

Title: Hadronic Mass Variation in Big Bang Nucleosynthesis

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Abstract: I will present a brief introduction to Big Bang Nucleosynthesis theory and observation. I will then discuss BBN as a probe of hadronic mass variation in the very early universe, including comparison with the observed Li7 discrepancy. Finally I want to explore the possibility of overproducing Li6 by three orders of magnitude in order to match reported observations.

Hadronic Mass Variation in Big Bang Nucleosynthesis

Julian Berengut

Perimeter Institute
16 July 2008



THE UNIVERSITY OF
NEW SOUTH WALES



SYDNEY • AUSTRALIA

Outline

- 1 Standard BBN
- 2 Deuterium Binding Energy
 - Sensitivity
 - Results
- 3 Variation of nuclear binding energy
 - Effect on BBN
 - Variation of light quark mass



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Overview

$$\begin{aligned}\frac{\dot{R}}{R} = H &= \sqrt{\frac{8\pi G_N}{3}\rho} \\ \frac{\ddot{R}}{R} &= -\frac{4\pi G_N}{3}(\rho + 3p) \\ \dot{\rho} &= -3H(\rho + p)\end{aligned}$$



Overview

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{8\pi G_N}{3}\rho}$$

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$$\dot{\rho} = -3H(\rho + p)$$

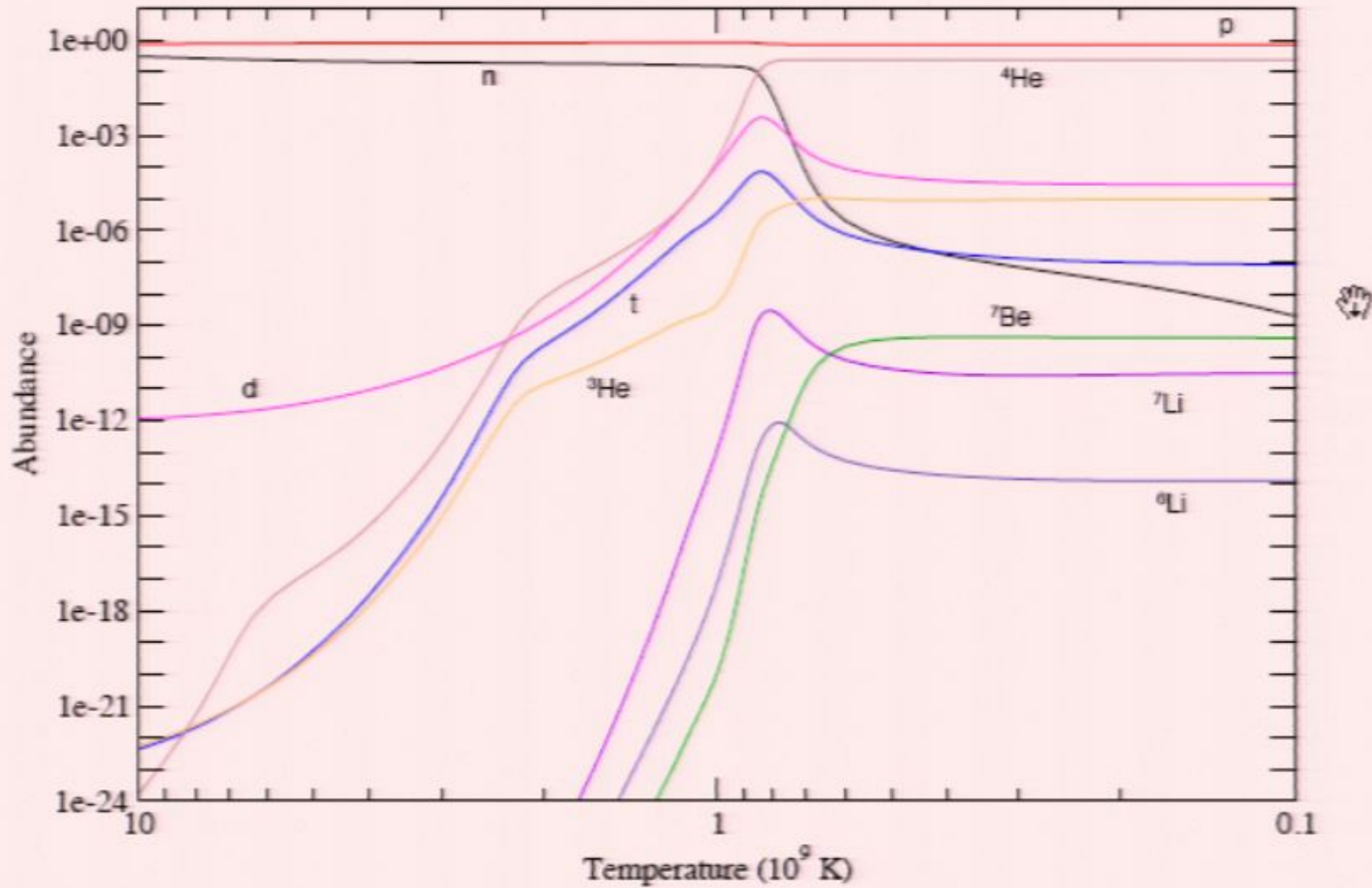
$$\dot{Y}_i = \sum_{j,k,l} N_i \left(\Gamma_{kl \rightarrow ij} \frac{Y_l Y_k}{N_l! N_k!} - \Gamma_{ij \rightarrow kl} \frac{Y_i Y_j}{N_i! N_j!} \right)$$

$$n_- - n_+ = n_B \sum_j Z_j Y_j$$

$$\frac{\dot{n}_B}{n_B} = -3H$$

$$(1 \text{ MeV} = 11.6 \times 10^9 \text{ K})$$

SBBN evolution of light abundances



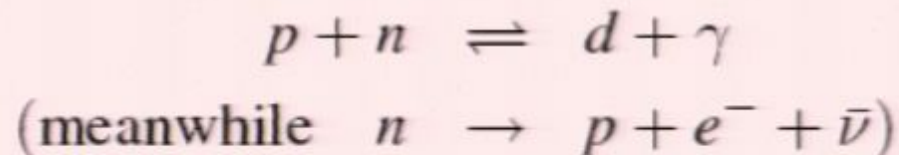
Stages of BBN

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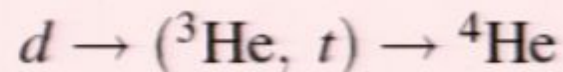


Freeze-out $t \sim 1 \text{ s} \sim 1 \text{ MeV} \sim 10^{10} \text{ K}$

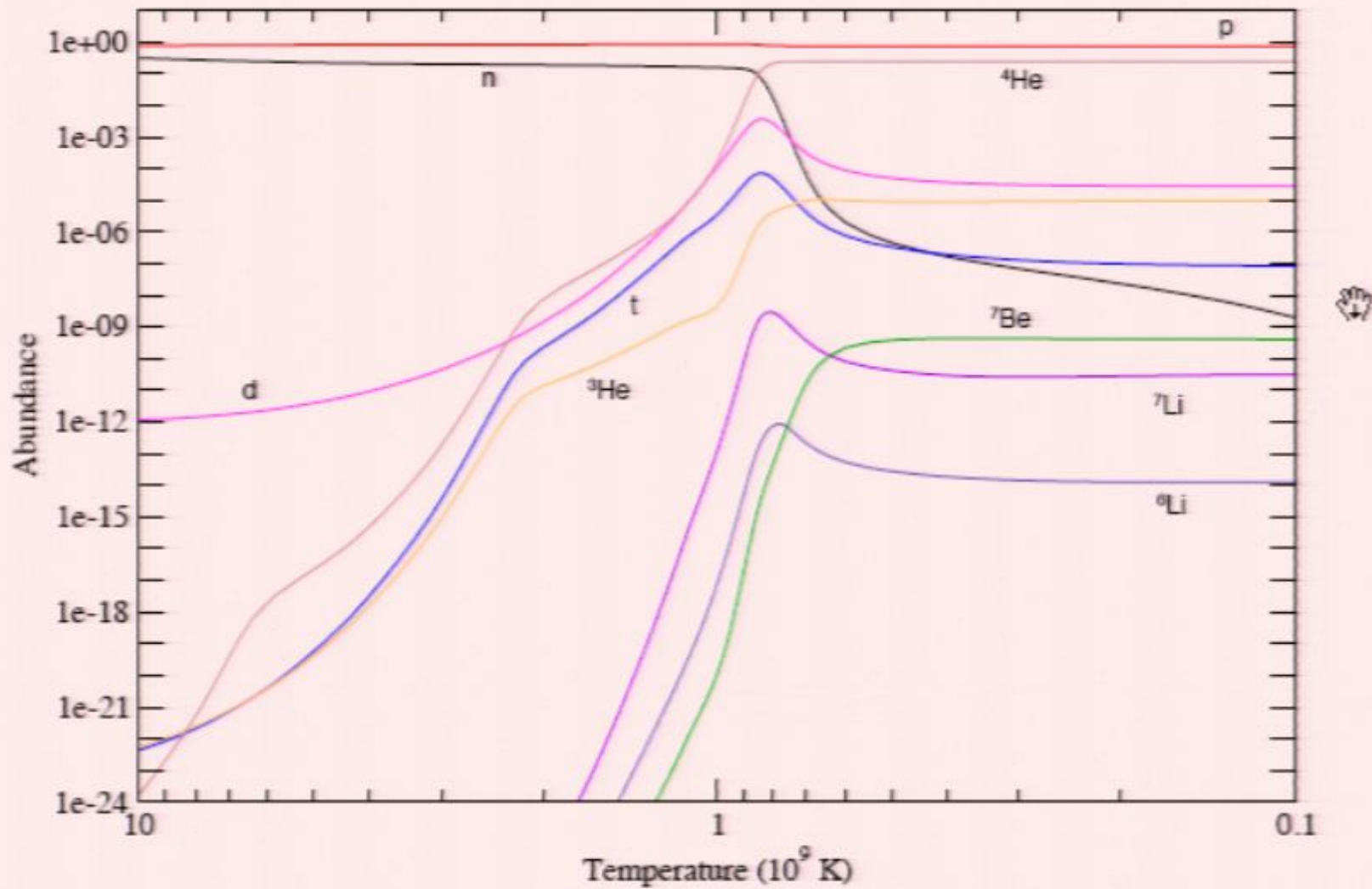
- D bottleneck:



- Nucleosynthesis from $t \sim 3 \rightarrow 10 \text{ min}$

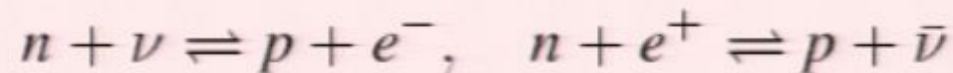


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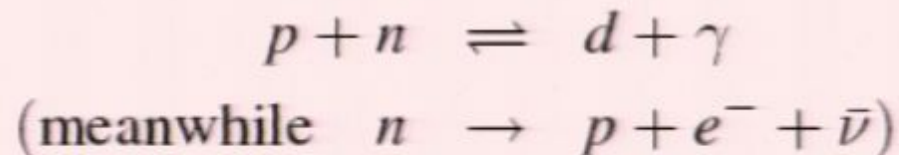
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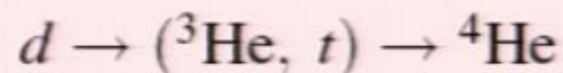


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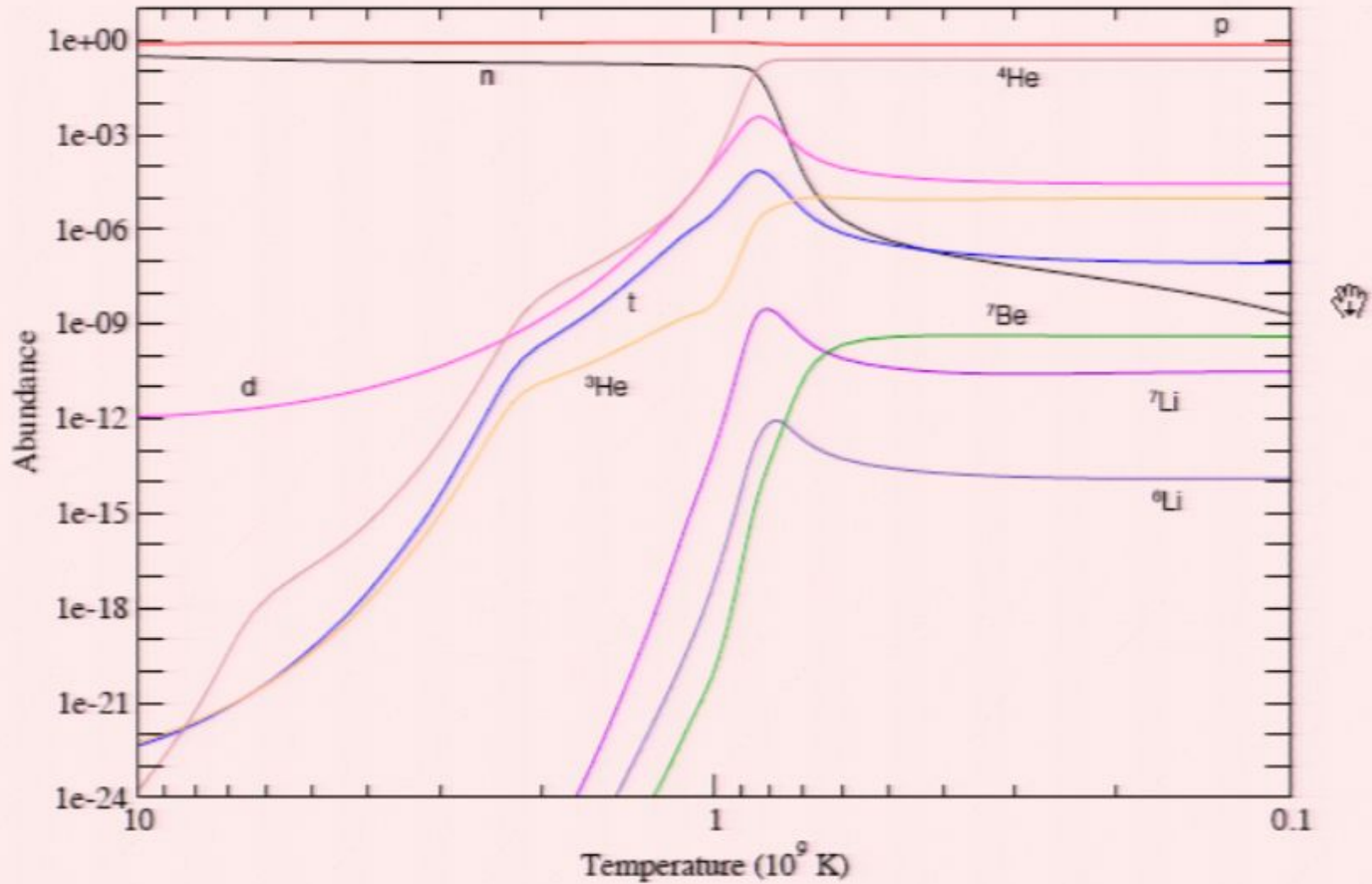
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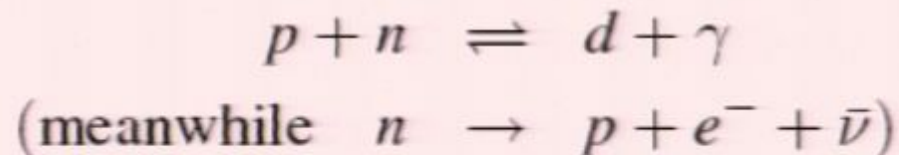
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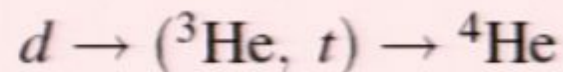


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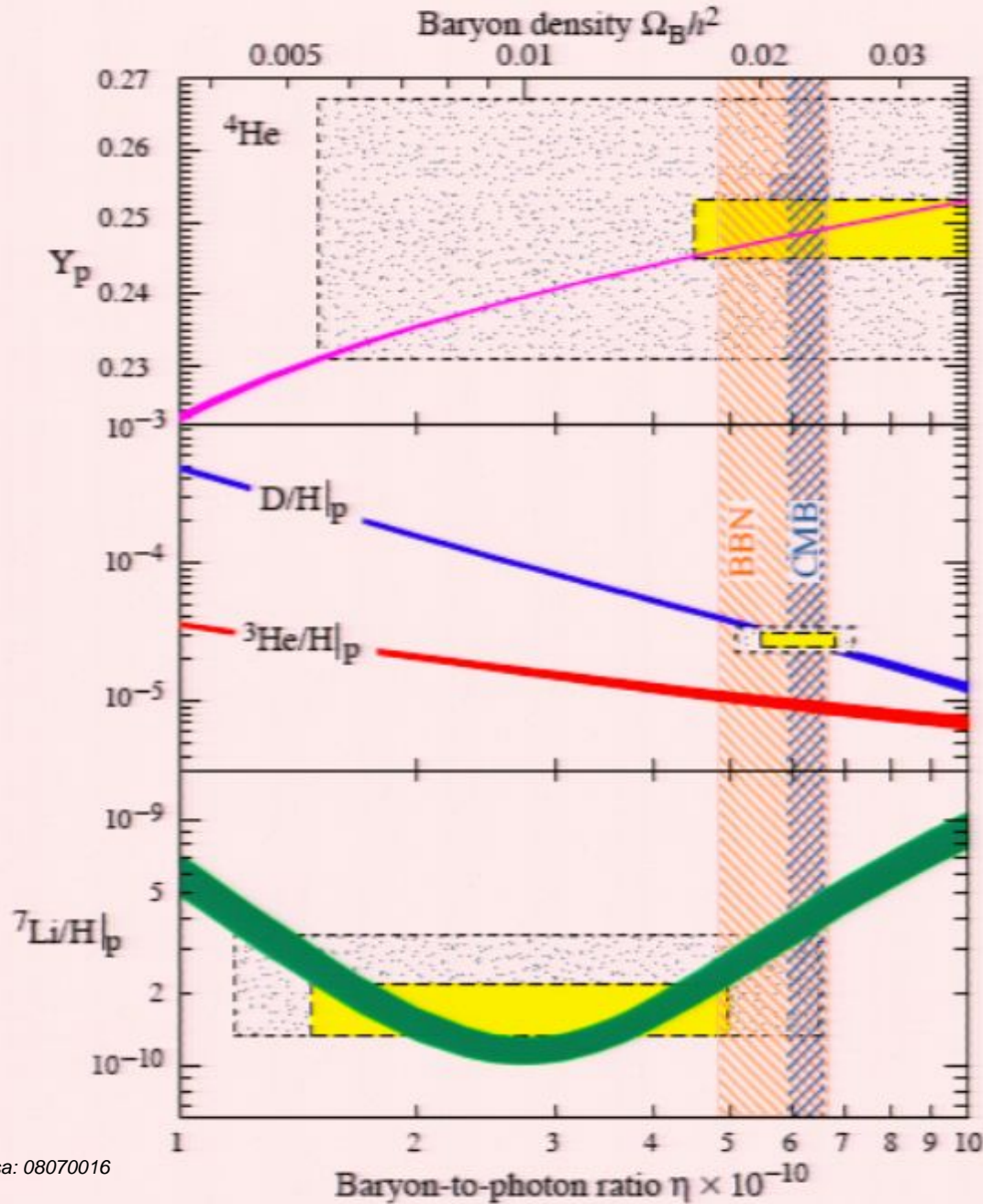
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Standard BBN
 Deuterium Binding Energy
 Variation of nuclear binding energy



