Title: Bell, Bayes, and 0.0331%

Date: Jun 26, 2008 02:00 PM

URL: http://pirsa.org/08060200

Abstract:

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S.J. van Enk
P. Lougovski
University of Oregon

Hirota and me

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PHYSICAL REVIEW A, VOLUME 64, 022313

Entangled coherent states: Teleportation and decoherence

S. J. van Enk[1] and O. Hirota[2]
[1]*Bell Laboratories, Room 2C-401, 600-700 Mountain Ave., Murray Hill, New Jersey 07974*
[2]*Research Center for Quantum Communications, Tamagawa University, Tokyo, Japan*
(Received 17 December 2000; published 13 July 2001; published 13 July 2001)

When a superposition ($|\alpha\rangle - |-\alpha\rangle$) of two coherent states with opposite phase falls upon a 50-50 beam splitter, the resulting state is entangled. Remarkably, the amount of entanglement is exactly 1 ebit, irrespective of α, as was recently discovered by Hirota and Sasaki [LANL e-print quant-ph/0101018]. Here we discuss decoherence properties of such states and give a simple protocol that teleports one qubit encoded in Schrödinger cat states.

Hirota and me

PHYSICAL REVIEW A **71**, 062322 (2005)

Entangled states of light and their robustness against photon absorption

S. J. van Enk[1,2] and O. Hirota[3]

[1]*Bell Labs, Lucent Technologies, 600-700 Mountain Avenue, Murray Hill, New Jersey 07974, USA*

[2]*Institute for Quantum Information, California Institute of Technology, Pasadena, California 91125, USA*

[3]*Research Center for Quantum Communications, Tamagawa University, Tokyo, Japan*

(Received 31 December 2004; published 17 June 2005)

We study how photon absorption losses degrade the bipartite entanglement of entangled states of light. We consider two questions: (i) What state contains the smallest average number of photons given a fixed amount of entanglement? (ii) What state is the most robust against photon absorption? We explain why the two-mode squeezed state is the answer to the first question but not quite to the second question.

Hirota and me

In this paper we discuss a third type of entangled states of two modes of the electromagnetic field. They are parametrized by a complex parameter α,

$$|H_\alpha\rangle_{1,2} = (|\alpha\rangle_1 |\alpha\rangle_2 - |-\alpha\rangle_1 |-\alpha\rangle_2)/\sqrt{N_\alpha}, \qquad (4)$$

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The Hirota states!

Bell inequalities

- Bell inequalities serve two purposes:
  - refute local hidden variable (LHV) models (but I don't care, I already believe in QM)
  - detect entanglement of qubits: Violating the Bell inequality

$$|B| := |X_A X'_B + X_A Y'_B + Y_A X'_B - Y_A Y'_B| \leq 2$$

implies entanglement

Better Bell inequalities

- Suppose we measure spin (on spin-1/2) in (locally) <u>orthogonal directons</u>

- Then there is a better (RUS) bound*: violating $B \le \sqrt{2}$ implies entanglement

  even though there is a LHV model when $B \le 2$ for all correlation measurement data

*S.M. Roy, Phys. Rev. Lett. **94**, 010402 (2005).

*J. Uffink and M. Seevinck, Phys. Lett. A **372**, 1205 (2008).

Proof & generalization

- There is a simple proof (someone else did the hard part*):

- Fix the quantum state ρ, and vary over all possible spin measurements in orthogonal directions: then** N:negativity F: fidelity

$$|Tr(\rho B)|_{max} \leq 2\sqrt{2}F(\rho) \leq \sqrt{2}(1+N(\rho))$$

*F. Verstraete and H. Verschelde, Phys. Rev. A **66**, 022307 (2005): **F≤(1+N)/2**

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- This includes both the RUS bound and Cirel'son's inequality
- Violating the RUS bound gives a lower bound on N

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Even better

- Bell correlation measurements allow one to detect entanglement
  - even if no Bell inequality is violated
  - and even if no RUS inequality is violated!
- How? Just use Bayesian updating, starting from a suitable prior

Bayesian probabilities

- Choose random test set of states, using uniform prior probability distribution $P_{prior}(\rho)$, w.r.t. some natural metric*

  *Zyczkowski et al., Phys. Rev. A 58 (1998) 883

  - Haar measure on SU(4), etc.

- While taking data d, update one's probability distribution by Bayes' rule

$$P_{posterior}(\rho|d) \propto P(d|\rho) \times P_{prior}(\rho)$$

- End up with $P(\rho) := P_{final,posterior}(\rho|\text{all } d)$

Using P(ρ)

- One gets a probability of entanglement:

$$P_{ent} = \int d\boldsymbol{\rho}_{ent}\, P(\boldsymbol{\rho}_{ent})$$

- One gets an estimate of entanglement

  or any function of $\boldsymbol{\rho}$ (+error bars), e.g.:

$$\underline{N} = \int d\boldsymbol{\rho}_{ent}\, P(\boldsymbol{\rho}_{ent})\, N(\boldsymbol{\rho}_{ent})$$

$$\underline{Pur} = \int d\boldsymbol{\rho}\, P(\boldsymbol{\rho})\, Tr(\boldsymbol{\rho}^2)$$

What data? Bell[4]

