Title: Quantum 2

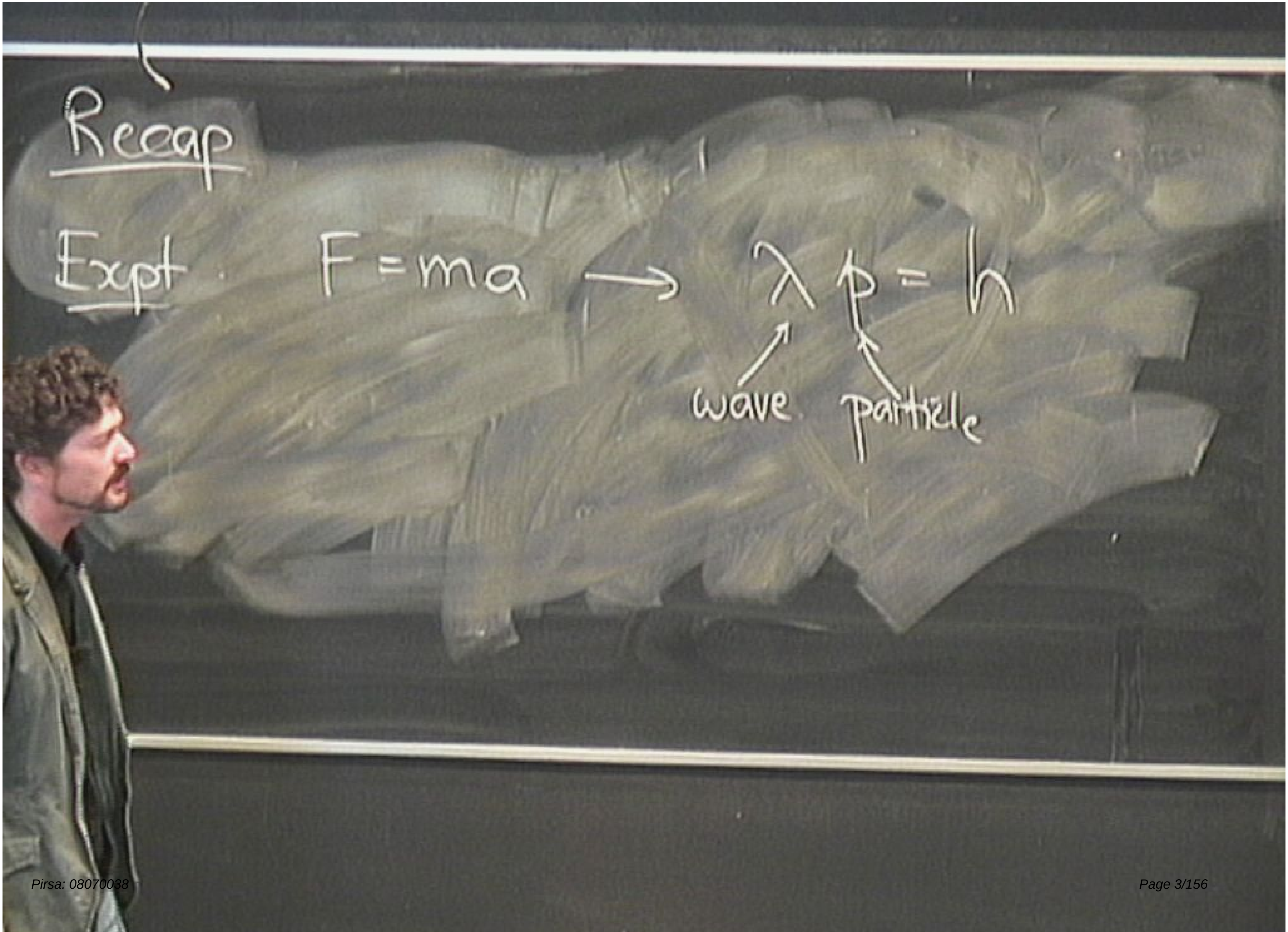
Date: Jul 25, 2008 10:30 AM

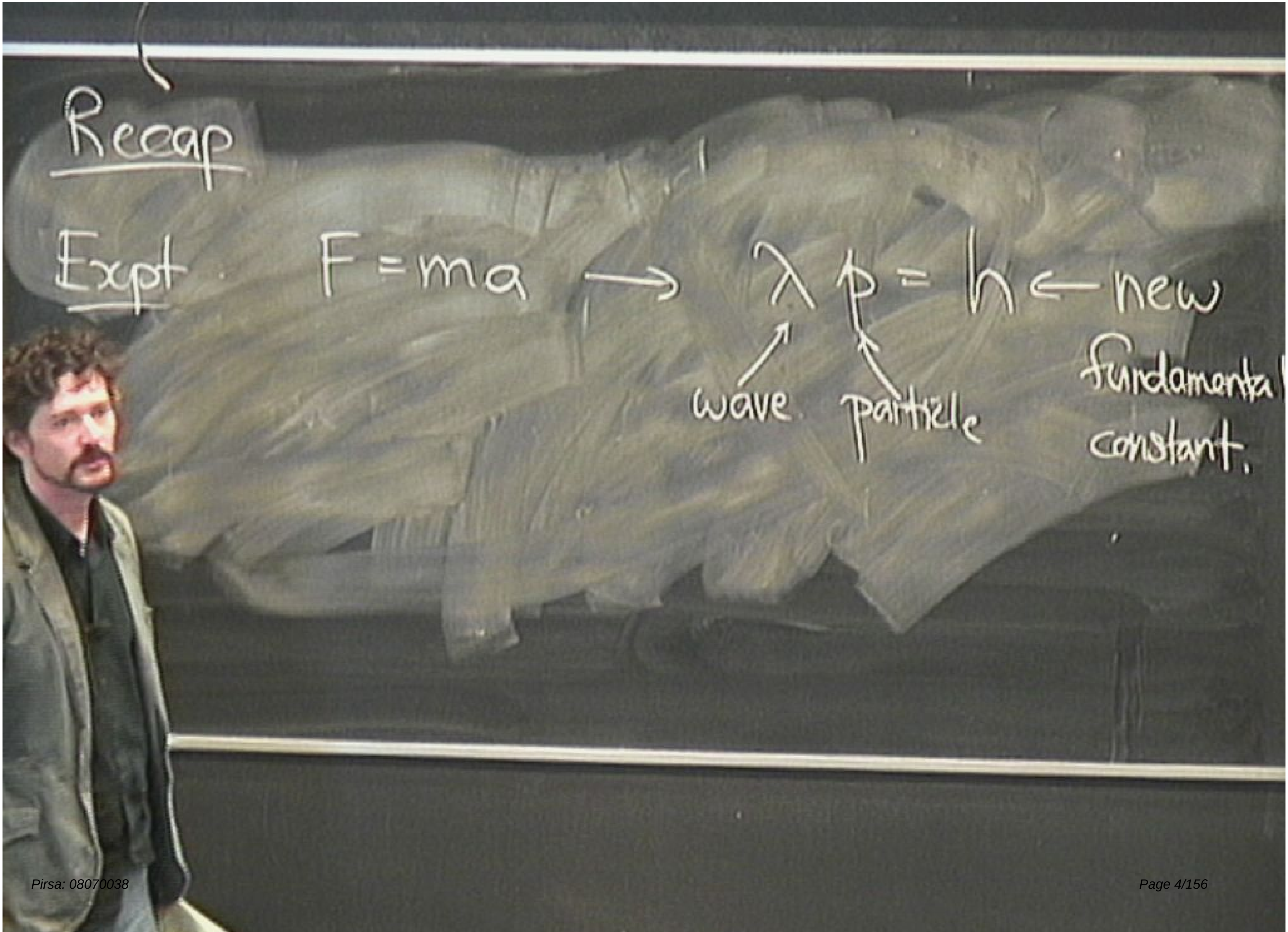
URL: http://pirsa.org/08070038

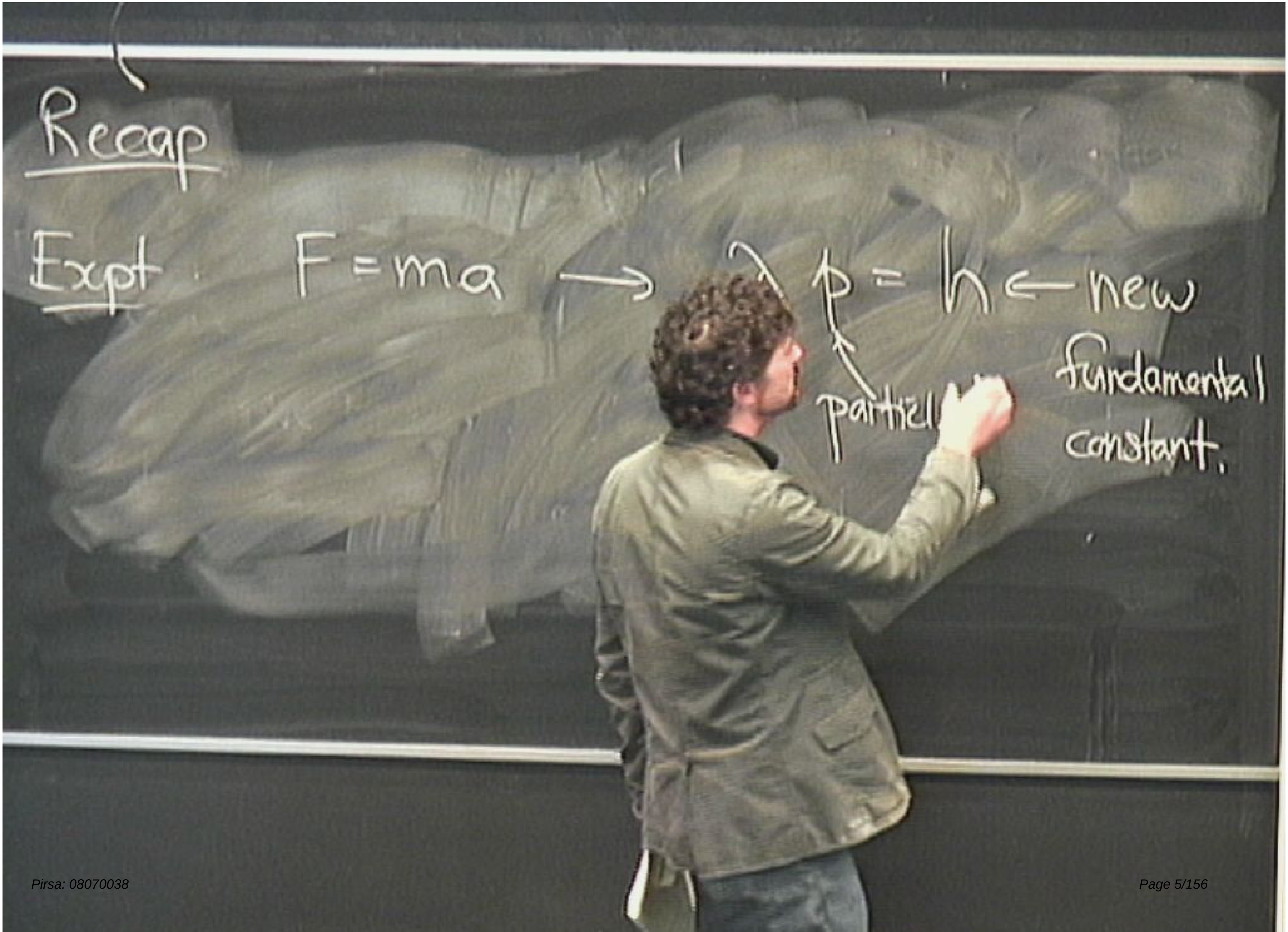
Abstract:

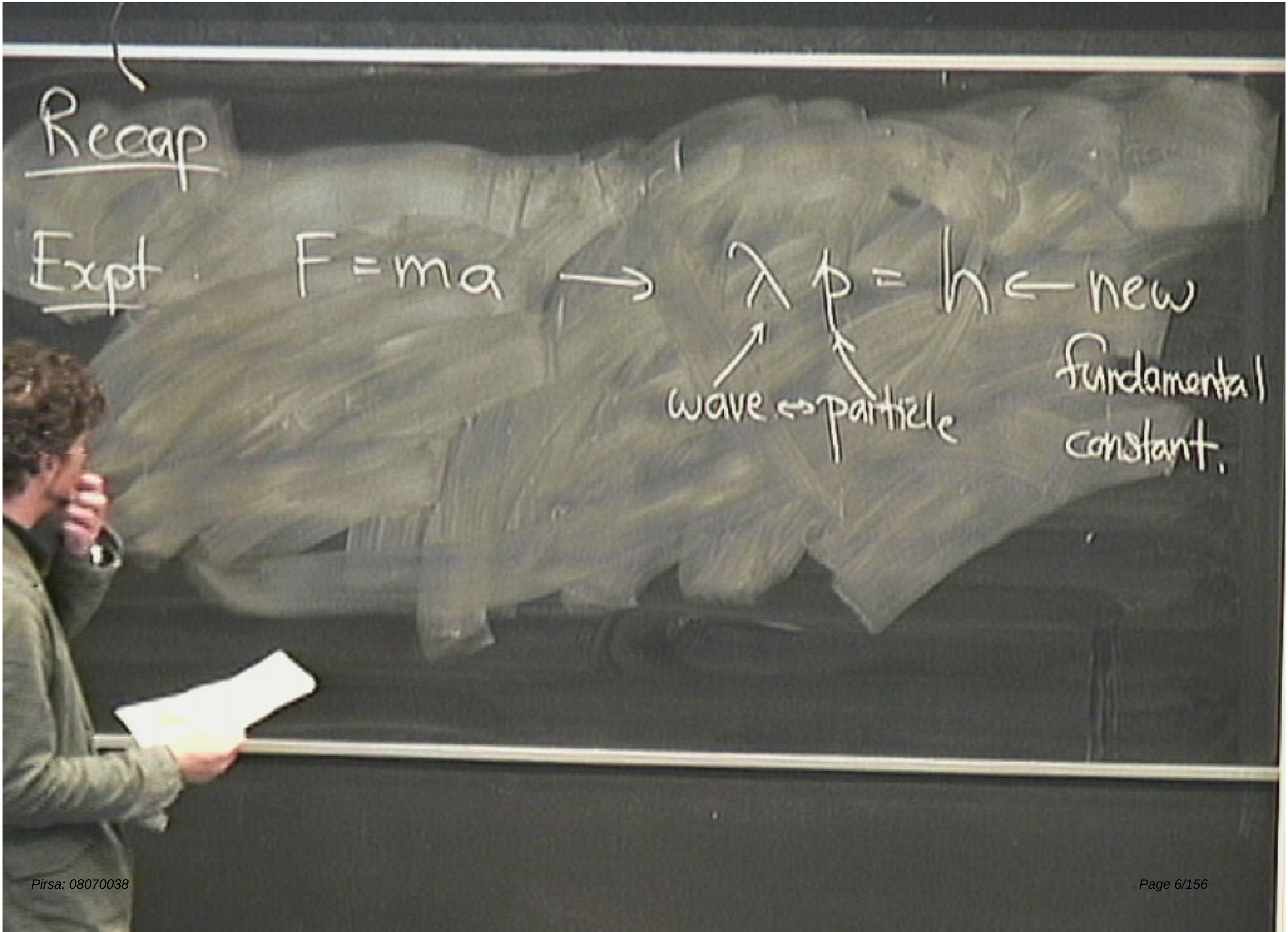
Recap

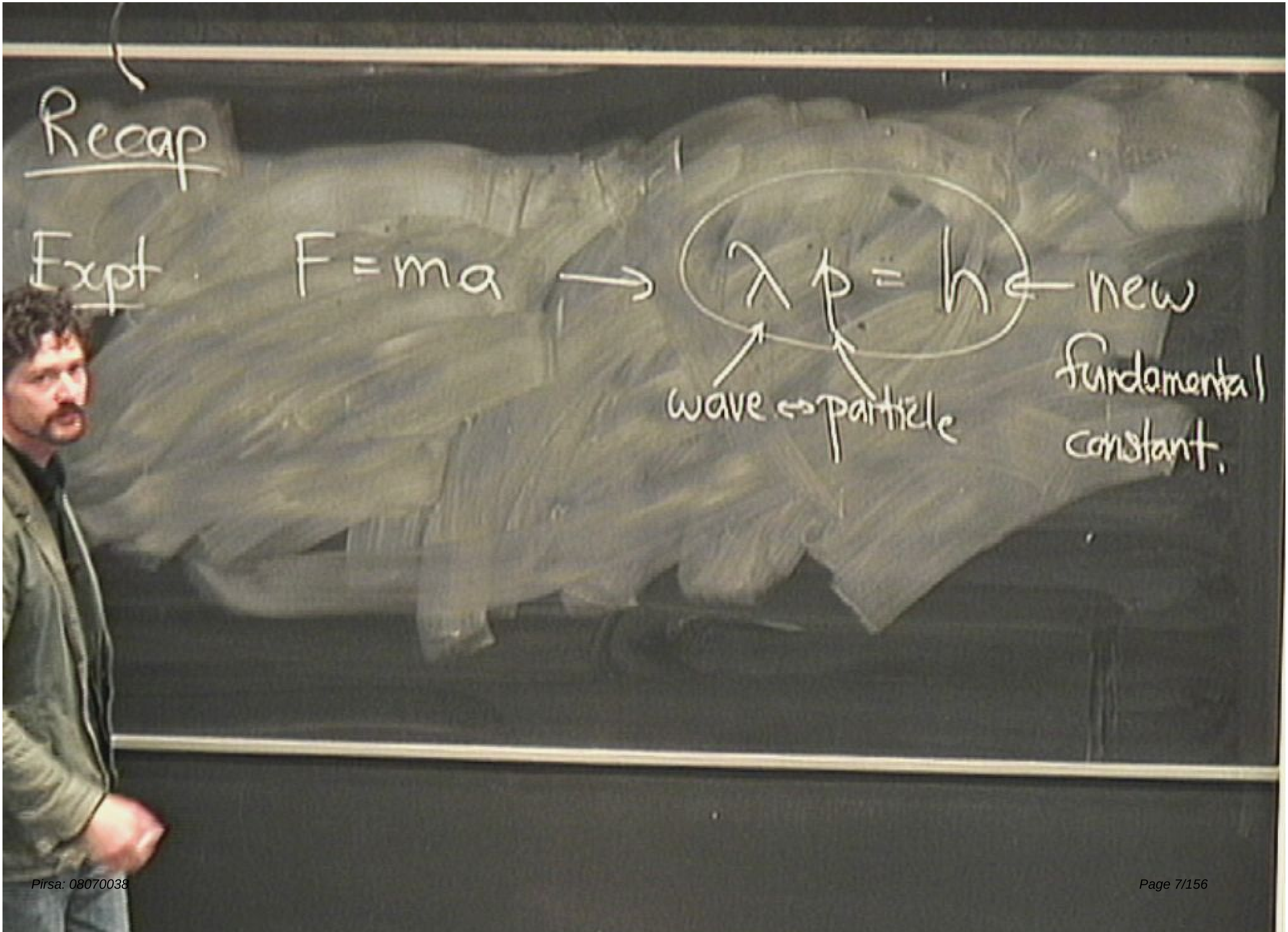
Expt $\quad F = ma \longrightarrow \lambda p = h$


Recap
Expt F = ma → λ p = h
wave
particle


Recap
Expt
F = ma → λp = h ← new
wave
particle
fundamental constant.


Recap
Expt
$F = ma \rightarrow \lambda p = h \leftarrow$ new
partic
fundamental
constant.


Recap
Expt
F = ma → λ p = h ← new
fundamental
constant.
wave ⇔ particle


Recap
Expt
F = ma → λp = h ← new fundamental constant.
wave ⇔ particle

Recap

Expt: $F = ma \longrightarrow \lambda p = h \longleftarrow$ new fundamental constant.

λ : wave $\Leftrightarrow$ p : particle

apply to 

Recap

Expt $F = ma \longrightarrow \lambda p = h \longleftarrow$ new fundamental constant.

wave $\Leftrightarrow$ particle

apply to


Energy


Energy
0

Recap

Expt $F = ma \rightarrow \lambda p = h$ ← new fundamental constant.

wave ⟺ particle

apply to

$$E = \frac{p^2}{2m}$$

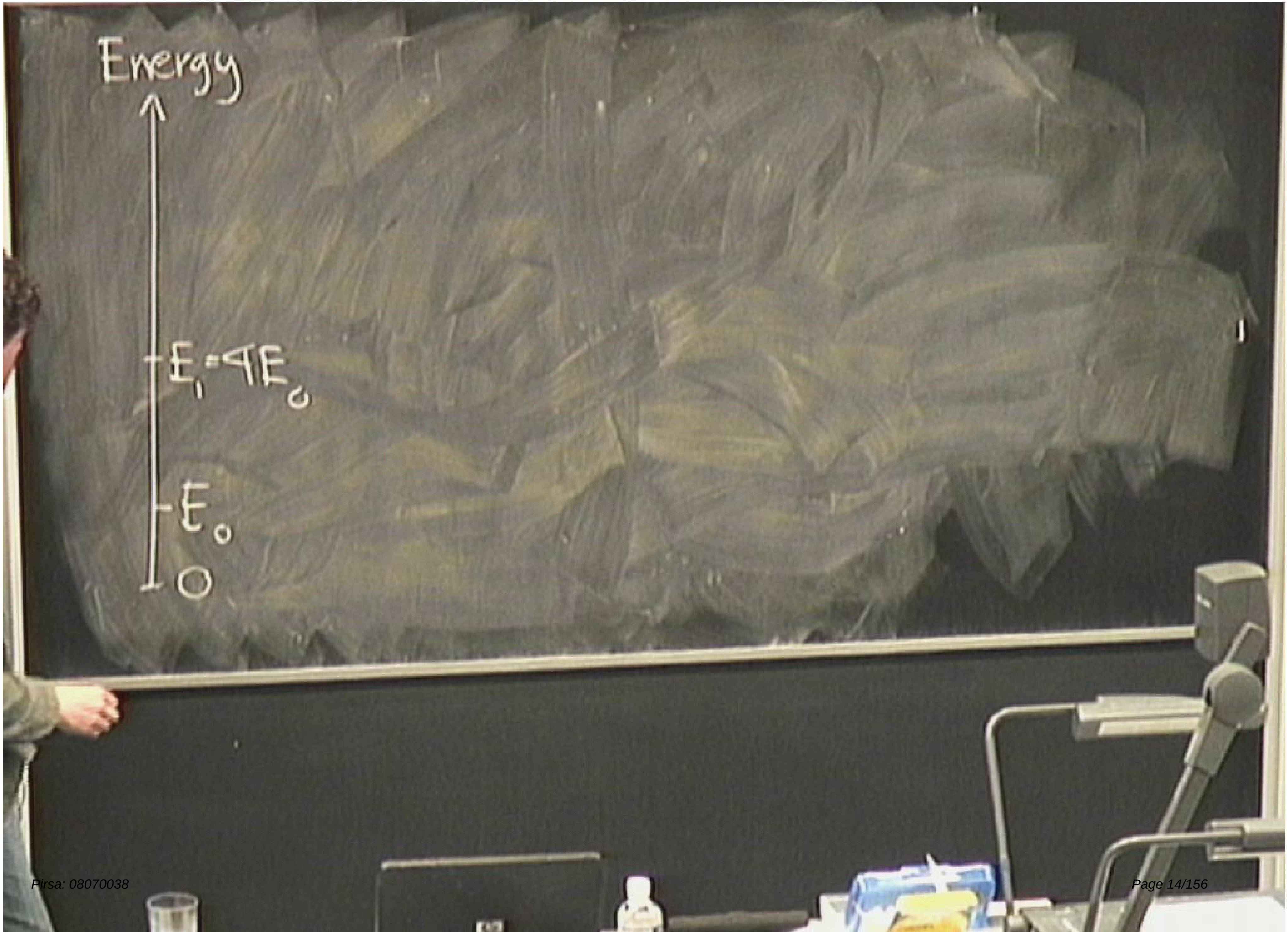
Recap

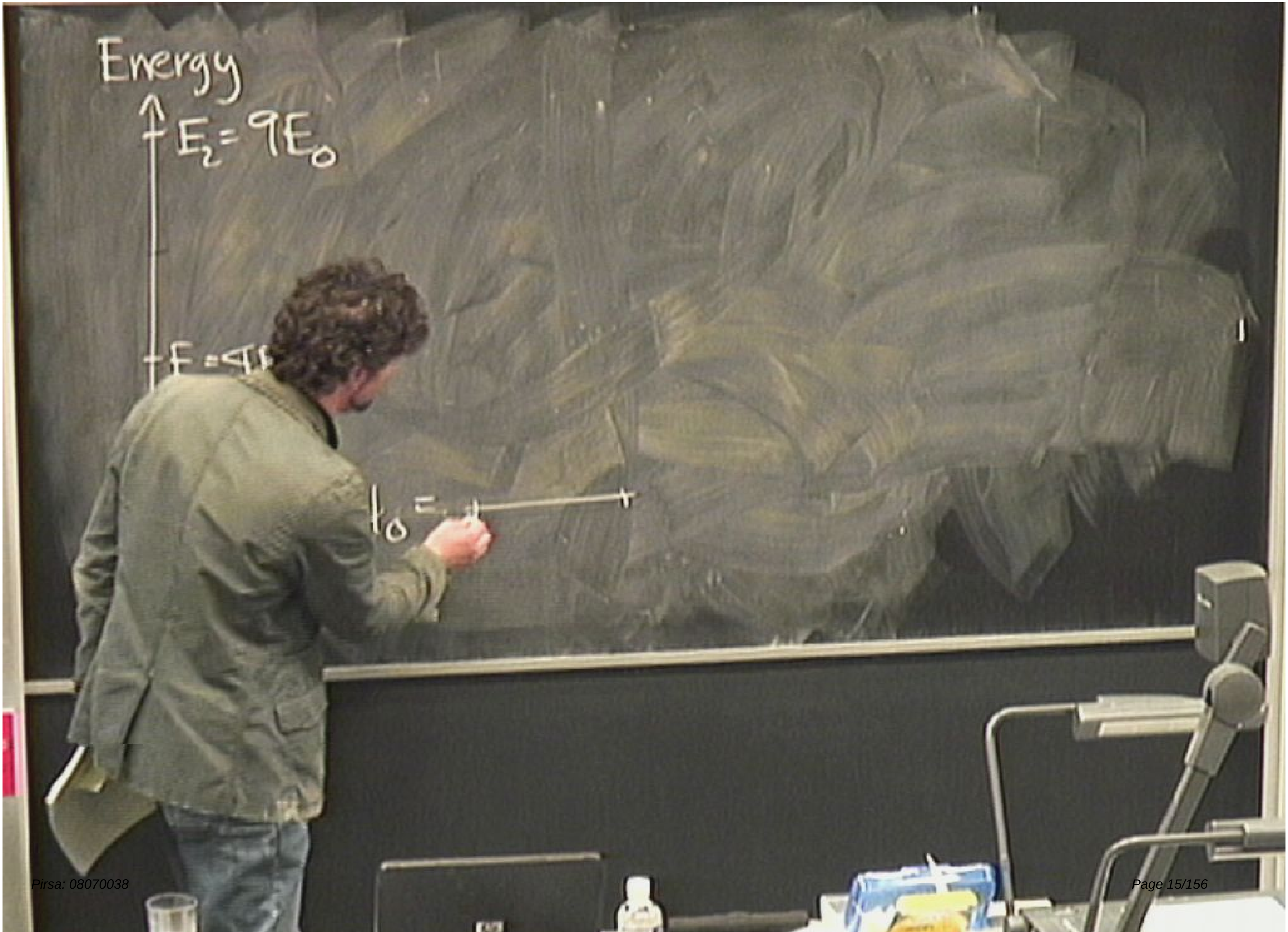
Expt $\quad F = ma \longrightarrow \lambda p = h \leftarrow$ new fundamental constant.

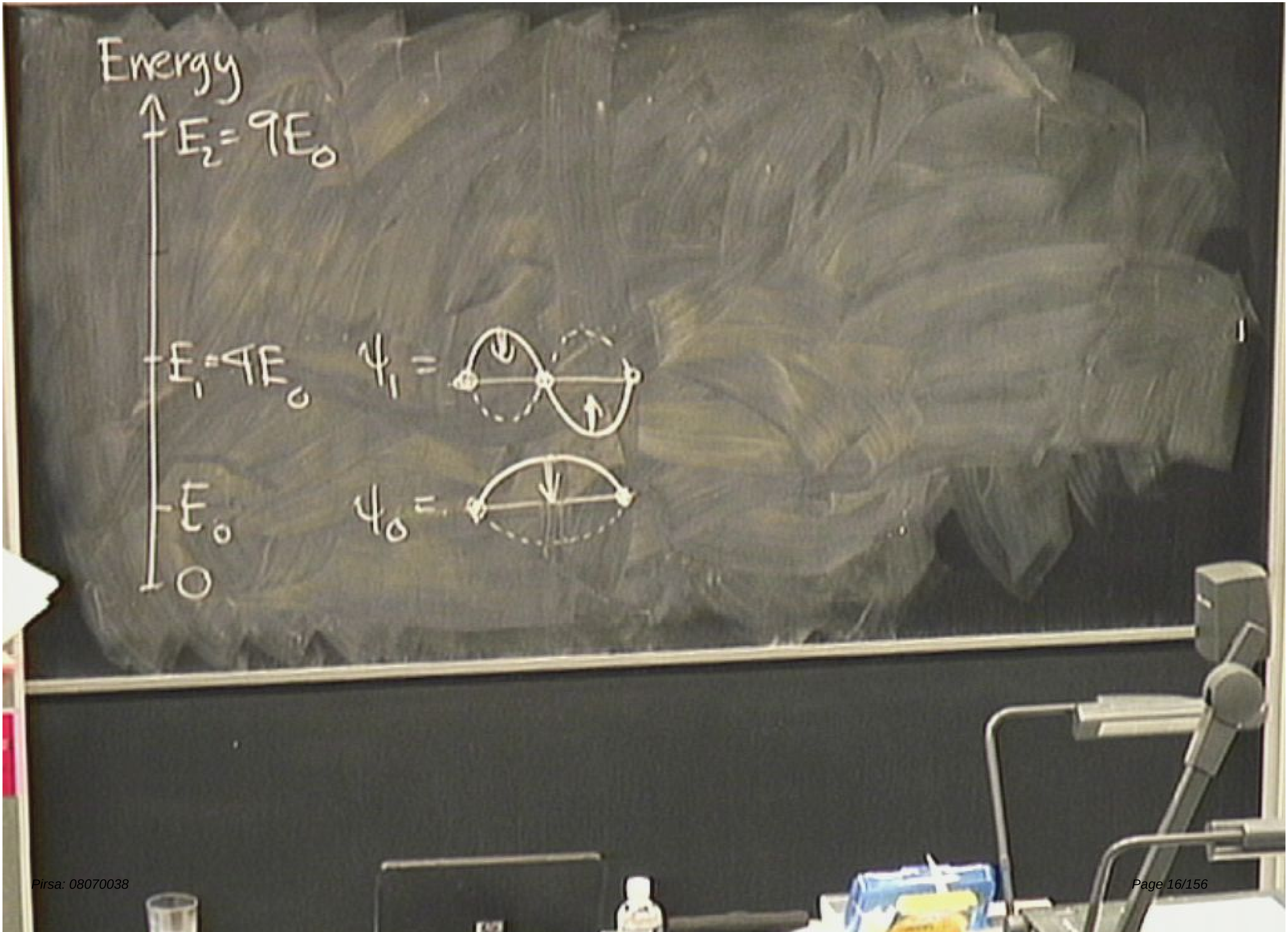
wave $\Leftrightarrow$ particle

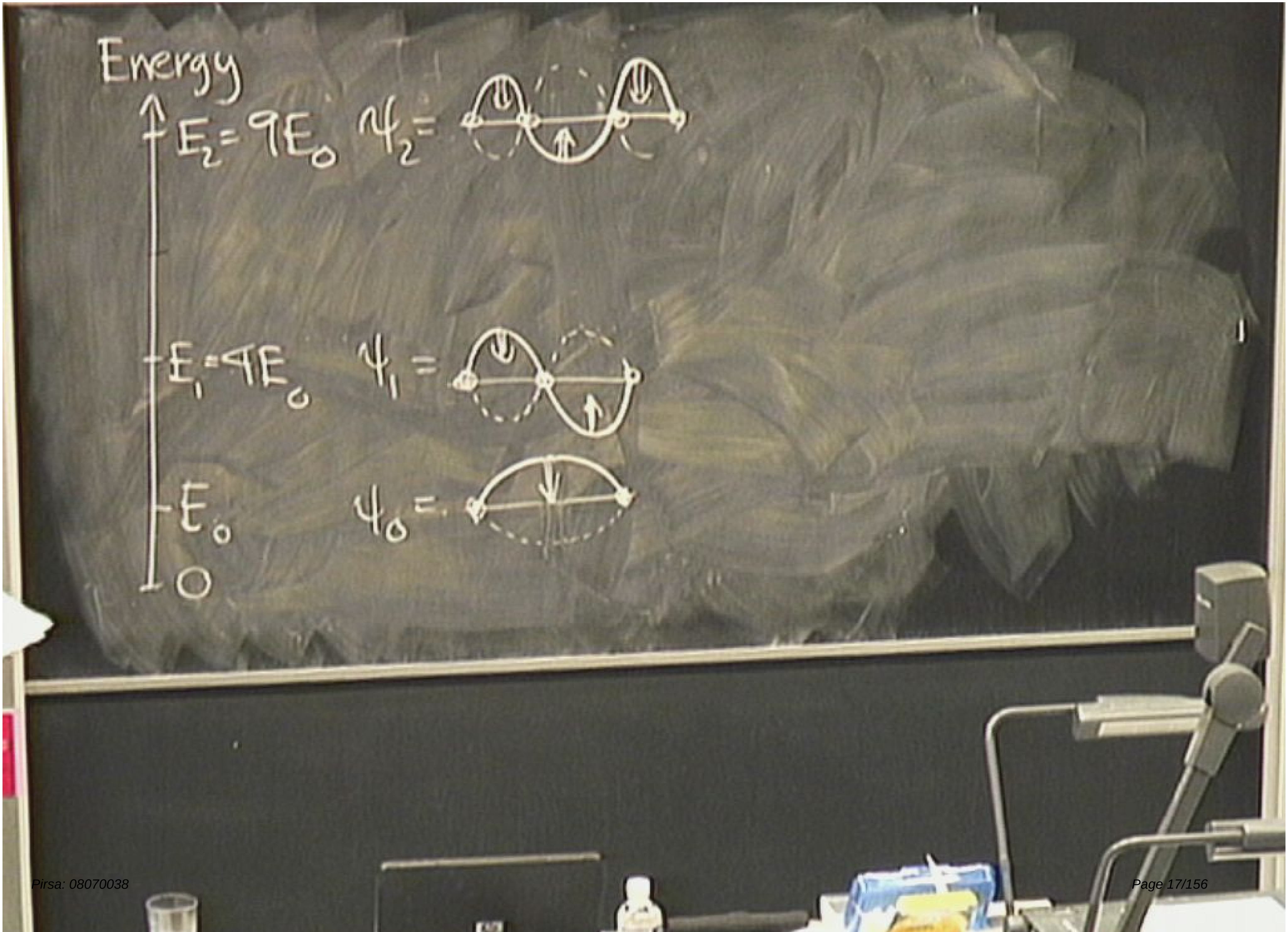
apply to 

$$E = \frac{p^2}{2m}$$

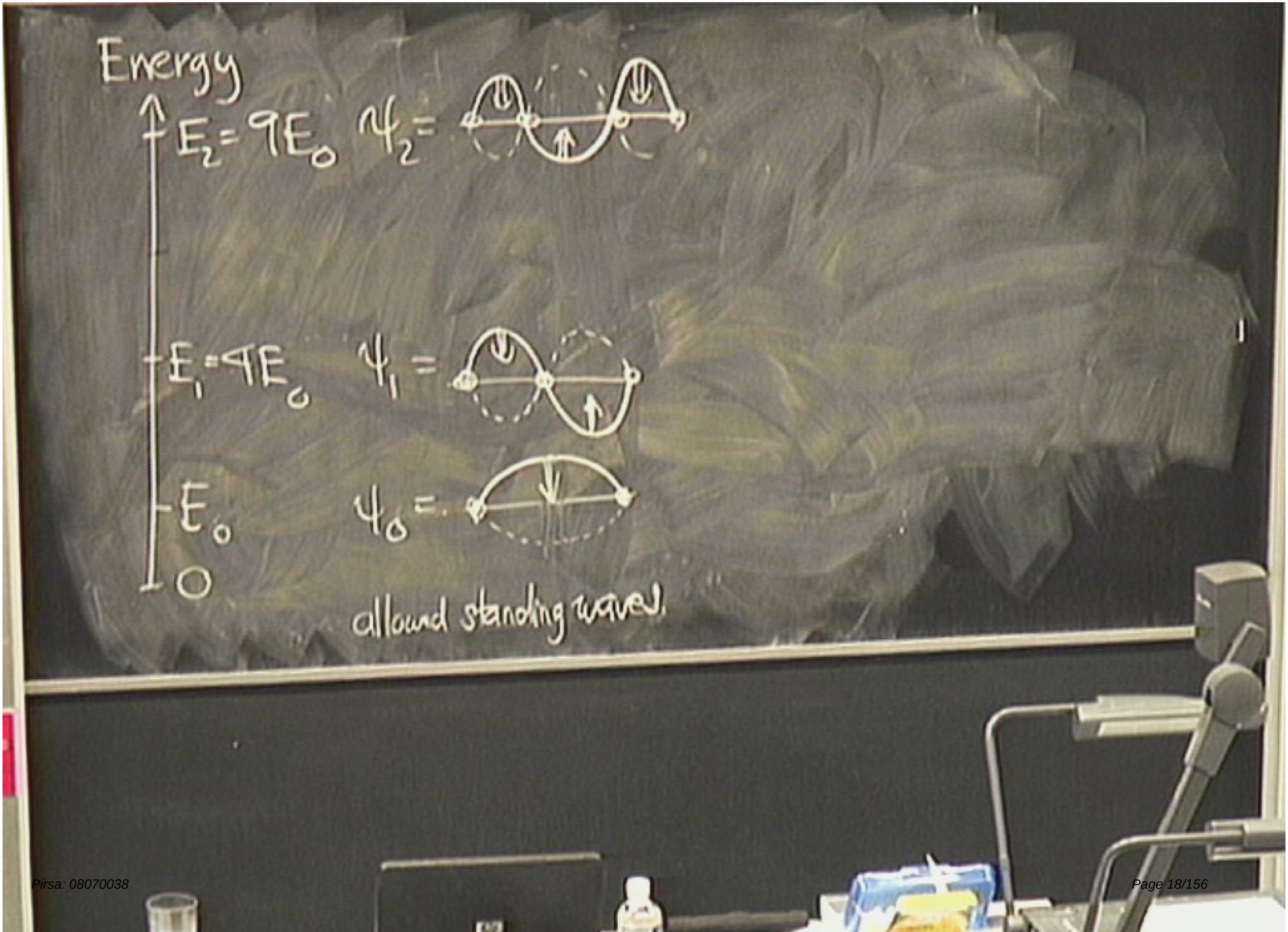

Energy
$E_1 = 4E_0$
E_0
0

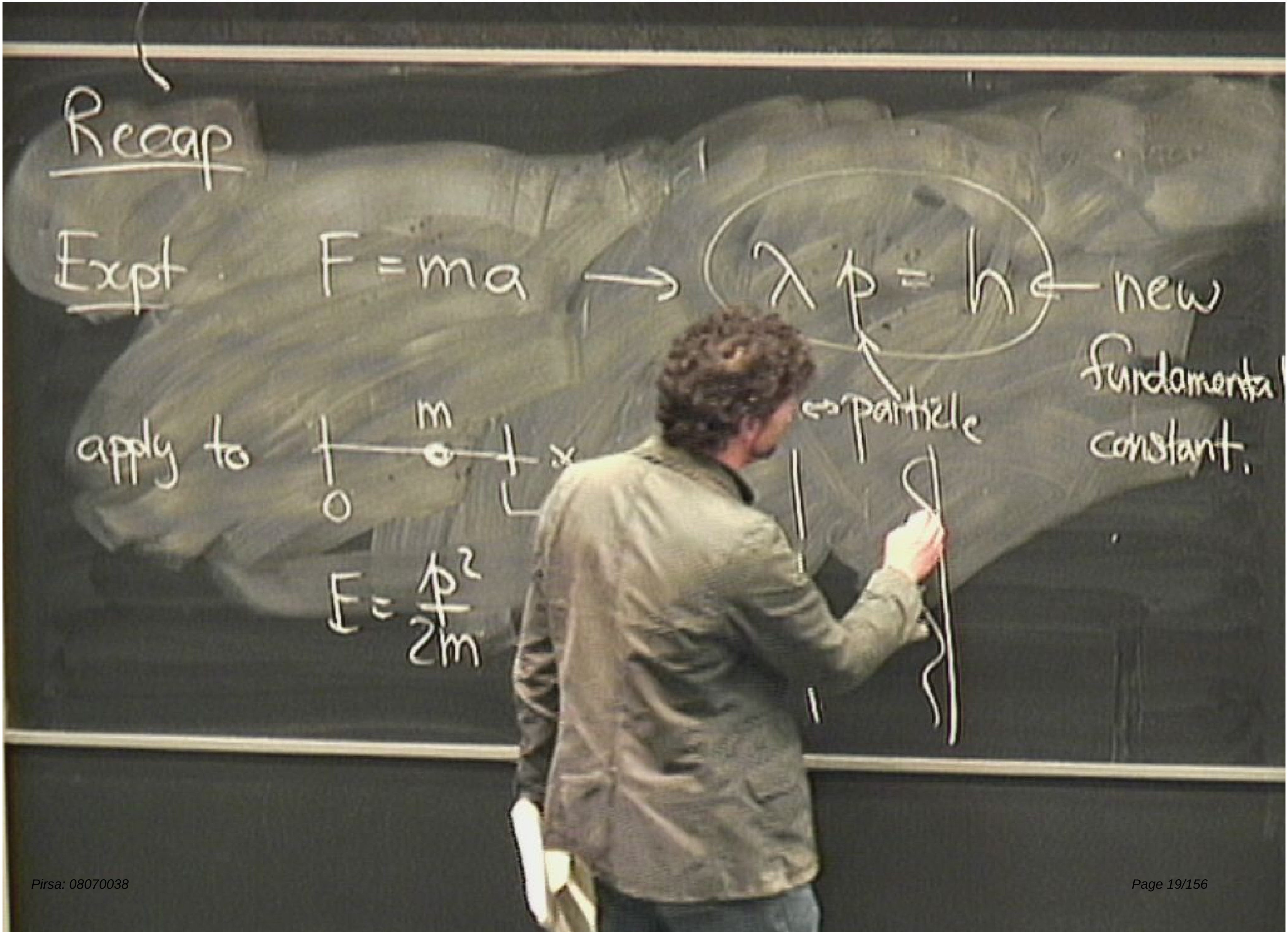

Energy
$E_2 = 9E_0$

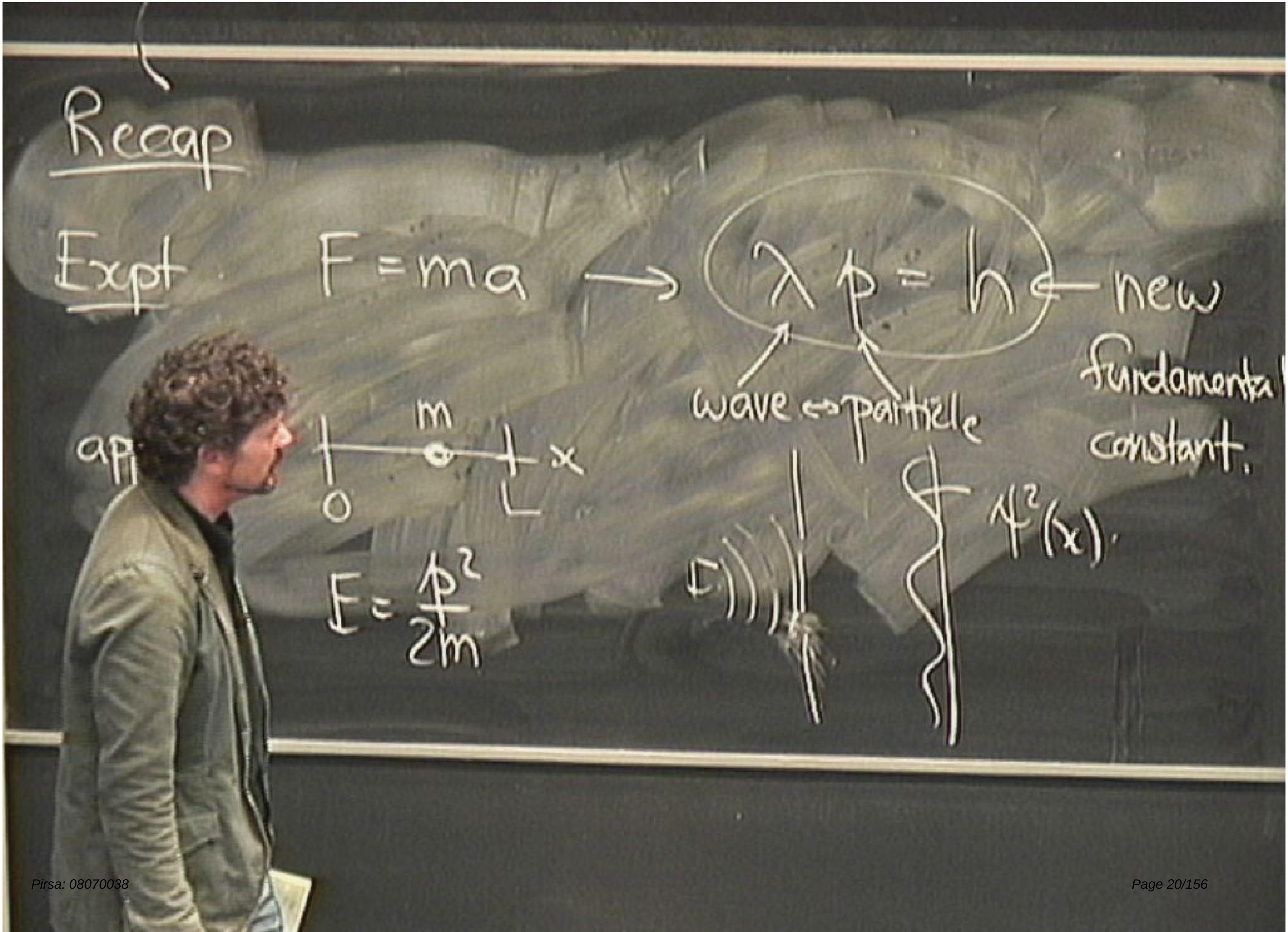

Energy
E2 = 9E0
E1 = 4E0
ψ1 =
E0
ψ0 =
0

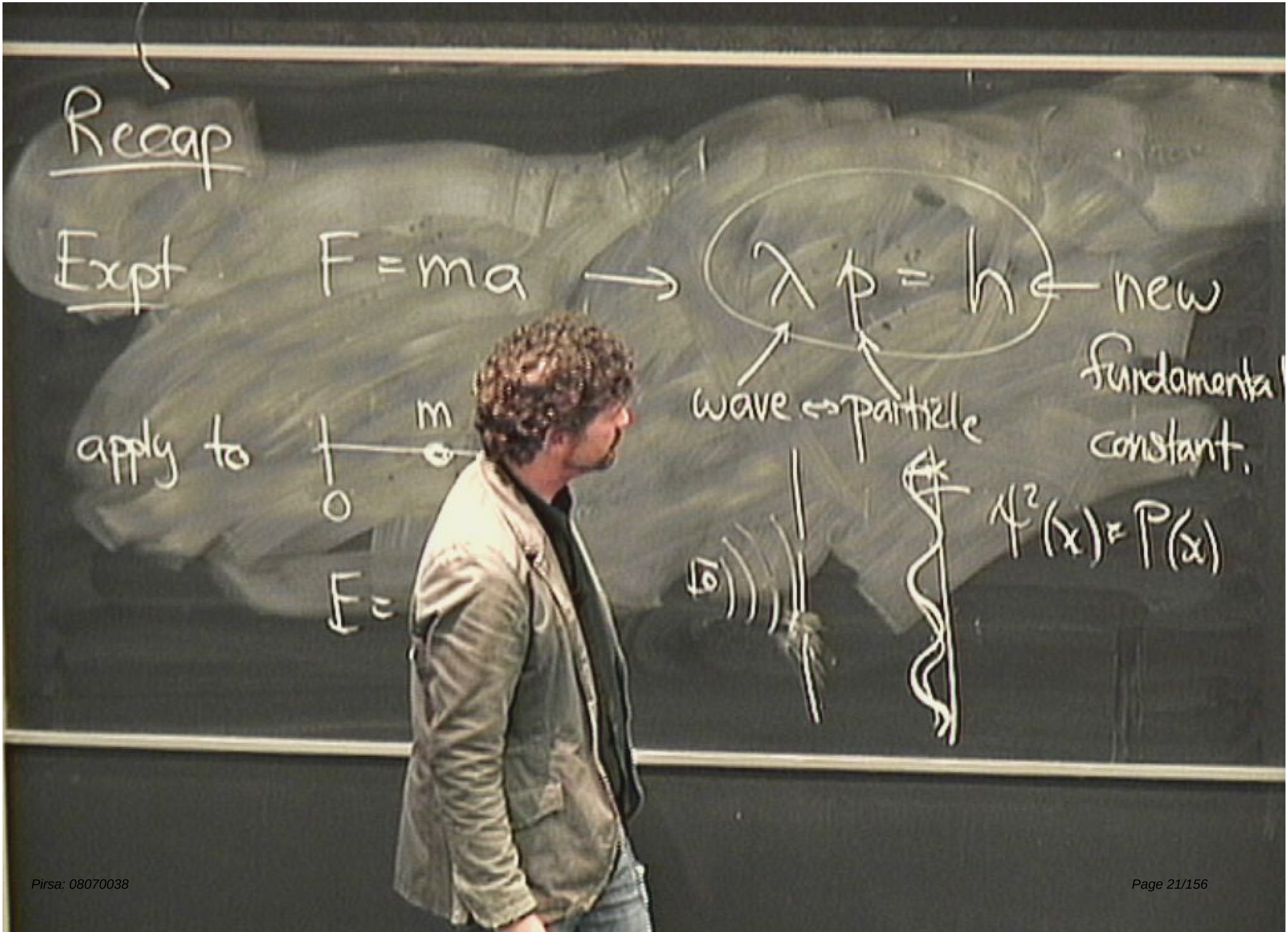


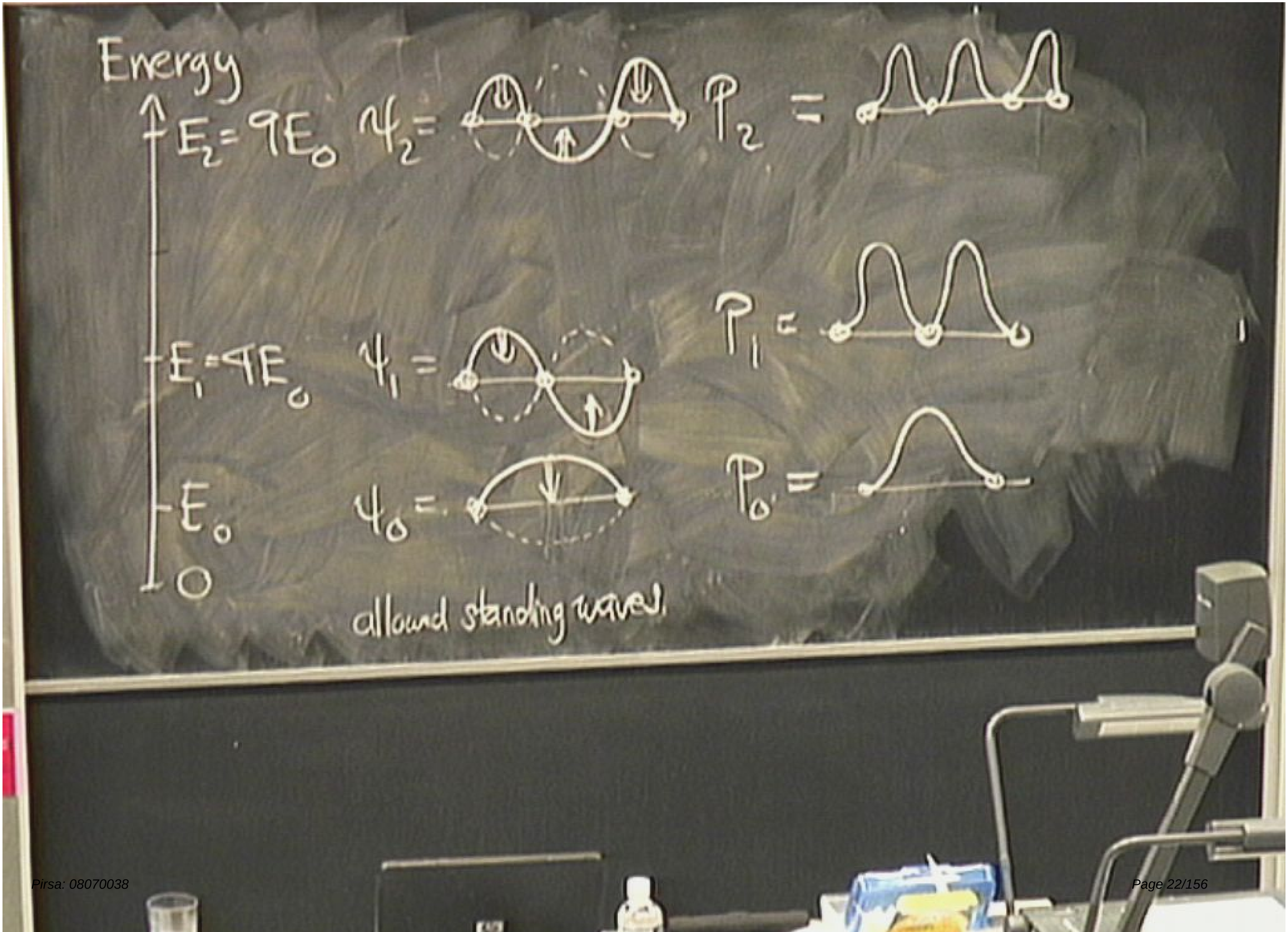
Energy
$E_2 = 9E_0$ $\psi_2 =$
$E_1 = 4E_0$ $\psi_1 =$
E_0 $\psi_0 =$
0

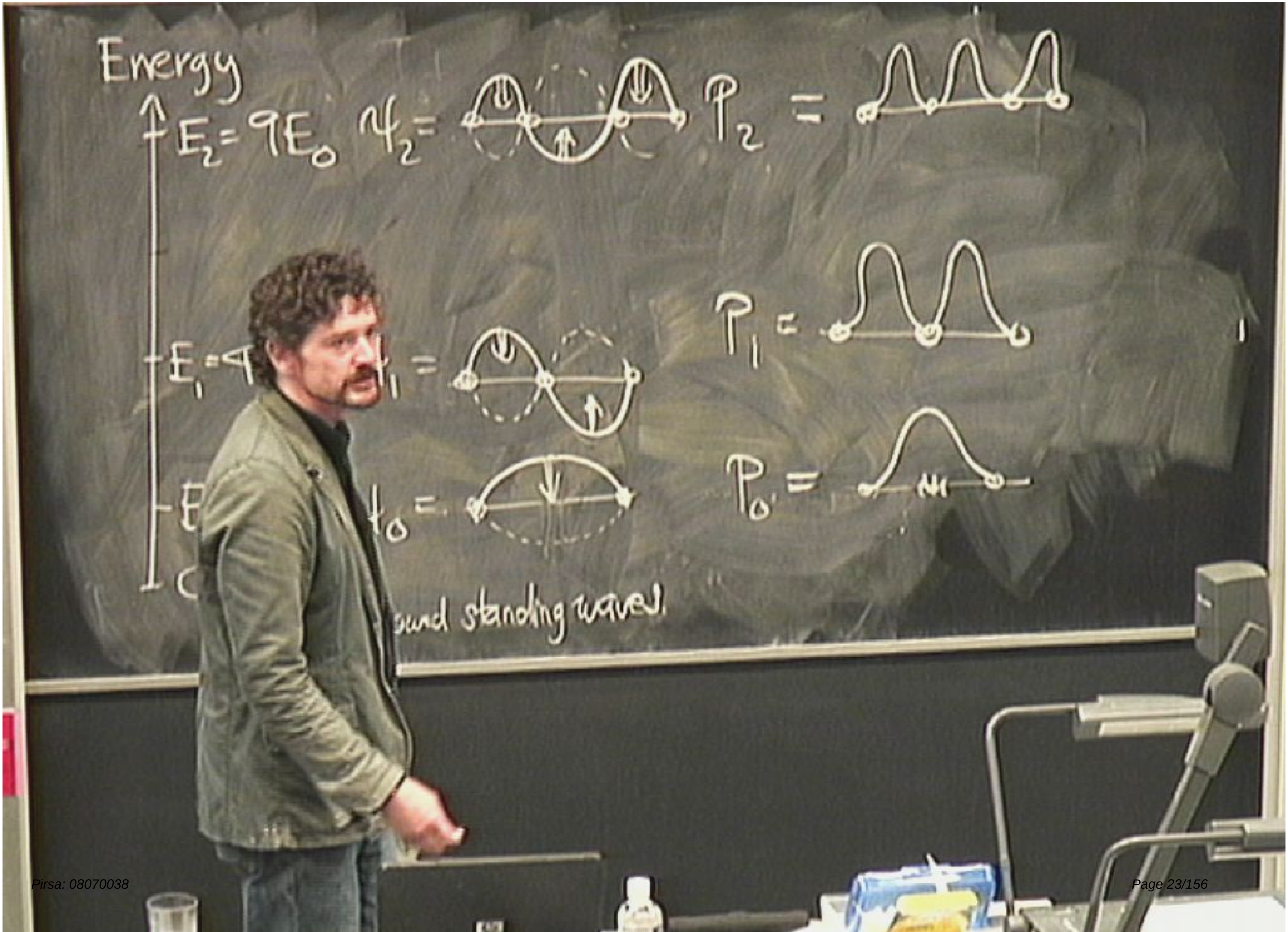

Energy
E2 = 9E0
ψ2 =
E1 = 4E0
ψ1 =
E0
ψ0 =
0
allowed standing waves.

