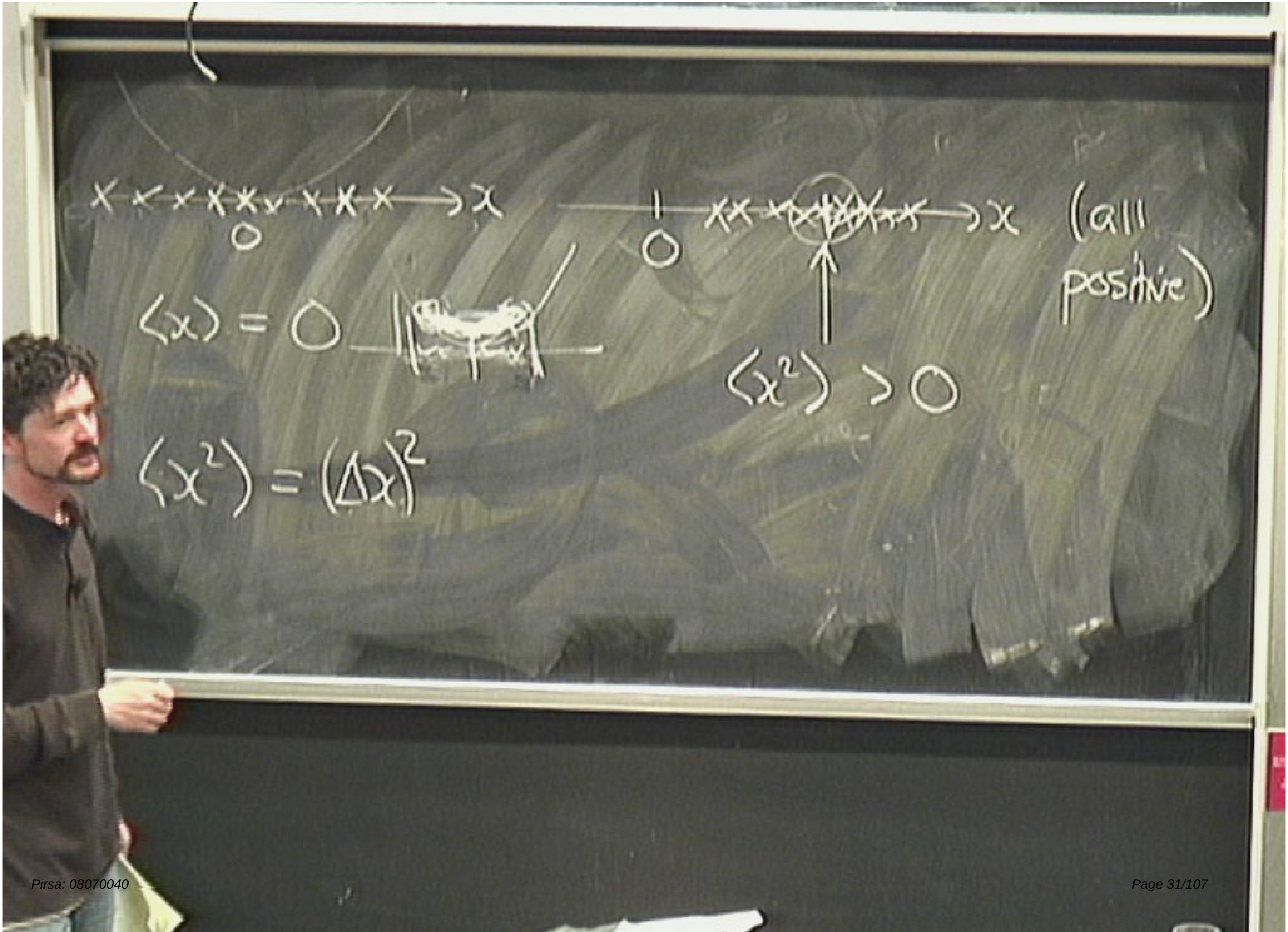
x — 0

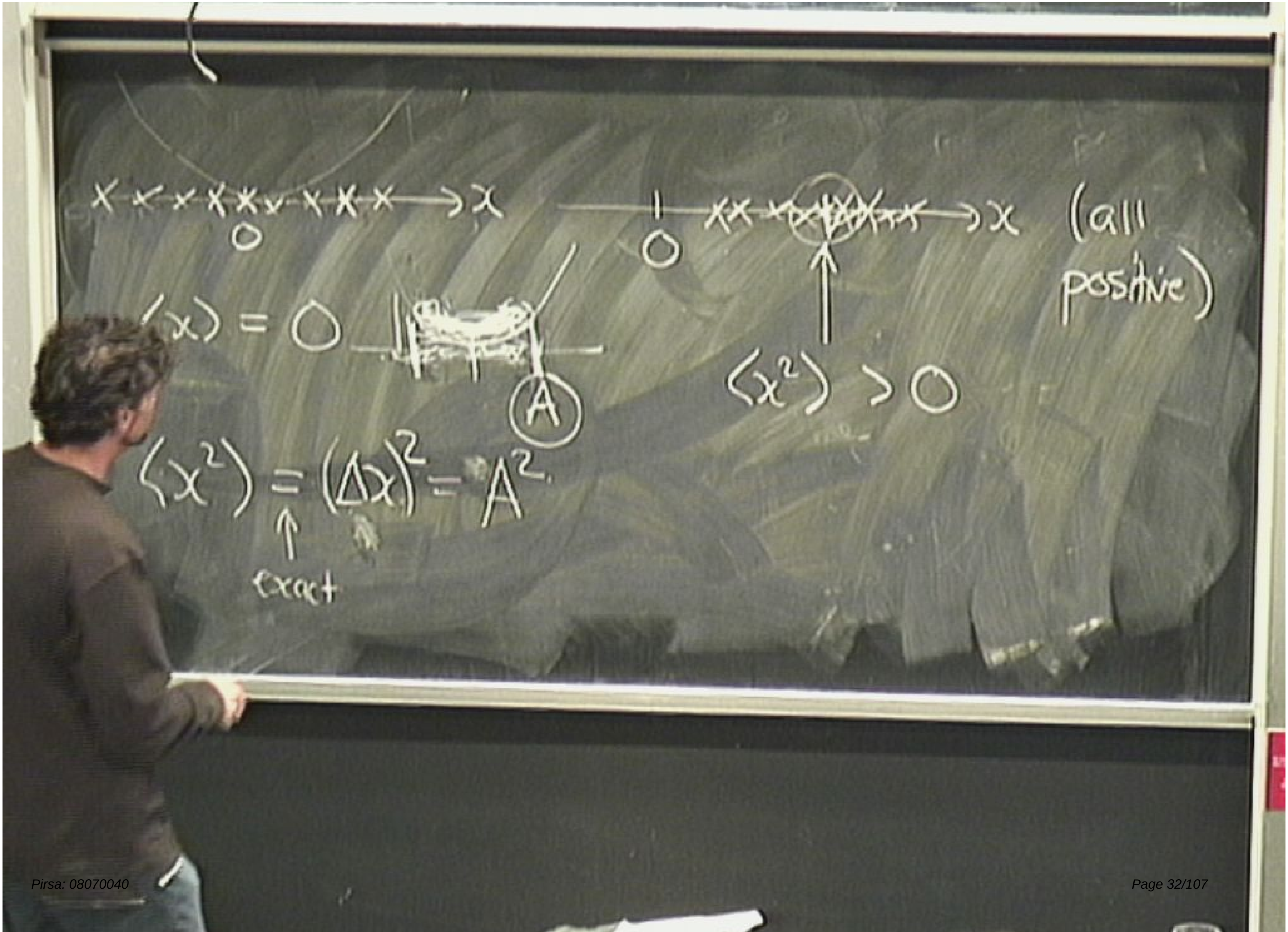
$\langle x \rangle = 0$

$\langle x^2 \rangle = (\Delta x)^2$

0 — x (all positive)

$\langle x^2 \rangle > 0$


x
0
$\langle x \rangle = 0$
$\langle x^2 \rangle = (\Delta x)^2$
x
(all positive)
0
$\langle x^2 \rangle > 0$


x
0
⟨x⟩ = 0
A
⟨x²⟩ = (Δx)² = A²
exact
0
x
(all
positive)
⟨x²⟩ > 0

(all positive)

$\langle x \rangle = 0$

$\langle x^2 \rangle > 0$

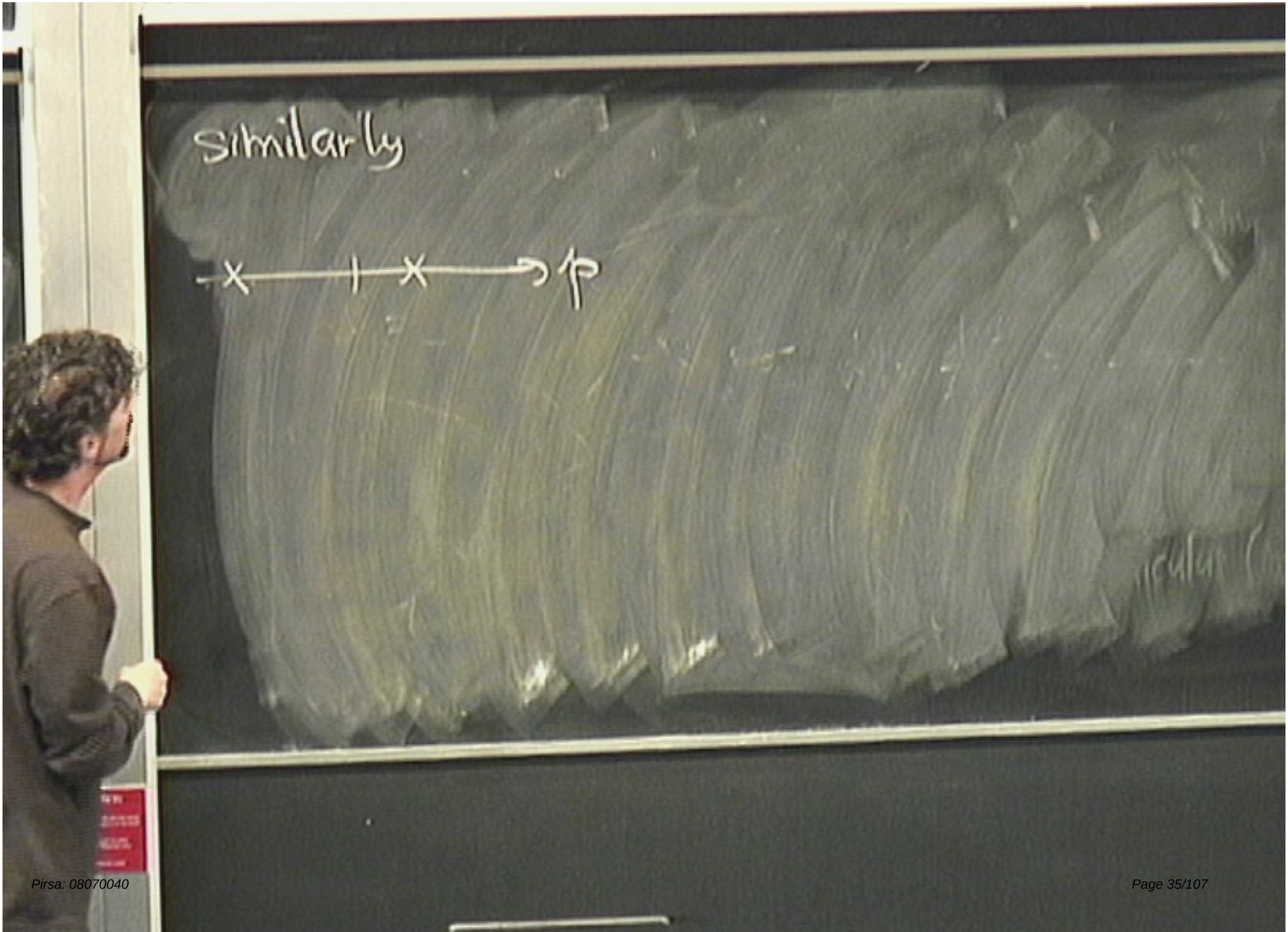
$\langle x^2 \rangle = (\Delta x)^2 \approx A^2$

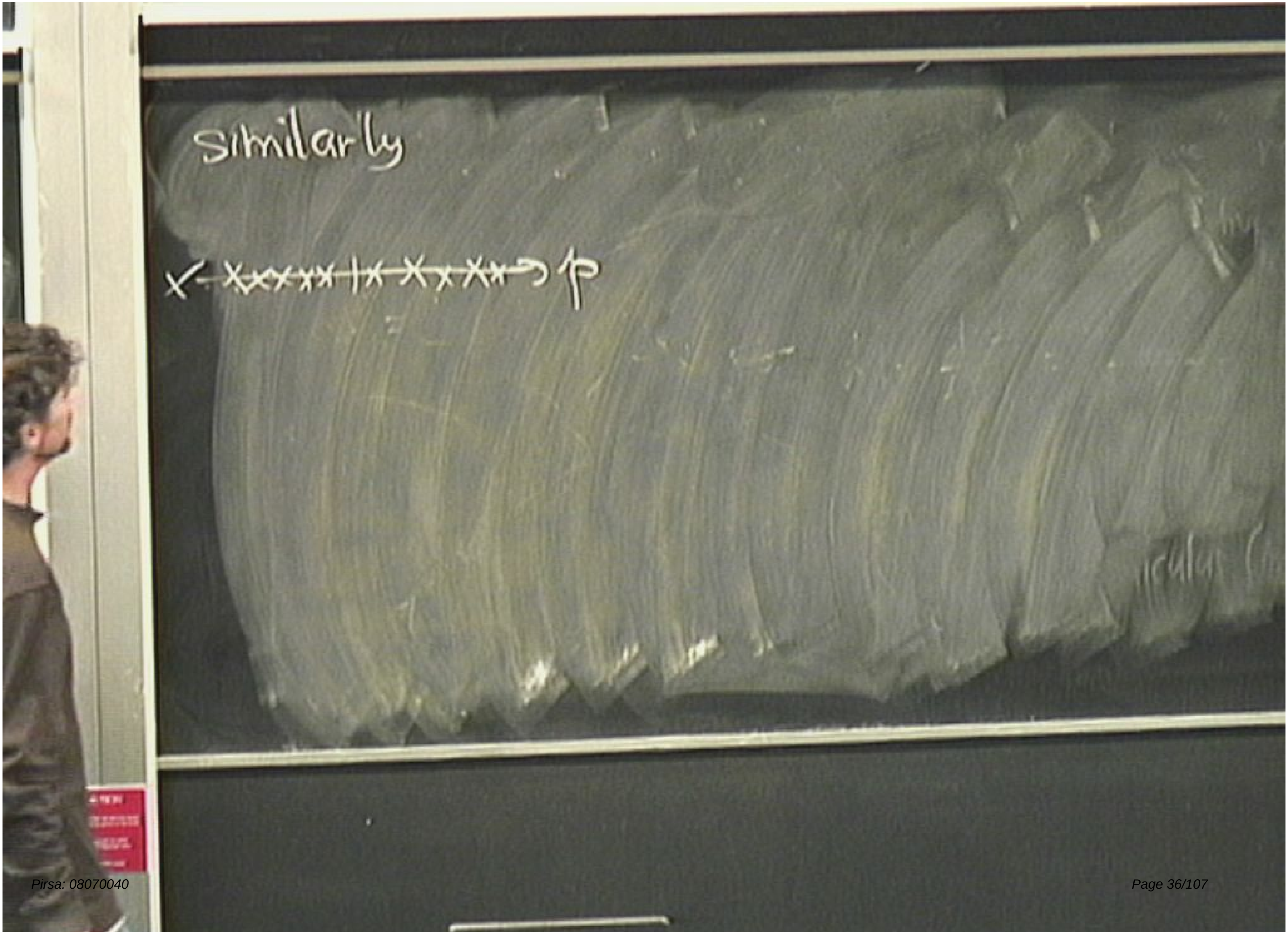
exact

A = amplitude of quantum fuzzy motion.

similarly

$\longrightarrow p$


similarly
p


similarly

similarly

$\langle p \rangle = 0$

$\langle p^2 \rangle > 0$

similarly

$\langle p \rangle = 0$

$\langle p^2 \rangle > 0$

$\langle p^2 \rangle =$

Similarly

$$\langle p \rangle = 0 \qquad\qquad \langle p^2 \rangle > 0$$

$$\langle p^2 \rangle = (\Delta p)^2$$

similarly

p $\qquad$ 0 $\quad p^2$

$\langle p \rangle = 0$ $\qquad\qquad$ $\langle p^2 \rangle > 0$

$\langle p^2 \rangle = (\Delta p)^2 \geq$

Similarly

p $\qquad$ p^2

0

$\langle p \rangle = 0$

$\langle p^2 \rangle > 0$

$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi \Delta x}\right)^2 \approx$$

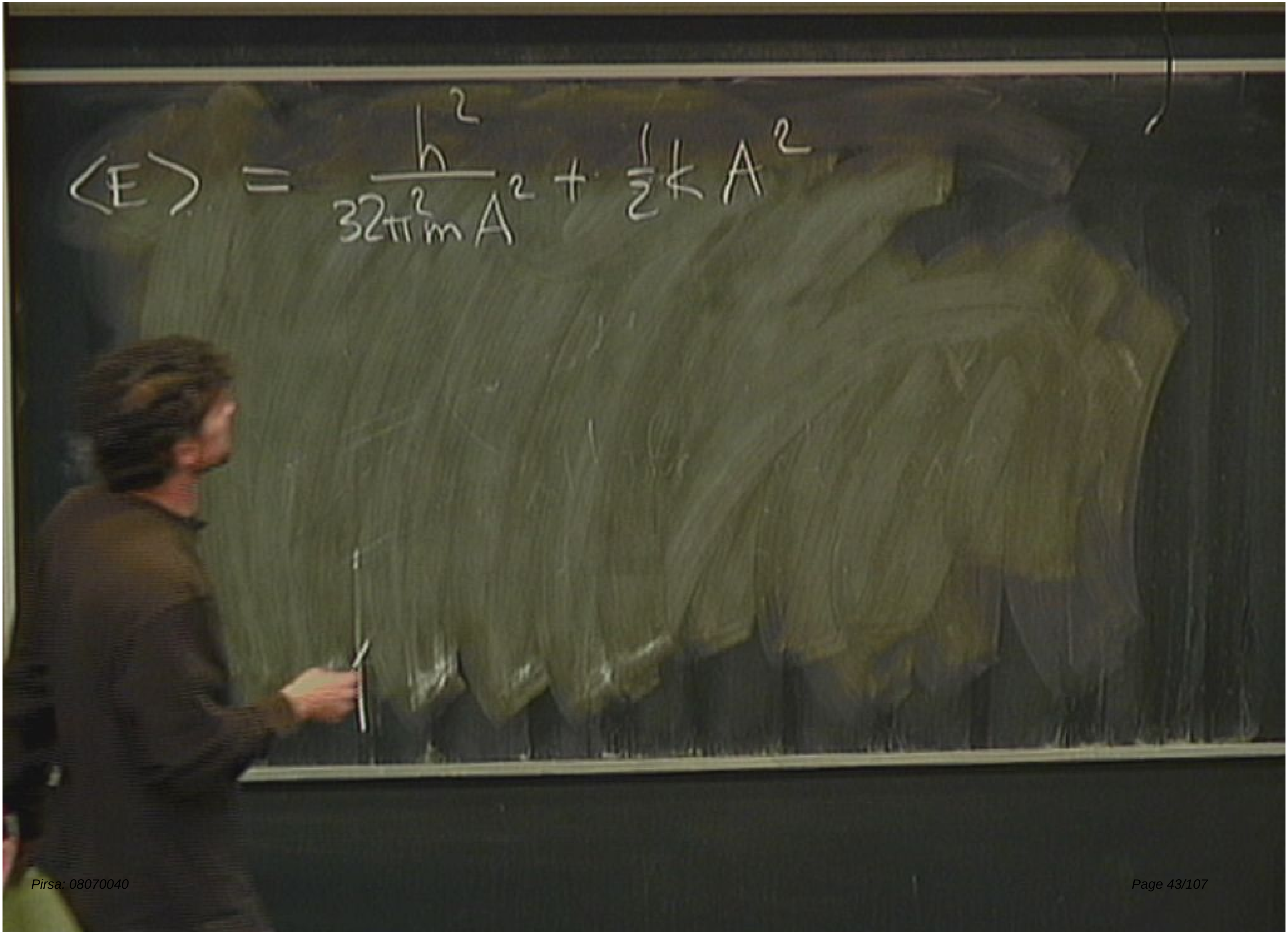
Similarly

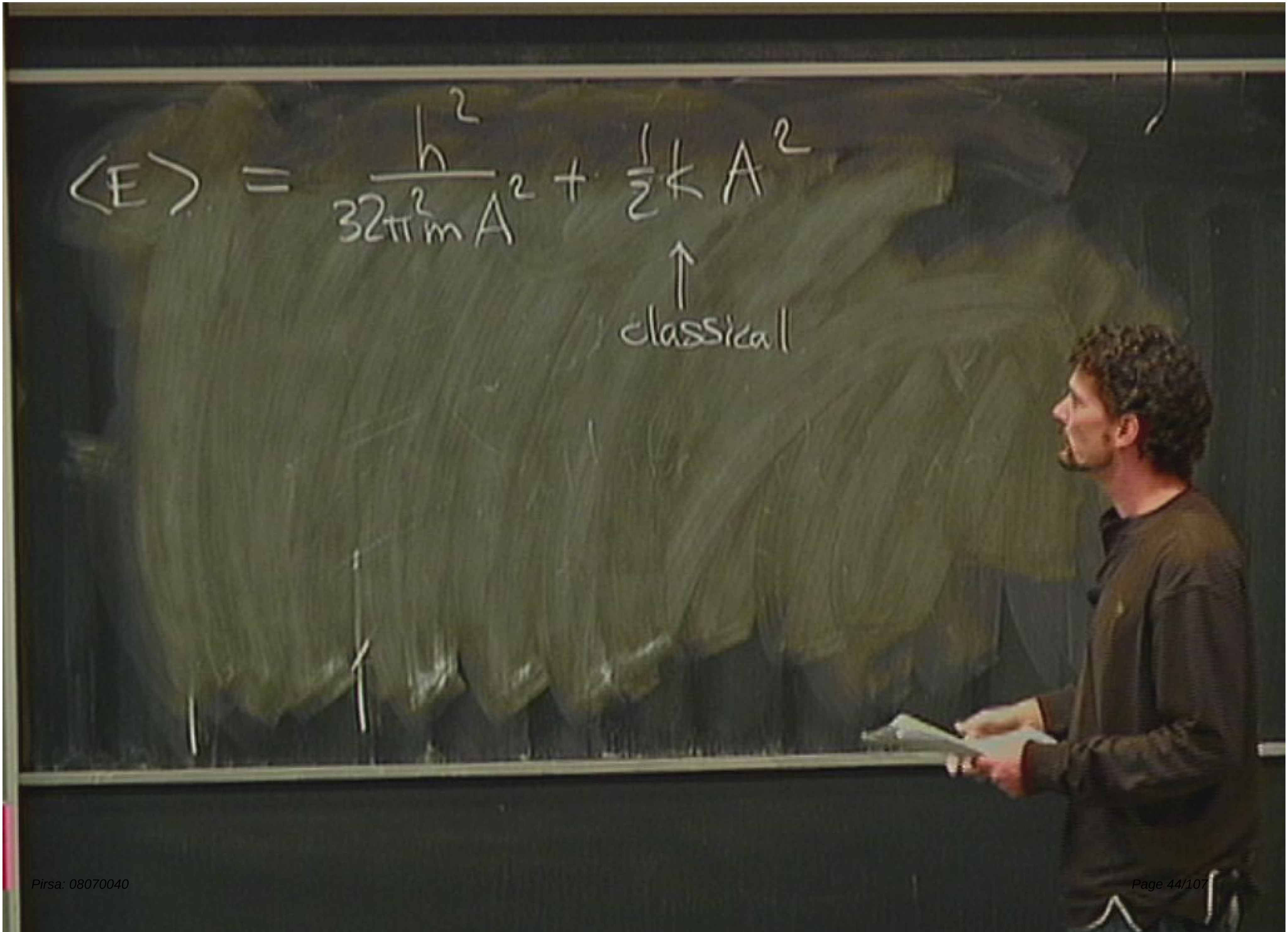
p $\qquad$ 0 $\qquad$ p^2

$\langle p \rangle = 0$

$\langle p^2 \rangle > 0$

$$\langle p^2 \rangle = (\Delta p)^2 \geq \left(\frac{h}{4\pi\Delta x}\right)^2 \approx \frac{h^2}{16\pi^2 A^2}$$


$\langle E \rangle = \frac{h^2}{32\pi^2 m A^2} + \frac{1}{2} k A^2$


$\langle E \rangle = \frac{h^2}{32\pi^2 m A^2} + \frac{1}{2} k A^2$
classical

$$\langle E \rangle = \frac{h^2}{32\pi^2 m A^2} + \frac{1}{2} k A^2$$

$\uparrow$

classical

$$\langle E \rangle = \frac{h^2}{32\pi^2 m A^2} + \frac{1}{2} k A^2$$

classical

purely quantum effect

$$\langle E \rangle = \frac{h^2}{32\pi^2 m A^2} + \frac{1}{2} k A^2$$

classical

purely quantum effect (disappears when $h=0$)

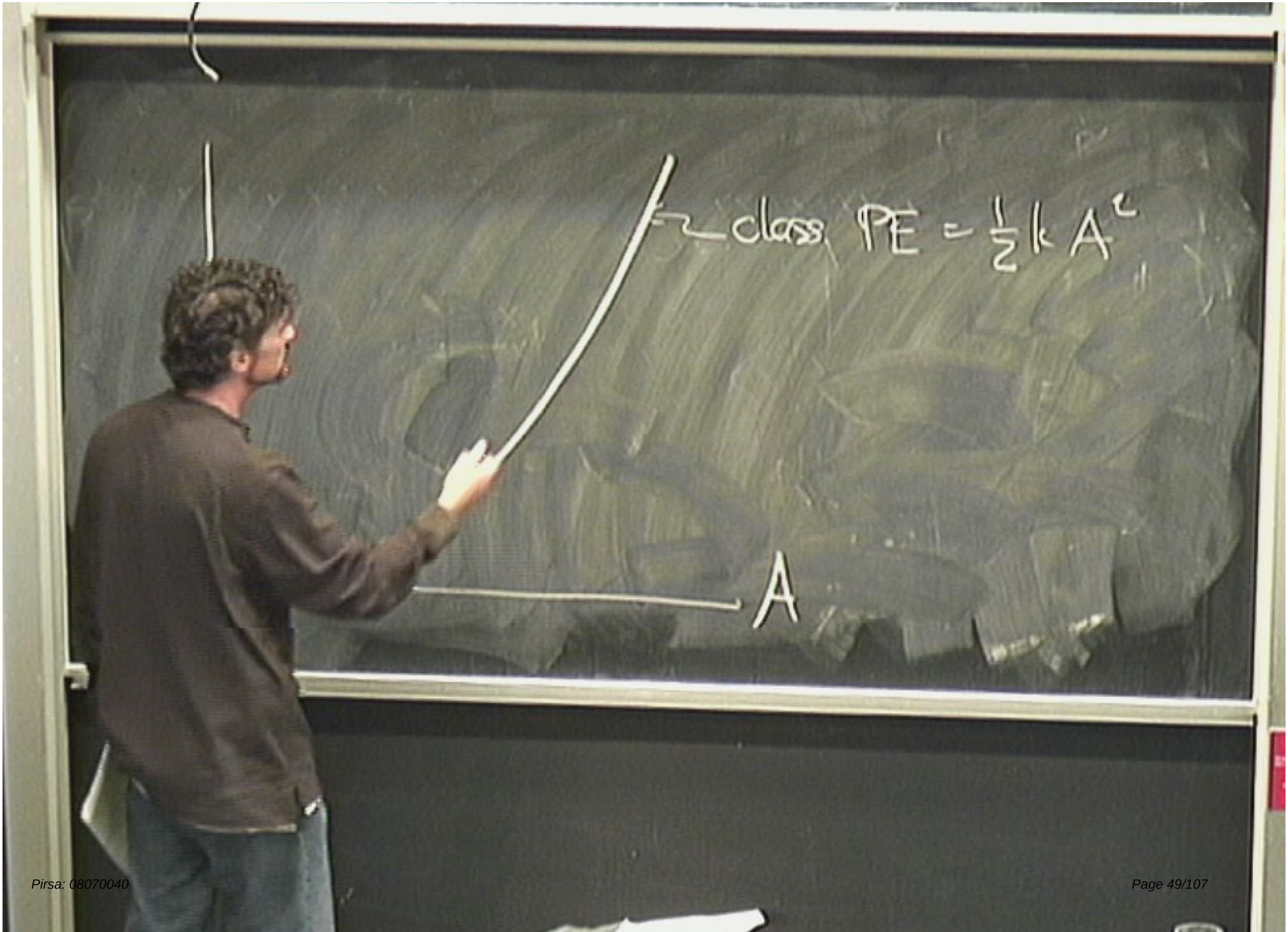
$$\langle E \rangle = \frac{h^2}{32\pi^2 m A^2} + \frac{1}{2} k A^2$$