- When testing the Bell inequality, one actually measures 4 correlations:

$$X_A X'_B \quad X_A Y'_B \quad Y_A X'_B \quad Y_A Y'_B$$

- From those correlations 4 Bell operators can be constructed:

$$B_1 := X_A X'_B + X_A Y'_B + Y_A X'_B - Y_A Y'_B$$

$$B_2 := X_A X'_B + X_A Y'_B - Y_A X'_B + Y_A Y'_B$$

$$B_3 := X_A X'_B - X_A Y'_B + Y_A X'_B + Y_A Y'_B$$

$$B_4 := -X_A X'_B + X_A Y'_B + Y_A X'_B + Y_A Y'_B$$

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$$B_4 := -X_A X'_B + X_A Y'_B + Y_A X'_B + Y_A Y'_B$$

Some a priori statistics

- 30 million states random 2-qubit states (w.r.t. Haar measure etc.)
- $P_{ent} = (36.437 \pm 0.010)\%$
  - $P_{B>2}(4) = (0.0331 \pm 0.0003)\%$
  - $P_{B>\sqrt{2}}(4) = (1.244 \pm 0.003)\%$
  - $P_{B>2}(36) = (0.2490 \pm 0.0008)\%$
  - $P_{B>\sqrt{2}}(36) = (5.690 \pm 0.004)\%$

One refinement

- We added one more correlation, $Z_A Z'_B$.
- This does not provide one with new Bell operators and/or inequalities
- This set is tomographically complete for <u>single</u>-party reduced density matrices
  - but not for full **ρ**, of course
- reminder: it's just a finite data set!

Example (I)

- Try 5x10,000 measurements on a Werner-like state that is barely entangled: N=0.01 (Purity=0.336)

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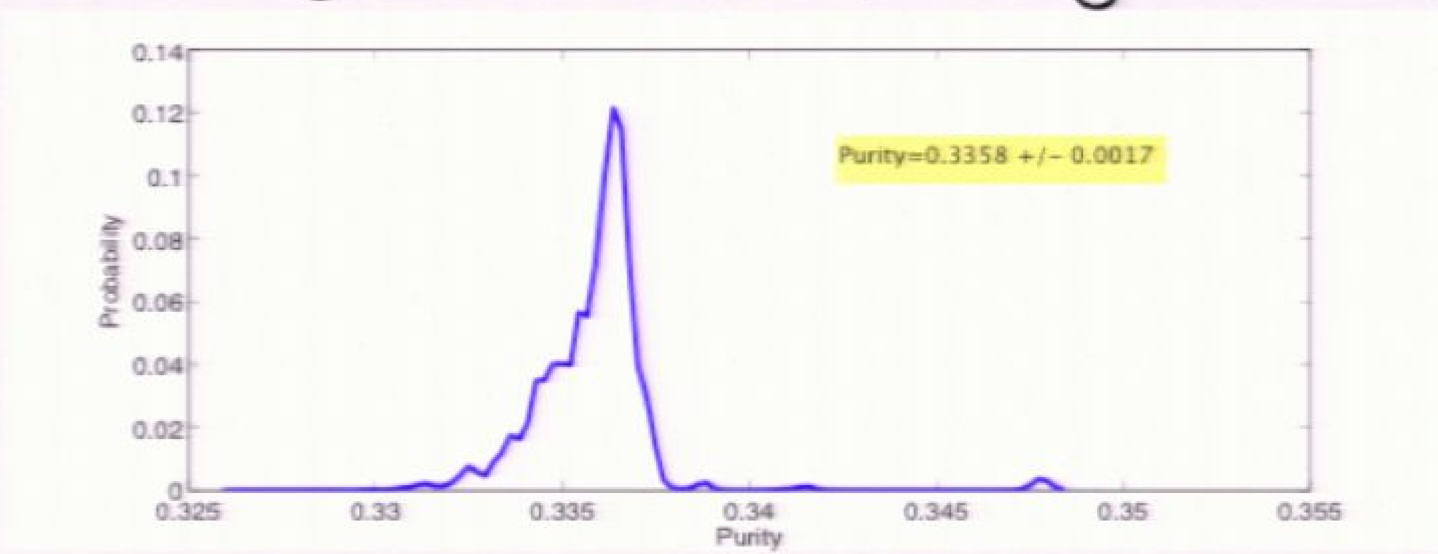

