Title: A candidate of a psi-epistemic theory

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Abstract: In deBroglie-Bohm theory the quantum state plays the role of a guiding agent. In this seminar we will explore if this is a universal feature shared by all hidden variable theories or merely a peculiar feature of deBroglie-Bohm theory. We present the bare bones of a model in which the quantum state represents a probability distribution and does not act as a guiding agent. The theory is also psi-epistemic according to Spekken\'s and Harrigan\'s definition. For simplicity we develop the model for a 1D discrete lattice but the generalization to higher dimensions is straightforward. The ontic state consists of a definite particle position and in addition possible non-local links between spatially separated lattice points. These non-local links comes in two types: directed links and non-directed links. Entanglement manifests itself through these links. Interestingly, this ontology seems to be the simplest possible and immediately suggested by the structure of quantum theory itself. For N lattice points there are N*3^(N(N-1)) ontic states growing exponentially with the Hilbert space dimension N as expected. We further require that the evolution of the probability distribution on the ontic state space is dictated by a master equation with non-negative transition rates. It is then easy to show that one can reproduce the Schroedinger equation if an only if there are positive solutions to a gigantic system of linear equations. This is a highly non-trivial problem and whether there exists such positive solutions or not is still not clear to me. Alternatively one can view this set of linear equations as constraints on the possible types of Hamiltonians. We end by speculating how one might incorporate gravity into this theory by requiring permutation invariance of the dynamical evolution law.

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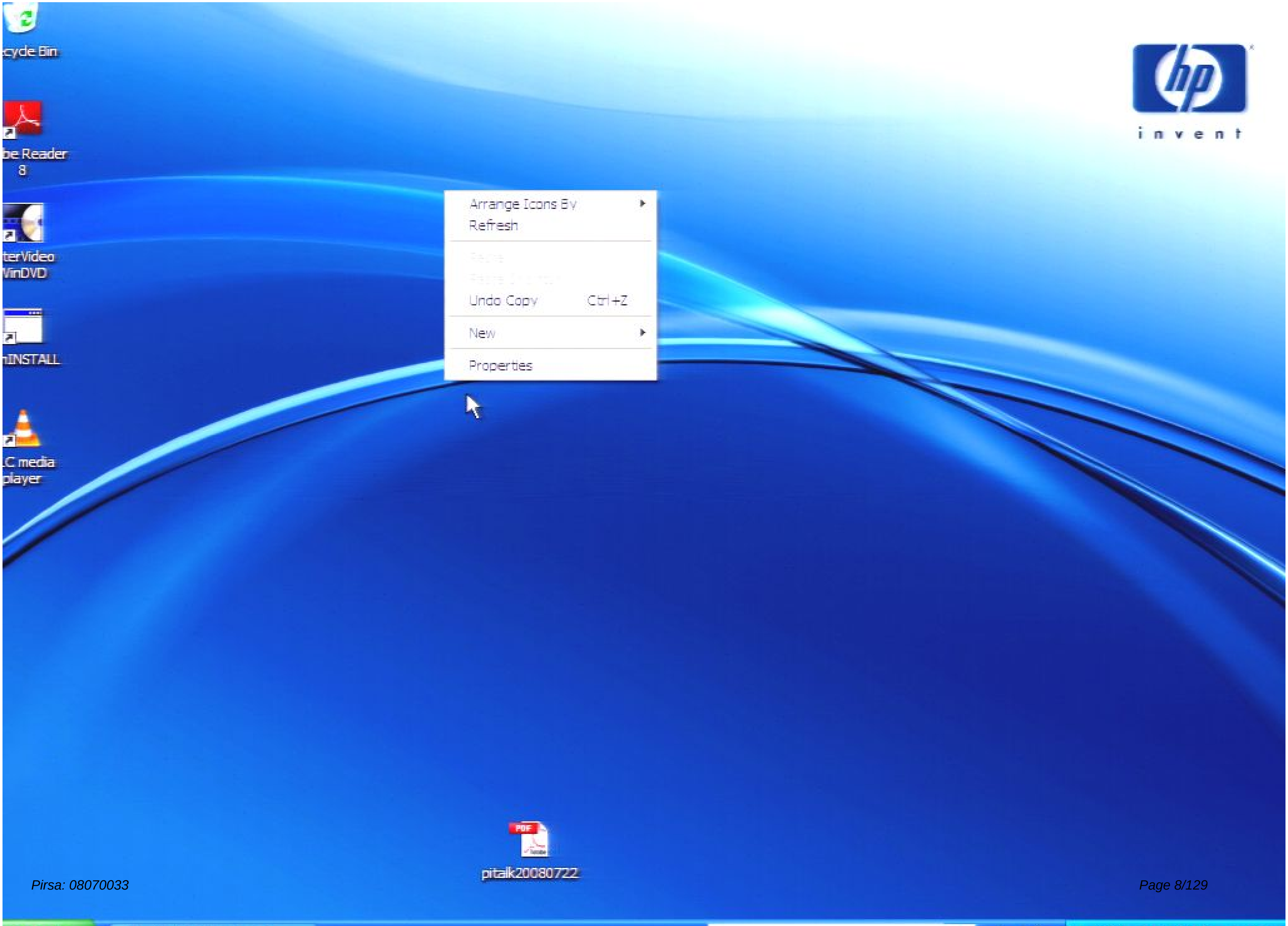
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Introduction & Heuristic Ideas

Quantum Branching

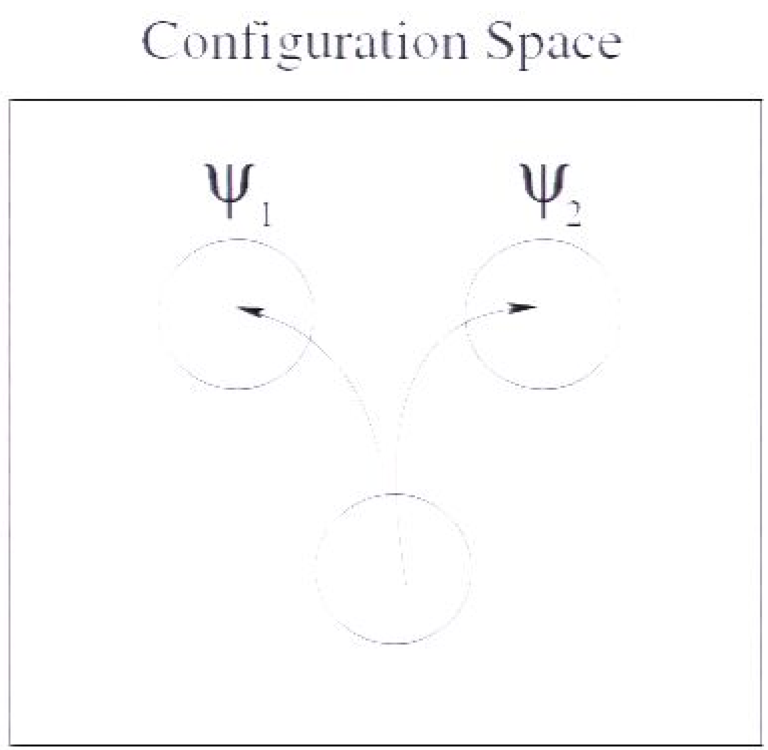


Figure: In a quantum measurement the quantum state branches into a superposition of macroscopically distinct states: $\psi = \psi_1 + \psi_2$.

The Quantum State Describing an Individual System

Figure: The probability distribution branches into a "superposition" of

Classical Branching

Phase Space

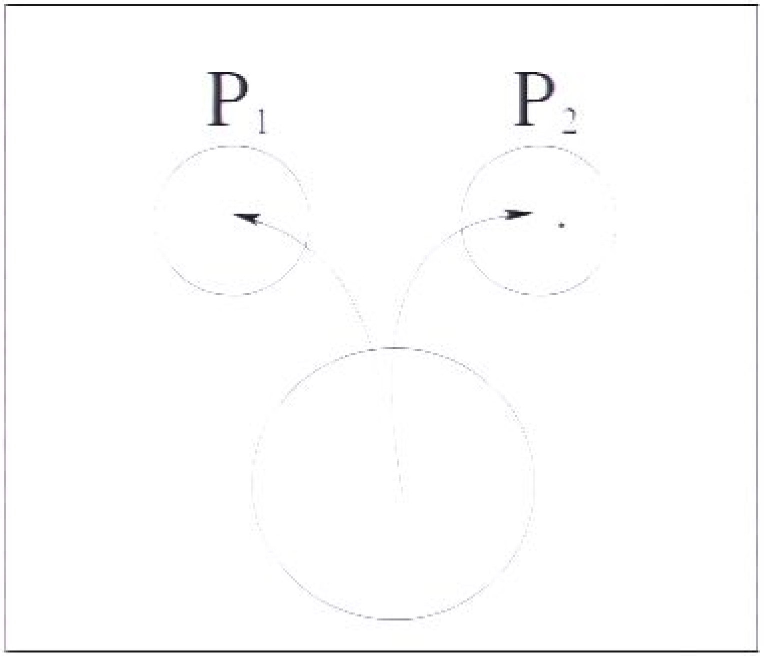
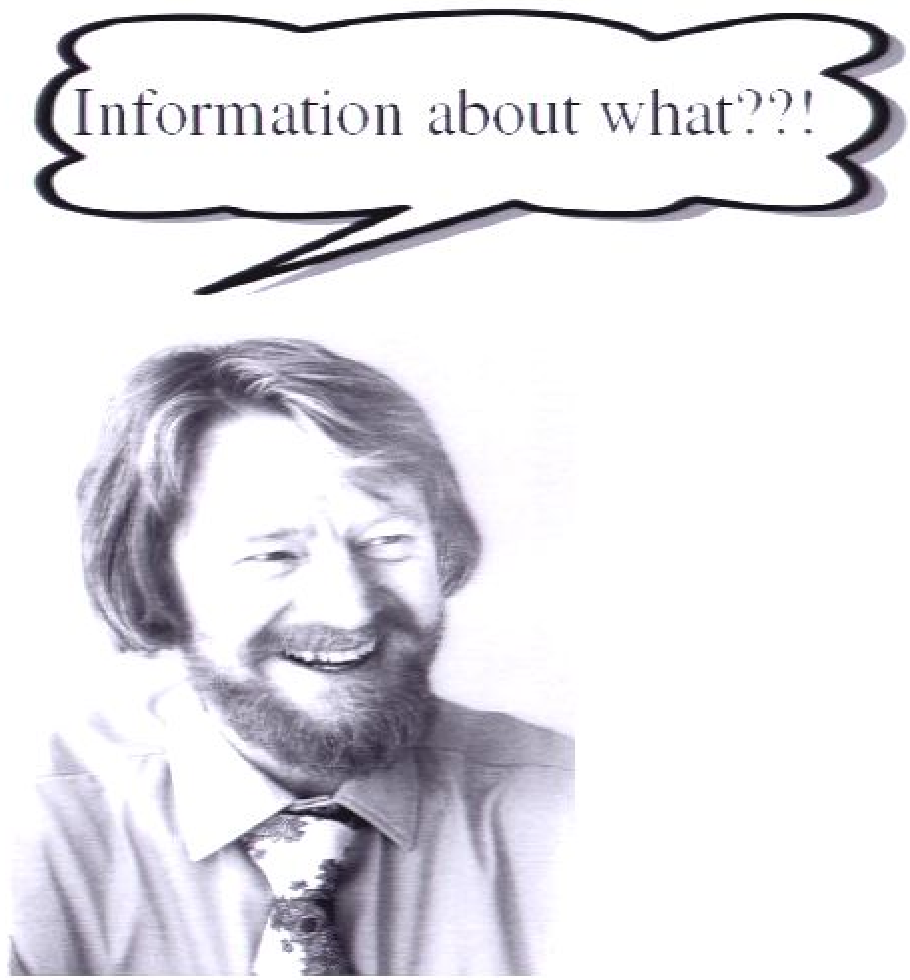


Figure: In a classical measurement the classical probability distribution branches into a "superposition" of macroscopically distinct states: $P \rightarrow P_1 + P_2$.

The Quantum State as an Ideal Observer's Information

Information about WHAT??!