PDG 2007

B. D. Fields and S. Sarkar
 2σ confidence intervals

Theory results

1σ ranges at $\eta = 6.14 \times 10^{-10}$ (WMAP)


	Dent ¹	Cyburt ²	This work
D/H	$2.61 (4) \times 10^{-5}$	$2.55^{+0.20}_{-0.20} \times 10^{-5}$	2.77×10^{-5}
³ He/H	$10.3 (3) \times 10^{-6}$	$10.12^{+0.67}_{-0.66} \times 10^{-6}$	10.09×10^{-5}
Y_p	0.2478 (2)	0.2485 (5)	0.2457
⁷ Li/H	$4.5 (4) \times 10^{-10}$	$4.26^{+0.91}_{-0.86} \times 10^{-10}$	4.43×10^{-10}

¹ Dent, Stern, Wetterich (2007); errors from Serpico *et al.* (2004)

² Cyburt (2004)


Observation

$Y_{4\text{He}}$ (usually denoted Y_p):

- Observed in H II regions, extrapolated to zero metallicity. 
Systematics debated.
- 0.249 (9) – Olive and Skillman, 2004
- 0.2477 (29) – Peimbert *et al.*, 2007
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Observation

D/H:

- Detected in QAS via isotope shifted damped Ly- α (no astrophysical sources) 
- Averaging six precise observations in QAS gives (PDG 2007):
 $2.84 (14) \times 10^{-5}$
- Including dispersion (systematic errors):
 $2.84 (26) \times 10^{-5}$

Observation

${}^7\text{Li}/\text{H}$:


- Detected in Pop II stars
- ${}^7\text{Li}$ does not vary in Pop II stars with low metallicity (Spite plateau)
- $1.23(6) \times 10^{-10}$ – Ryan *et al.*, 2000
- Others: $2.34(32)^1$, $1.26(26)^2 \times 10^{-10}$
- $1.7(2) \times 10^{-10}$ – PDG 2007
- Upper bound: 3.0×10^{-10}

possibility of Li destruction by mixing of stellar atmospheres

¹ Meléndez and Ramirez, 2004 ² Bonifacio *et al.*, 2007


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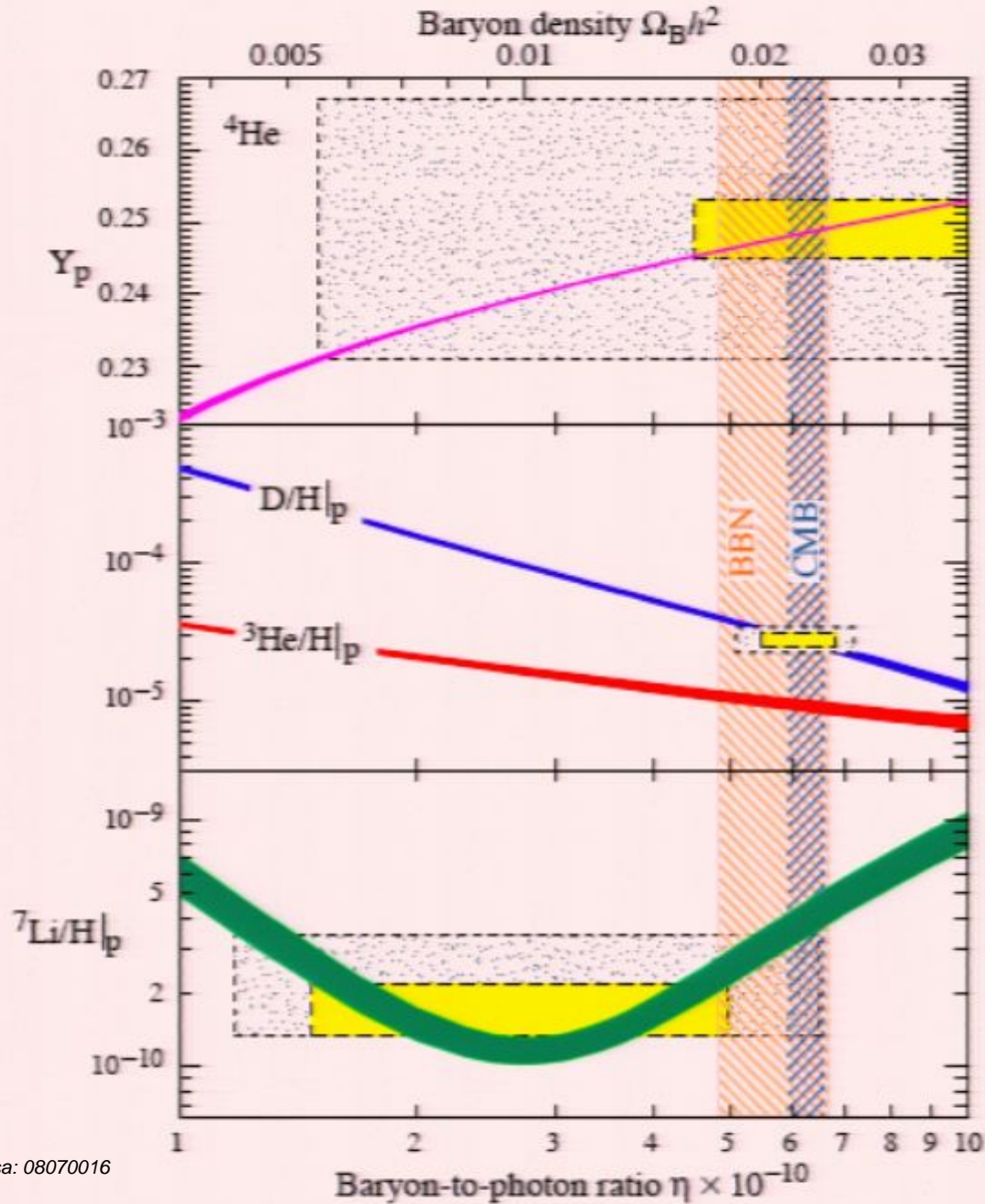
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BBN modifications

- n freeze-out temperature depends on expansion rate
- more light species \rightarrow higher freeze-out temperature
 \rightarrow more neutrons will survive after decoupling \rightarrow more ${}^4\text{He}$
- neutron lifetime $\tau_n = 886$ s. Freeze-out is at ~ 1 s.
 D bottleneck opens at ~ 200 s.
 $\therefore \tau_n$ affects neutron survival until BBN and therefore ${}^4\text{He}$ abundance.

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Variation of mass scales

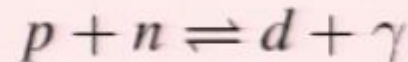
GUTs consistently predicts much larger variation of strong interaction than electromagnetic:

$$\frac{\delta m_q}{m_q} \sim 35 \frac{\delta \alpha}{\alpha}$$

(Marciano 1984, Calmet and Fritzsche 2002, Wetterich 2003, ...)

d production sensitivity

- The first step in nucleosynthesis is



with binding energy $B_d = 2.22 \text{ MeV} = 25.82 \times 10^9 \text{ K}$.

- The rate $\Gamma_{p(n,\gamma)d}$ is sensitive to Q (Dmitriev, Flambaum, Webb 2004)