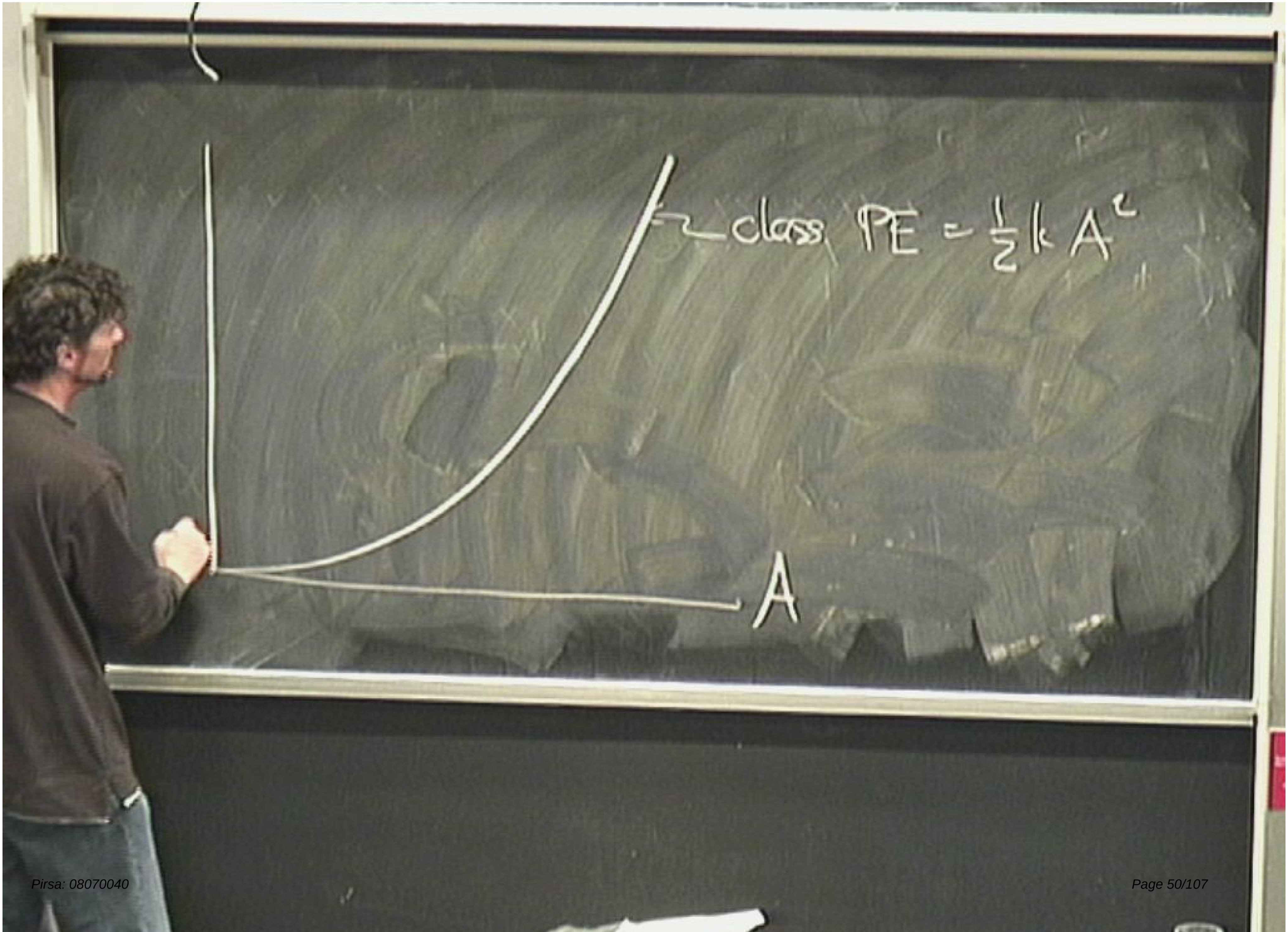
classical

purely <u>quantum</u> effect (disappears when $h=0$)

