Title: Why Quantum?

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Abstract:

Why Quantum?

Giacomo Mauro D'Ariano

University of Pavia

Osamu Hirota, a True Quantum Communication Channel
June 25-27, Perimeter Institute, Waterloo, CA

Quantum Mechanics as Quantum Information (and only a little more)

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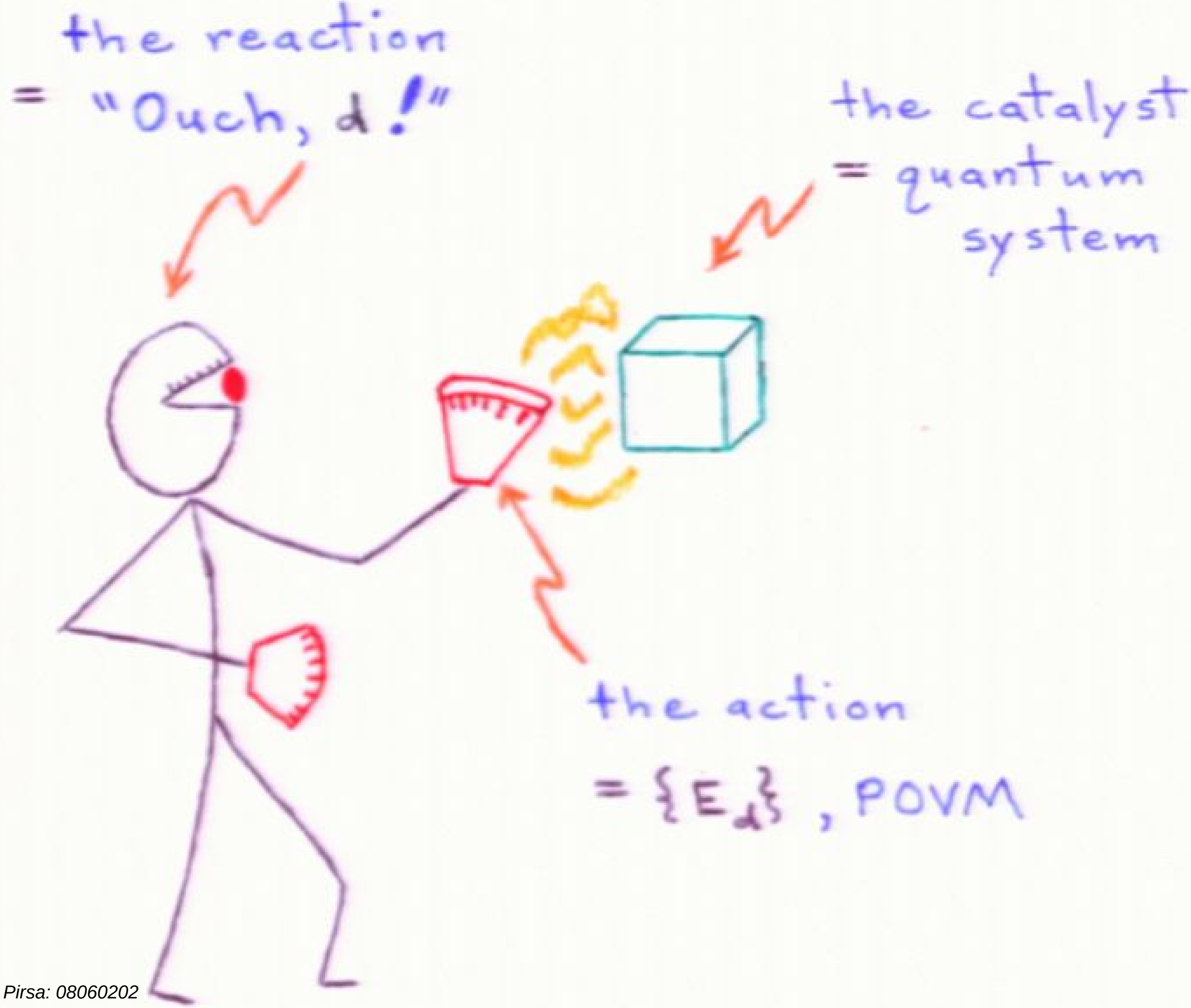
Abstract

In this paper, I try once again to cause some good-natured trouble. The issue remains, when will we ever stop burdening the taxpayer with conferences devoted to the quantum foundations? The suspicion is expressed that no end will be in sight until a means is found to reduce quantum theory to two or three statements of crisp physical (rather than abstract, axiomatic) significance. In this regard, no tool appears better calibrated for a direct assault than quantum information theory. Far from a strained application of the latest fad to a time-honored problem, this method holds promise precisely because a large part—*but not all*—of the structure of quantum theory has always concerned information. It is just that the physics community needs reminding.

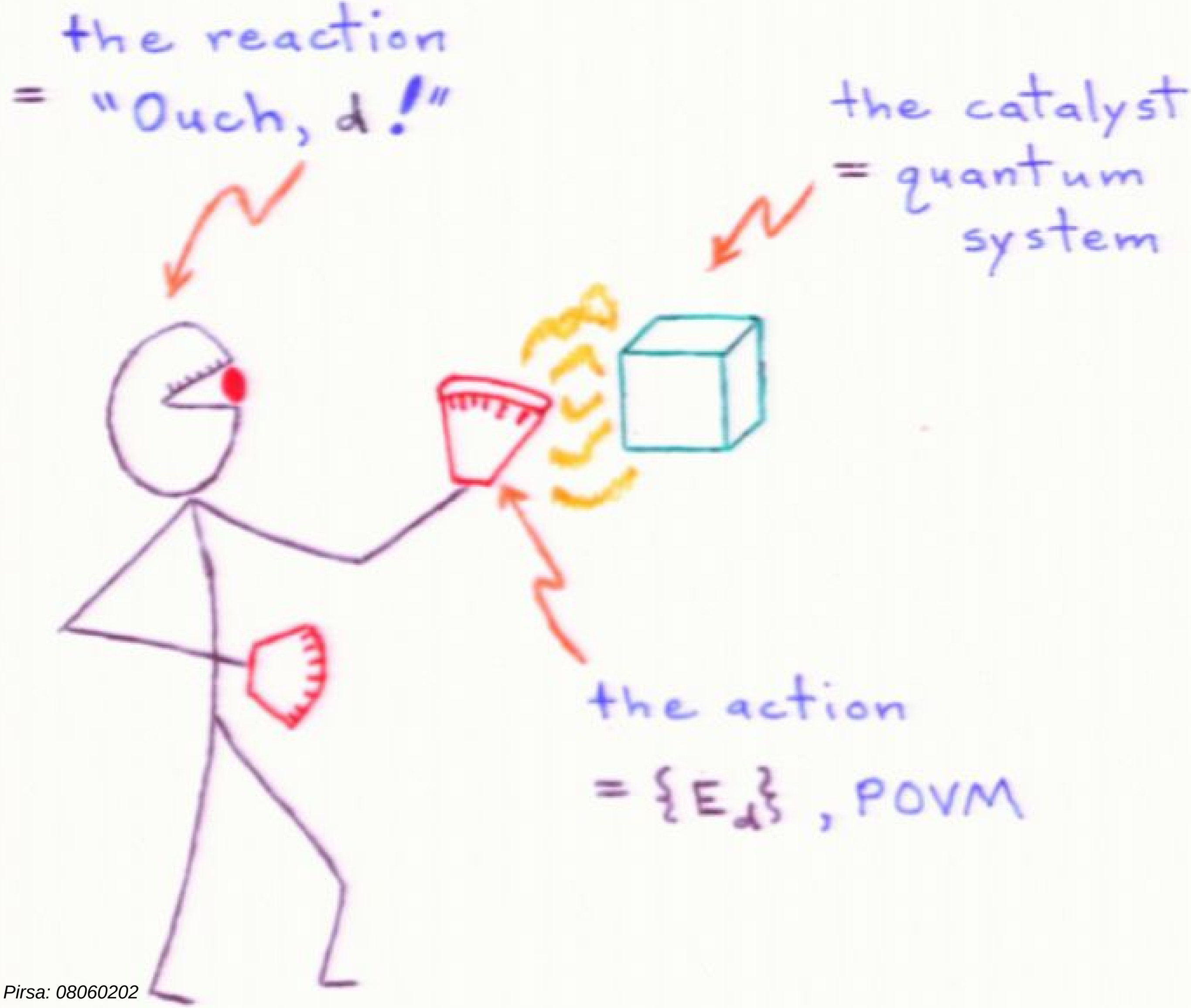
This paper, though taking `quant-ph/0106166` as its core, corrects one mistake and offers several observations beyond the previous version. In particular, I identify one element of quantum mechanics that I would *not* label a subjective term in the theory—it is the integer parameter D traditionally ascribed to a quantum system via its Hilbert-space dimension.

Operationalism



Chris Fuchs ©

Operationalism



Operationalism

Quantum Theory From Five Reasonable Axioms

Lucien Hardy*
Centre for Quantum Computation,
The Clarendon Laboratory,
Parks road, Oxford OX1 3PU, UK

June 17, 2002

Abstract

The usual formulation of quantum theory is based on rather obscure axioms (employing complex Hilbert spaces, Hermitean operators, and the trace formula for calculating probabilities). In this paper it is shown that quantum theory can be derived from five very reasonable axioms. The first four of these axioms are obviously consistent with both quantum theory and classical probability theory. Axiom 5 (which requires that there exist continuous reversible transformations between pure states) rules out classical probability theory. If Axiom 5 (or even just the word "continuous" from Axiom 5) is dropped then we obtain classical probability theory instead. This work provides some insight into the reasons why quantum theory is the way it is. For example, it explains the need for complex numbers and where the trace formula comes from. We also gain insight into the relationship between quantum theory and classical probability theory.

theory is simply a new type of probability theory. Like classical probability theory it can be applied to a wide range of phenomena. However, the rules of classical probability theory can be determined by pure thought alone without any particular appeal to experiment (though, of course, to develop classical probability theory, we do employ some basic intuitions about the nature of the world). Is the same true of quantum theory? Put another way, could a 19th century theorist have developed quantum theory without access to the empirical data that later became available to his 20th century descendants? In this paper it will be shown that quantum theory follows from five very reasonable axioms which might well have been posited without any particular access to empirical data. We will not recover any specific form of the Hamiltonian from the axioms since that belongs to particular applications of quantum theory (for example - a set of interacting spins or the motion of a particle in one dimension). Rather we will recover the basic structure of quantum theory along with the most general type of quantum evo-

Operationalism

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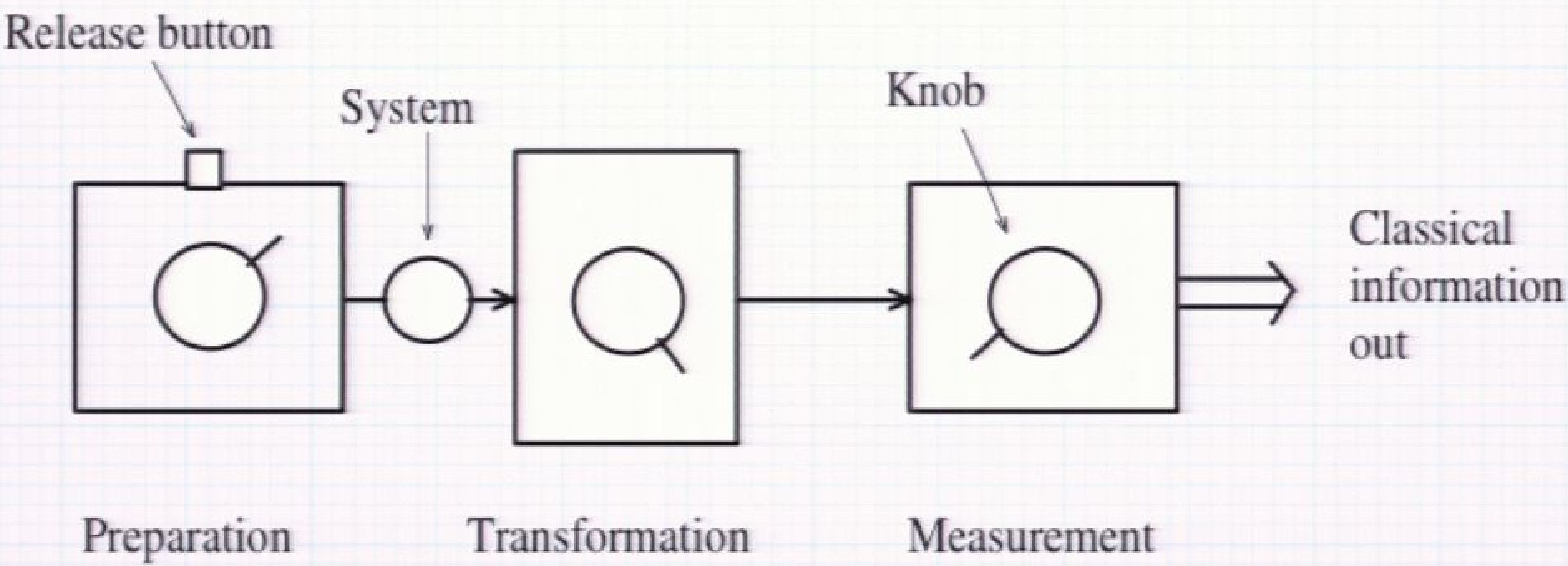

Figure 1: The situation considered consists of a preparation device with a knob for varying the state of the system produced and a release button for releasing the system, a transformation device for transforming the state (and a knob to vary this transformation), and a measuring apparatus for measuring the state (with a knob to vary what is measured) which outputs a classical number.