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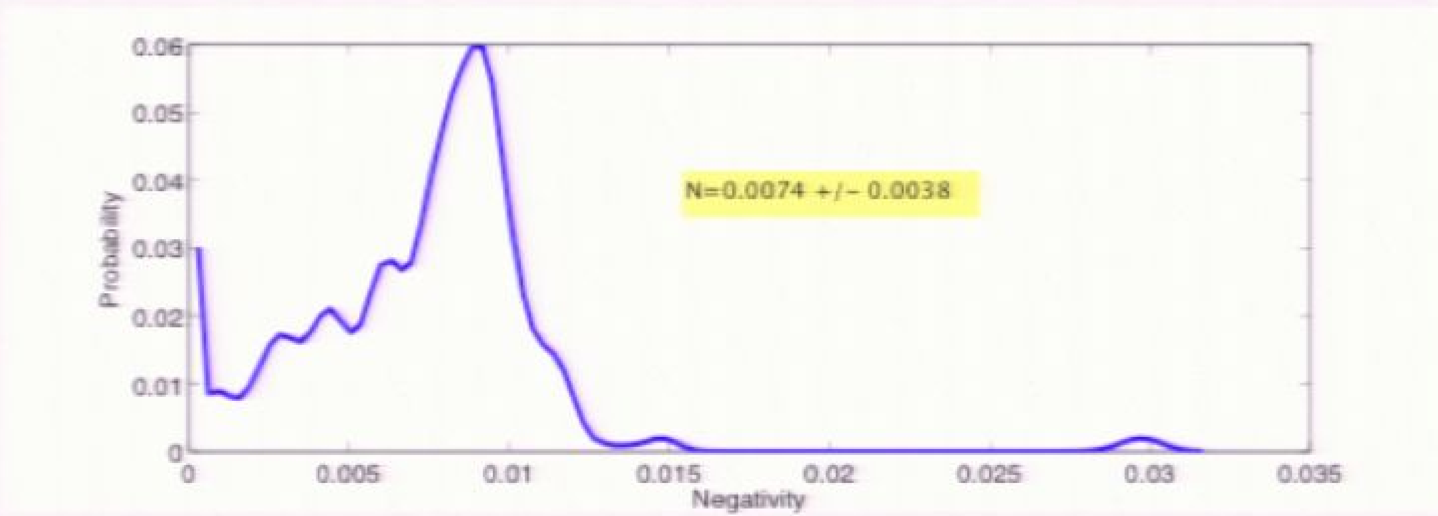

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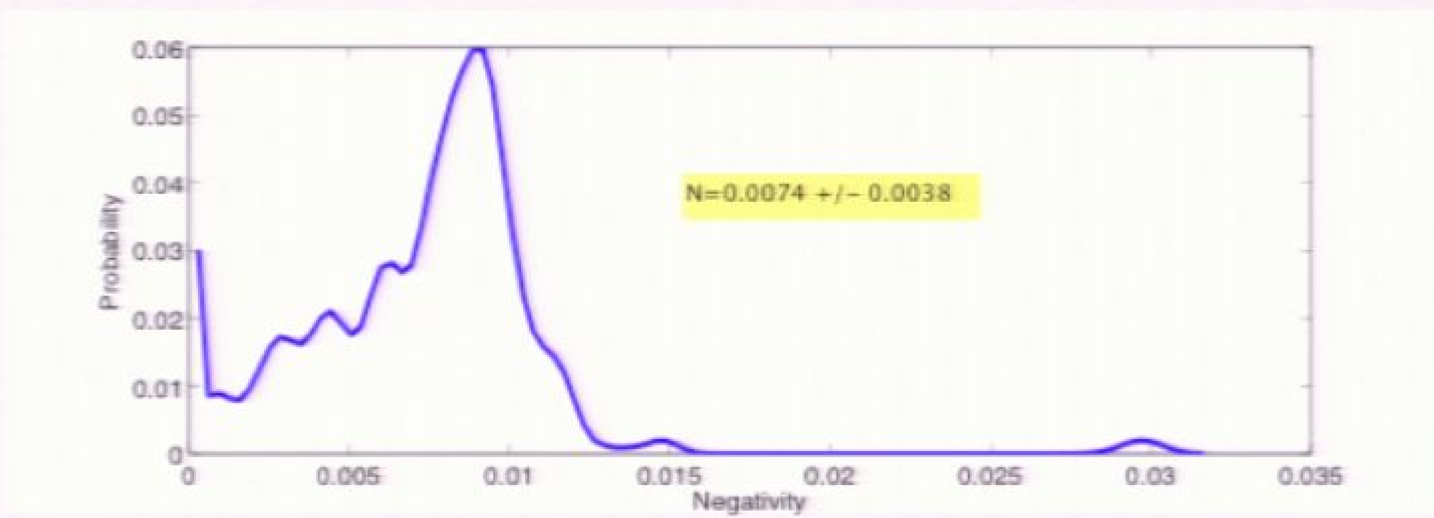