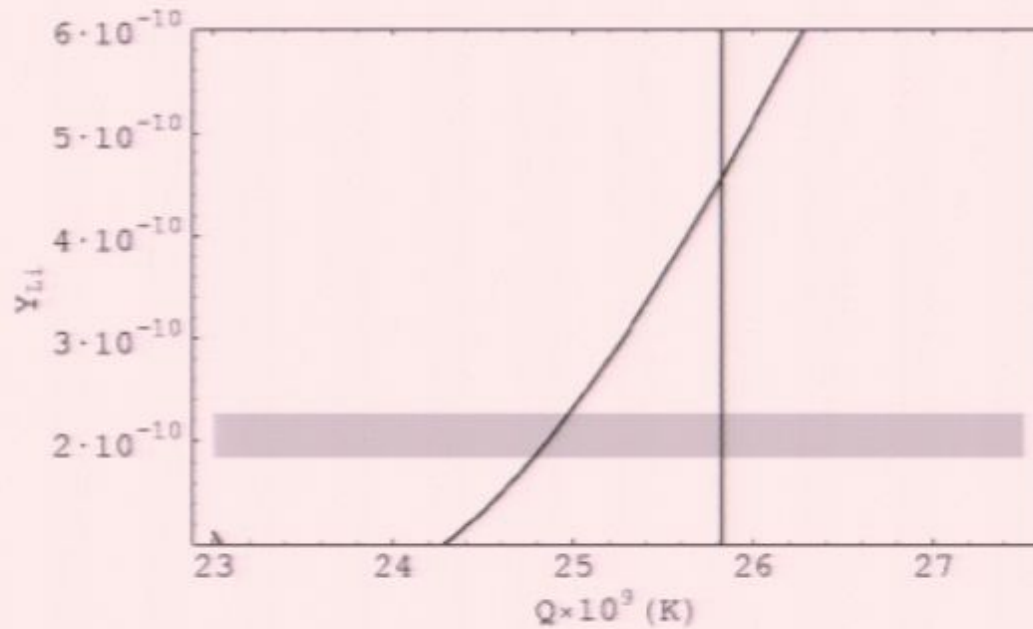
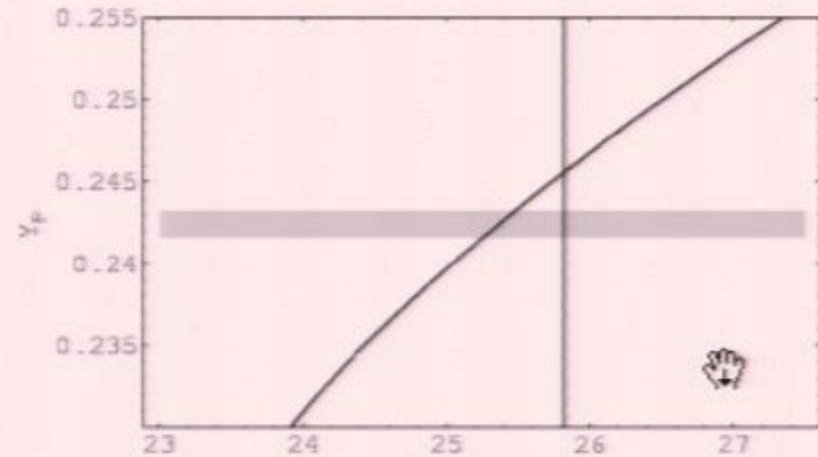
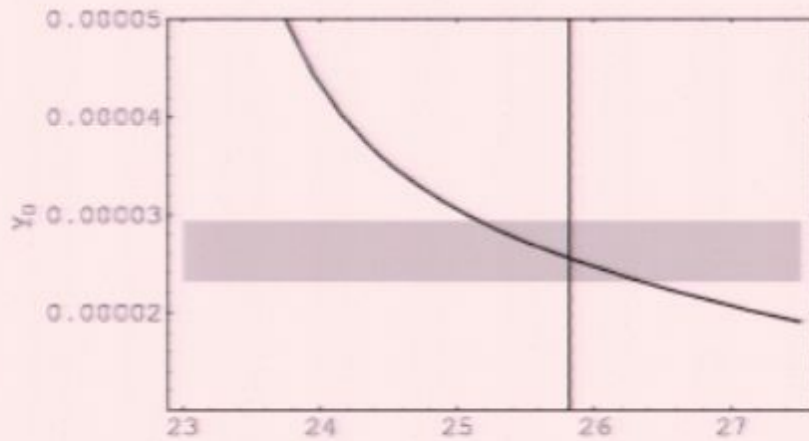
$$\Gamma_{p(n,\gamma)d} \sim 1 + \left(\frac{5}{2} + \sqrt{\frac{Q}{\epsilon_v}} \right) \frac{\delta Q}{Q}$$

where the second term accounts for variation in the position of the virtual level with energy $\epsilon_v \sim 0.07 \text{ MeV}$

- The inverse reaction

$$\Gamma_{d(\gamma,n)p} \sim e^{-Q/T}$$

Q variation results



Q variation results

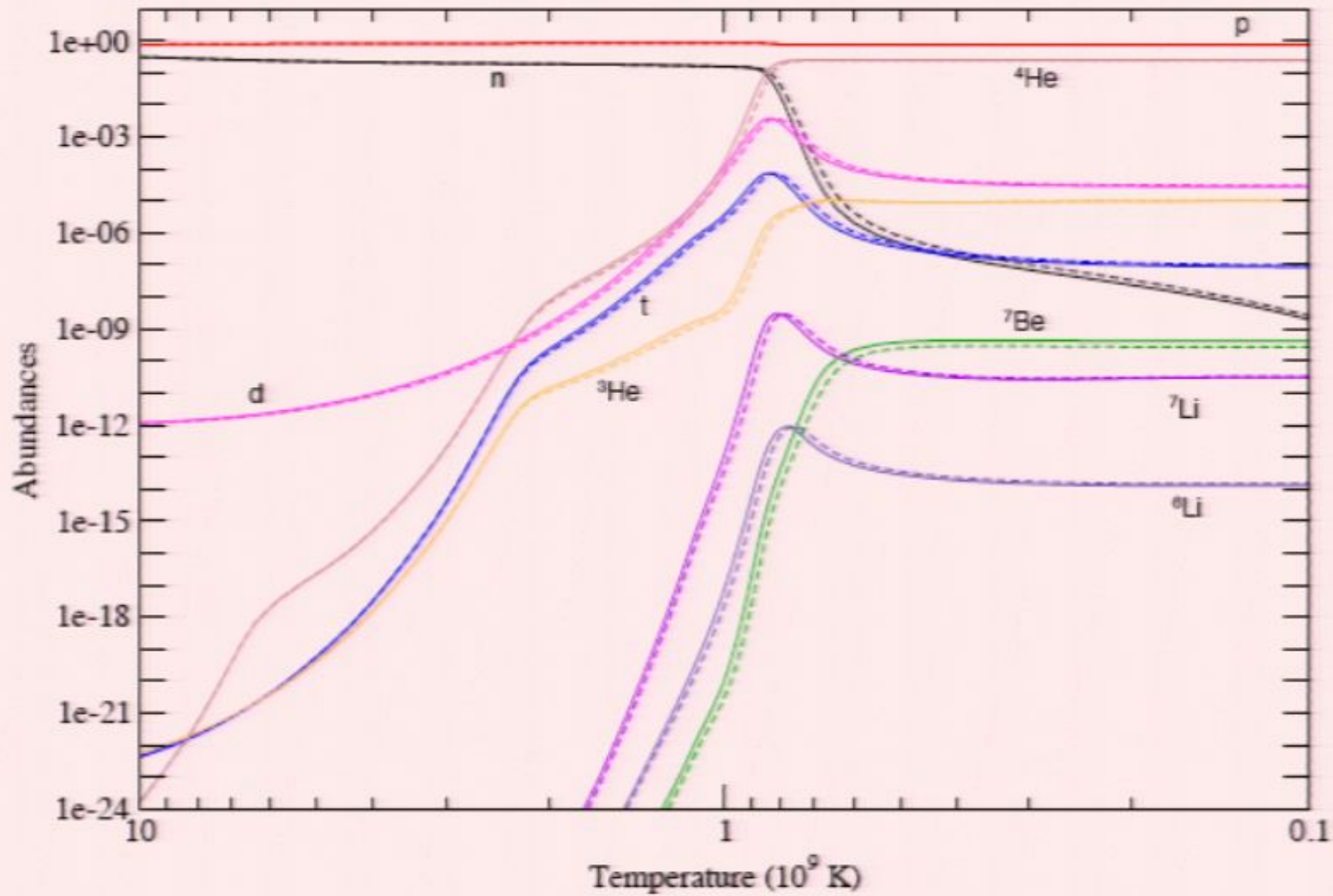
Concordance with experiment (DFW 2004):

$$\delta Q/Q = -0.019 (5)$$

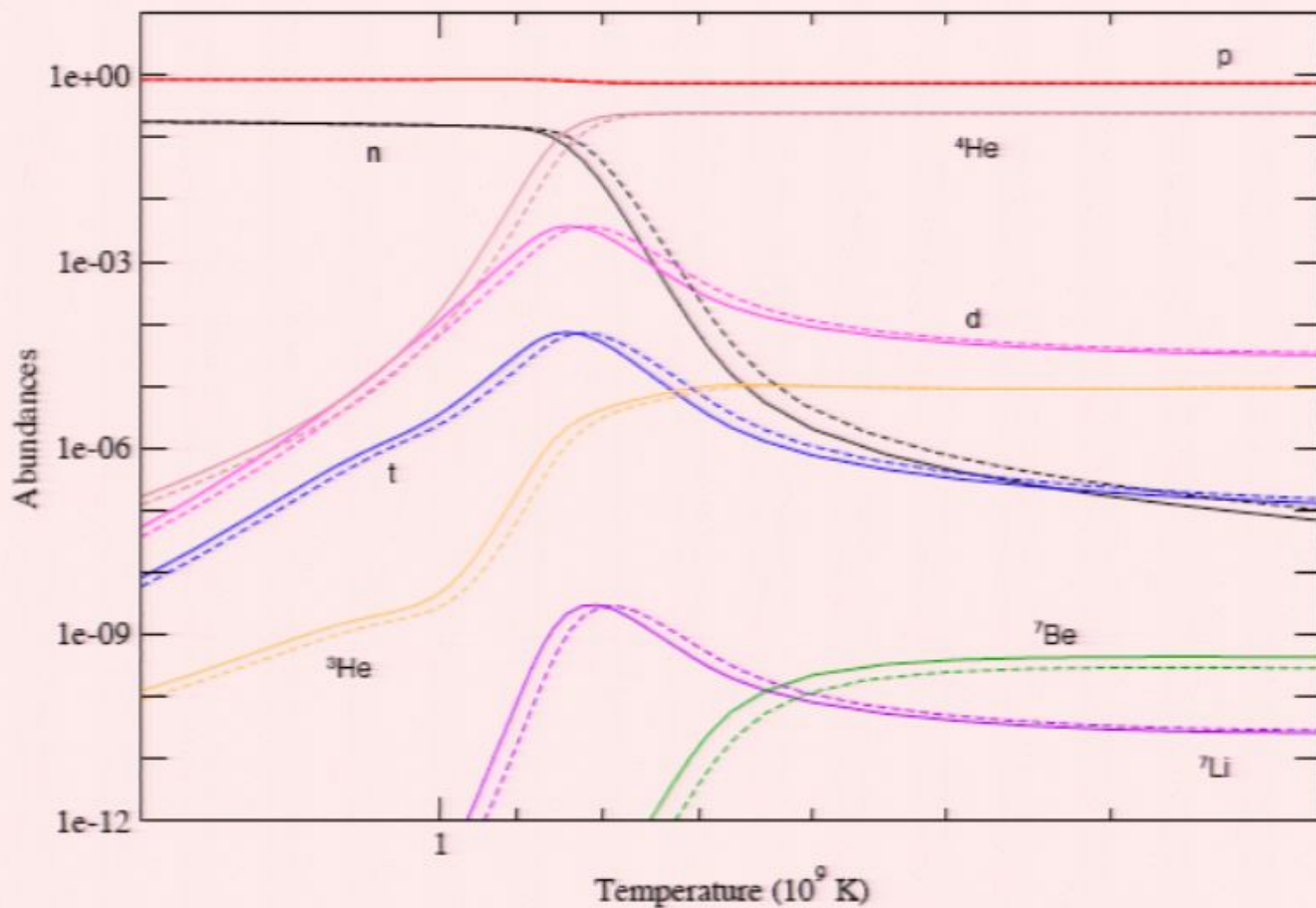
- Q reduced, D bottleneck opens later
- More n , p , D, ${}^3\text{He}$, ${}^6\text{Li}$, ${}^7\text{Li}$
- Less ${}^4\text{He}$, less ${}^7\text{Be}$



