

Title: Limits on variation of fundamental constants from microwave and infrared transitions in atoms and molecules

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Abstract:

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# The constants which can be probed with atomic & molecular spectra:

- the fine-structure constant  $\alpha = e^2/hc$
- The proton-to-electron mass ratio  $\mu = m_p/m_e$
- the nuclear gyromagnetic ratio  $g_n$

*Reported in the literature optical data concerning the relative variation of constants  $\delta\mu/\mu$  and  $\delta\alpha/\alpha$  at redshifts  $z \sim 1-3$  are controversial at the level of a few ppm (1 ppm =  $10^{-6}$ ).*

# Astronomical estimates of the physical constants

- These estimates are based on the comparison of the line centers in the spectra of astronomical objects with corresponding laboratory values.
- In order to disentangle the line shifts caused by the motion of the object and by the putative effect of the variability of constants, lines with different sensitivities to the variation of fundamental constants should be used.
- The accuracy of the method depends on the linewidths and the respective sensitivity coefficients.

## Sensitivity coefficients to the variation of fundamental constants

The observed linewidth  $\Gamma$  in astrophysical spectra is usually determined by the Doppler broadening effect, i.e.

$$\frac{\Gamma}{\omega} = \frac{\Delta v}{c},$$

where  $\Delta v$  is the velocity dispersion,  $c$  is the speed of light, and  $\omega$  the transition frequency. For extragalactic observations the typical values of  $\Delta v$  are about  $1 - 10$  km/s, which means that:

$$\frac{\Gamma}{\omega} \sim 10^{-5}.$$

The dimensionless sensitivity coefficients can be defined as:

$$\frac{\delta\omega}{\omega} = K_\alpha \frac{\delta\alpha}{\alpha} + K_\mu \frac{\delta\mu}{\mu} + K_g \frac{\delta g_n}{g_n}$$

If we observe two lines with different sensitivities and the same *actual* redshift, the apparent redshifts will differ by

$$\frac{\delta z'}{1+z'} = -\Delta K_\alpha \frac{\delta \alpha}{\alpha} - \Delta K_\mu \frac{\delta \mu}{\mu} - \Delta K_g \frac{\delta g_n}{g_n}.$$

From the observation of one pair of lines it is impossible to distinguish between variation of different constants. Thus, we express difference in apparent redshift in terms of the variation of a following combination of fundamental constants:

$$\frac{\delta z'}{1+z'} = -\frac{\delta F}{F}, \quad F = \alpha^{\Delta K_\alpha} \mu^{\Delta K_\mu} g_n^{\Delta K_g}.$$

Obviously we should maximize either  $\Delta K_\alpha$ ,  $\Delta K_\mu$ , or  $\Delta K_g$ . Note that differences  $\Delta K_i$  are independent on the frequency units.

## Sensitivity coefficients for different wavebands

Transition	$K_\alpha$	$K_\mu$	$K_g$
<i>Optical and UV range</i>			
typical E1-transition in atom	$10^{-2} - 10^{-1}$	$10^{-3}$	$10^{-7}$
electronic transition in light molecule	$10^{-2}$	$10^{-2}$	$10^{-7}$
<i>Microwave and infrared range</i>			
fine-structure M1-transition	<b>2</b>	0.0	0.0
vibrational transition	0.0	-0.5	0.0
rotational transition	0.0	-1.0	0.0
21-cm hyperfine line in hydrogen	<b>2.0</b>	-1.0	<b>1.0</b>
18-cm $\Lambda$ -doublet line in OH	<b>-2</b>	<b>-3</b>	$10^{-1}$
1.25-cm inversion line in NH <sub>3</sub>	0.0	<b>-4.5</b>	0.0

We see that:

- Sensitivity coefficients in microwave and infrared ranges are several orders of magnitude larger, than in optical and UV ranges.
- There are lines of different types here, and sensitivity coefficients change drastically from one type to another.
- For example, the splitting between the components of the  $\Lambda$ -doublet of the ground  $\Pi_{3/2}$  state in the OH molecule appears in the 3rd order in Coriolis interaction and is extremely sensitive to fundamental constants  $\alpha$  and  $\mu$  [Kanekar *et al*].
- Inversion line in ammonia corresponds to the tunneling transition of three hydrogen atoms from one minimum of a double-well potential to another. The tunneling frequency exponentially depends on the reduced mass for the respective vibrational mode and is, therefore, extremely sensitive to  $\mu$ -variation [van Veldhoven *et al*].

Limits on variation of fundamental  
constants at redshifts  $z \sim 1$   
*(timescale of few Gyr)*  
from microwave and IR spectra

In 1996 Varshalovich & Potekhin compared apparent redshifts of rotational and optical lines to place following bound at  $z=1.9$ :

$$\delta\mu/\mu = (70 \pm 100) \times 10^{-6}$$

In 2001 Murphy et al compared redshifts of 21 cm hydrogen line and a number of rotational lines for the object B0218+357 at the redshift  $z=0.68$  to get the bound on variation of the product:

$$\delta F'/F' = (1.6 \pm 5.4) \times 10^{-6}, \quad F' = \alpha^2 g_n$$

$\Lambda$ -doublet OH line from the same object B0218+357 was recently used to place very stringent bound on the variation of different combination of constants [Kanerar et al, 2005]:

$$\delta F/F = (3.5 \pm 4.0) \times 10^{-6}, \text{ where } F = \alpha^{3.14} \mu^{1.57} g_n$$

Finally,  $\text{NH}_3$  inversion line from B0218+357 allows to place limit on the variation of  $\mu$  [Flambaum and Kozlov, 2007]:

$$\delta\mu/\mu = (0.6 \pm 1.9) \times 10^{-6}$$

The above limits are based on the analysis of different microwave lines of the same object  $B0218+357$  at  $z=0.68$ . Simultaneous analysis of all these lines allow to have a complete experiment, i.e. to study all three fundamental constants relevant to atomic physics and place three model-independent limits on their time-variation:

$$\begin{cases} \delta\mu/\mu = (0.6 \pm 1.9) \times 10^{-6}, \\ \delta\alpha/\alpha = (0.9 \pm 6.4) \times 10^{-6}, \\ \delta g_n/g_n = (0 \pm 17) \times 10^{-6}. \end{cases}$$

*It would be extremely interesting to get new high precision data for this object for a dedicated and comprehensive analysis.*

## Using fine-structure lines to place bounds on time-variation at very high redshifts

The redshift of the fine-structure [C II] 158  $\mu\text{m}$  line is compared to that of the rotational CO line. Both lines are observed in emission for the quasars J1148+5251 and BR 1202-0725 with respective redshifts  $z=6.42$  and  $z=4.69$ . The absence of the meaningful differences in apparent redshifts allowed to place bounds on the variation of the parameter