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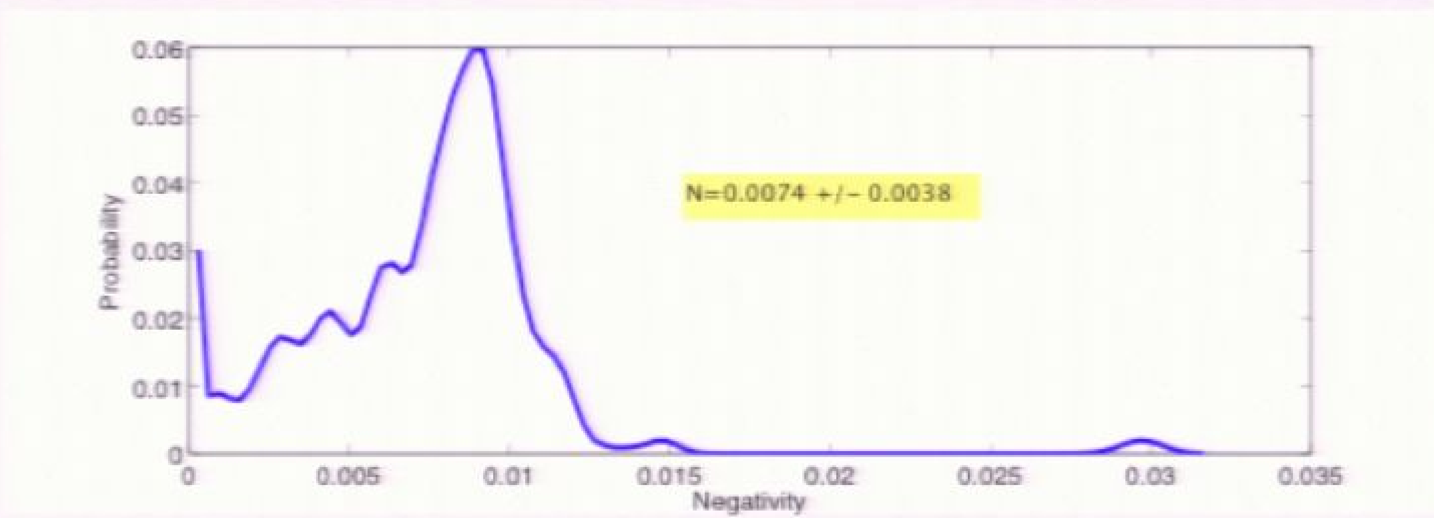

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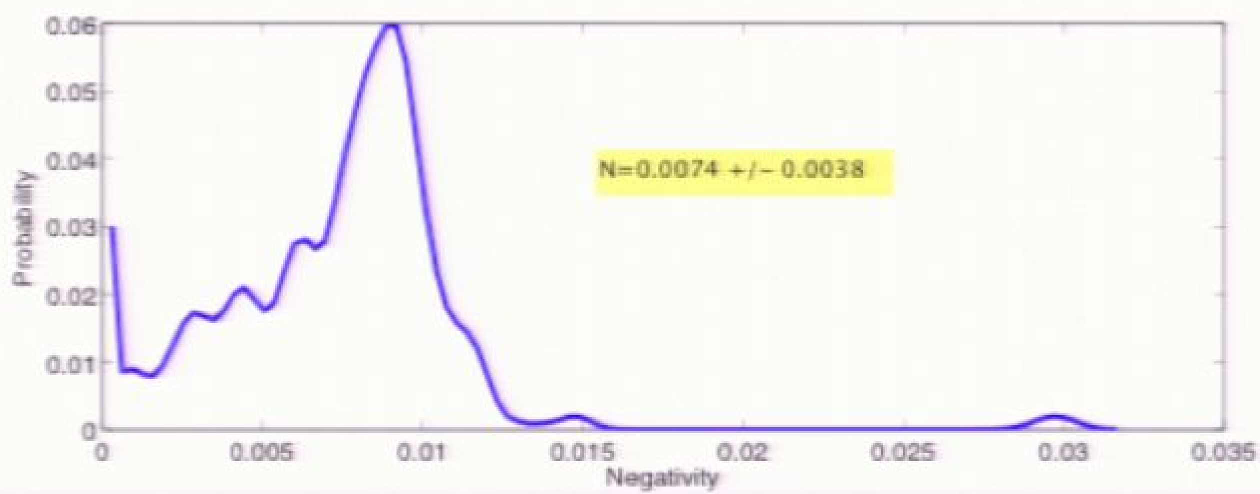

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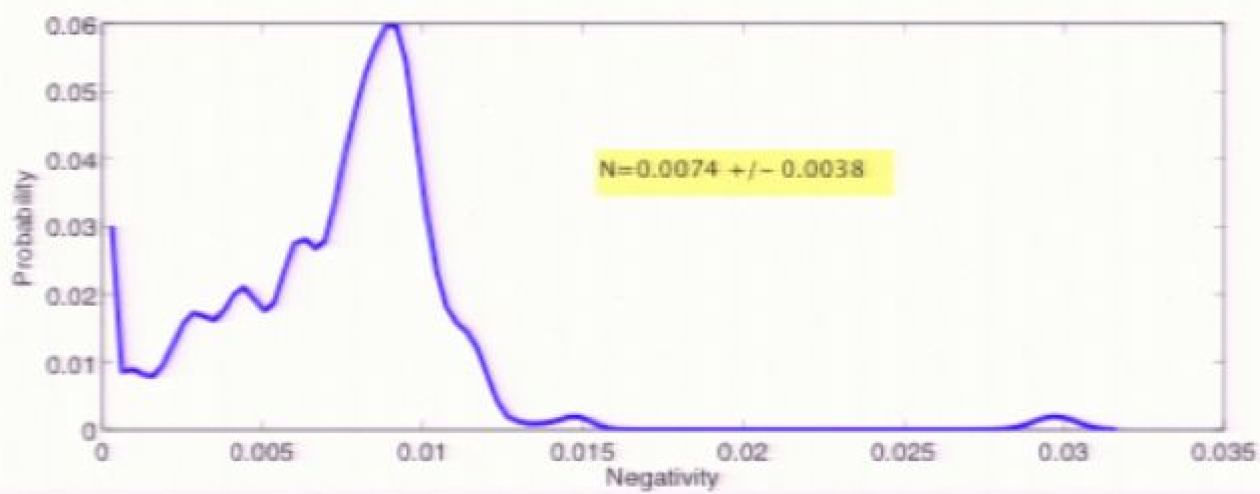