Q variation results



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Mechanisms for change

Modification of binding energies \rightarrow change in $Q \rightarrow$ reaction rates

- For $2 \rightarrow 2$ inelastic scattering reactions,

$$\sigma \sim (Q + E)^{1/2}$$

- For $E1$ radiative capture reactions,

$$\sigma \sim (Q + E)^3$$

- Reverse reaction $\sim e^{-Q/T}$

(Dent *et al.* 2007)

Linear variation

$\partial \ln Y_a / \partial \ln X_i$	D	${}^3\text{He}$	${}^4\text{He}$	${}^6\text{Li}$	${}^7\text{Li}$
G	0.94	0.33	0.36	1.4	-0.72
α	2.3	0.79	0.00	4.6	-8.1
τ_n	0.41	0.15	0.73	1.4	0.43
m_e	-0.16	-0.02	-0.71	-1.1	-0.82
Q_N	0.83	0.31	1.55	2.9	1.00
m_N	3.5	0.11	-0.07	2.0	-12
B_D	-2.8	-2.1	0.68	-6.8	8.8
B_T	-0.22	-1.4	0	-0.20	-2.5
$B_{{}^3\text{He}}$	-2.1	3.0	0	-3.1	-9.5
$B_{{}^4\text{He}}$	-0.01	-0.57	0	-59	-57
$B_{{}^6\text{Li}}$	0	0	0	69	0
$B_{{}^7\text{Li}}$	0	0	0	0	-6.9
$B_{{}^7\text{Be}}$	0	0	0	0	81
η	-1.6	-0.57	0.04	-1.5	2.1

Linear variation of B_D

- 1 Dent *et al.* 2007; ϵ_ν resonance not considered
- 2 Resonance variation considered:

$$\Gamma_{p(n,\gamma)d} \sim 1 + \left(\frac{5}{2} + \sqrt{\frac{Q}{\epsilon_\nu}} \right) \frac{\delta Q}{Q}$$

Q changed only for $p(n, \gamma)d$; same as DFW 2004

- 3 Resonance variation considered; Q changed for all rates



$\partial \ln Y_a / \partial \ln B_D$	D	^3He	^4He	^6Li	^7Li	^7Be
1.	-2.8	-2.1	0.68	-6.8	8.8	
2.	-4.98	-1.39	0.71	-4.08	17.88	
3.	-3.99	-1.78	0.70	-7.63	16.72	

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Reanalysis of B_D variation

Linear reanalysis of DFW 2004 work:

$$1 + \frac{\partial \ln Y_a}{\partial \ln B_D} x = \frac{Y_a (\text{obs})}{Y_a (\text{theory})}$$

where $x = \delta B_D / B_D$.

D:	$1 - 4.98x = 1.09 \pm 0.10$		-0.018	(20)
^4He :	$1 + 0.71x = 1.005 \pm 0.036$	\rightarrow	0.007	(51)
^7Li :	$1 + 17.88x = 0.33 \pm 0.11$		-0.037	(6)

Consistent solution: $x = -0.035$ (6) dominated by ^7Li
c.f. original paper $x = -0.019$ (5)

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Strong upper bound of ^7Li (3.0×10^{-10}) gives $x < -0.018$

Not consistent with tight bounds on ^4He (1.8σ deviation).

Dependence of nuclear binding on hadronic mass variation

Flambaum and Wiringa, 2007

- Calculate dependence of binding energies w.r.t. hadrons:
 N , π , Δ , ρ - and ω -mesons
- Use Argonne models of nuclear Hamiltonian
+ Urbana model IX for three-nucleon potential.

Result is tabulated values of

$$\frac{\partial(\ln B)}{\partial(\ln m_H)}$$



Dependence of nuclear binding on hadronic mass variation

- Use Dyson-Schwinger equation (DSE) to map variation of light quark mass, $m_q = (m_u + m_d)/2$, to hadron masses (Flambaum *et al.* 2006).

$$\frac{\delta m_H}{m_H} = \frac{\sigma_H}{m_H} \frac{\delta m_q}{m_q}$$

with $\frac{\sigma_H}{m_H}$ value of 0.498 for pion, etc.



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- Put it together:

$$\frac{\partial B/B}{\partial m_q/m_q} = \sum_H \frac{\partial(\ln B)}{\partial(\ln m_H)} \frac{\sigma_H}{m_H}$$

tabulated for all light species (up to ${}^7\text{Be}$).

Linear variation of m_q

Now $x = \delta m_q / m_q$

$$\begin{array}{ll} \text{D:} & 1 + 9.34x = 1.09 \pm 0.10 & 0.009 \text{ (11)} \\ {}^4\text{He:} & 1 - 1.00x = 1.005 \pm 0.036 & \rightarrow -0.005 \text{ (36)} \\ {}^7\text{Li:} & 1 - 60x = 0.33 \pm 0.11 & 0.011 \text{ (2)} \end{array}$$

Consistent solution: $x = 0.011 \text{ (2)}$ dominated by ${}^7\text{Li}$
c.f. Flambaum and Wiringa, 2007: $x = 0.013 \text{ (2)}$



$$\frac{M_g}{\Lambda_{\text{QCD}}}$$



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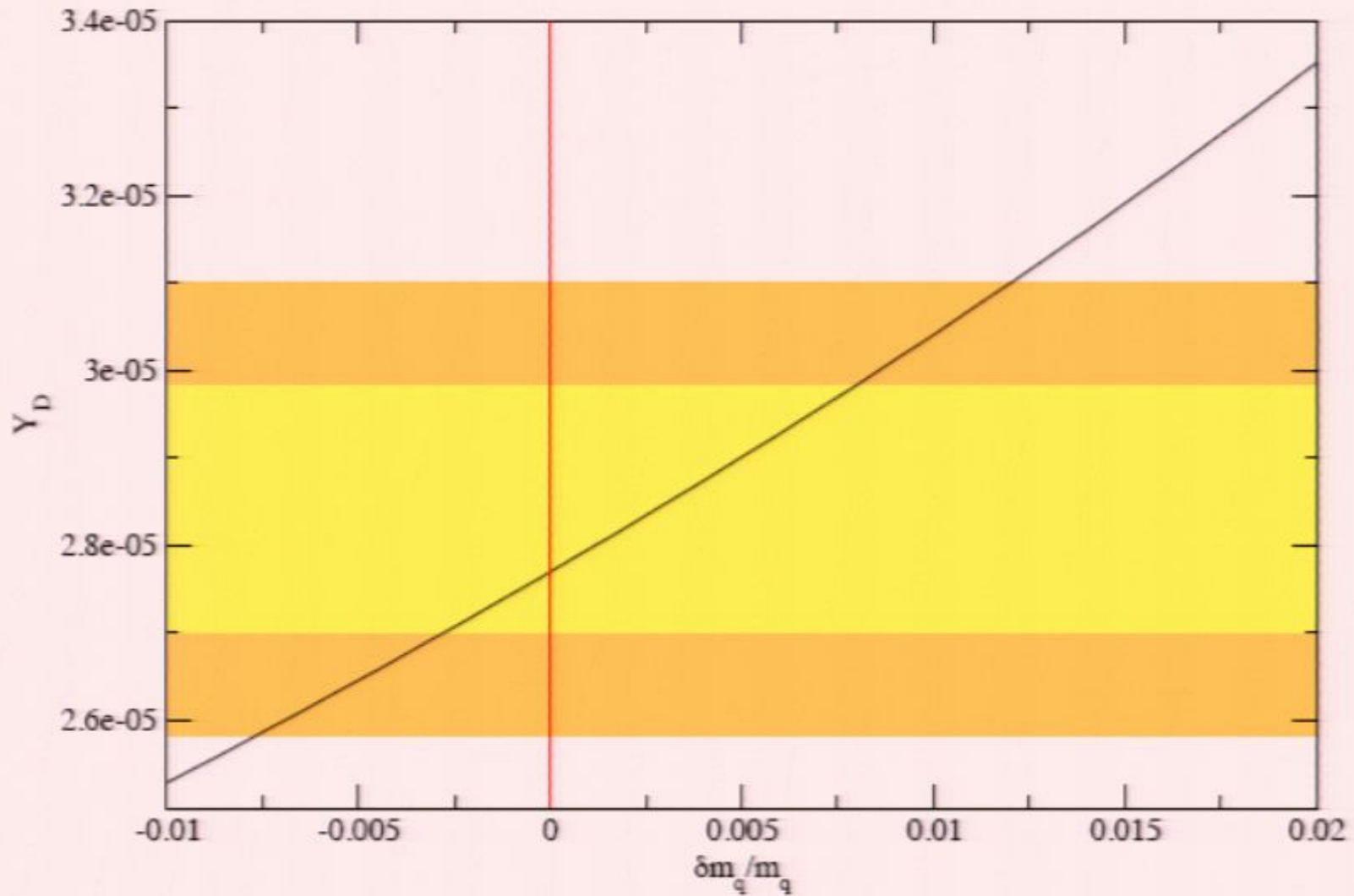
Now $x = \delta m_q / m_q$

$$\begin{array}{ll} \text{D:} & 1 + 9.34x = 1.09 \pm 0.10 & 0.009 \text{ (11)} \\ {}^4\text{He:} & 1 - 1.00x = 1.005 \pm 0.036 & \rightarrow -0.005 \text{ (36)} \\ {}^7\text{Li:} & 1 - 60x = 0.33 \pm 0.11 & 0.011 \text{ (2)} \end{array}$$