Operationalism

- Test/experiment: $\mathbb{A} \equiv \{\mathscr{A}_j\}$ set of possible events $\mathscr{A}_j$
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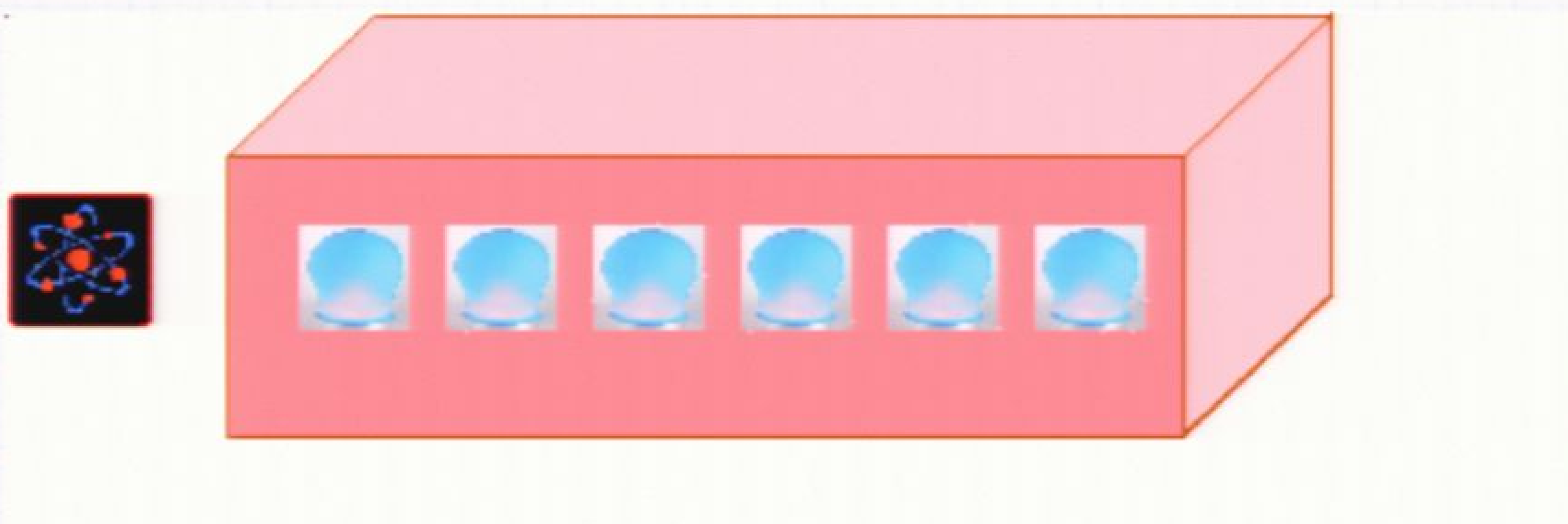
- States: compendia of probabilities for all tests



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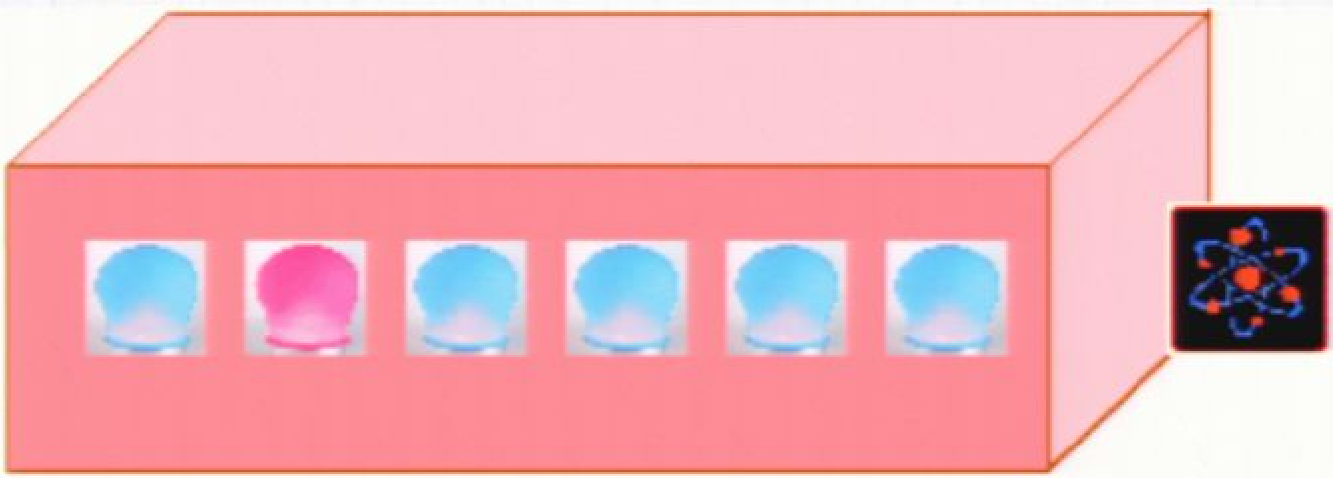

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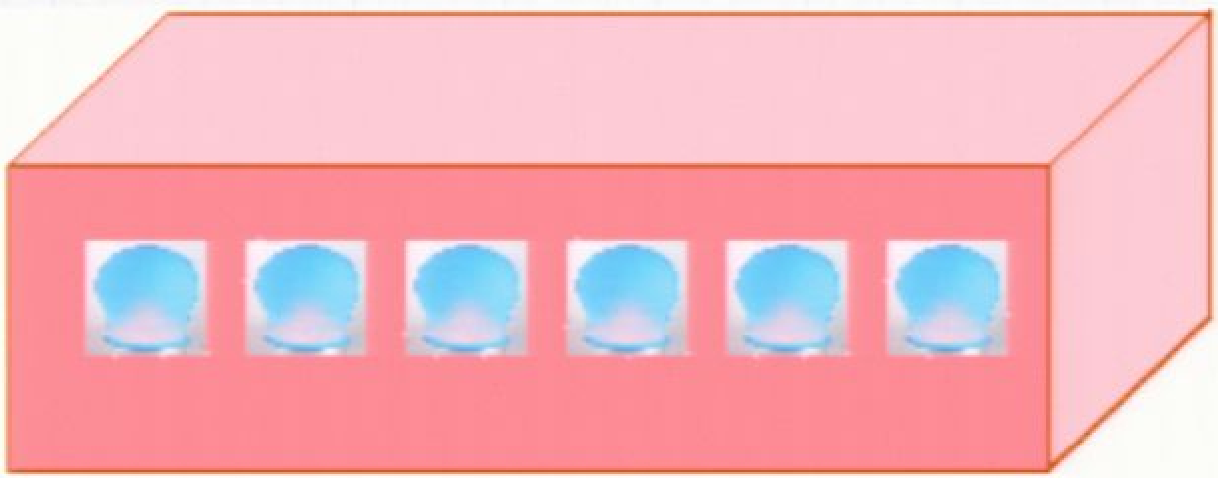

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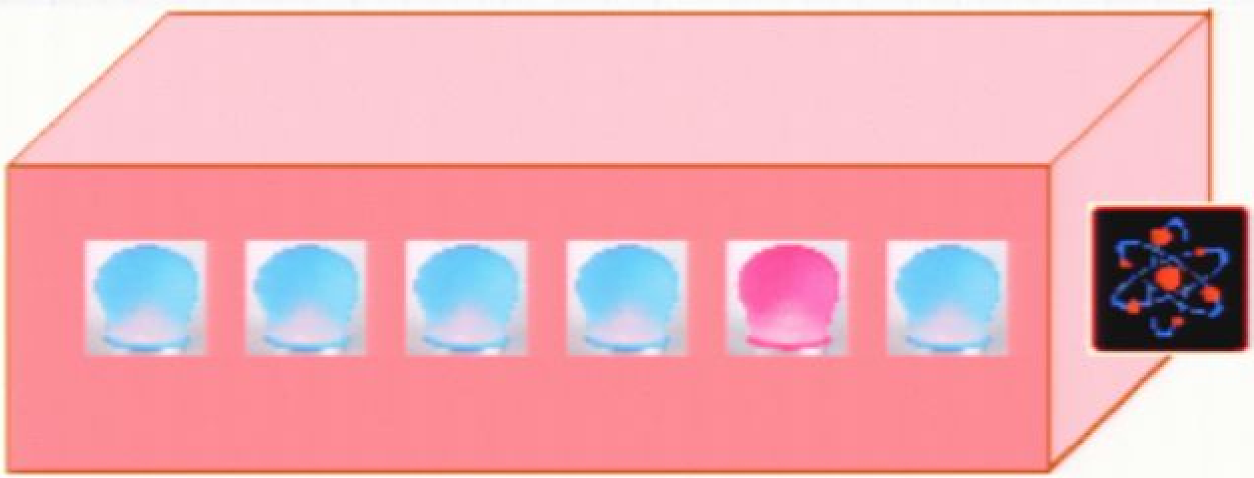

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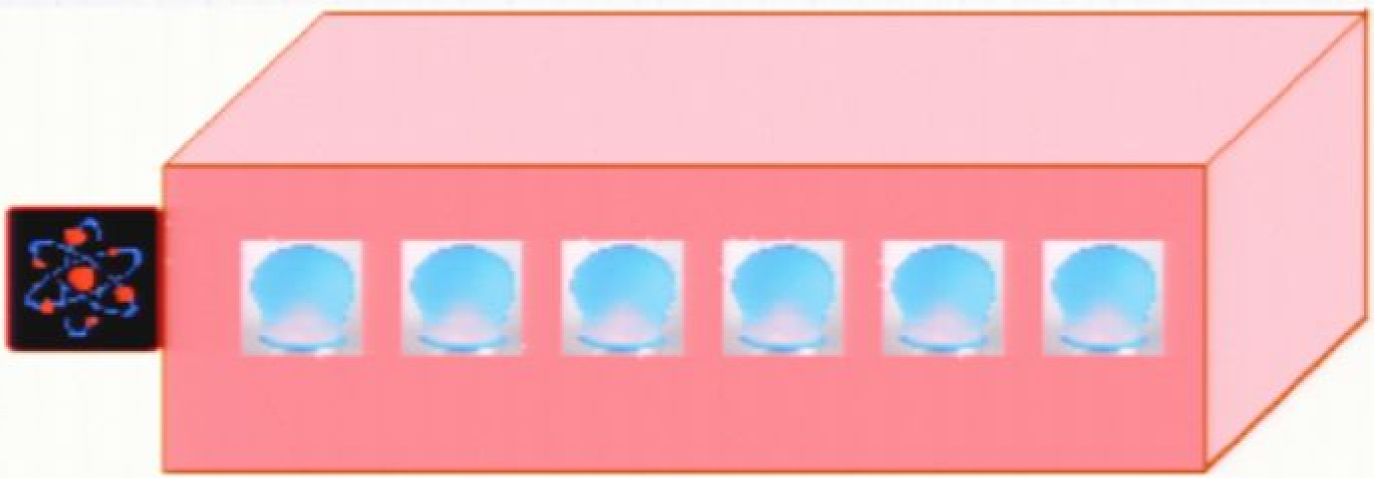

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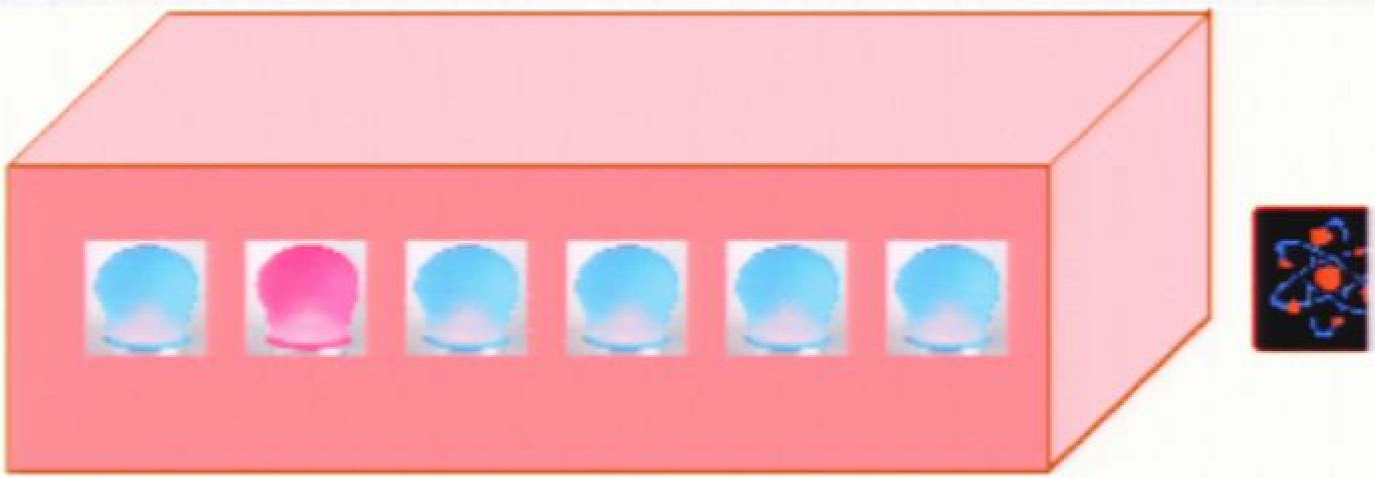

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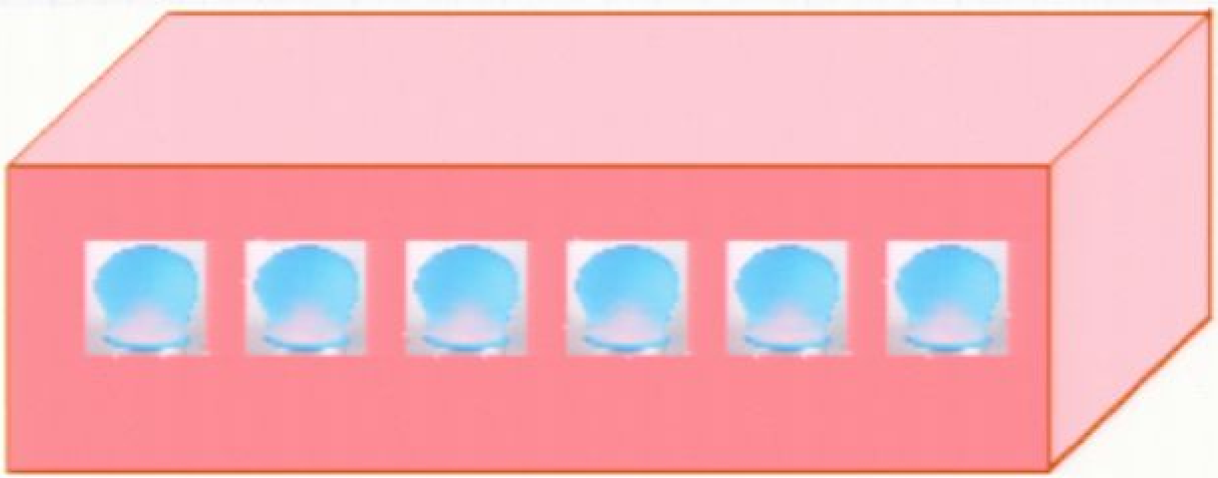

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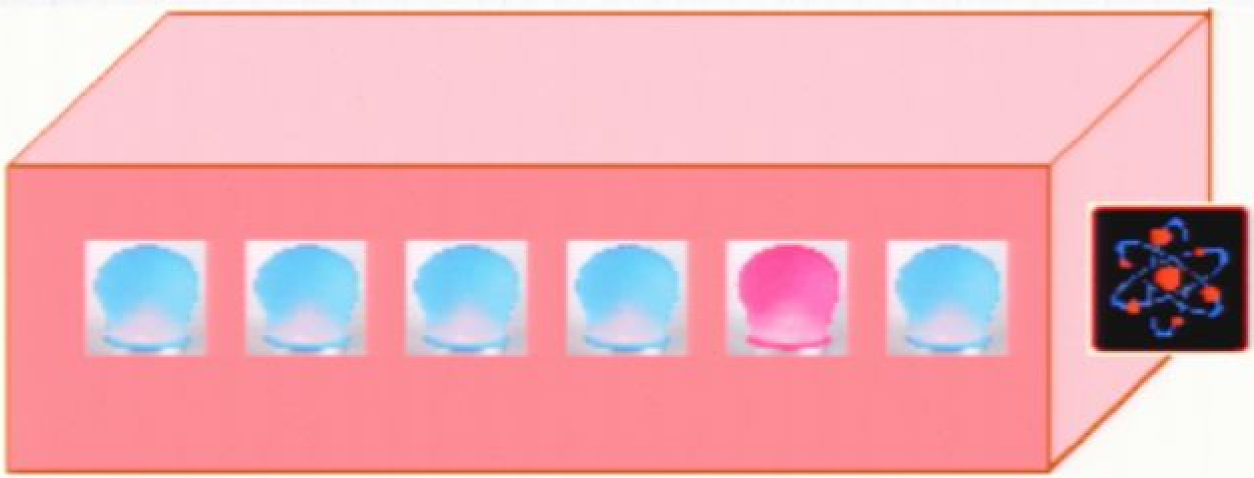