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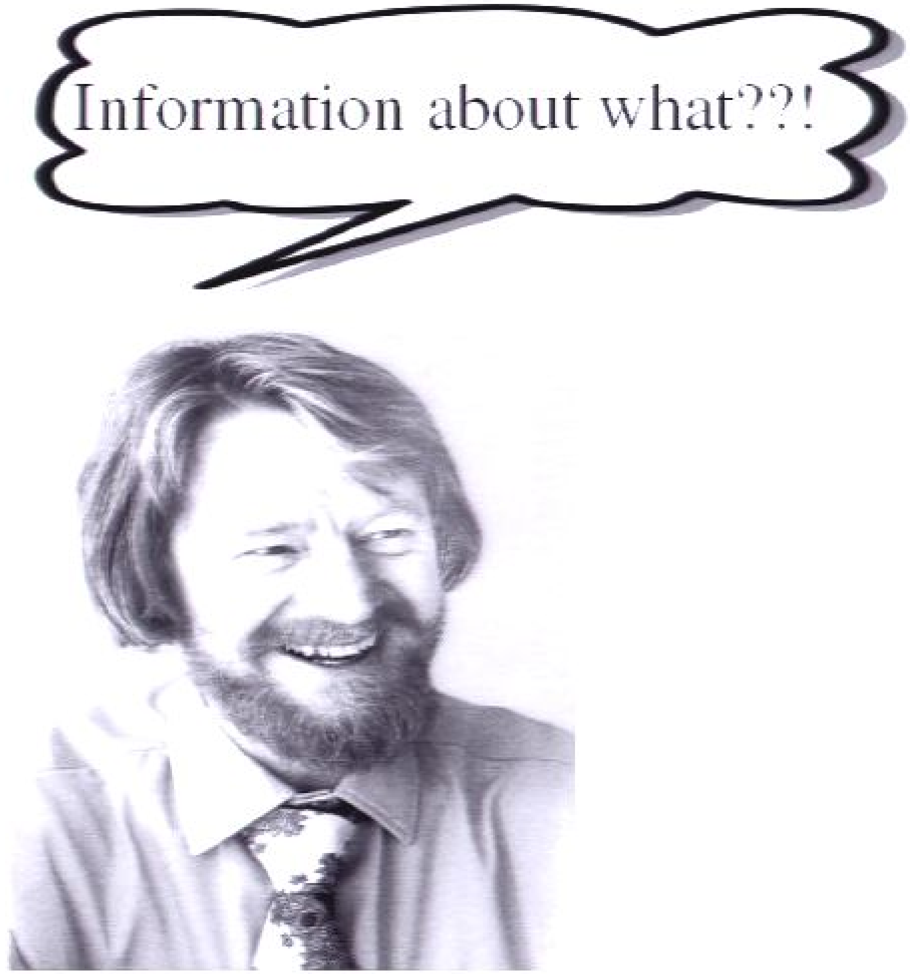


Avoiding Vague Concepts

- Information about possible 'outcomes' of a future 'measurement'?!
- But what **precisely** constitutes an 'outcome' or a 'measurement' in the physical world?? How many atoms do we need??
- As Bell repeatedly stressed, concepts like 'outcome' and 'measurement' are inherently vague and should not appear in a fundamental theory of nature.

Cows Appearing in the Fundamental Laws of Physics???

The "Hidden Variable" Program

- To develop empirically adequate theories in which such vague notions do not appear at the fundamental level is the basic idea behind the "hidden variable" program.
- "Hidden variables" is a fantastically stupid name for variables that are meant to represent tables, chairs, outcomes...
- Better word for 'hidden variable' is 'ontic variable', or 'beables'.

The Quantum State as Information of Actual Properties

The ψ-Epistemic Program

- The ψ-epistemic program is about finding an ontological model in which the quantum state just represents a probability distribution.
- It is also about developing exact mathematical criteria for deciding when a ontological model could be regarded as ψ-epistemic.
- For example, the deBroglie-Bohm theory should not be ψ-epistemic according to any reasonable definition. The quantum state is primarily a guiding agent choreographing the motion of point-particles (or field configurations in QFT).
- Many of the features of deBroglie-Bohm theory are universal so it could be that there are no ψ-epistmic theories compatible with the quantum statistics.

Exploring the Dynamical Dimension

- Spekkens, Barett, Hardy, Harrigan, et. al. have introduced a reasonable mathematical constraint that ψ-epistemic theories should satisfy.
- We would now like to introduce a new logically independent constraint by considering dynamics.
- The intuition is that we would like to exclude models like deBroglie-Bohm theory in which the quantum state guides an individual system, thus not acting as a probability distribution.
- This will not be a kinematical constraint but rather a mathematically sharp constraint on the dynamics of an ontological model.

Dynamical Evolution of the Ontic State

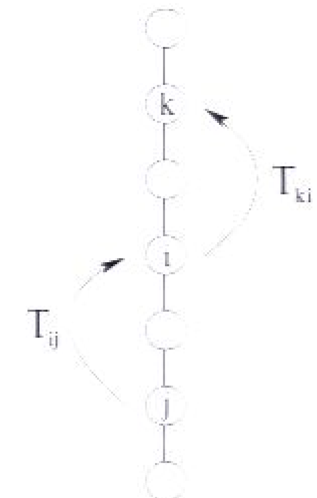
Figure: Evolution of an ontic state

Remarks

- Notice that we are assuming that the world can be decomposed into a sequence of 'instants'.
- This assumption can of course be false. But notice that quantum theory has the same structure.
- Let us now discuss the evolution of ensembles, or probability distributions on Λ.

The Master Equation

Master Equation: $$\dot{\rho}_i = \sum_j T_{ij}\rho_j - T_{ji}\rho_i$$



Conditions on the transition rates T_{ij}:

- Off-diagonal components are non-negative: $T_{ij} \geq 0,\ i \neq j$
- Diagonal components T_{ii} do not enter the master equation and are therefore arbitrary.
- The transition rates are allowed to be infinite (Georgi&Tumulka '03, math/0312294).

Figure: The transition rates T_{ij} represents the probility per unit time for a transition $j \to i$.

Example: Light Bulb

$$\begin{aligned} \dot{\rho}_w &= T_{wb}\rho_b + T_{ww}\rho_w - T_{bw}\rho_w - T_{ww}\rho_w = -k\rho_w \\ &\Rightarrow \rho_w(t) = Ce^{-kt} \end{aligned}$$

- Note that k can be as large as we want but not negative.
- Also note that no matter how large k is the light bulb will never be definitely broken.
- In order to reach the definitely broken state k needs to become infinite. For example, if $T_{bw}(t) = \frac{1}{\sqrt{|t|}}$ then the light bulb will definitely be in the broken state for $t > 0$.

Dynamical Constraint on ψ-epistemic Theories

- In deBroglie-Bohm theory the transition rates can be shown to depend on the quantum state.
- This means that the quantum state in that theory does not play the role of a probability distribution.
- Thus, we should impose the following the constraint:

Dynamical Constraint on ψ-epistemic models:

The transition rates T_{ij} in a ψ-epistemic ontological model has to be independent on the quantum state.

Bell-Type Ontological Models

Preliminaries

- Consider an N-dimensional quantum system with the quantum state $\hat{\rho}$ and Hamiltonian $\hat{H}$.
- In a specific basis $|i\rangle$ the quantum state and Hamiltonian can be represented as a Hermitian matrices $\hat{\rho} \sim \rho_{ij}$ and $\hat{H} \sim H_{ij}$.
- In this basis the Schrödinger equation reads:

$$\dot{\rho}_{ij} = i \sum_k \rho_{ik} H_{kj} - H_{ik} \rho_{kj}.$$

Introducing ontic properties

- Think of $i = 1.\ldots.N$ as an ontic property and $\rho_i = \rho_{ii}$ as the probability of the system of having that property.
- Thus $\lambda = i$ and $\rho(\lambda) = \rho_i$.
- The evolution of the probability distribution ρ_i is given by

$$\dot{\rho}_i = \dot{\rho}_{ii} = i \sum_j \rho_{ij} H_{ji} - H_{ij} \rho_{ji}.$$

Introducing transition rates

- In order for the model to be an ontological model the probability distribution ρ_i has to obey a master equation:

$$\dot{\rho}_i = \sum_j T_{ij}\rho_j - T_{ji}\rho_i$$

where $T_{ij} \geq 0$ for $i \neq j$.

- Let $J_{ij} = i(\rho_{ij}H_{ji} - H_{ij}\rho_{ji})$ and notice the anti-symmetry: $J_{ij} = -J_{ji}$.
- Our task is now to find transition rates T_{ij} so that

$$\sum_j T_{ij}\rho_j - T_{ji}\rho_i = \sum_j J_{ij}$$

Finding transition rates

- One solution is to take (Bell '87)

$$T_{ij} = \begin{cases} \frac{J_{ij}}{\rho_j} & \text{if } J_{ij} \geq 0 \\ 0 & \text{if } J_{ij} < 0 \end{cases}$$

- Note that with this particular choice the transition rates depend on the quatum state. Therefore, the theory does not count as a genuine ψ-epistemic theory.
- Is it possible, within the framework of Bell-type ontological models, to find transition rates that does not depend on the quantum state?

Proof by Contradiction

- We proceed by proof by contradiction. Therefore assume that T_{ij} does not depend on the quantum state.
- This implies in particular that $\sum_j T_{ij}\rho_j - T_{ji}\rho_i$ cannot depend on off-diagonal component $\rho_{i\neq j}$ of the quantum state.
- But since $\sum_j T_{ij}\rho_j - T_{ji}\rho_i = \sum_j J_{ij}$ we have that $\sum_j J_{ij}$ cannot depend on the off-diagonal components either.
- The only way that could happen is if $\sum_j J_{ij} = 0$ which immediately implies that $\dot{\rho} = 0$.
- This is absurd and we have to give up the assumption that the transition rates T_{ij} do not depend on the quantum state.

Introducing Generalized Bell-Type Models

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Introducing Generalized Bell-Type Models

Key Idea

- It is important to note that we could derive a contradiction precisely because the probability distribution ρ_i did not uniquely determine the quantum state.

$$\sum_j i(\rho_{ij}H_{ji} - H_{ij}\rho_{ji}) = \sum_j T_{ij}\rho_j - T_{ji}\rho_i$$

- It is therefore seems necessary to develop a new class of ontological models so that the epistemic probability distribution $\rho(\lambda)$ uniquly determines the quantum state.
- To do this it is necessary to introduce more ontological properties.

Today's Misleading Statement

Not all 'obervables' can be given beable status, for they do not all have simulateneous eigenvalues, i.e. o not all commute. J. S. Bell '84.

Generalized Bell-Type Models

- First we outline the general structure of generalized Bell-type models. Then we are going to consider a specific Bell-type model which seems to be the simplest possible with a minimal number of ontic states.
- The density matrix and the Hamitonian are Hermitian operators, therefore, we can express any quantum state and any Hamilton as

$$\hat{\rho} = \frac{1}{N} + \sum_A c_A T_A \qquad \hat{H} = b \cdot 1 + \sum_A b_A T_A .$$

where the operators T_A are $N^2 - 1$ generators of $SU(N)$.

- Together with the identity the generators form a basis of the space of Hermitian operators.

Properties of the Generators

- The generators satisfy

$$[T_A, T_B] = \sum_C i f_{ABC} T_C \qquad \{T_A, T_B\} = \frac{1}{N}\delta_{AB} + \sum_C d_{ABC} T_C$$

$$Tr T_A = 0 \qquad T_A T_B = \frac{1}{2N}\delta_{AB} + \sum_C \frac{1}{2}(i f_{ABC} + d_{ABC}) T_C$$

$$Tr(T_A T_B) = \frac{1}{2}\delta_{AB}.$$

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The Schrödinger Equation

- Using these relations we see that

$$\frac{c_A}{2} = Tr(\hat{\rho} T_A)$$

- We can also rewrite the Schrödinger equation in terms to the c-values:

$$\dot{c}_C = -\sum_{A.B} f_{ABC} c_A b_B$$

Introducing Ontic States

- We now regard each A as representing an ontic property $i_A = 1, \ldots, N^2 - 1$.
- Our ontic state λ we take to be the collection of these properties, i.e. $\lambda = (i_1, \ldots, i_A, \ldots, i_{N^2-1})$.
- The number of ontic states M grows exponentially with the dimension: $M = N^{N^2-1}$.
- For $N = 2$ we have 8 ontic states, for $N = 3$ $M = 6561$, $N = 4$ $M \approx 10^9$, $N = 5$ $M \approx 0.6\ 10^{18}$.

Introducing Ontic States

- We will later see in how the number of ontic properties can gratly be reduced by choosing a special representation of the operators T_A.
- In addition we can make use of the fact that a subset of them commute (the Cartan subalgebra). This means that we only need to introduce one property for all operators in the commuting subalgebra.
- In the specific model we will present below the number of ontic states is $N \cdot 3^{(N(N-1))}$ which is far less than N^{N^2-1}!