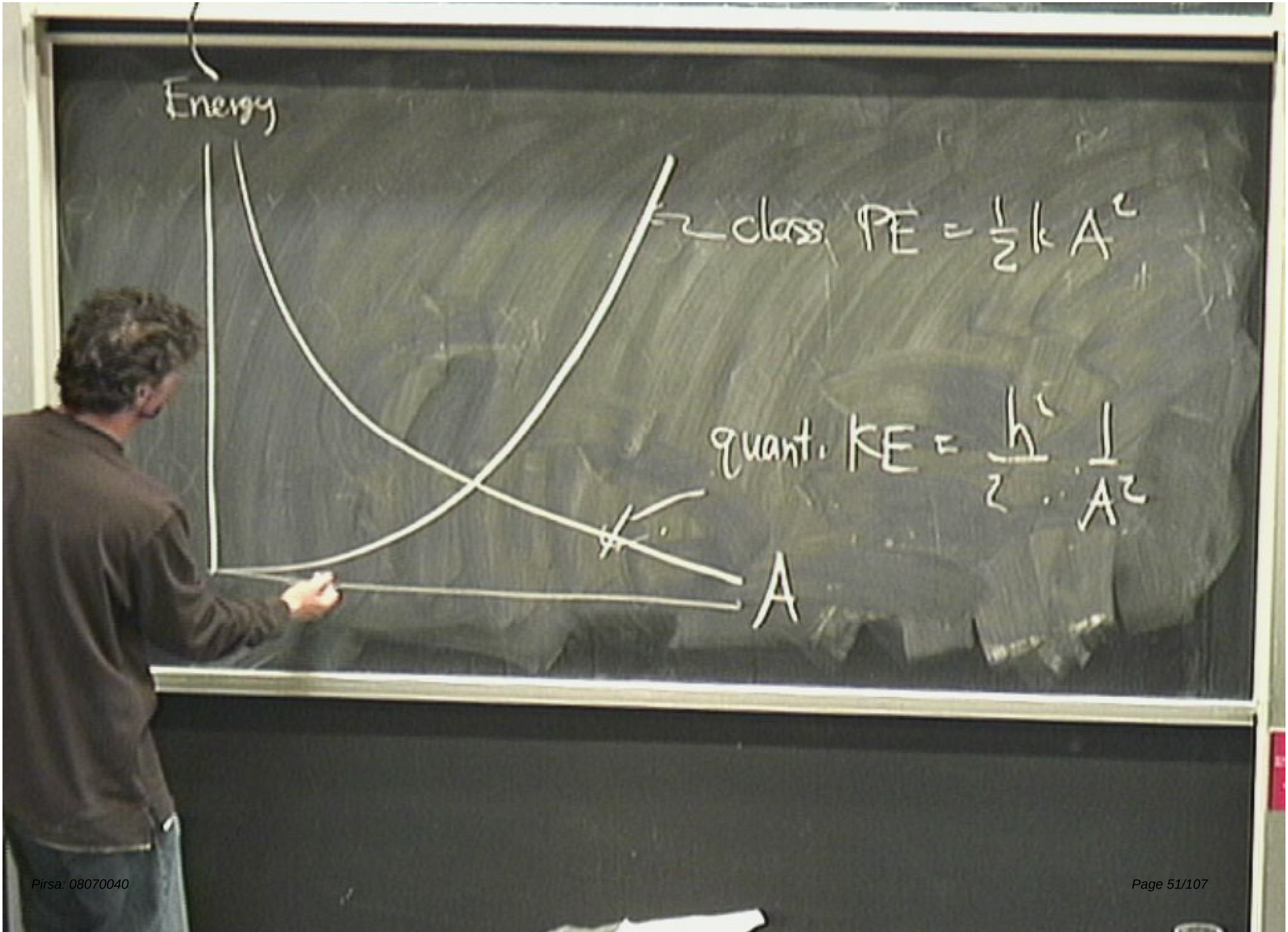
class. PE $\downarrow$ $A\downarrow$

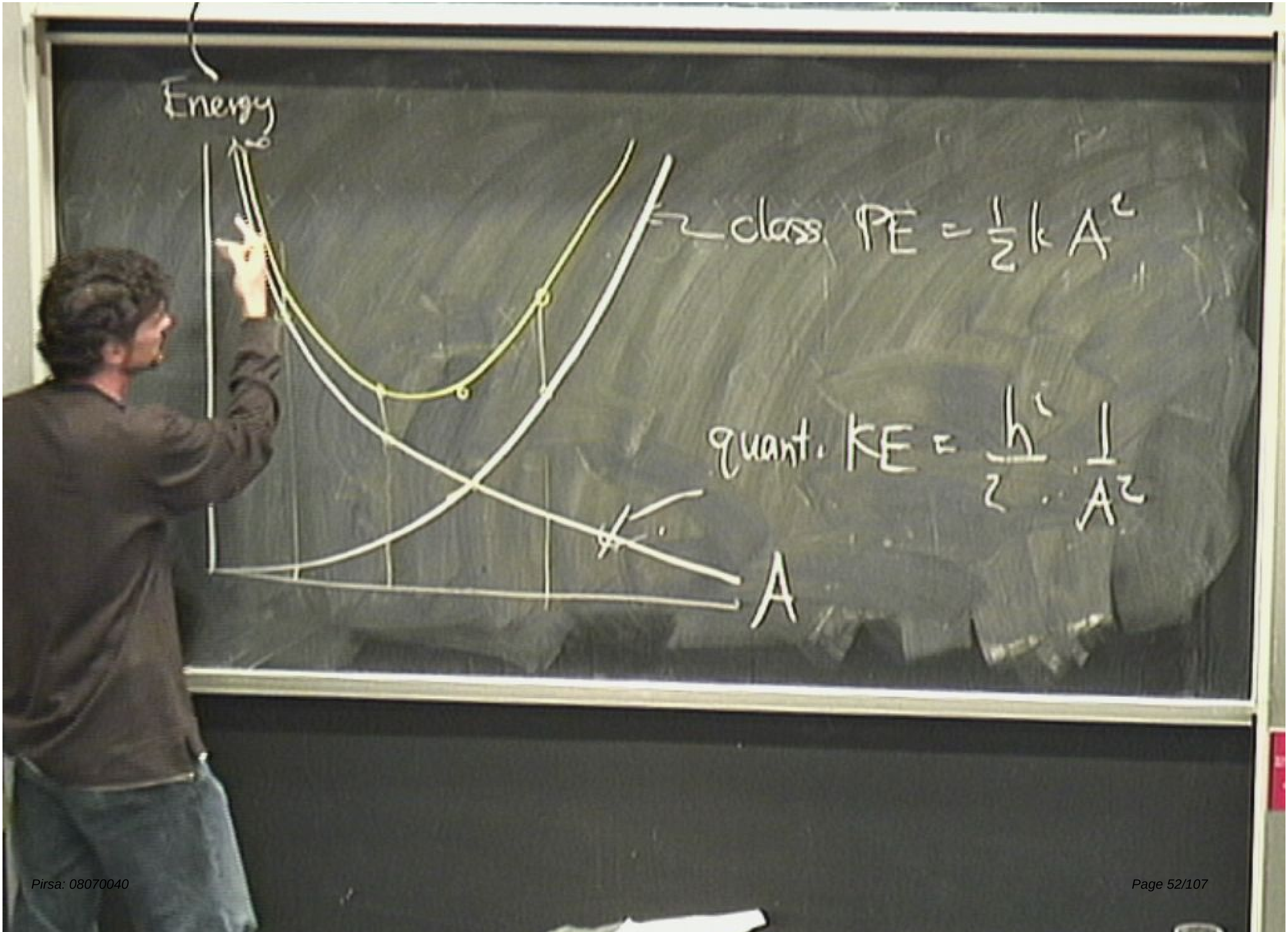
quant. KE $\uparrow$ $A\downarrow$

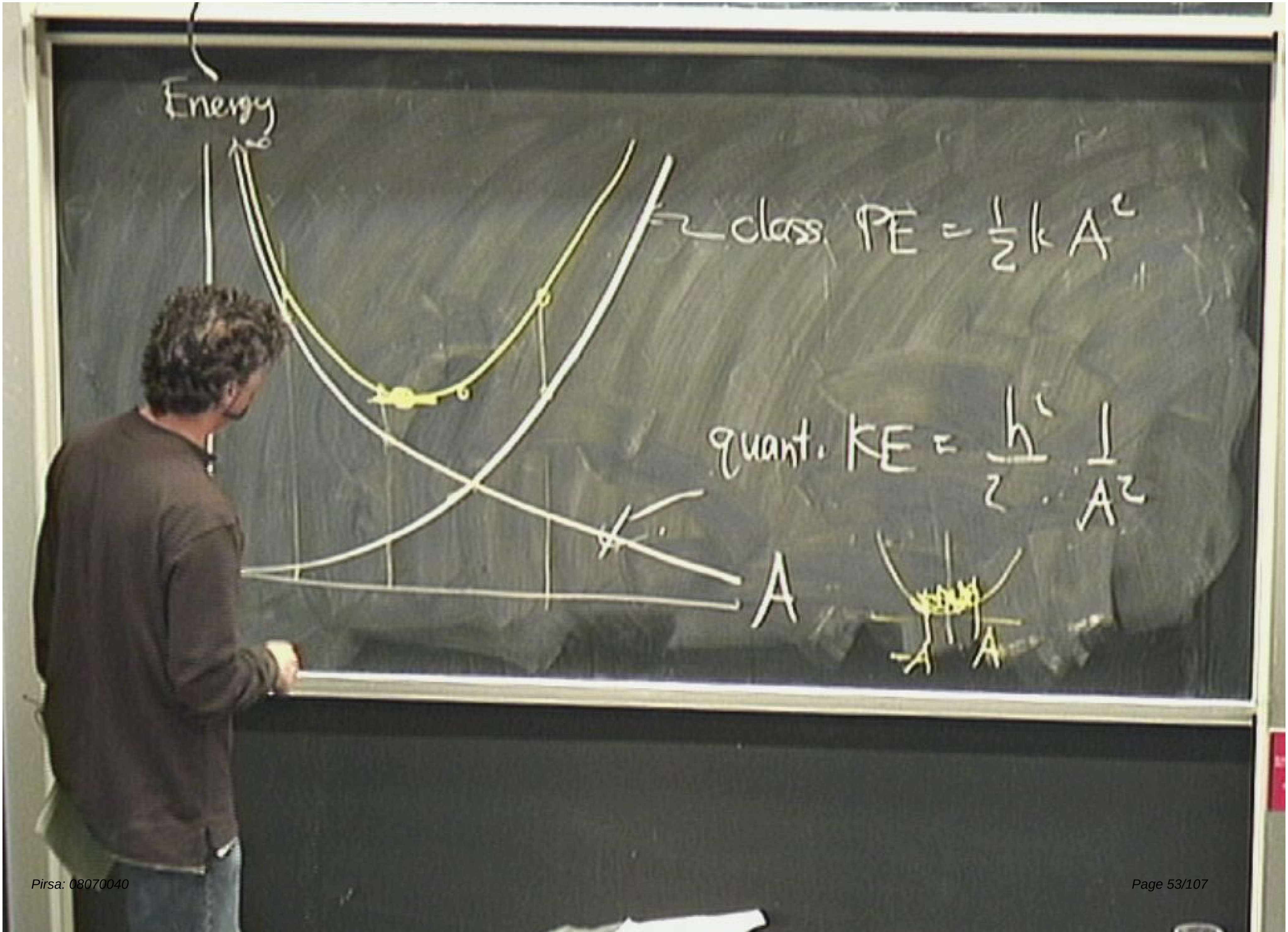

class PE = $\frac{1}{2} k A^2$
A

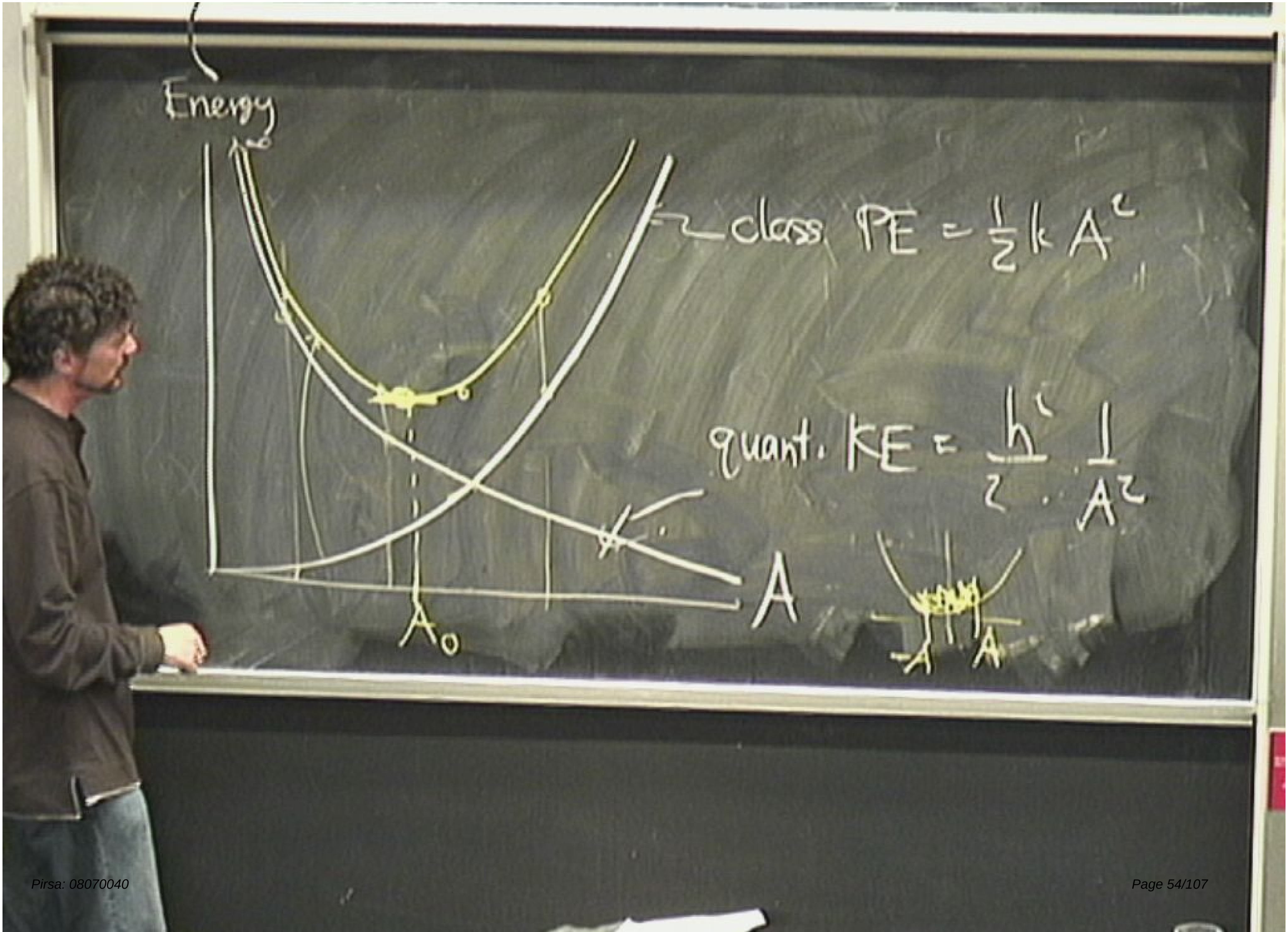


class $PE = \frac{1}{2} k A^2$
A

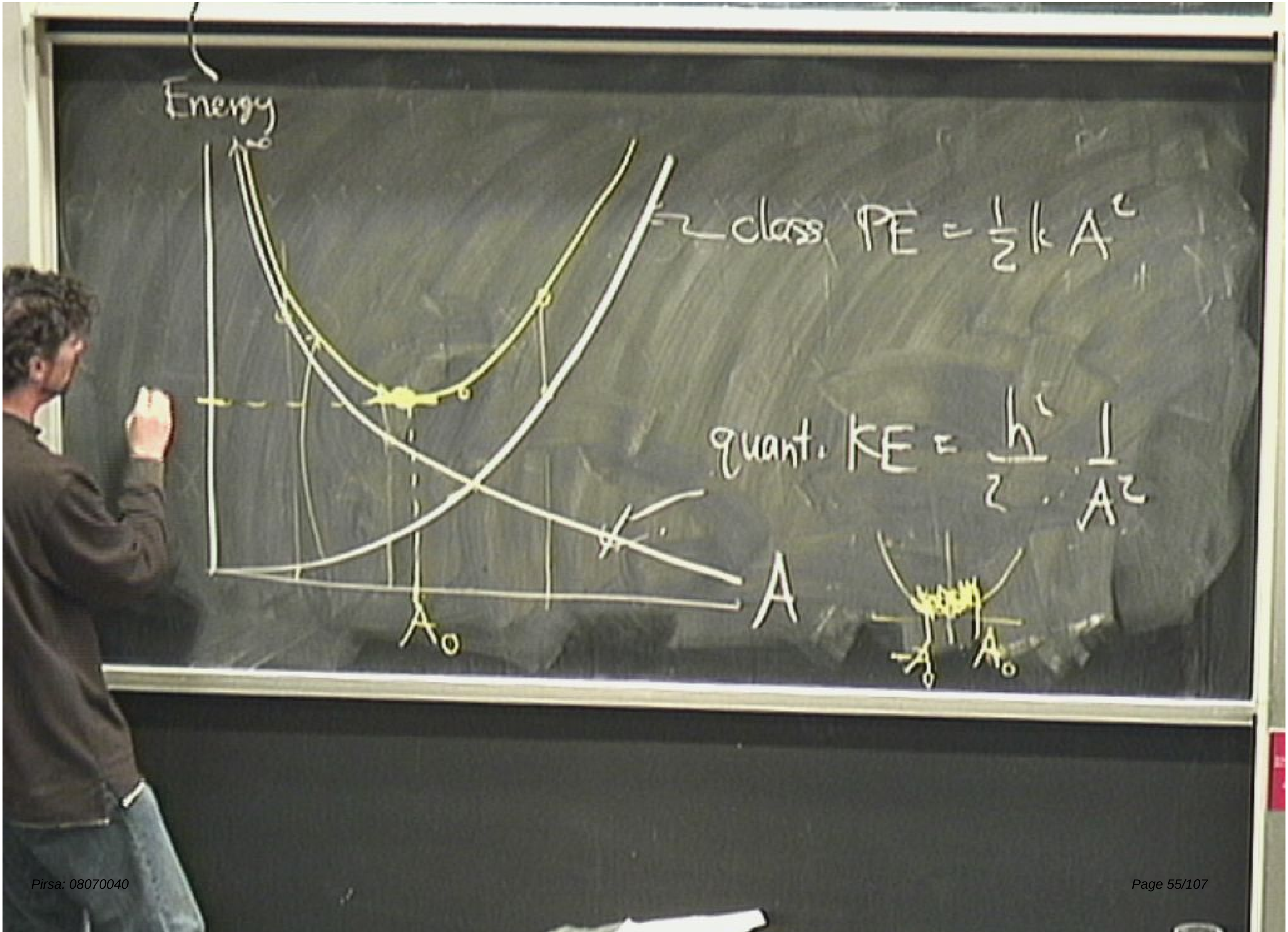

Energy
class. PE = ½ k A²
quant. KE = ħ²/2 · 1/A²
A


Energy
class. PE = ½ k A²
quant. KE = ℏ²/2 · 1/A²
A

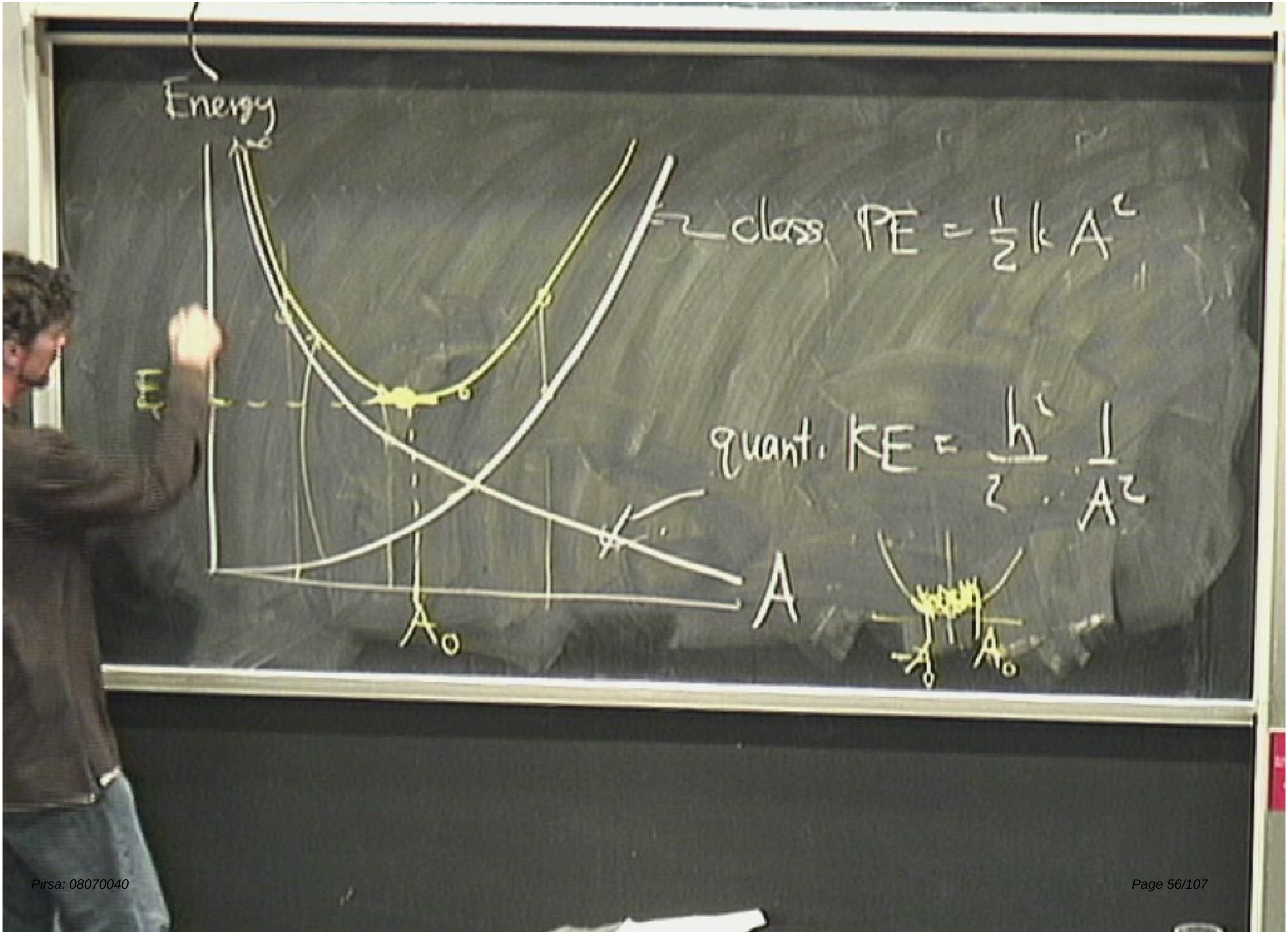

Energy
class. PE = ½ k A²
quant. KE = ℏ²/2 · 1/A²
A
-A   A

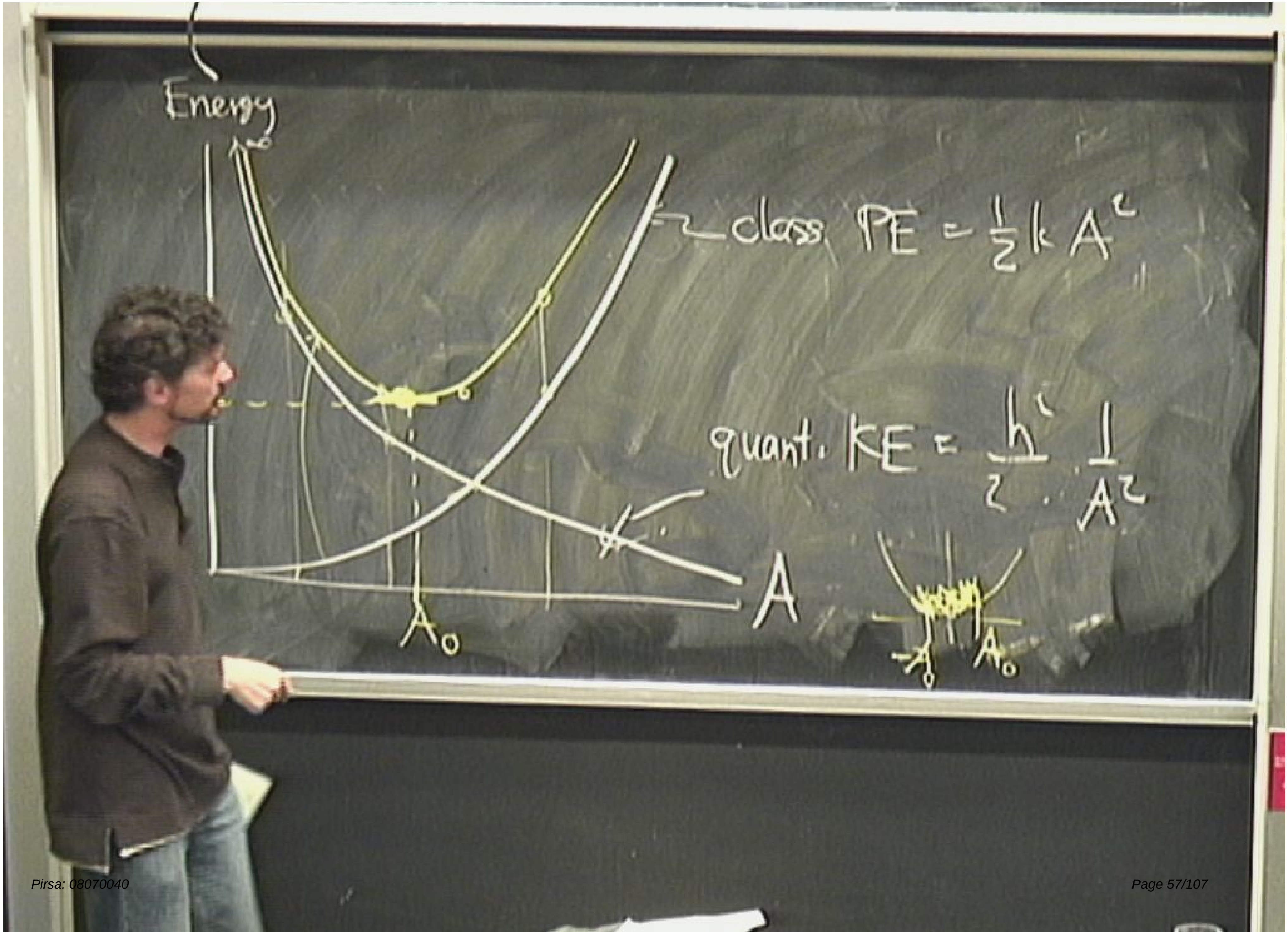


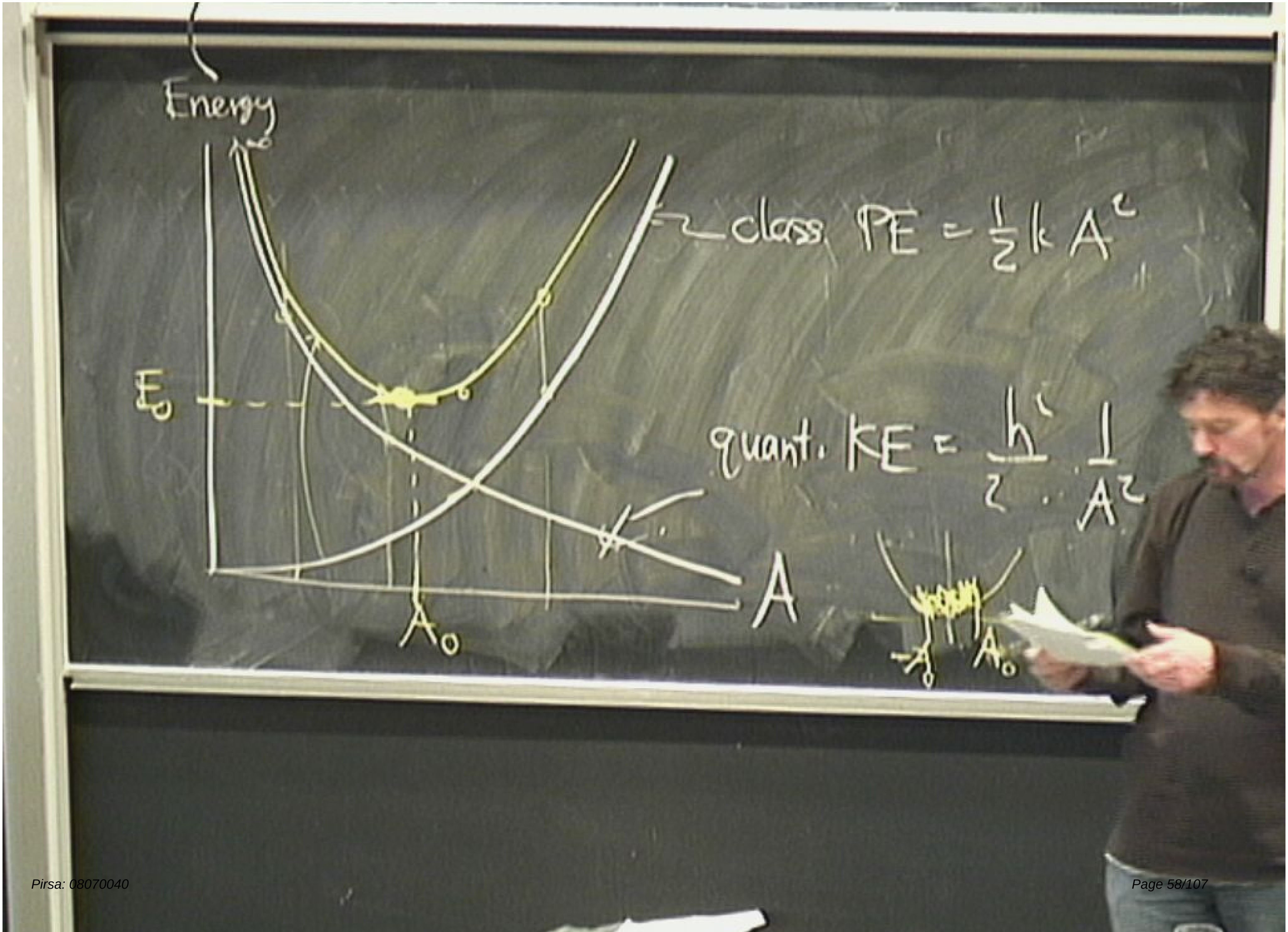
Energy
class. PE = ½ k A²
quant. KE = ħ²/2 · 1/A²
A
A₀
-A
A



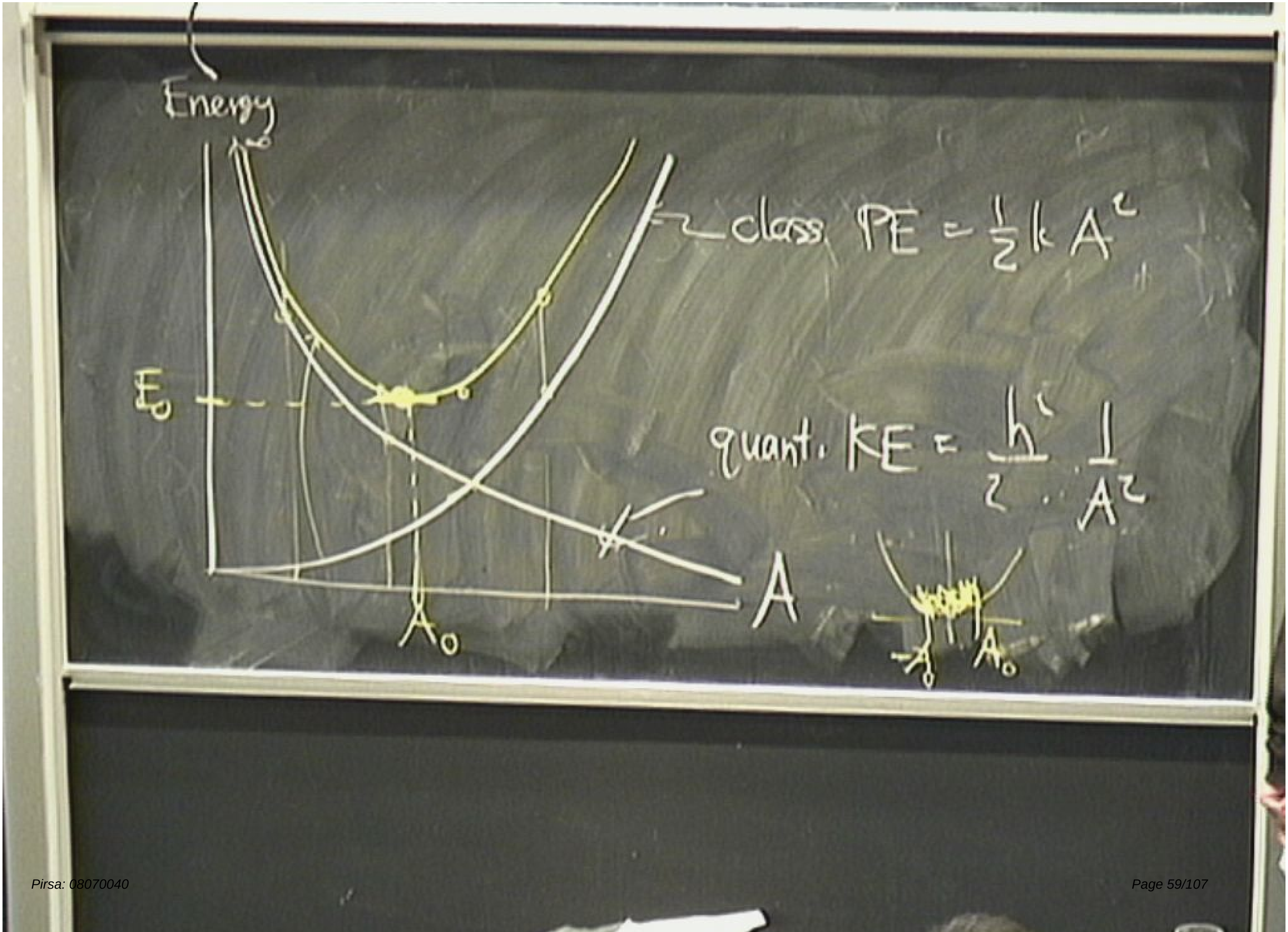
Energy
class. PE = 1/2 k A^2
quant. KE = h^2/2 · 1/A^2
A
A0
-A0
A0


Energy
class. PE = 1/2 k A^2
quant. KE = h^2/2 · 1/A^2
E
A
A0
-A0
A0

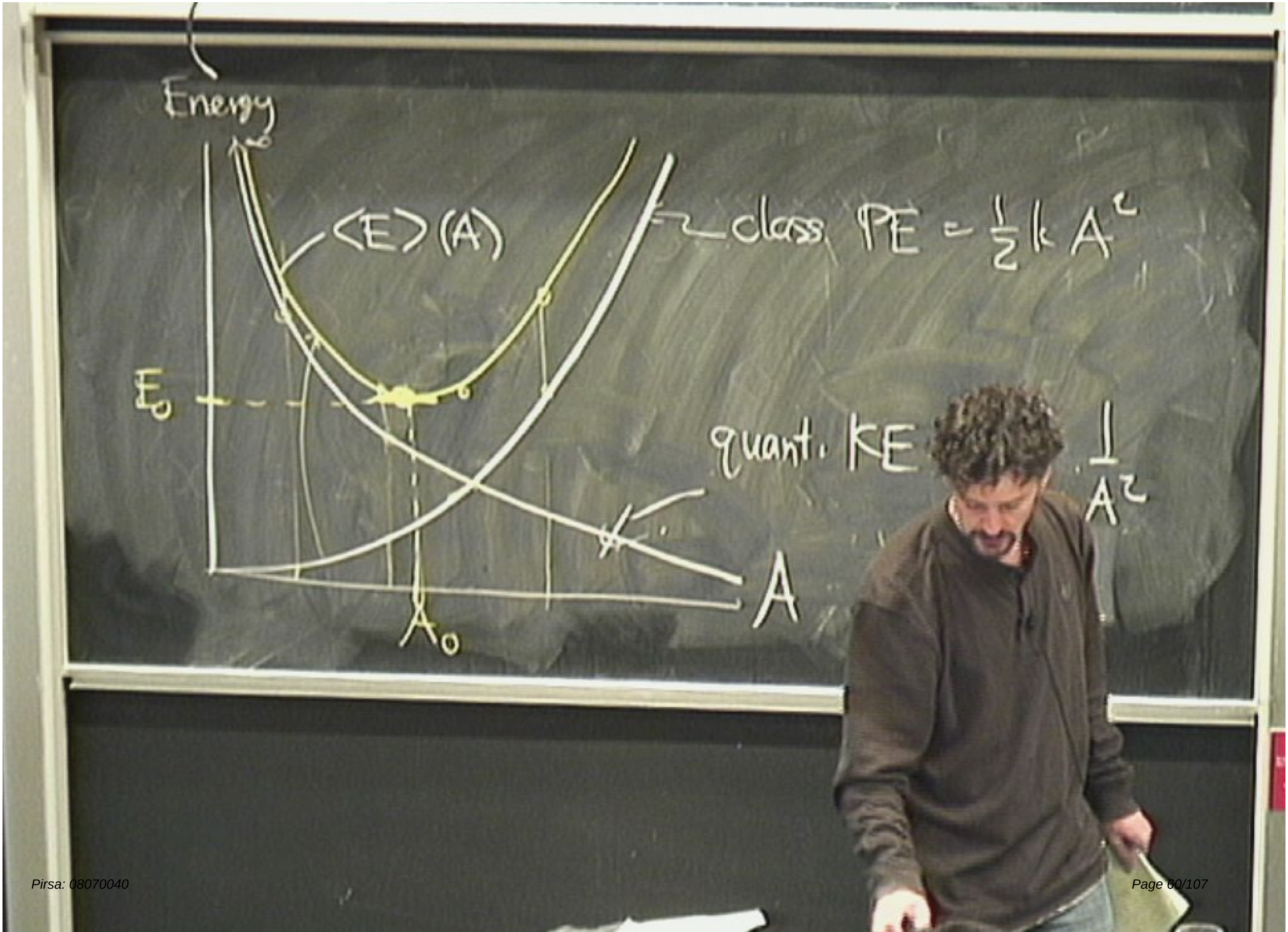

Energy
class. PE = ½ k A²
quant. KE = h²/2 · 1/A²
A
A₀
-A₀ A₀

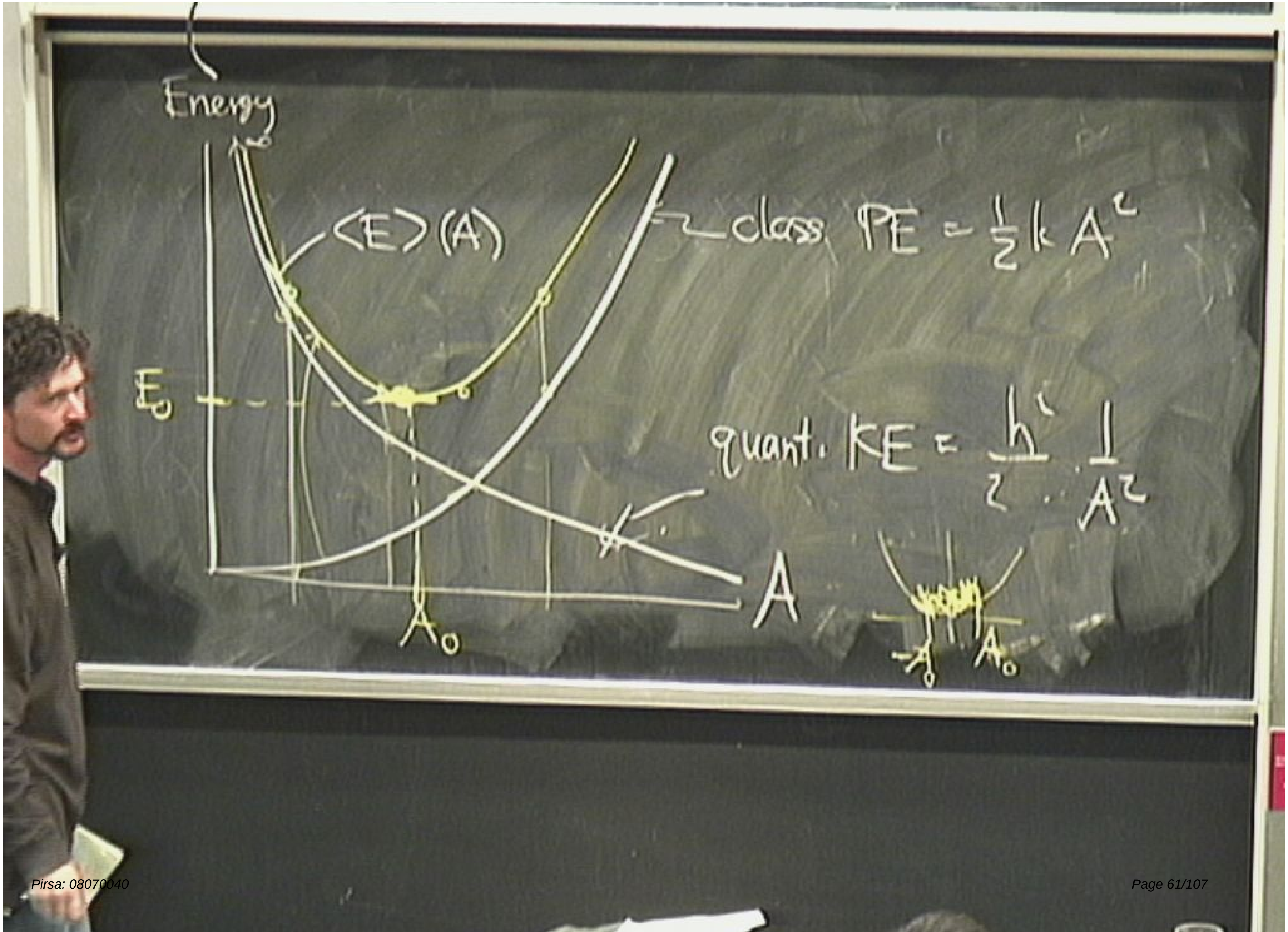


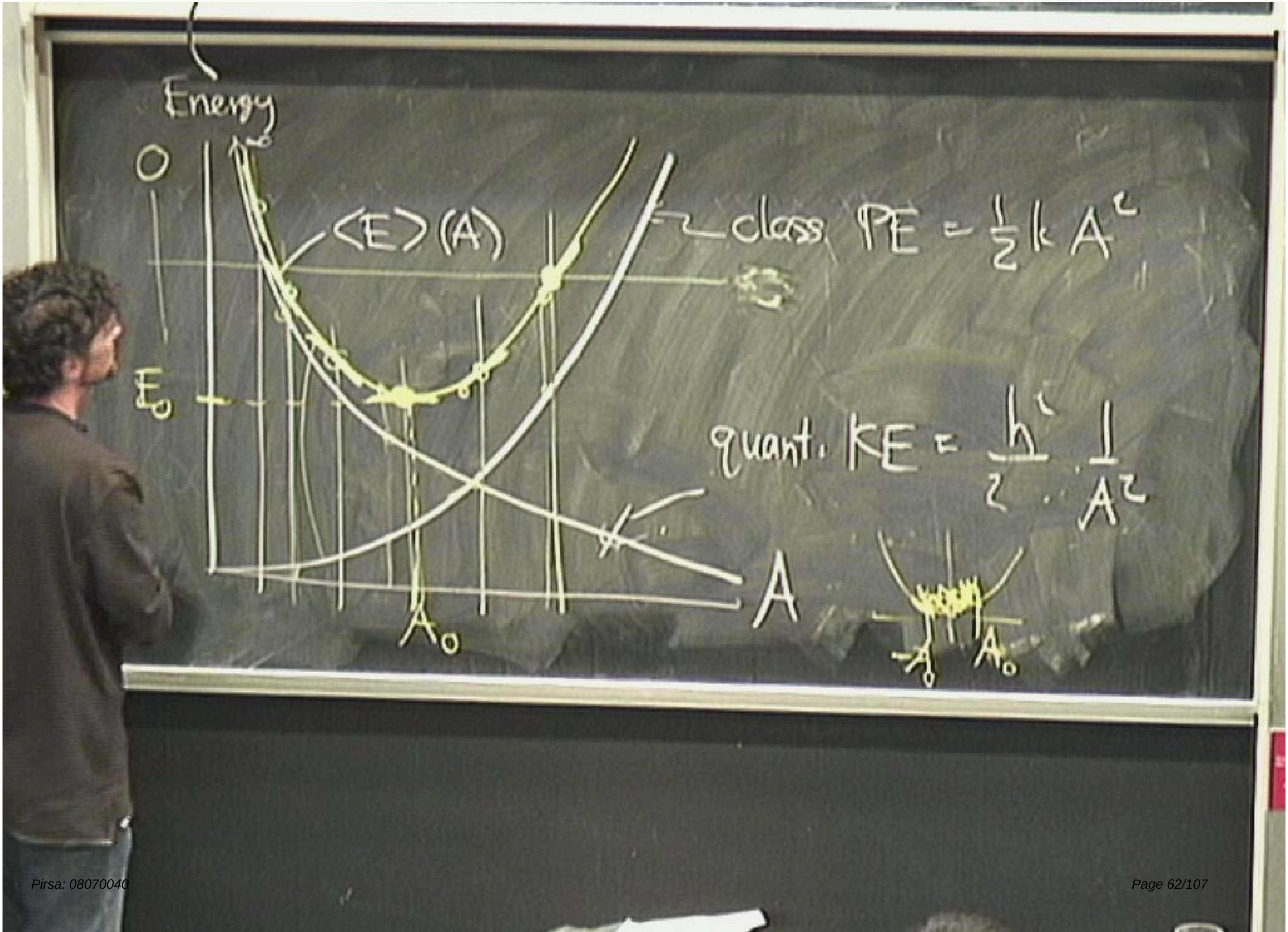
Energy
E_0
class. $PE = \frac{1}{2} k A^2$
quant. $KE = \frac{h^2}{2} \cdot \frac{1}{A^2}$
A
A_0
$-A_0$
A_0

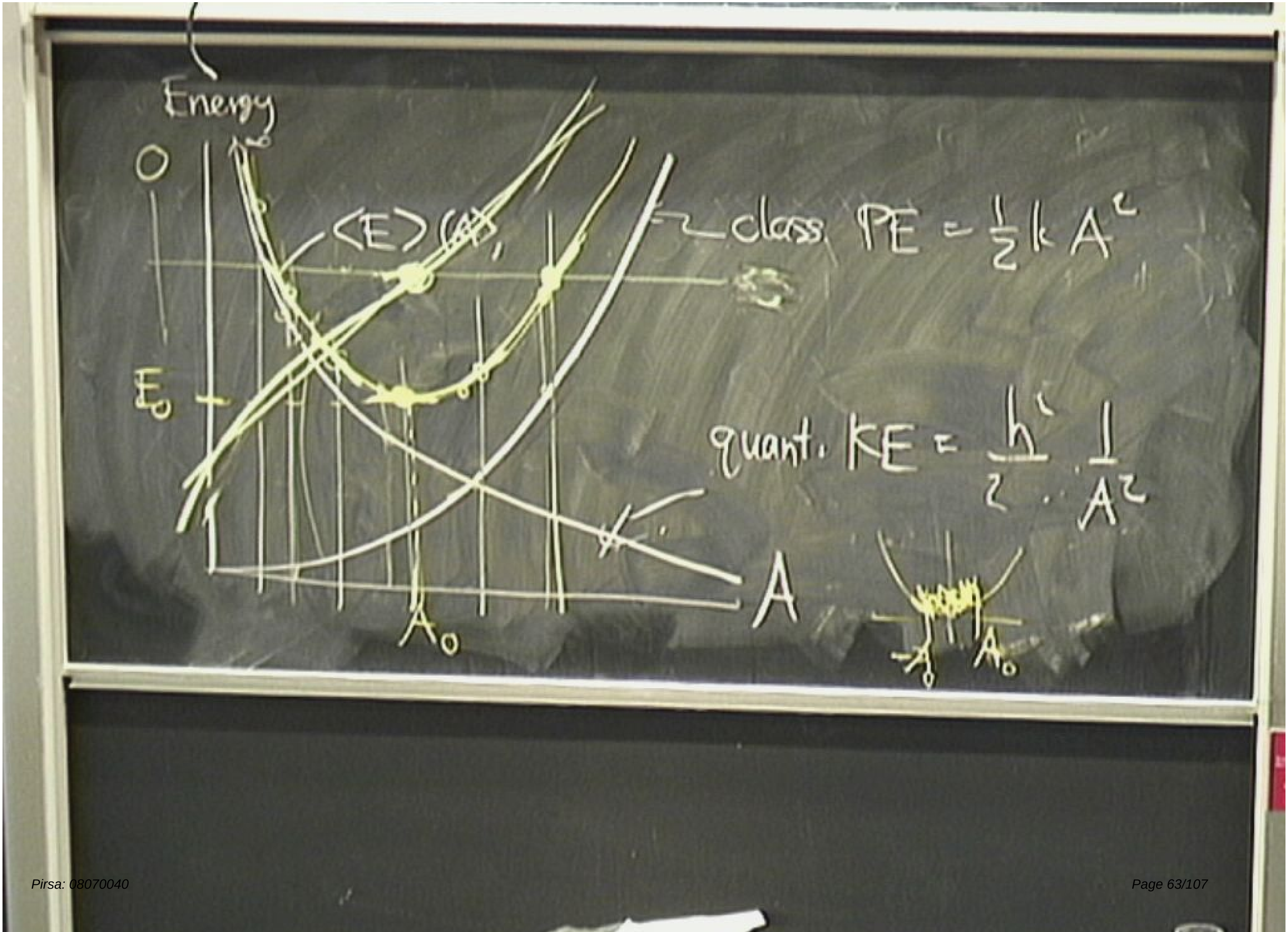


Energy
E_0
class. $PE = \frac{1}{2} k A^2$
quant. $KE = \frac{\hbar^2}{2} \cdot \frac{1}{A^2}$
A
A_0
$-A_0$
A_0


