Title: Nuclear binding and the light quark masses â€“ Dynamics and constraints

Date: Jul 17, 2008 02:40 PM

URL: http://pirsa.org/08070028

Abstract: I will describe a method of understanding how the nuclear binding energies depend on the masses of the light quarks. This is useful in applications ranging from anthropic constraints to equivalence principle tests and bounds on the time variation on the quark masses.

Nuclear binding and light quark masses: Dynamics and constraints

Advertisement: (OT) Limits on spatial variation of Λ... from CMB

with Evan Donoghue

Main talk

1) Nuclear binding –heavy nuclei

2) Quark mass dependence

3) Anthropic constraints on quark masses

4) Oklo constraint (very rough and preliminary)

5) Equivalence principle (sketchy)

4 papers on related topics
2 solo
2 with Thibault Damour (one still virtual)

PI, July17, 2008

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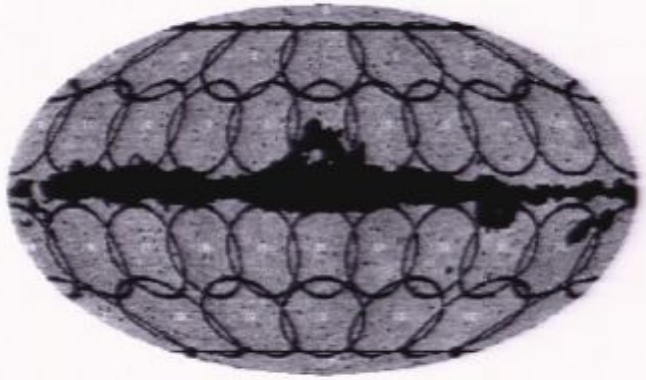
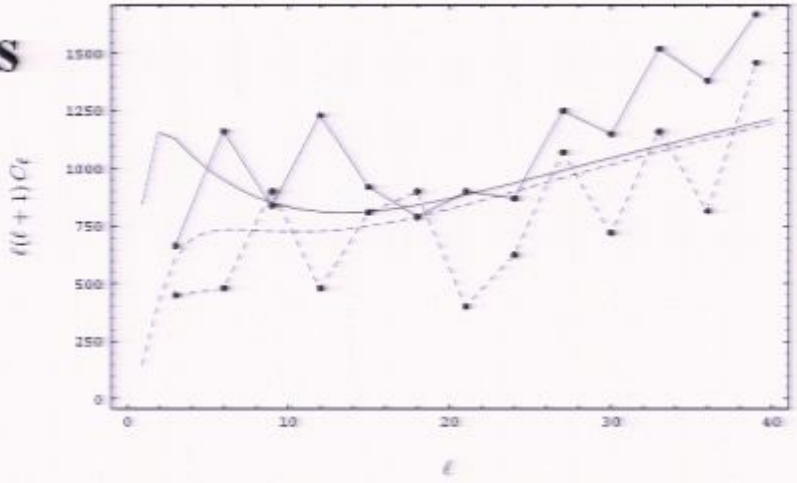
Eriksen et al demonstrate potential asymmetry in CMB power spectrum
- -stable, ~3σ
- -occurs in low ℓ data – up to $\ell \sim 40$



Can this be due to spatial variations of some parameters?
- especially cosmological constant?



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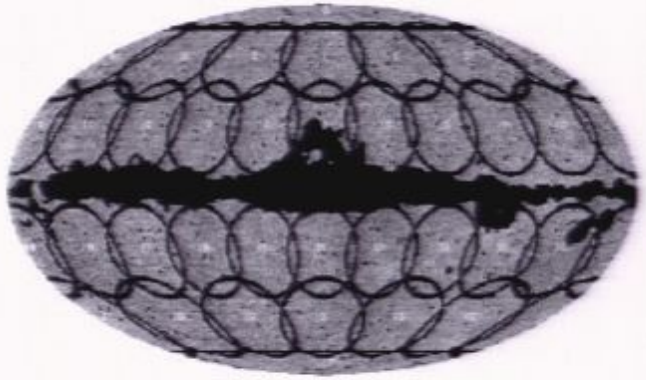

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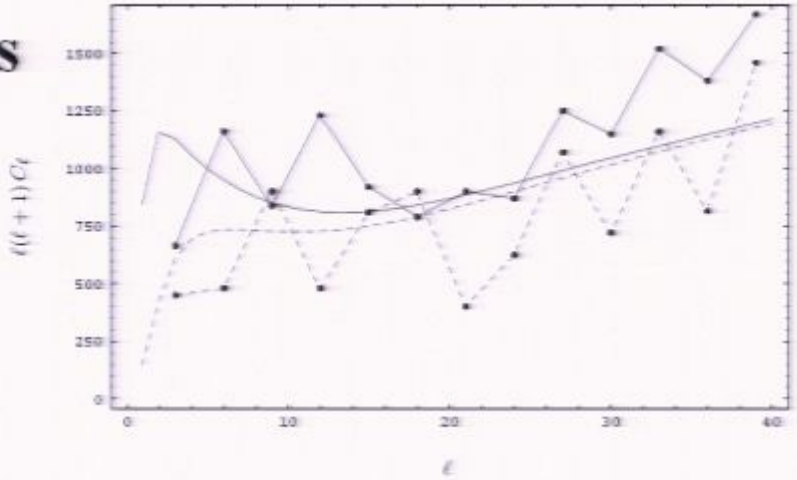

Answer = NO

Reason: **First acoustic peak** shows no spatial asymmetry
- variation of any parameters would shift peak

co – latitude	longitude	ℓ location	amplitude
0	0	223.3	5551
35	0	217.0	5235
35	60	215.8	5515
35	120	218.1	5909
35	180	207.5	6211
35	240	231.8	5231
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65	0	212.3	5957
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65	72	230.9	5410
65	108	224.5	5099
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65	180	221.5	5888
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115	36	222.3	5415
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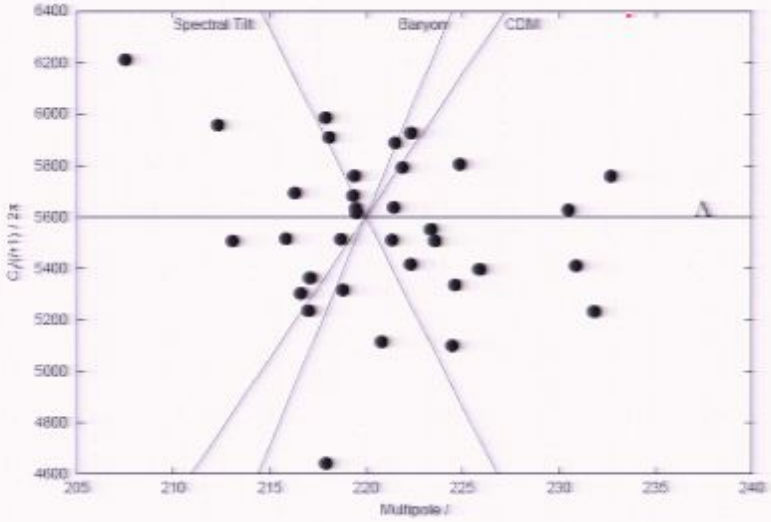
Table 2: HBG data on the height and location of the acoustic peak on patches centered on different locations on the sky. We thank the authors of



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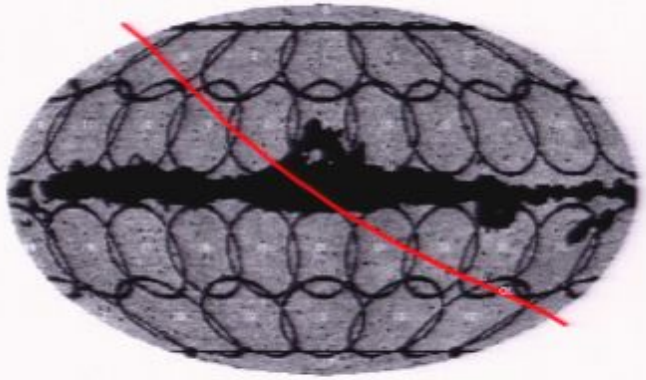

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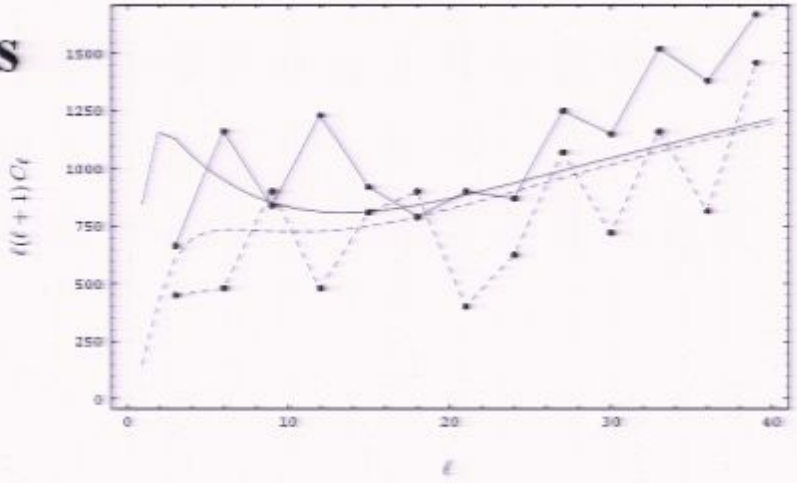

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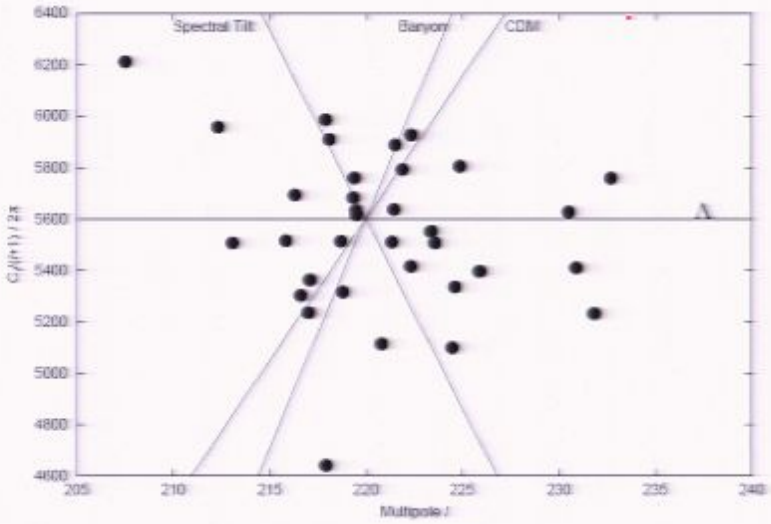

Result: Limits on spatial variation of Λ etc

lever arm ~ 2 x 13 Gly

parameter	Baryon	CDM	Λ	amplitude	tilt
η fit	0.076 ± 0.048	0.004 ± 0.11	0.055 ± 0.035	0.027 ± 0.017	0.03 ± 0.02
2σ bound	0.17	0.28	0.125	0.061	0.057

Table 3: Isotropy bounds when interpreted as variations of the underlying parameters. The first row of results gives the fit value of the fractional variation of each parameter, while the second row reports the 2 σ upper limit on the possible variation.

Need to seek explanation elsewhere

- transient effect in inflation – kinetic energy or extra field

JFD, Dutta, Ross and others

Nuclear binding

a) EFT and contact interactions
b) What is important?
c) Dispersive representation
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Traditionally described in terms of meson exchange

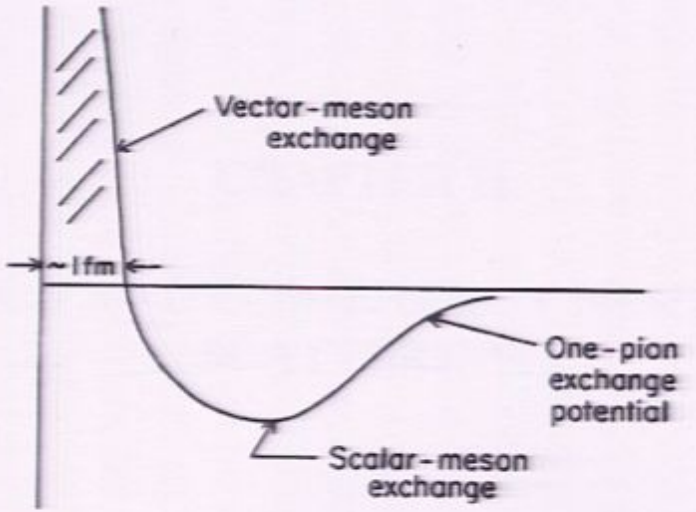

Fig. 2. Schematization of the nucleon-nucleon potential.

$$V(r) = \pm \frac{g^2}{4\pi} \frac{e^{-mr}}{r}$$

One pion exchange is not important for heavy nuclei
- key features are "sigma" and "omega" exchanges

I. Potentials → "contact" interactions

Nucleons are at very low energy ~ 10 MeV/nucleon

Effective field theory-

High energy effects (e.g. ρ,ω exchange) appear local at low energy

Example:

$$V(r) = -g_S^2 \frac{e^{-m_S r}}{4\pi r} \rightarrow -\frac{g_S^2}{m_S^2}\delta^3(r)$$

Then use contact interactions instead of potentials

- more general
- no assumptions about shape or origin

Description of contact interactions

Serot
Furnstahl
-alsoWalecka
Madland
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Nucleon densities:

$$\tilde{\rho}_{\rm s} \equiv \frac{\rho_{\rm s}}{f_\pi^2\Lambda} \equiv \frac{\overline{\psi}\psi}{f_\pi^2\Lambda}\,, \quad \longleftarrow \text{scalar}$$

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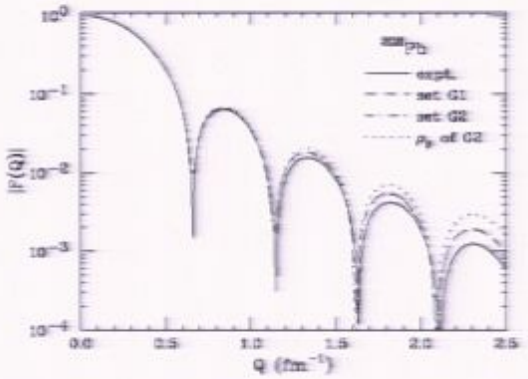
Some sample results:



Charge form factor of ^{208}Pb. The solid line is taken from experiment [77]. Form factors for the G1 and G2 parameter sets from Table I.