Introducing a Probability Distribtion $\rho(I)$

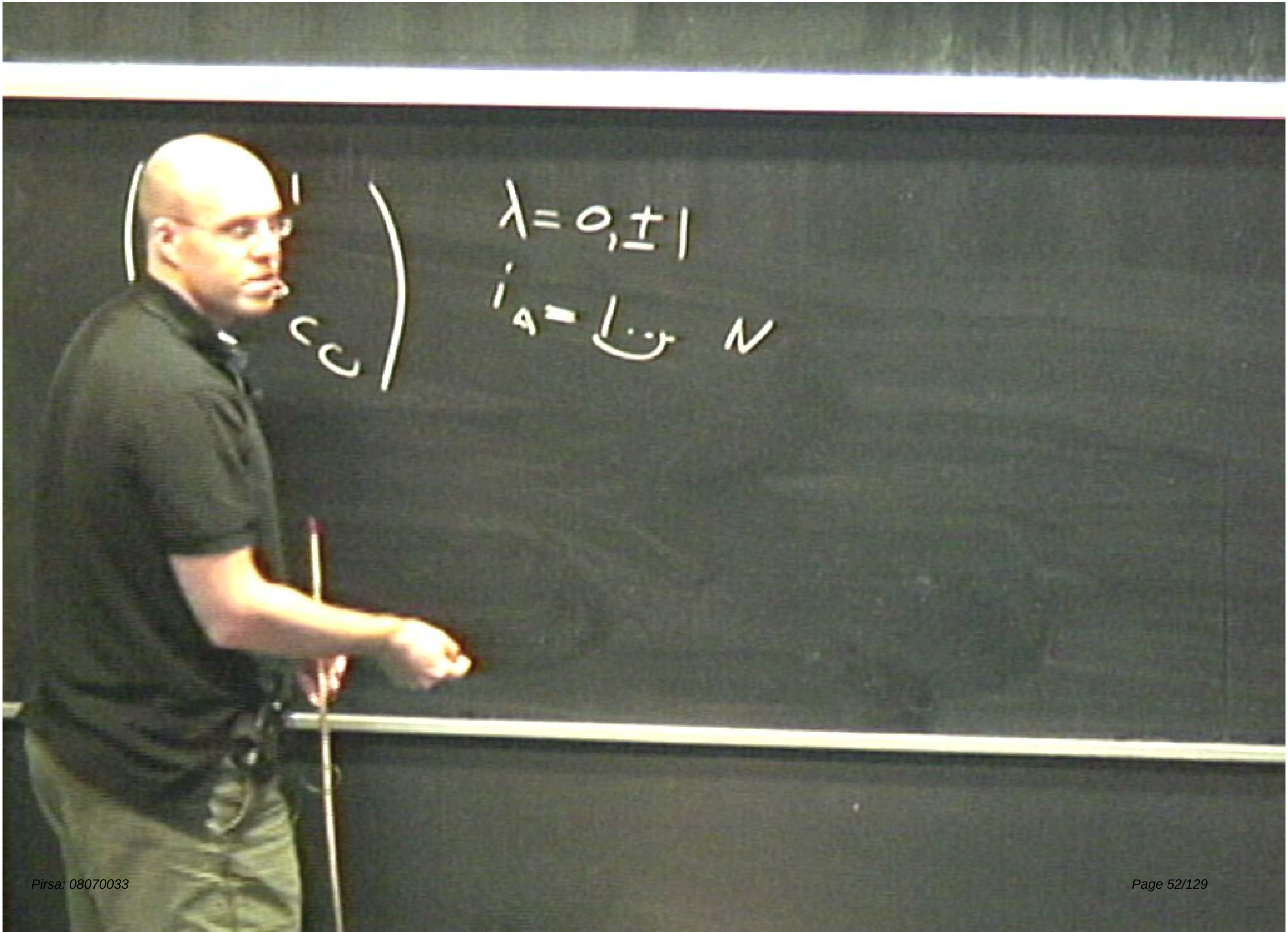
- We now introduce an epistemic distribution $\rho(i_1 \ldots \ldots i_A \ldots \ldots i_{N^2-1})$ so that c_A is the expectation value of the property i_A.

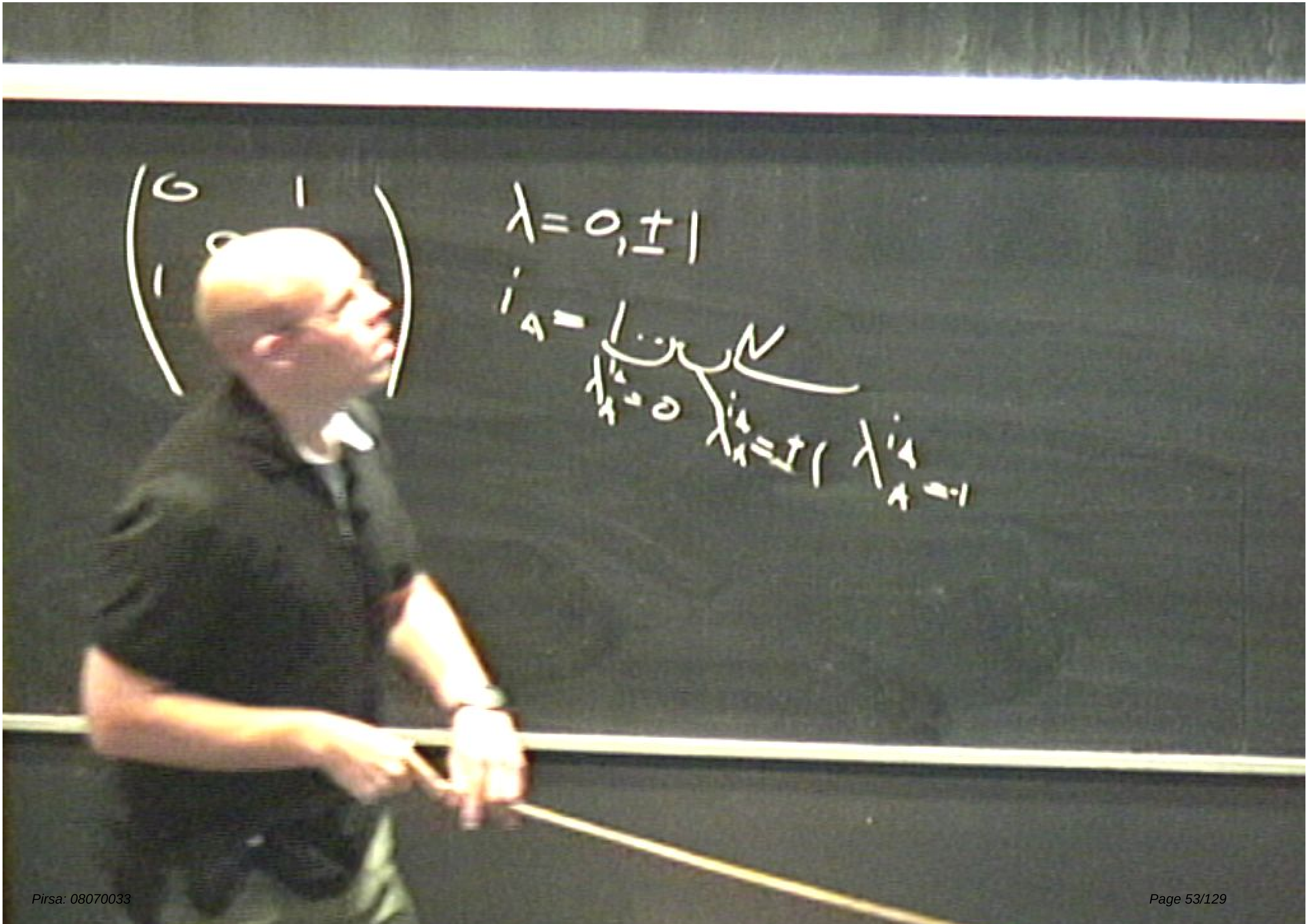
$$\frac{c_A}{2} = \sum_I \lambda_A^{i_A} \rho(I)$$

where $I = (i_1 \ldots \ldots i_{N^2-1})$ is a multi-index.

- We note that the quantum state is uniquely determined by specifying the distribution $\rho(I)$ and that not all distributions give rise to a proper density matrix. (These are examples of quantum non-equilibrium distributions.)

$$\begin{pmatrix} 0 & & 1 & & \\ & 0 & & & \\ 1 & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \qquad \lambda = 0, \pm 1$$


λ = 0, ±1
N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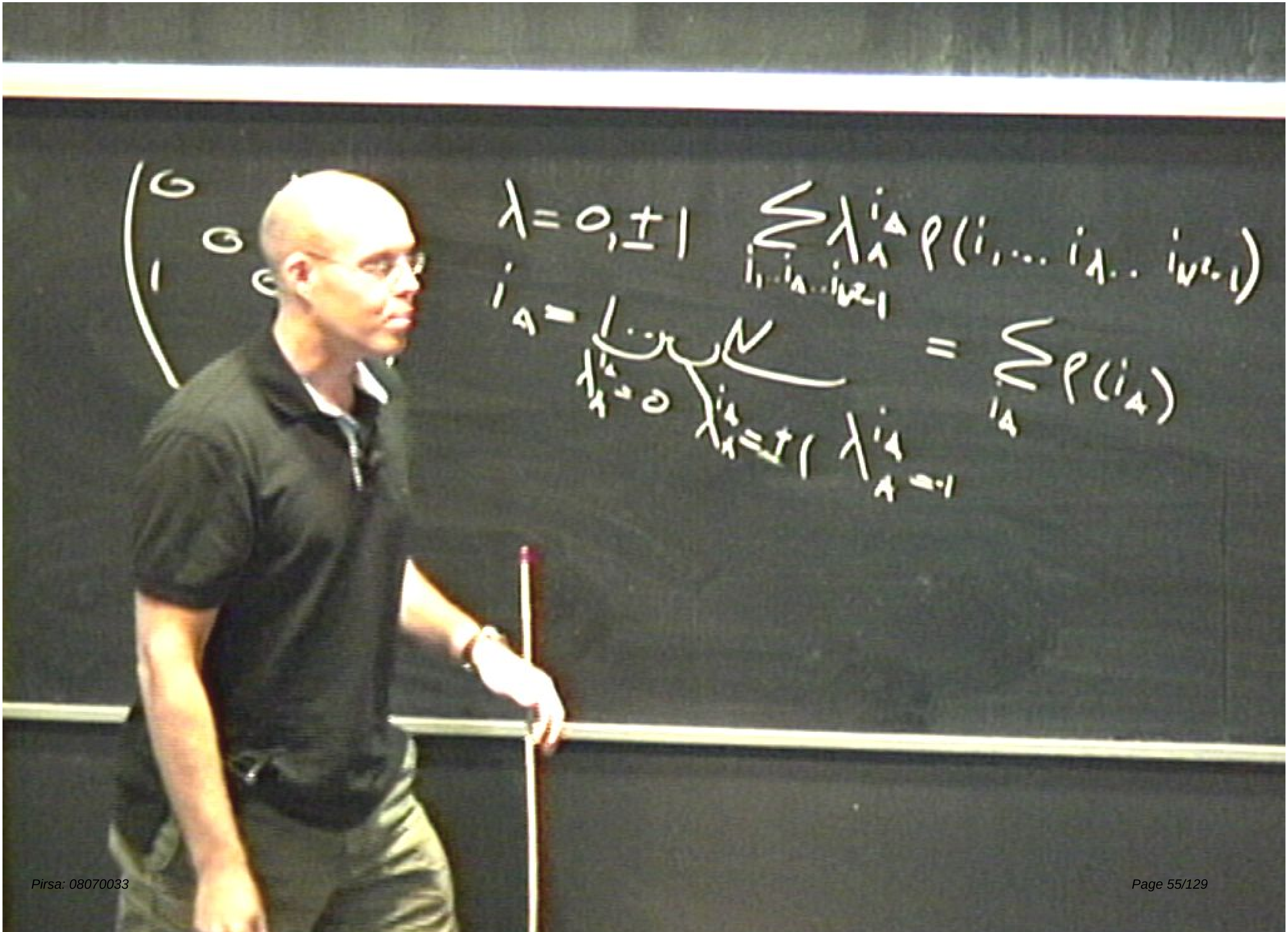
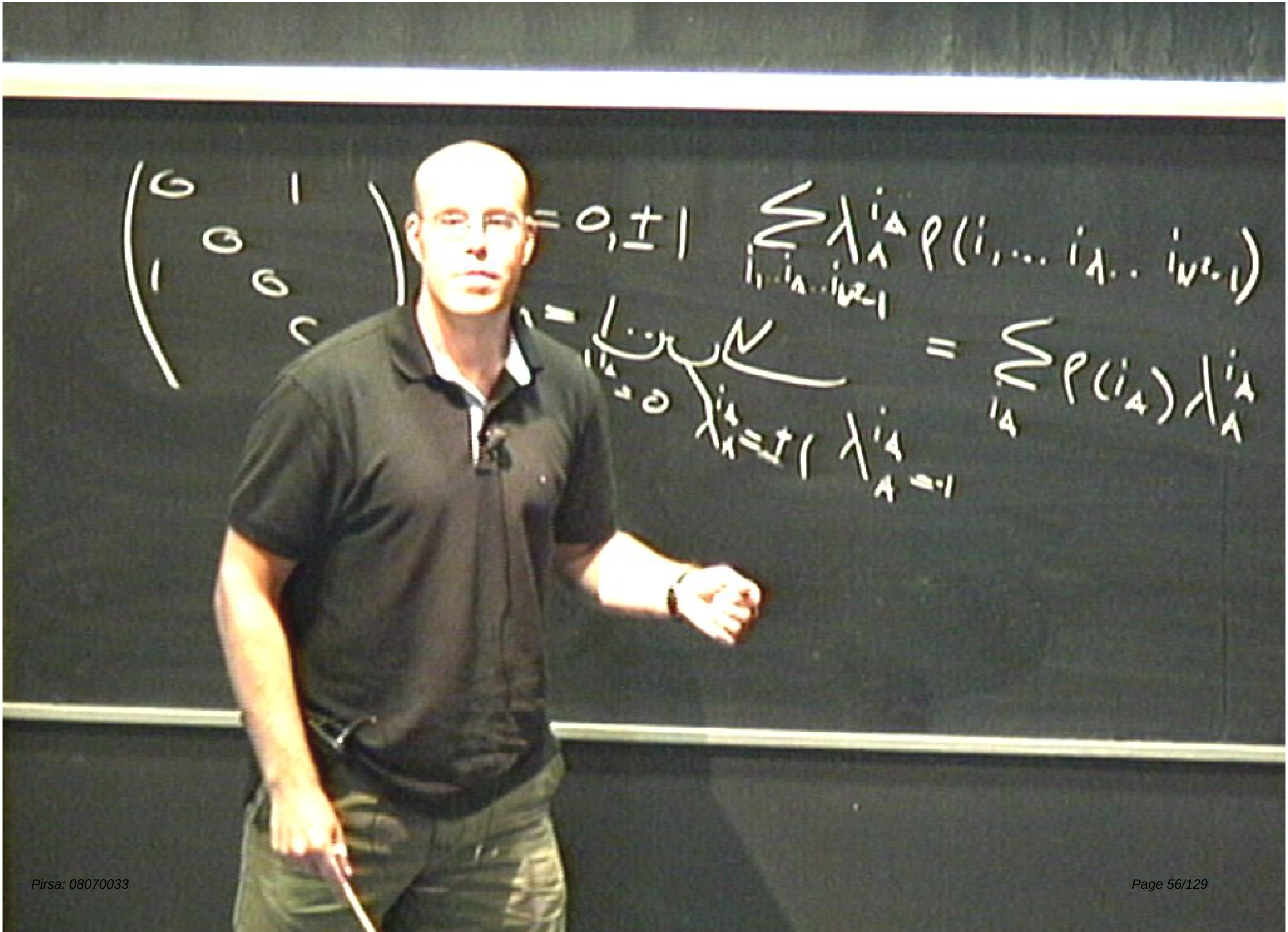