Energy
<E>(A)
class. PE = ½ k A²
E₀
quant. KE
1/A²
A
A₀


Energy
<E>(A)
class. PE = ½ k A²
quant. KE = ħ²/2 · 1/A²
E₀
A
A₀
-A₀
A₀


Energy
O
⟨E⟩(A)
class. PE = ½ k A²
E₀
quant. KE = ħ²/2 · 1/A²
A
A₀
-A₀
A₀


Energy
O
⟨E⟩(A),
class. PE = ½ k A²
E₀
quant. KE = h²/2 · 1/A²
A
A₀
−A₀
A₀

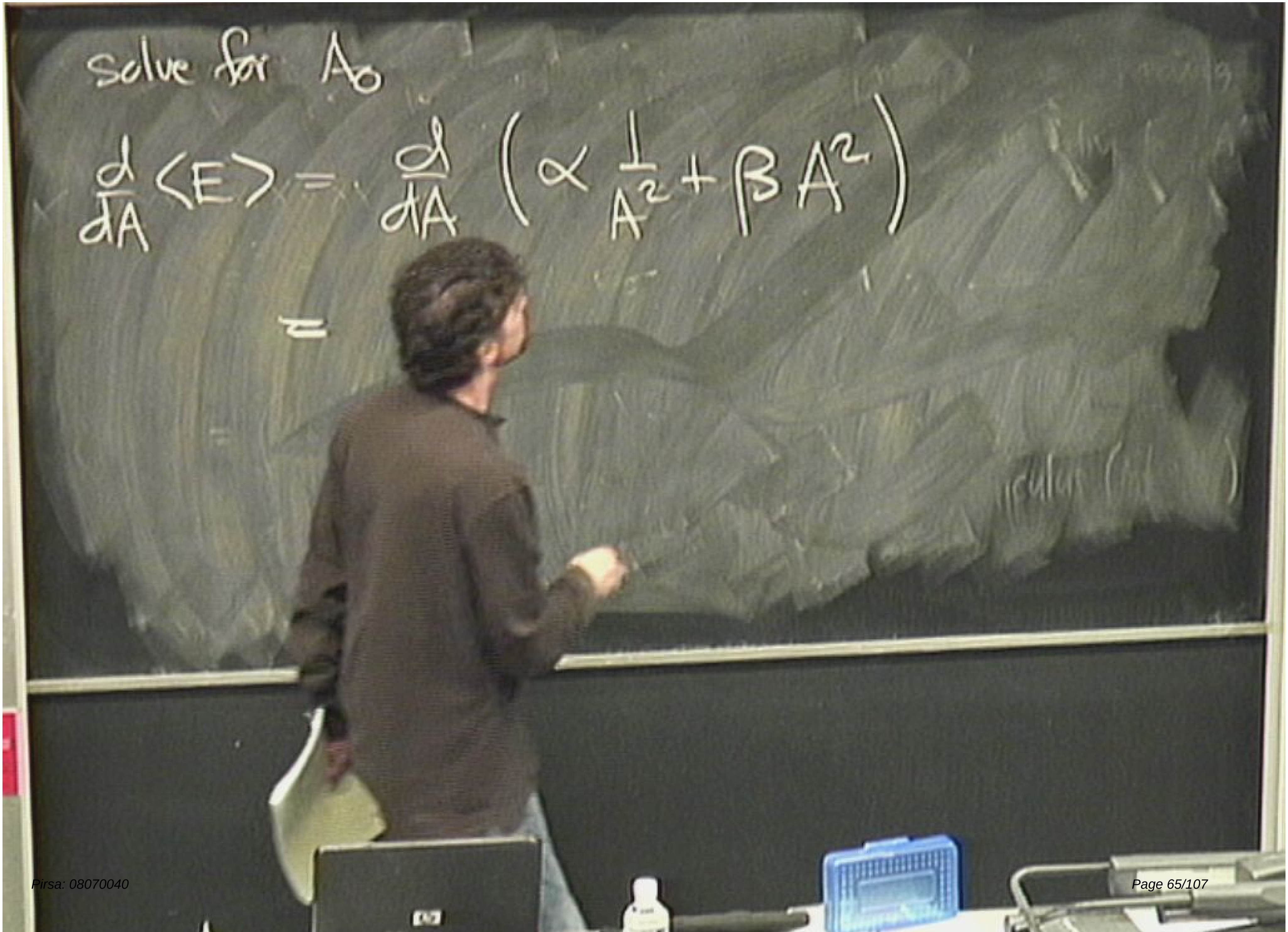
$$\langle E \rangle = \underbrace{\frac{h^2}{32\pi^2 m A^2}}_{\alpha} + \underbrace{\tfrac{1}{2}k}_{\beta} A^2$$

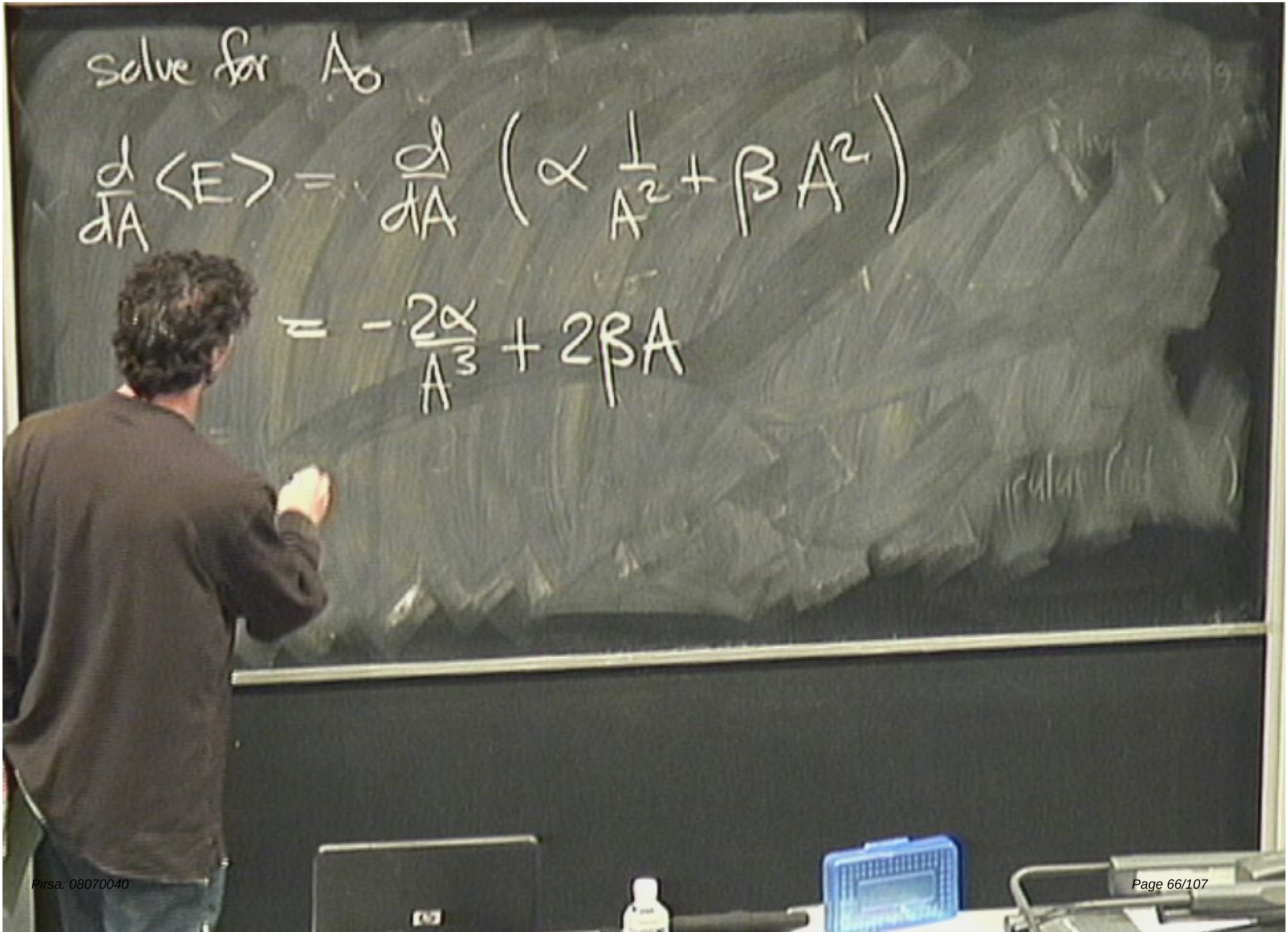
classical

purely quantum effect (disappears when $h=0$)

class. PE $\downarrow$ $A\downarrow$

quant. KE $\uparrow$ $A\downarrow$


solve for A_0
$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha\frac{1}{A^2}+\beta A^2\right)$
$=$

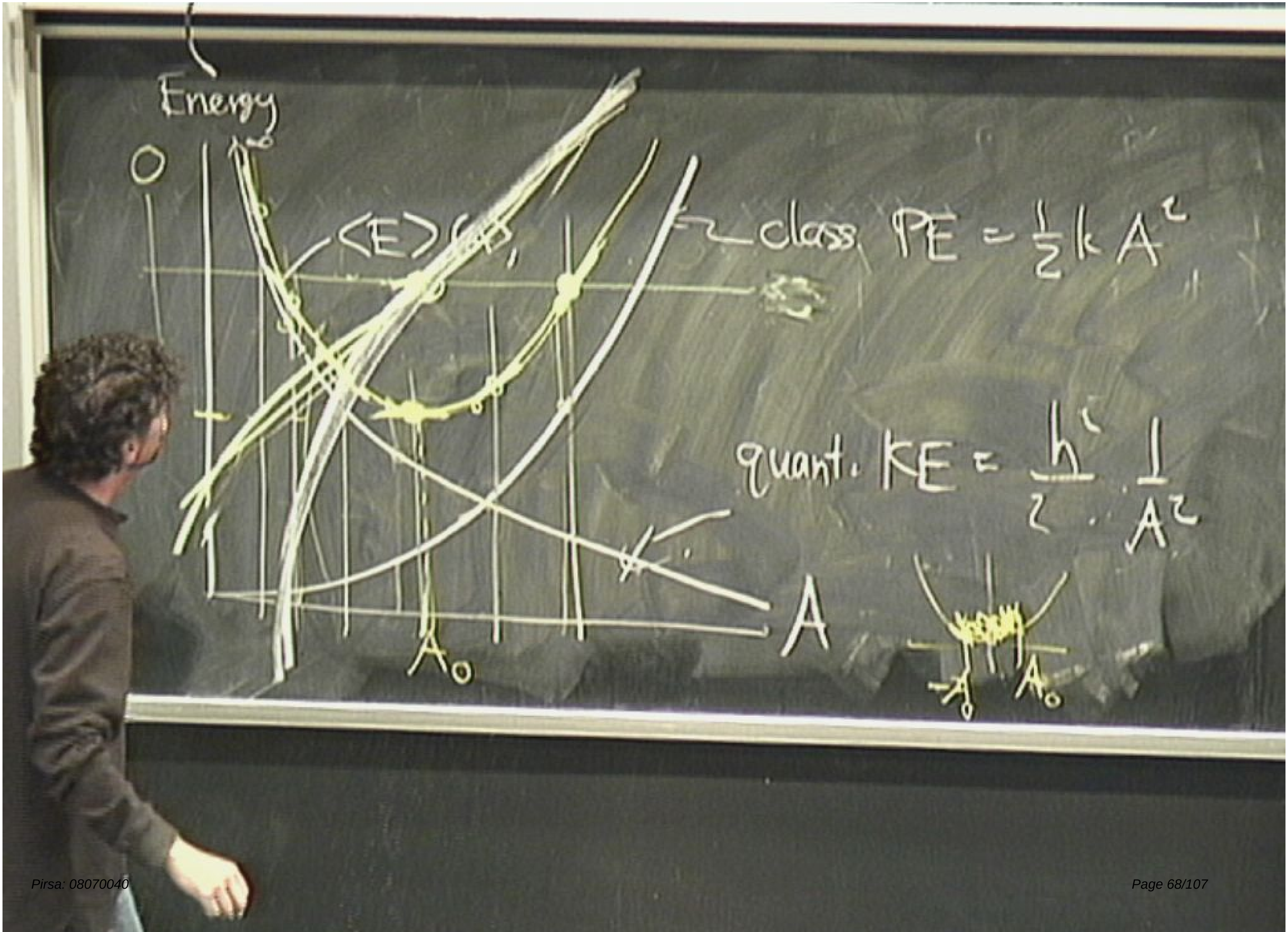

solve for A_0
$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha \frac{1}{A^2} + \beta A^2\right)$
$= -\frac{2\alpha}{A^3} + 2\beta A$

solve for A_0

$$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha\frac{1}{A^2}+\beta A^2\right)$$

$$= -\frac{2\alpha}{A^3}+2\beta A$$

$$= 0$$


Energy
⟨E⟩
class. PE = ½ k A²
quant. KE = ħ²/2 · 1/A²
A
A₀
-A₀
A₀

solve for A_0

$$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha\frac{1}{A^2}+\beta A^2\right)$$

$$= \left.-\frac{2\alpha}{A^3}+2\beta A\right|_{A_0}$$

$$= 0$$

$$A_0^2 = \sqrt{\frac{\alpha}{\beta}} =$$

solve for A_0

$$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha\frac{1}{A^2}+\beta A^2\right)$$

$$= -\frac{2\alpha}{A^3}+2\beta A\Big|_{A_0}$$

$$= 0$$

$$A_0^2 = \sqrt{\frac{\alpha}{\beta}} = \frac{h}{4\pi\sqrt{mk}}$$

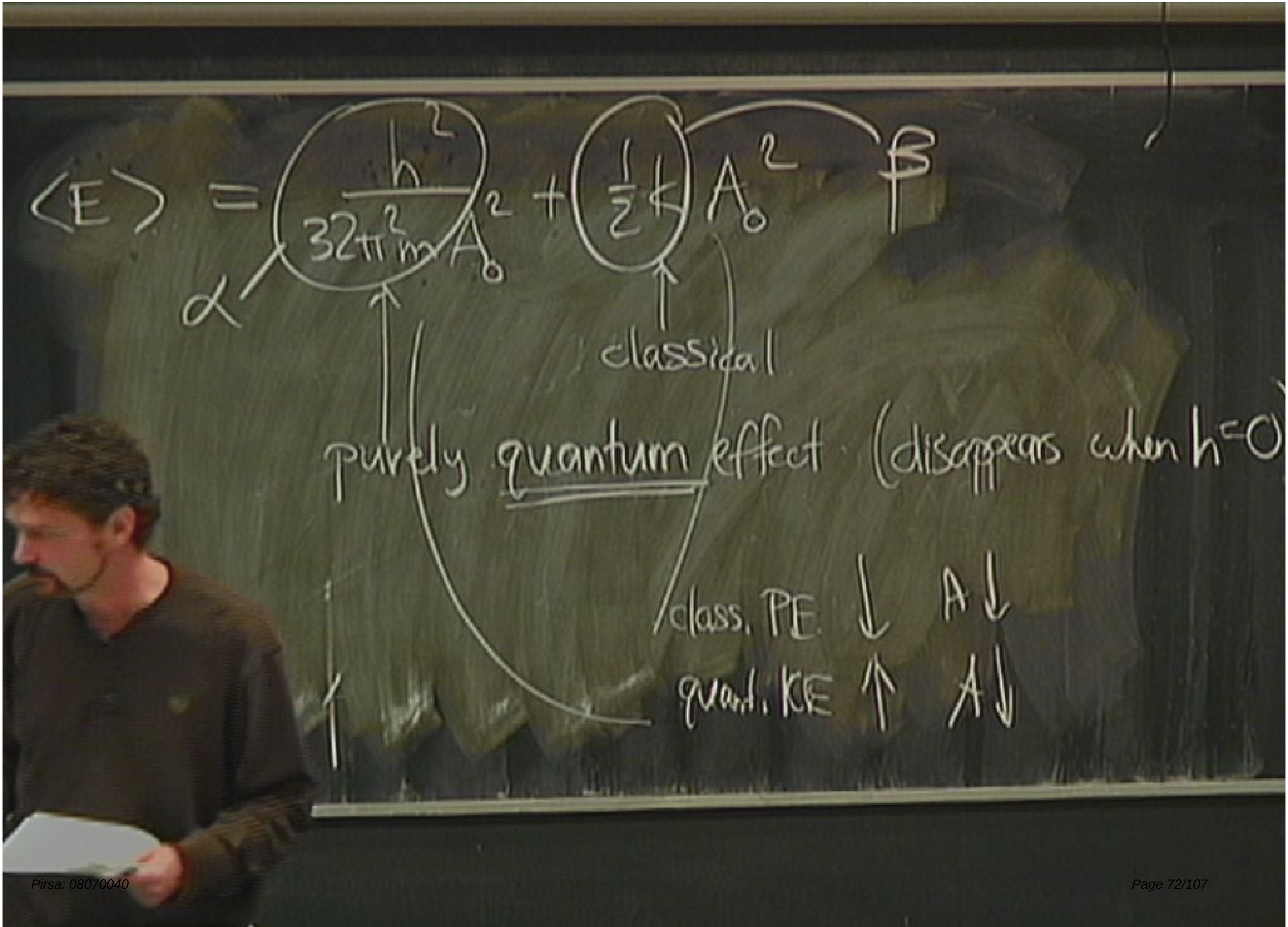
Solve for A_0

$$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha\frac{1}{A^2}+\beta A^2\right)$$

$$= -\frac{2\alpha}{A^3}+2\beta A\,\Big|_{A_0}$$

$$= 0$$

$$A_0^2 = \sqrt{\frac{\alpha}{\beta}} = \frac{h}{4\pi\sqrt{mk}}$$

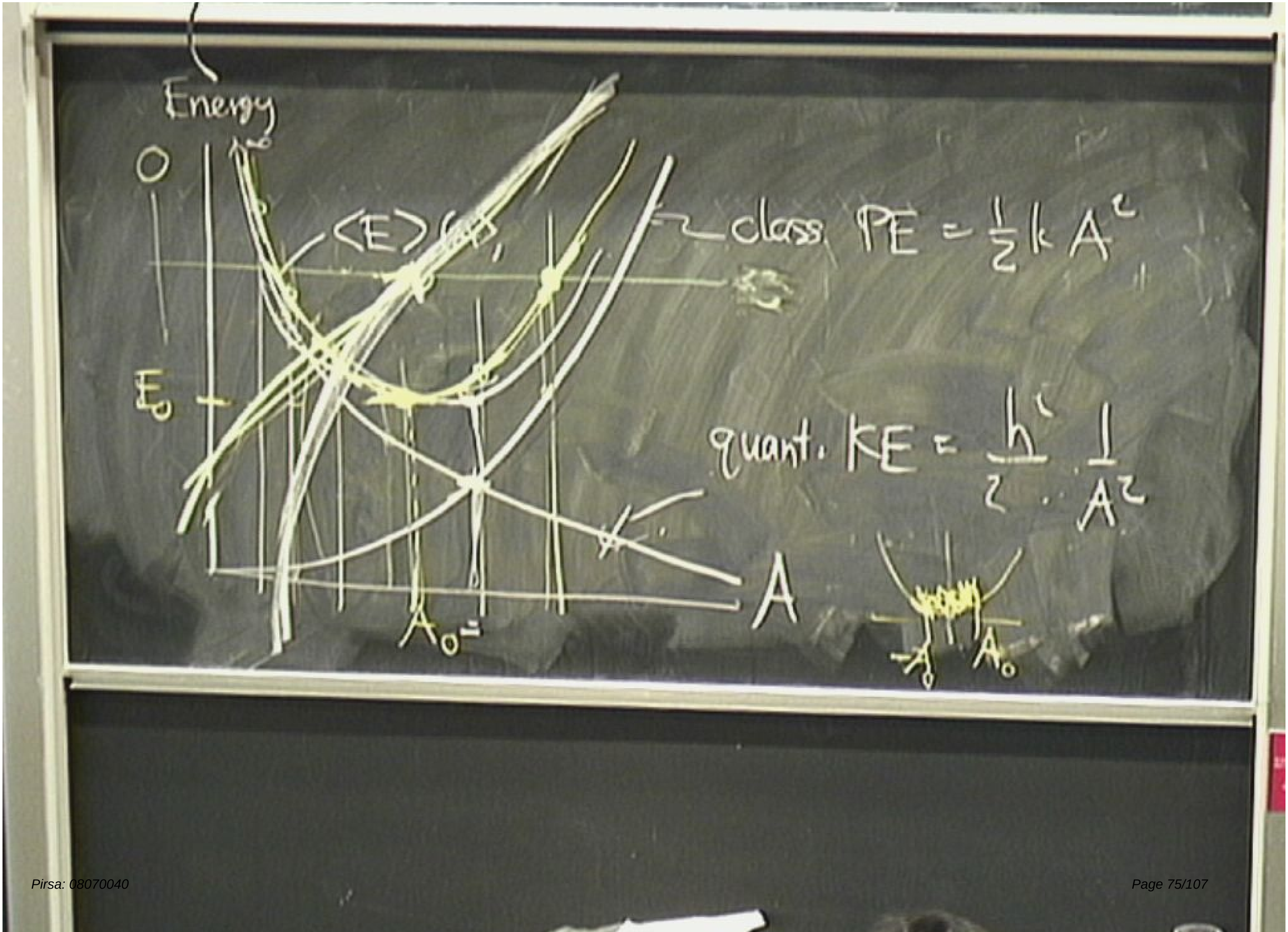

$\langle E \rangle = \frac{h^2}{32\pi^2 m A_0^2} + \frac{1}{2} k A_0^2$
α
β
classical
purely quantum effect (disappears when h=0)
class. PE ↓ A↓
quant. KE ↑ A↓

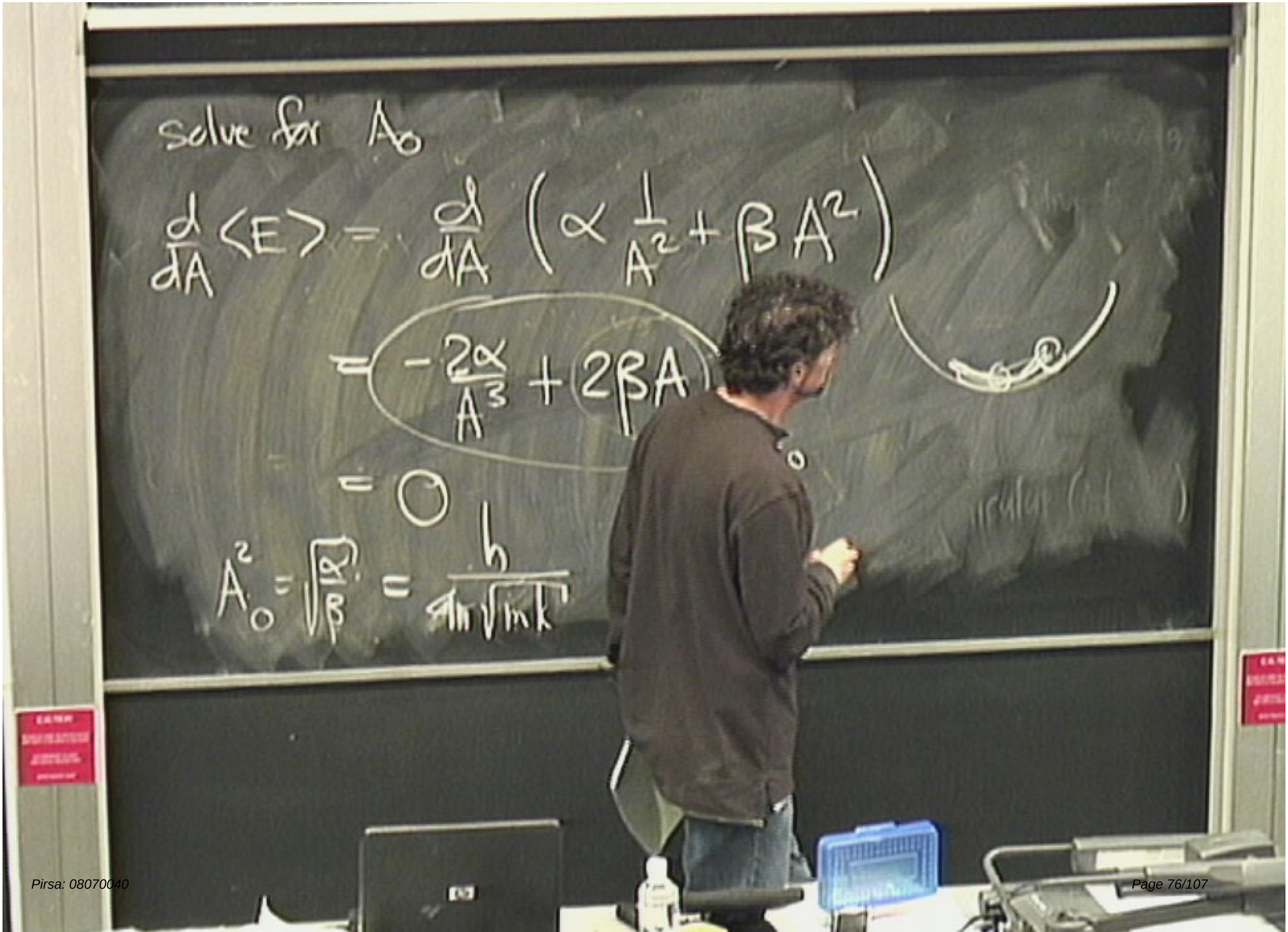
$$E_0 = \langle E \rangle \Big|_{A=A_0}$$

$$= \tfrac{1}{4} hf + \tfrac{1}{4} hf$$

$$E_0 = \langle E \rangle \big|_{A=A_0}$$

$$= \underset{\substack{\uparrow \\ KE}}{\tfrac{1}{4} hf} + \underset{\substack{\uparrow \\ PE}}{\tfrac{1}{4} hf}$$

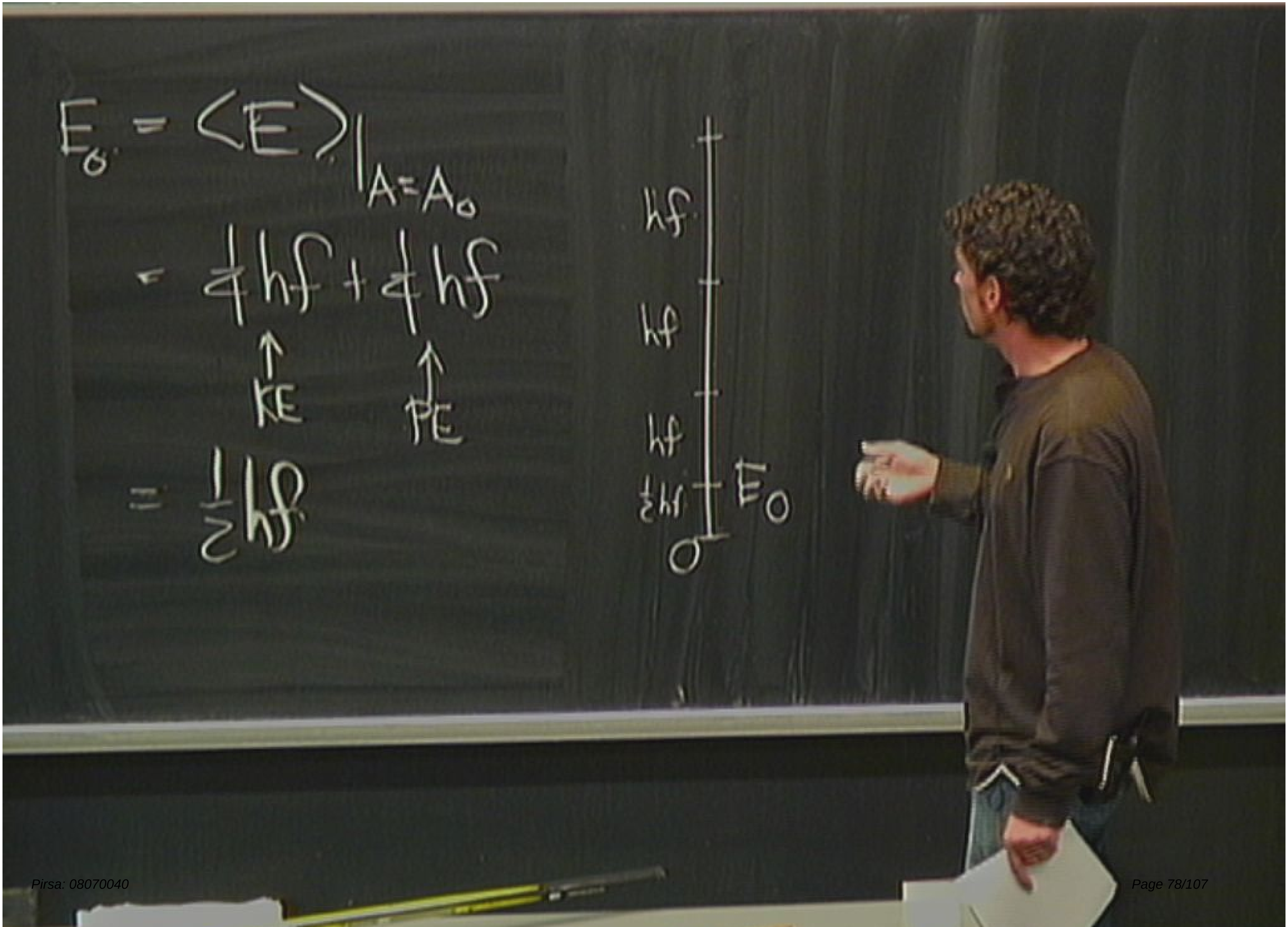

Energy
$\langle E \rangle$
class. $PE = \frac{1}{2} k A^2$
E_0
quant. $KE = \frac{\hbar^2}{2} \cdot \frac{1}{A^2}$
A
A_0
$-A_0$
A_0