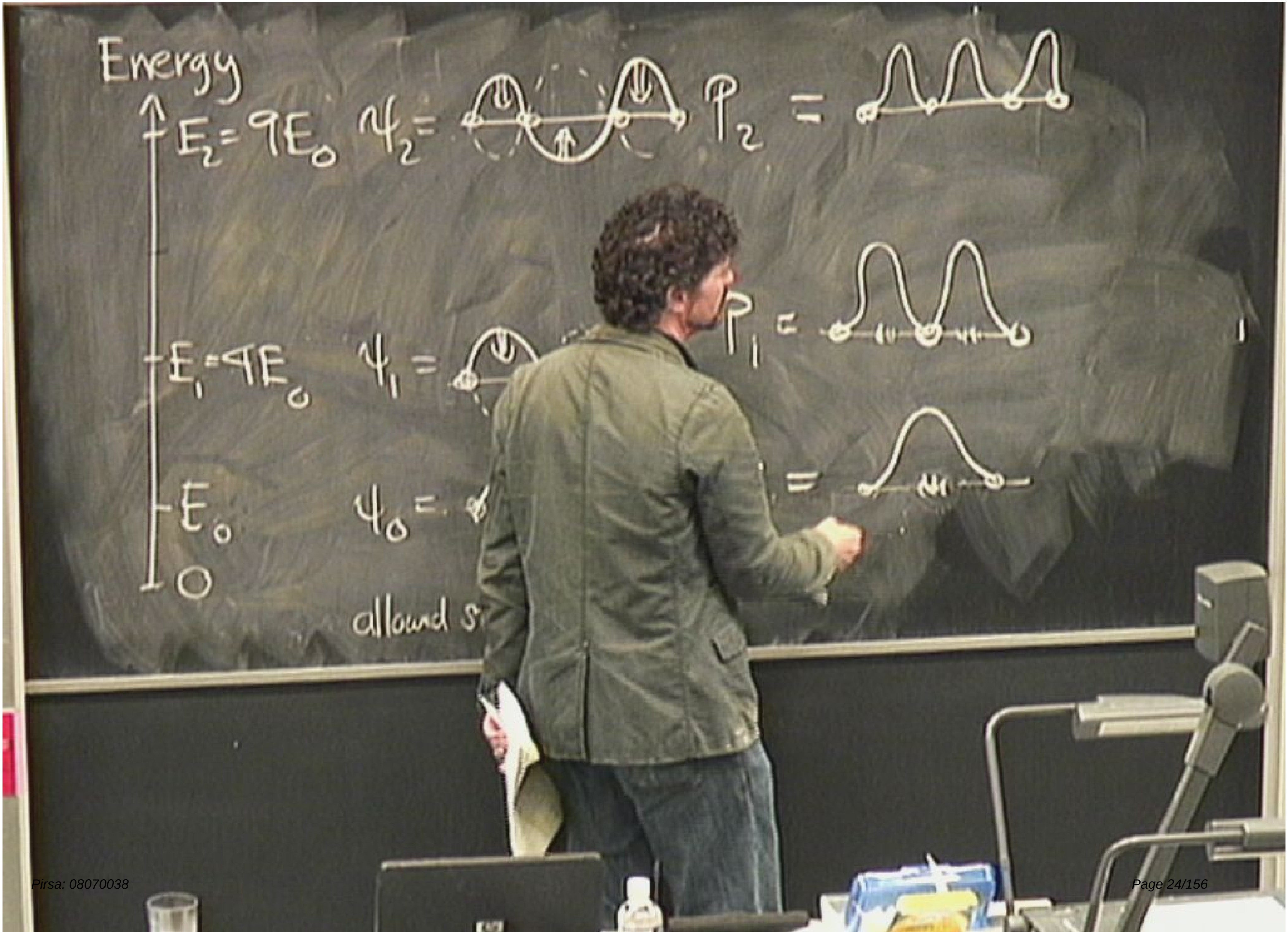

Recap
Expt
F = ma
λ p = h
new
fundamental
constant.
particle
apply to
m
x
0
L
E = p²/2m

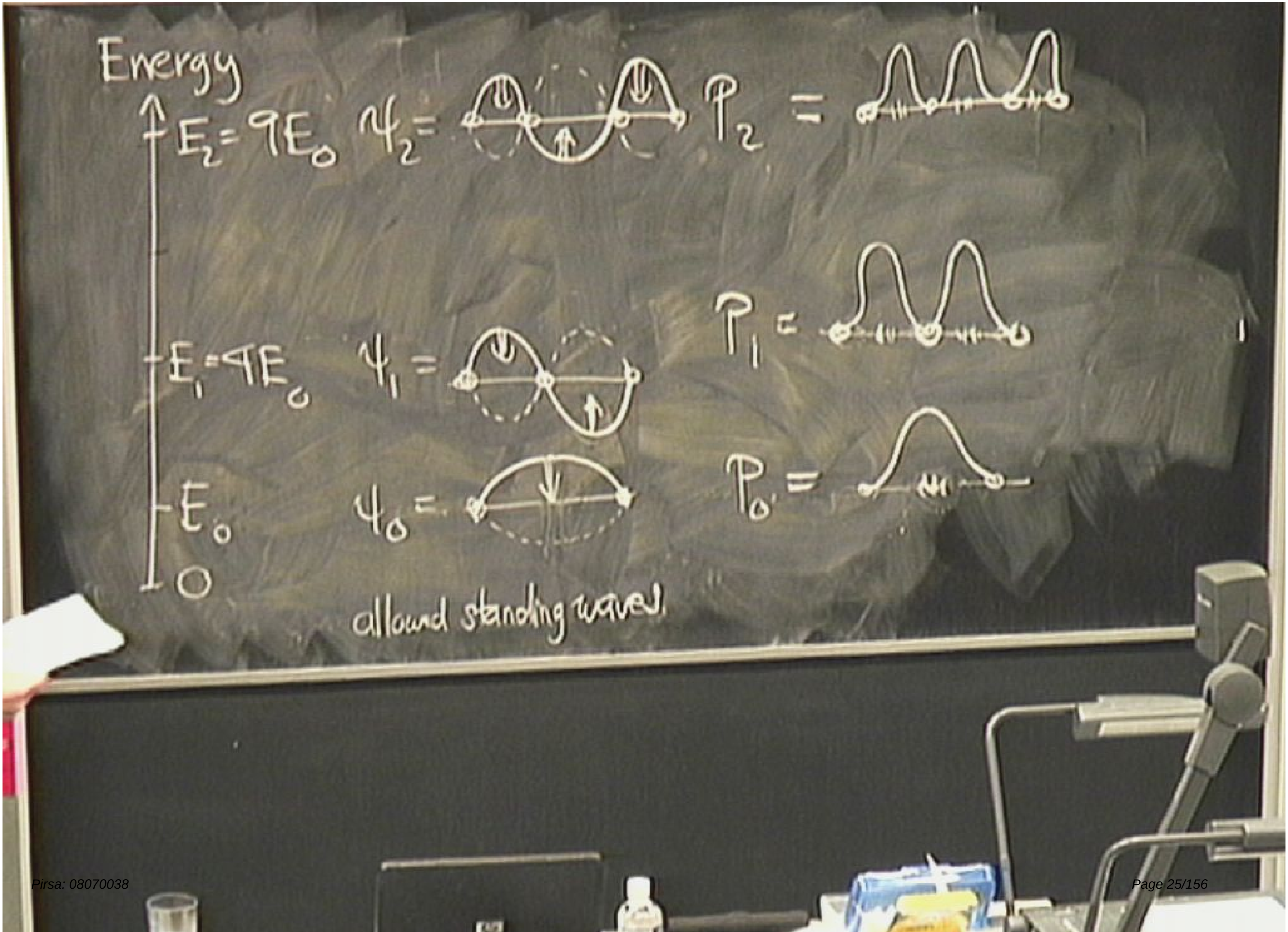

Recap
Expt
F = ma
λp = h
new
fundamental constant.
wave ↔ particle
m
x
0
L
E = p²/2m
ψ²(x).

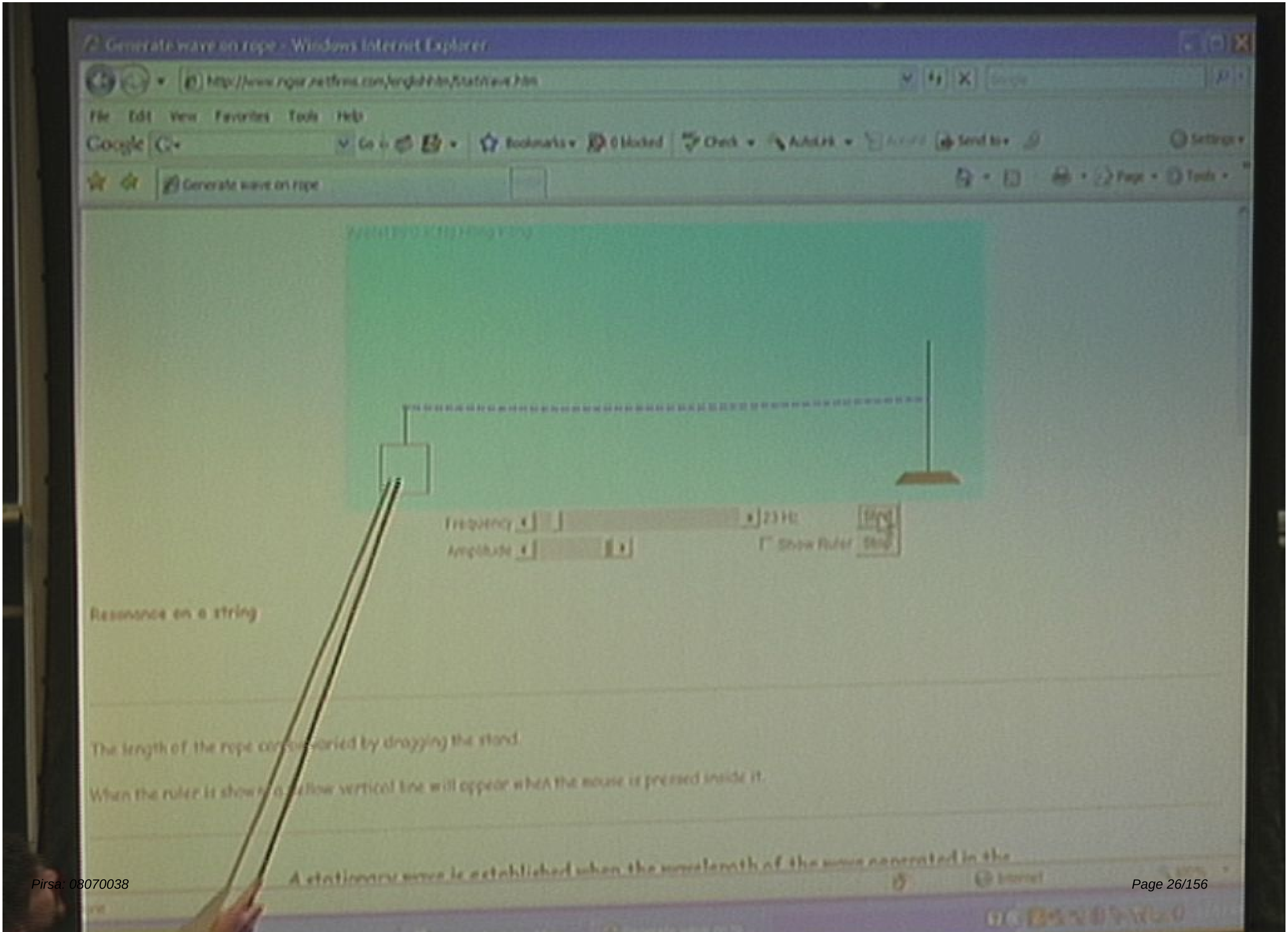

Recap
Expt
F = ma
λp = h
new
fundamental
constant.
wave ⇔ particle
apply to
m
0
E =
ψ²(x) = P(x)

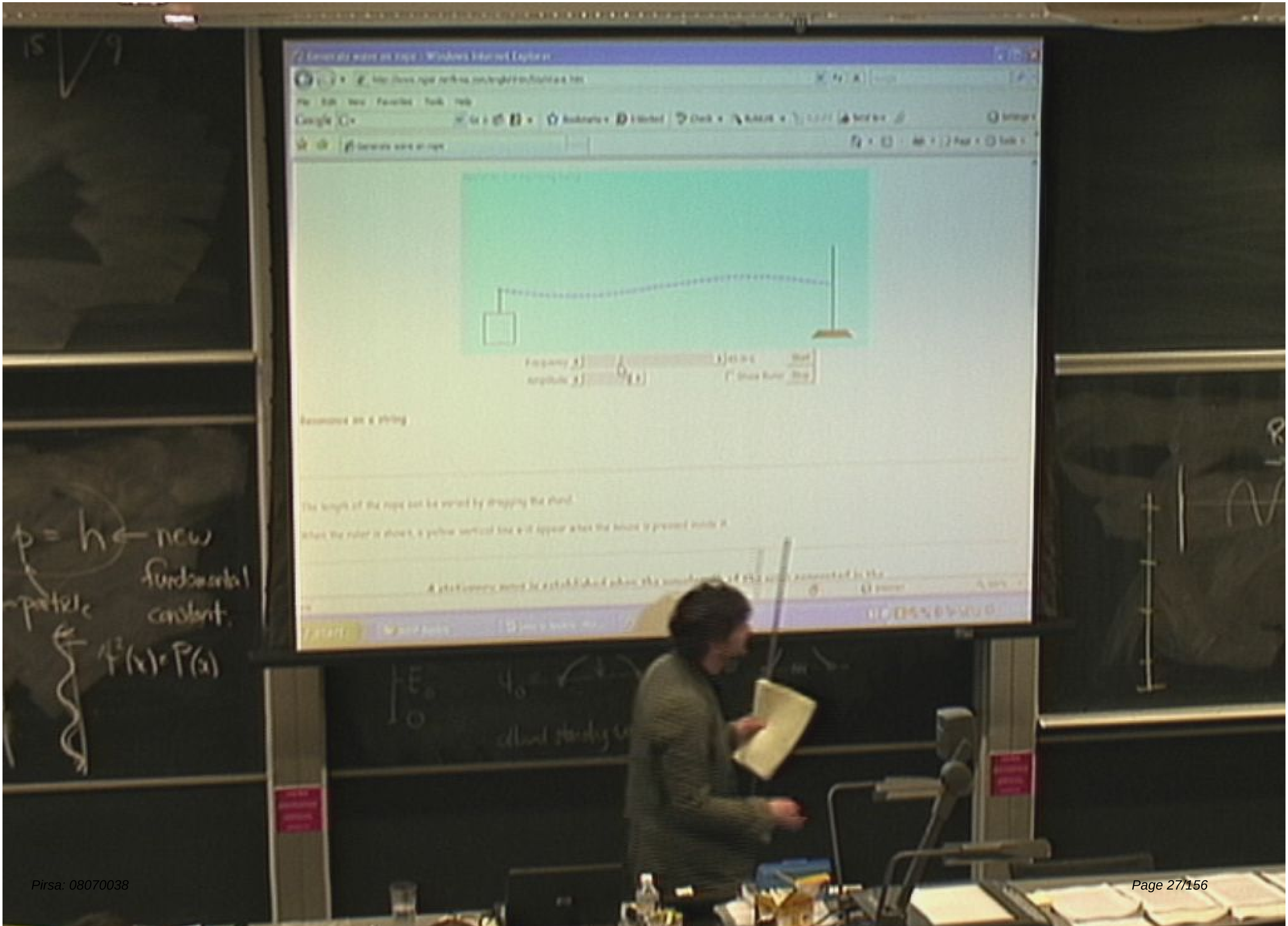

Energy
$E_2 = 9E_0$
$\psi_2 =$
$P_2 =$
$P_1 =$
$E_1 = 4E_0$
$\psi_1 =$
$P_0 =$
E_0
$\psi_0 =$
0
allowed standing waves.

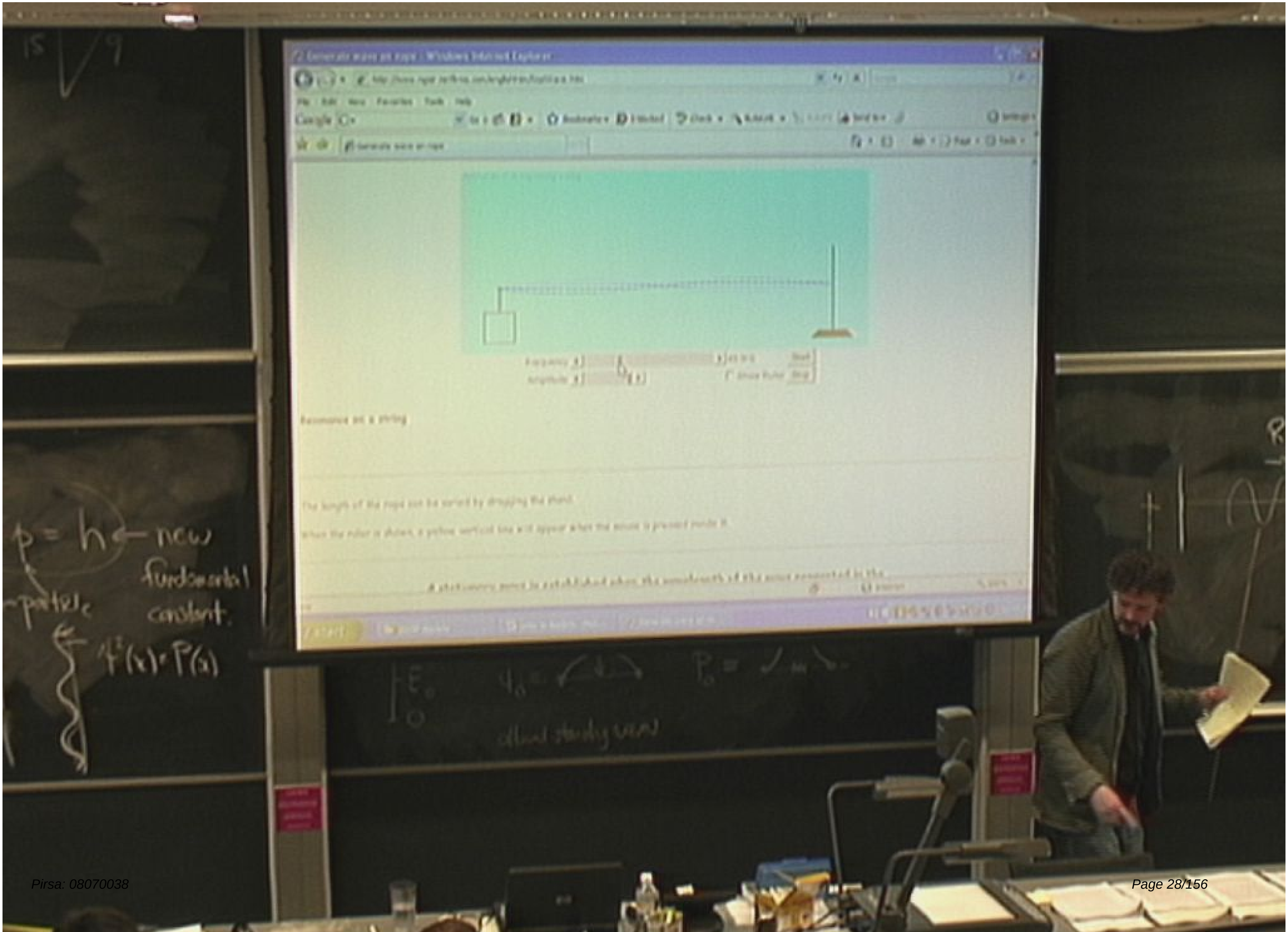

Energy
$E_2 = 9E_0$ $\psi_2 =$
$P_2 =$
$P_1 =$
$E_1 = 4$
$\psi_1 =$
$P_0 =$
$\psi_0 =$
und standing waves.

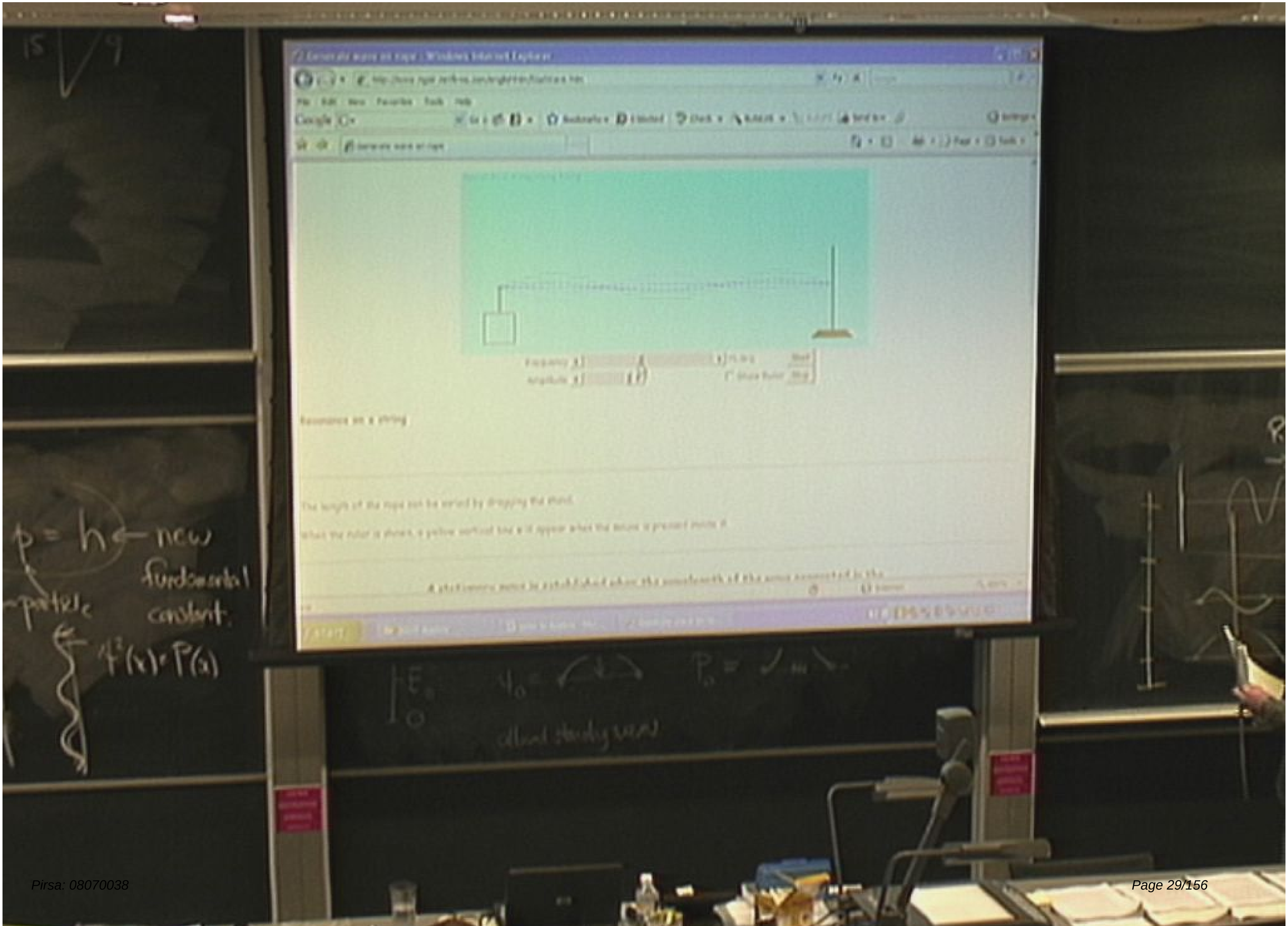

Energy
E2 = 9E0
ψ2 =
P2 =
E1 = 4E0
ψ1 =
P1 =
E0
ψ0 =
0

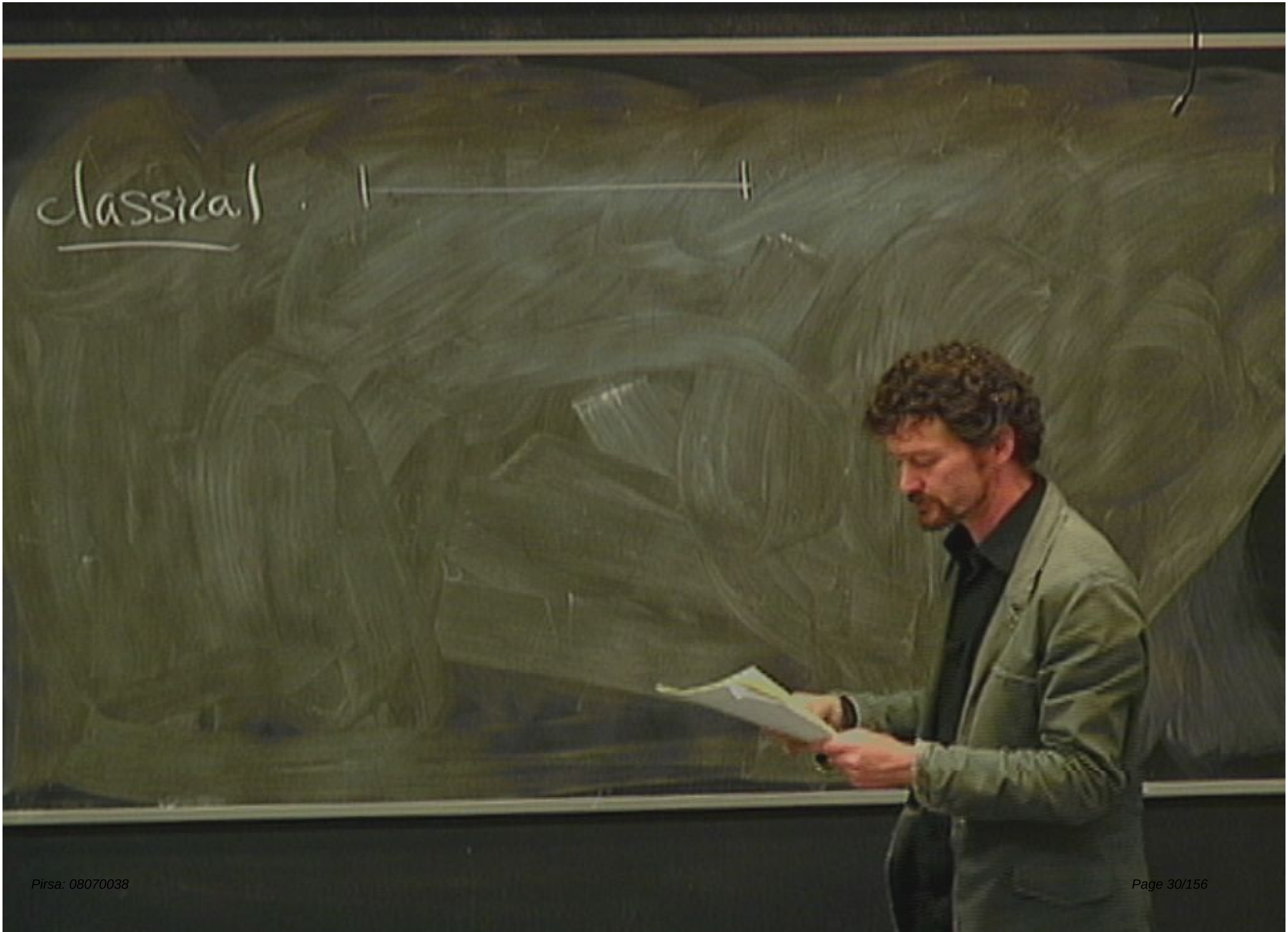

Energy
$E_2 = 9E_0$
$\psi_2 =$
$P_2 =$
$E_1 = 4E_0$
$\psi_1 =$
$P_1 =$
E_0
$\psi_0 =$
$P_0 =$
0
allowd standing waves.

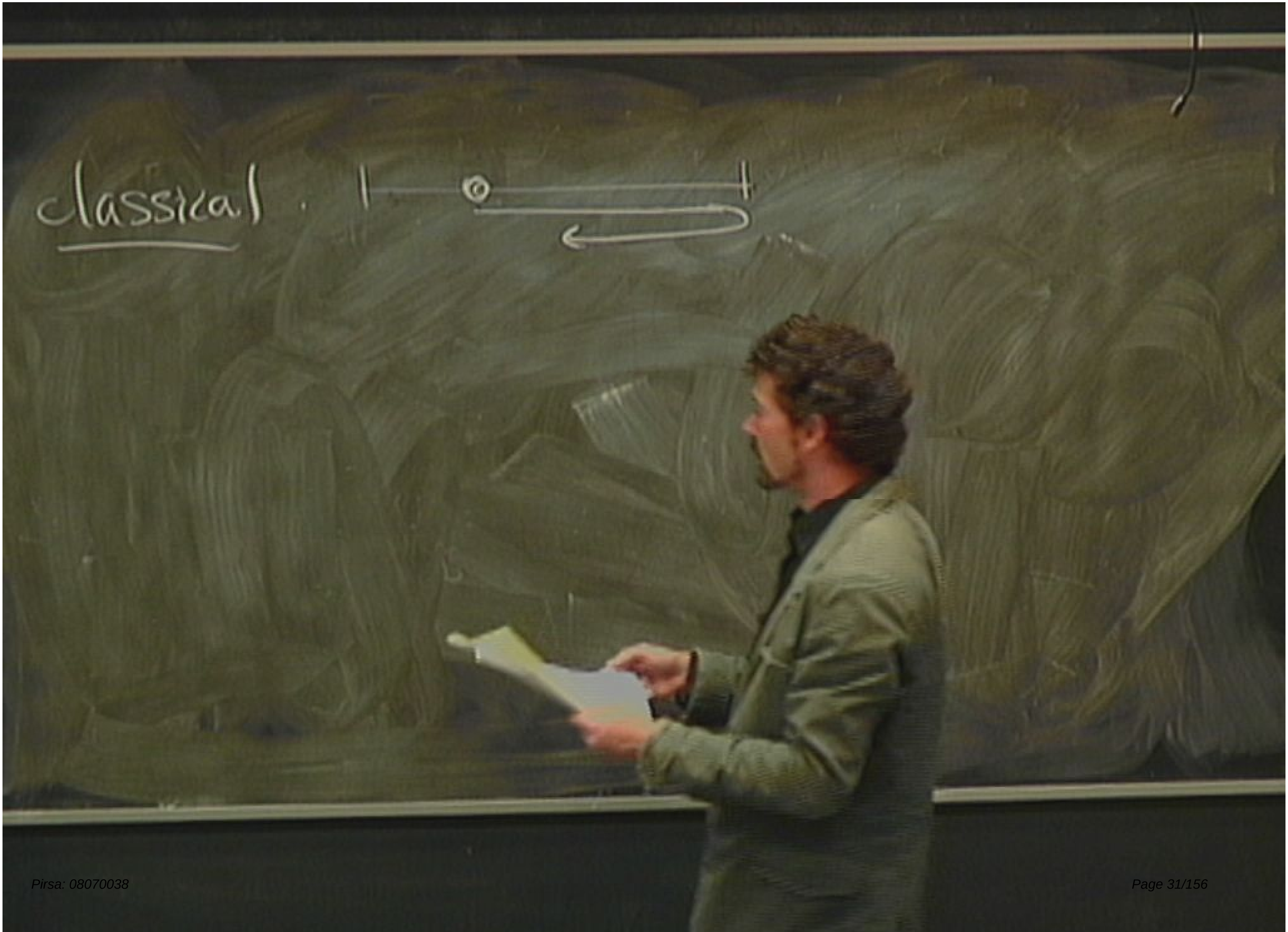

Generate wave on rope - Windows Internet Explorer
File Edit View Favorites Tools Help
Generate wave on rope
Frequency
Amplitude
Show Ruler
Resonance on a string
The length of the rope can be varied by dragging the stand.
When the ruler is shown a yellow vertical line will appear when the mouse is pressed inside it.
A stationary wave is established when the wavelength of the wave generated in the

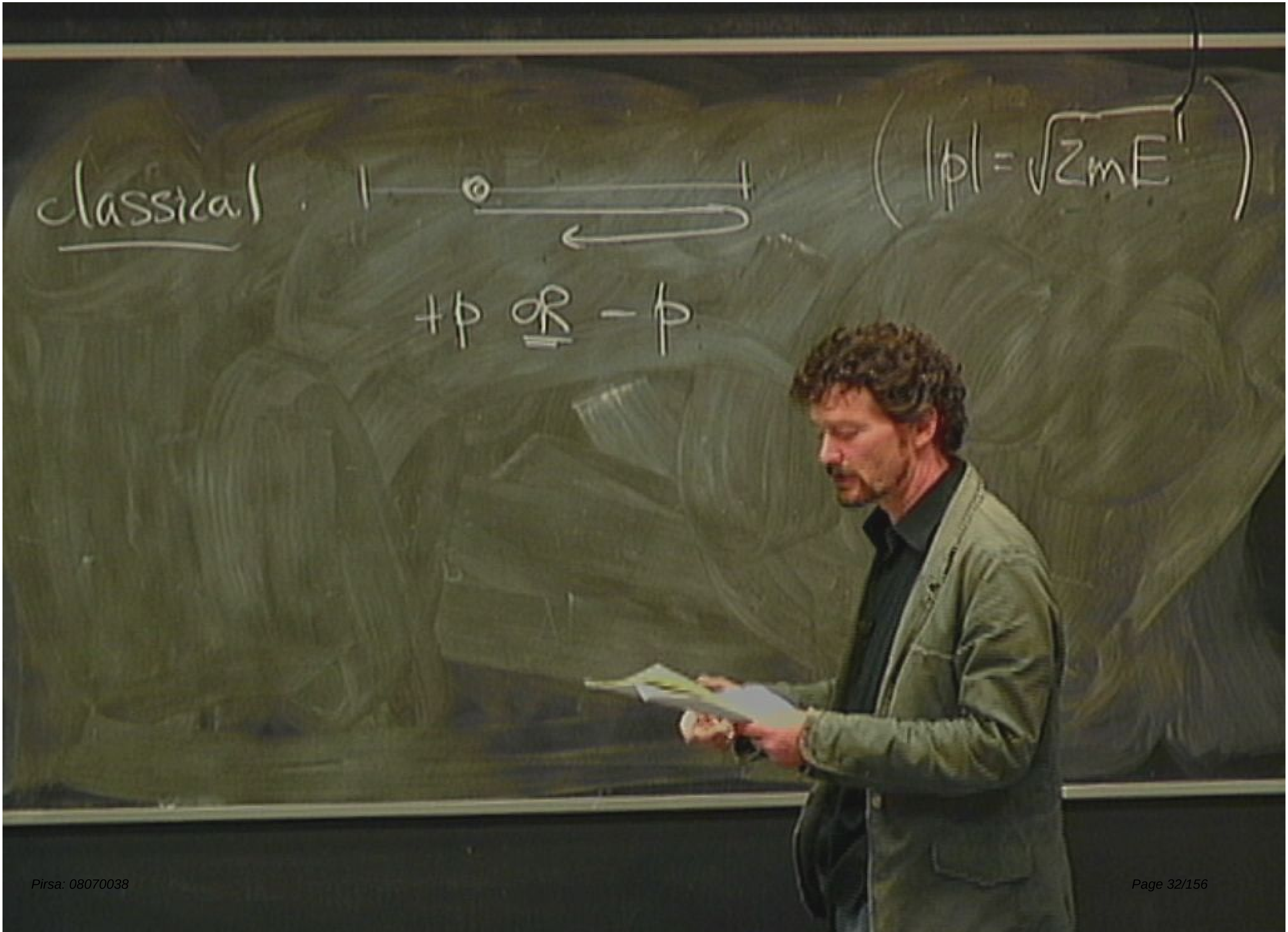

Resonance on a string
p = h ← new
fundamental
constant
particle

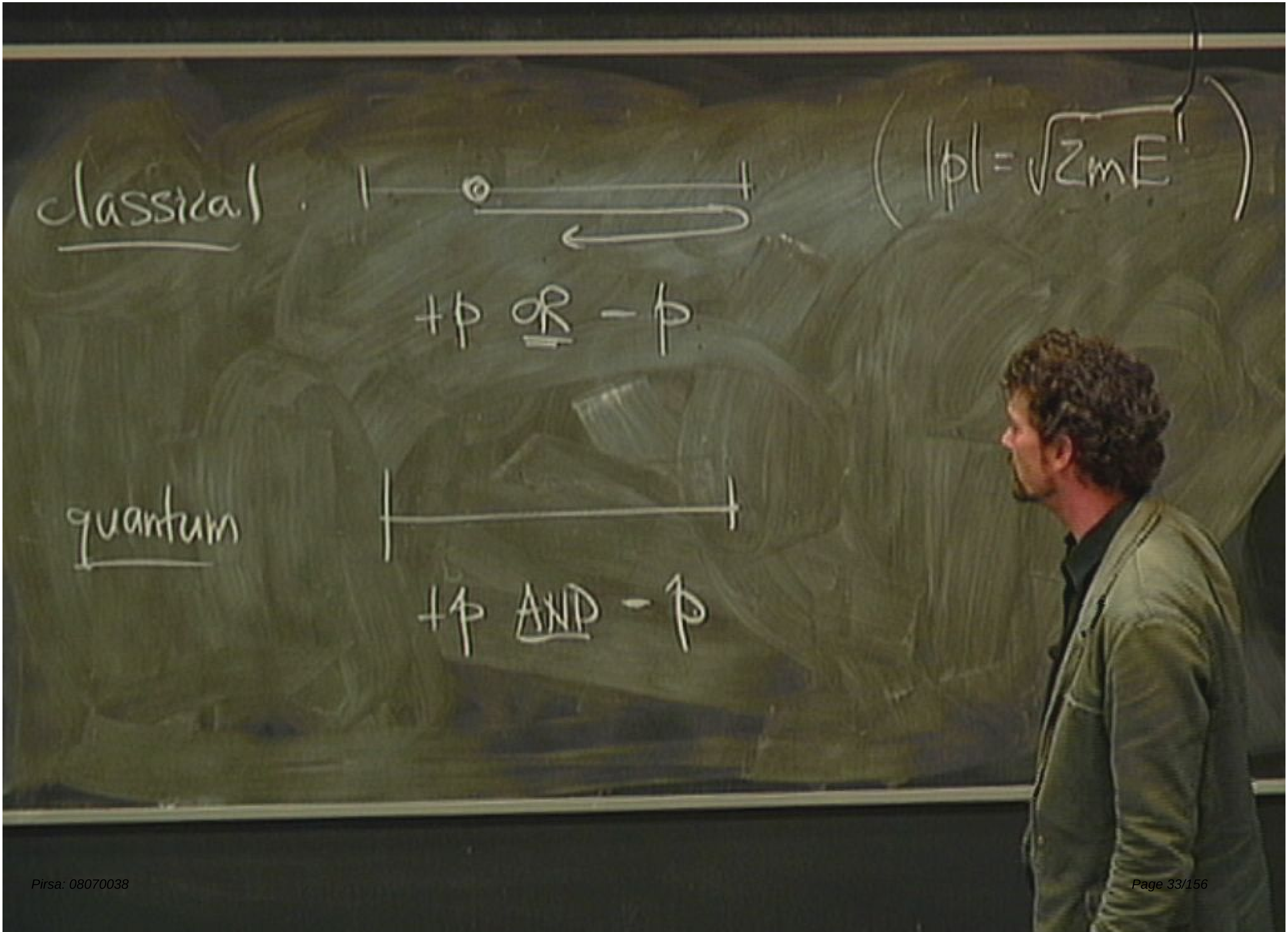


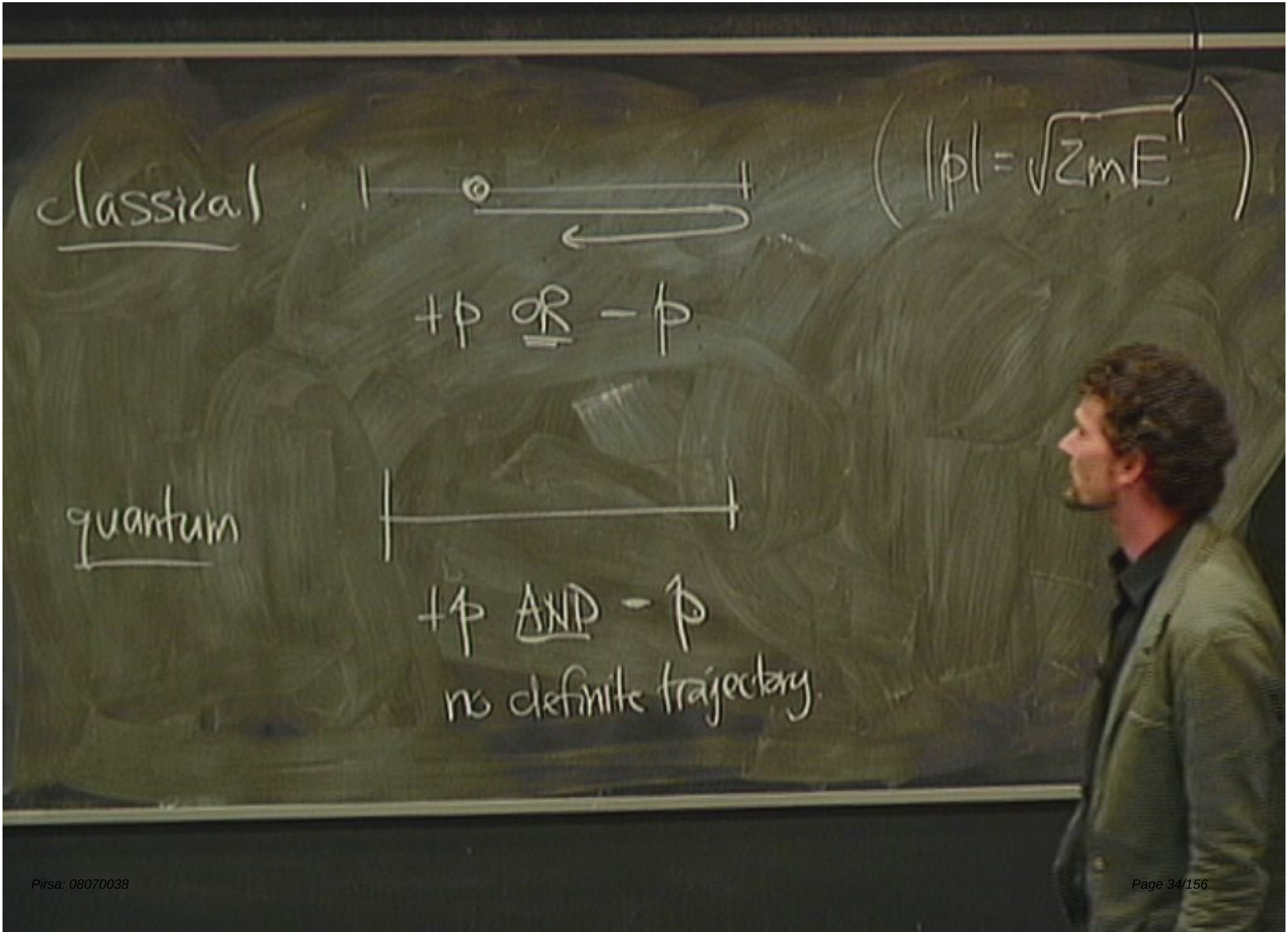

p = h ← new
fundamental
constant
particle


classical


classical


classical
|p| = √2mE
+p OR −p


classical
+p OR −p
quantum
+p AND −p
$(|p| = \sqrt{2mE})$


classical
(|p| = √2mE)
+p OR − p
quantum
+p AND − p
no definite trajectory.

classical

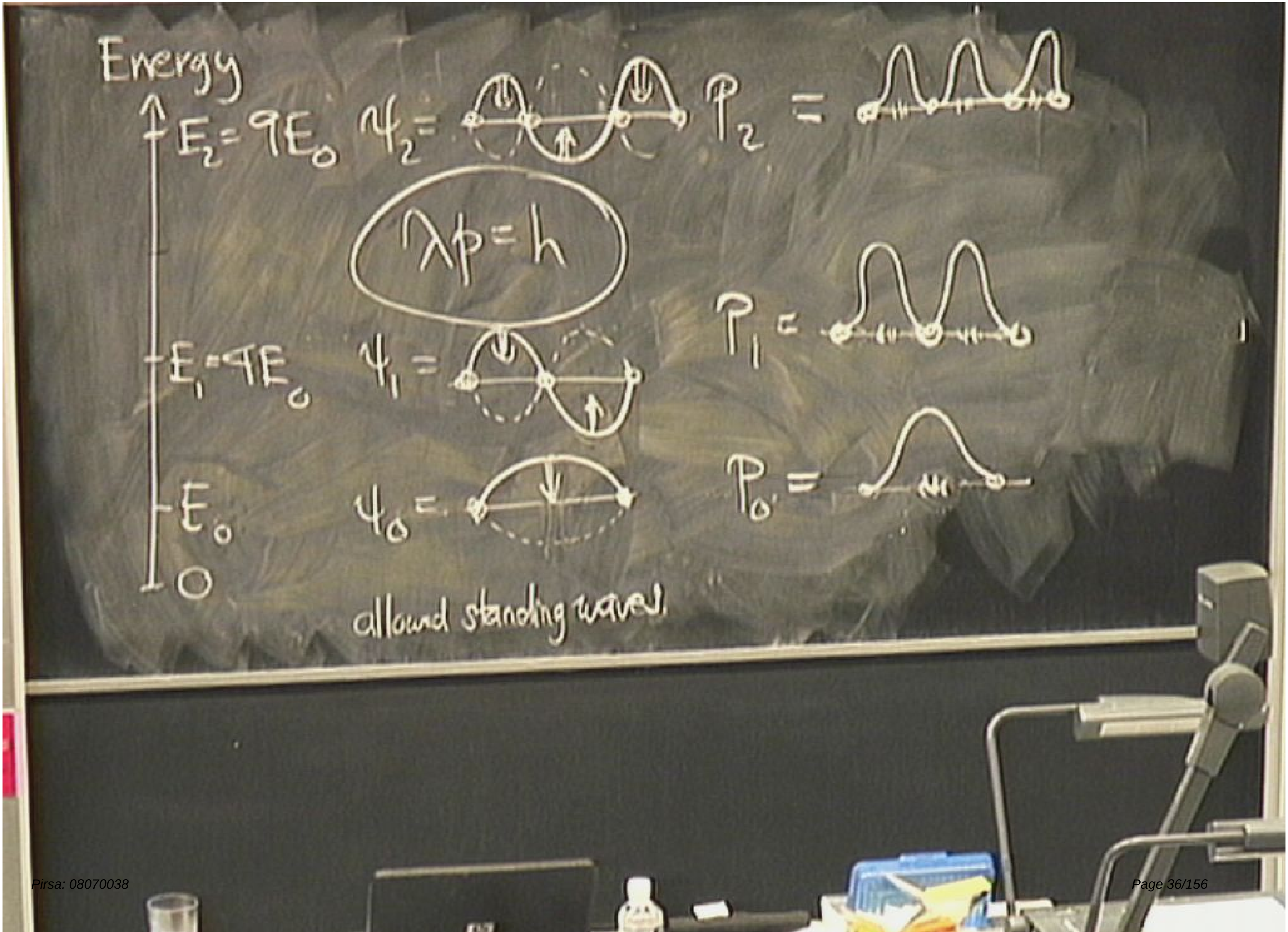
$$\left(|p| = \sqrt{2mE} \right)$$

$+p$ OR $-p$

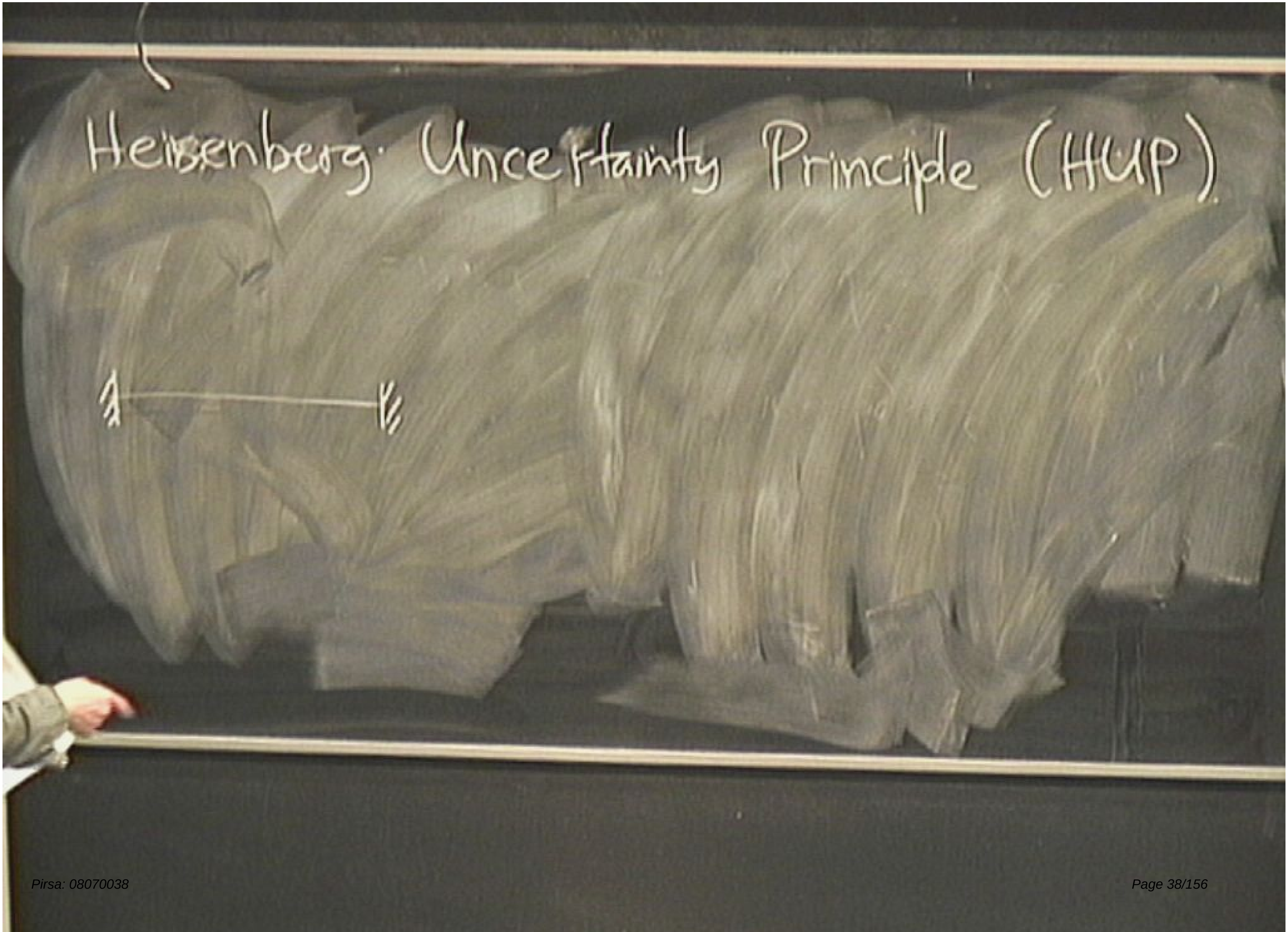
quantum

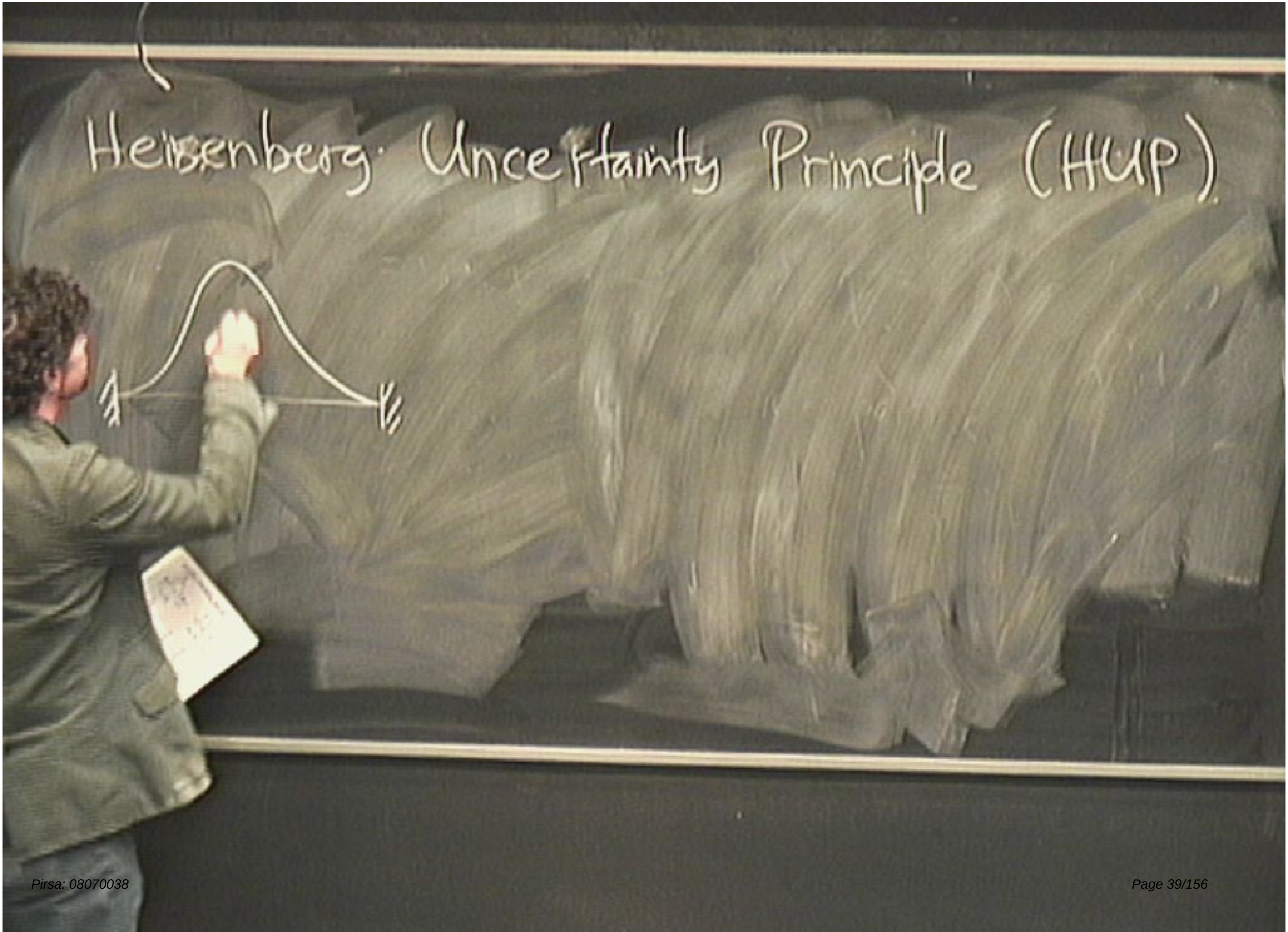
$+p$ AND $-p$

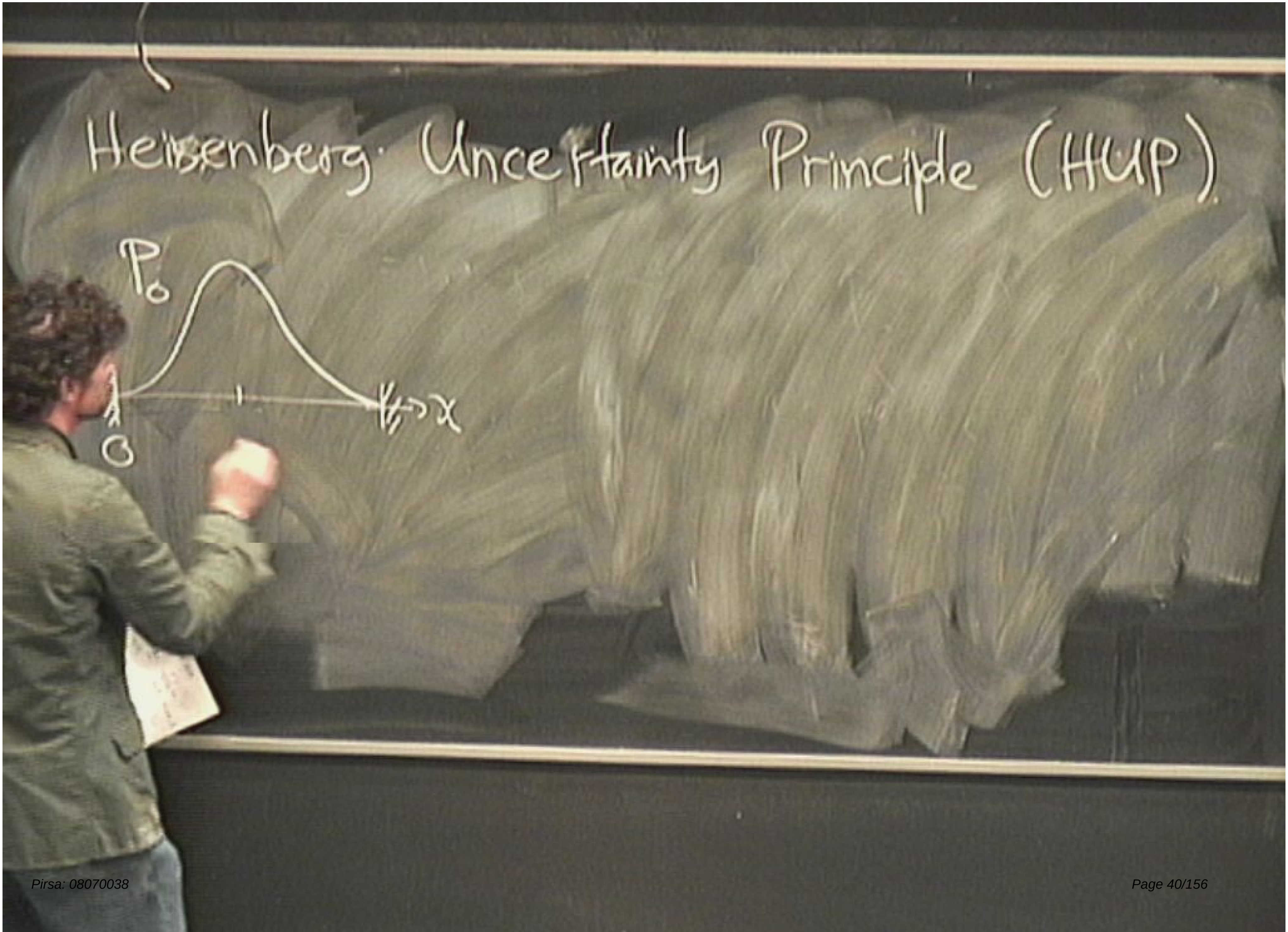
no definite trajectory.
just probability.


Energy
E2 = 9E0
ψ2 =
P2 =
λp = h
E1 = 4E0
ψ1 =
P1 =
E0
ψ0 =
P0 =
0
allowd standing waves.