λ = 0, ±1
Σ λ_A^{i_A} ρ(i_1 ... i_A .. i_{N²-1})
i_1..i_A..i_{N²-1}
i_A = 1 ... N
= Σ ρ(i_A)
i_A
λ_A^{i_A} = 0
λ_A^{i_A} = ±1
λ_A^{i_A} = -1



$$\begin{pmatrix} 0 & & 1 & \\ & 0 & & \\ 1 & & 0 & \\ & & & 0 \ldots 0 \end{pmatrix}$$

$$\lambda = 0, \pm 1 \qquad \sum_{i_1 \ldots i_A \ldots i_{N^2-1}} \lambda_A^{i_A} \rho(i_1 \ldots i_A \ldots i_{N^2-1})$$

$$i_A = 1 \ldots N \qquad = \sum_{i_A} \rho(i_A) \lambda_A^{i_A}$$

$$\lambda_A^{i_A} = 0 \qquad \lambda_A^{i_A} = \pm 1 \qquad \lambda_A^{i_A} = -1$$

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Evolution of the Probability Distribution $\rho(I)$

- Next assume that the evolution of $\rho(I)$ is governed by a master equation $\dot{\rho}(I) = \sum_J T(I, J)\rho(J) - T(J, I)\rho(I)$.
- We now have two distinct ways of computing $\dot{\rho}(I)$: one through the Schrödinger equation and one through the master equation.

Evolution of the Distribution $\rho(I)$

- The two expressions are

$$\frac{\dot{c}_C}{2} = \sum_I \lambda_C^{i_C} \dot{\rho}(I) = \sum_I \lambda_C^{i_C} \sum_J T(I,J)\rho(J) - T(J,I)\rho(I)$$

$$= \sum_{I,J} (\lambda_C^{i_C} - \lambda_C^{j_C}) T(I,J)\rho(J)$$

$$\frac{\dot{c}_C}{2} = -\sum_{A,B} f_{ABC} \frac{c_A}{2} b_B = -\sum_{A,B} f_{ABC} \sum_J \lambda_C^{j_C} \rho(I) b_B.$$

- These have to agree!

Constraint Imposed by the Schrödinger Equation

- Equating both sides yields:

$$\sum_J \left[\sum_I (\lambda_C^{i_C} - \lambda_C^{j_C}) T(I, J) + f_{ABC} \lambda_C^{j_C} b_B \right] \rho(J) = 0$$

- This is an equation for the transition rates $T(I, J)$.

Constraint Imposed by the Schrödinger Equation

- The most obvious way to satistfy the equation is to put

$$V_C(J) \equiv \sum_I (\lambda_C^{i_C} - \lambda_C^{j_C}) T(I, J) + f_{ABC} \lambda_C^{j_C} b_B = 0$$

- But on reflection this is way too restrictive. It would imply that the Schrödinger evolution was valid for all distributions $\rho(J)$. But we can only safely assume the Schrödinger equation to be satisfied for equilibiurm distributions $\rho_{eq.}(I)$.
- The equilibrium distributions are parameterized by the c-values: $\rho_{eq.}(I; c_A)$.

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Imposing that the $T(I,J)$'s are Independent of c_A

- Equating both sides yields:

$$\sum_J \left[\sum_I (\lambda_C^{ic} - \lambda_C^{jc}) T(I,J) + f_{ABC} \lambda_C^{jc} b_B \right] \rho(J; c_A) = 0$$

- Thus, the $N^2 - 1$ vectors $V_C(J)$ have to be orthogonal to all equilibrium distributions $\rho(J; c_A)$.

Space of probability distributions

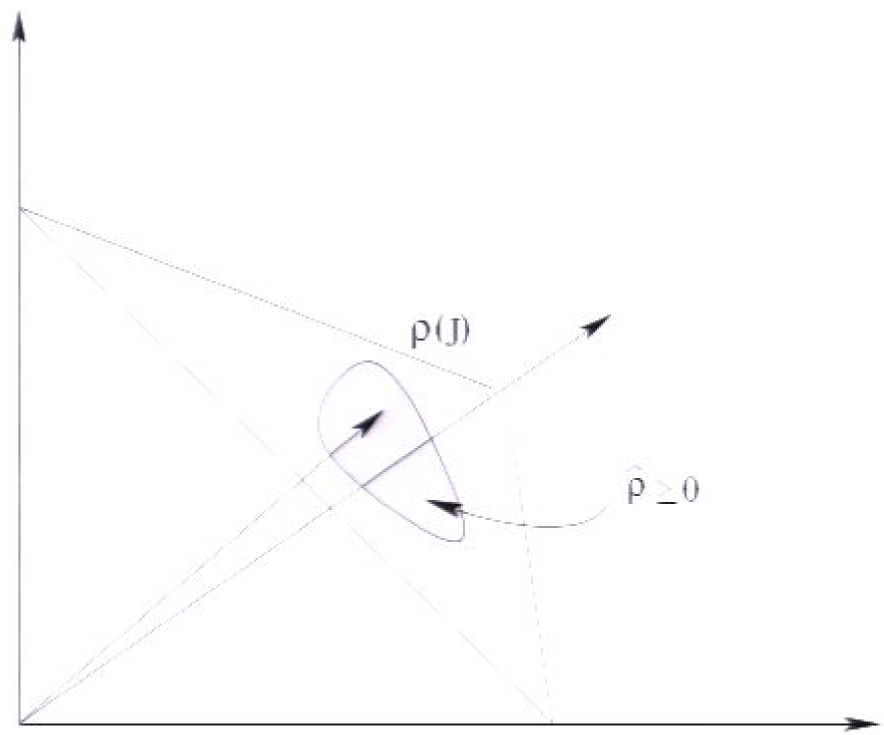


Figure:

Fibre bundle structure of the space of probability distributions $\rho(I; c_A)$

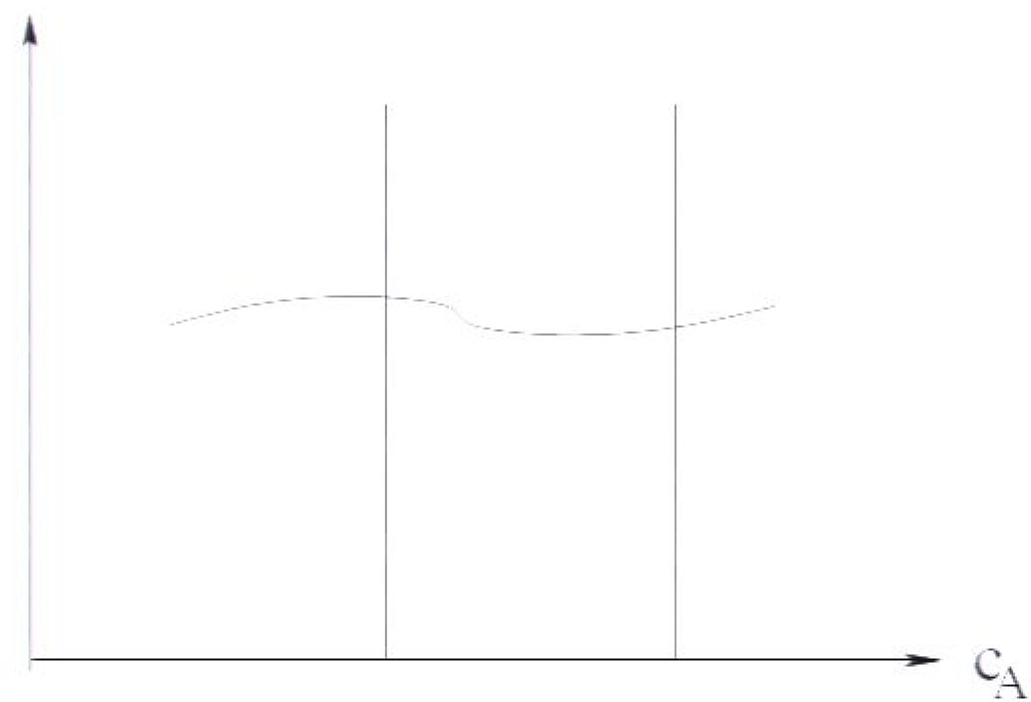


Figure: The space of probability distributions has a natural fibre bundle structure. To each value of c_A there is an equivalence class of different distributions $\rho(I; c_A)$.

Selecting a set of equilibrium distributions

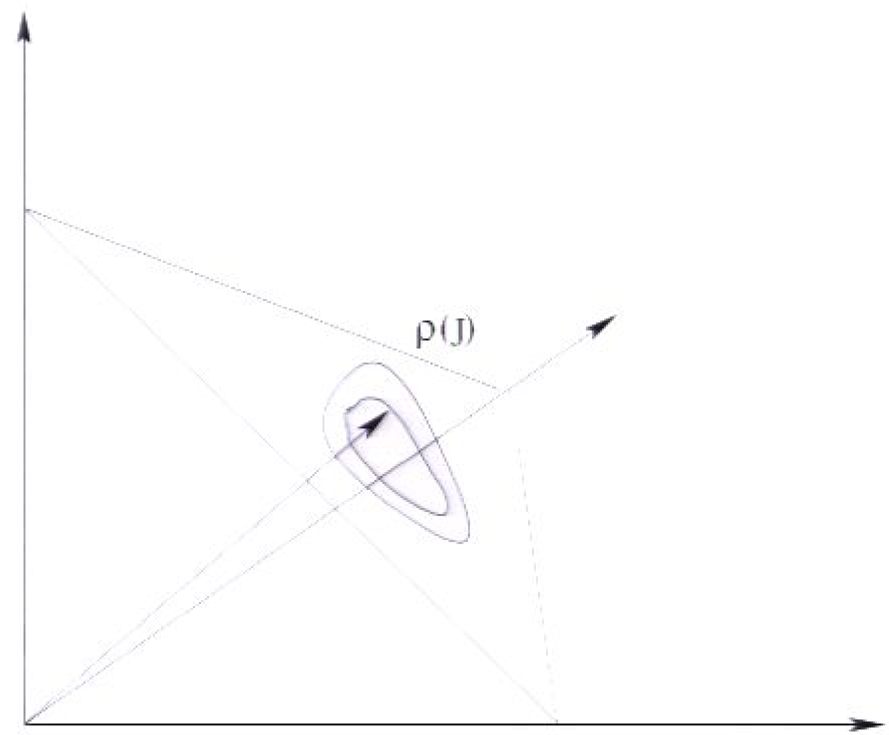


Figure: If the section is "curved" then there will be no vectors $V_C(J)$ orthogonal to all equilibrium distributions on the line.

Can a section be contained within a subplane?