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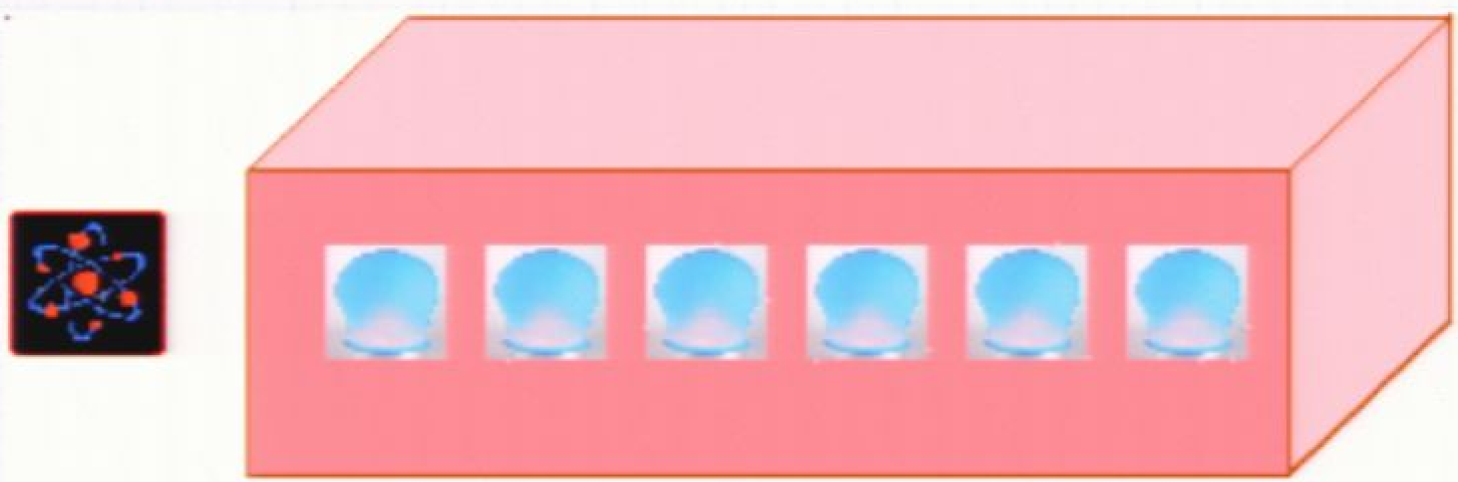

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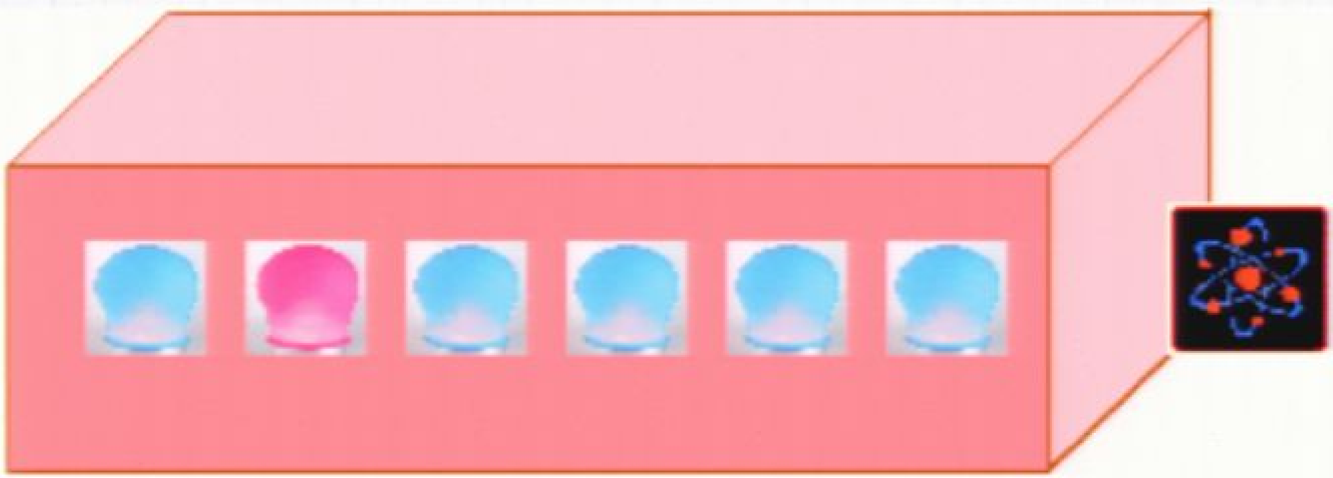

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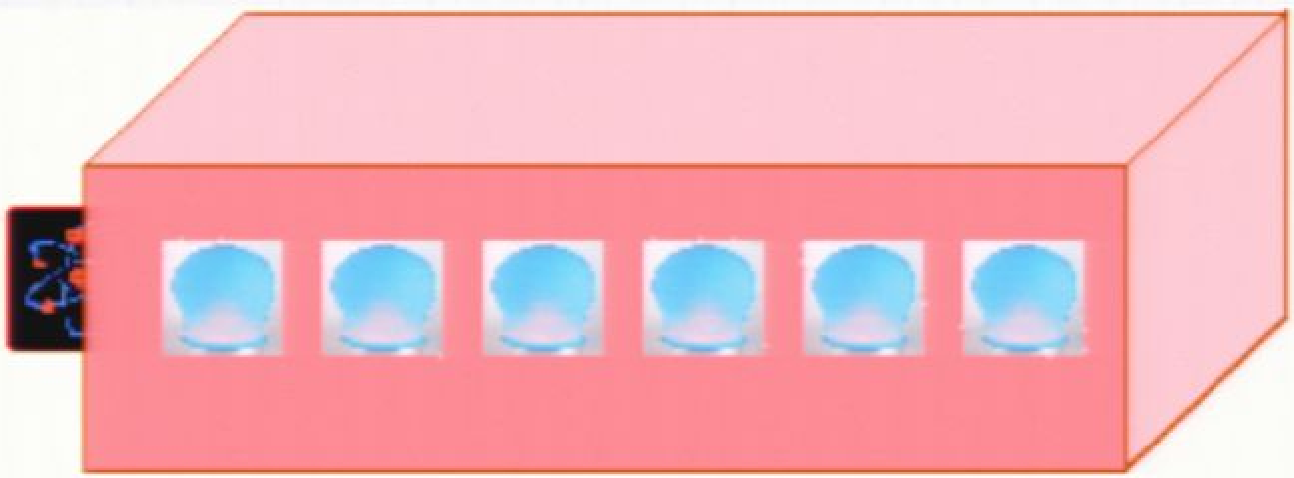

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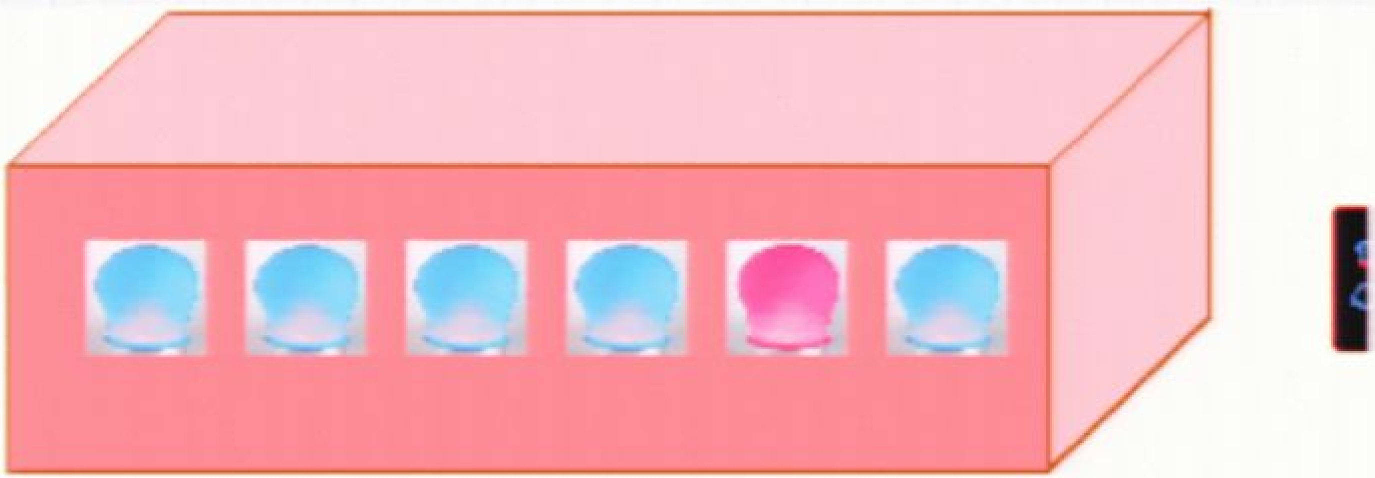

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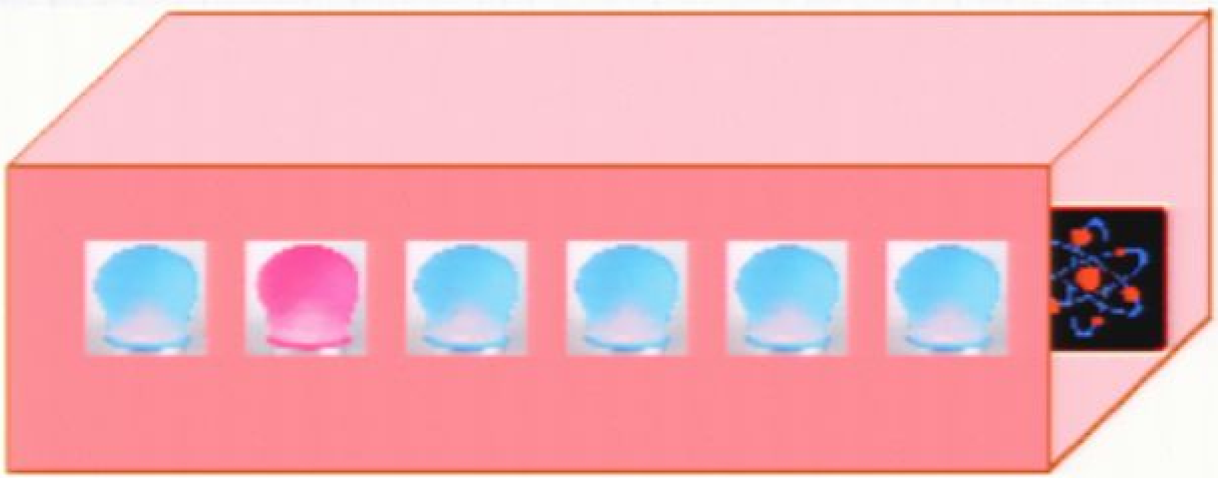

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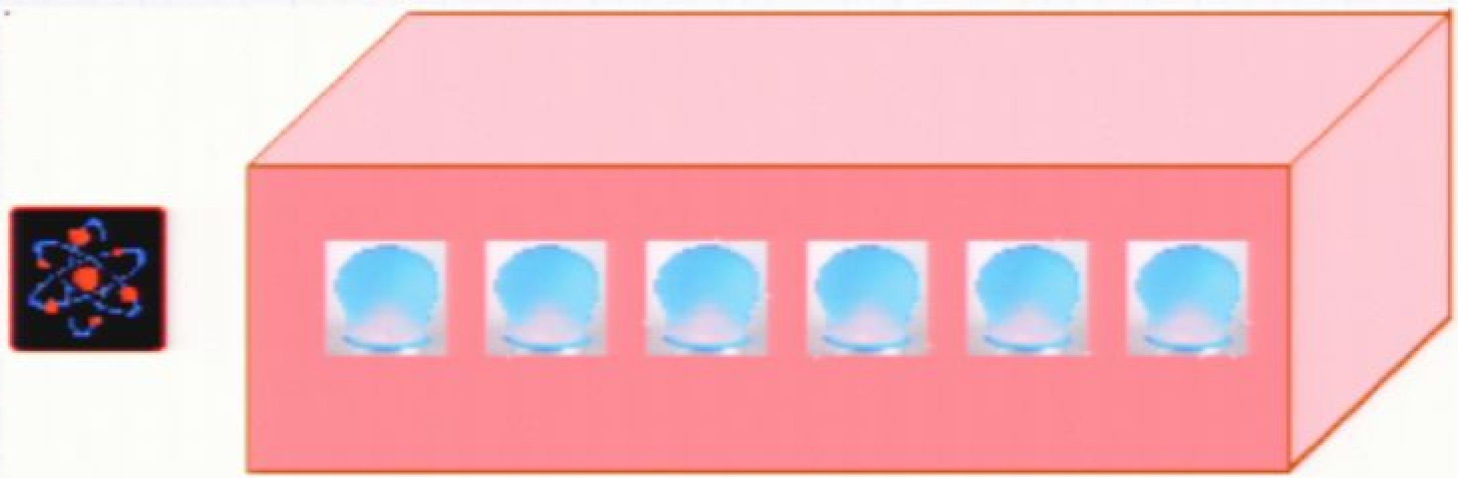

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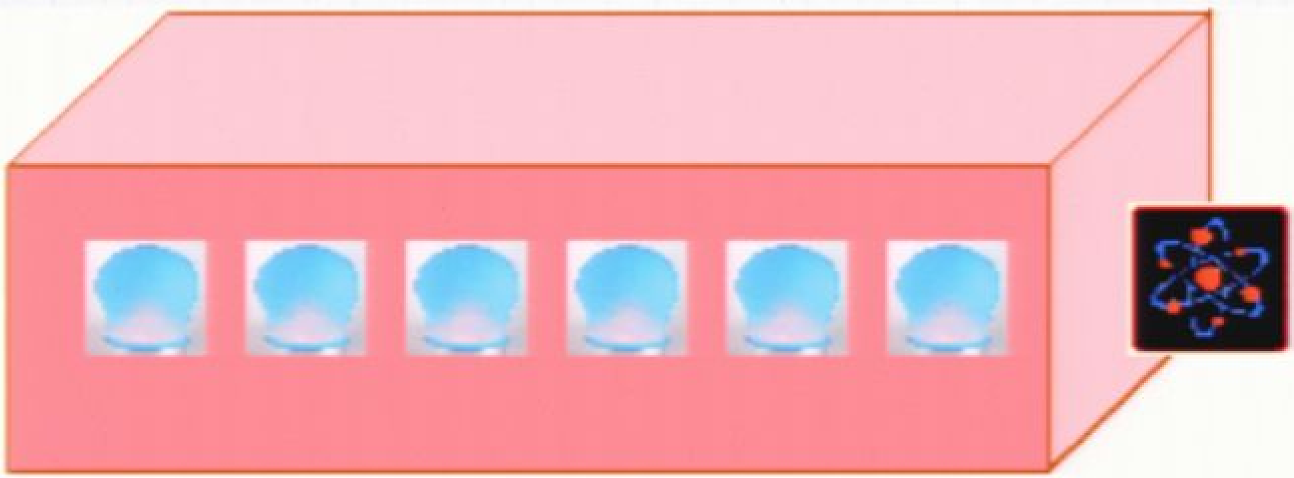

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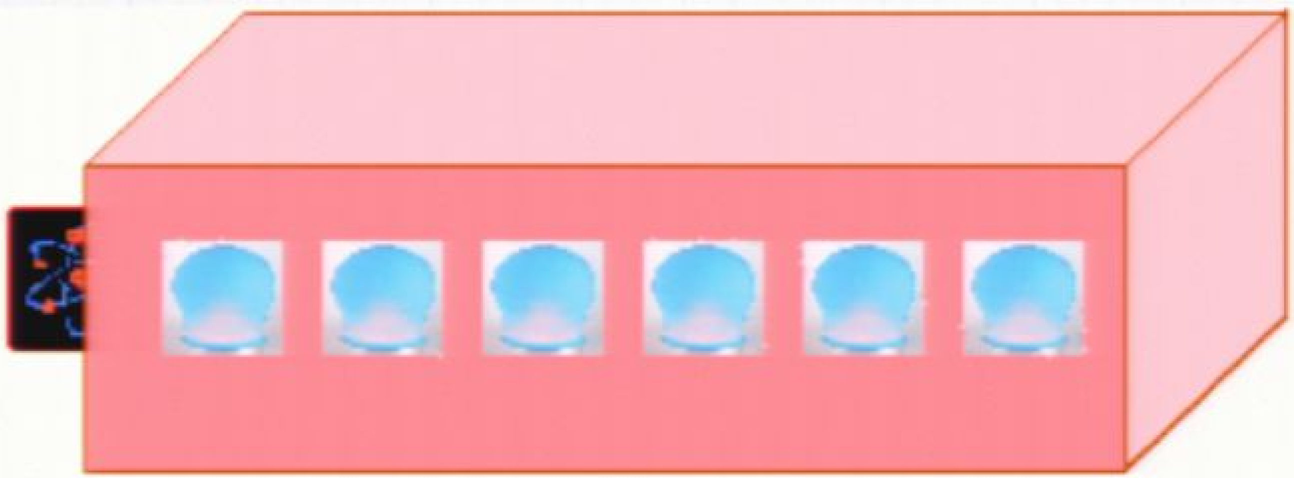