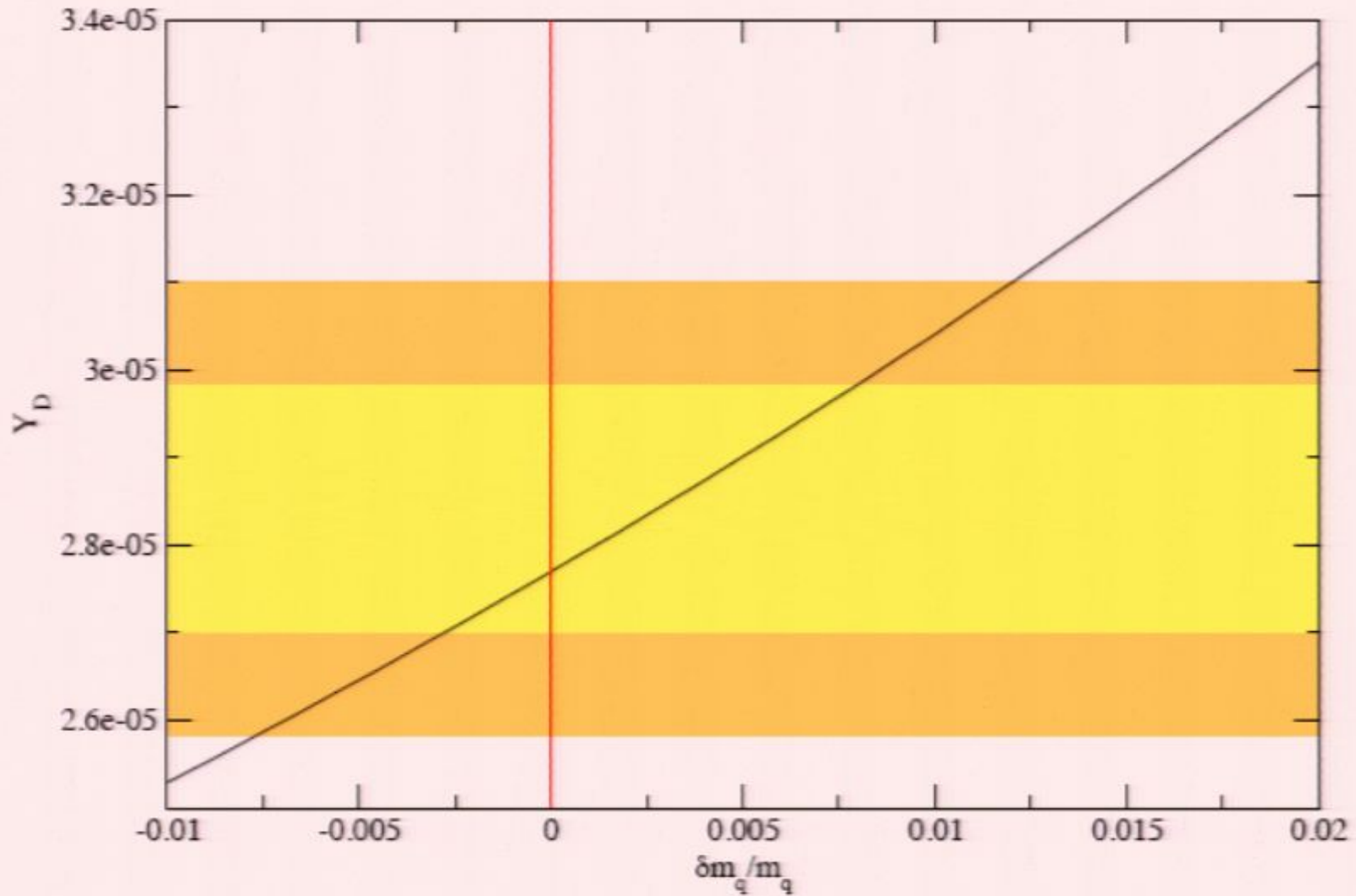
Consistent solution: $x = 0.011 \text{ (2)}$ dominated by ${}^7\text{Li}$
c.f. Flambaum and Wiringa, 2007: $x = 0.013 \text{ (2)}$



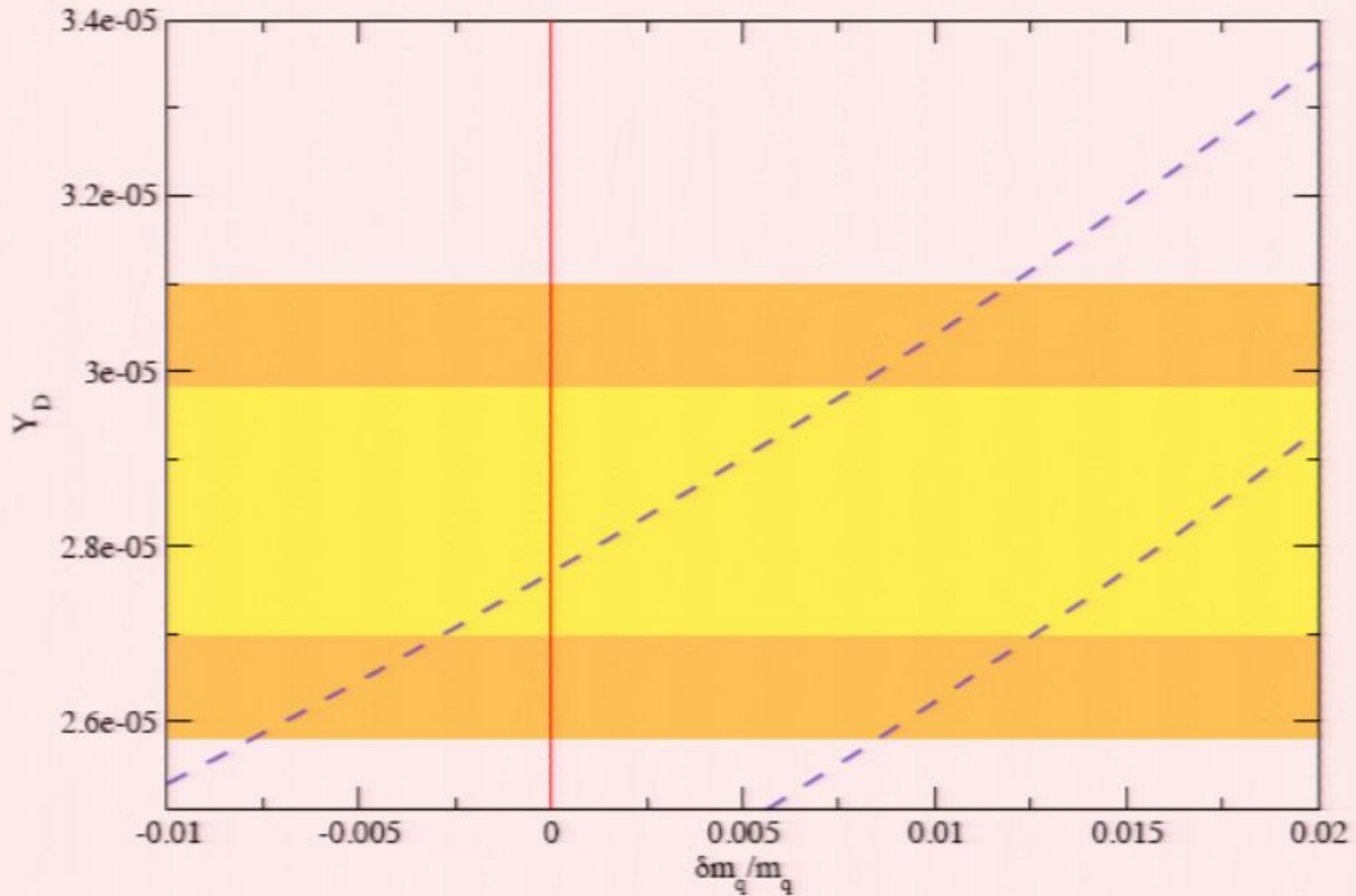
Nonlinear variation of m_q : D constraint