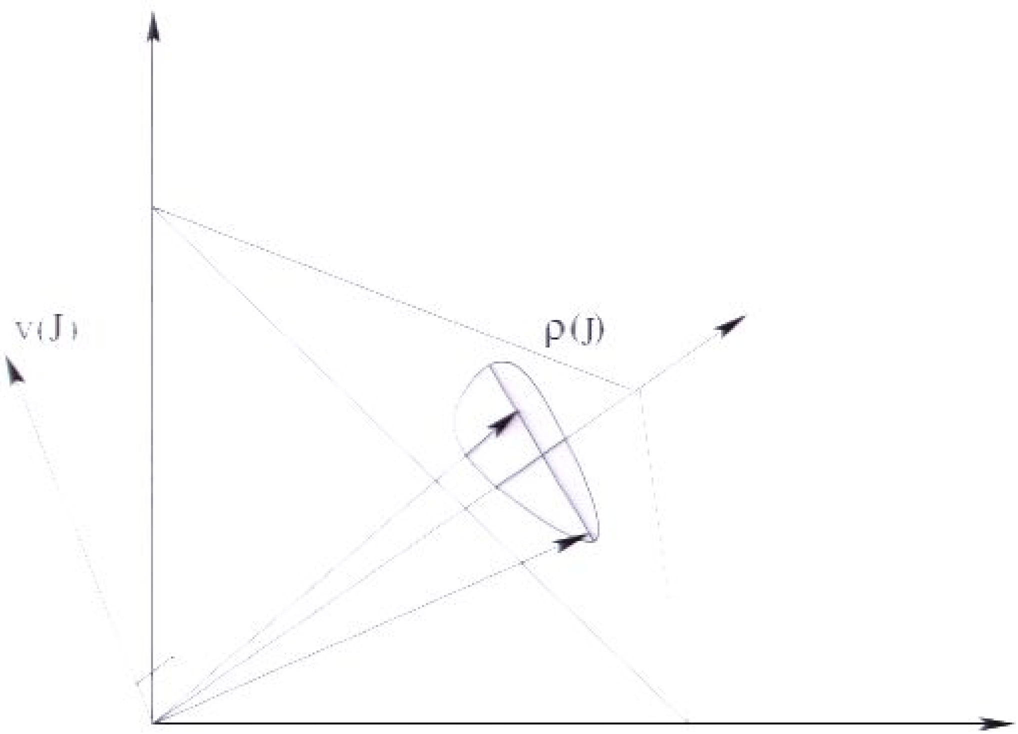
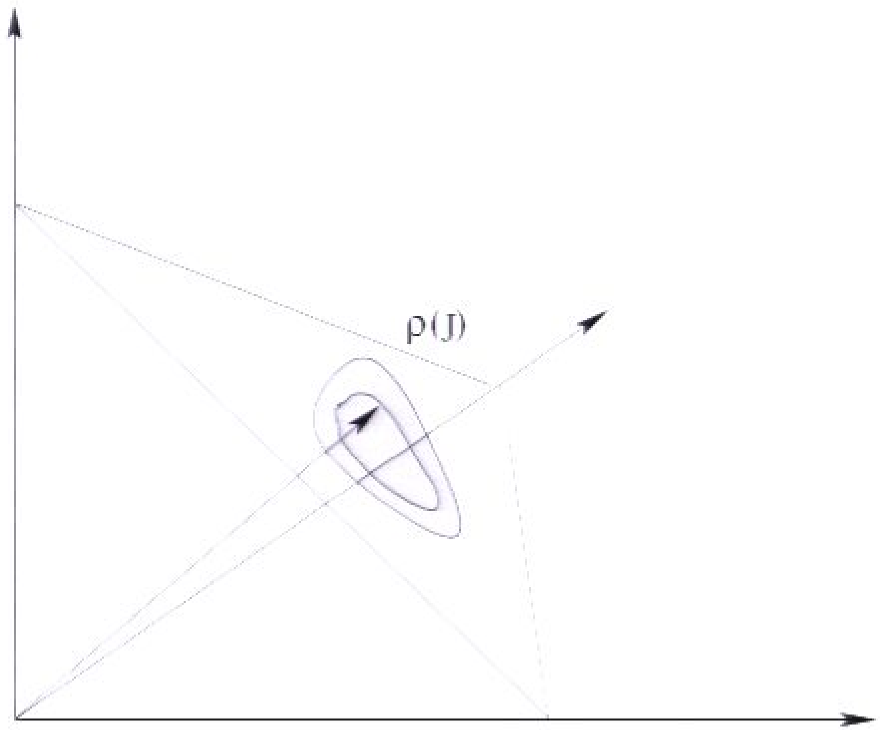


Figure: If it is possible to choose a section to that all equilibrium distributions are contained within a plane then there (might!) exist vectors a $V_C(J)$ orthogonal to all equilibrium distributions.

Selecting a set of equilibrium distributions



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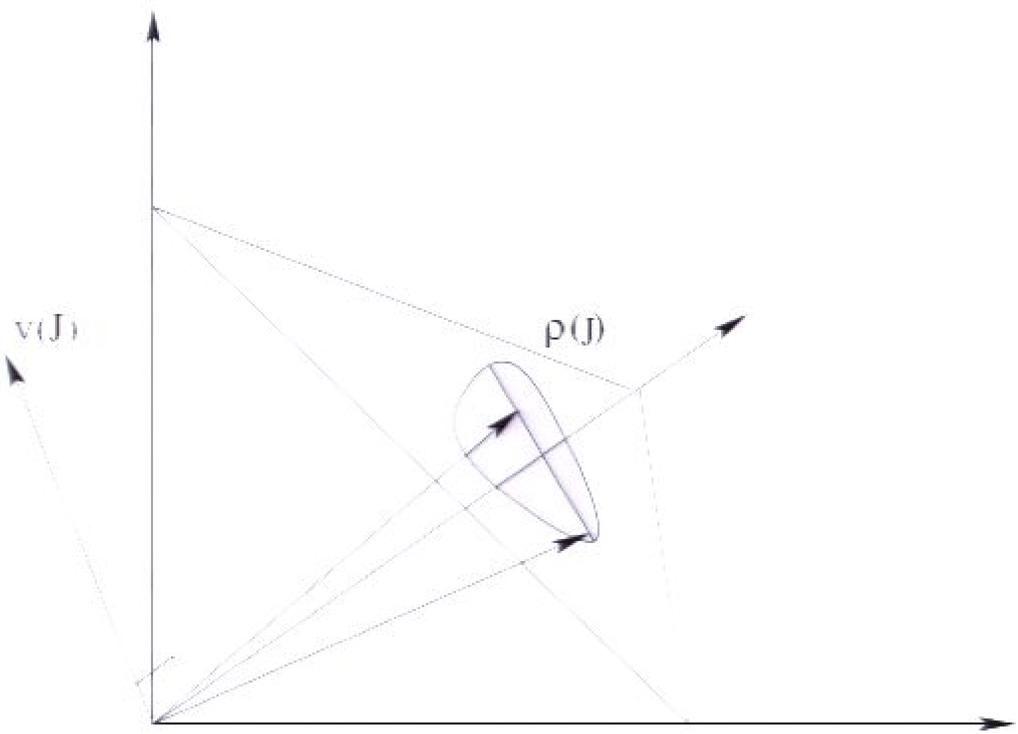

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Modelling a Measurement Along any Direction of a Qubit

Modelling a General Spin Measurement

- Consider two qubits. One we regard as a model of a measurement apparetus and the other the system, i.e. the qubit we are performing the measurement on.
- We shall interpret the z property of the measuring qubit as revealing the outcome.
- A measurement constitutes an interaction between the apparatus and system, $\hat{H}_I$ say.

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Mathematical Modelling

- The operator $\sigma_z \otimes 1$ and T_{15} are simultaneously diagonal in the standard matrix representation. The $A = 15$ property has 4 distinct states 1, 2, 3, 4. The first two corresponds to eigenvalue $+1$ of $\sigma_z \otimes 1$ and the last two eigenvalue -1.
- It is therefore natural to take the $A = 15$ property as the measurement 'needle'.
- Summarizing: If we find the $A = 15$ property to be 1 or 2 the we have outcome $+1$ and if it is 3 or 4 the outcome is -1.

Introducing a Specific Model

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Empirical Adequacy of the Spin Measurement Model

- But how close can we get to quantum predictions in our model? A recent paper (quant-ph/0805.1728) examines how many measurements can be modelled by M ontic states. They claim that the relationship is roughly linear. I.e. with M ontic states we can reproduce the statistics of M distinct measurements.
- In our spin measurement model there are 10^9 ontic states so in principle one should be able to account for 10^9 distinct quantum measurements!!!
- This could mean that this model, even if it does not exactly reproduce the quantum predictions, will be empirically adequate for carefully chosen transition rates $T(I, J)$.

Introducing a Specific Model

- What we outlined above is a framework of a specific type of models that we have denoted generalized Bell-type models.
- In order to produce a specific theory we need to choose a specific basis T_A on the space of Hermitian operators.

Introducing a Specific Model

- One simple choice of basis is

$$M_{ij}^{ab} = \frac{1}{2}(\delta_{ia}\delta_{jb} + \delta_{ib}\delta_{ja}) \qquad N_{ij}^{ab} = \frac{i}{2}(\delta_{ia}\delta_{jb} - \delta_{ib}\delta_{ja})$$

- Notice first the symmetry properties: $\hat{M}^{ab} = \hat{M}^{ba}$ and $\hat{N}^{ab} = -\hat{N}^{ba}$.
- This means that there are

$$\frac{N(N-1)}{2} + \frac{N(N+1)}{2} = N^2$$

operators as required.

Properties of the Operators $\hat{M}^{ab}$ and $\hat{N}^{ab}$.

- The diagonal operators $\hat{M}^{aa}$ constitute a set of N commuting projection operators and have eigenvalues $0, 1$. Since they are commuting it is only necessary to associate one ontological property to all these projectors. ($\hat{N}^{aa} \equiv 0$ because of antisymmetry.)
- The off-diagonal ones $\hat{M}^{a \neq b}$ are permutation operators permuting the points $a \leftarrow b$. The eigenvalue spectra is $0, \pm\frac{1}{2}$.
- The antismmetric operators $\hat{N}^{ab}$ are also a type of permutation operators with eigenvalue spectra is $0, \pm\frac{1}{2}$. They are there because we are dealing with a Hilbert space over the complex numbers.

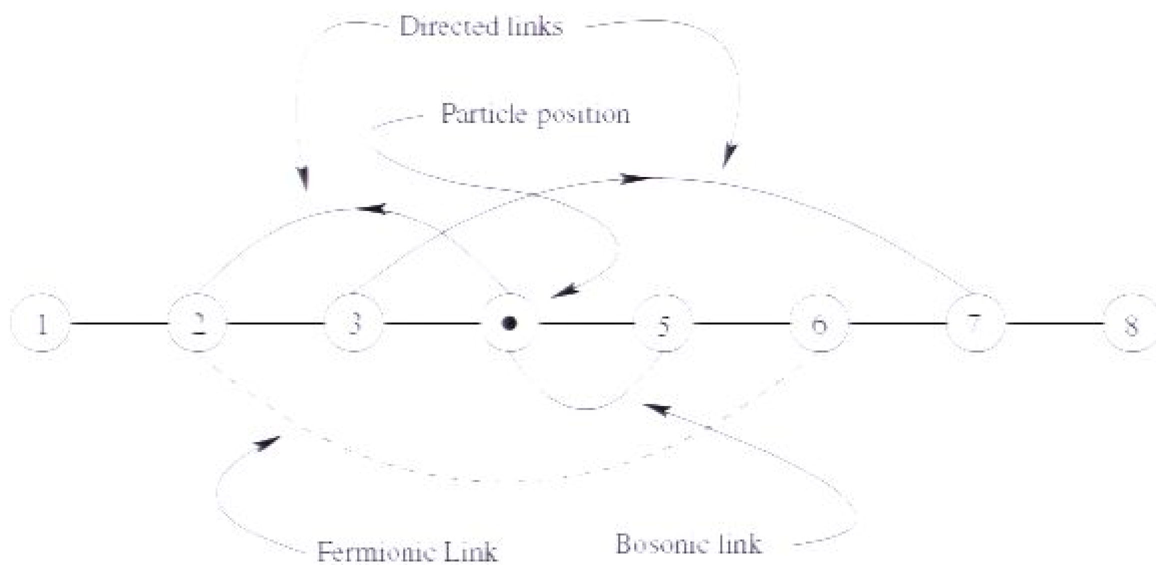
Natural Ontology Associated to the Choice of Basis

- There is a natural ontology associated with this choice of basis.
- The N commuting projection operators $\hat{M}^{aa}$ could represent the position PVM on a discrete lattice with N points.
- Just like in deBroglie-Bohm theory we will assume that the only one of the projectors $\hat{M}^{aa}$ "has value 1" and the rest "have value 0".
- Thus, part of the ontology constists of a particle occupying a definite position on the lattice. The position property can take on the values $1, \ldots, N$.