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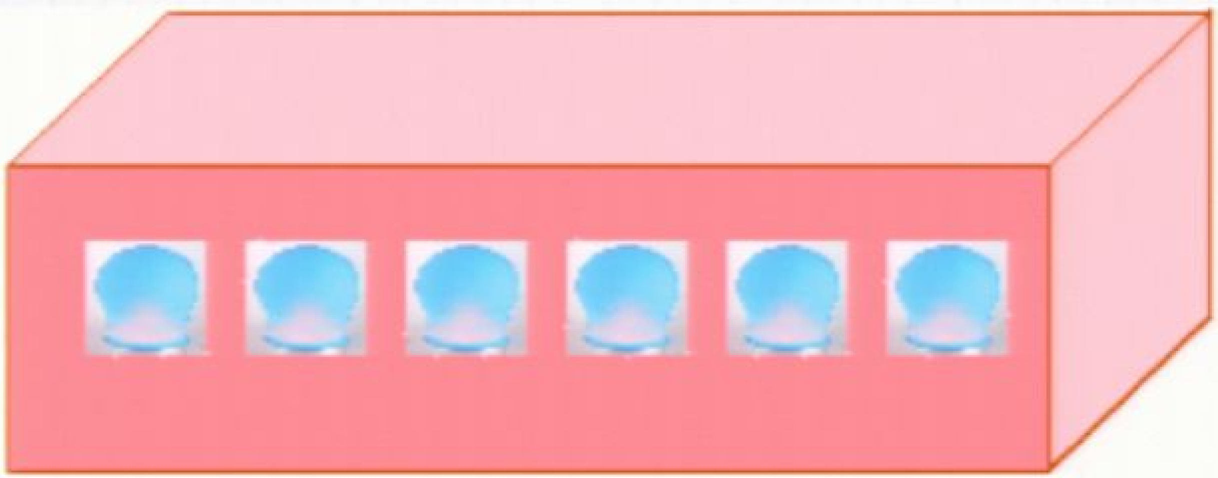

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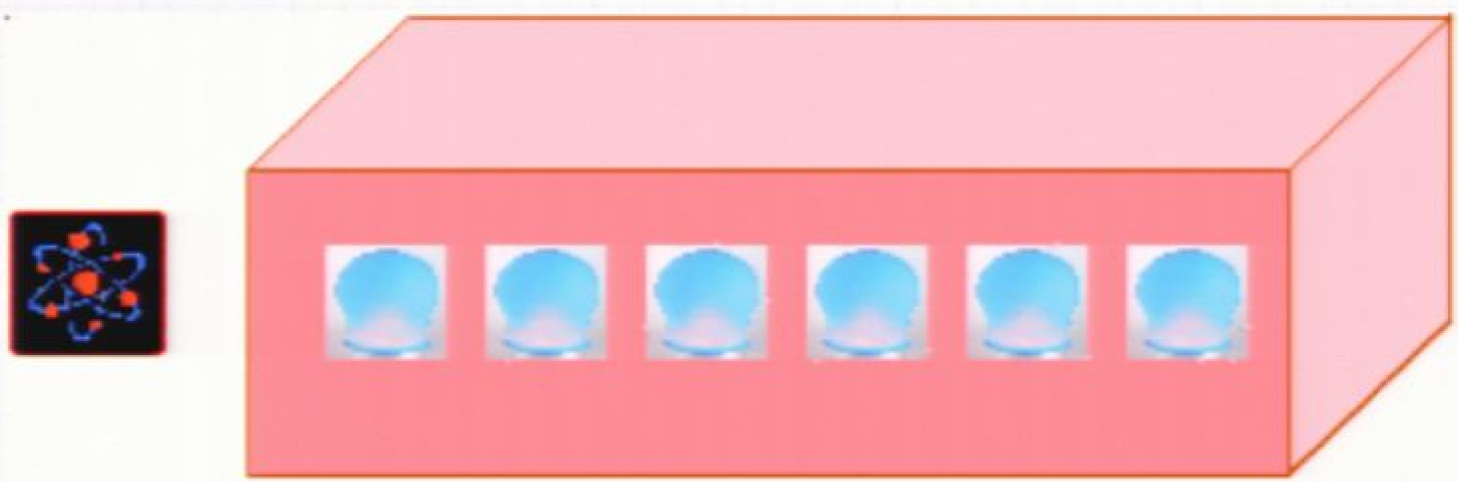

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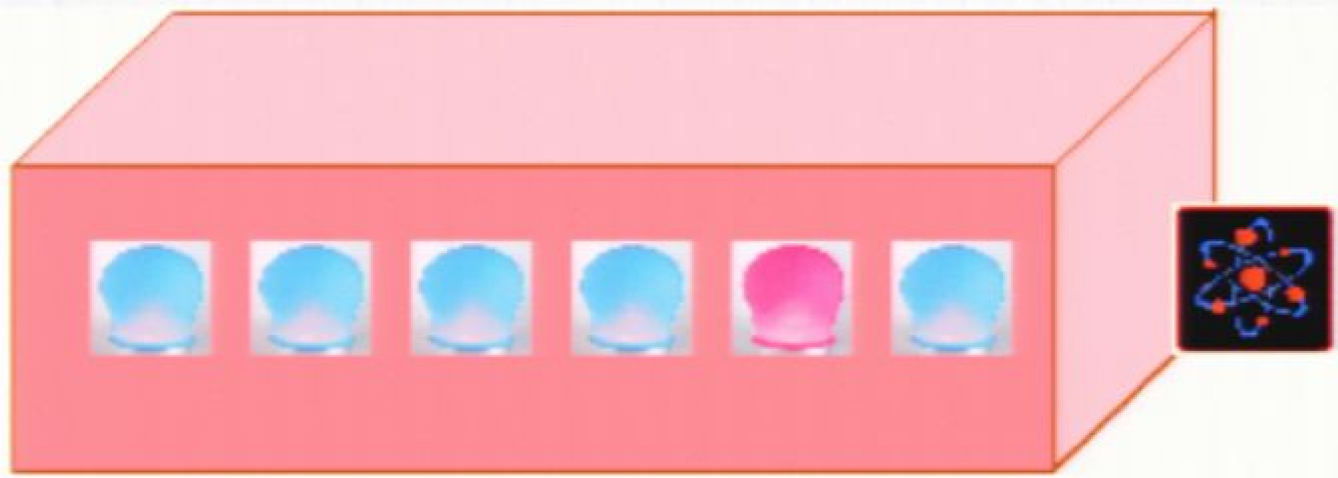

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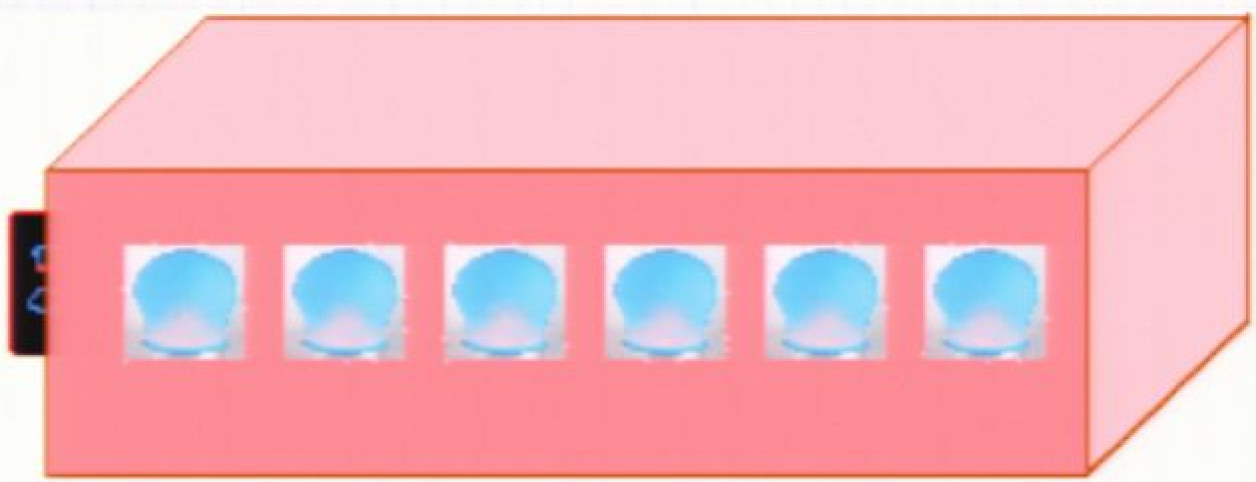

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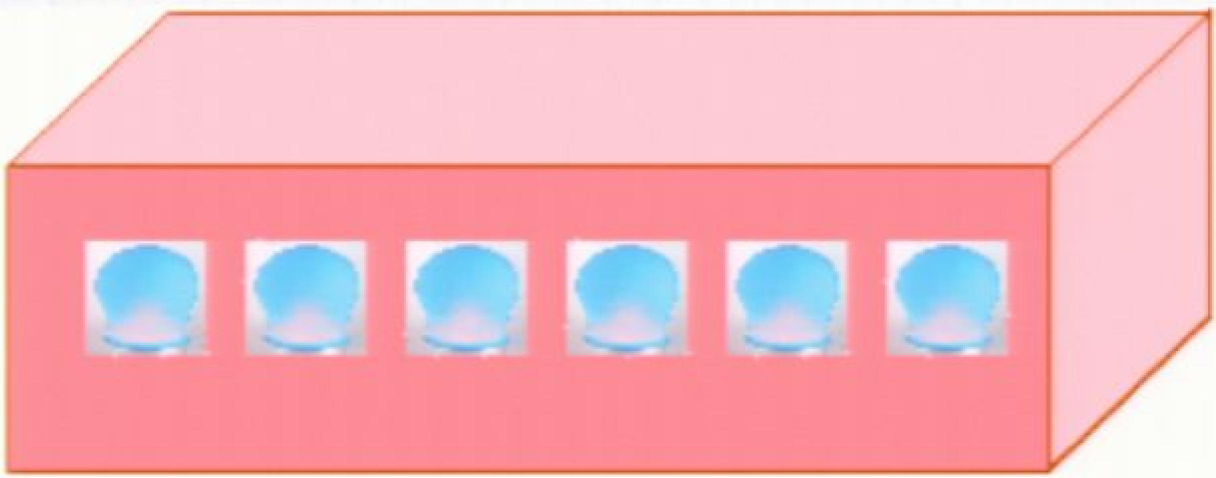

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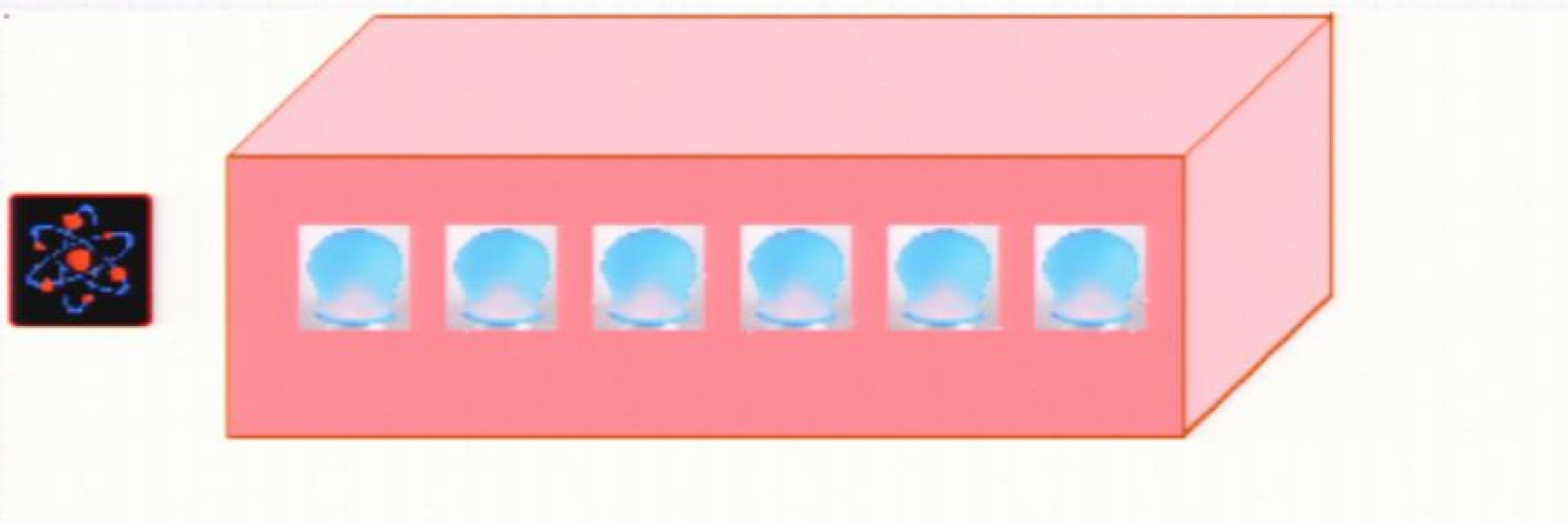

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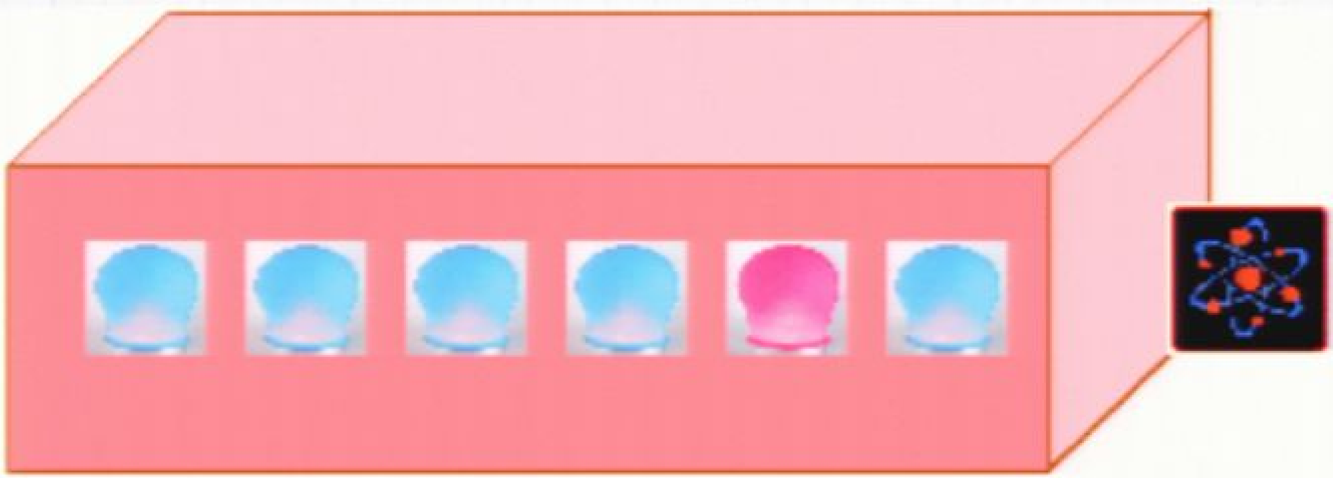