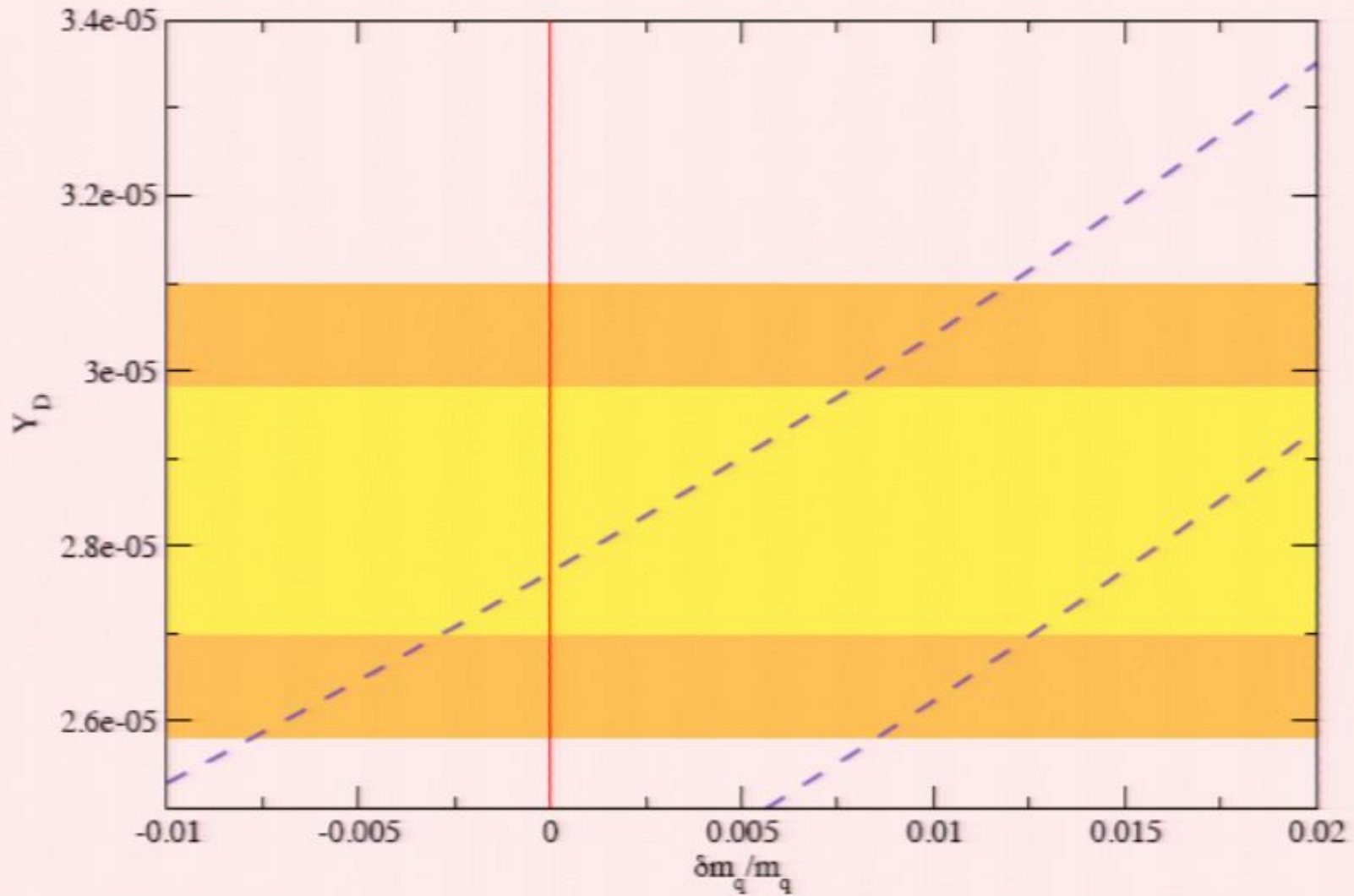
Nonlinear variation of m_q : D constraint



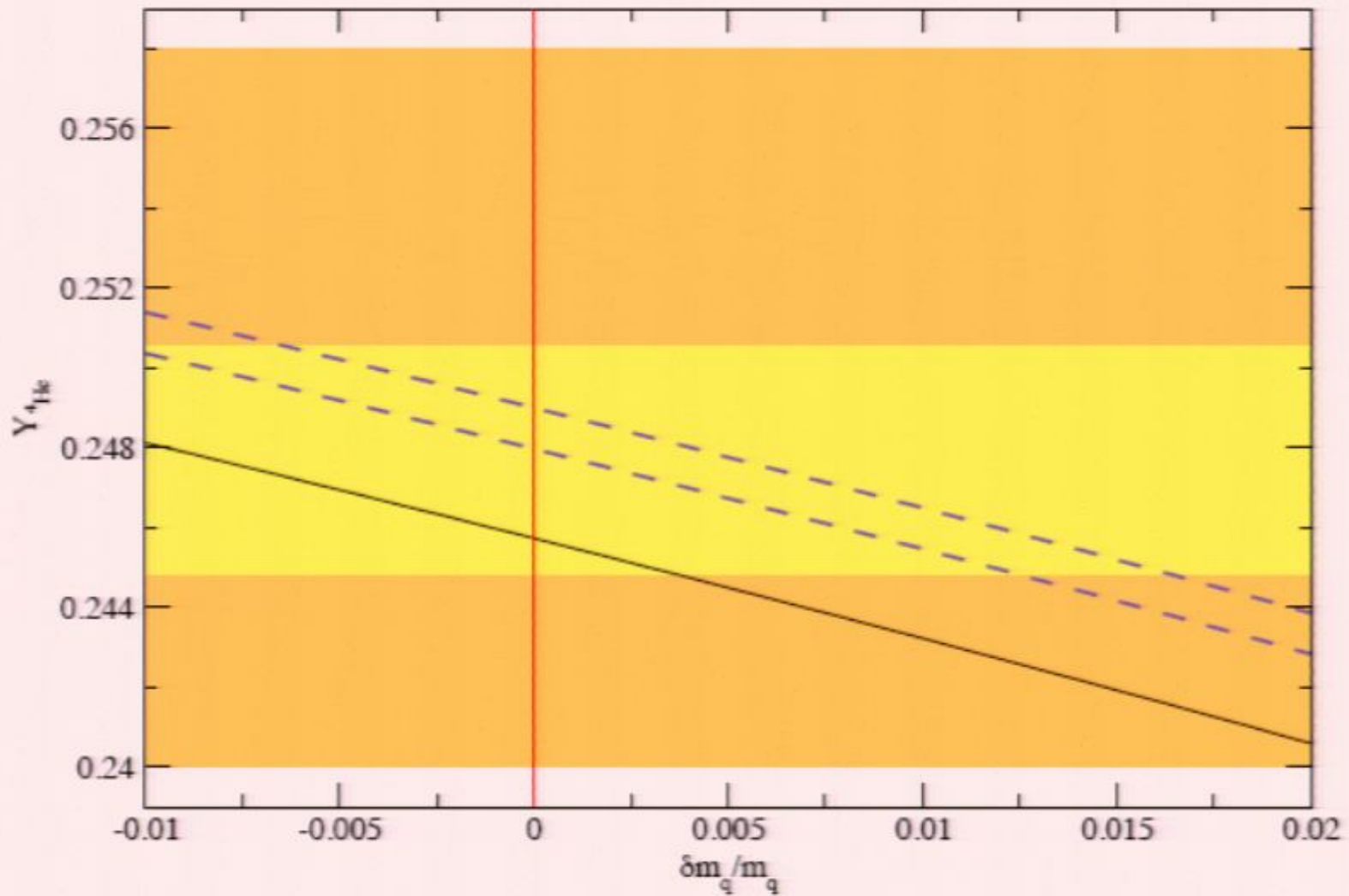
Nonlinear variation of m_q : D constraint



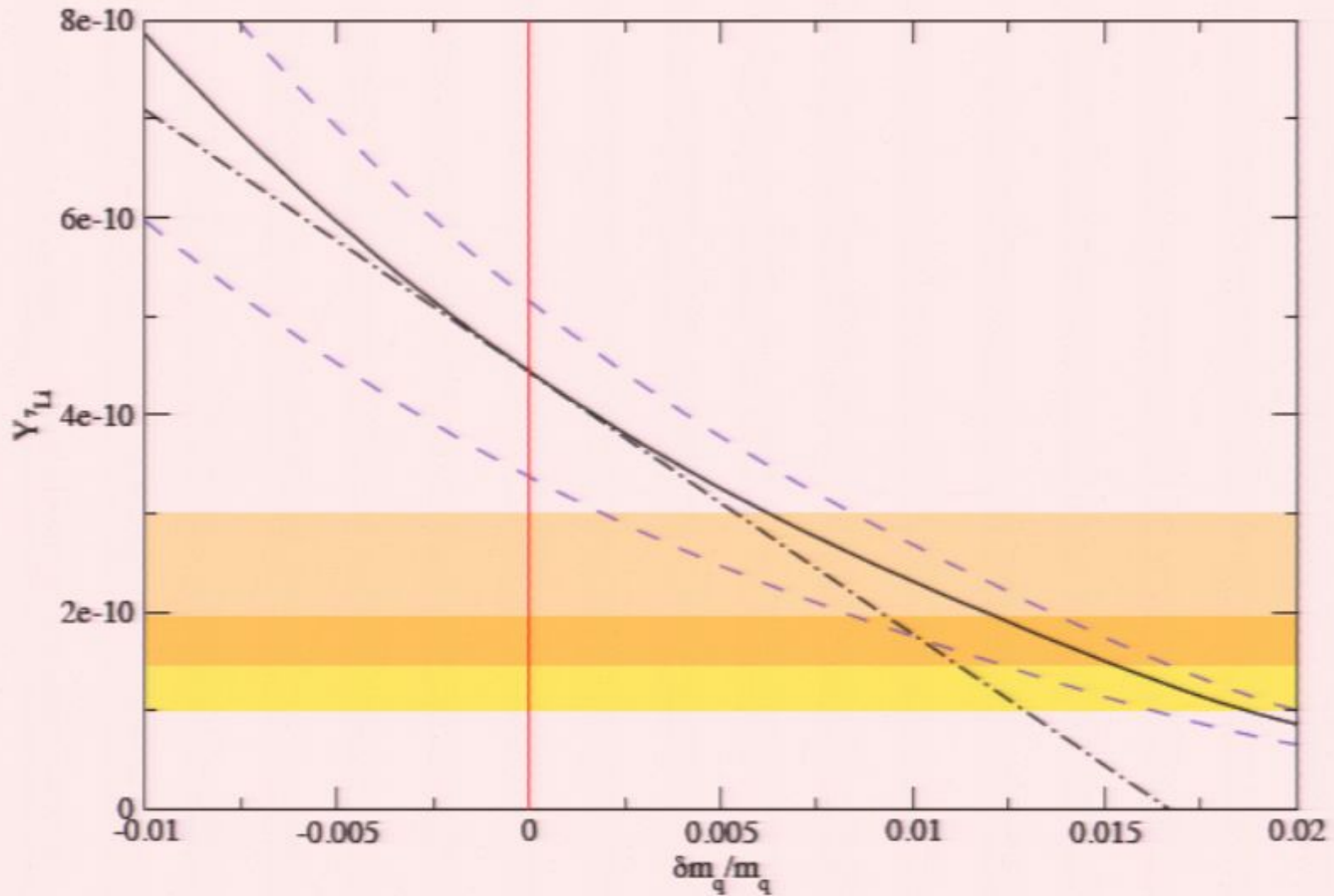
Nonlinear variation of m_q : D constraint