Natural Ontology Associated to the Choice of Basis

- The off-diagonal "symmetric" operator $\hat{M}^{ab}$ (for some $a \neq b$) have eigenvalues 0 and $\pm\frac{1}{2}$. Since we only have three distinct eigenvalues the property μ_{ab} corresponding to this operator only needs three distinct attainable values: $\mu_{ab} = 0, \pm\frac{1}{2}$.
- We interpret $\mu_{ab} = 0$ to mean that there is no link between the points a and b. Eigenvalue $\mu_{ab} = +\frac{1}{2}$ correspond to a non-directed "bosonic" link and $\mu_{ab} = -\frac{1}{2}$ to a non-directed "fermionic" link.
- The antisymmetry of the operators $\hat{N}^{ab}$ immediately suggest that these represent directed links. Eigenvalue $\nu_{ab} = 0$ means no link, eigenvalue $\nu_{ab} = +\frac{1}{2}$ means that there is a link starting from a and ending up at b, and $\nu_{ab} = -\frac{1}{2}$ that there is a link starting from b and ending up at a.

Ontology Illustrated



The ontic state in this particular case is: $\mu_{aa} = \delta_{4a}$, $\nu_{24} = -1/2$, $\mu_{26} = -1/2$, $\mu_{45} = +1/2$, $\nu_{37} = +1/2$, and the rest equal to zero.

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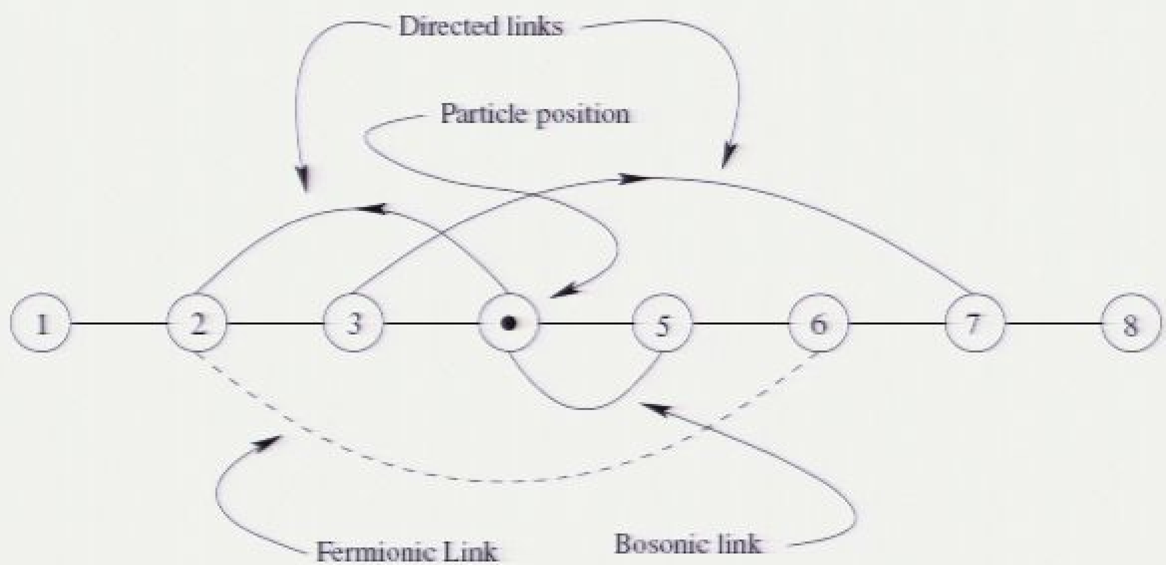

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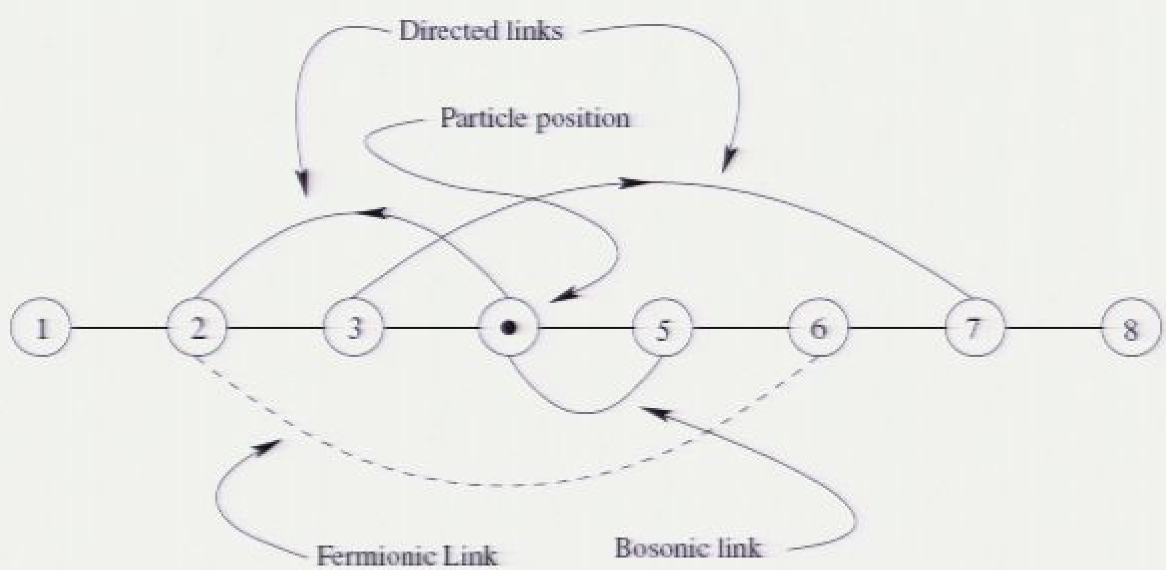

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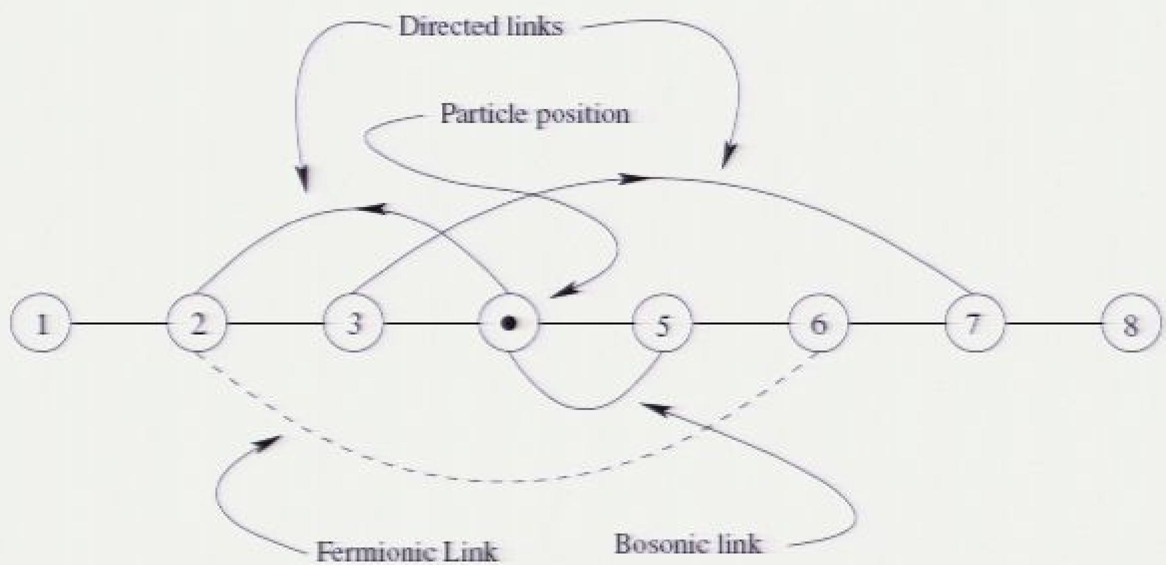

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Expectation Values

- As before the quantum state and Hamiltonian can be expressed in the basis

$$\hat{\rho} = \sum_{a,b} c_{ab}\hat{M}^{ab} + d_{ab}\hat{N}^{ab} \qquad \hat{H} = \sum_{a,b} a_{ab}\hat{M}^{ab} + b_{ab}\hat{N}^{ab}$$

  where $c_{ab} = c_{ba}$ and $d_{ab} = -d_{ba}$.
- We also have

$$\langle \hat{M}^{ab} \rangle = Tr(\hat{\rho}\hat{M}^{ab}) = c_{ab} \qquad \langle \hat{N}^{ab} \rangle = Tr(\hat{\rho}\hat{N}^{ab}) = d_{ab}.$$

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The Schrödinger Equation

- The Schrödinger equation can be shown to take the form

$$\dot{c}_{ef} = \sum_d c_{ed} b_{fd} - d_{ed} a_{fd} + c_{fd} b_{ed} - d_{fd} a_{ed}$$

$$\dot{d}_{ef} = \sum_d c_{ed} a_{fd} + d_{ed} b_{fd} - c_{fd} a_{ed} - d_{fd} b_{ed}$$

Introducing an Probability Distribution

- We now introduce a probability distribution $\rho(\mu_{11}, \mu_{12}, \ldots, \nu_{12}, \ldots)$ such that we reproduce the expectation values

$$c_{ab} = \sum_{\mu,\nu} \mu_{ab}\rho(\mu,\nu)$$

$$d_{ab} = \sum_{\mu,\nu} \mu_{ab}\rho(\nu,\nu)$$

where μ and ν are multi-indices.

- Recall that $\mu_{aa} = \delta_{a\bar{a}}$ for some $\bar{a}$. This implies that $\sum_a c_{aa} = \sum_{\mu,\nu} \rho(\mu,\nu) = 1$ since ρ is assumed to be a normalized probability distribution. Thus $Tr(\hat{\rho}) = 1$ just means that the probability distribution $\rho(\mu,\nu)$ is normalized.

Condition Imposed by the Schrödinger Equation

- As before we require the probability distribution $\rho(\lambda) = \rho(\mu, \nu)$ evolves according to a master equation.
- Introduce the symbols

$$u_{ab}(\bar{\mu}, \bar{\nu}) = u_{ba}(\bar{\mu}, \bar{\nu})$$

$$\sum_{\mu,\nu} (\mu_{ab} - \bar{\mu}_{ab}) T(\mu, \nu | \bar{\nu}, \bar{\nu}) - \sum_{d} \bar{\mu}_{ed} b_{fd} - \bar{\nu}_{ed} a_{fd} + \bar{\mu}_{fd} b_{ed} - \bar{\nu}_{fd} a_{ed}$$

$$v_{ab}(\bar{\mu}, \bar{\nu}) = -v_{ba}(\bar{\mu}, \bar{\nu})$$

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Condition Imposed by the Schrödinger Equation

- The Schrödinger equation then gives rise to the following N^2 conditions:

$$\sum_{\bar{\mu},\bar{\nu}} u_{ab}(\bar{\mu},\bar{\nu})\rho_{eq.}(\bar{\mu},\bar{\nu}) = 0$$

$$\sum_{\bar{\mu},\bar{\nu}} v_{ab}(\bar{\mu},\bar{\nu})\rho_{eq.}(\bar{\mu},\bar{\nu}) = 0$$

- As before these conditions should hold for all equilibrium distributions in order for the quantum state to be a purely epistemic object.

Quantum Equilibrium vs Quantum Non-Equilibrium Distributions

- A distribution $\rho_{eq.}(\mu,\nu)$ that satisfies

$$\sum_{\mu,\nu} u_{ab}(\mu,\nu)\rho_{eq.}(\mu,\nu) = 0$$

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$$\hat{\rho} \;=\; \sum_{a,b}\left(\sum_{\mu,\nu}\mu_{ab}\hat{M}^{ab} + \nu_{ab}\hat{N}^{ab}\right)\rho(\mu,\nu) \geq 0$$

  we denote a “quantum” equilibrium distribution.
- Just as in deBroglie-Bohm theory we can have non-quantum distributions $\rho(\mu,\nu)$.

Quantum Equilibrium and Quantum Non-Equilibrium Distributions

- An example of a quantum non-equilibrium distribution $\rho(\mu,\nu)$ is one which doesn't yield a positive density matrix $\hat{\rho} \ngeq 0$.
- A positive density matrix is necessary to have a quantum equilibrium but not sufficient. There are many distributions $\rho(\mu,\nu)$ that yields the same density matrix. Not all distributions need to satisfy the constraints imposed by the Schrödinger equation.
- If it does not satisfy the Schrödinger equation it is also to be regarded as a non-equilibrium distribution.