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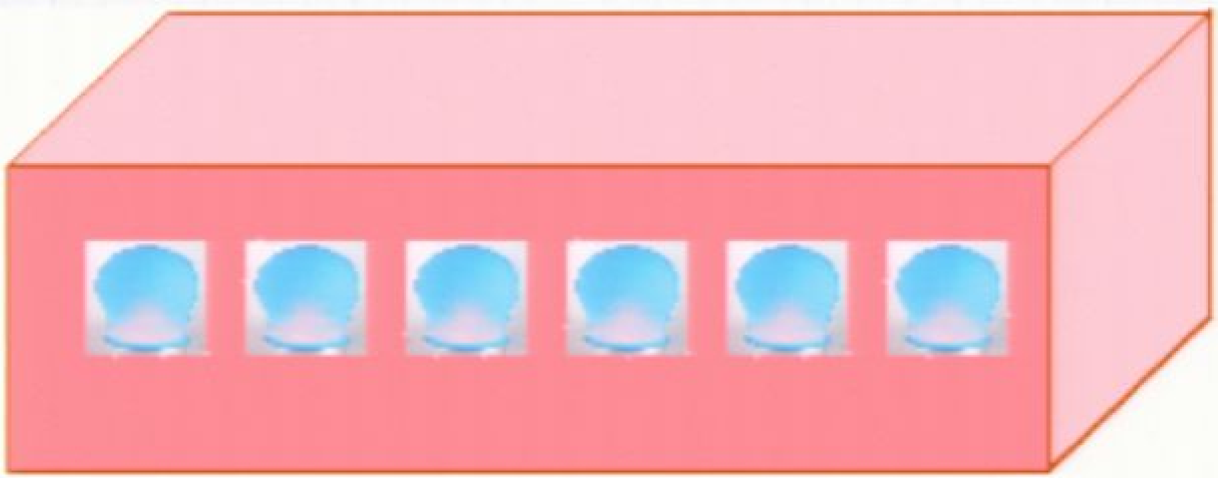

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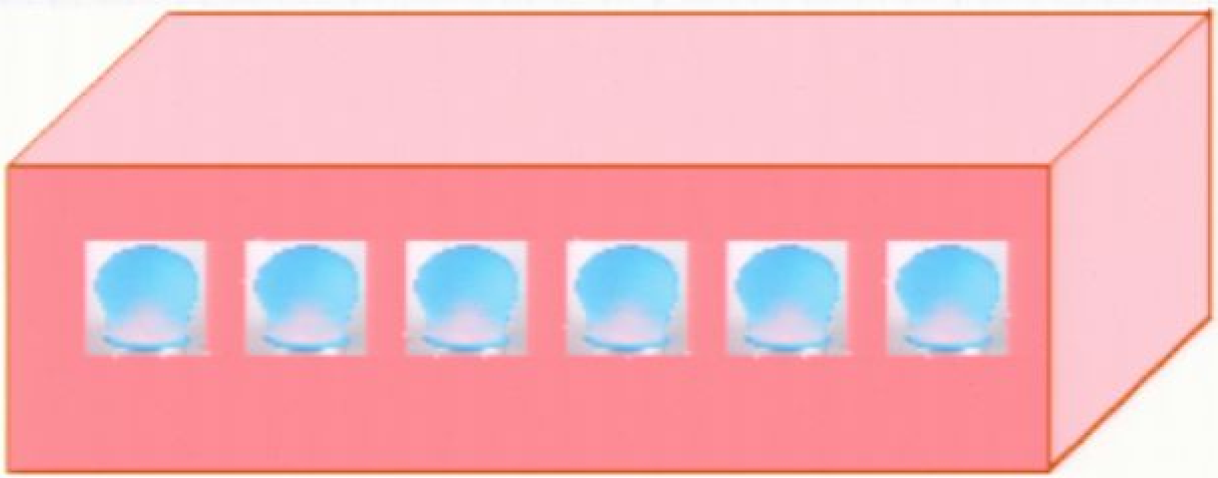

Earlier operationalism

Quantum Logic: QM as a propositional calculus

- Birkhoff-von Neumann ('36), von Neuman, Jordan, Wigner ('34)
- Mackey ('63) (orthomodular lattice)
- Jauch-Piron ('76),...

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Focus on single systems, little on multipartite systems

Earlier operationalism

Algebraic formulations

- von Neumann, Jordan, Wigner ('34)
  - Jordan product: $a \circ b = (a+b)^2 - a^2 - b^2$
    - commutative, defined in terms of sums and squares
    - Problems:
      - non associative
      - non distributive
      - no consistent operational definition of sum

- Alfsen-Shultz derived $\circ$ from geometry of convex of states

Earlier operationalism

Algebraic formulations

- Segal ('47)
  - Operational framework allowing non-distributive algebras of observables
    - Problems:
      - framework largely more general than QM
      - still no consistent definition of sum of observables

Earlier operationalism

Algebraic formulations

Earlier operationalism

Algebraic formulations

- C*-algebra of observables
  - Quantum-classical hybrid (= QM+super+selection rules)
  - Representation independent formulation
  - Non operational: still no meaning for sums and products of observables
  - Or start from Jordan product (see Alfsen and Shultz): however, assumptions for embedding $a \circ b = (a+b)^2 - a^2 - b^2$ in an associative algebra as $a \circ b = ab + ba$ are still non operational.

Recent operationalism
Info-theoretical motivations

- Fuchs ('02)(info-theoretical motivations for QM)
- Clifton Bub Halvorson (01') [assume C*-algebra]
- Hardy ('01) (simplicity of axioms)
- Barnum, Wilce, GMD, et al. (convex operational approach)

Operationalism

Main problem:
to derive the C*-algebraic formulation
for the classical-quantum hybrid
versus other probabilistic theories

Operationalism

Working hypotesis:

Quantum Mechanics as a mathematical representation of

Operationalism

Operationalism

Main problem:
to derive the C*-algebraic formulation
for the classical-quantum hybrid
versus other probabilistic theories
(PR boxes)

Operationalism

Working hypotesis:

Quantum Mechanics as a mathematical representation of "fair" operational framework

Fair operational framework

Set of rules which allows one to make correct predictions on future events on the basis of suitable tests

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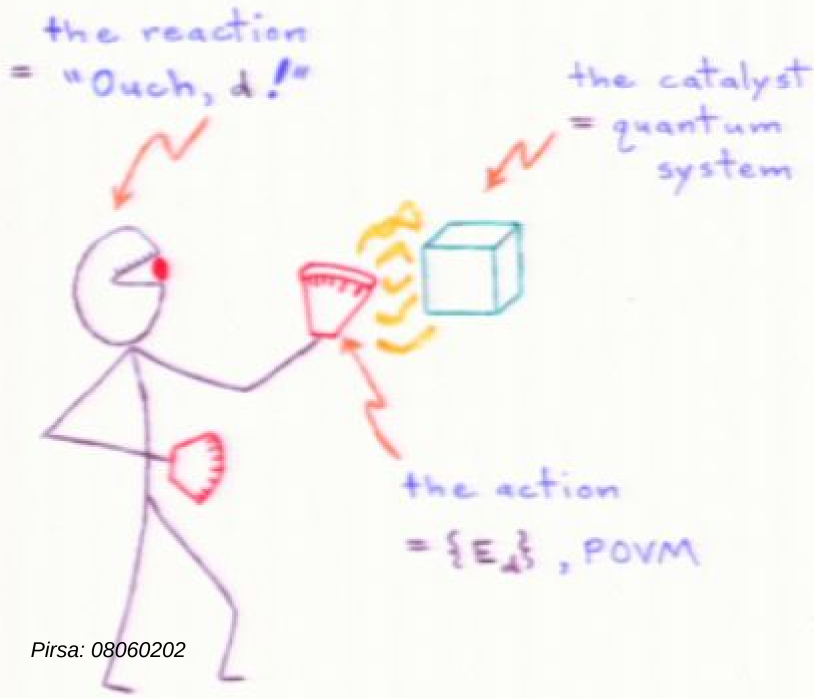

A fair game between Observer and Mother Nature

Examples of fair rules

Examples of fair rules

- No-signaling from regions of space-time not under control, e.g. at space-like separation or from the future ...

Examples of fair rules

Fair operational framework

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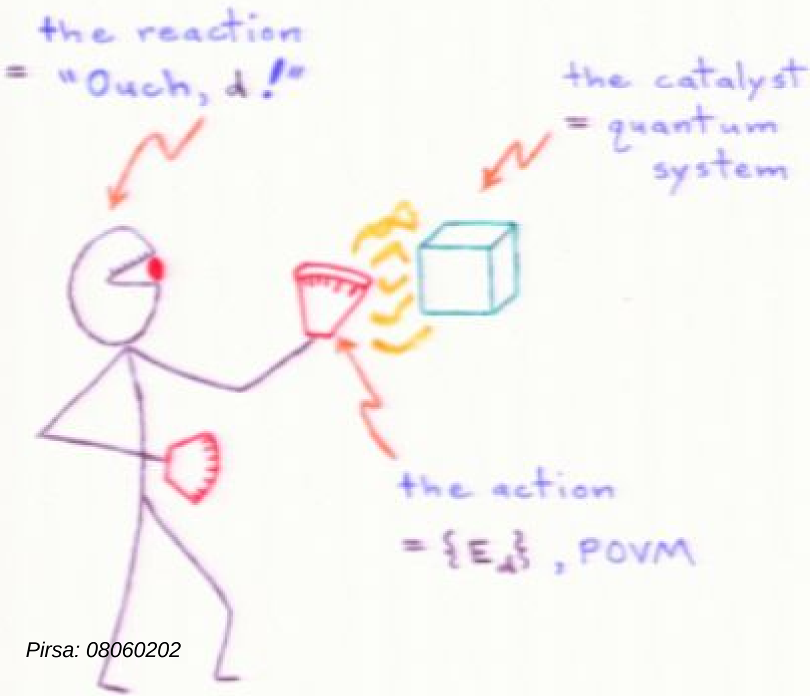

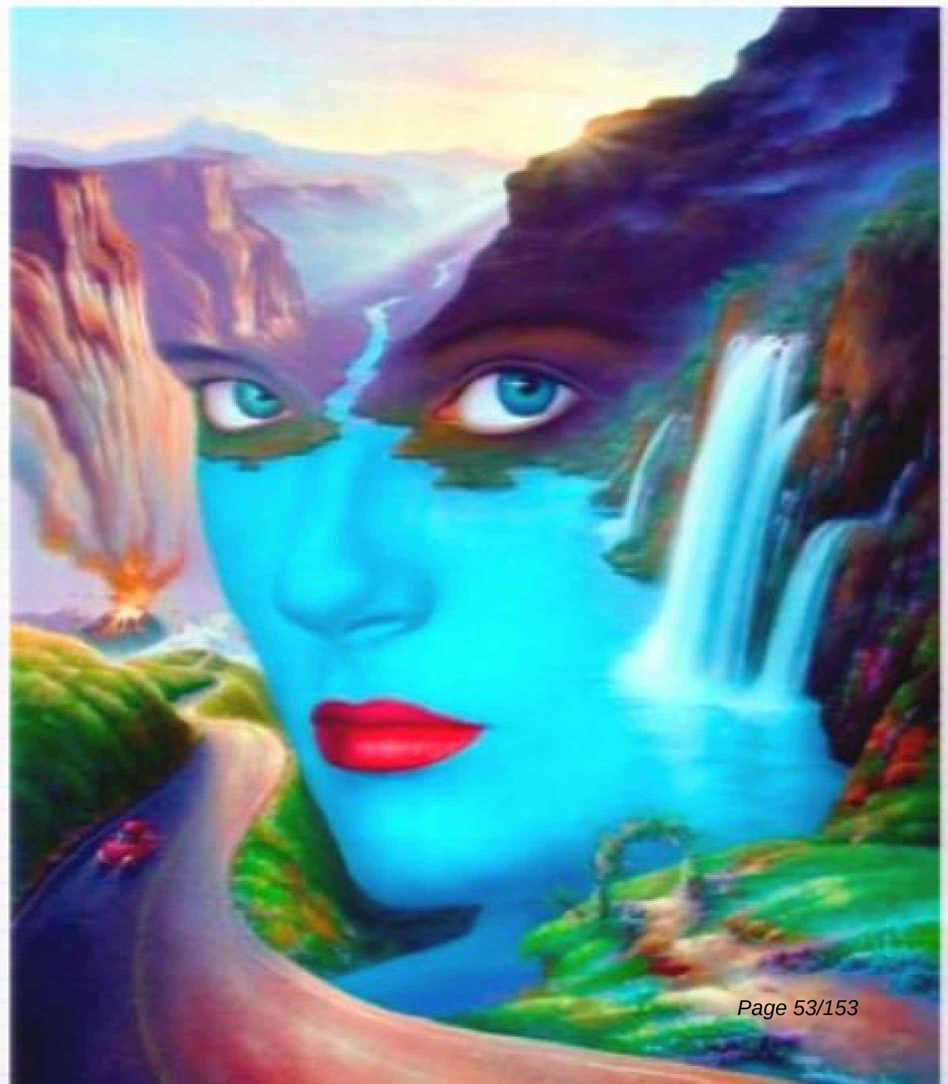

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- Possibility of calibrating tests
- Global observability by local tests
- PLEASE, GIVE ME MORE

Operational framework

Axioms:

- probability, events, ...
- test: $\mathbb{A} \equiv \{\mathscr{A}_j\}$ set of possible events $\mathscr{A}_j$
- independent systems

Operational framework

Requirements for fairness:

Operational framework

Requirements for fairness:

- **NSF:** No signaling from the future.
- **NS:** No signaling (=existence of independent systems)
- **FAITH:** There exists faithful states (calibrability)
- **CJ: a "little more"**

Operational framework

General principles for a mathematical representation of the operational framework

- all mathematical objects must be defined operationally
- completion for mathematical convenience (e.g. algebraic closure, norm closure, linear span, etc.)

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General principles for a mathematical representation of the operational framework

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In this talk we will consider only:

a) finite dimension

b) identical systems

States

State ω**:** probability rule $\omega(\mathscr{A})$ for any possible event $\mathscr{A}$ in any test $\mathbb{A}$

Normalization: $$\sum_{\mathscr{A}_j \in \mathbb{A}} \omega(\mathscr{A}_j) = 1$$

Deterministic evolution (e.g. free): $\omega(\mathscr{D}) = 1$

Convex set of states of a system: $\mathfrak{S}$, cone: $\mathfrak{S}_+$

Transformations

Cascade of tests: $\mathbb{B}$ **following** $\mathbb{A}$

Event $\mathscr{B} \circ \mathscr{A}$: event $\mathscr{B} \in \mathbb{B}$ followed by $\mathscr{A} \in \mathbb{A}$

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NSF
(Ozawa)

$$\sum_{\mathscr{B}_j \in \mathbb{B}} \omega(\mathscr{B}_j \circ \mathscr{A}) = \omega(\mathscr{A}), \quad \forall \mathbb{B},\ \forall \mathscr{A},\ \forall \omega$$

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$\Rightarrow$ conditional probability: $p(\mathscr{B}|\mathscr{A}) = \omega(\mathscr{B} \circ \mathscr{A})/\omega(\mathscr{A})$

Transformations

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$\Rightarrow$ events ≡ transformations   Convex monoid of transformations: $\mathfrak{T}$

Dynamical and informational equivalence

Dynamical equivalence of transformations: two transformations $\mathscr{A}$ and $\mathscr{B}$ are dynamically equivalent if

$$\omega_{\mathscr{A}} = \omega_{\mathscr{B}} \qquad \forall \omega \in \mathfrak{S}$$

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A transformation is completely specified by the two classes

i.e. $\mathscr{A}$ is completely specified by the probabilities :

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No information without disturbance
(detectability of the eavesdropper)

Zero-postulate theorems

Analogous of Wigner theorem for QM:

The only transformations that are inverted by another transformation must send pure states to pure states and are isometries of $\mathfrak{S}$

Effects

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Duality: effects positive linear functionals over states (bounded by 1)

Cones and Linear spaces

Convex set of states: $\mathfrak{S}$, cone: $\mathfrak{S}_+$

Convex set of effects: $\mathfrak{E}$, cone: $\mathfrak{E}_+$

Convex monoid of transformations: $\mathfrak{T}$, cone: $\mathfrak{T}_+$

Cones and Linear spaces

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Linear spaces:

$\mathfrak{S}_{\mathbb{R}} = \mathsf{Span}_{\mathbb{R}}\mathfrak{S}$

$\mathfrak{S}_{\mathbb{C}} = \mathsf{Span}_{\mathbb{C}}\mathfrak{S}$

$\mathfrak{E}_{\mathbb{R}}, \mathfrak{E}_{\mathbb{C}}, \mathfrak{T}_{\mathbb{R}}, \mathfrak{T}_{\mathbb{C}}$

Observables

Observable $\mathbb{L} = \{l_i\}$ **:** complete set of effects of a test

Normalization: $$\sum_{i \in \mathbb{L}} l_i = e$$

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Informationally complete observable $\mathbb{L}$ **:**

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$\circ, +$ distributive

Banach spaces

Natural norms:

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Norm closure (mathematical convenience)

$\Longrightarrow \mathfrak{S}_{\mathbb{R}}, \mathfrak{E}_{\mathbb{R}}$ **dual Banach pair** under the pairing $(a, \omega) := \omega(a$

$\Longrightarrow \mathfrak{T}_{\mathbb{R}}$ **Banach algebra**

C*-algebra of transformations (finite dim.)