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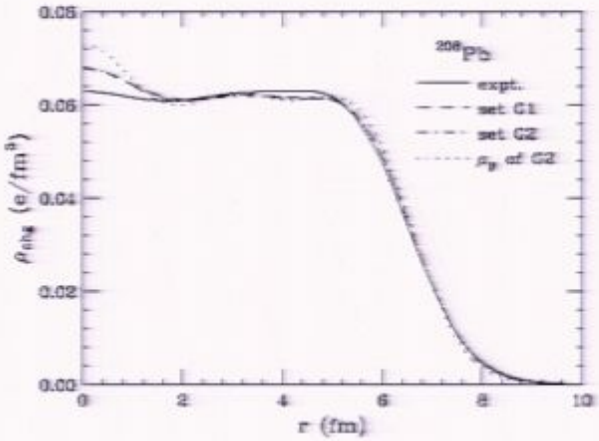
expt.	2.74	2.76	3.45	3.85	3.45	3.96	4.20	4.99	5.50	6.78



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Madland et al

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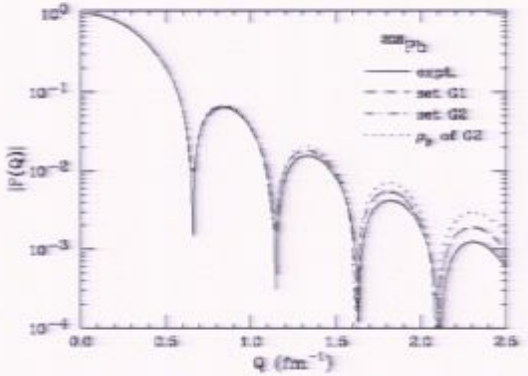

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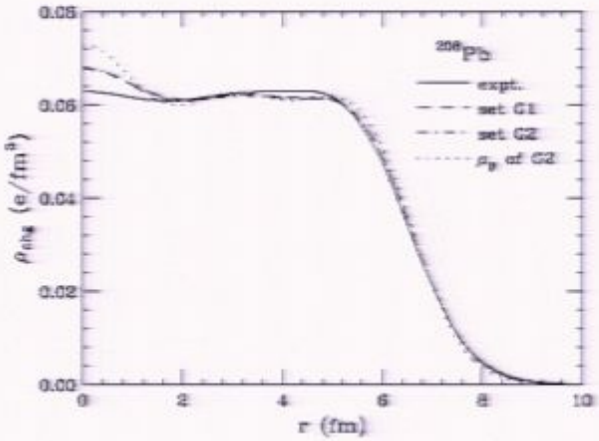

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Madland et al

Only a small number of terms important:

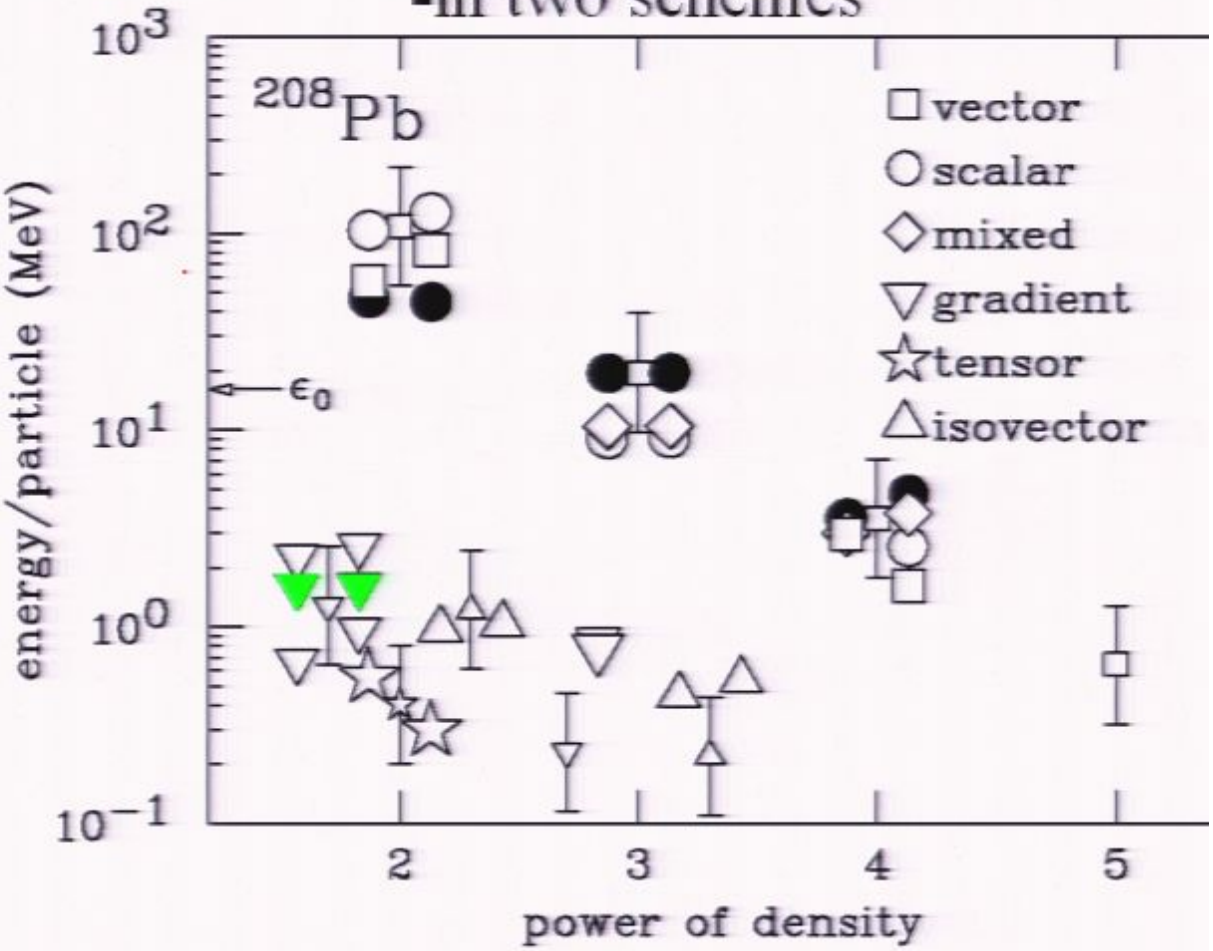
Serot
Furnstahl

Most contact terms have small effects

Most important are the "σ,ω" terms:

Scalar ~-100 MeV
Vector ~ +60 MeV

Magnitude of contributions:
-in two schemes



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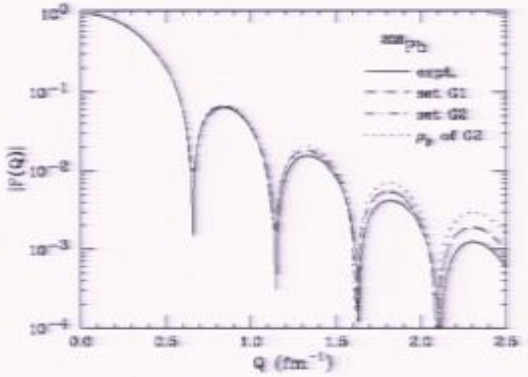

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G1	2.72	2.77	3.46	3.84	3.45	3.95	4.19	4.99	5.50	6.80
G2	2.73	2.77	3.46	3.84	3.45	3.95	4.19	5.00	5.50	6.80
PC	2.73		3.45		3.48		4.21		5.50	
expt.	2.74	2.76	3.45	3.85	3.45	3.96	4.20	4.99	5.50	6.78

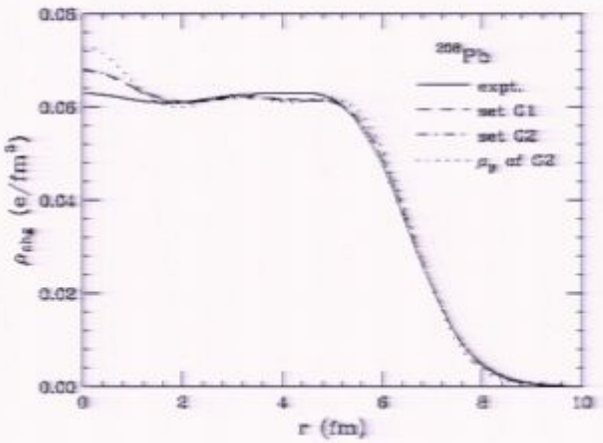

FIG. 7. Charge density of ^{208}Pb. The solid line is taken from experiment [77]. Charge densities are shown for the G1 and G2 parameter sets from Table I. Also shown is the point-proton density for set G2.

* PC = "point coupling" = contact interactions

Madland et al

Only a small number of terms important:

Serot
Furnstahl

Most contact terms have small effects

Most important are the "σ,ω" terms:

Scalar ~-100 MeV
Vector ~ +60 MeV

Magnitude of contributions:
-in two schemes

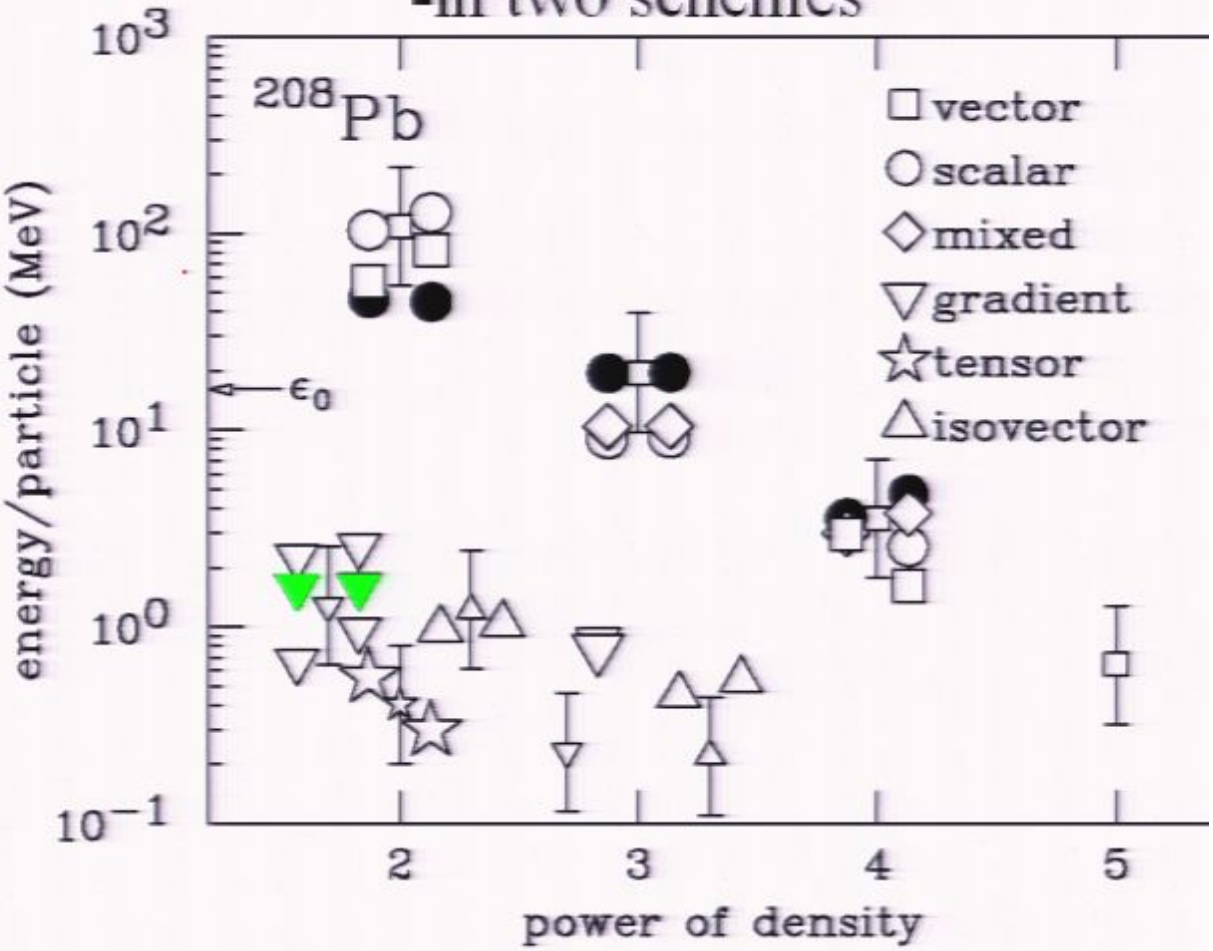

FIG. 2. Same as Fig. 1 for ^{208}Pb.