$$F'' = \alpha^2 \mu \quad [\text{Levshakov et al, 2008}]:$$

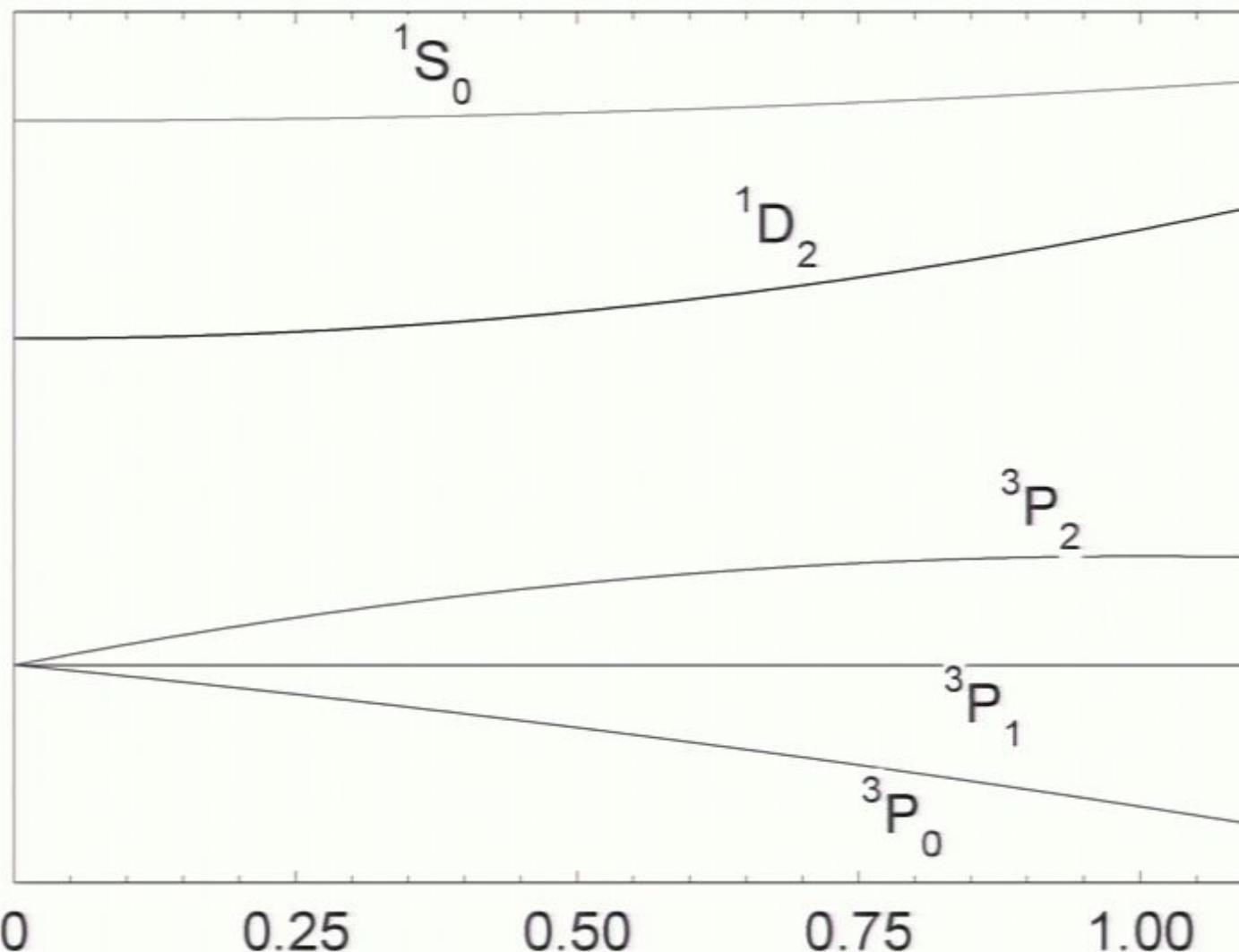
$$\begin{cases} \delta F''/F'' = (0.1 \pm 1.0) \times 10^{-4}, z = 6.42, \\ \delta F''/F'' = (1.4 \pm 1.5) \times 10^{-4}, z = 4.69. \end{cases}$$

*Note that  $z=6.42$  corresponds to the look-back time of approximately 12.9 Gyr, which constitutes 93% of the age of the Universe.*

## Using transitions in the same species to reduce Doppler noise

- In order to reduce systematic errors from the Doppler noise it is desirable to compare redshifts for transitions in the same species.
- We need transitions with different sensitivities.
- Usually lines of the same nature have very close sensitivities.  
Therefore, one has to compare apparent redshifts of, say, inversion line and rotational lines of  $\text{NH}_3$ , etc.
- Some fine-structure transitions may have different sensitivities due to the strong interactions between multiplets [Dzuba & Flambaum, 2005]. For light atoms such differences are small, but they rapidly grow with nuclear charge  $Z$ .

## Multiplet structure of configuration $ns^2np^2$



### Examples:

C I, O III, Ne V,  
Si I, S III, Ar V,  
etc

To a first approximation the frequencies of the fine-structure transitions obey the Landé-rule:

$$\omega_{J,J-1}/\omega_{J-1,J-2} = J/(J-1)$$

In this approximation sensitivities of both transitions are the same,  $K_\alpha = 2$ . Interaction between levels of different multiplets leads to violation of the Landé-rule and to deviation of the sensitivities from 2. The following relation still hold:

$$\Delta K_\alpha \equiv K_\alpha(J, J-1) - K_\alpha(J-1, J-2) = 2 \left[ \frac{J-1}{J} \frac{\omega_{J,J-1}}{\omega_{J-1,J-2}} - 1 \right]$$

For the most interesting case of the multiplet  ${}^3P_J$ ,  $J = 0, 1, 2$ , this relation reduces to:

$$\Delta K_\alpha \equiv K_\alpha(2,1) - K_\alpha(1,0) = \frac{\omega_{2,1}}{\omega_{1,0}} - 2$$

## Differences in sensitivity coefficients for fine-structure transitions in light ions

Ion	Transition <i>a</i>			Transition <i>b</i>			$\Delta K_a$
	$(J_a' J_a)$	$\lambda_a$ ( $\mu\text{m}$ )	$\omega_a$ ( $\text{cm}^{-1}$ )	$(J_b' J_b)$	$\lambda_b$ ( $\mu\text{m}$ )	$\omega_b$ ( $\text{cm}^{-1}$ )	
C I	(1,0)	609.1	16.40	(2,1)	370.4	27.00	-0.016
O I	(0,1)	145.5	68.73	(1,2)	63.2	158.27	0.042
Si I	(1,0)	129.7	77.11	(2,1)	68.5	146.05	-0.11
S I	(0,1)	56.3	177.59	(1,2)	25.3	396.06	0.23
Ti I	(2,3)	58.8	170.13	(3,4)	46.1	216.74	-0.090
Fe I	(2,3)	34.7	288.07	(3,4)	24.0	415.93	0.17
	(1,2)	54.3	184.13	(2,3)	34.7	288.07	0.086
	(0,1)	111.2	89.94	(1,2)	54.3	184.13	0.048

Ion	Transition <i>a</i>			Transition <i>b</i>			$\Delta K_\alpha$
	$(J_a' J_a)$	$\lambda_a$ ( $\mu\text{m}$ )	$\omega_a$ ( $\text{cm}^{-1}$ )	$(J_b' J_b)$	$\lambda_b$ ( $\mu\text{m}$ )	$\omega_b$ ( $\text{cm}^{-1}$ )	
N II	(1,0)	205.3	48.70	(2,1)	121.8	82.10	-0.032
Fe II	(5/2,7/2)	35.3	282.89	(7/2,9/2)	26.0	384.79	0.12
	(3/2,5/2)	51.3	194.93	(5/2,7/2)	35.3	282.89	0.074
	(1/2,3/2)	87.4	114.44	(3/2,5/2)	51.3	194.93	0.044
O III	(1,0)	88.4	113.18	(2,1)	51.8	193.00	-0.054
Ne III	(0,1)	36.0	277.67	(1,2)	15.6	642.88	0.11
S III	(1,0)	33.5	298.69	(2,1)	18.7	534.39	-0.21
Ar III	(0,1)	21.9	458.05	(1,2)	9.0	1112.18	0.42
Fe III	(2,3)	33.0	302.7	(3,4)	22.9	436.2	0.16
	(1,2)	51.7	193.5	(2,3)	33.0	302.7	0.086
	(0,1)	105.4	94.9	(1,2)	51.7	193.5	0.038
Ne V	(1,0)	24.3	411.23	(2,1)	14.3	698.24	-0.12
Mg V	(0,1)	13.5	738.7	(1,2)	5.6	1783.1	0.41
Ca V	(0,1)	11.5	870.9	(1,2)	4.2	2404.7	0.76