Transformations/events are linear maps over effects, i.e. they make a matrix algebra over effects

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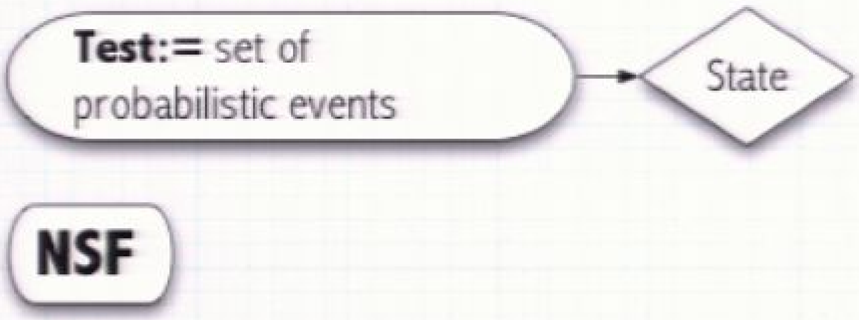
One can introduce a scalar product over effects ...
$\Rightarrow$ transformations become a C*-algebra ...

Summary of probabilistic theories

Test:= set of probabilistic events

Summary of probabilistic theories



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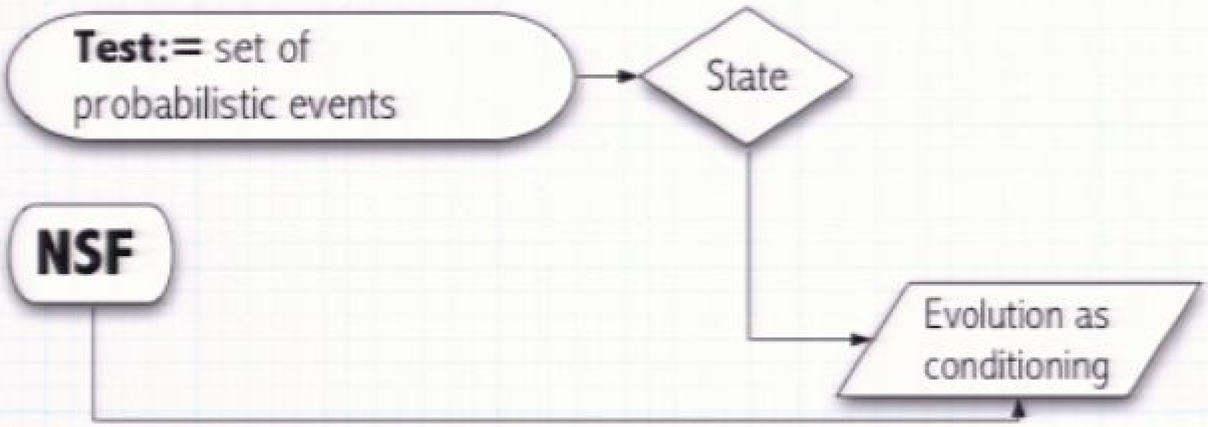

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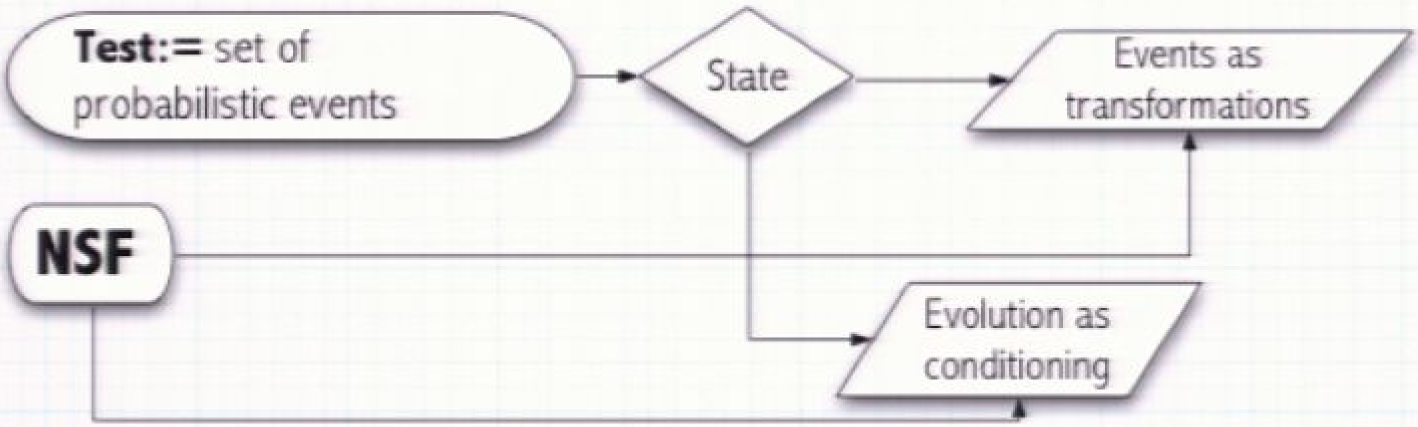

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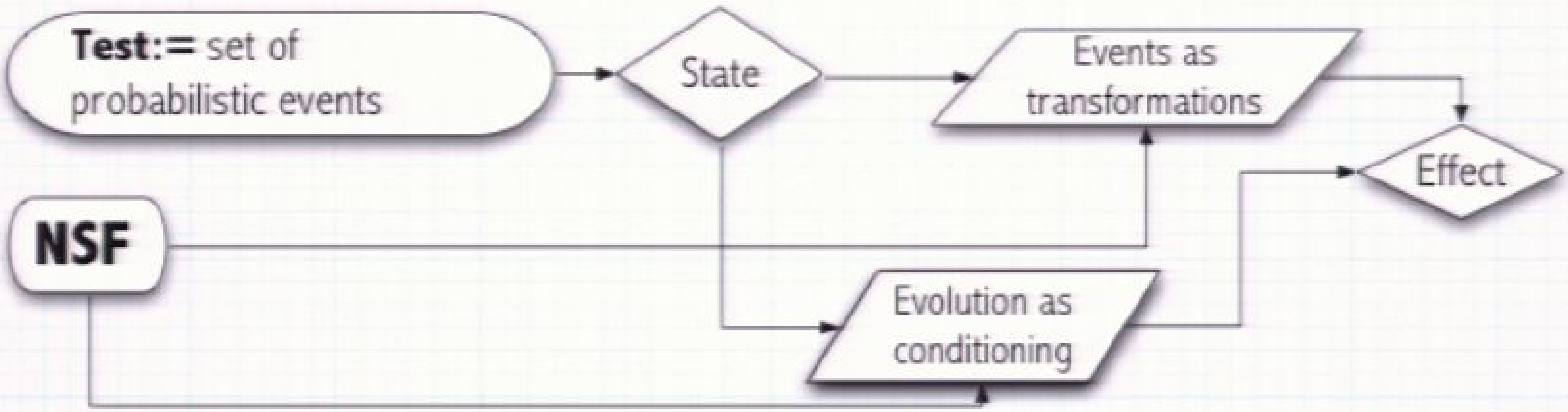

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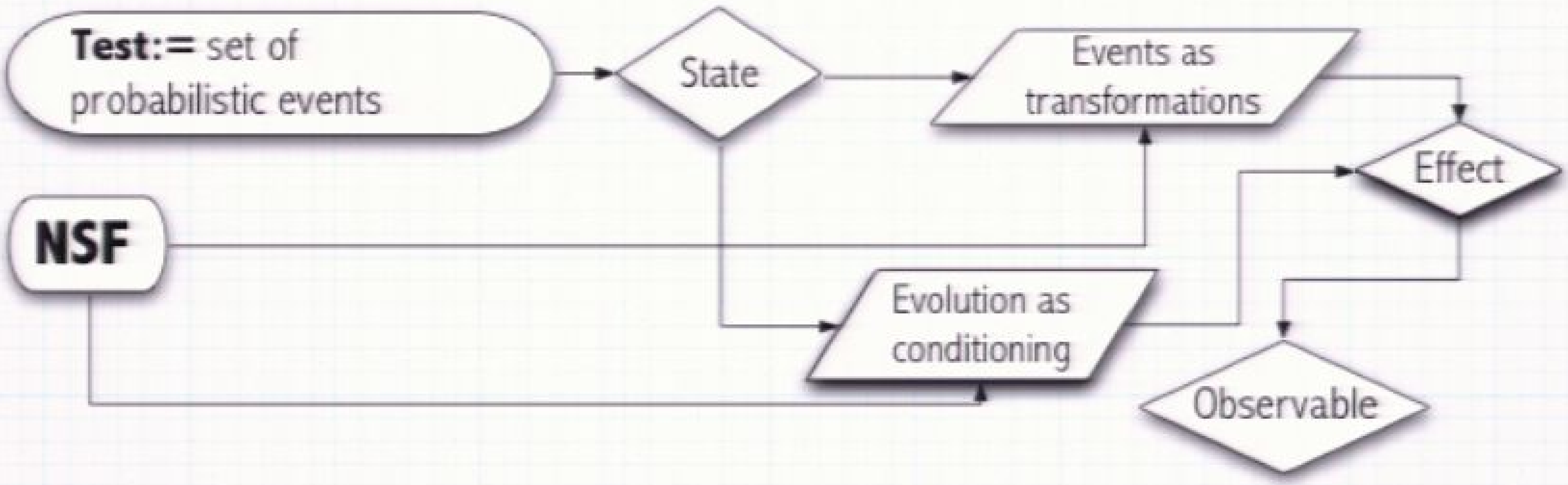

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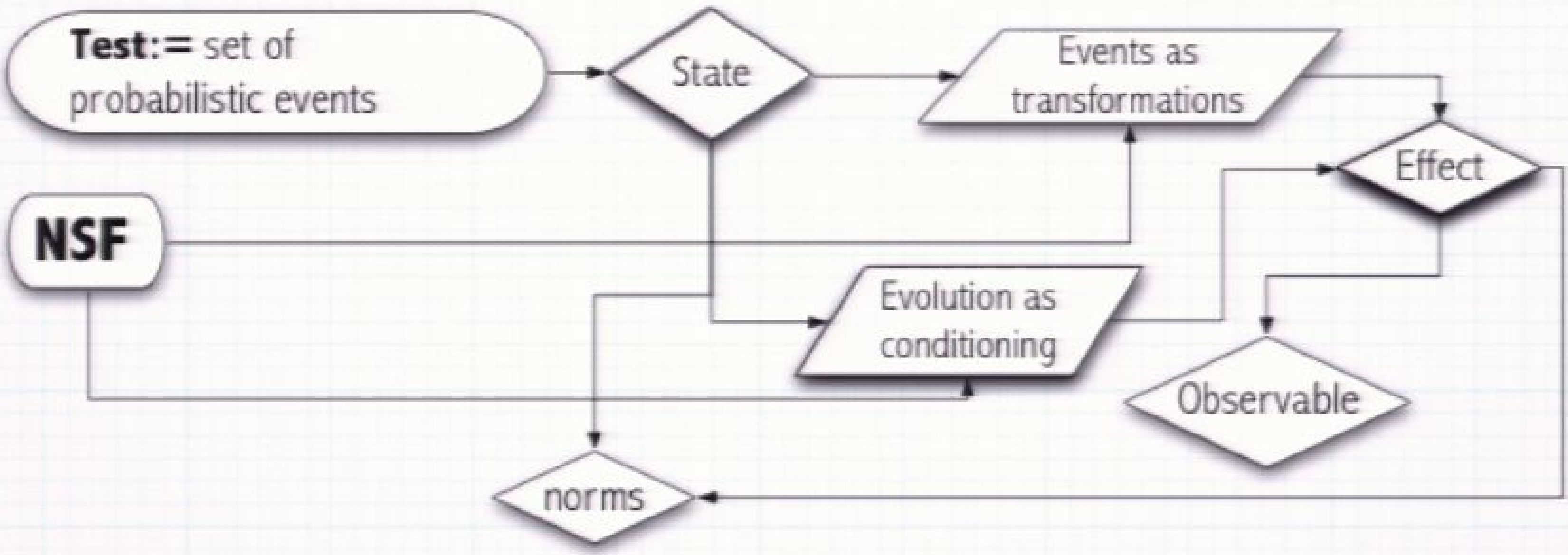

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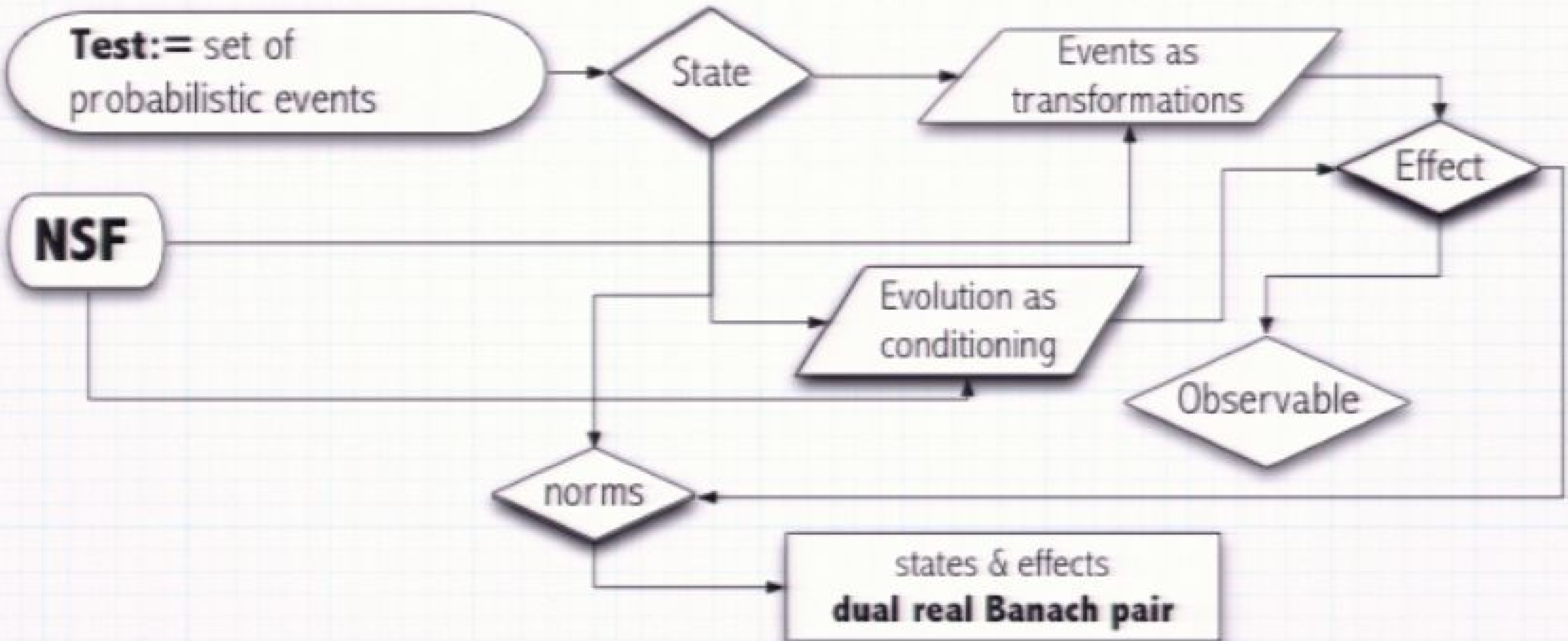