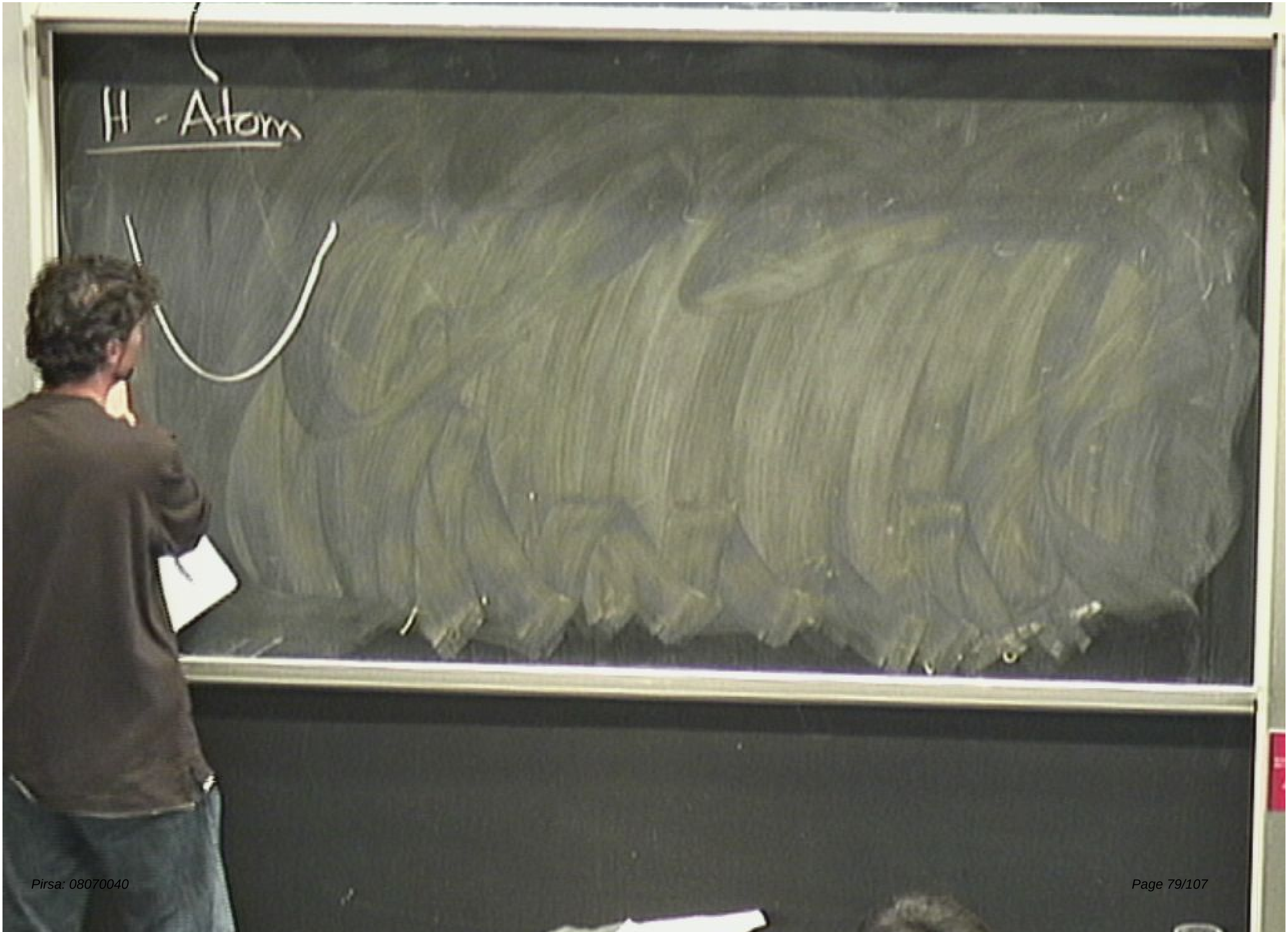

solve for A_0
$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha\frac{1}{A^2}+\beta A^2\right)$
$= -\frac{2\alpha}{A^3}+2\beta A$
$= 0$
$A_0^2 = \sqrt{\frac{\alpha}{\beta}} = \frac{h}{4\pi\sqrt{mk}}$

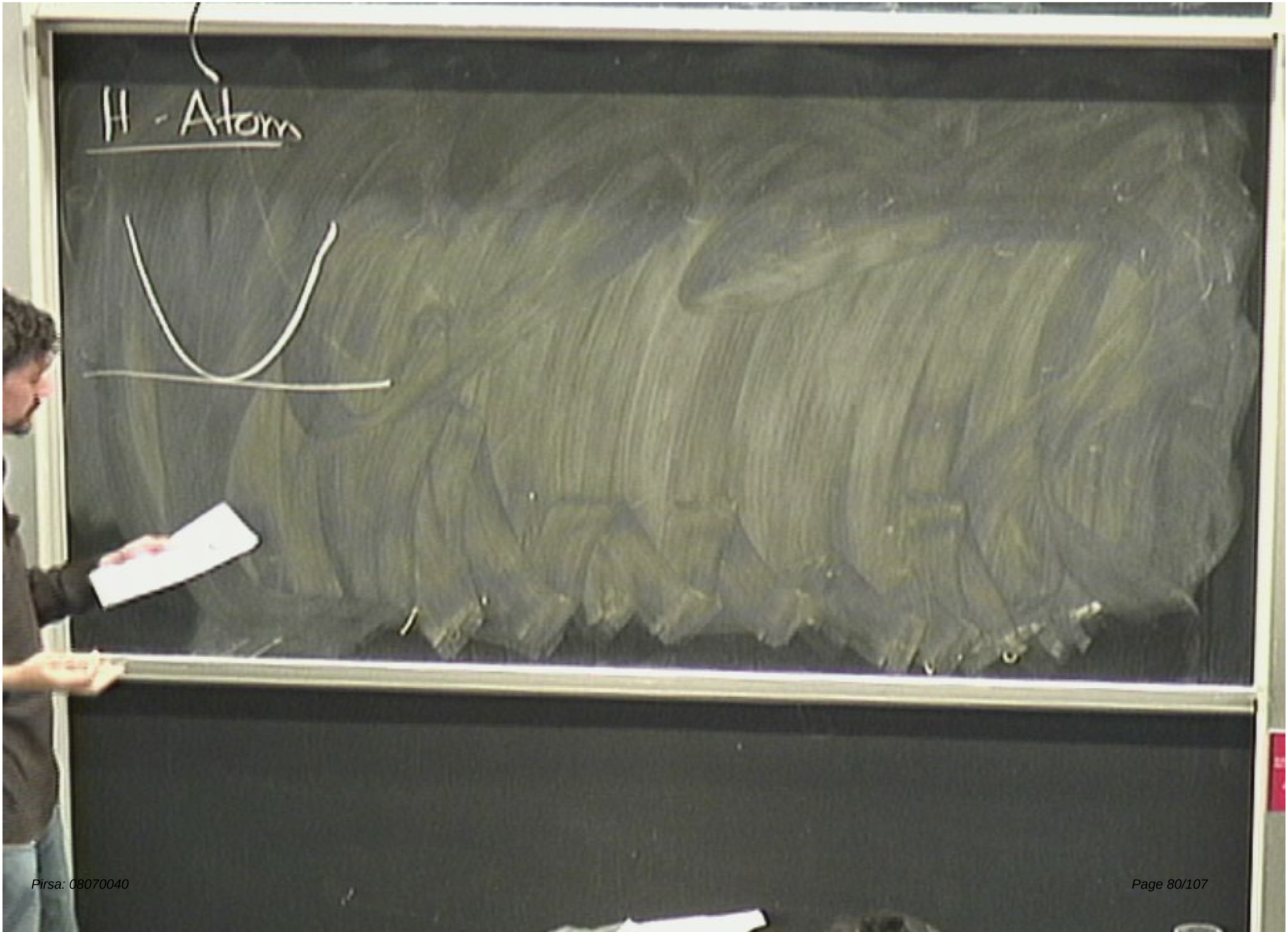
$$E_0 = \langle E \rangle \Big|_{A=A_0}$$

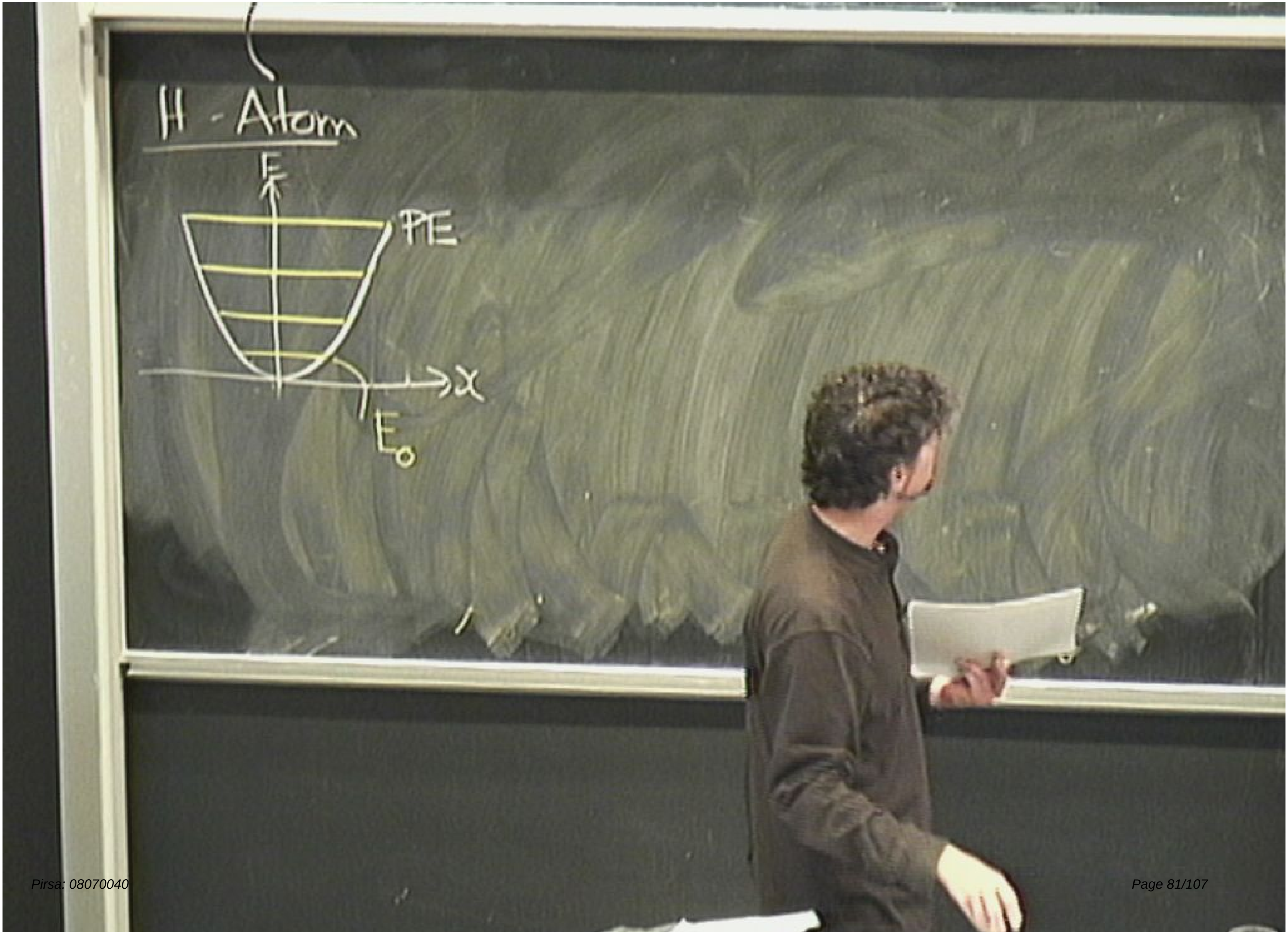
$$= \underset{\substack{\uparrow \\ KE}}{\tfrac{1}{4}hf} + \underset{\substack{\uparrow \\ PE}}{\tfrac{1}{4}hf}$$