Results to be extracted:

Two main interactions: -both ρ^2

Attractive scalar contact term

Repulsive vector contact term

$$H = G_S\left(\overline{N}N\right)\left(\overline{N}N\right) + G_V\left(\overline{N}\gamma^{\mu}N\right)\left(\overline{N}\gamma_{\mu}N\right)$$

Known contributions from each

-of order 100 MeV/nucleon

Significant cancellation occurs

Primary effects in nuclear binding

- know the main contributions to binding energy

$$\eta_v = \frac{G_V}{G_V^{\text{physical}}}$$

Let:

$$\eta_s = \frac{G_S}{G_S^{\text{physical}}}$$

Then for ^{208}Pb:

$$\frac{\text{B.E.}}{A} \sim -104\eta_s + 57\eta_v + 18 + \text{K.E}$$

or in general:

$$\frac{\text{B.E.}}{A} = -(120 - \frac{97}{A^{1/3}})\eta_s + (67 - \frac{57}{A^{1/3}})\eta_v + \text{residual terms}$$

Constructing the contact interactions

Recall:

$\delta^3(r)$ is Fourier transform of a constant

V(q=0) gives the coefficient of the contact potential

$-q^2$ dependence gives the gradient expansion

$$H = G_S\left(\overline{N}N\right)\left(\overline{N}N\right) + G_V\left(\overline{N}\gamma^{\mu}N\right)\left(\overline{N}\gamma_{\mu}N\right)$$

Consider the scattering amplitude in different channels

$M_S(q^2) = G_s + H_s\, q^2 + \ldots.$

Need to understand the low energy scattering processes

Rigorous "old" technology -dispersion relations

Cottingha
Vinh Mau

NN $\longrightarrow$NN amplitude satisfies t-channel dispersion relation

-cross reaction to $N\bar{N} \to N\bar{N}$

-decompose in partial waves

- write dispersion relation

-first cuts in isoscalar S,P waves at $4\,m_\pi^2$, $9\,m_\pi^2$

$$M(s,t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} d\mu^2 \; \frac{ImM(s,\mu^2)}{\mu^2 - t - i\epsilon}$$

Take limit of low energy ($s \longrightarrow 4\,m_N^2$) to define potential in each channel and use:

$$\rho_S(\mu) = ImV_S(q = i\mu)$$

Using dispersive representation:

Momentum space potential:

$$V_S(q^2) = \frac{2}{\pi}\int_{2m_\pi}^{\infty} d\mu\ \mu \frac{\rho_S(\mu)}{\mu^2 + q^2}$$

Contact interaction:

$$G_{S,V} = \frac{2}{\pi}\int_{(2m_\pi, 3m_\pi)}^{\infty} \frac{d\mu}{\mu}\ \rho_{S,V}(\mu^2)$$

For orientation: sigma potential from $\rho(\mu) = -\pi g_\sigma^2 \delta(\mu^2 - m_\sigma^2)$

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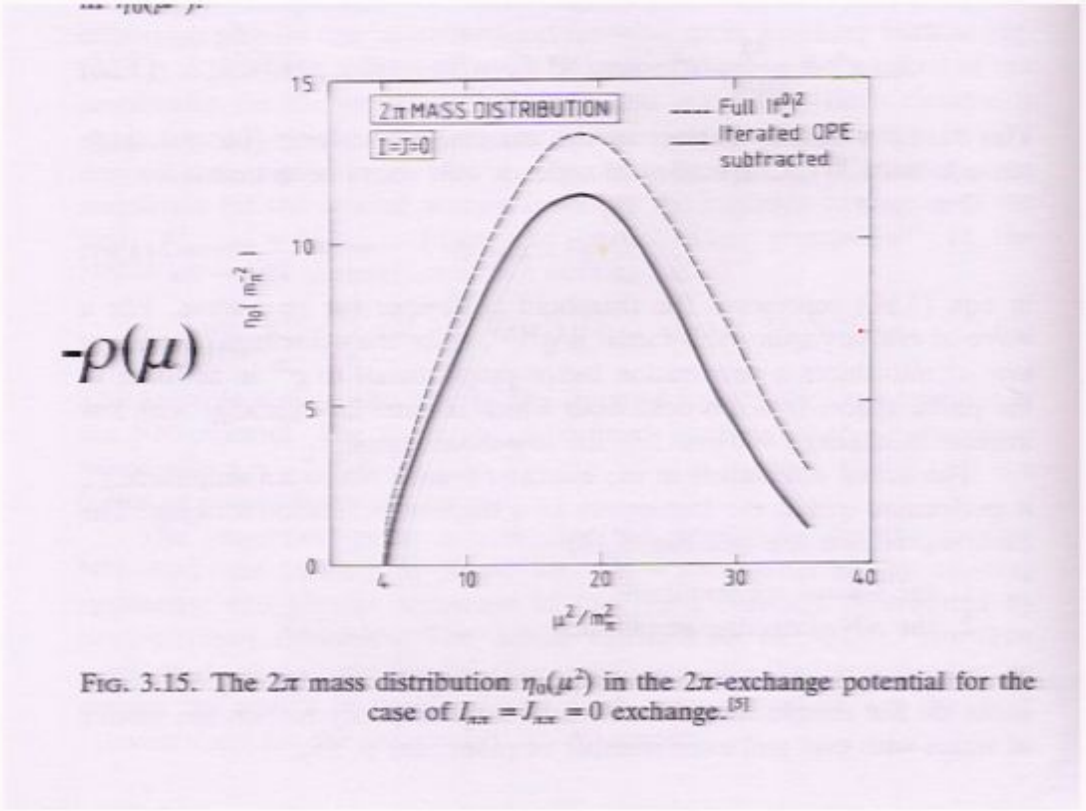
The experimental information on I=0, S=0 channel



$-\rho(\mu)$

FIG. 3.15. The 2π mass distribution $\eta_0(\mu^2)$ in the 2π-exchange potential for the case of $I_{\pi\pi} = J_{\pi\pi} = 0$ exchange.[51]

from "Paris potential"

fits to scatterin data

This is the "sigma"

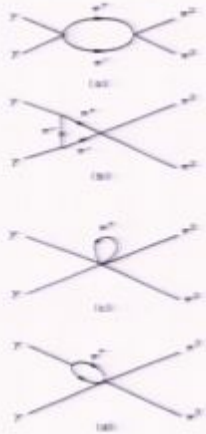
We can calculate this in Chiral Perturbation Theory

We have had reasonable success calculating pionic processes

Sample: $\gamma\gamma \Longrightarrow \pi^0\,\pi^0$

General structure:

- photons couple to charged pions
- charged pions rescatter to neutral pions
- rescattering determined by $\pi\pi$ scattering phases



$$Amp(\gamma\gamma \to \pi\pi) = P(\mu)\Omega(\mu)$$

$$\Omega(\mu) = exp[\frac{\mu^2}{\pi}\int \frac{ds}{s}\,\frac{\delta(s)}{s-\mu^2}]$$

Omnes function

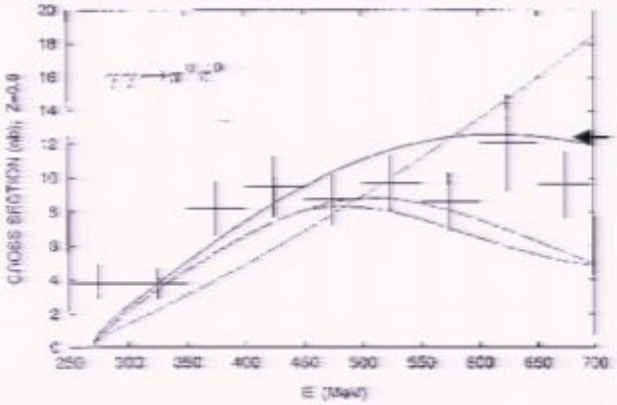
$\gamma\gamma \Longrightarrow \pi^0 \pi^0$

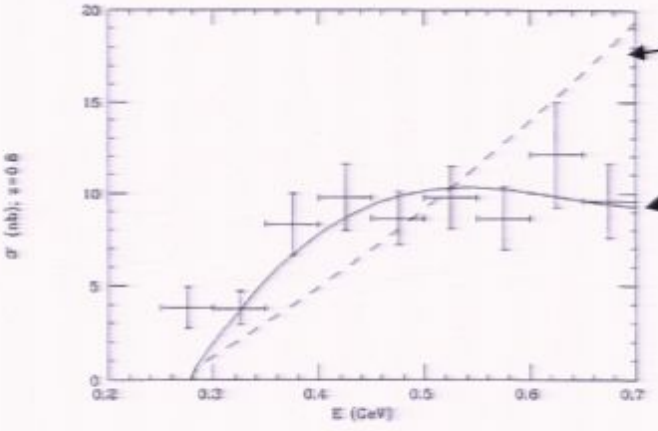
JFD and Holstein

Starts at one loop
- -finite – no free parameters
- -result grows with energy
- -violates unitarity bound around 600 MeV

Onmes solution
- -respects unitarity bound
- -agrees with data
- -also closely matches 2 loop result

Gasser et al





This is the analogue of the nuclear calculation to follo

Interactions of pions with nucleons:

$$= \frac{g_A}{F_\pi}\gamma_\mu\gamma_5$$

$$= \frac{1}{F_\pi^2}\left[2c_1 m_\pi^2 + c_2 E_1 E_2 + c_3 q_1 \cdot q_2\right]$$

- c_1 , c_3 are known low energy constants from NN$\pi\pi$ interactions

$c_1 = -(0.5 \longrightarrow 0.9)$ GeV^{-1} ,

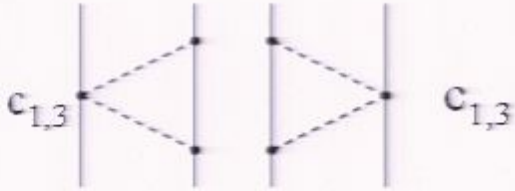
$c_2 = 3.3 \pm 0.2$

$c_3 = -(3.9 \longrightarrow 5.3)$ GeV^{-1}

Chiral calculation via spectral function:

Epelbaum, Glöckle, Meissn
Kaiser Brockmann, Weise

Leading chiral loop – called NNLO



$$\rho_S^{\rm NNLO}(\mu) = \frac{3g_A^2}{64F_\pi^4}\left[4c_1 m_\pi^2 + c_3(\mu^2 - 2m_\pi^2)\right]\frac{(\mu^2 - 2m_\pi^2)}{\mu}\,\theta(\mu - 2m_\pi)$$

Kaiser

Also next contribution: "NNNLO"

$$\rho_S^{\rm NNNLO}(\mu) = -\frac{3}{32\pi F_\pi^4}\sqrt{1-\frac{4m_\pi^2}{\mu^2}}\,\theta(\mu - 2m_\pi) \qquad (38)$$
$$\left(\left[4c_1 m_\pi^2 + \frac{c_2}{6}(\mu^2 - 4m_\pi^2) + c_3(\mu^2 - 2m_\pi^2)\right]^2 + \frac{c_2^2}{45}(\mu^2 - 4m_\pi^2)^2\right)$$

Including rescattering:

Gasser JFD Leutwyl

- elastic rescattering
- single channel problem

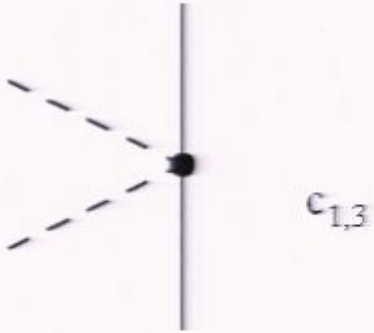
reaction $N\bar{N} \to \pi\pi$ at $2m_\pi < E < 1\text{GeV}$

rescattering only in elastic channel $\pi\pi \longrightarrow \pi\pi$

Solution:

$$\mathcal{M}(N\bar{N} \to \pi\pi) = P(\mu)\Omega(\mu)$$

$$\Omega(\mu) = exp[\frac{\mu^2}{\pi}\int \frac{ds}{s}\,\frac{\delta(s)}{s-\mu^2}]$$ Omnes function



Approximation method:

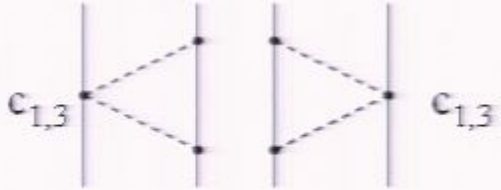
Match chiral prediction to Omnes solution order by order

Experience – this provides extrapolation to higher energy

Matching to the chiral calculations:

Full result must agree with chiral calculation at low energy
-NN$\pi\pi$ vertex determined by $c_{1,3}$

Modify Im$V(\mu)$ calculations:

NNLO 

$$\Omega(\mu) \quad + \quad \Omega^{\dagger}(\mu)$$

"NNNLO" 

$$|\Omega(\mu)|^2$$

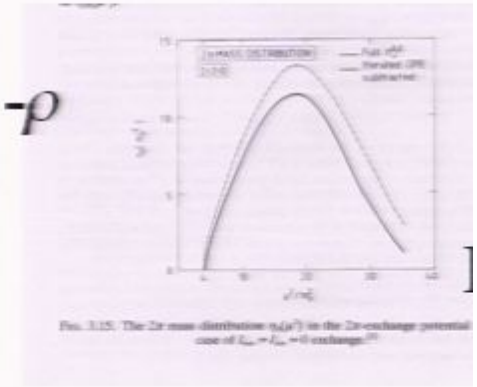
Result:

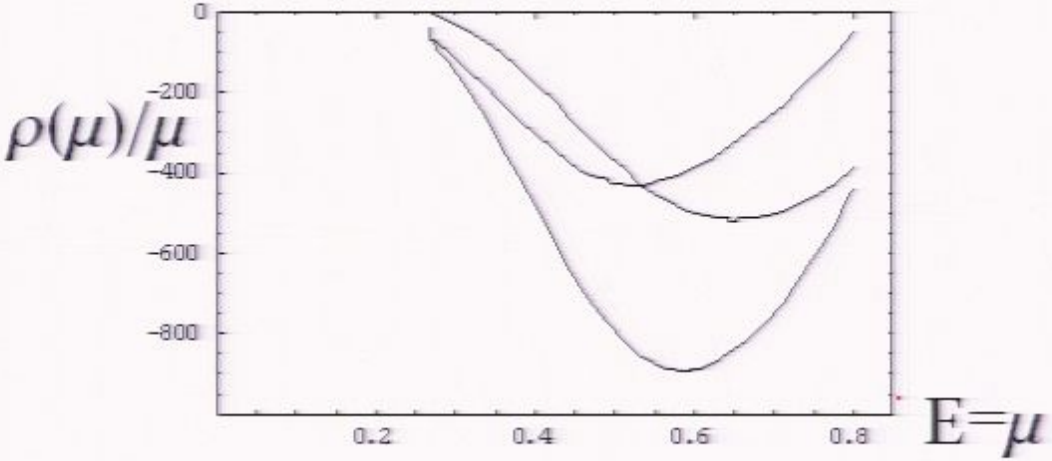
$$\rho_S(\mu) = \rho_S^{NNLO} Re\Omega(\mu) + \rho_S^{NNNLO} |\Omega(\mu)|^2$$

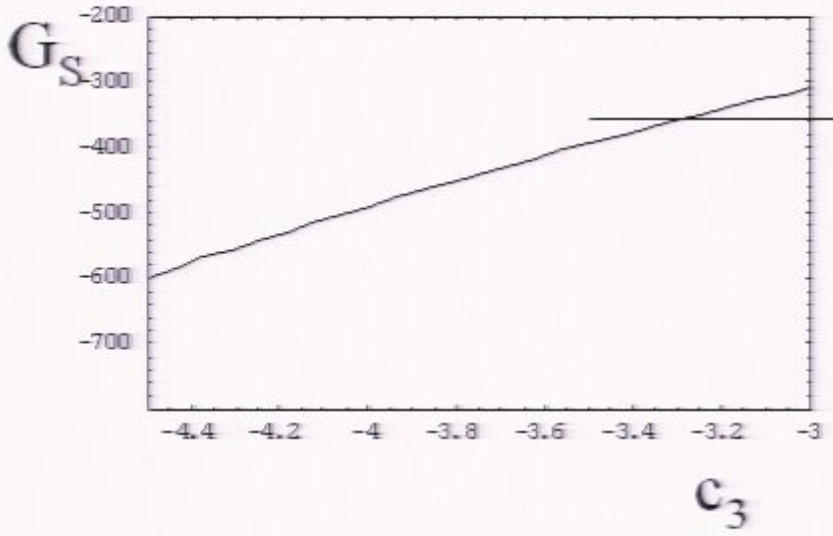
Results:

Recall:

$-\rho$







favored value

Easily reproduces correct shape and strength!

Reproduces "σ potential" very closely

Recall momentum space dispersion relation:

$$V_S(q^2) = \frac{2}{\pi}\int_{2m_\pi}^{\infty} d\mu\ \mu \frac{\rho_S(\mu)}{\mu^2 + q^2}$$

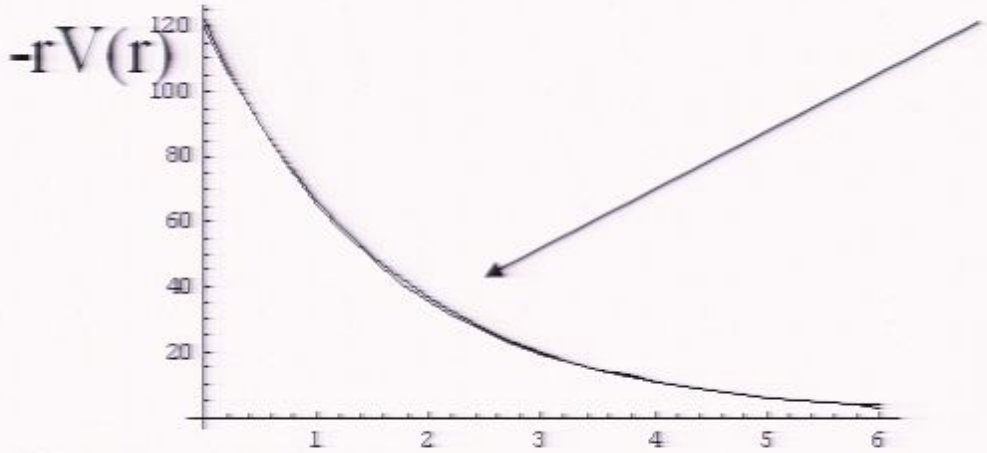
Define coordinate space potential by F.T.

$$V_S(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\ \mu\, e^{-\mu r}\ \rho_S(\mu)$$

Sigma potential

$$V(r) = \pm\frac{g^2}{4\pi}\frac{e^{-mr}}{r}$$

vs ChPTh $\rho(\mu)$



The modern story of the sigma:

Consequence of Goldstone dynamics

Caprini, Colangelo Leutwyler

ChPTh describes $\pi\pi$ scattering using only F_π, m_π

- analytic continuation reveals pole far from real axis

$$M_\sigma = 441\ ^{+16}_{-8}\ \text{MeV}\,, \quad \Gamma_\sigma = 544\ ^{+18}_{-25}\ \text{MeV}$$

-would appear in **any** chiral theory

- sigma does not drive $\pi\pi$ scattering, it is secondary effect

In nuclei:

- similar result – fundamental sigma is not driving feature
- chiral dynamics control the spectral function
- uses $\pi\pi$ phase shifts, so sigma is "present" far from real axis
- but key features are couplings of pions and the pion mass

Overall: More understandable picture

Only a few numbers are important
- contact interactions
- effective field theory treatment

Contact interactions have simple ingredients
- dispersive representation

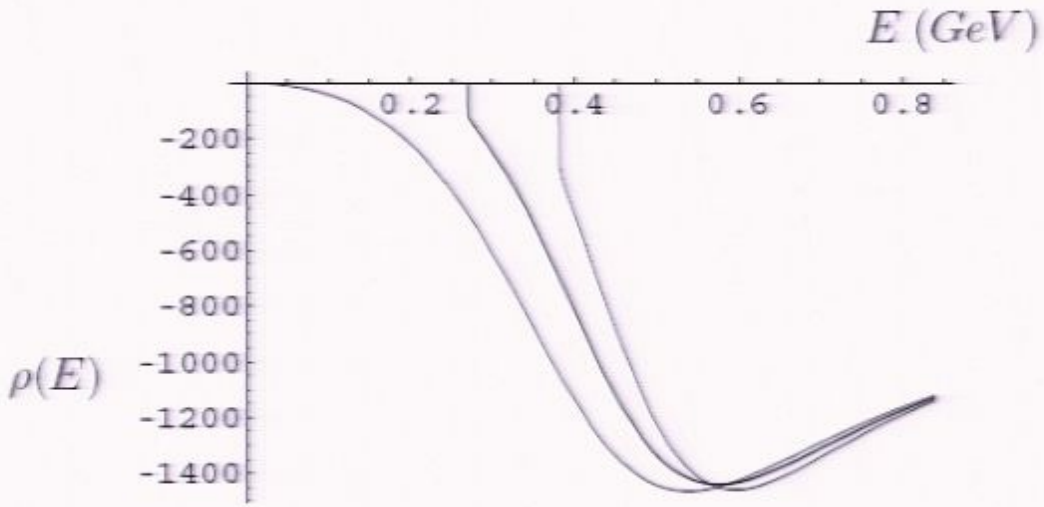
"Sigma" channel is calculable with reasonable control
- no need to postulate a resonance at 600 MeV
- strength is reasonable

Application: m_q dependence of binding

Reasonable control over the ingredients to this calculation:

Main difference is kinematic

-in region where we trust calculation



Note: Only discuss scalar channel – other effects studied but sma

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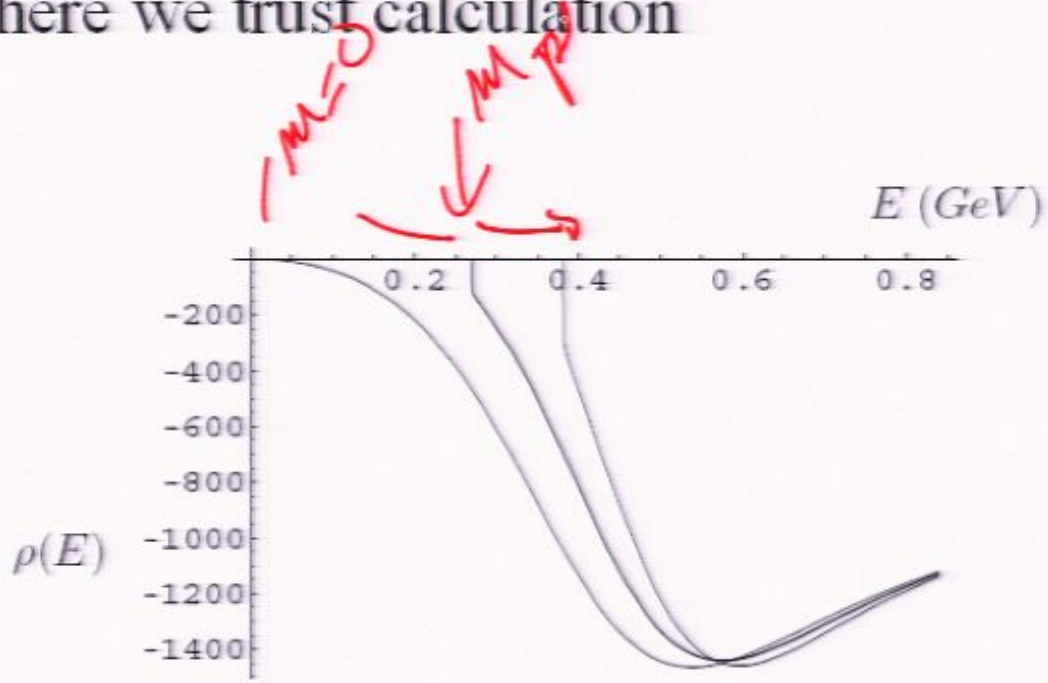


Note: Only discuss scalar channel – other effects studied but sma

Two possible pathways:

1) **"Pure" ChPTh**

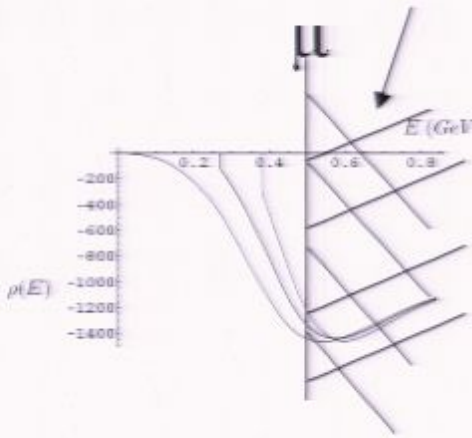
-treat low energy region with chiral methods

- find **enhanced** sensitivity

-use local constant for high energy region

- assume normal sensitivity m_π^2/Λ^2

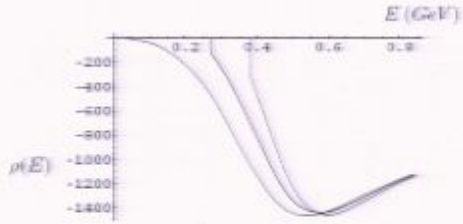
$G_0(\mu$



2) **Full treatment through "sigma" region**

-include effects up up to 0.85 GeV

-include all known mass effects



Essentially identical results

Spectral functions depend on quark masses:

Epelbaum, Glöckle, Meissn
Kaiser Brockmann, Weise

NNLO

$$\rho_S^{\rm NNLO}(\mu) = \frac{3g_A^2}{64F_\pi^4}\left[4c_1 m_\pi^2 + c_3(\mu^2 - 2m_\pi^2)\right]\frac{(\mu^2 - 2m_\pi^2)}{\mu}\,\theta(\mu - 2m_\pi)$$

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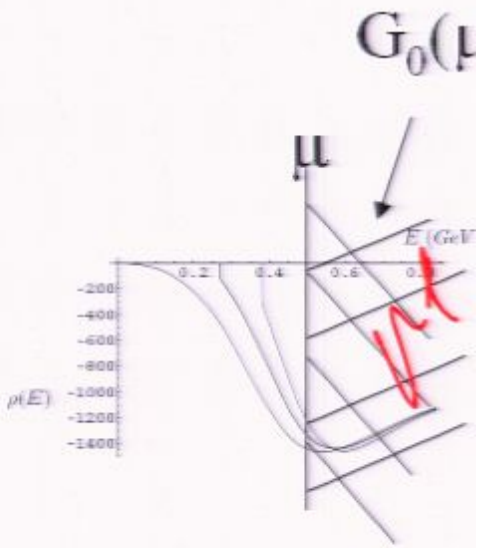