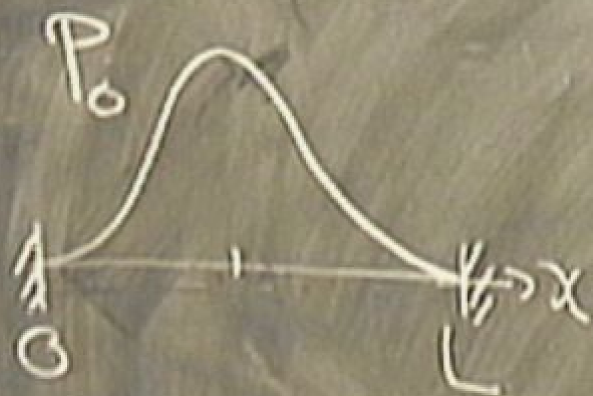
Heisenberg Uncertainty Principle (HUP)

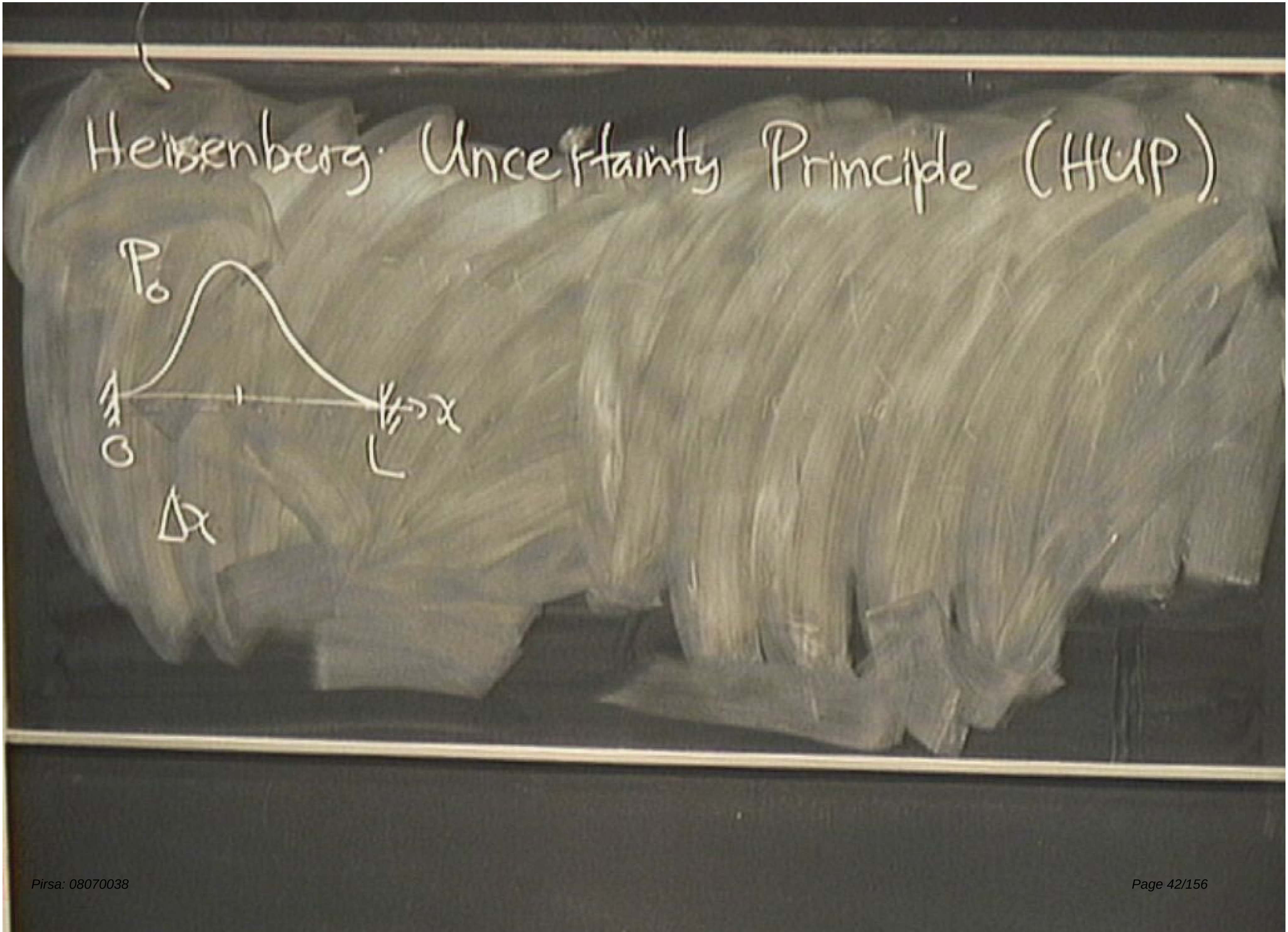

Heisenberg Uncertainty Principle (HUP)

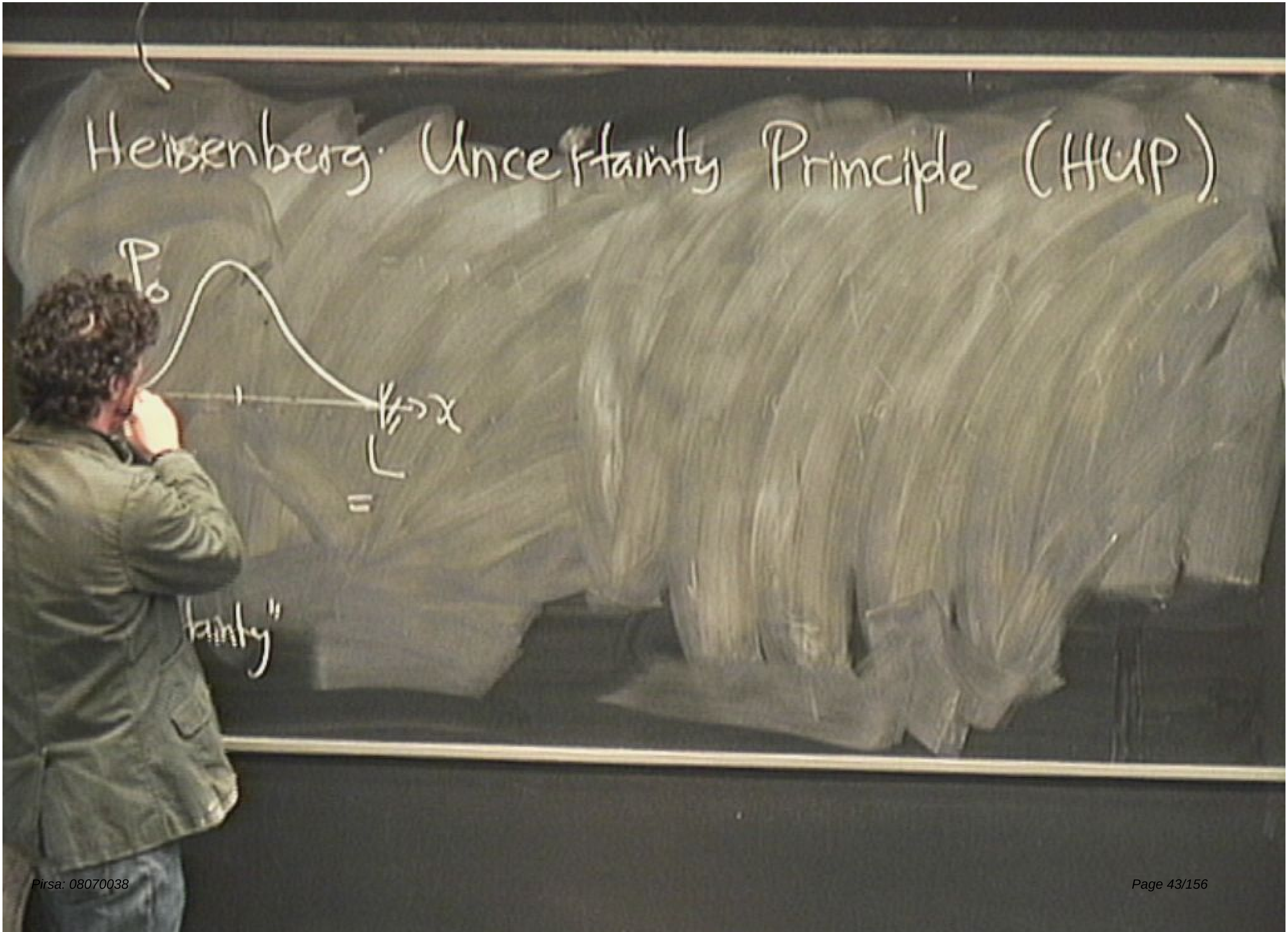

Heisenberg Uncertainty Principle (HUP)


Heisenberg Uncertainty Principle (HUP)
P0
0
x

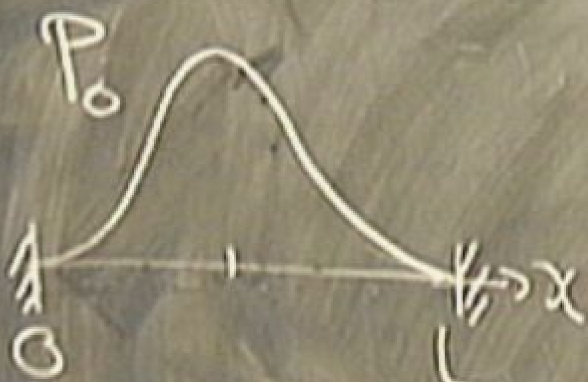
Heisenberg Uncertainty Principle (HUP)




Heisenberg Uncertainty Principle (HUP)
P_0
0
L
x
Δx


Heisenberg Uncertainty Principle (HUP)
P0
x
L
=
tainty"

Heisenberg Uncertainty Principle (HUP)

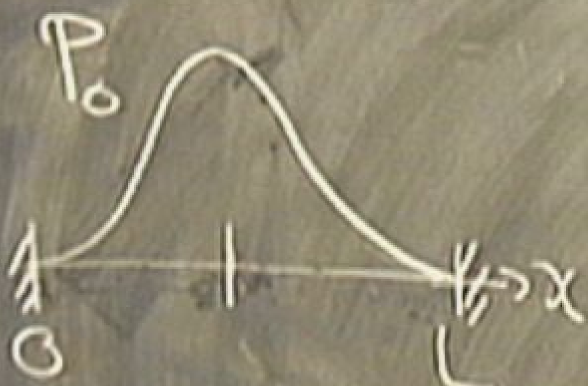


$= \lambda/2$

$$\Delta x = \frac{\lambda}{2}$$

= "uncertainty"

Heisenberg Uncertainty Principle (HUP)

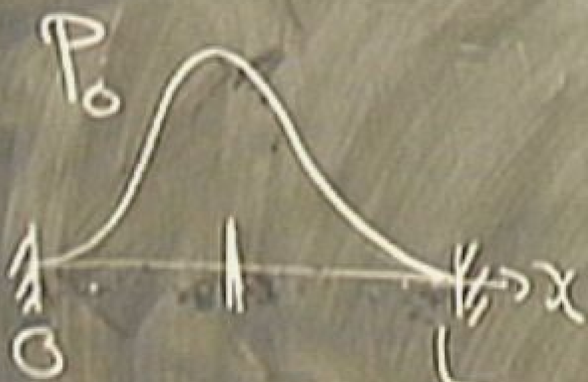


$= \lambda/2$

$$\Delta x = \frac{\lambda}{4}$$

$\Delta =$ "uncertainty"

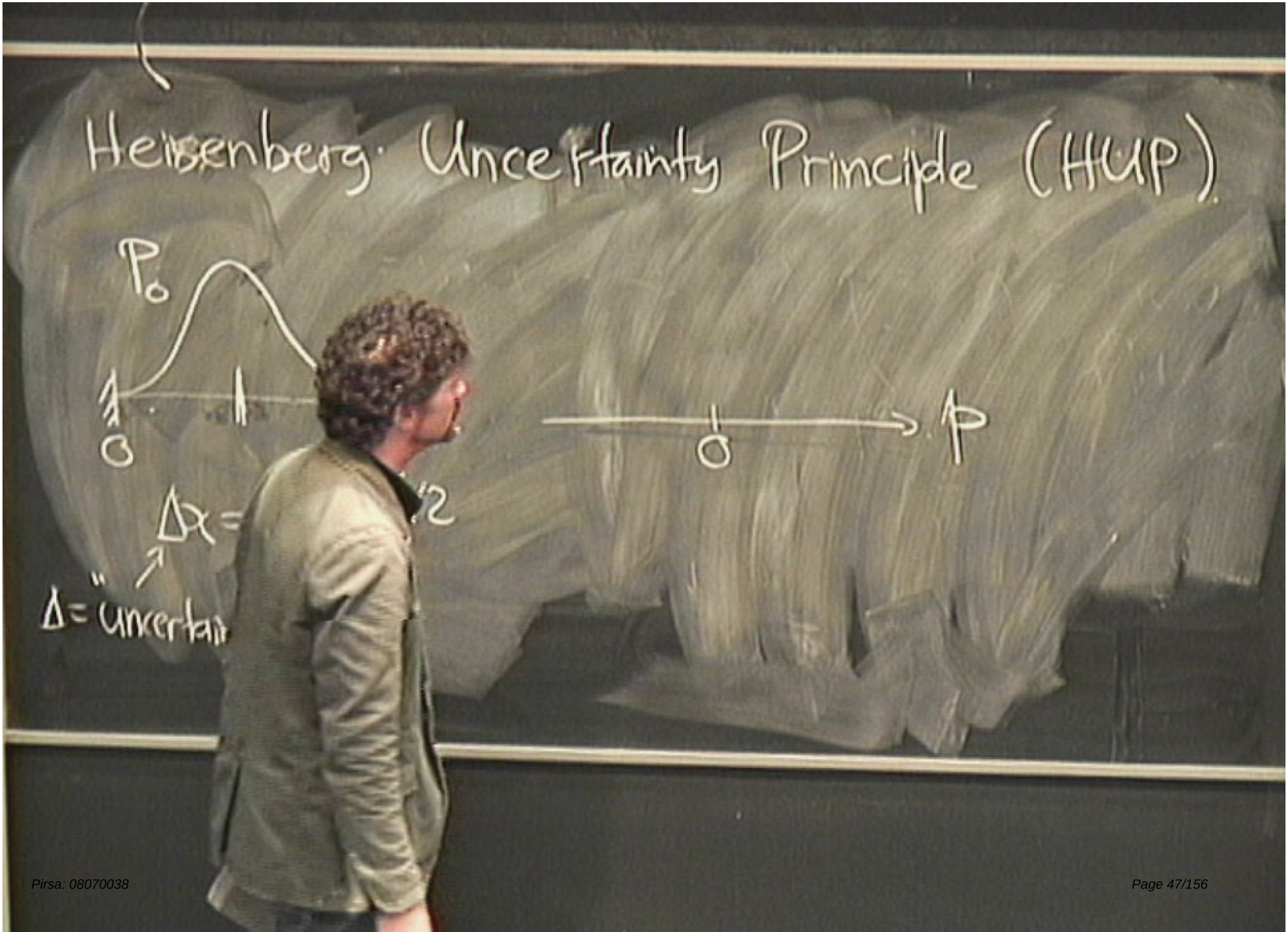
Heisenberg Uncertainty Principle (HUP)

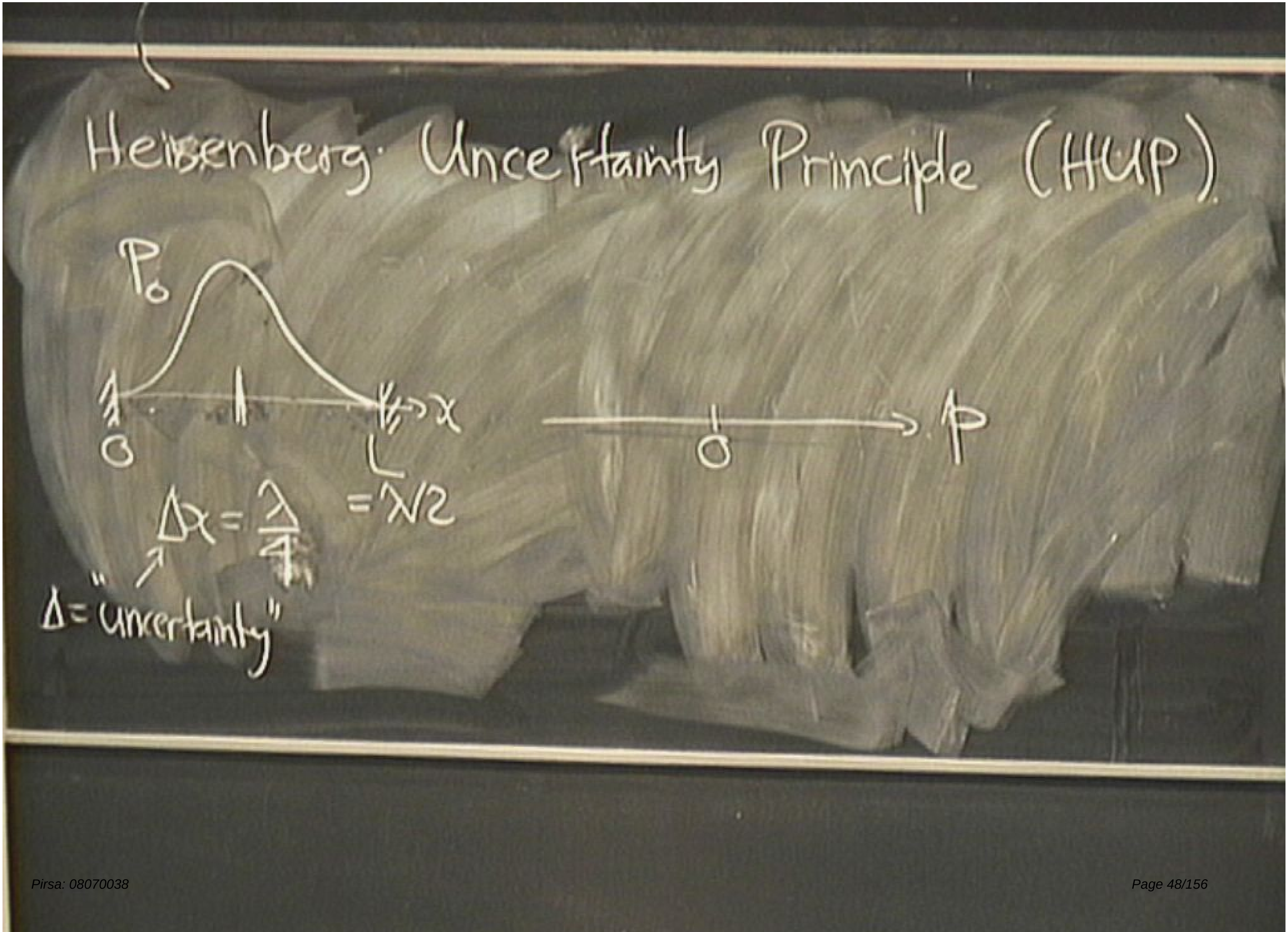


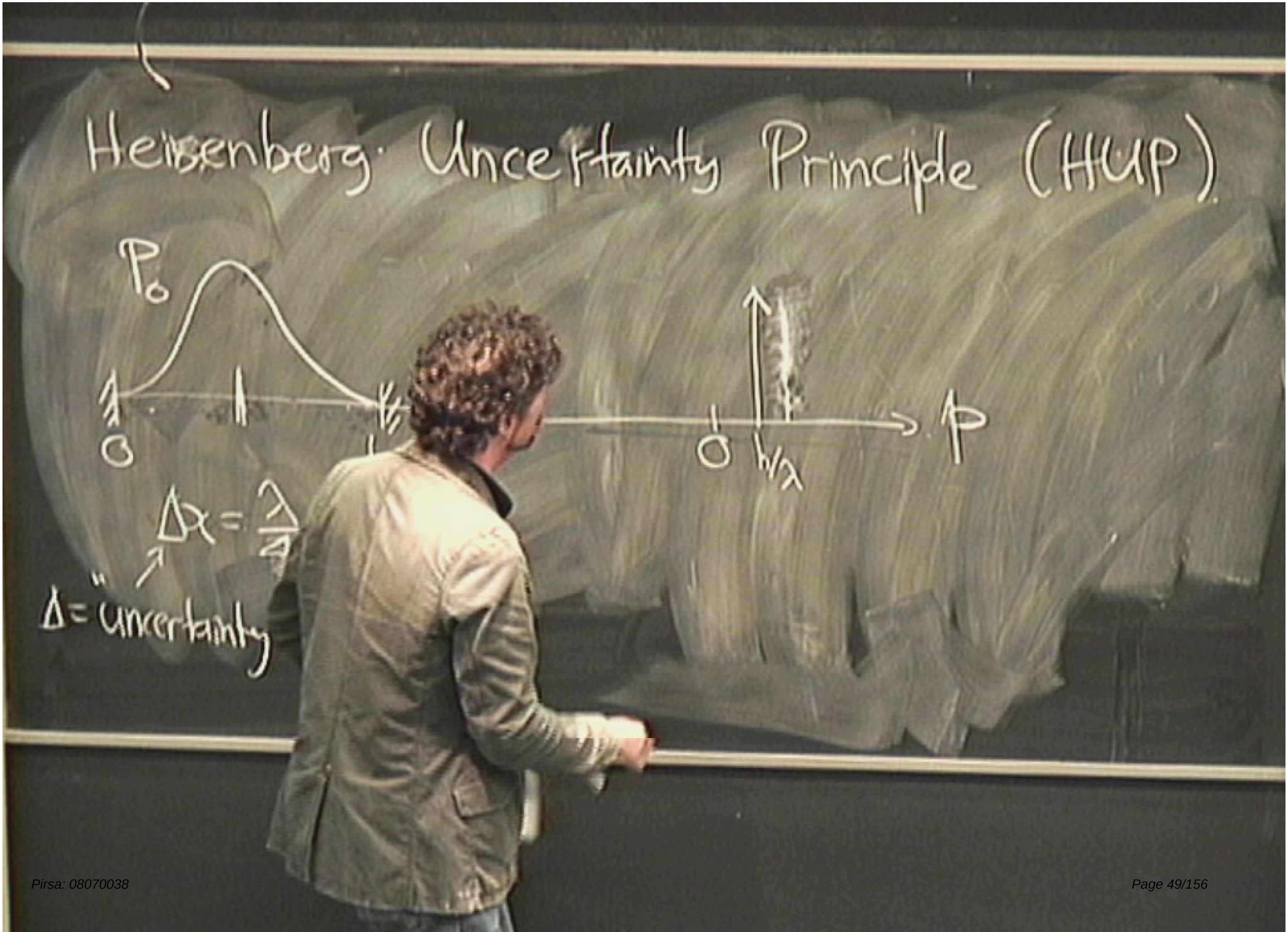
$= \lambda/2$

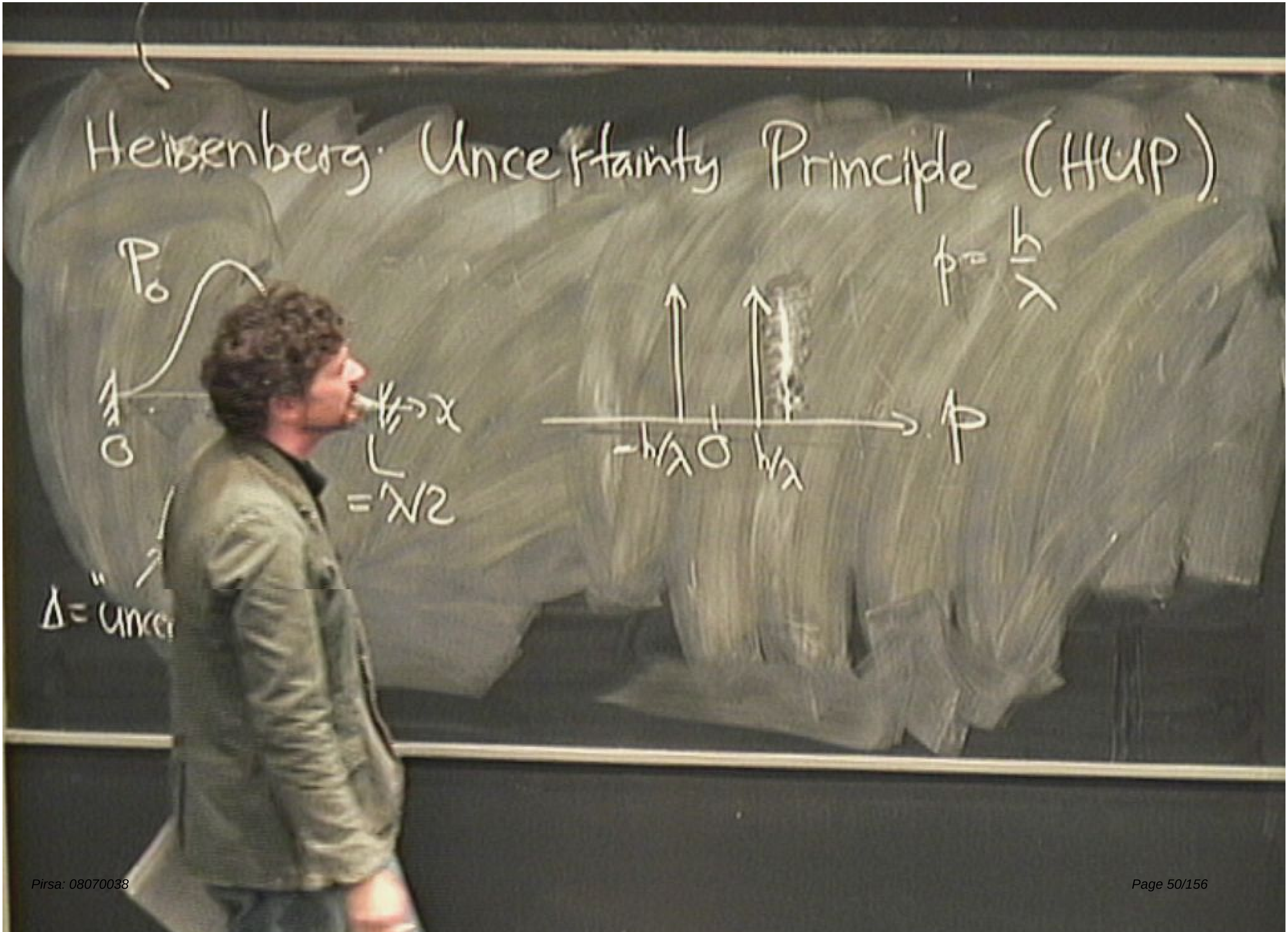
$$\Delta x = \frac{\lambda}{4}$$

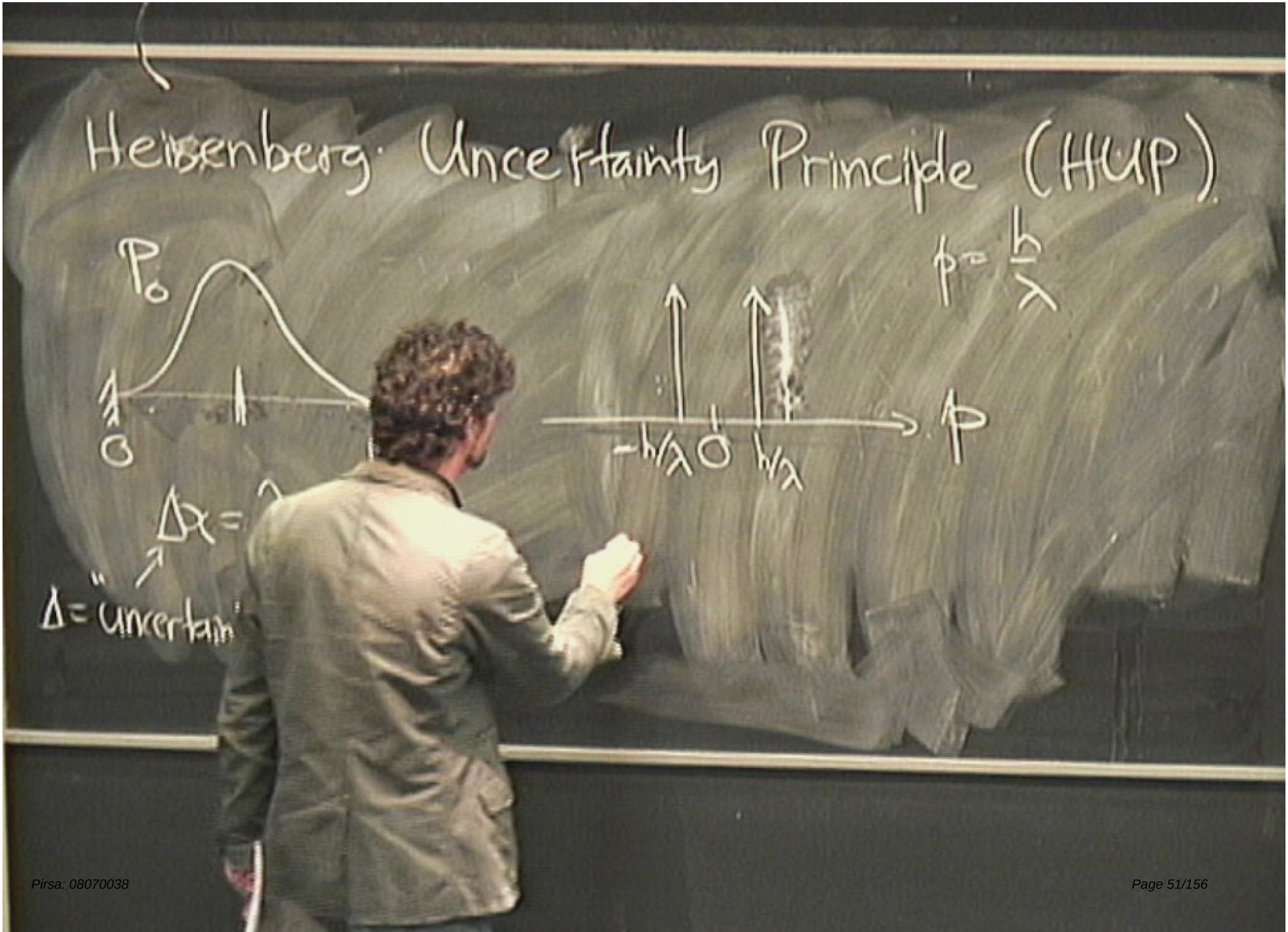
Δ = "uncertainty"


Heisenberg Uncertainty Principle (HUP)
P0
0
0
p
Δx =
/2
Δ = "uncertai

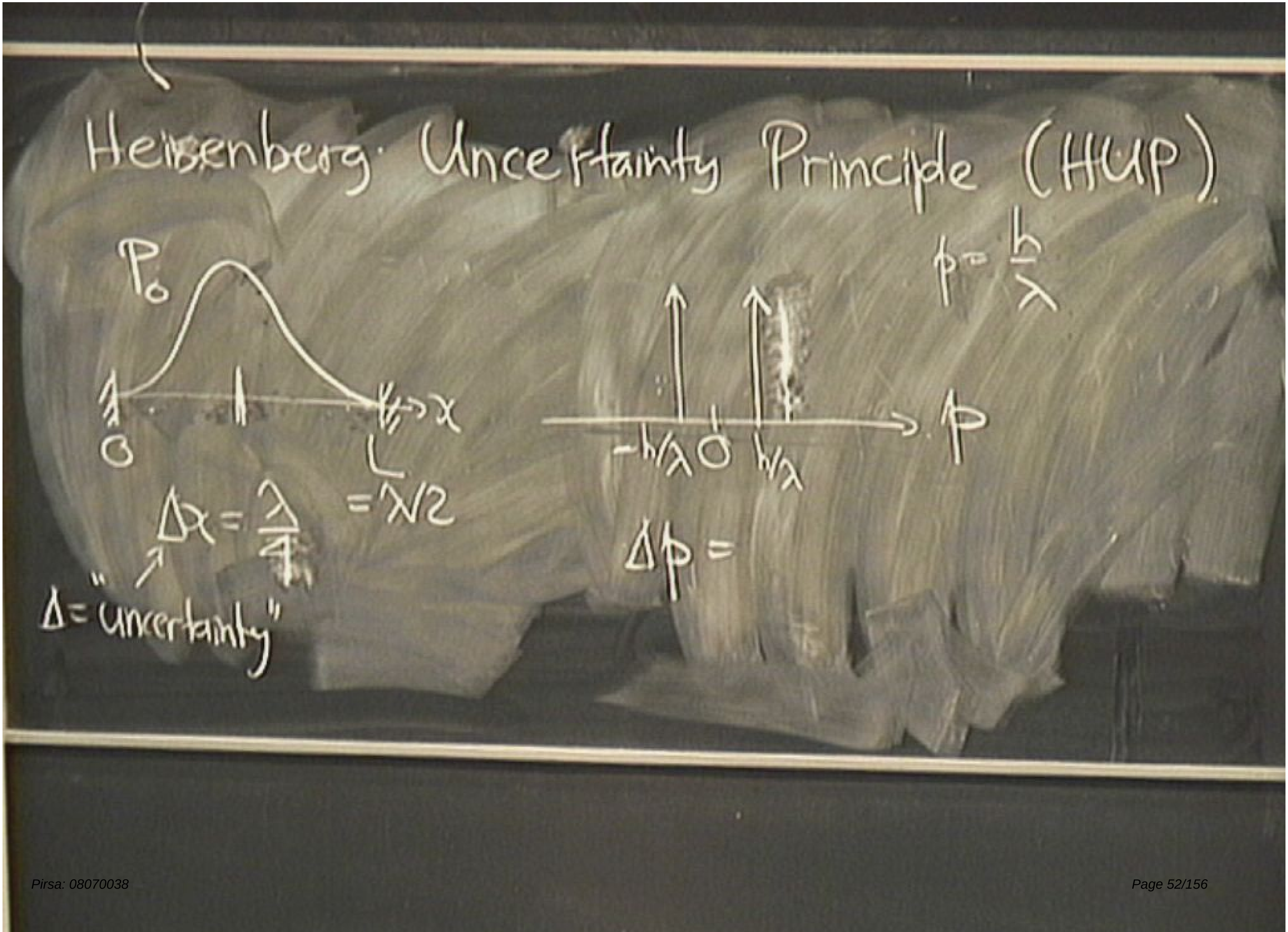

Heisenberg Uncertainty Principle (HUP)
P_0
x
0
L
$\Delta x = \frac{\lambda}{4}$
$= \lambda/2$
p
0
Δ = "uncertainty"

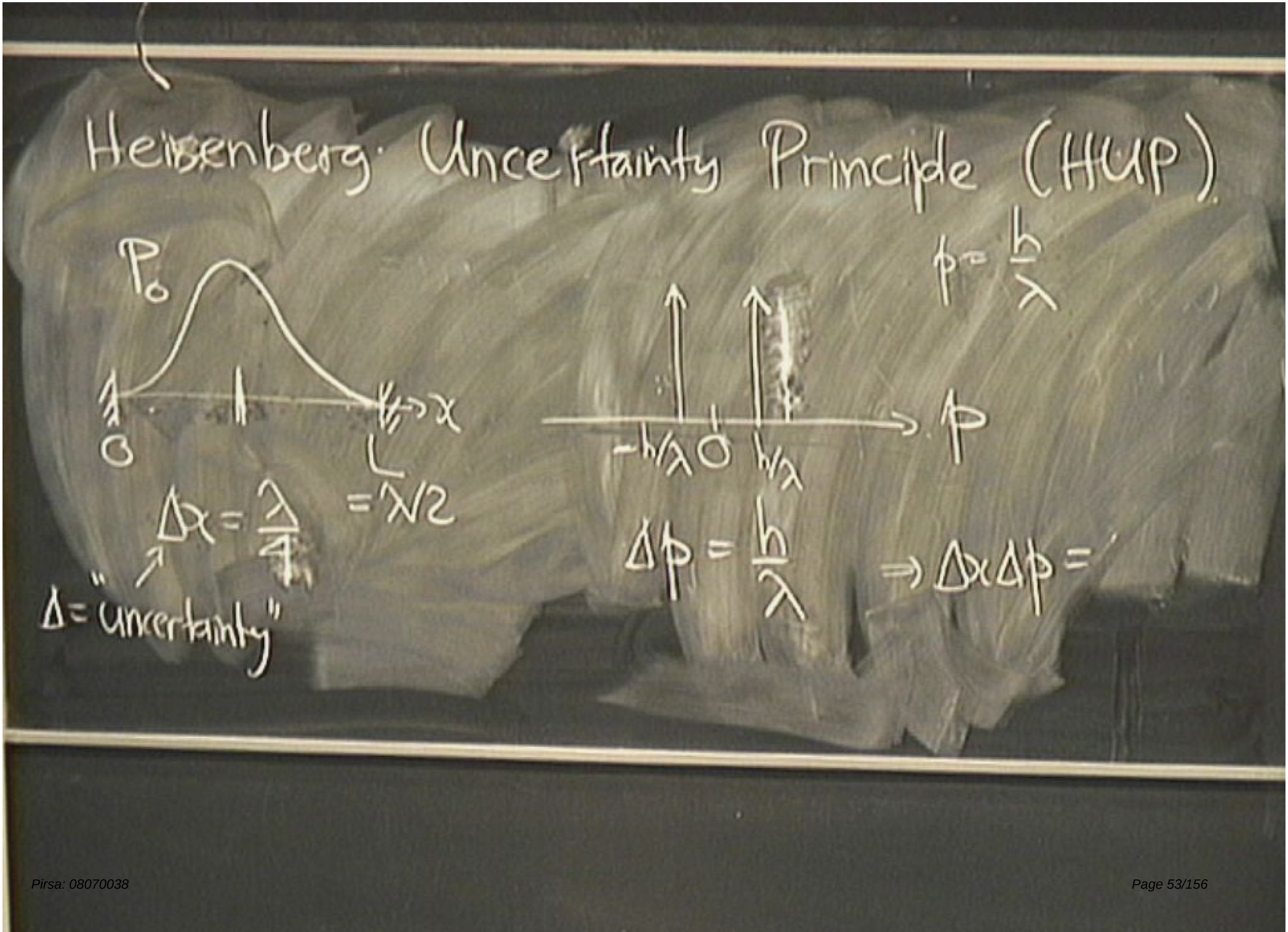

Heisenberg Uncertainty Principle (HUP)
p
Δ = "uncertainty"

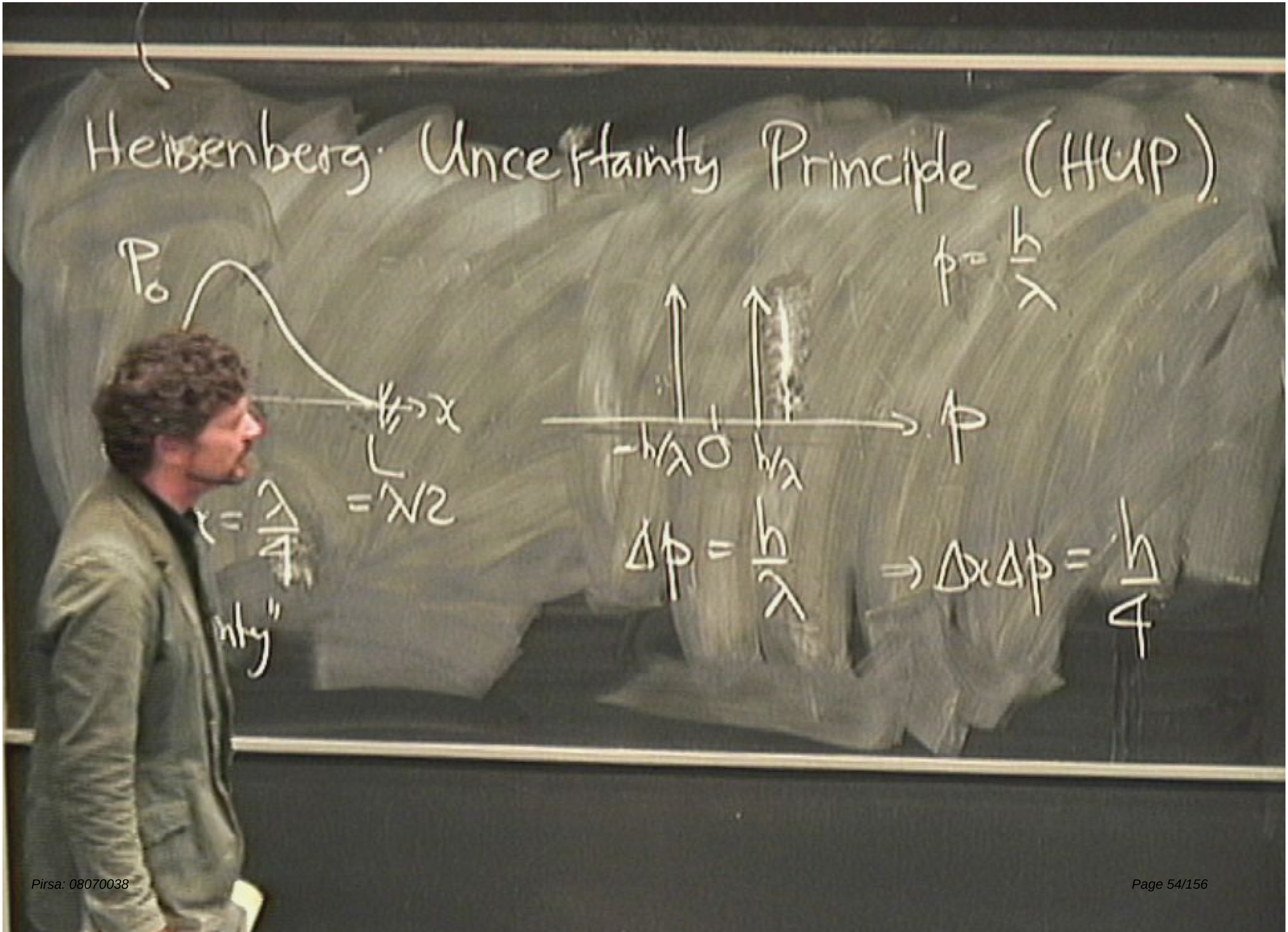

Heisenberg Uncertainty Principle (HUP)
P₀
x
L
= λ/2
−h/λ
0
h/λ
p
p = h/λ

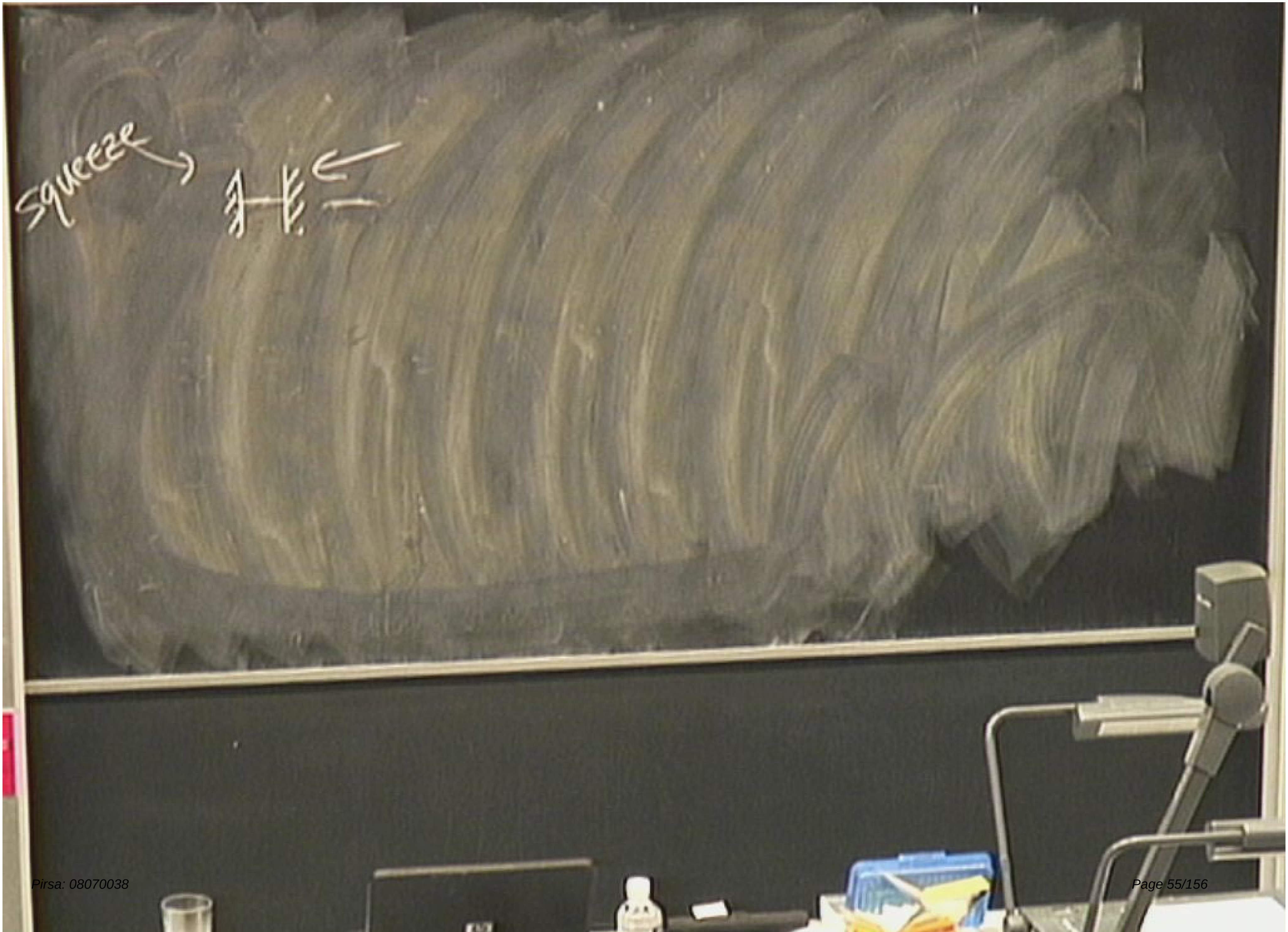


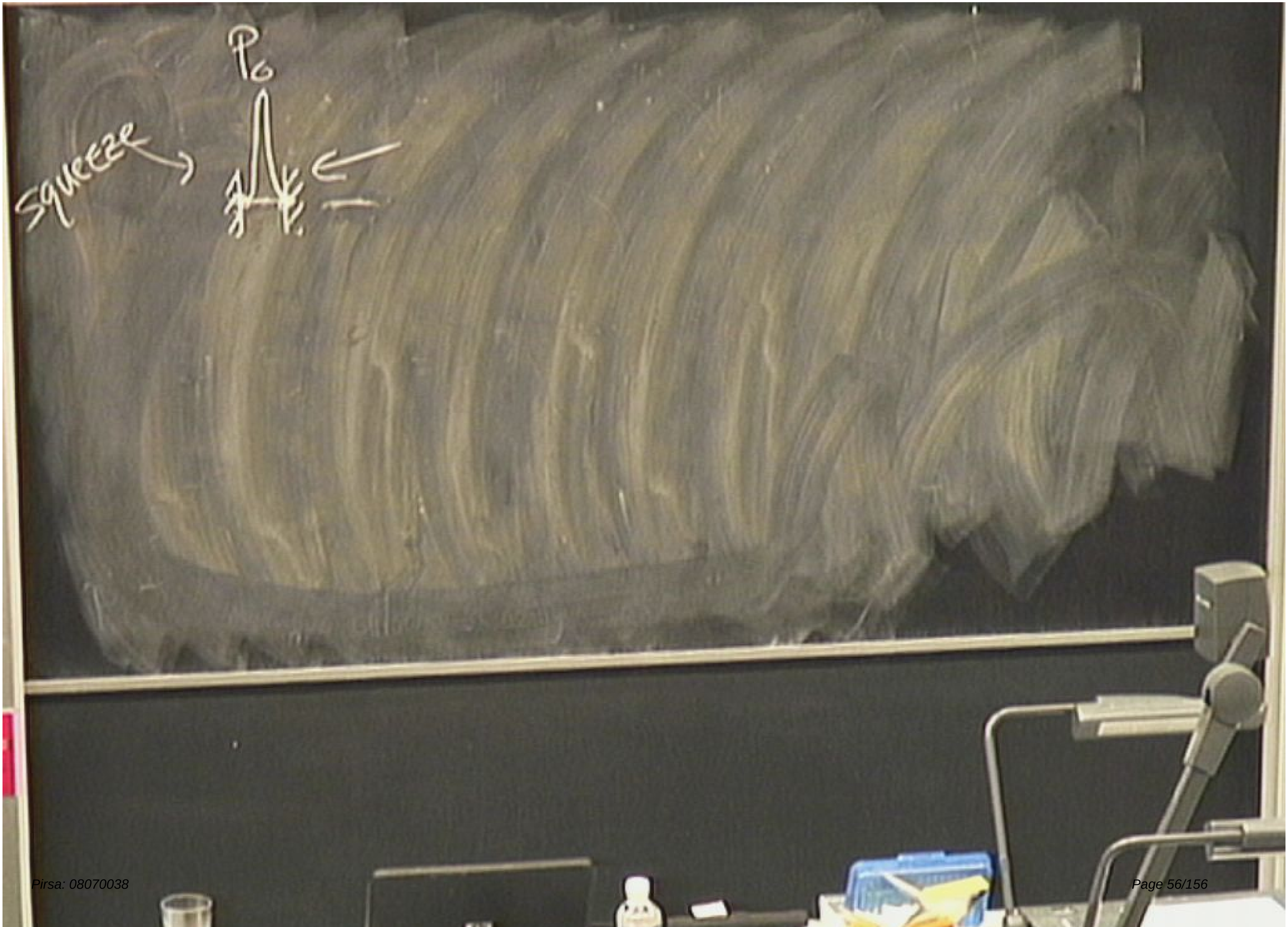
Heisenberg Uncertainty Principle (HUP)
P_0
0
$\Delta x = $
Δ = "uncertain
$p = \frac{h}{\lambda}$
$-h/\lambda$ 0 h/λ
p

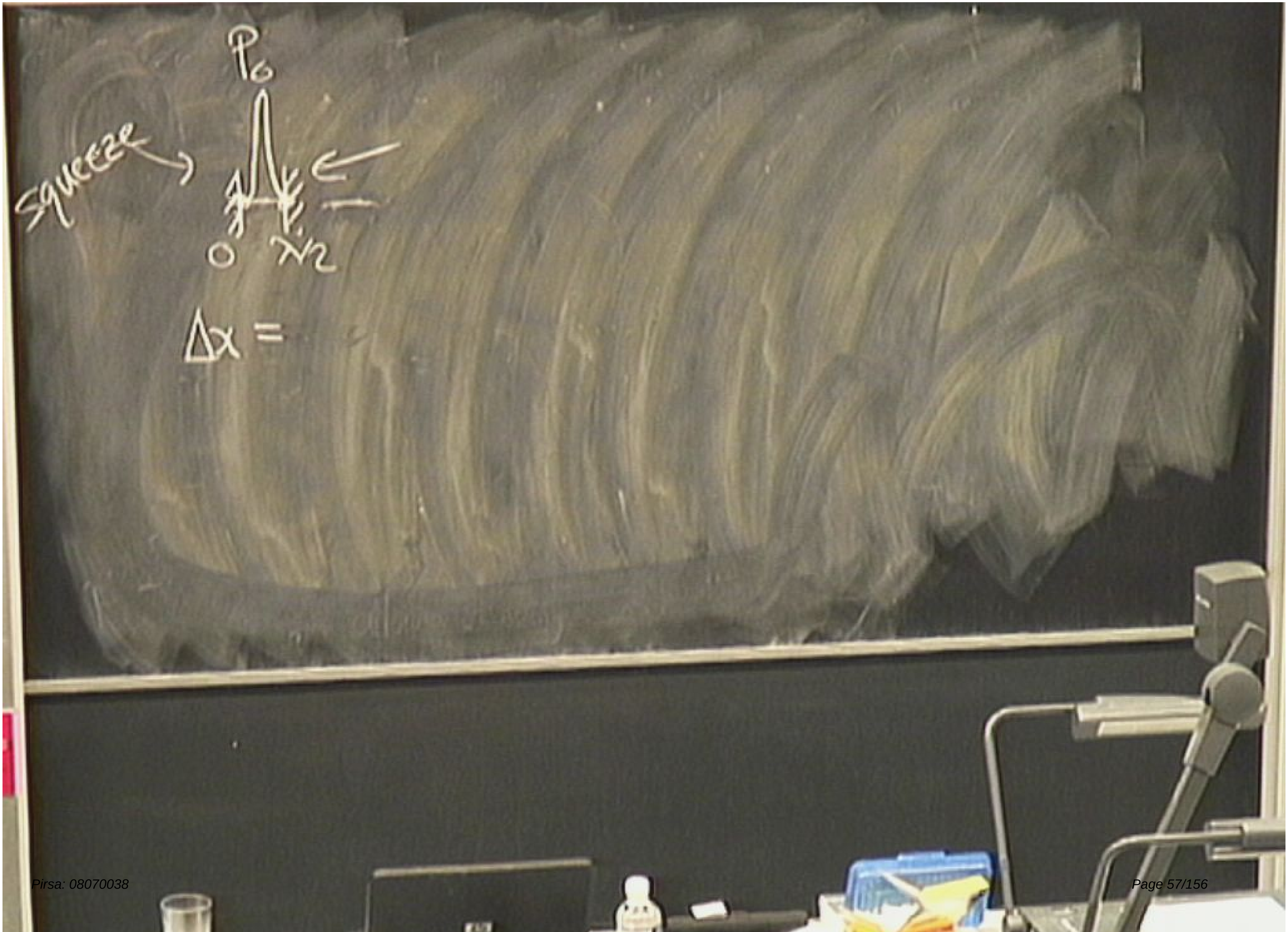

Heisenberg Uncertainty Principle (HUP)
P_0
0
L
x
$\Delta x = \frac{\lambda}{4}$
$= \lambda/2$
Δ = "uncertainty"
$p = \frac{h}{\lambda}$
$-h/\lambda$
0
h/λ
p
$\Delta p =$

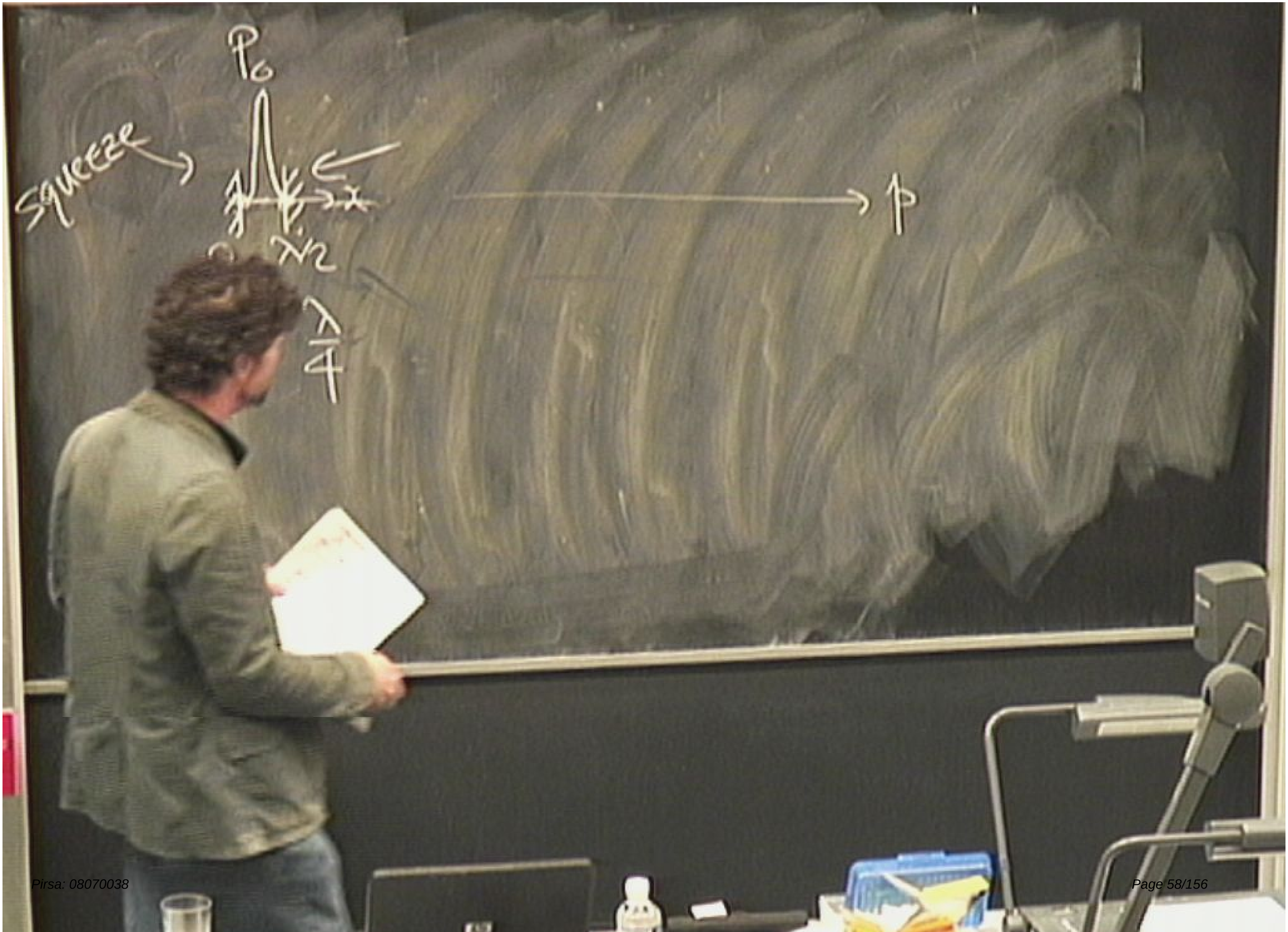

Heisenberg Uncertainty Principle (HUP)
P0
x
0
L
Δx = λ/4 = λ/2
"Δ = uncertainty"
p = h/λ
-h/λ 0 h/λ
p
Δp = h/λ
⇒ ΔxΔp =