## Conclusions

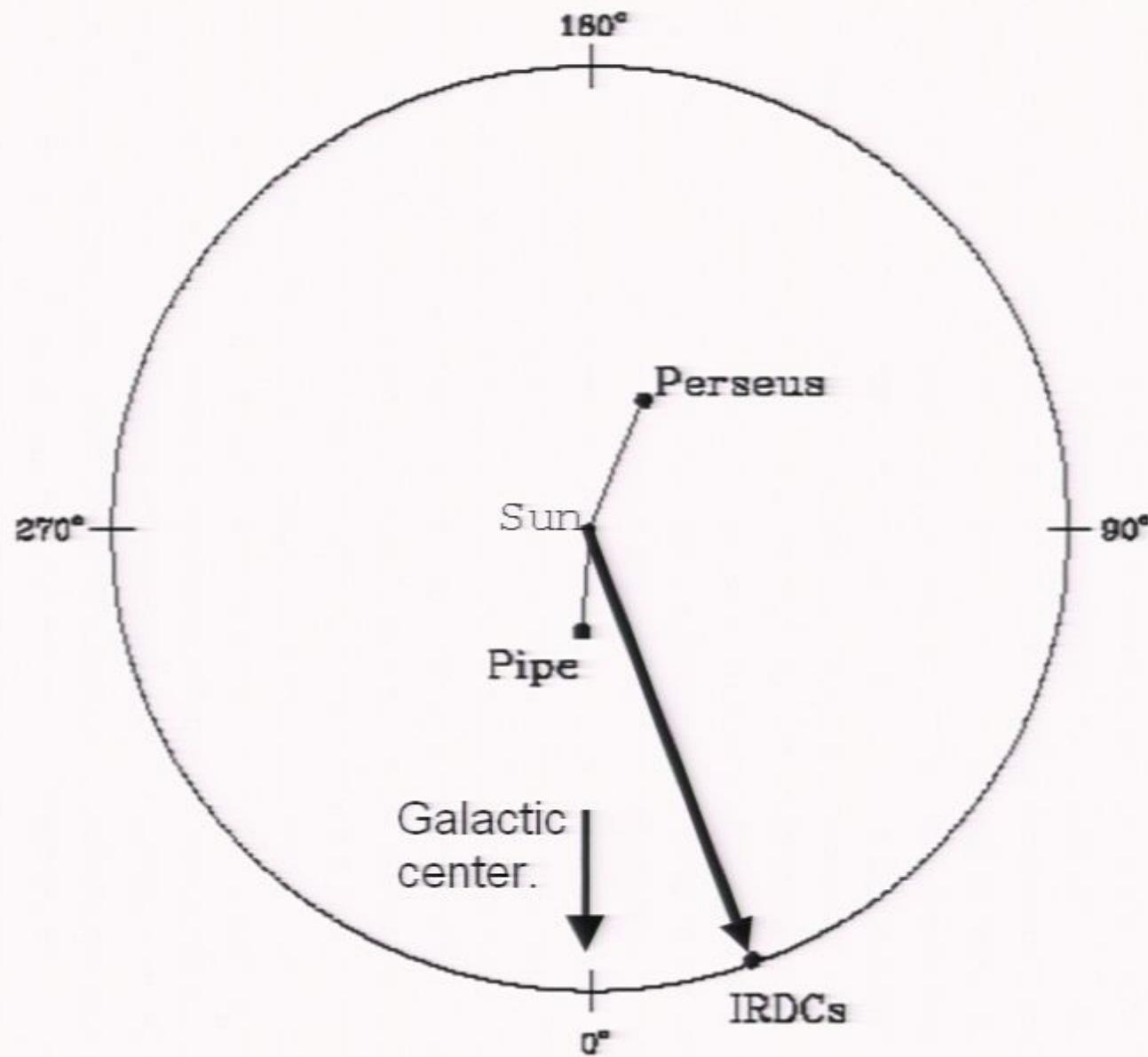
Microwave and IR lines have *higher sensitivity* to time-variation of fundamental constants compared to optical and UV bands.

Lines of different nature (fine-structure, hyperfine, rotational,  $\Lambda$ -doublet, inversion, etc.) are sensitive to different combinations of fundamental constants. If several lines of different types are observed for one object, *it is possible to determine the time-variation of all three constants*,  $a$ ,  $\mu$ , and  $g_n$ .

Some fine-structure and rotational lines are observed in emission for *extremely high redshifts*, up to  $z \approx 10$ . This allows to probe fundamental constants at very early epochs of the evolution of the Universe.

Observing lines of the same species one can *suppress systematic errors* caused by non-identical spatial distribution of different species in cold molecular gas clouds. For example, one can use 18 cm  $\Lambda$ -doublet line and rotational lines of OH. Similarly, the 1.2 cm inversion line can be used in combination with rotational lines of ammonia.

High precision data on variation of constants from microwave spectra of cold molecular clouds in our galaxy