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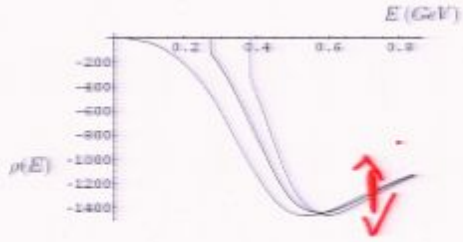
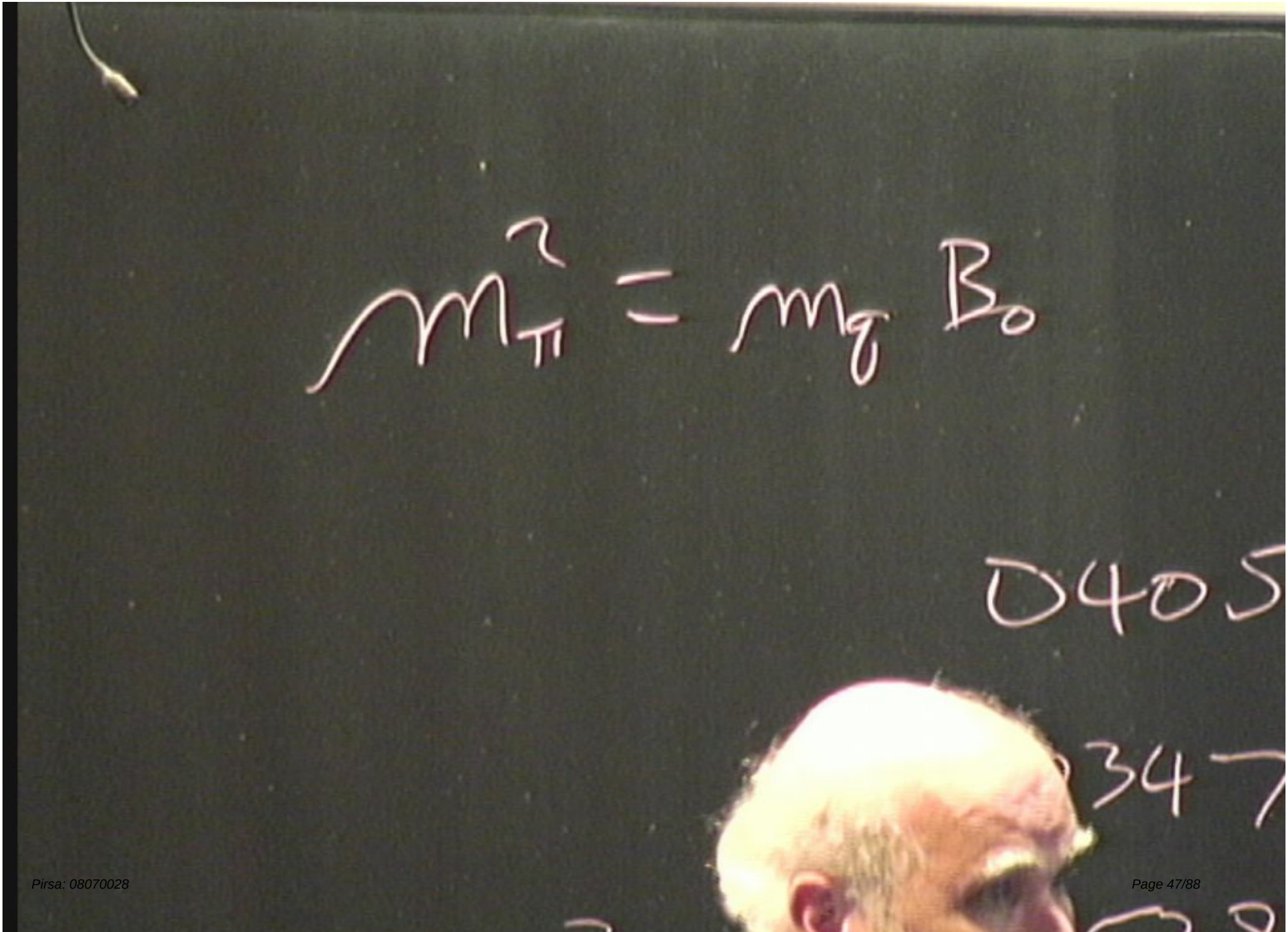


Essentially identical results


$m_\pi^2 = m_q B_0$
0405
347

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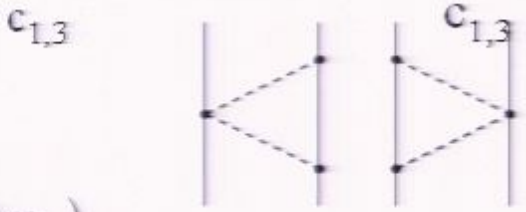

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Kaiser

"NNNLO"

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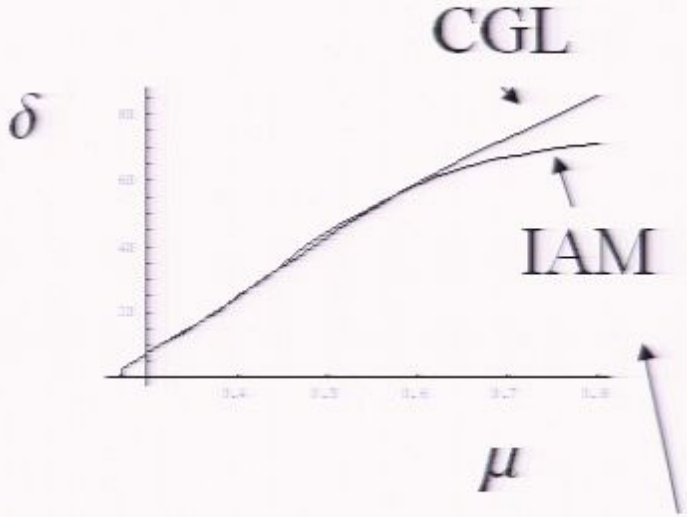
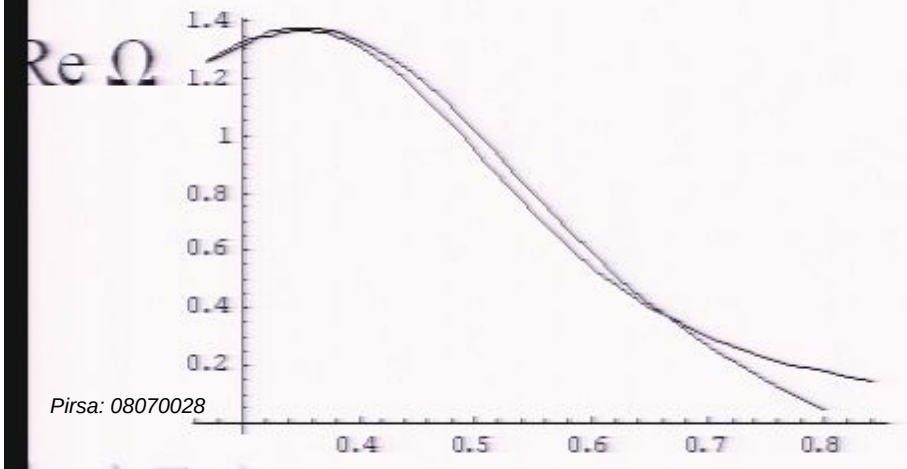
Pion scattering

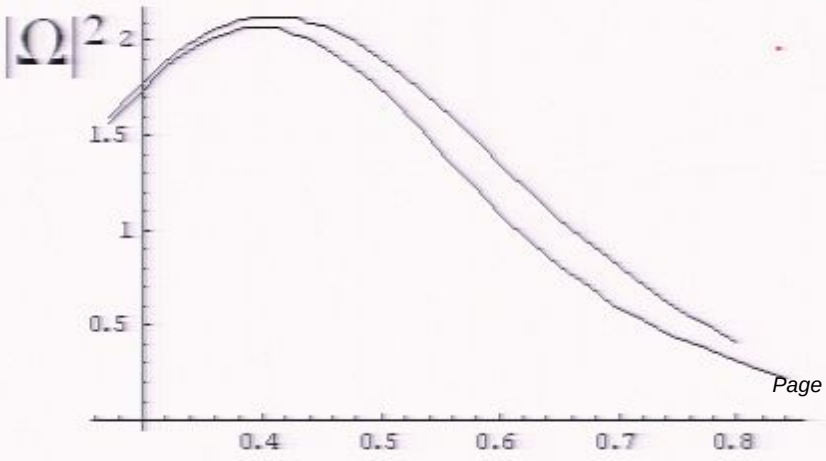
In order to vary mass, I use "inverse amplitude method" for scattering information

$$T_{00} = t_2 + t_4 + ... \rightarrow \frac{t_2^2}{t_2 - t_4}$$

exactly satisfies unitarity
reasonable representation of CGL results

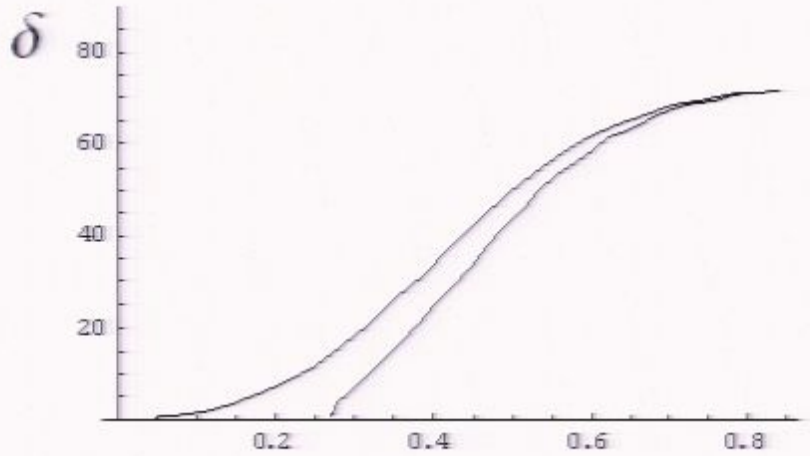




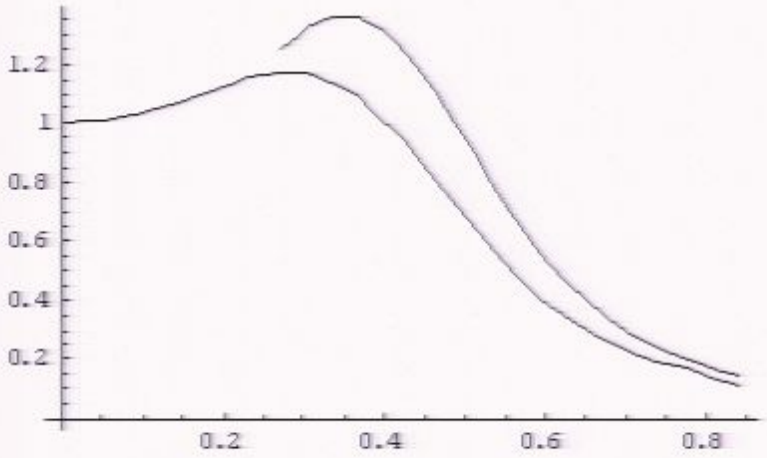


Omnes function – physical vs chiral limit

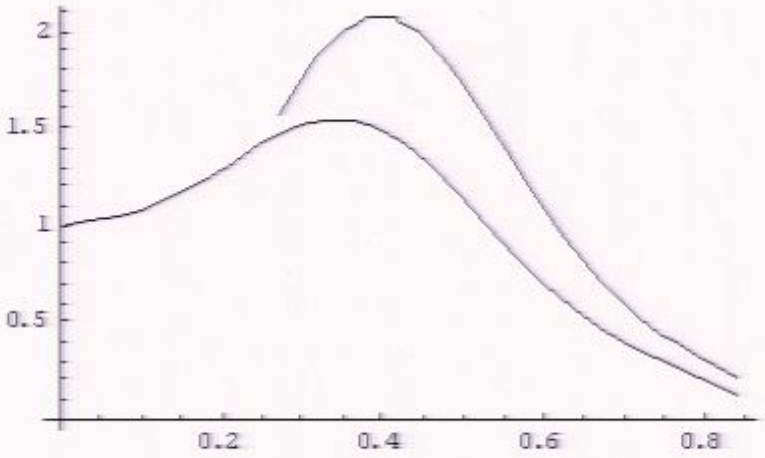
Phase shift



Re Ω



$|\Omega|^2$



major difference is due to lower threshold

Dependence of pion coupling on quark mass

Pion coupling is g_A/F_π, with known behavior

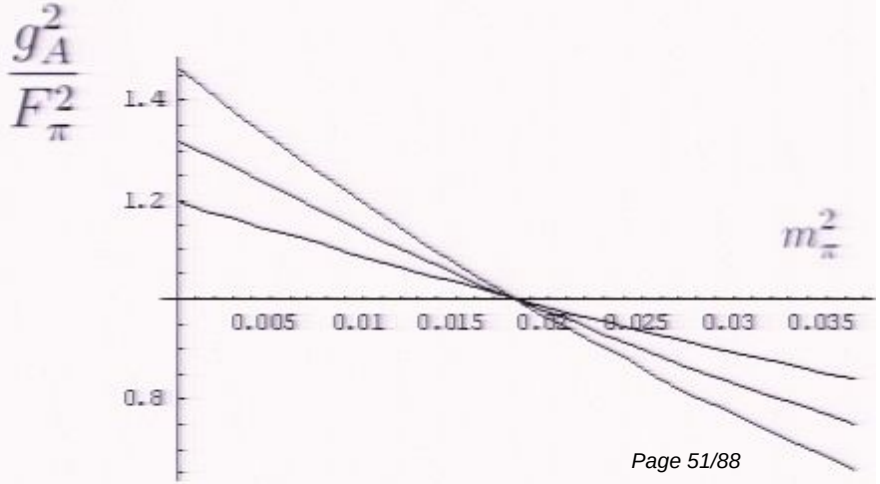
$$F_\pi = F_0\left[1-\frac{1}{16\pi^2F_0^2}m_\pi^2\left(\ln\frac{m_\pi^2}{m_{ph}^2}+\bar{l}_4\right)\right]$$