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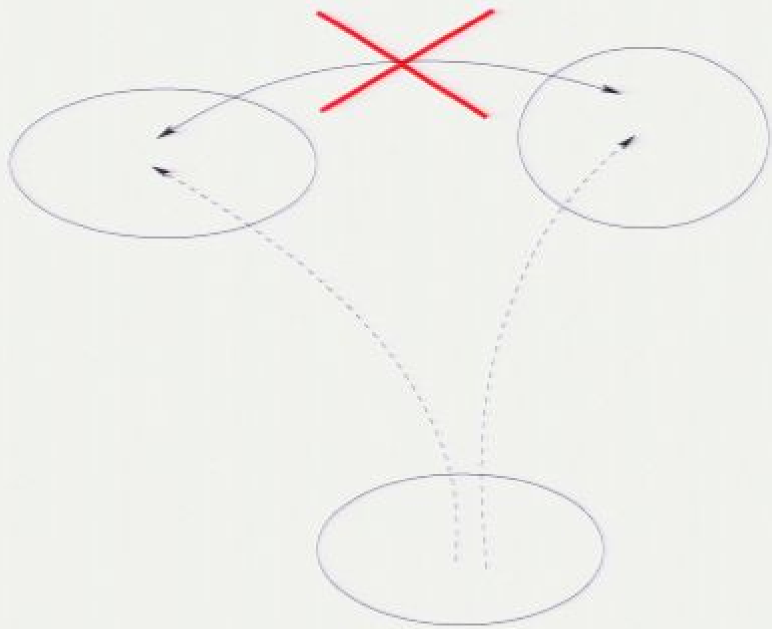
Generalization to Many Body Systems

- In order to generalize this construction to n particles in three dimensions we simply regards the indices a, b, i, j in the operators M_{ij}^{ab} and N_{ij}^{ab} as multi-indices: $a = \{(a_x^1, a_y^1, a_z^1), (a_x^2, a_y^2, a_z^2), \ldots, (a_x^n, a_y^n, a_z^n)\}$ and similarly for b, i, j.
- It is now clear that the non-local links are in fact links between different points in configuration space and not physical 3D space.
- Thus it seems that, just as in deBroglie-Bohm theory, configuration space is the real arena in this theory and not physical space.
- Although it is possible that we may excape the conclusion that the quantum state is not "real", it seems that configuration space seems to be the fundamental arena.

Measurement Theory

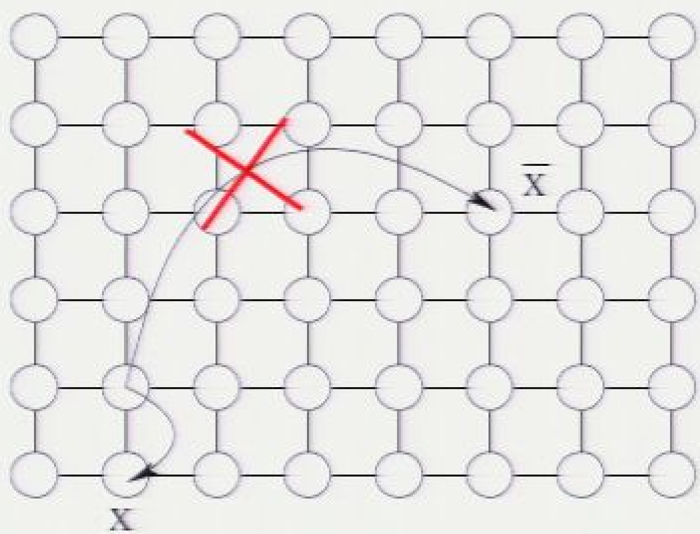
- Since the position of paricles is a definite property we will have no problem in developing models of quantum measurements. As Bell repeatedly stressed, as long as we get the correct statistical distribution for positions, then we will immediately reproduce all the quantum statistics for any measurement.
- Contextuality will work out in the same way as in deBroglie-Bohm theory, i.e. two incompatible measurements of a degenerate obsevable $\hat{A}$ can yields two outcomes simply because the "measurement" is not a passive intervention.

Further Necessary Requirements: Stability of Records



Stability of records

- Mathematically we can guarantee the stability of records if we forbid non-local jumps on configuration space: i.e. the corresponding transition rates are zero:
$T(X; \mu_{a\neq b}, \nu_{ab}|\bar{X}; \bar{\mu}_{a\neq b}, \bar{\nu}_{ab}) = 0.$



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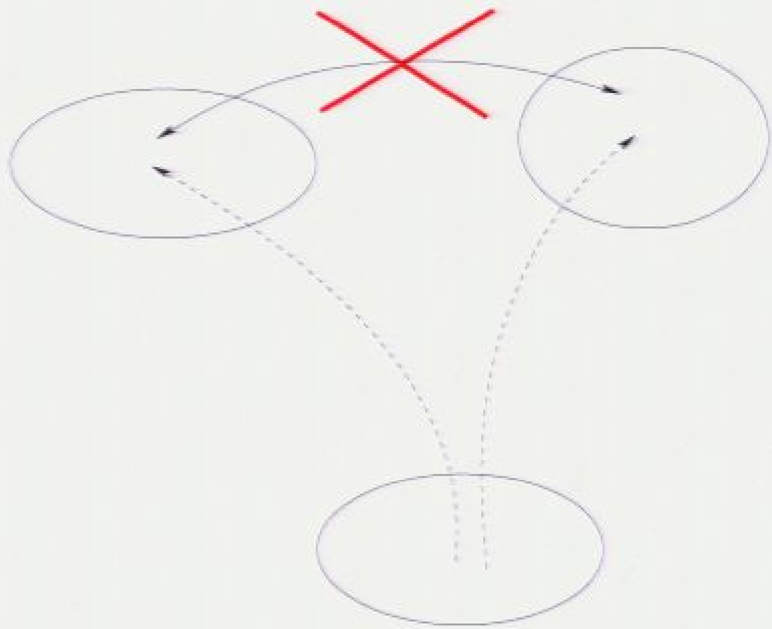

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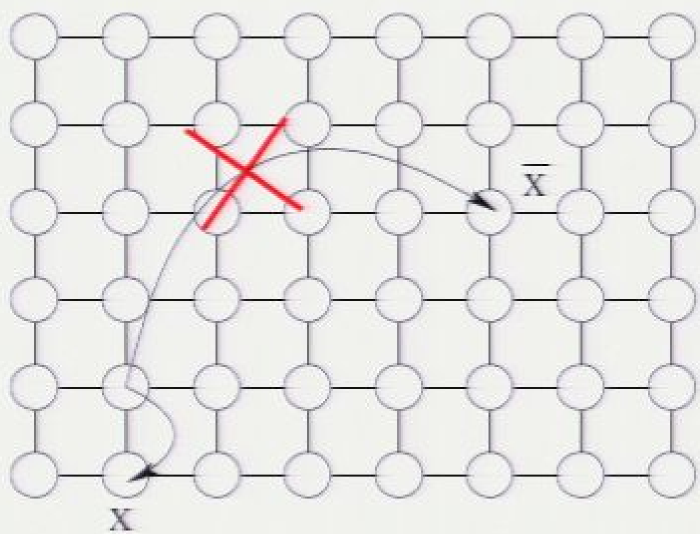

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Further Speculations

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The Absolute Spatial Structure in the Non-Relativistic Hamiltonian

- Where did all this structure of configuration space come from?
- It is all encoded in the non-relatistic Hamiltonian $\hat{H} = -\sum_k \frac{\hat{p}_k^2}{2m_k} + V(\hat{X})$.
- Let us consider a 1D lattice for simplicity. Then the discretized non-relativistic Hamiltonian looks like.

$$\hat{H} = -\frac{\hbar^2}{ml^2}\sum_a (\hat{M}^{a+1,a} - \hat{M}^{aa}) + \sum_a V_a \hat{M}^{aa}$$

where l is the lattice spacing.

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Mathematical Modelling

- First note that modelling two qubits require a Hilbert space of 4 dimensions. This means that we are going to have $4^{15} \approx 10^9$ ontic states in our model of the spin measurement.
- We implement different measurements by choosing different interaction Hamiltonians, i.e. different values of b_A.
- We can read off the b_A values from the equation

$$|\psi\rangle = c_+|z+\rangle \otimes |\hat{n} + c_-|z-\rangle \otimes |\hat{n}-\rangle$$

- We also need to mathematically precise about which property it is that will reveal the outcome. Above we took the z-property of the measurement qubit.

Modelling a General Spin Measurement

- Consider two qubits. One we regard as a model of a measurement apparetus and the other the system, i.e. the qubit we are performing the measurement on.
- We shall interpret the z property of the measuring qubit as revealing the outcome.
- A measurement constitutes an interaction between the apparatus and system, $\hat{H}_I$ say.

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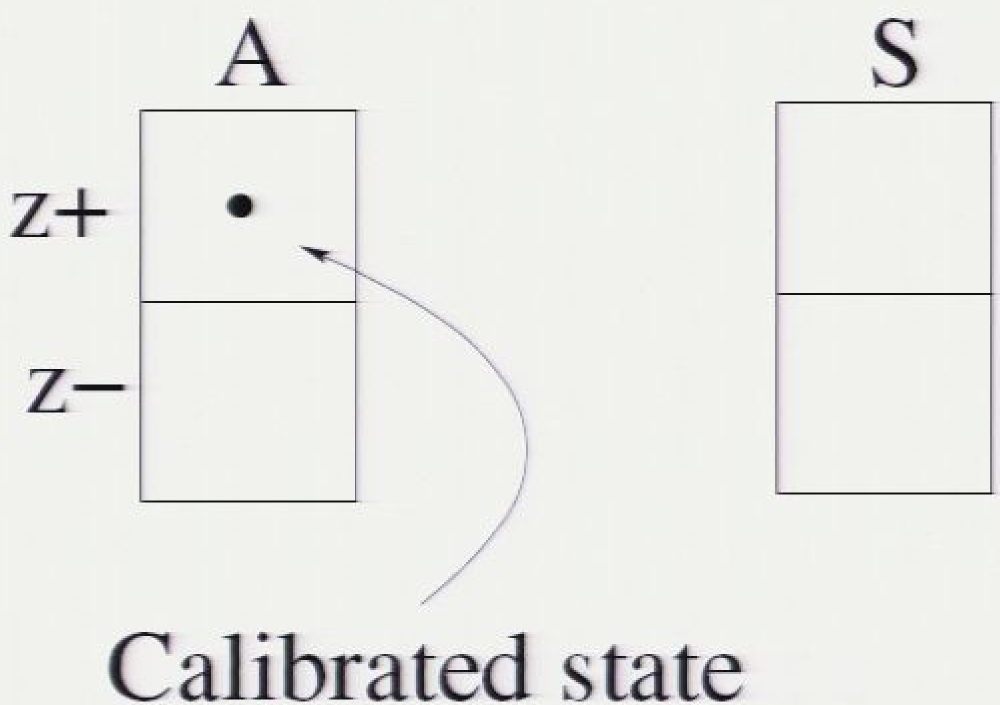
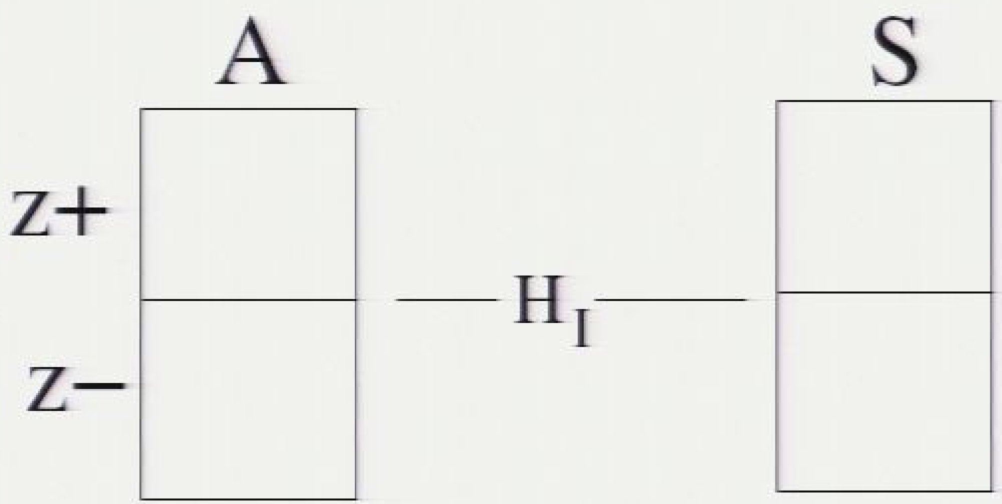