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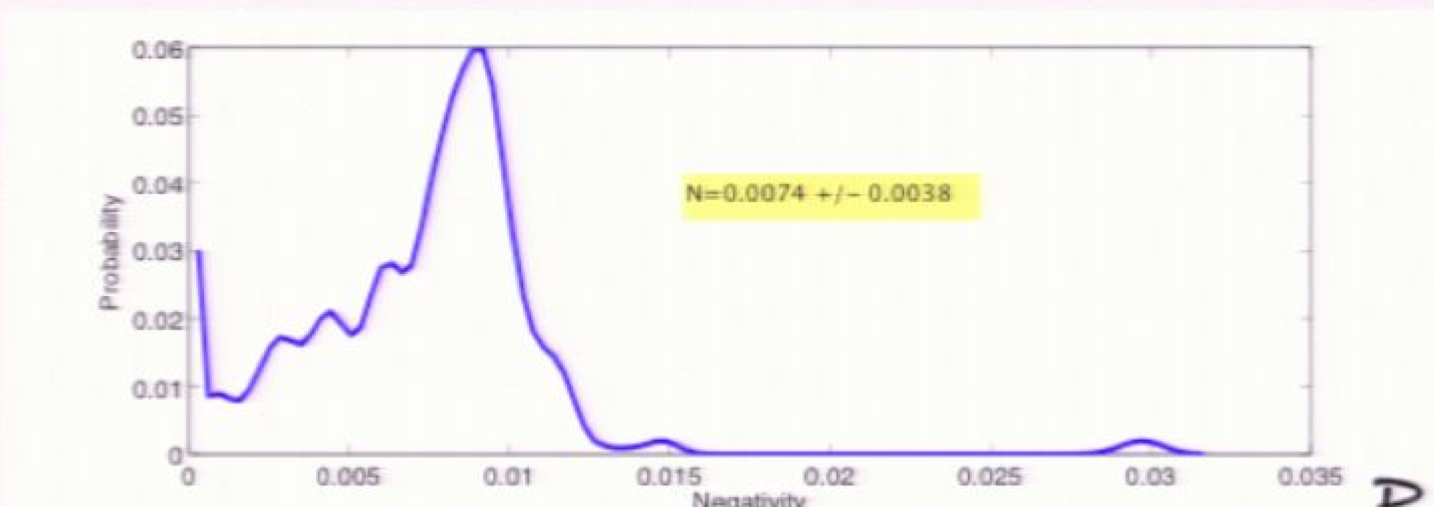


More details

- Choose a test set of 25,000 states
- Calculate likelihood function
- Keep the top 1%
- Add 99% new random states around top with spread Δ in parameters
- Repeat 30 times, with decreasing Δ
- Repeat all of this 120 times (improvements certainly possible!)

Interpreting P_{ent}, $\underline{N}$...

- Meaning of P_{ent}:
  - if one were to do a tomographically complete measurement, then P_{ent} gives the best estimate of probability to end up with an entangled state IF state is drawn from initial prior distribution
  - Similarly, $\underline{N}$ is the average amount of entanglement expected after full tomography

Interpreting P_{ent}, $\underline{N}$...

- Note, for most tasks one does need to know more than just the amount of entanglement
- $\underline{\rho}=\int d\rho\, P(\rho)\, \rho$ would be the best* (conservative) estimate for ρ before full tomography, but that's not what we're using here

*R. Blume-Kohout, quant-ph/0611080

- For example: $N(\underline{\rho})\leq\underline{N}$; $Pur(\underline{\rho})\leq\underline{Pur}$