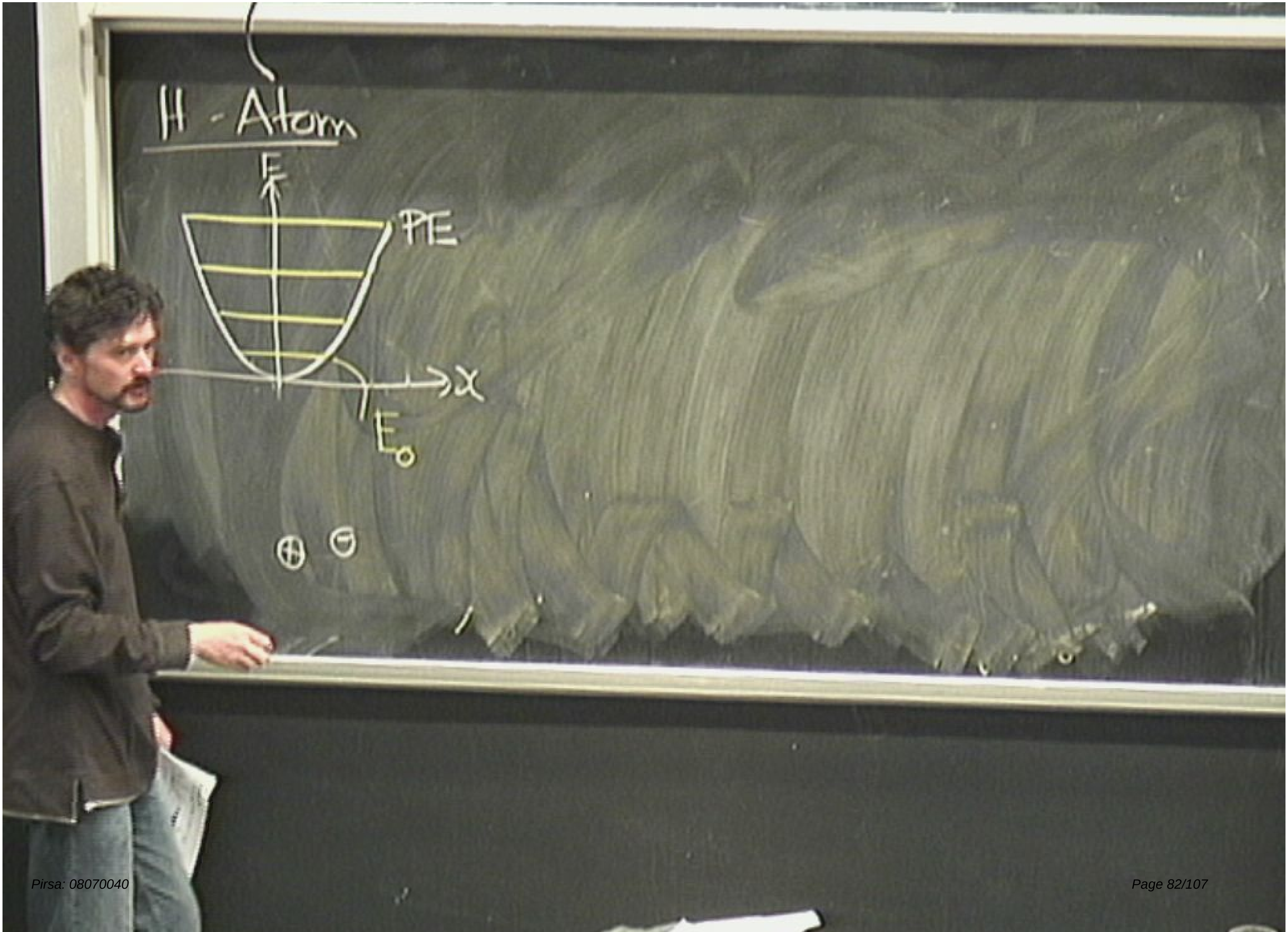
$$= \frac{1}{2}hf$$


E0 = ⟨E⟩|A=A0
= 1/4 hf + 1/4 hf
KE
PE
= 1/2 hf
hf
hf
hf
1/2 hf
E0
0

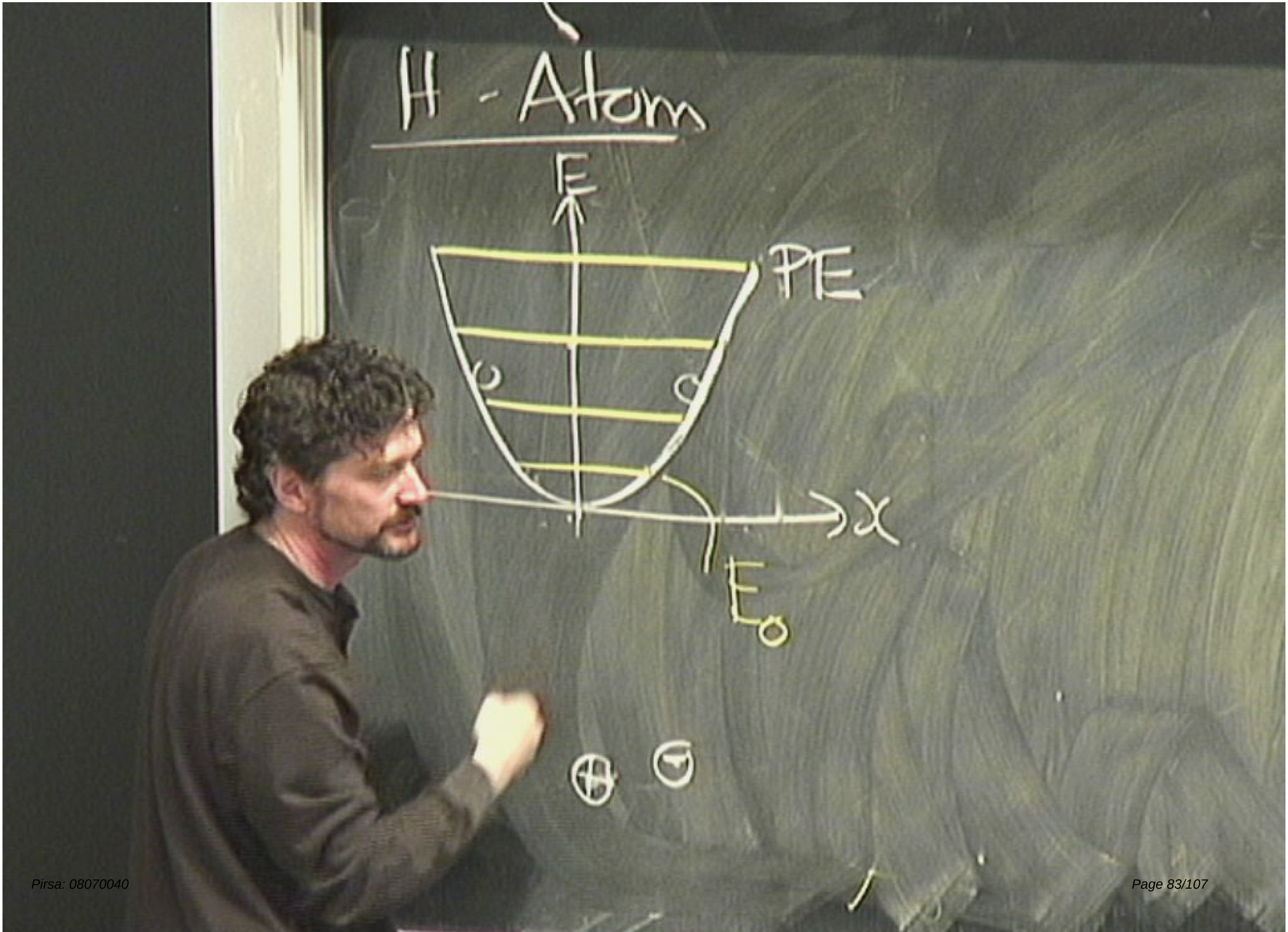

H - Atom

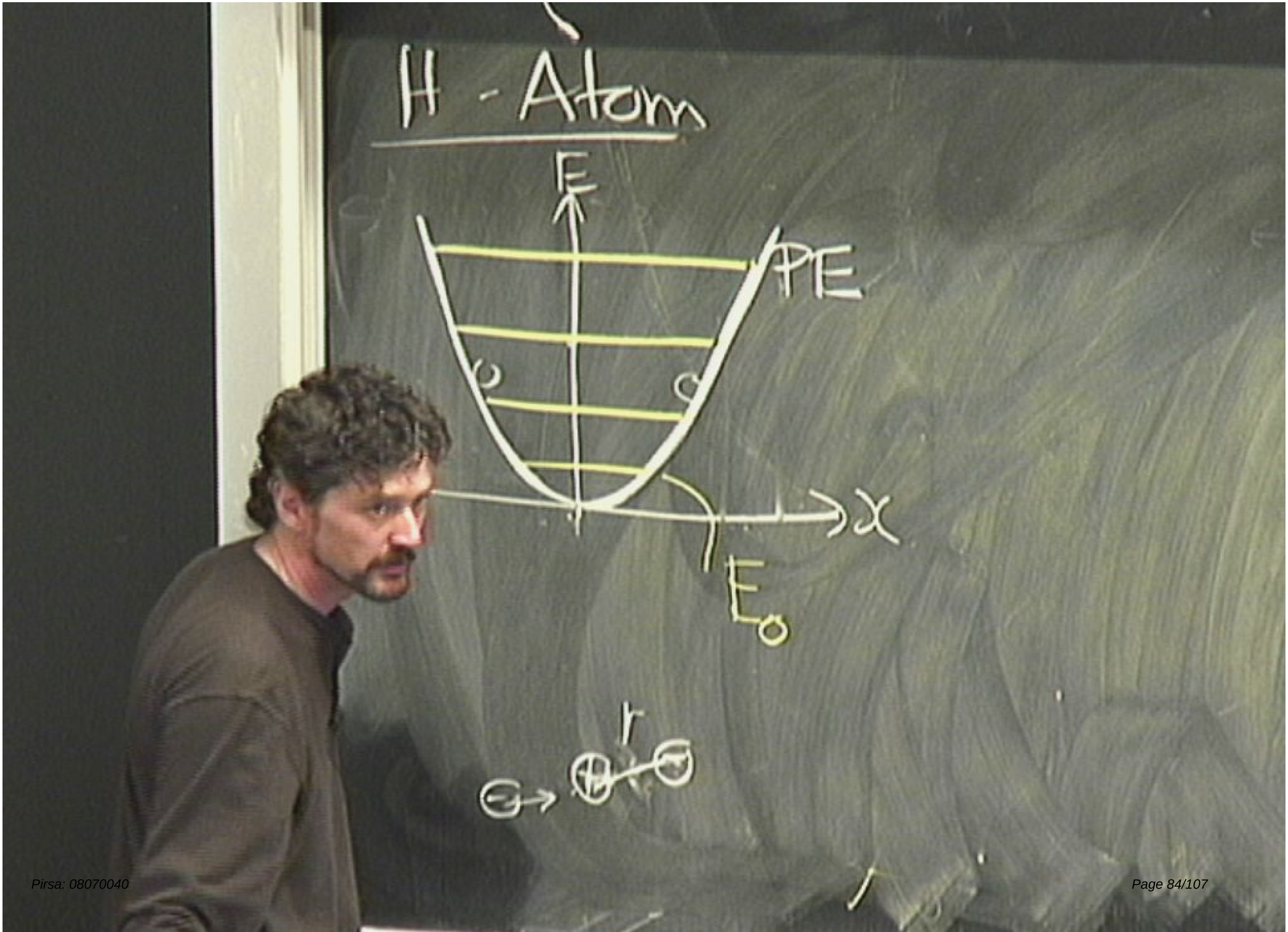

H - Atom


H - Atom
E
PE
x
E0

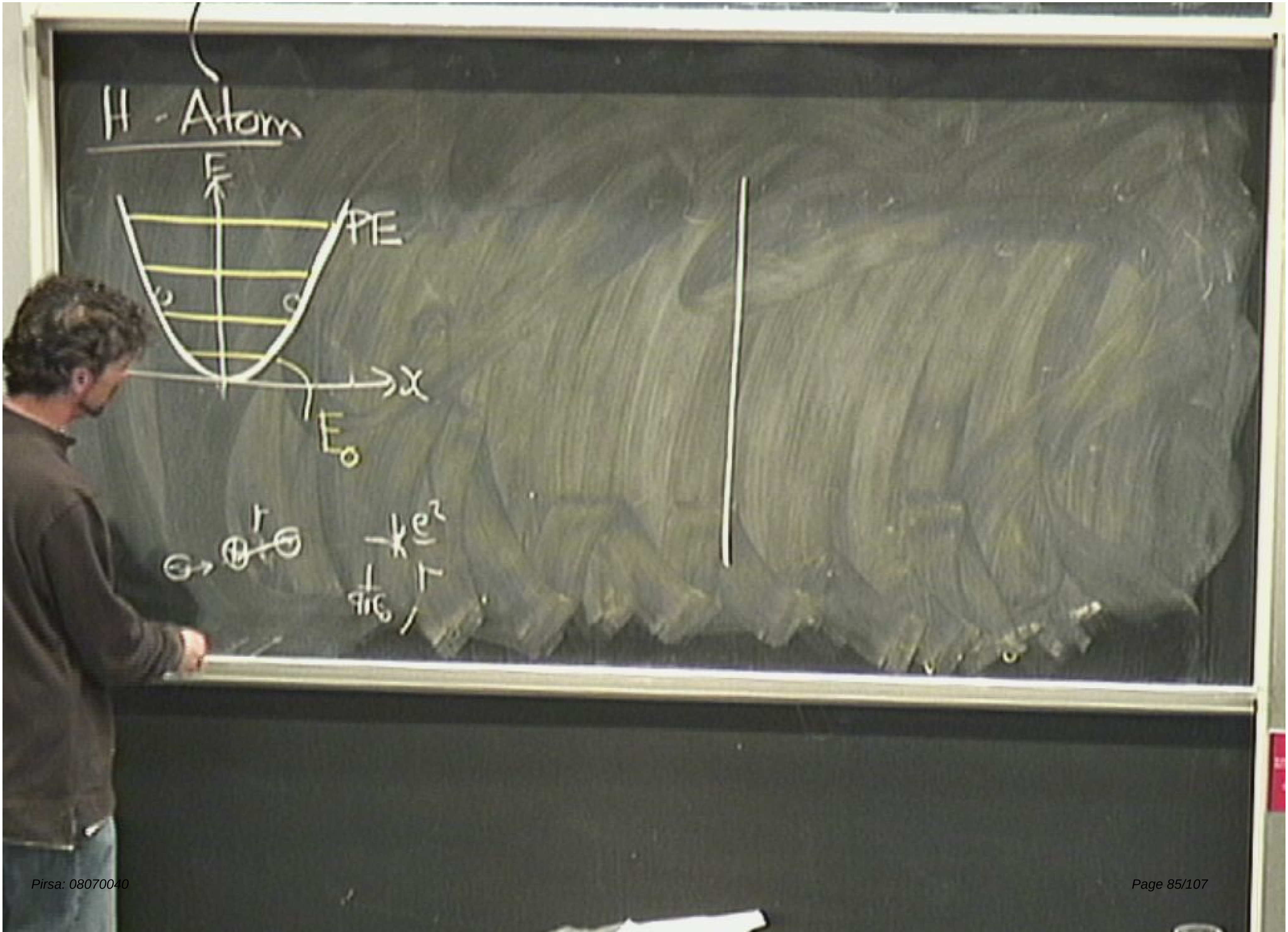


H - Atom
E
PE
x
E_0

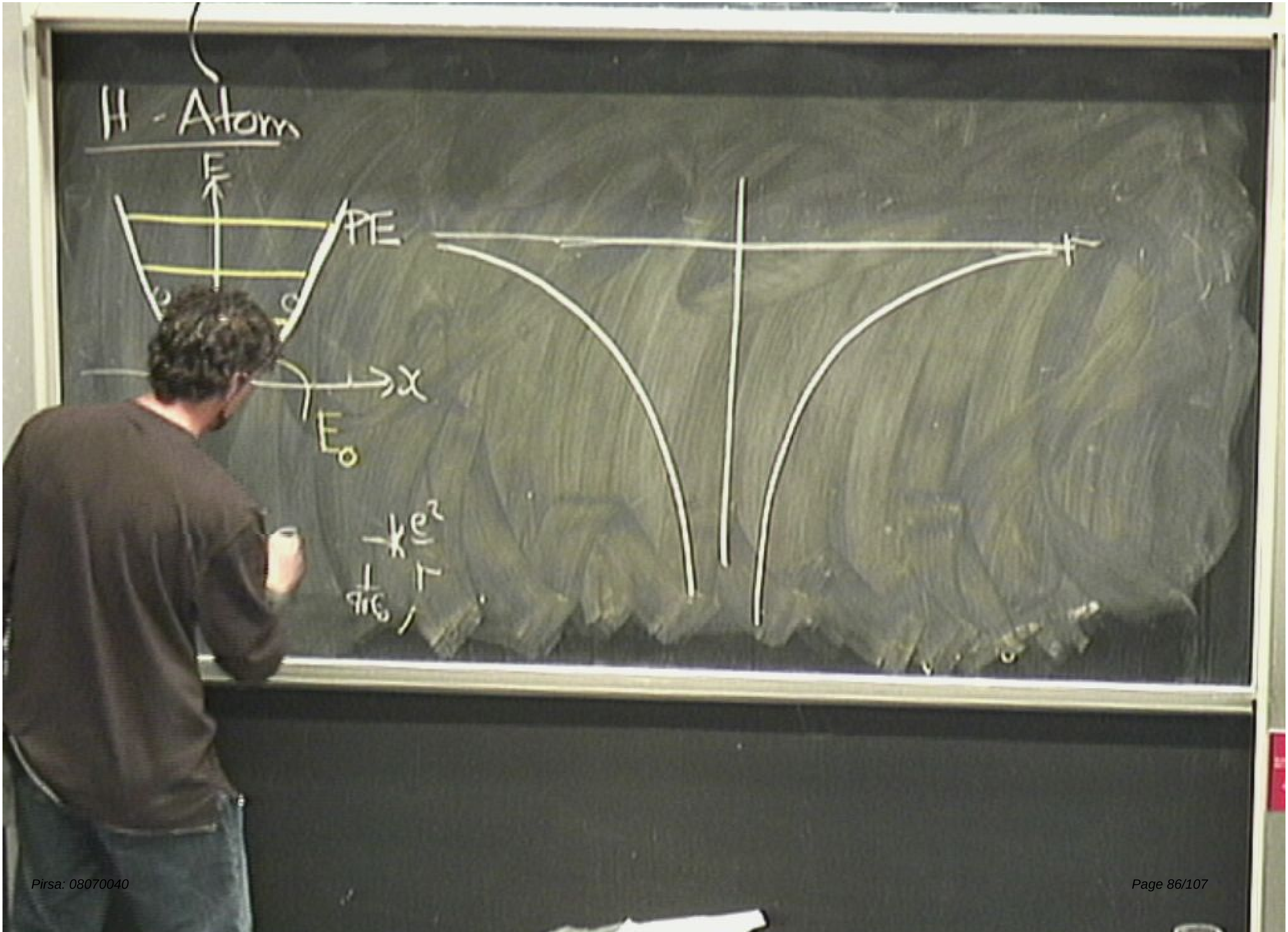

H - Atom
E
PE
x
E_0

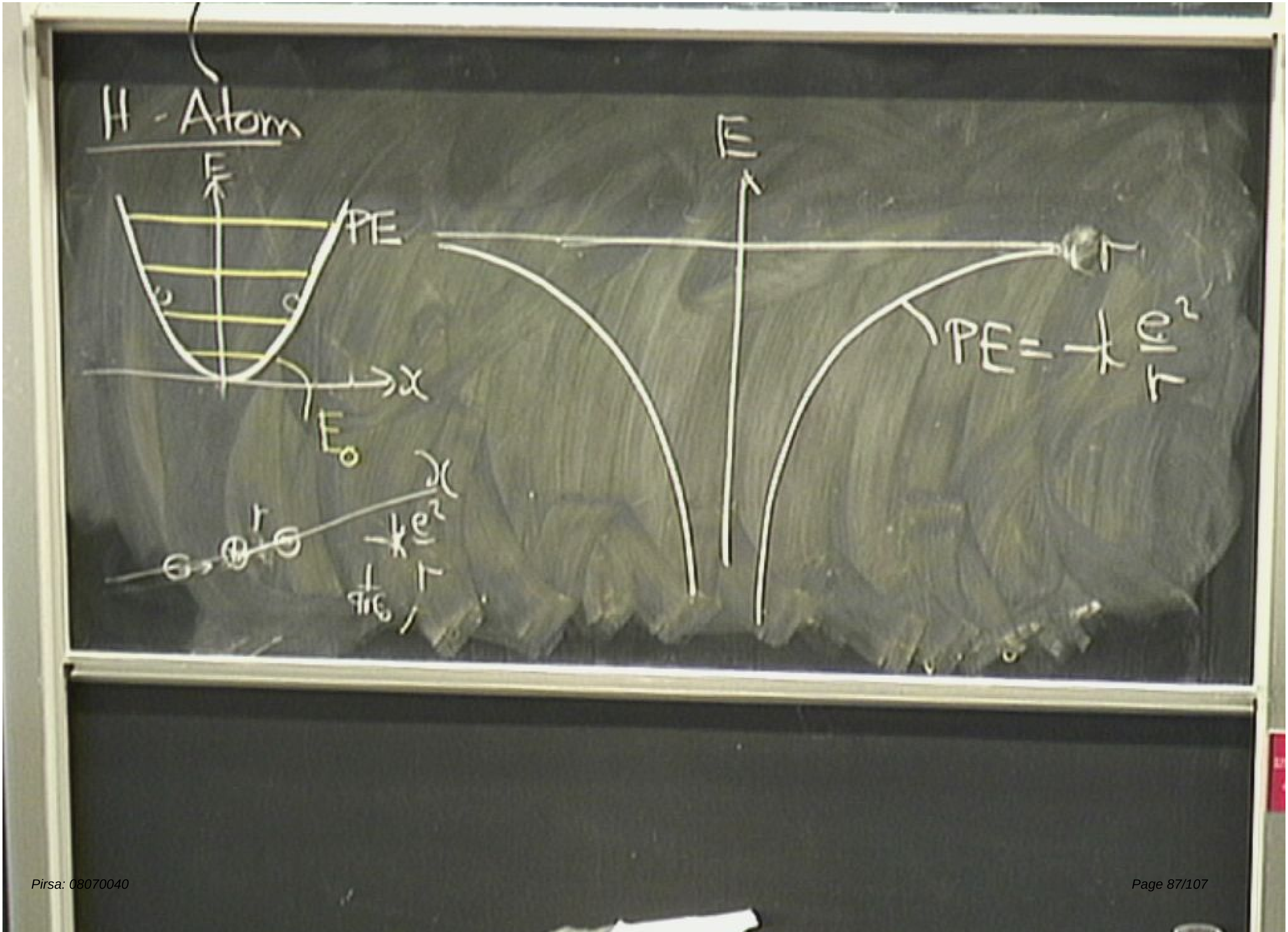


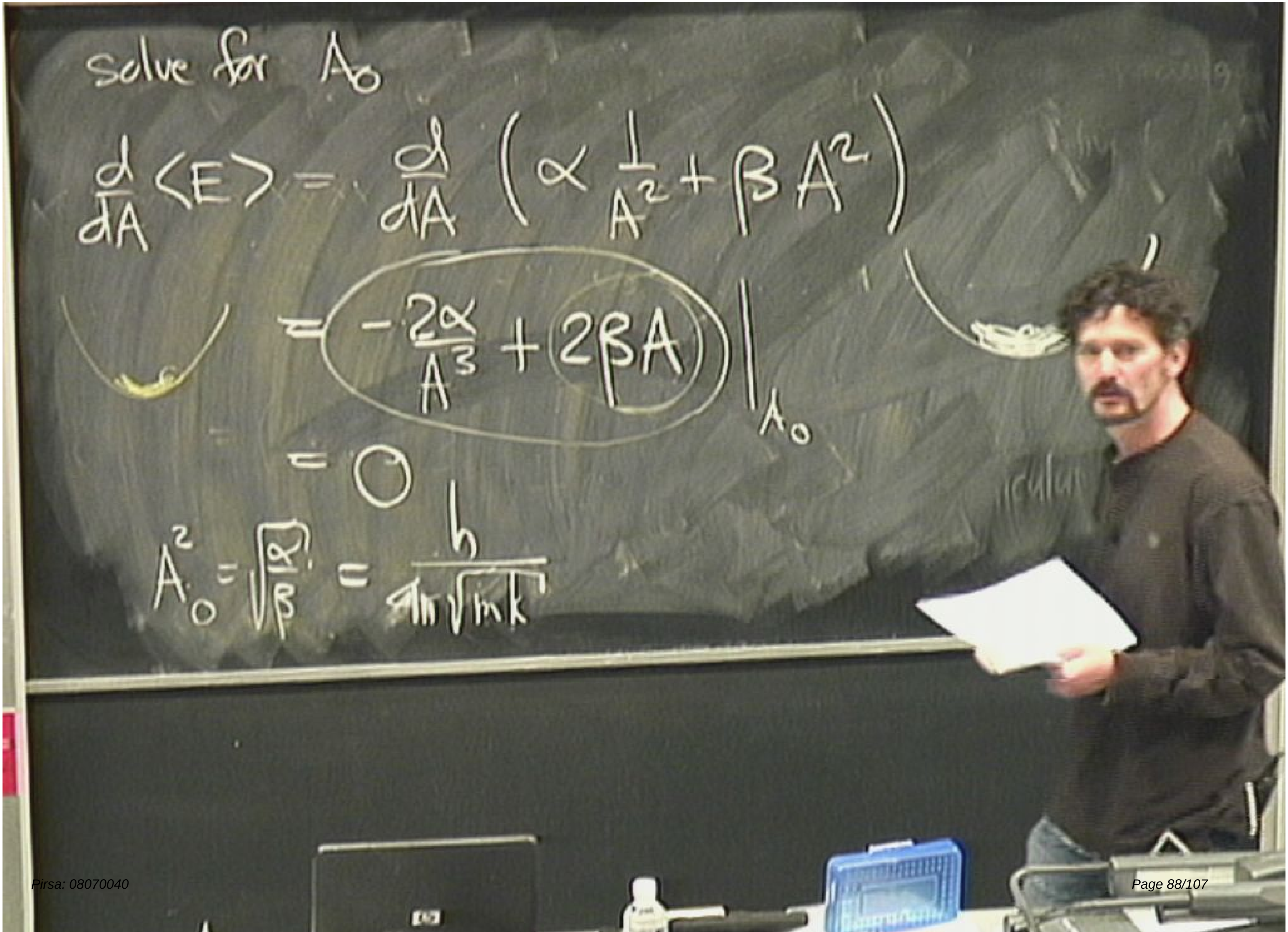
H - Atom
E
PE
x
E_0
r

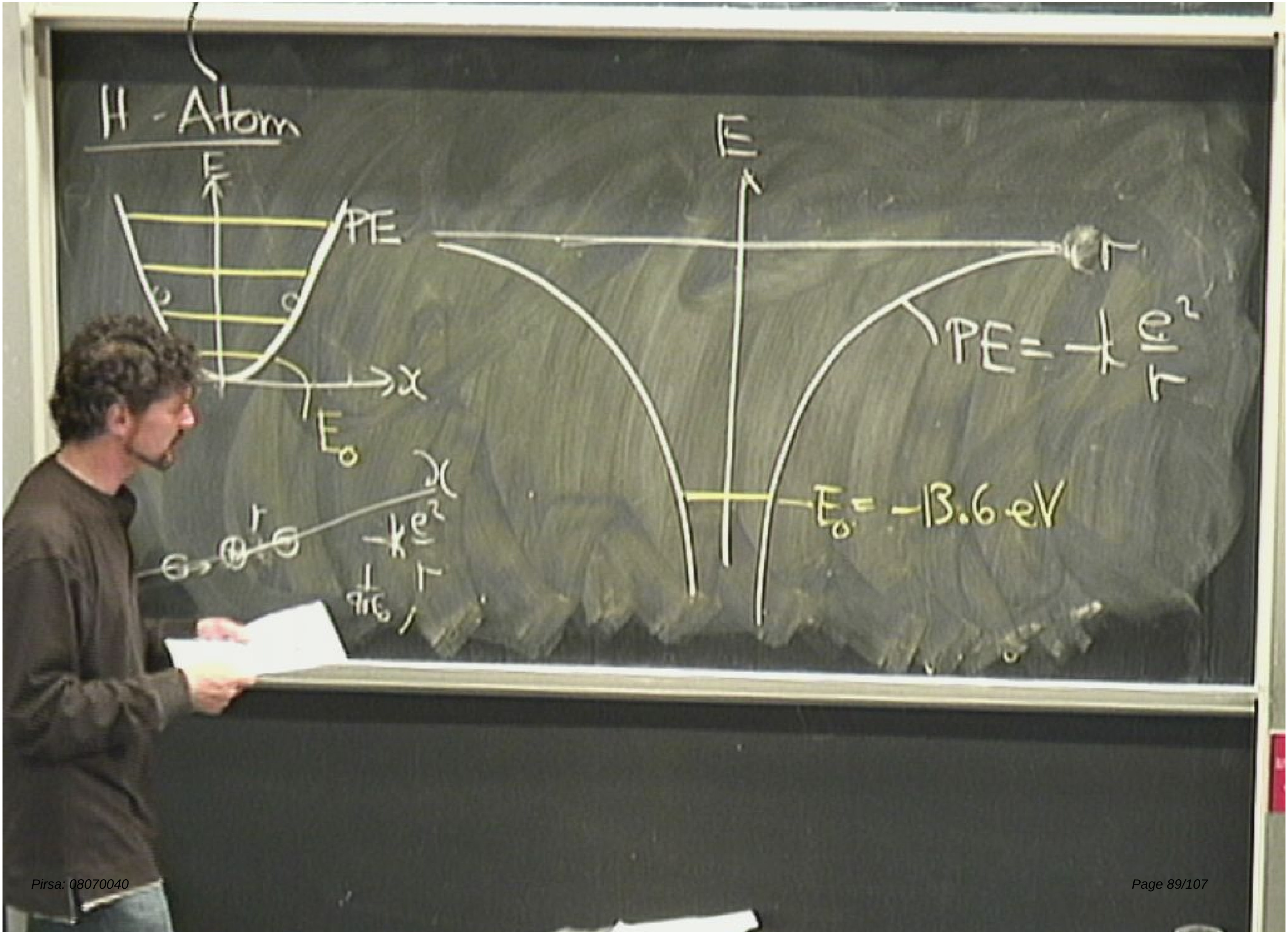


H - Atom
E
PE
x
E_0
r
$-k\frac{e^2}{r}$
$\frac{1}{4\pi\epsilon_0}$

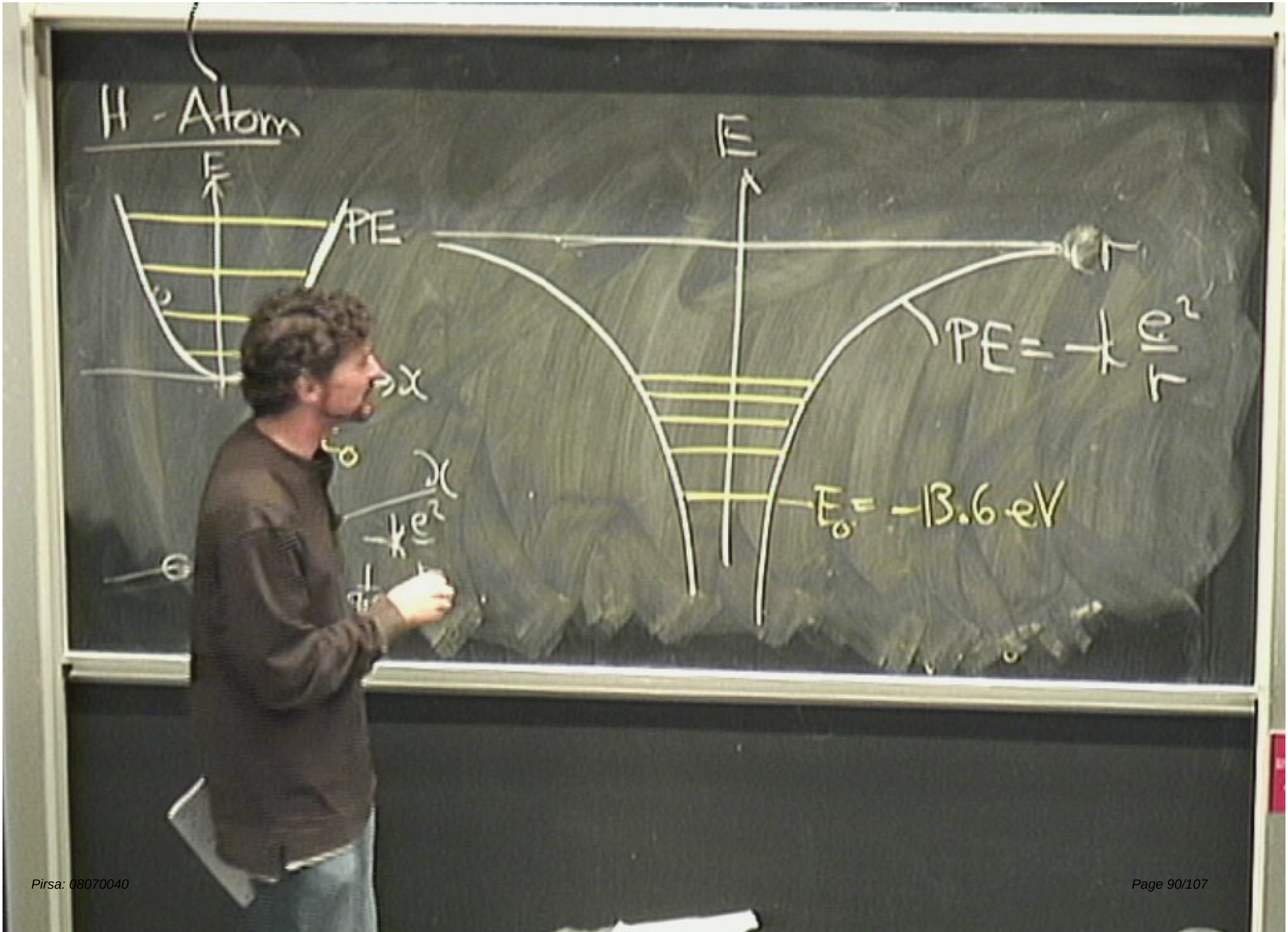

H - Atom
E
PE
x
E₀
-k e²/r
1/(4πε₀)

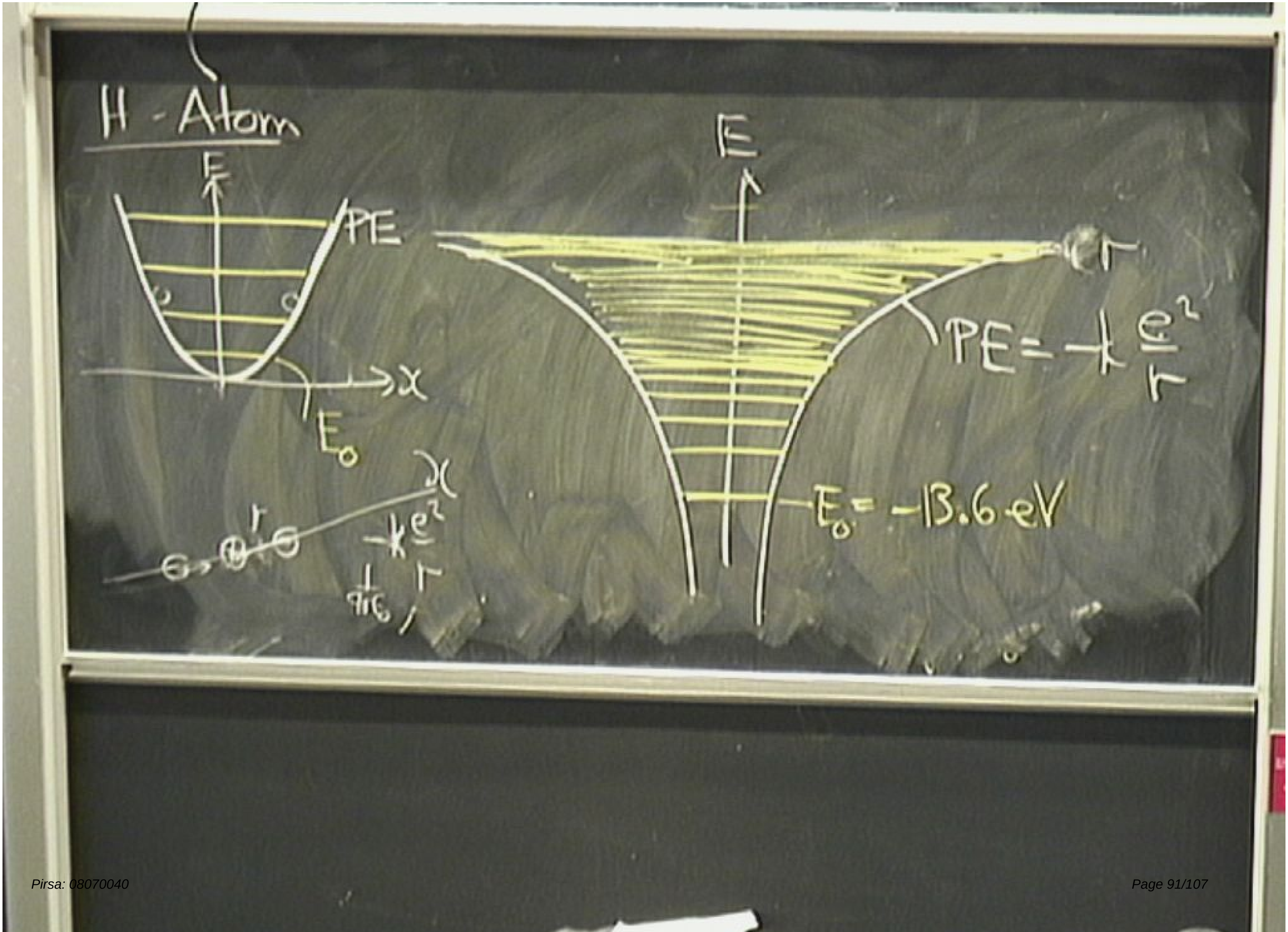

H - Atom
E
PE
x
E_0
E
r
$PE = -k\frac{e^2}{r}$
r
x
$-k\frac{e^2}{r}$
$\frac{1}{4\pi\epsilon_0}$


solve for A_0
$\frac{d}{dA}\langle E\rangle = \frac{d}{dA}\left(\alpha \frac{1}{A^2} + \beta A^2\right)$
$= -\frac{2\alpha}{A^3} + 2\beta A \Big|_{A_0}$
$= 0$
$A_0^2 = \sqrt{\frac{\alpha}{\beta}} = \frac{\hbar}{\sqrt{mk}}$

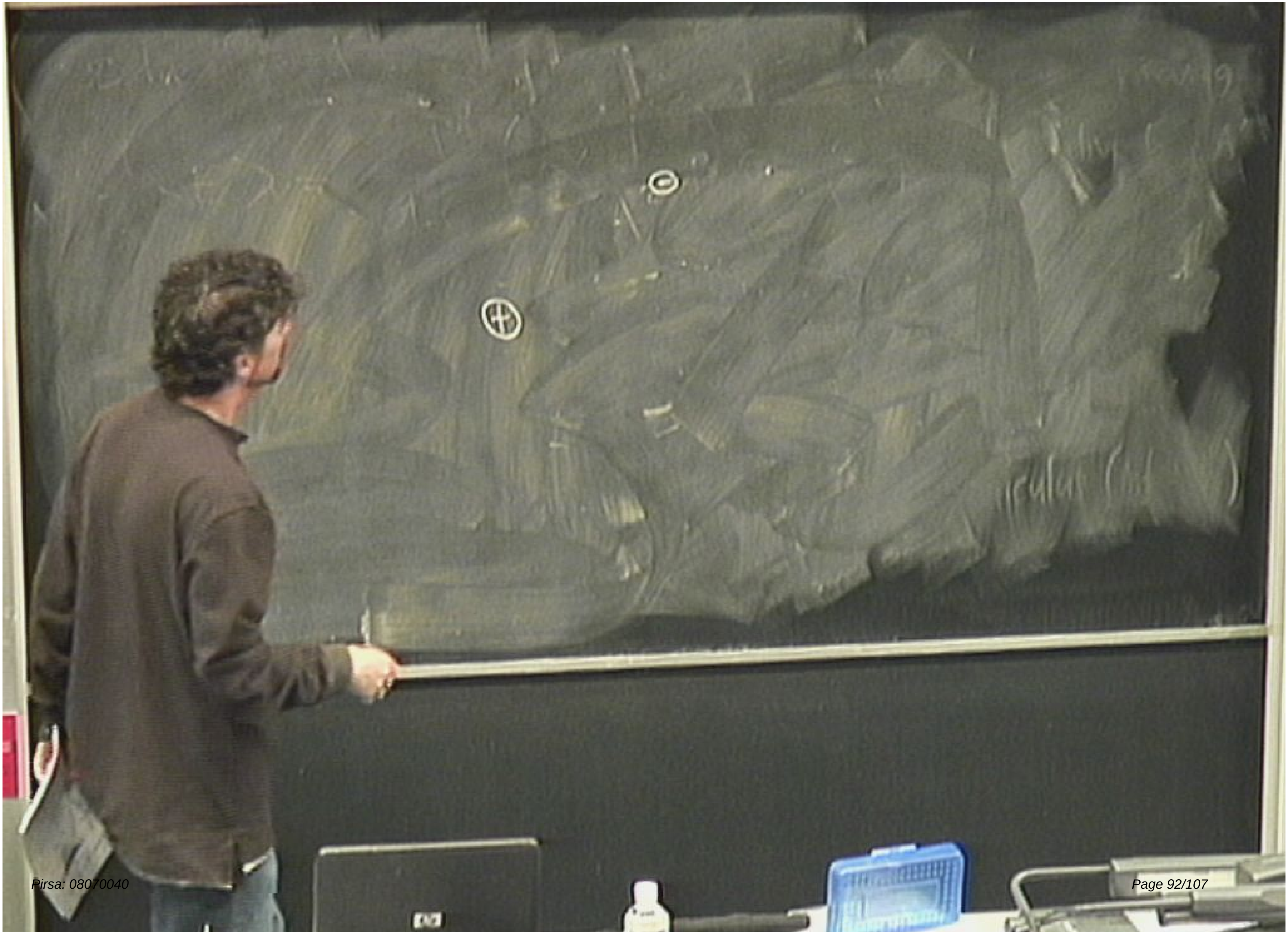


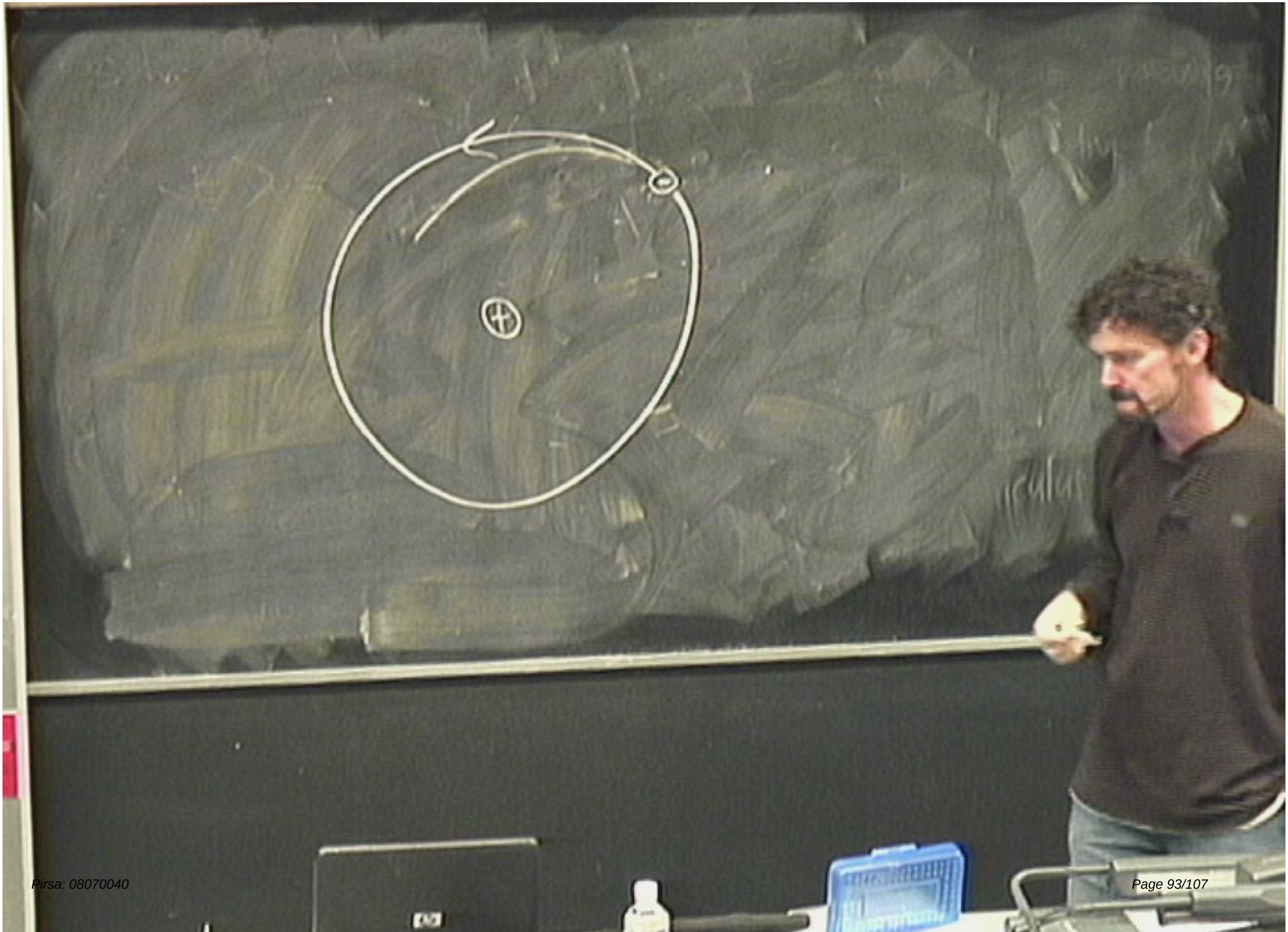
H - Atom
E
PE
x
E₀
E
r
PE = -k e²/r
E₀ = -13.6 eV
r
-k e²/r