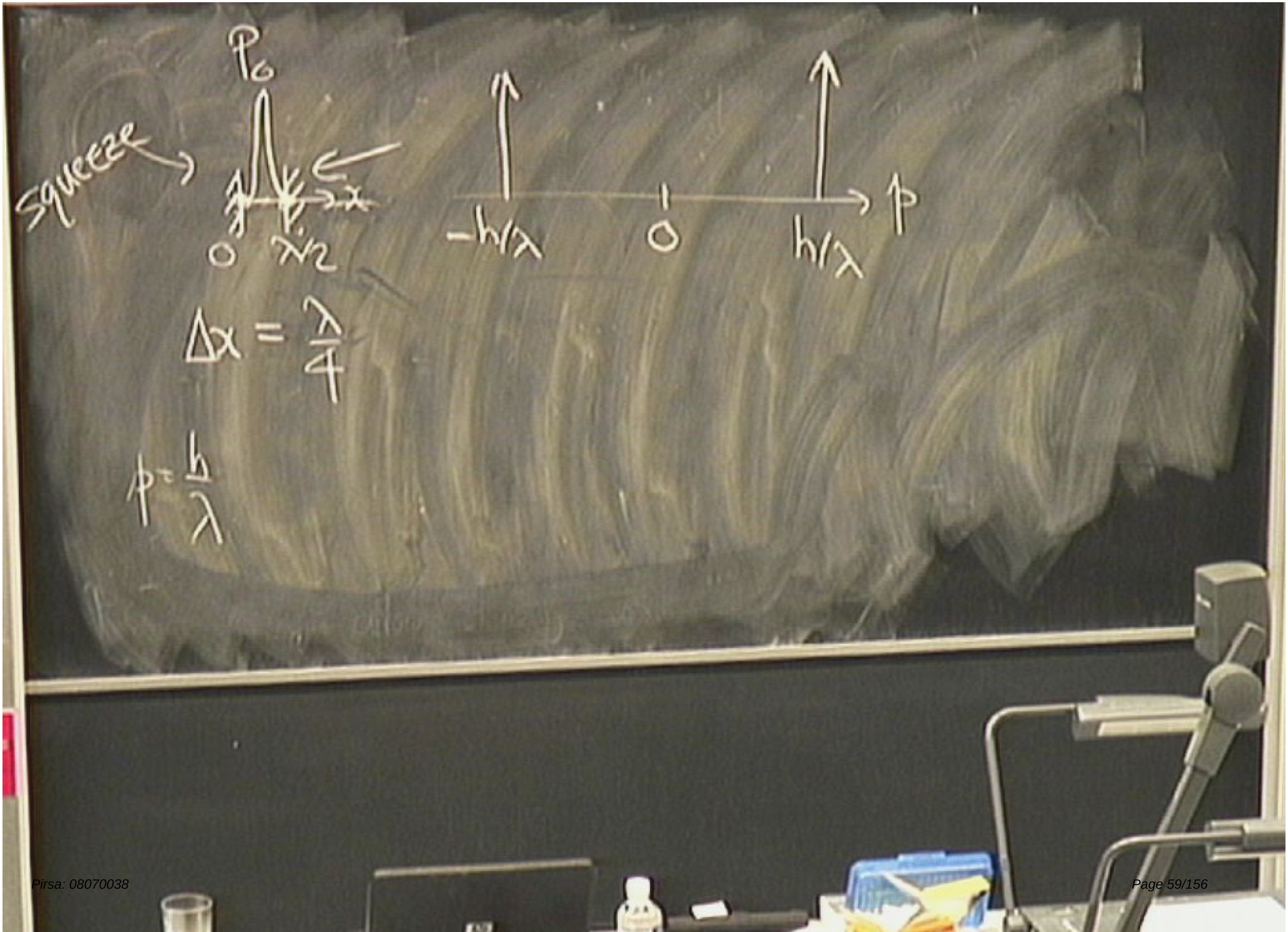

Heisenberg Uncertainty Principle (HUP)
P_0
x
L
$= \lambda/2$
$x = \frac{\lambda}{4}$
$p = \frac{h}{\lambda}$
$-h/\lambda$ 0 h/λ
p
$\Delta p = \frac{h}{\lambda}$
$\Rightarrow \Delta x \Delta p = \frac{h}{4}$

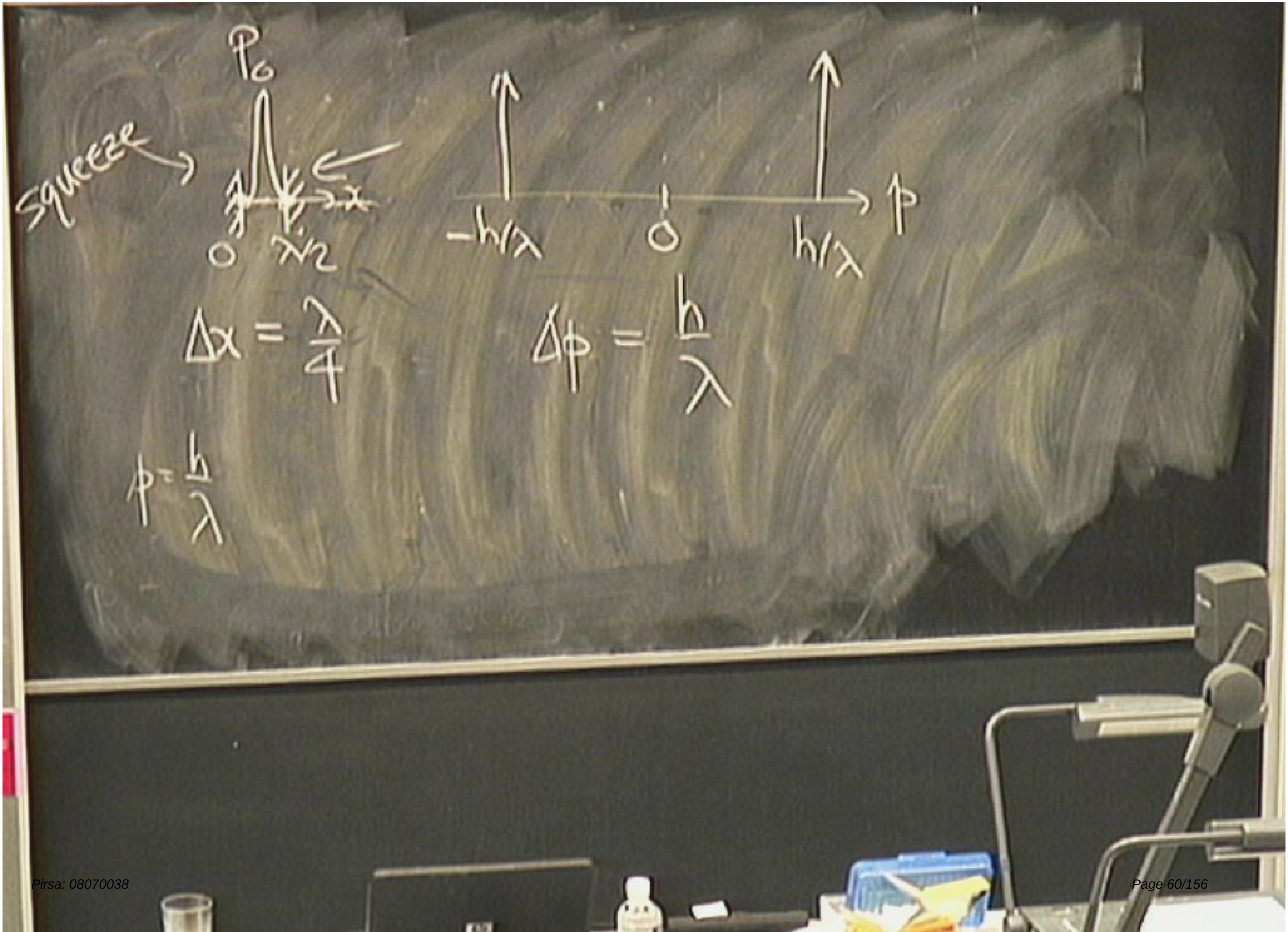

squeeze


squeeze

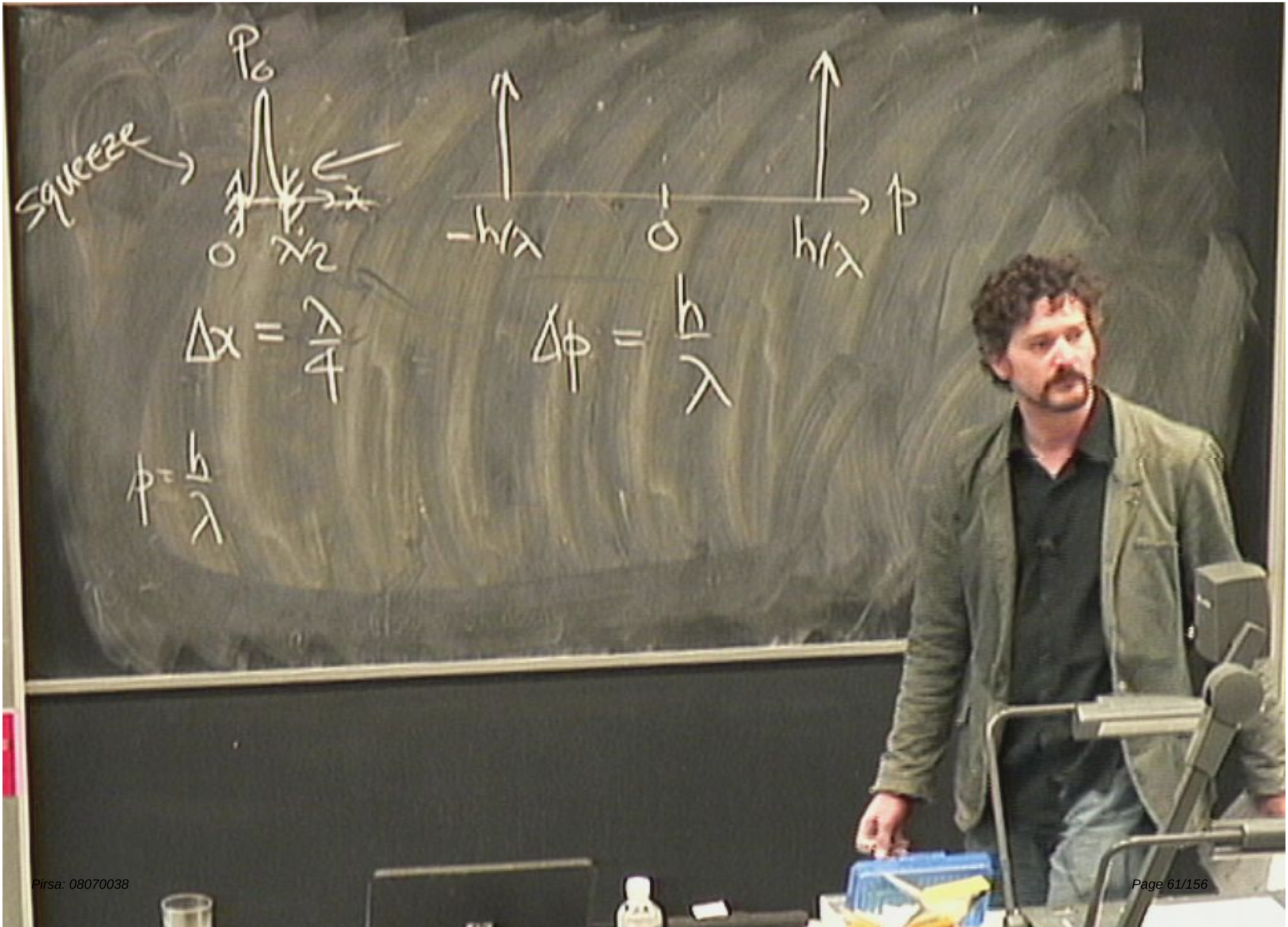

Pθ
SQUEEZE
0
π/2
Δx =

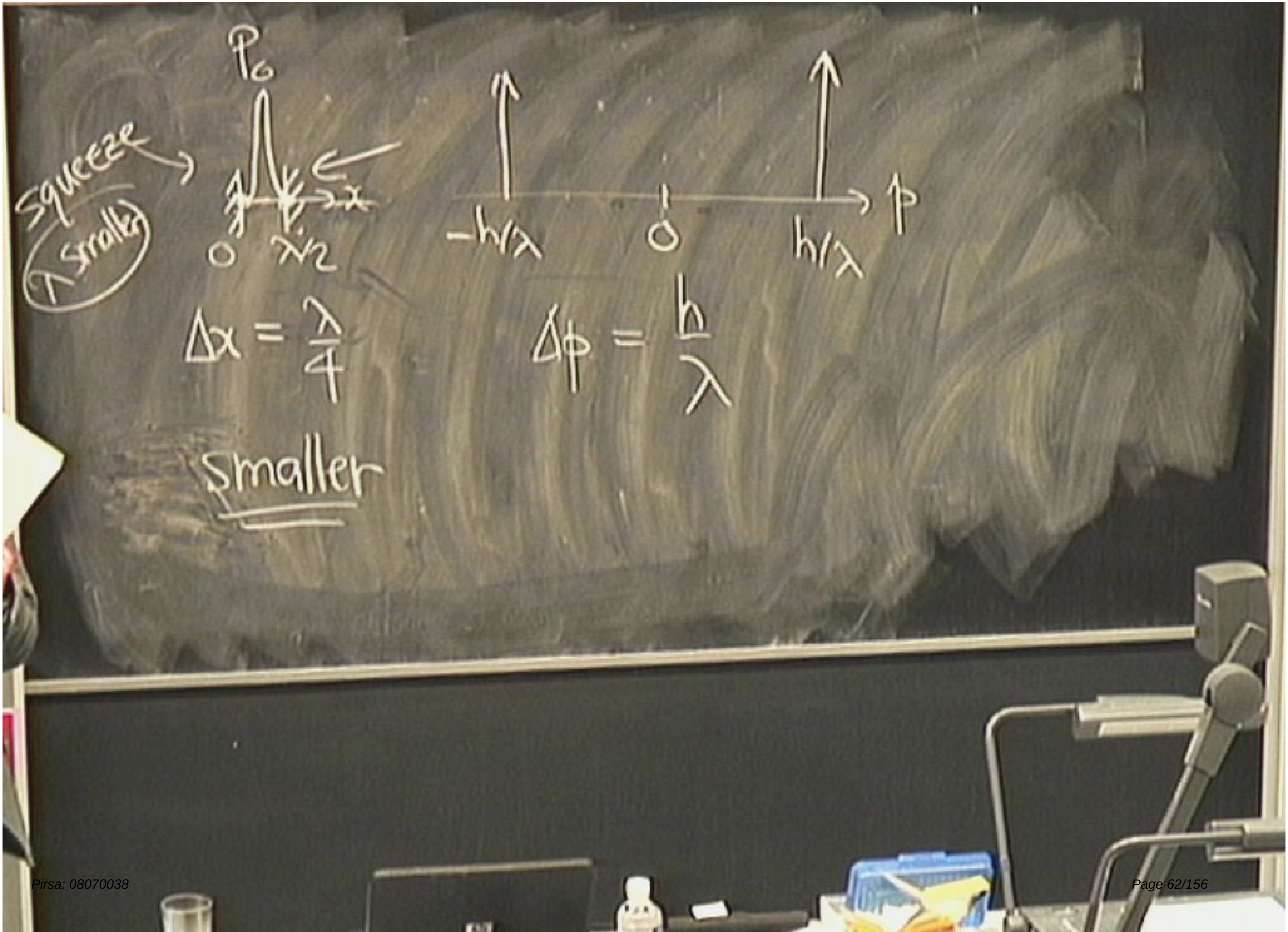

squeeze
p

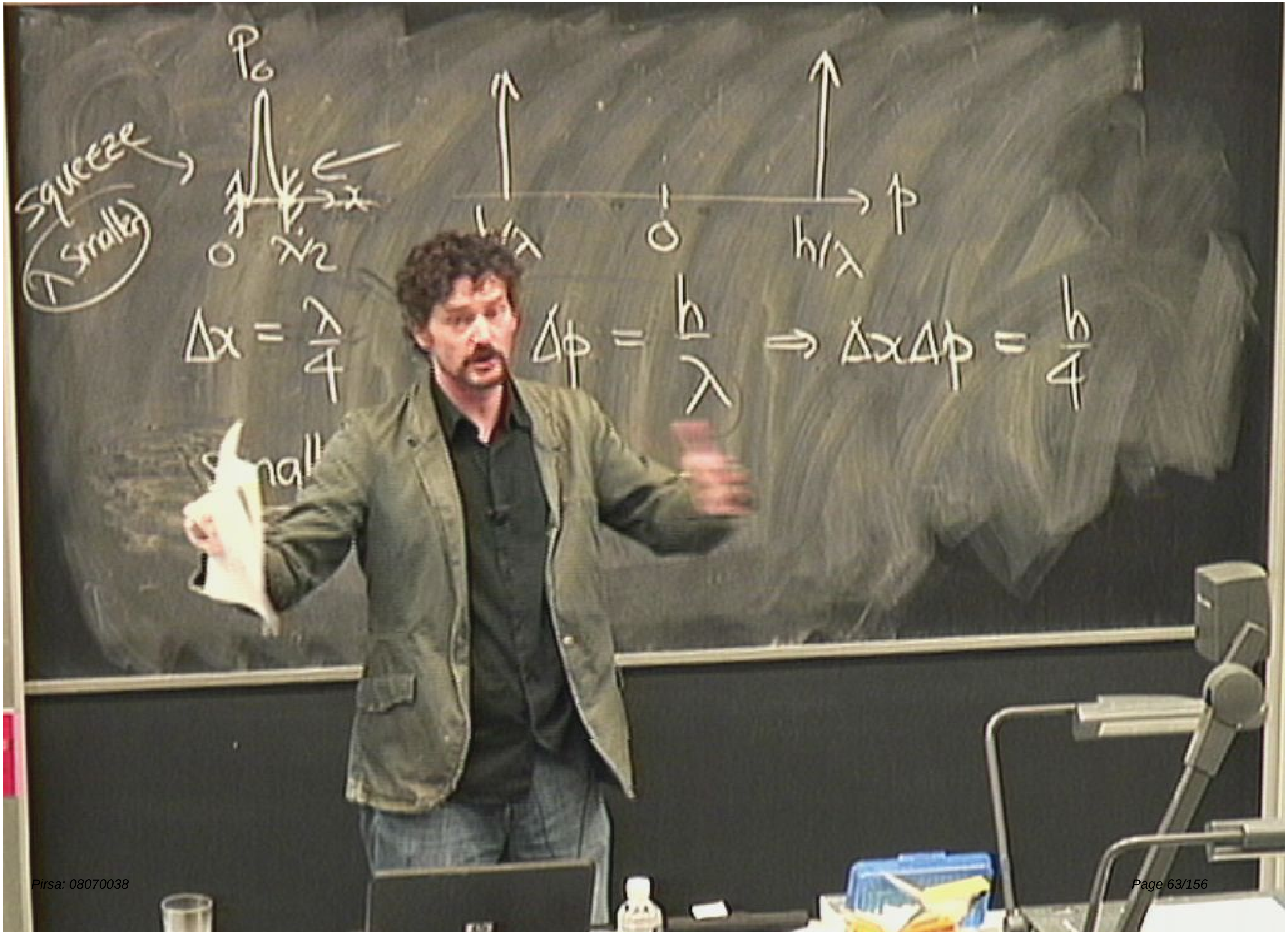

squeeze
P_0
0
$\lambda/2$
x
$-h/\lambda$
0
h/λ
p
$\Delta x = \frac{\lambda}{4}$
$p = \frac{h}{\lambda}$



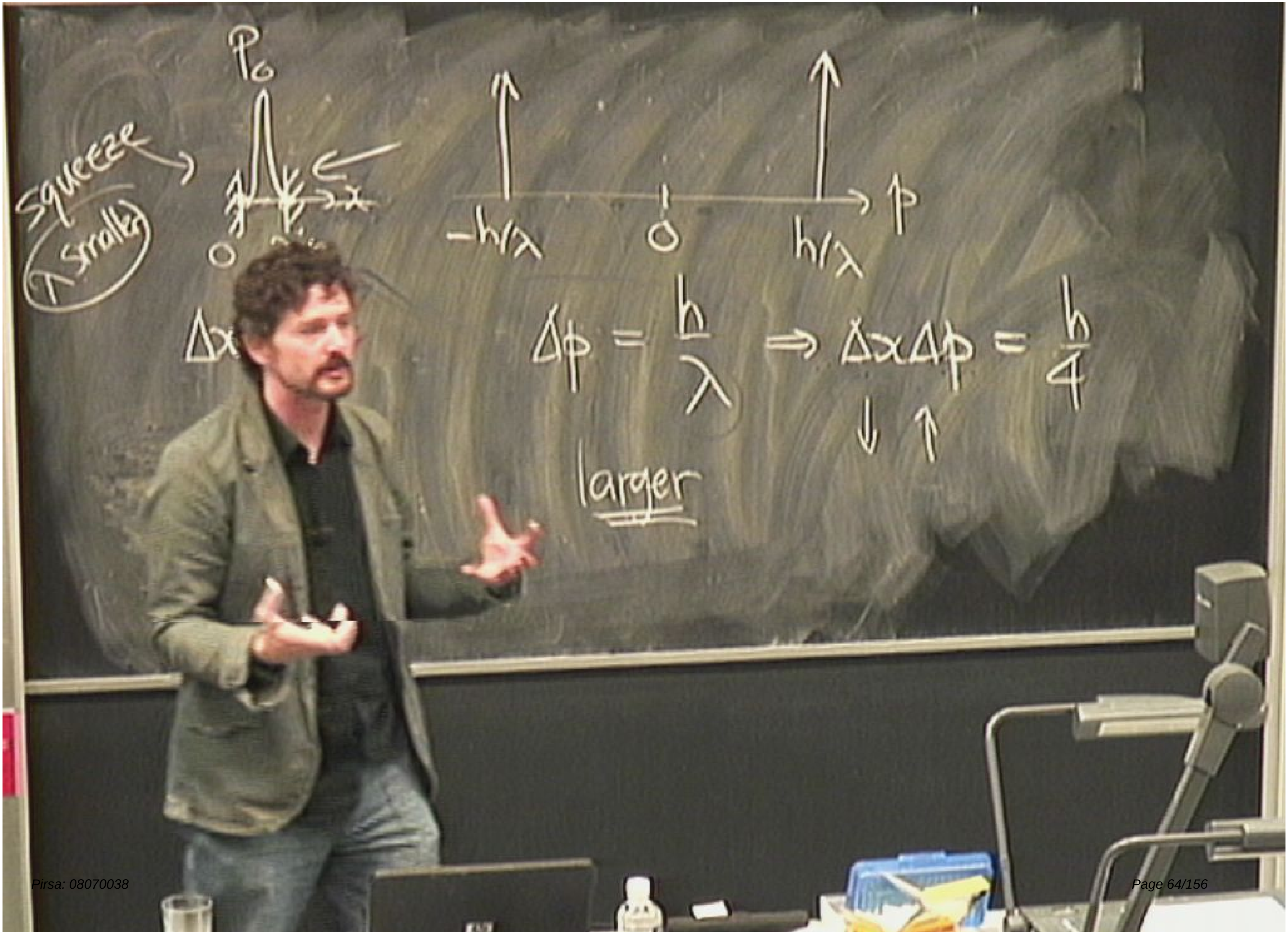
squeeze
P_0
Δx
0
$\lambda/2$
$-h/\lambda$
0
h/λ
p
$\Delta x = \frac{\lambda}{4}$
$\Delta p = \frac{h}{\lambda}$
$p = \frac{h}{\lambda}$


SQUEEZE
P_0
0
$\lambda/2$
x
$-h/\lambda$
0
h/λ
p
$\Delta x = \frac{\lambda}{4}$
$\Delta p = \frac{h}{\lambda}$
$p = \frac{h}{\lambda}$

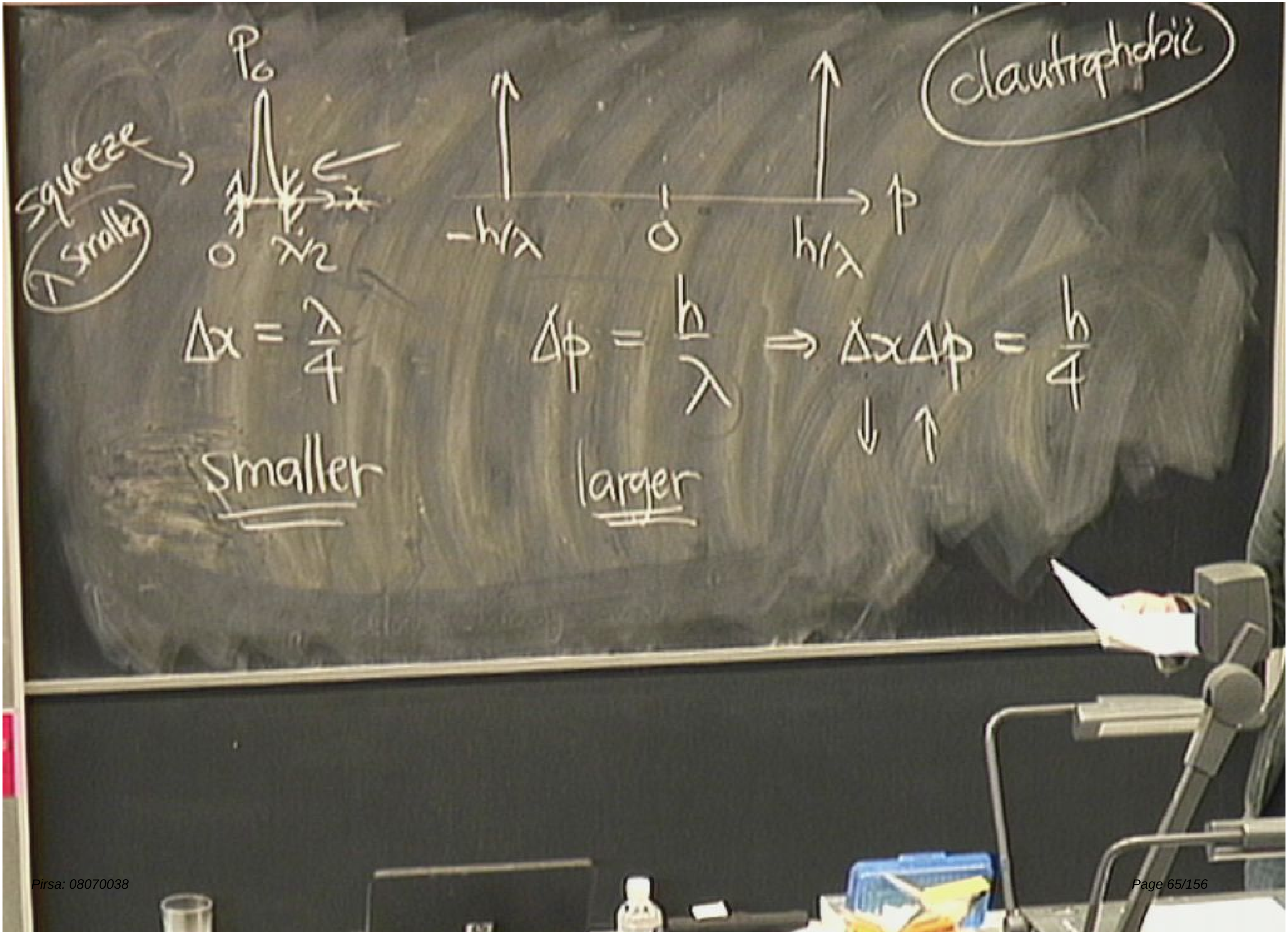

Squeeze
λ smaller
P0
x
0
λ/2
-h/λ
0
h/λ
p
Δx = λ/4
Δp = h/λ
Smaller

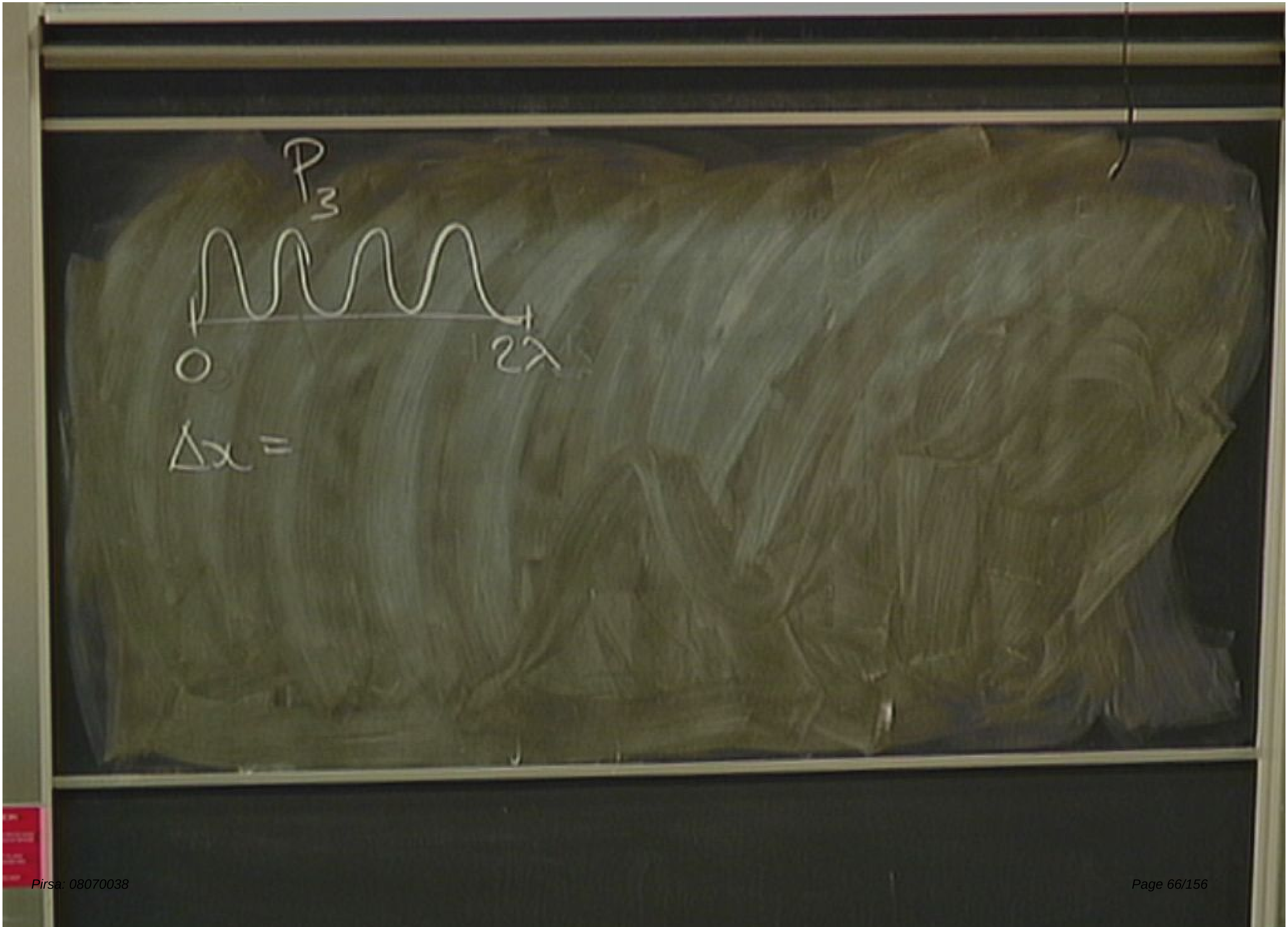


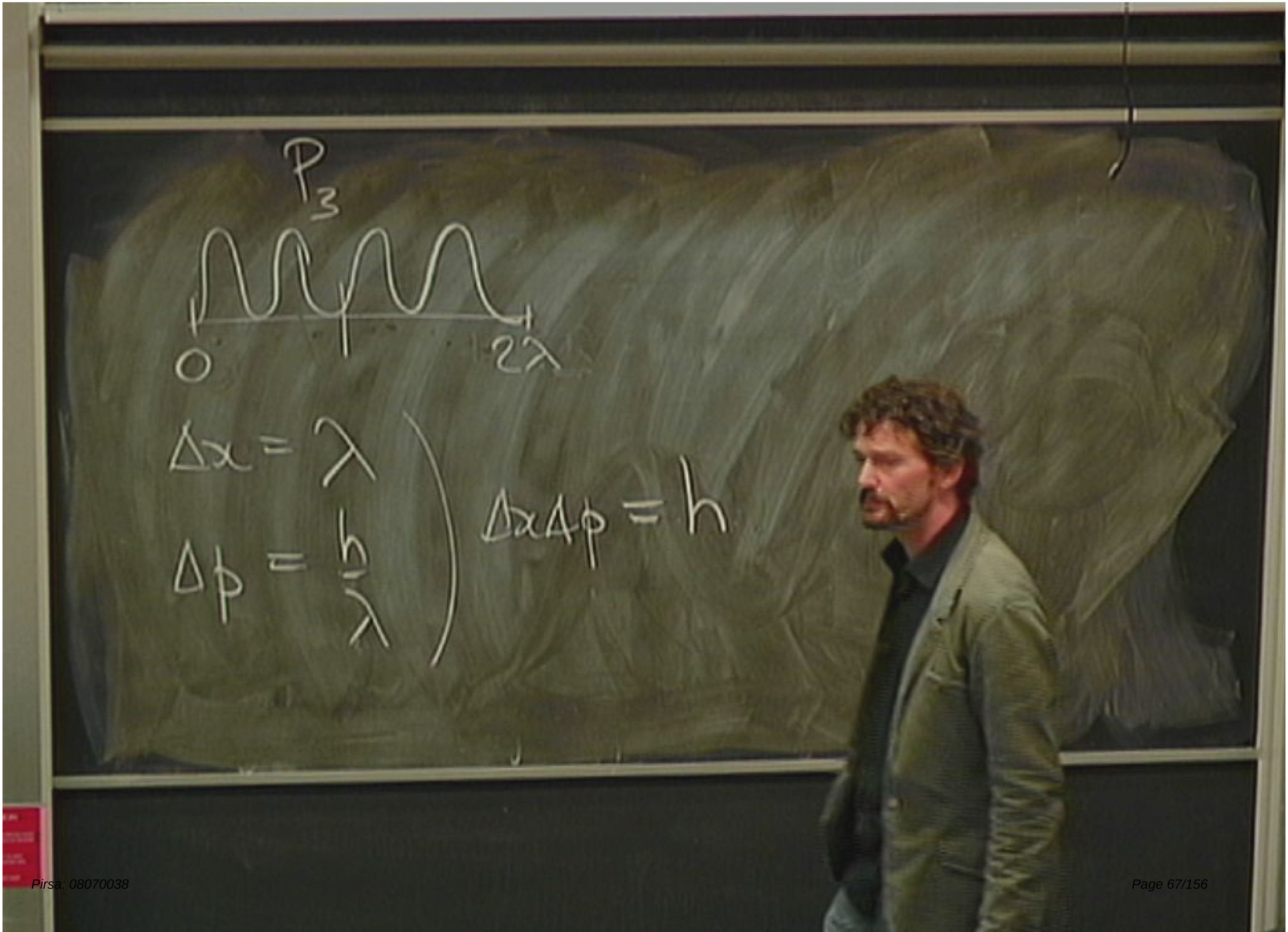
P_0
squeeze
Δ small
Δx
0
$\lambda/2$
h/λ
0
h/λ
p
$\Delta x = \frac{\lambda}{4}$
$\Delta p = \frac{h}{\lambda}$
$\Rightarrow \Delta x \Delta p = \frac{h}{4}$

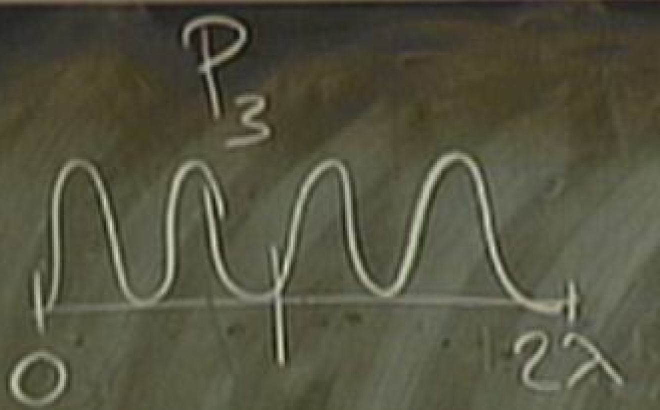


squeeze
(smaller)
p_0
0
x
$-h/\lambda$
0
h/λ
p
Δx
$\Delta p = \frac{h}{\lambda} \Rightarrow \Delta x \Delta p = \frac{h}{4}$
larger


claustrophobic
squeeze
λ smaller
P_0
x
0
$\lambda/2$
$-h/\lambda$
0
h/λ
p
$\Delta x = \frac{\lambda}{4}$
$\Delta p = \frac{h}{\lambda}$
$\Rightarrow \Delta x \Delta p = \frac{h}{4}$
$\downarrow \quad \uparrow$
smaller
larger


P3
0
2λ
Δx =


P_3
0
2λ
$\Delta x = \lambda$
$\Delta p = \frac{h}{\lambda}$
$\Delta x \Delta p = h$

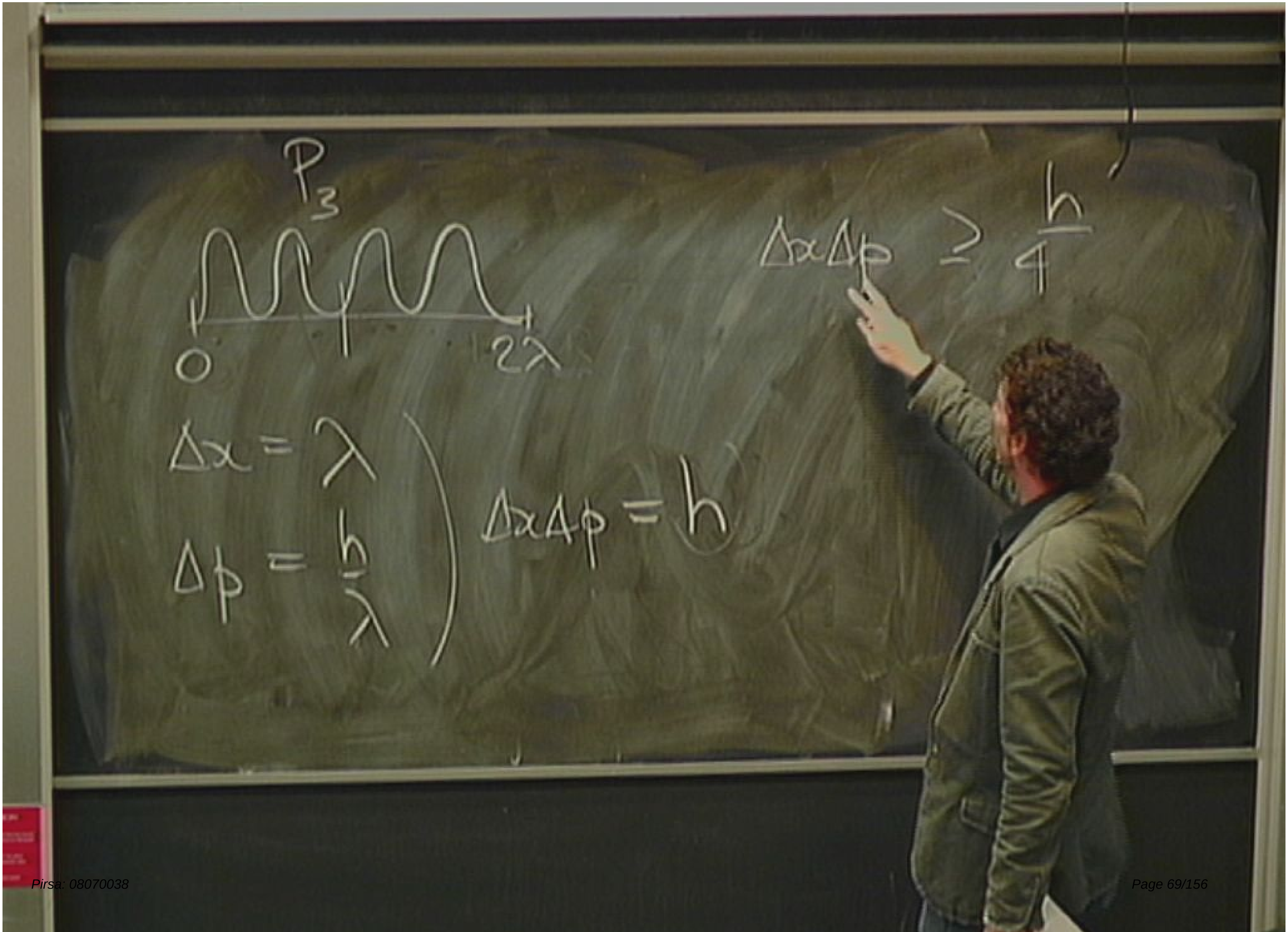


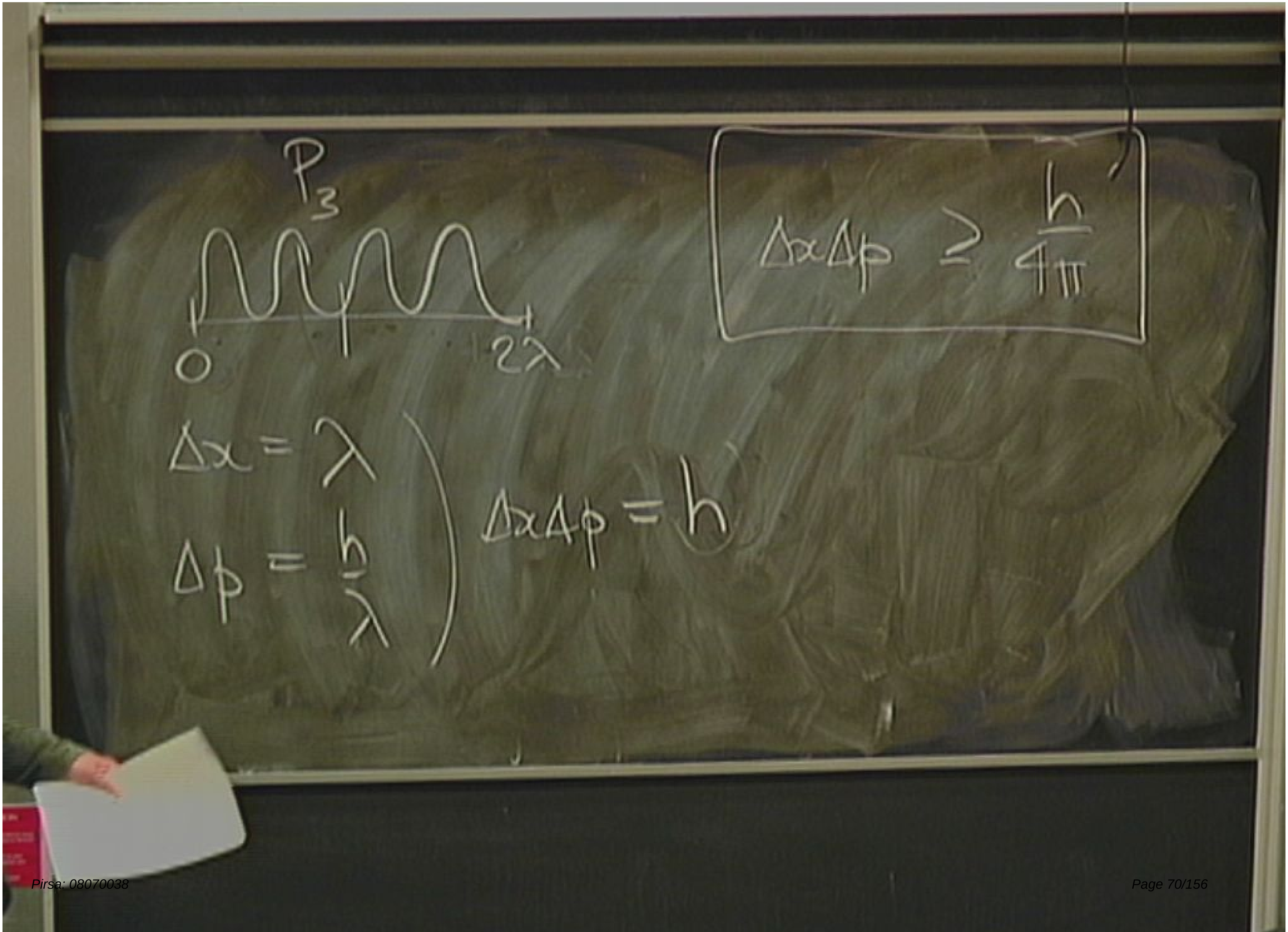
$$\Delta x \Delta p \geq \frac{h}{4}$$

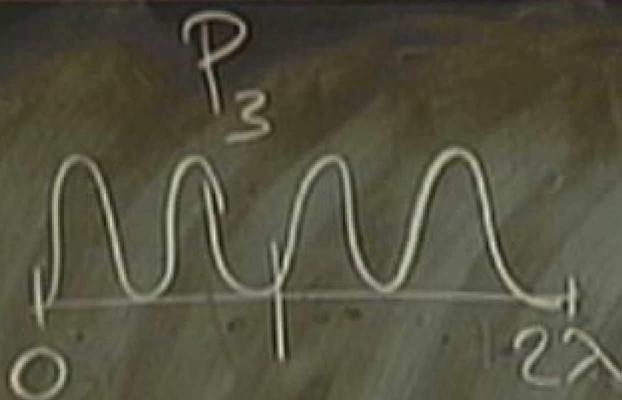
$$\Delta x = \lambda$$

$$\Delta p = \frac{h}{\lambda}$$

$$\Delta x \Delta p = h$$

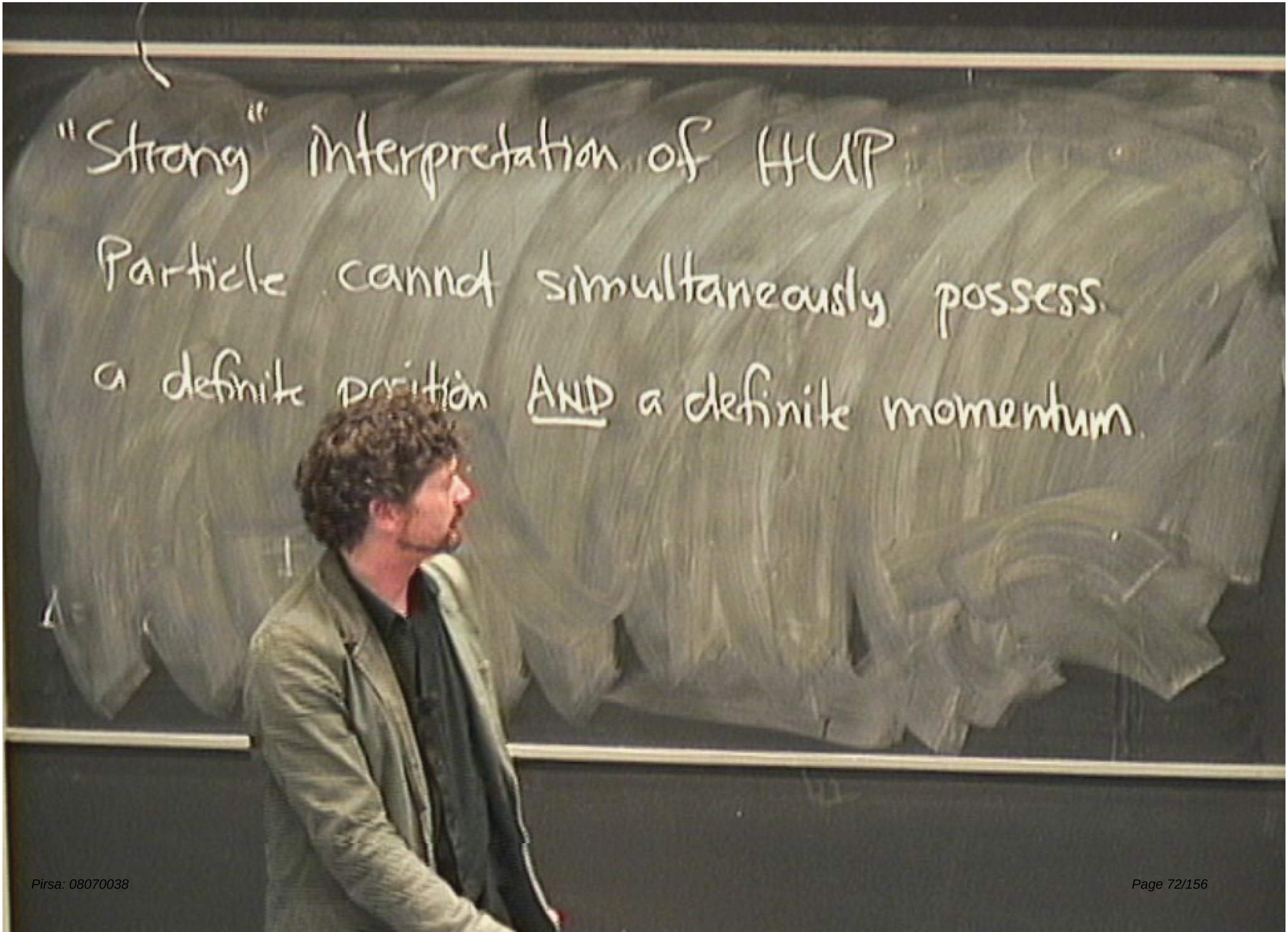

P_3
0
2λ
$\Delta x = \lambda$
$\Delta p = \frac{h}{\lambda}$
$\Delta x \Delta p = h$
$\Delta x \Delta p \geq \frac{h}{4}$

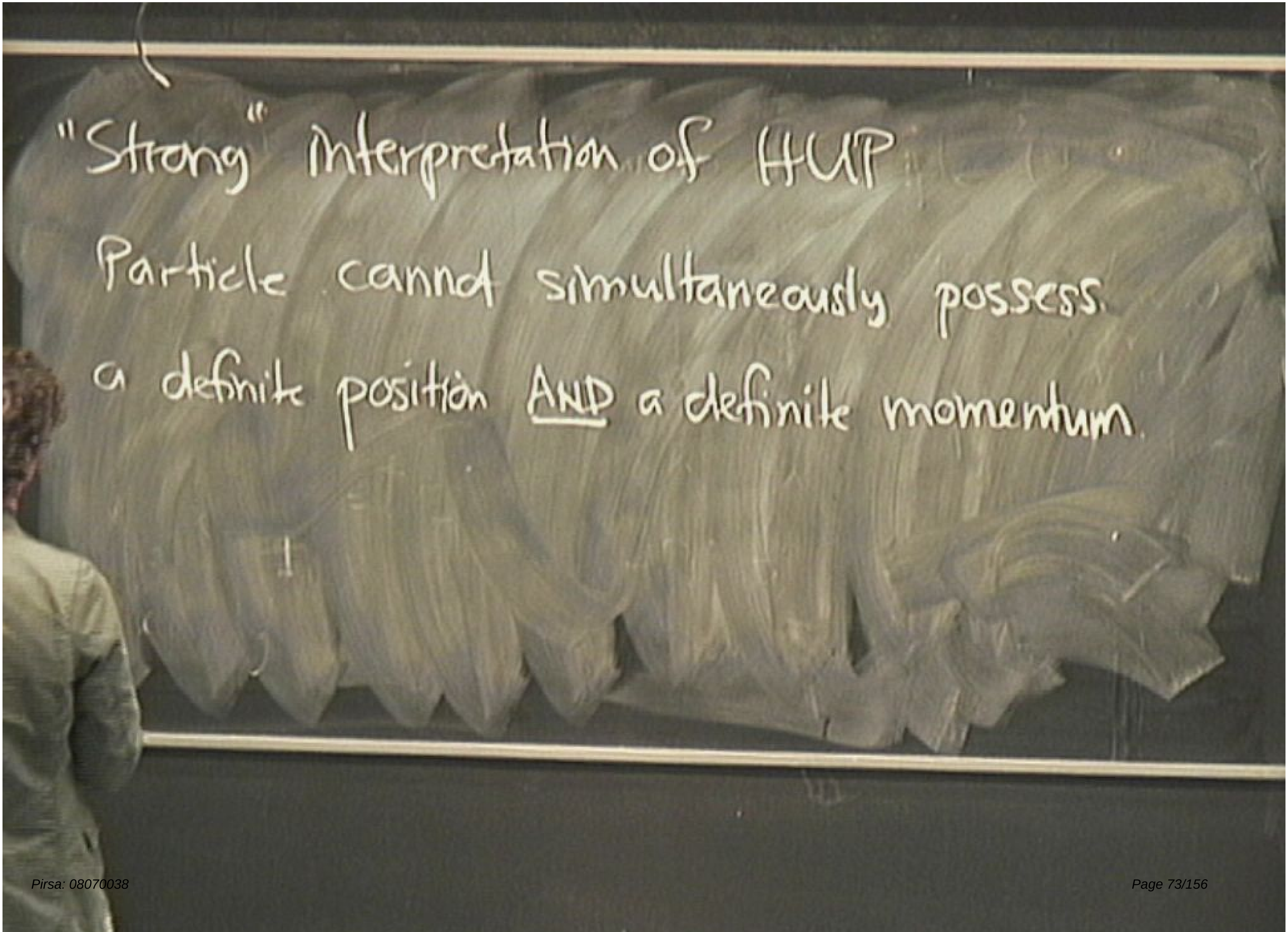

P_3
0
2λ
$\Delta x \Delta p \geq \frac{h}{4\pi}$
$\Delta x = \lambda$
$\Delta p = \frac{h}{\lambda}$
$\Delta x \Delta p = h$



$$\boxed{\Delta x \Delta p \geq \frac{h}{4\pi}}$$

$$\left.\begin{aligned} \Delta x &= \lambda \\ \Delta p &= \frac{h}{\lambda} \end{aligned}\right) \quad \Delta x \Delta p = h$$


"Strong" interpretation of HUP:
Particle cannot simultaneously possess
a definite position AND a definite momentum


"Strong" interpretation of HUP
Particle cannot simultaneously possess
a definite position AND a definite momentum

"Strong" interpretation of HUP

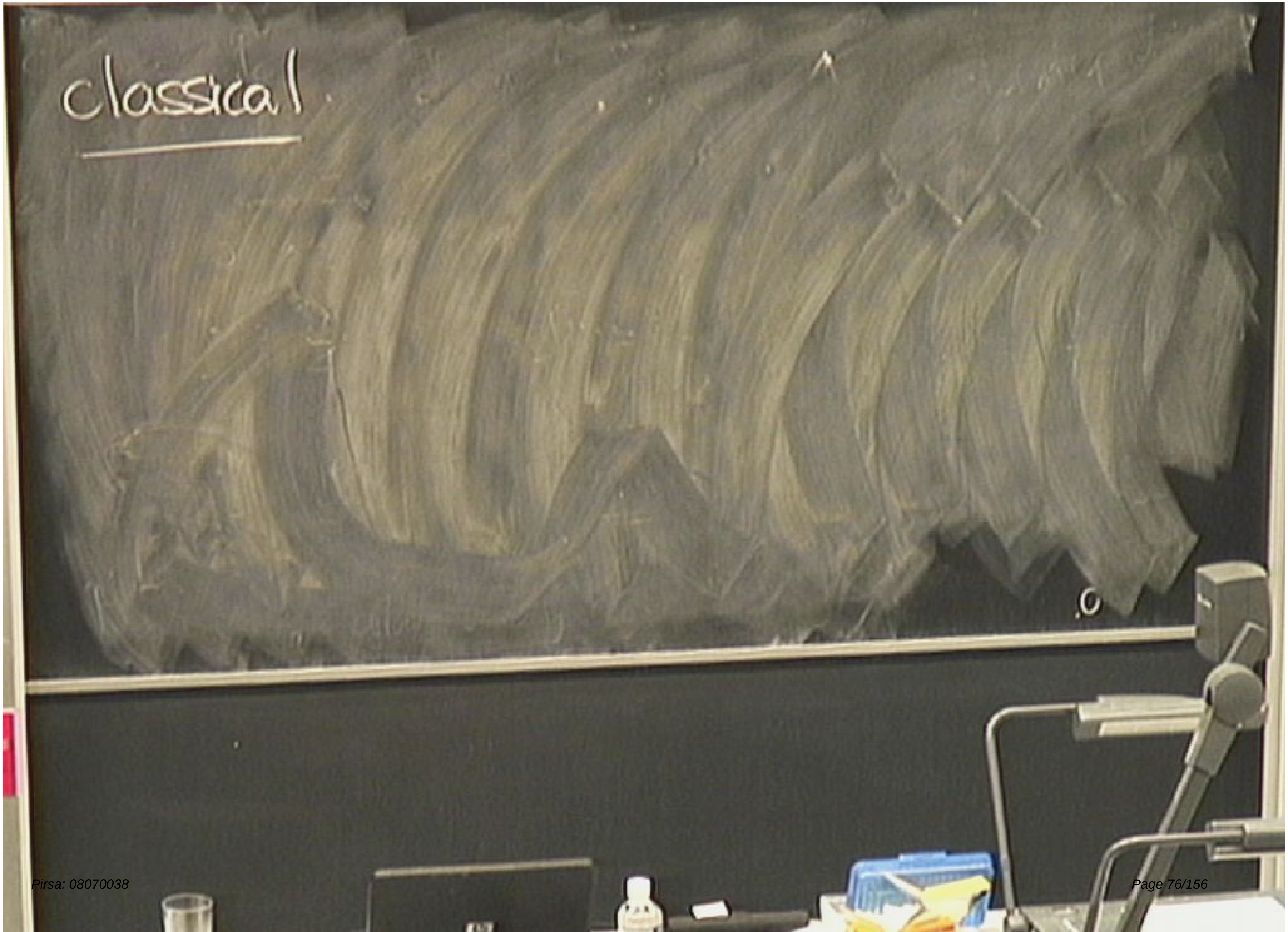
Particle cannot simultaneously possess a definite position <u>AND</u> a definite momentum