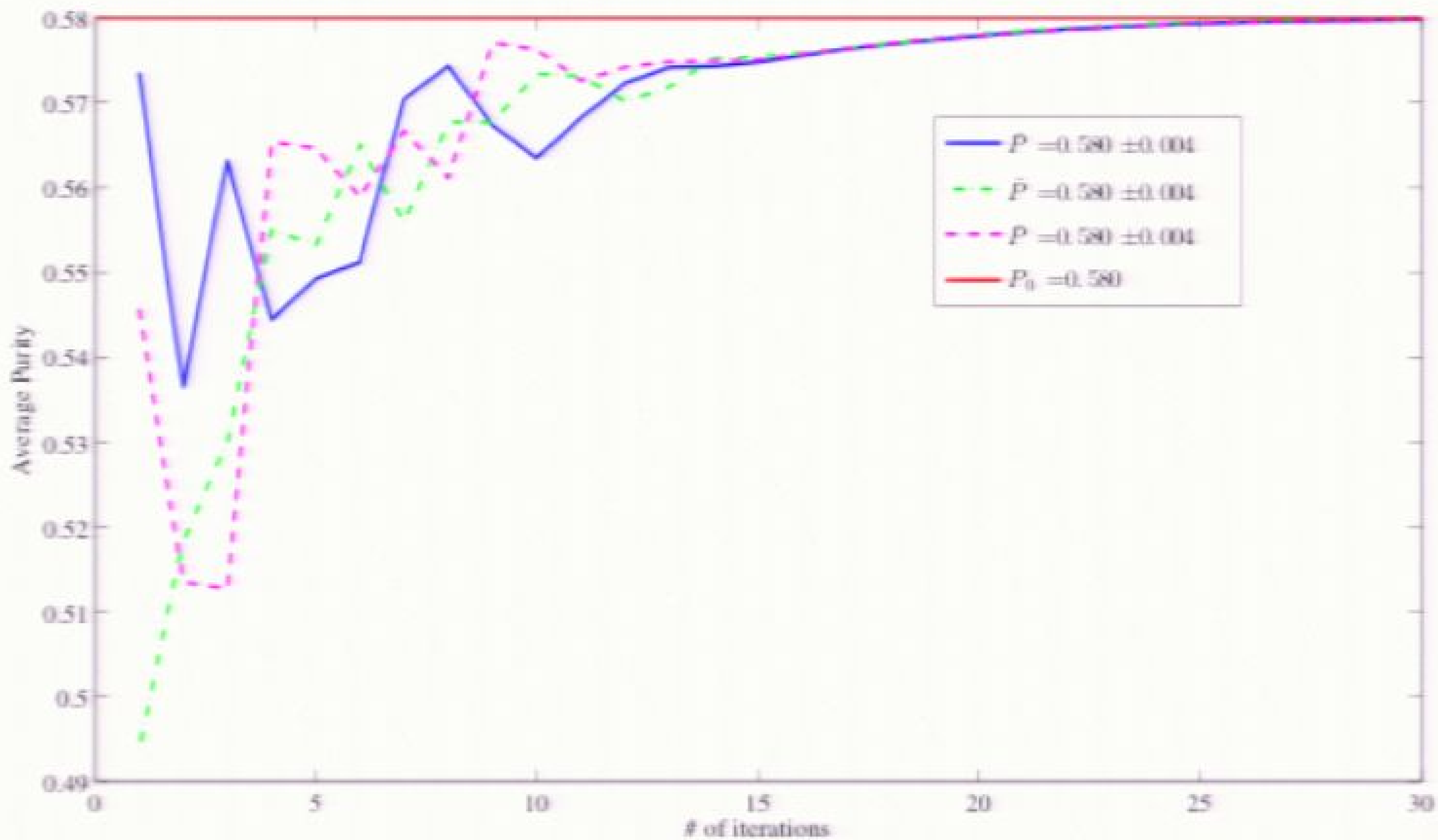
Completing the story

- The 15-D parameter space of 2-qubit density matrices is annoyingly large
- In general, one will find a large (~4-D) subset of states fitting the data well:
  - broad distribution of purity
  - broad distribution of negativity
- One can (should) add more correlation measurements

Example (II)

- state: 50/50 mixture of two states: $[1,2,3,4]/\sqrt{30}$ and $[4,3,2,-1]/\sqrt{30}$
- 4 million test states
- 3 runs of 30 iterations (not enough!)
- 5x5,000 correlation measurements

Example (II)



0.58
0.57
0.56
0.55
0.54
0.53
0.52
0.51
0.5
0.49
Average Purity
0
5
10
15
20
25
30
# of iterations
P =0.580 ±0.004
$\bar{P}$ =0.580 ±0.004
P =0.580 ±0.004
P_0 =0.580