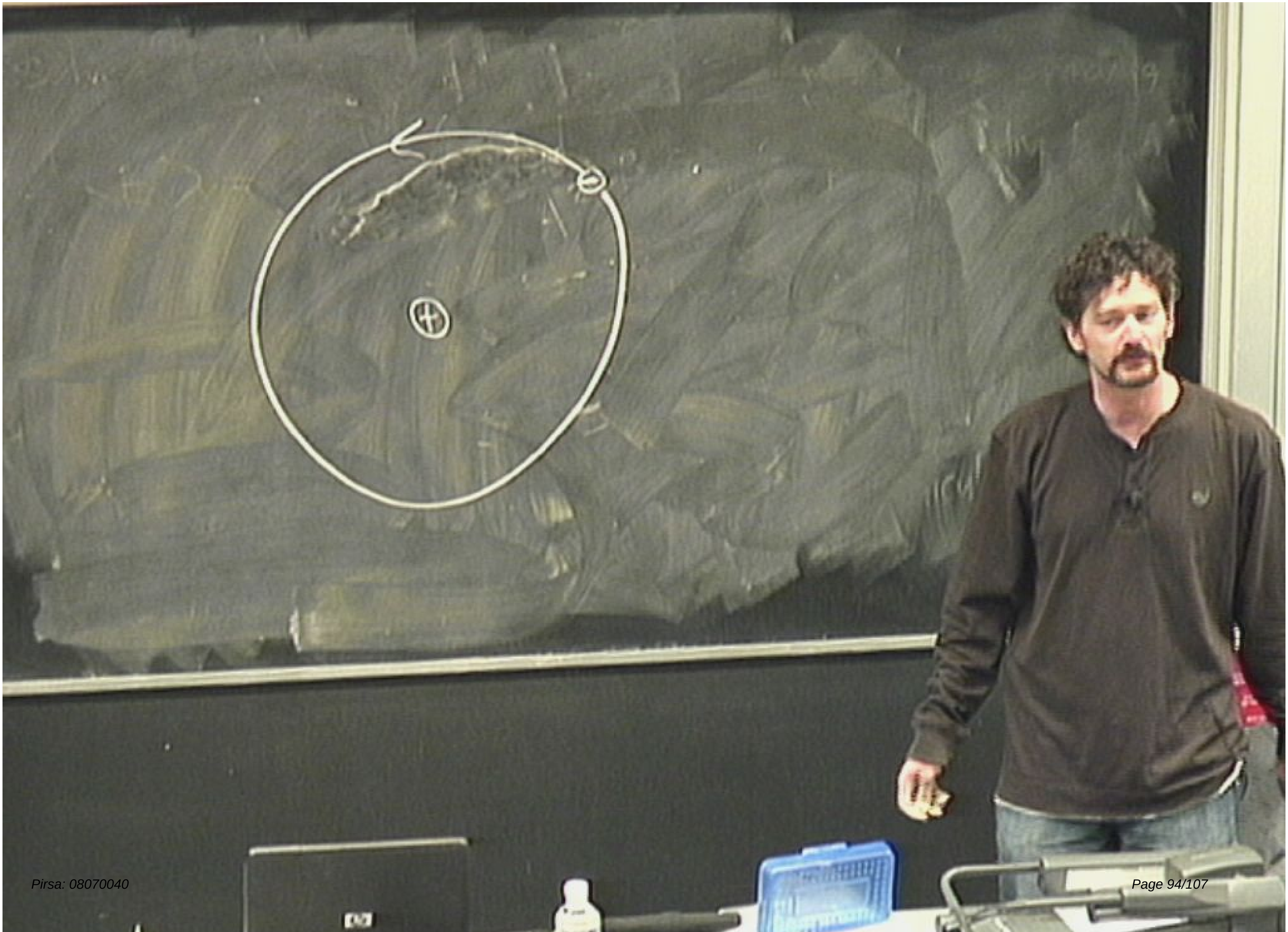

H - Atom
E
PE
x
E
r
PE = -k e²/r
E₀ = -13.6 eV

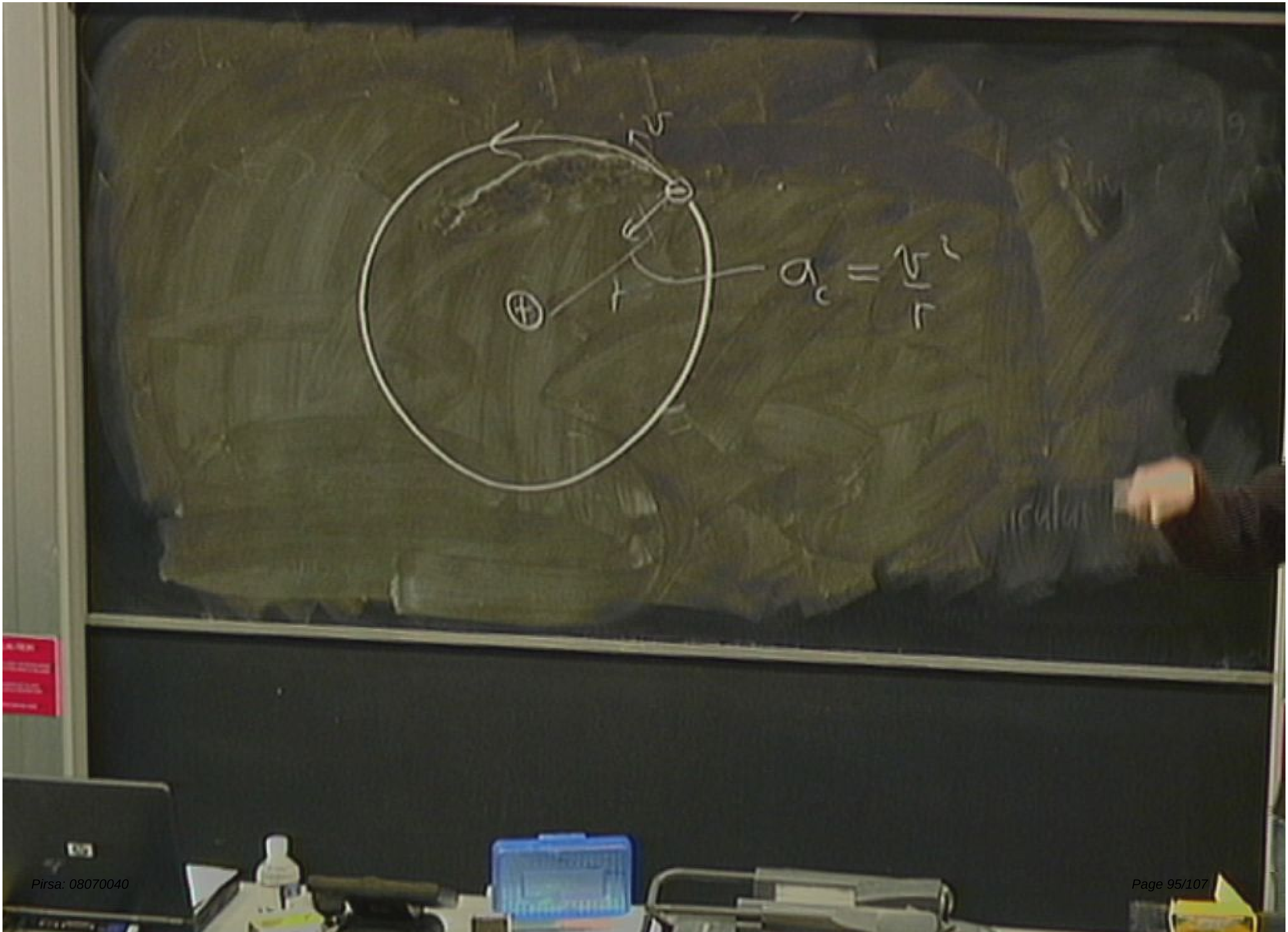


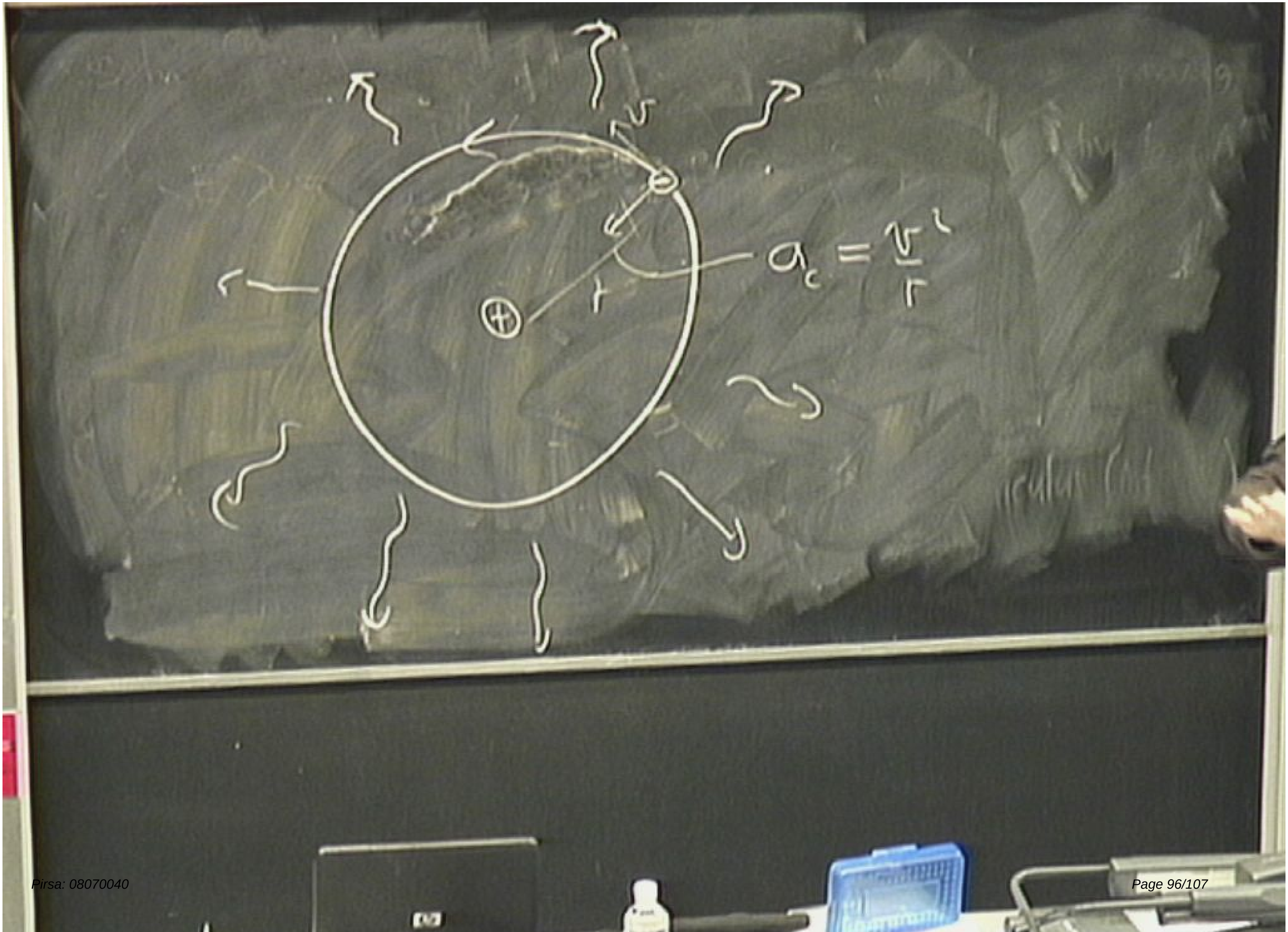
H - Atom
E
PE
x
E₀
E
r
PE = -k e²/r
E₀ = -13.6 eV
-k e²/r