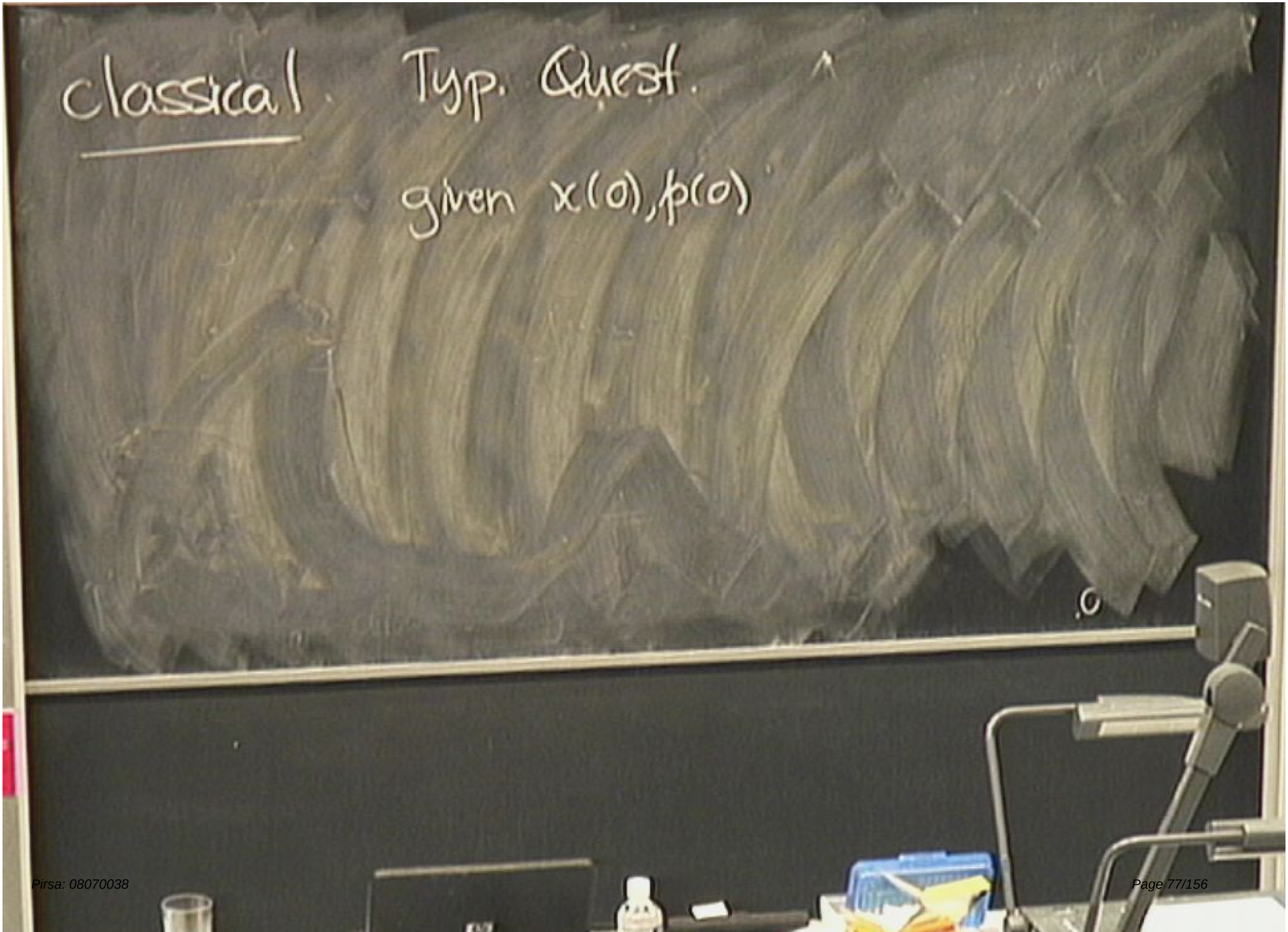
$\Delta x = 0$ $\Delta p = 0$

"Strong" interpretation of HUP

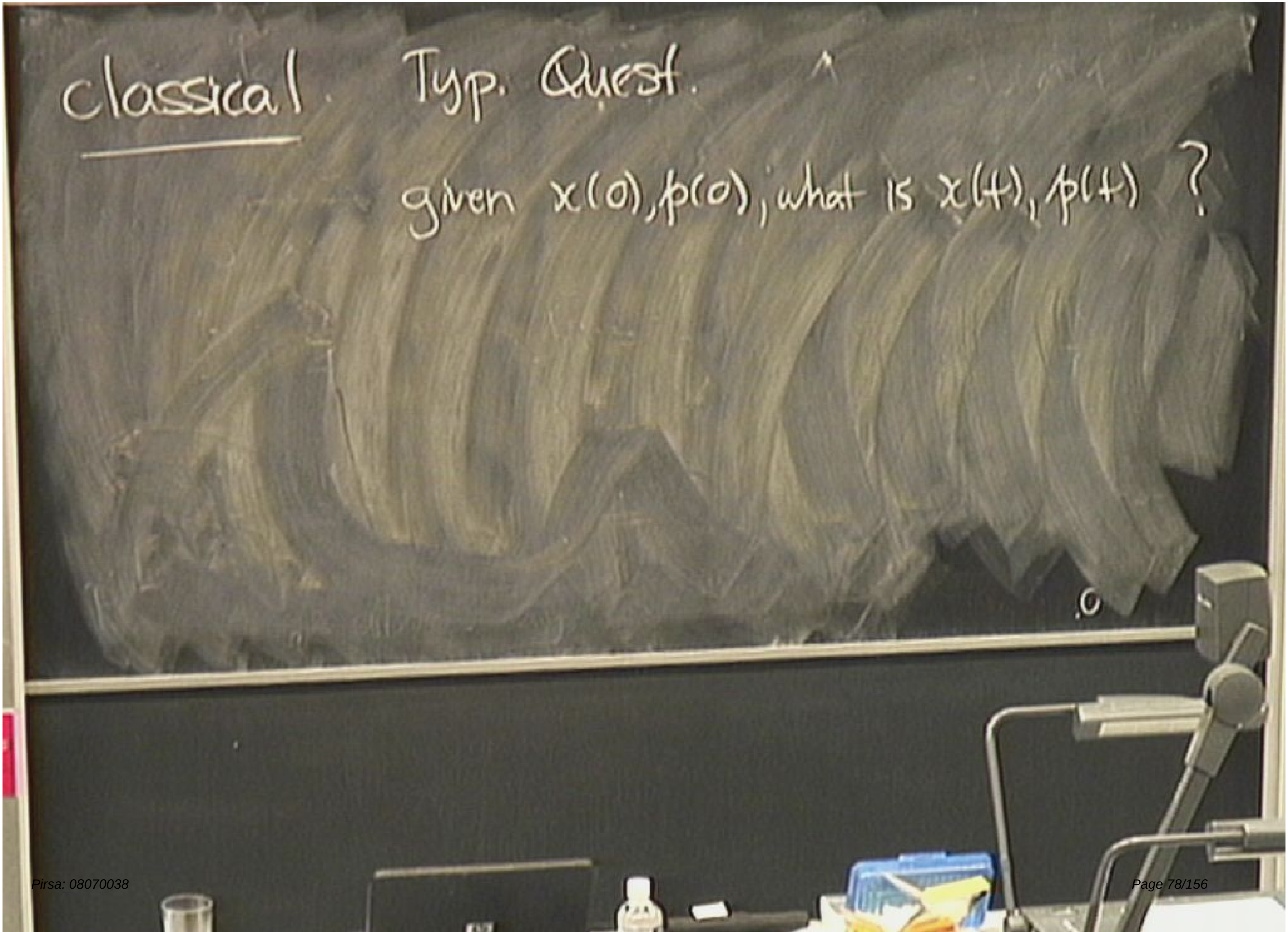
Particle cannot simultaneously possess a definite position **AND** a definite momentum

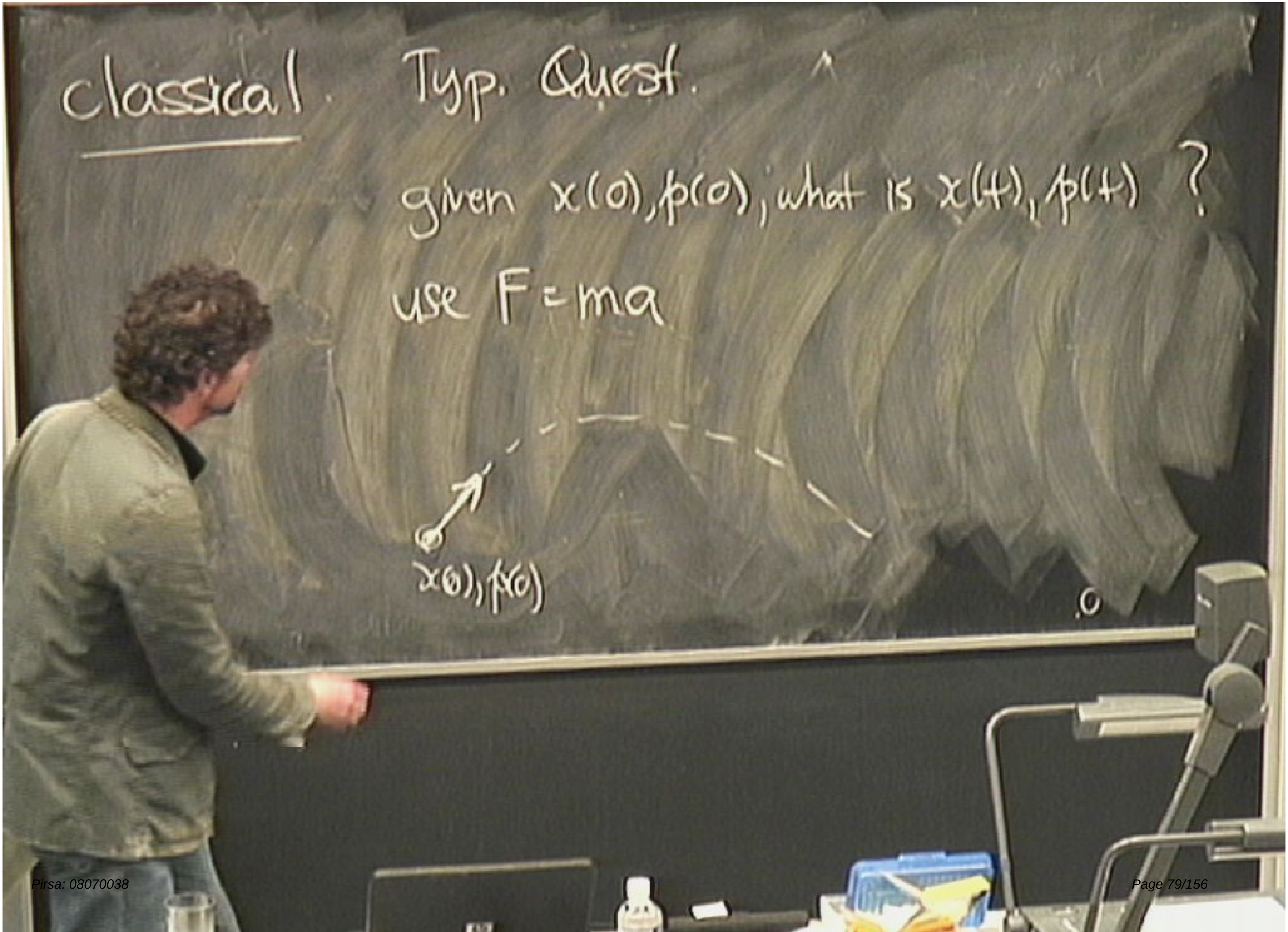
$\Delta x = 0$ $\Delta p = 0$

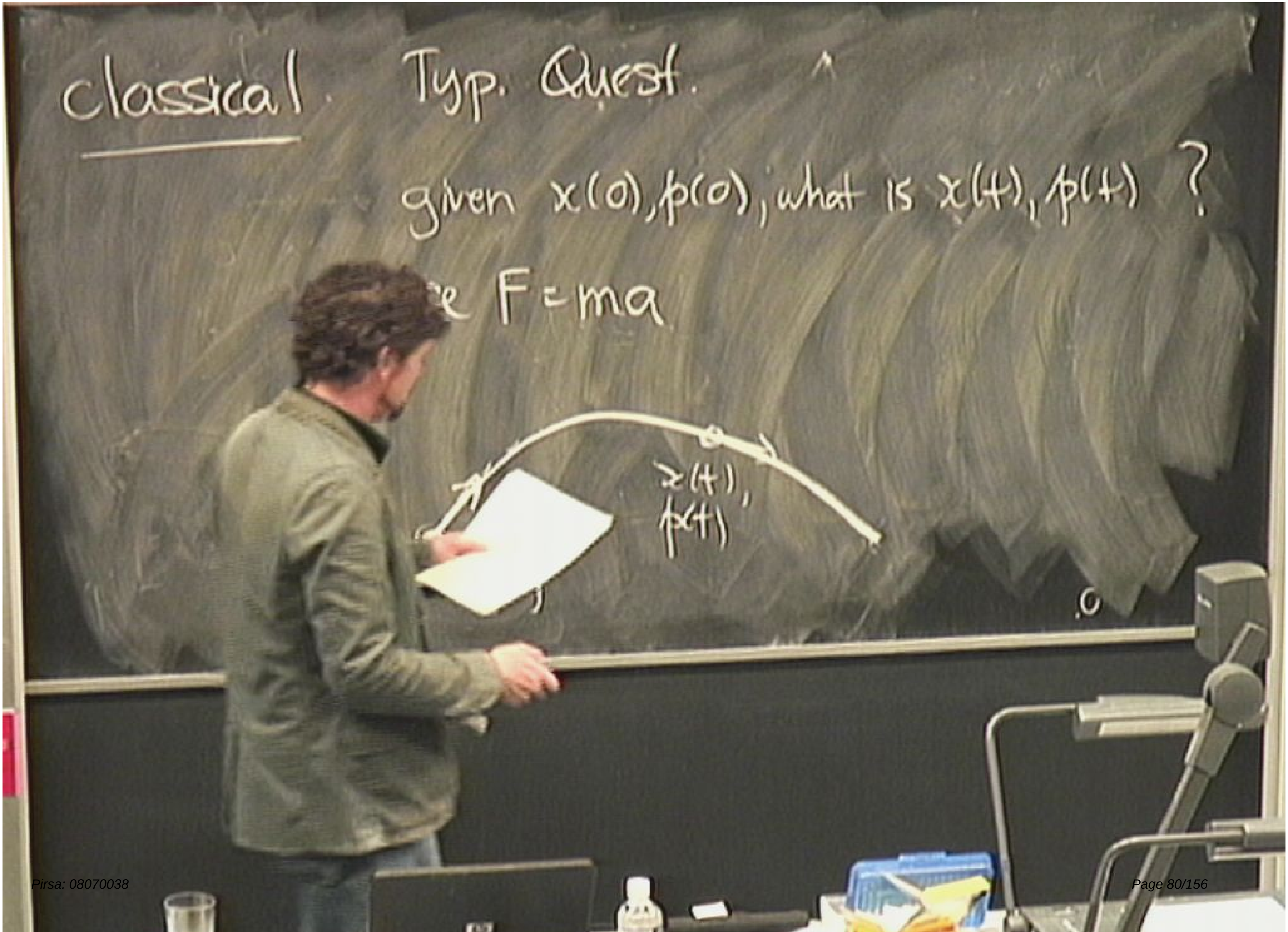

classical

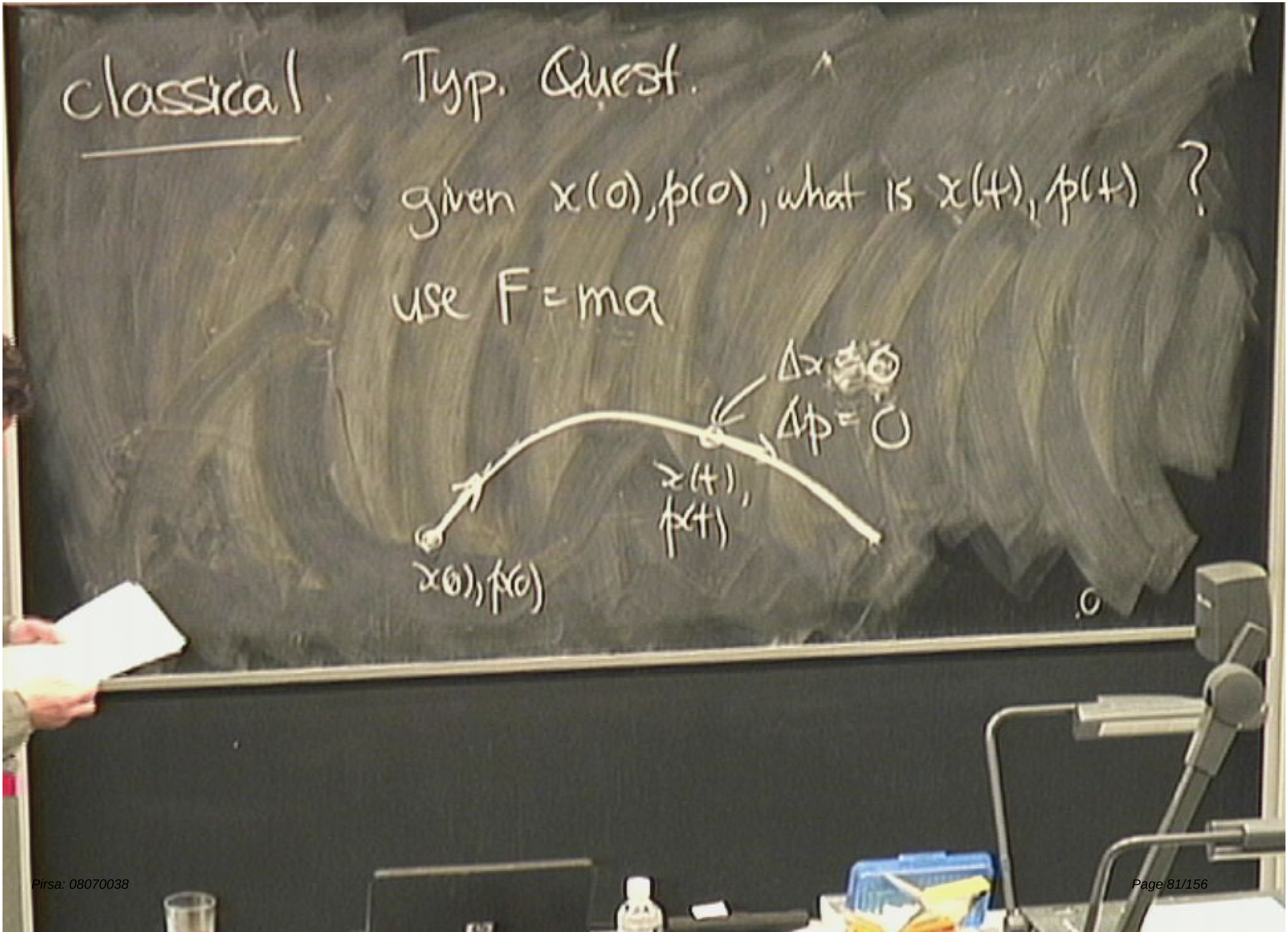


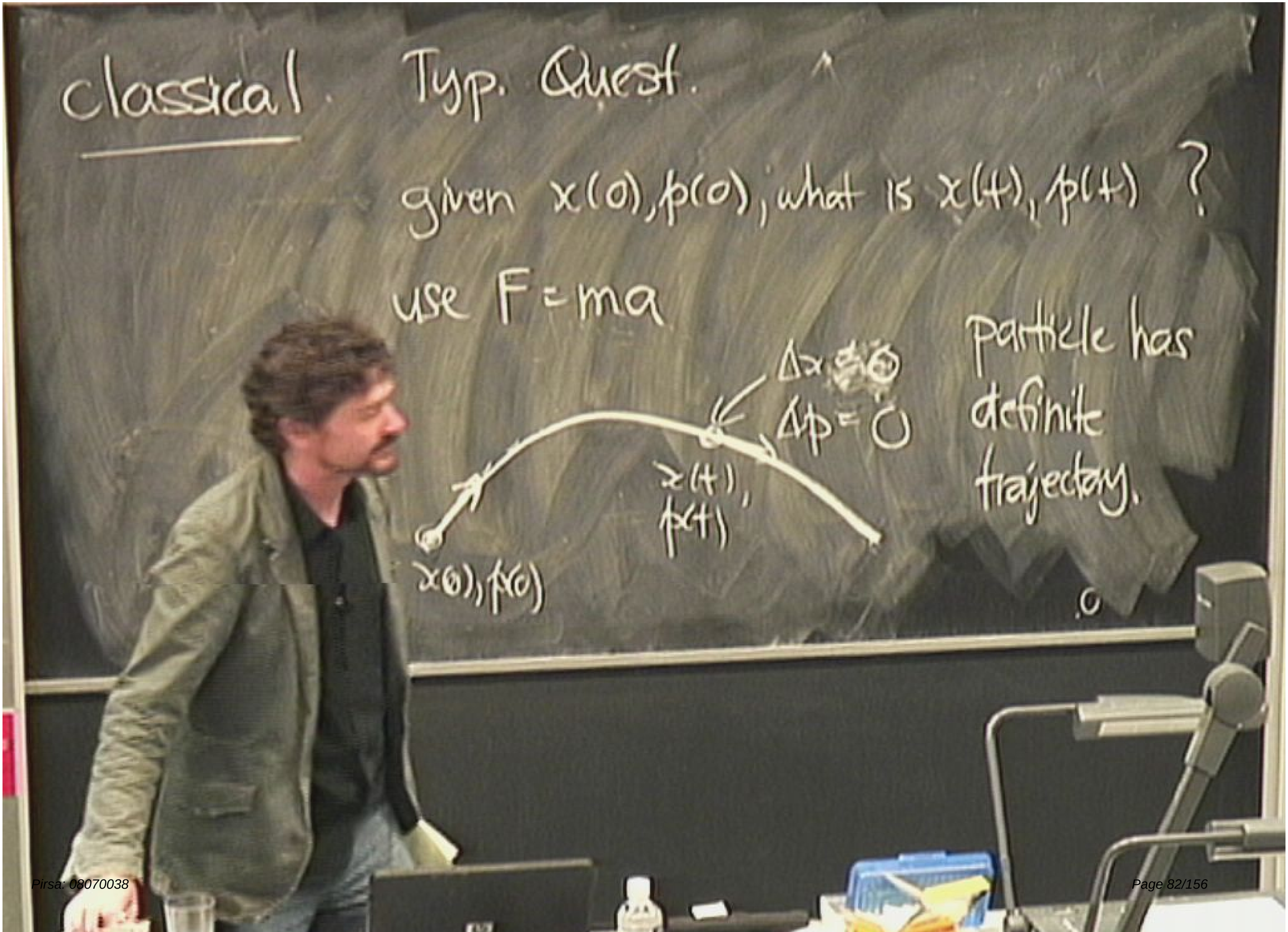
classical
Typ. Quest.
given x(0), p(0)

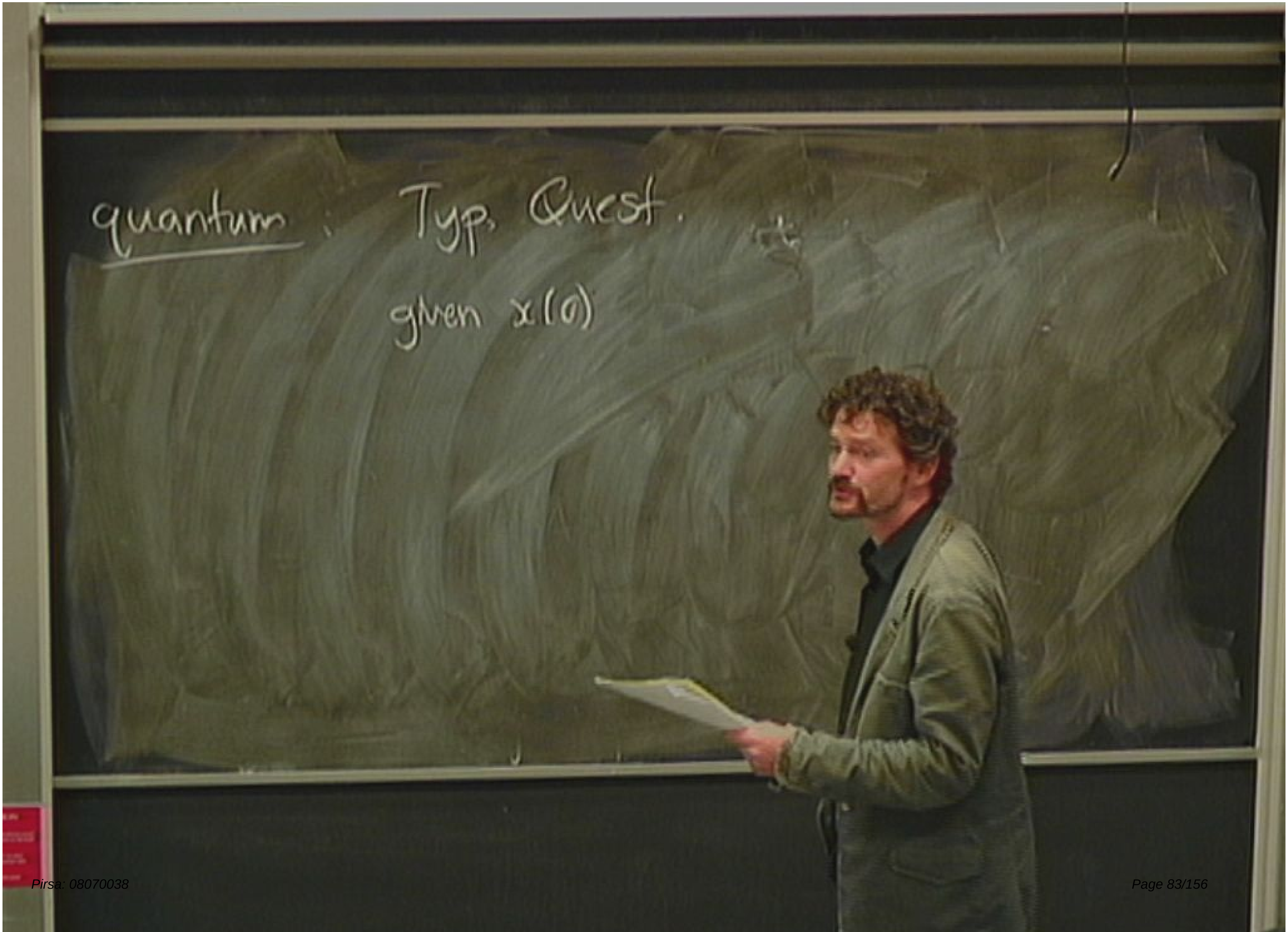

Classical
Typ. Quest.
given x(0), p(0), what is x(t), p(t) ?

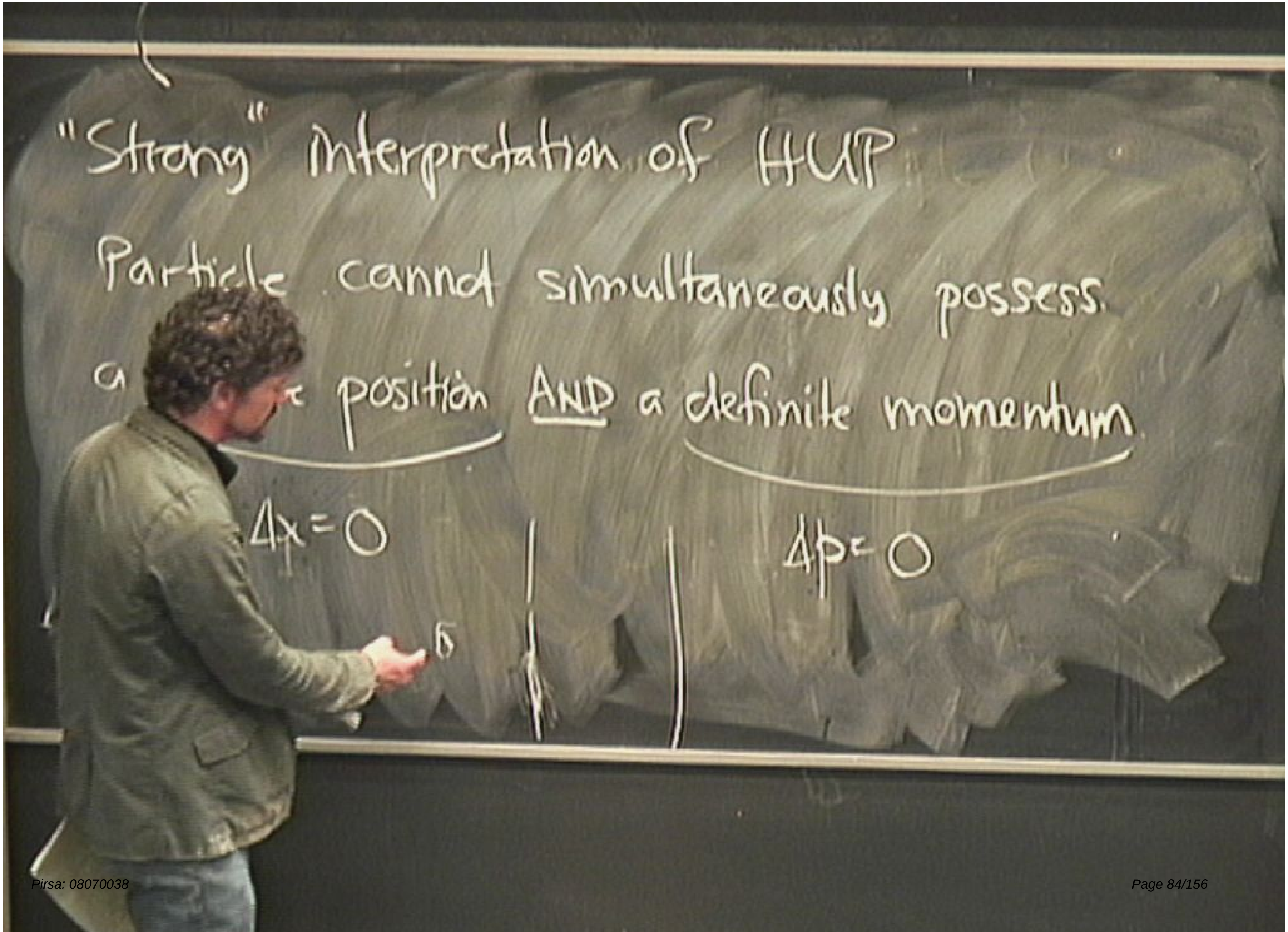

Classical
Typ. Quest.
given x(0), p(0), what is x(t), p(t) ?
use F = ma
x(0), p(0)


Classical
Typ. Quest.
given x(0), p(0), what is x(t), p(t) ?
F=ma
x(t),
p(t)


classical
Typ. Quest.
given x(0), p(0), what is x(t), p(t) ?
use F = ma
Δx
Δp = 0
x(t),
p(t)
x(0), p(0)

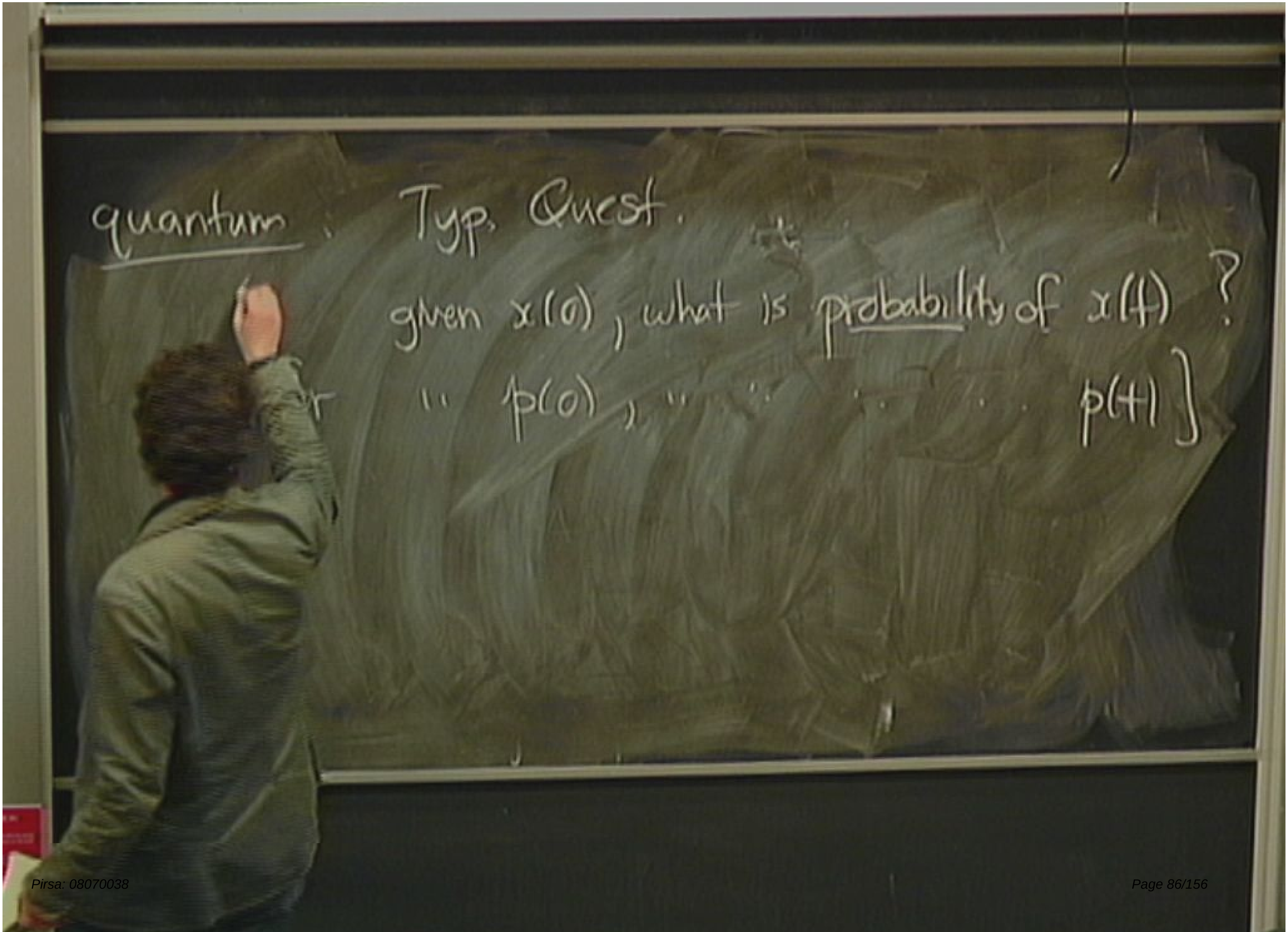

classical
Typ. Quest.
given x(0), p(0), what is x(t), p(t) ?
use F = ma
particle has definite trajectory.
Δx = 0
Δp = 0
x(t), p(t)
x(0), p(0)

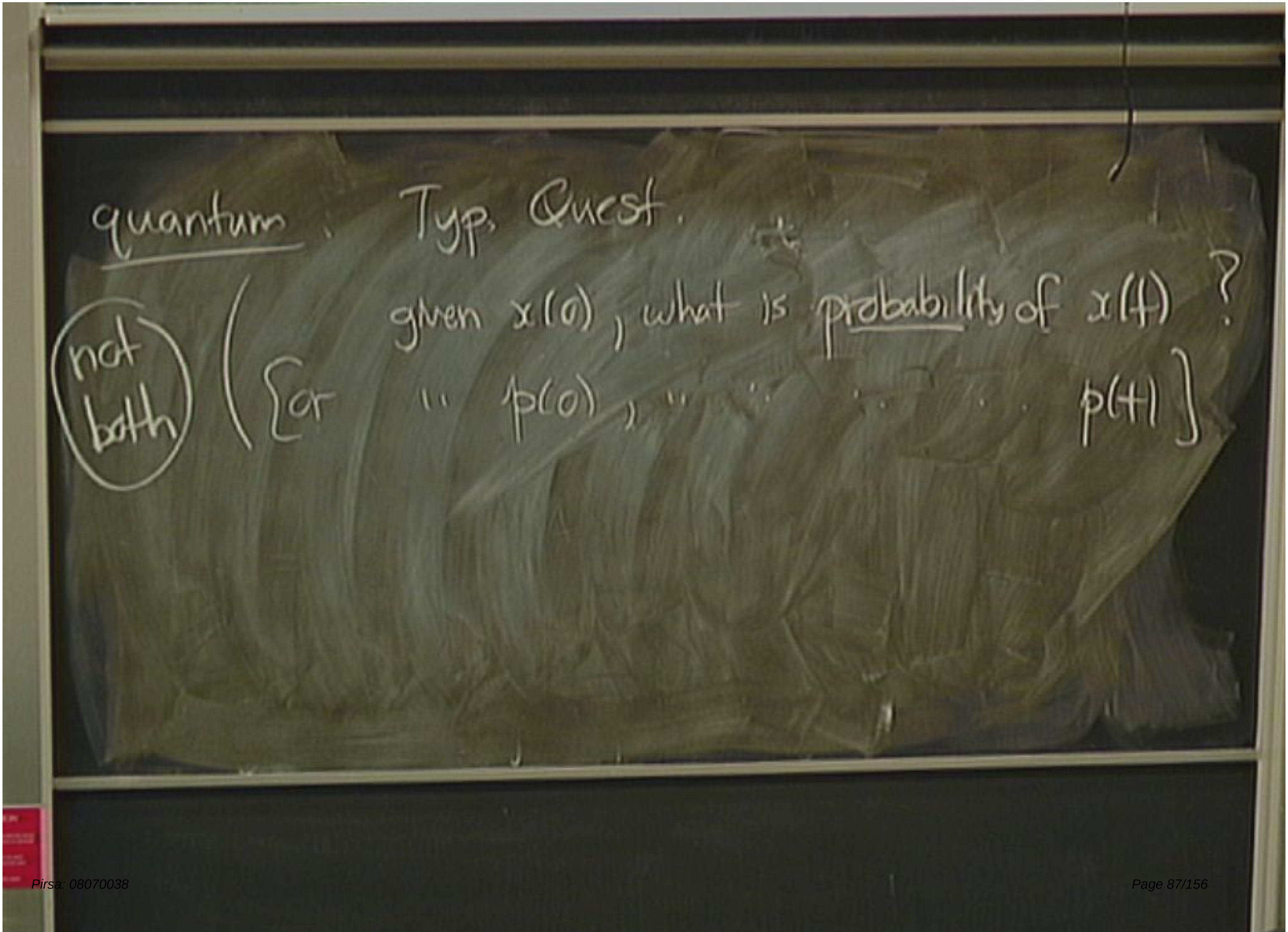

quantum : Typ. Quest.
given x(0)

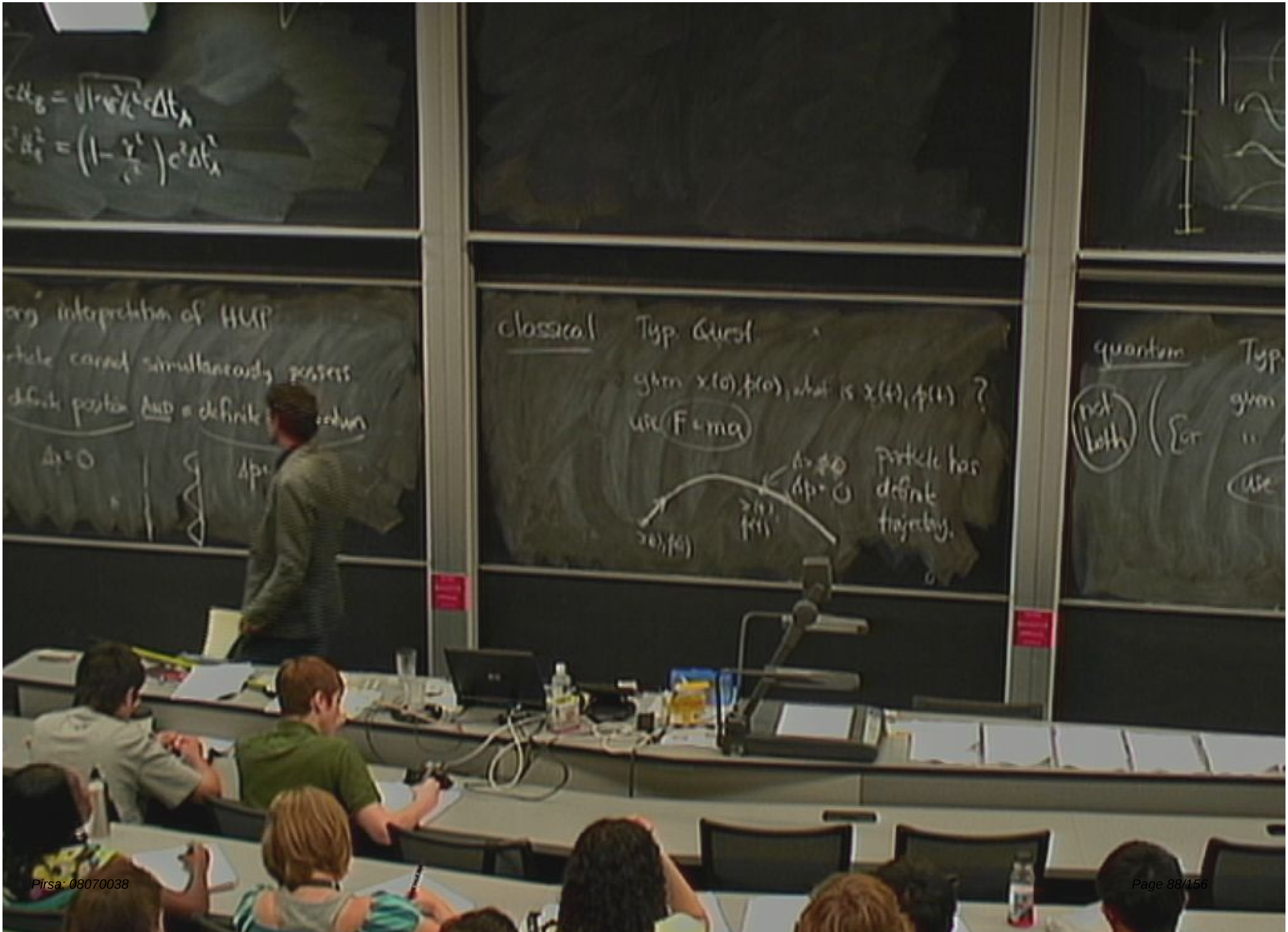

"Strong" interpretation of HUP
Particle cannot simultaneously possess
a position AND a definite momentum
Δx = 0
Δp = 0

"Strong" interpretation of HUP

Particle cannot simultaneously possess a $\underbrace{\text{definite position}}_{\Delta x = 0}$ **AND** a $\underbrace{\text{definite momentum}}_{\Delta p = 0}$


quantum
Typ. Quest.
given x(0), what is probability of x(t) ?
p(0) ,
p(t)


quantum. Typ. Quest.
given x(0), what is probability of x(t) ?
not both
or
p(0),
p(t)


interpretation of HUP
classical
Typ. Quest
given x(0), p(0), what is x(t), p(t) ?
use F = ma
particle has
definite
trajectory.
quantum
not
both

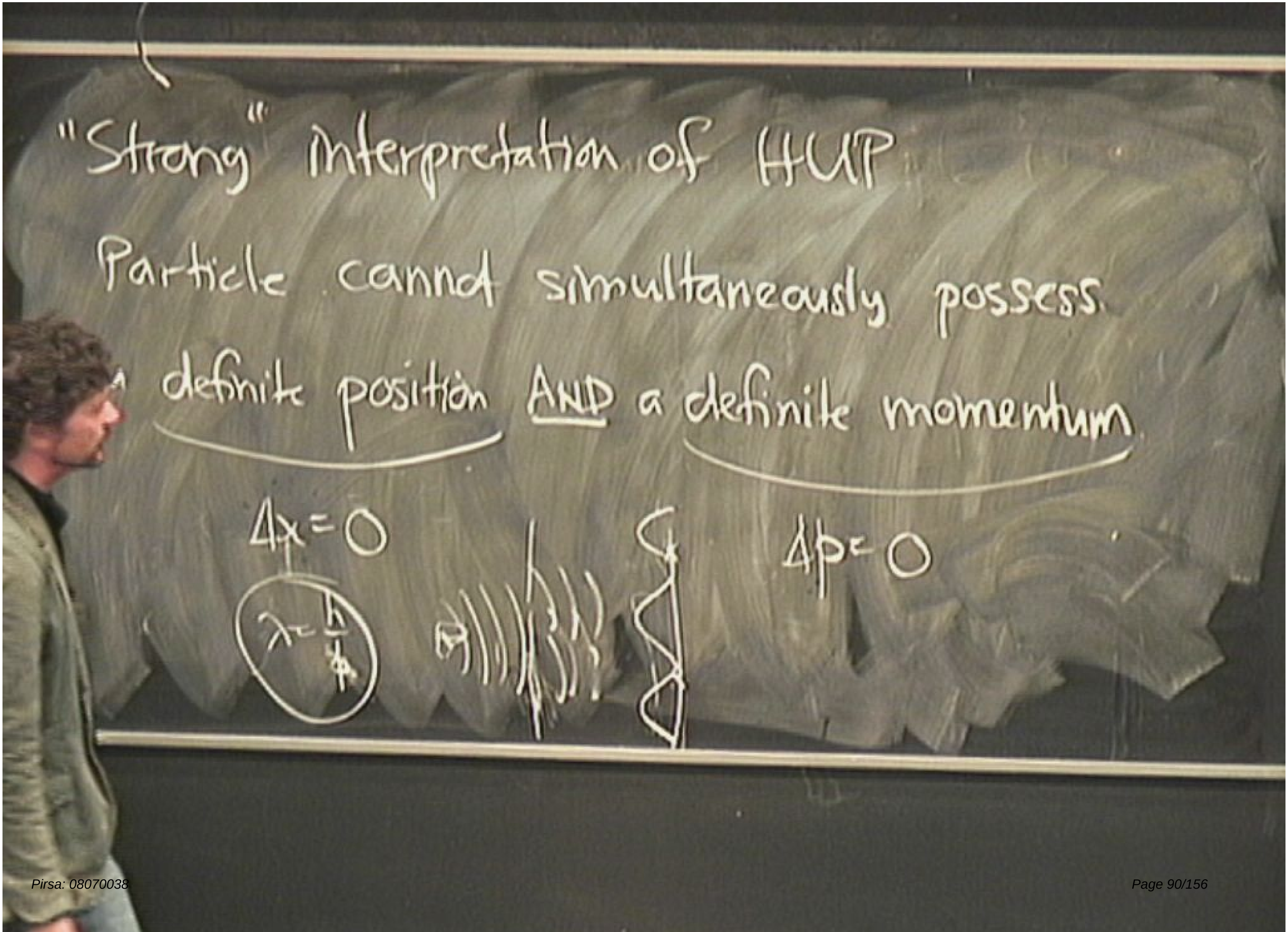
"Strong" Interpretation of HUP

Particle cannot simultaneously possess a definite position **AND** a definite momentum

$\Delta x = 0$

$\Delta p = 0$

$\lambda = \frac{h}{p}$

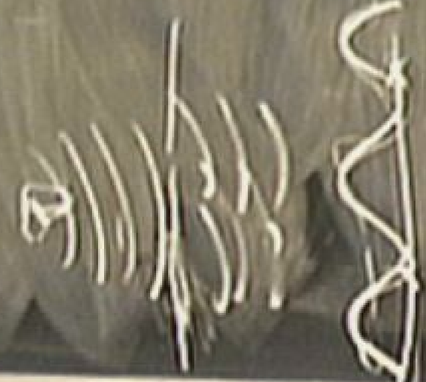

"Strong" interpretation of HUP
Particle cannot simultaneously possess
a definite position AND a definite momentum
Δx = 0
λ = h/p
Δp = 0

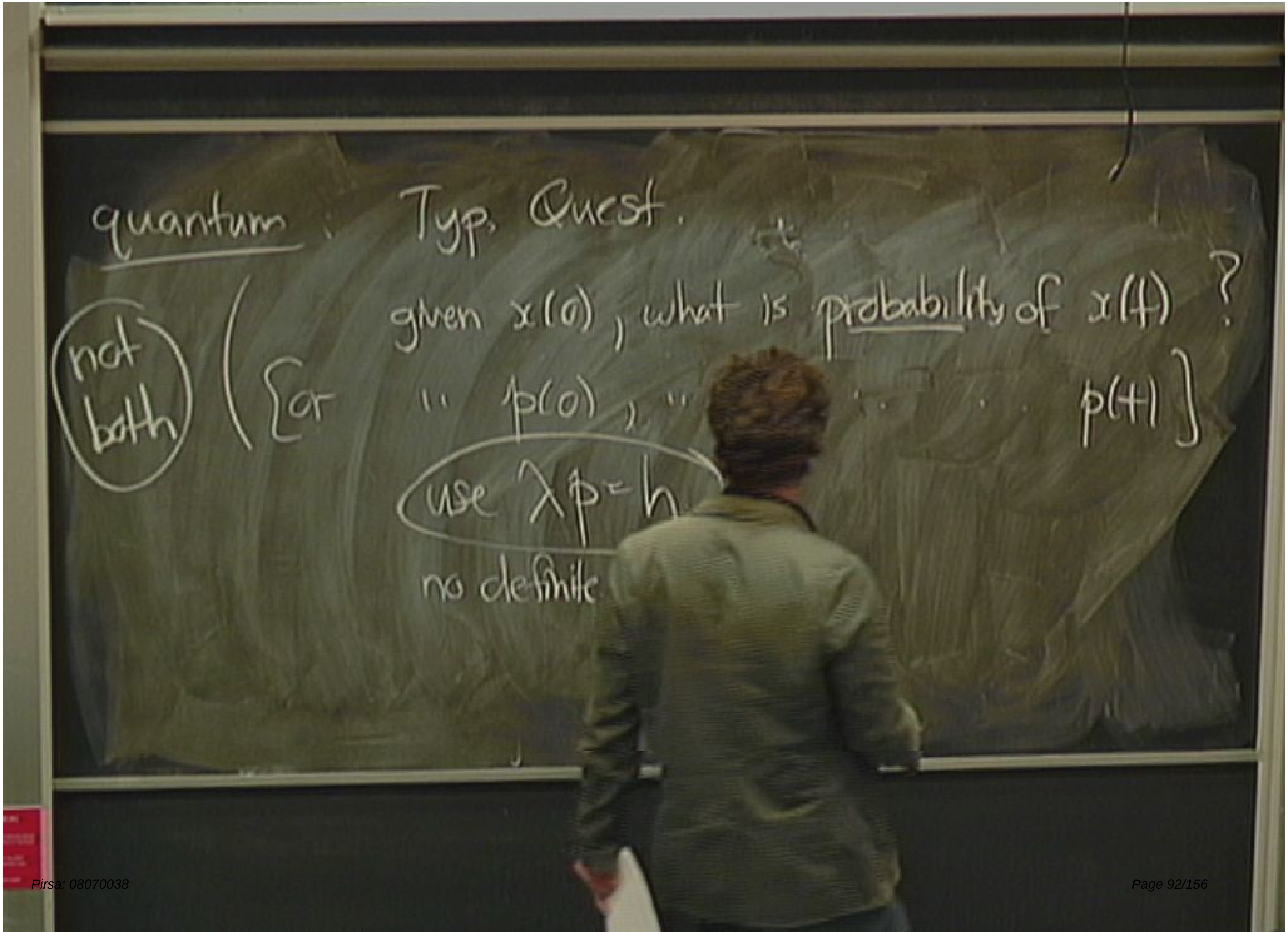
"Strong" interpretation of HUP

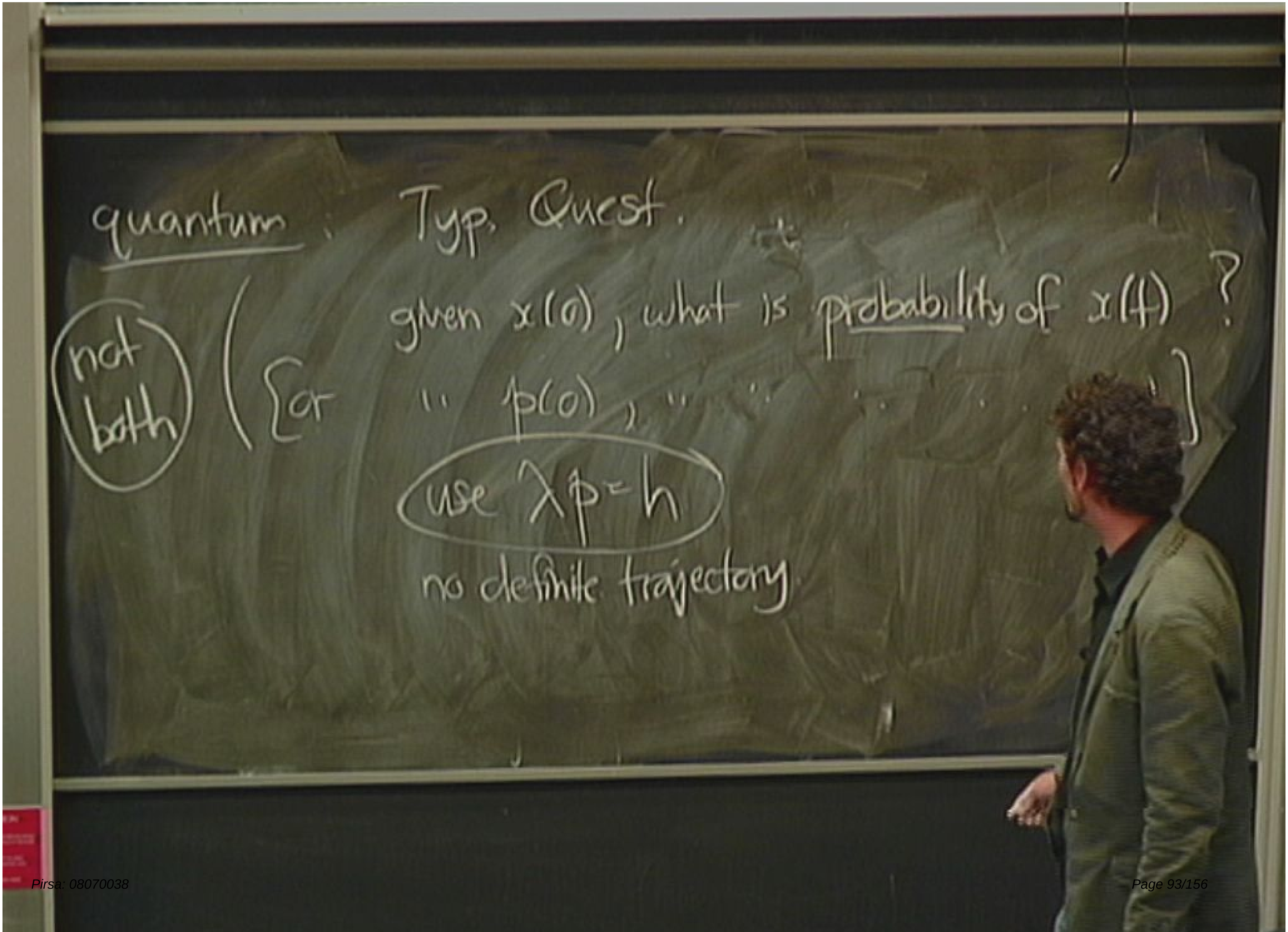
Particle cannot simultaneously possess a definite position **AND** a definite momentum

$\Delta x = 0$ $\Delta p = 0$

$\lambda = \frac{h}{p}$




quantum
Typ. Quest.
given x(0), what is probability of x(t) ?
not both
or
p(0)
p(t)
use λp = h
no definite


quantum : Typ. Quest.
given x(0), what is probability of x(t) ?
not both
{or .. p(0), ..
use λp=h
no definite trajectory

quantum : Typ. Quest.

given $x(0)$, what is probability of $x(t)$?

not both ({or '' $p(0)$, '' '' . . . $p(t)$ }

use $\lambda p = h$

no definite trajectory

quantum : Typ. Quest.

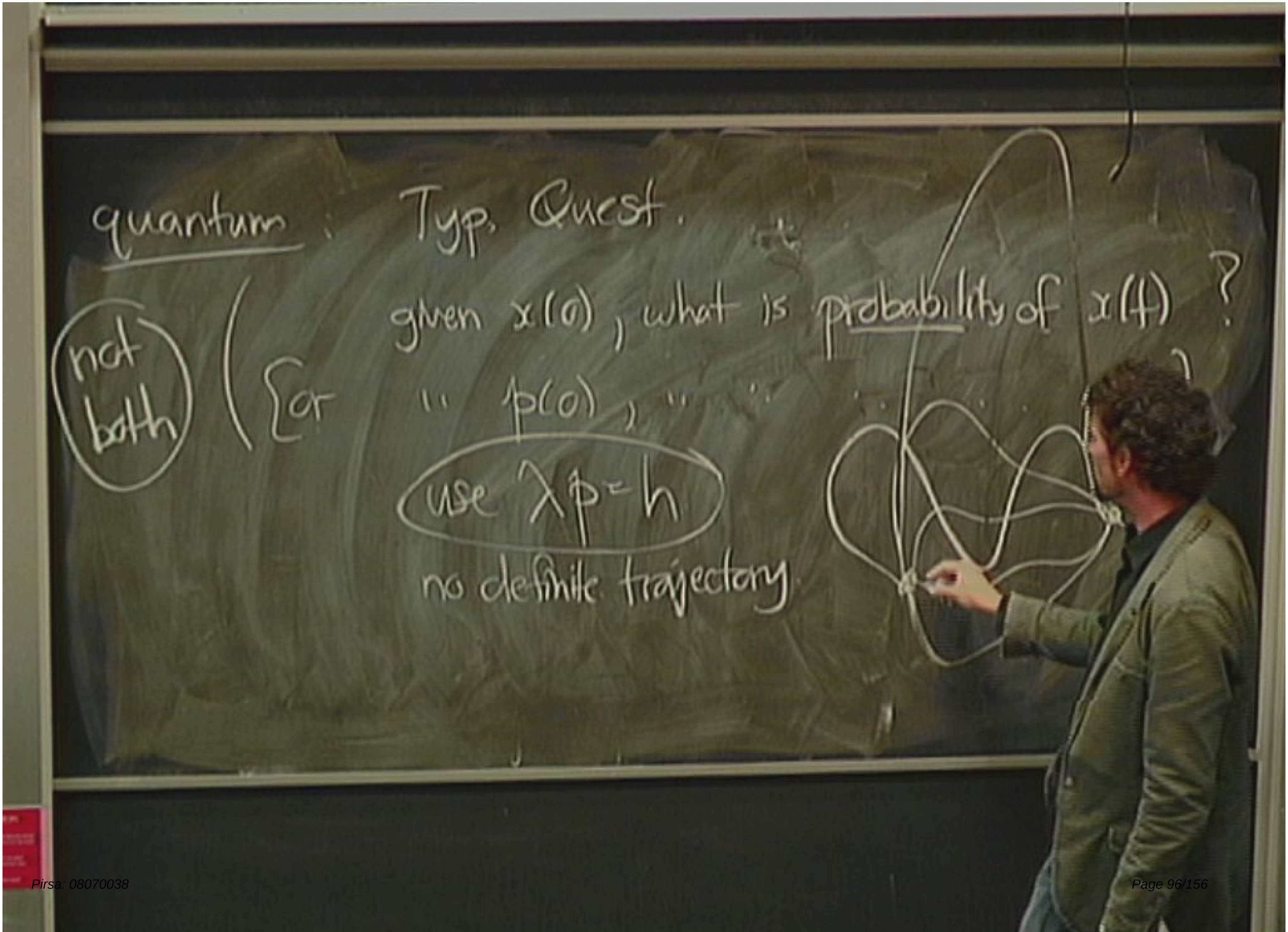
not both

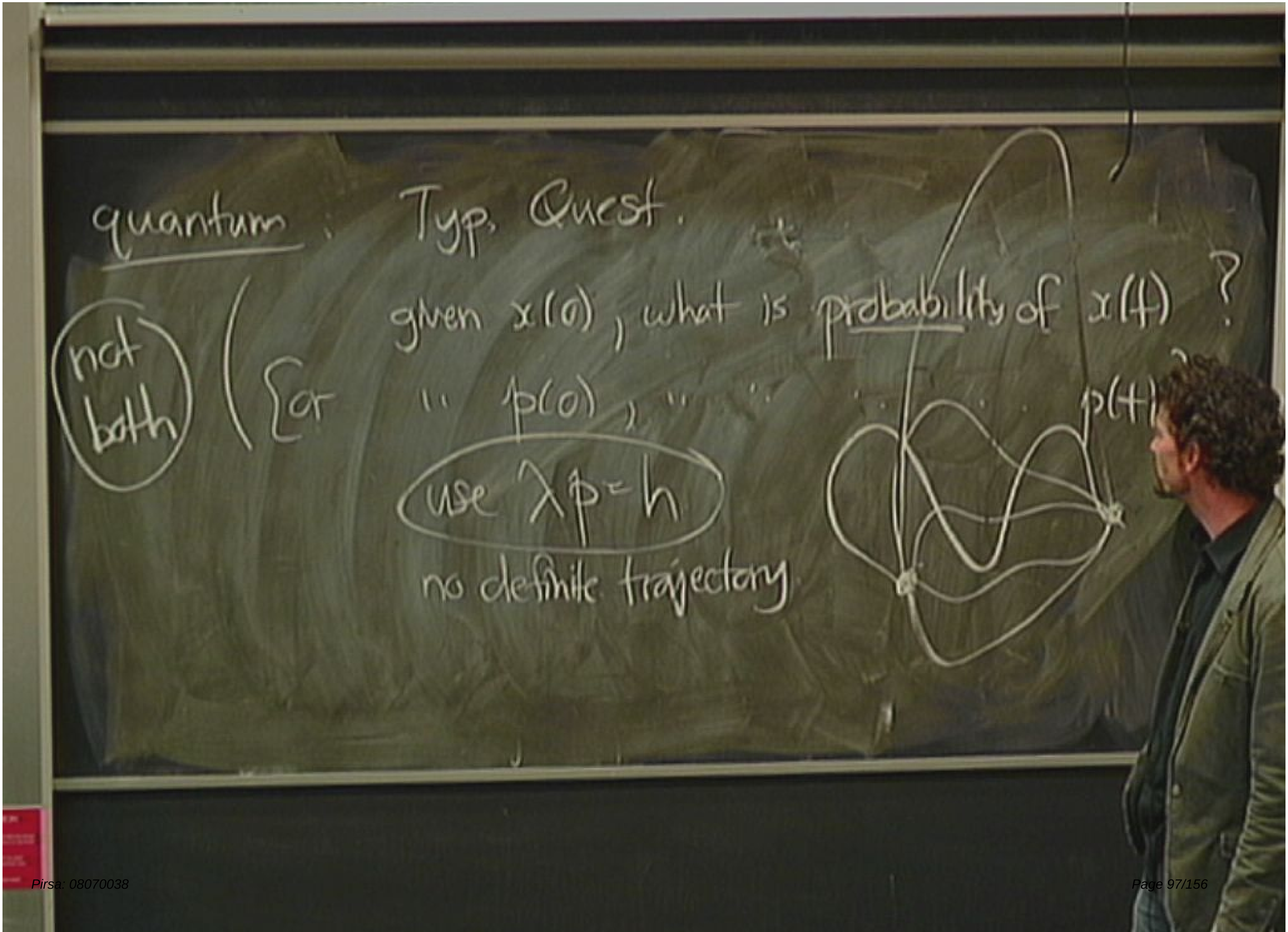
given $x(0)$, what is **probability** of $x(t)$?