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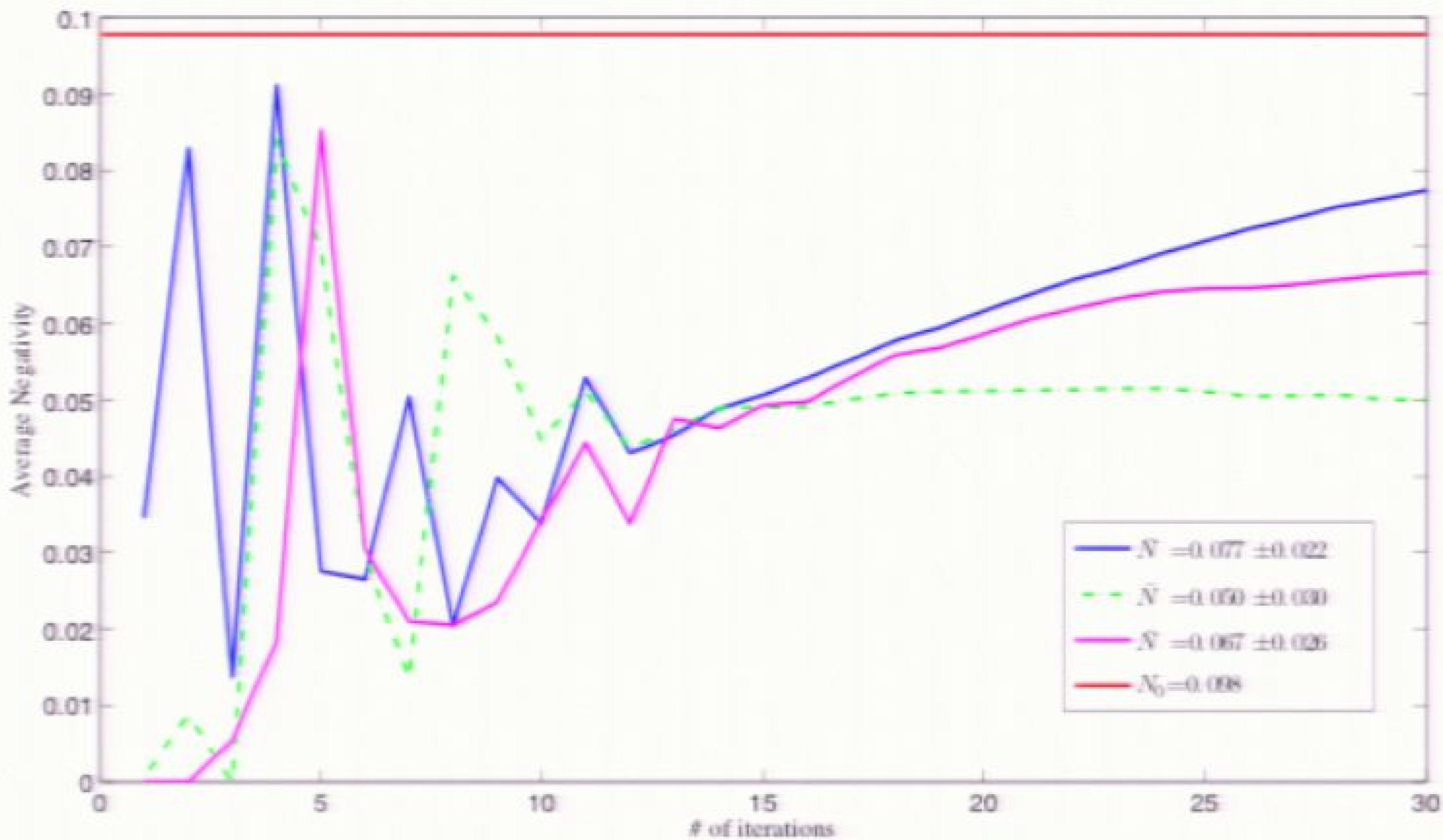
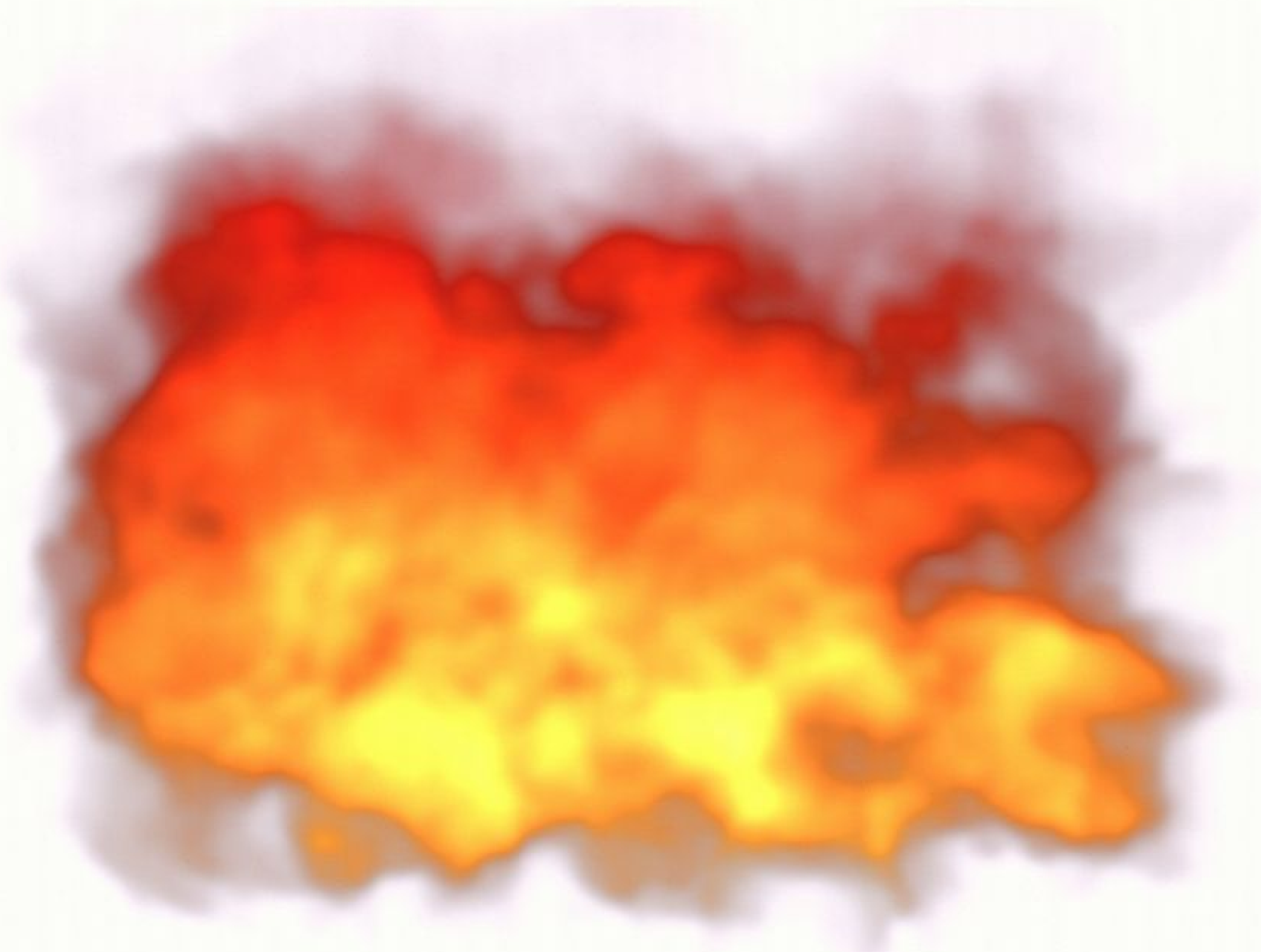