**Fig.1**

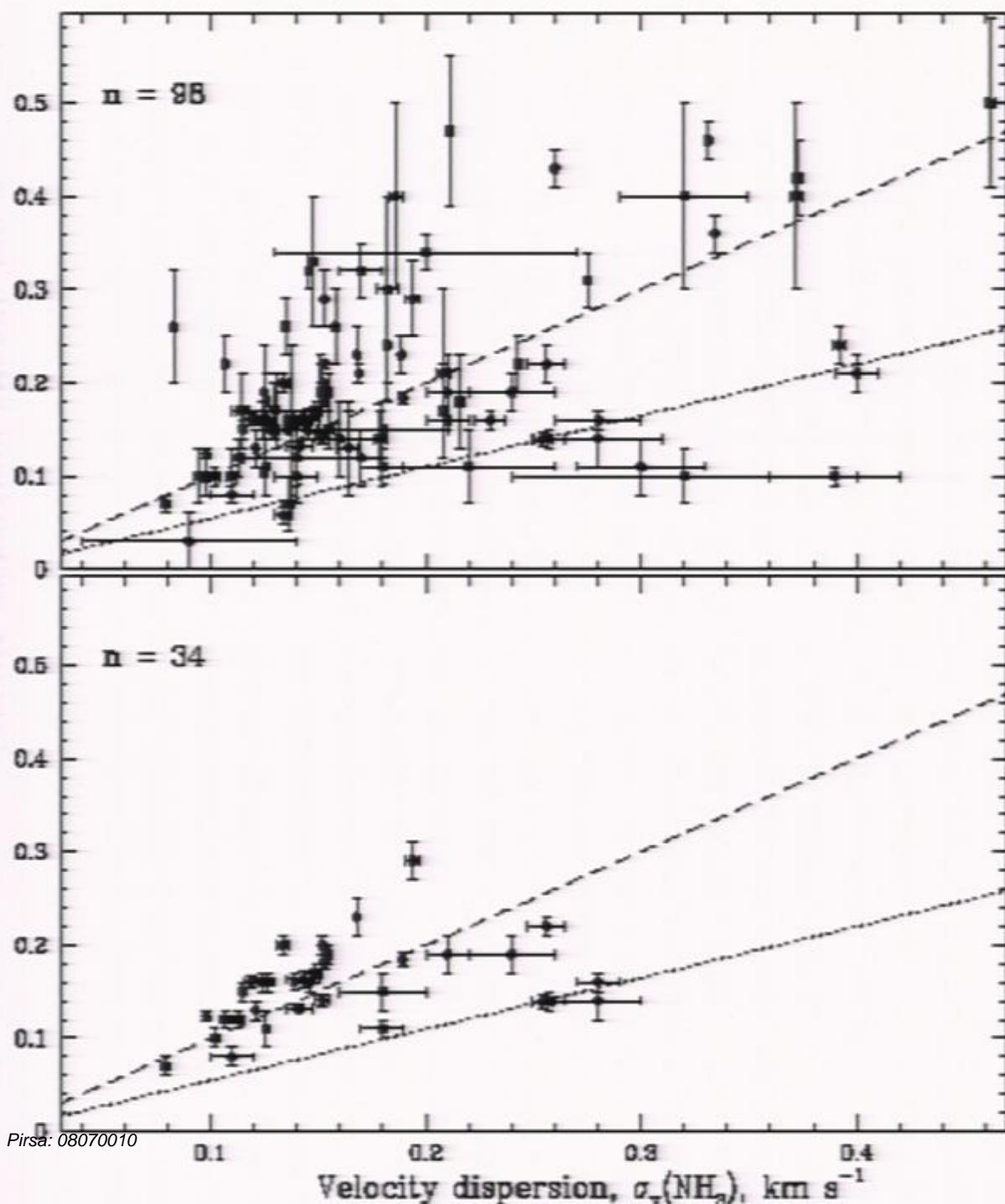
## Perseus Cloud

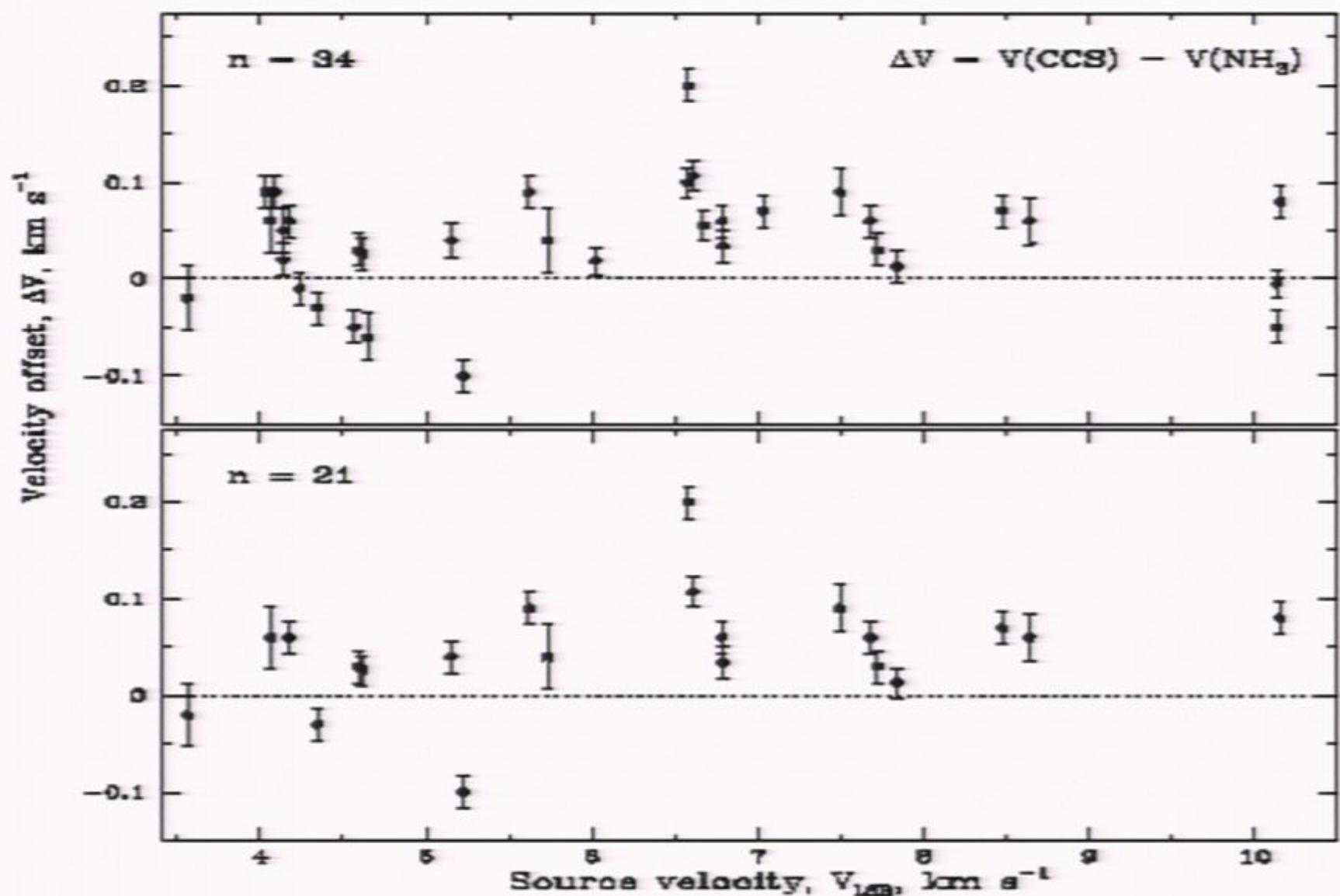
### Upper panel:

$\text{C}_2\text{S}$  ( $2_1 - 1_0$ ) versus  $\text{NH}_3$  (1,1) linewidths for cores in the Perseus molecular cloud from the total sample of Rosolowsky et al. (2008).

### Lower panel:

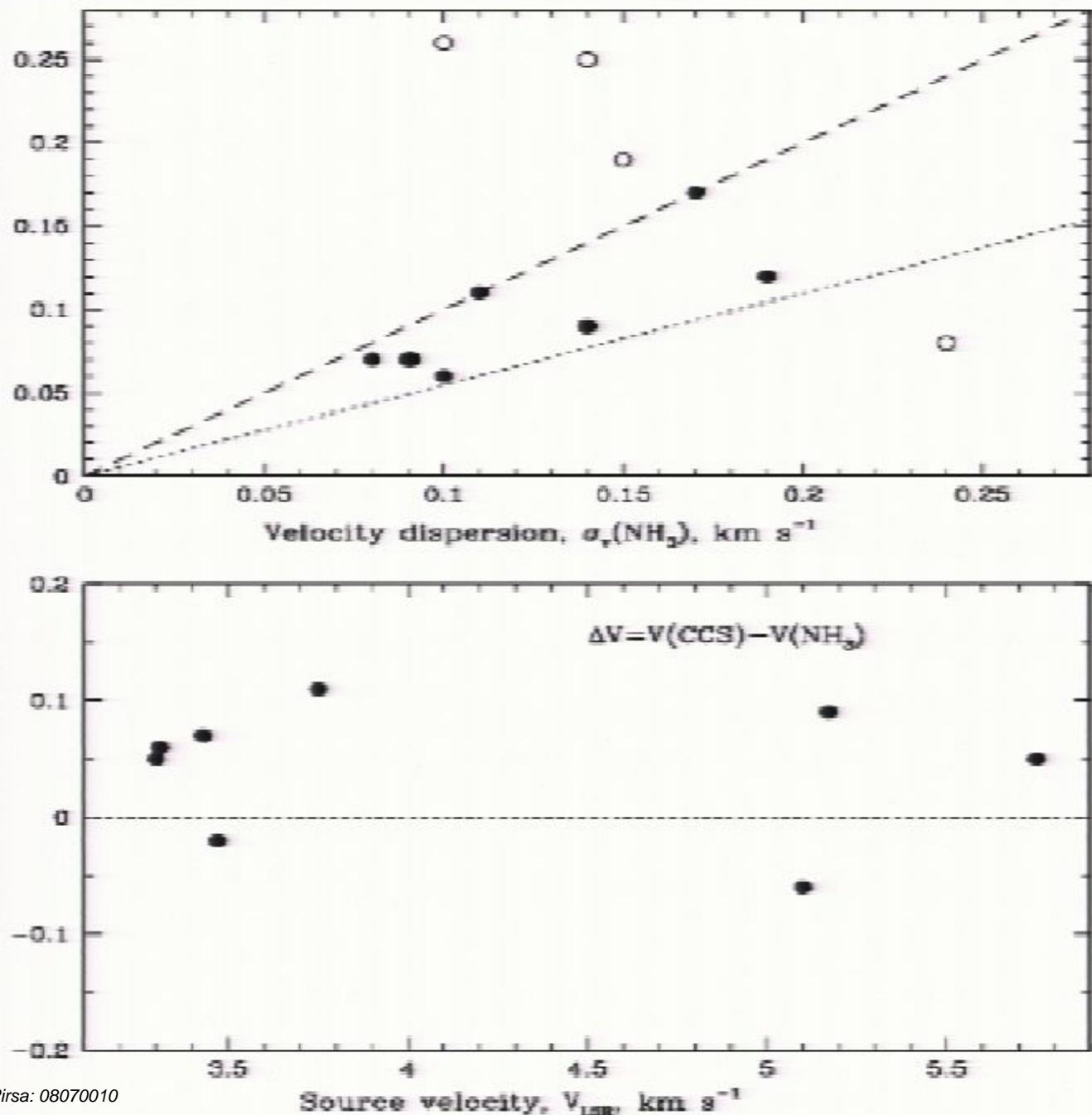
Subsample of the best single-component profiles of  $\text{C}_2\text{S}$  and  $\text{NH}_3$  selected from the full set.