$$g_A = g_0\left[1-\frac{2g_0^2+1}{16\pi^2F_0^2}m_\pi^2\ln\frac{m_\pi^2}{m_{ph}^2}-\frac{g_0^2m_\pi^2}{16\pi^2F_0^2}+\frac{4m_\pi^2\bar{d}_{16}}{g_0}\right]$$

many references

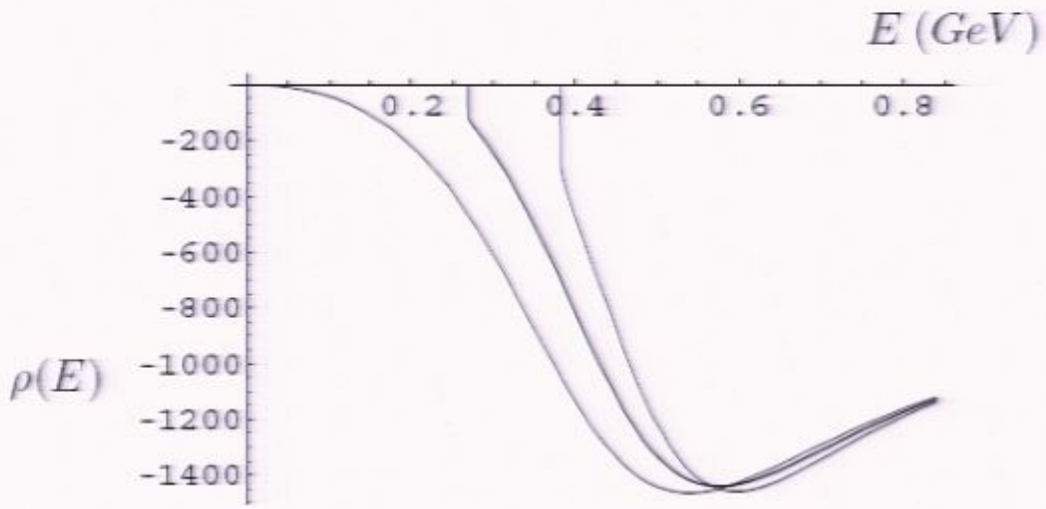
with $\bar{l}_4 = 4.4 \pm 0.2$ and $\bar{d}_{16} = -1.0 \pm 0.7$

Uncertainty of d_{16} is largest uncertainty



Result is mostly related to kinematic threshold

Threshold region is highly reliable:



Significant effect

Numerical results (c_3= -3.6) :

Chiral limit: G_S = - 427±26

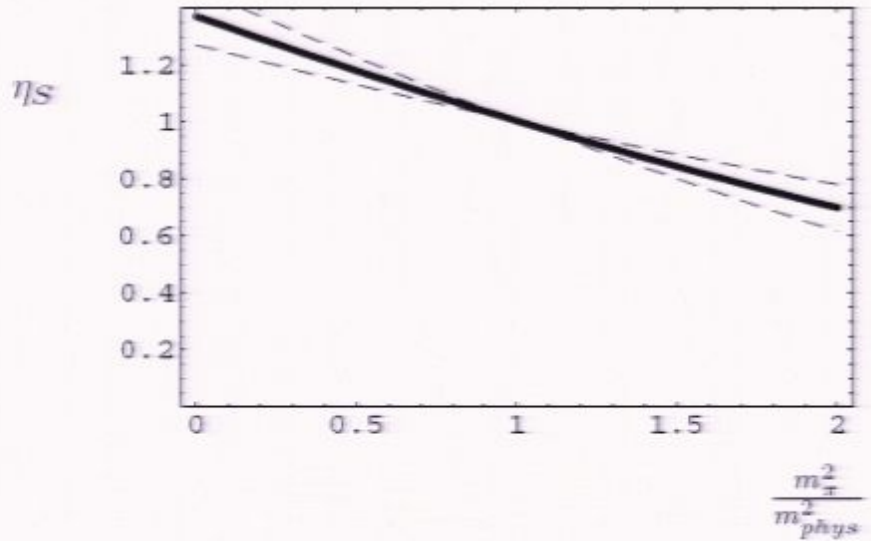
Physical limit: G_S = -380

$\mathbf{m_\pi^2 = 2\, m_{phys}^2}$: G_S= - 343±15

12% change

10% change

quoted error bar due to uncertainty in m_q dependence in g_A



Dependence of binding energy on pion mass

Recall: Large scalar contribution

$$\frac{\text{B.E.}}{A} = -(120 - \frac{97}{A^{1/3}})\eta_s + (67 - \frac{57}{A^{1/3}})\eta_v + \text{residual terms}$$

Cancellation amplifies relative effect:

^{208}Pb: $$\frac{\text{B.E.}}{A} = [-8 + 12.2\frac{m_\pi^2 - m_{ph}^2}{m_{ph}^2}]\ \text{MeV}$$

In general:

$$\begin{aligned}\text{Total B.E.} &= B(A) + (14.1A - 11.3A^{2/3})\frac{m_\pi^2 - m_{ph}^2}{m_{ph}^2}\text{MeV} \\ &\sim B(A) + (2.4A - 1.8A^{2/3})(\hat{m} - \hat{m}_{ph})\end{aligned}$$

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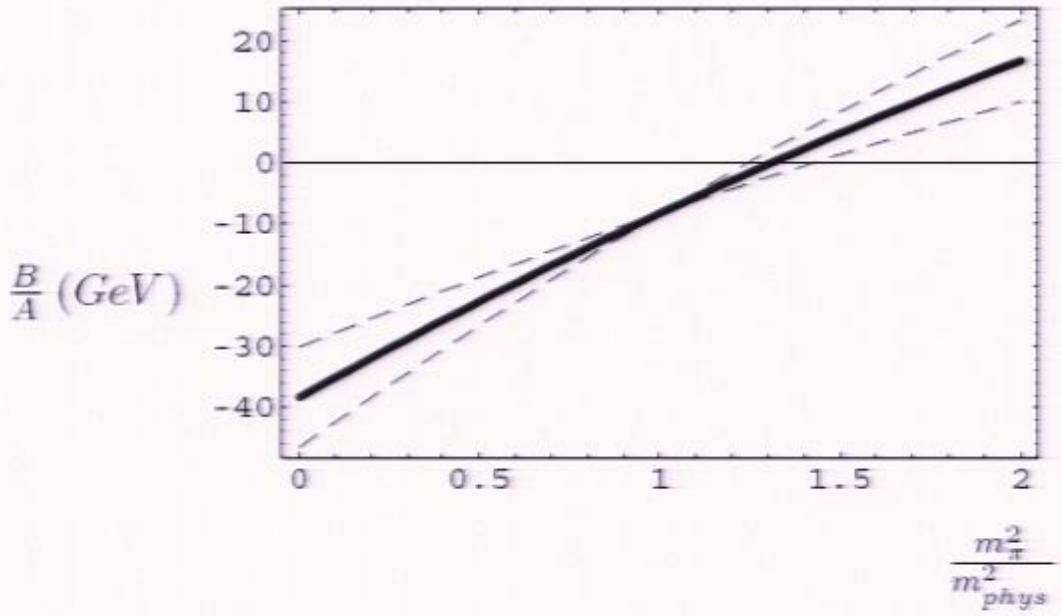

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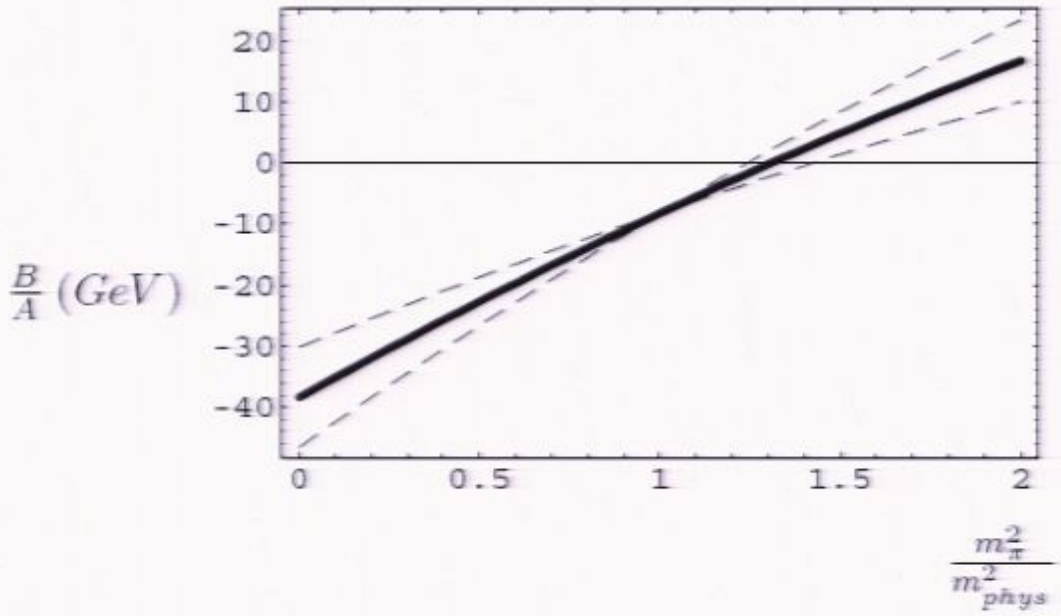

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-no evidence that m_s plays significant role in nuclear binding

-maybe shift in Λ_{QCD}

-dispersive threshold is far away

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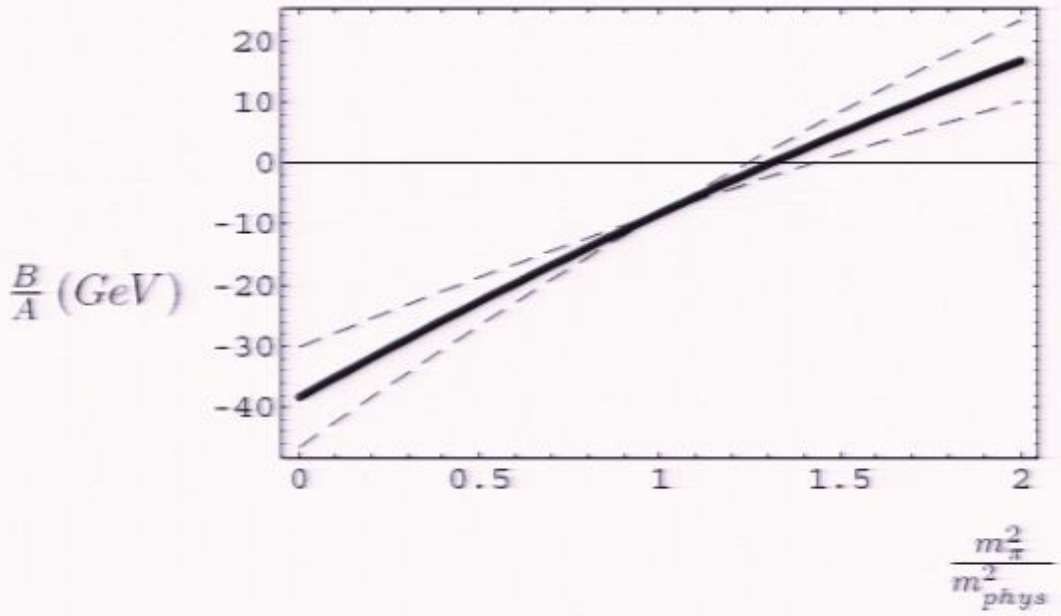

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Anthropic constraints on quark masses:

Damour
Donoghue

Basic constraint is **existence of atoms:**

Older work wi
Agrawal
Barr
Seckel

A) Changing quark masses can eliminate nuclear binding

B) Summary of constraints

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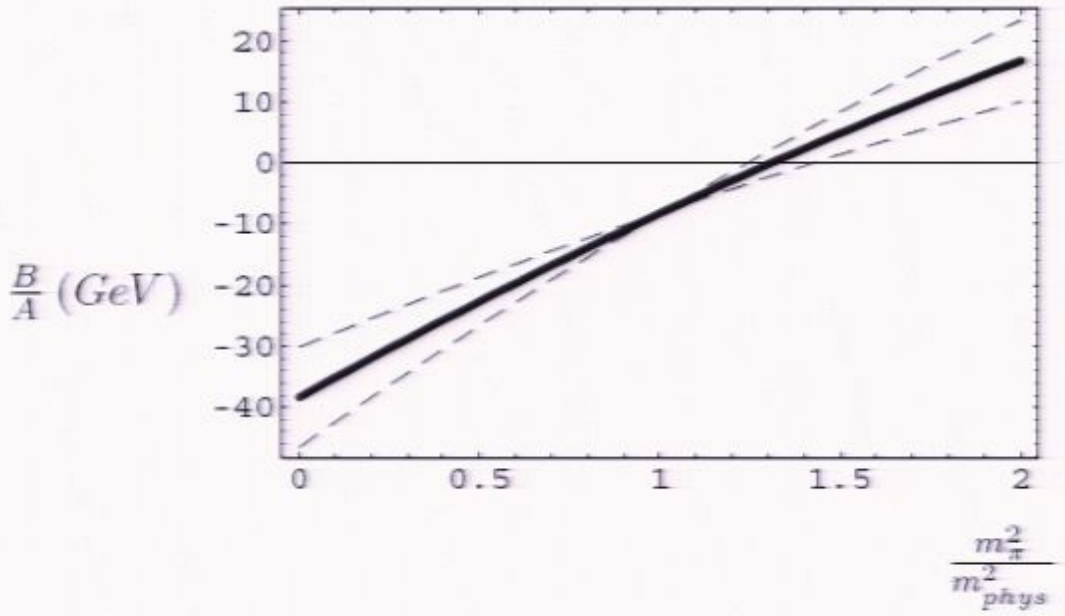