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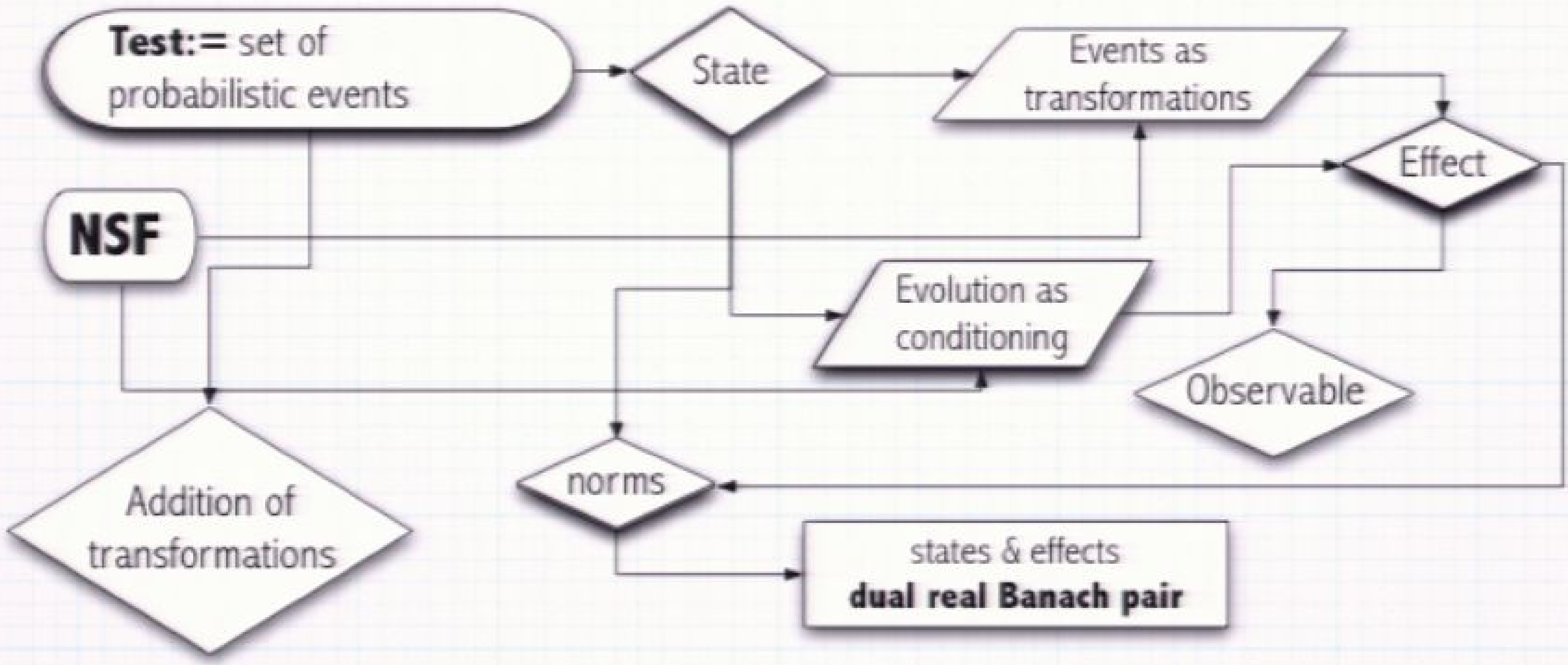

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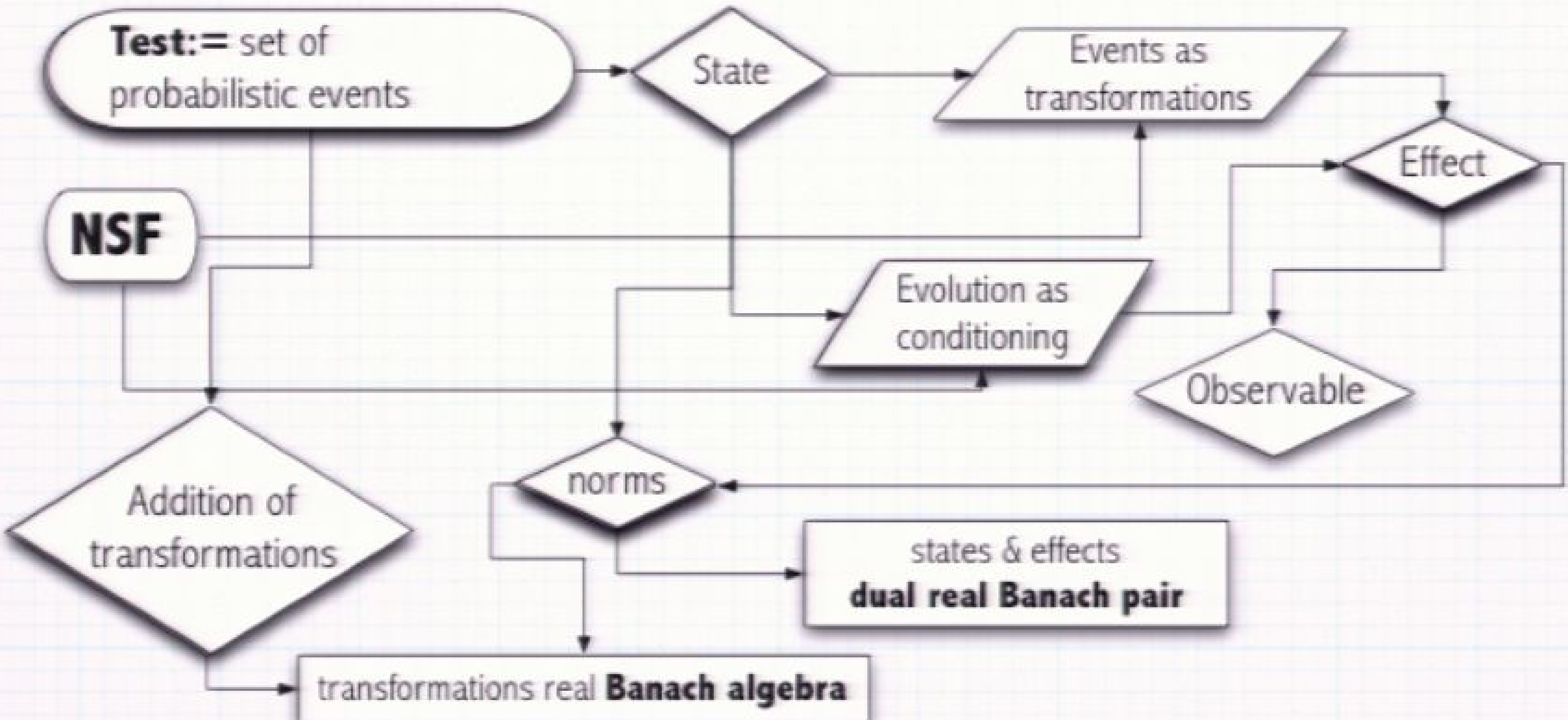

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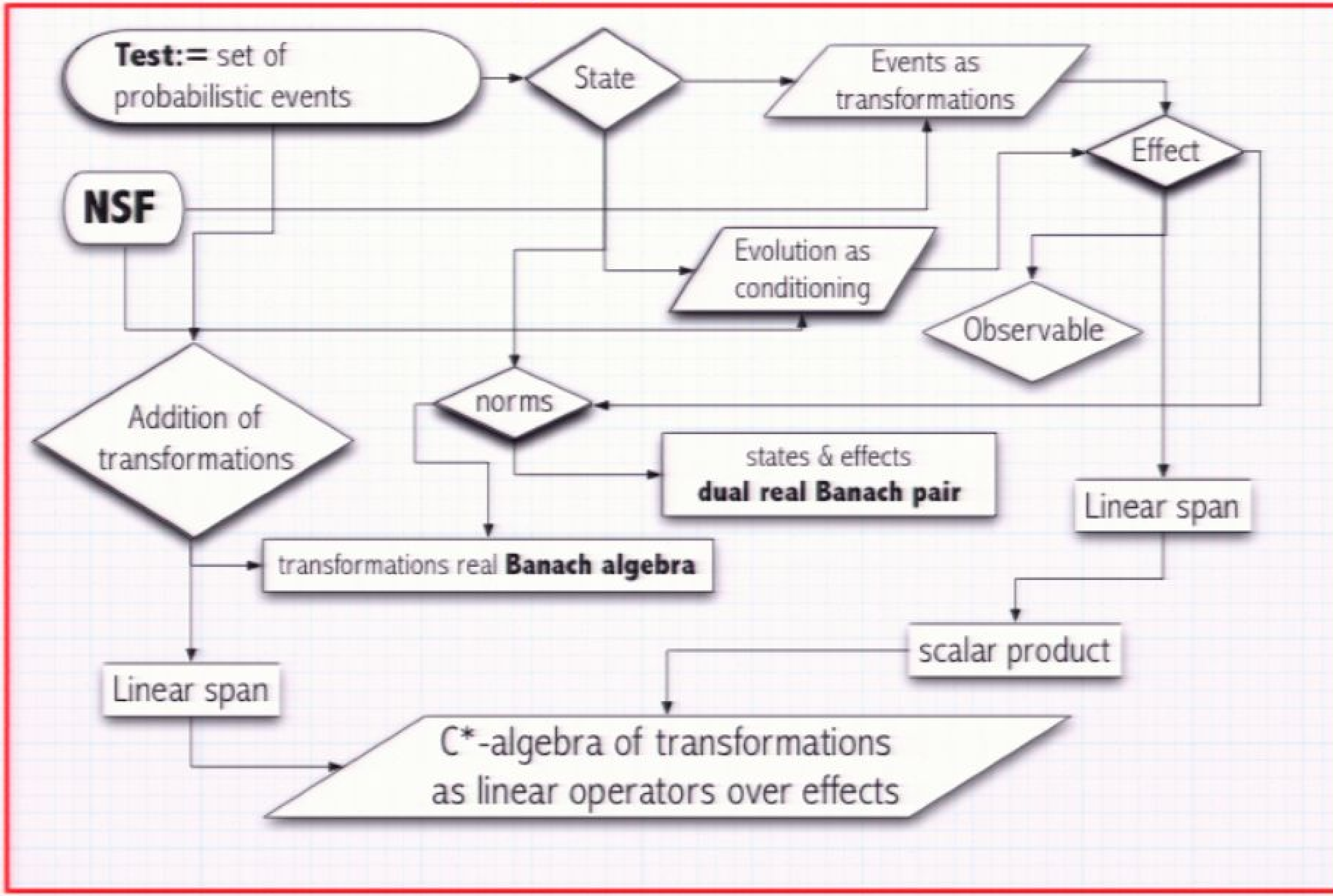

ndependent systems and local transformations

Two systems are **independent** if on each system it is possible to perform **local tests** for which on every joint state one has the commutativity of the pertaining transformations

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Local (marginal) state

For a multipartite system we define the local state $\Omega|_n$ of the n-th system the state that gives the probability of any local transformation $\mathscr{A}$ on the n-th system with all other systems untouched, namely

$$\Omega|_n(\mathscr{A}) := \Omega(\mathscr{I}, \ldots, \mathscr{I}, \underbrace{\mathscr{A}}_{n-th}, \mathscr{I}, \ldots)$$

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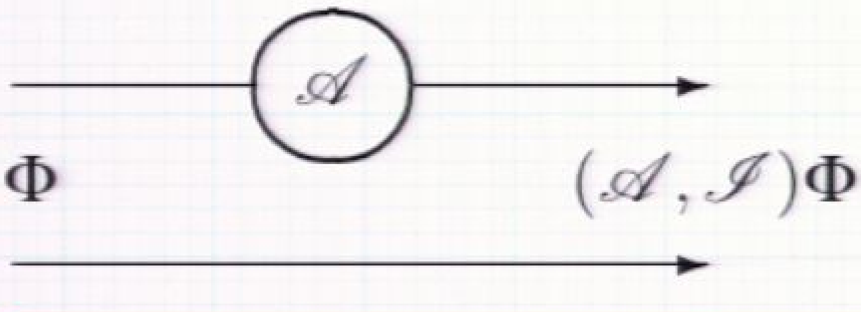
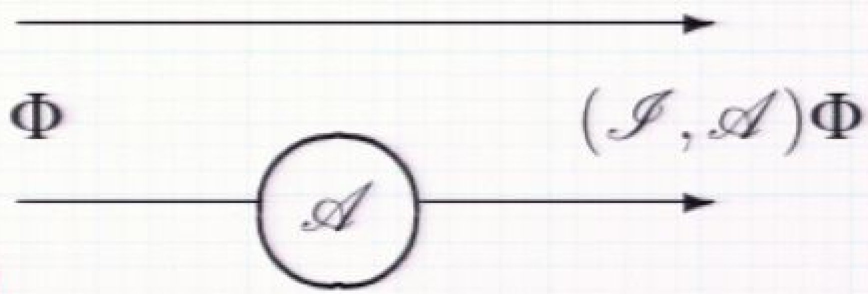
no-signaling: any local test on a system is equivalent to no-test on an another independent system.

no-signaling → existence of equivalent "incompatible" mixtures

Faithful state

A state Φ of a bipartite system is **dynamically faithful** when the output state $(\mathscr{A},\mathscr{I})\Phi$ from a local transformation $\mathscr{A}$ on one system is in 1-to-1 correspondence with the transformation $\mathscr{A}$





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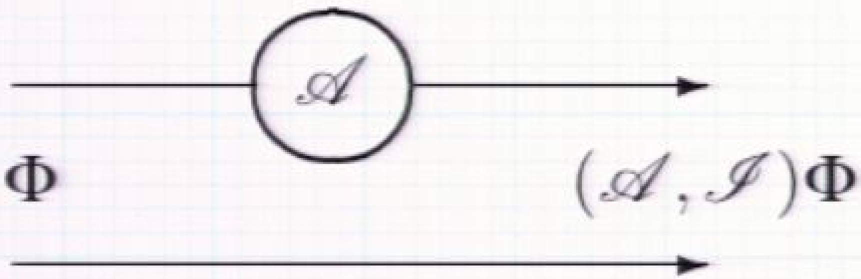
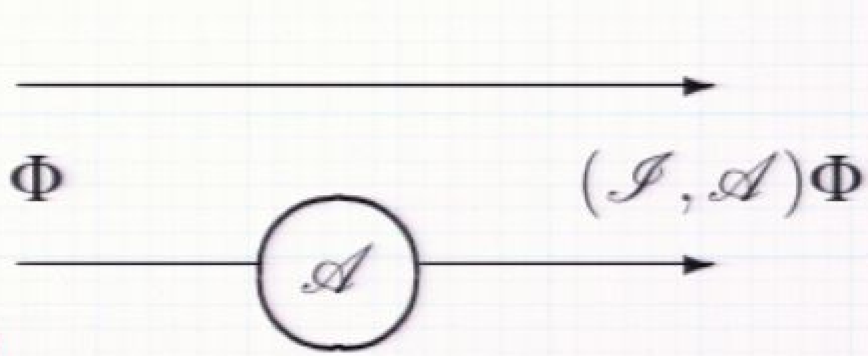
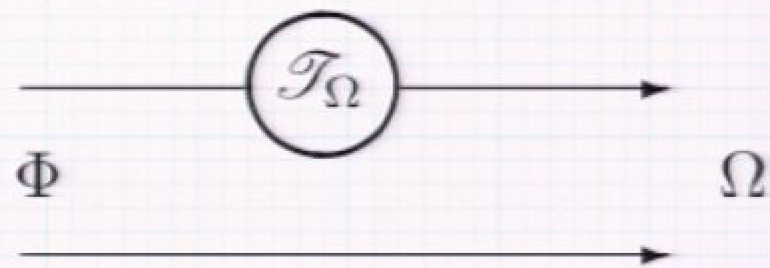