**Upper panel:** Velocity offset  $\Delta V_{\text{CCS-NH}_3}$  versus the source radial velocity for points shown in the lower panel of Fig. 1.

**Lower panel:** Same as the upper panel but for the points, which lie in allowed region



## Pipe Nebular

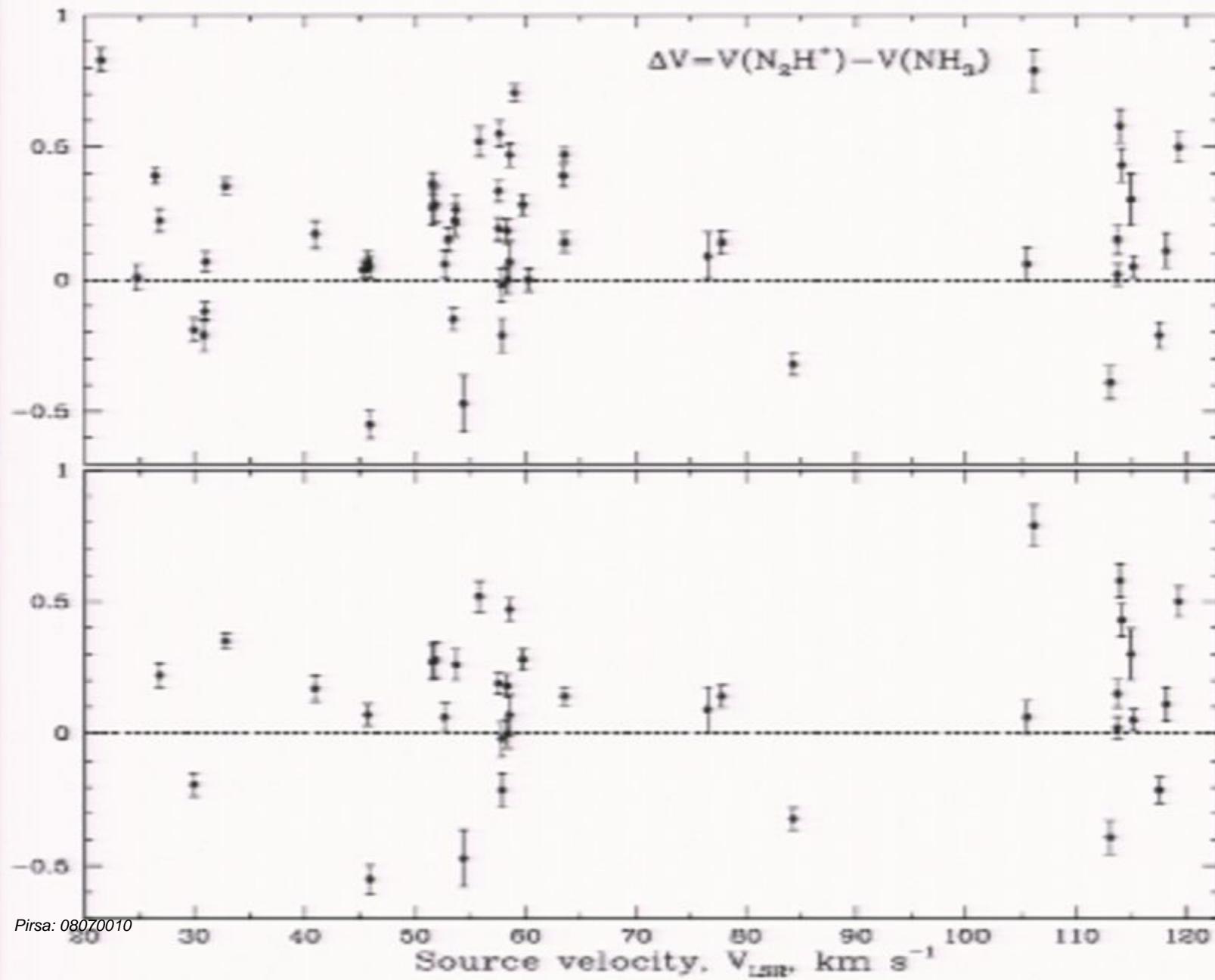
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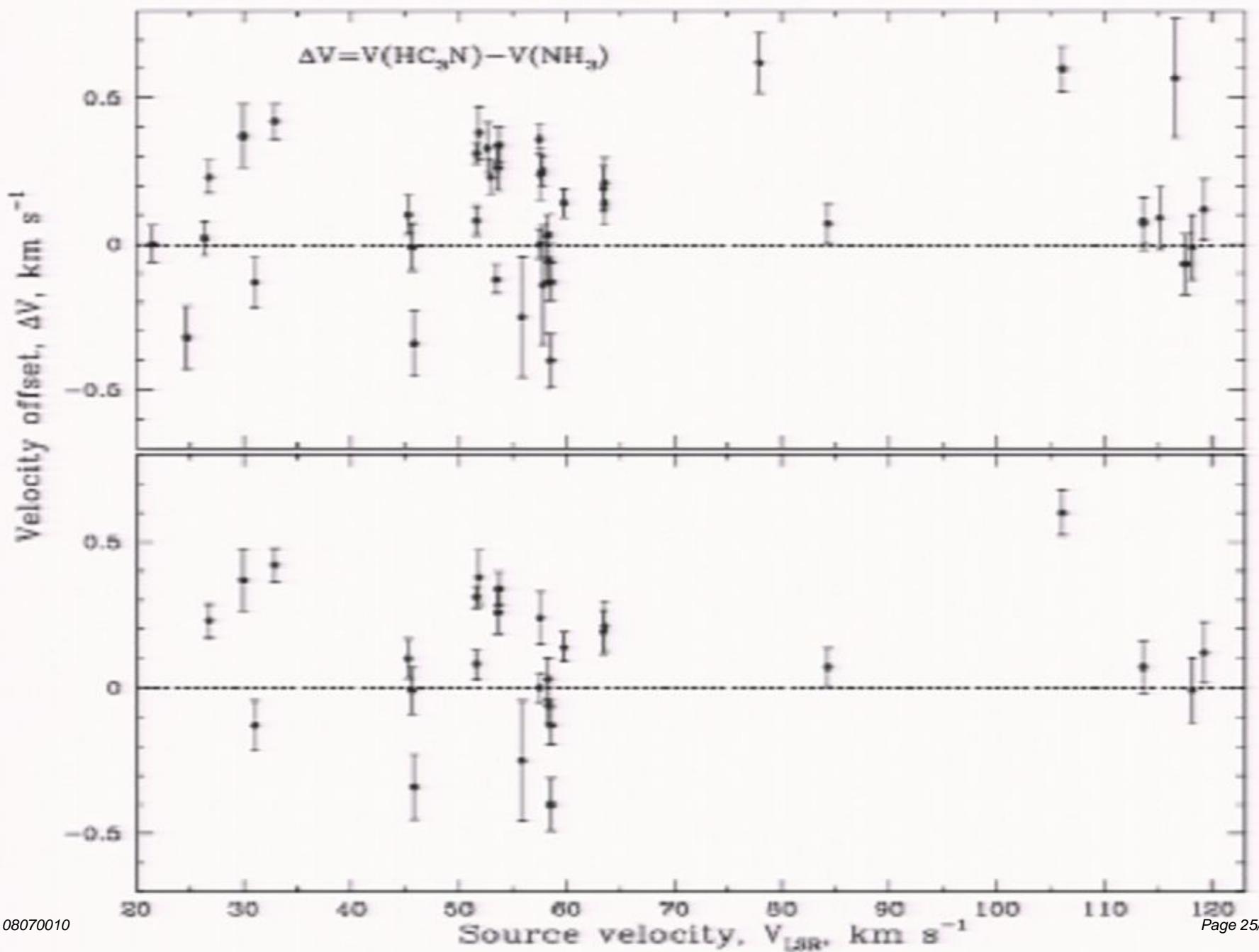
$\text{C}_2\text{S} (2_1 - 1_0)$  versus  
 $\text{NH}_3 (1,1)$  linewidths  
*Rathborn et al. (2008).*