Nonlinear variation of m_q : ${}^4\text{He}$ constraint



Nonlinear variation of m_q : ${}^7\text{Li}$ constraint



Results from ${}^7\text{Li}$

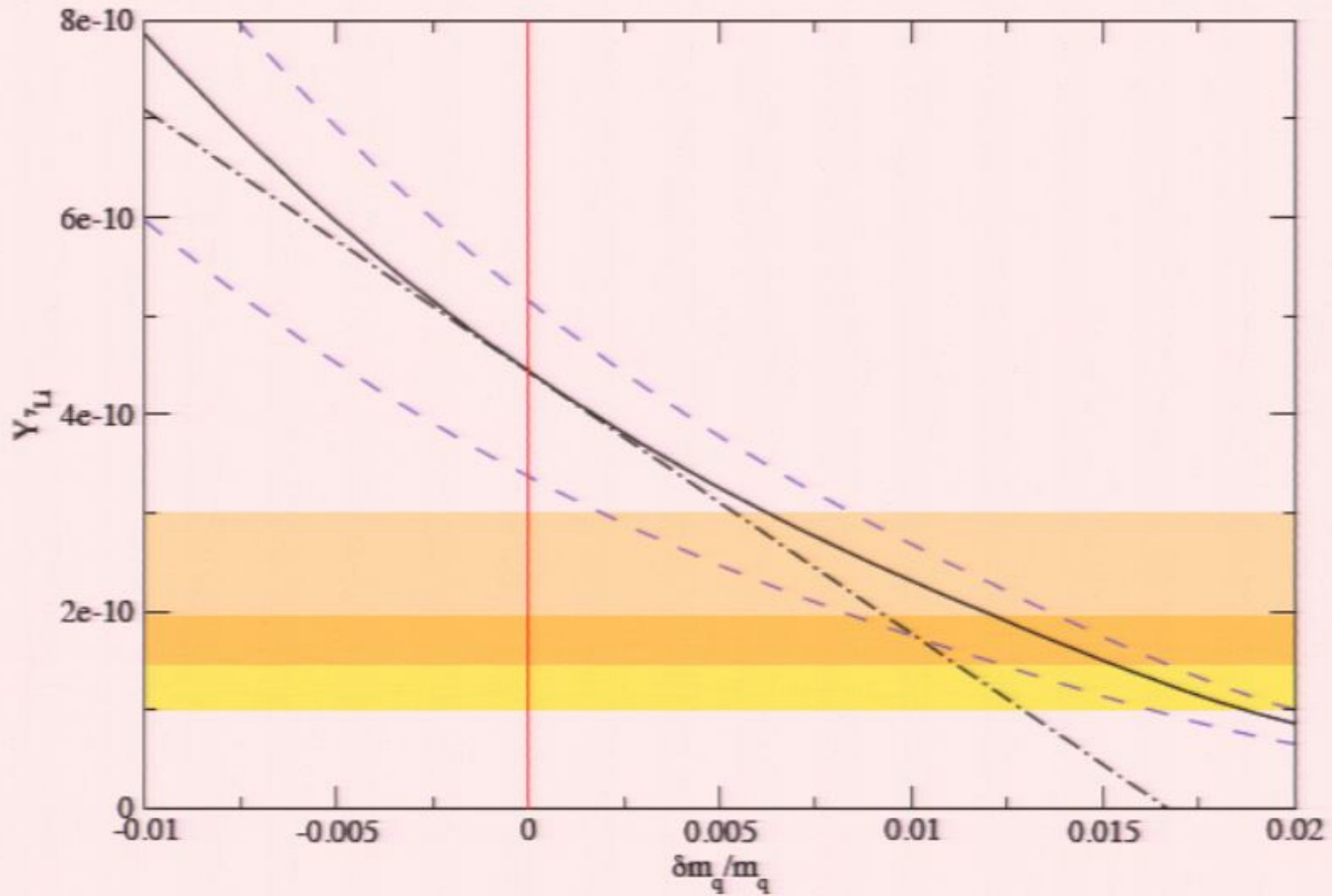
Only observational uncertainty

Bonifacio <i>et al.</i> 2007	$\delta m_q/m_q = 0.017$ (2)
PDG 2007	$\delta m_q/m_q = 0.013$ (2)
Bonifacio + PDG	$\delta m_q/m_q = 0.015$ (3)
Lower limit	$\delta m_q/m_q > 0.006$

- Large theory errors (Cyburt) increase uncertainty to ~ 0.007
- Nonlinear effect is ~ 0.005



Nonlinear variation of m_q : ${}^7\text{Li}$ constraint



Results from ${}^7\text{Li}$

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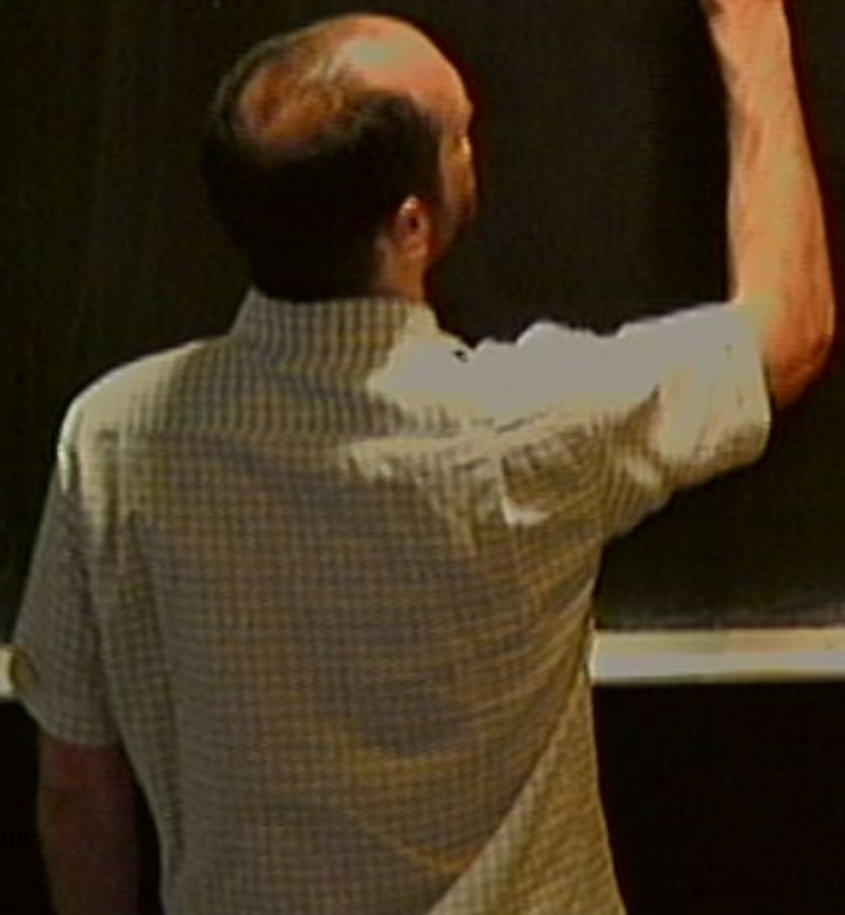
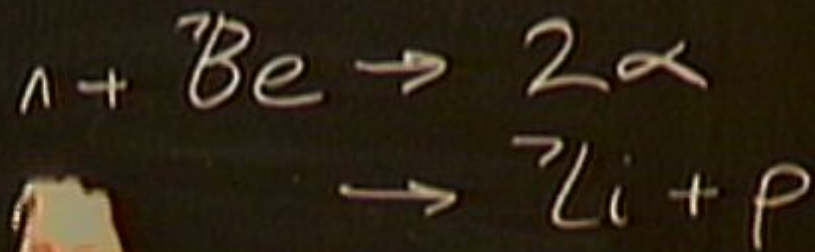
- Large theory errors (Cyburt) increase uncertainty to ~ 0.007
- Nonlinear effect is ~ 0.005
- $\delta m_q/m_q$ is not small!
e.g. Q goes negative for some minor reactions
 - ${}^{10}\text{Be} (p, \alpha) {}^7\text{Be}$
 - ${}^9\text{Be} (p, d\alpha) {}^4\text{He}$

Summary

- BBN is a useful probe of hadronic mass variation in the early universe
- Improvements in observational constraints (incl. ^3He) and nuclear reactions will further constrain hadron mass variation
- Nonlinear effects have some importance



$$\frac{M_g}{\Lambda_{\text{QCD}}}$$



$\frac{Mg}{\Lambda_{QCD}}$