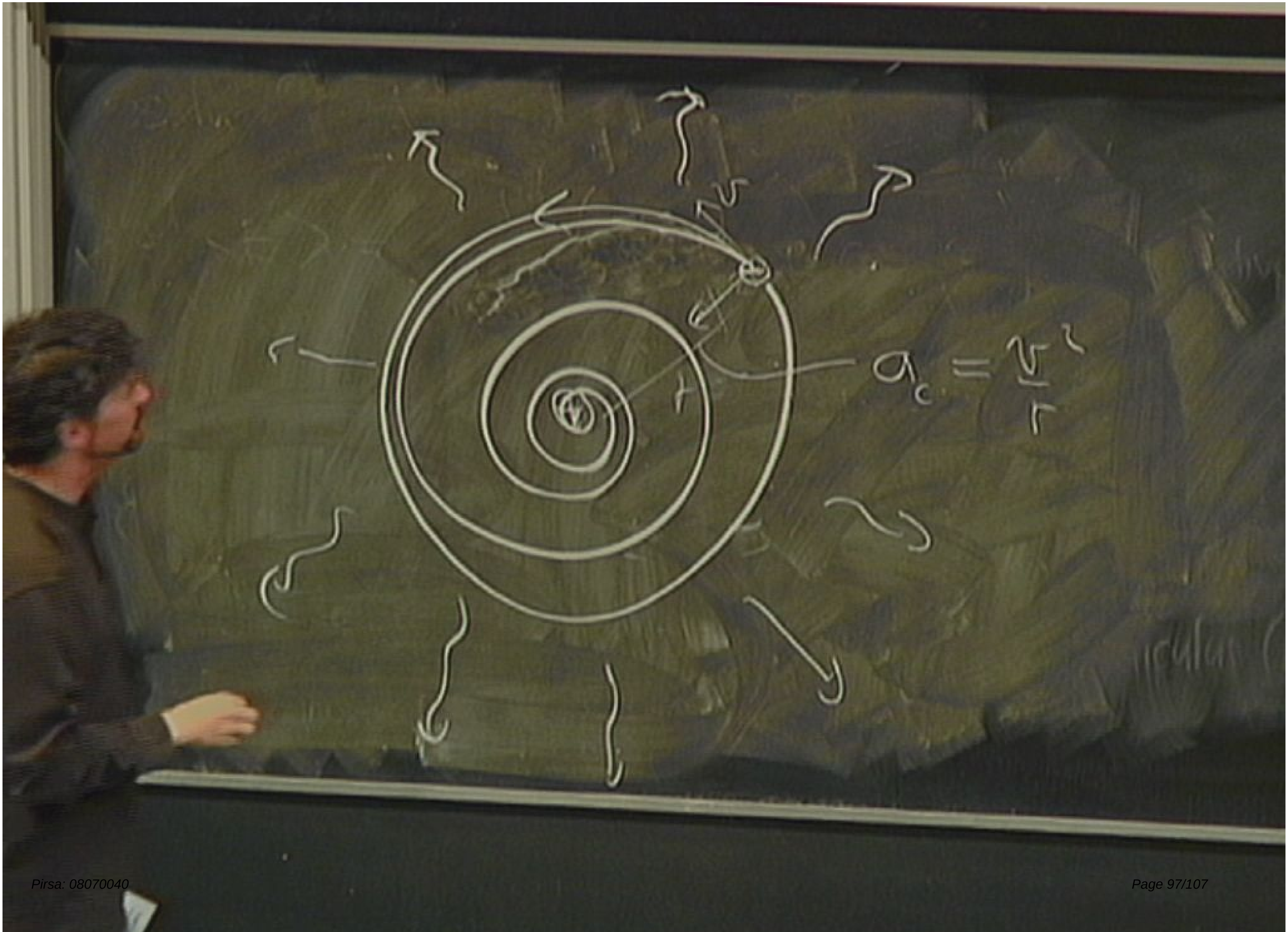


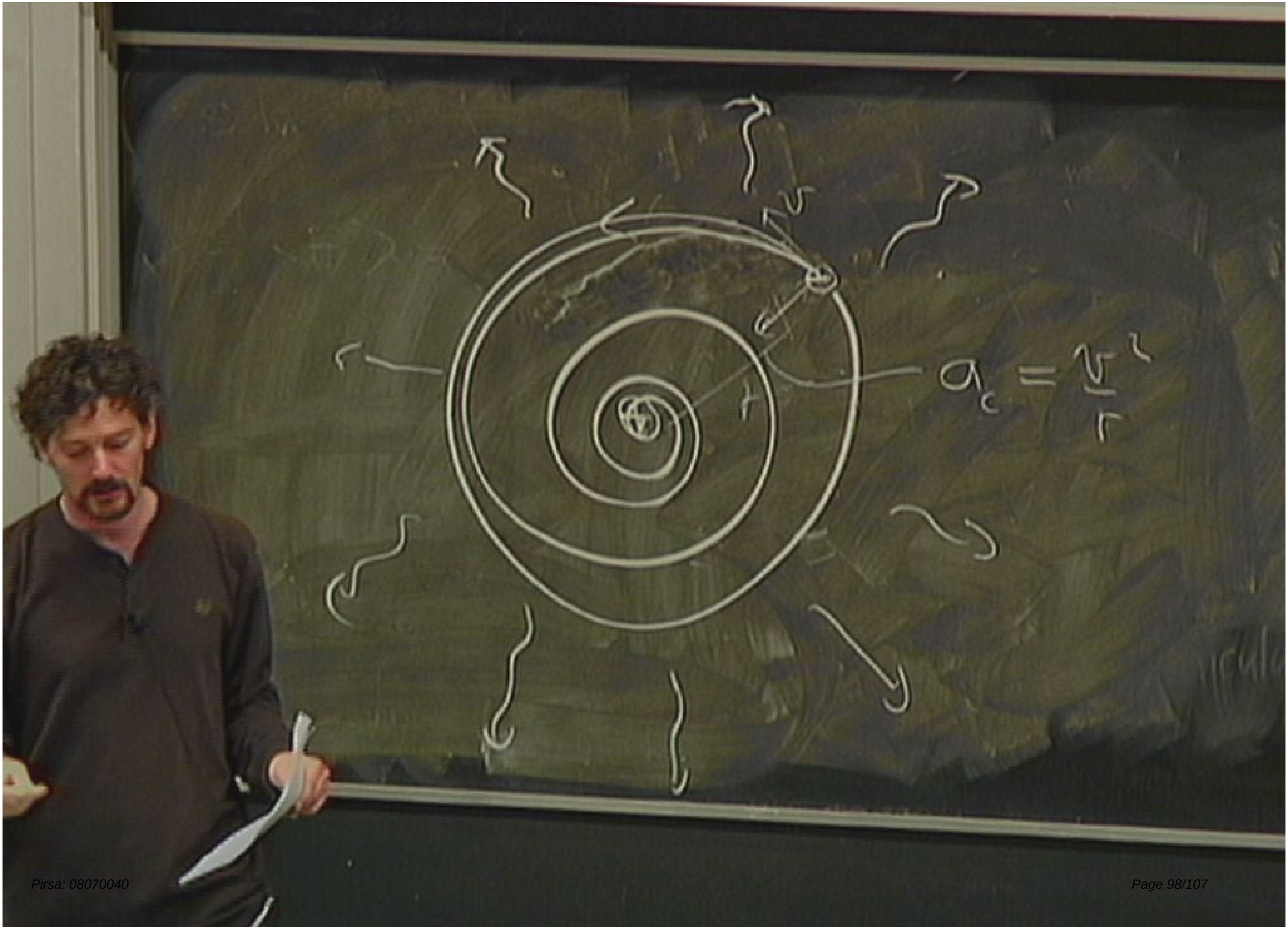




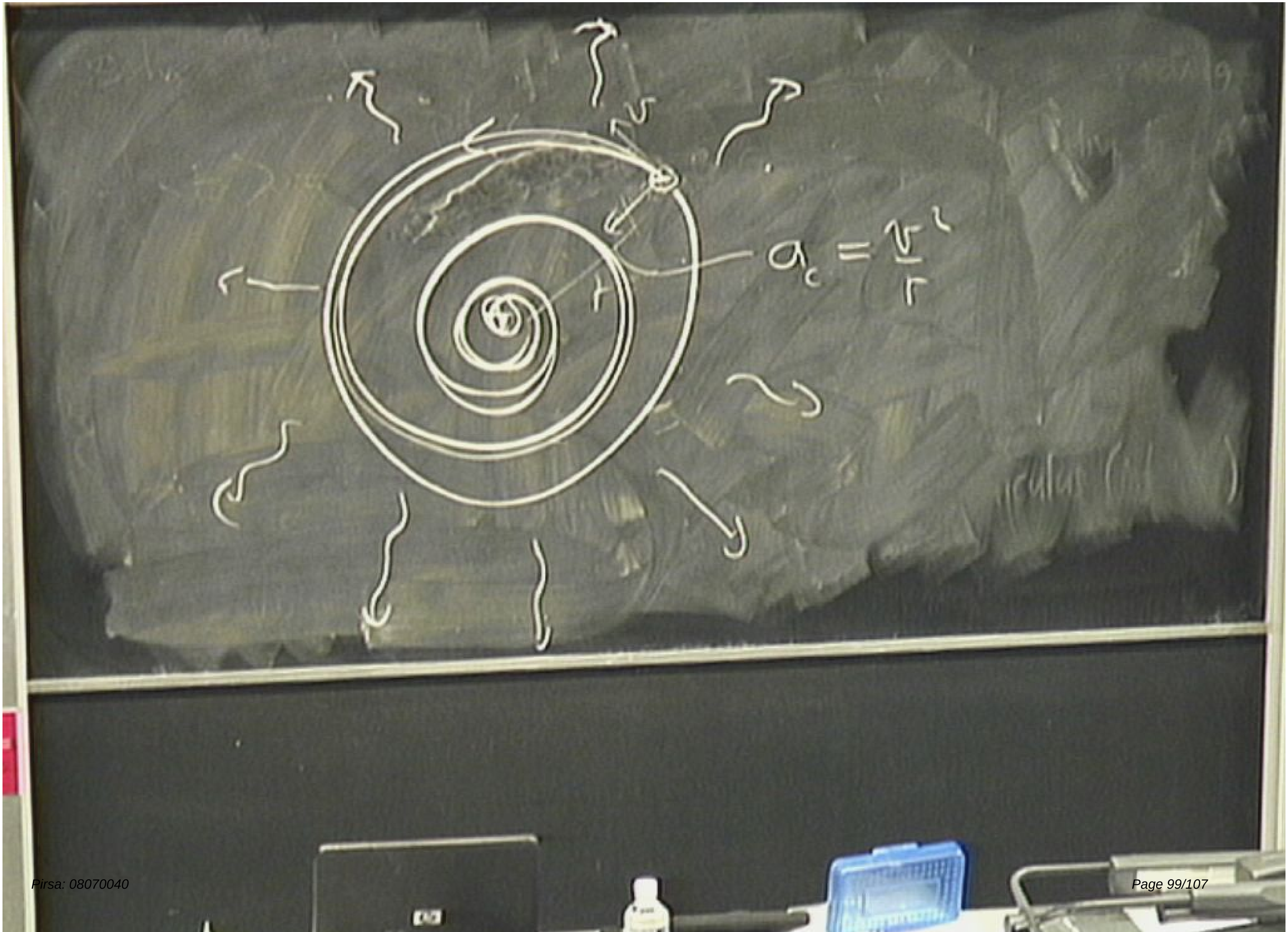

v
r
$a_c = \frac{v^2}{r}$

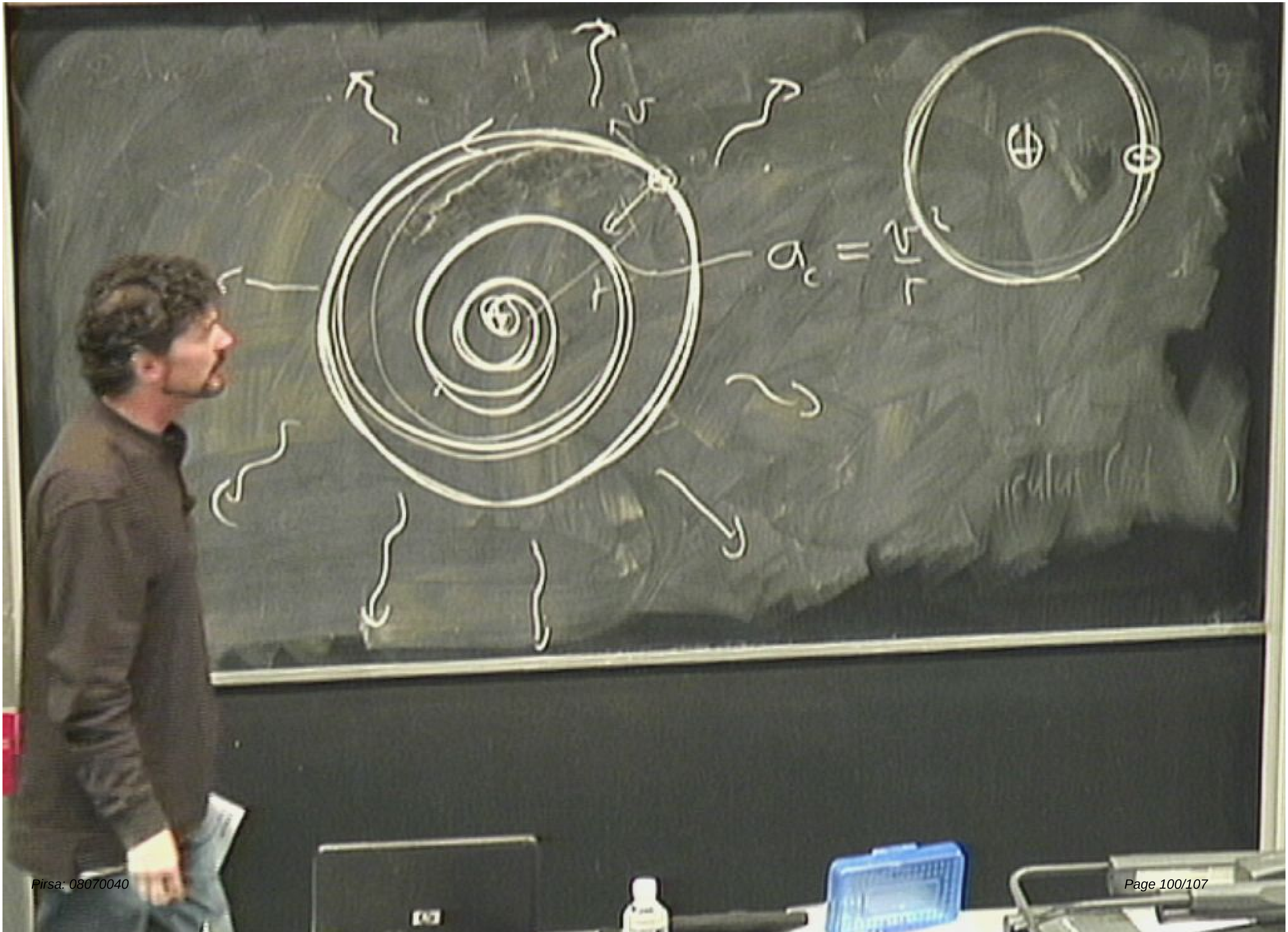

v
r
$a_c = \frac{v^2}{r}$

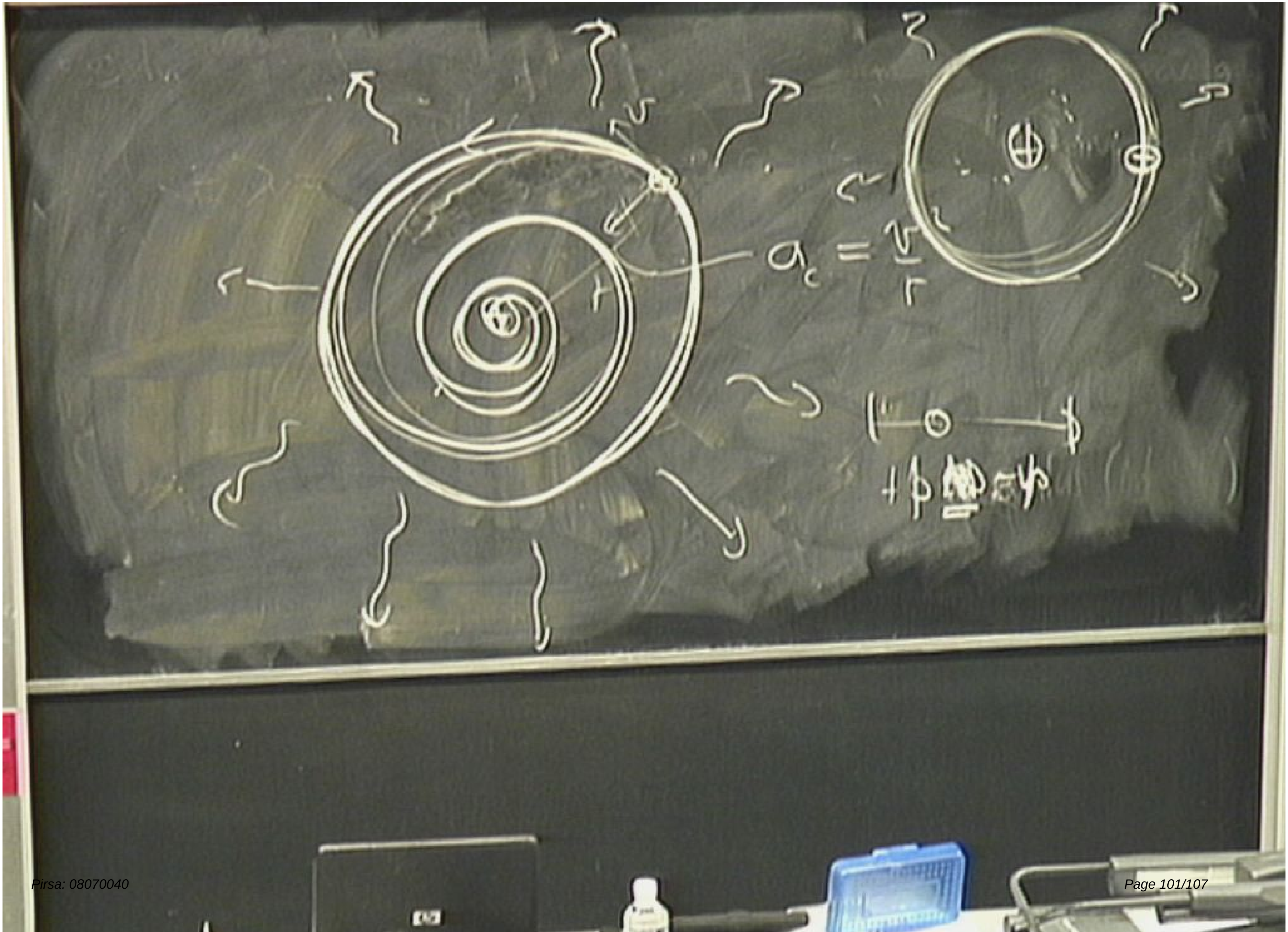

v
r
$a_c = \frac{v^2}{r}$

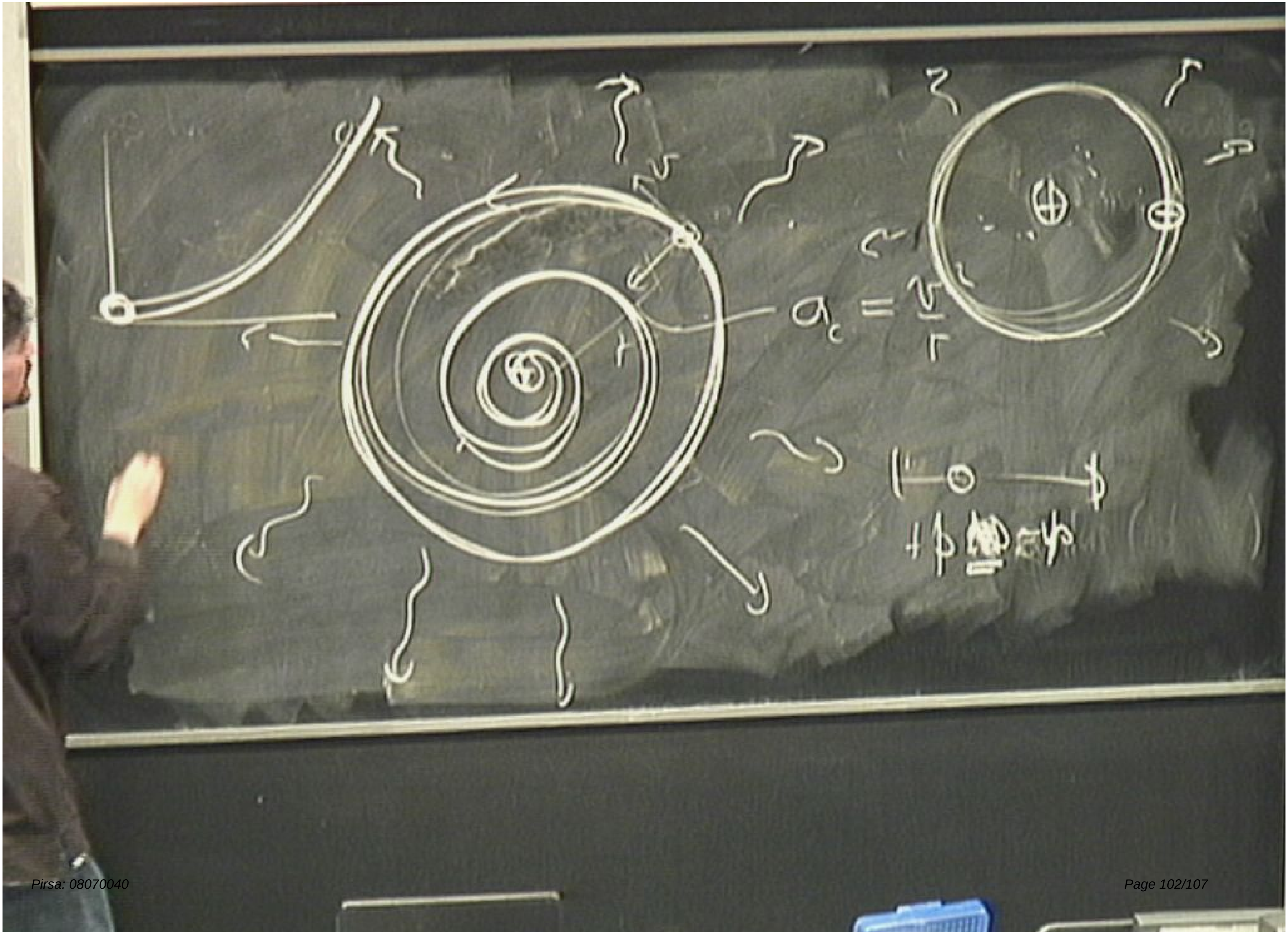


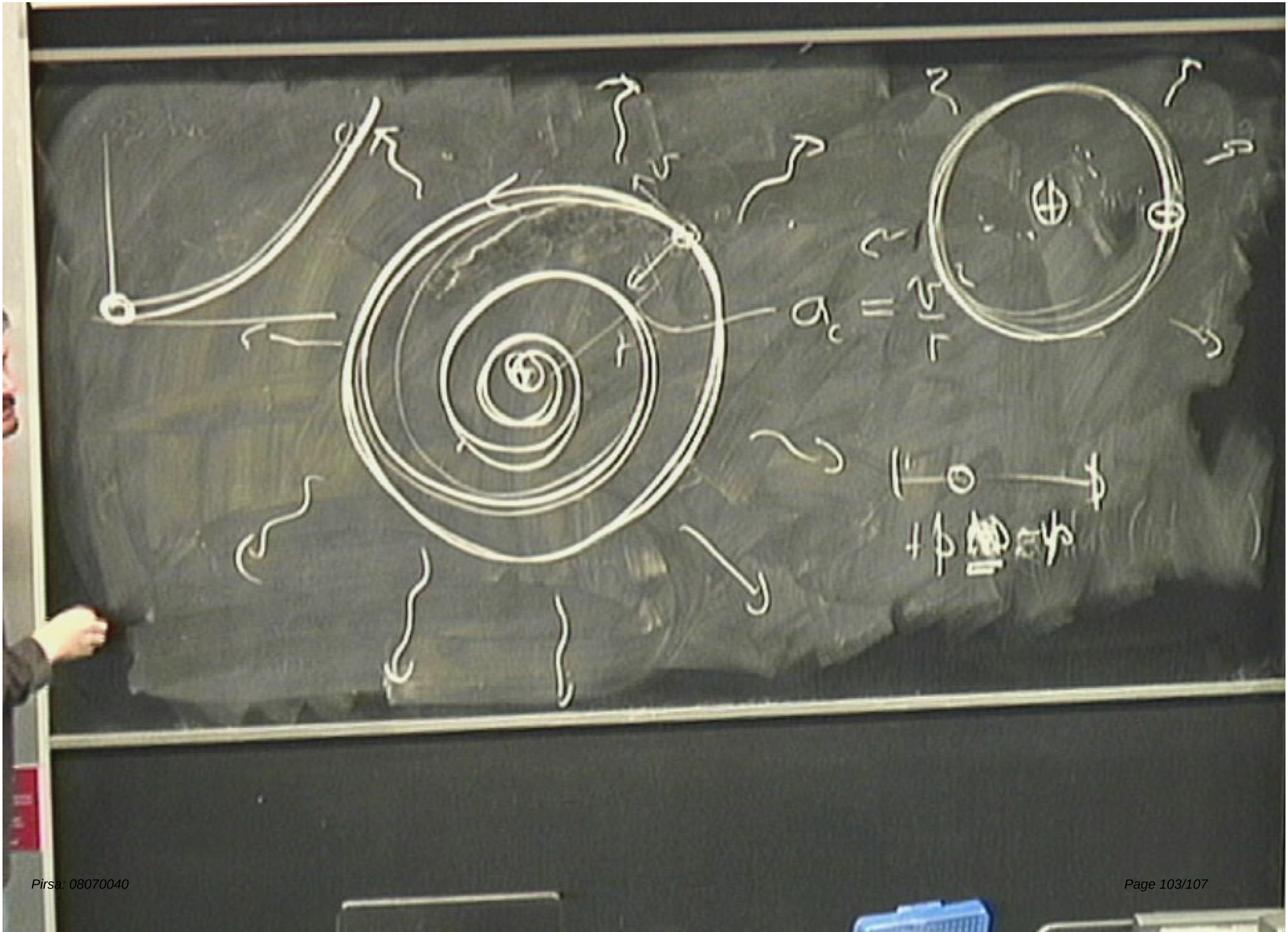
v
r
$a_c = \frac{v^2}{r}$

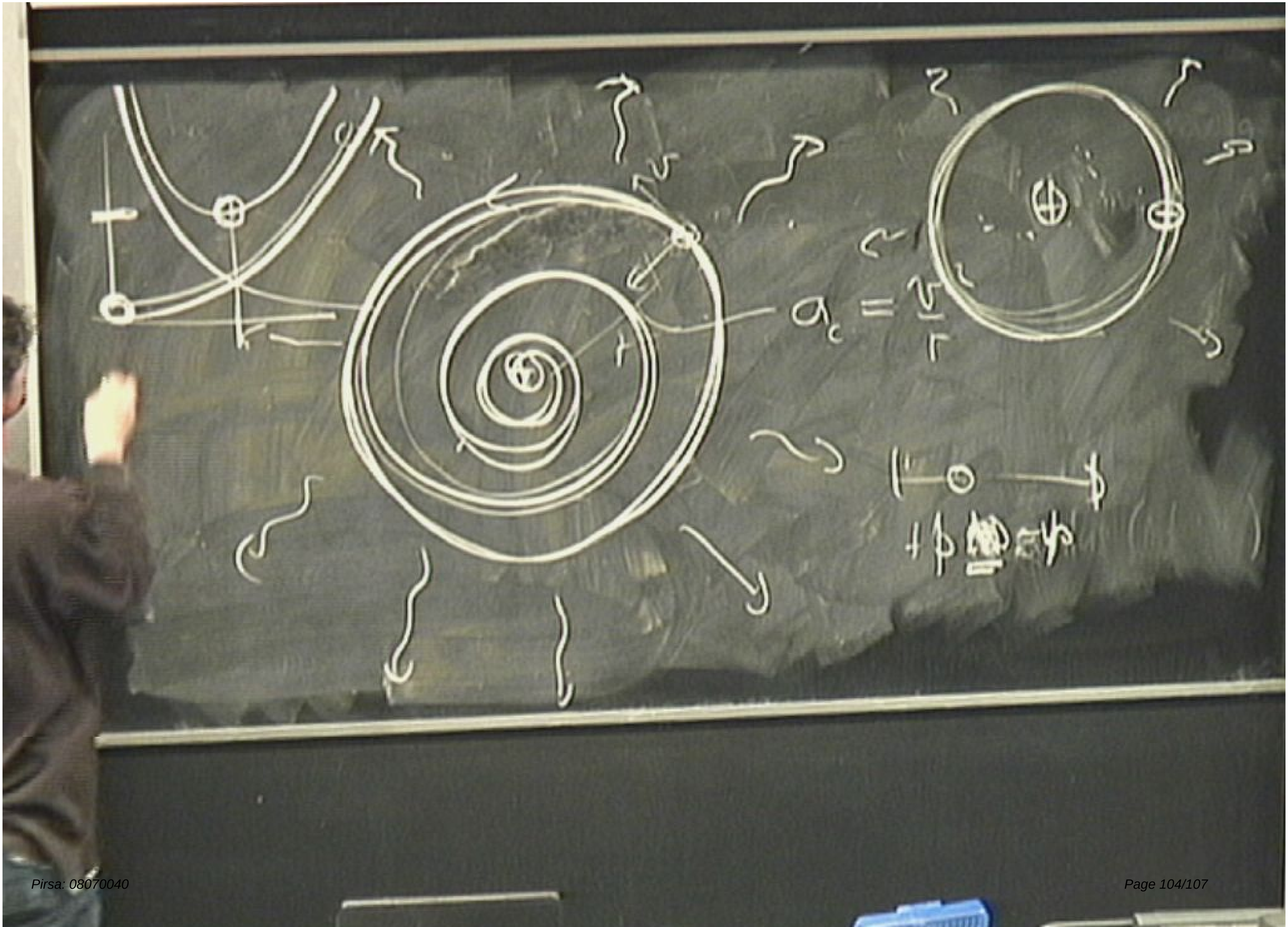

v
r
$a_c = \frac{v^2}{r}$


v
r
$a_c = \frac{v^2}{r}$

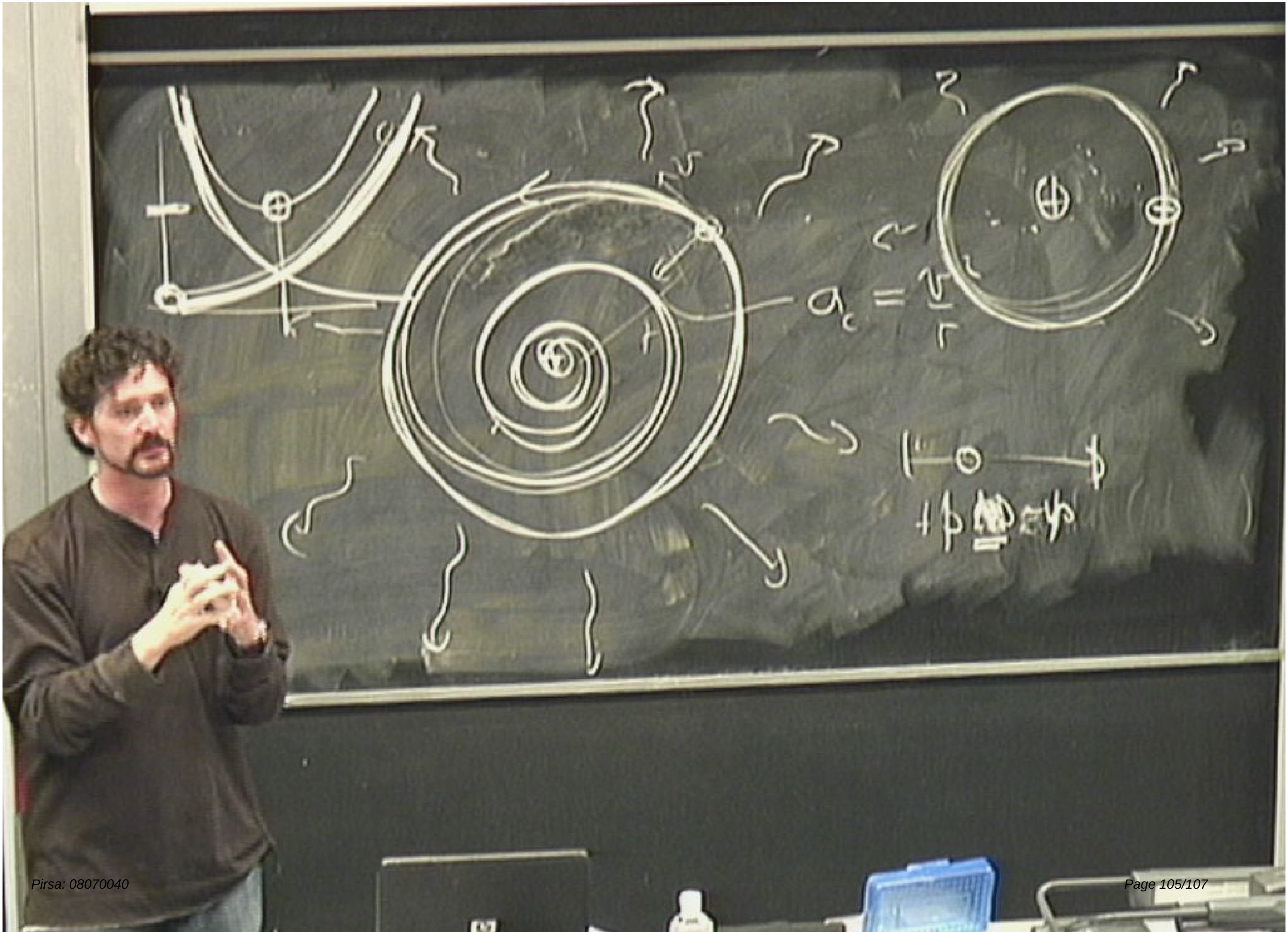

$a_c = \frac{v^2}{r}$

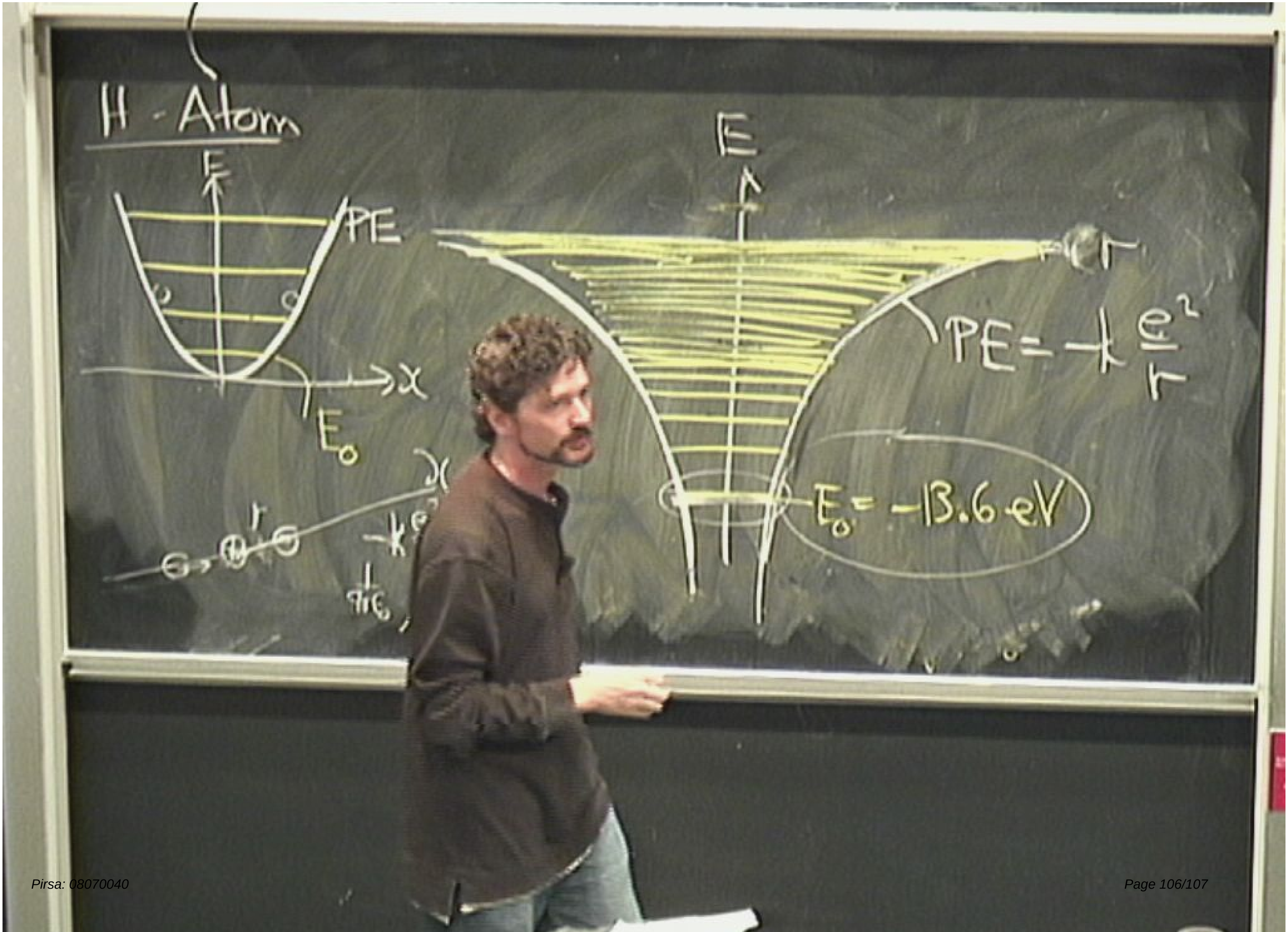

$a_c = \frac{v^2}{r}$

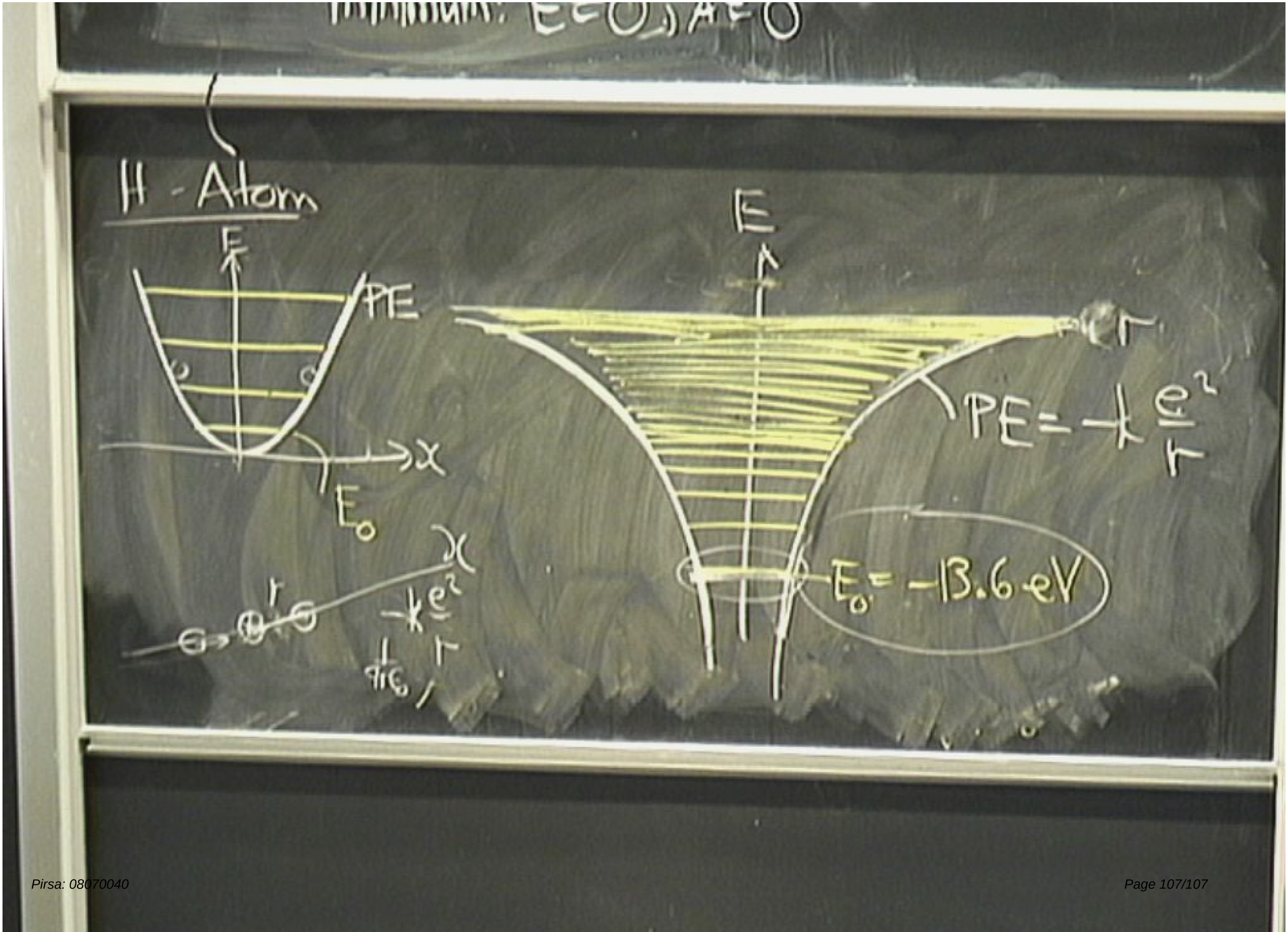

$a_c = \frac{v^2}{r}$



$a_c = \frac{v^2}{r}$


$a_c = \frac{v^2}{r}$


H - Atom
E
PE
x
E
E
E_0
PE= $-k\frac{e^2}{r}$
r
$E_0 = -13.6\ eV$



H - Atom
E
PE
x
E_0
$-k\frac{e^2}{r}$
$\frac{1}{4\pi\epsilon_0}$
E
r
$PE = -k\frac{e^2}{r}$
$E_0 = -13.6\,eV$