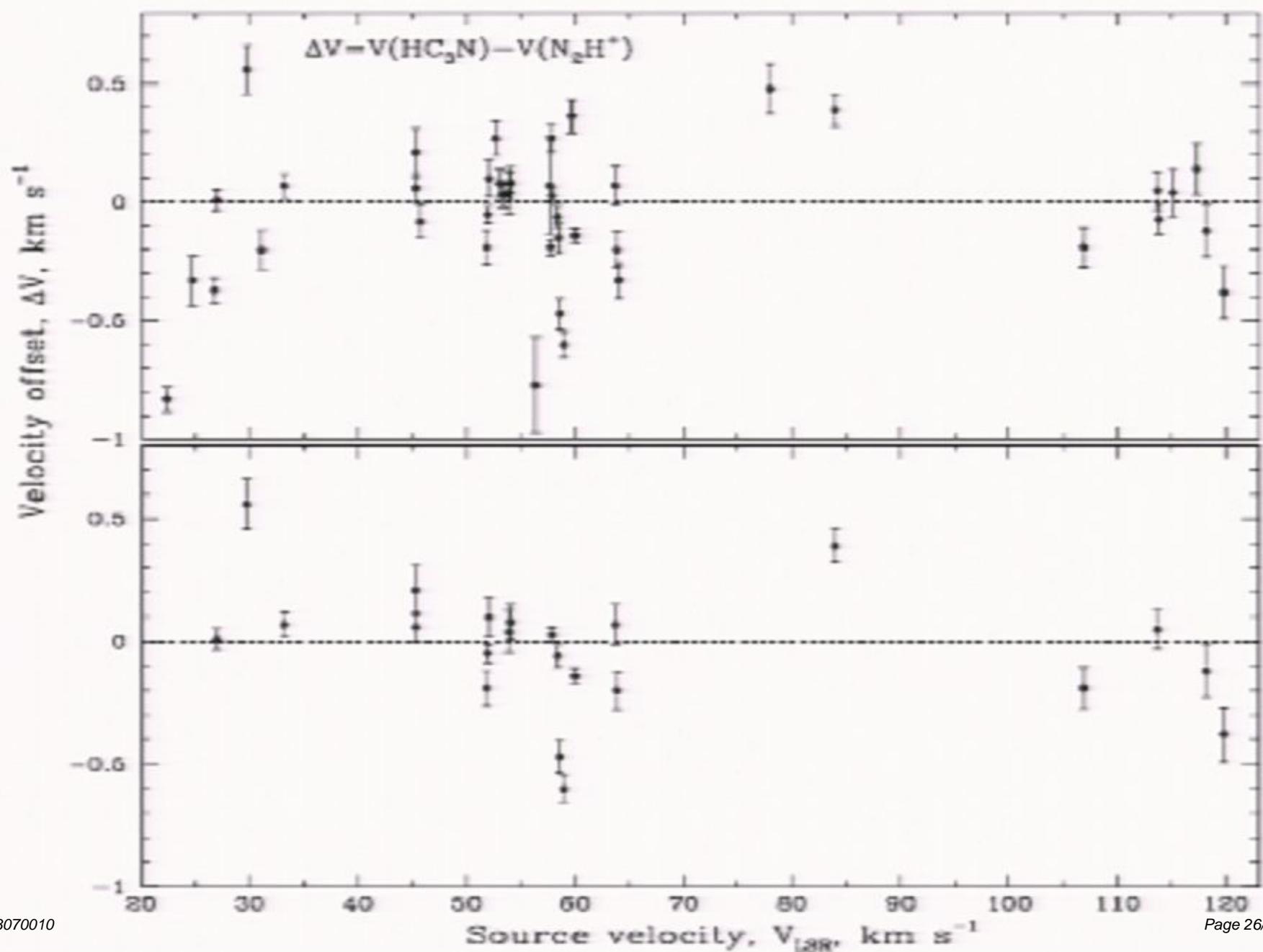
### Lower panel:

Velocity offset  
 $\Delta V_{\text{CCS-NH}_3}$  versus the  
source radial veloc-  
ity  $V_{\text{LSR}}(\text{NH}_3)$ .

## Infrared Dark Clouds (IRDCs)







ample mean values (unweighted)  $\Delta V$ , standard deviations  $\sigma_{\text{rms}}$ , and robust M-estimates of the sample deduced from the original data and from the subsamples of molecular lines showing self-consistent linewidths.

Object (1)	Molecular pair (2)	Sample size, $n$ (3)	$\Delta V^{\dagger}$ , km s $^{-1}$ (4)	$\sigma_{\text{rms}}$ , km s $^{-1}$ (5)	$\Delta V_M^{\dagger}$ , km s $^{-1}$ (6)
Orion Bar	$\text{NH}_3/\text{CCS}$	98	$0.044 \pm 0.013$	0.129	$0.040 \pm 0.007$
		34	$0.039 \pm 0.010$	0.058	$0.045 \pm 0.007$
		21	$0.048 \pm 0.013$	0.060	$0.052 \pm 0.007$
Serpens	$\text{NH}_3/\text{CCS}$	12	$0.087 \pm 0.039$	0.135	$0.039 \pm 0.023$
		8	$0.044 \pm 0.020$	0.057	$0.069 \pm 0.011$
L1544	$\text{NH}_3/\text{N}_2\text{H}^+$	54	$0.157 \pm 0.040$	0.294	$0.160 \pm 0.030$
		36	$0.122 \pm 0.049$	0.294	$0.160 \pm 0.032$
	$\text{NH}_3/\text{HC}_3\text{N}$	43	$0.138 \pm 0.043$	0.282	$0.110 \pm 0.032$
		27	$0.105 \pm 0.045$	0.234	$0.120 \pm 0.037$
Taurus	$\text{N}_2\text{H}^+/\text{HC}_3\text{N}$	41	$-0.056 \pm 0.047$	0.301	$-0.020 \pm 0.037$
		22	$-0.033 \pm 0.055$	0.258	$-0.017 \pm 0.034$