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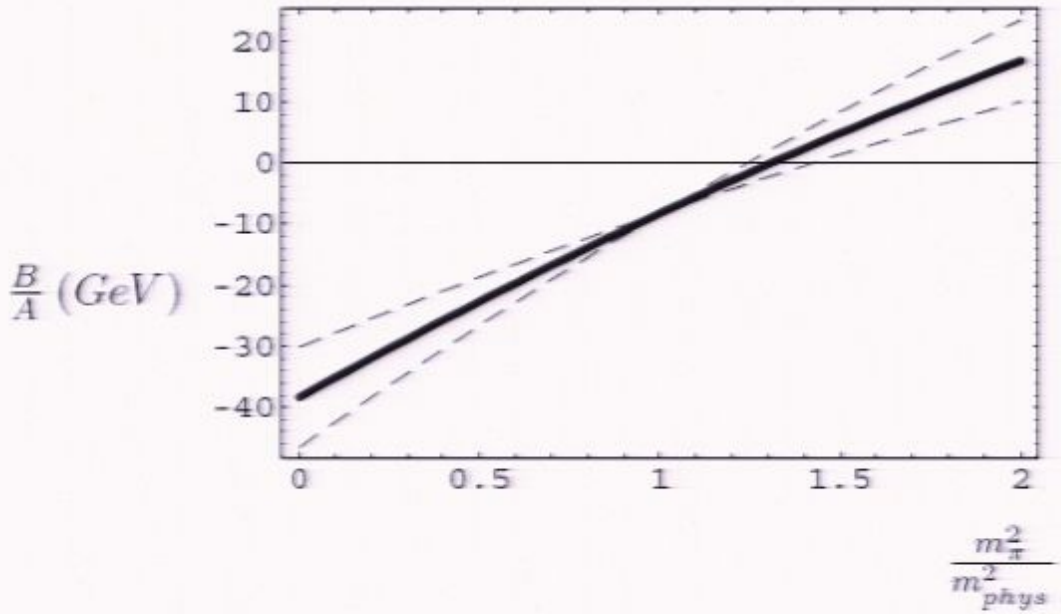

Anthropic constraint – existence of atoms

$$\frac{m_u + m_d}{(m_u + m_d)_{\rm phys}} < 1.36 \pm 0.14$$

At 95% C.L.→ $\frac{m_u + m_d}{(m_u + m_d)_{\rm phys}} < 1.64$ or $m_u + m_d \leq 18$ MeV

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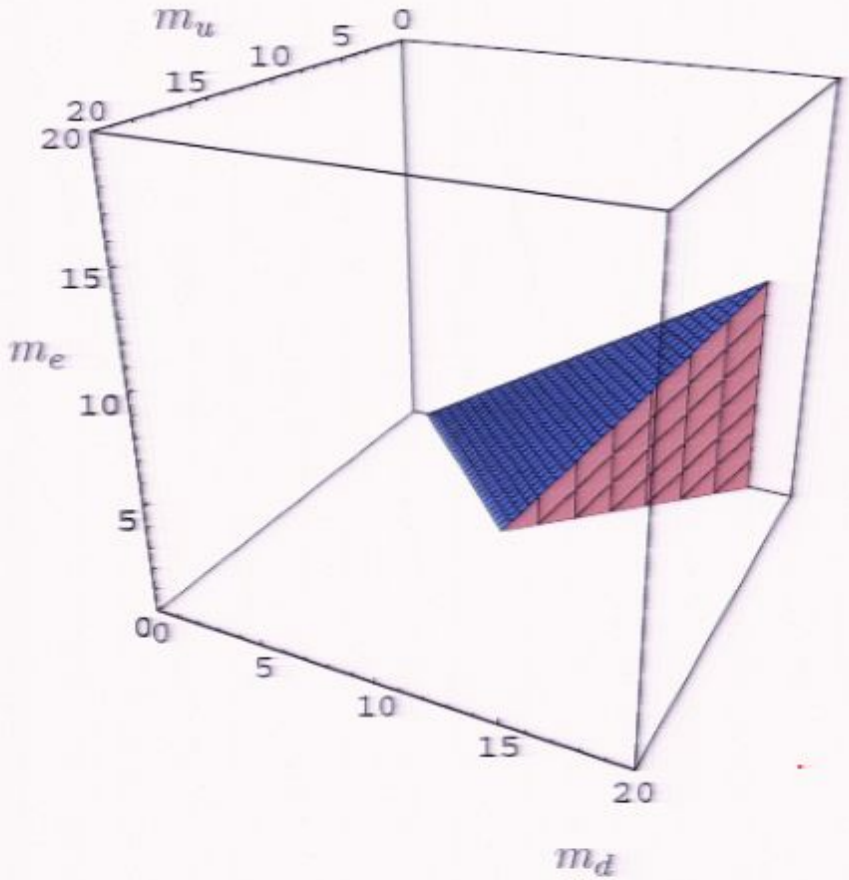

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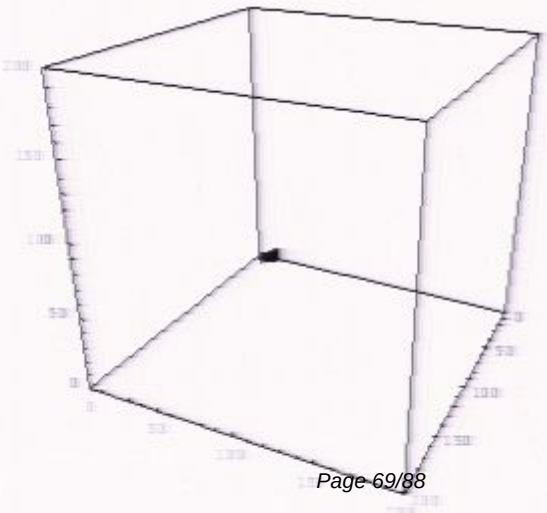

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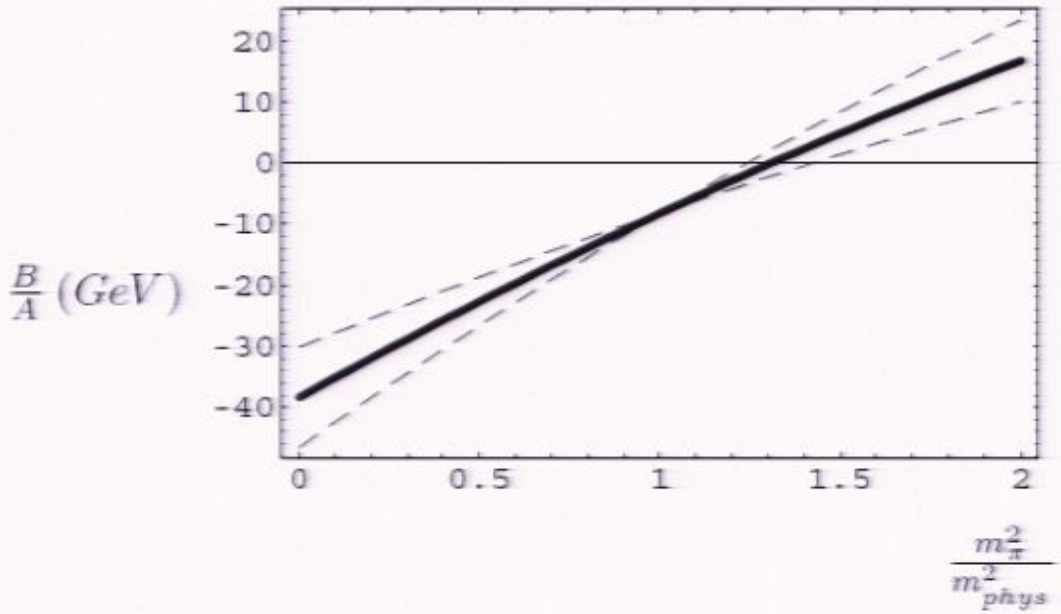

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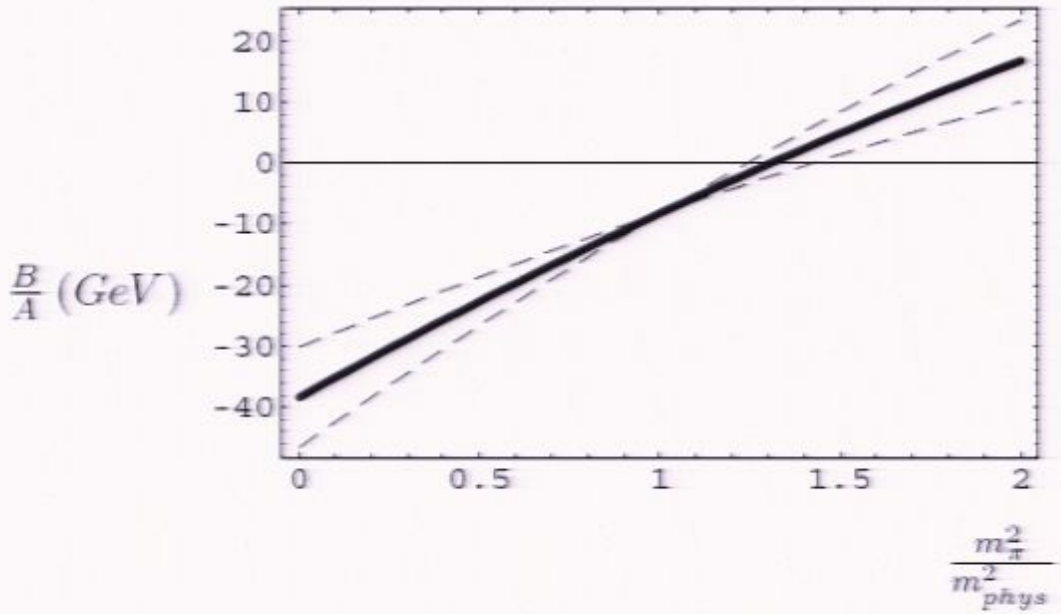

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Second Anthropic constraint:

- atoms do not decay into neutron world

$$m_P + m_e \leq m_N$$

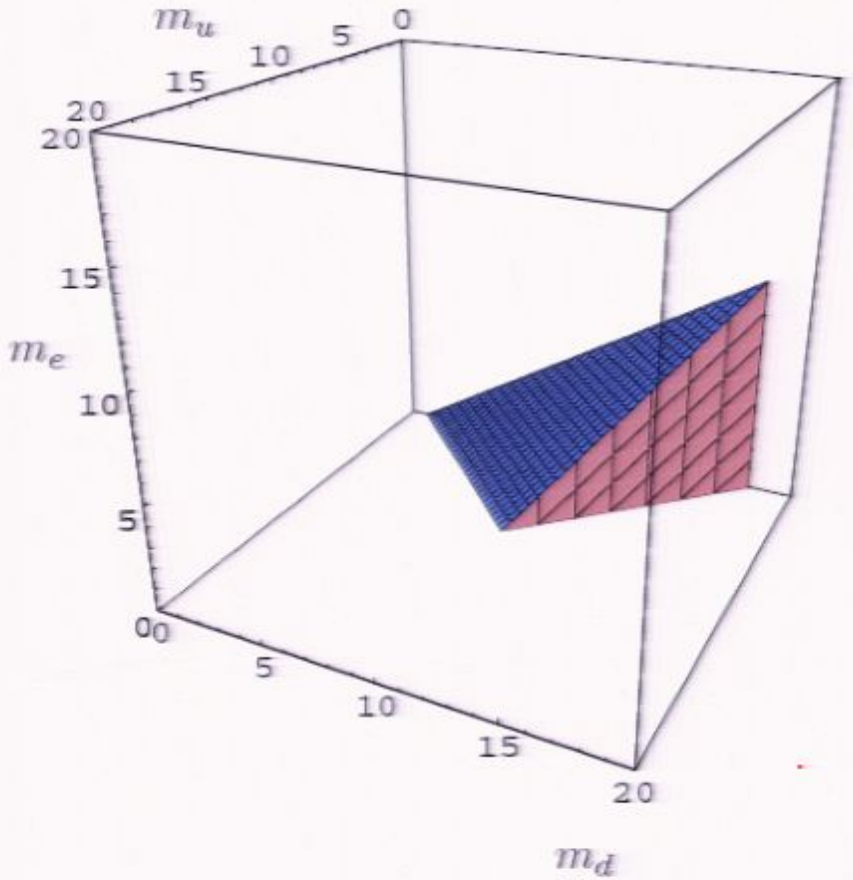
Here we need to account for E&M also:

$$m_N - m_P = Z_0(m_d - m_u) - \epsilon_{EM}$$

My estimates:

$$\boxed{m_d - m_u - 1.6m_e \geq 0.85 \text{ MeV}}$$

Overall constraints:



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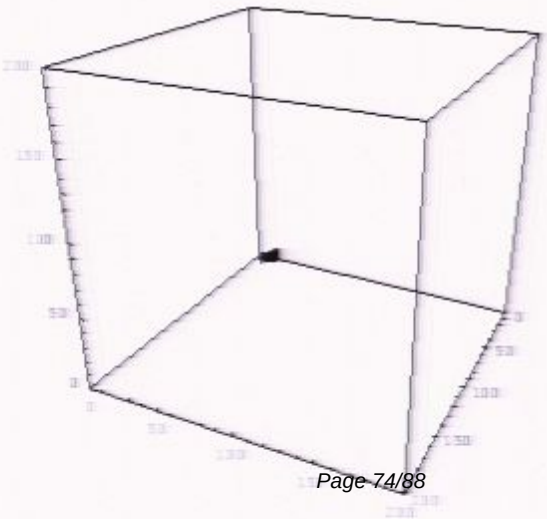


Oklo constraint – time variation of quark masses

Shlyakhter
Damour, Dyson

$^{149}Sm + n \rightarrow {}^{150}Sm + \gamma$

$-0.12\ eV < \Delta(Q\text{-}E_*) < 0.09\ eV$

Very roughly:
- estimate $\Delta Q \sim \eta\, \Delta\langle V\rangle_1$ with $\eta \sim 1/2$
- with previous work, translates to

η=1 is result of Flambaum and Shuryak

$$\frac{|\Delta m_q|}{m_q} \leq 2 \times 10^{-8}$$

over a time scale of 1.8×10^9 yrs

or $$\frac{|\dot{m}_q|}{m_q} \leq 3.6 \times 10^{-25}\ \mathrm{sec}^{-1}$$ if linear

Aside: Strongest constraint on time variation:

- Assumption – Cosmological constant has not varied by more than 100% since z~1 (~8 x 10^9 yrs)
  - agreement of CMB and supernova data
  - non-closure of universe
  - will be refined

$$\Lambda = \Lambda_0 + m_q B_0 + m_q^2 C_{0+\ldots}$$

- Consider **only** quark mass variation
  - only first order term linear in the quark mass – **rigorous**

$$\Delta\Lambda = \left(\frac{\Delta m_\pi^2}{m_\pi^2}\right) F_\pi^2 m_\pi^2 = \left(\frac{\Delta m_q}{m_q}\right) \times (112\ \mathrm{MeV})^4$$

Result:

$$\frac{|\Delta m_q|}{m_q} \leq 1.7 \times 10^{-43} \quad \text{or} \quad \frac{|\dot{m}_q|}{m_q} \leq 7 \times 10^{-61}\ \mathrm{sec}^{-1}$$

All variations cause trouble with Λ – also true for $\Delta\alpha$

Application to equivalence principle tests:

- our original motivation for getting into this topic
- B.E. contribution is least understood ingredient

Scalar couplings:

$\phi\bar{\psi}\psi$ like $m\bar{\psi}\psi$

Leading quark mass effect is in the masses of nucleons
- reasonably well understood

EP tests compare acceleration of different elements
- need to compare BE for different elements

Generic scalar models

Coupling to any scalar invariant:

-generally suppressed by powers of some heavy mass

$$S = \int d^4x\sqrt{g}\left[\frac{B_g(\phi)}{16\pi G}\,R - \frac{k}{4}B_F(\phi)F^2 - B_m(\phi)m\bar{\psi}\psi + ...\right]$$

Specific models have predictions for the couplings

Two sample physical consequences:

1) All violate the equivalence principle at some level

2) If ϕ changes in time, the coupling constants change

Differences in acceleration

With massless scalar: $G_{AB} = G(1 + \alpha_A \alpha_B)$

with $\alpha_A = \frac{1}{m_A}\frac{\partial m_A}{\partial \phi}$

Then: $\left(\frac{\Delta a}{a}\right)_{AB} \equiv 2\frac{a_A - a_B}{a_A + a_B} \simeq (\alpha_A - \alpha_B)\,\alpha_E$

But if all energies proportional to Λ_{QCD}, then α_i universal
- need to know contributions of quark masses and EM

Example – runaway dilaton (ignoring quark masses)

$\left(\frac{\Delta a}{a}\right)_{AB} \simeq 2 \times 10^{-5}\,\alpha_{had}^2 \left[\left(\frac{E}{M}\right)_A - \left(\frac{E}{M}\right)_B\right]$ $\qquad E \equiv Z(Z-1)/(N+Z)^{1/3}$

What is missing?

Reasonable understanding of:

-Quark mass effects on nucleons:

-EM effects on nucleons

-EM in nuclear binding

$$m_p = m_\Lambda + b_u m_u + b_d m_d + C_p \alpha$$
$$m_n = m_\Lambda + b_d m_u + b_u m_d + C_n \alpha$$

$$m(\text{Atom}) = Z m_p + N m_n + Z m_e + E_3 + E_1$$

QCD EM

Missing:

-Quark mass effects on nuclear binding $E_3 = E_\Lambda + E_m$

$\Longrightarrow$ Need to understand $\dfrac{\partial E_3}{\partial m}$

The dependence on Nuclear number:

Pair interactions depend on near neighbors:

-nuclear density nearly constant in central region $r \sim A^{1/3}$

-central interactions # $\sim r^3 \sim A$ = # of nucleons

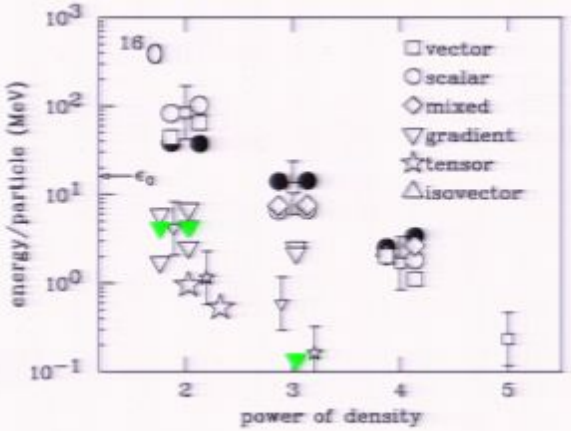
-surface interaction correction $\sim r^2 \sim A^{2/3}$

B.E. $\sim a\,(A - b\,A^{2/3})$

(like semi-empirical mass formula)

fits give b~1

predicts nuclear matter limit to 6%



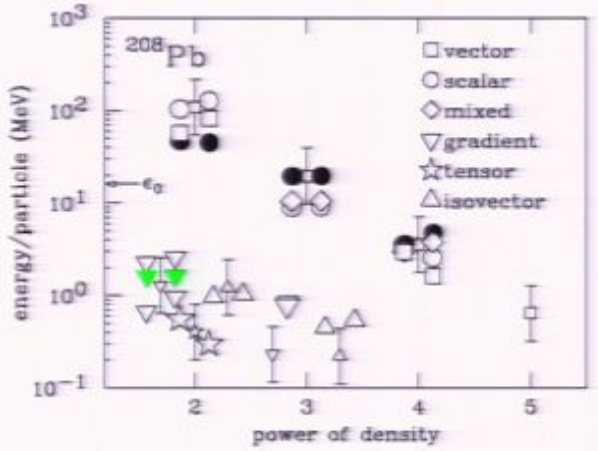


FIG. 2. Same as Fig. 1 for ^{208}Pb.

Qualitative conclusion:

- B.E. differences not large in EP tests
- reason is clear

  For most test materials, BE per nucleon is roughly constant
  BE contribution then roughly proportional to mass
  quark mass dependence shares this property
  then – scalar coupling to BE roughly proportional to mass

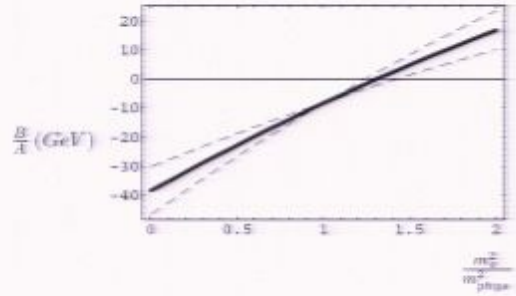
- details to be forthcoming

Summary

Reasonable control over the key ingredient in nuclear binding

Quark mass dependence comes from region where the calculation is the most reliable

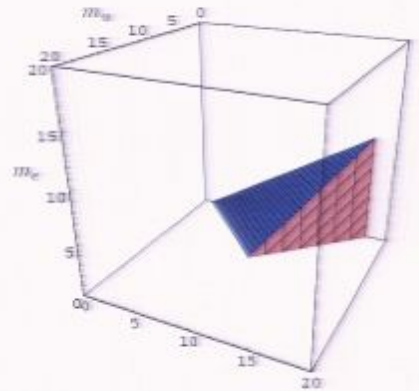
Reasons for strong dependence are well understood



Applications to:

- Anthropics
- Oklo
- Equivalence Principle

.....



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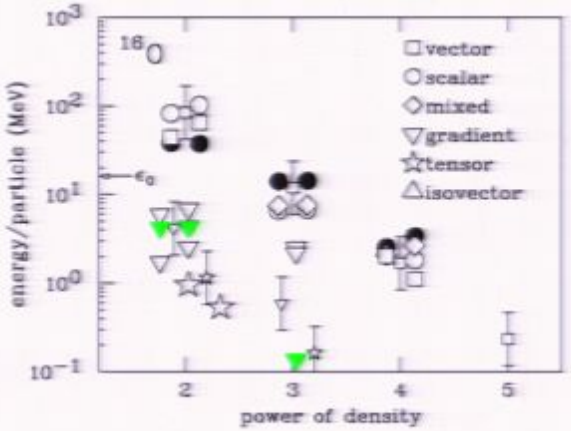

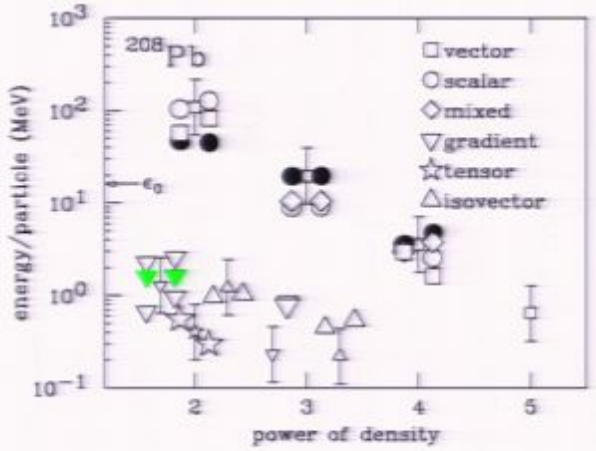

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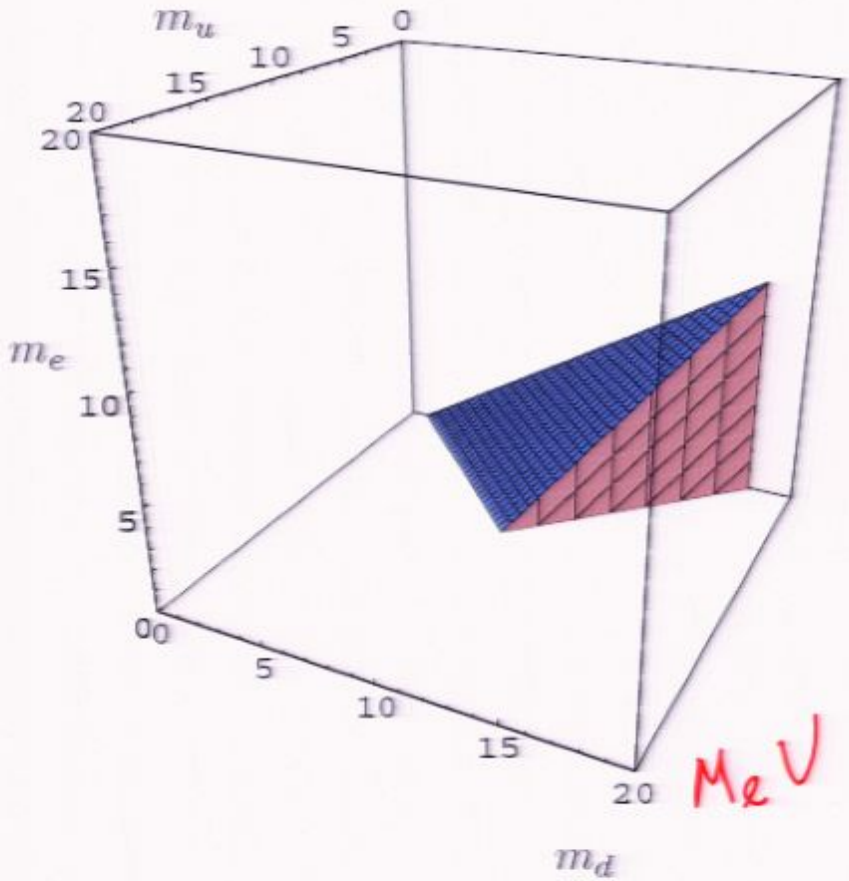

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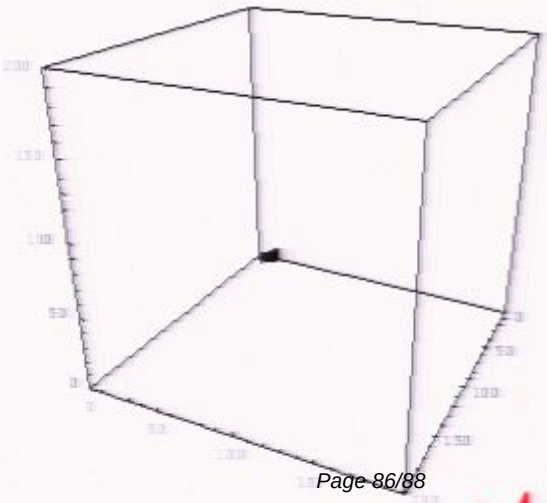

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