Measurement Interaction

$$|\psi\rangle \rightarrow U(t)|\psi\rangle$$



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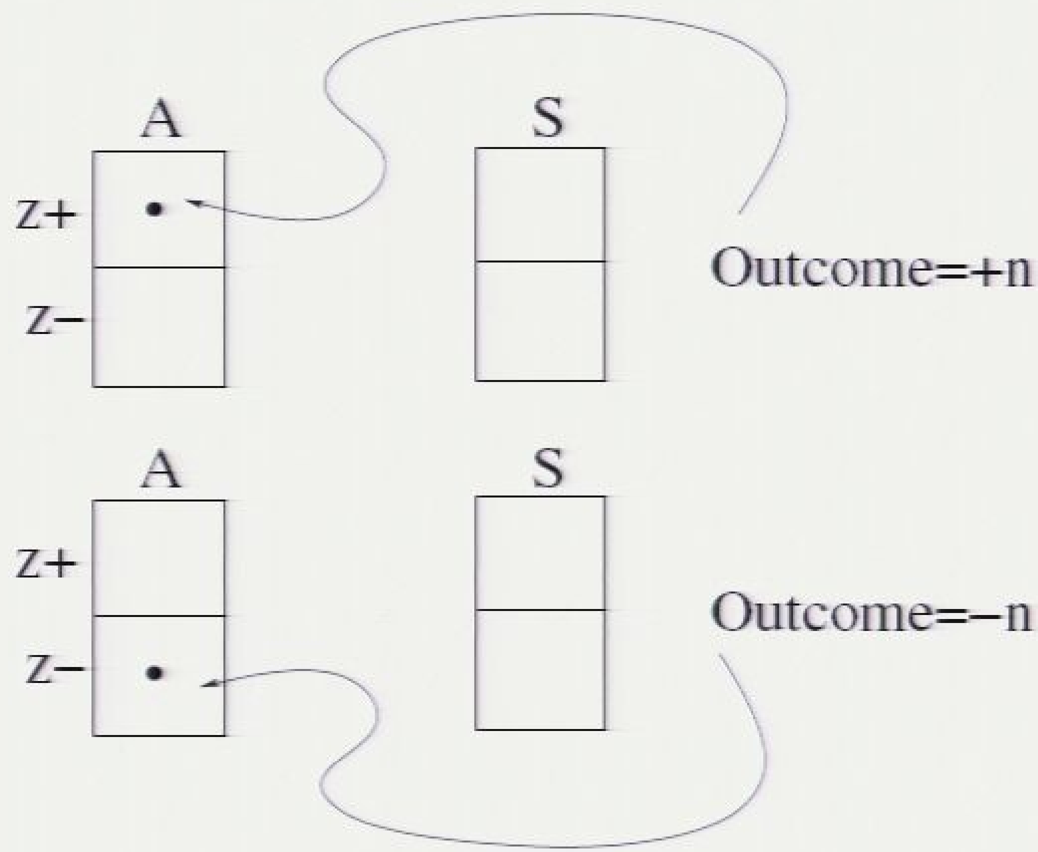

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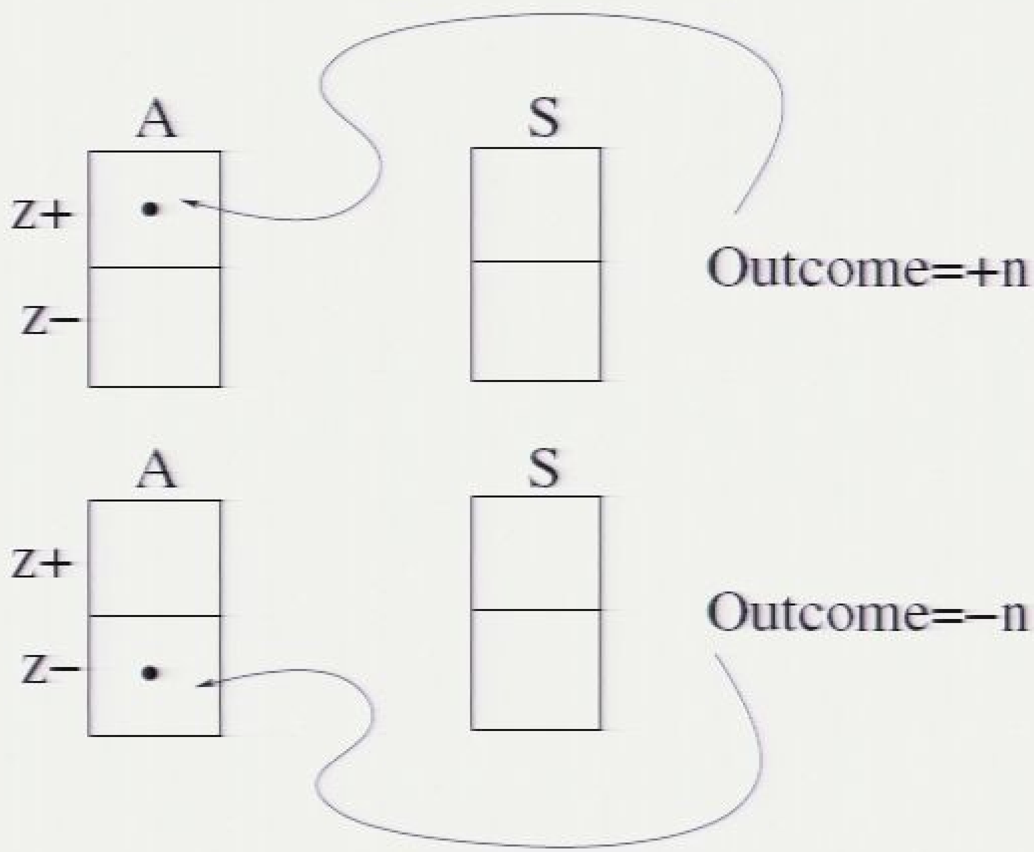

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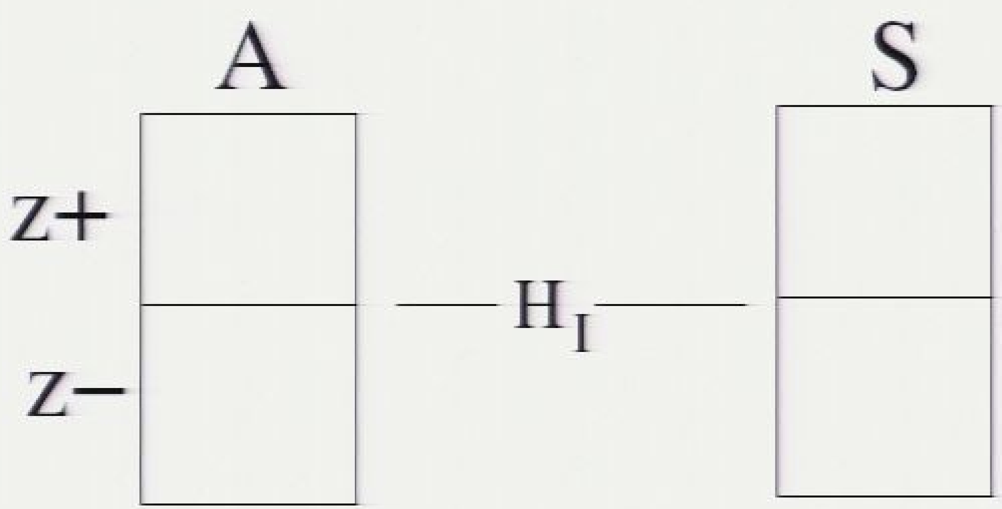

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Mathematical Modelling

- The operator $\sigma_z \otimes 1$ and T_{15} are simultaneously diagonal in the standard matrix representation. The $A = 15$ property has 4 distinct states $1, 2, 3, 4$. The first two corresponds to eigenvalue $+1$ of $\sigma_z \otimes 1$ and the last two eigenvalue -1.
- It is therefore natural to take the $A = 15$ property as the measurement 'needle'.
- Summarizing: If we find the $A = 15$ property to be 1 or 2 the we have outcome $+1$ and if it is 3 or 4 the outcome is -1.

Empirical Adequacy of the Spin Measurement Model

- Due to Hardy's ontological excess baggage theory we know that no ontological model wih finitely many ontic states ($\approx 10^9$ in our case) can reproduce all measurement statistics on a qubit.
- Somehow it must be the case that one cannot reproduce the Schödinger equation for all interaction Hamitonians.
- However, it seems like the larger Hilbert space dimension N we consider the more Hamiltonian's will be allowed. This is so because the number of transition rates grow much much faster than the number of constraints imposed by the Schrödinger equation.

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- But how close can we get to quantum predictions in our model? A recent paper (quant-ph/0805.1728) examines how many measurements can be modelled by M ontic states. They claim that the relationship is roughly linear. I.e. with M ontic states we can reproduce the statistics of M distinct measurements.
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Fibre bundle structure of the space of probability distributions $\rho(I; c_A)$

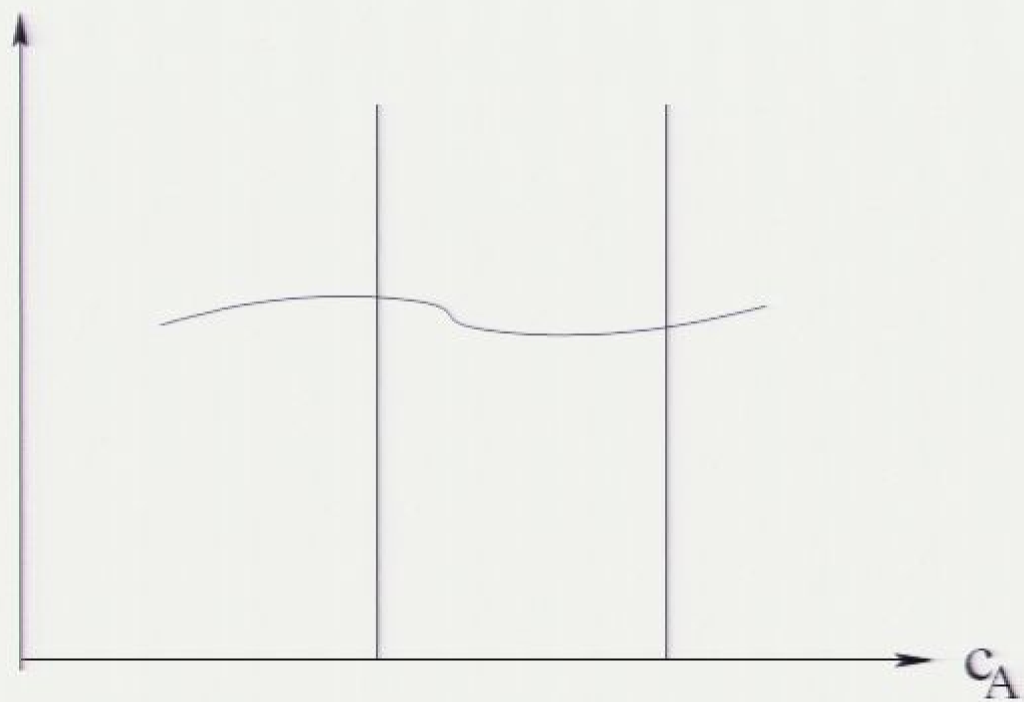

Figure: The space of probability distributions has a natural fibre bundle structure. To each value of c_A there is an equivalence class of different distributions $\rho(I; c_A)$.

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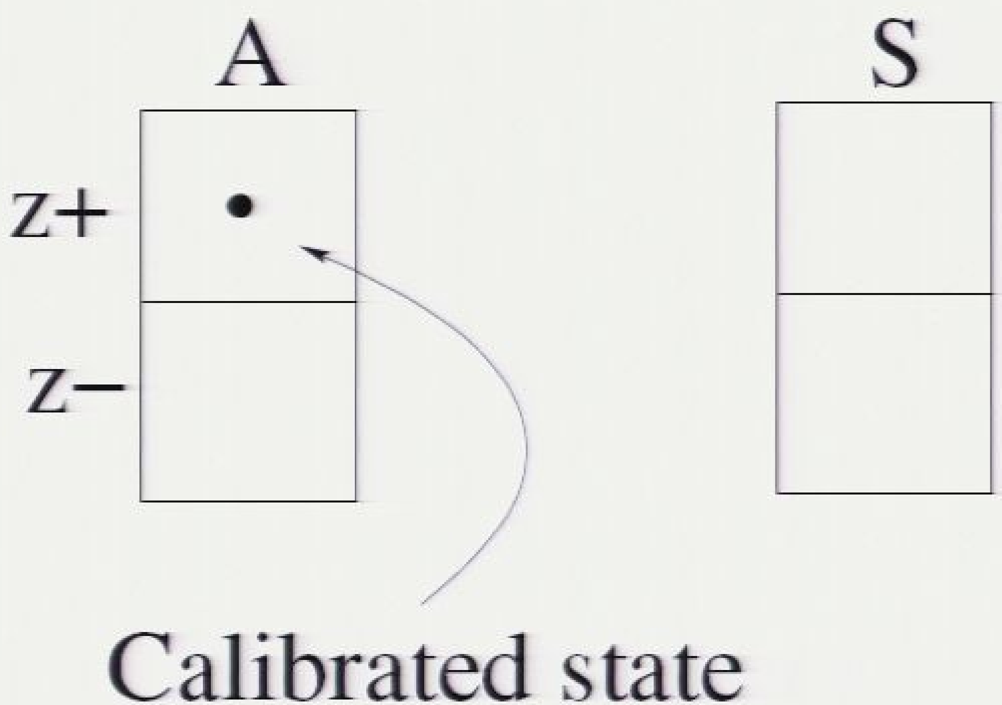

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