[or " $p(0)$, " " . . . . $p(t)$]

use $\lambda p = h$

no definite trajectory


quantum :
Typ. Quest.
given x(0), what is probability of x(t) ?
not both
for .. p(0), ...
use λp = h
no definite trajectory


quantum : Typ. Quest.
not both
for
given x(0), what is probability of x(t) ?
p(0) , p(t) ?
use λp = h
no definite trajectory

quantum : Typ. Quest.

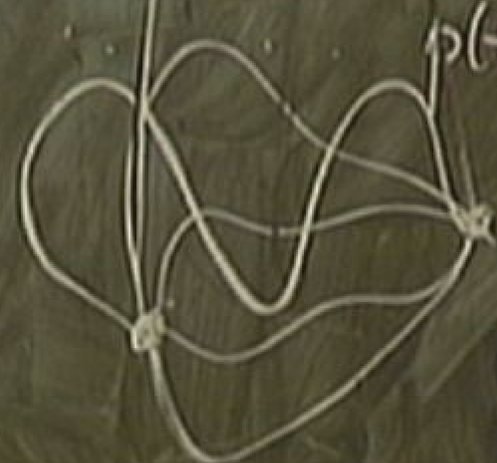
not both ({or

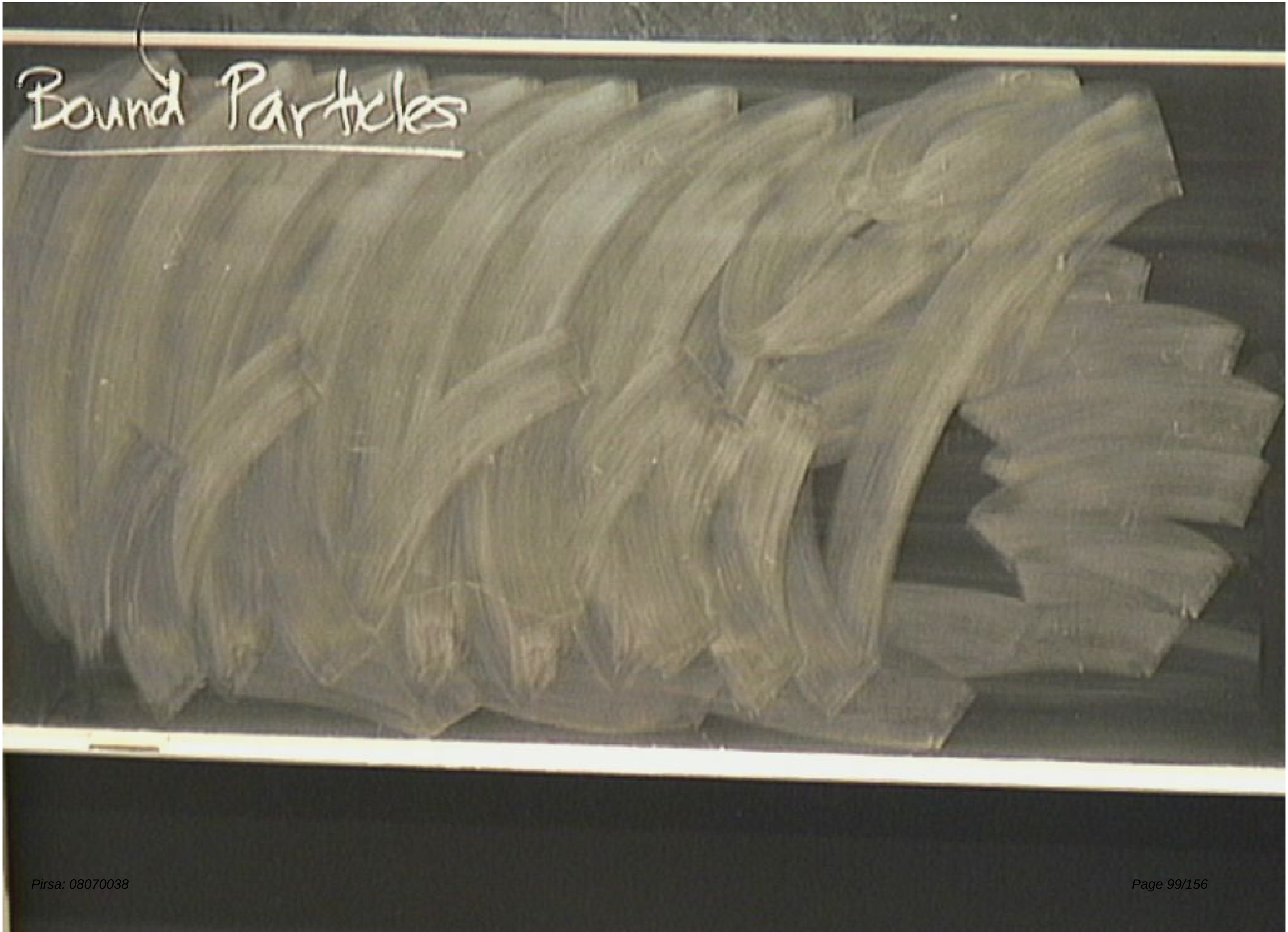
given $x(0)$, what is probability of $x(t)$?

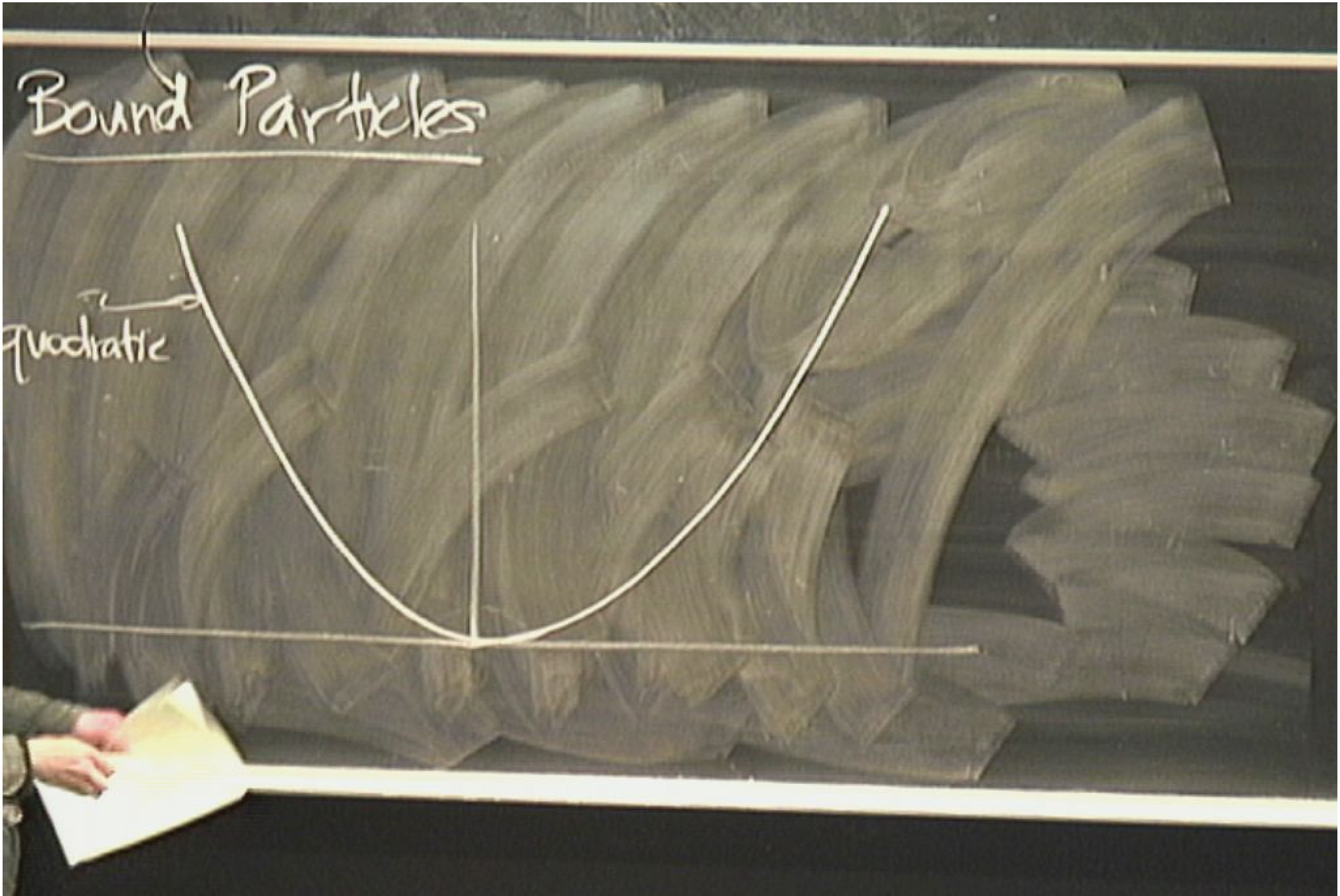
.. $p(0)$, ... $p(t)$ }

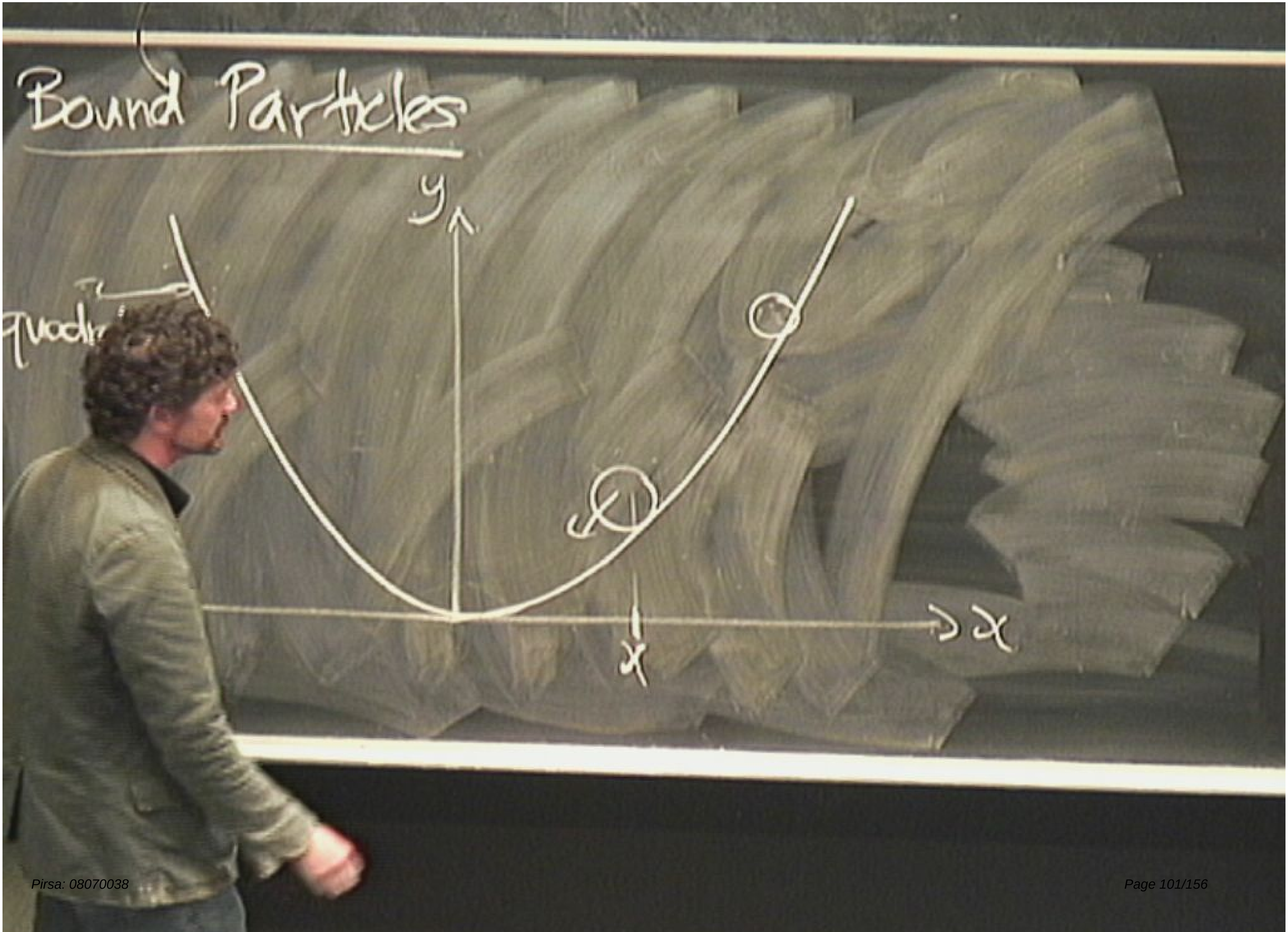
use $\lambda p = h$

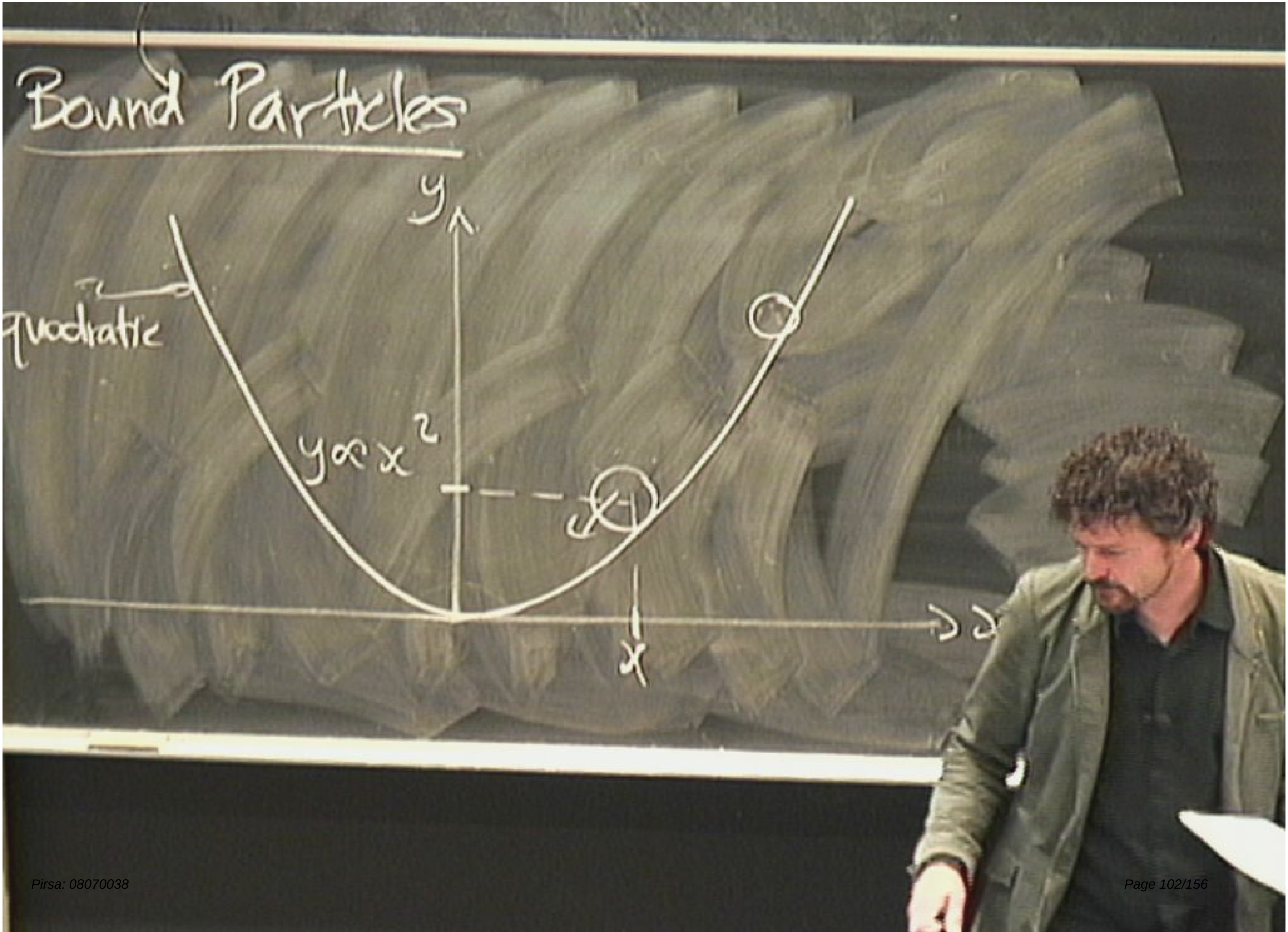
no definite trajectory




Bound Particles


Bound Particles
quadratic

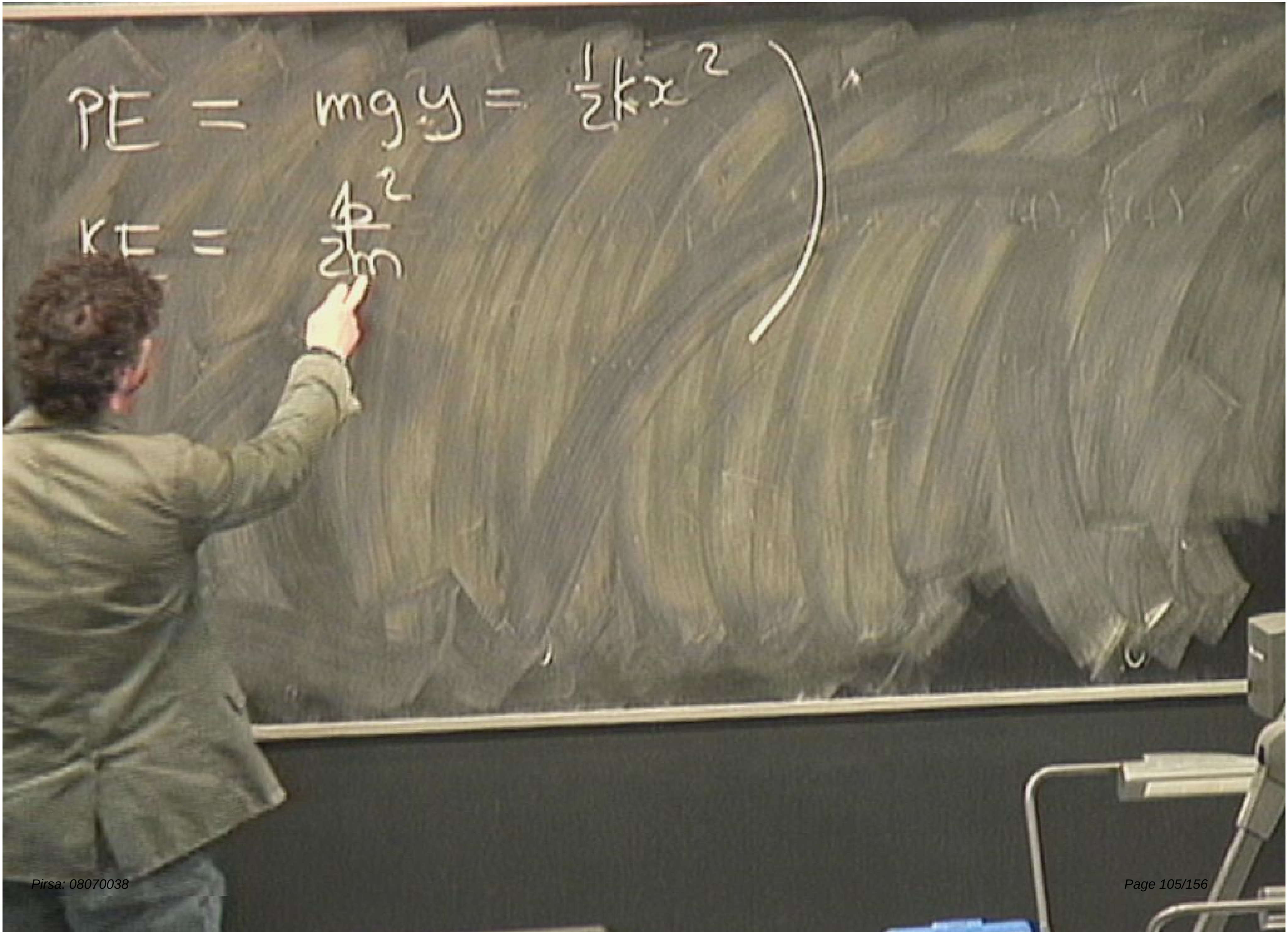

Bound Particles
y
x
x


Bound Particles
y
quadratic
$y \propto x^2$
x
x

$$PE = mgy = \frac{1}{2}kx^2$$

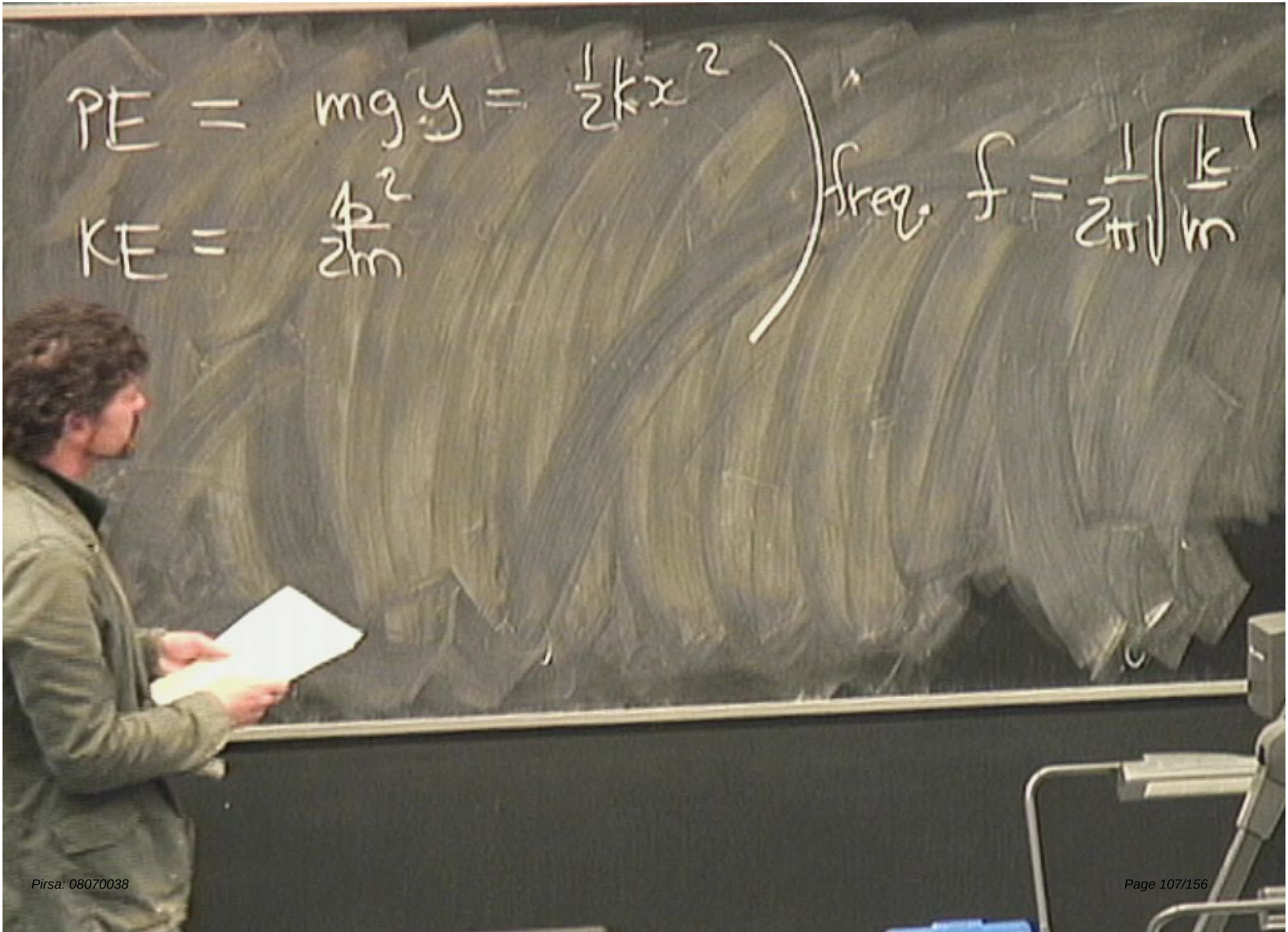
$$PE = mgy = \frac{1}{2}kx^2$$

$$KE =$$


PE = mgy = ½kx²
KE = p²/2m

$$PE = mgy = \frac{1}{2}kx^2$$

$$KE = \frac{p^2}{2m}$$

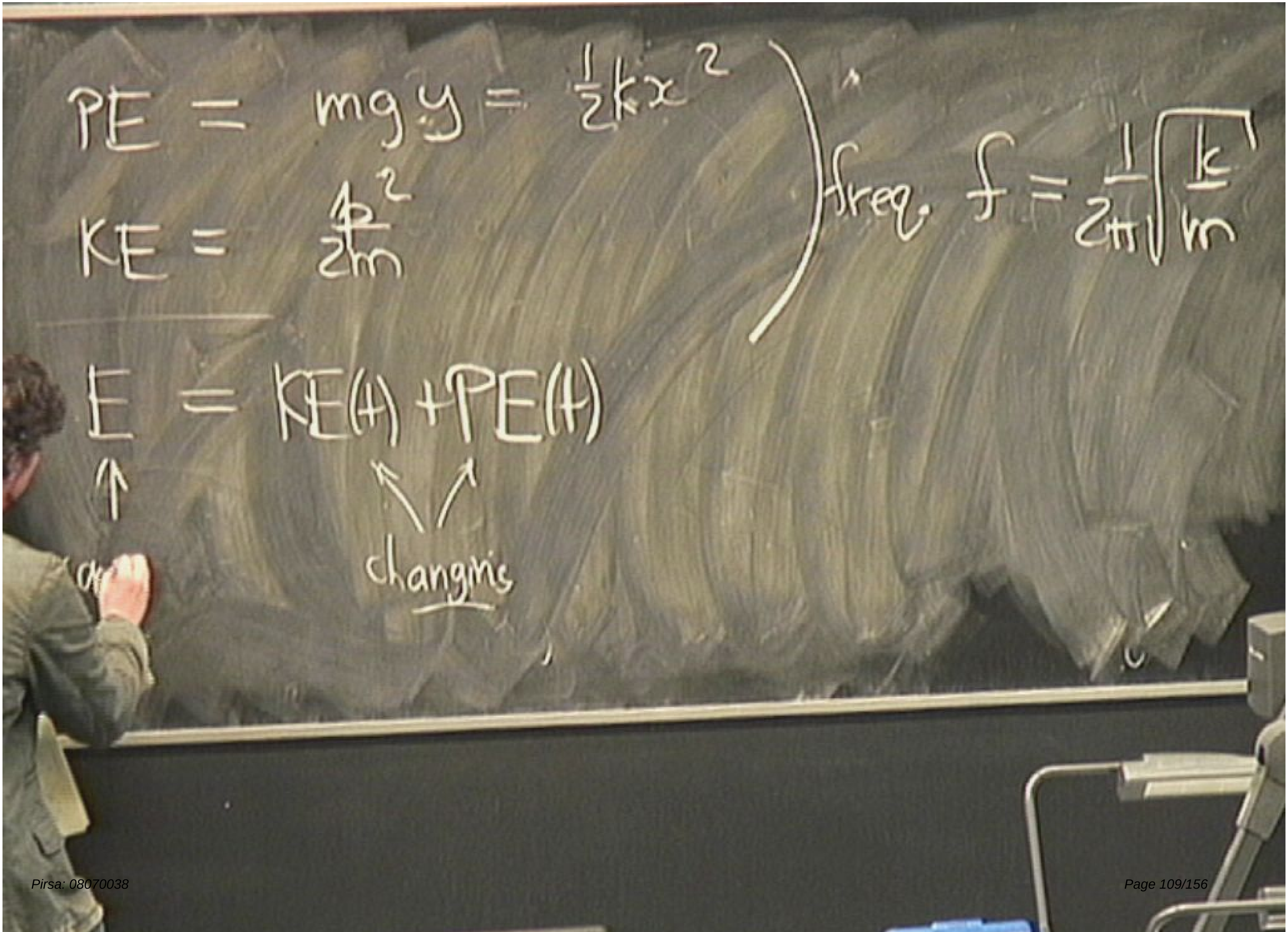

$PE = mgy = \frac{1}{2}kx^2$
$KE = \frac{p^2}{2m}$
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$PE = mgy = \tfrac{1}{2}kx^2$$

$$KE = \frac{p^2}{2m}$$

freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$E = KE(t) + PE(t)$$


$PE = mgy = \frac{1}{2}kx^2$
$KE = \frac{p^2}{2m}$
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
$E = KE(t) + PE(t)$
changing

$$PE = mgy = \frac{1}{2}kx^2$$

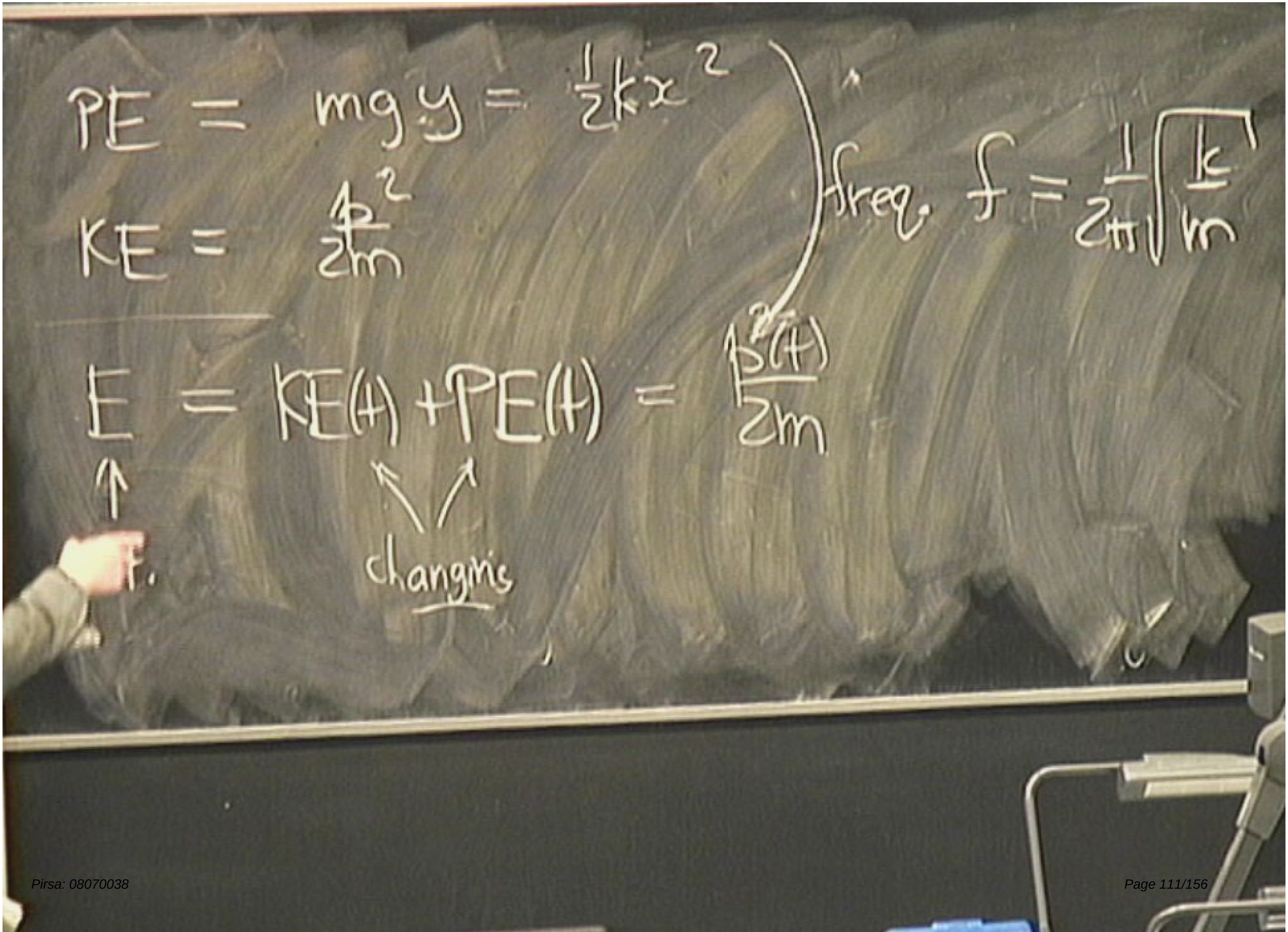
$$KE = \frac{p^2}{2m}$$

freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$E = KE(t) + PE(t)$$

const.

changing


$PE = mgy = \frac{1}{2}kx^2$
$KE = \frac{p^2}{2m}$
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
$E = KE(t) + PE(t) = \frac{p^2(t)}{2m}$
changing

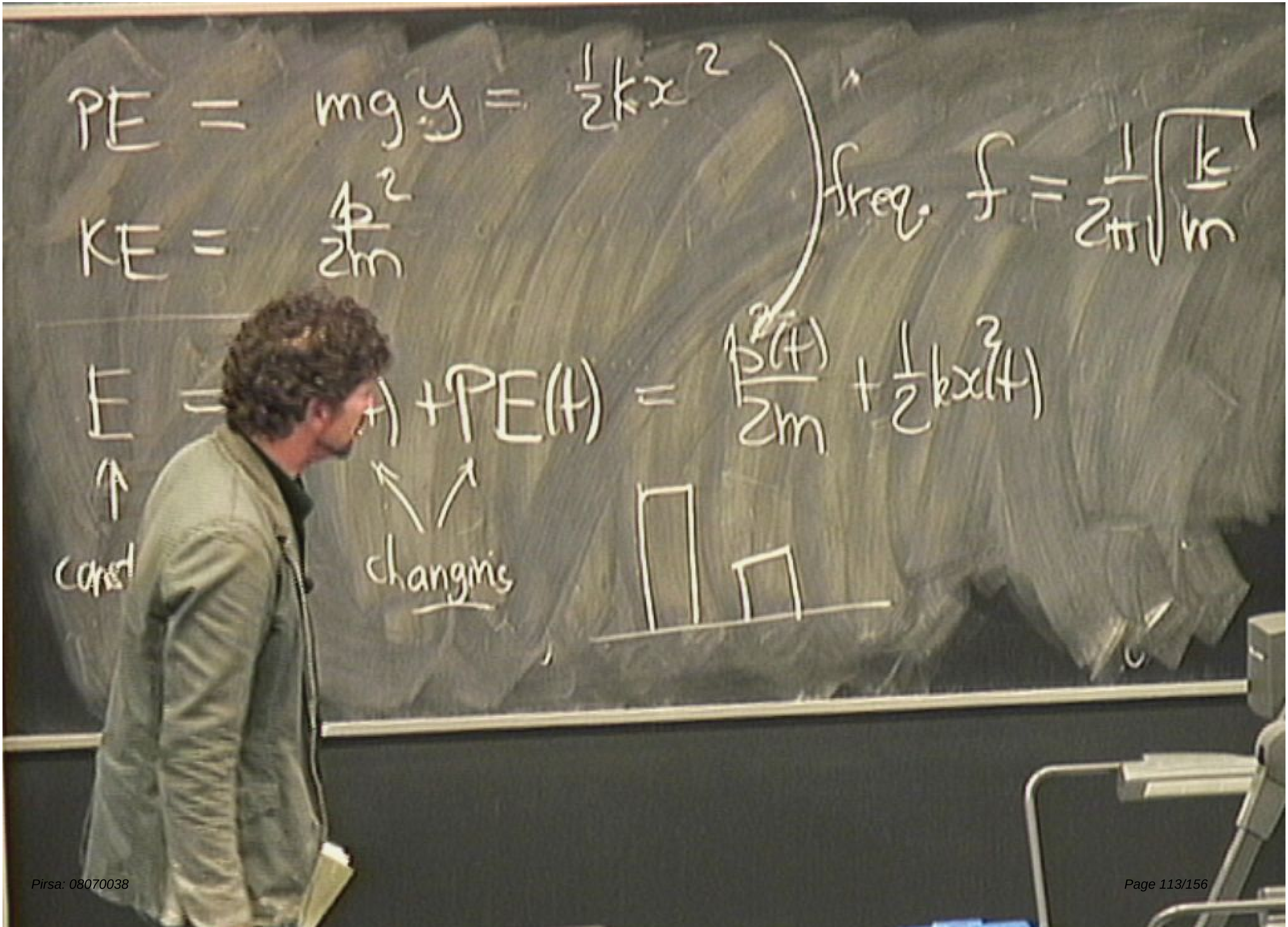
$$PE = mgy = \frac{1}{2}kx^2$$

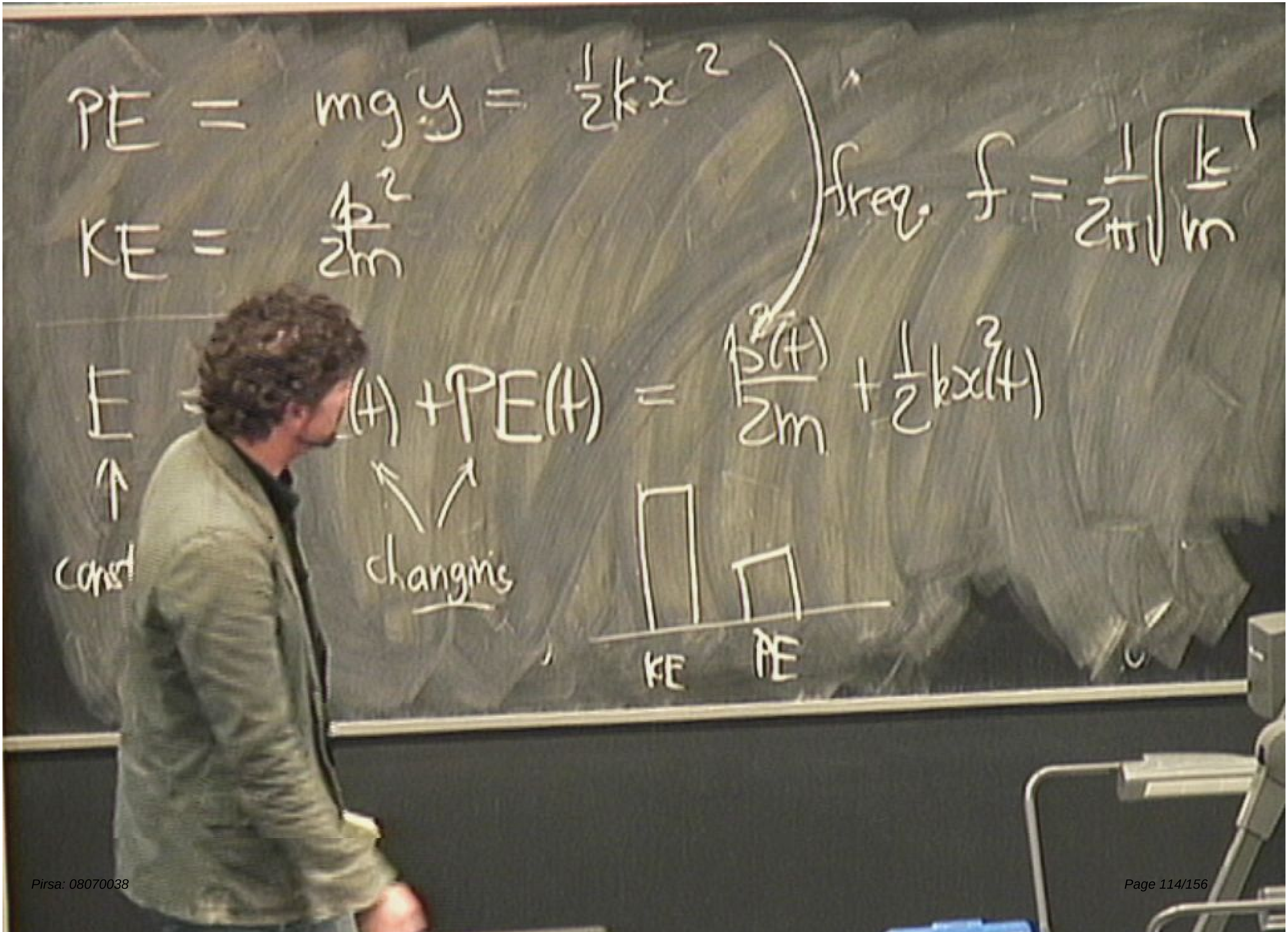
$$KE = \frac{p^2}{2m}$$

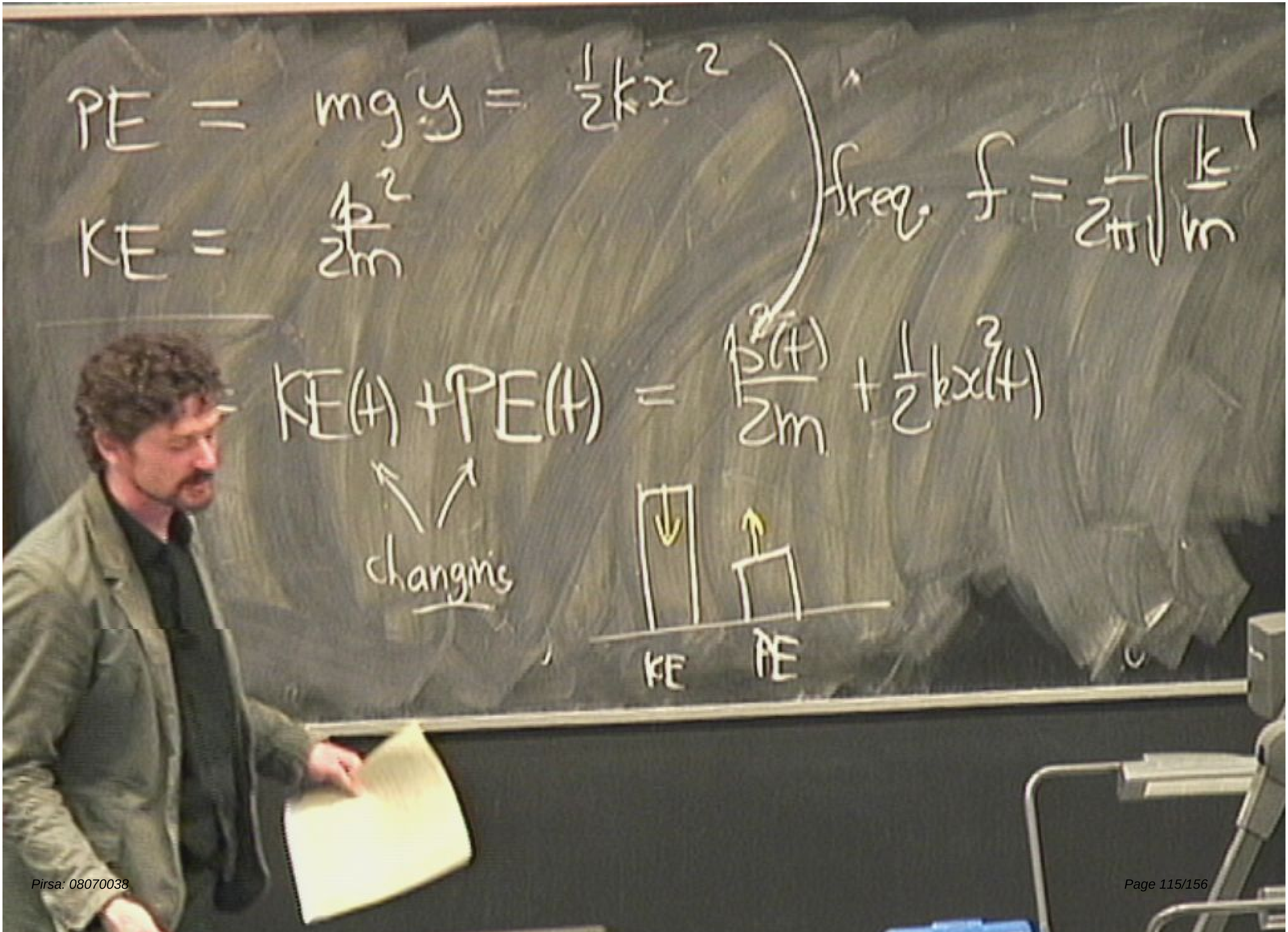
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$E = KE(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$$

$\uparrow$ const.   changing


$PE = mgy = \frac{1}{2}kx^2$
$KE = \frac{p^2}{2m}$
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
$E = \ldots) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$
const
changing


$PE = mgy = \frac{1}{2}kx^2$
$KE = \frac{p^2}{2m}$
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
E
$(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$
const
changing
KE
PE


PE = mgy = ½kx²
KE = p²/2m
freq. f = 1/2π √(k/m)
= KE(t) + PE(t) = p²(t)/2m + ½kx²(t)
changing
KE
PE

$$PE = mgy = \frac{1}{2}kx^2$$

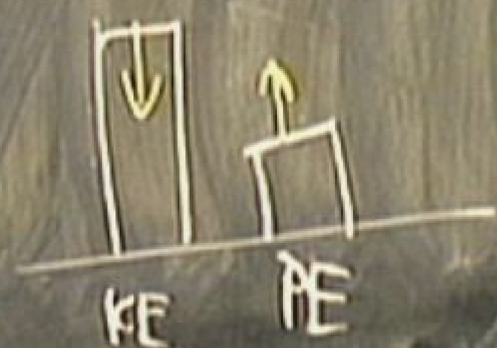
$$KE = \frac{p^2}{2m}$$

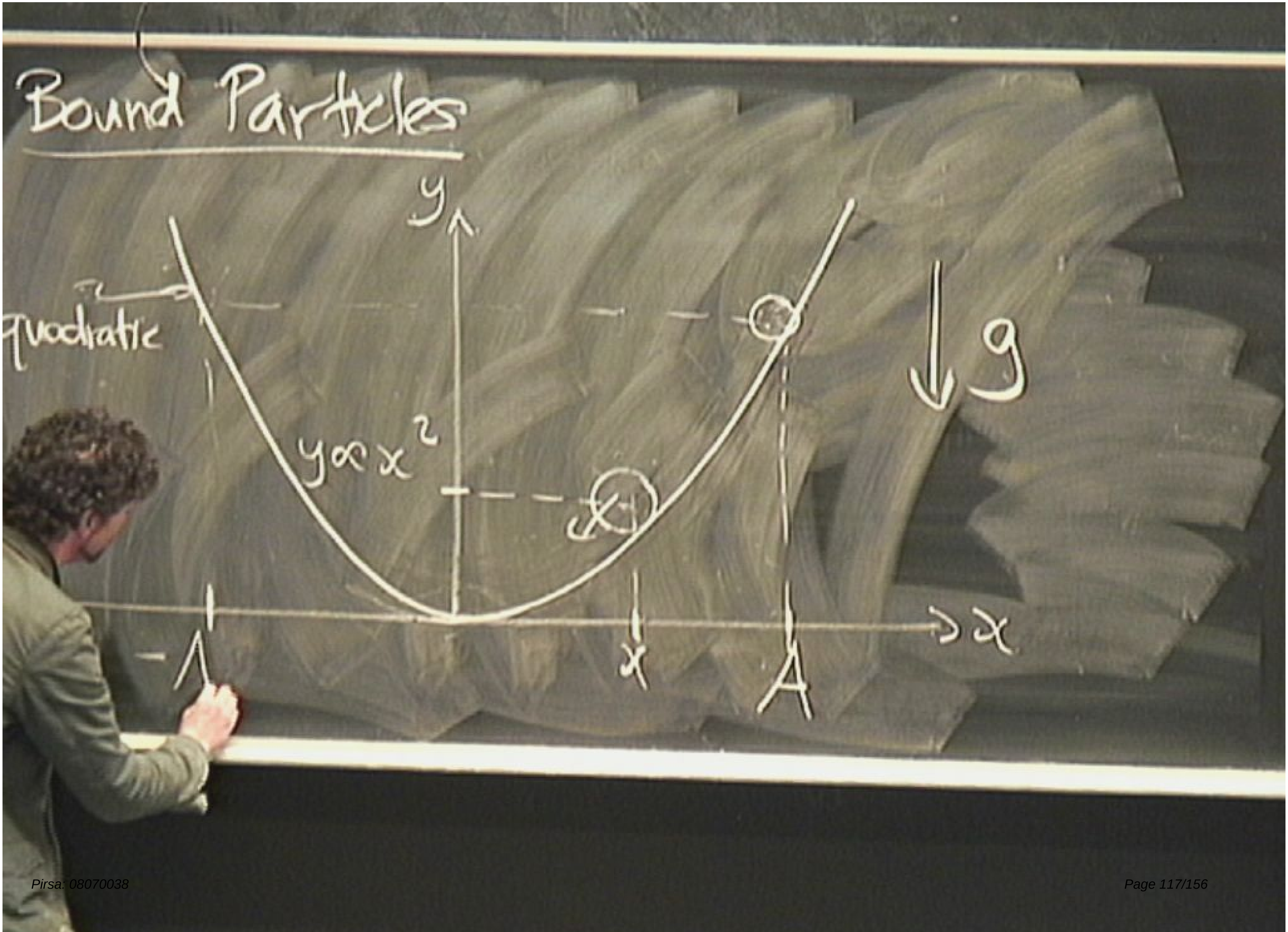
freq. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$E = KE(t) + PE(t) = \frac{p^2(t)}{2m} + \frac{1}{2}kx^2(t)$$

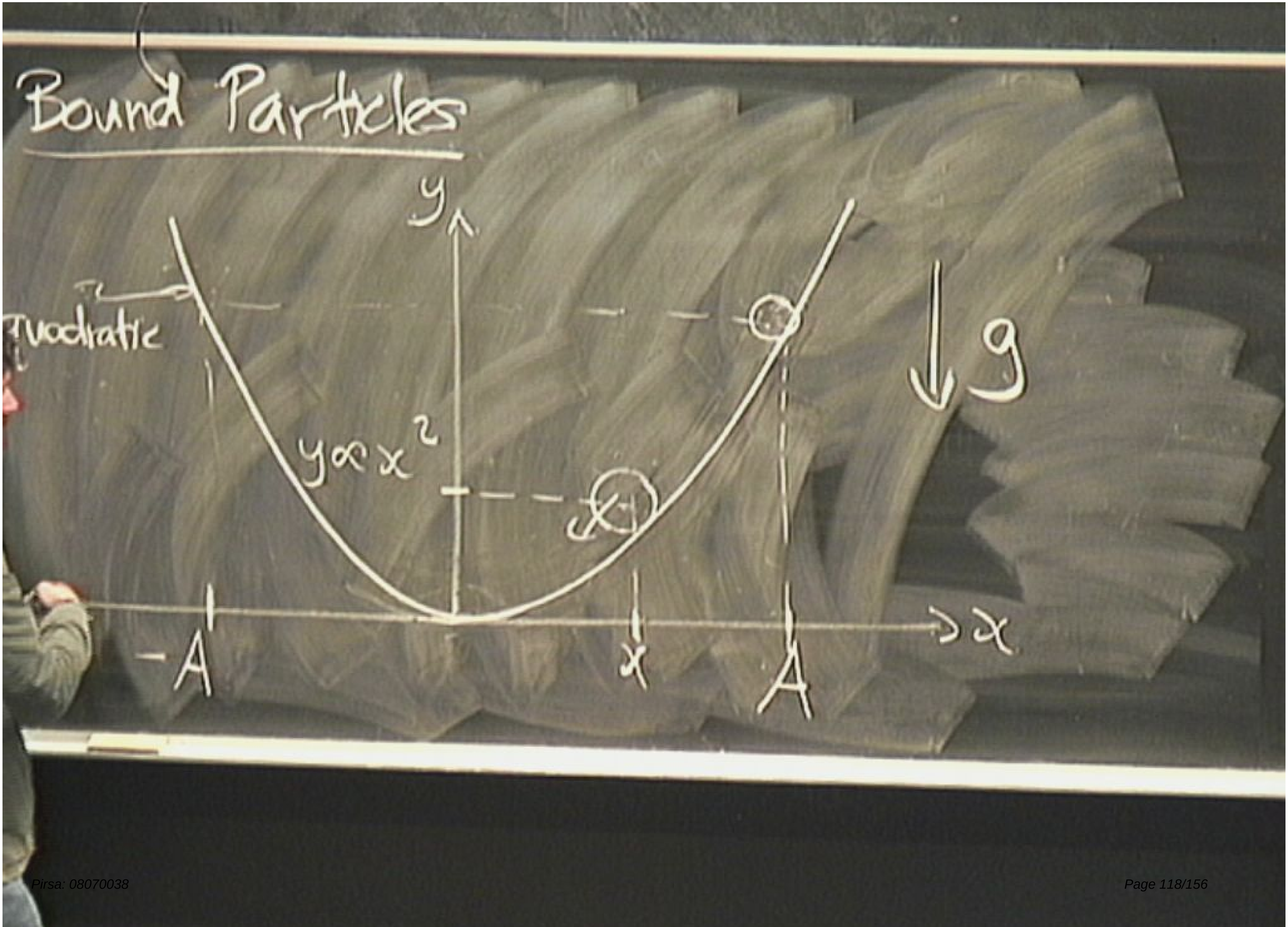
const.