Our final results for velocity offsets of NH<sub>3</sub> inversion line versus rotational lines of other molecules for Perseus cloud, Pipe Nebulae (C<sub>2</sub>S), and IRDCs (N<sub>2</sub>H<sup>+</sup> & HC<sub>3</sub>N) are:

$$\Delta V_{Perseus} = 52 \pm 7_{stat} \pm 14_{sys}$$

$$\Delta V_{Pipe} = 69 \pm 11_{stat} \pm 14_{sys}$$

$$\Delta V_{IRDCs} = 160 \pm 32_{stat} \pm 4_{sys}$$

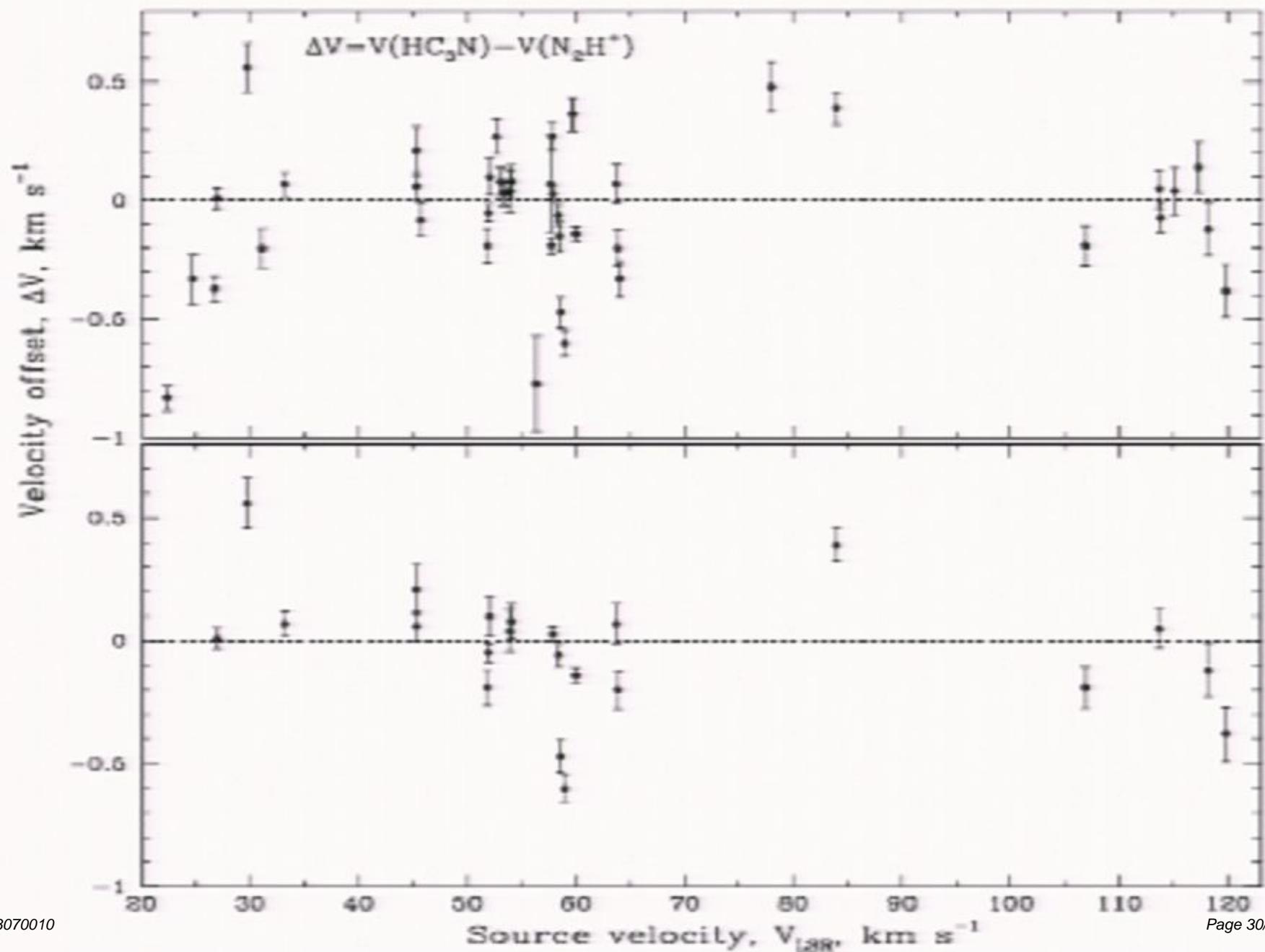
$$\Delta V_{IRDCs} = 120 \pm 37_{stat} \pm 7_{sys}$$

If we interpret these results in terms of  $\mu$ -variation, we get:

$$\Delta\mu/\mu = (-5 - 15) \times 10^{-8}$$

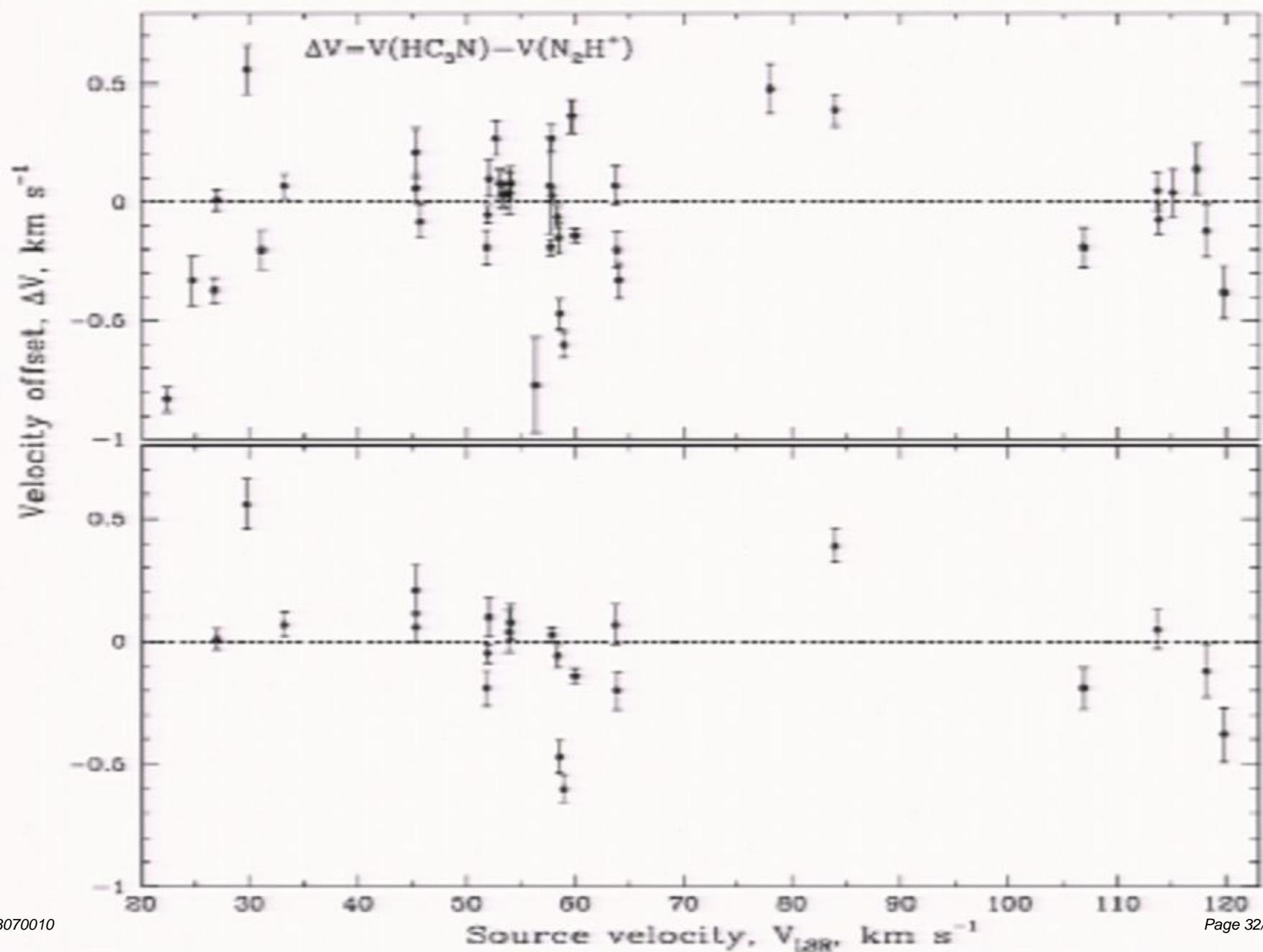
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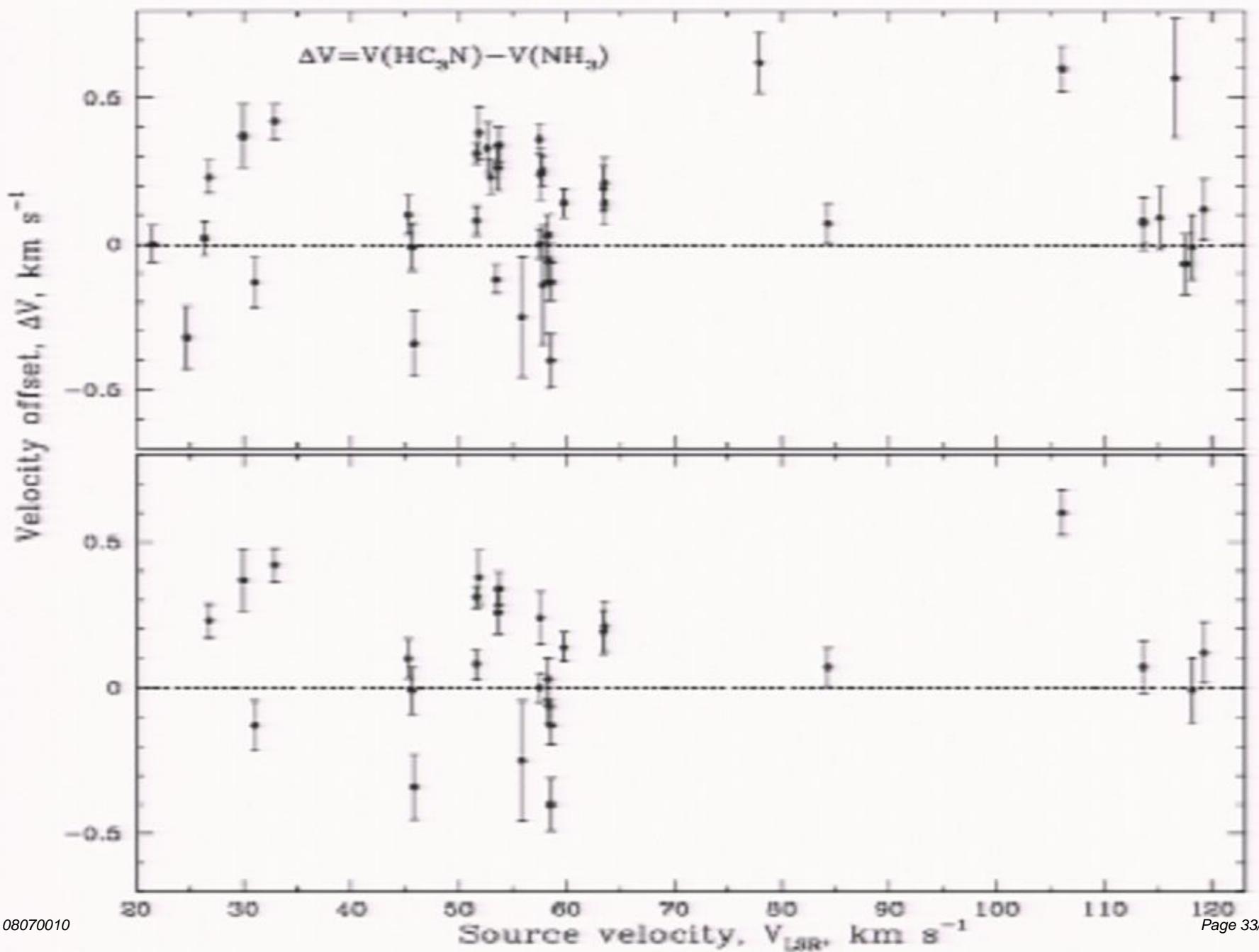
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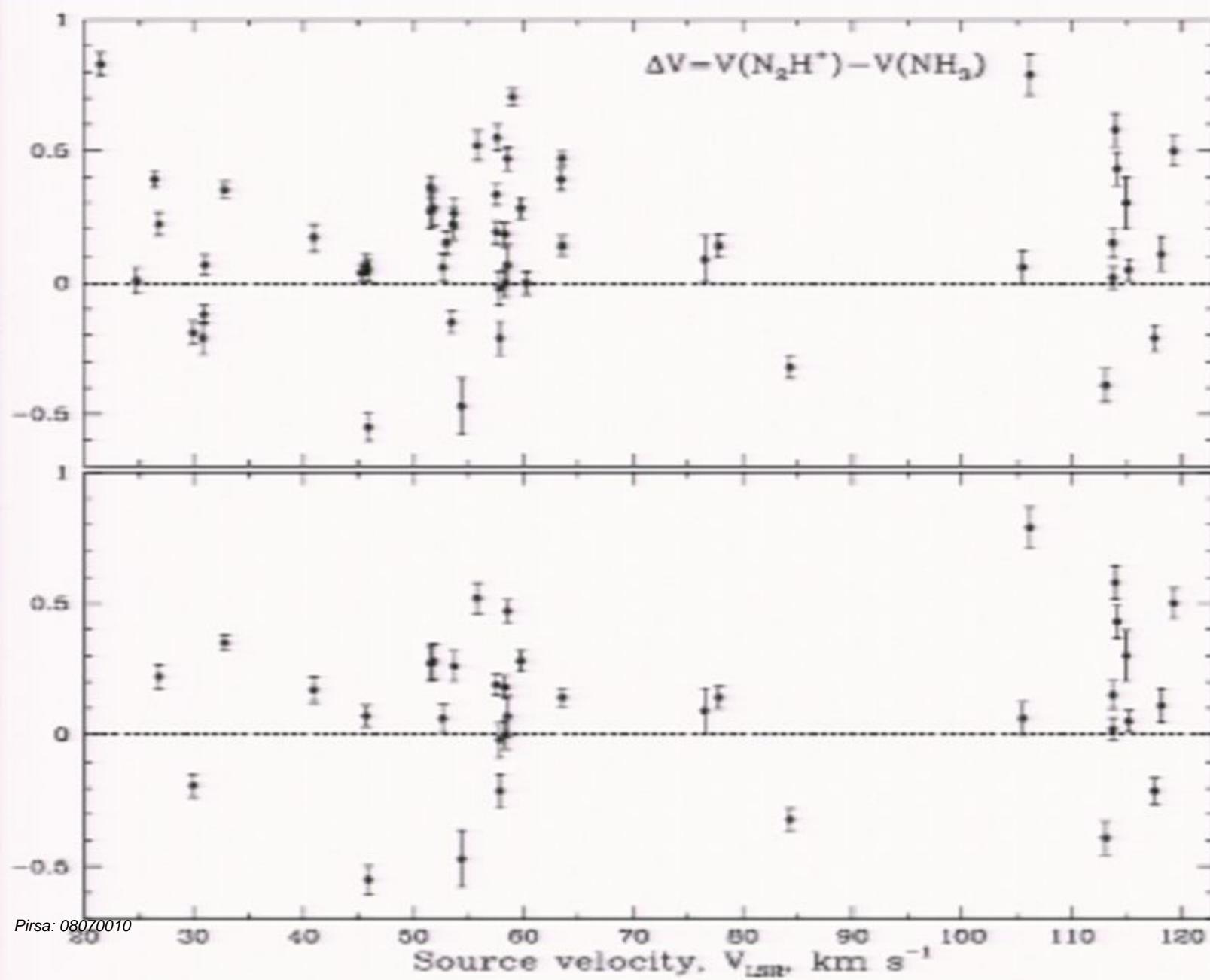
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- These observations correspond to the time intervals from 400 years for Pipe Nebulae to  $\sim$ 10000 years for IRDCs. Such time-variation contradicts both laboratory and extragalactic observations.
- For the same reasons it can not be linked with gravitational potential.
- It is possible to link such variation to the local matter density as suggested in some chameleon-type scalar field models.

Our final results for velocity offsets of NH<sub>3</sub> inversion line versus rotational lines of other molecules for Perseus cloud, Pipe Nebulae (C<sub>2</sub>S), and IRDCs (N<sub>2</sub>H<sup>+</sup> & HC<sub>3</sub>N) are:

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