Average Negativity
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
5
10
15
20
25
30
# of iterations
$\bar{N}$ =0.077 ±0.022
$\bar{N}$ =0.050 ±0.030
$\bar{N}$ =0.067 ±0.026
N_0=0.098

Conclusions

- Detecting entanglement does not require violating Bell's or RUS inequality
  - Indeed, very few entangled states violate a given Bell inequality, or a given RUS inequality
- Detecting entanglement also does not require accurate state estimation
- Works for any finite data set from "any" finite set of measurements


HAPPY
BIRTHDAY

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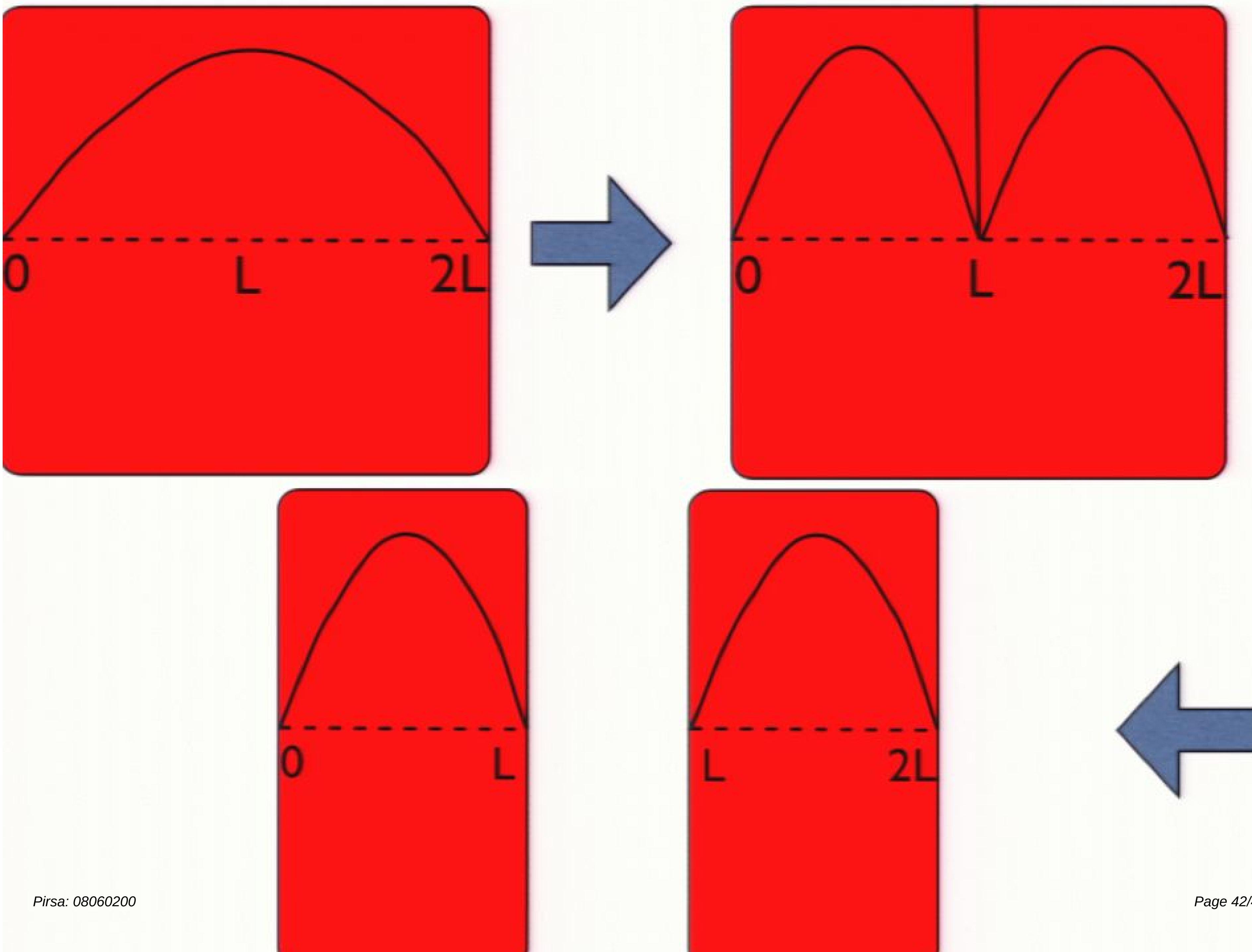
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