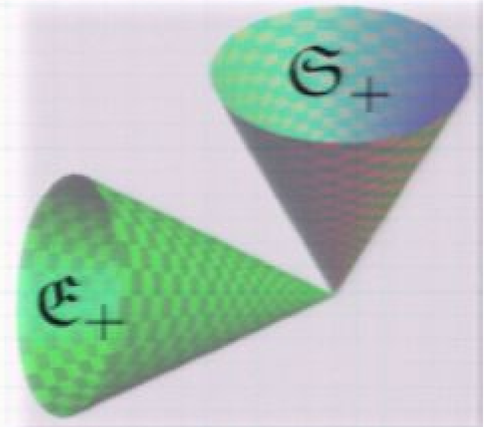


$$(\mathscr{A},\mathscr{I})\Phi = 0 \Longrightarrow \mathscr{A} = 0, \quad \forall \mathscr{A} \in \mathfrak{T}_{\mathbb{R}}$$

A state Φ of a bipartite system is **preparationally faithful** if every joint state Ω can be achieved by a suitable local transformation $\mathscr{T}_{\Omega}$ on one system occurring with nonzero probability



Faithful state



FAITH

Weak self-duality:

State and effect cones are isomorphic: $\mathfrak{E}_+ \ni a \mapsto \omega_a = \Phi(a, \cdot) \in \mathfrak{S}_+$

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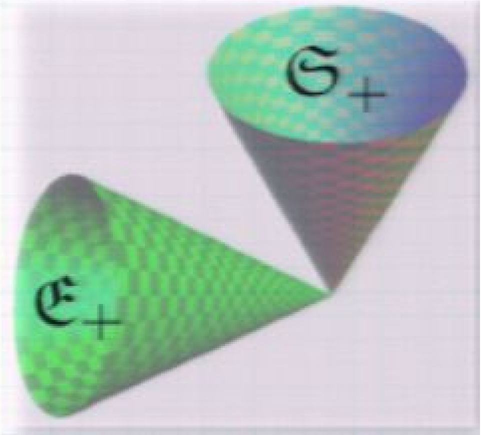

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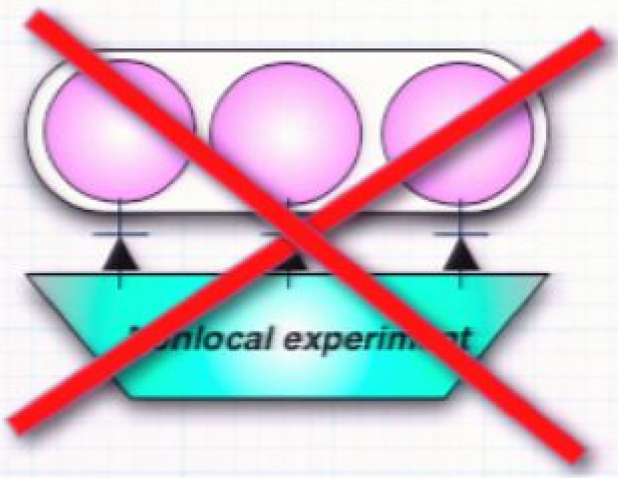
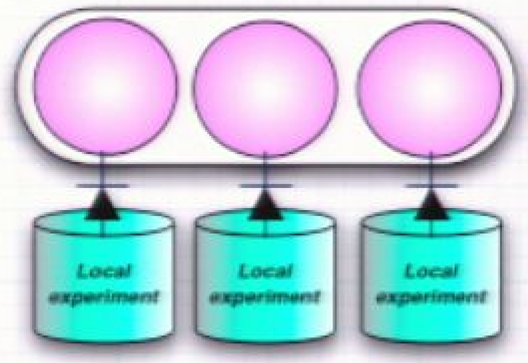
Bipartite systems can be represented with tensor product, i.e.

$$\mathfrak{S}_{\mathbb{C}}^{\times 2} \subseteq \mathfrak{S}_{\mathbb{C}}^{\otimes 2} \qquad \mathfrak{E}_{\mathbb{C}}^{\times 2} \subseteq \mathfrak{E}_{\mathbb{C}}^{\otimes 2}$$

Theorem: local observability principle

For composite systems local info-complete observables provide global info-complete observables.





Holism  **Reductionism**

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We have derived a C*-algebra representation of the **transformation algebra** $\mathfrak{T}_{\mathbb{C}}$

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Question: when such representation is Quantum?

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Answer: When **effects make a C*-algebra**

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Operational theories:

- effects are a linear space:

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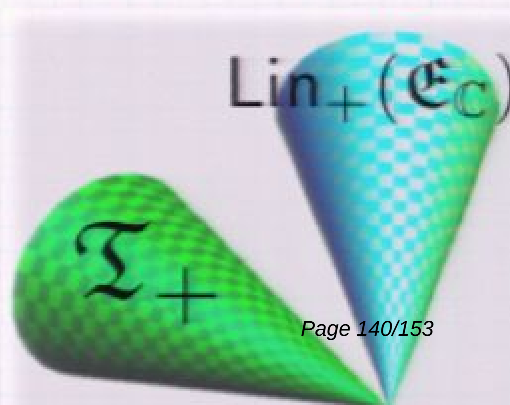
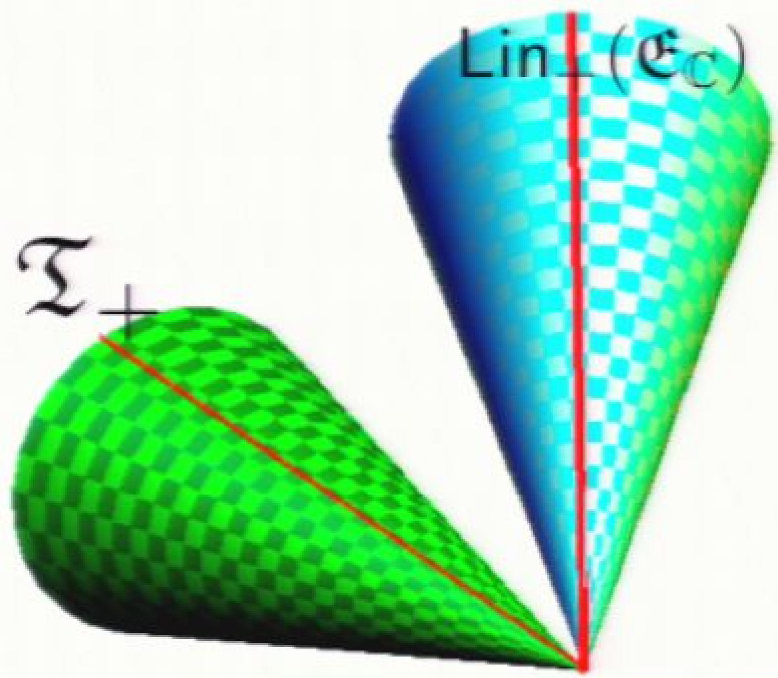
Answer: (the "little" more) Choi-Jamiolkowski isomorphism

$$\mathrm{I} : \mathfrak{T}_+ \simeq \mathrm{Lin}_+(\mathfrak{E}_\mathbb{C})$$



CJ Isomorphism ⇒ composition of effects

Effects are identified with "elementary" events
(apart from a phase) i.e. events that cannot be written as sum of other events



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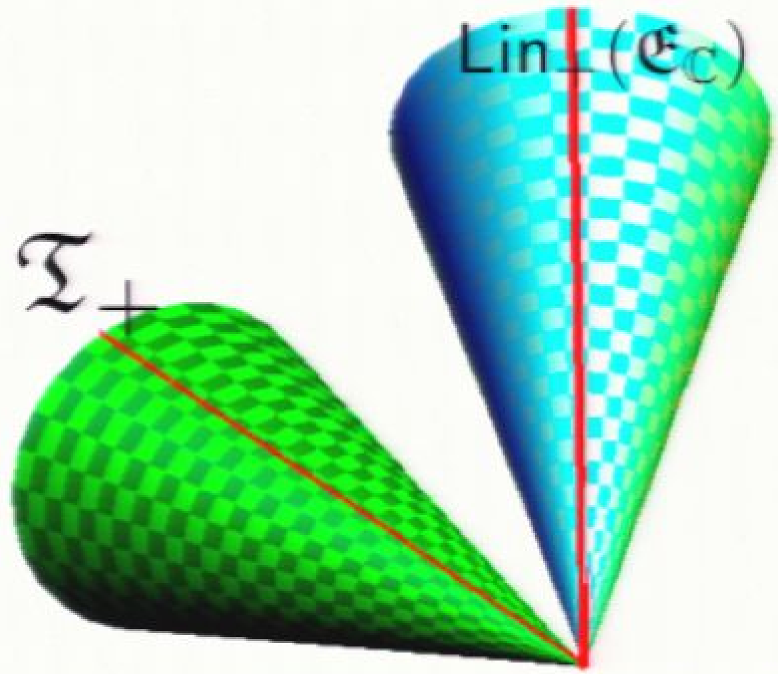

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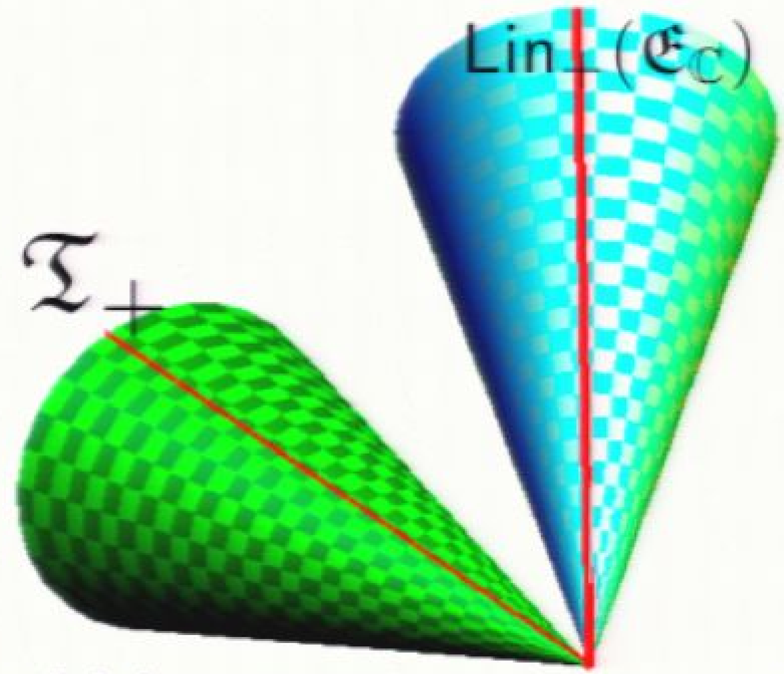

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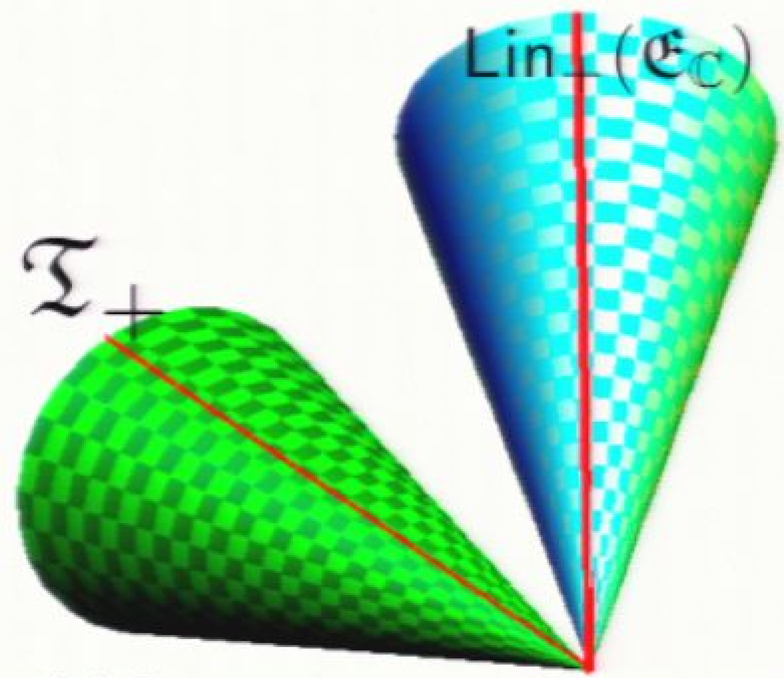

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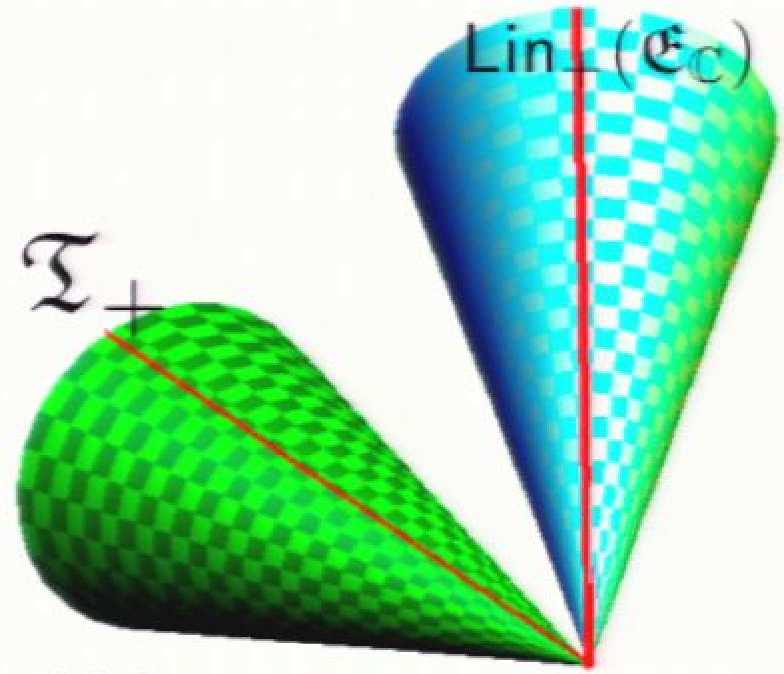

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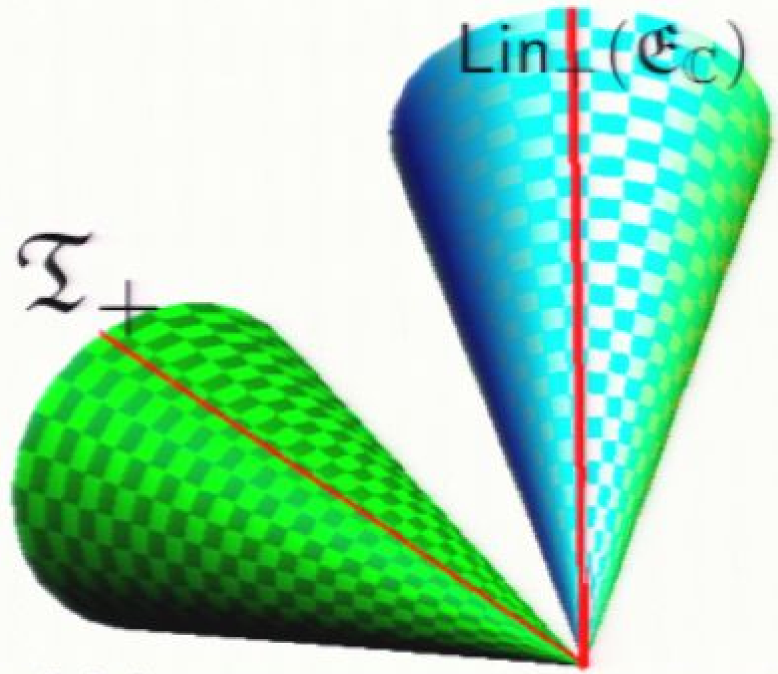

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→ quantum C*-algebra of effects

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- Explain "the little more": what is the operational meaning of the Choi Jamiolkowski isomorphism?

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- Please, tell me your "fair rules"