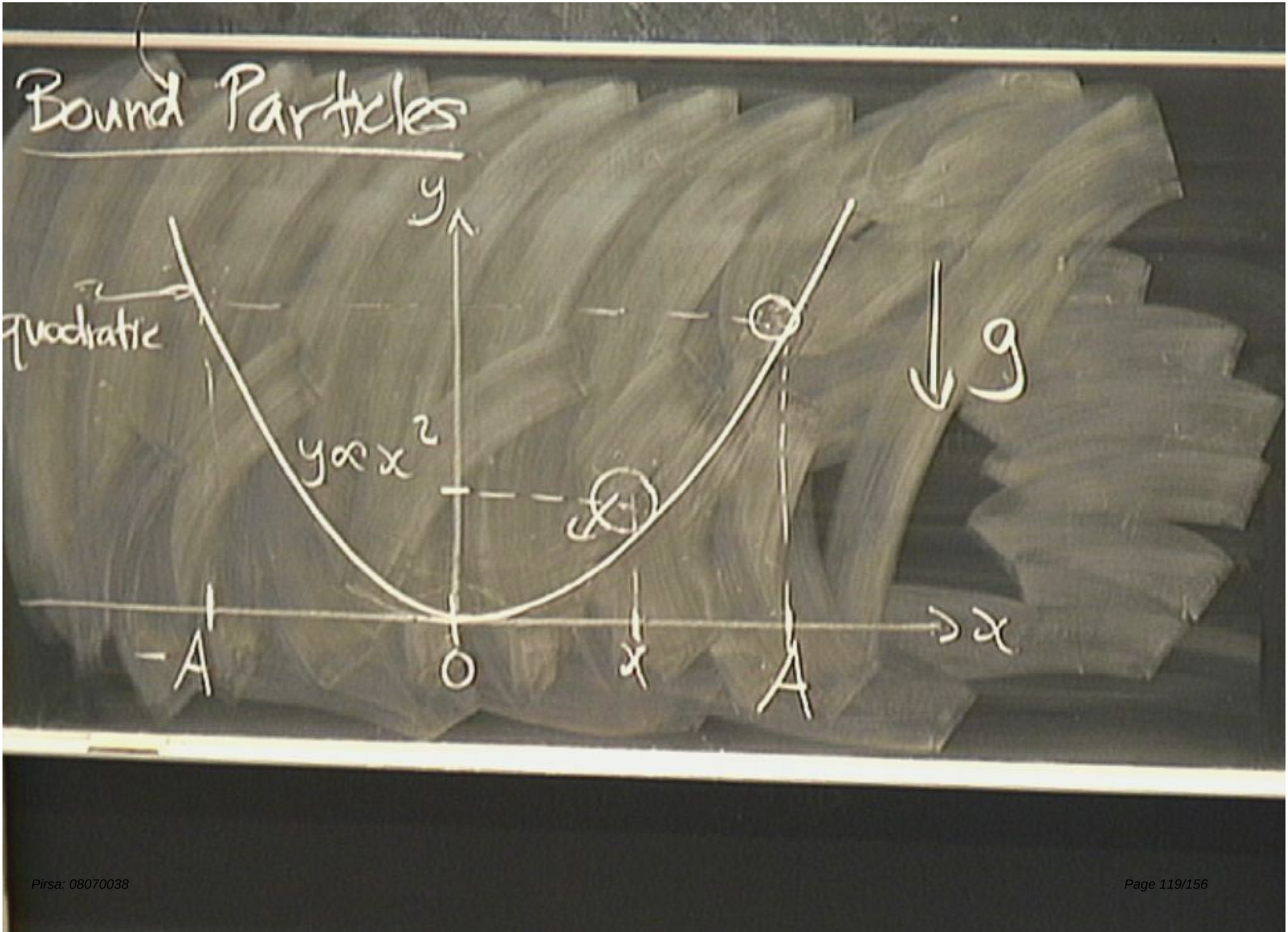
changing

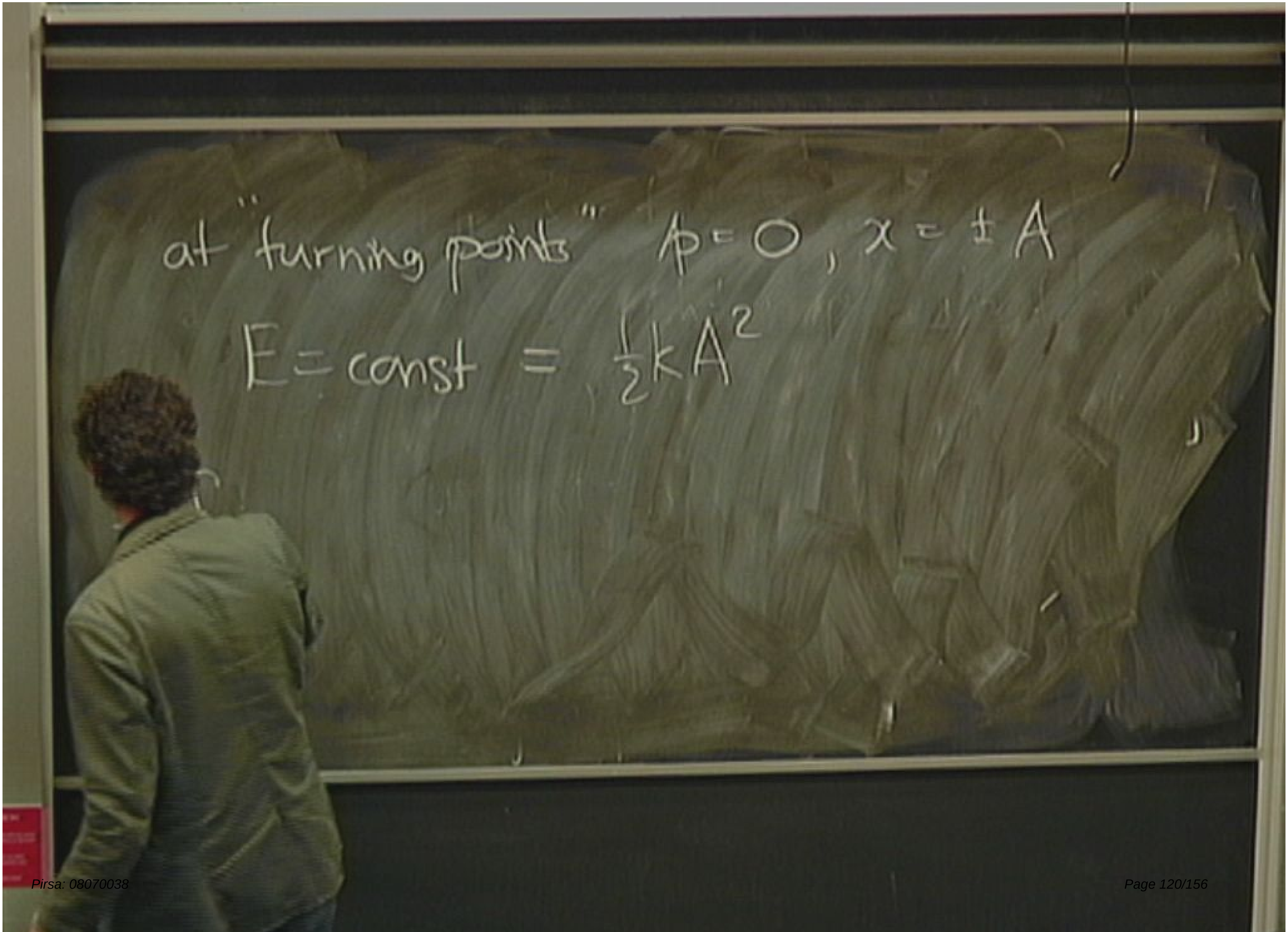




Bound Particles
y
quadratic
$y \propto x^2$
x
g
x
$-\Lambda$
x
A


Bound Particles
Quadratic
y
$y \propto x^2$
g
$-A$
x
A
x


Bound Particles
y
quadratic
g
$y \propto x^2$
x
$-A$
0
x
A


at "turning points" $p=0$, $x=\pm A$
$E = \text{const} = \frac{1}{2}kA^2$

at "turning points" $p = 0$, $x = \pm A$

$$E = \text{const} = \frac{1}{2} k A^2$$

* for given E, particle <u>bound</u> to $-A \leq x \leq$

at "turning points" $p=0$, $x=\pm A$

$$E = \text{const} = \tfrac{1}{2}kA^2$$

* for given E, particle <u>bound</u> to "$-A \le x \le A$"

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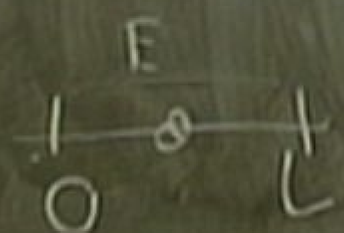
$$E = \text{const} = \frac{1}{2}kA^2$$

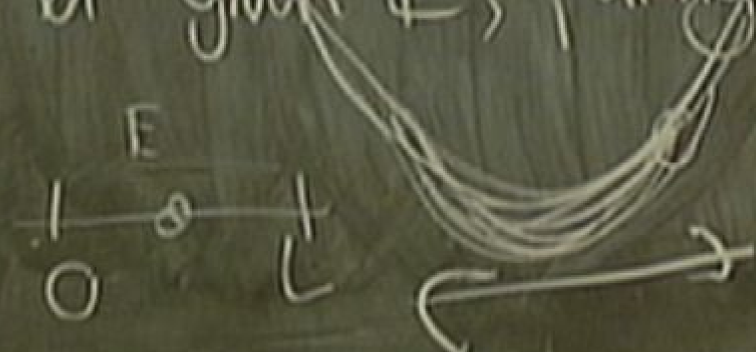
* for given E, particle bound to "$-A \leq x \leq A$"

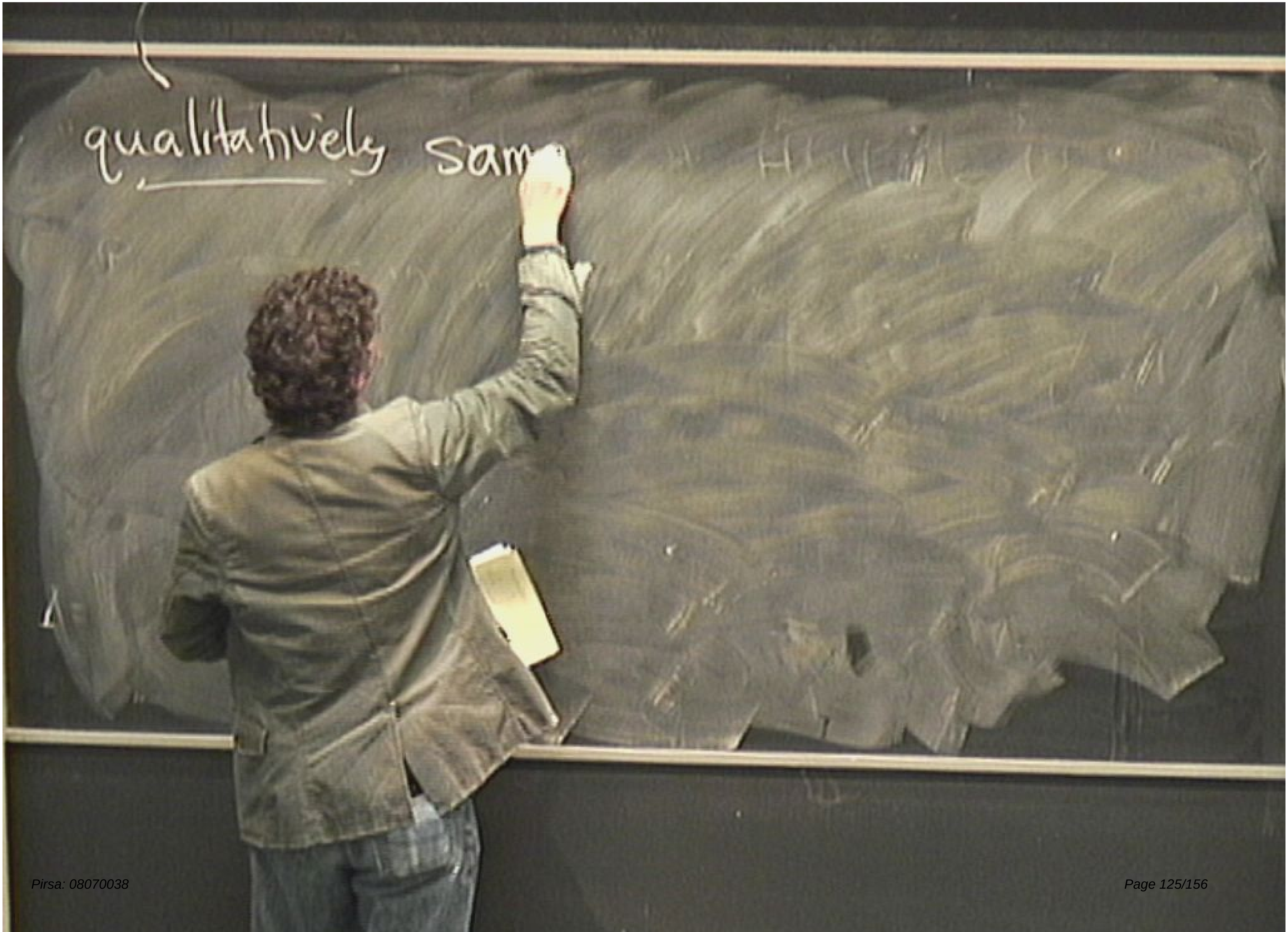


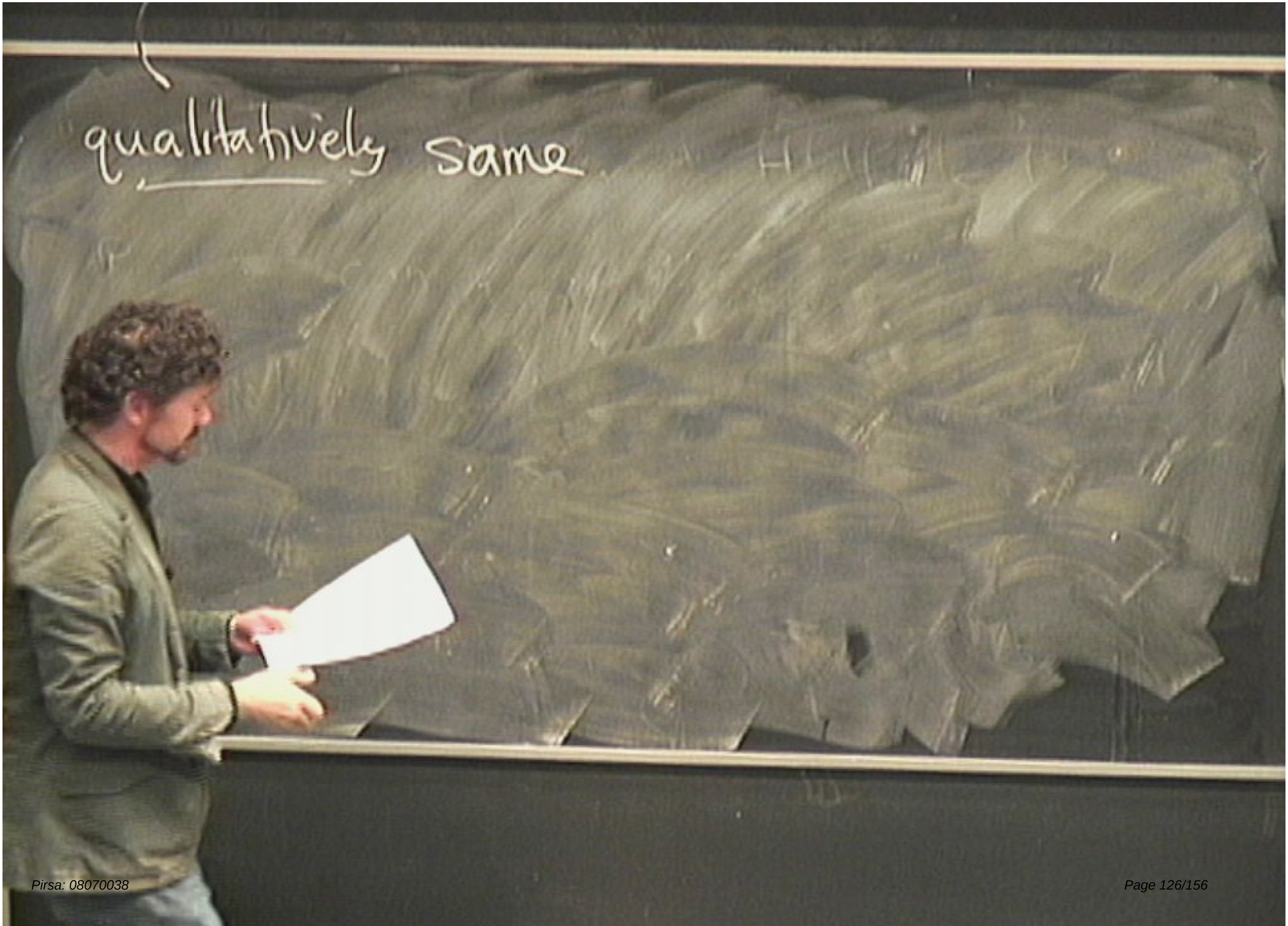
at "turning points" $p = 0$, $x = \pm A$

$$E = \text{const} = \frac{1}{2} k A^2$$

* for given E, particle <u>bound</u> to "$-A \le x \le A$"

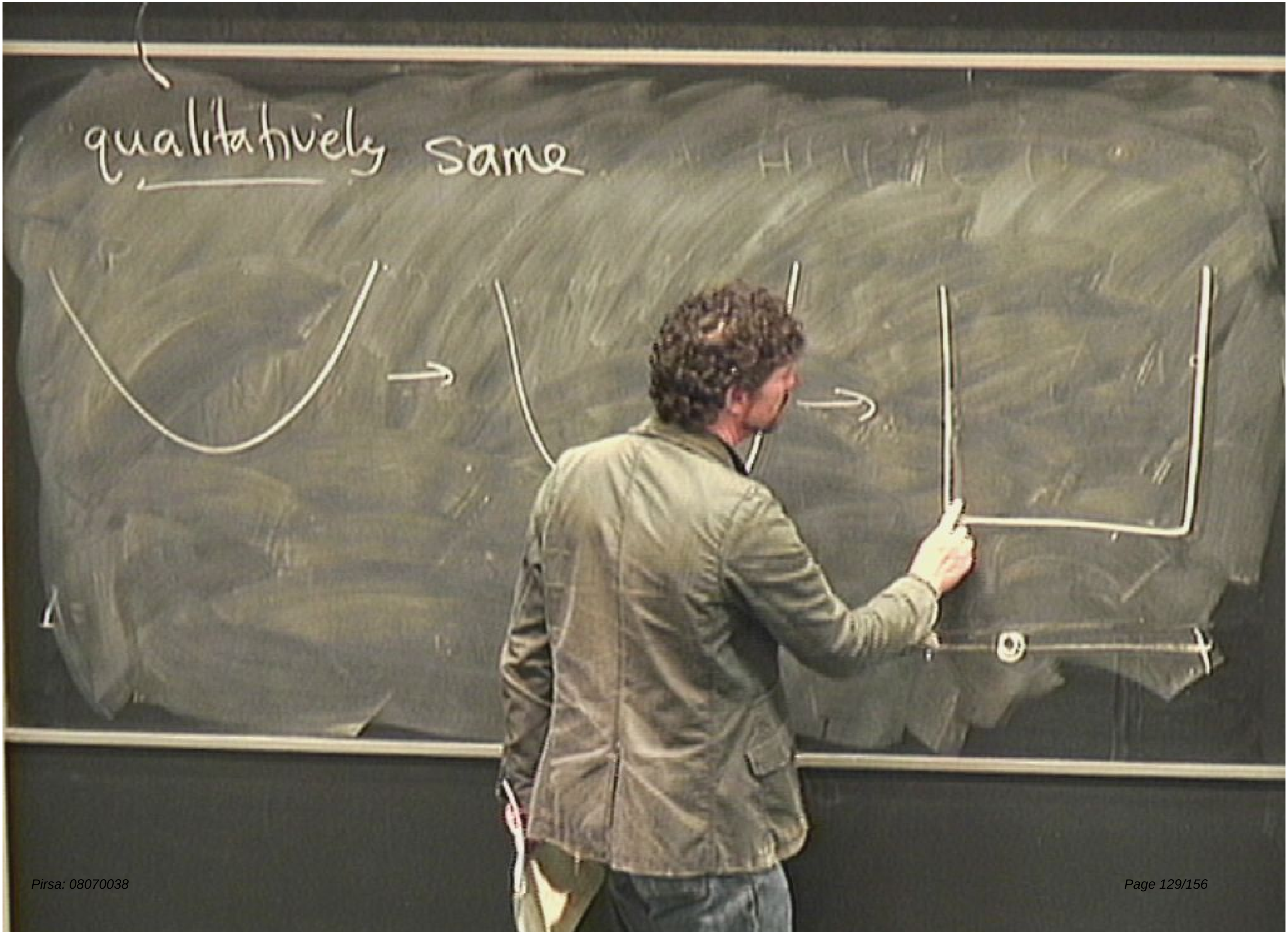


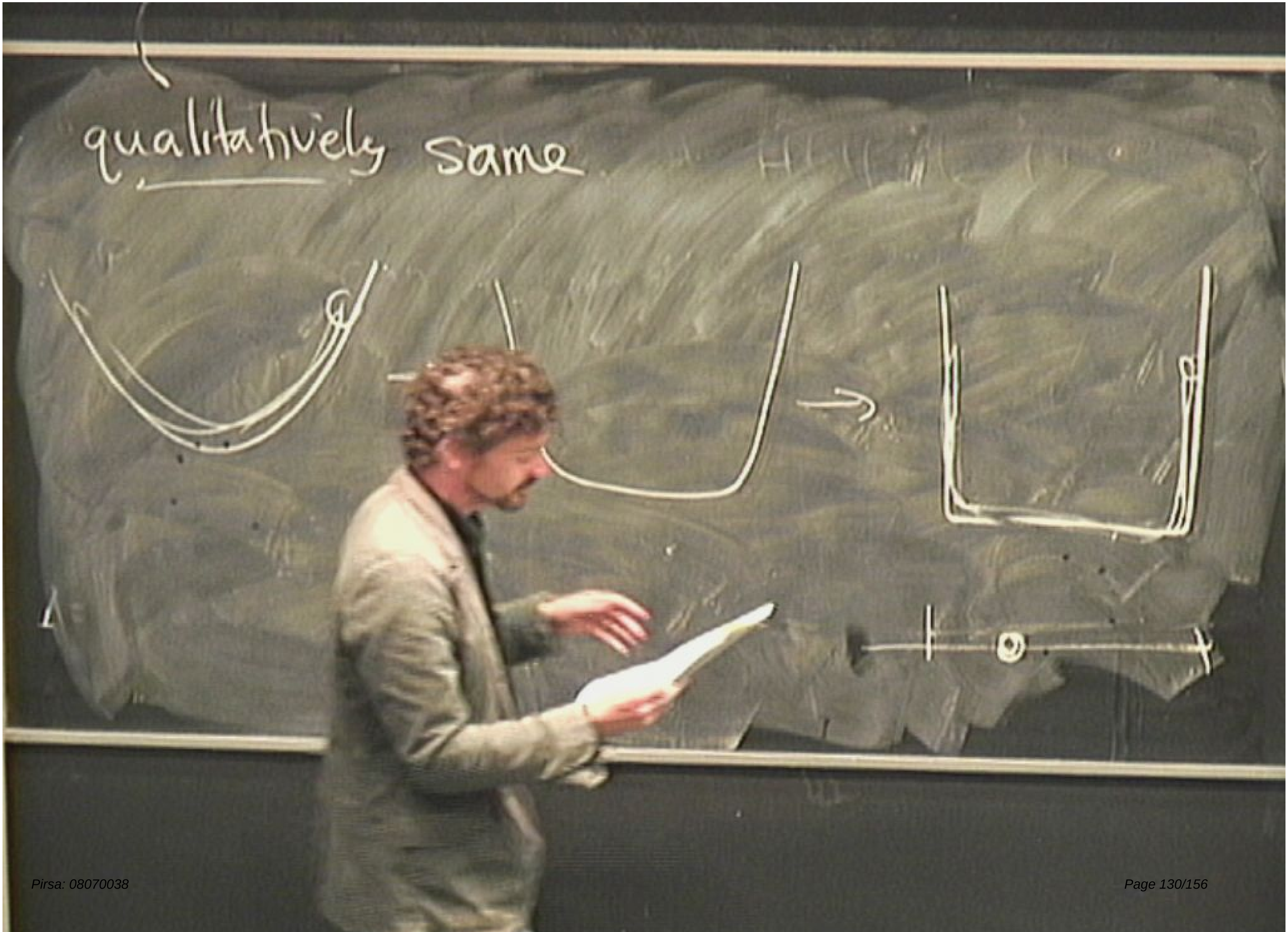

qualitatively sam

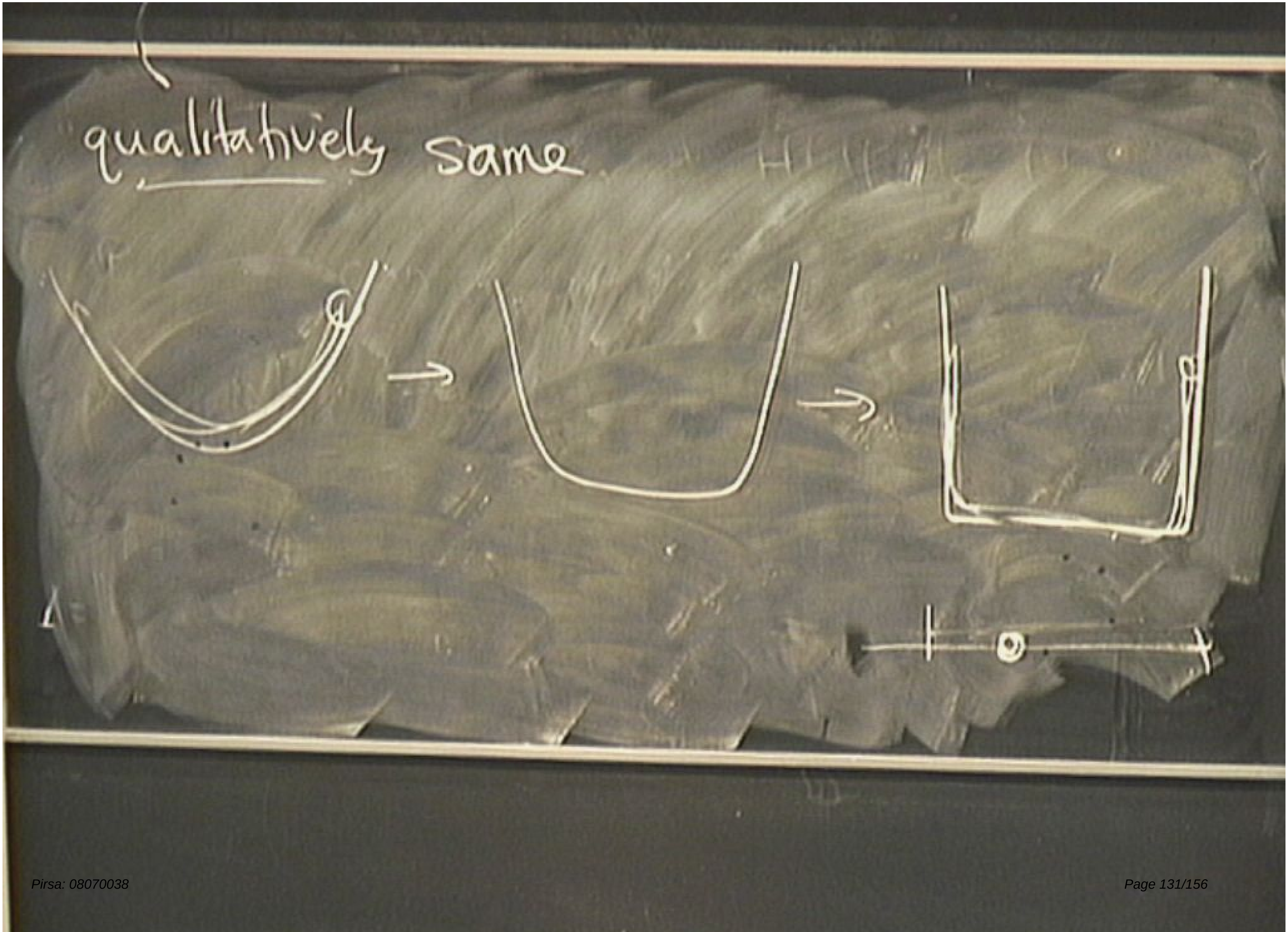

qualitatively same

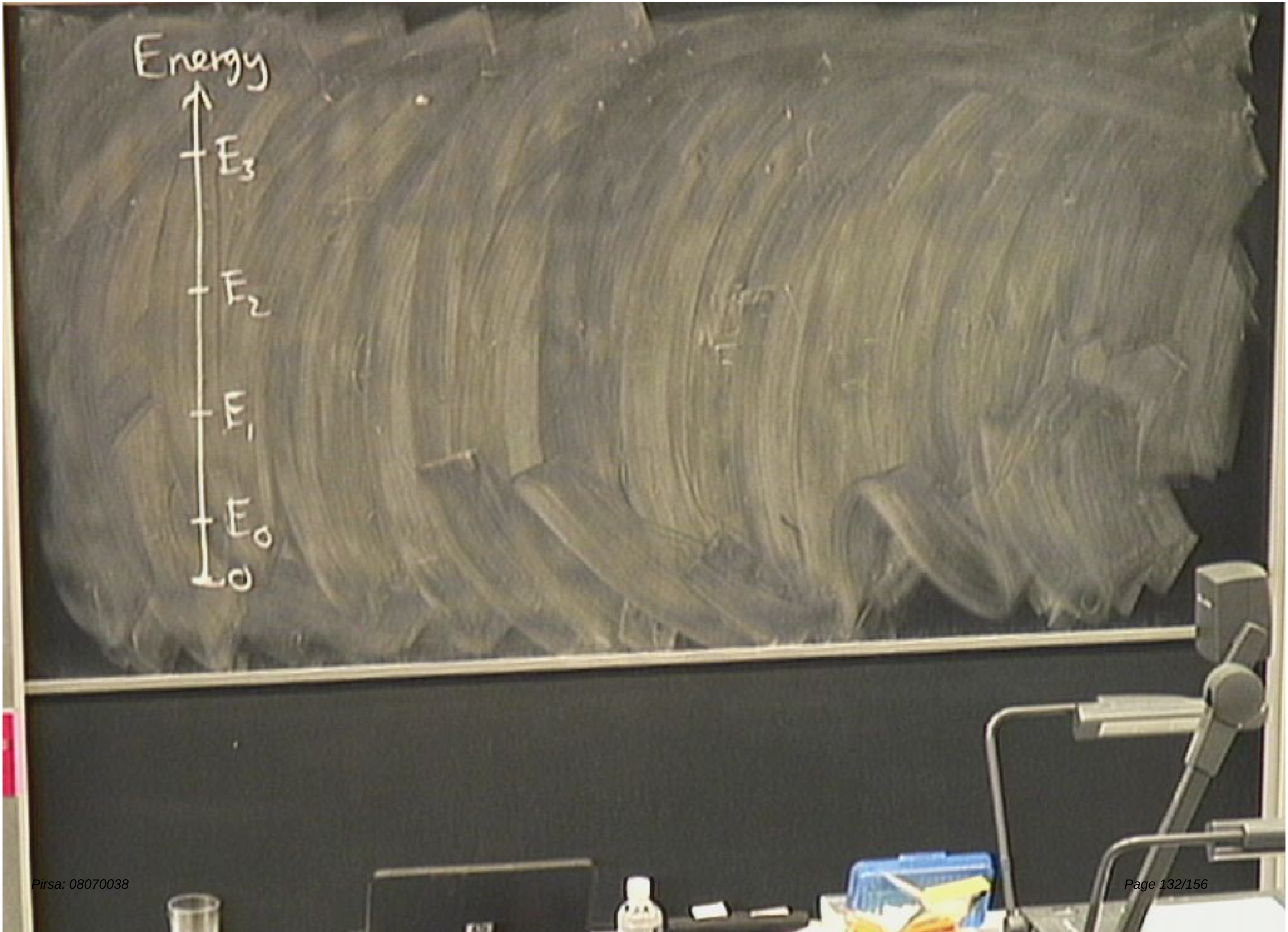
qualitatively same

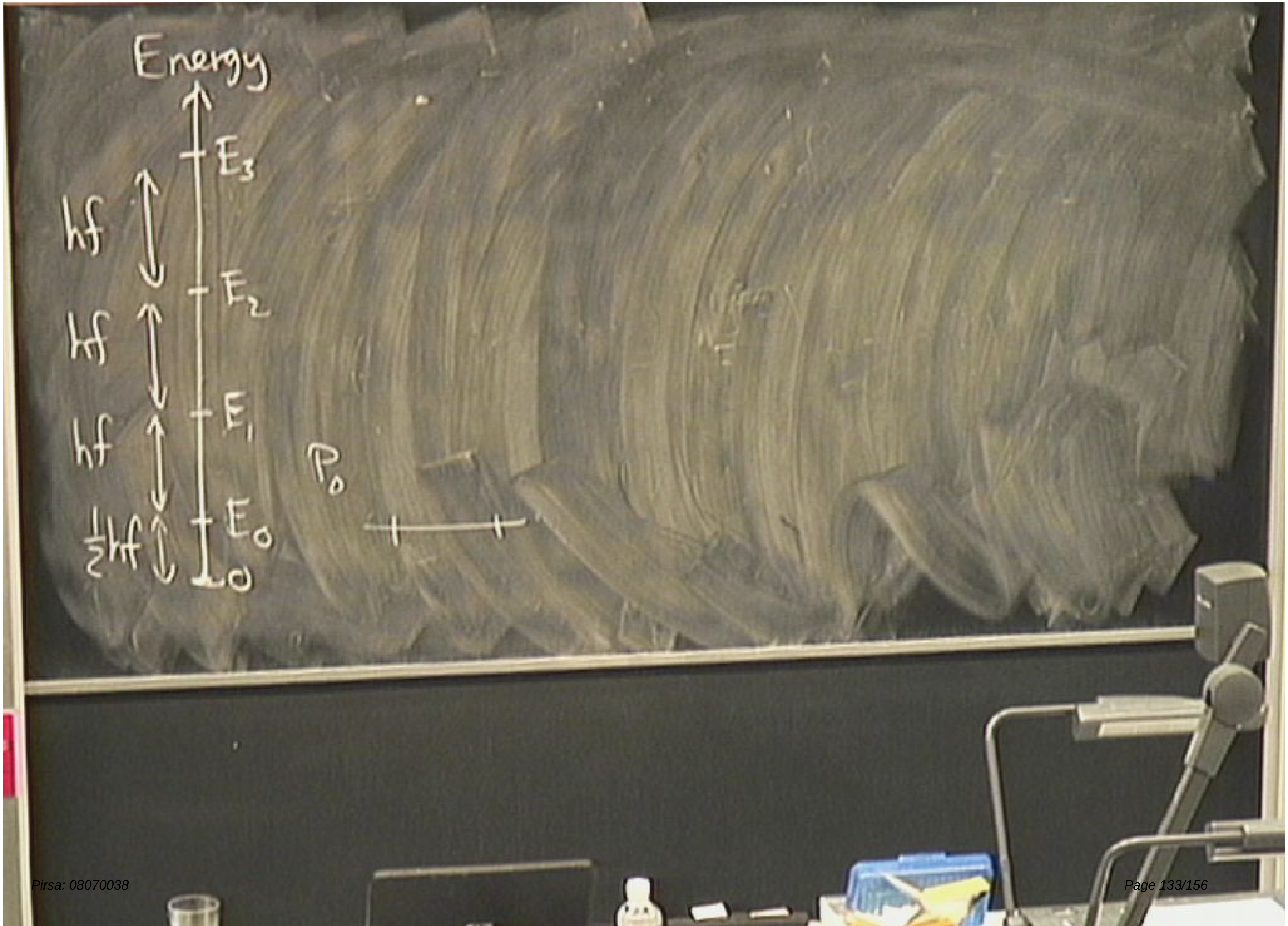
qualitatively same

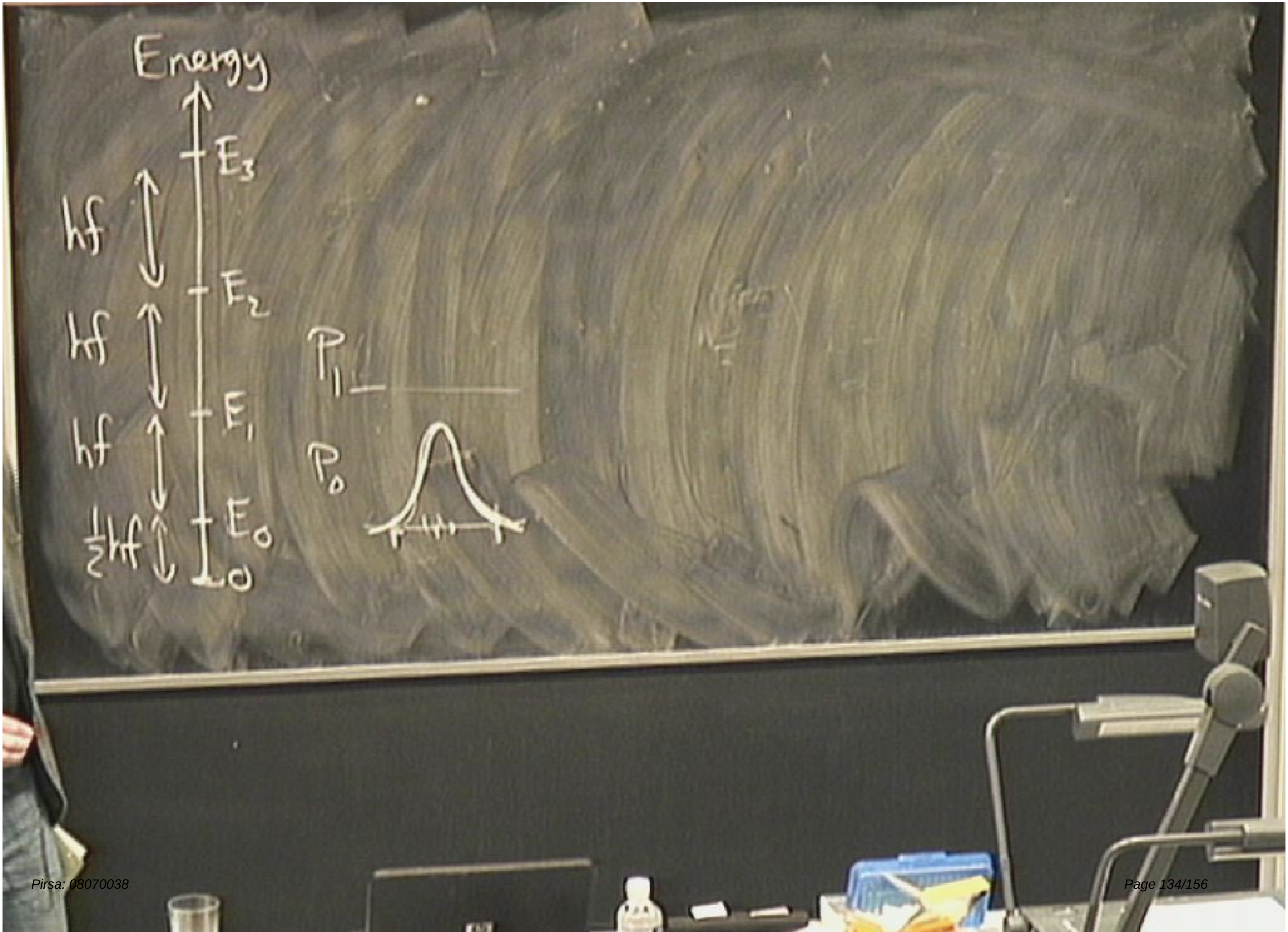

qualitatively same


qualitatively same

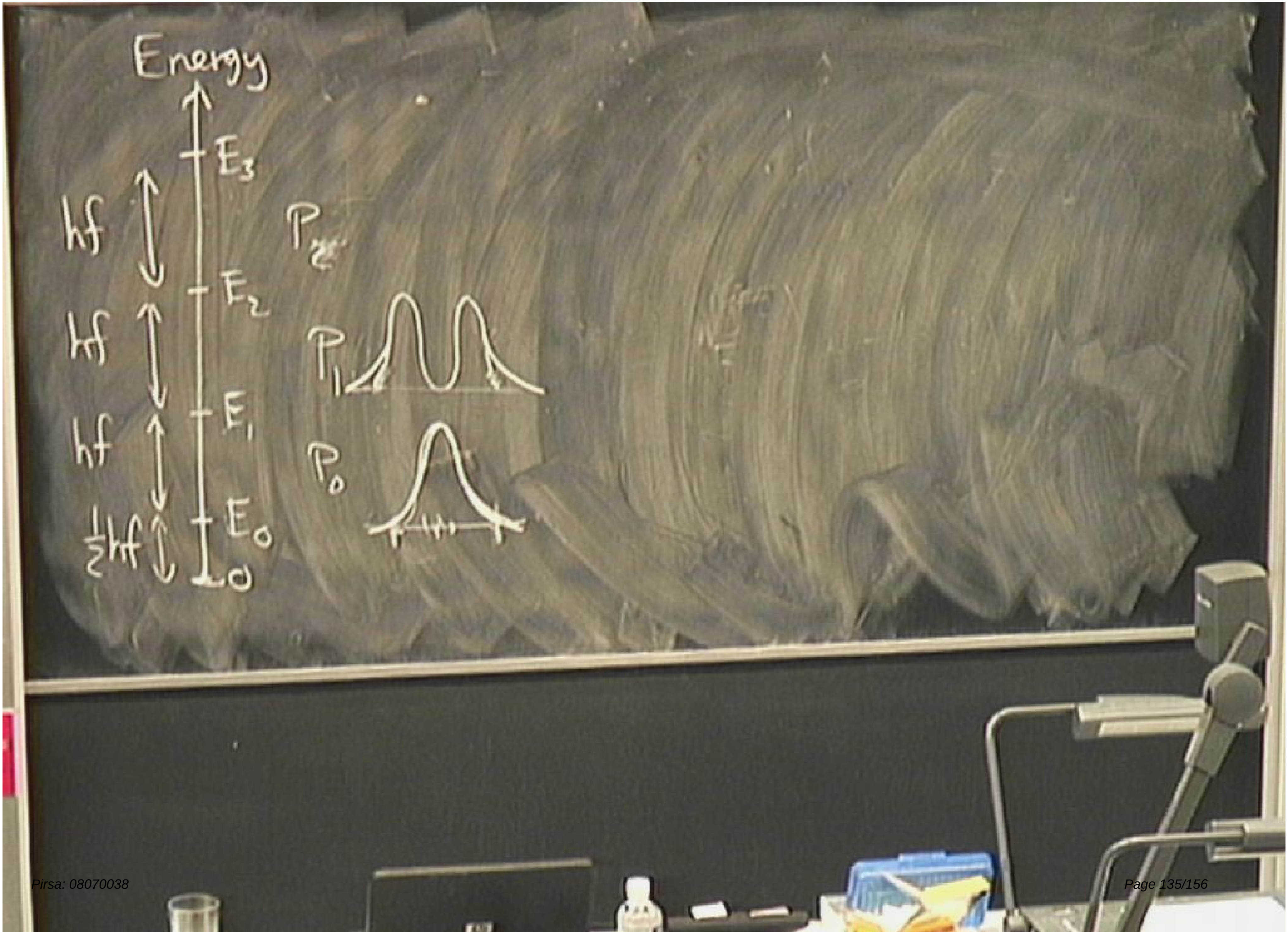

qualitatively same

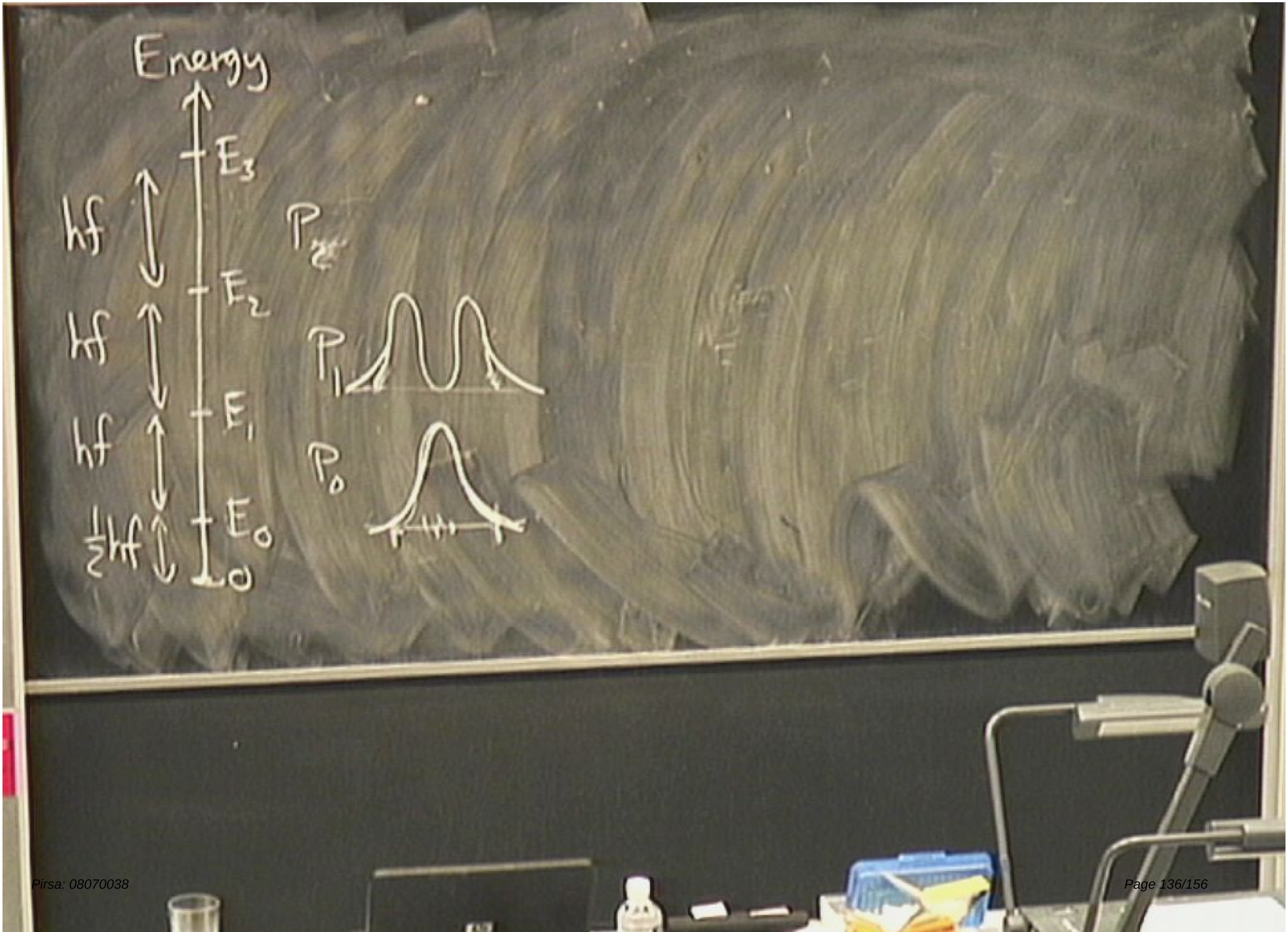

Energy
E_3
E_2
E_1
E_0
0

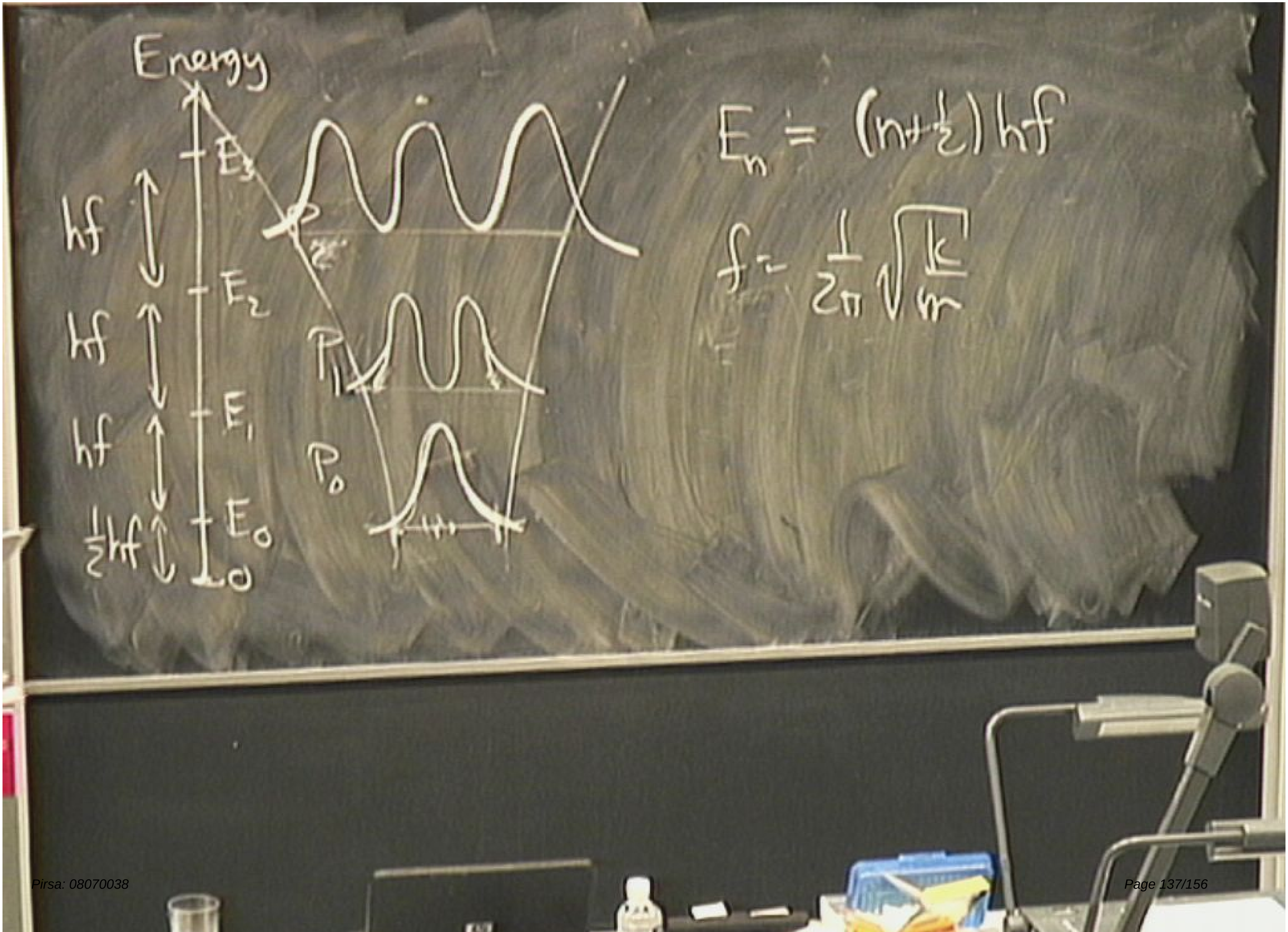

Energy
E3
E2
E1
E0
hf
hf
hf
½hf
0
P0

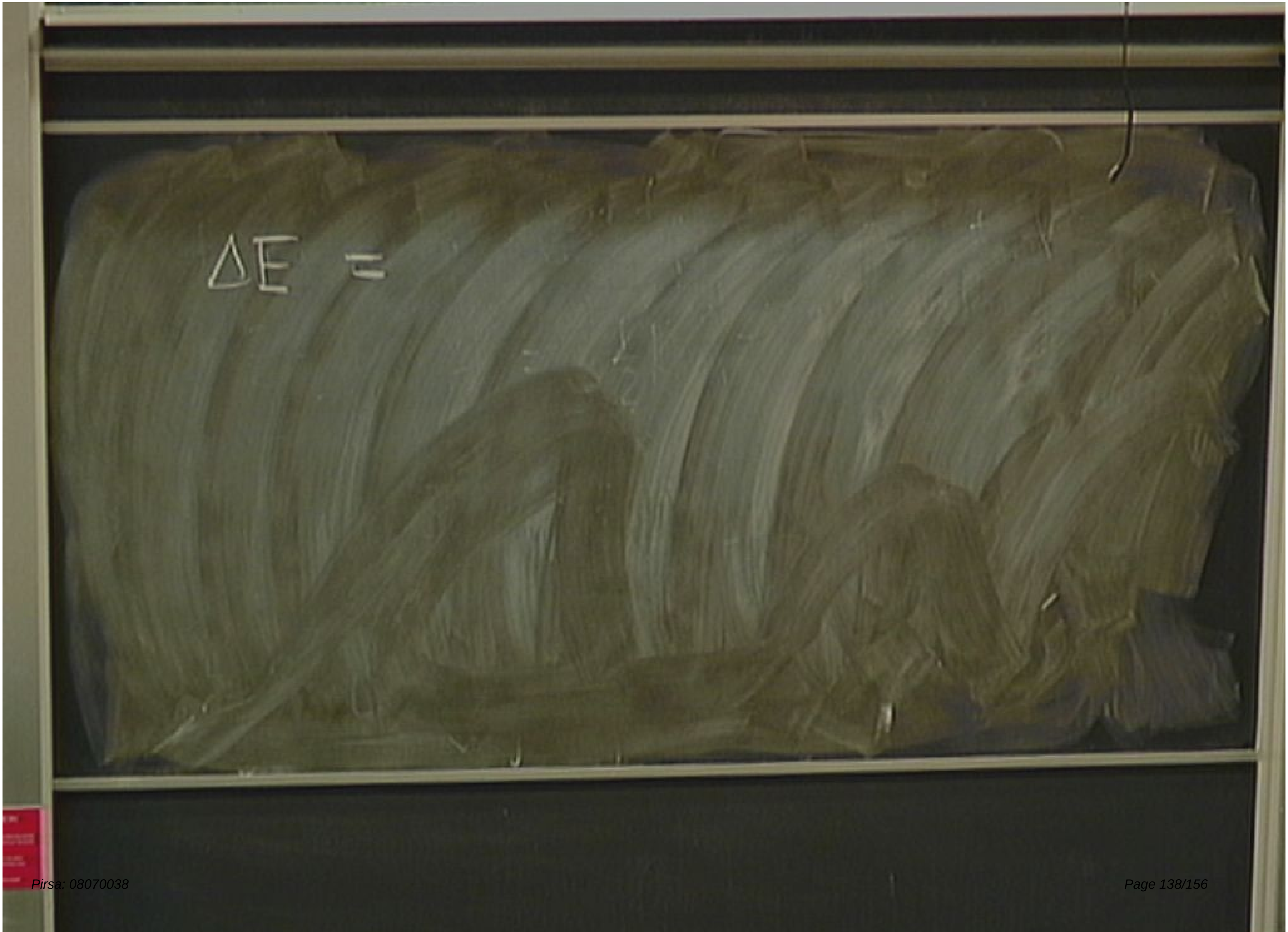


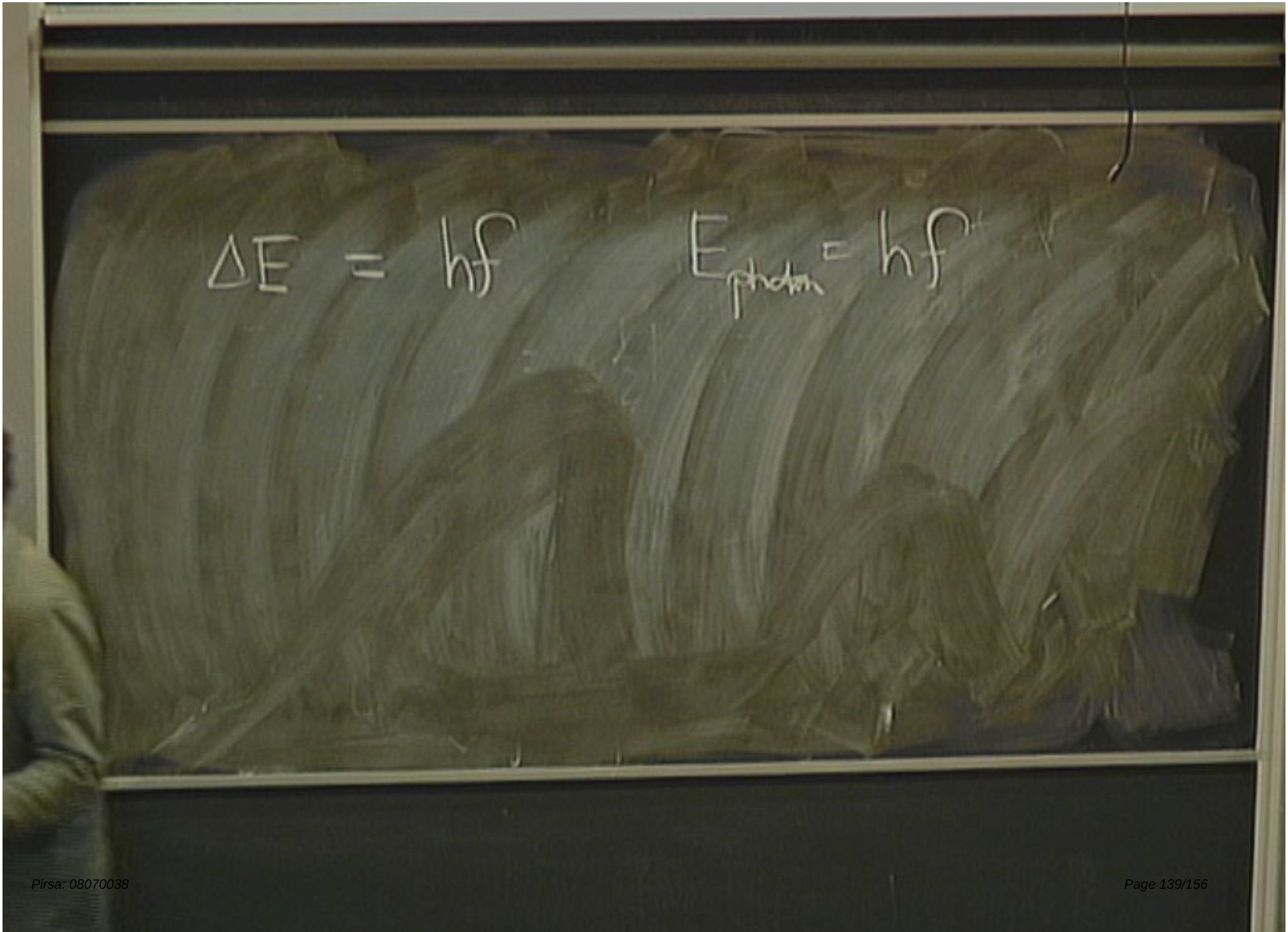
Energy
E_3
E_2
E_1
E_0
0
hf
hf
hf
$\frac{1}{2}$hf
P_1
P_0

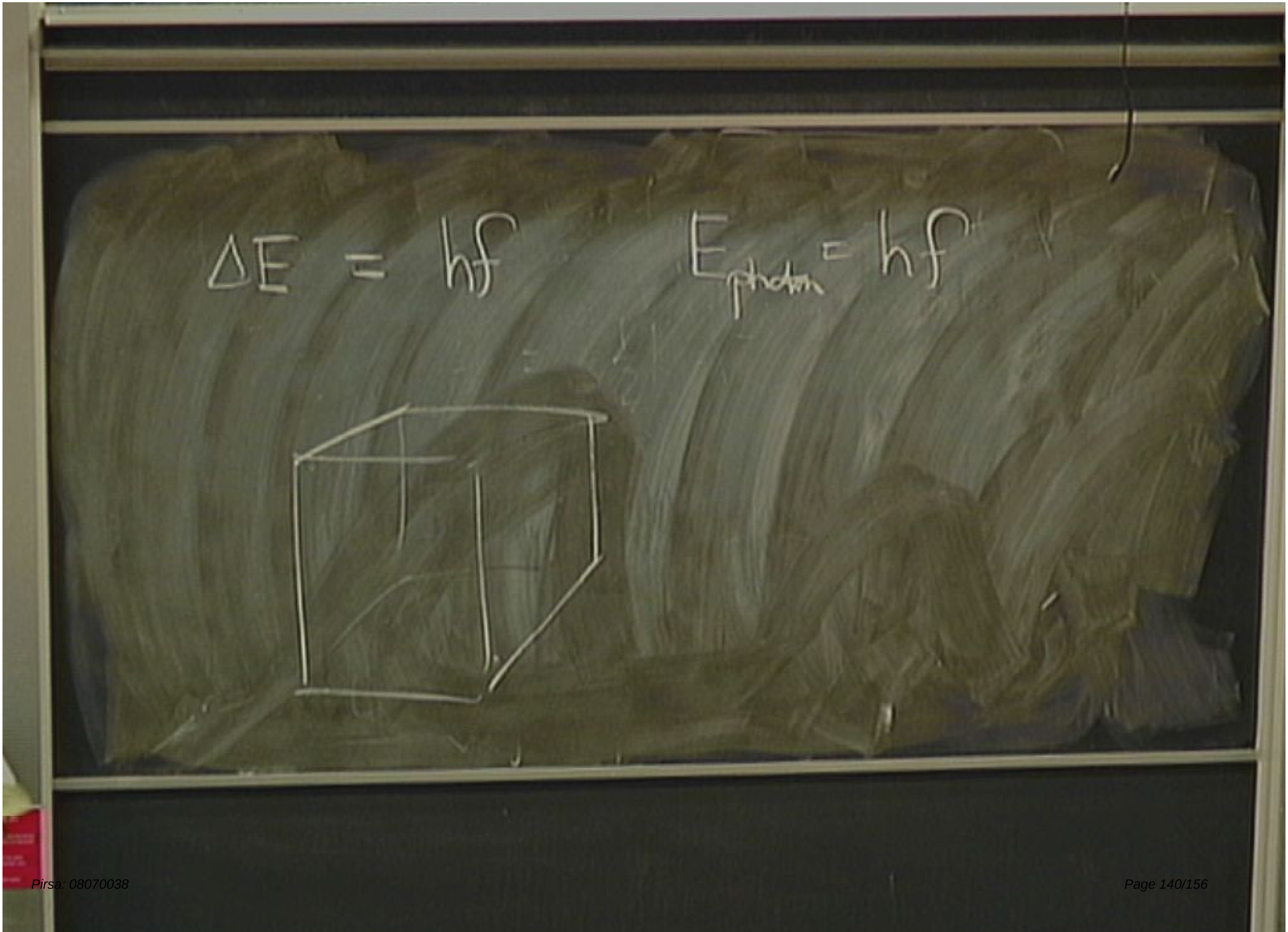

Energy
E_3
E_2
E_1
E_0
0
hf
hf
hf
$\frac{1}{2}$hf
P_2
P_1
P_0


Energy
E_3
E_2
E_1
E_0
0
hf
hf
hf
$\frac{1}{2}$hf
P_2
P_1
P_0

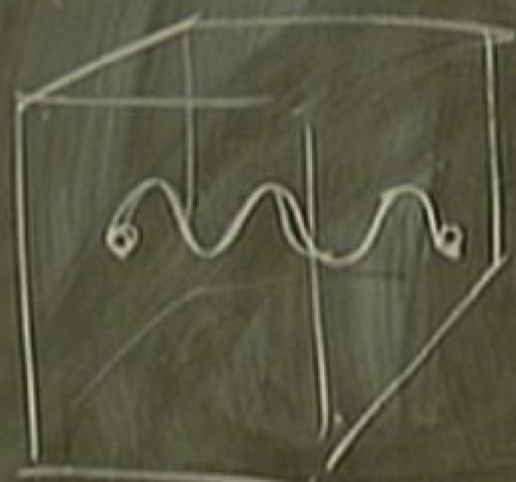

Energy
E_3
E_2
E_1
E_0
hf
hf
hf
$\frac{1}{2}hf$
0
P_1
P_0
$E_n = (n+\frac{1}{2})hf$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

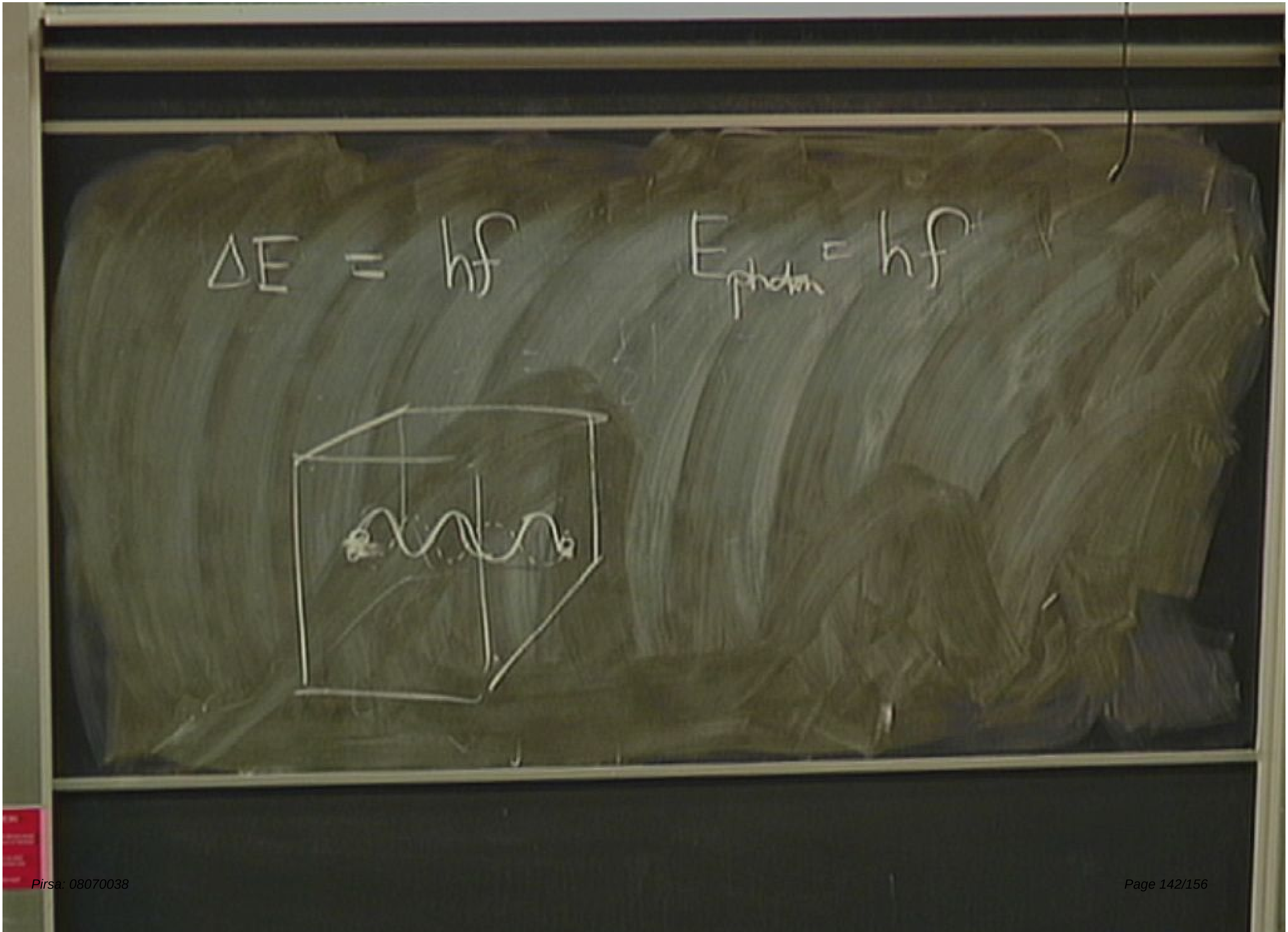

ΔE =

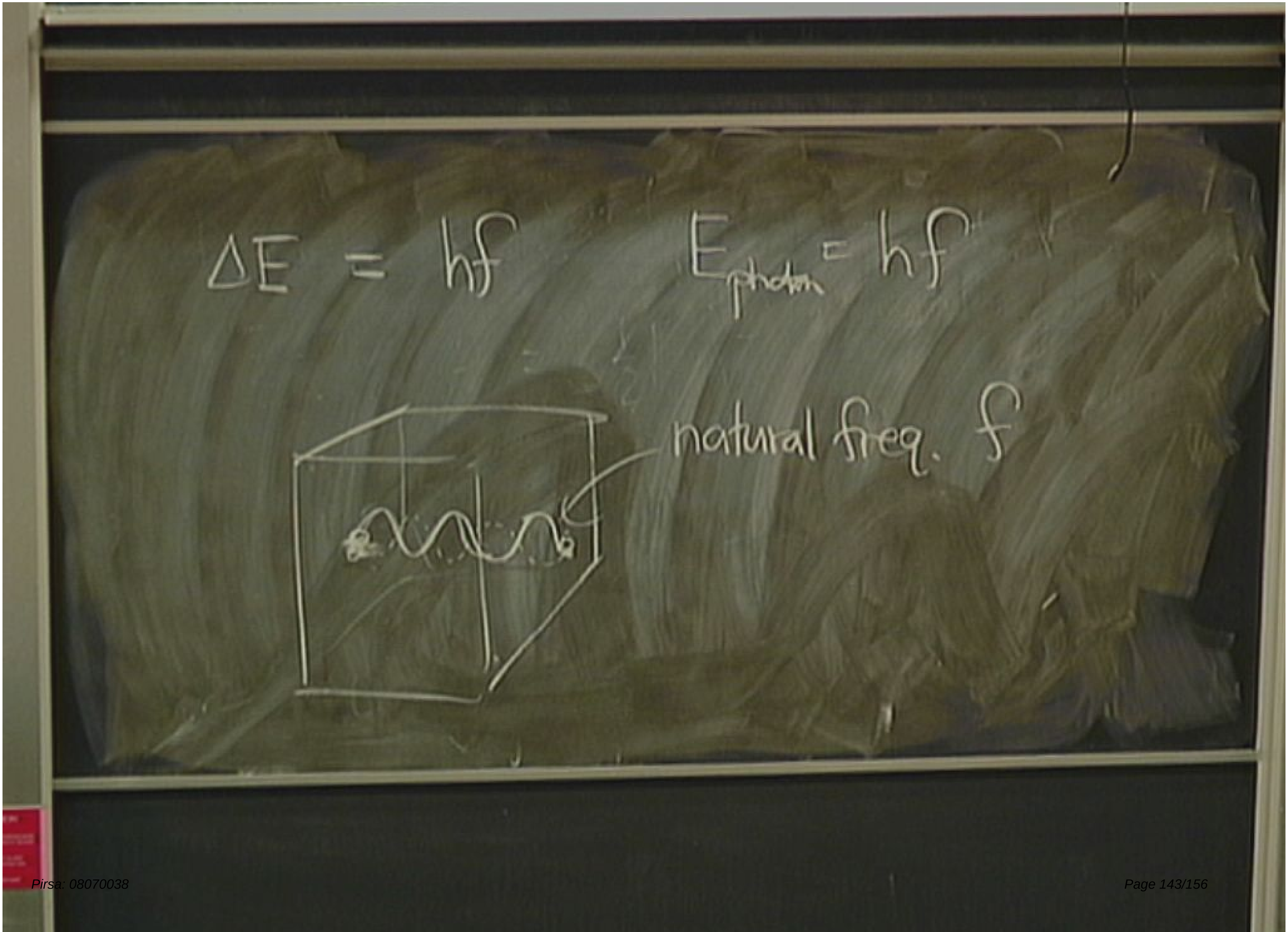

$\Delta E = hf$
$E_{photon} = hf$

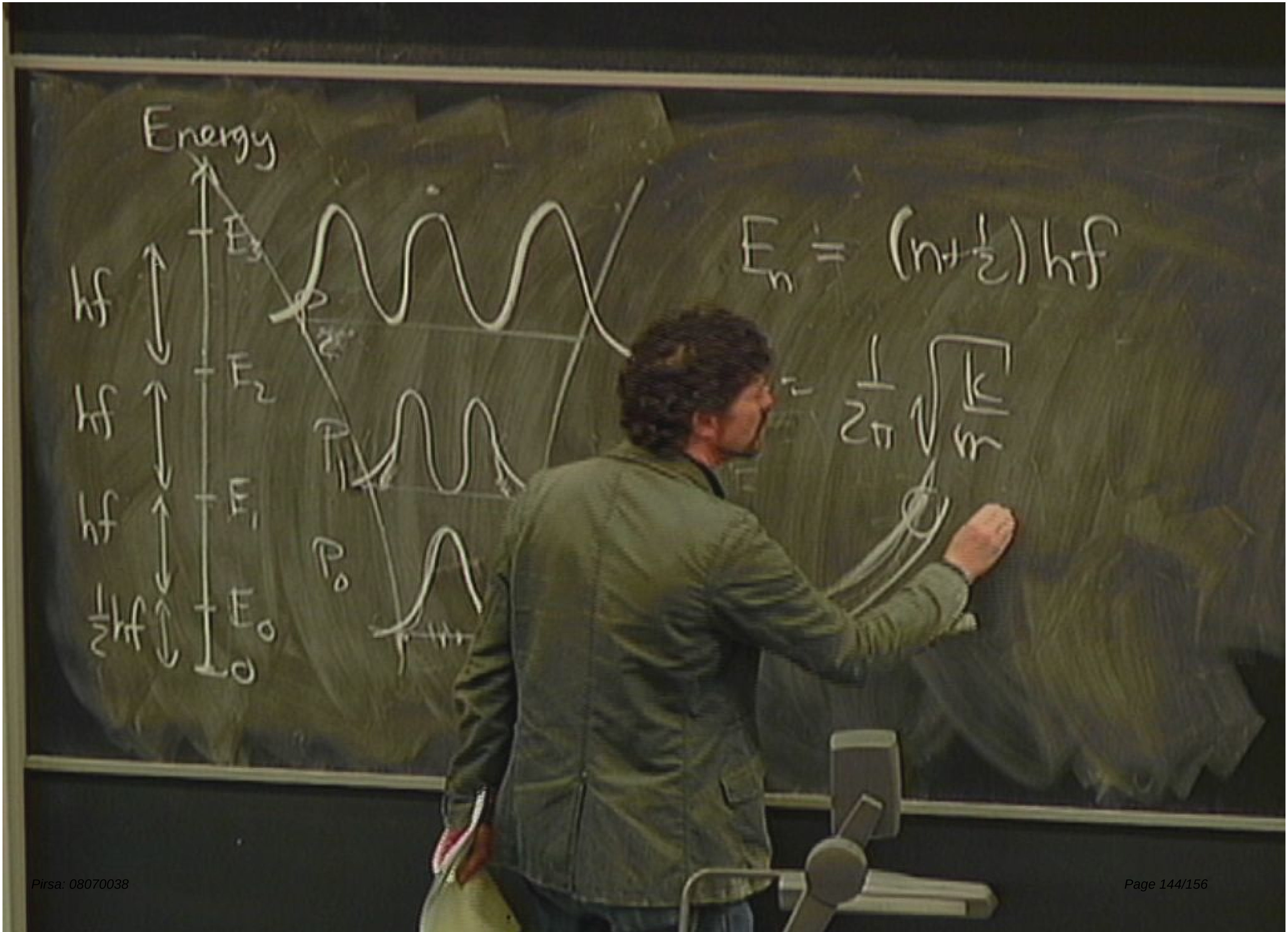

$\Delta E = hf$
$E_{photon} = hf$

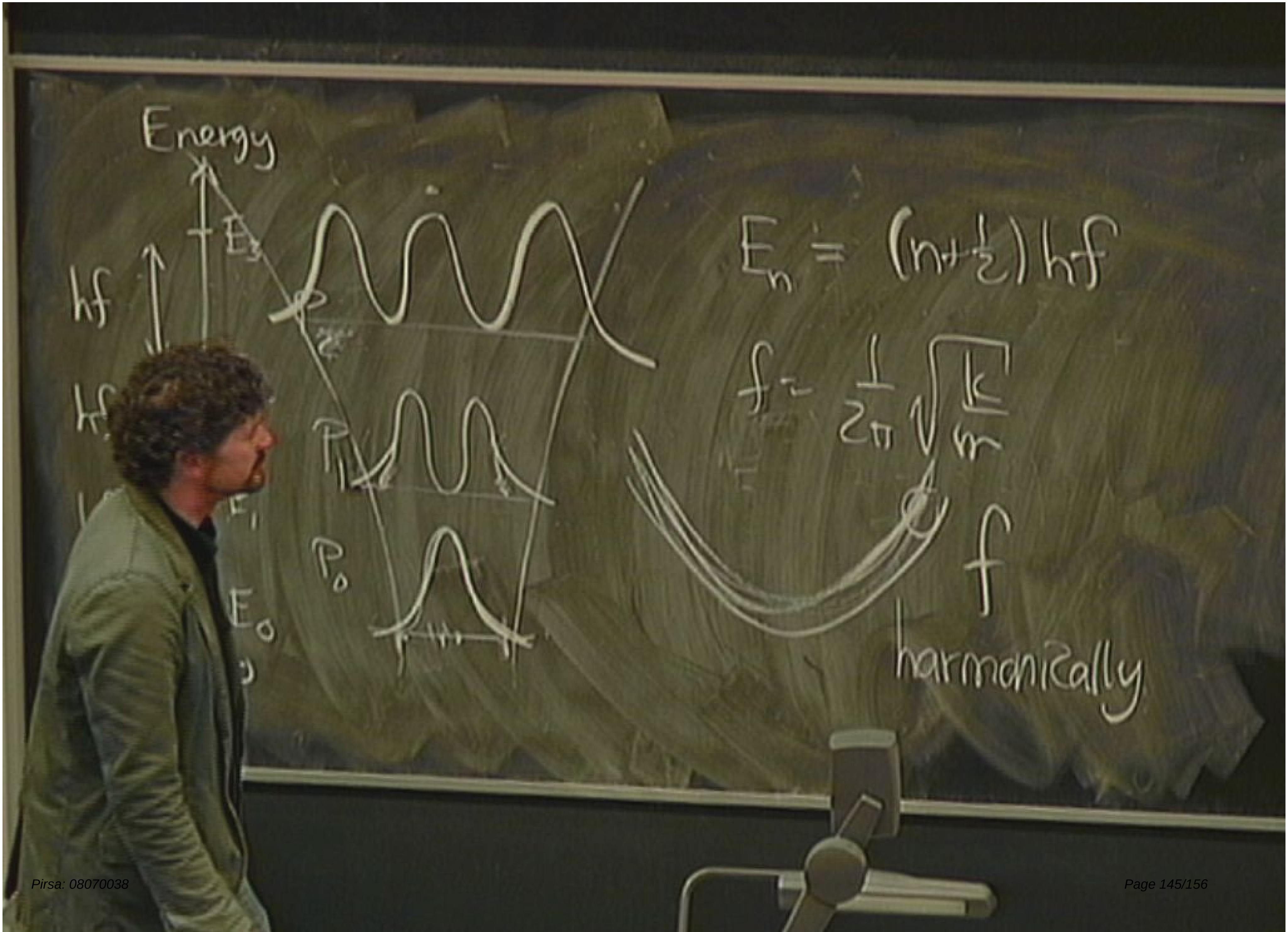
$\Delta E = hf$ $\qquad E_{photon} = hf$

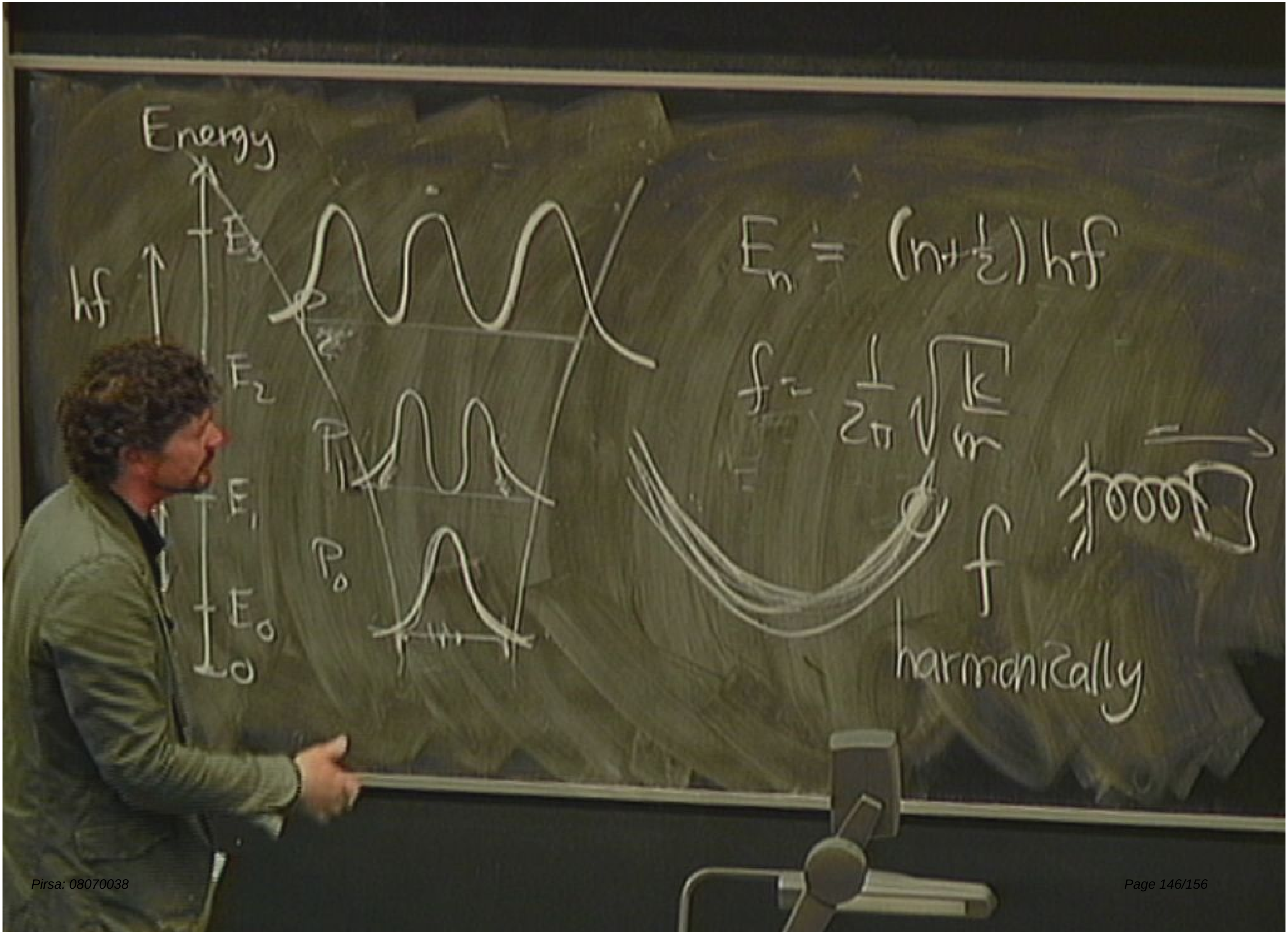


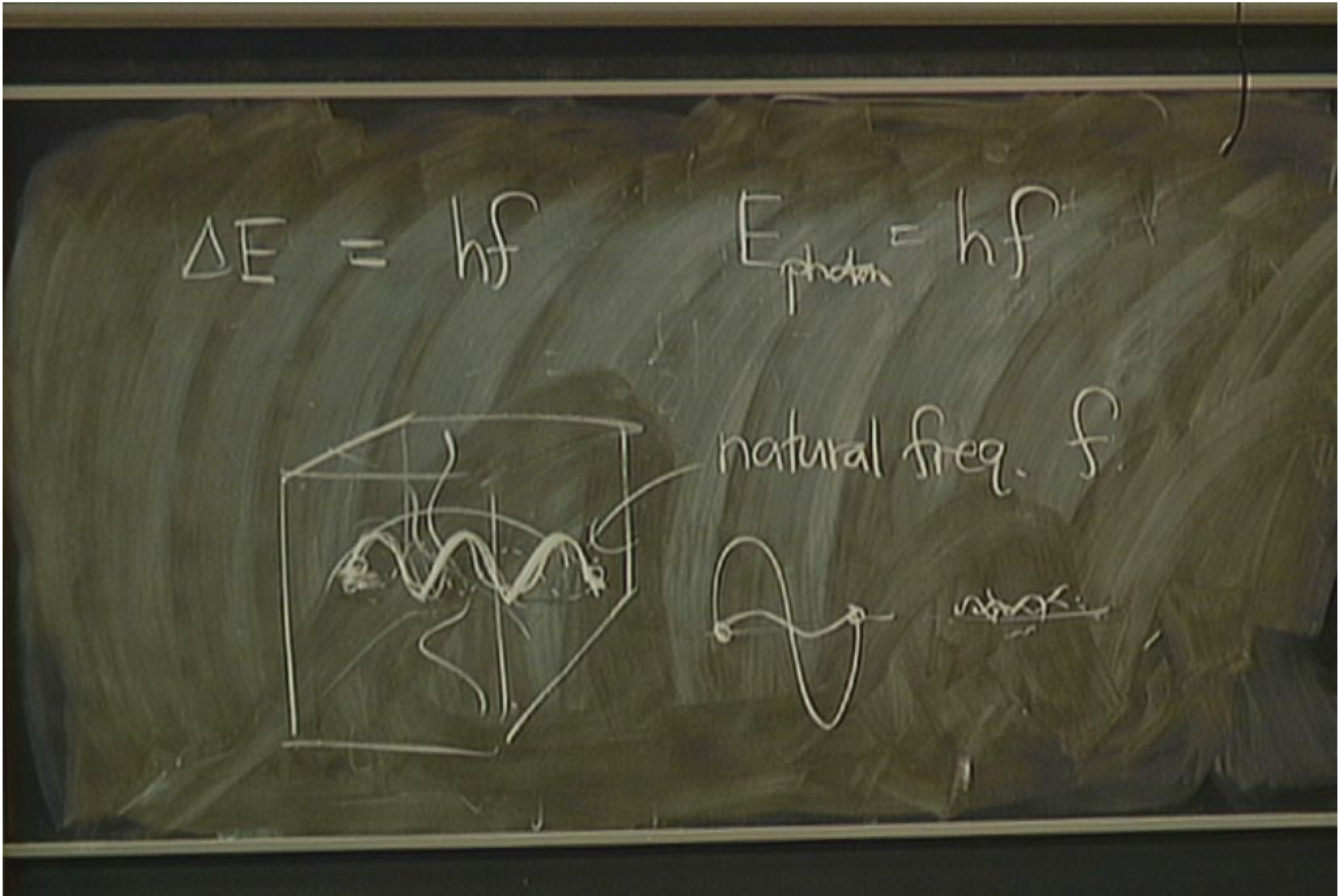

$\Delta E = hf$
$E_{photon} = hf$

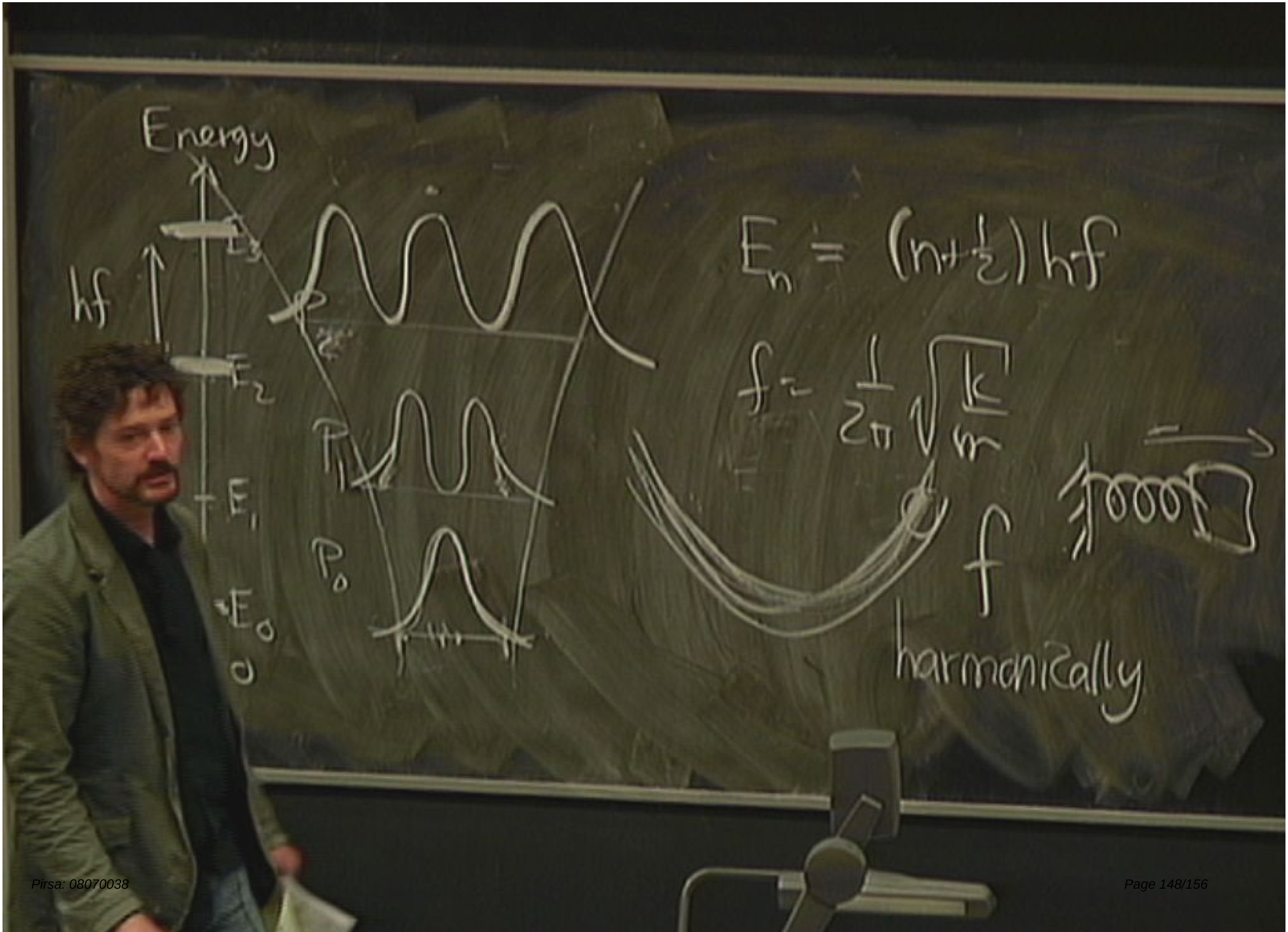

$\Delta E = hf$
$E_{photon} = hf$
natural freq. f

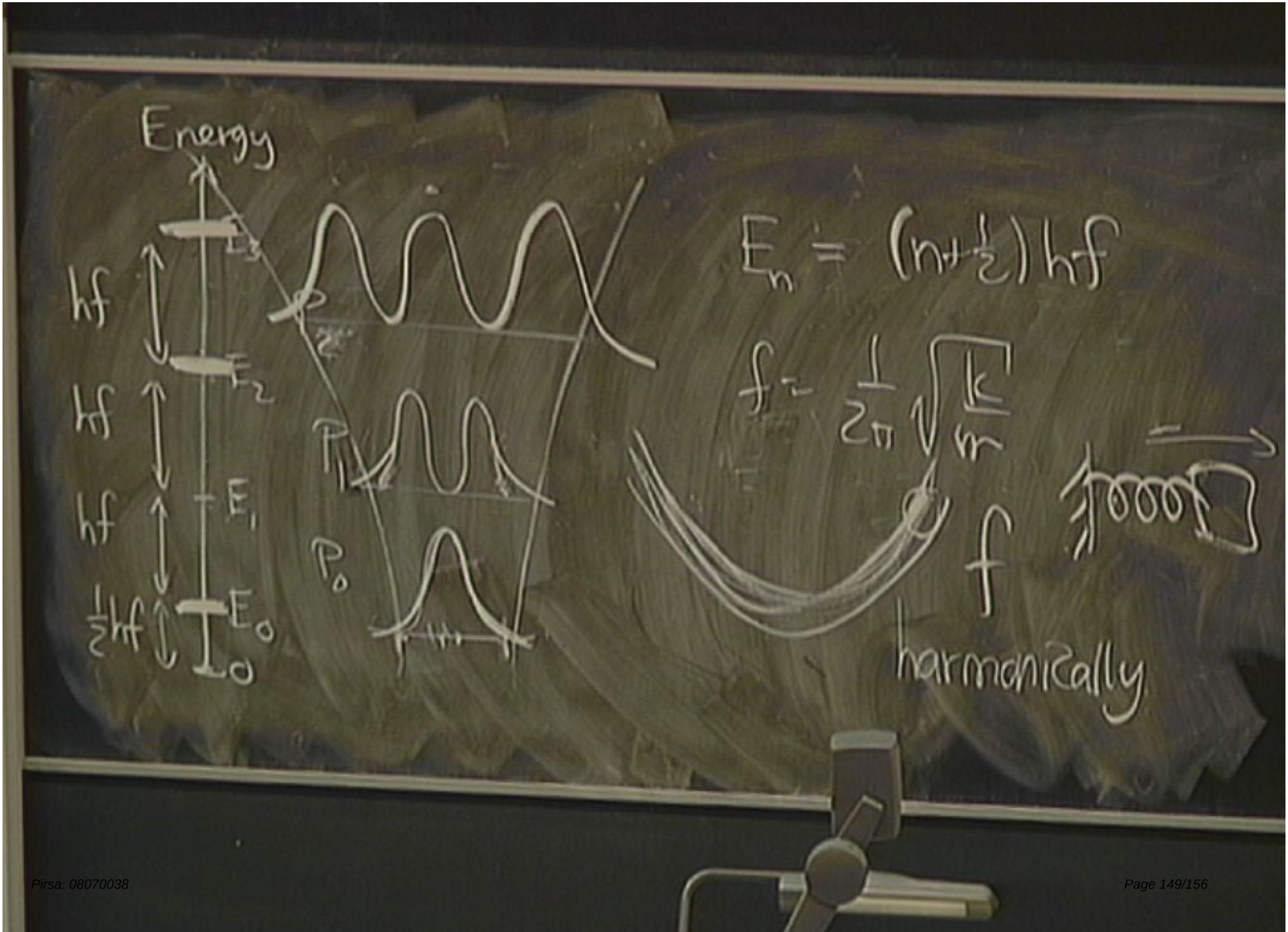

Energy
hf
hf
hf
½hf
E3
E2
E1
E0
0
P2
P1
P0
$E_n = (n+\frac{1}{2})hf$
$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

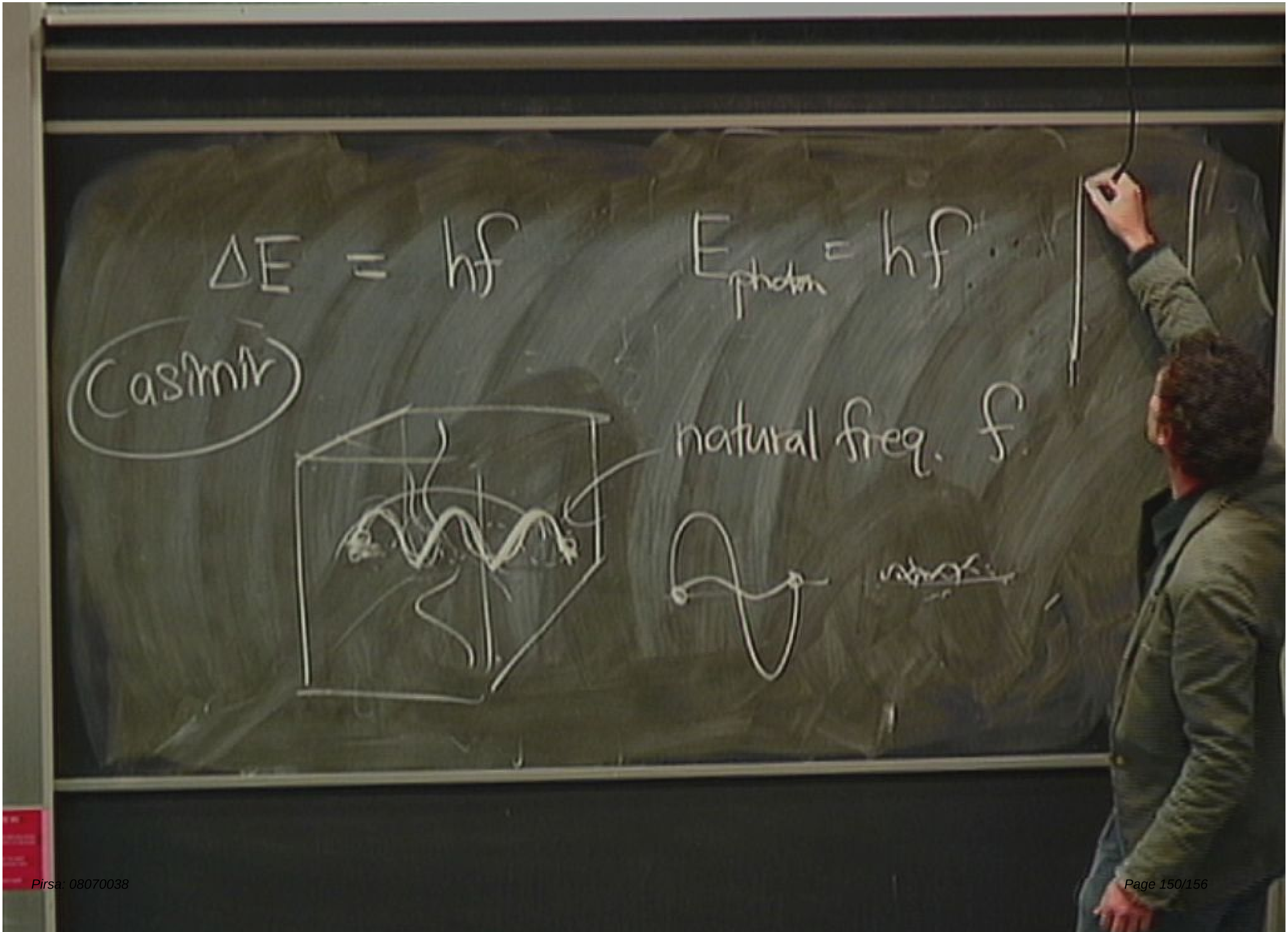

Energy
hf
hf
E_3
P_1
E_1
P_0
E_0
$E_n = (n+\frac{1}{2}) hf$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
f
harmonically

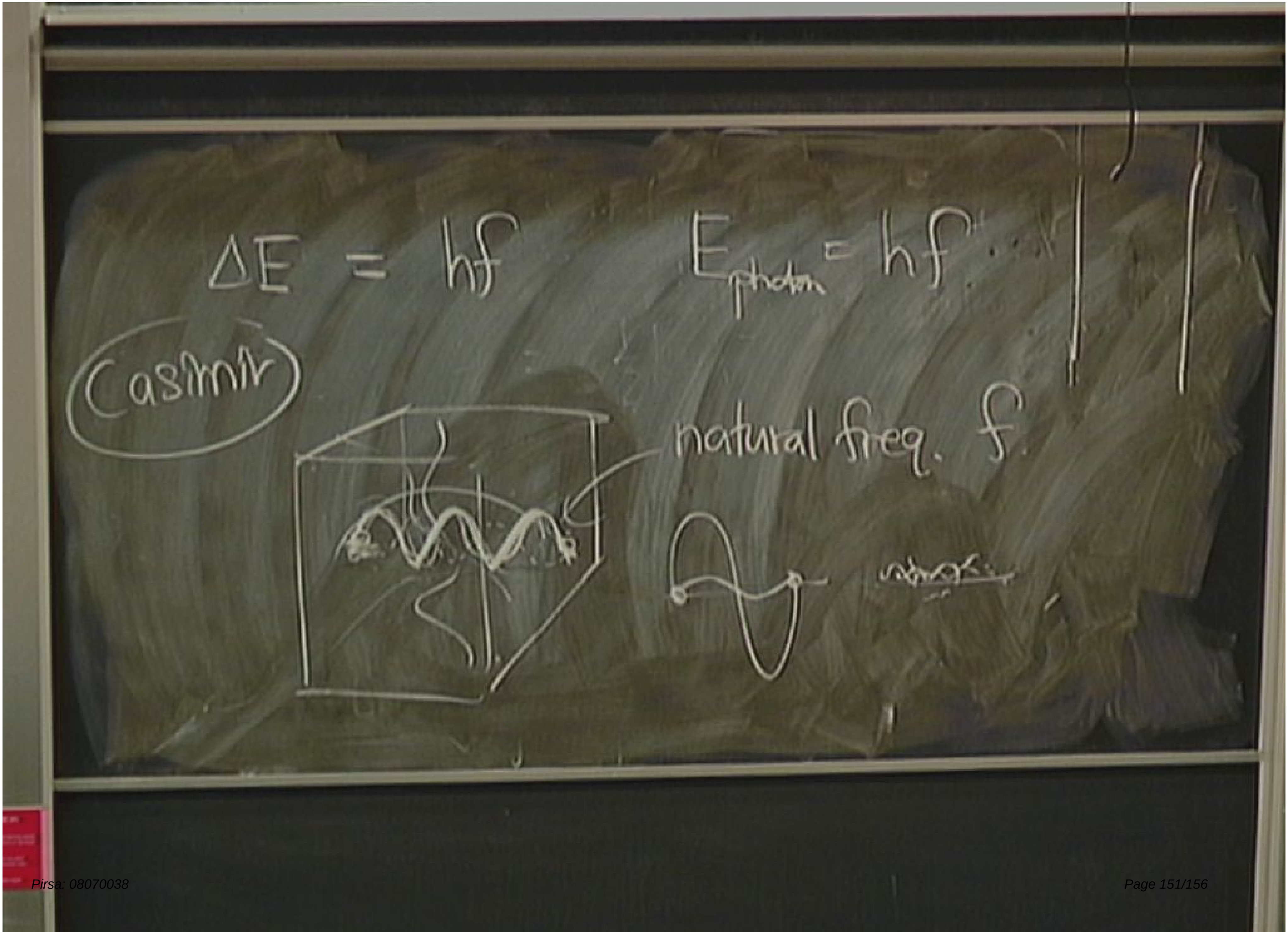

Energy
hf
E_3
E_2
E_1
E_0
0
P_1
P_0
$E_n = (n+\frac{1}{2})\,h\cdot f$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
f
harmonically

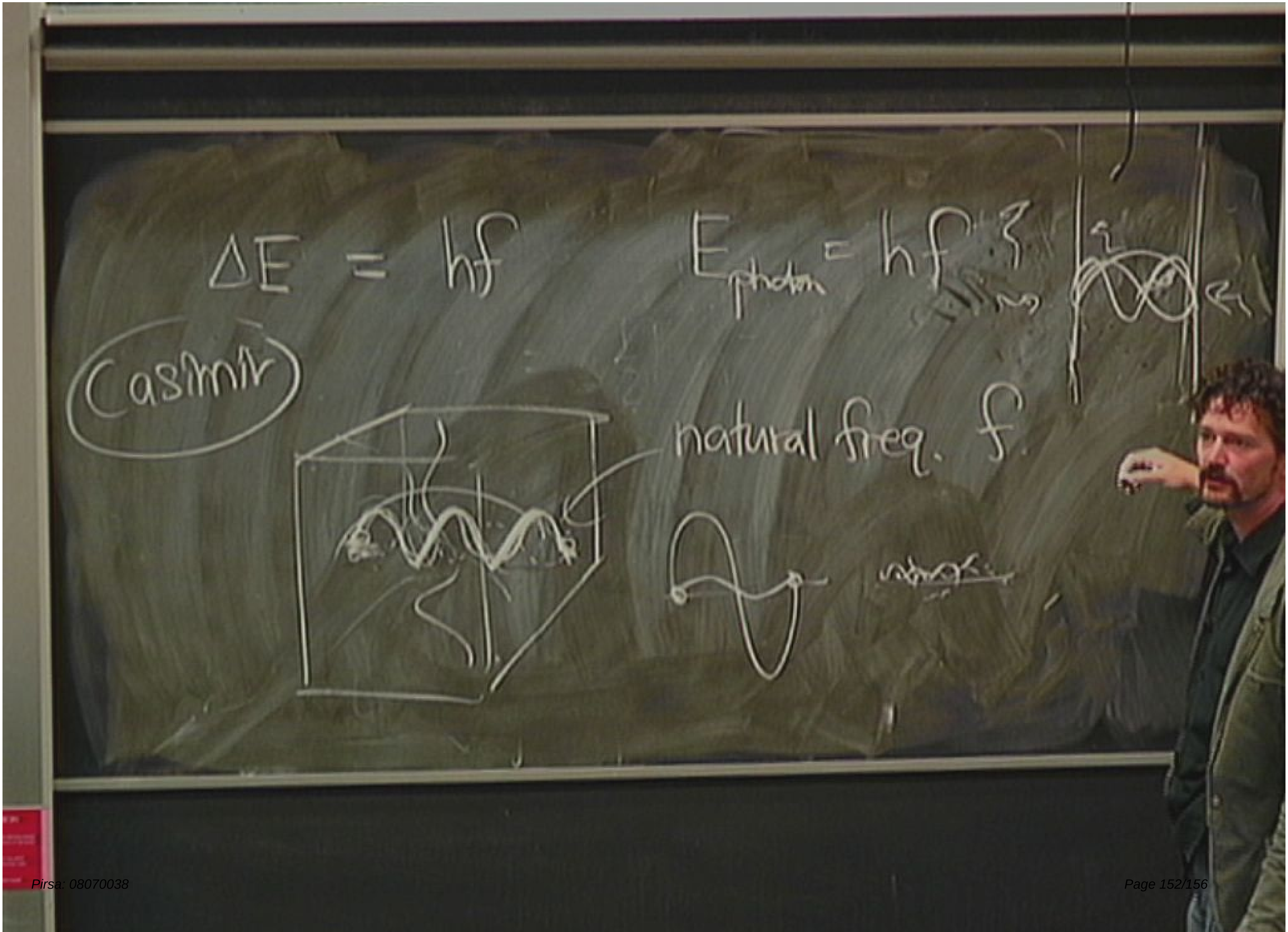

ΔE = hf
E photon = hf
natural freq. f

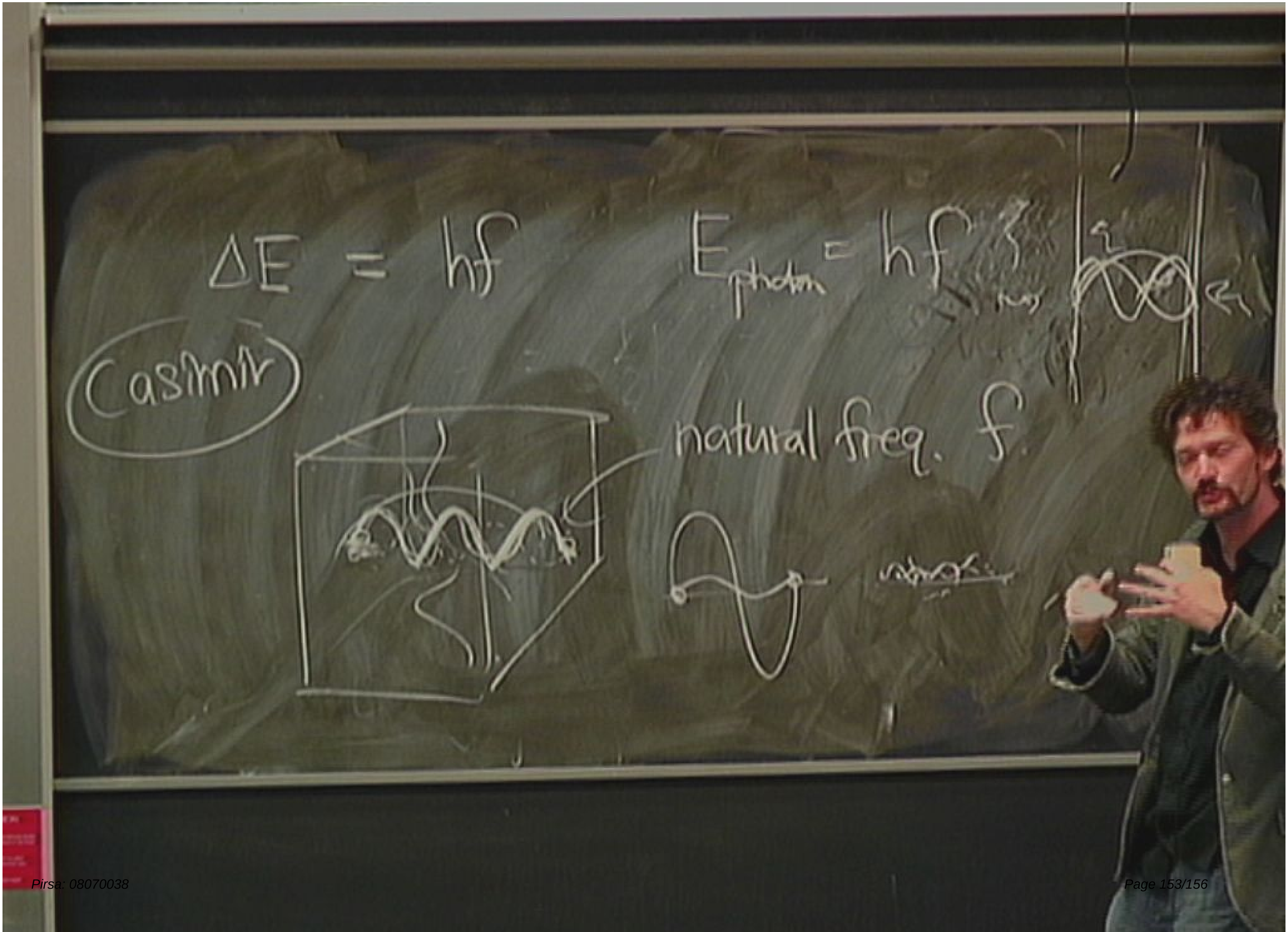

Energy
hf
E_3
E_2
P_1
E_1
P_0
E_0
0
$E_n = (n+\frac{1}{2})hf$
$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
f
harmonically

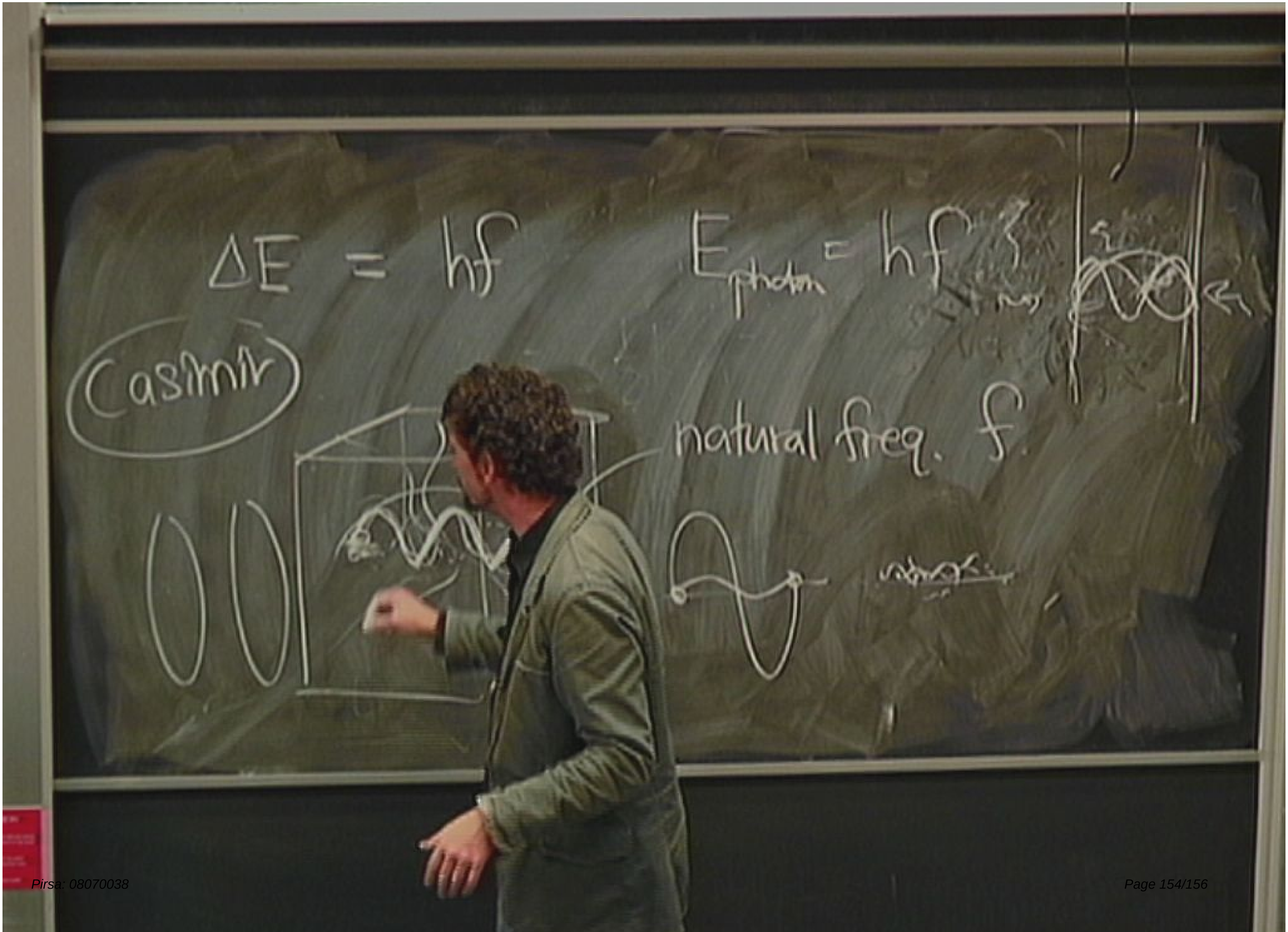

Energy
hf
hf
hf
½hf
E3
E2
E1
E0
0
P1
P0
En = (n+½)hf
f = 1/2π √(k/m)
f
harmonically

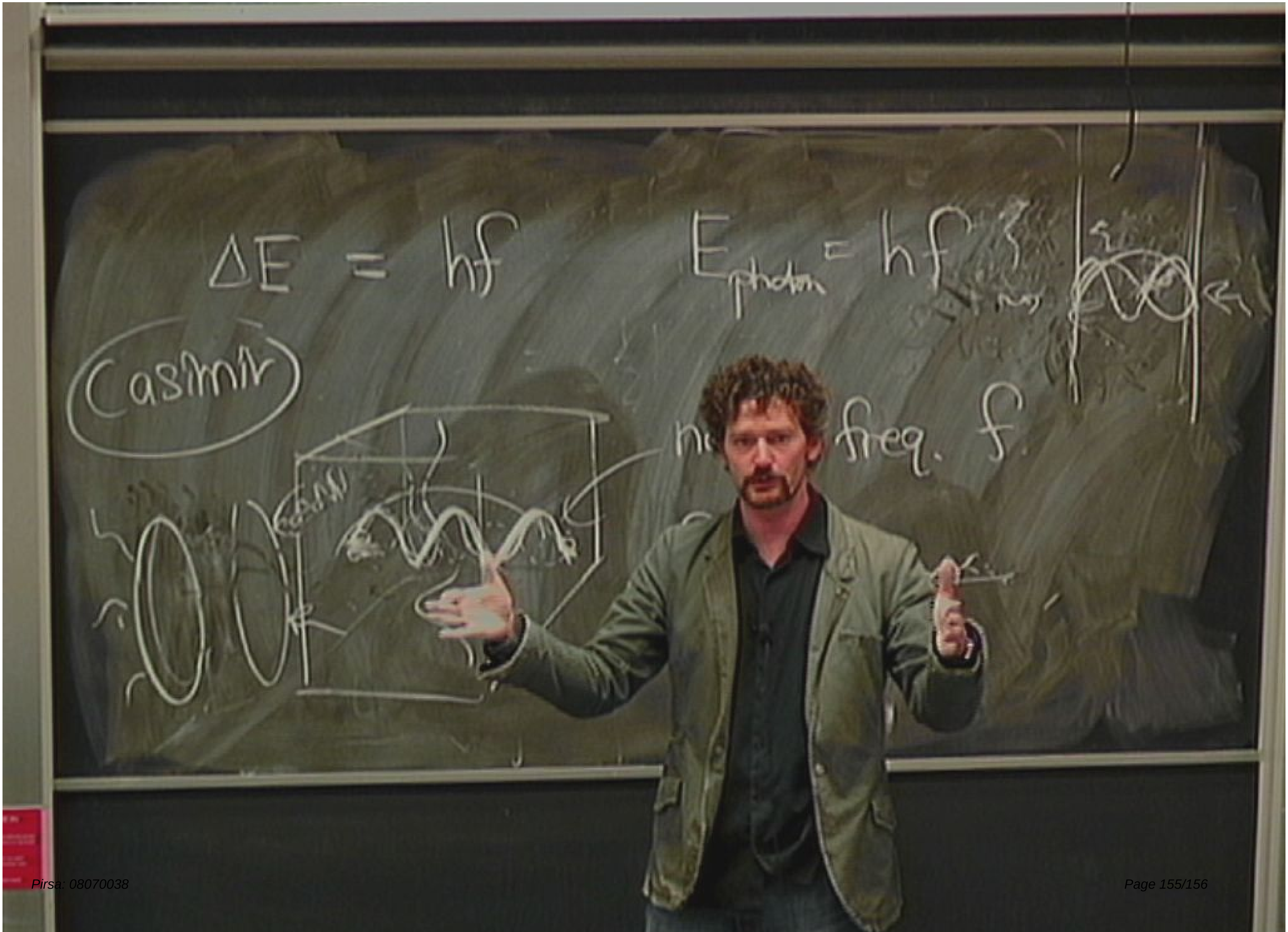

ΔE = hf
E photon = hf
Casimir
natural freq. f

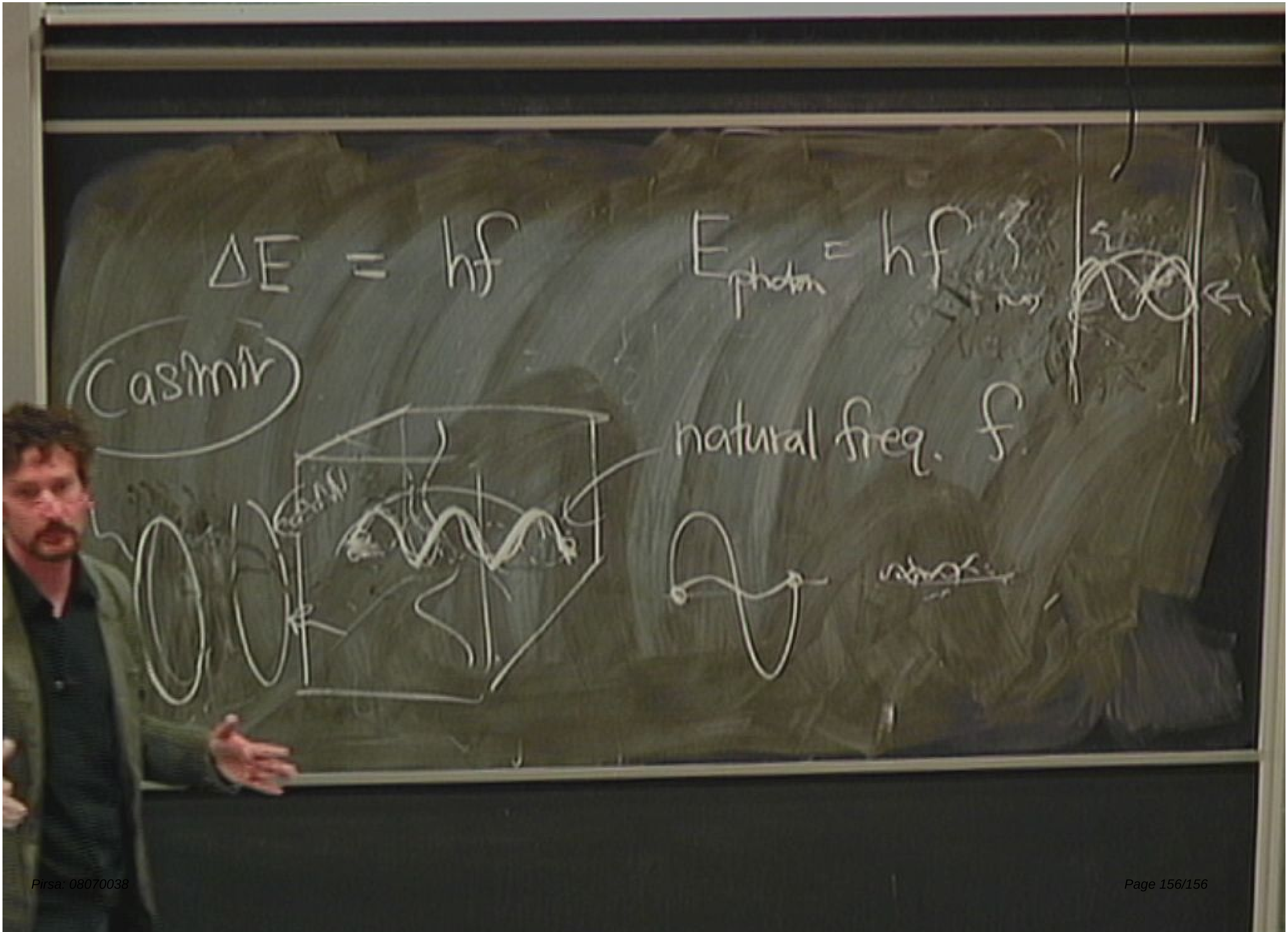

ΔE = hf
E photon = hf
Casimir
natural freq. f


ΔE = hf
E photon = hf
Casimir
natural freq. f


$\Delta E = hf$
$E_{photon} = hf$
Casimir
natural freq. f


ΔE = hf
E photon = hf
Casimir
natural freq. f


ΔE = hf
E photon = hf
Casimir
freq. f


ΔE = hf
E photon = hf
Casimir
natural freq. f